Sensor Systems Fundamentals and Applications 1st edition Silva Solutions Manual

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Chapter 2 Component Interconnection

Solution 2.1

(a):

$$\text{Electrical Impedance} = \frac{\text{Voltage Output}}{\text{Current Input}}$$

$$\text{Mechanical Impedance} = \frac{\text{Force Output}}{\text{Velocity Input}}$$

(b): Both these impedances are frequency response functions (defined in the frequency domain). Both define the resistance provided by the load against the driving force. High-impedance devices need high levels of effort (voltage or force) to drive them (i.e., to pass current through electrical impedances, or to cause movement of mechanical loads). Note that voltage is an across-variable whereas force is a through-variable. Hence, there is an inconsistency in the definitions of "impedance," with respect to the force-current analogy.

(c): To avoid this inconsistency, we may use the force-voltage analogy, in which voltage and force are termed "effort" variables and velocity and current are "flow" variables, as in the "bond graph" nomenclature.

Note, however, that in order to use general relations for interconnecting basic elements (in forming multicomponent devices or circuits), it is the across-variable and through-variable nomenclature that is applicable. Specifically, when two elements are connected in series, the through-variable is common and the across-variables add; when two elements are connected in parallel, the across-variable is common and the through-variables add. Hence, it is the through- and across-variable nomenclature that is natural with regard to component interconnection. In this context we may define a generalized series element or generalized impedance (to include electrical impedance or mechanical mobility) and a generalized parallel element (to include electrical admittance or mechanical impedance).

(d): The input impedance has to be comparatively high for a measuring device that is connected in parallel, to measure an across variable, whereas the input impedance has to be quite low for a device that connected in series, to measures a through variable. This is essential to reduce loading errors. The output impedance of a measuring device has to be low in order to maintain a high *sensitivity*, and get acceptable signal levels for processing, actuating or recording.

When cascading two devices, in order to reduce the "loading" of one device by the other, and to maintain good frequency characteristics, the output impedance of the first device (which provides the signal) has to be smaller in comparison to the input impedance of the second device (which receives the signal). Otherwise, the signal will be distorted by the second device (the load). If power transfer characteristics are important, however, one impedance should be the complex conjugate of the other. Different matching criteria are used depending on the applications.

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Solution 2.2

1. Maximum power transfer

2. Power transfer at maximum efficiency

3. Reflection prevention in signal transmission

4. Loading reduction

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Solution 2.3

When a measuring device is connected to a system, the conditions in the system itself will change, as the measured signal flows through the measuring device. For example, in electrical measurements, a current may pass through the measuring device, thereby altering the voltages and currents in the original system. This is called "electrical loading," and will introduce an error, as the measurand itself is distorted. Similarly, in mechanical measurements, due to the mass of the measuring device, the mechanical condition (forces, motions) of the original system will change, thereby affecting the measurand and causing an error. This is called mechanical loading.

Now consider the system shown in Figure P2.3. We have: $v_o = K\left[\dfrac{Z_i}{Z_s + Z_i}\right] v_i$

For a voltage follower, $K = 1$ and $Z_o << Z_i$. Hence, $v_o = \left[\dfrac{Z_i}{Z_s + Z_i}\right] v_i$, or

$$\frac{v_o}{v_i} = \frac{(Z_i / Z_s)}{1 + (Z_i / Z_s)}.$$

This relationship is sketched in Figure S2.3.

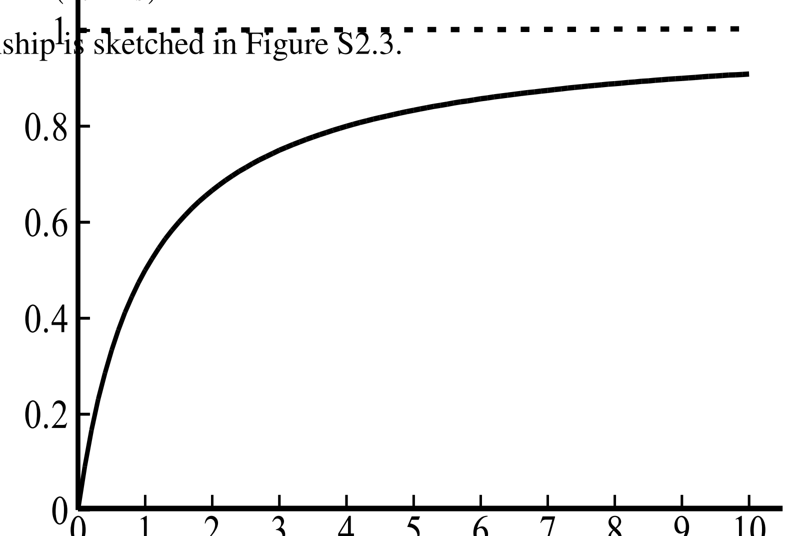

Figure S2.3: Non-dimensional curve of loading performance.

Some representative values of the curve are tabulated below.

Z_i / Z_s	v_o / v_i
0.1	0.091
0.5	0.55
1	0.5
2	0.667
5	0.855
7	0.875
10	0.909

Note: Performance improves with the impedance ratio Z_i / Z_s.

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Solution 2.4

Open-circuit voltage at the output port is (in the frequency domain)

$$v_{oc} = \frac{\left[R_2 + \dfrac{1}{j\omega C}\right]}{\left[R_1 + R_2 + j\omega L + \dfrac{1}{j\omega C}\right]} = v_{eq} \qquad \text{(i)}$$

Note: Equivalent source v_{eq} is expressed here as a function of frequency. Its corresponding time function $v_{eq}(t)$ is obtained by using inverse Fourier transform. Alternatively, first replace $j\omega$ by the Laplace variable s: $v_{eq}(s) = \dfrac{\left[R_2 + \dfrac{1}{sC}\right]}{\left[R_1 + R_2 + sL + \dfrac{1}{sC}\right]} v(s)$. Then obtain the inverse Laplace transform, for a given $v(s)$, using Laplace transform tables.

Now, in order to determine Z_{eq}, note from Figure P2.4(b) that if the output port is shorted, the resulting short circuit current i_{SC} is given by: $i_{SC} = \frac{v_{eq}}{Z_{eq}}$. Hence,

$$Z_{eq} = \frac{v_{eq}}{i_{sc}} = \frac{v_{oc}}{i_{sc}} \qquad \text{(ii)}$$

Since we know v_{oc} (or v_{eq}) from equation (i) we only have to determine i_{sc}. Using the actual circuit with shorted output, we see that there is no current through the parallel impedance $R_2 + \frac{1}{j\omega C}$ because the potential difference across it is zero. Thus,

$$i_{sc} = \frac{v}{\left(R_1 + j\omega L\right)} \qquad \text{(iii)}$$

Now substituting Equations (i) and (iii) in (ii) we have:

$$Z_{eq} = \left[\frac{\left[R_2 + \frac{1}{j\omega C}\right]\left[R_1 + j\omega L\right]}{\left[R_1 + R_2 + j\omega L + \frac{1}{j\omega C}\right]}\right]$$

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Solution 2.5

(a) Load power efficiency $\eta = \frac{R_l}{(R_l + R_s)} = \frac{R_l / R_s}{(R_l / R_s + 1)}$

(b) Load power $p_l = \frac{{v_s}^2 R_l}{[R_l + R_s]^2}$; Maximum load power (occurs at $R_l = R_s$) $p_{\max} = \frac{{v_s}^2}{4R_s}$

➔ $p_l / p_{\max} = \frac{4R_l / R_s}{[R_l / R_s + 1]^2}$

We use the following MATLAB script (.m file) to generate the two curves:

```
% Efficiency and load power curves
lr=[]; eff=[]; pw=[];  % declare vectors
lr=0; eff=0; pw=0; %initialize variables
for i=1:100
a=0.1*i; %load resistance ratio
lr(end+1)=a; % store load resistance
eff(end+1)=a/(a+1); % efficiency
pw(end+1)=4*a/(a+1)^2;  % load power
end
plot(lr,eff,'-',lr,pw,'-',lr,pw,'x')
```

The two curves are plotted in Figure S2.5.

It is seen that maximum efficiency does not correspond to maximum power. In particular, the efficiency increases monotonically with the load resistance while the maximum power occurs when $R_l = R_s$. Hence, a reasonable trade-off in matching the resistances would be needed when both considerations are important.

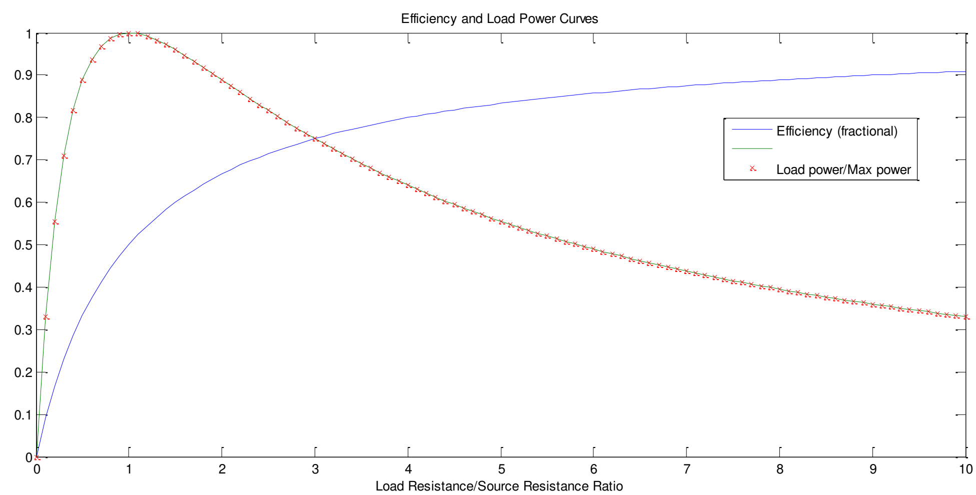

Figure S2.5: Variation of efficiency and maximum power with load resistance.

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<u>Solution 2.6</u>

Voltage is an across variable. In order to reduce loading effects, the resistance of a voltmeter should be much larger than the output impedance of system or load impedance. Then, the voltmeter will not draw a significant part of the signal current (and will not distort the signal). Current is a through variable. The resistance of an ammeter should be much smaller than the output impedance of system or load impedance. Then, the ammeter will not provide a significant voltage drop (and will not distort the signal).

Voltmeter should be able to operate with a low current (due to its high resistance) and associated low torque, in conventional electromagnetic deflection type meters. Low torque means, a torsional spring having low stiffness has to be used to get an adequate meter reading. This makes the meter slow, less robust, and more nonlinear, even though high sensitivity is realized.

Ammeter should be able to carry a large current because of its low resistance. Hence meter torque would be high in conventional designs. This can create thermal problems, magnetic hysteresis, and other nonlinearities. The device can be made fast, robust, and mechanically linear, however, while obtaining sufficient sensitivity.
Note: The torque is not a factor in modern digital multi-meters.

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Solution 2.7

(a) The input impedance of the amplifier = 500 MΩ.

Estimated error $= \dfrac{10}{(500+10)} \times 100\% = 2\%$

(b) Impedance of the speaker = 4 Ω.

Estimated error $= \dfrac{0.1}{(4+0.1)} \times 100\% = 2.4\%$

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Solution 2.8

$$v_o = F_1(f_o, f_i) \quad \Rightarrow \quad \delta v_o = \frac{\partial F_1}{\partial f_o}\,\delta f_o + \frac{\partial F_1}{\partial f_i}\,\delta f_i \qquad \text{(i)}$$

$$v_i = F_2(f_o, f_i) \quad \Rightarrow \quad \delta v_i = \frac{\partial F_2}{\partial f_o}\,\delta f_o + \frac{\partial F_2}{\partial f_i}\,\delta f_i \qquad \text{(ii)}$$

In terms of incremental variables about an operating point, we can define the input impedance Z_i and the output impedance Z_o as

$$Z_i = \frac{\delta v_i}{\delta f_i} \qquad \text{with} \quad \delta f_o = 0 \qquad \text{(iii)}$$

$$Z_o = \frac{\delta v_o \text{ with } \delta f_o = 0}{\delta f_o \text{ with } \delta v_o = 0} \qquad \text{(iv)}$$

Note: $\delta f_o = 0$ corresponds to incremental open-circuit condition and $\delta v_o = 0$ corresponds to incremental short-circuit condition.

From (ii) with $\delta f_o = 0$ (i.e., open circuit at output) we get $Z_i = \dfrac{\partial F_2}{\partial f_i}$.

Now using the open-circuit by subscript "*oc*" and the short-circuit by subscript "*sc*" we have:

From (i):

$$\delta v_o \Big|_{oc} = \frac{\partial F_1}{\partial f_i}\,\delta f_i \Big|_{oc} \qquad \text{(v)}$$

From (ii):

$$\delta v_i \Big|_{oc} = \frac{\partial F_2}{\partial f_i}\,\delta f_i \Big|_{oc} \qquad \text{(vi)}$$

Note: δv_i is an independent increment, which does not depend on whether *oc* or *sc* condition exists at the output. But δf_i will change depending on the output condition.
From (v) and (vi):

$$\delta v_o \Big|_{oc} = \frac{\partial F_1}{\partial f_i} \Big/ \frac{\partial F_2}{\partial f_i} \qquad \delta v_i \tag{vii}$$

From (i):

$$0 = \frac{\partial F_1}{\partial f_o}\, \delta f_o \Big|_{sc} + \frac{\partial F_1}{\partial f_i}\, \delta f_i \Big|_{sc} \tag{viii}$$

From (ii):

$$\delta v_i = \frac{\partial F_2}{\partial f_o}\, \delta f_o \Big|_{sc} + \frac{\partial F_2}{\partial f_i}\, \delta f_i \Big|_{sc} \tag{ix}$$

Eliminating $\delta f_i \Big|_{sc}$ from Equations(viii) and (ix) we get,

$$\delta f_o \Big|_{sc} = \frac{1}{\left[\dfrac{\partial F_2}{\partial f_o} - \dfrac{\partial F_1}{\partial f_o} \cdot \dfrac{\partial F_2}{\partial f_i} \Big/ \dfrac{\partial F_1}{\partial f_i}\right]}\ \delta v_i \tag{x}$$

Substitute (vii) and (x) in (iv):

$$Z_o = \left[\frac{\partial F_1}{\partial f_i} \Big/ \frac{\partial F_2}{\partial f_i}\right]\left[\frac{\partial F_2}{\partial f_o} - \frac{\partial F_1}{\partial f_o} \cdot \frac{\partial F_2}{\partial f_i} \Big/ \frac{\partial F_1}{\partial f_i}\right] = \frac{\partial F_1}{\partial f_i} \cdot \frac{\partial F_2}{\partial f_o} \Big/ \frac{\partial F_1}{\partial f_i} - \frac{\partial F_1}{\partial f_o}$$

One way to experimentally determine Z_i and Z_o (under static conditions) is to first experimentally determine the two sets of operating curves given by $v_o = F_1(f_o, f_i)$ and $v_i = F_2(f_o, f_i)$ under steady-state conditions. For example f_o is kept constant and f_i is changed in increments to measure v_o and v_i once the steady state is reached. This will give two curves f_i versus v_o and v_i versus v_i for a particular value of f_o. Next f_o is incremented and another pair of curves is obtained. Once these two sets of curves are obtained for the required range for f_i and f_o, the particular derivatives are determined from using the general method shown in Figure S2.8, for the case $z = F(x,y)$ with: $\dfrac{\partial z}{\partial x} \cong \alpha$ and $\dfrac{\partial z}{\partial y} \cong \dfrac{\Delta z}{\Delta y}$.

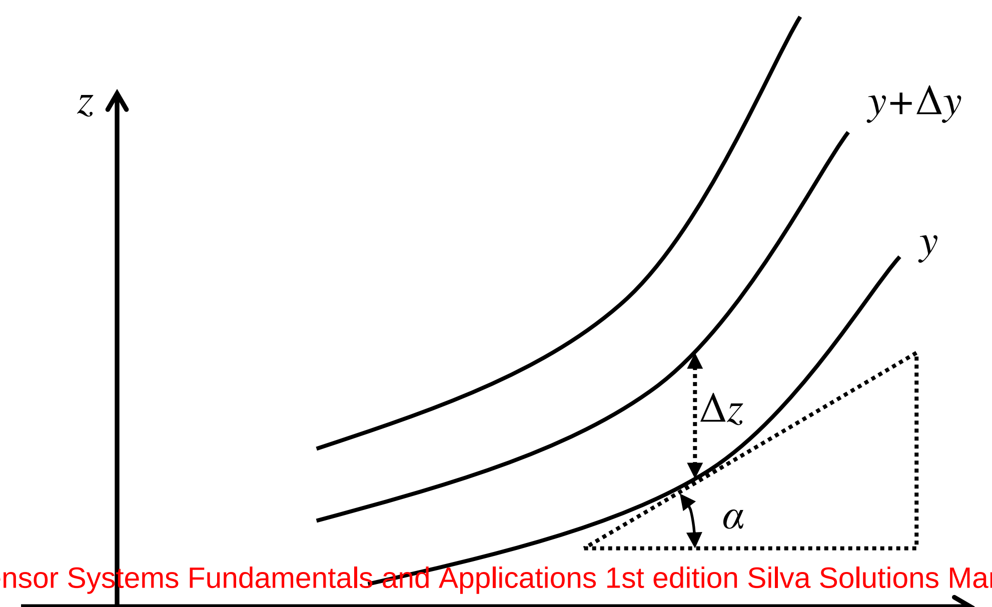

Figure S2.8: Computation of local slopes.

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Solution 2.9

(a)
We have: $v_i = Z_c i_i$, $v_t = Z_l i_t$, $v_r = -Z_c i_r$, $v_t = v_i + v_r$, and $i_t = i_i + i_r$, where "*i*" denotes current and the subscript "*r*" denotes "reflected."
Substitute:

$$v_t = Z_l i_t = Z_l(i_i + i_r) = Z_l\left(\frac{v_i}{Z_c} - \frac{v_r}{Z_c}\right) = \frac{Z_l}{Z_c}(v_i - v_r) = \frac{Z_l}{Z_c}(v_i - (v_t - v_i)) = \frac{Z_l}{Z_c}(2v_i - v_t)$$

$$\Rightarrow \ \left(1 + \frac{Z_l}{Z_c}\right)v_t = 2\frac{Z_l}{Z_c}v_i \ \Rightarrow \ v_t = \frac{2Z_l}{(Z_l + Z_c)}v_i$$

(b) We need $v_t = v_i \Rightarrow \dfrac{2Z_l}{(Z_l + Z_c)} = 1 \Rightarrow Z_l = Z_c$

(c) Use a transformer with the required impedance ratio = (turns ratio)^2

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Solution 2.10

For the given system, $\omega_n = \sqrt{\dfrac{1\times 10^6}{100}}$ rad/s = 100 rad/s and $\omega \geq 200$ rad/s. Hence, we have the frequency ratio $r \geq 2.0$.

For $r = 2.0$ and $\left|T_f\right| = 0.5$ we have $0.5 = \sqrt{\dfrac{1 + 16\zeta^2}{9 + 16\zeta^2}}$ or, $\zeta = \sqrt{\dfrac{5}{48}}$. Hence,

$$b = 2\zeta\omega_n m = 2\sqrt{\frac{5}{48}} \times 100 \times 100 \text{ N.s/m} \rightarrow b = 6.455 \times 10^3 \text{ N.s/m}.$$

With this damping constant, for $r \geq 2$, we will have $\left|T_f\right| \leq 0.5$. Decreasing b will decrease $\left|T_f\right|$ in this frequency range.

To plot the Bode diagram using MATLAB, first note that:

$$2\zeta\omega_n = b/m = 6.455\times10^3/100 = 64.55 \text{ rad/s} \text{ and } \omega_n^{\,2} = 10^4 \text{ (rad/s)}^2$$

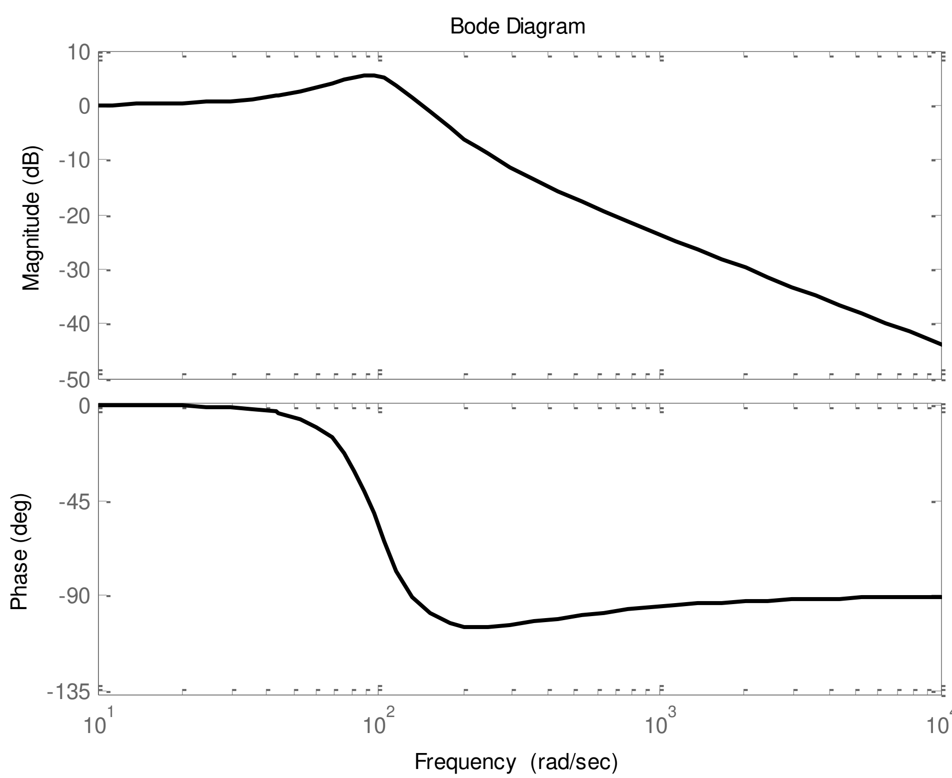
The corresponding transmissibility function is $T_f = \dfrac{64.55s+10^4}{s^2+64.55s+10^4}$ with $s = j\omega$

The following MATLAB script will plot the required Bode diagram:

```
% Plotting of transmissibility function
clear;
m=100.0;
k=1.0e6;
b=6.455e3;
sys=tf([b/m k/m],[1 b/m k/m]);
bode(sys);
```

The resulting Bode diagram is shown in Figure S2.10. A transmissibility magnitude of 0.5 corresponds to $20\log_{10}0.5$ dB = -6.02 dB.

Note from the Bode magnitude curve in Figure S2.10.4 that at the frequency 200 rad/s the transmissibility magnitude is less than -6 dB and it decreases continuously for higher frequencies. This confirms that the designed system meets the design specification.



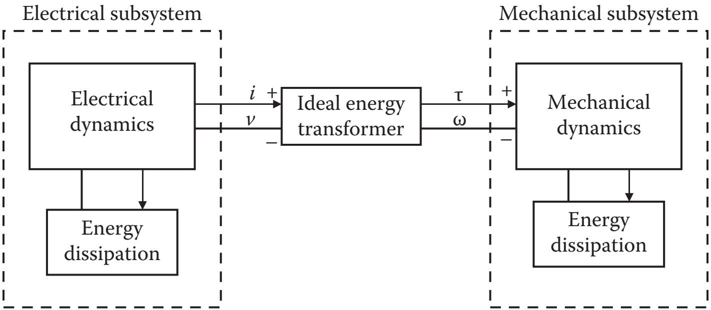


Figure 2.1
A model for mixed-domain (electromechanical) component interconnection.

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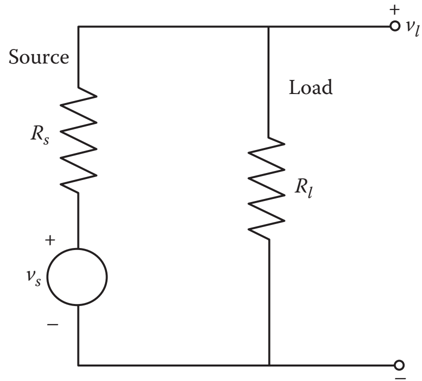

Figure 2.2
A DC circuit with a source and a load.

002x002.eps

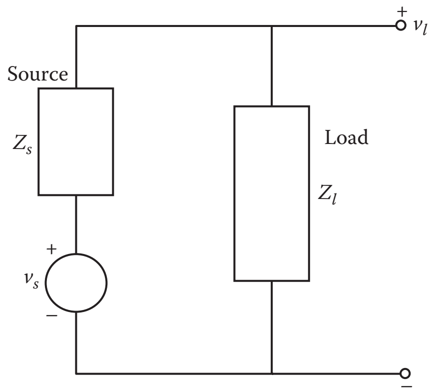

Figure 2.3
An impedance (AC) circuit with a source and a load.

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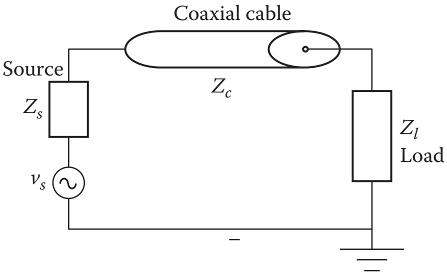

Figure 2.4
A source and a load connected by a cable.

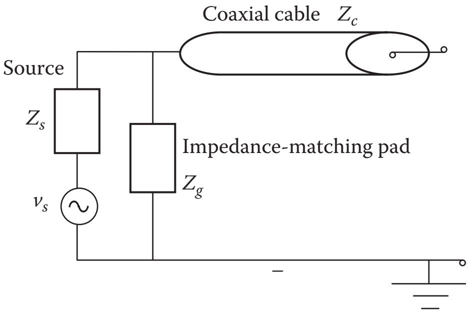

Figure 2.5
Application of an impedance-matching pad.

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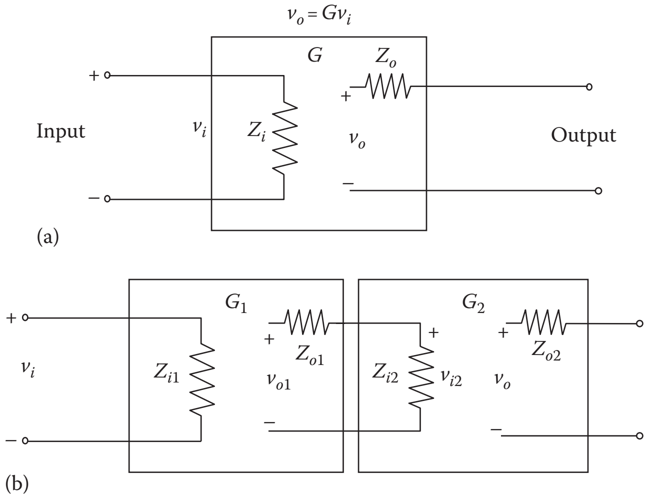

Figure 2.6
(a) Schematic representation of input impedance and output impedance and (b) cascade connection of two two-port devices.

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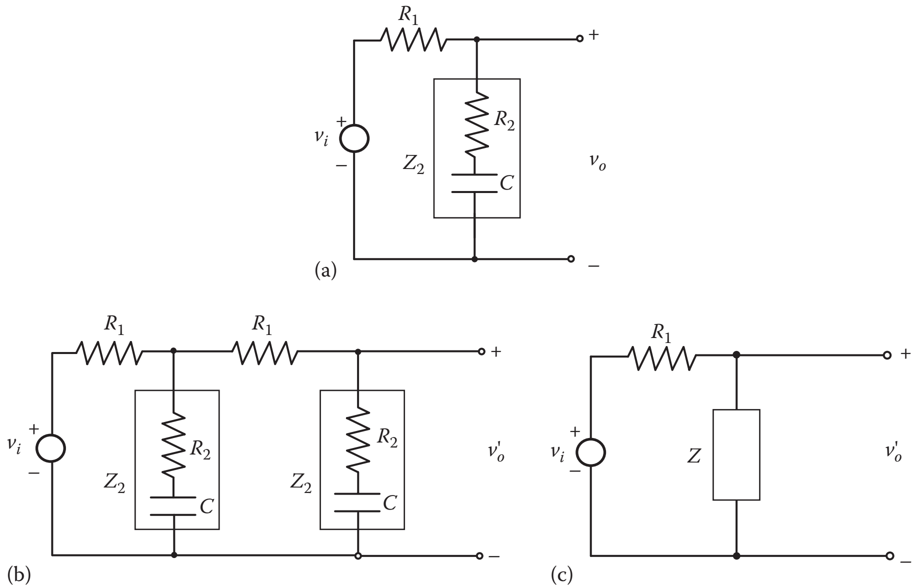

Figure 2.7
(a) A single-circuit module, (b) cascade connection of two modules, and (c) an equivalent circuit for (b).

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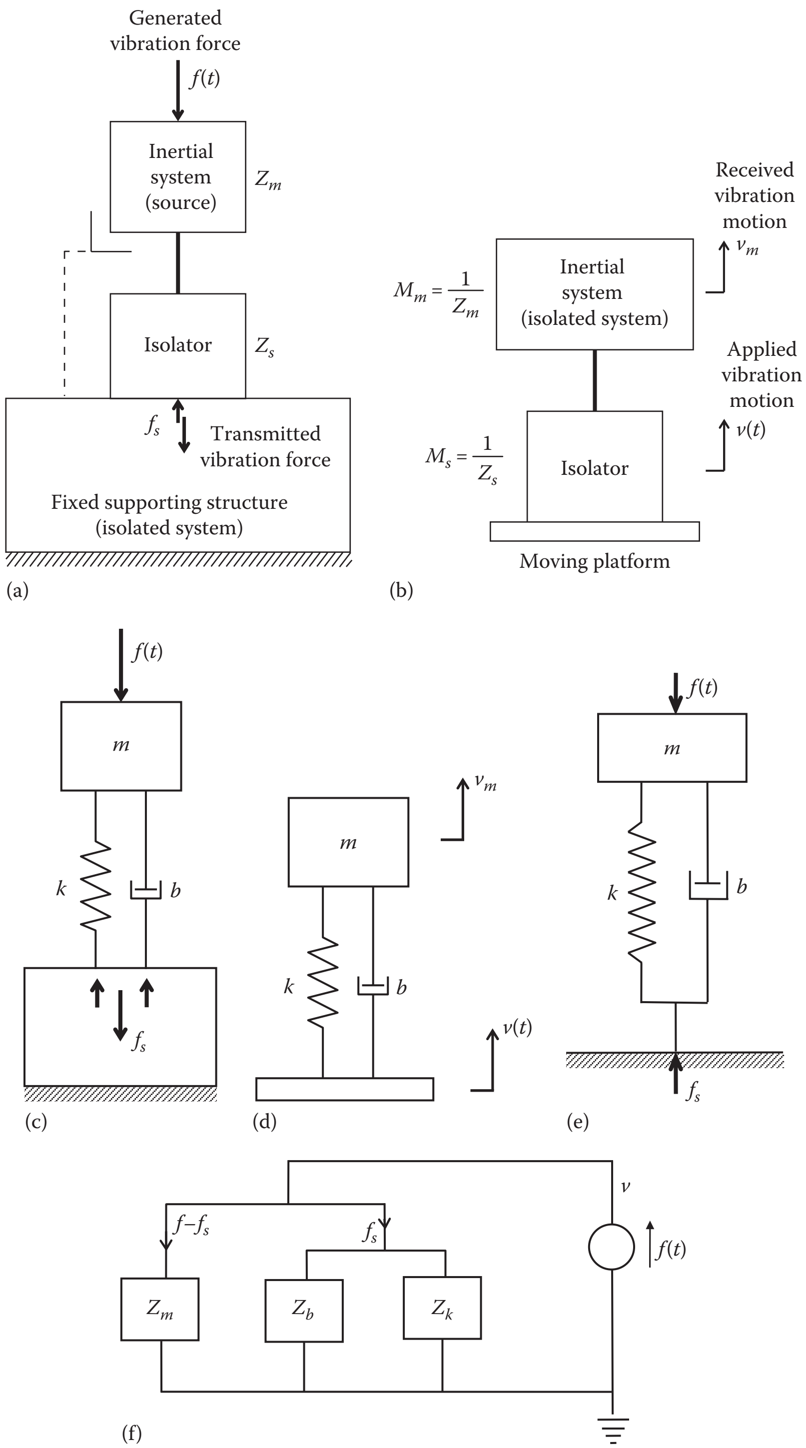

Figure 2.8
(a) Force isolation, (b) motion isolation, (c) force isolation example, (d) motion isolation example, (e) simplified model of a machine tool and its supporting structure, (f) and mechanical impedance circuit of the force isolation problem.

002x008a.eps 002x008b.eps 002x008c.eps

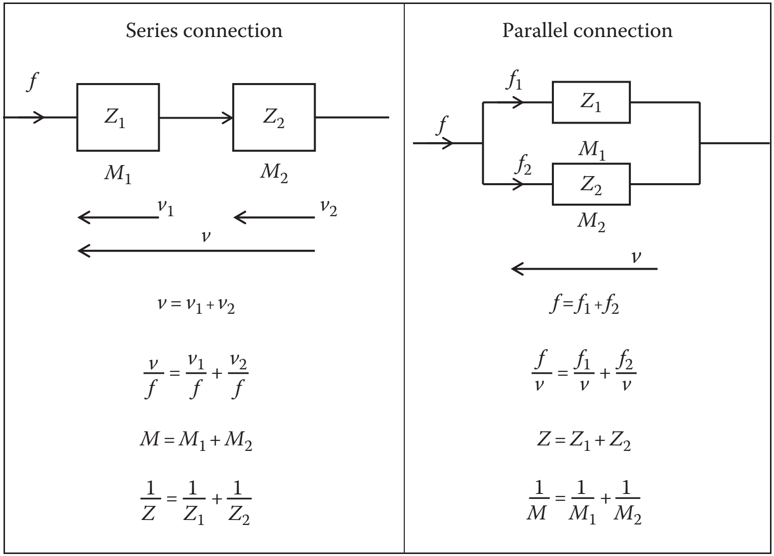

Figure 2.9
Interconnection laws for mechanical impedance and mobility.

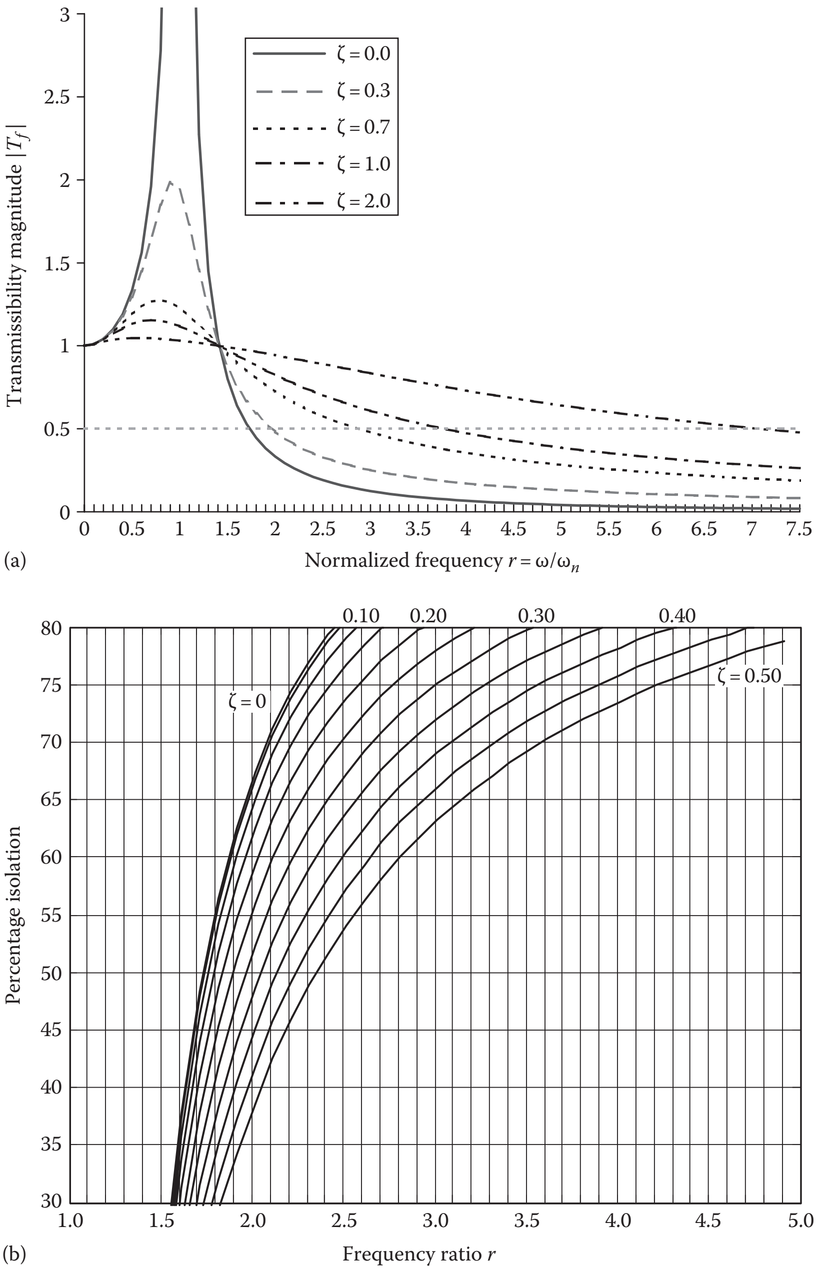

Figure 2.10
(a) Transmissibility curves for a simple oscillator model. (b) Curves of vibration isolation.

002x010a.eps 002x010b.eps

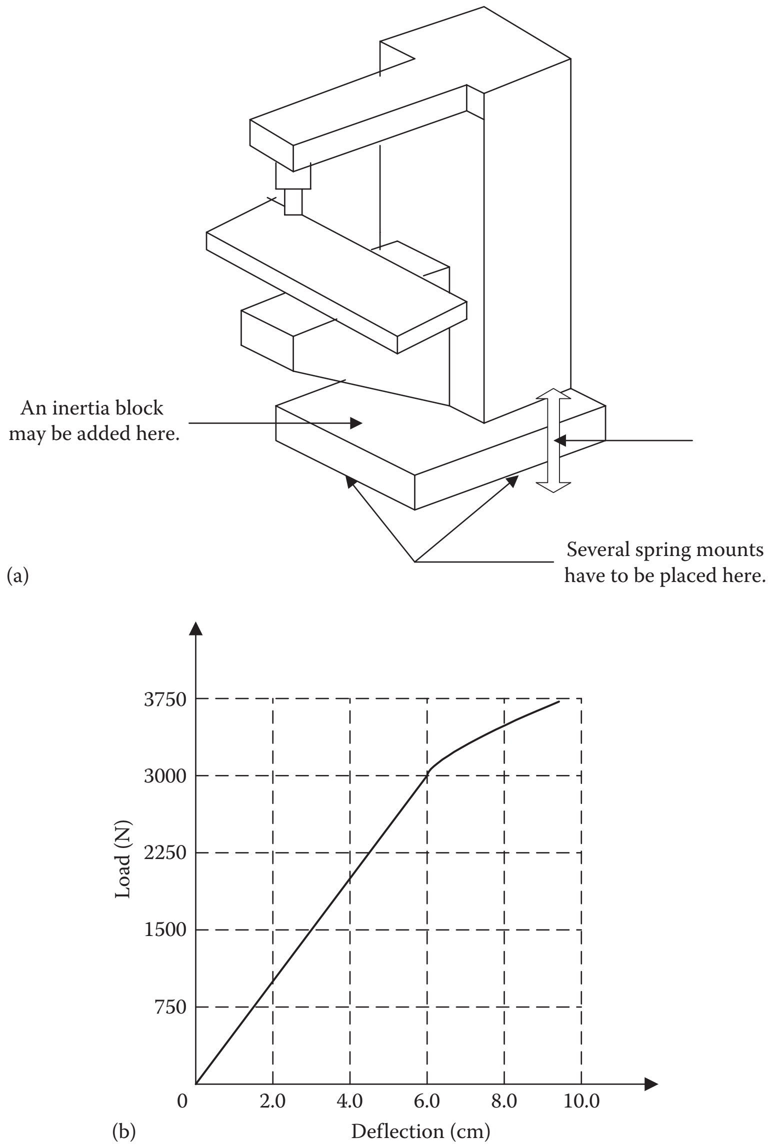

Figure 2.11
(a) A machine tool. (b) Load–deflection characteristic of a spring mount.

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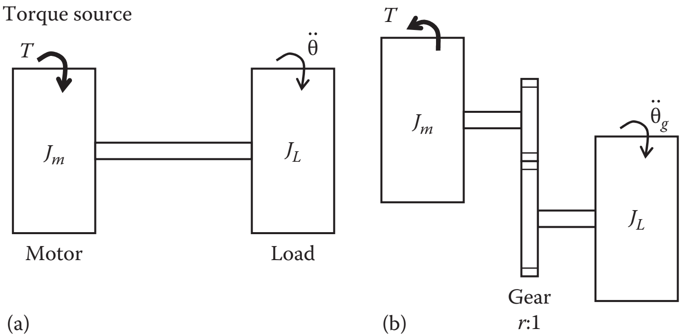

Figure 2.12
An inertial load driven by a motor: (a) without a gear transmission and (b) with a gear transmission.

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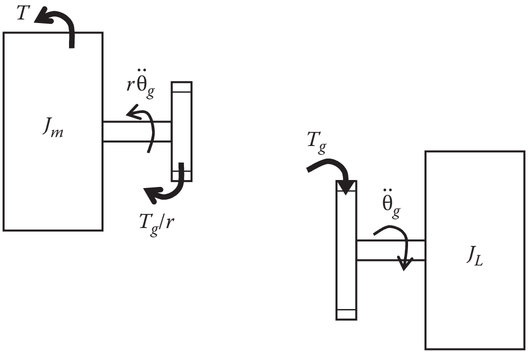

Figure 2.13
Free-body diagram.

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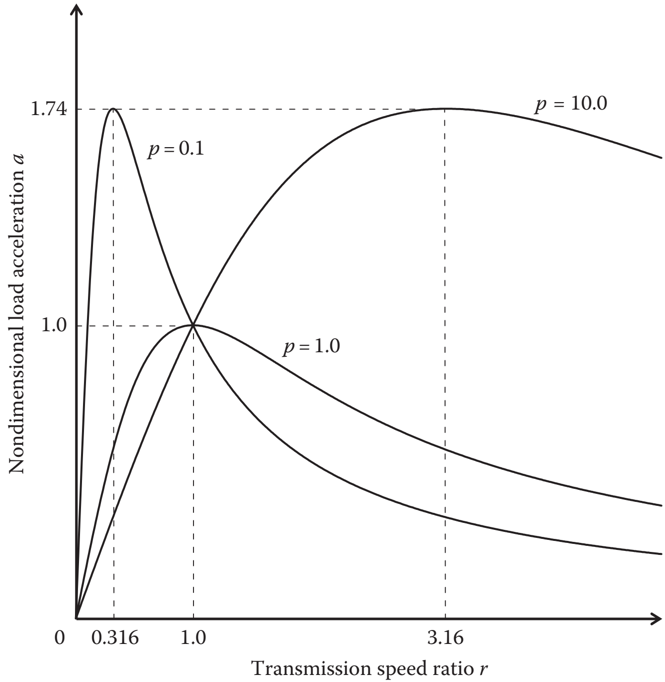

Figure 2.14
Normalized acceleration versus speed ratio.

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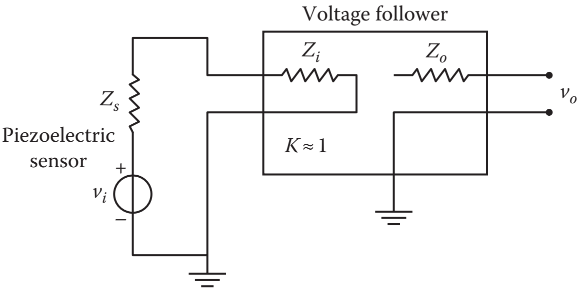

Figure P2.3
System with a piezoelectric sensor.

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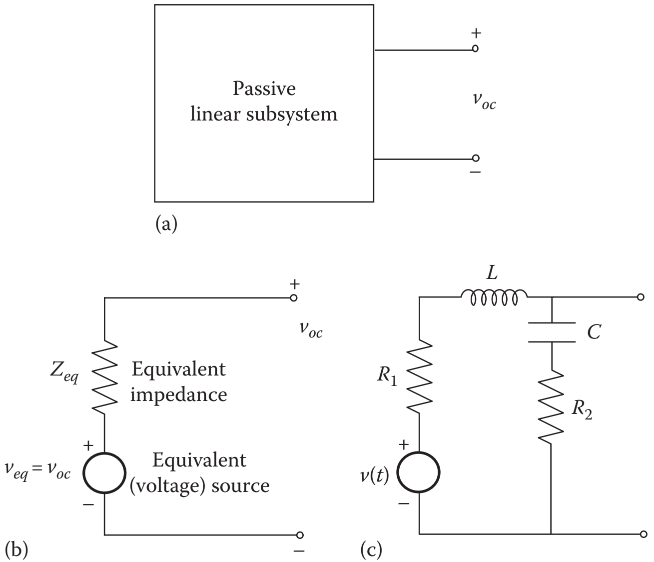

Figure P2.4
Illustration of Thevenin's theorem. (a) Unknown linear subsystem. (b) Equivalent representation. (c) Example.

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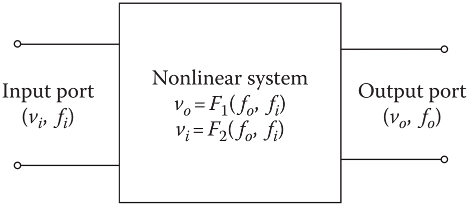

Figure P2.8
Impedance characteristics of a nonlinear system.

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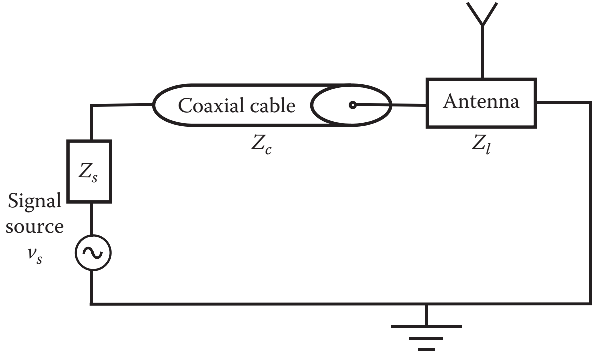

Figure P2.9
Impedance matching of transmission cable and antenna.

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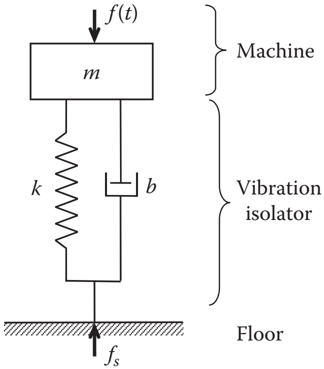

Figure P2.10
Model of a machine mounted on a vibration isolator.

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