Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

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Chapter 2. Equations of Motion

1. Determine the lift coefficient of an aircraft with a 180 ft^2 wing area and a mass of 3,200 kg in a cruising flight when flying at sea level with a speed of

a. 80 knot

b. 130 knot.

$S1 := 180 \cdot ft^2$ $\quad m1 := 3200 \cdot kg$ $\quad g = 9.807 m \cdot s^{-2}$ $\quad \rho_o := 1.225 \cdot \frac{kg}{m^3}$

- 80 knot

$V1 := 80 \cdot knot \quad V1 = 41.156 \frac{m}{s}$

$$C_L := \frac{2 \cdot m1 \cdot g}{\rho_o \cdot V1^2 \cdot S1} = 1.809$$ (Equ 2.4)

- 130 knot

$V1 := 130 \cdot knot \quad V1 = 66.878 \frac{m}{s}$

$$C_L := \frac{2 \cdot m1 \cdot g}{\rho_o \cdot V1^2 \cdot S1} = 0.685$$ (Equ 2.4)

2. An aircraft with a mass of 1,200 kg and a wing area of 14 m^2 is cruising at 3,000 ft altitude. Determine its lift coefficient when the true air speed is 100 knot.

$S1 := 14 \cdot m^2$ $\quad m1 := 1200 \cdot kg$ $\quad g = 9.807 \frac{m}{s^2}$ $\quad h := 3000 \cdot ft$

$\rho_3 := 0.002175 \cdot \frac{slug}{ft^3}$ $\quad \rho_3 = 1.121 \frac{kg}{m^3}$

$V1 := 100 \cdot knot$ $\quad V1 = 51.444 \frac{m}{s}$

$C_L := \frac{2 \cdot m1 \cdot g}{\rho_3 \cdot V1^2 \cdot S1} = 0.567$ (Equ 2.4)

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3. Assume that the aircraft in problem 2 has drag coefficient of 0.05. How much thrust the engine is producing?

$S1 := 14 m^2$ $\quad m1 := 1200 kg$ $\quad g = 9.807 \frac{m}{s^2}$ $\quad h := 3000 ft$ $\quad C_D := 0.05$

$\rho_3 := 0.002175 \frac{slug}{ft^3}$ $\quad \rho_3 = 1.121 \frac{kg}{m^3}$

$V1 := 100 knot$ $\quad V1 = 51.444 \frac{m}{s}$

$D := \frac{1}{2} \cdot \rho_3 \cdot V1^2 \cdot S1 \cdot C_D = 1038.3 N$ (Equ 2.5)

$T1 := D = 1038.3 N$ (Equ 2.17)

4. Determine lift curve slope (in 1/rad) of a wing with aspect ratio of 12.5. Then, calculate the lift coefficient of this wing when its angle of attack is 5 degrees. Assume that the zero lift angle of attack is zero, and a_o is 2π 1/rad.

$AR := 12.5 \qquad \alpha := 5 \cdot deg \qquad a_o := 2 \cdot \pi \qquad \alpha_o := 0$

$$C_{L\alpha} := \frac{a_o}{1 + \frac{a_o}{\pi \cdot AR}} = 5.417 \qquad \text{(Equ 2.13)}$$

$$C_{L\alpha} = \frac{C_L}{(\alpha - \alpha_o)} \qquad C_L := C_{L\alpha} \cdot (\alpha - \alpha_o) = 0.473 \qquad \text{(Equ 2.12)}$$

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5. Calculate the true and equivalent stall speeds of the aircraft in problem 2, when the maximum lift coefficient is 1.6.

$S1 := 14 \cdot m^2 \qquad m1 := 1200 \cdot kg \qquad g = 9.807 \cdot \frac{m}{s^2} \qquad h := 3000 \cdot ft \qquad C_{Lmax} := 1.6$

$\rho_3 := 0.002175 \cdot \frac{slug}{ft^3} \qquad \rho_3 = 1.121 \cdot \frac{kg}{m^3} \qquad \rho_o := 1.225 \cdot \frac{kg}{m^3}$

$$V_s := \sqrt{\frac{2 \cdot m1 \cdot g}{(\rho_3 \cdot C_{Lmax} \cdot S1)}} = 30.616 \frac{m}{s} \qquad V_s = 59.513\, knot \qquad \text{(Equ 2.51)} \qquad \text{True}$$

$$\sigma := \frac{\rho_3}{\rho_o} = 0.915 \qquad \text{(Equ 1.25)}$$

$$V_T = \frac{V_E}{\sqrt{\sigma}} \qquad V_E := V_s \cdot \sqrt{\sigma} = 29.287 \frac{m}{s} \qquad V_E = 56.929\, knot \qquad \text{(Equ 2.49)} \qquad \text{Equivalent}$$

6. An aircraft is required to climb with a 10 degrees of climb angle. The aircraft has a mass of 30,000 kg and producing 50,000 N of drag. Assume zero angle of attack and zero thrust setting angle.

a. How much lift this aircraft must generate?
b. How much thrust the aircraft engine must produce?

$\gamma := 10\,deg \qquad m1 := 30000\,kg \qquad D1 := 50000\,N \qquad \alpha := 0 \qquad i_T := 0$

$W1 := m1 \cdot g = 294200\,N$

$L1 := W1 \cdot \cos(\gamma) = 289730\,N$ (Equ 2.33)

$T1 := D1 + W1 \cdot \sin(\gamma) = 101087\,N$ (Equ 2.32)

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7. An aircraft which is initially at rest is accelerating on a runway with an acceleration of 10 m/sec^2. Consider a moment when other features of this aircraft are:

$$S = 30\ m^2,\ m = 6{,}000\ kg,\ C_L = 0.7,\ C_D = 0.1,\ V = 60\ knot$$

Calculate the engine thrust, assuming that the friction force is constant and equal to 2% of the aircraft weight.

$S1 := 30\,m^2 \qquad m1 := 6000\,kg \qquad C_L := 0.7 \qquad C_D := 0.1 \qquad a1 := 10\,\frac{m}{s^2}$

$\rho := 1.225\,\frac{kg}{m^3} \qquad V1 := 60\,knot \qquad V1 = 30.9\,\frac{m}{s}$

$W1 := m1 \cdot g = 58839.9\,N$

$F_f := 0.02 \cdot W1 = 1176.8\,N$

$D1 := \frac{1}{2} \cdot \rho \cdot (V1)^2 \cdot S1 \cdot C_D = 1750.7\,N$ (Equ 2.5)

$T1 := m1 \cdot a1 + D1 + F_f$ (Equ 2.34)

8. A cargo aircraft with a weight of 145,000 lb and wing area of 1,318 ft^2 has a maximum lift coefficient of 2.5. Is this aircraft able to cruise at an altitude of 25,000 ft and ISA+15 condition with a speed of 150 KTAS?

$W1 := 145000\,lbf$ $\quad S1 := 1318\,ft^2$ $\quad C_{Lmax} := 2.5$ $\quad h := 25000\,ft$ $\quad$ ISA+15

$V1 := 150\,knot$ $\quad L1 := 2\cdot\frac{K}{1000\,ft}$ $\quad$ Chapter 1 $\quad R1 := 287\cdot\frac{J}{kg\cdot K}$

Sea level: $\quad P_o := 101325\,Pa$ $\quad T_o := (15 + 15 + 273)\cdot K = 303\,K$

25000 ft: $\quad T_{25} := T_o - L1\cdot h = 253\,K$ $\quad$ (Equ 1.6)

Appendix B: $\quad P_{25} := 786.3\frac{lbf}{ft^2}$

$$\rho_{25} := \frac{P_{25}}{R1\cdot(T_{25})} = 0.518\frac{kg}{m^3} \qquad \rho_{25} = 0.00101\frac{slug}{ft^3}$$ (Equ 1.23)

$$V_{s15} := \sqrt{\frac{2\cdot W1}{\rho_{25}\cdot S1\cdot C_{Lmax}}} = 90.153\frac{m}{s} \qquad V_{s15} = 175.243\,knot$$ (Equ 2.51)

This aircraft is not able to cruise at an altitude of 25,000 ft and ISA+15 condition with a speed of 150 KTAS, since this speed is less than the stall speed at that altitude.

9. A hang glider (*Nimbus*) has a mass (structure plus pilot) of 138 kg and wing area of 16.2 m^2 and stall speed of 16 knot. What is the maximum lift coefficient?

$S1 := 16.2 \cdot m^2$ $\quad m1 := 138 \cdot kg$ $\quad g = 9.807 \cdot \frac{m}{s^2}$ $\quad \rho_o := 1.225 \cdot \frac{kg}{m^3}$

$V_s := 16 \cdot knot$ $\quad V_s = 8.231 \cdot \frac{m}{s}$

$$C_{Lmax} := \frac{2 \cdot m1 \cdot g}{\rho_o \cdot V_s^2 \cdot S1} = 2.013$$ (Equ 2.51)

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10. Calculate the wing area of a hang glider *Volmer VJ-23*. The aircraft geometry and weight information may be taken from Table 2.2. If the pilot mass is 75 kg, what is the mass of the aircraft structure?

$g = 9.807 \cdot \frac{m}{s^2}$ $\quad \rho_o := 1.225 \cdot \frac{kg}{m^3}$ $\quad m_P := 75 \cdot kg$

From Table 2.2, row 4: $V_s := 13 \cdot knot$ $\quad C_{Lmax} := 2.9$ $\quad m1 := 136 \cdot kg$

$$S1 := \frac{2 \cdot m1 \cdot g}{\left(\rho_o \cdot C_{Lmax} \cdot V_s^2\right)} = 16.616 m^2$$ (Equ 2.51)

$m_{st} := m1 - m_P = 61 kg$

11. The sport aircraft *Butterworth* has the following characteristic

$$m = 635 \text{ kg}, S = 10.4 \text{ m}^2, V_s = 56 \text{ knot (at sea level)}$$

Assume that the maximum speed of this aircraft at every altitude is 126 knot (TAS). At what altitude maximum true airspeed and stall true airspeed will be the same?

$m1 := 635 \cdot kg$ $\quad S1 := 10.4 \cdot m^2$ $\quad V_s := 56 \cdot knot$ $\quad \rho_o := 1.225 \cdot \frac{kg}{m^3}$ $\quad V_{s\_alt} := 126 \cdot knot$

$$C_{Lmax} := \frac{(2 \cdot m1 \cdot g)}{\left(\rho_o \cdot S1 \cdot V_s^2\right)} = 1.178$$ (Equ 2.51)

Aircraft CLmax is constant.

$$\rho := \frac{2 \cdot m1 \cdot g}{S1 \cdot C_{Lmax} \cdot V_{s\_alt}^2} = 0.242 \frac{kg}{m^3}$$ (Equ 2.51)

From Appendix A, this air density belongs to an altitude of 13700 m or 45000 ft.

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12. The cargo aircraft *C-130* has an empty mass of 13,000 kg, a wing area of 85 m^2, and a stall speed of 94 knot (EAS). If maximum lift coefficient is 2.2, determine maximum the mass of payload (cargo and crew) plus fuel to satisfy this stall speed.

$m_E := 13000 \cdot kg$ $\quad S1 := 85 \cdot m^2$ $\quad V_s := 94 \cdot knot$ $\quad \rho_o := 1.225 \cdot \frac{kg}{m^3}$ $\quad C_{Lmax} := 2.2$

$$m \cdot g = \frac{1}{2} \cdot \rho \cdot V_s^2 \cdot S \cdot C_{Lmax}$$ (Equ 2.51)

$$m_{TO} := \frac{\rho_o \cdot V_s^2 \cdot S1 \cdot C_{Lmax}}{2 \cdot g} = 27312 kg$$

$$m_{pf} := m_{TO} - m_E = 14312 kg$$

13. The bomber *B-1B* has a maximum take-off mass of 216,367 kg, a 181 m^2 wing area, and a maximum velocity of Mach 2.2. Assume drag coefficient of this aircraft at cruise is 0.03, how much thrust the four engines are generating for this flight condition?

$m1 := 216367 \cdot kg$ $\quad S1 := 181 \cdot m^2$ $\quad g = 9.807 \cdot \frac{m}{s^2}$ $\quad C_D := 0.03$ $\quad M1 := 2.2$

$\rho_o := 1.225 \cdot \frac{kg}{m^3}$ $\quad a := 340 \cdot \frac{m}{s}$ speed of sound at sea level

$V1 := M1 \cdot a = 748 \frac{m}{s}$ (Equ 1.35)

$D := \frac{1}{2} \cdot \rho_o \cdot V1^2 \cdot S1 \cdot C_D = 1860840.4\ N$ (Equ 2.5)

$T1 := D = 1860.8 \cdot kN$ (Equ 2.17)

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14. The trainer aircraft *PC-7* with a mass of 2,700 kg and a wing area of 16.6 m^2 has a cruising speed of 330 km/hr.

a. What is the lift coefficient when cruising at 5,000 m altitude, ISA condition?
b. In a summer day, the temperature at sea level is 42 ºC. How much lift coefficient must be increased when cruising at this day and at the same altitude?

$m1 := 2700kg$ $\quad S1 := 16.6m^2$ $\quad h := 5000m$ $\quad V_C := 330\cdot\frac{km}{hr}$ $\quad V_C = 178.186knot$ $\quad V_C = 91.667\frac{m}{s}$

a .... 5000 m, ISA $\quad \rho_5 := 0.7364\frac{kg}{m^3}$

$$C_{Lc1} := \frac{2\cdot m1\cdot g}{\rho_5\cdot V_C^{\,2}\cdot S1} = 0.516 \qquad \text{(Equ 2.4)}$$

b ....Non-ISA

$L1 := 6.5\frac{K}{1000m}$ $\quad$ Chapter 1 $\quad R1 := 287\cdot\frac{J}{kg\cdot K}$

Sea level: $\quad P_o := 101325Pa$ $\quad T_o := (42 + 273)\cdot K = 315K$

5000 m: $\quad T_5 := T_o - L1\cdot h = 282.5K$ $\quad$ (Equ 1.6)

Appendix B: $\quad P_5 := 54048Pa$

$$\rho_5 := \frac{P_5}{R1\cdot T_5} = 0.667\frac{kg}{m^3} \qquad \text{(Equ 1.23)}$$

$$C_{Lc2} := \frac{2\cdot m1\cdot g}{\rho_5\cdot V_C^{\,2}\cdot S1} = 0.57 \qquad \text{(Equ 2.4)}$$

$$\Delta C_L := C_{Lc2} - C_{Lc1} = 0.054$$

$$\%\Delta C_L := \frac{C_{Lc2} - C_{Lc1}}{C_{Lc1}} = 10.5\%$$

15. A maneuverable aircraft has a mass of 6,800 kg, a wing area of 32 m^2 and a drag coefficient of 0.02. The aircraft is required to climb vertically with the speed of 100 knot. How much thrust the engine needs to produce?

In a vertical climb, T = D + W

$m1 := 6800 \cdot kg$ $\quad S1 := 32 \cdot m^2$ $\quad C_D := 0.02$ $\quad V1 := 100 \cdot knot$ $\quad V1 = 51.444 \frac{m}{s}$

$\rho_o := 1.225 \cdot \frac{kg}{m^3}$ $\quad a := 340 \cdot \frac{m}{s}$ speed of sound at sea level $\quad g = 9.807 \cdot \frac{m}{s^2}$

$W1 := m1 \cdot g = 66685.2\,N$

$D := \frac{1}{2} \cdot \rho_o \cdot V1^2 \cdot S1 \cdot C_D = 1037.4 \cdot N$ (Equ 2.5)

$T1 := D + W1 = 67.7 \cdot kN$ (Equ 2.32)

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16. Calculate the wing area of the aircraft *EMB-121A1*. The aircraft geometry and weight data may be taken from Table 2.2.

From Table 2.2, row 6: $\quad V_s := 76 \cdot knot$ $\quad C_{Lmax} := 2.16$ $\quad m1 := 5670 \cdot kg$

$\rho_o := 1.225 \cdot \frac{kg}{m^3}$ $\quad g = 9.807 \cdot \frac{m}{s^2}$

$m \cdot g = \frac{1}{2} \cdot \rho \cdot V_s^2 \cdot S \cdot C_{Lmax}$ (Equ 2.51)

$S1 := \frac{2 \cdot m1 \cdot g}{\left(\rho_o \cdot C_{Lmax} \cdot V_s^2\right)} = 27.49\,m^2$ (Equ 2.51)

17. Is fighter aircraft *F-14* able to fly vertically? The aircraft data may be taken from table 2.2. Assume drag coefficient to be 0.03.

From Table 2.2, row 21:

$m1 := 33724 \cdot kg$ $\quad C_{Lmax} := 2.9$ $\quad V_s := 115 \cdot knot$ $\quad V_s = 59.161 \frac{m}{s}$ $\quad T_{max} := 2 \cdot 93 \cdot kN$

$W1 := m1 \cdot g = 330.7 kN$

$T_{max} = 186 kN$

The maximum engine thrust is less than aircraft weight, so, it is unable, even with the lowest possible speed.

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18. The dynamic pressure of an aircraft that is cruising at an altitude is 9,000 N/m^2.

a. Determine the altitude, if the aircraft speed is 389 KTAS.
b. Calculate aircraft equivalent airspeed in terms of KEAS.

- Altitude

$q := 9000 \frac{N}{m^2}$ $\quad V_T := 389 \cdot knot$ $\quad V_T = 200.119 \frac{m}{s}$ $\quad \rho_o := 1.225 \frac{kg}{m^3}$

$q = \frac{1}{2} \cdot \rho \cdot V^2$ $\quad \rho := \frac{2 \cdot q}{V_T^2} = 0.449 \frac{kg}{m^3}$ (Equ 2.47)

The altitude for this air density is 9,300 m or 30,500 ft.

- Equivalent airspeed in terms of KEAS

$V_E := V_T \cdot \sqrt{\frac{\rho}{\rho_o}} = 235.63 knot$ (Equ 2.49)

19. A transport aircraft is cruising at 20,000 ft altitude with a speed of Mach 0.5. If a 50 m/sec head-wind is blowing, what is the ground speed and true airspeed in terms of knot?

- **Ground speed**

$h := 20000\,ft$ $M1 := 0.5$ $V_W := 50 \cdot \frac{m}{s}$ $V_W = 97.192\,knot$ $R1 := 287 \cdot \frac{J}{kg \cdot K}$ $\gamma := 1.4$

From Appendix B: $T1 := 447.4\,R$ $T1 = 248.556\,K$

$a1 := \sqrt{\gamma \cdot R1 \cdot T1} = 316.022\frac{m}{s}$ (Equ 1.34)

$V1 := M1 \cdot a1$ $V1 = 158.011\frac{m}{s}$ (Equ 1.35)

$V_G := V1 - V_W = 108.011\frac{m}{s}$ $V_G = 210\,knot$

- **True airspeed**

$V_T := V1 = 307.148\,knot$

20. The aircraft *Voyager* is able to fly around the globe without refueling. In one mission the aircraft is flying at the equator at an altitude of 15,000 ft with the speed of 110 knot. Assume there is a 15 m/sec wind from West to East all the time.

a. How many days does it take to do this mission if cruising from West to East?
b. How many days does it take to do this mission if cruising from East to West?

Note: The Earth has a diameter of 12,800 km.

- From West to East

$h := 15000ft \qquad h = 4572m \qquad V_T := 110 \cdot knot \qquad V_T = 56.6\frac{m}{s} \qquad V_w := 15 \cdot \frac{m}{sec} \qquad D_E := 12800km$

Total Distance traveled; Circumference: $X := \pi \cdot (D_E + 2 \cdot h) = 40241.1km$

$V_{G1} := V_T + V_w = 71.589\frac{m}{s} \qquad V_{G1} = 139.158knot$

$V = \frac{X}{t} \qquad t1 := \frac{X}{V_{G1}} = 6.506day \qquad t1 = 5.621 \times 10^5 s \qquad t1 = 156.1hr$

- From East to West

$V_{G2} := V_T - V_w = 41.589\frac{m}{s} \qquad V_{G2} = 80.842knot$

$V = \frac{X}{t} \qquad t2 := \frac{X}{V_{G2}} = 11.199day \qquad t2 = 9.676 \times 10^5 s \qquad t2 = 268.8hr$

21. The aircraft *Cessna Citation II* is climbing with a 3 degrees of angle of attack. The geometry and weight data of this aircraft may be taken from Table 2.2.

a. If the drag coefficient is 0.035, determine its climb angle, when climbing with a speed of 160 knot.
b. Determine the ratio between lift to weight at this climbing flight.

- **Climb angle**

From Table 2.2, row 18: $m3 := 6033 kg$ $V_s := 82 \cdot knot$ $C_{Lmax} := 1.73$ $T3 := 2 \cdot 11.1 \cdot kN$

$V_{climb} := 160 \cdot knot$ $C_D := 0.035$ $\alpha := 3 \cdot deg$ $\rho_o := 1.225 \frac{kg}{m^3}$

$W3 := m3 g = 59163.5 N$

$$S3 := \frac{2 \cdot W3}{\rho_o \cdot V_s^2 \cdot C_{Lmax}} = 31.376 m^2 \qquad \text{(Equ 2.51)}$$

$$D3 := \frac{1}{2} \cdot \rho_o \cdot V_{climb}^2 \cdot S3 \cdot C_D = 4557.1 N \qquad \text{(Equ 2.5)}$$

$$T \cdot \cos(\alpha) = D + W \cdot \sin(\gamma) \qquad \text{(Equ 2.32)}$$

$$\gamma := asin\left(\frac{T3 \cdot \cos(\alpha) - D3}{W3}\right) = 0.302 rad \qquad \gamma = 17.319 deg$$

- **Ratio between lift to weight**

$$L = W \cdot \cos(\gamma) - T \cdot \sin(\alpha) \qquad \text{(Equ 2.33)}$$

$$L3 := W3 \cdot \cos(\gamma) - T3 \sin(\alpha) = 55319.3 N$$

$$\frac{L3}{W3} = 0.935$$

22. A transport aircraft with a wing area of 200 m^2 is cruising with a speed of Mach 0.6 at 35,000 ft altitude, ISA condition.

a. determine the mass of aircraft, if lift coefficient is 0.24.

b. determine the engine thrust, if drag coefficient is 0.035.

- **Mass of aircraft**

$S1 := 200 \cdot m^2$ $\quad M1 := 0.6$ $\quad h := 35000 ft$ $\quad C_L := 0.24$ $\quad R1 := 287 \cdot \frac{J}{kg \cdot K}$ $\quad \gamma := 1.4$ $\quad C_D := 0.035$

$g = 9.807 \frac{m}{s^2}$

From Appendix B: $\quad T1 := 394.1R$ $\quad T1 = 218.944K$ $\quad \rho_{35} := 0.000738 \frac{slug}{ft^3}$ $\quad \rho_{35} = 0.38 \frac{kg}{m^3}$

$a1 := \sqrt{\gamma \cdot R1 \cdot T1} = 296.601 \frac{m}{s}$ (Equ 1.34)

$V1 := M1 \cdot a1$ $\quad V1 = 177.96 \frac{m}{s}$ $\quad V1 = 345.927 knot$ (Equ 1.35)

$m1 := \frac{\rho_{35} \cdot V1^2 \cdot S1}{2 \cdot g} = 122831.2 kg$ (Equ 2.4)

- **Engine thrust**

$D1 := \frac{1}{2} \cdot \rho_{35} \cdot V1^2 \cdot S1 \cdot C_D = 42159.7N$ (Equ 2.5)

$T1 := D1 = 4.216 \times 10^4 N$ (Equ 2.17)

23. A transport aircraft with a wing area of 420 m^2 is cruising with a constant speed of 550 knot (KTAS) at 38,000 ft altitude. The aircraft has a mass of 390,000 kg at the beginning of a cruising flight and consumes 150,000 kg of fuel at the end of the cruise. Determine wing angle of attack at the beginning and at the end of the cruise. Also assume:

$$AR = 8.5;\ a_o = 2\pi\ 1/\text{rad};\ \alpha_o = -1\ \text{deg};$$

- **Wing angle of attack at the beginning of the cruise**

$S1 := 420 \cdot m^2$ $\quad V1 := 550 \cdot knot$ $\quad V1 = 282.944 \frac{m}{s}$ $\quad h := 38000 ft$ $\quad m1 := 390000 kg$

$m_f := 150000 kg$ $\quad AR := 8.5$ $\quad a_o := 2 \cdot \pi \cdot \frac{1}{rad}$ $\quad \alpha_o := -1 \cdot deg$

Appendix B: $\quad \rho_{38} := 6.4629 \cdot 10^{-4} \cdot \frac{slug}{ft^3}$ $\quad \rho_{38} = 0.333 \frac{kg}{m^3}$

$$C_{L1} := \frac{2 \cdot m1 \cdot g}{\rho_{38} \cdot (V1)^2 \cdot S1} = 0.683 \qquad \text{(Equ 2.4)}$$

$$C_{L\alpha w} := \frac{a_o}{1 + \frac{a_o}{\pi \cdot AR}} = 5.086 \qquad \text{(Equ 2.13)}$$

$$C_{L\alpha} = \frac{C_L}{(\alpha - \alpha_o)} \qquad \alpha_1 := \frac{C_{L1}}{C_{L\alpha w}} + \alpha_o = 0.117 rad \qquad \alpha_1 = 6.693 deg \qquad \text{(Equ 2.12)}$$

- Wing angle of attack at the end of the cruise

b.
$$C_{L2} := \frac{2 \cdot [(m1 - m_f) g]}{\rho_{38} \cdot (V1)^2 \cdot S1} = 0.42$$

$$\alpha_2 := \frac{C_{L2}}{C_{L\alpha w}} + \alpha_o = 0.065 rad \qquad \alpha_2 = 3.734 deg$$

24. The aircraft *Falco 900* is going to take-off from a runway in a winter day (ISA-20). It starts from rest and after a few seconds, when speed reaches 0.5 V_s, friction force is 1% of aircraft weight, and drag coefficient is 0.1.

a. Determine aircraft acceleration for this moment.
b. How long does it take to come to this point? Assume the acceleration is constant during this period.
Note: Aircraft geometry and weight data may be taken from Table 2.2.

- **Acceleration**

From Table 2.2, row 20: $m1 := 20640kg$ $V_s := 82 \cdot knot$ $C_{Lmax} := 2.25$ $Th := 3 \cdot 20 \cdot kN$

$F_f = 0.01 \cdot W1$ $C_D := 0.1$ $V1 := 0.5 \cdot V_s$ $g = 9.807 \frac{m}{s^2}$ $R1 := 287 \cdot \frac{J}{kg \cdot K}$ $\rho_o := 1.225 \frac{kg}{m^3}$

From Appendix B: $P1 := 101325Pa$

$T1 := (15 - 20 + 273) \cdot K$ (Equ 2.5)

$W1 := m1 \cdot g = 202409.3N$

$\rho := \frac{P1}{R1 \cdot (T1)} = 1.317 \frac{kg}{m^3}$ (Equ 1.23)

$S1 := \frac{2 \cdot W1}{\rho_o \cdot V_s^2 \cdot C_{Lmax}} = 82.535m^2$ (Equ 2.51)

$D1 := \frac{1}{2} \cdot \rho \cdot (0.5V_s)^2 \cdot S1 \cdot C_D = 2418.5N$ (Equ 2.5)

$F_f := 0.01 \cdot W1 = 2024.1N$

$a_{TO} := \frac{Th - D1 - F_f}{m1} = 2.692 \frac{m}{s^2}$ (Equ 2.34)

- **Duration**

$V_{TO} := 0.5V_s = 21.092 \frac{m}{s}$

$Time := \frac{V_{TO}}{a_{TO}} = 7.836s$

25. Repeat problem 24, assuming the aircraft is taking off in a summer day (ISA+20).

- **Acceleration**

From Table 2.2, row 20: $m1 := 20640 \cdot kg$ $V_s := 82 \cdot knot$ $C_{Lmax} := 2.25$ $Th := 3 \cdot 20 \cdot kN$

$F_f = 0.01 \cdot W1$ $C_D := 0.1$ $V1 := 0.5 \cdot V_s$ $g = 9.807 \frac{m}{s^2}$ $R1 := 287 \cdot \frac{J}{kg \cdot K}$ $\rho_o := 1.225 \frac{kg}{m^3}$

From Appendix B: $P1 := 101325 Pa$

$T1 := (15 + 20 + 273) \cdot K = 308 K$ (Equ 2.5)

$W1 := m1 \cdot g = 202409.3 N$

$$\rho := \frac{P1}{R1 \cdot (T1)} = 1.146 \frac{kg}{m^3}$$ (Equ 1.23)

$$S1 := \frac{2 \cdot W1}{\rho_o \cdot V_s^2 \cdot C_{Lmax}} = 82.535 m^2$$ (Equ 2.51)

$$D1 := \frac{1}{2} \cdot \rho \cdot \left(0.5 V_s\right)^2 \cdot S1 \cdot C_D = 2104.4 N$$ (Equ 2.5)

$F_f := 0.01 \cdot W1 = 2024.1 N$

$$a_{TO} := \frac{Th - D1 - F_f}{m1} = 2.707 \frac{m}{s^2}$$ (Equ 2.34)

- **Duration**

$$V_{TO} := 0.5 V_s = 21.092 \frac{m}{s}$$

$$Time := \frac{V_{TO}}{a_{TO}} = 7.792 s$$

26. An aircraft that is initially at rest is accelerating on a runway for a take-off operation. When the aircraft speed is 35 KTAS, the acceleration is 10 m/sec^2. Other features of this aircraft at this time are:

$$S = 35\ m^2,\ m = 6400\ kg,\ C_L = 0.8,\ C_D = 0.037.$$

If the friction coefficient is 0.02, calculate the engine thrust. Assume sea-level ISA condition.

$S1 := 35 \cdot m^2$ $\quad m1 := 6400 kg$ $\quad C_L := 0.8$ $\quad C_D := 0.037$ $\quad V_{TO} := 35 \cdot knot$ $\quad V_{TO} = 18 \frac{m}{s}$

$a := 10 \cdot \frac{m}{s^2}$ $\quad \mu := 0.02$ $\quad g = 9.807 \frac{m}{s^2}$ $\quad \rho_o := 1.225 \frac{kg}{m^3}$

$W1 := m1 \cdot g = 62762.6 N$

$L1 := \frac{1}{2} \cdot \rho_o \cdot (V_{TO})^2 \cdot S1 \cdot C_L = 5560 N$ (Equ 2.4)

$D1 := \frac{1}{2} \cdot \rho_o \cdot (V_{TO})^2 \cdot S1 \cdot C_D = 257.2 N$ (Equ 2.5)

Normal force: $N1 := W1 - L1 = 57202.5 N$

$F_f := \mu \cdot N1 = 1144.1 N$

$T1 := m1 \cdot a + D1 + F_f = 65.4 kN$ (Equ 2.34)

---

27. The following aircraft (Figure 2.24) is descending with a constant airspeed and a descent angle of 20 degrees. Aircraft has an angle of attack of 5 degrees, and engines have 3 degrees of setting angle. Draw forces (weight, aerodynamic forces, and engine thrust) on the aircraft and derive the governing equations for this flight phase.

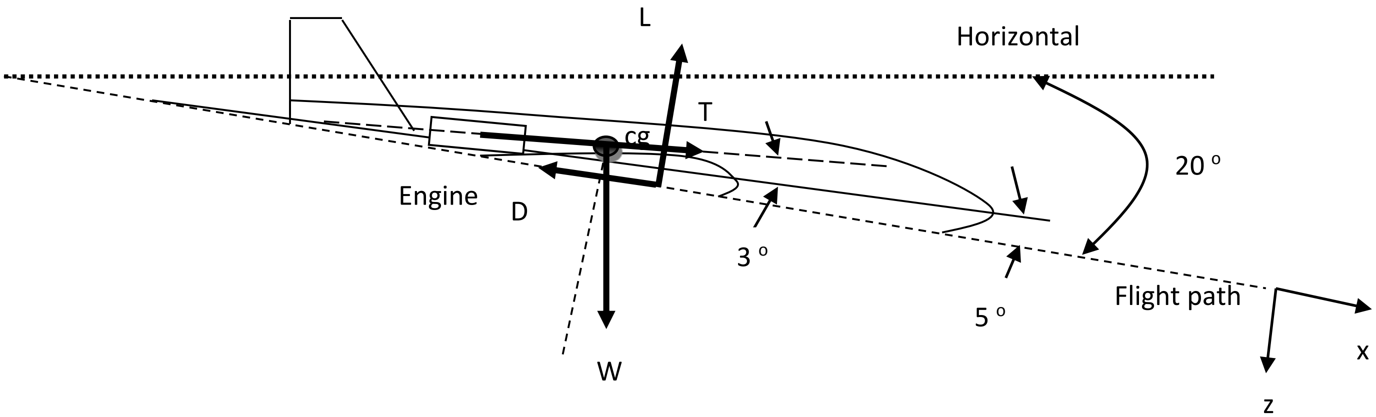


Figure 2.1. An aircraft is descending flight

Governing equations:

$$\sum F_x = 0 \Rightarrow T.\cos(i_t + \alpha) + W.\sin(\gamma) = D$$

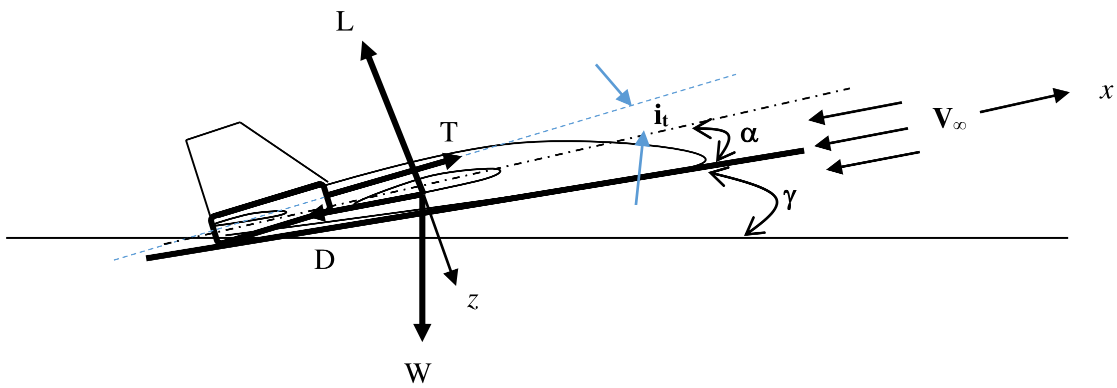
$$\sum F_z = 0 \Rightarrow L + T.\sin(i_t + \alpha) = W.\cos(\gamma)$$

Thus:

$$\sum F_x = 0 \Rightarrow T.\cos(3+5) + W.\sin(20) = D$$

$$\sum F_z = 0 \Rightarrow L + T.\sin(3+5) = W.\cos(20)$$

28. A fighter aircraft is climbing with an arbitrary climb angle. The aircraft has two turbofan engines; both engines have a positive i_t degrees of setting angle. Draw side-view of the aircraft with an arbitrary angle of attack. Then, derive the governing equations of motion for this climbing flight.

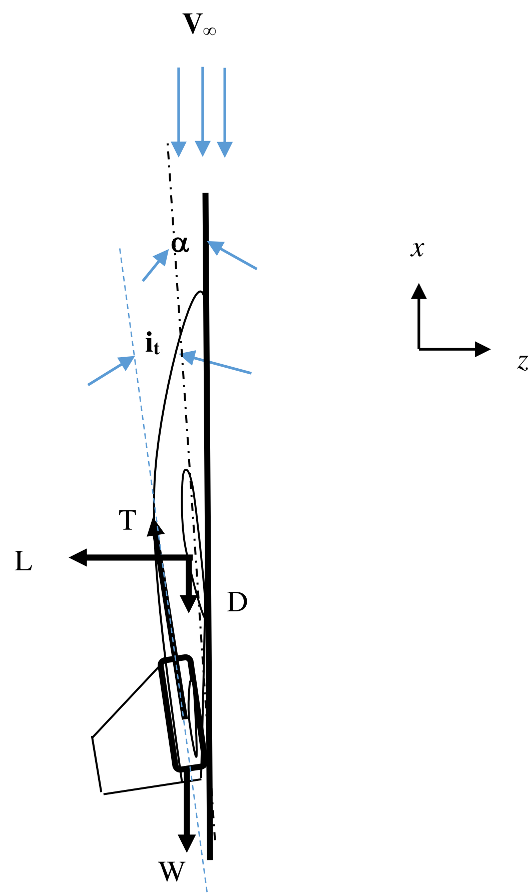


Governing equations:

$$\sum F_x = 0 \Rightarrow T.\cos(i_t + \alpha) = D + W.\sin(\gamma)$$

$$\sum F_z = 0 \Rightarrow L + T.\sin(i_t + \alpha) = W.\cos(\gamma)$$

29. A non-VTOL fighter aircraft is climbing vertically at sea level. The aircraft has two turbofan engines; both engines have a positive i_t degrees of setting angle. Draw side-view of the aircraft with an arbitrary angle of attack. Then, derive the governing equations of motion for this climbing flight.

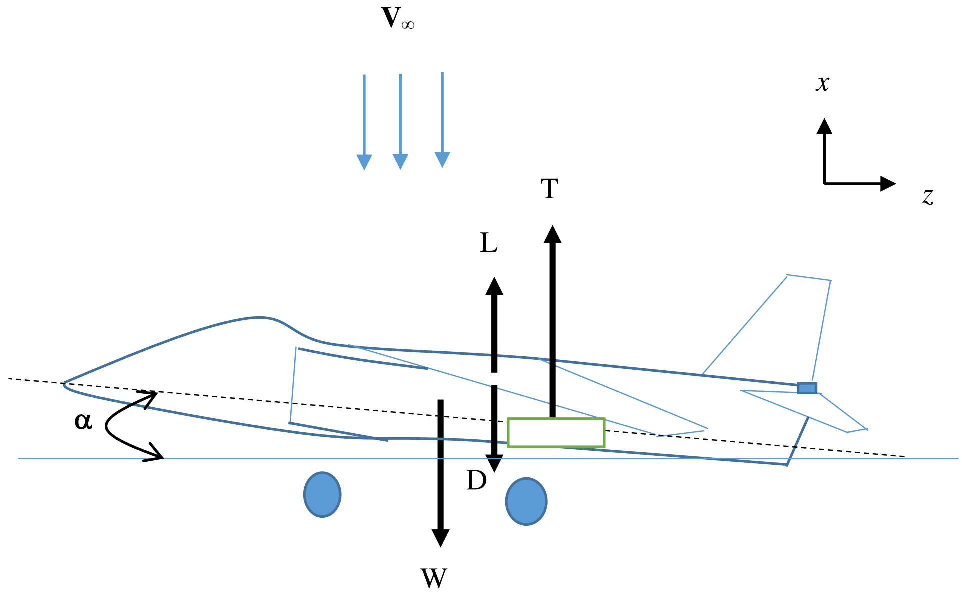


Governing equations:

$$\sum F_x = 0 \Rightarrow T.\cos(i_t + \alpha) = D + W$$

$$\sum F_z = 0 \Rightarrow L + T.\sin(i_t + \alpha) = 0$$

30. A VTOL aircraft is climbing vertically at sea level. The aircraft has two turbofan engines; where during take-off they are arranged such that they produce a thrust which is upward. Draw side-view of the aircraft with an arbitrary angle of attack. Then, derive the governing equations of motion for this climbing flight.



Governing equations:

$$\sum F_x = 0 \Rightarrow T + L = D + W$$

$$\sum F_z = 0$$

31. A pilot is planning to fly east to west at 5,000 m altitude such that he can watch the sunset for a couple of hours. What must be the flight speed (in Mach number) in order to achieve such objective? Assume there is a 30 knot headwind during this flight. The radius of Earth at sea level is 6400 km.

If the aircraft can fly with the speed of air (i.e., earth) at 5,000 m, the pilot can watch the sunset as long as he/she has fuel to fly.

$h := 5000\,m$ $\quad V_W := 30\,knot$ $\quad V_W = 15.433\,\frac{m}{s}$ $\quad R_E := 6400\,km$ $\quad \gamma := 1.4$ $\quad R1 := 287\cdot\frac{J}{kg\cdot K}$

The Earth is revolving around itself in each 24 hours.

$t := 24\,hr$ $\quad t = 8.64\times 10^{4}\,s$

If the aircraft is flying at sea level, the aircraft velocity should be the same as the earth linear speed at the sea level. For the 5,000 m altitude:

Total Distance traveled; Circumference: $\quad X := 2\pi\cdot(R_E + h) = 40243.8\,km$

$V_A := \frac{X}{t} = 465.785\,\frac{m}{s}$

$V_G := V_A + V_W = 481.218\,\frac{m}{s}$

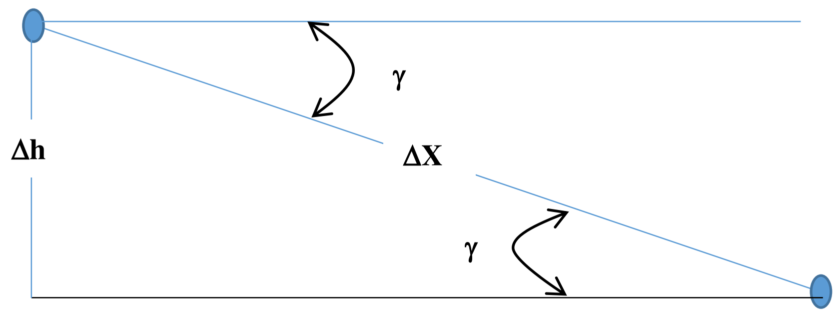
From Appendix A: $\quad T_5 := 255.69\,K$

$a1 := \sqrt{\gamma\cdot R1\cdot T_5} = 320.525\,\frac{m}{s}$ $\qquad$ (Equ 1.34)

$M1 := \frac{V_G}{a1} = 1.501$ $\qquad$ (Equ 1.35)

32. A transport aircraft Boeing 777 (Figure 7.21) is descending with a velocity of 318 mph at 8,200 ft.

a. If at After one minute, the altitude is 4,000 ft, and the airspeed is 234 mph, determine the average descend angle.
b. If at the touchdown (sea level altitude), the airspeed is 180 mph, determine the average descend angle and deceleration. This phase takes one minute.



- **a………**

$V1 := 318 \cdot \frac{mile}{hr}$ $\quad V2 := 234 \cdot \frac{mile}{hr}$ $\quad h1 := 8200 ft$ $\quad h2 := 4000 ft$ $\quad t := 1 \cdot min$

$\Delta h := h1 - h2 = 4200 ft$ $\quad V_{avg} := \frac{V1 + V2}{2} = 276 \frac{mile}{hr}$

Distance traveled $\quad \Delta X := V_{avg} \cdot t = 24288 ft$ $\quad \Delta X = 7403 m$

$\sin(\gamma) = \frac{\Delta h}{\Delta X}$ $\quad \gamma_{avg} := asin\left(\frac{\Delta h}{\Delta X}\right)$ $\quad \gamma_{avg} = 9.958 deg$

- **b…………**

$V1 := 234 \cdot \frac{mile}{hr}$ $\quad V2 := 180 \cdot \frac{mile}{hr}$ $\quad h1 := 8200 ft$ $\quad h2 := 4000 ft$ $\quad t := 1 \cdot min$

$\Delta h := h1 - h2 = 4200 ft$ $\quad V_{avg} := \frac{V1 + V2}{2} = 207 \frac{mile}{hr}$

Distance traveled $\quad \Delta X := V_{avg} \cdot t = 18216 ft$

$\sin(\gamma) = \frac{\Delta h}{\Delta X}$ $\quad \gamma_{avg} := asin\left(\frac{\Delta h}{\Delta X}\right)$ $\quad \gamma_{avg} = 13.33 deg$

$a := \frac{V1 - V2}{t} = 1.32 \frac{ft}{s^2}$ $\quad a = 0.402 \frac{m}{s^2}$

33. The earth is moving in a circular orbit about the sun, with a radius of 147 × 10^9 m. The duration of one turn is one year. Determine the velocity of earth in terms of speed of sound (i.e., Mach number) at sea level.

$X_{ES} := 147 \cdot 10^9 \cdot m$ $\quad X_{ES} = 1.47 \times 10^8\, km$

$t := 1 \cdot yr$ $\quad t = 3.156 \times 10^7\, s$ $\quad t = 365.242\, day$ $\quad t = 8.766 \times 10^3\, hr$

Total Distance traveled; Circumference: $\quad X := 2\pi \cdot (X_{ES}) = 9.2 \times 10^8\, km$

$V_{Earth} := \frac{X}{t} = 2.927 \times 10^4\, \frac{m}{s}$ $\quad V_{Earth} = 29.269\, \frac{km}{s}$

Speed of sound: $\quad a := 340 \cdot \frac{m}{s}$

$M1 := \frac{V_{Earth}}{a} = 86.084$

---

34. In 24 Mar 1960, the maximum speed of a Tu-114 – World's fastest propeller-driven aircraft on a 1,000 km closed circuit with payloads of 0 to 25,000 kg was recorded to be 871.38 km/hr. Determine this velocity in terms of Mach number. Assume sea level.

$V_A := 871.38\, \frac{km}{hr}$ $\quad V_A = 242.05\, \frac{m}{s}$

Speed of sound: $\quad a := 340 \cdot \frac{m}{s}$

$M1 := \frac{V_A}{a} = 0.712$ $\quad$ (Equ 1.35)

CHAPTER 2

Equations of Motion

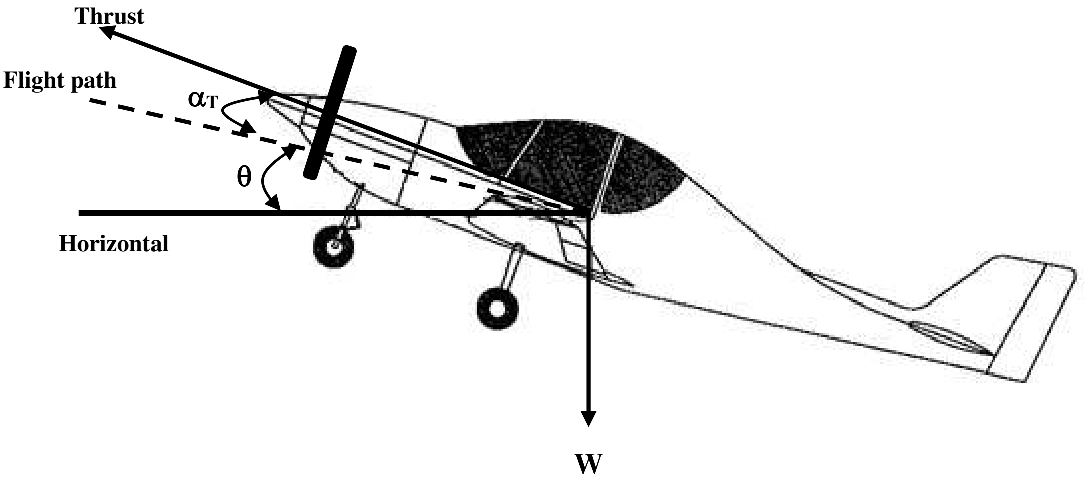


Figure 2.1. An aircraft with two non-aerodynamic forces (weight and thrust)

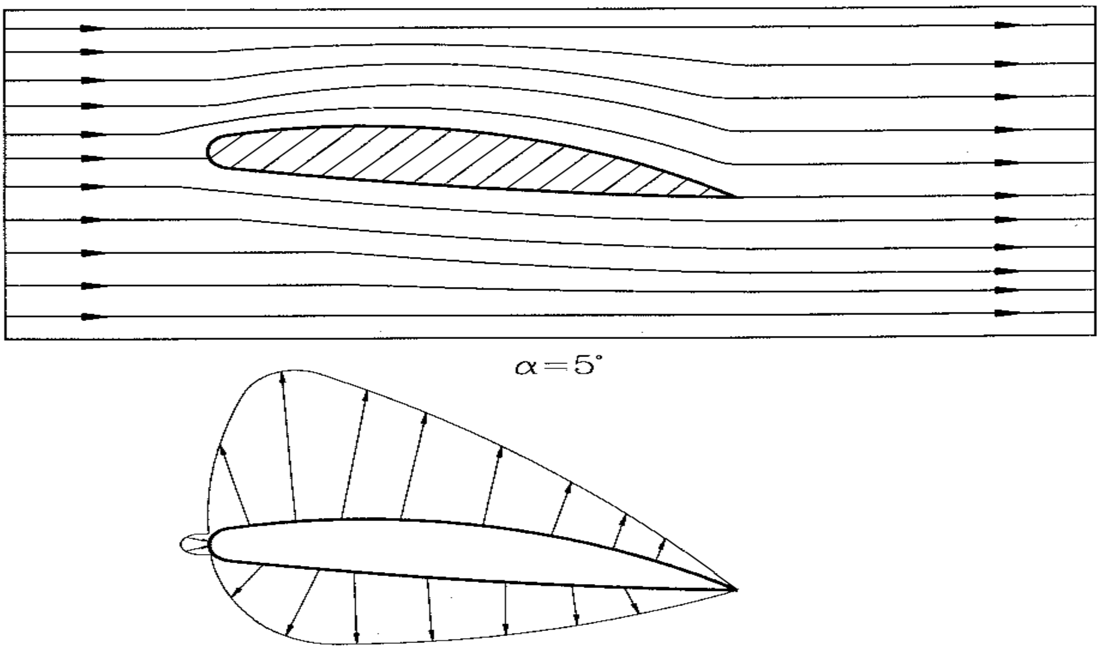


Figure 2.2. A typical pressure distribution over an airfoil with 5 degrees of angle of attack

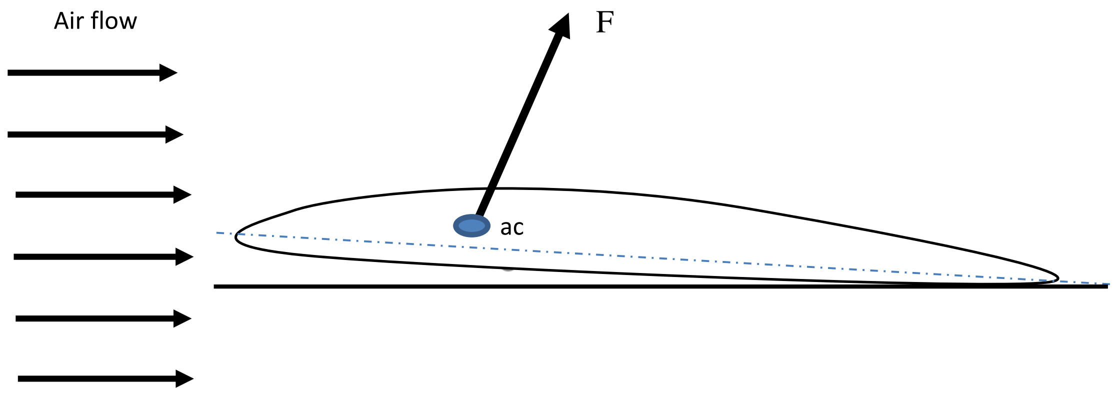
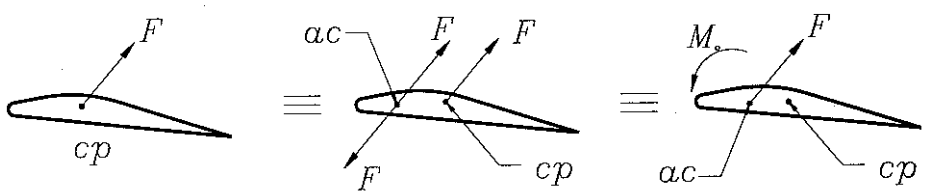


Figure 2.3. The resultant force out of integration of pressure distribution



a. original force b. adding two equal and opposite forces c. resultant force

Figure 2.4. The movement of resultant force to aerodynamic center

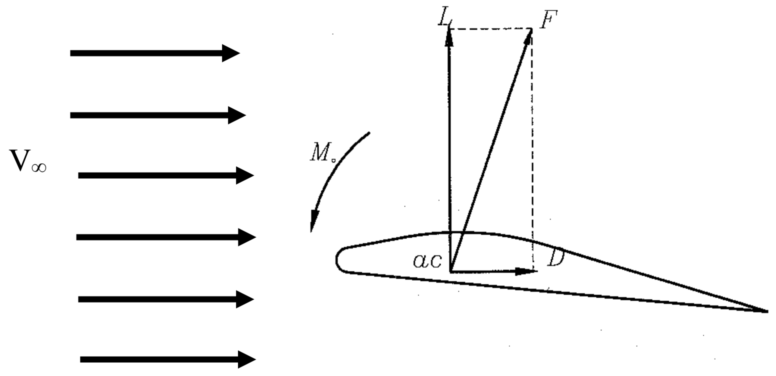


Figure 2.5. The definitions of lift, drag, and pitching moment

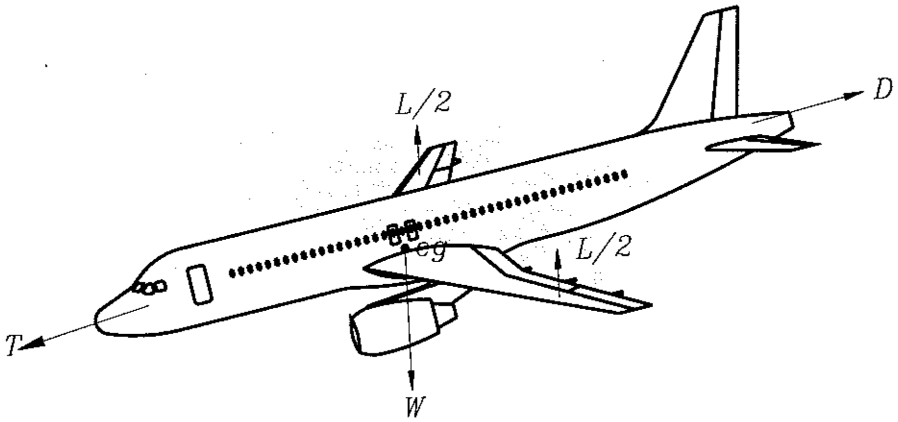


Figure 2.6. The major forces on an airplane

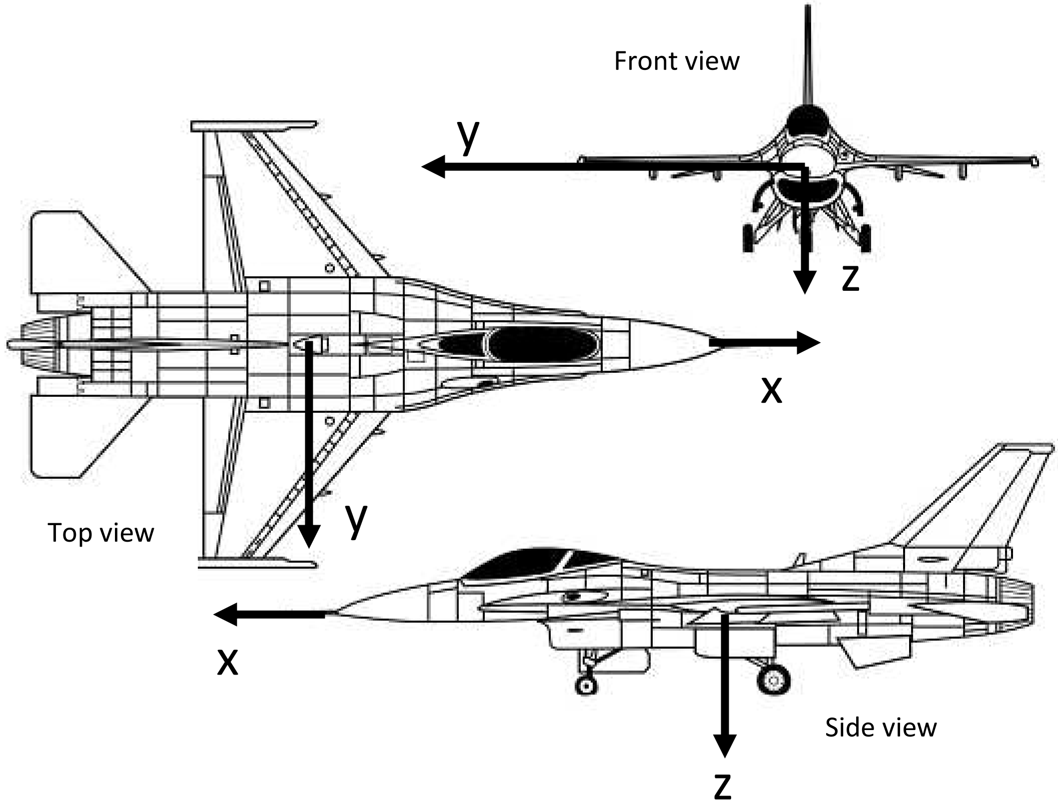


Figure 2.7. Aircraft body-fixed coordinate system (F-16)

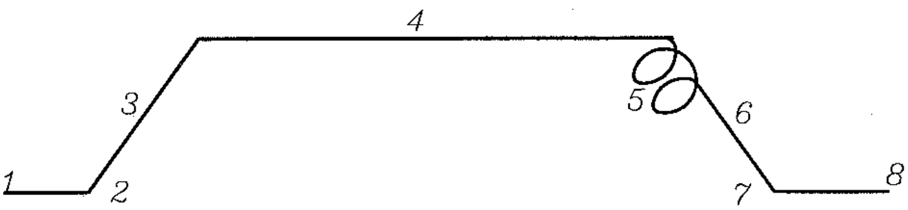


Figure 2.8. A basic and typical flight operation of an aircraft

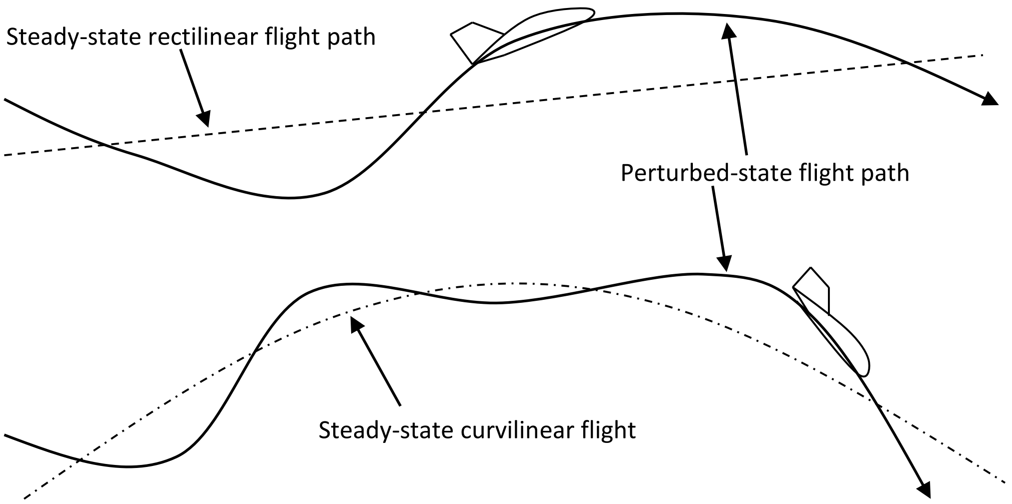
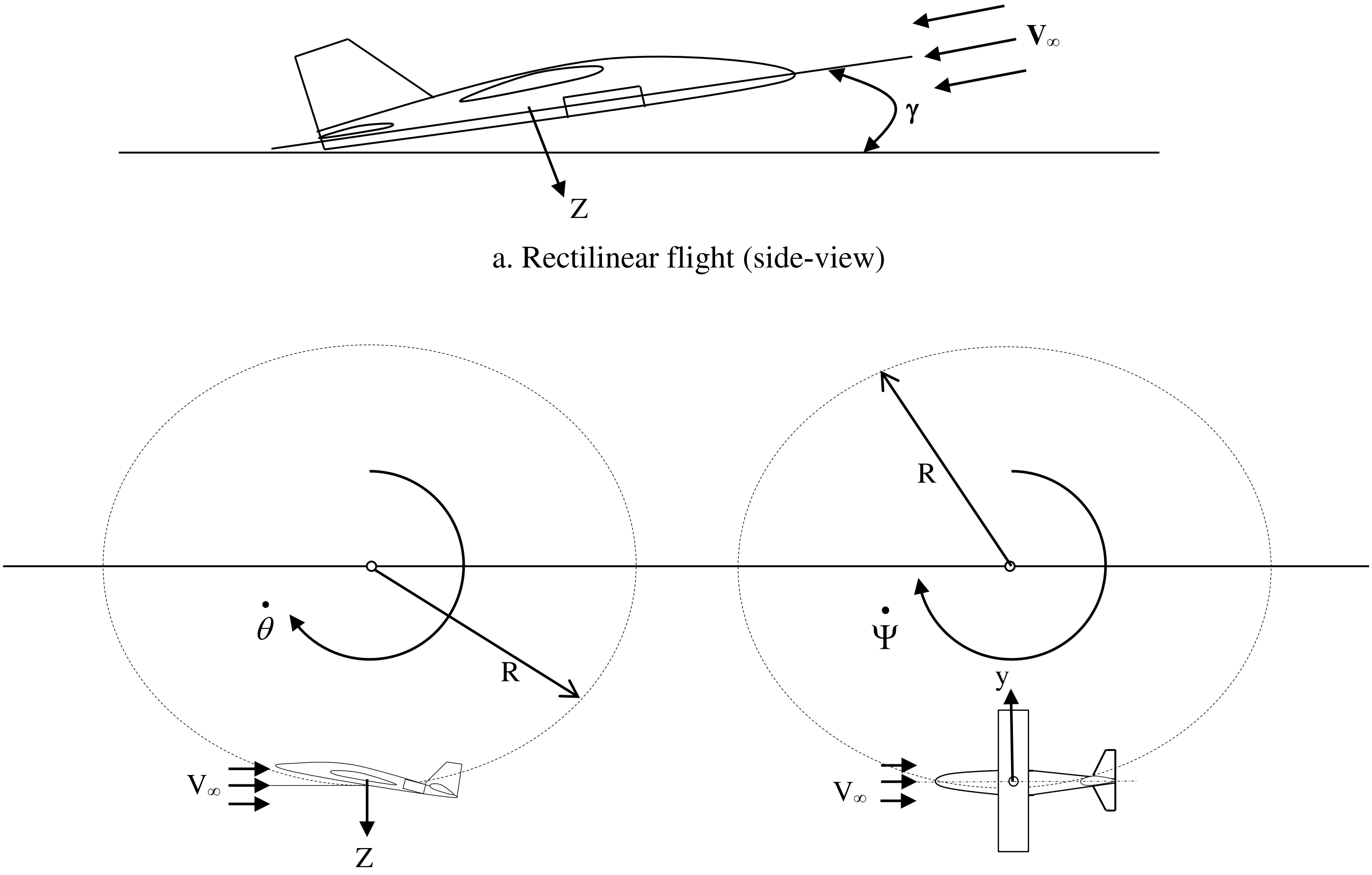


Figure 2.9. Examples of perturbed-state flight paths



a. Rectilinear flight (side-view)

b. Symmetrical pull-up (side-view)

c. Level turn (Top-view)

Figure 2.10. Examples of steady state flight conditions

Figure 2.11. NASA Global Hawk in a cruising flight

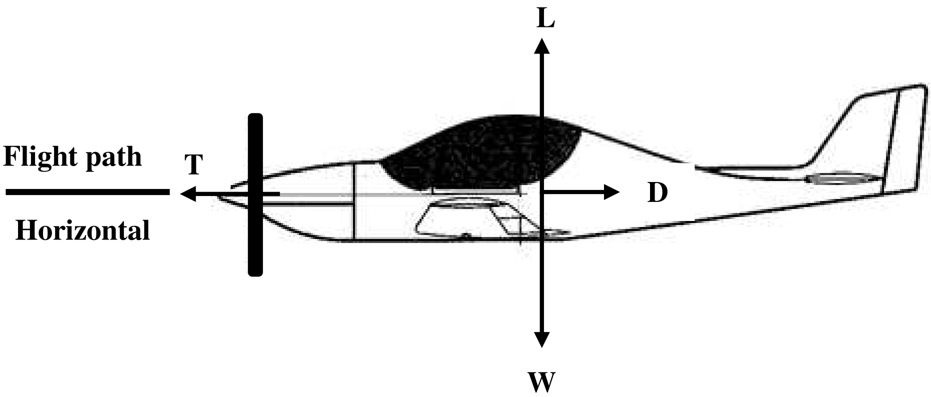


Figure 2.12. Straight line flight

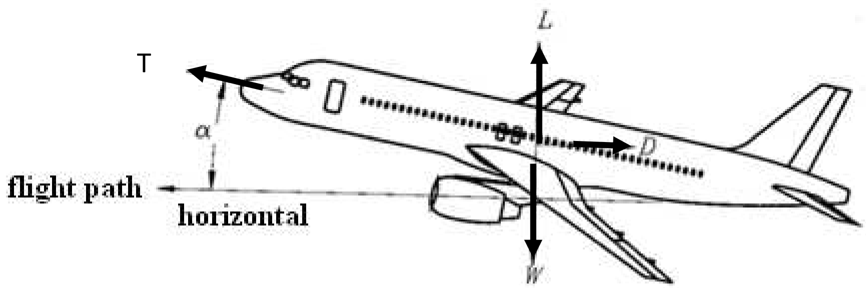


Figure 2.13. An aircraft with an angle of attack in a straight line horizontal flight

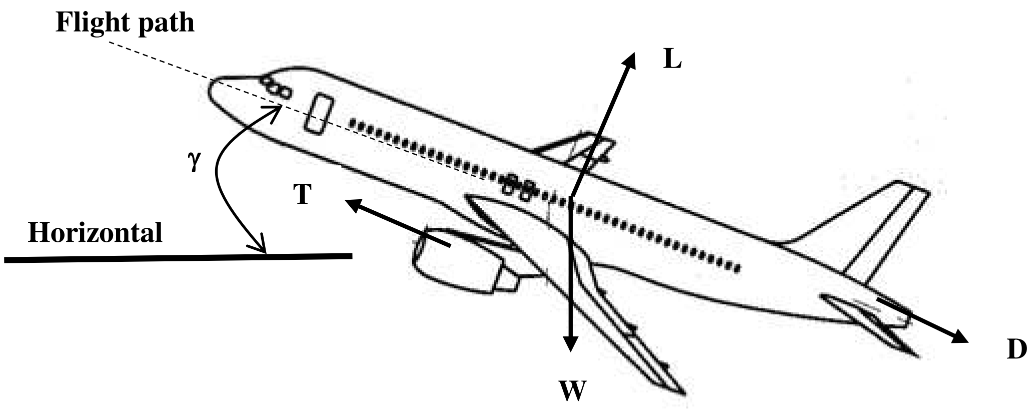


Figure 2.14. An aircraft in climb (assuming zero angle of attack)

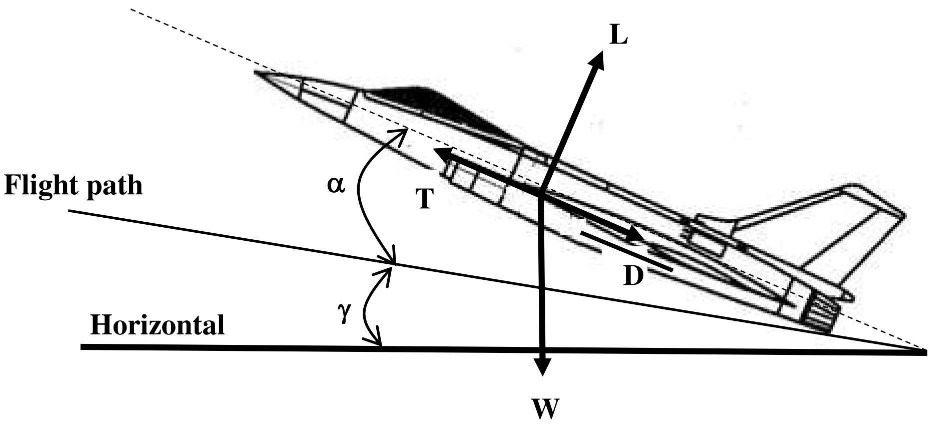


Figure 2.15. An aircraft in a climbing flight with an angle of attack, α

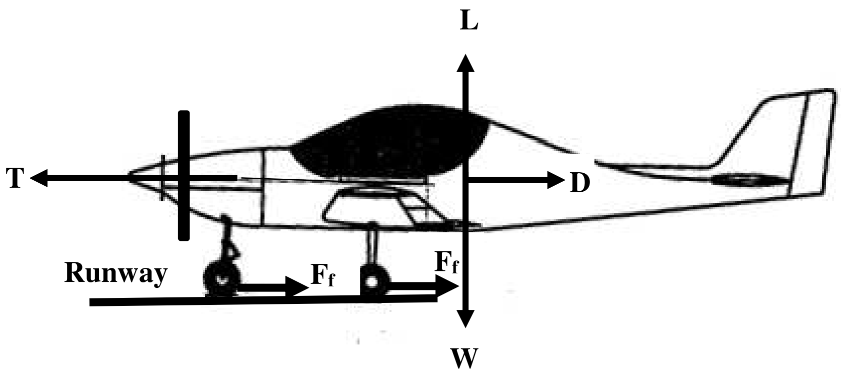
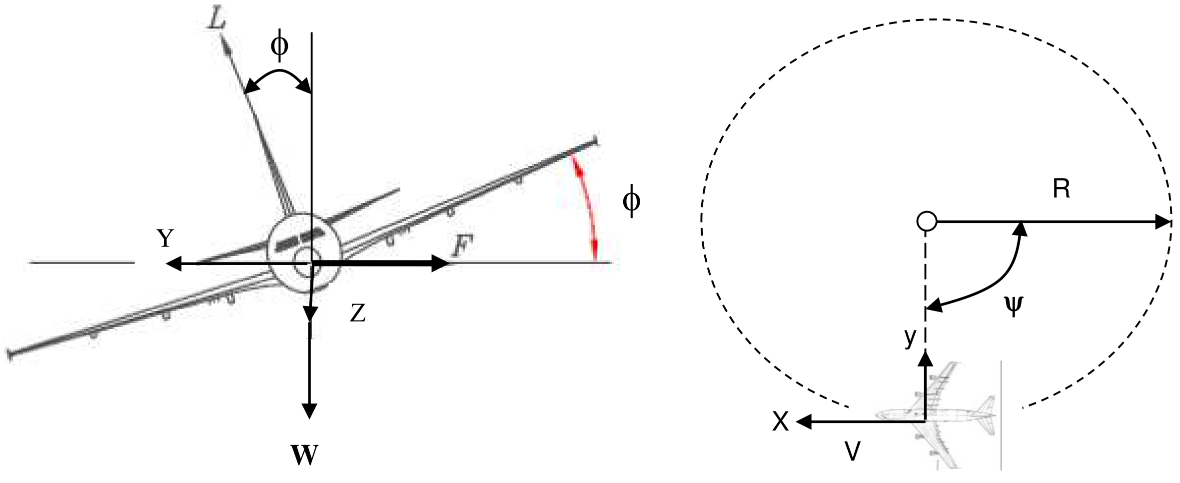


Figure 2.16. An aircraft in take off



a. Front view b. Top-view

Figure 2.17. An aircraft in a coordinated turning flight

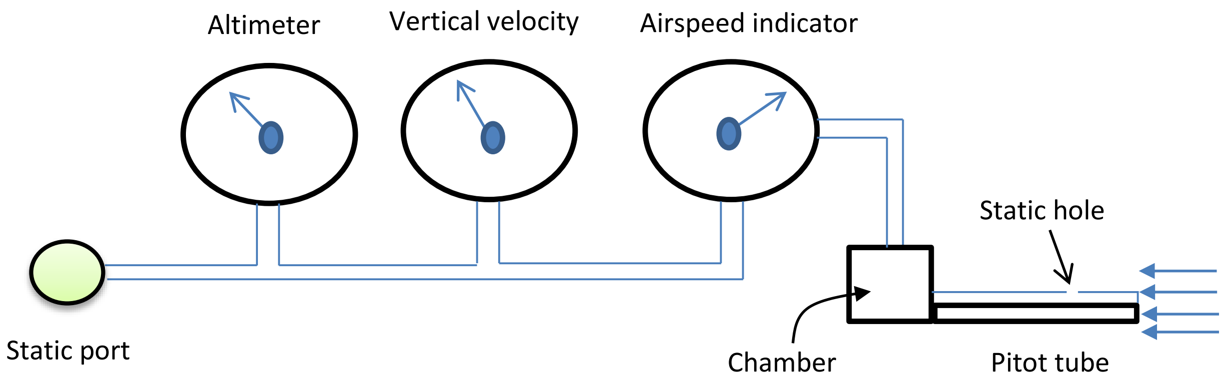


Figure 2.18. Pitot-static measurement device

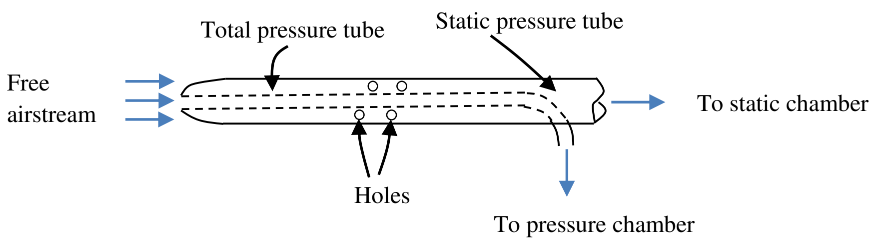
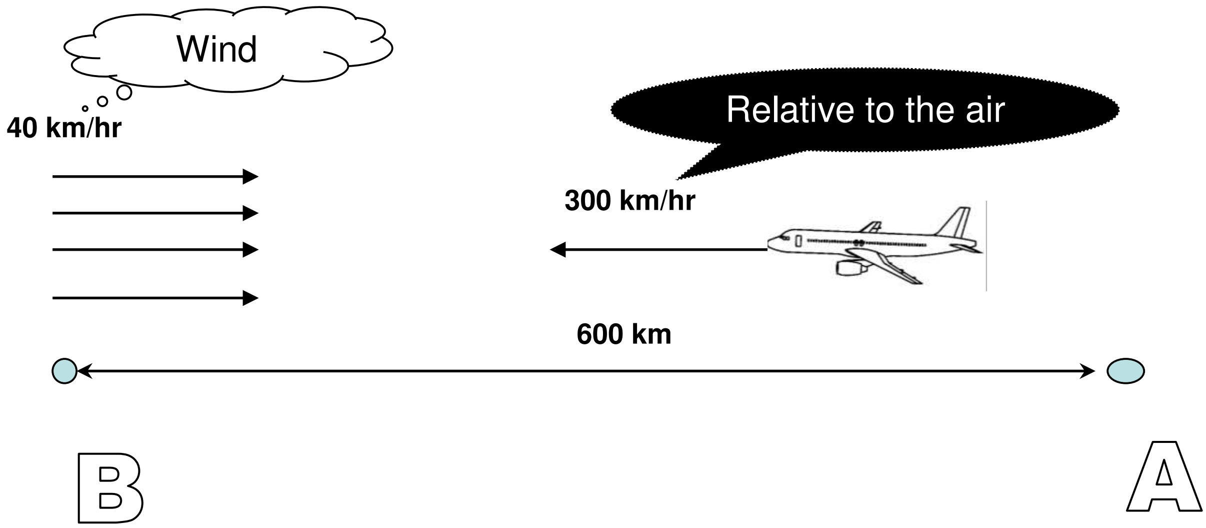
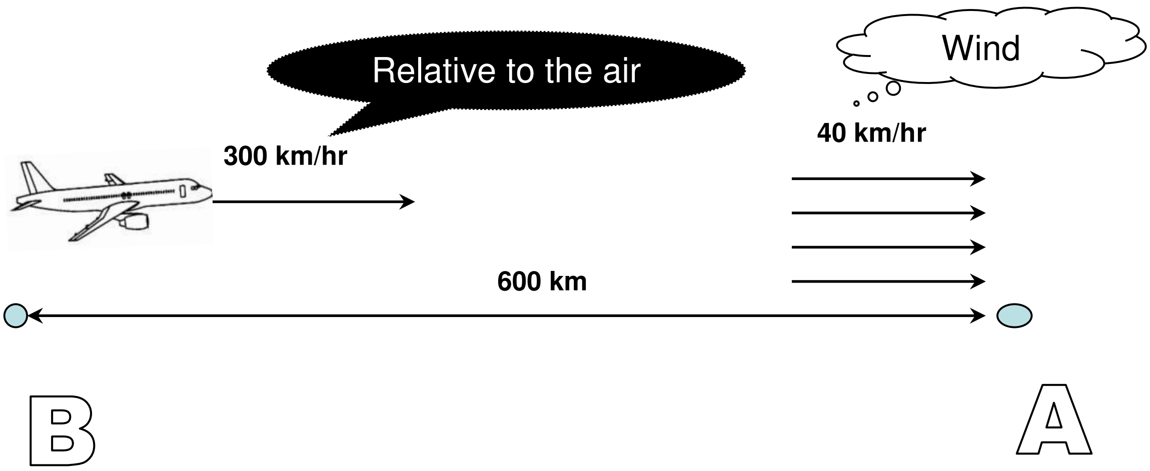


Figure 2.19. A pitot-static tube



a. Headwind



b. Tailwind
Figure 2.20. Airspeed and wind speed

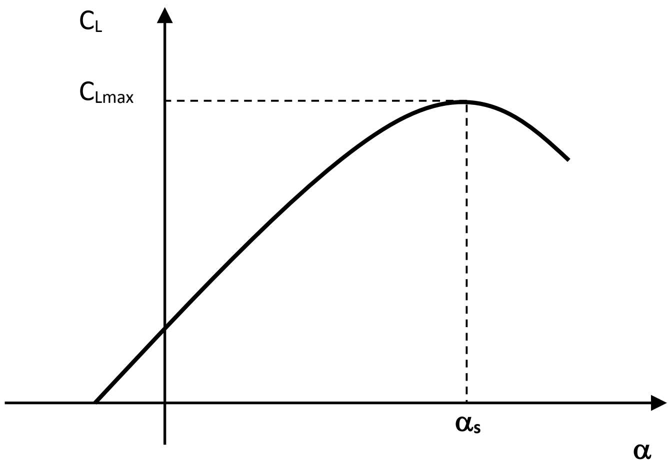


Figure 2.21. Wing lift curve slope

Figure 2.22. Aircraft Raytheon Hawker 800XP with a C_{Lmax} of 2.28 (Courtesy of Gustavo Corujo)

Figure 2.23. Eurofighter EF-2000 Typhoon, a single-seat fighter in an angle beyond stall angle (Courtesy of Fabrizio Capenti)

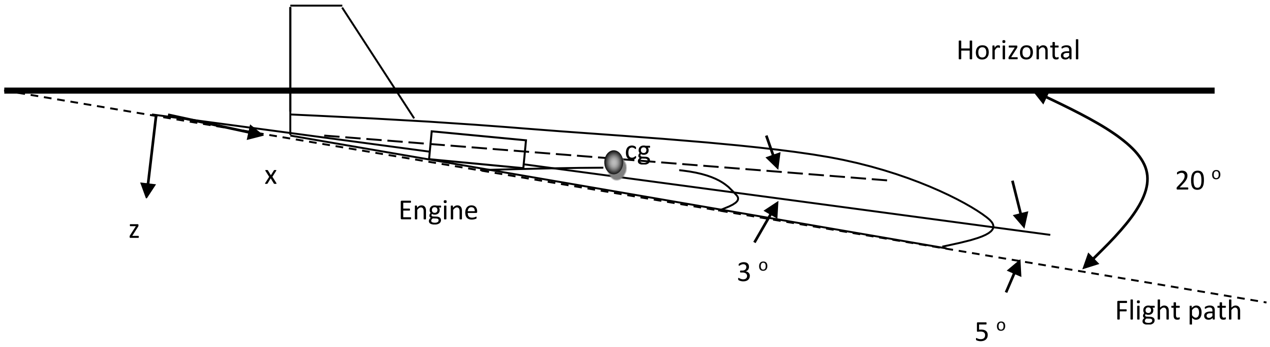


Figure 2.24. An aircraft is a descending flight