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```
% take the first time step

x(1) = 0; t(1) = 0; t(2) = deltat;
x(2) = x(1) + deltat - b * deltat * sign(x(1)) * abs(x(1))^beta;
sum = x(1) + x(2);

% take the remaining time steps

  for k = 2:K
    t(k) = t(k-1) + deltat;
    x(k) = x(k-1) + deltat ...
       - b * deltat * sign(x(k-1)) * abs(x(k-1))^beta ...
       - deltat * deltat * sum;
    sum = sum + x(k);
  end

% plot the results

subplot(2,2,n), plot(t,x); xlabel('time','Fontsize',20);
ylabel('x(t)','Fontsize',20); legend(['B = ',num2str(b)])

end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Section 2.0

1. Since the second solution is $y_2(x) = u(x)$, $y_2'(x) = u'(x)$ and $y_2''(x) = u''(x)$. Consequently, substituting these values of $y_2(x)$, $y_2'(x)$, and $y_2''(x)$ into the differential equation, we find that $xu'' + 2u' = 0$. Therefore,

$$\frac{u''(x)}{u'(x)} = -\frac{2}{x}, \qquad u'(x) = Cx^{-2}.$$

Thus, $u(x) = A/x$, and the second solution is $y_2(x) = A/x$.

2. Since the second solution is $y_2(x) = u(x)e^x$,

$$y_2'(x) = u'(x)e^x + u(x)e^x, \quad \text{and} \quad y_2''(x) = u''(x)e^x + 2u'(x)e^x + u(x)e^x.$$

Consequently, substituting these values of $y_2(x)$, $y_2'(x)$, and $y_2''(x)$ into the differential equation, we find that $u'' + 3u' = 0$. Therefore,

$$\frac{u''(x)}{u'(x)} = -3, \qquad u'(x) = Ce^{-3x}.$$

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Thus, $u(x) = Ae^{-3x}$, and the second solution is $y_2(x) = Ae^{-2x}$.

3. Since the second solution is $y_2(x) = xu(x)$,

$$y_2'(x) = xu'(x) + u(x), \quad \text{and} \quad y_2''(x) = xu''(x) + 2u'(x).$$

Consequently, substituting these values of $y_2(x)$, $y_2'(x)$, and $y_2''(x)$ into the differential equation, we find that $xu'' + 6u' = 0$. Therefore,

$$\frac{u''(x)}{u'(x)} = -\frac{6}{x}, \qquad u'(x) = Cx^{-6}.$$

Thus, $u(x) = Ax^{-5}$, and the second solution is $y_2(x) = Ax^{-4}$.

4. Since the second solution is $y_2(x) = e^x u(x)$,

$$y_2'(x) = e^x u(x) + e^x u'(x), \quad \text{and} \quad y_2''(x) = e^x u(x) + 2e^x u'(x) + e^x u''(x).$$

Consequently, substituting these values of $y_2(x)$, $y_2'(x)$, and $y_2''(x)$ into the differential equation, we find that $xu'' + (x-1)u' = 0$. Therefore,

$$\frac{u''(x)}{u'(x)} = \frac{1}{x} - 1, \qquad u'(x) = -Axe^{-x}.$$

Thus, $u(x) = A(x+1)e^{-x}$, and the second solution is $y_2(x) = A(x+1)$.

5. Since the second solution is $y_2(x) = (x-1)u(x)$,

$$y_2'(x) = (x-1)u'(x) + u(x), \quad \text{and} \quad y_2''(x) = (x-1)u''(x) + 2u'(x).$$

Consequently, substituting these values of $y_2(x)$, $y_2'(x)$, and $y_2''(x)$ into the differential equation, we find that $x(2-x)(x-1)u'' + 2u' = 0$. Therefore,

$$\frac{u''(x)}{u'(x)} = \frac{2}{x(x-2)(x-1)} = \frac{1}{x} + \frac{1}{x-2} - \frac{2}{x-1},$$

and

$$u'(x) = A\frac{x(x-2)}{(x-1)^2} = A\left[1 - \frac{1}{(x-1)^2}\right].$$

Thus, $u(x) = Ax + A/(x-1)$, and the second solution is $y_2(x) = A(x^2 - x + 1)$.

6. Since the second solution is $y_2(x) = u(x)\sin^3(x)$,

$$y_2'(x) = 3\sin^2(x)\cos(x)u(x) + \sin^3(x)u'(x),$$

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and

$$y_2''(x) = 6\sin(x)\cos^2(x)u(x) - 3\sin^3(x)u(x) + 6\sin^2(x)\cos(x)u'(x) + \sin^3(x)u''(x).$$

Consequently, substituting these values of $y_2(x)$, $y_2'(x)$, and $y_2''(x)$ into the differential equation, we find that

$$\sin(x)\cos(x)u'' + [6\cos^2(x) + \sin^2(x)]u' = 0.$$

Therefore,

$$\frac{u''(x)}{u'(x)} = -6\cot(x) - \tan(x), \qquad u'(x) = -\frac{A\cos(x)}{5\sin^6(x)}.$$

Thus, $u(x) = A/\sin^5(x)$, and the second solution is $y_2(x) = A/\sin^2(x)$.

7. Since the second solution is $y_2(x) = u(x)\cos(x)/\sqrt{x}$,

$$y_2'(x) = -\frac{\sin(x)}{\sqrt{x}}u(x) - \frac{\cos(x)}{2x\sqrt{x}}u(x) + \frac{\cos(x)}{\sqrt{x}}u'(x)$$

and

$$y_2''(x) = -\frac{\cos(x)}{\sqrt{x}}u(x) + \frac{3\cos(x)}{4x^2\sqrt{x}}u(x) + \frac{\sin(x)}{x\sqrt{x}}u(x) - \frac{\cos(x)}{x\sqrt{x}}u'(x) - \frac{2\sin(x)}{\sqrt{x}}u'(x) + \frac{\cos(x)}{\sqrt{x}}u''(x).$$

Consequently, substituting these values of $y_2(x)$, $y_2'(x)$, and $y_2''(x)$ into the differential equation, we find that $\cos(x)u'' - 2\sin(x)u' = 0$. Therefore,

$$\frac{u''(x)}{u'(x)} = \frac{2\sin(x)}{\cos(x)}, \qquad u'(x) = A\sec^2(x).$$

Thus, $u(x) = A\tan(x)$, and the second solution is $y_2(x) = A\sin(x)/\sqrt{x}$.

8. Since the second solution is $y_2(x) = u(x)e^{-bx^2/2}$

$$y_2'(x) = -bxe^{-bx^2/2}u(x) + e^{-bx^2/2}u'(x),$$

and

$$y_2''(x) = (b^2x^2 - b)e^{-bx^2/2}u(x) - 2bxe^{-bx^2/2}u'(x) + u''(x)e^{-bx^2/2}.$$

Consequently, substituting these values of $y_2(x)$, $y_2'(x)$, and $y_2''(x)$ into the differential equation, we find that $u'' + (a - 2bx)u' = 0$. Therefore,

$$\frac{u''(x)}{u'(x)} = 2bx - a.$$

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Thus, $u(x) = \int^x e^{b\xi^2 - a\xi}\, d\xi$, and the second solution is

$$y_2(x) = e^{-bx^2/2} \int^x e^{b\xi^2 - a\xi}\, d\xi.$$

9. Letting $v = y'$, we can rewrite the differential equation $y(dv/dy) = v^2$. Assuming $v \neq 0$, we can integrate this equation to give $v = C_1 y$. Therefore, $dy/dx = C_1 y$. Integrating this, we obtain the final answer $y(x) = C_2 e^{C_1 x}$.

10. Letting $v = y'$, we can rewrite the differential equation $dv/dy = 2y$. Assuming $v \neq 0$, we can integrate this equation to give $v = y^2 + C_1$. Because $v(1) = 1$, $C_1 = 0$ and $dy/dx = y^2$. Integrating this equation, $1/y = C_2 - x$. Again, using the initial conditions, $y(x) = 1/(1-x)$.

11. Letting $v = y'$, we can rewrite the differential equation $yv(dv/dy) = v + v^2$. Assuming $v \neq 0$, we can integrate this equation to give $1 + v = C_1 y$. Substituting for v, $dy/dx = C_1 y - 1$. Integrating this equation, $y = \left(1 + C_2 e^{C_1 x}\right)/C_1$.

12. Letting $v = y'$, we can rewrite the differential equation $2yv(dv/dy) = 1 + v^2$. Assuming $v \neq 0$, we can integrate this equation to give $1 + v^2 = C_1 y$. Substituting for v, $dy/dx = \sqrt{C_1 y - 1}$. Integrating this equation, $y = \left[1 + (C_1 x + C_2)^2/4\right]/C_1$.

13. Letting $v = y'$, we can rewrite the differential equation $v(dv/dy) = e^{2y}$ with $v(0) = 1$. We can integrate this equation and find $v = e^y$. Substituting for v, $dy/dx = e^y$. Integrating this equation, $e^{-y} = C - x$, or $y = -\ln|1 - x|$.

14. We begin by integrating once and using the initial conditions, $y'' = \frac{3}{2}y^2$. Letting $v = y'$, we can rewrite this differential equation $v(dv/dy) = \frac{3}{2}y^2$. Integrating this equation, we have $v = y^{3/2}$. Substituting for v, $dy/dx = y^{3/2}$. Integrating this equation, $y = 4/(2-x)^2 = 4/(x-2)^2$.

15. If we define $z = 1/v$, the Bernoulli equation becomes $z' + z/x = -\frac{1}{2}$. Its solution is $z = -A^2/x - x/4$. Therefore, $y' = v = -4x/(x^2 + 4A^2)$. Integration yields the final answer $y(x) = B - 2\ln(x^2 + 4A^2)$.

16. First we compute

$$y'(x) = u'(x) \exp\left[-\frac{1}{2}\int^x \frac{a_1(\xi)}{a_2(\xi)}\, d\xi\right] - \frac{a_1}{2a_2} u(x) \exp\left[-\frac{1}{2}\int^x \frac{a_1(\xi)}{a_2(\xi)}\, d\xi\right],$$

and

$$y''(x) = u''(x) \exp\left[-\frac{1}{2}\int^x \frac{a_1(\xi)}{a_2(\xi)}\, d\xi\right] - \frac{1}{2}\frac{d}{dx}\left(\frac{a_1}{a_2}\right) u(x) \exp\left[-\frac{1}{2}\int^x \frac{a_1(\xi)}{a_2(\xi)}\, d\xi\right]$$
$$- \frac{a_1}{a_2} u'(x) \exp\left[-\frac{1}{2}\int^x \frac{a_1(\xi)}{a_2(\xi)}\, d\xi\right] + \frac{a_1^2}{4a_2^2} u(x) \exp\left[-\frac{1}{2}\int^x \frac{a_1(\xi)}{a_2(\xi)}\, d\xi\right].$$

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Substituting $y(x)$, $y'(x)$, and $y''(x)$ into the original ordinary differential equation yields the final answer.

Section 2.1

1. The characteristic equation is $m^2+6m+5 = (m+1)(m+5) = 0$. Therefore, the general solution is $y(x) = C_1e^{-x} + C_2e^{-5x}$.

2. The characteristic equation is $m^2 - 6m + 10 = (m - 3 + i)(m - 3 - i) = 0$. Therefore, the general solution is $y(x) = C_1e^{3x}\cos(x) + C_2e^{3x}\sin(x)$.

3. The characteristic equation is $m^2 - 2m + 1 = (m - 1)^2 = 0$. Therefore, the general solution is $y(x) = C_1e^x + C_2xe^x$.

4. The characteristic equation is $m^2-3m+2 = (m-1)(m-2) = 0$. Therefore, the general solution is $y(x) = C_1e^{2x} + C_2e^x$.

5. The characteristic equation is $m^2 - 4m + 8 = (m - 2)^2 + 4 = 0$. Therefore, the general solution is $y(x) = C_1e^{2x}\cos(2x) + C_2e^{2x}\sin(2x)$.

6. The characteristic equation is $m^2 + 6m + 9 = (m + 3)^2 = 0$. Therefore, the general solution is $y(x) = C_1e^{-3x} + C_2xe^{-3x}$.

7. The characteristic equation is $m^2 + 6m - 40 = (m + 10)(m - 4) = 0$. Therefore, the general solution is $y(x) = C_1e^{-10x} + C_2e^{4x}$.

8. The characteristic equation is $m^2 + 4m + 5 = (m + 2)^2 + 1 = 0$. Therefore, the general solution is $y(x) = C_1e^{-2x}\cos(x) + C_2e^{-2x}\sin(x)$.

9. The characteristic equation is $m^2 + 8m + 25 = (m+4)^2 + 9 = 0$. Therefore, the general solution is $y(x) = e^{-4x}\left[C_1\cos(3x) + C_2\sin(3x)\right]$.

10. The characteristic equation is $4m^2 - 12m + 9 = (2m - 3)^2 = 0$. Therefore, the general solution is $y(x) = e^{3x/2}\left(C_1 + C_2x\right)$.

11. The characteristic equation is $m^2 + 8m + 16 = (m + 4)^2 = 0$. Therefore, the general solution is $y(x) = C_1e^{-4x} + C_2xe^{-4x}$.

12. The characteristic equation is $m^3 + 4m^2 = m^2(m + 4) = 0$. Therefore, the general solution is $y(x) = C_1 + C_2x + C_3e^{-4x}$.

13. The characteristic equation is $m^4 + 4m^2 = m^2(m^2 + 4) = 0$. Therefore, the general solution is $y(x) = C_1 + C_2x + C_3\cos(2x) + C_4\sin(2x)$.

14. The characteristic equation is $m^4 + 2m^3 + m^2 = m^2(m + 1)^2 = 0$. Therefore, the general solution is $y(x) = C_1 + C_2x + C_3e^{-x} + C_4xe^{-x}$.

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15. The characteristic equation is $m^3+8=(m-2)(m+1+\sqrt{3})(m+1-\sqrt{3})=0$. Therefore, the general solution is

$$y(x)=C_1e^{2x}+C_2e^{-x}\cos(\sqrt{3}\,x)+C_3e^{-x}\sin(\sqrt{3}\,x).$$

16. The characteristic equation is $m^4-3m^3+3m^2-m=m(m-1)^3=0$. Therefore, the general solution is $y(x)=C_1+(C_2+C_3x+C_4x^2)e^x$.

17. Taking the derivative of the integro-differential equation,

$$\frac{d^2y}{dt^2}=\frac{A}{2\tau^2}\int_{-\infty}^{t}e^{-(t-x)/\tau}y(x)\,dx-\frac{A}{2\tau}y(t),$$

or

$$\frac{d^2y}{dt^2}+\frac{1}{\tau}\frac{dy}{dt}+\frac{A}{2\tau}y(t)=0.$$

Try the solution $y(t)=Ce^{-\lambda t/2}$. Then

$$\frac{\lambda^2}{4}-\frac{\lambda}{2\tau}+\frac{A}{2\tau}=\frac{1}{4}\left[\left(\lambda-\frac{1}{\tau}\right)^2+\frac{2A}{\tau}-\frac{1}{\tau^2}\right]=0.$$

The roots to this equation are $\lambda_\pm=(1\pm\sqrt{1-2A\tau}\,)/\tau$. Therefore, the general solution is

$$y(t)=e^{-t/(2\tau)}\left\{A\exp\left[t\sqrt{1-2A\tau}/(2\tau)\right]+B\exp\left[-t\sqrt{1-2A\tau}/(2\tau)\right]\right\}.$$

Section 2.2

1. The homogeneous solution to the equation is $x(t)=A\cos(5t)+B\sin(5t)$. Immediately, we have that $x(0)=A=10$. Because $x'(0)=5B=-10$, then $B=-2$. Therefore, $x(t)=10\cos(5t)-2\sin(5t)$. If we write the solution $x(t)=C\sin(5t+\varphi)$, then $C=\sqrt{100+4}=\sqrt{104}=2\sqrt{26}$. On the other hand, $\varphi=\tan^{-1}(10/-2)=\tan^{-1}(-5)=1.7682$. Our choice of angle is dictated by $A=C\sin(\varphi)$ and $B=C\cos(\varphi)$. Therefore the final answer is $x(t)=2\sqrt{26}\,\sin(5t+1.7682)$.

2. The solution in this case is $x(t)=A\cos(3t/2+\varphi)$, and $x'(t)=-3A\sin(3t/2+\varphi)/2$. Using the initial conditions, $x(0)=A\cos(\varphi)=2\pi$, and $x'(0)=-3A\sin(\varphi)/2=3\pi$. Therefore, $A=2\sqrt{2}\,\pi$, and $\varphi=-\pi/4$.

3. The solution in this case is $x(t)=A\cos(\pi t+\varphi)$, and $x'(t)=-A\pi\sin(\pi t+\varphi)$. Using the initial conditions, $x(0)=A\cos(\varphi)=1$, and $x'(0)=-\pi A\sin(\varphi)/2=\sqrt{3}\pi$. Therefore, $A=2$, and $\varphi=-\pi/3$.

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4. The differential equation for this problem is $4x'' + 100x = 0$, where $x(t)$ is the displacement (given in m) and t is time (measured in seconds). The auxiliary equation is then $m^2 + 25 = 0$ or $m = \pm 5i$. Therefore, the general solution is $x(t) = A\cos(5t) + B\sin(5t)$. Since $x(0) = 0$, $A = 0$. Next, $x'(t) = 5B\cos(5t)$. Because $x'(0) = 5B = 5$, $B = 1$. The final solution is $x(t) = 5\cos(5t)$.

5. From the information provided by the original weight, we have that $Mg = kL$. Therefore, the differential equation is $m\,d^2x/dt^2 = -Mgx/L$. Let $\omega^2 = Mg/mL$. Then the problem can be written $x'' + \omega^2 x = 0$, $x(0) = s_0$, $x'(0) = v_0$. The general solution is $x(t) = A\cos(\omega t) + B\sin(\omega t)$. From the $x(0)$ initial condition, $A = s_0$. From the $x'(0)$ initial condition, $\omega B = v_0$. Therefore, the final answer is $x(t) = s_0\cos(\omega t) + v_0\sin(\omega t)/\omega$, and $v(t) = v_0\cos(\omega t) - \omega s_0\sin(\omega t)$.

6. If $x(t)$ denotes the distance from the origin, the differential equation is $mx'' - kx = 0$. Its general solution is $x(t) = Ae^t\sqrt{k/m} + Be^{-t}\sqrt{k/m}$, where k is the constant of proportionality.

Because $x(0) = a$, we have that $x(0) = a = A + B$. Next, we compute $x'(t)$ or

$$x'(t) = \sqrt{k}\left(A\,e^t\sqrt{k/m} - B\,e^{-t}\sqrt{k/m}\right)/\sqrt{m}.$$

Therefore, $x'(0) = a\sqrt{k} = \sqrt{k}(A - B)/\sqrt{m}$, or $A - B = a\sqrt{m}$. Solving for A and B, the final solution is

$$x(t) = \frac{a}{2}\left[(1 + \sqrt{m}\,)e^t\sqrt{k/m} + (1 - \sqrt{m}\,)e^{-t}\sqrt{k/m}\right].$$

Section 2.3

1. The differential equation is $\frac{1}{2}x'' + 3x' + 4x = 0$, which has the characteristic polynomial $m^2 + 6m + 8 = (m + 4)(m + 2) = 0$. The general solution is $x(t) = Ae^{-2t} + Be^{-4t}$. Because $x(0) = A + B = 2$, and $x'(0) = -2A - 4B = 0$, $A = -2B$ or $B = -2$ and $A = 4$. Therefore, the final answer is $x(t) = 4e^{-2t} - 2e^{-4t}$.

2. The differential equation is $x'' + 10x' + 125x = 0$, which has the characteristic polynomial $m^2 + 10m + 125 = (m + 5)^2 + 100 = 0$. The general solution is $x(t) = e^{-5t}\left[A\cos(10t) + B\sin(10t)\right]$. Because $x(0) = A = 3$ and $x'(0) = -5A + 10B = 25$ or $B = 4$, the final answer is $x(t) = e^{-5t}\left[3\cos(10t) + 4\sin(10t)\right]$.

3. The differential equation is $4x'' + 20x' + 169x = 0$, which has the characteristic polynomial $4m^2 + 20m + 169 = \left(m + \frac{1}{2}\right)^2 + 36 = 0$. The general solution

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is $x(t) = e^{-5t/2}\left[A\cos(6t) + B\sin(6t)\right]$. Because $x(0) = A = 4$ and $6B = 16 + 10 = 26$ or $B = 13/3$, the final answer is $x(t) = e^{-5t/2}\left[4\cos(6t) + \frac{13}{3}\sin(6t)\right]$.

4. The amplitude of the oscillations decays by 50% by the time $t = \ln(2)/\lambda$. The period of one oscillation is $2\pi/\sqrt{\omega^2 - \lambda^2}$. Therefore, the minimum number of oscillations before the amplitude decays 50% is the first integer equal to or greater than $\ln(2)\sqrt{1 - (\lambda/\omega)^2}/(2\pi\lambda/\omega)$.

5. The characteristic polynomial is $m^2 + cm + 4 = 0$. The roots are equal when $c = 4$ when $m = -2$.

6. The characteristic polynomial is $m^2 + \frac{1}{4}cm + \frac{9}{4} = \left(m + \frac{1}{8}c\right)^2 + \frac{9}{4} - \frac{1}{64}c^2 = 0$. Therefore, we are overdamped when $c > 12$, underdamped when $c < 12$, and critically damped when $c = 12$.

7. For an overdamped system, $x(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$, $r_{1,2} = (-c \pm \sqrt{c^2 - 4km})/(2m)$ with $c^2 > 4km$.

Let us find the value of t at which $x(t) = 0$. This occurs when $(r_1 - r_2)t - \ln(-C_2/C_1)$. If $C_2/C_1 > 0$, there is no solution. If $C_2/C_1 < 0$, there is only one solution if $r_1 > r_2$. If $r_1 < r_2$, there is no crossing.

Section 2.4

1. To find the homogeneous solution, we solve $y_H'' + 4y_H' + 3y_H = 0$. Its solution is $y_H(x) = Ae^{-3x} + Be^{-x}$. For the particular solution, we guess $y_p(x) = Cx + D$, so that $y_p(x) = C$, and $y_p''(x) = 0$. Substitution into the differential equation yields $4C + 3Cx + 3D = x + 1$, and $C = \frac{1}{3}$ and $D = -\frac{1}{9}$. The general solution is therefore $y(x) = Ae^{-3x} + Be^{-x} + \frac{1}{3}x - \frac{1}{9}$.

2. To find the homogeneous solution, we solve $y_H'' - y_H = 0$. Its solution is $y_H(x) = Ae^{x} + Be^{-x}$. For the particular solution, we guess $y_p(x) = Cxe^{x} + De^{-2x}$, yielding $y_p'(x) = Ce^{x} + Cxe^{x} - 2De^{-2x}$, and $y_p''(x) = 2Ce^{x} + Cxe^{x} + 4De^{-2x}$. Substitution into the differential equation gives $2Ce^{x} + 3De^{-2x} = e^{x} - 2e^{-2x}$, or $C = \frac{1}{2}$ and $D = -\frac{2}{3}$. The general solution is therefore $y(x) = Ae^{x} + Be^{-x} + \frac{1}{2}xe^{x} - \frac{2}{3}e^{-2x}$.

3. To find the homogeneous solution, we solve $y_H'' + 2y_H' + 2y_H = 0$. Its solution is $y_H(x) = e^{-x}[A\cos(x) + B\sin(x)]$. For the particular solution, we guess $y_p(x) = Cx^2 + Dx + E$, yielding $y_p'(x) = 2Cx + D$, and $y_p''(x) = 2C$. Substitution into the differential equation gives $2C + 4Cx + 2D + 2Cx^2 + 2Dx + 2E = 2x^2 + 2x + 4$, or $C = 1$, $D = -1$ and $E = 2$. The general solution is therefore $y(x) = e^{-x}[A\cos(x) + B\sin(x)] + x^2 - x + 2$.

4. To find the homogeneous solution, we solve $y_H'' + y_H' = 0$. Its solution is $y_H(x) = A + Be^{-x}$. For the particular solution, we guess $y_p(x) = Cx^3 + Dx^2 +$

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Ex, yielding $y_p'(x) = 3Cx^2+2Dx+E$ and $y_p''(x) = 6Cx+2D$. Substitution into the differential equation gives $6Cx+2D+3Cx^2+2Dx+E = x^2+2x$, or $C = \frac{1}{3}$ and $D = E = 0$. The general solution is therefore $y(x) = Ax + Be^{-x} + \frac{1}{3}x^3$.

5. To find the homogeneous solution, we solve $y_H'' + 2y_H' = 0$. Its solution is $y_H(x) = A + Be^{-2x}$. For the particular solution, we guess $y_p(x) = Cx^2 + Dx + Exe^{-2x}$, yielding $y_p'(x) = 2Cx + D + Ee^{-2x} - 2Exe^{-2x}$, and $y_p''(x) = 2C + 4Exe^{-2x} - 4Ee^{-2x}$. Substitution into the differential equation gives $4Cx + 2C + 2D + 2Ee^{-2x} = 2x + 5 - e^{-2x}$, or $C = \frac{1}{2}$, $D = 2$, and $E = \frac{1}{2}$. The general solution is therefore $y(x) = A + Be^{-2x} + \frac{1}{2}x^2 + 2x + \frac{1}{2}e^{-2x}$.

6. To find the homogeneous solution, we solve $y_H''-4y_H'+4y_H = 0$. Its solution is $y_H(x) = Ae^{2x} + Bxe^{2x}$. For the particular solution, we guess $y_p(x) = Cx^3e^{2x} + Dx^2e^{2x}$, yielding $y_p'(x) = 3Cx^2e^{2x} + 2Cx^3e^{2x} + 2Dxe^{2x} + 2Dx^2e^{2x}$, and $y_p''(x) = 6Cxe^{2x} + 12Cx^2e^{2x} + 4Cx^3e^{2x} + 2De^{2x} + 8Dxe^{2x} + 4Dx^2e^{2x}$. Substitution into the differential equation gives $6Cxe^{2x} + 2De^{2x} = xe^{2x} + e^{2x}$, or $C = \frac{1}{6}$ and $D = \frac{1}{2}$. The general solution is therefore $y(x) = Ae^{2x} + Bxe^{2x} + \left(\frac{1}{2}x^2 + \frac{1}{6}x^3\right)e^{2x}$.

7. To find the homogeneous solution, we solve $y_H'' + 4y_H' + 4y_H = 0$. Its solution is $y_H(x) = (A + Bx)e^{-2x}$. For the particular solution, we guess $y_p(x) = Ce^x + Dxe^x$, yielding $y_p'(x) = Ce^x + De^x + Dxe^x$, and $y_p''(x) = Ce^x + 2De^x + Dxe^x$. Substitution into the differential equation gives $(9C + 6D)e^x + 9Dxe^x = xe^x$, or $C = -\frac{2}{27}$ and $D = \frac{1}{9}$. The general solution is therefore $y(x) = (A + Bx)e^{-2x} + \left(\frac{1}{9}x - \frac{2}{27}\right)e^x$.

8. To find the homogeneous solution, we solve $y_H'' - 4y_H = 0$. Its solution is $y_H(x) = A\cosh(2x) + B\sinh(2x)$. For the particular solution, we guess $y_p(x) = Cx\cosh(2x) + Dx\sinh(2x)$, yielding $y_p'(x) = C\cosh(2x) + D\sinh(2x) + 2Cx\sinh(2x) + 2Dx\cosh(2x)$, and $y_p''(x) = 4C\sinh(2x) + 4D\cosh(2x) + 4Cx\cosh(2x) + 4Dx\sinh(2x)$. Substitution into the differential equation gives $y_p'' - 4y_p = 4C\sinh(2x) + 4D\cosh(2x) = 4\sinh(2x)$, or $D = 0$ and $C = 1$. The general solution is therefore $y(x) = A\cosh(2x)+B\sinh(2x)+x\cosh(2x)$.

9. To find the homogeneous solution, we solve $y_H'' + 9y_H = 0$. Its solution is $y_H(x) = A\cos(3x) + B\sin(3x)$. For the particular solution, we guess $y_p(x) = Cx^2\cos(3x)+Dx^2\sin(3x)+Ex\cos(3x)+Fx\sin(3x)$, yielding $y_p'(x) = (2Cx+3Dx^2 + E + 3Fx)\cos(3x) + (2Dx - 3Cx^2 + F - 3Ex)\sin(3x)$, and $y_p''(x) = (2C - 9Cx^2 - 9Ex + 12Dx + 6F)\cos(3x) + (2D - 9Dx^2 - 12Cx - 6E - 9Fx)\sin(3x)$. Substitution into the differential equation gives $y_p''+9y_p = (2C+12Dx+6F)\cos(3x)+(2D-12Cx-6E)\sin(3x) = x\cos(3x)$, or $2C+6F = 0$, $2D - 6E = 0$, $-12C = 0$, and $12D = 1$. The general solution is therefore $y(x) = A\cos(3x) + B\sin(3x) + \frac{1}{12}x^2\sin(3x) + \frac{1}{36}x\cos(3x)$.

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10. To find the homogeneous solution, we solve $y_H'' + y_H = 0$. Its solution is $y_H(x) = A\cos(x) + B\sin(x)$. For the particular solution, we guess $y_p(x) = Cx^2\cos(x)+Dx^2\sin(x)+Ex\cos(x)+Fx\sin(x)$, yielding $y_p'(x) = (2Cx+Dx^2+E+Fx)\cos(x) + (2Dx - Cx^2 + F - Ex)\sin(x)$, and $y_p''(x) = (2C - Cx^2 - Ex + 4Dx + 2F)\cos(x) + (2D - Dx^2 - 4Cx - 2E - Fx)\sin(x)$. Substitution into the differential equation gives $y_p''+9y_p = (2C+4Dx+2F)\cos(x)+(2D-4Cx-2E)\sin(x) = \sin(x)+x\cos(x)$, or $2C+2F = 0$, $2D-2E = 1$, $-4C = 0$, and $4D = 1$. The general solution is therefore $y(x) = A\cos(x) + B\sin(x) + \frac{1}{4}\left[x^2\sin(x) - x\cos(x)\right]$.

11. Using the method of undetermined coefficients, the homogeneous differential equation is $y_H''+2ay_H = 0$. Its solution is $y_H(x) = A+Be^{-2ax}$. For the particular solution, we guess $y_p(x) = Cx + D\sin(2\omega x) + E\cos(2\omega x)$, yielding $y_p'(x) = C + 2\omega D\cos(2\omega x) - 2\omega E\sin(2\omega x)$, and $y_p''(x) = -4\omega^2 D\sin(2\omega x) - 4\omega^2 E\cos(2\omega x)$. The trick here is to recognize that the differential equation can be rewritten as $y'' + 2ay = \frac{1}{2}[1 - \cos(2\omega x)]$. The form of the particular solution follows directly. Substitution into the differential equation gives $y_p'' + 2ay_p' = 2aC - 4\left(a\omega E + \omega^2 D\right)\sin(2\omega x) + 4\left(a\omega D - \omega^2 E\right)\cos(2\omega x)$. Matching up terms, we find that

$$C = \frac{1}{4a}, \quad aE = -\omega D, \quad D = -\frac{a}{8\omega(\omega^2+a^2)} \qquad \text{and} \qquad E = \frac{1}{8(\omega^2+a^2)}.$$

Therefore, the general solution is

$$y(x) = A + Be^{-2ax} + \frac{x}{4a} + \frac{\omega\cos(2\omega x) - a\sin(2\omega x)}{8\omega(a^2+\omega^2)}.$$

To find A and B, we use the initial conditions:

$$y(0) = A+B+\frac{1}{8(a^2+\omega^2)} = 0, \quad \text{and} \quad y'(0) = -2aB+\frac{1}{4a}-\frac{a}{4(a^2+\omega^2)} = 0.$$

Solving for A and B, $A = -1/(8a^2)$ and $B = \omega^2/[8a^2(a^2+\omega^2)]$. Therefore, the complete solution is

$$y(x) = \frac{2ax-1}{8a^2} + \frac{\omega^2 e^{-2ax}}{8a^2(a^2+\omega^2)} - \frac{a\sin(2\omega x) - \omega\cos(2\omega x)}{8\omega(a^2+\omega^2)}.$$

Turning to the integration technique, we begin by noting that

$$\int_0^x y''\,d\xi + 2a\int_0^x y'\,d\xi = \tfrac{1}{2}\int_0^x [1-\cos(2\omega\xi)]\,d\xi,$$

or

$$y'(x) - y'(0) + 2a[y(x) - y(0)] = \frac{x}{2} - \frac{\sin(2ax)}{4a}.$$

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Substituting the initial conditions yields

$$y' + 2ay = \frac{x}{2} - \frac{\sin(2ax)}{4a}.$$

Since this differential equation is already in canonical form, $P(x) = 2a$, and $\mu(x) = e^{2ax}$. Multiplying by the integrating factor, we have that

$$\frac{d}{dx}\left[e^{2ax}y(x)\right] = \frac{x}{2}e^{2ax} - \frac{sin(2\omega x)}{4\omega}e^{2ax}.$$

Integrating both sides of this equation,

$$e^{2ax}y(x) - y(0) = \tfrac{1}{2}\int_0^x \xi e^{2a\xi}\,d\xi - \frac{1}{4\omega}\int_0^x \sin(2\omega\xi)e^{2a\xi}\,d\xi$$

or

$$e^{2ax}y(x) = \left.\frac{e^{2a\xi}}{8a^2}\left(2a\xi - 1\right)\right|_0^x - \left.\frac{e^{2a\xi}}{8\omega(a^2+\omega^2)}\left[a\sin(2\omega\xi) - \omega\cos(2\omega\xi)\right]\right|_0^x.$$

Solving for $y(x)$,

$$y(x) = \frac{2ax-1}{8a^2} + \frac{\omega^2 e^{-2ax}}{8a^2(a^2+\omega^2)} - \frac{a\sin(2\omega x) - \omega\cos(2\omega x)}{8\omega(a^2+\omega^2)}.$$

Section 2.5

1. Because the characteristic polynomial is $m^2+6m+18 = (m+3)^2+9 = 0$, the homogeneous solution is $y_H(t) = e^{-3t}\left[A\cos(3t) + B\sin(3t)\right]$. Therefore, resonance will occur when $\gamma = 3$.

2. Because the characteristic polynomial is $m^2+2m+2 = (m+1)^2+1 = 0$, the homogeneous solution is $x_H(t) = e^{-t}\left[A\cos(t) + B\sin(t)\right]$. For the particular solution we guess $x_p(t) = C\cos(2t) + D\sin(2t)$ along with $x_p'(t) = -2C\sin(2t) + 2D\cos(2t)$, and $x_p''(t) = -4C\cos(2t) - 4D\sin(2t)$. Substitution into the differential equation yields two linear equations for C and D: $-2C + 4D = 0$, $-2D - 4C = 10$, or $C = -2$ and $D = -1$. Therefore, the general solution is $x(t) = e^{-t}\left[A\cos(t) + B\sin(t)\right] - 2\cos(2t) - \sin(2t)$ along with $x'(t) = -e^{-t}\left[A\cos(t) + B\sin(t)\right] + e^{-t}\left[-A\sin(t) + B\cos(t)\right] + 4\sin(2t) - 2\cos(2t)$. Substituting the initial conditions, $x'(0) = -A + B - 2 = 0$, $x(0) = A - 2 = x_0$. Solving these equations and substituting back into the general solution yields the final answer $x(t) = e^{-t}\left[(2+x_0)\cos(t) + (4+x_0)\sin(t)\right] - 2\cos(2t) - \sin(2t)$.

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3. If x is taken as positive in the downward directions,

$$m\frac{d^2x}{dt^2} = \sum \text{ forces} = mg - kx,$$

or

$$m\frac{d^2x}{dt^2} + kx = mg.$$

Because the system is initially at rest and the coordinate system is chosen so that $x(0) = 0$, $x(0) = x'(0) = 0$.

The general solution to this differential equation is $x(t) = A\cos(\omega t) + B\sin(\omega t) + mg/k$, where $\omega^2 = k/m$. Since $x'(0) = 0$, $B = 0$. Because $x(0) = 0$, $A = -mg/k$. Therfore, $x(t) = mg[1 - \cos(\omega t)]/k$.

4. Starting with the differential equation

$$L\frac{dI}{dt} + \frac{1}{C}\int I\,dt = E_0[1 - \cos(\omega t)],$$

we take its derivative with respect to time and obtain

$$L\frac{d^2I}{dt^2} + \frac{I}{C} = \omega E_0 \sin(\omega t).$$

The general solution is

$$I(t) = A\sin(\omega_1 t + \theta) - \frac{\omega E_0 \sin(\omega t)}{L\omega^2 - 1/C},$$

where $\omega_1 = 1/\sqrt{LC}$. The initial condition $I(0) = 0$ yields $\theta = 0$. On the other hand, the initial condition $I'(0) = 0$ gives $A = \omega_1\omega^2 E_0/(L\omega^2 - 1/C)$. Therefore, the final solution is

$$I(t) = \frac{\omega_1\omega^2 E_0}{L\omega^2 - 1/C}\sin(\omega_1 t) - \frac{\omega E_0 \sin(\omega t)}{L\omega^2 - 1/C}.$$

5. Because the characteristic polynomial is $mp^2 + cp + k = 0$ or $[p + c/(2m)]^2 + \omega_0^2 = 0$, $\omega_0^2 = k/m - c^2/(4m^2)$, the homogeneous solution is $x_H(t) = e^{-ct/(2m)}$ $[A\cos(\omega_0 t) + B\sin(\omega_0 t)]$. The particular solution is $x_p(t) = C\sin(\omega t - \varphi) = C\cos(\varphi)\sin(\omega t) - C\sin(\varphi)\cos(\omega t)$, along with $x_p'(t) = C\omega\cos(\varphi)\cos(\omega t) + C\omega\sin(\varphi)\sin(\omega t)$, and $x_p''(t) = -C\omega^2\cos(\varphi)\sin(\omega t) + C\omega^2\sin(\varphi)\cos(\omega t)$. Substituting into the differential equation, we obtain the following system of linear equations: $c\omega C\sin(\varphi) + (k - m\omega^2)C\cos(\varphi) = F_0$, and $c\omega C\cos(\varphi) - (k - m\omega^2)C\sin(\varphi) = 0$. The second equation immediately yields the relationship $\tan(\varphi) = c\omega/(k - m\omega^2)$. Solving for C, we find that

$$C = F_0/\sqrt{c^2\omega^2 + (k - m\omega^2)^2}.$$

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Therefore, the general solution is

$$x(t) = e^{-ct/(2m)}\left[A\cos(\omega_0 t) + B\sin(\omega_0 t)\right] + \frac{F_0\,\sin(\omega t - \varphi)}{\sqrt{c^2\omega^2 + (k - m\omega^2)^2}}.$$

6. The characteristic polynomial is $m^2 + Rm/L + 1/(LC) = [m + R/(2L)]^2 + \left[1/(LC) - R^2/(4L^2)\right] = 0$. If $\omega^2 = 1/(LC) - R^2/(4L^2) > 0$, then $Q(t) = e^{-Rt/(2L)}\left[A\cos(\omega t) + B\sin(\omega t)\right]$. As $t \to \infty$, $Q(t) \to 0$.

If $\omega^2 = 0$, then $Q(t) = e^{-Rt/(2L)}\,(A + Bt)$. Again, as $t \to \infty$, $Q(t) \to 0$.

Finally, if $\omega^2 < 0$, then $Q(t) = Ae^{[\Gamma - R/(2L)]t} + Be^{[-\Gamma - R/(2L)]t}$, where $\Gamma^2 = -\omega^2$ and $\Gamma > 0$. Because $\Gamma < R/(2L)$, $Q(t) \to 0$ as $t \to \infty$.

Section 2.6

1. First we find the homogeneous solution to the differential equation $y_H'' - 4y_H' + 3y_H = 0$. Its solution is $y_H(x) = Ae^x + Be^{3x}$. According to variation of parameters, the particular solution is of the form $y_p(x) = u_1(x)e^x + u_2(x)e^{3x}$. Substitution into the differential equation yields the following system of equations $e^x u_1'(x) + e^{3x}u_2'(x) = 0$ and $e^x u_1'(x) + 3e^{3x}u_2'(x) = e^{-x}$. Solving for $u_1'(x)$ and $u_2'(x)$, $u_1'(x) = -\frac{1}{2}e^{-2x}$ or $u_1(x) = \frac{1}{4}e^{-2x}$, and $u_2'(x) = \frac{1}{2}e^{-4x}$ or $u_2(x) = -\frac{1}{8}e^{-4x}$. The general solution is $y(x) = Ae^x + Be^{3x} + u_1(x)e^x + u_2(x)e^{3x} = Ae^x + Be^{3x} + \frac{1}{8}e^{-x}$.

2. First we find the homogeneous solution to the differential equation $y_H'' - y_H' - 2y_H = 0$. Its solution is $y_H(x) = Ae^{2x} + Be^{-x}$. According to variation of parameters, the particular solution is of the form $y_p(x) = u_1(x)e^{2x} + u_2(x)e^{-x}$. Substitution into the differential equation yields the following system of equations $e^{2x}u_1'(x) + e^{-x}u_2'(x) = 0$ and $2e^{2x}u_1'(x) - e^{-x}u_2'(x) = x$. Solving for $u_1'(x)$ and $u_2'(x)$, $u_1'(x) = \frac{1}{3}xe^{-2x}$ or $u_1(x) = -\frac{1}{12}(2x + 1)e^{-2x}$, and $u_2'(x) = -\frac{1}{3}xe^x$ or $u_2(x) = -\frac{1}{3}(x - 1)e^x$. The general solution is $y(x) = Ae^{2x} + Be^{-x} + u_1(x)e^{2x} + u_2(x)e^{-x} = Ae^x + Be^{3x} - \frac{1}{2}x + \frac{1}{4}$.

3. First we find the homogeneous solution to the differential equation $y_H'' - 4y_H = 0$. Its solution is $y_H(x) = Ae^{2x} + Be^{-2x}$. According to variation of parameters, the particular solution is of the form $y_p(x) = u_1(x)e^{2x} + u_2(x)e^{-2x}$. Substitution into the differential equation yields the following system of equations $e^{2x}u_1'(x) + e^{-2x}u_2'(x) = 0$ and $2e^{2x}u_1'(x) - 2e^{-2x}u_2'(x) = xe^x$. Solving for $u_1'(x)$ and $u_2'(x)$, $u_1'(x) = \frac{1}{4}xe^{-x}$ or $u_1(x) = -\frac{1}{4}(x + 1)e^{-x}$, and $u_2'(x) = -\frac{1}{4}xe^{3x}$ or $u_2(x) = -\frac{1}{36}(3x - 1)e^{3x}$. The general solution is $y(x) = Ae^{2x} + Be^{-2x} + u_1(x)e^{2x} + u_2(x)e^{-2x} = Ae^{2x} + Be^{-2x} - (3x + 2)e^x/9$.

4. First we find the homogeneous solution to the differential equation $y_H'' + 9y_H' = 0$. Its solution is $y_H(x) = A\cos(3x) + B\sin(3x)$. According to variation

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of parameters, the particular solution is of the form $y_p(x) = u_1(x)\cos(3x) + u_2(x)\sin(3x)$. Substitution into the differential equation yields the following system of equations $\cos(3x)u_1'(x) + \sin(3x)u_2'(x) = 0$ and $-3\sin(3x)u_1'(x) + 3\cos(3x)u_2'(x) = 2\sec(3x)$. Solving for $u_1'(x)$ and $u_2'(x)$, $u_2'(x) = \frac{2}{3}$ or $u_2(x) = 2x/3$. Then $u_1'(x) = -\frac{2\sin(3x)}{3\cos(3x)}$ or $u_1(x) = \frac{2}{9}\ln|\cos(3x)|$. The final answer is $y(x) = A\cos(3x) + B\sin(3x) + \frac{2}{9}\ln|\cos(3x)|\cos(3x) + \frac{2}{3}x\sin(3x)$.

5. First we find the homogeneous solution to the differential equation $y_H'' + 4y_H' + 4y_H = 0$. Its solution is $y_H(x) = Ae^{-2x} + Bxe^{-2x}$. According to variation of parameters, the particular solution is of the form $y_p(x) = u_1(x)e^{-2x} + u_2(x)xe^{-2x}$. Substitution into the differential equation yields the following system of equations $e^{-2x}u_1'(x) + xe^{-2x}u_2'(x) = 0$ and $-2e^{-2x}u_1'(x) + (1-2x)e^{-2x}u_2'(x) = xe^{-2x}$. Solving for $u_1'(x)$ and $u_2'(x)$, $u_1'(x) = -x^2$, or $u_1(x) = -\frac{1}{3}x^3$, and $u_2'(x) = x$ or $u_2(x) = \frac{1}{2}x^2$. The general solution is $y(x) = (A+Bx)e^{-2x} + u_1(x)e^{-2x} + u_2(x)xe^{-2x} = (A+Bx)e^{-2x} + \frac{1}{6}x^3e^{-2x}$.

6. First we find the homogeneous solution to the differential equation $y_H'' + 2ay_H' = 0$. Its solution is $y_H(x) = A' + Bxe^{-2ax}$. According to variation of parameters, the particular solution is of the form $y_p(x) = u_1(x) + u_2(x)e^{-2ax}$. Substitution into the differential equation yields the following system of equations $u_1'(x) + e^{-2ax}u_2'(x) = 0$, and $-2ae^{-2ax}u_2' = \sin^2(\omega x)$. Solving for $u_1'(x)$ and $u_2'(x)$,

$$u_2'(x) = -\frac{e^{2ax}}{4a}\left[1 - \cos(2\omega x)\right],$$

or

$$u_2(x) = -\frac{e^{2ax}}{8a^2} + \frac{e^{2ax}}{8a(a^2+\omega^2)}\left[a\cos(2\omega x) + \omega\sin(2\omega x)\right].$$

Then

$$u_1'(x) = \frac{1-\cos(2\omega x)}{4a}, \qquad \text{or} \qquad u_1(x) = \frac{x}{4a} - \frac{\sin(2\omega x)}{4a\omega}.$$

Therefore,

$$\begin{aligned} y(x) &= A' + Bxe^{-2ax} + \frac{x}{4a} - \frac{\sin(2\omega x)}{8a\omega} - \frac{1}{8a^2} + \frac{a\cos(2\omega x) + \omega\sin(2\omega x)}{8a(a^2+\omega^2)} \\ &= A + Bxe^{-2ax} + \frac{x}{4a} - \frac{a\sin(2\omega x) - \omega\cos(2\omega x)}{8\omega(a^2+\omega^2)}. \end{aligned}$$

7. First we find the homogeneous solution to the differential equation $y_H'' - 4y_H' + 4y_H = 0$. Its solution is $y_H(x) = Ae^{2x} + Bxe^{2x}$. According to variation of parameters, the particular solution is of the form $y_p(x) = u_1(x)e^{2x} + u_2(x)xe^{2x}$. Substitution into the differential equation yields the following system of equations $e^{2x}u_1'(x) + xe^{2x}u_2'(x) = 0$, and $2e^{2x}u_1'(x) + (2x+1)e^{2x}u_2'(x) =$

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$(x+1)e^{2x}$. Solving for $u_1'(x)$ and $u_2'(x)$, $u_1'(x) = -x - x^2$ or $u_1(x) = -\frac{1}{2}x^2 - \frac{1}{3}x^3$, and $u_2'(x) = 1 + x$ or $u_2(x) = x + \frac{1}{2}x^2$. The general solution is $y(x) = Ae^{2x} + Bxe^{2x} + u_1(x)e^{2x} + u_2(x)xe^{2x} = Ae^{2x} + Bxe^{2x} + \left(\frac{1}{2}x^2 + \frac{1}{6}x^3\right)e^{2x}$.

8. First we find the homogeneous solution to the differential equation $y_H'' - 4y_H = 0$. Its solution is $y_H(x) = Ae^x + Be^{-x}$. According to variation of parameters, the particular solution is of the form $y_p(x) = u_1(x)e^x + u_2(x)e^{-x}$. Substitution into the differential equation yields the following system of equations $e^x u_1'(x) + e^{-x}u_2'(x) = 0$, and $e^x u_1'(x) - e^{-x}u_2'(x) = \sin^2(x)$. Solving for $u_1'(x)$ and $u_2'(x)$, $u_1'(x) = \frac{1}{2}e^{-x}\sin^2(x)$, or $u_1(x) = -\frac{1}{10}e^{-x}\left[2 + \sin(x) + 2\cos(x)\right]$, and $u_2'(x) = -\frac{1}{2}e^x\sin^2(x)$, or $u_2(x) = -\frac{1}{10}e^x\left[2 + \sin(x) - 2\cos(x)\right]$. The general solution is $y(x) = Ae^x + Be^{-x} + u_1(x)e^x + u_2(x)e^{-x} = Ae^x + Be^{-x} - \frac{1}{5}\left[2 + \sin^2(x)\right]$.

9. First we find the homogeneous solution to the differential equation $y_H'' - 2y_H' + y_H = 0$. Its solution is $y_H(x) = Ae^x + Cxe^x$. According to variation of parameters, the particular solution is of the form $y_p(x) = u_1(x)e^x + u_2(x)xe^x$. Substitution into the differential equation yields the following system of equations $e^x u_1'(x) + xe^x u_2'(x) = 0$ and $e^x u_1'(x) + (e^x + xe^x)u_2'(x) = e^x/x$. Solving for $u_1'(x)$ and $u_2'(x)$, $u_1'(x) = -1$, or $u_1(x) = -x$, and $u_2'(x) = 1/x$, or $u_2(x) = \ln(x)$. The general solution is $y(x) = Ae^x + Cxe^x + u_1(x)e^x + u_2(x)xe^x = Ae^x + Bxe^x + x\ln(x)e^x$.

10. First we find the homogeneous solution to the differential equation $y_H'' + y_H = 0$. Its solution is $y_H(x) = A\cos(x) + B\sin(x)$. According to variation of parameters, the particular solution is of the form $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$. Substitution into the differential equation yields the following system of equations $\cos(x)u_1'(x) + \sin(x)u_2'(x) = 0$, and $-\sin(x)u_1'(x) + \cos(x)u_2'(x) = \tan(x)$. Solving for $u_1'(x)$ and $u_2'(x)$, $u_1'(x) = -\sin^2(x)/\cos(x)$, or $u_1(x) = -\ln|\sec(x) + \tan(x)| + \sin(x)$, and $u_2'(x) = \sin(x)$, or $u_2(x) = -\cos(x)$. The general solution is

$$\begin{aligned} y(x) &= A\cos(x) + B\sin(x) + u_1(x)\cos(x) + u_2(x)\sin(x) \\ &= A\cos(x) + B\sin(x) - \ln\left|\frac{1+\sin(x)}{\cos(x)}\right|\cos(x) \\ &= A\cos(x) + B\sin(x) + \ln\left|\frac{1-\sin(x)}{\cos(x)}\right|\cos(x). \end{aligned}$$

Section 2.7

1. The auxiliary equation is $m(m-1)+m-1 = (m-1)(m+1) = 0$. Therefore, $y(x) = C_1x + C_2x^{-1}$.

2. The auxiliary equation is $m(m-1) + 2m - 2 = (m-1)(m+2) = 0$. Therefore, $y(x) = C_1x + C_2x^{-2}$.

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3. The auxiliary equation is $m(m-1)-2=(m-2)(m+1)=0$. Therefore, $y(x)=C_1x^2+C_2x^{-1}$.

4. The auxiliary equation is $m(m-1)-m+1=(m-1)^2=0$. Therefore, $y(x)=C_1x+C_2x\ln(x)$.

5. The auxiliary equation is $m(m-1)+3m+1=(m+1)^2=0$. Therefore, $y(x)=C_1x^{-1}+C_2x^{-1}\ln(x)$.

6. The auxiliary equation is $m(m-1)-3m+4=(m-2)^2=0$. Therefore, $y(x)=C_1x^2+C_2x^2\ln(x)$.

7. The auxiliary equation is $m(m-1)-m+5=(m-1)^2+4=0$ so that $m=-1\pm 2i$. Therefore, $y(x)=C_1x\cos[2\ln(x)]+C_2x\sin[\ln(x)]$.

8. The auxiliary equation is $4m(m-1)+8m+5=(2m+1)^2+4=0$ so that $m=-\frac{1}{2}\pm i$. Therefore, $y(x)=C_1x^{-1/2}\cos[\ln(x)]+C_2x^{-1/2}\sin[\ln(x)]$.

9. The auxiliary equation is $m(m-1)+m+1=m^2+1=0$ so that $m=\pm i$. Therefore, $y(x)=C_1\cos[\ln(x)]+C_2\sin[\ln(x)]$.

10. The auxiliary equation is $m(m-1)-3m+13=(m-2)^2+9=0$ so that $m=2\pm 3i$. Therefore, $y(x)=C_1x^2\cos[3\ln(x)]+C_2x^2\sin[3\ln(x)]$.

11. The auxiliary equation is $m(m-1)(m-2)-2m(m-1)-2m+8=m^3-5m^2+2m+8=(m-2)(m-4)(m+1)=0$. Therefore, $y(x)=C_1x^2+C_2x^4+C_3x^{-1}$.

12. Let $t=\ln(x)$ and $y(x)=Y(t)$, then $Y''-3Y'-4Y=e^t$. The homogeneous solution is $Y_H(t)=Ae^{4t}+Be^{-t}$. From the method of undetermined coefficients, $Y_p(t)=-\frac{1}{6}e^t$. Transforming back, $y(x)=Ax^4+B/x-x/6$.

Section 2.8

1. The solution to the differential equation is $x(t)=C_1e^t+C_2e^{2t}$. Therefore, any point on the phase diagram except for $x=v=0$ has a trajectory off to infinity.

2. Multiplying the differential equation by x' and setting $v=x'$ on the left side, $vv'=x^3x'-xx'$. Integration yields $\frac{1}{2}v^2-\frac{1}{4}x^4+\frac{1}{2}x^2=C$. If $C=0$, $x=v=0$ and we have a equilibrium point where the energy is at a minimum. If $C=\frac{1}{4}$, we have another equilibrium point corresponding to $x=\pm 1$ and $v=0$. Note that $C\leq\frac{1}{4}$.

3. The curves on the phase diagram are given by $v=2x+C$. The equilibrium points are $v=0$ for all x. Therefore the entire abscissa contain all of

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the equilibra. Any displacement from this equilibrium moves off to infinity. Therefore, the equilibrium is unstable.

4. The curves on the phase diagram are given by $v^2 + \text{sgn}(x)x^2 = C$. The equilibrium points are $v = 0$ and $v' = -\text{sgn}(x)x = 0$. Therefore, the point $(0, 0)$ is the equibrium point. Any displacement from this equilibrium moves off to infinity. Therefore, the equilbrium is unstable.

5. The curves on the phase diagram are given by $v^2 - x = C$ if $|x| > 2$ and $v = C$ if $|x| < 2$. The equilibrium points are $v = 0$ and $|x| < 2$. Therefore, there are infinite number of equilibrium points, all located along the x-axis for $|x| < 2$. Any displacement from these equilbria moves off to infinity and all of the equilibria are unstable.

Section 2.9

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%                MATLAB Code for Pendulum Problem
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% initialize parameters

clear; b = 0.22; g = 9.8; k = 0.02; L = 1; omega_0_sq = g/L;
omega_sq = omega_0_sq - b*b/4; omega = sqrt(omega_sq);
delta_t = 0.005; t = [0:delta_t:50]; % set up the time
theta(1) = 10*pi/180; thetap(1) = 0; % initial conditions

% time step to find position and velocity
%     of the pendulum at later times

for i = 1:length(t)-1
  theta_g = theta(i) + delta_t*thetap(i);
  thetap_g = thetap(i) - b*delta_t*thetap(i) ...
           - omega_0_sq*delta_t*theta(i);
  avg_1 = (theta(i) + theta_g) / 2;
  avg_2 = (thetap(i) + thetap_g) / 2;
  theta(i+1) = theta(i) + delta_t*avg_2;
  thetap(i+1) = thetap(i) - b*delta_t*avg_2 ...
              - omega_0_sq*delta_t*avg_1;
  if ((abs(theta(i)) < delta_t/2) & (thetap(i) > 0))
      thetap(i+1) = thetap(i+1) + k; end % impulse forcing
end
```

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```
% plot the results

plot(theta,thetap,'k'); xlabel('\theta','FontSize',25);
ylabel('\theta^\prime','FontSize',25)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Section 3.1

1.
$$A + B = \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix} = B + A$$

2.
$$A - B = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$$
$$B - A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} -2 & -3 \\ 1 & 0 \end{pmatrix}$$

3.
$$3A - 2B = \begin{pmatrix} 9 & 12 \\ 3 & 6 \end{pmatrix} - \begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ -1 & 2 \end{pmatrix}$$
$$3(2A - B) = 3\left[\begin{pmatrix} 6 & 8 \\ 2 & 4 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}\right] = 3\begin{pmatrix} 5 & 7 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 15 & 21 \\ 0 & 6 \end{pmatrix}$$

4.
$$A^T = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}, \qquad B^T = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, \qquad (B^T)^T = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$$

5.
$$(A + B)^T = \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}^T = \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}$$
$$A^T + B^T = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}$$

6.
$$B + B^T = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$$
$$B - B^T = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

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