

***Precalculus Mathematics for Calculus International
Metric Edition 8th edition Stewart Solutions Manual***

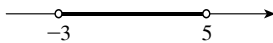
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CHAPTER 1 **FUNDAMENTALS**

- 1.1** Real Numbers 1
- 1.2** Exponents and Radicals 6
- 1.3** Algebraic Expressions 13
- 1.4** Rational Expressions 19
- 1.5** Equations 27
- 1.6** Complex Numbers 36
- 1.7** Modeling with Equations 39
- 1.8** Inequalities 50
- 1.9** The Coordinate Plane; Graphs of Equations; Circles 72
- 1.10** Lines 91
- 1.11** Solving Equations and Inequalities Graphically 101
- 1.12** Modeling Variation 111
- Chapter 1 Review 115
- Chapter 1 Test 132
- **FOCUS ON MODELING:** Fitting Lines to Data 138
- Errata in Exercises and Answers in First Printing 141

1 FUNDAMENTALS

1.1 REAL NUMBERS

1. (a) The natural numbers are $\{1, 2, 3, \dots\}$.
 (b) The numbers $\{\dots, -3, -2, -1, 0\}$ are integers but not natural numbers.
 (c) Any irreducible fraction $\frac{p}{q}$ with $q \neq 1$ is rational but is not an integer. Examples: $\frac{3}{2}$, $-\frac{5}{12}$, $\frac{1729}{23}$.
 (d) Any number which cannot be expressed as a ratio $\frac{p}{q}$ of two integers is irrational. Examples are $\sqrt{2}$, $\sqrt{3}$, π , and e .
2. (a) $ab = ba$; Commutative Property of Multiplication
 (b) $a + (b + c) = (a + b) + c$; Associative Property of Addition
 (c) $a(b + c) = ab + ac$; Distributive Property
3. (a) In set-builder notation: $\{x \mid -3 < x < 5\}$
 (b) In interval notation: $(-3, 5)$
 (c) As a graph: 
4. The symbol $|x|$ stands for the *absolute value* of the number x . If x is not 0, then the sign of $|x|$ is always *positive*.
5. The distance between a and b on the real line is $d(a, b) = |b - a|$. So the distance between -5 and 2 is $|2 - (-5)| = 7$.
6. (a) If $a < b$, then any interval between a and b (whether or not it contains either endpoint) contains infinitely many numbers—including, for example $a + \frac{b-a}{2^n}$ for every positive n . (If an interval extends to infinity in either or both directions, then it obviously contains infinitely many numbers.)
 (b) No, because $(5, 6)$ does not include 5 .
7. (a) No: $a - b = -(b - a) \neq b - a$ in general.
 (b) No; by the Distributive Property, $-2(a - 5) = -2a + -2(-5) = -2a + 10 \neq -2a - 10$.
8. (a) Yes, absolute values (such as the distance between two different numbers) are always positive.
 (b) Yes, $|b - a| = |a - b|$.
9. (a) Natural number: 100
 (b) Integers: 0, 100, -8
 (c) Rational numbers: -1.5 , 0 , $\frac{5}{2}$, 2.71 , $3.1\overline{4}$, 100 , -8
 (d) Irrational numbers: $\sqrt{7}$, $-\pi$
10. (a) Natural numbers: 2 , $\sqrt{9} (= 3)$, 10
 (b) Integers: 2 , $-\frac{100}{2} (= -50)$, $\sqrt{9} (= 3)$, 10
 (c) Rational numbers: $4.5 \left(= \frac{9}{2}\right)$, $\frac{1}{3}$, $1.6666\dots \left(= \frac{5}{3}\right)$, 2 , $-\frac{100}{2}$, $\sqrt{9} (= 3)$, 10
 (d) Irrational numbers: $\sqrt{2}$, $\sqrt{3.14}$
11. Commutative Property of addition
13. Associative Property of addition
15. Distributive Property
17. Commutative Property of multiplication
19. $x + 3 = 3 + x$
21. $4(A + B) = 4A + 4B$
12. Commutative Property of multiplication
14. Distributive Property
16. Distributive Property
18. Distributive Property
20. $7(3x) = (7 \cdot 3)x$
22. $5x + 5y = 5(x + y)$

23. $-2(x + y) = -2x - 2y$

25. $5(2xy) = (5 \cdot 2)xy = 10xy$

27. $-\frac{5}{2}(2x - 4y) = -\frac{5}{2}(2x) + \frac{5}{2}(4y) = -5x + 10y$

29. (a) $\frac{2}{3} + \frac{5}{7} = \frac{14}{21} + \frac{15}{21} = \frac{29}{21}$

(b) $\frac{5}{12} - \frac{3}{8} = \frac{10}{24} - \frac{9}{24} = \frac{1}{24}$

31. (a) $\frac{2}{3}\left(6 - \frac{3}{2}\right) = \frac{2}{3} \cdot 6 - \frac{2}{3} \cdot \frac{3}{2} = 4 - 1 = 3$

(b) $\left(3 + \frac{1}{4}\right)\left(1 - \frac{4}{5}\right) = \left(\frac{12}{4} + \frac{1}{4}\right)\left(\frac{5}{5} - \frac{4}{5}\right) = \frac{13}{4} \cdot \frac{1}{5} = \frac{13}{20}$

33. (a) $2 \cdot 3 = 6$ and $2 \cdot \frac{7}{2} = 7$, so $3 < \frac{7}{2}$

(b) $-3 > -\frac{7}{2}$

(c) $3.5 = \frac{7}{2}$

35. (a) False

(b) True

37. (a) True

(b) False

39. (a) $x > 0$

(b) $t < 4$

(c) $a \geq \pi$

(d) $-5 < x < \frac{1}{3}$

(e) $|3 - p| \leq 5$

41. (a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(b) $A \cap B = \{2, 4, 6\}$

43. (a) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(b) $A \cap C = \{7\}$

45. (a) $B \cup C = \{x \mid x \leq 5\}$

(b) $B \cap C = \{x \mid -1 < x < 4\}$

47. $(-3, 0) = \{x \mid -3 < x < 0\}$



49. $[2, 8) = \{x \mid 2 \leq x < 8\}$



24. $(a - b)(-5) = -5a + 5b$

26. $\frac{4}{3}(-6y) = \left[\frac{4}{3}(-6)\right]y = -8y$

28. $(3a)(b + c - 2d) = 3ab + 3ac - 6ad$

30. (a) $\frac{2}{5} - \frac{3}{8} = \frac{16}{40} - \frac{15}{40} = \frac{1}{40}$

(b) $\frac{3}{2} - \frac{5}{8} + \frac{1}{6} = \frac{36}{24} - \frac{15}{24} + \frac{4}{24} = \frac{25}{24}$

32. (a) $\frac{2}{\frac{2}{3}} - \frac{\frac{3}{2}}{2} = 2 \cdot \frac{3}{2} - \frac{2}{3} \cdot \frac{1}{2} = 3 - \frac{1}{3} = \frac{9}{3} - \frac{1}{3} = \frac{8}{3}$

(b) $\frac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{3}{15}} = \frac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{1}{5}} = \frac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{1}{5}} \cdot \frac{10}{10} = \frac{4+5}{1+2} = \frac{9}{3} = 3$

34. (a) $3 \cdot \frac{2}{3} = 2$ and $3 \cdot 0.67 = 2.01$, so $\frac{2}{3} < 0.67$

(b) $\frac{2}{3} > -0.67$

(c) $|0.6| = |-0.6|$

36. (a) False: $\sqrt{3} \approx 1.73205 < 1.7325$.

(b) False

38. (a) True

(b) True

40. (a) $y < 0$

(b) $z \geq 3$

(c) $b \leq 8$

(d) $0 < w \leq 17$

(e) $|y - \pi| \geq 2$

42. (a) $B \cup C = \{2, 4, 6, 7, 8, 9, 10\}$

(b) $B \cap C = \{8\}$

44. (a) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(b) $A \cap B \cap C = \emptyset$

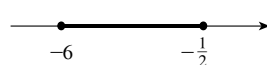
46. (a) $A \cap C = \{x \mid -1 < x \leq 5\}$

(b) $A \cap B = \{x \mid -2 \leq x < 4\}$

48. $(2, 8] = \{x \mid 2 < x \leq 8\}$



50. $\left[-6, -\frac{1}{2}\right] = \left\{x \mid -6 \leq x \leq -\frac{1}{2}\right\}$



51. $[2, \infty) = \{x \mid x \geq 2\}$



52. $(-\infty, 1) = \{x \mid x < 1\}$



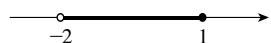
53. $x \leq 1 \Leftrightarrow x \in (-\infty, 1]$



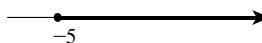
54. $1 \leq x \leq 2 \Leftrightarrow x \in [1, 2]$



55. $-2 < x \leq 1 \Leftrightarrow x \in (-2, 1]$



56. $x \geq -5 \Leftrightarrow x \in [-5, \infty)$



57. $x > -1 \Leftrightarrow x \in (-1, \infty)$



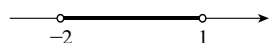
58. $-5 < x < 2 \Leftrightarrow x \in (-5, 2)$



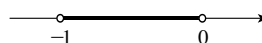
59. (a) $[-3, 5]$ (b) $(-3, 5]$ (c) $(-3, \infty)$

60. (a) $[0, 2)$ (b) $(-2, 0]$ (c) $(-\infty, 0]$

61. $(-2, 0) \cup (-1, 1) = (-2, 1)$



62. $(-2, 0) \cap (-1, 1) = (-1, 0)$



63. $[-4, 6] \cap [0, 8] = [0, 6]$



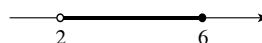
64. $[-4, 6] \cup [0, 8] = [-4, 8]$



65. $(-\infty, -4) \cup (4, \infty)$



66. $(-\infty, 6] \cap (2, 10) = (2, 6]$



67. (a) $|50| = 50$

(b) $|-13| = 13$

68. (a) $|2 - 8| = |-6| = 6$

(b) $|8 - |-2|| = |8 - 2| = |6| = 6$

69. (a) $||-6| - |-4|| = |6 - 4| = |2| = 2$

(b) $\left|\frac{-1}{|-1|}\right| = \left|\frac{-1}{1}\right| = -1$

70. (a) $|2 - |-12|| = |2 - 12| = |-10| = 10$

(b) $-1 - |1 - |-1|| = -1 - |1 - 1| = -1 - |0| = -1$

71. (a) $|(-2) \cdot 6| = |-12| = 12$

(b) $\left|(-\frac{1}{3})(-15)\right| = |5| = 5$

72. (a) $\left|\frac{-6}{24}\right| = \left|-\frac{1}{4}\right| = \frac{1}{4}$

(b) $\left|\frac{7-12}{12-7}\right| = \left|-\frac{5}{5}\right| = |-1| = 1$

73. $|(-2) - 3| = |-5| = 5$

74. $|-2.5 - 1.5| = |-4| = 4$

75. (a) $|17 - 2| = 15$

(b) $|21 - (-3)| = |21 + 3| = |24| = 24$

(c) $\left| -\frac{3}{10} - \frac{11}{8} \right| = \left| -\frac{12}{40} - \frac{55}{40} \right| = \left| -\frac{67}{40} \right| = \frac{67}{40}$

76. (a) $\left| \frac{7}{15} - \left(-\frac{1}{21} \right) \right| = \left| \frac{49}{105} + \frac{5}{105} \right| = \left| \frac{54}{105} \right| = \left| \frac{18}{35} \right| = \frac{18}{35}$

(b) $|-38 - (-57)| = |-38 + 57| = |19| = 19.$

(c) $|-2.6 - (-1.8)| = |-2.6 + 1.8| = |-0.8| = 0.8.$

77. (a) Let $x = 0.777 \dots$. So $10x = 7.777 \dots \Leftrightarrow x = 0.7777 \dots \Leftrightarrow 9x = 7$. Thus, $x = \frac{7}{9}$.

(b) Let $x = 0.2888 \dots$. So $100x = 28.8888 \dots \Leftrightarrow 10x = 2.8888 \dots \Leftrightarrow 90x = 26$. Thus, $x = \frac{26}{90} = \frac{13}{45}$.

(c) Let $x = 0.575757 \dots$. So $100x = 57.5757 \dots \Leftrightarrow x = 0.5757 \dots \Leftrightarrow 99x = 57$. Thus, $x = \frac{57}{99} = \frac{19}{33}$.

78. (a) Let $x = 5.2323 \dots$. So $100x = 523.2323 \dots \Leftrightarrow 1x = 5.2323 \dots \Leftrightarrow 99x = 518$. Thus, $x = \frac{518}{99}$.

(b) Let $x = 1.3777 \dots$. So $100x = 137.7777 \dots \Leftrightarrow 10x = 13.7777 \dots \Leftrightarrow 90x = 124$. Thus, $x = \frac{124}{90} = \frac{62}{45}$.

(c) Let $x = 2.13535 \dots$. So $1000x = 2135.3535 \dots \Leftrightarrow 10x = 21.3535 \dots \Leftrightarrow 990x = 2114$. Thus, $x = \frac{2114}{990} = \frac{1057}{495}$.

79. $\pi > 3$, so $|\pi - 3| = \pi - 3$.

80. $\sqrt{2} > 1$, so $|1 - \sqrt{2}| = \sqrt{2} - 1$.

81. $a < b$, so $|a - b| = -(a - b) = b - a$.

82. $a + b + |a - b| = a + b + b - a = 2b$

83. (a) $-a$ is negative because a is positive.

(b) bc is positive because the product of two negative numbers is positive.

(c) $a - b = a + (-b)$ is positive because it is the sum of two positive numbers.

(d) $ab + ac$ is negative: Each summand is the product of a positive number and a negative number, and the sum of two negative numbers is negative.

84. (a) $-b$ is positive because b is negative.

(b) $a + bc$ is positive because it is the sum of two positive numbers.

(c) $c - a = c + (-a)$ is negative because c and $-a$ are both negative.

(d) ab^2 is positive because both a and b^2 are positive.

85. Distributive Property

86. (a) When $L = 150$, $x = 20$, and $y = 15$, we have $L + 2(x + y) = 150 + 2(20 + 15) = 150 + 70 = 220$. Because $220 \leq 274$, the post office will accept this package.

When $L = 120$, $x = 60$, and $y = 60$, we have $L + 2(x + y) = 120 + 2(60 + 60) = 120 + 240 = 360$, and since $360 \not\leq 274$, the post office will *not* accept this package.

(b) If $x = y = 22.5$, then $L + 2(22.5 + 22.5) \leq 274 \Leftrightarrow L + 90 \leq 274 \Leftrightarrow L \leq 184$. So the length can be as long as $184 \text{ cm} = 1.84 \text{ m}$.

87. Let $x = \frac{m_1}{n_1}$ and $y = \frac{m_2}{n_2}$ be rational numbers. Then $x + y = \frac{m_1}{n_1} + \frac{m_2}{n_2} = \frac{m_1n_2 + m_2n_1}{n_1n_2}$,

$$x - y = \frac{m_1}{n_1} - \frac{m_2}{n_2} = \frac{m_1n_2 - m_2n_1}{n_1n_2}, \text{ and } x \cdot y = \frac{m_1}{n_1} \cdot \frac{m_2}{n_2} = \frac{m_1m_2}{n_1n_2}.$$

This shows that the sum, difference, and product of two rational numbers are again rational numbers. However the product of two irrational numbers is not necessarily irrational; for example, $\sqrt{2} \cdot \sqrt{2} = 2$, which is rational. Also, the sum of two irrational numbers is not necessarily irrational; for example, $\sqrt{2} + (-\sqrt{2}) = 0$ which is rational.

88. $\frac{1}{2} + \sqrt{2}$ is irrational. If it were rational, then by Exercise 6(a), the sum $\left(\frac{1}{2} + \sqrt{2}\right) + \left(-\frac{1}{2}\right) = \sqrt{2}$ would be rational, but this is not the case.

Similarly, $\frac{1}{2} \cdot \sqrt{2}$ is irrational.

(a) Following the hint, suppose that $r + t = q$, a rational number. Then by Exercise 6(a), the sum of the two rational numbers $r + t$ and $-r$ is rational. But $(r + t) + (-r) = t$, which we know to be irrational. This is a contradiction, and hence our original premise—that $r + t$ is rational—was false.

- (b) r is a nonzero rational number, so $r = \frac{a}{b}$ for some nonzero integers a and b . Let us assume that $rt = q$, a rational number. Then by definition, $q = \frac{c}{d}$ for some integers c and d . But then $rt = q \Leftrightarrow \frac{a}{b}t = \frac{c}{d}$, whence $t = \frac{bc}{ad}$, implying that t is rational. Once again we have arrived at a contradiction, and we conclude that the product of a rational number and an irrational number is irrational.

89.

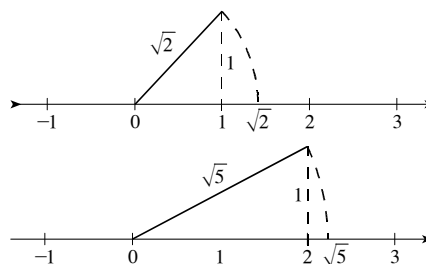
x	1	2	10	100	1000
$\frac{1}{x}$	1	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

As x gets large, the fraction $1/x$ gets small. Mathematically, we say that $1/x$ goes to zero.

x	1	0.5	0.1	0.01	0.001
$\frac{1}{x}$	1	$\frac{1}{0.5} = 2$	$\frac{1}{0.1} = 10$	$\frac{1}{0.01} = 100$	$\frac{1}{0.001} = 1000$

As x gets small, the fraction $1/x$ gets large. Mathematically, we say that $1/x$ goes to infinity.

90. We can construct the number $\sqrt{2}$ on the number line by transferring the length of the hypotenuse of a right triangle with legs of length 1 and 1. Similarly, to locate $\sqrt{5}$, we construct a right triangle with legs of length 1 and 2. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{1^2 + 2^2} = \sqrt{5}$. Then transfer the length of the hypotenuse to the number line.



The square root of any rational number can be located on a number line in this fashion.

The circle in the second figure in the text has circumference π , so if we roll it along a number line one full rotation, we have found π on the number line. Similarly, any rational multiple of π can be found this way.

91. (a) Suppose that $a > b$, so $\max(a, b) = a$ and $|a - b| = a - b$. Then $\frac{a + b + |a - b|}{2} = \frac{a + b + a - b}{2} = a$. On the other hand, if $b > a$, then $\max(a, b) = b$ and $|a - b| = -(a - b) = b - a$. In this case, $\frac{a + b + |a - b|}{2} = \frac{a + b + b - a}{2} = b$. If $a = b$, then $|a - b| = 0$ and the result is trivial.
- (b) If $a < b$, then $\min(a, b) = a$ and $|a - b| = b - a$. In this case $\frac{a + b - |a - b|}{2} = \frac{a + b - (b - a)}{2} = a$. Similarly, if $b < a$, then $\frac{a + b - |a - b|}{2} = b$; and if $a = b$, the result is trivial.

92. Answers will vary.

93. (a) Subtraction is not commutative. For example, $5 - 1 \neq 1 - 5$.
 (b) Division is not commutative. For example, $5 \div 1 \neq 1 \div 5$.
 (c) Putting on your socks and putting on your shoes are not commutative. If you put on your socks first, then your shoes, the result is not the same as if you proceed the other way around.
 (d) Putting on your hat and putting on your coat are commutative. They can be done in either order, with the same result.
 (e) Washing laundry and drying it are not commutative.

94. (a) If $x = 2$ and $y = 3$, then $|x + y| = |2 + 3| = |5| = 5$ and $|x| + |y| = |2| + |3| = 5$.

If $x = -2$ and $y = -3$, then $|x + y| = |-5| = 5$ and $|x| + |y| = 5$.

If $x = -2$ and $y = 3$, then $|x + y| = |-2 + 3| = 1$ and $|x| + |y| = 5$.

In each case, $|x + y| \leq |x| + |y|$ and the Triangle Inequality is satisfied.

- (b) *Case 0:* If either x or y is 0, the result is equality, trivially.

Case 1: If x and y have the same sign, then $|x + y| = \begin{cases} x + y & \text{if } x \text{ and } y \text{ are positive} \\ -(x + y) & \text{if } x \text{ and } y \text{ are negative} \end{cases} = |x| + |y|$.

Case 2: If x and y have opposite signs, then suppose without loss of generality that $x < 0$ and $y > 0$. Then $|x + y| < |-x + y| = |x| + |y|$.

1.2 EXPONENTS AND RADICALS

1. (a) Using exponential notation we can write the product $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ as 5^6 .

(b) In the expression 3^4 , the number 3 is called the *base* and the number 4 is called the *exponent*.

2. (a) When we multiply two powers with the same base, we *add* the exponents. So $3^4 \cdot 3^5 = 3^9$.

(b) When we divide two powers with the same base, we *subtract* the exponents. So $\frac{3^5}{3^2} = 3^3$.

3. To move a number raised to a power from numerator to denominator or from denominator to numerator change the sign of the *exponent*. So $a^{-2} = \frac{1}{a^2}$, $\frac{1}{b^{-2}} = b^2$, $\frac{a^{-3}}{b^2} = \frac{1}{a^3 b^2}$, and $\frac{6a^2}{b^{-3}} = 6a^2 b^3$.

4. (a) Using exponential notation we can write $\sqrt[3]{5}$ as $5^{1/3}$.

(b) Using radicals we can write $5^{1/2}$ as $\sqrt{5}$.

(c) No. $\sqrt{5^2} = (5^2)^{1/2} = 5^{2(1/2)} = 5$ and $(\sqrt{5})^2 = (5^{1/2})^2 = 5^{(1/2)2} = 5$.

5. $(4^{1/2})^3 = 2^3 = 8$; $(4^3)^{1/2} = 64^{1/2} = 8$

6. Because the denominator is of the form $\sqrt{a}$, we multiply numerator and denominator by $\sqrt{a}$: $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

7. $5^{1/3} \cdot 5^{2/3} = 5^1 = 5$

8. (a) Yes, $(-5)^4 = 625$, while $-5^4 = -(5^4) = -625$.

(b) No, $(x^2)^3 = x^{2 \cdot 3} = x^6$.

(c) No, $(2x^4)^3 = 2^3 x^{4 \cdot 3} = 8x^{12}$.

(d) No; if a is negative, then $\sqrt{4a^2} = -2a$.

9. (a) $-2^6 = -(2^6) = -64$

10. (a) $(-5)^3 = -125$

(b) $(-2)^6 = 64$

(b) $-5^3 = -(5^3) = -125$

(c) $(\frac{1}{5})^2 \cdot (-3)^3 = \frac{(-3)^3}{5^2} = -\frac{27}{25}$

(c) $(-5)^2 \cdot (\frac{2}{5})^2 = 4$

11. (a) $(\frac{5}{3})^0 \cdot 2^{-1} = \frac{1}{2}$

12. (a) $-2^3 \cdot (-2)^0 = -(2^3) \cdot 1 = -8$

(b) $\frac{2^{-3}}{3^0} = \frac{1}{2^3} = \frac{1}{8}$

(b) $-2^{-3} \cdot (-2)^0 = -\frac{1}{2^3} \cdot 1 = -\frac{1}{8}$

(c) $(\frac{2}{3})^{-2} = (\frac{3}{2})^2 = \frac{9}{4}$

(c) $(\frac{-3}{5})^{-3} = \frac{5^3}{(-3)^3} = -\frac{125}{27}$

13. (a) $5^3 \cdot 5 = 5^{3+1} = 625$

(b) $5^4 \cdot 5^{-2} = 5^{4-2} = 25$

(c) $(2^2)^3 = 2^{2 \cdot 3} = 64$

15. (a) $3\sqrt[3]{16} = 3\sqrt[3]{8 \cdot 2} = 3\sqrt[3]{8}\sqrt[3]{2} = 6\sqrt[3]{2}$

(b) $\frac{\sqrt{18}}{\sqrt{81}} = \sqrt{\frac{18}{81}} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$

(c) $\sqrt{\frac{27}{4}} = \frac{3\sqrt{3}}{2} = \frac{\sqrt{9 \cdot 3}}{\sqrt{4}} = \frac{3\sqrt{3}}{2}$

17. (a) $\sqrt{3}\sqrt{15} = \sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$

(b) $\frac{\sqrt{48}}{\sqrt{3}} = \sqrt{\frac{48}{3}} = \sqrt{16} = 4$

(c) $\sqrt[3]{24}\sqrt[3]{18} = \sqrt[3]{24 \cdot 18} = \sqrt[3]{6^3 \cdot 2} = 6\sqrt[3]{2}$

19. (a) $t^5 t^2 = t^{5+2} = t^7$

(b) $(4z^3)^2 = 4^2 (z^3)^2 = 16z^{3 \cdot 2} = 16z^6$

(c) $x^{-3} x^5 = x^{-3+5} = x^2$

21. (a) $x^{-5} \cdot x^3 = x^{-5+3} = x^{-2} = \frac{1}{x^2}$

(b) $w^{-2} w^{-4} w^5 = w^{-2-4+5} = w^{-1} = \frac{1}{w}$

(c) $\frac{x^{16}}{x^{10}} = x^{16-10} = x^6$

23. (a) $\frac{a^9 a^{-2}}{a} = a^{9-2-1} = a^6$

(b) $(a^2 a^4)^3 = (a^{2+4})^3 = (a^6)^3 = a^{6 \cdot 3} = a^{18}$

(c) $\left(\frac{x}{2}\right)^3 (5x^6) = \frac{x^3 \cdot 5x^6}{2^3} = \frac{5x^9}{8}$

25. (a) $(3x^3 y^2)(2y^3) = 3 \cdot 2x^3 y^2 y^3 = 6x^3 y^5$

(b) $(5w^2 z^{-2})^2 (z^3) = \left(\frac{5w^2}{z^2}\right)^2 (z^3) = \frac{5^2 (w^2)^2 z^3}{(z^2)^2} = \frac{25w^4}{z}$

26. (a) $(8m^{-2} n^4)\left(\frac{1}{2}n^{-2}\right) = 8 \cdot \frac{1}{2} m^{-2} n^{4-2} = \frac{4n^2}{m^2}$

(b) $(3a^4 b^{-2})^3 (a^2 b^{-1}) = (3^3 a^{4 \cdot 3} b^{-2 \cdot 3})(a^2 b^{-1}) = 3^3 a^{12+2} b^{-6-1} = \frac{27a^{14}}{b^7}$

27. (a) $\frac{x^2 y^{-1}}{x^{-5}} = x^{2-(-5)} y^{-1} = \frac{x^7}{y}$

(b) $\left(\frac{a^3}{2b^2}\right)^3 = \frac{a^{3 \cdot 3}}{2^3 (b^2)^3} = \frac{a^9}{8b^6}$

14. (a) $3^8 \cdot 3^5 = 3^{8+5} = 3^{13}$

(b) $\frac{10^7}{10^4} = 10^{7-4} = 1000$

(c) $(3^5)^4 = 3^{5 \cdot 4} = 3^{20}$

16. (a) $2\sqrt[3]{81} = 2\sqrt[3]{27 \cdot 3} = 2\sqrt[3]{27}\sqrt[3]{3} = 6\sqrt[3]{3}$

(b) $\frac{\sqrt{18}}{\sqrt{25}} = \frac{\sqrt{9 \cdot 2}}{5} = \frac{3\sqrt{2}}{5}$

(c) $\sqrt{\frac{12}{49}} = \frac{\sqrt{4 \cdot 3}}{\sqrt{49}} = \frac{2\sqrt{3}}{7}$

18. (a) $\sqrt{10}\sqrt{32} = \sqrt{320} = \sqrt{64 \cdot 5} = 8\sqrt{5}$

(b) $\frac{\sqrt{54}}{\sqrt{6}} = \sqrt{\frac{54}{6}} = 3$

(c) $\sqrt[3]{15}\sqrt[3]{75} = \sqrt[3]{15 \cdot 75} = \sqrt[3]{5^3 \cdot 9} = 5\sqrt[3]{9}$

20. (a) $a^4 a^6 = a^{4+6} = a^{10}$

(b) $(-2b^{-3})^3 = (-2)^3 (b^{-3})^3 = -8b^{-3 \cdot 3} = -\frac{8}{b^9}$

(c) $-2y^{10} y^{-11} = -2y^{10+(-11)} = -2y^{-1} = -\frac{2}{y}$

22. (a) $y^2 \cdot y^{-5} = y^{2-5} = y^{-3} = \frac{1}{y^3}$

(b) $z^5 z^{-3} z^{-4} = z^{5-3-4} = z^{-2} = \frac{1}{z^2}$

(c) $\frac{y^7 y^0}{y^{10}} = y^{7+0-10} = y^{-3} = \frac{1}{y^3}$

24. (a) $\frac{z^2 z^4}{z^3 z^{-1}} = z^{2+4-3-(-1)} = z^4$

(b) $(2a^3 a^2)^4 = 2^4 (a^{3+2})^4 = 16 (a^5)^4 = 16a^{20}$

(c) $(-3z^2)^3 (2z^3) = (-3)^3 \cdot z^{2 \cdot 3} \cdot 2z^3 = -54z^9$

$$28. (a) \frac{y^{-2}z^{-3}}{y^{-1}} = \frac{y}{y^2z^3} = \frac{1}{yz^3}$$

$$(b) \left(\frac{x^3y^{-2}}{x^{-3}y^2} \right)^{-2} = \left[x^{3-(-3)}y^{-2-2} \right]^{-2} = \left(x^6y^{-4} \right)^{-2} = \frac{y^8}{x^{12}}$$

$$29. (a) \left(\frac{a^2}{b} \right)^5 \left(\frac{a^3b^2}{c^3} \right)^3 = a^{2 \cdot 5 + 3 \cdot 3} b^{-5 + 2 \cdot 3} c^{-3 \cdot 3} = \frac{a^{19}b}{c^9}$$

$$(b) \frac{(u^{-1}v^2)^2}{(u^3v^{-2})^3} = u^{-1 \cdot 2 - 3 \cdot 3} v^{2 \cdot 2 - (-2) \cdot 3} = \frac{v^{10}}{u^{11}}$$

$$30. (a) \left(\frac{x^4z^2}{4y^5} \right) \left(\frac{2x^3y^2}{z^3} \right)^2 = \frac{2^2}{4} x^{4+3 \cdot 2} y^{-5+2 \cdot 2} z^{2+(-3) \cdot 2} = \frac{x^{10}}{yz^4}$$

$$(b) \frac{(rs^2)^3}{(r^{-3}s^2)^2} = r^{3-(-3) \cdot 2} s^{2 \cdot 3 - 2 \cdot 2} = r^9 s^2$$

$$31. (a) \frac{8a^3b^{-4}}{2a^{-5}b^5} = 4a^{3-(-5)}b^{-4-5} = \frac{4a^8}{b^9}$$

$$(b) \left(\frac{y}{5x^{-2}} \right)^{-3} = 5^{-1(-3)} x^{-(-2)(-3)} y^{-3} = \frac{125}{x^6y^3}$$

$$32. (a) \frac{5xy^{-2}}{x^{-1}y^{-3}} = 5x^{1-(-1)}y^{-2-(-3)} = 5x^2y$$

$$(b) \left(\frac{2a^{-1}b}{a^2b^{-3}} \right)^{-3} = 2^{-3} a^{-1(-3)-2(-3)} b^{-3-(-3)(-3)} = \frac{a^9}{8b^{12}}$$

$$33. (a) \sqrt[4]{x^4} = |x|$$

$$(b) \sqrt[4]{16x^8} = \sqrt[4]{2^4x^8} = 2x^2$$

$$34. (a) \sqrt[5]{x^{10}} = (x^{10})^{1/5} = x^2$$

$$(b) \sqrt[3]{x^3y^6} = (x^3y^6)^{1/3} = xy^2$$

$$35. (a) \sqrt[3]{8x^9y^3} = \sqrt[3]{2^3(x^3)^3y^3} = 2x^3y$$

$$(b) \sqrt[4]{8x^6y^2} \sqrt[4]{2x^2y^2} = \sqrt[4]{8x^6y^2 \cdot 2x^2y^2} = \sqrt[4]{16x^8y^4} = \sqrt[4]{2^4(x^2)^4y^4} = 2x^2|y|$$

$$36. (a) \sqrt{16x^4y^6z^2} = \sqrt{4^2(x^2)^2(y^3)^2z^2} = 4x^2|y|^3|z| \quad (b) \sqrt[3]{\sqrt[3]{512x^9}} = (512x^9)^{1/9} = 2x$$

$$37. (a) \sqrt{32} + \sqrt{18} = \sqrt{16 \cdot 2} + \sqrt{9 \cdot 2} = \sqrt{16}\sqrt{2} + \sqrt{9}\sqrt{2} = 4\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$$

$$(b) \sqrt{75} + \sqrt{48} = \sqrt{25 \cdot 3} + \sqrt{16 \cdot 3} = \sqrt{25}\sqrt{3} + \sqrt{16}\sqrt{3} = 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$$

$$38. (a) \sqrt{125} + \sqrt{45} = \sqrt{25 \cdot 5} + \sqrt{9 \cdot 5} = 5\sqrt{5} + 3\sqrt{5} = 8\sqrt{5}$$

$$(b) \sqrt[3]{54} - \sqrt[3]{16} = \sqrt[3]{27 \cdot 2} - \sqrt[3]{8 \cdot 2} = \sqrt[3]{27}\sqrt[3]{2} - \sqrt[3]{8}\sqrt[3]{2} = \sqrt[3]{2} - \sqrt[3]{2} = 0$$

$$39. (a) \sqrt{9a^3} + \sqrt{a} = \sqrt{9 \cdot a^2 \cdot a} + \sqrt{a} = 3a\sqrt{a} + \sqrt{a} = (3a + 1)\sqrt{a}$$

$$(b) \sqrt{16x} + \sqrt{x^5} = \sqrt{16}\sqrt{x} + \sqrt{x^4}\sqrt{x} = (x^2 + 4)\sqrt{x}$$

$$40. (a) \sqrt[3]{x^4} + \sqrt[3]{8x} = \sqrt[3]{x^3 \cdot x} + \sqrt[3]{2^3x} = (x + 2)\sqrt[3]{x}$$

$$(b) 4\sqrt{18rt^3} + 5\sqrt{32r^3t^5} = 4\sqrt{(3^2 \cdot 2)r(t^2 \cdot t)} + 5\sqrt{(4^2 \cdot 2)(r^2 \cdot r)(t^4 \cdot t)} = 4(3)(t)\sqrt{2rt} + 5(4)(r)(t^2)\sqrt{2rt} \\ = (20rt^2 + 12t)\sqrt{2rt}$$

$$41. (a) \sqrt{36x^2 + 36x^4} = \sqrt{6^2x^2(1 + x^2)} = 6x\sqrt{1 + x^2}$$

$$(b) \sqrt{81x^2 + 81y^2} = \sqrt{9^2(x^2 + y^2)} = 9\sqrt{x^2 + y^2}$$

$$42. (a) \sqrt{27a^3 + 63a^2} = \sqrt{3^2a^2(3a + 7)} = 3a\sqrt{3a + 7}$$

$$(b) \sqrt{25t^2 + 100t^2} = \sqrt{125t^2} = 5\sqrt{5}t, t \geq 0$$

$$43. \sqrt{10} = 10^{1/2}$$

$$44. \sqrt[5]{6} = 6^{1/5}$$

45. $7^{3/5} = (7^3)^{1/5} = \sqrt[5]{7^3}$

46. $6^{-5/2} = \frac{1}{6^{5/2}} = \frac{1}{(6^5)^{1/2}} = \frac{1}{\sqrt{6^5}}$

47. $\frac{1}{\sqrt{5}} = \frac{1}{5^{1/2}} = 5^{-1/2}$

48. $5^{-3/4} = \frac{1}{5^{3/4}} = \frac{1}{\sqrt[4]{5^3}} = \frac{1}{\sqrt[4]{125}}$

49. $y^{-1.5} = y^{-3/2} = \frac{1}{\sqrt{y^3}}$

50. $\frac{1}{\sqrt[3]{x^2}} = \frac{1}{x^{2/3}} = x^{-2/3}$

51. (a) $16^{1/4} = \sqrt[4]{16} = 2$

(b) $-8^{1/3} = \sqrt[3]{-8} = -2$

(c) $9^{-1/2} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

52. (a) $27^{1/3} = \sqrt[3]{27} = 3$

(b) $(-8)^{1/3} = \sqrt[3]{-8} = -2$

(c) $-\left(\frac{1}{8}\right)^{1/3} = -\frac{1}{\sqrt[3]{8}} = -\frac{1}{2}$

53. (a) $32^{2/5} = \left(\sqrt[5]{32}\right)^2 = 2^2 = 4$

(b) $\left(\frac{4}{9}\right)^{-1/2} = \frac{1}{\sqrt{4/9}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

(c) $\left(\frac{16}{81}\right)^{3/4} = \left(\frac{\sqrt[4]{16}}{\sqrt[4]{81}}\right)^3 = \frac{8}{27}$

54. (a) $125^{2/3} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

(b) $\left(\frac{25}{64}\right)^{3/2} = \left(\frac{\sqrt{25}}{\sqrt{64}}\right)^3 = \left(\frac{5}{8}\right)^3 = \frac{125}{512}$

(c) $27^{-4/3} = \frac{1}{\left(\sqrt[3]{27}\right)^4} = \frac{1}{3^4} = \frac{1}{81}$

55. (a) $5^{2/3} \cdot 5^{1/3} = 5^{(2/3)+(1/3)} = 5^1 = 5$

(b) $\frac{3^{3/5}}{3^{2/5}} = 3^{(3/5)-(2/5)} = 3^{1/5} = \sqrt[5]{3}$

(c) $\left(\sqrt[3]{4}\right)^3 = \left(4^{1/3}\right)^3 = 4^{(1/3) \cdot 3} = 4^1 = 4$

56. (a) $3^{2/7} \cdot 3^{12/7} = 3^{2/7+12/7} = 3^2 = 9$

(b) $\frac{7^{2/3}}{7^{5/3}} = 7^{(2/3)-(5/3)} = 7^{-1} = \frac{1}{7}$

(c) $\left(\sqrt[5]{6}\right)^{-10} = \left(6^{1/5}\right)^{-10} = 6^{(1/5)(-10)} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

57. (a) $x^{3/4}x^{5/4} = x^{3/4+5/4} = x^2$

(b) $y^{2/3}y^{4/3} = y^{2/3+4/3} = y^2$

58. (a) $(4b)^{1/2} \left(8b^{1/4}\right) = 4^{1/2} \cdot 8b^{1/2+1/4} = 16b^{3/4}$

(b) $\left(3a^{3/4}\right)^2 \left(5a^{1/2}\right) = 3^2 \cdot 5a^{(3/4) \cdot 2 + (1/2)} = 45a^2$

59. (a) $\frac{w^{4/3}w^{2/3}}{w^{1/3}} = w^{4/3+2/3-1/3} = w^{5/3}$

(b) $\left(3x^{1/2}y^{1/3}\right)^6 = 3^6x^{(1/2)6}y^{(1/3)6} = 3^6 = 729x^3y^2$

60. (a) $\left(8y^3\right)^{-2/3} = 8^{-2/3}y^{3(-2/3)} = \frac{1}{4y^2}$

(b) $\left(u^4v^6\right)^{-1/3} = u^{4(-1/3)}v^{6(-1/3)} = \frac{1}{u^{4/3}v^2}$

61. (a) $\left(8a^6b^{3/2}\right)^{2/3} = 8^{2/3}a^{6(2/3)}b^{(3/2)(2/3)} = 4a^4b$

(b) $\left(4a^{-4}b^6\right)^{3/2} = 4^{3/2}a^{-4(3/2)}b^{6(3/2)} = 8a^{-6}b^9 = \frac{8b^9}{a^6}$

62. (a) $\left(x^{-5}y^{1/3}\right)^{-3/5} = x^{-5(-3/5)}y^{(1/3)(-3/5)} = \frac{x^3}{y^{1/5}}$

(b) $\left(u^3v^2\right)^{1/3} \left(16u^6v^{2/3}\right)^{1/2} = u^{3(1/3)}v^{2(1/3)}16^{1/2}u^{6(1/2)}v^{(2/3)(1/2)} = 4uv^{2/3}u^3v^{1/3} = 4u^4v$

63. (a) $\left(\frac{3x^{1/4}}{y}\right)^2 \left(\frac{x^{-1}}{y^4}\right)^{1/2} = \frac{3^2 x^{(1/4)2} x^{-1(1/2)}}{y^2 y^{4(1/2)}} = \frac{9x^{1/2} x^{-1/2}}{y^2 y^2} = \frac{9}{y^4}$
 (b) $\left(\frac{3w^{-1/3}}{z^{-1}}\right)^{-4} \left(\frac{2w^{1/3}}{z^{1/2}}\right)^2 = \frac{3^{-4} w^{(-1/3)(-4)} 2^2 w^{(1/3)2}}{z^{-1(-4)} z^{(1/2)2}} = \frac{w^{4/3} 4w^{2/3}}{81z^4 z} = \frac{4w^{(4/3)+(2/3)}}{81z^5} = \frac{4w^2}{81z^5}$
64. (a) $\left(\frac{y^{2/3}}{x^{-1}}\right)^3 \left(\frac{x}{y^{-2}}\right)^{-1} = \frac{(y^{2/3})^3 x^{-1}}{(x^{-1})^3 (y^{-2})^{-1}} = \frac{y^2 x^{-1}}{x^{-3} y^2} = x^2$
 (b) $\left(\frac{4y^3 z^{2/3}}{x^{1/2}}\right)^2 \left(\frac{x^{-3} y^6}{8z^4}\right)^{1/3} = \frac{4^2 (y^3)^2 (z^{2/3})^2 (x^{-3})^{1/3} (y^6)^{1/3}}{(x^{1/2})^2 8^{1/3} (z^4)^{1/3}} = \frac{16y^6 z^{4/3} x^{-1} y^2}{x 2z^{4/3}} = \frac{8y^8}{x^2}$
65. (a) $\sqrt{x^3} = (x^3)^{1/2} = x^{3(1/2)} = x^{3/2}$
 (b) $\sqrt[5]{x^6} = (x^6)^{1/5} = x^{6(1/5)} = x^{6/5}$
66. (a) $\sqrt{x^5} = (x^5)^{1/2} = x^{5(1/2)} = x^{5/2}$
 (b) $\sqrt[4]{x^6} = (x^6)^{1/4} = x^{6(1/4)} = x^{3/2}$
67. (a) $\sqrt[6]{y^5} \sqrt[3]{y^2} = y^{5/6} \cdot y^{2/3} = y^{5/6+2/3} = y^{3/2}$
 (b) $(5\sqrt[3]{x})(2\sqrt[4]{x}) = 5 \cdot 2x^{1/3+1/4} = 10x^{7/12}$
68. (a) $\sqrt[4]{b^3} \sqrt{b} = b^{3/4+1/2} = b^{5/4}$
 (b) $(2\sqrt{a})(\sqrt[3]{a^2}) = 2a^{1/2+2/3} = 2a^{7/6}$
69. (a) $\sqrt[4]{4st^3} \sqrt[6]{s^3 t^2} = 4^{1/2} s^{1/2+3/6} t^{3/2+2/6} = 2st^{11/6}$
 (b) $\frac{\sqrt[4]{x^7}}{\sqrt[4]{x^3}} = x^{7/4-3/4} = x$
70. (a) $\sqrt[5]{x^3 y^2} \sqrt[10]{x^4 y^{16}} = x^{3/5+4/10} y^{2/5+16/10} = xy^2$
 (b) $\frac{\sqrt[3]{8x^2}}{\sqrt{x}} = 8^{1/3} x^{2/3-1/2} = 2x^{1/6}$
71. (a) $\sqrt[3]{y\sqrt{y}} = (y^{1+1/2})^{1/3} = y^{(3/2)(1/3)} = y^{1/2}$
 (b) $\sqrt{\frac{18u^5 v}{2u^3 v^3}} = \sqrt{\frac{9u^2}{v^2}} = \frac{3u}{v}$
72. (a) $\sqrt{s\sqrt{s^3}} = (s^{1+3/2})^{1/2} = s^{5/4}$
 (b) $\sqrt[3]{\frac{54x^2 y^4}{2x^5 y}} = \sqrt[3]{\frac{27y^3}{x^3}} = \frac{3y}{x}$
73. (a) $\frac{1}{\sqrt{6}} = \frac{1}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{6}$
 (b) $\sqrt{\frac{3}{2}} = \sqrt{\frac{3}{2}} \sqrt{\frac{2}{2}} = \frac{\sqrt{6}}{2}$
 (c) $\frac{9}{\sqrt[4]{2}} = \frac{9}{2^{1/4}} \cdot \frac{2^{3/4}}{2^{3/4}} = \frac{9\sqrt[4]{8}}{2}$
74. (a) $\frac{12}{\sqrt{3}} = \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$
 (b) $\sqrt{\frac{12}{5}} = \sqrt{\frac{4 \cdot 3}{5}} \sqrt{\frac{5}{5}} = \frac{2\sqrt{15}}{5}$
 (c) $\frac{8}{\sqrt[3]{5^2}} = \frac{8}{5^{2/3}} \cdot \frac{5^{1/3}}{5^{1/3}} = \frac{8\sqrt[3]{5}}{5}$
75. (a) $\frac{1}{\sqrt{5x}} = \frac{1}{\sqrt{5x}} \cdot \frac{\sqrt{5x}}{\sqrt{5x}} = \frac{\sqrt{5x}}{5x}$
 (b) $\sqrt{\frac{x}{5}} = \sqrt{\frac{x}{5}} \sqrt{\frac{5}{5}} = \frac{\sqrt{5x}}{5}$
 (c) $\sqrt[5]{\frac{1}{x^3}} = \frac{1}{x^{3/5}} \cdot \frac{x^{2/5}}{x^{2/5}} = \frac{\sqrt[5]{x^2}}{x}$
76. (a) $\sqrt{\frac{s}{3t}} = \sqrt{\frac{s}{3t}} \sqrt{\frac{3t}{3t}} = \frac{\sqrt{3st}}{3t}$
 (b) $\frac{a}{\sqrt[6]{b^2}} = \frac{a}{\sqrt[3]{b} \sqrt[3]{b^2}} = \frac{a\sqrt[3]{b^2}}{b}$
 (c) $\frac{1}{c^{3/5}} = \frac{1}{c^{3/5}} \cdot \frac{c^{2/5}}{c^{2/5}} = \frac{\sqrt[5]{c^2}}{c}$
77. (a) $\sqrt[4]{\frac{1}{4}} \sqrt[4]{\frac{1}{64}} = \sqrt[4]{\frac{1}{4 \cdot 64}} = \frac{1}{4}$
 (b) $\frac{\sqrt{5}}{\sqrt{40}} = \sqrt{\frac{5}{40}} = \sqrt{\frac{1}{8}} = \sqrt{\frac{2}{16}} = \frac{\sqrt{2}}{4}$
78. (a) $\left(\frac{x^{3/2}}{y^{-1/2}}\right)^4 = \frac{x^{(3/2)4}}{y^{(-1/2)(4)}} = \frac{x^6}{y^{-2}} = x^6 y^2$
 (b) $(25u^2 v^{-4})^{1/2} = 25^{1/2} u^{2(1/2)} v^{-4(1/2)} = 5|u| v^{-2} = \frac{5|u|}{v^2}$. Since $u < 0$, this is equivalent to $-\frac{5u}{v^2}$.

79. (a) $\sqrt{y}\sqrt[4]{y^2} = y^{1/2}y^{2/4} = y$

(b) $\left(81\sqrt[4]{w^8z^8}\right)^{1/2} = 81^{1/2}w^{(8/4)(1/2)}z^{(8/4)(1/2)} = 9|w||z|$. Since $w > 0$ and $z < 0$, this is equivalent to $-9wz$.

80. (a) $\left(\frac{x^{3/2}}{y^{-1/2}}\right)^4 \left(\frac{x^{-2}}{y^3}\right) = \frac{(x^{3/2})^4}{(y^{-1/2})^4} \cdot \frac{x^{-2}}{y^3} = \frac{x^6 \cdot x^{-2}}{y^{-2} \cdot y^3} = \frac{x^4}{y}$

(b) $\left(\frac{w^4}{\sqrt[3]{w^3z^6}}\right)^{1/2} = \frac{w^{4(1/2)}}{w^{1/2}z^{2(1/2)}} = \frac{w^2}{\sqrt{wz}} = \frac{w^{3/2}}{z}$

81. (a) $69,300,000 = 6.93 \times 10^7$

(b) $7,200,000,000,000 = 7.2 \times 10^{12}$

(c) $0.000028536 = 2.8536 \times 10^{-5}$

(d) $0.0001213 = 1.213 \times 10^{-4}$

82. (a) $129,540,000 = 1.2954 \times 10^8$

(b) $7,259,000,000 = 7.259 \times 10^9$

(c) $0.0000000014 = 1.4 \times 10^{-9}$

(d) $0.0007029 = 7.029 \times 10^{-4}$

83. (a) $3.19 \times 10^5 = 319,000$

(b) $2.721 \times 10^8 = 272,100,000$

(c) $2.670 \times 10^{-8} = 0.00000002670$

(d) $9.999 \times 10^{-9} = 0.000000009999$

84. (a) $7.1 \times 10^{14} = 710,000,000,000,000$

(b) $6 \times 10^{12} = 6,000,000,000,000$

(c) $8.55 \times 10^{-3} = 0.00855$

(d) $6.257 \times 10^{-10} = 0.0000000006257$

85. (a) $9,460,000,000,000 \text{ km} = 9.5 \times 10^{12} \text{ km}$

(b) $0.0000000000004 \text{ cm} = 4 \times 10^{-13} \text{ cm}$

(c) $33 \text{ billion billion molecules} = 33 \times 10^9 \times 10^9 = 3.3 \times 10^{19} \text{ molecules}$

86. (a) $150,000,000 \text{ km} = 1.5 \times 10^8 \text{ km}$

(b) $0.0000000000000000000000053 \text{ g} = 5.3 \times 10^{-23} \text{ g}$

(c) $5,970,000,000,000,000,000,000,000 \text{ kg} = 5.97 \times 10^{24} \text{ kg}$

87. $(7.2 \times 10^{-9})(1.806 \times 10^{-12}) = 7.2 \times 1.806 \times 10^{-9} \times 10^{-12} \approx 13.0 \times 10^{-21} = 1.3 \times 10^{-20}$

88. $(1.062 \times 10^{24})(8.61 \times 10^{19}) = 1.062 \times 8.61 \times 10^{24} \times 10^{19} \approx 9.14 \times 10^{43}$

89. $\frac{1.295643 \times 10^9}{(3.610 \times 10^{-17})(2.511 \times 10^6)} = \frac{1.295643}{3.610 \times 2.511} \times 10^{9+17-6} \approx 0.1429 \times 10^{19} = 1.429 \times 10^{19}$

90. $\frac{(73.1)(1.6341 \times 10^{28})}{0.0000000019} = \frac{(7.31 \times 10)(1.6341 \times 10^{28})}{1.9 \times 10^{-9}} = \frac{7.31 \times 1.6341}{1.9} \times 10^{1+28-(-9)} \approx 6.3 \times 10^{38}$

91. $\frac{(0.0000162)(0.01582)}{(594,621,000)(0.0058)} = \frac{(1.62 \times 10^{-5})(1.582 \times 10^{-2})}{(5.94621 \times 10^8)(5.8 \times 10^{-3})} = \frac{1.62 \times 1.582}{5.94621 \times 5.8} \times 10^{-5-2-8+3} = 0.074 \times 10^{-12} = 7.4 \times 10^{-14}$

92. $\frac{(3.542 \times 10^{-6})^9}{(5.05 \times 10^4)^{12}} = \frac{(3.542)^9 \times 10^{-54}}{(5.05)^{12} \times 10^{48}} = \frac{87,747.96}{275,103,767.10} \times 10^{-54-48} \approx 3.19 \times 10^{-4} \times 10^{-102} \approx 3.19 \times 10^{-106}$

93. (a) b^5 is negative since a negative number raised to an odd power is negative.

(b) b^{10} is positive since a negative number raised to an even power is positive.

(c) ab^2c^3 we have (positive) (negative)² (negative)³ = (positive) (positive) (negative) which is negative.

(d) Since $b - a$ is negative, $(b - a)^3 = (\text{negative})^3$ which is negative.

(e) Since $b - a$ is negative, $(b - a)^4 = (\text{negative})^4$ which is positive.

(f) $\frac{a^3 c^3}{b^6 c^6} = \frac{(\text{positive})^3 (\text{negative})^3}{(\text{negative})^6 (\text{negative})^6} = \frac{(\text{positive}) (\text{negative})}{(\text{positive}) (\text{positive})} = \frac{\text{negative}}{\text{positive}}$ which is negative.

94. (a) Since $\frac{1}{2} > \frac{1}{3}$, $2^{1/2} > 2^{1/3}$.

(b) $\left(\frac{1}{2}\right)^{1/2} = 2^{-1/2}$ and $\left(\frac{1}{2}\right)^{1/3} = 2^{-1/3}$. Since $-\frac{1}{2} < -\frac{1}{3}$, we have $\left(\frac{1}{2}\right)^{1/2} < \left(\frac{1}{2}\right)^{1/3}$.

(c) We find a common root: $7^{1/4} = 7^{3/12} = (7^3)^{1/12} = 343^{1/12}$; $4^{1/3} = 4^{4/12} = (4^4)^{1/12} = 256^{1/12}$. So $7^{1/4} > 4^{1/3}$.

(d) We find a common root: $\sqrt[3]{5} = 5^{1/3} = 5^{2/6} = (5^2)^{1/6} = 25^{1/6}$; $\sqrt{3} = 3^{1/2} = 3^{3/6} = (3^3)^{1/6} = 27^{1/6}$. So $\sqrt[3]{5} < \sqrt{3}$.

95. Since one light year is 9.5×10^{12} km, Centauri is about $4.3 \times 9.5 \times 10^{12} \approx 4.06 \times 10^{13}$ km, or 40,600,000,000,000 kilometers away.

96. $9.3 \times 10^7 \text{ mi} = 186,000 \frac{\text{mi}}{\text{s}} \times t \text{ s} \Leftrightarrow t = \frac{9.3 \times 10^7}{186,000} \text{ s} = 500 \text{ s} = 8\frac{1}{3} \text{ min.}$

97. Volume = (Average depth) (Area) = $(3.7 \times 10^3 \text{ m}) (3.6 \times 10^{14} \text{ m}^2) \left(\frac{10^3 \text{ liters}}{\text{m}^3}\right) \approx 1.33 \times 10^{21} \text{ liters}$

98. Each person's share is equal to $\frac{\text{National debt}}{\text{Population}} = \frac{2.670 \times 10^{13}}{3.3145 \times 10^8} \approx 8.0555 \times 10^4 \approx \$80,555$.

99. First, we estimate the total mass of the stars in the observable universe:

$$\begin{aligned} \text{Total star mass} &\approx (\text{Number of galaxies}) \left(\frac{\text{Number of stars}}{\text{Galaxy}} \right) (\text{Mass of typical star}) = (1.5 \cdot 2 \cdot 1.77) (10^{12} \cdot 10^{11} \cdot 10^{30}) \\ &\approx 5.31 \times 10^{53} \text{ kg} \end{aligned}$$

Thus, the number of hydrogen atoms in the observable universe is

$$\frac{\text{Total star mass}}{\text{Mass of a single hydrogen atom}} \approx \frac{5.31 \times 10^{53}}{1.67 \times 10^{-27}} \approx 3.18 \times 10^{80}$$

100. First convert 346 meters to kilometers. This gives $346 \text{ m} = 346 \cdot \frac{1 \text{ kilometer}}{1000 \text{ meters}} = 0.346 \text{ km}$. Thus the distance you can see

is given by $D = \sqrt{2rh + h^2} = \sqrt{2(6340)(0.346) + (0.346)^2} \approx \sqrt{4387.4} \approx 66.2 \text{ kilometers}$.

101. (a) Using $f = 0.4$ and substituting $d = 65$, we obtain $s = \sqrt{30fd} = \sqrt{30 \times 0.4 \times 65} \approx 28 \text{ km/h}$.

(b) Using $f = 0.5$ and substituting $s = 50$, we find d . This gives $s = \sqrt{30fd} \Leftrightarrow 50 = \sqrt{30 \cdot (0.5)d} \Leftrightarrow 50 = \sqrt{15d} \Leftrightarrow 2500 = 15d \Leftrightarrow d = \frac{500}{3} \approx 167 \text{ m}$.

102. Since 1 day = 86,400 s, 365.25 days = 31,557,600 s. Substituting, we obtain $d = \left(\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{4\pi^2} \right)^{1/3} \cdot (3.15576 \times 10^7)^{2/3} \approx 1.5 \times 10^{11} \text{ m} = 1.5 \times 10^8 \text{ km}$.

103. Since $10^6 = 10^3 \cdot 10^3$ it would take 1000 days ≈ 2.74 years to spend the million dollars.

Since $10^9 = 10^3 \cdot 10^6$ it would take $10^6 = 1,000,000$ days ≈ 2739.72 years to spend the billion dollars.

104. (a)

n	1	2	5	10	100
$2^{1/n}$	$2^{1/1} = 2$	$2^{1/2} = 1.414$	$2^{1/5} = 1.149$	$2^{1/10} = 1.072$	$2^{1/100} = 1.007$

So when n gets large, $2^{1/n}$ decreases to 1.

(b)

n	1	2	5	10	100
$\left(\frac{1}{2}\right)^{1/n}$	$\left(\frac{1}{2}\right)^{1/1} = 0.5$	$\left(\frac{1}{2}\right)^{1/2} = 0.707$	$\left(\frac{1}{2}\right)^{1/5} = 0.871$	$\left(\frac{1}{2}\right)^{1/10} = 0.933$	$\left(\frac{1}{2}\right)^{1/100} = 0.993$

So when n gets large, $\left(\frac{1}{2}\right)^{1/n}$ increases to 1.

105. (a) $\frac{18^5}{9^5} = \left(\frac{18}{9}\right)^5 = 2^5 = 32$

(b) $20^6 \cdot (0.5)^6 = (20 \cdot 0.5)^6 = 10^6 = 1,000,000$

106. (a) $\frac{a^m}{a^n} = \frac{\overbrace{a \cdot a \cdot \dots \cdot a}^{m \text{ factors}}}{\underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ factors}}}$. Because $m > n$, we can cancel n factors of a from numerator and denominator and are left with

 $m - n$ factors of a in the numerator. Thus, $\frac{a^m}{a^n} = a^{m-n}$.

(b) $\left(\frac{a}{b}\right)^n = \frac{\overbrace{a \cdot a \cdot \dots \cdot a}^{n \text{ factors}}}{\underbrace{b \cdot b \cdot \dots \cdot b}_{n \text{ factors}}} = \frac{a^n}{b^n}$

107. (a) $\left(\frac{a}{b}\right)^{-n} = \frac{1}{(a/b)^n} = \frac{1}{a^n/b^n} = \frac{b^n}{a^n}$

(b) $\frac{a^{-n}}{b^{-m}} = \frac{1/a^n}{1/b^m} = \frac{1}{a^n} \cdot b^m = \frac{b^m}{a^n}$

1.3 ALGEBRAIC EXPRESSIONS

- The greatest common factor in the expression $18x^3 + 30x$ is $6x$, and the expression factors as $6x(3x^2 + 5)$.
- (a) The polynomial $2x^3 + 3x^2 + 10x$ has three terms: $2x^3$, $3x^2$, and $10x$.
(b) The factor x is common to each term, so $2x^3 + 3x^2 + 10x = x(2x^2 + 3x + 10)$.
- To factor the trinomial $x^2 + 8x + 12$ we look for two integers whose product is 12 and whose sum is 8. These integers are 6 and 2, so the trinomial factors as $(x + 6)(x + 2)$.
- The Special Product Formula for the “square of a sum” is $(A + B)^2 = A^2 + 2AB + B^2$. So $(2x + 3)^2 = (2x)^2 + 2(2x)(3) + 3^2 = 4x^2 + 12x + 9$.
- The Special Product Formula for the “product of the sum and difference of terms” is $(A + B)(A - B) = A^2 - B^2$. So $(6 + x)(6 - x) = 6^2 - x^2 = 36 - x^2$.
- The Special Factoring Formula for the “difference of squares” is $A^2 - B^2 = (A - B)(A + B)$. So $49x^2 - 9 = (7x - 3)(7x + 3)$.
- The Special Factoring Formula for a “perfect square” is $A^2 + 2AB + B^2 = (A + B)^2$. So $x^2 + 10x + 25 = (x + 5)^2$.
- (a) No; $(x + 5)^2 = x^2 + 2(5x) + 25 \neq x^2 + 25$.
(b) Yes; $(x + a)^2 = x^2 + 2xa + a^2$.
(c) Yes; by a Special Product Formula, $(x + 5)(x - 5) = x^2 - 25$.
(d) No, $(x + a)(x - a) = x^2 - a^2$, by a Special Product Formula.

9. Type: binomial. Terms: $5x^3$ and 6. Degree: 3.
10. Type: trinomial. Terms: $-2x^2$, $5x$, and -3 . Degree: 2.
11. Type: monomial. Term: -8 . Degree: 0.
12. Type: monomial. Terms: $\frac{1}{2}x^7$. Degree: 7.
13. Type: four-term polynomial. Terms: x , $-x^2$, x^3 , and $-x^4$. Degree: 4.
14. Type: binomial. Terms: $\sqrt{2}x$ and $-\sqrt{3}$. Degree: 1.
15. $(12x - 7) - (5x - 12) = 12x - 7 - 5x + 12 = 7x + 5$
16. $(5 - 3x) + (2x - 8) = -x - 3$
17. $(-2x^2 - 3x + 1) + (3x^2 + 5x - 4) = -2x^2 - 3x + 1 + 3x^2 + 5x - 4 = x^2 + 2x - 3$
18. $(3x^2 + x + 1) - (2x^2 - 3x - 5) = 3x^2 + x + 1 - 2x^2 + 3x + 5 = x^2 + 4x + 6$
19. $(5x^3 + 4x^2 - 3x) - (x^2 + 7x + 2) = 5x^3 + 4x^2 - 3x - x^2 - 7x - 2 = 5x^3 + 3x^2 - 10x - 2$
20. $3(x - 1) + 4(x + 2) = 3x - 3 + 4x + 8 = 7x + 5$
21. $8(2x + 5) - 7(x - 9) = 16x + 40 - 7x + 63 = 9x + 103$
22. $4(x^2 - 3x + 5) - 3(x^2 - 2x + 1) = 4x^2 - 12x + 20 - 3x^2 + 6x - 3 = x^2 - 6x + 17$
23. $x^3(x^2 + 3x) - 2x(x^4 - 3x^2) = x^5 + 3x^4 - 2x^5 + 6x^3 = -x^5 + 3x^4 + 6x^3$
24. $4x(1 - x^3) + 3x^3(x^3 - x) = 4x - 4x^4 + 3x^6 - 3x^4 = 3x^6 - 7x^4 + 4x$
25. $4x(x - 2) - 2(x^2 - 4x) + 2x^2(x - 1) = 4x^2 - 8x - 2x^2 + 8x + 2x^3 - 2x^2 = 2x^3$
26. $6x^3(x^2 - 1) - 2x(2 + 3x^2) + 2(2x - 4) = 6x^5 - 6x^3 - 4x - 6x^3 + 4x - 8 = 6x^5 - 12x^3 - 8$
27. $(3t - 2)(7t - 4) = 21t^2 - 12t - 14t + 8 = 21t^2 - 26t + 8$
28. $(4s - 1)(2s + 5) = 8s^2 + 18s - 5$
29. $(3x + 5)(2x - 1) = 6x^2 + 10x - 3x - 5 = 6x^2 + 7x - 5$
30. $(7y - 3)(2y - 1) = 14y^2 - 13y + 3$
31. $(x + 3y)(2x - y) = 2x^2 + 5xy - 3y^2$
32. $(4x - 5y)(3x - y) = 12x^2 - 19xy + 5y^2$
33. $(4x + 3)^2 = 16x^2 + 24x + 9$
34. $(2 - 7y)^2 = 49y^2 - 28y + 4$
35. $(y - 3x)^2 = y^2 - 6xy + 9x^2$
36. $(5x - y)^2 = 25x^2 - 10xy + y^2$
37. $(2x + 3y)^2 = 4x^2 + 12xy + 9y^2$
38. $(r - 2s)^2 = r^2 - 4rs + 4s^2$
39. $(w + 7)(w - 7) = w^2 - 49$
40. $(5 - y)(5 + y) = 25 - y^2$
41. $(3x - 4)(3x + 4) = (3x)^2 - 4^2 = 9x^2 - 16$
42. $(2y + 5)(2y - 5) = 4y^2 - 25$
43. $(\sqrt{x} + 2)(\sqrt{x} - 2) = x - 4$
44. $(\sqrt{y} + \sqrt{2})(\sqrt{y} - \sqrt{2}) = y - 2$
45. $(y + 2)^3 = y^3 + 3y^2(2) + 3y(2^2) + 2^3 = y^3 + 6y^2 + 12y + 8$
46. $(x - 3)^3 = x^3 - 9x^2 + 27x - 27$
47. $(x + 2)(x^2 + 2x + 3) = x^3 + 2x^2 + 3x + 2x^2 + 4x + 6 = x^3 + 4x^2 + 7x + 6$
48. $(x + 1)(2x^2 - x + 1) = 2x^3 + x^2 + 1$
49. $(2x - 5)(x^2 - x + 1) = 2x^3 - 2x^2 + 2x - 5x^2 + 5x - 5 = 2x^3 - 7x^2 + 7x - 5$
50. $(1 + 2x)(x^2 - 3x + 1) = 2x^3 - 5x^2 - x + 1$

51. $\sqrt{x}(x - \sqrt{x}) = x\sqrt{x} - (\sqrt{x})^2 = x\sqrt{x} - x$
52. $x^{3/2}(\sqrt{x} - 1/\sqrt{x}) = x^2 - x$
53. $y^{1/3}(y^{2/3} + y^{5/3}) = y^{1/3+2/3} + y^{1/3+5/3} = y^2 + y$
54. $x^{1/4}(2x^{3/4} - x^{1/4}) = 2x - \sqrt{x}$
55. $(x^{1/2} - y^{1/2})^2 = x - 2\sqrt{xy} + y$
56. $\left(\sqrt{u} + \frac{1}{\sqrt{u}}\right)^2 = u + 2 + \frac{1}{u}$
57. $(x^2 - a^2)(x^2 + a^2) = x^4 - a^4$
58. $(x^{1/2} + y^{1/2})(x^{1/2} - y^{1/2}) = x - y$
59. $(\sqrt{a} - b)(\sqrt{a} + b) = a - b^2$
60. $(\sqrt{h^2 + 1} + 1)(\sqrt{h^2 + 1} - 1) = h^2$
61. $((x - 1) + x^2)((x - 1) - x^2) = (x - 1)^2 - (x^2)^2 = x^2 - 2x + 1 - x^4 = -x^4 + x^2 - 2x + 1$
62. $(x + (2 + x^2))(x - (2 + x^2)) = -x^4 - 3x^2 - 4$
63. $(2x + y - 3)(2x + y + 3) = (2x + y)^2 - 3^2 = 4x^2 + 4xy + y^2 - 9$
64. $(x + y + z)(x - y - z) = x^2 - y^2 - z^2 - 2yz$
65. $4x^4 - x^2 = x^2(4x^2 - 1) = x^2(2x + 1)(2x - 1)$
66. $3x^4 - 6x^3 - x^2 = x^2(3x^2 - 6x - 1)$
67. $y(y - 6) + 9(y - 6) = (y - 6)(y + 9)$
68. $(z + 2)^2 - 5(z + 2) = (z + 2)[(z + 2) - 5] = (z + 2)(z - 3)$
69. $4x^3y^2 - 6xy^3 + 8x^2y^4 = 2xy^2(2x^2 - 3y + 4xy^2)$
70. $-7x^4y^2 + 14xy^3 + 21xy^4 = 7xy^2(-x^3 + 2y + 3y^2)$
71. $(x + 3)^5(x + 2)^2 - (x + 3)^4(x + 2)^3 = (x + 3)^4(x + 2)^2[(x + 3) - (x + 2)] = (x + 3)^4(x + 2)^2$
72. $3(2x - 1)^3(x^2 + 1)^4 - (2x - 1)^4(x^2 + 1)^3 = (2x - 1)^3(x^2 + 1)^3[3(x^2 + 1) - (2x - 1)]$
 $= (2x - 1)^3(x^2 + 1)^3(3x^2 - 2x + 4)$
73. $z^2 - 11z + 18 = (z - 9)(z - 2)$
74. $x^2 + 4x - 5 = (x + 5)(x - 1)$
75. $10x^2 - 19x + 6 = (5x - 2)(2x - 3)$
76. $6y^2 + 11y - 21 = (y + 3)(6y - 7)$
77. $3x^2 - 16x + 5 = (3x - 1)(x - 5)$
78. $5x^2 - 7x - 6 = (5x + 3)(x - 2)$
79. $(3x + 2)^2 + 8(3x + 2) + 12 = [(3x + 2) + 2][(3x + 2) + 6] = (3x + 4)(3x + 8)$
80. $2(a + b)^2 + 5(a + b) - 3 = [(a + b) + 3][2(a + b) - 1] = (a + b + 3)(2a + 2b - 1)$
81. $36a^2 - 49 = (6a + 7)(6a - 7)$
82. $(x + 3)^2 - 4 = (x + 3)^2 - 2^2 = [(x + 3) - 2][(x + 3) + 2] = (x + 1)(x + 5)$
83. $27x^3 + y^3 = (3x)^3 + y^3 = (3x + y)[(3x)^2 + 3xy + y^2] = (3x + y)(9x^2 + 3xy + y^2)$
84. $a^3 - b^6 = a^3 - (b^2)^3 = (a - b^2)[a^2 + ab^2 + (b^2)^2] = (a - b^2)(a^2 + ab^2 + b^4)$
85. $8s^3 - 125t^3 = (2s)^3 - (5t)^3 = (2s - 5t)[(2s)^2 + (2s)(5t) + (5t)^2] = (2s - 5t)(4s^2 + 10st + 25t^2)$
86. $1 + 1000y^3 = 1 + (10y)^3 = (1 + 10y)[1 - 10y + (10y)^2] = (1 + 10y)(1 - 10y + 100y^2)$
87. $x^2 + 12x + 36 = x^2 + 2(6x) + 6^2 = (x + 6)^2$

$$88. 16z^2 - 24z + 9 = (4z)^2 - 2(4z)(3) + 3^2 = (4z - 3)^2$$

$$89. x^3 + 4x^2 + x + 4 = x^2(x + 4) + 1(x + 4) = (x + 4)(x^2 + 1)$$

$$90. 3x^3 - x^2 + 6x - 2 = x^2(3x - 1) + 2(3x - 1) = (3x - 1)(x^2 + 2)$$

$$91. 5x^3 + x^2 + 5x + 1 = x^2(5x + 1) + (5x + 1) = (x^2 + 1)(5x + 1)$$

$$92. 18x^3 + 9x^2 + 2x + 1 = 9x^2(2x + 1) + (2x + 1) = (9x^2 + 1)(2x + 1)$$

$$93. x^3 + x^2 + x + 1 = x^2(x + 1) + 1(x + 1) = (x + 1)(x^2 + 1)$$

$$94. x^5 + x^4 + x + 1 = x^4(x + 1) + 1(x + 1) = (x + 1)(x^4 + 1)$$

$$95. \text{Start by factoring out the power of } x \text{ with the smallest exponent, that is, } x^{2/3}.$$

$$\text{Thus, } x^{2/3} + 3x^{5/3} = x^{2/3}(1 + 3x)$$

$$96. \text{Start by factoring out the power of } x \text{ with the smallest exponent, that is, } x^{-1/4}.$$

$$\text{Thus, } x^{3/4} - 5x^{-1/4} = x^{-1/4}(x - 5)$$

$$97. \text{Start by factoring out the power of } x \text{ with the smallest exponent, that is, } x^{-3/2}.$$

$$\text{Thus, } x^{-3/2} - x^{-1/2} + x^{1/2} = x^{-3/2}(1 - x + x^2) = x^{-3/2}(x^2 - x + 1)$$

$$98. \text{Start by factoring out the power of } x \text{ with the smallest exponent, that is, } x^{-1/3}.$$

$$\text{Thus, } x^{5/3} + x^{2/3} + 2x^{-1/3} = x^{-1/3}(x^2 + x + 2)$$

$$99. \text{Start by factoring out the power of } x^2 + 1 \text{ with the smallest exponent, that is, } (x^2 + 1)^{-1/2}.$$

$$\text{Thus, } (x^2 + 1)^{1/2} + 2(x^2 + 1)^{-1/2} = (x^2 + 1)^{-1/2}[(x^2 + 1) + 2] = (x^2 + 1)^{-1/2}(x^2 + 3) = \frac{x^2 + 3}{\sqrt{x^2 + 1}}.$$

$$100. x^{-1/2}(x + 1)^{1/2} + x^{1/2}(x + 1)^{-1/2} = x^{-1/2}(x + 1)^{-1/2}[(x + 1) + x] = x^{-1/2}(x + 1)^{-1/2}(2x + 1) = \frac{2x + 1}{\sqrt{x}\sqrt{x + 1}}$$

$$101. 2x + 12x^3 = 2x(1 + 6x^2)$$

$$102. 12x^2 + 3x^3 = 3x^2(4 + x)$$

$$103. x^2 - 2x - 8 = (x - 4)(x + 2)$$

$$104. x^2 - 14x + 48 = (x - 8)(x - 6)$$

$$105. 2x^2 + 5x + 3 = (2x + 3)(x + 1)$$

$$106. 2x^2 + 7x - 4 = (2x - 1)(x + 4)$$

$$107. 9x^2 - 36x - 45 = 9(x^2 - 4x - 5) = 9(x - 5)(x + 1)$$

$$108. 8x^2 + 10x + 3 = (4x + 3)(2x + 1)$$

$$109. 49 - 4y^2 = (7 - 2y)(7 + 2y)$$

$$110. 4t^2 - 9s^2 = (2t - 3s)(2t + 3s)$$

$$111. t^2 - 6t + 9 = (t - 3)^2$$

$$112. x^2 + 10x + 25 = (x + 5)^2$$

$$113. y^2 - 10yz + 25z^2 = (y - 5z)^2$$

$$114. r^2 - 6rs + 9s^2 = (r - 3s)^2$$

$$115. (a + b)^2 - (a - b)^2 = [(a + b) - (a - b)][(a + b) + (a - b)] = (2b)(2a) = 4ab$$

$$116. \left(1 + \frac{1}{x}\right)^2 - \left(1 - \frac{1}{x}\right)^2 = \left[\left(1 + \frac{1}{x}\right) - \left(1 - \frac{1}{x}\right)\right]\left[\left(1 + \frac{1}{x}\right) + \left(1 - \frac{1}{x}\right)\right]$$

$$= \left(1 + \frac{1}{x} - 1 + \frac{1}{x}\right)\left(1 + \frac{1}{x} + 1 - \frac{1}{x}\right) = \left(\frac{2}{x}\right)(2) = \frac{4}{x}$$

$$117. x^2(x^2 - 1) - 9(x^2 - 1) = (x^2 - 1)(x^2 - 9) = (x - 1)(x + 1)(x - 3)(x + 3)$$

$$118. (a^2 - 1)(b - 2)^2 - 4(a^2 - 1) = (a^2 - 1)[(b - 2)^2 - 4] = (a + 1)(a - 1)[(b - 2) + 2][(b - 2) - 2]$$

$$= (a + 1)(a - 1)b(b - 4)$$

$$119. 8x^3 - 125 = (2x)^3 - 5^3 = (2x - 5) \left[(2x)^2 + (2x)(5) + 5^2 \right] = (2x - 5) (4x^2 + 10x + 25)$$

$$120. x^6 + 64 = x^6 + 2^6 = (x^2)^3 + (4)^3 = (x^2 + 4) \left[(x^2)^2 - 4(x^2) + (4)^2 \right] = (x^2 + 4) (x^4 - 4x^2 + 16)$$

$$121. x^3 + 2x^2 + x = x (x^2 + 2x + 1) = x (x + 1)^2$$

$$122. 3x^3 - 27x = 3x (x^2 - 9) = 3x (x - 3) (x + 3)$$

$$123. x^4 y^3 - x^2 y^5 = x^2 y^3 (x^2 - y^2) = x^2 y^3 (x + y) (x - y)$$

$$124. 18y^3 x^2 - 2xy^4 = 2xy^3 (9x - y)$$

$$125. 3x^3 - x^2 - 12x + 4 = 3x (x^2 - 4) - (x^2 - 4) = (3x - 1) (x^2 - 4) = (3x - 1) (x + 2) (x - 2)$$

$$126. 9x^3 + 18x^2 - x - 2 = 9x^2 (x + 2) - (x + 2) = (9x^2 - 1) (x + 2) = (3x + 1) (3x - 1) (x + 2)$$

127. Start by factoring out the power of x with the smallest exponent, that is, $x^{-3/2}$. So

$$x^{-3/2} + 2x^{-1/2} + x^{1/2} = x^{-3/2} (1 + 2x + x^2) = x^{-3/2} (1 + x)^2 = \frac{(1 + x)^2}{x^{3/2}}.$$

$$128. (x - 1)^{7/2} - (x - 1)^{3/2} = (x - 1)^{3/2} [(x - 1)^2 - 1] = (x - 1)^{3/2} [(x - 1) - 1][(x - 1) + 1] \\ = (x - 1)^{3/2} (x - 2) (x)$$

$$129. (x - 1) (x + 2)^2 - (x - 1)^2 (x + 2) = (x - 1) (x + 2) [(x + 2) - (x - 1)] = 3 (x - 1) (x + 2)$$

$$130. y^4 (y + 2)^3 + y^5 (y + 2)^4 = y^4 (y + 2)^3 [(1) + y (y + 2)] = y^4 (y + 2)^3 (y^2 + 2y + 1) = y^4 (y + 2)^3 (y + 1)^2$$

131. Start by factoring $y^2 - 7y + 10$, and then substitute $a^2 + 1$ for y . This gives

$$(a^2 + 1)^2 - 7(a^2 + 1) + 10 = [(a^2 + 1) - 2][(a^2 + 1) - 5] = (a^2 - 1)(a^2 - 4) = (a - 1)(a + 1)(a - 2)(a + 2)$$

$$132. (a^2 + 2a)^2 - 2(a^2 + 2a) - 3 = [(a^2 + 2a) - 3][(a^2 + 2a) + 1] = (a^2 + 2a - 3)(a^2 + 2a + 1) \\ = (a - 1)(a + 3)(a + 1)^2$$

$$133. (x^2 + 3)^2 (x - 1)^3 - (4x + 1)^2 (x - 1)^3 = (x - 1)^3 [(x^2 + 3)^2 - (4x + 1)^2] \\ = (x - 1)^3 [(x^2 + 3) + (4x + 1)][(x^2 + 3) - (4x + 1)] = (x - 1)^3 (x^2 + 4x + 4)(x^2 - 4x + 2) \\ = (x - 1)^3 (x + 2)^2 (x^2 - 4x + 2)$$

$$134. (x + 1)^2 (x + 2) - 6(x + 1)(x + 2) + 9(x + 2) = (x + 2) [(x + 1)^2 - 6(x + 1) + 9] = (x + 2) [(x + 1) - 3]^2 = (x + 2) (x - 2)^2$$

$$135. 5(x^2 + 4)^4 (2x)(x - 2)^4 + (x^2 + 4)^5 (4)(x - 2)^3 = 2(x^2 + 4)^4 (x - 2)^3 [(5)(x)(x - 2) + (x^2 + 4)(2)] \\ = 2(x^2 + 4)^4 (x - 2)^3 (5x^2 - 10x + 2x^2 + 8) = 2(x^2 + 4)^4 (x - 2)^3 (7x^2 - 10x + 8)$$

$$136. 3(2x - 1)^2 (2)(x + 3)^{1/2} + (2x - 1)^3 \left(\frac{1}{2}\right)(x + 3)^{-1/2} = (2x - 1)^2 (x + 3)^{-1/2} [6(x + 3) + (2x - 1)\left(\frac{1}{2}\right)] \\ = (2x - 1)^2 (x + 3)^{-1/2} \left(6x + 18 + x - \frac{1}{2}\right) = (2x - 1)^2 (x + 3)^{-1/2} \left(7x + \frac{35}{2}\right)$$

$$137. (x^2 + 3)^{-1/3} - \frac{2}{3}x^2 (x^2 + 3)^{-4/3} = (x^2 + 3)^{-4/3} [(x^2 + 3) - \frac{2}{3}x^2] = (x^2 + 3)^{-4/3} \left(\frac{1}{3}x^2 + 3\right) = \frac{\frac{1}{3}x^2 + 3}{(x^2 + 3)^{4/3}}$$

$$138. \frac{1}{2}x^{-1/2} (3x + 4)^{1/2} - \frac{3}{2}x^{1/2} (3x + 4)^{-1/2} = \frac{1}{2}x^{-1/2} (3x + 4)^{-1/2} [(3x + 4) - 3x] = \frac{1}{2}x^{-1/2} (3x + 4)^{-1/2} (4) \\ = 2x^{-1/2} (3x + 4)^{-1/2}$$

139. (a) $\frac{1}{2}[(a+b)^2 - (a^2 + b^2)] = \frac{1}{2}[a^2 + 2ab + b^2 - a^2 - b^2] = \frac{1}{2}(2ab) = ab.$

(b) $(a^2 + b^2)^2 - (a^2 - b^2)^2 = [(a^2 + b^2) - (a^2 - b^2)][(a^2 + b^2) + (a^2 - b^2)]$
 $= (a^2 + b^2 - a^2 + b^2)(a^2 + b^2 + a^2 - b^2) = (2b^2)(2a^2) = 4a^2b^2$

140. LHS $= (a^2 + b^2)(c^2 + d^2) = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2.$

RHS $= (ac + bd)^2 + (ad - bc)^2 = a^2c^2 + 2abcd + b^2d^2 + a^2d^2 - 2abcd + b^2c^2 = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2.$

So LHS = RHS, that is, $(a^2 + b^2)(c^2 + d^2) = (ac + bd)^2 + (ad - bc)^2.$

141. $4a^2c^2 - (c^2 - b^2 + a^2)^2 = (2ac)^2 - (c^2 - b^2 + a^2)$
 $= [(2ac) - (c^2 - b^2 + a^2)][(2ac) + (c^2 - b^2 + a^2)]$ (difference of squares)
 $= (2ac - c^2 + b^2 - a^2)(2ac + c^2 - b^2 + a^2)$
 $= [b^2 - (c^2 - 2ac + a^2)][(c^2 + 2ac + a^2) - b^2]$ (regrouping)
 $= [b^2 - (c - a)^2][(c + a)^2 - b^2]$ (perfect squares)
 $= [b - (c - a)][b + (c - a)][(c + a) - b][(c + a) + b]$ (each factor is a difference of squares)
 $= (b - c + a)(b + c - a)(c + a - b)(c + a + b)$
 $= (a + b - c)(-a + b + c)(a - b + c)(a + b + c)$

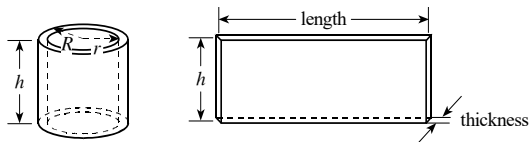
142. (a) $x^4 + x^2 - 2 = (x^2 - 1)(x^2 + 2) = (x - 1)(x + 1)(x^2 + 2)$

(b) $x^4 + 2x^2 + 9 = (x^4 + 6x^2 + 9) - 4x^2 = (x^2 + 3)^2 - (2x)^2 = [(x^2 + 3) - 2x][(x^2 + 3) + 2x]$
 $= (x^2 - 2x + 3)(x^2 + 2x + 3)$

(c) $x^4 + 4x^2 + 16 = (x^4 + 8x^2 + 16) - 4x^2 = (x^2 + 4)^2 - (2x)^2$
 $= [(x^2 + 4) - 2x][(x^2 + 4) + 2x] = (x^2 - 2x + 4)(x^2 + 2x + 4)$

(d) $x^4 + 2x^2 + 1 = (x^2 + 1)^2$

143. The volume of the shell is the difference between the volumes of the outside cylinder (with radius R) and the inside cylinder (with radius r). Thus $V = \pi R^2 h - \pi r^2 h = \pi(R^2 - r^2)h = \pi(R - r)(R + r)h = 2\pi \cdot \frac{R + r}{2} \cdot h \cdot (R - r)$. The average radius is $\frac{R + r}{2}$ and $2\pi \cdot \frac{R + r}{2}$ is the average circumference (length of the rectangular box), h is the height, and $R - r$ is the thickness of the rectangular box. Thus $V = \pi R^2 h - \pi r^2 h = 2\pi \cdot \frac{R + r}{2} \cdot h \cdot (R - r) = 2\pi \cdot (\text{average radius}) \cdot (\text{height}) \cdot (\text{thickness})$



144. (a) Moved portion = field - habitat

(b) Using the difference of squares, we get $b^2 - (b - 2x)^2 = [b - (b - 2x)][b + (b - x)] = 2x(2b - 2x) = 4x(b - x).$

145. (a) The degree of the product is the sum of the degrees.

- (b) The degree of a sum is at most the largest of the degrees — it could be smaller than either. For example, the degree of $(x^3) + (-x^3 + x) = x$ is 1.

146. (a) $528^2 - 527^2 = (528 - 527)(528 + 527) = 1(1055) = 1055$

(b) $1020^2 - 1010^2 = (1020 - 1010)(1020 + 1010) = 10(2030) = 20,300$

(c) $501 \cdot 499 = (500 + 1)(500 - 1) = 500^2 - 1^2 = 250,000 - 1 = 249,999$

(d) $1002 \cdot 998 = (1000 + 2)(1000 - 2) = 1000^2 - 2^2 = 1,000,000 - 4 = 999,996$

147. (a)

$$\begin{array}{r} A + 1 \\ \times \quad A - 1 \\ \hline -A - 1 \\ A^2 + A \\ \hline A^2 \quad - 1 \end{array} \qquad \begin{array}{r} A^2 + A + 1 \\ \times \quad A - 1 \\ \hline -A^2 - A - 1 \\ A^3 + A^2 + A \\ \hline A^3 \quad - 1 \end{array} \qquad \begin{array}{r} A^3 + A^2 + A + 1 \\ \times \quad A - 1 \\ \hline -A^3 - A^2 - A - 1 \\ A^4 + A^3 + A^2 + A \\ \hline A^4 \quad - 1 \end{array}$$

- (b) Based on the pattern in part (a), we suspect that $A^5 - 1 = (A - 1)(A^4 + A^3 + A^2 + A + 1)$. Check:

$$\begin{array}{r} A^4 + A^3 + A^2 + A + 1 \\ \times \quad A - 1 \\ \hline -A^4 - A^3 - A^2 - A - 1 \\ A^5 + A^4 + A^3 + A^2 + A \\ \hline A^5 \quad - 1 \end{array}$$

The general pattern is $A^n - 1 = (A - 1)(A^{n-1} + A^{n-2} + \cdots + A^2 + A + 1)$, where n is a positive integer.

148. (a) $(A - B)(A^2 + AB + B^2) = A^3 + A^2B + AB^2 - A^2B - AB^2 - B^3 = A^3 - B^3$

(b) $(A + B)(A^2 - AB + B^2) = A^3 - A^2B + AB^2 + A^2B - AB^2 + B^3 = A^3 + B^3$

1.4 RATIONAL EXPRESSIONS

- A rational expression has the form $\frac{P(x)}{Q(x)}$, where P and Q are polynomials.
 - $\frac{3x}{x^2 - 1}$ is a rational expression.
 - $\frac{\sqrt{x+1}}{2x+3}$ is not a rational expression. A rational expression must be a polynomial divided by a polynomial, and the numerator of the expression is $\sqrt{x+1}$, which is not a polynomial.
 - $\frac{x(x^2 - 1)}{x + 3} = \frac{x^3 - x}{x + 3}$ is a rational expression.
- To simplify a rational expression we cancel factors that are common to the *numerator* and *denominator*. So, the expression $\frac{(x+1)(x+2)}{(x+3)(x+2)}$ simplifies to $\frac{x+1}{x+3}$.
- To multiply two rational expressions we multiply their *numerators* together and multiply their *denominators* together. So $\frac{2}{x+1} \cdot \frac{x}{x+3}$ is the same as $\frac{2 \cdot x}{(x+1) \cdot (x+3)} = \frac{2x}{x^2 + 4x + 3}$.
- (a) $\frac{1}{x} - \frac{2}{(x+1)} - \frac{x}{(x+1)^2}$ has three terms.
 (b) The least common denominator of all the terms is $x(x+1)^2$.

$$\begin{aligned} \text{(c)} \quad \frac{1}{x} - \frac{2}{(x+1)} - \frac{x}{(x+1)^2} &= \frac{(x+1)^2}{x(x+1)^2} - \frac{2x(x+1)}{(x+1)^2} - \frac{x(x)}{(x+1)^2} = \frac{(x+1)^2 - 2x(x+1) - x^2}{x(x+1)^2} \\ &= \frac{x^2 + 2x + 1 - 2x^2 - 2x - x^2}{x(x+1)^2} = \frac{-2x^2 + 1}{x(x+1)^2} \end{aligned}$$

5. (a) Yes. Cancelling $x+1$, we have $\frac{x(x+1)}{(x+1)^2} = \frac{x}{x+1}$.

(b) No; $(x+5)^2 = x^2 + 10x + 25 \neq x^2 + 25$, so $x+5 = \sqrt{x^2 + 10x + 25} \neq \sqrt{x^2 + 25}$.

6. (a) Yes, $\frac{3+a}{3} = \frac{3}{3} + \frac{a}{3} = 1 + \frac{a}{3}$.

(b) No. We cannot “separate” the denominator in this way; only the numerator, as in part (a). (See also Exercise 101.)

7. The domain of $4x^2 - 10x + 3$ is all real numbers.

8. The domain of $-x^4 + x^3 + 9x$ is all real numbers.

9. Since $x-3 \neq 0$ we have $x \neq 3$. Domain: $\{x \mid x \neq 3\}$

10. Since $3t+6 \neq 0$ we have $t \neq -2$. Domain: $\{t \mid t \neq -2\}$

11. Since $x+3 \geq 0$, $x \geq -3$. Domain: $\{x \mid x \geq -3\}$

12. Since $x-1 > 0$, $x > 1$. Domain: $\{x \mid x > 1\}$

13. $x^2 - x - 2 = (x+1)(x-2) \neq 0 \Leftrightarrow x \neq -1$ or 2 , so the domain is $\{x \mid x \neq -1, 2\}$.

14. $\frac{x}{x^2-4} = \frac{x}{(x+2)(x-2)}$. $(x+2)(x-2) \neq 0 \Leftrightarrow x \neq \pm 2$, so the domain is $\{x \mid x \neq \pm 2\}$.

15. $\frac{\sqrt{x-2}}{x+3}$ is defined for $\{x \mid x \geq 2, x \neq -3\}$, so its domain is $\{x \mid x \geq 2\}$.

16. $\frac{\sqrt{x-2}}{x^2-9}$ is defined for $\{x \mid x \geq 2, x \neq \pm 3\}$, so its domain is $\{x \mid x \geq 2, x \neq 3\}$.

17. $\frac{(x-5)(x+5)}{2x-10} = \frac{(x-5)(x+5)}{2(x-5)} = \frac{1}{2}(x+5)$

18. $\frac{x^3-2x}{x^2+x} = \frac{x(x^2-2)}{x(x+1)} = \frac{x^2-2}{x+1}$

19. $\frac{x-2}{x^2-4} = \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2}$

20. $\frac{x^2-x-2}{x^2-1} = \frac{(x-2)(x+1)}{(x-1)(x+1)} = \frac{x-2}{x-1}$

21. $\frac{x^2-7x-8}{x^2-10x+16} = \frac{(x-8)(x+1)}{(x-8)(x-2)} = \frac{x+1}{x-2}$

22. $\frac{x^2-x-12}{x^2+5x+6} = \frac{(x-4)(x+3)}{(x+2)(x+3)} = \frac{x-4}{x+2}$

23. $\frac{y^2+y}{y^2-1} = \frac{y(y+1)}{(y-1)(y+1)} = \frac{y}{y-1}$

24. $\frac{y^3-5y^2-24y}{y^3-9y} = \frac{y(y^2-5y-24)}{y(y^2-9)} = \frac{y(y-8)(y+3)}{y(y+3)(y-3)} = \frac{y-8}{y-3}$

25. $\frac{2x^3-x^2-6x}{2x^2-7x+6} = \frac{x(2x^2-x-6)}{(2x-3)(x-2)} = \frac{x(2x+3)(x-2)}{(2x-3)(x-2)} = \frac{x(2x+3)}{2x-3}$

26. $\frac{1-x^2}{x^3-1} = \frac{(1-x)(1+x)}{(x-1)(x^2+x+1)} = \frac{-(x-1)(1+x)}{(x-1)(x^2+x+1)} = \frac{-(x+1)}{x^2+x+1}$

27. $\frac{4x}{x^2-4} \cdot \frac{x+2}{16x} = \frac{4x}{(x-2)(x+2)} \cdot \frac{x+2}{16x} = \frac{1}{4(x-2)}$

28. $\frac{x^2-25}{x^2-16} \cdot \frac{x+4}{x+5} = \frac{(x-5)(x+5)}{(x-4)(x+4)} \cdot \frac{x+4}{x+5} = \frac{x-5}{x-4}$

29. $\frac{x^2+2x-15}{x^2-25} \cdot \frac{x-5}{x+2} = \frac{(x+5)(x-3)(x-5)}{(x+5)(x-5)(x+2)} = \frac{x-3}{x+2}$

30. $\frac{x^2 + 2x - 3}{x^2 - 2x - 3} \cdot \frac{3 - x}{3 + x} = \frac{(x + 3)(x - 1)}{(x - 3)(x + 1)} \cdot \frac{-(x - 3)}{x + 3} = \frac{-(x - 1)}{x + 1} = \frac{-x + 1}{x + 1} = \frac{1 - x}{1 + x}$
31. $\frac{2t + 3}{t^2 + 9} \cdot \frac{2t - 3}{4t^2 - 9} = \frac{(2t + 3)(2t - 3)}{(t^2 + 9)(2t + 3)(2t - 3)} = \frac{1}{t^2 + 9}$
32. $\frac{3y^2 + 9y}{y^3 - 9y} \cdot \frac{y^2 - 9}{2y^2 + 3y - 9} = \frac{3y(y + 3)(y + 3)(y - 3)}{y(y^2 - 9)(y + 3)(2y - 3)} = \frac{3}{2y - 3}$
33. $\frac{x^3 - 2x^2 - 8x}{x^2 + 8x + 12} \cdot \frac{x^2 + 2x - 24}{x^3 - 16x} = \frac{x(x + 2)(x - 4)(x - 4)(x + 6)}{(x + 6)(x + 2)x(x + 4)(x - 4)} = \frac{x - 4}{x + 4}$
34. $\frac{2x^2 + xy - y^2}{x^2 + xy - 2y^2} \cdot \frac{x^2 - 2xy + y^2}{2x^2 - 3xy + y^2} = \frac{(2x - y)(x + y)(x - y)^2}{(x - y)(x + 2y)(2x - y)(x - y)} = \frac{x + y}{x + 2y}$
35. $\frac{x + 3}{4x^2 - 9} \div \frac{x^2 + 7x + 12}{2x^2 + 7x - 15} = \frac{x + 3}{4x^2 - 9} \cdot \frac{2x^2 + 7x - 15}{x^2 + 7x + 12} = \frac{x + 3}{(2x - 3)(2x + 3)} \cdot \frac{(x + 5)(2x - 3)}{(x + 3)(x + 4)} = \frac{x + 5}{(2x + 3)(x + 4)}$
36. $\frac{2x + 1}{2x^2 + x - 15} \div \frac{6x^2 - x - 2}{x + 3} = \frac{2x + 1}{(x + 3)(2x - 5)} \cdot \frac{x + 3}{(2x + 1)(3x - 2)} = \frac{1}{(2x - 5)(3x - 2)}$
37. $\frac{\frac{x^3}{x + 1}}{\frac{x}{x^2 + 2x + 1}} = \frac{x^3}{x + 1} \cdot \frac{x^2 + 2x + 1}{x} = \frac{x^3(x + 1)(x + 1)}{(x + 1)x} = x^2(x + 1)$
38. $\frac{\frac{2x^2 - 3x - 2}{x^2 - 1}}{\frac{x^2 + x - 2}{2x^2 + 5x + 2}} = \frac{2x^2 - 3x - 2}{x^2 - 1} \cdot \frac{x^2 + x - 2}{2x^2 + 5x + 2} = \frac{(x - 2)(2x + 1)}{(x - 1)(x + 1)} \cdot \frac{(x - 1)(x + 2)}{(x + 2)(2x + 1)} = \frac{x - 2}{x + 1}$
39. $\frac{x/y}{z} = \frac{x}{y} \cdot \frac{1}{z} = \frac{x}{yz}$
40. $\frac{x}{y/z} = x \div \frac{y}{z} = \frac{x}{1} \cdot \frac{z}{y} = \frac{xz}{y}$
41. $1 + \frac{1}{x + 3} = \frac{x + 3}{x + 3} + \frac{1}{x + 3} = \frac{x + 4}{x + 3}$
42. $\frac{3x - 2}{x + 1} - 2 = \frac{3x - 2}{x + 1} - \frac{2(x + 1)}{x + 1} = \frac{3x - 2 - 2x - 2}{x + 1} = \frac{x - 4}{x + 1}$
43. $\frac{1}{x + 5} + \frac{2}{x - 3} = \frac{x - 3}{(x + 5)(x - 3)} + \frac{2(x + 5)}{(x + 5)(x - 3)} = \frac{x - 3 + 2x + 10}{(x + 5)(x - 3)} = \frac{3x + 7}{(x + 5)(x - 3)}$
44. $\frac{1}{x + 1} + \frac{1}{x - 1} = \frac{x - 1}{(x + 1)(x - 1)} + \frac{x + 1}{(x + 1)(x - 1)} = \frac{x - 1 + x + 1}{(x + 1)(x - 1)} = \frac{2x}{(x + 1)(x - 1)}$
45. $\frac{3}{x + 1} - \frac{1}{x + 2} = \frac{3(x + 2)}{(x + 1)(x + 2)} - \frac{1(x + 1)}{(x + 1)(x + 2)} = \frac{3x + 6 - x - 1}{(x + 1)(x + 2)} = \frac{2x + 5}{(x + 1)(x + 2)}$
46. $\frac{x}{x - 4} - \frac{3}{x + 6} = \frac{x(x + 6)}{(x - 4)(x + 6)} + \frac{-3(x - 4)}{(x - 4)(x + 6)} = \frac{x^2 + 6x - 3x + 12}{(x - 4)(x + 6)} = \frac{x^2 + 3x + 12}{(x - 4)(x + 6)}$
47. $\frac{5}{2x - 3} - \frac{3}{(2x - 3)^2} = \frac{5(2x - 3)}{(2x - 3)^2} - \frac{3}{(2x - 3)^2} = \frac{10x - 15 - 3}{(2x - 3)^2} = \frac{10x - 18}{(2x - 3)^2} = \frac{2(5x - 9)}{(2x - 3)^2}$
48. $\frac{x}{(x + 1)^2} + \frac{2}{x + 1} = \frac{x}{(x + 1)^2} + \frac{2(x + 1)}{(x + 1)(x + 1)} = \frac{x + 2x + 2}{(x + 1)^2} = \frac{3x + 2}{(x + 1)^2}$
49. $u + 1 + \frac{u}{u + 1} = \frac{(u + 1)(u + 1)}{u + 1} + \frac{u}{u + 1} = \frac{u^2 + 2u + 1 + u}{u + 1} = \frac{u^2 + 3u + 1}{u + 1}$
50. $\frac{2}{a^2} - \frac{3}{ab} + \frac{4}{b^2} = \frac{2b^2}{a^2b^2} - \frac{3ab}{a^2b^2} + \frac{4a^2}{a^2b^2} = \frac{2b^2 - 3ab + 4a^2}{a^2b^2}$

$$51. \frac{1}{x^2} + \frac{1}{x^2 + x} = \frac{1}{x^2} + \frac{1}{x(x+1)} = \frac{x+1}{x^2(x+1)} + \frac{x}{x^2(x+1)} = \frac{2x+1}{x^2(x+1)}$$

$$52. \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} = \frac{x^2}{x^3} + \frac{x}{x^3} + \frac{1}{x^3} = \frac{x^2 + x + 1}{x^3}$$

$$53. \frac{2}{x+3} - \frac{1}{x^2 + 7x + 12} = \frac{2}{x+3} - \frac{1}{(x+3)(x+4)} = \frac{2(x+4)}{(x+3)(x+4)} + \frac{-1}{(x+3)(x+4)} \\ = \frac{2x+8-1}{(x+3)(x+4)} = \frac{2x+7}{(x+3)(x+4)}$$

$$54. \frac{x}{x^2 - 4} + \frac{1}{x-2} = \frac{x}{(x-2)(x+2)} + \frac{1}{x-2} = \frac{x}{(x-2)(x+2)} + \frac{x+2}{(x-2)(x+2)} \\ = \frac{2x+2}{(x-2)(x+2)} = \frac{2(x+1)}{(x-2)(x+2)}$$

$$55. \frac{1}{x+3} + \frac{1}{x^2 - 9} = \frac{1}{x+3} + \frac{1}{(x-3)(x+3)} = \frac{x-3}{(x-3)(x+3)} + \frac{1}{(x-3)(x+3)} = \frac{x-2}{(x-3)(x+3)}$$

$$56. \frac{x}{x^2 + x - 2} - \frac{2}{x^2 - 5x + 4} = \frac{x}{(x-1)(x+2)} + \frac{-2}{(x-1)(x-4)} \\ = \frac{x(x-4)}{(x-1)(x+2)(x-4)} + \frac{-2(x+2)}{(x-1)(x+2)(x-4)} = \frac{x^2 - 4x - 2x - 4}{(x-1)(x+2)(x-4)} = \frac{x^2 - 6x - 4}{(x-1)(x+2)(x-4)}$$

$$57. \frac{2}{x} + \frac{3}{x-1} - \frac{4}{x^2 - x} = \frac{2}{x} + \frac{3}{x-1} - \frac{4}{x(x-1)} = \frac{2(x-1)}{x(x-1)} + \frac{3x}{x(x-1)} + \frac{-4}{x(x-1)} = \frac{2x-2+3x-4}{x(x-1)} = \frac{5x-6}{x(x-1)}$$

$$58. \frac{1}{x^3 y^3 z} + \frac{1}{x y^3 z^3} + \frac{1}{x^3 y z^3} = \frac{z^2}{x^3 y^3 z^3} + \frac{x^2}{x^3 y^3 z^3} + \frac{y^2}{x^3 y^3 z^3} = \frac{x^2 + y^2 + z^2}{x^3 y^3 z^3}$$

$$59. \frac{1}{x^2(x+1)} + \frac{1}{x^2(x+1)^2} + \frac{1}{x^3(x+1)^2} = \frac{x(x+1)}{x^3(x+1)^2} + \frac{x}{x^3(x+1)^2} + \frac{1}{x^3(x+1)^2} = \frac{x^2 + 2x + 1}{x^3(x+1)^2} \\ = \frac{(x+1)^2}{x^3(x+1)^2} = \frac{1}{x^3}$$

$$60. \frac{1}{x+1} - \frac{2}{(x+1)^2} + \frac{3}{x^2 - 1} = \frac{1}{x+1} + \frac{-2}{(x+1)^2} + \frac{3}{(x-1)(x+1)} \\ = \frac{(x+1)(x-1)}{(x-1)(x+1)^2} + \frac{-2(x-1)}{(x-1)(x+1)^2} + \frac{3(x+1)}{(x-1)(x+1)^2} \\ = \frac{x^2 - 1}{(x-1)(x+1)^2} + \frac{-2x + 2}{(x-1)(x+1)^2} + \frac{3x + 3}{(x-1)(x+1)^2} = \frac{x^2 - 1 - 2x + 2 + 3x + 3}{(x-1)(x+1)^2} = \frac{x^2 + x + 4}{(x-1)(x+1)^2}$$

$$61. \frac{1 + \frac{1}{x}}{\frac{1}{x} - 2} = \frac{x \left(1 + \frac{1}{x}\right)}{x \left(\frac{1}{x} - 2\right)} = \frac{x+1}{1-2x}$$

$$62. \frac{1 - \frac{2}{y}}{\frac{3}{y} - 1} = \frac{y \left(1 - \frac{2}{y}\right)}{y \left(\frac{3}{y} - 1\right)} = \frac{y-2}{3-y}$$

$$63. \frac{1 + \frac{1}{x+2}}{1 - \frac{1}{x+2}} = \frac{(x+2) \left(1 + \frac{1}{x+2}\right)}{(x+2) \left(1 - \frac{1}{x+2}\right)} = \frac{(x+2) + 1}{(x+2) - 1} = \frac{x+3}{x+1}$$

$$64. \frac{1 + \frac{1}{c-1}}{1 - \frac{1}{c-1}} = \frac{c-1+1}{c-1-1} = \frac{c}{c-2}$$

$$65. \frac{\frac{1}{x-1} + \frac{1}{x+3}}{x+1} = \frac{(x-1)(x+3) \left(\frac{1}{x-1} + \frac{1}{x+3} \right)}{(x-1)(x+3)(x+1)} = \frac{(x+3) + (x-1)}{(x-1)(x+1)(x+3)} = \frac{2(x+1)}{(x-1)(x+1)(x+3)} \\ = \frac{2}{(x-1)(x+3)}$$

$$66. \frac{\frac{x-3}{x-4} - \frac{x+2}{x+1}}{x+3} = \frac{(x-3)(x+1) - (x+2)(x-4)}{(x-4)(x+3)(x+1)} = \frac{x^2 - 2x - 3 - (x^2 - 2x - 8)}{(x-4)(x+3)(x+1)} = \frac{5}{(x-4)(x+3)(x+1)}$$

$$67. \frac{x - \frac{x}{y}}{y - \frac{y}{x}} = \frac{xy \left(x - \frac{x}{y} \right)}{xy \left(y - \frac{y}{x} \right)} = \frac{x^2y - x^2}{xy^2 - y^2} = \frac{x^2(y-1)}{y^2(x-1)}$$

$$68. \frac{x + \frac{y}{x}}{y + \frac{x}{y}} = \frac{xy \left(x + \frac{y}{x} \right)}{xy \left(y + \frac{x}{y} \right)} = \frac{x^2y + y^2}{xy^2 + x^2} = \frac{y(y+x^2)}{x(x+y^2)}$$

$$69. \frac{\frac{x}{y} - \frac{y}{x}}{\frac{1}{x^2} - \frac{1}{y^2}} = \frac{\frac{x^2 - y^2}{xy}}{\frac{y^2 - x^2}{x^2y^2}} = \frac{x^2 - y^2}{xy} \cdot \frac{x^2y^2}{y^2 - x^2} = \frac{xy}{-1} = -xy. \text{ An alternative method is to multiply the}$$

numerator and denominator by the common denominator of both the numerator and denominator, in this case x^2y^2 :

$$\frac{\frac{x}{y} - \frac{y}{x}}{\frac{1}{x^2} - \frac{1}{y^2}} = \frac{\left(\frac{x}{y} - \frac{y}{x} \right)}{\left(\frac{1}{x^2} - \frac{1}{y^2} \right)} \cdot \frac{x^2y^2}{x^2y^2} = \frac{x^3y - xy^3}{y^2 - x^2} = \frac{xy(x^2 - y^2)}{y^2 - x^2} = -xy.$$

$$70. x - \frac{y}{\frac{x}{y} + \frac{y}{x}} = x - \frac{y}{\frac{x}{y} + \frac{y}{x}} \cdot \frac{xy}{xy} = x - \frac{xy^2}{x^2 + y^2} = \frac{x(x^2 + y^2)}{x^2 + y^2} - \frac{xy^2}{x^2 + y^2} = \frac{x^3 + xy^2 - xy^2}{x^2 + y^2} = \frac{x^3}{x^2 + y^2}$$

$$71. \frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}} = \frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x} + \frac{1}{y}} = \frac{\frac{y^2}{x^2y^2} - \frac{x^2}{x^2y^2}}{\frac{y}{xy} + \frac{x}{xy}} = \frac{y^2 - x^2}{x^2y^2} \cdot \frac{xy}{y+x} = \frac{(y-x)(y+x)xy}{x^2y^2(y+x)} = \frac{y-x}{xy}$$

$$\text{Alternatively, } \frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}} = \frac{\left(\frac{1}{x^2} - \frac{1}{y^2} \right)}{\left(\frac{1}{x} + \frac{1}{y} \right)} \cdot \frac{x^2y^2}{x^2y^2} = \frac{y^2 - x^2}{xy^2 + x^2y} = \frac{(y-x)(y+x)}{xy(y+x)} = \frac{y-x}{xy}.$$

$$72. \frac{1}{1+u} + \frac{1}{1+\frac{1}{u}} = \frac{1}{1+u} + \frac{u}{u+1} = 1$$

$$73. 1 - \frac{1}{1 - \frac{1}{x}} = 1 - \frac{x}{x-1} = \frac{x-1-x}{x-1} = \frac{1}{1-x}$$

$$74. 1 + \frac{1}{1 + \frac{1}{1+x}} = 1 + \frac{1+x}{(1+x)+1} = 1 + \frac{x+1}{x+2} = \frac{x+2+x+1}{x+2} = \frac{2x+3}{x+2}$$

$$75. \frac{\frac{1}{1+x+h} - \frac{1}{1+x}}{h} = \frac{(1+x) - (1+x+h)}{h(1+x)(1+x+h)} = -\frac{1}{(1+x)(1+x+h)}$$

76. In calculus it is necessary to eliminate the h in the denominator, and we do this by rationalizing the numerator:

- $$\frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} = \frac{x - (x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} = -\frac{1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}.$$
- $$77. \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \frac{x^2 - (x+h)^2}{hx^2(x+h)^2} = \frac{x^2 - (x^2 + 2xh + h^2)}{hx^2(x+h)^2} = -\frac{2x+h}{x^2(x+h)^2}$$
- $$78. \frac{(x+h)^2 + 3(x+h) - x^2 - 3x}{h} = \frac{x^2 + h^2 + 2xh + 3x + 3h - x^2 - 3x}{h} = \frac{h(h+2x+3)}{h} = 2x+h+3$$
- $$79. \sqrt{1 + \left(\frac{x}{\sqrt{1-x^2}}\right)^2} = \sqrt{1 + \frac{x^2}{1-x^2}} = \sqrt{\frac{1-x^2}{1-x^2} + \frac{x^2}{1-x^2}} = \sqrt{\frac{1}{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$
- $$80. \sqrt{1 + \left(x^3 - \frac{1}{4x^3}\right)^2} = \sqrt{1 + x^6 - \frac{2x^3}{4x^3} + \frac{1}{16x^6}} = \sqrt{1 + x^6 - \frac{1}{2} + \frac{1}{16x^6}} = \sqrt{x^6 + \frac{1}{2} + \frac{1}{16x^6}}$$
- $$= \sqrt{\left(x^3 + \frac{1}{4x^3}\right)^2} = \left|x^3 + \frac{1}{4x^3}\right|$$
- $$81. \frac{2(x-3)(x+5)^3 - 3(x+5)^2(x-3)^2}{(x+5)^6} = \frac{(x-3)(x+5)^2[2(x+5) - 3(x-3)]}{(x+5)^6} = \frac{(x-3)(19-x)}{(x+5)^4}$$
- $$82. \frac{4x^3(1-x)^3 - 3(1-x)^2(-1)(x^4)}{(1-x)^6} = \frac{x^3(1-x)^2[4(1-x) + 3x]}{(1-x)^6} = \frac{x^3(4-x)}{(1-x)^4}$$
- $$83. \frac{2(1+x)^{1/2} - x(1+x)^{-1/2}}{1+x} = \frac{(1+x)^{-1/2}[2(1+x) - x]}{1+x} = \frac{x+2}{(1+x)^{3/2}}$$
- $$84. \frac{(1-x^2)^{1/2} + x^2(1-x^2)^{-1/2}}{1-x^2} = \frac{(1-x^2)^{-1/2}(1-x^2+x^2)}{1-x^2} = \frac{1}{(1-x^2)^{3/2}}$$
- $$85. \frac{3(1+x)^{1/3} - x(1+x)^{-2/3}}{(1+x)^{2/3}} = \frac{(1+x)^{-2/3}[3(1+x) - x]}{(1+x)^{2/3}} = \frac{2x+3}{(1+x)^{4/3}}$$
- $$86. \frac{(7-3x)^{1/2} + \frac{3}{2}x(7-3x)^{-1/2}}{7-3x} = \frac{(7-3x)^{-1/2}(7-3x + \frac{3}{2}x)}{7-3x} = \frac{7 - \frac{3}{2}x}{(7-3x)^{3/2}}$$
- $$87. \frac{1}{3+\sqrt{10}} = \frac{3-\sqrt{10}}{(3+\sqrt{10})(3-\sqrt{10})} = \frac{3-\sqrt{10}}{9-10} = \sqrt{10}-3$$
- $$88. \frac{3}{2-\sqrt{5}} = \frac{(2+\sqrt{5})^3}{(2+\sqrt{5})(2-\sqrt{5})} = \frac{6+3\sqrt{5}}{4-5} = -6-3\sqrt{5}$$
- $$89. \frac{2}{\sqrt{5}-\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3} = \sqrt{5}+\sqrt{3}$$
- $$90. \frac{1}{\sqrt{x}+1} = \frac{1}{\sqrt{x}+1} \cdot \frac{\sqrt{x}-1}{\sqrt{x}-1} = \frac{\sqrt{x}-1}{x-1}$$
- $$91. \frac{y}{\sqrt{3}+\sqrt{y}} = \frac{y}{\sqrt{3}+\sqrt{y}} \cdot \frac{\sqrt{3}-\sqrt{y}}{\sqrt{3}-\sqrt{y}} = \frac{y(\sqrt{3}-\sqrt{y})}{3-y} = \frac{y\sqrt{3}-y\sqrt{y}}{3-y}$$
- $$92. \frac{2(x-y)}{\sqrt{x}-\sqrt{y}} = \frac{2(x-y)}{\sqrt{x}-\sqrt{y}} \cdot \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{2(x-y)(\sqrt{x}+\sqrt{y})}{x-y} = 2(\sqrt{x}+\sqrt{y}) = 2\sqrt{x}+2\sqrt{y}$$

$$93. \frac{2 - \sqrt{5}}{5} = \frac{(2 - \sqrt{5})(2 + \sqrt{5})}{5(2 + \sqrt{5})} = \frac{4 - 5}{5(2 + \sqrt{5})} = \frac{-1}{5(2 + \sqrt{5})}$$

$$94. \frac{\sqrt{3} + \sqrt{5}}{2} = \frac{\sqrt{3} + \sqrt{5}}{2} \cdot \frac{\sqrt{3} - \sqrt{5}}{\sqrt{3} - \sqrt{5}} = \frac{3 - 5}{2(\sqrt{3} - \sqrt{5})} = \frac{-2}{2(\sqrt{3} - \sqrt{5})} = \frac{-1}{\sqrt{3} - \sqrt{5}}$$

$$95. \frac{\sqrt{r} + \sqrt{2}}{5} = \frac{\sqrt{r} + \sqrt{2}}{5} \cdot \frac{\sqrt{r} - \sqrt{2}}{\sqrt{r} - \sqrt{2}} = \frac{r - 2}{5(\sqrt{r} - \sqrt{2})}$$

$$96. \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} = \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} = \frac{x - (x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} = \frac{-h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} = \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

$$97. \sqrt{x^2 + 1} - x = \frac{\sqrt{x^2 + 1} - x}{1} \cdot \frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1} + x} = \frac{x^2 + 1 - x^2}{\sqrt{x^2 + 1} + x} = \frac{1}{\sqrt{x^2 + 1} + x}$$

$$98. \sqrt{x+1} - \sqrt{x} = \frac{\sqrt{x+1} - \sqrt{x}}{1} \cdot \frac{\sqrt{x+1} + \sqrt{x}}{\sqrt{x+1} + \sqrt{x}} = \frac{x+1-x}{\sqrt{x+1} + \sqrt{x}} = \frac{1}{\sqrt{x+1} + \sqrt{x}}$$

$$99. (a) R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \cdot \frac{R_1 R_2}{R_1 R_2} = \frac{R_1 R_2}{R_2 + R_1}$$

$$(b) \text{ Substituting } R_1 = 10 \text{ ohms and } R_2 = 20 \text{ ohms gives } R = \frac{(10)(20)}{(20) + (10)} = \frac{200}{30} \approx 6.7 \text{ ohms.}$$

$$100. (a) \text{ The average cost } A = \frac{\text{Cost}}{\text{number of shirts}} = \frac{500 + 6x + 0.01x^2}{x}.$$

(b)

x	10	20	50	100	200	500	1000
Average cost	\$56.10	\$31.20	\$16.50	\$12.00	\$10.50	\$12.00	\$16.50

101.

x	2.80	2.90	2.95	2.99	2.999	3	3.001	3.01	3.05	3.10	3.20
$\frac{x^2 - 9}{x - 3}$	5.80	5.90	5.95	5.99	5.999	?	6.001	6.01	6.05	6.10	6.20

From the table, we see that the expression $\frac{x^2 - 9}{x - 3}$ approaches 6 as x approaches 3. We simplify the expression:

$$\frac{x^2 - 9}{x - 3} = \frac{(x - 3)(x + 3)}{x - 3} = x + 3, x \neq 3. \text{ Clearly as } x \text{ approaches } 3, x + 3 \text{ approaches } 6. \text{ This explains the result in the table.}$$

$$102. \text{ No, squaring } \frac{2}{\sqrt{x}} \text{ changes its value by a factor of } \frac{2}{\sqrt{x}}.$$

103. Answers will vary.

Algebraic Error	Counterexample
$\frac{1}{a} + \frac{1}{b} \neq \frac{1}{a+b}$	$\frac{1}{2} + \frac{1}{2} \neq \frac{1}{2+2}$
$(a+b)^2 \neq a^2 + b^2$	$(1+3)^2 \neq 1^2 + 3^2$
$\sqrt{a^2 + b^2} \neq a + b$	$\sqrt{5^2 + 12^2} \neq 5 + 12$
$\frac{a+b}{a} \neq b$	$\frac{2+6}{2} \neq 6$
$\frac{a}{a+b} \neq \frac{1}{b}$	$\frac{1}{1+1} \neq 1$
$\frac{a^m}{a^n} \neq a^{m/n}$	$\frac{3^5}{3^2} \neq 3^{5/2}$

104. (a) $\frac{5+a}{5} = \frac{5}{5} + \frac{a}{5} = 1 + \frac{a}{5}$, so the statement is true.(b) This statement is false. For example, take $x = 5$ and $y = 2$. Then $\text{LHS} = \frac{x+1}{y+1} = \frac{5+1}{2+1} = \frac{6}{3} = 2$, while

$$\text{RHS} = \frac{x}{y} = \frac{5}{2}, \text{ and } 2 \neq \frac{5}{2}.$$

(c) This statement is false. For example, take $x = 0$ and $y = 1$. Then $\text{LHS} = \frac{x}{x+y} = \frac{0}{0+1} = 0$, while

$$\text{RHS} = \frac{1}{1+y} = \frac{1}{1+1} = \frac{1}{2}, \text{ and } 0 \neq \frac{1}{2}.$$

(d) This statement is false. For example, take $x = 1$ and $y = 1$. Then $\text{LHS} = 2\left(\frac{a}{b}\right) = 2\left(\frac{1}{1}\right) = 2$, while

$$\text{RHS} = \frac{2a}{2b} = \frac{2}{2} = 1, \text{ and } 2 \neq 1.$$

(e) This statement is true: $\frac{-a}{b} = (-a)\left(\frac{1}{b}\right) = (-1)(a)\left(\frac{1}{b}\right) = (-1)\left(\frac{a}{b}\right) = -\frac{a}{b}$.(f) This statement is true: $\frac{1+x+x^2}{x} = \frac{1}{x} + \frac{x}{x} + \frac{x^2}{x} = \frac{1}{x} + 1 + x$.

105. (a)

x	1	3	$\frac{1}{2}$	$\frac{9}{10}$	$\frac{99}{100}$	$\frac{999}{1000}$	$\frac{9999}{10,000}$
$x + \frac{1}{x}$	2	3.333	2.5	2.011	2.0001	2.000001	2.00000001

It appears that the smallest possible value of $x + \frac{1}{x}$ is 2.(b) Because $x > 0$, we can multiply both sides by x and preserve the inequality: $x + \frac{1}{x} \geq 2 \Leftrightarrow x\left(x + \frac{1}{x}\right) \geq 2x \Leftrightarrow$
 $x^2 + 1 \geq 2x \Leftrightarrow x^2 - 2x + 1 \geq 0 \Leftrightarrow (x-1)^2 \geq 0$. The last statement is true for all $x > 0$, and because each step is reversible, we have shown that $x + \frac{1}{x} \geq 2$ for all $x > 0$.

1.5 EQUATIONS

1. (a) Yes. If $a = b$, then $a + x = b + x$, and vice versa.
 (b) Yes. If $a = b$, then $ma = mb$ for $m \neq 0$, and vice versa.
 (c) No. For example, $-5 \neq 5$, but $(-5)^2 = 5^2 = 25$.
2. (a) Take positive and negative square roots of both sides: $(x + 5)^2 = 64 \Leftrightarrow x + 5 = \pm 8$.
 (b) Subtract 5 from both sides: $(x + 5)^2 + 5 = 64 \Leftrightarrow (x + 5)^2 = 59$.
 (c) Subtract 2 from both sides: $x^2 + x = 2 \Leftrightarrow x^2 + x - 2 = 0$.
3. (a) To solve the equation $x^2 - 6x - 16 = 0$ by factoring, we write $x^2 - 6x - 16 = (x - 8)(x + 2) = 0$ if $x = 8$ or $x = -2$.
 (b) To solve by completing the square, we write $x^2 - 6x - 16 = 0 \Leftrightarrow x^2 - 6x = 16 \Leftrightarrow x^2 - 6x + 9 = 16 + 9 \Leftrightarrow (x - 3)^2 = 25 \Leftrightarrow x - 3 = \pm 5 \Leftrightarrow x = 8$ or $x = -2$.
 (c) To solve using the Quadratic Formula, we write $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-16)}}{2(1)} = \frac{6 \pm \sqrt{100}}{2} = 3 \pm 5 \Leftrightarrow x = 8$ or $x = -2$.
4. (a) The Zero-Product Property says that if $a \cdot b = 0$ then either a or b must be 0.
 (b) The solutions of the equation $x^2(x - 4) = 0$ are $x = 0$ and $x = 4$.
 (c) To solve the equation $x^3 - 4x^2 = 0$ we factor the left-hand side: $x^2(x - 4) = 0$, as above.
5. (a) Isolating the radical in $\sqrt{2x} + x = 0$, we obtain $\sqrt{2x} = -x$.
 (b) Now square both sides: $(\sqrt{2x})^2 = (-x)^2 \Rightarrow 2x = x^2$.
 (c) Solving the resulting quadratic equation, we find $2x = x^2 \Rightarrow x^2 - 2x = x(x - 2) = 0$, so the solutions are $x = 0$ and $x = 2$.
 (d) We substitute these possible solutions into the original equation: $\sqrt{2 \cdot 0} + 0 = 0$, so $x = 0$ is a solution, but $\sqrt{2 \cdot 2} + 2 = 4 \neq 0$, so $x = 2$ is not a solution. The only real solution is $x = 0$.
6. The equation $(x + 1)^2 - 5(x + 1) + 6 = 0$ is of *quadratic* type. To solve the equation we set $W = x + 1$. The resulting quadratic equation is $W^2 - 5W + 6 = 0 \Rightarrow (W - 3)(W - 2) = 0 \Rightarrow W = 2$ or $W = 3 \Rightarrow x + 1 = 2$ or $x + 1 = 3 \Rightarrow x = 1$ or $x = 2$. You can verify that these are both solutions to the original equation.
7. To eliminate the denominators in the equation $\frac{3}{x} + \frac{5}{x+2} = 2$, multiply each side by the lowest common denominator $x(x + 2)$ to get the equivalent equation $3(x + 2) + 5x = 2x(x + 2)$.
8. To eliminate the square root in the equation $2x + 1 = \sqrt{x + 1}$, *square* each side to get the equation $(2x + 1)^2 = x + 1$. (But don't forget that squaring sometimes introduces extraneous solutions.)
9. (a) When $x = -2$, LHS = $4(-2) + 7 = -8 + 7 = -1$ and RHS = $9(-2) - 3 = -18 - 3 = -21$. Since LHS $\neq$ RHS, $x = -2$ is not a solution.
 (b) When $x = 2$, LHS = $4(-2) + 7 = 8 + 7 = 15$ and RHS = $9(2) - 3 = 18 - 3 = 15$. Since LHS = RHS, $x = 2$ is a solution.
10. (a) When $x = 2$, LHS = $1 - [2 - (3 - (2))]$ = $1 - [2 - 1]$ = $1 - 1 = 0$ and RHS = $4(2) - (6 + (2)) = 8 - 8 = 0$. Since LHS = RHS, $x = 2$ is a solution.
 (b) When $x = 4$ LHS = $1 - [2 - (3 - (4))]$ = $1 - [2 - (-1)]$ = $1 - 3 = -2$ and RHS = $4(4) - (6 + (4)) = 16 - 10 = 6$. Since LHS $\neq$ RHS, $x = 4$ is not a solution.
11. (a) When $x = 2$, LHS = $\frac{1}{2} - \frac{1}{2-4} = \frac{1}{2} - \frac{1}{-2} = \frac{1}{2} + \frac{1}{2} = 1$ and RHS = 1. Since LHS = RHS, $x = 2$ is a solution.

- (b) When $x = 4$ the expression $\frac{1}{4-4}$ is not defined, so $x = 4$ is not a solution.
12. (a) When $x = 4$, $\text{LHS} = \frac{4^{3/2}}{4-6} = \frac{2^3}{-2} = \frac{8}{-2} = -4$ and $\text{RHS} = (4) - 8 = -4$. Since $\text{LHS} = \text{RHS}$, $x = 4$ is a solution.
- (b) When $x = 8$, $\text{LHS} = \frac{8^{3/2}}{8-6} = \frac{(2^3)^{3/2}}{2} = \frac{2^{9/2}}{2} = 2^{7/2}$ and $\text{RHS} = (8) - 8 = 0$. Since $\text{LHS} \neq \text{RHS}$, $x = 8$ is not a solution.
13. $8x + 13 = 5 \Leftrightarrow 8x = 5 - 13 \Leftrightarrow 8x = -8 \Leftrightarrow x = -1$
14. $1 - x = 10 \Leftrightarrow 1 - 10 = x \Leftrightarrow x = -9$
15. $\frac{1}{2}x - 8 = 1 \Leftrightarrow \frac{1}{2}x = 9 \Leftrightarrow x = 18$
16. $3 + \frac{1}{3}x = 5 \Leftrightarrow \frac{1}{3}x = 2 \Leftrightarrow x = 6$
17. $-x + 3 = 4x \Leftrightarrow -x + 3 + x = 4x + x \Leftrightarrow 3 = 5x \Leftrightarrow x = \frac{3}{5}$
18. $2x + 3 = 7 - 3x \Leftrightarrow 5x = 4 \Leftrightarrow x = \frac{4}{5}$
19. $\frac{x}{3} - 2 = \frac{5}{3}x + 7 \Leftrightarrow 3\left(\frac{x}{3} - 2\right) = 3\left(\frac{5}{3}x + 7\right) \Leftrightarrow x - 6 = 5x + 21 \Leftrightarrow -27 = 4x \Leftrightarrow x = -\frac{27}{4}$
20. $\frac{2}{5}x - 1 = \frac{3}{10}x + 3 \Leftrightarrow 4x - 10 = 3x + 30 \Leftrightarrow x = 40$
21. $2(1 - x) = 3(1 + 2x) + 5 \Leftrightarrow 2 - 2x = 3 + 6x + 5 \Leftrightarrow 2 - 2x = 8 + 6x \Leftrightarrow -6 = 8x \Leftrightarrow x = -\frac{3}{4}$
22. $(3y + 2) - 5(2y - 1) = 2(3 - y) + 1 \Leftrightarrow 3y + 2 - 10y + 5 = 6 - 2y + 1 \Leftrightarrow -7y + 7 = 7 - 2y - 7y = -2y \Leftrightarrow y = 0$.
23. $x - \frac{1}{3}x - \frac{1}{2}x - 5 = 0 \Leftrightarrow 6x - 2x - 3x - 30 = 0$ (multiply both sides by the LCD, 6) $\Leftrightarrow x = 30$
24. $2x - \frac{x}{2} + \frac{x+1}{4} = 6x \Leftrightarrow 8x - 2x + x + 1 = 24x \Leftrightarrow 7x + 1 = 24x \Leftrightarrow 1 = 17x \Leftrightarrow x = \frac{1}{17}$
25. $\frac{2x-1}{x+2} = \frac{4}{5} \Rightarrow 5(2x-1) = 4(x+2) \Leftrightarrow 10x-5 = 4x+8 \Leftrightarrow 6x = 13 \Leftrightarrow x = \frac{13}{6}$
26. $\frac{3}{x-1} = \frac{5}{x+2} - \frac{1}{x-1} \Leftrightarrow \frac{3}{x-1} + \frac{1}{x-1} = \frac{5}{x+2} \Leftrightarrow \frac{4}{x-1} = \frac{5}{x+2} \Leftrightarrow 4(x+2) = 5(x-1) \Leftrightarrow 4x+8 = 5x-5 \Leftrightarrow x = 13$
27. $\frac{1}{x} = \frac{4}{3x} + 1 \Rightarrow 3 = 4 + 3x$ (multiply both sides by the LCD, $3x$) $\Leftrightarrow -1 = 3x \Leftrightarrow x = -\frac{1}{3}$
28. $\frac{3}{x+1} - \frac{1}{2} = \frac{1}{3x+3} \Rightarrow 3(6) - (3x+3) = 2$ [multiply both sides by the LCD, $6(x+1)$] $\Leftrightarrow 18 - 3x - 3 = 2 \Leftrightarrow -3x + 15 = 2 \Leftrightarrow -3x = -13 \Leftrightarrow x = \frac{13}{3}$
29. $E = \frac{1}{2}mv^2 \Rightarrow m = \frac{2E}{v^2}$
30. $F = G\frac{mM}{r^2} \Leftrightarrow m = \frac{Fr^2}{GM}$
31. $P = 2l + 2w \Leftrightarrow 2w = P - 2l \Leftrightarrow w = \frac{P-2l}{2}$
32. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Leftrightarrow R_1R_2 = RR_2 + RR_1$ (multiply both sides by the LCD, RR_1R_2). Thus $R_1R_2 - RR_1 = RR_2 \Leftrightarrow R_1(R_2 - R) = RR_2 \Leftrightarrow R_1 = \frac{RR_2}{R_2 - R}$
33. $\frac{ax+b}{cx+d} = 2 \Leftrightarrow ax+b = 2(cx+d) \Leftrightarrow ax+b = 2cx+2d \Leftrightarrow ax-2cx = 2d-b \Leftrightarrow (a-2c)x = 2d-b \Leftrightarrow x = \frac{2d-b}{a-2c}$
34. $a - 2[b - 3(c - x)] = 6 \Leftrightarrow a - 2(b - 3c + 3x) = 6 \Leftrightarrow a - 2b + 6c - 6x = 6 \Leftrightarrow -6x = 6 - a + 2b - 6c \Leftrightarrow x = \frac{6-a+2b-6c}{-6} = -\frac{6-a+2b-6c}{6}$

35. $a^2x + (a - 1) = (a + 1)x \Leftrightarrow a^2x - (a + 1)x = -(a - 1) \Leftrightarrow (a^2 - (a + 1))x = -a + 1 \Leftrightarrow (a^2 - a - 1)x = -a + 1$
 $\Leftrightarrow x = \frac{-a + 1}{a^2 - a - 1}$
36. $\frac{a + 1}{b} = \frac{a - 1}{b} + \frac{b + 1}{a} \Leftrightarrow a(a + 1) = a(a - 1) + b(b + 1) \Leftrightarrow a^2 + a = a^2 - a + b^2 + b \Leftrightarrow 2a = b^2 + b \Leftrightarrow$
 $a = \frac{1}{2}(b^2 + b)$
37. $V = \frac{1}{3}\pi r^2 h \Leftrightarrow r^2 = \frac{3V}{\pi h} \Rightarrow r = \pm\sqrt{\frac{3V}{\pi h}}$
38. $F = G\frac{mM}{r^2} \Leftrightarrow r^2 = G\frac{mM}{F} \Rightarrow r = \pm\sqrt{G\frac{mM}{F}}$
39. $a^2 + b^2 = c^2 \Leftrightarrow b^2 = c^2 - a^2 \Rightarrow b = \pm\sqrt{c^2 - a^2}$
40. $A = P\left(1 + \frac{i}{100}\right)^2 \Leftrightarrow \frac{A}{P} = \left(1 + \frac{i}{100}\right)^2 \Rightarrow 1 + \frac{i}{100} = \pm\sqrt{\frac{A}{P}} \Leftrightarrow \frac{i}{100} = -1 \pm \sqrt{\frac{A}{P}} \Leftrightarrow i = -100 \pm 100\sqrt{\frac{A}{P}}$
41. $x^2 + x = 12 \Leftrightarrow x^2 + x - 12 = 0 \Leftrightarrow (x - 3)(x + 4) = 0 \Leftrightarrow x - 3 = 0$ or $x + 4 = 0$. Thus, $x = 3$ or $x = -4$.
42. $x^2 + 9x = -20 \Leftrightarrow x^2 + 9x + 20 = 0 \Leftrightarrow (x + 4)(x + 5) = 0 \Leftrightarrow x + 4 = 0$ or $x + 5 = 0$. Thus, $x = -4$ or $x = -5$.
43. $x^2 + 13x - 30 = 0 \Leftrightarrow (x + 15)(x - 2) = 0 \Leftrightarrow x + 15 = 0$ or $x - 2 = 0$. Thus, $x = -15$ or $x = 2$.
44. $x^2 - 13x + 30 = 0 \Leftrightarrow (x - 10)(x - 3) = 0 \Leftrightarrow x - 10 = 0$ or $x - 3 = 0$. Thus, $x = 3$ or $x = 10$.
45. $4x^2 - 4x - 15 = 0 \Leftrightarrow (2x + 3)(2x - 5) = 0 \Leftrightarrow 2x + 3 = 0$ or $2x - 5 = 0$. Thus, $x = -\frac{3}{2}$ or $x = \frac{5}{2}$.
46. $2y^2 + 7y + 3 = 0 \Leftrightarrow (y + 3)(2y + 1) = 0 \Leftrightarrow y + 3 = 0$ or $2y + 1 = 0$. Thus, $y = -3$ or $y = -\frac{1}{2}$.
47. $2x^2 = 8 \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2$.
48. $3x^2 - 27 = 0 \Leftrightarrow x^2 = 9 \Leftrightarrow x = \pm 3$.
49. $(2x - 5)^2 = 81 \Leftrightarrow 2x - 5 = \pm 9 \Leftrightarrow 2x = -4$ or $2x = 14 \Leftrightarrow x = -2$ or $x = 7$
50. $(5x + 1)^2 + 3 = 10 \Leftrightarrow (5x + 1)^2 = 7 \Leftrightarrow 5x + 1 = \pm\sqrt{7} \Leftrightarrow 5x = -1 \pm \sqrt{7} \Leftrightarrow x = \frac{-1 \pm \sqrt{7}}{5}$
51. $8(x + 5)^2 = 16 \Leftrightarrow (x + 5)^2 = 2 \Leftrightarrow x + 5 = \pm\sqrt{2} \Leftrightarrow x = -5 \pm \sqrt{2}$
52. $2(3x - 1)^2 - 3 = 11 \Leftrightarrow 2(3x - 1)^2 = 14 \Leftrightarrow (3x - 1)^2 = 7 \Leftrightarrow 3x - 1 = \pm\sqrt{7} \Leftrightarrow x = \frac{1 \pm \sqrt{7}}{3}$
53. $x^2 + 10x + 2 = 0 \Leftrightarrow x^2 + 10x = -2 \Leftrightarrow x^2 + 10x + 25 = -2 + 25 \Leftrightarrow (x + 5)^2 = 23 \Leftrightarrow x + 5 = \pm\sqrt{23} \Leftrightarrow x = -5 \pm \sqrt{23}$
54. $x^2 - 4x + 2 = 0 \Leftrightarrow x^2 - 4x = -2 \Leftrightarrow x^2 - 4x + 4 = -2 + 4 \Leftrightarrow (x - 2)^2 = 2 \Rightarrow x - 2 = \pm\sqrt{2} \Leftrightarrow x = 2 \pm \sqrt{2}$
55. $x^2 - 6x - 11 = 0 \Leftrightarrow x^2 - 6x = 11 \Leftrightarrow x^2 - 6x + 9 = 11 + 9 \Leftrightarrow (x - 3)^2 = 20 \Rightarrow x - 3 = \pm 2\sqrt{5} \Leftrightarrow x = 3 \pm 2\sqrt{5}$
56. $x^2 + 3x - \frac{7}{4} = 0 \Leftrightarrow x^2 + 3x = \frac{7}{4} \Leftrightarrow x^2 + 3x + \frac{9}{4} = \frac{7}{4} + \frac{9}{4} \Leftrightarrow \left(x + \frac{3}{2}\right)^2 = \frac{16}{4} = 4 \Rightarrow x + \frac{3}{2} = \pm 2 \Leftrightarrow x = -\frac{3}{2} \pm 2 \Leftrightarrow$
 $x = \frac{1}{2}$ or $x = -\frac{7}{2}$.
57. $5x^2 + 10x = 1 \Leftrightarrow 5(x^2 + 2x) = 1 \Leftrightarrow 5(x^2 + 2x + 1) = 1 + 5 \Leftrightarrow (x + 1)^2 = \frac{6}{5} \Leftrightarrow x + 1 = \pm\sqrt{\frac{6}{5}} \Leftrightarrow x = -1 \pm \frac{\sqrt{30}}{5}$
58. $3x^2 - 6x - 1 = 0 \Leftrightarrow 3x^2 - 6x = 1 \Leftrightarrow 3(x^2 - 2x) = 1 \Leftrightarrow x^2 - 2x + 1 = \frac{1}{3} + 1 \Leftrightarrow (x - 1)^2 = \frac{4}{3} \Leftrightarrow x - 1 = \pm\sqrt{\frac{4}{3}} \Leftrightarrow$
 $x = 1 \pm \frac{2\sqrt{3}}{3}$
59. $4x^2 - x = 0 \Leftrightarrow x(4x - 1) = 0 \Leftrightarrow x = 0$ or $x = \frac{1}{4}$.
60. $x^2 = \frac{3}{4}x - \frac{1}{8} \Leftrightarrow x^2 - \frac{3}{4}x = -\frac{1}{8} \Leftrightarrow x^2 - \frac{3}{4}x + \frac{9}{64} = -\frac{1}{8} + \frac{9}{64} \Leftrightarrow \left(x - \frac{3}{8}\right)^2 = \frac{1}{64} \Rightarrow x - \frac{3}{8} = \pm\frac{1}{8} \Leftrightarrow x = \frac{3}{8} \pm \frac{1}{8}$, so
 $x = \frac{1}{2}$ or $x = \frac{1}{4}$.
61. $x^2 - 2x - 15 = 0 \Leftrightarrow (x + 3)(x - 5) = 0 \Leftrightarrow x + 3 = 0$ or $x - 5 = 0$. Thus, $x = -3$ or $x = 5$.

62. $x^2 - 13x + 42 = 0 \Leftrightarrow (x - 6)(x - 7) = 0 \Leftrightarrow x - 6 = 0$ or $x - 7 = 0$. Thus, $x = 6$ or $x = 7$.

63. $2x^2 + x - 3 = 0 \Leftrightarrow (x - 1)(2x + 3) = 0 \Leftrightarrow x - 1 = 0$ or $2x + 3 = 0$. If $x - 1 = 0$, then $x = 1$; if $2x + 3 = 0$, then $x = -\frac{3}{2}$.

64. $3x^2 + 7x + 4 = 0 \Leftrightarrow (3x + 4)(x + 1) = 0 \Leftrightarrow 3x + 4 = 0$ or $x + 1 = 0$. Thus, $x = -\frac{4}{3}$ or $x = -1$.

65. $3x^2 + 6x - 5 = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(3)(-5)}}{2(3)} = \frac{-6 \pm \sqrt{96}}{6} = \frac{-3 \pm 2\sqrt{6}}{3}$

66. $x^2 - 6x + 1 = 0 \Rightarrow$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(1)}}{2(1)} = \frac{6 \pm \sqrt{36 - 4}}{2} = \frac{6 \pm \sqrt{32}}{2} = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

67. $9x^2 + 12x + 4 = 0 \Leftrightarrow (3x + 2)^2 = 0 \Leftrightarrow x = -\frac{2}{3}$

68. $4x^2 - 4x + 1 = 0 \Leftrightarrow (2x - 1)^2 = 0 \Leftrightarrow x = \frac{1}{2}$

69. $4x^2 + 16x = 9 \Leftrightarrow 4x^2 + 16x - 9 = 0 \Leftrightarrow (2x - 1)(2x + 9) = 0 \Leftrightarrow 2x - 1 = 0$ or $2x + 9 = 0$. If $2x - 1 = 0$, then $x = \frac{1}{2}$; if $2x + 9 = 0$, then $x = -\frac{9}{2}$.

70. $4x = x^2 + 1 \Leftrightarrow x^2 - 4x + 1 = 0 \Leftrightarrow$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

71. $7x^2 = 2x - 4 \Leftrightarrow 7x^2 - 2x + 4 = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(7)(4)}}{2(7)} = \frac{2 \pm \sqrt{4 - 112}}{-4}$, so there is no real solution.

72. $z(z - 3) = 5 \Leftrightarrow z^2 - 3z - 5 = 0 \Leftrightarrow z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4(1)(-5)}}{2} = \frac{3 \pm \sqrt{29}}{2}$.

73. $D = b^2 - 4ac = (-6)^2 - 4(1)(1) = 32$. Since D is positive, this equation has two real solutions.

74. $3x^2 = 6x - 9 \Leftrightarrow 3x^2 - 6x + 9 = 0$. $D = b^2 - 4ac = (-6)^2 - 4(3)(9) = 36 - 108 = -72$. Since D is negative, this equation has no real solution.

75. $D = b^2 - 4ac = (2.20)^2 - 4(1)(1.21) = 4.84 - 4.84 = 0$. Since $D = 0$, this equation has one real solution.

76. $D = b^2 - 4ac = (2.21)^2 - 4(1)(1.21) = 4.8841 - 4.84 = 0.0441$. Since $D \neq 0$, this equation has two real solutions.

77. $D = b^2 - 4ac = (5)^2 - 4(4)\left(\frac{13}{8}\right) = 25 - 26 = -1$. Since D is negative, this equation has no real solution.

78. $D = b^2 - 4ac = (r)^2 - 4(1)(-s) = r^2 + 4s$. Since D is positive, this equation has two real solutions.

79. $\frac{x^2}{x + 100} = 50 \Rightarrow x^2 = 50(x + 100) = 50x + 5000 \Leftrightarrow x^2 - 50x - 5000 = 0 \Leftrightarrow (x - 100)(x + 50) = 0 \Leftrightarrow x - 100 = 0$ or $x + 50 = 0$. Thus $x = 100$ or $x = -50$. The solutions are 100 and -50.

80. $\frac{1}{x-1} - \frac{2}{x^2} = 0 \Leftrightarrow x^2 - 2(x-1) = 0 \Leftrightarrow x^2 - 2x + 2 = 0 \Rightarrow$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{4 - 8}}{2} = \frac{2 \pm \sqrt{-4}}{2}$$
. Since the radicand is negative, there is no real solution.

81. $\frac{y+1}{y^2+1} = \frac{2}{y+2} \Leftrightarrow (y+1)(y+2) = (y^2+1)(2) \Leftrightarrow y^2+3y+2 = 2y^2+2 \Leftrightarrow y^2-3y=0 \Leftrightarrow y(y-3)=0 \Leftrightarrow y=0$ or 3 .

82. $\frac{3w+1}{w} = \frac{w-3}{w-1} \Leftrightarrow (3w+1)(w-1) = w(w-3) \Leftrightarrow 3w^2-2w-1 = w^2-3w \Leftrightarrow 2w^2+w-1=0 \Leftrightarrow$
 $(w+1)(2w-1)=0 \Leftrightarrow w=-1$ or $\frac{1}{2}$.

83. $\frac{1}{x-1} + \frac{1}{x+2} = \frac{5}{4} \Leftrightarrow 4(x-1)(x+2)\left(\frac{1}{x-1} + \frac{1}{x+2}\right) = 4(x-1)(x+2)\left(\frac{5}{4}\right) \Leftrightarrow$
 $4(x+2) + 4(x-1) = 5(x-1)(x+2) \Leftrightarrow 4x+8+4x-4 = 5x^2+5x-10 \Leftrightarrow 5x^2-3x-14 = 0 \Leftrightarrow$
 $(5x+7)(x-2) = 0$. If $5x+7=0$, then $x = -\frac{7}{5}$; if $x-2=0$, then $x = 2$. The solutions are $-\frac{7}{5}$ and 2.
84. $\frac{x+5}{x-2} = \frac{5}{x+2} + \frac{28}{x^2-4} \Rightarrow (x+2)(x+5) = 5(x-2) + 28 \Leftrightarrow x^2+7x+10 = 5x-10+28 \Leftrightarrow x^2+2x-8 = 0 \Leftrightarrow$
 $(x-2)(x+4) = 0 \Leftrightarrow x-2=0$ or $x+4=0 \Leftrightarrow x=2$ or $x=-4$. However, $x=2$ is inadmissible since we can't divide by 0 in the original equation, so the only solution is -4 .
85. $5 = \sqrt{4x-3} \Rightarrow 5^2 = (\sqrt{4x-3})^2 \Leftrightarrow 25 = 4x-3 \Leftrightarrow 4x = 28 \Leftrightarrow x = 7$
86. $\sqrt{8x-1} = 3 \Rightarrow 8x-1 = 9 \Leftrightarrow 8x = 10 \Leftrightarrow x = \frac{5}{4}$
87. $\sqrt{2x-1} = \sqrt{3x-5} \Leftrightarrow 2x-1 = 3x-5 \Leftrightarrow x = 4$
88. $\sqrt{3+x} = \sqrt{x^2+1} \Leftrightarrow 3+x = x^2+1 \Leftrightarrow x^2-x-2 = 0 \Leftrightarrow (x+1)(x-2) = 0 \Leftrightarrow x = -1$ or $x = 2$.
89. $\sqrt{2x+1} + 1 = x \Leftrightarrow \sqrt{2x+1} = x-1 \Rightarrow 2x+1 = (x-1)^2 \Leftrightarrow 2x+1 = x^2-2x+1 \Leftrightarrow 0 = x^2-4x = x(x-4)$.
 Potential solutions are $x = 0$ and $x = 4 \Leftrightarrow x = 4$. These are only potential solutions since squaring is not a reversible operation. We must check each potential solution in the original equation. Checking $x = 0$: $\sqrt{2(0)+1} + 1 \stackrel{?}{=} (0) \Leftrightarrow$
 $\sqrt{1} + 1 \stackrel{?}{=} 0$ is false. Checking $x = 4$: $\sqrt{2(4)+1} + 1 \stackrel{?}{=} (4)$, $\sqrt{9} + 1 \stackrel{?}{=} 4$, $3 + 1 \stackrel{?}{=} 4$ is true. Thus, the only solution is $x = 4$.
90. $2x + \sqrt{x+1} = 8 \Leftrightarrow \sqrt{x+1} = 8-2x \Rightarrow x+1 = (8-2x)^2 \Leftrightarrow x+1 = 64-32x+4x^2 \Leftrightarrow$
 $0 = 4x^2-33x+63 = (4x-21)(x-3)$. Potential solutions are $x = \frac{21}{4}$ and $x = 3$. Substituting each of these solutions into the original equation, we see that $x = 3$ is a solution, but $x = \frac{21}{4}$ is not. Thus 3 is the only solution.
91. Let $w = x^2$. Then $x^4 - 13x^2 + 40 = (x^2)^2 - 13x^2 + 40 = 0$ becomes $w^2 - 13w + 40 = 0 \Leftrightarrow (w-5)(w-8) = 0$.
 So $w-5=0 \Leftrightarrow w=5$, and $w-8=0 \Leftrightarrow w=8$. When $w=5$, we have $x^2=5 \Rightarrow x = \pm\sqrt{5}$. When $w=8$, we have $x^2=8 \Rightarrow x = \pm\sqrt{8} = \pm 2\sqrt{2}$. The solutions are $\pm\sqrt{5}$ and $\pm 2\sqrt{2}$.
92. $0 = x^6 - 2x^3 - 3 = (x^3-3)(x^3+1)$. Let $W = x^3$, so the equation becomes $(W-3)(W+1) = 0$. If $W = 3$, then $x^3 = 3 \Leftrightarrow x = \sqrt[3]{3}$, and if $W = -1$, then $x^3 = -1 \Leftrightarrow x = -1$. Thus $x = \sqrt[3]{3}$ or $x = -1$. The solutions are $\sqrt[3]{3}$ and -1 .
93. $(x+2)^4 - 5(x+2)^2 + 4 = 0 \Leftrightarrow [(x+2)^2-4][(x+2)^2-1] = 0$. If $(x+2)^2-4 = 0$, then $(x+2)^2 = 4 \Leftrightarrow$
 $x+2 = \pm 2 \Leftrightarrow x = 0$ or $x = -4$. If $(x+2)^2-1 = 0$, then $(x+2)^2 = 1 \Leftrightarrow x+2 = \pm 1 \Leftrightarrow x = -1$ or $x = -3$. Thus, the solutions are $x = -4, -3, -1$, and 0.
94. $2x^4 + 4x^2 + 1 = 0$. The LHS is the sum of two nonnegative numbers and a positive number, so $2x^4 + 4x^2 + 1 \geq 1 > 0$. This equation has no real solution.
95. Let $u = x^{2/3}$. Then $0 = x^{4/3} - 5x^{2/3} + 6$ becomes $u^2 - 5u + 6 = 0 \Leftrightarrow (u-3)(u-2) = 0 \Leftrightarrow u-3=0$ or $u-2=0$. If $u-3=0$, then $x^{2/3}-3=0 \Leftrightarrow x^{2/3}=3 \Rightarrow x = \pm 3^{3/2}$. If $u-2=0$, then $x^{2/3}-2=0 \Leftrightarrow x^{2/3}=2 \Rightarrow x = \pm 2^{3/2}$. The solutions are $\pm 3^{3/2}$ and $\pm 2^{3/2}$; that is, $x = \pm 2\sqrt{2}$ and $x = \pm 3\sqrt{3}$.
96. Let $W = (x+1)^{1/2}$. Then $4(x+1)^{1/2} - 5(x+1)^{3/2} + (x+1)^{5/2} = 0 \Leftrightarrow 4W - 5W^3 + W^5 = 0 \Leftrightarrow$
 $W(W^4 - 5W^2 + 4) = 0 \Leftrightarrow W(W^2-4)(W^2-1) = 0 \Leftrightarrow W(W-2)(W+2)(W-1)(W+1) = 0 \Leftrightarrow W = -2,$
 $-1, 0, 1$, or 2 . If $W < 0$, then $W = \sqrt{x+1}$ has no solution. If $W = 0$, then $x = -1$; if $W = 1$, then $\sqrt{x+1} = 1 \Leftrightarrow x = 1$, and if $W = 2$, then $\sqrt{x+1} = 2 \Leftrightarrow x = 3$. The solutions are $-1, 0$, and 3.
97. $x^{1/2} + 3x^{-1/2} = 10x^{-3/2} \Leftrightarrow 0 = x^{1/2} + 3x^{-1/2} - 10x^{-3/2} = x^{-3/2}(x^2 + 3x - 10) = x^{-3/2}(x-2)(x+5)$. Now $x^{-3/2}$ never equals 0, and no solution can be negative, because we cannot take the square root of a negative number. Thus 2 is the only solution.

98. Let $u = \sqrt{x}$. Then $0 = x - 5\sqrt{x} + 6$ becomes $u^2 - 5u + 6 = (u - 3)(u - 2) = 0$. If $u - 3 = 0$, then $\sqrt{x} - 3 = 0 \Leftrightarrow \sqrt{x} = 3 \Rightarrow x = 9$. If $u - 2 = 0$, then $\sqrt{x} - 2 = 0 \Leftrightarrow \sqrt{x} = 2 \Rightarrow x = 4$. The solutions are 9 and 4.
99. $|3x + 5| = 1$. So $3x + 5 = 1 \Leftrightarrow 3x = -4 \Leftrightarrow x = -\frac{4}{3}$ or $3x + 5 = -1 \Leftrightarrow 3x = -6 \Leftrightarrow x = -2$.
100. $|2x| = 3$. So $2x = 3 \Leftrightarrow x = \frac{3}{2}$ or $2x = -3 \Leftrightarrow x = -\frac{3}{2}$.
101. $|x - 4| = 0.01$. So $x - 4 = 0.01 \Leftrightarrow x = 4.01$ or $x - 4 = -0.01 \Leftrightarrow x = 3.99$.
102. $|x - 6| = -1$. Since absolute value is never negative, this equation has no real solution.
103. $2x + \frac{1}{2} = \frac{1}{4}x - 3 \Leftrightarrow 4\left(2x + \frac{1}{2}\right) = 4\left(\frac{1}{4}x - 3\right) \Leftrightarrow 8x + 2 = x - 12 \Leftrightarrow 7x = -14 \Leftrightarrow x = -2$
104. $x^2 - 7x + 12 = 0 \Leftrightarrow (x - 4)(x - 3) = 0 \Leftrightarrow x - 4 = 0$ or $x - 3 = 0$. Thus, $x = 4$ or $x = 3$.
105. $\frac{4-a}{3a-2} = a \Leftrightarrow 4-a = a(3a-2) \Leftrightarrow 4-a = 3a^2-2a \Leftrightarrow 3a^2-a-4 = 0 \Leftrightarrow (a+1)(3a-4) = 0$. Thus, $a = -1$ or $a = \frac{4}{3}$.
106. $\frac{3x-5}{x+5} = 8 \Leftrightarrow 3x-5 = 8(x+5) \Leftrightarrow 3x-5 = 8x+40 \Leftrightarrow 5x = -45 \Leftrightarrow x = -9$
107. $\frac{2}{3}y + \frac{1}{2}(y-3) = \frac{y+1}{4} \Leftrightarrow 8y + 6(y-3) = 3(y+1) \Leftrightarrow 8y + 6y - 18 = 3y + 3 \Leftrightarrow 14y - 18 = 3y + 3 \Leftrightarrow 11y = 21 \Leftrightarrow y = \frac{21}{11}$
108. $6x(x-1) = 21-x \Leftrightarrow 6x^2-6x = 21-x \Leftrightarrow 6x^2-5x-21 = 0 \Leftrightarrow (2x+3)(3x-7) = 0 \Leftrightarrow 2x+3 = 0$ or $3x-7 = 0$. If $2x+3 = 0$, then $x = -\frac{3}{2}$; if $3x-7 = 0$, then $x = \frac{7}{3}$.
109. $x - \sqrt{9-3x} = 0 \Leftrightarrow x = \sqrt{9-3x} \Rightarrow x^2 = 9-3x \Leftrightarrow x^2 + 3x - 9 = 0 \Leftrightarrow$
 $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-3 \pm \sqrt{3^2-4(1)(-9)}}{2(1)} = \frac{-3 \pm \sqrt{9+36}}{2} = \frac{-3 \pm 3\sqrt{5}}{2}$. However, $\frac{-3-3\sqrt{5}}{2}$ is not a
 solution because $\frac{-3-3\sqrt{5}}{2} - \sqrt{9-3\left(\frac{-3-3\sqrt{5}}{2}\right)}$ is the sum of two negative nonzero numbers and hence is nonzero.
 Thus, the only solution is $x = \frac{-3+3\sqrt{5}}{2}$.
110. $\sqrt{3}x + \sqrt{12} = \frac{x+5}{\sqrt{3}} \Leftrightarrow 3x + 6 = x + 5$ (multiply both sides by $\sqrt{3}$) $\Leftrightarrow 2x = -1 \Leftrightarrow x = -\frac{1}{2}$
111. Let $u = \sqrt[4]{x}$. Then $0 = \sqrt{x} - 3\sqrt[4]{x} - 4 = u^2 - 3u - 4 = (u-4)(u+1)$. So $u-4 = \sqrt[4]{x}-4 = 0 \Leftrightarrow \sqrt[4]{x} = 4 \Rightarrow x = 4^4 = 256$, or $u+1 = \sqrt[4]{x}+1 = 0 \Leftrightarrow \sqrt[4]{x} = -1$. However, $\sqrt[4]{x}$ is the positive fourth root, so this cannot equal -1 . The only solution is 256.
112. $x^6 - x^3 - 6 = 0 \Leftrightarrow (x^3 + 2)(x^3 - 3) = 0 \Leftrightarrow x^3 = -2$ or $x^3 = 3 \Leftrightarrow x = -\sqrt[3]{2}$ or $\sqrt[3]{3}$
113. $|3x - 10| = 29 \Leftrightarrow 3x - 10 = 29$ or $3x - 10 = -29 \Leftrightarrow 3x = 39$ or $3x = -19 \Leftrightarrow x = 13$ or $x = -\frac{19}{3}$
114. Let $u = x^{1/6}$. (We choose the exponent $\frac{1}{6}$ because the LCD of 2, 3, and 6 is 6.) Then $x^{1/2} - 3x^{1/3} = 3x^{1/6} - 9 \Leftrightarrow$
 $x^{3/6} - 3x^{2/6} = 3x^{1/6} - 9 \Leftrightarrow u^3 - 3u^2 = 3u - 9 \Leftrightarrow 0 = u^3 - 3u^2 - 3u + 9 = u^2(u-3) - 3(u-3) = (u-3)(u^2-3)$.
 So $u-3 = 0$ or $u^2-3 = 0$. If $u-3 = 0$, then $x^{1/6} - 3 = 0 \Leftrightarrow x^{1/6} = 3 \Leftrightarrow x = 3^6 = 729$. If $u^2-3 = 0$, then $x^{1/3} - 3 = 0 \Leftrightarrow x^{1/3} = 3 \Leftrightarrow x = 3^3 = 27$. The solutions are 729 and 27.
115. $(z-1)^2 - 8(z-1) + 15 = 0 \Leftrightarrow [(z-1)-5][(z-1)-3] = 0 \Leftrightarrow (z-6)(z-4) = 0 \Leftrightarrow z = 4$ or 6
116. $(t-4)^2 = (t+4)^2 + 32 \Leftrightarrow (t-4)^2 - (t+4)^2 = 32 \Leftrightarrow [(t-4)+(t+4)][(t-4)-(t+4)] = 32 \Leftrightarrow -16t = 32 \Leftrightarrow t = -2$
117. $|3x^2 - 1| - 4 = 7 \Leftrightarrow |3x^2 - 1| = 11 \Leftrightarrow 3x^2 - 1 = 11$ or $3x^2 - 1 = -10 \Leftrightarrow 3x^2 = 12 \Leftrightarrow x = \pm 2$ (since $3x^2 - 1 = -10$ has no real solution).

118. $\left(\frac{x}{x+1}\right)^2 - 6\left(\frac{x}{x+1}\right) + 8 = 0 \Leftrightarrow \left(\frac{x}{x+1} - 4\right)\left(\frac{x}{x+1} - 2\right) = 0 \Leftrightarrow \frac{x}{x+1} = 4$ or $\frac{x}{x+1} = 2 \Leftrightarrow x = 4x + 4$ or $x = 2x + 2 \Leftrightarrow 3x = -4$ or $x = -2 \Leftrightarrow x = -2$ or $-\frac{4}{3}$.
119. $\frac{1}{x^3} + \frac{4}{x^2} + \frac{4}{x} = 0 \Leftrightarrow 1 + 4x + 4x^2 = 0 \Leftrightarrow (1 + 2x)^2 = 0 \Leftrightarrow 1 + 2x = 0 \Leftrightarrow 2x = -1 \Leftrightarrow x = -\frac{1}{2}$. The solution is $-\frac{1}{2}$.
120. $0 = 4x^{-4} - 16x^{-2} + 4$. Multiplying by $\frac{x^4}{4}$ we get, $0 = 1 - 4x^2 + x^4$. Substituting $u = x^2$, we get $0 = 1 - 4u + u^2$, and using the Quadratic Formula, we get $u = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$. Substituting back, we have $x^2 = 2 \pm \sqrt{3}$, and since $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are both positive we have $x = \pm\sqrt{2 + \sqrt{3}}$ or $x = \pm\sqrt{2 - \sqrt{3}}$. Thus the solutions are $-\sqrt{2 - \sqrt{3}}$, $\sqrt{2 - \sqrt{3}}$, $-\sqrt{2 + \sqrt{3}}$, and $\sqrt{2 + \sqrt{3}}$.
121. $\sqrt{x+5} + x = 5$. Squaring both sides, we get $\sqrt{x+5} + x = 25 \Leftrightarrow \sqrt{x+5} = 25 - x$. Squaring both sides again, we get $x + 5 = (25 - x)^2 \Leftrightarrow x + 5 = 625 - 50x + x^2 \Leftrightarrow 0 = x^2 - 51x + 620 = (x - 20)(x - 31)$. Potential solutions are $x = 20$ and $x = 31$. We must check each potential solution in the original equation.
 Checking $x = 20$: $\sqrt{\sqrt{20+5}+20} = 5 \Leftrightarrow \sqrt{\sqrt{25}+20} = 5 \Leftrightarrow \sqrt{5+20} = 5$, which is true, and hence $x = 20$ is a solution.
 Checking $x = 31$: $\sqrt{\sqrt{31+5}+31} = 5 \Leftrightarrow \sqrt{\sqrt{36}+31} = 5 \Leftrightarrow \sqrt{37} = 5$, which is false, and hence $x = 31$ is not a solution. The only real solution is $x = 20$.
122. $\sqrt[3]{4x^2 - 4x} = x \Leftrightarrow 4x^2 - 4x = x^3 \Leftrightarrow 0 = x^3 - 4x^2 + 4x = x(x^2 - 4x + 4) = x(x - 2)^2$. So $x = 0$ or $x = 2$. The solutions are 0 and 2.
123. $x^2\sqrt{x+3} = (x+3)^{3/2} \Leftrightarrow 0 = x^2\sqrt{x+3} - (x+3)^{3/2} \Leftrightarrow 0 = \sqrt{x+3}[(x^2) - (x+3)] \Leftrightarrow 0 = \sqrt{x+3}(x^2 - x - 3)$.
 If $(x+3)^{1/2} = 0$, then $x+3 = 0 \Leftrightarrow x = -3$. If $x^2 - x - 3 = 0$, then by the Quadratic Formula, $x = \frac{1 \pm \sqrt{13}}{2}$. The solutions are -3 and $\frac{1 \pm \sqrt{13}}{2}$.
124. Let $u = \sqrt{11 - x^2}$. By definition of u we require it to be nonnegative. Now $\sqrt{11 - x^2} - \frac{2}{\sqrt{11 - x^2}} = 1 \Leftrightarrow u - \frac{2}{u} = 1$.
 Multiplying both sides by u we obtain $u^2 - 2 = u \Leftrightarrow 0 = u^2 - u - 2 = (u - 2)(u + 1)$. So $u = 2$ or $u = -1$. But since u must be nonnegative, we only have $u = 2 \Leftrightarrow \sqrt{11 - x^2} = 2 \Leftrightarrow 11 - x^2 = 4 \Leftrightarrow x^2 = 7 \Leftrightarrow x = \pm\sqrt{7}$. The solutions are $\pm\sqrt{7}$.
125. $0 = x^4 - 5ax^2 + 4a^2 = (a - x^2)(4a - x^2)$. Since a is positive, $a - x^2 = 0 \Leftrightarrow x^2 = a \Leftrightarrow x = \pm\sqrt{a}$. Again, since a is positive, $4a - x^2 = 0 \Leftrightarrow x^2 = 4a \Leftrightarrow x = \pm 2\sqrt{a}$. Thus the four solutions are $\pm\sqrt{a}$ and $\pm 2\sqrt{a}$.
126. $0 = a^3x^3 + b^3 = (ax + b)(a^2x^2 - abx + b^2)$. So $ax + b = 0 \Leftrightarrow ax = -b \Leftrightarrow x = -\frac{b}{a}$ or

$$x = \frac{-(-ab) \pm \sqrt{(-ab)^2 - 4(a^2)(b^2)}}{2(a^2)} = \frac{ab \pm \sqrt{-3a^2b^2}}{2a^2}$$
, but this gives no real solution. Thus, the solution is $x = -\frac{b}{a}$.
127. $\sqrt{x+a} + \sqrt{x-a} = \sqrt{2}\sqrt{x+6}$. Squaring both sides, we have
 $x + a + 2(\sqrt{x+a})(\sqrt{x-a}) + x - a = 2(x+6) \Leftrightarrow 2x + 2(\sqrt{x+a})(\sqrt{x-a}) = 2x + 12 \Leftrightarrow 2(\sqrt{x+a})(\sqrt{x-a}) = 12$
 $\Leftrightarrow (\sqrt{x+a})(\sqrt{x-a}) = 6$. Squaring both sides again we have $(x+a)(x-a) = 36 \Leftrightarrow x^2 - a^2 = 36 \Leftrightarrow x^2 = a^2 + 36$
 $\Leftrightarrow x = \pm\sqrt{a^2 + 36}$. Checking these answers, we see that $x = -\sqrt{a^2 + 36}$ is not a solution (for example, try substituting $a = 8$), but $x = \sqrt{a^2 + 36}$ is a solution.

128. Let $w = x^{1/6}$. Then $x^{1/3} = w^2$ and $x^{1/2} = w^3$, and so
 $0 = w^3 - aw^2 + bw - ab = w^2(w - a) + b(w - a) = (w^2 + b)(w - a) = (\sqrt[6]{x} + b)(\sqrt[6]{x} - a)$. So $\sqrt[6]{x} - a = 0 \Leftrightarrow a = \sqrt[6]{x} \Leftrightarrow x = a^6$ is one solution. Setting the first factor equal to zero, we have $\sqrt[6]{x} + b = 0 \Leftrightarrow \sqrt[6]{x} = -b \Leftrightarrow x = -b^6$. However, the original equation includes the term $b\sqrt[6]{x}$, and we cannot take the sixth root of a negative number, so this is not a solution. The only solution is $x = a^6$.
129. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**
 Using $h_0 = 88$, we solve $0 = -4.8t^2 + 88$, for $t \geq 0$. So $0 = -4.8t^2 + 88 \Leftrightarrow 4.8t^2 = 88 \Leftrightarrow t^2 \approx 18.33 \Leftrightarrow t \approx \pm 4.28$. Thus it takes 4.28 seconds for the ball to hit the ground.
130. (a) Using $h_0 = 29$, half the distance is 14.5, so we solve the equation $14.5 = -4.8t^2 + 29 \Leftrightarrow -14.5 = -4.8t^2 \Leftrightarrow t^2 = \frac{14.5}{4.8} \approx 3 \Rightarrow t \approx \pm\sqrt{3}$. Since $t \geq 0$, it takes $\sqrt{3} \approx 1.732$ s.
 (b) The ball hits the ground when $h = 0$, so we solve the equation $0 = -4.8t^2 + 29 \Leftrightarrow 4.8t^2 = 29 \Leftrightarrow t^2 = 6 \Rightarrow t = \pm\sqrt{6}$. Since $t \geq 0$, it takes $\sqrt{6} \approx 2.449$ s.
131. We are given that $v_0 = 12$ m/s.
 (a) Setting $h = 7.2$, we have $7.2 = -4.8t^2 + 12t \Leftrightarrow 4.8t^2 - 12t + 7.2 = 0 \Leftrightarrow 2.4(2t^2 - 5t + 3) = 0 \Leftrightarrow 2.4(2t - 3)(t - 1) = 0 \Leftrightarrow t = 1$ or $t = 1\frac{1}{2}$. Therefore, the ball reaches 7.2 meters in 1 second (ascending) and again after $1\frac{1}{2}$ seconds (descending).
 (b) Setting $h = 14.4$, we have $14.4 = -4.8t^2 + 12t \Leftrightarrow 4.8t^2 - 12t + 14.4 = 2.4(2t^2 - 5t + 6) = 0 \Leftrightarrow t = \frac{5 \pm \sqrt{25 - 48}}{4} = \frac{5 \pm \sqrt{-23}}{4}$. However, since the discriminant $D < 0$, there is no real solution, and hence the ball never reaches a height of 14.4 meters.
 (c) The greatest height h is reached only once. So $h = -4.8t^2 + 12t \Leftrightarrow 4.8t^2 - 12t + h = 0$ has only one solution. Thus $D = (-12)^2 - 4(4.8)(h) = 0 \Leftrightarrow 144 - 19.2h = 0 \Leftrightarrow h = 7.5$. So the greatest height reached by the ball is 7.5 meters.
 (d) Setting $h = 7.5$, we have $7.5 = -4.8t^2 + 12t \Leftrightarrow 4.8t^2 - 12t + 7.5 = 0 \Leftrightarrow t = 1\frac{1}{4}$. Thus the ball reaches the highest point of its path after $1\frac{1}{4}$ seconds.
 (e) Setting $h = 0$ (ground level), we have $0 = -4.8t^2 + 12t \Leftrightarrow 2t^2 - 5t = 0 \Leftrightarrow t(2t - 5) = 0 \Leftrightarrow t = 0$ (start) or $t = 2\frac{1}{2}$. So the ball hits the ground in $2\frac{1}{2}$ s.
132. If the maximum height is 30 meters, then the discriminant of the equation $4.8t^2 - v_0t + 30 = 0$ must equal zero. So $0 = b^2 - 4ac = (-v_0)^2 - 4(4.8)(30) \Leftrightarrow v_0^2 = 576 \Rightarrow v_0 = \pm 24$. Since $v_0 = -24$ does not make sense, we must have $v_0 = 24$ m/s.
133. (a) The shrinkage factor when $w = 250$ is $S = \frac{0.032(250) - 2.5}{10,000} = \frac{8 - 2.5}{10,000} = 0.00055$. So the beam shrinks $0.00055 \times 12.025 \approx 0.007$ m, so when it dries it will be $12.025 - 0.007 = 12.018$ m long.
 (b) Substituting $S = 0.00050$ we get $0.00050 = \frac{0.032w - 2.5}{10,000} \Leftrightarrow 5 = 0.032w - 2.5 \Leftrightarrow 7.5 = 0.032w \Leftrightarrow w = \frac{7.5}{0.032} \approx 234.375$. So the water content should be 234.375 kg/m³.
134. Let d be the distance from the lens to the object. Then the distance from the lens to the image is $d - 4$. So substituting $F = 4.8$, $x = d$, and $y = d - 4$, and then solving for x , we have $\frac{1}{4.8} = \frac{1}{d} + \frac{1}{d - 4}$. Now we multiply by the LCD, $4.8d(d - 4)$, to get $d(d - 4) = 4.8(d - 4) + 4.8d \Leftrightarrow d^2 - 4d = 9.6d - 19.2 \Leftrightarrow 0 = d^2 - 13.6d + 19.2 \Rightarrow d = \frac{13.6 \pm 10.4}{2}$. So $d = 1.6$ or $d = 12$. Since $d - 4$ must also be positive, the object is 12 cm from the lens.

135. (a) $C = \frac{1}{2} (1000) (2^2) + 1000 (9.81) (1) + 200,000 = 211,810$

(b) $\frac{1}{2} \rho v^2 + \rho gh + P = C \Leftrightarrow P = C - \frac{1}{2} \rho v^2 - \rho gh.$

If the height increases to 5 m, then $P = 211,810 - \frac{1}{2} (1000) (2^2) - 1000 (9.81) (5) = 160,760$ Pa.

If $h = 1$ m and $v = 4$ m/s, then $P = 211,810 - \frac{1}{2} (1000) (4^2) - 1000 (9.81) (1) = 194,000$ Pa.

136. Setting $P = 1250$ and solving for x , we have $1250 = \frac{1}{10}x(300 - x) = 30x - \frac{1}{10}x^2 \Leftrightarrow \frac{1}{10}x^2 - 30x + 1250 = 0$. Using the

quadratic formula, $x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4\left(\frac{1}{10}\right)(1250)}}{2\left(\frac{1}{10}\right)} = \frac{30 \pm \sqrt{900 - 500}}{0.2} = \frac{30 \pm 20}{0.2}$. Thus $x = \frac{30 - 20}{0.2} = 50$

or $x = \frac{30 + 20}{0.2} = 250$. Since he must have $0 \leq x \leq 200$, he should make 50 ovens per week.

137. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

Let x be the distance from the center of the earth to the dead spot (in thousands of kilometers). Now setting $F = 0$, we have

$$0 = -\frac{K}{x^2} + \frac{0.012K}{(382 - x)^2} \Leftrightarrow \frac{K}{x^2} = \frac{0.012K}{(382 - x)^2} \Leftrightarrow K(382 - x)^2 = 0.012Kx^2 \Leftrightarrow 145,924 - 764x + x^2 = 0.012x^2 \Leftrightarrow$$

$0.988x^2 - 764x + 145,924 = 0$. Using the Quadratic Formula, we obtain

$$x = \frac{-(-764) \pm \sqrt{(-764)^2 - 4(0.988)(145,924)}}{2(0.988)} = \frac{764 \pm \sqrt{583,696 - 576,691.648}}{1.976}$$

$$= \frac{764 \pm \sqrt{7004.352}}{1.976} \approx \frac{764 \pm 83.692}{1.976} \approx 386.64 \pm 42.35$$

So either $x \approx 386.64 + 42.35 \approx 429$ or $x \approx 386.64 - 42.35 \approx 344$. Since 429,000 is greater than the distance from the earth to the moon, we reject the first root; thus $x \approx 344,000$ kilometers.

138. Since the total time is 3 s, we have $3 = \frac{\sqrt{d}}{2.21} + \frac{d}{332}$. Letting $w = \sqrt{d}$, we have $3 = \frac{1}{2.21}w + \frac{1}{332}w^2 \Leftrightarrow$

$$\frac{1}{332}w^2 + \frac{1}{2.21}w - 3 = 0 \Leftrightarrow w^2 + 150.23w - 996 = 0 \Rightarrow w = \frac{-150.23 \pm 162.95}{2}$$

Since $w \geq 0$, we have $\sqrt{d} = w \approx 6.36$, so $d = 40.46$. The well is 40.5 m deep.

139. (a) $3(0) + k - 5 = k(0) - k + 1 \Leftrightarrow k - 5 = -k + 1 \Leftrightarrow 2k = 6 \Leftrightarrow k = 3$

(b) $3(1) + k - 5 = k(1) - k + 1 \Leftrightarrow 3 + k - 5 = k - k + 1 \Leftrightarrow k - 2 = 1 \Leftrightarrow k = 3$

(c) $3(2) + k - 5 = k(2) - k + 1 \Leftrightarrow 6 + k - 5 = 2k - k + 1 \Leftrightarrow k + 1 = k + 1$. Since both sides of this equation are equal, $x = 2$ is a solution for every value of k . That is, $x = 2$ is a solution to every member of this family of equations.

140. When we multiplied by x , we introduced $x = 0$ as a solution. When we divided by $x - 1$, we are really dividing by 0, since $x = 1 \Leftrightarrow x - 1 = 0$.

141. (a) $x^2 - 9x + 20 = 0 \Leftrightarrow (x - 4)(x - 5) = 0 \Leftrightarrow x = 4$ or $x = 5$. The product of the solutions is $4 \cdot 5 = 20$, the constant term in the original equation; and their sum is $4 + 5 = 9$, the negative of the coefficient of x in the original equation.

(b) In general, the equation $x^2 + bx + c = 0$ has solutions $r_1 = \frac{-b + \sqrt{b^2 - 4c}}{2}$ and $r_2 = \frac{-b - \sqrt{b^2 - 4c}}{2}$.

Thus, $r_1 r_2 = \frac{-b + \sqrt{b^2 - 4c}}{2} \cdot \frac{-b - \sqrt{b^2 - 4c}}{2} = \frac{1}{4} [(-b)^2 - (\sqrt{b^2 - 4c})^2] = \frac{1}{4} (b^2 - b^2 + 4c) = c$ and

$$r_1 + r_2 = \frac{-b + \sqrt{b^2 - 4c}}{2} + \frac{-b - \sqrt{b^2 - 4c}}{2} = \frac{-2b}{2} = -b.$$

142. (a) We make the substitution $x = u - \frac{b}{2}$: $x^2 + bx + c = 0 \Leftrightarrow \left(u - \frac{b}{2}\right)^2 + b\left(u - \frac{b}{2}\right) + c = 0 \Leftrightarrow$
 $u^2 - bu + \frac{b^2}{4} + bu - \frac{b^2}{2} + c = 0 \Leftrightarrow u^2 - \left(\frac{b^2}{4} - c\right) = 0.$
- (b) Here $b = 5$ and $c = -6$, so the substitution $x = u - \frac{5}{2}$ results in the equation $u^2 - \left(\frac{25}{4} + 6\right) = 0 \Leftrightarrow u^2 = \frac{49}{4} \Leftrightarrow$
 $u = \pm \frac{7}{2} \Leftrightarrow x = -\frac{7}{2} - \frac{5}{2} = -6$ or $x = \frac{7}{2} - \frac{5}{2} = 1.$
143. (a) *Method 1:* Let $u = \sqrt{x}$, so $u^2 = x$. Thus $x - \sqrt{x} - 2 = 0$ becomes $u^2 - u - 2 = 0 \Leftrightarrow (u - 2)(u + 1) = 0$. So $u = 2$ or $u = -1$. If $u = 2$, then $\sqrt{x} = 2 \Rightarrow x = 4$. If $u = -1$, then $\sqrt{x} = -1 \Rightarrow x = 1$. So the possible solutions are 4 and 1. Checking $x = 4$ we have $4 - \sqrt{4} - 2 = 4 - 2 - 2 = 0$. Checking $x = 1$ we have $1 - \sqrt{1} - 2 = 1 - 1 - 2 \neq 0$. The only solution is 4.
- Method 2:* $x - \sqrt{x} - 2 = 0 \Leftrightarrow x - 2 = \sqrt{x} \Rightarrow x^2 - 4x + 4 = x \Leftrightarrow x^2 - 5x + 4 = 0 \Leftrightarrow (x - 4)(x - 1) = 0$. So the possible solutions are 4 and 1. Checking will result in the same solution.
- (b) *Method 1:* Let $u = \frac{1}{x-3}$, so $u^2 = \frac{1}{(x-3)^2}$. Thus $\frac{12}{(x-3)^2} + \frac{10}{x-3} + 1 = 0$ becomes $12u^2 + 10u + 1 = 0$. Using the quadratic formula, we have $u = \frac{-10 \pm \sqrt{10^2 - 4(12)(1)}}{2(12)} = \frac{-10 \pm \sqrt{52}}{24} = \frac{-10 \pm 2\sqrt{13}}{24} = \frac{-5 \pm \sqrt{13}}{12}$. If $u = \frac{-5 - \sqrt{13}}{12}$, then $\frac{1}{x-3} = \frac{-5 - \sqrt{13}}{12} \Leftrightarrow x - 3 = \frac{12}{-5 - \sqrt{13}} \cdot \frac{-5 + \sqrt{13}}{-5 + \sqrt{13}} = \frac{12(-5 + \sqrt{13})}{-25 + 13} = -5 + \sqrt{13}$. So $x = -2 + \sqrt{13}$.
- If $u = \frac{-5 + \sqrt{13}}{12}$, then $\frac{1}{x-3} = \frac{-5 + \sqrt{13}}{12} \Leftrightarrow x - 3 = \frac{12}{-5 + \sqrt{13}} \cdot \frac{-5 - \sqrt{13}}{-5 - \sqrt{13}} = \frac{12(-5 - \sqrt{13})}{-25 + 13} = -5 - \sqrt{13}$. So $x = -2 - \sqrt{13}$.
- The solutions are $-2 \pm \sqrt{13}$.
- Method 2:* Multiplying by the LCD, $(x-3)^2$, we get $(x-3)^2 \left(\frac{12}{(x-3)^2} + \frac{10}{x-3} + 1\right) = 0 \cdot (x-3)^2 \Leftrightarrow$
 $12 + 10(x-3) + (x-3)^2 = 0 \Leftrightarrow 12 + 10x - 30 + x^2 - 6x + 9 = 0 \Leftrightarrow x^2 + 4x - 9 = 0$. Using the Quadratic Formula, we have $u = \frac{-4 \pm \sqrt{4^2 - 4(1)(-9)}}{2} = \frac{-4 \pm \sqrt{52}}{2} = \frac{-4 \pm 2\sqrt{13}}{2} = -2 \pm \sqrt{13}$. The solutions are $-2 \pm \sqrt{13}$.

1.6 COMPLEX NUMBERS

- The imaginary number i has the property that $i^2 = -1$.
- For the complex number $3 + 4i$ the real part is 3 and the imaginary part is 4.
- (a) The complex conjugate of $3 + 4i$ is $\overline{3 + 4i} = 3 - 4i$.
 (b) $(3 + 4i)(\overline{3 + 4i}) = 3^2 + 4^2 = 25$
- If $3 + 4i$ is a solution of a quadratic equation with real coefficients, then $\overline{3 + 4i} = 3 - 4i$ is also a solution of the equation.
- Yes, every real number a is a complex number of the form $a + 0i$.
- Yes. For any complex number z , $z + \bar{z} = (a + bi) + \overline{(a + bi)} = a + bi + a - bi = 2a$, which is a real number.
- $3 - 8i$: real part 3, imaginary part -8 .
 $8 - (5 - i) = -5 + i$: real part -5 , imaginary part 1.
- $\frac{-2 - 5i}{3} = -\frac{2}{3} - \frac{5}{3}i$: real part $-\frac{2}{3}$, imaginary part $-\frac{5}{3}$.
 $\frac{4 + 7i}{2} = 2 + \frac{7}{2}i$: real part 2, imaginary part $\frac{7}{2}$.
- 3: real part 3, imaginary part 0.
 $- \frac{1}{2}$: real part $-\frac{1}{2}$, imaginary part 0.
- $-\frac{2}{3}i$: real part 0, imaginary part $-\frac{2}{3}$.
 $\sqrt{3}i$: real part 0, imaginary part $\sqrt{3}$.

15. $\sqrt{3} + \sqrt{-4} = \sqrt{3} + 2i$: real part $\sqrt{3}$, imaginary part 2. 16. $2 - \sqrt{-5} = 2 - i\sqrt{5}$: real part 2, imaginary part $-\sqrt{5}$.
17. $(3 + 2i) + 5i = 3 + (2 + 5)i = 3 + 7i$ 18. $3i - (2 - 3i) = -2 + [3 - (-3)]i = -2 + 6i$
19. $(5 - 3i) + (-4 - 7i) = (5 - 4) + (-3 - 7)i = 1 - 10i$ 20. $(-3 + 4i) - (2 - 5i) = (-3 - 2) + [4 - (-5)]i = -5 + 9i$
21. $(-6 + 6i) + (9 - i) = (-6 + 9) + (6 - 1)i = 3 + 5i$ 22. $(3 - 2i) + \left(-5 - \frac{1}{3}i\right) = (3 - 5) + \left(-2 - \frac{1}{3}\right)i = -2 - \frac{7}{3}i$
23. $\left(7 - \frac{1}{2}i\right) - \left(5 + \frac{3}{2}i\right) = (7 - 5) + \left(-\frac{1}{2} - \frac{3}{2}\right)i = 2 - 2i$
24. $(-4 + i) - (2 - 5i) = -4 + i - 2 + 5i = (-4 - 2) + (1 + 5)i = -6 + 6i$
25. $(-12 + 8i) - (7 + 4i) = -12 + 8i - 7 - 4i = (-12 - 7) + (8 - 4)i = -19 + 4i$
26. $6i - (4 - i) = 6i - 4 + i = (-4) + (6 + 1)i = -4 + 7i$
27. $4(-1 + 2i) = -4 + 8i$ 28. $-2(3 - 4i) = -6 + 8i$
29. $(3 - 4i)(2 + 5i) = 6 + 15i - 8i - 20i^2 = 6 + 7i - 20(-1) = 26 + 7i$
30. $(-5 + i)(6 - 2i) = -30 + 10i + 6i - 2i^2 = -30 + 16i - 2(-1) = -28 + 16i$
31. $(6 + 5i)(2 - 3i) = 12 - 18i + 10i - 15i^2 = (12 + 15) + (-18 + 10)i = 27 - 8i$
32. $(-2 + i)(3 - 7i) = -6 + 14i + 3i - 7i^2 = (-6 + 7) + (14 + 3)i = 1 + 17i$
33. $(3 - 2i)(3 + 2i) = 3^2 - (2i)^2 = 13$
34. $(10 + i)(10 - i) = 10^2 - i^2 = 101$
35. $(3 - 2i)^2 = 9 - 12i + 4(-1) = 5 - 12i$
36. $(10 + i)^2 = 100 + 20i - 1 = 99 + 20i$
37. $\frac{1}{i} = \frac{1}{i} \cdot \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$
38. $\frac{1}{1+i} = \frac{1}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i}{1-i^2} = \frac{1-i}{1+1} = \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i$
39. $\frac{1-3i}{1+2i} = \frac{(1-3i)(1-2i)}{(1+2i)(1-2i)} = \frac{-5+5i}{5} = -1+i$
40. $\frac{2-i}{1-3i} = \frac{(2-i)(1+3i)}{(1-3i)(1+3i)} = \frac{5+5i}{10} = \frac{1}{2} + \frac{1}{2}i$
41. $\frac{10i}{1-2i} = \frac{10i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{10i+20i^2}{1-4i^2} = \frac{-20+10i}{1+4} = \frac{5(-4+2i)}{5} = -4+2i$
42. $(2-3i)^{-1} = \frac{1}{2-3i} = \frac{1}{2-3i} \cdot \frac{2+3i}{2+3i} = \frac{2+3i}{4-9i^2} = \frac{2+3i}{4+9} = \frac{2+3i}{13} = \frac{2}{13} + \frac{3}{13}i$
43. $\frac{4+6i}{3i} = \frac{4+6i}{3i} \cdot \frac{3i}{3i} = \frac{12i+18i^2}{9i^2} = \frac{-18+12i}{-9} = \frac{-18}{-9} + \frac{12}{-9}i = 2 - \frac{4}{3}i$
44. $\frac{-3+5i}{15i} = \frac{-3+5i}{15i} \cdot \frac{15i}{15i} = \frac{-45i+75i^2}{225i^2} = \frac{-75-45i}{-225} = \frac{-75}{-225} + \frac{-45}{-225}i = \frac{1}{3} + \frac{1}{5}i$
45. $\frac{1}{1+i} - \frac{1}{1-i} = \frac{1}{1+i} \cdot \frac{1-i}{1-i} - \frac{1}{1-i} \cdot \frac{1+i}{1+i} = \frac{1-i}{1-i^2} - \frac{1+i}{1-i^2} = \frac{1-i}{2} - \frac{1+i}{2} = \frac{1-i-1-i}{2} = -i$
46. $\frac{(1+2i)(3-i)}{2+i} = \frac{3-i+6i-2i^2}{2+i} = \frac{5+5i}{2+i} \cdot \frac{2-i}{2-i} = \frac{10-5i+10i-5i^2}{4-i^2} = \frac{(10+5)+(-5+10)i}{5}$
 $= \frac{15+5i}{5} = \frac{15}{5} + \frac{5}{5}i = 3+i$
47. $i^3 = i^2i = -i$ 48. $i^{10} = (i^2)^5 = (-1)^5 = -1$
49. $(3i)^5 = 3^5(i^2)^2i = 243(-1)^2i = 243i$ 50. $(2i)^4 = 2^4i^4 = 16(1) = 16$

51. $i^{1000} = (i^4)^{250} = 1^{250} = 1$

52. $i^{1002} = (i^4)^{250} i^2 = 1i^2 = -1$

53. $\sqrt{-25} = \sqrt{25}\sqrt{-1} = 5i$

54. $\frac{\sqrt{-8}}{\sqrt{2}} = \sqrt{4}\sqrt{-1} = 2i$

55. $\sqrt{-4}\sqrt{-9} = (2i)(3i) = -6$

56. $\sqrt{\frac{1}{2}}\sqrt{-32} = \sqrt{16}\sqrt{-1} = 4i$

57. $(2 + \sqrt{-1})(3 - \sqrt{-3}) = 6 - 2\sqrt{3}i + 3i - \sqrt{3} = 6 + \sqrt{3} + (3 - 2\sqrt{3})i$

58. $(\sqrt{3} - \sqrt{-4})(\sqrt{6} - \sqrt{-8}) = (\sqrt{3} - 2i)(\sqrt{6} - 2\sqrt{2}i) = \sqrt{18} - 2\sqrt{6}i - 2\sqrt{6}i + 4\sqrt{2}i^2$
 $= (3\sqrt{2} - 4\sqrt{2}) + (-2\sqrt{6} - 2\sqrt{6})i = -\sqrt{2} - 4\sqrt{6}i$

59. $\frac{2 + \sqrt{-8}}{1 + \sqrt{-2}} = \frac{2 + 2\sqrt{2}i}{1 + \sqrt{2}i} = \frac{2(1 + \sqrt{2}i)}{1 + \sqrt{2}i} = 2$

60. $\frac{\sqrt{-36}}{\sqrt{-2}\sqrt{-9}} = \frac{6i}{\sqrt{2}i \cdot 3i} = \frac{2}{\sqrt{2}i} \cdot \frac{\sqrt{2}i}{\sqrt{2}i} = \frac{2\sqrt{2}i}{2i^2} = \frac{\sqrt{2}i}{-1} = -\sqrt{2}i$

61. $x^2 + 25 = 0 \Leftrightarrow x^2 = -25 \Leftrightarrow x = \pm 5i$

62. $2x^2 + 5 = 0 \Leftrightarrow x^2 = -\frac{5}{2} \Leftrightarrow x = \pm \frac{\sqrt{10}}{2}i$

63. $x^2 - 6x + 13 = 0 \Leftrightarrow x = \frac{6 \pm \sqrt{36 - 4(13)}}{2} = 3 \pm \frac{1}{2}\sqrt{-16} = 3 \pm 2i$

Or: $x^2 - 6x + 13 = 0 \Leftrightarrow x^2 - 6x + 9 = -13 + 9 \Leftrightarrow (x - 3)^2 = -4 \Leftrightarrow x - 3 = \pm 2i \Leftrightarrow x = 3 \pm 2i$

64. $x^2 + 2x + 2 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)} = \frac{-2 \pm \sqrt{4 - 8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$

65. $2x^2 - 2x + 5 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4 - 4(2)(5)}}{2(2)} = \frac{2 \pm \sqrt{-36}}{4} = \frac{1}{2} \pm \frac{3}{2}i$

66. $8x^2 - 12x + 5 = 0 \Leftrightarrow x = \frac{12 \pm \sqrt{(-12)^2 - 4(8)(5)}}{2(8)} = \frac{12 \pm \sqrt{-16}}{16} = \frac{3}{4} \pm \frac{1}{4}i$

67. $x^2 + x + 1 = 0 \Rightarrow x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{1 - 4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

68. $x^2 - 3x + 3 = 0 \Rightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(3)}}{2(1)} = \frac{3 \pm \sqrt{9 - 12}}{2} = \frac{3 \pm \sqrt{-3}}{2} = \frac{3 \pm \sqrt{3}i}{2} = \frac{3}{2} \pm \frac{\sqrt{3}}{2}i$

69. $9x^2 - 4x + 4 = 0 \Leftrightarrow x = \frac{4 \pm \sqrt{(-4)^2 - 4(9)(4)}}{2(9)} = \frac{4 \pm \sqrt{-128}}{18} = \frac{2}{9} \pm \frac{4\sqrt{2}}{9}i$

70. $t + 2 + \frac{6}{t} = 0 \Leftrightarrow t^2 + 2t + 6 = 0 \Leftrightarrow t = \frac{-2 \pm \sqrt{4 - 4(1)(6)}}{2} = \frac{-2 \pm \sqrt{-20}}{2} = -1 \pm \sqrt{5}i$

71. $6x^2 + 12x + 7 = 0 \Rightarrow$

$$x = \frac{-(12) \pm \sqrt{(12)^2 - 4(6)(7)}}{2(6)} = \frac{-12 \pm \sqrt{144 - 168}}{12} = \frac{-12 \pm \sqrt{-24}}{12} = \frac{-12 \pm 2\sqrt{6}i}{12} = \frac{-12}{12} \pm \frac{2\sqrt{6}i}{12} = -1 \pm \frac{\sqrt{6}}{6}i$$

72. $x^2 + \frac{1}{2}x + 1 = 0 \Rightarrow$

$$x = \frac{-\left(\frac{1}{2}\right) \pm \sqrt{\left(\frac{1}{2}\right)^2 - 4(1)(1)}}{2(1)} = \frac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4}}{2} = \frac{-\frac{1}{2} \pm \sqrt{-\frac{15}{4}}}{2} = \frac{-\frac{1}{2} \pm \frac{1}{2}\sqrt{15}i}{2} = -\frac{1}{4} \pm \frac{\sqrt{15}}{4}i$$

73. $\bar{z} + \bar{w} = \overline{3 - 4i} + \overline{5 + 2i} = 3 + 4i + 5 - 2i = 8 + 2i$

74. $\overline{z + w} = \overline{3 - 4i + 5 + 2i} = \overline{8 - 2i} = 8 + 2i$

75. $z \cdot \bar{z} = (3 - 4i)(3 + 4i) = 3^2 + 4^2 = 25$

76. $\bar{z} \cdot \bar{w} = (3 + 4i)(5 - 2i) = 15 - 6i + 20i - 8i^2 = 23 + 14i$
77. LHS $= \bar{z} + \bar{w} = \overline{(a + bi)} + \overline{(c + di)} = a - bi + c - di = (a + c) + (-b - d)i = (a + c) - (b + d)i$.
 RHS $= \overline{z + w} = \overline{(a + bi) + (c + di)} = \overline{(a + c) + (b + d)i} = (a + c) - (b + d)i$.
 Since LHS = RHS, this proves the statement.
78. LHS $= \overline{zw} = \overline{(a + bi)(c + di)} = \overline{ac + adi + bci + bdi^2} = \overline{(ac - bd) + (ad + bc)i} = (ac - bd) - (ad + bc)i$.
 RHS $= \bar{z} \cdot \bar{w} = \overline{a + bi} \cdot \overline{c + di} = (a - bi)(c - di) = ac - adi - bci + bdi^2 = (ac - bd) - (ad + bc)i$.
 Since LHS = RHS, this proves the statement.
79. LHS $= (\bar{z})^2 = \overline{(a + bi)^2} = \overline{a^2 - 2abi + b^2i^2} = \overline{(a^2 - b^2) - 2abi} = (a^2 - b^2) + 2abi$.
 RHS $= \overline{z^2} = \overline{(a + bi)^2} = \overline{a^2 + 2abi + b^2i^2} = \overline{(a^2 - b^2) + 2abi} = (a^2 - b^2) - 2abi$.
 Since LHS = RHS, this proves the statement.
80. $\bar{\bar{z}} = \overline{a + bi} = \overline{a - bi} = a + bi = z$.
81. $z + \bar{z} = (a + bi) + \overline{(a + bi)} = a + bi + a - bi = 2a$, which is a real number.
82. $z - \bar{z} = (a + bi) - \overline{(a + bi)} = a + bi - (a - bi) = a + bi - a + bi = 2bi$, which is a pure imaginary number.
83. $z \cdot \bar{z} = (a + bi) \cdot \overline{(a + bi)} = (a + bi) \cdot (a - bi) = a^2 - b^2i^2 = a^2 + b^2$, which is a real number.
84. Suppose $z = \bar{z}$. Then we have $(a + bi) = \overline{(a + bi)} \Rightarrow a + bi = a - bi \Rightarrow 0 = -2bi \Rightarrow b = 0$, so z is real. Now if z is real, then $z = a + 0i$ (where a is real). Since $\bar{z} = a - 0i$, we have $z = \bar{z}$.
85. Using the Quadratic Formula, the solutions to the equation are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Since both solutions are nonreal, we have $b^2 - 4ac < 0 \Leftrightarrow 4ac - b^2 > 0$, so the solutions are $x = \frac{-b}{2a} \pm \frac{\sqrt{4ac - b^2}}{2a}i$, where $\sqrt{4ac - b^2}$ is a real number. Thus the solutions are complex conjugates of each other.
86. $i = i, i^5 = i^4 \cdot i = i, i^9 = i^8 \cdot i = i; i^2 = -1, i^6 = i^4 \cdot i^2 = -1, i^{10} = i^8 \cdot i^2 = -1; i^3 = -i, i^7 = i^4 \cdot i^3 = -i, i^{11} = i^8 \cdot i^3 = -i; i^4 = 1, i^8 = i^4 \cdot i^4 = 1, i^{12} = i^8 \cdot i^4 = 1$.
 Because $i^4 = 1$, we have $i^n = i^r$, where r is the remainder when n is divided by 4, that is, $n = 4 \cdot k + r$, where k is an integer and $0 \leq r < 4$. Since $4446 = 4 \cdot 1111 + 2$, we must have $i^{4446} = i^2 = -1$.

1.7 MODELING WITH EQUATIONS

- An equation modeling a real-world situation can be used to help us understand a real-world problem using mathematical methods. We translate real-world ideas into the language of algebra to construct our model, and translate our mathematical results back into real-world ideas in order to interpret our findings.
- In the formula $I = Prt$ for simple interest, P stands for *principal*, r for *interest rate*, and t for *time (in years)*.
- (a) A square of side x has area $A = x^2$.
 (b) A rectangle of length l and width w has area $A = lw$.
 (c) A circle of radius r has area $A = \pi r^2$.
- Balsamic vinegar contains 5% acetic acid, so a 960 mL bottle of balsamic vinegar contains $960 \cdot 5\% = 960 \cdot \frac{5}{100} = 48$ milliliters of acetic acid.
- A painter paints a wall in x hours, so the fraction of the wall she paints in one hour is $\frac{1 \text{ wall}}{x \text{ hours}} = \frac{1}{x}$.
- Solving $d = rt$ for r , we find $\frac{d}{t} = \frac{rt}{t} \Rightarrow r = \frac{d}{t}$. Solving $d = rt$ for t , we find $\frac{d}{r} = \frac{rt}{r} \Rightarrow t = \frac{d}{r}$.
- If n is the first integer, then $n + 1$ is the middle integer, and $n + 2$ is the third integer. So the sum of the three consecutive integers is $n + (n + 1) + (n + 2) = 3n + 3$.

8. If n is the middle integer, then $n - 1$ is the first integer, and $n + 1$ is the third integer. So the sum of the three consecutive integers is $(n - 1) + n + (n + 1) = 3n$.
9. If n is the first even integer, then $n + 2$ is the second even integer and $n + 4$ is the third. So the sum of three consecutive even integers is $n + (n + 2) + (n + 4) = 3n + 6$.
10. If n is the first integer, then the next integer is $n + 1$. The sum of their squares is

$$n^2 + (n + 1)^2 = n^2 + (n^2 + 2n + 1) = 2n^2 + 2n + 1.$$
11. If s is the third test score, then since the other test scores are 78 and 82, the average of the three test scores is

$$\frac{78 + 82 + s}{3} = \frac{160 + s}{3}.$$
12. If q is the fourth quiz score, then since the other quiz scores are 8, 8, and 8, the average of the four quiz scores is

$$\frac{8 + 8 + 8 + q}{4} = \frac{24 + q}{4}.$$
13. If x dollars are invested at $2\frac{1}{2}\%$ simple interest, then the first year you will receive $0.025x$ dollars in interest.
14. If n is the number of months the apartment is rented, and each month the rent is \$945, then the total rent paid is $945n$.
15. Since w is the width of the rectangle, the length is four times the width, or $4w$. Then

$$\text{area} = \text{length} \times \text{width} = 4w \times w = 4w^2 \text{ m}^2.$$
16. Since w is the width of the rectangle, the length is $w + 6$. The perimeter is

$$2 \times \text{length} + 2 \times \text{width} = 2(w + 6) + 2(w) = 4w + 12$$
17. If d is the given distance, in kilometers, and $\text{distance} = \text{rate} \times \text{time}$, we have $\text{time} = \frac{\text{distance}}{\text{rate}} = \frac{d}{55}$.
18. Since $\text{distance} = \text{rate} \times \text{time}$ we have $\text{distance} = s \times (45 \text{ min}) \frac{1 \text{ h}}{60 \text{ min}} = \frac{3}{4}s \text{ km}$.
19. If x is the quantity of pure water added, the mixture will contain 750 g of salt and $3 + x$ liters of water. Thus the concentration is $\frac{750}{3 + x}$.
20. If p is the number of pennies in the purse, then the number of nickels is $2p$, the number of dimes is $4 + 2p$, and the number of quarters is $(2p) + (4 + 2p) = 4p + 4$. Thus the value (in cents) of the change in the purse is

$$1 \cdot p + 5 \cdot 2p + 10 \cdot (4 + 2p) + 25 \cdot (4p + 4) = p + 10p + 40 + 20p + 100p + 100 = 131p + 140.$$
21. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**
 If d is the number of days and m the number of kilometers, then the cost of a rental is $C = 65d + 0.40m$. In this case, $d = 3$ and $C = 283$, so we solve for m : $283 = 65 \cdot 3 + 0.40m \Leftrightarrow 283 = 195 + 0.40m \Leftrightarrow 0.40m = 88 \Leftrightarrow m = \frac{88}{0.4} = 220$. Thus, the truck was driven 220 kilometers.
22. The cost of speaking for m minutes on this plan is $C = \begin{cases} 100 & \text{if } m \leq 250 \\ 100 + 0.25(m - 250) & \text{if } m > 250 \end{cases}$ In this case $m > 250$
 and $C = 120.50$, so we solve $100 + 0.25(m - 250) = 120.5 \Leftrightarrow 0.25(m - 250) = 20.5 \Leftrightarrow m - 250 = 4(20.5) = 82 \Leftrightarrow m = 250 + 82 = 332$. Therefore, the tourist used 332 minutes of talk in the month of June.
23. If x is the student's score on their final exam, then because the final counts twice as much as each midterm, the average score is $\frac{82 + 75 + 71 + 2x}{3(100) + 200} = \frac{228 + 2x}{500} = \frac{114 + x}{250}$. For the average to be 80%, we must have $\frac{114 + x}{250} = 80\% = 0.8 \Leftrightarrow 114 + x = 250(0.8) = 200 \Leftrightarrow x = 86$. So the student scored 86% on their final exam.
24. Six students scored 100 and three students scored 60. Let x be the average score of the remaining $25 - 6 - 3 = 16$ students. Because the overall average is $84\% = 0.84$, we have $\frac{6(100) + 3(60) + 16x}{25(100)} = 0.84 \Leftrightarrow 780 + 16x = 0.84(2500) = 2100 \Leftrightarrow 16x = 1320 \Leftrightarrow x = \frac{1320}{16} = 82.5$. Thus, the remaining 16 students' average score was 82.5%.

25. Let m be the amount invested at $2\frac{1}{2}\%$. Then $12,000 - m$ is the amount invested at 3% .
 Since the total interest is equal to the interest earned at 2.5% plus the interest earned at 3% , we have

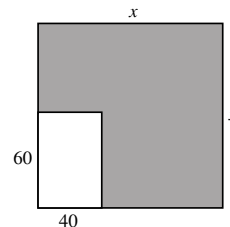
$$318 = 0.025m + 0.03(12,000 - m) \Leftrightarrow 318 = 0.025m + 360 - 0.03m \Leftrightarrow 0.005m = 42 \Leftrightarrow m = \frac{42}{0.005} = 8400.$$
 Thus, \$8400 is invested at $2\frac{1}{2}\%$ and $12,000 - 8400 = \$3600$ is invested at 3% .
26. Let m be the amount invested at 5% . Then $8000 + m$ is the total amount invested. Thus

$$4\% \text{ of the total investment} = \text{interest earned at } 3\frac{1}{2}\% + \text{interest earned at } 5\%$$
 So $0.04(8000 + m) = 0.035(8000) + 0.05m \Leftrightarrow 320 + 0.04m = 280 + 0.05m \Leftrightarrow m = \frac{40}{0.01} = 4000$. Thus, \$4000 must be invested at 5% .
27. Using the formula $I = Prt$ and solving for r , we get $262.50 = 3500 \cdot r \cdot 1 \Leftrightarrow r = \frac{262.5}{3500} = 0.075$ or 7.5% .
28. If \$3000 is invested at an interest rate $a\%$, then \$5000 is invested at $(a + \frac{1}{2})\%$, so, remembering that a is expressed as a percentage, the total interest is $I = 3000 \cdot \frac{a}{100} \cdot 1 + 5000 \cdot \frac{a + \frac{1}{2}}{100} \cdot 1 = 30a + 50a + 25 = 80a + 25$. Since the total interest is \$265, we have $265 = 80a + 25 \Leftrightarrow 80a = 240 \Leftrightarrow a = 3$. Thus, \$3000 is invested at 3% interest.
29. Let x be the monthly salary. Since annual salary = $12 \times (\text{monthly salary}) + (\text{Christmas bonus})$, we have
 $180,100 = 12x + 7300 \Leftrightarrow 172,800 = 12x \Leftrightarrow x \approx 14,400$. The monthly salary is \$14,400.
30. Let s be the assistant's annual salary. Then the foreman's annual salary is $1.15s$. Their total income is the sum of their salaries, so $s + 1.15s = 113,305 \Leftrightarrow 2.15s = 113,305 \Leftrightarrow s = 52,700$. Thus, the assistant's annual salary is \$52,700.
31. Let x be the overtime hours worked. Since gross pay = regular salary + overtime pay, we obtain the equation
 $814 = 18.50 \cdot 35 + 18.50 \cdot 1.5 \cdot x \Leftrightarrow 814 = 647.50 + 27.75x \Leftrightarrow 166.50 = 27.75x \Leftrightarrow x = \frac{166.50}{27.75} = 6$. Thus, the lab technician worked 6 hours of overtime.
32. Let x be the number of hours worked by the plumber. Then the cabinet maker works for $9x$ hours. The total labor charge is the sum of their charges, so $2610 = 150x + 80(9x) \Leftrightarrow 2610 = 150x + 720x \Leftrightarrow x = \frac{2610}{870} = 3$. Thus, the plumber works for 3 hours and the cabinet maker works for $3 \cdot 9 = 27$ hours.
33. All ages are in terms of the daughter's age 7 years ago. Let y be age of the daughter 7 years ago. Then $11y$ is the age of the movie star 7 years ago. Today, the daughter is $y + 7$, and the movie star is $11y + 7$. But the movie star is also 4 times his daughter's age today. So $4(y + 7) = 11y + 7 \Leftrightarrow 4y + 28 = 11y + 7 \Leftrightarrow 21 = 7y \Leftrightarrow y = 3$. Thus, today the movie star is $11(3) + 7 = 40$ years old.
34. Let h be number of home runs Babe Ruth hit. Then $h + 41$ is the number of home runs that Hank Aaron hit. So
 $1469 = h + h + 41 \Leftrightarrow 1428 = 2h \Leftrightarrow h = 714$. Thus Babe Ruth hit 714 home runs.
35. Let n be the number of nickels. Then there are also n dimes and n quarters. The total value of the coins in the purse is the sum of the values of nickels, dimes, and quarters, so $2.80 = 0.05n + 0.10n + 0.25n \Leftrightarrow 2.8 = 0.4p \Leftrightarrow p = \frac{2.8}{0.4} = 7$. So the purse contains 7 nickels, 7 dimes, and 7 quarters.
36. Let q be the number of quarters. Then $2q$ is the number of dimes, and $2q + 5$ is the number of nickels. Thus $3.00 = \text{value of nickels} + \text{value of dimes} + \text{value of quarters}$, so
 $3.00 = 0.05(2q + 5) + 0.10(2q) + 0.25q \Leftrightarrow 3.00 = 0.10q + 0.25 + 0.20q + 0.25q \Leftrightarrow 2.75 = 0.55q \Leftrightarrow q = \frac{2.75}{0.55} = 5$.
 Thus you have 5 quarters, $2(5) = 10$ dimes, and $2(5) + 5 = 15$ nickels.
37. Let l be the length of the garden. Since area = width $\cdot$ length, we obtain the equation $1125 = 25l \Leftrightarrow l = \frac{1125}{25} = 45$ m. So the garden is 45 meters long.
38. Let w be the width of the pasture. Then the length of the pasture is $2w$. Since area = length $\times$ width, we have
 $115,200 = w(2w) = 2w^2 \Leftrightarrow w^2 = \frac{115,200}{2} = 57,600 \Rightarrow w = \pm 240$. Thus the width of the pasture is 240 meters.

39. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

Let w be the width of the building lot. Then the length of the building lot is $5w$. Since a half-hectare is $\frac{1}{2} \cdot 10,000 = 5000$ and area is length times width, we have $5000 = w(5w) = 5w^2 \Leftrightarrow w^2 = 1000 \Rightarrow w = \pm 31.6$. Thus the width of the building lot is 31.6 meters and the length of the building lot is $5(31.6) = 158$ meters.

40. Let x be the length of a side of the square plot. As shown in the figure, area of the plot = area of the building + area of the parking lot. Thus,
 $x^2 = 60(40) + 12,000 = 2,400 + 12,000 = 14,400 \Rightarrow x = \pm 120$. So the plot of land measures 120 meters by 120 meters.



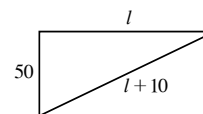
41. Let w be the width of the garden in meters. Then the length is $w + 10$. Thus $875 = w(w + 10) \Leftrightarrow w^2 + 10w - 875 = 0 \Leftrightarrow (w + 35)(w - 25) = 0$. If $w + 35 = 0$, then $w = -35$, which is impossible. Therefore $w - 25 = 0$, and so $w = 25$. The garden is 25 meters wide and 35 meters long.
42. Let w be the width of the bedroom. Then its length is $w + 4$. Since area is length times width, we have $21 = (w + 4)w = w^2 + 4w \Leftrightarrow w^2 + 4w - 21 = 0 \Leftrightarrow (w + 7)(w - 3) = 0 \Leftrightarrow w = -7$ or $w = 3$. Since the width must be positive, the width is 3 meters.
43. Let w be the width of the garden in meters. We use the perimeter to express the length l of the garden in terms of width. Since the perimeter is twice the width plus twice the length, we have $200 = 2w + 2l \Leftrightarrow 2l = 200 - 2w \Leftrightarrow l = 100 - w$. Using the formula for area, we have $2400 = w(100 - w) = 100w - w^2 \Leftrightarrow w^2 - 100w + 2400 = 0 \Leftrightarrow (w - 40)(w - 60) = 0$. So $w - 40 = 0 \Leftrightarrow w = 40$, or $w - 60 = 0 \Leftrightarrow w = 60$. If $w = 40$, then $l = 100 - 40 = 60$. And if $w = 60$, then $l = 100 - 60 = 40$. So the length is 60 meters and the width is 40 meters.
44. Let w be the width of the lot in meters. Then the length is $w + 6$. Using the Pythagorean Theorem, we have $w^2 + (w + 6)^2 = (174)^2 \Leftrightarrow w^2 + w^2 + 12w + 36 = 30276 \Leftrightarrow 2w^2 + 12w - 30240 = 0 \Leftrightarrow w^2 + 6w - 15120 = 0 \Leftrightarrow (w + 126)(w - 120) = 0$. Since widths must be positive, the width is 120 meters and the length is 126 meters.

45. Let l be the length of the lot in meters. Then the length of the diagonal is $l + 10$.

We apply the Pythagorean Theorem with the hypotenuse as the diagonal. So

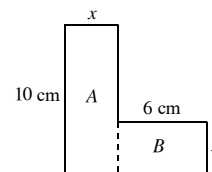
$$l^2 + 50^2 = (l + 10)^2 \Leftrightarrow l^2 + 2500 = l^2 + 20l + 100 \Leftrightarrow 20l = 2400 \Leftrightarrow l = 120.$$

Thus the length of the lot is 120 meters.

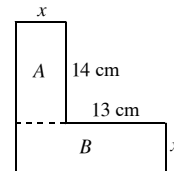


46. Let r be the radius of the running track. The running track consists of two semicircles and two straight sections 110 meters long, so we get the equation $2\pi r + 220 = 440 \Leftrightarrow 2\pi r = 220 \Leftrightarrow r = \frac{110}{\pi} = 35.03$. Thus the radius of the semicircle is about 35 meters.

47. (a) First we write a formula for the area of the figure in terms of x . Region A has dimensions 10 cm and x cm and region B has dimensions 6 cm and x cm. So the shaded region has area $(10 \cdot x) + (6 \cdot x) = 16x \text{ cm}^2$. We are given that this is equal to 144 cm^2 , so $144 = 16x \Leftrightarrow x = \frac{144}{16} = 9 \text{ cm}$.

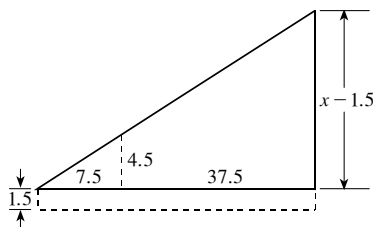


- (b) First we write a formula for the area of the figure in terms of x . Region A has dimensions 14 cm and x cm and region B has dimensions $(13 + x)$ cm and x cm. So the area of the figure is $(14 \cdot x) + [(13 + x)x] = 14x + 13x + x^2 = x^2 + 27x$. We are given that this is equal to 160 cm^2 , so $160 = x^2 + 27x \Leftrightarrow x^2 + 27x - 160 = 0 \Leftrightarrow (x + 32)(x - 5) \Leftrightarrow x = -32$ or $x = 5$. x must be positive, so $x = 5$ cm.



48. (a) The shaded area is the sum of the area of a square and the area of a triangle. So $A = y^2 + \frac{1}{2}(y)(y) = \frac{3}{2}y^2$. We are given that the area is 120 cm^2 , so $120 = \frac{3}{2}y^2 \Leftrightarrow y^2 = 80 \Leftrightarrow y = \pm\sqrt{80} = \pm 4\sqrt{5}$. y is positive, so $y = 4\sqrt{5} \approx 8.94$ cm.
- (b) The shaded area is the sum of the area of a rectangle and the area of a triangle. So $A = y(1) + \frac{1}{2}(y)(y) = \frac{1}{2}y^2 + y$. We are given that the area is 1200 cm^2 , so $1200 = \frac{1}{2}y^2 + y \Leftrightarrow y^2 + 2y - 2400 = 0 \Leftrightarrow (y + 50)(y - 48) = 0$. y is positive, so $y = 48$ cm.
49. Let x be the width of the strip. Then the length of the mat is $50 + 2x$, and the width of the mat is $38 + 2x$. Now the perimeter is twice the length plus twice the width, so $256 = 2(50 + 2x) + 2(38 + 2x) \Leftrightarrow 256 = 100 + 4x + 76 + 4x \Leftrightarrow 256 = 176 + 8x \Leftrightarrow 80 = 8x \Leftrightarrow x = 10$. Thus the strip of mat is 10 centimeters wide.
50. Let x be the width of the strip. Then the width of the poster is $100 + 2x$ and its length is $140 + 2x$. The perimeter of the printed area is $2(100) + 2(140) = 480$, and the perimeter of the poster is $2(100 + 2x) + 2(140 + 2x)$. Now we use the fact that the perimeter of the poster is $1\frac{1}{2}$ times the perimeter of the printed area: $2(100 + 2x) + 2(140 + 2x) = \frac{3}{2} \cdot 480 \Leftrightarrow 480 + 8x = 720 \Leftrightarrow 8x = 240 \Leftrightarrow x = 30$. The blank strip is thus 30 cm wide.
51. Let h be the height the ladder reaches (in meters). Using the Pythagorean Theorem we have $(2.3)^2 + h^2 = (6)^2 \Leftrightarrow h^2 = (6)^2 - (2.3)^2 = 36 - 5.29 \approx 30.71$. So $h = \sqrt{30.71} \approx 5.54$ meters.
52. Let h be the height of the flagpole, in meters. Then the length of each guy wire is $h + 1.5$. Since the distance between the points where the wires are fixed to the ground is equal to one guy wire, the triangle is equilateral, and the flagpole is the perpendicular bisector of the base. Thus from the Pythagorean Theorem, we get $\left[\frac{1}{2}(h + 1.5)\right]^2 + h^2 = (h + 1.5)^2 \Leftrightarrow h^2 + 3h + 2.25 + 4h^2 = 4h^2 + 12h + 9 \Leftrightarrow h^2 - 9h - 6.75 = 0 \Rightarrow h = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(-6.75)}}{2(1)} = \frac{9 \pm \sqrt{81 + 27}}{2} = \frac{9 \pm \sqrt{98}}{2} = \frac{9 \pm 7\sqrt{2}}{2}$. Since $\frac{9 - 7\sqrt{2}}{2} < 0$, we reject it. Thus the height is $h = \frac{9 + 7\sqrt{2}}{2} \approx 9.45$ m.
53. Let x be the length of the person's shadow, in meters. Using similar triangles, $\frac{10 + x}{6} = \frac{x}{2} \Leftrightarrow 20 + 2x = 6x \Leftrightarrow 4x = 20 \Leftrightarrow x = 5$. Thus the person's shadow is 5 meters long.

54. Let x be the height of the tall tree. Here we use the property that corresponding sides in similar triangles are proportional. The base of the similar triangles starts at eye level of the woodcutter, 1.5 meters. Thus we obtain the proportion $\frac{x - 1.5}{4.5} = \frac{45}{7.5} \Leftrightarrow 7.5(x - 1.5) = 4.5(45) \Leftrightarrow 7.5x - 11.25 = 202.5 \Leftrightarrow 7.5x = 213.75 \Leftrightarrow x = 28.5$. Thus the tree is 28.5 meters tall.



55. Let x be the amount (in mL) of 60% acid solution to be used. Then $300 - x$ mL of 30% solution would have to be used to yield a total of 300 mL of solution.

	60% acid	30% acid	Mixture
mL	x	$300 - x$	300
Rate (% acid)	0.60	0.30	0.50
Value	$0.60x$	$0.30(300 - x)$	$0.50(300)$

Thus the total amount of pure acid used is $0.60x + 0.30(300 - x) = 0.50(300) \Leftrightarrow 0.3x + 90 = 150 \Leftrightarrow x = \frac{60}{0.3} = 200$.

So 200 mL of 60% acid solution must be mixed with 100 mL of 30% solution to get 300 mL of 50% acid solution.

56. The amount of pure acid in the original solution is $300(50\%) = 150$. Let x be the number of mL of pure acid added. Then the final volume of solution is $300 + x$. Because its concentration is to be 60%, we must have $\frac{150 + x}{300 + x} = 60\% = 0.6 \Leftrightarrow 150 + x = 0.6(300 + x) \Leftrightarrow 150 + x = 180 + 0.6x \Leftrightarrow 0.4x = 30 \Leftrightarrow x = \frac{30}{0.4} = 75$. Thus, 75 mL of pure acid must be added.

57. Let x be the number of grams of silver added. The weight of the rings is $5 \times 18 \text{ g} = 90 \text{ g}$.

	5 rings	Pure silver	Mixture
Grams	90	x	$90 + x$
Rate (% gold)	0.90	0	0.75
Value	$0.90(90)$	$0x$	$0.75(90 + x)$

So $0.90(90) + 0x = 0.75(90 + x) \Leftrightarrow 81 = 67.5 + 0.75x \Leftrightarrow 0.75x = 13.5 \Leftrightarrow x = \frac{13.5}{0.75} = 18$. Thus 18 grams of silver must be added to get the required mixture.

58. Let x be the number of liters of water to be boiled off. The result will contain $6 - x$ liters.

	Original	Water	Final
Liters	6	$-x$	$6 - x$
Concentration	120	0	200
Amount	$120(6)$	0	$200(6 - x)$

So $120(6) + 0 = 200(6 - x) \Leftrightarrow 720 = 1200 - 200x \Leftrightarrow 200x = 480 \Leftrightarrow x = 2.4$. Thus 2.4 liters need to be boiled off.

59. Let x be the number of liters of coolant removed and replaced by water.

	60% antifreeze	60% antifreeze (removed)	Water	Mixture
Liters	3.6	x	x	3.6
Rate (% antifreeze)	0.60	0.60	0	0.50
Value	$0.60(3.6)$	$-0.60x$	$0x$	$0.50(3.6)$

So $0.60(3.6) - 0.60x + 0x = 0.50(3.6) \Leftrightarrow 2.16 - 0.6x = 1.8 \Leftrightarrow -0.6x = -0.36 \Leftrightarrow x = \frac{-0.36}{-0.6} = 0.6$. Thus 0.6 liters must be removed and replaced by water.

60. Let x be the number of liters of 2% bleach removed from the tank. This is also the number of liters of pure bleach added to make the 5% mixture.

	Original 2%	Pure bleach	5% mixture
Liters	$100 - x$	x	100
Concentration	0.02	1	0.05
Bleach	$0.02(100 - x)$	$1x$	$0.05 \cdot 100$

So $0.02(100 - x) + x = 0.05 \cdot 100 \Leftrightarrow 2 - 0.02x + x = 5 \Leftrightarrow 0.98x = 3 \Leftrightarrow x = 3.06$. Thus 3.06 liters need to be removed and replaced with pure bleach.

61. Let c be the concentration of fruit juice in the cheaper brand. The new mixture consists of 650 mL of the original fruit punch and 100 mL of the cheaper fruit punch.

	Original Fruit Punch	Cheaper Fruit Punch	Mixture
mL	650	100	750
Concentration	0.50	c	0.48
Juice	$0.50 \cdot 650$	$100c$	$0.48 \cdot 750$

So $0.50 \cdot 650 + 100c = 0.48 \cdot 750 \Leftrightarrow 325 + 100c = 360 \Leftrightarrow 100c = 35 \Leftrightarrow c = 0.35$. Thus the cheaper brand is only 35% fruit juice.

62. Let x be the number of packets of \$3.00/packet tea. Then $80 - x$ is the number of packets of \$2.75/packet tea.

	\$3.00 tea	\$2.75 tea	Mixture
Packets	x	$80 - x$	80
Rate (cost per packet)	3.00	2.75	2.90
Value	$3.00x$	$2.75(80 - x)$	$2.90(80)$

So $3.00x + 2.75(80 - x) = 2.90(80) \Leftrightarrow 3.00x + 220 - 2.75x = 232 \Leftrightarrow 0.25x = 12 \Leftrightarrow x = 48$. The mixture uses 48 packets of \$3.00/packet tea and $80 - 48 = 32$ packets of \$2.75/packet tea.

63. Let t be the time in minutes it would take to wash the car if the friends worked together. Friend 1 washes $\frac{1}{25}$ of the car per minute, while Friend 2 washes $\frac{1}{35}$ of the car per minute. The sum of these fractions is equal to the fraction of the job they can do working together, so we have $\frac{1}{t} = \frac{1}{25} + \frac{1}{35} \Leftrightarrow \frac{875t}{t} = \frac{875t}{25} + \frac{875t}{35} \Leftrightarrow 875 = 35t + 25t = 60t \Leftrightarrow t = \frac{875}{60} \Leftrightarrow t = 14\frac{35}{60}$ minutes, or 14 minutes 35 seconds.
64. Let t be the time, in minutes, it takes the landscaper to mow the lawn. Since the assistant is half as fast, it would take them $2t$ minutes to mow the lawn alone. Thus, $\frac{1}{10} = \frac{1}{t} + \frac{1}{2t} \Leftrightarrow \frac{10t}{10} = \frac{10t}{t} + \frac{10t}{2t} \Leftrightarrow t = 10 + 5 = 15$. Thus, it would take the assistant $2(15) = 30$ minutes to mow the lawn alone.
65. Let t be the number of hours it would take your friend to paint a house alone. Then working together, it takes $\frac{2}{3}t$ hours. Because it takes you 7 hours, we have $\frac{1}{7} + \frac{1}{t} = \frac{1}{\frac{2}{3}t} \Leftrightarrow \frac{14t}{7} + \frac{14t}{t} = \frac{14t}{\frac{2}{3}t} = \frac{3}{2}(14t) \Leftrightarrow 2t + 14 = 21 \Leftrightarrow t = \frac{7}{2}$. Thus, it would take your friend 3.5 h to paint a house alone.
66. Let h be the time, in hours, to fill the swimming pool using the smaller hose alone. Since the larger hose takes 20% less time, it takes $0.8h$ to fill the pool alone. Thus $16 \cdot \frac{1}{h} + 16 \cdot \frac{1}{0.8h} = 1 \Leftrightarrow 16 \cdot 0.8 + 16 = 0.8h \Leftrightarrow 12.8 + 16 = 0.8h \Leftrightarrow h = \frac{28.8}{0.8} = 36$. Thus, the smaller hose takes 36 hours to fill the pool alone, and the larger hose takes $0.8(36) = 28.8$ hours.

67. Let t be the time in hours that it takes you to wash all the windows. Then it takes your roommate $t + \frac{3}{2}$ hours to wash all the windows, and the sum of the fractions of the job you can do individually per hour equals the fraction of the job you can do together. Since 1 hour 48 minutes $= 1 + \frac{48}{60} = 1 + \frac{4}{5} = \frac{9}{5}$, we have $\frac{1}{t} + \frac{1}{t + \frac{3}{2}} = \frac{1}{\frac{9}{5}} \Leftrightarrow$

$$\frac{1}{t} + \frac{2}{2t + 3} = \frac{5}{9} \Rightarrow 9(2t + 3) + 2(9t) = 5t(2t + 3) \Leftrightarrow 18t + 27 + 18t = 10t^2 + 15t \Leftrightarrow 10t^2 - 21t - 27 = 0$$

$$\Leftrightarrow t = \frac{-(-21) \pm \sqrt{(-21)^2 - 4(10)(-27)}}{2(10)} = \frac{21 \pm \sqrt{441 + 1080}}{20} = \frac{21 \pm 39}{20}. \text{ So } t = \frac{21 - 39}{20} = -\frac{9}{10}$$

or $t = \frac{21 + 39}{20} = 3$. Since $t < 0$ is impossible, you can wash the windows alone in 3 hours, and it takes your roommate

$$3 + \frac{3}{2} = 4\frac{1}{2} \text{ hours.}$$

68. Let t be the time, in hours, it takes your manager to deliver all the flyers alone. Then it takes the assistant $t + 1$ hours to deliver all the flyers alone, and it takes the group $0.4t$ hours to do it together. Thus $\frac{1}{4} + \frac{1}{t} + \frac{1}{t + 1} = \frac{1}{0.4t}$
- $$\Leftrightarrow \frac{1}{4}(0.4t) + \frac{1}{t}(0.4t) + \frac{1}{t + 1}(0.4t) = 1 \Leftrightarrow t + 4 + \frac{4t}{t + 1} = 10 \Leftrightarrow t(t + 1) + 4(t + 1) + 4t = 10(t + 1) \Leftrightarrow$$
- $$t^2 + t + 4t + 4 + 4t = 10t + 10 \Leftrightarrow t^2 - t - 6 = 0 \Leftrightarrow (t - 3)(t + 2) = 0. \text{ So } t = 3 \text{ or } t = -2. \text{ Since } t = -2 \text{ is impossible, it takes your manager 3 hours to deliver all the flyers alone.}$$

69. Let t be the time in hours that the commuter spent on the train. Then $\frac{11}{2} - t$ is the time in hours that commuter spent on the bus. We construct a table:

	Rate	Time	Distance
By train	64	t	$64t$
By bus	96	$\frac{11}{2} - t$	$96\left(\frac{11}{2} - t\right)$

The total distance traveled is the sum of the distances traveled by bus and by train, so $480 = 64t + 96\left(\frac{11}{2} - t\right) \Leftrightarrow$

$$480 = 64t + 528 - 96t \Leftrightarrow -48 = -32t \Leftrightarrow t = \frac{-48}{-32} = 1.5 \text{ hours. So the time spent on the train is } 5.5 - 1.5 = 4 \text{ hours.}$$

70. Let r be the speed of the slower cyclist, in km/h. Then the speed of the faster cyclist is $2r$.

	Rate	Time	Distance
Slower cyclist	r	2	$2r$
Faster cyclist	$2r$	2	$4r$

When they meet, they will have traveled a combined total of 90 kilometers, so $2r + 4r = 90 \Leftrightarrow 6r = 90 \Leftrightarrow r = 15$. The speed of the slower cyclist is 15 km/h, while the speed of the faster cyclist is $2(15) = 30$ km/h.

71. Let r be the speed of the plane from Montreal to Los Angeles. Then $r + 0.20r = 1.20r$ is the speed of the plane from Los Angeles to Montreal.

	Rate	Time	Distance
Montreal to L.A.	r	$\frac{4000}{r}$	4000
L.A. to Montreal	$1.2r$	$\frac{4000}{1.2r}$	4000

The total time is the sum of the times each way, so $9\frac{1}{6} = \frac{4000}{r} + \frac{4000}{1.2r} \Leftrightarrow \frac{55}{6} = \frac{4000}{r} + \frac{4000}{1.2r} \Leftrightarrow$

$$55 \cdot 1.2r = 4000 \cdot 6 \cdot 1.2 + 4000 \cdot 6 \Leftrightarrow 66r = 28,800 + 24,000 \Leftrightarrow 66r = 52,800 \Leftrightarrow r = \frac{52,800}{66} = 800. \text{ Thus the plane flew at a speed of 800 km/h on the trip from Montreal to Los Angeles.}$$

72. Let x be the speed of the car in km/h. Since a kilometer contains 1000 m and an hour contains 3600 s,
 $1 \text{ km/h} = \frac{1000 \text{ m}}{3600 \text{ s}} = \frac{5}{18} \text{ m/s}$. The truck is traveling at $80 \cdot \frac{5}{18} = \frac{200}{9} \text{ m/s}$. So in 6 seconds, the truck travels
 $6 \cdot \frac{200}{9} \approx 133$ meters. Thus the back end of the car must travel the length of the car, the length of the truck, and
the 133 meters in 6 seconds, so its speed must be $\frac{4+10+133}{6} = \frac{147}{6} \text{ m/s}$. Converting to km/h, the speed of the car is
 $\frac{147}{6} \cdot \frac{18}{5} \approx 88.2 \text{ km/h}$.

73. Let x be the rate, in km/h, at which the salesperson drove between Ajax and Barrington.

Cities	Distance	Rate	Time
Ajax → Barrington	192	x	$\frac{192}{x}$
Barrington → Collins	240	$x + 16$	$\frac{240}{x + 16}$

We have used the equation $\text{time} = \frac{\text{distance}}{\text{rate}}$ to fill in the “Time” column of the table. Since the second part of the trip took 6 minutes (or $\frac{1}{10}$ hour) more than the first, we can use the time column to get the equation $\frac{192}{x} + \frac{1}{10} = \frac{240}{x + 16} \Rightarrow 192(10)(x + 16) + x(x + 16) = 240(10x) \Leftrightarrow 1920x + 30,720 + x^2 + 16x = 2400x \Leftrightarrow x^2 - 464x + 30,720 = 0 \Leftrightarrow x = \frac{-(-464) \pm \sqrt{(-464)^2 - 4(1)(30,720)}}{2} = \frac{464 \pm \sqrt{215,296 - 122,880}}{2} = \frac{464 \pm \sqrt{92,416}}{2} = \frac{464 \pm 304}{2} = 232 \pm 152$. Hence, the salesman drove either 80 km/h or 384 km/h between Ajax and Barrington. (The former seems likelier.)

74. Let x be the rate, in km/h, at which the trucker drove from Tortula to Cactus.

Cities	Distance	Rate	Time
Tortula → Cactus	400	x	$\frac{400}{x}$
Cactus → Dry Junction	576	$x + 16$	$\frac{576}{x + 16}$

We have used $\text{time} = \frac{\text{distance}}{\text{rate}}$ to fill in the time column of the table. We are given that

the sum of the times is 11 hours. Thus we get the equation $\frac{400}{x} + \frac{576}{x + 16} = 11 \Leftrightarrow$

$$400(x + 16) + 576x = 11x(x + 16) \Leftrightarrow 400x + 6400 + 576x = 11x^2 + 176x \Leftrightarrow 11x^2 - 800x - 6400 = 0 \Rightarrow$$

$$x = \frac{-(-800) \pm \sqrt{(-800)^2 - 4(11)(-6400)}}{2(11)} = \frac{800 \pm \sqrt{640,000 + 281,600}}{22} = \frac{800 \pm \sqrt{921,600}}{22} = \frac{800 \pm 960}{22}.$$

Hence, the trucker drove either -7.27 km/h (impossible) or 80 km/h between Tortula and Cactus.

75. Let r be the rowing rate in km/h of the crew in still water. Then their rate upstream was $r - 3$ km/h, and their rate downstream was $r + 3$ km/h.

	Distance	Rate	Time
Upstream	6	$r - 3$	$\frac{6}{r - 3}$
Downstream	6	$r + 3$	$\frac{6}{r + 3}$

Since the time to row upstream plus the time to row downstream was 2 hours 40 minutes $= \frac{8}{3}$ hour, we get the equation

$$\frac{6}{r - 3} + \frac{6}{r + 3} = \frac{8}{3} \Leftrightarrow 6(3)(r + 3) + 6(3)(r - 3) = 8(r - 3)(r + 3) \Leftrightarrow 18r + 54 + 18r - 54 = 8r^2 - 72 \Leftrightarrow 0 = 8r^2 - 36r - 72 = 4(2r^2 - 9r - 18) = 4(2r + 3)(r - 6).$$

Since $2r + 3 = 0 \Leftrightarrow r = -\frac{3}{2}$ is impossible, the solution is $r - 6 = 0 \Leftrightarrow r = 6$. So the rate of the rowing crew in still water is 6 km/h.

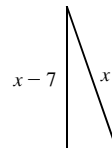
76. Let r be the speed of the southbound boat. Then $r + 3$ is the speed of the eastbound boat. In two hours the southbound boat has traveled $2r$ kilometers and the eastbound boat has traveled $2(r + 3) = 2r + 6$ kilometers. Since they are traveling in directions with are 90° apart, we can use the Pythagorean Theorem to get $(2r)^2 + (2r + 6)^2 = 30^2 \Leftrightarrow 4r^2 + 4r^2 + 24r + 36 = 900 \Leftrightarrow 8r^2 + 24r - 864 = 0 \Leftrightarrow 8(r^2 + 3r - 108) = 0 \Leftrightarrow 8(r + 12)(r - 9) = 0$. So $r = -12$ or $r = 9$. Since speed is positive, the speed of the southbound boat is 9 km/h.

77. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

Let x be the distance from the fulcrum to where the 56-kg friend sits. Then substituting the known values into the formula given, we have $45(2) = 56x \Leftrightarrow 90 = 56x \Leftrightarrow x = 1.6$. So the 56-kg friend should sit 1.6 meters from the fulcrum.

78. Let w be the largest weight that can be hung. In this exercise, the edge of the building acts as the fulcrum, so the 110-kg man is sitting 7.5 meters from the fulcrum. Then substituting the known values into the formula given in Exercise 77, we have $110(7.5) = 1.5w \Leftrightarrow 825 = 1.5w \Leftrightarrow w = 550$. Therefore, 550 kilograms is the largest weight that can be hung.
79. The volume is 180 m^3 , so $x(x - 4)(x + 9) = 180 \Leftrightarrow x^3 + 5x^2 - 36x = 180 \Leftrightarrow x^3 + 5x^2 - 36x - 180 = 0 \Leftrightarrow x^2(x + 5) - 36(x + 5) = 0 \Leftrightarrow (x + 5)(x^2 - 36) = 0 \Leftrightarrow (x + 5)(x + 6)(x - 6) = 0 \Rightarrow x = 6$ is the only positive solution. So the box is 2 meters by 6 meters by 15 meters.
80. Let r be the radius of the larger sphere, in mm. Equating the volumes, we have $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2^3 + 3^3 + 4^3) \Leftrightarrow r^3 = 2^3 + 3^3 + 4^3 \Leftrightarrow r^3 = 99 \Leftrightarrow r = \sqrt[3]{99} \approx 4.63$. Therefore, the radius of the larger sphere is about 4.63 mm.
81. Let x be the length of one side of the cardboard, so we start with a piece of cardboard x by x . When 4 centimeters are removed from each side, the base of the box is $x - 8$ by $x - 8$. Since the volume is 100 cm^3 , we get $4(x - 8)^2 = 100 \Leftrightarrow x^2 - 16x + 64 = 25 \Leftrightarrow x^2 - 16x + 39 = 0 \Leftrightarrow (x - 3)(x - 13) = 0$. So $x = 3$ or $x = 13$. But $x = 3$ is not possible, since then the length of the base would be $3 - 8 = -5$, and all lengths must be positive. Thus $x = 13$, and the piece of cardboard is 13 centimeters by 13 centimeters.
82. Let r be the radius of the can. Now using the formula $V = \pi r^2 h$ with $V = 40\pi \text{ cm}^3$ and $h = 10$, we solve for r . Thus $40\pi = \pi r^2(10) \Leftrightarrow 4 = r^2 \Rightarrow r = \pm 2$. Since r represents radius, $r > 0$. Thus $r = 2$, and the diameter is 4 cm.
83. Let r be the radius of the tank, in meters. The volume of the spherical tank is $\frac{4}{3}\pi r^3$ and is also $2840 \times 0.001 = 2.84$. So $\frac{4}{3}\pi r^3 = 2.84 \Leftrightarrow r^3 = 0.678 \Leftrightarrow r = 0.88$ meters.

84. Let x be the length of the hypotenuse of the triangle, in meters. Then one of the other sides has length $x - 7$ meters, and since the perimeter is 392 meters, the remaining side must have length $392 - x - (x - 7) = 399 - 2x$. From the



Pythagorean Theorem, we get $(x - 7)^2 + (399 - 2x)^2 = x^2 \Leftrightarrow$

$4x^2 - 1610x + 159,250 = 0$. Using the Quadratic Formula, we get

$x = \frac{1610 \pm \sqrt{1610^2 - 4(4)(159,250)}}{2(4)} = \frac{1610 \pm \sqrt{44,100}}{8} = \frac{1610 \pm 210}{8}$, and so $x = 227.5$ or $x = 175$. But if $x = 227.5$, then the side of length $x - 7$ combined with the hypotenuse already exceeds the perimeter of 392 meters, and so we must have $x = 175$. Thus the other sides have length $175 - 7 = 168$ and $399 - 2(175) = 49$. The lot has sides of length 49 meters, 168 meters, and 175 meters.

85. Let x be the length, in kilometers, of the abandoned road to be used. Then the length of the abandoned road not used is $40 - x$, and the length of the new road is $\sqrt{10^2 + (40 - x)^2}$ kilometers, by the Pythagorean Theorem. Since the cost of the road is cost per kilometer $\times$ number of kilometers, we have $100,000x + 200,000\sqrt{x^2 - 80x + 1700} = 6,800,000$
 $\Leftrightarrow 2\sqrt{x^2 - 80x + 1700} = 68 - x$. Squaring both sides, we get $4x^2 - 320x + 6800 = 4624 - 136x + x^2 \Leftrightarrow$
 $3x^2 - 184x + 2176 = 0 \Leftrightarrow x = \frac{184 \pm \sqrt{33856 - 26112}}{6} = \frac{184 \pm 88}{6} \Leftrightarrow x = \frac{136}{3}$ or $x = 16$. Since $45\frac{1}{3}$ is longer than the existing road, 16 kilometers of the abandoned road should be used. A completely new road would have length $\sqrt{10^2 + 40^2}$ (let $x = 0$) and would cost $\sqrt{1700} \times 200,000 \approx 8.3$ million dollars. So no, it would not be cheaper.

86. Let x be the distance, in meters, that you go on the boardwalk before veering off onto the sand. The distance along the boardwalk from where you started to the point on the boardwalk closest to the umbrella is $\sqrt{229^2 - 64^2} \approx 220$ m. Thus the distance you walk on the sand is $\sqrt{(220 - x)^2 + 64^2} = \sqrt{48,400 - 440x + x^2 + 4096} = \sqrt{x^2 - 440x + 52,496}$.

	Distance	Rate	Time
Along boardwalk	x	1.2	$\frac{x}{1.2}$
Across sand	$\sqrt{x^2 - 440x + 52,496}$	0.6	$\frac{\sqrt{x^2 - 440x + 52,496}}{0.6}$

Since 4 minutes 45 seconds = 285 seconds, we equate the time it takes to walk along the boardwalk and across the sand to

the total time to get $285 = \frac{x}{1.2} + \frac{\sqrt{x^2 - 440x + 52,496}}{0.6} \Leftrightarrow 342 - x = 2\sqrt{x^2 - 440x + 52,496}$. Squaring both sides, we get

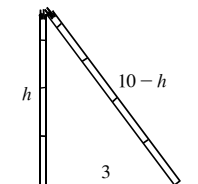
$(342 - x)^2 = 4(x^2 - 440x + 52,496) \Leftrightarrow 116,964 - 684x + x^2 = 4x^2 - 1760x + 209,984 \Leftrightarrow 0 = 3x^2 - 1076x + 93,020$

$\Leftrightarrow x \approx 213.3$ or 145.4 . Checking $x = 213.3$: The distance across the sand is 64 m, so $\frac{213.3}{1.2} + \frac{64}{0.6} \approx 284.4$ s. Checking

$x = 145.4$: The distance across the sand is $\sqrt{(220 - 145.4)^2 + 64^2} \approx 98.3$ m, so $\frac{145.4}{1.2} + \frac{98.3}{0.6} = 285.0$ s. Since both solutions are less than or equal to 220 m, we have two solutions: Either you walk 145.4 meters down the boardwalk and then head towards the umbrella, or you walk 213.3 meters down the boardwalk and then head towards the umbrella.

87. Let x be the height of the pile in meters. Then the diameter is $3x$ and the radius is $\frac{3}{2}x$ meters. Since the volume of the cone is 28 m^3 , we have $\frac{\pi}{3} \left(\frac{3x}{2}\right)^2 x = 28 \Leftrightarrow \frac{3\pi x^3}{4} = 28 \Leftrightarrow x^3 = \frac{112}{3\pi} \Leftrightarrow x = \sqrt[3]{\frac{112}{3\pi}} \approx 2.28$ meters.

88. Let h be the height of the screens in centimeters. The width of the smaller screen is $h + 17.5$ centimeters, and the width of the bigger screen is $1.8h$ centimeters. The diagonal measure of the smaller screen is $\sqrt{h^2 + (h + 17.5)^2}$, and the diagonal measure of the larger screen is $\sqrt{h^2 + (1.8h)^2} = \sqrt{4.24h^2} \approx 2.06h$. Thus $\sqrt{h^2 + (h + 17.5)^2} + 7.5 = 2.06h \Leftrightarrow \sqrt{h^2 + (h + 17.5)^2} = 2.06h - 7.5$. Squaring both sides gives $h^2 + h^2 + 35h + 306.25 = 4.24h^2 - 30.9h + 56.25 \Leftrightarrow 0 = 2.24h^2 - 65.9h - 362.5$. Applying the Quadratic Formula, we obtain $h = \frac{65.9 \pm \sqrt{(-65.9)^2 - 4(2.24)(-362.5)}}{2(2.24)} = \frac{65.9 \pm \sqrt{7590.81}}{4.48} \approx \frac{65.9 \pm 87.13}{4.48}$. So $h \approx \frac{65.9 + 87.13}{4.48} \approx 34.16$. Thus, the screens are approximately 34.2 centimeters high.
89. Let h be the height in meters of the structure. The structure is composed of a right cylinder with radius 3 and height $\frac{2}{3}h$ and a cone with base radius 3 and height $\frac{1}{3}h$. Using the formulas for the volume of a cylinder and that of a cone, we obtain the equation $40\pi = \pi(3)^2\left(\frac{2}{3}h\right) + \frac{1}{3}\pi(3)^2\left(\frac{1}{3}h\right) \Leftrightarrow 40\pi = 6\pi h + \pi h \Leftrightarrow 40 = 7h \Leftrightarrow h \approx 5.7$. Thus the height of the structure is 5.7 meters.
90. Let y be the circumference of the circle, so $360 - y$ is the perimeter of the square. Use the circumference to find the radius, r , in terms of y : $y = 2\pi r \Rightarrow r = y/(2\pi)$. Thus the area of the circle is $\pi[y/(2\pi)]^2 = y^2/(4\pi)$. Now if the perimeter of the square is $360 - y$, the length of each side is $\frac{1}{4}(360 - y)$, and the area of the square is $\left[\frac{1}{4}(360 - y)\right]^2$. Setting these areas equal, we obtain $y^2/(4\pi) = \left[\frac{1}{4}(360 - y)\right]^2 \Leftrightarrow y/(2\sqrt{\pi}) = \frac{1}{4}(360 - y) \Leftrightarrow 2y = 360\sqrt{\pi} - \sqrt{\pi}y \Leftrightarrow (2 + \sqrt{\pi})y = 360\sqrt{\pi}$. Therefore, $y = 360\sqrt{\pi}/(2 + \sqrt{\pi}) \approx 169.1$. Thus one wire is 169.1 cm long and the other is 190.9 cm long.
91. Let h be the height of the break, in meters. Then the portion of the bamboo above the break is $10 - h$. Applying the Pythagorean Theorem, we obtain $h^2 + 3^2 = (10 - h)^2 \Leftrightarrow h^2 + 9 = 100 - 20h + h^2 \Leftrightarrow -91 = -20h \Leftrightarrow h = \frac{91}{20} = 4.55$. Thus the break is 4.55 m above the ground.
92. Answers will vary.
93. Let x equal the original length of the reed in cubits. Then $x - 1$ is the piece that fits 60 times along the length of the field, that is, the length is $60(x - 1)$. The width is $30x$. Then converting cubits to ninda, we have $375 = 60(x - 1) \cdot 30x \cdot \frac{1}{12^2} = \frac{25}{2}x(x - 1) \Leftrightarrow 30 = x^2 - x \Leftrightarrow x^2 - x - 30 = 0 \Leftrightarrow (x - 6)(x + 5) = 0$. So $x = 6$ or $x = -5$. Since x must be positive, the original length of the reed is 6 cubits.



1.8 INEQUALITIES

1. (a) If $x < 5$, then $x - 3 < 5 - 3 \Rightarrow x - 3 < 2$.
- (b) If $x \leq 5$, then $3 \cdot x \leq 3 \cdot 5 \Rightarrow 3x \leq 15$.
- (c) If $x \geq 2$, then $-3 \cdot x \leq -3 \cdot 2 \Rightarrow -3x \leq -6$.
- (d) If $x < -2$, then $-x > 2$.

2. To solve the nonlinear inequality $\frac{x+1}{x-2} \leq 0$ we

first observe that the numbers -1 and 2 are zeros of the numerator and denominator. These numbers divide the real line into the three intervals $(-\infty, -1)$, $(-1, 2)$, and $(2, \infty)$.

5	$(-\infty, -1)$	$(-1, 2)$	$(2, \infty)$
Sign of $x + 1$	−	+	+
Sign of $x - 2$	−	−	+
Sign of $(x + 1) / (x - 2)$	+	−	+

The endpoint -1 satisfies the inequality, because $\frac{-1+1}{-1-2} = 0 \leq 0$, but 2 fails to satisfy the inequality because $\frac{2+1}{2-2}$ is not defined.

Thus, referring to the table, we see that the solution of the inequality is $[-1, 2)$.

3. (a) The solution of the inequality $|x| \leq 3$ is the interval $[-3, 3]$. Any number that lies inside this interval satisfies the inequality.



- (b) The solution of the inequality $|x| \geq 3$ is the union of intervals $(-\infty, -3] \cup [3, \infty)$. Any number that lie inside one of these intervals satisfies the inequality.



4. (a) The set of all points on the real line whose distance from zero is less than 3 can be described by the absolute value inequality $|x| < 3$.
- (b) The set of all points on the real line whose distance from zero is greater than 3 can be described by the absolute value inequality $|x| > 3$.
5. (a) No. For example, if $x = -2$, then $x(x+1) = -2(-1) = 2 > 0$, but $x < 0$.
- (b) No. For example, if $x = 2$, then $x(x+1) = 2(3) = 6 > 5$, but $x < 5$.
6. (a) To solve $3x \leq 7$, start by dividing both sides of the inequality by 3.
- (b) To solve $5x - 2 \geq 1$, start by adding 2 to both sides of the inequality.
- (c) To solve $|3x + 2| \leq 8$, start by using Property 2 of Absolute Value Inequalities to rewrite the inequality as $-8 \leq 3x + 2 \leq 8$.

7.

x	$-2 + 3x \geq \frac{1}{3}$
-5	$-17 \geq \frac{1}{3}$; no
-1	$-5 \geq \frac{1}{3}$; no
0	$-2 \geq 0$; no
$\frac{2}{3}$	$0 \geq \frac{1}{3}$; no
$\frac{5}{6}$	$\frac{1}{2} \geq \frac{1}{3}$; yes
1	$1 \geq \frac{1}{3}$; yes
$\sqrt{5}$	$4.7 \geq \frac{1}{3}$; yes
3	$7 \geq \frac{1}{3}$; yes
5	$13 \geq \frac{1}{3}$; yes

The elements $\frac{5}{6}$, 1, $\sqrt{5}$, 3, and 5 satisfy the inequality.

8.

x	$1 - 2x \geq 5x$
-5	$11 \geq -25$; yes
-1	$3 \geq -5$; yes
0	$1 \geq 0$; yes
$\frac{2}{3}$	$-\frac{1}{3} \geq \frac{10}{3}$; no
$\frac{5}{6}$	$-\frac{2}{3} \geq \frac{25}{6}$; no
1	$-1 \geq 5$; no
$\sqrt{5}$	$-3.47 \geq 11.18$; no
3	$-5 \geq 15$; no
5	$-9 \geq 25$; no

The elements -5 , -1 , and 0 satisfy the inequality.

9.

x	$1 < 2x - 4 \leq 7$
-5	$1 < -14 \leq 7$; no
-1	$1 < -6 \leq 7$; no
0	$1 < -4 \leq 7$; no
$\frac{2}{3}$	$1 < -\frac{8}{3} \leq 7$; no
$\frac{5}{6}$	$1 < -\frac{7}{3} \leq 7$; no
1	$1 < -2 \leq 7$; no
$\sqrt{5}$	$1 < 0.47 \leq 7$; no
3	$1 < 2 \leq 7$; yes
5	$1 < 6 \leq 7$; yes

The elements 3 and 5 satisfy the inequality.

10.

x	$-2 \leq 3 - x < 2$
-5	$-2 \leq 8 < 2$; no
-1	$-2 \leq 4 < 2$; no
0	$-2 \leq 3 < 2$; no
$\frac{2}{3}$	$-2 \leq \frac{7}{3} < 2$; no
$\frac{5}{6}$	$-2 < \frac{13}{6} < 2$; no
1	$-2 \leq 2 < 2$; no
$\sqrt{5}$	$-2 \leq 0.76 < 2$; yes
3	$-2 \leq 0 < 2$; yes
5	$-2 \leq -2 < 2$; yes

The elements $\sqrt{5}$, 3, and 5 satisfy the inequality.

11.

x	$\frac{1}{x} \leq \frac{1}{2}$
-5	$-\frac{1}{5} \leq \frac{1}{2}$; yes
-1	$-1 \leq \frac{1}{2}$; yes
0	$\frac{1}{0}$ is undefined; no
$\frac{2}{3}$	$\frac{3}{2} \leq \frac{1}{2}$; no
$\frac{5}{6}$	$\frac{6}{5} \leq \frac{1}{2}$; no
1	$1 \leq \frac{1}{2}$; no
$\sqrt{5}$	$0.45 \leq \frac{1}{2}$; yes
3	$\frac{1}{3} \leq \frac{1}{2}$; yes
5	$\frac{1}{5} \leq \frac{1}{2}$; yes

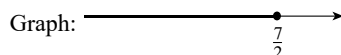
The elements -5, -1, $\sqrt{5}$, 3, and 5 satisfy the inequality.

12.

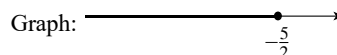
x	$x^2 + 2 < 4$
-5	$27 < 4$; no
-1	$3 < 4$; yes
0	$2 < 4$; yes
$\frac{2}{3}$	$\frac{22}{9} < 4$; yes
$\frac{5}{6}$	$\frac{97}{36} < 4$; yes
1	$3 < 4$; yes
$\sqrt{5}$	$7 < 4$; no
3	$11 < 4$; no
5	$27 < 4$; no

The elements -1, 0, $\frac{2}{3}$, $\frac{5}{6}$, and 1 satisfy the inequality.

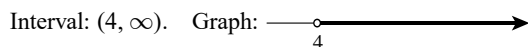
13. $2x \leq 7 \Leftrightarrow x \leq \frac{7}{2}$. Interval: $(-\infty, \frac{7}{2}]$



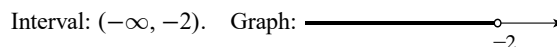
14. $-4x \geq 10 \Leftrightarrow x \leq -\frac{5}{2}$. Interval: $(-\infty, -\frac{5}{2}]$



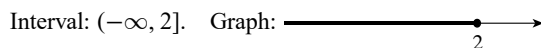
15. $2x - 5 > 3 \Leftrightarrow 2x > 8 \Leftrightarrow x > 4$



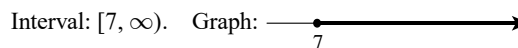
16. $3x + 11 < 5 \Leftrightarrow 3x < -6 \Leftrightarrow x < -2$



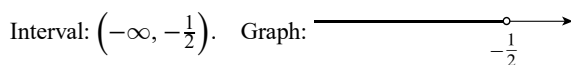
17. $7 - x \geq 5 \Leftrightarrow -x \geq -2 \Leftrightarrow x \leq 2$



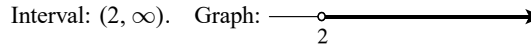
18. $5 - 3x \leq -16 \Leftrightarrow -3x \leq -21 \Leftrightarrow x \geq 7$



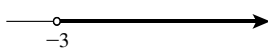
19. $2x + 1 < 0 \Leftrightarrow 2x < -1 \Leftrightarrow x < -\frac{1}{2}$



20. $0 < 4x - 8 \Leftrightarrow 0 < x - 2 \Leftrightarrow x > 2$



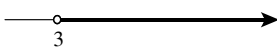
21. $4x - 7 < 8 + 9x \Leftrightarrow -5x < 15 \Leftrightarrow x > -3.$

Interval: $(-3, \infty)$. Graph: 

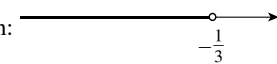
22. $5 - 3x \geq 8x - 7 \Leftrightarrow -11x \geq -12 \Leftrightarrow x \leq \frac{12}{11}$

Interval: $(-\infty, \frac{12}{11}]$. Graph: 

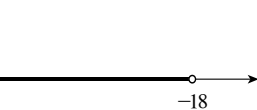
23. $x - \frac{3}{2} > \frac{1}{2}x \Leftrightarrow \frac{1}{2}x > \frac{3}{2} \Leftrightarrow x > 3$

Interval: $(3, \infty)$. Graph: 

24. $\frac{2}{3}x + 1 < \frac{1}{3} - 2x \Leftrightarrow \frac{12}{5}x < -\frac{4}{5} \Leftrightarrow x < -\frac{1}{3}$

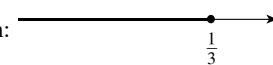
Interval: $(-\infty, -\frac{1}{3})$. Graph: 

25. $\frac{1}{3}x + 2 < \frac{1}{6}x - 1 \Leftrightarrow \frac{1}{6}x < -3 \Leftrightarrow x < -18$

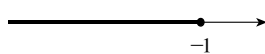
Interval: $(-\infty, -18)$. Graph: 

26. $\frac{2}{3} - \frac{1}{2}x \geq \frac{1}{6} + x$ (multiply both sides by 6) $\Leftrightarrow$

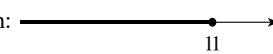
$4 - 3x \geq 1 + 6x \Leftrightarrow 3 \geq 9x \Leftrightarrow \frac{1}{3} \geq x$

Interval: $(-\infty, \frac{1}{3}]$. Graph: 

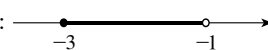
27. $4 - 3x \leq -(1 + 8x) \Leftrightarrow 4 - 3x \leq -1 - 8x \Leftrightarrow 5x \leq -5 \Leftrightarrow x \leq -1$

Interval: $(-\infty, -1]$. Graph: 

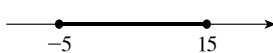
28. $2(7x - 3) \leq 12x + 16 \Leftrightarrow 14x - 6 \leq 12x + 16 \Leftrightarrow 2x \leq 22 \Leftrightarrow x \leq 11$

Interval: $(-\infty, 11]$. Graph: 

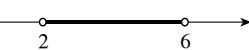
29. $2 \leq x + 5 < 4 \Leftrightarrow -3 \leq x < -1$

Interval: $[-3, -1)$. Graph: 

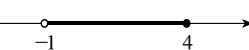
30. $-8 \leq x - 3 \leq 12 \Leftrightarrow -5 \leq x \leq 15$

Interval: $[-5, 15]$. Graph: 

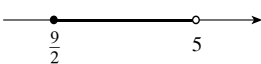
31. $-1 < 2x - 5 < 7 \Leftrightarrow 4 < 2x < 12 \Leftrightarrow 2 < x < 6$

Interval: $(2, 6)$. Graph: 

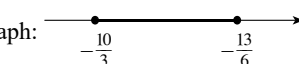
32. $1 < 3x + 4 \leq 16 \Leftrightarrow -3 < 3x \leq 12 \Leftrightarrow -1 < x \leq 4$

Interval: $(-1, 4]$. Graph: 

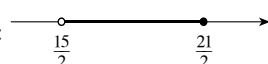
33. $-2 < 8 - 2x \leq -1 \Leftrightarrow -10 < -2x \leq -9 \Leftrightarrow 5 > x \geq \frac{9}{2} \Leftrightarrow \frac{9}{2} \leq x < 5$

Interval: $[\frac{9}{2}, 5)$. Graph: 

34. $-3 \leq 3x + 7 \leq \frac{1}{2} \Leftrightarrow -10 \leq 3x \leq -\frac{13}{2} \Leftrightarrow -\frac{10}{3} \leq x \leq -\frac{13}{6}$

Interval: $[-\frac{10}{3}, -\frac{13}{6}]$. Graph: 

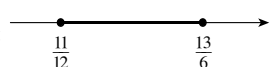
35. $\frac{1}{6} < \frac{2x - 13}{12} \leq \frac{2}{3} \Leftrightarrow 2 < 2x - 13 \leq 8$ (multiply each expression by 12) $\Leftrightarrow 15 < 2x \leq 21 \Leftrightarrow \frac{15}{2} < x \leq \frac{21}{2}$.

Interval: $(\frac{15}{2}, \frac{21}{2}]$. Graph: 

36. $-\frac{1}{2} \leq \frac{4 - 3x}{5} \leq \frac{1}{4} \Leftrightarrow$ (multiply each expression by 20)

$-10 \leq 4(4 - 3x) \leq 5 \Leftrightarrow -10 \leq 16 - 12x \leq 5 \Leftrightarrow$

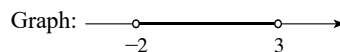
$-26 \leq -12x \leq -11 \Leftrightarrow \frac{13}{6} \geq x \geq \frac{11}{12} \Leftrightarrow \frac{11}{12} \leq x \leq \frac{13}{6}.$

Interval: $[\frac{11}{12}, \frac{13}{6}]$. Graph: 

37. $(x + 2)(x - 3) < 0$. The expression on the left of the inequality changes sign where $x = -2$ and where $x = 3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 3)$	$(3, \infty)$
Sign of $x + 2$	—	+	+
Sign of $x - 3$	—	—	+
Sign of $(x + 2)(x - 3)$	+	—	+

From the table, the solution set is $\{x \mid -2 < x < 3\}$. Interval: $(-2, 3)$.



38. $(x - 5)(x + 4) \geq 0$. The expression on the left of the inequality changes sign when $x = 5$ and $x = -4$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -4)$	$(-4, 5)$	$(5, \infty)$
Sign of $x - 5$	—	—	+
Sign of $x + 4$	—	+	+
Sign of $(x - 5)(x + 4)$	+	—	+

From the table, the solution set is $\{x \mid x \leq -4 \text{ or } 5 \leq x\}$.

Interval: $(-\infty, -4] \cup [5, \infty)$.

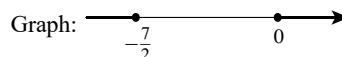


39. $x(2x + 7) \geq 0$. The expression on the left of the inequality changes sign where $x = 0$ and where $x = -\frac{7}{2}$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -\frac{7}{2})$	$(-\frac{7}{2}, 0)$	$(0, \infty)$
Sign of x	—	—	+
Sign of $2x + 7$	—	+	+
Sign of $x(2x + 7)$	+	—	+

From the table, the solution set is $\{x \mid x \leq -\frac{7}{2} \text{ or } 0 \leq x\}$.

Interval: $(-\infty, -\frac{7}{2}] \cup [0, \infty)$.

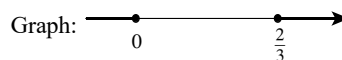


40. $x(2 - 3x) \leq 0$. The expression on the left of the inequality changes sign when $x = 0$ and $x = \frac{2}{3}$. Thus we must check the intervals in the following table.

Interval	$(-\infty, 0)$	$(0, \frac{2}{3})$	$(\frac{2}{3}, \infty)$
Sign of x	—	+	+
Sign of $2 - 3x$	+	+	—
Sign of $x(2 - 3x)$	—	+	—

From the table, the solution set is $\{x \mid x \leq 0 \text{ or } \frac{2}{3} \leq x\}$.

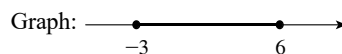
Interval: $(-\infty, 0] \cup [\frac{2}{3}, \infty)$.



41. $x^2 - 3x - 18 \leq 0 \Leftrightarrow (x + 3)(x - 6) \leq 0$. The expression on the left of the inequality changes sign where $x = 6$ and where $x = -3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, 6)$	$(6, \infty)$
Sign of $x + 3$	—	+	+
Sign of $x - 6$	—	—	+
Sign of $(x + 3)(x - 6)$	+	—	+

From the table, the solution set is $\{x \mid -3 \leq x \leq 6\}$. Interval: $[-3, 6]$.



42. $x^2 - 8x + 7 > 0 \Leftrightarrow (x - 7)(x - 1) > 0$. The expression on the left of the inequality changes sign where $x = 7$ and where $x = 1$. Thus we must check the intervals in the following table.

Interval	$(-\infty, 1)$	$(1, 7)$	$(7, \infty)$
Sign of $x - 7$	-	-	+
Sign of $x - 1$	-	+	+
Sign of $(x - 7)(x - 1)$	+	-	+

From the table, the solution set is $\{x \mid x < 1 \text{ or } x > 7\}$. Interval: $(-\infty, 1) \cup (7, \infty)$.



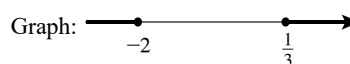
43. $3x^2 + 5x \geq 2 \Leftrightarrow 3x^2 + 5x - 2 \geq 0 \Leftrightarrow (x + 2)(3x - 1) \geq 0$. The expression on the left of the inequality changes sign where $x = -2$ and where $x = \frac{1}{3}$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, \frac{1}{3})$	$(\frac{1}{3}, \infty)$
Sign of $x + 2$	-	+	+
Sign of $3x - 1$	-	-	+
Sign of $(x + 2)(3x - 1)$	+	-	+

From the table, the solution set is

$$\{x \mid x \leq -2 \text{ or } x \geq \frac{1}{3}\}.$$

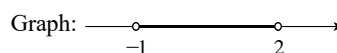
Interval: $(-\infty, -2] \cup [\frac{1}{3}, \infty)$.



44. $x^2 < x + 2 \Leftrightarrow x^2 - x - 2 < 0 \Leftrightarrow (x + 1)(x - 2) < 0$. The expression on the left of the inequality changes sign when $x = -1$ and $x = 2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, 2)$	$(2, \infty)$
Sign of $x + 1$	-	+	+
Sign of $x - 2$	-	-	+
Sign of $(x + 1)(x - 2)$	+	-	+

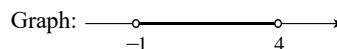
From the table, the solution set is $\{x \mid -1 < x < 2\}$. Interval: $(-1, 2)$.



45. $3x^2 - 3x < 2x^2 + 4 \Leftrightarrow x^2 - 3x - 4 < 0 \Leftrightarrow (x + 1)(x - 4) < 0$. The expression on the left of the inequality changes sign where $x = -1$ and where $x = 4$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
Sign of $x + 1$	-	+	+
Sign of $x - 4$	-	-	+
Sign of $(x + 1)(x - 4)$	+	-	+

From the table, the solution set is $\{x \mid -1 < x < 4\}$. Interval: $(-1, 4)$.



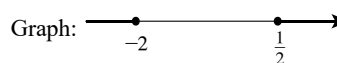
46. $5x^2 + 3x \geq 3x^2 + 2 \Leftrightarrow 2x^2 + 3x - 2 \geq 0 \Leftrightarrow (2x - 1)(x + 2) \geq 0$. The expression on the left of the inequality changes sign when $x = \frac{1}{2}$ and $x = -2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, \frac{1}{2})$	$(\frac{1}{2}, \infty)$
Sign of $2x - 1$	-	-	+
Sign of $x + 2$	-	+	+
Sign of $(2x - 1)(x + 2)$	+	-	+

From the table, the solution set is

$$\{x \mid x \leq -2 \text{ or } \frac{1}{2} \leq x\}.$$

Interval: $(-\infty, -2] \cup [\frac{1}{2}, \infty)$.

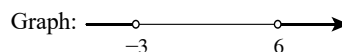


47. $x^2 > 3(x + 6) \Leftrightarrow x^2 - 3x - 18 > 0 \Leftrightarrow (x + 3)(x - 6) > 0$. The expression on the left of the inequality changes sign where $x = 6$ and where $x = -3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, 6)$	$(6, \infty)$
Sign of $x + 3$	-	+	+
Sign of $x - 6$	-	-	+
Sign of $(x + 3)(x - 6)$	+	-	+

From the table, the solution set is $\{x \mid x < -3 \text{ or } 6 < x\}$.

Interval: $(-\infty, -3) \cup (6, \infty)$.

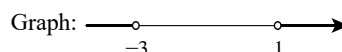


48. $x^2 + 2x > 3 \Leftrightarrow x^2 + 2x - 3 > 0 \Leftrightarrow (x + 3)(x - 1) > 0$. The expression on the left of the inequality changes sign when $x = -3$ and $x = 1$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, 1)$	$(1, \infty)$
Sign of $x + 3$	-	+	+
Sign of $x - 1$	-	-	+
Sign of $(x + 3)(x - 1)$	+	-	+

From the table, the solution set is $\{x \mid x < -3 \text{ or } 1 < x\}$.

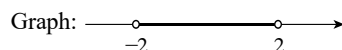
Interval: $(-\infty, -3) \cup (1, \infty)$.



49. $x^2 < 4 \Leftrightarrow x^2 - 4 < 0 \Leftrightarrow (x + 2)(x - 2) < 0$. The expression on the left of the inequality changes sign where $x = -2$ and where $x = 2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Sign of $x + 2$	-	+	+
Sign of $x - 2$	-	-	+
Sign of $(x + 2)(x - 2)$	+	-	+

From the table, the solution set is $\{x \mid -2 < x < 2\}$. Interval: $(-2, 2)$.

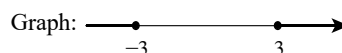


50. $x^2 \geq 9 \Leftrightarrow x^2 - 9 \geq 0 \Leftrightarrow (x + 3)(x - 3) \geq 0$. The expression on the left of the inequality changes sign when $x = -3$ and $x = 3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, 3)$	$(3, \infty)$
Sign of $x + 3$	-	+	+
Sign of $x - 3$	-	-	+
Sign of $(x + 3)(x - 3)$	+	-	+

From the table, the solution set is $\{x \mid x \leq -3 \text{ or } 3 \leq x\}$.

Interval: $(-\infty, -3] \cup [3, \infty)$.



51. $(x + 2)(x - 1)(x - 3) \leq 0$. The expression on the left of the inequality changes sign when $x = -2$, $x = 1$, and $x = 3$. Thus we must check the intervals in the following table.

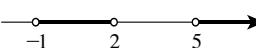
Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, 3)$	$(3, \infty)$
Sign of $x + 2$	-	+	+	+
Sign of $x - 1$	-	-	+	+
Sign of $x - 3$	-	-	-	+
Sign of $(x + 2)(x - 1)(x - 3)$	-	+	-	+

From the table, the solution set is $\{x \mid x \leq -2 \text{ or } 1 \leq x \leq 3\}$. Interval: $(-\infty, -2] \cup [1, 3]$.



52. $(x - 5)(x - 2)(x + 1) > 0$. The expression on the left of the inequality changes sign when $x = 5$, $x = 2$, and $x = -1$. Thus we must check the intervals in the following table.

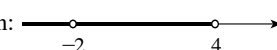
Interval	$(-\infty, -1)$	$(-1, 2)$	$(2, 5)$	$(5, \infty)$
Sign of $x - 5$	—	—	—	+
Sign of $x - 2$	—	—	+	+
Sign of $x + 1$	—	+	+	+
Sign of $(x - 5)(x - 2)(x + 1)$	—	+	—	+

From the table, the solution set is $\{x \mid -1 < x < 2 \text{ or } 5 < x\}$. Interval: $(-1, 2) \cup (5, \infty)$. Graph: 

53. $(x - 4)(x + 2)^2 < 0$. Note that $(x + 2)^2 > 0$ for all $x \neq -2$, so the expression on the left of the original inequality changes sign only when $x = 4$. We check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 4)$	$(4, \infty)$
Sign of $x - 4$	—	—	+
Sign of $(x + 2)^2$	+	+	+
Sign of $(x - 4)(x + 2)^2$	—	—	+

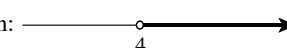
From the table, the solution set is $\{x \mid x \neq -2 \text{ and } x < 4\}$. We exclude the endpoint -2 since the original expression cannot be 0. Interval: $(-\infty, -2) \cup (-2, 4)$.

Graph: 

54. $(x - 4)(x + 2)^2 > 0$. Note that $(x + 2)^2 > 0$ for all $x \neq -2$, so the expression on the left of the original inequality changes sign only at $x = 4$. We check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 4)$	$(4, \infty)$
Sign of $x - 4$	—	—	+
Sign of $(x + 2)^2$	+	+	+
Sign of $(x - 4)(x + 2)^2$	—	—	+

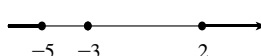
From the table, the solution set is $\{x \mid x > 4\}$. (The point -2 is already excluded.) Interval: $(4, \infty)$.

Graph: 

55. $(x + 3)^2(x - 2)(x + 5) \geq 0$. The left-hand side is 0 when $x = -5$, $x = -3$, or $x = 2$. We check the intervals in the following table.

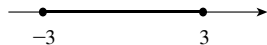
Interval	$(-\infty, -5)$	$(-5, -3)$	$(-3, 2)$	$(2, \infty)$
Sign of $(x + 3)^2$	+	+	+	+
Sign of $x + 5$	—	+	+	+
Sign of $x - 2$	—	—	—	+
Sign of $(x + 3)^2(x - 2)(x + 5)$	+	—	—	+

When $x = -3$, the left-hand side is equal to 0 and the inequality is satisfied. Thus, the solution set is

$\{x \mid x \leq -5, x = -3, \text{ or } x \geq 2\}$. Interval: $(-\infty, -5] \cup \{-3\} \cup [2, \infty)$. Graph: 

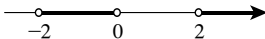
56. $4x^2(x^2 - 9) \leq 0 \Leftrightarrow 4x^2(x + 3)(x - 3) \geq 0$. The expression on the left of the inequality changes sign when $x = \pm 3$ and $x = 0$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, 0)$	$(0, 3)$	$(3, \infty)$
Sign of $4x^2$	+	+	+	+
Sign of $x + 3$	-	+	+	+
Sign of $x - 3$	-	-	-	+
Sign of $4x^2(x^2 - 9)$	+	-	-	+

From the table, the solution set is $\{x \mid -3 \leq x \leq 3\}$. (The endpoint 0 is included since the original expression is allowed to be 0.) Interval: $[-3, 3]$. Graph: 

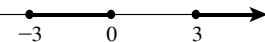
57. $x^3 - 4x > 0 \Leftrightarrow x(x^2 - 4) > 0 \Leftrightarrow x(x + 2)(x - 2) > 0$. The expression on the left of the inequality changes sign where $x = 0$, $x = -2$ and where $x = 4$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
Sign of x	-	-	+	+
Sign of $x + 2$	-	+	+	+
Sign of $x - 2$	-	-	-	+
Sign of $x(x + 2)(x - 2)$	-	+	-	+

From the table, the solution set is $\{x \mid -2 < x < 0 \text{ or } x > 2\}$. Interval: $(-2, 0) \cup (2, \infty)$. Graph: 

58. $9x \leq x^3 \Leftrightarrow 0 \leq x^3 - 9x = x(x^2 - 9) = x(x - 3)(x + 3)$. The expression on the left of the inequality changes sign when $x = -3$, $x = 0$, and $x = 3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, 0)$	$(0, 3)$	$(3, \infty)$
Sign of $x + 3$	-	+	+	+
Sign of x	-	-	+	+
Sign of $x - 3$	-	-	-	+
Sign of $x(x + 3)(x - 3)$	-	+	-	+

From the table, the solution set is $\{x \mid -3 \leq x \leq 0 \text{ or } 3 \leq x\}$. Interval: $[-3, 0] \cup [3, \infty)$. Graph: 

59. $x^4 > x^2 \Leftrightarrow x^4 - x^2 > 0 \Leftrightarrow x^2(x^2 - 1) > 0 \Leftrightarrow x^2(x-1)(x+1) > 0$. The expression on the left of the inequality changes sign where $x = 0$, where $x = 1$, and where $x = -1$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of x^2	+	+	+	+
Sign of $x - 1$	-	-	-	+
Sign of $x + 1$	-	+	+	+
Sign of $x^2(x-1)(x+1)$	+	-	-	+

From the table, the solution set is $\{x \mid x < -1 \text{ or } 1 < x\}$. Interval: $(-\infty, -1) \cup (1, \infty)$. Graph: 

60. $x^5 > x^2 \Leftrightarrow x^5 - x^2 > 0 \Leftrightarrow x^2(x^3 - 1) > 0 \Leftrightarrow x^2(x-1)(x^2+x+1) > 0$. The expression on the left of the inequality

changes sign when $x = 0$ and $x = 1$. But the solution of $x^2 + x + 1 = 0$ are $x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2}$.

Since these are not real solutions. The expression $x^2 + x + 1$ does not change signs, so we must check the intervals in the following table.

Interval	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
Sign of x^2	+	+	+
Sign of $x - 1$	-	-	+
Sign of $x^2 + x + 1$	+	+	+
Sign of $x^2(x-1)(x^2+x+1)$	-	-	+

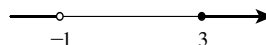
From the table, the solution set is $\{x \mid 1 < x\}$. Interval: $(1, \infty)$. Graph: 

61. $\frac{x-3}{x+1} \geq 0$. The expression on the left of the inequality changes sign where $x = -1$ and where $x = 3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, 3)$	$(3, \infty)$
Sign of $x + 1$	-	+	+
Sign of $x - 3$	-	-	+
Sign of $\frac{x-3}{x+1}$	+	-	+

From the table, the solution set is $\{x \mid x < -1 \text{ or } x \leq 3\}$. Since the denominator cannot equal 0 we must have $x \neq -1$.

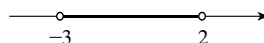
Interval: $(-\infty, -1) \cup [3, \infty)$.

Graph: 

62. $\frac{2x+6}{x-2} < 0$. The expression on the left of the inequality changes sign when $x = -3$ and $x = 2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, 2)$	$(2, \infty)$
Sign of $2x + 6$	-	+	+
Sign of $x - 2$	-	-	+
Sign of $\frac{2x+6}{x-2}$	+	-	+

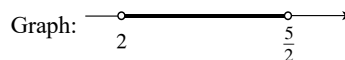
From the table, the solution set is $\{x \mid -3 < x < 2\}$. Interval: $(-3, 2)$.

Graph: 

63. $\frac{x}{x-2} > 5 \Leftrightarrow \frac{x}{x-2} - 5 > 0 \Leftrightarrow \frac{x}{x-2} - \frac{5(x-2)}{x-2} > 0 \Leftrightarrow \frac{-4x+10}{x-2} > 0 \Leftrightarrow \frac{-2x+5}{x-2} > 0$. The expression on the left of the inequality changes sign when $x = 2$ and $x = \frac{5}{2}$. Thus we must check the intervals in the following table.

Interval	$(-\infty, 2)$	$(2, \frac{5}{2})$	$(\frac{5}{2}, \infty)$
Sign of $-2x + 5$	+	+	-
Sign of $x - 2$	-	+	+
Sign of $\frac{-2x+5}{x-2}$	-	+	-

From the table, the solution set is $\{x \mid 2 < x < \frac{5}{2}\}$. Interval: $(2, \frac{5}{2})$.

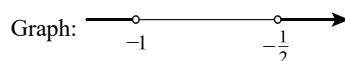


64. $\frac{x-4}{2x+1} < 5 \Leftrightarrow \frac{x-4}{2x+1} - 5 < 0 \Leftrightarrow \frac{x-4-5(2x+1)}{2x+1} < 0 \Leftrightarrow \frac{-9x-9}{2x+1} < 0$. The expression on the left of the inequality changes sign when $x = -1$ and $x = -\frac{1}{2}$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, -\frac{1}{2})$	$(-\frac{1}{2}, \infty)$
Sign of $-9x - 9$	+	-	-
Sign of $2x + 1$	-	-	+
Sign of $\frac{-9x-9}{2x+1}$	-	+	-

From the table, the solution set is $\{x \mid x < -1 \text{ or } -\frac{1}{2} < x < \infty\}$.

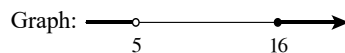
Interval: $(-\infty, -1) \cup (-\frac{1}{2}, \infty)$.



65. $\frac{2x+1}{x-5} \leq 3 \Leftrightarrow \frac{2x+1}{x-5} - 3 \leq 0 \Leftrightarrow \frac{2x+1-3(x-5)}{x-5} \leq 0 \Leftrightarrow \frac{-x+16}{x-5} \leq 0$. The expression on the left of the inequality changes sign where $x = 16$ and where $x = 5$. Thus we must check the intervals in the following table.

Interval	$(-\infty, 5)$	$(5, 16)$	$(16, \infty)$
Sign of $-x + 16$	+	+	-
Sign of $x - 5$	-	+	+
Sign of $\frac{-x+16}{x-5}$	-	+	-

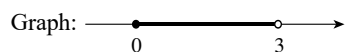
From the table, the solution set is $\{x \mid x < 5 \text{ or } x \geq 16\}$. Since the denominator cannot equal 0, we must have $x \neq 5$. Interval: $(-\infty, 5) \cup [16, \infty)$.



66. $\frac{3+x}{3-x} \geq 1 \Leftrightarrow \frac{3+x}{3-x} - 1 \geq 0 \Leftrightarrow \frac{3+x-3+x}{3-x} \geq 0 \Leftrightarrow \frac{2x}{3-x} \geq 0$. The expression on the left of the inequality changes sign when $x = 0$ and $x = 3$. Thus we must check the intervals in the following table.

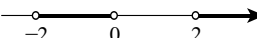
Interval	$(-\infty, 0)$	$(0, 3)$	$(3, \infty)$
Sign of $3 - x$	+	+	-
Sign of $2x$	-	+	+
Sign of $\frac{2x}{3-x}$	-	+	-

Since the denominator cannot equal 0, we must have $x \neq 3$. The solution set is $\{x \mid 0 \leq x < 3\}$. Interval: $[0, 3)$.



67. $\frac{4}{x} < x \Leftrightarrow \frac{4}{x} - x < 0 \Leftrightarrow \frac{4}{x} - \frac{x \cdot x}{x} < 0 \Leftrightarrow \frac{4 - x^2}{x} < 0 \Leftrightarrow \frac{(2 - x)(2 + x)}{x} < 0$. The expression on the left of the inequality changes sign where $x = 0$, where $x = -2$, and where $x = 2$. Thus we must check the intervals in the following table.

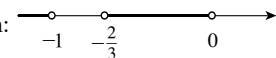
Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
Sign of $2 + x$	−	+	+	+
Sign of x	−	−	+	+
Sign of $2 - x$	+	+	+	−
Sign of $\frac{(2 - x)(2 + x)}{x}$	+	−	+	−

From the table, the solution set is $\{x \mid -2 < x < 0 \text{ or } 2 < x\}$. Interval: $(-2, 0) \cup (2, \infty)$. Graph: 

68. $\frac{x}{x+1} > 3x \Leftrightarrow \frac{x}{x+1} - 3x > 0 \Leftrightarrow \frac{x}{x+1} - \frac{3x(x+1)}{x+1} > 0 \Leftrightarrow \frac{-2x - 3x^2}{x+1} > 0 \Leftrightarrow \frac{-x(2+3x)}{x+1} > 0$. The expression on the left of the inequality changes sign when $x = 0$, $x = -\frac{2}{3}$, and $x = -1$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, -\frac{2}{3})$	$(-\frac{2}{3}, 0)$	$(0, \infty)$
Sign of $-x$	+	+	+	−
Sign of $2 + 3x$	−	−	+	+
Sign of $x + 1$	−	+	+	+
Sign of $\frac{-x(2+3x)}{x+1}$	+	−	+	−

From the table, the solution set is $\{x \mid x < -1 \text{ or } -\frac{2}{3} < x < 0\}$. Interval: $(-\infty, -1) \cup (-\frac{2}{3}, 0)$.

Graph: 

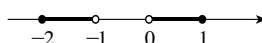
$$69. 1 + \frac{2}{x+1} \leq \frac{2}{x} \Leftrightarrow 1 + \frac{2}{x+1} - \frac{2}{x} \leq 0 \Leftrightarrow \frac{x(x+1)}{x(x+1)} + \frac{2x}{x(x+1)} - \frac{2(x+1)}{x(x+1)} \leq 0 \Leftrightarrow \frac{x^2 + x + 2x - 2x - 2}{x(x+1)} \leq 0 \Leftrightarrow$$

$$\frac{x^2 + x - 2}{x(x+1)} \leq 0 \Leftrightarrow \frac{(x+2)(x-1)}{x(x+1)} \leq 0. \text{ The expression on the left of the inequality changes sign where } x = -2, \text{ where}$$

$x = -1$, where $x = 0$, and where $x = 1$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of $x + 2$	−	+	+	+	+
Sign of $x - 1$	−	−	−	−	+
Sign of x	−	−	−	+	+
Sign of $x + 1$	−	−	+	+	+
Sign of $\frac{(x+2)(x-1)}{x(x+1)}$	+	−	+	−	+

Since $x = -1$ and $x = 0$ yield undefined expressions, we cannot include them in the solution. From the table, the solution

set is $\{x \mid -2 \leq x < -1 \text{ or } 0 < x \leq 1\}$. Interval: $[-2, -1) \cup (0, 1]$. Graph: 

$$70. \frac{3}{x-1} - \frac{4}{x} \geq 1 \Leftrightarrow \frac{3}{x-1} - \frac{4}{x} - 1 \geq 0 \Leftrightarrow \frac{3x}{x(x-1)} - \frac{4(x-1)}{x(x-1)} - \frac{x(x-1)}{x(x-1)} \geq 0 \Leftrightarrow \frac{3x - 4x + 4 - x^2 + x}{x(x-1)} \geq 0 \Leftrightarrow$$

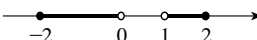
$$\frac{4 - x^2}{x(x-1)} \geq 0 \Leftrightarrow \frac{(2-x)(2+x)}{x(x-1)} \geq 0. \text{ The expression on the left of the inequality changes sign when } x = 2, x = -2,$$

$x = 0$, and $x = 1$.

Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 1)$	$(1, 2)$	$(2, \infty)$
Sign of $2 - x$	+	+	+	+	−
Sign of $2 + x$	−	+	+	+	+
Sign of x	−	−	+	+	+
Sign of $x - 1$	−	−	−	+	+
Sign of $\frac{(2-x)(2+x)}{x(x-1)}$	−	+	−	+	−

Since $x = 0$ and $x = 1$ give undefined expressions, we cannot include them in the solution. From the table, the solution set

is $\{x \mid -2 \leq x < 0 \text{ or } 1 < x \leq 2\}$. Interval: $[-2, 0) \cup (1, 2]$. Graph: 

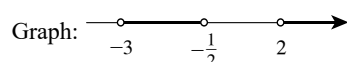
$$71. \frac{x+2}{x+3} < \frac{x-1}{x-2} \Leftrightarrow \frac{x+2}{x+3} - \frac{x-1}{x-2} < 0 \Leftrightarrow \frac{(x+2)(x-2)}{(x+3)(x-2)} - \frac{(x-1)(x+3)}{(x-2)(x+3)} < 0 \Leftrightarrow$$

$$\frac{x^2 - 4 - x^2 - 2x + 3}{(x+3)(x-2)} < 0 \Leftrightarrow \frac{-2x-1}{(x+3)(x-2)} < 0. \text{ The expression on the left of the inequality}$$

changes sign where $x = -\frac{1}{2}$, where $x = -3$, and where $x = 2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, -\frac{1}{2})$	$(-\frac{1}{2}, 2)$	$(2, \infty)$
Sign of $-2x - 1$	+	+	-	-
Sign of $x + 3$	-	+	+	+
Sign of $x - 2$	-	-	-	+
Sign of $\frac{-2x-1}{(x+3)(x-2)}$	+	-	+	-

From the table, the solution set is $\{x \mid -3 < x < -\frac{1}{2} \text{ or } 2 < x\}$. Interval: $(-3, -\frac{1}{2}) \cup (2, \infty)$.



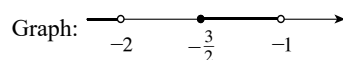
$$72. \frac{1}{x+1} + \frac{1}{x+2} \leq 0 \Leftrightarrow \frac{x+2}{(x+1)(x+2)} + \frac{x+1}{(x+1)(x+2)} \leq 0 \Leftrightarrow \frac{x+2+x+1}{(x+1)(x+2)} \leq 0 \Leftrightarrow \frac{2x+3}{(x+1)(x+2)} \leq 0. \text{ The}$$

expression on the left of the inequality changes sign when $x = -\frac{3}{2}$, $x = -1$, and $x = -2$. Thus we must check the intervals in the following table.

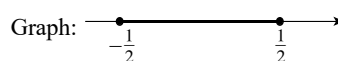
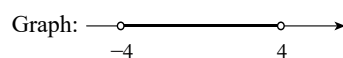
Interval	$(-\infty, -2)$	$(-2, -\frac{3}{2})$	$(-\frac{3}{2}, -1)$	$(-1, \infty)$
Sign of $2x + 3$	-	-	+	+
Sign of $x + 1$	-	-	-	+
Sign of $x + 2$	-	+	+	+
Sign of $\frac{2x+3}{(x+1)(x+2)}$	-	+	-	+

From the table, the solution set is $\{x \mid x < -2 \text{ or } -\frac{3}{2} \leq x < -1\}$. The points $x = -2$ and $x = -1$ are

excluded from the solution because the expression is undefined at those values. Interval: $(-\infty, -2) \cup [-\frac{3}{2}, -1)$.

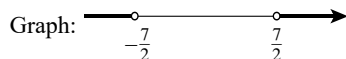


$$73. |5x| < 20 \Leftrightarrow |x| < 4 \Leftrightarrow -4 < x < 4. \text{ Interval: } (-4, 4). \quad 74. |16x| \leq 8 \Leftrightarrow |x| \leq \frac{1}{2} \Leftrightarrow -\frac{1}{2} \leq x \leq \frac{1}{2}. \text{ Interval: } [-\frac{1}{2}, \frac{1}{2}].$$

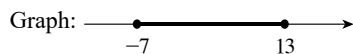


75. $|2x| > 7$ is equivalent to $2x > 7 \Leftrightarrow x > \frac{7}{2}$; or $2x < 7 \Leftrightarrow x < \frac{7}{2}$.

Interval: $(-\infty, -\frac{7}{2}) \cup (\frac{7}{2}, \infty)$.

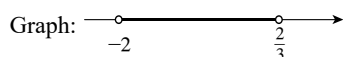


77. $|x - 3| \leq 10 \Leftrightarrow -10 \leq x - 3 \leq 10 \Leftrightarrow -7 \leq x \leq 13$.
Interval: $[-7, 13]$.



79. $|3x + 2| < 4 \Leftrightarrow -4 < 3x + 2 < 4 \Leftrightarrow -6 < 3x < 2 \Leftrightarrow -2 < x < \frac{2}{3}$.

Interval: $(-2, \frac{2}{3})$.



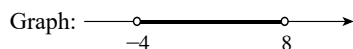
81. $|3x - 2| \geq 5 \Leftrightarrow |x - \frac{2}{3}| \geq \frac{5}{3}$, so either $x - \frac{2}{3} \geq \frac{5}{3} \Leftrightarrow x \geq \frac{7}{3}$ or $x - \frac{2}{3} \leq -\frac{5}{3} \Leftrightarrow x \leq -1$.

Interval: $(-\infty, -1] \cup [\frac{7}{3}, \infty)$.

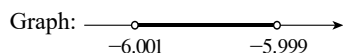


83. $|\frac{x-2}{3}| < 2 \Leftrightarrow -2 < \frac{x-2}{3} < 2 \Leftrightarrow -6 < x-2 < 6 \Leftrightarrow -4 < x < 8$.

Interval: $(-4, 8)$.



85. $|x + 6| < 0.001 \Leftrightarrow -0.001 < x + 6 < 0.001 \Leftrightarrow -6.001 < x < -5.999$.
Interval: $(-6.001, -5.999)$.



76. $\frac{1}{2}|x| \geq 1 \Leftrightarrow |x| \geq 2$ is equivalent to $x \geq 2$ or $x \leq -2$.

Interval: $(-\infty, -2] \cup [2, \infty)$.



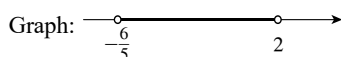
78. $|x + 1| \geq 1$ is equivalent to $x + 1 \geq 1 \Leftrightarrow x \geq 0$; or $x + 1 \leq -1 \Leftrightarrow x \leq -2$.

Interval: $(-\infty, -2] \cup [0, \infty)$.



80. $|5x - 2| < 8 \Leftrightarrow -8 < 5x - 2 < 8 \Leftrightarrow -6 < 5x < 10 \Leftrightarrow -\frac{6}{5} < x < 2$.

Interval: $(-\frac{6}{5}, 2)$.



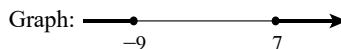
82. $|3x - 4| \geq 5 \Leftrightarrow |x - \frac{4}{3}| \geq \frac{5}{3}$, so either $x - \frac{4}{3} \geq \frac{5}{3} \Leftrightarrow x \geq 3$ or $x - \frac{4}{3} \leq -\frac{5}{3} \Leftrightarrow x \leq -\frac{1}{3}$.

Interval: $(-\infty, -\frac{1}{3}] \cup [3, \infty)$.



84. $|\frac{x+1}{2}| \geq 4 \Leftrightarrow |\frac{1}{2}(x+1)| \geq 4 \Leftrightarrow \frac{1}{2}|x+1| \geq 4 \Leftrightarrow |x+1| \geq 8$ which is equivalent either to $x+1 \geq 8 \Leftrightarrow x \geq 7$, or to $x+1 \leq -8 \Leftrightarrow x \leq -9$.

Interval: $(-\infty, -9] \cup [7, \infty)$.



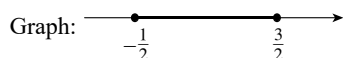
86. $3 - |2x + 4| \leq 1 \Leftrightarrow -|2x + 4| \leq -2 \Leftrightarrow |2x + 4| \geq 2$ which is equivalent to either $2x + 4 \geq 2 \Leftrightarrow 2x \geq -2 \Leftrightarrow x \geq -1$; or $2x + 4 \leq -2 \Leftrightarrow 2x \leq -6 \Leftrightarrow x \leq -3$.

Interval: $(-\infty, -3] \cup [-1, \infty)$.



$$87. 8 - |2x - 1| \geq 6 \Leftrightarrow -|2x - 1| \geq -2 \Leftrightarrow |2x - 1| \leq 2 \Leftrightarrow -2 \leq 2x - 1 \leq 2 \Leftrightarrow -1 \leq 2x \leq 3 \Leftrightarrow -\frac{1}{2} \leq x \leq \frac{3}{2}.$$

Interval: $\left[-\frac{1}{2}, \frac{3}{2}\right]$.



$$88. 7|x + 2| + 5 > 4 \Leftrightarrow 7|x + 2| > -1 \Leftrightarrow |x + 2| > -\frac{1}{7}.$$

Since the absolute value is always nonnegative, the inequality is true for all real numbers. In interval notation, this is $(-\infty, \infty)$.

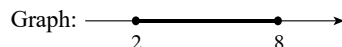


$$89. 0 < 5 - 2x \Leftrightarrow 2x < 5 \Leftrightarrow x < \frac{5}{2}$$

Interval: $(-\infty, \frac{5}{2})$. Graph:

$$90. |x - 5| \leq 3 \Leftrightarrow -3 \leq x - 5 \leq 3 \Leftrightarrow 2 \leq x \leq 8.$$

Interval: $[2, 8]$.



91. $16x \leq x^3 \Leftrightarrow 0 \leq x^3 - 16x = x(x^2 - 16) = x(x - 4)(x + 4)$. The expression on the left of the inequality changes sign when $x = -4$, $x = 0$, and $x = 4$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -4)$	$(-4, 0)$	$(0, 4)$	$(4, \infty)$
Sign of $x + 4$	-	+	+	+
Sign of x	-	-	+	+
Sign of $x - 4$	-	-	-	+
Sign of $x(x + 4)(x - 4)$	-	+	-	+

From the table, the solution set is $\{x \mid -4 \leq x \leq 0 \text{ or } 4 \leq x\}$. Interval: $[-4, 0] \cup [4, \infty)$. Graph:

$$92. \frac{6}{x-1} - \frac{6}{x} \geq 1 \Leftrightarrow \frac{6}{x-1} - \frac{6}{x} - 1 \geq 0 \Leftrightarrow \frac{6x}{x(x-1)} - \frac{6(x-1)}{x(x-1)} - \frac{x(x-1)}{x(x-1)} \geq 0 \Leftrightarrow$$

$$\frac{6x - 6x + 6 - x^2 + x}{x(x-1)} \geq 0 \Leftrightarrow \frac{-x^2 + x + 6}{x(x-1)} \geq 0 \Leftrightarrow \frac{(-x+3)(x+2)}{x(x-1)} \geq 0. \text{ The}$$

expression on the left of the inequality changes sign where $x = 3$, where $x = -2$, where $x = 0$, and where $x = 1$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 1)$	$(1, 3)$	$(3, \infty)$
Sign of $-x + 3$	+	+	+	+	-
Sign of $x + 2$	-	+	+	+	+
Sign of x	-	-	+	+	+
Sign of $x - 1$	-	-	-	+	+
Sign of $\frac{(-x+3)(x+2)}{x(x-1)}$	-	+	-	+	-

From the table, the solution set is $\{x \mid -2 \leq x < 0 \text{ or } 1 < x \leq 3\}$. The points $x = 0$ and $x = 1$ are excluded from the

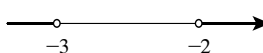
solution set because they make the denominator zero. Interval: $[-2, 0) \cup (1, 3]$. Graph:

93. $x^2 + 5x + 6 > 0 \Leftrightarrow (x + 3)(x + 2) > 0$. The expression on the left of the inequality changes sign when $x = -3$ and $x = -2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, -2)$	$(-2, \infty)$
Sign of $x + 3$	—	+	+
Sign of $x + 2$	—	—	+
Sign of $(x + 3)(x + 2)$	+	—	+

From the table, the solution set is $\{x \mid x < -3 \text{ or } -2 < x\}$.

Interval: $(-\infty, -3) \cup (-2, \infty)$.

Graph: 

94. $5 \leq 3x - 4 \leq 14 \Leftrightarrow 9 \leq 3x \leq 18 \Leftrightarrow 3 \leq x \leq 6$. Interval: $[3, 6]$. Graph: 

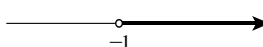
95. $(x + 3)^2(x + 1) > 0$. Note that $(x + 3)^2 > 0$ for all $x \neq -3$, so the expression on the left of the original inequality changes sign only when $x = -1$. We check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, -1)$	$(-1, \infty)$
Sign of $(x + 3)^2$	+	+	+
Sign of $x + 1$	—	—	+
Sign of $(x + 3)^2(x + 1)$	—	—	+

From the table, the solution set is $\{x \mid x > -1\}$.

(The endpoint -3 is already excluded.)

Interval: $(-1, \infty)$.

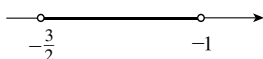
Graph: 

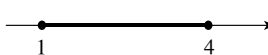
96. $\frac{x}{x+1} > 3 \Leftrightarrow \frac{x}{x+1} - 3 > 0 \Leftrightarrow \frac{x}{x+1} - \frac{3(x+1)}{x+1} > 0 \Leftrightarrow \frac{-2x-3}{x+1} > 0$. The expression on the left of the inequality changes sign when $x = -\frac{3}{2}$ and $x = -1$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -\frac{3}{2})$	$(-\frac{3}{2}, -1)$	$(-1, \infty)$
Sign of $-2x - 3$	+	—	—
Sign of $x + 1$	—	—	+
Sign of $\frac{-2x-3}{x+1}$	—	+	—

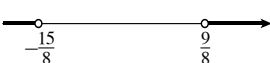
From the table, the solution set is

$\{x \mid -\frac{3}{2} < x < -1\}$. Interval: $(-\frac{3}{2}, -1)$.

Graph: 

97. $1 \geq 3 - 2x \geq -5 \Leftrightarrow -2 \geq -2x \geq -8 \Leftrightarrow 1 \leq x \leq 4$. Interval: $[1, 4]$. Graph: 

98. $|8x + 3| > 12 \Leftrightarrow 8x + 3 > 12 \text{ or } 8x + 3 < -12 \Leftrightarrow 8x > 9 \text{ or } 8x < -15 \Leftrightarrow x > \frac{9}{8} \text{ or } x < -\frac{15}{8}$. Intervals:

$(-\infty, -\frac{15}{8}) \cup (\frac{9}{8}, \infty)$. Graph: 

99. $|x| < 3$

100. $|x| > 2$

101. $|x - 7| \geq 5$

102. $|x - 2| \leq 4$

103. $|x| \leq 2$

104. $|x| \geq 1$

105. $|x| > 3$

106. $|x| < 4$

107. $|x - 1| \leq 3$

108. $|x + 2| > 1$

109. For $\sqrt{x^2 - 9}$ to be defined as a real number we must have $x^2 - 9 \geq 0 \Leftrightarrow (x + 3)(x - 3) \geq 0$. The expression in the inequality changes sign at $x = -3$ and $x = 3$.

Interval	$(-\infty, -3)$	$(-3, 3)$	$(3, \infty)$
Sign of $x + 3$	−	+	+
Sign of $x - 3$	−	−	+
Sign of $(x + 3)(x - 3)$	+	−	+

Thus $x \leq -3$ or $x \geq 3$.

110. For $\sqrt{x^2 - 5x - 50}$ to be defined as a real number, we must have $x^2 - 5x - 50 \geq 0 \Leftrightarrow (x + 5)(x - 10) \geq 0$. The expression on the left of the inequality changes sign when $x = -5$ and $x = 10$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -5)$	$(-5, 10)$	$(10, \infty)$
Sign of $x + 5$	−	+	+
Sign of $x - 10$	−	−	+
Sign of $(x + 5)(x - 10)$	+	−	+

Thus $x \leq -5$ or $x \geq 10$.

111. For $\left(\frac{1}{x^2 - 3x - 10}\right)^{1/2}$ to be defined as a real number we must have $x^2 - 3x - 10 > 0 \Leftrightarrow (x + 2)(x - 5) > 0$. The expression in the last inequality changes sign at $x = -2$ and $x = 5$.

Interval	$(-\infty, -2)$	$(-2, 5)$	$(5, \infty)$
Sign of $x + 2$	−	+	+
Sign of $x - 5$	−	−	+
Sign of $(x + 2)(x - 5)$	+	−	+

Thus $x < -2$ or $x > 5$, and the solution set is $(-\infty, -2) \cup (5, \infty)$.

112. For $\sqrt[4]{\frac{1-x}{2+x}}$ to be defined as a real number we must have $\frac{1-x}{2+x} \geq 0$. The expression on the left of the inequality changes sign when $x = 1$ and $x = -2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
Sign of $1 - x$	+	+	−
Sign of $2 + x$	−	+	+
Sign of $\frac{1-x}{2+x}$	−	+	−

Thus $-2 < x \leq 1$ and the solution set is $(-2, 1]$. Note that $x = -2$ has been excluded from the solution set because the expression is undefined at that value.

$$113. a(bx - c) \geq bc \text{ (where } a, b, c > 0) \Leftrightarrow bx - c \geq \frac{bc}{a} \Leftrightarrow bx \geq \frac{bc}{a} + c \Leftrightarrow x \geq \frac{1}{b} \left(\frac{bc}{a} + c \right) = \frac{c}{a} + \frac{c}{b} \Leftrightarrow x \geq \frac{cb + ca}{ab} = \frac{(a+b)c}{ab}.$$

$$114. \text{ We have } a \leq bx + c < 2a, \text{ where } a, b, c > 0 \Leftrightarrow a - c \leq bx < 2a - c \Leftrightarrow \frac{a - c}{b} \leq x < \frac{2a - c}{b}.$$

$$115. a|bx - c| + d \geq 4a, \text{ where } a, b, c > 0 \Leftrightarrow |bx - c| \geq \frac{4a - d}{a} \Leftrightarrow -(bx - c) \geq \frac{4a - d}{a} \text{ or } bx - c \geq \frac{4a - d}{a} \Leftrightarrow -bx \geq \frac{4a - d}{a} - c \text{ or } bx \geq \frac{4a - d}{a} + c \Leftrightarrow x \leq \frac{ac - 4a + d}{ab} \text{ or } x \geq \frac{ac + 4a - d}{ab}.$$

$$116. \left| \frac{bx + c}{a} \right| > 5a \Leftrightarrow -bx - c > 5a^2 \text{ or } bx + c > 5a^2 \Leftrightarrow x < \frac{-5a^2 - c}{b} \text{ or } x > \frac{5a^2 - c}{b}$$

$$117. \text{ Inserting the relationship } C = \frac{5}{9}(F - 32), \text{ we have } 20 \leq C \leq 30 \Leftrightarrow 20 \leq \frac{5}{9}(F - 32) \leq 30 \Leftrightarrow 36 \leq F - 32 \leq 54 \Leftrightarrow 68 \leq F \leq 86.$$

$$118. \text{ Inserting the relationship } F = \frac{9}{5}C + 32, \text{ we have } 50 \leq F \leq 95 \Leftrightarrow 50 \leq \frac{9}{5}C + 32 \leq 95 \Leftrightarrow 18 \leq \frac{9}{5}C \leq 63 \Leftrightarrow 10 \leq C \leq 35.$$

119. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

Let x be the average number of kilometers driven per day. Each day the cost of Plan A is $95 + 0.40x$, and the cost of Plan B is 135. Plan B saves money when $135 < 95 + 0.40x \Leftrightarrow 40 < 0.4x \Leftrightarrow 100 < x$. So Plan B saves money when you average more than 100 kilometers a day.

120. Let m be the number of minutes of international calls placed per month. Then under Plan A, the cost will be $25 + 0.05m$, and under Plan B, the cost will be $5 + 0.12m$. To determine when Plan B is advantageous, we must solve $25 + 0.05m > 5 + 0.12m \Leftrightarrow 20 > 0.07m \Leftrightarrow 285.7 > m$. So Plan B is advantageous if a person places fewer than 286 minutes of international calls per month.

121. We need to solve $6400 \leq 0.35m + 2200 \leq 7100$ for m . So $6400 \leq 0.35m + 2200 \leq 7100 \Leftrightarrow 4200 \leq 0.35m \leq 4900 \Leftrightarrow 12,000 \leq m \leq 14,000$. She plans on driving between 12,000 and 14,000 miles.

122. (a) $T = 20 - \frac{h}{100}$, where T is the temperature in $^{\circ}\text{C}$, and h is the height in meters.
 (b) Solving the expression in part (a) for h , we get $h = 100(20 - T)$. So $0 \leq h \leq 5000 \Leftrightarrow 0 \leq 100(20 - T) \leq 5000 \Leftrightarrow 0 \leq 20 - T \leq 50 \Leftrightarrow -20 \leq -T \leq 30 \Leftrightarrow 20 \geq T \geq -30$. Thus the range of temperature is from 20°C down to -30°C .

123. (a) Let x be the number of \$3 increases. Then the number of seats sold is $120 - x$. So $P = 200 + 3x$
 $\Leftrightarrow 3x = P - 200 \Leftrightarrow x = \frac{1}{3}(P - 200)$. Substituting for x we have that the number of seats sold is
 $120 - x = 120 - \frac{1}{3}(P - 200) = -\frac{1}{3}P + \frac{560}{3}$.
 (b) $90 \leq -\frac{1}{3}P + \frac{560}{3} \leq 115 \Leftrightarrow 270 \leq 360 - P + 200 \leq 345 \Leftrightarrow 270 \leq -P + 560 \leq 345 \Leftrightarrow -290 \leq -P \leq -215 \Leftrightarrow 290 \geq P \geq 215$. Putting this into standard order, we have $215 \leq P \leq 290$. So the ticket prices are between \$215 and \$290.

124. If the customer buys x kilograms of coffee at \$6.50 per kilograms, then their cost c will be $6.50x$. Thus $x = \frac{c}{6.5}$. Since the scale's accuracy is ± 0.03 kg, and the scale shows 3 kg, we have $3 - 0.03 \leq x \leq 3 + 0.03 \Leftrightarrow 2.97 \leq \frac{c}{6.5} \leq 3.03 \Leftrightarrow (6.50)2.97 \leq c \leq (6.50)3.03 \Leftrightarrow 19.305 \leq c \leq 19.695$. Since the customer paid \$19.50, he could have been over- or undercharged by as much as 19.5 cents.

125. $0.0004 \leq \frac{4,000,000}{d^2} \leq 0.01$. Since $d^2 \geq 0$ and $d \neq 0$, we can multiply each expression by d^2 to obtain
 $0.0004d^2 \leq 4,000,000 \leq 0.01d^2$. Solving each pair, we have $0.0004d^2 \leq 4,000,000 \Leftrightarrow d^2 \leq 10,000,000,000$
 $\Rightarrow d \leq 100,000$ (recall that d represents distance, so it is always nonnegative). Solving $4,000,000 \leq 0.01d^2 \Leftrightarrow 400,000,000 \leq d^2 \Rightarrow 20,000 \leq d$. Putting these together, we have $20,000 \leq d \leq 100,000$.

126. $\frac{600,000}{x^2 + 300} < 500 \Leftrightarrow 600,000 < 500(x^2 + 300)$ (Note that $x^2 + 300 \geq 300 > 0$, so we can multiply both sides by the denominator and not worry that we might be multiplying both sides by a negative number or by zero.) $1200 < x^2 + 300 \Leftrightarrow 0 < x^2 - 900 \Leftrightarrow 0 < (x - 30)(x + 30)$. The expression in the inequality changes sign at $x = 30$ and $x = -30$. However, since x represents distance, we must have $x > 0$.

Interval	$(0, 30)$	$(30, \infty)$
Sign of $x - 30$	−	+
Sign of $x + 30$	+	+
Sign of $(x - 30)(x + 30)$	−	+

So $x > 30$ and you must stand at least 30 meters from the center of the fire.

127. **Note: In the first printing of the text, the answer given for part (a) of this exercise is incorrect.**

- (a) We substitute $d = 402$ m and $t = 10$ s into the given formula and find the value of a that results in a quarter-mile time of exactly 10 s: $402 = \frac{1}{2}a(10)^2 \Leftrightarrow 402 = 50a \Leftrightarrow a = \frac{402}{50} = 8.04 \text{ m/s}^2$. Thus, the quarter-mile time will be less than 10 s if $a > 8.04 \text{ m/s}^2$.

- (b) We substitute $a = g = 9.8 \text{ m/s}^2$ and solve for t : $402 = \frac{1}{2}(9.8)t^2 \Leftrightarrow t^2 = \frac{2(402)}{9.8}$. Taking the positive root, we find that the quarter-mile time for a (downward) quarter-mile under Earth's gravity is $\sqrt{\frac{2(402)}{9.8}} \approx 9.1 \text{ s}$.

128. Solve $13 \leq 4.2 + 0.4v - 0.004v^2$ for $15 \leq v \leq 120$. We have $13 \leq 4.2 + 0.4v - 0.004v^2 \Leftrightarrow 0.004v^2 - 0.4v + 8.8 \leq 0 \Leftrightarrow 0.004(v - 67.32)(v - 32.68) \leq 0$. The possible endpoints are $v = 67.32$ and $v = 32.68$.

Interval	(15, 32.68)	(32.68, 67.32)	(67.32, 120)
Sign of $v - 32.68$	−	+	+
Sign of $v - 67.32$	−	−	+
Sign of $(v - 67.32)(v - 32.68)$	+	−	+

Thus, the condition is satisfied when the car is driven between roughly 33 and 67 km/h.

129. $70 \geq v + \frac{v^2}{35} \Leftrightarrow \frac{1}{35}v^2 + v - 70 \leq 0 \Leftrightarrow \left(\frac{1}{35}v - 1\right)(v + 70) \leq 0$. The expression in the inequality changes sign at $v = 35$ and $v = -70$. However, since v represents the speed, we must have $v \geq 0$.

Interval	(0, 35)	(35, ∞)
Sign of $\frac{1}{35}v - 1$	−	+
Sign of $v + 70$	+	+
Sign of $\left(\frac{1}{35}v - 1\right)(v + 70)$	−	+

So you must drive between 0 and 35 km/h.

130. Solve $2400 \leq 20x - (2000 + 8x + 0.0025x^2) \Leftrightarrow 2400 \leq 20x - 2000 - 8x - 0.0025x^2 \Leftrightarrow 0.0025x^2 - 12x + 4400 \leq 0 \Leftrightarrow (0.0025x - 1)(x - 4400) \leq 0$. The expression on the left of the inequality changes sign when $x = 400$ and $x = 4400$.

Since the manufacturer can only sell positive units, we check the intervals in the following table.

Interval	(0, 400)	(400, 4400)	(4400, ∞)
Sign of $0.0025x - 1$	−	+	+
Sign of $x - 4400$	−	−	+
Sign of $(0.0025x - 1)(x - 4400)$	+	−	+

So the manufacturer must sell between 400 and 4400 units to enjoy a profit of at least \$2400.

131. Let x be the length of the garden and w its width. Using the fact that the perimeter is 120 m, we must have $2x + 2w = 120$
 $\Leftrightarrow w = 60 - x$. Now since the area must be at least 800 m^2 , we have $800 < x(60 - x) \Leftrightarrow 800 < 60x - x^2 \Leftrightarrow$
 $x^2 - 60x + 800 < 0 \Leftrightarrow (x - 20)(x - 40) < 0$. The expression in the inequality changes sign at $x = 20$ and $x = 40$.
 However, since x represents length, we must have $x > 0$.

Interval	$(0, 20)$	$(20, 40)$	$(40, \infty)$
Sign of $x - 20$	−	+	+
Sign of $x - 40$	−	−	+
Sign of $(x - 20)(x - 40)$	+	−	+

The length of the garden should be between 20 and 40 meters.

132. (a) Let x be the thickness of the laminate. Then $|x - 0.05| \leq 0.0075$.
 (b) $|x - 0.05| \leq 0.0075 \Leftrightarrow -0.0075 \leq x - 0.05 \leq 0.0075 \Leftrightarrow 0.0425 \leq x \leq 0.0575$.
133. $\left| \frac{h - 170.5}{7.25} \right| \leq 2 \Leftrightarrow -2 \leq \frac{h - 170.5}{7.25} \leq 2 \Leftrightarrow -14.5 \leq h - 170.5 \leq 14.5 \Leftrightarrow 156 \leq h \leq 185$. Thus 95% of the adult males are between 156 cm and 185 cm.

134. *Case 1:* $a < b < 0$ We have $a \cdot a > a \cdot b$, since $a < 0$, and $b \cdot a > b \cdot b$, since $b < 0$. So $a^2 > a \cdot b > b^2$, that is $a < b < 0 \Rightarrow a^2 > b^2$. Continuing, we have $a \cdot a^2 < a \cdot b^2$, since $a < 0$ and $b^2 \cdot a < b^2 \cdot b$, since $b^2 > 0$. So $a^3 < ab^2 < b^3$. Thus $a < b < 0 \Rightarrow a^3 > b^3$. So $a < b < 0 \Rightarrow a^n > b^n$, if n is even, and $a^n < b^n$, if n is odd.
- Case 2:* $0 < a < b$ We have $a \cdot a < a \cdot b$, since $a > 0$, and $b \cdot a < b \cdot b$, since $b > 0$. So $a^2 < a \cdot b < b^2$. Thus $0 < a < b \Rightarrow a^2 < b^2$. Likewise, $a^2 \cdot a < a^2 \cdot b$ and $b \cdot a^2 < b \cdot b^2$, thus $a^3 < b^3$. So $0 < a < b \Rightarrow a^n < b^n$, for all positive integers n .
- Case 3:* $a < 0 < b$ If n is odd, then $a^n < b^n$, because a^n is negative and b^n is positive. If n is even, then we could have either $a^n < b^n$ or $a^n > b^n$. For example, $-1 < 2$ and $(-1)^2 < 2^2$, but $-3 < 2$ and $(-3)^2 > 2^2$.

135. The rule we want to apply here is “ $a < b \Rightarrow ac < bc$ if $c > 0$ and $a < b \Rightarrow ac > bc$ if $c < 0$ ”. Thus we cannot simply multiply by x , since we don’t yet know if x is positive or negative, so in solving $1 < \frac{3}{x}$, we must consider two cases.

Case 1: $x > 0$ Multiplying both sides by x , we have $x < 3$. Together with our initial condition, we have $0 < x < 3$.

Case 2: $x < 0$ Multiplying both sides by x , we have $x > 3$. But $x < 0$ and $x > 3$ have no elements in common, so this gives no additional solution. Hence, the only solutions are $0 < x < 3$.

136. $|x - 1|$ is the distance between x and 1; $|x - 3|$ is the distance between x and 3. So $|x - 1| < |x - 3|$ represents those points closer to 1 than to 3, and the solution is $x < 2$, since 2 is the point halfway between 1 and 3. If $a < b$, then the solution to $|x - a| < |x - b|$ is $x < \frac{a + b}{2}$.

137. $a < b$, so by Rule 1, $a + c < b + c$. Using Rule 1 again, $b + c < b + d$, and so by transitivity, $a + c < b + d$.

138. $\frac{a}{b} < \frac{c}{d}$, so by Rule 3, $d\frac{a}{b} < d\frac{c}{d} \Leftrightarrow \frac{ad}{b} < c$. Adding a to both sides, we have $\frac{ad}{b} + a < c + a$. Rewriting the left-hand side as $\frac{ad}{b} + \frac{ab}{b} = \frac{a(b+d)}{b}$ and dividing both sides by $b+d$ gives $\frac{a}{b} < \frac{a+c}{b+d}$.

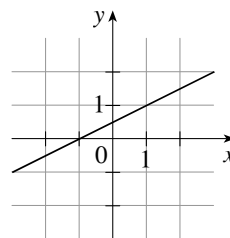
Similarly, $a + c < \frac{cb}{d} + c = \frac{c(b+d)}{d}$, so $\frac{a+c}{b+d} < \frac{c}{d}$.

139. (a) Because x is nonnegative, $x \leq y \Leftrightarrow x^2 \leq xy$, and because y is nonnegative, $x \leq y \Leftrightarrow xy \leq y^2$. Thus, by transitivity, $x^2 \leq y^2$.
- (b) By part (a), $\sqrt{xy} \leq \frac{x+y}{2} \Leftrightarrow xy \leq \frac{(x+y)^2}{4}$. Expanding, this becomes $4xy \leq (x+y)^2 = x^2 + y^2 + 2xy \Leftrightarrow x^2 + y^2 - 2xy \geq 0 \Leftrightarrow (x-y)^2 \geq 0$. This is true for any x and y , so the original inequality is true for all nonnegative x and y .

1.9 THE COORDINATE PLANE; GRAPHS OF EQUATIONS; CIRCLES

1. (a) The point that is 3 units to the right of the y -axis and 5 units below the x -axis has coordinates $(3, -5)$.
- (b) The point $(2, 7)$ is 2 units to the right of the y -axis and 7 units above the x -axis, so it is closer to the y -axis.
2. The distance between the points (a, b) and (c, d) is $\sqrt{(c-a)^2 + (d-b)^2}$. So the distance between $(1, 2)$ and $(7, 10)$ is $\sqrt{(7-1)^2 + (10-2)^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$.
3. The point midway between (a, b) and (c, d) is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$. So the point midway between $(1, 2)$ and $(7, 10)$ is $\left(\frac{1+7}{2}, \frac{2+10}{2}\right) = \left(\frac{8}{2}, \frac{12}{2}\right) = (4, 6)$.
4. If the point $(2, 3)$ is on the graph of an equation in x and y , then the equation is satisfied when we replace x by 2 and y by 3. We check whether $2(3) \stackrel{?}{=} 2+1 \Leftrightarrow 6 \stackrel{?}{=} 3$. This is false, so the point $(2, 3)$ is not on the graph of the equation $2y = x + 1$. To complete the table, we express y in terms of x : $2y = x + 1 \Leftrightarrow y = \frac{1}{2}(x + 1) = \frac{1}{2}x + \frac{1}{2}$.

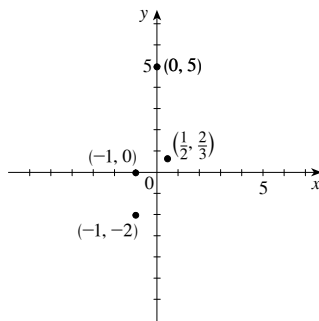
x	y	(x, y)
-2	$-\frac{1}{2}$	$\left(-2, -\frac{1}{2}\right)$
-1	0	$(-1, 0)$
0	$\frac{1}{2}$	$\left(0, \frac{1}{2}\right)$
1	1	$(1, 1)$
2	$\frac{3}{2}$	$\left(2, \frac{3}{2}\right)$



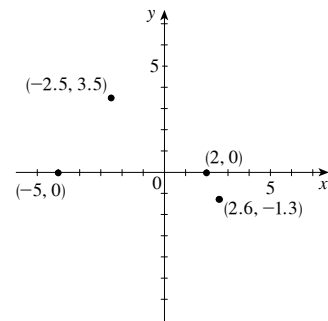
5. (a) To find the x -intercept(s) of the graph of an equation we set y equal to 0 in the equation and solve for x : $2(0) = x + 1 \Leftrightarrow x = -1$, so the x -intercept of $2y = x + 1$ is -1 .
- (b) To find the y -intercept(s) of the graph of an equation we set x equal to 0 in the equation and solve for y : $2y = 0 + 1 \Leftrightarrow y = \frac{1}{2}$, so the y -intercept of $2y = x + 1$ is $\frac{1}{2}$.
6. (a) The graph of the equation $(x-1)^2 + (y-2)^2 = 9$ is a circle with center $(1, 2)$ and radius $\sqrt{9} = 3$.
- (b) In order to just touch the y -axis, a circle centered at $(3, 4)$ must have radius 3 and equation $(x-3)^2 + (y-4)^2 = 9$. It touches the y -axis at the point $(0, 4)$.
7. (a) If a graph is symmetric with respect to the x -axis and (a, b) is on the graph, then $(a, -b)$ is also on the graph.

- (b) If a graph is symmetric with respect to the y -axis and (a, b) is on the graph, then $(-a, b)$ is also on the graph.
- (c) If a graph is symmetric about the origin and (a, b) is on the graph, then $(-a, -b)$ is also on the graph.
8. (a) The x -intercepts are -5 and 3 , and the y -intercepts are ± 2 .
- (b) The graph is symmetric about the x -axis.
9. Yes. If (a, b) is on the graph, then by symmetry about the x -axis, the point $(a, -b)$ is on the graph. Then by symmetry about the y -axis, the point $(-a, -b)$ is on the graph. Thus, the graph is symmetric with respect to the origin.
10. No, the graph is not necessarily symmetric with respect to either axis. For example, the graph of $y = x^3$ is symmetric with respect to the origin, but not with respect to either axis.
11. The points have coordinates $A(5, 1)$, $B(1, 2)$, $C(-2, 6)$, $D(-6, 2)$, $E(-4, -1)$, $F(-2, 0)$, $G(-1, -3)$, and $H(2, -2)$.
12. A and B lie in Quadrant I and E and G lie in Quadrant III.

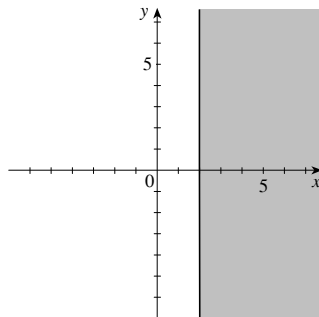
13.



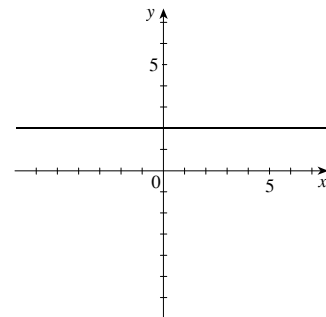
14.



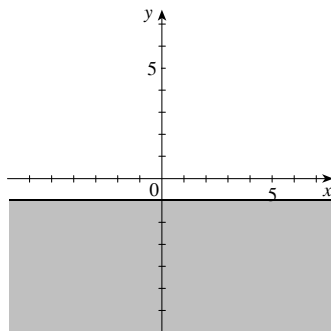
15. (a)



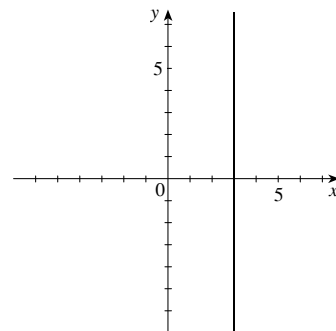
(b)



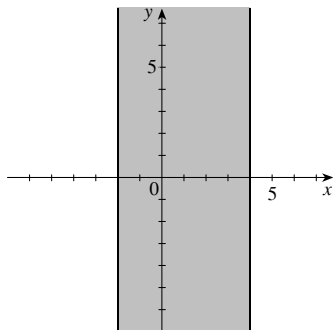
16. (a)



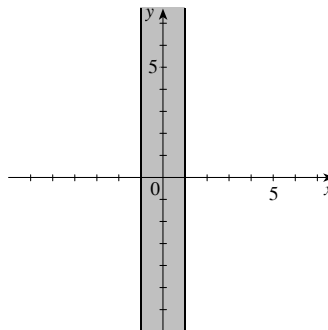
(b)



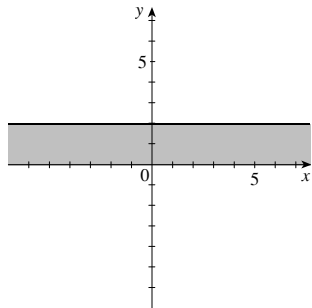
17. (a)



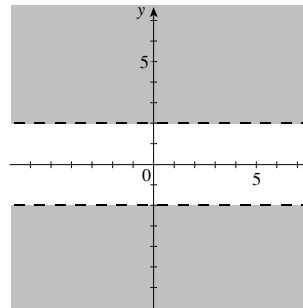
(b)



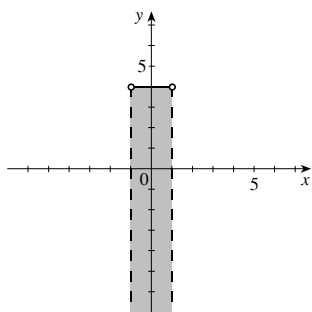
18. (a)



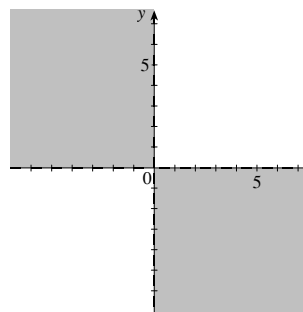
(b)



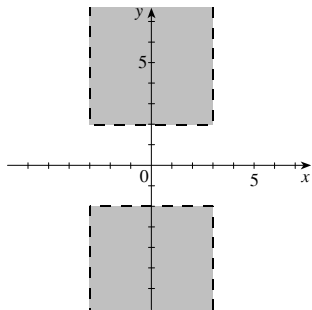
19. (a)



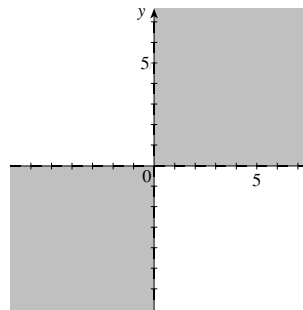
(b)



20. (a)



(b)

21. The two points are $(0, 2)$ and $(3, 0)$.

$$(a) d = \sqrt{(3-0)^2 + (0-2)^2} = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$$

$$(b) \text{ midpoint: } \left(\frac{3+0}{2}, \frac{0+2}{2} \right) = \left(\frac{3}{2}, 1 \right)$$

22. The two points are $(-2, -1)$ and $(2, 2)$.

$$(a) d = \sqrt{(-2-2)^2 + (-1-2)^2} = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$(b) \text{ midpoint: } \left(\frac{-2+2}{2}, \frac{-1+2}{2} \right) = \left(0, \frac{1}{2} \right)$$

23. The two points are $(-3, 3)$ and $(5, -3)$.

(a) $d = \sqrt{(-3 - 5)^2 + (3 - (-3))^2} = \sqrt{(-8)^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10$

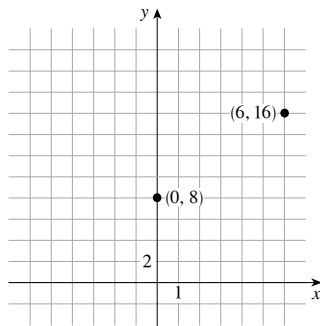
(b) midpoint: $\left(\frac{-3 + 5}{2}, \frac{3 + (-3)}{2}\right) = (1, 0)$

24. The two points are $(-2, -3)$ and $(4, -1)$.

(a) $d = \sqrt{(-2 - 4)^2 + (-3 - (-1))^2} = \sqrt{(-6)^2 + (-2)^2} = \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$

(b) midpoint: $\left(\frac{-2 + 4}{2}, \frac{-3 + (-1)}{2}\right) = (1, -2)$

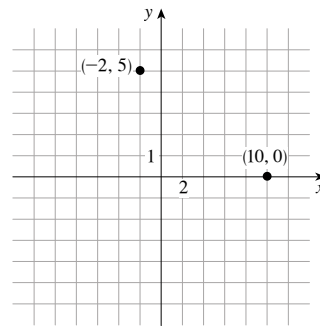
25. (a)



(b) $d = \sqrt{(0 - 6)^2 + (8 - 16)^2}$
 $= \sqrt{(-6)^2 + (-8)^2} = \sqrt{100} = 10$

(c) Midpoint: $\left(\frac{0 + 6}{2}, \frac{8 + 16}{2}\right) = (3, 12)$

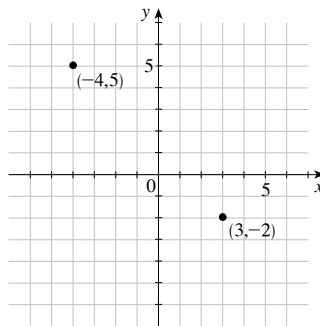
26. (a)



(b) $d = \sqrt{(-2 - 10)^2 + (5 - 0)^2}$
 $= \sqrt{(-12)^2 + (5)^2} = \sqrt{169} = 13$

(c) Midpoint: $\left(\frac{-2 + 10}{2}, \frac{5 + 0}{2}\right) = \left(4, \frac{5}{2}\right)$

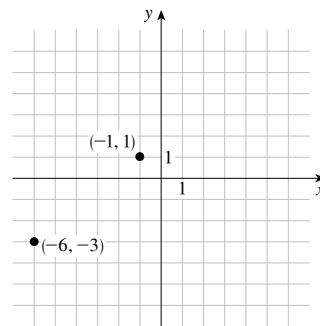
27. (a)



(b) $d = \sqrt{(-3 - (-4))^2 + (-2 - 5)^2}$
 $= \sqrt{7^2 + (-7)^2} = \sqrt{49 + 49} = \sqrt{98} = 7\sqrt{2}$

(c) Midpoint: $\left(\frac{-4 + 3}{2}, \frac{5 - 2}{2}\right) = \left(-\frac{1}{2}, \frac{3}{2}\right)$

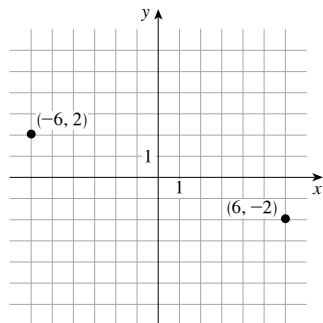
28. (a)



(b) $d = \sqrt{(-1 - (-6))^2 + (1 - (-3))^2}$
 $= \sqrt{5^2 + 4^2} = \sqrt{41}$

(c) Midpoint: $\left(\frac{-6 - 1}{2}, \frac{-3 + 1}{2}\right) = \left(-\frac{7}{2}, -1\right)$

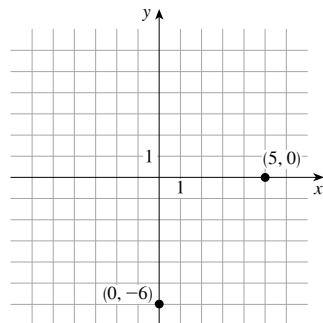
29. (a)



$$(b) d = \sqrt{(6 - (-6))^2 + (-2 - 2)^2} = \sqrt{12^2 + (-4)^2} \\ = \sqrt{144 + 16} = \sqrt{160} = 4\sqrt{10}$$

$$(c) \text{Midpoint: } \left(\frac{6 - 6}{2}, \frac{-2 + 2}{2} \right) = (0, 0)$$

30. (a)



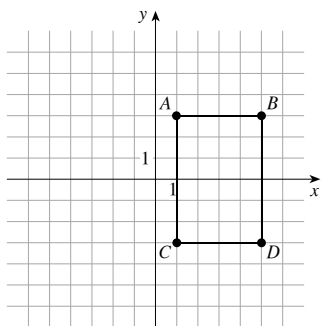
$$(b) d = \sqrt{(0 - 5)^2 + (-6 - 0)^2} \\ = \sqrt{5^2 + (-6)^2} = \sqrt{25 + 36} = \sqrt{61}$$

$$(c) \text{Midpoint: } \left(\frac{0 + 5}{2}, \frac{-6 + 0}{2} \right) = \left(\frac{5}{2}, -3 \right)$$

$$31. d(A, B) = \sqrt{(1 - 5)^2 + (3 - 3)^2} = \sqrt{(-4)^2} = 4.$$

$$d(A, C) = \sqrt{(1 - 1)^2 + (3 - (-3))^2} = \sqrt{(6)^2} = 6. \text{ So}$$

the area is $4 \cdot 6 = 24$.



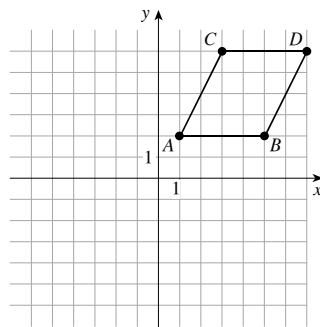
32. The area of a parallelogram is its base times its height.

Since two sides are parallel to the x -axis, we use the length of one of these as the base. Thus, the base is

$$d(A, B) = \sqrt{(1 - 5)^2 + (2 - 2)^2} = \sqrt{(-4)^2} = 4. \text{ The}$$

height is the change in the y coordinates, thus, the height is $6 - 2 = 4$. So the area of the parallelogram is

$$\text{base} \cdot \text{height} = 4 \cdot 4 = 16.$$



33. From the graph, the quadrilateral $ABCD$ has a pair of parallel sides, so $ABCD$ is a trapezoid. The area is

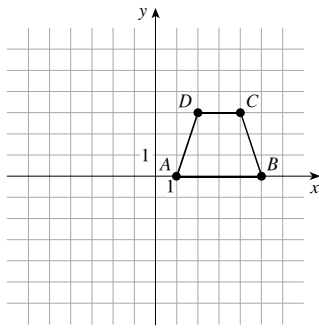
$$\left(\frac{b_1 + b_2}{2}\right)h. \text{ From the graph we see that}$$

$$b_1 = d(A, B) = \sqrt{(1-5)^2 + (0-0)^2} = \sqrt{4^2} = 4;$$

$$b_2 = d(C, D) = \sqrt{(4-2)^2 + (3-3)^2} = \sqrt{2^2} = 2; \text{ and}$$

h is the difference in y -coordinates is $|3-0| = 3$. Thus

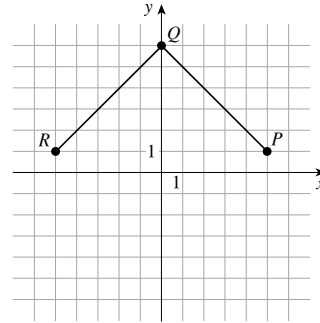
$$\text{the area of the trapezoid is } \left(\frac{4+2}{2}\right)3 = 9.$$



34. The point S must be located at $(0, -4)$. To find the area, we find the length of one side and square it. This gives

$$\begin{aligned} d(Q, R) &= \sqrt{(-5-0)^2 + (1-6)^2} \\ &= \sqrt{(-5)^2 + (-5)^2} \\ &= \sqrt{25+25} = \sqrt{50} \end{aligned}$$

$$\text{So the area is } (\sqrt{50})^2 = 50.$$



$$35. d(0, A) = \sqrt{(6-0)^2 + (7-0)^2} = \sqrt{6^2 + 7^2} = \sqrt{36+49} = \sqrt{85}.$$

$$d(0, B) = \sqrt{(-5-0)^2 + (8-0)^2} = \sqrt{(-5)^2 + 8^2} = \sqrt{25+64} = \sqrt{89}.$$

Thus point $A(6, 7)$ is closer to the origin.

$$36. d(E, C) = \sqrt{(-6-(-2))^2 + (3-1)^2} = \sqrt{(-4)^2 + 2^2} = \sqrt{16+4} = \sqrt{20}.$$

$$d(E, D) = \sqrt{(3-(-2))^2 + (0-1)^2} = \sqrt{5^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26}.$$

Thus point C is closer to point E .

$$37. d(P, R) = \sqrt{(-1-3)^2 + (-1-1)^2} = \sqrt{(-4)^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}.$$

$$d(Q, R) = \sqrt{(-1-(-1))^2 + (-1-3)^2} = \sqrt{0 + (-4)^2} = \sqrt{16} = 4. \text{ Thus point } Q(-1, 3) \text{ is closer to point } R.$$

38. (a) The distance from $(7, 3)$ to the origin is $\sqrt{(7-0)^2 + (3-0)^2} = \sqrt{7^2 + 3^2} = \sqrt{49+9} = \sqrt{58}$. The distance from $(3, 7)$ to the origin is $\sqrt{(3-0)^2 + (7-0)^2} = \sqrt{3^2 + 7^2} = \sqrt{9+49} = \sqrt{58}$. So the points are the same distance from the origin.

- (b) The distance from (a, b) to the origin is $\sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$. The distance from (b, a) to the origin is $\sqrt{(b-0)^2 + (a-0)^2} = \sqrt{b^2 + a^2} = \sqrt{a^2 + b^2}$. So the points are the same distance from the origin.

39. Since we do not know which pair are isosceles, we find the length of all three sides.

$$d(A, B) = \sqrt{(-3-0)^2 + (-1-2)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}.$$

$$d(C, B) = \sqrt{(-3-(-4))^2 + (-1-3)^2} = \sqrt{1^2 + (-4)^2} = \sqrt{1+16} = \sqrt{17}.$$

$$d(A, C) = \sqrt{(0-(-4))^2 + (2-3)^2} = \sqrt{4^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}. \text{ So sides } AC \text{ and } CB \text{ have the same length.}$$

40. Since the side AB is parallel to the x -axis, we use this as the base in the formula $\text{area} = \frac{1}{2}(\text{base} \cdot \text{height})$. The height is the change in the y -coordinates. Thus, the base is $|-2 - 4| = 6$ and the height is $|4 - 1| = 3$. So the area is $\frac{1}{2}(6 \cdot 3) = 9$.

41. (a) Here we have $A = (2, 2)$, $B = (3, -1)$, and $C = (-3, -3)$. So

$$d(A, B) = \sqrt{(3-2)^2 + (-1-2)^2} = \sqrt{1^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10};$$

$$d(C, B) = \sqrt{(3-(-3))^2 + (-1-(-3))^2} = \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10};$$

$$d(A, C) = \sqrt{(-3-2)^2 + (-3-2)^2} = \sqrt{(-5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}.$$

Since $[d(A, B)]^2 + [d(C, B)]^2 = [d(A, C)]^2$, we conclude that the triangle is a right triangle.

- (b) The area of the triangle is $\frac{1}{2} \cdot d(C, B) \cdot d(A, B) = \frac{1}{2} \cdot \sqrt{10} \cdot 2\sqrt{10} = 10$.

42. $d(A, B) = \sqrt{(11-6)^2 + (-3-(-7))^2} = \sqrt{5^2 + 4^2} = \sqrt{25+16} = \sqrt{41};$

$$d(A, C) = \sqrt{(2-6)^2 + (-2-(-7))^2} = \sqrt{(-4)^2 + 5^2} = \sqrt{16+25} = \sqrt{41};$$

$$d(B, C) = \sqrt{(2-11)^2 + (-2-(-3))^2} = \sqrt{(-9)^2 + 1^2} = \sqrt{81+1} = \sqrt{82}.$$

Since $[d(A, B)]^2 + [d(A, C)]^2 = [d(B, C)]^2$, we conclude that the triangle is a right triangle. The area is

$$\frac{1}{2}(\sqrt{41} \cdot \sqrt{41}) = \frac{41}{2}.$$

43. We show that all sides are the same length (its a rhombus) and then show that the diagonals are equal. Here we have

$A = (-2, 9)$, $B = (4, 6)$, $C = (1, 0)$, and $D = (-5, 3)$. So

$$d(A, B) = \sqrt{(4-(-2))^2 + (6-9)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45};$$

$$d(B, C) = \sqrt{(1-4)^2 + (0-6)^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{9+36} = \sqrt{45};$$

$$d(C, D) = \sqrt{(-5-1)^2 + (3-0)^2} = \sqrt{(-6)^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45};$$

$$d(D, A) = \sqrt{(-2-(-5))^2 + (9-3)^2} = \sqrt{3^2 + 6^2} = \sqrt{9+36} = \sqrt{45}. \text{ So the points form a}$$

rhombus. Also $d(A, C) = \sqrt{(1-(-2))^2 + (0-9)^2} = \sqrt{3^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$,

and $d(B, D) = \sqrt{(-5-4)^2 + (3-6)^2} = \sqrt{(-9)^2 + (-3)^2} = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$. Since the diagonals are equal, the rhombus is a square.

44. $d(A, B) = \sqrt{(3-(-1))^2 + (11-3)^2} = \sqrt{4^2 + 8^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}.$

$$d(B, C) = \sqrt{(5-3)^2 + (15-11)^2} = \sqrt{2^2 + 4^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}.$$

$d(A, C) = \sqrt{(5-(-1))^2 + (15-3)^2} = \sqrt{6^2 + 12^2} = \sqrt{36+144} = \sqrt{180} = 6\sqrt{5}$. So $d(A, B) + d(B, C) = d(A, C)$, and the points are collinear.

45. Let $P = (0, y)$ be such a point. Setting the distances equal we get

$$\sqrt{(0-5)^2 + (y-(-5))^2} = \sqrt{(0-1)^2 + (y-1)^2} \Leftrightarrow$$

$\sqrt{25 + y^2 + 10y + 25} = \sqrt{1 + y^2 - 2y + 1} \Rightarrow y^2 + 10y + 50 = y^2 - 2y + 2 \Leftrightarrow 12y = -48 \Leftrightarrow y = -4$. Thus, the point is $P = (0, -4)$. Check:

$$\sqrt{(0-5)^2 + (-4-(-5))^2} = \sqrt{(-5)^2 + 1^2} = \sqrt{25+1} = \sqrt{26};$$

$$\sqrt{(0-1)^2 + (-4-1)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{25+1} = \sqrt{26}.$$

46. The midpoint of AB is $C' = \left(\frac{1+3}{2}, \frac{0+6}{2}\right) = (2, 3)$. So the length of the median CC' is $d(C, C') = \sqrt{(2-8)^2 + (3-2)^2} = \sqrt{37}$. The midpoint of AC is $B' = \left(\frac{1+8}{2}, \frac{0+2}{2}\right) = \left(\frac{9}{2}, 1\right)$. So the length of the median BB' is $d(B, B') = \sqrt{\left(\frac{9}{2}-3\right)^2 + (1-6)^2} = \frac{\sqrt{109}}{2}$. The midpoint of BC is $A' = \left(\frac{3+8}{2}, \frac{6+2}{2}\right) = \left(\frac{11}{2}, 4\right)$. So the length of the median AA' is $d(A, A') = \sqrt{\left(\frac{11}{2}-1\right)^2 + (4-0)^2} = \frac{\sqrt{145}}{2}$.

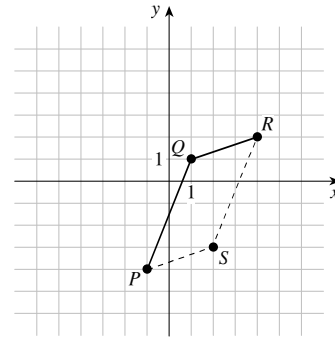
47. As indicated by Example 3, we must find a point $S(x_1, y_1)$ such that the midpoints of PR and of QS are the same. Thus

$$\left(\frac{4+(-1)}{2}, \frac{2+(-4)}{2}\right) = \left(\frac{x_1+1}{2}, \frac{y_1+1}{2}\right).$$

Setting the x -coordinates equal, we get $\frac{4+(-1)}{2} = \frac{x_1+1}{2} \Leftrightarrow 4-1 = x_1+1 \Leftrightarrow x_1 = 2$. Setting the

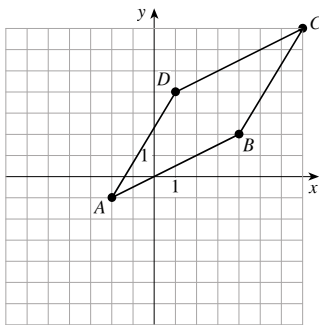
y -coordinates equal, we get $\frac{2+(-4)}{2} = \frac{y_1+1}{2} \Leftrightarrow 2-4 = y_1+1 \Leftrightarrow y_1 = -3$.

Thus $S = (2, -3)$.



48. We solve the equation $6 = \frac{2+x}{2}$ to find the x coordinate of B . This gives $6 = \frac{2+x}{2} \Leftrightarrow 12 = 2+x \Leftrightarrow x = 10$. Likewise, $8 = \frac{3+y}{2} \Leftrightarrow 16 = 3+y \Leftrightarrow y = 13$. Thus, $B = (10, 13)$.

49. (a)



- (b) The midpoint of AC is $\left(\frac{-2+7}{2}, \frac{-1+7}{2}\right) = \left(\frac{5}{2}, 3\right)$, the midpoint of BD is $\left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$.

- (c) Since they have the same midpoint, we conclude that the diagonals bisect each other.

50. We have $M = \left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$. Thus,

$$d(C, M) = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(\frac{b}{2}-0\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2};$$

$$d(A, M) = \sqrt{\left(\frac{a}{2}-a\right)^2 + \left(\frac{b}{2}-0\right)^2} = \sqrt{\left(-\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2};$$

$$d(B, M) = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(\frac{b}{2}-b\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2}.$$

51. $(0, 5)$: $3(0) - 5 + 5 \stackrel{?}{=} 0 \Leftrightarrow 0 - 5 + 5 \stackrel{?}{=} 0$. Yes.

$$(2, 1): 3(2) - 1 + 5 \stackrel{?}{=} 0 \Leftrightarrow 6 - 1 + 5 \stackrel{?}{=} 0. \text{ No.}$$

$$(-2, -1): 3(-2) - (-1) + 5 \stackrel{?}{=} 0 \Leftrightarrow -6 + 1 + 5 \stackrel{?}{=} 0. \text{ Yes.}$$

So $(0, 5)$ and $(-2, -1)$ are points on the graph of this equation.

52. $(1, 1)$: $(1) \left[(1)^2 + 1 \right] \stackrel{?}{=} 1 \Leftrightarrow 1 \stackrel{?}{=} 1$. No.

$\left(1, \frac{1}{2}\right)$: $\left(\frac{1}{2}\right) \left[(1)^2 + 1 \right] \stackrel{?}{=} 1 \Leftrightarrow \frac{1}{2} \stackrel{?}{=} 1$. Yes.

$\left(-1, \frac{1}{2}\right)$: $\left(\frac{1}{2}\right) \left[(-1)^2 + 1 \right] \stackrel{?}{=} 1 \Leftrightarrow \frac{1}{2} \stackrel{?}{=} 1$. Yes.

So both $\left(1, \frac{1}{2}\right)$ and $\left(-1, \frac{1}{2}\right)$ are points on the graph of this equation.

53. $(0, -2)$: $(0)^2 + (0)(-2) + (-2)^2 \stackrel{?}{=} 4 \Leftrightarrow 0 + 0 + 4 \stackrel{?}{=} 4$. Yes.

$(1, -2)$: $(1)^2 + (1)(-2) + (-2)^2 \stackrel{?}{=} 4 \Leftrightarrow 1 - 2 + 4 \stackrel{?}{=} 4$. No.

$(2, -2)$: $(2)^2 + (2)(-2) + (-2)^2 \stackrel{?}{=} 4 \Leftrightarrow 4 - 4 + 4 \stackrel{?}{=} 4$. Yes.

So $(0, -2)$ and $(2, -2)$ are points on the graph of this equation.

54. $(0, 1)$: $(0)^2 + (1)^2 - 1 \stackrel{?}{=} 0 \Leftrightarrow 0 + 1 - 1 \stackrel{?}{=} 0$. Yes.

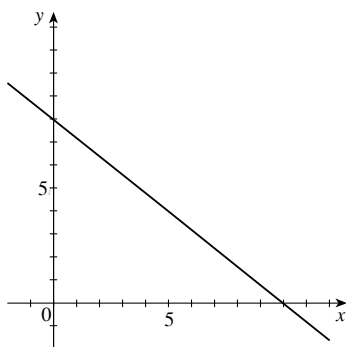
$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$: $\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1 \stackrel{?}{=} 0 \Leftrightarrow \frac{1}{2} + \frac{1}{2} - 1 \stackrel{?}{=} 0$. Yes.

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$: $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - 1 \stackrel{?}{=} 0 \Leftrightarrow \frac{3}{4} + \frac{1}{4} - 1 \stackrel{?}{=} 0$. Yes.

So $(0, 1)$, $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, and $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ are all points on the graph of this equation.

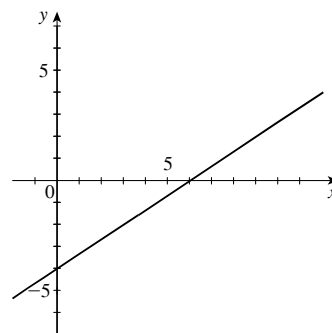
55. $4x + 5y = 40 \Leftrightarrow y = 8 - \frac{4}{5}x$

x	y
-2	9.6
0	8
2	6.4
4	4.8
6	3.2
8	1.6
10	0
12	-1.6



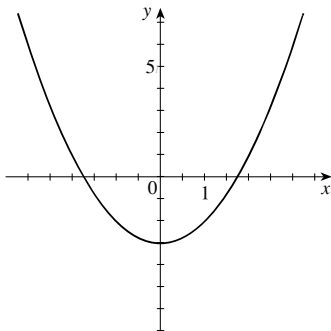
56. $2x - 3y = 12 \Leftrightarrow y = \frac{2}{3}x - 4$

x	y
-3	-6
-1	-4.7
0	-4
1	-3.3
3	-2
6	0
9	2
12	4



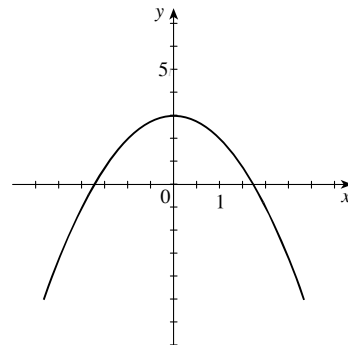
57. $y = x^2 - 3$

x	y
-3	6
-2	1
-1	-2
0	-3
1	-2
2	1
3	6



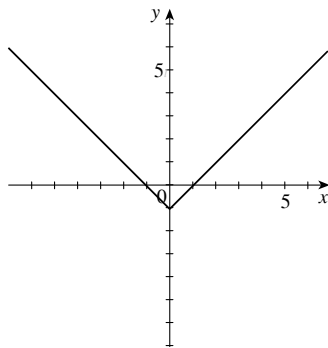
58. $y = 3 - x^2$

x	y
-3	-6
-2	-1
-1	2
0	3
1	2
2	-1
3	-6



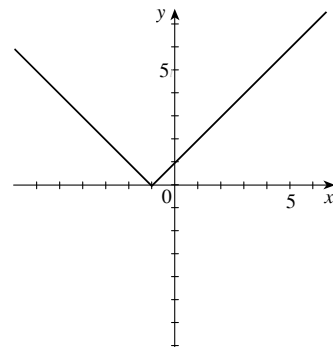
59. $y = |x| - 1$

x	y
-5	4
-3	2
-1	0
0	-1
1	0
3	2
5	4



60. $y = |x + 1|$

x	y
-5	4
-3	2
-1	0
0	1
1	2
3	4
5	6



61. (a) Solve for y : $2x - y = 6 \Leftrightarrow y = 2x - 6$.

x	y
-2	-10
-1	-8
0	-6
1	-4
2	-2
3	0
4	2

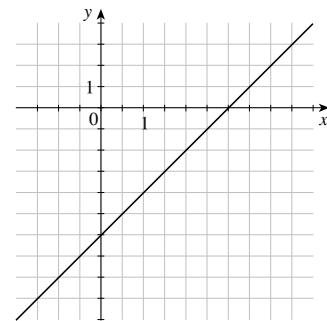
$y = 0 \Rightarrow 0 = 2x - 6 \Leftrightarrow x = 3$, so the x -intercept is 3, and

$x = 0 \Rightarrow y = 2(0) - 6 = -6$, so the y -intercept is -6.

x -axis symmetry: $2x + y = 6$, which is not the same as $2x - y = 6$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $-2x - y = 6$, which is not the same as $2x - y = 6$, so the graph is not symmetric with respect to the y -axis.

Origin symmetry: $-2x + y = 6$, which is not the same as $2x - y = 6$, so the graph is not symmetric with respect to the origin.



(b) $y = 2(x - 1)^2$

x	y
-2	18
-1	8
0	2
1	0
2	2
3	8

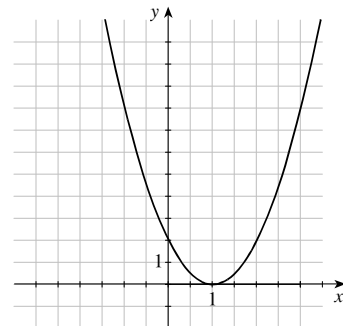
$y = 0 \Rightarrow 0 = 2(x - 1)^2 \Rightarrow x = 1$, so the x -intercept is 1,

and $x = 0 \Rightarrow y = 2(0 - 1)^2 = 2$, so the y -intercept is 2.

x -axis symmetry: $-y = 2(x - 1)^2$, which is not the same as $y = 2(x - 1)^2$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = 2(-x - 1)^2$, which is not the same as $y = 2(x - 1)^2$, so the graph is not symmetric with respect to the y -axis.

Origin symmetry: $-y = 2(-x - 1)^2 \Leftrightarrow y = -2(x + 1)^2$, which is not the same as $y = 2(x - 1)^2$, so the graph is not symmetric with respect to the origin.



62. (a) Solve for y : $x - 4y = 8 \Leftrightarrow y = \frac{1}{4}x - 2$.

x	y
-2	$-\frac{5}{2}$
0	-2
2	$-\frac{3}{2}$
4	-1
8	0
12	1

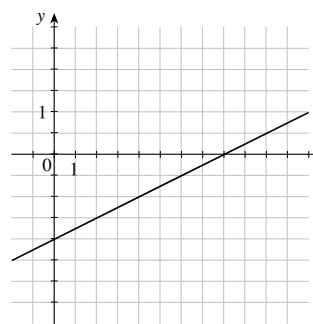
$y = 0 \Rightarrow 0 = \frac{1}{4}x - 2 \Leftrightarrow x = 8$, so the x -intercept is 8, and

$x = 0 \Rightarrow y = \frac{1}{4}(0) - 2 = -2$, so the y -intercept is -2.

x -axis symmetry: $-y = 2x - 6$, which is not the same as $y = 2x - 6$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = 2(-x) - 6 \Leftrightarrow y = -2x - 6$, which is not the same as $y = 2x - 6$, so the graph is not symmetric with respect to the y -axis.

Origin symmetry: $-y = 2(-x) - 6 \Leftrightarrow y = 2x + 6$, which is not the same as $y = 2x - 6$, so the graph is not symmetric with respect to the origin.



(b) $y = -x^2 + 4$

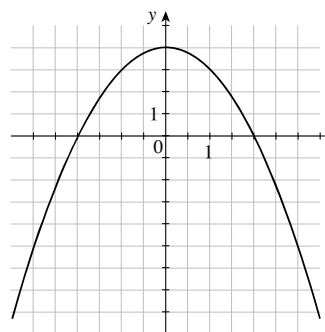
x	y
-3	-5
-2	0
-1	3
0	4
1	3
2	0
3	-5

$y = 0 \Rightarrow 0 = -x^2 + 4 \Leftrightarrow x = \pm 2$, so the x -intercepts are ± 2 , and $x = 0 \Rightarrow y = -0^2 + 4 = 4$, so the y -intercept is 4.

x -axis symmetry: $-y = -x^2 + 4$, which is not the same as $y = -x^2 + 4$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = -(-x)^2 + 4 = -x^2 + 4$, so the graph is symmetric with respect to the y -axis.

Origin symmetry: $-y = -x^2 + 4$, which is not the same as $y = -x^2 + 4$, so the graph is not symmetric with respect to the origin.



63. (a) $y = \sqrt{x} + 2$

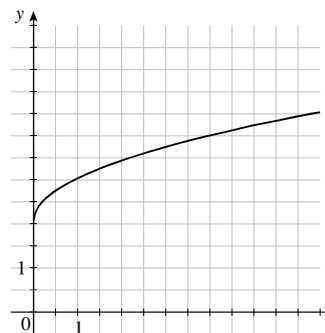
x	y
0	2
1	3
2	$1 + \sqrt{2}$
3	$1 + \sqrt{3}$
4	3
6	$1 + \sqrt{6}$
9	5

$y = 0 \Rightarrow 0 = \sqrt{x} + 2$, so there is no x -intercept, and $x = 0 \Rightarrow y = \sqrt{0} + 2 = 2$, so the y -intercept is 2.

x -axis symmetry: $-y = \sqrt{x} + 2$, which is not the same as $y = \sqrt{x} + 2$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = \sqrt{-x} + 2$, which is not the same as $y = \sqrt{x} + 2$, so the graph is not symmetric with respect to the y -axis.

Origin symmetry: $-y = \sqrt{-x} + 2 \Leftrightarrow y = -\sqrt{-x} - 2$, which is not the same as $y = \sqrt{x} + 2$, so the graph is not symmetric with respect to the origin.



(b) $y = -|x|$

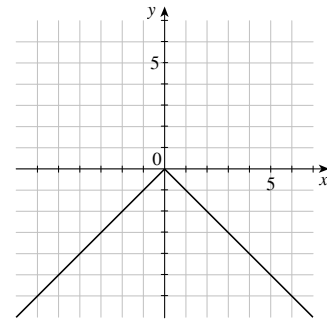
x	y
-5	-5
-3	-3
-1	-1
0	0
1	-1
3	-3
5	-5

$y = 0 \Rightarrow -|x| = 0 \Rightarrow x = 0$, so the x -intercept is 0, and
 $x = 0 \Rightarrow y = -|0| = 0$, so the y -intercept is 0.

x -axis symmetry: $-y = -|x| \Leftrightarrow y = |x|$, which is not the same as $y = -|x|$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = -|-x| = -|x|$, so the graph is symmetric with respect to the y -axis.

Origin symmetry: $-y = -|-x| \Leftrightarrow y = |x|$, which is not the same as $y = -|x|$, so the graph is not symmetric with respect to the origin.



64. (a) $y = \sqrt{x-4}$

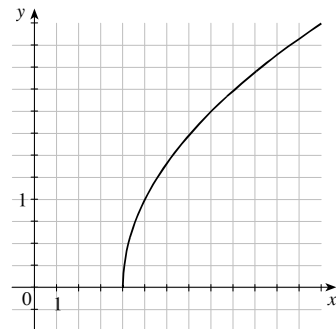
x	y
4	0
5	1
6	$\sqrt{2}$
8	2
13	3

$y = 0 \Rightarrow 0 = \sqrt{x-4} \Leftrightarrow x = 4$, so the x -intercept is 4, and
 $x = 0 \Rightarrow y = \sqrt{0-4}$, so there is no y -intercept.

x -axis symmetry: $-y = \sqrt{x-4}$, which is not the same as $y = \sqrt{x-4}$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = \sqrt{-x-4}$, which is not the same as $y = \sqrt{x-4}$, so the graph is not symmetric with respect to the y -axis.

Origin symmetry: $-y = \sqrt{-x-4} \Leftrightarrow y = -\sqrt{-x-4}$, which is not the same as $y = \sqrt{x-4}$, so the graph is not symmetric with respect to the origin.



(b) $x = |y|$. Here y is not a function of x .

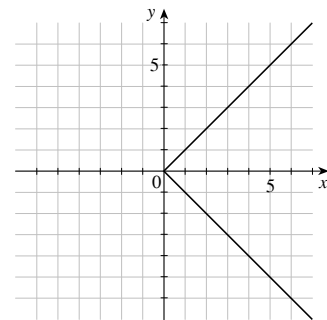
y	x
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

$y = 0 \Rightarrow x = |0| = 0$, so the x -intercept is 0, and $x = 0 \Rightarrow |y| = 0 \Rightarrow y = 0$, so the y -intercept is 0.

x -axis symmetry: $x = |-y| = |y|$, which is the same as $x = |y|$, so the graph is symmetric with respect to the x -axis.

y -axis symmetry: $-x = |y|$, which is not the same, so the graph is not symmetric with respect to the y -axis.

Origin symmetry: $-x = |-y| = |y|$, which is not the same, so the graph is not symmetric with respect to the origin.



65. (a) $y = \sqrt{4 - x^2}$

x	y
-2	0
-1	$\sqrt{3}$
0	2
1	$\sqrt{3}$
2	0

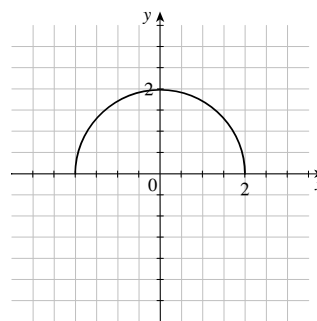
$y = 0 \Rightarrow \sqrt{4 - x^2} = 0 \Leftrightarrow x = \pm 2$, so the x -intercepts are ± 2 , and $x = 0 \Rightarrow y = \sqrt{4 - 0^2} = 2$, so the y -intercept is 2.

x -axis symmetry: $-y = \sqrt{4 - x^2}$, which is not the same as $y = \sqrt{4 - x^2}$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = \sqrt{4 - (-x)^2} = \sqrt{4 - x^2}$, so the graph is symmetric with respect to the y -axis.

Origin symmetry: $-y = \sqrt{4 - (-x)^2} = \sqrt{4 - x^2} \Leftrightarrow$

$y = -\sqrt{4 - x^2}$, which is not the same as $y = \sqrt{4 - x^2}$, so the graph is not symmetric with respect to the origin.



(b) $y = x^3 - 4x$

y	x
-3	15
-2	0
-1	3
0	0
1	3
2	0
3	15

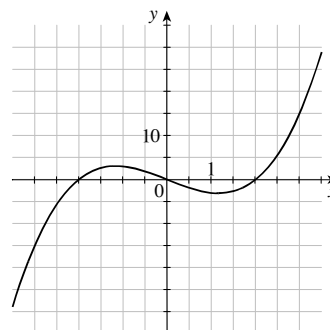
$y = 0 \Rightarrow 0 = x^3 - 4x = x(x^2 - 4) = x(x + 2)(x - 2)$, so the x -intercepts are 0 and ± 2 , and $x = 0 \Rightarrow$

$y = 0^3 - 4(0) = 0$, so the y -intercept is 0.

x -axis symmetry: $(-y) = x^3 - 4x \Leftrightarrow y = -x^3 + 4x$, which is not the same as $y = x^3 - 4x$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = (-x)^3 - 4(-x) = -x^3 + 4x$, which is not the same as $x = x^3 - 4x$, so the graph is not symmetric with respect to the y -axis.

Origin symmetry: $-y = (-x)^3 - 4(-x) \Leftrightarrow y = x^3 - 4x$, so the graph is symmetric with respect to the origin.



66. (a) $y = -\sqrt{4 - x^2}$

x	y
-2	0
-1	$-\sqrt{3}$
0	-2
1	$-\sqrt{3}$
2	0

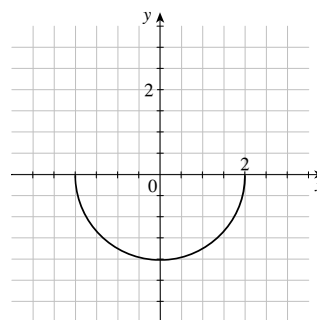
$y = 0 \Rightarrow -\sqrt{4 - x^2} = 0 \Leftrightarrow x = \pm 2$, so the x -intercepts are ± 2 , and $x = 0 \Rightarrow y = -\sqrt{4} = -2$, so the y -intercept is -2.

x -axis symmetry: $-y = -\sqrt{4 - x^2} \Leftrightarrow y = \sqrt{4 - x^2}$, which is not the same as $y = -\sqrt{4 - x^2}$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = -\sqrt{4 - (-x)^2} = -\sqrt{4 - x^2}$, so the graph is symmetric with respect to the y -axis.

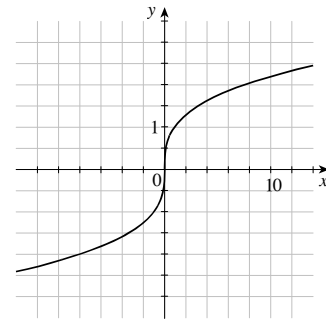
Origin symmetry: $-y = -\sqrt{4 - (-x)^2} \Leftrightarrow y = \sqrt{4 - x^2}$,

which is not the same as $y = -\sqrt{4 - x^2}$, so the graph is not symmetric with respect to the origin.



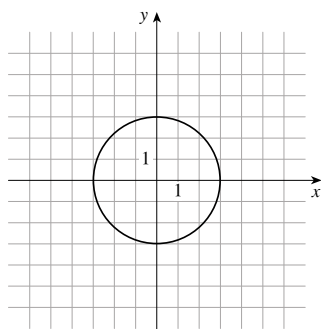
(b) $x = y^3 \Leftrightarrow y = \sqrt[3]{x}$.

y	x
-8	-2
-4	$-\sqrt[3]{4}$
-1	-1
0	0
1	1
4	$\sqrt[3]{4}$
8	2

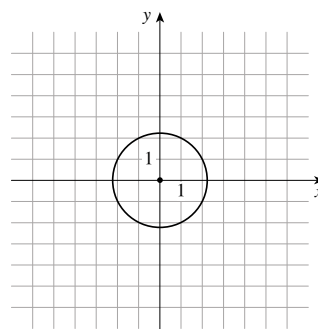
 $y = 0 \Rightarrow x = 0^3 = 0$, so the x -intercept is 0, and $x = 0 \Rightarrow$ $0 = y^3 \Rightarrow y = 0$, so the y -intercept is 0. x -axis symmetry: $x = (-y)^3 = -y^3$, which is not the same as $x = y^3$, so the graph is not symmetric with respect to the x -axis. y -axis symmetry: $-x = y^3 \Leftrightarrow x = -y^3$, which is not the same as $x = y^3$, so the graph is not symmetric with respect to the y -axis.Origin symmetry: $-x = (-y)^3 \Leftrightarrow x = y^3$, so the graph is symmetric with respect to the origin.

67. (a) To find x -intercepts, set $y = 0$. This gives $0 = x + 6 \Leftrightarrow x = -6$, so the x -intercept is -6 . To find y -intercepts, set $x = 0$. This gives $y = 0 + 6 = 6$, so the y -intercept is 6.
- (b) To find x -intercepts, set $y = 0$. This gives $0 = x^2 - 5 \Leftrightarrow x^2 = 5 \Leftrightarrow x = \pm\sqrt{5}$, so the x -intercepts are $\pm\sqrt{5}$. To find y -intercepts, set $x = 0$. This gives $y = 0^2 - 5 = -5$, so the y -intercept is -5 .
68. (a) To find x -intercepts, set $y = 0$. This gives $4x^2 + 25(0)^2 = 100 \Leftrightarrow x^2 = 25 \Leftrightarrow x = \pm 5$, so the x -intercepts are ± 5 . To find y -intercepts, set $x = 0$. This gives $4(0)^2 + 25y^2 = 100 \Leftrightarrow y^2 = 4 \Leftrightarrow y = \pm 2$, so the y -intercepts are ± 2 .
- (b) To find x -intercepts, set $y = 0$. This gives $x^2 - x(0) + 3(0) = 1 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$, so the x -intercepts are ± 1 . To find y -intercepts, set $x = 0$. This gives $0^2 - 0y + 3y = 1 \Leftrightarrow y = \frac{1}{3}$, so the y -intercept is $\frac{1}{3}$.
69. (a) To find x -intercepts, set $y = 0$. This gives $9x^2 - 4(0)^2 = 36 \Leftrightarrow 9x^2 = 36 \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2$, so the x -intercepts are ± 2 . To find y -intercepts, set $x = 0$. This gives $9(0)^2 - 4y^2 = 36 \Leftrightarrow y^2 = -9$, so there is no y -intercept.
- (b) To find x -intercepts, set $y = 0$. This gives $0 - 2x(0) + 4x = 1 \Leftrightarrow x = \frac{1}{4}$, so the x -intercept is $\frac{1}{4}$. To find y -intercepts, set $x = 0$. This gives $y - 2(0)y + 4(0) = 1$, so the y -intercept is 1.
70. (a) To find x -intercepts, set $y = 0$. This gives $0 = \sqrt{x^2 - 16} \Leftrightarrow x^2 = 16 \Leftrightarrow x = \pm 4$, so the x -intercepts are ± 4 . To find y -intercepts, set $x = 0$. This gives $y = \sqrt{0^2 - 16}$, so there is no y -intercept.
- (b) To find x -intercepts, set $y = 0$. This gives $0 = \sqrt{64 - x^3} \Leftrightarrow x^3 = 64 \Leftrightarrow x = 4$, so the x -intercept is 4. To find y -intercepts, set $x = 0$. This gives $y = \sqrt{64 - 0^3} \Leftrightarrow y = 8$, so the y -intercept is 8.
71. To find x -intercepts, set $y = 0$. This gives $0 = 4x - x^2 \Leftrightarrow 0 = x(4 - x) \Leftrightarrow 0 = x$ or $x = 4$, so the x -intercepts are 0 and 4. To find y -intercepts, set $x = 0$. This gives $y = 4(0) - 0^2 \Leftrightarrow y = 0$, so the y -intercept is 0.
72. To find x -intercepts, set $y = 0$. This gives $\frac{x^2}{9} + \frac{0^2}{4} = 1 \Leftrightarrow \frac{x^2}{9} = 1 \Leftrightarrow x^2 = 9 \Leftrightarrow x = \pm 3$, so the x -intercepts are -3 and 3 .
To find y -intercepts, set $x = 0$. This gives $\frac{0^2}{9} + \frac{y^2}{4} = 1 \Leftrightarrow \frac{y^2}{4} = 1 \Leftrightarrow y^2 = 4 \Leftrightarrow y = \pm 2$, so the y -intercepts are -2 and 2 .
73. To find x -intercepts, set $y = 0$. This gives $x^4 + 0^2 - x(0) = 16 \Leftrightarrow x^4 = 16 \Leftrightarrow x = \pm 2$. So the x -intercepts are -2 and 2 .
To find y -intercepts, set $x = 0$. This gives $0^4 + y^2 - (0)y = 16 \Leftrightarrow y^2 = 16 \Leftrightarrow y = \pm 4$. So the y -intercepts are -4 and 4 .
74. To find x -intercepts, set $y = 0$. This gives $x^2 + 0^3 - x^2(0)^2 = 64 \Leftrightarrow x^2 = 64 \Leftrightarrow x = \pm 8$. So the x -intercepts are -8 and 8 .
To find y -intercepts, set $x = 0$. This gives $0^2 + y^3 - (0)^2 y^2 = 64 \Leftrightarrow y^3 = 64 \Leftrightarrow y = 4$. So the y -intercept is 4.

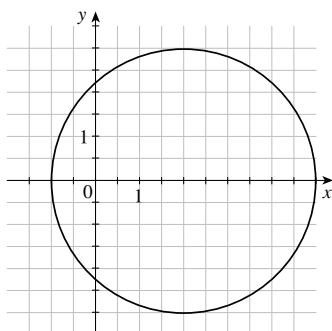
75. $x^2 + y^2 = 9$ has center $(0, 0)$ and radius 3.



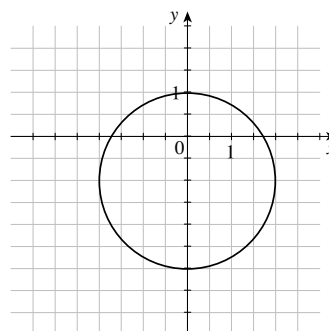
76. $x^2 + y^2 = 5$ has center $(0, 0)$ and radius $\sqrt{5}$.



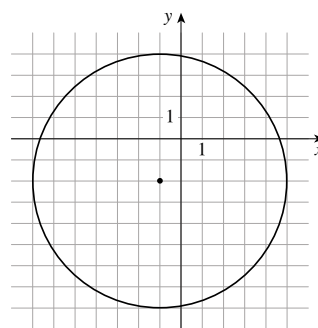
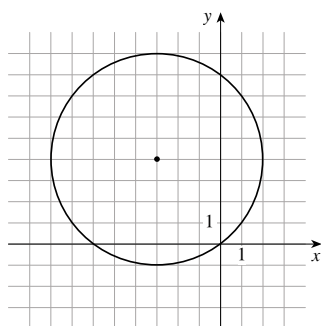
77. $(x - 2)^2 + y^2 = 9$ has center $(2, 0)$ and radius 3.



78. $x^2 + (y + 1)^2 = 4$ has center $(0, -1)$ and radius 2.



79. $(x + 3)^2 + (y - 4)^2 = 25$ has center $(-3, 4)$ and radius 5. 80. $(x + 1)^2 + (y + 2)^2 = 36$ has center $(-1, -2)$ and radius 6.



81. Using $h = -3$, $k = 1$, and $r = 2$, we get $(x - (-3))^2 + (y - 1)^2 = 2^2 \Leftrightarrow (x + 3)^2 + (y - 1)^2 = 4$.

82. Using $h = 2$, $k = -5$, and $r = 3$, we get $(x - 2)^2 + (y - (-5))^2 = 3^2 \Leftrightarrow (x - 2)^2 + (y + 5)^2 = 9$.

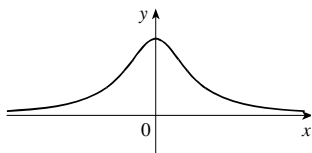
83. The equation of a circle centered at the origin is $x^2 + y^2 = r^2$. Using the point $(4, 7)$ we solve for r^2 . This gives $(4)^2 + (7)^2 = r^2 \Leftrightarrow 16 + 49 = 65 = r^2$. Thus, the equation of the circle is $x^2 + y^2 = 65$.

84. Using $h = -1$ and $k = 5$, we get $(x - (-1))^2 + (y - 5)^2 = r^2 \Leftrightarrow (x + 1)^2 + (y - 5)^2 = r^2$. Next, using the point $(-4, -6)$, we solve for r^2 . This gives $(-4 + 1)^2 + (-6 - 5)^2 = r^2 \Leftrightarrow 130 = r^2$. Thus, an equation of the circle is $(x + 1)^2 + (y - 5)^2 = 130$.

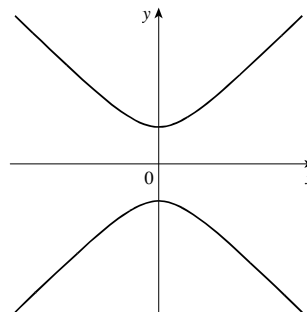
85. The center is at the midpoint of the line segment, which is $\left(\frac{-1+5}{2}, \frac{1+9}{2}\right) = (2, 5)$. The radius is one half the diameter, so $r = \frac{1}{2}\sqrt{(-1-5)^2 + (1-9)^2} = \frac{1}{2}\sqrt{36+64} = \frac{1}{2}\sqrt{100} = 5$. Thus, the equation of the circle is $(x-2)^2 + (y-5)^2 = 5^2$ or $(x-2)^2 + (y-5)^2 = 25$.
86. The center is at the midpoint of the line segment, which is $\left(\frac{-1+7}{2}, \frac{3+(-5)}{2}\right) = (3, -1)$. The radius is one half the diameter, so $r = \frac{1}{2}\sqrt{(-1-7)^2 + (3-(-5))^2} = 4\sqrt{2}$. Thus, an equation of the circle is $(x-3)^2 + (y+1)^2 = 32$.
87. Since the circle is tangent to the x -axis, it must contain the point $(7, 0)$, so the radius is the change in the y -coordinates. That is, $r = |-3-0| = 3$. So the equation of the circle is $(x-7)^2 + (y-(-3))^2 = 3^2$, which is $(x-7)^2 + (y+3)^2 = 9$.
88. Since the circle with $r = 5$ lies in the first quadrant and is tangent to both the x -axis and the y -axis, the center of the circle is at $(5, 5)$. Therefore, the equation of the circle is $(x-5)^2 + (y-5)^2 = 25$.
89. From the figure, the center of the circle is at $(-2, 2)$. The radius is the change in the y -coordinates, so $r = |2-0| = 2$. Thus the equation of the circle is $(x-(-2))^2 + (y-2)^2 = 2^2$, which is $(x+2)^2 + (y-2)^2 = 4$.
90. From the figure, the center of the circle is at $(-1, 1)$. The radius is the distance from the center to the point $(2, 0)$. Thus $r = \sqrt{(-1-2)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10}$, and the equation of the circle is $(x+1)^2 + (y-1)^2 = 10$.
91. Completing the square gives $x^2 + y^2 + 4x - 6y + 12 = 0 \Leftrightarrow x^2 + 4x + \left(\frac{4}{2}\right)^2 + y^2 - 6y + \left(\frac{6}{2}\right)^2 = -12 + \left(\frac{4}{2}\right)^2 + \left(\frac{6}{2}\right)^2$
 $\Leftrightarrow x^2 + 4x + 4 + y^2 - 6y + 9 = -12 + 4 + 9 \Leftrightarrow (x+2)^2 + (y-3)^2 = 1$. Thus, the circle has center $(-2, 3)$ and radius 1.
92. Completing the square gives $x^2 + y^2 + 8x + 5 = 0 \Leftrightarrow x^2 + 8x + \left(\frac{8}{2}\right)^2 + y^2 = -5 + \left(\frac{8}{2}\right)^2 \Leftrightarrow x^2 + 8x + 16 + y^2 = -5 + 16$
 $\Leftrightarrow (x+4)^2 + y^2 = 11$. Thus, the circle has center $(-4, 0)$ and radius $\sqrt{11}$.
93. Completing the square gives $x^2 + y^2 - \frac{1}{2}x + \frac{1}{2}y = \frac{1}{8} \Leftrightarrow x^2 - \frac{1}{2}x + \left(\frac{-1/2}{2}\right)^2 + y^2 + \frac{1}{2}y + \left(\frac{1/2}{2}\right)^2 = \frac{1}{8} + \left(\frac{-1/2}{2}\right)^2 + \left(\frac{1/2}{2}\right)^2$
 $\Leftrightarrow x^2 - \frac{1}{2}x + \frac{1}{16} + y^2 + \frac{1}{2}y + \frac{1}{16} = \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{2}{8} = \frac{1}{4} \Leftrightarrow \left(x - \frac{1}{4}\right)^2 + \left(y + \frac{1}{4}\right)^2 = \frac{1}{4}$. Thus, the circle has center $\left(\frac{1}{4}, -\frac{1}{4}\right)$ and radius $\frac{1}{2}$.
94. Completing the square gives $x^2 + y^2 + \frac{1}{2}x + 2y + \frac{1}{16} = 0 \Leftrightarrow x^2 + \frac{1}{2}x + \left(\frac{1/2}{2}\right)^2 + y^2 + 2y + \left(\frac{2}{2}\right)^2 = -\frac{1}{16} + \left(\frac{1/2}{2}\right)^2 + \left(\frac{2}{2}\right)^2$
 $\Leftrightarrow \left(x + \frac{1}{4}\right)^2 + (y+1)^2 = 1$.
 Thus, the circle has center $\left(-\frac{1}{4}, -1\right)$ and radius 1.
95. Completing the square gives $2x^2 + 2y^2 - 3x = 0 \Leftrightarrow x^2 + y^2 - \frac{3}{2}x = 0 \Leftrightarrow \left(x - \frac{3}{4}\right)^2 + y^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$.
 Thus, the circle has center $\left(\frac{3}{4}, 0\right)$ and radius $\frac{3}{4}$.
96. Completing the square gives $3x^2 + 3y^2 + 6x - y = 0 \Leftrightarrow x^2 + y^2 + 2x - \frac{1}{3}y = 0 \Leftrightarrow (x+1)^2 + \left(y - \frac{1}{6}\right)^2 = 1 + \frac{1}{36} = \frac{37}{36}$.
 Thus, the circle has center $\left(-1, \frac{1}{6}\right)$ and radius $\frac{\sqrt{37}}{6}$.
97. x -axis symmetry: $(-y) = x^4 + x^2 \Leftrightarrow y = -x^4 - x^2$, which is not the same as $y = x^4 + x^2$, so the graph is not symmetric with respect to the x -axis.
 y -axis symmetry: $y = (-x)^4 + (-x)^2 = x^4 + x^2$, so the graph is symmetric with respect to the y -axis.
 Origin symmetry: $(-y) = (-x)^4 + (-x)^2 \Leftrightarrow -y = x^4 + x^2$, which is not the same as $y = x^4 + x^2$, so the graph is not symmetric with respect to the origin.

- 98.** x -axis symmetry: $x = (-y)^4 - (-y)^2 = y^4 - y^2$, so the graph is symmetric with respect to the x -axis.
 y -axis symmetry: $(-x) = y^4 - y^2$, which is not the same as $x = y^4 - y^2$, so the graph is not symmetric with respect to the y -axis.
 Origin symmetry: $(-x) = (-y)^4 - (-y)^2 \Leftrightarrow -x = y^4 - y^2$, which is not the same as $x = y^4 - y^2$, so the graph is not symmetric with respect to the origin.
- 99.** x -axis symmetry: $x^2(-y)^4 + x^4(-y)^2 = 2 \Leftrightarrow x^2y^4 + x^4y^2 = 2$, so the graph is symmetric with respect to the x -axis.
 y -axis symmetry: $(-x)^2y^4 + (-x)^4y^2 = 2 \Leftrightarrow x^2y^4 + x^4y^2 = 2$, so the graph is symmetric with respect to the y -axis.
 Origin symmetry: $(-x)^2(-y)^4 + (-x)^4(-y)^2 = 2 \Leftrightarrow x^2y^4 + x^4y^2 = 2$, so the graph is symmetric with respect to the origin.
 (Note that if a graph is symmetric with respect to each coordinate axis, it is symmetric with respect to the origin. The converse is not true, as shown in the next exercise.)
- 100.** x -axis symmetry: $x^3(-y) + x(-y)^3 = 1 \Leftrightarrow -x^3y - xy^3 = 1 \Leftrightarrow x^3y + xy^3 = -1 \neq 1$, so the graph is not symmetric with respect to the x -axis.
 y -axis symmetry: $(-x)^3y + (-x)y^3 = 1 \Leftrightarrow -x^3y - xy^3 = 1 \Leftrightarrow x^3y + xy^3 = -1 \neq 1$, so the graph is not symmetric with respect to the y -axis.
 Origin symmetry: $(-x)^3(-y) + (-x)(-y)^3 = 1 \Leftrightarrow x^3y + xy^3 = 1$, so the graph is symmetric with respect to the origin.
- 101.** x -axis symmetry: $(-y) = x^3 + 10x \Leftrightarrow y = -x^3 - 10x$, which is not the same as $y = x^3 + 10x$, so the graph is not symmetric with respect to the x -axis.
 y -axis symmetry: $y = (-x)^3 + 10(-x) \Leftrightarrow y = -x^3 - 10x$, which is not the same as $y = x^3 + 10x$, so the graph is not symmetric with respect to the y -axis.
 Origin symmetry: $(-y) = (-x)^3 + 10(-x) \Leftrightarrow -y = -x^3 - 10x \Leftrightarrow y = x^3 + 10x$, so the graph is symmetric with respect to the origin.
- 102.** x -axis symmetry: $(-y) = x^2 + |x| \Leftrightarrow y = -x^2 - |x|$, which is not the same as $y = x^2 + |x|$, so the graph is not symmetric with respect to the x -axis.
 y -axis symmetry: $y = (-x)^2 + |-x| \Leftrightarrow y = x^2 + |x|$, so the graph is symmetric with respect to the y -axis. Note that $|-x| = |x|$.
 Origin symmetry: $(-y) = (-x)^2 + |-x| \Leftrightarrow -y = x^2 + |x| \Leftrightarrow y = -x^2 - |x|$, which is not the same as $y = x^2 + |x|$, so the graph is not symmetric with respect to the origin.

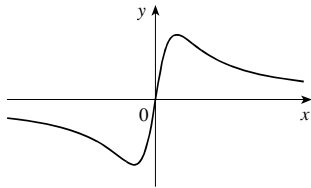
103. Symmetric with respect to the y -axis.



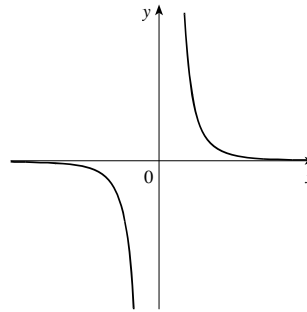
104. Symmetric with respect to the x -axis.



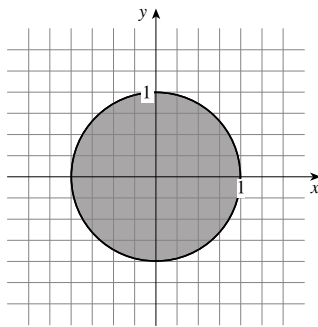
105. Symmetric with respect to the origin.



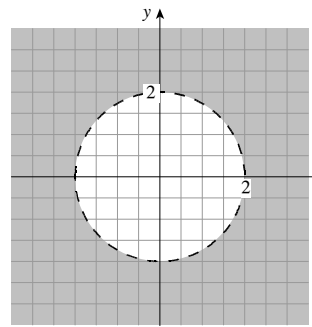
106. Symmetric with respect to the origin.



107. $\{(x, y) \mid x^2 + y^2 \leq 1\}$. This is the set of points inside (and on) the circle $x^2 + y^2 = 1$.



108. $\{(x, y) \mid x^2 + y^2 > 4\}$. This is the set of points outside the circle $x^2 + y^2 = 4$.

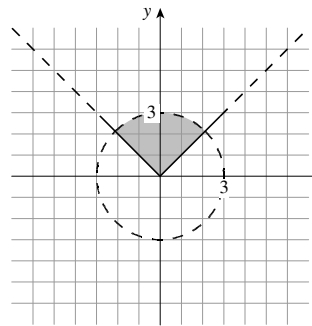


109. Completing the square gives $x^2 + y^2 - 4y - 12 = 0$

$$\Leftrightarrow x^2 + y^2 - 4y + \left(\frac{-4}{2}\right)^2 = 12 + \left(\frac{-4}{2}\right)^2 \Leftrightarrow$$

$x^2 + (y - 2)^2 = 16$. Thus, the center is $(0, 2)$, and the radius is 4. So the circle $x^2 + y^2 = 4$, with center $(0, 0)$ and radius 2, sits completely inside the larger circle. Thus, the area is $\pi 4^2 - \pi 2^2 = 16\pi - 4\pi = 12\pi$.

110. This is the top quarter of the circle of radius 3. Thus, the area is $\frac{1}{4}(9\pi) = \frac{9\pi}{4}$.



111. (a) The point $(5, 3)$ is shifted to $(5 + 3, 3 + 2) = (8, 5)$.

(b) The point (a, b) is shifted to $(a + 3, b + 2)$.

(c) Let (x, y) be the point that is shifted to $(3, 4)$. Then $(x + 3, y + 2) = (3, 4)$. Setting the x -coordinates equal, we get $x + 3 = 3 \Leftrightarrow x = 0$. Setting the y -coordinates equal, we get $y + 2 = 4 \Leftrightarrow y = 2$. So the point is $(0, 2)$.

(d) $A = (-5, -1)$, so $A' = (-5 + 3, -1 + 2) = (-2, 1)$; $B = (-3, 2)$, so $B' = (-3 + 3, 2 + 2) = (0, 4)$; and $C = (2, 1)$, so $C' = (2 + 3, 1 + 2) = (5, 3)$.

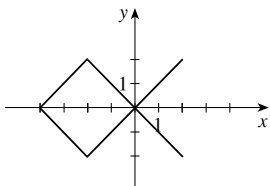
112. (a) The point $(3, 7)$ is reflected to the point $(-3, 7)$.

(b) The point (a, b) is reflected to the point $(-a, b)$.

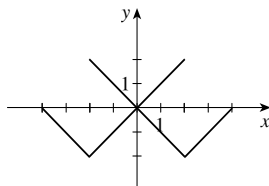
(c) Since the point $(-a, b)$ is the reflection of (a, b) , the point $(-4, -1)$ is the reflection of $(4, -1)$.

(d) $A = (3, 3)$, so $A' = (-3, 3)$; $B = (6, 1)$, so $B' = (-6, 1)$; and $C = (1, -4)$, so $C' = (-1, -4)$.

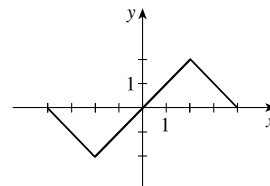
113. (a) Symmetric about the x -axis:



(b) Symmetric about the y -axis:



(c) Symmetric about the origin:



114. (a) From the graph, it appears that the closest the satellite gets to the center of the moon is approximately 5 Mm and farthest approximately 70 Mm.

Setting $y = 0$ in the equation (which obviously corresponds to the desired distances), we have $\frac{(x - 32.5)^2}{1420} = 1 \Leftrightarrow$

$(x - 32.5)^2 = 1420 \Leftrightarrow x - 32.5 = \pm\sqrt{1420} \approx \pm 37.7 \Leftrightarrow x \approx -5.2$ or 70.2 . Thus, the smallest and largest distances are 5.2 Mm and 70.2 Mm.

(b) Setting $y = 10$, we have $\frac{(x - 32.5)^2}{1420} + \frac{10^2}{213} = 1 \Leftrightarrow \frac{(x - 32.5)^2}{1420} = 1 - \frac{100}{213} \Leftrightarrow (x - 32.5)^2 = 1420 \left(\frac{113}{213}\right)$
 $\Leftrightarrow x - 32.5 \approx \pm\sqrt{753.33} \Leftrightarrow x \approx 5.05$ or 59.95 . The distances of these points from the center of the moon are
 $d_1 \approx \sqrt{(5.05)^2 + (10)^2} \approx 11.2$ Mm and $d_2 \approx \sqrt{(59.95)^2 + (10)^2} \approx 60.8$ Mm.

115. (a) $d(A, B) = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

(b) We want the distances from $C = (4, 2)$ to $D = (11, 26)$. The walking distance is
 $|4 - 11| + |2 - 26| = 7 + 24 = 31$ blocks. Straight-line distance is

$$\sqrt{(4 - 11)^2 + (2 - 26)^2} = \sqrt{7^2 + 24^2} = \sqrt{625} = 25 \text{ blocks.}$$

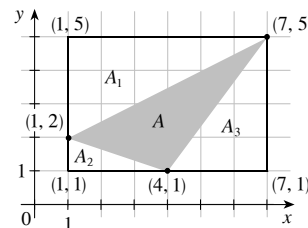
(c) The two points are on the same avenue or the same street.

116. Let the area of the triangle be A . As hinted, we draw a rectangle circumscribing the triangle. This rectangle is $7 - 1 = 6$ units long and $5 - 1 = 4$ units high, so its area is $A_R = 6 \cdot 4 = 24$. The areas of the three right triangles inside the rectangle but outside the original triangle are

$$A_1 = \frac{1}{2}(7 - 1)(5 - 2) = 9, A_2 = \frac{1}{2}(4 - 1)(2 - 1) = \frac{3}{2}, \text{ and}$$

$$A_3 = \frac{1}{2}(7 - 4)(5 - 1) = 6. \text{ Since the area of the rectangle is the sum of the areas of the right triangles and the original triangle, we have}$$

$$A = A_R - A_1 - A_2 - A_3 = 24 - 9 - \frac{3}{2} - 6 = \frac{15}{2}.$$



117. Completing the square gives $x^2 + y^2 + ax + by + c = 0 \Leftrightarrow x^2 + ax + \left(\frac{a}{2}\right)^2 + y^2 + by + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2$
 $\Leftrightarrow \left(x + \frac{a}{2}\right)^2 + \left(y + \frac{b}{2}\right)^2 = -c + \frac{a^2 + b^2}{4}$. This equation represents a circle only when $-c + \frac{a^2 + b^2}{4} > 0$. This equation represents a point when $-c + \frac{a^2 + b^2}{4} = 0$, and this equation represents the empty set when $-c + \frac{a^2 + b^2}{4} < 0$.

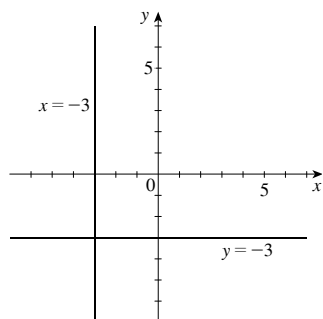
When the equation represents a circle, the center is $\left(-\frac{a}{2}, -\frac{b}{2}\right)$, and the radius is $\sqrt{-c + \frac{a^2 + b^2}{4}} = \frac{1}{2}\sqrt{a^2 + b^2 - 4ac}$.

118. Following the hint, we consider an equilateral triangle with side length 1 anywhere in the plane. Each of its vertices is either red or blue, and each is 1 unit away from the other two. So at least two have the same color and are exactly 1 unit apart.

1.10 LINES

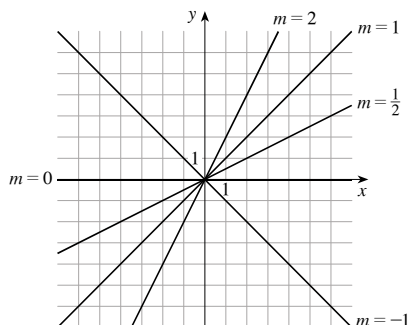
- We find the “steepness” or slope of a line passing through two points by dividing the difference in the y -coordinates of these points by the difference in the x -coordinates. So the line passing through the points $(0, 1)$ and $(2, 5)$ has slope $\frac{5-1}{2-0} = 2$.
- The line with equation $y = 3x + 2$ has slope 3.
 - Any line parallel to this line has slope 3.
 - Any line perpendicular to this line has slope $-\frac{1}{3}$.
- The point-slope form of the equation of the line with slope 3 passing through the point $(1, 2)$ is $y - 2 = 3(x - 1)$.
- For the linear equation $2x + 3y - 12 = 0$, the x -intercept is 6 and the y -intercept is 4. The equation in slope-intercept form is $y = -\frac{2}{3}x + 4$. The slope of the graph of this equation is $-\frac{2}{3}$.
- The slope of a horizontal line is 0. The equation of the horizontal line passing through $(2, 3)$ is $y = 3$.
- The slope of a vertical line is undefined. The equation of the vertical line passing through $(2, 3)$ is $x = 2$.
- Yes, the graph of $y = -3$ is a horizontal line 3 units below the x -axis.
 - Yes, the graph of $x = -3$ is a vertical line 3 units to the left of the y -axis.
 - No, a line perpendicular to a horizontal line is vertical and has undefined slope.
 - Yes, a line perpendicular to a vertical line is horizontal and has slope 0.

8.

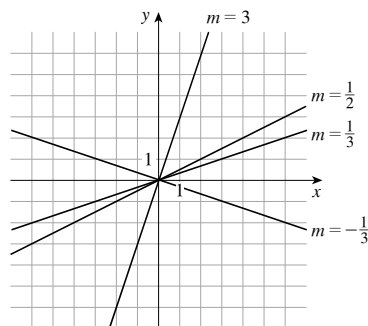
Yes, the graphs of $y = -3$ and $x = -3$ are perpendicular lines.

- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2}{0 - (-1)} = \frac{-2}{1} = -2$
- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 2}{3 - (-3)} = \frac{-5}{6}$
- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{0 - 5} = 0$
- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{6 - 8} = \frac{2}{-2} = -1$
- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 0}{3 - 0} = \frac{-1}{3} = -\frac{1}{3}$
- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 1}{3 - (-5)} = \frac{-3}{8} = -\frac{3}{8}$
- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-1)}{-2 - 4} = \frac{-2}{-6} = \frac{1}{3}$
- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-2)}{6 - 3} = 0$
- For ℓ_1 , we find two points, $(-1, 2)$ and $(0, 0)$ that lie on the line. Thus the slope of ℓ_1 is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{-1 - 0} = -2$.
For ℓ_2 , we find two points $(0, 2)$ and $(2, 3)$. Thus, the slope of ℓ_2 is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{2 - 0} = \frac{1}{2}$. For ℓ_3 we find the points $(2, -2)$ and $(3, 1)$. Thus, the slope of ℓ_3 is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{3 - 2} = 3$. For ℓ_4 , we find the points $(-2, -1)$ and $(2, -2)$. Thus, the slope of ℓ_4 is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-1)}{2 - (-2)} = \frac{-1}{4} = -\frac{1}{4}$.

18. (a)



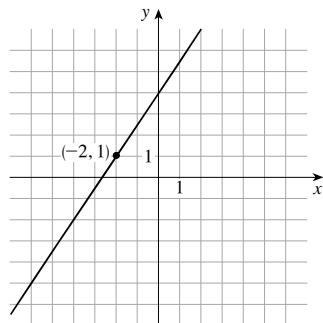
(b)



19. First we find two points $(0, 4)$ and $(4, 0)$ that lie on the line. So the slope is $m = \frac{0-4}{4-0} = -1$. Since the y -intercept is 4, the equation of the line is $y = mx + b = -1x + 4$. So $y = -x + 4$, or $x + y - 4 = 0$.
20. We find two points on the graph, $(0, 4)$ and $(-2, 0)$. So the slope is $m = \frac{0-4}{-2-0} = 2$. Since the y -intercept is 4, the equation of the line is $y = mx + b = 2x + 4$, so $y = 2x + 4 \Leftrightarrow 2x - y + 4 = 0$.
21. We choose the two intercepts as points, $(0, -3)$ and $(2, 0)$. So the slope is $m = \frac{0-(-3)}{2-0} = \frac{3}{2}$. Since the y -intercept is -3 , the equation of the line is $y = mx + b = \frac{3}{2}x - 3$, or $3x - 2y - 6 = 0$.
22. We choose the two intercepts, $(0, -4)$ and $(-3, 0)$. So the slope is $m = \frac{0-(-4)}{-3-0} = -\frac{4}{3}$. Since the y -intercept is -4 , the equation of the line is $y = mx + b = -\frac{4}{3}x - 4 \Leftrightarrow 4x + 3y + 12 = 0$.
23. Using $y = mx + b$, we have $y = 3x + (-2)$ or $3x - y - 2 = 0$.
24. Using $y = mx + b$, we have $y = \frac{2}{5}x + 4 \Leftrightarrow 2x - 5y + 20 = 0$.
25. Using the equation $y - y_1 = m(x - x_1)$, we get $y - 1 = 3(x - 4) \Leftrightarrow y - 1 = 3x - 12 \Leftrightarrow 3x - y - 11 = 0$.
26. Using the equation $y - y_1 = m(x - x_1)$, we get $y - 4 = -1(x - (-2)) \Leftrightarrow y - 4 = -x - 2 \Leftrightarrow x + y - 2 = 0$.
27. Using the equation $y - y_1 = m(x - x_1)$, we get $y - 7 = \frac{2}{3}(x - 1) \Leftrightarrow 3y - 21 = 2x - 2 \Leftrightarrow -2x + 3y = 19 \Leftrightarrow 2x - 3y + 19 = 0$.
28. Using the equation $y - y_1 = m(x - x_1)$, we get $y - (-5) = -\frac{7}{2}(x - (-3)) \Leftrightarrow 2y + 10 = -7x - 21 \Leftrightarrow 7x + 2y + 31 = 0$.
29. First we find the slope, which is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-1}{1-2} = \frac{5}{-1} = -5$. Substituting into $y - y_1 = m(x - x_1)$, we get $y - 6 = -5(x - 1) \Leftrightarrow y - 6 = -5x + 5 \Leftrightarrow 5x + y - 11 = 0$.
30. First we find the slope, which is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{4-(-1)} = \frac{5}{5} = 1$. Substituting into $y - y_1 = m(x - x_1)$, we get $y - 3 = 1(x - 4) \Leftrightarrow y - 3 = x - 4 \Leftrightarrow x - y - 1 = 0$.
31. We are given two points, $(2, -5)$ and $(5, 1)$. Thus, the slope is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-5)}{5-2} = \frac{6}{3} = 2$. Substituting into $y - y_1 = m(x - x_1)$, we get $y - (-5) = 2(x - 2) \Leftrightarrow y = 2x - 9$ or $2x - y - 9 = 0$.
32. We are given two points, $(1, 7)$ and $(4, 7)$. Thus, the slope is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-7}{4-1} = 0$. Substituting into $y - y_1 = m(x - x_1)$, we get $y - 7 = 0(x - 1) \Leftrightarrow y = 7$ or $y - 7 = 0$.
33. We are given two points, $(1, 0)$ and $(0, -3)$. Thus, the slope is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-0}{0-1} = \frac{-3}{-1} = 3$. Using the y -intercept, we have $y = 3x + (-3) \Leftrightarrow y = 3x - 3$ or $3x - y - 3 = 0$.
34. We are given two points, $(-8, 0)$ and $(0, 6)$. Thus, the slope is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{0-(-8)} = \frac{6}{8} = \frac{3}{4}$. Using the y -intercept we have $y = \frac{3}{4}x + 6 \Leftrightarrow 3x - 4y + 24 = 0$.

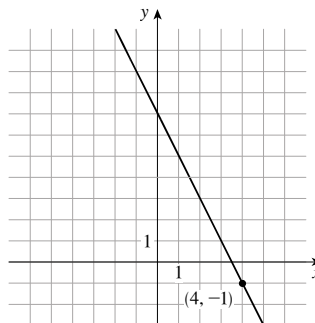
35. Since the equation of a line with slope 0 passing through (a, b) is $y = b$, the equation of this line is $y = 3$.
36. Since the equation of a line with undefined slope passing through (a, b) is $x = a$, the equation of this line is $x = 3$.
37. Since the equation of a line with undefined slope passing through (a, b) is $x = a$, the equation of this line is $x = 2$.
38. Since the equation of a line with slope 0 passing through (a, b) is $y = b$, the equation of this line is $y = 1$.
39. Any line parallel to $y = 2x + 8$ has slope 2. The desired line passes through $(-1, 4)$, so substituting into $y - y_1 = m(x - x_1)$, we get $y - 4 = 2[x - (-1)] \Leftrightarrow y = 2x + 6$ or $2x - y + 6 = 0$.
40. Any line perpendicular to $y = -\frac{1}{2}x + 7$ has slope $-\frac{1}{-1/2} = 2$. The desired line passes through $(-3, 2)$, so substituting into $y - y_1 = m(x - x_1)$, we get $y - 2 = 2[x - (-3)] \Leftrightarrow y = 2x + 8$ or $2x - y + 8 = 0$.
41. Since the equation of a horizontal line passing through (a, b) is $y = b$, the equation of the horizontal line passing through $(4, 5)$ is $y = 5$.
42. Any line parallel to the y -axis has undefined slope and an equation of the form $x = a$. Since the graph of the line passes through the point $(4, 5)$, the equation of the line is $x = 4$.
43. Since $3x + 2y = 4 \Leftrightarrow 2y = -3x + 4 \Leftrightarrow y = -\frac{3}{2}x + 2$, the slope of this line is $-\frac{3}{2}$. Thus, the line we seek is given by $y - (-4) = -\frac{3}{2}(x - (-3)) \Leftrightarrow y + 4 = -\frac{3}{2}x - \frac{9}{2} \Leftrightarrow y = -\frac{3}{2}x - \frac{17}{2} \Leftrightarrow 3x + 2y + 17 = 0$.
44. Since $2x + 3y + 4 = 0 \Leftrightarrow 3y = -2x - 4 \Leftrightarrow y = -\frac{2}{3}x - \frac{4}{3}$, the slope of this line is $m = -\frac{2}{3}$. Substituting $m = -\frac{2}{3}$ and $b = 6$ into the slope-intercept formula, the line we seek is given by $y = -\frac{2}{3}x + 6 \Leftrightarrow 2x + 3y - 18 = 0$.
45. Any line parallel to $x = 5$ has undefined slope and an equation of the form $x = a$. Thus, an equation of the line is $x = -1$.
46. Any line perpendicular to $y = 1$ has undefined slope and an equation of the form $x = a$. Since the graph of the line passes through the point $(2, 6)$, an equation of the line is $x = 2$.
47. First find the slope of $3x + 4y + 7 = 0 \Leftrightarrow 4y = -3x - 7 \Leftrightarrow y = -\frac{3}{4}x - \frac{7}{4}$. Thus, the slope of any line perpendicular to $3x + 4y + 7 = 0$ is $m = -\frac{1}{-3/4} = \frac{4}{3}$. Since it passes through $(-2, 1)$, the equation of the line we seek is $y - 1 = \frac{4}{3}(x - (-2)) \Leftrightarrow 3y - 3 = 4x + 8 \Leftrightarrow 4x - 3y + 11 = 0$.
48. First find the slope of the line $4x - 8y = 1$. This gives $4x - 8y = 1 \Leftrightarrow -8y = -4x + 1 \Leftrightarrow y = \frac{1}{2}x - \frac{1}{8}$. So the slope of the line that is perpendicular to $4x - 8y = 1$ is $m = -\frac{1}{1/2} = -2$. The equation of the line we seek is $y - \left(-\frac{2}{3}\right) = -2\left(x - \frac{1}{2}\right) \Leftrightarrow y + \frac{2}{3} = -2x + 1 \Leftrightarrow 6x + 3y - 1 = 0$.
49. First find the slope of the line passing through $(2, 5)$ and $(-2, 1)$. This gives $m = \frac{1 - 5}{-2 - 2} = \frac{-4}{-4} = 1$, and so the equation of the line we seek is $y - 7 = 1(x - 1) \Leftrightarrow x - y + 6 = 0$.
50. First find the slope of the line passing through $(-3, 5)$ and $(6, 2)$: $m = \frac{2 - 5}{6 - (-3)} = \frac{-3}{9} = -\frac{1}{3}$, and so the slope of the line that is perpendicular is $m = -\frac{1}{-1/3} = 3$. Thus an equation of the line we seek is $y + 10 = 3(x - 4) \Leftrightarrow 3x - y - 22 = 0$.

51. (a)



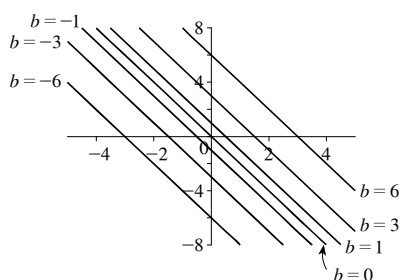
(b) $y - 1 = \frac{3}{2}(x - (-2)) \Leftrightarrow 2y - 2 = 3(x + 2) \Leftrightarrow$
 $2y - 2 = 3x + 6 \Leftrightarrow 3x - 2y + 8 = 0.$

52. (a)



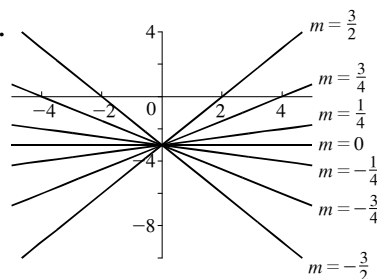
(b) $y - (-1) = -2(x - 4) \Leftrightarrow y + 1 = -2x + 8 \Leftrightarrow$
 $2x + y - 7 = 0.$

53.



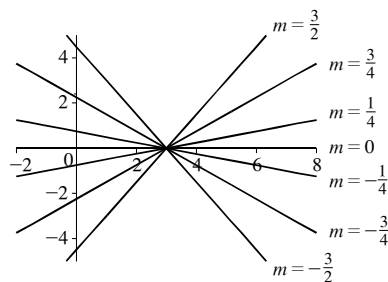
$y = -2x + b$, $b = 0, \pm 1, \pm 3, \pm 6$. They have the same slope, so they are parallel.

54.



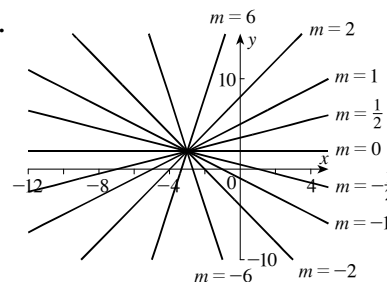
$y = mx - 3$, $m = 0, \pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{3}{2}$. Each of the lines contains the point $(0, -3)$ because the point $(0, -3)$ satisfies each equation $y = mx - 3$. Since $(0, -3)$ is on the y -axis, they all have the same y -intercept.

55.



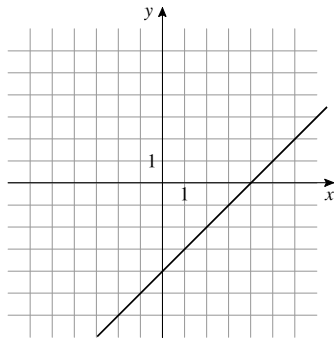
$y = m(x - 3)$, $m = 0, \pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{3}{2}$. Each of the lines contains the point $(3, 0)$ because the point $(3, 0)$ satisfies each equation $y = m(x - 3)$. Since $(3, 0)$ is on the x -axis, we could also say that they all have the same x -intercept.

56.

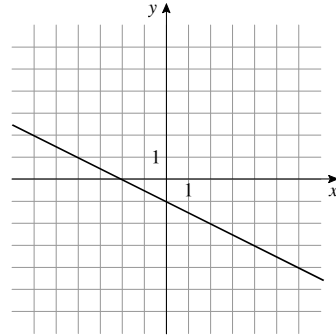


$y = 2 + m(x + 3)$, $m = 0, \pm \frac{1}{2}, \pm 1, \pm 2, \pm 6$. Each of the lines contains the point $(-3, 2)$ because the point $(-3, 2)$ satisfies each equation $y = 2 + m(x + 3)$.

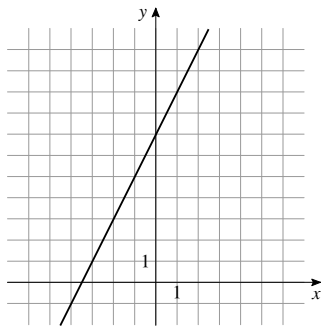
57. $y = x - 4$. So the slope is 1 and the y -intercept is -4 .



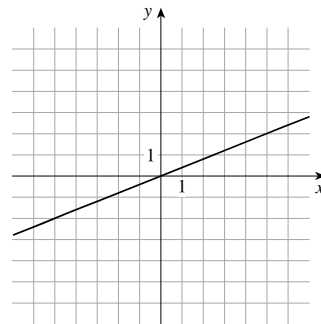
58. $y = -\frac{1}{2}x - 1$. So the slope is $-\frac{1}{2}$ and the y -intercept is -1 .



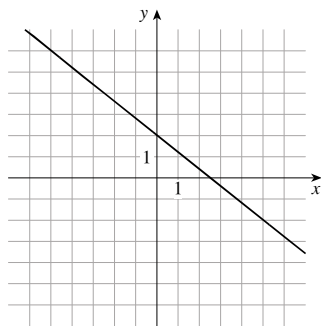
59. $-2x + y = 7 \Leftrightarrow y = 2x + 7$. So the slope is 2 and the y -intercept is 7.



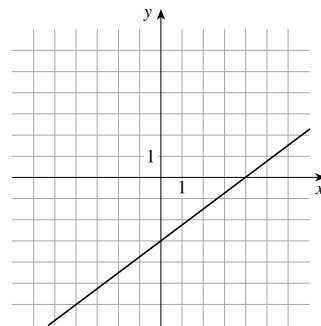
60. $2x - 5y = 0 \Leftrightarrow -5y = -2x \Leftrightarrow y = \frac{2}{5}x$. So the slope is $\frac{2}{5}$ and the y -intercept is 0.



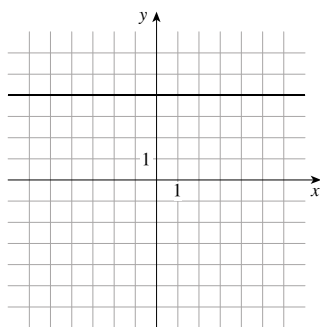
61. $4x + 5y = 10 \Leftrightarrow 5y = -4x + 10 \Leftrightarrow y = -\frac{4}{5}x + 2$. So the slope is $-\frac{4}{5}$ and the y -intercept is 2.



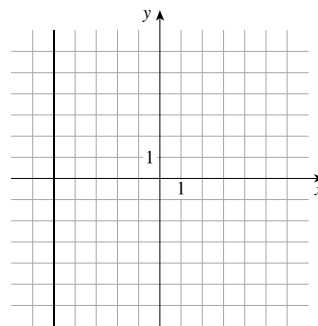
62. $3x - 4y = 12 \Leftrightarrow -4y = -3x + 12 \Leftrightarrow y = \frac{3}{4}x - 3$. So the slope is $\frac{3}{4}$ and the y -intercept is -3 .



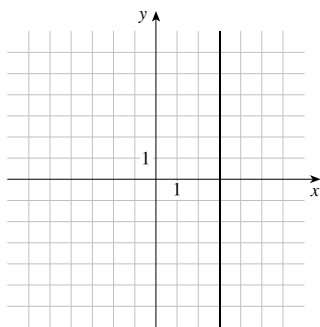
63. $y = 4$ can also be expressed as $y = 0x + 4$. So the slope is 0 and the y -intercept is 4.



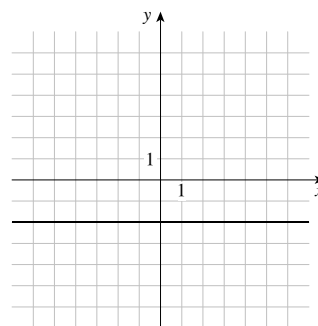
64. $x = -5$ cannot be expressed in the form $y = mx + b$. So the slope is undefined, and there is no y -intercept. This is a vertical line.



65. $x = 3$ cannot be expressed in the form $y = mx + b$. So the slope is undefined, and there is no y -intercept. This is a vertical line.

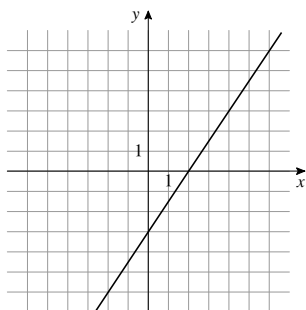


66. $y = -2$ can also be expressed as $y = 0x - 2$. So the slope is 0 and the y -intercept is -2 .



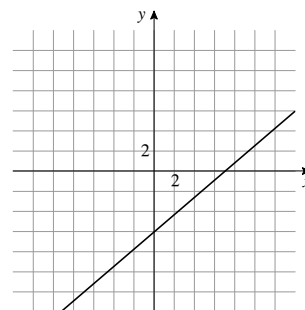
67. $3x - 2y - 6 = 0$. To find x -intercepts, we set $y = 0$ and solve for x : $3x - 2(0) - 6 = 0 \Leftrightarrow 3x = 6 \Leftrightarrow x = 2$, so the x -intercept is 2.

To find y -intercepts, we set $x = 0$ and solve for y : $3(0) - 2y - 6 = 0 \Leftrightarrow 2y = -6 \Leftrightarrow y = -3$, so the y -intercept is -3 .



68. $6x - 7y - 42 = 0$. To find x -intercepts, we set $y = 0$ and solve for x : $6x - 7(0) - 42 = 0 \Leftrightarrow 6x = 42 \Leftrightarrow x = 7$, so the x -intercept is 7.

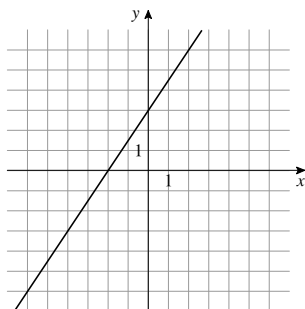
To find y -intercepts, we set $x = 0$ and solve for y : $6(0) - 7y - 42 = 0 \Leftrightarrow 7y = -42 \Leftrightarrow y = -6$, so the y -intercept is -6 .



69. $\frac{1}{2}x - \frac{1}{3}y + 1 = 0$. To find x -intercepts, we set $y = 0$ and solve for x : $\frac{1}{2}x - \frac{1}{3}(0) + 1 = 0 \Leftrightarrow \frac{1}{2}x = -1 \Leftrightarrow x = -2$, so the x -intercept is -2 .

To find y -intercepts, we set $x = 0$ and solve for y :

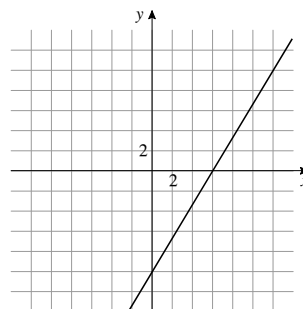
$$\frac{1}{2}(0) - \frac{1}{3}y + 1 = 0 \Leftrightarrow \frac{1}{3}y = 1 \Leftrightarrow y = 3, \text{ so the } y\text{-intercept is } 3.$$



70. $\frac{1}{3}x - \frac{1}{5}y - 2 = 0$. To find x -intercepts, we set $y = 0$ and solve for x : $\frac{1}{3}x - \frac{1}{5}(0) - 2 = 0 \Leftrightarrow \frac{1}{3}x = 2 \Leftrightarrow x = 6$, so the x -intercept is 6 .

To find y -intercepts, we set $x = 0$ and solve for y :

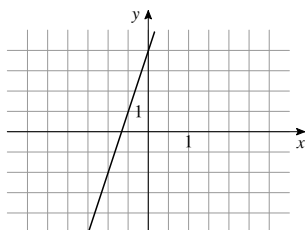
$$\frac{1}{3}(0) - \frac{1}{5}y - 2 = 0 \Leftrightarrow \frac{1}{5}y = -2 \Leftrightarrow y = -10, \text{ so the } y\text{-intercept is } -10.$$



71. $y = 6x + 4$. To find x -intercepts, we set $y = 0$ and solve for x : $0 = 6x + 4 \Leftrightarrow 6x = -4 \Leftrightarrow x = -\frac{2}{3}$, so the x -intercept is $-\frac{2}{3}$.

To find y -intercepts, we set $x = 0$ and solve for y :

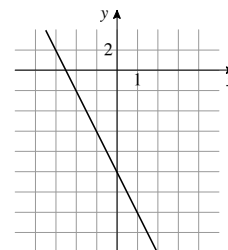
$$y = 6(0) + 4 = 4, \text{ so the } y\text{-intercept is } 4.$$



72. $y = -4x - 10$. To find x -intercepts, we set $y = 0$ and solve for x : $0 = -4x - 10 \Leftrightarrow 4x = -10 \Leftrightarrow x = -\frac{5}{2}$, so the x -intercept is $-\frac{5}{2}$.

To find y -intercepts, we set $x = 0$ and solve for y :

$$y = -4(0) - 10 = -10, \text{ so the } y\text{-intercept is } -10.$$



73. To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $y = 2x + 3$ has slope 2 . The line with equation $2y - 4x - 5 = 0 \Leftrightarrow 2y = 4x + 5 \Leftrightarrow y = 2x + \frac{5}{2}$ also has slope 2 , and so the lines are parallel.

74. To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $y = \frac{1}{2}x + 4$ has slope $\frac{1}{2}$. The line with equation $2x + 4y = 1 \Leftrightarrow 4y = -2x - 1 \Leftrightarrow y = -\frac{1}{2}x - \frac{1}{4}$ has slope $-\frac{1}{2} \neq -\frac{1}{1/2}$, and so the lines are neither parallel nor perpendicular.

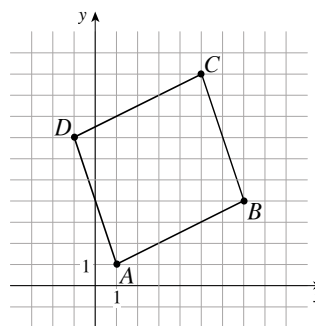
75. To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $2x - 5y = 8 \Leftrightarrow 5y = 2x - 8 \Leftrightarrow y = \frac{2}{5}x - \frac{8}{5}$ has slope $\frac{2}{5}$. The line with equation $10x + 4y = 1 \Leftrightarrow 4y = -10x + 1 \Leftrightarrow y = -\frac{5}{2}x + \frac{1}{4}$ has slope $-\frac{5}{2} = -\frac{1}{2/5}$, and so the lines are perpendicular.

76. To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $15x - 9y = 2 \Leftrightarrow 9y = 15x - 2 \Leftrightarrow y = \frac{5}{3}x - \frac{2}{9}$ has slope $\frac{5}{3}$. The line with equation $3y - 5x = 5 \Leftrightarrow 3y = 5x + 5 \Leftrightarrow y = \frac{5}{3}x + \frac{5}{3}$ has slope $\frac{5}{3}$, and so the lines are parallel.

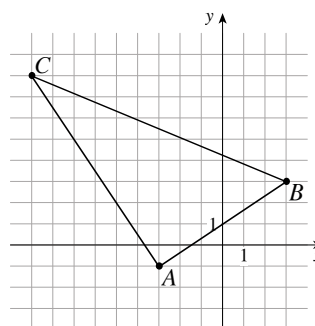
77. To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $7x - 3y = 2 \Leftrightarrow 3y = 7x - 2 \Leftrightarrow y = \frac{7}{3}x - \frac{2}{3}$ has slope $\frac{7}{3}$. The line with equation $9y + 21x = 1 \Leftrightarrow 9y = -21x - 1 \Leftrightarrow y = -\frac{7}{3} - \frac{1}{9}$ has slope $-\frac{7}{3} \neq -\frac{1}{7/3}$, and so the lines are neither parallel nor perpendicular.

78. To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $6y - 2x = 5 \Leftrightarrow 6y = 2x + 5 \Leftrightarrow y = \frac{1}{3}x + \frac{5}{6}$ has slope $\frac{1}{3}$. The line with equation $2y + 6x = 1 \Leftrightarrow 2y = -6x - 1 \Leftrightarrow y = -3x - \frac{1}{2}$ has slope $-3 = -\frac{1}{1/3}$, and so the lines are perpendicular.

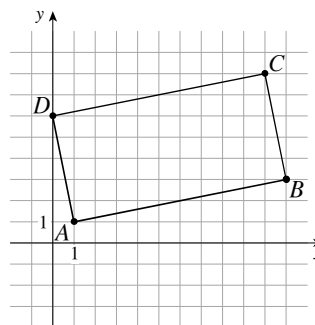
79. We first plot the points to find the pairs of points that determine each side. Next we find the slopes of opposite sides. The slope of AB is $\frac{4-1}{7-1} = \frac{3}{6} = \frac{1}{2}$, and the slope of DC is $\frac{10-7}{5-(-1)} = \frac{3}{6} = \frac{1}{2}$. Since these slope are equal, these two sides are parallel. The slope of AD is $\frac{7-1}{-1-1} = \frac{6}{-2} = -3$, and the slope of BC is $\frac{10-4}{5-7} = \frac{6}{-2} = -3$. Since these slope are equal, these two sides are parallel. Hence $ABCD$ is a parallelogram.



80. We first plot the points to determine the perpendicular sides. Next find the slopes of the sides. The slope of AB is $\frac{3-(-1)}{3-(-3)} = \frac{4}{6} = \frac{2}{3}$, and the slope of AC is $\frac{8-(-1)}{-9-(-3)} = \frac{9}{-6} = -\frac{3}{2}$. Since $(\text{slope of } AB) \times (\text{slope of } AC) = \left(\frac{2}{3}\right) \left(-\frac{3}{2}\right) = -1$, the sides are perpendicular, and ABC is a right triangle.



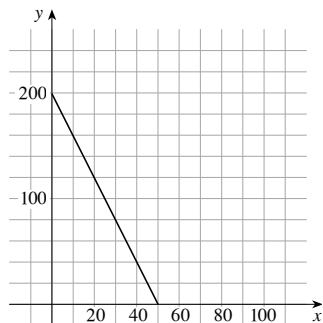
81. We first plot the points to find the pairs of points that determine each side. Next we find the slopes of opposite sides. The slope of AB is $\frac{3-1}{11-1} = \frac{2}{10} = \frac{1}{5}$ and the slope of DC is $\frac{6-8}{0-10} = \frac{-2}{-10} = \frac{1}{5}$. Since these slope are equal, these two sides are parallel. Slope of AD is $\frac{6-1}{0-1} = \frac{5}{-1} = -5$, and the slope of BC is $\frac{3-8}{11-10} = \frac{-5}{1} = -5$. Since these slope are equal, these two sides are parallel. Since $(\text{slope of } AB) \times (\text{slope of } AD) = \frac{1}{5} \times (-5) = -1$, the first two sides are each perpendicular to the second two sides. So the sides form a rectangle.



82. (a) The slope of the line passing through $(1, 1)$ and $(3, 9)$ is $\frac{9-1}{3-1} = \frac{8}{2} = 4$. The slope of the line passing through $(1, 1)$ and $(6, 21)$ is $\frac{21-1}{6-1} = \frac{20}{5} = 4$. Since the slopes are equal, the points are collinear.

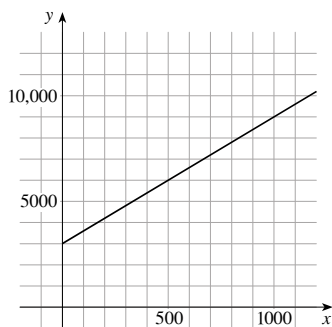
- (b) The slope of the line passing through $(-1, 3)$ and $(1, 7)$ is $\frac{7-3}{1-(-1)} = \frac{4}{2} = 2$. The slope of the line passing through $(-1, 3)$ and $(4, 15)$ is $\frac{15-3}{4-(-1)} = \frac{12}{5}$. Since the slopes are not equal, the points are not collinear.
83. We need the slope and the midpoint of the line AB . The midpoint of AB is $\left(\frac{1+7}{2}, \frac{4-2}{2}\right) = (4, 1)$, and the slope of AB is $m = \frac{-2-4}{7-1} = \frac{-6}{6} = -1$. The slope of the perpendicular bisector will have slope $\frac{-1}{m} = \frac{-1}{-1} = 1$. Using the point-slope form, the equation of the perpendicular bisector is $y - 1 = 1(x - 4)$ or $x - y - 3 = 0$.
84. We find the intercepts (the length of the sides). When $x = 0$, we have $2y + 3(0) - 6 = 0 \Leftrightarrow 2y = 6 \Leftrightarrow y = 3$, and when $y = 0$, we have $2(0) + 3x - 6 = 0 \Leftrightarrow 3x = 6 \Leftrightarrow x = 2$. Thus, the area of the triangle is $\frac{1}{2}(3)(2) = 3$.
85. (a) We start with the two points $(a, 0)$ and $(0, b)$. The slope of the line that contains them is $\frac{b-0}{0-a} = -\frac{b}{a}$. So the equation of the line containing them is $y = -\frac{b}{a}x + b$ (using the slope-intercept form). Dividing by b (since $b \neq 0$) gives $\frac{y}{b} = -\frac{x}{a} + 1 \Leftrightarrow \frac{x}{a} + \frac{y}{b} = 1$.
- (b) Setting $a = 6$ and $b = -8$, we get $\frac{x}{6} + \frac{y}{-8} = 1 \Leftrightarrow 4x - 3y = 24 \Leftrightarrow 4x - 3y - 24 = 0$.
86. (a) The line tangent at $(3, -4)$ will be perpendicular to the line passing through the points $(0, 0)$ and $(3, -4)$. The slope of this line is $\frac{-4-0}{3-0} = -\frac{4}{3}$. Thus, the slope of the tangent line will be $-\frac{1}{(-4/3)} = \frac{3}{4}$. Then the equation of the tangent line is $y - (-4) = \frac{3}{4}(x - 3) \Leftrightarrow 4(y + 4) = 3(x - 3) \Leftrightarrow 3x - 4y - 25 = 0$.
- (b) Since diametrically opposite points on the circle have parallel tangent lines, the other point is $(-3, 4)$.
87. Using the diagram provided, we calculate the coordinates of the midpoints: m_1 has coordinates $\left(\frac{0+a}{2}, \frac{0+b}{2}\right) = \left(\frac{1}{2}a, \frac{1}{2}b\right)$ and m_2 has coordinates $\left(\frac{c+a}{2}, \frac{0+b}{2}\right) = \left(\frac{1}{2}(a+c), \frac{1}{2}b\right)$.
- The line joining the midpoints thus has slope $\frac{\frac{1}{2}b - \frac{1}{2}b}{\frac{1}{2}(a+c) - \frac{1}{2}a} = 0$, and so it is parallel to the third side (a segment of the x -axis).
- The line joining the midpoints has length $\sqrt{\left[\frac{1}{2}(a+c) - \frac{1}{2}a\right]^2 + \left(\frac{1}{2}b - \frac{1}{2}b\right)^2} = \sqrt{\frac{1}{4}c^2} = \frac{1}{2}c$, half the length of the third side of the triangle; and so we have verified both conclusions of the theorem.
88. (a) The slope represents the increase in the average surface temperature in $^{\circ}\text{C}$ per year. The T -intercept is the average surface temperature in 1950, or 15°C .
- (b) In 2050, $t = 2050 - 1950 = 100$, so $T = 0.02(100) + 15 = 17$ degrees Celsius.
89. (a) The slope is $0.0417D = 0.0417(200) = 8.34$. It represents the increase in dosage for each one-year increase in the child's age.
- (b) When $a = 0$, $c = 8.34(0 + 1) = 8.34$ mg.

90. (a)



(b) The slope, -4 , represents the decline in number of spaces sold for each \$1 increase in rent. The y -intercept is the number of spaces at the flea market, 200, and the x -intercept is the cost per space when the manager rents no spaces, \$50.

91. (a)



(b) The slope is the cost per toaster oven, \$6. The y -intercept, \$3000, is the monthly fixed cost—the cost that is incurred no matter how many toaster ovens are produced.

92. (a)

C	-30°	-20°	-10°	0°	10°	20°	30°
F	-22°	-4°	14°	32°	50°	68°	86°

(b) Substituting a for both F and C, we have

$$a = \frac{9}{5}a + 32 \Leftrightarrow -\frac{4}{5}a = 32 \Leftrightarrow$$

$$a = -40^\circ. \text{ Thus both scales agree at } -40^\circ.$$

93. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

(a) Using n in place of x and t in place of y , we find that the slope is $\frac{t_2 - t_1}{n_2 - n_1} = \frac{27 - 21}{168 - 120} = \frac{6}{48} = \frac{1}{8}$. So the linear equation is $t - 27 = \frac{1}{8}(n - 168) \Leftrightarrow t - 27 = \frac{1}{8}n - 21 \Leftrightarrow t = \frac{1}{8}n + 6$.

(b) When $n = 150$, the temperature is approximately given by $t = \frac{1}{8}(150) + 6 = 24.75^\circ \text{C} \approx 25^\circ \text{C}$.

94. (a) Using t in place of x and V in place of y , we find the slope of the line using the points $(0, 4000)$ and $(4, 200)$. Thus, the slope is

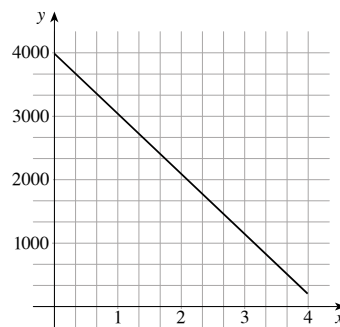
$$m = \frac{200 - 4000}{4 - 0} = \frac{-3800}{4} = -950. \text{ Using the } V\text{-intercept, the}$$

linear equation is $V = -950t + 4000$.

(c) The slope represents a decrease in the value of the computer of \$950 each year. The V -intercept is the original price of the computer.

(d) When $t = 3$, the value of the computer is given by $V = -950(3) + 4000 = 1150$.

(b)

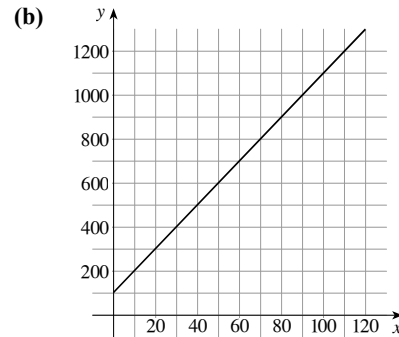


95. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

(a) We are given $\frac{\text{change in pressure}}{\text{3-meter change in depth}} = \frac{29.9}{3} = 9.97$. Using P for pressure and d for depth, and using the point $P = 103$ when $d = 0$, we have $P - 103 = 9.97(d - 0) \Leftrightarrow P = 9.97d + 103$.

(c) The slope represents the increase in pressure per meter of descent. The y -intercept represents the pressure at the surface.

(d) When $P = 690$, we have $690 = 9.97d + 103 \Leftrightarrow 9.97d = 587 \Leftrightarrow d = 58.9$ m. Thus the pressure is 690 kPa at a depth of approximately 59 m.



96. Slope is the rate of change of one variable per unit change in another variable. So if the slope is positive, then the temperature is rising. Likewise, if the slope is negative then the temperature is decreasing. If the slope is 0, then the temperature is not changing.

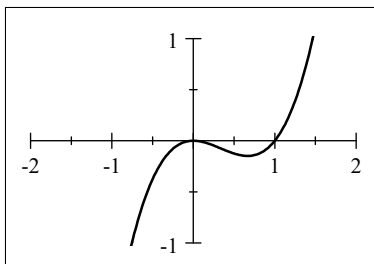
97. We label the three points A , B , and C . If the slope of the line segment $\overline{AB}$ is equal to the slope of the line segment $\overline{BC}$, then the points A , B , and C are collinear. Using the distance formula, we find the distance between A and B , between B and C , and between A and C . If the sum of the two smaller distances equals the largest distance, the points A , B , and C are collinear.

Another method: Find an equation for the line through A and B . Then check if C satisfies the equation. If so, the points are collinear.

1.11 SOLVING EQUATIONS AND INEQUALITIES GRAPHICALLY

- The solutions of the equation $x^2 - 2x - 3 = 0$ are the x -intercepts of the graph of $y = x^2 - 2x - 3$.
- The solutions of the inequality $x^2 - 2x - 3 > 0$ are the x -coordinates of the points on the graph of $y = x^2 - 2x - 3$ that lie above the x -axis.
- (a) From the graph, it appears that the graph of $y = x^4 - 3x^3 - x^2 + 3x$ has x -intercepts -1 , 0 , 1 , and 3 , so the solutions to the equation $x^4 - 3x^3 - x^2 + 3x = 0$ are $x = -1$, $x = 0$, $x = 1$, and $x = 3$.
(b) From the graph, we see that where $-1 \leq x \leq 0$ or $1 \leq x \leq 3$, the graph lies below the x -axis. Thus, the inequality $x^4 - 3x^3 - x^2 + 3x \leq 0$ is satisfied for $\{x \mid -1 \leq x \leq 0 \text{ or } 1 \leq x \leq 3\} = [-1, 0] \cup [1, 3]$.
- (a) The graphs of $y = 5x - x^2$ and $y = 4$ intersect at $x = 1$ and at $x = 4$, so the equation $5x - x^2 = 4$ has solutions $x = 1$ and $x = 4$.
(b) The graph of $y = 5x - x^2$ lies strictly above the graph of $y = 4$ when $1 < x < 4$, so the inequality $5x - x^2 > 4$ is satisfied for those values of x , that is, for $\{x \mid 1 < x < 4\} = (1, 4)$.

5. (a)

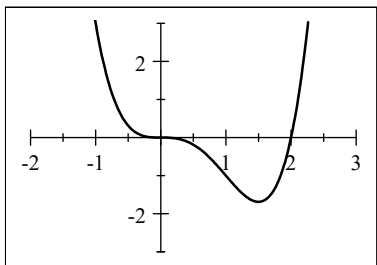


$$y = x^3 - x^2; [-2, 2] \text{ by } [-1, 1]$$

(b) x -intercepts 0 , 1 ; y -intercept 0 . $y = x^3 - x^2 = x^2(x - 1)$, so $y = 0 \Leftrightarrow x = 0$ or $x = 1$ and $x = 0 \Leftrightarrow y = 0$.

(c) The graph is not symmetric with respect to either axis or the origin.

6. (a)

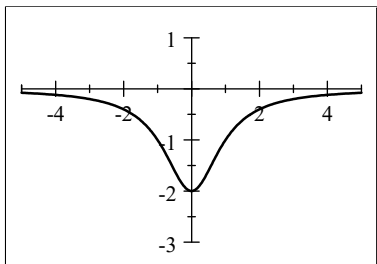


$$y = x^4 - 2x^3; [-2, 3] \text{ by } [-3, 3]$$

(b) x -intercepts 0, 2; y -intercept 0. $y = x^4 - 2x^3 = x^3(x - 2)$, so $y = 0 \Rightarrow x = 0$ or $x = 2$; $x = 0 \Rightarrow y = 0$.

(c) The graph is not symmetric with respect to either axis or the origin.

7. (a)



$$y = -\frac{2}{x^2 + 1}; [-5, 5] \text{ by } [-3, 1]$$

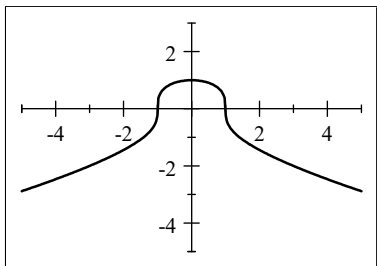
(b) No x -intercept; y -intercept -2 . $y = 0$ has no solution; $x = 0 \Rightarrow$

$$y = -\frac{2}{1} = -2.$$

(c) The graph is symmetric with respect to the y -axis:

$$-\frac{2}{(-x)^2 + 1} = -\frac{2}{x^2 + 1}.$$

8. (a)



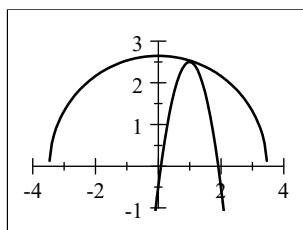
$$y = \sqrt[3]{1 - x^2}; [-5, 5] \text{ by } [-5, 3]$$

(b) x -intercepts ± 1 , y -intercept 1. $y = 0 \Rightarrow \sqrt[3]{1 - x^2} = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$; $x = 0 \Rightarrow y = \sqrt[3]{1 - 0^2} = 1$.

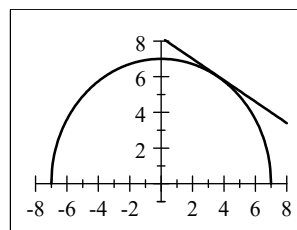
(c) The graph is symmetric with respect to the y -axis:

$$\sqrt[3]{1 - (-x)^2} = \sqrt[3]{1 - x^2}.$$

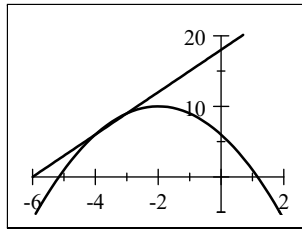
9. Although the graphs of $y = -3x^2 + 6x - \frac{1}{2}$ and $y = \sqrt{7 - \frac{7}{12}x^2}$ appear to intersect in the viewing rectangle $[-4, 4]$ by $[-1, 3]$, there is no point of intersection. You can verify this by zooming in.



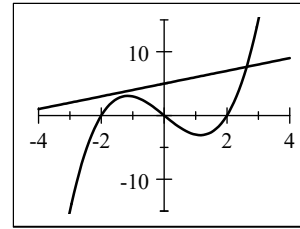
10. Although the graphs of $y = \sqrt{49 - x^2}$ and $y = \frac{1}{5}(41 - 3x)$ appear to intersect in the viewing rectangle $[-8, 8]$ by $[-1, 8]$, there is no point of intersection. You can verify this by zooming in.



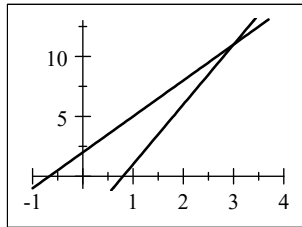
11. The graphs of $y = 6 - 4x - x^2$ and $y = 3x + 18$ appear to have two points of intersection in the viewing rectangle $[-6, 2]$ by $[-5, 20]$. You can verify that $x = -4$ and $x = -3$ are exact solutions.



12. The graphs of $y = x^3 - 4x$ and $y = x + 5$ appear to have one point of intersection in the viewing rectangle $[-4, 4]$ by $[-15, 15]$. The solution is $x \approx 2.627$.

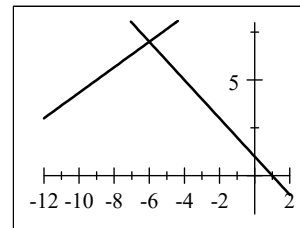


13. Algebraically: $3x + 2 = 5x - 4 \Leftrightarrow 6 = 2x \Leftrightarrow x = 3$.
Graphically: We graph the two equations $y_1 = 3x + 2$ and $y_2 = 5x - 4$ in the viewing rectangle $[1, 4]$ by $[-1, 13]$.
Zooming in, we see that the solution is $x = 3$.



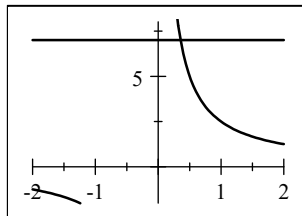
14. Algebraically: $\frac{2}{3}x + 11 = 1 - x \Leftrightarrow \frac{5}{3}x = -10 \Leftrightarrow x = -6$.

Graphically: We graph the two equations $y_1 = \frac{2}{3}x + 11$ and $y_2 = 1 - x$ in the viewing rectangle $[-12, 2]$ by $[-2, 8]$. Zooming in, we see that the solution is $x = -6$.



15. Algebraically: $\frac{2}{x} + \frac{1}{2x} = 7 \Leftrightarrow 2x \left(\frac{2}{x} + \frac{1}{2x} \right) = 2x(7) \Leftrightarrow 4 + 1 = 14x \Leftrightarrow x = \frac{5}{14}$.

Graphically: We graph the two equations $y_1 = \frac{2}{x} + \frac{1}{2x}$ and $y_2 = 7$ in the viewing rectangle $[-2, 2]$ by $[-2, 8]$.
Zooming in, we see that the solution is $x \approx 0.36$.

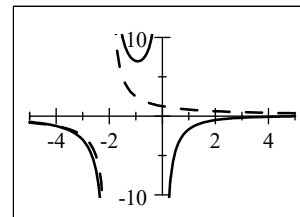


16. Algebraically: $\frac{4}{x+2} - \frac{6}{2x} = \frac{5}{2x+4} \Leftrightarrow$

$$2x(x+2) \left(\frac{4}{x+2} - \frac{6}{2x} \right) = 2x(x+2) \left(\frac{5}{2x+4} \right) \Leftrightarrow 2x(4) - (x+2)(6) = x(5) \Leftrightarrow 8x - 6x - 12 = 5x \Leftrightarrow -12 = 3x \Leftrightarrow -4 = x.$$

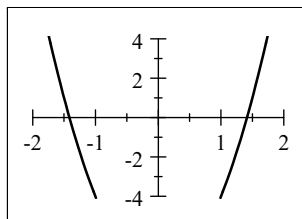
Graphically: We graph the two equations

$y_1 = \frac{4}{x+2} - \frac{6}{2x}$ and $y_2 = \frac{5}{2x+4}$ in the viewing rectangle $[-5, 5]$ by $[-10, 10]$. Zooming in, we see that there is only one solution at $x = -4$.



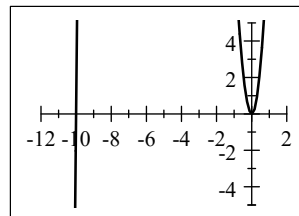
17. Algebraically: $4x^2 - 8 = 0 \Leftrightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$.

Graphically: We graph the equation $y_1 = 4x^2 - 8$ and determine where this curve intersects the x -axis. We use the viewing rectangle $[-2, 2]$ by $[-4, 4]$. Zooming in, we see that solutions are $x \approx 1.41$ and $x \approx -1.41$.



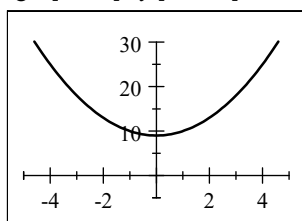
18. Algebraically: $x^3 + 10x^2 = 0 \Leftrightarrow x^2(x + 10) = 0 \Leftrightarrow x = 0$ or $x = -10$.

Graphically: We graph the equation $y = x^3 + 10x^2$ and determine where this curve intersects the x -axis. We use the viewing rectangle $[-12, 2]$ by $[-5, 5]$. We see that the solutions are $x = 0$ and $x = -10$.



19. Algebraically: $x^2 + 9 = 0 \Leftrightarrow x^2 = -9$, which has no real solution.

Graphically: We graph the equation $y = x^2 + 9$ and see that this curve does not intersect the x -axis. We use the viewing rectangle $[-5, 5]$ by $[-5, 30]$.

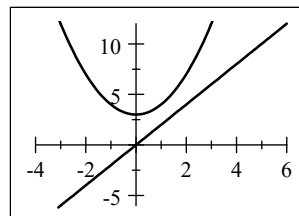


20. Algebraically: $x^2 + 3 = 2x \Leftrightarrow x^2 - 2x + 3 = 0 \Leftrightarrow$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)} = \frac{2 \pm \sqrt{-8}}{2(1)}.$$

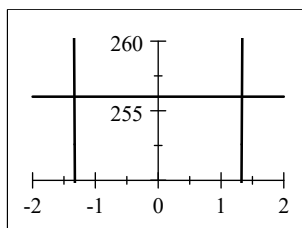
Because the discriminant is negative, there is no real solution.

Graphically: We graph the two equations $y_1 = x^2 + 3$ and $y_2 = 2x$ in the viewing rectangle $[-4, 6]$ by $[-6, 12]$, and see that the two curves do not intersect.



21. Algebraically: $81x^4 = 256 \Leftrightarrow x^4 = \frac{256}{81} \Rightarrow x = \pm\frac{4}{3}$.

Graphically: We graph the two equations $y_1 = 81x^4$ and $y_2 = 256$ in the viewing rectangle $[-2, 2]$ by $[250, 260]$. Zooming in, we see that solutions are $x \approx \pm 1.33$.

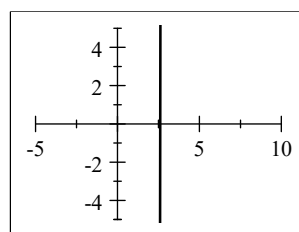


22. Algebraically: $2x^5 - 243 = 0 \Leftrightarrow 2x^5 = 243 \Leftrightarrow x^5 = \frac{243}{2}$

$$\Leftrightarrow x = \sqrt[5]{\frac{243}{2}} = \frac{3}{2}\sqrt[5]{16}.$$

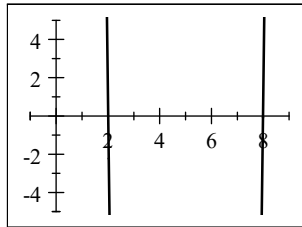
Graphically: We graph the equation $y = 2x^5 - 243$ and determine where this curve intersects the x -axis. We use the viewing rectangle $[-5, 10]$ by $[-5, 5]$.

Zooming in, we see that the solution is $x \approx 2.61$.



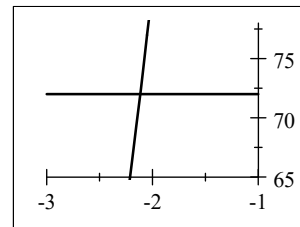
23. Algebraically: $(x - 5)^4 - 80 = 0 \Leftrightarrow (x - 5)^4 = 80 \Rightarrow x - 5 = \pm \sqrt[4]{80} = \pm 2\sqrt[4]{5} \Leftrightarrow x = 5 \pm 2\sqrt[4]{5}$.

Graphically: We graph the equation $y_1 = (x - 5)^4 - 80$ and determine where this curve intersects the x -axis. We use the viewing rectangle $[-1, 9]$ by $[-5, 5]$. Zooming in, we see that solutions are $x \approx 2.01$ and $x \approx 7.99$.

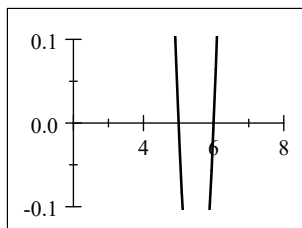


24. Algebraically: $3(x + 5)^3 = 72 \Leftrightarrow (x + 5)^3 = \frac{72}{3} = 24 \Leftrightarrow x + 5 = \sqrt[3]{24} \Leftrightarrow x = -5 + \sqrt[3]{24}$.

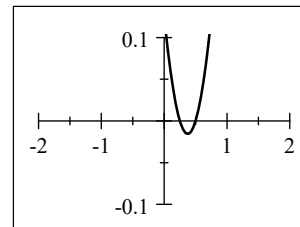
Graphically: We graph the two equations $y_1 = 3(x + 5)^3$ and $y_2 = 72$ in the viewing rectangle $[-3, -1]$ by $[65, 78]$. Zooming in, we see that the solution is $x \approx -2.12$.



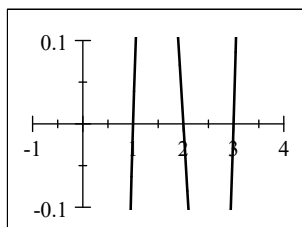
25. We graph $y = x^2 - 11x + 30$ in the viewing rectangle $[2, 8]$ by $[-0.1, 0.1]$. The solutions appear to be exactly $x = 5$ and $x = 6$. [In fact $x^2 - 11x + 30 = (x - 5)(x - 6)$.]



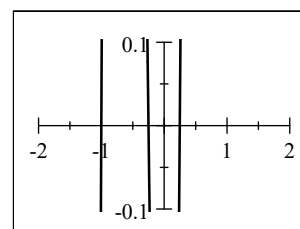
26. We graph $y = x^2 - 0.75x + 0.125$ in the viewing rectangle $[-2, 2]$ by $[-0.1, 0.1]$. The solutions are $x = 0.25$ and $x = 0.50$.



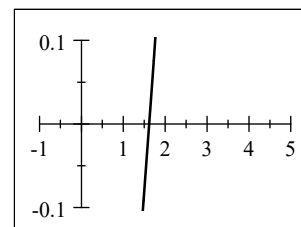
27. We graph $y = x^3 - 6x^2 + 11x - 6$ in the viewing rectangle $[-1, 4]$ by $[-0.1, 0.1]$. The solutions are $x = 1.00$, $x = 2.00$, and $x = 3.00$.



28. Since $16x^3 + 16x^2 = x + 1 \Leftrightarrow 16x^3 + 16x^2 - x - 1 = 0$, we graph $y = 16x^3 + 16x^2 - x - 1$ in the viewing rectangle $[-2, 2]$ by $[-0.1, 0.1]$. The solutions are: $x = -1.00$, $x = -0.25$, and $x = 0.25$.

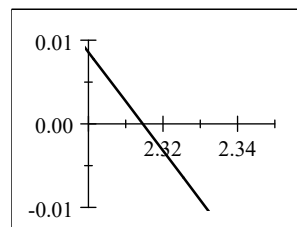
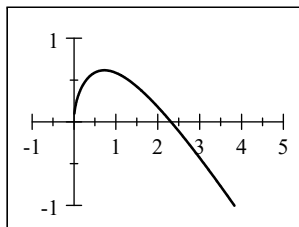


29. We first graph $y = x - \sqrt{x + 1}$ in the viewing rectangle $[-1, 5]$ by $[-0.1, 0.1]$ and find that the solution is near 1.6. Zooming in, we see that solutions is $x \approx 1.62$.

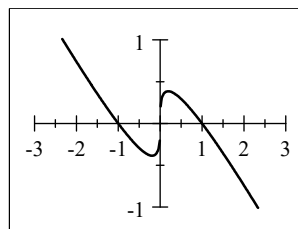


30. $1 + \sqrt{x} = \sqrt{1 + x^2} \Leftrightarrow$

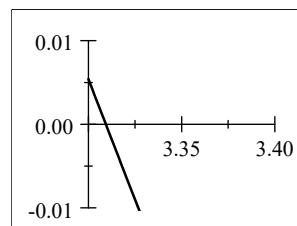
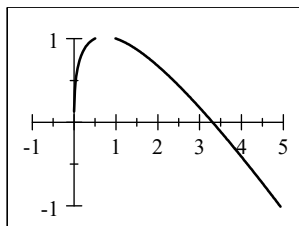
$1 + \sqrt{x} - \sqrt{1 + x^2} = 0$. Since $\sqrt{x}$ is only defined for $x \geq 0$, we start with the viewing rectangle $[-1, 5]$ by $[-1, 1]$. In this rectangle, there appears to be an exact solution at $x = 0$ and another solution between $x = 2$ and $x = 2.5$. We then use the viewing rectangle $[2.3, 2.35]$ by $[-0.01, 0.01]$, and isolate the second solution as $x \approx 2.314$. Thus the solutions are $x = 0$ and $x \approx 2.31$.



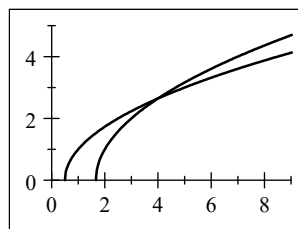
31. We graph $y = x^{1/3} - x$ in the viewing rectangle $[-3, 3]$ by $[-1, 1]$. The solutions are $x = -1$, $x = 0$, and $x = 1$, as can be verified by substitution.



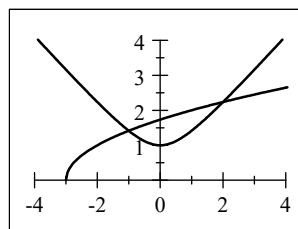
32. Since $x^{1/2}$ is defined only for $x \geq 0$, we start by graphing $y = x^{1/2} + x^{1/3} - x$ in the viewing rectangle $[-1, 5]$ by $[-1, 1]$. We see a solution at $x = 0$ and another one between $x = 3$ and $x = 3.5$. We then use the viewing rectangle $[3.3, 3.4]$ by $[-0.01, 0.01]$, and isolate the second solution as $x \approx 3.31$. Thus, the solutions are $x = 0$ and $x \approx 3.31$.



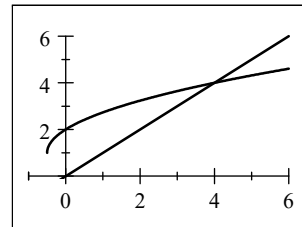
33. We graph $y = \sqrt{2x - 1}$ and $y = \sqrt{3x - 5}$ in the viewing rectangle $[0, 9]$ by $[0, 5]$ and see that the only solution to the equation $\sqrt{2x - 1} = \sqrt{3x - 5}$ is $x = 4$, which can be verified by substitution.



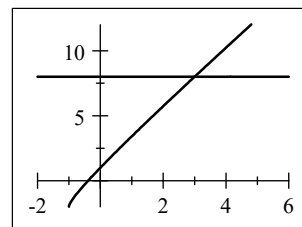
34. We graph $y = \sqrt{3 + x}$ and $y = \sqrt{x^2 + 1}$ in the viewing rectangle $[-4, 4]$ by $[0, 4]$ and see that the equation $\sqrt{3 + x} = \sqrt{x^2 + 1}$ has solutions $x = -1$ and $x = 2$, which can be verified by substitution.



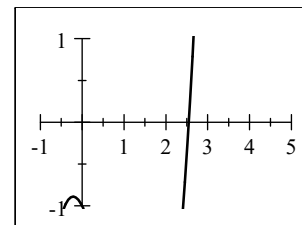
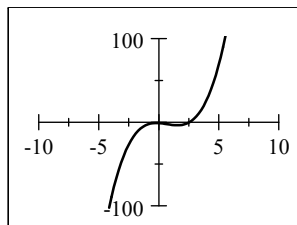
35. We graph $y = \sqrt{2x+1} + 1$ and $y = x$ in the viewing rectangle $[-1, 6]$ by $[0, 6]$ and see that the only solution to the equation $\sqrt{2x+1} + 1 = x$ is $x = 4$, which can be verified by substitution.



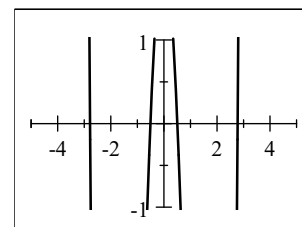
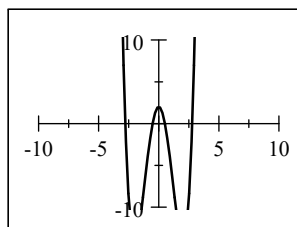
36. We graph $y = 2x + \sqrt{x+1}$ and $y = 8$ in the viewing rectangle $[-2, 6]$ by $[-2, 12]$ and see that the only solution to the equation $2x + \sqrt{x+1} = 8$ is $x = 3$, which can be verified by substitution.



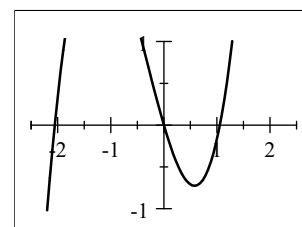
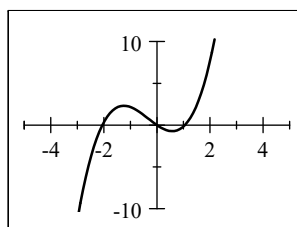
37. $x^3 - 2x^2 - x - 1 = 0$, so we start by graphing the function $y = x^3 - 2x^2 - x - 1$ in the viewing rectangle $[-10, 10]$ by $[-100, 100]$. There appear to be two solutions, one near $x = 0$ and another one between $x = 2$ and $x = 3$. We then use the viewing rectangle $[-1, 5]$ by $[-1, 1]$ and zoom in on the only solution, $x \approx 2.55$.



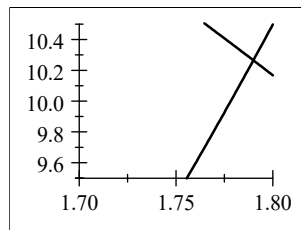
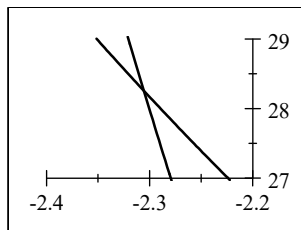
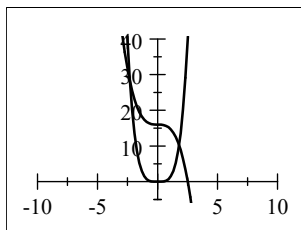
38. $x^4 - 8x^2 + 2 = 0$. We start by graphing the function $y = x^4 - 8x^2 + 2$ in the viewing rectangle $[-10, 10]$ by $[-10, 10]$. There appear to be four solutions between $x = -3$ and $x = 3$. We then use the viewing rectangle $[-5, 5]$ by $[-1, 1]$, and zoom to find the four solutions $x \approx -2.78$, $x \approx -0.51$, $x \approx 0.51$, and $x \approx 2.78$.



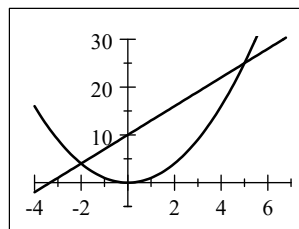
39. $x(x-1)(x+2) = \frac{1}{6}x \Leftrightarrow x(x-1)(x+2) - \frac{1}{6}x = 0$. We start by graphing the function $y = x(x-1)(x+2) - \frac{1}{6}x$ in the viewing rectangle $[-5, 5]$ by $[-10, 10]$. There appear to be three solutions. We then use the viewing rectangle $[-2.5, 2.5]$ by $[-1, 1]$ and zoom into the solutions at $x \approx -2.05$, $x = 0.00$, and $x \approx 1.05$.



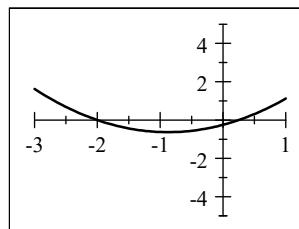
40. $x^4 = 16 - x^3$. We start by graphing the functions $y_1 = x^4$ and $y_2 = 16 - x^3$ in the viewing rectangle $[-10, 10]$ by $[-5, 40]$. There appears to be two solutions, one near $x = -2$ and another one near $x = 2$. We then use the viewing rectangle $[-2.4, -2.2]$ by $[27, 29]$, and zoom in to find the solution at $x \approx -2.31$. We then use the viewing rectangle $[1.7, 1.8]$ by $[9.5, 10.5]$, and zoom in to find the solution at $x \approx 1.79$.



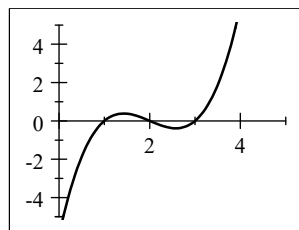
41. We graph $y = x^2$ and $y = 3x + 10$ in the viewing rectangle $[-4, 7]$ by $[-5, 30]$. The solution to the inequality is $[-2, 5]$.



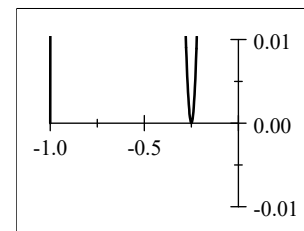
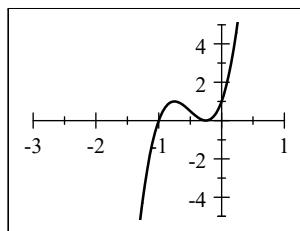
42. Since $0.5x^2 + 0.875x \leq 0.25 \Leftrightarrow 0.5x^2 + 0.875x - 0.25 \leq 0$, we graph $y = 0.5x^2 + 0.875x - 0.25$ in the viewing rectangle $[-3, 1]$ by $[-5, 5]$. Thus the solution to the inequality is $[-2, 0.25]$.



43. Since $x^3 + 11x \leq 6x^2 + 6 \Leftrightarrow x^3 - 6x^2 + 11x - 6 \leq 0$, we graph $y = x^3 - 6x^2 + 11x - 6$ in the viewing rectangle $[0, 5]$ by $[-5, 5]$. The solution set is $(-\infty, 1.00] \cup [2.00, 3.00]$.

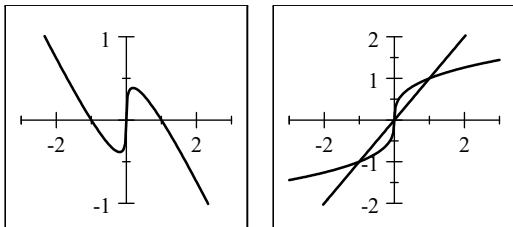


44. Since $16x^3 + 24x^2 > -9x - 1 \Leftrightarrow 16x^3 + 24x^2 + 9x + 1 > 0$, we graph $y = 16x^3 + 24x^2 + 9x + 1$ in the viewing rectangle $[-3, 1]$ by $[-5, 5]$. From this rectangle, we see that $x = -1$ is an x -intercept, but it is unclear what is occurring between $x = -0.5$ and $x = 0$. We then use the viewing rectangle $[-1, 0]$ by $[-0.01, 0.01]$. It shows $y = 0$ at $x = -0.25$. Thus in interval notation, the solution is $(-1, -0.25) \cup (-0.25, \infty)$.

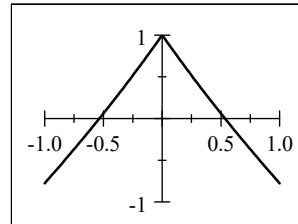


45. Since $x^{1/3} < x \Leftrightarrow x^{1/3} - x < 0$, we graph $y = x^{1/3} - x$ in the viewing rectangle $[-3, 3]$ by $[-1, 1]$. From this, we find that the solution set is $(-1, 0) \cup (1, \infty)$.

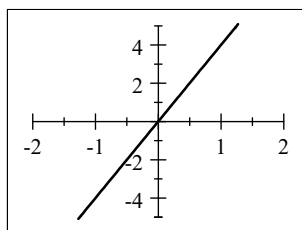
Another Method: As in Example 7, we graph $y_1 = x^{1/3}$ and $y_2 = x$ in the same viewing rectangle, and see that $x^{1/3} < x$ for $-1 < x < 0$ and for $1 < x$.



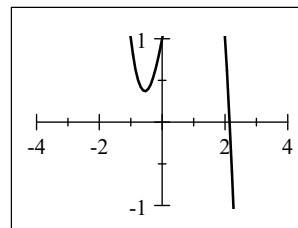
46. Since $\sqrt{0.5x^2 + 1} \leq 2|x| \Leftrightarrow \sqrt{0.5x^2 + 1} - 2|x| \leq 0$, we graph $y = \sqrt{0.5x^2 + 1} - 2|x|$ in the viewing rectangle $[-1, 1]$ by $[-1, 1]$. We locate the x -intercepts at $x \approx \pm 0.535$. Thus in interval notation, the solution is approximately $(-\infty, -0.535] \cup [0.535, \infty)$.



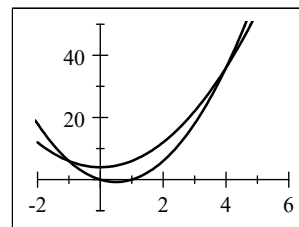
47. Since $(x+1)^2 < (x-1)^2 \Leftrightarrow (x+1)^2 - (x-1)^2 < 0$, we graph $y = (x+1)^2 - (x-1)^2$ in the viewing rectangle $[-2, 2]$ by $[-5, 5]$. The solution set is $(-\infty, 0)$.



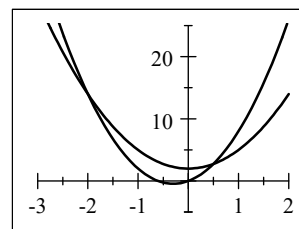
48. Since $(x+1)^2 \leq x^3 \Leftrightarrow (x+1)^2 - x^3 \leq 0$, we graph $y = (x+1)^2 - x^3$ in the viewing rectangle $[-4, 4]$ by $[-1, 1]$. The x -intercept is close to $x = 2$. Using a trace function, we obtain $x \approx 2.148$. Thus the solution is $[2.148, \infty)$.



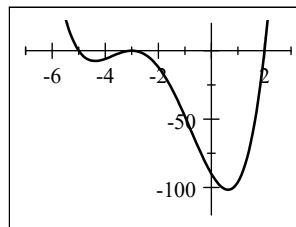
49. We graph the equations $y = 3x^2 - 3x$ and $y = 2x^2 + 4$ in the viewing rectangle $[-2, 6]$ by $[-10, 50]$. We see that the two curves intersect at $x = -1$ and at $x = 4$, and that the first curve is lower than the second for $-1 < x < 4$. Thus, we see that the inequality $3x^2 - 3x < 2x^2 + 4$ has solution set $(-1, 4)$.



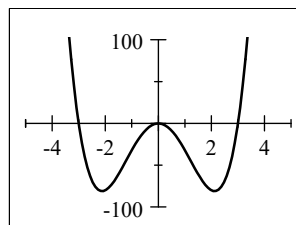
50. We graph the equations $y = 5x^2 + 3x$ and $y = 3x^2 + 2$ in the viewing rectangle $[-3, 2]$ by $[-5, 25]$. We see that the two curves intersect at $x = -2$ and at $x = \frac{1}{2}$, which can be verified by substitution. The first curve is higher than the second for $x < -2$ and for $x > \frac{1}{2}$, so the solution set of the inequality $5x^2 + 3x \geq 3x^2 + 2$ is $(-\infty, -2] \cup [\frac{1}{2}, \infty)$.



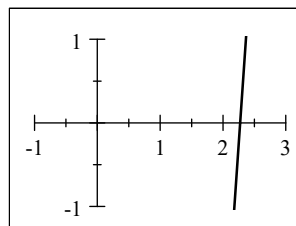
51. We graph the equation $y = (x + 3)^2(x - 2)(x + 5)$ in the viewing rectangle $[-7, 3]$ by $[-120, 20]$ and see that the inequality $(x + 3)^2(x - 2)(x + 5) \geq 0$ has the solution set $(-\infty, -5] \cup \{-3\} \cup [2, \infty)$.



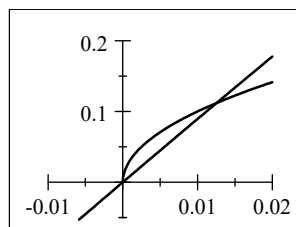
52. We graph the equation $y = 4x^2(x^2 - 9)$ in the viewing rectangle $[-5, 5]$ by $[-100, 100]$ and see that the inequality $4x^2(x^2 - 9) \leq 0$ has the solution set $[-3, 3]$.



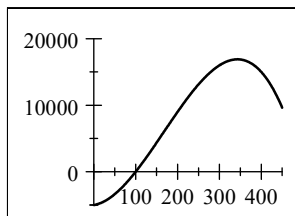
53. To solve $5 - 3x = 8x - 20$ by drawing the graph of a single equation, we isolate all terms on the left-hand side: $5 - 3x = 8x - 20 \Leftrightarrow 5 - 3x - 8x + 20 = 8x - 20 - 8x + 20 \Leftrightarrow -11x + 25 = 0$ or $11x - 25 = 0$. We graph $y = 11x - 25$, and see that the solution is $x \approx 2.27$, as in Example 2.



54. Graphing $y = x^3 - 6x^2 + 9x$ and $y = \sqrt{x}$ in the viewing rectangle $[-0.01, 0.02]$ by $[-0.05, 0.2]$, we see that $x = 0$ and $x = 0.01$ are solutions of the equation $x^3 - 6x^2 + 9x = \sqrt{x}$.

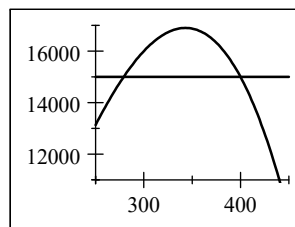


55. (a) We graph the equation $y = 10x + 0.5x^2 - 0.001x^3 - 5000$ in the viewing rectangle $[0, 450]$ by $[-5000, 20000]$.



- (b) From the graph it appears that $0 < 10x + 0.5x^2 - 0.001x^3 - 5000$ for $100 < x < 500$, and so 101 cooktops must be produced to *begin* to make a profit.

- (c) We graph the equations $y = 15,000$ and $y = 10x + 0.5x^2 - 0.001x^3 - 5000$ in the viewing rectangle $[250, 450]$ by $[11000, 17000]$. We use a zoom or trace function on a graphing calculator, and find that the company's profits are greater than \$15,000 for $279 < x < 400$.



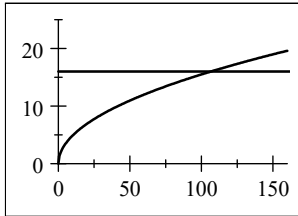
56. **Note:** On page 123 of the first printing of the text, the displayed equation in this exercise should read

$$y = \sqrt{2.4x + \left(\frac{x}{1000}\right)^2}$$

and part (a) of the exercise should read as follows:

- (a) Graph the equation for $0 \leq x \leq 160$.

(a)



- (b) Using a zoom or trace function, we find that $y \geq 16$ for $x \geq 106.7$. We

could estimate this since if $x < 160$, then $\left(\frac{x}{1000}\right)^2 \leq 0.0256$. So for

$x < 160$ we have $\sqrt{2.4x + \left(\frac{x}{1000}\right)^2} \approx \sqrt{2.4x}$. Solving $\sqrt{2.4x} > 16$ we get $2.4x > 256$ or $x > \frac{256}{2.4} = 106.7$ km.

57. Answers will vary.

58. Calculators perform operations in the following order: Exponents are applied before division, and division is applied before addition. Therefore, $Y_1 = x^{1/3}$ is interpreted as $y = \frac{x^1}{3} = \frac{x}{3}$, which is the equation of a line. Likewise, $Y_2 = x/x + 4$ is interpreted as $y = \frac{x}{x} + 4 = 1 + 4 = 5$. Instead, enter the following: $Y_1 = x^{(1/3)}$, $Y_2 = x/(x+4)$.

1.12 MODELING VARIATION

- If the quantities x and y are related by the equation $y = 5x$ then we say that y is *directly proportional* to x , and the constant of *proportionality* is 5.
- If the quantities x and y are related by the equation $y = \frac{5}{x}$ then we say that y is *inversely proportional* to x , and the constant of *proportionality* is 5.
- If the quantities x , y , and z are related by the equation $z = 5\frac{x}{y}$ then we say that z is *directly proportional* to x and *inversely proportional* to y .
- Because z is jointly proportional to x and y , we must have $z = kxy$. Substituting the given values, we get $10 = k(4)(5) = 20k \Leftrightarrow k = \frac{1}{2}$. Thus, x , y , and z are related by the equation $z = \frac{1}{2}xy$.
- (a) In the equation $y = 3x$, y is directly proportional to x .
(b) In the equation $y = 3x + 1$, y is not proportional to x .
- (a) In the equation $y = \frac{3}{x+1}$, y is not proportional to x .
(b) In the equation $y = \frac{3}{x}$, y is inversely proportional to x .
- $T = kx$, where k is constant.
- $P = kw$, where k is constant.
- $v = \frac{k}{z}$, where k is constant.
- $w = kmn$, where k is constant.
- $y = \frac{ks}{t}$, where k is constant.
- $P = \frac{k}{T}$, where k is constant.
- $z = k\sqrt{y}$, where k is constant.
- $A = \frac{kx^2}{t^3}$, where k is constant.

15. $V = klwh$, where k is constant.

16. $S = kr^2\theta^2$, where k is constant.

17. $R = \frac{kP^2t^2}{b^3}$, where k is constant.

18. $A = k\sqrt{xy}$, where k is constant.

19. Since y is directly proportional to x , $y = kx$. Since $y = 32$ when $x = 8$, we have $32 = k(8) \Leftrightarrow k = 4$. So $y = 4x$.

20. w is inversely proportional to t , so $w = \frac{k}{t}$. Since $w = 3$ when $t = 8$, we have $3 = \frac{k}{8} \Leftrightarrow k = 24$, so $w = \frac{24}{t}$.

21. A varies inversely as r , so $A = \frac{k}{r}$. Since $A = 15$ when $r = 5$, we have $15 = \frac{k}{5} \Leftrightarrow k = 75$. So $A = \frac{75}{r}$.

22. P is directly proportional to T , so $P = kT$. Since $P = 60$ when $T = 72$, we have $60 = k(72) \Leftrightarrow k = \frac{5}{6}$. So $P = \frac{5}{6}T$.

23. Since A is directly proportional to x and inversely proportional to t , $A = \frac{kx}{t}$. Since $A = 42$ when $x = 7$ and $t = 3$, we have $42 = \frac{k(7)}{3} \Leftrightarrow k = 18$. Therefore, $A = \frac{18x}{t}$.

24. $S = kpq$. Since $S = 350$ when $p = 7$ and $q = 20$, we have $350 = k(7)(20) \Leftrightarrow 350 = 140k \Leftrightarrow k = \frac{5}{2}$. So $S = \frac{5}{2}pq$.

25. Since W is inversely proportional to the square of r , $W = \frac{k}{r^2}$. Since $W = 24$ when $r = 3$, we have $24 = \frac{k}{(3)^2} \Leftrightarrow k = 216$.
So $W = \frac{216}{r^2}$.

26. $t = k\frac{xy}{r}$. Since $t = 125$ when $x = 10$, $y = 15$, and $r = 12$, we have $125 = k\frac{10 \cdot 15}{12} \Leftrightarrow k = 10$. So $t = 10\frac{xy}{r}$.

27. Since C is jointly proportional to l , w , and h , we have $C = klwh$. Since $C = 128$ when $l = w = h = 2$, we have $128 = k(2)(2)(2) \Leftrightarrow 128 = 8k \Leftrightarrow k = 16$. Therefore, $C = 16lwh$.

28. $H = kl^2w^2$. Since $H = 36$ when $l = 2$ and $w = \frac{1}{3}$, we have $36 = k(2)^2\left(\frac{1}{3}\right)^2 \Leftrightarrow 36 = \frac{4}{9}k \Leftrightarrow k = 81$. So $H = 81l^2w^2$.

29. $R = \frac{k}{\sqrt{x}}$. Since $R = 2.5$ when $x = 121$, $2.5 = \frac{k}{\sqrt{121}} = \frac{k}{11} \Leftrightarrow k = 27.5$. Thus, $R = \frac{27.5}{\sqrt{x}}$.

30. $M = k\frac{abc}{d}$. Since $M = 128$ when $a = d$ and $b = c = 2$, we have $128 = k\frac{a(2)(2)}{a} = 4k \Leftrightarrow k = 32$. So $M = 32\frac{abc}{d}$.

31. (a) $z = k\frac{x^3}{y^2}$

(b) If we replace x with $3x$ and y with $2y$, then $z = k\frac{(3x)^3}{(2y)^2} = \frac{27}{4}\left(k\frac{x^3}{y^2}\right)$, so z changes by a factor of $\frac{27}{4}$.

32. (a) $z = k\frac{x^2}{y^4}$

(b) If we replace x with $3x$ and y with $2y$, then $z = k\frac{(3x)^2}{(2y)^4} = \frac{9}{16}\left(k\frac{x^2}{y^4}\right)$, so z changes by a factor of $\frac{9}{16}$.

33. (a) $z = kx^3y^5$

(b) If we replace x with $3x$ and y with $2y$, then $z = k(3x)^3(2y)^5 = 864kx^3y^5$, so z changes by a factor of 864.

34. (a) $z = \frac{k}{x^2y^3}$

(b) If we replace x with $3x$ and y with $2y$, then $z = \frac{k}{(3x)^2(2y)^3} = \frac{1}{72}\frac{k}{x^2y^3}$, so z changes by a factor of $\frac{1}{72}$.

35. (a) The force F needed is $F = kx$.

(b) Since $F = 30$ N when $x = 9$ cm and the spring's natural length is 5 cm, we have $30 = k(9 - 5) \Leftrightarrow k = 7.5$ N/cm.

(c) From part (b), we have $F = 7.5x$. Substituting $x = 11 - 5 = 6$ into $F = 7.5x$ gives $F = 7.5(6) = 45$ N.

36. (a) $C = kpm$

(b) Since $C = 60,000$ when $p = 120$ and $m = 4000$, we get $60,000 = k(120)(4000) \Leftrightarrow k = \frac{1}{8}$. So $C = \frac{1}{8}pm$.

(c) Substituting $p = 92$ and $m = 5000$, we get $C = \frac{1}{8}(92)(5000) = \$57,500$.

37. (a) $P = ks^3$.

(b) Since $P = 96$ when $s = 20$, we get $96 = k \cdot 20^3 \Leftrightarrow k = 0.012$. So $P = 0.012s^3$.

(c) Substituting $x = 30$, we get $P = 0.012 \cdot 30^3 = 324$ watts.

38. Let the amount of soluble CO_2 be x and the temperature of the water be T . Because these quantities are inversely proportional, $x = \frac{k}{T}$. We are given that when $T = 273$ K, $x \approx 3$ g, so $3 \approx \frac{k}{273} \Leftrightarrow k \approx 819$. Thus, if $T = 298$, then $x \approx \frac{819}{298} \approx 2.75$ g.

39. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

$D = ks^2$. Since $D = 45$ when $s = 60$, we have $45 = k(60)^2$, so $k = 0.0125$. Thus, $D = 0.0125s^2$. If $D = 60$, then $60 = 0.0125s^2 \Leftrightarrow s^2 = 4800$, so $s \approx 69$ km/h (for safety reasons we round down).

40. $L = ks^2A$. Since $L = 5950$ when $s = 80$ and $A = 46$, we have $5950 = k(80^2)(46) \Leftrightarrow k = 0.02021$. Thus

$L = 0.02021s^2A$. When $A = 56$ and $s = 64$, the lift is $L = 0.02021(64^2)(56) \approx 4635.7$ N.

41. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

$F = kAs^2$. Since $F = 980$ when $A = 4$ and $s = 8$, we can solve for k : $980 = k(4)(8)^2 \Leftrightarrow 980 = 256k \Leftrightarrow k = 3.83$. Now when $A = 2.5$ and $F = 780$ we get $780 = 3.83(2.5)s^2 \Leftrightarrow 81.4621 = s^2$, so $s = \sqrt{81.4621} \approx 9.03$ km/h.

42. (a) $T^2 = kd^3$

(b) Substituting $T = 365$ and $d = 149 \times 10^6$, we get $365^2 = k \cdot (149 \times 10^6)^3 \Leftrightarrow k = 4.03 \times 10^{-20}$.

(c) $T^2 = 4.03 \times 10^{-20} (4.46 \times 10^9)^3 = 3.58 \times 10^9 \Rightarrow T = 5.98 \times 10^4$. Hence the period of Neptune is 5.98×10^4 days ≈ 164 years.

43. (a) $P = \frac{kT}{V}$.

(b) Substituting $P = 33.2$, $T = 400$, and $V = 100$, we get $33.2 = \frac{k(400)}{100} \Leftrightarrow k = 8.3$. Thus $k = 8.3$ and the equation is

$$P = \frac{8.3T}{V}.$$

(c) Substituting $T = 500$ and $V = 80$, we have $P = \frac{8.3(500)}{80} = 51.875$ kPa. Hence the pressure of the sample of gas is about 51.9 kPa.

44. (a) $F = k \frac{ws^2}{r}$

(b) For the first car we have $w_1 = 768$ and $s_1 = 100$ and for the second car we have $w_2 = 1200$. Since the forces are equal

we have $k \frac{768 \cdot 100^2}{r} = k \frac{1200 \cdot s_2^2}{r} \Leftrightarrow \frac{16 \cdot 100^2}{25} = s_2^2$, so $s_2 = 80$ km/h.

45. (a) The loudness L is inversely proportional to the square of the distance d , so $L = \frac{k}{d^2}$.

(b) Substituting $d = 10$ and $L = 70$, we have $70 = \frac{k}{10^2} \Leftrightarrow k = 7000$.

- (c) Substituting $2d$ for d , we have $L = \frac{k}{(2d)^2} = \frac{1}{4} \left(\frac{k}{d^2} \right)$, so the loudness is changed by a factor of $\frac{1}{4}$.
- (d) Substituting $\frac{1}{2}d$ for d , we have $L = \frac{k}{\left(\frac{1}{2}d\right)^2} = 4 \left(\frac{k}{d^2} \right)$, so the loudness is changed by a factor of 4.
46. (a) The power P is jointly proportional to the area A and the cube of the velocity v , so $P = kAv^3$.
- (b) Substituting $2v$ for v and $\frac{1}{2}A$ for A , we have $P = k \left(\frac{1}{2}A \right) (2v)^3 = 4kAv^3$, so the power is changed by a factor of 4.
- (c) Substituting $\frac{1}{2}v$ for v and $3A$ for A , we have $P = k(3A) \left(\frac{1}{2}v \right)^3 = \frac{3}{8}kAv^3$, so the power is changed by a factor of $\frac{3}{8}$.
47. (a) $R = \frac{kL}{d^2}$
- (b) Since $R = 140$ when $L = 1.2$ and $d = 0.005$, we get $140 = \frac{k(1.2)}{(0.005)^2} \Leftrightarrow k = \frac{7}{2400} = 0.00291\bar{6}$.
- (c) Substituting $L = 3$ and $d = 0.008$, we have $R = \frac{7}{2400} \cdot \frac{3}{(0.008)^2} = \frac{4375}{32} \approx 137$ ohms.
- (d) If we substitute $2d$ for d and $3L$ for L , then $R = \frac{k(3L)}{(2d)^2} = \frac{3}{4} \frac{kL}{d^2}$, so the resistance is changed by a factor of $\frac{3}{4}$.
48. Let S be the final size of the cabbage, in kilograms, let N be the amount of nutrients it receives, in grams, and let c be the number of other cabbages around it. Then $S = k \frac{N}{c}$. When $N = 600$ and $c = 12$, we have $S = 15$, so substituting, we have $15 = k \frac{600}{12} \Leftrightarrow k = 0.3$. Thus $S = 0.3 \frac{N}{c}$. When $N = 300$ and $c = 5$, the final size is $S = 0.3 \left(\frac{300}{5} \right) = 18$ kg.
49. **Note: In the first printing of the text, the answer given for part (b) of this exercise is incorrect.**
- (a) For the sun, $E_S = k6000^4$ and for earth, $E_E = k300^4$. Thus $\frac{E_S}{E_E} = \frac{k6000^4}{k300^4} = \left(\frac{6000}{300} \right)^4 = 20^4 = 160,000$. So the sun produces 160,000 times the radiation energy per unit area than the Earth.
- (b) The surface area of the sun is $4\pi (696,000)^2$ and the surface area of the Earth is $4\pi (6340)^2$. So the sun has $\frac{4\pi (696,000)^2}{4\pi (6340)^2} = \left(\frac{696,000}{6340} \right)^2$ times the surface area of the Earth. Thus the total radiation emitted by the sun is $160,000 \times \left(\frac{696,000}{6340} \right)^2 = 1,928,234,931$ times the total radiation emitted by the Earth.
50. (a) Let T and l be the period and the length of the pendulum, respectively. Then $T = k\sqrt{l}$.
- (b) $T = k\sqrt{l} \Rightarrow T^2 = k^2l \Leftrightarrow l = \frac{T^2}{k^2}$. If the period is doubled, the new length is $\frac{(2T)^2}{k^2} = 4 \frac{T^2}{k^2} = 4l$. So we would quadruple the length l to double the period T .
51. (a) Since f is inversely proportional to L , we have $f = \frac{k}{L}$, where k is a positive constant.
- (b) If we replace L by $2L$ we have $\frac{k}{2L} = \frac{1}{2} \cdot \frac{k}{L} = \frac{1}{2}f$. So the frequency of the vibration is cut in half.
52. (a) Since r is jointly proportional to x and $P - x$, we have $r = kx(P - x)$, where k is a positive constant.
- (b) When 10 people are infected the rate is $r = k10(5000 - 10) = 49,900k$. When 1000 people are infected the rate is $r = k \cdot 1000 \cdot (5000 - 1000) = 4,000,000k$. So the rate is much higher when 1000 people are infected. Comparing these rates, we find that $\frac{1000 \text{ people infected}}{10 \text{ people infected}} = \frac{4,000,000k}{49,900k} \approx 80$. So the infection rate when 1000 people are infected is about 80 times as large as when 10 people are infected.

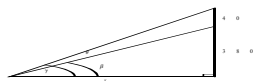
- (c) When the entire population is infected the rate is $r = k(5000)(5000 - 5000) = 0$. This makes sense since there are no more people who can be infected.
53. We substitute $e = 0.2$ kWh/km and $v = 100$ km/h into the second given equation: $0.2 = k(100)^2 \Leftrightarrow k = \frac{0.2}{100^2} = 2 \times 10^{-5}$.
Now, because the range is $R = 500$ km for these values of e and v , we can use the first equation to calculate the value of C :
 $500 = \frac{C}{0.2} \Leftrightarrow C = 100$, so a full charge for this vehicle is 100 kWh.
Substituting $e = kv^2$ into the first given equation, we have $R = \frac{C}{kv^2}$.
So at 130 km/h, its range is $R \approx \frac{100}{2 \times 10^{-5} (130)^2} \approx 296$ km; and at 80 km/h, its range is $R \approx \frac{100}{2 \times 10^{-5} (80)^2} \approx 781$ km.
54. (a) We are given that the mass flow rate is $m = A_0 v_0 \rho$. Because m is constant, we can equate the flow rates for the wider and narrower sections of pipe: $A_0 v_0 \rho = A v \rho \Leftrightarrow v = \frac{A_0 v_0}{A}$. The velocity of the fluid is indeed inversely proportional to the cross-sectional area of the pipe.
- (b) We substitute the given values into our expression for v : $v = \frac{A_0 v_0}{A} = \frac{\pi r_0^2 v_0}{\pi r^2} = \frac{(0.6 \text{ m})^2 (5 \text{ m/s})}{(0.2 \text{ m})^2} = 45 \text{ m/s}$. Thus, the fluid flows through the constricted section of pipe at 45 m/s.
55. Using $B = k \frac{L}{d^2}$ with $k = 0.080$, $L = 2.5 \times 10^{26}$, and $d = 2.4 \times 10^{19}$, we have $B = 0.080 \frac{2.5 \times 10^{26}}{(2.4 \times 10^{19})^2} \approx 3.47 \times 10^{-14}$.
The star's apparent brightness is about 3.47×10^{-14} W/m².
56. First, we solve $B = k \frac{L}{d^2}$ for d : $d^2 = k \frac{L}{B} \Rightarrow d = \sqrt{k \frac{L}{B}}$ because d is positive. Substituting $k = 0.080$, $L = 5.8 \times 10^{30}$, and $B = 8.2 \times 10^{-16}$, we find $d = \sqrt{0.080 \frac{5.8 \times 10^{30}}{8.2 \times 10^{-16}}} \approx 2.38 \times 10^{22}$, so the star is approximately 2.38×10^{22} m from earth.
57. Examples include radioactive decay and exponential growth in biology.

CHAPTER 1 REVIEW

1. Commutative Property for addition.

3. Distributive Property.

5. $[-2, 6) = \{x \mid -2 \leq x < 6\}$



7. $x \geq 5 \Leftrightarrow x \in [5, \infty)$



9. $|1 - |-4|| = |1 - 4| = |-3| = 3$

11. $2^{1/2} 8^{1/2} = \sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$

13. $216^{-1/3} = \frac{1}{216^{1/3}} = \frac{1}{\sqrt[3]{216}} = \frac{1}{6}$

2. Commutative Property for multiplication.

4. Distributive Property.

6. $(-\infty, 4] = \{x \mid x \leq 4\}$



8. $-1 < x \leq 5 \Leftrightarrow x \in (-1, 5]$



10. $5 - |10 - |-4|| = 5 - |10 - 4| = 5 - 6 = -1$

12. $2^{-3} - 3^{-2} = \frac{1}{8} - \frac{1}{9} = \frac{9}{72} - \frac{8}{72} = \frac{1}{72}$

14. $64^{2/3} = (4^3)^{2/3} = 4^2 = 16$

$$15. \frac{\sqrt{242}}{\sqrt{2}} = \sqrt{\frac{242}{2}} = \sqrt{121} = 11$$

$$16. \sqrt{2}\sqrt{50} = \sqrt{100} = 10$$

$$17. (a) (a^2)^{-3} (a^3b)^2 (b^3)^4 = a^{-6} \cdot a^6b^2 \cdot b^{12} = a^{-6+6}b^{2+12} = b^{14}$$

$$(b) (3xy^2)^3 \left(\frac{2}{3}x^{-1}y\right)^2 = 27x^3y^6 \cdot \frac{4}{9}x^{-2}y^2 = 27 \cdot \frac{4}{9}x^{3-2}y^{6+2} = 12xy^8$$

$$18. (a) \frac{x^4(3x)^2}{x^3} = \frac{x^4 \cdot 9x^2}{x^3} = 9x^{4+2-3} = 9x^3$$

$$(b) \left(\frac{r^2s^{4/3}}{r^{1/3}s}\right)^6 = \frac{r^{12}s^8}{r^2s^6} = r^{12-2}s^{8-6} = r^{10}s^2$$

$$19. (a) \sqrt[3]{(x^3y)^2y^4} = \sqrt[3]{x^6y^4y^2} = \sqrt[3]{x^6y^6} = x^2y^2$$

$$(b) \sqrt{w^8z^{10}} = \sqrt{w^8} \cdot \sqrt{z^{10}} = w^4|z|^5$$

$$20. (a) \frac{8r^{1/2}s^{-3}}{2r^{-2}s^4} = 4r^{(1/2)-(-2)}s^{-3-4} = 4r^{5/2}s^{-7} = \frac{4r^{5/2}}{s^7}$$

$$(b) \left(\frac{ab^2c^{-3}}{2a^3b^{-4}}\right)^{-2} = \frac{a^{-2}b^{-4}c^6}{2^{-2}a^{-6}b^8} = 2^2a^{-2-(-6)}b^{-4-8}c^6 = 4a^4b^{-12}c^6 = \frac{4a^4c^6}{b^{12}}$$

$$21. 78,250,000,000 = 7.825 \times 10^{10}$$

$$22. 2.08 \times 10^{-8} = 0.000\,000\,020\,8$$

$$23. \frac{ab}{c} \approx \frac{(0.00000293)(1.582 \times 10^{-14})}{2.8064 \times 10^{12}} = \frac{(2.93 \times 10^{-6})(1.582 \times 10^{-14})}{2.8064 \times 10^{12}} = \frac{2.93 \cdot 1.582}{2.8064} \times 10^{-6-14-12} \\ \approx 1.65 \times 10^{-32}$$

$$24. 80 \frac{\text{times}}{\text{minute}} \cdot \frac{60 \text{ minutes}}{\text{hour}} \cdot \frac{24 \text{ hours}}{\text{day}} \cdot \frac{365 \text{ days}}{\text{year}} \cdot 90 \text{ years} \approx 3.8 \times 10^9 \text{ times}$$

$$25. x^2 + 5x - 14 = (x + 7)(x - 2)$$

$$26. 12x^2 + 10x - 8 = 2(6x^2 + 5x - 4) = 2(3x + 4)(2x - 1)$$

$$27. x^4 - 2x^2 + 1 = (x^2 - 1)^2 = [(x - 1)(x + 1)]^2 = (x - 1)^2(x + 1)^2$$

$$28. 4y^2z^3 + 10y^3z - 12y^5z^2 = 2y^2z(2z^2 + 5y - 6y^3z)$$

$$29. 16 - 4t^2 = 4(4 - t^2) = 4(2 - t)(2 + t) = -4(t - 2)(t + 2)$$

$$30. 2y^6 - 32y^2 = 2y^2(y^4 - 16) = 2y^2(y^2 + 4)(y^2 - 4) = 2y^2(y^2 + 4)(y + 2)(y - 2)$$

$$31. x^6 - 1 = (x^3 - 1)(x^3 + 1) = (x - 1)(x^2 + x + 1)(x + 1)(x^2 - x + 1)$$

$$32. 16a^4b^2 + 2ab^5 = 2ab^2(8a^3 + b^3) = 2ab^2(2a + b)(4a^2 - 2ab + b^2)$$

$$33. -3x^{-1/2} + 2x^{1/2} + 5x^{3/2} = x^{-1/2}(5x^3 + 2x^2 - 3x) = x^{-1/2}(5x - 3)(x + 1)$$

$$34. 7x^{-3/2} - 8x^{-1/2} + x^{1/2} = x^{-3/2}(x^2 - 8x^2 + 7) = x^{-3/2}(x - 1)(x - 7)$$

$$35. 5x^3 + 15x^2 - x - 3 = 5x^2(x + 3) - (x + 3) = (5x^2 - 1)(x + 3)$$

$$36. w^3 - 3w^2 - 4w + 12 = w^2(w - 3) - 4(w - 3) = (w^2 - 4)(w - 3) = (w - 3)(w - 2)(w + 2)$$

$$37. (a + b)^2 - 3(a + b) - 10 = (a + b - 5)(a + b + 2)$$

38. $(3x + 2)^2 - (3x + 2) - 6 = [(3x + 2) - 3][(3x + 2) + 2] = (3x - 1)(3x + 4)$
39. $(2y - 7)(2y + 7) = 4y^2 - 49$
40. $(1 + x)(2 - x) - (3 - x)(3 + x) = 2 + x - x^2 - (9 - x^2) = 2 + x - x^2 - 9 + x^2 = -7 + x$
41. $x^2(x - 2) + x(x - 2)^2 = x^3 - 2x^2 + x(x^2 - 4x + 4) = x^3 - 2x^2 + x^3 - 4x^2 + 4x = 2x^3 - 6x^2 + 4x$
42. $\sqrt{x}(\sqrt{x} + 1)(2\sqrt{x} - 1) = (x + \sqrt{x})(2\sqrt{x} - 1) = 2x\sqrt{x} - x + 2x - \sqrt{x} = 2x^{3/2} + x - x^{1/2}$
43. $\frac{x^2 - 4x - 5}{x^2 - 25} \cdot \frac{x^2 + 12x + 36}{x^2 + 7x + 6} = \frac{(x - 5)(x + 1)}{(x + 5)(x - 5)} \cdot \frac{(x + 6)^2}{(x + 6)(x + 1)} = \frac{x + 1}{x + 5} \cdot \frac{x + 6}{x + 1} = \frac{x + 6}{x + 5}$
44. $\frac{x^2 - 2x - 15}{x^2 - 6x + 5} \div \frac{x^2 - x - 12}{x^2 - 1} = \frac{(x - 5)(x + 3)}{(x - 5)(x - 1)} \cdot \frac{(x - 1)(x + 1)}{(x - 4)(x + 3)} = \frac{x + 1}{x - 4}$
45. $\frac{2}{x} + \frac{1}{x - 2} + \frac{3}{(x - 2)^2} = \frac{2(x - 2)^2}{x(x - 2)^2} + \frac{x(x - 2)}{x(x - 2)^2} + \frac{3x}{x(x - 2)^2}$
 $= \frac{2(x^2 - 4x + 4) + x^2 - 2x + 3x}{x(x - 2)^2} = \frac{2x^2 - 8x + 8 + x^2 - 2x + 3x}{x(x - 2)^2} = \frac{3x^2 - 7x + 8}{x(x - 2)^2}$
46. $\frac{1}{x + 2} + \frac{1}{x^2 - 4} - \frac{2}{x^2 - x - 2} = \frac{1}{x + 2} + \frac{1}{(x - 2)(x + 2)} - \frac{2}{(x - 2)(x + 1)}$
 $= \frac{(x - 2)(x + 1)}{(x - 2)(x + 1)(x + 2)} + \frac{x + 1}{(x - 2)(x + 1)(x + 2)} - \frac{2(x + 2)}{(x - 2)(x + 1)(x + 2)}$
 $= \frac{x^2 - x - 2 + x + 1 - 2x - 4}{(x - 2)(x + 1)(x + 2)} = \frac{x^2 - 2x - 5}{(x - 2)(x + 1)(x + 2)}$
47. $\frac{\frac{1}{x} - \frac{1}{2}}{x - 2} = \frac{\frac{2}{2x} - \frac{x}{2x}}{x - 2} = \frac{2 - x}{2x} \cdot \frac{1}{x - 2} = \frac{-1(x - 2)}{2x} \cdot \frac{1}{x - 2} = \frac{-1}{2x}$
48. $\frac{\frac{1}{x} - \frac{1}{x + 1}}{\frac{1}{x} + \frac{1}{x + 1}} = \frac{\frac{1}{x} - \frac{1}{x + 1}}{\frac{1}{x} + \frac{1}{x + 1}} \cdot \frac{x(x + 1)}{x(x + 1)} = \frac{(x + 1) - x}{(x + 1) + x} = \frac{1}{2x + 1}$
49. $\frac{\sqrt{x} - 2}{\sqrt{x} + 2} = \frac{\sqrt{x} - 2}{\sqrt{x} + 2} \cdot \frac{\sqrt{x} - 2}{\sqrt{x} - 2} = \frac{x + 4 - 4\sqrt{x}}{x - 4}$
50. $\frac{\sqrt{x + h} - \sqrt{x}}{h} = \frac{\sqrt{x + h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x + h} + \sqrt{x}}{\sqrt{x + h} + \sqrt{x}} = \frac{(x + h) - x}{h(\sqrt{x + h} + \sqrt{x})} = \frac{h}{h(\sqrt{x + h} + \sqrt{x})} = \frac{1}{\sqrt{x + h} + \sqrt{x}}$
51. $\frac{1}{\sqrt{11}} = \frac{1}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{\sqrt{11}}{11}$
52. $\frac{3}{\sqrt{6}} = \frac{3}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{2}$
53. $\frac{5}{1 + \sqrt{2}} = \frac{5(1 - \sqrt{2})}{(1 + \sqrt{2})(1 - \sqrt{2})} = \frac{5 - 5\sqrt{2}}{1 - 2} = 5\sqrt{2} - 5$
54. $\frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}} = \frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}} \cdot \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \frac{3\sqrt{2} - 2\sqrt{3}}{3 - 2} = 3\sqrt{2} - 2\sqrt{3}$
55. $7x - 6 = 4x + 9 \Leftrightarrow 3x = 15 \Leftrightarrow x = 5$
56. $8 - 2x = 14 + x \Leftrightarrow -3x = 6 \Leftrightarrow x = -2$
57. $\frac{3x + 6}{x + 3} = \frac{3x}{x + 1} \Leftrightarrow (3x + 6)(x + 1) = 3x(x + 3) \Leftrightarrow 3x^2 + 9x + 6 = 3x^2 + 9x \Leftrightarrow 6 = 0$. This is false, so there is no solution to the original equation.

58. $(x+2)^2 = (x-4)^2 \Leftrightarrow (x+2)^2 - (x-4)^2 = 0 \Leftrightarrow [(x+2) - (x-4)][(x+2) + (x-4)] = 0 \Leftrightarrow [x+2-x+4][x+2+x-4] = 6(2x-2) = 0 \Leftrightarrow 2x-2 = 0 \Leftrightarrow x = 1.$
59. $x^2 - 9x + 14 = 0 \Leftrightarrow (x-7)(x-2) = 0 \Leftrightarrow x = 7$ or $x = 2.$
60. $x^2 + 24x + 144 = 0 \Leftrightarrow (x+12)^2 = 0 \Leftrightarrow x+12 = 0 \Leftrightarrow x = -12.$
61. $2x^2 + x = 1 \Leftrightarrow 2x^2 + x - 1 = 0 \Leftrightarrow (2x-1)(x+1) = 0.$ So either $2x-1 = 0 \Leftrightarrow 2x = 1 \Leftrightarrow x = \frac{1}{2};$ or $x+1 = 0 \Leftrightarrow x = -1.$
62. $\sqrt{x-1} = \sqrt{x^2-3} \Rightarrow x-1 = x^2-3 \Leftrightarrow x^2-x-2 = 0 \Leftrightarrow (x+1)(x-2) = 0 \Leftrightarrow x = -1$ or $x = 2.$ However, $x = -1$ fails to satisfy the original equation because $\sqrt{(-1)^2-3} = \sqrt{-2},$ which is undefined. Thus, the only solution is $x = 2.$
63. $0 = 4x^3 - 25x = x(4x^2 - 25) = x(2x-5)(2x+5) = 0.$ So either $x = 0;$ or $2x-5 = 0 \Leftrightarrow 2x = 5 \Leftrightarrow x = \frac{5}{2};$ or $2x+5 = 0 \Leftrightarrow 2x = -5 \Leftrightarrow x = -\frac{5}{2}.$
64. $x^3 - 2x^2 - 5x + 10 = 0 \Leftrightarrow x^2(x-2) - 5(x-2) = 0 \Leftrightarrow (x-2)(x^2-5) = 0 \Leftrightarrow x = 2$ or $x = \pm\sqrt{5}.$
65. $3x^2 + 4x - 1 = 0 \Rightarrow$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(4) \pm \sqrt{(4)^2 - 4(3)(-1)}}{2(3)} = \frac{-4 \pm \sqrt{16+12}}{-6} = \frac{-4 \pm \sqrt{28}}{-6} = \frac{-4 \pm 2\sqrt{7}}{6} = \frac{2(-2 \pm \sqrt{7})}{6} = \frac{-2 \pm \sqrt{7}}{3}.$$
66. $\frac{1}{x} + \frac{2}{x-1} = 3 \Leftrightarrow (x-1) + 2(x) = 3(x)(x-1) \Leftrightarrow x-1+2x = 3x^2-3x \Leftrightarrow 0 = 3x^2-6x+1 \Rightarrow$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)} = \frac{6 \pm \sqrt{36-12}}{6} = \frac{6 \pm \sqrt{24}}{6} = \frac{6 \pm 2\sqrt{6}}{6} = \frac{2(3 \pm \sqrt{6})}{6} = \frac{3 \pm \sqrt{6}}{3}.$$
67. $\frac{x}{x-2} + \frac{1}{x+2} = \frac{8}{x^2-4} \Leftrightarrow x(x+2) + (x-2) = 8 \Leftrightarrow x^2+2x+x-2 = 8 \Leftrightarrow x^2+3x-10 = 0 \Leftrightarrow (x-2)(x+5) = 0$
 $\Leftrightarrow x = 2$ or $x = -5.$ However, since $x = 2$ makes the expression undefined, we reject this solution. Hence the only solution is $x = -5.$
68. $x^4 - 8x^2 - 9 = 0 \Leftrightarrow (x^2-9)(x^2+1) = 0 \Leftrightarrow (x-3)(x+3)(x^2+1) = 0 \Rightarrow x-3 = 0 \Leftrightarrow x = 3,$ or $x+3 = 0 \Leftrightarrow x = -3,$ however $x^2+1 = 0$ has no real solution. The solutions are $x = \pm 3.$
69. $(x-3)^2 - 3(x-3) - 4 = 0 \Leftrightarrow [(x-3)-4][(x-3)+1] = 0 \Leftrightarrow (x-7)(x-2) = 0 \Leftrightarrow x = 2$ or $x = 7.$
70. $9x^{-1/2} - 6x^{1/2} + x^{3/2} = 0 \Leftrightarrow x^{-1/2}(9 - 6x + x^2) = 0 \Leftrightarrow x^{-1/2}(x-3)^2 = 0 \Leftrightarrow x = 3.$
71. $|x-7| = 4 \Leftrightarrow x-7 = \pm 4 \Leftrightarrow x = 7 \pm 4,$ so $x = 11$ or $x = 3.$
72. $|2x-5| = 9$ is equivalent to $2x-5 = \pm 9 \Leftrightarrow 2x = 5 \pm 9 \Leftrightarrow x = \frac{5 \pm 9}{2}.$ So $x = -2$ or $x = 7.$
73. (a) $(2-3i) + (1+4i) = (2+1) + (-3+4)i = 3+i$
 (b) $(2+i)(3-2i) = 6-4i+3i-2i^2 = 6-i+2 = 8-i$
74. (a) $(3-6i) - (6-4i) = (3-6) + [-6-(-4)]i = -3-2i$
 (b) $4i\left(2-\frac{1}{2}i\right) = 8i-2i^2 = 8i+2 = 2+8i$
75. (a) $\frac{4+2i}{2-i} = \frac{4+2i}{2-i} \cdot \frac{2+i}{2+i} = \frac{8+8i+2i^2}{4-i^2} = \frac{8+8i-2}{4+1} = \frac{6+8i}{5} = \frac{6}{5} + \frac{8}{5}i$
 (b) $(1-\sqrt{-1})(1+\sqrt{-1}) = (1-i)(1+i) = 1+i-i-i^2 = 1+1 = 2$
76. (a) $\frac{2-3i}{1+i} = \frac{(2-3i)(1-i)}{(1+i)(1-i)} = \frac{2-5i+3i^2}{1-i^2} = \frac{-1-5i}{2} = -\frac{1}{2} - \frac{5}{2}i$
 (b) $\sqrt{-10} \cdot \sqrt{-40} = i\sqrt{10} \cdot 2i\sqrt{10} = 20i^2 = -20$
77. $x^2 + 16 = 0 \Leftrightarrow x^2 = -16 \Leftrightarrow x = \pm 4i$
78. $x^2 = -12 \Leftrightarrow x = \pm\sqrt{-12} = \pm 2\sqrt{3}i$

$$79. x^2 + 6x + 10 = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 40}}{2} = -3 \pm i$$

$$80. 2x^2 - 3x + 2 = 0 \Leftrightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(2)}}{2(2)} = \frac{3 \pm \sqrt{-7}}{4} = \frac{3}{4} \pm \frac{\sqrt{7}}{4}i$$

$$81. x^4 - 256 = 0 \Leftrightarrow (x^2 - 16)(x^2 + 16) = 0 \Leftrightarrow x = \pm 4 \text{ or } x = \pm 4i$$

$$82. x^3 - 2x^2 + 4x - 8 = 0 \Leftrightarrow (x - 2)(x^2 + 4) \Leftrightarrow x = 2 \text{ or } x = \pm 2i$$

83. Let x be the number of kilograms of raisins. Then the number of kilograms of nuts is $50 - x$.

	Raisins	Nuts	Mixture
Weight	x	$50 - x$	50
Cost per kilogram	3.20	2.40	2.72

So $3.20x + 2.40(50 - x) = 2.72(50) \Leftrightarrow 3.20x + 120 - 2.40x = 136 \Leftrightarrow 0.8x = 16 \Leftrightarrow x = 20$. Thus the mixture uses 20 kilograms of raisins and $50 - 20 = 30$ kilograms of nuts.

84. Let t be the number of hours that the district supervisor drives. Then the store manager drives for $t - \frac{1}{4}$ hours.

	Rate	Time	Distance
Supervisor	72	t	$72t$
Manager	64	$t - \frac{1}{4}$	$64\left(t - \frac{1}{4}\right)$

When they pass each other, they will have traveled a total of 256 kilometers. So $72t + 64\left(t - \frac{1}{4}\right) = 256 \Leftrightarrow$

$72t + 64t - 16 = 256 \Leftrightarrow 136t = 272 \Leftrightarrow t = 2$. Since the supervisor leaves at 2:00 P.M. and travels for 2 hours, they pass each other at 4:00 P.M.

85. Let r be the athlete's running speed, in mi/h. Then they cycle at $r + 8$ km/h.

	Rate	Time	Distance
Cycle	$r + 8$	$\frac{4}{r + 8}$	4
Run	r	$\frac{2.5}{r}$	2.5

Since the total time of the workout is 1 hour, we have $\frac{4}{r + 8} + \frac{2.5}{r} = 1$. Multiplying by $2r(r + 8)$, we

get $4(2r) + 2.5(2)(r + 8) = 2r(r + 8) \Leftrightarrow 8r + 5r + 40 = 2r^2 + 16r \Leftrightarrow 0 = 2r^2 + 3r - 40 \Rightarrow$

$r = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-40)}}{2(2)} = \frac{-3 \pm \sqrt{9 + 320}}{4} = \frac{-3 \pm \sqrt{329}}{4}$. Since $r \geq 0$, we reject the negative value. The athlete's running

speed is $r = \frac{-3 + \sqrt{329}}{4} \approx 3.78$ km/h.

86. Let x be the length of one side in cm. Then $28 - x$ is the length of the other side. Using the Pythagorean Theorem, we have $x^2 + (28 - x)^2 = 20^2 \Leftrightarrow x^2 + 784 - 56x + x^2 = 400 \Leftrightarrow 2x^2 - 56x + 384 = 0 \Leftrightarrow 2(x^2 - 28x + 192) = 0 \Leftrightarrow 2(x - 12)(x - 16) = 0$. So $x = 12$ or $x = 16$. If $x = 12$, then the other side is $28 - 12 = 16$. Similarly, if $x = 16$, then the other side is 12. The sides are 12 cm and 16 cm.

87. Let t be the time it would take the interior decorator to paint a living room if they work alone. It would take the assistant $2t$ hours alone, and it would take the apprentice $3t$ hours alone. Thus, the decorator does $\frac{1}{t}$ of the job per hour, the assistant does $\frac{1}{2t}$ of the job per hour, and the apprentice does $\frac{1}{3t}$ of the job per hour. So $\frac{1}{t} + \frac{1}{2t} + \frac{1}{3t} = 1 \Leftrightarrow 6 + 3 + 2 = 6t \Leftrightarrow 6t = 11 \Leftrightarrow t = \frac{11}{6}$. Thus, it would take the decorator 1 hour 50 minutes to paint the living room alone.

88. Let l be length of each garden plot. The width of each plot is then $\frac{80}{l}$ and the total amount of fencing material is

$$4(l) + 6\left(\frac{80}{l}\right) = 88. \text{ Thus } 4l + \frac{480}{l} = 88 \Leftrightarrow 4l^2 + 480 = 88l \Leftrightarrow 4l^2 - 88l + 480 = 0 \Leftrightarrow 4(l^2 - 22l + 120) = 0 \Leftrightarrow$$

$4(l - 10)(l - 12) = 0$. So $l = 10$ or $l = 12$. If $l = 10$ m, then the width of each plot is $\frac{80}{10} = 8$ m. If $l = 12$ m, then the width of each plot is $\frac{80}{12} = 6.67$ m. Both solutions are possible.

89. $3x - 2 > -11 \Leftrightarrow 3x > -9 \Leftrightarrow x > -3$.

Interval: $(-3, \infty)$.

Graph: 

90. $-7 \leq 3x - 1 \leq -1 \Leftrightarrow -6 \leq 3x \leq 0 \Leftrightarrow -2 \leq x \leq 0$

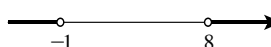
Interval: $[-2, 0]$.

Graph: 

91. $x^2 - 7x - 8 > 0 \Leftrightarrow (x - 8)(x + 1) > 0$. The expression on the left of the inequality changes sign where $x = 8$ and where $x = -1$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, 8)$	$(8, \infty)$
Sign of $x - 8$	-	-	+
Sign of $x + 1$	-	+	+
Sign of $(x - 8)(x + 1)$	+	-	+

Interval: $(-\infty, -1) \cup (8, \infty)$.

Graph: 

92. $x^2 \leq 1 \Leftrightarrow x^2 - 1 \leq 0 \Leftrightarrow (x - 1)(x + 1) \leq 0$. The expression on the left of the inequality changes sign when $x = -1$ and $x = 1$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Sign of $x - 1$	-	-	+
Sign of $x + 1$	-	+	+
Sign of $(x - 1)(x + 1)$	+	-	+

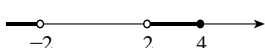
Interval: $[-1, 1]$

Graph: 

93. $\frac{x - 4}{x^2 - 4} \leq 0 \Leftrightarrow \frac{x - 4}{(x - 2)(x + 2)} \leq 0$. The expression on the left of the inequality changes sign where $x = -2$, where $x = 2$, and where $x = 4$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, 4)$	$(4, \infty)$
Sign of $x - 4$	-	-	-	+
Sign of $x - 2$	-	-	+	+
Sign of $x + 2$	-	+	+	+
Sign of $\frac{x - 4}{(x - 2)(x + 2)}$	-	+	-	+

Since the expression is not defined when $x = \pm 2$, we exclude these values and the solution is $(-\infty, -2) \cup (2, 4]$.

Graph: 

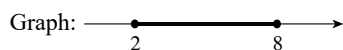
94. $\frac{5}{x^3 - x^2 - 4x + 4} < 0 \Leftrightarrow \frac{5}{x^2(x-1) - 4(x-1)} < 0 \Leftrightarrow \frac{5}{(x-1)(x^2-4)} < 0 \Leftrightarrow \frac{5}{(x-1)(x-2)(x+2)} < 0$. The expression on the left of the inequality changes sign when -2 , 1 , and 2 . Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, 2)$	$(2, \infty)$
Sign of $x - 1$	$-$	$-$	$+$	$+$
Sign of $x - 2$	$-$	$-$	$-$	$+$
Sign of $x + 2$	$-$	$+$	$+$	$+$
Sign of $\frac{5}{(x-1)(x-2)(x+2)}$	$-$	$+$	$-$	$+$

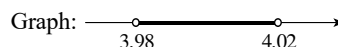
Interval: $(-\infty, -2) \cup (1, 2)$



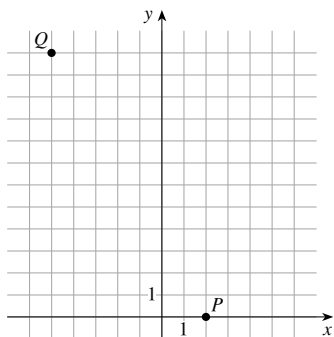
95. $|x - 5| \leq 3 \Leftrightarrow -3 \leq x - 5 \leq 3 \Leftrightarrow 2 \leq x \leq 8$.
Interval: $[2, 8]$



96. $|x - 4| < 0.02 \Leftrightarrow -0.02 < x - 4 < 0.02 \Leftrightarrow 3.98 < x < 4.02$
Interval: $(3.98, 4.02)$



97. (a)

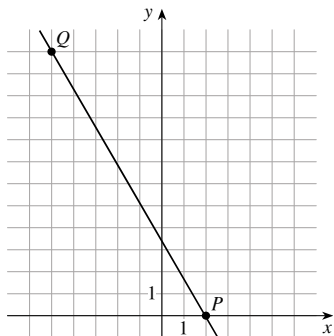


(b) The distance from P to Q is

$$\begin{aligned} d(P, Q) &= \sqrt{(-5-2)^2 + (12-0)^2} \\ &= \sqrt{49 + 144} = \sqrt{193} \end{aligned}$$

(c) The midpoint is $\left(\frac{-5+2}{2}, \frac{12+0}{2}\right) = \left(-\frac{3}{2}, 6\right)$.

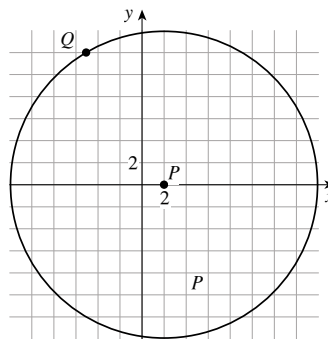
(d) The line has slope $m = \frac{12-0}{-5-2} = -\frac{12}{7}$, and has equation $y - 0 = -\frac{12}{7}(x - 2) \Leftrightarrow y = -\frac{12}{7}x + \frac{24}{7} \Leftrightarrow 12x + 7y - 24 = 0$.



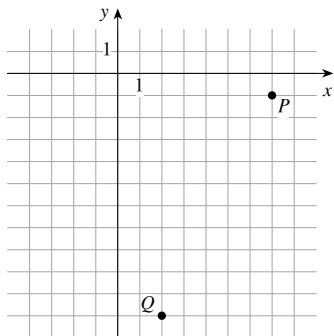
(e) The radius of this circle was found in part (b). It is

$r = d(P, Q) = \sqrt{193}$. So an equation is

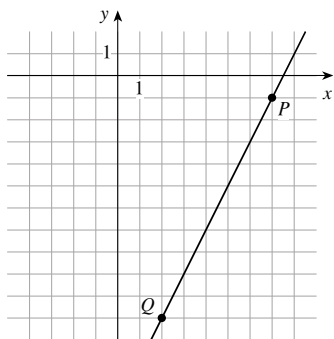
$$(x-2)^2 + (y-0)^2 = (\sqrt{193})^2 \Leftrightarrow (x-2)^2 + y^2 = 193.$$



98. (a)



- (d) The line has slope $m = \frac{-11 + 1}{2 - 7} = \frac{-10}{-5} = 2$,
and its equation is $y + 11 = 2(x - 2) \Leftrightarrow$
 $y + 11 = 2x - 4 \Leftrightarrow y = 2x - 15$.

(b) The distance from P to Q is

$$d(P, Q) = \sqrt{(2 - 7)^2 + (-11 + 1)^2} \\ = \sqrt{25 + 100} = \sqrt{125} = 5\sqrt{5}$$

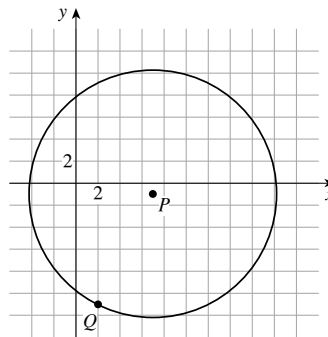
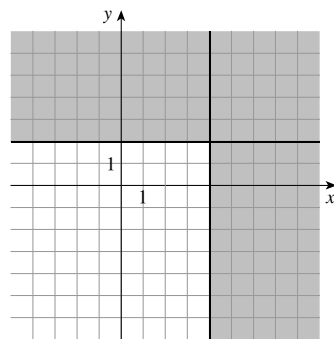
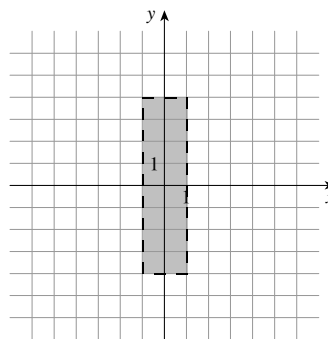
(c) The midpoint is $\left(\frac{2+7}{2}, \frac{-11+1}{2}\right) = \left(\frac{9}{2}, -6\right)$.

(e) The radius of this circle was found in part (b). It is

$$r = d(P, Q) = 5\sqrt{5}. \text{ So an equation is}$$

$$(x - 7)^2 + (y + 1)^2 = (5\sqrt{5})^2 \Leftrightarrow$$

$$(x - 7)^2 + (y + 1)^2 = 125.$$

99. $\{(x, y) \mid x \geq 4 \text{ or } y \geq 2\}$ 100. $\{(x, y) \mid |x| < 1 \text{ and } |y| < 4\}$ 

$$101. d(A, C) = \sqrt{(4 - (-1))^2 + (4 - (-3))^2} = \sqrt{(4 + 1)^2 + (4 + 3)^2} = \sqrt{74} \text{ and}$$

$$d(B, C) = \sqrt{(5 - (-1))^2 + (3 - (-3))^2} = \sqrt{(5 + 1)^2 + (3 + 3)^2} = \sqrt{72}. \text{ Therefore, } B \text{ is closer to } C.$$

102. The circle with center $(-3, 4)$ and radius $\sqrt{6}$ has equation $(x + 3)^2 + (y - 4)^2 = (\sqrt{6})^2 \Leftrightarrow (x + 3)^2 + (y - 4)^2 = 6$.

103. The center is $C = (-5, -1)$, and the point $P = (0, 0)$ is on the circle. The radius of the circle is

$$r = d(P, C) = \sqrt{(0 - (-5))^2 + (0 - (-1))^2} = \sqrt{(0 + 5)^2 + (0 + 1)^2} = \sqrt{26}. \text{ Thus, the equation of the circle is}$$

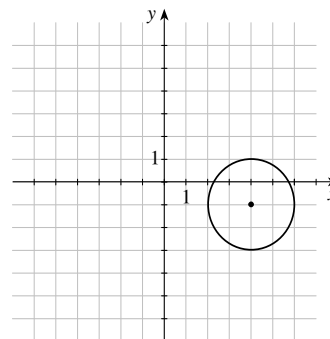
$$(x + 5)^2 + (y + 1)^2 = 26.$$

104. The midpoint of segment PQ is $\left(\frac{2-1}{2}, \frac{3+8}{2}\right) = \left(\frac{1}{2}, \frac{11}{2}\right)$, and the radius is $\frac{1}{2}$ of the distance from P to Q , or

$$r = \frac{1}{2} \cdot d(P, Q) = \frac{1}{2} \sqrt{(2 - (-1))^2 + (3 - 8)^2} = \frac{1}{2} \sqrt{(2 + 1)^2 + (3 - 8)^2} \Leftrightarrow r = \frac{1}{2} \sqrt{34}. \text{ Thus the equation is } \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{11}{2}\right)^2 = \frac{17}{2}.$$

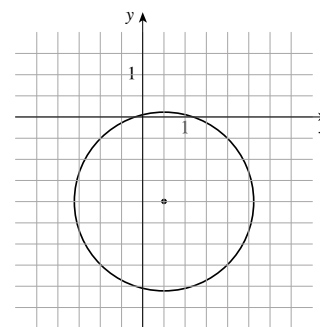
105. (a) $x^2 + y^2 - 8x + 2y + 13 = 0 \Leftrightarrow (x^2 - 8x) + (y^2 + 2y) = -13 \Leftrightarrow (x^2 - 8x + 16) + (y^2 + 2y + 1) = -13 + 16 + 1 \Leftrightarrow (x - 4)^2 + (y + 1)^2 = 4$, an equation of a circle.

(b) The circle has center $(4, -1)$ and radius 2.



106. (a) $2x^2 + 2y^2 - 2x + 8y = \frac{1}{2} \Leftrightarrow x^2 - x + y^2 + 4y = \frac{1}{4} \Leftrightarrow \left(x^2 - x + \frac{1}{4}\right) + (y^2 + 4y + 4) = \frac{1}{4} + \frac{1}{4} + 4 \Leftrightarrow \left(x - \frac{1}{2}\right)^2 + (y + 2)^2 = \frac{9}{2}$, an equation of a circle.

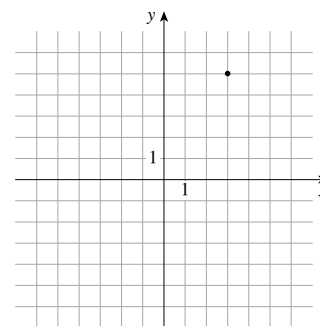
(b) The circle has center $\left(\frac{1}{2}, -2\right)$ and radius $\frac{3\sqrt{2}}{2}$.



107. (a) $x^2 + y^2 + 72 = 12x \Leftrightarrow (x^2 - 12x) + y^2 = -72 \Leftrightarrow (x^2 - 12x + 36) + y^2 = -72 + 36 \Leftrightarrow (x - 6)^2 + y^2 = -36$. Since the left side of this equation must be greater than or equal to zero, this equation has no graph.

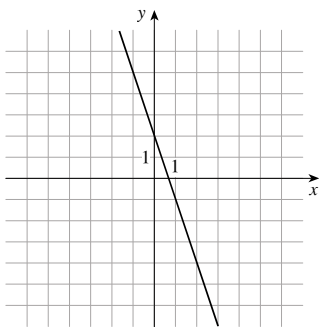
108. (a) $x^2 + y^2 - 6x - 10y + 34 = 0 \Leftrightarrow x^2 - 6x + y^2 - 10y = -34 \Leftrightarrow (x^2 - 6x + 9) + (y^2 - 10y + 25) = -34 + 9 + 25 \Leftrightarrow (x - 3)^2 + (y - 5)^2 = 0$, an equation of a point.

(b) This is the equation of the point $(3, 5)$.



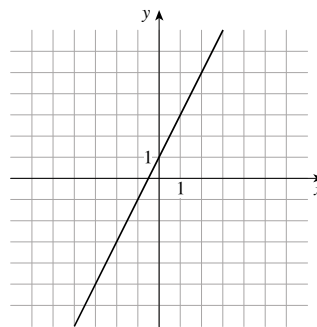
109. $y = 2 - 3x$

x	y
-2	8
0	2
$\frac{2}{3}$	0



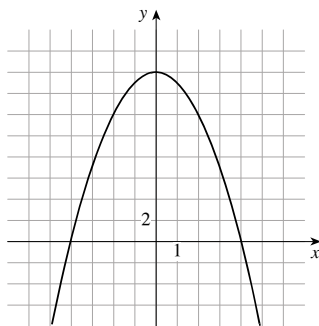
110. $2x - y + 1 = 0 \Leftrightarrow y = 2x + 1$

x	y
-2	-3
0	1
$-\frac{1}{2}$	0



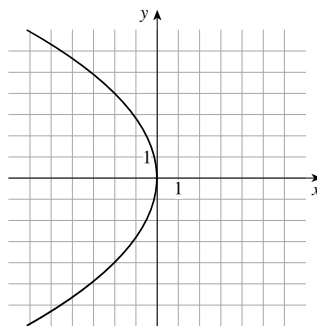
111. $y = 16 - x^2$

x	y
-3	7
-1	15
0	16
1	15
3	7



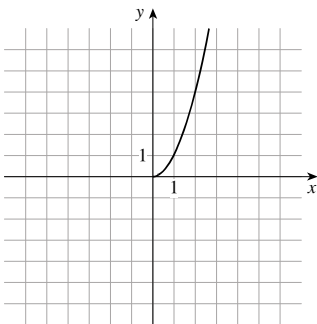
112. $8x + y^2 = 0 \Leftrightarrow y^2 = -8x$

x	y
-8	± 8
-2	± 4
0	0



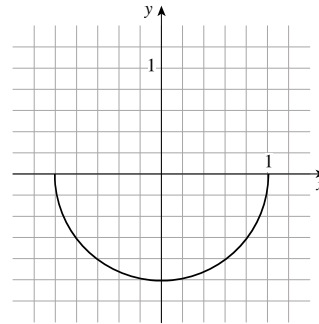
113. $x = \sqrt{y}$

x	y
0	0
1	1
2	4
3	9



114. $y = -\sqrt{1 - x^2}$

x	y
-1	0
$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
0	-1
1	0



115. $x = 16 - y^2$

(a) x -axis symmetry: replacing y by $-y$ gives $x = 16 - (-y)^2 = 16 - y^2$, which is the same as the original equation, so the graph is symmetric about the x -axis.

y -axis symmetry: replacing x by $-x$ gives $-x = 16 - y^2 \Leftrightarrow x = y^2 - 16$, which is not the same as the original equation, so the graph is not symmetric about the y -axis.

Origin symmetry: replacing x by $-x$ and y by $-y$ gives $-x = 16 - (-y)^2 \Leftrightarrow x = -16 - y^2$, which is not the same as the original equation, so the graph is not symmetric about the origin.

(b) To find x -intercepts, we set $y = 0$ and solve for x : $x = 16 - 0^2 = 16$, so the x -intercept is 16.

To find y -intercepts, we set $x = 0$ and solve for y : $0 = 16 - y^2 \Leftrightarrow y = \pm 4$, so the y -intercepts are 4 and -4.

116. $x^2 + 4y^2 = 9$

- (a) x -axis symmetry: replacing y by $-y$ gives $x^2 + 4(-y)^2 = 9 \Leftrightarrow x^2 + 4y^2 = 9$, which is the same as the original equation, so the graph is symmetric about the x -axis.
 y -axis symmetry: replacing x by $-x$ gives $(-x)^2 + 4(-y)^2 = 9 \Leftrightarrow x^2 + 4y^2 = 9$, which is the same as the original equation, so the graph is symmetric about the y -axis.
 Origin symmetry: replacing x by $-x$ and y by $-y$ gives $(-x)^2 + 4(-y)^2 = 9 \Leftrightarrow x^2 + 4y^2 = 9$, the same as the original equation, so the graph is symmetric about the origin.
- (b) To find x -intercepts, we set $y = 0$ and solve for x : $x^2 + 4(0)^2 = 9 \Leftrightarrow x = \pm 3$, so the x -intercepts are 3 and -3 .
 To find y -intercepts, we set $x = 0$ and solve for y : $0^2 + 4y^2 = 9 \Leftrightarrow y = \pm \frac{3}{2}$, so the y -intercepts are $\frac{3}{2}$ and $-\frac{3}{2}$.

117. $x^2 - 9y = 9$

- (a) x -axis symmetry: replacing y by $-y$ gives $x^2 - 9(-y) = 9 \Leftrightarrow x^2 + 9y = 9$, so the graph is not symmetric about the x -axis.
 y -axis symmetry: replacing x by $-x$ gives $(-x)^2 - 9y = 9 \Leftrightarrow x^2 - 9y = 9$, so the graph is symmetric about the y -axis.
 Origin symmetry: replacing x by $-x$ and y by $-y$ gives $(-x)^2 - 9(-y) = 9 \Leftrightarrow x^2 + 9y = 9$, so the graph is not symmetric about the origin.
- (b) To find x -intercepts, we set $y = 0$ and solve for x : $x^2 - 9(0) = 9 \Leftrightarrow x = \pm 3$, so the x -intercepts are 3 and -3 .
 To find y -intercepts, we set $x = 0$ and solve for y : $0^2 - 9y = 9 \Leftrightarrow y = -1$, so the y -intercept is -1 .

118. $(x + 1)^2 + y^2 = 4$

- (a) x -axis symmetry: replacing y by $-y$ gives $(x + 1)^2 + (-y)^2 = 4 \Leftrightarrow (x + 1)^2 + y^2 = 4$, so the graph is symmetric about the x -axis.
 y -axis symmetry: replacing x by $-x$ gives $(-x + 1)^2 + y^2 = 4 \Leftrightarrow (1 - x)^2 + y^2 = 4$, so the graph is not symmetric about the y -axis.
 Origin symmetry: replacing x by $-x$ and y by $-y$ gives $(-x + 1)^2 + (-y)^2 = 4 \Leftrightarrow (1 - x)^2 + y^2 = 4$, so the graph is not symmetric about the origin.
- (b) To find x -intercepts, we set $y = 0$ and solve for x : $(x + 1)^2 + 0^2 = 4 \Leftrightarrow (x + 1)^2 = 4 \Leftrightarrow x = -3$ or 1 , so the x -intercepts are -3 and 1 .
 To find y -intercepts, we set $x = 0$ and solve for y : $(0 + 1)^2 + y^2 = 4 \Leftrightarrow y^2 = 3 \Leftrightarrow y = \pm\sqrt{3}$, so the y -intercepts are $\sqrt{3}$ and $-\sqrt{3}$.

119. $x^2 + 4xy + y^2 = 1$

- (a) x -axis symmetry: replacing y by $-y$ gives $x^2 + 4x(-y) + (-y)^2 = 1$, which is different from the original equation, so the graph is not symmetric about the x -axis.
 y -axis symmetry: replacing x by $-x$ gives $(-x)^2 + 4(-x)y + y^2 = 1$, which is different from the original equation, so the graph is not symmetric about the y -axis.
 Origin symmetry: replacing x by $-x$ and y by $-y$ gives $(-x)^2 + 4(-x)(-y) + (-y)^2 = 1 \Leftrightarrow x^2 + 4xy + y^2 = 1$, so the graph is symmetric about the origin.
- (b) To find x -intercepts, we set $y = 0$ and solve for x : $x^2 + 4x(0) + 0^2 = 1 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$, so the x -intercepts are -1 and 1 .
 To find y -intercepts, we set $x = 0$ and solve for y : $0^2 + 4(0)y + y^2 = 1 \Leftrightarrow y^2 = 1 \Leftrightarrow y = \pm 1$, so the y -intercepts are -1 and 1 .

120. $x^3 + xy^2 = 5$

(a) x -axis symmetry: replacing y by $-y$ gives $x^3 + x(-y)^2 = 5 \Leftrightarrow x^3 + xy^2 = 5$, so the graph is symmetric about the x -axis.

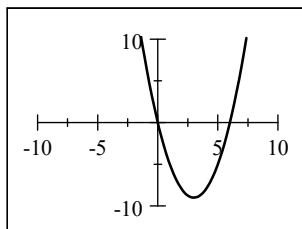
y -axis symmetry: replacing x by $-x$ gives $(-x)^3 + (-x)y^2 = 5$, which is different from the original equation, so the graph is not symmetric about the y -axis.

Origin symmetry: replacing x by $-x$ and y by $-y$ gives $(-x)^3 + (-x)(-y)^2 = 5$, which is different from the original equation, so the graph is not symmetric about the origin.

(b) To find x -intercepts, we set $y = 0$ and solve for x : $x^3 + x(0)^2 = 5 \Leftrightarrow x^3 = 5 \Leftrightarrow x = \sqrt[3]{5}$, so the x -intercept is $\sqrt[3]{5}$.

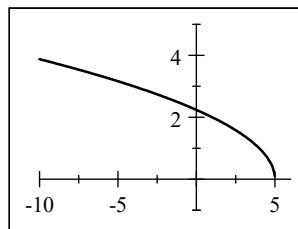
To find y -intercepts, we set $x = 0$ and solve for y : $0^3 + 0y^2 = 5$ has no solution, so there is no y -intercept.

121. (a) We graph $y = x^2 - 6x$ in the viewing rectangle $[-10, 10]$ by $[-10, 10]$.



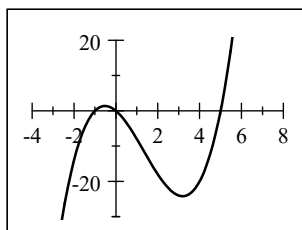
(b) From the graph, we see that the x -intercepts are 0 and 6 and the y -intercept is 0.

122. (a) We graph $y = \sqrt{5-x}$ in the viewing rectangle $[-10, 6]$ by $[-1, 5]$.



(b) From the graph, we see that the x -intercept is 5 and the y -intercept is approximately 2.24.

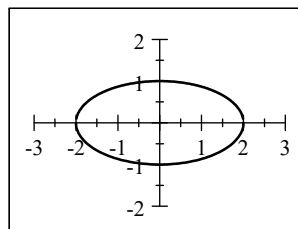
123. (a) We graph $y = x^3 - 4x^2 - 5x$ in the viewing rectangle $[-4, 8]$ by $[-30, 20]$.



(b) From the graph, we see that the x -intercepts are -1 , 0 , and 5 and the y -intercept is 0 .

124. (a) We graph $\frac{x^2}{4} + y^2 = 1 \Leftrightarrow y^2 = 1 - \frac{x^2}{4} \Rightarrow$

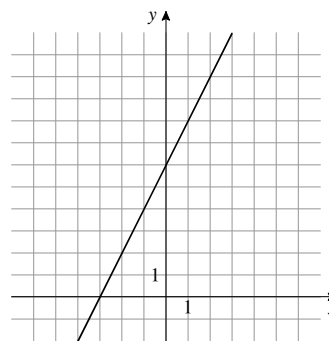
$y = \pm \sqrt{1 - \frac{x^2}{4}}$ in the viewing rectangle $[-3, 3]$ by $[-2, 2]$.



(b) From the graph, we see that the x -intercepts are -2 and 2 and the y -intercepts are -1 and 1 .

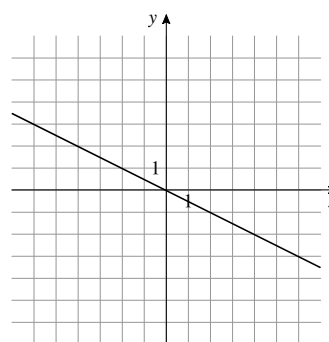
125. (a) The line that has slope 2 and y-intercept 6 has the slope-intercept equation (c)
 $y = 2x + 6$.

(b) An equation of the line in general form is $2x - y + 6 = 0$.



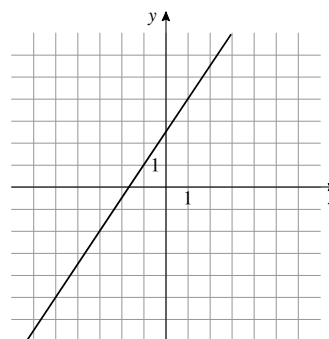
126. (a) The line that has slope $-\frac{1}{2}$ and passes through the point $(6, -3)$ has (c)
 equation $y - (-3) = -\frac{1}{2}(x - 6) \Leftrightarrow y + 3 = -\frac{1}{2}(x - 6) \Leftrightarrow y = -\frac{1}{2}x$.

(b) $-\frac{1}{2}x + 3 = y + 3 \Leftrightarrow x - 6 = -2y - 6 \Leftrightarrow x + 2y = 0$.



127. (a) The line that passes through the points $(-3, -2)$ and $(1, 4)$ has slope (c)
 $m = \frac{4 - (-2)}{1 - (-3)} = \frac{3}{2}$, so, using the second point for convenience, an
 equation of the line is $y - 4 = \frac{3}{2}(x - 1) \Leftrightarrow y = \frac{3}{2}x + \frac{5}{2}$.

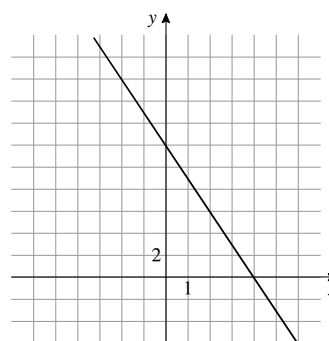
(b) $y = \frac{3}{2}x + \frac{5}{2} \Leftrightarrow 2y = 3x + 5 \Leftrightarrow 3x - 2y + 5 = 0$.



128. (a) The line that has x-intercept 4 and y-intercept 12 passes through the points (c)
 $(4, 0)$ and $(0, 12)$, so $m = \frac{12 - 0}{0 - 4} = -3$ and the equation is

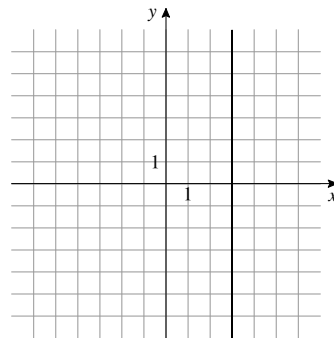
$$y - 0 = -3(x - 4) \Leftrightarrow y = -3x + 12.$$

(b) $y = -3x + 12 \Leftrightarrow 3x + y - 12 = 0$.



129. (a) The vertical line that passes through the point $(3, -2)$ has equation $x = 3$. (c)

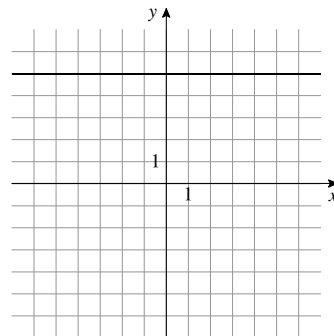
(b) $x = 3 \Leftrightarrow x - 3 = 0$.



130. (a) The horizontal line with y -intercept 5 has equation $y = 5$.

(b) $y = 5 \Leftrightarrow y - 5 = 0$.

(c)



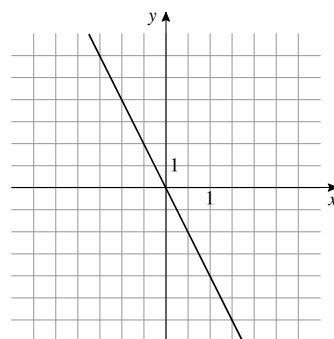
131. (a) The line containing $(2, 4)$ and $(4, -4)$ has slope

$$m = \frac{-4 - 4}{4 - 2} = \frac{-8}{2} = -4, \text{ and the line passing through the origin with}$$

this slope has equation $y = -4x$.

(b) $y = -4x \Leftrightarrow 4x + y = 0$.

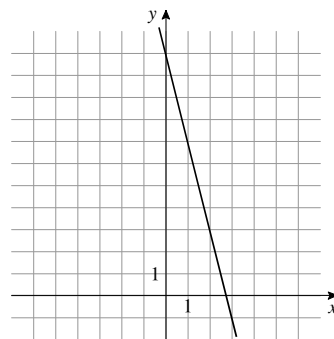
(c)



132. (a) The line with equation $x - 4y + 7 = 0 \Leftrightarrow 4y = x + 7 \Leftrightarrow y = \frac{1}{4}x + \frac{7}{4}$ has (c)

slope $\frac{1}{4}$, so any line perpendicular to it has slope $-\frac{1}{1/4} = -4$. Since the desired line passes through $(2, 3)$, it has equation $y - 3 = -4(x - 2) \Leftrightarrow y = -4x + 11$.

(b) $y = -4x + 11 \Leftrightarrow 4x + y - 11 = 0$.



133. (a) The slope, 0.3, represents the increase in length of the spring for each unit increase in weight w . The s -intercept is the resting or natural length of the spring.

(b) When $w = 5$, $s = 0.3(5) + 2.5 = 1.5 + 2.5 = 4.0$ centimeters.

134. (a) We use the information to find two points, $(0, 60000)$ and $(3, 70500)$. Then the slope is

$$m = \frac{70,500 - 60,000}{3 - 0} = \frac{10,500}{3} = 3,500. \text{ So } S = 3,500t + 60,000.$$

- (b) The slope represents the accountant's annual increase in salary, \$3500, and the S -intercept represents the accountant's initial salary, \$60,000.

- (c) When $t = 12$, the accountant's salary will be $S = 3500(12) + 60,000 = 42,000 + 60,000 = \$102,000$.

135. From the graph, we see that the graphs of $y = x^2 - 4x$ and $y = x + 6$ intersect at $x = -1$ and $x = 6$, so these are the solutions of the equation $x^2 - 4x = x + 6$.

136. From the graph, we see that the graph of $y = x^2 - 4x$ crosses the x -axis at $x = 0$ and $x = 4$, so these are the solutions of the equation $x^2 - 4x = 0$.

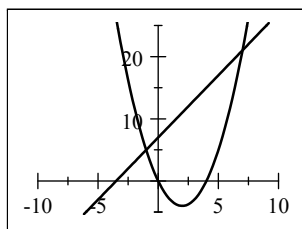
137. From the graph, we see that the graph of $y = x^2 - 4x$ lies below the graph of $y = x + 6$ for $-1 < x < 6$, so the inequality $x^2 - 4x \leq x + 6$ is satisfied on the interval $[-1, 6]$.

138. From the graph, we see that the graph of $y = x^2 - 4x$ lies above the graph of $y = x + 6$ for $-\infty < x < -1$ and $6 < x < \infty$, so the inequality $x^2 - 4x \geq x + 6$ is satisfied on the intervals $(-\infty, -1]$ and $[6, \infty)$.

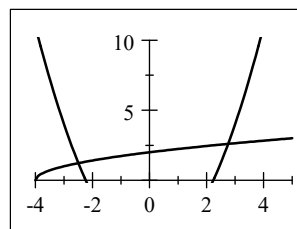
139. From the graph, we see that the graph of $y = x^2 - 4x$ lies above the x -axis for $x < 0$ and for $x > 4$, so the inequality $x^2 - 4x \geq 0$ is satisfied on the intervals $(-\infty, 0]$ and $[4, \infty)$.

140. From the graph, we see that the graph of $y = x^2 - 4x$ lies below the x -axis for $0 < x < 4$, so the inequality $x^2 - 4x \leq 0$ is satisfied on the interval $[0, 4]$.

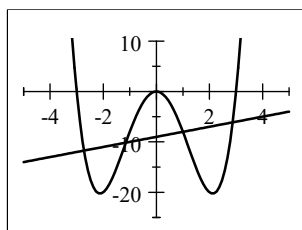
141. $x^2 - 4x = 2x + 7$. We graph the equations $y_1 = x^2 - 4x$ and $y_2 = 2x + 7$ in the viewing rectangle $[-10, 10]$ by $[-5, 25]$. Using a zoom or trace function, we get the solutions $x = -1$ and $x = 7$.



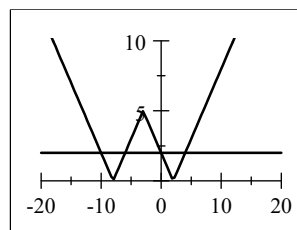
142. $\sqrt{x+4} = x^2 - 5$. We graph the equations $y_1 = \sqrt{x+4}$ and $y_2 = x^2 - 5$ in the viewing rectangle $[-4, 5]$ by $[0, 10]$. Using a zoom or trace function, we get the solutions $x \approx -2.50$ and $x \approx 2.76$.



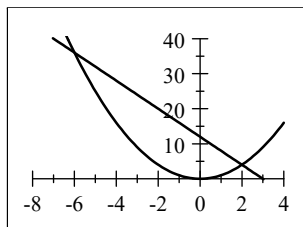
143. $x^4 - 9x^2 = x - 9$. We graph the equations $y_1 = x^4 - 9x^2$ and $y_2 = x - 9$ in the viewing rectangle $[-5, 5]$ by $[-25, 10]$. Using a zoom or trace function, we get the solutions $x \approx -2.72$, $x \approx -1.15$, $x = 1.00$, and $x \approx 2.87$.



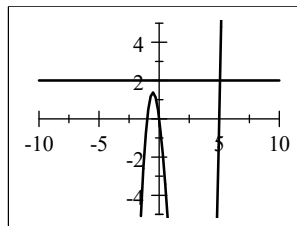
144. $||x + 3| - 5| = 2$. We graph the equations $y_1 = ||x + 3| - 5|$ and $y_2 = 2$ in the viewing rectangle $[-20, 20]$ by $[0, 10]$. Using Zoom and/or Trace, we get the solutions $x = -10$, $x = -6$, $x = 0$, and $x = 4$.



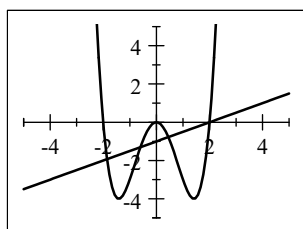
145. $x^2 > 12 - 4x$. We graph the equations $y_1 = x^2$ and $y_2 = 12 - 4x$ in the viewing rectangle $[-8, 4]$ by $[0, 40]$. Using a zoom or trace function, we find the points of intersection are at $x = -6$ and $x = 2$. Since we want $x^2 > 12 - 4x$, the solution is the union of intervals $(-\infty, -6) \cup (2, \infty)$.



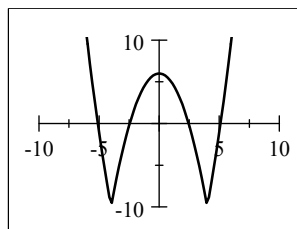
146. $x^3 - 4x^2 - 5x > 2$. We graph the equations $y_1 = x^3 - 4x^2 - 5x$ and $y_2 = 2$ in the viewing rectangle $[-10, 10]$ by $[-5, 5]$. We find that the point of intersection is at $x \approx 5.07$. Since we want $x^3 - 4x^2 - 5x > 2$, the solution is the interval $(5.07, \infty)$.



147. $x^4 - 4x^2 < \frac{1}{2}x - 1$. We graph the equations $y_1 = x^4 - 4x^2$ and $y_2 = \frac{1}{2}x - 1$ in the viewing rectangle $[-5, 5]$ by $[-5, 5]$. We find the points of intersection are at $x \approx -1.85$, $x \approx -0.60$, $x \approx 0.45$, and $x = 2.00$. Since we want $x^4 - 4x^2 < \frac{1}{2}x - 1$, the solution is $(-1.85, -0.60) \cup (0.45, 2.00)$.



148. $|x^2 - 16| - 10 \geq 0$. We graph the equation $y = |x^2 - 16| - 10$ in the viewing rectangle $[-10, 10]$ by $[-10, 10]$. Using a zoom or trace function, we find that the x -intercepts are $x \approx \pm 5.10$ and $x \approx \pm 2.45$. Since we want $|x^2 - 16| - 10 \geq 0$, the solution is approximately $(-\infty, -5.10] \cup [-2.45, 2.45] \cup [5.10, \infty)$.



149. Here the center is at $(0, 0)$, and the circle passes through the point $(-5, 12)$, so the radius is $r = \sqrt{(-5 - 0)^2 + (12 - 0)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$. The equation of the circle is $x^2 + y^2 = 13^2 \Leftrightarrow x^2 + y^2 = 169$. The line shown is the tangent that passes through the point $(-5, 12)$, so it is perpendicular to the line through the points $(0, 0)$ and $(-5, 12)$. This line has slope $m_1 = \frac{12 - 0}{-5 - 0} = -\frac{12}{5}$. The slope of the line we seek is $m_2 = -\frac{1}{m_1} = -\frac{1}{-12/5} = \frac{5}{12}$. Thus, an equation of the tangent line is $y - 12 = \frac{5}{12}(x + 5) \Leftrightarrow y - 12 = \frac{5}{12}x + \frac{25}{12} \Leftrightarrow y = \frac{5}{12}x + \frac{169}{12} \Leftrightarrow 5x - 12y + 169 = 0$.
150. Because the circle is tangent to the x -axis at the point $(5, 0)$ and tangent to the y -axis at the point $(0, 5)$, the center is at $(5, 5)$ and the radius is 5. Thus an equation is $(x - 5)^2 + (y - 5)^2 = 5^2 \Leftrightarrow (x - 5)^2 + (y - 5)^2 = 25$. The slope of the line passing through the points $(8, 1)$ and $(5, 5)$ is $m = \frac{5 - 1}{5 - 8} = \frac{4}{-3} = -\frac{4}{3}$, so an equation of the line we seek is $y - 1 = -\frac{4}{3}(x - 8) \Leftrightarrow 4x + 3y - 35 = 0$.
151. Since M varies directly as z we have $M = kz$. Substituting $M = 120$ when $z = 15$, we find $120 = k(15) \Leftrightarrow k = 8$. Therefore, $M = 8z$.

152. Since z is inversely proportional to y , we have $z = \frac{k}{y}$. Substituting $z = 12$ when $y = 16$, we find $12 = \frac{k}{16} \Leftrightarrow k = 192$.

Therefore $z = \frac{192}{y}$.

153. (a) The intensity I varies inversely as the square of the distance d , so $I = \frac{k}{d^2}$.

(b) Substituting $I = 1000$ when $d = 8$, we get $1000 = \frac{k}{(8)^2} \Leftrightarrow k = 64,000$.

(c) From parts (a) and (b), we have $I = \frac{64,000}{d^2}$. Substituting $d = 20$, we get $I = \frac{64,000}{(20)^2} = 160$ candles.

154. Let f be the frequency of the string and l be the length of the string. Since the frequency is inversely proportional to the length, we have $f = \frac{k}{l}$. Substituting $l = 12$ when $k = 440$, we find $440 = \frac{k}{12} \Leftrightarrow k = 5280$. Therefore $f = \frac{5280}{l}$. For $f = 660$, we must have $660 = \frac{5280}{l} \Leftrightarrow l = \frac{5280}{660} = 8$. So the string needs to be shortened to 8 centimeters.

155. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

Let v be the terminal velocity of the parachutist in km/h and w be their weight in kilograms. Since the terminal velocity is directly proportional to the square root of the weight, we have $v = k\sqrt{w}$. Substituting $v = 14$ when $w = 70$, we solve for k . This gives $14 = k\sqrt{70} \Leftrightarrow k = \frac{14}{\sqrt{70}} \approx 1.673$. Thus $v = 1.673\sqrt{w}$. When $w = 105$, the terminal velocity is $v = 1.673\sqrt{105} \approx 17$ km/h.

156. Let r be the maximum range of the baseball and v be the velocity of the baseball. Since the maximum range is directly proportional to the square of the velocity, we have $r = lv^2$. Substituting $v = 96$ and $r = 74$, we find $74 = k(96)^2 \Leftrightarrow k \approx 0.00803$. If $v = 112$, then we have a maximum range of $r = 0.00803(112)^2 = 100.7$ meters.

157. The speed v is inversely proportional to the square root of the density d , so $v = \frac{k}{\sqrt{d}}$. In fresh water with density

$d_1 = 1 \text{ g/cm}^3$, the speed is $v_1 = 1480 \text{ m/s}$, so $1480 = \frac{k}{\sqrt{1}} \Leftrightarrow k = 1480$. Thus, in seawater with density 1.0273 g/cm^3 , we

have $v = \frac{1480}{\sqrt{1.0273}} \approx 1460 \text{ m/s}$.

158. (a) Because $\theta = 1.22 \frac{\lambda}{D}$, for fixed D the angular distance θ is directly proportional to the wavelength λ . Thus, the angular distance is smaller for shorter wavelengths.

- (b) For fixed λ , if we substitute $D_1 = 2D$ in the formula $\theta = 1.22 \frac{\lambda}{D}$ then the angular distance becomes

$\theta_1 = 1.22 \frac{\lambda}{D_1} = 1.22 \frac{\lambda}{2D} = \frac{1}{2} \left(1.22 \frac{\lambda}{D} \right) = \frac{1}{2} \theta$. Thus, for a fixed wavelength, if the diameter of the mirror is doubled, the angular distance is halved.

159. We solve the first equation for v : $1 + z = \sqrt{\frac{c+v}{c-v}} \Rightarrow (1+z)^2 = \frac{c+v}{c-v} \Rightarrow (c-v)(1+z)^2 = c+v \Rightarrow$

$$c(1+z)^2 - v(1+z)^2 = c+v \Rightarrow v[(1+z)^2 + 1] = c[(1+z)^2 - 1] \Rightarrow v = \frac{c[(1+z)^2 - 1]}{(1+z)^2 + 1}.$$

$$\text{Substituting } z = 2 \text{ and } c = 3 \times 10^5, \text{ we find } v = \frac{3 \times 10^5 [(1+2)^2 - 1]}{(1+2)^2 + 1} = 3 \times 10^5 \left(\frac{8}{10} \right) = 2.4 \times 10^5 \text{ km/s.}$$

By Hubble's Law, $v = H_0 D \Rightarrow D = \frac{v}{H_0}$, so substituting v from above and $H_0 \approx 20.8$ Mly/km/s, we have

$$D \approx \frac{2.4 \times 10^5}{20.8} \approx 11,538 \text{ megalight-years.}$$

160. (a) $y = 2|x| - 3$ is the graph of the absolute value function $y = |x|$, stretched vertically by a factor of 2 and shifted downward 3 units. It has graph III

(b) $2y - 3x = -2 \Leftrightarrow y = \frac{3}{2}x - 1$ represents a line with slope $\frac{3}{2}$, x -intercept $\frac{2}{3}$, and y -intercept -1 . It has graph V.

(c) $y = x^4$ has $y(0) = 0$ and $y > 0$ elsewhere. It is an even function, and has graph II.

(d) $(x+1)^2 + (y+1)^2 = 9$ is the equation of a circle with center $(-1, -1)$ and radius 3. It has graph IV.

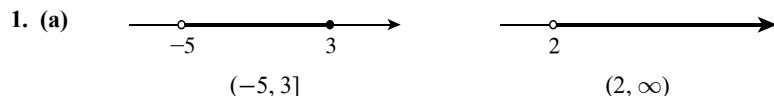
(e) $6y + x = 3 \Leftrightarrow y = -\frac{1}{6}x + \frac{1}{2}$ represents a line with slope $-\frac{1}{6}$, x -intercept 3, and y -intercept $\frac{1}{2}$. It has graph I.

(f) $y = \frac{6x}{1+x^4}$ is an odd function with $y(0) = 0$ and horizontal asymptote the x -axis. It has graph VII.

(g) $x = y^3$ is an odd function whose graph contains the points $(1, 1)$ and $(-1, -1)$. It has graph VIII.

(h) $x^2 - 2x + y^2 - 4y + 1 = 0 \Leftrightarrow (x-1)^2 + (y-2)^2 = -1 + 1 + 4 = 4$ is the equation of a circle with center $(1, 2)$ and radius 2. It has graph VI.

CHAPTER 1 TEST



(b) $x \leq 3 \Leftrightarrow x \in (-\infty, 3]$; $-1 \leq x < 4 \Leftrightarrow x \in [-1, 4)$

(c) Distance $= |-7 - 9| = |-16| = 16$

2. (a) $(-3)^4 = 81$

(b) $-3^4 = -81$

(c) $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

(d) $\frac{3^{75}}{3^{72}} = 3^{75-72} = 3^3 = 27$

(e) $\left(\frac{2}{3}\right)^{-2} = \frac{3^2}{2^2} = \frac{9}{4}$

(f) $16^{-3/4} = (2^4)^{-3/4} = 2^{-3} = \frac{1}{8}$

3. (a) $186,000,000,000 = 1.86 \times 10^{11}$

(b) $0.000\,000\,396\,5 = 3.965 \times 10^{-7}$

4. (a) $\sqrt{200} - \sqrt{32} = 10\sqrt{2} - 4\sqrt{2} = 6\sqrt{2}$

(b) $(3a^3b^3)(4ab^2)^2 = 3a^3b^3 \cdot 4^2a^2b^4 = 48a^5b^7$

(c) $\left(\frac{4x^9y^3}{xy^7}\right)^{-1/2} = \left(\frac{4x^8}{y^4}\right)^{-1/2} = \left(\frac{y^4}{4x^8}\right)^{1/2} = \frac{y^2}{2x^4}$

5. (a) $z(4z-3) + 2z(3-2z) = 4z^2 - 3z + 6z - 4z^2 = 3z$

(b) $(x+3)(4x-5) = 4x^2 - 5x + 12x - 15 = 4x^2 + 7x - 15$

- (c) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$
- (d) $(2x + 3)^2 = (2x)^2 + 2(2x)(3) + (3)^2 = 4x^2 + 12x + 9$
- (e) $(x + 2)^3 = (x)^3 + 3(x)^2(2) + 3(x)(2)^2 + (2)^3 = x^3 + 6x^2 + 12x + 8$
6. (a) $4x^2 - 25 = (2x - 5)(2x + 5)$
- (b) $2x^2 + 5x - 12 = (2x - 3)(x + 4)$
- (c) $x^3 - 3x^2 - 4x + 12 = x^2(x - 3) - 4(x - 3) = (x - 3)(x^2 - 4) = (x - 3)(x - 2)(x + 2)$
- (d) $x^4 + 27x = x(x^3 + 27) = x(x + 3)(x^2 - 3x + 9)$
- (e) $2x^{3/2} + 8x^{1/2} - 10x^{-1/2} = 2x^{-1/2}(x^2 + 4x - 5) = 2x^{-1/2}(x + 5)(x - 1)$
- (f) $x^4y^2 - 9x^2y^2 = x^2y^2(x^2 - 9) = x^2y^2(x - 3)(x + 3)$
7. (a) $\frac{w^2 + 4w + 3}{w^2 - 2w - 3} = \frac{(w + 3)(w + 1)}{(w - 3)(w + 1)} = \frac{w + 3}{w - 3}$
- (b) $\frac{x^2}{x^2 - 4} - \frac{x + 1}{x + 2} = \frac{x^2}{(x - 2)(x + 2)} - \frac{x + 1}{x + 2} = \frac{x^2}{(x - 2)(x + 2)} + \frac{-(x + 1)(x - 2)}{(x - 2)(x + 2)}$
 $= \frac{x^2 - (x^2 - x - 2)}{(x - 2)(x + 2)} = \frac{x + 2}{(x - 2)(x + 2)} = \frac{1}{x - 2}$
- (c) $\frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}} = \frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}} \cdot \frac{xy}{xy} = \frac{\frac{y^2 - x^2}{xy}}{\frac{y - x}{xy}} = \frac{y^2 - x^2}{x - y} = \frac{(y - x)(y + x)}{x - y} = \frac{-(x - y)(y + x)}{x - y} = -(x + y)$
8. $\frac{\sqrt{2}}{4 - \sqrt{2}} = \frac{\sqrt{2}(4 + \sqrt{2})}{(4 - \sqrt{2})(4 + \sqrt{2})} = \frac{4\sqrt{2} + 2}{14} = \frac{1 + 2\sqrt{2}}{7}$
9. (a) $x + 5 = 14 - \frac{1}{2}x \Leftrightarrow 2x + 10 = 28 - x \Leftrightarrow 3x = 18 \Leftrightarrow x = 6$
- (b) $\frac{2x}{x + 1} = \frac{2x - 1}{x} \Leftrightarrow (2x)(x) = (2x - 1)(x + 1) (x \neq -1, x \neq 0) \Leftrightarrow 2x^2 = 2x^2 + x - 1 \Leftrightarrow 0 = x - 1 \Leftrightarrow x = 1$
- (c) $x^2 - x - 12 = 0 \Leftrightarrow (x - 4)(x + 3) = 0$. So $x = 4$ or $x = -3$.
- (d) $2x^2 + 4x + 1 = 0 \Rightarrow x = \frac{-4 \pm \sqrt{4^2 - 4(2)(1)}}{2(2)} = \frac{-4 \pm \sqrt{16 - 8}}{4} = \frac{-4 \pm \sqrt{8}}{4} = \frac{-4 \pm 2\sqrt{2}}{4} = \frac{-2 \pm \sqrt{2}}{2}$.
- (e) $\sqrt{3 - \sqrt{x + 5}} = 2 \Rightarrow 3 - \sqrt{x + 5} = 4 \Leftrightarrow -1 = \sqrt{x + 5}$. (Note that this is impossible, so there can be no solution.) Squaring both sides again, we get $1 = x + 5 \Leftrightarrow x = -4$. But this does not satisfy the original equation, so there is no solution. (You must always check your final answers if you have squared both sides when solving an equation, since extraneous answers may be introduced, as here.)
- (f) $x^4 - 3x^2 + 2 = 0 \Leftrightarrow (x^2 - 1)(x^2 - 2) = 0$. So $x^2 - 1 = 0 \Leftrightarrow x = \pm 1$ or $x^2 - 2 = 0 \Leftrightarrow x = \pm\sqrt{2}$. Thus the solutions are $x = -1, x = 1, x = -\sqrt{2}$, and $x = \sqrt{2}$.
- (g) $3|x - 4| - 10 = 0 \Leftrightarrow 3|x - 4| = 10 \Leftrightarrow |x - 4| = \frac{10}{3} \Leftrightarrow x - 4 = \pm\frac{10}{3} \Leftrightarrow x = 4 \pm \frac{10}{3}$. So $x = 4 - \frac{10}{3} = \frac{2}{3}$ or $x = 4 + \frac{10}{3} = \frac{22}{3}$. Thus the solutions are $x = \frac{2}{3}$ and $x = \frac{22}{3}$.
10. (a) $(3 - 2i) + (4 + 3i) = 3 + 4 + (-2i + 3i) = 7 + i$
- (b) $(3 - 2i) - (4 + 3i) = (3 - 4) + (-2i - 3i) = -1 - 5i$
- (c) $(3 - 2i)(4 + 3i) = 3 \cdot 4 + 3 \cdot 3i - 2i \cdot 4 - 2i \cdot 3i = 12 + 9i - 8i - 6i^2 = 12 + i - 6(-1) = 18 + i$

$$(d) \frac{3-2i}{4+3i} = \frac{3-2i}{4+3i} \cdot \frac{4-3i}{4-3i} = \frac{12-17i+6i^2}{16-9i^2} = \frac{12-17i-6}{16+9} = \frac{6}{25} - \frac{17}{25}i$$

$$(e) i^{48} = (i^2)^{24} = (-1)^{24} = 1$$

$$(f) (\sqrt{2} - \sqrt{-2})(\sqrt{8} + \sqrt{-2}) = \sqrt{2} \cdot 8 + \sqrt{2}(-2) - \sqrt{(-2)8} - (\sqrt{-2})^2 = 4 + 2i - 4i - (-2) = 6 - 2i$$

$$11. \text{ Using the Quadratic Formula, } 2x^2 + 4x + 3 = 0 \Leftrightarrow x = \frac{-4 \pm \sqrt{4^2 - 4(2)(3)}}{2(2)} = \frac{-4 \pm \sqrt{-8}}{4} = -1 \pm \frac{\sqrt{2}}{2}i.$$

12. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

Let t be the time (in hours) it took the trucker to drive from Amity to Belleville. Then $4.4 - t$ is the time it took the trucker to drive from Belleville to Amity. Since the distance from Amity to Belleville equals the distance from Belleville to Amity, we have $80t = 96(4.4 - t) \Leftrightarrow 80t = 422.4 - 96t \Leftrightarrow 176t = 422.4 \Leftrightarrow t = 2.4$ hours. Thus the distance is $80(2.4) = 192$ km.

13. Let w be the width of the parcel of land. Then $w + 70$ is the length of the parcel of land. Then $w^2 + (w + 70)^2 = 130^2 \Leftrightarrow w^2 + w^2 + 140w + 4900 = 16,900 \Leftrightarrow 2w^2 + 140w - 12,000 = 0 \Leftrightarrow w^2 + 70w - 6000 = 0 \Leftrightarrow (w - 50)(w + 120) = 0$. So $w = 50$ or $w = -120$. Since $w \geq 0$, the width is $w = 50$ m and the length is $w + 70 = 120$ m.

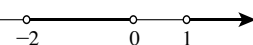
14. (a) $-4 < 5 - 3x \leq 17 \Leftrightarrow -9 < -3x \leq 12 \Leftrightarrow 3 > x \geq -4$. Expressing in standard form we have: $-4 \leq x < 3$.

Interval: $[-4, 3)$. Graph: 

(b) $x(x - 1)(x + 2) > 0$. The expression on the left of the inequality changes sign when $x = 0$, $x = 1$, and $x = -2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 1)$	$(1, \infty)$
Sign of x	-	-	+	+
Sign of $x - 1$	-	-	-	+
Sign of $x + 2$	-	+	+	+
Sign of $x(x - 1)(x + 2)$	-	+	-	+

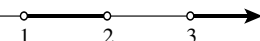
From the table, the solution set is $\{x \mid -2 < x < 0 \text{ or } 1 < x\}$. Interval: $(-2, 0) \cup (1, \infty)$.

Graph: 

(c) $|x - 4| < 3$ is equivalent to $-3 < x - 4 < 3 \Leftrightarrow 1 < x < 7$. Interval: $(1, 7)$. Graph: 

- (d) $\frac{3-2x}{2-x} < x \Leftrightarrow x - \frac{3-2x}{2-x} > 0 \Leftrightarrow \frac{x(2-x)}{2-x} - \frac{3-2x}{2-x} > 0 \Leftrightarrow \frac{2x-x^2-3+2x}{2-x} > 0 \Leftrightarrow \frac{x^2-4x+3}{x-2} = \frac{(x-1)(x-3)}{x-2} > 0$. The expression on the left of the inequality changes sign at $x = 1$, $x = 2$, and $x = 3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, 1)$	$(1, 2)$	$(2, 3)$	$(3, \infty)$
Sign of $x - 1$	−	+	+	+
Sign of $x - 3$	−	−	−	+
Sign of $x - 2$	−	−	+	+
Sign of $\frac{(x-1)(x-3)}{x-2}$	−	+	−	+

Intervals: $(1, 2) \cup (3, \infty)$ Graph: 

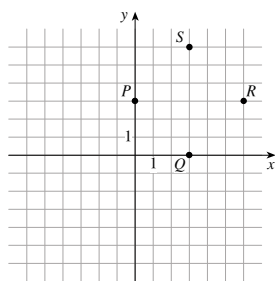
15. $5 \leq \frac{5}{9}(F - 32) \leq 10 \Leftrightarrow 9 \leq F - 32 \leq 18 \Leftrightarrow 41 \leq F \leq 50$. Thus the medicine is to be stored at a temperature between 41°F and 50°F .

16. For $\sqrt{6x - x^2}$ to be defined as a real number $6x - x^2 \geq 0 \Leftrightarrow x(6 - x) \geq 0$. The expression on the left of the inequality changes sign when $x = 0$ and $x = 6$. Thus we must check the intervals in the following table.

Interval	$(-\infty, 0)$	$(0, 6)$	$(6, \infty)$
Sign of x	−	+	+
Sign of $6 - x$	+	+	−
Sign of $x(6 - x)$	−	+	−

From the table, we see that $\sqrt{6x - x^2}$ is defined when $0 \leq x \leq 6$.

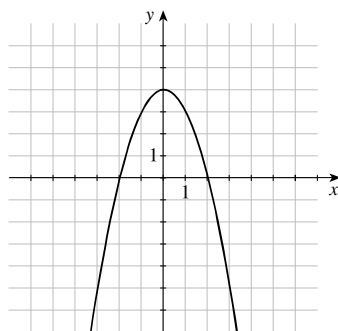
17. (a)



There are several ways to determine the coordinates of S . The diagonals of a square have equal length and are perpendicular. The diagonal PR is horizontal and has length 6 units, so the diagonal QS is vertical and also has length 6. Thus, the coordinates of S are $(3, 6)$.

- (b) The length of PQ is $\sqrt{(0-3)^2 + (3-0)^2} = \sqrt{18} = 3\sqrt{2}$. So the area of $PQRS$ is $(3\sqrt{2})^2 = 18$.

18. (a)



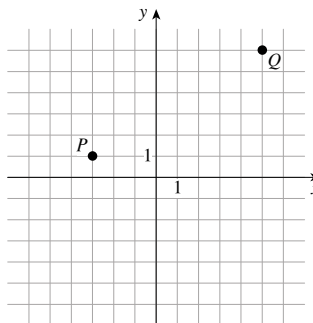
- (b) The x -intercepts occur when $y = 0$, so $0 = 4 - x^2 \Leftrightarrow x^2 = 4 \Rightarrow x = \pm 2$. The y -intercept occurs when $x = 0$, so $y = 4$.

- (c) x -axis symmetry: $(-y) = 4 - x^2 \Leftrightarrow y = x^2 - 4$, which is not the same as the original equation, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = 4 - (-x)^2 = 4 - x^2$, which is the same as the original equation, so the graph is symmetric with respect to the y -axis.

Origin symmetry: $(-y) = 4 - (-x)^2 \Leftrightarrow y = -4 - x^2$, which is not the same as the original equation, so the graph is not symmetric with respect to the origin.

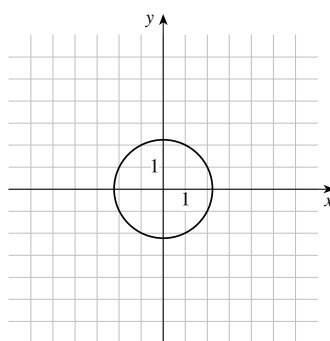
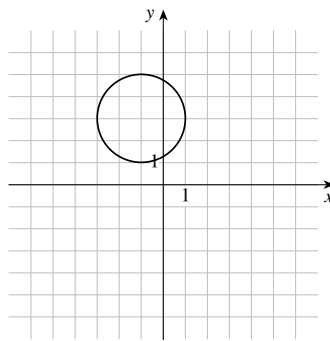
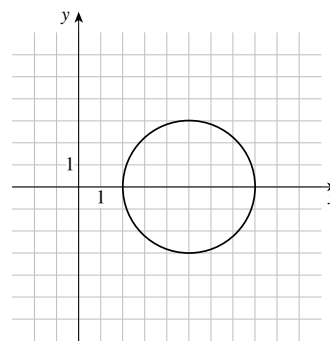
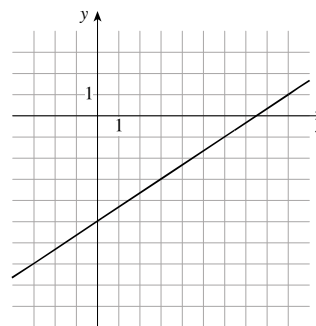
19. (a)

(b) The distance between P and Q is

$$d(P, Q) = \sqrt{(-3 - 5)^2 + (1 - 6)^2} = \sqrt{64 + 25} = \sqrt{89}.$$

(c) The midpoint is $\left(\frac{-3 + 5}{2}, \frac{1 + 6}{2}\right) = \left(1, \frac{7}{2}\right)$.(d) The center of the circle is the midpoint, $\left(1, \frac{7}{2}\right)$, and the length of the radius is $\frac{1}{2}\sqrt{89}$. Thus the equation of the circle whose diameter is PQ is

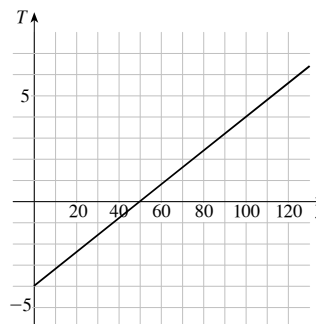
$$(x - 1)^2 + \left(y - \frac{7}{2}\right)^2 = \left(\frac{1}{2}\sqrt{89}\right)^2 \Leftrightarrow (x - 1)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{89}{4}.$$

20. (a) $x^2 + y^2 = 5$ has center $(0, 0)$ and radius $\sqrt{5}$.(b) $(x + 1)^2 + (y - 3)^2 = 4 = 2^2$ has center $(-1, 3)$ and radius 2.(c) $x^2 + y^2 - 10x + 16 = 0 \Leftrightarrow$ $(x - 5)^2 + y^2 = -16 + 25 = 9$ has center $(5, 0)$ and radius 3.21. $2x - 3y = 15 \Leftrightarrow -3y = -2x + 15 \Leftrightarrow y = \frac{2}{3}x - 5$. The slope is $\frac{2}{3}$ and the y -intercept is -5 .22. (a) The line through $(6, 7)$ and $(1, -3)$ has slope $\frac{-3 - 7}{1 - 6} = 2$, so an equation is $y - 7 = 2(x - 6) \Leftrightarrow y = 2x - 5 \Leftrightarrow 2x - y - 5 = 0$.(b) $3x + y - 10 = 0 \Leftrightarrow y = -3x + 10$, so the slope of the line we seek is -3 . Using the point-slope form, $y - (-6) = -3(x - 3) \Leftrightarrow y + 6 = -3x + 9 \Leftrightarrow 3x + y - 3 = 0$.(c) Using the intercept form we get $\frac{x}{6} + \frac{y}{4} = 1 \Leftrightarrow 2x + 3y = 12 \Leftrightarrow 2x + 3y - 12 = 0$.

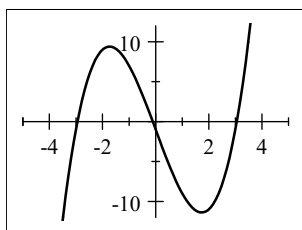
23. (a) When $x = 100$ we have $T = 0.08(100) - 4 = 8 - 4 = 4$, so the temperature at one meter is 4°C .

(c) The slope represents the raise in temperature as the depth increase. The T -intercept is the surface temperature of the soil and the x -intercept represents the depth of the “frost line”, where the soil below is not frozen.

(b)



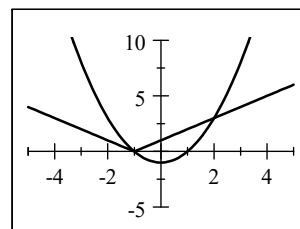
24. (a) $x^3 - 9x - 1 = 0$. We graph the equation $y = x^3 - 9x - 1$ in the viewing rectangle $[-5, 5]$ by $[-12, 12]$. We find that the points of intersection occur at $x \approx -2.94, -0.11, 3.05$.



- (b) $x^2 - 1 \leq |x + 1|$. We graph the equations

$y_1 = x^2 - 1$ and $y_2 = |x + 1|$ in the viewing rectangle $[-5, 5]$ by $[-5, 10]$. We find that the points of intersection occur at $x = -1$ and $x = 2$.

Since we want $x^2 - 1 \leq |x + 1|$, the solution is the interval $[-1, 2]$.



25. **Note: In the first printing of the text, the answers given for parts (b) and (c) of this exercise are incorrect.**

(a) $M = k \frac{wh^2}{L}$

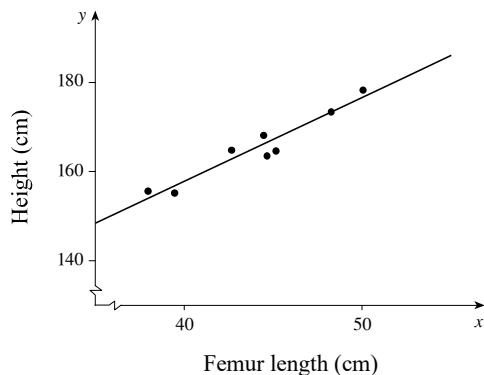
- (b) Substituting $w = 10$, $h = 15$, $L = 30$, and $M = 21,000$, we have $21,000 = k \frac{(10)(15^2)}{30} \Leftrightarrow k = 280$. Thus

$$M = 280 \frac{wh^2}{L}.$$

- (c) Now if $L = 25$, $w = 7.5$, and $h = 25$, then $M = 280 \frac{(7.5)(25^2)}{25} = 52,500$. So the beam can support 52,500 newtons.

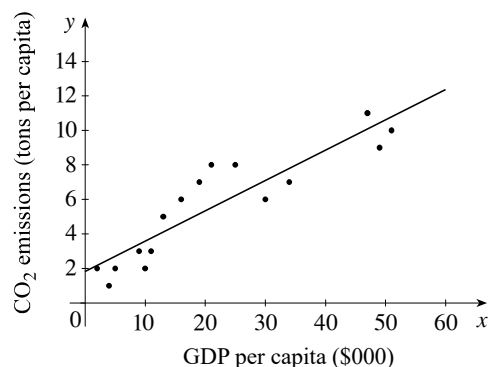
FOCUS ON MODELING Fitting Lines to Data

1. (a) Using a graphing calculator, we obtain the regression line $y = 1.8807x + 82.65$.



- (b) Using $x = 58$ in the equation $y = 1.8807x + 82.65$, we get $y = 1.8807(58) + 82.65 \approx 191.7$ cm.

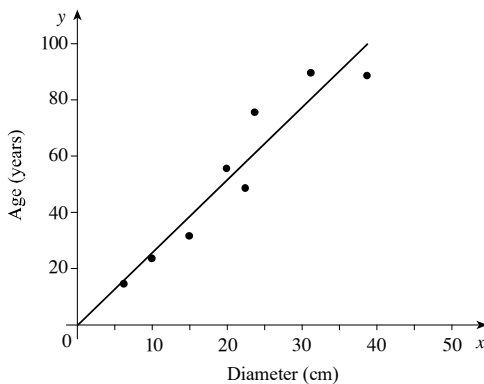
2. (a) Using a graphing calculator, we obtain the regression line $y = 0.17565x + 1.8265$.



- (b) For a per capita GDP of \$80,000, the model predicts carbon emissions of $y = 0.17565(80) + 1.8265 \approx 15.9$ tons per capita. For \$32,000, it predicts $y = 0.17565(32) + 1.8265 \approx 7.4$ tons per capita.
- (c) A linear model is reasonable for these data. One limitation is that there are no data points beyond per-capita income above \$51,000, so the model cannot reliably make predictions for GDPs much larger than \$51,000.

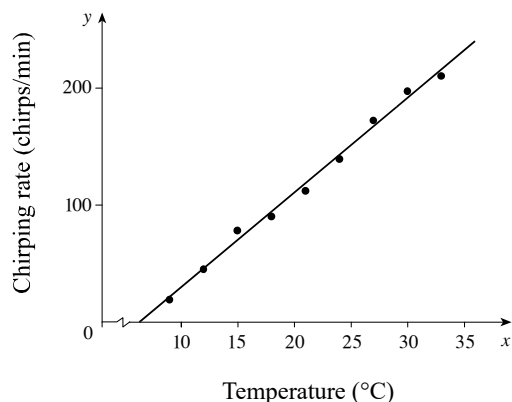
3. **Note: In the first printing of the text, the answer given for part (a) of this exercise is incorrect.**

- (a) Using a graphing calculator, we obtain the regression line $y = 2.579x - 0.1783$.



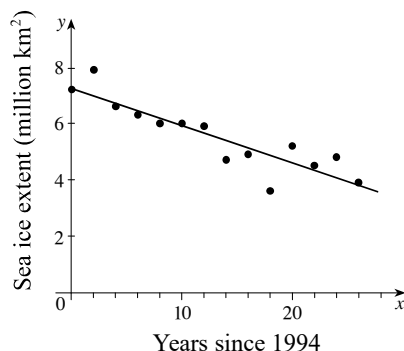
- (b) Using $x = 45$ in the equation $y = 2.579x - 0.1783$, we get $y = 2.579(45) - 0.1783 \approx 116$ years.

4. (a) Using a graphing calculator, we obtain the regression line $y = 8.278x - 53.93$.



- (b) Using $x = 38^\circ \text{C}$ in the equation $y = 8.278x - 53.93$, we get $y \approx 261$ chirps per minute.

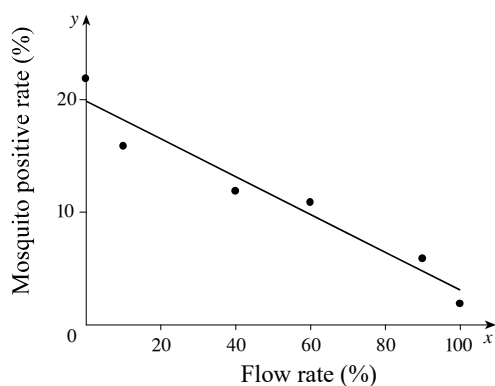
5. (a) Using a graphing calculator, we obtain the regression line $y = -0.13198x + 7.2514$



- (b) Using $x = 25$ in the regression line equation, we get $y = -0.13198(25) + 7.2514 \approx 3.95$ million km^2 . This is roughly 10% less than the actual figure of 4.4 million km^2 .

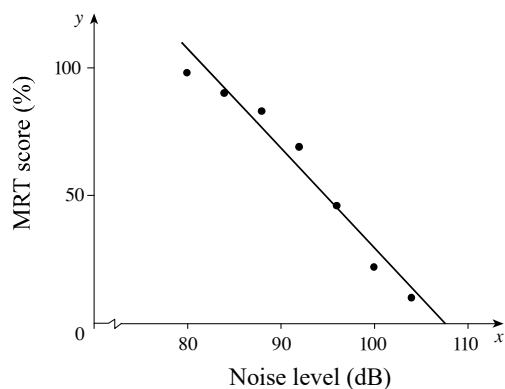
- (c) Despite fluctuations over brief periods, the model seems fairly accurate. If external circumstances change (reduced or increased CO_2 emissions, for example), it may become less reliable. It is unlikely to be accurate far into the future.

6. (a) Using a graphing calculator, we obtain the regression line $y = -0.168x + 19.89$.



- (b) Using the regression line equation $y = -0.168x + 19.89$, we get $y \approx 8.13\%$ when $x = 70\%$.

7. (a) Using a graphing calculator, we obtain
 $y = -3.9018x + 419.7$.



8. Students should find a fairly strong correlation between shoe size and height.
 9. Results will depend on student surveys in each class.

- (b) The correlation coefficient is $r = -0.98$, so linear model is appropriate for x between 80 dB and 104 dB.

- (c) Substituting $x = 94$ into the regression equation, we get $y = -3.9018(94) + 419.7 \approx 53$. So the intelligibility is about 53%.

1 ERRATA in Exercises and Answers in First Printing

Page 123 1.11.56 Displayed equation should read $y = \sqrt{2.4x + \left(\frac{x}{1000}\right)^2}$

Page 123 1.11.56 (a) Graph the equation for $0 \leq x \leq 160$.

Page A2 1.5.129 (Answer) ≈ 4.24 s

Page A2 1.5.137 (Answer) 344, 000 km

Page A3 1.7.21 (Answer) 220 km

Page A3 1.7.39 (Answer) 31.6 m by 158 m

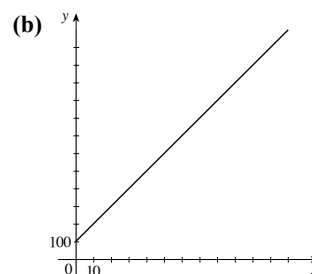
Page A3 1.7.77 (Answer) 1.6 m from the fulcrum

Page A4 1.8.119 (Answer) More than 100 km

Page A4 1.8.127 (Answer) (a) Acceleration greater than 8.04 m/s^2

Page A7 1.10.93 (Answer) (a) $t = \frac{1}{8}n + 6$ (b) 25°C

Page A7 1.10.95 (Answer) (a) $P = 9.97d + 103$, where P is pressure in kPa and d is depth in meters
 (c) The slope represents an increase of 9.97 kPa in pressure for each one meter increase in depth, and the d -intercept is the air pressure at the surface. (d) ≈ 59 m



Page A7 1.12.39 (Answer) ≈ 69 km/h

Page A7 1.12.41 (Answer) ≈ 9.03 km/h

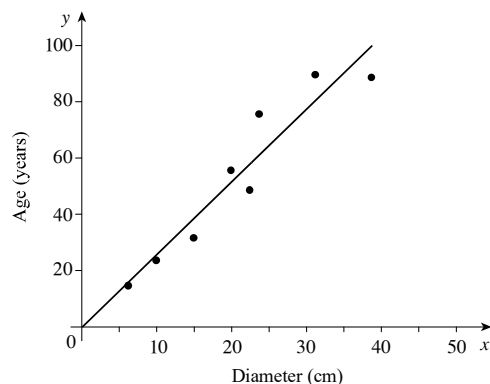
Page A7 1.12.49 (Answer) (b) 1, 928, 234, 931

Page A8 1.Review.155 (Answer) 17 km/h

Page A9 1.Test.12 (Answer) 192 km

Page A9 1.Test.25 (Answer) (b) 280 (c) 52,500 N

Page A10 1.Focus.3 (Answer) (a) $y = 2.579x - 0.1783$



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- 1.1** Real Numbers 1
 - 1.2** Exponents and Radicals 3
 - 1.3** Algebraic Expressions 7
 - 1.4** Rational Expressions 10
 - 1.5** Equations 14
 - 1.6** Complex Numbers 20
 - 1.7** Modeling with Equations 21
 - 1.8** Inequalities 27
 - 1.9** The Coordinate Plane; Graphs of Equations; Circles 37
 - 1.10** Lines 47
 - 1.11** Solving Equations and Inequalities Graphically 52
 - 1.12** Modeling Variation 57
 - Chapter 1 Review 59
 - Chapter 1 Test 68
 - **FOCUS ON MODELING:** Fitting Lines to Data 70
 - Errata in Exercises and Answers in First Printing 72

1 FUNDAMENTALS

1.1 REAL NUMBERS

1. (a) The natural numbers are $\{1, 2, 3, \dots\}$.
 (b) The numbers $\{\dots, -3, -2, -1, 0\}$ are integers but not natural numbers.
 (c) Any irreducible fraction $\frac{p}{q}$ with $q \neq 1$ is rational but is not an integer. Examples: $\frac{3}{2}, -\frac{5}{12}, \frac{1729}{23}$.
 (d) Any number which cannot be expressed as a ratio $\frac{p}{q}$ of two integers is irrational. Examples are $\sqrt{2}, \sqrt{3}, \pi$, and e .
3. (a) In set-builder notation: $\{x \mid -3 < x < 5\}$ (c) As a graph: 
 (b) In interval notation: $(-3, 5)$
5. The distance between a and b on the real line is $d(a, b) = |b - a|$. So the distance between -5 and 2 is $|2 - (-5)| = 7$.
7. (a) No: $a - b = -(b - a) \neq b - a$ in general.
 (b) No; by the Distributive Property, $-2(a - 5) = -2a + -2(-5) = -2a + 10 \neq -2a - 10$.
9. (a) Natural number: 100
 (b) Integers: 0, 100, -8
 (c) Rational numbers: $-1.5, 0, \frac{5}{2}, 2.71, 3.1\bar{4}, 100, -8$
 (d) Irrational numbers: $\sqrt{7}, -\pi$
11. Commutative Property of addition
13. Associative Property of addition
15. Distributive Property
17. Commutative Property of multiplication
19. $x + 3 = 3 + x$
21. $4(A + B) = 4A + 4B$
23. $-2(x + y) = -2x - 2y$
25. $5(2xy) = (5 \cdot 2)xy = 10xy$
27. $-\frac{5}{2}(2x - 4y) = -\frac{5}{2}(2x) + \frac{5}{2}(4y) = -5x + 10y$
29. (a) $\frac{2}{3} + \frac{5}{7} = \frac{14}{21} + \frac{15}{21} = \frac{29}{21}$
 (b) $\frac{5}{12} - \frac{3}{8} = \frac{10}{24} - \frac{9}{24} = \frac{1}{24}$
31. (a) $\frac{2}{3}\left(6 - \frac{3}{2}\right) = \frac{2}{3} \cdot 6 - \frac{2}{3} \cdot \frac{3}{2} = 4 - 1 = 3$
 (b) $\left(3 + \frac{1}{4}\right)\left(1 - \frac{4}{5}\right) = \left(\frac{12}{4} + \frac{1}{4}\right)\left(\frac{5}{5} - \frac{4}{5}\right) = \frac{13}{4} \cdot \frac{1}{5} = \frac{13}{20}$
33. (a) $2 \cdot 3 = 6$ and $2 \cdot \frac{7}{2} = 7$, so $3 < \frac{7}{2}$
 (b) $-3 > -\frac{7}{2}$
 (c) $3.5 = \frac{7}{2}$
35. (a) False
 (b) True
37. (a) True
 (b) False
39. (a) $x > 0$ (b) $t < 4$
 (c) $a \geq \pi$ (d) $-5 < x < \frac{1}{3}$
 (e) $|3 - p| \leq 5$
41. (a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$
 (b) $A \cap B = \{2, 4, 6\}$

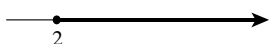
43. (a) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(b) $A \cap C = \{7\}$

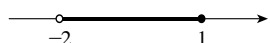
47. $(-3, 0) = \{x \mid -3 < x < 0\}$



51. $[2, \infty) = \{x \mid x \geq 2\}$



55. $-2 < x \leq 1 \Leftrightarrow x \in (-2, 1]$



59. (a) $[-3, 5]$ (b) $(-3, 5]$ (c) $(-3, \infty)$

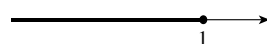
45. (a) $B \cup C = \{x \mid x \leq 5\}$

(b) $B \cap C = \{x \mid -1 < x < 4\}$

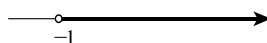
49. $[2, 8) = \{x \mid 2 \leq x < 8\}$



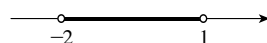
53. $x \leq 1 \Leftrightarrow x \in (-\infty, 1]$



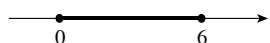
57. $x > -1 \Leftrightarrow x \in (-1, \infty)$



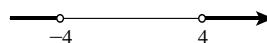
61. $(-2, 0) \cup (-1, 1) = (-2, 1)$



63. $[-4, 6] \cap [0, 8) = [0, 6]$



65. $(-\infty, -4) \cup (4, \infty)$



67. (a) $|50| = 50$

(b) $|-13| = 13$

71. (a) $|(-2) \cdot 6| = |-12| = 12$

(b) $\left| \left(-\frac{1}{3}\right)(-15) \right| = |5| = 5$

75. (a) $|17 - 2| = 15$

(b) $|21 - (-3)| = |21 + 3| = |24| = 24$

(c) $\left| -\frac{3}{10} - \frac{11}{8} \right| = \left| -\frac{12}{40} - \frac{55}{40} \right| = \left| -\frac{67}{40} \right| = \frac{67}{40}$

69. (a) $||-6| - |-4|| = |6 - 4| = |2| = 2$

(b) $\frac{-1}{|-1|} = \frac{-1}{1} = -1$

73. $|(-2) - 3| = |-5| = 5$

77. (a) Let $x = 0.777 \dots$. So $10x = 7.777 \dots \Leftrightarrow x = 0.777 \dots \Leftrightarrow 9x = 7$. Thus, $x = \frac{7}{9}$.

(b) Let $x = 0.2888 \dots$. So $100x = 28.888 \dots \Leftrightarrow 10x = 2.8888 \dots \Leftrightarrow 90x = 26$. Thus, $x = \frac{26}{90} = \frac{13}{45}$.

(c) Let $x = 0.575757 \dots$. So $100x = 57.5757 \dots \Leftrightarrow x = 0.5757 \dots \Leftrightarrow 99x = 57$. Thus, $x = \frac{57}{99} = \frac{19}{33}$.

79. $\pi > 3$, so $|\pi - 3| = \pi - 3$.

81. $a < b$, so $|a - b| = -(a - b) = b - a$.

83. (a) $-a$ is negative because a is positive.

(b) bc is positive because the product of two negative numbers is positive.

(c) $a - b = a + (-b)$ is positive because it is the sum of two positive numbers.

(d) $ab + ac$ is negative: Each summand is the product of a positive number and a negative number, and the sum of two negative numbers is negative.

85. Distributive Property

87. Let $x = \frac{m_1}{n_1}$ and $y = \frac{m_2}{n_2}$ be rational numbers. Then $x + y = \frac{m_1}{n_1} + \frac{m_2}{n_2} = \frac{m_1n_2 + m_2n_1}{n_1n_2}$,
 $x - y = \frac{m_1}{n_1} - \frac{m_2}{n_2} = \frac{m_1n_2 - m_2n_1}{n_1n_2}$, and $x \cdot y = \frac{m_1}{n_1} \cdot \frac{m_2}{n_2} = \frac{m_1m_2}{n_1n_2}$. This shows that the sum, difference, and product of two rational numbers are again rational numbers. However the product of two irrational numbers is not necessarily irrational; for example, $\sqrt{2} \cdot \sqrt{2} = 2$, which is rational. Also, the sum of two irrational numbers is not necessarily irrational; for example, $\sqrt{2} + (-\sqrt{2}) = 0$ which is rational.

89.

x	1	2	10	100	1000
$\frac{1}{x}$	1	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

As x gets large, the fraction $1/x$ gets small. Mathematically, we say that $1/x$ goes to zero.

x	1	0.5	0.1	0.01	0.001
$\frac{1}{x}$	1	$\frac{1}{0.5} = 2$	$\frac{1}{0.1} = 10$	$\frac{1}{0.01} = 100$	$\frac{1}{0.001} = 1000$

As x gets small, the fraction $1/x$ gets large. Mathematically, we say that $1/x$ goes to infinity.

91. (a) Suppose that $a > b$, so $\max(a, b) = a$ and $|a - b| = a - b$. Then $\frac{a + b + |a - b|}{2} = \frac{a + b + a - b}{2} = a$.
 On the other hand, if $b > a$, then $\max(a, b) = b$ and $|a - b| = -(a - b) = b - a$. In this case,
 $\frac{a + b + |a - b|}{2} = \frac{a + b + b - a}{2} = b$.
 If $a = b$, then $|a - b| = 0$ and the result is trivial.
- (b) If $a < b$, then $\min(a, b) = a$ and $|a - b| = b - a$. In this case $\frac{a + b - |a - b|}{2} = \frac{a + b - (b - a)}{2} = a$.
 Similarly, if $b < a$, then $\frac{a + b - |a - b|}{2} = b$; and if $a = b$, the result is trivial.
93. (a) Subtraction is not commutative. For example, $5 - 1 \neq 1 - 5$.
 (b) Division is not commutative. For example, $5 \div 1 \neq 1 \div 5$.
 (c) Putting on your socks and putting on your shoes are not commutative. If you put on your socks first, then your shoes, the result is not the same as if you proceed the other way around.
 (d) Putting on your hat and putting on your coat are commutative. They can be done in either order, with the same result.
 (e) Washing laundry and drying it are not commutative.

1.2 EXPONENTS AND RADICALS

1. (a) Using exponential notation we can write the product $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ as 5^6 .
 (b) In the expression 3^4 , the number 3 is called the *base* and the number 4 is called the *exponent*.
3. To move a number raised to a power from numerator to denominator or from denominator to numerator change the sign of the *exponent*. So $a^{-2} = \frac{1}{a^2}$, $\frac{1}{b^{-2}} = b^2$, $\frac{a^{-3}}{b^2} = \frac{1}{a^3b^2}$, and $\frac{6a^2}{b^{-3}} = 6a^2b^3$.
5. $(4^{1/2})^3 = 2^3 = 8$; $(4^3)^{1/2} = 64^{1/2} = 8$
7. $5^{1/3} \cdot 5^{2/3} = 5^1 = 5$

9. (a) $-2^6 = -(2^6) = -64$

(b) $(-2)^6 = 64$

(c) $\left(\frac{1}{5}\right)^2 \cdot (-3)^3 = \frac{(-3)^3}{5^2} = -\frac{27}{25}$

13. (a) $5^3 \cdot 5 = 5^{3+1} = 625$

(b) $5^4 \cdot 5^{-2} = 5^{4-2} = 25$

(c) $(2^2)^3 = 2^{2 \cdot 3} = 64$

17. (a) $\sqrt{3}\sqrt{15} = \sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$

(b) $\frac{\sqrt{48}}{\sqrt{3}} = \sqrt{\frac{48}{3}} = \sqrt{16} = 4$

(c) $\sqrt[3]{24}\sqrt[3]{18} = \sqrt[3]{24 \cdot 18} = \sqrt[3]{6^3 \cdot 2} = 6\sqrt[3]{2}$

21. (a) $x^{-5} \cdot x^3 = x^{-5+3} = x^{-2} = \frac{1}{x^2}$

(b) $w^{-2}w^{-4}w^5 = w^{-2-4+5} = w^{-1} = \frac{1}{w}$

(c) $\frac{x^{16}}{x^{10}} = x^{16-10} = x^6$

25. (a) $(3x^3y^2)(2y^3) = 3 \cdot 2x^3y^2y^3 = 6x^3y^5$

(b) $(5w^2z^{-2})^2(z^3) = \left(\frac{5w^2}{z^2}\right)^2(z^3) = \frac{5^2(w^2)^2z^3}{(z^2)^2} = \frac{25w^4}{z}$

27. (a) $\frac{x^2y^{-1}}{x^{-5}} = x^{2-(-5)}y^{-1} = \frac{x^7}{y}$

(b) $\left(\frac{a^3}{2b^2}\right)^3 = \frac{a^{3 \cdot 3}}{2^3(b^2)^3} = \frac{a^9}{8b^6}$

29. (a) $\left(\frac{a^2}{b}\right)^5 \left(\frac{a^3b^2}{c^3}\right)^3 = a^{2 \cdot 5+3 \cdot 3}b^{-5+2 \cdot 3}c^{-3 \cdot 3} = \frac{a^{19}b}{c^9}$

(b) $\frac{(u^{-1}v^2)^2}{(u^3v^{-2})^3} = u^{-1 \cdot 2-3 \cdot 3}v^{2 \cdot 2-(-2)3} = \frac{v^{10}}{u^{11}}$

31. (a) $\frac{8a^3b^{-4}}{2a^{-5}b^5} = 4a^{3-(-5)}b^{-4-5} = \frac{4a^8}{b^9}$

33. (a) $\sqrt[4]{x^4} = |x|$

35. (a) $\sqrt[3]{8x^9y^3} = \sqrt[3]{2^3(x^3)^3y^3} = 2x^3y$

(b) $\sqrt[4]{8x^6y^2}\sqrt[4]{2x^2y^2} = \sqrt[4]{8x^6y^2 \cdot 2x^2y^2} = \sqrt[4]{16x^8y^4} = \sqrt[4]{2^4(x^2)^4y^4} = 2x^2|y|$

37. (a) $\sqrt{32} + \sqrt{18} = \sqrt{16 \cdot 2} + \sqrt{9 \cdot 2} = \sqrt{16}\sqrt{2} + \sqrt{9}\sqrt{2} = 4\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$

11. (a) $\left(\frac{5}{3}\right)^0 \cdot 2^{-1} = \frac{1}{2}$

(b) $\frac{2^{-3}}{3^0} = \frac{1}{2^3} = \frac{1}{8}$

(c) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

15. (a) $3\sqrt[3]{16} = 3\sqrt[3]{8 \cdot 2} = 3\sqrt[3]{8}\sqrt[3]{2} = 6\sqrt[3]{2}$

(b) $\frac{\sqrt{18}}{\sqrt{81}} = \sqrt{\frac{18}{81}} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$

(c) $\sqrt{\frac{27}{4}} = \frac{3\sqrt{3}}{2} = \frac{\sqrt{9 \cdot 3}}{\sqrt{4}} = \frac{3\sqrt{3}}{2}$

19. (a) $t^5t^2 = t^{5+2} = t^7$

(b) $(4z^3)^2 = 4^2(z^3)^2 = 16z^{3 \cdot 2} = 16z^6$

(c) $x^{-3}x^5 = x^{-3+5} = x^2$

23. (a) $\frac{a^9a^{-2}}{a} = a^{9-2-1} = a^6$

(b) $(a^2a^4)^3 = (a^{2+4})^3 = (a^6)^3 = a^{6 \cdot 3} = a^{18}$

(c) $\left(\frac{x}{2}\right)^3(5x^6) = \frac{x^3 \cdot 5x^6}{2^3} = \frac{5x^9}{8}$

(b) $\left(\frac{y}{5x^{-2}}\right)^{-3} = 5^{-1(-3)}x^{-(-2)(-3)}y^{-3} = \frac{125}{x^6y^3}$

(b) $\sqrt[4]{16x^8} = \sqrt[4]{2^4x^8} = 2x^2$

- (b) $\sqrt{75} + \sqrt{48} = \sqrt{25 \cdot 3} + \sqrt{16 \cdot 3} = \sqrt{25}\sqrt{3} + \sqrt{16}\sqrt{3} = 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$
39. (a) $\sqrt{9a^3} + \sqrt{a} = \sqrt{9}\sqrt{a^2 \cdot a} + \sqrt{a} = 3a\sqrt{a} + \sqrt{a} = (3a + 1)\sqrt{a}$
 (b) $\sqrt{16x} + \sqrt{x^5} = \sqrt{16}\sqrt{x} + \sqrt{x^4}\sqrt{x} = (x^2 + 4)\sqrt{x}$
41. (a) $\sqrt{36x^2 + 36x^4} = \sqrt{6^2x^2(1 + x^2)} = 6x\sqrt{1 + x^2}$ (b) $\sqrt{81x^2 + 81y^2} = \sqrt{9^2(x^2 + y^2)} = 9\sqrt{x^2 + y^2}$
43. $\sqrt{10} = 10^{1/2}$ 45. $7^{3/5} = (7^3)^{1/5} = \sqrt[5]{7^3}$
47. $\frac{1}{\sqrt{5}} = \frac{1}{5^{1/2}} = 5^{-1/2}$ 49. $y^{-1.5} = y^{-3/2} = \frac{1}{\sqrt{y^3}}$
51. (a) $16^{1/4} = \sqrt[4]{16} = 2$ (b) $-8^{1/3} = \sqrt[3]{-8} = -2$ (c) $9^{-1/2} = \frac{1}{\sqrt{9}} = \frac{1}{3}$
53. (a) $32^{2/5} = (\sqrt[5]{32})^2 = 2^2 = 4$ (b) $\left(\frac{4}{9}\right)^{-1/2} = \frac{1}{\sqrt{4/9}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$ (c) $\left(\frac{16}{81}\right)^{3/4} = \left(\frac{\sqrt[4]{16}}{\sqrt[4]{81}}\right)^3 = \frac{8}{27}$
55. (a) $5^{2/3} \cdot 5^{1/3} = 5^{(2/3)+(1/3)} = 5^1 = 5$
 (b) $\frac{3^{3/5}}{3^{2/5}} = 3^{(3/5)-(2/5)} = 3^{1/5} = \sqrt[5]{3}$
 (c) $(\sqrt[3]{4})^3 = (4^{1/3})^3 = 4^{(1/3) \cdot 3} = 4^1 = 4$
57. (a) $x^{3/4}x^{5/4} = x^{3/4+5/4} = x^2$
 (b) $y^{2/3}y^{4/3} = y^{2/3+4/3} = y^2$
59. (a) $\frac{w^{4/3}w^{2/3}}{w^{1/3}} = w^{4/3+2/3-1/3} = w^{5/3}$ 61. (a) $(8a^6b^{3/2})^{2/3} = 8^{2/3}a^{6(2/3)}b^{(3/2)(2/3)} = 4a^4b$
 (b) $(3x^{1/2}y^{1/3})^6 = 3^6x^{(1/2)6}y^{(1/3)6} = 3^6 = 729x^3y^2$ (b) $(4a^{-4}b^6)^{3/2} = 4^{3/2}a^{-4(3/2)}b^{6(3/2)} = 8a^{-6}b^9 = \frac{8b^9}{a^6}$
63. (a) $\left(\frac{3x^{1/4}}{y}\right)^2 \left(\frac{x^{-1}}{y^4}\right)^{1/2} = \frac{3^2x^{(1/4)2}}{y^2} \frac{x^{-1(1/2)}}{y^{4(1/2)}} = \frac{9x^{1/2}}{y^2} \frac{x^{-1/2}}{y^2} = \frac{9}{y^4}$
 (b) $\left(\frac{3w^{-1/3}}{z^{-1}}\right)^{-4} \left(\frac{2w^{1/3}}{z^{1/2}}\right)^2 = \frac{3^{-4}w^{(-1/3)(-4)}}{z^{-1(-4)}} \frac{2^2w^{(1/3)2}}{z^{(1/2)2}} = \frac{w^{4/3}}{81z^4} \frac{4w^{2/3}}{z} = \frac{4w^{(4/3)+(2/3)}}{81z^5} = \frac{4w^2}{81z^5}$
65. (a) $\sqrt{x^3} = (x^3)^{1/2} = x^{3(1/2)} = x^{3/2}$ 67. (a) $\sqrt[6]{y^5}\sqrt[3]{y^2} = y^{5/6} \cdot y^{2/3} = y^{5/6+2/3} = y^{3/2}$
 (b) $\sqrt[5]{x^6} = (x^6)^{1/5} = x^{6(1/5)} = x^{6/5}$ (b) $(5\sqrt[3]{x})(2\sqrt[4]{x}) = 5 \cdot 2x^{1/3+1/4} = 10x^{7/12}$
69. (a) $\sqrt[4]{4st^3}\sqrt[6]{s^3t^2} = 4^{1/2}s^{1/2+3/6}t^{3/2+2/6} = 2st^{11/6}$ 71. (a) $\sqrt[3]{y\sqrt{y}} = (y^{1+1/2})^{1/3} = y^{(3/2)(1/3)} = y^{1/2}$
 (b) $\frac{\sqrt[4]{x^7}}{\sqrt[4]{x^3}} = x^{7/4-3/4} = x$ (b) $\sqrt{\frac{18u^5v}{2u^3v^3}} = \sqrt{\frac{9u^2}{v^2}} = \frac{3u}{v}$
73. (a) $\frac{1}{\sqrt{6}} = \frac{1}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{6}$ 75. (a) $\frac{1}{\sqrt{5x}} = \frac{1}{\sqrt{5x}} \cdot \frac{\sqrt{5x}}{\sqrt{5x}} = \frac{\sqrt{5x}}{5x}$
 (b) $\sqrt{\frac{3}{2}} = \sqrt{\frac{3}{2}} \sqrt{\frac{2}{2}} = \frac{\sqrt{6}}{2}$ (b) $\sqrt{\frac{x}{5}} = \sqrt{\frac{x}{5}} \sqrt{\frac{5}{5}} = \frac{\sqrt{5x}}{5}$
 (c) $\frac{9}{\sqrt[4]{2}} = \frac{9}{2^{1/4}} \cdot \frac{2^{3/4}}{2^{3/4}} = \frac{9\sqrt[4]{8}}{2}$ (c) $\sqrt[5]{\frac{1}{x^3}} = \frac{1}{x^{3/5}} \cdot \frac{x^{2/5}}{x^{2/5}} = \frac{\sqrt[5]{x^2}}{x}$

$$77. \text{ (a) } \sqrt[4]{\frac{1}{4}} \sqrt[4]{\frac{1}{64}} = \sqrt[4]{\frac{1}{4 \cdot 4^3}} = \frac{1}{4}$$

$$\text{ (b) } \frac{\sqrt{5}}{\sqrt{40}} = \sqrt{\frac{5}{40}} = \sqrt{\frac{1}{8}} = \sqrt{\frac{2}{16}} = \frac{\sqrt{2}}{4}$$

$$79. \text{ (a) } \sqrt{y} \sqrt[4]{y^2} = y^{1/2} y^{2/4} = y$$

$$\text{ (b) } \left(81 \sqrt[4]{w^8 z^8}\right)^{1/2} = 81^{1/2} w^{(8/4)(1/2)} z^{(8/4)(1/2)} = 9 |w| |z|. \text{ Since } w > 0 \text{ and } z < 0, \text{ this is equivalent to } -9wz.$$

$$81. \text{ (a) } 69,300,000 = 6.93 \times 10^7$$

$$\text{ (b) } 7,200,000,000,000 = 7.2 \times 10^{12}$$

$$\text{ (c) } 0.000\,028\,536 = 2.8536 \times 10^{-5}$$

$$\text{ (d) } 0.000\,121\,3 = 1.213 \times 10^{-4}$$

$$83. \text{ (a) } 3.19 \times 10^5 = 319,000$$

$$\text{ (b) } 2.721 \times 10^8 = 272,100,000$$

$$\text{ (c) } 2.670 \times 10^{-8} = 0.000\,000\,026\,70$$

$$\text{ (d) } 9.999 \times 10^{-9} = 0.000\,000\,009\,999$$

$$85. \text{ (a) } 9,460,000,000,000 \text{ km} = 9.5 \times 10^{12} \text{ km}$$

$$\text{ (b) } 0.000\,000\,000\,000\,4 \text{ cm} = 4 \times 10^{-13} \text{ cm}$$

$$\text{ (c) } 33 \text{ billion billion molecules} = 33 \times 10^9 \times 10^9 = 3.3 \times 10^{19} \text{ molecules}$$

$$87. \left(7.2 \times 10^{-9}\right) \left(1.806 \times 10^{-12}\right) = 7.2 \times 1.806 \times 10^{-9} \times 10^{-12} \approx 13.0 \times 10^{-21} = 1.3 \times 10^{-20}$$

$$89. \frac{1.295643 \times 10^9}{(3.610 \times 10^{-17})(2.511 \times 10^6)} = \frac{1.295643}{3.610 \times 2.511} \times 10^{9+17-6} \approx 0.1429 \times 10^{19} = 1.429 \times 10^{19}$$

$$91. \frac{(0.000\,016\,2)(0.015\,82)}{(594,621,000)(0.005\,8)} = \frac{(1.62 \times 10^{-5})(1.582 \times 10^{-2})}{(5.94621 \times 10^8)(5.8 \times 10^{-3})} = \frac{1.62 \times 1.582}{5.94621 \times 5.8} \times 10^{-5-2-8+3} = 0.074 \times 10^{-12} = 7.4 \times 10^{-14}$$

93. (a) b^5 is negative since a negative number raised to an odd power is negative.

(b) b^{10} is positive since a negative number raised to an even power is positive.

(c) ab^2c^3 we have (positive)(negative)²(negative)³ = (positive)(positive)(negative) which is negative.

(d) Since $b - a$ is negative, $(b - a)^3 = (\text{negative})^3$ which is negative.

(e) Since $b - a$ is negative, $(b - a)^4 = (\text{negative})^4$ which is positive.

(f) $\frac{a^3c^3}{b^6c^6} = \frac{(\text{positive})^3(\text{negative})^3}{(\text{negative})^6(\text{negative})^6} = \frac{(\text{positive})(\text{negative})}{(\text{positive})(\text{positive})} = \frac{\text{negative}}{\text{positive}}$ which is negative.

95. Since one light year is 9.5×10^{12} km, Centauri is about $4.3 \times 9.5 \times 10^{12} \approx 4.06 \times 10^{13}$ km, or 40,600,000,000,000 kilometers away.

$$97. \text{ Volume} = (\text{Average depth})(\text{Area}) = \left(3.7 \times 10^3 \text{ m}\right) \left(3.6 \times 10^{14} \text{ m}^2\right) \left(\frac{10^3 \text{ liters}}{\text{m}^3}\right) \approx 1.33 \times 10^{21} \text{ liters}$$

99. First, we estimate the total mass of the stars in the observable universe:

$$\begin{aligned} \text{Total star mass} &\approx (\text{Number of galaxies}) \left(\frac{\text{Number of stars}}{\text{Galaxy}}\right) (\text{Mass of typical star}) = (1.5 \cdot 2 \cdot 1.77) (10^{12} \cdot 10^{11} \cdot 10^{30}) \\ &\approx 5.31 \times 10^{53} \text{ kg} \end{aligned}$$

Thus, the number of hydrogen atoms in the observable universe is

$$\frac{\text{Total star mass}}{\text{Mass of a single hydrogen atom}} \approx \frac{5.31 \times 10^{53}}{1.67 \times 10^{-27}} \approx 3.18 \times 10^{80}$$

101. (a) Using $f = 0.4$ and substituting $d = 65$, we obtain $s = \sqrt{30fd} = \sqrt{30 \times 0.4 \times 65} \approx 28$ km/h.
 (b) Using $f = 0.5$ and substituting $s = 50$, we find d . This gives $s = \sqrt{30fd} \Leftrightarrow 50 = \sqrt{30 \cdot (0.5)d} \Leftrightarrow 50 = \sqrt{15d} \Leftrightarrow 2500 = 15d \Leftrightarrow d = \frac{500}{3} \approx 167$ m.
103. Since $10^6 = 10^3 \cdot 10^3$ it would take 1000 days ≈ 2.74 years to spend the million dollars.
 Since $10^9 = 10^3 \cdot 10^6$ it would take $10^6 = 1,000,000$ days ≈ 2739.72 years to spend the billion dollars.
105. (a) $\frac{18^5}{9^5} = \left(\frac{18}{9}\right)^5 = 2^5 = 32$
 (b) $20^6 \cdot (0.5)^6 = (20 \cdot 0.5)^6 = 10^6 = 1,000,000$
107. (a) $\left(\frac{a}{b}\right)^{-n} = \frac{1}{(a/b)^n} = \frac{1}{a^n/b^n} = \frac{b^n}{a^n}$ (b) $\frac{a^{-n}}{b^{-m}} = \frac{1/a^n}{1/b^m} = \frac{1}{a^n} \cdot b^m = \frac{b^m}{a^n}$

1.3 ALGEBRAIC EXPRESSIONS

- The greatest common factor in the expression $18x^3 + 30x$ is $6x$, and the expression factors as $6x(3x^2 + 5)$.
- To factor the trinomial $x^2 + 8x + 12$ we look for two integers whose product is 12 and whose sum is 8. These integers are 6 and 2, so the trinomial factors as $(x + 6)(x + 2)$.
- The Special Product Formula for the “product of the sum and difference of terms” is $(A + B)(A - B) = A^2 - B^2$. So $(6 + x)(6 - x) = 6^2 - x^2 = 36 - x^2$.
- The Special Factoring Formula for a “perfect square” is $A^2 + 2AB + B^2 = (A + B)^2$. So $x^2 + 10x + 25 = (x + 5)^2$.
- Type: binomial. Terms: $5x^3$ and 6. Degree: 3.
- Type: monomial. Term: -8 . Degree: 0.
- Type: four-term polynomial. Terms: x , $-x^2$, x^3 , and $-x^4$. Degree: 4.
- $(12x - 7) - (5x - 12) = 12x - 7 - 5x + 12 = 7x + 5$
- $(-2x^2 - 3x + 1) + (3x^2 + 5x - 4) = -2x^2 - 3x + 1 + 3x^2 + 5x - 4 = x^2 + 2x - 3$
- $(5x^3 + 4x^2 - 3x) - (x^2 + 7x + 2) = 5x^3 + 4x^2 - 3x - x^2 - 7x - 2 = 5x^3 + 3x^2 - 10x - 2$
- $8(2x + 5) - 7(x - 9) = 16x + 40 - 7x + 63 = 9x + 103$
- $x^3(x^2 + 3x) - 2x(x^4 - 3x^2) = x^5 + 3x^4 - 2x^5 + 6x^3 = -x^5 + 3x^4 + 6x^3$
- $4x(x - 2) - 2(x^2 - 4x) + 2x^2(x - 1) = 4x^2 - 8x - 2x^2 + 8x + 2x^3 - 2x^2 = 2x^3$
- $(3t - 2)(7t - 4) = 21t^2 - 12t - 14t + 8 = 21t^2 - 26t + 8$ 29. $(3x + 5)(2x - 1) = 6x^2 + 10x - 3x - 5 = 6x^2 + 7x - 5$
- $(x + 3y)(2x - y) = 2x^2 + 5xy - 3y^2$ 33. $(4x + 3)^2 = 16x^2 + 24x + 9$
- $(y - 3x)^2 = y^2 - 6xy + 9x^2$ 37. $(2x + 3y)^2 = 4x^2 + 12xy + 9y^2$
- $(w + 7)(w - 7) = w^2 - 49$ 41. $(3x - 4)(3x + 4) = (3x)^2 - 4^2 = 9x^2 - 16$

43. $(\sqrt{x} + 2)(\sqrt{x} - 2) = x - 4$

45. $(y + 2)^3 = y^3 + 3y^2(2) + 3y(2^2) + 2^3 = y^3 + 6y^2 + 12y + 8$

47. $(x + 2)(x^2 + 2x + 3) = x^3 + 2x^2 + 3x + 2x^2 + 4x + 6 = x^3 + 4x^2 + 7x + 6$

49. $(2x - 5)(x^2 - x + 1) = 2x^3 - 2x^2 + 2x - 5x^2 + 5x - 5 = 2x^3 - 7x^2 + 7x - 5$

51. $\sqrt{x}(x - \sqrt{x}) = x\sqrt{x} - (\sqrt{x})^2 = x\sqrt{x} - x$

53. $y^{1/3}(y^{2/3} + y^{5/3}) = y^{1/3+2/3} + y^{1/3+5/3} = y^2 + y$

55. $(x^{1/2} - y^{1/2})^2 = x - 2\sqrt{xy} + y$

57. $(x^2 - a^2)(x^2 + a^2) = x^4 - a^4$

59. $(\sqrt{a} - b)(\sqrt{a} + b) = a - b^2$

61. $((x - 1) + x^2)((x - 1) - x^2) = (x - 1)^2 - (x^2)^2 = x^2 - 2x + 1 - x^4 = -x^4 + x^2 - 2x + 1$

63. $(2x + y - 3)(2x + y + 3) = (2x + y)^2 - 3^2 = 4x^2 + 4xy + y^2 - 9$

65. $4x^4 - x^2 = x^2(4x^2 - 1) = x^2(2x + 1)(2x - 1)$

67. $y(y - 6) + 9(y - 6) = (y - 6)(y + 9)$

69. $4x^3y^2 - 6xy^3 + 8x^2y^4 = 2xy^2(2x^2 - 3y + 4xy^2)$

71. $(x + 3)^5(x + 2)^2 - (x + 3)^4(x + 2)^3 = (x + 3)^4(x + 2)^2[(x + 3) - (x + 2)] = (x + 3)^4(x + 2)^2$

73. $z^2 - 11z + 18 = (z - 9)(z - 2)$

75. $10x^2 - 19x + 6 = (5x - 2)(2x - 3)$

77. $3x^2 - 16x + 5 = (3x - 1)(x - 5)$

79. $(3x + 2)^2 + 8(3x + 2) + 12 = [(3x + 2) + 2][(3x + 2) + 6] = (3x + 4)(3x + 8)$

81. $36a^2 - 49 = (6a + 7)(6a - 7)$

83. $27x^3 + y^3 = (3x)^3 + y^3 = (3x + y)[(3x)^2 + 3xy + y^2] = (3x + y)(9x^2 - 3xy + y^2)$

85. $8s^3 - 125t^3 = (2s)^3 - (5t)^3 = (2s - 5t)[(2s)^2 + (2s)(5t) + (5t)^2] = (2s - 5t)(4s^2 + 10st + 25t^2)$

87. $x^2 + 12x + 36 = x^2 + 2(6x) + 6^2 = (x + 6)^2$

89. $x^3 + 4x^2 + x + 4 = x^2(x + 4) + 1(x + 4) = (x + 4)(x^2 + 1)$

91. $5x^3 + x^2 + 5x + 1 = x^2(5x + 1) + (5x + 1) = (x^2 + 1)(5x + 1)$

93. $x^3 + x^2 + x + 1 = x^2(x + 1) + 1(x + 1) = (x + 1)(x^2 + 1)$

95. Start by factoring out the power of x with the smallest exponent, that is, $x^{2/3}$.

Thus, $x^{2/3} + 3x^{5/3} = x^{2/3}(1 + 3x)$

97. Start by factoring out the power of x with the smallest exponent, that is, $x^{-3/2}$.

Thus, $x^{-3/2} - x^{-1/2} + x^{1/2} = x^{-3/2}(1 - x + x^2) = x^{-3/2}(x^2 - x + 1)$

99. Start by factoring out the power of $x^2 + 1$ with the smallest exponent, that is, $(x^2 + 1)^{-1/2}$.

Thus, $(x^2 + 1)^{1/2} + 2(x^2 + 1)^{-1/2} = (x^2 + 1)^{-1/2}[(x^2 + 1) + 2] = (x^2 + 1)^{-1/2}(x^2 + 3) = \frac{x^2 + 3}{\sqrt{x^2 + 1}}$

101. $2x + 12x^3 = 2x(1 + 6x^2)$

103. $x^2 - 2x - 8 = (x - 4)(x + 2)$

105. $2x^2 + 5x + 3 = (2x + 3)(x + 1)$

107. $9x^2 - 36x - 45 = 9(x^2 - 4x - 5) = 9(x - 5)(x + 1)$

$$109. 49 - 4y^2 = (7 - 2y)(7 + 2y)$$

$$111. t^2 - 6t + 9 = (t - 3)^2$$

$$113. y^2 - 10yz + 25z^2 = (y - 5z)^2$$

$$115. (a + b)^2 - (a - b)^2 = [(a + b) - (a - b)][(a + b) + (a - b)] = (2b)(2a) = 4ab$$

$$117. x^2(x^2 - 1) - 9(x^2 - 1) = (x^2 - 1)(x^2 - 9) = (x - 1)(x + 1)(x - 3)(x + 3)$$

$$119. 8x^3 - 125 = (2x)^3 - 5^3 = (2x - 5)[(2x)^2 + (2x)(5) + 5^2] = (2x - 5)(4x^2 + 10x + 25)$$

$$121. x^3 + 2x^2 + x = x(x^2 + 2x + 1) = x(x + 1)^2$$

$$123. x^4y^3 - x^2y^5 = x^2y^3(x^2 - y^2) = x^2y^3(x + y)(x - y)$$

$$125. 3x^3 - x^2 - 12x + 4 = 3x(x^2 - 4) - (x^2 - 4) = (3x - 1)(x^2 - 4) = (3x - 1)(x + 2)(x - 2)$$

127. Start by factoring out the power of x with the smallest exponent, that is, $x^{-3/2}$. So

$$x^{-3/2} + 2x^{-1/2} + x^{1/2} = x^{-3/2}(1 + 2x + x^2) = x^{-3/2}(1 + x)^2 = \frac{(1 + x)^2}{x^{3/2}}.$$

$$129. (x - 1)(x + 2)^2 - (x - 1)^2(x + 2) = (x - 1)(x + 2)[(x + 2) - (x - 1)] = 3(x - 1)(x + 2)$$

131. Start by factoring $y^2 - 7y + 10$, and then substitute $a^2 + 1$ for y . This gives

$$(a^2 + 1)^2 - 7(a^2 + 1) + 10 = [(a^2 + 1) - 2][(a^2 + 1) - 5] = (a^2 - 1)(a^2 - 4) = (a - 1)(a + 1)(a - 2)(a + 2)$$

$$\begin{aligned} 133. (x^2 + 3)^2(x - 1)^3 - (4x + 1)^2(x - 1)^3 &= (x - 1)^3[(x^2 + 3)^2 - (4x + 1)^2] \\ &= (x - 1)^3[(x^2 + 3) + (4x + 1)][(x^2 + 3) - (4x + 1)] = (x - 1)^3(x^2 + 4x + 4)(x^2 - 4x + 2) \\ &= (x - 1)^3(x + 2)^2(x^2 - 4x + 2) \end{aligned}$$

$$\begin{aligned} 135. 5(x^2 + 4)^4(2x)(x - 2)^4 + (x^2 + 4)^5(4)(x - 2)^3 &= 2(x^2 + 4)^4(x - 2)^3[(5)(x)(x - 2) + (x^2 + 4)(2)] \\ &= 2(x^2 + 4)^4(x - 2)^3(5x^2 - 10x + 2x^2 + 8) = 2(x^2 + 4)^4(x - 2)^3(7x^2 - 10x + 8) \end{aligned}$$

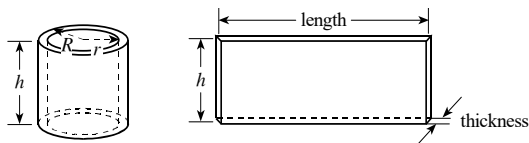
$$137. (x^2 + 3)^{-1/3} - \frac{2}{3}x^2(x^2 + 3)^{-4/3} = (x^2 + 3)^{-4/3}[(x^2 + 3) - \frac{2}{3}x^2] = (x^2 + 3)^{-4/3}(\frac{1}{3}x^2 + 3) = \frac{\frac{1}{3}x^2 + 3}{(x^2 + 3)^{4/3}}$$

$$139. (a) \frac{1}{2}[(a + b)^2 - (a^2 + b^2)] = \frac{1}{2}[a^2 + 2ab + b^2 - a^2 - b^2] = \frac{1}{2}(2ab) = ab.$$

$$\begin{aligned} (b) (a^2 + b^2)^2 - (a^2 - b^2)^2 &= [(a^2 + b^2) - (a^2 - b^2)][(a^2 + b^2) + (a^2 - b^2)] \\ &= (a^2 + b^2 - a^2 + b^2)(a^2 + b^2 + a^2 - b^2) = (2b^2)(2a^2) = 4a^2b^2 \end{aligned}$$

$$\begin{aligned} 141. 4a^2c^2 - (c^2 - b^2 + a^2)^2 &= (2ac)^2 - (c^2 - b^2 + a^2)^2 \\ &= [(2ac) - (c^2 - b^2 + a^2)][(2ac) + (c^2 - b^2 + a^2)] \text{ (difference of squares)} \\ &= (2ac - c^2 + b^2 - a^2)(2ac + c^2 - b^2 + a^2) \\ &= [b^2 - (c^2 - 2ac + a^2)][(c^2 + 2ac + a^2) - b^2] \text{ (regrouping)} \\ &= [b^2 - (c - a)^2][(c + a)^2 - b^2] \text{ (perfect squares)} \\ &= [b - (c - a)][b + (c - a)][(c + a) - b][(c + a) + b] \text{ (each factor is a difference of squares)} \\ &= (b - c + a)(b + c - a)(c + a - b)(c + a + b) \\ &= (a + b - c)(-a + b + c)(a - b + c)(a + b + c) \end{aligned}$$

143. The volume of the shell is the difference between the volumes of the outside cylinder (with radius R) and the inside cylinder (with radius r). Thus $V = \pi R^2 h - \pi r^2 h = \pi (R^2 - r^2) h = \pi (R - r)(R + r) h = 2\pi \cdot \frac{R + r}{2} \cdot h \cdot (R - r)$. The average radius is $\frac{R + r}{2}$ and $2\pi \cdot \frac{R + r}{2}$ is the average circumference (length of the rectangular box), h is the height, and $R - r$ is the thickness of the rectangular box. Thus $V = \pi R^2 h - \pi r^2 h = 2\pi \cdot \frac{R + r}{2} \cdot h \cdot (R - r) = 2\pi \cdot (\text{average radius}) \cdot (\text{height}) \cdot (\text{thickness})$



145. (a) The degree of the product is the sum of the degrees.
 (b) The degree of a sum is at most the largest of the degrees — it could be smaller than either. For example, the degree of $(x^3) + (-x^3 + x) = x$ is 1.

147. (a)

$$\begin{array}{r} A + 1 \\ \times \quad A - 1 \\ \hline -A - 1 \\ A^2 + A \\ \hline A^2 \quad - 1 \end{array}$$

$$\begin{array}{r} A^2 + A + 1 \\ \times \quad A - 1 \\ \hline -A^2 - A - 1 \\ A^3 + A^2 + A \\ \hline A^3 \quad - 1 \end{array}$$

$$\begin{array}{r} A^3 + A^2 + A + 1 \\ \times \quad A - 1 \\ \hline -A^3 - A^2 - A - 1 \\ A^4 + A^3 + A^2 + A \\ \hline A^4 \quad - 1 \end{array}$$

- (b) Based on the pattern in part (a), we suspect that $A^5 - 1 = (A - 1)(A^4 + A^3 + A^2 + A + 1)$. Check:

$$\begin{array}{r} A^4 + A^3 + A^2 + A + 1 \\ \times \quad A - 1 \\ \hline -A^4 - A^3 - A^2 - A - 1 \\ A^5 + A^4 + A^3 + A^2 + A \\ \hline A^5 \quad - 1 \end{array}$$

The general pattern is $A^n - 1 = (A - 1)(A^{n-1} + A^{n-2} + \cdots + A^2 + A + 1)$, where n is a positive integer.

1.4 RATIONAL EXPRESSIONS

1. A rational expression has the form $\frac{P(x)}{Q(x)}$, where P and Q are polynomials.

(a) $\frac{3x}{x^2 - 1}$ is a rational expression.

(b) $\frac{\sqrt{x+1}}{2x+3}$ is not a rational expression. A rational expression must be a polynomial divided by a polynomial, and the numerator of the expression is $\sqrt{x+1}$, which is not a polynomial.

(c) $\frac{x(x^2 - 1)}{x + 3} = \frac{x^3 - x}{x + 3}$ is a rational expression.

3. To multiply two rational expressions we multiply their *numerators* together and multiply their *denominators* together. So

$$\frac{2}{x+1} \cdot \frac{x}{x+3} \text{ is the same as } \frac{2 \cdot x}{(x+1) \cdot (x+3)} = \frac{2x}{x^2 + 4x + 3}.$$

5. (a) Yes. Cancelling $x + 1$, we have $\frac{x(x+1)}{(x+1)^2} = \frac{x}{x+1}$.
- (b) No; $(x+5)^2 = x^2 + 10x + 25 \neq x^2 + 25$, so $x+5 = \sqrt{x^2 + 10x + 25} \neq \sqrt{x^2 + 25}$.
7. The domain of $4x^2 - 10x + 3$ is all real numbers.
9. Since $x - 3 \neq 0$ we have $x \neq 3$. Domain: $\{x \mid x \neq 3\}$
11. Since $x + 3 \geq 0$, $x \geq -3$. Domain: $\{x \mid x \geq -3\}$
13. $x^2 - x - 2 = (x+1)(x-2) \neq 0 \Leftrightarrow x \neq -1$ or 2 , so the domain is $\{x \mid x \neq -1, 2\}$.
15. $\frac{\sqrt{x-2}}{x+3}$ is defined for $\{x \mid x \geq 2, x \neq -3\}$, so its domain is $\{x \mid x \geq 2\}$.
17. $\frac{(x-5)(x+5)}{2x-10} = \frac{(x-5)(x+5)}{2(x-5)} = \frac{1}{2}(x+5)$
19. $\frac{x-2}{x^2-4} = \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2}$
21. $\frac{x^2-7x-8}{x^2-10x+16} = \frac{(x-8)(x+1)}{(x-8)(x-2)} = \frac{x+1}{x-2}$
23. $\frac{y^2+y}{y^2-1} = \frac{y(y+1)}{(y-1)(y+1)} = \frac{y}{y-1}$
25. $\frac{2x^3-x^2-6x}{2x^2-7x+6} = \frac{x(2x^2-x-6)}{(2x-3)(x-2)} = \frac{x(2x+3)(x-2)}{(2x-3)(x-2)} = \frac{x(2x+3)}{2x-3}$
27. $\frac{4x}{x^2-4} \cdot \frac{x+2}{16x} = \frac{4x}{(x-2)(x+2)} \cdot \frac{x+2}{16x} = \frac{1}{4(x-2)}$
29. $\frac{x^2+2x-15}{x^2-25} \cdot \frac{x-5}{x+2} = \frac{(x+5)(x-3)(x-5)}{(x+5)(x-5)(x+2)} = \frac{x-3}{x+2}$
31. $\frac{2t+3}{t^2+9} \cdot \frac{2t-3}{4t^2-9} = \frac{(2t+3)(2t-3)}{(t^2+9)(2t+3)(2t-3)} = \frac{1}{t^2+9}$
33. $\frac{x^3-2x^2-8x}{x^2+8x+12} \cdot \frac{x^2+2x-24}{x^3-16x} = \frac{x(x+2)(x-4)(x-4)(x+6)}{(x+6)(x+2)x(x+4)(x-4)} = \frac{x-4}{x+4}$
35. $\frac{x+3}{4x^2-9} \div \frac{x^2+7x+12}{2x^2+7x-15} = \frac{x+3}{4x^2-9} \cdot \frac{2x^2+7x-15}{x^2+7x+12} = \frac{x+3}{(2x-3)(2x+3)} \cdot \frac{(x+5)(2x-3)}{(x+3)(x+4)} = \frac{x+5}{(2x+3)(x+4)}$
37. $\frac{\frac{x^3}{x+1}}{\frac{x^2+2x+1}{x}} = \frac{x^3}{x+1} \cdot \frac{x}{x^2+2x+1} = \frac{x^3(x+1)(x+1)}{(x+1)x} = x^2(x+1)$
39. $\frac{x/y}{z} = \frac{x}{y} \cdot \frac{1}{z} = \frac{x}{yz}$
41. $1 + \frac{1}{x+3} = \frac{x+3}{x+3} + \frac{1}{x+3} = \frac{x+4}{x+3}$
43. $\frac{1}{x+5} + \frac{2}{x-3} = \frac{x-3}{(x+5)(x-3)} + \frac{2(x+5)}{(x+5)(x-3)} = \frac{x-3+2x+10}{(x+5)(x-3)} = \frac{3x+7}{(x+5)(x-3)}$
45. $\frac{3}{x+1} - \frac{1}{x+2} = \frac{3(x+2)}{(x+1)(x+2)} - \frac{1(x+1)}{(x+1)(x+2)} = \frac{3x+6-x-1}{(x+1)(x+2)} = \frac{2x+5}{(x+1)(x+2)}$
47. $\frac{5}{2x-3} - \frac{3}{(2x-3)^2} = \frac{5(2x-3)}{(2x-3)^2} - \frac{3}{(2x-3)^2} = \frac{10x-15-3}{(2x-3)^2} = \frac{10x-18}{(2x-3)^2} = \frac{2(5x-9)}{(2x-3)^2}$
49. $u+1 + \frac{u}{u+1} = \frac{(u+1)(u+1)}{u+1} + \frac{u}{u+1} = \frac{u^2+2u+1+u}{u+1} = \frac{u^2+3u+1}{u+1}$
51. $\frac{1}{x^2} + \frac{1}{x^2+x} = \frac{1}{x^2} + \frac{1}{x(x+1)} = \frac{x+1}{x^2(x+1)} + \frac{x}{x^2(x+1)} = \frac{2x+1}{x^2(x+1)}$

$$53. \frac{2}{x+3} - \frac{1}{x^2+7x+12} = \frac{2}{x+3} - \frac{1}{(x+3)(x+4)} = \frac{2(x+4)}{(x+3)(x+4)} + \frac{-1}{(x+3)(x+4)}$$

$$= \frac{2x+8-1}{(x+3)(x+4)} = \frac{2x+7}{(x+3)(x+4)}$$

$$55. \frac{1}{x+3} + \frac{1}{x^2-9} = \frac{1}{x+3} + \frac{1}{(x-3)(x+3)} = \frac{x-3}{(x-3)(x+3)} + \frac{1}{(x-3)(x+3)} = \frac{x-2}{(x-3)(x+3)}$$

$$57. \frac{2}{x} + \frac{3}{x-1} - \frac{4}{x^2-x} = \frac{2}{x} + \frac{3}{x-1} - \frac{4}{x(x-1)} = \frac{2(x-1)}{x(x-1)} + \frac{3x}{x(x-1)} + \frac{-4}{x(x-1)} = \frac{2x-2+3x-4}{x(x-1)} = \frac{5x-6}{x(x-1)}$$

$$59. \frac{1}{x^2(x+1)} + \frac{1}{x^2(x+1)^2} + \frac{1}{x^3(x+1)^2} = \frac{x(x+1)}{x^3(x+1)^2} + \frac{x}{x^3(x+1)^2} + \frac{1}{x^3(x+1)^2} = \frac{x^2+2x+1}{x^3(x+1)^2}$$

$$= \frac{(x+1)^2}{x^3(x+1)^2} = \frac{1}{x^3}$$

$$61. \frac{1+\frac{1}{x}}{\frac{1}{x}-2} = \frac{x\left(1+\frac{1}{x}\right)}{x\left(\frac{1}{x}-2\right)} = \frac{x+1}{1-2x}$$

$$63. \frac{1+\frac{1}{x+2}}{1-\frac{1}{x+2}} = \frac{(x+2)\left(1+\frac{1}{x+2}\right)}{(x+2)\left(1-\frac{1}{x+2}\right)} = \frac{(x+2)+1}{(x+2)-1} = \frac{x+3}{x+1}$$

$$65. \frac{\frac{1}{x-1} + \frac{1}{x+3}}{x+1} = \frac{(x-1)(x+3)\left(\frac{1}{x-1} + \frac{1}{x+3}\right)}{(x-1)(x+3)(x+1)} = \frac{(x+3)+(x-1)}{(x-1)(x+1)(x+3)} = \frac{2(x+1)}{(x-1)(x+1)(x+3)}$$

$$= \frac{2}{(x-1)(x+3)}$$

$$67. \frac{x-\frac{x}{y}}{y-\frac{y}{x}} = \frac{xy\left(x-\frac{x}{y}\right)}{xy\left(y-\frac{y}{x}\right)} = \frac{x^2y-x^2}{xy^2-y^2} = \frac{x^2(y-1)}{y^2(x-1)}$$

$$69. \frac{\frac{x}{y} - \frac{y}{x}}{\frac{1}{x^2} - \frac{1}{y^2}} = \frac{\frac{x^2-y^2}{xy}}{\frac{y^2-x^2}{x^2y^2}} = \frac{x^2-y^2}{xy} \cdot \frac{x^2y^2}{y^2-x^2} = \frac{xy}{-1} = -xy. \text{ An alternative method is to multiply the}$$

numerator and denominator by the common denominator of both the numerator and denominator, in this case x^2y^2 :

$$\frac{\frac{x}{y} - \frac{y}{x}}{\frac{1}{x^2} - \frac{1}{y^2}} = \frac{\left(\frac{x}{y} - \frac{y}{x}\right)}{\left(\frac{1}{x^2} - \frac{1}{y^2}\right)} \cdot \frac{x^2y^2}{x^2y^2} = \frac{x^3y - xy^3}{y^2 - x^2} = \frac{xy(x^2 - y^2)}{y^2 - x^2} = -xy.$$

$$71. \frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}} = \frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x} + \frac{1}{y}} = \frac{\frac{y^2}{x^2y^2} - \frac{x^2}{x^2y^2}}{\frac{y}{xy} + \frac{x}{xy}} = \frac{y^2 - x^2}{x^2y^2} \cdot \frac{xy}{y+x} = \frac{(y-x)(y+x)xy}{x^2y^2(y+x)} = \frac{y-x}{xy}$$

$$\text{Alternatively, } \frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}} = \frac{\left(\frac{1}{x^2} - \frac{1}{y^2}\right)}{\left(\frac{1}{x} + \frac{1}{y}\right)} \cdot \frac{x^2y^2}{x^2y^2} = \frac{y^2 - x^2}{xy^2 + x^2y} = \frac{(y-x)(y+x)}{xy(y+x)} = \frac{y-x}{xy}.$$

$$73. 1 - \frac{1}{1 - \frac{1}{x}} = 1 - \frac{x}{x-1} = \frac{x-1-x}{x-1} = \frac{1}{1-x}$$

$$75. \frac{\frac{1}{1+x+h} - \frac{1}{1+x}}{h} = \frac{(1+x) - (1+x+h)}{h(1+x)(1+x+h)} = -\frac{1}{(1+x)(1+x+h)}$$

$$77. \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \frac{x^2 - (x+h)^2}{hx^2(x+h)^2} = \frac{x^2 - (x^2 + 2xh + h^2)}{hx^2(x+h)^2} = -\frac{2x+h}{x^2(x+h)^2}$$

$$79. \sqrt{1 + \left(\frac{x}{\sqrt{1-x^2}}\right)^2} = \sqrt{1 + \frac{x^2}{1-x^2}} = \sqrt{\frac{1-x^2}{1-x^2} + \frac{x^2}{1-x^2}} = \sqrt{\frac{1}{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

$$81. \frac{2(x-3)(x+5)^3 - 3(x+5)^2(x-3)^2}{(x+5)^6} = \frac{(x-3)(x+5)^2[2(x+5) - 3(x-3)]}{(x+5)^6} = \frac{(x-3)(19-x)}{(x+5)^4}$$

$$83. \frac{2(1+x)^{1/2} - x(1+x)^{-1/2}}{1+x} = \frac{(1+x)^{-1/2}[2(1+x) - x]}{1+x} = \frac{x+2}{(1+x)^{3/2}}$$

$$85. \frac{3(1+x)^{1/3} - x(1+x)^{-2/3}}{(1+x)^{2/3}} = \frac{(1+x)^{-2/3}[3(1+x) - x]}{(1+x)^{2/3}} = \frac{2x+3}{(1+x)^{4/3}}$$

$$87. \frac{1}{3+\sqrt{10}} = \frac{3-\sqrt{10}}{(3+\sqrt{10})(3-\sqrt{10})} = \frac{3-\sqrt{10}}{9-10} = \sqrt{10}-3$$

$$89. \frac{2}{\sqrt{5}-\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3} = \sqrt{5}+\sqrt{3}$$

$$91. \frac{y}{\sqrt{3}+\sqrt{y}} = \frac{y}{\sqrt{3}+\sqrt{y}} \cdot \frac{\sqrt{3}-\sqrt{y}}{\sqrt{3}-\sqrt{y}} = \frac{y(\sqrt{3}-\sqrt{y})}{3-y} = \frac{y\sqrt{3}-y\sqrt{y}}{3-y}$$

$$93. \frac{2-\sqrt{5}}{5} = \frac{(2-\sqrt{5})(2+\sqrt{5})}{5(2+\sqrt{5})} = \frac{4-5}{5(2+\sqrt{5})} = \frac{-1}{5(2+\sqrt{5})}$$

$$95. \frac{\sqrt{r}+\sqrt{2}}{5} = \frac{\sqrt{r}+\sqrt{2}}{5} \cdot \frac{\sqrt{r}-\sqrt{2}}{\sqrt{r}-\sqrt{2}} = \frac{r-2}{5(\sqrt{r}-\sqrt{2})}$$

$$97. \sqrt{x^2+1} - x = \frac{\sqrt{x^2+1}-x}{1} \cdot \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x} = \frac{x^2+1-x^2}{\sqrt{x^2+1}+x} = \frac{1}{\sqrt{x^2+1}+x}$$

$$99. (a) R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \cdot \frac{R_1 R_2}{R_1 R_2} = \frac{R_1 R_2}{R_2 + R_1}$$

$$(b) \text{ Substituting } R_1 = 10 \text{ ohms and } R_2 = 20 \text{ ohms gives } R = \frac{(10)(20)}{(20)+(10)} = \frac{200}{30} \approx 6.7 \text{ ohms.}$$

101.

x	2.80	2.90	2.95	2.99	2.999	3	3.001	3.01	3.05	3.10	3.20
$\frac{x^2-9}{x-3}$	5.80	5.90	5.95	5.99	5.999	?	6.001	6.01	6.05	6.10	6.20

From the table, we see that the expression $\frac{x^2-9}{x-3}$ approaches 6 as x approaches 3. We simplify the expression:

$\frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3} = x+3, x \neq 3$. Clearly as x approaches 3, $x+3$ approaches 6. This explains the result in the table.

103. Answers will vary.

Algebraic Error	Counterexample
$\frac{1}{a} + \frac{1}{b} \neq \frac{1}{a+b}$	$\frac{1}{2} + \frac{1}{2} \neq \frac{1}{2+2}$
$(a+b)^2 \neq a^2 + b^2$	$(1+3)^2 \neq 1^2 + 3^2$
$\sqrt{a^2 + b^2} \neq a + b$	$\sqrt{5^2 + 12^2} \neq 5 + 12$
$\frac{a+b}{a} \neq b$	$\frac{2+6}{2} \neq 6$
$\frac{a}{a+b} \neq \frac{1}{b}$	$\frac{1}{1+1} \neq 1$
$\frac{a^m}{a^n} \neq a^{m/n}$	$\frac{3^5}{3^2} \neq 3^{5/2}$

105. (a)

x	1	3	$\frac{1}{2}$	$\frac{9}{10}$	$\frac{99}{100}$	$\frac{999}{1000}$	$\frac{9999}{10,000}$
$x + \frac{1}{x}$	2	3.333	2.5	2.011	2.0001	2.000001	2.00000001

It appears that the smallest possible value of $x + \frac{1}{x}$ is 2.

- (b) Because $x > 0$, we can multiply both sides by x and preserve the inequality: $x + \frac{1}{x} \geq 2 \Leftrightarrow x \left(x + \frac{1}{x} \right) \geq 2x \Leftrightarrow x^2 + 1 \geq 2x \Leftrightarrow x^2 - 2x + 1 \geq 0 \Leftrightarrow (x-1)^2 \geq 0$. The last statement is true for all $x > 0$, and because each step is reversible, we have shown that $x + \frac{1}{x} \geq 2$ for all $x > 0$.

1.5 EQUATIONS

- (a) Yes. If $a = b$, then $a + x = b + x$, and vice versa.

(b) Yes. If $a = b$, then $ma = mb$ for $m \neq 0$, and vice versa.

(c) No. For example, $-5 \neq 5$, but $(-5)^2 = 5^2 = 25$.
- (a) To solve the equation $x^2 - 6x - 16 = 0$ by factoring, we write $x^2 - 6x - 16 = (x-8)(x+2) = 0$ if $x = 8$ or $x = -2$.

(b) To solve by completing the square, we write $x^2 - 6x - 16 = 0 \Leftrightarrow x^2 - 6x = 16 \Leftrightarrow x^2 - 6x + 9 = 16 + 9 \Leftrightarrow (x-3)^2 = 25 \Leftrightarrow x-3 = \pm 5 \Leftrightarrow x = 8$ or $x = -2$.

(c) To solve using the Quadratic Formula, we write $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-16)}}{2(1)} = \frac{6 \pm \sqrt{100}}{2} = 3 \pm 5 \Leftrightarrow x = 8$ or $x = -2$.
- (a) Isolating the radical in $\sqrt{2x} + x = 0$, we obtain $\sqrt{2x} = -x$.

(b) Now square both sides: $(\sqrt{2x})^2 = (-x)^2 \Rightarrow 2x = x^2$.

(c) Solving the resulting quadratic equation, we find $2x = x^2 \Rightarrow x^2 - 2x = x(x-2) = 0$, so the solutions are $x = 0$ and $x = 2$.

(d) We substitute these possible solutions into the original equation: $\sqrt{2 \cdot 0} + 0 = 0$, so $x = 0$ is a solution, but $\sqrt{2 \cdot 2} + 2 = 4 \neq 0$, so $x = 2$ is not a solution. The only real solution is $x = 0$.

7. To eliminate the denominators in the equation $\frac{3}{x} + \frac{5}{x+2} = 2$, multiply each side by the lowest common denominator $x(x+2)$ to get the equivalent equation $3(x+2) + 5x = 2x(x+2)$.
9. (a) When $x = -2$, LHS $= 4(-2) + 7 = -8 + 7 = -1$ and RHS $= 9(-2) - 3 = -18 - 3 = -21$. Since LHS $\neq$ RHS, $x = -2$ is not a solution.
- (b) When $x = 2$, LHS $= 4(-2) + 7 = 8 + 7 = 15$ and RHS $= 9(2) - 3 = 18 - 3 = 15$. Since LHS = RHS, $x = 2$ is a solution.
11. (a) When $x = 2$, LHS $= \frac{1}{2} - \frac{1}{2-4} = \frac{1}{2} - \frac{1}{-2} = \frac{1}{2} + \frac{1}{2} = 1$ and RHS $= 1$. Since LHS = RHS, $x = 2$ is a solution.
- (b) When $x = 4$ the expression $\frac{1}{4-4}$ is not defined, so $x = 4$ is not a solution.
13. $8x + 13 = 5 \Leftrightarrow 8x = 5 - 13 \Leftrightarrow 8x = -8 \Leftrightarrow x = -1$
15. $\frac{1}{2}x - 8 = 1 \Leftrightarrow \frac{1}{2}x = 9 \Leftrightarrow x = 18$
17. $-x + 3 = 4x \Leftrightarrow -x + 3 + x = 4x + x \Leftrightarrow 3 = 5x \Leftrightarrow x = \frac{3}{5}$
19. $\frac{x}{3} - 2 = \frac{5}{3}x + 7 \Leftrightarrow 3\left(\frac{x}{3} - 2\right) = 3\left(\frac{5}{3}x + 7\right) \Leftrightarrow x - 6 = 5x + 21 \Leftrightarrow -27 = 4x \Leftrightarrow x = -\frac{27}{4}$
21. $2(1-x) = 3(1+2x) + 5 \Leftrightarrow 2 - 2x = 3 + 6x + 5 \Leftrightarrow 2 - 2x = 8 + 6x \Leftrightarrow -6 = 8x \Leftrightarrow x = -\frac{3}{4}$
23. $x - \frac{1}{3}x - \frac{1}{2}x - 5 = 0 \Leftrightarrow 6x - 2x - 3x - 30 = 0$ (multiply both sides by the LCD, 6) $\Leftrightarrow x = 30$
25. $\frac{2x-1}{x+2} = \frac{4}{5} \Rightarrow 5(2x-1) = 4(x+2) \Leftrightarrow 10x-5 = 4x+8 \Leftrightarrow 6x = 13 \Leftrightarrow x = \frac{13}{6}$
27. $\frac{1}{x} = \frac{4}{3x} + 1 \Rightarrow 3 = 4 + 3x$ (multiply both sides by the LCD, $3x$) $\Leftrightarrow -1 = 3x \Leftrightarrow x = -\frac{1}{3}$
29. $E = \frac{1}{2}mv^2 \Rightarrow m = \frac{2E}{v^2}$
31. $P = 2l + 2w \Leftrightarrow 2w = P - 2l \Leftrightarrow w = \frac{P-2l}{2}$
33. $\frac{ax+b}{cx+d} = 2 \Leftrightarrow ax+b = 2(cx+d) \Leftrightarrow ax+b = 2cx+2d \Leftrightarrow ax-2cx = 2d-b \Leftrightarrow (a-2c)x = 2d-b \Leftrightarrow x = \frac{2d-b}{a-2c}$
35. $a^2x + (a-1) = (a+1)x \Leftrightarrow a^2x - (a+1)x = -(a-1) \Leftrightarrow (a^2 - (a+1))x = -a+1 \Leftrightarrow (a^2 - a - 1)x = -a+1$
 $\Leftrightarrow x = \frac{-a+1}{a^2-a-1}$
37. $V = \frac{1}{3}\pi r^2 h \Leftrightarrow r^2 = \frac{3V}{\pi h} \Rightarrow r = \pm\sqrt{\frac{3V}{\pi h}}$
39. $a^2 + b^2 = c^2 \Leftrightarrow b^2 = c^2 - a^2 \Rightarrow b = \pm\sqrt{c^2 - a^2}$
41. $x^2 + x = 12 \Leftrightarrow x^2 + x - 12 = 0 \Leftrightarrow (x-3)(x+4) = 0 \Leftrightarrow x-3 = 0$ or $x+4 = 0$. Thus, $x = 3$ or $x = -4$.
43. $x^2 + 13x - 30 = 0 \Leftrightarrow (x+15)(x-2) = 0 \Leftrightarrow x+15 = 0$ or $x-2 = 0$. Thus, $x = -15$ or $x = 2$.
45. $4x^2 - 4x - 15 = 0 \Leftrightarrow (2x+3)(2x-5) = 0 \Leftrightarrow 2x+3 = 0$ or $2x-5 = 0$. Thus, $x = -\frac{3}{2}$ or $x = \frac{5}{2}$.
47. $2x^2 = 8 \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2$.
49. $(2x-5)^2 = 81 \Leftrightarrow 2x-5 = \pm 9 \Leftrightarrow 2x = -4$ or $2x = 14 \Leftrightarrow x = -2$ or $x = 7$
51. $8(x+5)^2 = 16 \Leftrightarrow (x+5)^2 = 2 \Leftrightarrow x+5 = \pm\sqrt{2} \Leftrightarrow x = -5 \pm \sqrt{2}$
53. $x^2 + 10x + 2 = 0 \Leftrightarrow x^2 + 10x = -2 \Leftrightarrow x^2 + 10x + 25 = -2 + 25 \Leftrightarrow (x+5)^2 = 23 \Leftrightarrow x+5 = \pm\sqrt{23} \Leftrightarrow x = -5 \pm \sqrt{23}$
55. $x^2 - 6x - 11 = 0 \Leftrightarrow x^2 - 6x = 11 \Leftrightarrow x^2 - 6x + 9 = 11 + 9 \Leftrightarrow (x-3)^2 = 20 \Rightarrow x-3 = \pm 2\sqrt{5} \Leftrightarrow x = 3 \pm 2\sqrt{5}$
57. $5x^2 + 10x = 1 \Leftrightarrow 5(x^2 + 2x) = 1 \Leftrightarrow 5(x^2 + 2x + 1) = 1 + 5 \Leftrightarrow (x+1)^2 = \frac{6}{5} \Leftrightarrow x+1 = \pm\sqrt{\frac{6}{5}} \Leftrightarrow x = -1 \pm \frac{\sqrt{30}}{5}$
59. $4x^2 - x = 0 \Leftrightarrow x(4x-1) = 0 \Leftrightarrow x = 0$ or $x = \frac{1}{4}$.

61. $x^2 - 2x - 15 = 0 \Leftrightarrow (x + 3)(x - 5) = 0 \Leftrightarrow x + 3 = 0$ or $x - 5 = 0$. Thus, $x = -3$ or $x = 5$.
63. $2x^2 + x - 3 = 0 \Leftrightarrow (x - 1)(2x + 3) = 0 \Leftrightarrow x - 1 = 0$ or $2x + 3 = 0$. If $x - 1 = 0$, then $x = 1$; if $2x + 3 = 0$, then $x = -\frac{3}{2}$.
65. $3x^2 + 6x - 5 = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(3)(-5)}}{2(3)} = \frac{-6 \pm \sqrt{96}}{6} = \frac{-3 \pm 2\sqrt{6}}{3}$
67. $9x^2 + 12x + 4 = 0 \Leftrightarrow (3x + 2)^2 = 0 \Leftrightarrow x = -\frac{2}{3}$
69. $4x^2 + 16x = 9 \Leftrightarrow 4x^2 + 16x - 9 = 0 \Leftrightarrow (2x - 1)(2x + 9) = 0 \Leftrightarrow 2x - 1 = 0$ or $2x + 9 = 0$. If $2x - 1 = 0$, then $x = \frac{1}{2}$; if $2x + 9 = 0$, then $x = -\frac{9}{2}$.
71. $7x^2 = 2x - 4 \Leftrightarrow 7x^2 - 2x + 4 = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(7)(4)}}{2(7)} = \frac{2 \pm \sqrt{4 - 112}}{-4}$, so there is no real solution.
73. $D = b^2 - 4ac = (-6)^2 - 4(1)(1) = 32$. Since D is positive, this equation has two real solutions.
75. $D = b^2 - 4ac = (2.20)^2 - 4(1)(1.21) = 4.84 - 4.84 = 0$. Since $D = 0$, this equation has one real solution.
77. $D = b^2 - 4ac = (5)^2 - 4(4)\left(\frac{13}{8}\right) = 25 - 26 = -1$. Since D is negative, this equation has no real solution.
79. $\frac{x^2}{x + 100} = 50 \Rightarrow x^2 = 50(x + 100) = 50x + 5000 \Leftrightarrow x^2 - 50x - 5000 = 0 \Leftrightarrow (x - 100)(x + 50) = 0 \Leftrightarrow x - 100 = 0$ or $x + 50 = 0$. Thus $x = 100$ or $x = -50$. The solutions are 100 and -50.
81. $\frac{y + 1}{y^2 + 1} = \frac{2}{y + 2} \Leftrightarrow (y + 1)(y + 2) = (y^2 + 1)(2) \Leftrightarrow y^2 + 3y + 2 = 2y^2 + 2 \Leftrightarrow y^2 - 3y = 0 \Leftrightarrow y(y - 3) = 0 \Leftrightarrow y = 0$ or 3 .
83. $\frac{1}{x - 1} + \frac{1}{x + 2} = \frac{5}{4} \Leftrightarrow 4(x - 1)(x + 2)\left(\frac{1}{x - 1} + \frac{1}{x + 2}\right) = 4(x - 1)(x + 2)\left(\frac{5}{4}\right) \Leftrightarrow 4(x + 2) + 4(x - 1) = 5(x - 1)(x + 2) \Leftrightarrow 4x + 8 + 4x - 4 = 5x^2 + 5x - 10 \Leftrightarrow 5x^2 - 3x - 14 = 0 \Leftrightarrow (5x + 7)(x - 2) = 0$. If $5x + 7 = 0$, then $x = -\frac{7}{5}$; if $x - 2 = 0$, then $x = 2$. The solutions are $-\frac{7}{5}$ and 2.
85. $5 = \sqrt{4x - 3} \Rightarrow 5^2 = (\sqrt{4x - 3})^2 \Leftrightarrow 25 = 4x - 3 \Leftrightarrow 4x = 28 \Leftrightarrow x = 7$
87. $\sqrt{2x - 1} = \sqrt{3x - 5} \Leftrightarrow 2x - 1 = 3x - 5 \Leftrightarrow x = 4$
89. $\sqrt{2x + 1} + 1 = x \Leftrightarrow \sqrt{2x + 1} = x - 1 \Rightarrow 2x + 1 = (x - 1)^2 \Leftrightarrow 2x + 1 = x^2 - 2x + 1 \Leftrightarrow 0 = x^2 - 4x = x(x - 4)$. Potential solutions are $x = 0$ and $x - 4 \Leftrightarrow x = 4$. These are only potential solutions since squaring is not a reversible operation. We must check each potential solution in the original equation. Checking $x = 0$: $\sqrt{2(0) + 1} + 1 \stackrel{?}{=} (0) \Leftrightarrow \sqrt{1} + 1 \stackrel{?}{=} 0$ is false. Checking $x = 4$: $\sqrt{2(4) + 1} + 1 \stackrel{?}{=} (4)$, $\sqrt{9} + 1 \stackrel{?}{=} 4$, $3 + 1 \stackrel{?}{=} 4$ is true. Thus, the only solution is $x = 4$.
91. Let $w = x^2$. Then $x^4 - 13x^2 + 40 = (x^2)^2 - 13x^2 + 40 = 0$ becomes $w^2 - 13w + 40 = 0 \Leftrightarrow (w - 5)(w - 8) = 0$. So $w - 5 = 0 \Leftrightarrow w = 5$, and $w - 8 = 0 \Leftrightarrow w = 8$. When $w = 5$, we have $x^2 = 5 \Rightarrow x = \pm\sqrt{5}$. When $w = 8$, we have $x^2 = 8 \Rightarrow x = \pm\sqrt{8} = \pm 2\sqrt{2}$. The solutions are $\pm\sqrt{5}$ and $\pm 2\sqrt{2}$.
93. $(x + 2)^4 - 5(x + 2)^2 + 4 = 0 \Leftrightarrow [(x + 2)^2 - 4][(x + 2)^2 - 1] = 0$. If $(x + 2)^2 - 4 = 0$, then $(x + 2)^2 = 4 \Leftrightarrow x + 2 = \pm 2 \Leftrightarrow x = 0$ or -4 . If $(x + 2)^2 - 1 = 0$, then $(x + 2)^2 = 1 \Leftrightarrow x + 2 = \pm 1 \Leftrightarrow x = -1$ or -3 . Thus, the solutions are $x = -4, -3, -1$, and 0 .
95. Let $u = x^{2/3}$. Then $0 = x^{4/3} - 5x^{2/3} + 6$ becomes $u^2 - 5u + 6 = 0 \Leftrightarrow (u - 3)(u - 2) = 0 \Leftrightarrow u - 3 = 0$ or $u - 2 = 0$. If $u - 3 = 0$, then $x^{2/3} - 3 = 0 \Leftrightarrow x^{2/3} = 3 \Rightarrow x = \pm 3^{3/2}$. If $u - 2 = 0$, then $x^{2/3} - 2 = 0 \Leftrightarrow x^{2/3} = 2 \Rightarrow x = \pm 2^{3/2}$. The solutions are $\pm 3^{3/2}$ and $\pm 2^{3/2}$; that is, $x = \pm 2\sqrt{2}$ and $x = \pm 3\sqrt{3}$.

97. $x^{1/2} + 3x^{-1/2} = 10x^{-3/2} \Leftrightarrow 0 = x^{1/2} + 3x^{-1/2} - 10x^{-3/2} = x^{-3/2}(x^2 + 3x - 10) = x^{-3/2}(x - 2)(x + 5)$. Now $x^{-3/2}$ never equals 0, and no solution can be negative, because we cannot take the square root of a negative number. Thus 2 is the only solution.
99. $|3x + 5| = 1$. So $3x + 5 = 1 \Leftrightarrow 3x = -4 \Leftrightarrow x = -\frac{4}{3}$ or $3x + 5 = -1 \Leftrightarrow 3x = -6 \Leftrightarrow x = -2$.
101. $|x - 4| = 0.01$. So $x - 4 = 0.01 \Leftrightarrow x = 4.01$ or $x - 4 = -0.01 \Leftrightarrow x = 3.99$.
103. $2x + \frac{1}{2} = \frac{1}{4}x - 3 \Leftrightarrow 4\left(2x + \frac{1}{2}\right) = 4\left(\frac{1}{4}x - 3\right) \Leftrightarrow 8x + 2 = x - 12 \Leftrightarrow 7x = -14 \Leftrightarrow x = -2$
105. $\frac{4-a}{3a-2} = a \Leftrightarrow 4-a = a(3a-2) \Leftrightarrow 4-a = 3a^2-2a \Leftrightarrow 3a^2-a-4 = 0 \Leftrightarrow (a+1)(3a-4) = 0$. Thus, $a = -1$ or $a = \frac{4}{3}$.
107. $\frac{2}{3}y + \frac{1}{2}(y-3) = \frac{y+1}{4} \Leftrightarrow 8y + 6(y-3) = 3(y+1) \Leftrightarrow 8y + 6y - 18 = 3y + 3 \Leftrightarrow 14y - 18 = 3y + 3 \Leftrightarrow 11y = 21 \Leftrightarrow y = \frac{21}{11}$
109. $x - \sqrt{9-3x} = 0 \Leftrightarrow x = \sqrt{9-3x} \Rightarrow x^2 = 9-3x \Leftrightarrow x^2 + 3x - 9 = 0 \Leftrightarrow$
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4(1)(-9)}}{2(1)} = \frac{-3 \pm \sqrt{9+36}}{2} = \frac{-3 \pm 3\sqrt{5}}{2}$. However, $\frac{-3-3\sqrt{5}}{2}$ is not a solution because $\frac{-3-3\sqrt{5}}{2} - \sqrt{9-3\left(\frac{-3-3\sqrt{5}}{2}\right)}$ is the sum of two negative nonzero numbers and hence is nonzero.
 Thus, the only solution is $x = \frac{-3+3\sqrt{5}}{2}$.
111. Let $u = \sqrt[4]{x}$. Then $0 = \sqrt{x} - 3\sqrt[4]{x} - 4 = u^2 - 3u - 4 = (u-4)(u+1)$. So $u-4 = \sqrt[4]{x} - 4 = 0 \Leftrightarrow \sqrt[4]{x} = 4 \Rightarrow x = 4^4 = 256$, or $u+1 = \sqrt[4]{x} + 1 = 0 \Leftrightarrow \sqrt[4]{x} = -1$. However, $\sqrt[4]{x}$ is the positive fourth root, so this cannot equal -1 . The only solution is 256.
113. $|3x - 10| = 29 \Leftrightarrow 3x - 10 = 29$ or $3x - 10 = -29 \Leftrightarrow 3x = 39$ or $3x = -19 \Leftrightarrow x = 13$ or $x = -\frac{19}{3}$
115. $(z-1)^2 - 8(z-1) + 15 = 0 \Leftrightarrow [(z-1)-5][(z-1)-3] = 0 \Leftrightarrow (z-6)(z-4) = 0 \Leftrightarrow z = 4$ or 6
117. $|3x^2 - 1| - 4 = 7 \Leftrightarrow |3x^2 - 1| = 11 \Leftrightarrow 3x^2 - 1 = 11$ or $3x^2 - 1 = -10 \Leftrightarrow 3x^2 = 12 \Leftrightarrow x = \pm 2$ (since $3x^2 - 1 = -10$ has no real solution).
119. $\frac{1}{x^3} + \frac{4}{x^2} + \frac{4}{x} = 0 \Leftrightarrow 1 + 4x + 4x^2 = 0 \Leftrightarrow (1+2x)^2 = 0 \Leftrightarrow 1+2x = 0 \Leftrightarrow 2x = -1 \Leftrightarrow x = -\frac{1}{2}$. The solution is $-\frac{1}{2}$.
121. $\sqrt{x+5} + x = 5$. Squaring both sides, we get $\sqrt{x+5} + x = 25 \Leftrightarrow \sqrt{x+5} = 25-x$. Squaring both sides again, we get $x+5 = (25-x)^2 \Leftrightarrow x+5 = 625-50x+x^2 \Leftrightarrow 0 = x^2-51x+620 = (x-20)(x-31)$. Potential solutions are $x = 20$ and $x = 31$. We must check each potential solution in the original equation.
 Checking $x = 20$: $\sqrt{\sqrt{20+5}+20} = 5 \Leftrightarrow \sqrt{\sqrt{25}+20} = 5 \Leftrightarrow \sqrt{5+20} = 5$, which is true, and hence $x = 20$ is a solution.
 Checking $x = 31$: $\sqrt{\sqrt{31+5}+31} = 5 \Leftrightarrow \sqrt{\sqrt{36}+31} = 5 \Leftrightarrow \sqrt{37} = 5$, which is false, and hence $x = 31$ is not a solution. The only real solution is $x = 20$.
123. $x^2\sqrt{x+3} = (x+3)^{3/2} \Leftrightarrow 0 = x^2\sqrt{x+3} - (x+3)^{3/2} \Leftrightarrow 0 = \sqrt{x+3}[(x^2) - (x+3)] \Leftrightarrow 0 = \sqrt{x+3}(x^2-x-3)$.
 If $(x+3)^{1/2} = 0$, then $x+3 = 0 \Leftrightarrow x = -3$. If $x^2-x-3 = 0$, then by the Quadratic Formula, $x = \frac{1 \pm \sqrt{13}}{2}$. The solutions are -3 and $\frac{1 \pm \sqrt{13}}{2}$.
125. $0 = x^4 - 5ax^2 + 4a^2 = (a-x^2)(4a-x^2)$. Since a is positive, $a-x^2 = 0 \Leftrightarrow x^2 = a \Leftrightarrow x = \pm\sqrt{a}$. Again, since a is positive, $4a-x^2 = 0 \Leftrightarrow x^2 = 4a \Leftrightarrow x = \pm 2\sqrt{a}$. Thus the four solutions are $\pm\sqrt{a}$ and $\pm 2\sqrt{a}$.

127. $\sqrt{x+a} + \sqrt{x-a} = \sqrt{2}\sqrt{x+6}$. Squaring both sides, we have
 $x+a+2(\sqrt{x+a})(\sqrt{x-a})+x-a=2(x+6) \Leftrightarrow 2x+2(\sqrt{x+a})(\sqrt{x-a})=2x+12 \Leftrightarrow 2(\sqrt{x+a})(\sqrt{x-a})=12$
 $\Leftrightarrow (\sqrt{x+a})(\sqrt{x-a})=6$. Squaring both sides again we have $(x+a)(x-a)=36 \Leftrightarrow x^2-a^2=36 \Leftrightarrow x^2=a^2+36$
 $\Leftrightarrow x=\pm\sqrt{a^2+36}$. Checking these answers, we see that $x=-\sqrt{a^2+36}$ is not a solution (for example, try substituting
 $a=8$), but $x=\sqrt{a^2+36}$ is a solution.

129. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

Using $h_0 = 88$, we solve $0 = -4.8t^2 + 88$, for $t \geq 0$. So $0 = -4.8t^2 + 88 \Leftrightarrow 4.8t^2 = 88 \Leftrightarrow t^2 \approx 18.33 \Leftrightarrow t \approx \pm 4.28$.
 Thus it takes 4.28 seconds for the ball to hit the ground.

131. We are given that $v_0 = 12$ m/s.

- (a) Setting $h = 7.2$, we have $7.2 = -4.8t^2 + 12t \Leftrightarrow 4.8t^2 - 12t + 7.2 = 0 \Leftrightarrow 2.4(2t^2 - 5t + 3) = 0 \Leftrightarrow$
 $2.4(2t-3)(t-1) = 0 \Leftrightarrow t = 1$ or $t = 1\frac{1}{2}$. Therefore, the ball reaches 7.2 meters in 1 second (ascending) and again
 after $1\frac{1}{2}$ seconds (descending).
 (b) Setting $h = 14.4$, we have $14.4 = -4.8t^2 + 12t \Leftrightarrow 4.8t^2 - 12t + 14.4 = 0 \Leftrightarrow 2.4(2t^2 - 5t + 6) = 0 \Leftrightarrow$
 $t = \frac{5 \pm \sqrt{25-48}}{4} = \frac{5 \pm \sqrt{-23}}{4}$. However, since the discriminant $D < 0$, there is no real solution, and hence the ball
 never reaches a height of 14.4 meters.
 (c) The greatest height h is reached only once. So $h = -4.8t^2 + 12t \Leftrightarrow 4.8t^2 - 12t + h = 0$ has only one solution. Thus
 $D = (-12)^2 - 4(4.8)(h) = 0 \Leftrightarrow 144 - 19.2h = 0 \Leftrightarrow h = 7.5$. So the greatest height reached by the ball is 7.5 meters.
 (d) Setting $h = 7.5$, we have $7.5 = -4.8t^2 + 12t \Leftrightarrow 4.8t^2 - 12t + 7.5 = 0 \Leftrightarrow t = 1\frac{1}{4}$. Thus the ball reaches the highest
 point of its path after $1\frac{1}{4}$ seconds.
 (e) Setting $h = 0$ (ground level), we have $0 = -4.8t^2 + 12t \Leftrightarrow 2t^2 - 5t = 0 \Leftrightarrow t(2t-5) = 0 \Leftrightarrow t = 0$ (start) or $t = 2\frac{1}{2}$.
 So the ball hits the ground in $2\frac{1}{2}$ s.

133. (a) The shrinkage factor when $w = 250$ is $S = \frac{0.032(250) - 2.5}{10,000} = \frac{8 - 2.5}{10,000} = 0.00055$. So the beam shrinks
 $0.00055 \times 12.025 \approx 0.007$ m, so when it dries it will be $12.025 - 0.007 = 12.018$ m long.

- (b) Substituting $S = 0.00050$ we get $0.00050 = \frac{0.032w - 2.5}{10,000} \Leftrightarrow 5 = 0.032w - 2.5 \Leftrightarrow 7.5 = 0.032w \Leftrightarrow$
 $w = \frac{7.5}{0.032} \approx 234.375$. So the water content should be 234.375 kg/m³.

135. (a) $C = \frac{1}{2}(1000)(2^2) + 1000(9.81)(1) + 200,000 = 211,810$

- (b) $\frac{1}{2}\rho v^2 + \rho gh + P = C \Leftrightarrow P = C - \frac{1}{2}\rho v^2 - \rho gh$.

If the height increases to 5 m, then $P = 211,810 - \frac{1}{2}(1000)(2^2) - 1000(9.81)(5) = 160,760$ Pa.

If $h = 1$ m and $v = 4$ m/s, then $P = 211,810 - \frac{1}{2}(1000)(4^2) - 1000(9.81)(1) = 194,000$ Pa.

137. Note: In the first printing of the text, the answer given for this exercise is incorrect.

Let x be the distance from the center of the earth to the dead spot (in thousands of kilometers). Now setting $F = 0$, we have

$$0 = -\frac{K}{x^2} + \frac{0.012K}{(382-x)^2} \Leftrightarrow \frac{K}{x^2} = \frac{0.012K}{(382-x)^2} \Leftrightarrow K(382-x)^2 = 0.012Kx^2 \Leftrightarrow 145,924 - 764x + x^2 = 0.012x^2 \Leftrightarrow$$

$0.988x^2 - 764x + 145,924 = 0$. Using the Quadratic Formula, we obtain

$$x = \frac{-(-764) \pm \sqrt{(-764)^2 - 4(0.988)(145,924)}}{2(0.988)} = \frac{764 \pm \sqrt{583,696 - 576,691.648}}{1.976}$$

$$= \frac{764 \pm \sqrt{7004.352}}{1.976} \approx \frac{764 \pm 83.692}{1.976} \approx 386.64 \pm 42.35$$

So either $x \approx 386.64 + 42.35 \approx 429$ or $x \approx 386.64 - 42.35 \approx 344$. Since 429,000 is greater than the distance from the earth to the moon, we reject the first root; thus $x \approx 344,000$ kilometers.

139. (a) $3(0) + k - 5 = k(0) - k + 1 \Leftrightarrow k - 5 = -k + 1 \Leftrightarrow 2k = 6 \Leftrightarrow k = 3$

(b) $3(1) + k - 5 = k(1) - k + 1 \Leftrightarrow 3 + k - 5 = k - k + 1 \Leftrightarrow k - 2 = 1 \Leftrightarrow k = 3$

(c) $3(2) + k - 5 = k(2) - k + 1 \Leftrightarrow 6 + k - 5 = 2k - k + 1 \Leftrightarrow k + 1 = k + 1$. Since both sides of this equation are equal, $x = 2$ is a solution for every value of k . That is, $x = 2$ is a solution to every member of this family of equations.

141. (a) $x^2 - 9x + 20 = 0 \Leftrightarrow (x - 4)(x - 5) = 0 \Leftrightarrow x = 4$ or $x = 5$. The product of the solutions is $4 \cdot 5 = 20$, the constant term in the original equation; and their sum is $4 + 5 = 9$, the negative of the coefficient of x in the original equation.

(b) In general, the equation $x^2 + bx + c = 0$ has solutions $r_1 = \frac{-b + \sqrt{b^2 - 4c}}{2}$ and $r_2 = \frac{-b - \sqrt{b^2 - 4c}}{2}$.

Thus, $r_1 r_2 = \frac{-b + \sqrt{b^2 - 4c}}{2} \cdot \frac{-b - \sqrt{b^2 - 4c}}{2} = \frac{1}{4} \left[(-b)^2 - (\sqrt{b^2 - 4c})^2 \right] = \frac{1}{4} (b^2 - b^2 + 4c) = c$ and

$r_1 + r_2 = \frac{-b + \sqrt{b^2 - 4c}}{2} + \frac{-b - \sqrt{b^2 - 4c}}{2} = \frac{-2b}{2} = -b$.

143. (a) Method 1: Let $u = \sqrt{x}$, so $u^2 = x$. Thus $x - \sqrt{x} - 2 = 0$ becomes $u^2 - u - 2 = 0 \Leftrightarrow (u - 2)(u + 1) = 0$. So $u = 2$ or $u = -1$. If $u = 2$, then $\sqrt{x} = 2 \Rightarrow x = 4$. If $u = -1$, then $\sqrt{x} = -1 \Rightarrow x = 1$. So the possible solutions are 4 and 1. Checking $x = 4$ we have $4 - \sqrt{4} - 2 = 4 - 2 - 2 = 0$. Checking $x = 1$ we have $1 - \sqrt{1} - 2 = 1 - 1 - 2 \neq 0$. The only solution is 4.

Method 2: $x - \sqrt{x} - 2 = 0 \Leftrightarrow x - 2 = \sqrt{x} \Rightarrow x^2 - 4x + 4 = x \Leftrightarrow x^2 - 5x + 4 = 0 \Leftrightarrow (x - 4)(x - 1) = 0$. So the possible solutions are 4 and 1. Checking will result in the same solution.

(b) Method 1: Let $u = \frac{1}{x-3}$, so $u^2 = \frac{1}{(x-3)^2}$. Thus $\frac{12}{(x-3)^2} + \frac{10}{x-3} + 1 = 0$ becomes $12u^2 + 10u + 1 = 0$. Using

the quadratic formula, we have $u = \frac{-10 \pm \sqrt{10^2 - 4(12)(1)}}{2(12)} = \frac{-10 \pm \sqrt{52}}{24} = \frac{-10 \pm 2\sqrt{13}}{24} = \frac{-5 \pm \sqrt{13}}{12}$. If $u = \frac{-5 - \sqrt{13}}{12}$, then

$\frac{1}{x-3} = \frac{-5 - \sqrt{13}}{12} \Leftrightarrow x - 3 = \frac{12}{-5 - \sqrt{13}} \cdot \frac{-5 + \sqrt{13}}{-5 + \sqrt{13}} = \frac{12(-5 + \sqrt{13})}{-25 + 13} = -5 + \sqrt{13}$. So $x = -2 + \sqrt{13}$.

If $u = \frac{-5 + \sqrt{13}}{12}$, then $\frac{1}{x-3} = \frac{-5 + \sqrt{13}}{12} \Leftrightarrow x - 3 = \frac{12}{-5 + \sqrt{13}} \cdot \frac{-5 - \sqrt{13}}{-5 - \sqrt{13}} = \frac{12(-5 - \sqrt{13})}{-25 + 13} = -5 - \sqrt{13}$. So

$x = -2 - \sqrt{13}$.

The solutions are $-2 \pm \sqrt{13}$.

Method 2: Multiplying by the LCD, $(x-3)^2$, we get $(x-3)^2 \left(\frac{12}{(x-3)^2} + \frac{10}{x-3} + 1 \right) = 0 \cdot (x-3)^2 \Leftrightarrow$

$12 + 10(x-3) + (x-3)^2 = 0 \Leftrightarrow 12 + 10x - 30 + x^2 - 6x + 9 = 0 \Leftrightarrow x^2 + 4x - 9 = 0$. Using the Quadratic

Formula, we have $u = \frac{-4 \pm \sqrt{4^2 - 4(1)(-9)}}{2} = \frac{-4 \pm \sqrt{52}}{2} = \frac{-4 \pm 2\sqrt{13}}{2} = -2 \pm \sqrt{13}$. The solutions are $-2 \pm \sqrt{13}$.

1.6 COMPLEX NUMBERS

1. The imaginary number i has the property that $i^2 = -1$.
3. (a) The complex conjugate of $3 + 4i$ is $\overline{3 + 4i} = 3 - 4i$.
(b) $(3 + 4i)(\overline{3 + 4i}) = 3^2 + 4^2 = 25$
5. Yes, every real number a is a complex number of the form $a + 0i$.
7. $3 - 8i$: real part 3, imaginary part -8 .
9. $\frac{-2 - 5i}{3} = -\frac{2}{3} - \frac{5}{3}i$: real part $-\frac{2}{3}$, imaginary part $-\frac{5}{3}$.
11. 3: real part 3, imaginary part 0.
13. $-\frac{2}{3}i$: real part 0, imaginary part $-\frac{2}{3}$.
15. $\sqrt{3} + \sqrt{-4} = \sqrt{3} + 2i$: real part $\sqrt{3}$, imaginary part 2.
17. $(3 + 2i) + 5i = 3 + (2 + 5)i = 3 + 7i$
19. $(5 - 3i) + (-4 - 7i) = (5 - 4) + (-3 - 7)i = 1 - 10i$
21. $(-6 + 6i) + (9 - i) = (-6 + 9) + (6 - 1)i = 3 + 5i$
23. $(7 - \frac{1}{2}i) - (5 + \frac{3}{2}i) = (7 - 5) + (-\frac{1}{2} - \frac{3}{2})i = 2 - 2i$
25. $(-12 + 8i) - (7 + 4i) = -12 + 8i - 7 - 4i = (-12 - 7) + (8 - 4)i = -19 + 4i$
27. $4(-1 + 2i) = -4 + 8i$
29. $(3 - 4i)(2 + 5i) = 6 + 15i - 8i - 20i^2 = 6 + 7i - 20(-1) = 26 + 7i$
31. $(6 + 5i)(2 - 3i) = 12 - 18i + 10i - 15i^2 = (12 + 15) + (-18 + 10)i = 27 - 8i$
33. $(3 - 2i)(3 + 2i) = 3^2 - (2i)^2 = 13$
35. $(3 - 2i)^2 = 9 - 12i + 4(-1) = 5 - 12i$
37. $\frac{1}{i} = \frac{1}{i} \cdot \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$
39. $\frac{1 - 3i}{1 + 2i} = \frac{(1 - 3i)(1 - 2i)}{(1 + 2i)(1 - 2i)} = \frac{5 + 5i}{5} = -1 - i$
41. $\frac{10i}{1 - 2i} = \frac{10i}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i} = \frac{10i + 20i^2}{1 - 4i^2} = \frac{-20 + 10i}{1 + 4} = \frac{5(-4 + 2i)}{5} = -4 + 2i$
43. $\frac{4 + 6i}{3i} = \frac{4 + 6i}{3i} \cdot \frac{3i}{3i} = \frac{12i + 18i^2}{9i^2} = \frac{-18 + 12i}{-9} = \frac{-18}{-9} + \frac{12}{-9}i = 2 - \frac{4}{3}i$
45. $\frac{1}{1 + i} - \frac{1}{1 - i} = \frac{1}{1 + i} \cdot \frac{1 - i}{1 - i} - \frac{1}{1 - i} \cdot \frac{1 + i}{1 + i} = \frac{1 - i}{1 - i^2} - \frac{1 + i}{1 - i^2} = \frac{1 - i}{2} + \frac{-1 - i}{2} = -i$
47. $i^3 = i^2 i = -i$
49. $(3i)^5 = 3^5 (i^2)^2 i = 243 (-1)^2 i = 243i$
51. $i^{1000} = (i^4)^{250} = 1^{250} = 1$
53. $\sqrt{-25} = \sqrt{25}\sqrt{-1} = 5i$
55. $\sqrt{-4}\sqrt{-9} = (2i)(3i) = -6$
57. $(2 + \sqrt{-1})(3 - \sqrt{-3}) = 6 - 2\sqrt{3}i + 3i - \sqrt{3} = 6 + \sqrt{3} + (3 - 2\sqrt{3})i$
59. $\frac{2 + \sqrt{-8}}{1 + \sqrt{-2}} = \frac{2 + 2\sqrt{2}i}{1 + \sqrt{2}i} = \frac{2(1 + \sqrt{2}i)}{1 + \sqrt{2}i} = 2$
61. $x^2 + 25 = 0 \Leftrightarrow x^2 = -25 \Leftrightarrow x = \pm 5i$
63. $x^2 - 6x + 13 = 0 \Leftrightarrow x = \frac{6 \pm \sqrt{36 - 4(13)}}{2} = 3 \pm \frac{1}{2}\sqrt{-16} = 3 \pm 2i$
Or: $x^2 - 6x + 13 = 0 \Leftrightarrow x^2 - 6x + 9 = -13 + 9 \Leftrightarrow (x - 3)^2 = -4 \Leftrightarrow x - 3 = \pm 2i \Leftrightarrow x = 3 \pm 2i$

65. $2x^2 - 2x + 5 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4 - 4(2)(5)}}{2(2)} = \frac{2 \pm \sqrt{-36}}{4} = \frac{1}{2} \pm \frac{3}{2}i$
67. $x^2 + x + 1 = 0 \Rightarrow x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{1 - 4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$
69. $9x^2 - 4x + 4 = 0 \Leftrightarrow x = \frac{4 \pm \sqrt{(-4)^2 - 4(9)(4)}}{2(9)} = \frac{4 \pm \sqrt{-128}}{18} = \frac{2}{9} \pm \frac{4\sqrt{2}}{9}i$
71. $6x^2 + 12x + 7 = 0 \Rightarrow$
 $x = \frac{-(12) \pm \sqrt{(12)^2 - 4(6)(7)}}{2(6)} = \frac{-12 \pm \sqrt{144 - 168}}{12} = \frac{-12 \pm \sqrt{-24}}{12} = \frac{-12 \pm 2\sqrt{6}i}{12} = \frac{-12}{12} \pm \frac{2\sqrt{6}i}{12} = -1 \pm \frac{\sqrt{6}}{6}i$
73. $\bar{z} + \bar{w} = \overline{3 - 4i} + \overline{5 + 2i} = 3 + 4i + 5 - 2i = 8 + 2i$
75. $z \cdot \bar{z} = (3 - 4i)(3 + 4i) = 3^2 + 4^2 = 25$
77. LHS $= \bar{z} + \bar{w} = \overline{(a + bi)} + \overline{(c + di)} = a - bi + c - di = (a + c) + (-b - d)i = (a + c) - (b + d)i$.
 RHS $= \overline{z + w} = \overline{(a + bi) + (c + di)} = \overline{(a + c) + (b + d)i} = (a + c) - (b + d)i$.
 Since LHS = RHS, this proves the statement.
79. LHS $= (\bar{z})^2 = \overline{(a + bi)^2} = \overline{a^2 - 2abi + b^2i^2} = \overline{a^2 - b^2 - 2abi} = (a^2 - b^2) - 2abi$.
 RHS $= \overline{z^2} = \overline{(a + bi)^2} = \overline{a^2 + 2abi + b^2i^2} = \overline{a^2 - b^2 + 2abi} = (a^2 - b^2) - 2abi$.
 Since LHS = RHS, this proves the statement.
81. $z + \bar{z} = (a + bi) + \overline{(a + bi)} = a + bi + a - bi = 2a$, which is a real number.
83. $z \cdot \bar{z} = (a + bi) \cdot \overline{(a + bi)} = (a + bi) \cdot (a - bi) = a^2 - b^2i^2 = a^2 + b^2$, which is a real number.
85. Using the Quadratic Formula, the solutions to the equation are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Since both solutions are nonreal, we have $b^2 - 4ac < 0 \Leftrightarrow 4ac - b^2 > 0$, so the solutions are $x = \frac{-b}{2a} \pm \frac{\sqrt{4ac - b^2}}{2a}i$, where $\sqrt{4ac - b^2}$ is a real number. Thus the solutions are complex conjugates of each other.

1.7 MODELING WITH EQUATIONS

- An equation modeling a real-world situation can be used to help us understand a real-world problem using mathematical methods. We translate real-world ideas into the language of algebra to construct our model, and translate our mathematical results back into real-world ideas in order to interpret our findings.
- (a) A square of side x has area $A = x^2$.
 (b) A rectangle of length l and width w has area $A = lw$.
 (c) A circle of radius r has area $A = \pi r^2$.
- A painter paints a wall in x hours, so the fraction of the wall she paints in one hour is $\frac{1 \text{ wall}}{x \text{ hours}} = \frac{1}{x}$.
- If n is the first integer, then $n + 1$ is the middle integer, and $n + 2$ is the third integer. So the sum of the three consecutive integers is $n + (n + 1) + (n + 2) = 3n + 3$.
- If n is the first even integer, then $n + 2$ is the second even integer and $n + 4$ is the third. So the sum of three consecutive even integers is $n + (n + 2) + (n + 4) = 3n + 6$.
- If s is the third test score, then since the other test scores are 78 and 82, the average of the three test scores is $\frac{78 + 82 + s}{3} = \frac{160 + s}{3}$.
- If x dollars are invested at $2\frac{1}{2}\%$ simple interest, then the first year you will receive $0.025x$ dollars in interest.

15. Since w is the width of the rectangle, the length is four times the width, or $4w$. Then
 $\text{area} = \text{length} \times \text{width} = 4w \times w = 4w^2 \text{ m}^2$.
17. If d is the given distance, in kilometers, and $\text{distance} = \text{rate} \times \text{time}$, we have $\text{time} = \frac{\text{distance}}{\text{rate}} = \frac{d}{55}$.
19. If x is the quantity of pure water added, the mixture will contain 750 g of salt and $3 + x$ liters of water. Thus the concentration is $\frac{750}{3 + x}$.
21. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**
 If d is the number of days and m the number of kilometers, then the cost of a rental is $C = 65d + 0.40m$. In this case, $d = 3$ and $C = 283$, so we solve for m : $283 = 65 \cdot 3 + 0.40m \Leftrightarrow 283 = 195 + 0.40m \Leftrightarrow 0.4m = 88 \Leftrightarrow m = \frac{88}{0.4} = 220$. Thus, the truck was driven 220 kilometers.
23. If x is the student's score on their final exam, then because the final counts twice as much as each midterm, the average score is $\frac{82 + 75 + 71 + 2x}{3(100) + 200} = \frac{228 + 2x}{500} = \frac{114 + x}{250}$. For the average to be 80%, we must have $\frac{114 + x}{250} = 80\% = 0.8 \Leftrightarrow 114 + x = 250(0.8) = 200 \Leftrightarrow x = 86$. So the student scored 86% on their final exam.
25. Let m be the amount invested at $2\frac{1}{2}\%$. Then $12,000 - m$ is the amount invested at 3%.
 Since the total interest is equal to the interest earned at 2.5% plus the interest earned at 3%, we have
 $318 = 0.025m + 0.03(12,000 - m) \Leftrightarrow 318 = 0.025m + 360 - 0.03m \Leftrightarrow 0.005m = 42 \Leftrightarrow m = \frac{42}{0.005} = 8400$. Thus, \$8400 is invested at $2\frac{1}{2}\%$ and $12,000 - 8400 = \$3600$ is invested at 3%.
27. Using the formula $I = Prt$ and solving for r , we get $262.50 = 3500 \cdot r \cdot 1 \Leftrightarrow r = \frac{262.5}{3500} = 0.075$ or 7.5%.
29. Let x be the monthly salary. Since $\text{annual salary} = 12 \times (\text{monthly salary}) + (\text{Christmas bonus})$, we have
 $180,100 = 12x + 7300 \Leftrightarrow 172,800 = 12x \Leftrightarrow x \approx 14,400$. The monthly salary is \$14,400.
31. Let x be the overtime hours worked. Since $\text{gross pay} = \text{regular salary} + \text{overtime pay}$, we obtain the equation
 $814 = 18.50 \cdot 35 + 18.50 \cdot 1.5 \cdot x \Leftrightarrow 814 = 647.50 + 27.75x \Leftrightarrow 166.50 = 27.75x \Leftrightarrow x = \frac{166.50}{27.75} = 6$. Thus, the lab technician worked 6 hours of overtime.
33. All ages are in terms of the daughter's age 7 years ago. Let y be age of the daughter 7 years ago. Then $11y$ is the age of the movie star 7 years ago. Today, the daughter is $y + 7$, and the movie star is $11y + 7$. But the movie star is also 4 times his daughter's age today. So $4(y + 7) = 11y + 7 \Leftrightarrow 4y + 28 = 11y + 7 \Leftrightarrow 21 = 7y \Leftrightarrow y = 3$. Thus, today the movie star is $11(3) + 7 = 40$ years old.
35. Let n be the number of nickels. Then there are also n dimes and n quarters. The total value of the coins in the purse is the sum of the values of nickels, dimes, and quarters, so $2.80 = 0.05n + 0.10n + 0.25n \Leftrightarrow 2.8 = 0.4p \Leftrightarrow p = \frac{2.8}{0.4} = 7$. So the purse contains 7 nickels, 7 dimes, and 7 quarters.
37. Let l be the length of the garden. Since $\text{area} = \text{width} \cdot \text{length}$, we obtain the equation $1125 = 25l \Leftrightarrow l = \frac{1125}{25} = 45$ m. So the garden is 45 meters long.
39. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**
 Let w be the width of the building lot. Then the length of the building lot is $5w$. Since a half-hectare is $\frac{1}{2} \cdot 10,000 = 5000$ and area is length times width, we have $5000 = w(5w) = 5w^2 \Leftrightarrow w^2 = 1000 \Rightarrow w = \pm 31.6$. Thus the width of the building lot is 31.6 meters and the length of the building lot is $5(31.6) = 158$ meters.
41. Let w be the width of the garden in meters. Then the length is $w + 10$. Thus $875 = w(w + 10) \Leftrightarrow w^2 + 10w - 875 = 0 \Leftrightarrow (w + 35)(w - 25) = 0$. If $w + 35 = 0$, then $w = -35$, which is impossible. Therefore $w - 25 = 0$, and so $w = 25$. The garden is 25 meters wide and 35 meters long.

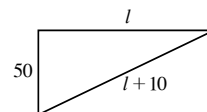
43. Let w be the width of the garden in meters. We use the perimeter to express the length l of the garden in terms of width. Since the perimeter is twice the width plus twice the length, we have $200 = 2w + 2l \Leftrightarrow 2l = 200 - 2w \Leftrightarrow l = 100 - w$. Using the formula for area, we have $2400 = w(100 - w) = 100w - w^2 \Leftrightarrow w^2 - 100w + 2400 = 0 \Leftrightarrow (w - 40)(w - 60) = 0$. So $w - 40 = 0 \Leftrightarrow w = 40$, or $w - 60 = 0 \Leftrightarrow w = 60$. If $w = 40$, then $l = 100 - 40 = 60$. And if $w = 60$, then $l = 100 - 60 = 40$. So the length is 60 meters and the width is 40 meters.

45. Let l be the length of the lot in meters. Then the length of the diagonal is $l + 10$.

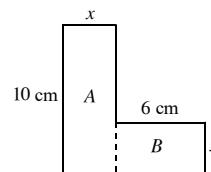
We apply the Pythagorean Theorem with the hypotenuse as the diagonal. So

$$l^2 + 50^2 = (l + 10)^2 \Leftrightarrow l^2 + 2500 = l^2 + 20l + 100 \Leftrightarrow 20l = 2400 \Leftrightarrow l = 120.$$

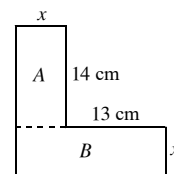
Thus the length of the lot is 120 meters.



47. (a) First we write a formula for the area of the figure in terms of x . Region A has dimensions 10 cm and x cm and region B has dimensions 6 cm and x cm. So the shaded region has area $(10 \cdot x) + (6 \cdot x) = 16x \text{ cm}^2$. We are given that this is equal to 144 cm^2 , so $144 = 16x \Leftrightarrow x = \frac{144}{16} = 9 \text{ cm}$.



- (b) First we write a formula for the area of the figure in terms of x . Region A has dimensions 14 cm and x cm and region B has dimensions $(13 + x)$ cm and x cm. So the area of the figure is $(14 \cdot x) + [(13 + x)x] = 14x + 13x + x^2 = x^2 + 27x$. We are given that this is equal to 160 cm^2 , so $160 = x^2 + 27x \Leftrightarrow x^2 + 27x - 160 = 0 \Leftrightarrow (x + 32)(x - 5) \Leftrightarrow x = -32$ or $x = 5$. x must be positive, so $x = 5 \text{ cm}$.



49. Let x be the width of the strip. Then the length of the mat is $50 + 2x$, and the width of the mat is $38 + 2x$. Now the perimeter is twice the length plus twice the width, so $256 = 2(50 + 2x) + 2(38 + 2x) \Leftrightarrow 256 = 100 + 4x + 76 + 4x \Leftrightarrow 256 = 176 + 8x \Leftrightarrow 80 = 8x \Leftrightarrow x = 10$. Thus the strip of mat is 10 centimeters wide.
51. Let h be the height the ladder reaches (in meters). Using the Pythagorean Theorem we have $(2.3)^2 + h^2 = (6)^2 \Leftrightarrow h^2 = (6)^2 - (2.3)^2 = 36 - 5.29 \approx 30.71$. So $h = \sqrt{30.71} \approx 5.54$ meters.
53. Let x be the length of the person's shadow, in meters. Using similar triangles, $\frac{10 + x}{6} = \frac{x}{2} \Leftrightarrow 20 + 2x = 6x \Leftrightarrow 4x = 20 \Leftrightarrow x = 5$. Thus the person's shadow is 5 meters long.
55. Let x be the amount (in mL) of 60% acid solution to be used. Then $300 - x$ mL of 30% solution would have to be used to yield a total of 300 mL of solution.

	60% acid	30% acid	Mixture
mL	x	$300 - x$	300
Rate (% acid)	0.60	0.30	0.50
Value	$0.60x$	$0.30(300 - x)$	$0.50(300)$

Thus the total amount of pure acid used is $0.60x + 0.30(300 - x) = 0.50(300) \Leftrightarrow 0.3x + 90 = 150 \Leftrightarrow x = \frac{60}{0.3} = 200$. So 200 mL of 60% acid solution must be mixed with 100 mL of 30% solution to get 300 mL of 50% acid solution.

57. Let x be the number of grams of silver added. The weight of the rings is $5 \times 18 \text{ g} = 90 \text{ g}$.

	5 rings	Pure silver	Mixture
Grams	90	x	$90 + x$
Rate (% gold)	0.90	0	0.75
Value	$0.90(90)$	$0x$	$0.75(90 + x)$

So $0.90(90) + 0x = 0.75(90 + x) \Leftrightarrow 81 = 67.5 + 0.75x \Leftrightarrow 0.75x = 13.5 \Leftrightarrow x = \frac{13.5}{0.75} = 18$. Thus 18 grams of silver must be added to get the required mixture.

59. Let x be the number of liters of coolant removed and replaced by water.

	60% antifreeze	60% antifreeze (removed)	Water	Mixture
Liters	3.6	x	x	3.6
Rate (% antifreeze)	0.60	0.60	0	0.50
Value	$0.60(3.6)$	$-0.60x$	$0x$	$0.50(3.6)$

So $0.60(3.6) - 0.60x + 0x = 0.50(3.6) \Leftrightarrow 2.16 - 0.6x = 1.8 \Leftrightarrow -0.6x = -0.36 \Leftrightarrow x = \frac{-0.36}{-0.6} = 0.6$. Thus 0.6 liters must be removed and replaced by water.

61. Let c be the concentration of fruit juice in the cheaper brand. The new mixture consists of 650 mL of the original fruit punch and 100 mL of the cheaper fruit punch.

	Original Fruit Punch	Cheaper Fruit Punch	Mixture
mL	650	100	750
Concentration	0.50	c	0.48
Juice	$0.50 \cdot 650$	$100c$	$0.48 \cdot 750$

So $0.50 \cdot 650 + 100c = 0.48 \cdot 750 \Leftrightarrow 325 + 100c = 360 \Leftrightarrow 100c = 35 \Leftrightarrow c = 0.35$. Thus the cheaper brand is only 35% fruit juice.

63. Let t be the time in minutes it would take to wash the car if the friends worked together. Friend 1 washes $\frac{1}{25}$ of the car per minute, while Friend 2 washes $\frac{1}{35}$ of the car per minute. The sum of these fractions is equal to the fraction of the job they can do working together, so we have $\frac{1}{t} = \frac{1}{25} + \frac{1}{35} \Leftrightarrow \frac{875t}{t} = \frac{875t}{25} + \frac{875t}{35} \Leftrightarrow 875 = 35t + 25t = 60t \Leftrightarrow t = \frac{875}{60} \Leftrightarrow t = 14\frac{35}{60}$ minutes, or 14 minutes 35 seconds

65. Let t be the number of hours it would take your friend to paint a house alone. Then working together, it takes $\frac{2}{3}t$ hours.

Because it takes you 7 hours, we have $\frac{1}{7} + \frac{1}{t} = \frac{1}{\frac{2}{3}t} \Leftrightarrow \frac{14t}{7} + \frac{14t}{t} = \frac{14t}{\frac{2}{3}t} = \frac{\frac{3}{2}(14t)}{t} \Leftrightarrow 2t + 14 = 21 \Leftrightarrow t = \frac{7}{2}$. Thus, it would take your friend 3.5 h to paint a house alone.

67. Let t be the time in hours that it takes you to wash all the windows. Then it takes your roommate $t + \frac{3}{2}$ hours to wash all the windows, and the sum of the fractions of the job you can do individually per hour equals the fraction of the job you can do together. Since $1 \text{ hour } 48 \text{ minutes} = 1 + \frac{48}{60} = 1 + \frac{4}{5} = \frac{9}{5}$, we have $\frac{1}{t} + \frac{1}{t + \frac{3}{2}} = \frac{1}{\frac{9}{5}} \Leftrightarrow \frac{1}{t} + \frac{2}{2t + 3} = \frac{5}{9} \Rightarrow 9(2t + 3) + 2(9t) = 5t(2t + 3) \Leftrightarrow 18t + 27 + 18t = 10t^2 + 15t \Leftrightarrow 10t^2 - 21t - 27 = 0$
- $$\Leftrightarrow t = \frac{-(-21) \pm \sqrt{(-21)^2 - 4(10)(-27)}}{2(10)} = \frac{21 \pm \sqrt{441 + 1080}}{20} = \frac{21 \pm 39}{20}. \text{ So } t = \frac{21 - 39}{20} = -\frac{9}{10}$$
- or $t = \frac{21 + 39}{20} = 3$. Since $t < 0$ is impossible, you can wash the windows alone in 3 hours, and it takes your roommate $3 + \frac{3}{2} = 4\frac{1}{2}$ hours.

69. Let t be the time in hours that the commuter spent on the train. Then $\frac{11}{2} - t$ is the time in hours that commuter spent on the bus. We construct a table:

	Rate	Time	Distance
By train	64	t	$64t$
By bus	96	$\frac{11}{2} - t$	$96\left(\frac{11}{2} - t\right)$

The total distance traveled is the sum of the distances traveled by bus and by train, so $480 = 64t + 96\left(\frac{11}{2} - t\right) \Leftrightarrow 480 = 64t + 528 - 96t \Leftrightarrow -48 = -32t \Leftrightarrow t = \frac{-48}{-32} = 1.5$ hours. So the time spent on the train is $5.5 - 1.5 = 4$ hours.

71. Let r be the speed of the plane from Montreal to Los Angeles. Then $r + 0.20r = 1.20r$ is the speed of the plane from Los Angeles to Montreal.

	Rate	Time	Distance
Montreal to L.A.	r	$\frac{4000}{r}$	4000
L.A. to Montreal	$1.2r$	$\frac{4000}{1.2r}$	4000

The total time is the sum of the times each way, so $9\frac{1}{6} = \frac{4000}{r} + \frac{4000}{1.2r} \Leftrightarrow \frac{55}{6} = \frac{4000}{r} + \frac{4000}{1.2r} \Leftrightarrow 55 \cdot 1.2r = 4000 \cdot 6 \cdot 1.2 + 4000 \cdot 6 \Leftrightarrow 66r = 28,800 + 24,000 \Leftrightarrow 66r = 52,800 \Leftrightarrow r = \frac{52,800}{66} = 800$. Thus the plane flew at a speed of 800 km/h on the trip from Montreal to Los Angeles.

73. Let x be the rate, in km/h, at which the salesperson drove between Ajax and Barrington.

Cities	Distance	Rate	Time
Ajax → Barrington	192	x	$\frac{192}{x}$
Barrington → Collins	240	$x + 16$	$\frac{240}{x + 16}$

We have used the equation $\text{time} = \frac{\text{distance}}{\text{rate}}$ to fill in the “Time” column of the table. Since the second part of the trip took 6 minutes (or $\frac{1}{10}$ hour) more than the first, we can use the time column to get the equation $\frac{192}{x} + \frac{1}{10} = \frac{240}{x + 16} \Rightarrow 192(10)(x + 16) + x(x + 16) = 240(10x) \Leftrightarrow 1920x + 30,720 + x^2 + 16x = 2400x \Leftrightarrow x^2 - 464x + 30,720 = 0 \Leftrightarrow x = \frac{-(-464) \pm \sqrt{(-464)^2 - 4(1)(30,720)}}{2} = \frac{464 \pm \sqrt{215,296 - 122,880}}{2} = \frac{464 \pm \sqrt{92,416}}{2} = \frac{464 \pm 304}{2} = 232 \pm 152$. Hence, the salesman drove either 80 km/h or 384 km/h between Ajax and Barrington. (The former seems likelier.)

75. Let r be the rowing rate in km/h of the crew in still water. Then their rate upstream was $r - 3$ km/h, and their rate downstream was $r + 3$ km/h.

	Distance	Rate	Time
Upstream	6	$r - 3$	$\frac{6}{r - 3}$
Downstream	6	$r + 3$	$\frac{6}{r + 3}$

Since the time to row upstream plus the time to row downstream was 2 hours 40 minutes $= \frac{8}{3}$ hour, we get the equation

$$\frac{6}{r - 3} + \frac{6}{r + 3} = \frac{8}{3} \Leftrightarrow 6(3)(r + 3) + 6(3)(r - 3) = 8(r - 3)(r + 3) \Leftrightarrow 18r + 54 + 18r - 54 = 8r^2 - 72 \Leftrightarrow 0 = 8r^2 - 36r - 72 = 4(2r^2 - 9r - 18) = 4(2r + 3)(r - 6).$$

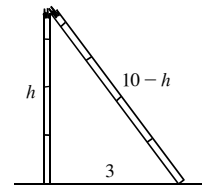
Since $2r + 3 = 0 \Leftrightarrow r = -\frac{3}{2}$ is impossible, the solution is $r - 6 = 0 \Leftrightarrow r = 6$. So the rate of the rowing crew in still water is 6 km/h.

77. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

Let x be the distance from the fulcrum to where the 56-kg friend sits. Then substituting the known values into the formula given, we have $45(2) = 56x \Leftrightarrow 90 = 56x \Leftrightarrow x = 1.6$. So the 56-kg friend should sit 1.6 meters from the fulcrum.

79. The volume is 180 m^3 , so $x(x - 4)(x + 9) = 180 \Leftrightarrow x^3 + 5x^2 - 36x = 180 \Leftrightarrow x^3 + 5x^2 - 36x - 180 = 0 \Leftrightarrow x^2(x + 5) - 36(x + 5) = 0 \Leftrightarrow (x + 5)(x^2 - 36) = 0 \Leftrightarrow (x + 5)(x + 6)(x - 6) = 0 \Rightarrow x = 6$ is the only positive solution. So the box is 2 meters by 6 meters by 15 meters.
81. Let x be the length of one side of the cardboard, so we start with a piece of cardboard x by x . When 4 centimeters are removed from each side, the base of the box is $x - 8$ by $x - 8$. Since the volume is 100 cm^3 , we get $4(x - 8)^2 = 100 \Leftrightarrow x^2 - 16x + 64 = 25 \Leftrightarrow x^2 - 16x + 39 = 0 \Leftrightarrow (x - 3)(x - 13) = 0$. So $x = 3$ or $x = 13$. But $x = 3$ is not possible, since then the length of the base would be $3 - 8 = -5$, and all lengths must be positive. Thus $x = 13$, and the piece of cardboard is 13 centimeters by 13 centimeters.
83. Let r be the radius of the tank, in meters. The volume of the spherical tank is $\frac{4}{3}\pi r^3$ and is also $2840 \times 0.001 = 2.84$. So $\frac{4}{3}\pi r^3 = 2.84 \Leftrightarrow r^3 = 0.678 \Leftrightarrow r = 0.88$ meters.

85. Let x be the length, in kilometers, of the abandoned road to be used. Then the length of the abandoned road not used is $40 - x$, and the length of the new road is $\sqrt{10^2 + (40 - x)^2}$ kilometers, by the Pythagorean Theorem. Since the cost of the road is cost per kilometer $\times$ number of kilometers, we have $100,000x + 200,000\sqrt{x^2 - 80x + 1700} = 6,800,000$
 $\Leftrightarrow 2\sqrt{x^2 - 80x + 1700} = 68 - x$. Squaring both sides, we get $4x^2 - 320x + 6800 = 4624 - 136x + x^2 \Leftrightarrow$
 $3x^2 - 184x + 2176 = 0 \Leftrightarrow x = \frac{184 \pm \sqrt{33856 - 26112}}{6} = \frac{184 \pm 88}{6} \Leftrightarrow x = \frac{136}{6}$ or $x = 16$. Since $45\frac{1}{3}$ is longer than the existing road, 16 kilometers of the abandoned road should be used. A completely new road would have length $\sqrt{10^2 + 40^2}$ (let $x = 0$) and would cost $\sqrt{1700} \times 200,000 \approx 8.3$ million dollars. So no, it would not be cheaper.
87. Let x be the height of the pile in meters. Then the diameter is $3x$ and the radius is $\frac{3}{2}x$ meters. Since the volume of the cone is 28 m^3 , we have $\frac{\pi}{3} \left(\frac{3x}{2}\right)^2 x = 28 \Leftrightarrow \frac{3\pi x^3}{4} = 28 \Leftrightarrow x^3 = \frac{112}{3\pi} \Leftrightarrow x = \sqrt[3]{\frac{112}{3\pi}} \approx 2.28$ meters.
89. Let h be the height in meters of the structure. The structure is composed of a right cylinder with radius 3 and height $\frac{2}{3}h$ and a cone with base radius 3 and height $\frac{1}{3}h$. Using the formulas for the volume of a cylinder and that of a cone, we obtain the equation $40\pi = \pi(3)^2 \left(\frac{2}{3}h\right) + \frac{1}{3}\pi(3)^2 \left(\frac{1}{3}h\right) \Leftrightarrow 40\pi = 6\pi h + \pi h \Leftrightarrow 40 = 7h \Leftrightarrow h \approx 5.7$. Thus the height of the structure is 5.7 meters.
91. Let h be the height of the break, in meters. Then the portion of the bamboo above the break is $10 - h$. Applying the Pythagorean Theorem, we obtain
 $h^2 + 3^2 = (10 - h)^2 \Leftrightarrow h^2 + 9 = 100 - 20h + h^2 \Leftrightarrow -91 = -20h \Leftrightarrow$
 $h = \frac{91}{20} = 4.55$. Thus the break is 4.55 m above the ground.
93. Let x equal the original length of the reed in cubits. Then $x - 1$ is the piece that fits 60 times along the length of the field, that is, the length is $60(x - 1)$. The width is $30x$. Then converting cubits to ninda, we have
 $375 = 60(x - 1) \cdot 30x \cdot \frac{1}{12^2} = \frac{25}{2}x(x - 1) \Leftrightarrow 30 = x^2 - x \Leftrightarrow x^2 - x - 30 = 0 \Leftrightarrow (x - 6)(x + 5) = 0$. So $x = 6$ or $x = -5$. Since x must be positive, the original length of the reed is 6 cubits.



1.8 INEQUALITIES

1. (a) If $x < 5$, then $x - 3 < 5 - 3 \Rightarrow x - 3 < 2$.
 (b) If $x \leq 5$, then $3 \cdot x \leq 3 \cdot 5 \Rightarrow 3x \leq 15$.
 (c) If $x \geq 2$, then $-3 \cdot x \leq -3 \cdot 2 \Rightarrow -3x \leq -6$.
 (d) If $x < -2$, then $-x > 2$.
3. (a) The solution of the inequality $|x| \leq 3$ is the interval $[-3, 3]$. Any number that lies inside this interval satisfies the inequality.
 (b) The solution of the inequality $|x| \geq 3$ is the union of intervals $(-\infty, -3] \cup [3, \infty)$. Any number that lie inside one of these intervals satisfies the inequality.
5. (a) No. For example, if $x = -2$, then $x(x + 1) = -2(-1) = 2 > 0$, but $x < 0$.
 (b) No. For example, if $x = 2$, then $x(x + 1) = 2(3) = 6 > 5$, but $x < 5$.



7.

x	$-2 + 3x \geq \frac{1}{3}$
-5	$-17 \geq \frac{1}{3}$; no
-1	$-5 \geq \frac{1}{3}$; no
0	$-2 \geq 0$; no
$\frac{2}{3}$	$0 \geq \frac{1}{3}$; no
$\frac{5}{6}$	$\frac{1}{2} \geq \frac{1}{3}$; yes
1	$1 \geq \frac{1}{3}$; yes
$\sqrt{5}$	$4.7 \geq \frac{1}{3}$; yes
3	$7 \geq \frac{1}{3}$; yes
5	$13 \geq \frac{1}{3}$; yes

The elements $\frac{5}{6}$, 1, $\sqrt{5}$, 3, and 5 satisfy the inequality.

9.

x	$1 < 2x - 4 \leq 7$
-5	$1 < -14 \leq 7$; no
-1	$1 < -6 \leq 7$; no
0	$1 < -4 \leq 7$; no
$\frac{2}{3}$	$1 < -\frac{8}{3} \leq 7$; no
$\frac{5}{6}$	$1 < -\frac{7}{3} \leq 7$; no
1	$1 < -2 \leq 7$; no
$\sqrt{5}$	$1 < 0.47 \leq 7$; no
3	$1 < 2 \leq 7$; yes
5	$1 < 6 \leq 7$; yes

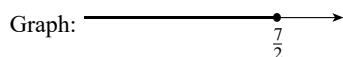
The elements 3 and 5 satisfy the inequality.

11.

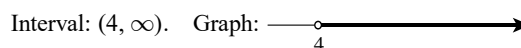
x	$\frac{1}{x} \leq \frac{1}{2}$
-5	$-\frac{1}{5} \leq \frac{1}{2}$; yes
-1	$-1 \leq \frac{1}{2}$; yes
0	$\frac{1}{0}$ is undefined; no
$\frac{2}{3}$	$\frac{3}{2} \leq \frac{1}{2}$; no
$\frac{5}{6}$	$\frac{6}{5} \leq \frac{1}{2}$; no
1	$1 \leq \frac{1}{2}$; no
$\sqrt{5}$	$0.45 \leq \frac{1}{2}$; yes
3	$\frac{1}{3} \leq \frac{1}{2}$; yes
5	$\frac{1}{5} \leq \frac{1}{2}$; yes

The elements -5, -1, $\sqrt{5}$, 3, and 5 satisfy the inequality.

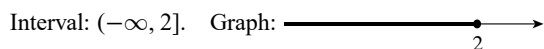
13. $2x \leq 7 \Leftrightarrow x \leq \frac{7}{2}$. Interval: $(-\infty, \frac{7}{2}]$



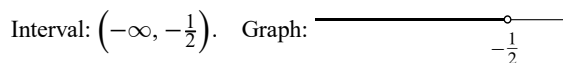
15. $2x - 5 > 3 \Leftrightarrow 2x > 8 \Leftrightarrow x > 4$



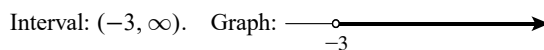
17. $7 - x \geq 5 \Leftrightarrow -x \geq -2 \Leftrightarrow x \leq 2$



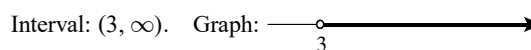
19. $2x + 1 < 0 \Leftrightarrow 2x < -1 \Leftrightarrow x < -\frac{1}{2}$



21. $4x - 7 < 8 + 9x \Leftrightarrow -5x < 15 \Leftrightarrow x > -3$.

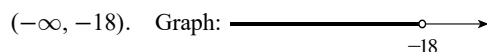


23. $x - \frac{3}{2} > \frac{1}{2}x \Leftrightarrow \frac{1}{2}x > \frac{3}{2} \Leftrightarrow x > 3$

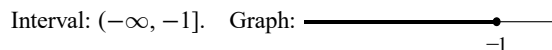


25. $\frac{1}{3}x + 2 < \frac{1}{6}x - 1 \Leftrightarrow \frac{1}{6}x < -3 \Leftrightarrow x < -18$

Interval:



27. $4 - 3x \leq -(1 + 8x) \Leftrightarrow 4 - 3x \leq -1 - 8x \Leftrightarrow 5x \leq -5 \Leftrightarrow x \leq -1$



29. $2 \leq x + 5 < 4 \Leftrightarrow -3 \leq x < -1$

Interval: $[-3, -1)$. Graph:

31. $-1 < 2x - 5 < 7 \Leftrightarrow 4 < 2x < 12 \Leftrightarrow 2 < x < 6$

Interval: $(2, 6)$. Graph:

33. $-2 < 8 - 2x \leq -1 \Leftrightarrow -10 < -2x \leq -9 \Leftrightarrow 5 > x \geq \frac{9}{2} \Leftrightarrow \frac{9}{2} \leq x < 5$

Interval: $[\frac{9}{2}, 5)$. Graph:

35. $\frac{1}{6} < \frac{2x - 13}{12} \leq \frac{2}{3} \Leftrightarrow 2 < 2x - 13 \leq 8$ (multiply each expression by 12) $\Leftrightarrow 15 < 2x \leq 21 \Leftrightarrow \frac{15}{2} < x \leq \frac{21}{2}$.

Interval: $(\frac{15}{2}, \frac{21}{2}]$. Graph: 37. $(x + 2)(x - 3) < 0$. The expression on the left of the inequality changes sign where $x = -2$ and where $x = 3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 3)$	$(3, \infty)$
Sign of $x + 2$	-	+	+
Sign of $x - 3$	-	-	+
Sign of $(x + 2)(x - 3)$	+	-	+

From the table, the solution set is $\{x \mid -2 < x < 3\}$. Interval: $(-2, 3)$.

Graph:

39. $x(2x + 7) \geq 0$. The expression on the left of the inequality changes sign where $x = 0$ and where $x = -\frac{7}{2}$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -\frac{7}{2})$	$(-\frac{7}{2}, 0)$	$(0, \infty)$
Sign of x	-	-	+
Sign of $2x + 7$	-	+	+
Sign of $x(2x + 7)$	+	-	+

From the table, the solution set is $\{x \mid x \leq -\frac{7}{2} \text{ or } 0 \leq x\}$.Interval: $(-\infty, -\frac{7}{2}] \cup [0, \infty)$.

Graph:

41. $x^2 - 3x - 18 \leq 0 \Leftrightarrow (x + 3)(x - 6) \leq 0$. The expression on the left of the inequality changes sign where $x = 6$ and where $x = -3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, 6)$	$(6, \infty)$
Sign of $x + 3$	-	+	+
Sign of $x - 6$	-	-	+
Sign of $(x + 3)(x - 6)$	+	-	+

From the table, the solution set is $\{x \mid -3 \leq x \leq 6\}$. Interval: $[-3, 6]$.

Graph:

43. $3x^2 + 5x \geq 2 \Leftrightarrow 3x^2 + 5x - 2 \geq 0 \Leftrightarrow (x + 2)(3x - 1) \geq 0$. The expression on the left of the inequality changes sign where $x = -2$ and where $x = \frac{1}{3}$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, \frac{1}{3})$	$(\frac{1}{3}, \infty)$
Sign of $x + 2$	-	+	+
Sign of $3x - 1$	-	-	+
Sign of $(x + 2)(3x - 1)$	+	-	+

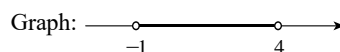
From the table, the solution set is $\{x \mid x \leq -2 \text{ or } x \geq \frac{1}{3}\}$.Interval: $(-\infty, -2] \cup [\frac{1}{3}, \infty)$.

Graph:

45. $3x^2 - 3x < 2x^2 + 4 \Leftrightarrow x^2 - 3x - 4 < 0 \Leftrightarrow (x + 1)(x - 4) < 0$. The expression on the left of the inequality changes sign where $x = -1$ and where $x = 4$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
Sign of $x + 1$	-	+	+
Sign of $x - 4$	-	-	+
Sign of $(x + 1)(x - 4)$	+	-	+

From the table, the solution set is $\{x \mid -1 < x < 4\}$. Interval: $(-1, 4)$.

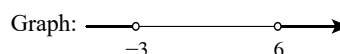


47. $x^2 > 3(x + 6) \Leftrightarrow x^2 - 3x - 18 > 0 \Leftrightarrow (x + 3)(x - 6) > 0$. The expression on the left of the inequality changes sign where $x = 6$ and where $x = -3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, 6)$	$(6, \infty)$
Sign of $x + 3$	-	+	+
Sign of $x - 6$	-	-	+
Sign of $(x + 3)(x - 6)$	+	-	+

From the table, the solution set is $\{x \mid x < -3 \text{ or } 6 < x\}$.

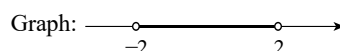
Interval: $(-\infty, -3) \cup (6, \infty)$.



49. $x^2 < 4 \Leftrightarrow x^2 - 4 < 0 \Leftrightarrow (x + 2)(x - 2) < 0$. The expression on the left of the inequality changes sign where $x = -2$ and where $x = 2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Sign of $x + 2$	-	+	+
Sign of $x - 2$	-	-	+
Sign of $(x + 2)(x - 2)$	+	-	+

From the table, the solution set is $\{x \mid -2 < x < 2\}$. Interval: $(-2, 2)$.



51. $(x + 2)(x - 1)(x - 3) \leq 0$. The expression on the left of the inequality changes sign when $x = -2$, $x = 1$, and $x = 3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, 3)$	$(3, \infty)$
Sign of $x + 2$	-	+	+	+
Sign of $x - 1$	-	-	+	+
Sign of $x - 3$	-	-	-	+
Sign of $(x + 2)(x - 1)(x - 3)$	-	+	-	+

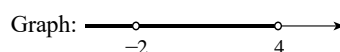
From the table, the solution set is $\{x \mid x \leq -2 \text{ or } 1 \leq x \leq 3\}$. Interval: $(-\infty, -2] \cup [1, 3]$.



53. $(x - 4)(x + 2)^2 < 0$. Note that $(x + 2)^2 > 0$ for all $x \neq -2$, so the expression on the left of the original inequality changes sign only when $x = 4$. We check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 4)$	$(4, \infty)$
Sign of $x - 4$	-	-	+
Sign of $(x + 2)^2$	+	+	+
Sign of $(x - 4)(x + 2)^2$	-	-	+

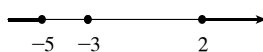
From the table, the solution set is $\{x \mid x \neq -2 \text{ and } x < 4\}$. We exclude the endpoint -2 since the original expression cannot be 0. Interval: $(-\infty, -2) \cup (-2, 4)$.



55. $(x + 3)^2 (x - 2) (x + 5) \geq 0$. The left-hand side is 0 when $x = -5, -3$, or 2 . We check the intervals in the following table.

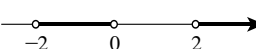
Interval	$(-\infty, -5)$	$(-5, -3)$	$(-3, 2)$	$(2, \infty)$
Sign of $(x + 3)^2$	+	+	+	+
Sign of $x + 5$	-	+	+	+
Sign of $x - 2$	-	-	-	+
Sign of $(x - 2)^2 (x - 3) (x + 1)$	+	-	-	+

When $x = -3$, the left-hand side is equal to 0 and the inequality is satisfied. Thus, the solution set is

$\{x \mid x \leq -5, x = -3, \text{ or } x \geq 2\}$. Interval: $(-\infty, -5] \cup \{-3\} \cup [2, \infty)$. Graph: 

57. $x^3 - 4x > 0 \Leftrightarrow x(x^2 - 4) > 0 \Leftrightarrow x(x + 2)(x - 2) > 0$. The expression on the left of the inequality changes sign where $x = 0, x = -2$ and where $x = 4$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
Sign of x	-	-	+	+
Sign of $x + 2$	-	+	+	+
Sign of $x - 2$	-	-	-	+
Sign of $x(x + 2)(x - 2)$	-	+	-	+

From the table, the solution set is $\{x \mid -2 < x < 0 \text{ or } x > 2\}$. Interval: $(-2, 0) \cup (2, \infty)$. Graph: 

59. $x^4 > x^2 \Leftrightarrow x^4 - x^2 > 0 \Leftrightarrow x^2(x^2 - 1) > 0 \Leftrightarrow x^2(x - 1)(x + 1) > 0$. The expression on the left of the inequality changes sign where $x = 0$, where $x = 1$, and where $x = -1$. Thus we must check the intervals in the following table.

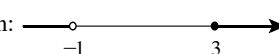
Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of x^2	+	+	+	+
Sign of $x - 1$	-	-	-	+
Sign of $x + 1$	-	+	+	+
Sign of $x^2(x - 1)(x + 1)$	+	-	-	+

From the table, the solution set is $\{x \mid x < -1 \text{ or } 1 < x\}$. Interval: $(-\infty, -1) \cup (1, \infty)$. Graph: 

61. $\frac{x - 3}{x + 1} \geq 0$. The expression on the left of the inequality changes sign where $x = -1$ and where $x = 3$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, 3)$	$(3, \infty)$
Sign of $x + 1$	-	+	+
Sign of $x - 3$	-	-	+
Sign of $\frac{x - 3}{x + 1}$	+	-	+

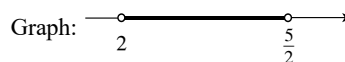
From the table, the solution set is $\{x \mid x < -1 \text{ or } x \leq 3\}$. Since the denominator cannot equal 0 we must have $x \neq -1$. Interval: $(-\infty, -1) \cup [3, \infty)$.

Graph: 

63. $\frac{x}{x-2} > 5 \Leftrightarrow \frac{x}{x-2} - 5 > 0 \Leftrightarrow \frac{x}{x-2} - \frac{5(x-2)}{x-2} > 0 \Leftrightarrow \frac{-4x+10}{x-2} > 0 \Leftrightarrow \frac{-2x+5}{x-2} > 0$. The expression on the left of the inequality changes sign when $x = 2$ and $x = \frac{5}{2}$. Thus we must check the intervals in the following table.

Interval	$(-\infty, 2)$	$(2, \frac{5}{2})$	$(\frac{5}{2}, \infty)$
Sign of $-2x + 5$	+	+	-
Sign of $x - 2$	-	+	+
Sign of $\frac{-2x+5}{x-2}$	-	+	-

From the table, the solution set is $\{x \mid 2 < x < \frac{5}{2}\}$. Interval: $(2, \frac{5}{2})$.

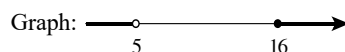


65. $\frac{2x+1}{x-5} \leq 3 \Leftrightarrow \frac{2x+1}{x-5} - 3 \leq 0 \Leftrightarrow \frac{2x+1}{x-5} - \frac{3(x-5)}{x-5} \leq 0 \Leftrightarrow \frac{-x+16}{x-5} \leq 0$. The expression on the left of the inequality changes sign where $x = 16$ and where $x = 5$. Thus we must check the intervals in the following table.

Interval	$(-\infty, 5)$	$(5, 16)$	$(16, \infty)$
Sign of $-x + 16$	+	+	-
Sign of $x - 5$	-	+	+
Sign of $\frac{-x+16}{x-5}$	-	+	-

From the table, the solution set is $\{x \mid x < 5 \text{ or } x \geq 16\}$. Since the denominator cannot equal 0, we must have $x \neq 5$.

Interval: $(-\infty, 5) \cup [16, \infty)$.



67. $\frac{4}{x} < x \Leftrightarrow \frac{4}{x} - x < 0 \Leftrightarrow \frac{4}{x} - \frac{x \cdot x}{x} < 0 \Leftrightarrow \frac{4-x^2}{x} < 0 \Leftrightarrow \frac{(2-x)(2+x)}{x} < 0$. The expression on the left of the inequality changes sign where $x = 0$, where $x = -2$, and where $x = 2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
Sign of $2 + x$	-	+	+	+
Sign of x	-	-	+	+
Sign of $2 - x$	+	+	+	-
Sign of $\frac{(2-x)(2+x)}{x}$	+	-	+	-

From the table, the solution set is $\{x \mid -2 < x < 0 \text{ or } 2 < x\}$. Interval: $(-2, 0) \cup (2, \infty)$. Graph:

69. $1 + \frac{2}{x+1} \leq \frac{2}{x} \Leftrightarrow 1 + \frac{2}{x+1} - \frac{2}{x} \leq 0 \Leftrightarrow \frac{x(x+1)}{x(x+1)} + \frac{2x}{x(x+1)} - \frac{2(x+1)}{x(x+1)} \leq 0 \Leftrightarrow \frac{x^2 + x + 2x - 2x - 2}{x(x+1)} \leq 0 \Leftrightarrow \frac{x^2 + x - 2}{x(x+1)} \leq 0 \Leftrightarrow \frac{(x+2)(x-1)}{x(x+1)} \leq 0$. The expression on the left of the inequality changes sign where $x = -2$, where $x = -1$, where $x = 0$, and where $x = 1$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -2)$	$(-2, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of $x + 2$	-	+	+	+	+
Sign of $x - 1$	-	-	-	-	+
Sign of x	-	-	-	+	+
Sign of $x + 1$	-	-	+	+	+
Sign of $\frac{(x+2)(x-1)}{x(x+1)}$	+	-	+	-	+

Since $x = -1$ and $x = 0$ yield undefined expressions, we cannot include them in the solution. From the table, the solution

set is $\{x \mid -2 \leq x < -1 \text{ or } 0 < x \leq 1\}$. Interval: $[-2, -1) \cup (0, 1]$. Graph:

71. $\frac{x+2}{x+3} < \frac{x-1}{x-2} \Leftrightarrow \frac{x+2}{x+3} - \frac{x-1}{x-2} < 0 \Leftrightarrow \frac{(x+2)(x-2)}{(x+3)(x-2)} - \frac{(x-1)(x+3)}{(x-2)(x+3)} < 0 \Leftrightarrow \frac{x^2 - 4 - x^2 - 2x + 3}{(x+3)(x-2)} < 0 \Leftrightarrow \frac{-2x - 1}{(x+3)(x-2)} < 0$. The expression on the left of the inequality

changes sign where $x = -\frac{1}{2}$, where $x = -3$, and where $x = 2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, -\frac{1}{2})$	$(-\frac{1}{2}, 2)$	$(2, \infty)$
Sign of $-2x - 1$	+	+	-	-
Sign of $x + 3$	-	+	+	+
Sign of $x - 2$	-	-	-	+
Sign of $\frac{-2x - 1}{(x+3)(x-2)}$	+	-	+	-

From the table, the solution set is $\{x \mid -3 < x < -\frac{1}{2} \text{ or } 2 < x\}$. Interval: $(-3, -\frac{1}{2}) \cup (2, \infty)$.

Graph:

73. $|5x| < 20 \Leftrightarrow |x| < 4 \Leftrightarrow -4 < x < 4$. Interval: $(-4, 4)$.

Graph:

75. $|2x| > 7$ is equivalent to $2x > 7 \Leftrightarrow x > \frac{7}{2}$; or $2x < -7 \Leftrightarrow x < -\frac{7}{2}$.

Interval: $(-\infty, -\frac{7}{2}) \cup (\frac{7}{2}, \infty)$.

Graph:

77. $|x - 3| \leq 10 \Leftrightarrow -10 \leq x - 3 \leq 10 \Leftrightarrow -7 \leq x \leq 13$.
Interval: $[-7, 13]$.

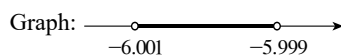


81. $|3x - 2| \geq 5 \Leftrightarrow |x - \frac{2}{3}| \geq \frac{5}{3}$, so either $x - \frac{2}{3} \geq \frac{5}{3} \Leftrightarrow x \geq \frac{7}{3}$ or $x - \frac{2}{3} \leq -\frac{5}{3} \Leftrightarrow x \leq -1$.

Interval: $(-\infty, -1] \cup [\frac{7}{3}, \infty)$.



85. $|x + 6| < 0.001 \Leftrightarrow -0.001 < x + 6 < 0.001 \Leftrightarrow -6.001 < x < -5.999$.
Interval: $(-6.001, -5.999)$.



89. $0 < 5 - 2x \Leftrightarrow 2x < 5 \Leftrightarrow x < \frac{5}{2}$

Interval: $(-\infty, \frac{5}{2})$. Graph:

91. $16x \leq x^3 \Leftrightarrow 0 \leq x^3 - 16x = x(x^2 - 16) = x(x - 4)(x + 4)$. The expression on the left of the inequality changes sign when $x = -4$, $x = 0$, and $x = 4$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -4)$	$(-4, 0)$	$(0, 4)$	$(4, \infty)$
Sign of $x + 4$	-	+	+	+
Sign of x	-	-	+	+
Sign of $x - 4$	-	-	-	+
Sign of $x(x + 4)(x - 4)$	-	+	-	+

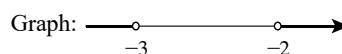
From the table, the solution set is $\{x \mid -4 \leq x \leq 0 \text{ or } 4 \leq x\}$. Interval: $[-4, 0] \cup [4, \infty)$. Graph:

93. $x^2 + 5x + 6 > 0 \Leftrightarrow (x + 3)(x + 2) > 0$. The expression on the left of the inequality changes sign when $x = -3$ and $x = -2$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -3)$	$(-3, -2)$	$(-2, \infty)$
Sign of $x + 3$	-	+	+
Sign of $x + 2$	-	-	+
Sign of $(x + 3)(x + 2)$	+	-	+

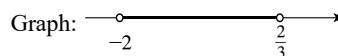
From the table, the solution set is $\{x \mid x < -3 \text{ or } -2 < x\}$.

Interval: $(-\infty, -3) \cup (-2, \infty)$.



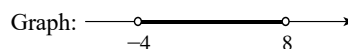
79. $|3x + 2| < 4 \Leftrightarrow -4 < 3x + 2 < 4 \Leftrightarrow -6 < 3x < 2 \Leftrightarrow -2 < x < \frac{2}{3}$.

Interval: $(-2, \frac{2}{3})$



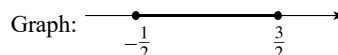
83. $|\frac{x-2}{3}| < 2 \Leftrightarrow -2 < \frac{x-2}{3} < 2 \Leftrightarrow -6 < x-2 < 6 \Leftrightarrow -4 < x < 8$.

Interval: $(-4, 8)$.



87. $8 - |2x - 1| \geq 6 \Leftrightarrow -|2x - 1| \geq -2 \Leftrightarrow |2x - 1| \leq 2 \Leftrightarrow -2 \leq 2x - 1 \leq 2 \Leftrightarrow -1 \leq 2x \leq 3 \Leftrightarrow -\frac{1}{2} \leq x \leq \frac{3}{2}$.

Interval: $[-\frac{1}{2}, \frac{3}{2}]$.



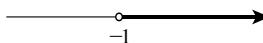
95. $(x+3)^2(x+1) > 0$. Note that $(x+3)^2 > 0$ for all $x \neq -3$, so the expression on the left of the original inequality changes sign only when $x = -1$. We check the intervals in the following table.

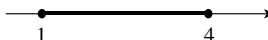
Interval	$(-\infty, -3)$	$(-3, -1)$	$(-1, \infty)$
Sign of $(x+3)^2$	+	+	+
Sign of $x+1$	-	-	+
Sign of $(x+3)^2(x+1)$	-	-	+

From the table, the solution set is $\{x \mid x > -1\}$.

(The endpoint -3 is already excluded.)

Interval: $(-1, \infty)$.

Graph: 

97. $1 \geq 3 - 2x \geq -5 \Leftrightarrow -2 \geq -2x \geq -8 \Leftrightarrow 1 \leq x \leq 4$. Interval: $[1, 4]$. Graph: 

99. $|x| < 3$ 101. $|x - 7| \geq 5$ 103. $|x| \leq 2$ 105. $|x| > 3$ 107. $|x - 1| \leq 3$

109. For $\sqrt{x^2 - 9}$ to be defined as a real number we must have $x^2 - 9 \geq 0 \Leftrightarrow (x+3)(x-3) \geq 0$. The expression in the inequality changes sign at $x = -3$ and $x = 3$.

Interval	$(-\infty, -3)$	$(-3, 3)$	$(3, \infty)$
Sign of $x+3$	-	+	+
Sign of $x-3$	-	-	+
Sign of $(x+3)(x-3)$	+	-	+

Thus $x \leq -3$ or $x \geq 3$.

111. For $\left(\frac{1}{x^2 - 3x - 10}\right)^{1/2}$ to be defined as a real number we must have $x^2 - 3x - 10 > 0 \Leftrightarrow (x+2)(x-5) > 0$. The expression in the last inequality changes sign at $x = -2$ and $x = 5$.

Interval	$(-\infty, -2)$	$(-2, 5)$	$(5, \infty)$
Sign of $x+2$	-	+	+
Sign of $x-5$	-	-	+
Sign of $(x+2)(x-5)$	+	-	+

Thus $x < -2$ or $x > 5$, and the solution set is $(-\infty, -2) \cup (5, \infty)$.

113. $a(bx - c) \geq bc$ (where $a, b, c > 0$) $\Leftrightarrow bx - c \geq \frac{bc}{a} \Leftrightarrow bx \geq \frac{bc}{a} + c \Leftrightarrow x \geq \frac{1}{b} \left(\frac{bc}{a} + c \right) = \frac{c}{a} + \frac{c}{b} \Leftrightarrow x \geq \frac{c}{a} + \frac{c}{b} \Leftrightarrow x \geq \frac{cb + ca}{ab} = \frac{(a+b)c}{ab}$.

115. $a|bx - c| + d \geq 4a$, where $a, b, c > 0 \Leftrightarrow |bx - c| \geq \frac{4a - d}{a} \Leftrightarrow -(bx - c) \geq \frac{4a - d}{a}$ or $bx - c \geq \frac{4a - d}{a} \Leftrightarrow -bx \geq \frac{4a - d}{a} - c$ or $bx \geq \frac{4a - d}{a} + c \Leftrightarrow x \leq \frac{ac - 4a + d}{ab}$ or $x \geq \frac{ac + 4a - d}{ab}$.

117. Inserting the relationship $C = \frac{5}{9}(F - 32)$, we have $20 \leq C \leq 30 \Leftrightarrow 20 \leq \frac{5}{9}(F - 32) \leq 30 \Leftrightarrow 36 \leq F - 32 \leq 54 \Leftrightarrow 68 \leq F \leq 86$.

119. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

Let x be the average number of kilometers driven per day. Each day the cost of Plan A is $95 + 0.40x$, and the cost of Plan B is 135. Plan B saves money when $135 < 95 + 0.40x \Leftrightarrow 40 < 0.4x \Leftrightarrow 100 < x$. So Plan B saves money when you average more than 100 kilometers a day.

121. We need to solve $6400 \leq 0.35m + 2200 \leq 7100$ for m . So $6400 \leq 0.35m + 2200 \leq 7100 \Leftrightarrow 4200 \leq 0.35m \leq 4900 \Leftrightarrow 12,000 \leq m \leq 14,000$. She plans on driving between 12,000 and 14,000 miles.

123. (a) Let x be the number of \$3 increases. Then the number of seats sold is $120 - x$. So $P = 200 + 3x$
 $\Leftrightarrow 3x = P - 200 \Leftrightarrow x = \frac{1}{3}(P - 200)$. Substituting for x we have that the number of seats sold is
 $120 - x = 120 - \frac{1}{3}(P - 200) = -\frac{1}{3}P + \frac{560}{3}$.
- (b) $90 \leq -\frac{1}{3}P + \frac{560}{3} \leq 115 \Leftrightarrow 270 \leq 360 - P + 200 \leq 345 \Leftrightarrow 270 \leq -P + 560 \leq 345 \Leftrightarrow -290 \leq -P \leq -215 \Leftrightarrow$
 $290 \geq P \geq 215$. Putting this into standard order, we have $215 \leq P \leq 290$. So the ticket prices are between \$215 and \$290.
125. $0.0004 \leq \frac{4,000,000}{d^2} \leq 0.01$. Since $d^2 \geq 0$ and $d \neq 0$, we can multiply each expression by d^2 to obtain
 $0.0004d^2 \leq 4,000,000 \leq 0.01d^2$. Solving each pair, we have $0.0004d^2 \leq 4,000,000 \Leftrightarrow d^2 \leq 10,000,000,000$
 $\Rightarrow d \leq 100,000$ (recall that d represents distance, so it is always nonnegative). Solving $4,000,000 \leq 0.01d^2 \Leftrightarrow$
 $400,000,000 \leq d^2 \Rightarrow 20,000 \leq d$. Putting these together, we have $20,000 \leq d \leq 100,000$.
127. **Note: In the first printing of the text, the answer given for part (a) of this exercise is incorrect.**
- (a) We substitute $d = 402$ m and $t = 10$ s into the given formula and find the value of a that results in a quarter-mile time of exactly 10 s: $402 = \frac{1}{2}a(10)^2 \Leftrightarrow 402 = 50a \Leftrightarrow a = \frac{402}{50} = 8.04$ m/s². Thus, the quarter-mile time will be less than 10 s if $a > 8.04$ m/s².
- (b) We substitute $a = g = 9.8$ m/s² and solve for t : $402 = \frac{1}{2}(9.8)t^2 \Leftrightarrow t^2 = \frac{2(402)}{9.8}$. Taking the positive root, we find that the quarter-mile time for a (downward) quarter-mile under Earth's gravity is $\sqrt{\frac{2(402)}{9.8}} \approx 9.1$ s.
129. $70 \geq v + \frac{v^2}{35} \Leftrightarrow \frac{1}{35}v^2 + v - 70 \leq 0 \Leftrightarrow \left(\frac{1}{35}v - 1\right)(v + 70) \leq 0$. The expression in the inequality changes sign at $v = 35$ and $v = -70$. However, since v represents the speed, we must have $v \geq 0$.

Interval	(0, 35)	(35, ∞)
Sign of $\frac{1}{35}v - 1$	−	+
Sign of $v + 70$	+	+
Sign of $\left(\frac{1}{35}v - 1\right)(v + 70)$	−	+

So you must drive between 0 and 35 km/h.

131. Let x be the length of the garden and w its width. Using the fact that the perimeter is 120 m, we must have $2x + 2w = 120$
 $\Leftrightarrow w = 60 - x$. Now since the area must be at least 800 m², we have $800 < x(60 - x) \Leftrightarrow 800 < 60x - x^2 \Leftrightarrow$
 $x^2 - 60x + 800 < 0 \Leftrightarrow (x - 20)(x - 40) < 0$. The expression in the inequality changes sign at $x = 20$ and $x = 40$.
 However, since x represents length, we must have $x > 0$.

Interval	(0, 20)	(20, 40)	(40, ∞)
Sign of $x - 20$	−	+	+
Sign of $x - 40$	−	−	+
Sign of $(x - 20)(x - 40)$	+	−	+

The length of the garden should be between 20 and 40 meters.

133. $\left|\frac{h - 170.5}{7.25}\right| \leq 2 \Leftrightarrow -2 \leq \frac{h - 170.5}{7.25} \leq 2 \Leftrightarrow -14.5 \leq h - 170.5 \leq 14.5 \Leftrightarrow 156 \leq h \leq 185$. Thus 95% of the adult males are between 156 cm and 185 cm.

135. The rule we want to apply here is “ $a < b \Rightarrow ac < bc$ if $c > 0$ and $a < b \Rightarrow ac > bc$ if $c < 0$ ”. Thus we cannot simply multiply by x , since we don’t yet know if x is positive or negative, so in solving $1 < \frac{3}{x}$, we must consider two cases.

Case 1: $x > 0$ Multiplying both sides by x , we have $x < 3$. Together with our initial condition, we have $0 < x < 3$.

Case 2: $x < 0$ Multiplying both sides by x , we have $x > 3$. But $x < 0$ and $x > 3$ have no elements in common, so this gives no additional solution. Hence, the only solutions are $0 < x < 3$.

137. $a < b$, so by Rule 1, $a + c < b + c$. Using Rule 1 again, $b + c < b + d$, and so by transitivity, $a + c < b + d$.

139. (a) Because x is nonnegative, $x \leq y \Leftrightarrow x^2 \leq xy$, and because y is nonnegative, $x \leq y \Leftrightarrow xy \leq y^2$. Thus, by transitivity, $x^2 \leq y^2$.

(b) By part (a), $\sqrt{xy} \leq \frac{x+y}{2} \Leftrightarrow xy \leq \frac{(x+y)^2}{4}$. Expanding, this becomes $4xy \leq (x+y)^2 = x^2 + y^2 + 2xy \Leftrightarrow x^2 + y^2 - 2xy \geq 0 \Leftrightarrow (x-y)^2 \geq 0$. This is true for any x and y , so the original inequality is true for all nonnegative x and y .

1.9 THE COORDINATE PLANE; GRAPHS OF EQUATIONS; CIRCLES

1. (a) The point that is 3 units to the right of the y -axis and 5 units below the x -axis has coordinates $(3, -5)$.

(b) The point $(2, 7)$ is 2 units to the right of the y -axis and 7 units above the x -axis, so it is closer to the y -axis.

3. The point midway between (a, b) and (c, d) is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$. So the point midway between $(1, 2)$ and $(7, 10)$ is

$$\left(\frac{1+7}{2}, \frac{2+10}{2}\right) = \left(\frac{8}{2}, \frac{12}{2}\right) = (4, 6).$$

5. (a) To find the x -intercept(s) of the graph of an equation we set y equal to 0 in the equation and solve for x : $2(0) = x + 1 \Leftrightarrow x = -1$, so the x -intercept of $2y = x + 1$ is -1 .

(b) To find the y -intercept(s) of the graph of an equation we set x equal to 0 in the equation and solve for y : $2y = 0 + 1 \Leftrightarrow y = \frac{1}{2}$, so the y -intercept of $2y = x + 1$ is $\frac{1}{2}$.

7. (a) If a graph is symmetric with respect to the x -axis and (a, b) is on the graph, then $(a, -b)$ is also on the graph.

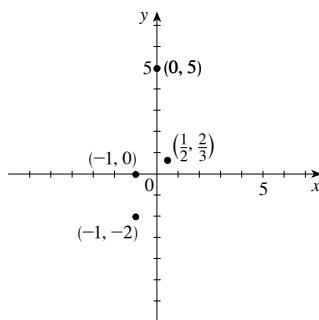
(b) If a graph is symmetric with respect to the y -axis and (a, b) is on the graph, then $(-a, b)$ is also on the graph.

(c) If a graph is symmetric about the origin and (a, b) is on the graph, then $(-a, -b)$ is also on the graph.

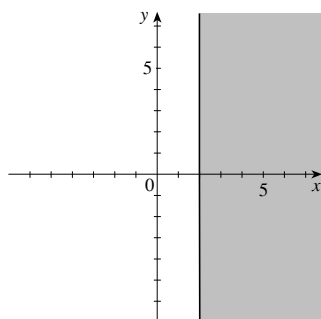
9. Yes. If (a, b) is on the graph, then by symmetry about the x -axis, the point $(a, -b)$ is on the graph. Then by symmetry about the y -axis, the point $(-a, -b)$ is on the graph. Thus, the graph is symmetric with respect to the origin.

11. The points have coordinates $A(5, 1)$, $B(1, 2)$, $C(-2, 6)$, $D(-6, 2)$, $E(-4, -1)$, $F(-2, 0)$, $G(-1, -3)$, and $H(2, -2)$.

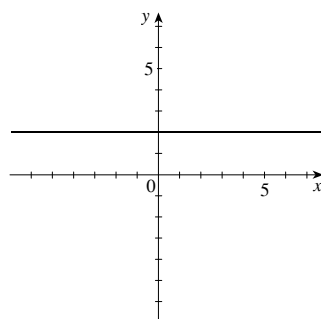
13.



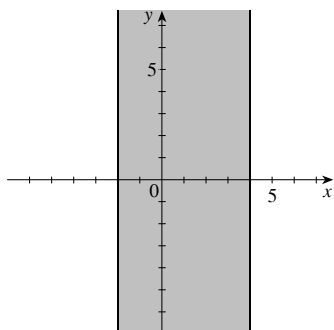
15. (a)



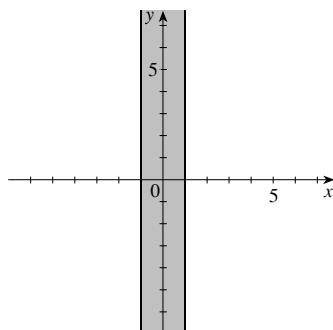
(b)



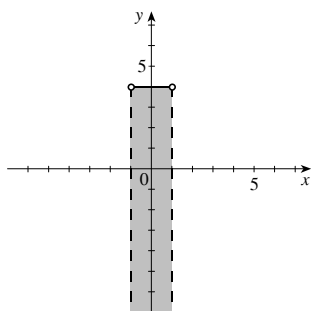
17. (a)



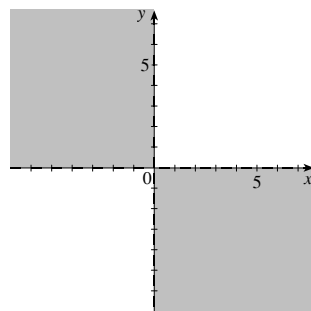
(b)



19. (a)



(b)

21. The two points are $(0, 2)$ and $(3, 0)$.

$$(a) d = \sqrt{(3-0)^2 + (0-2)^2} = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$$

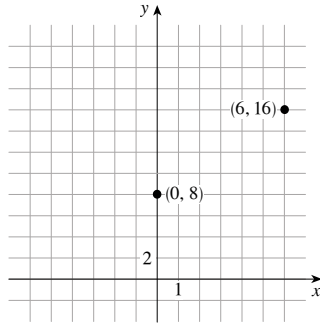
$$(b) \text{ midpoint: } \left(\frac{3+0}{2}, \frac{0+2}{2} \right) = \left(\frac{3}{2}, 1 \right)$$

23. The two points are $(-3, 3)$ and $(5, -3)$.

$$(a) d = \sqrt{(-3-5)^2 + (3-(-3))^2} = \sqrt{(-8)^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10$$

$$(b) \text{ midpoint: } \left(\frac{-3+5}{2}, \frac{3+(-3)}{2} \right) = (1, 0)$$

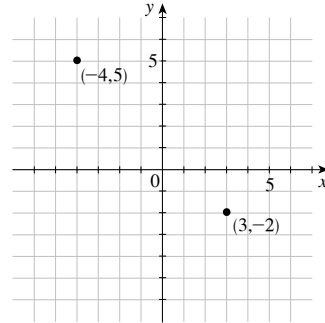
25. (a)



$$\begin{aligned} \text{(b)} \quad d &= \sqrt{(0-6)^2 + (8-16)^2} \\ &= \sqrt{(-6)^2 + (-8)^2} = \sqrt{100} = 10 \end{aligned}$$

$$\text{(c) Midpoint: } \left(\frac{0+6}{2}, \frac{8+16}{2} \right) = (3, 12)$$

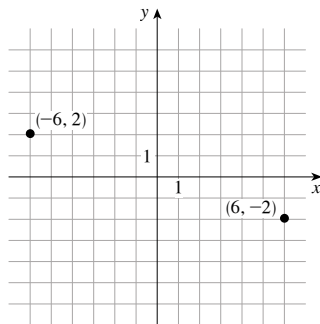
27. (a)



$$\begin{aligned} \text{(b)} \quad d &= \sqrt{(-3-(-4))^2 + (-2-5)^2} \\ &= \sqrt{7^2 + (-7)^2} = \sqrt{49+49} = \sqrt{98} = 7\sqrt{2} \end{aligned}$$

$$\text{(c) Midpoint: } \left(\frac{-4+3}{2}, \frac{5-2}{2} \right) = \left(-\frac{1}{2}, \frac{3}{2} \right)$$

29. (a)

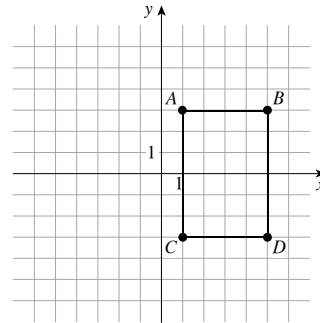


$$\begin{aligned} \text{(b)} \quad d &= \sqrt{(6-(-6))^2 + (-2-2)^2} = \sqrt{12^2 + (-4)^2} \\ &= \sqrt{144+16} = \sqrt{160} = 4\sqrt{10} \end{aligned}$$

$$\text{(c) Midpoint: } \left(\frac{6-6}{2}, \frac{-2+2}{2} \right) = (0, 0)$$

$$31. \quad d(A, B) = \sqrt{(1-5)^2 + (3-3)^2} = \sqrt{(-4)^2} = 4.$$

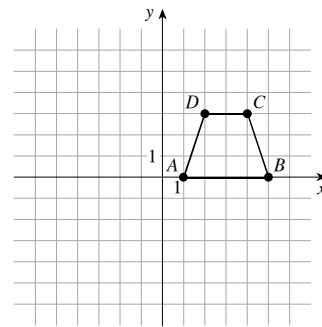
$$d(A, C) = \sqrt{(1-1)^2 + (3-(-3))^2} = \sqrt{(6)^2} = 6. \text{ So the area is } 4 \cdot 6 = 24.$$



33. From the graph, the quadrilateral $ABCD$ has a pair of parallel sides, so $ABCD$ is a trapezoid. The area is $\left(\frac{b_1 + b_2}{2} \right) h$. From the graph we see that

$$b_1 = d(A, B) = \sqrt{(1-5)^2 + (0-0)^2} = \sqrt{4^2} = 4;$$

$$b_2 = d(C, D) = \sqrt{(4-2)^2 + (3-3)^2} = \sqrt{2^2} = 2; \text{ and } h \text{ is the difference in } y\text{-coordinates is } |3-0| = 3. \text{ Thus the area of the trapezoid is } \left(\frac{4+2}{2} \right) 3 = 9.$$



$$35. \quad d(0, A) = \sqrt{(6-0)^2 + (7-0)^2} = \sqrt{6^2 + 7^2} = \sqrt{36+49} = \sqrt{85}.$$

$$d(0, B) = \sqrt{(-5-0)^2 + (8-0)^2} = \sqrt{(-5)^2 + 8^2} = \sqrt{25+64} = \sqrt{89}.$$

Thus point $A(6, 7)$ is closer to the origin.

$$37. d(P, R) = \sqrt{(-1-3)^2 + (-1-1)^2} = \sqrt{(-4)^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}.$$

$$d(Q, R) = \sqrt{(-1-(-1))^2 + (-1-3)^2} = \sqrt{0+(-4)^2} = \sqrt{16} = 4. \text{ Thus point } Q(-1, 3) \text{ is closer to point } R.$$

39. Since we do not know which pair are isosceles, we find the length of all three sides.

$$d(A, B) = \sqrt{(-3-0)^2 + (-1-2)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}.$$

$$d(C, B) = \sqrt{(-3-(-4))^2 + (-1-3)^2} = \sqrt{1^2 + (-4)^2} = \sqrt{1+16} = \sqrt{17}.$$

$$d(A, C) = \sqrt{(0-(-4))^2 + (2-3)^2} = \sqrt{4^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}. \text{ So sides } AC \text{ and } CB \text{ have the same length.}$$

41. (a) Here we have $A = (2, 2)$, $B = (3, -1)$, and $C = (-3, -3)$. So

$$d(A, B) = \sqrt{(3-2)^2 + (-1-2)^2} = \sqrt{1^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10};$$

$$d(C, B) = \sqrt{(3-(-3))^2 + (-1-(-3))^2} = \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10};$$

$$d(A, C) = \sqrt{(-3-2)^2 + (-3-2)^2} = \sqrt{(-5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}.$$

Since $[d(A, B)]^2 + [d(C, B)]^2 = [d(A, C)]^2$, we conclude that the triangle is a right triangle.

(b) The area of the triangle is $\frac{1}{2} \cdot d(C, B) \cdot d(A, B) = \frac{1}{2} \cdot \sqrt{10} \cdot 2\sqrt{10} = 10$.

43. We show that all sides are the same length (its a rhombus) and then show that the diagonals are equal. Here we have

$A = (-2, 9)$, $B = (4, 6)$, $C = (1, 0)$, and $D = (-5, 3)$. So

$$d(A, B) = \sqrt{(4-(-2))^2 + (6-9)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45};$$

$$d(B, C) = \sqrt{(1-4)^2 + (0-6)^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{9+36} = \sqrt{45};$$

$$d(C, D) = \sqrt{(-5-1)^2 + (3-0)^2} = \sqrt{(-6)^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45};$$

$$d(D, A) = \sqrt{(-2-(-5))^2 + (9-3)^2} = \sqrt{3^2 + 6^2} = \sqrt{9+36} = \sqrt{45}. \text{ So the points form a}$$

rhombus. Also $d(A, C) = \sqrt{(1-(-2))^2 + (0-9)^2} = \sqrt{3^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$,

and $d(B, D) = \sqrt{(-5-4)^2 + (3-6)^2} = \sqrt{(-9)^2 + (-3)^2} = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$. Since the diagonals are equal, the rhombus is a square.

45. Let $P = (0, y)$ be such a point. Setting the distances equal we get

$$\sqrt{(0-5)^2 + (y-(-5))^2} = \sqrt{(0-1)^2 + (y-1)^2} \Leftrightarrow$$

$$\sqrt{25 + y^2 + 10y + 25} = \sqrt{1 + y^2 - 2y + 1} \Rightarrow y^2 + 10y + 50 = y^2 - 2y + 2 \Leftrightarrow 12y = -48 \Leftrightarrow y = -4. \text{ Thus, the point is } P = (0, -4). \text{ Check:}$$

$$\sqrt{(0-5)^2 + (-4-(-5))^2} = \sqrt{(-5)^2 + 1^2} = \sqrt{25+1} = \sqrt{26};$$

$$\sqrt{(0-1)^2 + (-4-1)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{25+1} = \sqrt{26}.$$

47. As indicated by Example 3, we must find a point $S(x_1, y_1)$ such that the midpoints of PR and of QS are the same. Thus

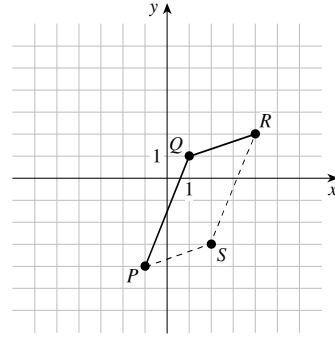
$$\left(\frac{4 + (-1)}{2}, \frac{2 + (-4)}{2}\right) = \left(\frac{x_1 + 1}{2}, \frac{y_1 + 1}{2}\right).$$

Setting the x -coordinates equal,

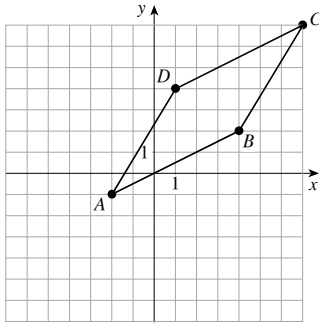
$$\text{we get } \frac{4 + (-1)}{2} = \frac{x_1 + 1}{2} \Leftrightarrow 4 - 1 = x_1 + 1 \Leftrightarrow x_1 = 2.$$

$$\text{Setting the } y\text{-coordinates equal, we get } \frac{2 + (-4)}{2} = \frac{y_1 + 1}{2} \Leftrightarrow 2 - 4 = y_1 + 1 \Leftrightarrow y_1 = -3.$$

Thus $S = (2, -3)$.



49. (a)



- (b) The midpoint of AC is $\left(\frac{-2 + 7}{2}, \frac{-1 + 7}{2}\right) = \left(\frac{5}{2}, 3\right)$, the midpoint of BD is $\left(\frac{4 + 1}{2}, \frac{2 + 4}{2}\right) = \left(\frac{5}{2}, 3\right)$.

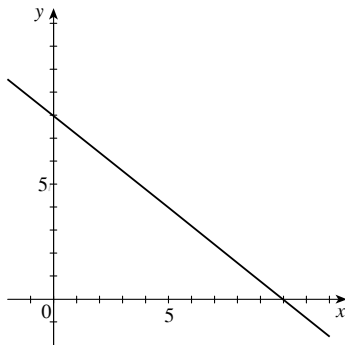
- (c) Since they have the same midpoint, we conclude that the diagonals bisect each other.

51. $(0, 5)$: $3(0) - 5 + 5 \stackrel{?}{=} 0 \Leftrightarrow 0 - 5 + 5 \stackrel{?}{=} 0$. Yes.
 $(2, 1)$: $3(2) - 1 + 5 \stackrel{?}{=} 0 \Leftrightarrow 6 - 1 + 5 \stackrel{?}{=} 0$. No.
 $(-2, -1)$: $3(-2) - (-1) + 5 \stackrel{?}{=} 0 \Leftrightarrow -6 + 1 + 5 \stackrel{?}{=} 0$. Yes.
 So $(0, 5)$ and $(-2, -1)$ are points on the graph of this equation.

53. $(0, -2)$: $(0)^2 + (0)(-2) + (-2)^2 \stackrel{?}{=} 4 \Leftrightarrow 0 + 0 + 4 \stackrel{?}{=} 4$. Yes.
 $(1, -2)$: $(1)^2 + (1)(-2) + (-2)^2 \stackrel{?}{=} 4 \Leftrightarrow 1 - 2 + 4 \stackrel{?}{=} 4$. No.
 $(2, -2)$: $(2)^2 + (2)(-2) + (-2)^2 \stackrel{?}{=} 4 \Leftrightarrow 4 - 4 + 4 \stackrel{?}{=} 4$. Yes.
 So $(0, -2)$ and $(2, -2)$ are points on the graph of this equation.

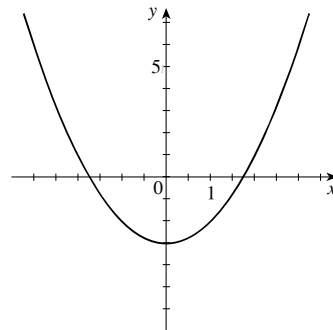
55. $4x + 5y = 40 \Leftrightarrow y = 8 - \frac{4}{5}x$

x	y
-2	9.6
0	8
2	6.4
4	4.8
6	3.2
8	1.6
10	0
12	-1.6



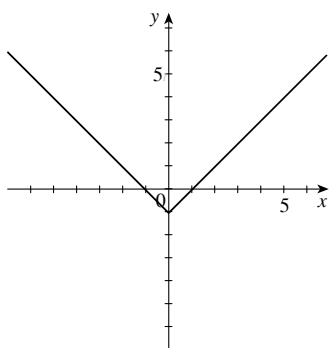
57. $y = x^2 - 3$

x	y
-3	6
-2	1
-1	-2
0	-3
1	-2
2	1
3	6



59. $y = |x| - 1$

x	y
-5	4
-3	2
-1	0
0	-1
1	0
3	2
5	4



61. (a) Solve for y : $2x - y = 6 \Leftrightarrow y = 2x - 6$.

x	y
-2	-10
-1	-8
0	-6
1	-4
2	-2
3	0
4	2

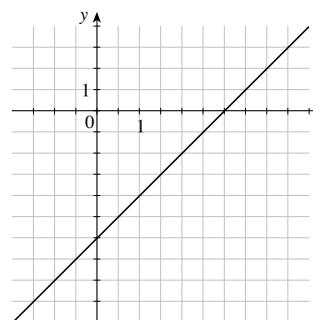
$y = 0 \Rightarrow 0 = 2x - 6 \Leftrightarrow x = 3$, so the x -intercept is 3, and

$x = 0 \Rightarrow y = 2(0) - 6 = -6$, so the y -intercept is -6.

x -axis symmetry: $2x + y = 6$, which is not the same as $2x - y = 6$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $-2x - y = 6$, which is not the same as $2x - y = 6$, so the graph is not symmetric with respect to the y -axis.

Origin symmetry: $-2x + y = 6$, which is not the same as $2x - y = 6$, so the graph is not symmetric with respect to the origin.



(b) $y = 2(x - 1)^2$

x	y
-2	18
-1	8
0	2
1	0
2	2
3	8

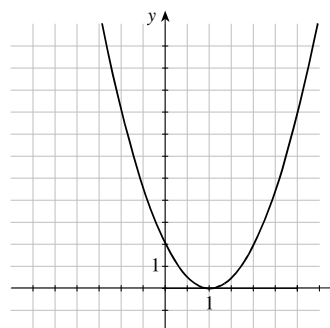
$y = 0 \Rightarrow 0 = 2(x - 1)^2 \Rightarrow x = 1$, so the x -intercept is 1,

and $x = 0 \Rightarrow y = 2(0 - 1)^2 = 2$, so the y -intercept is 2.

x -axis symmetry: $-y = 2(x - 1)^2$, which is not the same as $y = 2(x - 1)^2$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = 2(-x - 1)^2$, which is not the same as $y = 2(x - 1)^2$, so the graph is not symmetric with respect to the y -axis.

Origin symmetry: $-y = 2(-x - 1)^2 \Leftrightarrow y = -2(x + 1)^2$, which is not the same as $y = 2(x - 1)^2$, so the graph is not symmetric with respect to the origin.



63. (a) $y = \sqrt{x} + 2$

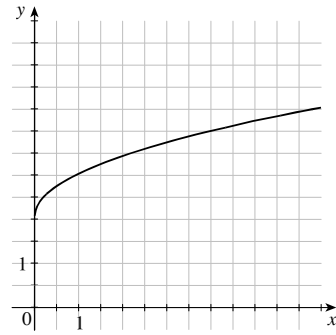
x	y
0	2
1	3
2	$1 + \sqrt{2}$
3	$1 + \sqrt{3}$
4	3
6	$1 + \sqrt{6}$
9	5

$y = 0 \Rightarrow 0 = \sqrt{x} + 2$, so there is no x -intercept, and $x = 0 \Rightarrow y = \sqrt{0} + 2 = 2$, so the y -intercept is 2.

x -axis symmetry: $-y = \sqrt{x} + 2$, which is not the same as $y = \sqrt{x} + 2$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = \sqrt{-x} + 2$, which is not the same as $y = \sqrt{x} + 2$, so the graph is not symmetric with respect to the y -axis.

Origin symmetry: $-y = \sqrt{-x} + 2 \Leftrightarrow y = -\sqrt{-x} - 2$, which is not the same as $y = \sqrt{x} + 2$, so the graph is not symmetric with respect to the origin.



(b) $y = -|x|$

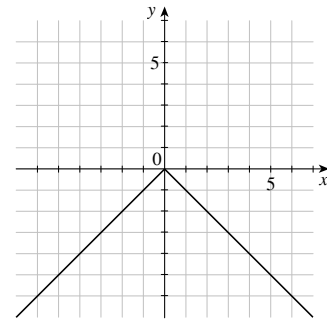
x	y
-5	-5
-3	-3
-1	-1
0	0
1	-1
3	-3
5	-5

$y = 0 \Rightarrow -|x| = 0 \Rightarrow x = 0$, so the x -intercept is 0, and $x = 0 \Rightarrow y = -|0| = 0$, so the y -intercept is 0.

x -axis symmetry: $-y = -|x| \Leftrightarrow y = |x|$, which is not the same as $y = -|x|$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = -|-x| = -|x|$, so the graph is symmetric with respect to the y -axis.

Origin symmetry: $-y = -|-x| \Leftrightarrow y = |x|$, which is not the same as $y = -|x|$, so the graph is not symmetric with respect to the origin.



65. (a) $y = \sqrt{4 - x^2}$

x	y
-2	0
-1	$\sqrt{3}$
0	2
1	$\sqrt{3}$
2	0

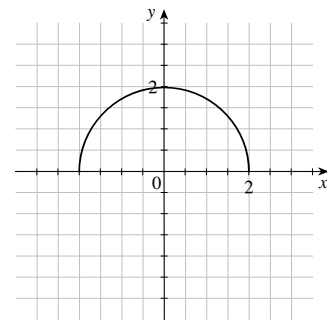
$y = 0 \Rightarrow \sqrt{4 - x^2} = 0 \Leftrightarrow x = \pm 2$, so the x -intercepts are ± 2 , and $x = 0 \Rightarrow y = \sqrt{4 - 0^2} = 2$, so the y -intercept is 2.

x -axis symmetry: $-y = \sqrt{4 - x^2}$, which is not the same as $y = \sqrt{4 - x^2}$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = \sqrt{4 - (-x)^2} = \sqrt{4 - x^2}$, so the graph is symmetric with respect to the y -axis.

Origin symmetry: $-y = \sqrt{4 - (-x)^2} = \sqrt{4 - x^2} \Leftrightarrow$

$y = -\sqrt{4 - x^2}$, which is not the same as $y = \sqrt{4 - x^2}$, so the graph is not symmetric with respect to the origin.



(b) $y = x^3 - 4x$

y	x
-3	15
-2	0
-1	3
0	0
1	3
2	0
3	15

$$y = 0 \Rightarrow 0 = x^3 - 4x = x(x^2 - 4) = x(x + 2)(x - 2), \text{ so}$$

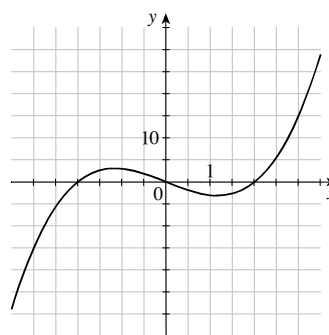
the x -intercepts are 0 and ± 2 , and $x = 0 \Rightarrow$

$$y = 0^3 - 4(0) = 0, \text{ so the } y\text{-intercept is } 0.$$

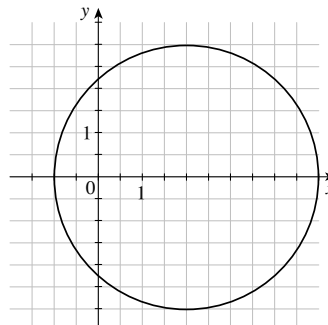
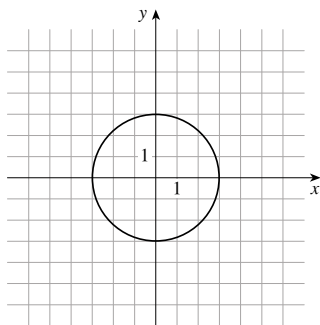
x -axis symmetry: $(-y) = x^3 - 4x \Leftrightarrow y = -x^3 + 4x$, which is not the same as $y = x^3 - 4x$, so the graph is not symmetric with respect to the x -axis.

y -axis symmetry: $y = (-x)^3 - 4(-x) = -x^3 + 4x$, which is not the same as $y = x^3 - 4x$, so the graph is not symmetric with respect to the y -axis.

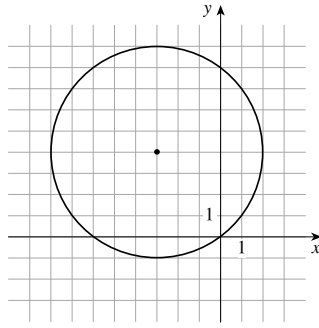
Origin symmetry: $-y = (-x)^3 - 4(-x) \Leftrightarrow y = x^3 - 4x$, so the graph is symmetric with respect to the origin.



67. (a) To find x -intercepts, set $y = 0$. This gives $0 = x + 6 \Leftrightarrow x = -6$, so the x -intercept is -6 . To find y -intercepts, set $x = 0$. This gives $y = 0 + 6 = 6$, so the y -intercept is 6 .
- (b) To find x -intercepts, set $y = 0$. This gives $0 = x^2 - 5 \Leftrightarrow x^2 = 5 \Leftrightarrow x = \pm\sqrt{5}$, so the x -intercepts are $\pm\sqrt{5}$. To find y -intercepts, set $x = 0$. This gives $y = 0^2 - 5 = -5$, so the y -intercept is -5 .
69. (a) To find x -intercepts, set $y = 0$. This gives $9x^2 - 4(0)^2 = 36 \Leftrightarrow 9x^2 = 36 \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2$, so the x -intercepts are ± 2 . To find y -intercepts, set $x = 0$. This gives $9(0)^2 - 4y^2 = 36 \Leftrightarrow y^2 = -9$, so there is no y -intercept.
- (b) To find x -intercepts, set $y = 0$. This gives $0 - 2x(0) + 4x = 1 \Leftrightarrow x = \frac{1}{4}$, so the x -intercept is $\frac{1}{4}$. To find y -intercepts, set $x = 0$. This gives $y - 2(0)y + 4(0) = 1$, so the y -intercept is 1 .
71. To find x -intercepts, set $y = 0$. This gives $0 = 4x - x^2 \Leftrightarrow 0 = x(4 - x) \Leftrightarrow 0 = x$ or $x = 4$, so the x -intercepts are 0 and 4 . To find y -intercepts, set $x = 0$. This gives $y = 4(0) - 0^2 \Leftrightarrow y = 0$, so the y -intercept is 0 .
73. To find x -intercepts, set $y = 0$. This gives $x^4 + 0^2 - x(0) = 16 \Leftrightarrow x^4 = 16 \Leftrightarrow x = \pm 2$. So the x -intercepts are -2 and 2 . To find y -intercepts, set $x = 0$. This gives $0^4 + y^2 - (0)y = 16 \Leftrightarrow y^2 = 16 \Leftrightarrow y = \pm 4$. So the y -intercepts are -4 and 4 .
75. $x^2 + y^2 = 9$ has center $(0, 0)$ and radius 3 .
77. $(x - 2)^2 + y^2 = 9$ has center $(2, 0)$ and radius 3 .



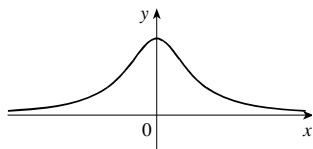
79. $(x + 3)^2 + (y - 4)^2 = 25$ has center $(-3, 4)$ and radius 5.



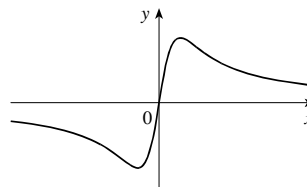
81. Using $h = -3$, $k = 1$, and $r = 2$, we get $(x - (-3))^2 + (y - 1)^2 = 2^2 \Leftrightarrow (x + 3)^2 + (y - 1)^2 = 4$.
83. The equation of a circle centered at the origin is $x^2 + y^2 = r^2$. Using the point $(4, 7)$ we solve for r^2 . This gives $(4)^2 + (7)^2 = r^2 \Leftrightarrow 16 + 49 = 65 = r^2$. Thus, the equation of the circle is $x^2 + y^2 = 65$.
85. The center is at the midpoint of the line segment, which is $\left(\frac{-1+5}{2}, \frac{1+9}{2}\right) = (2, 5)$. The radius is one half the diameter, so $r = \frac{1}{2}\sqrt{(-1-5)^2 + (1-9)^2} = \frac{1}{2}\sqrt{36+64} = \frac{1}{2}\sqrt{100} = 5$. Thus, the equation of the circle is $(x - 2)^2 + (y - 5)^2 = 5^2$ or $(x - 2)^2 + (y - 5)^2 = 25$.
87. Since the circle is tangent to the x -axis, it must contain the point $(7, 0)$, so the radius is the change in the y -coordinates. That is, $r = |-3 - 0| = 3$. So the equation of the circle is $(x - 7)^2 + (y - (-3))^2 = 3^2$, which is $(x - 7)^2 + (y + 3)^2 = 9$.
89. From the figure, the center of the circle is at $(-2, 2)$. The radius is the change in the y -coordinates, so $r = |2 - 0| = 2$. Thus the equation of the circle is $(x - (-2))^2 + (y - 2)^2 = 2^2$, which is $(x + 2)^2 + (y - 2)^2 = 4$.
91. Completing the square gives $x^2 + y^2 + 4x - 6y + 12 = 0 \Leftrightarrow x^2 + 4x + \left(\frac{4}{2}\right)^2 + y^2 - 6y + \left(\frac{6}{2}\right)^2 = -12 + \left(\frac{4}{2}\right)^2 + \left(\frac{6}{2}\right)^2 \Leftrightarrow x^2 + 4x + 4 + y^2 - 6y + 9 = -12 + 4 + 9 \Leftrightarrow (x + 2)^2 + (y - 3)^2 = 1$. Thus, the circle has center $(-2, 3)$ and radius 1.
93. Completing the square gives $x^2 + y^2 - \frac{1}{2}x + \frac{1}{2}y = \frac{1}{8} \Leftrightarrow x^2 - \frac{1}{2}x + \left(\frac{-1/2}{2}\right)^2 + y^2 + \frac{1}{2}y + \left(\frac{1/2}{2}\right)^2 = \frac{1}{8} + \left(\frac{-1/2}{2}\right)^2 + \left(\frac{1/2}{2}\right)^2 \Leftrightarrow x^2 - \frac{1}{2}x + \frac{1}{16} + y^2 + \frac{1}{2}y + \frac{1}{16} = \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{2}{8} = \frac{1}{4} \Leftrightarrow \left(x - \frac{1}{4}\right)^2 + \left(y + \frac{1}{4}\right)^2 = \frac{1}{4}$. Thus, the circle has center $\left(\frac{1}{4}, -\frac{1}{4}\right)$ and radius $\frac{1}{2}$.
95. Completing the square gives $2x^2 + 2y^2 - 3x = 0 \Leftrightarrow x^2 + y^2 - \frac{3}{2}x = 0 \Leftrightarrow \left(x - \frac{3}{4}\right)^2 + y^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$. Thus, the circle has center $\left(\frac{3}{4}, 0\right)$ and radius $\frac{3}{4}$.
97. x -axis symmetry: $(-y) = x^4 + x^2 \Leftrightarrow y = -x^4 - x^2$, which is not the same as $y = x^4 + x^2$, so the graph is not symmetric with respect to the x -axis.
 y -axis symmetry: $y = (-x)^4 + (-x)^2 = x^4 + x^2$, so the graph is symmetric with respect to the y -axis.
 Origin symmetry: $(-y) = (-x)^4 + (-x)^2 \Leftrightarrow -y = x^4 + x^2$, which is not the same as $y = x^4 + x^2$, so the graph is not symmetric with respect to the origin.
99. x -axis symmetry: $x^2(-y)^4 + x^4(-y)^2 = 2 \Leftrightarrow x^2y^4 + x^4y^2 = 2$, so the graph is symmetric with respect to the x -axis.
 y -axis symmetry: $(-x)^2y^4 + (-x)^4y^2 = 2 \Leftrightarrow x^2y^4 + x^4y^2 = 2$, so the graph is symmetric with respect to the y -axis.
 Origin symmetry: $(-x)^2(-y)^4 + (-x)^4(-y)^2 = 2 \Leftrightarrow x^2y^4 + x^4y^2 = 2$, so the graph is symmetric with respect to the origin.
 (Note that if a graph is symmetric with respect to each coordinate axis, it is symmetric with respect to the origin. The converse is not true, as shown in the next exercise.)

- 101.** x -axis symmetry: $(-y) = x^3 + 10x \Leftrightarrow y = -x^3 - 10x$, which is not the same as $y = x^3 + 10x$, so the graph is not symmetric with respect to the x -axis.
 y -axis symmetry: $y = (-x)^3 + 10(-x) \Leftrightarrow y = -x^3 - 10x$, which is not the same as $y = x^3 + 10x$, so the graph is not symmetric with respect to the y -axis.
 Origin symmetry: $(-y) = (-x)^3 + 10(-x) \Leftrightarrow -y = -x^3 - 10x \Leftrightarrow y = x^3 + 10x$, so the graph is symmetric with respect to the origin.

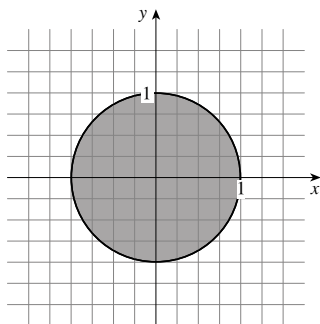
- 103.** Symmetric with respect to the y -axis.



- 105.** Symmetric with respect to the origin.



- 107.** $\{(x, y) \mid x^2 + y^2 \leq 1\}$. This is the set of points inside (and on) the circle $x^2 + y^2 = 1$.



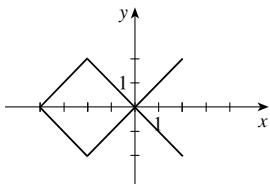
- 109.** Completing the square gives $x^2 + y^2 - 4y - 12 = 0$

$$\Leftrightarrow x^2 + y^2 - 4y + \left(\frac{-4}{2}\right)^2 = 12 + \left(\frac{-4}{2}\right)^2 \Leftrightarrow$$

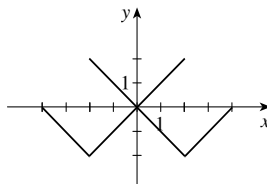
$x^2 + (y - 2)^2 = 16$. Thus, the center is $(0, 2)$, and the radius is 4. So the circle $x^2 + y^2 = 4$, with center $(0, 0)$ and radius 2, sits completely inside the larger circle. Thus, the area is $\pi 4^2 - \pi 2^2 = 16\pi - 4\pi = 12\pi$.

- 111.** (a) The point $(5, 3)$ is shifted to $(5 + 3, 3 + 2) = (8, 5)$.
 (b) The point (a, b) is shifted to $(a + 3, b + 2)$.
 (c) Let (x, y) be the point that is shifted to $(3, 4)$. Then $(x + 3, y + 2) = (3, 4)$. Setting the x -coordinates equal, we get $x + 3 = 3 \Leftrightarrow x = 0$. Setting the y -coordinates equal, we get $y + 2 = 4 \Leftrightarrow y = 2$. So the point is $(0, 2)$.
 (d) $A = (-5, -1)$, so $A' = (-5 + 3, -1 + 2) = (-2, 1)$; $B = (-3, 2)$, so $B' = (-3 + 3, 2 + 2) = (0, 4)$; and $C = (2, 1)$, so $C' = (2 + 3, 1 + 2) = (5, 3)$.

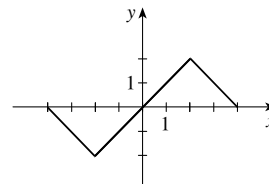
- 113.** (a) Symmetric about the x -axis:



- (b) Symmetric about the y -axis:



- (c) Symmetric about the origin:



- 115.** (a) $d(A, B) = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.
 (b) We want the distances from $C = (4, 2)$ to $D = (11, 26)$. The walking distance is $|4 - 11| + |2 - 26| = 7 + 24 = 31$ blocks. Straight-line distance is $\sqrt{(4 - 11)^2 + (2 - 26)^2} = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$ blocks.

(c) The two points are on the same avenue or the same street.

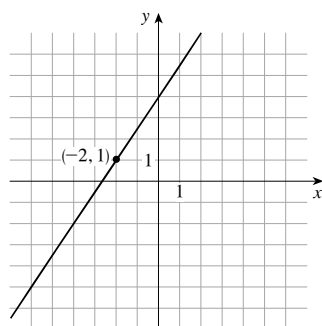
117. Completing the square gives $x^2 + y^2 + ax + by + c = 0 \Leftrightarrow x^2 + ax + \left(\frac{a}{2}\right)^2 + y^2 + by + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2$
 $\Leftrightarrow \left(x + \frac{a}{2}\right)^2 + \left(y + \frac{b}{2}\right)^2 = -c + \frac{a^2 + b^2}{4}$. This equation represents a circle only when $-c + \frac{a^2 + b^2}{4} > 0$. This
 equation represents a point when $-c + \frac{a^2 + b^2}{4} = 0$, and this equation represents the empty set when $-c + \frac{a^2 + b^2}{4} < 0$.
 When the equation represents a circle, the center is $\left(-\frac{a}{2}, -\frac{b}{2}\right)$, and the radius is $\sqrt{-c + \frac{a^2 + b^2}{4}} = \frac{1}{2}\sqrt{a^2 + b^2 - 4ac}$.

1.10 LINES

- We find the “steepness” or slope of a line passing through two points by dividing the difference in the y -coordinates of these points by the difference in the x -coordinates. So the line passing through the points $(0, 1)$ and $(2, 5)$ has slope $\frac{5-1}{2-0} = 2$.
- The point-slope form of the equation of the line with slope 3 passing through the point $(1, 2)$ is $y - 2 = 3(x - 1)$.
- The slope of a horizontal line is 0. The equation of the horizontal line passing through $(2, 3)$ is $y = 3$.
- (a) Yes, the graph of $y = -3$ is a horizontal line 3 units below the x -axis.
 (b) Yes, the graph of $x = -3$ is a vertical line 3 units to the left of the y -axis.
 (c) No, a line perpendicular to a horizontal line is vertical and has undefined slope.
 (d) Yes, a line perpendicular to a vertical line is horizontal and has slope 0.
- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2}{0 - (-1)} = \frac{-2}{1} = -2$
- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 2}{3 - (-3)} = \frac{-5}{6}$
- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{0 - 5} = 0$
- $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{6 - 8} = \frac{2}{-2} = -1$
- For ℓ_1 , we find two points, $(-1, 2)$ and $(0, 0)$ that lie on the line. Thus the slope of ℓ_1 is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{-1 - 0} = -2$.
 For ℓ_2 , we find two points $(0, 2)$ and $(2, 3)$. Thus, the slope of ℓ_2 is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{2 - 0} = \frac{1}{2}$. For ℓ_3 we find the points $(2, -2)$ and $(3, 1)$. Thus, the slope of ℓ_3 is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{3 - 2} = 3$. For ℓ_4 , we find the points $(-2, -1)$ and $(2, -2)$. Thus, the slope of ℓ_4 is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-1)}{2 - (-2)} = \frac{-1}{4} = -\frac{1}{4}$.
- First we find two points $(0, 4)$ and $(4, 0)$ that lie on the line. So the slope is $m = \frac{0 - 4}{4 - 0} = -1$. Since the y -intercept is 4, the equation of the line is $y = mx + b = -1x + 4$. So $y = -x + 4$, or $x + y - 4 = 0$.
- We choose the two intercepts as points, $(0, -3)$ and $(2, 0)$. So the slope is $m = \frac{0 - (-3)}{2 - 0} = \frac{3}{2}$. Since the y -intercept is -3 , the equation of the line is $y = mx + b = \frac{3}{2}x - 3$, or $3x - 2y - 6 = 0$.
- Using $y = mx + b$, we have $y = 3x + (-2)$ or $3x - y - 2 = 0$.
- Using the equation $y - y_1 = m(x - x_1)$, we get $y - 1 = 3(x - 4) \Leftrightarrow y - 1 = 3x - 12 \Leftrightarrow 3x - y - 11 = 0$.
- Using the equation $y - y_1 = m(x - x_1)$, we get $y - 7 = \frac{2}{3}(x - 1) \Leftrightarrow 3y - 21 = 2x - 2 \Leftrightarrow -2x + 3y = 19 \Leftrightarrow 2x - 3y + 19 = 0$.
- First we find the slope, which is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 1}{1 - 2} = \frac{5}{-1} = -5$. Substituting into $y - y_1 = m(x - x_1)$, we get $y - 6 = -5(x - 1) \Leftrightarrow y - 6 = -5x + 5 \Leftrightarrow 5x + y - 11 = 0$.

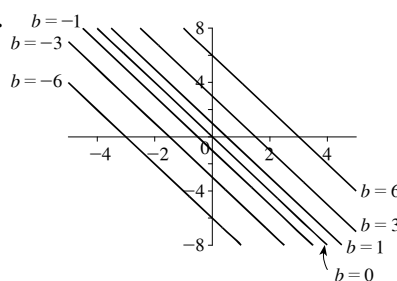
31. We are given two points, $(2, -5)$ and $(5, 1)$. Thus, the slope is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-5)}{5 - 2} = \frac{6}{3} = 2$. Substituting into $y - y_1 = m(x - x_1)$, we get $y - (-5) = 2(x - 2) \Leftrightarrow y = 2x - 9$ or $2x - y - 9 = 0$.
33. We are given two points, $(1, 0)$ and $(0, -3)$. Thus, the slope is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{0 - 1} = \frac{-3}{-1} = 3$. Using the y -intercept, we have $y = 3x + (-3) \Leftrightarrow y = 3x - 3$ or $3x - y - 3 = 0$.
35. Since the equation of a line with slope 0 passing through (a, b) is $y = b$, the equation of this line is $y = 3$.
37. Since the equation of a line with undefined slope passing through (a, b) is $x = a$, the equation of this line is $x = 2$.
39. Any line parallel to $y = 2x + 8$ has slope 2. The desired line passes through $(-1, 4)$, so substituting into $y - y_1 = m(x - x_1)$, we get $y - 4 = 2[x - (-1)] \Leftrightarrow y = 2x + 6$ or $2x - y + 6 = 0$.
41. Since the equation of a horizontal line passing through (a, b) is $y = b$, the equation of the horizontal line passing through $(4, 5)$ is $y = 5$.
43. Since $3x + 2y = 4 \Leftrightarrow 2y = -3x + 4 \Leftrightarrow y = -\frac{3}{2}x + 2$, the slope of this line is $-\frac{3}{2}$. Thus, the line we seek is given by $y - (-4) = -\frac{3}{2}(x - (-3)) \Leftrightarrow y + 4 = -\frac{3}{2}x - \frac{9}{2} \Leftrightarrow y = -\frac{3}{2}x - \frac{17}{2} \Leftrightarrow 3x + 2y + 17 = 0$.
45. Any line parallel to $x = 5$ has undefined slope and an equation of the form $x = a$. Thus, an equation of the line is $x = -1$.
47. First find the slope of $3x + 4y + 7 = 0 \Leftrightarrow 4y = -3x - 7 \Leftrightarrow y = -\frac{3}{4}x - \frac{7}{4}$. Thus, the slope of any line perpendicular to $3x + 4y + 7 = 0$ is $m = -\frac{1}{-\frac{3}{4}} = \frac{4}{3}$. Since it passes through $(-2, 1)$, the equation of the line we seek is $y - 1 = \frac{4}{3}(x - (-2)) \Leftrightarrow 3y - 3 = 4x + 8 \Leftrightarrow 4x - 3y + 11 = 0$.
49. First find the slope of the line passing through $(2, 5)$ and $(-2, 1)$. This gives $m = \frac{1 - 5}{-2 - 2} = \frac{-4}{-4} = 1$, and so the equation of the line we seek is $y - 7 = 1(x - 1) \Leftrightarrow x - y + 6 = 0$.

51. (a)



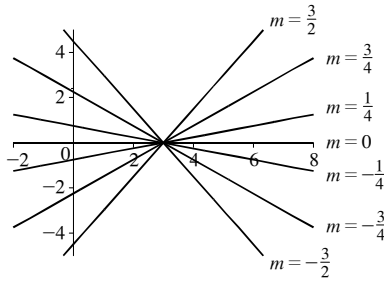
(b) $y - 1 = \frac{3}{2}(x - (-2)) \Leftrightarrow 2y - 2 = 3(x + 2) \Leftrightarrow 2y - 2 = 3x + 6 \Leftrightarrow 3x - 2y + 8 = 0$.

53.



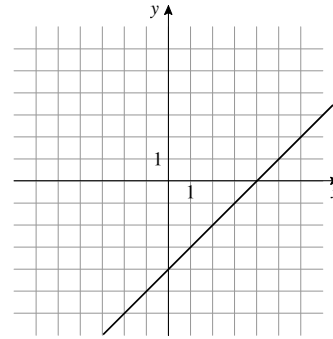
$y = -2x + b$, $b = 0, \pm 1, \pm 3, \pm 6$. They have the same slope, so they are parallel.

55.

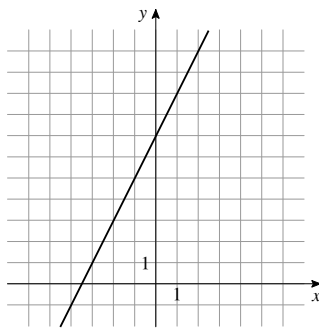


$y = m(x - 3)$, $m = 0, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{3}{2}$. Each of the lines contains the point $(3, 0)$ because the point $(3, 0)$ satisfies each equation $y = m(x - 3)$. Since $(3, 0)$ is on the x -axis, we could also say that they all have the same x -intercept.

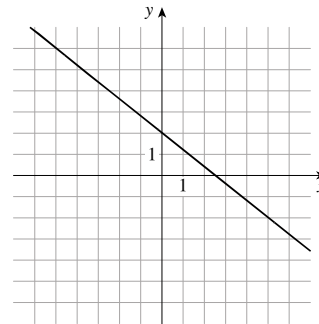
57. $y = x - 4$. So the slope is 1 and the y -intercept is -4 .



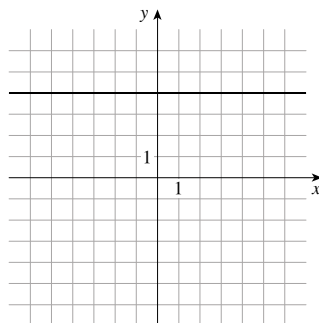
59. $-2x + y = 7 \Leftrightarrow y = 2x + 7$. So the slope is 2 and the y -intercept is 7.



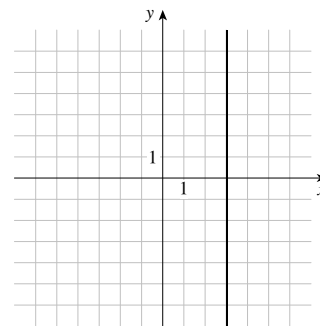
61. $4x + 5y = 10 \Leftrightarrow 5y = -4x + 10 \Leftrightarrow y = -\frac{4}{5}x + 2$. So the slope is $-\frac{4}{5}$ and the y -intercept is 2.



63. $y = 4$ can also be expressed as $y = 0x + 4$. So the slope is 0 and the y -intercept is 4.

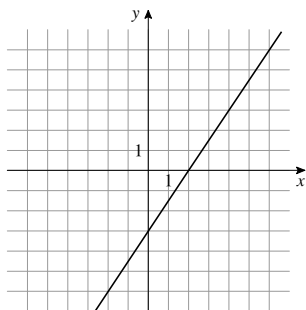


65. $x = 3$ cannot be expressed in the form $y = mx + b$. So the slope is undefined, and there is no y -intercept. This is a vertical line.



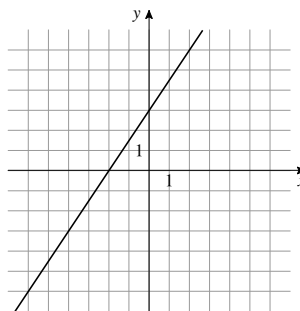
67. $3x - 2y - 6 = 0$. To find x -intercepts, we set $y = 0$ and solve for x : $3x - 2(0) - 6 = 0 \Leftrightarrow 3x = 6 \Leftrightarrow x = 2$, so the x -intercept is 2.

To find y -intercepts, we set $x = 0$ and solve for y : $3(0) - 2y - 6 = 0 \Leftrightarrow 2y = -6 \Leftrightarrow y = -3$, so the y -intercept is -3 .



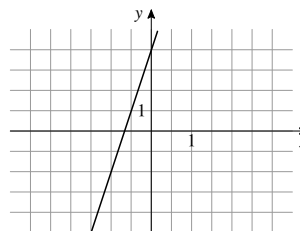
69. $\frac{1}{2}x - \frac{1}{3}y + 1 = 0$. To find x -intercepts, we set $y = 0$ and solve for x : $\frac{1}{2}x - \frac{1}{3}(0) + 1 = 0 \Leftrightarrow \frac{1}{2}x = -1 \Leftrightarrow x = -2$, so the x -intercept is -2 .

To find y -intercepts, we set $x = 0$ and solve for y : $\frac{1}{2}(0) - \frac{1}{3}y + 1 = 0 \Leftrightarrow \frac{1}{3}y = 1 \Leftrightarrow y = 3$, so the y -intercept is 3.



71. $y = 6x + 4$. To find x -intercepts, we set $y = 0$ and solve for x : $0 = 6x + 4 \Leftrightarrow 6x = -4 \Leftrightarrow x = -\frac{2}{3}$, so the x -intercept is $-\frac{2}{3}$.

To find y -intercepts, we set $x = 0$ and solve for y : $y = 6(0) + 4 = 4$, so the y -intercept is 4.

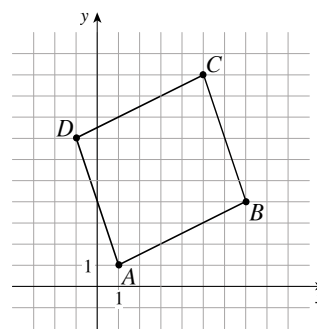


73. To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $y = 2x + 3$ has slope 2. The line with equation $2y - 4x - 5 = 0 \Leftrightarrow 2y = 4x + 5 \Leftrightarrow y = 2x + \frac{5}{2}$ also has slope 2, and so the lines are parallel.

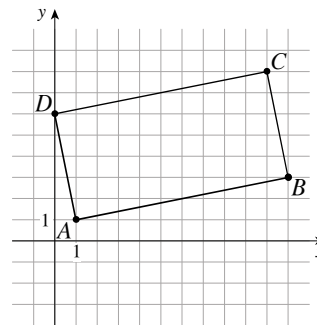
75. To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $2x - 5y = 8 \Leftrightarrow 5y = 2x - 8 \Leftrightarrow y = \frac{2}{5}x - \frac{8}{5}$ has slope $\frac{2}{5}$. The line with equation $10x + 4y = 1 \Leftrightarrow 4y = -10x + 1 \Leftrightarrow y = -\frac{5}{2}x + \frac{1}{4}$ has slope $-\frac{5}{2} = -\frac{1}{2/5}$, and so the lines are perpendicular.

77. To determine if the lines are parallel or perpendicular, we find their slopes. The line with equation $7x - 3y = 2 \Leftrightarrow 3y = 7x - 2 \Leftrightarrow y = \frac{7}{3}x - \frac{2}{3}$ has slope $\frac{7}{3}$. The line with equation $9y + 21x = 1 \Leftrightarrow 9y = -21x + 1 \Leftrightarrow y = -\frac{7}{3}x + \frac{1}{9}$ has slope $-\frac{7}{3} \neq -\frac{1}{7/3}$, and so the lines are neither parallel nor perpendicular.

79. We first plot the points to find the pairs of points that determine each side. Next we find the slopes of opposite sides. The slope of AB is $\frac{4-1}{7-1} = \frac{3}{6} = \frac{1}{2}$, and the slope of DC is $\frac{10-7}{5-(-1)} = \frac{3}{6} = \frac{1}{2}$. Since these slope are equal, these two sides are parallel. The slope of AD is $\frac{7-1}{-1-1} = \frac{6}{-2} = -3$, and the slope of BC is $\frac{10-4}{5-7} = \frac{6}{-2} = -3$. Since these slope are equal, these two sides are parallel. Hence $ABCD$ is a parallelogram.

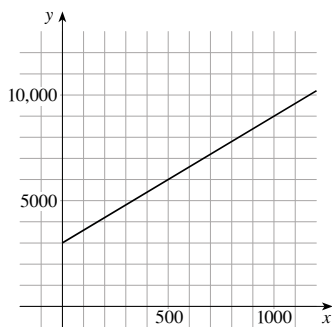


- 81.** We first plot the points to find the pairs of points that determine each side. Next we find the slopes of opposite sides. The slope of AB is $\frac{3-1}{11-1} = \frac{2}{10} = \frac{1}{5}$ and the slope of DC is $\frac{6-8}{0-10} = \frac{-2}{-10} = \frac{1}{5}$. Since these slope are equal, these two sides are parallel. Slope of AD is $\frac{6-1}{0-1} = \frac{5}{-1} = -5$, and the slope of BC is $\frac{3-8}{11-10} = \frac{-5}{1} = -5$. Since these slope are equal, these two sides are parallel. Since $(\text{slope of } AB) \times (\text{slope of } AD) = \frac{1}{5} \times (-5) = -1$, the first two sides are each perpendicular to the second two sides. So the sides form a rectangle.



- 83.** We need the slope and the midpoint of the line AB . The midpoint of AB is $\left(\frac{1+7}{2}, \frac{4-2}{2}\right) = (4, 1)$, and the slope of AB is $m = \frac{-2-4}{7-1} = \frac{-6}{6} = -1$. The slope of the perpendicular bisector will have slope $\frac{-1}{m} = \frac{-1}{-1} = 1$. Using the point-slope form, the equation of the perpendicular bisector is $y - 1 = 1(x - 4)$ or $x - y - 3 = 0$.
- 85. (a)** We start with the two points $(a, 0)$ and $(0, b)$. The slope of the line that contains them is $\frac{b-0}{0-a} = -\frac{b}{a}$. So the equation of the line containing them is $y = -\frac{b}{a}x + b$ (using the slope-intercept form). Dividing by b (since $b \neq 0$) gives $\frac{y}{b} = -\frac{x}{a} + 1 \Leftrightarrow \frac{x}{a} + \frac{y}{b} = 1$.
- (b)** Setting $a = 6$ and $b = -8$, we get $\frac{x}{6} + \frac{y}{-8} = 1 \Leftrightarrow 4x - 3y = 24 \Leftrightarrow 4x - 3y - 24 = 0$.
- 87.** Using the diagram provided, we calculate the coordinates of the midpoints: m_1 has coordinates $\left(\frac{0+a}{2}, \frac{0+b}{2}\right) = \left(\frac{1}{2}a, \frac{1}{2}b\right)$ and m_2 has coordinates $\left(\frac{c+a}{2}, \frac{0+b}{2}\right) = \left(\frac{1}{2}(a+c), \frac{1}{2}b\right)$. The line joining the midpoints thus has slope $\frac{\frac{1}{2}b - \frac{1}{2}b}{\frac{1}{2}(a+c) - \frac{1}{2}a} = 0$, and so it is parallel to the third side (a segment of the x -axis). The line joining the midpoints has length $\sqrt{\left[\frac{1}{2}(a+c) - \frac{1}{2}a\right]^2 + \left(\frac{1}{2}b - \frac{1}{2}b\right)^2} = \sqrt{\frac{1}{4}c^2} = \frac{1}{2}c$, half the length of the third side of the triangle; and so we have verified both conclusions of the theorem.
- 89. (a)** The slope is $0.0417D = 0.0417(200) = 8.34$. It represents the increase in dosage for each one-year increase in the child's age.
- (b)** When $a = 0$, $c = 8.34(0 + 1) = 8.34$ mg.

91. (a)



(b) The slope is the cost per toaster oven, \$6. The y -intercept, \$3000, is the monthly fixed cost—the cost that is incurred no matter how many toaster ovens are produced.

93. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

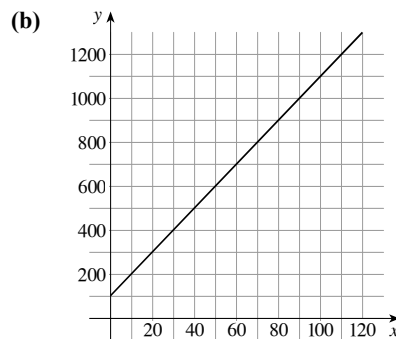
- (a) Using n in place of x and t in place of y , we find that the slope is $\frac{t_2 - t_1}{n_2 - n_1} = \frac{27 - 21}{168 - 120} = \frac{6}{48} = \frac{1}{8}$. So the linear equation is $t - 27 = \frac{1}{8}(n - 168) \Leftrightarrow t - 27 = \frac{1}{8}n - 21 \Leftrightarrow t = \frac{1}{8}n + 6$.
- (b) When $n = 150$, the temperature is approximately given by $t = \frac{1}{8}(150) + 6 = 24.75^\circ \text{C} \approx 25^\circ \text{C}$.

95. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

- (a) We are given $\frac{\text{change in pressure}}{3\text{-meter change in depth}} = \frac{29.9}{3} = 9.97$. Using P for pressure and d for depth, and using the point $P = 103$ when $d = 0$, we have $P - 103 = 9.97(d - 0) \Leftrightarrow P = 9.97d + 103$.

- (c) The slope represents the increase in pressure per meter of descent. The y -intercept represents the pressure at the surface.

- (d) When $P = 690$, we have $690 = 9.97d + 103 \Leftrightarrow 9.97d = 587 \Leftrightarrow d = 58.9$ m. Thus the pressure is 690 kPa at a depth of approximately 59 m.



97. We label the three points A , B , and C . If the slope of the line segment $\overline{AB}$ is equal to the slope of the line segment $\overline{BC}$, then the points A , B , and C are collinear. Using the distance formula, we find the distance between A and B , between B and C , and between A and C . If the sum of the two smaller distances equals the largest distance, the points A , B , and C are collinear.

Another method: Find an equation for the line through A and B . Then check if C satisfies the equation. If so, the points are collinear.

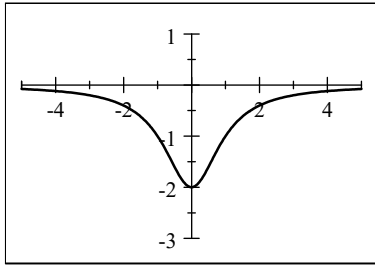
1.11 SOLVING EQUATIONS AND INEQUALITIES GRAPHICALLY

1. The solutions of the equation $x^2 - 2x - 3 = 0$ are the x -intercepts of the graph of $y = x^2 - 2x - 3$.
3. (a) From the graph, it appears that the graph of $y = x^4 - 3x^3 - x^2 + 3x$ has x -intercepts -1 , 0 , 1 , and 3 , so the solutions to the equation $x^4 - 3x^3 - x^2 + 3x = 0$ are $x = -1$, $x = 0$, $x = 1$, and $x = 3$.
- (b) From the graph, we see that where $-1 \leq x \leq 0$ or $1 \leq x \leq 3$, the graph lies below the x -axis. Thus, the inequality $x^4 - 3x^3 - x^2 + 3x \leq 0$ is satisfied for $\{x \mid -1 \leq x \leq 0 \text{ or } 1 \leq x \leq 3\} = [-1, 0] \cup [1, 3]$.

5. (a)
- (b) x -intercepts 0 , 1 ; y -intercept 0 . $y = x^3 - x^2 = x^2(x - 1)$, so $y = 0 \Rightarrow x = 0$ or $x = 1$ and $x = 0 \Leftrightarrow y = 0$.
- (c) The graph is not symmetric with respect to either axis or the origin.

$$y = x^3 - x^2; [-2, 2] \text{ by } [-1, 1]$$

7. (a)



$$y = -\frac{2}{x^2 + 1}; [-5, 5] \text{ by } [-3, 1]$$

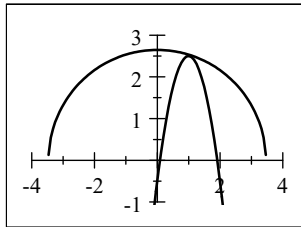
(b) No x -intercept; y -intercept -2 . $y = 0$ has no solution; $x = 0 \Rightarrow$

$$y = -\frac{2}{1} = -2.$$

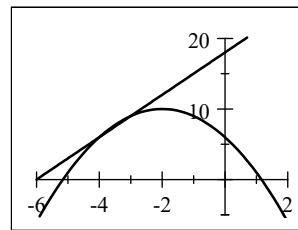
(c) The graph is symmetric with respect to the y -axis:

$$-\frac{2}{(-x)^2 + 1} = -\frac{2}{x^2 + 1}.$$

9. Although the graphs of $y = -3x^2 + 6x - \frac{1}{2}$ and $y = \sqrt{7 - \frac{7}{12}x^2}$ appear to intersect in the viewing rectangle $[-4, 4]$ by $[-1, 3]$, there is no point of intersection. You can verify this by zooming in.



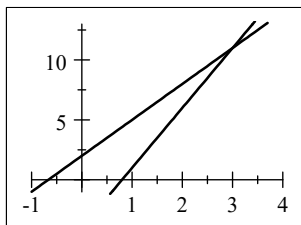
11. The graphs of $y = 6 - 4x - x^2$ and $y = 3x + 18$ appear to have two points of intersection in the viewing rectangle $[-6, 2]$ by $[-5, 20]$. You can verify that $x = -4$ and $x = -3$ are exact solutions.



13. Algebraically: $3x + 2 = 5x - 4 \Leftrightarrow 6 = 2x \Leftrightarrow x = 3$.

Graphically: We graph the two equations $y_1 = 3x + 2$ and $y_2 = 5x - 4$ in the viewing rectangle $[1, 4]$ by $[-1, 13]$.

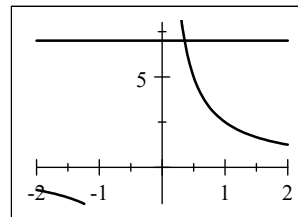
Zooming in, we see that the solution is $x = 3$.



15. Algebraically: $\frac{2}{x} + \frac{1}{2x} = 7 \Leftrightarrow 2x \left(\frac{2}{x} + \frac{1}{2x} \right) = 2x(7)$

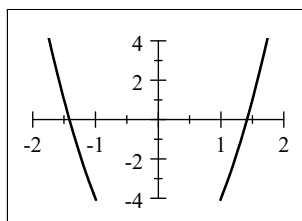
$$\Leftrightarrow 4 + 1 = 14x \Leftrightarrow x = \frac{5}{14}.$$

Graphically: We graph the two equations $y_1 = \frac{2}{x} + \frac{1}{2x}$ and $y_2 = 7$ in the viewing rectangle $[-2, 2]$ by $[-2, 8]$. Zooming in, we see that the solution is $x \approx 0.36$.



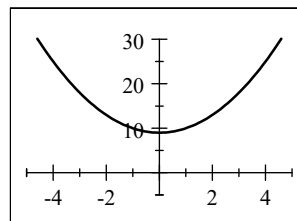
17. Algebraically: $4x^2 - 8 = 0 \Leftrightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$.

Graphically: We graph the equation $y_1 = 4x^2 - 8$ and determine where this curve intersects the x -axis. We use the viewing rectangle $[-2, 2]$ by $[-4, 4]$. Zooming in, we see that solutions are $x \approx 1.41$ and $x \approx -1.41$.



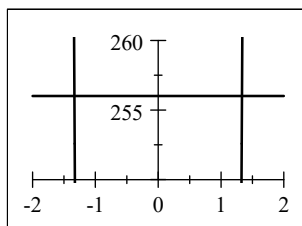
19. Algebraically: $x^2 + 9 = 0 \Leftrightarrow x^2 = -9$, which has no real solution.

Graphically: We graph the equation $y = x^2 + 9$ and see that this curve does not intersect the x -axis. We use the viewing rectangle $[-5, 5]$ by $[-5, 30]$.



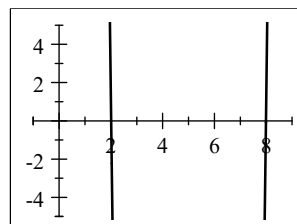
21. Algebraically: $81x^4 = 256 \Leftrightarrow x^4 = \frac{256}{81} \Rightarrow x = \pm\frac{4}{3}$.

Graphically: We graph the two equations $y_1 = 81x^4$ and $y_2 = 256$ in the viewing rectangle $[-2, 2]$ by $[250, 260]$. Zooming in, we see that solutions are $x \approx \pm 1.33$.

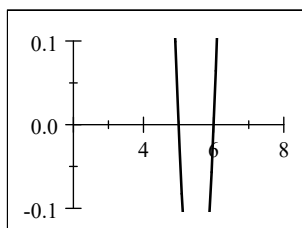


23. Algebraically: $(x - 5)^4 - 80 = 0 \Leftrightarrow (x - 5)^4 = 80 \Rightarrow x - 5 = \pm\sqrt[4]{80} = \pm 2\sqrt[4]{5} \Leftrightarrow x = 5 \pm 2\sqrt[4]{5}$.

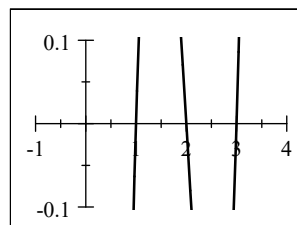
Graphically: We graph the equation $y_1 = (x - 5)^4 - 80$ and determine where this curve intersects the x -axis. We use the viewing rectangle $[-1, 9]$ by $[-5, 5]$. Zooming in, we see that solutions are $x \approx 2.01$ and $x \approx 7.99$.



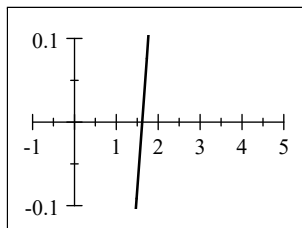
25. We graph $y = x^2 - 11x + 30$ in the viewing rectangle $[2, 8]$ by $[-0.1, 0.1]$. The solutions appear to be exactly $x = 5$ and $x = 6$. [In fact $x^2 - 11x + 30 = (x - 5)(x - 6)$.]



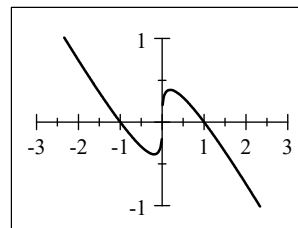
27. We graph $y = x^3 - 6x^2 + 11x - 6$ in the viewing rectangle $[-1, 4]$ by $[-0.1, 0.1]$. The solutions are $x = 1.00$, $x = 2.00$, and $x = 3.00$.



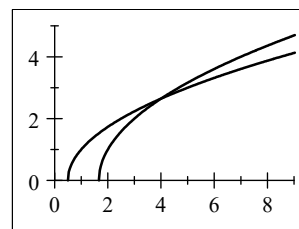
29. We first graph $y = x - \sqrt{x+1}$ in the viewing rectangle $[-1, 5]$ by $[-0.1, 0.1]$ and find that the solution is near 1.6. Zooming in, we see that solutions is $x \approx 1.62$.



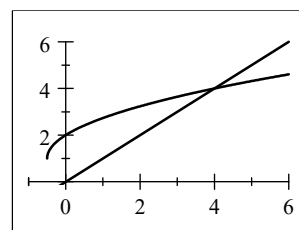
31. We graph $y = x^{1/3} - x$ in the viewing rectangle $[-3, 3]$ by $[-1, 1]$. The solutions are $x = -1$, $x = 0$, and $x = 1$, as can be verified by substitution.



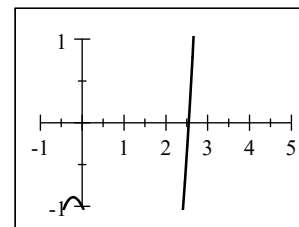
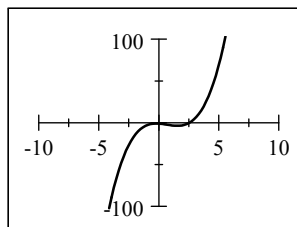
33. We graph $y = \sqrt{2x-1}$ and $y = \sqrt{3x-5}$ in the viewing rectangle $[0, 9]$ by $[0, 5]$ and see that the only solution to the equation $\sqrt{2x-1} = \sqrt{3x-5}$ is $x = 4$, which can be verified by substitution.



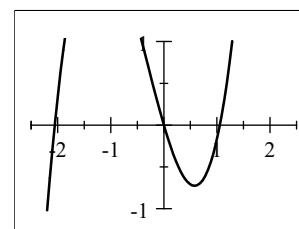
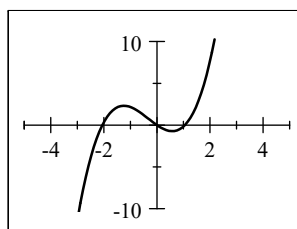
35. We graph $y = \sqrt{2x+1} + 1$ and $y = x$ in the viewing rectangle $[-1, 6]$ by $[0, 6]$ and see that the only solution to the equation $\sqrt{2x+1} + 1 = x$ is $x = 4$, which can be verified by substitution.



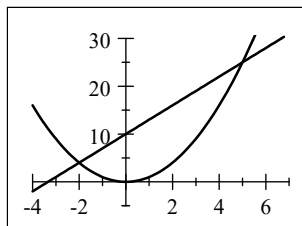
37. $x^3 - 2x^2 - x - 1 = 0$, so we start by graphing the function $y = x^3 - 2x^2 - x - 1$ in the viewing rectangle $[-10, 10]$ by $[-100, 100]$. There appear to be two solutions, one near $x = 0$ and another one between $x = 2$ and $x = 3$. We then use the viewing rectangle $[-1, 5]$ by $[-1, 1]$ and zoom in on the only solution, $x \approx 2.55$.



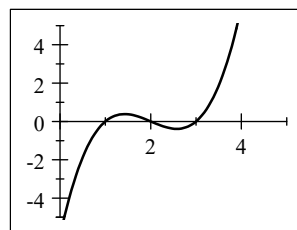
39. $x(x-1)(x+2) = \frac{1}{6}x \Leftrightarrow x(x-1)(x+2) - \frac{1}{6}x = 0$. We start by graphing the function $y = x(x-1)(x+2) - \frac{1}{6}x$ in the viewing rectangle $[-5, 5]$ by $[-10, 10]$. There appear to be three solutions. We then use the viewing rectangle $[-2.5, 2.5]$ by $[-1, 1]$ and zoom into the solutions at $x \approx -2.05$, $x = 0.00$, and $x \approx 1.05$.



41. We graph $y = x^2$ and $y = 3x + 10$ in the viewing rectangle $[-4, 7]$ by $[-5, 30]$. The solution to the inequality is $[-2, 5]$.

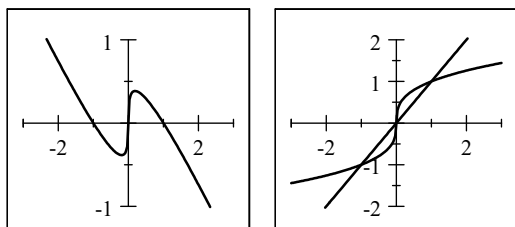


43. Since $x^3 + 11x \leq 6x^2 + 6 \Leftrightarrow x^3 - 6x^2 + 11x - 6 \leq 0$, we graph $y = x^3 - 6x^2 + 11x - 6$ in the viewing rectangle $[0, 5]$ by $[-5, 5]$. The solution set is $(-\infty, 1.00] \cup [2.00, 3.00]$.

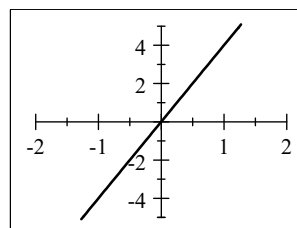


45. Since $x^{1/3} < x \Leftrightarrow x^{1/3} - x < 0$, we graph $y = x^{1/3} - x$ in the viewing rectangle $[-3, 3]$ by $[-1, 1]$. From this, we find that the solution set is $(-1, 0) \cup (1, \infty)$.

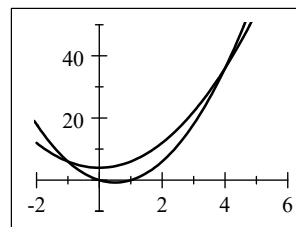
Another Method: As in Example 7, we graph $y_1 = x^{1/3}$ and $y_2 = x$ in the same viewing rectangle, and see that $x^{1/3} < x$ for $-1 < x < 0$ and for $1 < x$.



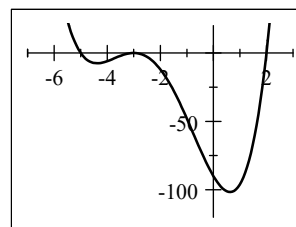
47. Since $(x + 1)^2 < (x - 1)^2 \Leftrightarrow (x + 1)^2 - (x - 1)^2 < 0$, we graph $y = (x + 1)^2 - (x - 1)^2$ in the viewing rectangle $[-2, 2]$ by $[-5, 5]$. The solution set is $(-\infty, 0)$.



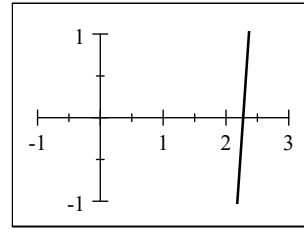
49. We graph the equations $y = 3x^2 - 3x$ and $y = 2x^2 + 4$ in the viewing rectangle $[-2, 6]$ by $[-10, 50]$. We see that the two curves intersect at $x = -1$ and at $x = 4$, and that the first curve is lower than the second for $-1 < x < 4$. Thus, we see that the inequality $3x^2 - 3x < 2x^2 + 4$ has solution set $(-1, 4)$.



51. We graph the equation $y = (x + 3)^2(x - 2)(x + 5)$ in the viewing rectangle $[-7, 3]$ by $[-120, 20]$ and see that the inequality $(x + 3)^2(x - 2)(x + 5) \geq 0$ has the solution set $(-\infty, -5] \cup \{-3\} \cup [2, \infty)$.

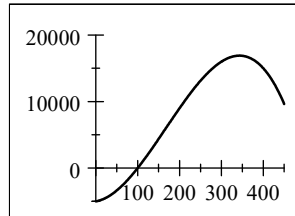


53. To solve $5 - 3x = 8x - 20$ by drawing the graph of a single equation, we isolate all terms on the left-hand side: $5 - 3x = 8x - 20 \Leftrightarrow 5 - 3x - 8x + 20 = 8x - 20 - 8x + 20 \Leftrightarrow -11x + 25 = 0$ or $11x - 25 = 0$. We graph $y = 11x - 25$, and see that the solution is $x \approx 2.27$, as in Example 2.



55. (a) We graph the equation

$y = 10x + 0.5x^2 - 0.001x^3 - 5000$ in the viewing rectangle $[0, 450]$ by $[-5000, 20000]$.

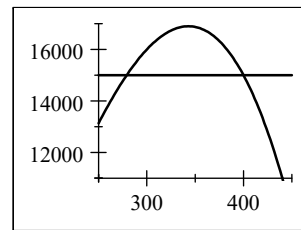


- (b) From the graph it appears that

$0 < 10x + 0.5x^2 - 0.001x^3 - 5000$ for $100 < x < 500$, and so 101 cooktops must be produced to *begin* to make a profit.

- (c) We graph the equations $y = 15,000$ and

$y = 10x + 0.5x^2 - 0.001x^3 - 5000$ in the viewing rectangle $[250, 450]$ by $[11000, 17000]$. We use a zoom or trace function on a graphing calculator, and find that the company's profits are greater than \$15,000 for $279 < x < 400$.



57. Answers will vary.

1.12 MODELING VARIATION

- If the quantities x and y are related by the equation $y = 5x$ then we say that y is *directly proportional* to x , and the constant of proportionality is 5.
- If the quantities x , y , and z are related by the equation $z = 5\frac{x}{y}$ then we say that z is *directly proportional* to x and *inversely proportional* to y .
- (a) In the equation $y = 3x$, y is directly proportional to x .
(b) In the equation $y = 3x + 1$, y is not proportional to x .
- $T = kx$, where k is constant.
- $v = \frac{k}{z}$, where k is constant.
- $y = \frac{ks}{t}$, where k is constant.
- $z = k\sqrt{y}$, where k is constant.
- $V = klwh$, where k is constant.
- $R = \frac{kP^2t^2}{b^3}$, where k is constant.
- Since y is directly proportional to x , $y = kx$. Since $y = 32$ when $x = 8$, we have $32 = k(8) \Leftrightarrow k = 4$. So $y = 4x$.
- A varies inversely as r , so $A = \frac{k}{r}$. Since $A = 15$ when $r = 5$, we have $15 = \frac{k}{5} \Leftrightarrow k = 75$. So $A = \frac{75}{r}$.
- Since A is directly proportional to x and inversely proportional to t , $A = \frac{kx}{t}$. Since $A = 42$ when $x = 7$ and $t = 3$, we have $42 = \frac{k(7)}{3} \Leftrightarrow k = 18$. Therefore, $A = \frac{18x}{t}$.

25. Since W is inversely proportional to the square of r , $W = \frac{k}{r^2}$. Since $W = 24$ when $r = 3$, we have $24 = \frac{k}{(3)^2} \Leftrightarrow k = 216$.

So $W = \frac{216}{r^2}$.

27. Since C is jointly proportional to l , w , and h , we have $C = klwh$. Since $C = 128$ when $l = w = h = 2$, we have $128 = k(2)(2)(2) \Leftrightarrow 128 = 8k \Leftrightarrow k = 16$. Therefore, $C = 16lwh$.

29. $R = \frac{k}{\sqrt{x}}$. Since $R = 2.5$ when $x = 121$, $2.5 = \frac{k}{\sqrt{121}} = \frac{k}{11} \Leftrightarrow k = 27.5$. Thus, $R = \frac{27.5}{\sqrt{x}}$.

31. (a) $z = k \frac{x^3}{y^2}$

(b) If we replace x with $3x$ and y with $2y$, then $z = k \frac{(3x)^3}{(2y)^2} = \frac{27}{4} \left(k \frac{x^3}{y^2} \right)$, so z changes by a factor of $\frac{27}{4}$.

33. (a) $z = kx^3y^5$

(b) If we replace x with $3x$ and y with $2y$, then $z = k(3x)^3(2y)^5 = 864kx^3y^5$, so z changes by a factor of 864.

35. (a) The force F needed is $F = kx$.

(b) Since $F = 30$ N when $x = 9$ cm and the spring's natural length is 5 cm, we have $30 = k(9 - 5) \Leftrightarrow k = 7.5$ N/cm.

(c) From part (b), we have $F = 7.5x$. Substituting $x = 11 - 5 = 6$ into $F = 7.5x$ gives $F = 7.5(6) = 45$ N.

37. (a) $P = ks^3$.

(b) Since $P = 96$ when $s = 20$, we get $96 = k \cdot 20^3 \Leftrightarrow k = 0.012$. So $P = 0.012s^3$.

(c) Substituting $s = 30$, we get $P = 0.012 \cdot 30^3 = 324$ watts.

39. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

$D = ks^2$. Since $D = 45$ when $s = 60$, we have $45 = k(60)^2$, so $k = 0.0125$. Thus, $D = 0.0125s^2$. If $D = 60$, then $60 = 0.0125s^2 \Leftrightarrow s^2 = 4800$, so $s \approx 69$ km/h (for safety reasons we round down).

41. **Note: In the first printing of the text, the answer given for this exercise is incorrect.**

$F = kAs^2$. Since $F = 980$ when $A = 4$ and $s = 8$, we can solve for k : $980 = k(4)(8)^2 \Leftrightarrow 980 = 256k \Leftrightarrow k = 3.83$. Now when $A = 2.5$ and $F = 780$ we get $780 = 3.83(2.5)s^2 \Leftrightarrow 81.4621 = s^2$, so $s = \sqrt{81.4621} \approx 9.03$ km/h.

43. (a) $P = \frac{kT}{V}$.

(b) Substituting $P = 33.2$, $T = 400$, and $V = 100$, we get $33.2 = \frac{k(400)}{100} \Leftrightarrow k = 8.3$. Thus $k = 8.3$ and the equation is

$$P = \frac{8.3T}{V}.$$

(c) Substituting $T = 500$ and $V = 80$, we have $P = \frac{8.3(500)}{80} = 51.875$ kPa. Hence the pressure of the sample of gas is about 51.9 kPa.

45. (a) The loudness L is inversely proportional to the square of the distance d , so $L = \frac{k}{d^2}$.

(b) Substituting $d = 10$ and $L = 70$, we have $70 = \frac{k}{10^2} \Leftrightarrow k = 7000$.

(c) Substituting $2d$ for d , we have $L = \frac{k}{(2d)^2} = \frac{1}{4} \left(\frac{k}{d^2} \right)$, so the loudness is changed by a factor of $\frac{1}{4}$.

(d) Substituting $\frac{1}{2}d$ for d , we have $L = \frac{k}{\left(\frac{1}{2}d\right)^2} = 4 \left(\frac{k}{d^2} \right)$, so the loudness is changed by a factor of 4.

47. (a) $R = \frac{kL}{d^2}$

(b) Since $R = 140$ when $L = 1.2$ and $d = 0.005$, we get $140 = \frac{k(1.2)}{(0.005)^2} \Leftrightarrow k = \frac{7}{2400} = 0.00291\bar{6}$.

(c) Substituting $L = 3$ and $d = 0.008$, we have $R = \frac{7}{2400} \cdot \frac{3}{(0.008)^2} = \frac{4375}{32} \approx 137$ ohms.

(d) If we substitute $2d$ for d and $3L$ for L , then $R = \frac{k(3L)}{(2d)^2} = \frac{3}{4} \frac{kL}{d^2}$, so the resistance is changed by a factor of $\frac{3}{4}$.

49. **Note: In the first printing of the text, the answer given for part (b) of this exercise is incorrect.**

(a) For the sun, $E_S = k6000^4$ and for earth, $E_E = k300^4$. Thus $\frac{E_S}{E_E} = \frac{k6000^4}{k300^4} = \left(\frac{6000}{300}\right)^4 = 20^4 = 160,000$. So the sun produces 160,000 times the radiation energy per unit area than the Earth.

(b) The surface area of the sun is $4\pi (696,000)^2$ and the surface area of the Earth is $4\pi (6340)^2$. So the sun has $\frac{4\pi (696,000)^2}{4\pi (6340)^2} = \left(\frac{696,000}{6340}\right)^2$ times the surface area of the Earth. Thus the total radiation emitted by the sun is $160,000 \times \left(\frac{696,000}{6340}\right)^2 = 1,928,234,931$ times the total radiation emitted by the Earth.

51. (a) Since f is inversely proportional to L , we have $f = \frac{k}{L}$, where k is a positive constant.

(b) If we replace L by $2L$ we have $\frac{k}{2L} = \frac{1}{2} \cdot \frac{k}{L} = \frac{1}{2}f$. So the frequency of the vibration is cut in half.

53. We substitute $e = 0.2$ kWh/km and $v = 100$ km/h into the second given equation: $0.2 = k(100)^2 \Leftrightarrow k = \frac{0.2}{100^2} = 2 \times 10^{-5}$.

Now, because the range is $R = 500$ km for these values of e and v , we can use the first equation to calculate the value of C : $500 = \frac{C}{0.2} \Leftrightarrow C = 100$, so a full charge for this vehicle is 100 kWh.

Substituting $e = kv^2$ into the first given equation, we have $R = \frac{C}{kv^2}$.

So at 130 km/h, its range is $R \approx \frac{100}{2 \times 10^{-5} (130)^2} \approx 296$ km; and at 80 km/h, its range is $R \approx \frac{100}{2 \times 10^{-5} (80)^2} \approx 781$ km.

55. Using $B = k \frac{L}{d^2}$ with $k = 0.080$, $L = 2.5 \times 10^{26}$, and $d = 2.4 \times 10^{19}$, we have $B = 0.080 \frac{2.5 \times 10^{26}}{(2.4 \times 10^{19})^2} \approx 3.47 \times 10^{-14}$.

The star's apparent brightness is about 3.47×10^{-14} W/m².

57. Examples include radioactive decay and exponential growth in biology.

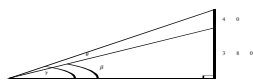
CHAPTER 1 REVIEW

1. Commutative Property for addition.

3. Distributive Property.

5. $[-2, 6) = \{x \mid -2 \leq x < 6\}$

7. $x \geq 5 \Leftrightarrow x \in [5, \infty)$



9. $|1 - |-4|| = |1 - 4| = |-3| = 3$

11. $2^{1/2} 8^{1/2} = \sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$

$$13. 216^{-1/3} = \frac{1}{216^{1/3}} = \frac{1}{\sqrt[3]{216}} = \frac{1}{6}$$

$$15. \frac{\sqrt{242}}{\sqrt{2}} = \sqrt{\frac{242}{2}} = \sqrt{121} = 11$$

$$17. (a) (a^2)^{-3} (a^3b)^2 (b^3)^4 = a^{-6} \cdot a^6b^2 \cdot b^{12} = a^{-6+6}b^{2+12} = b^{14}$$

$$(b) (3xy^2)^3 \left(\frac{2}{3}x^{-1}y\right)^2 = 27x^3y^6 \cdot \frac{4}{9}x^{-2}y^2 = 27 \cdot \frac{4}{9}x^{3-2}y^{6+2} = 12xy^8$$

$$19. (a) \sqrt[3]{(x^3y)^2y^4} = \sqrt[3]{x^6y^4y^2} = \sqrt[3]{x^6y^6} = x^2y^2$$

$$(b) \sqrt{w^8z^{10}} = \sqrt{w^8} \cdot \sqrt{z^{10}} = w^4|z|^5$$

$$21. 78,250,000,000 = 7.825 \times 10^{10}$$

$$23. \frac{ab}{c} \approx \frac{(0.00000293)(1.582 \times 10^{-14})}{2.8064 \times 10^{12}} = \frac{(2.93 \times 10^{-6})(1.582 \times 10^{-14})}{2.8064 \times 10^{12}} = \frac{2.93 \cdot 1.582}{2.8064} \times 10^{-6-14-12}$$

$$\approx 1.65 \times 10^{-32}$$

$$25. x^2 + 5x - 14 = (x + 7)(x - 2)$$

$$27. x^4 - 2x^2 + 1 = (x^2 - 1)^2 = [(x - 1)(x + 1)]^2 = (x - 1)^2(x + 1)^2$$

$$29. 16 - 4t^2 = 4(4 - t^2) = 4(2 - t)(2 + t) = -4(t - 2)(t + 2)$$

$$31. x^6 - 1 = (x^3 - 1)(x^3 + 1) = (x - 1)(x^2 + x + 1)(x + 1)(x^2 - x + 1)$$

$$33. -3x^{-1/2} + 2x^{1/2} + 5x^{3/2} = x^{-1/2}(5x^3 + 2x^2 - 3x) = x^{-1/2}(5x - 3)(x + 1)$$

$$35. 5x^3 + 15x^2 - x - 3 = 5x^2(x + 3) - (x + 3) = (5x^2 - 1)(x + 3)$$

$$37. (a + b)^2 - 3(a + b) - 10 = (a + b - 5)(a + b + 2)$$

$$39. (2y - 7)(2y + 7) = 4y^2 - 49$$

$$41. x^2(x - 2) + x(x - 2)^2 = x^3 - 2x^2 + x(x^2 - 4x + 4) = x^3 - 2x^2 + x^3 - 4x^2 + 4x = 2x^3 - 6x^2 + 4x$$

$$43. \frac{x^2 - 4x - 5}{x^2 - 25} \cdot \frac{x^2 + 12x + 36}{x^2 + 7x + 6} = \frac{(x - 5)(x + 1)}{(x + 5)(x - 5)} \cdot \frac{(x + 6)^2}{(x + 6)(x + 1)} = \frac{x + 1}{x + 5} \cdot \frac{x + 6}{x + 1} = \frac{x + 6}{x + 5}$$

$$45. \frac{2}{x} + \frac{1}{x - 2} + \frac{3}{(x - 2)^2} = \frac{2(x - 2)^2}{x(x - 2)^2} + \frac{x(x - 2)}{x(x - 2)^2} + \frac{3x}{x(x - 2)^2}$$

$$= \frac{2(x^2 - 4x + 4) + x^2 - 2x + 3x}{x(x - 2)^2} = \frac{2x^2 - 8x + 8 + x^2 - 2x + 3x}{x(x - 2)^2} = \frac{3x^2 - 7x + 8}{x(x - 2)^2}$$

$$47. \frac{\frac{1}{x} - \frac{1}{2}}{x - 2} = \frac{\frac{2x - x}{2x}}{x - 2} = \frac{2 - x}{2x} \cdot \frac{1}{x - 2} = \frac{-1(x - 2)}{2x} \cdot \frac{1}{x - 2} = \frac{-1}{2x}$$

$$49. \frac{\sqrt{x} - 2}{\sqrt{x} + 2} = \frac{\sqrt{x} - 2}{\sqrt{x} + 2} \cdot \frac{\sqrt{x} - 2}{\sqrt{x} - 2} = \frac{x + 4 - 4\sqrt{x}}{x - 4}$$

$$51. \frac{1}{\sqrt{11}} = \frac{1}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{\sqrt{11}}{11}$$

$$53. \frac{5}{1 + \sqrt{2}} = \frac{5(1 - \sqrt{2})}{(1 + \sqrt{2})(1 - \sqrt{2})} = \frac{5 - 5\sqrt{2}}{1 - 2} = 5\sqrt{2} - 5$$

$$55. 7x - 6 = 4x + 9 \Leftrightarrow 3x = 15 \Leftrightarrow x = 5$$

$$57. \frac{3x + 6}{x + 3} = \frac{3x}{x + 1} \Leftrightarrow (3x + 6)(x + 1) = 3x(x + 3) \Leftrightarrow 3x^2 + 9x + 6 = 3x^2 + 9x \Leftrightarrow 6 = 0. \text{ This is false, so there is no solution to the original equation.}$$

59. $x^2 - 9x + 14 = 0 \Leftrightarrow (x - 7)(x - 2) = 0 \Leftrightarrow x = 7$ or $x = 2$.
61. $2x^2 + x = 1 \Leftrightarrow 2x^2 + x - 1 = 0 \Leftrightarrow (2x - 1)(x + 1) = 0$. So either $2x - 1 = 0 \Leftrightarrow 2x = 1 \Leftrightarrow x = \frac{1}{2}$; or $x + 1 = 0 \Leftrightarrow x = -1$.
63. $0 = 4x^3 - 25x = x(4x^2 - 25) = x(2x - 5)(2x + 5) = 0$. So either $x = 0$; or $2x - 5 = 0 \Leftrightarrow 2x = 5 \Leftrightarrow x = \frac{5}{2}$; or $2x + 5 = 0 \Leftrightarrow 2x = -5 \Leftrightarrow x = -\frac{5}{2}$.
65. $3x^2 + 4x - 1 = 0 \Rightarrow$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(-1)}}{2(-3)} = \frac{-4 \pm \sqrt{16 + 12}}{-6} = \frac{-4 \pm \sqrt{28}}{-6} = \frac{-4 \pm 2\sqrt{7}}{-6} = \frac{2(-2 \pm \sqrt{7})}{-6} = \frac{-2 \pm \sqrt{7}}{3}.$$
67. $\frac{x}{x-2} + \frac{1}{x+2} = \frac{8}{x^2-4} \Leftrightarrow x(x+2) + (x-2) = 8 \Leftrightarrow x^2 + 2x + x - 2 = 8 \Leftrightarrow x^2 + 3x - 10 = 0 \Leftrightarrow (x-2)(x+5) = 0$
 $\Leftrightarrow x = 2$ or $x = -5$. However, since $x = 2$ makes the expression undefined, we reject this solution. Hence the only solution is $x = -5$.
69. $(x-3)^2 - 3(x-3) - 4 = 0 \Leftrightarrow [(x-3) - 4][(x-3) + 1] = 0 \Leftrightarrow (x-7)(x-2) = 0 \Leftrightarrow x = 2$ or $x = 7$.
71. $|x-7| = 4 \Leftrightarrow x-7 = \pm 4 \Leftrightarrow x = 7 \pm 4$, so $x = 11$ or $x = 3$.
73. (a) $(2-3i) + (1+4i) = (2+1) + (-3+4)i = 3+i$
 (b) $(2+i)(3-2i) = 6-4i+3i-2i^2 = 6-i+2 = 8-i$
75. (a) $\frac{4+2i}{2-i} = \frac{4+2i}{2-i} \cdot \frac{2+i}{2+i} = \frac{8+8i+2i^2}{4-i^2} = \frac{8+8i-2}{4+1} = \frac{6+8i}{5} = \frac{6}{5} + \frac{8}{5}i$
 (b) $(1-\sqrt{-1})(1+\sqrt{-1}) = (1-i)(1+i) = 1+i-i-i^2 = 1+1 = 2$
77. $x^2 + 16 = 0 \Leftrightarrow x^2 = -16 \Leftrightarrow x = \pm 4i$
79. $x^2 + 6x + 10 = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 40}}{2} = -3 \pm i$
81. $x^4 - 256 = 0 \Leftrightarrow (x^2 - 16)(x^2 + 16) = 0 \Leftrightarrow x = \pm 4$ or $x = \pm 4i$
83. Let x be the number of kilograms of raisins. Then the number of kilograms of nuts is $50 - x$.

	Raisins	Nuts	Mixture
Weight	x	$50 - x$	50
Cost per kilogram	3.20	2.40	2.72

So $3.20x + 2.40(50 - x) = 2.72(50) \Leftrightarrow 3.20x + 120 - 2.40x = 136 \Leftrightarrow 0.8x = 16 \Leftrightarrow x = 20$. Thus the mixture uses 20 kilograms of raisins and $50 - 20 = 30$ kilograms of nuts.

85. Let r be the athlete's running speed, in mi/h. Then they cycle at $r + 8$ km/h.

	Rate	Time	Distance
Cycle	$r + 8$	$\frac{4}{r + 8}$	4
Run	r	$\frac{2.5}{r}$	2.5

Since the total time of the workout is 1 hour, we have $\frac{4}{r+8} + \frac{2.5}{r} = 1$. Multiplying by $2r(r+8)$, we

get $4(2r) + 2.5(2)(r+8) = 2r(r+8) \Leftrightarrow 8r + 5r + 40 = 2r^2 + 16r \Leftrightarrow 0 = 2r^2 + 3r - 40 \Rightarrow$

$r = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-40)}}{2(2)} = \frac{-3 \pm \sqrt{9 + 320}}{4} = \frac{-3 \pm \sqrt{329}}{4}$. Since $r \geq 0$, we reject the negative value. The athlete's running speed is $r = \frac{-3 + \sqrt{329}}{4} \approx 3.78$ km/h.

87. Let t be the time it would take the interior decorator to paint a living room if they work alone. It would take the assistant $2t$ hours alone, and it would take the apprentice $3t$ hours alone. Thus, the decorator does $\frac{1}{t}$ of the job per hour, the assistant does $\frac{1}{2t}$ of the job per hour, and the apprentice does $\frac{1}{3t}$ of the job per hour. So $\frac{1}{t} + \frac{1}{2t} + \frac{1}{3t} = 1 \Leftrightarrow 6 + 3 + 2 = 6t \Leftrightarrow 6t = 11 \Leftrightarrow t = \frac{11}{6}$. Thus, it would take the decorator 1 hour 50 minutes to paint the living room alone.

89. $3x - 2 > -11 \Leftrightarrow 3x > -9 \Leftrightarrow x > -3$.

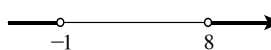
Interval: $(-3, \infty)$.

Graph: 

91. $x^2 - 7x - 8 > 0 \Leftrightarrow (x - 8)(x + 1) > 0$. The expression on the left of the inequality changes sign where $x = 8$ and where $x = -1$. Thus we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, 8)$	$(8, \infty)$
Sign of $x - 8$	-	-	+
Sign of $x + 1$	-	+	+
Sign of $(x - 8)(x + 1)$	+	-	+

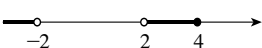
Interval: $(-\infty, -1) \cup (8, \infty)$.

Graph: 

93. $\frac{x - 4}{x^2 - 4} \leq 0 \Leftrightarrow \frac{x - 4}{(x - 2)(x + 2)} \leq 0$. The expression on the left of the inequality changes sign where $x = -2$, where $x = 2$, and where $x = 4$. Thus we must check the intervals in the following table.

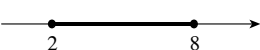
Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, 4)$	$(4, \infty)$
Sign of $x - 4$	-	-	-	+
Sign of $x - 2$	-	-	+	+
Sign of $x + 2$	-	+	+	+
Sign of $\frac{x - 4}{(x - 2)(x + 2)}$	-	+	-	+

Since the expression is not defined when $x = \pm 2$, we exclude these values and the solution is $(-\infty, -2) \cup (2, 4]$.

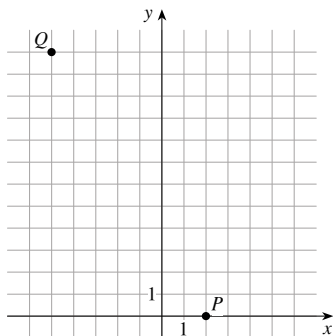
Graph: 

95. $|x - 5| \leq 3 \Leftrightarrow -3 \leq x - 5 \leq 3 \Leftrightarrow 2 \leq x \leq 8$.

Interval: $[2, 8]$

Graph: 

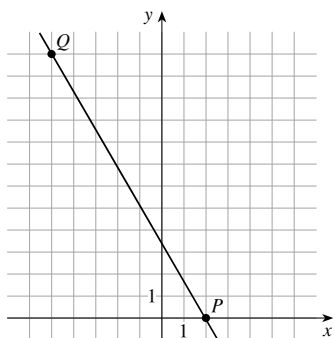
97. (a)

(b) The distance from P to Q is

$$\begin{aligned} d(P, Q) &= \sqrt{(-5 - 2)^2 + (12 - 0)^2} \\ &= \sqrt{49 + 144} = \sqrt{193} \end{aligned}$$

(c) The midpoint is $\left(\frac{-5 + 2}{2}, \frac{12 + 0}{2}\right) = \left(-\frac{3}{2}, 6\right)$.

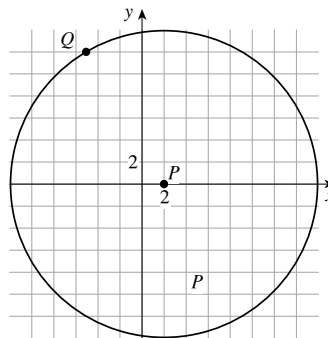
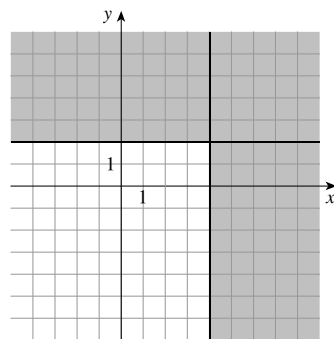
(d) The line has slope $m = \frac{12 - 0}{-5 - 2} = -\frac{12}{7}$, and has equation $y - 0 = -\frac{12}{7}(x - 2) \Leftrightarrow y = -\frac{12}{7}x + \frac{24}{7} \Leftrightarrow 12x + 7y - 24 = 0$.



(e) The radius of this circle was found in part (b). It is

 $r = d(P, Q) = \sqrt{193}$. So an equation is

$$(x - 2)^2 + (y - 0)^2 = (\sqrt{193})^2 \Leftrightarrow (x - 2)^2 + y^2 = 193.$$

99. $\{(x, y) \mid x \geq 4 \text{ or } y \geq 2\}$ 

$$101. d(A, C) = \sqrt{(4 - (-1))^2 + (4 - (-3))^2} = \sqrt{(4 + 1)^2 + (4 + 3)^2} = \sqrt{74} \text{ and}$$

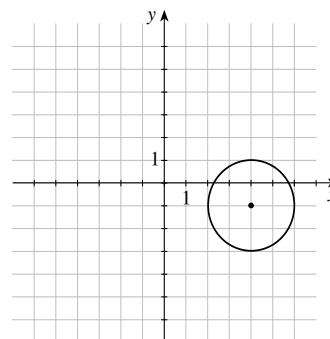
$$d(B, C) = \sqrt{(5 - (-1))^2 + (3 - (-3))^2} = \sqrt{(5 + 1)^2 + (3 + 3)^2} = \sqrt{72}. \text{ Therefore, } B \text{ is closer to } C.$$

103. The center is $C = (-5, -1)$, and the point $P = (0, 0)$ is on the circle. The radius of the circle is

$$\begin{aligned} r = d(P, C) &= \sqrt{(0 - (-5))^2 + (0 - (-1))^2} = \sqrt{(0 + 5)^2 + (0 + 1)^2} = \sqrt{26}. \text{ Thus, the equation of the circle is} \\ (x + 5)^2 + (y + 1)^2 &= 26. \end{aligned}$$

105. (a) $x^2 + y^2 - 8x + 2y + 13 = 0 \Leftrightarrow (x^2 - 8x) + (y^2 + 2y) = -13 \Leftrightarrow$
 $(x^2 - 8x + 16) + (y^2 + 2y + 1) = -13 + 16 + 1 \Leftrightarrow$
 $(x - 4)^2 + (y + 1)^2 = 4$, an equation of a circle.

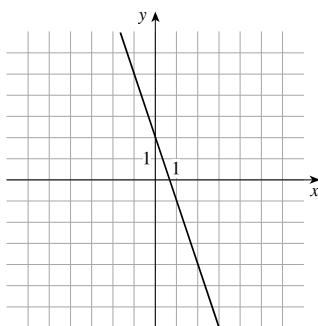
(b) The circle has center $(4, -1)$ and radius 2.



107. (a) $x^2 + y^2 + 72 = 12x \Leftrightarrow (x^2 - 12x) + y^2 = -72 \Leftrightarrow (x^2 - 12x + 36) + y^2 = -72 + 36 \Leftrightarrow (x - 6)^2 + y^2 = -36$.
 Since the left side of this equation must be greater than or equal to zero, this equation has no graph.

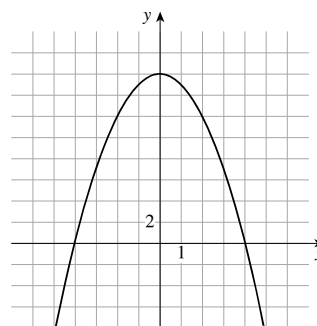
109. $y = 2 - 3x$

x	y
-2	8
0	2
$\frac{2}{3}$	0



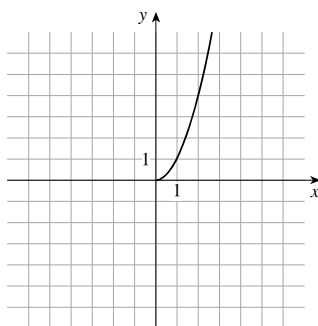
111. $y = 16 - x^2$

x	y
-3	7
-1	15
0	16
1	15
3	7



113. $x = \sqrt{y}$

x	y
0	0
1	1
2	4
3	9



115. $x = 16 - y^2$

(a) x -axis symmetry: replacing y by $-y$ gives $x = 16 - (-y)^2 = 16 - y^2$, which is the same as the original equation, so the graph is symmetric about the x -axis.

y -axis symmetry: replacing x by $-x$ gives $-x = 16 - y^2 \Leftrightarrow x = y^2 - 16$, which is not the same as the original equation, so the graph is not symmetric about the y -axis.

Origin symmetry: replacing x by $-x$ and y by $-y$ gives $-x = 16 - (-y)^2 \Leftrightarrow x = -16 - y^2$, which is not the same as the original equation, so the graph is not symmetric about the origin.

- (b) To find x -intercepts, we set $y = 0$ and solve for x : $x = 16 - 0^2 = 16$, so the x -intercept is 16.
To find y -intercepts, we set $x = 0$ and solve for y : $0 = 16 - y^2 \Leftrightarrow y = \pm 4$, so the y -intercepts are 4 and -4 .

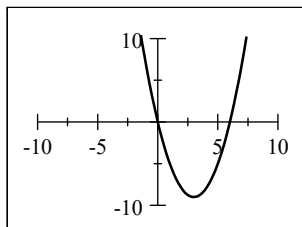
117. $x^2 - 9y = 9$

- (a) x -axis symmetry: replacing y by $-y$ gives $x^2 - 9(-y) = 9 \Leftrightarrow x^2 + 9y = 9$, so the graph is not symmetric about the x -axis.
 y -axis symmetry: replacing x by $-x$ gives $(-x)^2 - 9y = 9 \Leftrightarrow x^2 - 9y = 9$, so the graph is symmetric about the y -axis.
Origin symmetry: replacing x by $-x$ and y by $-y$ gives $(-x)^2 - 9(-y) = 9 \Leftrightarrow x^2 + 9y = 9$, so the graph is not symmetric about the origin.
- (b) To find x -intercepts, we set $y = 0$ and solve for x : $x^2 - 9(0) = 9 \Leftrightarrow x = \pm 3$, so the x -intercepts are 3 and -3 .
To find y -intercepts, we set $x = 0$ and solve for y : $0^2 - 9y = 9 \Leftrightarrow y = -1$, so the y -intercept is -1 .

119. $x^2 + 4xy + y^2 = 1$

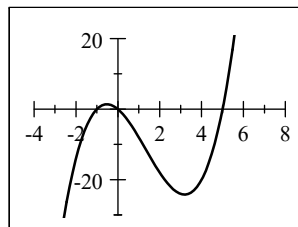
- (a) x -axis symmetry: replacing y by $-y$ gives $x^2 + 4x(-y) + (-y)^2 = 1$, which is different from the original equation, so the graph is not symmetric about the x -axis.
 y -axis symmetry: replacing x by $-x$ gives $(-x)^2 + 4(-x)y + y^2 = 1$, which is different from the original equation, so the graph is not symmetric about the y -axis.
Origin symmetry: replacing x by $-x$ and y by $-y$ gives $(-x)^2 + 4(-x)(-y) + (-y)^2 = 1 \Leftrightarrow x^2 + 4xy + y^2 = 1$, so the graph is symmetric about the origin.
- (b) To find x -intercepts, we set $y = 0$ and solve for x : $x^2 + 4x(0) + 0^2 = 1 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$, so the x -intercepts are -1 and 1.
To find y -intercepts, we set $x = 0$ and solve for y : $0^2 + 4(0)y + y^2 = 1 \Leftrightarrow y^2 = 1 \Leftrightarrow y = \pm 1$, so the y -intercepts are -1 and 1.

121. (a) We graph $y = x^2 - 6x$ in the viewing rectangle $[-10, 10]$ by $[-10, 10]$.



- (b) From the graph, we see that the x -intercepts are 0 and 6 and the y -intercept is 0.

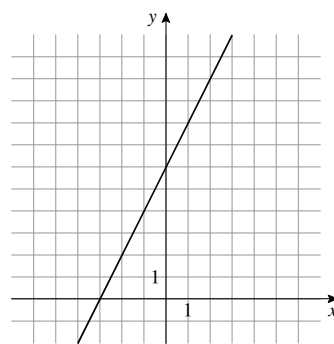
123. (a) We graph $y = x^3 - 4x^2 - 5x$ in the viewing rectangle $[-4, 8]$ by $[-30, 20]$.



- (b) From the graph, we see that the x -intercepts are -1 , 0, and 5 and the y -intercept is 0.

125. (a) The line that has slope 2 and y -intercept 6 has the slope-intercept equation $y = 2x + 6$.

- (b) An equation of the line in general form is $2x - y + 6 = 0$.



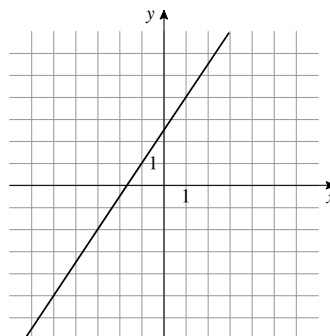
127. (a) The line that passes through the points $(-3, -2)$ and $(1, 4)$ has slope

$$m = \frac{4 - (-2)}{1 - (-3)} = \frac{3}{2}, \text{ so, using the second point for convenience, an}$$

$$\text{equation of the line is } y - 4 = \frac{3}{2}(x - 1) \Leftrightarrow y = \frac{3}{2}x + \frac{5}{2}.$$

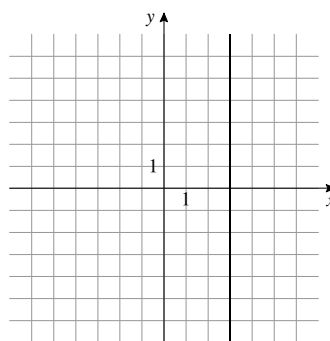
(b) $y = \frac{3}{2}x + \frac{5}{2} \Leftrightarrow 2y = 3x + 5 \Leftrightarrow 3x - 2y + 5 = 0.$

(c)



129. (a) The vertical line that passes through the point $(3, -2)$ has equation $x = 3$. (c)

(b) $x = 3 \Leftrightarrow x - 3 = 0.$



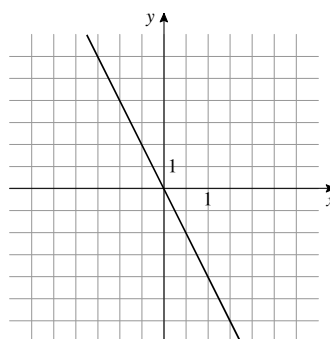
131. (a) The line containing $(2, 4)$ and $(4, -4)$ has slope

$$m = \frac{-4 - 4}{4 - 2} = \frac{-8}{2} = -4, \text{ and the line passing through the origin with}$$

$$\text{this slope has equation } y = -4x.$$

(b) $y = -4x \Leftrightarrow 4x + y = 0.$

(c)



133. (a) The slope, 0.3, represents the increase in length of the spring for each unit increase in weight w . The s -intercept is the resting or natural length of the spring.

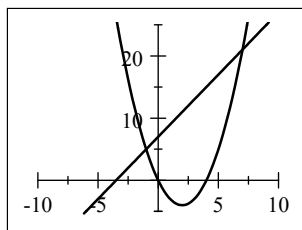
(b) When $w = 5$, $s = 0.3(5) + 2.5 = 1.5 + 2.5 = 4.0$ centimeters.

135. From the graph, we see that the graphs of $y = x^2 - 4x$ and $y = x + 6$ intersect at $x = -1$ and $x = 6$, so these are the solutions of the equation $x^2 - 4x = x + 6$.

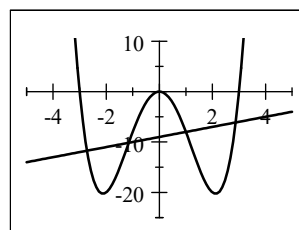
137. From the graph, we see that the graph of $y = x^2 - 4x$ lies below the graph of $y = x + 6$ for $-1 < x < 6$, so the inequality $x^2 - 4x \leq x + 6$ is satisfied on the interval $[-1, 6]$.

139. From the graph, we see that the graph of $y = x^2 - 4x$ lies above the x -axis for $x < 0$ and for $x > 4$, so the inequality $x^2 - 4x \geq 0$ is satisfied on the intervals $(-\infty, 0]$ and $[4, \infty)$.

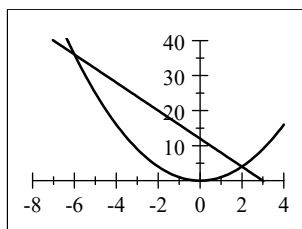
141. $x^2 - 4x = 2x + 7$. We graph the equations $y_1 = x^2 - 4x$ and $y_2 = 2x + 7$ in the viewing rectangle $[-10, 10]$ by $[-5, 25]$. Using a zoom or trace function, we get the solutions $x = -1$ and $x = 7$.



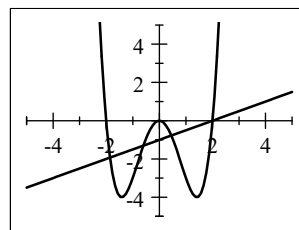
143. $x^4 - 9x^2 = x - 9$. We graph the equations $y_1 = x^4 - 9x^2$ and $y_2 = x - 9$ in the viewing rectangle $[-5, 5]$ by $[-25, 10]$. Using a zoom or trace function, we get the solutions $x \approx -2.72$, $x \approx -1.15$, $x = 1.00$, and $x \approx 2.87$.



145. $x^2 > 12 - 4x$. We graph the equations $y_1 = x^2$ and $y_2 = 12 - 4x$ in the viewing rectangle $[-8, 4]$ by $[0, 40]$. Using a zoom or trace function, we find the points of intersection are at $x = -6$ and $x = 2$. Since we want $x^2 > 12 - 4x$, the solution is the union of intervals $(-\infty, -6) \cup (2, \infty)$.



147. $x^4 - 4x^2 < \frac{1}{2}x - 1$. We graph the equations $y_1 = x^4 - 4x^2$ and $y_2 = \frac{1}{2}x - 1$ in the viewing rectangle $[-5, 5]$ by $[-5, 5]$. We find the points of intersection are at $x \approx -1.85$, $x \approx -0.60$, $x \approx 0.45$, and $x = 2.00$. Since we want $x^4 - 4x^2 < \frac{1}{2}x - 1$, the solution is $(-1.85, -0.60) \cup (0.45, 2.00)$.



149. Here the center is at $(0, 0)$, and the circle passes through the point $(-5, 12)$, so the radius is

$r = \sqrt{(-5-0)^2 + (12-0)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$. The equation of the circle is $x^2 + y^2 = 13^2 \Leftrightarrow x^2 + y^2 = 169$. The line shown is the tangent that passes through the point $(-5, 12)$, so it is perpendicular to the line through the points $(0, 0)$ and $(-5, 12)$. This line has slope $m_1 = \frac{12-0}{-5-0} = -\frac{12}{5}$. The slope of the line we seek is $m_2 = -\frac{1}{m_1} = -\frac{1}{-12/5} = \frac{5}{12}$. Thus, an equation of the tangent line is $y - 12 = \frac{5}{12}(x + 5) \Leftrightarrow y - 12 = \frac{5}{12}x + \frac{25}{12} \Leftrightarrow y = \frac{5}{12}x + \frac{169}{12} \Leftrightarrow 5x - 12y + 169 = 0$.

151. Since M varies directly as z we have $M = kz$. Substituting $M = 120$ when $z = 15$, we find $120 = k(15) \Leftrightarrow k = 8$. Therefore, $M = 8z$.

153. (a) The intensity I varies inversely as the square of the distance d , so $I = \frac{k}{d^2}$.

(b) Substituting $I = 1000$ when $d = 8$, we get $1000 = \frac{k}{(8)^2} \Leftrightarrow k = 64,000$.

(c) From parts (a) and (b), we have $I = \frac{64,000}{d^2}$. Substituting $d = 20$, we get $I = \frac{64,000}{(20)^2} = 160$ candles.

155. Note: In the first printing of the text, the answer given for this exercise is incorrect.

Let v be the terminal velocity of the parachutist in km/h and w be their weight in kilograms. Since the terminal velocity is directly proportional to the square root of the weight, we have $v = k\sqrt{w}$. Substituting $v = 14$ when $w = 70$, we solve for k . This gives $14 = k\sqrt{70} \Leftrightarrow k = \frac{14}{\sqrt{70}} \approx 1.673$. Thus $v = 1.673\sqrt{w}$. When $w = 105$, the terminal velocity is $v = 1.673\sqrt{105} \approx 17$ km/h.

157. The speed v is inversely proportional to the square root of the density d , so $v = \frac{k}{\sqrt{d}}$. In fresh water with density

$d_1 = 1$ g/cm³, the speed is $v_1 = 1480$ m/s, so $1480 = \frac{k}{\sqrt{1}} \Leftrightarrow k = 1480$. Thus, in seawater with density 1.0273 g/cm³, we have $v = \frac{1480}{\sqrt{1.0273}} \approx 1460$ m/s.

159. We solve the first equation for v : $1 + z = \sqrt{\frac{c+v}{c-v}} \Rightarrow (1+z)^2 = \frac{c+v}{c-v} \Rightarrow (c-v)(1+z)^2 = c+v \Rightarrow$

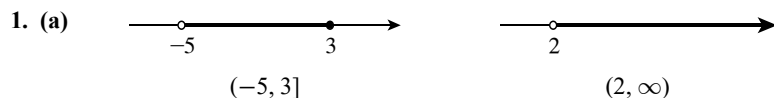
$$c(1+z)^2 - v(1+z)^2 = c+v \Rightarrow v[(1+z)^2 + 1] = c[(1+z)^2 - 1] \Rightarrow v = \frac{c[(1+z)^2 - 1]}{(1+z)^2 + 1}.$$

Substituting $z = 2$ and $c = 3 \times 10^5$, we find $v = \frac{3 \times 10^5 [(1+2)^2 - 1]}{(1+2)^2 + 1} = 3 \times 10^5 \left(\frac{8}{10} \right) = 2.4 \times 10^5$ km/s.

By Hubble's Law, $v = H_0 D \Rightarrow D = \frac{v}{H_0}$, so substituting v from above and $H_0 \approx 20.8$ Mly/km/s, we have

$$D \approx \frac{2.4 \times 10^5}{20.8} \approx 11,538 \text{ megalight-years.}$$

CHAPTER 1 TEST



(b) $x \leq 3 \Leftrightarrow x \in (-\infty, 3]$; $-1 \leq x < 4 \Leftrightarrow x \in [-1, 4)$

(c) Distance $= |-7 - 9| = |-16| = 16$

3. (a) $186,000,000,000 = 1.86 \times 10^{11}$

(b) $0.000\,000\,396\,5 = 3.965 \times 10^{-7}$

5. (a) $z(4z - 3) + 2z(3 - 2z) = 4z^2 - 3z + 6z - 4z^2 = 3z$

(b) $(x + 3)(4x - 5) = 4x^2 - 5x + 12x - 15 = 4x^2 + 7x - 15$

(c) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$

(d) $(2x + 3)^2 = (2x)^2 + 2(2x)(3) + (3)^2 = 4x^2 + 12x + 9$

(e) $(x + 2)^3 = (x)^3 + 3(x)^2(2) + 3(x)(2)^2 + (2)^3 = x^3 + 6x^2 + 12x + 8$

7. (a) $\frac{w^2 + 4w + 3}{w^2 - 2w - 3} = \frac{(w + 3)(w + 1)}{(w - 3)(w + 1)} = \frac{w + 3}{w - 3}$

(b) $\frac{x^2}{x^2 - 4} - \frac{x + 1}{x + 2} = \frac{x^2}{(x - 2)(x + 2)} - \frac{x + 1}{x + 2} = \frac{x^2}{(x - 2)(x + 2)} + \frac{-(x + 1)(x - 2)}{(x - 2)(x + 2)}$

$$= \frac{x^2 - (x^2 - x - 2)}{(x - 2)(x + 2)} = \frac{x + 2}{(x - 2)(x + 2)} = \frac{1}{x - 2}$$

$$(c) \frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}} = \frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}} \cdot \frac{xy}{xy} = \frac{y^2 - x^2}{x - y} = \frac{(y - x)(y + x)}{x - y} = \frac{-(x - y)(y + x)}{x - y} = -(x + y)$$

$$9. (a) x + 5 = 14 - \frac{1}{2}x \Leftrightarrow 2x + 10 = 28 - x \Leftrightarrow 3x = 18 \Leftrightarrow x = 6$$

$$(b) \frac{2x}{x+1} = \frac{2x-1}{x} \Leftrightarrow (2x)(x) = (2x-1)(x+1) (x \neq -1, x \neq 0) \Leftrightarrow 2x^2 = 2x^2 + x - 1 \Leftrightarrow 0 = x - 1 \Leftrightarrow x = 1$$

$$(c) x^2 - x - 12 = 0 \Leftrightarrow (x - 4)(x + 3) = 0. \text{ So } x = 4 \text{ or } x = -3.$$

$$(d) 2x^2 + 4x + 1 = 0 \Rightarrow x = \frac{-4 \pm \sqrt{4^2 - 4(2)(1)}}{2(2)} = \frac{-4 \pm \sqrt{16 - 8}}{4} = \frac{-4 \pm \sqrt{8}}{4} = \frac{-4 \pm 2\sqrt{2}}{4} = \frac{-2 \pm \sqrt{2}}{2}.$$

$$(e) \sqrt{3 - \sqrt{x+5}} = 2 \Rightarrow 3 - \sqrt{x+5} = 4 \Leftrightarrow -1 = \sqrt{x+5}. \text{ (Note that this is impossible, so there can be no solution.)}$$

Squaring both sides again, we get $1 = x + 5 \Leftrightarrow x = -4$. But this does not satisfy the original equation, so there is no solution. (You must always check your final answers if you have squared both sides when solving an equation, since extraneous answers may be introduced, as here.)

$$(f) x^4 - 3x^2 + 2 = 0 \Leftrightarrow (x^2 - 1)(x^2 - 2) = 0. \text{ So } x^2 - 1 = 0 \Leftrightarrow x = \pm 1 \text{ or } x^2 - 2 = 0 \Leftrightarrow x = \pm\sqrt{2}. \text{ Thus the solutions are } x = -1, x = 1, x = -\sqrt{2}, \text{ and } x = \sqrt{2}.$$

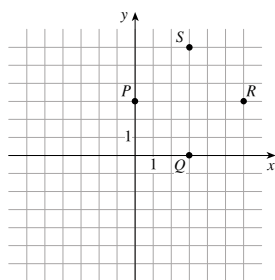
$$(g) 3|x - 4| - 10 = 0 \Leftrightarrow 3|x - 4| = 10 \Leftrightarrow |x - 4| = \frac{10}{3} \Leftrightarrow x - 4 = \pm\frac{10}{3} \Leftrightarrow x = 4 \pm \frac{10}{3}. \text{ So } x = 4 - \frac{10}{3} = \frac{2}{3} \text{ or } x = 4 + \frac{10}{3} = \frac{22}{3}. \text{ Thus the solutions are } x = \frac{2}{3} \text{ and } x = \frac{22}{3}.$$

$$11. \text{ Using the Quadratic Formula, } 2x^2 + 4x + 3 = 0 \Leftrightarrow x = \frac{-4 \pm \sqrt{4^2 - 4(2)(3)}}{2(2)} = \frac{-4 \pm \sqrt{-8}}{4} = -1 \pm \frac{\sqrt{2}}{2}i.$$

$$13. \text{ Let } w \text{ be the width of the parcel of land. Then } w + 70 \text{ is the length of the parcel of land. Then } w^2 + (w + 70)^2 = 130^2 \Leftrightarrow w^2 + w^2 + 140w + 4900 = 16,900 \Leftrightarrow 2w^2 + 140w - 12,000 = 0 \Leftrightarrow w^2 + 70w - 6000 = 0 \Leftrightarrow (w - 50)(w + 120) = 0. \text{ So } w = 50 \text{ or } w = -120. \text{ Since } w \geq 0, \text{ the width is } w = 50 \text{ m and the length is } w + 70 = 120 \text{ m.}$$

$$15. 5 \leq \frac{5}{9}(F - 32) \leq 10 \Leftrightarrow 9 \leq F - 32 \leq 18 \Leftrightarrow 41 \leq F \leq 50. \text{ Thus the medicine is to be stored at a temperature between } 41^\circ \text{ F and } 50^\circ \text{ F.}$$

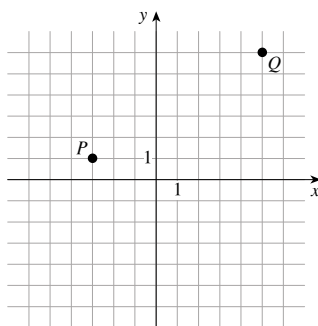
17. (a)



There are several ways to determine the coordinates of S . The diagonals of a square have equal length and are perpendicular. The diagonal PR is horizontal and has length 6 units, so the diagonal QS is vertical and also has length 6. Thus, the coordinates of S are $(3, 6)$.

$$(b) \text{ The length of } PQ \text{ is } \sqrt{(0 - 3)^2 + (3 - 0)^2} = \sqrt{18} = 3\sqrt{2}. \text{ So the area of } PQRS \text{ is } (3\sqrt{2})^2 = 18.$$

19. (a)

(b) The distance between P and Q is

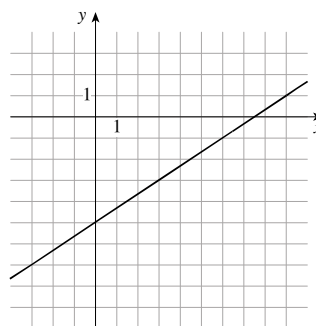
$$d(P, Q) = \sqrt{(-3 - 1)^2 + (-5 - 6)^2} = \sqrt{64 + 25} = \sqrt{89}.$$

$$(c) \text{ The midpoint is } \left(\frac{-3 + 1}{2}, \frac{-5 + 6}{2} \right) = \left(-1, \frac{1}{2} \right).$$

$$(d) \text{ The center of the circle is the midpoint, } \left(-1, \frac{1}{2} \right), \text{ and the length of the radius is } \frac{1}{2}\sqrt{89}. \text{ Thus the equation of the circle whose diameter is } PQ \text{ is}$$

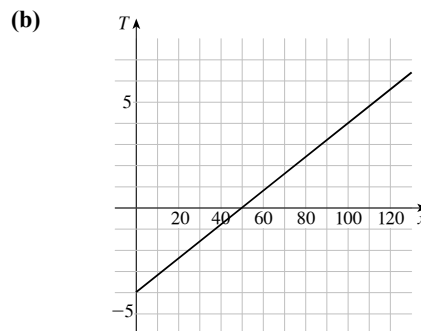
$$(x + 1)^2 + \left(y - \frac{1}{2} \right)^2 = \left(\frac{1}{2}\sqrt{89} \right)^2 \Leftrightarrow (x + 1)^2 + \left(y - \frac{1}{2} \right)^2 = \frac{89}{4}.$$

21. $2x - 3y = 15 \Leftrightarrow -3y = -2x + 15 \Leftrightarrow y = \frac{2}{3}x - 5$. The slope is $\frac{2}{3}$ and the y -intercept is -5 .



23. (a) When $x = 100$ we have $T = 0.08(100) - 4 = 8 - 4 = 4$, so the temperature at one meter is 4°C .

- (c) The slope represents the raise in temperature as the depth increase. The T -intercept is the surface temperature of the soil and the x -intercept represents the depth of the “frost line”, where the soil below is not frozen.



25. **Note: In the first printing of the text, the answers given for parts (b) and (c) of this exercise are incorrect.**

(a) $M = k \frac{wh^2}{L}$

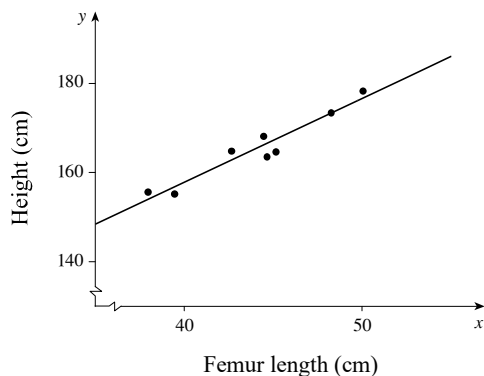
- (b) Substituting $w = 10$, $h = 15$, $L = 30$, and $M = 21,000$, we have $21,000 = k \frac{(10)(15^2)}{30} \Leftrightarrow k = 280$. Thus

$$M = 280 \frac{wh^2}{L}.$$

- (c) Now if $L = 25$, $w = 7.5$, and $h = 25$, then $M = 280 \frac{(7.5)(25^2)}{25} = 52,500$. So the beam can support 52,500 newtons.

FOCUS ON MODELING Fitting Lines to Data

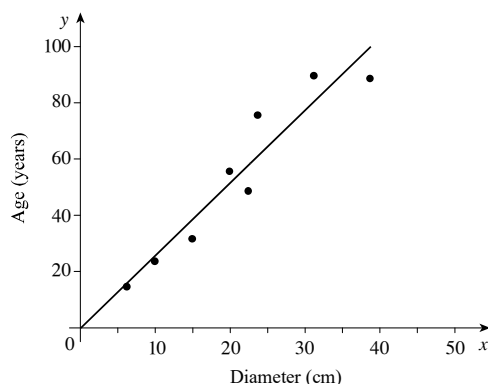
1. (a) Using a graphing calculator, we obtain the regression line $y = 1.8807x + 82.65$.



- (b) Using $x = 58$ in the equation $y = 1.8807x + 82.65$, we get $y = 1.8807(58) + 82.65 \approx 191.7$ cm.

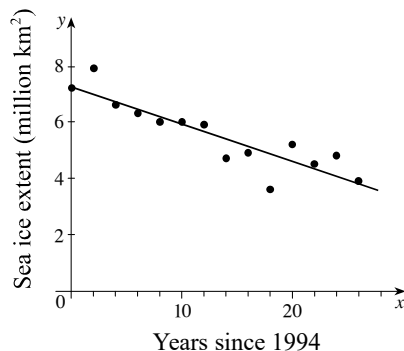
3. **Note: In the first printing of the text, the answer given for part (a) of this exercise is incorrect.**

(a) Using a graphing calculator, we obtain the regression line $y = 2.579x - 0.1783$.



(b) Using $x = 45$ in the equation $y = 2.579x - 0.1783$, we get $y = 2.579(45) - 0.1783 \approx 116$ years.

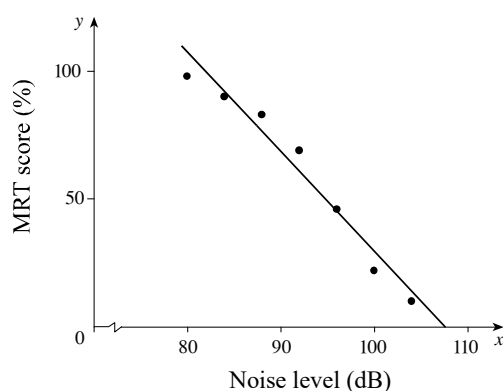
5. (a) Using a graphing calculator, we obtain the regression line $y = -0.13198x + 7.2514$.



(b) Using $x = 25$ in the regression line equation, we get $y = -0.13198(25) + 7.2514 \approx 3.95$ million km². This is roughly 10% less than the actual figure of 4.4 million km².

(c) Despite fluctuations over brief periods, the model seems fairly accurate. If external circumstances change (reduced or increased CO₂ emissions, for example), it may become less reliable. It is unlikely to be accurate far into the future.

7. (a) Using a graphing calculator, we obtain $y = -3.9018x + 419.7$.



(b) The correlation coefficient is $r = -0.98$, so linear model is appropriate for x between 80 dB and 104 dB.

(c) Substituting $x = 94$ into the regression equation, we get $y = -3.9018(94) + 419.7 \approx 53$. So the intelligibility is about 53%.

9. Results will depend on student surveys in each class.

1 ERRATA in Exercises and Answers in First Printing

Page 123 **1.11.56** **Displayed equation should read** $y = \sqrt{2.4x + \left(\frac{x}{1000}\right)^2}$

Page 123 **1.11.56** **(a)** Graph the equation for $0 \leq x \leq 160$.

Page A2 **1.5.129 (Answer)** ≈ 4.24 s

Page A2 **1.5.137 (Answer)** 344, 000 km

Page A3 **1.7.21 (Answer)** 220 km

Page A3 **1.7.39 (Answer)** 31.6 m by 158 m

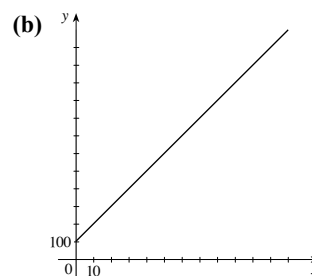
Page A3 **1.7.77 (Answer)** 1.6 m from the fulcrum

Page A4 **1.8.119 (Answer)** More than 100 km

Page A4 **1.8.127 (Answer)** **(a)** Acceleration greater than 8.04 m/s^2

Page A7 **1.10.93 (Answer)** **(a)** $t = \frac{1}{8}n + 6$ **(b)** 25°C

Page A7 **1.10.95 (Answer)** **(a)** $P = 9.97d + 103$, where P is pressure in kPa and d is depth in meters
(c) The slope represents an increase of 9.97 kPa in pressure for each one meter increase in depth, and the d -intercept is the air pressure at the surface. **(d)** ≈ 59 m



Page A7 **1.12.39 (Answer)** ≈ 69 km/h

Page A7 **1.12.41 (Answer)** ≈ 9.03 km/h

Page A7 **1.12.49 (Answer)** **(b)** 1, 928, 234, 931

Page A8 **1.Review.155 (Answer)** 17 km/h

Page A9 **1.Test.12 (Answer)** 192 km

Page A9 **1.Test.25 (Answer)** **(b)** 280 **(c)** 52,500 N

Page A10 **1.Focus.3 (Answer)** **(a)** $y = 2.579x - 0.1783$

