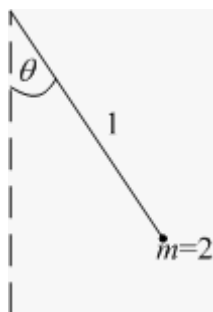


***Calculus Early Transcendentals 4th edition Rogawski
Test Bank***

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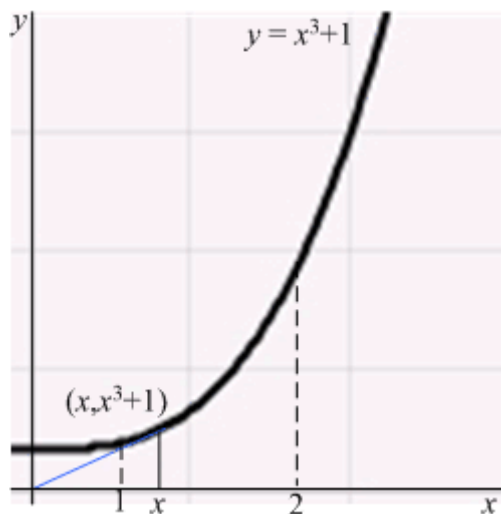
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1. The potential energy V of a pendulum of length 1 and mass 2, relative to its rest position is $V = 2g(1 - \cos \theta)$. Compute the average rate of change of the potential energy over the angle interval $\left[\frac{\pi}{4}, \frac{\pi}{3}\right]$.



ANSWER: $\frac{12g(\sqrt{2}-1)}{\pi} \approx 1.58g$

2. Let $S(x)$ denote the slope of the line segment connecting the origin to the point (x, y) on the graph of the equation $y = x^3 + 1$. Calculate the average rate of change of $S(x)$ for $1 \leq x \leq 2$.



ANSWER: $2\frac{1}{2}$

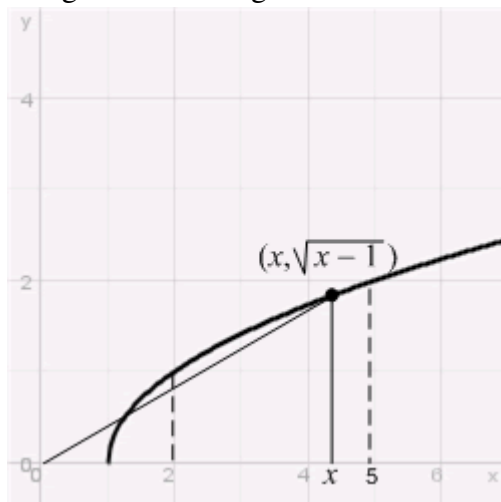
3. The flight time of a shell shot at an angle θ and initial velocity V is $T_f = \frac{2V}{g} \sin \theta$. Compute the average rate of change of the flight time for θ in the interval $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$.

ANSWER: $\frac{12(\sqrt{2}-1)V}{\pi g}$

4. Let $S(x)$ denote the slope of the line segment connecting the origin to the point (x, y) on the graph of the

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equation $y = \sqrt{x-1}$. Calculate the average rate of change of $S(x)$ for $2 \leq x \leq 5$.



ANSWER: $-\frac{1}{30}$

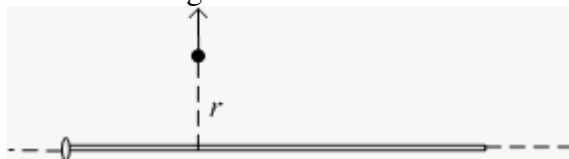
5. The volume of a cone of radius R and height H is $V = \frac{\pi R^2 H}{3}$.

What is the average rate of change of V if the radius increases from 1 to 3 and the height remains unchanged?

ANSWER: $\frac{4\pi H}{3}$

6. The electrical field due to an infinite rod at a point at distance r from the rod is perpendicular to the rod and has a magnitude of $E(r) = \frac{1}{2\pi\epsilon_0} \cdot \frac{\lambda}{r}$ (ϵ_0 is a constant and λ is the longitudinal charge density).

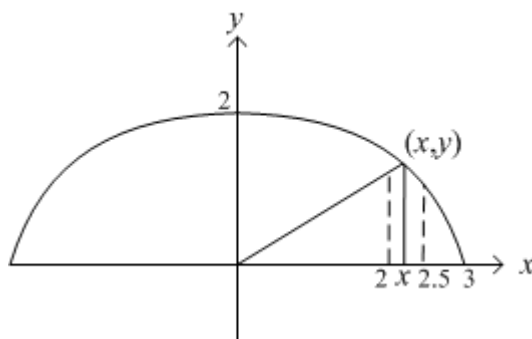
Find the average rate of change of the field along the interval $2 \leq r \leq 4$.



ANSWER: $-\frac{\lambda}{16\pi\epsilon_0}$

7. Let $S(x)$ denote the slope of the line segment connecting the origin to the point (x, y) on the graph of the semi-ellipse $y = 2\sqrt{1 - \frac{x^2}{9}}$. Calculate the average rate of change of $S(x)$ for $2 \leq x \leq 2.5$.

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ANSWER: -0.206 per unit length

8. The electrical field caused by an electrical charge q at a point at distance r is $E = \frac{kq}{r^2}$ (k is a constant).

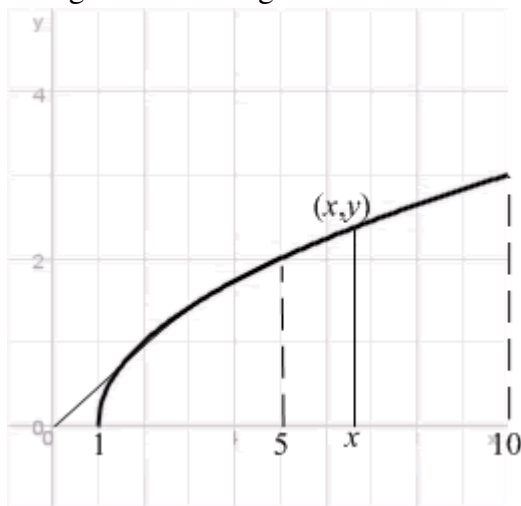
Find the average rate of change of the field along the interval $1 \leq r \leq 3$.

ANSWER: $-\frac{4kq}{9}$

9. The volume of a sphere of radius R is $V = \frac{4\pi R^3}{3}$. What is the average rate of change of the volume when the radius increases from $R = 1$ to $R = 3$?

ANSWER: $\frac{52\pi}{3}$

10. Let $S(x)$ denote the slope of the line segment connecting the origin to the point (x, y) on the graph of the equation $y = \sqrt{x-1}$. Calculate the average rate of change of $S(x)$ for $5 \leq x \leq 10$.



ANSWER: $-\frac{1}{50}$

11. The position of a particle is given by $g(t) = 2t^2 + 5$. Compute the average velocity over the time interval

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$[4, 6]$. Estimate the instantaneous velocity at $t = 4$.

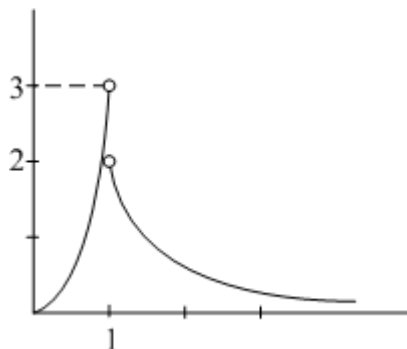
ANSWER: Average velocity over $[4, 6]$: 20

Instantaneous velocity at $t = 4$: 16

12. A balloon is blown up and takes the shape of a sphere. What is the average rate of change of the surface area of the balloon as the radius increases from 3 to 4 cm?

ANSWER: 28π

13. Determine $\lim_{x \rightarrow 1+} f(x)$ and $\lim_{x \rightarrow 1-} f(x)$ for the function shown in the figure.



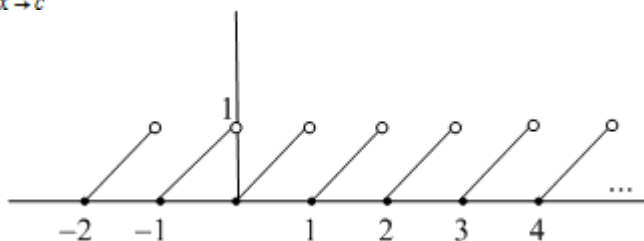
ANSWER: $\lim_{x \rightarrow 1+} f(x) = 2$ $\lim_{x \rightarrow 1-} f(x) = 3$

14. The greatest integer function is defined by $[x] = n$, where n is the unique integer such that $n \leq x < n+1$. The graph of $f(x) = x - [x]$ is shown in the figure.

A) For which values of c does $\lim_{x \rightarrow c-} f(x)$ exist?

B) For which values of c does $\lim_{x \rightarrow c+} f(x)$ exist?

C) For which values of c does $\lim_{x \rightarrow c} f(x)$ exist?



ANSWER: A) All c

B) All c

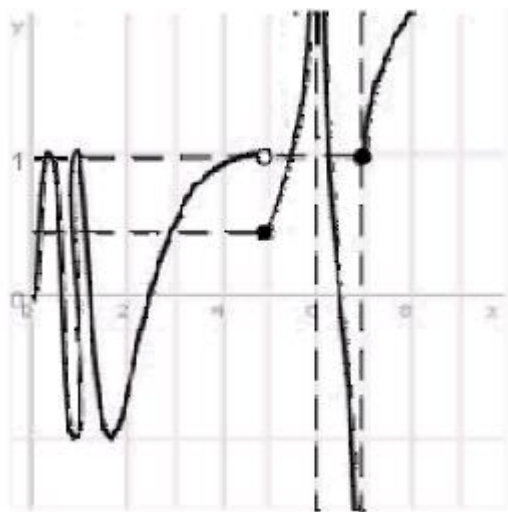
C) Every real number c that is not an integer

15. The graph of a function $y = f(x)$ is shown in the figure.

Determine the following limits or state that the limit does not exist (if the limit is infinite, write ∞ or $-\infty$):

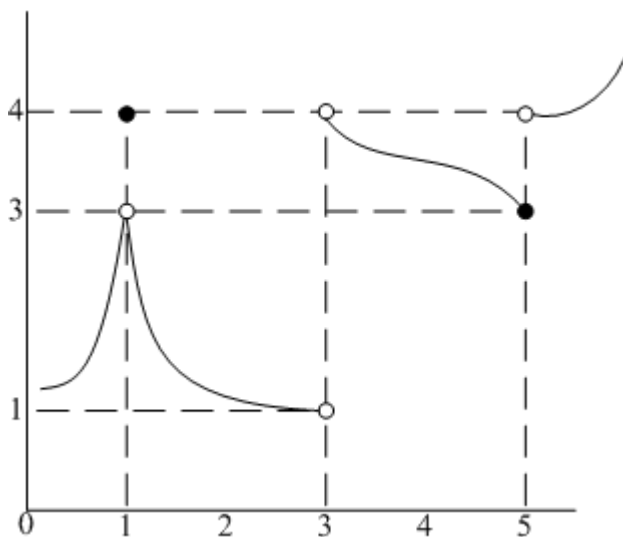
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- A) $\lim_{x \rightarrow 0+} f(x)$ B) $\lim_{x \rightarrow 5-} f(x)$ C) $\lim_{x \rightarrow 5+} f(x)$ D) $\lim_{x \rightarrow 5} f(x)$ E) $\lim_{x \rightarrow 6-} f(x)$
 F) $\lim_{x \rightarrow 6+} f(x)$ G) $\lim_{x \rightarrow 6} f(x)$ H) $\lim_{x \rightarrow 7-} f(x)$ I) $\lim_{x \rightarrow 7+} f(x)$ J) $\lim_{x \rightarrow 7} f(x)$



ANSWER: A) 0 B) 1 C) $\frac{1}{2}$ D) Does not exist E) ∞
 F) ∞ G) ∞ H) $-\infty$ I) 1 J) Does not exist

16. Determine the one-sided limits at $c = 1, 3, 5$ of the function $f(x)$ shown in the figure and state whether the limit exists at these points.



ANSWER: $\lim_{x \rightarrow 1-} f(x) = \lim_{x \rightarrow 1+} f(x) = 3$; limit exists
 $\lim_{x \rightarrow 3-} f(x) = 1$, $\lim_{x \rightarrow 3+} f(x) = 4$; limit does not exist
 $\lim_{x \rightarrow 5-} f(x) = 3$, $\lim_{x \rightarrow 5+} f(x) = 4$; limit does not exist

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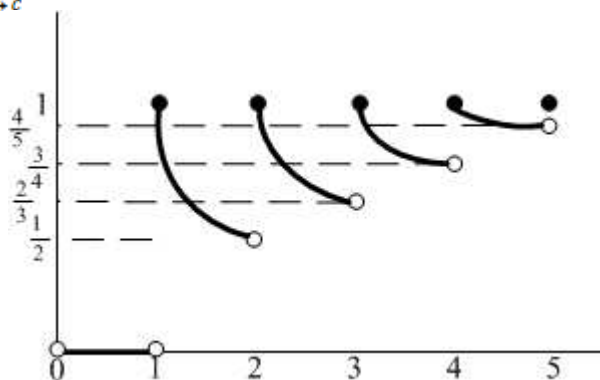
17. Consider the function $f(x) = \frac{[x]}{x}$ for $x > 0$. (Here, $[x]$ denotes the greatest integer function.)

A) Write $f(x)$ in piecewise form.

What is $f(n)$ for positive integers n ?

B) Determine $\lim_{x \rightarrow 3^-} f(x)$ and $\lim_{x \rightarrow 3^+} f(x)$.

C) For which values of c does $\lim_{x \rightarrow c} f(x)$ exist?



ANSWER:

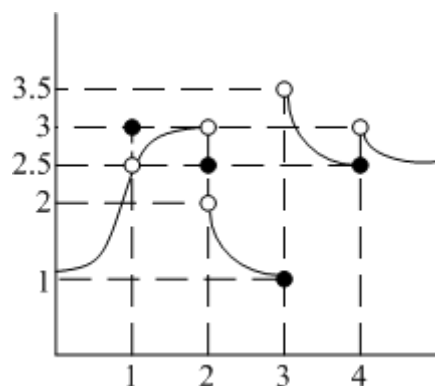
$$A) f(x) = \begin{cases} 0 & 0 < x < 1 \\ \frac{n}{x} & n \leq x < n+1 \quad n = 1, 2, \dots \end{cases}$$

$$f(n) = 1 \quad n = 1, 2, \dots$$

$$B) \frac{2}{3}, 1$$

C) The limit exists for all positive real numbers that are not integers.

18. Determine the one-sided limits at $c = 1, 2, 3, 4$ of the function shown in the figure and state whether the limit exists at these points.



ANSWER:

c	Left-sided	Right-sided	Limit
1	2.5	2.5	Exists (2.5)
2	3	2	Does not exist
3	1	3.5	Does not exist
4	2.5	3	Does not exist

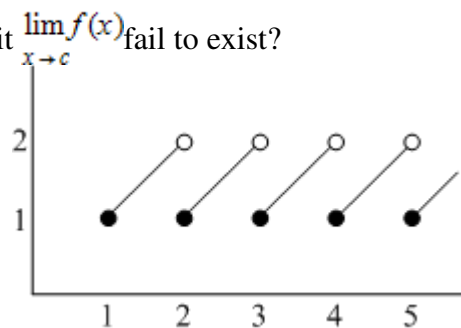
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19. Consider the function $f(x) = x + 1 - [x]$ for $x \geq 1$. (Here, $[x]$ denotes the greatest integer function.)

A) Write f in piecewise form. What is $f(n)$ for positive integers $n \geq 1$?

B) Find $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$.

C) For which values of c does the limit $\lim_{x \rightarrow c} f(x)$ fail to exist?

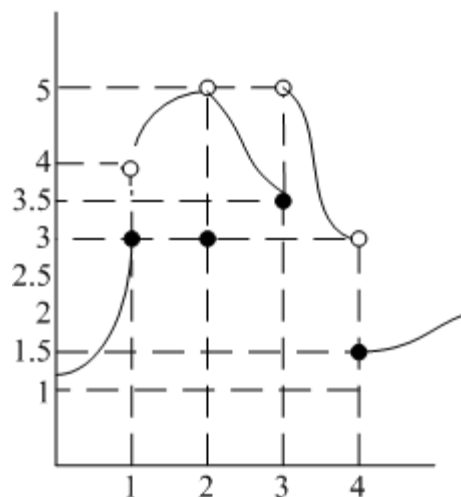


ANSWER: A) $f(x) = x + 1 - n$ for $n \leq x < n + 1$, $f(n) = 1$

B) 2, 1

C) The limit fails to exist for all positive integers.

20. Determine the one-sided limits at $c = 1, 2, 3, 4$ of the function shown in the figure and state whether the limit exists at these points.



ANSWER:	c	Left-sided	Right-sided	Limit
	1	3	4	Does not exist
	2	5	5	Exists (5)
	3	3.5	5	Does not exist
	4	3	1.5	Does not exist

21. Let $f(x)$ be the following function defined for $-0.5 \leq x \leq 4.5$:

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$$f(x) = \begin{cases} 1, & \text{if } \sin\left(\frac{\pi x}{2}\right) > 0 \\ -1, & \text{if } \sin\left(\frac{\pi x}{2}\right) < 0 \\ 0, & \text{if } \sin\left(\frac{\pi x}{2}\right) = 0 \end{cases}$$

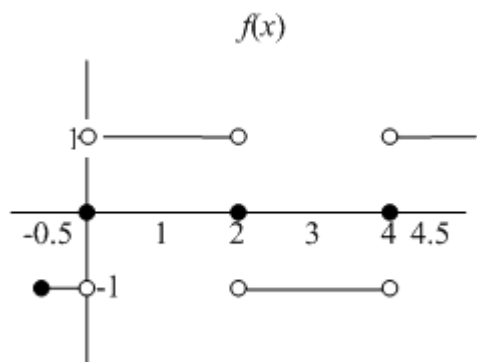
Write $f(x)$ as a piecewise-defined function where the intervals are in terms of x instead of $\sin\left(\frac{\pi x}{2}\right)$, sketch its graph, and determine the points where the limit of $f(x)$ does not exist. Find the one-sided limits at these points.

ANSWER:

$$f(x) = \begin{cases} -1 & -0.5 \leq x < 0 \\ 0 & x = 0 \\ 1 & 0 < x < 2 \\ 0 & x = 2 \\ -1 & 2 < x < 4 \\ 0 & x = 4 \\ 1 & 4 < x \leq 4.5 \end{cases}$$

$$x = -0.5, 0, 2, 4, 4.5$$

$$\begin{array}{cccc} \lim_{x \rightarrow -0.5+} f(x) = -1 & \lim_{x \rightarrow 0-} f(x) = -1 & \lim_{x \rightarrow 0+} f(x) = 1 & \lim_{x \rightarrow 2-} f(x) = 1 \\ \lim_{x \rightarrow 2+} f(x) = -1 & \lim_{x \rightarrow 4-} f(x) = -1 & \lim_{x \rightarrow 4+} f(x) = 1 & \lim_{x \rightarrow 4.5-} f(x) = 1 \end{array}$$



22. Find a real number c such that $\lim_{x \rightarrow 1} f(x)$ exists and compute the limit.

$$f(x) = \begin{cases} x - \frac{3}{x-2} & x < 1 \\ 10 & x = 1 \\ \frac{c}{(x+1)^2} & x > 1 \end{cases}$$

Chapter 02ANSWER: $c = 16$; $\lim_{x \rightarrow 1} f(x) = 4$

23. Let $\lim_{x \rightarrow a} f(x) = L$. Determine whether each of the following statements is always true, never true, or sometimes true.

A) $\lim_{x \rightarrow a} f(x) = L$

B) $\lim_{x \rightarrow a} f(x) = 3L$

C) $\lim_{x \rightarrow a-} f(x) - \lim_{x \rightarrow a+} f(x) < 0$

D) $\frac{\lim_{x \rightarrow a-} f(x)}{\lim_{x \rightarrow a+} f(x)} = 1$

ANSWER: A) Always

B) Sometimes

C) Never

D) Sometimes (note the case when $\lim_{x \rightarrow a} f(x) = 0$)

24. Compute the following one-sided limits:

A) $\lim_{x \rightarrow 2-} \frac{\sqrt{2-x}}{x^2+5x}$

B) $\lim_{\theta \rightarrow 0+} \frac{\theta^3 \cos^2 \theta}{\sin \theta}$

C) $\lim_{\theta \rightarrow \frac{\pi}{2}+} \frac{\theta^3}{\tan \theta}$

ANSWER: A) 0

B) 0

C) 0

25. Evaluate the limits using the Limit Laws:

A) $\lim_{x \rightarrow -1} (x^3 - 2x^2 + 1)$

B) $\lim_{t \rightarrow 1} \frac{t^2 - t}{t + 1}$

C) $\lim_{x \rightarrow 0} \frac{1 + \cos x}{x^3 + 2}$

D) $\lim_{t \rightarrow 0} \frac{3 \sin t}{2t}$

ANSWER: A) -2

B) 0

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- C) $\frac{1}{3}$
 D) $\frac{3}{2}$

26. Which of the following functions are examples of the existence of the limit $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$, although the limits of $f(x)$ and $g(x)$ as $x \rightarrow 0$ do not exist?

- a. $f(x) = x$ $g(x) = \frac{1}{x}$
 b. $f(x) = \frac{\sin x}{x}$ $g(x) = \frac{1}{x}$
 c. $f(x) = \frac{1}{x}$ $g(x) = \frac{1}{x^3}$
 d. $f(x) = x^2$ $g(x) = \cos x$
 e. $f(x) = \frac{x}{\sin x}$ $g(x) = \frac{1}{x}$

ANSWER: c

27. Let $f(x)$, $g(x)$ be functions and let $F(x) = f(x) - g(x)$. Consider the following statement: If $\lim_{x \rightarrow x_0} F(x)$ and $\lim_{x \rightarrow x_0} g(x)$ exist, then $\lim_{x \rightarrow x_0} f(x)$ also exists. To prove this statement, we should use which of the following?

- a. The statement is not true.
 b. The Product Rule applied to $\frac{F+g}{g}$ and g .
 c. The Quotient Rule applied to $(F+g)g$ and g .
 d. The Sum Rule applied to F and g .

ANSWER: d

28. Evaluate the limits using the Limit Laws:

- A) $\lim_{t \rightarrow (-2)} (2t+1)(t^2+2)$
 B) $\lim_{x \rightarrow (-1)} \frac{x^2+3x}{x-1}$
 C) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x + \cos x}{2 \tan x}$

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D) $\lim_{x \rightarrow 4} \frac{2x^{-1} + x^{-\frac{1}{2}}}{x+3}$

ANSWER: A) -18
 B) 1
 C) $\frac{\sqrt{2}}{2}$
 D) $\frac{1}{7}$

29. Determine whether the following statement is correct: If $\lim_{x \rightarrow 0} xg(x) = 0$, then $\lim_{x \rightarrow 0} g(x)$ exists. If yes, prove it; otherwise, give a counter example

ANSWER: False; $g(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$

30. Evaluate the limits using the Limit Laws:

A) $\lim_{t \rightarrow 3} (t^2 + t - 1) \sin \frac{\pi t}{2}$

B) $\lim_{x \rightarrow -1} \frac{x^3 + 5}{x^2 + 2x - 1}$

C) $\lim_{y \rightarrow 4} \frac{y^{-\frac{1}{2}} \tan \left(\frac{\pi y}{16} \right)}{\sqrt{y^2 + 9}}$

ANSWER: A) -11
 B) -2
 C) $\frac{1}{10}$

31. A) Can the Product Rule be used to compute the limit $\lim_{x \rightarrow 0} [x]x$? (Here, $[x]$ denotes the greatest integer function.) Explain.

B) Show that $\lim_{x \rightarrow 0} [x]x$ exists and find it. *Hint:* Compute the one-sided limits.

ANSWER: A) No. The limit $\lim_{x \rightarrow 0} [x]$ does not exist.
 B) 0

32. Let $f(x)$, $g(x)$, and $F(x) = f(x) + g(x)$. To prove that if $\lim_{x \rightarrow x_0} f(x)$ and $\lim_{x \rightarrow x_0} F(x)$ exist then also $\lim_{x \rightarrow x_0} g(x)$ exists, we should use which of the following?

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- a. The Product Rule applied to $\frac{F-f}{f}$ and f .
- b. The Quotient Rule applied to $(F-f)f$ and f .
- c. The Sum Rule applied to F and $-f$.
- d. The statement is not true.
- e. Both A and C

ANSWER: c

33. Evaluate the limits using the Limit Laws:

- A) $\lim_{x \rightarrow -1} \frac{x+2}{x^3-x-1}$
- B) $\lim_{x \rightarrow 1} (x^2-x^{-3}+x)(x+1)$
- C) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x + \cos x}{x}$

ANSWER: A) -1
 B) 2
 C) $\frac{4\sqrt{2}}{\pi}$

34. Consider this statement: If $\lim_{x \rightarrow x_0} f(x) = c \neq 0$ and $\lim_{x \rightarrow x_0} g(x) = 0$, then $\frac{f(x)}{g(x)}$ does not converge to a finite limit as $x \rightarrow x_0$.

To prove this statement, we assume that $\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = M$ exists and is finite. Then, by the Quotient Rule,

$$\lim_{x \rightarrow x_0} \frac{g(x)}{f(x)} = 0 \text{ and by the Product Rule, } \lim_{x \rightarrow x_0} \left(\frac{f(x)}{g(x)} \cdot \frac{g(x)}{f(x)} \right) = 0.$$

Which of the following statements completes the proof?

- a. From $\lim_{x \rightarrow x_0} \left(\frac{f(x)}{g(x)} \cdot \frac{g(x)}{f(x)} \right) = 0$, it follows that $1 = 0$, which is a contradiction.
- b. From $\lim_{x \rightarrow x_0} \frac{g(x)}{f(x)} = 0$, we can conclude that $\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = 0$, which contradicts our assumption.
- c. From $\lim_{x \rightarrow x_0} \frac{g(x)}{f(x)} = 0$, we can conclude that $\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \infty$, which contradicts our assumption.
- d. From $\lim_{x \rightarrow x_0} \left(\frac{f(x)}{g(x)} \cdot \frac{g(x)}{f(x)} \right) = 0$, we can conclude that $\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = 1$, which contradicts our assumption.

ANSWER: a

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35. Which of the following functions are examples of the existence of the limit $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$, although the limits

$\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 0} g(x)$ do not exist?

a. $f(x) = \frac{1}{x}$, $g(x) = x^2$

b. $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{\sin x}$

c. $f(x) = [x]$, $g(x) = x$ (Here, $[x]$ denotes the greatest integer function.)

d. $f(x) = \frac{1}{x^2}$, $g(x) = \frac{1}{x}$

e. $f(x) = \frac{1}{1-x}$, $g(x) = \frac{1}{1-\sin x}$

ANSWER: b

36. Assume a and L are nonzero real numbers. If $\lim_{x \rightarrow a} 2f(x) = L$ and $\lim_{x \rightarrow a} \frac{g(x)}{4} = 0$, calculate the following limits, if possible. If not, state that it is not possible.

A) $\lim_{x \rightarrow a} f(x) \cdot g(x)$

B) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$

C) $\lim_{x \rightarrow a} \frac{f(x) + x^2}{g(x) + a}$

ANSWER: A) 0

B) Not possible

C) $\frac{\frac{L}{2} + a^2}{a}$

37. Determine the points at which the following functions are not continuous and state the type of discontinuity: removable, jump, infinite, or none of these.

A) $f(x) = \frac{x^2 - 1}{|x - 3|}$

B) $g(x) = \frac{\sin x}{x}$

C) $h(x) = x - [x]$ (Here, $[x]$ denotes the greatest integer function.)

D) $j(x) = \left| \sin \frac{1}{x} \right|$

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$$\text{E) } k(x) = \frac{3x^2 - 27}{3 + x}$$

- ANSWER: A) $x = 3$; infinite
 B) $x = 0$; removable
 C) Integers; jump
 D) $x = 0$; none of these
 E) $x = -3$; removable

38. At each point of discontinuity, state whether the function is left or right continuous:

$$\text{A) } f(x) = \begin{cases} \sin\left(\frac{\pi x}{2}\right) + 4 & |x| \leq 2 \\ |x - 2| & |x| > 2 \end{cases}$$

$$\text{B) } f(x) = \begin{cases} 1 & x \leq 0 \\ \frac{\sin x}{x} & 0 < x \leq \frac{\pi}{2} \\ \frac{2x}{\pi - x} & \frac{\pi}{2} < x < \pi \\ x - \pi & \pi \leq x \end{cases}$$

- ANSWER: A) $x = 2$; left continuous
 B) $x = \frac{\pi}{2}$; left continuous
 $x = \pi$; right continuous

39. Determine real numbers a , b , and c that make the function continuous:

$$f(t) = \begin{cases} a & t < 0 \\ \frac{1}{4}t(t+8) & 0 \leq t < b \\ t+3 & b \leq t < 4 \\ c & 4 \leq t \end{cases}$$

ANSWER: $a = 0$, $b = 2$, $c = 7$

40. Find the points of discontinuity for each of these functions and state the type of discontinuity: removable, jump, infinite, or none of these.

$$\text{A) } f(x) = \frac{|4+x|}{4+x}$$

$$\text{B) } g(x) = \frac{[x]}{x}, \quad x > 0 \quad (\text{Here, } [x] \text{ denotes the greatest integer function.})$$

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$$C) h(x) = \frac{1-x}{x^2+4x-5}$$

ANSWER: A) $x = -4$; jump

B) $x =$ positive integer; jump

C) $x = 1$ removable; $x = -5$; infinite

41. Determine whether the function is left or right continuous at each of its points of discontinuity:

$$A) f(x) = \begin{cases} \cos \pi x & |x| \leq \frac{1}{2} \\ x - \frac{1}{2} & |x| > \frac{1}{2} \end{cases}$$

$$B) f(x) = x^2[x], x \geq 0 \quad (\text{Here, } [x] \text{ denotes the greatest integer function.})$$

ANSWER:

A) $x = -\frac{1}{2}$ right continuous

B) Right continuous at the positive integers

42. Determine real numbers a , b , and c that make the following function continuous:

$$f(t) = \begin{cases} t+a & t < 0 \\ t^2+t+b+\frac{a}{2} & 0 \leq t < 1 \\ t-b & 1 \leq t < 2 \\ c & 2 \leq t \end{cases}$$

$$ANSWER: a = -\frac{2}{3}, b = -\frac{1}{3}, c = \frac{7}{3}$$

43. Determine the points where the function is not continuous and state the type of the discontinuity: removable, jump, infinite, or none of these.

$$A) f(x) = \frac{x^2+2x-8}{|x-2|}$$

$$B) g(x) = \frac{x}{[x]}, x \geq 1 \quad (\text{Here, } [x] \text{ denotes the greatest integer function.})$$

$$C) h(x) = \frac{(x^3-3x+2) \sin 2x}{x}$$

$$D) j(x) = \frac{4}{|x|-3}$$

ANSWER: A) $x = 2$, jump

B) $x = 2, 3, 4, \dots$; jump

C) $x = 0$; removable

D) $x = 3, x = -3$; infinite

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44. At each point of discontinuity, state whether the function is left or right continuous.

$$A) f(x) = \begin{cases} \sin \frac{1}{x} & x < 0 \\ 1+x^2 & 0 \leq x < 2 \\ (x+1)^2 - 4 & 2 \leq x < 3 \\ 10 & 3 \leq x \end{cases}$$

$$B) f(x) = \begin{cases} |x-1| & x \leq 2 \\ x^2-3 & 2 < x \leq 4 \\ \frac{1}{x-4} & 4 < x < 5 \\ 6 & 5 \leq x \end{cases}$$

ANSWER: A) $x=0$; right continuous
 $x=3$; right continuous
 B) $x=4$; left continuous
 $x=5$; right continuous

45. Determine real numbers a , b , and c that make the following function continuous:

$$f(x) = \begin{cases} a & x \leq -1 \\ \frac{x}{[x]} & -1 < x < 0 \\ \sin\left(\frac{\pi x}{2}\right) + b + c & 0 \leq x < 1 \\ \frac{x^2+1}{b} & 1 \leq x \end{cases}$$

(Here, $[x]$ denotes the greatest integer function.)

ANSWER: $a=1$; $b=\frac{1}{2}$; $c=-\frac{1}{2}$

46. Determine the points where the function is not continuous and state the type of discontinuity: removable, jump, infinite, or none of these:

$$A) f(x) = \frac{x^2+x-6}{x-2}$$

$$B) g(x) = \frac{1}{x-2} + \sin \frac{1}{x}$$

$$C) h(x) = \frac{[x]x}{x^2+x-6} \text{ (Here, } [x] \text{ denotes the greatest integer function.)}$$

$$D) j(x) = \frac{x^2+x-6}{x-3}$$

ANSWER: A) $x=2$; removable
 B) $x=2$; infinite

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- $x = 0$; none of these
 C) Nonzero integers; jump
 D) $x = 3$; infinite

47. At each point of discontinuity state whether the function is left continuous, right continuous, or neither

$$\text{A) } f(x) = \begin{cases} \frac{1}{x-2} & x < 1 \\ \cos \pi x & 1 \leq x \leq 2 \\ \frac{1+x}{(x-3)^2} & 2 < x \end{cases}$$

$$\text{B) } f(x) = \begin{cases} 0 & x < 0 \\ \cos \pi x & 0 \leq x \leq 1 \\ 2 \cos \pi x & 1 < x \leq 2 \\ 2 & 2 < x \end{cases}$$

ANSWER: A) $x = 2$; left continuous
 $x = 3$; none of these
 B) $x = 0$; right continuous
 $x = 1$; left continuous

48. Determine real numbers a , b , and c that make the function continuous:

$$f(x) = \begin{cases} a & t < 0 \\ x^2 + 1 & 0 \leq t < b \\ 5x - c & b \leq t < 7 \\ 42 & 7 \leq t \end{cases}$$

ANSWER: $a = 1$; $b = 6$; $c = -7$

49. Consider the function

$$f(x) = \begin{cases} -1 & x \leq 0 \\ 1 & x > 0 \end{cases}$$

The function $f(x) + g(x)$ is continuous for which of the following functions g ?

- $g(x) = 2$ if $x \neq 0$, $g(0) = 0$
- $g(x) = 0$ if $x \neq 0$, $g(0) = 2$
- $g(x) = 2$ if $x \leq 0$, $g(x) = 0$ if $x > 0$
- $g(x) = 2$ if $x < 0$, $g(x) = 0$ if $x \geq 0$
- A and C both correct

ANSWER: c

50. Let $f(x)$ be a discontinuous function. Is it possible to find a continuous function $g(x)$ such that $f(x) + g(x)$ is continuous? Explain.

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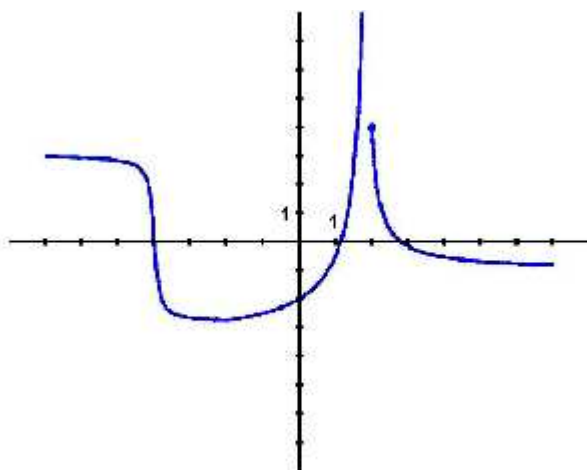
ANSWER: No. If $F(x) = f(x) + g(x)$ is continuous, then $f(x) = F(x) - g(x)$ is continuous by the continuity laws.

51. Sketch the graph of a function $f(x)$ that satisfies all of the following conditions:

$$\lim_{x \rightarrow 2^-} f(x) = \infty, \quad \lim_{x \rightarrow 2^+} f(x) = 4, \quad \lim_{x \rightarrow 0} f(x) = -2,$$

$$\lim_{x \rightarrow \infty} f(x) = -1, \quad \lim_{x \rightarrow -\infty} f(x) = 3$$

ANSWER:



52. Evaluate each limit or state that it does not exist:

A) $\lim_{x \rightarrow 3} \frac{x^4 - 3x^3 + x^2 - 9}{x - 3}$

B) $\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{x^2 + 3x - 1} - \sqrt{2x + 1}}$

C) $\lim_{x \rightarrow 1} \frac{\sqrt{x^2 - 2x + 1}}{x - 1}$

ANSWER: A) 33

B) $\frac{2\sqrt{3}}{3}$

C) Does not exist

53. Evaluate each limit or state that it does not exist:

A) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x}{(1 + \cos 2x)(2 + \cos 2x)}$

B) $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{\sqrt{1+x^2}}{x^2} \right)$

C) $\lim_{\theta \rightarrow 0} \frac{1 + \sin \theta - \cos \theta}{\sin \theta}$

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ANSWER:

A) $\frac{1}{2}$

B) $-\frac{1}{2}$

C) 1

54. Evaluate the limits in terms of the constants involved:

A) $\lim_{x \rightarrow 0} \frac{\sqrt{(x+h)^2+1} - \sqrt{h^2+1}}{x}$

B) $\lim_{h \rightarrow a} \frac{\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{h}}}{h-a}, a > 0$

ANSWER:

A) $\frac{h}{\sqrt{1+h^2}}$

B) $\frac{1}{2a\sqrt{a}}$

55. Evaluate each limit or state that it does not exist:

A) $\lim_{x \rightarrow 2} \frac{x^4 - x^2 - 12}{x-2}$

B) $\lim_{x \rightarrow 1} \frac{\sqrt{x^2+1} - \sqrt{x+1}}{x^2+x-2}$

C) $\lim_{x \rightarrow 2} \frac{\sqrt{x^2-4x+4}}{x-2}$

ANSWER: A) 28

B) $\frac{\sqrt{2}}{12}$

C) Does not exist

56. Evaluate the limit:

$\lim_{x \rightarrow 1} \frac{\sqrt{x+\sqrt{x}} - \sqrt{x+1}}{\sqrt{x}-1}$

ANSWER: $\frac{\sqrt{2}}{4}$

57. Evaluate each limit or state that it does not exist:

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A) $\lim_{x \rightarrow 2} \frac{x^3 - 2x^2 + 3x - 6}{x - 2}$

B) $\lim_{x \rightarrow -1} \frac{\sqrt{3x+4} + x}{x^2 - x - 2}$

C) $\lim_{x \rightarrow 1} \frac{|x^2 + x - 2|}{x - 1}$

ANSWER: A) 7

B) $-\frac{5}{6}$

C) Does not exist

58. Determine a real number c for which the limit exists and then compute the limit:

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{\sqrt{x+c^2}} - \frac{1}{\sqrt{x^2+x}} \right)$$

ANSWER: $c = 0$, the limit is 0

59. Evaluate each limit or state that it does not exist:

A) $\lim_{x \rightarrow 2} \left(\frac{x+1}{x-2} - \frac{x-5}{x^2-5x+6} \right)$

B) $\lim_{x \rightarrow 5} \frac{\sqrt{x^2-6} - \sqrt{4x-1}}{x-5}$

C) $\lim_{x \rightarrow 8} \frac{\sqrt{x+1} - 3}{x-8}$

ANSWER: A) (-1)

B) $\frac{3\sqrt{19}}{19}$

C) $\frac{1}{6}$

60. Determine a real number a for which the limit exists and then compute the limit:

$$\lim_{x \rightarrow 2} \frac{x^2 + ax + 6}{\sqrt{x^2 + 2x - 4} - \sqrt{x+2}}$$

ANSWER: $a = -5$; limit is $-\frac{4}{5}$ 61. Let $f(x) = 2x + 3$. Compute $\lim_{h \rightarrow 0} \frac{f(-3+h) - f(-3)}{h}$.

ANSWER: 2

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62. Compute $\lim_{\theta \rightarrow 0} (\cot^2 \theta - \csc^2 \theta)$.

ANSWER: -1

63. Compute $\lim_{\theta \rightarrow \frac{7\pi}{6}} \frac{2 \sin^2 \theta - 5 \sin \theta - 3}{2 \sin \theta + 1}$.

ANSWER: $-\frac{7}{2}$

64. Evaluate the limits:

A) $\lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin x^2}$

B) $\lim_{x \rightarrow 0} x \cos \frac{1}{x^3}$

C) $\lim_{x \rightarrow 0} \left| \sin x \right| \left(1 - \cos \frac{1}{x} \right)$

ANSWER: A) 1

B) 0

C) 0

65. Show that $0 \leq x - [x] < 1$ for all x . (Here, $[x]$ denotes the greatest integer function.)Then use the above inequality and the Squeeze Theorem to evaluate $\lim_{x \rightarrow 0} x(x - [x])$.

ANSWER: 0

66. Evaluate the limits in terms of the constants involved:

A) $\lim_{x \rightarrow h} \frac{\sin(x-h)}{x^2 + (1-h)x - h}$

B) $\lim_{x \rightarrow a} \frac{\frac{1}{x^2} - \frac{1}{a^2}}{x - a}$

ANSWER: A) $\frac{1}{h+1}$

B) $\left(-\frac{2}{a^3} \right)$

67. Evaluate the limits using the Squeeze Theorem, trigonometric identities, and trigonometric limits, as necessary:

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$$\text{A) } \lim_{x \rightarrow 0} \frac{\sin \frac{1}{x} \sin^2 \frac{x}{2}}{x}$$

$$\text{B) } \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x}$$

$$\text{C) } \lim_{x \rightarrow 0} \frac{\sin^3 x}{\sin(x^3)}$$

ANSWER: A) 0

B) 1

C) 1

68. Show that $0 \leq x - [x] < 1$ for all x . (Here, $[x]$ denotes the greatest integer function.) Then use this inequality with the Squeeze Theorem to evaluate $\lim_{x \rightarrow \pi} (x - [x]) \tan x$.

ANSWER: 0

69. Determine a real number c such that the following limit exists, and then evaluate the limit for this value:

$$\lim_{x \rightarrow 0} \frac{3 \sin \frac{x}{2} + (c-1)^2}{\sin x - \cos x + 1}$$

ANSWER: $c = 1$; limit is $\frac{3}{2}$

70. Evaluate each limit or state that it does not exist:

$$\text{A) } \lim_{t \rightarrow 0} \frac{1 - \cos t}{\sqrt{t}}$$

$$\text{B) } \lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin 2x \sin x}$$

$$\text{C) } \lim_{x \rightarrow 0} \frac{\sin 2x - \sin x}{x}$$

ANSWER: A) 0

B) $\frac{1}{4}$

C) 1

71. Evaluate the limits:

$$\text{A) } \lim_{x \rightarrow 0} \frac{1 - \cos^4 x}{x^2}$$

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$$B) \lim_{x \rightarrow 0} \frac{\cos\left(x + \frac{\pi}{4}\right) - \sin\left(x + \frac{\pi}{4}\right)}{x}$$

ANSWER: A) 2
 B) $-\sqrt{2}$

72. Evaluate the limits:

$$A) \lim_{x \rightarrow 0} \frac{\sin 5x}{x^2 - x}$$

Hint: Factor the denominator.

$$B) \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{2x^2 - x}$$

Hint: Factor the two expressions.

$$C) \lim_{x \rightarrow 0} \frac{\sin 3x \sin 8x}{x^3 + x^2}$$

ANSWER: A) -5
 B) 0
 C) 24

73. Use the Squeeze Theorem to evaluate the limit

$$\lim_{x \rightarrow \pi} (1 + \cos x) \sin \frac{1}{x}$$

ANSWER: 0

74. If $3x^2 - 4 \leq f(x) \leq x$ on the interval $[0, 4]$, then $\lim_{x \rightarrow 1} f(x)$ must exist.

- a. True
- b. False

ANSWER: b

75. Calculate the limits:

$$A) \lim_{x \rightarrow \infty} \frac{2x^5 - x^4 + 1}{8x^5 + x^3 + x - 2}$$

$$B) \lim_{x \rightarrow -\infty} \frac{3x^2 + x - 1}{4x - 7}$$

$$C) \lim_{x \rightarrow \infty} \left(\frac{6x^3}{2x^2 + 1} - 3x \right)$$

ANSWER: A) $\frac{1}{4}$
 B) $-\infty$

Chapter 02C) 0

76. Calculate the limits:

A) $\lim_{x \rightarrow -\infty} \frac{\sqrt[3]{2x^3 - x + 1}}{\sqrt{x^2 + x - 2}}$

B) $\lim_{x \rightarrow \infty} \frac{(4x + 1)^{15}(3x - 1)^{10}}{(9x + 7)^5(4x + 11)^{20}}$

C) $\lim_{x \rightarrow -\infty} \frac{\sqrt{|x^2 - 5|}}{x}$

ANSWER: A) $-\sqrt[3]{2}$

B) $\frac{1}{1024}$

C) -1

77. Calculate the following limits:

A) $\lim_{x \rightarrow -\infty} \frac{x^2 + 1}{\sqrt{x^4 - 2}}$

B) $\lim_{x \rightarrow -\infty} \frac{2x - 1}{\sqrt[3]{x^3 + 1}}$

C) $\lim_{x \rightarrow \infty} (\sqrt{x^2 - x} - \sqrt{x^2 + 5x})$

ANSWER: A) 1

B) 2

C) -3

78. Compute the following limits:

A) $\lim_{x \rightarrow \infty} \frac{2x^2 - 6x + 1}{x^2 - 3}$

B) $\lim_{x \rightarrow -\infty} (-x^5 + 2x^4 - x^2 + 1)$

C) $\lim_{x \rightarrow -\infty} \frac{x^7 - 6x^3 + 1}{2x^7 + x^2 - 2}$

ANSWER: A) 2

B) ∞

C) $\frac{1}{2}$

79. Compute the following limits:

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A) $\lim_{x \rightarrow \infty} \frac{3(x+7)^3 - (x-7)^3}{2(x+2)^3 - (x-2)^3}$

B) $\lim_{x \rightarrow -\infty} \frac{x-1}{2x^2+1}$

C) $\lim_{x \rightarrow -\infty} \frac{2x^2-6x+1}{x-3}$

ANSWER: A) 2
B) 0
C) $-\infty$

80. Compute the following limits:

A) $\lim_{x \rightarrow \infty} x(\sqrt{4x^2-1} - 2x)$

Hint: Multiply and divide by the conjugate expression.

B) $\lim_{x \rightarrow -\infty} \frac{2x+7}{\sqrt{x^2-1}}$

Hint: For $x < 0$, $\sqrt{x^2} = -x$.

C) $\lim_{x \rightarrow \infty} \frac{x^{\frac{5}{3}} - 3x^{\frac{2}{3}}}{x^{\frac{18}{5}} + x}$

ANSWER: A) $-\frac{1}{4}$
B) -2
C) 0

81. Compute the following limits:

A) $\lim_{x \rightarrow \infty} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right)$

B) $\lim_{x \rightarrow -\infty} \frac{x^3+x^2-2x+1}{1-2x^3}$

C) $\lim_{x \rightarrow -\infty} \frac{x^4+2x-1}{x^3+x}$

ANSWER: A) 0
B) $-\frac{1}{2}$
C) $-\infty$

82. Compute the following limits:

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A) $\lim_{x \rightarrow -\infty} \frac{1 - \sqrt{3 + x^2}}{1 + \sqrt{4x^2 + 1}}$

B) $\lim_{x \rightarrow \infty} \frac{x + 3 - \sqrt{x^2 + 2}}{\sqrt{x^2 + 1} - 5}$

(C) $\lim_{x \rightarrow \infty} (\sqrt{x^2 - 3x + 7} - x)$

ANSWER: A) $-\frac{1}{2}$

B) 0

C) $-\frac{3}{2}$

83. The Intermediate Value Theorem guarantees that the equation $x \cos x - \sin x = 0$ has a solution in which of the following intervals?

a. $(2\pi, 3\pi)$

b. $\left(\frac{\pi}{2}, \pi\right)$

c. $\left(\frac{3\pi}{2}, 2\pi\right)$

d. $\left(\frac{\pi}{4}, \pi\right)$

e. $(\pi, 3\pi)$

ANSWER: a

84. The polynomial $P(x) = x^3 - x - 5$ must have a root in which of the following intervals?

a. $(3, 4)$

b. $(1, 2)$

c. $(0, 1)$

d. $\left(\frac{1}{2}, 1\right)$

e. $(-1, 1)$

ANSWER: b

85. Which of the following functions is a counterexample for the converse of the Intermediate Value Theorem, which states: If $f(x)$ assumes all the values between $f(a)$ and $f(b)$ in the interval $[a, b]$, then f is continuous on $[a, b]$.

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a. $f(x) = x - 1$ on $[0, 2]$

b. $f(x) = \frac{1}{x-1}$ on $[0, 2]$

c. $f(x) = \frac{\sin x}{x}$ on $(0, 2), f(0) = 1$ $f(x) = \begin{cases} \frac{\sin x}{x}, & 0 < x \leq 2 \\ 1, & x = 0 \end{cases}$ on $[0, 2]$

d. $f(x) = [x]$ on $[0, 2]$ (Here, $[x]$ denotes the greatest integer function.)

e. $f(x) = \frac{1}{(x-1)^2}$ on $[-3, 2]$

ANSWER: e

86. Which of the following functions has a zero in the interval $[-1, 3]$?

a. $f(x) = \frac{x}{x-4}$

b. $f(x) = x^2 - 3x + 3$

c. $f(x) = \frac{x^2}{x-2}$

d. $f(x) = \cos \frac{x}{\pi}$

e. Both A and C

ANSWER: e

87. The Intermediate Value Theorem guarantees that the equation $\tan x = x$ has a solution in which of the following intervals?

a. $\left[-\frac{\pi}{8}, \frac{3\pi}{8}\right]$

b. $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

c. $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

d. $\left[\frac{\pi}{4}, \pi\right]$

e. Both A and C

ANSWER: e

88. Which of the following functions is a counterexample for the converse of the Intermediate Value Theorem, which states: If $f(x)$ assumes all the values between $f(a)$ and $f(b)$ in the interval $[a, b]$, then f is continuous on

Chapter 02 $[a, b]$

- a. $f(x) = x^2$ for $x \in (1, 3)$, $f(1) = 9$, $f(3) = 1$ on $[1, 3]$
- b. $f(x) = \frac{1}{x-2}$ on $[1, 3]$
- c. $f(x) = \frac{1 - \cos x}{x}$ on $(0, \frac{\pi}{2}]$, $f(0) = 0$ on $[0, \frac{\pi}{2}]$
- d. $f(x) = [x]$ on $[1, 3]$ (Here, $[x]$ denotes the greatest integer function.)
- e. Both A and C

ANSWER: a

89. Which of the following functions is a counterexample for the converse of the Intermediate Value Theorem: If f assumes all the values between $f(a)$ and $f(b)$ in the interval $[a, b]$, then f is continuous on $[a, b]$.

- a. $f(x) = \frac{1}{x-1}$ if $1 < x \leq 3$, $f(1) = 2$ on $[1, 3]$
- b. $f(x) = [x]$ on $1 < x \leq 3$ (Here, $[x]$ denotes the greatest integer function.)
- c. $f(x) = \frac{1}{x-4}$ on $1 < x \leq 3$
- d. $f(x) = x^2$ for $1 < x \leq 3$ and $x \neq 2$, $f(2) = 1$
- e. Both A and D

ANSWER: a

90. Assume $g(x)$ is continuous on $[-3, 9]$, $g(-3) = 14$, and $g(9) = 72$. Determine whether each of the following statements is always true, never true, or sometimes true.

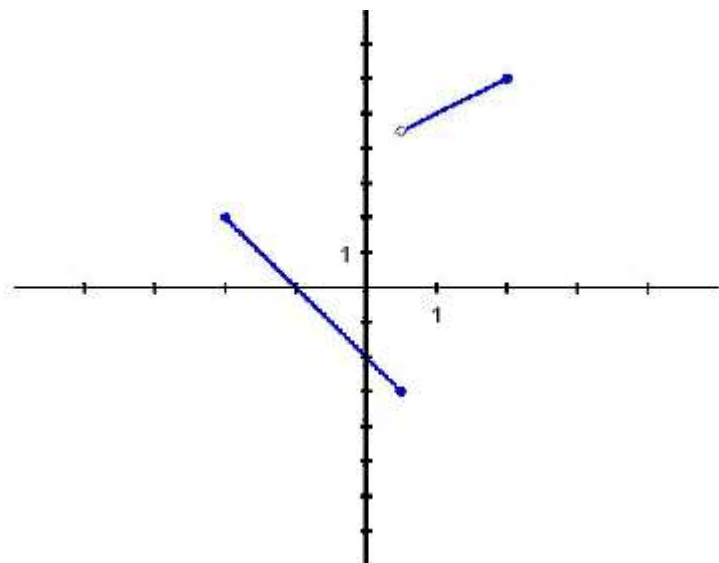
- A) $g(c) = 0$: no solution with $c \in [-3, 9]$
- B) $g(c) = 60$: no solution with $c \in [-2, 9]$
- C) $g(c) = 21$: no solution with $c \in [-3, 9]$
- D) $g(c) = -1,000,000$: exactly one solution with $c \in [-2, 9]$
- E) $g(c) = 49.5$: a solution with $c \in [-3, 9]$

ANSWER: A) Sometimes true
 B) Sometimes true
 C) Never true
 D) Sometimes true
 E) Always true

91. Draw the graph of a function $g(x)$ on $[-2, 2]$ such that the graph does not satisfy the conclusion of the Intermediate Value Theorem.

ANSWER: Answers may vary. A sample answer is:

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92. Which of the following properties can be used to prove that $f(x) = \cos x$ is continuous for all x ?

- a. $|\cos x| \leq 1$ for all x
- b. $|\cos x - \cos y| \leq |x - y|$ for all x and y
- c. $\cos x - \cos y \leq x - y$ for all x and y
- d. The limit $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$ exists
- e. $|\cos x - \cos y| \geq |x - y|$ for all x and y

ANSWER: b

93. Which of the following statements imply that $\frac{1}{x}$ is not continuous at $x = 0$?

- a. $\frac{1}{x}$ has opposite signs on the two sides of $x = 0$.
- b. $|\frac{1}{x}| < 0.01$ implies that $x > 100$.
- c. For any $\varepsilon > 0$, $|\frac{1}{x}| < \varepsilon$ implies that $|x| > \frac{1}{\varepsilon}$.
- d. If $x < \varepsilon$, then $|\frac{1}{x}| < \frac{1}{\varepsilon}$.
- e. A and C are correct.

ANSWER: c

94. To show that L is not the limit of $f(x)$ as $x \rightarrow x_0$, we should show that:

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- For any $\varepsilon > 0$, there exists $\delta > 0$ such that if $|x - x_0| > \delta$ then $|f(x) - L| < \varepsilon$.
- For any $\varepsilon > 0$, there exists $\delta > 0$ such that if $|x - x_0| > \delta$ then $|f(x) - L| > \varepsilon$.
- There exists $\varepsilon > 0$, such that for any $\delta > 0$ the inequalities $0 < |x - x_0| < \delta$ and $|f(x) - L| \geq \varepsilon$ have a solution x .
- There exist $\varepsilon > 0$ and $\delta > 0$ such that if $0 < |x - x_0| < \delta$, then $|f(x) - L| > \varepsilon$.
- A and C are both correct.

ANSWER: c

95. Suppose there exists a value of $\varepsilon > 0$ so that for any value of $\delta > 0$, we can find a value of x satisfying $0 < |x - x_0| < \delta$ and $|f(x) - L| > \varepsilon$. We may conclude that:

- L is the limit of f as $x \rightarrow x_0$.
- L is not the limit of f as $x \rightarrow x_0$.
- The limit of f as $x \rightarrow x_0$ does not exist.
- The limit of f as $x \rightarrow x_0$ exists but is not equal to L .
- None of the above.

ANSWER: b

96. To show that L is not the limit of $f(x)$ as $x \rightarrow x_0$, we should show that:

- There exists $\varepsilon > 0$ such that for any $\delta > 0$ there exists a solution to the inequalities $|x - x_0| < \delta$ and $|f(x) - L| > \varepsilon$.
- There exists $\varepsilon > 0$ such that for any $\delta > 0$ there exists a solution to the inequalities $|x - x_0| > \delta$ and $|f(x) - L| < \varepsilon$.
- There exists $\delta > 0$ such that for any $\varepsilon > 0$, if $|f(x) - L| < \varepsilon$, then $|x - x_0| < \delta$.
- For any $\varepsilon > 0$ and $\delta > 0$, if $|x - x_0| < \delta$, then $|f(x) - L| > \varepsilon$.
- A and D are both correct.

ANSWER: a