Explorations 10th edition Arny Solutions Manual

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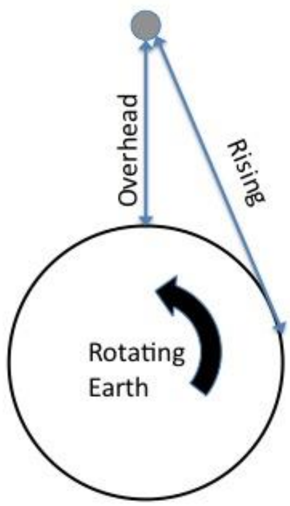
CHAPTER 2 THE RISE OF ASTRONOMY

Answers to Thought Questions

1. Despite the Moon illusion, you are actually closest to the Moon when it is at its highest point in the sky. As seen from over the North Pole,



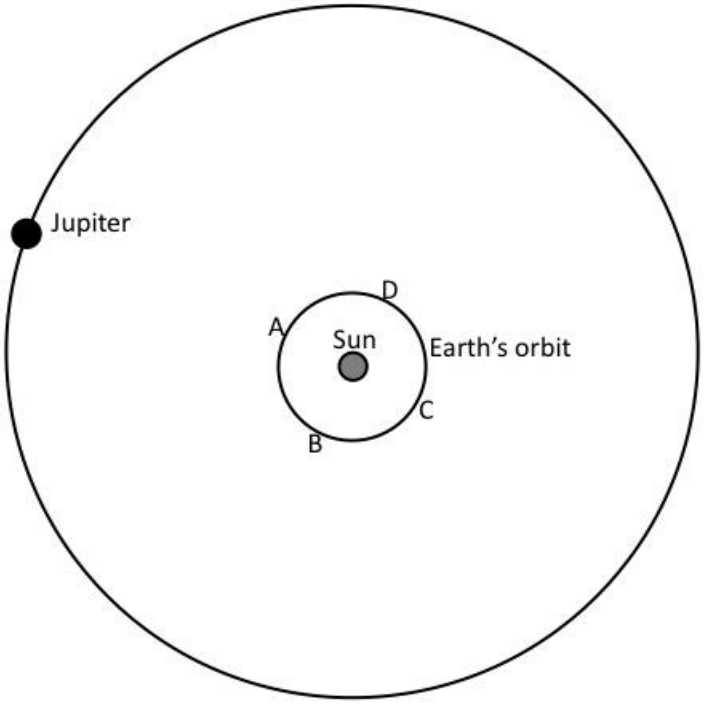
2. If the stars were much closer than they really are, Aristarchus would have been able to demonstrate the stellar parallax caused by Earth's orbital motion around the Sun.

3. The phases of Venus are caused by Venus orbiting the Sun, so keeping all distances and periods constant, it wouldn't matter whether Earth and Venus were orbiting the Sun or Venus was orbiting the Sun and the Sun (and Venus) were orbiting Earth.

4. The Sun has a slightly larger angular diameter in January than in July because Earth's orbit is an ellipse. The idea that orbits are ellipses is Kepler's First Law. Additionally, the Earth is closest to the sun in January (perihelion).

5. The apparent motion of Jupiter is primarily a result of Earth's motion around the Sun, not Jupiter's motion. A sketch like the one below can help show this—consider the view from Earth over the year, with Jupiter moving only a little along its orbit. At position A, Jupiter is at opposition, and rises when the Sun sets. By the time Earth is at B, Jupiter has not moved

very far (so its motion is neglected in this sketch), but now it is only 90 degrees away from the Sun—already high in the sky at Sunset. By position C, Jupiter would be "up" in the daytime. Moving to position D, Jupiter is moving past the Sun on the opposite side as before.



6. If the same comet is only visible every 50 years or more (or much, much more!) then the comet must have a much longer p than Earth's orbit. Consequently by Kepler's third law it must have a corresponding larger a. If the comet needs to be close to Earth and Sun to be seen, then at least some of the time it must be at only 1 or 2 AU from the Sun; for this to fit with a large a and p, it must have an elliptical orbit with high eccentricity.

7. This should be less about technology and more about philosophy; and about getting students to look up contemporary work. One key difference is less personal, government and patron-sponsored *religious/mystic* motivation—astronomy is no longer in the game of predicting planetary positions as portents of war and peace. Also, astronomy has become increasingly based on fact and less influenced by philosophy (Ptolemy's and Kepler's motivations looking for perfection in the heavens; Kepler was conflicted but notably broke with his philosophical position (circular orbits) when confronted with scientific evidence). Increased technology has revealed entire branches of astronomy previously invisible to scientists (radio, x-ray, infrared, gravitational waves, etc.).

8. (Students must research the astronomers in question).

Name	Birth	Death	Notable Events
Eudoxus	408 BCE	355 BCE	Studied eclipses, lunisolar-

Name	Birth	Death	Notable Events
			Venus cycle, planetary motion, and Spherical astronomy
Aristotle	384 BCE	322 BCE	Refuted Democritus's claim that the Milky Way was made up of "those stars which are shaded by the earth from the sun's rays
Aristarchus	310 BCE	230 BCE	Studied/calculated sizes and distances of the Sun and Moon
Eratosthenes	276 BCE	194 BCE	Studied/calculated the Earth's size (circumference)
Ptolemy	100 ACE	170 ACE	Wrote the Almagest, the only surviving comprehensive ancient treatise on astronomy
Copernicus	1473 ACE	1543 ACE	Planetary motion and proponent of Heliocentrism
Tycho (Brahe)	1546 ACE	1601 ACE	Observational astronomy, accurate and numerous measurements of the sky
Kepler	1571 ACE	1630 ACE	Heliocentric astronomy, discovered laws of planetary motion
Galileo	1564 ACE	1642 ACE	Improved the telescope, observed Sunspots, Moon, Jupiter, phases of Venus, Neptune, and Saturn
Newton	1642 ACE	1727 ACE	Significant contribution to physics and mathematics

Answers to Problems

1. This problem is a modern version of the method Eratosthenes used to measure the size of Earth. Given that the shadow length is 15 degrees, the distance in latitude between the two points on the asteroid must be 15 degrees, or $15° / 360° = 1/24\text{th}$ the circumference of the asteroid. If the 15 degrees corresponds to 10 km, then the total distance around the asteroid must be 10 km × 24 = 240 km. The radius, R, of the asteroid is related to its circumference, C, by C = 2πR. Thus $R = C / 2\pi = 240 / 2\pi = 38\,\text{km}$.

2. This can be directly calculated, but it is easier to just look at the proportionalities involved. Since $C = 360° / \text{angle} \times \text{distance}$, the circumference is directly proportional to the distance between Alexandria and Syene. If the distance were three times as much, and the angle the same, Eratosthenes would have calculated that the circumference of Earth was three times larger: 250,000 stadia × 3 = 750,000 stadia or 25,000 miles × 3 = 75,000 miles. ("Three times larger" should be an acceptable answer.)

3. From Earth, the Sun has an angular diameter of 0.5°. Angular size varies as the inverse of the distance, so if Mercury is 0.387 times as far, the Sun is $0.5°/0.387 = 1.29°$. Pluto is 39.53 times Earth's distance, so the Sun is $0.5°/39.53 = 0.0126°$.

4. The Andromeda galaxy has an angular diameter of 5 degrees at a distance of 2.2×10^6 ly. We can find its true size by using the angular diameter formula $L = 2\pi dA / 360°$, where d = distance, A = angle subtended, and L = linear diameter. Thus, $L = 2\pi \times 2.2 \times 10^6 \text{ ly} \times 5° / 360° = 1.92 \times 10^5 \text{ ly}$.

5. If P = 64 years, you can use Kepler's 3rd Law to estimate its distance from the Sun. $P^2 = a^3$, where P = period in years and a = average orbital radius in AU. We can find a by taking the cube root of both sides to get $P^{2/3} = a$, or $a = P^{2/3} = (64)^{2/3} = (64 \times 64)^{1/3} = (8 \times 8 \times 8 \times 8)^{1/3} = 2 \times 2 \times 2 \times 2 = 16 \text{ AU}$. This is the radius of the orbit if the orbit is circular.

6. $a^3 = P^2 \cdot (526)^3 = 1.4553 \times 10^8 = P^2$, so $P = 12{,}063$ years.

7. This is an application of Kepler's third law, $P^2 = a^3$, where a is in AU and P is in years. If P = 125 yrs, then $a^3 = 125^2$. Solving for a, we take the cube root of both sides to get $a = \left(125^2\right)^{1/3}$, where we have used the fact that the cube root of a number is the number to the $1/3$ power. This can be solved with a calculator or by noticing that $(125 \times 125)^{1/3} = (25 \times 5 \times 5 \times 25)^{1/3} = (25 \times 25 \times 25)^{1/3} = (25 \wedge 3) \wedge (1/3) = 25$, so a = 25 AU. If the planet's orbit is circular, then that is also the planet's orbital radius.

8. This problem is another application of Kepler's third law, $P^2 = a^3$, where a is in AU and P is in years. In this case, we are given a, and are asked to find P. Thus $P^2 = a^3 = 16^3$. Solving for P by taking the square root and recalling that the square root is the number to the $1/2$ power, we find that $P = \left(16^3\right)^{1/2} = 64 \text{ yrs}$. (Note: in solving this problem, you can simplify the math by reversing the order of the power and the square root. That is, $\left(16^3\right)^{1/2} = \left(16^{1/2}\right)^3 = 4^3 = 64$.)

Answers to Test Yourself

1. (d) Angular size is inversely proportional to distance so $L_M / L_S = 1/400$.
2. (b) Retrograde motion causes planets to stop their regular eastward motion with respect to the stars and move westward for a time.

3. (b) simplicity of models
4. (d) $P^2 = a^3 \cdot 4^3 = 4 \times 4 \times 4 = 4 \times 2 \times 2 \times 4 = 8^2$
5. (a) Kepler's 3rd Law relates a planet's orbital period to the size of its orbit.
6. (e) Venus orbits the Sun.
7. (c) Parallax was conjectured but impossible to measure without high quality telescopes.

ESSAY 2 – Special and General Relativity

Answers to Thought Questions

1. STUDENTS SHOULD PROVIDE A REASONED RESPONSE.

2. LOOK FOR REASONED ANSWERS AS THESE QUESTIONS ARE OPEN TO SOME INTERPRETATION. Formally, at c, the Lorentz factor becomes infinite, suggesting time would be infinitely slowed down for a photon. Likewise, the time dilation effect somewhere where the escape velocity is equal to the speed of light—like at the event horizon of a black hole—would be as extreme as possible and hence to an outside observer, time would appear to slow to a stop at the event horizon (and any light emitting from such a region is redshifted to zero energy). However, the point of view of an observer at these locations is harder to imagine clearly: if a space traveler traveling at a high Lorentz factor essentially considers the universe moving past her at high speed, and does notice the time dilation, it is difficult to say what the *photon* traveling at *c* actually experiences, whether it is trapped at the event horizon or winging through space.

3. Student's ideas should reflect the ideas of special relativity accurately. In particular, the times messages might be sent and received by a group that goes out, turns around, and comes back can be interesting.

Answers to Problems

1. To travel 100,000 light years in a time of 10 years, you would need to travel such that the 100,000 light years was contracted to an apparent distance of approximately 10 light years—a Lorentz factor of 10,000. Solving the equation for v/c,

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = 10{,}000 \text{ so } \sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{1}{10000} = 10^{-4} \text{ and } 1-\left(\frac{v}{c}\right)^2 = 10^{-8}.$$

So $\left(\frac{v}{c}\right)^2 = 1-10^{-8}$ and therefore $\left(\frac{v}{c}\right) = \sqrt{1-10^{-8}}$ which is very, very, very close to 1—you have to be traveling at approximately the speed of light. Numerically, if you use a calculator with more than 8 places, you can solve this out as:

$$\left(1-10^{-8}\right)^{1/2} = (0.999999990)^{1/2} = 0.999999994999999987499999375\ldots$$

Alternatively, there is an algebraic solution: $(1-\text{A})^{1/2} = 1-\tfrac{1}{2}\text{A}$. So,

$$\left(1-10^{-8}\right)^{1/2} = 1-0.5\times10^{-8} = 0.999999995.$$

2. Mercury orbits the Sun at speeds from $59\ \text{km/s}$ to $39\ \text{km/s}$ at its nearest and farthest distances from the Sun. The fractions of the speed of light are:

$$59\ \text{km/s}/300{,}000\ \text{km/s} = 1.967\times10^{-4}$$

$39\ \text{km/s}/300{,}000\ \text{km/s} = 1.3\times 10^{-4}$

The Lorentz factors are both approximately 1; the speeds are fairly slow (consult the Lorentz factor figure in the text). For the larger value:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\left(1.967\times 10^{-4}\right)^2}} = \frac{1}{\sqrt{1-\left(3.8691\times 10^{-8}\right)}} = \frac{1}{\sqrt{0.999999961309}}$$

Using a calculator with more than 8 decimals (e.g., many of the moderately advanced pocket calculators or the calculators on computers), you can solve this out as

=1.0000000193455005613725734750552

For the smaller value,

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\left(1.3\times 10^{-4}\right)^2}} = \frac{1}{\sqrt{1-\left(1.69\times 10^{-8}\right)}} = \frac{1}{\sqrt{0.9999999831}}$$

= 1.0000000084500001071037515083778

It is a small, but astrophysically important and measurable, difference between 1.00000001934 and 1.00000000845.

There would be a slightly increased Lorentz contraction of space, and time dilation, when nearer the Sun.

Analytically, the ratio of the values would look something like this:

$$\frac{near}{far} = \frac{\sqrt{1-\frac{39\,\frac{km}{s}^2}{c^2}}}{\sqrt{1-\frac{59\,\frac{km}{s}^2}{c^2}}} = \sqrt{\frac{c^2-39^2}{c^2-59^2}}$$

3. The escape velocity from the Sun is $76\ \text{km/s}$ at Mercury's closest distance to the Sun and is $62\ \text{km/s}$ at Mercury's farthest distance. Using the gravitational time dilation formula, find how slow time runs in these two places.

The answers are again extremely close to 1 for both numbers and require more than an 8 digit calculator (note that on some computers, the number of decimals can be set in the calculator application's options).

$\frac{1}{\sqrt{1-\frac{v_{esc}^{\ 2}}{c^2}}}$ gives $\frac{1}{\sqrt{1-\frac{76\,\frac{km}{s}^2}{c^2}}}$ for the larger dilation near the Sun, and $\frac{1}{\sqrt{1-\frac{62\,\frac{km}{s}^2}{c^2}}}$ for less dilation farther away.

Solving this out:
$v/c = 76\text{ km/s}/300{,}000\text{ km/s} = 2.533\times10^{-4}$

$$\gamma = \frac{1}{\sqrt{1-\frac{v_{esc}^2}{c^2}}} = \frac{1}{\sqrt{1-\left(\frac{v_{esc}}{c}\right)^2}} = \frac{1}{\sqrt{1-\left(2.533\times10^{-4}\right)^2}} = \frac{1}{\sqrt{1-\left(6.416089\times10^{-8}\right)}} = \frac{1}{\sqrt{0.99999993\ 583911}}$$

$= 1.0000000320804465437325096364142 = 1.000000032$
$v/c = 62\text{ km/s}/300{,}000\text{ km/s} = 2.067\times10^{-4}$

$$\gamma = \frac{1}{\sqrt{1-\frac{v_{esc}^2}{c^2}}} = \frac{1}{\sqrt{1-\left(\frac{v_{esc}}{c}\right)^2}} = \frac{1}{\sqrt{1-\left(2.067\times10^{-4}\right)^2}} = \frac{1}{\sqrt{1-\left(4.272489\times10^{-8}\right)}} = \frac{1}{\sqrt{0.99999995\ 727511}} =$$

$1.0000000213624456845311089391345 = 1.000000021$

This is again a small but clearly non-zero difference. Notice that in fact there is some time dilation happening at both the near and far points of the orbit—Mercury is close enough to the Sun we need to use General Relativity to understand even the farther part of the orbit. A few conclusions can be taken away from these numbers. First, if you compare the results here to problem 2, we notice that the General Relativistic effects we are looking at here are of the same approximate size as the Special Relativistic effects we calculated in problem 2. It's necessary to include both SR and GR to correctly predict Mercury's orbit. Second, time is running at different speeds in the near and far parts of Mercury's orbit—the difference in the correction factors is $1.00000001934 - 1.00000000845 = 1.089\times10^{-8}$ from problem 2 and 1.1×10^{-8} from problem 3, together a few parts in 10^8. That means every for every 10^8 seconds that passes far away from Mercury, 2.2 seconds less pass in Mercury's orbit. 10^8 seconds is only about 3.17 years, so Mercury's timing would be off by a minute in only 86 years. Over astronomical time scales, Mercury's timing would be quite out of sync with, say, Earth's. Moreover, if Mercury is moving (from our perspective) at 59 km/s in its orbit, a difference in timing of two seconds would put the planet 120 km away from where you would expect it to be!
Additionally, the SR and GR corrections will affect the apparent mass of Mercury, it will be more massive closer to the Sun. Newton's laws predict elliptical orbits, but Mercury itself would seem like a different object on a different path if you look at it close to the Sun or far from the Sun. Observationally, it has been known for centuries that the ellipse of Mercury's orbit precesses around the Sun (the perihelion and aphelion points move); and that the measured precession of Mercury's orbit cannot be explained by Newtonian mechanics; only since Einstein do we know it happens because of the relativistic effects.

<u>Answers to Test Yourself</u>

1. (e) Everything seems normal to the astronaut (remember, motion is relative).
2. (c) All observers always measure the speed of light in a vacuum to be *c*.
3. (b) His sister will be older (it can be shown that Bob has accelerated compared to Earth when he turns around, and you can tell he is the one with the dilated time on the round-trip).
4. (c) The principle of equivalence states that gravity is the same as being in an accelerating reference frame.

5. (d, a, b, c) The time dilation is stronger near more massive bodies.