Abstract Algebra An Interactive Approach 2nd edition Paulsen Solutions Manual

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24) 491.
26) 353.

Section 1.3

2) Yes, this is a group.
4) Not closed, no identity, hence no inverses.
6) Yes, this is a group.
8) No additive inverses.
10) Not closed, no identity, hence no inverses.
12) 2 has no inverse.
14) Yes, this is a group, $e = -3$.
16) If e_1 and e_2 are two identity elements, then $e_1 = e_1 \cdot e_2 = e_2$.
18) Note that $(x \cdot y) \cdot (y^{-1} \cdot x^{-1}) = x \cdot x^{-1} = e$, so $y^{-1} \cdot x^{-1}$ is the inverse of $x \cdot y$
20) Consider the set $\{a \cdot x \mid x \in S\}$ which is a subset of S, yet must contain the same number of elements.
22) To show $x \cdot y = y \cdot x$, start with $(x \cdot y)^2 = x \cdot y \cdot x \cdot y = e$.
24) If $a^3 = e$ then $(a^{-1})^3 = e$. Furthermore, if $a \neq e$, then $a^{-1} \neq a$. So the non-identity solutions pair off, and with the identity we have an odd number of solutions.
26)

$\cdot$	a	b	c	d
a	b	a	d	c
b	a	b	c	d
c	d	c	a	b
d	c	d	b	a

28) $9 \to 6$, $27 \to 18$, $81 \to 54$, $243 \to 162$, $5 \to 4$, $25 \to 20$, $125 \to 100$. Conjecture $(p-1)n/p$.
30) If m and n are coprime, $|Z^*_{mn}| = |Z^*_m| \cdot |Z^*_n|$.

Section 2.1

2) 1, 3, 5, 9, 11, and 13.
4) 1, 5, 7, 11, 13, 17, 19, and 23.
6) 2, 6, 7, and 8.
8) 3 and 5.
10) No generators
12) No generators
14) 96.
16) 320.
18) 1210.
20) 1680.

22) For $\phi(n) = 14$, either $p_i - 1$ or $p_i^{(r_i-1)}$ must be a multiple of 7 for some prime p_i. In the first case, $p_i \geq 29$, so $\phi(n) \geq 28$. In the latter case, $p_i = 7$ and $r_i \geq 2$, so $\phi(n) \geq 42$.
24) Yes, 8 elements are generators: 2, 3, 8, 12, 13, 17, 22, and 23.
26) Z_n^* is cyclic if n is twice the power of an odd prime.

Section 2.2

2) $b^2 \cdot a = b \cdot (a \cdot b^2) = (a \cdot b^2) \cdot b^2 = a \cdot b \cdot b^3 = a \cdot b$.
4) Answers will vary depending on how the elements are labelled. The group will be isomorphic to A_4.
6) $a \cdot b \cdot c^3$.
8) $a \cdot b^2 \cdot c^3$.
10) $a \cdot b^2 \cdot c^2$.
12) $a \cdot c$.
14) $a \cdot b \cdot c$.
16) $a \cdot c^3$.
18) The group has 20 elements.
20)

```
InitGroup("e")
AddGroupVar("a", "b")
Define(a^3, e)
Define(b^5, e)
Define((a*b)^2, e)
G = Group(a, b)
len(G)
    60
```

Section 2.3

2) $\{0\}$, $\{0,2,4,6,8,10,12,14,16,18\}$, $\{0,4,8,12,16\}$, $\{0,5,10,15\}$, $\{0,10\}$, and the whole group.
4) $\{1\}$, $\{1,8\}$, $\{1,4,7\}$, and the whole group.
6) $\{1\}$, $\{1,2,4,8\}$, $\{1,4\}$, $\{1,4,7,13\}$, $\{1,11\}$, $\{1,14\}$, $\{1,4,11,14\}$, and the whole group.
8) $R_2(G) = 10$, $R_3(G) = 9$, $R_4(G) = 16$, and $R_6(G) = 18$. For these examples, $R_k(G)$ is a multiple of k.
10) $R_9(G) = 9$, and $R_3(G) = 3$, so six elements of order 9.
12) When $n = k$, an element is of order k if, and only if, it is a generator. If k is a divisor of n, and m is a divisor of k, then $R_m(Z_k) = R_m(Z_n)$. Thus, computing the elements of order k in both Z_k and Z_n will give the same results.
14) If g is a generator, than only g and g^{-1} have finite order.
16) If a and b are of finite order, then $a^m = b^n = e$ for some $m > 0$ and $n > 0$. Then $(a \cdot b^{-1})^{mn} = e$, so $a \cdot b^{-1}$ is of finite order.
18) $(y \cdot x \cdot y^{-1})^2 = e$, but $y \cdot x \cdot y^{-1} \neq e$, so $y \cdot x \cdot y^{-1} = x$.
20) $x^2 \neq e$ if, and only if, $x^{-1} \neq x$, so these elements pair off, leaving an even number of elements. Thus, there is an even number of solutions to $x^2 = e$.

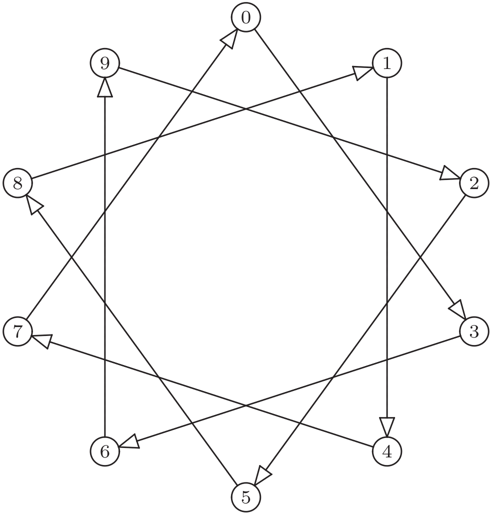

FIGURE 2.1: Circle graph of adding 3 **mod** 10

002x001.eps

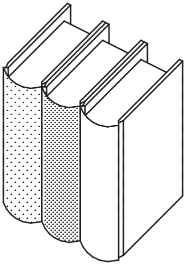

FIGURE 2.2: Visualizing arrangements of three books

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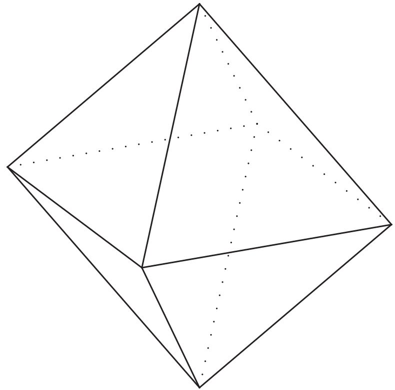

FIGURE 2.3: Octahedron with eight equilateral triangles

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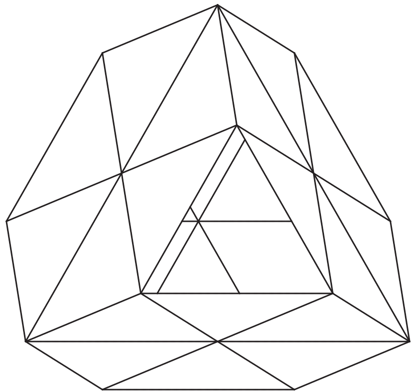

FIGURE 2.4: The Pyraminx$^{\text{TM}}$ puzzle without tips

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