MatLab Companion to Complex Variables 1st edition Wunsch Solutions Manual

SKU: 9781498755672-SOLUTIONS

Solutions Manual for A MATLAB Companion to Complex Variables.

Chapter 1

SECTION 1.1

```
SEC 1.1
Problem 1
format short
solution='PROBLEM 1'
a=(3-4i)/(1+2i)+2i
b=(2+i)/(3+5i)-1-i/2
c=(1+2*i/3)^3-i/(3-4i)^3
d=(1+2/(3i))^11
e=(2i/(3+4i)+conj(1/(5-4i)))^5
```

OUTPUT FROM THE ABOVE PROGRAM

solution =

PROBLEM 1

a =

-1

b =

-0.6765 - 0.7059i

c =

-0.3305 + 1.7112i

d =

7.4284 - 1.3889i

e =

0.0003 + 0.0216i

solution =

1

```
solution='PROBLEM 2'
%note that (1+i)^2=2i and (2i)^18=-2^18 since i raised to an even
%integer
%is plus or minus one;the minus sign holds if the integer not divisible
%by4

%so, without matlab, the  final answer is -2^18*(1+i)=-2^18-i2^18=
%-262144-i262144
%provided you get 2^18 off a pocket calculator or you can notice that
%2^18=8*32^3

format long
(1+i)^37

solution='PROBLEM 3a'
%part a
sa=0;
for n=0:10
    z=(i/2)^n;
    sa=sa+z;
end
sa

solution='PROBLEM 3b'
%part b
sb=(1-(i/2)^11)/(1-i/2)
%EXERCISES SEC 1.1
format short
solution='PROBLEM 1'
a=(3-4i)/(1+2i)+2i
b=(2+i)/(3+5i)-1-i/2
c=(1+2*i/3)^3-i/(3-4i)^3
d=(1+2/(3i))^11
e=(2i/(3+4i)+conj(1/(5-4i)))^5
```

PROBLEM 2

Without MATLAB

$$(1+i)^{37} = (1+i)^{36}(1+i) = \left((1+i)^2\right)^{18}(1+i) = 2^{18}i^{18}(1+i)$$

$$= -2^{18}(1+i) = -262144 - i262144$$

You can compute $2^{18} = 32*32*32*8$ =262144

With MATLAB

\>> format long

\>> (1+i)^37

ans =

-2.621440000000000e+05 - 2.621440000000000e+05i

```
3.

solution='PROBLEM 3'
%part a
sa=0;
for n=0:10
    z=(i/2)^n;
    sa=sa+z;
end
sa

solution='PROBLEM 3b'
%part b
sb=(1-(i/2)^11)/(1-i/2)
```

3.

PROBLEM 3a

```
%problem 3 section1.1
format long
s=1;
for n=1:10
    s=s+(i/2)^n;
end
s
```

WHOSE OUTPUT IS

sa =

0.79980468750000 + 0.40039062500000i

solution =

PROBLEM 3b

>> format long

>> (1-(i/2)^11)/(1-i/2)

ans =

0.799804687500000 + 0.400390625000000i

SECTION 1.2

Problem 1.

```
%PROBLEMS 1 %solution='problem 1'
%parta
a=(1+i)*(sqrt(3)+i)
angle_product=angle(a)
angle_first=angle(1+i)
angle_second=angle(sqrt(3)+i)
angle_sum=angle_first+angle_second
%partbb=(1+i)*(-sqrt(3)+i)
angle_product=angle(b)
angle_first=angle(1+i)
angle_second=angle(-sqrt(3)+i)
angle_sum=angle_first+angle_second
```

OUTPUT FROM THE ABOVE PROGRAM

solution =

problem 1

a =

0.73205080756888 + 2.73205080756888i

angle_product =

1.30899693899575

angle_first =

0.78539816339745

angle_second =

0.52359877559830

angle_sum =

1.30899693899575

```
b =

 -2.73205080756888 - 0.73205080756888i


angle_product =

  -2.87979326579064


angle_first =

   0.78539816339745


angle_second =

   2.61799387799149


angle_sum =

   3.40339204138894
% explanation of discrepancy;the principal angle of (1+i) is pi/4 while
%the
% principal angle of (-sqrt(3)+i)is 5pi/6. Their sum is 13pi/12 which
%is
% not a principal angle as it exceeds pi. MATLAB yields the principal
%angle of the product
% which is -11pi/12.
```

PROBLEM 2

```
%PROBLEM 2

%solution ='problem 2'
%parta
a=angle(cos(197)+i*sin(197))
b=angle(1/(4*cos(79)+4*i*sin(79)))
c=angle((1+2*i)^27)
theta=53.3*pi/180;
d= angle((cos(theta)+i*sin(theta))^8);
d=d/pi*180
```

OUTPUT OF THE ABOVE

a =

   2.22125547743282

b =

   2.68140899333462

c =

  -1.52291115545749

d =

  66.39999999999999

PROBLEM 3
part(a)
>> [x,y]=pol2cart(-207,3)

x =

   2.82310952232897

y =

   1.01491508262293

part(b)
[x,y]=pol2cart(50,4)

x =

3.85986411396845

y =

-1.04949941481572

```
4. function [x,y]=pold2cart(th,r);
%a function that will convert the polar form of a complex number to the
%rectangular form,the input angle is in degrees, x and y are real and
% imag parts
x=r*(cos(th*pi/180))
y=r*sin(th*pi/180)
clear

a)
>> pold2cart(5,60)

x =

   2.500000000000000


y =

   4.330127018922193

b)
>> pold2cart(2,-320)

x =

   1.532088886237956


y =

   1.285575219373079

>>
```

SECTION 1.3

PROBLEM 1

```
>> %part a
>> (-27*i)^(2/3)

ans =
```

```
   4.5000 - 7.7942i

>> %partb
>> (1+i)^(1/15)

ans =

   1.0220 + 0.0536i

>> %partc
>> (-1-i)^(1/9)

ans =

   1.0038 - 0.2690i
```

PROBLEM 2

a) $(i^{1/2})^2 = i$ must be true since $z^{1/m}$ is defined in such a way that $\left(z^{1/m}\right)^m$ will always yield *z when m is a nonzero integer*. Verify this using De Moivre's Theorem.

b) MATLAB will compute $i^{3/2}$ as $\left(\sqrt{1}\right)^3 cis(\frac{3\pi}{4}) =$ .Raising this to the 2/3 power using the MATLAB convention(we have $\left(\left(\sqrt{1}\right)^3 cis(\frac{3\pi}{4})\right)^{2/3} = cis(\frac{\pi}{2}) = i,$ since $3\pi/4$ is a principle angle.

c) MATLAB will compute $-1^{4/3}$ as $\left(\sqrt[3]{1}\right)^4 cis(\frac{4\pi}{3}) = \frac{-1}{2} - i\frac{\sqrt{3}}{2}$ .Raising this to the 3/4 power using the MATLAB convention (the principle angle is $-2\pi/3$) we have $\left(cis(\frac{-2\pi}{3})\right)^{3/4} = cis(\frac{-\pi}{2}) = -i,$. We do not recover -1.

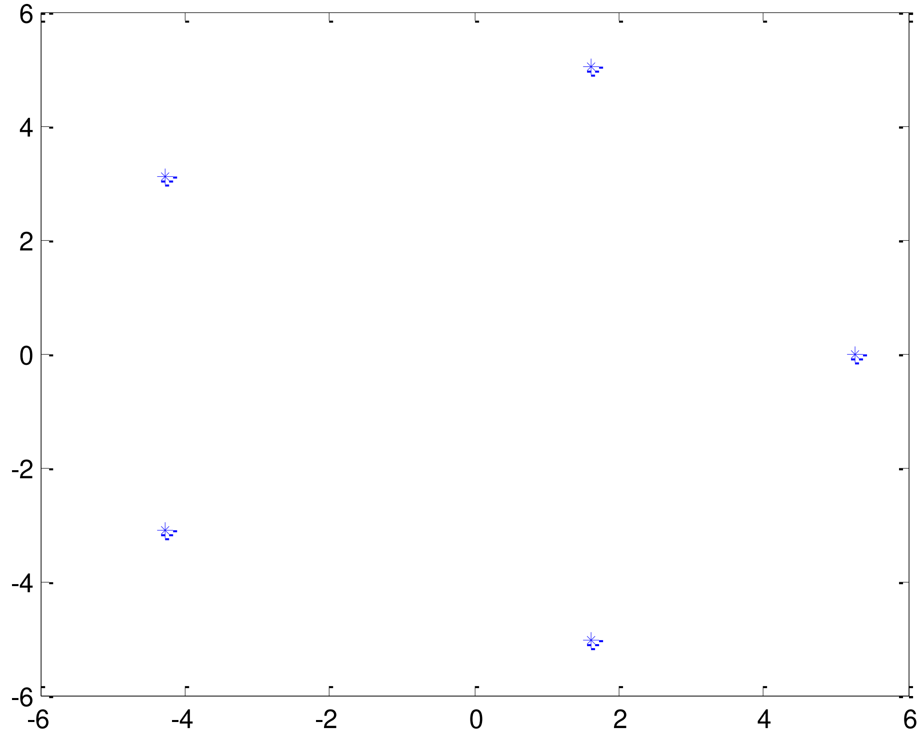
d) MATLAB will compute $-1^{3/4}$ as $\left(\sqrt[4]{1}\right)^3 cis(\frac{3\pi}{4}) = \frac{-1}{\sqrt{2}} + i\frac{\sqrt{1}}{2}$ .Raising this to the 4/3 power using the MATLAB convention (the principle angle is $3\pi/4$) we have $\left(cis(\frac{3\pi}{4})\right)^{4/3} = cis(\pi) = -1,$. We do recover -1.

e) MATLAB will compute $(-1+i)^{5/3}$ as $\left(\sqrt[3]{\sqrt{2}}\right)^5 cis(\frac{3\pi}{4}\frac{5}{3}) = \left(\sqrt[3]{\sqrt{2}}\right)^5 cis(\frac{5\pi}{4})$ .Raising this to the 3/5 power using the MATLAB convention (the principle angle is $-3\pi/4$) we have $\left(\left(\sqrt[3]{\sqrt{2}}\right)^5 cis(\frac{-3\pi}{4})\right)^{3/5} = \sqrt{2}cis(\frac{-9\pi}{20}) =$ 0.2212 - 1.3968i . We do not recover -1+i.

```
f) verification of part e) above
>> a=(-1+i)^(5/3)

a =

 -1.259921049894873 - 1.259921049894873i

>> a^(3/5)

ans =

   0.221231742082474 - 1.396802246667421i (which checks)
```

3.a)

```
>> coeffs=zeros(1,6);
>> coeffs(1)=1;
>> coeffs(6)=-1-i;
>> roots(coeffs)

ans =

  -0.9550 + 0.4866i
  -0.7579 - 0.7579i
   0.1677 + 1.0586i
   0.4866 - 0.9550i
   1.0586 + 0.1677i


>> ans.^24

ans =

   1.6310 + 5.0197i
   5.2780 - 0.0000i
  -4.2700 + 3.1023i
   1.6310 - 5.0197i
  -4.2700 - 3.1023i
```



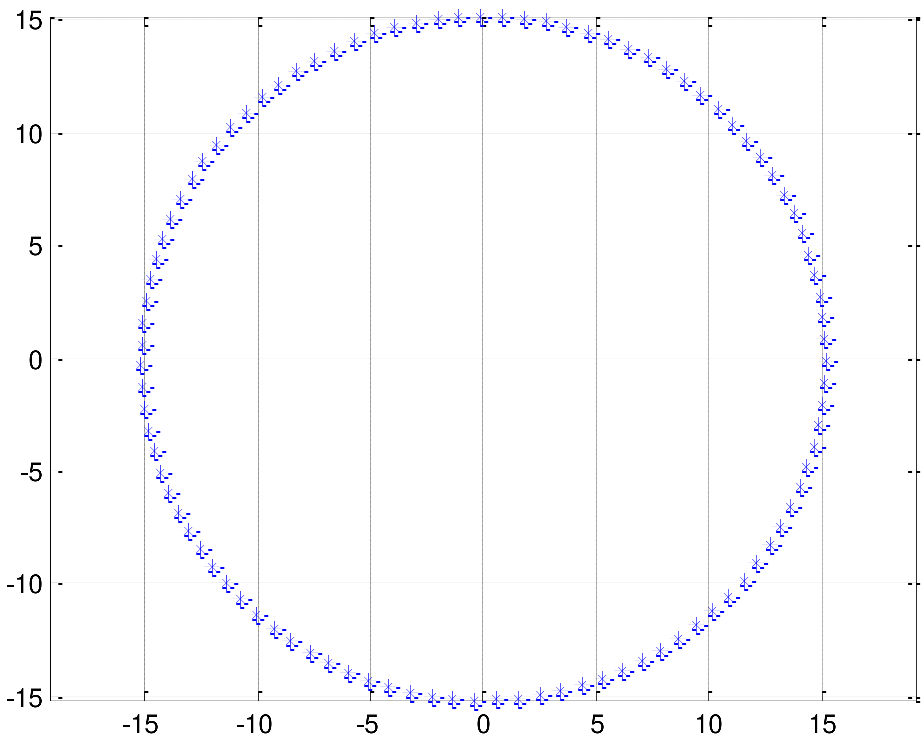
The above is obtained if we use the line of code plot(ans,'*')

b)

```
>> coeffs=zeros(1,8);
>> coeffs(1)=1;
>> coeffs(8)=-3+4*i;
>> roots(coeffs)

ans =

  -0.4396 - 1.1792i
  -1.1961 - 0.3915i
  -1.0518 + 0.6910i
  -0.1155 + 1.2532i
   0.6478 - 1.0790i
   0.9077 + 0.8717i
   1.2475 - 0.1662i

>> ans.^5

ans =

  -3.0852 + 0.6691i
   0.0342 - 3.1567i
```

3.0700 + 0.7358i
-1.4005 + 2.8293i
1.3388 + 2.8590i
-2.4467 - 1.9950i
2.4894 - 1.9414i

c) >> coeffs=zeros(1,100);
>> coeffs(1)=1;coeffs(100)=-13-7*i;
>> a=roots(coeffs);
>> b=a.^100;
>> plot(b,'*');axis equal;grid



The 99 values of $(13+7i)^{100/99}$.

PROBLEM 4

a) Refer to the code written for Example 2 and add an additional line
`sum_of_roots=sum(w)`
which will give you the sum of all the values of $z^{n/m}$.

b) sum_of_roots =

-1.143529715363911e-014 +6.661338147750939e-016i

The preceding is about 14 orders of magnitude less than any of the individual roots.

Had there been no rounding error we would have gotten zero.

b)
>> coeffs=zeros(1,8);
>> coeffs(1)=1;coeffs(8)=-1-i;
>> a=roots(coeffs);
>> b=a.^11;
>> sum(b)

ans =

1.0991e-14 - 1.7319e-14i

Note that this result is small but not zero.

Problem 5

```
clf; clear
x=linspace (-3, 3, 301);
w=(x-1).^(2/5);
figure(1)
plot(x,w);grid;
figure(2)
plot(x, real(w),'-',x,imag(w),'*')
grid;figure(2)
```

The above results in these 2 figures:

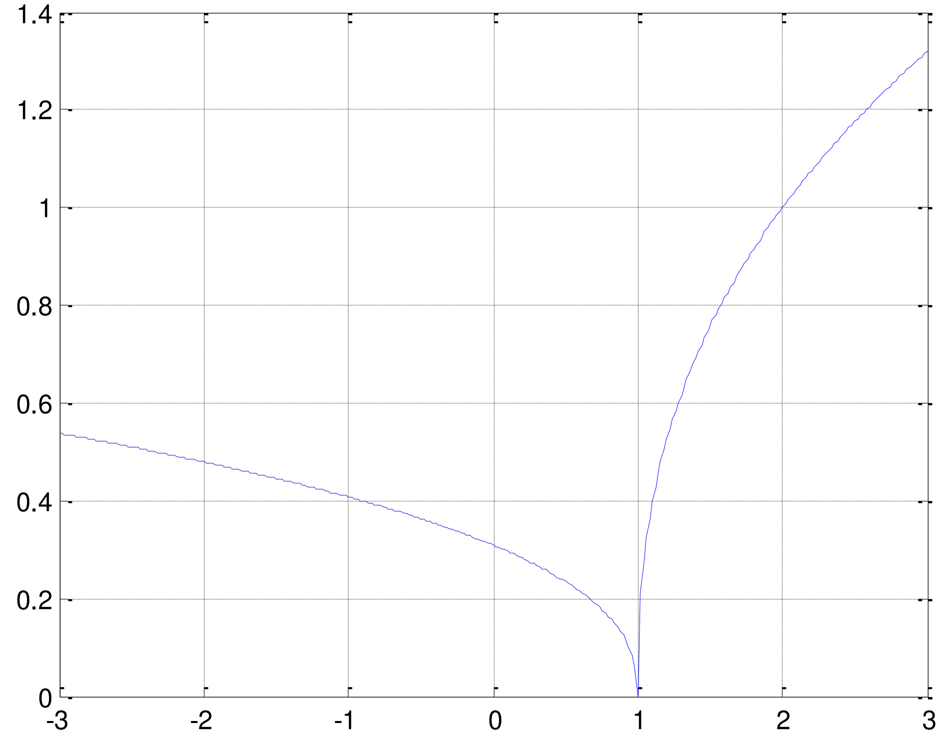


Figure 1

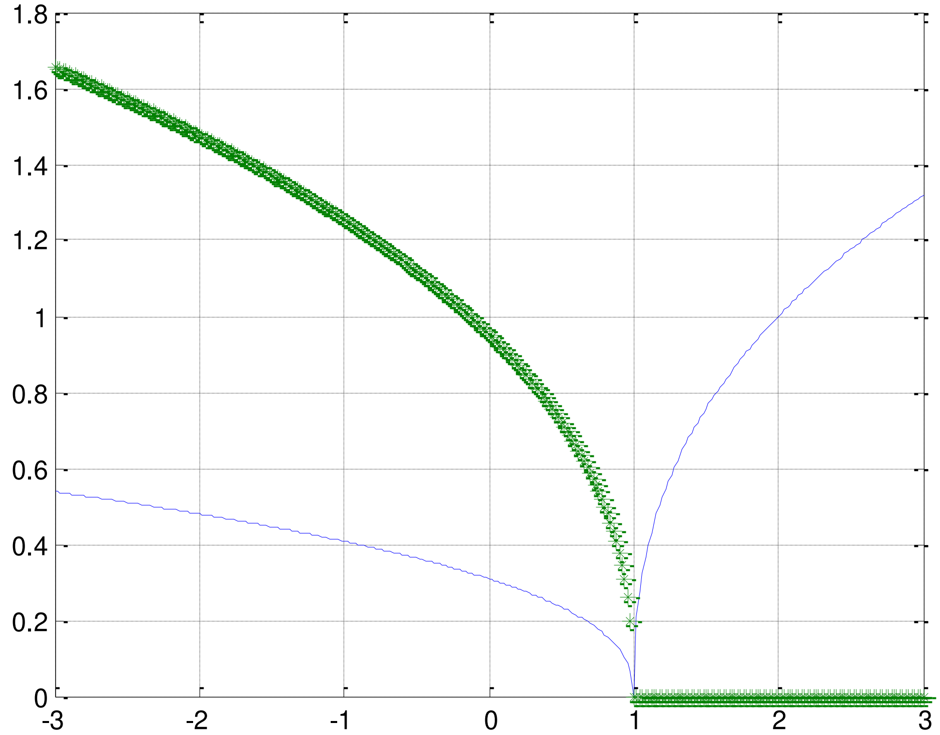


Figure 2 (Note that real part is in solid line while imag part consists of *

Explanation:

Figure 1 above is a plot of the real part of $(z-1)^{2/5}$ where z = x satisfies -3 $\leq$ x $\leq$3. Notice that the function of z has a branch cut at z = 1 . The principle value of this function is $f(z)=\left(\sqrt[5]{|z-1|}\right)^2 \angle\left(\frac{2}{5}\theta\right)$ where $-\pi<\arg(z-1)\leq\pi$ and $\arg(z-1)=\theta$.

If $z=x>1$, then $\arg(z\text{-}1)=0$ and for $x>1$ our plot is simply a graph of $f(z)=\left(\sqrt[5]{|x-1|}\right)^2$ for $x>1$. If $z=x<1$, then $\arg(z\text{-}1)=\pi$ and $w=f(z)=\left(\sqrt[5]{|x-1|}\right)^2[\cos(2\pi/5)+i\sin(2\pi/5)]$. Now MATLAB *plots only the real part* of this because of the line of code: `plot(x,w);grid;`

We obtain a graph of $f(x)=\left(\sqrt[5]{|x-1|}\right)^2[\cos(2\pi/5)]\approx\left(\sqrt[5]{|x-1|}\right)^2(.309)$ for x<1. Study the graphs in Figure 1 and you'll see that for values of x equally displaced on either side of x = 1 that the plot to the left of x=1 achieves a height that is about .31 times the height on the right.

Turn now to Figure 2. If $z = x \geq 1$, the function f(z) is real, and a plot of the real part reveals the behavior of the function. If $z = x \leq 1$ then as we have seen above the real part of *f(z)* when *z*=*x* is $f(x) \approx \left(\sqrt[5]{|x-1|}\right)^2 (.309)$ and the imaginary part is

$f(z) = \left(\sqrt[5]{|x-1|}\right)^2 [i\sin(2\pi/5)] \approx \left(\sqrt[5]{|x-1|}\right)^2 (.9511)$. The imaginary part of *w*, for $-3 < x < 3$ appears in the heavy* line in Figure 2.

PROBLEM 6

```
z=input('the complex value of z to be considered')
m=input('input the positive integer m')
n=input('the integer n')
v=zeros(1,m+1);%creates a row vector length m+1
%elements are zero.
v(1,1)=1;% the first coeff is 1, the coeff of w^m
v(1,m+1)=-z;% the constant term (the last coeff) is -z
%the following solves the equation w^m-z=0 for w=z^(1/m)
the_roots =roots(v);
final_answer=the_roots.^n
```

SECTION 1.4

PROBLEM 1

```
%sec1.4_prob1
syms x y real
w=1/x+1/(i*y)+i/((x-1)+i*y)^2;
u=real(w)
v=imag(w)
'pretty answers'
'u'
pretty(u)
'v'
pretty (v)
 u=subs(u,[x y],[1 1])
 v=subs(v,[x y],[1 1])
```

OUTPUT IS

>> sec1_4prob1

u =

1/x + (2*y*(x - 1))/(((x - 1)^2 - y^2)^2 + 4*y^2*(x - 1)^2)

v =

((x - 1)^2 - y^2)/(((x - 1)^2 - y^2)^2 + 4*y^2*(x - 1)^2) - 1/y ans =

pretty answers

ans =

u

$$\frac{1}{x}+\frac{2y(x-1)}{((x-1)^2-y^2)^2+4y^2(x-1)^2}$$

ans =

v

$$\frac{(x-1)-y}{((x-1)^2-y^2)^2+4y^2(x-1)^2}-\frac{1}{y}$$

u =

1

v =

-2

>>

PROBLEM 2

2
```
%sec1_4problem 2
syms r1 r2 x1 x2 real
Z=1/(1/(r1+i*x1)+1/(r2+i*x2))
r=real(Z)
x=imag(Z)
 R=subs(r,[r1 r2 x1 x2],[2 2 4 4])
 X=subs(x,[r1 r2 x1 x2],[2 2 4 4])
 R=subs(r, [r1 r2 x1 x2],[2 2 4 -4])
 X=subs(x, [r1 r2 x1 x2],[2 2 4 -4])

WHOSE OUTPUT IS
```

```
>> sec1_4prob2

Z =

1/(1/(r1 + x1*i) + 1/(r2 + x2*i))


r =

(r1/(r1^2 + x1^2) + r2/(r2^2 + x2^2))/((r1/(r1^2 + x1^2) +
r2/(r2^2 + x2^2))^2 + (x1/(r1^2 + x1^2) + x2/(r2^2 +
x2^2))^2)


x =

(x1/(r1^2 + x1^2) + x2/(r2^2 + x2^2))/((r1/(r1^2 + x1^2) +
r2/(r2^2 + x2^2))^2 + (x1/(r1^2 + x1^2) + x2/(r2^2 +
x2^2))^2)


R =

    1.0000


X =

    2.0000


R =

    5.0000


X =

     0

>>
```

PROBLEM 3

3.
```
%sec1.4_prob3
syms x y real
w=1/(x+i*y)^3;
u=real(w)
v=imag(w)
```

```
'pretty answers'
'u'
pretty(u)
'v'
pretty (v)
```

OUTPUT IS

>> sec1_4prob3

u =

(x*(x^2 - y^2) - 2*x*y^2)/((x*(x^2 - y^2) - 2*x*y^2)^2 + (y*(x^2 - y^2) + 2*x^2*y)^2)

v =

-(y*(x^2 - y^2) + 2*x^2*y)/((x*(x^2 - y^2) - 2*x*y^2)^2 + (y*(x^2 - y^2) + 2*x^2*y)^2)

ans =

pretty answers

ans =

u

$$\frac{x(x^2-y^2)-2xy^2}{(x(x^2-y^2)-2xy^2)^2+(y(x^2-y^2)+2x^2y)^2}$$

ans=
v

$$-\frac{y(x^2-y^2)+2x^2y}{(x(x^2-y^2)-2xy^2)^2+(y(x^2-y^2)+2x^2y)^2}$$

PROBLEM 4

```
%sec1.4_prob4
syms x y real
w=((x-i*y)/(x+i*y))^6;
u=real(w);
u=simplify(u)
v=imag(w);
v=simplify(v)
mag2=u^2+v^2;
'simplified u^2+v^2'
```

```
simplify(mag2)
```

OUTPUT IS
>> sec1_4prob4

u =

1 - (72*x^10*y^2 - 480*x^8*y^4 + 944*x^6*y^6 - 480*x^4*y^8 + 72*x^2*y^10)/(x^2 + y^2)^6

v =

-(4*x*y*(x^2 - 3*y^2)*(3*x^2 - y^2)*(x^6 - 15*x^4*y^2 + 15*x^2*y^4 - y^6))/(x^2 + y^2)^6

ans =

simplified u^2+v^2

ans =

1

The reason we must get one is this

$\left|\frac{\bar{z}}{z}\right| = 1$, thus $\left|\left(\frac{\bar{z}}{z}\right)^6\right| = 1$ which means that $\sqrt{u^2 + v^2} = \left|\left(\frac{\bar{z}}{z}\right)^6\right| = 1$

5.

```
%sec1.4_prob5
syms x y real
w=(x+i*y+1/(x+i*y))^8
u=real(w);
u=simplify(u)
v=imag(w);
v=simplify(v)
u1=subs(u,[x y],[0,1])
v1=subs(v,[x y],[0,1])
```

OUTPUT IS

u =

120*x^2*y^4 - 168*x^2*y^2 - 120*x^4*y^2 - 28*x^2*y^6 + 70*x^4*y^4 - 28*x^6*y^2 - (112*y^2 - 28)/(x^2 + y^2)^2 - (224*y^2 - 8)/(x^2 + y^2)^3 + 56/(x^2 + y^2) + (224*y^4 - 144*y^2 + 1)/(x^2 +

y^2)^4 - (256*y^6)/(x^2 + y^2)^7 + (128*y^8)/(x^2 + y^2)^8 + 56*x^2 + 28*x^4 + 8*x^6 + x^8 - 56*y^2 + 28*y^4 - 8*y^6 + y^8 + (32*y^2*(12*y^2 - 1))/(x^2 + y^2)^5 - (32*y^4*(8*y^2 - 5))/(x^2 + y^2)^6 + 70

v =

(8*x*y*(x^2 + y^2 - 1)*(x^2 + y^2 + 1)*(x^3 + x^2*y + x*y^2 + x + y^3 - y)*(x^3 - x^2*y + x*y^2 + x - y^3 + y)*(x^6 - 2*x^5*y + x^4*y^2 + 2*x^4 - 4*x^3*y^3 - x^2*y^4 + 4*x^2*y^2 + x^2 - 2*x*y^5 + 2*x*y - y^6 + 2*y^4 - y^2)*(x^6 + 2*x^5*y + x^4*y^2 + 2*x^4 + 4*x^3*y^3 - x^2*y^4 + 4*x^2*y^2 + x^2 + 2*x*y^5 - 2*x*y - y^6 + 2*y^4 - y^2))/(x^2 + y^2)^8

u1 =

0

v1 =

0

Notice that at z=0 y=1 that *z=i*. .Thus (z+1/z) =0 at z=i and the real and imaginary parts of the given expression must vanish.

6.
```
a=sym(sqrt(2));
b=sym(sqrt(3));
c=(a+i*b)^17
c1=expand(c)
cr=real(c1)
ci=imag(c1)
 % this is for part b below
d=1/(a-i*b)^13
d1=expand(d)
dr=real(d1)
di=imag(d1)
```

The numerical answers
c =

(2^(1/2) + 3^(1/2)*1i)^17

c1 =

- 493679*2^(1/2) + 3^(1/2)*303041i

cr =

-493679*2^(1/2)

ci =

303041*3^(1/2)

d =

1/(2^(1/2) - 3^(1/2)*1i)^13

d1 =

1/(12349*2^(1/2) + 3^(1/2)*17471i)

dr =

(12349*2^(1/2))/1220703125

di =

-(17471*3^(1/2))/1220703125

Thus the answers to part a) are cr and ci above
And the answers to part b) are dr and di above.

CHAPTER 2

SECTION 2.1

Note: in the following codes, you can insert the command `colormap(gray)` after using **mesh, meshz** or **surf** in order to obtain plots in shades of gray, with no coloring. For the most part we provide color solutions here.

PROBLEM 1

```
clf
%probl 2.1 number 1
x=linspace(.5,2,50);
y=x;
[x,y]=meshgrid(x,y);
z=x+i*y;
Z=z+1./z;
figure(1)
mesh(x,y,Z);xlabel('x');ylabel('y');title('using mesh');
figure(2)
 meshz(x,y,Z);xlabel('x');ylabel('y');title('using meshz');
 figure(3)
surf(x,y,Z);xlabel('x');ylabel('y');title('using surf');
% view(-50,30); %try this view for part d
```