

***Calculus Early Transcendentals 12th edition Anton
Test Bank***

SKU: 9781119777984-TEST-BANK

1. Find the average rate of change of y with respect to x for $y = f(x) = \frac{4}{x^2}$ over the interval $[1, 8]$.

A) 0.438 B) -0.563 C) -0.580 D) -27.562 E) 3.938

Ans: B

Difficulty: Easy

Section: 2.1

2. Find the average rate of change of y with respect to x over the interval $[2, 7]$. $y = f(x) = 2x^3$

A) 74 B) 140 C) 134 D) 96 E) 335

Ans: C

Difficulty: Easy

Section: 2.1

3. Find the instantaneous rate of change of $y = 3x^2$ with respect to x at $x_0 = 5$.

A) 6 B) 30 C) 10 D) 15 E) 16

Ans: B

Difficulty: Easy

Section: 2.1

4. Find the instantaneous rate of change of $y = \frac{7}{x}$ with respect to x at $x_0 = 5$.

A) -175 B) -6.9600 C) 0.2800 D) -0.2800 E) -0.1020

Ans: D

Difficulty: Medium

Section: 2.1

5. Find the instantaneous rate of change of $y = 11x^5$ with respect to x at a general point x_0 .

A) $55x_0^5$ B) $11x_0$ C) $11x_0^5$ D) $11x_0^4$ E) $55x_0^4$

Ans: E

Difficulty: Easy

Section: 2.1

6. Find the instantaneous rate of change of $y = \frac{-6}{x^3}$ with respect to x at a general point x_0 .

A) $\frac{18}{x_0^3}$ B) $\frac{-6}{x_0^4}$ C) $\frac{18}{x_0^4}$ D) $\frac{-18}{x_0^4}$ E) $\frac{-18}{x_0^3}$

Ans: C

Difficulty: Medium

Section: 2.1

7. Find the slope of the tangent line to the graph of $f(x) = 5x^5 - 7$ at a general point x_0 .
 A) $25x_0^4 - 7$ B) $5x_0^4$ C) $25x_0^4$ D) $5x_0^4 - 1$ E) $5x_0^4 - 7$

Ans: C

Difficulty: Easy

Section: 2.1

8. Answer true or false. The slope of the tangent line to the graph of $f(x) = 2x^2 - 1$ at $x_0 = 5$ is 19.

Ans: False

Difficulty: Easy

Section: 2.1

9. Answer true or false. Use a graphing utility to graph $y = 4t^3$ on $[0, 3]$. If this graph represents a position versus time curve for a particle, the instantaneous velocity of the particle is increasing over the graphed domain.

Ans: True

Difficulty: Easy

Section: 2.1

10. Use a graphing utility to graph $y = t^2 - 5t + 4$ on $[0, 10]$. If this graph represents a position versus time curve for a particle, the instantaneous velocity of the particle is zero at what time? Assume time is in seconds.

A) 4.5s B) 3s C) 2.5s D) 1.5s E) 5s

Ans: C

Difficulty: Medium

Section: 2.1

11. A rock is dropped from a height of 828.1 meters and falls toward earth in a straight line. In t seconds the rock drops a distance of $4.9t^2$ meters. What is the instantaneous velocity downward when it hits the ground?

A) 3,360,173.089 meters/s D) 9.8 meters/s
 B) 127.4 meters/s E) 26 meters/s
 C) 63.7 meters/s

Ans: B

Difficulty: Easy

Section: 2.1

12. Answer true or false. The magnitude of the instantaneous velocity is always less than the magnitude of the average velocity.

Ans: False

Difficulty: Easy

Section: 2.1

13. Answer true or false. If a rock is thrown straight upward to a height of 25 feet from the ground, when it returns to earth its average velocity will be its initial velocity.

Ans: False

Difficulty: Easy

Section: 2.1

14. Answer true or false. If an object is thrown straight upward with an instantaneous velocity of 20 m/s, its instantaneous velocity at the point where it stops rising is 0.

Ans: True

Difficulty: Easy

Section: 2.1

15. An object moves in a straight line so that after t s its distance in mm from its original position is given by $s = 3t^3 + 4t$. Its instantaneous velocity at $t = 3$ s is

A) 81 mm B) 247 mm C) 733 mm D) 85 mm E) 31 mm

Ans: D

Difficulty: Medium

Section: 2.1

16. Find the instantaneous rate of change of y with respect to x at $x_0 = 4$. $y = 4x^2 - 8$

A) 32 B) 24 C) 16 D) 40 E) 64

Ans: A

Difficulty: Easy

Section: 2.1

17. Find the instantaneous rate of change of y with respect to x at $x_0 = 49$. $y = \sqrt{x} - 6$

A) $\frac{1}{14}$ B) $\frac{1}{7}$ C) $\frac{11}{7}$ D) $\frac{14}{17}$ E) $\frac{1}{49}$

Ans: A

Difficulty: Hard

Section: 2.1

18. Let $f(x) = \frac{1}{x^2}$. Find the average rate of change of y with respect to x over the interval $[3, 5]$.

Ans: $-\frac{8}{225}$

Difficulty: Easy

Section: 2.1

19. Let $f(x) = \frac{1}{x^2}$. Find the instantaneous rate of change of y with respect to x at the point $x = 5$.

Ans: $-\frac{2}{125}$

Difficulty: Easy

Section: 2.1

20. Let $y = x^2 + 9$. Find the average rate of change of y with respect to x over the interval $[-3, -2]$.

Ans: -5

Difficulty: Easy

Section: 2.1

21. Let $y = x^2 + 12$. Find the instantaneous rate of change of y with respect to x at the point $x = -1$.

Ans: -2

Difficulty: Easy

Section: 2.1

22. Let $y = \frac{1}{x-2}$. Find the average rate of change of y with respect to x over the interval $[4, 7]$.

Ans: $-\frac{1}{10}$

Difficulty: Medium

Section: 2.1

23. Let $y = \frac{1}{x-3}$. Find the instantaneous rate of change of y with respect to x at the point $x = 7$.

Ans: $-\frac{1}{16}$

Difficulty: Medium

Section: 2.1

24. Let $y = \frac{1}{x+4}$. Find the average rate of change of y with respect to x over the given interval $[3,4]$.

Ans: $-\frac{1}{56}$

Difficulty: Medium

Section: 2.1

25. Let $y = \frac{4}{x+4}$. Find the instantaneous rate of change of y with respect to x at the point $x = 2$.

Ans: $-\frac{1}{9}$

Difficulty: Medium

Section: 2.1

26. Let $f(x) = \frac{2}{4-x}$. Find the slope of the tangent to the graph of f at a general point x_0 using limits and find the slope of the tangent line at $x_0 = 2$

Ans: $\lim_{x_1 \rightarrow x_0} \frac{2}{(4-x_1)(4-x_0)} = \frac{2}{(4-x_0)^2}$

The slope of the tangent line at $x_0 = 2$ is $\frac{1}{2}$.

Difficulty: Medium

Section: 2.1

27. Let $f(x) = \frac{1}{x-2}$. Find the slope of the tangent to the graph of f at a general point x_0 using limits and find the slope of the tangent at $x_0 = 5$.

Ans: $\lim_{x_1 \rightarrow x_0} \frac{-1}{(x_1-2)(x_0-2)} = \frac{-1}{(x_0-2)^2}$

The slope of the tangent line at $x_0 = 5$ is $-\frac{1}{9}$.

Difficulty: Medium

Section: 2.1

28. Let $f(x) = \frac{4}{x^6}$. Find the slope of the tangent to the graph of f at a general point x_0 using limits and find the slope of the tangent at $x_0 = -3$.

$$\text{Ans: } \lim_{x_1 \rightarrow -3} \frac{\frac{4}{x_1^6} - \frac{4}{-3^6}}{x_1 + 3} = \lim_{x_1 \rightarrow -3} \frac{4(-3^6 - x_1^6)}{729x_1^6(x_1 + 3)} = -\frac{24}{x_0^7}$$

The slope of the tangent line at $x_0 = -3$ is $-\frac{8}{729}$.

Difficulty: Medium

Section: 2.1

29. Let $f(x) = 3x^2$. Find the slope of the tangent to the graph of f at a general point x_0 using limits and find the slope of the tangent at $x_0 = 3$.

$$\text{Ans: } \lim_{x_1 \rightarrow x_0} 3(x_1 + x_0) = 6x_0$$

Slope of tangent at $x_0 = 3$ is 18

Difficulty: Easy

Section: 2.1

30. A rock is dropped from a height of 400 feet and falls toward the earth in a straight line. In t seconds, the rock drops a distance of $s = 16t^2$ feet. What is the average velocity of the rock while it is falling? Use limits to find the instantaneous velocity of the rock when it hits the ground.

Ans: Average velocity: 80 feet per second

Instantaneous velocity at ground = 160 feet per second

Difficulty: Medium

Section: 2.1

31. A particle moves in a straight line from its initial position so that after t seconds, its distance is given by $s = t^2 + t$ feet from its initial position. Find the average velocity of the particle over the interval $[3,5]$ seconds. Use limits to find the instantaneous velocity of the particle at $t = 1$ second.

Ans: Average velocity = 9 feet per second

The instantaneous velocity at $t = 1$ second is 3 feet per second.

Difficulty: Medium

Section: 2.1

32. A particle moves in a straight line from its initial position so that after t seconds, its

distance is given by $s = \frac{t}{t+2}$ feet from its initial position. Find the average velocity of the particle over the interval $[1,5]$ seconds. Use limits to find the instantaneous velocity of the particle at $t = 1$ seconds.

Ans: Average velocity = $\frac{2}{21}$ feet per second.

The instantaneous velocity at $t = 1$ seconds is $\frac{2}{9}$ feet per second.

Difficulty: Medium

Section: 2.1

33. Let $f(x) = ax^2 + b$, where a and b are constant. Use the method of Section 3.1 to show that the slope of the tangent to the graph of f at $x = x_0$ is $2ax_0$.

$$m_{\tan} = \lim_{x_1 \rightarrow x_0} \frac{(ax_1^2 + b) - (ax_0^2 + b)}{x_1 - x_0} = \lim_{x_1 \rightarrow x_0} \frac{a(x_1^2 - x_0^2)}{x_1 - x_0} = \lim_{x_1 \rightarrow x_0} a(x_1 + x_0) = 2ax_0$$

Ans:

Difficulty: Hard

Section: 2.1

34. Let $f(x) = ax^3 + b$, where a and b are constants. Use the method of Section 3.1 to show

that the slope of the tangent to the graph of f at $x = x_0$ is $3ax_0^2$.

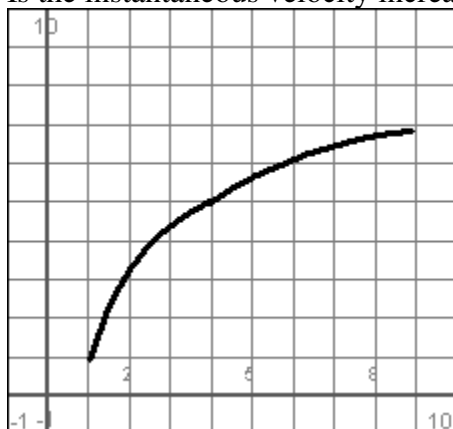
Ans:

$$m_{\tan} = \lim_{x_1 \rightarrow x_0} \frac{(ax_1^3 + b) - (ax_0^3 + b)}{x_1 - x_0} = \lim_{x_1 \rightarrow x_0} \frac{ax_1^3 - ax_0^3}{x_1 - x_0} = \lim_{x_1 \rightarrow x_0} a(x_1^2 + x_1x_0 + x_0^2) = 3ax_0^2$$

Difficulty: Medium

Section: 2.1

35. The graph shows the position versus time curve for a particle moving on a straight line. Is the instantaneous velocity increasing or decreasing with time?

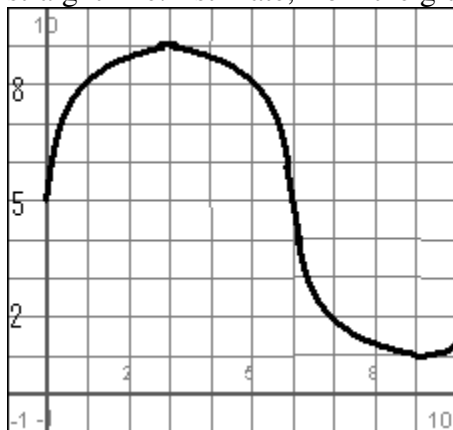


Ans: increasing

Difficulty: Easy

Section: 2.1

36. The figure shows the position versus time curve for a certain particle moving along a straight line. Estimate, from the graph, the instantaneous velocity at 8.



Ans: 1/3

Difficulty: Easy

Section: 2.1

37. Given $f(x) = x^4 - 3$, find the slope of the graph of f at the x -value $x_0 = 3$.

Ans: 108

Difficulty: Medium

Section: 2.1

38. Given $f(x) = 11\sqrt[4]{x}$, find the slope of the graph of f at $x_0 = 16$.

Ans: $\frac{11}{32}$

Difficulty: Medium

Section: 2.1

39. Find the instantaneous rate of change of $f(x) = \frac{11}{x^5}$ at $x_0 = 2$.

Ans: $-\frac{55}{64}$

Difficulty: Medium

Section: 2.1

40. Find the instantaneous rate of change of $f(x) = -2x^4 - 11$ at $x_0 = 2$.

Ans: -64

Difficulty: Medium

Section: 2.1

41. Find the instantaneous rate of change of $f(x) = -2x^2 - 3x + 11$ at $x_0 = 4$.

Ans: -19

Difficulty: Medium

Section: 2.1

1. Find the equation of the tangent line to $y = f(x) = 4x$ at $x = 5$.

- A) $y = 4x - 5$ B) $y = 4x$ C) $y = -4x$ D) $y = 4x + 5$ E) $y = 4x - 20$

Ans: B

Difficulty: Easy

Section: 2.2

2. Find the equation of the tangent line to $y = f(x) = \sqrt[3]{x} + 4$ at $x = 8$.

A) $y = \frac{x}{12} + \frac{64}{4}$

D) $y = \frac{x}{12} + \frac{4}{3}$

B) $y = \frac{x}{12} + \frac{16}{3}$

E) $y = \frac{x}{3} + \frac{64}{12}$

C) $y = \frac{x}{3} + \frac{64}{3}$

Ans: B

Difficulty: Hard

Section: 2.2

3. Given $y = 11\sqrt{x}$, find $\frac{dy}{dx}$.

A) $\frac{11\sqrt{x}}{x}$

B) $\frac{\sqrt{x}}{2x}$

C) $\frac{11}{2x}$

D) $\frac{11\sqrt{x}}{2x}$

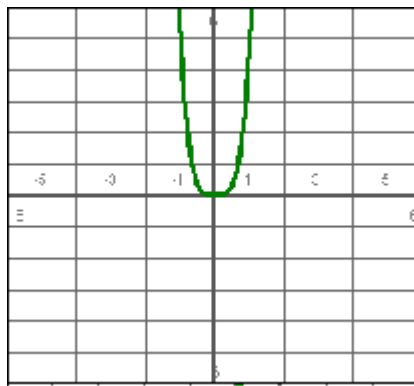
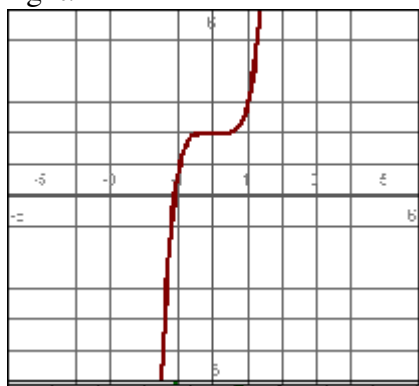
E) $\frac{11\sqrt{x}}{2}$

Ans: D

Difficulty: Medium

Section: 2.2

4. Answer true or false. The derivative of the function graphed on the left is graphed on the right.



Ans: True

Difficulty: Medium

Section: 2.2

5. Answer true or false. Use a graphing utility to help in obtaining the graph of $y = f(x) = |x - 11|$. The derivative $f'(x)$ is not defined at $x = 11$.

Ans: True

Difficulty: Medium

Section: 2.2

6. Find $f'(t)$ if $f(t) = 4t^6 - 8$.

A) $24t^5 - 8$ B) $24t^6$ C) $24t^6 - 8$ D) $4t^5 - 8$ E) $24t^5$

Ans: E

Difficulty: Easy

Section: 2.2

7. $\lim_{h \rightarrow 0} \frac{(8+h)^2 - 64}{h}$ represents the derivative of $f(x) = x^2$ at $x = a$. Find a .

A) 64 B) 8 C) 1 D) 0 E) 56

Ans: B

Difficulty: Easy

Section: 2.2

8. $\lim_{h \rightarrow 0} \frac{\sqrt[4]{81+h} - 3}{h}$ represents the derivative of $f(x) = \sqrt[4]{x}$ at $x = a$. Find a .

A) 3 B) 81 C) 78 D) 1 E) 0

Ans: B

Difficulty: Easy

Section: 2.2

9. Find an equation for the tangent line to the curve $y = x^4 - 11$ at $(2, 5)$.

A) $y = 32x$ D) $y = 32x - 59$
 B) $y = 32x - 11$ E) $y = 32x^3 - 11$
 C) $y = 32x^3 - 59$

Ans: D

Difficulty: Medium

Section: 2.2

10. Let $f(x) = \sin x$. Estimate $f'\left(\frac{3\pi}{4}\right)$ by using a graphing utility.

A) $\frac{\sqrt{2}}{2}$ B) $-\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $-\frac{1}{2}$ E) $\frac{1}{2}$

Ans: B

Difficulty: Medium

Section: 2.2

11. An air source constantly increases the air supply rate of a balloon. The volume V in cubic feet is given by $V(t) = 6t + 1$, $[0 \leq t \leq 5]$, where t is time in seconds. How fast is the volume of the balloon increasing at $t = 5$ s?

A) $30 \text{ ft}^3/\text{s}$ B) $31 \text{ ft}^3/\text{s}$ C) $1 \text{ ft}^3/\text{s}$ D) $0 \text{ ft}^3/\text{s}$ E) $6 \text{ ft}^3/\text{s}$

Ans: E

Difficulty: Medium

Section: 2.2

12. Answer true or false. Using a graphing utility it can be shown that $f(x) = \sqrt[3]{|x-7|}$ is differentiable everywhere on $[-10, 10]$.

Ans: False

Difficulty: Easy

Section: 2.2

13. Answer true or false. A graphing utility can be used to determine that

$f(x) = \begin{cases} x^8 + 8, & x \leq 0 \\ x^2 + 8, & x > 0 \end{cases}$ is differentiable at $x = 0$.

Ans: True

Difficulty: Medium

Section: 2.2

14. Answer true or false. A graphing utility will show that differentiable at $x = -1$.

$$f(x) = \begin{cases} 3x^6 - 12, & x \leq -1 \\ 3x^{12} - 12, & x > -1 \end{cases} \text{ is}$$

Ans: False

Difficulty: Medium

Section: 2.2

15. Find $f'(x)$ if $f(x) = 5x^4$.

A) $20x^4$ B) $20x^3$ C) $5x^3$ D) $4x^3$ E) $\frac{5x^3}{4}$

Ans: B

Difficulty: Easy

Section: 2.2

16. Find $f'(x)$ if $f(x) = 2x^3 - 3x$.

A) $6x^2 - 3$ B) $6x^2$ C) $6x^3 - 3$ D) $2x^2 - 3$ E) $6x^3$

Ans: A

Difficulty: Easy

Section: 2.2

17. Use the definition of the derivative to calculate $f'(x)$ if $f(x) = 2x^2 - 5x$ and find the equation of the tangent to the graph of f at $x = 2$.

Ans: *Derivative:* $f'(x) = 4x - 5$

Tangent line: $y = 3x - 8$

Difficulty: Medium

Section: 2.2

18. Use the definition of the derivative to calculate $f'(x)$ if $f(x) = 5x^2 - 8x + 6$.

Ans: $10x - 8$

Difficulty: Easy

Section: 2.2

19. Use the definition of the derivative to calculate $f'(x)$ if $f(x) = 4x^4 + 7$ and find the equations of the tangent line and the normal line to the graph of f at $x = -4$.

Ans: *Derivative:* $y = 16x^3$

Tangent line: $y = -1,024x - 3,065$

Normal line: $y = \frac{x}{1,024} + \frac{263,937}{256}$

Difficulty: Medium

Section: 2.2

20. Use the definition of the derivative to calculate $f'(x)$ if $f(x) = 2x^2 - 2x$ and find the equation of the tangent line and the normal line to the graph of f at $x = 2$.

Ans: $f'(x) = 4x - 2$

Tangent line: $y = 6x - 8$

Normal line: $y = -\frac{1}{6}x + \frac{11}{3}$

Difficulty: Medium

Section: 2.2

21. Use the definition of the derivative to calculate $f'(x)$ if $f(x) = \sqrt{2x}$ and find the equations of the tangent line and the normal line to the graph of f at $x = 2$.

Ans: $f'(x) = \frac{\sqrt{2}}{2\sqrt{x}}$

Tangent: $y = \frac{1}{2}x + 1$

Normal: $y = -2x + 6$

Difficulty: Medium

Section: 2.2

22. Let $y = \sqrt{6x+6}$. Use the definition of the derivative to find $\frac{dy}{dx}$.

Ans: $\frac{dy}{dx} = \frac{3}{\sqrt{6x+6}}$

Difficulty: Medium

Section: 2.2

23. Let $y = \frac{-3}{8x+7}$. Use the definition of the derivative to find $\frac{dy}{dx}$.

Ans: $\frac{dy}{dx} = \frac{24}{(8x+7)^2}$

Ans:

Difficulty: Medium

Section: 2.2

24. Use the definition of the derivative to calculate $f'(x)$ if $f(x) = \frac{1}{3-x}$.

Ans: $f'(x) = \frac{1}{(3-x)^2}$

Ans:

Difficulty: Medium

Section: 2.2

25. Given that $f(2) = -2$ and $f'(2) = -2$, find an equation for the tangent line to the graph of $y = f(x)$ at the point where $x = 2$.

Ans: $y = -2x + 2$

Difficulty: Hard

Section: 2.2

26. Given that $f(1) = 5$ and $f'(1) = 6$, find an equation for the tangent line to the graph of $y = f(x)$ at the point where $x = 1$.

Ans: $y = 6x - 1$

Difficulty: Hard

Section: 2.2

27. Use the definition of the derivative to calculate $f'(x)$ if $f(x) = \frac{2}{\sqrt{3x}}$ and find the equations of the tangent line and the normal line to the graph of f at $x = 12$.

$$f'(x) = \frac{-3}{\sqrt{(3x)^3}}$$

Ans:

$$\text{Tangent line: } y = -\frac{1}{72}x + \frac{1}{2}$$

$$\text{Normal line: } y = 72x + \frac{2,593}{3}$$

Difficulty: Hard

Section: 2.2

28. The volume of a sphere is given by $\frac{4}{3}\pi r^3$ where r is the radius of the sphere. Use the method of Section 3.2 to find the instantaneous rate of change of V with respect to r when $r = 6$.

Ans: The instantaneous rate of change = 144π

Difficulty: Easy

Section: 2.2

29. The surface area of a sphere is given by $S = 4\pi r^2$ where r is the radius of the sphere. Use the method of Section 3.2 to find the instantaneous rate of change of S with respect to r when $r = 2$.

Ans: The instantaneous rate of change of S with respect to r at $r = 2$ is **16π** .

Difficulty: Easy

Section: 2.2

30. The volume of a sphere is given by $V = \frac{\pi}{6}D^3$ where D is the diameter of the sphere. Use the method of Section 3.2 to find the instantaneous rate of change of V with respect to D when $D = 2$.

Ans: The instantaneous rate at which V changes with respect to D when $D = 2$ is **2π** .

Difficulty: Easy

Section: 2.2

31. Show that $f(x) = \begin{cases} x^2 - 5, & x \leq 1 \\ x - 5, & x > 1 \end{cases}$ is continuous but not differentiable at $x = 1$.

Ans: Continuous: $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x^2 - 5) = -4$ and $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x - 5) = -4$, so it is continuous at $x = 1$.

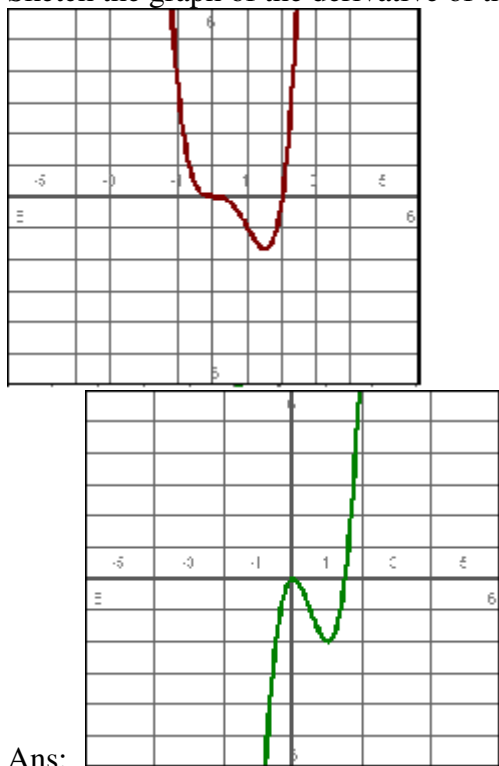
Differentiable: $\lim_{h \rightarrow 0^-} \frac{f(1+h) - f(1)}{(1+h) - 1} = \lim_{h \rightarrow 0^-} \frac{2h + h^2}{h} = \lim_{h \rightarrow 0^-} (2 + h) = 2$
 $\neq \lim_{h \rightarrow 0^+} \frac{f(1+h) - f(1)}{(1+h) - 1} = \lim_{h \rightarrow 0^+} \frac{h}{h} = 1$

, so it is NOT differentiable at $x = 1$.

Difficulty: Hard

Section: 2.2

32. Sketch the graph of the derivative of the function whose graph is shown.



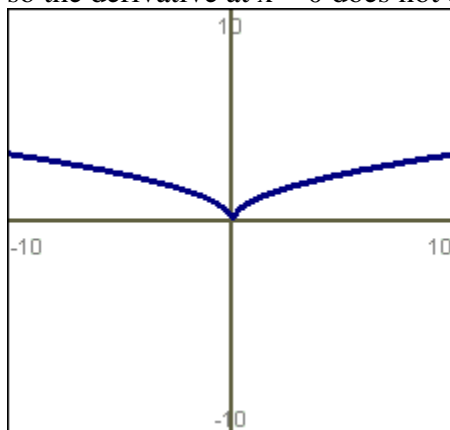
Ans:

Difficulty: Medium

Section: 2.2

33. Use a graphing utility to show that $y = \sqrt[4]{x^2}$ does not have a derivative at $x = 0$.

Ans: To the left of 0, the tangent is negative and to the right of 0, the tangent is positive, so the derivative at $x = 0$ does not exist.

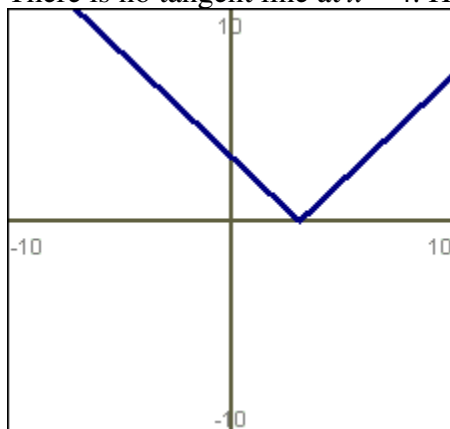


Difficulty: Easy

Section: 2.2

34. Use a graphing utility to show that $y = |x - 4|$ does not have a derivative everywhere.

Ans: There is no tangent line at $x = 4$. Hence, there is no derivative at $x = 4$.



Difficulty: Easy

Section: 2.2

35. Let $f(x) = 2^x$. Estimate $f'(2)$ using a graphing utility.

Ans: 2.773

Difficulty: Hard

Section: 2.2

36. Let $f(x) = \cos x$. Estimate $f'(1)$ using a graphing utility.

Ans: -0.841

Difficulty: Easy

Section: 2.2

37. The cost of manufacturing an object is given by $C = f(x)$ dollars. Estimate the cost of making the 310th item if $f'(309) = \$74$.

Ans: \$74

Difficulty: Easy

Section: 2.2

1. Find dy/dx if $y = 5x^8$.

- A) $40x^8$ B) $5x^7$ C) $40x^7$ D) $40x$ E) x^7

Ans: C

Difficulty: Easy

Section: 2.3

2. Find dy/dx if $y = \sqrt{2\pi}$.

- A) $\frac{2\sqrt{\pi}}{\pi}$ B) $-\frac{\sqrt{2\pi}}{4\pi}$ C) 0 D) 1 E) $\frac{\sqrt{2\pi}}{2\pi}$

Ans: C

Difficulty: Easy

Section: 2.3

3. Find dy/dx if $y = 3(x^6 - 6x + 1)$.

- A) $3x^5 - 6x - 18$ B) $6x^5 - 18$ C) $18x^5 - 18$ D) $18x^5 - 6$ E) $3x^5 - 6$

Ans: C

Difficulty: Easy

Section: 2.3

4. Answer true or false. If $f(x) = \sqrt[4]{x} + 7x$, $f'(x) = \frac{\sqrt[4]{x}}{4} + 7$.

Ans: False

Difficulty: Medium

Section: 2.3

5. $y = 4x^2 + 9x + 2$. Find d^2y/dx^2 .

- A) 8 B) $8x + 9$ C) 4 D) $4x + 2$ E) 2

Ans: A

Difficulty: Medium

Section: 2.3

6. $y = 2x^{-3} + 2x$. Find y''' .

- A) 12 B) -120 C) $-120x^{-6}$ D) $-120x^{-6} + 2$ E) $12x^{-6} + 2$

Ans: C

Difficulty: Hard

Section: 2.3

7. Answer true or false. $y = y''' + 5y' - 2$ is satisfied by $y = x$.

Ans: False

Difficulty: Hard

Section: 2.3

8. Use a graphing utility to locate all horizontal tangent lines to the curve $y = x^3 + 3x^2 + 12$.

A) $x = 0$ B) $x = -3$ and $x = 0$ C) $x = -2$ and $x = 0$ D) $x = -2$ E) $x = -3$

Ans: C

Difficulty: Medium

Section: 2.3

9. Find the x -coordinate of the point on the graph of $y = x^4 + 4$ where the tangent line is parallel to the secant line that cuts the curve at $x = 2$ and at $x = 3$.

A) $\sqrt[3]{\frac{65}{4}}$ B) 85 C) 20 D) $\sqrt[4]{\frac{65}{4}}$ E) $\frac{65}{4}$

Ans: A

Difficulty: Hard

Section: 2.3

10. The position of a moving particle is given by $s(t) = 3t^5 - 1$ where t is time in seconds. The velocity in m/s is given by ds/dt . Find the velocity at $t = 3$.

A) 3,645 m/s B) 243 m/s C) 405 m/s D) 1,215 m/s E) 1,214 m/s

Ans: D

Difficulty: Medium

Section: 2.3

11. Find $\frac{dy}{dx}$, if $y = \frac{5x^6 - 6}{6}$.

A) $30x^5$ B) $6x^5 - 6$ C) $\frac{30x^5 - 6}{6}$ D) $5x^5$ E) $\frac{30x^5}{6} + 6$

Ans: D

Difficulty: Easy

Section: 2.3

12. Find $\frac{dy}{dx}$ if $y = \frac{5x^4 + 8x^2 + \sqrt{x}}{x}$.

$\frac{dy}{dx} = 15x^2 + 8 - \frac{1}{2\sqrt{x^3}}$

Ans:

Difficulty: Easy

Section: 2.3

13. Find $\frac{dy}{dx}$ if $y = \frac{x^2 + 5x}{7 - 4x}$.

$$\frac{dy}{dx} = \frac{-4x^2 + 14x + 35}{(7 - 4x)^2}$$

Ans:

Difficulty: Medium

Section: 2.3

14. Find $f''(2)$ if $f(x) = \frac{-32}{x^2} + \frac{x^5}{5}$.

$$f'(x) = \frac{64}{x^3} + x^4$$

Ans: $f'(x) =$

$$f''(x) = \frac{-192}{x^4} + 4x^3$$

$$\text{So: } f''(2) = 20.$$

Difficulty: Hard

Section: 2.3

15. Find $\frac{d^2y}{dt^2}$ if $y = -\frac{2}{t} - \frac{7}{t^2}$.

$$f'(t) = \frac{2}{t^2} + \frac{14}{t^3}$$

Ans:

$$f''(t) = -\frac{4}{t^3} - \frac{42}{t^4}$$

Difficulty: Hard

Section: 2.3

16. Find $\frac{dv}{dh}$ if $v = 2\pi \left(ah^3 - \frac{1}{4}h^4 \right)$ and **a** is a constant.

$$\text{Ans: } \frac{dv}{dh} = 2\pi (3ah^2 - h^3)$$

Difficulty: Easy

Section: 2.3

17. Find equations for the tangents to the graph of $y = 35 - 2x - x^2$ at those points where the curve intersects the x -axis.

Ans: $y' = -2 - 2x$

The tangent lines are:

At $x = -7$, the tangent line is: $y = 12x + 84$.

At $x = 5$, the tangent line is: $y = -12x + 60$.

Difficulty: Hard

Section: 2.3

18. Find equations for the tangents and normals to the graph of $y = 30 - x - x^2$ at the points where the curve intersects the x -axis.

Ans: $y' = -1 - 2x$

So the tangent lines are:

At $x = 5$, the equation of the tangent line is: $y = -11x + 55$.

At $x = -6$, the equation of the tangent line is: $y = 11x + 66$.

The normal lines are:

At $x = 5$, the equation of the normal line is: $y = \frac{1}{11}x - \frac{5}{11}$.

At $x = -6$, the equation of the normal line is: $y = -\frac{1}{11}x - \frac{6}{11}$.

Difficulty: Hard

Section: 2.3

19. Find the points on the graph of $y = 2x^3 + 21x^2 + 60x + 30$ at which the tangent is parallel to the x -axis.

Ans: Equations of tangent that are parallel to the x -axis:

at $x = -5$, the equation of the tangent is: $y = 5$

at $x = -2$, the equation of the tangent is: $y = -22$

Difficulty: Medium

Section: 2.3

20. Show that the parabola $y = -x^2$ and the line $x - 10y - 255 = 0$ intersect at right angles at one of their points of intersection.

Ans: For the line, the slope is described by $1 - 10 \frac{dy}{dx} = 0$ or $\frac{dy}{dx} = \frac{1}{10}$.

For the parabola, the slope is described by $\frac{dy}{dx} = -2x$.

At $x = 5$, the slope of the line is $\frac{1}{10}$.

And the slope of the parabola is -10 .

So the (tangent line to the) parabola is perpendicular to the line at $x = 5$.

Difficulty: Hard

Section: 2.3

21. Show that the parabola $y = x^2$ and the line $x + 6y - 57 = 0$ intersect at right angles at one of their points of intersection.

Ans: For the line, the slope is described by $1 + 6 \frac{dy}{dx} = 0$ or $\frac{dy}{dx} = -\frac{1}{6}$.

For the parabola, the slope is described by $\frac{dy}{dx} = 2x$.

At $x = 3$, the slope of the line is $-\frac{1}{6}$.

And the slope of the parabola is 6 .

So the (tangent line to the) parabola is perpendicular to the line at $x = 3$.

Difficulty: Hard

Section: 2.3

22. Find $\frac{dy}{dx}$ if $y = -8x^{10}$.

Ans: $-80x^9$

Difficulty: Easy

Section: 2.3

23. Find $\frac{dy}{dx}$ if $y = \alpha^{11}$. (Alpha is a constant.)

Ans: 0

Difficulty: Easy

Section: 2.3

24. Find $\frac{dy}{dx}$ if $y = 3\sqrt{x} - 9x$.

Ans: $\frac{3}{2\sqrt{x}} - 9$ or $\frac{3\sqrt{x}}{2x} - 9$

Difficulty: Medium

Section: 2.3

25. If $y = x^8 + 4x^3$, find $\frac{d^2y}{dx^2}$.

Ans: $\frac{dy}{dx} = 8x^7 + 12x^2$

$\frac{d^2y}{dx^2} = 56x^6 + 24x$

Difficulty: Medium

Section: 2.3

26. If $y = x^4 - 4x + 5$, find $\frac{d^2y}{dx^2}$.

Ans: $\frac{dy}{dx} = 4x^3 - 4$,

$\frac{d^2y}{dx^2} = 12x^2$

Difficulty: Easy

Section: 2.3

27. Find $y'''(3)$ if $y = x^5$.

Ans: 540

Difficulty: Easy

Section: 2.3

28. Find $f'(s)$ if $f(s) = (6s^2 + 8)(s^2 - 7s)$.

Ans: $f'(s) = 12s(s^2 - 7s) + (2s - 7)(6s^2 + 8) = 24s^3 - 126s^2 + 16s - 56$

Difficulty: Medium

Section: 2.3

1. Answer true or false. If $y = \frac{1}{8x-2}$, $y' = \frac{1}{8}$.

Ans: False

Difficulty: Easy

Section: 2.4

2. If $y = \frac{6x}{x-12}$, find $\frac{dy}{dx}\bigg|_3$.

A) $\frac{8}{9}$ B) $-\frac{8}{9}$ C) $-\frac{6}{9}$ D) $\frac{6}{9}$ E) $-\frac{8}{225}$

Ans: B

Difficulty: Medium

Section: 2.4

3. If $y = \frac{6}{x+7}$, $y'(2) =$

A) 0 B) $\frac{2}{27}$ C) $-\frac{14}{27}$ D) $-\frac{2}{27}$ E) $-\frac{7}{27}$

Ans: D

Difficulty: Medium

Section: 2.4

4. $g(x) = x^4 f(x)$. Find $g'(4)$, given that $f(4) = 3$ and $f'(4) = 6$.

A) 1,792 B) 1,024 C) 2,304 D) 4,608 E) 1,536

Ans: C

Difficulty: Hard

Section: 2.4

5. Answer true or false. If f , g , and h are differentiable functions, and $h \neq 0$ anywhere on its

domain, then $\left(\frac{fg}{h}\right)' = \frac{fhg' + ghf' - fgh'}{h^2}$.

Ans: True

Difficulty: Medium

Section: 2.4

6. If $y = \frac{1}{\sqrt{x}-4}$, find $y'(25)$.
 A) $-\frac{1}{4}$ B) $\frac{1}{4}$ C) $\frac{1}{10}$ D) $-\frac{1}{25}$ E) $-\frac{1}{10}$

Ans: E

Difficulty: Hard

Section: 2.4

7. Find $\frac{dy}{dx}$ if $y = 3(x^2 - 2x)(6 + x^5)$.
 Ans: $\frac{dy}{dx} = 3(2x - 2)(6 + x^5) + 15x^4(x^2 - 2x) = 21x^6 - 36x^5 + 36x - 36$

Difficulty: Medium

Section: 2.4

8. Find $f'(x)$ if $f(x) = \frac{6x^2 + 6}{3x^2 + 8x}$.

$$f'(x) = \frac{12x(3x^2 + 8x) - (6x + 8)(6x^2 + 6)}{(3x^2 + 8x)^2} = \frac{48x^2 - 36x - 48}{(3x^2 + 8x)^2}$$

Ans:

Difficulty: Medium

Section: 2.4

9. If $f(3) = 4$, $f'(3) = -5$, $g(3) = 2$, $g'(3) = -4$, find $F'(3)$ if $F(x) = 2f(x) - g(x)$.
 Ans: -6
 Difficulty: Easy
 Section: 2.4

10. If $f(4) = 7$, $f'(4) = 2$, $g(4) = 2$, $g'(4) = -2$, find $F'(4)$ if $F(x) = 6f(x)g(x)$.
 Ans: -60
 Difficulty: Easy
 Section: 2.4

11. If $f(6) = 48$, $f'(6) = 5$, $g(6) = 4$, $g'(6) = -3$, find $F'(6)$ if $F(x) = \frac{4f(x)}{4g(x)}$.
 Ans: $\frac{41}{4}$ of $10\frac{1}{4}$
 Difficulty: Easy
 Section: 2.4

12. Find $f'(u)$ if $f(u) = \frac{u^2 - 7}{4u^2 - 4}$.

$$f'(u) = \frac{2u(4u^2 - 4) - 8u(u^2 - 7)}{(4u^2 - 4)^2} = \frac{48u}{(4u^2 - 4)^2}$$

Ans:

Difficulty: Medium

Section: 2.4

13. Find $f'(t)$ if $f(t) = \frac{3t^3 - 12t^2}{3t^4}$.

Ans: $y = t^{-1} - 4t^{-2}$ So $y' = -t^{-2} + 8t^{-3}$.

Difficulty: Medium

Section: 2.4

14. Find $\frac{dy}{dx}$ if $y = (x^2 - 1)(x^3 + 3x)$.

Ans: $\frac{dy}{dx} = 2x(x^3 + 3x) + (3x^2 + 3)(x^2 - 1) = 5x^4 + 6x^2 - 3$

Difficulty: Medium

Section: 2.4

15. Find $f'(x)$ if $f(x) = (x^2 + 5)(x^3 - 4x^2 + x)$.

Ans: $f'(x) = 5x^4 - 16x^3 + 18x^2 - 40x + 5$

Difficulty: Medium

Section: 2.4

16. Find the equations of the tangent and normal to the graph of $y = \frac{x+1}{x-1}$ at $x = 4$.

Ans: Tangent line: $y = -\frac{2}{9}x + \frac{23}{9}$

Normal line: $y = \frac{9}{2}x - \frac{49}{3}$

Difficulty: Hard

Section: 2.4

1. Find $f'(x)$ if $f(x) = x^4 \cos x$.

A) $4x^3 \cos x - x^4 \sin x$

B) $4x^3 \cos x$

C) $4x^3 \sin x$

D) $-4x^3 \cos x$

E) $-4x^3 \sin x$

Ans: A

Difficulty: Easy

Section: 2.5

2. Find $f'(x)$ if $f(x) = \cos x \tan x$.

A) $\sec x$ B) $-\cos x$ C) $-\sin x \tan x + 1$ D) $\cos x$ E) $-\tan x$

Ans: D

Difficulty: Medium

Section: 2.5

3. Find $f'(x)$ if $f(x) = \csc^2 x + \cot^2 x$.

A) $2 \csc x + 2 \cot x$ B) $2 \csc x - 2 \cot x$ C) -1 D) 1 E) 0

Ans: E

Difficulty: Easy

Section: 2.5

4. Find d^2y/dx^2 if $y = x \tan x$.

A) $2x \tan x \sec x$

B) $x \sec x$

C) $2 \sec^2 x + 2x \tan x \sec^2 x$

D) $\sec x$

E) $-2x \tan x \sec x$

Ans: C

Difficulty: Medium

Section: 2.5

5. Answer true or false. If $y = \sin x$, $d^2y/dx^2 = \sin x$.

Ans: False

Difficulty: Medium

Section: 2.5

6. Find the equation of the line tangent to the graph of $y = \sin x$ at the point where $x = 0$.

A) $y = \cos x$ B) $y = x$ C) $y = -x$ D) $y = x$ E) $y = x$

Ans: E

Difficulty: Medium

Section: 2.5

7. Find the x -coordinates of all points in the interval $[-2\pi, 2\pi]$ at which the graph of $f(x) = \csc 2x$ has a horizontal tangent line.

- A) $\left(\pm\frac{\pi}{4}, \pm\frac{\pi}{2}\right)$ D) $\left(\pm\frac{\pi}{4}, \pm\frac{\pi}{2}, \pm\frac{3\pi}{4}, \pm\frac{3\pi}{2}\right)$
 B) $\left(\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}\right)$ E) $\left(\pm\pi, \pm\frac{\pi}{2}, \pm\frac{3\pi}{4}, \pm 2\pi\right)$
 C) $\left(\pm\frac{3\pi}{4}, \pm\frac{3\pi}{2}\right)$

Ans: B

Difficulty: Easy

Section: 2.5

8. Find d^{75} of $\cos x/dx^{75}$.

- A) $-\cos x$ B) $-\sin x$ C) $\cos x$ D) 0 E) $\sin x$

Ans: E

Difficulty: Medium

Section: 2.5

9. Find all x -values on $[0, 2\pi]$ where $f(x)$ is not differentiable. $f(x) = \cos x \cot x$

- A) $\frac{\pi}{2}, \frac{3\pi}{2}$ B) $0, \pi, 2\pi$ C) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ D) 0 E) none

Ans: B

Difficulty: Medium

Section: 2.5

10. Answer true or false. If x is given in radians, the derivative formula for $y = \sin x$ in

degrees is $y' = \frac{\pi}{180} \cos x$.

Ans: False

Difficulty: Easy

Section: 2.5

11. A rock at an elevation angle of θ is falling in a straight line. If at a given instant it has an angle of elevation of $\theta = \frac{\pi}{3}$ and is a horizontal distance s from an observer, find the rate at which the rock is falling with respect to θ .

- A) $\frac{\sec^2\left(\frac{\pi}{3}\right)}{s}$ D) $\sec^2\left(\frac{s\pi}{3}\right)$
 B) $s \times \sec^2\left(\frac{\pi}{3}\right)$ E) $s \times \sec\left(\frac{\pi}{3}\right)$
 C) $\sec^2\left(\frac{\pi}{3}\right)$

Ans: B

Difficulty: Medium

Section: 2.5

12. Answer true or false. If $f(x) = \cos x \sec x - \sin x$, $f'(x) = 1 - \cos x$.

Ans: False

Difficulty: Medium

Section: 2.5

13. Answer true or false. If $f(x) = \frac{1}{\sin x}$, $f'(x) = \frac{1}{\cos x}$.

Ans: False

Difficulty: Easy

Section: 2.5

14. Answer true or false. $f(x) = \frac{\csc x}{1 - \csc x}$ is differentiable everywhere.

Ans: False

Difficulty: Easy

Section: 2.5

15. If $y = x^6 \cos x$, find d^2y/dx^2 .

- A) $30x^4 \cos x - 12x^5 \sin x + x^6 \cos x$ D) $30x^4 \cos x - 12x^5 \sin x$
 B) $30x^4 \cos x - 6x^5 \sin x - x^6 \cos x$ E) $6x^5 \cos x - x^6 \sin x$
 C) $30x^4 \cos x - 12x^5 \sin x - x^6 \cos x$

Ans: C

Difficulty: Hard

Section: 2.5

16. If $y = x^7 \cos x$, find dy/dx .

A) $7x^6 \cos x + x^7 \sin x$

B) $7x^6 \sin x - x^7 \cos x$

C) $42x^5 \cos x - 14x^6 \sin x - x^7 \cos x$

D) $42x^5 \cos x - 14x^6 \sin x$

E) $7x^6 \cos x - x^7 \sin x$

Ans: E

Difficulty: Hard

Section: 2.5

17. Find d^6y/dx^6 if $y = \cos x$.

A) $-\cos x$

B) $\cos x$

C) $\sin x$

D) $-\sin x$

E) $\sin x \cos x$

Ans: A

Difficulty: Easy

Section: 2.5

18. Find $f'(x)$ if $f(x) = x \tan x$.

Ans: $x \sec^2 x + \tan x$

Difficulty: Medium

Section: 2.5

19. Find $f''(x)$ if $f(x) = x \sin x$.

Ans: $-x \sin x + 2 \cos x$

Difficulty: Medium

Section: 2.5

20. Find $\frac{dy}{dx}$ if $y = \frac{\cos x}{x^4}$.

$\frac{dy}{dx} = \frac{-x \sin x - 4 \cos x}{x^5}$

Ans:

Difficulty: Medium

Section: 2.5

21. Find $\frac{dy}{dx}$ if $y = \sec x \tan x$.

Ans: $\sec x \tan^2 x + \sec^3 x$

Difficulty: Hard

Section: 2.5

22. Find $f'(x)$ if $f(x) = \frac{\cot x}{1 + \csc x}$.

$$f'(x) = \frac{-\csc x}{(1 + \csc x)}$$

Ans:

Difficulty: Medium

Section: 2.5

23. Find $f'(x)$ if $f(x) = (7x^4 + 9) \cos x$.

Ans: $28x^3 \cos x - (7x^4 + 9) \sin x$

Difficulty: Medium

Section: 2.5

24. Differentiate $y = \frac{\csc x}{\sqrt{x}}$.

$$\frac{dy}{dx} = \frac{-2x \csc x \cot x - \csc x}{2\sqrt{x^3}}$$

Ans:

Difficulty: Hard

Section: 2.5

25. Find $\frac{dy}{dx}$ if $y = \frac{\sin x}{1 - \cos x}$.

$$\frac{dy}{dx} = \frac{-1}{1 - \cos x}$$

Ans:

Difficulty: Medium

Section: 2.5

26. Find $\frac{dy}{dx}$ if $y = (3x^2 + 5x) \cot x$.

Ans: $\frac{dy}{dx} = (6x + 5) \cot x - (3x^2 + 5x) \csc^2 x$

Difficulty: Medium

Section: 2.5

27. Find $y''(x)$ if $y = 8 \cos x + 5 \sin x + \frac{x^6}{6}$.

$$\frac{dy}{dx} = -8 \cos x - 5 \sin x + 5x^4$$

Ans:

Difficulty: Hard

Section: 2.5

28. Find $f'(\theta)$ if $f(\theta) = \frac{1}{1-5\sin\theta}$.

$$f'(\theta) = \frac{5\cos\theta}{(1-5\sin\theta)^2}$$

Ans:

Difficulty: Medium

Section: 2.5

29. Find $\frac{dy}{dx}$ if $y = 8x \sin x - 3 \cos x + x^2 \cos x$.

$$\frac{dy}{dx} = 11 \sin x - x^2 \sin x + 10x \cos x$$

Ans:

Difficulty: Hard

Section: 2.5

30. Find $f'(\theta)$ if $f(\theta) = \frac{1+\sin\theta}{1-\sin\theta}$.

$$f'(\theta) = \frac{2\cos\theta}{(1-\sin\theta)^2}$$

Ans:

Difficulty: Medium

Section: 2.5

31. Find $\frac{dy}{dt}$ if $y = \frac{1+\tan t}{1-\tan t}$.

$$\frac{dy}{dt} = \frac{2\sec^2 t}{(1-\tan t)^2}$$

Ans:

Difficulty: Medium

Section: 2.5

32. Show by use of a trigonometric identity that $\frac{d}{dx} |\tan x - x| = \tan^2 x$.

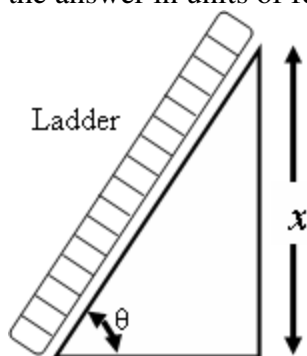
$$\frac{d}{dx} |\tan x - x| = \sec^2 x - 1 = \tan^2 x$$

Ans:

Difficulty: Easy

Section: 2.5

33. A 16 foot long ladder leans against a wall at an angle θ with the horizontal as shown in the figure. The top of the ladder is x feet above the ground. If the bottom of the ladder is pushed toward the wall, find the rate at which x changes with θ when $\theta = 45^\circ$. Express the answer in units of feet/degree.



Ans: $\sin \theta = \frac{x}{16}$
 $x = 16 \sin \theta$
 $\frac{dx}{d\theta} = 16 \cos \theta$

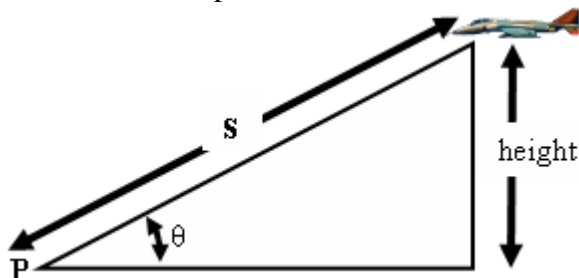
Since $\theta = 45^\circ$

$$\left. \frac{dx}{d\theta} \right|_{\theta=45^\circ} = 16 \cos 45^\circ = 16 \times 0.7071 = 11.3137 \text{ ft / degree}$$

Difficulty: Medium

Section: 2.5

34. An airplane is flying on a horizontal path at a height of 6000ft, as shown in the figure. At what rate is the distance s between the airplane and the fixed point P changing with θ when $\theta = 20^\circ$. Express the answer in units of feet/degree.



$$\begin{aligned} \text{Ans: } \csc \theta &= \frac{s}{6000} \\ s &= 6000 \csc \theta \\ \frac{ds}{d\theta} &= -6000 \csc \theta \cot \theta \\ \text{Since } \theta &= 20^\circ \quad \left. \frac{ds}{d\theta} \right|_{\theta=20^\circ} = \end{aligned}$$

$$6000 \csc 20^\circ \cot 20^\circ = 48,198.519 \text{ ft/degree}$$

Difficulty: Medium
Section: 2.5

35. $f(x) = 4 \cos x + 7 \sin x$. Find $f'(x)$.

$$\text{Ans: } f'(x) = -4 \sin x + 7 \cos x$$

Difficulty: Medium
Section: 2.5

36. $f(x) = x^2 \cos x + \sin x$. Find $f'(x)$.

$$\text{Ans: } -x^2 \sin x + 2x \cos x + \cos x$$

Difficulty: Medium
Section: 2.5

37. $f(x) = 3x \cot x$. Find $f'(x)$.

$$\text{Ans: } 3 \cot x - 3x \csc^2 x$$

Difficulty: Medium
Section: 2.5

38. $f(x) = \frac{\csc x}{x^5}$. Find $f'(x)$.

$$f'(x) = \frac{-x \csc x \cot x - 5 \csc x}{x^6}$$

Ans:

Difficulty: Medium

Section: 2.5

39. Find $\frac{dy}{dx}$ if $y = \tan x$.

Ans: $\sec^2 x$

Difficulty: Easy

Section: 2.5

1. $f(x) = \sqrt{x^5 + 3}$. $f'(x) =$

A) $\frac{5x^4}{\sqrt{x^5 + 3}}$ B) $\frac{5x^4 + 3}{2\sqrt{x^5 + 3}}$ C) $\frac{5x^4}{2}$ D) $\frac{5x^4}{2\sqrt{x^5 + 3}}$ E) $\frac{5x^5}{2}$

Ans: D

Difficulty: Medium

Section: 2.6

2. $f(x) = (x^4 - 3)^{38}$. $f'(x) =$

A) $38x^3 (x^4 - 3)^{37}$ D) $152(x^4 - 3)^{37}$
 B) $4x^3 (x^4 - 3)^{37}$ E) $38(x^4 - 3)^{37}$
 C) $152x^3 (x^4 - 3)^{37}$

Ans: C

Difficulty: Medium

Section: 2.6

3. $f(x) = \sin(6x)$. $f'(x) =$

A) $6 \cos(6x)$ B) $\cos(6x)$ C) $6 \cos(x)$ D) $6 \sin(6x)$ E) $\sin(6x)$

Ans: A

Difficulty: Medium

Section: 2.6

4. Answer true or false. If $f(x) = \sqrt{\cos^2 x + 11}$, $f'(x) = \frac{1}{2\sqrt{\cos^2 x + 11}}$.

Ans: False

Difficulty: Hard

Section: 2.6

5. If $f(x) = x^7 \sqrt{x^3 + 6}$, then $f'(x) =$

A) $7x^3 \sqrt{x^3 + 6} + \frac{3x^9}{2\sqrt{x^3 + 6}}$ D) $\frac{3x^9}{2\sqrt{x^3 + 6}}$
 B) $7x^6 \sqrt{x^3 + 6} + \frac{3x^9}{\sqrt{x^3 + 6}}$ E) $7x^6 \sqrt{x^3 + 6} + \frac{3x^9}{2\sqrt{x^3 + 6}}$
 C) $7x^6 \sqrt{x^3 + 6}$

Ans: E

Difficulty: Hard

Section: 2.6

6. $y = \cot(\sin x)$. Find dy/dx .

- A) $\cos x \csc^2(\sin x)$ D) $\sin x \csc^2(\cos x)$
 B) $-\cos x \csc^2(\sin x)$ E) $-\sin x \csc^2(\sin x)$
 C) $-\sin x \csc^2(\cos x)$

Ans: B

Difficulty: Medium

Section: 2.6

7. $y = x^4 \sin(5x)$. Find dy/dx .

- A) $5x^4 \cos(5x) - 4x^3 \sin(5x)$ D) $5x^4 \cos(5x)$
 B) $5x^4 \cos(5x) + x^3 \sin(5x)$ E) $4x^3 \sin(5x)$
 C) $5x^4 \cos(5x) + 4x^3 \sin(5x)$

Ans: C

Difficulty: Medium

Section: 2.6

8. If $y = \frac{1 - \sin^2 x}{\cos x}$, $\frac{dy}{dx} =$

- A) $\cos x$ B) $-\sin x$ C) $-\cos x$ D) $\sin x$ E) $\sin x \cos x$

Ans: B

Difficulty: Easy

Section: 2.6

9. If $y = \left(\frac{1 - x^2 \cos^2 x}{\sin x} \right)$, $\frac{dy}{dx} =$ (Do not reduce any trig ratio identities.)

- A) $\frac{2x \cos^2 x \sin x - 2x^2 \cos x \sin^2 x + \cos x - x^2 \cos^2 x}{\sin^2 x}$
 B) $\frac{-2x \cos^2 x \sin x + 2x^2 \cos x \sin^2 x - \cos x + x^2 \cos^3 x}{\sin^2 x}$
 C) $\frac{2x \cos^2 x \sin x + 2x^2 \cos x \sin^2 x - x^2 \cos^2 x}{\sin^2 x}$
 D) $\frac{2x \cos^2 x \sin x + 2x^2 \cos x \sin^2 x - \cos x - x^2 \cos^2 x}{\sin x}$
 E) $\frac{2x \cos^2 x \sin x + 2x^2 \sin^2 x - \cos x - x^2 \cos^2 x}{\sin^2 x}$

Ans: B

Difficulty: Hard

Section: 2.6

10. Answer true or false. If $y = \cos(x^6)$, $d^2y/dx^2 = -\cos(x^6)$.

Ans: False

Difficulty: Easy

Section: 2.6

11. Answer true or false. If $y = \sin 6x - \cos x^3$, $d^2y/dx^2 = 18 \cos x$.

Ans: False

Difficulty: Easy

Section: 2.6

12. Find an equation for the tangent line to the graph of $y = x \sin x$ at $x = \pi$.

A) $y = -x + \pi$

D) $y = x + \pi$

B) $y = x - \pi$

E) $y = -\pi x + \pi^2$

C) $y = \pi x - \pi^2$

Ans: E

Difficulty: Medium

Section: 2.6

13. $y = \sin^3(\pi - 9\theta)$. Find $dy/d\theta$.

A) $-27 \cos(\pi - 9\theta) \sin^2(\pi - 9\theta)$

D) $-\cos(\pi - 9\theta) \sin^2(\pi - 9\theta)$

B) $-27 \cos(\pi + 9\theta) \sin^2(\pi + 9\theta)$

E) $-27 \sin^2(\pi - 9\theta)$

C) $-27 \cos^2(\pi - 9\theta) \sin(\pi - 9\theta)$

Ans: A

Difficulty: Medium

Section: 2.6

14. Use a graphing utility to obtain the graph of $f(x) = x^8 \sqrt[4]{x}$. Determine the slope of the tangent line to the graph at $x = 0$.

A) $\frac{\sqrt{7}}{4}$ B) 0 C) 3 D) 16 E) 8

Ans: B

Difficulty: Medium

Section: 2.6

15. Find the value of the constant A in $y = A \sin 3t$ so that $d^2y/dt^2 + 7y = \sin 3t$.

A) $\frac{1}{2}$ B) $\frac{1}{16}$ C) $-\frac{1}{2}$ D) 2 E) 16

Ans: C

Difficulty: Hard

Section: 2.6

16. Answer true or false. Given $f'(x) = 5x$ and $g(x) = \sqrt{5x}$, then $F'(x) = 5x\sqrt{5x}$ if $F(x) = f(g(x))$.

Ans: False

Difficulty: Medium

Section: 2.6

17. Find $f'(x)$ if $f(x) = (x^5 - 11)^{10}$.

A) $10x^4(x^5 - 11)^9$

D) $50x^4(x^5 - 11)^9$

B) $5x^4(x^5 - 11)^9$

E) $10(x^5 - 11)^9$

C) $50(x^5 - 11)^9$

Ans: D

Difficulty: Medium

Section: 2.6

18. Find dy/dx if $y = \cos(\sin x)$.

A) $-\cos x \sin(\cos x)$

D) $-\cos x \sin(\sin x)$

B) $-\sin(\sin x)$

E) $\sin(\cos x)$

C) $\cos x \sin(\sin x)$

Ans: D

Difficulty: Medium

Section: 2.6

19. Find $f'(x)$ where $f(x) = x^2(\sin 8x)^6$.

Ans: $f'(x) = 2x (\sin 8x)^6 + 48x^2 (\cos 8x)(\sin 8x)^5$

Difficulty: Medium

Section: 2.6

20. Find $f'(x)$ where $f(x) = \frac{5}{(x^2 - 2x + 5)^5}$.

$f'(x) = \frac{-25(2x - 2)}{(x^2 - 2x + 5)^6}$

Ans:

Difficulty: Medium

Section: 2.6

21. Find $f'(x)$ where $f(x) = \cos(\tan 4x)$.

Ans: $-4 \sin(\tan 4x) \sec^2 4x$

Difficulty: Medium

Section: 2.6

22. Find $f'(\theta)$ where $f(\theta) = (\theta + \sin 2\theta)^2$.

Ans: $2(\theta + \sin 2\theta)(1 + 2 \cos 2\theta)$

Difficulty: Medium

Section: 2.6

23. Find $f'(\theta)$ where $f(\theta) = \cos^2(6\theta^6 - \theta)^6$.

Ans: $-12(6\theta^6 - \theta)^5(36\theta^5 - 1)\cos(6\theta^6 - \theta)^6 \sin(6\theta^6 - \theta)^6$

OR:

$-12(6\theta^6 - \theta)^5(36\theta^5 - 1)\cos 2(6\theta^6 - \theta)^6$

Difficulty: Hard

Section: 2.6

24. Find $f'\left(\frac{\pi}{20}\right)$ where $f(x) = \sin^6 5x$.

Ans: $f'(x) = 30\sin^5 5x \cos 5x$

$f'\left(\frac{\pi}{20}\right) = \frac{15}{4}$

Difficulty: Medium

Section: 2.6

25. Find $f'(x)$ where $f(x) = \csc^4 4x$.

Ans: $-16 \csc^4 4x \cot 4x$

Difficulty: Medium

Section: 2.6

26. Find $f'(x)$ where $f(x) = \sec^2(7x - x^2)$.

Ans: $2(7 - 2x)\sec^2(7x - x^2)\tan(7x - x^2)$

Difficulty: Medium

Section: 2.6

27. Find $f'(x)$ where $f(x) = (x^5 - 3)^7(x^5 + 2)^4$.

Ans: $35x^4(x^5 - 3)^6(x^5 + 2)^4 + 20x^4(x^5 + 2)^3(x^5 - 3)^7 = 5x^4(x^5 - 3)^6(x^5 + 2)^3(11x^5 + 2)$

Difficulty: Hard

Section: 2.6

28. Find $\frac{dy}{dx}$ where $y = (6x + 3)^6(2x + 3)^4$.
 Ans: $36(6x + 3)^5(2x + 3)^4 + 8(6x + 3)^6(2x + 3)^3 = 4(6x + 3)^5(2x + 3)^3(30x + 33)$
 Difficulty: Medium
 Section: 2.6

29. Find $\frac{dy}{dx}$ where $y = \left(\frac{x+5}{x-5}\right)^2$
 $\frac{20(x+5)}{(x-5)^3}$
 Ans:
 Difficulty: Medium
 Section: 2.6

30. Find $y'(\pi)$ where $y = \left(\frac{1}{x} + \sin x\right)^{-1}$.
 Ans: $\pi^2 + 1$
 Difficulty: Hard
 Section: 2.6

31. Find $f'(t)$ where $f(t) = \left(\frac{1}{t} + \frac{1}{t^2}\right)^4$.
 $f'(t) = 4\left(\frac{1}{t} + \frac{1}{t^2}\right)^3\left(-\frac{1}{t^2} - \frac{2}{t^3}\right)$
 Ans:
 Difficulty: Hard
 Section: 2.6

32. Find equations for the tangent and normal lines to the graph of $f(x) = \sin(16 - x^2)$ at $x = 4$.
 Ans: Tangent: $y = -8x + 32$
 Normal: $y = \frac{1}{8}x - \frac{1}{2}$
 Difficulty: Hard
 Section: 2.6

33. Find equations for the tangent and normal lines to the graph of $f(x) = \cos(4 - x^2)$ at $x = 2$.
 Ans: Tangent: $y = 1$
 Normal: $x = 2$
 Difficulty: Easy
 Section: 2.6

34. Find equations for the tangent and normal lines to the graph of $f(x) = x \cos 6x$ at $x = \pi/6$.

Ans: Tangent: $y = -x$

Normal: $y = x - \frac{\pi}{3}$

Difficulty: Medium

Section: 2.6

35. Find $f'(x)$ where $f(x) = (x^6 + 6x)^{11}$.

Ans: $11(x^6 + 6x)^{10}(6x^5 + 6)$

Difficulty: Medium

Section: 2.6

36. Find $f'(x)$ where $f(x) = \sqrt{x^6 + 7x + 8}$.

$$f'(x) = \frac{6x^5 + 7}{2\sqrt{x^6 + 7x + 8}}$$

Ans:

Difficulty: Medium

Section: 2.6

37. $y = x^3 \cos(3x)$. Find dy/dx .

Ans: $3x^2 \cos 3x - 3x^3 \sin 3x$

Difficulty: Medium

Section: 2.6

38. Find $f'(x)$ if $f(x) = \sqrt{x^8 + 7x^4 + 7}$.

$$f'(x) = \frac{8x^7 + 28x^3}{2\sqrt{x^8 + 7x^4 + 7}}$$

Ans:

Difficulty: Medium

Section: 2.6

39. Find $f'(x)$ if $f(x) = \cos(x^6)$.

Ans: $-6x^5 \sin(x^6)$

Difficulty: Medium

Section: 2.6

40. Given that $f(1) = 5$, $f'(1) = 4$ and $g(x) = (f(x))^{-4}$, find $g'(1)$.

$$-\frac{16}{3,125}$$

Ans:

Difficulty: Hard

Section: 2.6

41. Find $(f \circ g)'(3)$ if $f'(3) = 5$, $g(3) = 3$ and $g'(3) = 2$.

Ans: $(f \circ g)'(x) = f'(g(x))g'(x) = 5 \cdot 2 = 10$

Difficulty: Medium

Section: 2.6

42. Find $f'(x)$ if $f(x) = \frac{1}{(x-1)^6}$.

$$f(x) = \frac{-6}{(x-1)^7}$$

Ans:

Difficulty: Easy

Section: 2.6