Calculus Single and Multivariable 8th edition Hughes-Hallett Test Bank

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***Calculus Single and Multivariable, 8e* (Hughes-Hallett)**
Chapter 2 Key Concept: The Derivative

2.1 How do We Measure Speed?

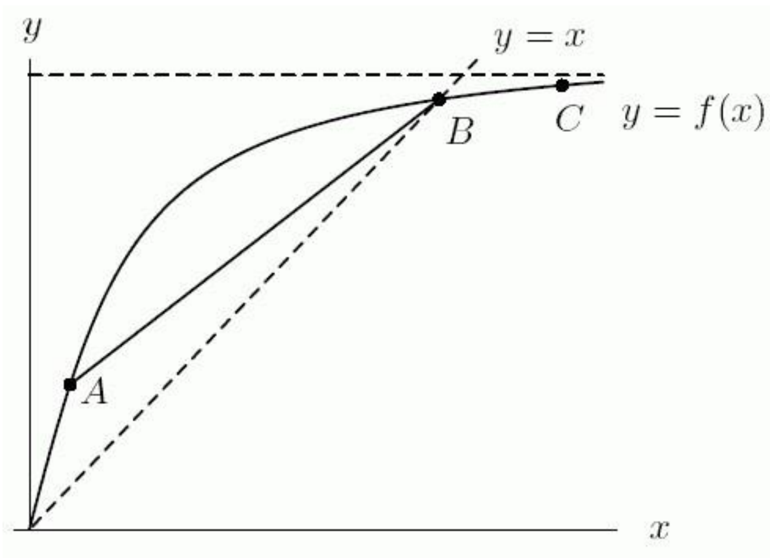
1) For any number *r*, let *m*(*r*) be the slope of the graph of the function $y = (2.1)^x$ at the point $x = r.$ Estimate *m*(7) to 2 decimal places.
Answer: 133.63
Diff: 2 Type: SA
Learning Objective: LO 2.1.4 Estimate slope of a curve.
Section: 2.1

2) If *x*(*V*) = $V^{1/3}$ is the length of the side of a cube in terms of its volume, *V*, calculate the average rate of change of *x* with respect to *V* over the interval $5 < V < 6$ to 2 decimal places.
Answer: 0.11
Diff: 1 Type: SA
Learning Objective: LO 2.1.5 Estimate limits of difference quotients as h approaches 0 numerically.
Section: 2.1

3) The length, *x*, of the side of a cube with volume *V* is given by $x(V) = V^{1/3}.$ Is the average rate of change of *x* with respect to *V* increasing or decreasing as the volume *V* decreases?
Answer: increasing
Diff: 2 Type: SA
Learning Objective: LO 2.1.4 Estimate slope of a curve.
Section: 2.1

4) If the graph of $y = f(x)$ is shown below, arrange the following in ascending order with 1 representing the smallest value and 6 the largest.

A. $f'(A)$ B. $f'(B)$ C. $f'(C)$ D. slope of $\overline{AB}$ E. 1 F. 0



Answer: Part A: 6
Part B: 3
Part C: 2
Part D: 4
Part E: 5
Part F: 1
Diff: 2 Type: ES
Learning Objective: LO 2.1.4 Estimate slope of a curve.
Section: 2.1

5) The height of an object in feet above the ground is given in the following table. Compute the average velocity over the interval $4 \le t \le 6$.

t (sec)	0	1	2	3	4	5	6
y (feet)	10	45	70	85	90	85	70

Answer: -10
Diff: 1 Type: SA
Learning Objective: LO 2.1.7 Find average velocity.
Section: 2.1

6) The height of an object in feet above the ground is given in the following table. If heights of the object are cut in half, how does the average velocity change, over a given interval?

t (sec)	0	1	2	3	4	5	6
y (feet)	10	45	70	85	90	85	70

A) It is cut in half.
B) It is doubled.
C) It remains the same.
D) It depends on the interval.
Answer: A
Diff: 2 Type: BI
Learning Objective: LO 2.1.7 Find average velocity.
Section: 2.1

7) The height of an object in feet above the ground is given in the following table, $y = f(t)$. Make a graph of $f(t)$. On your graph, what does the average velocity over the interval $1 \leq t \leq 4$ represent?

t (sec)	0	1	2	3	4	5	6
y (feet)	10	45	70	85	90	85	70

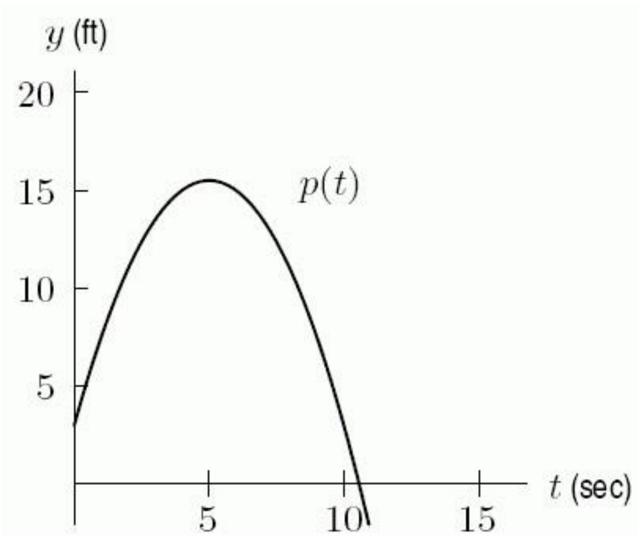
A) The average height between $f(1)$ and $f(4)$.
B) The slope of the line between the points $(1, f(1))$, and $(4, f(4))$.
C) The average of the slopes of the tangent lines to the points $(1, f(1))$, and $(4, f(4))$.
D) The distance between the points $(1, f(1))$, and $(4, f(4))$.
Answer: B
Diff: 2 Type: BI
Learning Objective: LO 2.1.7 Find average velocity.
Section: 2.1

8) The graph of $p(t)$, in the following figure, gives the position of a particle p at time t. List the following quantities in order, smallest to largest with 1 representing the smallest value.

A. Average velocity on $1 \le t \le 10$.

B. Instantaneous velocity at $t = 1$.

C. Instantaneous velocity at $t = 10$.



Answer: Part A: 2
Part B: 3
Part C: 1
Diff: 2 Type: ES
Learning Objective: LO 2.1.2 Explain how to interpret instantaneous velocity as a slope on the position-versus-time graph.
Section: 2.1

9) Estimate $\lim\limits_{h \to 0} \frac{(5+h)^2 - 25}{h}$ to 2 decimal places by substituting smaller and smaller values of h.
Answer: 10
Diff: 1 Type: SA
Learning Objective: LO 2.1.5 Estimate limits of difference quotients as h approaches 0 numerically.
Section: 2.1

10) Estimate $\lim_{h \to 0} \frac{\sin(h^2)}{h}$ to 2 decimal places by substituting smaller and smaller values of h (use radians).
Answer: 0
Diff: 1 Type: SA
Learning Objective: LO 2.1.6 Find limits of difference quotients as h approaches 0 algebraically.
Section: 2.1

11) A runner planned her strategy for running a half marathon, a distance of 13.1 miles. She planned to run negative splits, faster speeds as time passed during the race. In the actual race, she ran the first 3 miles in 18 minutes, the second 7 miles in 49 minutes and the last 3.1 miles in 25 minutes. What was her average velocity over the first 3 miles? What was her average velocity over the entire race? Did she run negative splits?
A) 10.00 mph for the first 5 miles, 8.54 mph for the race, No
B) 8.54 mph for the first 5 miles, 10.00 mph for the race, No
C) 8.12 mph for the first 5 miles, 7.35 mph for the race, Yes
D) 7.35 mph for the first 5 miles, 8.12 mph for the race, No
Answer: A
Diff: 2 Type: BI
Learning Objective: LO 2.1.7 Find average velocity.
Section: 2.1

12) Let $f(x) = x^{2/3}$. Use a graph to decide which one of the following statements is true.
A) When $x = -10$, the derivative is negative; when $x = 10$, the derivative is positive; and as x approaches infinity, the derivative approaches 0.
B) When $x = -8$, the derivative is positive; when $x = 8$, the derivative is also positive, and as x approaches infinity, the derivative approaches 0.
C) When $x = -2$, the derivative is negative; when $x = 2$, the derivative is positive, and as x approaches infinity, the derivative approaches infinity.
D) The derivative is positive at at all values of x.
Answer: A
Diff: 1 Type: MC
Learning Objective: LO 2.1.1 Derive instantaneous velocity as a limit of average velocities.
Section: 2.1

2.2 The Derivative at a Point

1) Given the following data about a function f, estimate $f'(5.75)$.

x	3	3.5	4	4.5	5	5.5	6
$f(x)$	10	8	7	4	2	0	-1

Answer: -2
Diff: 2 Type: SA
Learning Objective: LO 2.2.5 Find derivatives algebraically.
Section: 2.2

2) Given the following data about a function $f(x)$, the equation of the tangent line at $x = 5$ is approximated by

x	3	3.5	4	4.5	5	5.5	6
$f(x)$	10	8	7	4	2	0	-1

A) $y - 5 = -4(x - 2)$
B) $y - 5 = -8(x - 2)$
C) $y - 2 = -4(x - 5)$
D) $y - 2 = -8(x - 5)$
Answer: C
Diff: 2 Type: BI
Learning Objective: LO 2.2.6 Find equation of tangent line.
Section: 2.2

3) For $f(x) = 5^{-x}$, estimate $f'(0)$ to 3 decimal places.
Answer: -1.609
Diff: 2 Type: SA
Learning Objective: LO 2.2.4 Estimate derivative numerically.
Section: 2.2

4) Let $f(x) = \log(\log(x))$. Estimate $f'(8)$ to 3 decimal places using any method.
Answer: 0.026
Diff: 3 Type: SA
Learning Objective: LO 2.2.4 Estimate derivative numerically.
Section: 2.2

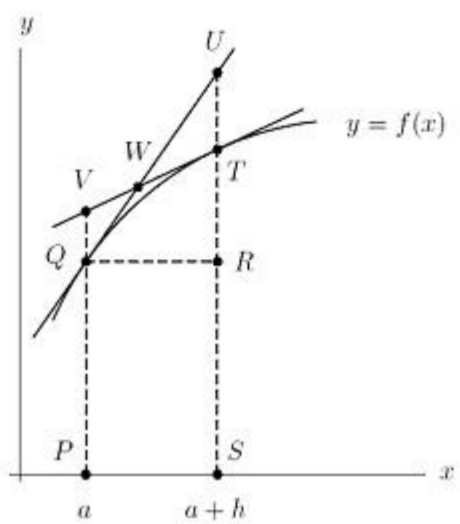
5) For $f(x) = \log(x)$, estimate $f'(2)$ to 3 decimal places by finding the average slope over intervals containing the value $x = 2$.
Answer: 0.217
Diff: 2 Type: SA
Learning Objective: LO 2.2.4 Estimate derivative numerically.
Section: 2.2

6) There is a function used by statisticians, called the error function, which is written $y = \mathrm{erf}(x)$. Suppose you have a statistical calculator, which has a button for this function. Playing with your calculator, you discover the following:

x	**erf(*x*)**
1	0.29793972
0.1	0.03976165
0.01	0.00398929
0.001	0.000398942
0	0

Using this information alone, give an estimate for erf′(0), the derivative of erf at $x = 0$ to 4 decimal places.
Answer: 0.3989
Diff: 2 Type: SA
Learning Objective: LO 2.2.4 Estimate derivative numerically.
Section: 2.2

7) In the picture



the quantity $f'(a+h)$ is represented by
A) the slope of the line TV
B) the area of the rectangle PQRS
C) the slope of the line *RU*.
D) the length of the line *TV*.
E) the slope of the line *QU*.
F) the length of the line *QU*.
Answer: A
Diff: 2 Type: BI
Learning Objective: LO 2.2.3 Visualize the derivative as the slope of the curve and the slope of the tangent line.
Section: 2.2

8) Given the following table of values for a Bessel function, $J_0(x)$, estimate the derivative at $x = 0.8$.

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$J_0(x)$	1.0	.9975	.9900	.9776	.9604	.9385	.9120	.8812	.8463	.8075	.7652

Hint: Average the left and right difference quotients.
Answer: -0.3685
Diff: 2 Type: SA
Learning Objective: LO 2.2.4 Estimate derivative numerically.
Section: 2.2

9) The data in the table report the average improvement in scores of six college freshmen who took a writing assessment before and again after they had x hours of tutoring by a tutor trained in a new method of instruction. When $f(x) > 0$ the group showed improvement on average.

x	2	3.5	5	6.5	8	9.5	11
$f(x)$	-2	-1	0	3	7	9	10

a) Find the average change in score from 6.5 to 9.5 hours of tutoring.
b) Estimate the instantaneous rate of change at 8 hours.
c) Approximate the equation of the tangent line at x = 8 hours.
d) Use the tangent line to estimate f (8.5).
Answer: a) 2.00 points; b) 2.00 points, but answers may vary; c) $y - 7 = 2.00(x - 8)$;
d) 8 points
Diff: 2 Type: ES
Learning Objective: LO 2.2.1 Explain the concept of average rate of change.; LO 2.2.6 Find equation of tangent line.
Section: 2.2

10) Use the graph of $3.5e^{3x}$ at the point (0, 3.5) to estimate $f'(0)$ to three decimal places.

Hint: Zoom in on the graph at the point $(0, 3.5)$.
A) 10.500
B) 23.433
C) 3.500
D) 0.363
Answer: A
Diff: 1 Type: BI
Learning Objective: LO 2.2.4 Estimate derivative numerically.
Section: 2.2

11) A horticulturist conducted an experiment to determine the effects of different amounts of fertilizer on the yield of a plot of green onions. He modeled his results with the function $Y(x) = -0.5(x - 2)^2 + 5$ where Y is the yield in bushels and x is the amount of fertilizer in pounds. What are $Y(1.25)$ and $Y'(1.25)$? Give your answers to two decimal places, specify units.
A) 4.72 bushels, 0.75 bushels/pound, respectively
B) 4.72 bushels, 5.75 bushels/pound, respectively
C) 0.75 bushels, 0.56 bushels/pound, respectively
D) 0.75 bushels, 4.72 bushels/pound, respectively
Answer: A
Diff: 2 Type: BI
Learning Objective: LO 2.2.4 Estimate derivative numerically.; LO 2.2.5 Find derivatives algebraically.
Section: 2.2

12) Use the limit of the difference quotient to find the derivative of $g(x) = \frac{3}{x + 1}$ at the point (1, 3/2).
A) $-\frac{3}{4}$

B) $\frac{3}{2}$

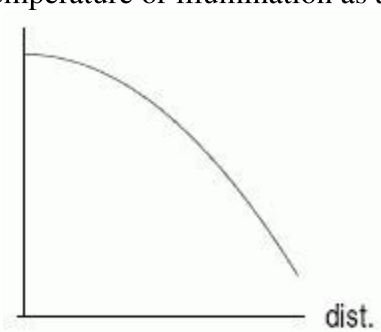
C) -3
D) $-\frac{3}{2}$

Answer: A
Diff: 2 Type: BI
Learning Objective: LO 2.2.1 Explain the concept of average rate of change.
Section: 2.2

13) Alone in your dim, unheated room you light one candle rather than curse the darkness. Disgusted by the mess, you walk directly away from the candle. The temperature (in $^\circ F$) and illumination (in % of one candle power) decrease as your distance (in feet) from the candle increases. The table below shows this information.

distance (feet)	**Temp. (° F)**	**illumination (%)**
0	55	100
1	54.5	85
2	53.5	75
3	52	67
4	50	60
5	47	56
6	43.5	53

Does the following graph show temperature or illumination as a function of distance?



Answer: temperature
Diff: 1 Type: SA
Learning Objective: LO 2.2.1 Explain the concept of average rate of change.
Section: 2.2

14) Alone in your dim, unheated room you light one candle rather than curse the darkness. Disgusted by the mess, you walk directly away from the candle. The temperature (in $^\circ F$) and illumination (in % of one candle power) decrease as your distance (in feet) from the candle increases. The table below shows this information.

distance(feet)	**Temp. (° F)**	**illumination (%)**
0	55	100
1	54.5	85
2	53.5	75
3	52	67
4	50	60
5	47	56
6	43.5	53

What is the average rate at which the temperature is changing (in degrees per foot) when the illumination drops from 67% to 53%? Round to 2 decimal places.
Answer: 2.83
Diff: 2 Type: SA
Learning Objective: LO 2.2.1 Explain the concept of average rate of change.
Section: 2.2

15) Alone in your dim, unheated room you light one candle rather than curse the darkness. Disgusted by the mess, you walk directly away from the candle. The temperature (in $^\circ F$) and illumination (in % of one candle power) decrease as your distance (in feet) from the candle increases. The table below shows this information.

distance(feet)	**Temp. (° F)**	**illumination (%)**
0	55	100
1	54.5	85
2	53.5	75
3	52	67
4	50	60
5	47	56
6	43.5	53

You can still read your watch when the illumination is about 64%, so somewhere between 3 and 4 feet. Can you read your watch at 3.5 feet?
A) yes
B) no
C) cannot tell
Answer: B
Diff: 2 Type: BI
Learning Objective: LO 2.2.1 Explain the concept of average rate of change.
Section: 2.2

16) Alone in your dim, unheated room you light one candle rather than curse the darkness. Disgusted by the mess, you walk directly away from the candle. The temperature (in $°F$) and illumination (in % of one candle power) decrease as your distance (in feet) from the candle increases. The table below shows this information.

distance(feet)	Temp. (° F)	illumination (%)
0	55	100
1	54.5	85
2	53.5	75
3	52	67
4	50	60
5	47	56
6	43.5	53

Suppose you know that at 6 feet the instantaneous rate of change of the illumination is -4.5 % candle power/ft. At 7 feet, the illumination is approximately _____ % candle power.
Answer: 48.5
Diff: 2 Type: SA
Learning Objective: LO 2.2.1 Explain the concept of average rate of change.
Section: 2.2

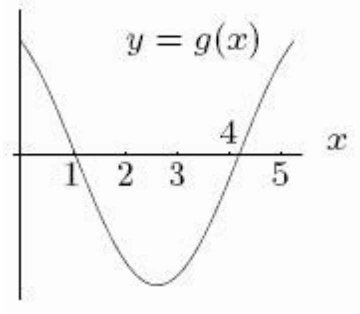
17) Alone in your dim, unheated room you light one candle rather than curse the darkness. Disgusted by the mess, you walk directly away from the candle. The temperature (in $°F$) and illumination (in % of one candle power) decrease as your distance (in feet) from the candle increases. The table below shows this information.

distance(feet)	Temp. (° F)	illumination (%)
0	55	100
1	54.5	85
2	53.5	75
3	52	67
4	50	60
5	47	56
6	43.5	53

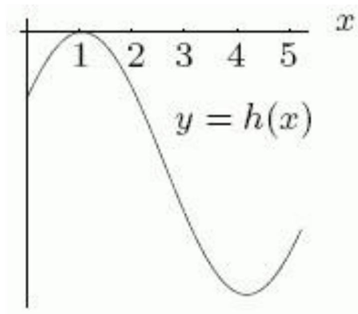
You are cold when the temperature is below 40° F. You are in the dark when the illumination is at most 50% of one candle power. Suppose you know that at 6 feet the instantaneous rate of change of the temperature is -4.5° F/ft and the instantaneous rate of change of illumination is -3% candle power/ft. Are you in the dark before you are cold, or cold before you are dark?
A) You are cold before you are in the dark.
B) You are in the dark before you are cold.
Answer: A
Diff: 2 Type: BI
Learning Objective: LO 2.2.1 Explain the concept of average rate of change.
Section: 2.2

2.3 The Derivative Function

1) Could the first graph, A be the derivative of the second graph, B?



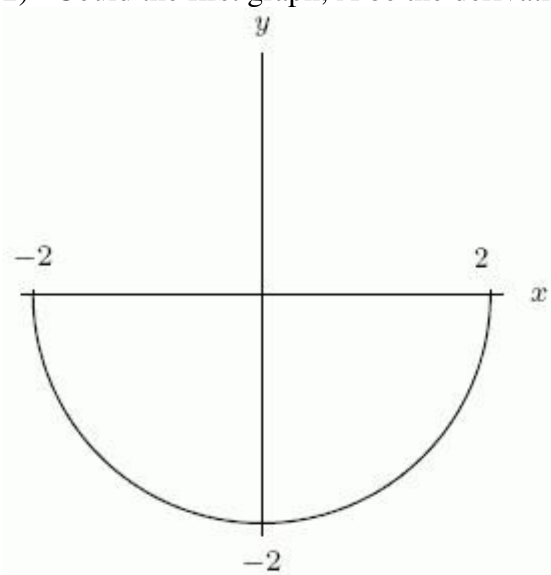
A



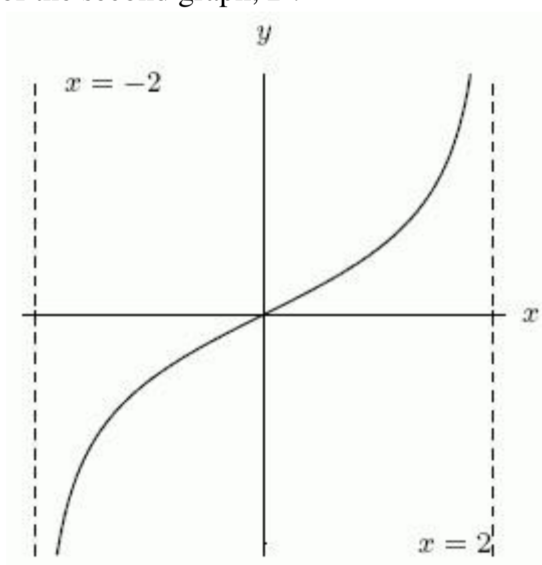
B

Answer: yes
Diff: 2 Type: SA
Learning Objective: LO 2.3.1 Understand the concept of the derivative function.
Section: 2.3

2) Could the first graph, A be the derivative of the second graph, B?



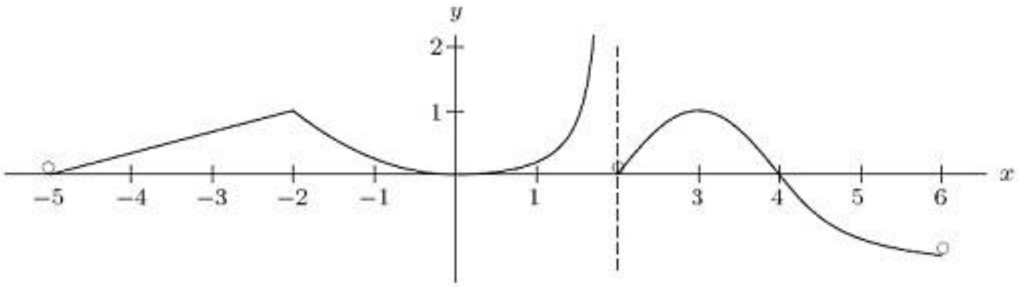
A



B

Answer: no
Diff: 2 Type: SA
Learning Objective: LO 2.3.1 Understand the concept of the derivative function.
Section: 2.3

3) Consider the function $y = f(x)$ graphed below. At the point $x = 3$, is $f'(x)$ positive, negative, 0, or undefined? Note: $f(x)$ is defined for $-5 < x < 6$, *except* $x = 2$.



Answer: 0
Diff: 2 Type: SA
Learning Objective: LO 2.3.6 Use the derivative to analyze a function.
Section: 2.3

4) Estimate a formula for $f'(x)$ for the function $f(x) = 6^x$. Round constants to 3 decimal places.

A) $(1.792)6^x$

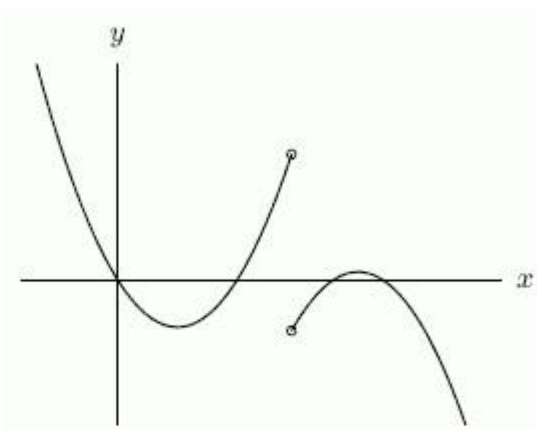
B) $(1.285)6^x$

C) $(x)6^{(x-1)}$

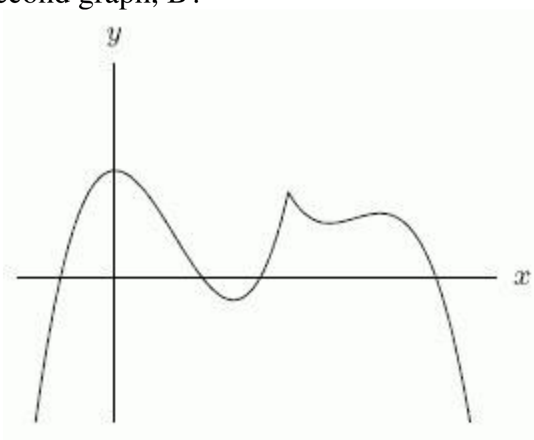
D) $(x-1)6^x$

Answer: A
Diff: 3 Type: BI
Learning Objective: LO 2.3.4 Find the formula of the derivative function by computing a limit.
Section: 2.3

5) Could the first graph, A be the derivative of the second graph, B?



A



B

Answer: yes
Diff: 2 Type: SA
Learning Objective: LO 2.3.2 Sketch the graph of the derivative from the graph of the function.
Section: 2.3

6) Find the derivative of $2x^2 + 4x - 7$ at $x = 6$ algebraically.

Answer: 28

Diff: 2 Type: SA

Learning Objective: LO 2.3.1 Understand the concept of the derivative function.

Section: 2.3

7) To find the derivative of $g(x) = 4x^2 + 3x - 3$ at $x = 3$ algebraically, you evaluate the following expression.

A) $\lim\limits_{h \to 0} \dfrac{4(3 + h)^2 + 3(3 + h) - 3 - (4 \cdot 3^2 + 3 \cdot 3 - 3)}{h}$

B) $\lim\limits_{h \to 0} \dfrac{g(3 + 1) + g(3)}{h}$

C) $\lim\limits_{h \to 0} \dfrac{g(h) - g(3)}{h}$

D) All of the above are correct.

E) None of the above is correct.

Answer: A

Diff: 2 Type: BI

Learning Objective: LO 2.3.4 Find the formula of the derivative function by computing a limit.

Section: 2.3

8) Find the derivative of $m(x) = 5x^3$ at $x = 5$ algebraically.

Answer: 375

Diff: 2 Type: SA

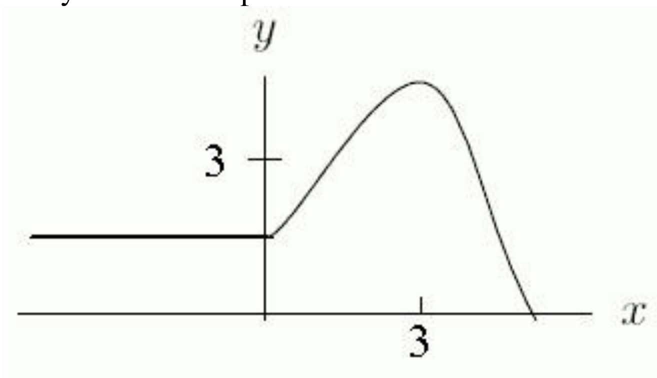
Learning Objective: LO 2.3.5 Apply the Power Rule.

Section: 2.3

9) Draw the graph of a continuous function $y = g(x)$ that satisfies the following three conditions:

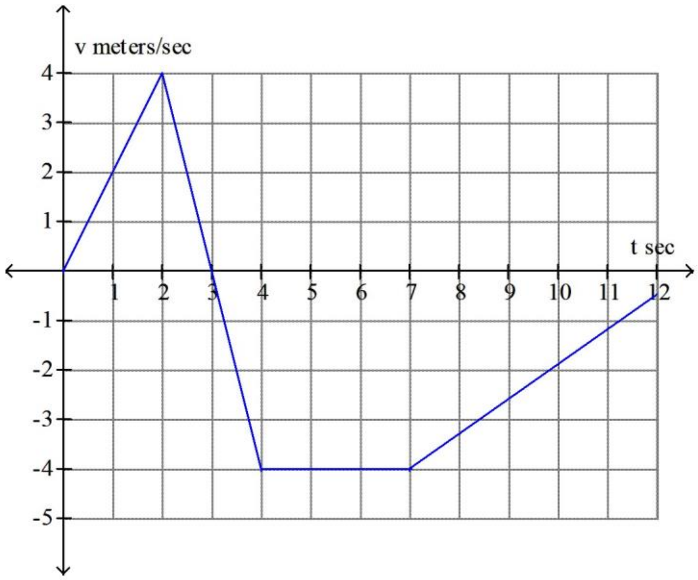
- $g'(x) = 0$ for $x < 0$
- $g'(x) > 0$ for $0 < x < 3$
- $g'(x) < 0$ for $x > 3$

Answer: Answers will vary. One example:



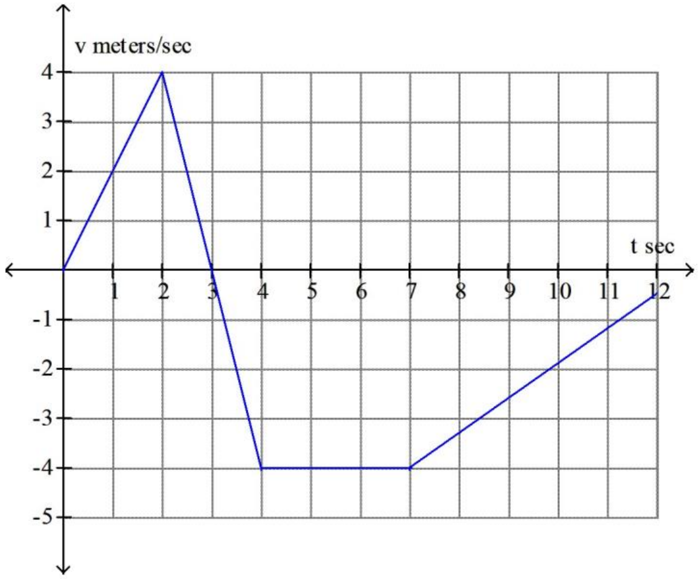
Diff: 2 Type: ES
Learning Objective: LO 2.3.3 Understand how the shape of the graph of a function affects the shape of the graph of its derivative.
Section: 2.3

10) The graph below shows the velocity of a bug traveling along a straight line on the classroom floor.



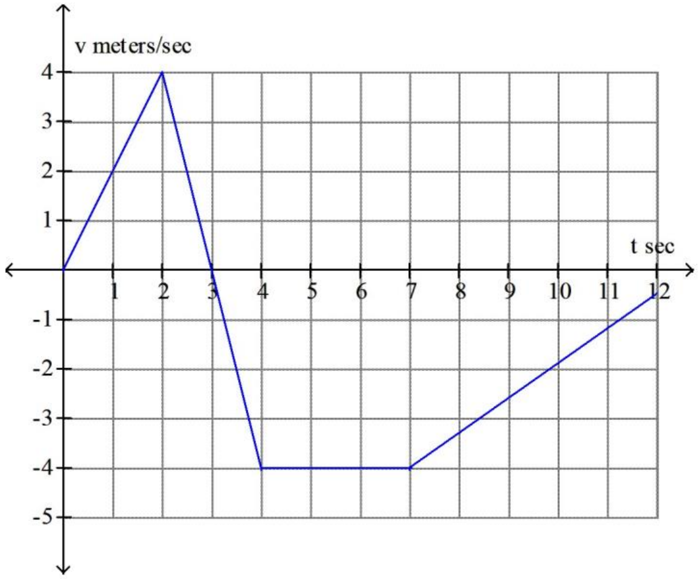
At what time(s) does the bug turn around?
A) At 3 seconds.
B) At 2 seconds and again at 7 seconds.
C) At 4 seconds and again at 7 seconds.
D) Never.
Answer: A
Diff: 1 Type: BI
Learning Objective: LO 2.3.6 Use the derivative to analyze a function.
Section: 2.3

11) The graph below shows the velocity of a bug traveling along a straight line on the classroom floor.

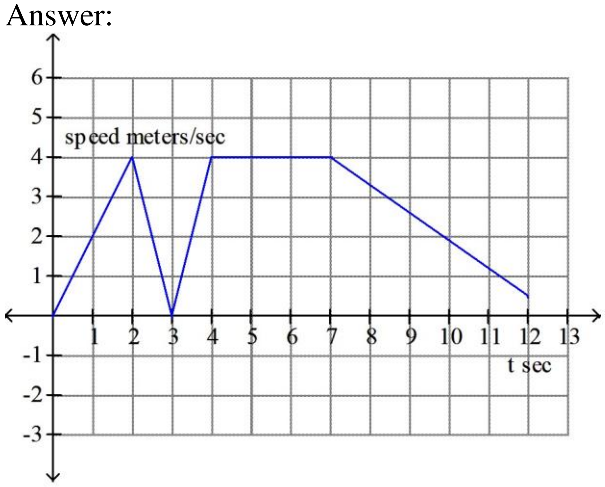


When is the bug moving at a constant speed?
A) Between 4 and 7 seconds.
B) Whenever the velocity is linear with a positive slope.
C) Whenever the velocity is linear with a negative slope.
D) When the velocity is equal to zero.
Answer: A
Diff: 1 Type: BI
Learning Objective: LO 2.3.6 Use the derivative to analyze a function.
Section: 2.3

12) The graph below shows the velocity of a bug traveling along a straight line on the classroom floor.



Graph the bug's speed at time, *t*. How does it differ from the bug's velocity?
Answer:



Speed is always non-negative, but has the same magnitude as the velocity.
Diff: 2 Type: ES
Learning Objective: LO 2.3.6 Use the derivative to analyze a function.
Section: 2.3

13) Use the definition of the derivative function to find a formula for the slope of the graph of $f(t) = \frac{1}{5t + 1}$.

A) $-\frac{5}{(5t + 1)^2}$

B) $\frac{5}{(5t + 1)^2}$

C) $-\frac{5}{(5t + 1)^{-2}}$

D) $\frac{5}{(5t + 1)^{-2}}$

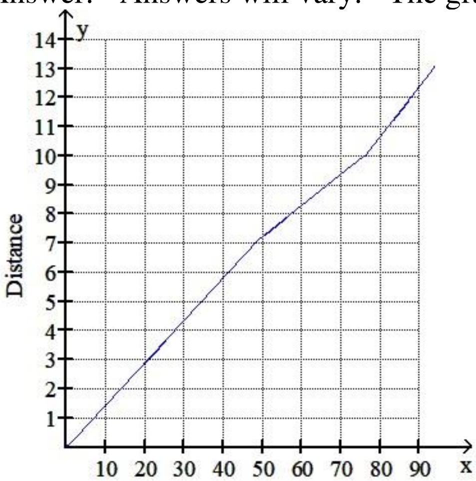
Answer: A
Diff: 3 Type: BI
Learning Objective: LO 2.3.4 Find the formula of the derivative function by computing a limit.
Section: 2.3

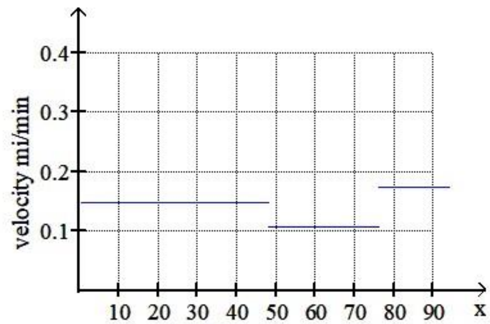
14) What is the equation of the tangent line to the graph of $f(x) = x^3$ at the point $(4, 64)$?
A) $y = 48x - 128$
B) $y = 4x + 64$
C) $y = 64x + 4$
D) $y = 16x + 4096$
Answer: A
Diff: 1 Type: BI
Learning Objective: LO 2.3.5 Apply the Power Rule.
Section: 2.3

15) The definition of the derivative function is $f'(x) = \frac{f(x + h) - f(x)}{h}$.

Answer: FALSE
Diff: 1 Type: TF
Learning Objective: LO 2.3.1 Understand the concept of the derivative function.
Section: 2.3

16) A runner competed in a half marathon in Anaheim, a distance of 13.1 miles. She ran the first 7 miles at a steady pace in 48 minutes, the second 3 miles at a steady pace in 28 minutes and the last 3.1 miles at a steady pace in 18 minutes.
a) Sketch a well-labeled graph of her distance completed with respect to time.
b) Sketch a well-labeled graph of her velocity with respect to time.
Answer: Answers will vary. The graphs below give one possibility.

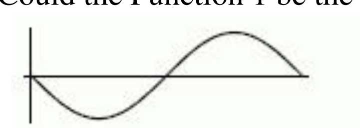




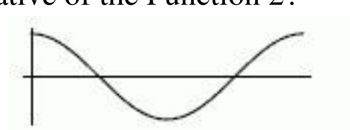
Diff: 2 Type: ES
Learning Objective: LO 2.3.2 Sketch the graph of the derivative from the graph of the function.
Section: 2.3

17) Which of the following is NOT a way to describe the derivative of a function at a point?
A) slope of the tangent line
B) slope of the curve
C) y-intercept of the tangent line
D) limit of the difference quotient
E) limit of the slopes of secant lines
F) limit of the average rates of change
Answer: C
Diff: 1 Type: BI
Learning Objective: LO 2.3.1 Understand the concept of the derivative function.
Section: 2.3

18) Could the Function 1 be the derivative of the Function 2?



Function 1



Function 2

Answer: yes
Diff: 2 Type: SA
Learning Objective: LO 2.3.6 Use the derivative to analyze a function.
Section: 2.3

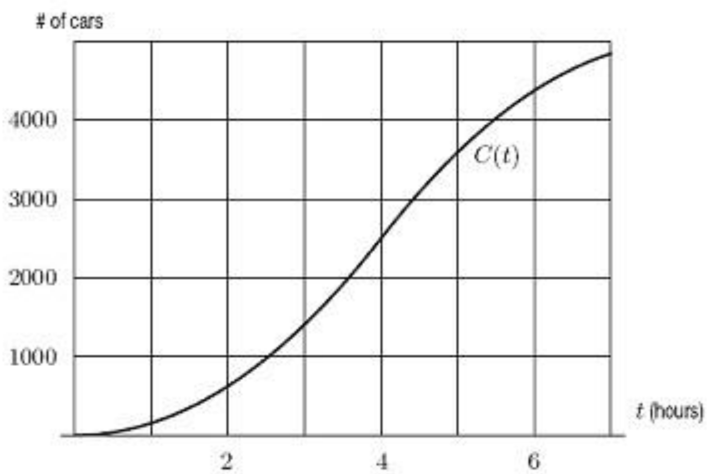
19) If $f(x) = x^5$, what is $f'(4)$?
A) 1024
B) 4
C) 1280
D) 20
E) 5
Answer: C
Diff: 1 Type: BI
Learning Objective: LO 2.2.5 Find derivatives algebraically.
Section: 2.3

20) The derivative of $f(t) = e^{\pi}$ is $\pi e^{\pi-1}$.
Answer: FALSE
Diff: 1 Type: TF
Learning Objective: LO 2.2.5 Find derivatives algebraically.
Section: 2.3

2.4 Interpretations of the Derivative

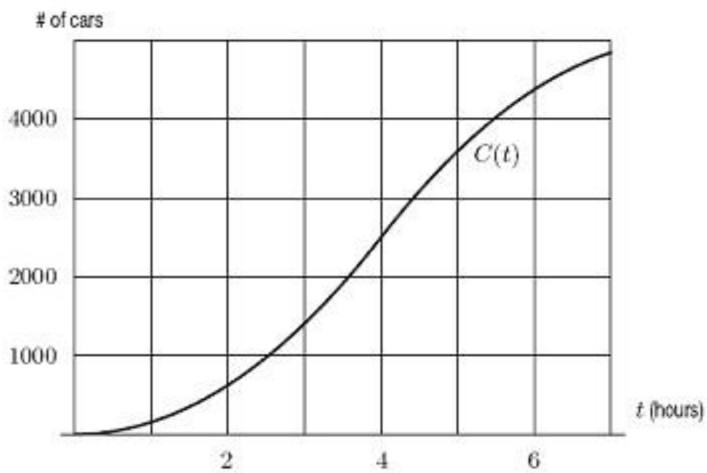
1) Suppose that $f(T)$ is the cost to heat my house, in dollars per day, when the outside temperature is $T° F$. If $f(22) = 9.59$ and $f'(22) = -0.18,$ approximately what is the cost to heat my house when the outside temperature is $19°F$?
Answer: $10.13
Diff: 1 Type: SA
Learning Objective: LO 2.4.1 Use the difference quotient to interpret the meaning of the derivative.
Section: 2.4

2) To study traffic flow along a major road, the city installs a device at the edge of the road at 3:00 a.m. The device counts the cars driving past, and records the total periodically. The resulting data is plotted on a graph, with time (in hours since installation) on the horizontal axis and the number of cars on the vertical axis. The graph is shown below; it is the graph of the function $C(t) = \text{Total}$ number of cars that have passed by after t hours. When is the traffic flow greatest?



A) 4:00 a.m.
B) 5:00 a.m.
C) 6:00 a.m.
D) 7:00 a.m.
Answer: D
Diff: 2 Type: BI
Learning Objective: LO 2.4.1 Use the difference quotient to interpret the meaning of the derivative.
Section: 2.4

3) To study traffic flow along a major road, the city installs a device at the edge of the road at 1:00 a.m. The device counts the cars driving past, and records the total periodically. The resulting data is plotted on a graph, with time (in hours since installation) on the horizontal axis and the number of cars on the vertical axis. The graph is shown below; it is the graph of the function $C(t) = \text{Total}$ number of cars that have passed by after t hours. From the graph, estimate $C'(5)$.



A) 900
B) 1200
C) 1500
D) 1800
Answer: A
Diff: 2 Type: BI
Learning Objective: LO 2.4.1 Use the difference quotient to interpret the meaning of the derivative.
Section: 2.4

4) Every day the Office of Undergraduate Admissions receives inquiries from eager high school students. They keep a running account of the number of inquiries received each day, along with the total number received until that point. Below is a table of *weekly* figures from about the end of August to about the end of October of a recent year.

Week of	**Inquiries That Week**	**Total for Year**
8/28-9/01	1085	11,928
9/04-9/08	1193	13,121
9/11-9/15	1312	14,433
9/18-9/22	1443	15,876
9/25-9/29	1588	17,464
10/02-10/06	1746	19,210
10/09-10/13	1921	21,131
10/16-10/20	2113	23,244
10/23-10/27	2325	25,569

One of these columns can be interpreted as a rate of change. Which one is it?
A) the first
B) the second
C) the third
Answer: B
Diff: 1 Type: BI
Learning Objective: LO 2.4.2 Use units to interpret the derivative.
Section: 2.4

5) Every day the Office of Undergraduate Admissions receives inquiries from eager high school students. They keep a running account of the number of inquiries received each day, along with the total number received until that point. Below is a table of *weekly* figures from about the end of August to about the end of October of a recent year.

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10/09-10/13	1921	21,131
10/16-10/20	2113	23,244
10/23-10/27	2325	25,569

Based on the table determine a formula that approximates the total number of inquiries received by a given week. Use your formula to estimate how many inquiries the admissions office will have received by November 24.

Hint: Use linear or exponential growth, whichever is appropriate.
Answer: 37,435
Diff: 2 Type: SA
Learning Objective: LO 2.4.2 Use units to interpret the derivative.
Section: 2.4

6) Let $L(r)$ be the amount of board-feet of lumber produced from a tree of radius r (measured in inches). What does $L(8)$ mean in practical terms?
A) The amount of board-feet of lumber produced from a tree with a radius of 8 inches.
B) The radius of a tree that will produce 8 board-feet of lumber.
C) The rate of change of the amount of lumber with respect to radius when the radius is 8 inches (in board-feet per inch).
D) The rate of change of the radius with respect to the amount of lumber produced when the amount is 8 board-feet (in inches per board-foot).
Answer: A
Diff: 1 Type: BI
Learning Objective: LO 2.4.1 Use the difference quotient to interpret the meaning of the derivative.
Section: 2.4

7) Let $f(d)$ be the noise level (in decibels) of a rock concert at a distance d (in meters) from the concert speakers. What does $f'(40)$ mean in practical terms?
A) The noise level (in decibels) of a rock concert at 40 meters from the concert speakers.
B) The distance (in meters) from the speakers that will result in a noise level of 40 decibels.
C) The rate of change of noise 40 meters from the concert speakers (in decibels per meter).
D) The rate of change of the distance from the concert speakers when the noise is 40decibels (in meters per decibel).
Answer: C
Diff: 1 Type: BI
Learning Objective: LO 2.4.1 Use the difference quotient to interpret the meaning of the derivative.
Section: 2.4

8) Let $t(h)$ be the temperature in degrees Celsius at a height h (in meters) above the surfaceof the earth. What does h such that $t(h) = 60$ mean in practical terms?
A) The temperature in degrees Celsius at a height 60 meters above the surface of the earth.
B) The height above the surface of the earth at which the temperature is 60 degrees Celsius.
C) The rate of change of temperature with respect to height at 60 meters above the surface of the earth (in degrees per meter).
D) The rate of change of height with respect to temperature when the temperature is 60 degrees Celsius (in meters per degree).
Answer: B
Diff: 1 Type: BI
Learning Objective: LO 2.4.1 Use the difference quotient to interpret the meaning of the derivative.
Section: 2.4

9) Let $L(r)$ be the amount of board-feet of lumber produced from a tree of radius r inches. What does $L(r) + 10$ mean in practical terms?
A) The amount of board-feet of lumber produced from a tree with a radius of r inches plus an additional 10 board-feet.
B) The radius of a tree that will produce r board-feet of lumber plus an additional 10 inches.
C) The rate of change of the amount of lumber with respect to radius when the radius is 10 additional inches in board-feet per inch.
D) The rate of change of the radius with respect to the amount of lumber produced when the amount is 10 additional board-feet in inches per board-foot.
Answer: A
Diff: 1 Type: BI
Learning Objective: LO 2.4.1 Use the difference quotient to interpret the meaning of the derivative.
Section: 2.4

10) A concert promoter estimates that the cost of printing p full color posters for a major concert is given by a function $\text{Cost} = c(p)$ where p is the number of posters produced.

a) Interpret the meaning of the statement $c(750) = 9000$.

b) Interpret the meaning of the statement $c'(750) = 10$.

Answer:

a) It costs \$9000.00 to produce 750 posters.

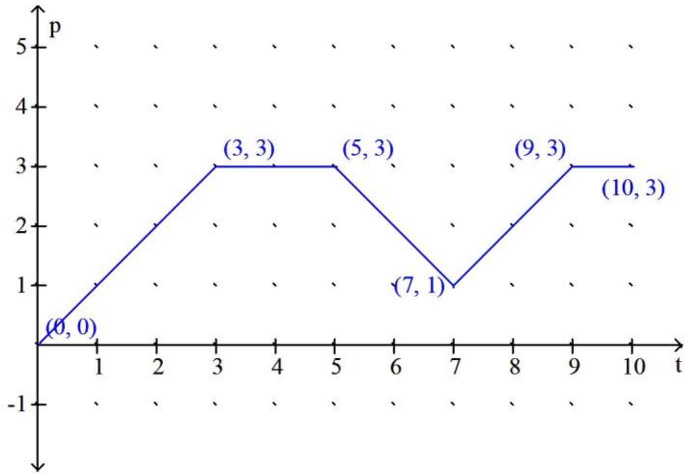
b) When 750 posters have been produced, it costs \$10.00 to produce an additional poster.

Diff: 1 Type: ES

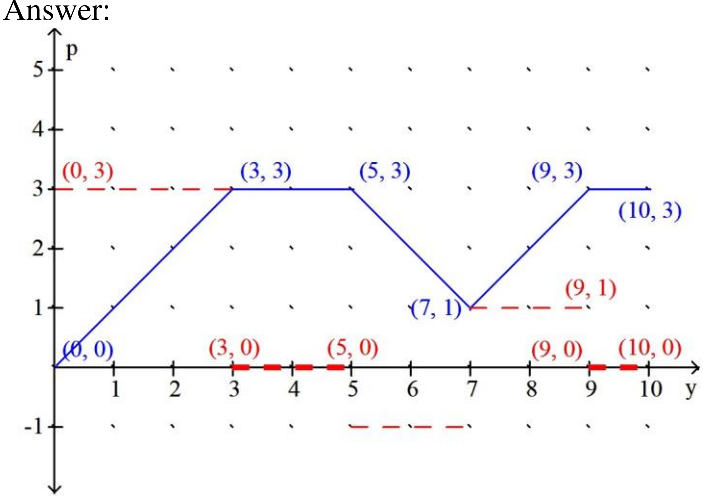
Learning Objective: LO 2.4.1 Use the difference quotient to interpret the meaning of the derivative.

Section: 2.4

11) The graph below gives the position of a spider moving along a straight line on the forest floor for 10 seconds. On the same axes, sketch a graph of the spider's velocity over the 10 seconds. Then write a description of the spider's movement for the 10 second period.



Answer:



The dashed line represents the spider's velocity.

For the first three seconds the spider moves forward at 3 feet/sec. It stops for the next 2 seconds, turns around and goes back in the opposite direction for at a speed of 1 ft/sec for 2 seconds. It turns around again and goes forward at 1 ft/sec for the next two seconds, then stops for the final second.

Diff: 2 Type: ES

Learning Objective: LO 2.4.2 Use units to interpret the derivative.

Section: 2.4

12) A typhoon is a tropical cyclone, like a hurricane, that forms in the northwestern Pacific Ocean. The wind speed of a typhoon is given by a function $W = w(r)$ where W is measured in meters/sec., and r is measured in kilometers from the center of the typhoon. What does the statement that $w'(50) < 0$ tell you about the typhoon?
A) At a distance of 50 kilometers from the center of the typhoon, the wind speed is decreasing.
B) At a distance of 50 kilometers from the center of the typhoon, the wind speed is negative.
C) The wind speed of the typhoon is 50 meters per second at any distance from the center of the typhoon.
Answer: A
Diff: 2 Type: BI
Learning Objective: LO 2.4.3 Interpret the sign of the derivative.
Section: 2.4

13) The cost in dollars to produce q bottles of a prescription skin treatment is given by the function $C(q) = 0.08q^2 + 55q + 900$. The manufacturing process is difficult and costly when large quantities are produced. The marginal cost of producing one additional bottle when q bottles have been produced is the derivative $\frac{dC}{dq}$.

a) Find the marginal cost function.

b) Compute $C(50)$ and explain what the number means in terms of cost and production.

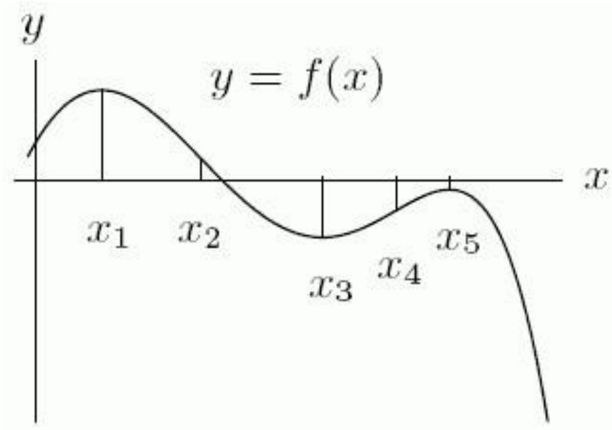
c) Compute $C'(50)$ and explain what the number means in terms of cost and production.
Answer:
a) $\frac{dC}{dq} = 0.16q + 55$

b) $C(50)$ = \$3850.00 is the cost of producing 50 bottles of the skin treatment.

c) $C'(50)$ = \$63.00 per bottle of the cost of producing an additional bottle when 50 have already been produced.
Diff: 2 Type: ES
Learning Objective: LO 2.4.3 Interpret the sign of the derivative.
Section: 2.4

2.5 The Second Derivative

1) The graph of $f(x)$ is given in the following figure. What happens to $f'(x)$ at the point x_1?



A) $f'(x)$ has an inflection point.
B) $f'(x)$ has a local minimum or maximum.
C) $f'(x)$ changes sign.
D) none of the above
Answer: C
Diff: 3 Type: BI
Learning Objective: LO 2.5.4 Use first and second derivatives to analyze a function.
Section: 2.5

2) Esther is a swimmer who prides herself in having a smooth backstroke. Let $s(t)$ be her position in an Olympic size (50-meter) pool, as a function of time ($s(t)$ is measured in meters, t is seconds). Below we list some values of $s(t)$ for a recent swim. Find Esther's average speed over the entire swim in meters per second. Round to 2 decimal places.

t	0	3.0	8.6	14.88	21.7	28.52	32.86	39.68	47.12	55.8	62
$s(t)$	0	10	20	30	40	50	40	30	20	10	0

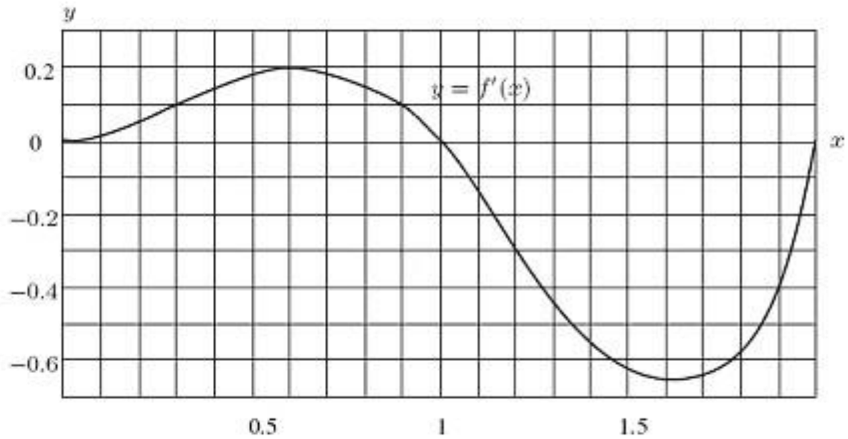
Answer: 1.61
Diff: 2 Type: SA
Learning Objective: LO 2.5.4 Use first and second derivatives to analyze a function.
Section: 2.5

3) Esther is a swimmer who prides herself in having a smooth backstroke. Let $s(t)$ be her position in an Olympic size (50-meter) pool, as a function of time ($s(t)$ is measured in meters, t is seconds). Below we list some values of $s(t)$, for a recent swim. Based on the data, was Esther's instantaneous speed ever greater than 3.2 meters/second?

t	0	3.0	8.6	14.6	20.8	27.6	31.9	38.1	45.8	53.9	60
$s(t)$	0	10	20	30	40	50	40	30	20	10	0

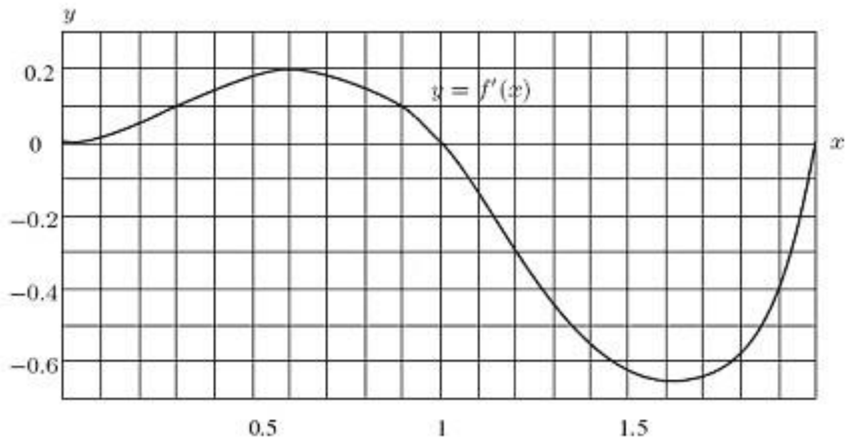
Answer: yes
Diff: 2 Type: SA
Learning Objective: LO 2.5.4 Use first and second derivatives to analyze a function.
Section: 2.5

4) The graph below represents the *rate of change* of a function f with respect to x; i.e., it is a graph of f'. You are told that $f(0) = 0$. What can you say about $f(x)$ at the point $x = 1.2$? Mark all that apply.



A) $f(x)$ is decreasing.
B) $f(x)$ is increasing.
C) $f(x)$ is concave up.
D) $f(x)$ is concave down.
Answer: A, D
Diff: 1 Type: MC
Learning Objective: LO 2.5.1 Interpret the second derivative as concavity.
Section: 2.5

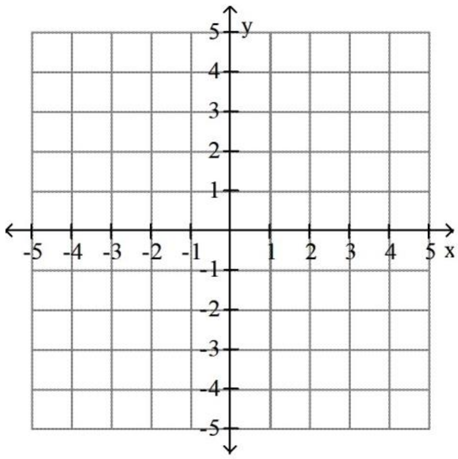
5) The graph below represents the *rate of change* of a function f with respect to x; i.e., it is a graph of f'. You are told that $f(0) = 1$. For approximately what value of x other than $x = 0$ in the interval $0 \le x \le 2$ does $f(x) = 1$?



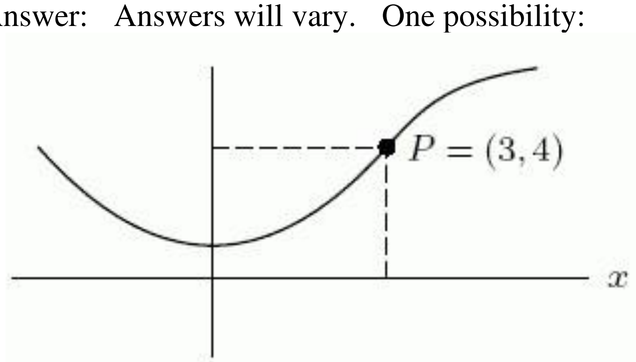
A) 0.6
B) 1
C) 1.4
D) 2
E) None of the above
Answer: C
Diff: 2 Type: BI
Learning Objective: LO 2.5.2 Interpret the second derivative as the rate of change of the first derivative.
Section: 2.5

6) On the axes below, sketch a smooth, continuous curve (i.e., no sharp corners, no breaks) which passes through the point $P(3, 4)$, and which clearly satisfies the following conditions:

- Concave up to the left of *P*
- Concave down to the right of *P*
- Increasing for $x > 0$
- Decreasing for $x < 0$
- Does *not* pass through the origin.



Answer: Answers will vary. One possibility:

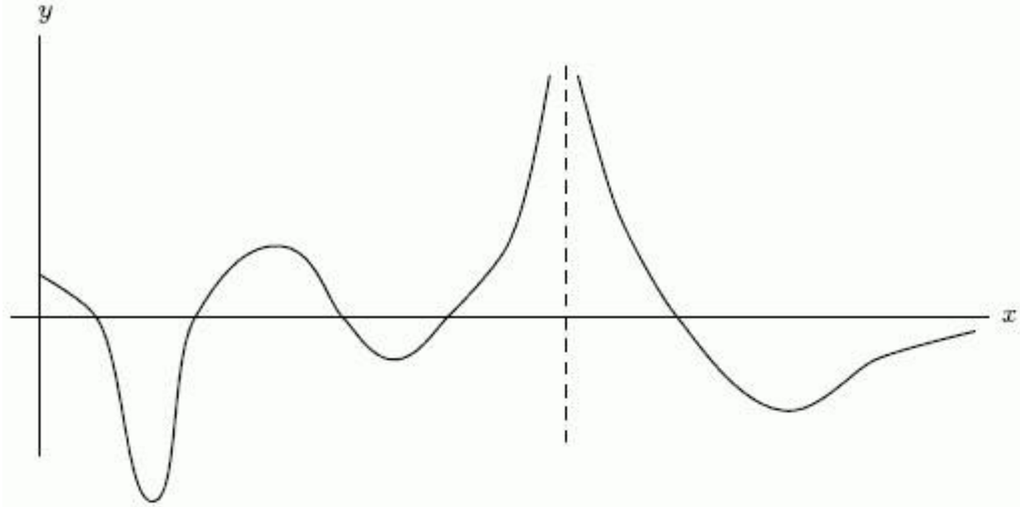


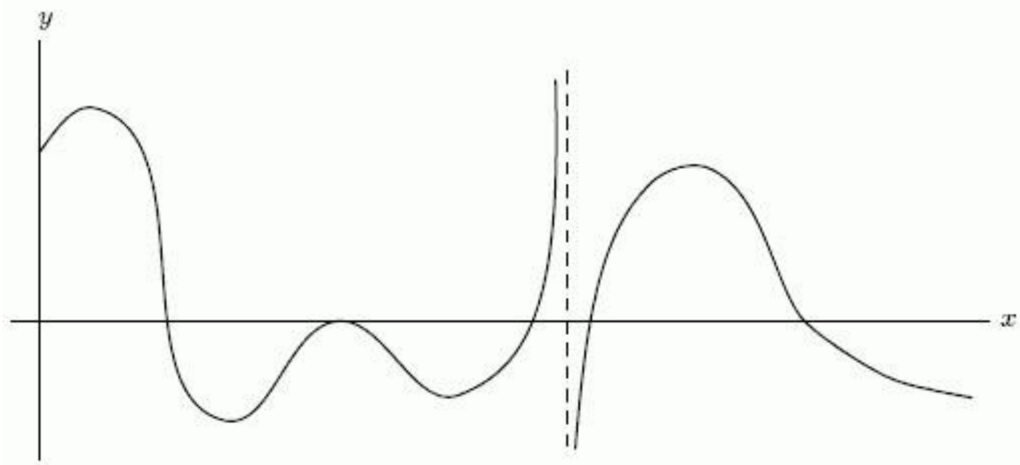
Diff: 1 Type: ES

Learning Objective: LO 2.5.1 Interpret the second derivative as concavity.

Section: 2.5

7) One of the following graphs is of $f(x)$, and the other is of $f'(x)$. Is $f(x)$ the first graph or the second graph?





Answer: second
Diff: 2 Type: SA
Learning Objective: LO 2.5.4 Use first and second derivatives to analyze a function.
Section: 2.5

8) Given the following data about a function f, estimate the rate of change of the derivative f' at $x = 4.5$.

x	3	3.5	4	4.5	5	5.5	6
$f(x)$	10	8	7	4	2	0	-1

Answer: 4
Diff: 2 Type: SA
Learning Objective: LO 2.5.2 Interpret the second derivative as the rate of change of the first derivative.
Section: 2.5

9) A function defined for all x has the following properties:

- f is increasing
- f is concave down
- $f(7) = 2$
- $f'(7) = 1/2$

How many zeros does $f(x)$ have in the interval $0 \le x \le 1$?
Answer: 0
Diff: 2 Type: SA
Learning Objective: LO 2.5.4 Use first and second derivatives to analyze a function.
Section: 2.5

10) A function defined for all x has the following properties:

- f is increasing
- f is concave down
- $f(6) = 2$
- $f'(6) = 1/2$

Is it possible that $f'(1) = \frac{1}{4}$?

Answer: no
Diff: 2 Type: SA
Learning Objective: LO 2.5.4 Use first and second derivatives to analyze a function.
Section: 2.5

11) Assume that f is a differentiable function defined on all of the real line. Is it possible that $f > 0$ everywhere, $f' > 0$ everywhere, and $f'' < 0$ everywhere?

Answer: no
Diff: 2 Type: SA
Learning Objective: LO 2.5.1 Interpret the second derivative as concavity.
Section: 2.5

12) Assume that f and g are differentiable functions defined on all of the real line. Is it possible that $f'(x) > g'(x)$ for all x and $f(x) < g(x)$ for all x?

Answer: yes
Diff: 2 Type: SA
Learning Objective: LO 2.5.1 Interpret the second derivative as concavity.
Section: 2.5

13) Assume that f and g are differentiable functions defined on all of the real line. If $f'(x) = g'(x)$ for all x and if $f(x_0) = g(x_0)$ for some x_0, then must $f(x) = g(x)$ for all x?

Answer: yes
Diff: 2 Type: SA
Learning Objective: LO 2.5.1 Interpret the second derivative as concavity.
Section: 2.5

14) Assume that f and g are differentiable functions defined on all of the real line. If $f'' < 0$ everywhere and $f' < 0$ everywhere then must $\lim_{x\to+\infty} f(x) = -\infty$?

Answer: yes
Diff: 2 Type: SA
Learning Objective: LO 2.5.4 Use first and second derivatives to analyze a function.
Section: 2.5

15) Suppose a function is given by a table of values as follows:

x	1.1	1.3	1.5	1.7	1.9	2.1
$f(x)$	11	14	20	22	23	24

Give your best estimate of $f''(1.3)$.
Answer: 75
Diff: 2 Type: SA
Learning Objective: LO 2.5.2 Interpret the second derivative as the rate of change of the first derivative.
Section: 2.5

16) If the Figure 1 is $f(x)$, could Figure 2 be $f''(x)$?

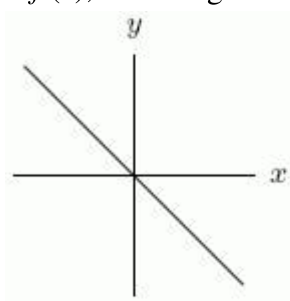


Figure 1

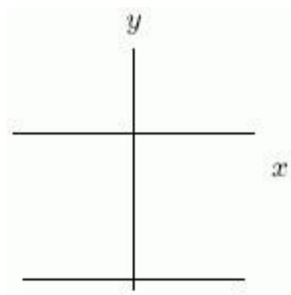


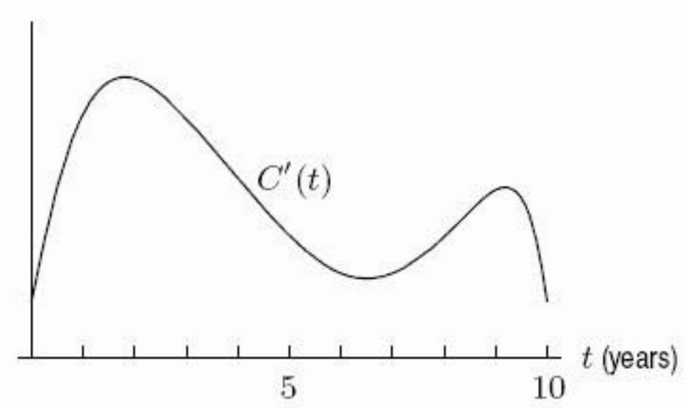
Figure 2

Answer: no
Diff: 2 Type: SA
Learning Objective: LO 2.5.5 Graph the second derivative.
Section: 2.5

17) The cost of mining a ton of coal is rising faster every year. Suppose $C(t)$ is the cost of mining a ton of coal at time t. Must $C''(t)$ be increasing?
Answer: no
Diff: 2 Type: SA
Learning Objective: LO 2.5.1 Interpret the second derivative as concavity.
Section: 2.5

18) Let $S(t)$ represent the number of students enrolled in school in the year t. If the number of students enrolling is increasing faster and faster, then is $S''(t)$ positive, negative, or 0?
Answer: positive
Diff: 2 Type: SA
Learning Objective: LO 2.5.1 Interpret the second derivative as concavity.
Section: 2.5

19) A company graphs $C'(t)$, the derivative of the number of pints of ice cream sold over the past ten years. At approximately what year was $C(t)$ greatest?



Answer: 10
Diff: 2 Type: SA
Learning Objective: LO 2.5.2 Interpret the second derivative as the rate of change of the first derivative.
Section: 2.5

20) A golf ball thrown directly upwards from the surface of the moon with an initial velocity of 14 meters per second and will attain a height of $s(t) = -0.8t^2 + 14t$ meters in t seconds. Find a formula for the velocity of the golf ball at time t.
A) $v(t) = -1.6t + 14$ meters/sec
B) $v(t) = -0.8t + 7$ meters/sec
C) $v(t) = -1.12$ meters/sec
D) $v(t) = -16t^2 + 14$ meters/sec
Answer: A
Diff: 2 Type: BI
Learning Objective: LO 2.5.3 Interpret the second derivative as acceleration.
Section: 2.5

21) A golf ball thrown directly upwards from the surface of the moon with an initial velocity of 17 meters per second and will attain a height of $s(t) = -0.8t^2 + 17t$ meters in t seconds. What is the acceleration of the golf ball at time t?
A) -1.6 meters/sec/sec
B) $-1.6t$ meters/sec^2
C) $-0.8t$ meters/sec/sec
D) 17 meters/sec^2
Answer: A
Diff: 2 Type: BI
Learning Objective: LO 2.5.3 Interpret the second derivative as acceleration.
Section: 2.5

22) A golf ball thrown directly upwards from the surface of the moon with an initial velocity of 14 meters per second and will attain a height of $s(t) = -0.8t^2 + 14t$ meters in t seconds. How fast is the golf ball going at its high point?
A) 0 meters/sec
B) -0.8 meters/sec
C) 14 meters/sec
D) -14 meters/sec
Answer: A
Diff: 1 Type: BI
Learning Objective: LO 2.5.3 Interpret the second derivative as acceleration.
Section: 2.5

23) A golf ball thrown directly upwards from the surface of the moon with an initial velocity of 18 meters per second and will attain a height of $s(t) = -0.8t^2 + 18t$ meters in t seconds. On Earth, its height would be given by $-4.9t^2 + 18t$. Compare the velocity and acceleration of the golf ball on the moon after two seconds with its velocity and acceleration on Earth after two seconds.
Answer: Comparisons will vary. The numerical results are:
On the moon: velocity -3.2 m/s and acceleration -1.6 m/s^2
On the Earth: velocity -19.6 m/s and acceleration - 9.8 m/s^2
Diff: 2 Type: ES
Learning Objective: LO 2.5.1 Interpret the second derivative as concavity.
Section: 2.5

24) The Chief Financial Officer of an insurance firm reports to the board of directors that the cost of claims is rising more slowly than last quarter. Let C(t) be the cost of claims. Select all statements that apply.
A) C is positive.
B) C is negative.
C) The first derivative of C is positive.
D) The first derivative of C is negative.
E) The second derivative of C is positive.
F) The second derivative of C is negative.
Answer: A, C, F
Diff: 2 Type: MC
Learning Objective: LO 2.5.2 Interpret the second derivative as the rate of change of the first derivative.
Section: 2.5

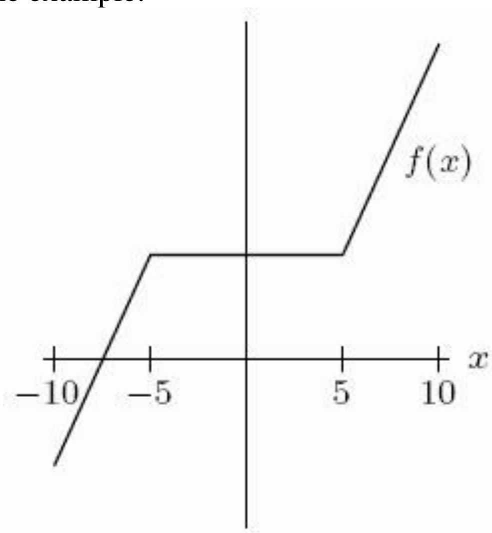
25) A husband and wife purchase life insurance policies. Over the next 40 years, one policy pays out when the husband dies, and the other pays out when both husband and wife die. Their life expectancy is 20 years, and the probability that both die before year t is given by the function $f_1(\mathrm{t}) = \dfrac{1}{1600}t^2$. How fast is the probability that both are dead increasing in 30 years?

A) 0.0375
B) 0.5625
C) 0.0600
D) 60.0006
Answer: A
Diff: 3 Type: BI
Learning Objective: LO 2.5.3 Interpret the second derivative as acceleration.
Section: 2.5

2.6 Differentiability

1) Sketch a graph $y = f(x)$ that is continuous everywhere on $-10 < x < 10$ but not differentiable at $x = -5$ or $x = 5$.

Answer: Many possible. One example:

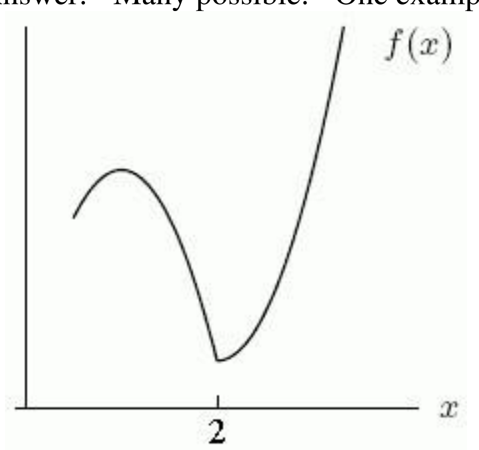


Diff: 1 Type: ES
Learning Objective: LO 2.6.1 Interpret differentiability graphically.
Section: 2.6

2) Sketch a graph of a continuous function $f(x)$ with the following properties:

- $f''(x) < 0$ for $x < 2$
- $f''(x) > 0$ for $x > 2$
- $f''(2)$ is undefined

Answer: Many possible. One example:



Diff: 1 Type: ES
Learning Objective: LO 2.6.1 Interpret differentiability graphically.
Section: 2.6

3) Is the graph of $f(x) = |x + 7|$ continuous at $x = -7$?
Answer: yes
Diff: 1 Type: SA
Learning Objective: LO 2.6.3 Determine if a function is differentiable at a point.
Section: 2.6

4) Is the graph of $f(x) = \dfrac{1}{x+4}$ continuous at $x = -4$?

Answer: no
Diff: 1 Type: SA
Learning Objective: LO 2.6.3 Determine if a function is differentiable at a point.
Section: 2.6

5) Given the function $h(r) = \begin{cases} 1 - \sin\dfrac{\pi r}{2} & -1 \le r \le 1 \\ 0 & r < -1, r > 1 \end{cases}$. Is $h(r)$ differentiable at $r = -1$?

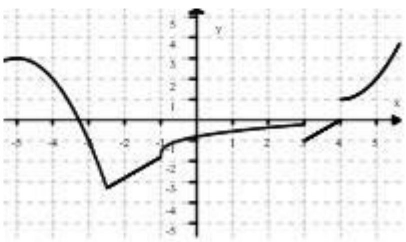
Answer: no
Diff: 2 Type: SA
Learning Objective: LO 2.6.3 Determine if a function is differentiable at a point.
Section: 2.6

6) Describe two ways that a continuous function can fail to have a derivative at a point, $x = a$. Illustrate your description with graphs.
Answer: Answers will vary but will describe two of: cusps, corners, vertical tangents.
Diff: 2 Type: ES
Learning Objective: LO 2.6.2 Explain the relationship between differentiability and continuity.
Section: 2.6

7) A function that has an instantaneous rate of change of 3 at a point (x, y) can fail to be continuous at that point.
Answer: FALSE
Diff: 1 Type: TF
Learning Objective: LO 2.6.2 Explain the relationship between differentiability and continuity.
Section: 2.6

8) Based on the graph of $f(x)$ below:

a) List all values of x for which f is NOT differentiable.
b) List all values of x for which f is NOT continuous.
c) List all values of x for which $f'(x) = 0$.



Answer:
a) Not differentiable at $x = -2.5, -1, 3, 4$
b) Not continuous at $x = 3, 4$
c) Derivative of zero at $x = -5$.

Diff: 1 Type: ES
Learning Objective: LO 2.6.1 Interpret differentiability graphically.
Section: 2.6

9) Let $f(x) = x^{\sin x}$. Using your calculator, estimate $f'(1)$ to 3 decimal places.

Answer: 0.841
Diff: 2 Type: SA
Learning Objective: LO 2.2.7 Use derivative to estimate change.
Section: 2.6

10) Is the function $f(x)\ \dfrac{x^2|2x - 2|}{x - 1}$ continuous at $x = -1$?

Answer: yes
Diff: 1 Type: SA
Learning Objective: LO 2.6.2 Explain the relationship between differentiability and continuity.; LO 2.6.3 Determine if a function is differentiable at a point.
Section: 2.6

11) Mark all TRUE statements.
A) $f(x) = |x - 3|$ is continuous at $x = 0$.
B) $g(x) = \sqrt[3]{x}$ fails to be differentiable at $x = 0$.
C) $h(x) = |x + 5|$ is not continuous at $x = -5$.
D) $r(x) = \dfrac{(x - 3)^2}{(x - 3)}$ is continuous for all values of x.
E) Any polynomial function is differentiable for all values of x.
Answer: A, E
Diff: 1 Type: MC
Learning Objective: LO 2.6.2 Explain the relationship between differentiability and continuity.; LO 2.6.3 Determine if a function is differentiable at a point.
Section: 2.6

12) In the lobby of a university mathematics building, there is a large bronze sculpture in the shape of a parabola. When the sun shines on the parabola at a certain time, its shadow falls on a mural with a coordinate plane that reveals the sculpture's height as the function $f(x) = -x^2 + 18$. A spider drops from its web onto the sculpture at the point $(3, 9)$. What is the slope of the parabola at the point where the spider lands?
A) -6
B) -24
C) 9
D) -3
E) None of the above
Answer: A
Diff: 1 Type: BI
Learning Objective: LO 2.1.4 Estimate slope of a curve.
Section: 2.6