Precalculus Functions and Graphs 13th edition Swokowski Solutions Manual

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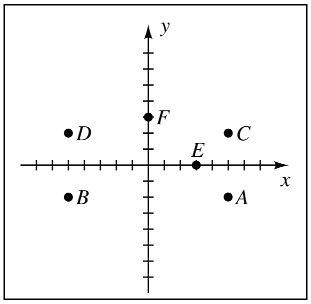
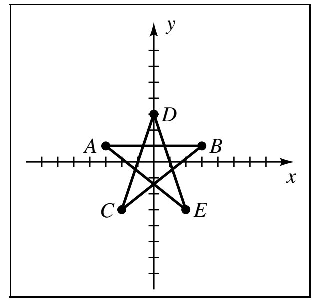
Chapter 2: Functions and Graphs

2.1 Exercises

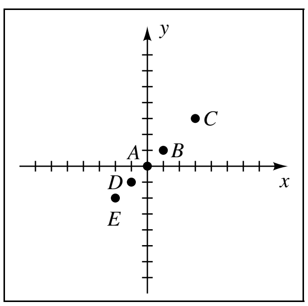
1 The points $A(5,-2)$, $B(-5,-2)$, $C(5,2)$, $D(-5,2)$, $E(3,0)$, and $F(0,3)$ are plotted in the figure.



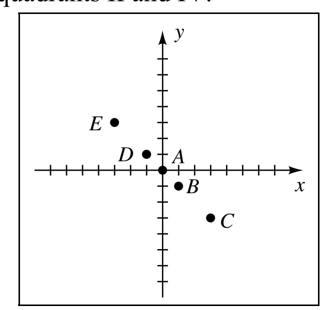
2 The points $A(-3,1)$, $B(3,1)$, $C(-2,-3)$, $D(0,3)$, and $E(2,-3)$, as well as $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EA}$, are plotted in the figure.



3 The points $A(0,0)$, $B(1,1)$, $C(3,3)$, $D(-1,-1)$, and $E(-2,-2)$ are plotted in the figure. The set of all points of the form (a,a) is the line bisecting quadrants I and III.



4 The points $A(0,0)$, $B(1,-1)$, $C(3,-3)$, $D(-1,1)$, and $E(-3,3)$ are plotted in the figure. The set of all points of the form $(a,-a)$ is the line bisecting quadrants II and IV.



5 The points are $A(3,3)$, $B(-3,3)$, $C(-3,-3)$, $D(3,-3)$, $E(1,0)$, and $F(0,3)$.

6 The points are $A(0,4)$, $B(-4,0)$, $C(0,-4)$, $D(4,0)$, $E(2,2)$, and $F(-2,-2)$.

7 **(a)** $x=-2$ is the line parallel to the y-axis that intersects the x-axis at $(-2,0)$.

(b) $y=5$ is the line parallel to the x-axis that intersects the y-axis at $(0,5)$.

(c) $x \geq 0$ $\{x$ is zero or positive$\}$ is the set of all points to the right of and on the y-axis.

(d) $xy > 0$ $\{x$ and y have the same sign, that is, either both are positive or both are negative$\}$ is the set of all points in quadrants I and III.

(e) $y < 0$ $\{y$ is negative$\}$ is the set of all points below the x-axis.

(f) $x=0$ is the set of all points on the y-axis.

8 **(a)** $y = -2$ is the line parallel to the x-axis that intersects the y-axis at $(0, -2)$.

(b) $x = 4$ is the line parallel to the y-axis that intersects the x-axis at $(4, 0)$.

(c) $x/y < 0$ is the set of all points in quadrants II and IV $\{x$ and y have opposite signs$\}$.

(d) $xy = 0$ is the set of all points on the x-axis or y-axis.

(e) $y > 1$ is the set of all points above the line parallel to the x-axis which intersects the y-axis at $(0, 1)$.

(f) $y = 0$ is the set of all points on the x-axis.

9 **(a)** $A(4, -3), B(6, 2) \quad \Rightarrow \quad d(A, B) = \sqrt{(6-4)^2 + [2-(-3)]^2} = \sqrt{4+25} = \sqrt{29}$

(b) $A(4, -3), B(6, 2) \quad \Rightarrow \quad M_{AB} = \left(\dfrac{4+6}{2}, \dfrac{-3+2}{2}\right) = \left(5, -\dfrac{1}{2}\right)$

10 **(a)** $A(-2, -5), B(4, 6) \quad \Rightarrow \quad d(A, B) = \sqrt{[4-(-2)]^2 + [6-(-5)]^2} = \sqrt{36+121} = \sqrt{157}$

(b) $A(-2, -5), B(4, 6) \quad \Rightarrow \quad M_{AB} = \left(\dfrac{-2+4}{2}, \dfrac{-5+6}{2}\right) = \left(1, \dfrac{1}{2}\right)$

11 **(a)** $A(-7, 0), B(-2, -4) \quad \Rightarrow \quad d(A, B) = \sqrt{[-2-(-7)]^2 + (-4-0)^2} = \sqrt{25+16} = \sqrt{41}$

(b) $A(-7, 0), B(-2, -4) \quad \Rightarrow \quad M_{AB} = \left(\dfrac{-7+(-2)}{2}, \dfrac{0+(-4)}{2}\right) = \left(-\dfrac{9}{2}, -2\right)$

12 **(a)** $A(5, 2), B(5, -2) \quad \Rightarrow \quad d(A, B) = \sqrt{(5-5)^2 + (-2-2)^2} = \sqrt{0+16} = 4$

(b) $A(5, 2), B(5, -2) \quad \Rightarrow \quad M_{AB} = \left(\dfrac{5+5}{2}, \dfrac{2+(-2)}{2}\right) = (5, 0)$

13 **(a)** $A(7, -3), B(3, -3) \quad \Rightarrow \quad d(A, B) = \sqrt{(3-7)^2 + [-3-(-3)]^2} = \sqrt{16+0} = 4$

(b) $A(7, -3), B(3, -3) \quad \Rightarrow \quad M_{AB} = \left(\dfrac{7+3}{2}, \dfrac{-3+(-3)}{2}\right) = (5, -3)$

14 **(a)** $A(-4, 7), B(0, -8) \quad \Rightarrow \quad d(A, B) = \sqrt{[0-(-4)]^2 + (-8-7)^2} = \sqrt{16+225} = \sqrt{241}$

(b) $A(-4, 7), B(0, -8) \quad \Rightarrow \quad M_{AB} = \left(\dfrac{-4+0}{2}, \dfrac{7+(-8)}{2}\right) = \left(-2, -\dfrac{1}{2}\right)$

15 The points are $A(6, 3)$, $B(1, -2)$, and $C(-3, 2)$. We need to show that the sides satisfy the Pythagorean theorem. Finding the distances, we have $d(A, B) = \sqrt{50}$, $d(B, C) = \sqrt{32}$, and $d(A, C) = \sqrt{82}$. Since $d(A, C)$ is the largest of the three values, it must be the hypotenuse, hence, we need to check if $d(A, C)^2 = d(A, B)^2 + d(B, C)^2$. Since $\left(\sqrt{82}\right)^2 = \left(\sqrt{50}\right)^2 + \left(\sqrt{32}\right)^2$, we know that $\triangle ABC$ is a right triangle. The area of a triangle is given by $A = \frac{1}{2}(\text{base})(\text{height})$. We can use $d(B, C)$ for the base and $d(A, B)$ for the height.
Hence, area $= \frac{1}{2}bh = \frac{1}{2}\left(\sqrt{32}\right)\left(\sqrt{50}\right) = \frac{1}{2}\left(4\sqrt{2}\right)\left(5\sqrt{2}\right) = \frac{1}{2}(20)(2) = 20$.

16 The points are $A(-6, 3)$, $B(3, -5)$, and $C(-1, 5)$.
Show that $d(A, B)^2 = d(A, C)^2 + d(B, C)^2$; that is, $\left(\sqrt{145}\right)^2 = \left(\sqrt{29}\right)^2 + \left(\sqrt{116}\right)^2$.
Area $= \frac{1}{2}bh = \frac{1}{2} \cdot d(A, C) \cdot d(B, C) = \frac{1}{2}\left(\sqrt{29}\right)\left(\sqrt{116}\right) = \frac{1}{2}\left(\sqrt{29}\right)\left(2\sqrt{29}\right) = 29$.

17 The points are $A(-4,2)$, $B(1,4)$, $C(3,-1)$, and $C(-2,-3)$. We need to show that all four sides are the same length. Checking, we find that $d(A,B)=d(B,C)=d(C,D)=d(D,A)=\sqrt{29}$. This guarantees that we have a rhombus {a parallelogram with 4 equal sides}. Thus, we also need to show that adjacent sides meet at right angles. This can be done by showing that two adjacent sides and a diagonal form a right triangle. Using $\triangle\, ABC$, we see that $d(A,C)=\sqrt{58}$ and hence $d(A,C)^2=d(A,B)^2+d(B,C)^2$. We conclude that $ABCD$ is a square.

18 The points are $A(-4,-1)$, $B(0,-2)$, $C(6,1)$, and $C(2,2)$.
Show that $d(A,D)=d(B,C)=\sqrt{45}$ and $d(A,B)=d(C,D)=\sqrt{17}$.

19 Let $B=(x,y)$. $A(-3,8) \;\Rightarrow\; M_{AB}=\left(\dfrac{-3+x}{2},\dfrac{8+y}{2}\right)$.
Since $M_{AB}=C(5,-10)$, we must have $\dfrac{-3+x}{2}=5$ and $\dfrac{8+y}{2}=-10 \;\Rightarrow$
$-3+x=2(5)$ and $8+y=2(-10) \;\Rightarrow\; x=13$ and $y=-28$. Thus, $B=(13,-28)$.

20 If Q is the midpoint of segment AB, then the midpoint of QB is the point that is three-fourths of the way from $A(5,-8)$ to $B(-6,2)$.
$Q=M_{AB}=\left(\dfrac{5+(-6)}{2},\dfrac{-8+2}{2}\right)=\left(-\dfrac{1}{2},-3\right)$. $\qquad M_{QB}=\left(\dfrac{(-1/2)+(-6)}{2},\dfrac{-3+2}{2}\right)=\left(-\dfrac{13}{4},-\dfrac{1}{2}\right)$.

21 The perpendicular bisector of AB is the line that passes through the midpoint of segment AB and intersects segment AB at a right angle. The points on the perpendicular bisector are all equidistant from A and B. Thus, we need to show that $d(A,C)=d(B,C)$, where $A=(-4,-3)$, $B=(6,1)$, and $C=(3,-6)$. Since each of these distances is $\sqrt{58}$, we conclude that C is on the perpendicular bisector of AB.

22 Show that $d(A,C)=d(B,C)=\sqrt{125}$, where $A=(-3,2)$, $B=(5,-4)$, and $C=(7,7)$.

23 The points are $A(-4,-3)$, $B(6,1)$, and $P(x,y)$. We must have $d(A,P)=d(B,P)$.
$\sqrt{(x+4)^2+(y+3)^2}=\sqrt{(x-6)^2+(y-1)^2} \;\Rightarrow$
$x^2+8x+16+y^2+6y+9=x^2-12x+36+y^2-2y+1$ {square both sides} $\Rightarrow$
$8x+6y+25=-12x-2y+37 \;\Rightarrow\; 20x+8y=12 \;\Rightarrow\; 5x+2y=3$

24 The points are $A(-3,2)$, $B(5,-4)$, and $P(x,y)$. We must have $d(A,P)=d(B,P)$.
$\sqrt{(x+3)^2+(y-2)^2}=\sqrt{(x-5)^2+(y+4)^2} \;\Rightarrow$
$x^2+6x+9+y^2-4y+4=x^2-10x+25+y^2+8y+16$ {square both sides} $\Rightarrow$
$6x-4y+13=-10x+8y+41 \;\Rightarrow\; 16x-12y=28 \;\Rightarrow\; 4x-3y=7$

25 Let $O(0,0)$ represent the origin. Applying the distance formula with O and $P(x,y)$, we have
$d(O,P)=5 \;\Rightarrow\; \sqrt{(x-0)^2+(y-0)^2}=5 \;\Rightarrow\; \sqrt{x^2+y^2}=5$.
This formula represents a circle of radius 5 with center at the origin.

26 $d(C,P)=r \;\Rightarrow\; \sqrt{(x-h)^2+(y-k)^2}=r$. This formula represents a circle of radius r and center (h,k).

27 Let $Q(0,y)$ be an arbitrary point on the y-axis. Applying the distance formula with Q and $P(5,3)$, we have
$6=d(P,Q) \;\Rightarrow\; 6=\sqrt{(0-5)^2+(y-3)^2} \;\Rightarrow\; 36=25+y^2-6y+9 \;\Rightarrow$
$y^2-6y-2=0 \;\Rightarrow\; y=3\pm\sqrt{11}$ {quadratic formula}. The points are $\left(0,3+\sqrt{11}\right)$ and $\left(0,3-\sqrt{11}\right)$.

28 Let $Q(0, y)$ be an arbitrary point on the y-axis. Applying the distance formula with Q and $P(12, 4)$, we have

$13 = d(P, Q) \Rightarrow 13 = \sqrt{(0-12)^2 + (y-4)^2} \Rightarrow 169 = 144 + y^2 - 8y + 16 \Rightarrow$

$y^2 - 8y - 9 = 0 \Rightarrow (y-9)(y+1) = 0 \Rightarrow y = 9, -1$. The points are $(0, 9)$ and $(0, -1)$.

29 Let $Q(x, 0)$ be an arbitrary point on the x-axis. Applying the distance formula with Q and $P(-2, 4)$, we have

$5 = d(P, Q) \Rightarrow 5 \Rightarrow \sqrt{(x+2)^2 + (0-4)^2} \Rightarrow 25 = x^2 + 4x + 4 + 16 \Rightarrow$

$x^2 + 4x - 5 = 0 \Rightarrow (x+5)(x-1) = 0 \Rightarrow x = -5, 1$. The points are $(1, 0)$ and $(-5, 0)$.

30 Let $Q(x, 0)$ be an arbitrary point on the x-axis. Applying the distance formula with Q and $P(-1, 6)$, we have

$7 = d(P, Q) \Rightarrow 7 \Rightarrow \sqrt{(x+1)^2 + (0-6)^2} \Rightarrow 49 = x^2 + 2x + 1 + 36 \Rightarrow x^2 + 2x - 12 = 0 \Rightarrow$

$x = -1 \pm \sqrt{13}$ {quadratic formula}. The points are $\left(-1 - \sqrt{13}, 0\right)$ and $\left(-1 + \sqrt{13}, 0\right)$.

31 $d = 5$ with points $(2a, a)$ and $(1, 3) \Rightarrow 5 = \sqrt{(2a-1)^2 + (a-3)^2} \Rightarrow$

$25 = 4a^2 - 4a + 1 + a^2 - 6a + 9 \Rightarrow 5a^2 - 10a - 15 = 0 \Rightarrow a^2 - 2a - 3 = 0 \Rightarrow$

$(a-3)(a+1) = 0 \Rightarrow a = 3, -1$.

Since the y-coordinate is negative in the third quadrant, $a = -1$, and $(2a, a) = (-2, -1)$.

32 $d = 3$ with points (a, a) and $(-2, 1) \Rightarrow 3 = \sqrt{(a+2)^2 + (a-1)^2} \Rightarrow$

$9 = a^2 + 4a + 4 + a^2 - 2a + 1 \Rightarrow 0 = 2a^2 + 2a - 4 \Rightarrow a^2 + a - 2 = 0 \Rightarrow (a+2)(a-1) = 0 \Rightarrow$

$a = -2, 1$. The points are $(-2, -2)$ and $(1, 1)$.

33 With $P(a, 3)$ and $Q(5, 2a)$, we get $d(P, Q) > \sqrt{26} \Rightarrow \sqrt{(5-a)^2 + (2a-3)^2} > \sqrt{26} \Rightarrow$

$25 - 10a + a^2 + 4a^2 - 12a + 9 > 26 \Rightarrow 5a^2 - 22a + 8 > 0 \Rightarrow (5a-2)(a-4) > 0$.

Interval	$(-\infty, \frac{2}{5})$	$(\frac{2}{5}, 4)$	$(4, \infty)$
Sign of $5a-2$	$-$	$+$	$+$
Sign of $a-4$	$-$	$-$	$+$
Resulting sign	$+$	$-$	$+$

From the sign chart, we see that $a < \frac{2}{5}$ or $a > 4$ will assure us that $d(P, Q) > \sqrt{26}$.

34 With $P(a, 4)$ and $Q(2, a)$, we get $d(P, Q) > \sqrt{10} \Rightarrow \sqrt{(2-a)^2 + (a-4)^2} > \sqrt{10} \Rightarrow$

$4 - 4a + a^2 + a^2 - 8a + 16 > 10 \Rightarrow 2a^2 - 12a + 10 > 0 \Rightarrow a^2 - 6a + 5 > 0 \Rightarrow (a-1)(a-5) > 0$.

Interval	$(-\infty, 1)$	$(1, 5)$	$(5, \infty)$
Sign of $a-1$	$-$	$+$	$+$
Sign of $a-5$	$-$	$-$	$+$
Resulting sign	$+$	$-$	$+$

From the sign chart, we see that $a < 1$ or $a > 5$ will assure us that $d(P, Q) > \sqrt{10}$.

35 Let $A(x, 0)$ be an arbitrary point on the x-axis. $Q(0, 2)$, $P(x, y)$ • $d(A, P) = d(Q, P) \Rightarrow$

$\sqrt{(x-x)^2 + (y-0)^2} = \sqrt{(x-0)^2 + (y-2)^2} \Rightarrow y^2 = x^2 + y^2 - 4y + 4 \Rightarrow x^2 = 4y - 4$

36 Let $A(0, y)$ be an arbitrary point on the y-axis. $Q(6, 0)$, $P(x, y)$ • $d(A, P) = d(Q, P)$ $\Rightarrow$

$$\sqrt{(x-0)^2 + (y-y)^2} = \sqrt{(x-6)^2 + (y-0)^2} \;\Rightarrow\; x^2 = x^2 - 12x + 36 + y^2 \;\Rightarrow\; y^2 = 12x - 36$$

37 $A(-2, 0)$, $B(2, 0)$, $P(x, y)$ • $d(A, P) + d(B, P) = 5$ $\Rightarrow$ $\sqrt{(x+2)^2 + y^2} + \sqrt{(x-2)^2 + y^2} = 5$ $\Rightarrow$

$$\sqrt{x^2 + 4x + 4 + y^2} = 5 - \sqrt{x^2 - 4x + 4 + y^2} \;\Rightarrow$$
$$x^2 + 4x + 4 + y^2 = 25 - 10\sqrt{x^2 - 4x + 4 + y^2} + x^2 - 4x + 4 + y^2 \;\Rightarrow$$
$$10\sqrt{x^2 - 4x + 4 + y^2} = 25 - 8x \;\Rightarrow\; 100(x^2 - 4x + 4 + y^2) = 625 - 400x + 64x^2 \;\Rightarrow$$
$$100x^2 - 400x + 400 + 100y^2 = 625 - 400x + 64x^2 \;\Rightarrow\; 36x^2 + 100y^2 = 225$$

38 $A(0, -1)$, $B(0, 1)$, $P(x, y)$ • $d(A, P) + d(B, P) = 3$ $\Rightarrow$ $\sqrt{x^2 + (y+1)^2} + \sqrt{x^2 + (y-1)^2} = 3$ $\Rightarrow$

$$\sqrt{x^2 + y^2 + 2y + 1} = 3 - \sqrt{x^2 + y^2 - 2y + 1} \;\Rightarrow$$
$$x^2 + y^2 + 2y + 1 = 9 - 6\sqrt{x^2 + y^2 - 2y + 1} + x^2 + y^2 - 2y + 1 \;\Rightarrow$$
$$6\sqrt{x^2 + y^2 - 2y + 1} = 9 - 4y \;\Rightarrow\; 36(x^2 + y^2 - 2y + 1) = 81 - 72y + 16y^2 \;\Rightarrow$$
$$36x^2 + 36y^2 - 72y + 36 = 81 - 72y + 16y^2 \;\Rightarrow\; 36x^2 + 20y^2 = 45$$

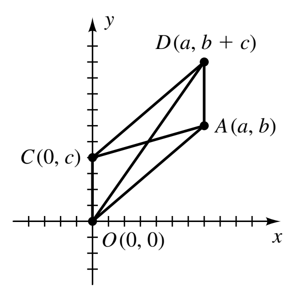
39 Let M be the midpoint of the hypotenuse. Then $M = \left(\frac{1}{2}a, \frac{1}{2}b\right)$. Since $A = (a, 0)$, we have

$$d(A, M) = \sqrt{\left(a - \tfrac{1}{2}a\right)^2 + \left(0 - \tfrac{1}{2}b\right)^2} = \sqrt{\left(\tfrac{1}{2}a\right)^2 + \left(-\tfrac{1}{2}b\right)^2} = \sqrt{\tfrac{1}{4}a^2 + \tfrac{1}{4}b^2} = \sqrt{\tfrac{1}{4}(a^2 + b^2)} = \tfrac{1}{2}\sqrt{a^2 + b^2}$$

In a similar fashion, show that $d(B, M) = d(O, M) = \frac{1}{2}\sqrt{a^2 + b^2}$.

40 Let $D(a, b + c)$ be the fourth vertex as shown in the figure. We need to show that the midpoint of OD is the same as the midpoint of AC.

$$M_{OD} = \left(\frac{0 + a}{2}, \frac{0 + b + c}{2}\right) = \left(\frac{a}{2}, \frac{b + c}{2}\right) \text{ and}$$
$$M_{CA} = \left(\frac{0 + a}{2}, \frac{c + b}{2}\right) = \left(\frac{a}{2}, \frac{b + c}{2}\right).$$



41 Plot the points $A(-5, -3.5)$, $B(-2, 2)$, $C(1, 0.5)$, $D(4, 1)$, and $E(7, 2.5)$.

$[-10, 10]$ by $[-10, 10]$

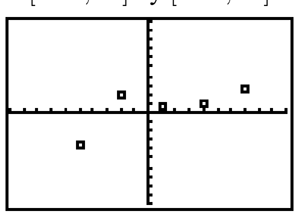


Figure 41

$[-12, 12]$ by $[-8, 8]$

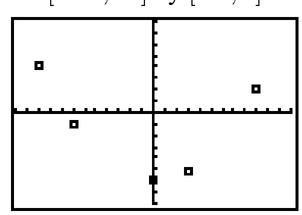


Figure 42

42 Plot the points $A(-10, 4)$, $B(-7, -1.1)$, $C(0, -6)$, $D(3, -5.1)$, and $E(9, 2.1)$.

$\boxed{43}$ **(a)** Plot $(1984, 87{,}073)$, $(1993, 98{,}736)$, $(2003, 113{,}126)$, and $(2009, 119{,}296)$.

(b) The number of U.S. households with a computer is increasing each year.

$[1982, 2012]$ by $[80\text{E}3, 120\text{E}3, 10\text{E}3]\{80\text{E}3 = 80 \times 10^3\}$

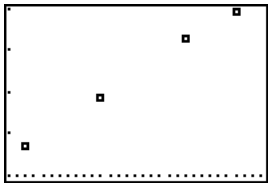


Figure 43

$[1895, 2005, 10]$ by $[0, 3000, 1000]$

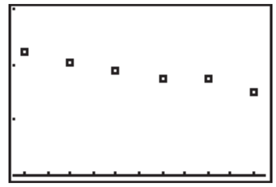


Figure 44

$\boxed{44}$ **(a)** Plot $(1900, 2226)$, $(1920, 2042)$, $(1940, 1878)$, $(1960, 1763)$, $(1980, 1745)$, and $(2000, 1480)$.

(b) Find the midpoint of $(1920, 2042)$ and $(1940, 1878)$. $\left(\dfrac{1920 + 1940}{2}, \dfrac{2042 + 1878}{2}\right) = (1930, 1960)$.

The midpoint formula predicts 1960 daily newspapers published in the year 1930 compared to the actual value of 1942 daily newspapers.

2.2 Exercises

$\boxed{1}$ As in Example 1, we expect the graph of $y = 2x - 3$ to be a line.

Creating a table of values similar to those in the text, we have:

x	-2	-1	0	1	2
y	-7	-5	-3	-1	1

By plotting these points and connecting them, we obtain the figure.

To find the x-intercept, let $y = 0$ in $y = 2x - 3$, and solve for x to get 1.5.

To find the y-intercept, let $x = 0$ in $y = 2x - 3$, and solve for y to get -3.

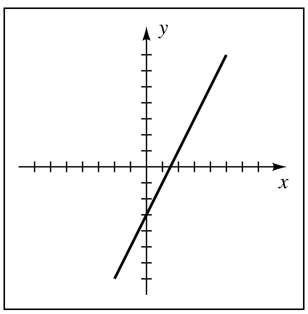


Figure 1

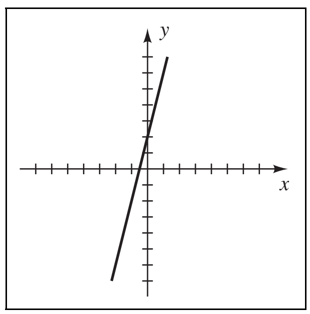


Figure 2

$\boxed{2}$ $y = 4x + 2$ • $y = 0$ gives the x-intercept $-\frac{1}{2}$; $x = 0$ gives the y-intercept 2

3 $y = -x + 2$ • x-intercept: $y = 0 \Rightarrow 0 = -x + 2 \Rightarrow x = 2$
y-intercept: $x = 0 \Rightarrow y = -0 + 2 = 2$

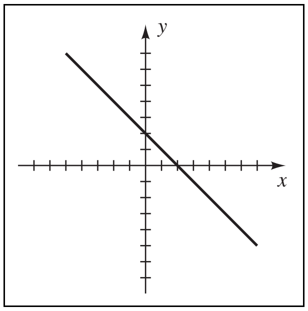


Figure 3

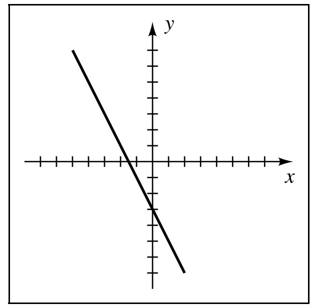


Figure 4

4 $y = -2x - 3$ • x-intercept: $y = 0 \Rightarrow x = -1.5$; y-intercept: $x = 0 \Rightarrow y = -3$

5 $y = -2x^2$ • Multiplying the y-values of $y = x^2$ by -2 gives us all negative y-values for $y = -2x^2$. The vertex of the parabola is at $(0, 0)$, so its x-intercept is 0 and its y-intercept is 0.

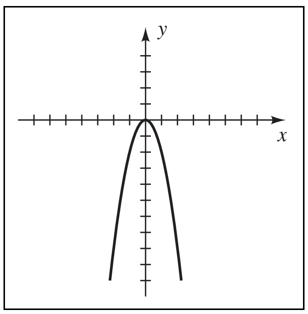


Figure 5

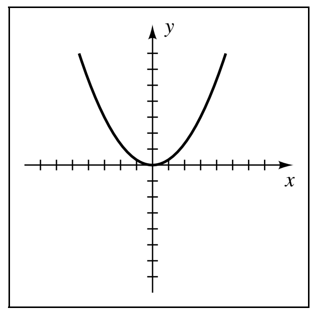


Figure 6

6 $y = \frac{1}{3}x^2$ • x-intercept 0; y-intercept 0

7 $y = 2x^2 - 1$ • x-intercepts: $y = 0 \Rightarrow 0 = 2x^2 - 1 \Rightarrow 1 = 2x^2 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\sqrt{\frac{1}{2}}$, which can be written as $\pm\frac{1}{\sqrt{2}}$ or $\pm\frac{\sqrt{2}}{2}$ or $\pm\frac{1}{2}\sqrt{2}$; y-intercept: $x = 0 \Rightarrow y = -1$

Since we can substitute $-x$ for x in the equation and obtain an equivalent equation, we know the graph is symmetric with respect to the y-axis. We will make use of this fact when constructing our table. As in Example 2, we obtain a parabola.

x	± 2	$\pm\frac{3}{2}$	± 1	$\pm\frac{1}{2}$	0
y	7	$\frac{7}{2}$	1	$-\frac{1}{2}$	-1

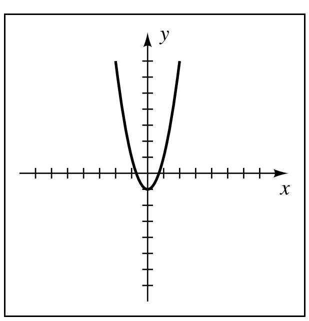


Figure 7

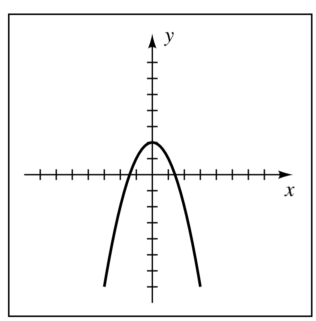


Figure 8

8 $y = -x^2 + 2$ • x-intercepts: $y = 0 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$; y-intercept 2

9 $x = \frac{1}{4}y^2$ • This graph is similar to the one in Example 5—multiplying by $\frac{1}{4}$ narrows the parabola $x = y^2$. The x-intercept is 0 and the y-intercept is 0.

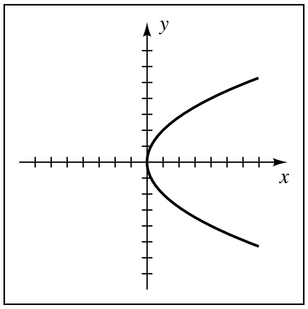


Figure 9

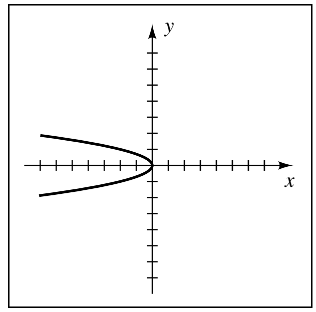


Figure 10

10 $x = -2y^2$ • x-intercept 0; y-intercept 0

11 $x = -y^2 + 5$ • x-intercept: $y = 0 \Rightarrow x = 5$;
y-intercepts: $x = 0 \Rightarrow 0 = -y^2 + 5 \Rightarrow y^2 = 5 \Rightarrow y = \pm\sqrt{5}$

Since we can substitute $-y$ for y in the equation and obtain an equivalent equation, we know the graph is symmetric with respect to the x-axis. We will make use of this fact when constructing our table. As in Example 5, we obtain a parabola.

x	-11	-4	1	4	5
y	± 4	± 3	± 2	± 1	0

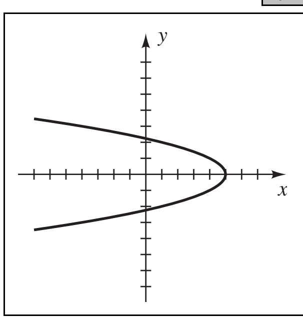


Figure 11

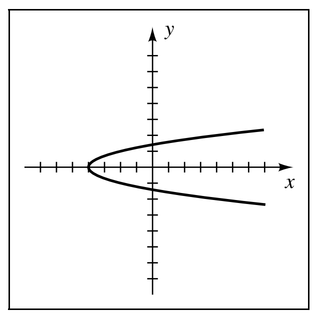


Figure 12

12 $x = 2y^2 - 4$ • x-intercept -4; y-intercepts: $x = 0 \Rightarrow y^2 = 4 \Rightarrow y = \pm\sqrt{2}$

13 $y = -\frac{1}{4}x^3$ • The graph is similar to the graph of $y = \frac{1}{4}x^3$ in Example 6. The negative has the effect of "flipping" the graph about the x-axis. The x-intercept is 0 and the y-intercept is 0.

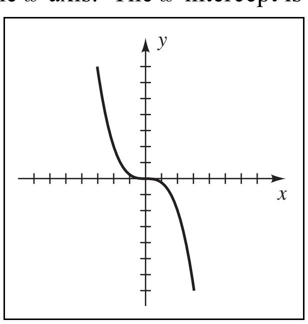


Figure 13

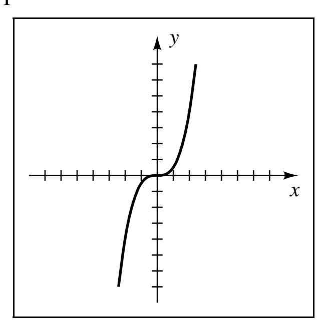


Figure 14

14 $y = \frac{1}{2}x^3$ • x-intercept 0; y-intercept 0

15 $y = x^3 - 8$ • x-intercept: $y = 0 \Rightarrow 0 = x^3 - 8 \Rightarrow x^3 = 8 \Rightarrow x = \sqrt[3]{8} = 2$
y-intercept: $x = 0 \Rightarrow y = -8$
The effect of the -8 is to shift the graph of $y = x^3$ down 8 units.

x	-2	-1	0	1	2
y	-16	-9	-8	-7	0

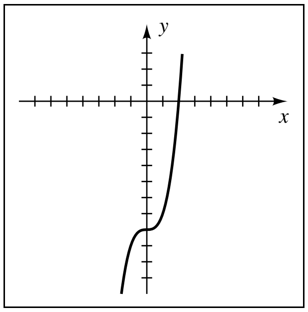


Figure 15

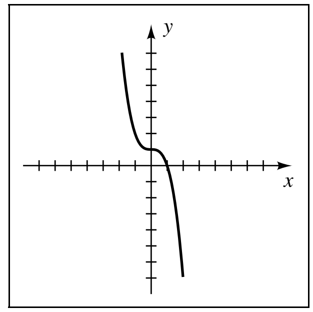


Figure 16

16 $y = -x^3 + 1$ • x-intercept: $y = 0 \Rightarrow x^3 = 1 \Rightarrow x = \sqrt[3]{1} = 1$; y-intercept 1

17 $y = \sqrt{x}$ • x-intercept 0; y-intercept 0. This is the top half of the parabola $x = y^2$. An equation of the bottom half is $y = -\sqrt{x}$.

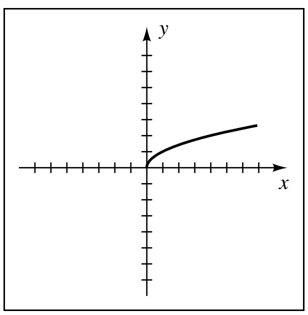


Figure 17

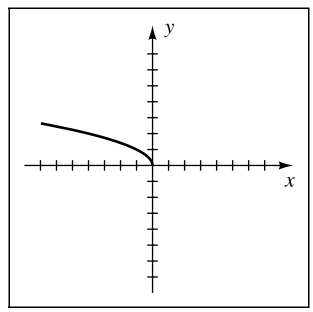


Figure 18

18 $y = \sqrt{-x}$ • The negative has the effect of "flipping" the graph of $y = \sqrt{x}$ about the y-axis. The x-intercept is 0 and the y-intercept is 0.

19 $y = \sqrt{x} - 4$ • x-intercept: $y = 0 \Rightarrow 0 = \sqrt{x} - 4 \Rightarrow 4 = \sqrt{x} \Rightarrow x = 16$ (not on graph)
y-intercept: $x = 0 \Rightarrow y = -4$

x	0	1	4	9	16
y	-4	-3	-2	-1	0

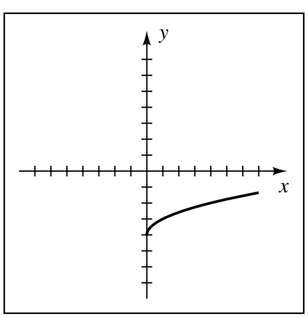


Figure 19

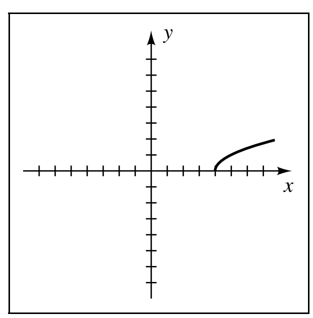


Figure 20

20 $y = \sqrt{x - 4}$ • x-intercept 4; y-intercept: None

$\boxed{21}$ You may be able to do this exercise mentally. For example (using Exercise 1 with $y = 2x - 3$), we see that substituting $-x$ for x gives us $y = -2x - 3$; substituting $-y$ for y gives us $-y = 2x - 3$ or, equivalently, $y = -2x + 3$; and substituting $-x$ for x and $-y$ for y gives us $-y = -2x - 3$ or, equivalently, $y = 2x + 3$. None of the resulting equations are equivalent to the original equation, so there is no symmetry with respect to the y-axis, x-axis, or the origin.

(a) The graphs of the equations in Exercises 5 and 7 are symmetric with respect to the y-axis.

(b) The graphs of the equations in Exercises 9 and 11 are symmetric with respect to the x-axis.

(c) The graph of the equation in Exercise 13 is symmetric with respect to the origin.

$\boxed{22}$ **(a)** $6, 8$ **(b)** $10, 12$ **(c)** 14

$\boxed{23}$ **(a)** As $x \to -1^-$, $f(x) \to \underline{2}$ **(b)** As $x \to 2^+$, $f(x) \to \underline{1}$ **(c)** As $x \to 3$, $f(x) \to \underline{4}$

(d) As $x \to \infty$, $f(x) \to \underline{\infty}$ **(e)** As $x \to -\infty$, $f(x) \to \underline{-\infty}$

$\boxed{24}$ **(a)** As $x \to 2^-$, $f(x) \to \underline{3}$ **(b)** As $x \to -1^+$, $f(x) \to \underline{-3}$ **(c)** As $x \to 0$, $f(x) \to \underline{-1}$

(d) As $x \to \infty$, $f(x) \to \underline{-\infty}$ **(e)** As $x \to -\infty$, $f(x) \to \underline{\infty}$

$\boxed{25}$ $x^2 + y^2 = 11 \quad \Leftrightarrow \quad (x-0)^2 + (y-0)^2 = \left(\sqrt{11}\right)^2$ is a circle of radius $\sqrt{11}$ with center at the origin.

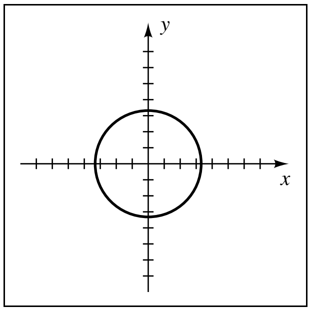


Figure 25

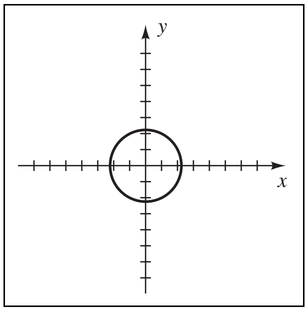


Figure 26

$\boxed{26}$ $x^2 + y^2 = 5 \quad \Leftrightarrow \quad (x-0)^2 + (y-0)^2 = \left(\sqrt{5}\right)^2$ is a circle of radius $\sqrt{5}$ with center at the origin.

27 $(x+3)^2+(y-2)^2=9$ is a circle of radius $r=\sqrt{9}=3$ with center $C(-3,2)$. To determine the center from the given equation, it may help to ask yourself "What values makes the expressions $(x+3)$ and $(y-2)$ equal to zero?" The answers are -3 and 2.

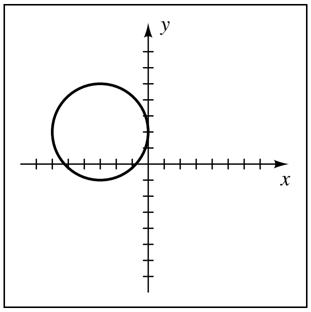


Figure 27

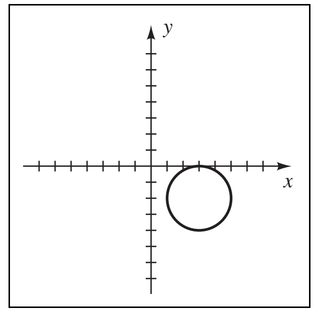


Figure 28

28 $(x-3)^2+(y+2)^2=4$ is a circle of radius $r=\sqrt{4}=2$ with center $C(3,-2)$.

29 $(x+3)^2+y^2=16$ is a circle of radius $r=\sqrt{16}=4$ with center $C(-3,0)$.

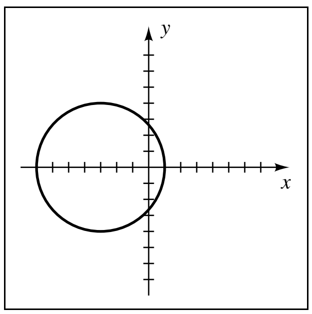


Figure 29

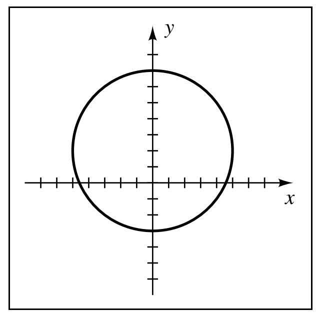


Figure 30

30 $x^2+(y-2)^2=25$ is a circle of radius $r=\sqrt{25}=5$ with center $C(0,2)$.

31 $4x^2+4y^2=1 \quad\Rightarrow\quad x^2+y^2=\frac{1}{4}$ is a circle of radius $r=\sqrt{\frac{1}{4}}=\frac{1}{2}$ with center $C(0,0)$.

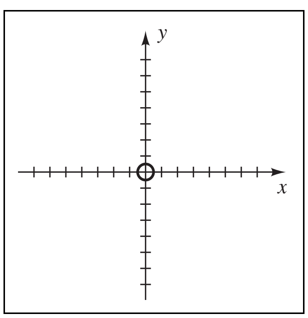


Figure 31

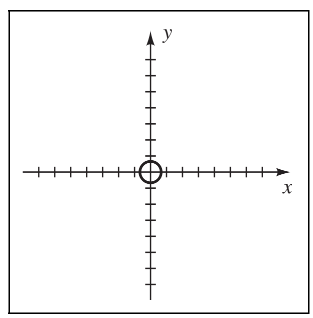


Figure 32

32 $9x^2+9y^2=4 \quad\Rightarrow\quad x^2+y^2=\frac{4}{9}$ is a circle of radius $r=\sqrt{\frac{4}{9}}=\frac{2}{3}$ with center $C(0,0)$.

33 As in Example 9, $y = -\sqrt{16 - x^2}$ is the lower half of the circle $x^2 + y^2 = 16$.

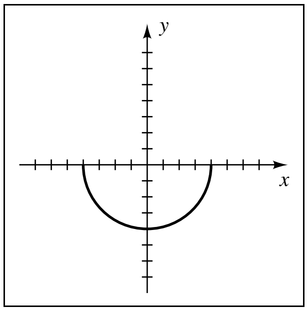


Figure 33

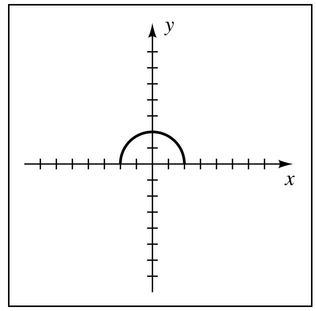


Figure 34

34 $y = \sqrt{4 - x^2}$ is the upper half of the circle $x^2 + y^2 = 4$.

35 $x = \sqrt{9 - y^2}$ is the right half of the circle $x^2 + y^2 = 9$.

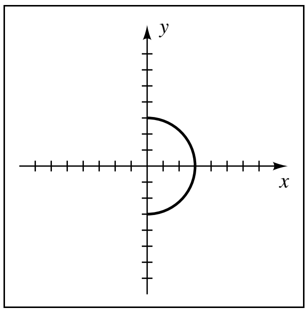


Figure 35

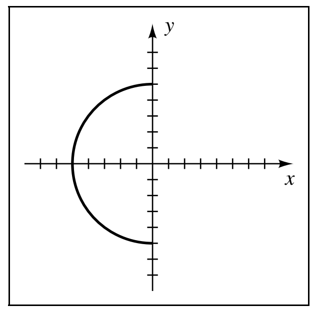


Figure 36

36 $x = -\sqrt{25 - y^2}$ is the left half of the circle $x^2 + y^2 = 25$.

37 Center $C(2, -3)$, radius 5 • $(x-2)^2 + [y-(-3)]^2 = 5^2 \quad \Leftrightarrow \quad (x-2)^2 + (y+3)^2 = 25$

38 Center $C(-5, 1)$, radius 3 • $[x-(-5)]^2 + (y-1)^2 = 3^2 \quad \Leftrightarrow \quad (x+5)^2 + (y-1)^2 = 9$

39 Center $C\left(\frac{1}{4}, 0\right)$, radius $\sqrt{5}$ • $\left(x - \frac{1}{4}\right)^2 + (y-0)^2 = \left(\sqrt{5}\right)^2 \quad \Leftrightarrow \quad \left(x - \frac{1}{4}\right)^2 + y^2 = 5$

40 Center $C\left(\frac{3}{4}, -\frac{2}{3}\right)$, radius $3\sqrt{2}$ • $\left(x - \frac{3}{4}\right)^2 + \left[y - \left(-\frac{2}{3}\right)\right]^2 = \left(3\sqrt{2}\right)^2 \quad \Leftrightarrow \quad \left(x - \frac{3}{4}\right)^2 + \left(y + \frac{2}{3}\right)^2 = 18$

41 An equation of a circle with center $C(-4, 6)$ is $(x+4)^2 + (y-6)^2 = r^2$.

Since the circle passes through $P(3, 1)$, we know that $x = 3$ and $y = 1$ is one solution of the general equation.

Letting $x = 3$ and $y = 1$ yields $7^2 + (-5)^2 = r^2 \quad \Rightarrow \quad r^2 = 74$. An equation is $(x+4)^2 + (y-6)^2 = 74$.

42 An equation of a circle with center at the origin is $x^2 + y^2 = r^2$.

Letting $x = 4$ and $y = -7$ yields $4^2 + (-7)^2 = r^2 \quad \Rightarrow \quad r^2 = 65$. $x^2 + y^2 = 65$

43 "Tangent to the y-axis" means that the circle will intersect the y-axis at exactly one point. The distance from the center $C(-3, 6)$ to this point of tangency is 3 units—this is the length of the radius of the circle.

An equation is $(x+3)^2 + (y-6)^2 = 9$.

44 "Tangent to the y-axis" means that the circle will intersect the y-axis at exactly one point. The distance from the center $C(-5, -7)$ to this point of tangency is 5 units—this is the length of the radius of the circle.

An equation is $(x+5)^2 + (y+7)^2 = 25$.

45 The circle is tangent to the x-axis and has center $C(4, -3)$.

Its radius, 3, is the distance from the x-axis to the y-value of the center. An equation is $(x-4)^2 + (y+3)^2 = 9$.

46 The circle is tangent to the x-axis and has center $C(2, 8)$.

Its radius, 8, is the distance from the x-axis to the y-value of the center. An equation is $(x-2)^2 + (y-8)^2 = 64$.

47 Since the radius is 2 and $C(h, k)$ is in QII, $h = -2$ and $k = 2$. An equation is $(x+2)^2 + (y-2)^2 = 4$.

48 Since the radius is 3 and $C(h, k)$ is in QIV, $h = 3$ and $k = -3$. An equation is $(x-3)^2 + (y+3)^2 = 9$.

49 The center of the circle is the midpoint M of $A(4, -3)$ and $B(-2, 7)$. $M_{AB} = (1, 2)$. The radius of the circle is $\frac{1}{2} \cdot d(A, B) = \frac{1}{2}\sqrt{[4-(-2)]^2 + (-3-7)^2} = \frac{1}{2}\sqrt{36+100} = \frac{1}{2}\sqrt{136} = \frac{1}{2}\sqrt{4 \cdot 34} = \sqrt{34}$.

An equation is $(x-1)^2 + (y-2)^2 = 34$.

Alternatively, once we know the center, $(1, 2)$, we also know that the equation has the form $(x-1)^2 + (y-2)^2 = r^2$. Now substitute 4 for x and -3 for y to obtain

$$3^2 + (-5)^2 = r^2 \quad \Rightarrow \quad 9 + 25 = r^2, \text{ or } r^2 = 34.$$

50 As in the solution to Exercise 49, $M_{AB} = (-1, 4)$ and $r = \frac{1}{2} \cdot d(A, B) = \frac{1}{2}\sqrt{80} = \sqrt{20}$.

An equation is $(x+1)^2 + (y-4)^2 = 20$.

51 $x^2 + y^2 - 4x + 6y - 36 = 0$ {complete the square on x and y} $\Rightarrow$

$x^2 - 4x + \underline{\ 4\ } + y^2 + 6y + \underline{\ 9\ } = 36 + \underline{\ 4\ } + \underline{\ 9\ } \quad \Rightarrow \quad (x-2)^2 + (y+3)^2 = 49$.

This is a circle with center $C(2, -3)$ and radius $r = 7$.

52 $x^2 + y^2 + 8x - 10y + 37 = 0 \quad \Rightarrow \quad x^2 + 8x + \underline{\ 16\ } + y^2 - 10y + \underline{\ 25\ } = -37 + \underline{\ 16\ } + \underline{\ 25\ } \quad \Rightarrow$

$(x+4)^2 + (y-5)^2 = 4$. $C(-4, 5)$; $r = 2$

53 $x^2 + y^2 + 4y - 7 = 0$ {complete the square on x and y} $\Rightarrow \quad x^2 + y^2 + 4y + \underline{\ 4\ } = 7 + \underline{\ 4\ } \quad \Rightarrow$

$x^2 + (y+2)^2 = 11$. This is a circle with center $C(0, -2)$ and radius $r = \sqrt{11}$.

54 $x^2 + y^2 - 10x + 18 = 0 \quad \Rightarrow \quad x^2 - 10x + \underline{\ 25\ } + y^2 = -18 + \underline{\ 25\ } \quad \Rightarrow \quad (x-5)^2 + y^2 = 7$. $C(5, 0)$; $r = \sqrt{7}$

55 $2x^2 + 2y^2 - 12x + 4y - 15 = 0$ {add 15 to both sides and divide by 2} $\Rightarrow$

$x^2 + y^2 - 6x + 2y = \frac{15}{2}$ {complete the square on x and y} $\Rightarrow$

$x^2 - 6x + \underline{\ 9\ } + y^2 + 2y + \underline{\ 1\ } = \frac{15}{2} + \underline{\ 9\ } + \underline{\ 1\ } \quad \Rightarrow \quad (x-3)^2 + (y+1)^2 = \frac{35}{2}$.

This is a circle with center $C(3, -1)$ and radius $r = \frac{1}{2}\sqrt{70}$.

56 $4x^2 + 4y^2 + 16x + 24y + 31 = 0 \quad \Rightarrow \quad x^2 + y^2 + 4x + 6y = -\frac{31}{4} \quad \Rightarrow$

$x^2 + 4x + \underline{\ 4\ } + y^2 + 6y + \underline{\ 9\ } = -\frac{31}{4} + \underline{\ 4\ } + \underline{\ 9\ } \quad \Rightarrow \quad (x+2)^2 + (y+3)^2 = \frac{21}{4}$. $C(-2, -3)$; $r = \frac{1}{2}\sqrt{21}$

57 $x^2 + y^2 + 4x - 2y + 5 = 0 \quad \Rightarrow \quad x^2 + 4x + \underline{\ 4\ } + y^2 - 2y + \underline{\ 1\ } = -5 + \underline{\ 4\ } + \underline{\ 1\ } \quad \Rightarrow$

$(x+2)^2 + (y-1)^2 = 0$. $C(-2, 1)$; $r = 0$ (a point)

58 $x^2 + y^2 - 6x - 4y + 13 = 0 \quad \Rightarrow \quad x^2 - 6x + \underline{\ 9\ } + y^2 - 4y + \underline{\ 4\ } = -13 + \underline{\ 9\ } + \underline{\ 4\ } \quad \Rightarrow$

$(x-3)^2 + (y-2)^2 = 0$. $C(3, 2)$; $r = 0$ (a point)

59 $x^2 + y^2 - 2x - 8y + 21 = 0 \quad \Rightarrow \quad x^2 - 2x + \underline{\ 1\ } + y^2 - 8y + \underline{\ 16\ } = -21 + \underline{\ 1\ } + \underline{\ 16\ } \quad \Rightarrow$

$(x-1)^2 + (y-4)^2 = -4$. This is not a circle since r^2 cannot equal -4.

$\boxed{60}$ $x^2+y^2+4x+6y+16=0 \Rightarrow x^2+4x+\underline{\ 4\ }+y^2+6y+\underline{\ 9\ }=-16+\underline{\ 4\ }+\underline{\ 9\ } \Rightarrow$
$(x+2)^2+(y+3)^2=-3$. This is not a circle since r^2 cannot equal -3.

$\boxed{61}$ To obtain equations for the upper and lower halves, we solve the given equation for y in terms of x.
$x^2+y^2=25 \Rightarrow y^2=25-x^2 \Rightarrow y=\pm\sqrt{25-x^2}$.
The upper half is $y=\sqrt{25-x^2}$ and the lower half is $y=-\sqrt{25-x^2}$.
To obtain equations for the right and left halves, we solve the given equation for x in terms of y.
$x^2+y^2=25 \Rightarrow x^2=25-y^2 \Rightarrow x=\pm\sqrt{25-y^2}$.
The right half is $x=\sqrt{25-y^2}$ and the left half is $x=-\sqrt{25-y^2}$.

$\boxed{62}$ $(x+3)^2+y^2=64 \Rightarrow y^2=64-(x+3)^2 \Rightarrow y=\pm\sqrt{64-(x+3)^2}$.
$(x+3)^2+y^2=64 \Rightarrow (x+3)^2=64-y^2 \Rightarrow x+3=\pm\sqrt{64-y^2} \Rightarrow x=-3\pm\sqrt{64-y^2}$.

$\boxed{63}$ To obtain equations for the upper and lower halves, we solve the given equation for y in terms of x.
$(x-2)^2+(y+1)^2=49 \Rightarrow (y+1)^2=49-(x-2)^2 \Rightarrow y+1=\pm\sqrt{49-(x-2)^2} \Rightarrow$
$y=-1\pm\sqrt{49-(x-2)^2}$.
The upper half is $y=-1+\sqrt{49-(x-2)^2}$ and the lower half is $y=-1-\sqrt{49-(x-2)^2}$.
To obtain equations for the right and left halves, we solve the given equation for x in terms of y.
$(x-2)^2+(y+1)^2=49 \Rightarrow (x-2)^2=49-(y+1)^2 \Rightarrow x-2=\pm\sqrt{49-(y+1)^2} \Rightarrow$
$x=2\pm\sqrt{49-(y+1)^2}$. The right half is $x=2+\sqrt{49-(y+1)^2}$ and the left half is $x=2-\sqrt{49-(y+1)^2}$.

$\boxed{64}$ $(x-3)^2+(y-5)^2=4 \Rightarrow (y-5)^2=4-(x-3)^2 \Rightarrow y-5=\pm\sqrt{4-(x-3)^2} \Rightarrow$
$y=5\pm\sqrt{4-(x-3)^2}$.
$(x-3)^2+(y-5)^2=4 \Rightarrow (x-3)^2=4-(y-5)^2 \Rightarrow x-3=\pm\sqrt{4-(y-5)^2} \Rightarrow$
$x=3\pm\sqrt{4-(y-5)^2}$.

$\boxed{65}$ From the figure, we see that the center of the circle is the origin, $(0,0)$, and the radius is 5. Using the standard form of an equation of a circle, $(x-h)^2+(y-k)^2=r^2$, we have $(x-0)^2+(y-0)^2=5^2$,
or $x^2+y^2=5^2$ {or use 25}.

$\boxed{66}$ From the figure, we see that the center of the circle is the origin, $(0,0)$, and the radius is 1. Using the standard form of an equation of a circle, $(x-h)^2+(y-k)^2=r^2$, we have $(x-0)^2+(y-0)^2=1^2$, or $x^2+y^2=1^2$ {or use 1}.

$\boxed{67}$ From the figure, we see that the diameter {look at the x-values of the rightmost and leftmost points} of the circle is $1-(-7)=8$ units, so the radius is $\frac{1}{2}(8)=4$. The center (h,k) is at the average of the extreme values; that is, $h=\dfrac{-7+1}{2}=-3$, and similarly, $k=\dfrac{-2+6}{2}=2$. Using the standard form of an equation of a circle, $(x-h)^2+(y-k)^2=r^2$, we have $[x-(-3)]^2+(y-2)^2=4^2$, or $(x+3)^2+(y-2)^2=4^2$ {or use 16}.

$\boxed{68}$ diameter $=4-(-2)=6$; radius $=\frac{1}{2}(6)=3$; $h=\dfrac{-2+4}{2}=1$ and $k=\dfrac{-5+1}{2}=-2$
An equation is $(x-1)^2+[y-(-2)]^2=3^2$, or $(x-1)^2+(y+2)^2=3^2$ {or use 9}.

69 The figure shows the lower semicircle of a circle centered at the origin having radius 4. The circle has equation $x^2 + y^2 = 4^2$. We want the lower semicircle, so we must have *negative* y-values, indicating that we should solve for y and use the negative sign. $x^2 + y^2 = 4^2 \Rightarrow y^2 = 4^2 - x^2 \Rightarrow y = \pm\sqrt{4^2 - x^2}$, so $y = -\sqrt{4^2 - x^2}$ is the desired equation.

70 $x^2 + y^2 = 3^2 \Rightarrow x^2 = 3^2 - y^2 \Rightarrow x = \pm\sqrt{3^2 - y^2}$, so $x = -\sqrt{3^2 - y^2}$ is the desired equation since we want *negative* x-values.

71 $x^2 + y^2 = 2^2 \Rightarrow y^2 = 2^2 - x^2 \Rightarrow y = \pm\sqrt{2^2 - x^2}$, so $y = \sqrt{2^2 - x^2}$ is the desired equation since we want *positive* y-values.

72 $x^2 + y^2 = 6^2 \Rightarrow x^2 = 6^2 - y^2 \Rightarrow x = \pm\sqrt{6^2 - y^2}$, so $x = \sqrt{6^2 - y^2}$ is the desired equation since we want *positive* x-values.

73 We need to determine if the distance from P to C is *less than* r, *greater than* r, or *equal to* r and hence, P will be *inside* the circle, *outside* the circle, or *on* the circle, respectively.

(a) $P(2,3), C(4,6) \Rightarrow d(P,C) = \sqrt{4+9} = \sqrt{13} < r\ \{r = 4\} \Rightarrow P$ is *inside* C.

(b) $P(4,2), C(1,-2) \Rightarrow d(P,C) = \sqrt{9+16} = 5 = r\ \{r = 5\} \Rightarrow P$ is *on* C.

(c) $P(-3,5), C(2,1) \Rightarrow d(P,C) = \sqrt{25+16} = \sqrt{41} > r\ \{r = 6\} \Rightarrow P$ is *outside* C.

74 **(a)** $P(3,8), C(-2,-4) \Rightarrow d(P,C) = \sqrt{25+144} = 13 = r\ \{r = 13\} \Rightarrow P$ is *on* C.

(b) $P(-2,5), C(3,7) \Rightarrow d(P,C) = \sqrt{25+4} = \sqrt{29} < r\ \{r = 6\} \Rightarrow P$ is *inside* C.

(c) $P(1,-2), C(6,-7) \Rightarrow d(P,C) = \sqrt{25+25} = \sqrt{50} > r\ \{r = 7\} \Rightarrow P$ is *outside* C.

75 **(a)** To find the x-intercepts of $x^2 + y^2 - 4x - 6y + 4 = 0$, let $y = 0$ and solve the resulting equation for x.

$$x^2 - 4x + 4 = 0 \Rightarrow (x-2)^2 = 0 \Rightarrow x - 2 = 0 \Rightarrow x = 2.$$

(b) To find the y-intercepts of $x^2 + y^2 - 4x - 6y + 4 = 0$, let $x = 0$ and solve the resulting equation for y.

$$y^2 - 6y + 4 = 0 \Rightarrow y = \frac{6 \pm \sqrt{36-16}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}.$$

76 **(a)** $x^2 + y^2 - 10x + 4y + 13 = 0 \bullet y = 0 \Rightarrow x^2 - 10x + 13 = 0 \Rightarrow x = \dfrac{10 \pm \sqrt{100-52}}{2} = 5 \pm 2\sqrt{3}$

(b) $x^2 + y^2 - 10x + 4y + 13 = 0 \bullet x = 0 \Rightarrow y^2 + 4y + 13 = 0 \Rightarrow y = \dfrac{-4 \pm \sqrt{16-52}}{2}$.

The negative discriminant implies that there are no real solutions to the equation and hence, no y-intercepts.

77 $x^2 + y^2 + 4x - 6y + 4 = 0 \Leftrightarrow (x+2)^2 + (y-3)^2 = 9$. This is a circle with center $C(-2,3)$ and radius 3. The circle we want has the same center, $C(-2,3)$, and radius that is equal to the distance from C to $P(2,6)$.

$$d(P,C) = \sqrt{16+9} = 5, \text{ so an equation is } (x+2)^2 + (y-3)^2 = 25.$$

78 By the Pythagorean theorem, the two stations are $d = \sqrt{100^2 + 80^2} \approx 128.06$ miles apart. The sum of their radii, $80 + 50 = 130$, is greater than d, indicating that the circles representing their broadcast ranges do overlap.

79 The equation of circle C_2 is $(x-h)^2 + (y-2)^2 = 2^2$. If we draw a line from the origin to the center of C_2, we form a right triangle with hypotenuse $5 - 2\ \{C_2 \text{ radius} - C_1 \text{ radius}\} = 3$ and sides of length 2 and h. Thus, $h^2 + 2^2 = 3^2 \Rightarrow h = \sqrt{5}$.

80 The equation of circle C_2 is $(x-h)^2+(y-3)^2=2^2$. If we draw a line from the origin to the center of C_2, we form a right triangle with hypotenuse $5+2$ $\{C_2 \text{ radius} + C_1 \text{ radius}\}=7$ and sides of length 3 and h. Thus, $h^2+3^2=7^2 \quad \Rightarrow \quad h=\sqrt{40}$.

81 The graph of y_1 is *below* the graph of y_2 to the left of $x=-3$ and to the right of $x=2$. Writing the x-values in interval notation gives us $(-\infty,-3)\cup(2,\infty)$.

Note that the y-values play no role in writing the answer in interval notation.

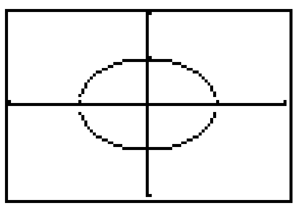
82 $y_1<y_2$ between $x=-8$ and $x=8$, so the interval is $(-8,8)$.

83 $y_1<y_2$ between $x=-1$ and $x=1$, excluding $x=0$ since $y_1=y_2$ at that value, so the interval is $(-1,0)\cup(0,1)$.

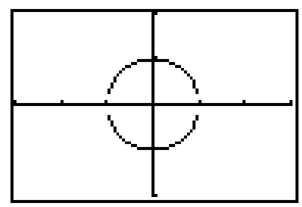
84 $y_1<y_2$ to the left of $x=-8$, between $x=-1$ and $x=1$, and to the right of $x=8$, so the interval notation is $(-\infty,-8)\cup(-1,1)\cup(8,\infty)$.

85 The viewing rectangles significantly affect the shape of the circle. The second viewing rectangle results in a graph that most looks like a circle.

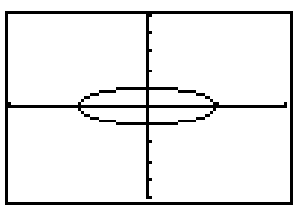
(1) $[-2,2]$ by $[-2,2]$



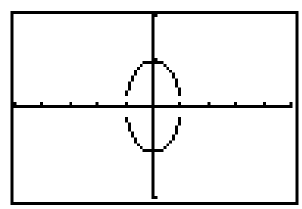
(2) $[-3,3]$ by $[-2,2]$



(3) $[-2,2]$ by $[-5,5]$

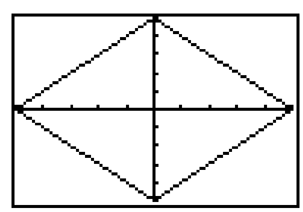


(4) $[-5,5]$ by $[-2,2]$



86 **(a)** From the graph, there are two x–intercepts and two y–intercepts.

(b) Using the free–moving cursor, one can conclude that $|x|+|y|<5$ is true whenever the point (x,y) is located inside the diamond shape.



87 Assign $x^3-\frac{9}{10}x^2-\frac{43}{25}x+\frac{24}{25}$ to Y_1. After trying a standard viewing rectangle, we see that the x-intercepts are near the origin and we choose the viewing rectangle $[-6,6]$ by $[-4,4]$. This is simply one choice, not necessarily the best choice. For most graphing calculator exercises, we have selected viewing rectangles that are in a $3:2$ proportion (horizontal : vertical) to maintain a true proportion. From the graph, there are three x-intercepts. Use a root feature to determine that they are -1.2, 0.5, and 1.6.

$[-6, 6]$ by $[-4, 4]$

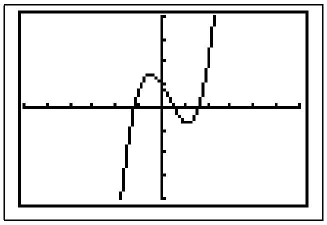


Figure 87

$[-6, 6]$ by $[-4, 4]$

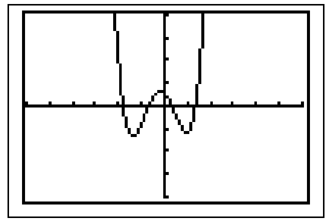


Figure 88

88 From the graph, there are four x-intercepts. They are approximately $-1.8, -0.7, 0.3$ and 1.35.

89 Make the assignments $\text{Y}_1 = x^3 + x$, $\text{Y}_2 = \sqrt{1 - x^2}$, and $\text{Y}_3 = -\text{Y}_2$. From the graph, there are two points of intersection. They are approximately $(0.6, 0.8)$ and $(-0.6, -0.8)$.

$[-3, 3]$ by $[-2, 2]$

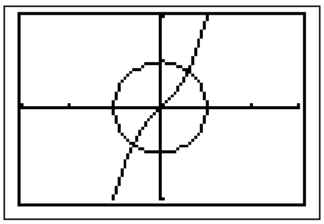


Figure 89

$[-3, 3]$ by $[-2, 2]$

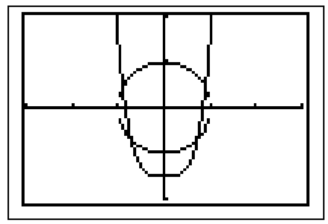


Figure 90

90 Make the assignments $\text{Y}_1 = 3x^4 - \frac{3}{2}$, $\text{Y}_2 = \sqrt{1 - x^2}$, and $\text{Y}_3 = -\text{Y}_2$. From the graph, there are four points of intersection. They are approximately $(\pm 0.9, 0.4)$ and $(\pm 0.7, -0.7)$.

91 Depending on the type of graphing utility used, you may need to solve for y first.

$x^2 + (y-1)^2 = 1 \quad \Rightarrow \quad y = 1 \pm \sqrt{1 - x^2}$; $\left(x - \frac{5}{4}\right)^2 + y^2 = 1 \quad \Rightarrow \quad y = \pm\sqrt{1 - \left(x - \frac{5}{4}\right)^2}$.

Make the assignments $\text{Y}_1 = \sqrt{1 - x^2}$, $\text{Y}_2 = 1 + \text{Y}_1$, $\text{Y}_3 = 1 - \text{Y}_1$, $\text{Y}_4 = \sqrt{1 - \left(x - \frac{5}{4}\right)^2}$, and $\text{Y}_5 = -\text{Y}_4$.

Be sure to "turn off" Y_1 before graphing. From the graph, there are two points of intersection.

They are approximately $(0.999, 0.968)$ and $(0.251, 0.032)$.

$[-3, 3]$ by $[-2, 2]$

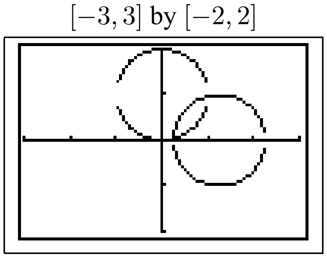


Figure 91

$[-3, 3]$ by $[-2, 2]$

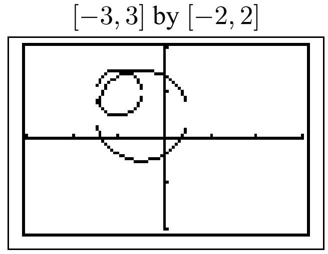


Figure 92

92 $(x+1)^2 + (y-1)^2 = \frac{1}{4} \quad \Rightarrow \quad y = 1 \pm \sqrt{\frac{1}{4} - (x+1)^2}$; $\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = 1 \quad \Rightarrow$

$y = \frac{1}{2} \pm \sqrt{1 - \left(x + \frac{1}{2}\right)^2}$. From the graph, there are two points of intersection.

They are approximately $(-0.79, 1.46)$ and $(-1.46, 0.79)$.

93 The cars are initially 4 miles apart. The distance between them decreases to 0 when they meet on the highway after 2 minutes. Then, the distance between them starts to increase until it is 4 miles after a total of 4 minutes.

$[0, 4]$ by $[0, 4]$

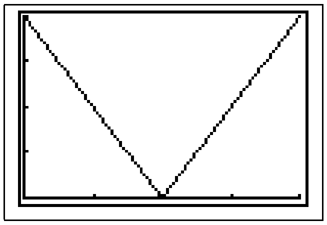


Figure 93

$[0, 6]$ by $[0, 20{,}000, 5000]$

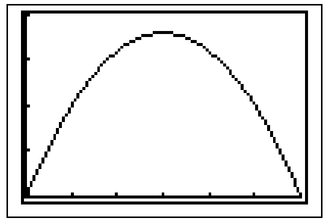


Figure 94

94 At noon on Sunday the pool is empty since when $x = 0$, $A = 0$. It is then filled with water, until at noon on Wednesday $(x = 3)$, it contains 18,000 gallons. It is then drained until at noon on Saturday $(x = 6)$, it is empty again.

95 **(a)** $v = 1087\sqrt{\dfrac{T+273}{273}}$ • $T = 20°\text{C} \;\Rightarrow\; v = 1087\sqrt{\dfrac{20+273}{273}} \approx 1126$ ft/sec.

(b) Algebraically: $v = 1000 \;\Rightarrow\; 1000 = 1087\sqrt{\dfrac{T+273}{273}} \;\Rightarrow\; \sqrt{\dfrac{T+273}{273}} = \dfrac{1000}{1087} \;\Rightarrow$

$\dfrac{T+273}{273} = \dfrac{1000^2}{1087^2} \;\Rightarrow\; T + 273 = \dfrac{273 \cdot 1000^2}{1087^2} \;\Rightarrow\; T = \dfrac{273 \cdot 1000^2}{1087^2} - 273 \approx -42°\text{C}.$

Graphically: Graph $\text{Y}_1 = 1087\sqrt{(T+273)/273}$ and $\text{Y}_2 = 1000$.

At the point of their intersection, $T \approx -42°\text{C}$.

$[-50, 50, 10]$ by $[900, 1200, 100]$

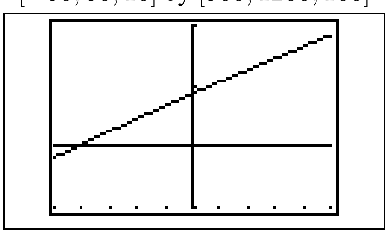


Figure 95

$[14, 16]$ by $[95, 105]$

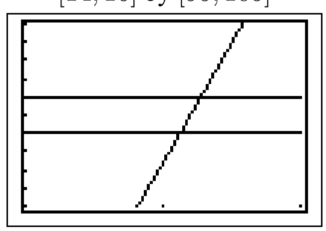


Figure 96

96 The horizontal lines $(y = 99$ and $y = 101)$ intersect the graph of $A = \left(\sqrt{3}/4\right)s^2$ at $s \approx 15.12, 15.27$.

Thus, if $15.12 \le s \le 15.27$, then $99 \le A \le 101$.

2.3 Exercises

1 $A(-3, 2), B(5, -4) \quad \Rightarrow \quad m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{(-4) - 2}{5 - (-3)} = \dfrac{-6}{8} = -\dfrac{3}{4}$

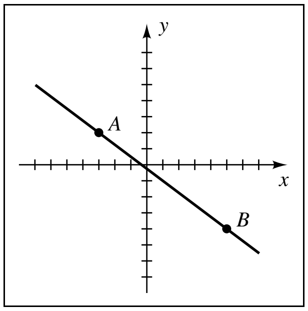


Figure 1

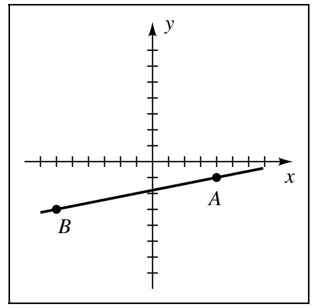


Figure 2

2 $A(4, -1), B(-6, -3) \quad \Rightarrow \quad m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3 - (-1)}{-6 - 4} = \dfrac{-2}{-10} = \dfrac{1}{5}$

3 $A(3, 4), B(-6, 4) \quad \Rightarrow \quad m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 4}{-6 - 3} = \dfrac{0}{-9} = 0$ {horizontal line}

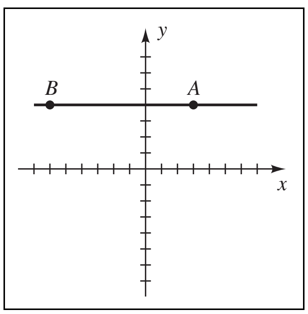


Figure 3

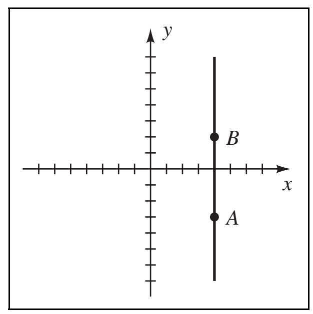


Figure 4

4 $A(4, -3), B(4, 2) \quad \Rightarrow \quad m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2 - (-3)}{4 - 4} = \dfrac{5}{0} \quad \Rightarrow \quad m$ is undefined {vertical line}

5 $A(-3, 2), B(-3, 5) \quad \Rightarrow \quad m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - 2}{-3 - (-3)} = \dfrac{3}{0} \quad \Rightarrow \quad m$ is undefined {vertical line}

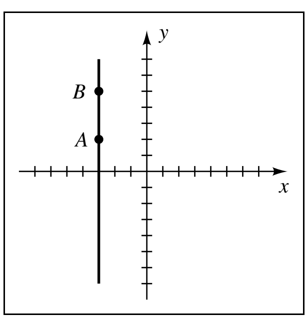


Figure 5

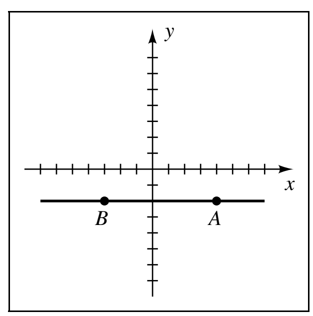


Figure 6

6 $A(4, -2), B(-3, -2) \quad \Rightarrow \quad m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2 - (-2)}{-3 - 4} = \dfrac{0}{-7} = 0$ {horizontal line}

7 To show that the polygon is a parallelogram, we must show that the slopes of opposite sides are equal.

$A(-2, 1), B(6, 3), C(4, 0), D(-4, -2) \quad \Rightarrow \quad m_{AB} = \frac{1}{4} = m_{DC}$ and $m_{DA} = \frac{3}{2} = m_{CB}$.

8 To show that the polygon is a trapezoid, we must show that the slopes of one pair of opposite sides are equal.

$$A(0,3),\ B(3,-1),\ C(-2,-6),\ D(-8,2) \quad\Rightarrow\quad m_{AB} = -\tfrac{4}{3} = m_{CD}.$$

9 To show that the polygon is a rectangle, we must show that the slopes of opposite sides are equal (parallel lines) and the slopes of two adjacent sides are negative reciprocals (perpendicular lines).

$$A(6,15),\ B(11,12),\ C(-1,-8),\ D(-6,-5) \quad\Rightarrow\quad m_{DA} = \tfrac{5}{3} = m_{CB} \text{ and } m_{AB} = -\tfrac{3}{5} = m_{DC}.$$

10 To show that the polygon is a right triangle, we must show that the slopes of two adjacent sides are negative reciprocals (perpendicular lines). $A(1,4),\ B(6,-4),\ C(-15,-6) \quad\Rightarrow\quad m_{AB} = -\frac{8}{5}$ and $m_{AC} = \frac{5}{8}$.

11 $A(-1,-3)$ is 5 units to the left and 5 units down from $B(4,2)$. The fourth vertex D will have the same relative position from $C(-7,5)$, that is, 5 units to the left and 5 units down from C. Its coordinates are $(-7-5, 5-5) = (-12, 0)$.

12 Let $E = M_{AB} = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$, $F = M_{BC} = \left(\dfrac{x_2+x_3}{2}, \dfrac{y_2+y_3}{2}\right)$, $G = M_{CD} = \left(\dfrac{x_3+x_4}{2}, \dfrac{y_3+y_4}{2}\right)$, and $H = M_{AD} = \left(\dfrac{x_1+x_4}{2}, \dfrac{y_1+y_4}{2}\right)$. The slopes of opposite sides are equal (or lines are vertical).

$$m_{EF} = m_{GH} = \frac{y_3-y_1}{x_3-x_1} \text{ and } m_{FG} = m_{EH} = \frac{y_4-y_2}{x_4-x_2}.$$

13 $m = 3,\ -2,\ \frac{2}{3},\ -\frac{1}{4}$ • Lines with equation $y = mx$ pass through the origin. Draw lines through the origin with slopes 3 {rise = 3, run = 1}, -2 {rise = -2, run = 1}, $\frac{2}{3}$ {rise = 2, run = 3}, and $-\frac{1}{4}$ {rise = -1, run = 4}. A negative "rise" can be thought of as a "drop."

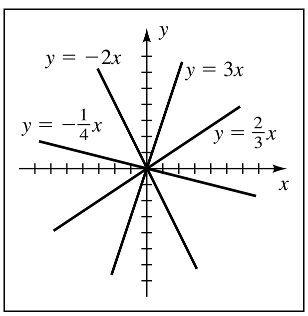


Figure 13

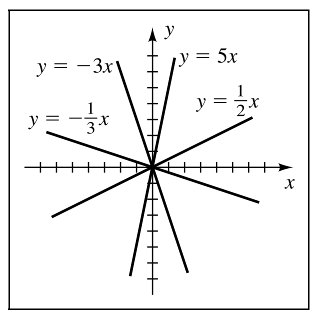


Figure 14

14 $m = 5,\ -3,\ \frac{1}{2},\ -\frac{1}{3}$ •

15 Draw lines through the point $P(3,1)$ with slopes $\frac{1}{2}$ {rise = 1, run = 2}, -1 {rise = -1, run = 1}, and $-\frac{1}{5}$ {rise = -1, run = 5}.

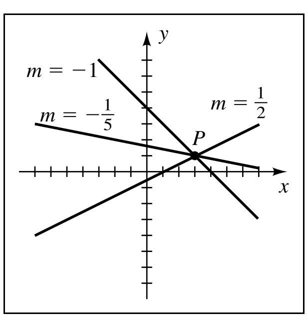


Figure 15

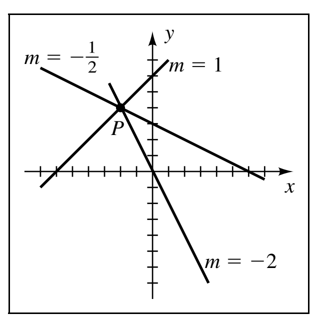


Figure 16

16 $P(-2,4);\ m = 1,\ -2,\ -\frac{1}{2}$ •

17 From the figure, the slope of one of the lines is $\dfrac{\Delta y}{\Delta x} = \dfrac{5}{4}$, so the slopes are $\pm\dfrac{5}{4}$.

Using the point-slope form for the equation of a line with slope $m = \pm\frac{5}{4}$ and point $(x_1, y_1) = (2, -3)$ gives us
$y - (-3) = \pm\frac{5}{4}(x - 2)$, or $y + 3 = \pm\frac{5}{4}(x - 2)$.

18 $m = \dfrac{\Delta y}{\Delta x} = \dfrac{3}{4}$, so the slopes are $\pm\dfrac{3}{4}$.

Using the point $(x_1, y_1) = (-1, 2)$ gives us $y - 2 = \pm\frac{3}{4}[x - (-1)]$, or $y - 2 = \pm\frac{3}{4}(x + 1)$.

19 The line $y = 1x + 3$ has slope 1 and y-intercept 3. The line $y = 1x + 1$ has slope 1 and y-intercept 1.
The line $y = -1x + 1$ has slope -1 and y-intercept 1. Check your graph for parallel and perpendicular lines.

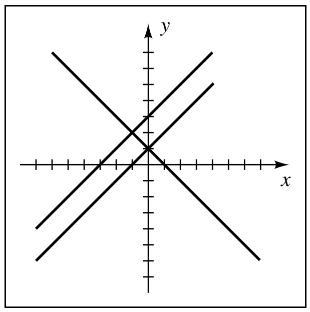


Figure 19

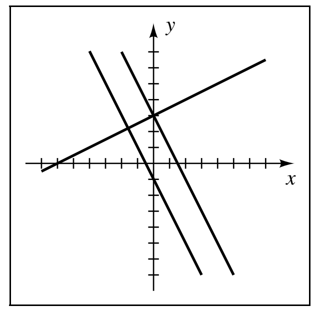


Figure 20

20 $y = -2x - 1$, $y = -2x + 3$, $y = \frac{1}{2}x + 3$ • Check your graph for parallel and perpendicular lines.

21 **(a)** "Parallel to the y-axis" implies the equation is of the form $x = k$.
The x-value of $A(3, -1)$ is 3, hence $x = 3$ is the equation.

(b) "Perpendicular to the y-axis" implies the equation is of the form $y = k$.
The y-value of $A(3, -1)$ is -1, hence $y = -1$ is the equation.

22 **(a)** The line through $A(-4, 2)$ and parallel to the x-axis is $y = 2$.

(b) The line through $A(-4, 2)$ and perpendicular to the x-axis is $x = -4$.

23 Using the point-slope form, the equation of the line through $A(5, -3)$ with slope -4 is
$$y - (-3) = -4(x - 5) \quad\Rightarrow\quad y + 3 = -4x + 20 \quad\Rightarrow\quad 4x + y = 17.$$

24 Using the point-slope form, the equation of the line through $A(-1, 4)$ with slope $\frac{2}{5}$ is
$$y - 4 = \tfrac{2}{5}[x - (-1)] \quad\Rightarrow\quad 5(y - 4) = 2(x + 1) \quad\Rightarrow\quad 5y - 20 = 2x + 2 \quad\Rightarrow\quad 2x - 5y = -22.$$

25 $A(4, 1)$; slope $-\frac{1}{3}$ {use the point-slope form of a line} $\Rightarrow$
$$y - 1 = -\tfrac{1}{3}(x - 4) \quad\Rightarrow\quad 3(y - 1) = -1(x - 4) \quad\Rightarrow\quad 3y - 3 = -x + 4 \quad\Rightarrow\quad x + 3y = 7.$$

26 $A(0, -2)$; slope 5 $\Rightarrow$ $y + 2 = 5(x - 0)$ $\Rightarrow$ $y + 2 = 5x$ $\Rightarrow$ $5x - y = 2$.

27 $A(4, -5)$, $B(-3, 6)$ $\Rightarrow$ $m_{AB} = -\frac{11}{7}$. By the point-slope form, with $A(4, -5)$, an equation of the line is
$$y + 5 = -\tfrac{11}{7}(x - 4) \quad\Rightarrow\quad 7(y + 5) = -11(x - 4) \quad\Rightarrow\quad 7y + 35 = -11x + 44 \quad\Rightarrow\quad 11x + 7y = 9.$$

28 $A(-3, 2)$, $B(4, -4)$ $\Rightarrow$ $m_{AB} = -\frac{6}{7}$. By the point-slope form, with $A(-3, 2)$, an equation of the line is
$$y - 2 = -\tfrac{6}{7}(x + 3) \quad\Rightarrow\quad 7(y - 2) = -6(x + 3) \quad\Rightarrow\quad 7y - 14 = -6x - 18 \quad\Rightarrow\quad 6x + 7y = -4.$$

29 $A(-3,5)$, y-intercept -2 • Use the slope-intercept form with $b=-2$ to get $y=mx-2$. The slope of the line through $A(-3,5)$ and $B(0,-2)$ is $-\frac{7}{3}$. $y=-\frac{7}{3}x-2 \;\Rightarrow\; 3y=-7x-6 \;\Rightarrow\; 7x+3y=-6$.

30 $A(1,8)$, y-intercept 5 • Use the slope-intercept form with $b=5$ to get $y=mx+5$. The slope of the line through $A(1,8)$ and $B(0,5)$ is 3. $y=3x+5 \;\Rightarrow\; 3x-y=-5$.

31 $A(2,10)$, $B(-3,0) \;\Rightarrow\; m_{AB}=2$. $y-0=2(x+3) \;\Rightarrow\; y=2x+6 \;\Rightarrow\; 2x-y=-6$.

32 $A(-1,6)$, $B(5,0) \;\Rightarrow\; m_{AB}=-1$. $y-0=-1(x-5) \;\Rightarrow\; y=-x+5 \;\Rightarrow\; x+y=5$.

33 $5x-2y=4 \;\Leftrightarrow\; 5x-4=2y \;\Leftrightarrow\; y=\frac{5}{2}x-2$. Using the same slope, $\frac{5}{2}$, with $A(3,-1)$, gives us

$$y+1=\tfrac{5}{2}(x-3) \;\Rightarrow\; 2(y+1)=5(x-3) \;\Rightarrow\; 2y+2=5x-15 \;\Rightarrow\; 5x-2y=17.$$

34 $x+3y=1 \;\Leftrightarrow\; 3y=-x+1 \;\Leftrightarrow\; y=-\frac{1}{3}x+\frac{1}{3}$. Using the same slope, $-\frac{1}{3}$, with $A(-3,5)$, gives us

$$y-5=-\tfrac{1}{3}(x+3) \;\Rightarrow\; 3y-15=-x-3 \;\Rightarrow\; x+3y=12.$$

35 $2x-5y=8 \;\Leftrightarrow\; 2x-8=5y \;\Leftrightarrow\; y=\frac{2}{5}x-\frac{8}{5}$. The slope of this line is $\frac{2}{5}$, so we'll use the negative reciprocal, $-\frac{5}{2}$, for the slope of the new line, with $A(7,-3)$.

$$y+3=-\tfrac{5}{2}(x-7) \;\Rightarrow\; 2(y+3)=-5(x-7) \;\Rightarrow\; 2y+6=-5x+35 \;\Rightarrow\; 5x+2y=29.$$

36 $3x+2y=7 \;\Leftrightarrow\; y=-\frac{3}{2}x+\frac{7}{2}$. Using the negative reciprocal of $-\frac{3}{2}$ for the slope, with $A(5,4)$,

$$y-4=\tfrac{2}{3}(x-5) \;\Rightarrow\; 3y-12=2x-10 \;\Rightarrow\; 2x-3y=-2.$$

37 $A(4,0)$, $B(0,-3) \;\Rightarrow\; m_{AB}=\frac{3}{4}$.

Since B is the y-intercept, we use the slope-intercept form with $b=-3$ to get $y=\frac{3}{4}x-3$.

38 $A(-6,0)$, $B(0,-1) \;\Rightarrow\; m_{AB}=-\frac{1}{6}$. Using the slope-intercept form with $b=-1$ gives us $y=-\frac{1}{6}x-1$.

39 $A(5,2)$, $B(-1,4) \;\Rightarrow\; m_{AB}=-\frac{1}{3}$. By the point-slope form, with $A(5,2)$, an equation of the line is

$$y-2=-\tfrac{1}{3}(x-5) \;\Rightarrow\; y=-\tfrac{1}{3}x+\tfrac{5}{3}+2 \;\Rightarrow\; y=-\tfrac{1}{3}x+\tfrac{11}{3}.$$

40 $A(-3,1)$, $B(2,7) \;\Rightarrow\; m_{AB}=\frac{6}{5}$. By the point-slope form, with $A(-3,1)$, an equation of the line is

$$y-1=\tfrac{6}{5}(x+3) \;\Rightarrow\; y=\tfrac{6}{5}x+\tfrac{18}{5}+1 \;\Rightarrow\; y=\tfrac{6}{5}x+\tfrac{23}{5}.$$

41 We need the line through the midpoint M of segment AB that is perpendicular to segment AB.

$A(3,-1)$, $B(-2,6) \;\Rightarrow\; M_{AB}=\left(\frac{1}{2},\frac{5}{2}\right)$ and $m_{AB}=-\frac{7}{5}$. Use M_{AB} and $m=\frac{5}{7}$ in the point-slope form.

$$y-\tfrac{5}{2}=\tfrac{5}{7}\left(x-\tfrac{1}{2}\right) \;\Rightarrow\; 7\left(y-\tfrac{5}{2}\right)=5\left(x-\tfrac{1}{2}\right) \;\Rightarrow\; 7y-\tfrac{35}{2}=5x-\tfrac{5}{2} \;\Rightarrow\; 5x-7y=-15.$$

42 $A(4,2)$, $B(-2,-6) \;\Rightarrow\; M_{AB}=(1,-2)$ and $m_{AB}=\frac{4}{3}$. Use M_{AB} and $m=-\frac{3}{4}$ in the point-slope form.

$$y+2=-\tfrac{3}{4}(x-1) \;\Rightarrow\; 4y+8=-3x+3 \;\Rightarrow\; 3x+4y=-5.$$

43 An equation of the line with slope -1 through the origin is $y-0=-1(x-0)$, or $y=-x$.

44 An equation of the line with slope 1 through the origin is $y-0=1(x-0)$, or $y=x$.

45 We can solve the given equation for y to obtain the slope-intercept form, $y = mx + b$.

$2x = 15 - 3y \quad \Rightarrow \quad 3y = -2x + 15 \quad \Rightarrow \quad y = -\frac{2}{3}x + 5;\ m = -\frac{2}{3},\ b = 5$

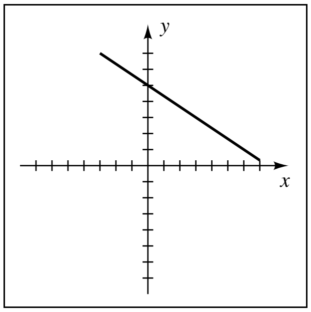


Figure 45

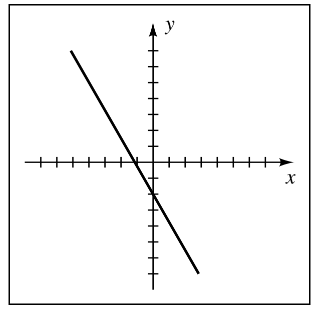


Figure 46

46 $7x = -4y - 8 \quad \Rightarrow \quad -4y = 7x + 8 \quad \Rightarrow \quad y = -\frac{7}{4}x - 2;\ m = -\frac{7}{4},\ b = -2$

47 $4x - 3y = 9 \quad \Rightarrow \quad -3y = -4x + 9 \quad \Rightarrow \quad y = \frac{4}{3}x - 3;\ m = \frac{4}{3},\ b = -3$

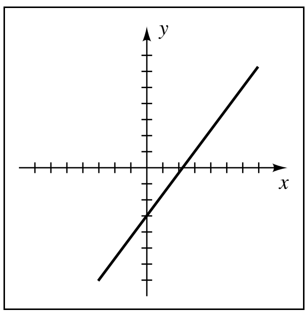


Figure 47

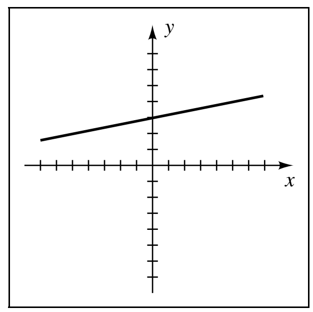


Figure 48

48 $x - 5y = -15 \quad \Rightarrow \quad -5y = -x - 15 \quad \Rightarrow \quad y = \frac{1}{5}x + 3;\ m = \frac{1}{5},\ b = 3$

49 **(a)** An equation of the horizontal line with y-intercept 3 is $y = 3$.

(b) An equation of the line through the origin with slope $-\frac{1}{2}$ is $y = -\frac{1}{2}x$.

(c) An equation of the line with slope $-\frac{3}{2}$ and y-intercept 1 is $y = -\frac{3}{2}x + 1$.

(d) An equation of the line through $(3, -2)$ with slope -1 is $y + 2 = -(x - 3)$.

Alternatively, we have a slope of -1 and a y-intercept of 1, i.e., $y = -x + 1$.

50 **(a)** An equation of the vertical line with x-intercept -2 is $x = -2$.

(b) An equation of the line through the origin with slope $\frac{4}{3}$ is $y = \frac{4}{3}x$.

(c) An equation of the line with slope $\frac{1}{3}$ and y-intercept -2 is $y = \frac{1}{3}x - 2$.

(d) An equation of the line through $(-2, -5)$ with slope 3 is $y + 5 = 3(x + 2)$.

Alternatively, we have a slope of 3 and a y-intercept of 1, i.e., $y = 3x + 1$.

51 Since we want to obtain a "1" on the right side of the equation, we will divide each term by -10.

$\frac{5x}{-10} - \frac{2y}{-10} = \frac{-10}{-10} \quad \Leftrightarrow \quad \frac{x}{-2} + \frac{y}{5} = 1.$ The x-intercept is -2 and the y-intercept is 5.

52 Since we want to obtain a "1" on the right side of the equation, we will divide each term by 12.

$\frac{4x}{12} - \frac{3y}{12} = \frac{12}{12} \quad \Leftrightarrow \quad \frac{x}{3} + \frac{y}{-4} = 1.$ The x-intercept is 3 and the y-intercept is -4.

53 Since we want to obtain a "1" on the right side of the equation, we will divide each term by 6.

$$\frac{4x}{6} - \frac{2y}{6} = \frac{6}{6} \quad \Leftrightarrow \quad \frac{2x}{3} - \frac{y}{3} = 1 \quad \Leftrightarrow \quad \frac{x}{\frac{3}{2}} + \frac{y}{-3} = 1.$$ The x-intercept is $\frac{3}{2}$ and the y-intercept is -3.

54 $\dfrac{x}{-2} - \dfrac{3y}{-2} = \dfrac{-2}{-2} \quad \Leftrightarrow \quad \dfrac{x}{-2} + \dfrac{y}{\frac{2}{3}} = 1.$ The x-intercept is -2 and the y-intercept is $\frac{2}{3}$.

55 The radius of the circle is the vertical distance from the center of the circle to the line $y = 5$, that is,

$r = 5 - (-2) = 7$. With $C(3, -2)$, an equation is $(x-3)^2 + (y+2)^2 = 49$.

56 The line through the origin and P is perpendicular to the desired line.

This line has equation $y = \frac{4}{3}x$, so the desired line has slope $-\frac{3}{4}$.

With $P(3, 4)$, an equation is $y - 4 = -\frac{3}{4}(x-3) \quad \Rightarrow \quad y = -\frac{3}{4}x + \frac{9}{4} + 4 \quad \Rightarrow \quad y = -\frac{3}{4}x + \frac{25}{4}$.

57 The y-intercept is 1 (in millions of dollars), so when no widgets are produced, production costs are \$1 million. The slope is shown as positive $\frac{1}{3}$, with the 3 in thousands, so for every 3000 widgets produced, production costs increase \$1 million.

58 The y-intercept is 15 (in gallons of gas remaining), so after filling the gas tank and driving 0 miles, the driver has 15 gallons in the tank. The slope is shown as $-\frac{1}{26}$, so for every 26 miles driven, the number of gallons remaining in the tank decreases 1 gallon.

59 **(a)** The L-intercept of the graph of the equation $L = 1.53t - 6.7$ is -6.7, which has no meaning since the length cannot be negative. When $t = 12$, $L = 1.53(12) - 6.7 = 11.66$ cm, which is the shortest length for which this model applies.

(b) The slope is 1.53, so for every increase of 1 week in time, there is an increase of 1.53 cm in length.

(c) $L = 28 \quad \Rightarrow \quad 1.53t - 6.7 = 28 \quad \Rightarrow \quad t = \dfrac{28 + 6.7}{1.53} \approx 22.68$, or approximately 23 weeks.

60 $S = 0.03 + 1.805C \;\bullet\; S = 0.35 \quad \Rightarrow \quad 0.03 + 1.805C = 0.35 \quad \Rightarrow \quad C = \dfrac{0.35 - 0.03}{1.805} \approx 0.177.$

61 **(a)** The W-intercept of the graph of the equation $W = 1.70L - 42.8$ is -42.8, which has no meaning since the weight cannot be negative. When $L = 30$, $W = 1.70(30) - 42.8 = 8.2$ tons, which is the smallest weight for which this model applies.

(b) The slope is 1.70, so for every increase of 1 foot in length, there is an increase of 1.7 tons in weight.

(c) $W = 1.70L - 42.8 \;\bullet\; L = 40 \quad \Rightarrow \quad W = 1.70(40) - 42.8 = 25.2$ tons

(d) Error in $L = \pm 2 \quad \Rightarrow \quad$ Error in $W = 1.70(\pm 2) = \pm 3.4$ tons

62 **(a)** $L = at + b$ and $L = 24$ when $t = 0 \quad \Rightarrow \quad L = at + 24$.

$L = 53$ when $t = 7 \quad \Rightarrow \quad 53 = 7a + 24 \quad \Rightarrow \quad a = \frac{29}{7}$ and $L = \frac{29}{7}t + 24$.

(b) From part (a), the slope is $\frac{29}{7}$ ft/month $= \frac{29}{210}$ ft/day ≈ 1.657 inches/day.

(c) $W = at + b$ and $W = 3$ when $t = 0 \quad \Rightarrow \quad W = at + 3$.

$W = 23$ when $t = 7 \quad \Rightarrow \quad 23 = 7a + 3 \quad \Rightarrow \quad a = \frac{20}{7}$ and $W = \frac{20}{7}t + 3$.

(d) From part (c), the slope is $\frac{20}{7}$ tons/month $= \frac{2}{21}$ tons/day ≈ 190.476 pounds/day.

63 **(a)** $y = mx = \dfrac{\text{change in } y \text{ from the beginning of the season}}{\text{change in } x \text{ from the beginning of the season}}(x) = \dfrac{5-0}{14-0}x = \dfrac{5}{14}x.$

(b) $x = 162 \quad \Rightarrow \quad y = \frac{5}{14}(162) \approx 58.$

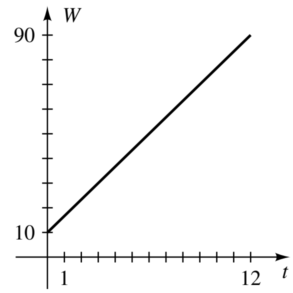
64 **(a)** $y = mx = \dfrac{\text{change in } y \text{ for the year}}{\text{change in } x \text{ for the year}}(x) = \dfrac{18{,}000 - 0}{(31+28+24)-0}x = \dfrac{18{,}000}{83}x.$

(b) $x = 365 \quad \Rightarrow \quad y = \frac{18{,}000}{83}(365) \approx 79{,}157.$

65 **(a)** Using the slope-intercept form, $W = mt + b = mt + 10.$

$W = 30$ when $t = 3 \quad \Rightarrow \quad 30 = 3m + 10 \quad \Rightarrow$

$m = \frac{20}{3}$ and $W = \frac{20}{3}t + 10.$



(b) $t = 6 \quad \Rightarrow \quad W = \frac{20}{3}(6) + 10 \quad \Rightarrow \quad W = 50$ lb

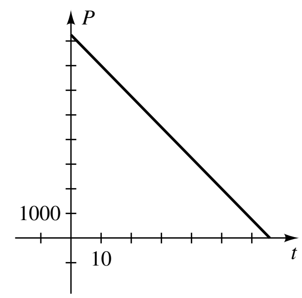
(c) $W = 70 \quad \Rightarrow \quad 70 = \frac{20}{3}t + 10 \quad \Rightarrow \quad 60 = \frac{20}{3}t \quad \Rightarrow \quad t = 9$ years old

(d) The graph has endpoints at $(0, 10)$ and $(12, 90)$.

66 **(a)** Using the slope-intercept form, $P = mt + b = mt + 8250.$

$P = 8125$ when $t = 1$ {after one payment} $\quad \Rightarrow$

$8125 = m + 8250 \quad \Rightarrow \quad m = -125$ and $P = -125t + 8250.$



(b) $P = 5000 \quad \Rightarrow \quad -125\text{t} + 8250 = 5000 \quad \Rightarrow$

$-125t = -3250 \quad \Rightarrow \quad t = 26$ months

(c) $\frac{8250}{125} = 66$ payments. The graph has endpoints at $(0, 8250)$ and $(66, 0)$.

67 Using $(10, 2480)$ and $(25, 2440)$ {since an increase of 15°C lowers H by 40}, we have

$H - 2440 = \dfrac{2440 - 2480}{25 - 10}(T - 25) \quad \Rightarrow \quad H - 2440 = \dfrac{-40}{15}(T - 25) \quad \Rightarrow \quad H - 2440 = -\dfrac{8}{3}(T - 25) \quad \Rightarrow$

$H - \frac{7320}{3} = -\frac{8}{3}T + \frac{200}{3} \quad \Rightarrow \quad H = -\frac{8}{3}T + \frac{7520}{3}.$

68 **(a)** Using $(1800, 100)$ and $(5000, 40)$, we have $P - 40 = \dfrac{40 - 100}{5000 - 1800}(h - 5000) \quad \Rightarrow$

$P - 40 = -\frac{3}{160}(h - 5000) \quad \Rightarrow \quad P - \frac{6400}{160} = -\frac{3}{160}h + \frac{15{,}000}{160} \quad \Rightarrow \quad P = -\frac{3}{160}h + \frac{21{,}400}{160} \quad \Rightarrow$

$P = -\frac{3}{160}h + \frac{535}{4}$ for $1800 \leq h \leq 5000.$

(b) $h = 2400 \quad \Rightarrow \quad P = -\frac{3}{160}(2400) + \frac{535}{4} = -\frac{180}{4} + \frac{535}{4} = \frac{355}{4}$, or 88.75%.

69 **(a)** Using the slope-intercept form with $m = 0.032$ and $b = 13.5$, we have $T = 0.032t + 13.5.$

(b) $t = 2020 - 1915 = 105 \quad \Rightarrow \quad T = 0.032(105) + 13.5 = 16.86$°C.

70 **(a)** Using $(1870, 11.8)$ and $(1969, 13.5)$, we have

$T - 13.5 = \dfrac{13.5 - 11.8}{1969 - 1870}(t - 1969)$, or $T = \dfrac{1.7}{99}(t - 1969) + 13.5.$

(b) $T = 12.5 \quad \Rightarrow \quad 12.5 - 13.5 = \dfrac{1.7}{99}(t - 1969) \quad \Rightarrow \quad -1 = \dfrac{1.7}{99}(t - 1969) \quad \Rightarrow \quad -\dfrac{99}{1.7} = t - 1969 \quad \Rightarrow$

$t \approx 1910.76$, or during the year 1910.

71 **(a)** Expenses (E) = (\$1000) + (5% of R) + (\$2600) + (50% of R) $\Rightarrow$

$$E = 1000 + 0.05R + 2600 + 0.50R \quad \Rightarrow \quad E = 0.55R + 3600.$$

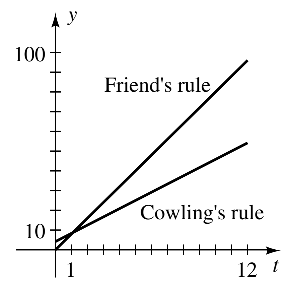
(b) Profit (P) = Revenue (R) − Expenses (E) $\Rightarrow$ $P = R - (0.55R + 3600)$ $\Rightarrow$

$$P = R - 0.55R - 3600 \quad \Rightarrow \quad P = 0.45R - 3600.$$

(c) *Break even* means P would be 0. $P = 0 \Rightarrow 0 = 0.45R - 3600 \Rightarrow$

$$0.45R = 3600 \quad \Rightarrow \quad \tfrac{45}{100}R = 3600 \quad \Rightarrow \quad R = 3600\left(\tfrac{100}{45}\right) = \$8000/\text{month}$$

72 **(a)** $a = 100$ gives us $y = \frac{25}{6}(t+1)$ with endpoints $\left(0, \frac{25}{6}\right)$ and $\left(12, \frac{325}{6}\right)$, and $y = 8t$ with endpoints $(0, 0)$ and $(12, 96)$.



(b) $\frac{1}{24}(t+1)a = \frac{2}{25}ta \Rightarrow \frac{1}{24}(t+1) = \frac{2}{25}t \Rightarrow$

$25(t+1) = 24(2t) \Rightarrow 25t + 25 = 48t \Rightarrow$

$$25 = 23t \quad \Rightarrow \quad t = \tfrac{25}{23}\text{ yr} \approx 13 \text{ months}$$

73 The targets are on the x-axis $\{$which is the line $y = 0\}$. To determine if a target is hit, set $y = 0$ and solve for x.

(a) $y - 2 = -1(x - 1) \Rightarrow x + y = 3.$ $y = 0 \Rightarrow x = 3$ and a creature is hit.

(b) $y - \frac{5}{3} = -\frac{4}{9}\left(x - \frac{3}{2}\right) \Rightarrow 4x + 9y = 21.$ $y = 0 \Rightarrow x = 5.25$ and no creature is hit.

74 **(a)** $C = F$ and $C = \frac{5}{9}(F - 32) \Rightarrow F = \frac{5}{9}(F - 32) \Rightarrow 9F = 5F - 160 \Rightarrow 4F = -160 \Rightarrow$

$$F = -40.$$

(b) $F = 2C$ and $C = \frac{5}{9}(F - 32) \Rightarrow C = \frac{5}{9}(2C - 32) \Rightarrow 9C = 10C - 160 \Rightarrow$

$$C = 160 \text{ and hence, } F = 320.$$

75 $s = \dfrac{v_2 - v_1}{h_2 - h_1} \Rightarrow 0.07 = \dfrac{v_2 - 22}{185 - 0} \Rightarrow 0.07(185) = v_2 - 22 \Rightarrow v_2 = 22 + 12.95 = 34.95$ mi/hr.

76 From Exercise 75, the average wind shear is $s = \dfrac{v_2 - v_1}{h_2 - h_1}$.

We know $v_1 = 32$ at $h_1 = 20$. We need to find v_2 at $h_2 = 200$. $\dfrac{v_1}{v_2} = \left(\dfrac{h_1}{h_2}\right)^P \Rightarrow \dfrac{v_2}{v_1} = \left(\dfrac{h_2}{h_1}\right)^P \Rightarrow$

$$v_2 = v_1\left(\frac{h_2}{h_1}\right)^P = 32\left(\frac{200}{20}\right)^{0.13}. \text{ Thus, } s = \frac{32(10^{0.13}) - 32}{200 - 20} \approx 0.062 \text{ (mi/hr)/ft.}$$

77 The slope of AB is $\dfrac{-1.11905 - (-1.3598)}{-0.55 - (-1.3)} = 0.321$. Similarly, the slopes of BC and CD are also 0.321.

Therefore, the four points all lie on the same line. Since the common slope is 0.321, let $a = 0.321$.

$y = 0.321x + b \Rightarrow -1.3598 = 0.321(-1.3) + b \Rightarrow b = -0.9425.$

Thus, the points are linearly related by the equation $y = 0.321x - 0.9425$.

78 The slopes of AB and BC are both -0.44, whereas the slope of CD is approximately -1.107.

Thus, the points do not lie on the same line.

79 $x - 3y = -58 \Leftrightarrow y = (x + 58)/3$ and $3x - y = -70 \Leftrightarrow y = 3x + 70$. Assign $(x + 58)/3$ to Y_1 and $3x + 70$ to Y_2. Using a standard viewing rectangle, we don't see the lines. Zooming out gives us an indication where the lines intersect and by using an intersect feature, we find that the lines intersect at $(-19, 13)$.

$[-30, 3, 2]$ by $[-2, 20, 2]$

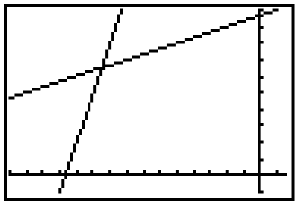


Figure 79

$[-12, 12]$ by $[-2, 14]$

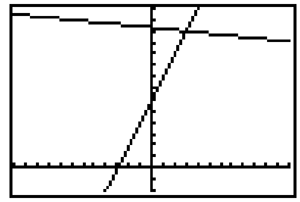


Figure 80

80 $x + 10y = 123 \quad \Leftrightarrow \quad y = (-x + 123)/10$ and $2x - y = -6 \quad \Leftrightarrow \quad y = 2x + 6$. Assign $(-x + 123)/10$ to Y_1 and $y = 2x + 6$ to Y_2. Similarly to Exercise 79, the lines intersect at $(3, 12)$.

81 From the graph, we can see that the points of intersection are $A(-0.8, -0.6)$, $B(4.8, -3.4)$, and $C(2, 5)$. The lines intersecting at A are perpendicular since they have slopes of 2 and $-\frac{1}{2}$. Since $d(A, B) = \sqrt{39.2}$ and $d(A, C) = \sqrt{39.2}$, the triangle is isosceles. Thus, the polygon is a right isosceles triangle.

$[-15, 15]$ by $[-10, 10]$

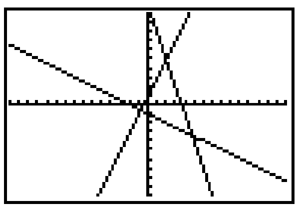


Figure 81

$[-3, 3]$ by $[-2, 2]$

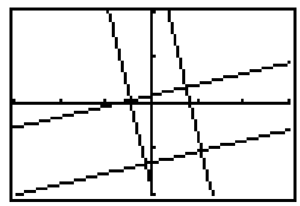


Figure 82

82 The equations of the lines can be rewritten as $y = \frac{1}{4.2}x + 0.17$, $y = -4.2x - 1.9$, $y = \frac{1}{4.2}x - 1.3$, and $y = -4.2x + 3.5$. From the graph, we can see that the points of intersection are approximately $A(0.75, 0.35)$, $B(1.08, -1.04)$, $C(-0.14, -1.33)$, and $D(-0.47, 0.059)$. The first and third lines are parallel as are the second and fourth lines. In addition, these pairs of lines are perpendicular to each other since their slopes are -4.2 and $\frac{1}{4.2}$. Since $d(A, B) \approx 1.43$ and $d(A, D) \approx 1.25$, it is not a square. Thus, the polygon is a rectangle.

83 The data appear to be linear. Using the two arbitrary points $(0.6, 1.3)$ and $(4.6, 8.5)$, the slope of the line is $\dfrac{8.5 - 1.3}{4.6 - 0.6} = 1.8$. An equation of the line is $y - 1.3 = 1.8(x - 0.6) \quad \Rightarrow \quad y = 1.8x + 0.22$.
If we find the regression line on a calculator, we get the model $y \approx 1.84589x + 0.21027$.

$[0, 5]$ by $[0, 10]$

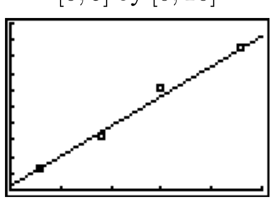


Figure 83

$[0, 5]$ by $[0, 3.5]$

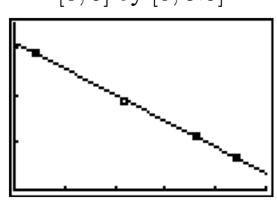


Figure 84

84 The data appear to be linear. Using the two arbitrary points $(0.4, 2.88)$ and $(4.4, 0.68)$, the slope of the line is $\dfrac{0.68 - 2.88}{4.4 - 0.4} = -0.55$. An equation of the line is $y - 2.88 = -0.55(x - 0.4) \quad \Rightarrow \quad y = -0.55x + 3.1$.
If we find the regression line on a calculator, we get the model $y \approx -0.54951x + 3.09621$.

$\boxed{85}$ **(a)** Plot the points with the form (Year, Distance): $(1911, 15.52)$, $(1932, 15.72)$, $(1955, 16.56)$, $(1975, 17.89)$, and $(1995, 18.29)$.

(b) To find a first approximation for the line use the arbitrary points $(1911, 15.52)$ and $(1995, 18.29)$. The resulting line is $D_1 \approx 0.033Y - 47.545$. Adjustments may be made to this equation. If we find the regression line on a calculator, we get the model $D \approx 0.03648Y - 54.47409$.

(c) Using D_1 from part (b), when $x = 1985$, $y = 17.96$ (so the distance is 17.96 meters)—almost equal to the actual record of 17.97 meters.

$[1900, 2010, 20]$ by $[15, 20]$

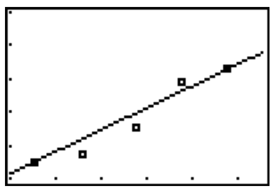


Figure 85

$[1900, 2010, 20]$ by $[210, 260, 10]$

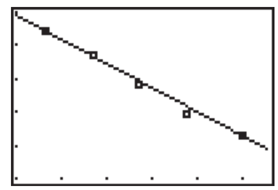


Figure 86

$\boxed{86}$ **(b)** For an approximation for the line, we'll use the first and last points given; that is, $(1913, 254.4)$ and $(1999, 223.1)$. These points determine the line $T - 254.4 = \dfrac{223.1 - 254.4}{1999 - 1913}(Y - 1913) \quad \Rightarrow$
$T \approx -0.36395Y + 950.643$. The regression line is $y \approx -0.37455x + 970.58262$.

(c) $T = -0.36395(1985) + 950.643 \approx 228.2$ seconds, which is off by 1.9 seconds.

(d) The slope of the line is approximately -0.4. This means that *on the average*, the record time for the mile has decreased by 0.4 sec/yr.

2.4 Exercises

$\boxed{1}$ $f(x) = -x^2 - x - 4 \quad \Rightarrow \quad f(-2) = -4 + 2 - 4 = -6$, $f(0) = -4$, and $f(4) = -16 - 4 - 4 = -24$.

$\boxed{2}$ $f(x) = -x^3 - x^2 + 3 \quad \Rightarrow \quad f(-3) = 27 - 9 + 3 = 21$, $f(0) = 3$, and $f(2) = -8 - 4 + 3 = -9$.

$\boxed{3}$ $f(x) = \sqrt{x-2} + 3x \quad \Rightarrow \quad f(1) = \sqrt{1-2} + 3(1) = \sqrt{-1} + 3$ is undefined for the real numbers.
Similarly, $f(6) = \sqrt{6-2} + 3(6) = \sqrt{4} + 18 = 2 + 18 = 20$
and $f(11) = \sqrt{11-2} + 3(11) = \sqrt{9} + 33 = 3 + 33 = 36$.
Note that $f(a)$, for any $a < 2$, would be undefined.

$\boxed{4}$ $f(x) = x - \sqrt{3-x} \quad \Rightarrow \quad f(-1) = -1 - \sqrt{3-(-1)} = -1 - \sqrt{4} = -1 - 2 = -3$
Similarly, $f(2) = 2 - \sqrt{3-2} = 2 - \sqrt{1} = 2 - 1 = 1$
and $f(5) = 5 - \sqrt{3-5} = 5 - \sqrt{-2}$ is undefined for the real numbers.
Note that $f(a)$, for any $a > 3$, would be undefined.

$\boxed{5}$ $f(x) = \dfrac{x}{x-3} \quad \Rightarrow \quad f(-2) = \dfrac{-2}{-5} = \dfrac{2}{5}$, $f(0) = \dfrac{0}{-3} = 0$, and $f(3) = \dfrac{3}{0}$ is undefined.

$\boxed{6}$ $f(x) = \dfrac{2x}{x+2} \quad \Rightarrow \quad f(-2) = \dfrac{-4}{0}$ is undefined, $f(0) = \dfrac{0}{2} = 0$, and $f(2) = \dfrac{4}{4} = 1$.

$\boxed{7}$ **(a)** $f(x) = 5x - 2 \quad \Rightarrow \quad f(a) = 5(a) - 2 = 5a - 2$ **(b)** $f(-a) = 5(-a) - 2 = -5a - 2$

(c) $-f(a) = -1 \cdot (5a - 2) = -5a + 2$ **(d)** $f(a+h) = 5(a+h) - 2 = 5a + 5h - 2$

(e) $f(a)+f(h)=(5a-2)+(5h-2)=5a+5h-4$

(f) Using parts (d) and (a), $\dfrac{f(a+h)-f(a)}{h}=\dfrac{(5a+5h-2)-(5a-2)}{h}=\dfrac{5h}{h}=5.$

$\boxed{8}$ **(a)** $f(x)=1-4x \Rightarrow f(a)=1-4(a)=1-4a$ **(b)** $f(-a)=1-4(-a)=1+4a$

(c) $-f(a)=-1\cdot(1-4a)=4a-1$ **(d)** $f(a+h)=1-4(a+h)=1-4a-4h$

(e) $f(a)+f(h)=(1-4a)+(1-4h)=2-4a-4h$

(f) Using parts (d) and (a), $\dfrac{f(a+h)-f(a)}{h}=\dfrac{(1-4a-4h)-(1-4a)}{h}=\dfrac{-4h}{h}=-4.$

$\boxed{9}$ **(a)** $f(x)=-x^2+3 \Rightarrow f(a)=-(a)^2+3=-a^2+3$

(b) $f(-a)=-(-a)^2+3=-a^2+3$ **(c)** $-f(a)=-1\cdot(-a^2+3)=a^2-3$

(d) $f(a+h)=-(a+h)^2+3=-(a^2+2ah+h^2)+3=-a^2-2ah-h^2+3$

(e) $f(a)+f(h)=(-a^2+3)+(-h^2+3)=-a^2-h^2+6$

(f) $\dfrac{f(a+h)-f(a)}{h}=\dfrac{(-a^2-2ah-h^2+3)-(-a^2+3)}{h}=\dfrac{-2ah-h^2}{h}=\dfrac{h(-2a-h)}{h}=-2a-h$

$\boxed{10}$ **(a)** $f(x)=3-x^2 \Rightarrow f(a)=3-(a)^2=3-a^2$ **(b)** $f(-a)=3-(-a)^2=3-a^2$

(c) $-f(a)=-1\cdot(3-a^2)=-3+a^2$

(d) $f(a+h)=3-(a+h)^2=3-(a^2+2ah+h^2)=3-a^2-2ah-h^2$

(e) $f(a)+f(h)=(3-a^2)+(3-h^2)=6-a^2-h^2$

(f) $\dfrac{f(a+h)-f(a)}{h}=\dfrac{(3-a^2-2ah-h^2)-(3-a^2)}{h}=\dfrac{-2ah-h^2}{h}=\dfrac{h(-2a-h)}{h}=-2a-h$

$\boxed{11}$ **(a)** $f(x)=x^2-x+3 \Rightarrow f(a)=(a)^2-(a)+3=a^2-a+3$

(b) $f(-a)=(-a)^2-(-a)+3=a^2+a+3$ **(c)** $-f(a)=-1\cdot(a^2-a+3)=-a^2+a-3$

(d) $f(a+h)=(a+h)^2-(a+h)+3=a^2+2ah+h^2-a-h+3$

(e) $f(a)+f(h)=(a^2-a+3)+(h^2-h+3)=a^2+h^2-a-h+6$

(f) $\dfrac{f(a+h)-f(a)}{h}=\dfrac{(a^2+2ah+h^2-a-h+3)-(a^2-a+3)}{h}=\dfrac{2ah+h^2-h}{h}=\dfrac{h(2a+h-1)}{h}$
$=2a+h-1$

$\boxed{12}$ **(a)** $f(x)=2x^2+3x-7 \Rightarrow f(a)=2(a)^2+3(a)-7=2a^2+3a-7$

(b) $f(-a)=2(-a)^2+3(-a)-7=2a^2-3a-7$

(c) $-f(a)=-1\cdot(2a^2+3a-7)=-2a^2-3a+7$

(d) $f(a+h)=2(a+h)^2+3(a+h)-7=2(a^2+2ah+h^2)+3a+3h-7=$
$2a^2+4ah+2h^2+3a+3h-7$

(e) $f(a)+f(h)=(2a^2+3a-7)+(2h^2+3h-7)=2a^2+2h^2+3a+3h-14$

(f) $\dfrac{f(a+h)-f(a)}{h}=\dfrac{(2a^2+4ah+2h^2+3a+3h-7)-(2a^2+3a-7)}{h}$
$=\dfrac{4ah+2h^2+3h}{h}=\dfrac{h(4a+2h+3)}{h}=4a+2h+3$

$\boxed{13}$ **(a)** $g(x)=4x^2 \Rightarrow g\left(\dfrac{1}{a}\right)=4\left(\dfrac{1}{a}\right)^2=4\cdot\dfrac{1}{a^2}=\dfrac{4}{a^2}$ **(b)** $g(a)=4a^2 \Rightarrow \dfrac{1}{g(a)}=\dfrac{1}{4a^2}$

(c) $g(\sqrt{a})=4(\sqrt{a})^2=4a$ **(d)** $\sqrt{g(a)}=\sqrt{4a^2}=2|a|=2a$ since $a>0$

$\boxed{14}$ **(a)** $g(x)=2x-7 \Rightarrow g\left(\dfrac{1}{a}\right)=2\left(\dfrac{1}{a}\right)-7=\dfrac{2}{a}-7=\dfrac{2-7a}{a}$ **(b)** $\dfrac{1}{g(a)}=\dfrac{1}{2(a)-7}=\dfrac{1}{2a-7}$

(c) $g(\sqrt{a})=2(\sqrt{a})-7=2\sqrt{a}-7$ **(d)** $\sqrt{g(a)}=\sqrt{2a-7}$

$\boxed{15}$ **(a)** $g(x)=\dfrac{2x}{x^2+1} \Rightarrow g\left(\dfrac{1}{a}\right)=\dfrac{2(1/a)}{(1/a)^2+1}=\dfrac{2/a}{1/a^2+1}\cdot\dfrac{a^2}{a^2}=\dfrac{2a}{1+a^2}=\dfrac{2a}{a^2+1}$

(b) $\dfrac{1}{g(a)}=\dfrac{1}{\dfrac{2a}{a^2+1}}=\dfrac{a^2+1}{2a}$ **(c)** $g(\sqrt{a})=\dfrac{2\sqrt{a}}{(\sqrt{a})^2+1}=\dfrac{2\sqrt{a}}{a+1}$

(d) $\sqrt{g(a)}=\sqrt{\dfrac{2a}{a^2+1}}\cdot\dfrac{\sqrt{a^2+1}}{\sqrt{a^2+1}}=\dfrac{\sqrt{2a(a^2+1)}}{a^2+1}$, or, equivalently, $\dfrac{\sqrt{2a^3+2a}}{a^2+1}$

$\boxed{16}$ **(a)** $g(x)=\dfrac{x^2}{x+1} \Rightarrow g\left(\dfrac{1}{a}\right)=\dfrac{\left(\dfrac{1}{a}\right)^2}{\dfrac{1}{a}+1}=\dfrac{\dfrac{1}{a^2}}{\dfrac{1+a}{a}}\cdot\dfrac{a^2}{a^2}=\dfrac{1}{a(a+1)}$ **(b)** $\dfrac{1}{g(a)}=\dfrac{1}{\dfrac{a^2}{a+1}}=\dfrac{a+1}{a^2}$

(c) $g(\sqrt{a})=\dfrac{(\sqrt{a})^2}{\sqrt{a}+1}\cdot\dfrac{\sqrt{a}-1}{\sqrt{a}-1}=\dfrac{a(\sqrt{a}-1)}{a-1}$ **(d)** $\sqrt{g(a)}=\sqrt{\dfrac{a^2}{a+1}}\cdot\sqrt{\dfrac{a+1}{a+1}}=\dfrac{a\sqrt{a+1}}{a+1}$

$\boxed{17}$ All vertical lines intersect the graph in at most one point,

so the graph *is* the graph of a function because it passes the Vertical Line Test.

$\boxed{18}$ At least one vertical line intersects the graph in more than one point,

so the graph *is not* the graph of a function because it fails the Vertical Line Test.

$\boxed{19}$ The domain D is the set of x-values; that is, $D=[-4,1]\cup[2,4)$. Note that the solid dots on the figure correspond to using brackets {including}, whereas the open dot corresponds to using parentheses {excluding}. The range R is the set of y-values; that is, $R=[-3,3)$.

$\boxed{20}$ Domain $D=\{x\text{-values}\}=[-4,4)$. Range $R=\{y\text{-values}\}=(-3,-1)\cup[1,3]$.

$\boxed{21}$ **(a)** The domain of a function f is the set of all x-values for which the function is defined. In this case, the graph extends from $x=-3$ to $x=4$. Hence, the domain is $[-3,4]$.

(b) The range of a function f is the set of all y-values that the function takes on. In this case, the graph includes all values from $y=-2$ to $y=2$. Hence, the range is $[-2,2]$.

(c) $f(1)$ is the y-value of f corresponding to $x=1$. In this case, $f(1)=0$.

(d) If we were to draw the horizontal line $y=1$ on the same coordinate plane, it would intersect the graph at $x=-1,\frac{1}{2}$, and 2. Hence, $f(x)=1 \Rightarrow x=-1,\frac{1}{2}$, and 2.

(e) The function is above 1 between $x=-1$ and $x=\frac{1}{2}$, and also to the right of $x=2$. Hence, $f(x)>1 \Rightarrow x\in\left(-1,\frac{1}{2}\right)\cup(2,4]$.

22 **(a)** $[-5, 7]$ **(b)** $[-1, 2]$ **(c)** $f(1) = -1$

(d) $f(x) = 1 \ \Rightarrow \ x = -3, -1, 3, 5$ **(e)** $f(x) > 1 \ \Rightarrow \ x \in (-3, -1) \cup (3, 5)$

Note: In Exercises 23–0, we need to make sure that the radicand {the expression under the radical sign} is greater than or equal to zero and that the denominator is not equal to zero.

23 $f(x) = \sqrt{2x+7}$ • $2x + 7 \geq 0 \ \Rightarrow \ 2x \geq -7 \ \Rightarrow \ x \geq -\frac{7}{2} \ \Leftrightarrow \ \left[-\frac{7}{2}, \infty\right)$

24 $f(x) = \sqrt{4-3x}$ • $4 - 3x \geq 0 \ \Rightarrow \ 4 \geq 3x \ \Rightarrow \ x \leq \frac{4}{3} \ \Leftrightarrow \ \left(-\infty, \frac{4}{3}\right]$

25 $f(x) = \sqrt{16-x^2}$ • $16 - x^2 \geq 0 \ \Rightarrow \ 16 \geq x^2 \ \Rightarrow \ x^2 \leq 16 \ \Rightarrow \ |x| \leq 4 \ \Rightarrow$
$-4 \leq x \leq 4$, or $[-4, 4]$ in interval notation.

26 $f(x) = \sqrt{x^2-25}$ • $x^2 - 25 \geq 0 \ \Rightarrow \ |x| \geq 5 \ \Rightarrow \ x \geq 5$ or $x \leq -5 \ \Leftrightarrow \ (-\infty, -5] \cup [5, \infty)$

27 $f(x) = \sqrt{4x^2-9}$ • $4x^2 - 9 \geq 0 \ \Rightarrow \ x^2 \geq \frac{9}{4} \ \Rightarrow \ |x| \geq \frac{3}{2} \ \Rightarrow \ x \geq \frac{3}{2}$ or $x \leq -\frac{3}{2}$,
or $\left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$ in interval notation.

28 $f(x) = \sqrt{16-25x^2}$ • $16 - 25x^2 \geq 0 \ \Rightarrow \ 16 \geq 25x^2 \ \Rightarrow \ 25x^2 \leq 16 \ \Rightarrow \ x^2 \leq \frac{16}{25} \ \Rightarrow \ |x| \leq \frac{4}{5} \ \Rightarrow$
$-\frac{4}{5} \leq x \leq \frac{4}{5}$, or $\left[-\frac{4}{5}, \frac{4}{5}\right]$ in interval notation.

29 $f(x) = \dfrac{x+1}{x^3 - 9x}$ • For this function we must have the denominator not equal to 0. The denominator is $x^3 - 9x = x(x^2 - 9) = x(x+3)(x-3)$, so $x \neq 0, -3, 3$. The solution is then all real numbers *except* $0, -3, 3$. In interval notation, we have $(-\infty, -3) \cup (-3, 0) \cup (0, 3) \cup (3, \infty)$. We could also denote this solution as $\mathbb{R} - \{\pm 3, 0\}$.

30 $f(x) = \dfrac{4x}{6x^2 + 13x - 5}$ • $6x^2 + 13x - 5 = 0 \ \Rightarrow \ (2x+5)(3x-1) = 0 \ \Rightarrow \ \mathbb{R} - \left\{-\frac{5}{2}, \frac{1}{3}\right\}$

31 $f(x) = \dfrac{\sqrt{2x-5}}{x^2 - 5x + 4}$ • For this function we must have the radicand greater than or equal to 0 *and* the denominator not equal to 0. The radicand is greater than or equal to 0 if $2x - 5 \geq 0$, or, equivalently, $x \geq \frac{5}{2}$. The denominator is $(x-1)(x-4)$, so $x \neq 1, 4$. The solution is then all real numbers greater than or equal to $\frac{5}{2}$, excluding 4. In interval notation, we have $\left[\frac{5}{2}, 4\right) \cup (4, \infty)$.

32 $f(x) = \dfrac{\sqrt{4x-3}}{x^2 - 4}$ • $x^2 - 4 = 0 \ \Rightarrow \ (x+2)(x-2) = 0 \ \Rightarrow \ x = \pm 2$;
$4x - 3 \geq 0 \ \Rightarrow \ x \geq \frac{3}{4}$, so the domain is $\left[\frac{3}{4}, 2\right) \cup (2, \infty)$

33 $f(x) = \dfrac{x-4}{\sqrt{x-2}}$ • For this function we must have $x - 2 > 0 \ \Rightarrow \ x > 2$.
Note that " $>$ " must be used since the denominator cannot *equal* 0. In interval notation, we have $(2, \infty)$.

34 $f(x) = \dfrac{1}{(x-3)\sqrt{x+3}}$ • $x + 3 > 0 \ \Rightarrow \ x > -3 \ \{x \neq 3\}$, so the domain is $(-3, 3) \cup (3, \infty)$

35 $f(x) = \sqrt{x+3} + \sqrt{3-x}$ • We must have $x + 3 \geq 0 \ \Rightarrow \ x \geq -3$ and $3 - x \geq 0 \ \Rightarrow \ x \leq 3$.
The domain is the intersection of $x \geq -3$ and $x \leq 3$, that is, $[-3, 3]$.

36 $f(x) = \sqrt{1-x} + \sqrt{x+5}$ • We must have $1 - x \geq 0 \;\; \Rightarrow \;\; x \leq 1$ and $x + 5 \geq 0 \;\; \Rightarrow \;\; x \geq -5$.
The domain is the intersection of $x \leq 1$ and $x \geq -5$, that is, $[-5, 1]$.

37 $f(x) = \sqrt{(x-2)(x-6)}$ • $(x-2)(x-6) \geq 0$ {see the sign chart} $\Rightarrow \;\; x \leq 2$ or $x \geq 6$.

Interval	$(-\infty, 2)$	$(2, 6)$	$(6, \infty)$
Sign of $x - 2$	$-$	$+$	$+$
Sign of $x - 6$	$-$	$-$	$+$
Resulting sign	$+$	$-$	$+$

In interval notation, we have $(-\infty, 2] \cup [6, \infty)$.

38 $f(x) = \sqrt{(x+2)(x+4)}$ • $(x+2)(x+4) \geq 0 \;\; \Rightarrow \;\; x \leq -4$ or $x \quad -2$ {use a sign chart} $\Rightarrow$
$(-\infty, -4] \cup [-2, \infty)$

39 **(a)** $D = \{x\text{-values}\} = [-5, -3) \cup (-1, 1] \cup (2, 4]$; $R = \{y\text{-values}\} = \{-3\} \cup [-1, 4]$.
Note that the notation for including the single value -3 in R uses braces.

(b) ***f* is increasing** on an interval if it goes up as we move from left to right,
so f is increasing on $[-4, -3) \cup [3, 4]$.

***f* is decreasing** on an interval if it goes down as we move from left to right, so f is decreasing on $[-5, -4] \cup (2, 3]$. Note that the values $x = -4$ and $x = 3$ are in intervals that are listed as increasing and in intervals that are listed as decreasing.

***f* is constant** on an interval if the y-values do not change, so f is constant on $(-1, 1]$.

40 **(a)** $D = \{x\text{-values}\} = [-5, -3) \cup (-2, -1] \cup 0, 2) \cup (3, 5]$; $R = \{y\text{-values}\} = [-3, 3] \cup \{4\}$

(b) f is increasing on $[-5, -3) \cup (3, 4]$. f is decreasing on $[0, 2)$. f is constant on $(-2, -1] \cup [4, 5]$.

41 The graph of the function is increasing on $(-\infty, -3]$ and is decreasing on $[-3, 2]$, so there must be a high point at $x = -3$. Now the graph of the function is increasing on $[2, \infty)$, so there must be a low point at $x = 2$.

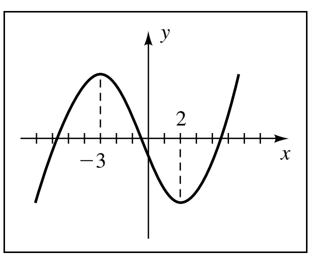


Figure 41

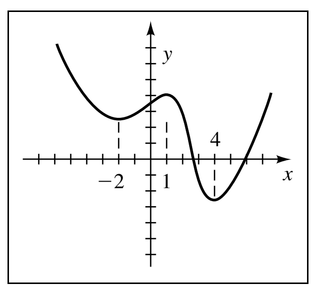


Figure 42

42 The graph of the function is decreasing on $(-\infty, -2]$ and $[1, 4]$, and is increasing on $[-2, 1]$ and $[4, \infty)$.

$\boxed{43}$ **(a)** $f(x) = -2x + 1$ • This is a line with slope -2 and y-intercept 1.

(b) The domain D and the range R are equal to $(-\infty, \infty)$.

(c) f is decreasing on its entire domain, that is, $(-\infty, \infty)$.

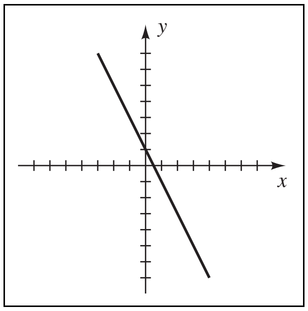


Figure 43

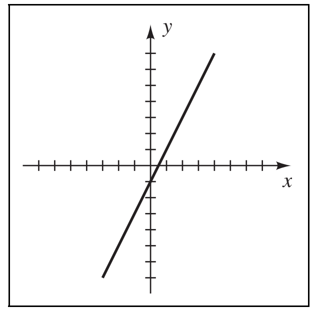


Figure 44

$\boxed{44}$ **(a)** $f(x) = 2x - 1$ • This is a line with slope 2 and y-intercept -1.

(b) $D = (-\infty, \infty)$, $R = (-\infty, \infty)$

(c) Increasing on $(-\infty, \infty)$

$\boxed{45}$ **(a)** To sketch the graph of $f(x) = 4 - x^2$, we can make use of the symmetry with respect to the y-axis.

x	± 4	± 3	± 2	± 1	0
y	-12	-5	0	3	4

(b) Since we can substitute any number for x, the domain is all real numbers, that is, $D = \mathbb{R}$. By examining the figure, we see that the values of y are at most 4. Hence, the range of f is all reals less than or equal to 4, that is, $R = (-\infty, 4]$.

(c) A common mistake is to confuse the function values, the y's, with the input values, the x's. We are not interested in the specific y-values for determining if the function is increasing, decreasing, or constant. We are only interested if the y-values are going up, going down, or staying the same. For the function $f(x) = 4 - x^2$, we say f *is increasing on* $(-\infty, 0]$ since the y-values are getting larger as we move from left to right over the x-values from $-\infty$ to 0. Also, f *is decreasing on* $[0, \infty)$ since the y-values are getting smaller as we move from left to right over the x-values from 0 to ∞. Note that this answer would have been the same if the function was $f(x) = 500 - x^2$, $f(x) = -300 - x^2$, or any function of the form $f(x) = a - x^2$, where a is any real number.

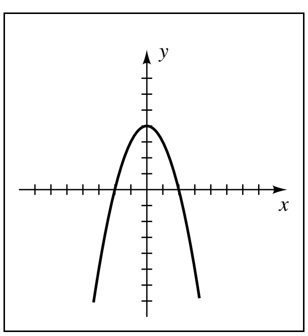


Figure 45

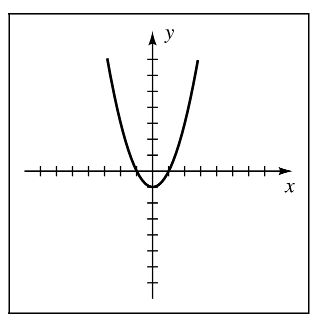


Figure 46

$\boxed{46}$ **(a)** $f(x) = x^2 - 1$ • **(b)** $D = (-\infty, \infty)$, $R = [-1, \infty)$

(c) Decreasing on $(-\infty, 0]$, increasing on $[0, \infty)$

$\boxed{47}$ **(a)** $f(x) = \sqrt{x-1}$ • This is half of a parabola opening to the right. It has an x-intercept of 1 and no y-intercept.

(b) $x - 1 \geq 0 \quad \Rightarrow \quad x \geq 1$, so the domain is $D = [1, \infty)$.

The y-values are all positive or zero, so the range is $R = [0, \infty)$.

(c) f is increasing on its entire domain, that is, on $[1, \infty)$.

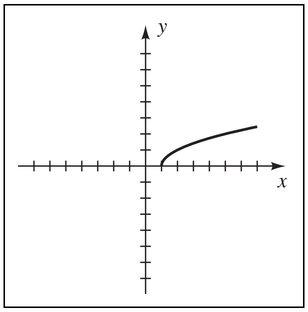


Figure 47

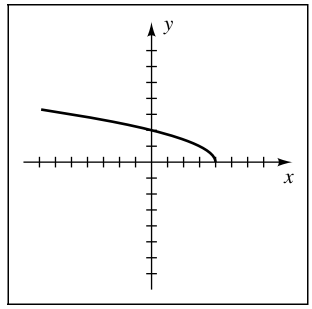


Figure 48

$\boxed{48}$ **(a)** $f(x) = x^2 - 1$ • **(b)** $D = (-\infty, 4]$, $R = [0, \infty)$ **(c)** Decreasing on $(-\infty, 4]$

$\boxed{49}$ **(a)** $f(x) = -4$ • This is a horizontal line with y-intercept -4.

(b) The domain is $D = (-\infty, \infty)$ and the range consists of a single value, so $R = \{-4\}$.

(c) f is constant on its entire domain, that is, $(-\infty, \infty)$.

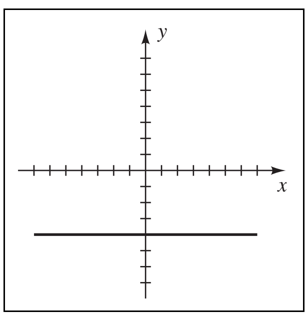


Figure 49

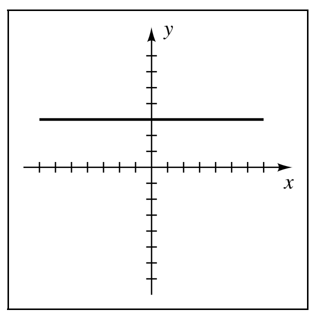


Figure 50

$\boxed{50}$ **(a)** $f(x) = 3$ • **(b)** $D = (-\infty, \infty)$, $R = \{3\}$ **(c)** Constant on $(-\infty, \infty)$

$\boxed{51}$ **(a)** We recognize $y = f(x) = -\sqrt{36 - x^2}$ as the lower half of the circle $x^2 + y^2 = 36$.

(b) To find the domain, we solve $36 - x^2 \geq 0$. $36 - x^2 \geq 0 \quad \Rightarrow \quad x^2 \leq 36 \quad \Rightarrow \quad |x| \leq 6 \quad \Rightarrow \quad D = [-6, 6]$.

From the figure, we see that the y-values vary from $y = -6$ to $y = 0$. Hence, the range R is $[-6, 0]$.

(c) As we move from left to right, for $x = -6$ to $x = 0$, the y-values are decreasing. From $x = 0$ to $x = 6$, the y-values increase. Hence, f is decreasing on $[-6, 0]$ and increasing on $[0, 6]$.

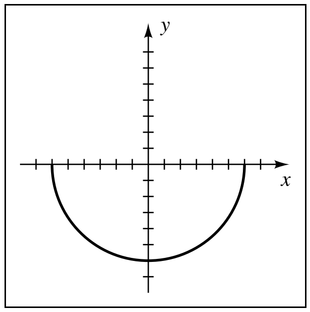


Figure 51

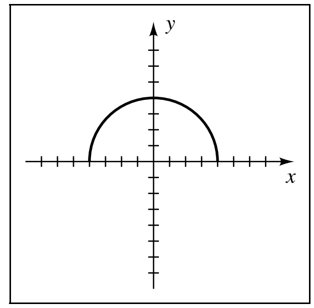


Figure 52

$\boxed{52}$ **(a)** $f(x) = \sqrt{16 - x^2}$ • **(b)** $D = [-4, 4]$, $R = [0, 4]$ **(c)** Increasing on $[-4, 0]$, decreasing on $[0, 4]$

$\boxed{53}$ $f(x) = x^2 - 6x$, so $f(2) = 2^2 - 6(2) = 4 - 12 = -8$.

$$\frac{f(2+h) - f(2)}{h} = \frac{\left[(2+h)^2 - 6(2+h)\right] - (-8)}{h} = \frac{4 + 4h + h^2 - 12 - 6h + 8}{h} = \frac{h - 2h}{h} = \frac{h(h-2)}{h}$$
$$= h - 2$$

$\boxed{54}$ $f(x) = -2x^2 + 5$, so $f(2) = -2(2)^2 + 5 = -2 \cdot 4 + 5 = -8 + 5 = -3$.

$$\frac{f(2+h) - f(2)}{h} = \frac{\left[-2(2+h)^2 + 5\right] - (-3)}{h} = \frac{-2(4 + 4h + h^2) + 5 + 3}{h} = \frac{-8 - 8h - 2h^2 + 8}{h}$$
$$= \frac{-8h - 2h^2}{h} = \frac{h(-8 - 2h)}{h} = -2h - 8$$

$\boxed{55}$ $$\frac{f(x+h) - f(x)}{h} = \frac{\dfrac{4}{x+h} - \dfrac{4}{x}}{h} = \frac{\dfrac{4x - 4(x+h)}{(x+h)x}}{h} = \frac{4x - 4x - 4h}{(x+h)xh} = \frac{-4h}{x(x+h)h} = -\frac{4}{x(x+h)}$$

$\boxed{56}$ $$\frac{f(x+h) - f(x)}{h} = \frac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h} = \frac{\dfrac{x^2 - (x+h)^2}{(x+h)^2 x^2}}{h} = \frac{x^2 - (x^2 + 2xh + h^2)}{hx^2(x+h)^2}$$
$$= \frac{-2xh - h^2}{hx^2(x+h)^2} = -\frac{h(2x+h)}{hx^2(x+h)^2} = -\frac{2x+h}{x^2(x+h)^2}$$

$\boxed{57}$ $$\frac{f(x) - f(a)}{x - a} = \frac{\sqrt{x-3} - \sqrt{a-3}}{x - a} = \frac{\sqrt{x-3} - \sqrt{a-3}}{x - a} \cdot \frac{\sqrt{x-3} + \sqrt{a-3}}{\sqrt{x-3} + \sqrt{a-3}}$$
$$= \frac{(x-3) - (a-3)}{(x-a)\left(\sqrt{x-3} + \sqrt{a-3}\right)} = \frac{x - a}{(x-a)\left(\sqrt{x-3} + \sqrt{a-3}\right)} = \frac{1}{\sqrt{x-3} + \sqrt{a-3}}$$

$\boxed{58}$ $$\frac{f(x) - f(a)}{x - a} = \frac{(x^3 - 2) - (a^3 - 2)}{x - a} = \frac{x^3 - a^3}{x - a} = \frac{(x-a)(x^2 + ax + a^2)}{x - a} = x^2 + ax + a^2$$

$\boxed{59}$ As in Example 7, $a = \dfrac{2 - 1}{3 - (-3)} = \dfrac{1}{6}$ and f has the form $f(x) = \dfrac{1}{6}x + b$.

$f(3) = \frac{1}{6}(3) + b = \frac{1}{2} + b$. But $f(3) = 2$, so $\frac{1}{2} + b = 2 \Rightarrow b = \frac{3}{2}$, and $f(x) = \frac{1}{6}x + \frac{3}{2}$.

$\boxed{60}$ As in Example 7, $a = \dfrac{-2 - 7}{4 - (-2)} = -\dfrac{9}{6} = -\dfrac{3}{2}$ and f has the form $f(x) = -\dfrac{3}{2}x + b$.

$f(-2) = -\frac{3}{2}(-2) + b = 3 + b$. But $f(-2) = 7$, so $3 + b = 7 \Rightarrow b = 4$, and $f(x) = -\frac{3}{2}x + 4$.

Note: For Exercises 61–70, a good question to consider is, "Given a particular value of x, can a unique value of y be found?" If the answer is yes, the value of y (general formula) is given. If not, two ordered pairs satisfying the relation having x in the first position are given.

61 $3y = x^2 + 7 \quad\Rightarrow\quad y = \dfrac{x^2+7}{3}$, which is a function

62 $x = 3y + 2 \quad\Rightarrow\quad x - 2 = 3y \quad\Rightarrow\quad y = \dfrac{x-2}{3}$, which is a function

63 $x^2 + y^2 = 4 \quad\Rightarrow\quad y^2 = 4 - x^2 \quad\Rightarrow\quad y = \pm\sqrt{4-x^2}$, not a function: $(0, \pm 2)$

64 $y^2 - x^2 = 4 \quad\Rightarrow\quad y^2 = 4 + x^2 \quad\Rightarrow\quad y = \pm\sqrt{4+x^2}$, not a function: $(0, \pm 2)$

65 $y = 5$ is a function since for any x, $(x, 5)$ is the only ordered pair in W having x in the first position.

66 $x = 3$ is not a function: $(3, 0)$ and $(3, 1)$

67 Any ordered pair with x-coordinate 0 satisfies $xy = 0$. Two such ordered pairs are $(0, 0)$ and $(0, 1)$. Not a function

68 $x + y = 0 \quad\Rightarrow\quad y = -x$, which is a function

69 $|y| = |x| \quad\Rightarrow\quad \pm y = \pm x \quad\Rightarrow\quad y = \pm x$, not a function: $(1, \pm 1)$

70 Many ordered pairs with x-coordinate 3 (or any other number) satisfy $y < x$.

Two such ordered pairs are $(3, 1)$ and $(3, 2)$. Not a function

71
$$\begin{aligned} V = lwh &= (30 - x - x)(20 - x - x)(x) \\ &= (30 - 2x)(20 - 2x)(x) = 2(15 - x) \cdot 2(10 - x)(x) = 4x(15 - x)(10 - x) \end{aligned}$$

72 $S = 2\pi rh + 2(2\pi r^2) = 2\pi r(10) + 4\pi r^2 = 20\pi r + 4\pi r^2 = 4\pi r(5 + r)$

73 **(a)** The formula for the area of a rectangle is $A = lw$ {Area = length $\times$ width}.

$$A = 500 \quad\Rightarrow\quad xy = 500 \quad\Rightarrow\quad y = \frac{500}{x}$$

(b) We need to determine the number of linear feet (P) first. There are two walls of length y, two walls of length $(x - 3)$, and one wall of length x, so $P =$ Linear feet of wall $= x + 2(y) + 2(x - 3) = 3x + 2\left(\dfrac{500}{x}\right) - 6$.

The cost C is 100 times P, so $C = 100P = 300x + \dfrac{100{,}000}{x} - 600$.

74 **(a)** $V = lwh \quad\Rightarrow\quad 6 = xy(1.5) \quad\Rightarrow\quad xy = 4 \quad\Rightarrow\quad y = \dfrac{4}{x}$

(b) Surface area $S = xy + 2(1.5)x + 2(1.5)y = x\left(\dfrac{4}{x}\right) + 3x + 3\left(\dfrac{4}{x}\right) = 4 + 3x + \dfrac{12}{x}$

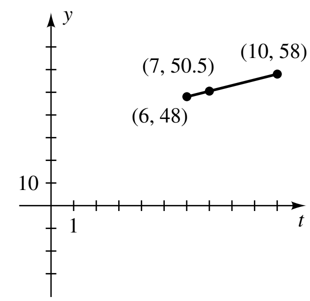
75 The expression $(h - 25)$ represents the number of feet *above* 25 feet.

$S(h) = 6(h - 25) + 100 = 6h - 150 + 100 = 6h - 50$.

76 $T(x) = \dfrac{125{,}000 \text{ BTUs}}{1 \text{ gallon of gas}} \cdot x \text{ gallons} \cdot \dfrac{\$0.342}{1{,}000{,}000 \text{ BTU}} = \$0.04275x$.

77 **(a)** Using $(6, 48)$ and $(7, 50.5)$, we have

$$y - 48 = \frac{50.5 - 48}{7 - 6}(t - 6), \text{ or } y = 2.5t + 33.$$



(b) The slope represents the yearly increase in height, 2.5 in./yr.

(c) $t = 10 \quad \Rightarrow \quad y = 2.5(10) + 33 = 58$ in.

78 Let A denote the area of the contamination. A is linearly related to t, so $A = at + b$.
$A = 0$ when $t = 0 \quad \Rightarrow \quad b = 0$. $A = 40{,}000$ when $t = 40 \quad \Rightarrow \quad a = 1000$.
Thus, $A = 1000t$. Since the contamination is circular, $A = \pi r^2$. Hence, $\pi r^2 = 1000t \quad \Rightarrow \quad r = \sqrt{\dfrac{1000t}{\pi}}$.

79 The height of the balloon is $2t$. Using the Pythagorean theorem,

$$d^2 = 100^2 + (2t)^2 = 10{,}000 + 4t^2 \quad \Rightarrow \quad d = \sqrt{4(2500 + t^2)} \quad \Rightarrow \quad d = 2\sqrt{t^2 + 2500}.$$

80 **(a)** By the Pythagorean theorem, $x^2 + y^2 = 15^2 \quad \Rightarrow \quad y = \sqrt{225 - x^2}$.

(b) $\mathcal{A} = \frac{1}{2}bh = \frac{1}{2}xy = \frac{1}{2}x\sqrt{225 - x^2}$.
The domain of this function is $-15 \le x \le 15$; however, only $0 < x < 15$ will form triangles.

81 **(a)** CTP forms a right angle, so the Pythagorean theorem may be applied.

$$(CT)^2 + (PT)^2 = (PC)^2 \quad \Rightarrow \quad r^2 + y^2 = (h + r)^2 \quad \Rightarrow$$
$$r^2 + y^2 = h^2 + 2hr + r^2 \quad \Rightarrow \quad y^2 = h^2 + 2hr \; \{y > 0\} \quad \Rightarrow \quad y = \sqrt{h^2 + 2hr}$$

(b) $y = \sqrt{(200)^2 + 2(4000)(200)} = \sqrt{(200)^2(1 + 40)} = 200\sqrt{41} \approx 1280.6$ mi

82 **(a)** The dimensions L, 50, and $x - 2$ form a right triangle 2 feet off the ground.
Thus, $L^2 = 50^2 + (x - 2)^2 \quad \Rightarrow \quad L = \sqrt{2500 + (x - 2)^2}$.

(b) $L = 75 \quad \Rightarrow \quad 75^2 = 50^2 + (x - 2)^2 \quad \Rightarrow \quad x - 2 = \pm\sqrt{3125} \quad \Rightarrow \quad x = 25\sqrt{5} + 2 \approx 57.9$ ft

83 Form a right triangle with the control booth and the beginning of the runway. Let y denote the distance from the control booth to the beginning of the runway and apply the Pythagorean theorem. $y^2 = 300^2 + 20^2 \quad \Rightarrow \quad y^2 = 90{,}400$. Now form a right triangle, in a different plane, with sides y and x and hypotenuse d. Then $d^2 = y^2 + x^2 \quad \Rightarrow \quad d^2 = 90{,}400 + x^2 \quad \Rightarrow \quad d = \sqrt{90{,}400 + x^2}$.

84 $\text{Time}_{\text{total}} = \text{Time}_{\text{rowing}} + \text{Time}_{\text{walking}}$ {use $t = d/r$ and $d(A, P) = 6 - x$} $\Rightarrow$

$$T = \frac{\sqrt{2^2 + (6 - x)^2}}{3} + \frac{x}{5} \quad \Rightarrow \quad T = \frac{\sqrt{x^2 - 12x + 40}}{3} + \frac{x}{5}$$

85 **(b)** The maximum y-value of 0.75 occurs when $x \approx 0.55$ and the minimum y-value of -0.75 occurs when $x \approx -0.55$. Therefore, the range of f is approximately $[-0.75, 0.75]$.

(c) f is decreasing on $[-2, -0.55]$ and on $[0.55, 2]$. f is increasing on $[-0.55, 0.55]$.

$[-2, 2]$ by $[-2, 2]$

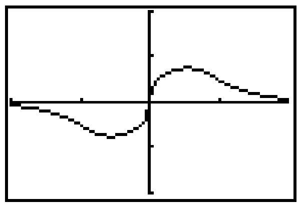


Figure 85

$[-1, 1]$ by $[-1, 1]$

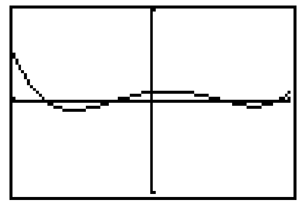


Figure 86

86 **(b)** The maximum y-value of 0.5 occurs when $x = -1$ and the minimum y-value of -0.094 occurs when $x \approx -0.56$. Therefore, the range of f is approximately $[-0.094, 0.5]$.

(c) f is decreasing on $[-1, -0.56]$ and on $[0.12, 0.75]$. f is increasing on $[-0.56, 0.12]$ and on $[0.75, 1]$.

87 **(b)** The maximum y-value of 1 occurs when $x = 0$ and the minimum y-value of -1.03 occurs when $x \approx 1.06$. Therefore, the range of f is approximately $[-1.03, 1]$.

(c) f is decreasing on $[0, 1.06]$. f is increasing on $[-0.7, 0]$ and on $[1.06, 1.4]$.

$[-0.7, 1.4]$ by $[-1.1, 1]$

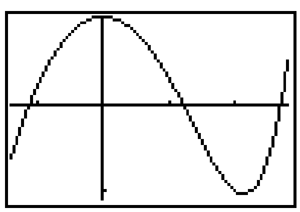


Figure 87

$[-4, -4]$ by $[-4, -4]$

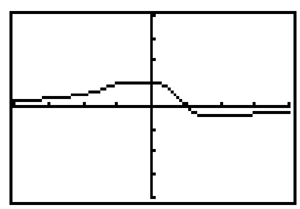


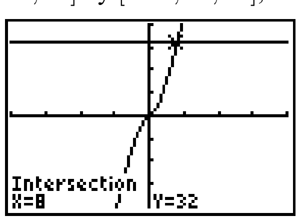
Figure 88

88 **(b)** The maximum y-value of 1.08 occurs when $x \approx -0.69$ and the minimum y-value of -0.42 occurs when $x \approx 1.78$. Therefore, the range of f is approximately $[-0.42, 1.08]$.

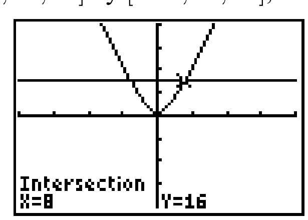
(c) f is decreasing on $[-0.69, 1.78]$. f is increasing on $[-4, -0.69]$ and on $[1.78, 4]$.

89 For each of (a)–(e), an assignment to Y_1, an appropriate viewing rectangle and the solution(s) are listed.

(a) $Y_1 = (x\hat{\ }5)\hat{\ }(1/3) = 32$,
VR: $[-40, 40, 10]$ by $[-40, 40, 10]$, $x = 8$



(b) $Y_1 = (x\hat{\ }4)\hat{\ }(1/3) = 16$,
VR: $[-40, 40, 10]$ by $[-40, 40, 10]$, $x = \pm 8$



(c) $Y_1 = (x\hat{\ }2)\hat{\ }(1/3) = -64$, VR: $[-40, 40, 10]$ by $[-40, 40, 10]$, no real solutions

(d) $Y_1 = (x\hat{\ }3)\hat{\ }(1/4) = 125$, VR: $[0, 800, 100]$ by $[0, 200, 100]$, $x = 625$

(e) $Y_1 = (x\hat{\ }3)\hat{\ }(1/2) = -27$, VR: $[-30, 30, 10]$ by $[-30, 30, 10]$, no real solutions

90 **(a)** $Y_1 = (x\hat{\ }3)\hat{\ }(1/5) = -27$, VR: $[-250, 250, 100]$ by $[-30, 30, 10]$, $x = -243$

(b) $Y_1 = (x\hat{\ }2)\hat{\ }(1/3) = 25$, VR: $[-130, 130, 50]$ by $[0, 30, 10]$, $x = \pm 125$

(c) $Y_1 = (x\hat{\ }4)\hat{\ }(1/3) = -49$, VR: $[-50, 50, 10]$ by $[-50, 50, 10]$, no real solutions

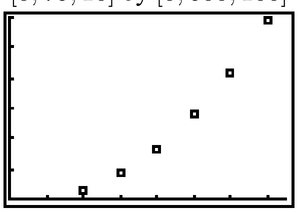
(d) $Y_1 = (x\hat{\ }3)\hat{\ }(1/2) = 64$, VR: $[-5, 30, 5]$ by $[-5, 30, 5]$, $x = 16$

(e) $Y_1 = (x\hat{\ }3)\hat{\ }(1/4) = -8$, VR: $[-10, 10,]$ by $[-10, 10]$, no real solutions

91 **(a)** There are $95 \times 63 = 5985$ total pixels in the screen.

(b) If a function is graphed in dot mode, only one pixel in each column of pixels on the screen can be darkened. Therefore, there are at most 95 pixels darkened. ***Note:*** In connected mode this may not be true.

92 **(a)** $[0, 75, 10]$ by $[0, 600, 100]$



(b) The data and plot show that stopping distance is not a linear function of the speed. The distance required to stop a car traveling at 30 mi/hr is 86 ft whereas the distance required to stop a car traveling at 60 mi/hr is 414 ft. $\frac{414}{86} \approx 4.81$ rather than double.

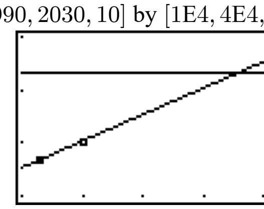
(c) If you double the speed of a car, it requires almost *five times* the stopping distance. If stopping distance were a linear function of speed, doubling the speed would require twice the stopping distance.

93 **(a)** First, we must determine an equation of the line that passes through the points $(1993, 16{,}871)$ and $(2000, 20{,}356)$.

$y - 16{,}871 = \frac{20{,}356 - 16{,}871}{2000 - 1993}(x - 1993) = \frac{3485}{7}(x - 1993) \Rightarrow$

$y = \frac{3485}{7}x - \frac{6{,}827{,}508}{7}$. Thus, let $f(x) = \frac{3485}{7}x - \frac{6{,}827{,}508}{7}$ and graph f.

$[1990, 2030, 10]$ by $[1E4, 4E4, 1E4]$



(b) The average annual increase in the price paid for a new car is equal to the slope: $\frac{3485}{7} \approx \$497.86$.

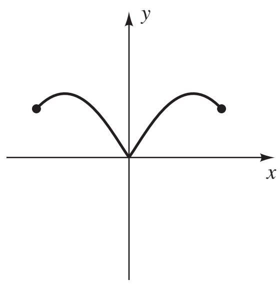
(c) Graph $y = \frac{3485}{7}x - \frac{6{,}827{,}508}{7}$ and $y = 33{,}000$ on the same coordinate axes. Their point of intersection is approximately $(2025.4, 33{,}000)$. Thus, according to this model, in the year 2025 the average price paid for a new car will be \$33,000.

2.5 Exercises

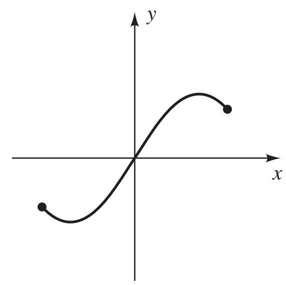
1 Since f is an even function, $f(-x) = f(x)$. From the table, we see that $f(2) = 7$, so $f(-2) = 7$. In general, if f is an even function, and (a, b) is a point on the graph of f, then the point $(-a, b)$ is also on the graph.
Since g is an odd function, $g(-x) = -g(x)$. From the table, we see that $g(2) = -6$, so $g(-2) = -(-6) = 6$. In general, if f is an odd function, and (a, b) is a point on the graph of f, then the point $(-a, -b)$ is also on the graph.

2 Since f is an even function, $f(-x) = f(x)$. From the table, we see that $f(3) = -5$, so $f(-3) = -5$.
Since g is an odd function, $g(-x) = -g(x)$. From the table, we see that $g(3) = 6$, so $g(-3) = -6$.

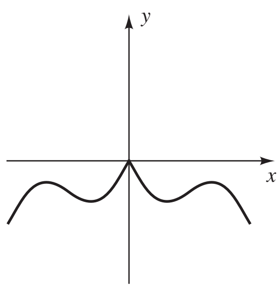
3 **(a)** f is an even function, so we can obtain the graph of f from the given partial graph by reflecting the partial graph through the y-axis.



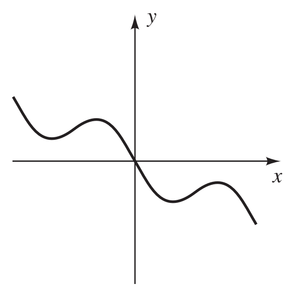
(b) f is an odd function, so we can obtain the graph of f from the given partial graph by reflecting the partial graph through the origin. You can also think of this as rotating the partial graph 180° (half of a circle).



4 **(a)** f is an even function, so we can obtain the graph of f from the given partial graph by reflecting the partial graph through the y-axis.



(b) f is an odd function, so we can obtain the graph of f from the given partial graph by reflecting the partial graph through the origin. You can also think of this as rotating the partial graph 180° (half of a circle).



5 $f(x) = 5x^3 + 2x \quad \Rightarrow \quad f(-x) = 5(-x)^3 + 2(-x) = -5x^3 - 2x$ and
$-f(x) = -1 \cdot f(x) = -(5x^3 + 2x) = -5x^3 - 2x$.
Since $f(-x) = -f(x)$, f is odd and its graph is symmetric with respect to the origin.
Note that this means if (a, b) is a point on the graph of f, then the point $(-a, -b)$ is also on the graph.

6 $f(x) = |x| - 3 \quad \Rightarrow \quad f(-x) = |-x| - 3 = |x| - 3 = f(x)$, so f is even

7 $f(x) = 3x^4 - 6x^2 - 5 \quad \Rightarrow \quad f(-x) = 3(-x)^4 - 6(-x)^2 - 5 = 3x^4 - 6x^2 - 5 = f(x)$.
Since $f(-x) = f(x)$, f is even and its graph is symmetric with respect to the y-axis.
Note that this means if (a, b) is a point on the graph of f, then the point $(-a, b)$ is also on the graph.

8 $f(x) = 7x^5 + 2x^3 \quad \Rightarrow \quad f(-x) = 7(-x)^5 + 2(-x)^3 = -7x^5 - 2x^3 = -(7x^5 + 2x^3) = -f(x)$, so f is odd

9 $f(x) = 8x^3 - 3x^2 \quad \Rightarrow \quad f(-x) = 8(-x)^3 - 3(-x)^2 = -8x^3 - 3x^2$
$-f(x) = -1 \cdot f(x) = -1(8x^3 - 3x^2) = -8x^3 + 3x^2$
Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, f is neither even nor odd.

10 $f(x) = \sqrt[3]{5} \quad \Rightarrow \quad f(-x) = \sqrt[3]{5} = f(x)$, so f is even

11 $f(x) = \sqrt{x^2+4} \quad \Rightarrow \quad f(-x) = \sqrt{(-x)^2+4} = \sqrt{x^2+4} = f(x)$.

Since $f(-x) = f(x)$, f is even and its graph is symmetric with respect to the y-axis.

12 $f(x) = 3x^2+2x-4 \quad \Rightarrow \quad f(-x) = 3(-x)^2+2(-x)-4 = 3x^2-2x-4 \neq \pm f(x)$,

so f is neither even nor odd

13 $f(x) = \sqrt[3]{x^3-x} \quad \Rightarrow$

$f(-x) = \sqrt[3]{(-x)^3-(-x)} = \sqrt[3]{-x^3+x} = \sqrt[3]{-1(x^3-x)} = \sqrt[3]{-1}\sqrt[3]{x^3-x} = -\sqrt[3]{x^3-x}$.

$-f(x) = -1 \cdot f(x) = -1 \cdot \sqrt[3]{x^3-x} = -\sqrt[3]{x^3-x}$.

Since $f(-x) = -f(x)$, f is odd and its graph is symmetric with respect to the origin.

14 $f(x) = x^3 - \dfrac{1}{x} \quad \Rightarrow \quad f(-x) = (-x)^3 - \dfrac{1}{-x} = -x^3 + \dfrac{1}{x} = -\left(x^3 - \dfrac{1}{x}\right) = -f(x)$, so f is odd

15 $f(x) = |x| + c$, $c = -3, 1, 3$ • Shift $g(x) = |x|$ {in Figure 1 in the text} down 3, up 1, and up 3 units, respectively.

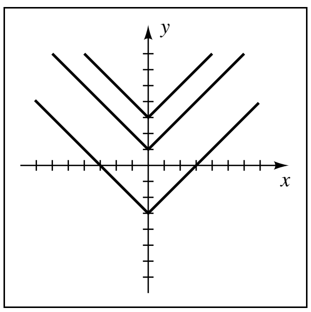


Figure 15

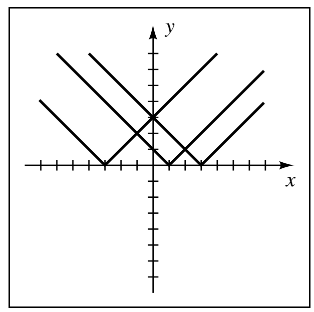


Figure 16

16 $f(x) = |x-c|$, $c = -3, 1, 3$ • Shift $g(x) = |x|$ left 3, right 1, and right 3 units, respectively.

17 $f(x) = -x^2 + c$, $c = -4, 2, 4$ • Shift $g(x) = -x^2$ {in Figure 5 in the text} down 4, up 2, up 4, respectively.

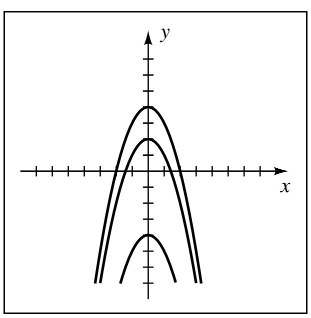


Figure 17

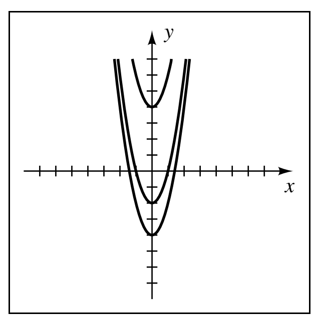


Figure 18

18 $f(x) = 2x^2 - c$, $c = -4, 2, 4$ • Shift $g(x) = 2x^2$ up 4, down 2, down 4, respectively.

19 $f(x) = 2\sqrt{x} + c$, $c = -3, 0, 2$ • The graph of $y^2 = x$ is shown in Figure 3 in Section 2.2 of the text. The top half of this graph is the graph of the square root function, $h(x) = \sqrt{x}$. The second value of c, 0, gives us the graph of $g(x) = 2\sqrt{x}$, which is a vertical stretching of h by a factor of 2. The effect of adding -3 and 2 is to vertically shift g down 3 units and up 2 units, respectively.

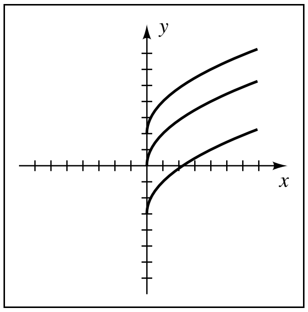


Figure 19

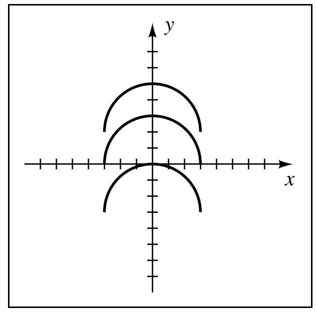


Figure 20

20 $f(x) = \sqrt{9 - x^2} + c$, $c = -3, 0, 2$ • Shift $g(x) = \sqrt{9 - x^2}$ down 3, up 2, respectively.

21 $f(x) = \frac{1}{2}\sqrt{x - c}$, $c = -3, 0, 4$ • The graph of $g(x) = \frac{1}{2}\sqrt{x}$ is a vertical compression of the square root function by a factor of $1/(1/2) = 2$. The effect of subtracting -3 and 4 from x will be to horizontally shift g left 3 units and right 4 units, respectively. If you forget which way to shift the graph, it is helpful to find the domain of the function. For example, if $h(x) = \sqrt{x - 2}$, then $x - 2$ must be nonnegative. $x - 2 \geq 0 \Rightarrow x \geq 2$, which also indicates a shift of 2 units to the right.

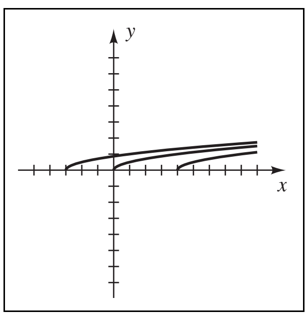


Figure 21

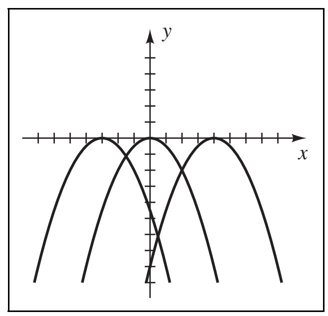


Figure 22

22 $f(x) = -\frac{1}{2}(x - c)^2$, $c = -3, 0, 4$ • Shift $g(x) = -\frac{1}{2}x^2$ left 3, right 4, respectively.

23 $f(x) = c\sqrt{4 - x^2}$, $c = -2, 1, 3$ • For $c = 1$, the graph of $g(x) = \sqrt{4 - x^2}$ is the upper half of the circle $x^2 + y^2 = 4$. For $c = -2$, reflect g through the x-axis and vertically stretch it by a factor of 2. For $c = 3$, vertically stretch g by a factor of 3.

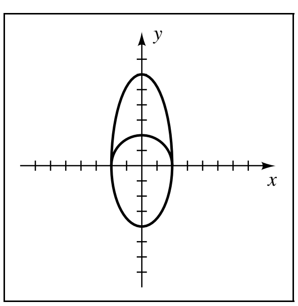


Figure 23

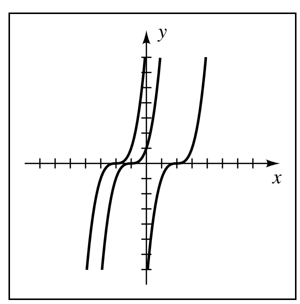


Figure 24

24 $f(x) = (x + c)^3$, $c = -2, 1, 2$ • Shift $g(x) = x^3$ right 2, left 1, and left 2, respectively.

25 $f(x) = cx^3$, $c = -\frac{1}{3}, 1, 2$ • For $c = 1$, see a graph of the cubing function in Appendix I of the text. For $c = -\frac{1}{3}$, reflect $g(x) = x^3$ through the x-axis and vertically compress it by a factor of $1/(1/3) = 3$. For $c = 2$, vertically stretch g by a factor of 2.

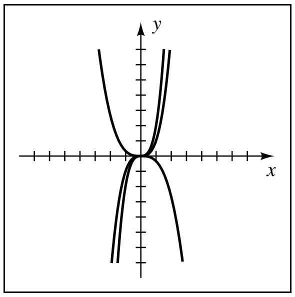


Figure 25

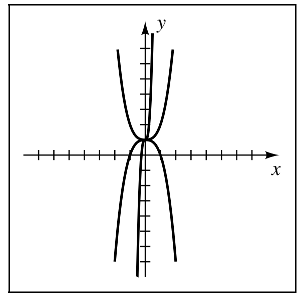


Figure 26

26 $f(x) = (cx)^3 + 1$, $c = -1, 1, 4$ • For $c = -1$, reflect $g(x) = x^3 + 1$ through the y-axis. For $c = 4$, horizontally compress g by a factor of 4 {this could also be considered as a vertical stretch by a factor of $4^3 = 64$}.

27 $f(x) = \sqrt{cx} - 1$, $c = -1, \frac{1}{9}, 4$ • If $c = 1$, then the graph of $g(x) = \sqrt{x} - 1$ is the graph of the square root function vertically shifted down one unit. For $c = -1$, reflect g through the y-axis. For $c = \frac{1}{9}$, horizontally stretch g by a factor $1/(1/9) = 9$ {x-intercept changes from 1 to 9}. For $c = 4$, horizontally compress g by a factor 4 {x-intercept changes from 1 to $\frac{1}{4}$}.

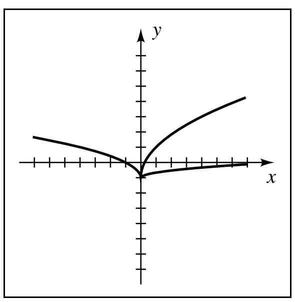


Figure 27

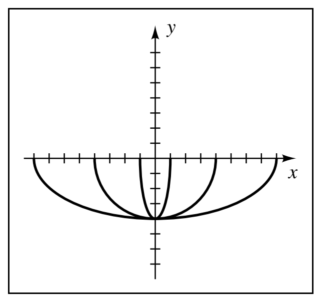


Figure 28

28 $f(x) = -\sqrt{16 - (cx)^2}$, $c = 1, \frac{1}{2}, 4$ • For $c = \frac{1}{2}$, horizontally stretch $g(x) = -\sqrt{16 - x^2}$ by a factor of $1/(1/2) = 2$ {x-intercepts change from ± 4 to ± 8}. For $c = 4$, horizontally compress g by a factor of 4 {x-intercepts change from ± 4 to ± 1}.

29 You know that $y = f(x + 2)$ is the graph of $y = f(x)$ shifted to the left 2 units, so the point $P(0, 5)$ would move to the point $(-2, 5)$. The graph of $y = f(x + 2) - 1$ is the graph of $y = f(x + 2)$ shifted down 1 unit, so the point $(-2, 5)$ moves to $(-2, 4)$. Summarizing these steps gives us the following:

$P(0, 5)$	$\{x + 2$	[subtract 2 from the x-coordinate]}	$\to (-2, 5)$
	$\{-1$	[subtract 1 from the y-coordinate]}	$\to (-2, 4)$

30 $y = 2f(x) + 4$ • $P(3, -1)$

$\{\times 2$	[multiply the y-coordinate by 2]}	$\to (3, -2)$
$\{+4$	[add 4 to the y-coordinate]}	$\to (3, 2)$

$\boxed{31}$ To determine what happens to a point P under this transformation, think of how you would evaluate $y = 2f(x-4)+1$ for a particular value of x. You would first subtract 4 from x and then put that value into the function, obtaining a corresponding y-value. Next, you would multiply that y-value by 2 and finally, add 1. Summarizing these steps using the given point P, we have the following:

$P(3,-2)$ $\{x-4$ [add 4 to the x-coordinate]$\}$ $\to (7,-2)$
$\{\times 2$ [multiply the y-coordinate by 2]$\}$ $\to (7,-4)$
$\{+1$ [add 1 to the y-coordinate]$\}$ $\to (7,-3)$

$\boxed{32}$ $y = \frac{1}{2}f(x-3)+3$ • $P(-5,8)$ $\{x-3$ [add 3 to the x-coordinate]$\}$ $\to (-2,8)$
$\{\times \frac{1}{2}$ [multiply the y-coordinate by $\frac{1}{2}$]$\}$ $\to (-2,4)$
$\{+3$ [add 3 to the y-coordinate]$\}$ $\to (-2,7)$

$\boxed{33}$ $y = \frac{1}{3}f\left(\frac{1}{2}x\right)-1$ • $P(4,9)$ $\{\frac{1}{2}x$ [multiply the x-coordinate by 2]$\}$ $\to (8,9)$
$\{\times \frac{1}{3}$ [multiply the y-coordinate by $\frac{1}{3}$]$\}$ $\to (8,3)$
$\{-1$ [subtract 1 from the y-coordinate]$\}$ $\to (8,2)$

$\boxed{34}$ $y = -3f(2x)-5$ • $P(-2,1)$ $\{2x$ [divide the x-coordinate by 2]$\}$ $\to (-1,1)$
$\{\times(-3)$ [multiply the y-coordinate by -3]$\}$ $\to (-1,-3)$
$\{-5$ [subtract 5 from the y-coordinate]$\}$ $\to (-1,-8)$

$\boxed{35}$ For $y = f(x-2)+3$, the graph of f is shifted 2 units to the right and 3 units up.

$\boxed{36}$ For $y = 3f(x-1)$, the graph of f is shifted 1 unit to the right and stretched vertically by a factor of 3.

$\boxed{37}$ For $y = f(-x)-4$, the graph of f is reflected through the y-axis and shifted 4 units down.

$\boxed{38}$ For $y = -f(x+2)$, the graph of f is shifted 2 units to the left and reflected through the x-axis.

$\boxed{39}$ For $y = -\frac{1}{2}f(x)$, the graph of f is compressed vertically by a factor of 2 and reflected through the x-axis.

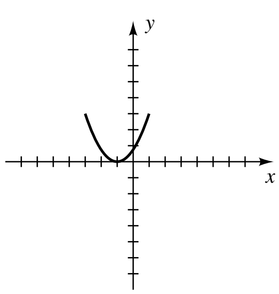
$\boxed{40}$ For $y = f\left(\frac{1}{2}x\right)-3$, the graph of f is stretched horizontally by a factor of 2 and shifted 3 units down.

$\boxed{41}$ For $y = -2f\left(\frac{1}{3}x\right)$, the graph of f is stretched horizontally by a factor of 3, stretched vertically by a factor of 2, and reflected through the x-axis.

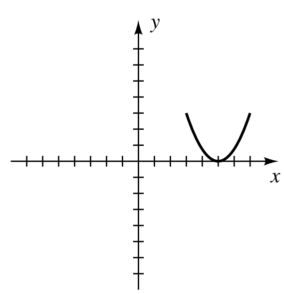
$\boxed{42}$ For $y = \frac{1}{3}|f(x)|$, the part of the graph of f below the x-axis is reflected through the x-axis and that graph is compressed vertically by a factor of 3.

$\boxed{43}$ When graphing the parts in this exercise, it may help to keep track of at least one of the points $(0,3)$, $(2,0)$, or $(4,3)$, on the given graph.

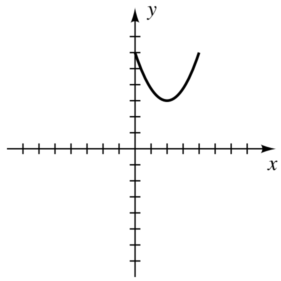
(a) $y = f(x+3)$ • shift f left 3 units



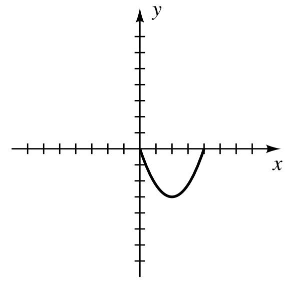
(b) $y = f(x-3)$ • shift f right 3 units



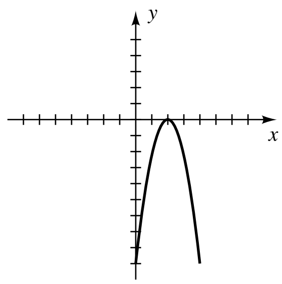
(c) $y = f(x) + 3$ • shift f up 3 units



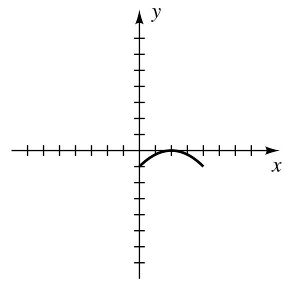
(d) $y = f(x) - 3$ • shift f down 3 units



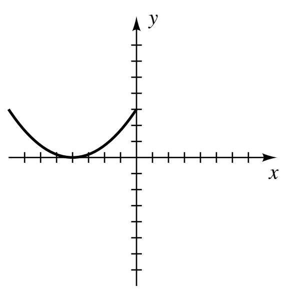
(e) $y = -3f(x)$ • reflect f through the x-axis {the effect of the negative sign in front of 3} and vertically stretch it by a factor of 3



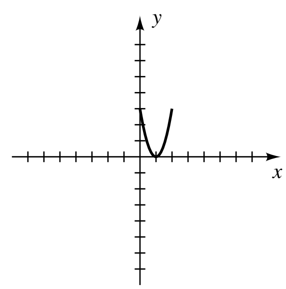
(f) $y = -\frac{1}{3}f(x)$ • reflect f through the x-axis {the effect of the negative sign in front of $\frac{1}{3}$} and vertically compress it by a factor of $1/(1/3) = 3$



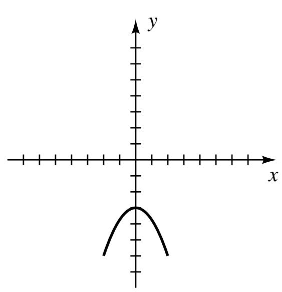
(g) $y = f\left(-\frac{1}{2}x\right)$ • reflect f through the y-axis {the effect of the negative sign inside the parentheses} and horizontally stretch it by a factor of $1/(1/2) = 2$



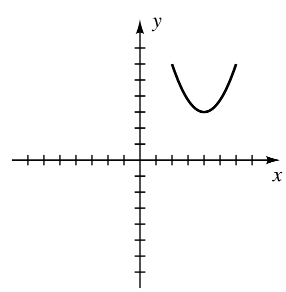
(h) $y = f(2x)$ • horizontally compress f by a factor of 2



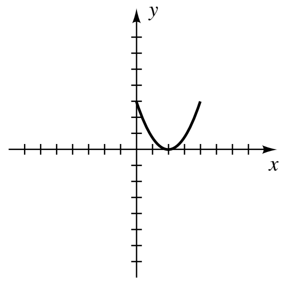
(i) $y = -f(x + 2) - 3$ • shift f left 2 units, reflect it through the x-axis, and then shift it down 3 units



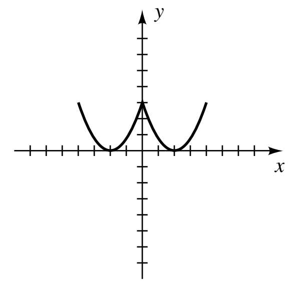
(j) $y = f(x - 2) + 3$ • shift f right 2 units and up 3



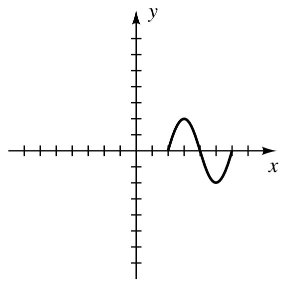
(k) $y = |f(x)|$ • since no portion of the graph lies below the x-axis, the graph is unchanged



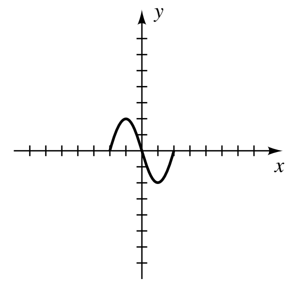
(l) $y = f(|x|)$ • include the reflection of the given graph through the y-axis since all points have positive x-coordinates



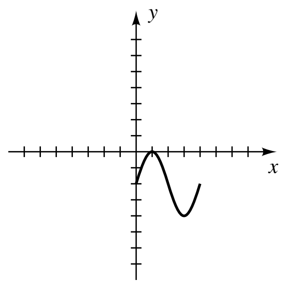
$\boxed{44}$ **(a)** $y = f(x - 2)$ • shift f right 2 units



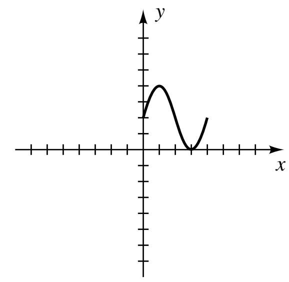
(b) $y = f(x + 2)$ • shift f left 2 units



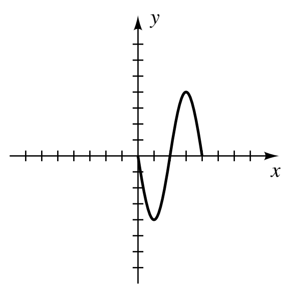
(c) $y = f(x) - 2$ • shift f down 2 units



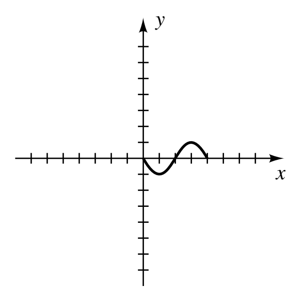
(d) $y = f(x) + 2$ • shift f up 2 units



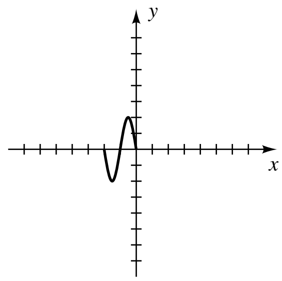
(e) $y = -2f(x)$ • reflect f through the x-axis and vertically stretch it by a factor of 2



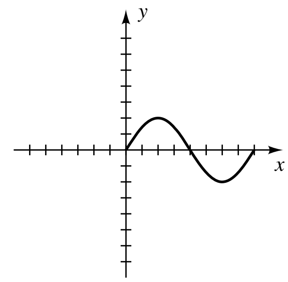
(f) $y = -\frac{1}{2}f(x)$ • reflect f through the x-axis and vertically compress it by a factor of $1/(1/2) = 2$



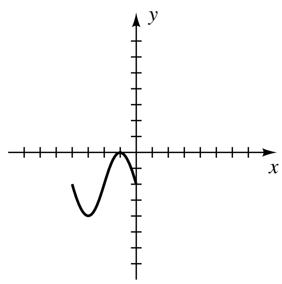
(g) $y = f(-2x)$ • reflect f through the y-axis and horizontally compress it by a factor of 2



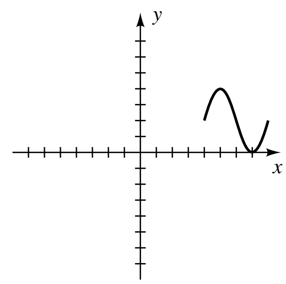
(h) $y = f\left(\frac{1}{2}x\right)$ • horizontally stretch f by a factor of $1/(1/2) = 2$



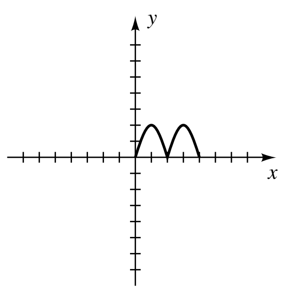
(i) $y = -f(x+4) - 2$ • reflect f about the x-axis, shift it left 4 units and down 2



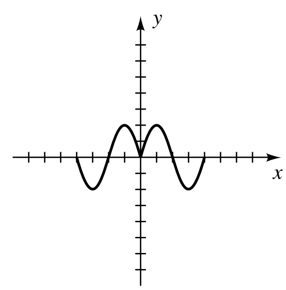
(j) $y = f(x-4) + 2$ • shift f right 4 units and up 2



(k) $y = |f(x)|$ • reflect the portion of the graph below the x-axis through the x-axis.



(l) $y = f(|x|)$ • include the reflection of the given graph through the y-axis since all points have positive x-coordinates



$\boxed{45}$ **(a)** The minimum point on $y = f(x)$ is $(2, -1)$. On the graph labeled (a), the minimum point is $(-7, 0)$. It has been shifted left 9 units and up 1. Hence, $y = f(x+9) + 1$.

(b) f is reflected through the x-axis $\Rightarrow$ $y = -f(x)$

(c) f is reflected through the x-axis and shifted left 7 units and down 1 $\Rightarrow$ $y = -f(x+7) - 1$

$\boxed{46}$ **(a)** f is shifted left 1 unit and up 1 $\Rightarrow$ $y = f(x+1) + 1$

(b) f is reflected through the x-axis $\Rightarrow$ $y = -f(x)$ {or $y = f(-x)$}

(c) f is reflected through the x-axis and shifted right 2 units $\Rightarrow$ $y = -f(x-2)$

47 **(a)** f is shifted left 4 units $\Rightarrow$ $y = f(x+4)$

(b) f is shifted up 1 unit $\Rightarrow$ $y = f(x) + 1$

(c) f is reflected through the y-axis $\Rightarrow$ $y = f(-x)$

48 **(a)** f is shifted right 2 units and up 2 $\Rightarrow$ $y = f(x-2) + 2$

(b) f is reflected through the x-axis $\Rightarrow$ $y = -f(x)$

(c) f is reflected through the x-axis and shifted left 4 units and up 2 $\Rightarrow$ $y = -f(x+4) + 2$

49 $f(x) = \begin{cases} 3 & \text{if } x \le -1 \\ -2 & \text{if } x > -1 \end{cases}$ • We can think of f as 2 functions: If $x \le -1$, then $y = 3$ {include the point $(-1, 3)$}, and if $x > -1$, then $y = -2$ {exclude the point $(-1, -2)$}.

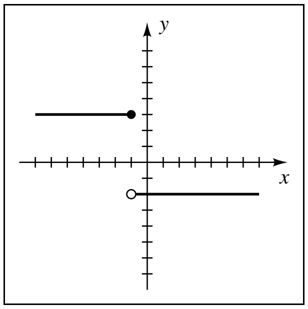


Figure 49

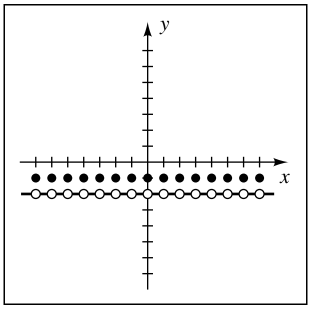


Figure 50

50 $f(x) = \begin{cases} -1 & \text{if } x \text{ is an integer} \\ -2 & \text{if } x \text{ is not an integer} \end{cases}$ • Solid dots at every integer
The line $y = -2$ with holes at every integer

51 $f(x) = \begin{cases} 3 & \text{if } x < -2 \\ -x + 1 & \text{if } |x| \le 2 \\ -4 & \text{if } x > 2 \end{cases}$ • For the second part of the function, we have $|x| \le 2$, or, equivalently, $-2 \le x \le 2$. On this part of the domain, we want to graph $f(x) = -x + 1$, a line with slope -1 and y-intercept 1. Include both endpoints, $(-2, 3)$ and $(2, -1)$.

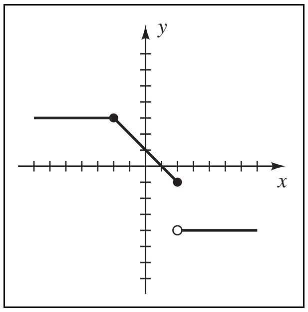


Figure 51

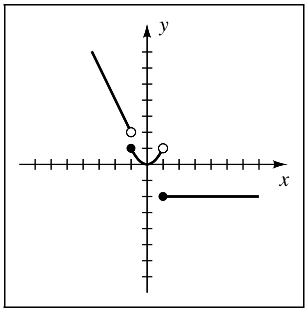


Figure 52

52 $f(x) = \begin{cases} -2x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ -2 & \text{if } x \ge 1 \end{cases}$ • A line with slope -2 and endpoint $(-1, 2)$
A parabola portion with open endpoint $(-1, 1)$ and endpoint $(1, 1)$
A horizontal line with endpoint $(1, -2)$

53 $f(x) = \begin{cases} x + 2 & \text{if } x \le -1 \\ x^3 & \text{if } |x| < 1 \\ -x + 3 & \text{if } x \ge 1 \end{cases}$ • If $x \le -1$, we want the graph of $y = x + 2$. To determine the endpoint of this part of the graph, merely substitute $x = -1$ in $y = x + 2$, obtaining $y = 1$. If $|x| < 1$, or, equivalently, $-1 < x < 1$, we want the graph of $y = x^3$. We do not include the endpoints $(-1, -1)$ and $(1, 1)$. If $x \ge 1$, we want the graph of $y = -x + 3$ and include its endpoint $(1, 2)$.

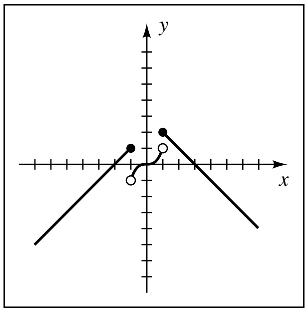


Figure 53

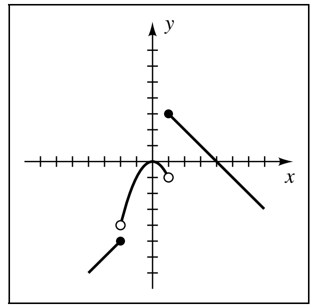
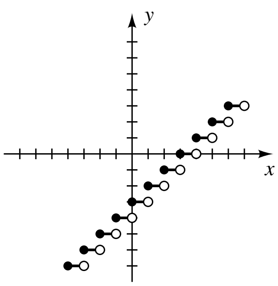


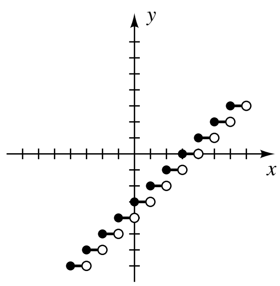
Figure 54

54 $f(x) = \begin{cases} x-3 & \text{if } x \le -2 \\ -x^2 & \text{if } -2 < x < 1 \\ -x+4 & \text{if } x \ge 1 \end{cases}$ • A line with slope 1 and endpoint $(-2,-5)$
A parabola portion with open endpoints $(-2,-4)$ and $(1,-1)$
A line with slope -1 and endpoint $(1,3)$

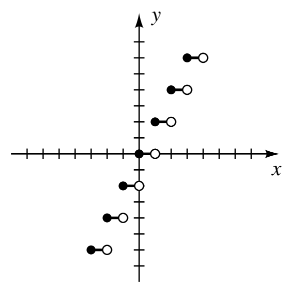
55 **(a)** $f(x) = [\![x-3]\!]$ • shift $g(x) = [\![x]\!]$ right 3 units {see Figure 10 on page 146 of the text}



(b) $f(x) = [\![x]\!] - 3$ • shift $g(x) = [\![x]\!]$ down 3 units, which is the same graph as in part (a).



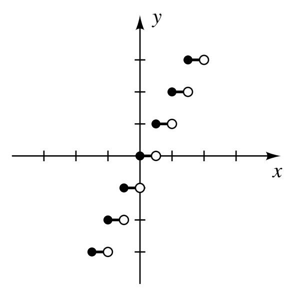
(c) $f(x) = 2[\![x]\!]$ • vertically stretch $g(x) = [\![x]\!]$ by a factor of 2



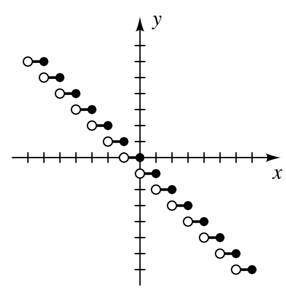
(d) $f(x) = [\![2x]\!]$ • horizontally compress $g(x) = [\![x]\!]$ by a factor of 2

Alternatively, we could determine the pattern of "steps" for this function by finding the values of x that make $f(x)$ change from 0 to 1, then from 1 to 2, etc. If $2x = 0$, then $x = 0$, and if $2x = 1$, then $x = \frac{1}{2}$.

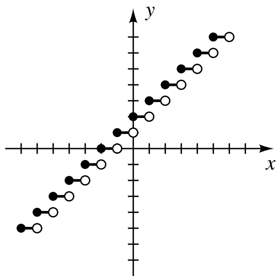
Thus, the function will equal 0 from $x = 0$ to $x = \frac{1}{2}$ and then jump to 1 at $x = \frac{1}{2}$. If $2x = 2$, then $x = 1$. The pattern is established: each step will be $\frac{1}{2}$ unit long.



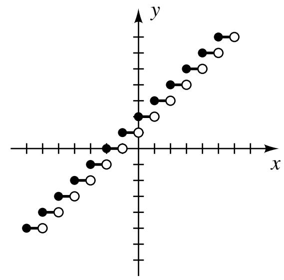
(e) $f(x) = [\![-x]\!]$ • reflect $g(x) = [\![x]\!]$ through the y-axis



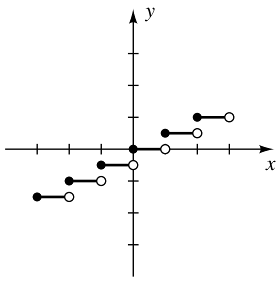
56 **(a)** $f(x) = [\![x+2]\!]$ • shift $g(x) = [\![x]\!]$ left 2 units



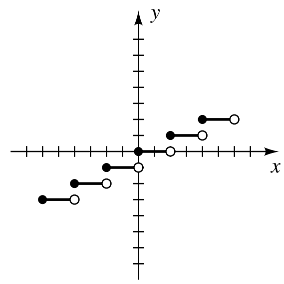
(b) $f(x) = [\![x]\!] + 2$ • shift $g(x) = [\![x]\!]$ up 2 units, which is the same graph as in part (a).



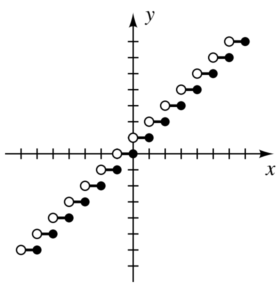
(c) $f(x) = \frac{1}{2}[\![x]\!]$ • vertically compress $g(x) = [\![x]\!]$ by a factor of $1/(1/2) = 2$



(d) $f(x) = [\![\frac{1}{2}x]\!]$ • horizontally stretch $g(x) = [\![x]\!]$ by a factor of $1/(1/2) = 2$



(e) $f(x) = -[\![-x]\!]$ • reflect $g(x) = [\![x]\!]$ through the y-axis and through the x-axis



57 **(a)** As $x \to 1^-$, $f(x) \to \underline{0}$ **(b)** As $x \to 1^+$, $f(x) \to \underline{2}$ **(c)** As $x \to -2$, $f(x) \to \underline{1}$

(d) As $x \to \infty$, $f(x) \to \underline{4}$ **(e)** As $x \to -\infty$, $f(x) \to \underline{-\infty}$

58 **(a)** As $x \to 2^-$, $f(x) \to \underline{3}$ **(b)** As $x \to 2^+$, $f(x) \to \underline{1}$ **(c)** As $x \to -1$, $f(x) \to \underline{-2}$

(d) As $x \to \infty$, $f(x) \to \underline{-\infty}$ **(e)** As $x \to -\infty$, $f(x) \to \underline{4}$

59 A question you can ask to help determine if a relationship is a function is "If x is a particular value, can I find a unique y-value?" In this case, if x was 16, then $16 = y^2 \Rightarrow y = \pm 4$. Since we cannot find a unique y-value, this is not a function. Graphically, {see Figure 3 in Section 2.2 in the text} given any x-value greater than 0, there are two points on the graph and a vertical line intersects the graph in more than one point.

60 The graph of $x = -|y|$ is not the graph of a function because if $x < 0$, two different points on the graph have x-coordinate x.

61 Reflect each portion of the graph that is below the x-axis through the x-axis.

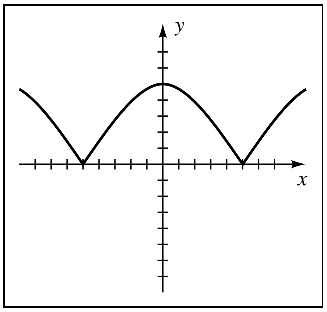


Figure 61

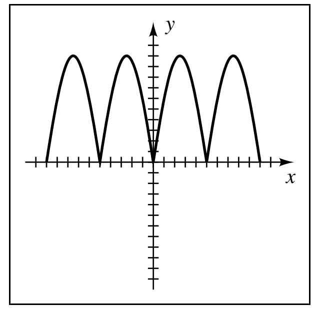


Figure 62

62 Reflect each portion of the graph that is below the x-axis through the x-axis.

63 $y = |4 - x^2|$ • First sketch $y = 4 - x^2$, then reflect the portions of the graph below the x-axis through the x-axis.

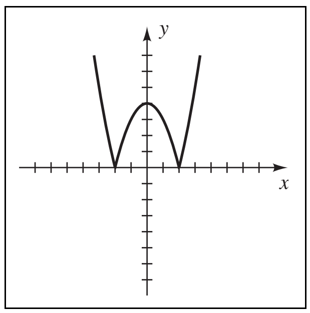


Figure 63

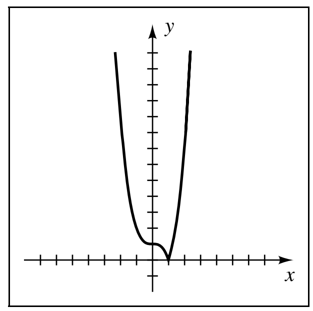


Figure 64

64 $y = |x^3 - 1|$ • First sketch $y = x^3 - 1$, then reflect the portions of the graph below the x-axis through the x-axis.

65 $y = \left|\sqrt{x} - 2\right|$ • First sketch $y = \sqrt{x} - 2$, which is the graph of the square root function shifted down 2 units. Then reflect the portions of the graph below the x-axis through the x-axis.

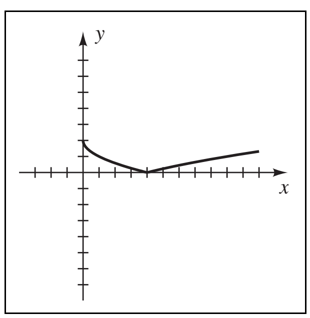


Figure 65

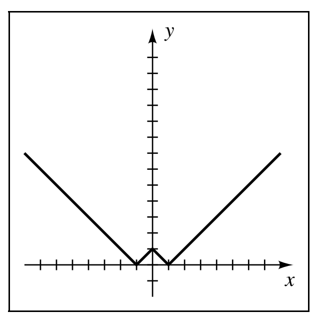


Figure 66

66 $y = ||x| - 1|$ • First sketch $y = |x| - 1$, which is the graph of the absolute value function shifted down 1 unit. Then reflect the portions of the graph below the x-axis through the x-axis.

$\boxed{67}$ **(a)** For $y = -2f(x)$, multiply the y-coordinates by -2. So the given range, $[-4, 8]$, becomes $[-16, 8]$ {note that we changed the order so that -16 appears first—listing the range as $[8, -16]$ is incorrect}. The domain {x-coordinates} remains the same. $D = [-2, 6]$, $R = [-16, 8]$

(b) For $y = f\left(\frac{1}{2}x\right)$, multiply the x-coordinates by 2. The range {y-coordinates} remains the same. $D = [-4, 12]$, $R = [-4, 8]$

(c) For $y = f(x-3)+1$, add 3 to the x-coordinates and add 1 to the y-coordinates. $D = [1, 9]$, $R = [-3, 9]$

(d) For $y = f(x+2)-3$, subtract 2 from the x-coordinates and subtract 3 from the y-coordinates. $D = [-4, 4]$, $R = [-7, 5]$

(e) For $y = f(-x)$, multiply all x-coordinates by -1. $D = [-6, 2]$, $R = [-4, 8]$

(f) For $y = -f(x)$, multiply all y-coordinates by -1. $D = [-2, 6]$, $R = [-8, 4]$

(g) $y = f(|x|)$ • Graphically, we can reflect all points with positive x-coordinates through the y-axis, so the domain $[-2, 6]$ becomes $[-6, 6]$. Algebraically, we are replacing x with $|x|$, so $-2 \leq x \leq 6$ becomes $-2 \leq |x| \leq 6$, which is equivalent to $|x| \leq 6$, or, equivalently, $-6 \leq x \leq 6$. The range stays the same because of the given assumptions: $f(2) = 8$ and $f(6) = -4$; that is, the full range is taken on for $x \geq 0$. Note that the range could not be determined if $f(-2)$ was equal to 8. $D = [-6, 6]$, $R = [-8, 4]$

(h) $y = |f(x)|$ • The points with y-coordinates having values from -4 to 0 will have values from 0 to 4, so the range will be $[0, 8]$. $D = [-2, 6]$, $R = [0, 8]$

$\boxed{68}$ **(a)** For $y = \frac{1}{2}f(x)$, multiply the y-coordinates by $\frac{1}{2}$. $D = [-6, -2]$, $R = [-5, -2]$

(b) For $y = f(2x)$, multiply the x-coordinates by $\frac{1}{2}$. $D = [-3, -1]$, $R = [-10, -4]$

(c) For $y = f(x-2)+5$, add 2 to the x-coordinates and add 5 to the y-coordinates. $D = [-4, 0]$, $R = [-5, 1]$

(d) For $y = f(x+4)-1$, subtract 4 from the x-coordinates and subtract 1 from the y-coordinates. $D = [-10, -6]$, $R = [-11, -5]$

(e) For $y = f(-x)$, negate all x-coordinates. $D = [2, 6]$, $R = [-10, -4]$

(f) For $y = -f(x)$, negate all y-coordinates. $D = [-6, -2]$, $R = [4, 10]$

(g) For $y = f(|x|)$, there is no graph since the domain of f consists of only negative values, -6 to -2, and $|x|$ is never negative.

(h) For $y = |f(x)|$, the negative y-coordinates having values from -10 to -4 will have values from 4 to 10. $D = [-6, -2]$, $R = [4, 10]$

69 If $x \le 20{,}000$, then $T(x) = 0.15x$. If $x > 20{,}000$, then the tax is 15% of the first 20,000, which is 3000, plus 20% of the amount over 20,000, which is $(x - 20{,}000)$. We may summarize and simplify as follows:

$$T(x) = \begin{cases} 0.15x & \text{if } 0 \le x \le 20{,}000 \\ 3000 + 0.20(x - 20{,}000) & \text{if } x > 20{,}000 \end{cases} = \begin{cases} 0.15x & \text{if } 0 \le x \le 20{,}000 \\ 3000 + 0.20x - 4000 & \text{if } x > 20{,}000 \end{cases}$$
$$= \begin{cases} 0.15x & \text{if } 0 \le x \le 20{,}000 \\ 0.20 - 1000 & \text{if } x > 20{,}000 \end{cases}$$

70 If $0 \le x \le 600{,}000$, then $T(x) = 0.01x$. If $x > 600{,}000$, then the tax is 1% of the first 600,000, which is 6000, plus 1.25% of the amount over 600,000.

$$T(x) = \begin{cases} 0.01x & \text{if } 0 \le x \le 600{,}000 \\ 6000 + 0.0125(x - 600{,}000) & \text{if } x > 600{,}000 \end{cases} = \begin{cases} 0.01x & \text{if } 0 \le x \le 600{,}000 \\ 0.0125x - 1500 & \text{if } x > 600{,}000 \end{cases}$$

71 The author receives \$1.20 on the first 10,000 copies, \$1.50 on the next 5000, and \$1.80 on each additional copy.

$$R(x) = \begin{cases} 1.20x & \text{if } 0 \le x \le 10{,}000 \\ 12{,}000 + 1.50(x - 10{,}000) & \text{if } 10{,}000 < x \le 15{,}000 \\ 19{,}500 + 1.80(x - 15{,}000) & \text{if } x > 15{,}000 \end{cases} = \begin{cases} 1.20x & \text{if } 0 \le x \le 10{,}000 \\ 1.50x - 3000 & \text{if } 10{,}000 < x \le 15{,}000 \\ 1.80x - 7500 & \text{if } x > 15{,}000 \end{cases}$$

72 The cost for 1000 kWh is \$57.70 and the cost for 5000 kWh is $\$57.70 + 4000(\$0.0532) = \$270.50$.

$$C(x) = \begin{cases} 0.0577x & \text{if } 0 \le x \le 1000 \\ 57.70 + 0.0532(x - 1000) & \text{if } 1000 < x \le 5000 \\ 270.50 + 0.0511(x - 5000) & \text{if } x > 5000 \end{cases} = \begin{cases} 0.0577x & \text{if } 0 \le x \le 1000 \\ 4.50 + 0.0532x & \text{if } 1000 < x \le 5000 \\ 15.00 + 0.0511x & \text{if } x > 5000 \end{cases}$$

73 Assign ABS$(1.3x + 2.8)$ to Y_1 and $1.2x + 5$ to Y_2. The viewing rectangle $[-10, 50, 10]$ by $[-10, 50, 10]$ shows intersection points at exactly -3.12 and -22. The solution of $|1.3x + 2.8| < 1.2x + 5$ is the interval $(-3.12, 22)$.

74 The solutions of $|0.3x| - 2 = 2.5 - 0.63x^2$ are approximately ± 2.45 {by symmetry}.

The solution of $|0.3x| - 2 > 2.5 - 0.63x^2$ is $(-\infty, -2.45) \cup (2.45, \infty)$.

75 Assign ABS$(1.2x^2 - 10.8)$ to Y_1 and $1.36x + 4.08$ to Y_2. The standard viewing rectangle $[-15, 15]$ by $[-10, 10]$ shows intersection points at exactly -3 and at approximately 1.87 and 4.13.

The solution of $|1.2x^2 - 10.8| > 1.36x + 4.08$ is $(-\infty, -3) \cup (-3, 1.87) \cup (4.13, \infty)$.

76 The solutions of $\left|\sqrt{16 - x^2} - 3\right| = 0.12x^2 - 0.3$ are approximately ± 3.60, ± 2.25 {by symmetry}.

The solution of $\left|\sqrt{16 - x^2} - 3\right| < 0.12x^2 - 0.3$ is $(-3.60, -2.25) \cup (2.25, 3.60)$.

77 $f(x) = (0.5x^3 - 4x - 5)$ and $g(x) = (0.5x^3 - 4x - 5) + 4 = f(x) + 4$, so the graph of g can be obtained by shifting the graph of f upward a distance of 4.

$[-12, 12]$ by $[-8, 8]$

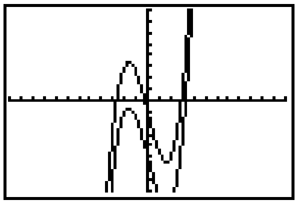


Figure 77

$[-12, 12]$ by $[-8, 8]$

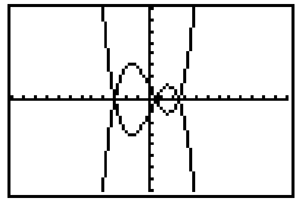


Figure 78

78 $f(x) = (0.25x^3 - 2x + 1)$ and $g(x) = (-0.25x^3 + 2x - 1) = -f(x)$, so the graph of g can be obtained reflecting the graph of f through the x–axis.

79 $f(x) = x^2 - 5$ and $g(x) = \left(\frac{1}{2}x\right)^2 - 5 = f\left(\frac{1}{2}x\right)$, so the graph of g can be obtained by stretching the graph of f horizontally by a factor of 2.

$[-12, 12]$ by $[-8, 8]$

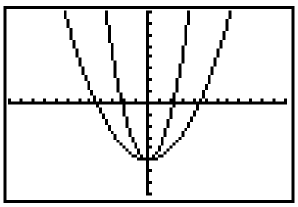


Figure 79

$[-12, 12]$ by $[-8, 8]$

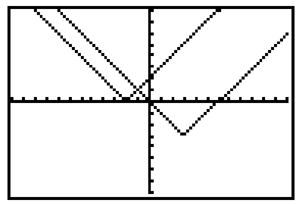


Figure 80

80 $f(x) = |x + 2|$ and $g(x) = |(x + 2) - 5| - 3 = f(x - 5) - 3$, so the graph of g can be obtained by shifting the graph of f horizontally to the right a distance of 5 and vertically downward a distance of 3.

81 $f(x) = x^3 - 5x$ and $g(x) = |x^3 - 5x| = |f(x)|$, so the graph of g is the same as the graph of f if f is nonnegative. If $f(x) < 0$, then the graph of f will be reflected through the x–axis.

$[-12, 12]$ by $[-8, 8]$

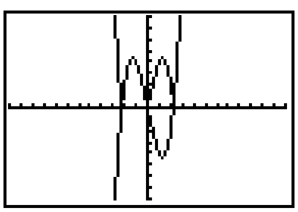


Figure 81

$[-12, 12]$ by $[-8, 8]$

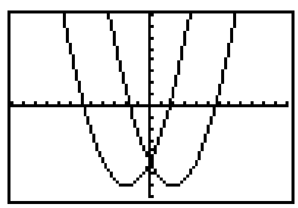


Figure 82

82 $f(x) = (0.5x^2 - 2x - 5)$ and $g(x) = (0.5x^2 + 2x - 5) = f(-x)$, so the graph of g can be obtained from the graph of f by reflecting the graph of f through the y–axis.

83 **(a)** Option I gives $C_1 = 4(\$45.00) + \$0.40(500 - 200) = 180.00 + 120.00 = \$300.00$.
Option II gives $C_2 = 4(\$58.75) + \$0.25(500) = 235.00 + 125.00 = \$360.00$.

(b) Let x represent the mileage. The cost function for Option I is the piecewise linear function

$$C_1(x) = \begin{cases} 180.00 & \text{if } 0 \le x \le 200 \\ 180.00 + 0.40(x - 200) & \text{if } x > 200 \end{cases}$$

Option II is the linear function $C_2(x) = 235.00 + 0.25x$ for $x \ge 0$.

(c) Let $C_1 = \mathrm{Y}_1$ and $C_2 = \mathrm{Y}_2$. Table $\mathrm{Y}_1 = 180.00 + 0.40(x - 200)$*$(x > 200)$ and $\mathrm{Y}_2 = 235.00 + 0.25x$

x	100	200	300	400	500	600	700	800	900	1000	1100	1200
Y_1	180	180	220	260	300	340	380	420	460	500	540	580
Y_2	260	285	310	335	360	385	410	435	460	485	510	535

(d) From the table, we see that the options are equal in cost for $x = 900$ miles. Option I is preferable if $x \in [0, 900)$ and Option II is preferable if $x > 900$.

$\boxed{84}$ **(a)** Since 1 mi = 5,280 ft and each car requires $(12 + d)$ ft, it follows directly that the bridge can hold $[\![5280/(12 + d)]\!]$ cars. The greatest integer function is necessary since a fraction of a car is not allowed.

(b) Since the bridge is 1 mile long, the car "density" is $\dfrac{5280}{12 + d}$ cars/mi. If each car is moving at v mi/hr, then the flow rate is $F = [\![5280v/(12 + d)]\!]$ cars/hr.

2.6 Exercises

$\boxed{1}$ Using the standard equation of a parabola with a vertical axis having vertex $V(-3, 1)$, we have $y = a[x - (-3)]^2 + 1$. The coefficient a determines whether the parabola opens upward {if a is positive} or opens downward {if a is negative}. If $|a| > 1$, then the parabola is narrower {steeper} than the graph of $y = x^2$; if $|a| < 1$, then the parabola is wider {flatter} than the graph of $y = x^2$. Simplifying the equation gives us $y = a(x + 3)^2 + 1$.

$\boxed{2}$ $V(5, -4)$ • $y = a(x - h)^2 + k \quad \Rightarrow \quad y = a(x - 5)^2 - 4$

$\boxed{3}$ $V(0, -2)$ • We can use the standard equation of a parabola with a vertical axis.

$$y = a(x - h)^2 + k \quad \Rightarrow \quad y = a(x - 0)^2 - 2 \quad \Rightarrow \quad y = ax^2 - 2.$$

$\boxed{4}$ $V(-7, 0)$ • $y = a(x - h)^2 + k \quad \Rightarrow \quad y = a\,[x - (-7)]^2 + 0 \quad \Rightarrow \quad y = a(x + 7)^2$

$\boxed{5}$ The approach shown here {like Solution 1 in Example 2} requires us to *factor out the leading coefficient.*

$$\begin{aligned} f(x) &= -x^2 - 4x - 5 && \text{\{given\}} \\ &= -\left(x^2 + 4x + \underline{\quad}\right) - 5 + \underline{\quad} && \text{\{factor out } -1 \text{ from } -x^2 - 4x\text{\}} \\ &= -\left(x^2 + 4x + \underline{\ 4\ }\right) - 5 + \underline{\ 4\ } && \text{\{complete the square for } x^2 + 4x\text{\}} \\ &= -(x + 2)^2 - 1 && \text{\{equivalent equation\}} \end{aligned}$$

$\boxed{6}$ $f(x) = x^2 - 6x + 11 = x^2 - 6x + \underline{\ 9\ } + 11 - \underline{\ 9\ } = (x - 3)^2 + 2$

$\boxed{7}$ The approach shown here {like Solution 2 in Example 2} requires us to *divide both sides by the leading coefficient*—remember to multiply both sides by the same coefficient in the end. Either method is fine.

$$\begin{aligned} f(x) &= 2x^2 - 16x + 35 && \{\text{given}\} \\ \tfrac{1}{2}f(x) &= (x^2 - 8x + \underline{\quad}) + \tfrac{35}{2} - \underline{\quad} && \{\text{divide the equation by } 2\} \\ &= \left(x^2 - 8x + \underline{\ 16\ }\right) + \tfrac{35}{2} - \underline{\ 16\ } && \{\text{complete the square for } x^2 - 8x\} \\ &= (x-4)^2 + \tfrac{3}{2} && \{\text{equivalent equation}\} \\ 2 \cdot \tfrac{1}{2}f(x) &= 2 \cdot \left[(x-4)^2 + \tfrac{3}{2}\right] && \{\text{multiply the equation by } 2\} \\ f(x) &= 2(x-4)^2 + 3 && \{\text{the desired standard form}\} \end{aligned}$$

$\boxed{8}$ $f(x) = 5x^2 + 20x + 14 \quad \Rightarrow \quad \frac{1}{5}f(x) = x^2 + 4x + \underline{\ 4\ } + \frac{14}{5} - \underline{\ 4\ } = (x+2)^2 - \frac{6}{5} \quad \Rightarrow$
$5 \cdot \frac{1}{5}f(x) = 5 \cdot \left[(x+2)^2 - \frac{6}{5}\right] \quad \Rightarrow \quad f(x) = 5(x+2)^2 - 6$

$\boxed{9}$ $f(x) = -3x^2 - 6x - 5$ {divide by -3 and proceed as in Exercise 7} $\Rightarrow$
$-\frac{1}{3}f(x) = x^2 + 2x + \underline{\ 1\ } + \frac{5}{3} - \underline{\ 1\ } = (x+1)^2 + \frac{2}{3} \quad \Rightarrow$
$-3 \cdot \left(-\frac{1}{3}\right)f(x) = -3 \cdot \left[(x+1)^2 + \frac{2}{3}\right] \quad \Rightarrow \quad f(x) = -3(x+1)^2 - 2$

$\boxed{10}$ $f(x) = -4x^2 + 16x - 13 \quad \Rightarrow \quad -\frac{1}{4}f(x) = x^2 - 4x + \underline{\ 4\ } + \frac{13}{4} - \underline{\ 4\ } = (x-2)^2 - \frac{3}{4} \quad \Rightarrow$
$-4 \cdot \left(-\frac{1}{4}\right)f(x) = -4 \cdot \left[(x-2)^2 - \frac{3}{4}\right] \quad \Rightarrow \quad f(x) = -4(x-2)^2 + 3$

$\boxed{11}$ $f(x) = -\frac{3}{4}x^2 + 9x - 34$ {divide by $-\frac{3}{4}$ and proceed as in Exercise 7} $\Rightarrow$
$-\frac{4}{3}f(x) = x^2 - 12x + \frac{136}{3} = x^2 - 12x + \underline{\ 36\ } + \frac{136}{3} - \underline{\ 36\ } = (x-6)^2 + \frac{28}{3} \quad \Rightarrow$
$\left(-\frac{3}{4}\right)\left(-\frac{4}{3}\right)f(x) = \left(-\frac{3}{4}\right)\left[(x-6)^2 + \frac{28}{3}\right] \quad \Rightarrow \quad f(x) = -\frac{3}{4}(x-6)^2 - 7$

$\boxed{12}$ $f(x) = \frac{2}{5}x^2 - \frac{12}{5}x + \frac{23}{5} \quad \Rightarrow \quad \frac{5}{2}f(x) = x^2 - 6x + \frac{23}{2} = x^2 - 6x + \underline{\ 9\ } + \frac{23}{2} - \underline{\ 9\ } = (x-3)^2 + \frac{5}{2} \quad \Rightarrow$
$\frac{2}{5} \cdot \frac{5}{2}f(x) = \frac{2}{5} \cdot \left[(x-3)^2 + \frac{5}{2}\right] \quad \Rightarrow \quad f(x) = \frac{2}{5}(x-3)^2 + 1$

$\boxed{13}$ **(a)** $x^2 - 6x = 0$ $\{a = 1, b = -6, c = 0\}$ $\Rightarrow \quad x = \dfrac{6 \pm \sqrt{36 - 0}}{2} = \dfrac{6 \pm 6}{2} = 0, 6$

(b) Using the theorem for locating the vertex of a parabola with $y = x^2 - 6x$ gives us x-coordinate $-\dfrac{b}{2a} = -\dfrac{-6}{2(1)} = 3$. The parabola opens upward since $a = 1 > 0$, so there is a minimum value of y. To find the minimum, substitute 3 for x in $f(x) = x^2 - 6x$ to get $f(3) = -9$.

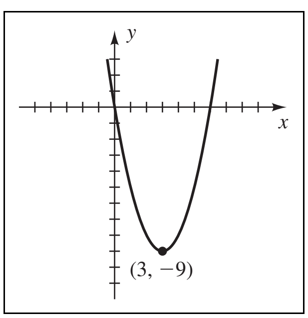


Figure 13

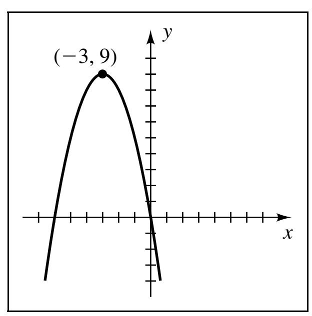


Figure 14

$\boxed{14}$ **(a)** $-x^2 - 6x = 0 \ \{a = -1, b = -6, c = 0\} \quad \Rightarrow \quad x = \dfrac{6 \pm \sqrt{36 - 0}}{-2} = -6, 0$

(b) $f(x) = -x^2 - 6x \quad \Rightarrow \quad -\dfrac{b}{2a} = -\dfrac{-6}{2(-1)} = -3.$ $f(-3) = 9$ is a maximum since $a < 0$.

$\boxed{15}$ **(a)** $f(x) = -12x^2 + 11x + 15 = 0 \quad \Rightarrow \quad x = \dfrac{-11 \pm \sqrt{121 + 720}}{-24} = \dfrac{-11 \pm 29}{-24} = -\dfrac{3}{4}, \dfrac{5}{3}$

(b) The x-coordinate of the vertex is given by $x = -\dfrac{b}{2a} = -\dfrac{11}{2(-12)} = \dfrac{11}{24}$. Note that this value is easily obtained from part (a). The y-coordinate of the vertex is then $f\left(\frac{11}{24}\right) = \frac{841}{48} \approx 17.52$. This is a maximum since $a = -12 < 0$.

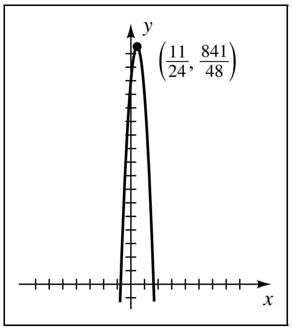


Figure 15

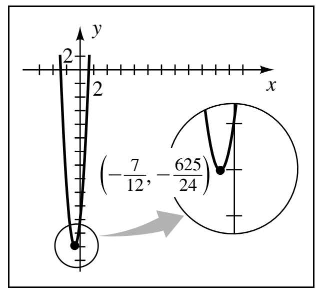


Figure 16

$\boxed{16}$ **(a)** $6x^2 + 7x - 24 = 0 \quad \Rightarrow \quad x = \dfrac{-7 \pm \sqrt{49 + 576}}{12} = \dfrac{-7 \pm 25}{12} = -\dfrac{8}{3}, \dfrac{3}{2}$

(b) $f(x) = 6x^2 + 7x - 24 \quad \Rightarrow \quad -\dfrac{b}{2a} = -\dfrac{7}{2(6)} = -\dfrac{7}{12}.$ $f\left(-\frac{7}{12}\right) = -\frac{625}{24} \approx -26.04$ is a minimum since $a > 0$.

$\boxed{17}$ **(a)** $9x^2 + 24x + 16 = 0 \quad \Rightarrow \quad x = \dfrac{-24 \pm \sqrt{576 - 576}}{18} = \dfrac{-24}{18} = -\dfrac{4}{3}$

(b) $f(x) = 9x^2 + 24x + 16 \quad \Rightarrow \quad -\dfrac{b}{2a} = -\dfrac{24}{2(9)} = -\dfrac{4}{3}.$ $f\left(-\frac{4}{3}\right) = 0$ is a minimum since $a > 0$.

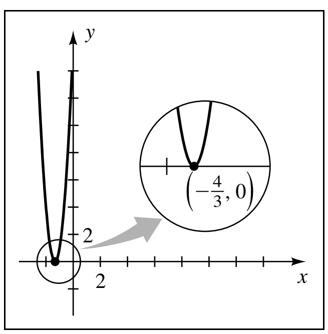


Figure 17

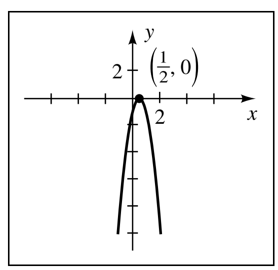


Figure 18

$\boxed{18}$ **(a)** $-4x^2 + 4x - 1 = 0 \quad \Rightarrow \quad x = \dfrac{-4 \pm \sqrt{16 - 16}}{-8} = \dfrac{-4}{-8} = \dfrac{1}{2}$

(b) $f(x) = -4x^2 + 4x - 1 \quad \Rightarrow \quad -\dfrac{b}{2a} = -\dfrac{4}{2(-4)} = \dfrac{1}{2}.$ $f\left(\frac{1}{2}\right) = 0$ is a maximum since $a < 0$.

19 **(a)** $x^2+4x+9=0 \Rightarrow x=\dfrac{-4\pm\sqrt{16-36}}{2}=\dfrac{-4\pm\sqrt{-20}}{2}=-2\pm\sqrt{5}\,i.$

The imaginary part indicates that there are no x-intercepts.

(b) $f(x)=x^2+4x+9 \Rightarrow -\dfrac{b}{2a}=-\dfrac{4}{2(1)}=-2.$ $f(-2)=5$ is a minimum since $a=1>0$.

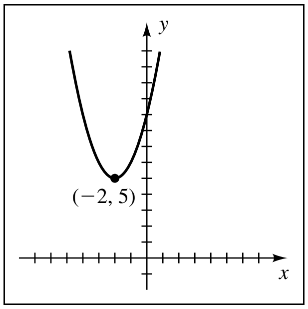


Figure 19

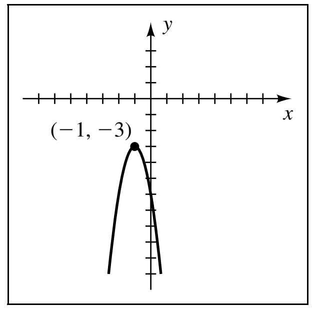


Figure 20

20 **(a)** $-3x^2-6x-6=0 \Rightarrow x=\dfrac{6\pm\sqrt{36-72}}{-6}=\dfrac{6\pm\sqrt{-36}}{-6}=\dfrac{6\pm 6i}{-6}=-1\pm i.$

The imaginary part indicates that there are no x-intercepts.

(b) $f(x)=-3x^2-6x-6 \Rightarrow -\dfrac{b}{2a}=-\dfrac{-6}{2(-3)}=-1.$ $f(-1)=-3$ is a maximum since $a<0$.

21 **(a)** $-2x^2+16x-26=0 \Rightarrow x=\dfrac{-16\pm\sqrt{256-208}}{-4}=4\pm\dfrac{1}{4}\sqrt{48}=4\pm\sqrt{3}\approx 5.73, 2.27$

(b) $f(x)=-2x^2+16x-26 \Rightarrow -\dfrac{b}{2a}=-\dfrac{16}{2(-2)}=4.$ $f(4)=6$ is a maximum since $a<0$.

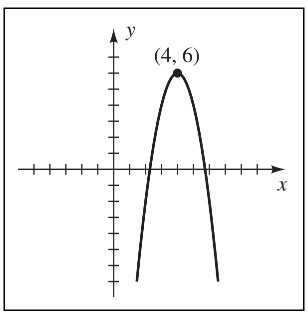


Figure 21

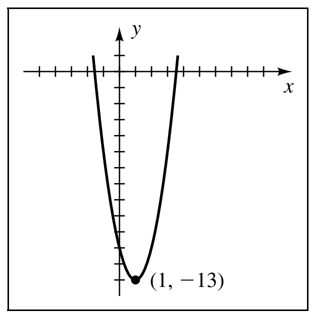


Figure 22

22 **(a)** $2x^2-4x-11=0 \Rightarrow x=\dfrac{4\pm\sqrt{16+88}}{4}=1\pm\dfrac{1}{2}\sqrt{26}\approx 3.55, -1.55$

(b) $f(x)=2x^2-4x-11 \Rightarrow -\dfrac{b}{2a}=-\dfrac{-4}{2(2)}=1.$ $f(1)=-13$ is a minimum since $a>0$.

23 $V(4,-1) \Rightarrow y=a(x-4)^2-1$. Substituting 0 for x and 1 for y gives us

$1=a(0-4)^2-1 \Rightarrow 2=16a \Rightarrow a=\frac{1}{8}$. Hence, $y=\frac{1}{8}(x-4)^2-1$.

24 $V(2,4) \Rightarrow y=a(x-2)^2+4$. Substituting 0 for x and 0 for y gives us

$0=a(0-2)^2+4 \Rightarrow -4=4a \Rightarrow a=-1$. Hence, $y=-(x-2)^2+4$.

25 $V(-2,5) \Rightarrow y=a(x+2)^2+5$. Substituting 2 for x and 0 for y gives us

$0=a(2+2)^2+5 \Rightarrow -5=16a \Rightarrow a=-\frac{5}{16}$. Hence, $y=-\frac{5}{16}(x+2)^2+5$.

26 $V(-1,-2) \Rightarrow y = a(x+1)^2 - 2$. Substituting 2 for x and 3 for y gives us
$3 = a(2+1)^2 - 2 \Rightarrow 5 = 9a \Rightarrow a = \frac{5}{9}$. Hence, $y = \frac{5}{9}(x+1)^2 - 2$.

27 From the figure, the x-intercepts are -2 and 4, so the equation must have the form
$y = a[x-(-2)](x-4) = a(x+2)(x-4)$. To find the value of a, use the point $(2,4)$.
$4 = a(2+2)(2-4) \Rightarrow 4 = a(4)(-2) \Rightarrow -8a = 4 \Rightarrow a = -\frac{1}{2}$,
so the equation is $y = -\frac{1}{2}(x+2)(x-4)$.

28 $y = a(x+3)(x-7)$ • $x = 5, y = -4 \Rightarrow -4 = a(8)(-2) \Rightarrow a = \frac{1}{4}$,
so the equation is $y = \frac{1}{4}(x+3)(x-7)$.

29 $V(0,-2) \Rightarrow (h,k) = (0,-2)$. $x = 3, y = 25 \Rightarrow 25 = a(3-0)^2 - 2 \Rightarrow 27 = 9a \Rightarrow a = 3$.
Hence, $y = 3(x-0)^2 - 2$, or $y = 3x^2 - 2$.

30 $V(0,7) \Rightarrow (h,k) = (0,7)$. $x = 2, y = -1 \Rightarrow -1 = a(2-0)^2 + 7 \Rightarrow -8 = 4a \Rightarrow a = -2$.
Hence, $y = -2(x-0)^2 + 7$, or $y = -2x^2 + 7$.

31 $V(3,1) \Rightarrow y = a(x-3)^2 + 1$ (*). If the x-intercept is 0, then the point $(0,0)$ is on the parabola. Substituting $x = 0$ and $y = 0$ into (*) gives us $0 = a(0-3)^2 + 1 \Rightarrow -1 = 9a \Rightarrow a = -\frac{1}{9}$. Hence, $y = -\frac{1}{9}(x-3)^2 + 1$.

32 $V(4,-7) \Rightarrow y = a(x-4)^2 - 7$. $x = -4, y = 0 \Rightarrow 0 = a(-4-4)^2 - 7 \Rightarrow 7 = 64a \Rightarrow a = \frac{7}{64}$.
Hence, $y = \frac{7}{64}(x-4)^2 - 7$.

33 Since the x-intercepts are -3 and 5, the x-coordinate of the vertex of the parabola is 1 {the average of the x-intercept values}. Since the highest point has y-coordinate 4, the vertex is $(1,4)$.
$V(1,4) \Rightarrow y = a(x-1)^2 + 4$. We can use the point $(-3,0)$ since there is an x-intercept of -3.
$x = -3, y = 0 \Rightarrow 0 = a(-3-1)^2 + 4 \Rightarrow -4 = 16a \Rightarrow a = -\frac{1}{4}$. Hence, $y = -\frac{1}{4}(x-1)^2 + 4$.

34 Since the x-intercepts are -2 and 12, the x-coordinate of the vertex of the parabola is 5 {the average of the x-intercept values}. Since the highest point has y-coordinate 11, the vertex is $(5,11)$.
$V(5,11) \Rightarrow y = a(x-5)^2 + 11$. We can use the point $(-2,0)$ since there is an x-intercept of -2.
$x = -2, y = 0 \Rightarrow 0 = a(-2-5)^2 + 11 \Rightarrow -11 = 49a \Rightarrow a = -\frac{11}{49}$. Hence, $y = -\frac{11}{49}(x-5)^2 + 11$.

35 Since the x-intercepts are 8 and 0, the x-coordinate of the vertex of the parabola is 4 {the average of the x-intercept values}. Since the lowest point has y-coordinate -48, the vertex is $(4,-48)$.
$V(4,-48) \Rightarrow y = a(x-4)^2 - 48$. $x = 0, y = 0 \Rightarrow 0 = a(0-4)^2 - 48 \Rightarrow 48 = 16a \Rightarrow a = 3$.
Hence, $y = 3(x-4)^2 - 48$.

36 Since the x-intercepts are -3 and 7, the x-coordinate of the vertex of the parabola is 2 {the average of the x-intercept values}. Since the lowest point has y-coordinate -6, the vertex is $(2,-6)$.
$V(2,-6) \Rightarrow y = a(x-2)^2 - 6$. $x = 7, y = 0 \Rightarrow 0 = a(7-2)^2 - 6 \Rightarrow 6 = 25a \Rightarrow a = \frac{6}{25}$.
Hence, $y = \frac{6}{25}(x-2)^2 - 6$.

37 Let d denote the distance between the parabola and the line.

$$d = (\text{parabola}) - (\text{line}) = \left(-2x^2 + 4x + 3\right) - (x-2) = -2x^2 + 3x + 5.$$

This relation is quadratic and the x-value of its maximum value is $-\dfrac{b}{2a} = -\dfrac{3}{2(-2)} = \dfrac{3}{4}$.

Thus, maximum $d = -2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right) + 5 = \frac{49}{8} = 6.125$. Note that the maximum value *of the parabola* is $f(1) = 5$, which is not the same as the maximum value of the distance d.

38 As in Exercise 37, $d = (\text{line}) - (\text{parabola}) = (-x+3) - (2x^2+8x+4) = -2x^2 - 9x - 1$, and

$-\dfrac{b}{2a} = -\dfrac{-9}{2(-2)} = -\dfrac{9}{4}$. Thus, maximum $d = -2\left(-\frac{9}{4}\right)^2 - 9\left(-\frac{9}{4}\right) - 1 = \frac{73}{8} = 9.125$.

Note: We may find the vertex of a parabola in the following problems using either:

(1) the complete the square method,

(2) the formula method, $-b/(2a)$, or

(3) the fact that the vertex lies halfway between the x-intercepts.

39 For $D(h) = -0.058h^2 + 2.867h - 24.239$, the vertex is located at $h = \dfrac{-b}{2a} = \dfrac{-2.867}{2(-0.058)} \approx 24.72$ km.

Since $a < 0$, this will produce a maximum value.

40 The vertex is located at $h = \dfrac{-b}{2a} = \dfrac{-3.811}{2(-0.078)} \approx 24.43$ km. Since $a < 0$, this will produce a maximum value.

41 Since the x-intercepts of $y = cx(21-x)$ are 0 and 21,

the maximum will occur halfway between them, that is, when the infant weighs 10.5 lb.

42 **(a)** $M = -\frac{1}{30}v^2 + \frac{5}{2}v$ • M will be a maximum when $v = \dfrac{-b}{2a} = \dfrac{-5/2}{2(-1/30)} = \dfrac{75}{2}$, or 37.5 mi/hr.

(b) $v = \frac{75}{2} \quad \Rightarrow \quad M = -\frac{1}{30}\left(\frac{75}{2}\right)^2 + \frac{5}{2}\left(\frac{75}{2}\right) = -\frac{375}{8} + \frac{375}{4} = \frac{375}{8} = 46.875$ mi/gal.

43 **(a)** $s(t) = -16t^2 + 144t + 100$ • s will be a maximum when $t = \dfrac{-b}{2a} = \dfrac{-144}{2(-16)} = \dfrac{9}{2}$.

$s\left(\frac{9}{2}\right) = -16\left(\frac{9}{2}\right)^2 + 144\left(\frac{9}{2}\right) + 100 = -324 + 648 + 100 = 424$ ft.

(b) When $t = 0$, $s(t) = 100$ ft, which is the height of the building.

44 **(a)** $s(t) = -16t^2 + v_0 t$ • $s = 0$ when $t = 12 \quad \Rightarrow \quad 0 = -16(12)^2 + v_0(12) \quad \Rightarrow \quad v_0 = 192$ ft/sec.

(b) Since the total flight is 12 seconds, the maximum height will occur when $t = 6$.

$s(6) = -16(6)^2 + 192(6) = 576$ ft.

45 Let x and $40 - x$ denote the numbers. Their product P is $P = x(40-x) = -x^2 + 40x$.

P has zeros at 0 and 40 and is a maximum (since $a < 0$) when $x = \dfrac{0+40}{2} = 20$.

The product will be a maximum when both numbers are 20.

46 Let x and $x - 60$ denote the numbers. Their product P is $P = x(x-60) = x^2 - 60x$.

P has zeros at 0 and 60 and is a minimum (since $a > 0$) when $x = \dfrac{0+60}{2} = 30$.

The product will be a minimum for $x = 30$ and $x - 60 = -30$.

47 **(a)** The 1000 ft of fence is made up of 3 sides of length x and 4 sides of length y.

To express y as a function of x, we need to solve $3x + 4y = 1000$ for y.

$3x + 4y = 1000 \quad \Rightarrow \quad 4y = 1000 - 3x \quad \Rightarrow \quad y = 250 - \frac{3}{4}x$.

(b) Using the value of y from part (a), $A = xy = x\left(250 - \frac{3}{4}x\right) = -\frac{3}{4}x^2 + 250x$.

(c) A will be a maximum when $x = \dfrac{-b}{2a} = \dfrac{-250}{2(-3/4)} = \dfrac{500}{3} = 166\frac{2}{3}$ ft.

Using part (a) to find the corresponding value of y, $y = 250 - \frac{3}{4}\left(\frac{500}{3}\right) = 250 - 125 = 125$ ft.

48 Represent the perimeter of the field by $2y + 4x = 1000$. $A = xy = x(500 - 2x)$.
A has zeros at 0 and 250 and will be a maximum when $x = \dfrac{0 + 250}{2} = 125$ yd. $y = 500 - 2(125) = 250$ yd.
The dimensions should be 125 yd by 250 yd with intermediate fences parallel to the short side.

49 The parabola has vertex $V\left(\frac{9}{2}, 3\right)$. Hence, the equation has the form $y = a\left(x - \frac{9}{2}\right)^2 + 3$.
Using the point $(9, 0)$ {or $(0, 0)$}, we have $0 = a\left(9 - \frac{9}{2}\right)^2 + 3 \quad \Rightarrow \quad a = -\frac{4}{27}$.
Thus, the path may be described by $y = -\frac{4}{27}\left(x - \frac{9}{2}\right)^2 + 3$.

50 **(a)** $y = ax^2 + x + c$ • Since $(0, 15)$ is on the graph, $c = 15$. Substituting $(175, 0)$ for (x, y) yields
$$0 = a(175)^2 + 175 + 15 \quad \Rightarrow \quad a = -\frac{190}{175^2} \text{ and } y = -\frac{190}{175^2}x^2 + x + 15.$$
(b) y will be a maximum when $x = \dfrac{-b}{2a} = \dfrac{-1}{2(-190/175^2)} = \dfrac{175^2}{380} \approx 80.59$.
The corresponding y-value is $y = -\dfrac{190}{175^2}\left(\dfrac{175^2}{380}\right)^2 + \dfrac{175^2}{380} + 15 = \dfrac{8405}{152} \approx 55.3$ ft.

51 **(a)** Since the vertex is at $(0, 10)$, an equation for the parabola is $y = ax^2 + 10$.
The points $(200, 90)$ and $(-200, 90)$ are on the parabola. Substituting $(200, 90)$ for (x, y) yields
$$90 = a(200)^2 + 10 \quad \Rightarrow \quad 80 = 40{,}000a \quad \Rightarrow \quad a = \tfrac{80}{40{,}000} = \tfrac{1}{500}. \text{ Hence, } y = \tfrac{1}{500}x^2 + 10.$$
(b) The cables are spaced 40 ft apart. Using $y = \frac{1}{500}x^2 + 10$ with $x = 40, 80, 120$, and 160, we get $y = \frac{66}{5}, \frac{114}{5}, \frac{194}{5}$, and $\frac{306}{5}$, respectively. There is one cable of length 10 ft and 2 cables of each of the other lengths. Thus, the total length is $10 + 2\left(\frac{66}{5} + \frac{114}{5} + \frac{194}{5} + \frac{306}{5}\right) = 282$ ft.

52 **(a)** Substituting $x = 0$ and $m = 0$ into $m = 2ax + b$ yields $b = 0$.
Substituting $x = 800$ and $m = \frac{1}{5}$ into $m = 2ax$ yields $a = \frac{1}{8000}$, hence, $y = \frac{1}{8000}x^2$.
(b) Substitute $x = 800$ into $y = \frac{1}{8000}x^2$ to get $y = 80$. Thus, $B = (800, 80)$.

53 An equation describing the doorway is $y = ax^2 + 9$. Since the doorway is 6 feet wide at the base, $x = 3$ when $y = 0 \quad \Rightarrow \quad 0 = a(3)^2 + 9 \quad \Rightarrow \quad 0 = 9a + 9 \quad \Rightarrow \quad 9a = -9 \quad \Rightarrow \quad a = -1$. Thus, the equation is $y = -x^2 + 9$. To fit an 8 foot high box through the doorway, we must find x when $y = 8$. If $y = 8$, then $8 = -x^2 + 9 \quad \Rightarrow \quad x^2 = 1 \quad \Rightarrow \quad x = \pm 1$. Hence, the box can only be $1 - (-1) = 2$ feet wide.

54 **(a)** The maximum height of the baseball occurs at the vertex of the parabola.
The horizontal (x) coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{3/10}{-3/4000} = 200$, so the corresponding vertical (y) coordinate is $y = -\frac{3}{4000}(200)^2 + \frac{3}{10}(200) + 3 = 33$. The maximum height is 33 feet.
(b) The height of the baseball when $x = 385$ is $y = -\frac{3}{4000}(385)^2 + \frac{3}{10}(385) + 3 \approx 7.33$, which is less than 8, so no, the baseball does not clear an 8-foot fence that is 385 feet from home plate.

55 Let x denote the number of pairs of shoes that are ordered. If $x < 50$, then the amount A of money that the company makes is $40x$. If $50 \le x \le 600$, then each pair of shoes is discounted $0.04x$, so the price per pair is $40 - 0.04x$, and the amount of money that the company makes is $(40 - 0.04x)x$. In piecewise form, we have
$$A(x) = \begin{cases} 40x & \text{if } x < 50 \\ (40 - 0.04x)x & \text{if } 50 \le x \le 600 \end{cases}$$
The maximum value of the first part of A is $(\$40)(49) = \$1960$. For the second part of A, $A = -0.04x^2 + 40x$ has a maximum when $x = \dfrac{-b}{2a} = \dfrac{-40}{2(-0.04)} = 500$ pairs.
$A(500) = 10{,}000 > 1960$, so $x = 500$ produces a maximum for both parts of A.

56 Let x denote the number of people in the group. The discount per person is $0.50(x-30)$. The amount of money taken in by the agency may be expressed as:

$$A(x) = \begin{cases} 60x & \text{if } x \le 30 \\ [60-0.50(x-30)]x & \text{if } 30 < x \le 90 \end{cases}$$

The maximum value of the first part of A is $(\$60)(30) = \$1800$. For the second part of A, $A = \left(75 - \frac{1}{2}x\right)x$ has a maximum when $x = 75$ {the x-intercepts are 0 and 150}. This value corresponds to each person receiving a discount of \$22.50 and hence, paying \$37.50 for the tour. $A(75) = 2812.50 > 1800$, so $x = 75$ produces a maximum for both parts of A.

57 **(a)** Let y denote the number of \$5 decreases in the monthly charge.

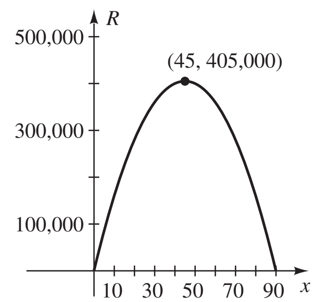
$$R(y) = (\#\text{ of customers})(\text{monthly charge per customer}) = (8000+1000y)(50-5y)$$

Now let x denote the monthly charge, which is $50-5y$. $x = 50-5y \;\Rightarrow\; 5y = 50-x \;\Rightarrow\; y = \dfrac{50-x}{5}$.

R becomes

$$\left[8000+1000\left(\frac{50-x}{5}\right)\right](x) = [8000+200(50-x)](x) = 200x[40+(50-x)] = 200x(90-x).$$

(b) R has x-intercepts at 0 and 90, and must have its vertex halfway between them at $x = 45$.
Note that this gives us $y = \dfrac{50-45}{5} = 1$, and we have $8000 + 1000(1) = 9000$ customers for a revenue of $(9000)(\$45) = \$405{,}000$.



58 Let x denote the number of \$25 increases in rent and $M(x)$ the monthly income. The number of occupied apartments is $218-5x$ and the rent per apartment is $940+25x$.

$$M(x) = (\#\text{ of occupied apartments})(\text{rent per apartment}) = (218-5x)(940+25x) = 5(218-5x)(188+5x).$$

The x-intercepts of M are $\frac{218}{5}$ and $-\frac{188}{5}$. Hence, the maximum of M will occur when
$x = \frac{1}{2}\left(-\frac{188}{5}+\frac{218}{5}\right) = \frac{1}{2}(6) = 3$. The rent charged should be $\$940 + \$25(3) = \$1015$.

59 From the graph of $f(x) = x^2 - x - \frac{1}{4}$ and $y = x^3 - x^{1/3}$, there are three points of intersection. Their coordinates are approximately $(-0.57, 0.64)$, $(0.02, -0.27)$, and $(0.81, -0.41)$.

$[-3,3]$ by $[-2,2]$

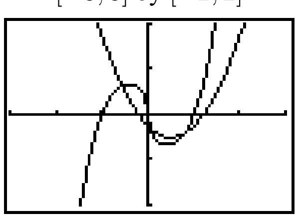


Figure 59

$[-6,6]$ by $[-4,4]$

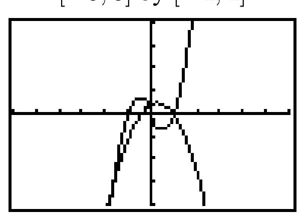


Figure 60

60 From the graph of $f(x) = -x^2 + 0.5x + 0.4$ and $y = x^3 - x^{1/3}$, there are three points of intersection. Their coordinates are approximately $(-1.61, -2.99)$, $(-0.05, 0.37)$, and $(0.98, -0.06)$.

61 $y = ax^2 + x + 1$ for $a = \frac{1}{4}, \frac{1}{2}, 1, 2,$ and 4 • Since $a > 0$, all parabolas open upward. From the graph, we can see that smaller values of a result in the parabola opening wider while larger values of a result in the parabola becoming narrower.

$[-8, 4]$ by $[-1, 7]$

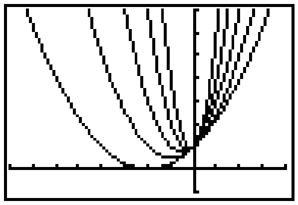


Figure 61

$[-6, 6]$ by $[-2, 6]$

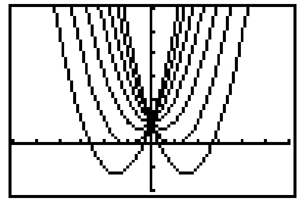


Figure 62

62 $y = x^2 + bx + 1$ for $b = 0, \pm 1, \pm 2,$ and ± 3 • As $|b|$ increases, the graph of each parabola shifts downward. Negative values of b shift the parabola to the right while positive values of b shift the parabola to the left.

63 **(a)** Let January correspond to 1, February to 2, ... , and December to 12.

(b) Let $f(x) = a(x - h)^2 + k$. The vertex appears to occur near $(7, 0.77)$ Thus, $h = 7$ and $k = 0.77$. Using trial and error, a reasonable value for a is 0.17. Thus, let $f(x) = 0.17(x - 7)^2 + 0.77$. {From the TI-83/4 Plus, the quadratic regression equation is $y \approx 0.1676x^2 - 2.1369x + 7.9471$.}

(c) $f(4) \approx 2.3$, compared to the actual value of 2.55 in.

$[0, 13]$ by $[0, 8]$

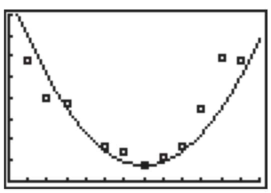


Figure 63

$[1980, 1995]$ by $[5, 15]$

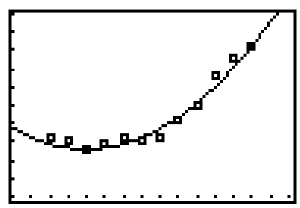


Figure 64

64 **(a)** Overall, the data decrease slightly and then start to increase, although there is an unexpected decrease in 1987.

(b) Start by choosing a vertex. The lowest data point is $(1984, 7.6)$, so let $h = 1984$ and $k = 7.6$ in the equation $f(x) = a(x - h)^2 + k$. Choosing one more point will determine the parabola.

$f(1993) = 13.3 \quad \Rightarrow \quad 13.3 = a(1993 - 1984)^2 + 7.6 \quad \Rightarrow \quad a \approx 0.07$.

Let $f(x) = 0.07(x - 1984)^2 + 7.6$. f can be adjusted to give a slightly better fit.

65 **(a)** The equation of the line passing through $A(-800, -48)$ and $B(-500, 0)$ is $y = \frac{4}{25}x + 80$.

The equation of the line passing through $D(500, 0)$ and $E(800, -48)$ is $y = -\frac{4}{25}x + 80$.

Let $y = a(x - h)^2 + k$ be the equation of the parabola passing through the points $B(-500, 0)$, $C(0, 40)$, and $D(500, 0)$. The vertex is located at $(0, 40)$ so $y = a(x - 0)^2 + 40$. Since $D(500, 0)$ is on the graph, $0 = a(500 - 0)^2 + 40 \quad \Rightarrow \quad a = -\frac{1}{6250}$ and $y = -\frac{1}{6250}x^2 + 40$. Thus, let

$$f(x) = \begin{cases} \frac{4}{25}x + 80 & \text{if } -800 \le x < -500 \\ -\frac{1}{6250}x^2 + 40 & \text{if } -500 \le x \le 500 \\ -\frac{4}{25}x + 80 & \text{if } 500 < x \le 800 \end{cases}$$

(b) Graph the equations: $\mathrm{Y}_1 = (4/25*x + 80)/(x < -500)$,
$\mathrm{Y}_2 = (-1/6250*x\hat{\ }2 + 40)/(x \geq -500 \text{ and } x \leq 500)$, $\mathrm{Y}_3 = (-4/25*x + 80)/(x > 500)$

$[-800, 800, 100]$ by $[-100, 200, 100]$

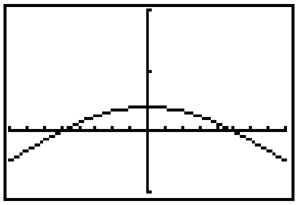


Figure 65

$[-500, 2000, 500]$ by $[0, 800, 100]$

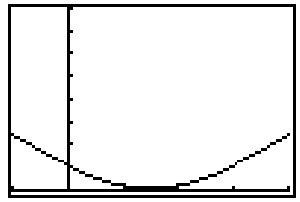


Figure 66

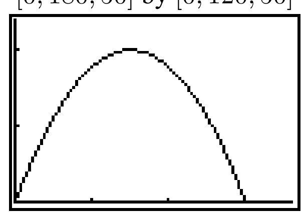
66 **(a)** The equation of the line passing through $A\left(-500, 243\frac{1}{3}\right)$ and $B(0, 110)$ is $y = -\frac{4}{15}x + 110$.
The equation of the line passing through $D(1500, 110)$ and $E\left(2000, 243\frac{1}{3}\right)$ is $y = \frac{4}{15}x - 290$.
Let $y = a(x-h)^2 + k$ be the equation of the parabola passing through the points $B(0, 110)$, $C(750, 10)$, and $D(1500, 110)$. The vertex is located at $(750, 10)$ so $y = a(x-750)^2 + 10$. Since $D(1500, 110)$ is on the graph, $110 = a(1500-750)^2 + 10 \Rightarrow a = \frac{1}{5625}$; $y = \frac{1}{5625}(x-750)^2 + 10$. Thus, let

$$f(x) = \begin{cases} -\frac{4}{15}x + 110 & \text{if } -500 \leq x < 0 \\ \frac{1}{5625}(x-750)^2 + 10 & \text{if } 0 \leq x \leq 1500 \\ \frac{4}{15}x - 290 & \text{if } 1500 < x \leq 2000 \end{cases}$$

(b) Graph the equations: $\mathrm{Y}_1 = (-4/15*x + 110)/(x < 0)$,
$\mathrm{Y}_2 = (1/5625*(x-750)\hat{\ }2 + 10)/(x \geq 0 \text{ and } x \leq 1500)$, $\mathrm{Y}_3 = (4/15*x - 290)/(x > 1500)$

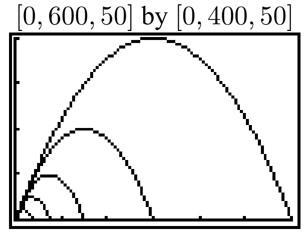
67 **(a)** f must have zeros of 0 and 150. Thus, $f(x) = a(x-0)(x-150)$.
Also, f will have a maximum of 100 occurring at $x = 75$.
(The vertex will be midway between the zeros of f.)
$a(75-0)(75-150) = 100 \Rightarrow a = \dfrac{100}{(75)(-75)} = -\dfrac{4}{225}$.
$f(x) = -\frac{4}{225}(x)(x-150) = -\frac{4}{225}x^2 + \frac{8}{3}x$, so $a = -\frac{4}{225}$ and $b = \frac{8}{3}$.

(b) $[0, 180, 50]$ by $[0, 120, 50]$



(c) $y = kax^2 + bx$ for $k = \frac{1}{4}, \frac{1}{2}, 1, 2,$ and 4 • The value of k affects both the distance and the height traveled by the object. The distance and height decrease by a factor of $\dfrac{1}{k}$ when $k > 1$ and increase by a factor of $\dfrac{1}{k}$ when $0 < k < 1$.

$[0, 600, 50]$ by $[0, 400, 50]$



2.7 Exercises

1 $f(x) = x + 3$, $g(x) = x^2$ • $f(3) = 3 + 3 = 6$, $g(3) = 3^2 = 9$

(a) $(f+g)(3) = f(3) + g(3) = 6 + 9 = 15$

(b) $(f-g)(3) = f(3) - g(3) = 6 - 9 = -3$

(c) $(fg)(3) = f(3) \cdot g(3) = 6 \cdot 9 = 54$

(d) $(f/g)(3) = \dfrac{f(3)}{g(3)} = \dfrac{6}{9} = \dfrac{2}{3}$

2 $f(x) = -x^2$, $g(x) = 2x - 1$ • $f(3) = -3^2 = -9$, $g(3) = 2(3) - 1 = 5$

(a) $(f+g)(3) = f(3) + g(3) = -9 + 5 = -4$

(b) $(f-g)(3) = f(3) - g(3) = -9 - 5 = -14$

(c) $(fg)(3) = f(3) \cdot g(3) = -9 \cdot 5 = -45$

(d) $(f/g)(3) = \dfrac{f(3)}{g(3)} = \dfrac{-9}{5} = -\dfrac{9}{5}$

3 **(a)** $(f+g)(x) = f(x) + g(x) = (x^2+2) + (2x^2-1) = 3x^2+1$;
$(f-g)(x) = f(x) - g(x) = (x^2+2) - (2x^2-1) = 3 - x^2$;
$(fg)(x) = f(x) \cdot g(x) = (x^2+2) \cdot (2x^2-1) = 2x^4 + 3x^2 - 2$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2+2}{2x^2-1}$

(b) The domain of $f+g$, $f-g$, and fg is the set of all real numbers, $\mathbb{R}$.

(c) The domain of f/g is the same as in (b), except we must exclude the zeros of g.
$2x^2 - 1 = 0 \quad \Rightarrow \quad x^2 = \frac{1}{2} \quad \Rightarrow \quad x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$ or $\pm\frac{1}{2}\sqrt{2}$.
Hence, the domain of f/g is all real numbers except $\pm\frac{1}{2}\sqrt{2}$.

4 **(a)** $(f+g)(x) = f(x) + g(x) = (x^2+x) + (x^2-4) = 2x^2 + x - 4$;
$(f-g)(x) = f(x) - g(x) = (x^2+x) - (x^2-4) = x + 4$;
$(fg)(x) = f(x) \cdot g(x) = (x^2+x) \cdot (x^2-4) = x^4 + x^3 - 4x^2 - 4x$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2+x}{x^2-4}$

(b) The domain of $f+g$, $f-g$, and fg is the set of all real numbers, $\mathbb{R}$.

(c) The domain of f/g is all real numbers except ± 2.

5 **(a)** $(f+g)(x) = f(x) + g(x) = \sqrt{x+5} + \sqrt{x+5} = 2\sqrt{x+5}$;
$(f-g)(x) = f(x) - g(x) = \sqrt{x+5} - \sqrt{x+5} = 0$; $(fg)(x) = f(x) \cdot g(x) = \sqrt{x+5} \cdot \sqrt{x+5} = x+5$;
$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x+5}}{\sqrt{x+5}} = 1$

(b) The radicand, $x+5$, must be nonnegative; that is $x + 5 \geq 0 \quad \Rightarrow \quad x \geq -5$.
Thus, the domain of $f+g$, $f-g$, and fg is $[-5, \infty)$.

(c) Now the radicand must be positive {can't have zero in the denominator}. Thus, the domain of f/g is $(-5, \infty)$.

6 **(a)** $(f+g)(x) = f(x) + g(x) = \sqrt{5-2x} + \sqrt{x+3}$; $(f-g)(x) = f(x) - g(x) = \sqrt{5-2x} - \sqrt{x+3}$;
$(fg)(x) = f(x) \cdot g(x) = \sqrt{5-2x} \cdot \sqrt{x+3} = \sqrt{(5-2x)(x+3)}$;
$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{5-2x}}{\sqrt{x+3}} = \sqrt{\dfrac{5-2x}{x+3}}$

(b) The radicands must be nonnegative. $5 - 2x \geq 0 \quad \Rightarrow \quad 2x \leq 5 \quad \Rightarrow \quad x \leq \frac{5}{2}$ and $x + 3 \geq 0 \quad \Rightarrow \quad x \geq -3$.
Thus, the domain of $f+g$, $f-g$, and fg is $\left[-3, \frac{5}{2}\right]$.

(c) Now $x+3$ must be positive {can't have zero in the denominator}. Thus, the domain of f/g is $\left(-3, \frac{5}{2}\right]$.

$\boxed{7}$ **(a)** $(f+g)(x) = f(x) + g(x) = \dfrac{2x}{x-4} + \dfrac{x}{x+5} = \dfrac{2x(x+5) + x(x-4)}{(x-4)(x+5)} = \dfrac{3x^2+6x}{(x-4)(x+5)}$;

$(f-g)(x) = f(x) - g(x) = \dfrac{2x}{x-4} - \dfrac{x}{x+5} = \dfrac{2x(x+5) - x(x-4)}{(x-4)(x+5)} = \dfrac{x^2+14x}{(x-4)(x+5)}$;

$(fg)(x) = f(x) \cdot g(x) = \dfrac{2x}{x-4} \cdot \dfrac{x}{x+5} = \dfrac{2x^2}{(x-4)(x+5)}$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x/(x-4)}{x/(x+5)} = \dfrac{2(x+5)}{x-4}$

(b) The domain of f is $\mathbb{R} - \{4\}$ and the domain of g is $\mathbb{R} - \{-5\}$. The intersection of these two domains, $\mathbb{R} - \{-5, 4\}$, is the domain of the three functions.

(c) To determine the domain of the quotient f/g, we also exclude any values that make the denominator g equal to zero. Hence, we exclude $x = 0$ and the domain of the quotient is all real numbers except -5, 0, and 4, that is, $\mathbb{R} - \{-5, 0, 4\}$.

$\boxed{8}$ **(a)** $(f+g)(x) = f(x) + g(x) = \dfrac{x}{x-2} + \dfrac{7x}{x+4} = \dfrac{x(x+4) + 7x(x-2)}{(x-2)(x+4)} = \dfrac{8x^2-10x}{(x-2)(x+4)}$;

$(f-g)(x) = f(x) - g(x) = \dfrac{x}{x-2} - \dfrac{7x}{x+4} = \dfrac{x(x+4) - 7x(x-2)}{(x-2)(x+4)} = \dfrac{-6x^2+18x}{(x-2)(x+4)}$;

$(fg)(x) = f(x) \cdot g(x) = \dfrac{x}{x-2} \cdot \dfrac{7x}{x+4} = \dfrac{7x^2}{(x-2)(x+4)}$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x/(x-2)}{7x/(x+4)} = \dfrac{x+4}{7(x-2)}$

(b) All real numbers except -4 and 2

(c) All real numbers except -4, 0, and 2

$\boxed{9}$ **(a)** $f(x) = 2x - 1$, $g(x) = -x^2$ • $(f \circ g)(x) = f(g(x)) = f(-x^2) = 2(-x^2) - 1 = -2x^2 - 1$

(b) $(g \circ f)(x) = g(f(x)) = g(2x-1) = -(2x-1)^2 = -(4x^2 - 4x + 1) = -4x^2 + 4x - 1$

(c) $(f \circ f)(x) = f(f(x)) = f(2x-1) = 2(2x-1) - 1 = (4x-2) - 1 = 4x - 3$

(d) $(g \circ g)(x) = g(g(x)) = g(-x^2) = -(-x^2)^2 = -(x^4) = -x^4$

$\boxed{10}$ **(a)** $(f \circ g)(x) = f(g(x)) = f(x-2) = 3(x-2)^2 = 3(x^2 - 4x + 4) = 3x^2 - 12x + 12$

(b) $(g \circ f)(x) = g(f(x)) = g(3x^2) = (3x^2) - 2 = 3x^2 - 2$

(c) $(f \circ f)(x) = f(f(x)) = f(3x^2) = 3(3x^2)^2 = 3(9x^4) = 27x^4$

(d) $(g \circ g)(x) = g(g(x)) = g(x-2) = (x-2) - 2 = x - 4$

Note: In Exercises 11–38, let $h(x) = (f \circ g)(x) = f(g(x))$ and $k(x) = (g \circ f)(x) = g(f(x))$.
$h(-2)$ and $k(3)$ could be worked two ways, as in Example 3(c) in the text.

$\boxed{11}$ **(a)** $h(x) = f(3x+4) = 2(3x+4) - 5 = 6x + 3$ **(b)** $k(x) = g(2x-5) = 3(2x-5) + 4 = 6x - 11$

(c) Using the result from part (a), $h(-2) = 6(-2) + 3 = -12 + 3 = -9$.

(d) Using the result from part (b), $k(3) = 6(3) - 11 = 18 - 11 = 7$.

$\boxed{12}$ **(a)** $h(x) = f(6x-3) = 5(6x-3) + 2 = 30x - 13$ **(b)** $k(x) = g(5x+2) = 6(5x+2) - 3 = 30x + 9$

(c) Using the result from part (a), $h(-2) = 30(-2) - 13 = -60 - 13 = -73$.

(d) Using the result from part (b), $k(3) = 30(3) + 9 = 90 + 9 = 99$.

$\boxed{13}$ **(a)** $h(x) = f(5x) = 3(5x)^2 + 4 = 75x^2 + 4$ **(b)** $k(x) = g(3x^2+4) = 5(3x^2+4) = 15x^2 + 20$

(c) $h(-2) = 75(-2)^2 + 4 = 300 + 4 = 304$ **(d)** $k(3) = 15(3)^2 + 20 = 135 + 20 = 155$

14 **(a)** $h(x) = f(4x^2) = 3(4x^2) - 1 = 12x^2 - 1$

(b) $k(x) = g(3x-1) = 4(3x-1)^2 = 4(9x^2 - 6x + 1) = 36x^2 - 24x + 4$

(c) $h(-2) = 12(-2)^2 - 1 = 48 - 1 = 47$

(d) $k(3) = 36(3)^2 - 24(3) + 4 = 324 - 72 + 4 = 256$

15 **(a)** $h(x) = f(2x-1) = 2(2x-1)^2 + 3(2x-1) - 4 = 8x^2 - 2x - 5$

(b) $k(x) = g(2x^2 + 3x - 4) = 2(2x^2 + 3x - 4) - 1 = 4x^2 + 6x - 9$

(c) $h(-2) = 8(-2)^2 - 2(-2) - 5 = 32 + 4 - 5 = 31$

(d) $k(3) = 4(3)^2 + 6(3) - 9 = 36 + 18 - 9 = 45$

16 **(a)** $h(x) = f(3x^2 - x + 2) = 5(3x^2 - x + 2) - 7 = 15x^2 - 5x + 3$

(b) $k(x) = g(5x-7) = 3(5x-7)^2 - (5x-7) + 2 = 75x^2 - 215x + 156$

(c) $h(-2) = 15(-2)^2 - 5(-2) + 3 = 60 + 10 + 3 = 73$

(d) $k(3) = 75(3)^2 - 215(3) + 156 = 675 - 645 + 156 = 186$

17 **(a)** $h(x) = f(2x^3 - 5x) = 4(2x^3 - 5x) = 8x^3 - 20x$ **(b)** $k(x) = g(4x) = 2(4x)^3 - 5(4x) = 128x^3 - 20x$

(c) $h(-2) = 8(-2)^3 - 20(-2) = -64 + 40 = -24$ **(d)** $k(3) = 128(3)^3 - 20(3) = 3456 - 60 = 3396$

18 **(a)** $h(x) = f(3x) = (3x)^3 + 2(3x)^2 = 27x^3 + 18x^2$ **(b)** $k(x) = g(x^3 + 2x^2) = 3(x^3 + 2x^2) = 3x^3 + 6x^2$

(c) $h(-2) = 27(-2)^3 + 18(-2)^2 = -216 + 72 = -144$ **(d)** $k(3) = 3(3)^3 + 6(3)^2 = 81 + 54 = 135$

19 **(a)** $h(x) = f(-7) = |-7| = 7$ **(b)** $k(x) = g(|x|) = -7$

(c) $h(-2) = 7$ since $h(\text{any value}) = 7$ **(d)** $k(3) = -7$ since $k(\text{any value}) = -7$

20 **(a)** $h(x) = f(x^2) = -5$ **(b)** $k(x) = g(-5) = (-5)^2 = 25$

(c) $h(-2) = -5$ since $h(\text{any value}) = -5$ **(d)** $k(3) = 25$ since $k(\text{any value}) = 25$

21 **(a)** $h(x) = f(\sqrt{x+2}) = (\sqrt{x+2})^2 - 3(\sqrt{x+2}) = x + 2 - 3\sqrt{x+2}$. The domain of $f \circ g$ is the set of all x in the domain of g, $x \geq -2$, such that $g(x)$ is in the domain of f. Since the domain of f is $\mathbb{R}$, any value of $g(x)$ is in its domain. Thus, the domain is all x such that $x \geq -2$. Note that *both* $g(x)$ *and* $f(g(x))$ are defined for x in $[-2, \infty)$.

(b) $k(x) = g(x^2 - 3x) = \sqrt{(x^2 - 3x) + 2} = \sqrt{x^2 - 3x + 2}$. The domain of $g \circ f$ is the set of all x in the domain of f, $\mathbb{R}$, such that $f(x)$ is in the domain of g. Since the domain of g is $x \geq -2$, we must solve $f(x) \geq -2$.

$x^2 - 3x \geq -2 \quad \Rightarrow \quad x^2 - 3x + 2 \geq 0 \quad \Rightarrow \quad (x-1)(x-2) \geq 0$

Interval	$(-\infty, 1)$	$(1, 2)$	$(2, \infty)$
Sign of $x - 2$	$-$	$-$	$+$
Sign of $x - 1$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

From the sign chart, $(x-1)(x-2) \geq 0 \quad \Rightarrow \quad x \in (-\infty, 1] \cup [2, \infty)$.

Thus, the domain is all x such that $x \in (-\infty, 1] \cup [2, \infty)$.

Note that *both* $f(x)$ *and* $g(f(x))$ are defined for x in $(-\infty, 1] \cup [2, \infty)$.

22 **(a)** $h(x) = f(x^2+2x) = \sqrt{(x^2+2x)-15} = \sqrt{x^2+2x-15}$.

Domain of $g = \mathbb{R}$. Domain of $f = [15, \infty)$. $g(x) \geq 15 \;\Rightarrow\; x^2+2x \geq 15 \;\Rightarrow\; x^2+2x-15 \geq 0 \;\Rightarrow$

$(x+5)(x-3) \geq 0 \;\Rightarrow\; x \in (-\infty, -5] \cup [3, \infty)$.

(b) $k(x) = g\left(\sqrt{x-15}\right) = \left(\sqrt{x-15}\right)^2 + 2\left(\sqrt{x-15}\right) = x - 15 + 2\sqrt{x-15}$.

Domain of $f = [15, \infty)$. Domain of $g = \mathbb{R}$. Since $f(x)$ is always in the domain

of g, the domain of $g \circ f$ is the same as the domain of f, $[15, \infty)$.

23 **(a)** $h(x) = f\left(\sqrt{3x}\right) = \left(\sqrt{3x}\right)^2 - 4 = 3x - 4$. Domain of $g = [0, \infty)$. Domain of $f = \mathbb{R}$.

Since $g(x)$ is always in the domain of f, the domain of $f \circ g$ is the same as the domain of g, $[0, \infty)$.

Note that *both* $g(x)$ *and* $f(g(x))$ are defined for x in $[0, \infty)$.

(b) $k(x) = g(x^2-4) = \sqrt{3(x^2-4)} = \sqrt{3x^2-12}$. Domain of $f = \mathbb{R}$. Domain of $g = [0, \infty)$.

$f(x) \geq 0 \;\Rightarrow\; x^2 - 4 \geq 0 \;\Rightarrow\; x^2 \geq 4 \;\Rightarrow\; |x| \geq 2 \;\Rightarrow\; x \in (-\infty, -2] \cup [2, \infty)$.

Note that *both* $f(x)$ *and* $g(f(x))$ are defined for x in $(-\infty, -2] \cup [2, \infty)$.

24 **(a)** $h(x) = f\left(\sqrt{x}\right) = -\left(\sqrt{x}\right)^2 + 1 = -x + 1$. Domain of $g = [0, \infty)$. Domain of $f = \mathbb{R}$.

Since $g(x)$ is always in the domain of f, the domain of $f \circ g$ is the same as the domain of g, $[0, \infty)$.

(b) $k(x) = g(-x^2+1) = \sqrt{-x^2+1}$. Domain of $f = \mathbb{R}$. Domain of $g = [0, \infty)$.

$f(x) \geq 0 \;\Rightarrow\; -x^2 + 1 \geq 0 \;\Rightarrow\; x^2 \leq 1 \;\Rightarrow\; |x| \leq 1 \;\Rightarrow\; x \in [-1, 1]$.

25 **(a)** $h(x) = f\left(\sqrt{x+5}\right) = \sqrt{\sqrt{x+5} - 2}$. Domain of $g = [-5, \infty)$. Domain of $f = [2, \infty)$.

$g(x) \geq 2 \;\Rightarrow\; \sqrt{x+5} \geq 2 \;\Rightarrow\; x + 5 \geq 4 \;\Rightarrow\; x \geq -1$ or $x \in [-1, \infty)$.

(b) $k(x) = g\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2} + 5}$. Domain of $f = [2, \infty)$. Domain of $g = [-5, \infty)$.

$f(x) \geq -5 \;\Rightarrow\; \sqrt{x-2} \geq -5$. This is always true since the result of a square root is nonnegative.

The domain is $[2, \infty)$.

26 **(a)** $h(x) = f\left(\sqrt{x+2}\right) = \sqrt{3 - \sqrt{x+2}}$. Domain of $g = [\,-2, \infty)$. Domain of $f = (-\infty, 3]$.

$g(x) \leq 3 \;\Rightarrow\; \sqrt{x+2} \leq 3 \;\Rightarrow\; x + 2 \leq 9 \;\Rightarrow\; x \leq 7$.

We must remember that $x \geq -2$, hence, $-2 \leq x \leq 7$.

(b) $k(x) = g\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x} + 2}$. Domain of $f = (-\infty, 3]$. Domain of $g = [-2, \infty)$.

$f(x) \geq -2 \;\Rightarrow\; \sqrt{3-x} \geq -2$. This is always true since the result of a square root is nonnegative.

The domain is $(-\infty, 3]$.

27 **(a)** $h(x) = f\left(\sqrt{x^2-16}\right) = \sqrt{3 - \sqrt{x^2-16}}$. Domain of $g = (-\infty, -4] \cup [4, \infty)$. Domain of $f = (-\infty, 3]$.

$g(x) \leq 3 \;\Rightarrow\; \sqrt{x^2-16} \leq 3 \;\Rightarrow\; x^2 - 16 \leq 9 \;\Rightarrow\; x^2 \leq 25 \;\Rightarrow\; x \in [-5, 5]$.

But $|x| \geq 4$ from the domain of g, so the domain of $f \circ g$ is $[-5, -4] \cup [4, 5]$.

(b) $k(x) = g\left(\sqrt{3-x}\right) = \sqrt{\left(\sqrt{3-x}\right)^2 - 16} = \sqrt{3 - x - 16} = \sqrt{-x-13}$. Domain of $f = (-\infty, 3]$.

Domain of $g = (-\infty, -4] \cup [4, \infty)$. $f(x) \geq 4$ $\{f(x)$ cannot be less than $0\}$ $\Rightarrow$

$\sqrt{3-x} \geq 4 \;\Rightarrow\; 3 - x \geq 16 \;\Rightarrow\; x \leq -13$.

28 **(a)** $h(x) = f\left(\sqrt[3]{x-5}\right) = \left(\sqrt[3]{x-5}\right)^3 + 5 = x - 5 + 5 = x$. Domain of $g = \mathbb{R}$. Domain of $f = \mathbb{R}$.

All values of $g(x)$ are in the domain of f. Hence, the domain of $f \circ g$ is $\mathbb{R}$.

(b) $k(x) = g(x^3+5) = \sqrt[3]{(x^3+5)-5} = \sqrt[3]{x^3} = x$. Domain of $f = \mathbb{R}$. Domain of $g = \mathbb{R}$.

All values of $f(x)$ are in the domain of g. Hence, the domain of $g \circ f$ is $\mathbb{R}$.

29 **(a)** $h(x) = f\left(\dfrac{5x-3}{2}\right) = \dfrac{2\left(\dfrac{5x-3}{2}\right)+3}{5} = \dfrac{5x-3+3}{5} = \dfrac{5x}{5} = x$. Domain of $g = \mathbb{R}$. Domain of $f = \mathbb{R}$.

All values of $g(x)$ are in the domain of f. Hence, the domain of $f \circ g$ is $\mathbb{R}$.

(b) $k(x) = g\left(\dfrac{2x+3}{5}\right) = \dfrac{5\left(\dfrac{2x+3}{5}\right)-3}{2} = \dfrac{2x+3-3}{2} = \dfrac{2x}{2} = x$. Domain of $f = \mathbb{R}$. Domain of $g = \mathbb{R}$.

All values of $f(x)$ are in the domain of g. Hence, the domain of $g \circ f$ is $\mathbb{R}$.

30 **(a)** $h(x) = f(x-1) = \dfrac{1}{(x-1)-1} = \dfrac{1}{x-2}$. Domain of $g = \mathbb{R}$. Domain of $f = \mathbb{R} - \{1\}$.

$g(x) \neq 1 \;\Rightarrow\; x - 1 \neq 1 \;\Rightarrow\; x \neq 2$. Hence, the domain of $f \circ g$ is $\mathbb{R} - \{2\}$.

(b) $k(x) = g\left(\dfrac{1}{x-1}\right) = \dfrac{1}{x-1} - 1 = \dfrac{1-(x-1)}{x-1} = \dfrac{2-x}{x-1}$. Domain of $f = \mathbb{R} - \{1\}$. Domain of $g = \mathbb{R}$.

All values of $f(x)$ are in the domain of g. The domain of $g \circ f$ is $\mathbb{R} - \{1\}$.

31 **(a)** $h(x) = f\left(\dfrac{1}{x^3}\right) = \left(\dfrac{1}{x^3}\right)^2 = \dfrac{1}{x^6}$. Domain of $g = \mathbb{R} - \{0\}$. Domain of $f = \mathbb{R}$.

All values of $g(x)$ are in the domain of f. Hence, the domain of $f \circ g$ is $\mathbb{R} - \{0\}$.

(b) $k(x) = g(x^2) = \dfrac{1}{(x^2)^3} = \dfrac{1}{x^6}$. Domain of $f = \mathbb{R}$. Domain of $g = \mathbb{R} - \{0\}$.

All values of $f(x)$ are in the domain of g except for 0. Since f is 0 when x is 0, the domain of $f \circ g$ is $\mathbb{R} - \{0\}$.

32 **(a)** $h(x) = f\left(\dfrac{3}{x}\right) = \dfrac{3/x}{(3/x)-2} \cdot \dfrac{x}{x} = \dfrac{3}{3-2x}$. Domain of $g = \mathbb{R} - \{0\}$. Domain of $f = \mathbb{R} - \{2\}$.

$g(x) \neq 2 \;\Rightarrow\; \dfrac{3}{x} \neq 2 \;\Rightarrow\; x \neq \frac{3}{2}$. Hence, the domain of $f \circ g$ is $\mathbb{R} - \left\{0, \frac{3}{2}\right\}$.

(b) $k(x) = g\left(\dfrac{x}{x-2}\right) = \dfrac{3}{x/(x-2)} = \dfrac{3x-6}{x}$. Domain of $f = \mathbb{R} - \{2\}$. Domain of $g = \mathbb{R} - \{0\}$.

$f(x) \neq 0 \;\Rightarrow\; \dfrac{x}{x-2} \neq 0 \;\Rightarrow\; x \neq 0$. Hence, the domain of $g \circ f$ is $\mathbb{R} - \{0, 2\}$.

33 **(a)** $h(x) = f\left(\dfrac{x-3}{x-4}\right) = \dfrac{\dfrac{x-3}{x-4} - 1}{\dfrac{x-3}{x-4} - 2} \cdot \dfrac{x-4}{x-4} = \dfrac{x-3-1(x-4)}{x-3-2(x-4)} = \dfrac{1}{5-x}$.

Domain of $g = \mathbb{R} - \{4\}$. Domain of $f = \mathbb{R} - \{2\}$.

$g(x) \neq 2 \;\Rightarrow\; \dfrac{x-3}{x-4} \neq 2 \;\Rightarrow\; x - 3 \neq 2x - 8 \;\Rightarrow\; x \neq 5$. The domain is $\mathbb{R} - \{4, 5\}$.

(b) $k(x) = g\left(\dfrac{x-1}{x-2}\right) = \dfrac{\dfrac{x-1}{x-2} - 3}{\dfrac{x-1}{x-2} - 4} \cdot \dfrac{x-2}{x-2} = \dfrac{x-1-3(x-2)}{x-1-4(x-2)} = \dfrac{-2x+5}{-3x+7}$.

Domain of $f = \mathbb{R} - \{2\}$. Domain of $g = \mathbb{R} - \{4\}$.

$f(x) \neq 4 \;\Rightarrow\; \dfrac{x-1}{x-2} \neq 4 \;\Rightarrow\; x - 1 \neq 4x - 8 \;\Rightarrow\; x \neq \frac{7}{3}$. The domain is $\mathbb{R} - \left\{2, \frac{7}{3}\right\}$.

34 **(a)** $h(x) = f\left(\dfrac{x-5}{x+4}\right) = \dfrac{\dfrac{x-5}{x+4}+2}{\dfrac{x-5}{x+4}-1} \cdot \dfrac{x+4}{x+4} = \dfrac{x-5+2(x+4)}{x-5-1(x+4)} = \dfrac{3x+3}{-9} = \dfrac{-x-1}{3}$.

Domain of $g = \mathbb{R} - \{-4\}$. Domain of $f = \mathbb{R} - \{1\}$. $g(x) \neq 1 \quad \Rightarrow \quad \dfrac{x-5}{x+4} \neq 1 \quad \Rightarrow \quad x-5 \neq x+4$.

This is always true—so no additional values need to be excluded, and thus, the domain is $\mathbb{R} - \{-4\}$.

(b) $k(x) = g\left(\dfrac{x+2}{x-1}\right) = \dfrac{\dfrac{x+2}{x-1}-5}{\dfrac{x+2}{x-1}+4} \cdot \dfrac{x-1}{x-1} = \dfrac{x+2-5(x-1)}{x+2+4(x-1)} = \dfrac{-4x+7}{5x-2}$.

Domain of $f = \mathbb{R} - \{1\}$. Domain of $g = \mathbb{R} - \{-4\}$. $f(x) \neq -4 \quad \Rightarrow \quad \dfrac{x+2}{x-1} \neq -4 \quad \Rightarrow$

$x+2 \neq -4x+4 \quad \Rightarrow \quad x \neq \frac{2}{5}$. The domain is $\mathbb{R} - \left\{\frac{2}{5}, 1\right\}$.

35 **(a)** $f(x) = \dfrac{1}{x-5}$, $g(x) = \sqrt{x-7}$ • $h(x) = f\left(\sqrt{x-7}\right) = \dfrac{1}{\sqrt{x-7}-5}$.

Domain of $g = [7, \infty)$. Domain of $f = \mathbb{R} - \{5\}$.

$g(x) \neq 5 \quad \Rightarrow \quad \sqrt{x-7} \neq 5 \quad \Rightarrow \quad x-7 \neq 25 \quad \Rightarrow \quad x \neq 32$. The domain is $[7, 32) \cup (32, \infty)$.

(b) $k(x) = g\left(\dfrac{1}{x-5}\right) = \sqrt{\dfrac{1}{x-5}-7}$. Domain of $f = \mathbb{R} - \{5\}$. Domain of $g = [7, \infty)$.

$\dfrac{1}{x-5} - 7 \geq 0 \quad \Rightarrow \quad \dfrac{1-7(x-5)}{x-5} \quad 0 \quad \Rightarrow \quad \dfrac{-7x+36}{x-5} \quad 0$ {see the sign chart} $\quad \Rightarrow \quad x \in (5, \frac{36}{7}]$.

Interval	$(-\infty, 5)$	$\left(5, \frac{36}{7}\right)$	$\left(\frac{36}{7}, \infty\right)$
Sign of $-7x+36$	$+$	$+$	$-$
Sign of $x-5$	$-$	$+$	$+$
Resulting sign	$-$	$+$	$-$

36 **(a)** $f(x) = \dfrac{1}{x-7}$, $g(x) = \sqrt{x-4}$ • $h(x) = f\left(\sqrt{x-4}\right) = \dfrac{1}{\sqrt{x-4}-7}$.

Domain of $g = [4, \infty)$. Domain of $f = \mathbb{R} - \{7\}$.

$g(x) \neq 7 \quad \Rightarrow \quad \sqrt{x-4} \neq 7 \quad \Rightarrow \quad x-4 \neq 49 \quad \Rightarrow \quad x \neq 53$. The domain is $[4, 53) \cup (53, \infty)$.

(b) $k(x) = g\left(\dfrac{1}{x-7}\right) = \sqrt{\dfrac{1}{x-7}-4}$. Domain of $f = \mathbb{R} - \{7\}$. Domain of $g = [4, \infty)$.

$\dfrac{1}{x-7} - 4 \geq 0 \quad \Rightarrow \quad \dfrac{1-4(x-7)}{x-7} \geq 0 \quad \Rightarrow \quad \dfrac{-4x+29}{x-7} \quad 0$ {see the sign chart} $\quad \Rightarrow \quad x \in \left(7, \frac{29}{4}\right]$.

Interval	$(-\infty, 7)$	$\left(7, \frac{29}{4}\right)$	$\left(\frac{29}{4}, \infty\right)$
Sign of $-4x+29$	$+$	$+$	$-$
Sign of $x-7$	$-$	$+$	$+$
Resulting sign	$-$	$+$	$-$

37 **(a)** $f(x) = \dfrac{1}{x+3}$, $g(x) = \sqrt{5-x}$ • $h(x) = f\left(\sqrt{5-x}\right) = \dfrac{1}{\sqrt{5-x}+3}$.

Domain of $g = (-\infty, 5]$. Domain of $f = \mathbb{R} - \{-3\}$.

The domain of g is $x \leq 5$ and $f(g(x))$ is defined for $x \leq 5$. The domain is $(-\infty, 5]$.

(b) $k(x) = g\left(\dfrac{1}{x+3}\right) = \sqrt{5 - \dfrac{1}{x+3}}$. Domain of $f = \mathbb{R} - \{-3\}$. Domain of $g = (-\infty, 5]$.

$5 - \dfrac{1}{x+3} \geq 0 \;\Rightarrow\; \dfrac{5(x+3)-1}{x+3} \geq 0 \;\Rightarrow\; \dfrac{5x+14}{x+3} \geq 0$ {see the sign chart} $\Rightarrow$

$x \in (-\infty, -3) \cup \left[-\frac{14}{5}, \infty\right)$.

Interval	$(-\infty, -3)$	$(-3, -\frac{14}{5})$	$(-\frac{14}{5}, \infty)$
Sign of $5x+14$	$-$	$-$	$+$
Sign of $x+3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

38 **(a)** $f(x) = \dfrac{2}{x+4}$, $g(x) = \sqrt{3-x}$ • $h(x) = f\left(\sqrt{3-x}\right) = \dfrac{2}{\sqrt{3-x}+4}$.

Domain of $g = (-\infty, 3]$. Domain of $f = \mathbb{R} - \{-4\}$.

The domain of g is $x \leq 3$ and $f(g(x))$ is defined for $x \leq 3$. The domain is $(-\infty, 3]$.

(b) $k(x) = g\left(\dfrac{2}{x+4}\right) = \sqrt{3 - \dfrac{2}{x+4}}$. Domain of $f = \mathbb{R} - \{-4\}$. Domain of $g = (-\infty, 3]$.

$3 - \dfrac{2}{x+4} \geq 0 \;\Rightarrow\; \dfrac{3(x+4)-2}{x+4} \geq 0 \;\Rightarrow\; \dfrac{3x+10}{x+4} \geq 0$ {see the sign chart} $\Rightarrow$

$x \in (-\infty, -4) \cup \left[\frac{10}{3}, \infty\right)$.

Interval	$(-\infty, -4)$	$(-4, -\frac{10}{3})$	$(-\frac{10}{3}, \infty)$
Sign of $3x+10$	$-$	$-$	$+$
Sign of $x+4$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

39 $f(x) = x^2 - 2$, $g(x) = x + 3$ • $(f \circ g)(x) = f(g(x)) = f(x+3) = (x+3)^2 - 2$.

$(f \circ g)(x) = 0 \;\Rightarrow\; (x+3)^2 - 2 = 0 \;\Rightarrow\; (x+3)^2 = 2 \;\Rightarrow\; x + 3 = \pm\sqrt{2} \;\Rightarrow\; x = -3 \pm \sqrt{2}$

40 $f(x) = x^2 - x - 2$, $g(x) = 2x - 5$ •

$(f \circ g)(x) = f(g(x)) = f(2x-5) = (2x-5)^2 - (2x-5) - 2 = 4x^2 - 22x + 28$.

$(f \circ g)(x) = 0 \;\Rightarrow\; 4x^2 - 22x + 28 = 0 \;\Rightarrow\; (2x-4)(2x-7) = 0 \;\Rightarrow\; x = 2, \frac{7}{2}$

41 **(a)** $(f \circ g)(6) = f(g(6)) = f(8) = 5$ **(b)** $(g \circ f)(6) = g(f(6)) = g(7) = 6$

(c) $(f \circ f)(6) = f(f(6)) = f(7) = 6$ **(d)** $(g \circ g)(6) = g(g(6)) = g(8) = 5$

(e) $(f \circ g)(9) = f(g(9)) = f(4)$, but $f(4)$ cannot be determined from the table.

42 **(a)** $(T \circ S)(1) = T(S(1)) = T(0) = 2$ **(b)** $(S \circ T)(1) = S(T(1)) = S(3) = 2$

(c) $(T \circ T)(1) = T(T(1)) = T(3) = 0$ **(d)** $(S \circ S)(1) = S(S(1)) = S(0) = 1$

(e) $(T \circ S)(4) = T(S(4)) = T(5)$, but $T(5)$ cannot be determined from the table.

43 $D(t) = \sqrt{400 + t^2}$, $R(x) = 20x$ • $(D \circ R)(x) = D(R(x)) = D(20x) = \sqrt{400 + (20x)^2} = \sqrt{400 + 400x^2}$
$= \sqrt{400(1 + x^2)} = \sqrt{400}\,\sqrt{x^2 + 1} = 20\sqrt{x^2 + 1}$

44 $S(r) = 4\pi r^2$, $D(t) = 2t + 5$ • $(S \circ D)(t) = S(D(t)) = S(2t + 5) = 4\pi(2t + 5)^2$

45 We need to examine $(fg)(-x)$—if we obtain $(fg)(x)$, then fg is an even function, whereas if we obtain $-(fg)(x)$, then fg is an odd function.

$$\begin{aligned} (fg)(-x) &= f(-x)g(-x) && \{\text{definition of the product of two functions}\} \\ &= -f(x)g(x) && \{f \text{ is odd, so } f(-x) = -f(x);\ g \text{ is even, so } g(-x) = g(x)\} \\ &= -(fg)(x) && \{\text{definition of the product of two functions}\} \end{aligned}$$

Since $(fg)(-x) = -(fg)(x)$, fg is an odd function.

46 If f is even and odd, then $f(-x) = f(x)$ and $f(-x) = -f(x)$.
Thus, $f(x) = -f(x) \Rightarrow 2f(x) = 0 \Rightarrow f(x) = 0$. Hence, $f(x) = 0$ is a function that is both even and odd.

47 $$\begin{aligned} (\text{ROUND2} \circ \text{SSTAX})(525.00) &= \text{ROUND2}(\text{SSTAX}(525.00)) \\ &= \text{ROUND2}(0.0765 \cdot 525.00) \\ &= \text{ROUND2}(40.1625) = 40.16 \end{aligned}$$

48 **(a)** $(\text{CHR} \circ \text{ORD})(\text{“C”}) = \text{CHR}(\text{ORD}(\text{“C”})) = \text{CHR}(67) = \text{“C”}$

(b) $\text{CHR}(\text{ORD}(\text{“A”}) + 3) = \text{CHR}(65 + 3) = \text{CHR}(68) = \text{“D”}$

49 $r = 5t$ and $A = \pi r^2 \Rightarrow A = \pi(5t)^2 = 25\pi t^2$ ft^2.

50 $V = \dfrac{4}{3}\pi r^3 \Rightarrow r^3 = \dfrac{3V}{4\pi} \Rightarrow r = \sqrt[3]{\dfrac{3V}{4\pi}}$. $V = \dfrac{9}{2}\pi t \Rightarrow r(t) = \sqrt[3]{\dfrac{3\left(\frac{9}{2}\pi t\right)}{4\pi}} = \sqrt[3]{\dfrac{27t}{8}} = \dfrac{3}{2}\sqrt[3]{t}$ ft.

51 The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$ $\{\text{since } h = r\}$ $\Rightarrow$
$r^3 = \dfrac{3V}{\pi} \Rightarrow r = \sqrt[3]{\dfrac{3V}{\pi}}$. $V = 243\pi t \Rightarrow r = \sqrt[3]{\dfrac{3 \cdot 243\pi t}{\pi}} = \sqrt[3]{729t} = 9\sqrt[3]{t}$ ft.

52 $x^2 + x^2 = y^2 \Rightarrow y^2 = 2x^2 \Rightarrow y = \sqrt{2}x$. $y^2 + x^2 = d^2 \Rightarrow d^2 = 3x^2 \Rightarrow d(x) = \sqrt{3}x$.

53 Let l denote the length of the rope. At $t = 0$, $l = 20$. At $t = 1$, $l = 25$.
At time t, $l = 20 + 5t$, *not* just $5t$. We have a right triangle with sides 20, h, and l. $h^2 + 20^2 = l^2 \Rightarrow$
$h = \sqrt{(20 + 5t)^2 - 20^2} = \sqrt{25t^2 + 200t} = \sqrt{25(t^2 + 8t)} = 5\sqrt{t^2 + 8t}$.

54 The triangle has sides of length 28, 50, and $\sqrt{28^2 + 50^2} = \sqrt{3284} = 2\sqrt{821}$.
Let $y = h - 2$. Using similar triangles and the fact that $d = 2t$,
$\dfrac{y}{d} = \dfrac{28}{2\sqrt{821}} \Rightarrow y = \dfrac{14}{\sqrt{821}}d \Rightarrow h(t) = \dfrac{28}{\sqrt{821}}t + 2$.

55 From Exercise 83 of Section 2.4, $d(x) = \sqrt{90{,}400 + x^2}$. The distance x of the plane from the control tower is 500 feet plus 150 feet per second, that is, $x(t) = 500 + 150t$. Thus,
$$\begin{aligned} d(t) &= \sqrt{90{,}400 + (500 + 150t)^2} = \sqrt{90{,}400 + 250{,}000 + 150{,}000t + 22{,}500t^2} \\ &= \sqrt{22{,}500t^2 + 150{,}000t + 340{,}400} = 10\sqrt{225t^2 + 1500t + 3404} \end{aligned}$$

56 Consider the cable to be a right circular cylinder.

$A = 2\pi rh = \pi dh = 1200\pi d$ $\{h = 1200 \text{ inches}\}$ $\Rightarrow$ $d = \dfrac{A}{1200\pi}$.

$A = -750t$, so $d(t) = \text{original value} + \text{change} = 4 - \dfrac{750t}{1200\pi} = 4 - \dfrac{5}{8\pi}t$ inches.

57 $y = (x^2 + 5x)^{1/3}$ • Suppose you were to find the value of y if x was equal to 3. Using a calculator, you might compute the value of $x^2 + 5x$ first, and then raise that result to the $\frac{1}{3}$ power. Thus, we would choose $y = u^{1/3}$ and $u = x^2 + 5x$.

58 For $y = \sqrt[4]{x^4 - 64}$, choose $u = x^4 - 64$ and $y = \sqrt[4]{u}$.

59 For $y = \dfrac{1}{(x-3)^6}$, choose $u = x - 3$ and $y = \dfrac{1}{u^6} = u^{-6}$.

60 For $y = 4 + \sqrt{x^2 + 1}$, choose $u = x^2 + 1$ and $y = 4 + \sqrt{u}$.

61 For $y = \left(x^4 - 2x^2 + 5\right)^5$, choose $u = x^4 - 2x^2 + 5$ and $y = u^5$.

62 For $y = \dfrac{1}{(x^2 + 3x - 5)^3}$, choose $u = x^2 + 3x - 5$ and $y = \dfrac{1}{u^3} = u^{-3}$.

63 For $y = \dfrac{\sqrt{x+4} - 2}{\sqrt{x+4} + 2}$, there is not a "simple" choice for y as in previous exercises.

One choice for u is $u = x + 4$. Then y would be $\dfrac{\sqrt{u} - 2}{\sqrt{u} + 2}$.

Another choice for u is $u = \sqrt{x+4}$. Then y would be $\dfrac{u-2}{u+2}$.

64 For $y = \dfrac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$, choose $u = \sqrt[3]{x}$ and $y = \dfrac{u}{1+u}$.

65 $(f \circ g)(x) = f(g(x)) = f(x^3 + 1) = \sqrt{x^3 + 1} - 1$. We will multiply this expression by $\dfrac{\sqrt{x^3+1} + 1}{\sqrt{x^3+1} + 1}$, treating it as though it was one factor of a difference of two squares.

$$\left(\sqrt{x^3+1} - 1\right) \times \frac{\sqrt{x^3+1} + 1}{\sqrt{x^3+1} + 1} = \frac{\left(\sqrt{x^3+1}\right)^2 - 1^2}{\sqrt{x^3+1} + 1} = \frac{x^3}{\sqrt{x^3+1} + 1}$$

Thus, $(f \circ g)(0.0001) = f\left(g\left(10^{-4}\right)\right) \approx \dfrac{\left(10^{-4}\right)^3}{2} = 5 \times 10^{-13}$.

66 $f(1.12) \approx 0.321170$, $g(1.12) \approx 0.280105$, $f(5.2) \approx 4.106542$, and $f(f(5.2)) \approx 3.014835$ $\Rightarrow$

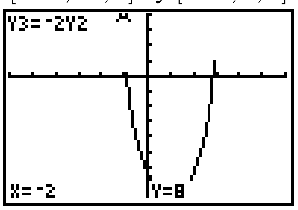
$$\frac{(f+g)(1.12) - (f/g)(1.12)}{[(f \circ f)(5.2)]^2} = \frac{[f(1.12) + g(1.12)] - f(1.12)/g(1.12)}{\{f[f(5.2)]\}^2} \approx -0.059997$$

67 ***Note:*** If we relate this problem to the composite function $g(f(u))$, then Y_2 can be considered to be f, Y_1 {which can be considered to be u} is the function of x we are substituting into f, and Y_3 {which can be considered to be g} is accepting Y_2's output values as its input values. Hence, Y_3 is a function of a function of a function.

Note: The graphs often do not show the correct endpoints—you need to change the viewing rectangle, or zoom in to actually view them on the screen.

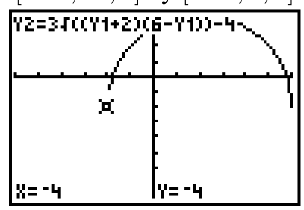
(a) $y = -2f(x)$; $\mathrm{Y}_1 = x$, $\mathrm{Y}_2 = 3\sqrt{(\mathrm{Y}_1+2)(6-\mathrm{Y}_1)} - 4$, graph $\mathrm{Y}_3 = -2\mathrm{Y}_2$ {turn off Y_1 and Y_2, leaving only Y_3 on}; $D = [-2, 6]$, $R = [-16, 8]$

$[-12, 12, 2]$ by $[-16, 8, 2]$



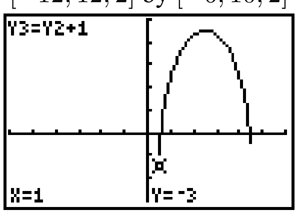
(b) $y = f\left(\frac{1}{2}x\right)$; $\mathrm{Y}_1 = 0.5x$, graph Y_2; $D = [-4, 12]$, $R = [-4, 8]$

$[-12, 12, 2]$ by $[-16, 8, 2]$



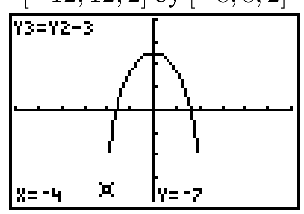
(c) $y = f(x-3) + 1$; $\mathrm{Y}_1 = x - 3$, graph $\mathrm{Y}_3 = \mathrm{Y}_2 + 1$; $D = [1, 9]$, $R = [-3, 9]$

$[-12, 12, 2]$ by $[-6, 10, 2]$



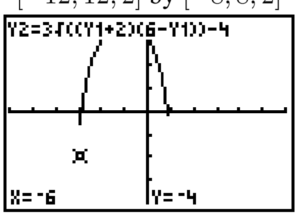
(d) $y = f(x+2) - 3$; $\mathrm{Y}_1 = x + 2$, graph $\mathrm{Y}_3 = \mathrm{Y}_2 - 3$; $D = [-4, 4]$, $R = [-7, 5]$

$[-12, 12, 2]$ by $[-8, 8, 2]$



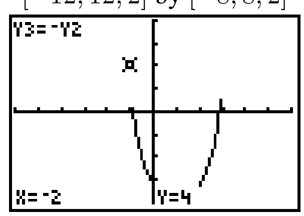
(e) $y = f(-x)$; $\mathrm{Y}_1 = -x$, graph Y_2; $D = [-6, 2]$, $R = [-4, 8]$

$[-12, 12, 2]$ by $[-8, 8, 2]$



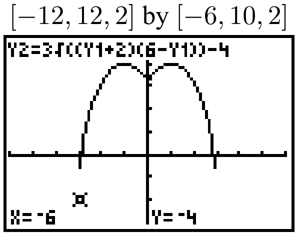
(f) $y = -f(x)$; $\mathrm{Y}_1 = x$, graph $\mathrm{Y}_3 = -\mathrm{Y}_2$; $D = [-2, 6]$, $R = [-8, 4]$

$[-12, 12, 2]$ by $[-8, 8, 2]$



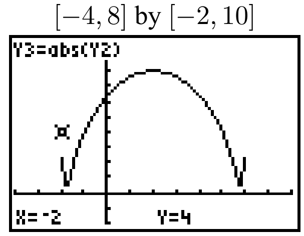
(g) $y = f(|x|)$; $\mathrm{Y}_1 = \mathrm{abs}\, x$, graph Y_2; $D = [-6, 6]$, $R = [-4, 8]$

$[-12, 12, 2]$ by $[-6, 10, 2]$



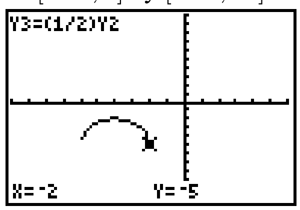
(h) $y = |f(x)|$; $\mathrm{Y}_1 = x$, graph $\mathrm{Y}_3 = \mathrm{abs}\, \mathrm{Y}_2$; $D = [-2, 6]$, $R = [0, 8]$

$[-4, 8]$ by $[-2, 10]$



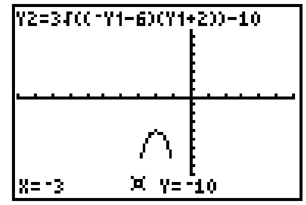
$\boxed{68}$ **(a)** $y = \frac{1}{2}f(x)$; $\mathrm{Y}_1 = x$, graph $\mathrm{Y}_3 = 0.5\mathrm{Y}_2$;
$D = [-6, -2]$, $R = [-5, -2]$

$[-10, 6]$ by $[-11, 10]$



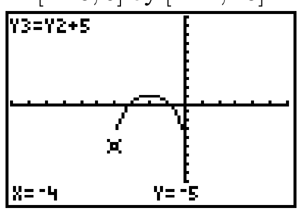
(b) $y = f(2x)$; $\mathrm{Y}_1 = 2x$, graph Y_2;
$D = [-3, -1]$, $R = [-10, -4]$

$[-10, 6]$ by $[-11, 10]$



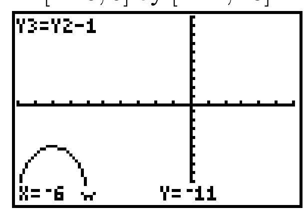
(c) $y = f(x-2) + 5$; $\mathrm{Y}_1 = x - 2$, graph
$\mathrm{Y}_3 = \mathrm{Y}_2 + 5$; $D = [-4, 0]$, $R = [-5, 1]$

$[-10, 6]$ by $[-11, 10]$



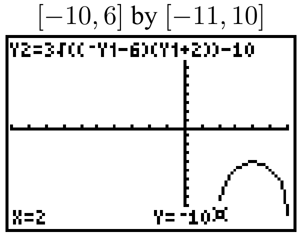
(d) $y = f(x+4) - 1$; $\mathrm{Y}_1 = x + 4$, graph
$\mathrm{Y}_3 = \mathrm{Y}_2 - 1$; $D = [-10, -6]$, $R = [-11, -5]$

$[-10, 6]$ by $[-11, 10]$



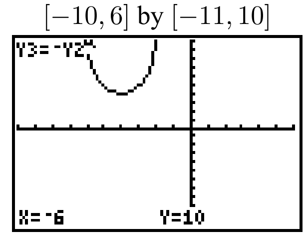
(e) $y = f(-x)$; $\mathrm{Y}_1 = -x$, graph Y_2;
$D = [2, 6]$, $R = [-10, -4]$

$[-10, 6]$ by $[-11, 10]$



(f) $y = -f(x)$; $\mathrm{Y}_1 = x$, graph $\mathrm{Y}_3 = -\mathrm{Y}_2$;
$D = [-6, -2]$, $R = [4, 10]$

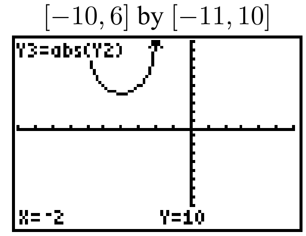
$[-10, 6]$ by $[-11, 10]$



(g) $y = f(|x|)$; No graph

(h) $y = |f(x)|$; $\mathrm{Y}_1 = x$, graph $\mathrm{Y}_3 = \text{abs}\,\mathrm{Y}_2$;
$D = [-6, -2]$, $R = [4, 10]$

$[-10, 6]$ by $[-11, 10]$



Chapter 2 Review Exercises

1 If $y/x < 0$, then y and x must have opposite signs, and hence, the set consists of all points in quadrants II and IV.

2 The points are $A(3,1)$, $B(-5,-3)$, and $C(4,-1)$.
Show that $d(A,B)^2 + d(A,C)^2 = d(B,C)^2$; that is, $\left(\sqrt{80}\right)^2 + \left(\sqrt{5}\right)^2 = \left(\sqrt{85}\right)^2$.
Area $= \frac{1}{2}bh = \frac{1}{2}\left(\sqrt{80}\right)\left(\sqrt{5}\right) = \frac{1}{2}\left(4\sqrt{5}\right)\left(\sqrt{5}\right) = 10$.

3 **(a)** $P(-5,9), Q(-8,-7) \Rightarrow d(P,Q) = \sqrt{[-8-(-5)]^2 + (-7-9)^2} = \sqrt{9+256} = \sqrt{265}$.

(b) $P(-5,9), Q(-8,-7) \Rightarrow M_{PQ} = \left(\dfrac{-5+(-8)}{2}, \dfrac{9+(-7)}{2}\right) = \left(-\dfrac{13}{2}, 1\right)$.

(c) Let $R = (x,y)$. $Q = M_{PR} \Rightarrow (-8,-7) = \left(\dfrac{-5+x}{2}, \dfrac{9+y}{2}\right) \Rightarrow -8 = \dfrac{-5+x}{2}$ and
$-7 = \dfrac{9+y}{2} \Rightarrow -5+x = -16$ and $9+y = -14 \Rightarrow x = -11$ and $y = -23 \Rightarrow R = (-11,-23)$.

4 Let $Q(0,y)$ be an arbitrary point on the y-axis.
$13 = d(P,Q)$ {with $P = (12,8)$} $\Rightarrow 13 = \sqrt{(0-12)^2 + (y-8)^2} \Rightarrow 169 = 144 + y^2 - 16y + 64 \Rightarrow$
$y^2 - 16y + 39 = 0 \Rightarrow (y-3)(y-13) = 0 \Rightarrow y = 3, 13$. The points are $(0,3)$ and $(0,13)$.

5 With $P(a,1)$ and $Q(-2,a)$, $d(P,Q) < 3 \Rightarrow \sqrt{(-2-a)^2 + (a-1)^2} < 3 \Rightarrow$
$4 + 4a + a^2 + a^2 - 2a + 1 < 9 \Rightarrow 2a^2 + 2a - 4 < 0 \Rightarrow a^2 + a - 2 < 0 \Rightarrow (a+2)(a-1) < 0$.

Interval	$(-\infty,-2)$	$(-2,1)$	$(1,\infty)$
Sign of $a+2$	$-$	$+$	$+$
Sign of $a-1$	$-$	$-$	$+$
Resulting sign	$+$	$-$	$+$

From the chart, we see that $-2 < a < 1$ will assure us that $d(P,Q) < 3$.

6 The equation of a circle with center $C(7,-4)$ is $(x-7)^2 + (y+4)^2 = r^2$.
Letting $x = -2$ and $y = 5$ yields $(-9)^2 + 9^2 = r^2 \Rightarrow r^2 = 162$. An equation is $(x-7)^2 + (y+4)^2 = 162$.

7 The center of the circle is the midpoint of $A(8,10)$ and $B(-2,-14)$, so the center is
$M_{AB} = \left(\dfrac{8+(-2)}{2}, \dfrac{10+(-14)}{2}\right) = (3,-2)$. The radius of the circle is
$\frac{1}{2} \cdot d(A,B) = \frac{1}{2}\sqrt{(-2-8)^2 + (-14-10)^2} = \frac{1}{2}\sqrt{100+576} = \frac{1}{2} \cdot 26 = 13$.
An equation is $(x-3)^2 + (y+2)^2 = 13^2 = 169$.

8 We need to solve the equation for x. $(x+2)^2 + y^2 = 7 \Rightarrow (x+2)^2 = 7 - y^2 \Rightarrow x + 2 = \pm\sqrt{7-y^2} \Rightarrow$
$x = -2 \pm \sqrt{7-y^2}$. Choose the term with the minus sign for the left half. $x = -2 - \sqrt{7-y^2}$

9 $C(11,-5), D(-6,8) \Rightarrow m_{CD} = \dfrac{8-(-5)}{-6-11} = \dfrac{13}{-17} = -\dfrac{13}{17}$.

10 Show that the slopes of one pair of opposite sides are equal.
$A(-3,1)$, $B(1,-1)$, $C(4,1)$, and $D(3,5) \Rightarrow m_{AD} = \frac{2}{3} = m_{BC}$.

11 **(a)** $6x + 2y + 5 = 0 \quad \Leftrightarrow \quad y = -3x - \frac{5}{2}$. Using the same slope, -3, with $A\left(\frac{1}{2}, -\frac{1}{3}\right)$, we have

$$y + \tfrac{1}{3} = -3\left(x - \tfrac{1}{2}\right) \quad \Rightarrow \quad 6\left(y + \tfrac{1}{3}\right) = -18\left(x - \tfrac{1}{2}\right) \quad \Rightarrow \quad 6y + 2 = -18x + 9 \quad \Rightarrow \quad 18x + 6y = 7.$$

(b) Using the negative reciprocal of -3 for the slope,

$$y + \tfrac{1}{3} = \tfrac{1}{3}\left(x - \tfrac{1}{2}\right) \quad \Rightarrow \quad 6\left(y + \tfrac{1}{3}\right) = 2\left(x - \tfrac{1}{2}\right) \quad \Rightarrow \quad 6y + 2 = 2x - 1 \quad \Rightarrow \quad 2x - 6y = 3.$$

12 Solving for y gives us: $8x + 3y - 15 = 0 \quad \Leftrightarrow \quad 3y = -8x + 15 \quad \Leftrightarrow \quad y = -\frac{8}{3}x + 5$

13 The radius of the circle is the distance from the line $x = 4$ to the x-value of the center $C(-5, -1)$; $r = 4 - (-5) = 9$. An equation is $(x + 5)^2 + (y + 1)^2 = 81$.

14 $x^2 + y^2 - 4x + 10y + 26 = 0 \quad \Rightarrow \quad x^2 - 4x + \underline{\ 4\ } + y^2 + 10y + \underline{\ 25\ } = -26 + \underline{\ 4\ } + \underline{\ 25\ } \quad \Rightarrow$
$(x - 2)^2 + (y + 5)^2 = 3 \quad \Rightarrow \quad C(2, -5)$. We want the equation of the line through $(-3, 0)$ and $(2, -5)$.

$$y - 0 = \frac{-5 - 0}{2 + 3}(x + 3) \quad \Rightarrow \quad y = -1(x + 3) \quad \Rightarrow \quad x + y = -3.$$

15 $P(3, -7)$ with $m = 4 \quad \Rightarrow \quad y + 7 = 4(x - 3) \quad \Rightarrow \quad y + 7 = 4x - 12 \quad \Rightarrow \quad 4x - y = 19.$

16 $A(-1, 2)$ and $B(3, -4) \quad \Rightarrow \quad M_{AB} = (1, -1)$ and $m_{AB} = -\frac{3}{2}$. We want the equation of the line through $(1, -1)$ with slope $\frac{2}{3}$ $\left\{\text{the negative reciprocal of } -\frac{3}{2}\right\}$. $y + 1 = \frac{2}{3}(x - 1) \quad \Rightarrow \quad 3y + 3 = 2x - 2 \quad \Rightarrow \quad 2x - 3y = 5.$

17 $x^2 + y^2 - 12y + 31 = 0 \quad \Rightarrow \quad x^2 + y^2 - 12y + \underline{\ 36\ } = -31 + \underline{\ 36\ } \quad \Rightarrow \quad x^2 + (y - 6)^2 = 5.$ $C(0, 6)$; $r = \sqrt{5}$

18 $4x^2 + 4y^2 + 24x - 16y + 41 = 0 \quad \Rightarrow \quad x^2 + y^2 + 6x - 4y + \frac{41}{4} = 0 \quad \Rightarrow$

$$x^2 + 6x + \underline{\ 9\ } + y^2 - 4y + \underline{\ 4\ } = -\tfrac{41}{4} + \underline{\ 9\ } + \underline{\ 4\ } \quad \Rightarrow \quad (x + 3)^2 + (y - 2)^2 = \tfrac{11}{4}.\ C(-3, 2);\ r = \tfrac{1}{2}\sqrt{11}$$

19 **(a)** $f(x) = \dfrac{x}{\sqrt{x + 3}} \quad \Rightarrow \quad f(1) = \dfrac{1}{\sqrt{4}} = \dfrac{1}{2}$ **(b)** $f(-1) = \dfrac{-1}{\sqrt{-1 + 3}} = -\dfrac{1}{\sqrt{2}}$

(c) $f(0) = \dfrac{0}{\sqrt{3}} = 0$ **(d)** $f(-x) = \dfrac{-x}{\sqrt{-x + 3}} = -\dfrac{x}{\sqrt{3 - x}}$

(e) $-f(x) = -1 \cdot f(x) = -\dfrac{x}{\sqrt{x + 3}}$ **(f)** $f\left(x^2\right) = \dfrac{x^2}{\sqrt{x^2 + 3}}$

(g) $[f(x)]^2 = \left(\dfrac{x}{\sqrt{x + 3}}\right)^2 = \dfrac{x^2}{x + 3}$

20 $f(x) = \dfrac{-32(x^2 - 4)}{(9 - x^2)^{5/3}} \quad \Rightarrow \quad f(4) = \dfrac{(-)(+)}{(-)} = +.$ $f(4)$ is positive.

Note that the factor -32 always contributes one negative sign to the quotient.

21 $f(x) = \dfrac{-2(x^2 - 20)(3 - x)}{(6 - x^2)^{4/3}} \quad \Rightarrow \quad f(4) = \dfrac{(-)(-)(-)}{(+)} = (-).$ $f(4)$ is negative.

Note that the factor -2 always contributes one negative sign to the quotient and that the denominator is always positive if $x \neq \pm\sqrt{6}$.

22 **(a)** $y = f(x) = \sqrt{3x - 4}$ • $3x - 4 \geq 0 \quad \Rightarrow \quad x \quad \frac{4}{3}$; $D = \left[\frac{4}{3}, \infty\right)$.

Since y is the result of a square root, $y \geq 0$; $R = [0, \infty)$.

(b) $y = f(x) = \dfrac{1}{(x + 4)^2}$ • $D =$ All real numbers except -4.

Since y is the square of the nonzero term $\dfrac{1}{x + 4}$, $y > 0$; $R = (0, \infty)$.

23 $\dfrac{f(a+h)-f(a)}{h} = \dfrac{\left[-(a+h)^2+(a+h)+5\right]-\left[-a^2+a+5\right]}{h}$

$= \dfrac{-a^2-2ah-h^2+a+h+5+a^2-a-5}{h}$

$= \dfrac{-2ah-h^2+h}{h} = \dfrac{h(-2a-h+1)}{h} = -2a-h+1$

24 $\dfrac{f(a+h)-f(a)}{h} = \dfrac{\dfrac{1}{a+h+4}-\dfrac{1}{a+4}}{h} = \dfrac{\dfrac{(a+4)-(a+h+4)}{(a+h+4)(a+4)}}{h}$

$= \dfrac{-h}{(a+h+4)(a+4)h} = -\dfrac{1}{(a+h+4)(a+4)}$

25 $f(x)=ax+b$ is the desired form. $f(1)=3$ and $f(4)=8 \quad\Rightarrow\quad a = \text{slope} = \dfrac{8-3}{4-1} = \dfrac{5}{3}$.

$f(x)=\frac{5}{3}x+b \quad\Rightarrow\quad f(1)=\frac{5}{3}+b$, but $f(1)=3$, so $\frac{5}{3}+b=3$, and $b=\frac{4}{3}$. Thus, $f(x)=\frac{5}{3}x+\frac{4}{3}$.

26 **(a)** $f(x)=\sqrt[3]{x^3+4x} \quad\Rightarrow\quad f(-x)=\sqrt[3]{(-x)^3+4(-x)}=\sqrt[3]{-1(x^3+4x)}=-\sqrt[3]{x^3+4x}=-f(x)$,

so f is odd

(b) $f(x)=\sqrt[3]{\pi x^2-x^3} \quad\Rightarrow\quad f(-x)=\sqrt[3]{\pi(-x)^2-(-x)^3}=\sqrt[3]{\pi x^2+x^3}\neq \pm f(x)$,

so f is neither even nor odd

(c) $f(x)=\sqrt[3]{x^4-3x^2+5} \quad\Rightarrow\quad f(-x)=\sqrt[3]{(-x)^4-3(-x)^2+5}=\sqrt[3]{x^4-3x^2+5}=f(x)$, so f is even

27 $x+5=0 \quad\Leftrightarrow\quad x=-5$, a vertical line; x-intercept -5; y-intercept: None

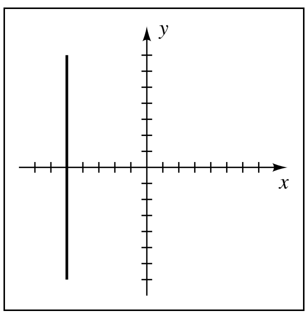


Figure 27

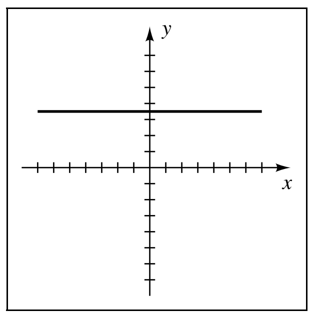


Figure 28

28 $2y-7=0 \quad\Leftrightarrow\quad y=\frac{7}{2}$, a horizontal line; x-intercept: None; y-intercept 3.5

29 $2y + 5x - 8 = 0 \iff y = -\frac{5}{2}x + 4$, a line with slope $-\frac{5}{2}$ and y-intercept 4.

x-intercept: $y = 0 \Rightarrow 2(0) + 5x - 8 = 0 \Rightarrow 5x = 8 \Rightarrow x = 1.6$

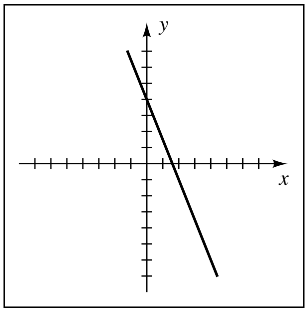


Figure 29

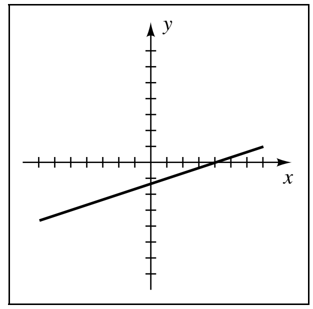


Figure 30

30 $x = 3y + 4 \iff y = \frac{1}{3}x - \frac{4}{3}$, a line with slope $\frac{1}{3}$ and y-intercept $-\frac{4}{3}$; x-intercept 4

31 $9y + 2x^2 = 0 \iff y = -\frac{2}{9}x^2$, a parabola opening down; x- and y-intercept 0

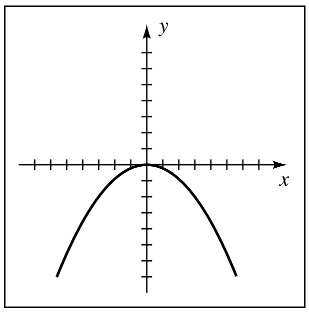


Figure 31

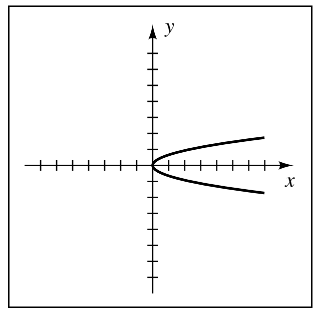


Figure 32

32 $3x - 7y^2 = 0 \iff x = \frac{7}{3}y^2$, a parabola opening to the right; x- and y-intercept 0

33 $y = \sqrt{1-x}$ • The radicand must be nonnegative for the radical to be defined. $1 - x \geq 0 \Rightarrow 1 \geq x$, or equivalently, $x \leq 1$.

The domain is $(-\infty, 1]$ and the range is $[0, \infty)$.

x-intercept: $y = 0 \Rightarrow 0 = \sqrt{1-x} \Rightarrow 0 = 1 - x \Rightarrow x = 1$

y-intercept: $x = 0 \Rightarrow y = \sqrt{1-0} = 1$

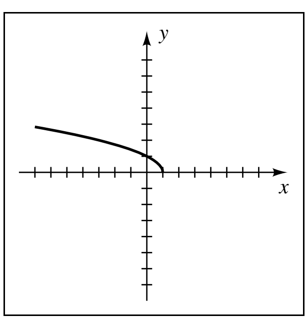


Figure 33

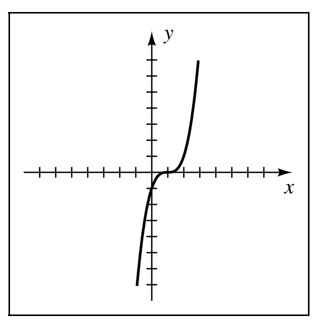


Figure 34

34 $y = (x-1)^3$ • shift $y = x^3$ right one unit; x-intercept 1; y-intercept -1

35 $y^2 = 16 - x^2 \Leftrightarrow x^2 + y^2 = 16$; x-intercepts ± 4; y-intercepts ± 4

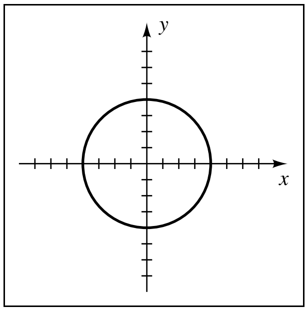


Figure 35

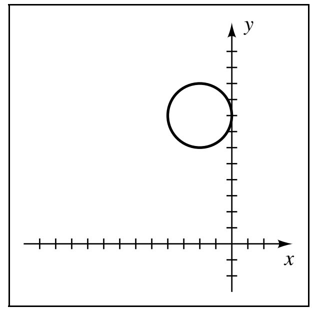


Figure 36

36 $x^2 + y^2 + 4x - 16y + 64 = 0 \Rightarrow x^2 + 4x + \underline{4} + y^2 - 16y + \underline{64} = -64 + \underline{4} + \underline{64} \Rightarrow$
$(x+2)^2 + (y-8)^2 = 4$; $C(-2, 8)$, $r = \sqrt{4} = 2$; x-intercept: None; y-intercept 8

37 $x^2 + y^2 - 8x = 0 \Leftrightarrow x^2 - 8x + \underline{16} + y^2 = \underline{16} \Leftrightarrow (x-4)^2 + y^2 = 16$. This is a circle with center $C(4, 0)$ and radius $r = \sqrt{16} = 4$. x-intercepts: $y = 0 \Rightarrow x^2 - 8x = 0 \Rightarrow x(x-8) = 0 \Rightarrow x = 0$ and 8;
y-intercept: 0

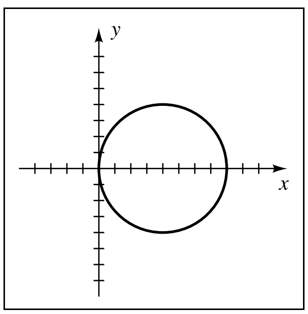


Figure 37

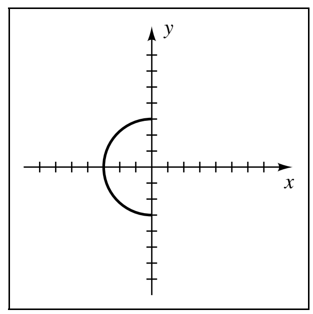


Figure 38

38 $x = -\sqrt{9 - y^2}$ is the left half of the circle $x^2 + y^2 = 9$; x-intercept -3; y-intercepts ± 3

39 $y = (x-3)^2 - 2$ is a parabola that opens upward and has vertex $(3, -2)$.
x-intercepts: $y = 0 \Rightarrow 0 = (x-3)^2 - 2 \Rightarrow (x-3)^2 = 2 \Rightarrow x - 3 = \pm\sqrt{2} \Rightarrow x = 3 \pm \sqrt{2}$
y-intercept: $x = 0 \Rightarrow y = (0-3)^2 - 2 = 9 - 2 = 7$

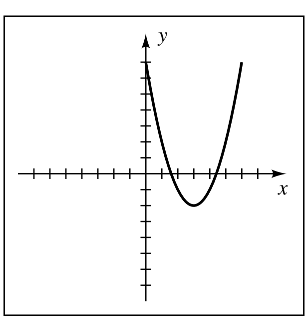


Figure 39

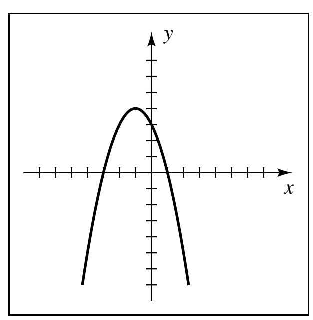


Figure 40

40 $y = -x^2 - 2x + 3 = -(x^2 + 2x + \underline{1}) + 3 + \underline{1} = -(x+1)^2 + 4$; $V(-1, 4)$; x-intercepts -3 and 1; y-intercept 3

41 The radius of the large circle is 3 and the radius of the small circle is 1. Since the center of the small circle is 4 units from the origin and lies on the line $y = x$, we must have $x^2 + y^2 = 4^2 \quad\Rightarrow\quad x^2 + x^2 = 16 \quad\Rightarrow\quad 2x^2 = 16 \quad\Rightarrow$ $x^2 = 8 \quad\Rightarrow\quad x = \sqrt{8}$. The center of the small circle is $\left(\sqrt{8}, \sqrt{8}\right)$.

42 The graph of $y = -f(x-2)$ is the graph of $y = f(x)$ shifted to the right 2 units and reflected through the x-axis.

43 **(a)** The graph of $f(x) = \dfrac{1-3x}{2} = -\dfrac{3}{2}x + \dfrac{1}{2}$ is a line with slope $-\dfrac{3}{2}$ and y-intercept $\dfrac{1}{2}$.

(b) The function is defined for all x, so the domain D is the set of all real numbers.

The range is the set of all real numbers, so $D = \mathbb{R}$ and $R = \mathbb{R}$.

(c) The function f is decreasing on $(-\infty, \infty)$.

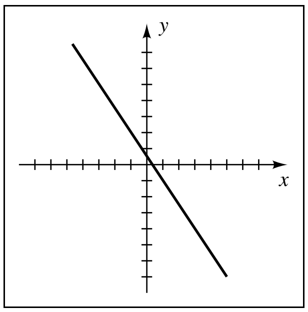


Figure 43

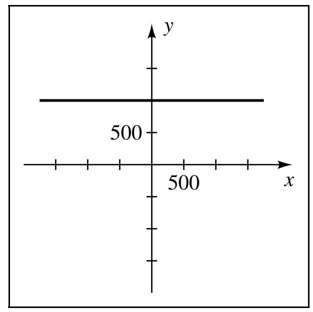


Figure 44

44 **(a)** $f(x) = 1000$ • **(b)** $D = \mathbb{R}$; $R = \{1000\}$ **(c)** Constant on $(-\infty, \infty)$

45 **(a)** The graph of $f(x) = |x+3|$ can be thought of as the graph of $g(x) = |x|$ shifted left 3 units.

(b) The function is defined for all x, so the domain D is the set of all real numbers. The range is the set of all nonnegative numbers, that is, $R = [0, \infty)$.

(c) The function f is decreasing on $(-\infty, -3]$ and is increasing on $[-3, \infty)$.

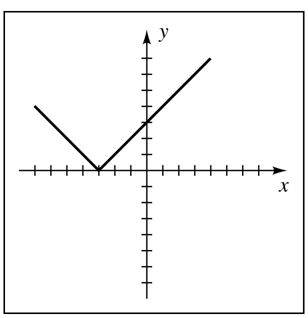


Figure 45

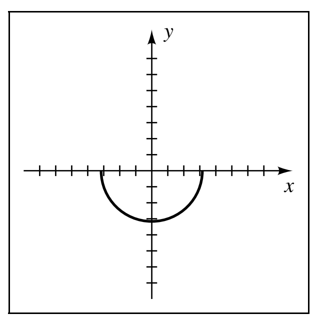


Figure 46

46 **(a)** $f(x) = -\sqrt{10 - x^2}$ • **(b)** $D = \left[-\sqrt{10}, \sqrt{10}\right]$; $R = \left[-\sqrt{10}, 0\right]$

(c) Decreasing on $\left[-\sqrt{10}, 0\right]$, increasing on $\left[0, \sqrt{10}\right]$

47 **(a)** $f(x) = 1 - \sqrt{x+1} = -\sqrt{x+1} + 1$ • We can think of this graph as the graph of $y = \sqrt{x}$ shifted left 1 unit, reflected through the x-axis, and shifted up 1 unit.

(b) For f to be defined, we must have $x + 1 \geq 0 \quad \Rightarrow \quad x \geq -1$, so $D = [-1, \infty)$. The y-values of f are all the values less than or equal to 1, so $R = (-\infty, 1]$.

(c) The function f is decreasing on $[-1, \infty)$.

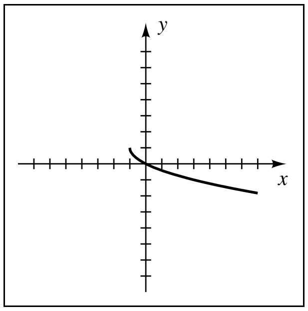


Figure 47

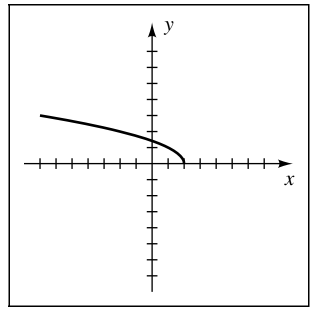


Figure 48

48 **(a)** $f(x) = \sqrt{2-x}$ • **(b)** $D = (-\infty, 2]$; $R = [0, \infty)$ **(c)** Decreasing on $(-\infty, 2]$

49 **(a)** $f(x) = 9 - x^2 = -x^2 + 9$ • We can think of this graph as the graph of $y = x^2$ reflected through the x-axis and shifted up 9 units.

(b) The function is defined for all x, so the domain D is the set of all real numbers. The y-values of f are all the values less than or equal to 9, so $R = (-\infty, 9]$.

(c) The function f is increasing on $(-\infty, 0]$ and is decreasing on $[0, \infty)$.

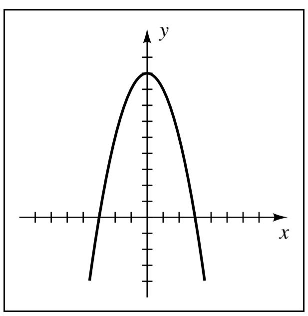


Figure 49

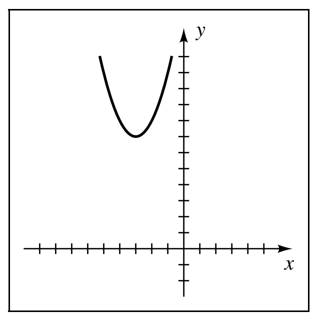


Figure 50

50 **(a)** $f(x) = x^2 + 6x + 16 = x^2 + 6x + 9 + 7 = (x+3)^2 + 7$. The vertex is $(-3, 7)$.

(b) $D = \mathbb{R}$; $R = [7, \infty)$

(c) Decreasing on $(-\infty, -3]$, increasing on $[-3, \infty)$

51 **(a)** $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 3x & \text{if } 0 \leq x < 2 \\ 6 & \text{if } x \geq 2 \end{cases}$

If $x < 0$, we want the graph of the parabola $y = x^2$. The endpoint of this part of the graph is $(0, 0)$, but it is not included. If $0 \leq x < 2$, we want the graph of $y = 3x$, a line with slope 3 and y-intercept 0. Now we do include the endpoint $(0, 0)$, but we don't include the endpoint $(2, 6)$. If $x \geq 2$, we want the graph of the horizontal line $y = 6$ and include its endpoint $(2, 6)$, so there are no open endpoints on the graph of f.

(b) The function is defined for all x, so the domain D is the set of all real numbers. The y-values of f are all the values greater than or equal to 0, so $R = [0, \infty)$.

(c) f is decreasing on $(-\infty, 0]$, increasing on $[0, 2]$, and constant on $[2, \infty)$.

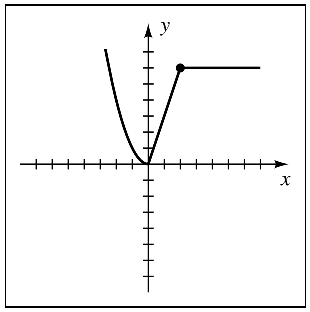


Figure 51

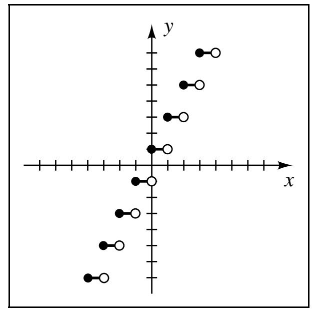


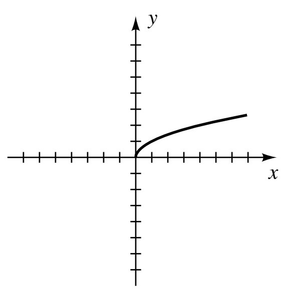
Figure 52

$\boxed{52}$ **(a)** $f(x) = 1 + 2[\![x]\!]$ • The "2" in front of $[\![x]\!]$ has the effect of doubling all the y-values of $g(x) = [\![x]\!]$. The "+1" has the effect of vertically shifting the graph of $h(x) = 2[\![x]\!]$ up 1 unit.

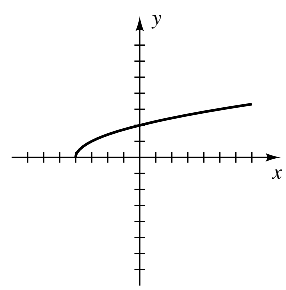
(b) The function is defined for all x, so the domain D is the set of all real numbers. The range of $g(x) = [\![x]\!]$ is the set of integers, that is, $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$. The range of $h(x) = 2[\![x]\!]$ is the set of even integers since we are doubling the values of g—that is, $\{\ldots, -4, -2, 0, 2, 4, \ldots\}$. Since $f(x) = 1 + h(x)$, the range R of f is $\{\ldots, -3, -1, 1, 3, \ldots\}$.

(c) The function f is constant on intervals such as $[0, 1)$, $[1, 2)$, and $[2, 3)$. In general, f is constant on $[n, n+1)$, where n is any integer.

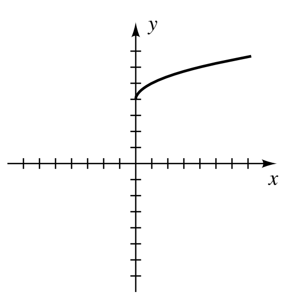
$\boxed{53}$ **(a)** $y = \sqrt{x}$ • This is the square root function, whose graph is a half-parabola.



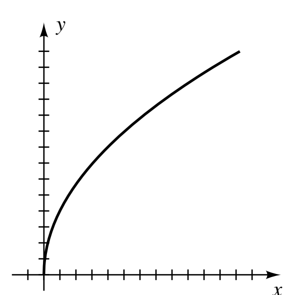
(b) $y = \sqrt{x+4}$ • shift $y = \sqrt{x}$ left 4 units



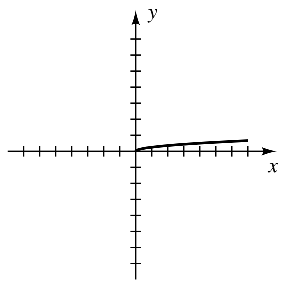
(c) $y = \sqrt{x} + 4$ • shift $y = \sqrt{x}$ up 4 units



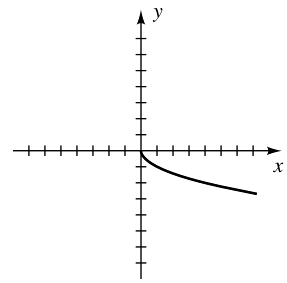
(d) $y = 4\sqrt{x}$ • vertically stretch the graph of $y = \sqrt{x}$ by a factor of 4



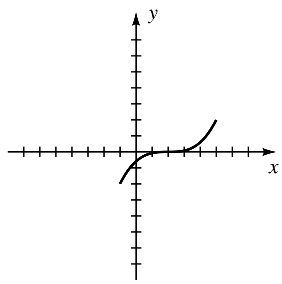
(e) $y = \frac{1}{4}\sqrt{x}$ • vertically compress the graph of $y = \sqrt{x}$ by a factor of $1/\frac{1}{4} = 4$



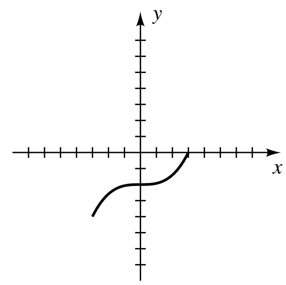
(f) $y = -\sqrt{x}$ • reflect the graph of $y = \sqrt{x}$ through the x-axis



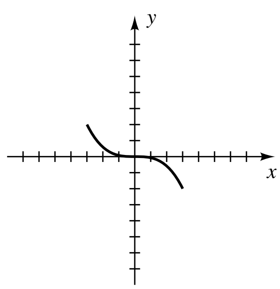
$\boxed{54}$ **(a)** $y = f(x - 2)$ • shift f right 2 units



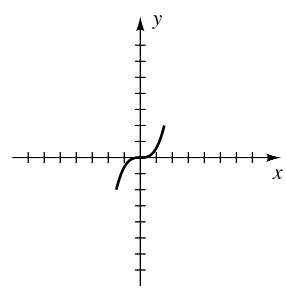
(b) $y = f(x) - 2$ • shift f down 2 units



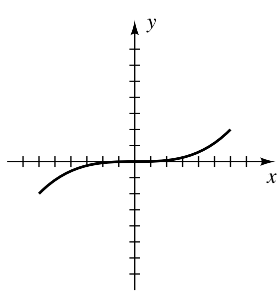
(c) $y = f(-x)$ • reflect f through the y-axis



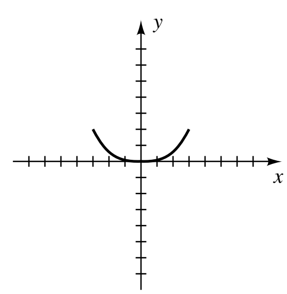
(d) $y = f(2x)$ • horizontally compress f by a factor of 2



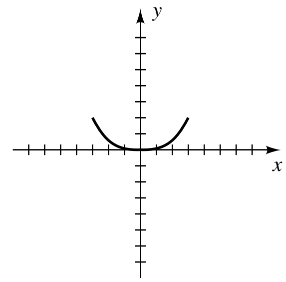
(e) $y = f\left(\frac{1}{2}x\right)$ • horizontally stretch f by a factor of $1/(1/2) = 2$



(f) $y = |f(x)|$ • reflect the portion of the graph below the x-axis through the x-axis.



(g) $y = f(|x|)$ • include the reflection of all points with positive x-coordinates through the y-axis—results in the same graph as in part (f).



$\boxed{55}$ Using the intercept form of a line with x-intercept 5 and y-intercept -2, an equation of the line is $\frac{x}{5} + \frac{y}{-2} = 1$. Multiplying by 10 gives us $\left[\frac{x}{5} + \frac{y}{-2} = 1\right] \cdot 10$, or, equivalently, $2x - 5y = 10$.

$\boxed{56}$ The midpoint of $(-7, 1)$ and $(3, 1)$ is $M(-2, 1)$. This point is 5 units from either of the given points. An equation of the circle is $(x + 2)^2 + (y - 1)^2 = 5^2 = 25$.

$\boxed{57}$ $V(2, -4)$ and $P(-2, 4)$ with $y = a(x - h)^2 + k \quad \Rightarrow \quad 4 = a(-2 - 2)^2 - 4 \quad \Rightarrow \quad 8 = 16a \quad \Rightarrow \quad a = \frac{1}{2}$. An equation of the parabola is $y = \frac{1}{2}(x - 2)^2 - 4$.

$\boxed{58}$ The graph could be made by taking the graph of $y = |x|$, reflecting it through the x-axis $\{y = -|x|\}$, then shifting that graph to the right 2 units $\{y = -|x - 2|\}$, and then shifting that graph down 1 unit, resulting in the graph of the equation $y = -|x - 2| - 1$.

$\boxed{59}$ $f(x) = 3x^2 - 24x + 46 \quad \Rightarrow \quad -\frac{b}{2a} = -\frac{-24}{2(3)} = 4$. $f(4) = -2$ is a minimum since $a = 3 > 0$.

$\boxed{60}$ $f(x) = -2x^2 - 12x - 24 \quad \Rightarrow \quad -\frac{b}{2a} = -\frac{-12}{2(-2)} = -3$. $f(-3) = -6$ is a maximum since $a = -2 < 0$.

$\boxed{61}$ $f(x) = -12(x + 4)^2 + 20$ is in the standard form. $f(-4) = 20$ is a maximum since $a = -12 < 0$.

$\boxed{62}$ $f(x) = 3(x + 2)(x - 10)$ has x-intercepts at -2 and 10. The vertex is halfway between them at $x = 4$. $f(4) = 3 \cdot 6 \cdot (-6) = -108$ is a minimum since $a = 3 > 0$.

$\boxed{63}$
$$\begin{aligned} f(x) = -2x^2 + 12x - 14 &= -2\big(x^2 - 6x + \underline{\quad}\big) - 14 + \underline{\quad} \quad \{\text{complete the square}\} \\ &= -2\big(x^2 - 6x + \underline{9}\big) - 14 + 18 = -2(x - 3)^2 + 4. \end{aligned}$$

$\boxed{64}$ $V(3, -2) \quad \Rightarrow \quad (h, k) = (3, -2)$ in $y = a(x - h)^2 + k$.
$x = 1, y = -5 \quad \Rightarrow \quad -5 = a(1 - 3)^2 - 2 \quad \Rightarrow \quad -3 = 4a \quad \Rightarrow \quad a = -\frac{3}{4}$. Hence, $y = -\frac{3}{4}(x - 3)^2 - 2$.

$\boxed{65}$ The domain of $f(x) = \sqrt{9 - x^2}$ is $[-3, 3]$. The domain of $g(x) = \sqrt{x}$ is $[0, \infty)$.

(a) The domain of fg is the intersection of those two domains, $[0, 3]$.

(b) The domain of f/g is the same as that of fg, excluding any values that make g equal to 0. Thus, the domain of f/g is $(0, 3]$.

$\boxed{66}$ **(a)** $f(x) = 8x - 3$ and $g(x) = \sqrt{x - 2} \quad \Rightarrow \quad (f \circ g)(2) = f(g(2)) = f(0) = -3$

(b) $(g \circ f)(2) = g(f(2)) = g(13) = \sqrt{11}$

$\boxed{67}$ **(a)** $f(x) = 2x^2 - 5x + 1$ and $g(x) = 3x + 2 \Rightarrow$
$(f \circ g)(x) = f(g(x)) = 2(3x+2)^2 - 5(3x+2) + 1 = 2(9x^2 + 12x + 4) - 15x - 10 + 1 = 18x^2 + 9x - 1$

(b) $(g \circ f)(x) = g(f(x)) = 3(2x^2 - 5x + 1) + 2 = 6x^2 - 15x + 3 + 2 = 6x^2 - 15x + 5$

$\boxed{68}$ **(a)** $f(x) = \sqrt{3x+2}$ and $g(x) = \dfrac{1}{x^2} \Rightarrow (f \circ g)(x) = f(g(x)) = \sqrt{3\left(\dfrac{1}{x^2}\right) + 2} = \sqrt{\dfrac{3 + 2x^2}{x^2}}$

(b) $(g \circ f)(x) = g(f(x)) = \dfrac{1}{\left(\sqrt{3x+2}\right)^2} = \dfrac{1}{3x+2}$

$\boxed{69}$ **(a)** $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x-3}\right) = \sqrt{25 - \left(\sqrt{x-3}\right)^2} = \sqrt{25 - (x-3)} = \sqrt{28 - x}$.

Domain of $g(x) = \sqrt{x-3} = [3, \infty)$. Domain of $f(x) = \sqrt{25 - x^2} = [-5, 5]$.

$g(x) \leq 5$ {$g(x)$ cannot be less than 0} $\Rightarrow \sqrt{x-3} \leq 5 \Rightarrow x - 3 \leq 25 \Rightarrow x \leq 28$.

$[3, \infty) \cap (-\infty, 28] = [3, 28]$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{25 - x^2}\right) = \sqrt{\sqrt{25 - x^2} - 3}$. Domain of $f = [-5, 5]$. Domain of $g = [3, \infty)$.

$f(x) \geq 3 \Rightarrow \sqrt{25 - x^2} \quad 3 \Rightarrow 25 - x^2 \geq 9 \Rightarrow x^2 \leq 16 \Rightarrow x \in [-4, 4]$.

$\boxed{70}$ **(a)** $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{2}{x}\right) = \dfrac{2/x}{3(2/x) + 2} \cdot \dfrac{x}{x} = \dfrac{2}{6 + 2x} = \dfrac{1}{x+3}$. Domain of $g = \mathbb{R} - \{0\}$.

Domain of $f = \mathbb{R} - \{-\frac{2}{3}\}$. $g(x) \neq -\frac{2}{3} \Rightarrow \dfrac{2}{x} \neq -\dfrac{2}{3} \Rightarrow x \neq -3$.

Hence, the domain of $f \circ g$ is $\mathbb{R} - \{-3, 0\}$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{x}{3x+2}\right) = \dfrac{2}{x/(3x+2)} = \dfrac{6x+4}{x}$. Domain of $f = \mathbb{R} - \left\{-\frac{2}{3}\right\}$. Domain of $g = \mathbb{R} - \{0\}$. $f(x) \neq 0 \Rightarrow \dfrac{x}{3x+2} \neq 0 \Rightarrow x \neq 0$. Hence, the domain of $g \circ f$ is $\mathbb{R} - \left\{-\frac{2}{3}, 0\right\}$.

$\boxed{71}$ **(a)** $f(x) = \dfrac{1}{x-3}$, $g(x) = \sqrt{4-x}$ • $h(x) = f\left(\sqrt{4-x}\right) = \dfrac{1}{\sqrt{4-x} - 3}$.

Domain of $g = (-\infty, 4]$. Domain of $f = \mathbb{R} - \{3\}$.

$g(x) \neq 3 \Rightarrow \sqrt{4-x} \neq 3 \Rightarrow 4 - x \neq 9 \Rightarrow x \neq -5$. The domain is $(-\infty, -5) \cup (-5, 4]$.

(b) $k(x) = g\left(\dfrac{1}{x-3}\right) = \sqrt{4 - \dfrac{1}{x-3}}$. Domain of $f = \mathbb{R} - \{3\}$. Domain of $g = (-\infty, 4]$.

$4 - \dfrac{1}{x-3} \quad 0 \Rightarrow \dfrac{4(x-3) - 1}{x-3} \quad 0 \Rightarrow \dfrac{4x - 13}{x-3} \geq 0$ {see the sign chart} $\Rightarrow$

$x \in (-\infty, 3) \cup \left[\frac{13}{4}, \infty\right)$.

Interval	$(-\infty, 3)$	$\left(3, \frac{13}{4}\right)$	$\left(\frac{13}{4}, \infty\right)$
Sign of $4x - 13$	$-$	$-$	$+$
Sign of $x - 3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

$\boxed{72}$ For $y = \sqrt[3]{x^2 - 5x}$, choose $u = x^2 - 5x$ and $y = \sqrt[3]{u}$.

$\boxed{73}$ The slope of the ramp should be between $\frac{1}{12}$ and $\frac{1}{20}$.

If the rise of the ramp is 3 feet, then the run should be between $3 \times 12 = 36$ ft and $3 \times 20 = 60$ ft.

The range of the ramp lengths should be from $L = \sqrt{3^2 + 36^2} \approx 36.1$ ft to $L = \sqrt{3^2 + 60^2} \approx 60.1$ ft.

$\boxed{74}$ **(a)** The year 2016 corresponds to $t = 2016 - 1948 = 68$. $d = 181 + 1.065(68) = 253.42 \approx 253$ ft

(b) $d = 265 \quad \Rightarrow \quad 265 = 181 + 1.065t \quad \Rightarrow \quad 84 = 1.065t \quad \Rightarrow \quad t = \frac{84}{1.065} \approx 78.9$ yr,

which corresponds to $79 + 1948 = 2027$ or Olympic year 2028.

$\boxed{75}$ **(a)** $V = at + b$ is the desired form. $V = 179{,}000$ when $t = 0 \quad \Rightarrow \quad V = at + 179{,}000$.

$V = 215{,}000$ when $t = 6 \quad \Rightarrow \quad 215{,}000 = 6a + 179{,}000 \quad \Rightarrow \quad a = \frac{36{,}000}{6} = 6000$ and hence,

$V = 6000t + 179{,}000$.

(b) $V = 193{,}000 \quad \Rightarrow \quad 193{,}000 = 6000t + 179{,}000 \quad \Rightarrow \quad t = \frac{14{,}000}{6000} = \frac{7}{3}$, or $2\frac{1}{3}$.

$\boxed{76}$ **(a)** $F = aC + b$ is the desired form. $F = 32$ when $C = 0 \quad \Rightarrow \quad F = aC + 32$. $F = 212$ when $C = 100 \quad \Rightarrow$

$212 = 100a + 32 \quad \Rightarrow \quad a = \frac{180}{100} = \frac{9}{5}$ and hence, $F = \frac{9}{5}C + 32$.

(b) If C increases $1°$, F increases $\left(\frac{9}{5}\right)°$, or $1.8°$.

$\boxed{77}$ **(a)** $C_1(x) = \left(3.00\,\dfrac{\text{dollars}}{\text{gallon}}\right) \div \left(20\,\dfrac{\text{miles}}{\text{gallon}}\right) \cdot x \text{ miles} = \dfrac{3}{20}x$, or $0.15x$.

(b) After the tune-up, the gasoline mileage will be 10% more than 20 mi/gal; that is, 22 mi/gal.

$C_2(x) = \frac{3}{22}x + 120 \approx 0.136x + 120$.

(c) $C_2 < C_1 \quad \Rightarrow \quad \left[\frac{3}{22}x + 120 < \frac{3}{20}x\right] \cdot 220 \quad \Rightarrow \quad 30x + 26{,}400 < 33x \quad \Rightarrow \quad 3x > 26{,}400 \quad \Rightarrow$

$x > 8800$ miles.

$\boxed{78}$ **(a)** The length across the top of the pen is $3x$ and the length across the bottom is $2y$. These lengths are equal, so $2y = 3x \quad \Rightarrow \quad y = \frac{3}{2}x$, or $y(x) = \frac{3}{2}x$.

(b) There are 6 long lengths (y) and 9 short lengths (x), so the perimeter P is given by $P = 6y + 9x$ and $C = 10P = 10(6y + 9x) = 10\left[(6 \cdot \frac{3}{2}x) + 9x\right] = 10(9x + 9x) = 10(18x) = 180x$. Thus, $C(x) = 180x$.

$\boxed{79}$ The distances covered by cars A and B in t seconds are $88t$ and $66t$, respectively. The vertical distance between car A and car B after t seconds is $(20 + 88t) - 66t = 20 + 22t$. Using the Pythagorean theorem, we have

$$d(t) = \sqrt{10^2 + (20 + 22t)^2} = \sqrt{100 + 400 + 880t + 484t^2} = 2\sqrt{121t^2 + 220t + 125}.$$

$\boxed{80}$ Surface area $S = 2(4)(x) + (4)(y) = 8x + 4y$. Cost $C = 2(8x) + 5(4y) = 16x + 20y$.

(a) $C = 400 \quad \Rightarrow \quad 16x + 20y = 400 \quad \Rightarrow \quad 20y = -16x + 400 \quad \Rightarrow \quad y = -\frac{4}{5}x + 20$

(b) $V = lwh = (y)(4)(x) = 4xy = 4x\left(-\frac{4}{5}x + 20\right)$

$\boxed{81}$ $V = \pi r^2 h$ and $V = 24\pi \quad \Rightarrow \quad h = \dfrac{24}{r^2}$. $S = \pi r^2 + 2\pi rh = \pi r^2 + 2\pi r \cdot \dfrac{24}{r^2} = \pi r^2 + \dfrac{48\pi}{r}$.

$$C = (0.30)(\pi r^2) + (0.10)\left(\frac{48\pi}{r}\right) = \frac{3\pi r^2}{10} + \frac{48\pi}{10r} = \frac{3\pi(r^3 + 16)}{10r}.$$

82 **(a)** $V = (10 \text{ ft}^3 \text{ per minute})(t \text{ minutes}) = 10t$

(b) The height and length of the bottom triangular region are in the proportion 6-60, or 1-10, and the length is 10 times the height. When $0 \le h \le 6$, the volume is

$$V = (\text{cross sectional area})(\text{pool width}) = \tfrac{1}{2}bh(40) = \tfrac{1}{2}(10h)(h)(40) = 200h^2 \text{ ft}^3.$$

When $6 < h \le 9$, the triangular region is full and $V = 200(6)^2 + (h-6)(80)(40) = 7200 + 3200(h-6)$.

(c) For $0 \le h \le 6$: $10t = 200h^2 \Rightarrow h^2 = \dfrac{10t}{200} \Rightarrow h = \sqrt{\dfrac{t}{20}}$;

$$0 \le h \le 6 \Rightarrow 0 \le \sqrt{\frac{t}{20}} \le 6 \Rightarrow 0 \le \frac{t}{20} \le 36 \Rightarrow 0 \le t \le 720.$$

For $6 < h \le 9$: $10t = 7200 + 3200(h-6) \Rightarrow h - 6 = \dfrac{10t - 7200}{3200} \Rightarrow h = 6 + \dfrac{t-720}{320}$;

$6 < h \le 9 \Rightarrow 6 < 6 + \dfrac{t-720}{320} \le 9 \Rightarrow 0 < \dfrac{t-720}{320} \le 3 \Rightarrow 0 < t - 720 \le 960 \Rightarrow$

$$720 < t \le 1680.$$

83 **(a)** Using similar triangles, $\dfrac{r}{x} = \dfrac{2}{4} \Rightarrow r = \dfrac{1}{2}x$.

(b) $\text{Volume}_{\text{cone}} + \text{Volume}_{\text{cup}} = \text{Volume}_{\text{total}} \Rightarrow \tfrac{1}{3}\pi r^2 h + \pi r^2 h = 5 \Rightarrow$

$$\tfrac{1}{3}\pi\left(\tfrac{1}{2}x\right)^2(x) + \pi(2)^2(y) = 5 \Rightarrow 4\pi y = 5 - \tfrac{\pi}{12}x^3 \Rightarrow y = \tfrac{5}{4\pi} - \tfrac{1}{48}x^3$$

84 **(a)** $\dfrac{y}{b} = \dfrac{y+h}{a} \Rightarrow ay = by + bh \Rightarrow ay - by = bh \Rightarrow y(a-b) = bh \Rightarrow y = \dfrac{bh}{a-b}$

(b) $V = \dfrac{1}{3}\pi a^2(y+h) - \dfrac{1}{3}\pi b^2 y = \dfrac{1}{3}\pi a^2 y + \dfrac{1}{3}\pi a^2 h - \dfrac{1}{3}\pi b^2 y = \dfrac{\pi}{3}\left[(a^2-b^2)y + a^2h\right]$

$$= \frac{\pi}{3}\left[(a^2-b^2)\frac{bh}{a-b} + a^2h\right] = \frac{\pi}{3}h\left[(a+b)b + a^2\right] = \frac{\pi}{3}h(a^2+ab+b^2)$$

(c) $a = 6, b = 3, V = 600 \Rightarrow 600 = \frac{\pi}{3}h(6^2 + 6\cdot 3 + 3^2) \Rightarrow h = \frac{1800}{63\pi} = \frac{200}{7\pi} \approx 9.1 \text{ ft}$

85 If $0 \le x \le 5000$, then $B(x) = \$3.61(x/1000)$. If $x > 5000$, then the charge is $\$3.61(5000/1000)$ for the first 5000 gallons, which is \$18.05, plus \$4.17/1000 for the number of gallons over 5000, which is $(x - 5000)$. We may summarize and simplify as follows:

$$B(x) = \begin{cases} 3.61\left(\frac{x}{1000}\right) & \text{if } 0 \le x \le 5000 \\ 3.61(5) + 4.17\left(\frac{x-5000}{1000}\right) & \text{if } x > 5000 \end{cases} = \begin{cases} 0.00361x & \text{if } 0 \le x \le 5000 \\ 18.05 + 0.00417(x-5000) & \text{if } x > 5000 \end{cases}$$

$$= \begin{cases} 0.00361x & \text{if } 0 \le x \le 5000 \\ 0.00417x - 2.8 & \text{if } x > 5000 \end{cases}$$

86 The high point of the path occurs halfway through the jump, that is, at $\frac{1}{2}(8.95) = 4.475$ meters from the beginning of the jump. Using the form $y = a(x-h)^2 + k$, we have $y = a(x - 4.475)^2 + 1$. Substituting 0 for x and 0 for y {or equivalently, 8.95 for x}, we get $0 = a(0 - 4.475)^2 + 1 \Rightarrow -1 = a(4.475)^2 \Rightarrow$

$$a = -\frac{1}{4.475^2}, \text{ and an equation is } y = -\frac{1}{4.475^2}(x - 4.475)^2 + 1.$$

87 **(a)** $P = 24 \Rightarrow 2x + 2y = 24 \Rightarrow y = 12 - x$ **(b)** $A = xy = x(12 - x)$

(c) A is zero at 0 and 12 and will be a maximum when $x = \dfrac{0+12}{2} = 6$.

Thus, the maximum value of A occurs if the rectangle is a square.

$\boxed{88}$ Let t denote the time (in hr) after 1:00 P.M. If the starting point for ship B is the origin, then the locations of A and B are $-30+15t$ and $-10t$, respectively. Using the Pythagorean theorem,

$$d^2 = (-30+15t)^2 + (-10t)^2 = 900 - 900t + 225t^2 + 100t^2 = 325t^2 - 900t + 900.$$

The time at which the distance between the ships is minimal is the same as the time at which the square of the distance between the ships is minimal. Thus, $t = -\dfrac{b}{2a} = -\dfrac{-900}{2(325)} = \dfrac{18}{13}$, or about 2:23 P.M.

$\boxed{89}$ Let r denote the radius of the semicircles and x the length of the rectangle. Perimeter = half-mile $\Rightarrow$ $2x + 2\pi r = \frac{1}{2}$ $\Rightarrow$ $x = -\pi r + \frac{1}{4}$. $A = 2rx = 2r\left(-\pi r + \frac{1}{4}\right) = -2\pi r^2 + \frac{1}{2}r$.

The maximum value of A occurs when $r = -\dfrac{b}{2a} = -\dfrac{1/2}{2(-2\pi)} = \dfrac{1}{8\pi}$ mi. $x = -\pi\left(\dfrac{1}{8\pi}\right) + \dfrac{1}{4} = \dfrac{1}{8}$ mi.

$\boxed{90}$ **(a)** $f(t) = -\frac{1}{2}gt^2 + 16t$ • $g = 32$ $\Rightarrow$ $f(t) = -16t^2 + 16t$.

Solving $f(t) = 0$ gives us $-16t(t-1)$ $\Rightarrow$ $t = 0, 1$. The player is in the air for 1 second.

(b) $t = -\dfrac{b}{2a} = -\dfrac{16}{2(-16)} = \dfrac{1}{2}$. $f\left(\dfrac{1}{2}\right) = 4$ $\Rightarrow$ the player jumps 4 feet high.

(c) $g = \dfrac{32}{6}$ $\Rightarrow$ $f(t) = -\dfrac{8}{3}t^2 + 16t$. Solving $f(t) = 0$ yields $t = 0$ or 6.

The player would be in the air for 6 seconds on the moon.

$t = -\dfrac{b}{2a} = -\dfrac{16}{2(-8/3)} = 3$. $f(3) = 24$ $\Rightarrow$ the player jumps 24 feet high.

$\boxed{91}$ **(a)** Solving $-0.016x^2 + 1.6x = \frac{1}{5}x$ for x represents the intersection between the parabola and the line.

$-0.08x^2 + 8x = x$ {multiply by 5} $\Rightarrow$ $7x - \frac{8}{100}x^2 = 0$ $\Rightarrow$ $x\left(7 - \frac{8}{100}x\right) = 0$ $\Rightarrow$ $x = 0, \frac{175}{2}$.

The rocket lands at $\left(\frac{175}{2}, \frac{35}{2}\right) = (87.5, 17.5)$.

(b) The *difference* d between the parabola and the line is to be maximized here.

$d = (-0.016x^2 + 1.6x) - \left(\frac{1}{5}x\right) = -0.016x^2 + 1.4x$. d obtains a maximum when

$x = -\dfrac{b}{2a} = -\dfrac{1.4}{2(-0.016)} = 43.75$. The maximum height of the rocket *above the ground* is

$d = -0.016(43.75)^2 + 1.4(43.75) = 30.625$ units.

Chapter 2 Discussion Exercises

$\boxed{1}$ Graphs of equations of the form $y = x^{p/q}$, where $x \geq 0$, and p and q are positive integers all pass through $(0,0)$ and $(1,1)$. If $p/q < 1$, the graph is above $y = x$ for $0 \leq x \leq 1$ and below $y = x$ for $x \geq 1$. The closer p/q is to 1, the closer $y = x^{p/q}$ is to $y = x$. If $p/q > 1$, the graph is below $y = x$ for $0 \leq x \leq 1$ and above $y = x$ for $x \geq 1$.

$\boxed{2}$ **(a)** About the x-axis • replace y with $-y$: $-y = \frac{1}{2}x - 3$ $\Rightarrow$ $g(x) = -\frac{1}{2}x + 3$

(b) About the y-axis • replace x with $-x$: $y = \frac{1}{2}(-x) - 3$ $\Rightarrow$ $g(x) = -\frac{1}{2}x - 3$

(c) About the line $y = 2$ • by examining the graphs of $f(x) = \frac{1}{2}x - 3$ and $y = 2$, we observe that the slope of the reflected line should be $-\frac{1}{2}$ and the y-intercept should be 5 units above $y = 2$ (since the y-intercept of f is 5 units below 2). Hence, $g(x) = -\frac{1}{2}x + 7$.

(d) About the line $x = 3$ • similar to part (c), the slope of g is $-\frac{1}{2}$. The x-intercept of f, 6, is 3 units to the right of $x = 3$, so the x-intercept of g should be 3 units to the left of $x = 3$. Hence, $g(x) = -\frac{1}{2}x$.

Note: An interesting generalization can be made for problems of the form of those in parts (a)–(d) which would be a worthwhile exploratory exercise in itself. It goes as follows: If the graph of $y = f(x)$ is reflected about the line $y = k$ (or $x = k$), how can you obtain the equation of the new graph? Answer: For $y = k$, replace y with $2k - y$; for $x = k$, replace x with $2k - x$.

$\boxed{3}$ For the graph of $g(x) = \sqrt{f(x)}$, where $f(x) = ax^2 + bx + c$, consider 2 cases:

(1) $(\boldsymbol{a > 0})$ If f has 0 or 1 x-intercept(s), the domain of g is $\mathbb{R}$ and its range is $\left[\sqrt{k}, \infty\right)$, where k is the y-value of the vertex of f. If f has 2 x-intercepts (say x_1 and x_2 with $x_1 < x_2$), then the domain of g is $(-\infty, x_1] \cup [x_2, \infty)$ and its range is $[0, \infty)$. The general shape is similar to the v-shape of the graph of $y = \sqrt{a}|x|$.

(2) $(\boldsymbol{a < 0})$ If f has no x-intercepts, there is no graph of g. If f has 1 x-intercept, the graph of g consists of that point. If f has 2 x-intercepts, the domain of g is $[x_1, x_2]$ and the range is $\left[0, \sqrt{k}\right]$. The shape of g is that of the top half of an oval.

The main advantage of graphing g as a composition (say $\mathrm{Y}_1 = f$ and $\mathrm{Y}_2 = \sqrt{\mathrm{Y}_1}$ on a graphing calculator) is to observe the relationship between the range of f and the domain of g.

$\boxed{4}$ $f(x) = ax^2 + bx + c \quad \Rightarrow$

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{\left[a(x+h)^2 + b(x+h) + c\right] - \left[ax^2 + bx + c\right]}{h} \\
&= \frac{ax^2 + 2ahx + ah^2 + bx + bh + c - ax^2 - bx - c}{h} \\
&= \frac{2ahx + ah^2 + bh}{h} = \frac{h(2ax + ah + b)}{h} = 2ax + ah + b
\end{aligned}$$

$\boxed{5}$ The expression $2x + h + 6$ represents the slope of the line between the points $P(x, f(x))$ and $Q(x + h, f(x + h))$. If $h = 0$, then $2x + 6$ represents the slope of the tangent line at the point $P(x, f(x))$.

$\boxed{6}$ To determine the x-coordinate of R, we want to start at x_1 and go $\dfrac{m}{n}$ of the way to x_2.

We could write this as $x_3 = x_1 + \dfrac{m}{n}\Delta x = x_1 + \dfrac{m}{n}(x_2 - x_1) = x_1 + \dfrac{m}{n}x_2 - \dfrac{m}{n}x_1 = \left(1 - \dfrac{m}{n}\right)x_1 + \dfrac{m}{n}x_2$.

Similarly, $y_3 = \left(1 - \dfrac{m}{n}\right)y_1 + \dfrac{m}{n}y_2$.

$\boxed{7}$ The values of the x-intercepts (if they exist) are found using the quadratic formula $x = -\dfrac{b}{2a} \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$. Hence, the distance d from the axis of symmetry, $x = -\dfrac{b}{2a}$, to either x-intercept is $d = \dfrac{\sqrt{b^2 - 4ac}}{2|a|}$ and $d^2 = \dfrac{b^2 - 4ac}{4a^2}$. From page 155 of the text, the y-coordinate of the vertex is $h = c - \dfrac{b^2}{4a} = \dfrac{4ac - b^2}{4a}$, so

$$\frac{h}{d^2} = \frac{\dfrac{4ac - b^2}{4a}}{\dfrac{b^2 - 4ac}{4a^2}} = \frac{4a^2(4ac - b^2)}{4a(b^2 - 4ac)} = -a.$$

Thus, $h = -ad^2$. Note that this relationship also reveals a connection between the discriminant $D = b^2 - 4ac$ and the y-coordinate of the vertex, namely $h = -D/(4a)$.

$\boxed{8}$ The graph of $y = -[\![-x]\!]$ in the solution to Exercise 56(e) of Section 2.5 illustrates the concept of one of the most common billing methods with the open and closed endpoints reversed from those of the greatest integer function. Starting with $y = -[\![-x]\!]$ and adjusting for jumps every 15 minutes gives us $y = -[\![-x/15]\!]$. Since each quarter-hour charge is \$20, we multiply by 20 to obtain $y = -20[\![-x/15]\!]$. Because of the initial \$40 charge, we must add 40 to obtain the function $f(x) = 40 - 20[\![-x/15]\!]$.

$\boxed{9}$ $D = 0.0833x^2 - 0.4996x + 3.5491 \quad \Rightarrow \quad 0.0833x^2 - 0.4996x + (3.5491 - D) = 0.$

Solving for x with the quadratic formula yields $x = \dfrac{0.4996 \pm \sqrt{(-0.4996)^2 - 4(0.0833)(3.5491 - D)}}{2(0.0833)}$,

or, equivalently, $x = \dfrac{4996 \pm \sqrt{33{,}320{,}000D - 93{,}295{,}996}}{1666}$.

From the figure (a graph of D), we see that $3 \le x \le 15$ corresponds to the right half of the parabola. Hence, we choose the plus sign in the equation for x.

$[-20, 20, 2]$ by $[0, 40, 2]$

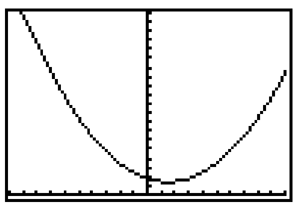


Figure 9

$[0.5, 12.5]$ by $[0, 5]$

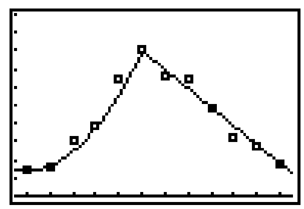


Figure 10

$\boxed{10}$ **(a)** Let January correspond to 1, February to 2, ..., and December to 12.

(b) The data points are (approximately) parabolic on the interval $[1, 6]$ and linear on $[6, 12]$.
Let $f_1(x) = a(x - h)^2 + k$ on $[1, 6]$ and $f_2(x) = mx + b$ on $[6, 12]$. On $[1, 6]$, let the vertex $(h, k) = (1, 0.7)$.
Since $(6, 4)$ is on the graph of f_1, $f_1(6) = a(6 - 1)^2 + 0.7 = 4 \quad \Rightarrow \quad a = 0.132$.
Thus, $f_1(x) = 0.132(x - 1)^2 + 0.7$ on $[1, 6]$. Now, let $f_2(x) = mx + b$ pass through the points $(6, 4)$ and $(12, 0.9)$. An equation of this line is approximately $(y - 4) = -0.517(x - 6)$.
Thus, let $f_2(x) = -0.517x + 7.102$ on $[6, 12]$.

$$f(x) = \begin{cases} 0.132(x-1)^2 + 0.7 & \text{if } 1 \le x \le 6 \\ -0.517x + 7.102 & \text{if } 6 < x \le 12 \end{cases}$$

(c) To plot the piecewise function, let

$$\mathrm{Y}_1 = (0.132(x-1)\text{^}2 + 0.7)/(x \le 6) \quad \text{and} \quad \mathrm{Y}_2 = (-0.517x + 7.102)/(x > 6).$$

These assignments use the concept of Boolean division. For example, when $(x \le 6)$ is false, the expression Y_1 will be undefined (division by 0) and the calculator will not plot any values.

Chapter 2 Test

1 To get to $B(1,2)$ from $A(-3,4)$, move 4 units right and 2 units down.
Since that represents one-tenth of the distance, 40 units right and 20 units down represents the whole distance to

$$P(x,y) = P(-3+40, 4-20) = P(37,-16).$$

2 $d(P,Q) > 5 \Rightarrow \sqrt{(6-2)^2 + (a-3)^2} > 5 \Rightarrow 4^2 + a^2 - 6a + 9 > 5^2 \Rightarrow a^2 - 6a > 0 \Rightarrow a(a-6) > 0 \Rightarrow a < 0$ or $a > 6$. Make a sign chart to establish the final answer.

Interval	$(-\infty,0)$	$(0,6)$	$(6,\infty)$
Sign of a	$-$	$+$	$+$
Sign of $a-6$	$-$	$-$	$+$
Resulting sign	$+$	$-$	$+$

3 The standard equation of the circle with center $(4,5)$ has the form $(x-4)^2 + (y-5)^2 = r^2$. Since an x-intercept is 0, a y-intercept is 0, so $(0-4)^2 + (0-5)^2 = r^2 \Rightarrow 16 + 25 = r^2 \Rightarrow 41 = r^2$, and the desired equation is $(x-4)^2 + (y-5)^2 = 41$.

4 To go from the center $(4,5)$ to the origin, it's 5 units down and 4 units left. By symmetry, to get to the other y-intercept from the center, move 5 units up and 4 units left to the point $(0,10)$; and to get to the other x-intercept, move 5 units down and 4 units right to the point $(8,0)$. Another method would be to find the standard equation of the circle, and then substitute 0 for x to find the y-intercept and then 0 for y to find the x-intercept.

5 The tangent line and the circle pass through the origin. The slope of the line from the origin to the center of the circle is $\frac{5}{4}$. Since the tangent line is perpendicular to that line, it has slope $-\frac{4}{5}$ and equation $y = -\frac{4}{5}x$.

6 $2x - 7y = 3 \Rightarrow -7y = -2x + 3 \Rightarrow y = \frac{2}{7}x - \frac{3}{7}$, so the slope of the given line is $\frac{2}{7}$ and the slope of the desired line is $-\frac{7}{2}$ $\{$the negative reciprocal of $\frac{2}{7}\}$. Since the desired line has x-intercept 4, its equation is $y - 0 = -\frac{7}{2}(x-4) \Rightarrow y = -\frac{7}{2}x + 14$.

7 The cost of the pizza plus the toppings is $9 + 0.80x$. Multiply by 1.10 to get the total cost including the tax. Thus, $T(x) = 1.10(9 + 0.80x) \Rightarrow T(x) = 9.9 + 0.88x$, or $T(x) = 0.88x + 9.9$.

8 $f(x) = \dfrac{\sqrt{-x}}{(x+2)(x-2)}$ • The numerator is defined for $x \leq 0$. The denominator is defined for all real numbers, but we must exclude ± 2 since the denominator is zero for those values.

Thus, the domain of f is $(-\infty,-2) \cup (-2,0]$.

9 For $f(x) = x^2 + 5x - 7$:

$$\begin{aligned}\frac{f(a+h)-f(a)}{h} &= \frac{\left[(a+h)^2 + 5(a+h) - 7\right] - (a^2 + 5a - 7)}{h} \\ &= \frac{[a^2 + 2ah + h^2 + 5a + 5h - 7] - (a^2 + 5a - 7)}{h} \\ &= \frac{2ah + h^2 + 5h}{h} = \frac{h(2a+h+5)}{h} = 2a + h + 5\end{aligned}$$

Based on that answer, a prediction of $2a + h - 7$ seems reasonable for the second function, $f(x) = x^2 - 7x + 5$, since it appears that only the coefficient of x is part of the answer.

10 $V = 2 \times y \times y = 2y^2 \quad \Rightarrow \quad y^2 = V/2 \quad \Rightarrow \quad y = \sqrt{V/2}$. There are four sides of area $2 \times y$ and the top and bottom each have area $y \times y$, so $S(y) = 4(2y) + 2(y^2) \quad \Rightarrow \quad S(V) = 8\sqrt{V/2} + 2(V/2) = V + 8\sqrt{V/2}$ or $V + 4\sqrt{2V}$.

11 Start with the point $P(3,-2)$ on f and find the corresponding point on the graph of $y = 2|f(x-3)| - 1$.

$P(3,-2)$	$\{\, x-3$ [add 3 to the x-coordinate] $\}$	$\to (6,-2)$
	$\{\, \lvert$absolute value$\rvert$ [make the y-coordinate positive] $\}$	$\to (6,2)$
	$\{\, \times 2$ [multiply the y-coordinate by 2] $\}$	$\to (6,4)$
	$\{\, -1$ [subtract 1 from the y-coordinate] $\}$	$\to (6,3)$

12 The salesman makes \$1.20 per cap on the first 1000 caps and \$1.80 on each additional cap.

$$C(x) = \begin{cases} 1.20x & \text{if } 0 \le x \le 1000 \\ 1.20(1000) + 1.80(x - 1000) & \text{if } x > 1000 \end{cases} = \begin{cases} 1.20x & \text{if } 0 \le x \le 1000 \\ 1.80x - 600 & \text{if } x > 1000 \end{cases}$$

13 The standard equation of a parabola with vertical axis and vertex $V(-2,1)$ is $y = a(x+2)^2 + 1$.
If $a < 0$, then the graph will have two x-intercepts, so the desired restriction is $a > 0$.

14 A parabola that has x-intercepts -2 and 4 has equation $f(x) = a(x+2)(x-4)$.
Since $(3,-15)$ is on the parabola, $-15 = a(3+2)(3-4) \quad \Rightarrow \quad -15 = -5a \quad \Rightarrow \quad a = 3$.
The vertex has x-coordinate that is halfway between the x-intercepts, that is, $x = 1$.
The minimum value is of $f(x) = 3(x+2)(x-4)$ is $f(1) = 3(1+2)(1-4) = 3(3)(-3) = -27$.

15 Let x and $2x - 9$ denote the numbers, so that the product is $p = x(2x-9)$. The intercepts of the graph of p are 0 and $\frac{9}{2}$, so the vertex is halfway between them at $x = \frac{9}{4}$, and the minimum value is $\frac{9}{4}\left[2\left(\frac{9}{4}\right) - 9\right] = \frac{9}{4}\left(-\frac{9}{2}\right) = -\frac{81}{8}$. We could also use $p = 2x^2 - 9x$ and note that $-\frac{b}{2a} = -\frac{-9}{2(2)} = \frac{9}{4}$.

16 Let x denote the number of people in the group. The cost per person can be represented by $4 - 0.01(x - 100) = 4 - 0.01x + 1 = 5 - 0.01x$. The total cost T of the group is the product of these two expressions, so $T = x(5 - 0.01x)$. The intercepts of the graph of T are 0 and 500, so the vertex is at $x = 250$. $T(250) = 250(5 - 2.5) = 625$. The tour is only offered for 100 to 300 people at a time, so we check the endpoints and get $T(100) = 100(5-1) = 400$ and $T(300) = 300(5-3) = 600$. Thus, the maximum total cost for the group is \$625 when 250 people are in the group and the minimum total cost for the group is \$400 when 100 people are in the group. Note that group sizes from 251 to 300 have a smaller total cost than \$625.

17 $f(x) = x^2$ and $g(x) = \sqrt{x-3} \quad \Rightarrow \quad (f \circ g)(x) = f(g(x)) = \left(\sqrt{x-3}\right)^2 = x - 3$, which is defined for all reals, but g is defined for only $x \ge 3$, so the domain of $(f \circ g)(x)$ is $[3, \infty)$.

18 $C = y^2 - 2y + 10$ and $y(t) = 5t$, so $(C \circ y)(t) = C(y(t)) = (5t)^2 - 2(5t) + 10 = 25t^2 - 10t + 10$.
The minimum cost occurs when $t = -\frac{b}{2a} = -\frac{-10}{2(25)} = \frac{1}{5}$.
This cost is $C\left(\frac{1}{5}\right) = 25\left(\frac{1}{25}\right) - 10\left(\frac{1}{5}\right) + 10 = 1 - 2 + 10 = 9$, which represents \$9000.