

***Advanced Engineering Mathematics 6th edition Zill
Test Bank***

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Chapter 2

1. Which of the following is an example of a separable differential equation?

A. $x \tan^{-1}(y) \frac{dy}{dx} = e^{x+y}$

B. $y' = \sin(xy)$

C. $(x^2 + 1) \frac{dy}{dx} = \sqrt{x + y}$

D. $y' = \frac{1}{x + y}$

Ans: A

2. The general solution of the differential equation $5 \frac{dy}{dx} + 30y = 6$ is

A. $y = Ce^{-6x} + \frac{1}{5}$

B. $y = \frac{1}{5}e^{6x} + C$

C. $y = Ce^{-6x} + \frac{1}{5}e^{6x}$

D. $y = \frac{1}{5}e^{12x} + Ce^{6x}$

Ans: A

3. Which of the following is a homogeneous differential equation?

A. $\frac{dy}{dx} = \frac{2x + 7y}{7x + 2y}$

B. $(x^2 + y^2)dy - x^4 dx = 0$

C. $\frac{dy}{dx} + x^2 y = xy^2$

D. $(y^2 + x)dx = (x + y^2)dy$

Ans: A

4. Which of the following differential equations is a Bernoulli equation?

A. $3dy = \sin(x)(y^2 - y)dx$

B. $(x^2 + y^2)dy - 2xydx = 0$

C. $e^y dx = 5 - \sin(y)dy$

D. $\frac{dy}{dx} = \ln(y)$

Ans: A

5. Initially, 500 milligrams of a radioactive substance was present. After 3 days, the mass had decreased by 145 milligrams. If the rate of decay is proportional to the amount of the substance present at time t , find the amount remaining after 10 days (rounded to the nearest milligram).

A. 160 mg

B. 7 mg

C. 0 mg

D. 173 mg

Ans: A

6. Suppose that a large mixing tank initially holds 400 gallons of water in which 65 pounds of salt have been dissolved. Another brine solution is pumped into the tank at a rate of 6 gal/min, and when the solution is well stirred, it is pumped out at a slower rate of 5 gal/min. If the concentration of the solution entering is 3 lb/gal, find the amount of salt in the tank after 10 minutes.

A. 95 pounds

B. 85 pounds

C. 90 pounds

D. 100 pounds

Ans: A

7. Suppose that a large mixing tank initially holds 300 gallons of water in which 50 pounds of salt have been dissolved. Another brine solution is pumped into the tank at a rate of 4 gal/min, and when the solution is well stirred, it is pumped out at a faster rate of 6 gal/min. If the concentration of the solution entering is 7 lb/gal, find the amount of salt in the tank after 6 minutes.

- A. 202.29 pounds
- B. 176.31 pounds
- C. 215.78 pounds
- D. 264.40 pounds

Ans: A

8. A thermometer reading 65° F is placed in an oven heated to a constant temperature. The thermometer reads 125° F after 1 minute and 170° F after 2 minutes. What is the temperature of the oven?

- A. 305° F
- B. 320° F
- C. 353° F
- D. 375° F

Ans: A

9. Find the critical point(s) of the differential equation $\frac{dV}{dt} = V\sqrt{c+mP}$ where c and m are nonzero constants.

- A. $1, -\frac{c}{m}$
- B. $0, \frac{c}{m}$
- C. $0, -\frac{c}{m}$
- D. $-\frac{c}{m}$

Ans: C

10. Suppose the slope field of a differential equation $\frac{dy}{dx} = f(x, y)$ has a lineal element at the point $(2, 3)$ that slopes up from left to right. What does this tell us about $y'(2)$?

- A. $y'(2) = 3$
- B. $y'(2) < 0$
- C. $y'(2) = 0$
- D. $y'(2) > 0$

Ans: D

11. Rewrite the linear differential equation $\cos x \, dy + (\sin x)y \, dx = dx$ in the standard form of a linear differential equation

- A. $\frac{dy}{dx} + (\tan x)y = \sec x$
- B. $\frac{dy}{dx} + (\sin x)y = \cos x$
- C. $\cos x \frac{dy}{dx} + (\sin x)y = 1$
- D. $\cos x \, dy = (1 - y \sin x) \, dx$

Ans: A

True/False

12. For the DE $y' = (y - 5)^3 (y^2 - 9)^2$, 5 is an unstable critical point while 3 and -3 are semi-stable critical points.

Ans: True

13. The general solution of the differential equation $2 \frac{dy}{dx} = -4y + 5$ is $y = Cx^{-2} + \frac{5}{4}$.

Ans: True

14. The integrating factor of the differential equation $-7(y + x^5)dx + x^2dy = 0$ is $\frac{-7}{x^2}$.

Ans: False

15. The population of a certain town is known to increase at a rate proportional to the current population. If the population doubles every 15 years, and the current population is 32,000, then the population in 11 years will be 53,199.

Ans: True

16. 0 is a critical number of the differential equation $\frac{dy}{dx} = \frac{1}{x}$.

Ans: False

17. A differential equation can be both separable and linear.

Ans: True

18. An integrating factor is another name for constant in an integral which can be factored out of the integrand.

Ans: False

19. The differential equation $2xy dx - (x^2 + 1)dy = 0$ is exact.

Ans: False

20. A differential equation $M(x) dx + N(y) dy = 0$ is homogenous if $M(x)$ and $N(y)$ are both polynomials of the same degree.

Ans: True

Short Answer

21. Solve the differential equation $e^{2x} \frac{dy}{dx} = e^{-2y} + e^{-2x-2y}$.

Ans: $y = \frac{1}{2} \ln \left(-e^{-2x} - \frac{1}{2} e^{-4x} + C \right)$

22. Find an implicit solution of the initial value problem $y' = \frac{y^2 - 4}{x^2 - 9}$, $y(0) = 3$.

Ans: $\frac{y-2}{y+2} = \frac{1}{5} \left(\frac{x-3}{x+3} \right)^{2/3}$

23. Solve the initial value problem $\frac{dx}{dt} = t(x-1)$, $x(0) = 7$.

Ans: $x = 1 + 6e^{-t^2/2}$

24. Find the general solution of the differential equation $(x+4) \frac{dy}{dx} + (x+5)y = 5xe^{-2x}$ and give the largest interval over which the general solution is defined.

Ans: $y = \frac{-5(x+1)e^{-2x}}{x+4} + \frac{ce^{-x}}{(x+4)}$, $(-4, \infty)$

25. Solve the following initial value problem:

$$\left[2xy^3 - \cos(x)e^y + x \sin(x)e^y + y \cos(xy) \right] dx + \left[3x^2y^2 - x \cos(x)e^y + x \cos(xy) \right] dy = 0, y(\pi) = 0$$

Ans: $x^2y^3 - x \cos(x)e^y + \sin(xy) = \pi$

26. Find the value of k so that the differential equation

$$\left[8yt^5 + kt^2e^{ty} \right] dy = \left[\frac{2}{t} - 20y^2t^4 - ktye^{ty} - 5e^{ty} \right] dt \text{ is exact.}$$

Ans: $k = 5$

27. Using an appropriate substitution, solve the differential equation $y' = -y + xy^5$.

Ans: $y = \left(x + \frac{1}{4} + ce^{4x} \right)^{-1/4}$

28. Using an appropriate substitution, solve the differential equation
 $ydx - 5(x - y)dy = 0$.

Ans: $y^5 = c(5y - 4x)$

29. For the DE $y' = -x - 2y + 2$, use Euler's Method to obtain a three-decimal approximation of $y(2.3)$ if $y(2) = 1$. Use $h = 0.1$.

Ans: 0.484

30. For the DE $y' = y \cos(x)$, use Euler's Method to obtain a four-decimal approximation of $y(0.4)$ if $y(0) = 3$. Use $h = 0.1$.

Ans: 4.3646

31. For the DE $y' = e^{x-y}$, use Euler's Method to obtain a four-decimal approximation of $y(-0.6)$ if $y(-1) = 2$. Use $h = 0.1$.

Ans: 2.0231

32. The population of a certain town is governed by the logistic equation

$\frac{dP}{dt} = P \left(0.5 - \frac{0.5}{105,000} P \right)$. What is the carrying capacity for the town? If the current population is 9000, when will the population reach 25,000? What will the population be in 5 years?

Ans: 105,000 people; 2.408 years; 55,983 people

33. Two very large tanks A and B are each partially filled with 200 gallons of brine. Initially, 25 pounds of salt are dissolved in Tank A while 75 pounds are dissolved in Tank B. One interconnecting pipe sends brine from Tank A to Tank B at a rate of 5 gal/min. Another interconnecting pipe sends brine from Tank B to Tank A at a rate of 2 gal/min. An input pipe feeds pure water to Tank A at a rate of 3 gal/min, while an output pipe drains brine from Tank B at a rate of 3 gal/min. Construct a mathematical model for the amount of salt $x_1(t)$ and $x_2(t)$ at time t in Tanks A and B, respectively.

$$x_1' = -\frac{x_1}{40} + \frac{x_2}{100}$$

Ans: $x_2' = \frac{x_1}{40} - \frac{x_2}{40}$

$$x_1(0) = 25, \quad x_2(0) = 75$$

34. Initially, two tanks A and B are each partially filled with 100 gallons of pure water. A feed valve lets brine into Tank A at a rate of 4 gal/min with a concentration of 3 lb/gal. An interconnecting pipe pumps brine from Tank A to Tank B at a rate of 2 gal/min, while another pipe pumps brine from Tank B to Tank A at a rate of 1 gal/min. Construct a mathematical model for the amount of salt $x_1(t)$ and $x_2(t)$ at time t in Tanks A and B, respectively.

$$x_1' = 12 - \frac{2x_1}{100+3t} + \frac{x_2}{100+t}$$

Ans: $x_2' = \frac{2x_1}{100+3t} - \frac{x_2}{100+t}$

$$x_1(0) = 0, \quad x_2(0) = 0$$

35. Determine whether $\left[\sin(xy) + xy \cos(xy) + ye^x \right] dx + \left[x^2 \cos(xy) + e^x \right] dy = 0$ is exact. If it is exact, solve it.

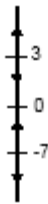
Ans: Yes, it is exact. The solution is $x \sin(xy) + ye^x = c$

36. Determine whether $\left[\pi y + 6xye^y + y^{-2} \right] dx = \left[2xy^{-3} - 3x^2e^y - \pi x - 3x^2ye^y \right] dy$ is exact. If it is exact, solve it

Ans: Yes, it is exact. The solution is $\pi xy + 3x^2ye^y + xy^{-2} = c$

37. Find the critical points and phase portrait of the autonomous first-order differential equation $\frac{dy}{dx} = -2y(y+7)(3-y)$.

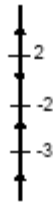
Ans: Critical Points: -7, 0, 3



Phase Portrait:

38. Find the critical points and phase portrait of the autonomous first-order differential equation $\frac{dy}{dx} = (y+3)^2(y^2-4)$.

Ans: Critical Points: -3, -2, 2



Phase Portrait:

39. Solve implicitly the differential equation $\frac{dy}{dx} = \frac{x}{y^2 + 1}$.

Ans: $\frac{y^3}{3} + y = \frac{x^2}{2} + C$

40. Find an implicit solution of the initial value problem $\frac{dy}{dx} = \frac{y^3 x}{y + 2}$, $y(1) = -2$

Ans: $-\frac{1}{y} - \frac{1}{y^2} = \frac{x^2}{2} - \frac{1}{4}$

41. Solve the initial-value problem $2\frac{dy}{dx} - 4xy - 3 = 0$, $y(0) = 4$.

Ans: $y = \frac{3\sqrt{\pi}}{4} e^{x^2} \operatorname{erf}(x) + 4e^{x^2}$

42. Solve the differential equation $\frac{dy}{dx} = \frac{4x + 6y}{10y - 6x}$

Ans: $2x^2 + 6xy - 5y^2 = c$

43. Solve the differential equation $\frac{dy}{dx} + 4y = 2x^2 y^3$

Ans: $y = \left(ce^{8x} + \frac{32x^2 + 8x + 1}{64} \right)$

45. Find the general solution of the differential equation $3y' + 2xy = 6x$.

Ans: $ce^{-x^2/3} + 3$

46. The population of a certain town is known to increase at a rate proportional to the current population. Suppose the population triples every 21 years. If the population is currently 150,000, what was the population 9 years ago?

Ans: 93,672

47. Consider the differential equation $M(x) dx + N(y) dy = 0$ where $M(x)$ and $N(y)$ are nonzero functions of x and y , respectively. Determine if this differential equation is separable. Justify your answer.

Ans: The differential equation is separable since it can be written in the form

$$\frac{dy}{dx} = -M(x) \cdot \frac{1}{N(y)}.$$

48. Let $y(t)$ = the size of a certain population at time t . Suppose that $y(t)$ is described by the initial value problem $y' = \sqrt{y}$, $y(0) = 0$. Find two different solutions to this initial value problem. In this context of this problem, which solution is correct? Briefly explain why.

Ans: By inspection, $y(t) = 0$ is a solution. By separation of variables, $y(t) = t^2 / 4$ is also a solution. Since $y(t)$ represents a population which starts at 0, it should remain 0 forever. Thus $y(t) = 0$ is the correct solution in this context.

