Applied Calculus 6th edition Hughes-Hallett Solutions Manual

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CHAPTER TWO

Solutions for Section 2.1

1. Recall that $f(x)$ gives the y-coordinate at a point while $f'(x)$ gives the slope at a point. We see that
   (a) $f(-2)$ is negative
   (b) $f(0)$ is positive
   (c) $f(4)$ is negative
   (d) $f(5)$ is zero
   (e) $f'(-2)$ is positive
   (f) $f'(2)$ is negative
   (g) $f'(4)$ is zero
   (h) $f'(5)$ is positive

2. Recall that $f(x)$ gives the y-coordinate at a point while $f'(x)$ gives the slope at a point.
   (a) We see that $f(1)$ is higher on the graph than $f(2)$ so $f(1)$ is larger.
   (b) We see that $f(6)$ is higher on the graph than $f(5)$ so $f(6)$ is larger.
   (c) We see that $f(3)$ is higher on the graph than $f(4)$ so $f(3)$ is larger.
   (d) We see that $f'(-2)$ is positive while $f'(2)$ is negative so $f'(-2)$ is larger.
   (e) We see that $f'(4)$ is approximately zero while $f'(5)$ is positive, so $f'(5)$ is larger.
   (f) We see that $f'(-1)$ and $f'(0)$ are both positive, but that the graph is steeper at $x = -1$, so the slope is greater there and $f'(-1)$ is larger.

3. Recall that $f(x)$ gives the y-coordinate at a point while $f'(x)$ gives the slope at a point. In this case, we are determining whether the y-coordinate at $x = 2$ is positive. We see that this is true only for the functions shown in A and B.

4. Recall that $f(x)$ gives the y-coordinate at a point while $f'(x)$ gives the slope at a point. In this case, we are determining whether the y-coordinate at $x = 4$ is negative. We see that this is true only for the functions shown in C and D.

5. Recall that $f(x)$ gives the y-coordinate at a point while $f'(x)$ gives the slope at a point. In this case, we are determining whether the slope at $x = 2$ is positive. We see that this is true only for the functions shown in A and B.

6. Recall that $f(x)$ gives the y-coordinate at a point while $f'(x)$ gives the slope at a point. In this case, we are determining whether the slope at $x = 4$ is negative. We see that this is true only for the functions shown in A and D.

7. **(a)** We have $s = f(t)$ where s is the height at time t and we are told that the height is $s = 96$ at time $t = 2$, so we have $f(2) = 96$.
   (b) We know that the derivative $f'(t)$ gives the instantaneous rate of change at time t, and we are told that the rate of change at $t = 2$ is 16. We have $f'(2) = 16$.

8. **(a)** We have $w = f(t)$ where w is the quantity at time t and we are told that the quantity is $w = 50$ at time $t = 20$, so we have $f(20) = 50$.
   (b) We know that the derivative $f'(t)$ gives the instantaneous rate of change at time t, and we are told that the rate of change at $t = 20$ is 3. We have $f'(20) = 3$.

9. **(a)** The function $N = f(t)$ is decreasing when $t = 1950$. Therefore, $f'(1950)$ is negative. That means that the number of farms in the US was decreasing in 1950.
   (b) The function $N = f(t)$ is decreasing in 1960 as well as in 1980 but it is decreasing faster in 1960 than in 1980. Therefore, $f'(1960)$ is more negative than $f'(1980)$.

10. **(a)** The average rate of change is the slope of the secant line in Figure 2.1, which shows that this slope is positive.
    (b) The instantaneous rate of change is the slope of the graph at $x = 3$, which we see from Figure 2.2 is negative.

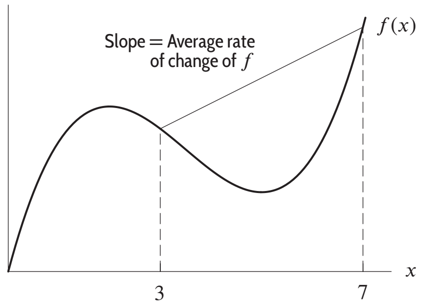


Figure 2.1

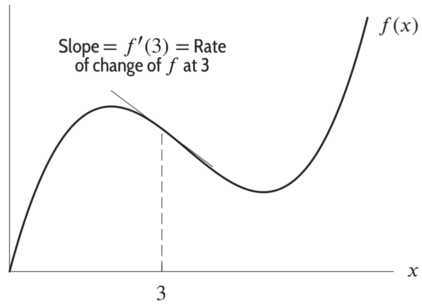


Figure 2.2

11. (a) For the interval $0 \le t \le 0.2$, we have

$$\left(\begin{array}{c}\text{Average velocity}\\ 0 \le t \le 0.2\end{array}\right) = \frac{s(0.2)-s(0)}{0.2-0} = \frac{0.5}{0.2} = 2.5 \text{ ft/sec.}$$

(b) For the interval $0.2 \le t \le 0.4$, we have

$$\left(\begin{array}{c}\text{Average velocity}\\ 0.2 \le t \le 0.4\end{array}\right) = \frac{s(0.4)-s(0.2)}{0.4-0.2} = \frac{1.3}{0.2} = 6.5 \text{ ft/sec.}$$

(c) To estimate the instantaneous velocity at $t = 0.2$, we can average the average velocities found on the left and the right of $t = 0.2$. So a reasonable estimate of the velocity at $t = 0.2$ is $\frac{1}{2}(6.5 + 2.5) = 4.5$ ft/sec.

12. (a) Let $s = f(t)$.

(i) We wish to find the average velocity between $t = 1$ and $t = 1.1$. We have

$$\text{Average velocity} = \frac{f(1.1)-f(1)}{1.1-1} = \frac{7.84-7}{0.1} = 8.4 \text{ m/sec.}$$

(ii) We have

$$\text{Average velocity} = \frac{f(1.01)-f(1)}{1.01-1} = \frac{7.0804-7}{0.01} = 8.04 \text{ m/sec.}$$

(iii) We have

$$\text{Average velocity} = \frac{f(1.001)-f(1)}{1.001-1} = \frac{7.008004-7}{0.001} = 8.004 \text{ m/sec.}$$

(b) We see in part (a) that as we choose a smaller and smaller interval around $t = 1$ the average velocity appears to be getting closer and closer to 8, so we estimate the instantaneous velocity at $t = 1$ to be 8 m/sec.

13. (a) The average rate of change of a function over an interval is represented graphically as the slope of the secant line to its graph over the interval. See Figure 2.3. Segment AB is the secant line to the graph in the interval from $x = 0$ to $x = 3$ and segment BC is the secant line to the graph in the interval from $x = 3$ to $x = 5$.

We can easily see that slope of $AB >$ slope of BC. Therefore, the average rate of change between $x = 0$ and $x = 3$ is greater than the average rate of change between $x = 3$ and $x = 5$.

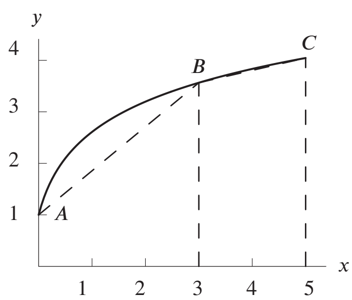


Figure 2.3

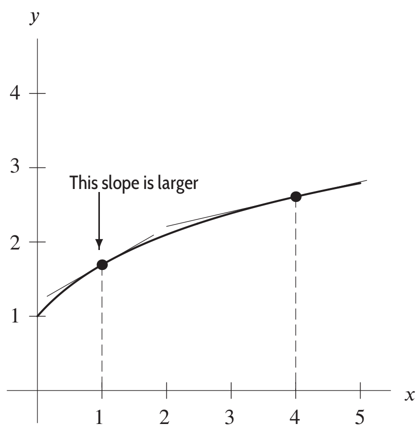


Figure 2.4

(b) We can see from the graph in Figure 2.4 that the function is increasing faster at $x = 1$ than at $x = 4$. Therefore, the instantaneous rate of change at $x = 1$ is greater than the instantaneous rate of change at $x = 4$.

(c) The units of rate of change are obtained by dividing units of cost by units of product: thousands of dollars/kilogram.

14. (a) The average velocity between $t = 2$ and $t = 5$ is

$$\frac{\text{Distance}}{\text{Time}} = \frac{s(5) - s(2)}{5 - 2} = \frac{25 - 4}{3} = \frac{21}{3} = 7 \text{ ft/sec.}$$

(b) Using an interval of size 0.1, we have

$$\begin{pmatrix}\text{Instantaneous velocity} \\ \text{at } t = 2\end{pmatrix} \approx \frac{s(2.1) - s(2)}{2.1 - 2} = \frac{4.41 - 4}{0.1} = 4.1.$$

Using an interval of size 0.01, we have

$$\begin{pmatrix}\text{Instantaneous velocity} \\ \text{at } t = 2\end{pmatrix} \approx \frac{s(2.01) - s(2)}{2.01 - 2} = \frac{4.0401 - 4}{0.01} = 4.01.$$

From this we guess that the instantaneous velocity at $t = 2$ is about 4 ft/sec.

15. First, draw a line joining the origin to any point in the first quadrant labeled $(5, f(5))$. This line represents the distance as a function of time if the particle had moved at a constant speed, the average velocity. The average velocity is the slope of the line between $(0, 0)$ and $(5, f(5))$.

Since the particle's velocity varies, it must sometimes be more than and sometimes less than the average velocity. The curve we want has instantaneous velocity at exactly two points equal to the average velocity. Thus, we want a curve whose endpoints are the origin and $(5, f(5))$ and with tangents at exactly two points that are parallel the line. To achieve this, the curve must be sometimes above, and sometimes below, the line.

For example, we can draw a smooth curve that is above the line for the first half of the interval from 0 to 5 and below the line for the second half of this interval. One possibility is in Figure 2.5. Other answers are possible.

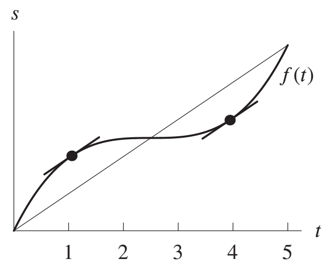


Figure 2.5

16. (a) We have

$$\text{Rate of change of population} = \frac{7.32 - 4.45}{2015 - 1980} = 0.082 \text{ billion people per year.}$$

(b) To estimate $f'(2010)$, we use the rate of change formula on an interval containing 2010:

$$f'(2010) \approx \frac{7.32 - 6.92}{2015 - 2010} = 0.080 \text{ billion people per year.}$$

17. (a) The size of the tumor when $t = 0$ months is $S(0) = 2^0 = 1$ cubic millimeter. The size of the tumor when $t = 6$ months is $S(6) = 2^6 = 64$ cubic millimeters. The total change in the size of the tumor is $S(6) - S(0) = 64 - 1 = 63$ mm^3.

(b) The average rate of change in the size of the tumor during the first six months is:

$$\text{Average rate of change} = \frac{S(6) - S(0)}{6 - 0} = \frac{64 - 1}{6} = \frac{63}{6} = 10.5 \text{ cubic millimeters/month.}$$

(c) We will consider intervals to the right of $t = 6$:

t (months)	6	6.001	6.01	6.1
S (cubic millimeters)	64	64.0444	64.4452	68.5935

$$\text{Average rate of change} = \frac{68.5935 - 64}{6.1 - 6} = \frac{4.5935}{0.1} = 45.935$$
$$\text{Average rate of change} = \frac{64.4452 - 64}{6.01 - 6} = \frac{0.4452}{0.01} = 44.52$$
$$\text{Average rate of change} = \frac{64.0444 - 64}{6.001 - 6} = \frac{0.0444}{0.001} = 44.4$$

We can continue taking smaller intervals but the value of the average rate will not change much. Therefore, we can say that a good estimate of the growing rate of the tumor at $t = 6$ months is about 44.4 cubic millimeters/month.

18. We use the interval $x = 1$ to $x = 1.01$:

$$g'(1) \approx \frac{f(1.01) - f(1)}{1.01 - 1} = \frac{4^{1.01} - 4^1}{0.01} = \frac{4.05583 - 4}{0.01} = 5.583.$$

For greater accuracy, we can use the smaller interval $x = 1$ to $x = 1.001$:

$$g'(1) \approx \frac{f(1.001) - f(1)}{1.001 - 1} = \frac{4^{1.001} - 4^1}{0.001} = \frac{4.005549 - 4}{0.001} = 5.549.$$

19. **(a)** Figure 2.6 shows that for $t = 2$ the function $g(t) = (0.8)^t$ is decreasing. Therefore, $g'(2)$ is negative.

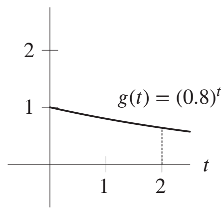


Figure 2.6

(b) To estimate $g'(2)$ we take the small interval between $t = 2$ and $t = 2.001$ to the right of $t = 2$.

$$g'(2) \approx \frac{g(2.001) - g(2)}{2.001 - 2} = \frac{0.8^{2.001} - 0.8^2}{0.001} = \frac{0.6399 - 0.64}{0.001} = \frac{-0.0001}{0.001} = -0.1$$

20. The slope is positive at A and D; negative at C and F. The slope is most positive at A; most negative at F.

21.

Slope	-3	-1	0	1/2	1	2
Point	F	C	E	A	B	D

22. Using the interval $1 \le x \le 1.001$, we estimate

$$f'(1) \approx \frac{f(1.001) - f(1)}{0.001} = \frac{3.0033 - 3.0000}{0.001} = 3.3$$

The graph of $f(x) = 3^x$ is concave up so we expect our estimate to be greater than $f'(1)$.

23. **(a)** Since the values of P go up as t goes from 0 to 2 and from 2 to 4, we see that $f'(2)$ appears to be positive. The number of households with cable television is increasing at $t = 2$ (in the year 2000).

(b) We estimate $f'(2)$ using the difference quotient for the interval to the right of $t = 2$, as follows:

$$f'(2) \approx \frac{\Delta P}{\Delta t} = \frac{66.472 - 66.250}{4 - 2} = \frac{0.222}{2} = 0.111.$$

The fact that $f'(2) = 0.111$ tells us that the number of households with cable television in the United States was increasing at a rate of 0.111 million per year when $t = 2$ (in the year 2000).

Similarly:

$$f'(10) \approx \frac{\Delta P}{\Delta t} = \frac{60.958 - 64.274}{12 - 10} = \frac{-3.316}{2} = -1.658.$$

The fact that $f'(10) = -1.658$ tells us that the number of households in the United States with cable television was decreasing at a rate of 1.658 million per year when $t = 10$ (in the year 2008).

24. **(a)** Since 75.2 percent live in the city in 1990 and 35.1 percent in 1890, we have

$$\text{Average rate of change} = \frac{75.2 - 35.1}{1990 - 1890} = \frac{40.1}{100} = 0.401.$$

Thus the average rate of change is 0.401 percent/year. (This rate is said to be 0.401 percentage points per year, to distinguish it from a percent change.)

(b) By looking at 1990 and 2000 we see that

$$\text{Average rate of change} = \frac{79.0 - 75.2}{2000 - 1990} = \frac{3.8}{10} = 0.38$$

which gives an average rate of change of 0.38 percent/year between 1990 and 2000. (That is, 0.38 percentage points per year.)

However, the definition of an urban area was changed for the 2000 data, so this estimate should be used with care. We could also look at 1980 and 1990 and see that

$$\text{Average rate of change} = \frac{75.2 - 73.7}{1990 - 1980} = \frac{1.5}{10} = 0.15$$

giving an average rate of change of 0.15 percent/year between 1980 and 1990. We see that the rate of change in the year 1990 is somewhere between 0.38 and 0.15 percent/year.

(c) By looking at 1830 and 1860 we see that

$$\text{Average rate of change} = \frac{19.8 - 9.0}{1860 - 1830} = \frac{10.8}{30} = 0.36,$$

giving an average rate of change of 0.36 percent/year between 1830 and 1860. (That is, 0.36 percentage points per year.)

Alternatively, we can look at 1800 and 1830 and see that

$$\text{Average rate of change} = \frac{9.0 - 6.0}{1830 - 1800} = \frac{3.0}{30} = 0.10.$$

giving an average rate of change of 0.10 percent/year between 1800 and 1830 . We see that the rate of change at the year 1830 is somewhere between 0.10 and 0.36.

This tells us that in the year 1830 the percent of the population in urban areas is increasing at a rate somewhere between 0.10 percent/year and 0.36 percent/year.

25. **(a)** The value 6.13 is the population (in billions of people) of the world in the year 2000, and is not a rate of change. The answer is (iii).

(b) The value 7.32 is the population (in billions of people) of the world in the year 2015, and is not a rate of change. The answer is (iii).

(c) The value 0.079 is the average rate of change in the world's population, in billions of people per year, during the 17 year period from the year 2000 to the year 2015. The answer is (i).

(d) The value 0.076 is the instantaneous rate of change in the world's population, in billions of people per year, right at the start of the year 2000. The answer is (ii).

(e) The value 0.082 is the instantaneous rate of change in the world's population, in billions of people per year, right at the start of the year 2015. The answer is (ii).

26. **(a)** The average velocity between $t = 2$ and $t = 3$ is given by

$$\frac{f(3) - f(2)}{3 - 2} = \frac{56 - 136}{1} = -80 \text{ ft/sec.}$$

(b) We use the average rate of change formula in each case:

(i) If $t = 0.1$, the average rate of change is

$$\frac{f(2.1) - f(2)}{2.1 - 2} = \frac{129.44 - 136}{0.1} = -65.6 \text{ ft/sec.}$$

(ii) If $t = 0.01$, the average rate of change is

$$\frac{f(2.01) - f(2)}{2.01 - 2} = \frac{135.3584 - 136}{0.01} = -64.16 \text{ ft/sec.}$$

(iii) If $t = 0.001$, the average rate of change is

$$\frac{f(2.001) - f(2)}{2.001 - 2} = \frac{135.9360 - 136}{0.001} = -64.016 \text{ ft/sec.}$$

(c) The values in part (b) appear to be getting closer to -64 so we estimate that the instantaneous velocity at time $t = 2$ is about -64 ft/sec.

(d) We see that $f(2) = 136$ ft and we estimated in part (c) that $f'(2)$ is approximately -64 ft/sec.

27.

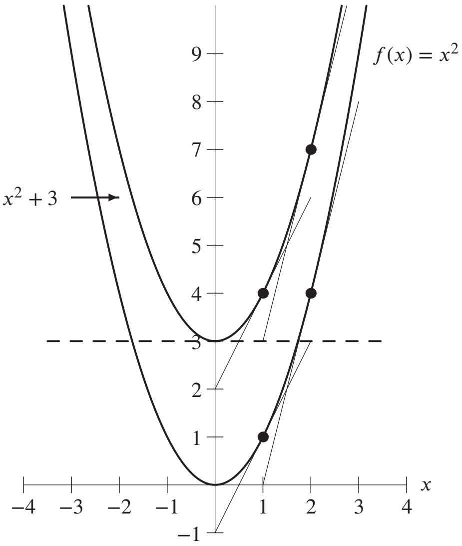


Figure 2.7

(a) The tangent line to the graph of $f(x) = x^2$ at $x = 0$ coincides with the x-axis and therefore is horizontal (slope = 0). The tangent line to the graph of $g(x) = x^2 + 3$ at $x = 0$ is the dashed line indicated in the figure and it also has a slope equal to zero. Therefore both tangent lines at $x = 0$ are parallel.

We see in Figure 2.7 that the tangent lines at $x = 1$ appear parallel, and the tangent lines at $x = 2$ appear parallel. The slopes of the tangent lines at any value $x = a$ will be equal.

(b) Adding a constant shifts the graph vertically, but does not change the slope of the curve.

28. **(a)** The velocity of the car represented by $f(t)$ at time t is equal to the slope of $f(t)$ at that point. Therefore, since the slope of $f(t)$ starts large and positive and then gets less steep as t increases, this car starts out fast and then slows down. Likewise, since the slope of $g(t)$ starts small and positive and then gets steeper as t increases, this car starts out slowly and then speeds up.

(b) The average velocity of the car represented by $f(t)$ for $a \le t \le b$ is the slope of the line joining the points on the graph of $f(t)$ corresponding to $t = a$ and $t = b$. Therefore, we are looking for an interval $a \le t \le b$ on which the average slopes of $g(t)$ and $f(t)$ are the same. The interval from $t = 0$ to $t = 1$ is one such interval. Other answers are possible.

(c) Item (i) is true. The cars are traveling at the same instantaneous velocity when the slopes of both curves are the same. This appears to occur near the end of the first half minute, or between $t = 0$ and $t = 1/2$. This also means that (iv) is not true.

Item (ii) is not true. For times between $t = 1/2$ and $t = 1$, the slopes of $f(t)$ appear to be less than the slopes of $g(t)$, so the car represented by $g(t)$ is going faster.

Item (iii) is not true. At $t = 1$ minute, the cars are at the same position, but the slopes are different, so the velocities are different.

29. The coordinates of A are $(4, 25)$. See Figure 2.8. The coordinates of B and C are obtained using the slope of the tangent line. Since $f'(4) = 1.5$, the slope is 1.5

From A to B, $\Delta x = 0.2$, so $\Delta y = 1.5(0.2) = 0.3$. Thus, at C we have $y = 25 + 0.3 = 25.3$. The coordinates of B are $(4.2, 25.3)$.

From A to C, $\Delta x = -0.1$, so $\Delta y = 1.5(-0.1) = -0.15$. Thus, at C we have $y = 25 - 0.15 = 24.85$. The coordinates of C are $(3.9, 24.85)$.

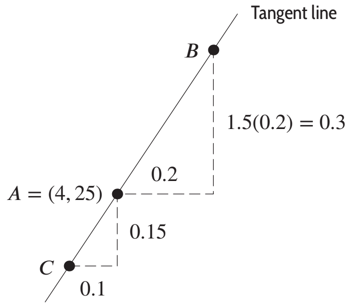


Figure 2.8

30. **(a)** Since the point $A = (7, 3)$ is on the graph of f, we have $f(7) = 3$.
(b) The slope of the tangent line touching the curve at $x = 7$ is given by

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{3.8 - 3}{7.2 - 7} = \frac{0.8}{0.2} = 4.$$

Thus, $f'(7) = 4$.

31. We have

$$\Delta y \approx f'(100)(\Delta x).$$

Taking $\Delta x = 1$ we have

$$\text{Change in } y = f(101) - f(100) \approx (0.4)(1) = 0.4.$$

32. We have

$$\Delta y \approx f'(12)(\Delta x).$$

Taking $\Delta x = 0.2$ we have

$$\text{Change in } y = f(12.2) - f(12) \approx (30)(0.2) = 6.$$

33. We have

$$\Delta y \approx g'(250)(\Delta x).$$

Taking $\Delta x = 1.5$ we have

$$\text{Change in } y = g(251.5) - g(250) \approx (-0.5)(1.5) = -0.75.$$

34. We have

$$\Delta y \approx p'(400)(\Delta x).$$

Taking $\Delta x = -2$ we have

$$\text{Change in } y = p(398) - p(400) \approx (2)(-2) = -4.$$

35. The answers to parts (a)–(d) are shown in Figure 2.9.

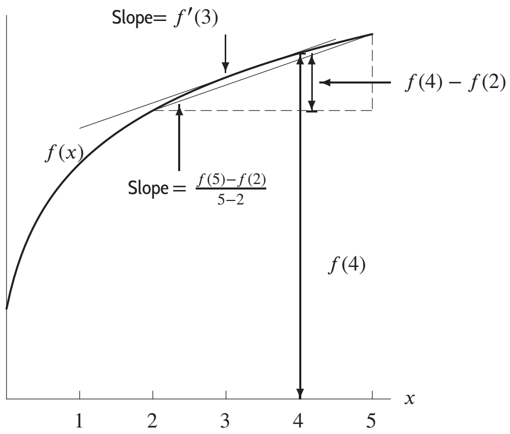


Figure 2.9

36. **(a)** Since f is increasing, $f(4) > f(3)$.

(b) From Figure 2.10, it appears that $f(2) - f(1) > f(3) - f(2)$.

(c) The quantity $\dfrac{f(2) - f(1)}{2 - 1}$ represents the slope of the secant line connecting the points on the graph at $x = 1$ and $x = 2$. This is greater than the slope of the secant line connecting the points at $x = 1$ and $x = 3$ which is $\dfrac{f(3) - f(1)}{3 - 1}$.

(d) The function is steeper at $x = 1$ than at $x = 4$ so $f'(1) > f'(4)$.

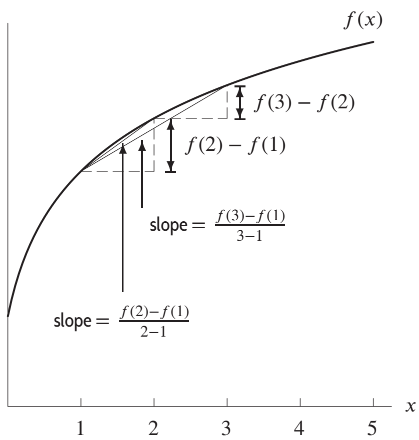


Figure 2.10

37. **(a)** The average rate of change from $x = a$ to $x = b$ is the slope of the line between the points on the curve with $x = a$ and $x = b$. Since the curve is concave up, the line from $x = 0$ to $x = 2$ has a smaller slope than the line from $x = 2$ to $x = 4$, and so the average rate of change between $x = 2$ and $x = 4$ is greater than that between $x = 0$ and $x = 2$.

(b) Since g is increasing, $g(4)$ is the greater.

(c) As in part (a), g is concave up and g' is increasing throughout so $g'(4)$ is the greater.

38. Using a difference quotient with $h = 0.001$, say, we find

$$f'(1) \approx \frac{1.001\ln(1.001) - 1\ln(1)}{1.001 - 1} = 1.0005$$

$$f'(2) \approx \frac{2.001\ln(2.001) - 2\ln(2)}{2.001 - 2} = 1.6934$$

The fact that f' is larger at $x = 2$ than at $x = 1$ suggests that f is concave up between $x = 1$ and $x = 2$.

39. We want to approximate $P'(0)$ and $P'(1)$. Since for small h

$$P'(0) \approx \frac{P(h) - P(0)}{h} \quad \text{and} \quad P'(1) \approx \frac{P(1+h) - P(1)}{h},$$

if we take $h = 0.01$, we get

$$\begin{aligned} P'(0) &\approx \frac{1.394(1.006)^{0.01} - 1.394}{0.01} = 0.00834 \text{ billion/year} \\ &= 8.34 \text{ million people/year in 2014}, \\ P'(1) &\approx \frac{1.394(1.006)^{1.01} - 1.394(1.006)^{1}}{0.01} = 0.00839 \text{ billion/year} \\ &= 8.39 \text{ million people/year in 2015} \end{aligned}$$

40. The quantity $f(0)$ represents the population on October 17, 2006, so $f(0) = 300$ million.

The quantity $f'(0)$ represents the rate of change of the population (in millions per year). Since

$$\frac{1 \text{ person}}{11 \text{ seconds}} = \frac{1/10^6 \text{ million people}}{11/(60 \cdot 60 \cdot 24 \cdot 365) \text{ years}} = 2.867 \text{ million people/year},$$

so we have $f'(0) = 2.867$.

41. **(a)** $f'(t)$ is negative or zero, because the average number of hours worked a week has been decreasing or constant over time.

$g'(t)$ is positive, because hourly wage has been increasing.

$h'(t)$ is positive, because average weekly earnings has been increasing.

(b) We use a difference quotient to the right for our estimates.

(i)

$$f'(1970) \approx \frac{36.0 - 37.0}{1975 - 1970} = -0.2 \quad \text{hours/year}$$
$$f'(1995) \approx \frac{34.3 - 34.3}{2000 - 1995} = 0 \quad \text{hours/year.}$$

In 1970, the average number of hours worked by a production worker in a week was decreasing at the rate of 0.2 hours per year. In 1995, the number of hours was not changing.

(ii)

$$g'(1970) \approx \frac{4.73 - 3.40}{1975 - 1970} = \$0.27 \quad \text{per year}$$
$$g'(1995) \approx \frac{14.00 - 11.64}{2000 - 1995} = \$0.47 \quad \text{per year.}$$

In 1970, the hourly wage was increasing at a rate of \$0.27 per year. In 1995, the hourly wage was increasing at a rate of \$0.47 per year.

(iii)

$$h'(1970) \approx \frac{170.28 - 125.80}{1975 - 1970} = \$8.90 \quad \text{per year}$$
$$h'(1995) \approx \frac{480.41 - 399.53}{2000 - 1995} = \$16.18 \quad \text{per year.}$$

In 1970, average weekly earnings were increasing at a rate of \$8.90 a year. In 1995, weekly earnings were increasing at a rate of \$16.18 a year.

Solutions for Section 2.2

1. Estimating the slope of the lines in Figure 2.11, we find that $f'(-2) \approx 1.0$, $f'(-1) \approx 0.3$, $f'(0) \approx -0.5$, and $f'(2) \approx -1$.

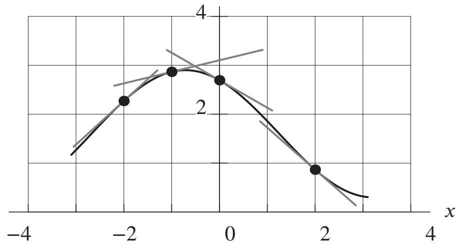


Figure 2.11

2. We visualize tangent lines on the graph at the given points, and estimate the slopes of the tangent lines. We find that $f'(1) \approx -2$, $f'(2) \approx -1$, $f'(3) \approx 0$, $f'(4) \approx 1$, and $f'(2) \approx 2$.

3. The graph is that of the line $y = -2x + 2$. The slope, and hence the derivative, is -2. See Figure 2.12.

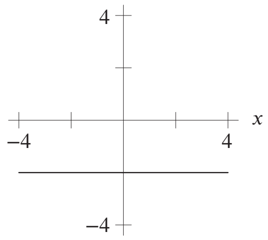


Figure 2.12

4. See Figure 2.13.

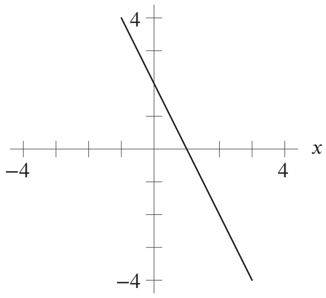


Figure 2.13

5. See Figure 2.14.

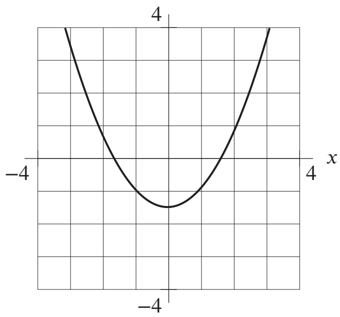


Figure 2.14

6. The slope of this curve is approximately -1 at $x = -4$ and at $x = 4$, approximately 0 at $x = -2.5$ and $x = 1.5$, and approximately 1 at $x = 0$. See Figure 2.15.

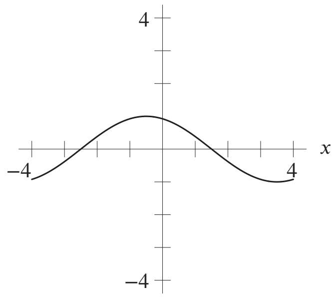


Figure 2.15

7. See Figure 2.16.

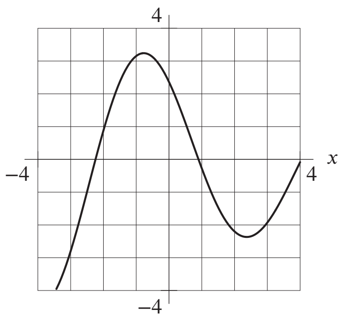


Figure 2.16

8. See Figure 2.17.

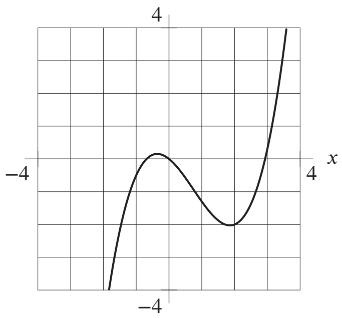


Figure 2.17

9. (a) x_3 (b) x_4 (c) x_5 (d) x_3

10. For $x = 0$, 5, 10, and 15, we use the interval to the right to estimate the derivative. For $x = 20$, we use the interval to the left. For $x = 0$, we have

$$f'(0) \approx \frac{f(5) - f(0)}{5 - 0} = \frac{70 - 100}{5 - 0} = \frac{-30}{5} = -6.$$

Similarly, we find the other estimates in Table 2.1.

Table 2.1

x	0	5	10	15	20
$f'(x)$	−6	−3	−1.8	−1.2	−1.2

11. The value of $R'(0)$ is the derivative of the function $R(x) = 100(1.1)^x$ at $x = 0$. This is the same as the rate of change of $R(x)$ at $x = 0$. We estimate this by computing the average rate of change over intervals near $x = 0$.

If we use the intervals $-0.001 \le x \le 0$ and $0 \le x \le 0.001$, we see that:

$$\begin{pmatrix}\text{Average rate of change}\\ \text{on } -0.001 \le x \le 0\end{pmatrix} = \frac{100(1.1)^0 - 100(1.1)^{-0.001}}{0-(-0.001)} = \frac{100 - 99.990469}{0.001} = 9.531,$$

$$\begin{pmatrix}\text{Average rate of change}\\ \text{on } 0 \le x \le 0.001\end{pmatrix} = \frac{100(1.1)^{0.001} - 100(1.1)^0}{0.001-0} = \frac{100.009531 - 100}{0.001} = 9.531.$$

It appears that the rate of change of $R(x)$ at $x = 0$ is 9.531, so we estimate $R'(0) = 9.531$.

12. See Figure 2.18.

Figure 2.18

13. This is a line with slope 1, so the derivative is the constant function $f'(x) = 1$. The graph is the horizontal line $y = 1$. See Figure 2.19.

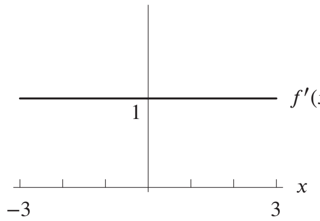


Figure 2.19

14. See Figure 2.20.

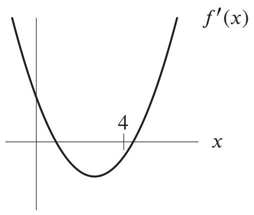


Figure 2.20

15. See Figure 2.21.

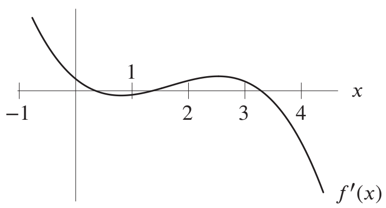


Figure 2.21

16. See Figure 2.22.

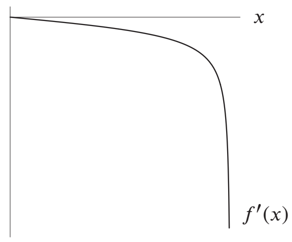


Figure 2.22

17. See Figure 2.23.

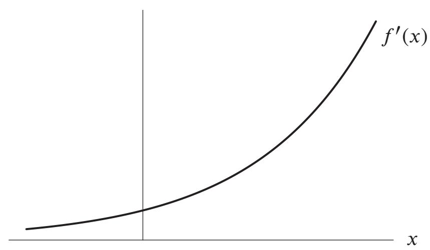


Figure 2.23

18. The function is decreasing for $x < -2$ and $x > 2$, and increasing for $-2 < x < 2$. The matching derivative must be negative (below the x-axis) for $x < -2$ and $x > 2$, positive (above the x-axis) for $-2 < x < 2$, and zero (on the x-axis) for $x = -2$ and $x = 2$. The matching derivative is in graph VIII.

19. The function is a line with negative slope, so $f'(x)$ is a negative constant, and the graph of $f'(x)$ is a horizontal line below the x-axis. The matching derivative is in graph IV.

20. The function is increasing for $x < 2$ and decreasing for $x > 2$. The corresponding derivative is positive (above the x-axis) for $x < 2$, negative (below the x-axis) for $x > 2$, and zero at $x = 2$. The matching derivative is in graph II.

21. The function is increasing for $x < -2$ and decreasing for $x > -2$. The corresponding derivative is positive (above the x-axis) for $x < -2$, negative (below the x-axis) for $x > -2$, and zero at $x = -2$. The derivatives in graphs VI and VII both satisfy these requirements. To decide which is correct, consider what happens as x gets large. The graph of $f(x)$ approaches an asymptote, gets more and more horizontal, and the slope gets closer and closer to zero. The derivative in graph VI meets this requirement and is the correct answer.

22. Since $f'(x) > 0$ for all x, we know that f is increasing for all x, which rules out choices (II) and (IV). Since $f'(x)$ reaches its highest value when $x = 0$, we know that f is the steepest when $x = 0$. Thus, the answer is (III).

23. Since $f'(x)$ is a constant function, the slope of f is constant; that is, f is linear. Therefore, our answer is (V).

24. Since $f'(x) > 0$ for all x, we know that f is increasing for all x, which rules out choices (II) and (IV). Since $f'(x)$ achieves its smallest value when $x = 0$, we know that f is the least steep at $x = 0$. Therefore, our answer is (I).

25. Since $f'(x) < 0$ when $x < 0$ and $f'(x) > 0$ when $x > 0$, we know that f is decreasing when $x < 0$ and increasing when $x > 0$. Thus, the choices are either (II) or (IV). Since the magnitude of $f'(x)$ increases as x moves away from 0, the graph of f becomes steeper as x moves away from 0. Thus, the answer is (II).

26. The graph is increasing for $0 < t < 15$ and is decreasing for $15 < t < 30$. One possible graph is shown in Figure 2.24. The units on the horizontal axis are years and the units on the vertical axis are people.

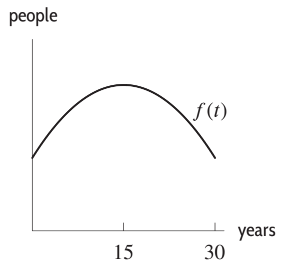


Figure 2.24

The derivative is positive for $0 < t < 15$ and negative for $15 < t < 30$. Two possible graphs are shown in Figure 2.25. The units on the horizontal axes are years and the units on the vertical axes are people per year.

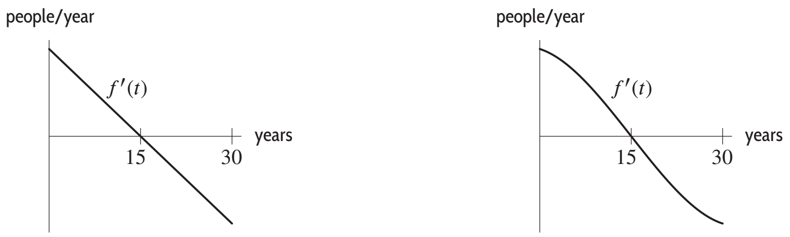


Figure 2.25

27. The value of $g(x)$ is increasing at a decreasing rate for $2.7 < x < 4.2$ and increasing at an increasing rate for $x > 4.2$.

$$\frac{\Delta y}{\Delta x} = \frac{7.4 - 6.0}{5.2 - 4.7} = 2.8 \qquad \text{between } x = 4.7 \text{ and } x = 5.2$$
$$\frac{\Delta y}{\Delta x} = \frac{9.0 - 7.4}{5.7 - 5.2} = 3.2 \qquad \text{between } x = 5.2 \text{ and } x = 5.7$$

Thus $g'(x)$ should be close to 3 near $x = 5.2$.

28. Notice that the x-values in the table are equally spaced 3 units apart, so we expect $f'(x)$ to be greatest on the interval where $f(x)$ has the largest increase. This occurs between $x = -9$ and $x = -6$, where $f(x)$ increases by 0.07. So the greatest value of $f'(x)$ likely occurs between $x = -9$ and $x = -6$.

To find the least value of $f'(x)$, we look for the interval where $f(x)$ has the largest decrease. This occurs between $x = 6$ and $x = 9$, where $f(x)$ decreases by 0.06. So the least value of $f'(x)$ on the interval $-12 \le x \le 9$ likely occurs between $x = 6$ and $x = 9$, possibly at $x = 9$.

29. Since $f'(x) > 0$ for $x < -1$, $f(x)$ is increasing on this interval.
Since $f'(x) < 0$ for $x > -1$, $f(x)$ is decreasing on this interval.
Since $f'(x) = 0$ at $x = -1$, the tangent to $f(x)$ is horizontal at $x = -1$.
One possible shape for $y = f(x)$ is shown in Figure 2.26.

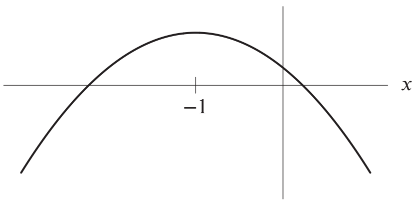


Figure 2.26

30. Since $f'(x) > 0$ for $1 < x < 3$, we see that $f(x)$ is increasing on this interval.
Since $f'(x) < 0$ for $x < 1$ and for $x > 3$, we see that $f(x)$ is decreasing on these intervals.
Since $f'(x) = 0$ for $x = 1$ and $x = 3$, the tangent to $f(x)$ will be horizontal at these x's.
One of many possible shapes of $y = f(x)$ is shown in Figure 2.27.

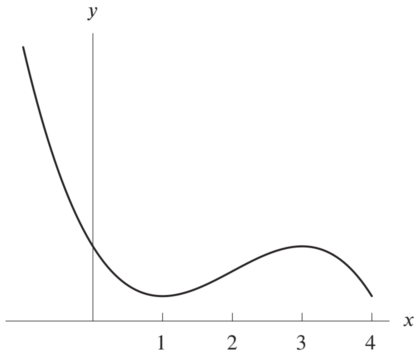


Figure 2.27

31. **(a)** Graph II
(b) Graph I
(c) Graph III

32. **(a)**
$$f'(1) \approx \frac{f(1.1) - f(1)}{0.1} = \frac{\ln(1.1) - \ln(1)}{0.1} \approx 0.95.$$
$$f'(2) \approx \frac{f(2.1) - f(2)}{0.1} = \frac{\ln(2.1) - \ln(2)}{0.1} \approx 0.49.$$
$$f'(3) \approx \frac{f(3.1) - f(3)}{0.1} = \frac{\ln(3.1) - \ln(3)}{0.1} \approx 0.33.$$
$$f'(4) \approx \frac{f(4.1) - f(4)}{0.1} = \frac{\ln(4.1) - \ln(4)}{0.1} \approx 0.25.$$
$$f'(5) \approx \frac{f(5.1) - f(5)}{0.1} = \frac{\ln(5.1) - \ln(5)}{0.1} \approx 0.20.$$
(b) It looks like the derivative of $\ln(x)$ is $1/x$.

33.

Table 2.2

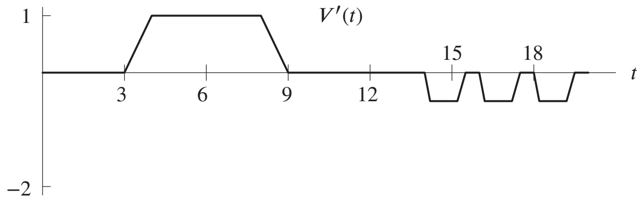
x	$f(x)$	x	$f(x)$	x	$f(x)$
1.998	2.6587	2.998	8.9820	3.998	21.3013
1.999	2.6627	2.999	8.9910	3.999	21.3173
2.000	2.6667	3.000	9.0000	4.000	21.3333
2.001	2.6707	3.001	9.0090	4.001	21.3493
2.002	2.6747	3.002	9.0180	4.002	21.3653

Near 2, the values of $f(x)$ seem to be increasing by 0.004 for each increase of 0.001 in x, so the derivative appears to be $\frac{0.004}{0.001} = 4$. Near 3, the values of $f(x)$ are increasing by 0.009 for each step of 0.001, so the derivative appears to be 9. Near 4, $f(x)$ increases by 0.016 for each step of 0.001, so the derivative appears to be 16. The pattern seems to be, then, that at a point x, the derivative of $f(x) = \frac{1}{3}x^3$ is $f'(x) = x^2$.

34. **(a)** The only graph in which the slope is 1 for all x is Graph (III).
(b) The only graph in which the slope is positive for all x is Graph (III).
(c) Graphs where the slope is 1 at $x = 2$ are Graphs (III) and (IV).
(d) Graphs where the slope is 2 at $x = 1$ are Graphs (II) and (IV).

35. **(a)** $t = 3$
(b) $t = 9$
(c) $t = 14$

(d)



Solutions for Section 2.3

1. In Leibniz notation the derivative is dD/dt and the units are feet per minute. This is because D is a function of t.

2. In Leibniz notation the derivative is dC/dW and the units are dollars per pound. This is because C is a function of W.

3. In Leibniz notation the derivative is dN/dD and the units are gallons per mile. This is because N is a function of D.

4. In Leibniz notation the derivative is dP/dH and the units are dollars per hour. This is because P is a function of H.

5. (a) The statement $f(5) = 18$ means that when 5 milliliters of catalyst are present, the reaction will take 18 minutes. Thus, the units for 5 are ml while the units for 18 are minutes.

(b) As in part (a), 5 is measured in ml. Since f' tells how fast T changes per unit a, we have f' measured in minutes/ml. If the amount of catalyst increases by 1 ml (from 5 to 6 ml), the reaction time decreases by about 3 minutes.

6. (a) If the price is \$150, then 2000 items will be sold.

(b) If the price goes up from \$150 by \$1 per item, about 25 fewer items will be sold. Equivalently, if the price is decreased from \$150 by \$1 per item, about 25 more items will be sold.

7. (a) The units of 0.073 are million square kilometers per day. This number tell us that on January 1, 2015, the area of Arctic Sea covered by ice was growing by about 73,000 km^2 per day.

(b) We know that, as long as Δt is small,

$$\frac{\Delta F}{\Delta t} \approx 0.073 \quad \text{so} \quad \Delta F \approx 0.073\Delta t.$$

In our case, $\Delta t = 5$ because there are five days between January 1 and January 6, so we have:

$$\Delta F = 0.073(5) = 0.365 \text{ million km}^2.$$

Thus, between January 1 and January 6 2015, the area of Arctic Sea covered by ice grew by about 365,000 km^2.

8. (a) The 12 represents the weight of the chemical; therefore, its units are pounds. The 5 represents the cost of the chemical; therefore, its units are dollars. The statement $f(12) = 5$ means that when the weight of the chemical is 12 pounds, the cost is 5 dollars.

(b) We expect the derivative to be positive since we expect the cost of the chemical to increase when the weight bought increases.

(c) Again, 12 is the weight of the chemical in pounds. The units of the 0.4 are dollars/pound since it is the rate of change of the cost as a function of the weight of the chemical bought. The statement $f'(12) = 0.4$ means that the cost is increasing at a rate of 0.4 dollars per pound when the weight is 12 pounds, or that an additional pound will cost about an extra 40 cents.

9. We use the interval from $t = 6$ to $t = 8$ to estimate $f'(6)$,

$$f'(6) \approx \frac{f(8) - f(6)}{8 - 6} = \frac{200 - 150}{2} = 25.$$

Thus, we have $f'(6) \approx 25$ megawatts per year. In 1996, the world solar energy output was increasing at a rate of about 25 megawatts per year.

10. (a) The turbine generates the most power when the wind speed is 10 meters per second. Maximum power is approximately 54,000 watts.

(b) Since $p'(7)$ is positive, the function p is increasing for wind speeds near 7 meters per second: for wind speeds near 7 m/sec, the turbine generates more power from faster winds.

Since $p'(15)$ is negative, the function p is decreasing for wind speeds near 15 meters per second: for wind speeds near 15 m/sec, the turbine generates less power from faster winds.

(c) The statement $p(7) = 17,500$ means that the turbine generates 17,500 watts when the wind is blowing at a speed of 7 meters per second.

The statement $p'(7) = 8000$ means that when the wind speed is around 7 meters per second, a 1 m/sec increase in wind speed corresponds to about an 8000 watt increase in power.

(d) The statement $p(15) = 50,000$ means means that the turbine generates 50,000 watts when the wind is blowing at a speed of 15 meters per second.

The statement $p'(15) = -800$ means that when the wind speed is around 15 meters per second, a 1 m/sec increase in wind speed corresponds to about an 800 watt decrease in power.

11. (a) The units of compliance are units of volume per units of pressure, or liters per centimeter of water.

(b) The increase in volume for a 5 cm reduction in pressure is largest between 10 and 15 cm. Thus, the compliance appears maximum between 10 and 15 cm of pressure reduction. The derivative is given by the slope, so

$$\text{Compliance} \approx \frac{0.70 - 0.49}{15 - 10} = 0.042 \text{ liters per centimeter.}$$

(c) When the lung is nearly full, it cannot expand much more to accommodate more air.

12. (a) The derivative $f'(r)$ is a function of the rainfall, r. Hence the units of 1500 are the units of rainfall, r, millimeters.

(b) Since $f'(r) = dw/dr$, we have

$$\text{Units of } f'(r) = \frac{\text{Units of } w}{\text{Units of } r} = \frac{\text{mm (of leaf width)}}{\text{mm (of rain)}}.$$

(c) We have

$$\begin{aligned}\Delta w &\approx f'(2000)\Delta x\\ &= (0.0218)(200) = 4.36 \text{ mm.}\end{aligned}$$

The average leaf width in the rainier forest is about 4.4 mm greater.

13. (a) The derivative $f'(x)$ is a function of the distance from the top of the slope, x. Hence the units of 15 are the units of distance, x, meters.

(b) Since $f'(h) = dh/dx$, we have

$$\text{Units of } f'(x) = \frac{\text{Units of } h}{\text{Units of } x} = \frac{\text{mm}}{\text{meters}}.$$

(c) We have

$$\begin{aligned}\Delta h &\approx f'(15)\Delta x\\ &= (0.02)(4) = 0.08 \text{ mm.}\end{aligned}$$

The runoff is about 0.08 mm deeper at a distance of 4 meters farther down the slope.

14. (a) We have

$$\text{Units of } \frac{dQ}{dV} = \frac{\text{Units of } Q}{\text{Units of } V} = \frac{\text{meters}^3/\text{sec}}{\text{million meters}^3}.$$

(b) Since

$$500{,}000 \text{ meters}^3 = 0.5 \text{ million meters}^3$$

we have

$$\begin{aligned}\Delta Q &\approx \frac{dQ}{dV}\Delta V\\ &= (22)(0.5) = 11 \frac{\text{meters}^3}{\text{second}}.\end{aligned}$$

Maximal outflow rate from the larger lake is about 11 meters^3/second greater.

15. Let p be the rating points earned by the CBS Evening News, let R be the revenue earned in millions of dollars, and let $R = f(p)$. When $p = 4.3$,

$$\text{Rate of change of revenue} \approx \frac{\$5.5 \text{ million}}{0.1 \text{ point}} = 55 \text{ million dollars/point.}$$

Thus

$$f'(4.3) \approx 55.$$

16. **(a)** The units of $S'(t)$ are acre-feet per week.
(b) For $S'(t) > 0$ to occur, the amount of water in the reservoir has to increase. This can happen if more water flows into the reservoir than flows out of the reservoir, for example after a recent storm or if the water agency reduces the water outflow to let the reservoir fill.

17. **(a)** The yam is cooling off so T is decreasing and $f'(t)$ is negative.
(b) Since $f(t)$ is measured in degrees Fahrenheit and t is measured in minutes, df/dt must be measured in units of $^\circ$F/min.

18. The units of $f'(x)$ are feet/mile. The derivative, $f'(x)$, represents the rate of change of elevation with distance from the source, so if the river is flowing downhill everywhere, the elevation is always decreasing and $f'(x)$ is always negative. (In fact, there may be some stretches where the elevation is more or less constant, so $f'(x) = 0$.)

19. **(a)** The units of lapse rate are the same as for the derivative dT/dz, namely (units of T)/(units of z) $= \ ^\circ C/\text{km}$.
(b) Since the lapse rate is 6.5, the derivative of T with respect to z is $dT/dz = -6.5^\circ C/\text{km}$. The air temperature drops about 6.5° for one more kilometer you go up.

20. **(a)** This means that investing the \$1000 at 2% would yield \$1221 after 10 years.
(b) Writing $g'(r)$ as dB/dr, we see that the units of dB/dr are dollars per percent (interest). We can interpret dB as the extra money earned if interest rate is increased by dr percent. Therefore $g'(2) = \frac{dB}{dr}|_{r=2} \approx 122$ means that the balance, at 2% interest, would increase by about \$122 if the interest rate were increased by 1%. In other words, $g(3) \approx g(2) + 122 = 1221 + 122 = 1343$.

21. **(a)** The derivative has units of people/second, so we find the rate of births, deaths, and migrations per second and combine them.

$$\begin{aligned} \text{Birth rate} &= \frac{1}{8} \text{ people per second} \\ \text{Death rate} &= \frac{1}{12} \text{ people per second} \\ \text{Migration rate} &= \frac{1}{32} \text{ people per second} \end{aligned}$$

Thus

$$f'(0) = \text{ Rate of change of population } = \frac{1}{8} - \frac{1}{12} + \frac{1}{32} = 0.0729 \text{ people/second.}$$

In other words, the population is increasing at 0.0729 people per second.
(b) From the answer to part (a), we see that it took $1/0.0729 = 13.71 \approx 14$ seconds to add one person.

22. **(a)** The derivative $f'(t)$ appears to be positive for most of the period 2011-2015, because according to the table, gold production is increasing.
(b) The derivative (or rate of change) appears to be greatest between 2013 and 2014.
(c) We have

$$f'(2015) \approx \frac{3000 - 2990}{2015 - 2014} = 10 \text{ metric tons/year.}$$

In 2015, gold production was increasing at a rate of approximately 10 metric tons per year.
(d) In 2015, gold production was 3000 metric tons and was increasing at a rate of 10 metric tons each year. Therefore, in 2016 (one year later), we have

$$f(2016) \approx 3000 + 10 = 3010 \text{ metric tons,}$$

and in 2020 (5 years later), we have

$$f(2020) \approx 3000 + 10(5) = 3050 \text{ metric tons.}$$

We estimate that gold production in 2015 is 3010 metric tons and gold production in 2020 is 3050 metric tons.

23. **(a)** Since $W = f(c)$ where W is weight in pounds and c is the number of Calories consumed per day:

$f(1800) = 155$	means that	consuming 1800 Calories per day results in a weight of 155 pounds.
$f'(2000) = 0$	means that	consuming 2000 Calories per day causes neither weight gain nor loss.

(b) The units of dW/dc are pounds/(Calories/day).

24. **(a)** The statement $f(200) = 1300$ means that it costs \$1300 to produce 200 gallons of the chemical.
(b) The statement $f'(200) = 6$ means that when the number of gallons produced is 200, costs are increasing at a rate of \$6 per gallon. In other words, it costs about \$6 to produce the next (the 201$^{\text{st}}$) gallon of the chemical.

25. **(a)** An additional dollar per year of government purchases increases national output for the year by about \$0.60. The derivative is called a fiscal policy multiplier because if government purchases increase by x dollars per year, then national output increases by about $0.60x$ dollars per year.
(b) An additional tax dollar collected per year decreases national output for the year by about \$0.26. The derivative is called a fiscal policy multiplier because if government tax revenues increase by x dollars per year, then national output decreases by about $0.26x$ dollars per year.

26. **(a)** Positive, since weight increases as the child gets older.
(b) $f(8) = 45$ tells us that when the child is 8 years old, the child weighs 45 pounds.
(c) The units of $f'(a)$ are lbs/year. The value of $f'(a)$ gives approximate increase in weight for a 1 year increase in age.
(d) $f'(8) = 4$ tells us that an 8-year-old child weighs about 4 more pounds after the next year.
(e) As a increases, $f'(a)$ will decrease since the rate of growth slows down as the child grows up.

27. Let $f(t)$ be the age of onset of Alzheimer's, in weeks, of a typical man who retires at age t years. The derivative $f'(t)$ has units weeks/year. The study reports that each additional year of employment is associated with about a six week later age of onset, that is, $f'(t) \approx 6$.

28. **(a)** Since $f'(c)$ is negative, the function $P = f(c)$ is decreasing: on average, pelican eggshells are thinner if the PCB concentration, c, in the eggshell is higher.
(b) The statement $f(200) = 0.28$ means that the thickness of pelican eggshells is 0.28 mm when the concentration of PCBs in the eggshell is 200 parts per million (ppm).

The statement $f'(200) = -0.0005$ means that when the PCB concentration is 200 ppm, a 1 ppm increase in the concentration typically corresponds to about a 0.0005 mm decrease in eggshell thickness.

29. Since we do not have information beyond $t = 25$, we will assume that the function will continue to change at the same rate. Therefore,

$$\begin{aligned} f(26) &\approx f(25) + f'(25) \\ &= 3.6 + (-0.2) = 3.4. \end{aligned}$$

Since $30 = 25 + 5$, then

$$\begin{aligned} f(30) &\approx f(25) + f'(25)(5) \\ &= 3.6 + (-0.2)(5) = 2.6. \end{aligned}$$

30. Using $\Delta x = 1$, we can say that

$$\begin{aligned} f(21) &= f(20) + \text{change in } f(x) \\ &\approx f(20) + f'(20)\Delta x \\ &= 68 + (-3)(1) \\ &= 65. \end{aligned}$$

Similarly, using $\Delta x = -1$,

$$\begin{aligned} f(19) &= f(20) + \text{change in } f(x) \\ &\approx f(20) + f'(20)\Delta x \\ &= 68 + (-3)(-1) \\ &= 71. \end{aligned}$$

Using $\Delta x = 5$, we can write

$$\begin{aligned} f(25) &= f(20) + \text{change in } f(x) \\ &\approx f(20) + (-3)(5) \\ &= 68 - 15 \\ &= 53. \end{aligned}$$

31. We have $\Delta x = 4.02 - 4 = 0.02$. From the tangent line approximation we get:

$$\Delta y \approx f'(4)\Delta x = 7(0.02) = 0.14.$$

Thus,

$$f(4.02) = f(4) + \Delta y \approx 5 + 0.14 = 5.14.$$

32. We have $\Delta x = 3.92 - 4 = -0.08$. From the tangent line approximation we get:

$$\Delta y \approx f'(4)\Delta x = 7(-0.08) = -0.56.$$

Thus,

$$f(3.92) = f(4) + \Delta y \approx 5 - 0.56 = 4.44.$$

33. We have $\Delta x = 5.03 - 5 = 0.03$. From the tangent line approximation we get:

$$\Delta y \approx f'(5)\Delta x = -2(0.03) = -0.06.$$

Thus,

$$f(5.03) = f(5) + \Delta y \approx 3 - 0.06 = 2.94.$$

34. We have $\Delta x = 1.95 - 2 = -0.05$. From the tangent line approximation we get:

$$\Delta y \approx f'(2)\Delta x = -3(-0.05) = 0.15.$$

Thus,

$$f(1.95) = f(2) + \Delta y \approx -4 + 0.15 = -3.85.$$

35. We have $\Delta x = -2.99 - (-3) = 0.01$. From the tangent line approximation we get:

$$\Delta y \approx f'(-3)\Delta x = 2(0.01) = 0.02.$$

Thus,

$$f(-2.99) = f(-3) + \Delta y \approx -4 + 0.02 = -3.98.$$

36. We have $\Delta x = 2.99 - 3 = -0.01$. From the tangent line approximation we get:

$$\Delta y \approx f'(3)\Delta x = -2(-0.01) = 0.02.$$

Thus,

$$f(2.99) = f(3) + \Delta y \approx -4 + 0.02 = -3.98.$$

37. **(a)** We have $\Delta V \approx 16\Delta r$.
(b) We have $\Delta V \approx 16(0.1) = 1.6$.
(c) Since $V = 32$ when $r = 2$ and $\Delta V \approx 1.6$, we have $V \approx 32 + 1.6 = 33.6$.

38. **(a)** We have $\Delta R \approx 3\Delta S$.
(b) We have $\Delta S = 10.2 - 10 = 0.2$. Therefore $\Delta R \approx 3(0.2) = 0.6$.
(c) Since $R = f(10) = 13$ and the change in $\Delta R = 0.6$, we have $R = f(10.2) \approx 13 + 0.6 = 13.6$.

39. **(a)** kilograms per week
(b) At week 24 the fetus is growing at a rate of 0.096 kg/week, or the fetus will weigh about 0.096 kilograms more in one week.

40. **(a)** The tangent line to the weight graph is steeper at 36 weeks then at 20 weeks, so $g'(36)$ is greater than $g'(20)$.
(b) The fetus increases its weight more rapidly at week 36 than at week 20.

41. Compare the secant line to the graph from week 0 to week 40 to the tangent lines at week 16 and week 36.

(a) At week 16 the secant line is steeper than the tangent line. The instantaneous weight growth rate is less than the average.
(b) At week 36 the tangent line is steeper than the secant line. The instantaneous weight growth rate is greater than the average.

42. **(a)** We use the interval from $t = 20$ to $t = 24$ to estimate $g'(20)$,

$$g'(20) \approx \frac{g(24) - g(20)}{24 - 20} = \frac{0.60 - 0.25}{4} = 0.0875 \text{ kg/week}.$$

(b) We use the interval from $t = 36$ to $t = 40$ to estimate $g'(36)$,

$$g'(36) \approx \frac{g(40) - g(36)}{40 - 36} = \frac{3.1 - 2.3}{4} = 0.20 \text{ kg/week}.$$

(c) The average rate of change is the slope of the secant line from $(0, 0)$ to $(40, 3.1)$. Thus,

$$\text{Average rate of change} = \frac{3.1 - 0}{40 - 0} = 0.078 \text{ kg/week}.$$

43. **(a)** The statement $f(5) = 7.39$ means that annual net sales for the Hershey Company were 7.39 billion dollars in 2015. The statement $f'(5) = -0.03$ tells us that in 2015, annual net sales were decreasing by about 0.03 billion dollars per year.

(b) Since sales were 7.39 billion in 2015 and decreasing at 0.03 billion dollars per year, we estimate that, three years later in 2018,

$$\text{Sales in 2018} \approx 7.39 + 3(-0.03) = 7.3 \text{ billion dollars}.$$

Thus $f(8) = 7.3$, so annual net sales for the Hershey Company are projected to be 7.3 billion dollars in 2018. This prediction assumes that the growth rate remains constant.

44. **(a)** The statement $f(140) = 120$ means that a patient weighing 140 pounds should receive a dose of 120 mg of the painkiller. The statement $f'(140) = 3$ tells us that if the weight of a patient increases by one pound (from 140 pounds), the dose should be increased by about 3 mg.

(b) Since the dose for a weight of 140 lbs is 120 mg and at this weight the dose goes up by about 3 mg for one pound, a 145 lb patient should get about an additional $3(5) = 15$ mg. Thus, for a 145 lb patient, the correct dose is approximately

$$f(145) \approx 120 + 3(5) = 135 \text{ mg}.$$

45. **(a)** The statement $f(5) = 23.7$ tells us that beef production was 23.7 billion pounds in 2015. The statement $f'(5) = -0.1$ means that in 2015, US beef production was decreasing by about 0.1 billion pounds per year.

(b) We assume that the growth rate remains constant until $t = 8$. Since production in 2015 is 23.7 billion pounds and is decreasing at 0.1 billion pounds a year, we expect production in 2018 to be approximately

$$f(8) \approx 23.7 + 3 \cdot (-0.1) = 23.7 - 0.3 = 23.4 \text{ billion pounds}.$$

46. **(a)** The statement $f(20) = 0.36$ means that 20 minutes after smoking a cigarette, there will be 0.36 mg of nicotine in the body. The statement $f'(20) = -0.002$ means that 20 minutes after smoking a cigarette, about 0.002 mg of nicotine leaves the body in the next minute. The units are 20 minutes, 0.36 mg, and -0.002 mg/minute.

(b)

$$\begin{aligned}
f(21) &\approx f(20) + \text{ change in } f \text{ in one minute} \\
&= 0.36 + (-0.002) \\
&= 0.358 \\
f(30) &\approx f(20) + \text{change in } f \text{ in 10 minutes} \\
&= 0.36 + (-0.002)(10) \\
&= 0.36 - 0.02 \\
&= 0.34
\end{aligned}$$

47. **(a)** Since t represents the number of days from now, we are told $f(0) = 80$ and $f'(0) = 0.50$.

(b)

$$\begin{aligned}
f(10) &\approx \text{value now} + \text{change in value in 10 days} \\
&= 80 + 0.50(10) \\
&= 80 + 5 \\
&= 85.
\end{aligned}$$

In 10 days, we expect that the mutual fund will be worth about \$85 a share.

48. **(a)** The slope of the tangent line at 2 kg can be approximated by the slope of the secant line passing through the points (2, 6) and (3, 2). So

$$\text{Slope of tangent line} \approx \frac{v(3) - v(2)}{3 - 2} = \frac{2 - 6}{1} = -4 \text{ (cm/sec)/kg.}$$

(b) Since 50 grams = 0.050 kg, the contraction velocity changes by about -4(cm/sec)/kg $\cdot$ 0.050kg $= -0.20$ cm/sec. The velocity is reduced by about 0.20 cm/sec or 2.0 mm/sec.

(c) Since $v(x)$ is the contraction velocity in cm/sec with a load of x kg, we have $v'(2) = -4$.

49. **(a)** The slope of the tangent line at 2 hours can be approximated by the slope of the secant line passing through the points (2, 4.2) and (2.5, 4.75). So

$$\text{Slope of tangent line} \approx \frac{g(2.5) - g(2)}{2.5 - 2} = \frac{4.75 - 4.2}{0.5} = 1.1 \text{ (liters/minute)/hour.}$$

(b) The rate of change of the pumping rate is the slope of the tangent line. One minute = 1/60 hour, so in one minute the

$$\text{Pumping rate increases by about } 1.1 \frac{\text{(liter/minute)}}{\text{hour}} \cdot \frac{1}{60} \text{ hour } = 0.018 \text{ liter/minute.}$$

(c) Since $g(t)$ is the pumping rate in liters/minute at time t hours, we have $g'(2) = 1.1$.

50. Units of $C'(r)$ are dollars/percent. Approximately, $C'(r)$ means the additional amount needed to pay off the loan when the interest rate is increased by 1%. The sign of $C'(r)$ is positive, because increasing the interest rate will increase the amount it costs to pay off a loan.

51. The consumption rates (kg/week) are the rates at which the quantities are decreasing, that is, -1 times the derivatives of the storage functions. To compare rates at a given time, compare the steepness of the tangent lines to the graphs at that time.

(a) At 3 weeks, the tangent line to the fat storage graph is steeper than the tangent line to the protein storage graph. During the third week, fat is consumed at a greater rate than protein.

(b) At 7 weeks, the protein storage graph is steeper than the fat storage graph. During the seventh week, protein is consumed at a greater rate than fat.

52. Where the graph is linear, the derivative of the fat storage function is constant. The derivative gives the rate of fat consumption (kg/week). Thus, for the first four weeks the body burns fat at a constant rate.

53. The fat consumption rate (kg/week) is the rate at which the quantity of fat is decreasing, that is, -1 times the derivative of the fat storage function. We estimate the derivatives at 3, 6, and 8 weeks by calculating the slope of the secant line.

(a) The tangent line at 3 weeks can be approximated by the secant line containing (3, 6) and (4, 4). So

$$\text{Slope of the tangent line } \approx \frac{4 - 12}{4 - 0} = -2.0 \text{ kg/week.}$$

The consumption rate is 2.0 kg/week.

(b) Two points on the secant line are (6, 1.2) and (7, 0.8). So

$$\text{Slope of the tangent line} = \frac{0.8 - 1.2}{7 - 6} = -0.4 \text{ kg/week.}$$

The consumption rate is 0.4 kg/week.

(c) Two points on the secant line are (8, 0.5) and (7, 0.8). So

$$\text{Slope of the tangent line} = \frac{0.8 - 0.5}{7 - 8} = -0.3 \text{ kg/week.}$$

The consumption rate is 0.3 kg/week.

54. The body changes from burning more fat than protein to burning more protein. This is done by reducing the rate at which it burns fat and simultaneously increasing the rate at which it burns protein. The physiological reason is that the body has begun to run out of fat.

55. The graph of fat storage is linear for four weeks, then becomes concave up. Thus, the derivative of fat storage is constant for four weeks, then increases. This matches graph I.

The graph of protein storage is concave up for three weeks, then becomes concave down. Thus, the derivative of protein storage is increasing for three weeks and then becomes decreasing. This matches graph II.

56. **(a)** See part (b).

(b) See Figure 2.28.

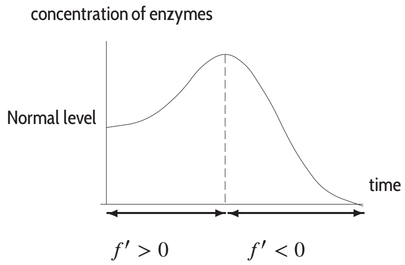


Figure 2.28

(c) The derivative, f', is the rate at which the concentration is increasing or decreasing. We see that f' is positive at the start of the disease and negative toward the end. In practice, of course, f' cannot be measured directly. Checking the value of C in blood samples taken on consecutive days allows us to estimate $f'(t)$:

$$f'(t) \approx f(t+1) - f(t) = \frac{f(t+1) - f(t)}{(t+1) - t}.$$

57. **(a)** The company hopes that increased advertising always brings in more customers instead of turning them away. Therefore, it hopes $f'(a)$ is always positive.

(b) If $f'(100) = 2$, it means that if the advertising budget is \$100,000, an extra dollar spent on advertising will bring in about \$2 worth of sales. If $f'(100) = 0.5$, an extra dollar above \$100 thousand spent on advertising will bring in about \$0.50 worth of sales.

(c) If $f'(100) = 2$, then as we saw in part (b), spending slightly more than \$100,000 will increase revenue by an amount greater than the additional expense, and thus more should be spent on advertising. If $f'(100) = 0.5$, then the increase in revenue is less than the additional expense, hence too much is being spent on advertising. The optimum amount to spend, a, is an amount that makes $f'(a) = 1$. At this point, the increases in advertising expenditures just pay for themselves. If $f'(a) < 1$, too much is being spent; if $f'(a) > 1$, more should be spent.

58. **(a)** If $f'(80{,}000) = 2$, it means that if the budget for materials is \$80,000, another dollar spent on materials will bring in about \$2 more in revenue. If $f'(80{,}000) = 0.5$, another dollar spent on materials will bring in about \$0.50 more in revenue.

(b) If $f'(100) = 2$, then as we saw in part (b), spending slightly more than \$80,000 will increase revenue by an amount greater than the additional expense, and thus more should be spent on materials. If $f'(100) = 0.5$, then the increase in revenue is less than the additional expense, and it does not make sense to spend more on materials.

59. **(a)** If $dA/dg = 1.3$, then producing 1 more gallon of biofuel requires about 1.3 more gallons of gasoline. This is not sustainable since we are using more gallons than we are producing.

(b) If $dA/dg = 0.2$, then producing 1 more gallon of biofuel requires about 0.2 more gallons of gasoline. This might be sustainable since we are able to use the gasoline to produce many more gallons of biofuel.

60. **(a)** Let $f(t)$ be the mass, in gigatons, of the Greenland Ice Sheet t years after 2002 (Alternatively, in year t). We are given information about $f'(t)$, which has units gigatons per year.

(b) If t is in years since 2002, we know $f'(11)$ is approximately -6 gt/year. Similarly, $f'(10)$ is approximately -474 gt/yr.

61. **(a)** The statement $f(5) = 5500$ tells us that there were 5,500 thousand square kilometers of rain forest in the Amazon in 2015. The derivative $f'(5) = -10.9$ tells us that in 2015, after one more year the Amazon's rain forests will have shrunk by about 10.9 thousand square kilometers.

(b) We have

$$\text{Relative rate of change } = \frac{f'(5)}{f(5)} = \frac{-10.9}{5500} = -0.00198.$$

The Amazon's rain forests are shrinking at a continuous rate of 0.198% per year.

62. Since $t = 12$ is the end of December 2015, we have $f(12) = 1.59$ billion users. We use a difference quotient with $t = 9$ for the end of September 2015 and $t = 12$ for the end of December 2015 to estimate the derivative:

$$f'(12) \approx \frac{f(12) - f(9)}{12 - 9} = \frac{1.59 - 1.55}{12 - 9} = 0.0133 \text{ billion users per month.}$$

To estimate the relative rate of change, we use

$$\text{Relative rate of change } = \frac{f'(12)}{f(12)} = \frac{0.0133}{1.59} = 0.0084 \text{ per month.}$$

The number of active Facebook users at the end of December 2015 was 1.59 billion. The number was increasing at 0.0133 billion, (13.3 million) users per month, which represents a relative growth rate of 0.84% per month.

63. Estimating the relative rate of change using $\Delta t = 0.01$, we have

$$\frac{1}{f}\frac{\Delta f}{\Delta t} \approx \frac{1}{f(4)}\frac{f(4.01)-f(4)}{0.01} = \frac{1}{4^2}\frac{4.01^2-4^2}{0.01} = 0.50.$$

64. Estimating the relative rate of change using $\Delta t = 0.01$, we have

$$\frac{1}{f}\frac{\Delta f}{\Delta t} \approx \frac{1}{f(10)}\frac{f(10.01)-f(10)}{0.01} = \frac{1}{10^2}\frac{10.01^2-10^2}{0.01} = 0.20.$$

65. Let $P = f(t)$. In 2020 we have $t = 4$. The relative rate of change of f in 2020 is $f'(4)/f(4)$. We estimate $f'(4)$ using a difference quotient.

(a) Estimating the relative rate of change using $\Delta t = 1$ at $t = 4$, we have

$$\frac{dP/dt}{P} = \frac{f'(4)}{f(4)} \approx \frac{1}{f(4)}\frac{f(5)-f(4)}{1} = 0.01076 = 1.076\% \text{ per year.}$$

(b) With $\Delta t = 0.1$ and $t = 4$, we have

$$\frac{dP/dt}{P} = \frac{f'(4)}{f(4)} \approx \frac{1}{f(4)}\frac{f(4.1)-f(4)}{0.1} = 0.01071 = 1.071\% \text{ per year.}$$

(c) With $\Delta t = 0.01$ and $t = 4$, we have

$$\frac{dP/dt}{P} = \frac{f'(4)}{f(4)} \approx \frac{1}{f(4)}\frac{f(4.01)-f(4)}{0.01} = 0.01070 = 1.07\% \text{ per year.}$$

The relative rate of change is approximately 1.07% per year. This is as we would expect since the function $P = 7.4e^{0.0107t}$ has a continuous rate of change of 1.07% per year for all t.

66. **(a)** At 2 1/2 months, the baby weighs 5.67 kilograms.

(b) At 2 1/2 months, the baby's weight is increasing at a relative rate of 13% per month.

67. **(a)** The statement $g(36) = 15$ tells us that 15 billion Apps had been downloaded by June 2011 (36 months after June, 2008). The statement $g'(36) = 0.93$ tells us that in June 2011 the number of downloaded Apps was increasing at a rate of 0.93 billion Apps per month.

(b) We have

$$\text{Relative rate of change of downloaded Apps } = \frac{g'(36)}{g(36)} = \frac{0.93}{15} = 0.062.$$

In June 2011, monthly downloaded Apps were increasing at a continuous rate of 6.2% per month.

68. March 2015 corresponds to $t = 14$. Let $B = f(t)$. The relative rate of change of f at $t = 14$ is $f'(14)/f(14)$. We estimate $f'(14)$ using a difference quotient.

(a) Estimating the relative rate of change using $\Delta t = 1$ at $t = 14$, we have

$$\frac{dB/dt}{B} = \frac{f'(14)}{f(14)} \approx \frac{1}{f(14)}\frac{f(15)-f(14)}{1} = 0.02163 = 2.163\% \text{ per month}$$

(b) With $\Delta t = 0.1$ and $t = 14$, we have

$$\frac{dB/dt}{B} = \frac{f'(14)}{f(14)} \approx \frac{1}{f(14)}\frac{f(14.1)-f(14)}{0.1} = 0.02142 = 2.142\% \text{ per month}$$

(c) With $\Delta t = 0.01$ and $t = 14$, we have

$$\frac{dB/dt}{B} = \frac{f'(14)}{f(14)} \approx \frac{1}{f(14)}\frac{f(14.01)-f(14)}{0.01} = 0.02140 = 2.140\% \text{ per month}$$

The relative rate of change is approximately 2.14% per month.

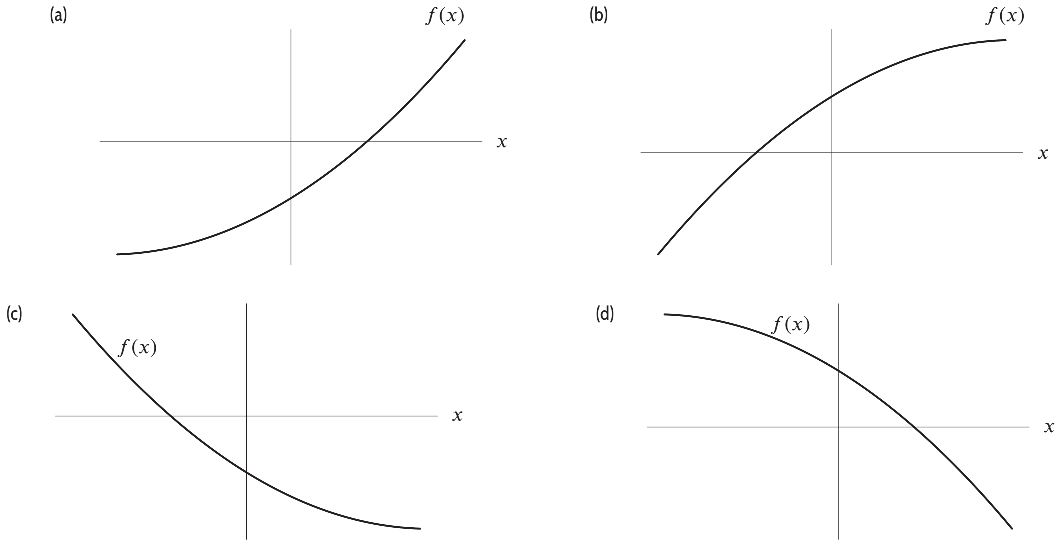
69. Since $O'(2016) = -1.5$, we know the ODGI is decreasing at 1.5 units per year. Assuming that it continues to decrease at the same rate, to reduce the ODGI from 81.4 to 0 will take $81.4/1.5 = 54.3$ years. Thus, the ozone hole is predicted to recover in $2016 + 54.3 = 2070.3$—in other words, by 2071.

Solutions for Section 2.4

1. **(a)** Since $g(x)$ is decreasing at $x = 0$, the value of $g'(0)$ is negative.
(b) Since $g(x)$ is concave down at $x = 0$, the value of $g''(0)$ is negative.

2. At E both dy/dx and d^2y/dx^2 could be negative because y is decreasing and the graph is concave down there. At all the other points one or both of the derivatives could not be negative.

3.



4. $f'(x) > 0$
$f''(x) > 0$

5. $f'(x) = 0$
$f''(x) = 0$

6. $f'(x) < 0$
$f''(x) = 0$

7. $f'(x) < 0$
$f''(x) > 0$

8. $f'(x) > 0$
$f''(x) < 0$

9. $f'(x) < 0$
$f''(x) < 0$

10. The slope of the graph is larger at $x = 0$ than at $x = 4$, so $f'(0) > f'(4)$.

11. The downward slope of the graph at $x = 2$ is steeper than at $x = 6$. Thus, $f'(2)$ is farther below zero than $f'(6)$, so $f'(2) < f'(6)$.

12. The graph is concave down at $x = 1$, so $f''(1)$ is negative. On the other hand the graph is concave up at $x = 3$, so $f''(3)$ is positive. Thus, $f''(1) < f''(3)$.

13. The derivative is positive on those intervals where the function is increasing and negative on those intervals where the function is decreasing. Therefore, the derivative is positive on the intervals $0 < t < 0.4$ and $1.7 < t < 3.4$, and negative on the intervals $0.4 < t < 1.7$ and $3.4 < t < 4$.

The second derivative is positive on those intervals where the graph of the function is concave up and negative on those intervals where the graph of the function is concave down. Therefore, the second derivative is positive on the interval $1 < t < 2.6$ and negative on the intervals $0 < t < 1$ and $2.6 < t < 4$.

14. The derivative is positive on those intervals where the function is increasing and negative on those intervals where the function is decreasing. Therefore, the derivative is positive on the interval $-2.3 < t < -0.5$ and negative on the interval $-0.5 < t < 4$.

The second derivative is positive on those intervals where the graph of the function is concave up and negative on those intervals where the graph of the function is concave down. Therefore, the second derivative is positive on the interval $0.5 < t < 4$ and negative on the interval $-2.3 < t < 0.5$.

15. The derivative of $w(t)$ appears to be negative since the function is decreasing over the interval given. The second derivative, however, appears to be positive since the function is concave up, i.e., it is decreasing at a decreasing rate.

16. The derivative, $s'(t)$, appears to be positive since $s(t)$ is increasing over the interval given. The second derivative also appears to be positive or zero since the function is concave up or possibly linear between $t = 1$ and $t = 3$, i.e., it is increasing at a non-decreasing rate.

17. The graph must be everywhere decreasing and concave up on some intervals and concave down on other intervals. One possibility is shown in Figure 2.29.

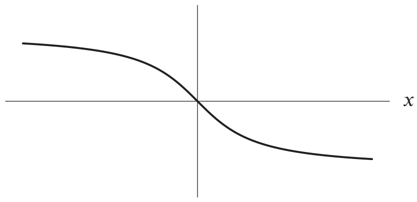


Figure 2.29

18. Since all advertising campaigns are assumed to produce an increase in sales, a graph of sales against time would be expected to have a positive slope.

A positive second derivative means the rate at which sales are increasing is increasing. If a positive second derivative is observed during a new campaign, it is reasonable to conclude that this increase in the rate sales are increasing is caused by the new campaign–which is therefore judged a success. A negative second derivative means a decrease in the rate at which sales are increasing, and therefore suggests the new campaign is a failure.

19. **(a)** The function appears to be decreasing and concave down, and so we conjecture that f' is negative and that f'' is negative.

(b) We use difference quotients to the right:
$f'(2) \approx \frac{137-145}{4-2} = -4$
$f'(8) \approx \frac{56-98}{10-8} = -21.$

20. **(a)** The derivative, $f'(t)$, appears to be positive since the number of cars is increasing. The second derivative, $f''(t)$, appears to be negative during the period 1975–1990 because the rate of change is increasing. For example, between 1975 and 1980, the rate of change is $(121.6 - 106.7)/5 = 2.98$ million cars per year, while between 1985 and 1990, the rate of change is 1.16 million cars per year.

(b) The derivative, $f'(t)$, appears to be negative between 1990 and 1995 since the number of cars is decreasing, but increasing between 1995 and 2000. The second derivative, $f''(t)$, appears to be positive during the period 1990–2000 because the rate of change is increasing. For example, between 1990 and 1995, the rate of change is $(128.4-133.7)/5 = -1.06$ million cars per year, while between 1995 and 2000, the rate of change is 1.04 million cars per year.

(c) To estimate $f'(2005)$ we consider the interval 2000–2005

$$f'(2005) \approx \frac{f(2005) - f(2000)}{2005 - 2000} \approx \frac{136.6 - 133.6}{5} = \frac{3}{5} = 0.6.$$

We estimate that $f'(2005) \approx 0.6$ million cars per year. The number of passenger cars in the US was increasing at a rate of about 600,000 cars per year in 2005.

21. This graph is increasing for all x, and is concave down to the left of 2 and concave up to the right of 2. One possible answer is shown in Figure 2.30:

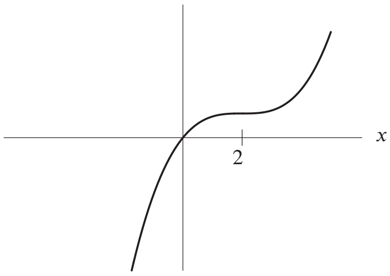


Figure 2.30

22. Since $f(2) = 5$, the graph goes through the point $(2, 5)$. Since $f'(2) = 1/2$, the slope of the curve is $1/2$ when it passes through this point. Since $f''(2) > 0$, the graph is concave up at this point. One possible graph is shown in Figure 2.31. Many other answers are also possible.

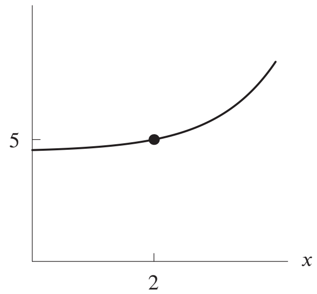


Figure 2.31

23. The two points at which $f' = 0$ are A and B. Since f' is nonzero at C and D and f'' is nonzero at all four points, we get the completed Table 2.3:

Table 2.3

Point	f	f'	f''
A	$-$	0	$+$
B	$+$	0	$-$
C	$+$	$-$	$-$
D	$-$	$+$	$+$

24. (b). The positive first derivative tells us that the temperature is increasing; the negative second derivative tells us that the rate of increase of the temperature is slowing.

25. (e). Since the smallest value of $f'(t)$ was $2°\text{C/hour}$, we know that $f'(t)$ was always positive. Thus, the temperature rose all day.

26. To the right of $x = 5$, the function starts by increasing, since $f'(5) = 2 > 0$ (though f may subsequently decrease) and is concave down, so its graph looks like the graph shown in Figure 2.32. Also, the tangent line to the curve at $x = 5$ has slope 2 and lies above the curve for $x > 5$. If we follow the tangent line until $x = 7$, we reach a height of 24. Therefore, $f(7)$ must be smaller than 24, meaning 22 is the only possible value for $f(7)$ from among the choices given.

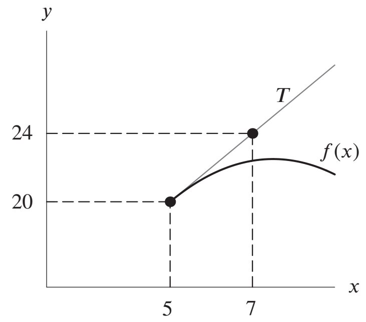


Figure 2.32

27. **(a)** The EPA will say that the rate of discharge is still rising. The industry will say that the rate of discharge is increasing less quickly, and may soon level off or even start to fall.

(b) The EPA will say that the rate at which pollutants are being discharged is leveling off, but not to zero—so pollutants will continue to be dumped in the lake. The industry will say that the rate of discharge has decreased significantly.

28. **(a)** Let $N(t)$ be the number of people below the poverty line. See Figure 2.33.

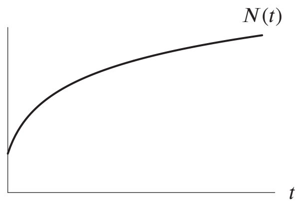
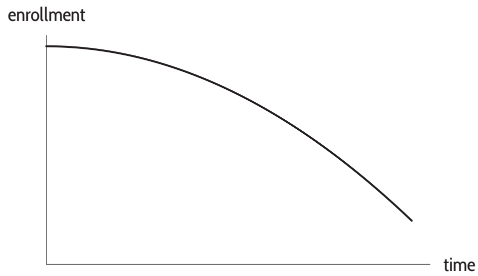


Figure 2.33

(b) dN/dt is positive, since people are still slipping below the poverty line. d^2N/dt^2 is negative, since the rate at which people are slipping below the poverty line, dN/dt, is decreasing.

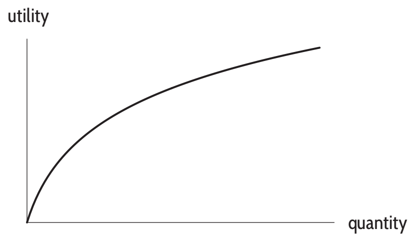
29. **(a)** The author is talking about the number of students enrolled in law schools in the United States over time. We could let $f(t)$ be the total number of students enrolled in US law schools in year t.

(b)



(c) Since law school enrollment is decreasing, $f'(t) < 0$. Since the slide accelerates, $f(t)$ is concave down, so $f''(t) < 0$.

30. **(a)**



(b) As a function of quantity, utility is increasing but at a decreasing rate; the graph is increasing but concave down. So the derivative of utility is positive, but the second derivative of utility is negative.

31. **(a)** $dP/dt > 0$ and $d^2P/dt^2 > 0$.

(b) $dP/dt < 0$ and $d^2P/dt^2 > 0$ (but dP/dt is close to zero).

32. (a) IV, (b) III, (c) II, (d) I, (e) IV, (f) II

33. **(a)** Given that Arctic Sea ice extent grows from November to March, $G(t)$ must be increasing over $0 < t < 4$, which means $G'(t) > 0$ over this same interval.

(b) The sign of the second derivative indicates G is concave down for $0 < t < 4$. Under this assumption, the Arctic Sea ice extent grows quickly at the beginning of the winter period and slower toward the end of the period, as warmer months approach.

(c) The graph of $G(t)$ must be increasing and concave down over the interval $0 < t < 4$. One possible graph[1] appears in Figure 2.34.

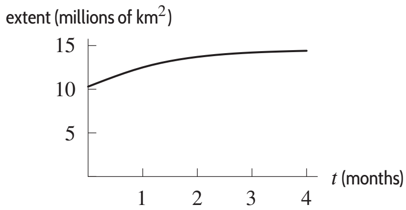


Figure 2.34: Arctic Sea ice extent from Nov. 1, 2014 to Mar. 1, 2015

34. **(a)** Since the sea level is rising, we know that $a'(t) > 0$ and $m'(t) > 0$. Since the rate is accelerating, we know that $a''(t) > 0$ and $m''(t) > 0$.

(b) The rate of change of sea level for the mid-Atlantic states is between 2 and 4, we know $2 < a'(t) < 4$. (Possibly also $a'(t) = 2$ or $a'(t) = 4$.)
Similarly, $2 < m'(t) < 10$. (Possibly also $m'(t) = 2$ or $m'(t) = 10$.)

(c) (i) If $a'(t) = 2$, then sea level rise $= 2 \cdot 100 = 200$ mm.
If $a'(t) = 4$, then sea level rise $= 4 \cdot 100 = 400$ mm.
So sea level rise is between 200 mm and 400 mm.

(ii) The shortest amount of time for the sea level in the Gulf of Mexico to rise 1 meter occurs when the rate is largest, 10 mm per year. Since 1 meter = 1000 mm,
shortest time to rise 1 meter $= 1000/10 = 100$ years.

35. **(a)** For each time interval we can calculate the average rate of change of the number of Facebook users per month over this interval. For example, for quarters ending September 2014 and December 2014, a period of 3 months,

$$\begin{array}{c}\text{Average rate}\\ \text{of change}\end{array} = \frac{\Delta N}{\Delta t} = \frac{1393 - 1350}{3} = \frac{43}{3} = 14.33.$$

Thus, between September 2014 and December 2014, there were approximately 14,330,000 more active users each month. Values of $\Delta N/\Delta t$ are listed in Table 2.4.

Table 2.4

Time interval	Sept–Dec 2014	Dec 2014–Mar 2015	Mar–June 2015	June–Sept 2015
Average rate of change, $\Delta N/\Delta t$ (millions/month)	14.33	16.0	16.33	18.33

(b) We assume the data lies on a smooth curve. Since the values of $\Delta N/\Delta t$ are increasing between September 2014–September 2015, this suggests that dN/dt also increases, so d^2N/dt^2 is positive for this period.

[1]An actual graph of the Arctic Sea ice extent over this period can be obtained from the National Snow and Ice Center, Boulder, CO, nsidc.org/arcticseaicenews/files/2015/03/Figure2.png. Accessed March 2015.

36. **(a)** For each time interval we can calculate the average rate of change of the number of yeast population per hour over this interval. For example, between 0 and 2 hours

$$\text{Average rate of change} = \frac{\Delta P}{\Delta t} = \frac{29.0 - 9.6}{2 - 0} = 9.7.$$

Values of $\Delta P/\Delta t$ are listed in Table 2.5.

Table 2.5

Time interval	0–2	2–4	4–6	6–8	8–10	10–12	12–14	14–16	16–18
Average rate of change, $\Delta P/\Delta t$	9.7	21.1	51.8	88.1	81.3	40.8	23.0	7.6	2.9

(b) We assume the data lies on a smooth curve. Since the values of $\Delta P/\Delta t$ are increasing for $0 < t < 8$, this suggests that dP/dt also increases, so d^2P/dt^2 is positive for this period. Since the values of $\Delta P/\Delta t$ are decreasing for $8 < t < 18$, this suggests that dP/dt also decreases, so d^2P/dt^2 is negative for this period.

Solutions for Section 2.5

1. The marginal cost is approximated by the difference quotient

$$MC \approx \frac{\Delta C}{\Delta q} = \frac{4830 - 4800}{1305 - 1295} = 3.$$

The marginal cost is approximately \$3 per item.

2. **(a)** Marginal cost is the derivative $C'(q)$, so its units are dollars/barrel.
(b) It costs about \$3 more to produce 101 barrels of olive oil than to produce 100 barrels.

3. Drawing in the tangent line at the point $(600, R(600))$, we get Figure 2.35.

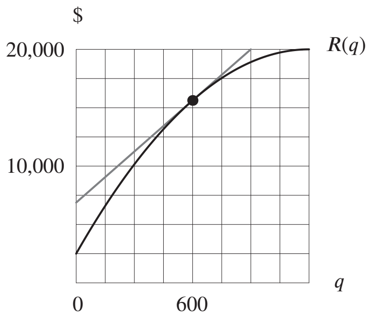


Figure 2.35

We see that each vertical increase of 2500 in the tangent line gives a corresponding horizontal increase of roughly 150. The marginal revenue at the production level of 600 units is

$$R'(600) = \text{Slope of tangent line to } R(q) \text{ at } q = 600 = \frac{2500}{150} = 16.67.$$

This tells us that after producing 600 units, the revenue for producing the 601$^{\text{st}}$ product will be roughly \$16.67.

4. Marginal cost $= C'(q)$. Therefore, marginal cost at q is the slope of the graph of $C(q)$ at q. We can see that the slope at $q = 5$ is greater than the slope at $q = 30$. Therefore, marginal cost is greater at $q = 5$. At $q = 20$, the slope is small, whereas at $q = 40$ the slope is larger. Therefore, marginal cost at $q = 40$ is greater than marginal cost at $q = 20$.

5. Drawing in the tangent line at the point $(10000, C(10000))$ we get Figure 2.36.

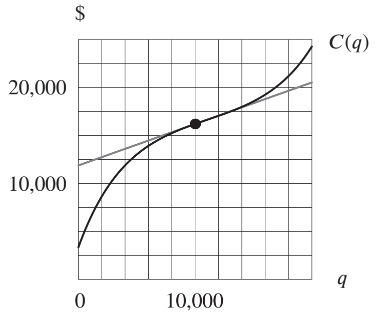


Figure 2.36

We see that each vertical increase of 2500 in the tangent line gives a corresponding horizontal increase of roughly 6000. Thus the marginal cost at the production level of 10,000 units is

$$C'(10{,}000) = \begin{array}{c}\text{Slope of tangent line}\\ \text{to } C(q) \text{ at } q = 10{,}000\end{array} = \frac{2500}{6000} = 0.42.$$

This tells us that after producing 10,000 units, it will cost roughly \$0.42 to produce one more unit.

6. **(a)** For $q = 500$

$$\text{Profit } = \pi(500) = R(500) - C(500) = 9400 - 7200 = 2200 \text{ dollars.}$$

(b) As production increases from $q = 500$ to $q = 501$,

$$\Delta R \approx R'(500)\Delta q = 20 \cdot 1 = 20 \text{ dollars,}$$
$$\Delta C \approx C'(500)\Delta q = 15 \cdot 1 = 15 \text{ dollars,}$$

Thus

$$\text{Change in profit } = \Delta\pi = \Delta R - \Delta C = 20 - 15 = 5 \text{ dollars.}$$

7. We have

$$C'(2000) \approx \frac{C(2500) - C(2000)}{2500 - 2000} = \frac{3825 - 3640}{500} = \$0.37/\text{ton}.$$

This means that recycling the 2001st ton of paper will cost around \$0.37. The marginal cost is smallest at the point where the derivative of the function is smallest. Thus the marginal cost appears to be smallest on the interval $2500 \leq q \leq 3000$.

8. **(a)** The fixed cost is \$8500.
(b) The marginal cost per item is \$4.65.
(c) The price per item is \$5.15.
(d) The company makes a profit when revenues exceed costs, so we find the values of q for which $R > C$.

$$\begin{aligned} R &> C \\ 5.15q &> 8500 + 4.65q \\ 0.50q &> 8500 \\ q &> 17000. \end{aligned}$$

The company makes a profit when it produces more than 17,000 units.
(e) Each additional unit costs the company \$4.65 and the company can sell it for \$5.15, so the company makes $5.15 - 4.65 = \$0.50$ on each additional unit sold.

9. Since marginal revenue is smaller than marginal cost around $q = 4500$, as you produce more of the product your revenue increases slower than your costs, so profit goes down. It does not pay to increase production, and maximal profit occurs at a production level below 4500.

10. The slope of the revenue curve is greater than the slope of the cost curve at both q_1 and q_2, so the marginal revenue is greater at both production levels.

11. **(a)** We can approximate $C(16)$ by adding $C'(15)$ to $C(15)$, since $C'(15)$ is an estimate of the cost of the 16^{th} item.

$$C(16) \approx C(15) + C'(15) = \$2300 + \$108 = \$2408.$$

(b) We approximate $C(14)$ by subtracting $C'(15)$ from $C(15)$, where $C'(15)$ is an approximation of the cost of producing the 15^{th} item.

$$C(14) \approx C(15) - C'(15) = \$2300 - \$108 = \$2192.$$

12. We know $MC \approx C(1{,}001) - C(1{,}000)$. Therefore, $C(1{,}001) \approx C(1{,}000) + MC$ or $C(1{,}001) \approx 5000 + 25 = 5025$ dollars.

Since we do not know $MC(999)$, we will assume that $MC(999) = MC(1{,}000)$. Therefore:

$$MC(999) = C(1{,}000) - C(999).$$

Then:

$$C(999) \approx C(1{,}000) - MC(999) = 5{,}000 - 25 = 4{,}975 \text{ dollars}.$$

Alternatively, we can reason that

$$MC(1{,}000) \approx C(1{,}000) - C(999),$$

so

$$C(999) \approx C(1{,}000) - MC(1{,}000) = 4{,}975 \text{ dollars}.$$

Now for $C(1{,}000)$, we have

$$C(1{,}100) \approx C(1{,}000) + MC \cdot 100.$$

Since $1{,}100 - 1{,}000 = 100$,

$$C(1{,}100) \approx 5{,}000 + 25 \times 100 = 5{,}000 + 2{,}500 = 7{,}500 \text{ dollars}.$$

13. **(a)** The cost to produce 50 units is \$4300 and the marginal cost to produce additional items is about \$24 per unit. Producing two more units (from 50 to 52) increases cost by \$48. We have

$$C(52) \approx 4300 + 24(2) = \$4348.$$

(b) When $q = 50$, the marginal cost is \$24 per item and the marginal revenue is \$35 per item. The profit on the 51^{st} item is approximately $35 - 24 = \$11$.

(c) When $q = 100$, the marginal cost is \$38 per item and the marginal revenue is \$35 per item, so the company loses about \$3 by producing the 101^{st} item. Since the company will lose money, it should not produce the 101^{st} item.

14. The company makes money when revenue exceeds cost and loses money when cost exceeds revenue. Profit is maximized when revenue exceeds cost and the vertical distance between the graphs is as large as possible. This occurs when $R(q) > C(q)$ and $R'(q) = C'(q)$. It appears that maximum profit in this case is at about $q = 190$.

(a) At $q = 75$, we see that the cost function is larger than the revenue function, so the company is losing money. The company needs to increase production in order to make a profit.

(b) At $q = 150$, the revenue function is larger than the cost function so the company is making money. Since marginal revenue is greater than marginal cost here, the company should increase production.

(c) At $q = 225$, the revenue function is larger than the cost function so the company is making money. Since marginal revenue is less than marginal cost here, the company should decrease production.

(d) At $q = 300$, the cost function is larger than the revenue function so the company is losing money. The company needs to decrease production in order to make a profit.

15. At $q = 50$, the slope of the revenue is larger than the slope of the cost. Thus, at $q = 50$, marginal revenue is greater than marginal cost and the 50^{th} bus should be added. At $q = 90$ the slope of revenue is less than the slope of cost. Thus, at $q = 90$ the marginal revenue is less than marginal cost and the 90^{th} bus should not be added.

16. **(a)** At $q = 2.1$ million,

$$\text{Profit } = \pi(2.1) = R(2.1) - C(2.1) = 6.9 - 5.1 = 1.8 \text{ million dollars}.$$

(b) If $\Delta q = 0.04$,

$$\text{Change in revenue, } \Delta R \approx R'(2.1)\Delta q = 0.7(0.04) = 0.028 \text{ million dollars } = \$28{,}000.$$

Thus, revenues increase by about \$28,000.

(c) If $\Delta q = -0.05$,

$$\text{Change in revenue, } \Delta R \approx R'(2.1)\Delta q = 0.7(-0.05) = -0.035 \text{ million dollars } = -\$35{,}000.$$

Thus, revenues decrease by about \$35,000.

(d) We find the change in cost by a similar calculation. For $\Delta q = 0.04$,

$$\text{Change in cost, } \Delta C \approx C'(2.1)\Delta q = 0.6(0.04) = 0.024 \text{ million dollars } = \$24{,}000$$
$$\text{Change in profit, } \Delta \pi \approx \$28{,}000 - \$24{,}000 = \$4000.$$

Thus, increasing production 0.04 million units increases profits by about \$4000.

For $\Delta q = -0.05$,

$$\text{Change in cost, } \Delta C \approx C'(2.1)\Delta q = 0.6(-0.05) = -0.03 \text{ million dollars } = -\$30{,}000$$
$$\text{Change in profit, } \Delta \pi \approx -\$35{,}000 - (-\$30{,}000) = -\$5000.$$

Thus, decreasing production 0.05 million units decreases profits by about \$5000.

17. **(a)** At $q = 2000$, we have

$$\text{Profit } = R(2000) - C(2000) = 7780 - 5930 = 1850 \text{ dollars.}$$

(b) If q increases from 2000 to 2001,

$$\Delta R \approx R'(2000) \cdot \Delta q = 2.5 \cdot 1 = 2.5 \text{ dollars,}$$
$$\Delta C \approx C'(2000) \cdot \Delta q = 2.1 \cdot 1 = 2.1 \text{ dollars,}$$

Thus,

$$\text{Change in profit } = \Delta R - \Delta C \approx 2.5 - 2.1 = 0.4 \text{ dollars.}$$

Since increasing production increases profit, the company should increase production.

(c) By a calculation similar to that in part (b), as q increases from 2000 to 2001,

$$\text{Change in profit } \approx 4.32 - 4.77 = -0.45 \text{ dollars.}$$

Since increasing production reduces the profit, the company should decrease production.

18. For each q, we calculate the average rate of change of the cost and the revenue over the interval to the right. For example, for $q = 0$

$$C'(0) \approx \begin{matrix}\text{Average rate} \\ \text{of change of } C(q)\end{matrix} = \frac{C(1) - C(0)}{1 - 0} = \frac{10 - 9}{1 - 0} = 1,$$

while

$$R'(0) \approx \begin{matrix}\text{Average rate} \\ \text{of change of } R(q)\end{matrix} = \frac{R(1) - R(0)}{1 - 0} = \frac{5 - 0}{1 - 0} = 5.$$

Estimates for $C'(q)$ and $R'(q)$ are in Table 2.6.

Table 2.6

q	0	1	2	3	4	5	6
$C'(q)$	1	2	3	4	5	6	7
$R'(q)$	5	5	5	5	5	5	5

19. For each value of q, we calculate the average rate of change of the cost and the revenue over the interval to the right. For example, for $q = 1$,

$$MC(1) \approx \begin{matrix}\text{Average rate} \\ \text{of change of } C(q)\end{matrix} = \frac{C(2) - C(1)}{2 - 1} = \frac{60 - 20}{2 - 1} = 40,$$

while

$$MR(1) \approx \begin{matrix}\text{Average rate} \\ \text{of change of } R(q)\end{matrix} = \frac{R(2) - R(1)}{2 - 1} = \frac{220 - 100}{2 - 1} = 120.$$

Estimates for $MC(q)$ and $MR(q)$ are in Table 2.7.

Table 2.7

q	1	2	3	4	5
$MC(q)$	40	60	80	100	120
$MR(q)$	120	110	80	40	30

STRENGTHEN YOUR UNDERSTANDING: Digital only

1. True, this is the definition of the derivative.
2. True, one way to see is that the graph of $f(x)$ is increasing for all $x > 0$. Since the graph is increasing at $x = 1$, the instantaneous rate of change, $f'(1)$, is positive.
3. True, since $g'(1) \approx (g(1.001) - g(1))/(1.001 - 1) = (2^{1.001} - 2)/(1.001 - 1)$.
4. False, since the function $H(x) = \sqrt{x}$ is increasing, the slope is always positive.
5. False, the function $f(x) = x^2$ is such a function, with $a = 0$.
6. False, the average rate of change between 0 and 1 is given by the slope of the line connecting the points $(0, f(0))$ and $(1, f(1))$. So, for example, the graph of $f(x) = x^2$ has slope 0 at $x = 0$, but the average rate of change between 0 and 1 is $(f(1) - f(0))/(1 - 0) = 1$.
7. False, the function r appears to be increasing, and therefore would have a positive derivative.
8. False, the derivative of f at 3 is approximated by a difference quotient on a small interval containing 3, and is not the same as the average rate of change on the interval $x = 0$ to $x = 3$.
9. False, $R(w)$ is increasing for all w so the derivative can not be negative at any point.
10. True, consider any constant function.
11. True.
12. False, the opposite is true: If the derivative of f is negative on an interval, then f is decreasing on that interval.
13. False. a function can be increasing while its slope is decreasing.
14. False, since g could be less than f at $x = 2$ but increasing faster than f near $x = 2$. For example, $f(x) = 5$ and $g(x) = x^2$ has $f(2) = 5 > 4 = g(2)$, but $f'(2) = 0$ since f is constant, and $g'(2) > 0$ since g is increasing near $x = 2$.
15. True, the derivative is the slope of the curve, which is always 0 for a horizontal line.
16. False, since a function $f(x)$ can be decreasing and concave up, which means the function is decreasing less rapidly as $x \to \infty$. Therefore, the derivative of the function is less and less negative, so the derivative is increasing.
17. False, the function $\ln(t)$ is an increasing function, and therefore has positive derivative.
18. False, if we let $f(x) = 3 - x$, then $f(3) = 0$, but f is linear, and so the slope is the derivative, which is -1.
19. True.
20. True.
21. True, since the derivative is the limit of the difference quotient $\frac{f(x+h)-f(x)}{h}$, the units of the derivative are the units of the numerator over the units of the denominator, which is dollars per student.
22. True, as long as it is understood that x is the independent variable, then the two quantities are equal.
23. True, this interpretation is as specified in the text.
24. True, the local linear approximation is given by $f(10) + f'(10)(1) = 20 + 3(1) = 23$.
25. True, dA/dB represents the change in A with respect to a change in B, and so the units are the units of A divided by the units of B.
26. False, since $f'(B) = dA/dB$, the units of $f'(B)$ are the units of A divided by the units of B.
27. True, since the derivative is the approximate change in f when the independent variable is increased by one.
28. True, since $f'(D) = dt/dD$ the units of $f'(D)$ are the units of t divided by the units of D, which is minutes per milligram.
29. True, since for the first 10 years, height is an increasing function of age.
30. True, since if the derivative is negative, then W is a decreasing function of R.
31. False. If we let $f(x) = x^2$ then $f''(x) > 0$ because f is concave up, but $f(x)$ is decreasing for $x < 0$.
32. True, if $f'' < 0$, then the derivative is decreasing, which means that the graph is concave down.
33. False, since $f'' > 0$ means only that the derivative is increasing, that is, the graph of f is concave up. However, f can still be decreasing, for example $f(x) = 1/x$ for $x > 0$.
34. True, if f' is decreasing, then $f'' < 0$ which means that f is concave down.
35. True. If the car is slowing down, then the derivative is decreasing, which means the second derivative is negative.
36. True. The function $f(x) = e^x$ is positive, and since it is increasing and concave up, f' and f'' are both positive.

37. True, any decaying exponential function has these properties, for example $f(x) = e^{-x}$.

38. True, any linear function f has $f' = constant$ and so also has $f'' = 0$.

39. True, since e^x is concave up everywhere.

40. False, since the graph of $f(x) = \ln(x)$ is always concave down, so $f''(x) < 0$.

41. True.

42. True.

43. True, if marginal revenue is greater than marginal cost, then the amount of revenue earned on producing an additional unit will be more than the amount it costs to produce, and so it will increase the profit.

44. True, since the total cost function never decreases, so its derivative is never negative.

45. False, if the revenue function is linear, then the marginal revenue is constant slope of the line, or 5.

46. False, the units of marginal cost are the units of cost divided by the units of quantity, which is dollars/unit.

47. True, both the marginal revenue and the marginal cost have dollars as the dependent variable and units as the dependent variable, so their units are the same.

48. False, most firms operate at a profit maximizing level. At this level, $P = R - C$ where P is profit, R is revenue, and C is cost. If the profit is maximal at q^*, the the derivative of the profit function must be zero at q^*. This gives $P'(q^*) = 0 = R(q^*) - C(q^*)$ which says that $R(q^*) = C(q^*)$.

49. False, the cost and revenue functions can be equal at q^* but have different slopes.

50. True. If two graphs intersect, then the values of the functions are equal at the intersection point.

51. True. If $P = f(t)$, then the relative rate of change is

$$\frac{f'(t)}{f(t)} = \frac{1}{P} \cdot \frac{dP}{dt}.$$

52. True, since the relative rate of change is $f'/f = 2/10 = 0.20 = 20\%$ per minute.

53. True, since 3% of 100 is 3 and the quantity is decreasing.

54. False. The rate of change is about 100 animals per month. To find the *relative* rate of change, we divide by the size of the population. The relative rate of change is $-100/500 = 0.20 = 20\%$ per month.

55. True. This is one of the reasons that linear functions and exponential functions are so important.

PROJECTS FOR CHAPTER TWO

1. (a) A possible graph is shown in Figure 2.37. At first, the yam heats up very quickly, since the difference in temperature between it and its surroundings is so large. As time goes by, the yam gets hotter and hotter, its rate of temperature increase slows down, and its temperature approaches the temperature of the oven as an asymptote. The graph is thus concave down. (We are considering the average temperature of the yam, since the temperature in its center and on its surface will vary in different ways.)

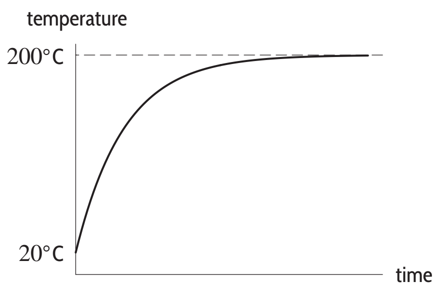


Figure 2.37

(b) If the rate of temperature increase were to remain 2°/min, in ten minutes the yam's temperature would increase 20°, from 120° to 140°. Since we know the graph is not linear, but concave down, the actual temperature is between 120° and 140°.

(c) In 30 minutes, we know the yam increases in temperature by 45° at an average rate of $45/30 = 1.5^\circ$/min. Since the graph is concave down, the temperature at $t = 40$ is therefore between $120 + 1.5(10) = 135^\circ$ and 140°.

(d) If the temperature increases at 2°/minute, it reaches 150° after 15 minutes, at $t = 45$. If the temperature increases at 1.5°/minute, it reaches 150° after 20 minutes, at $t = 50$. So t is between 45 and 50 minutes.

2. (a) Since illumination is concave up and temperature is concave down, the graph on the left side corresponds to the graph of illumination as a function of distance and the graph on the right side corresponds to the graph of temperature as a function of distance.

(b) The illumination drops from 75% at $d = 2$ to 56% at $d = 5$. Since $T = 47$ when $d = 5$ and $T = 53.5$ when $d = 2$,

$$\text{Average rate of change of temperature} = \frac{47 - 53.5}{5 - 2} = \frac{-6.5}{3} = -2.17^\circ \text{ per foot.}$$

(c) A good estimate of the illumination when the distance is 3.5 feet is the average of the values of the illumination at 3 feet and at 4 feet. Therefore:

$$\text{Illumination at 3.5 feet} = \frac{67 + 60}{2} = 63.5\% < 65\%.$$

Since illumination is concave up, the 63.5% is likely to be an overestimate, so you are not likely to be able to read the watch.

(d) Let's represent the illumination as a function of the distance by $I(d)$ and the temperature as a function of the distance by $T(d)$. Therefore:

$$\begin{aligned} I(7) &= I(6) + \text{change in } I(d) \\ &\approx I(6) + \text{Average rate of change of } I(d) \\ &= 53\% + (-3\%) = 50\%. \end{aligned}$$

And

$$\begin{aligned} T(7) &= T(6) + \text{ change in } T(d) \\ &\approx T(6) + \text{Average rate of change of } T(d) \\ &= 43.5^\circ + (-4.5^\circ) = 39^\circ F. \end{aligned}$$

(e) Let's calculate the distance when $T(d) = 40$ (when we are cold):

$$\begin{aligned} T(d) &= T(6) + T'(6) \cdot (d - 6) \\ 40 &= 43.5 + (-4.5)(d - 6) \\ 40 &= 43.5 - 4.5d + 27 \\ 40 &= 70.5 - 4.5d \\ -30.5 &= -4.5d \\ d &= \frac{-30.5}{-4.5} = 6.78 \text{ feet.} \end{aligned}$$

From part (d) we know that the illumination is 50% (darkness) when the distance is 7 feet. Therefore, as we walk away from the candle, we first get cold and then we are in darkness.

3. (a) From the information given, $C(2000) = 1915$ ppt and $C(2010) = 1640$ ppt.

(b) Since the change has been approximately linear, the rate of change is constant:

$$C'(2000) = C'(2014) = \frac{1915 - 1640}{2014 - 2000} = -19.643 \text{ ppt per year .}$$

(c) The slope is -19.643 and $C(2000) = 1915$.
If t is the year, we have

$$C(t) = 1915 - 19.643(t - 2000).$$

(d) We solve

$$\begin{aligned} 1500 &= 1915 - 19.643(t-2000) \\ 19.643(t-2000) &= 1915 - 1500 \\ t &= 2000 + \frac{1915-1500}{19.643} = 2021.13. \end{aligned}$$

The CFC level in the atmosphere above the US is predicted to return to the original level in 2021.

(e) Since $C''(t) > 0$, the graph bends upward, so the answer to part (d) is too early. The CFCs are expected to reach their original level later than 2021.

Solutions to Problems on Limits and the Definition of the Derivative

1. The answers to parts (a)–(f) are marked in Figure 2.38.

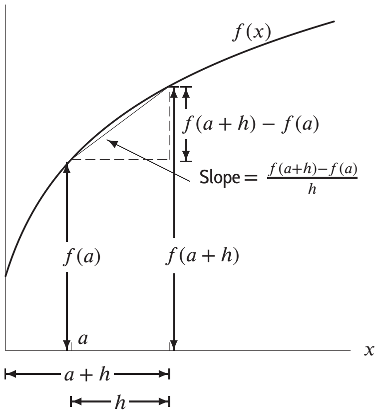


Figure 2.38

HINT INAPPROPRIATE

2. The answers to parts (a)–(f) are marked in Figure 2.39.

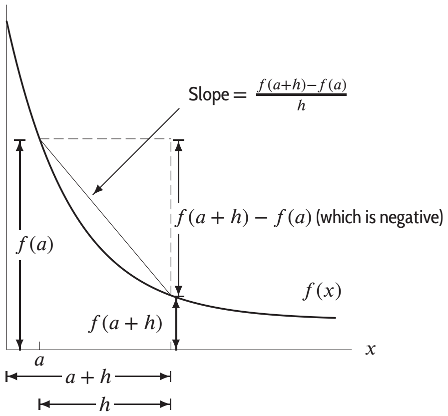


Figure 2.39

3. **(a)** When we substitute $h = 0$,we get $0/0$. The expression is undefined at $h = 0$.
(b) We have

$$\text{Substituting } h = 0.1: \quad \frac{e^{5(0.1)} - 1}{0.1} = 6.487.$$
$$\text{Substituting } h = 0.01: \quad \frac{e^{5(0.01)} - 1}{0.01} = 5.127.$$
$$\text{Substituting } h = 0.001: \quad \frac{e^{5(0.001)} - 1}{0.001} = 5.013.$$
$$\text{Substituting } h = 0.0001: \quad \frac{e^{5(0.0001)} - 1}{0.0001} = 5.001.$$

(c) It appears that

$$\lim_{h\to 0} \frac{e^{5h} - 1}{h} \approx 5.$$

4. **(a)** When we substitute $h = 0$,we get $0/0$. The expression is undefined at $h = 0$.
(b) We have

$$\text{Substituting } h = 0.1: \quad \frac{2e^{3(0.1)} - 2}{0.1} = 6.997.$$
$$\text{Substituting } h = 0.01: \quad \frac{2e^{3(0.01)} - 2}{0.01} = 6.091.$$
$$\text{Substituting } h = 0.001: \quad \frac{2e^{3(0.001)} - 2}{0.001} = 6.009.$$
$$\text{Substituting } h = 0.0001: \quad \frac{2e^{3(0.0001)} - 2}{0.0001} = 6.001.$$

(c) It appears that

$$\lim_{h\to 0} \frac{2e^{3h} - 2}{h} \approx 6.$$

5. **(a)** When we substitute $h = 0$,we get $0/0$. The expression is undefined at $h = 0$.
(b) We have

$$\text{Substituting } h = 0.1: \quad \frac{\ln(1.1)}{0.1} = 0.953.$$
$$\text{Substituting } h = 0.01: \quad \frac{\ln(1.01)}{0.01} = 0.995.$$
$$\text{Substituting } h = 0.001: \quad \frac{\ln(1.001)}{0.001} = 0.9995.$$
$$\text{Substituting } h = 0.0001: \quad \frac{\ln(1.0001)}{0.0001} = 0.99995.$$

(c) It appears that

$$\lim_{h\to 0} \frac{\ln(h+1)}{h} \approx 1.$$

6. **(a)** When we substitute $h = 0$,we get $0/0$. The expression is undefined at $h = 0$.
(b) We have

$$\text{Substituting } h = 0.1: \quad \frac{\sin(3(0.1))}{0.1} = 2.955.$$
$$\text{Substituting } h = 0.01: \quad \frac{\sin(3(0.01))}{0.01} = 2.9996.$$
$$\text{Substituting } h = 0.001: \quad \frac{\sin(3(0.001))}{0.001} = 2.999996.$$
$$\text{Substituting } h = 0.0001: \quad \frac{\sin(3(0.0001))}{0.0001} = 2.99999996.$$

(c) It appears that

$$\lim_{h\to 0} \frac{\sin(3h)}{h} \approx 3.$$

7. Figure 2.40 shows that as x approaches 0 from either side, the values of $\dfrac{5^x - 1}{x}$ appear to approach 1.6, suggesting that

$$\lim_{x\to 0} \frac{5^x - 1}{x} \approx 1.6.$$

Zooming in on the graph near $x = 0$ provides further support for this conclusion. Notice that $\dfrac{5^x - 1}{x}$ is undefined at $x = 0$.

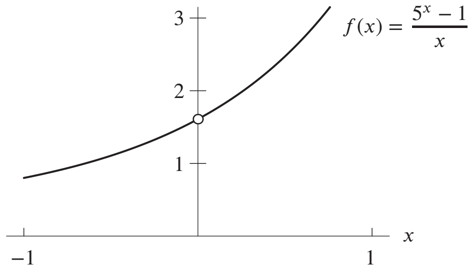


Figure 2.40

8. Figure 2.41 shows that as x approaches 0 from either side, the value of $\dfrac{\sin x}{x}$ appears to approach 1, suggesting that $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$. Zooming in on the graph near $x = 0$ provides further support for this conclusion. Notice that $\dfrac{\sin x}{x}$ is undefined at $x = 0$.

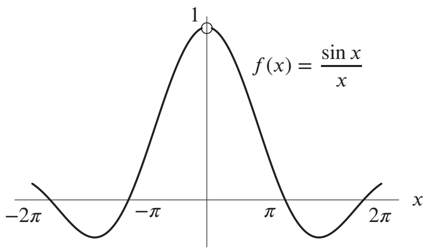


Figure 2.41

9. Using $h = 0.1, 0.01, 0.001$, we see

$$\frac{7^{0.1} - 1}{0.1} = 2.148$$
$$\frac{7^{0.01} - 1}{0.01} = 1.965$$
$$\frac{7^{0.001} - 1}{0.001} = 1.948$$
$$\frac{7^{0.0001} - 1}{0.0001} = 1.946.$$

This suggests that $\lim_{h\to 0} \dfrac{7^h - 1}{h} \approx 1.9$.

10. Using $h = 0.1, 0.01, 0.001$, we see

$$\frac{(3 + 0.1)^3 - 27}{0.1} = 27.91$$
$$\frac{(3 + 0.01)^3 - 27}{0.01} = 27.09$$
$$\frac{(3 + 0.001)^3 - 27}{0.001} = 27.009.$$

These calculations suggest that $\lim_{h\to 0} \dfrac{(3 + h)^3 - 27}{h} = 27$.

11. Using radians,

h	$(\cos h - 1)/h$
0.01	-0.005
0.001	-0.0005
0.0001	-0.00005

These values suggest that $\lim_{h\to 0} \dfrac{\cos h - 1}{h} = 0$.

12. Using $h = 0.1, 0.01, 0.001$, we see

h	$(e^{1+h} - e)/h$
0.01	2.7319
0.001	2.7196
0.0001	2.7184

These values suggest that $\lim_{h\to 0} \dfrac{e^{1+h} - e}{h} = 2.7$. In fact, this limit is e.

13. No, $f(x)$ is not continuous on $0 \le x \le 2$, but it is continuous on the interval $0 \le x \le 0.5$.

14. Yes, $f(x)$ is continuous on $0 \le x \le 2$.

15. No, $f(x)$ is not continuous on $0 \le x \le 2$, but it is continuous on $0 \le x \le 0.5$.

16. No, $f(x)$ is not continuous on $0 \le x \le 2$, but it is continuous on $0 \le x \le 0.5$.

17. **(a)** Yes, $f(x)$ is continuous on $1 \le x \le 3$.
(b) Yes, $f(x)$ is continuous on $0.5 \le x \le 1.5$.
(c) No, $f(x)$ is not continuous on $3 \le x \le 5$ because of the jump at $x = 4$.
(d) No, $f(x)$ is not continuous on $2 \le x \le 6$ because of the jump at $x = 4$.

18. **(a)** No, $f(x)$ is not continuous on $1 \le x \le 3$ because of the jump at $x = 2$.
(b) No, $f(x)$ is not continuous on $0.5 \le x \le 1.5$ because of the jump at $x = 2$.
(c) Yes, $f(x)$ is continuous on $3 \le x \le 5$.
(d) Yes, $f(x)$ is continuous on $4 \le x \le 5$.

19. We have

$$\lim_{h\to 0} \frac{(5+2h)-5}{h} = \lim_{h\to 0} \frac{2h}{h} = \lim_{h\to 0} 2 = 2.$$

20. We have

$$\lim_{h\to 0} \frac{(3h-7)+7}{h} = \lim_{h\to 0} \frac{3h}{h} = \lim_{h\to 0} 3 = 3.$$

21. We have

$$\lim_{h\to 0} \frac{(h+1)^2-1}{h} = \lim_{h\to 0} \frac{(h^2+2h+1)-1}{h} = \lim_{h\to 0} \frac{h^2+2h}{h} = \lim_{h\to 0}(h+2) = 0+2 = 2.$$

22. We have

$$\lim_{h\to 0} \frac{(h-2)^2-4}{h} = \lim_{h\to 0} \frac{(h^2-4h+4)-4}{h} = \lim_{h\to 0} \frac{h^2-4h}{h} = \lim_{h\to 0}(h-4) = 0-4 = -4.$$

23. Yes: $f(x) = x + 2$ is continuous for all values of x.

24. Yes: $f(x) = 2^x$ is continuous function for all values of x.

25. Yes: $f(x) = x^2 + 2$ is a continuous function for all values of x.

26. Yes: $f(x) = 1/(x-1)$ is a continuous function on any interval that does not contain $x = 1$.

27. No: $f(x) = 1/(x-1)$ is not continuous on any interval containing $x = 1$.

28. Yes: $f(x) = 1/(x^2+1)$ is continuous for all values of x because the denominator is never 0.

29. This function is not continuous. Each time someone is born or dies, the number jumps by one.

30. Even though the car is stopping and starting, the distance traveled is a continuous function of time.

31. Continuous

32. Since we can't make a fraction of a pair of pants, the number increases in jumps, so the function is not continuous.

33. The time is not a continuous function of position as distance from your starting point, because every time you cross from one time zone into the next, the time jumps by 1 hour.

34. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{5(x+h)-5x}{h} \\ &= \lim_{h\to 0} \frac{5x+5h-5x}{h} \\ &= \lim_{h\to 0} \frac{5h}{h}. \end{aligned}$$

As long as we let h get close to zero without actually equaling zero, we can cancel the h in the numerator and denominator, and we are left with $f'(x) = 5$.

35. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(3(x+h)-2)-(3x-2)}{h} \\ &= \lim_{h\to 0} \frac{3x+3h-2-3x+2}{h} \\ &= \lim_{h\to 0} \frac{3h}{h}. \end{aligned}$$

As h gets very close to zero without actually equaling zero, we can cancel the h in the numerator and denominator to obtain

$$f'(x) = \lim_{h\to 0}(3) = 3.$$

36. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{((x+h)^2+4)-(x^2+4)}{h} \\ &= \lim_{h\to 0} \frac{x^2+2xh+h^2+4-x^2-4}{h} \\ &= \lim_{h\to 0} \frac{2xh+h^2}{h} \\ &= \lim_{h\to 0} \frac{h(2x+h)}{h}. \end{aligned}$$

As h gets very close to zero without actually equaling zero, we can cancel the h in the numerator and denominator to obtain

$$f'(x) = \lim_{h\to 0}(2x+h) = 2x.$$

37. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2-3x^2}{h} \\ &= \lim_{h\to 0} \frac{3(x^2+2xh+h^2)-3x^2}{h} \end{aligned}$$

$$= \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$$
$$= \lim_{h\to 0} \frac{6xh + 3h^2}{h} = \lim_{h\to 0} \frac{h(6x + 3h)}{h}.$$

As h gets very close to zero (but not equal to zero), we can cancel the h in the numerator and denominator to leave the following:

$$f'(x) = \lim_{h\to 0}(6x + 3h).$$

As $h \to 0$, we have $f'(x) = 6x$.

38. Using the definition of the derivative, we have

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{((x+h) - (x+h)^2) - (x - x^2)}{h} \\
&= \lim_{h\to 0} \frac{(x + h - (x^2 + 2xh + h^2)) - x + x^2}{h} \\
&= \lim_{h\to 0} \frac{x + h - x^2 - 2xh - h^2 - x + x^2}{h} \\
&= \lim_{h\to 0} \frac{h - 2xh - h^2}{h}.
\end{aligned}$$

As long as we let h get close to zero without actually equaling zero, we can cancel the h in the numerator and denominator, and we are left with $1 - 2x - h$. Taking the limit as h goes to zero, we get $f'(x) = 1 - 2x$ since the other term goes to zero.

39. Using the definition of the derivative, we have

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(5(x+h)^2 + 1) - (5x^2 + 1)}{h} \\
&= \lim_{h\to 0} \frac{5(x^2 + 2xh + h^2) + 1 - 5x^2 - 1}{h} \\
&= \lim_{h\to 0} \frac{5x^2 + 10xh + 5h^2 + 1 - 5x^2 - 1}{h} \\
&= \lim_{h\to 0} \frac{10xh + 5h^2}{h} \\
&= \lim_{h\to 0} \frac{h(10x + 5h)}{h}.
\end{aligned}$$

As h gets very close to zero without actually equaling zero, we can cancel the h in the numerator and denominator to obtain

$$f'(x) = \lim_{h\to 0}(10x + 5h) = 10x.$$

40. Using the definition of the derivative, we have

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(2(x+h)^2 + (x+h)) - (2x^2 + x)}{h} \\
&= \lim_{h\to 0} \frac{2(x^2 + 2xh + h^2) + (x+h) - 2x^2 - x}{h} \\
&= \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 + x + h - 2x^2 - x}{h} \\
&= \lim_{h\to 0} \frac{4xh + 2h^2 + h}{h} \\
&= \lim_{h\to 0} \frac{h(4x + 2h + 1)}{h}.
\end{aligned}$$

As h gets very close to zero without actually equaling zero, we can cancel the h in the numerator and denominator to obtain

$$f'(x) = \lim_{h\to 0}(4x + 2h + 1) = 4x + 1.$$

41. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{-2(x+h)^3 - (-2x^3)}{h} \\ &= \lim_{h\to 0} \frac{-2(x^3 + 3x^2h + 3xh^2 + h^3) + 2x^3}{h} \\ &= \lim_{h\to 0} \frac{-2x^3 - 6x^2h - 6xh^2 - 2h^3 + 2x^3}{h} \\ &= \lim_{h\to 0} \frac{-6x^2h - 6xh^2 - 2h^3}{h}. \end{aligned}$$

As long as we let h get close to zero without actually equaling zero, we can cancel the h in the numerator and denominator, and we are left with $-6x^2 - 6xh - 2h^2$. Taking the limit as h goes to zero, we get $f'(x) = -6x^2$ since the other two terms go to zero.

42. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{(1 - (x+h)^3) - (1 - x^3)}{h} \\ &= \lim_{h\to 0} \frac{(1 - (x^3 + 3x^2h + 3xh^2 + h^3)) - 1 + x^3}{h} \\ &= \lim_{h\to 0} \frac{1 - x^3 - 3x^2h - 3xh^2 - h^3 - 1 + x^3}{h} \\ &= \lim_{h\to 0} \frac{-3x^2h - 3xh^2 - h^3}{h}. \end{aligned}$$

As long as we let h get close to zero without actually equaling zero, we can cancel the h in the numerator and denominator, and we are left with $-3x^2 - 3xh - h^2$. Taking the limit as h goes to zero, we get $f'(x) = -3x^2$ since the other two terms go to zero.

43. Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{1/(x+h) - 1/x}{h}. \end{aligned}$$

Writing the numerator over a common denominator and simplifying, we get

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{(x - (x+h))/(x(x+h))}{h} \\ &= \lim_{h\to 0} \frac{-h/(x(x+h))}{h} \\ &= \lim_{h\to 0} \frac{-h}{hx(x+h)}. \end{aligned}$$

As long as we let h get close to zero without actually equaling zero, we can cancel the h in the numerator and denominator, and we are left with $-1/(x(x+h))$. Taking the limit as h goes to zero, we get $f'(x) = -1/x^2$ since h goes to zero.