

***Statics and Mechanics of Materials SI 1st edition
Goodno Solutions Manual***

SKU: 9781133364412-SOLUTIONS

Chapter 2 Solutions

Problem 2.1-1

Part (a)

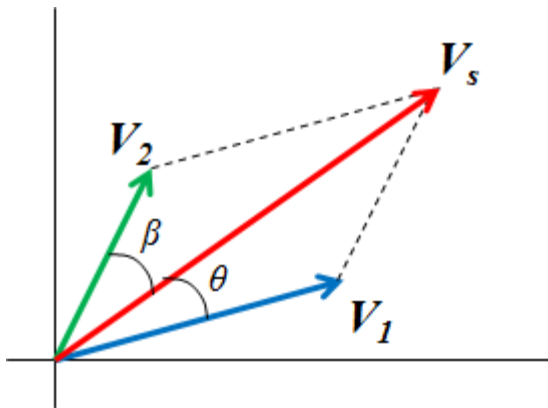
$$V_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad V_2 = \begin{pmatrix} 2 \\ \alpha \end{pmatrix} \quad V_1 \cdot V_2 = 0 \text{ solve, } \alpha \rightarrow -8$$

Part (b)

$$\gamma = \text{atan}\left(\frac{1}{4}\right) = 14.036 \text{ deg} \quad \varphi = 30 \text{ deg} \quad \beta = \varphi + \gamma = 44.036 \text{ deg}$$

$$\tan(\beta) = \frac{\alpha}{2} \left| \begin{array}{l} \text{solve, } \alpha \\ \text{float, 4} \end{array} \right. \rightarrow 1.934$$

Part (c)



$$V_1 = 4 \quad V_2 = 8 \quad V_s = 12$$

Law of Cosines

$$\theta = \text{acos}\left(\frac{V_1^2 + V_s^2 - V_2^2}{2 \cdot V_1 \cdot V_s}\right) = 0 \cdot \text{deg}$$

Next, Law of Sines

$$\beta = \text{asin}\left(\frac{V_1}{V_2} \cdot \sin(\theta)\right) = 0 \cdot \text{deg}$$

$$\phi = \theta + \beta = 0 \cdot \text{deg}$$

Problem 2.1-2

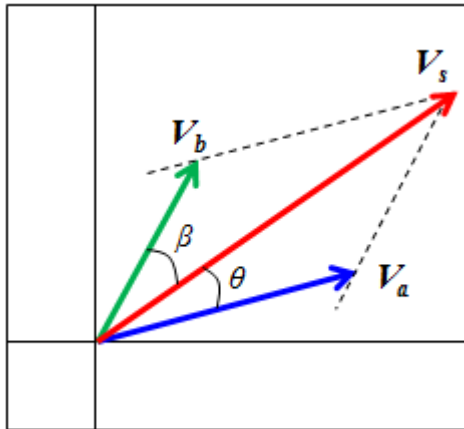


Fig. 2 – Vectors V_a and V_b in Part (c)

$$V_a = 5 \quad V_b = 7 \quad V_s = 10$$

Law of Cosines

$$\theta = \arccos\left(\frac{V_a^2 + V_s^2 - V_b^2}{2 \cdot V_a \cdot V_s}\right) = 40.5^\circ$$

Next, Law of Sines

$$\beta = \arcsin\left(\frac{V_a}{V_b} \cdot \sin(\theta)\right) = 27.7^\circ$$

$$\phi = \theta + \beta = 68.2^\circ$$

Problem 2.1-3

$$F = 2.25 \text{ kN}$$

$$\text{ORIGIN} = 1$$

Part (a)

$$F_{xy} = F \cdot \begin{pmatrix} \cos(60^\circ) \\ \sin(60^\circ) \end{pmatrix} = \begin{pmatrix} 1.125 \\ 1.949 \end{pmatrix} \cdot \text{kN}$$

$$F_x = F_{xy_1} = 1.125 \cdot \text{kN}$$

$$F_y = F_{xy_2} = 1.949 \cdot \text{kN}$$

Part (b) - components

parallelogram law

Law of Sines

$$\frac{F_\alpha}{\sin(90^\circ)} = \frac{F}{\sin(60^\circ)}$$

$$F_\alpha = F \cdot \left(\frac{\sin(90^\circ)}{\sin(60^\circ)} \right)$$

$$F_\alpha = 2.598 \text{ kN}$$

$$\frac{F_\beta}{\sin(30^\circ)} = \frac{F}{\sin(60^\circ)}$$

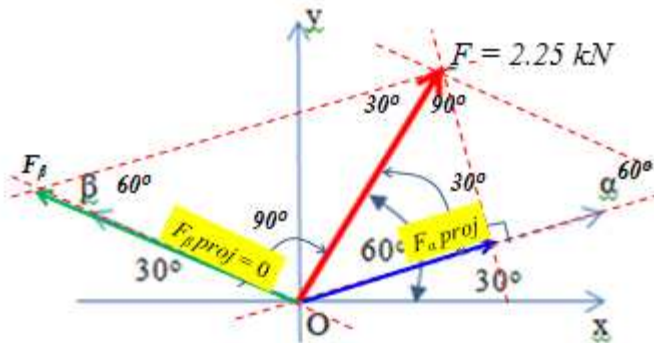
$$F_\beta = F \cdot \left(\frac{\sin(30^\circ)}{\sin(60^\circ)} \right)$$

$$F_\beta = 1.299 \text{ kN}$$

Law of cosines check

$$\sqrt{F_\alpha^2 + F_\beta^2 - 2 \cdot F_\alpha \cdot F_\beta \cdot \cos(60^\circ)} = 2.25 \text{ kN}$$

Part (c) - projections



$$F_{\alpha \text{proj}} = F \cdot \cos(30^\circ) = 1.949 \text{ kN}$$

$$F_{\beta \text{proj}} = F \cdot \cos(90^\circ) = 0 \cdot \text{kN}$$

Problem 2.1-4

Move the coordinate system from O to A then find component of F in x-y or n-t systems

$$F = 2.5\text{kN} \quad \alpha = 45\text{deg}$$

Part (a)

$$F_x = -F \cdot \cos(\alpha) = -1.768 \cdot \text{kN}$$

$$F_y = F \cdot \sin(\alpha) = 1.768 \cdot \text{kN}$$

Part (b)

$$\theta = 60\text{deg} - 45\text{deg} = 15 \cdot \text{deg}$$

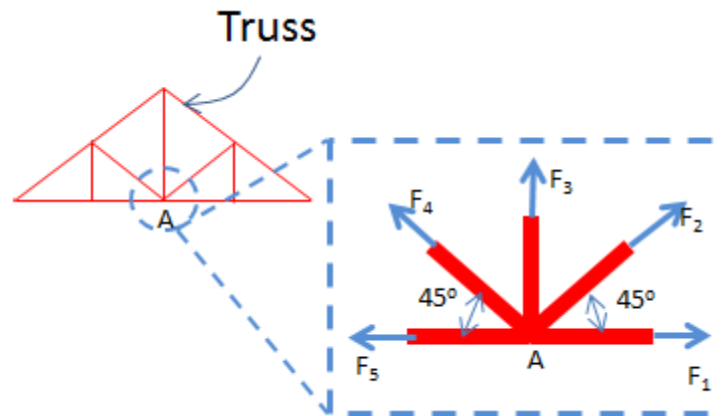
$$F_n = -F \cdot \cos(\theta) = -2.415 \cdot \text{kN}$$

$$F_t = -F \cdot \sin(\theta) = -0.647 \cdot \text{kN}$$

$$(a) \quad F_x = -1.768 \cdot \text{kN} \quad F_y = 1.768 \cdot \text{kN}$$

$$(b) \quad F_n = -2.415 \cdot \text{kN} \quad F_t = -0.647 \cdot \text{kN}$$

Problem 2.1-5



$$F_1 = 450\text{N} \quad F_3 = 90\text{N} \quad F_5 = 670\text{N} \quad \alpha = 45^\circ$$

$$\Sigma F_x = 0$$

$$F_1 + F_2 \cdot \cos(\alpha) - F_4 \cdot \cos(\alpha) - F_5 = 0 \quad \dots (1) \quad \text{so} \quad F_4 = \frac{1}{\cos(\alpha)} \cdot (F_1 + F_2 \cdot \cos(\alpha) - F_5)$$

$$\Sigma F_y = 0$$

$$F_2 \cdot \sin(\alpha) + F_4 \cdot \sin(\alpha) + F_3 = 0 \quad \dots (2) \quad \text{so} \quad F_2 = \frac{1}{\sin(\alpha)} \cdot (-F_3 - F_4 \cdot \sin(\alpha))$$

Substitute expression for F_4 into Eq. (2)

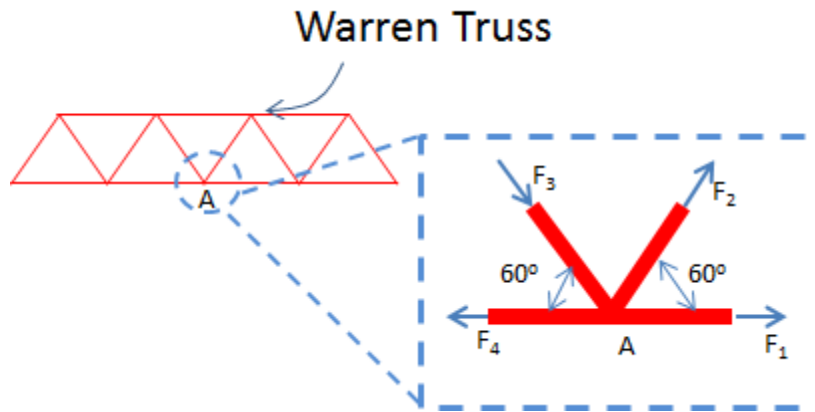
$$F_2 = \frac{F_5 - F_1 - F_3}{(\cos(\alpha) + \sin(\alpha))} = 91.9\text{N} \quad \text{and} \quad F_4 = \frac{-F_3 - F_2 \cdot \sin(\alpha)}{\sin(\alpha)} = -219.2\text{N}$$

CHECK that resultant is zero

$$\Sigma F_x = F_1 + F_2 \cdot \cos(\alpha) - F_4 \cdot \cos(\alpha) - F_5 = 0 \cdot \text{lb}$$

$$\Sigma F_y = F_2 \cdot \sin(\alpha) + F_3 + F_4 \cdot \sin(\alpha) = 0 \cdot \text{lb}$$

Problem 2.1-6



Part (a): units = kN

$$F_1 = \begin{pmatrix} 100 \\ 0 \end{pmatrix} \quad F_2 = \begin{pmatrix} 60 \cdot \cos(60^\circ) \\ 60 \cdot \sin(60^\circ) \end{pmatrix} = \begin{pmatrix} 30 \\ 51.962 \end{pmatrix} \quad R_{12} = F_1 + F_2 = \begin{pmatrix} 130 \\ 52 \end{pmatrix} \quad |R_{12}| = 140$$

Law of sines: β_{12} = angle that R_{12} makes with x axis

$$\beta_{12} = \text{asin}\left(\frac{|F_2|}{|R_{12}|} \cdot \sin(120^\circ)\right) = 21.79^\circ \quad \beta_{12} = 21.8^\circ$$

Part (b)

$$F_3 = \begin{pmatrix} 60 \cdot \cos(60^\circ) \\ -60 \cdot \sin(60^\circ) \end{pmatrix} = \begin{pmatrix} 30 \\ -51.962 \end{pmatrix} \quad F_4 = \begin{pmatrix} -100 \\ 0 \end{pmatrix} \quad R_{34} = F_3 + F_4 = \begin{pmatrix} -70 \\ -51.96 \end{pmatrix} \quad |R_{34}| = 87.18$$

γ_{34} = angle that R_{34} makes with (-x) axis

$$\gamma_{34} = \text{asin}\left(\frac{|F_3|}{|R_{34}|} \cdot \sin(60^\circ)\right) = 36.587^\circ \quad \text{so} \quad \beta_{34} = 180^\circ + \gamma_{34} = 217^\circ$$

Part (c)

$$F_1 + F_2 + F_3 + F_4 = \begin{pmatrix} 60 \\ 0 \end{pmatrix} \quad R_{12} + R_{34} = \begin{pmatrix} 60 \\ 0 \end{pmatrix} \quad |F_1 + F_2 + F_3 + F_4| = 60$$

Problem 2.1-6 - cont'd**Solution #2**

$$F_1 = 100\text{kN} \quad F_2 = 60\text{kN} \quad \alpha = 60\text{deg} \quad F_4 = 100\text{kN}$$

Part (a)

Use parallelogram rule & law of cosines

$$R_{12} = \sqrt{F_1^2 + F_2^2 + 2 \cdot F_1 \cdot F_2 \cdot \cos(\alpha)} = 140 \cdot \text{kN} \quad \beta = \arcsin\left(\frac{F_2}{R_{12}} \cdot \sin(120\text{deg})\right) = 21.787 \cdot \text{deg}$$

Part (b)

$$F_3 = 60\text{kN} \quad F_4 = 100\text{kN} \quad \alpha = 120\text{deg} \quad 60\text{deg} - \beta = 38.213 \cdot \text{deg}$$

$$R_{34} = \sqrt{F_3^2 + F_4^2 + 2 \cdot F_3 \cdot F_4 \cdot \cos(\alpha)} = 87.2 \cdot \text{kN} \quad \gamma = \arcsin\left(\frac{F_3}{R_{34}} \cdot \sin(60\text{deg})\right) = 36.587 \cdot \text{deg}$$

Part (c): Sum of resultant forces at point A $\alpha = 60\text{deg}$

$$\Sigma F_x = F_1 - F_4 + F_2 \cdot \cos(\alpha) + F_3 \cdot \cos(\alpha) = 60 \cdot \text{kN}$$

$$\Sigma F_y = F_2 \cdot \sin(\alpha) - F_3 \cdot \sin(\alpha) = 0 \text{ N}$$

$$\Sigma F = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 60 \cdot \text{kN}$$

Problem 2.1-7

$$T = 350\text{N} \quad r = 100\text{mm}$$

The tension force of the cable will induce no moment resultant at O.

The resultant of the cable forces is

$$R = \sqrt{T^2 + T^2 + 2 \cdot T \cdot T \cdot \cos(30^\circ)} = 676.1\text{N}$$

$$\alpha = \text{atan}\left(\frac{T \cdot \sin(30^\circ)}{T + T \cdot \cos(30^\circ)}\right) = 15^\circ$$

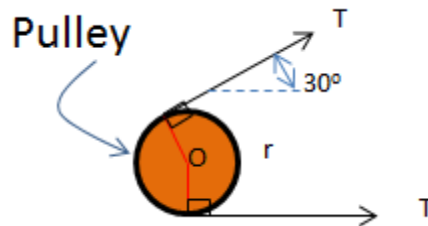
or

$$\Sigma F_x = T \cdot \cos(30^\circ) + T = 653.109\text{N}$$

$$\Sigma F_y = T \cdot \sin(30^\circ) = 175\text{N}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 676.1\text{N}$$

$$\alpha = \text{atan}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = 15^\circ$$



Problem 2.1-8

Part (a) magnitudes > $N_1 = 200\text{N}$ $N_2 = 347\text{N}$

vectors:
$$N_1 = N_1 \cdot \begin{pmatrix} \cos(30\text{deg}) \\ \sin(30\text{deg}) \\ 0 \end{pmatrix} = \begin{pmatrix} 173.2 \\ 100 \\ 0 \end{pmatrix} \text{N}$$

$$N_2 = N_2 \cdot \begin{pmatrix} -\sin(30\text{deg}) \\ \cos(30\text{deg}) \\ 0 \end{pmatrix} = \begin{pmatrix} -173.5 \\ 300.5 \\ 0 \end{pmatrix} \text{N}$$

Resultant:
$$R = N_1 + N_2 = \begin{pmatrix} -0.3 \\ 400.5 \\ 0 \end{pmatrix} \text{N}$$

angle with x axis:

$$180\text{-deg} + \text{atan}\left(\frac{R_2}{R_1}\right) = 90.042\text{-deg}$$

Part (b)

$$n = \begin{pmatrix} -\cos(60\text{deg}) \\ \sin(60\text{deg}) \\ 0 \end{pmatrix} = \begin{pmatrix} -0.5 \\ 0.866 \\ 0 \end{pmatrix}$$

$$t = \begin{pmatrix} \cos(30\text{deg}) \\ \sin(30\text{deg}) \\ 0 \end{pmatrix} = \begin{pmatrix} 0.866 \\ 0.5 \\ 0 \end{pmatrix}$$

$$R_n = R \cdot n = 347 \text{ N} = N_2$$

$$R_t = R \cdot t = 200 \text{ N} = N_1$$

Problem 2.1-9

Cartesian vectors

$$T_1 = 3.6\text{kN} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad T_2 = 2.7\text{kN} \cdot \begin{pmatrix} \cos(30\text{deg}) \\ \sin(30\text{deg}) \\ 0 \end{pmatrix} = \begin{pmatrix} 2.3 \\ 1.4 \\ 0 \end{pmatrix} \cdot \text{kN} \quad T_3 = 5.3\text{kN} \cdot \begin{pmatrix} \sin(30\text{deg}) \\ -\cos(30\text{deg}) \\ 0 \end{pmatrix} = \begin{pmatrix} 2.7 \\ -4.6 \\ 0 \end{pmatrix} \cdot \text{kN}$$

Resultant

$$R = T_1 + T_2 + T_3 = \begin{pmatrix} 4.988 \\ 0.36 \\ 0 \end{pmatrix} \cdot \text{kN} \quad |R| = 5.001 \cdot \text{kN}$$

Angle with x axis

$$i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \theta_x = \cos^{-1} \left(\frac{R}{|R|} \cdot i \right) = 4.13^\circ \quad \text{or direction cosines of } R: \quad n_R = \frac{R}{|R|} = \begin{pmatrix} 1 \\ 0.07 \\ 0 \end{pmatrix}$$
$$\theta_x = \cos^{-1}(n_{R_1}) = 4.13^\circ$$
$$\tan^{-1} \left(\frac{0.36}{4.988} \right) = 4.13^\circ$$

Problem 2.1-10

Use Law of Cosines and Law of Sines to find lengths of each cable segment & angles between cables and line OB

$$L_1 = \sqrt{2.5^2 + 1.5^2 - 2(2.5)(1.5)\cos(60\text{deg})} = 2.179 \text{ m} \quad L_2 = \sqrt{2.5^2 + 3^2 - 2(2.5)(3)\cos(60\text{deg})} = 2.784 \text{ m}$$

$$\frac{L_1}{\sin(60\text{deg})} = \frac{2.5}{\sin(\theta_1)} \quad \theta_1 = \sin^{-1}\left(\frac{2.5}{L_1} \cdot \sin(60\text{deg})\right) = 83.413\text{deg} \quad \theta_2 = \sin^{-1}\left(\frac{2.5}{L_2} \cdot \sin(60\text{deg})\right) = 51.052\text{deg}$$

Subtract 30 deg. from each angle theta to find angles between each cable segment & x axis.

$$\theta_{1x} = \theta_1 - 30\text{deg} = 53.413\text{deg} \quad \theta_{2x} = \theta_2 - 30\text{deg} = 21.052\text{deg}$$

Cartesian components for each cable

$$T_1 = (440\text{N}) \cdot \begin{pmatrix} -\cos(\theta_{1x}) \\ \sin(\theta_{1x}) \end{pmatrix} = \begin{pmatrix} -262.257 \\ 353.3 \end{pmatrix} \text{N} \quad T_2 = (560\text{N}) \cdot \begin{pmatrix} -\cos(\theta_{2x}) \\ \sin(\theta_{2x}) \end{pmatrix} = \begin{pmatrix} -522.624 \\ 201.158 \end{pmatrix} \text{N}$$

Resultant

$$R = T_1 + T_2 = \begin{pmatrix} -785 \\ 554 \end{pmatrix} \text{N}$$

Direction cosines of R

$$n_R = \frac{R}{|R|} = \begin{pmatrix} -0.817 \\ 0.577 \end{pmatrix}$$

$$\theta_x = \cos^{-1}(n_{R_1}) = 144.8\text{deg}$$

$$180\text{deg} - \theta_x = 35.238\text{deg}$$

^ between θ_{1x} and θ_{2x}

Problem 2.1-11

$$\theta = 80\text{deg} \quad \alpha = 25\text{deg} \quad T = 22.25\text{kN} \quad a = 4.6\text{m} \quad b = 7.6\text{m}$$

Geometry

$$\beta = 180\text{deg} - (\theta + \alpha) = 75\text{deg}$$

Resultant acts in y dir. so $T \cdot \sin(\beta_1) = T \cdot \sin(\beta_2)$ and $\beta_2 = \beta_1$ which gives $\beta_1 = \frac{\beta}{2} = 37.5\text{deg}$

Resultant of cable force T = 22.25 kN at B (in -y dir.)

$$R = T \cdot (\cos(\beta_1) + \cos(\beta_2)) \quad R = 35.304\text{kN}$$

Check that resultant component in x-dir. is zero $R_x = T \cdot (-\sin(\beta_1) + \sin(\beta_2)) = 0\text{lb}$

$$-T \cdot \sin(\beta_1) = -14\text{kN} \quad T \cdot \sin(\beta_2) = 14\text{kN}$$

Problem 2.1-12

$$F_1 = 250\text{N} \cdot \begin{pmatrix} -\cos(30\text{deg}) \\ \sin(30\text{deg}) \end{pmatrix} = \begin{pmatrix} -216.506 \\ 125 \end{pmatrix} \text{N}$$

$$F_3 = 250\text{N} \cdot \begin{pmatrix} \cos(30\text{deg}) \\ -\sin(30\text{deg}) \end{pmatrix} = \begin{pmatrix} 216.506 \\ -125 \end{pmatrix} \text{N}$$

$$F_2 = 180\text{N} \cdot \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$R = F_1 + F_2 + F_3 = \begin{pmatrix} 0 \\ -180 \end{pmatrix} \text{N}$$

Problem 2.1-13

$$F_{EB} = 20\text{kN} \quad \alpha = 80\text{deg} \quad \beta = 60\text{deg}$$

Part (a) - Cartesian components

$$F_{EB} = F_{EB} \begin{pmatrix} -\cos(\alpha) \\ \sin(\alpha) \\ 0 \end{pmatrix} = \begin{pmatrix} -3.47 \\ 19.7 \\ 0 \end{pmatrix} \cdot \text{kN}$$

Part (b) - projection on CA

$$n_{CA} = \begin{pmatrix} -\cos(\beta) \\ \sin(\beta) \\ 0 \end{pmatrix} = \begin{pmatrix} -0.5 \\ 0.866 \\ 0 \end{pmatrix} \quad F_{CA} = F_{EB} \cdot n_{CA} = 18.79\text{kN}$$

Problem 2.1-14

$$F = 8\text{N} \quad T = 9\text{N}$$

Fig. (a)
$$A = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} \quad E = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$n_{AE} = \frac{E - A}{|E - A|} = \begin{pmatrix} -0.333 \\ 0.667 \\ -0.667 \end{pmatrix} \quad R_a = F \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + T \cdot n_{AE} = \begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix} \text{N}$$

Fig. (b)
$$R_b = F \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + T \cdot (-n_{AE}) = \begin{pmatrix} -5 \\ -6 \\ 6 \end{pmatrix} \text{N}$$

Fig. (c)
$$E = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

$$n_{AE} = \frac{E - A}{|E - A|} = \begin{pmatrix} -0.728 \\ 0.485 \\ -0.485 \end{pmatrix} \quad R_c = F \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + T \cdot n_{AE} = \begin{pmatrix} 1.45 \\ 4.37 \\ -4.37 \end{pmatrix} \text{N}$$

Answers

$$R_a = \begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix} \text{N} \quad R_b = \begin{pmatrix} -5 \\ -6 \\ 6 \end{pmatrix} \text{N} \quad R_c = \begin{pmatrix} 1.45 \\ 4.37 \\ -4.37 \end{pmatrix} \text{N}$$

Problem 2.1-15

$$\mathbf{A} = \begin{pmatrix} 1.5 \\ 6 \\ 9 \end{pmatrix} \cdot \text{m} \quad \mathbf{n}_{OA} = \frac{\mathbf{A}}{|\mathbf{A}|} = \begin{pmatrix} 0.137 \\ 0.549 \\ 0.824 \end{pmatrix} \quad T = 12.5 \text{ kN} \quad \mathbf{T} = T \cdot \mathbf{n}_{OA} = \begin{pmatrix} 1.72 \\ 6.87 \\ 10.3 \end{pmatrix} \cdot \text{kN} \quad |\mathbf{T}| = 12.5 \text{ kN}$$

Problem 2.1-16

$$\alpha = \operatorname{atan}\left(\frac{34}{27}\right) = 51.546 \cdot \text{deg} \qquad \beta = \operatorname{atan}\left(\frac{17}{27}\right) = 32.196 \cdot \text{deg}$$

For resultant to be vertical, horizontal components of cable forces must sum to zero

$$T = \frac{6 \text{ kN} \cdot (\cos(\alpha)) + 4 \text{ kN} \cdot (\cos(\beta)) - 3.6 \text{ kN} \cdot (\cos(\beta))}{\cos(\alpha)} = 6.54 \cdot \text{kN}$$

Problem 2.1-17

$$T_{CE} = 1.67 \text{ kN} \quad a = 6 \text{ m} \quad b = 2.4 \text{ m} \quad c = 1.8 \text{ m}$$

Position and unit vectors

$$\mathbf{r}_{CE} = \begin{pmatrix} \frac{-a}{2} \\ c \\ -b \end{pmatrix} = \begin{pmatrix} -3 \\ 1.8 \\ -2.4 \end{pmatrix} \cdot \text{m} \quad \mathbf{n}_{CE} = \frac{\mathbf{r}_{CE}}{|\mathbf{r}_{CE}|} = \begin{pmatrix} -0.707 \\ 0.424 \\ -0.566 \end{pmatrix} \quad \mathbf{r}_{CA} = \begin{pmatrix} 0 \\ 0 \\ 0 - b \end{pmatrix} \quad \mathbf{n}_{CA} = \frac{\mathbf{r}_{CA}}{|\mathbf{r}_{CA}|} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$T_{CE} = T_{CE} \cdot \mathbf{n}_{CE} = \begin{pmatrix} -1.181 \\ 0.709 \\ -0.945 \end{pmatrix} \cdot \text{kN} \quad |T_{CE}| = 1.67 \cdot \text{kN}$$

$$\text{Angle between CE and line CA (= -z axis)} \quad \theta = \cos^{-1}(\mathbf{n}_{CE} \cdot \mathbf{n}_{CA}) = 124.45^\circ \quad \cos^{-1}\left[\mathbf{n}_{CE} \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}\right] = 55.55^\circ$$

$$124.45 + 55.55 = 180$$

Projection onto line CA

$$T_{CE} \cdot \cos(55.55^\circ) = 0.945 \cdot \text{kN}$$

^ in this case, projection = component (except for sign)

Problem 2.1-18

$$a = 10\text{m} \quad b = 4\text{m} \quad c = 3\text{m} \quad T_{CE} = 1700\text{N} \quad T_{CF} = T_{CE}$$

Symmetry about yz plane so R will lie in yz plane (x component is zero)

Joint coordinates

$$C = \begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \quad E = \begin{pmatrix} \frac{-a}{2} \\ \frac{c}{2} \\ 0 \end{pmatrix} \quad F = \begin{pmatrix} \frac{a}{2} \\ \frac{c}{2} \\ 0 \end{pmatrix}$$

unit vectors

$$n_{CE} = \frac{E - C}{|E - C|} = \begin{pmatrix} -0.707 \\ 0.424 \\ -0.566 \end{pmatrix} \quad n_{CF} = \frac{F - C}{|F - C|} = \begin{pmatrix} 0.707 \\ 0.424 \\ -0.566 \end{pmatrix}$$

cable force vectors and resultant

$$T_{CE} = T_{CE} \cdot n_{CE} = \begin{pmatrix} -1202 \\ 721 \\ -962 \end{pmatrix} \text{N} \quad T_{CF} = T_{CF} \cdot n_{CF} = \begin{pmatrix} 1202 \\ 721 \\ -962 \end{pmatrix} \text{N} \quad R = T_{CE} + T_{CF} = \begin{pmatrix} 0 \\ 1442 \\ -1923 \end{pmatrix} \text{N}$$

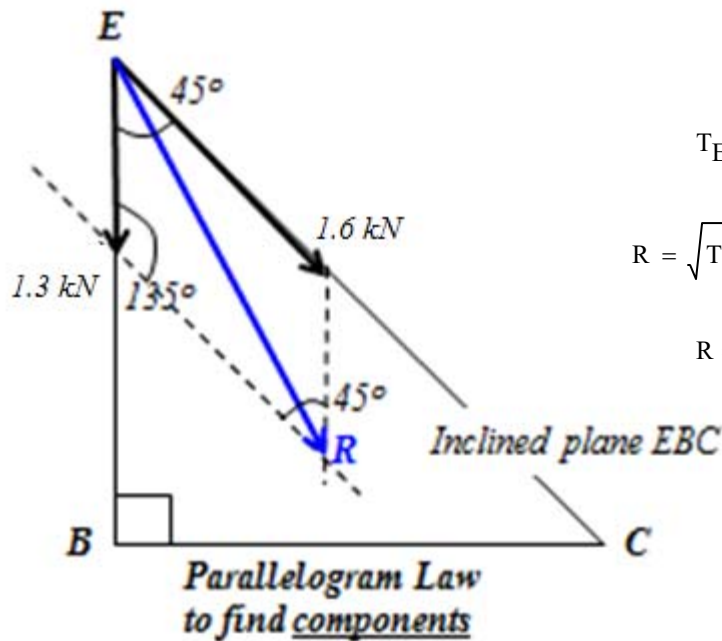
Problem 2.1-19

1. Note that EBC is a right angle
2. Length of EB is 3 m = a/2 so angle CEB is 45 deg.
3. Use Parallelogram Law to find R (start with sketch)

Dimensions (m)

$$a = 6\text{m} \quad b = 2.4\text{m} \quad c = 1.8\text{m}$$

PART (a) - Parallelogram Law and Law of Cosines



$$T_{EB} = 1.3\text{kN} \quad T_{EC} = 1.6\text{kN}$$

$$R = \sqrt{T_{EB}^2 + T_{EC}^2 - 2 \cdot (T_{EC}) \cdot (T_{EB}) \cdot \cos(135\text{deg})} = 2.682 \cdot \text{kN}$$

$$R = 2.682 \cdot \text{kN} \quad < \text{magnitude of resultant}$$

Vectors

$$\begin{aligned} \mathbf{r}_{EB} &= \begin{pmatrix} 0 \\ -c \\ b \end{pmatrix} & \mathbf{r}_{EC} &= \begin{pmatrix} \frac{a}{2} \\ -c \\ b \end{pmatrix} & \mathbf{n}_{EB} &= \frac{\mathbf{r}_{EB}}{|\mathbf{r}_{EB}|} = \begin{pmatrix} 0 \\ -0.6 \\ 0.8 \end{pmatrix} & \mathbf{n}_{EC} &= \frac{\mathbf{r}_{EC}}{|\mathbf{r}_{EC}|} = \begin{pmatrix} 0.707 \\ -0.424 \\ 0.566 \end{pmatrix} \\ \mathbf{r}_{FC} &= \begin{pmatrix} \frac{-a}{2} \\ -c \\ b \end{pmatrix} & \mathbf{r}_{FD} &= \begin{pmatrix} 0 \\ -c \\ b \end{pmatrix} & \mathbf{n}_{FC} &= \frac{\mathbf{r}_{FC}}{|\mathbf{r}_{FC}|} = \begin{pmatrix} -0.707 \\ -0.424 \\ 0.566 \end{pmatrix} & \mathbf{n}_{FD} &= \frac{\mathbf{r}_{FD}}{|\mathbf{r}_{FD}|} = \begin{pmatrix} 0 \\ -0.6 \\ 0.8 \end{pmatrix} \end{aligned}$$

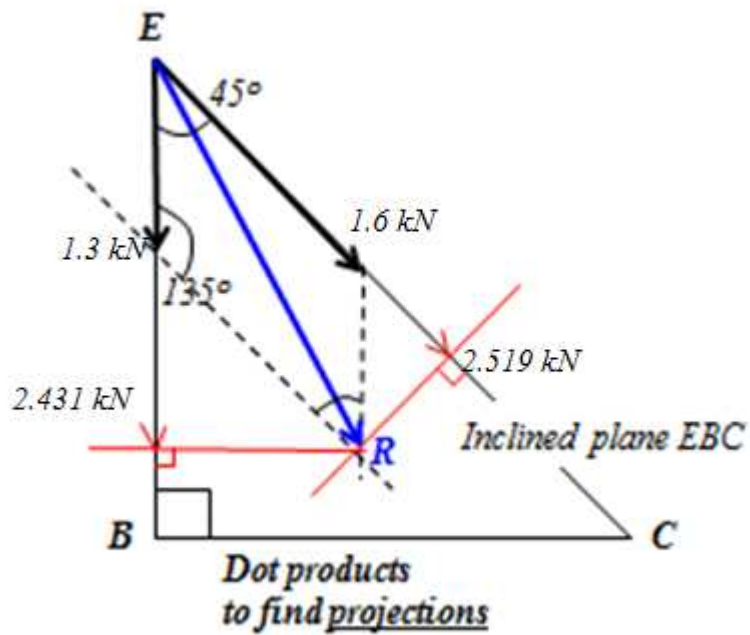
$$\begin{aligned} T_{EB} &= T_{EB} \mathbf{n}_{EB} & T_{EC} &= T_{EC} \mathbf{n}_{EC} & \mathbf{R} &= T_{EB} + T_{EC} = \begin{pmatrix} 1.131 \\ -1.459 \\ 1.945 \end{pmatrix} \cdot \text{kN} & |\mathbf{R}| &= 2.682 \text{kN} & < \text{agrees with above result} \end{aligned}$$

$$|T_{EB}| = 1.3 \cdot \text{kN} \quad |T_{EC}| = 1.6 \cdot \text{kN}$$

Problem 2.1-19 - PART (b) - Projections of R onto lines EB and EC - use dot product

$$R_{EB} = R \cdot n_{EB} = 2.431 \cdot \text{kN} \quad < \text{not equal to component of } R \text{ along } EB \text{ (i.e., } 1.3 \text{ kN)}$$

$$R_{EC} = R \cdot n_{EC} = 2.519 \cdot \text{kN} \quad < \text{not equal to component of } R \text{ along } EC \text{ (i.e., } 1.6 \text{ kN)}$$



Problem 2.1-20

$$F_s = 154\text{N} \quad h = 660\text{mm} \quad d = 150\text{mm} \quad H = 1041\text{mm} \quad c = 506\text{mm}$$

Joint coordinates

$$A = \begin{pmatrix} h \\ h - H \\ d \end{pmatrix} = \begin{pmatrix} 660 \\ -381 \\ 150 \end{pmatrix} \cdot \text{mm} \quad B = \begin{pmatrix} h \\ h \\ c \end{pmatrix} = \begin{pmatrix} 660 \\ 660 \\ 506 \end{pmatrix} \cdot \text{mm}$$

Unit vector

$$n_{AB} = \frac{B - A}{|B - A|} = \begin{pmatrix} 0 \\ 0.946 \\ 0.324 \end{pmatrix}$$

Part (a) $F_s = F_s \cdot n_{AB} = \begin{pmatrix} 0 \\ 145.7 \\ 49.8 \end{pmatrix} \text{N} \quad |F_s| = 154 \text{ N}$

Part (b) $n_{AD} = \frac{-A}{|A|} = \begin{pmatrix} -0.85 \\ 0.491 \\ -0.193 \end{pmatrix} \quad F_{AD} = F_s \cdot n_{AD} = 61.9 \text{ N} \quad \theta = \arccos\left(\frac{F_s \cdot n_{AD}}{|F_s|}\right) = 66.3 \cdot \text{deg}$

Problem 2.1-21

$$F = 1.5 \text{ kN} \quad r = 0.75 \text{ m} \quad h = 2.25 \text{ m} \quad A = \begin{pmatrix} -r \cdot \sin(60^\circ) \\ r \cdot \cos(60^\circ) \\ h \end{pmatrix} = \begin{pmatrix} -0.65 \\ 0.375 \\ 2.25 \end{pmatrix} \cdot \text{m}$$

Part (a)

$$n_{OA} = \frac{A}{|A|} = \begin{pmatrix} -0.274 \\ 0.158 \\ 0.949 \end{pmatrix} \quad F = F \cdot n_{OA} = \begin{pmatrix} -410.79 \\ 237.17 \\ 1423.02 \end{pmatrix} \cdot \text{N} \quad |F| = 1.5 \text{ kN}$$

Part (b)

$$n_{BC} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad F_{BC} = F \cdot n_{BC} = 237.2 \text{ N} \quad \theta = \arccos\left(\frac{F \cdot n_{BC}}{|F|}\right) = 80.9^\circ$$

Problem 2.1-22

$$T_B = 3700\text{N}$$

Joint coordinates

$$B = \begin{pmatrix} -0.5 \\ -1 \\ 0 \end{pmatrix} \cdot \text{m} \quad C = \begin{pmatrix} 0 \\ 0 \\ 2.5 \end{pmatrix} \cdot \text{m}$$

Unit vector along cable B:

$$\mathbf{n}_{BC} = \frac{C - B}{|C - B|} = \begin{pmatrix} 0.183 \\ 0.365 \\ 0.913 \end{pmatrix}$$

Cable force vector:

$$T_B = T_B \cdot \mathbf{n}_{BC} = \begin{pmatrix} 676 \\ 1351 \\ 3378 \end{pmatrix} \cdot \text{N} \quad |T_B| = 3700 \cdot \text{N}$$

Problem 2.1-23

$$T_{AB} = 67\text{N}$$

Joint coordinates:
$$A = \begin{pmatrix} 200\text{mm} \\ 500\text{mm} - 500\text{mm}\cos(60\text{deg}) \\ -500\text{mm}\sin(60\text{deg}) \end{pmatrix} = \begin{pmatrix} 200 \\ 250 \\ -433.013 \end{pmatrix} \cdot \text{mm} \quad B = \begin{pmatrix} 150 \\ 0 \\ 0 \end{pmatrix} \cdot \text{mm} \quad C = \begin{pmatrix} 0 \\ 500 \\ 0 \end{pmatrix} \cdot \text{mm}$$

Part (a)
$$n_{AB} = \frac{B - A}{|B - A|} = \begin{pmatrix} -0.1 \\ -0.498 \\ 0.862 \end{pmatrix} \quad T_{AB} = T_{AB} \cdot n_{AB} = \begin{pmatrix} -6.667 \\ -33.334 \\ 57.736 \end{pmatrix} \cdot \text{N}$$

Part (b)
$$n_{CD} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad T_{CD} = T_{AB} \cdot n_{CD} = -6.667\text{N}$$

Part (c)
$$n_{CB} = \frac{B - C}{|B - C|} = \begin{pmatrix} 0.287 \\ -0.958 \\ 0 \end{pmatrix} \quad T_{CB} = T_{AB} \cdot n_{CB} = 30.01\text{N}$$

Problem 2.1-24

Find the intersection line between OD and AB/ coordinate F in X-Z plane

line OD

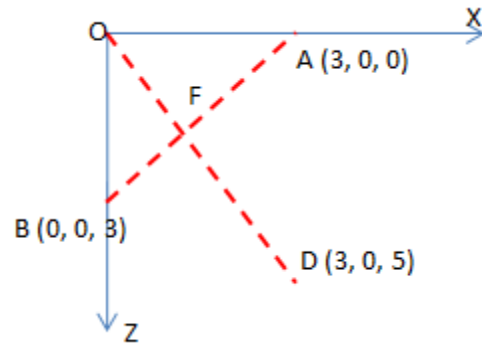
$$z(x) = \frac{5}{3} \cdot x \quad \dots (1)$$

line AB

$$z(x) = 3 - x \quad \dots (2)$$

equating equation (1) & (2)

$$\text{Hence,} \quad x = 3 \cdot \frac{3}{8} = 1.125 \quad z(x) = 1.875$$



Part (a)

$$F = \begin{pmatrix} 1.125 \\ 0 \\ 1.875 \end{pmatrix} \quad C = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \quad E = \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \quad A = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$$

$$n_{CE} = \frac{E - C}{|E - C|} = \begin{pmatrix} 0.514 \\ 0 \\ 0.857 \end{pmatrix} \quad n_{FE} = \frac{E - F}{|E - F|} = \begin{pmatrix} 0.397 \\ 0.636 \\ 0.662 \end{pmatrix}$$

$$P = (3 \text{ kN}) \cdot n_{CE} = \begin{pmatrix} 1.543 \\ 0 \\ 2.572 \end{pmatrix} \cdot \text{kN} \quad Q = (4 \text{ kN}) \cdot n_{FE} = \begin{pmatrix} 1.589 \\ 2.542 \\ 2.648 \end{pmatrix} \cdot \text{kN}$$

Part (b)

$$R = P + Q = \begin{pmatrix} 3.13 \\ 2.54 \\ 5.22 \end{pmatrix} \cdot \text{kN} \quad Y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \theta_Y = \arccos\left(\frac{R \cdot Y}{|R|}\right) = 67.3^\circ$$

Part (c)

$$n_{CA} = \frac{A - C}{|A - C|} = \begin{pmatrix} 0.707 \\ -0.707 \\ 0 \end{pmatrix} \quad P_{CApar} = P \cdot n_{CA} = 1.091 \cdot \text{kN} \quad P_{CApar} \cdot n_{CA} = \begin{pmatrix} 0.772 \\ -0.772 \\ 0 \end{pmatrix} \cdot \text{kN}$$

$$P_{CAper} = P - P_{CApar} \cdot n_{CA} = \begin{pmatrix} 0.77 \\ 0.77 \\ 2.57 \end{pmatrix} \cdot \text{kN}$$

Problem 2.1-24 - cont'd

Part (d)

$$n_{BA} = \frac{A - B}{|A - B|} = \begin{pmatrix} 0.707 \\ 0 \\ -0.707 \end{pmatrix} \quad Q_{BApar} = Q \cdot n_{BA} = -0.749 \cdot \text{kN} \quad Q_{BApar} \cdot n_{BA} = \begin{pmatrix} -0.53 \\ 0 \\ 0.53 \end{pmatrix} \cdot \text{kN}$$

$$Q_{BAper} = Q - Q_{BApar} \cdot n_{BA} = \begin{pmatrix} 2.12 \\ 2.54 \\ 2.12 \end{pmatrix} \cdot \text{kN}$$

Part (e)

$$\theta = \arccos\left(\frac{P \cdot Q}{|P| \cdot |Q|}\right) = 39.5 \cdot \text{deg}$$

Problem 2.1-25

$$T_2 = 23\text{kN} \quad B = \begin{pmatrix} 3.6 \\ 0 \\ 0 \end{pmatrix} \cdot \text{m} \quad D = \begin{pmatrix} 1.5 \\ 3.6 \\ 0 \end{pmatrix} \cdot \text{m} \quad Q = \begin{pmatrix} 1.5 \\ 1.5 \\ 2.1 \end{pmatrix} \cdot \text{m}$$

$$\text{Unit vector along cable 2} \quad n_{BQ} = \frac{Q - B}{|Q - B|} = \begin{pmatrix} -0.631 \\ 0.451 \\ 0.631 \end{pmatrix}$$

$$\text{Components of vector } T_2 \quad T_2 = T_2 \cdot n_{BQ} = \begin{pmatrix} -14.52 \\ 10.37 \\ 14.52 \end{pmatrix} \cdot \text{kN} \quad |T_2| = 23 \cdot \text{kN}$$

Projection on line BD

$$n_{BD} = \frac{D - B}{|D - B|} = \begin{pmatrix} -0.504 \\ 0.864 \\ 0 \end{pmatrix} \quad T_{BD} = T_2 \cdot n_{BD} = 16.27 \cdot \text{kN}$$

Problem 2.1-26

$$O = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad A = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} \quad x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$r_{OA} = A - O = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} \quad n_{OA} = \frac{r_{OA}}{|r_{OA}|} = \begin{pmatrix} 0.514 \\ 0.514 \\ 0.686 \end{pmatrix}$$

Part (a)

$$F = (25\text{kN}) \cdot n_{OA} = \begin{pmatrix} 12.86 \\ 12.86 \\ 17.15 \end{pmatrix} \cdot \text{kN}$$

Part (b)

$$n_{OA} = \begin{pmatrix} 0.514 \\ 0.514 \\ 0.686 \end{pmatrix}$$

Part (c)

$$x_p = \begin{pmatrix} \cos(30\text{deg}) \\ -\sin(30\text{deg}) \\ 0 \end{pmatrix} \quad y_p = \begin{pmatrix} \sin(30\text{deg}) \\ \cos(30\text{deg}) \\ 0 \end{pmatrix} \quad z_p = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$F_{xp} = F \cdot x_p = 4.71 \cdot \text{kN}$$

$$F_{yp} = F \cdot y_p = 17.57 \cdot \text{kN}$$

$$F_{zp} = F \cdot z_p = 17.15 \cdot \text{kN}$$

Problem 2.1-27

$$H = \begin{pmatrix} 0 \\ 0 \\ 75 \end{pmatrix} \cdot \text{mm} \quad B = \begin{pmatrix} 100 \\ 125 \\ 0 \end{pmatrix} \text{mm} \quad C = \begin{pmatrix} 100 \\ 0 \\ 0 \end{pmatrix} \cdot \text{mm} \quad P = 45 \text{kN}$$

$$r_{HB} = B - H = \begin{pmatrix} 100 \\ 125 \\ -75 \end{pmatrix} \cdot \text{mm} \quad n_{HB} = \frac{r_{HB}}{|r_{HB}|} = \begin{pmatrix} 0.566 \\ 0.707 \\ -0.424 \end{pmatrix} \quad x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Part (a)

$$P = P \cdot n_{HB} = \begin{pmatrix} 25.456 \\ 31.82 \\ -19.092 \end{pmatrix} \cdot \text{kN} \quad P = \begin{pmatrix} 25.46 \\ 31.82 \\ -19.09 \end{pmatrix} \cdot \text{kN}$$

Part (b)

$$n_{HC} = \frac{C - H}{|C - H|} = \begin{pmatrix} 0.8 \\ 0 \\ -0.6 \end{pmatrix} \quad P_{HC\text{par}} = P \cdot n_{HC} = 31.82 \text{kN}$$

$$P_{HC\text{per}} = P - P_{HC\text{par}} \cdot n_{HC} = \begin{pmatrix} 0 \\ 31.82 \\ 0 \end{pmatrix} \cdot \text{kN} \quad |P_{HC\text{per}}| = 31.82 \text{kN}$$

Part (c)

$$D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad G = \begin{pmatrix} 100 \\ 0 \\ 75 \end{pmatrix} \cdot \text{mm} \quad n_{DG} = \frac{G - D}{|G - D|} = \begin{pmatrix} 0.8 \\ 0 \\ 0.6 \end{pmatrix}$$

$$P_{DG} = P \cdot n_{DG} = 8.91 \cdot \text{kN} \quad \theta = \cos^{-1} \left(\frac{P \cdot n_{DG}}{|P|} \right) = 78.6 \text{deg}$$

Problem 2.1-28

$$O = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad A = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 3 \\ 6 \end{pmatrix} \quad C = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$$

$$x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$n_{BC} = \frac{C - B}{|C - B|} = \begin{pmatrix} 0.442 \\ 0.147 \\ -0.885 \end{pmatrix}$$

Part (a)

$$P = \begin{pmatrix} 30 \\ 25 \\ -75 \end{pmatrix} \cdot \text{kN}$$

Part (b)

$$P \cdot n_{BC} = 83.3 \cdot \text{kN}$$

Part (c)

$$\theta = \arccos\left(\frac{P \cdot n_{BC}}{|P|}\right) = 9.88 \cdot \text{deg}$$

Problem 2.1-29

$$E = \begin{pmatrix} 1.2 \\ 0 \\ 0 \end{pmatrix} \text{ m} \quad F = \begin{pmatrix} 1.2 \\ 0 \\ 1.2 \end{pmatrix} \text{ m} \quad D = \begin{pmatrix} 1.2 \\ 0.6 \\ 0.6 \end{pmatrix} \text{ m} \quad C = \begin{pmatrix} 0.3 \\ 1.2 \\ 1.2 \end{pmatrix} \text{ m} \quad B = \begin{pmatrix} 0.3 \\ 1.2 \\ 0 \end{pmatrix} \text{ m}$$

$$n_{DB} = \frac{B - D}{|B - D|} = \begin{pmatrix} -0.728 \\ 0.485 \\ -0.485 \end{pmatrix} \quad n_{DC} = \frac{C - D}{|C - D|} = \begin{pmatrix} -0.728 \\ 0.485 \\ 0.485 \end{pmatrix} \quad n_{DE} = \frac{E - D}{|E - D|} = \begin{pmatrix} 0 \\ -0.707 \\ -0.707 \end{pmatrix}$$

$$n_{DF} = \frac{F - D}{|F - D|} = \begin{pmatrix} 0 \\ -0.707 \\ 0.707 \end{pmatrix}$$

$$T_{DB} = 45 \text{ kN} \quad T_{DC} = 45 \text{ kN} \quad T_{DE} = 67 \text{ kN} \quad T_{DF} = 67 \text{ kN}$$

Part (a)

$$T_{DB} = T_{DB} \cdot n_{DB} = \begin{pmatrix} -32.7 \\ 21.8 \\ -21.8 \end{pmatrix} \cdot \text{kN} \quad T_{DC} = T_{DC} \cdot n_{DC} = \begin{pmatrix} -32.7 \\ 21.8 \\ 21.8 \end{pmatrix} \cdot \text{kN}$$

$$T_{DE} = T_{DE} \cdot n_{DE} = \begin{pmatrix} 0 \\ -47.4 \\ -47.4 \end{pmatrix} \cdot \text{kN} \quad T_{DF} = T_{DF} \cdot n_{DF} = \begin{pmatrix} 0 \\ -47.4 \\ 47.4 \end{pmatrix} \cdot \text{kN}$$

Part (b)

$$R = T_{DB} + T_{DC} + T_{DE} + T_{DF} = \begin{pmatrix} -65.5 \\ -51.1 \\ 0 \end{pmatrix} \cdot \text{kN} \quad |R| = 83.1 \text{ kN}$$

Problem 2.1-30

$$\text{mass} = 450\text{kg} \quad P = 20\text{kN} \quad D = \begin{pmatrix} 0 \\ 9 \\ 9 \cdot \tan(55^\circ) \end{pmatrix} \cdot \text{m} = \begin{pmatrix} 0 \\ 9 \\ 12.853 \end{pmatrix} \cdot \text{m} \quad E = \begin{pmatrix} 3 \\ 13 \\ 0 \end{pmatrix} \cdot \text{m}$$

Part (a)

$$W = \text{mass} \cdot g = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -4413 \end{pmatrix} \cdot \text{N} \quad n_{DE} = \frac{E - D}{|E - D|} = \begin{pmatrix} 0.218 \\ 0.29 \\ -0.932 \end{pmatrix} \quad P = P \cdot n_{DE} = \begin{pmatrix} 4350 \\ 5801 \\ -18639 \end{pmatrix} \cdot \text{N}$$

Part (b)

$$R = W + P = \begin{pmatrix} 4350 \\ 5801 \\ -23052 \end{pmatrix} \cdot \text{N} \quad |R| = 24.2 \cdot \text{kN}$$

Problem 2.2-1

$$P = 45\text{kN} \quad M = 2.8\text{kN}\cdot\text{m} \quad L = 1.8\text{m} \quad \theta = 45^\circ$$

Resultant force

$$R_A = P \cdot \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + P \cdot \begin{pmatrix} \cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 31.82 \\ -76.82 \\ 0 \end{pmatrix} \cdot \text{kN} \quad |R_A| = 83.15\text{kN} \quad \alpha = \text{atan}\left(\frac{R_{A0}}{R_{A1}}\right) = -22.5^\circ$$

Resultant moment

$$M_A = \begin{pmatrix} \frac{3 \cdot L}{4} \\ 0 \\ 0 \end{pmatrix} \times \left[P \cdot \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \right] + M \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} \frac{3 \cdot L}{2} \\ 0 \\ 0 \end{pmatrix} \times \left[P \cdot \begin{pmatrix} \cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix} \right] = \begin{pmatrix} 0 \\ 0 \\ -149 \end{pmatrix} \cdot \text{kN}\cdot\text{m}$$

x intercept Given $x = L$

$$\begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} \times R_A = M_A$$

$$x = \text{Find}(x) = 1.95\text{m}$$

$$d = \frac{|M_A|}{|R_A|} = 1.798\text{m} \quad \frac{d}{\cos(|\alpha|)} = 1.95\text{m}$$

$$x = \frac{|M_A|}{|R_y|} = 1.946\text{m}$$

Problem 2.2-2

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

$$P_B = 25 \text{ kN} \cdot \begin{pmatrix} \cos(45^\circ - \theta) \\ -\sin(45^\circ - \theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 24.749 \\ -3.536 \\ 0 \end{pmatrix} \cdot \text{kN}$$

$$P_H = \begin{pmatrix} 0 \\ -50 \text{ kN} \\ 0 \end{pmatrix} \quad P_F = \begin{pmatrix} 0 \\ -75 \text{ kN} \\ 0 \end{pmatrix}$$

$$P_D = 50 \text{ kN} \cdot \begin{pmatrix} \cos(60^\circ - \theta) \\ \sin(60^\circ - \theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 45.981 \\ 19.641 \\ 0 \end{pmatrix} \cdot \text{kN}$$

Resultants

$$R_A = P_B + P_D + P_H + P_F = \begin{pmatrix} 70.7 \\ -108.9 \\ 0 \end{pmatrix} \cdot \text{kN} \quad |R_A| = 129.8 \cdot \text{kN}$$

$$M_A = \begin{pmatrix} 2 \text{ m} \\ \frac{3}{4} \cdot 2 \text{ m} \\ 0 \end{pmatrix} \times P_B + \begin{pmatrix} 2 \text{ m} \\ 0 \\ 0 \end{pmatrix} \times P_H + \begin{pmatrix} 6 \text{ m} \\ 0 \\ 0 \end{pmatrix} \times P_F + \begin{pmatrix} 6 \text{ m} \\ \frac{3}{4} \cdot 2 \text{ m} \\ 0 \end{pmatrix} \times P_D = \begin{pmatrix} 0 \\ 0 \\ -545 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad |M_A| = 545 \cdot \text{kN} \cdot \text{m}$$

y intercept

$$y = \frac{|M_A|}{R_x} = 7.71 \text{ m}$$

Problem 2.2-3

$$L = 3.6\text{m} \quad \theta = 60\text{deg}$$

$$R_A = \begin{pmatrix} 90 - 333\cos(\theta) \\ -200 - 333\sin(\theta) \\ 0 \end{pmatrix} \cdot \text{kN} = \begin{pmatrix} -76.5 \\ -488.4 \\ 0 \end{pmatrix} \cdot \text{kN} \quad |R_A| = 494.3\text{kN}$$

$$M_A = \begin{pmatrix} 0 \\ 0 \\ -200\text{kN} \cdot \frac{L}{2} - 333\text{kN} \cdot \sin(\theta) \cdot \frac{3 \cdot L}{4} - 135\text{kN} \cdot \text{m} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1273.643 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Problem 2.2-4

$$\theta = 60\text{deg}$$

$$R_O = \begin{pmatrix} 2 - 1 \\ 2 - 1 \\ 0 \end{pmatrix} \cdot F \quad R_O = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} F$$

$$M_{OZ} = -2 \cdot F \cdot (r \cdot \sin(60\text{deg})) - 2 \cdot F \cdot (r \cdot \cos(60\text{deg})) - F \cdot r$$

$$M_{OZ} = -(2 + \sqrt{3}) \cdot F \cdot r$$

Problem 2.2-5

$$F_{EB} = 11\text{kN} \quad \alpha = 80\text{deg} \quad \beta = 45\text{deg} \quad BC = 5.4\text{m}$$

$$F_{EB} = F_{EB} \begin{pmatrix} -\cos(\alpha) \\ \sin(\alpha) \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 11 \\ 0 \end{pmatrix} \cdot \text{kN}$$

$$r_{CB} = \begin{pmatrix} -BC \cdot \cos(\beta) \\ BC \cdot \sin(\beta) \\ 0 \end{pmatrix} = \begin{pmatrix} -3.818 \\ 3.818 \\ 0 \end{pmatrix} \cdot \text{m}$$

Part (a)

$$M_C = r_{CB} \times F_{EB} = \begin{pmatrix} 0 \\ 0 \\ -34 \end{pmatrix} \cdot \text{kN}\cdot\text{m}$$

Part (b)

$$M_{Cz} = (|F_{EB}|) [\cos(\alpha) \cdot (BC \cdot \sin(\beta)) - \sin(\alpha) \cdot (BC \cdot \cos(\beta))] = -34 \text{ kN}\cdot\text{m}$$

Problem 2.2-6

$$F = 110\text{kN} \quad A = \begin{pmatrix} -3 \\ -1 \\ 0 \end{pmatrix} \cdot \text{m} \quad B = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \cdot \text{m} \quad r_{OA} = A \quad r_{OB} = B$$

$$F_A = F \cdot \begin{pmatrix} -\cos(30\text{deg}) \\ -\sin(30\text{deg}) \\ 0 \end{pmatrix} = \begin{pmatrix} -95.263 \\ -55 \\ 0 \end{pmatrix} \cdot \text{kN} \quad F_B = F \cdot \begin{pmatrix} \cos(30\text{deg}) \\ \sin(30\text{deg}) \\ 0 \end{pmatrix} \quad R_O = F_A + F_B = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{N}$$

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B = \begin{pmatrix} 0 \\ 0 \\ 139.5 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Problem 2.2-7

$$F = 45 \text{ kN} \quad A = \begin{pmatrix} 75 \\ 75 \\ 100 \end{pmatrix} \cdot \text{mm} \quad B = \begin{pmatrix} 125 \\ 0 \\ 0 \end{pmatrix} \cdot \text{mm} \quad C = \begin{pmatrix} 0 \\ 0 \\ 75 \end{pmatrix} \cdot \text{mm}$$

Part (a)

$$n_{OA} = \frac{A}{|A|} = \begin{pmatrix} 0.514 \\ 0.514 \\ 0.686 \end{pmatrix} \quad F_{OA} = F \cdot n_{OA} = \begin{pmatrix} 23.152 \\ 23.152 \\ 30.87 \end{pmatrix} \cdot \text{kN} \quad r_{BO} = -B$$

$$M_B = r_{BO} \times F_{OA} = \begin{pmatrix} 0 \\ 3.9 \\ -2.9 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad |M_B| = 4.8 \cdot \text{kN} \cdot \text{m}$$

Part (b)

$$r_{CO} = -C \quad M_C = r_{CO} \times F_{OA} = \begin{pmatrix} 1.74 \\ -1.74 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Part (c)

$$n_{CB} = \frac{B - C}{|B - C|} = \begin{pmatrix} 0.857 \\ 0 \\ -0.514 \end{pmatrix} \quad M_{CB} = M_B \cdot n_{CB} = 1.49 \cdot \text{kN} \cdot \text{m} = M_C \cdot n_{CB} = 1.49 \cdot \text{kN} \cdot \text{m}$$

$$\text{Vectors} \quad M_{CB\text{par}} = M_B \cdot n_{CB} \cdot n_{CB} = \begin{pmatrix} 1.28 \\ 0 \\ -0.77 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad M_{CB\text{per}} = M_B - M_{CB\text{par}} = \begin{pmatrix} -1.28 \\ 3.86 \\ -2.13 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$|M_{CB\text{par}}| = 1.49 \cdot \text{kN} \cdot \text{m}$$

Problem 2.2-8

$$P = \begin{pmatrix} 30 \\ 25 \\ -75 \end{pmatrix} \cdot \text{kN} \quad A = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} \cdot \text{m} \quad B = \begin{pmatrix} 0 \\ 3 \\ 6 \end{pmatrix} \cdot \text{m} \quad C = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \cdot \text{m}$$

Part (a)

$$r_{BA} = A - B = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} \text{m} \quad M_B = r_{BA} \times P = \begin{pmatrix} 225 \\ 0 \\ 90 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Part (b)

$$M_{Bx} = (75 \text{ kN}) \cdot (3 \text{ m}) = 225 \cdot \text{kN} \cdot \text{m} \quad M_{By} = 0 \quad M_{Bz} = (30 \text{ kN}) \cdot (3 \text{ m}) = 90 \cdot \text{kN} \cdot \text{m}$$

Part (c)

$$n_{BC} = \frac{C - B}{|C - B|} = \begin{pmatrix} 0.442 \\ 0.147 \\ -0.885 \end{pmatrix} \quad M_{BC} = M_B \cdot n_{BC} = 19.9 \cdot \text{kN} \cdot \text{m}$$

$$\text{Vectors} \quad M_{BC\text{par}} = M_B \cdot n_{BC} \cdot n_{BC} = \begin{pmatrix} 8.8 \\ 2.93 \\ -17.61 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad M_{BC\text{per}} = M_B - M_{BC\text{par}} = \begin{pmatrix} 216.2 \\ -2.9 \\ 107.6 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$|M_{BC\text{par}}| = 19.9 \cdot \text{kN} \cdot \text{m}$$

Part (d)

$$r_{OA} = A \quad M_O = r_{OA} \times P = \begin{pmatrix} -150 \\ 180 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad n_{OB} = \frac{B}{|B|} = \begin{pmatrix} 0 \\ 0.447 \\ 0.894 \end{pmatrix}$$

$$M_{OB\text{par}} = M_O \cdot n_{OB} \cdot n_{OB} = \begin{pmatrix} 0 \\ 36 \\ 72 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad M_{OB\text{per}} = M_O - M_{OB\text{par}} = \begin{pmatrix} -150 \\ 144 \\ -72 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Problem 2.2-9

Part (a)

$$P = \begin{pmatrix} 25 \\ 50 \\ -75 \end{pmatrix} \text{ kN} \quad A = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} \text{ m} \quad D = \begin{pmatrix} x_D \\ y_D \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} 4.5 \\ 3 \\ 0 \end{pmatrix} \text{ m} \quad r_{OA} = A \quad r_{OD} = D \quad M_O = r_{OA} \times P = \begin{pmatrix} -300 \\ 150 \\ 0 \end{pmatrix} \cdot \text{kN}\cdot\text{m}$$

$$\text{Also} \quad r_{OD} \times P = M_O \quad r_{OD} \times P \rightarrow \begin{pmatrix} -75 \cdot \text{kN} \cdot y_D \\ 75 \cdot \text{kN} \cdot x_D \\ 50 \cdot \text{kN} \cdot x_D - 25 \cdot \text{kN} \cdot y_D \end{pmatrix} \quad \text{so} \quad \begin{aligned} x_D &= \frac{150 \text{ kN}\cdot\text{m}}{75 \text{ kN}} = 2 \text{ m} \\ y_D &= \frac{-300 \text{ kN}\cdot\text{m}}{-75 \text{ kN}} = 4 \text{ m} \end{aligned}$$

(or use a geometry solution)

$$50 \text{ kN} \cdot x_D - 25 \text{ kN} \cdot y_D = 0 \cdot \text{kN}\cdot\text{m}$$

Part (b)

$$P_A = P \quad P_O = -P \quad D = \begin{pmatrix} x_D \\ y_D \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} \cdot \text{m} \quad |M_O| = 335 \text{ kN}\cdot\text{m}$$

$$M_A = -r_{OA} \times P_O \quad |M_A| = 335 \text{ kN}\cdot\text{m}$$

$$r_{DO} = -D \quad M_D = r_{DO} \times P_O \quad |M_D| = 335 \text{ kN}\cdot\text{m}$$

$$r_{CO} = -C \quad r_{CD} = D - C \quad M_C = r_{CD} \times P_A + r_{CO} \times P_O \quad |M_C| = 335 \text{ kN}\cdot\text{m}$$

Part (c)

$$d = \frac{|M_O|}{|P|} = 3.59 \text{ m}$$

Part (d)

Varignon's theorem

$$M_{Ox} = -(50 \text{ kN}) \cdot (6 \text{ m}) = -300 \text{ kN}\cdot\text{m}$$

$$M_{Oy} = (25 \text{ kN}) \cdot (6 \text{ m}) = 150 \text{ kN}\cdot\text{m}$$

$$M_O = \begin{pmatrix} -300 \\ 150 \\ 0 \end{pmatrix} \cdot \text{kN}\cdot\text{m}$$

$$M_{Oz} = 0 \quad \text{since } P \text{ has line of action through } z \text{ axis}$$

Part (e)

$$P = \begin{pmatrix} 25 \\ 50 \\ -75 \end{pmatrix} \cdot \text{kN} \quad Q = 0.8 \cdot P = \begin{pmatrix} 20 \\ 40 \\ -60 \end{pmatrix} \cdot \text{kN} \quad |P| = 93.541 \cdot \text{kN} \quad |Q| = 74.833 \cdot \text{kN} \quad \frac{|Q|}{|P|} = 0.8$$

$$B = \begin{pmatrix} 0 \\ 0 \\ z_Q \end{pmatrix} \quad r_{OB} = B \quad r_{OB} \times Q = M_O \quad |M_O| = 335.41 \cdot \text{kN}\cdot\text{m}$$

$$\left| \begin{pmatrix} 0 \\ 0 \\ z_Q \end{pmatrix} \times \begin{pmatrix} 20 \\ 40 \\ -60 \end{pmatrix} \right| = 335.41 \quad \left| \begin{pmatrix} 0 \\ 0 \\ z_Q \end{pmatrix} \times \begin{pmatrix} 20 \\ 40 \\ -60 \end{pmatrix} \right| \rightarrow 20 \cdot \sqrt{5} \cdot |z_Q| = 335.41 \quad \frac{335.41}{20\sqrt{5}} = 7.5 \text{ m}$$

Problem 2.2-10

$$P_A = \begin{pmatrix} 30 \\ 25 \\ -75 \end{pmatrix} \text{ kN} \quad P_O = -P_A = \begin{pmatrix} -30 \\ -25 \\ 75 \end{pmatrix} \cdot \text{kN} \quad |P_A| = 84.558 \cdot \text{kN} \quad F_C = |P_A| \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -84.558 \\ 0 \\ 0 \end{pmatrix} \cdot \text{kN}$$

$$A = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} \text{ m} \quad C = \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix} \text{ m} \quad r_{OA} = A \quad r_{OC} = C$$

Resultants at O

$$R_O = P_A + P_O + F_C = \begin{pmatrix} -84.6 \\ 0 \\ 0 \end{pmatrix} \cdot \text{kN} \quad M_O = r_{OA} \times P_A + r_{OC} \times F_C = \begin{pmatrix} -150 \\ 180 \\ 338 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Wrench resultant - components of M_O (kN·m) along & perpend. to R_O

$$n_R = \frac{R_O}{|R_O|} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \quad M_{R\text{par}} = M_O \cdot n_R \cdot n_R = \begin{pmatrix} -150 \\ 0 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad M_{R\text{per}} = M_O - M_{R\text{par}} = \begin{pmatrix} 0 \\ 180 \\ 338 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

positive wrench resultant in (-x) direction

Coordinates (meters) of pt at which wrench pierces yz plane

$$\begin{pmatrix} 0 \\ y \\ z \end{pmatrix} \times R_O = M_{O\text{per}} \text{ float, 5} \rightarrow 0.0$$

$$y = \frac{338230.0 \cdot \text{N} \cdot \text{m}}{84558.0 \cdot \text{N}} = 4 \text{ m}$$

$$z = \frac{180000 \cdot \text{N} \cdot \text{m}}{-84558.0 \cdot \text{N}} = -2.13 \text{ m}$$

$$d = \frac{|M_{R\text{per}}|}{|R_O|} = 4.53 \text{ m} \quad < \text{perpend. dist. (m) from O to line of action of wrench}$$

$$\sqrt{z^2 + y^2} = 4.53 \text{ m} \quad < \text{same as d since wrench is perpendicular to yz plane}$$

Answers

$$R_O = \begin{pmatrix} -84.6 \\ 0 \\ 0 \end{pmatrix} \cdot \text{kN} \quad M_{R\text{par}} = \begin{pmatrix} -150 \\ 0 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad y = 4 \text{ m} \quad z = -2.13 \text{ m}$$

Problem 2.2-11

$$F = 1.5\text{kN} \quad r = 0.75\text{m} \quad h = 2.25\text{m} \quad A = \begin{pmatrix} -r \cdot \sin(60\text{deg}) \\ r \cdot \cos(60\text{deg}) \\ h \end{pmatrix} = \begin{pmatrix} -0.65 \\ 0.375 \\ 2.25 \end{pmatrix} \cdot \text{m} \quad B = \begin{pmatrix} 0 \\ -r \\ h \end{pmatrix} \quad C = \begin{pmatrix} 0 \\ r \\ h \end{pmatrix} \quad D = \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix}$$

$$n_{OA} = \frac{A}{|A|} = \begin{pmatrix} -0.274 \\ 0.158 \\ 0.949 \end{pmatrix}$$

Part (a)

$$F = F \cdot n_{OA} = \begin{pmatrix} -0.411 \\ 0.237 \\ 1.423 \end{pmatrix} \cdot \text{kN} \quad r_{BO} = -B \quad r_{BA} = A - B \quad F_x = F_1 \quad F_y = F_2 \quad F_z = F_3$$

$$M_B = r_{BO} \times F = \begin{pmatrix} 1.601 \\ 0.924 \\ 0.308 \end{pmatrix} \cdot \text{kN} \cdot \text{m} = \quad r_{BA} \times F = \begin{pmatrix} 1.601 \\ 0.924 \\ 0.308 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Part (b)

$$M_{Bx} = F_y \cdot h + F_z \cdot r = 1.601 \text{ kN} \cdot \text{m} \quad M_{By} = -F_x \cdot h = 0.924 \text{ kN} \cdot \text{m} \quad M_{Bz} = -F_x \cdot r = 0.308 \text{ kN} \cdot \text{m}$$

Part (c)

$$r_{DO} = -D \quad r_{DA} = A - D \quad M_D = r_{DO} \times F = \begin{pmatrix} -1.067 \\ 0 \\ -0.308 \end{pmatrix} \cdot \text{kN} \cdot \text{m} = \quad r_{DA} \times F = \begin{pmatrix} -1.067 \\ 0 \\ -0.308 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$n_{BD} = \frac{D - B}{|D - B|} = \begin{pmatrix} 0 \\ 0.555 \\ -0.832 \end{pmatrix} \quad M_{BD\text{par}} = M_D \cdot n_{BD} \cdot n_{BD} = \begin{pmatrix} 0 \\ 0.142 \\ -0.213 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$M_{BD\text{per}} = M_D - M_{BD\text{par}} = \begin{pmatrix} -1.067 \\ -0.142 \\ -0.095 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Problem 2.2-12

$$F = 520\text{N} \quad A = \begin{pmatrix} 0 \\ 9 \\ 4 \end{pmatrix} \text{m} \quad B = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \text{m} \quad C = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \text{m} \quad D = \begin{pmatrix} 5 \\ 9 \\ 0 \end{pmatrix} \text{m} \quad n_{CA} = \frac{A - C}{|A - C|} = \begin{pmatrix} -0.453 \\ 0.815 \\ 0.362 \end{pmatrix}$$

Part (a)

$$F = F \cdot n_{CA} = \begin{pmatrix} -235.39 \\ 423.71 \\ 188.31 \end{pmatrix} \text{N} \quad r_{BC} = C - B = \begin{pmatrix} 5 \\ 0 \\ -4 \end{pmatrix} \text{m} \quad r_{BA} = A - B = \begin{pmatrix} 0 \\ 9 \\ 0 \end{pmatrix} \text{m}$$

$$M_B = r_{BC} \times F = \begin{pmatrix} 1695 \\ 0 \\ 2119 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad \text{check} \quad r_{BA} \times F = \begin{pmatrix} 1695 \\ 0 \\ 2119 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Part (b)

$$F_x = -235.39\text{N} \quad F_y = 423.71\text{N} \quad F_z = 188.31\text{N}$$

$$M_{Bx} = F_y \cdot (4\text{m}) = 1695 \cdot \text{N} \cdot \text{m} \quad M_{By} = -F_z \cdot (5\text{m}) - F_x \cdot (4\text{m}) = 0 \cdot \text{N} \cdot \text{m} \quad M_{Bz} = F_y \cdot (5\text{m}) = 2119 \cdot \text{N} \cdot \text{m}$$

Part (c)

$$n_{BD} = \frac{D - B}{|D - B|} = \begin{pmatrix} 0.453 \\ 0.815 \\ -0.362 \end{pmatrix} \quad M_{BD\text{par}} = M_B \cdot n_{BD} \cdot n_{BD} = \begin{pmatrix} -0 \\ -0 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad M_{BD\text{per}} = M_B - M_{BD\text{par}} = \begin{pmatrix} 1695 \\ 0 \\ 2119 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Part (d)

$$r_{OC} = C \quad M_O = r_{OC} \times F = \begin{pmatrix} 0 \\ -942 \\ 2119 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

$$n_{OD} = \frac{D}{|D|} = \begin{pmatrix} 0.486 \\ 0.874 \\ 0 \end{pmatrix} \quad M_{OD\text{par}} = M_O \cdot n_{OD} \cdot n_{OD} = \begin{pmatrix} -400 \\ -720 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad M_{OD\text{per}} = M_O - M_{OD\text{par}} = \begin{pmatrix} 400 \\ -222 \\ 2119 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Problem 2.2-13

$$T_{AB} = 67\text{N}$$

Joint coordinates: $A = \begin{pmatrix} 200 \\ 500 - 500\cos(60\text{deg}) \\ -500\sin(60\text{deg}) \end{pmatrix} \cdot \text{mm} = \begin{pmatrix} 200 \\ 250 \\ -433.013 \end{pmatrix} \cdot \text{mm}$ $B = \begin{pmatrix} 150 \\ 0 \\ 0 \end{pmatrix} \cdot \text{mm}$ $C = \begin{pmatrix} 0 \\ 500 \\ 0 \end{pmatrix} \cdot \text{mm}$

Part (a) $n_{AB} = \frac{B - A}{|B - A|} = \begin{pmatrix} -0.1 \\ -0.498 \\ 0.862 \end{pmatrix}$ $T_{AB} = T_{AB} \cdot n_{AB} = \begin{pmatrix} -6.667 \\ -33.334 \\ 57.736 \end{pmatrix} \cdot \text{N}$ $r_{OB} = B$ $r_{OA} = A$

$$M_O = r_{OB} \times T_{AB} = \begin{pmatrix} 0 \\ -8.66 \\ -5 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad \text{or as a check} \quad r_{OA} \times T_{AB} = \begin{pmatrix} -0 \\ -8.66 \\ -5 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Part (b) $T_{ABx} = -6.66\text{N}$ $T_{ABy} = -33.33\text{N}$ $T_{ABz} = 57.73\text{N}$

$$M_{Ox} = 0 \quad M_{Oy} = -T_{ABz} \cdot (200\text{mm}) - T_{ABx} \cdot (500\text{mm} \sin(60\text{deg})) = -8.66 \text{ N} \cdot \text{m}$$

$$M_{Oz} = -T_{ABx} \cdot (500\text{mm} - 500\text{mm} \cos(60\text{deg})) + T_{ABy} \cdot (200\text{mm}) = -5 \cdot \text{N} \cdot \text{m}$$

Part (c)

$$r_{CA} = A - C = \begin{pmatrix} 200 \\ -250 \\ -433.013 \end{pmatrix} \cdot \text{mm} \quad M_C = r_{CA} \times T_{AB} = \begin{pmatrix} -28.9 \\ -8.7 \\ -8.3 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad n_{CD} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$M_{CD\text{par}} = M_C \cdot n_{CD} \cdot n_{CD} = \begin{pmatrix} -29 \\ 0 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad M_{CD\text{per}} = M_C - M_{CD\text{par}} = \begin{pmatrix} 0 \\ -8.7 \\ -8.3 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Problem 2.2-14

$$F_{DO} = 2.5\text{N} \quad T_{AE} = 4\text{N} \quad M_y = 5\text{N}\cdot\text{m} \quad A = \begin{pmatrix} 1.8 \\ 0 \\ 2.1 \end{pmatrix} \text{m} \quad E = \begin{pmatrix} 2.6 \\ 0 \\ 0 \end{pmatrix} \text{m} \quad r_{OA} = A$$

$$n_{AE} = \frac{E - A}{|E - A|} = \begin{pmatrix} 0.356 \\ 0 \\ -0.934 \end{pmatrix} \quad T_{AE} = T_{AE} \cdot n_{AE} = \begin{pmatrix} 1.424 \\ 0 \\ -3.738 \end{pmatrix} \text{N} \quad T_{AEx} = 1.424\text{N} \quad T_{AEy} = 0 \quad T_{AEz} = -3.738\text{N}$$

Part (a)

$$M_O = M_y \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + r_{OA} \times T_{AE} = \begin{pmatrix} 0 \\ 14.72 \\ 0 \end{pmatrix} \cdot \text{N}\cdot\text{m}$$

Part (b)

$$M_{Ox} = 0 \quad M_{Oy} = M_y + T_{AEx} \cdot (2.1\text{m}) - T_{AEz} \cdot (1.8\text{m}) = 14.72 \cdot \text{N}\cdot\text{m} \quad M_{Oz} = 0$$

Problem 2.2-15

Part (a)

$$P = 175\text{N} \quad F_C = P \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad R_O = F_C = \begin{pmatrix} 0 \\ 0 \\ -175 \end{pmatrix} \cdot \text{N} \quad C = \begin{pmatrix} 125\text{mm} \cos(40\text{deg}) \\ 31.25\text{mm} + 50\text{mm} \\ 125\text{mm} \sin(40\text{deg}) \end{pmatrix} = \begin{pmatrix} 95.756 \\ 81.25 \\ 80.348 \end{pmatrix} \cdot \text{mm}$$

$$r_{OC} = C \quad M_O = r_{OC} \times F_C = \begin{pmatrix} -14.2 \\ 16.8 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Part (b)

$$A = \begin{pmatrix} 0 \\ 31.25\text{mm} \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 125\text{mm} \cos(40\text{deg}) \\ 31.25\text{mm} \\ 125\text{mm} \sin(40\text{deg}) \end{pmatrix} = \begin{pmatrix} 95.756 \\ 31.25 \\ 80.348 \end{pmatrix} \cdot \text{mm} \quad n_{AB} = \frac{B - A}{|B - A|} = \begin{pmatrix} 0.766 \\ 0 \\ 0.643 \end{pmatrix}$$

$$r_{AC} = C - A$$

$$M_A = r_{AC} \times F_C = \begin{pmatrix} -8.8 \\ 16.8 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad M_{AB} = M_A \cdot n_{AB} \cdot n_{AB} = \begin{pmatrix} -5.1 \\ 0 \\ -4.3 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad M_A \cdot n_{AB} = -6.7 \cdot \text{N} \cdot \text{m}$$

$$|M_{AB}| = 6.7 \cdot \text{N} \cdot \text{m}$$

Problem 2.2-16

$$F_{DO} = 2.5\text{N} \quad T_{AB} = 4\text{N} \quad M_y = 5\text{N}\cdot\text{m}$$

$$A = \begin{pmatrix} 1.8 \\ 0 \\ 2.1 \end{pmatrix} \text{m} \quad B = \begin{pmatrix} 2.6 \\ 1.4 \\ 0 \end{pmatrix} \text{m} \quad C = \begin{pmatrix} 1.8 \\ 1.4 \\ 0 \end{pmatrix} \text{m} \quad D = \begin{pmatrix} 0 \\ 0 \\ 2.1 \end{pmatrix} \text{m} \quad r_{OA} = A$$

$$n_{AB} = \frac{B - A}{|B - A|} = \begin{pmatrix} 0.302 \\ 0.529 \\ -0.793 \end{pmatrix} \quad T_{AB} = T_{AB} \cdot n_{AB} = \begin{pmatrix} 1.209 \\ 2.115 \\ -3.173 \end{pmatrix} \text{N} \quad \text{so} \quad \begin{aligned} T_{ABx} &= 1.209\text{N} & T_{ABz} &= -3.173\text{N} \\ T_{ABz} &= -3.173\text{N} & T_{ABx} &= 1.209\text{N} \end{aligned}$$

Part (a)

$$M_O = M_y \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + r_{OA} \times T_{AB} = \begin{pmatrix} -4.44 \\ 13.25 \\ 3.81 \end{pmatrix} \cdot \text{N}\cdot\text{m}$$

Part (b)

$$M_{Ox} = -T_{ABz} \cdot (2.1\text{m}) = -4.44\text{N}\cdot\text{m} \quad M_{Oy} = M_y + T_{ABx} \cdot (2.1\text{m}) + (-T_{ABz}) \cdot (1.8\text{m}) = 13.25\text{N}\cdot\text{m}$$

$$M_{Oz} = T_{ABz} \cdot (1.8\text{m}) = 3.81\text{N}\cdot\text{m}$$

Part (c)

$$n_{OC} = \frac{C}{|C|} = \begin{pmatrix} 0.789 \\ 0.614 \\ 0 \end{pmatrix} \quad M_{OC\text{par}} = M_O \cdot n_{OC} \cdot n_{OC} = \begin{pmatrix} 3.65 \\ 2.84 \\ 0 \end{pmatrix} \cdot \text{N}\cdot\text{m} \quad M_{OC\text{per}} = M_O - M_{OC\text{par}} = \begin{pmatrix} -8.09 \\ 10.41 \\ 3.81 \end{pmatrix} \cdot \text{N}\cdot\text{m}$$

Part (d)

$$M_O = M_O - M_y \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -4.442 \\ 8.249 \\ 3.807 \end{pmatrix} \text{N}\cdot\text{m}$$

$$d = \frac{|M_O|}{|T_{AB}|} = 2.53\text{m}$$

Problem 2.2-17

$$F = 67\text{N} \quad A = \begin{pmatrix} 540 \\ 0 \\ 630 \end{pmatrix} \cdot \text{mm} \quad D = \begin{pmatrix} 0 \\ 0 \\ 630 \end{pmatrix} \cdot \text{mm} \quad d = 540\text{mm}$$

Varignon's theorem

$$M_{Ay} = -F \cdot d = -36.18 \text{ N} \cdot \text{m} \quad M_{Dy} = M_{Ay} \quad M_{Oy} = M_{Ay} \quad < \text{ same moment about any point for force couple}$$

Vector algebra

$$F_A = F \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad F_D = -F_A$$

$$r_{AD} = D - A = \begin{pmatrix} -0.54 \\ 0 \\ 0 \end{pmatrix} \cdot \text{m} \quad M_A = r_{AD} \times F_D = \begin{pmatrix} 0 \\ -36.18 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

$$r_{DA} = -r_{AD} = \begin{pmatrix} 0.54 \\ 0 \\ 0 \end{pmatrix} \cdot \text{m} \quad M_D = r_{DA} \times F_A = \begin{pmatrix} 0 \\ -36.18 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

$$r_{OA} = A \quad M_O = r_{OA} \times F_A = \begin{pmatrix} 0 \\ -36.18 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Problem 2.2-18

$$\mathbf{r}_{CB} = \begin{pmatrix} -3 \\ -8 \\ 3 \end{pmatrix} \cdot \text{m} \quad \mathbf{r}_{CA} = \begin{pmatrix} -3 \\ 0 \\ 9 \end{pmatrix} \cdot \text{m} \quad \mathbf{r}_{BA} = \begin{pmatrix} 0 \\ 8 \\ 6 \end{pmatrix} \cdot \text{m} \quad \mathbf{r}_{DC} = \begin{pmatrix} -12 \\ 8 \\ -9 \end{pmatrix} \cdot \text{m}$$

$$F = 500\text{N} \quad F_{BA} = F \cdot \frac{r_{BA}}{|r_{BA}|} = \begin{pmatrix} 0 \\ 400 \\ 300 \end{pmatrix} \cdot \text{N}$$

a) Find moment of F about point C

$$\mathbf{M}_C = \mathbf{r}_{CB} \times \mathbf{F}_{BA} = \begin{pmatrix} -3600 \\ 900 \\ -1200 \end{pmatrix} \cdot \text{N}\cdot\text{m} \quad \mathbf{r}_{CA} \times \mathbf{F}_{BA} = \begin{pmatrix} -3600 \\ 900 \\ -1200 \end{pmatrix} \cdot \text{N}\cdot\text{m} \quad < \text{can use any pt along line of action of F}$$

b) Find moment of F about line CD using a cross product

$$\mathbf{n}_{CD} = \frac{\mathbf{r}_{DC}}{|\mathbf{r}_{DC}|} = \begin{pmatrix} -0.706 \\ 0.471 \\ -0.529 \end{pmatrix}$$

$$\mathbf{M}_{CD} = \mathbf{M}_C \cdot \mathbf{n}_{CD} \cdot \mathbf{n}_{CD} = \begin{pmatrix} -2541 \\ 1694 \\ -1906 \end{pmatrix} \cdot \text{N}\cdot\text{m} \quad |\mathbf{M}_{CD}| = 3600 \cdot \text{N}\cdot\text{m}$$

c) Find the shortest distance d between component F_{per} (perpendicular to line CD) and line CD

$$\mathbf{F}_{\text{parCD}} = \mathbf{F}_{BA} \cdot \mathbf{n}_{CD} \cdot \mathbf{n}_{CD} = \begin{pmatrix} -20.761 \\ 13.841 \\ -15.571 \end{pmatrix} \cdot \text{N} \quad \mathbf{F}_{\text{perCD}} = \mathbf{F}_{BA} - \mathbf{F}_{\text{parCD}} = \begin{pmatrix} 20.761 \\ 386.159 \\ 315.571 \end{pmatrix} \cdot \text{N} \quad \mathbf{r}_{DC} \times \mathbf{F}_{\text{parCD}} = \begin{pmatrix} -0 \\ 0 \\ 0 \end{pmatrix} \cdot \text{N}\cdot\text{m}$$

$$d = \frac{|\mathbf{M}_{CD}|}{|\mathbf{F}_{\text{perCD}}|} = 7.21 \cdot \text{m} \quad |\mathbf{F}_{\text{perCD}}| \cdot d = 3600 \cdot \text{N}\cdot\text{m} \quad < \text{same as } |\mathbf{M}_{CD}|$$

^ component of F which is parallel to CD has no moment about CD

Problem 2.2-19

$$F = 90\text{N} \quad M = 5.7\text{N}\cdot\text{m} \quad \mathbf{r}_{DB} = \begin{pmatrix} 75 \\ -100 \\ 0 \end{pmatrix} \cdot \text{mm} \quad \mathbf{n}_{DB} = \frac{\mathbf{r}_{DB}}{|\mathbf{r}_{DB}|} = \begin{pmatrix} 0.6 \\ -0.8 \\ 0 \end{pmatrix} \quad \mathbf{F}_{DB} = F \cdot \mathbf{n}_{DB} = \begin{pmatrix} 54 \\ -72 \\ 0 \end{pmatrix} \cdot \text{N}$$

Find unit vector normal to plane ABC

$$\mathbf{r}_{BC} = \begin{pmatrix} 0 \\ 100 \\ 0 \end{pmatrix} \cdot \text{mm} \quad \mathbf{r}_{BA} = \begin{pmatrix} -75 \\ 0 \\ 125 \end{pmatrix} \cdot \text{mm} \quad \mathbf{r}_{AB} = -\mathbf{r}_{BA}$$

$$\mathbf{n}_{BCA} = \frac{\mathbf{r}_{BC} \times \mathbf{r}_{BA}}{|\mathbf{r}_{BC} \times \mathbf{r}_{BA}|} = \begin{pmatrix} 0.857 \\ 0 \\ 0.514 \end{pmatrix} \quad \mathbf{M} = M \cdot \mathbf{n}_{BCA} = \begin{pmatrix} 4.888 \\ 0 \\ 2.933 \end{pmatrix} \cdot \text{N}\cdot\text{m}$$

a) Find M_A due to both F and M

$$\mathbf{M}_A = \mathbf{M} + \mathbf{r}_{AB} \times \mathbf{F}_{DB} = \begin{pmatrix} -4.1 \\ -6.8 \\ -2.5 \end{pmatrix} \cdot \text{N}\cdot\text{m}$$

b) Find moment about line AC

$$\mathbf{r}_{AC} = \begin{pmatrix} 75 \\ 100 \\ -125 \end{pmatrix} \cdot \text{mm} \quad \mathbf{n}_{AC} = \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = \begin{pmatrix} 0.424 \\ 0.566 \\ -0.707 \end{pmatrix} \quad \mathbf{M}_{AC} = \mathbf{M}_A \cdot \mathbf{n}_{AC} \cdot \mathbf{n}_{AC} = \begin{pmatrix} -1.62 \\ -2.16 \\ 2.7 \end{pmatrix} \cdot \text{N}\cdot\text{m}$$

c) What is the perpendicular distance d (mm) from force F to line AC?

$$d = \frac{|(\mathbf{M}_A) \cdot \mathbf{n}_{AC} \cdot \mathbf{n}_{AC}|}{|F|} = 42.426 \text{ mm} \quad < \text{as expected distance is less than } 1/2 \text{ of diagonal dist. at base } (= 125/2 = 62.5)$$

Problem 2.2-20

Find unit vectors in directions of three couples Let $b = 1$

$$n_1 = \frac{\begin{pmatrix} 0 \\ \frac{3 \cdot b}{2} \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \cdot b \\ 0 \\ 3 \cdot b \end{pmatrix}}{\left| \begin{pmatrix} 0 \\ \frac{3 \cdot b}{2} \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \cdot b \\ 0 \\ 3 \cdot b \end{pmatrix} \right|} \rightarrow \begin{pmatrix} \frac{3 \cdot \sqrt{13}}{13} \\ 0 \\ \frac{2 \cdot \sqrt{13}}{13} \end{pmatrix} = \begin{pmatrix} 0.832 \\ 0 \\ 0.555 \end{pmatrix}$$

$$n_2 = \frac{\begin{pmatrix} -2 \cdot b \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \cdot b \\ \frac{-3 \cdot b}{2} \\ 3 \cdot b \end{pmatrix}}{\left| \begin{pmatrix} -2 \cdot b \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \cdot b \\ \frac{-3 \cdot b}{2} \\ 3 \cdot b \end{pmatrix} \right|} \rightarrow \begin{pmatrix} 0 \\ \frac{2 \cdot \sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} \end{pmatrix} = \begin{pmatrix} 0 \\ 0.894 \\ 0.447 \end{pmatrix}$$

$$n_3 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad \wedge \text{ by inspection}$$

$$M_1 = M_0 \cdot n_1 = \begin{pmatrix} 0.832 \\ 0 \\ 0.555 \end{pmatrix} M_0 \quad M_2 = M_0 \cdot n_2 = \begin{pmatrix} 0 \\ 0.894 \\ 0.447 \end{pmatrix} M_0 \quad M_3 = M_0 \cdot n_3 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} M_0$$

Single couple is sum of three vectors

$$M_{SE} = M_1 + M_2 + M_3 = \begin{pmatrix} 0.832 \\ 0.894 \\ 0.002 \end{pmatrix} M_0$$

Problem 2.2-21

(a) Moment of force $F_1 = 54 \text{ N}$ about E

$$\mathbf{r}_{EG} = \begin{pmatrix} 1.2 \\ 1.2 \\ -0.9 \end{pmatrix} \cdot \text{m} \quad \mathbf{F}_1 = 54 \text{ N} \cdot \frac{\begin{pmatrix} 0 \\ -1.2 \\ 0.9 \end{pmatrix}}{\left| \begin{pmatrix} 0 \\ -1.2 \\ 0.9 \end{pmatrix} \right|} = \begin{pmatrix} 0 \\ -43.2 \\ 32.4 \end{pmatrix} \cdot \text{N} \quad \mathbf{M}_{EF1} = \mathbf{r}_{EG} \times \mathbf{F}_1 = \begin{pmatrix} 0 \\ -38.88 \\ -51.84 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

(b) Moment of force $F_2 = 80 \text{ N}$ about E

$$\mathbf{F}_2 = 80 \text{ N} \cdot \frac{\begin{pmatrix} -1.2 \\ 0 \\ 0.9 \end{pmatrix}}{\left| \begin{pmatrix} -1.2 \\ 0 \\ 0.9 \end{pmatrix} \right|} = \begin{pmatrix} -64 \\ 0 \\ 48 \end{pmatrix} \cdot \text{N} \quad \mathbf{M}_{EF2} = \mathbf{r}_{EG} \times \mathbf{F}_2 = \begin{pmatrix} 57.6 \\ 0 \\ 76.8 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

(c) Moment of couple $C = 19 \text{ N} \cdot \text{m}$ about E

$$\mathbf{r}_{OG} = \begin{pmatrix} 1.2 \\ 1.2 \\ 0 \end{pmatrix} \cdot \text{m} \quad \mathbf{r}_{OB} = \begin{pmatrix} 0 \\ 1.2 \\ 0.9 \end{pmatrix} \cdot \text{m}$$

$$\mathbf{C} = 19 \text{ N} \cdot \text{m} \cdot \frac{\mathbf{r}_{OG} \times \mathbf{r}_{OB}}{|\mathbf{r}_{OG} \times \mathbf{r}_{OB}|} = \begin{pmatrix} 9.775 \\ -9.775 \\ 13.034 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad \mathbf{M}_{EC} = \mathbf{C} = \begin{pmatrix} 9.78 \\ -9.78 \\ 13.03 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

(d) Moment of resultant of F_1 , F_2 and C about line EG

resultant moment about pt. E

$$\mathbf{n}_{EG} = \frac{\mathbf{r}_{EG}}{|\mathbf{r}_{EG}|} = \begin{pmatrix} 0.625 \\ 0.625 \\ -0.469 \end{pmatrix} \quad \mathbf{r}_{EG} = \begin{pmatrix} 1.2 \\ 1.2 \\ -0.9 \end{pmatrix} \cdot \text{m}$$

$$\mathbf{M}_{ER} = \mathbf{M}_{EF1} + \mathbf{M}_{EF2} + \mathbf{M}_{EC} = \begin{pmatrix} 67.4 \\ -48.7 \\ 38 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad \mathbf{M}_{EG} = \mathbf{M}_{ER} \cdot \mathbf{n}_{EG} \cdot \mathbf{n}_{EG} = \begin{pmatrix} -3.81 \\ -3.81 \\ 2.86 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

(e) Moment of resultant of F_1 , F_2 and C about line EB

$$\mathbf{n}_{EB} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\mathbf{M}_{EB} = \mathbf{M}_{ER} \cdot \mathbf{n}_{EB} \cdot \mathbf{n}_{EB} = \begin{pmatrix} 0 \\ -48.7 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad < \text{y component of } \mathbf{M}_{ER} \text{ above}$$

$$\text{and} \quad (\mathbf{M}_{EF1} + \mathbf{C}) \cdot \mathbf{n}_{EB} \cdot \mathbf{n}_{EB} = \begin{pmatrix} 0 \\ -48.7 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad < F_2 \text{ passes through line EB so offers no contribution to } \mathbf{M}_{EB}$$

Problem 2.2-22

Part (a) Resultant force (N)

$$R = (50 + 75 + 95 + 65 + 70) \cdot N \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 355 \\ 0 \end{pmatrix} N$$

Part (b)

Moment (N.m) about pt. B: note that $r \times F = -(F \times r)$

$$M_B = - \left[\begin{pmatrix} 0 \\ 50 \\ 0 \end{pmatrix} N \times \begin{pmatrix} 0 \\ 0 \\ \frac{3}{2} \end{pmatrix} m + \begin{pmatrix} 0 \\ 75 \\ 0 \end{pmatrix} N \times \begin{pmatrix} 0 \\ \frac{6}{2} \\ 0 \end{pmatrix} m + \begin{pmatrix} 0 \\ 95 \\ 0 \end{pmatrix} N \times \begin{pmatrix} \frac{5}{2} \\ 6 \\ 0 \end{pmatrix} m + \begin{pmatrix} 0 \\ 65 \\ 0 \end{pmatrix} N \times \begin{pmatrix} 5 \\ 6 \\ \frac{4}{2} \end{pmatrix} m + \begin{pmatrix} 0 \\ 70 \\ 0 \end{pmatrix} N \times \begin{pmatrix} 5 \\ 6-2 \\ 4 \end{pmatrix} m \right]$$

$$M_B = \begin{pmatrix} -485 \\ 0 \\ 913 \end{pmatrix} \cdot N \cdot m \quad \text{< all forces are parallel to y-axis so moment has x and z components only, as expected}$$

Part (c) $|R| = 355 \text{ N}$ $|M_B| = 1033 \cdot N \cdot m$ $d = \frac{|M_B|}{|R|} = 2.91 \text{ m}$

$$\begin{pmatrix} x \\ 0 \\ z \end{pmatrix} \times R = M_B \quad \left| \begin{array}{l} \text{solve, x, z} \\ \text{float, 4} \end{array} \right. \rightarrow (2.57 \cdot m \quad 1.366 \cdot m) \quad \sqrt{2.57^2 + 1.366^2} = 2.9105$$

^ move resultant R (acting in +y-dir.) a dist. d from B so that it pierces (x, z) plane at coordinates x = 2.57m, z = 1.366m

Problem 2.2-23

Part (a)

$$R_{Ax} = -22.5\text{N} \quad R_{Ay} = -13.5\text{N} + 9\text{N} = -4.5\text{N} \quad R_{Az} = 0$$

$$M_{Ax} = 13.5\text{N} \cdot (100\text{mm}) = 1.35\text{N} \cdot \text{m} \quad M_{Ay} = -22.5\text{N} \cdot (100\text{mm}) = -2.25\text{N} \cdot \text{m}$$

$$M_{Az} = (9 - 13.5)\text{N} \cdot (25\text{mm}) = -0.113\text{N} \cdot \text{m}$$

Part (b)

$$\mathbf{r}_{AB} = \begin{pmatrix} 25 \\ 0 \\ 100 \end{pmatrix} \text{mm} \quad \mathbf{F}_B = \begin{pmatrix} -22.5 \\ -13.5 \\ 0 \end{pmatrix} \text{N} \quad \mathbf{R}_A = \mathbf{F}_B + \begin{pmatrix} 0 \\ 9\text{N} \\ 0 \end{pmatrix} = \begin{pmatrix} -22.5 \\ -4.5 \\ 0 \end{pmatrix} \cdot \text{N}$$

$$\mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F}_B + \begin{pmatrix} 0 \\ 0 \\ 9 \cdot 25 \end{pmatrix} \text{N} \cdot \text{mm} = \begin{pmatrix} 1.35 \\ -2.25 \\ -0.113 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Part (c)

$$\mathbf{n}_R = \frac{\mathbf{R}_A}{|\mathbf{R}_A|} = \begin{pmatrix} -0.981 \\ -0.196 \\ 0 \end{pmatrix} \quad \mathbf{M}_{R\text{par}} = \mathbf{M}_A \cdot \mathbf{n}_R \cdot \mathbf{n}_R = \begin{pmatrix} 0.865 \\ 0.173 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad \mathbf{M}_{R\text{per}} = \mathbf{M}_A - \mathbf{M}_{R\text{par}} = \begin{pmatrix} 0.485 \\ -2.423 \\ -0.113 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

$$\begin{pmatrix} 0 \\ y \\ z \end{pmatrix} \times \mathbf{R}_A = \mathbf{M}_{R\text{per}} \quad \begin{pmatrix} 0 \\ y \\ z \end{pmatrix} \times \mathbf{R}_A \rightarrow \begin{pmatrix} 4.5 \cdot \text{N} \cdot z \\ -22.5 \cdot \text{N} \cdot z \\ 22.5 \cdot \text{N} \cdot y \end{pmatrix}$$

$$y = \frac{-0.113\text{N} \cdot \text{m}}{22.5\text{N}} = -5.022\text{mm} \quad z = \frac{-2.423\text{N} \cdot \text{m}}{-22.5\text{N}} = 107.689\text{mm}$$

$$\begin{pmatrix} 0 \\ -5.022 \\ 107.689 \end{pmatrix} \text{mm} \times \mathbf{R}_A = \begin{pmatrix} 0.485 \\ -2.423 \\ -0.113 \end{pmatrix} \cdot \text{N} \cdot \text{m} = \mathbf{M}_{R\text{per}}$$

Problem 2.2-24

Position and unit vectors

$$\begin{aligned} \mathbf{r}_{AC} &= \begin{pmatrix} 3 \\ 7 \\ 0 \end{pmatrix} \text{m} & \mathbf{r}_{AB} &= \begin{pmatrix} 0 \\ 7 \\ -5 \end{pmatrix} \text{m} & \mathbf{r}_{CD} &= \begin{pmatrix} 0 \\ -7 \\ -5 \end{pmatrix} \text{m} & \mathbf{n}_{AC} &= \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = \begin{pmatrix} 0.394 \\ 0.919 \\ 0 \end{pmatrix} & \mathbf{n}_{AB} &= \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \begin{pmatrix} 0 \\ 0.814 \\ -0.581 \end{pmatrix} \\ & & & & & & \mathbf{n}_{CD} &= \frac{\mathbf{r}_{CD}}{|\mathbf{r}_{CD}|} = \begin{pmatrix} 0 \\ -0.814 \\ -0.581 \end{pmatrix} \end{aligned}$$

Resultant force (N) and unit vector along R

$$\mathbf{R} = 75\text{N} \cdot \mathbf{n}_{AC} + 50\text{N} \cdot \mathbf{n}_{AB} + 90\text{N} \cdot \mathbf{n}_{CD} = \begin{pmatrix} 29.5 \\ 36.4 \\ -81.4 \end{pmatrix} \text{N} \quad \mathbf{n}_R = \frac{\mathbf{R}}{|\mathbf{R}|} = \begin{pmatrix} 0.315 \\ 0.387 \\ -0.867 \end{pmatrix}$$

Moment (N.m) about pt. O

$$\mathbf{M}_O = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \text{m} \times (75\text{N} \cdot \mathbf{n}_{AC} + 50\text{N} \cdot \mathbf{n}_{AB}) + \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \text{m} \times (90\text{N} \cdot \mathbf{n}_{CD}) = \begin{pmatrix} -548.1 \\ 304.7 \\ -219.7 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Components of $\mathbf{M}_O$ along & perpend. to R

$$\begin{aligned} M_{R\text{par}} &= \mathbf{M}_O \cdot \mathbf{n}_R = \begin{pmatrix} 42.8 \\ 52.7 \\ -117.8 \end{pmatrix} \cdot \text{N} \cdot \text{m} & M_{R\text{per}} &= \mathbf{M}_O - M_{R\text{par}} \mathbf{n}_R = \begin{pmatrix} -590.9 \\ 252 \\ -101.9 \end{pmatrix} \cdot \text{N} \cdot \text{m} & |M_{R\text{per}}| &= 650.4 \cdot \text{N} \cdot \text{m} \end{aligned}$$

Coordinates (meters) of pt P where wrench pierces xy plane

$$\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \times \mathbf{R} = M_{R\text{per}} \text{float}, 5 \rightarrow \begin{pmatrix} -81.373 \cdot \text{N} \cdot y \\ 81.373 \cdot \text{N} \cdot x \\ 36.387 \cdot \text{N} \cdot x + -29.544 \cdot \text{N} \cdot y \end{pmatrix} = \begin{pmatrix} -590.9 \cdot \text{J} \\ 251.96 \cdot \text{J} \\ -101.87 \cdot \text{J} \end{pmatrix}$$

$$x = \frac{251.96 \text{N} \cdot \text{m}}{81.373 \text{N}} = 3.1 \text{m} \quad y = \frac{590.9 \text{N} \cdot \text{m}}{81.373 \text{N}} = 7.26 \text{m}$$

$$36.387 \text{N} \cdot x + -29.544 \text{N} \cdot y = -101.87 \cdot \text{N} \cdot \text{m}$$

$$d = \frac{|M_{R\text{per}}|}{|\mathbf{R}|} = 6.93 \text{m} \quad \text{< perpend. dist. (m) from O to line of action of wrench (note that R is not perpendicular to xy plane)}$$

Problem 2.2-25

$$P = \begin{pmatrix} 25 \\ 50 \\ -75 \end{pmatrix} \text{ kN} \quad P_A = P \quad P_O = -P \quad R_O = P_A + P_O = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \cdot \text{kN}$$

$$M_C = (275 \text{ kN} \cdot \text{m}) \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -275 \\ 0 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$M_O = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} \text{ m} \times P_A + M_C = \begin{pmatrix} -575 \\ 150 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Problem 2.2-26

$$R_A = \begin{pmatrix} -140 \\ 80 \\ -120 \end{pmatrix} \text{ N} \quad M_A = \begin{bmatrix} -80 \cdot (2) \\ 120 \cdot (5) - 140 \cdot (2) + 30 - 25 \\ 80 \cdot (5) + 140 \cdot (7) \end{bmatrix} = \begin{pmatrix} -160 \\ 325 \\ 1380 \end{pmatrix} \text{ N} \cdot \text{m} \quad n_R = \frac{R_A}{|R_A|} = \begin{pmatrix} -0.697 \\ 0.398 \\ -0.597 \end{pmatrix}$$

Wrench resultant (here, a negative wrench)

$$M_{R_{\text{par}}} = M_A \cdot n_R \cdot n_R = \begin{pmatrix} 406 \\ -232 \\ 348 \end{pmatrix} \quad M_{R_{\text{per}}} = M_A - M_{R_{\text{par}}} = \begin{pmatrix} -566 \\ 557 \\ 1032 \end{pmatrix}$$

$$d = \frac{|M_{R_{\text{per}}}|}{|R_A|} = 6.479 \text{ m} \quad \begin{pmatrix} 0 \\ y \\ z \end{pmatrix} \times R_A = M_{R_{\text{per}}} \text{ float}, 5 \rightarrow \begin{pmatrix} -120 \cdot y - 80 \cdot z \\ -140 \cdot z \\ 140 \cdot y \end{pmatrix} = \begin{pmatrix} -566.14 \\ 557.08 \\ 1031.9 \end{pmatrix}$$

$$y_P = \frac{1031.9}{140} = 7.37 \text{ m} \quad z_P = \frac{557.08}{-140} = -3.98 \text{ m}$$

$$-120 \cdot y_P - 80 \cdot z_P = -566.154$$

Problem 2.2-27

$$\mathbf{R}_O = (5) \cdot (775\text{N}) \cdot \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -3.875 \times 10^3 \\ 0 \end{pmatrix} \cdot \text{N} \quad \beta = 120\text{deg}$$

Use Varignon's theorem

$$M_{Ox} = 775\text{N} \cdot (1.2\text{m} + 3.6\text{m} + 6\text{m}) - 775\text{N} \cdot (3\text{m} \cdot \sin(\beta - 90\text{deg}) + 6.6\text{m} \cdot \sin(\beta - 90\text{deg})) = 4650\text{N} \cdot \text{m}$$

$$M_{Oy} = 0 \quad \text{since all forces are parallel to y axis}$$

$$M_{Oz} = -775\text{N} \cdot (3\text{m} \cdot \cos(\beta - 90\text{deg}) + 6.6\text{m} \cdot \cos(\beta - 90\text{deg})) = -6443\text{N} \cdot \text{m}$$

$$\mathbf{M}_O = \begin{pmatrix} M_{Ox} \\ M_{Oy} \\ M_{Oz} \end{pmatrix} = \begin{pmatrix} 4650 \\ 0 \\ -6443 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Problem 2.2-28

Joint coordinates (meters)

$$A = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} \quad E = \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix} \quad F = \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} \quad G = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} \quad D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad n_{DB} = \frac{B}{|B|} = \begin{pmatrix} 0.625 \\ 0.781 \\ 0 \end{pmatrix}$$

Resultants

$$R_D = \begin{pmatrix} 25 \\ 30 \\ -10 \end{pmatrix} + 15 \cdot n_{DB} = \begin{pmatrix} 34.4 \\ 41.7 \\ -10 \end{pmatrix} \text{ kN} \quad M_D = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + A \times \begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix} + E \times \begin{pmatrix} 25 \\ 0 \\ 0 \end{pmatrix} + G \times \begin{pmatrix} 0 \\ 30 \\ 0 \end{pmatrix} = \begin{pmatrix} -140 \\ 75 \\ 0 \end{pmatrix} \text{ kN}\cdot\text{m}$$

Moments about and perpendicular to line DF

$$n_{DF} = \frac{F}{|F|} = \begin{pmatrix} 0.566 \\ 0.707 \\ 0.424 \end{pmatrix} \quad M_{DF\text{par}} = M_D \cdot n_{DF} \cdot n_{DF} = \begin{pmatrix} -14.8 \\ -18.5 \\ -11.1 \end{pmatrix} \text{ kN}\cdot\text{m} \quad M_{DF\text{per}} = M_D - M_{DF\text{par}} = \begin{pmatrix} -125.2 \\ 93.5 \\ 11.1 \end{pmatrix} \text{ kN}\cdot\text{m}$$

Wrench resultant (have negative wrench here)

$$n_R = \frac{R_D}{|R_D|} = \begin{pmatrix} 0.625 \\ 0.759 \\ -0.182 \end{pmatrix} \quad M_{R\text{par}} = M_D \cdot n_R \cdot n_R = \begin{pmatrix} -19.15 \\ -23.24 \\ 5.57 \end{pmatrix} \text{ kN}\cdot\text{m} \quad M_{R\text{per}} = M_D - M_{R\text{par}} = \begin{pmatrix} -120.8 \\ 98.2 \\ -5.6 \end{pmatrix} \text{ kN}\cdot\text{m}$$

$$d = \frac{|M_{R\text{per}}|}{|R_D|} = 2.84 \text{ m} \quad \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \times R_D = M_{R\text{per}} \text{ float, 5} \rightarrow \begin{pmatrix} -10 \cdot y \\ 10 \cdot x \\ 41.713 \cdot x + -34.37 \cdot y \end{pmatrix} = \begin{pmatrix} -120.85 \\ 98.241 \\ -5.5717 \end{pmatrix}$$

$$x = 9.82 \text{ m} \quad y = 12.08 \text{ m}$$

$$41.713 \cdot x + -34.37 \cdot y = -5.568$$

Answers

$$R_D = \begin{pmatrix} 34.4 \\ 41.7 \\ -10 \end{pmatrix} \text{ kN} \quad M_D = \begin{pmatrix} -140 \\ 75 \\ 0 \end{pmatrix} \text{ kN}\cdot\text{m} \quad M_{DF\text{par}} = \begin{pmatrix} -14.8 \\ -18.5 \\ -11.1 \end{pmatrix} \text{ kN}\cdot\text{m}$$

$$M_{DF\text{per}} = \begin{pmatrix} -125.2 \\ 93.5 \\ 11.1 \end{pmatrix} \text{ kN}\cdot\text{m} \quad x = 9.82 \text{ m} \quad y = 12.08 \text{ m}$$

Problem 2.2-29

$$A = \begin{pmatrix} 32 \\ 0 \\ 0 \end{pmatrix} \text{ mm} \quad B = \begin{pmatrix} 32 \\ 190 \\ 0 \end{pmatrix} \text{ mm} \quad F = 112\text{N} \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -112 \end{pmatrix} \cdot \text{N} \quad N_A = 67\text{N} \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -67 \\ 0 \\ 0 \end{pmatrix} \cdot \text{N}$$

$$r_{OB} = B$$

Part (a)

$$R_O = F + N_A = \begin{pmatrix} -67 \\ 0 \\ -112 \end{pmatrix} \cdot \text{N} \quad M_O = r_{OB} \times F = \begin{pmatrix} -21.28 \\ 3.584 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Part (b)

$$n_R = \frac{R_O}{|R_O|} = \begin{pmatrix} -0.513 \\ 0 \\ -0.858 \end{pmatrix}$$

$$M_{Rpar} = M_O \cdot n_R \cdot n_R = \begin{pmatrix} -5.6 \\ 0 \\ -9.4 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad M_{Rper} = M_O - M_{Rpar} = \begin{pmatrix} -15.672 \\ 3.584 \\ 9.375 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

$$\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \times R_O = M_{Rper} \quad \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \times R_O \text{ float}, 5 \rightarrow \begin{pmatrix} -112.0\text{N} \cdot y \\ 112.0\text{N} \cdot x \\ 67.0\text{N} \cdot y \end{pmatrix}$$

$$x = \frac{3.584\text{N} \cdot \text{m}}{112.0\text{N}} = 32\text{mm}$$

$$y = \frac{-15.672\text{N} \cdot \text{m}}{-112\text{N}} = 139.929\text{mm}$$

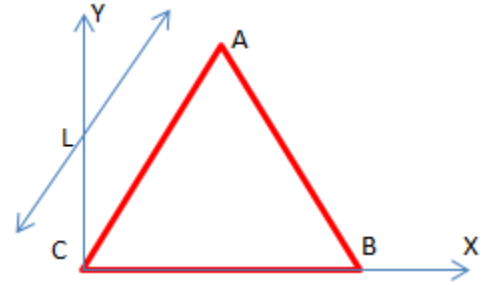
$$\frac{9.375\text{N} \cdot \text{m}}{67\text{N}} = 139.925\text{mm}$$

$$d = \frac{|M_{Rper}|}{|R_O|} = 142.6\text{mm}$$

Problem 2.2-30

$$F_1 = 150\text{N} \quad F_2 = F_1 \quad F_3 = 75\text{N}$$

ABC is an equilateral triangle



$$R = F_1 + F_2 + F_3 = 375\text{N} \quad R = R \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Joint coordinates (assume that $L = 1\text{m}$)

$$C = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad A = \begin{pmatrix} \cos(60^\circ) \\ \sin(60^\circ) \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.866 \\ 0 \end{pmatrix}$$

$$M_C = B \times \begin{pmatrix} 0 \\ 0 \\ F_2 \end{pmatrix} + A \times \begin{pmatrix} 0 \\ 0 \\ F_3 \end{pmatrix} \quad M_C \rightarrow \begin{pmatrix} 64.951905283832895 \cdot \text{N} \\ -37.50000000000000825 \cdot \text{N} - 150 \cdot \text{N} \\ 0 \end{pmatrix} = \begin{pmatrix} 64.952 \\ -187.5 \\ 0 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \times R = M_C \quad \left| \begin{array}{l} \text{solve, x, y} \\ \text{float, 4} \end{array} \right. \rightarrow (0.5 \quad 0.1732) \quad \text{so resultant R passes through point with coordinates}$$

$$x = \frac{L}{2} \quad y = \frac{\sqrt{3}}{10} \cdot L$$

Problem 2.2-31

Joint coordinates

$$A = \begin{pmatrix} 0 \\ 0 \\ 300 \end{pmatrix} \text{ mm} \quad B = \begin{pmatrix} 0 \\ 87.5 \\ 300 \end{pmatrix} \text{ mm} \quad C = \begin{pmatrix} 87.5 \\ 87.5 \\ 300 \end{pmatrix} \text{ mm} \quad r_{OA} = A \quad r_{OC} = C$$

Part (a)

$$R_{Ox} = -45\text{N} \quad R_{Oy} = (31.5 - 22.5)\text{N} = 9\text{N} \quad R_{Oz} = 0$$

$$M_{Ox} = -31.5\text{N} \cdot (150\text{mm}) + 22.5\text{N} \cdot (300\text{mm}) = 2.025\text{N} \cdot \text{m} \quad M_{Oy} = -45\text{N} \cdot (300\text{mm}) = -13.5\text{N} \cdot \text{m}$$

$$M_{Oz} = 1.6\text{N} \cdot \text{m} - 22.5\text{N} \cdot (87.5\text{mm}) = -0.369\text{N} \cdot \text{m}$$

Part (b)

$$R_O = \begin{pmatrix} -45 \\ 31.5 - 22.5 \\ 0 \end{pmatrix} \cdot \text{N} = \begin{pmatrix} -45 \\ 9 \\ 0 \end{pmatrix} \cdot \text{N}$$

$$M_O = \left[\begin{pmatrix} 0 \\ 0 \\ 150 \end{pmatrix} \text{ mm} \right] \times \left[\begin{pmatrix} 0 \\ 31.5 \\ 0 \end{pmatrix} \text{ N} \right] + r_{OA} \times \left[\begin{pmatrix} -45 \\ 0 \\ 0 \end{pmatrix} \text{ N} \right] + r_{OC} \times \begin{pmatrix} 0 \\ -22.5 \\ 0 \end{pmatrix} \text{ N} + \begin{pmatrix} 0 \\ 0 \\ 1.6 \end{pmatrix} \text{ N} \cdot \text{m} = \begin{pmatrix} 2.025 \\ -13.5 \\ -0.369 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Problem 2.2-32

$$\mathbf{R}_O = \begin{bmatrix} 0 \\ -2 \cdot (300\text{N}) \\ 2 \cdot (50\text{N}) - 2 \cdot (50\text{N}) \end{bmatrix} = \begin{pmatrix} 0 \\ -600 \\ 0 \end{pmatrix} \text{N}$$

$$M_{Ox} = 4 \cdot (50\text{N}) \cdot (1.5\text{m}) - 2 \cdot (800\text{N} \cdot \text{m}) = -1300 \cdot \text{N} \cdot \text{m} \quad M_{Oy} = 0 \quad M_{Oz} = 0 \quad \mathbf{M}_O = \begin{pmatrix} M_{Ox} \\ M_{Oy} \\ M_{Oz} \end{pmatrix} = \begin{pmatrix} -1300 \\ 0 \\ 0 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Problem 2.2-33

Joint coordinates

$$C = \begin{pmatrix} 3.6 \\ 1.2 \\ 1.2 \end{pmatrix} \text{m}$$

$$D = \begin{pmatrix} 0 \\ 0.9 \\ 0 \end{pmatrix} \text{m}$$

Unit vector

$$n_{CD} = \frac{D - C}{|D - C|} = \begin{pmatrix} -0.946 \\ -0.079 \\ -0.315 \end{pmatrix}$$

Force F

$$F_{CD} = (110 \text{ kN}) \cdot n_{CD} = \begin{pmatrix} -104.03 \\ -8.67 \\ -34.68 \end{pmatrix} \cdot \text{kN}$$

$$R_O = F_{CD} = \begin{pmatrix} -104.03 \\ -8.67 \\ -34.68 \end{pmatrix} \cdot \text{kN} \quad M_x = (11 \cdot \text{kN} \cdot \text{m}) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad r_{OD} = D$$

$$M_O = M_x + r_{OD} \times F_{CD} = \begin{pmatrix} -20.2 \\ 0 \\ 93.6 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Part (b) - select x and z components of M_O

Part (a)

$$M_{OApar} = M_O \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -20.2 \\ 0 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$M_{OAper} = M_O - M_{OApar} = \begin{pmatrix} 0 \\ 0 \\ 94 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Problem 2.2-34

Joint coordinates

$$F = \begin{pmatrix} 1.125 \\ 0 \\ 1.875 \end{pmatrix} \text{ m} \quad C = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \text{ m} \quad E = \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \text{ m} \quad A = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \text{ m} \quad B = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \text{ m} \quad D = \begin{pmatrix} 3 \\ 0 \\ 5 \end{pmatrix} \text{ m}$$

$$n_{CE} = \frac{E - C}{|E - C|} = \begin{pmatrix} 0.514 \\ 0 \\ 0.857 \end{pmatrix} \quad n_{FE} = \frac{E - F}{|E - F|} = \begin{pmatrix} 0.397 \\ 0.636 \\ 0.662 \end{pmatrix}$$

$$P = (3\text{kN}) \cdot n_{CE} = \begin{pmatrix} 1.543 \\ 0 \\ 2.572 \end{pmatrix} \cdot \text{kN} \quad Q = (4\text{kN}) \cdot n_{FE} = \begin{pmatrix} 1.589 \\ 2.542 \\ 2.648 \end{pmatrix} \cdot \text{kN}$$

Part (a)

$$R_O = P + Q = \begin{pmatrix} 3.13 \\ 2.54 \\ 5.22 \end{pmatrix} \cdot \text{kN} \quad r_{OE} = E \quad M_O = r_{OE} \times (P + Q) = \begin{pmatrix} 2.95 \\ 0 \\ -1.77 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Part (b)

$$r_{AE} = E - A = \begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix} \text{ m} \quad M_A = r_{AE} \times (P + Q) = \begin{pmatrix} 2.95 \\ 15.66 \\ -9.4 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$n_{AD} = \frac{D - A}{|D - A|} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad M_{AD\text{par}} = M_A \cdot n_{AD} \cdot n_{AD} = \begin{pmatrix} 0 \\ 0 \\ -9.4 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad M_{AD\text{per}} = M_A - M_{AD\text{par}} = \begin{pmatrix} 2.95 \\ 15.66 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Answers

$$\text{a) } R_O = \begin{pmatrix} 3.13 \\ 2.54 \\ 5.22 \end{pmatrix} \cdot \text{kN} \quad M_O = \begin{pmatrix} 2.95 \\ 0 \\ -1.77 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$\text{b) } M_{AD\text{par}} = \begin{pmatrix} 0 \\ 0 \\ -9.4 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad M_{AD\text{per}} = \begin{pmatrix} 2.95 \\ 15.66 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Problem 2.2-35

$$H = \begin{pmatrix} 0 \\ 0 \\ 75 \end{pmatrix} \cdot \text{mm} \quad B = \begin{pmatrix} 100 \\ 125 \\ 0 \end{pmatrix} \cdot \text{mm} \quad C = \begin{pmatrix} 100 \\ 0 \\ 0 \end{pmatrix} \cdot \text{mm} \quad G = \begin{pmatrix} 100 \\ 0 \\ 75 \end{pmatrix} \cdot \text{mm} \quad A = \begin{pmatrix} 0 \\ 125 \\ 0 \end{pmatrix} \cdot \text{mm} \quad P = 45 \text{ kN}$$

$$n_{HB} = \frac{B - H}{|B - H|} = \begin{pmatrix} 0.566 \\ 0.707 \\ -0.424 \end{pmatrix} \quad n_{DG} = \frac{G}{|G|} = \begin{pmatrix} 0.8 \\ 0 \\ 0.6 \end{pmatrix} \quad r_{DH} = H \quad r_{DB} = B$$

Part (a)

$$P = P \cdot n_{HB} = \begin{pmatrix} 25.456 \\ 31.82 \\ -19.092 \end{pmatrix} \cdot \text{kN} \quad R_D = P = \begin{pmatrix} 25.46 \\ 31.82 \\ -19.09 \end{pmatrix} \cdot \text{kN}$$

$$M_D = r_{DH} \times P = \begin{pmatrix} -2.39 \\ 1.91 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad < \text{no } z \text{ component since } P \text{ acts through } z \text{ axis} \quad \text{and} \quad r_{DB} \times P = \begin{pmatrix} -2.39 \\ 1.91 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Part (b)

$$M_{DG\text{par}} = M_D \cdot n_{DG} \cdot n_{DG} = \begin{pmatrix} -1.53 \\ 0 \\ -1.15 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad |M_{DG\text{par}}| = 1.91 \cdot \text{kN} \cdot \text{m}$$

$$M_{DG\text{per}} = M_D - M_{DG\text{par}} = \begin{pmatrix} -0.86 \\ 1.91 \\ 1.15 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Part (c)

$$r_{AB} = \begin{pmatrix} 100 \\ 0 \\ 0 \end{pmatrix} \cdot \text{mm} \quad M_A = r_{AB} \times P = \begin{pmatrix} 0 \\ 1.909 \\ 3.182 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad d = \frac{|M_A|}{|P|} = 82.46 \text{ mm}$$

Problem 2.2-36

$$T_{CE} = 50 \text{ kN} \quad a = 10 \text{ m} \quad b = 4 \text{ m} \quad c = 3 \text{ m}$$

Position and unit vectors

$$\mathbf{r}_{CE} = \begin{pmatrix} -a \\ 2 \\ c \\ -b \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \\ -4 \end{pmatrix} \cdot \text{m} \quad \mathbf{n}_{CE} = \frac{\mathbf{r}_{CE}}{|\mathbf{r}_{CE}|} = \begin{pmatrix} -0.707 \\ 0.424 \\ -0.566 \end{pmatrix} \quad \mathbf{r}_{CA} = \begin{pmatrix} 0 \\ 0 \\ 0 - b \end{pmatrix} \quad \mathbf{n}_{CA} = \frac{\mathbf{r}_{CA}}{|\mathbf{r}_{CA}|} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$\mathbf{T}_{CE} = T_{CE} \cdot \mathbf{n}_{CE} = \begin{pmatrix} -35.4 \\ 21.2 \\ -28.3 \end{pmatrix} \cdot \text{kN} \quad |T_{CE}| = 50 \cdot \text{kN}$$

Part (a)

$$\mathbf{R}_A = \mathbf{T}_{CE} = \begin{pmatrix} -35.4 \\ 21.2 \\ -28.3 \end{pmatrix} \cdot \text{kN} \quad \mathbf{r}_{AC} = \begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \quad \mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{T}_{CE} = \begin{pmatrix} -84.9 \\ -141.4 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Part (b)

$$\mathbf{D} = \begin{pmatrix} a \\ 2 \\ 0 \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix} \cdot \text{m} \quad \mathbf{n}_{AD} = \frac{\mathbf{D}}{|\mathbf{D}|} = \begin{pmatrix} 0.781 \\ 0 \\ 0.625 \end{pmatrix}$$

$$\mathbf{M}_{AD\text{par}} = \mathbf{M}_A \cdot \mathbf{n}_{AD} \cdot \mathbf{n}_{AD} = \begin{pmatrix} -51.7 \\ 0 \\ -41.4 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad \mathbf{M}_{AD\text{per}} = \mathbf{M}_A - \mathbf{M}_{AD\text{par}} = \begin{pmatrix} -33.1 \\ -141.4 \\ 41.4 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$|M_{AD\text{par}}| = 66.3 \cdot \text{kN} \cdot \text{m}$$

Answers

$$\text{a) } \mathbf{R}_A = \begin{pmatrix} -35.4 \\ 21.2 \\ -28.3 \end{pmatrix} \cdot \text{kN} \quad \mathbf{M}_A = \begin{pmatrix} -84.9 \\ -141.4 \\ 0 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$\text{b) } \mathbf{M}_{AD\text{par}} = \begin{pmatrix} -51.7 \\ 0 \\ -41.4 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad \mathbf{M}_{AD\text{per}} = \begin{pmatrix} -33.1 \\ -141.4 \\ 41.4 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Problem 2.2-37

Solution

$$E = \begin{pmatrix} 1.2 \\ 0 \\ 0 \end{pmatrix} \text{ m} \quad F = \begin{pmatrix} 1.2 \\ 0 \\ 1.2 \end{pmatrix} \text{ m} \quad D = \begin{pmatrix} 1.2 \\ 0.6 \\ 0.6 \end{pmatrix} \text{ m} \quad C = \begin{pmatrix} 0.3 \\ 1.2 \\ 1.2 \end{pmatrix} \text{ m} \quad B = \begin{pmatrix} 0.3 \\ 1.2 \\ 0 \end{pmatrix} \text{ m} \quad A = \begin{pmatrix} 0.3 \\ 0 \\ 0.6 \end{pmatrix} \text{ m} \quad G = \begin{pmatrix} 0.3 \\ 0.6 \\ 0.6 \end{pmatrix} \text{ m}$$

$$n_{DB} = \frac{B - D}{|B - D|} = \begin{pmatrix} -0.728 \\ 0.485 \\ -0.485 \end{pmatrix} \quad n_{DC} = \frac{C - D}{|C - D|} = \begin{pmatrix} -0.728 \\ 0.485 \\ 0.485 \end{pmatrix} \quad n_{DE} = \frac{E - D}{|E - D|} = \begin{pmatrix} 0 \\ -0.707 \\ -0.707 \end{pmatrix}$$

$$n_{DF} = \frac{F - D}{|F - D|} = \begin{pmatrix} 0 \\ -0.707 \\ 0.707 \end{pmatrix} \quad n_{AG} = \frac{G - A}{|G - A|} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$T_{DB} = 45 \text{ kN} \quad T_{DC} = 45 \text{ kN} \quad T_{DE} = 67 \text{ kN} \quad T_{DF} = 67 \text{ kN}$$

Part (a)

$$T_{DB} = T_{DB} \cdot n_{DB} = \begin{pmatrix} -32.74 \\ 21.83 \\ -21.83 \end{pmatrix} \cdot \text{kN} \quad T_{DC} = T_{DC} \cdot n_{DC} = \begin{pmatrix} -32.74 \\ 21.83 \\ 21.83 \end{pmatrix} \cdot \text{kN}$$

$$T_{DE} = T_{DE} \cdot n_{DE} = \begin{pmatrix} 0 \\ -47.38 \\ -47.38 \end{pmatrix} \cdot \text{kN} \quad T_{DF} = T_{DF} \cdot n_{DF} = \begin{pmatrix} 0 \\ -47.38 \\ 47.38 \end{pmatrix} \cdot \text{kN}$$

$$R_A = T_{DB} + T_{DC} + T_{DE} + T_{DF} = \begin{pmatrix} -65.48 \\ -51.1 \\ 0 \end{pmatrix} \cdot \text{kN} \quad |R_A| = 83.1 \text{ kN}$$

$$r_{AD} = D - A = \begin{pmatrix} 0.9 \\ 0.6 \\ 0 \end{pmatrix} \text{ m} \quad M_A = r_{AD} \times R_A = \begin{pmatrix} 0 \\ 0 \\ -6.7 \end{pmatrix} \cdot \text{kN}\cdot\text{m} \quad |M_A| = 6.7 \text{ kN}\cdot\text{m}$$

Part (b)

$$M_A \cdot n_{AG} = 0 \cdot \text{kN}\cdot\text{m} \quad T_{DB} = T_{DC} \text{ and } T_{DE} = T_{DF} \text{ so **symmetry** of loading (with respect to a vertical plane through ADG) means there is no moment component about AG axis}$$

Problem 2.2-38

$$F_s = 154\text{N} \quad h = 660\text{mm} \quad d = 150\text{mm} \quad H = 1041\text{mm} \quad c = 506\text{mm} \quad a = 760\text{mm} \quad h_o = 490\text{mm} \quad b = 254\text{mm}$$

Joint coordinates

$$\text{mass} = 20\text{kg} \quad g = 9.807 \frac{\text{m}}{\text{s}^2}$$

$$A = \begin{pmatrix} h \\ h - H \\ d \end{pmatrix} = \begin{pmatrix} 660 \\ -381 \\ 150 \end{pmatrix} \cdot \text{mm} \quad B = \begin{pmatrix} h \\ h \\ c \end{pmatrix} = \begin{pmatrix} 660 \\ 660 \\ 506 \end{pmatrix} \cdot \text{mm} \quad C = \begin{pmatrix} 0 \\ 0 \\ a + b + c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1520 \end{pmatrix} \cdot \text{mm} \quad D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{mm} \quad O = \begin{pmatrix} h_o \\ \text{unknown} \\ b + c \end{pmatrix}$$

Unit vectors

$$n_{AB} = \frac{B - A}{|B - A|} = \begin{pmatrix} 0 \\ 0.946 \\ 0.324 \end{pmatrix} \quad n_{CD} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad n_{AC} = \frac{C - A}{|C - A|} = \begin{pmatrix} -0.421 \\ 0.243 \\ 0.874 \end{pmatrix}$$

Part (a)

$$W = \text{mass} \cdot g \cdot \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -196.13 \\ 0 \end{pmatrix} \text{N} \quad F_s = F_s \cdot n_{AB} = \begin{pmatrix} 0 \\ 145.71 \\ 49.83 \end{pmatrix} \text{N} \quad R_A = W + F_s = \begin{pmatrix} 0 \\ -50.4 \\ 49.8 \end{pmatrix} \text{N} \quad |R_A| = 70.9 \text{N}$$

$$M_{Ax} = \text{mass} \cdot g \cdot (b + c - d) = 119.6 \cdot \text{N} \cdot \text{m} \quad M_{Ay} = 0 \quad M_{Az} = -\text{mass} \cdot g \cdot (h - h_o) = -33.3 \cdot \text{N} \cdot \text{m}$$

$$M_A = \begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = \begin{pmatrix} 119.6 \\ 0 \\ -33.3 \end{pmatrix} \cdot \text{N} \cdot \text{m}$$

Part (b)

$$r_{CA} = A - C = \begin{pmatrix} 660 \\ -381 \\ -1370 \end{pmatrix} \cdot \text{mm} \quad M_C = r_{CA} \times F_s + \begin{bmatrix} -\text{mass} \cdot g \cdot (a) \\ 0 \\ -\text{mass} \cdot g \cdot (h_o) \end{bmatrix} = \begin{pmatrix} 31.58 \\ -32.89 \\ 0.07 \end{pmatrix} \cdot \text{N} \cdot \text{m} \quad n_{CD} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$M_{CD} = M_C \cdot n_{CD} = -0.067 \cdot \text{N} \cdot \text{m}$$

Problem 2.2-39

$$P = 20\text{kN} \quad W = 450\text{kg} \cdot g = 4.413\text{kN}$$

$$D = \begin{pmatrix} 0 \\ 9 \\ 9 \cdot \tan(55^\circ) \end{pmatrix} \cdot \text{m} = \begin{pmatrix} 0 \\ 9 \\ 12.853 \end{pmatrix} \text{m} \quad E = \begin{pmatrix} 3 \\ 13 \\ 0 \end{pmatrix} \text{m} \quad C = \begin{pmatrix} 0 \\ 5 \\ 5 \cdot \tan(55^\circ) \end{pmatrix} \text{m} = \begin{pmatrix} 0 \\ 5 \\ 7.141 \end{pmatrix} \text{m} \quad r_{OC} = C$$

$$n_{DE} = \frac{E - D}{|E - D|} = \begin{pmatrix} 0.218 \\ 0.29 \\ -0.932 \end{pmatrix} \quad P = P \cdot n_{DE} = \begin{pmatrix} 4 \\ 6 \\ -19 \end{pmatrix} \cdot \text{kN} \quad W = W \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -4.413 \end{pmatrix} \cdot \text{kN} \quad r_{OE} = E$$

Resultants

$$R_O = P + W = \begin{pmatrix} 4.35 \\ 5.8 \\ -23.05 \end{pmatrix} \cdot \text{kN} \quad M_O = r_{OC} \times W + r_{OE} \times P = \begin{pmatrix} -264.38 \\ 55.92 \\ -39.15 \end{pmatrix} \cdot \text{kN} \cdot \text{m} \quad M_O = \begin{pmatrix} -264.4 \\ 55.9 \\ -39.2 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$|R_O| = 24\text{kN} \quad |M_O| = 273\text{kN} \cdot \text{m}$$