

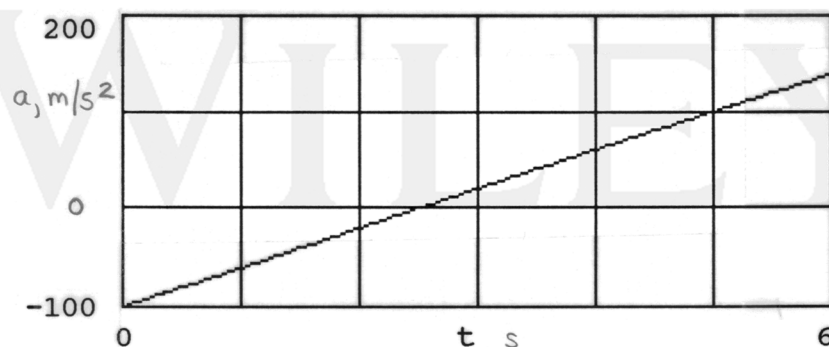
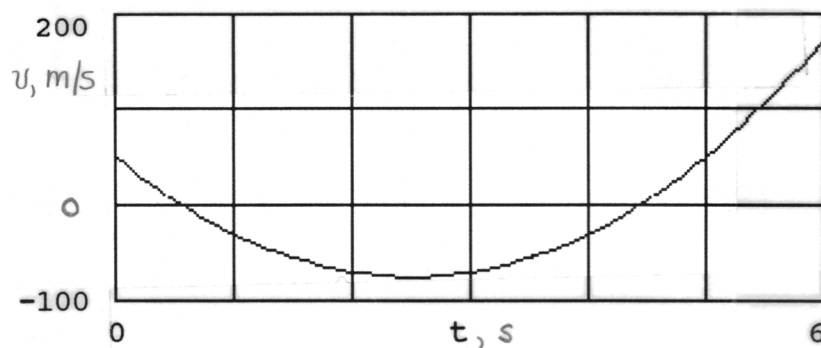
***Engineering Mechanics Dynamics 9th edition
Meriam Solutions Manual***

SKU: 9781119391081-SOLUTIONS

$$\begin{aligned} \boxed{2/1} \quad & v = 20t^2 - 100t + 50 \\ & a = \frac{dv}{dt} = 40t - 100 \end{aligned} \quad \left. \vphantom{\begin{aligned} v = 20t^2 - 100t + 50 \\ a = \frac{dv}{dt} = 40t - 100 \end{aligned}} \right\} \text{See plots}$$

$$a = 0 : 40t - 100 = 0, \quad t = 2.5 \text{ s}$$

$$\begin{aligned} \text{At } t = 2.5 \text{ s, } v &= 20(2.5)^2 - 100(2.5) + 50 \\ &= \underline{-75 \text{ m/s}} \end{aligned}$$



2/2

$$s = 0.27t^3 - 0.65t^2 - 2.35t + 4.4 \quad \text{ft}$$

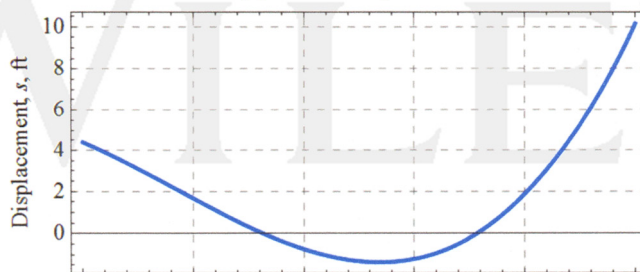
$$\begin{cases} v = \frac{ds}{dt} = 0.81t^2 - 1.3t - 2.35 & \text{ft/sec} \\ a = \frac{dv}{dt} = 1.62t - 1.3 & \text{ft/sec}^2 \end{cases}$$

(SEE CORRESPONDING PLOTS)

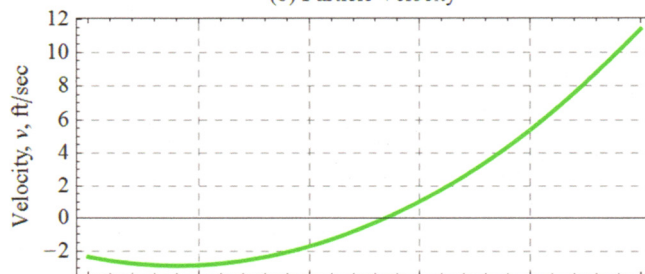
FOR A CHANGE IN DIRECTION, $v=0$ TO CHANGE SIGN.

$$v = 0 = 0.81t^2 - 1.3t - 2.35 \rightarrow \begin{cases} t = -1.080 \text{ sec} \\ t = 2.69 \text{ sec} \end{cases}$$

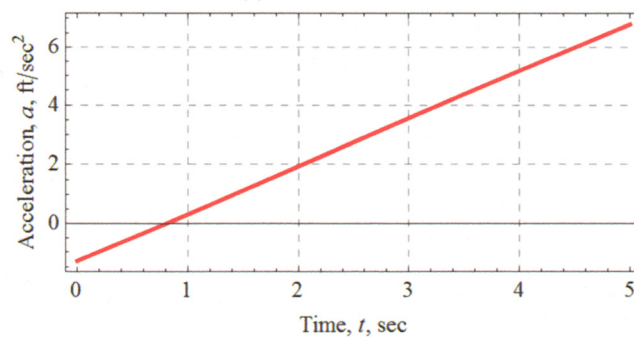
(a) Particle Displacement



(b) Particle Velocity



(c) Particle Acceleration



$$\begin{aligned} \boxed{2/3} \quad v &= 2 + 5t^{3/2} \\ a &= \frac{dv}{dt} = \frac{3}{2} \cdot 5t^{1/2} = \frac{15}{2} \sqrt{t} \\ \frac{ds}{dt} &= 2 + 5t^{3/2} \\ \int_{s_0=0}^s ds &= \int_0^t (2 + 5t^{3/2}) dt \\ s &= 2t + 2t^{5/2} \\ \text{At } t = 4 \text{ s} : &\begin{cases} s = 72 \text{ m}, & v = 42 \text{ m/s} \\ a = 15 \text{ m/s}^2 \end{cases} \end{aligned}$$

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$$\frac{2/4}{\quad} \quad a = \frac{dv}{dt} = 2t - 10$$

$$\int_{v_0=3}^v dv = \int_0^t (2t - 10) dt, \quad \underline{v = 3 - 10t + t^2 \text{ (m/s)}}$$

$$\frac{ds}{dt} = 3 - 10t + t^2$$

$$\int_{s_0=-4}^s ds = \int_0^t (3 - 10t + t^2) dt$$

$$\underline{s = -4 + 3t - 5t^2 + \frac{1}{3}t^3 \text{ (m)}}$$

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$$\begin{aligned} \frac{2}{5} \quad a &= v \frac{dv}{ds} = -ks^2 \\ \int_{v_0}^v v dv &= - \int_{s_0}^s ks^2 ds \Rightarrow v^2 = v_0^2 - \frac{2}{3}k(s^3 - s_0^3) \\ \text{Taking positive sign: } v &= \left[v_0^2 - \frac{2}{3}k(s^3 - s_0^3) \right]^{1/2} \\ \text{Numbers: } v &= \left[10^2 - \frac{2}{3}(0.1)(5^3 - 3^3) \right]^{1/2} \\ &= \underline{9.67 \text{ m/s}} \end{aligned}$$

WILEY

2/6

At $t=0$, $s=s_0$ AND $v=v_0$

$$a = c_1 + c_2 v = \frac{dv}{dt} \rightarrow \int_{t_0=0}^t dt = \int_{v_0}^v \frac{dv}{c_1 + c_2 v} \quad \begin{cases} u = c_1 + c_2 v \\ du = c_2 dv \end{cases}$$

$$t = \frac{1}{c_2} \int \frac{du}{u} = \frac{\ln u}{c_2} = \frac{\ln(c_1 + c_2 v)}{c_2} \bigg|_{v_0}^v = \frac{1}{c_2} \ln \left(\frac{c_1 + c_2 v}{c_1 + c_2 v_0} \right)$$

SOLVING... $v = \frac{1}{c_2} \left[(c_1 + c_2 v_0) e^{c_2 t} - c_1 \right] = \frac{ds}{dt}$

SO... $\int_{s_0}^s ds = \frac{1}{c_2} \int_{t_0=0}^t \left[(c_1 + c_2 v_0) e^{c_2 t} - c_1 \right] dt = \frac{1}{c_2} \left((c_1 + c_2 v_0) e^{c_2 t} - \frac{c_1}{c_2} t \right) \bigg|_0^t$

$$s = s_0 + \frac{c_1 + c_2 v_0}{c_2^2} (e^{c_2 t} - 1) - \frac{c_1}{c_2} t$$

$$a = c_1 + c_2 v = \frac{v dv}{ds} \rightarrow \int_{s_0}^s ds = \int_{v_0}^v \frac{v dv}{c_1 + c_2 v}$$

FROM APPENDIX C/10... $\int_{v_0}^v \frac{v dv}{c_1 + c_2 v} = \frac{1}{c_2^2} \left[c_1 + c_2 v - c_1 \ln(c_1 + c_2 v) \right] \bigg|_{v_0}^v$

$$s - s_0 = \frac{1}{c_2^2} \left[c_1 + c_2 v - c_1 \ln(c_1 + c_2 v) \right] - \frac{1}{c_2^2} \left[c_1 + c_2 v_0 - c_1 \ln(c_1 + c_2 v_0) \right]$$

$$s = s_0 + \frac{1}{c_2^2} \left[c_2(v - v_0) + c_1 \ln \left(\frac{c_1 + c_2 v}{c_1 + c_2 v_0} \right) \right]$$

$$\begin{aligned} \boxed{2/7} \quad v^2 - v_0^2 &= 2a(s - s_0) \\ 0 - \left[60 \frac{5280}{3600} \right]^2 &= 2a(120) \\ a &= -32.3 \text{ ft/sec}^2 \end{aligned}$$

From 80 mi/hr :

$$\begin{aligned} 0 - \left[80 \frac{5280}{3600} \right]^2 &= 2(-32.3)s \\ s &= 213 \text{ ft} \end{aligned}$$

(Note: Braking tests support the notion of decelerations whose magnitudes are greater than $1g$!)

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2/8

$$k = 0.2 \text{ mm}^{1/2} \text{ s}^{-1}, \quad v_0 = 3 \text{ mm/s}$$

$$v = k s^{1/2} = \frac{ds}{dt} \rightarrow \int_{t_0=0}^t k dt = \int_{s_0}^s \frac{ds}{s^{1/2}} \rightarrow kt = 2(s^{1/2} - s_0^{1/2})$$

Solving... $s = \frac{1}{4} (kt + 2s_0^{1/2})^2$

$$v_0 = k s_0^{1/2} \rightarrow 3 = 0.2 s_0^{1/2} \rightarrow s_0 = 225 \text{ mm}$$

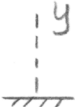
So... $s = \frac{1}{4} (0.2t + 30)^2 \text{ mm}$

$$\begin{cases} v = \frac{ds}{dt} \rightarrow v = \frac{k}{2} (kt + 2s_0^{1/2}) & \text{or... } v = \frac{1}{10} (0.2t + 30) \text{ mm/s} \\ a = \frac{dv}{dt} \rightarrow a = \frac{1}{2} k^2 & \text{or... } a = 0.02 \text{ mm/s}^2 \end{cases}$$

For $v = 15 \text{ mm/s}$...

$$\begin{cases} 15 = \frac{1}{10} (0.2t + 30) \rightarrow t = 600 \text{ s} \\ 15 = 0.2 s^{1/2} \rightarrow s = 5625 \text{ mm} \end{cases}$$

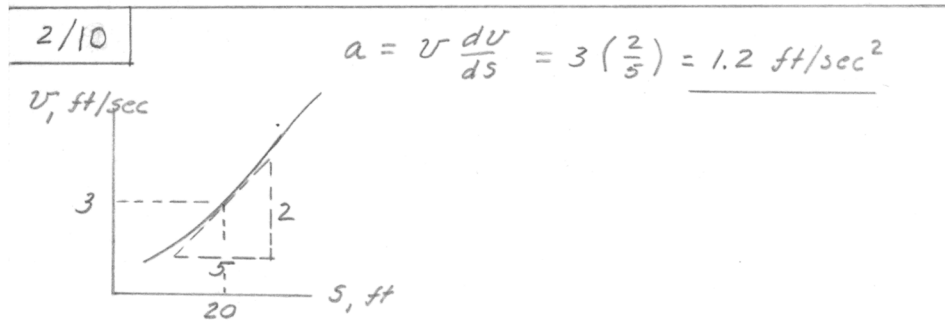
2/9 | ①: $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ @ 300 ft:

y

 $300 = 0 + 160t - 16.1t^2$
 $t = 2.51, 7.43 \text{ sec}$
 So impact must be when ball 1 is descending.

②: $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ @ 300 ft:

$300 = 0 + v_2(7.43 - 3) - 16.1(7.43 - 3)^2$
 $v_2 = \underline{139.0 \text{ ft/sec}}$

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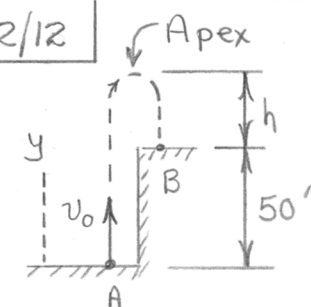


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$$\begin{aligned} 2/11 \quad v_B^2 - v_A^2 &= 2a(s_B - s_A) \\ v_B^2 - 0 &= 2(2.75)(3), \quad v_B = 4.06 \text{ m/s} \\ s_B &= s_A + v_0 t + \frac{1}{2}at^2: \quad 3 = \frac{1}{2}(2.75)t_{AB}^2 \\ t_{AB} &= 1.477 \text{ s} \\ s_C &= s_B + v_B t + \frac{1}{2}(0)t^2: \quad 4 = 4.06 t_{BC} \\ t_{BC} &= 0.985 \text{ s} \\ t_{AC} &= t_{AB} + t_{BC} = \underline{2.46 \text{ s}} \end{aligned}$$

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2/12



Evaluate $v^2 = v_0^2 - 2g(y - y_0)$
at apex:

$$0 = 80^2 - 2(32.2)(50 + h - 0)$$

$$h = \underline{49.4 \text{ ft}}$$

Evaluate $y = y_0 + v_0 t - \frac{1}{2} g t^2$ at B:

$$50 = 0 + 80t - 16.1t^2 \text{ or } 16.1t^2 - 80t + 50 = 0$$

$$t = \frac{80 \pm \sqrt{80^2 - 4(16.1)(50)}}{2(16.1)} = 0.733, 4.26 \text{ sec}$$

$t = 4.24 \text{ sec}$ represents the second time
at which $y = 50 \text{ ft}$.

$$v_B = v_0 - gt = 80 - 32.2(4.24) = -56.4 \text{ ft/sec}$$

(or 56.4 ft/sec downward)

$$\boxed{2/13} \quad v_0 = 100/3.6 = 27.8 \text{ m/s}$$

$$a = -g \sin \theta = -9.81 \sin \left[\tan^{-1} \frac{6}{100} \right] = -0.588 \text{ m/s}^2$$

$$(a) \quad v = v_0 + at = 27.8 - 0.588(10) = \underline{21.9 \text{ m/s}}$$

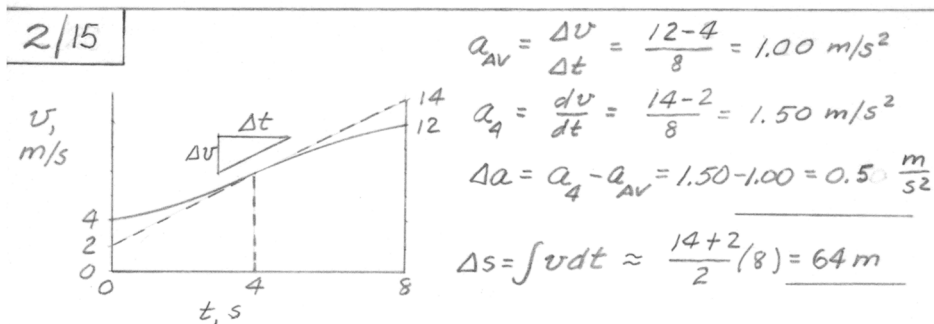
$$(b) \quad v^2 = v_0^2 + 2a(s-s_0) = 27.8^2 + 2(-0.588)(100)$$

$$v = \underline{25.6 \text{ m/s}}$$

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$$\begin{aligned} 2/14 \quad v^2 &= v_0^2 + 2a(s-s_0) \\ \left(\frac{200}{3.6}\right)^2 &= 0^2 + 2(0.4 \cdot 9.81)s \\ s &= 393 \text{ m} \\ v &= v_0 + at : \left(\frac{200}{3.6}\right) = 0 + 0.4(9.81)t \\ t &= 14.16 \text{ s} \end{aligned}$$

WILEY



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$$\begin{array}{|l} 2/16 \end{array} \quad v^2 = v_0^2 + 2as, \text{ where } a = g/6$$
$$v^2 = 2^2 + 2\left(\frac{9.81}{6}\right)5, \quad \underline{v = 4.51 \text{ m/s}}$$

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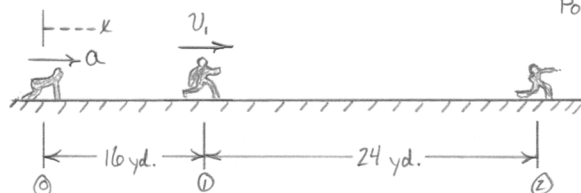
$$\begin{aligned} 2/17 \quad v^2 &= v_0^2 + 2a(s-s_0) \\ 0 &= 4^2 + 2\left(-\frac{9.81}{4}\right)(s), \quad \underline{s = 3.26 \text{ m}} \\ v &= v_0 + at : 0 = 4 + \left(-\frac{9.81}{4}\right)t_{up}, \quad t_{up} = 1.63 \text{ s} \\ t &= 2t_{up} = 2(1.63) = \underline{3.26 \text{ s}} \end{aligned}$$

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2/18

$$t_2 = 4.25 \text{ sec}$$

v_1 IS CONSTANT BETWEEN
POINTS ① AND ②



$$v_1 = v_0' + at_1 \rightarrow v_1 = at_1$$

$$x_1 = x_0' + v_0't_1 + \frac{1}{2}at_1^2 \rightarrow 16(3) = \frac{1}{2}at_1^2$$

$$x_2 = x_1 + v_1t_{1-2} \rightarrow 40(3) = 16(3) + v_1t_{1-2}$$

$$t_1 + t_{1-2} = t_2 = 4.25$$

Solving...

$$\left\{ \begin{array}{l} v_1 = 39.5 \text{ ft/sec (OR 27.0 mph!)} \\ t_1 = 2.43 \text{ sec} \\ t_{1-2} = 1.821 \text{ sec} \\ a = 16.28 \text{ ft/sec}^2 \end{array} \right.$$

2/19 The area under the a-s curve is

$$\int_0^v v dv = \frac{1}{2} v^2$$

$$\text{Area} \Big|_0^{200 \text{ m}} = \frac{3+6}{2} (100) + \frac{6+4}{2} (100) = 950 \text{ m}^2/\text{s}^2$$

$$\text{So } \frac{1}{2} v^2 = 950, \quad \underline{v = 43.6 \text{ m/s}}$$

$$\frac{dv}{ds} = \frac{a}{v} = \frac{4}{43.6} = \underline{0.0918 \text{ s}^{-1}}$$

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$$\boxed{2/20} \quad \text{Train: } v^2 = v_1^2 + 2as : 60^2 = 80^2 + 2a\left(\frac{1}{2}\right)$$

$$a = -2800 \text{ mi/hr}^2$$

$$s = v_1 t + \frac{1}{2}at^2 : 1 = 80t - \frac{2800}{2}t^2$$

$$t = 0.01847 \text{ hr} \quad \text{or} \quad t = 0.03867 \text{ hr (disregard)}$$

$$s_0 \quad t = 0.01847 (60)^2 = 66.5 \text{ sec}$$

$$\text{Car: } t = 66.5 - 4 = 62.5 \text{ sec}$$

$$s = v_1 t + \frac{1}{2}at^2 : 1.3(5280) = 50 \frac{44}{30}(62.5) + \frac{a}{2}(62.5)^2$$

$$a = 1.168 \text{ ft/sec}^2$$

$$v = v_1 + at = 50 + 1.168 \frac{3600^2}{5280} \frac{62.5}{3600} = 99.8 \frac{\text{mi}}{\text{hr}}$$

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2/21 $s = s_0 + v_0 t + \frac{1}{2} g t^2$

Sphere ①: $H = \frac{1}{2} g t_2^2$

Sphere ②:

$(H-h) = \frac{1}{2} g \left(t_2 - \frac{1}{2}\right)^2$

With $H = 3 \text{ m}$ & $g = 9.81 \text{ m/s}^2$,

eliminate t_2 between the

2 equations & obtain $h = 2.61 \text{ m}$.

The diagram illustrates the motion of two spheres, labeled ① and ②, falling from a height $H = 3 \text{ m}$. Sphere ① starts at $t = 0$ and falls the full height H . Sphere ② starts at $t = \frac{1}{2} \text{ s}$ and falls a height h . At a common time t_2 , Sphere ① has fallen a distance H and Sphere ② has fallen a distance h . The diagram shows the initial positions, the starting times, and the positions at time t_2 .

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$$\frac{2}{22} \quad v_A = \frac{130}{3.6} = 36.1 \text{ m/s} = \text{constant}$$

$$s_A = v_A t = 36.1 t$$

$$(s_P)_{acc} = \frac{1}{2} a_P t^2 = \frac{1}{2} (6) t^2 = 3 t^2$$

$$(v_P)_{acc} = a_P t; \quad t_{acc} = \frac{(v_P)_{acc}}{a_P} = \frac{160/3.6}{6} = 7.4 \text{ s}$$

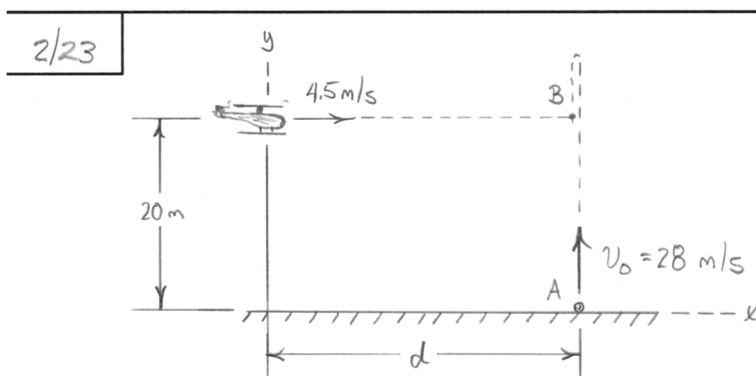
$$\text{So } (s_P)_{acc} = 3 (7.4)^2 = 164.6 \text{ m}$$

$$\begin{aligned} s_P &= (s_P)_{acc} + (s_P)_{cr} = 164.6 + \frac{160}{3.6} (t - 7.4) \\ &= 44.4 t - 164.6 \end{aligned}$$

$$\begin{aligned} \text{Now set } s_A &= s_P: 36.1 t = 44.4 t - 164.6 \\ t &= 19.75 \text{ s} \end{aligned}$$

$$\text{Then } s = 36.1 (19.75) = \underline{713 \text{ m}}$$

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$$\begin{cases} y_B = y_A + v_y t - \frac{1}{2} g t^2 \\ 20 = 0 + 28t - \frac{1}{2}(9.8)t^2 \end{cases}$$

so... $t = 0.837 \text{ s}$ or 4.87 s

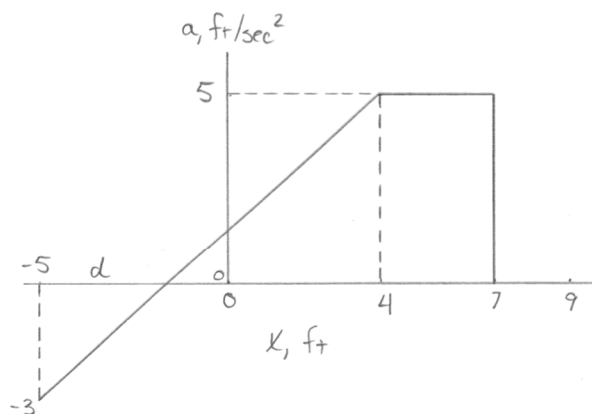
(CHOOSE 4.87 FOR DOWNWARD MOTION)

$$\begin{cases} x_B = x_0 + v_x t \\ d = 0 + 4.5(4.87) \end{cases}$$

so... $\underline{d = 21.9 \text{ m}}$

2/24

At $x = -5 \text{ ft}$, $v = 4 \text{ ft/sec}$

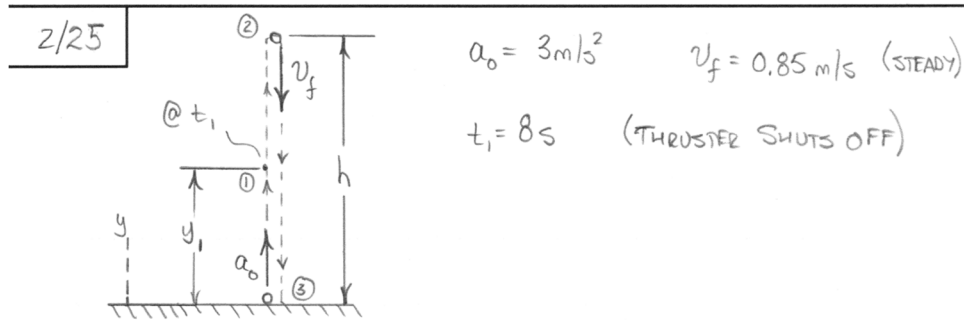


$$d = \frac{3}{8/9} \rightarrow d = 3.375 \text{ ft}$$

$$\frac{1}{2}(v_2^2 - v_1^2) = \text{AREA UNDER } a(x) \text{ GRAPH.}$$

$$\frac{1}{2}(v_2^2 - (4)^2) = -\frac{1}{2}(3.375)(3) + \frac{1}{2}(5)(9 - 3.375) + 5(3)$$

$$\therefore \underline{v_2 = 8 \text{ ft/sec}}$$



$$y_1 = y_0' + v_0' t_1 + \frac{1}{2} a_0 t_1^2 = \frac{1}{2} (3) (8)^2 \rightarrow y_1 = 96 \text{ m}$$

$$v_1 = v_0' + a_0 t_1 = 3(8) \rightarrow v_1 = 24 \text{ m/s}$$

$$\begin{cases} y_2 = y_1 + v_1 t_{1-2} - \frac{1}{2} g t_{1-2}^2 \\ h = 96 + 24 t_{1-2} - \frac{1}{2} (9.81) t_{1-2}^2 \end{cases} \quad \begin{cases} v_2 = v_1 - g t_{1-2} \\ 0 = 24 - 9.81 t_{1-2} \\ t_{1-2} = 2.45 \text{ s} \end{cases}$$

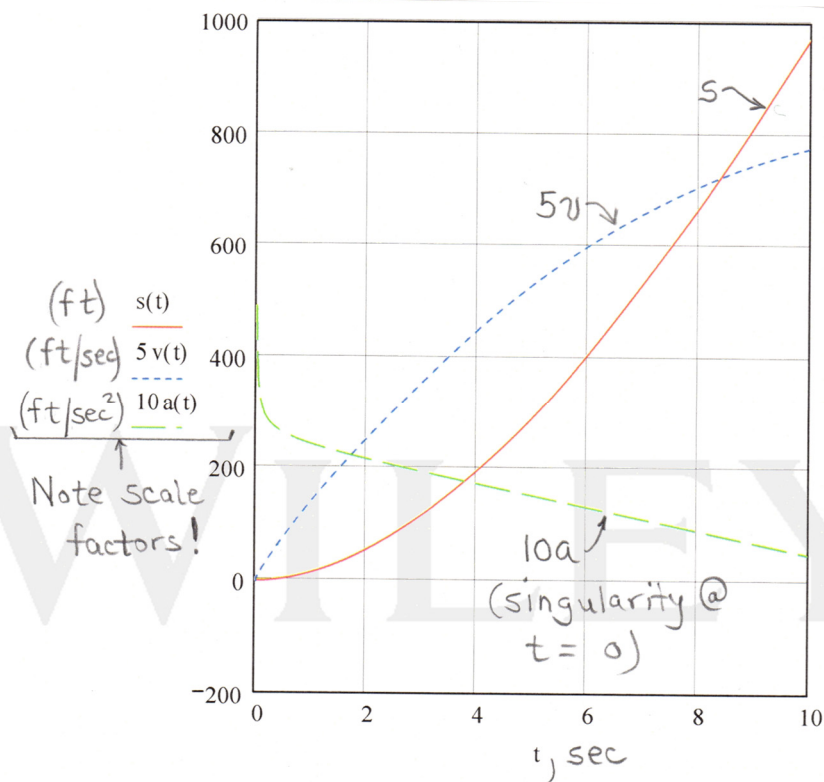
Solving... $h = 125.4 \text{ m}$

$$h = v_f t_{2-3} \rightarrow 125.4 = 0.85 t_{2-3} \rightarrow t_{2-3} = 147.5 \text{ s}$$

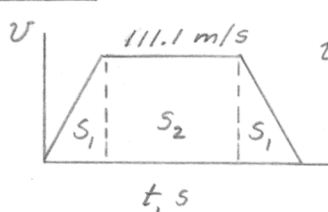
$$t_{\text{total}} = t_1 + t_{1-2} + t_{2-3} = 8 + 2.45 + 147.5$$

$$\therefore t_{\text{total}} = 157.9 \text{ s}$$

$$\begin{aligned} \frac{2}{26} \quad v &= \frac{ds}{dt} = 24t - t^2 + 5\sqrt{t} \\ \int_{s_0=0}^s ds &= \int_0^t (24t - t^2 + 5\sqrt{t}) dt \\ s &= 12t^2 - \frac{1}{3}t^3 + \frac{10}{3}t^{3/2} \text{ ft} \\ s(10) &= 972 \text{ ft} \end{aligned}$$



2/27 $400 \text{ km/h} \equiv \frac{400}{3.6} = 111.1 \text{ m/s}$



$v^2 = 2as, S_1 = \frac{(111.1)^2}{2(0.6)(9.81)} = 1049 \text{ m}$

$S_2 = 10000 - 2(1049) = 7903 \text{ m}$

$t_1 = \frac{v}{a} = \frac{111.1}{0.6(9.81)} = 18.88 \text{ s}$

$t_2 = \frac{S_2}{v} = \frac{7903}{111.1} = 71.13 \text{ s}$

$t = 2t_1 + t_2 = 2(18.88) + 71.13 = 108.9 \text{ s}$

or $t = 1.81 \text{ min}$

WILEY

2/28

$$a = -0.0003v^2 = -kv^2 = v \frac{dv}{ds}$$

$$-k ds = \frac{dv}{v}$$

$$-ks = \ln\left(\frac{v_f}{v_0}\right)$$

$$-0.0003s = \ln\left(\frac{35}{200}\right)$$

$$\underline{s = 5810 \text{ ft}}$$

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2/29 (a) $200 \rightarrow 100 \text{ mi/hr} : a = -k_1 v^2 = v \frac{dv}{ds}$

$$-k_1 ds = \frac{dv}{v}$$

$$-k_1 s = \ln\left(\frac{v_{100}}{v_{200}}\right) : -0.00035 = \ln\left(\frac{100}{200}\right)$$

$$s = 2310 \text{ ft}$$

(b) $100 \rightarrow 35 \text{ mi/hr} : a = -k_1 v^2 - k_2 = v \frac{dv}{ds}$

$$ds = -\frac{v dv}{k_2 + k_1 v^2}$$

$$s = \frac{1}{2k_1} \ln \left[\frac{k_2 + k_1 v_0^2}{k_2 + k_1 v_f^2} \right]$$

$$= \frac{1}{2(0.0003)} \ln \left[\frac{5 + 0.0003 \left(100 \cdot \frac{88}{60}\right)^2}{5 + 0.0003 \left(35 \cdot \frac{88}{60}\right)^2} \right]$$

$$= 1137 \text{ ft}$$

(c) $35 \rightarrow 0 \text{ mi/hr} :$

$$v_f^2 - v_0^2 = 2a(s - s_0)$$

$$0 - \left(35 \cdot \frac{88}{60}\right)^2 = 2(-5)(s)$$

$$s = 264 \text{ ft}$$

Total: $D = 2310 + 1137 + 264 = \underline{3710 \text{ ft}}$

2/30

$$\begin{cases} v_0 = 3.25 \text{ m/s} & x_{\max} = 475 \text{ mm} \\ v_{\text{mid}} = 2.85 \text{ m/s} & \text{At } x = \frac{1}{2} x_{\max} \end{cases}$$

$$a = -k_1 x - k_2 x^3 = v \frac{dv}{dx}$$

$$\int_0^x (-k_1 x - k_2 x^3) dx = \int_{v_0}^v v dv$$

$$-\frac{k_1}{2} x^2 - \frac{k_2}{4} x^4 = \frac{1}{2} (v^2 - v_0^2)$$

$$\begin{cases} \text{At } x_{\max} \dots & -\frac{k_1}{2} (0.475)^2 - \frac{k_2}{4} (0.475)^4 = \frac{1}{2} (0^2 - 3.25^2) \\ v = 0 \end{cases}$$

$$\begin{cases} \text{At } \frac{x_{\max}}{2} \dots & -\frac{k_1}{2} \left(\frac{0.475}{2}\right)^2 - \frac{k_2}{4} \left(\frac{0.475}{2}\right)^4 = \frac{1}{2} (2.85^2 - 3.25^2) \\ v = 2.85 \text{ m/s} \end{cases}$$

Solving...

$$\begin{cases} k_1 = 42.1 \text{ s}^{-2} \\ k_2 = 42.0 \text{ m} \cdot \text{s}^{-2} \end{cases}$$

$$\begin{aligned} \underline{2/31} \quad (a) \quad (g_m)_0 &= 5.32 \text{ ft/sec}^2 = \text{constant} \\ v^2 &= v_0^2 + 2(g_m)_0(s-s_0) : v^2 = 0^2 + 2(5.32)(750 \cdot 5280) \\ v &= \underline{6490 \text{ ft/sec}} \end{aligned}$$

$$\begin{aligned} (b) \quad a &= -(g_m)_0 \frac{R_m^2}{r^2} = v \frac{dv}{dr} \quad (R_m = \text{moon radius}) \\ -(g_m)_0 R_m^2 \int_{R_m+h}^{R_m} \frac{dr}{r^2} &= \int_{v_0=0}^v v dv \\ -(g_m)_0 R_m^2 \left(-\frac{1}{r} \right) \bigg|_{R_m+h}^{R_m} &= \frac{1}{2} v^2 \bigg|_0^v \\ v &= \sqrt{\frac{2(g_m)_0 R_m h}{R_m+h}} = \sqrt{\frac{2(5.32)\left(\frac{2160}{2}\right)(750)(5280)^2}{\left(\frac{2160}{2}+750\right)(5280)}} \\ &= \underline{4990 \text{ ft/sec}} \end{aligned}$$

WILEY

*2/32 $k_1 = 12 \text{ sec}^{-1}$, $k_2 = 24 \text{ sec}^{-2}$, $v_0 = 25 \text{ in./sec}$

$$a = g - k_1 v - k_2 y \rightarrow a + k_1 v + k_2 y = g \text{ or } y'' + k_1 y' + k_2 y = g$$

$$y(t) = y_H(t) + y_p(t) \quad (\text{HOMOGENEOUS AND PARTICULAR SOLUTIONS})$$

For $y_H(t)$ ASSUME A SOLUTION OF FORM $y(t) = e^{\lambda t}$

THIS YIELDS... $\lambda^2 e^{\lambda t} + k_1 \lambda e^{\lambda t} + k_2 e^{\lambda t} = 0$ ($e^{\lambda t} \neq 0$ FOR ALL t)

SOLVING FOR THE ROOTS OF THE RESULTING POLYNOMIAL IN λ ...

$$\lambda_1 = \frac{-k_1}{2} + \frac{1}{2}\sqrt{k_1^2 - 4k_2} \quad \text{AND} \quad \lambda_2 = \frac{-k_1}{2} - \frac{1}{2}\sqrt{k_1^2 - 4k_2}$$

SO... $y_H(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$

For $y_p(t)$ ASSUME A SOLUTION OF FORM $y_p(t) = C_3$

THIS YIELDS... $0 + k_1(0) + k_2(C_3) = g$ SO... $C_3 = \frac{g}{k_2}$

SO... $y(t) = \frac{g}{k_2} + C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$ $y(0) = 0$ & $y'(0) = v_0$

SUBSTITUTING... $y(0) = 0 = \frac{g}{k_2} + C_1 + C_2$

$$y'(0) = v_0 = C_1 \lambda_1 + C_2 \lambda_2$$

SOLVING...

$$C_1 = \frac{k_2 v_0 + g \lambda_2}{k_2(\lambda_1 - \lambda_2)} \quad \text{AND} \quad C_2 = \frac{k_2 v_0 + g \lambda_1}{k_2(\lambda_2 - \lambda_1)}$$

PLUGGING INTO $y(t) = y_H(t) + y_p(t)$...

$$y(t) = \frac{g}{k_2} + \frac{(-gk_1 + 2k_2 v_0 - g\sqrt{k_1^2 - 4k_2})}{2k_2\sqrt{k_1^2 - 4k_2}} e^{\left(\frac{-k_1}{2} + \frac{1}{2}\sqrt{k_1^2 - 4k_2}\right)t} - \frac{(-gk_1 + 2k_2 v_0 + g\sqrt{k_1^2 - 4k_2})}{2k_2\sqrt{k_1^2 - 4k_2}} e^{\left(\frac{-k_1}{2} - \frac{1}{2}\sqrt{k_1^2 - 4k_2}\right)t}$$

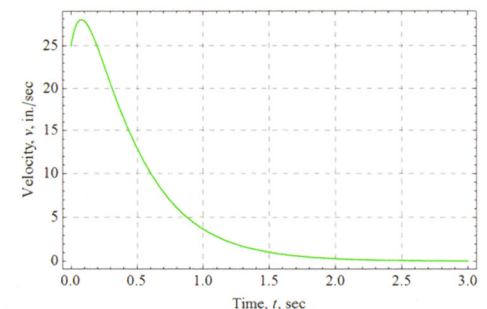
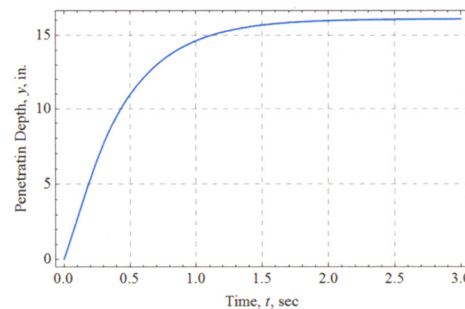
WITH NUMBERS... $y(t) = 16.1 - 18.38 e^{-2.54t} + 2.28 e^{-9.46t}$ in.

$$v(t) = y'(t) = \frac{(-gk_1 + 2k_2 v_0 - g\sqrt{k_1^2 - 4k_2})\left(\frac{-k_1}{2} + \frac{1}{2}\sqrt{k_1^2 - 4k_2}\right)}{2k_2\sqrt{k_1^2 - 4k_2}} e^{\left(\frac{-k_1}{2} + \frac{1}{2}\sqrt{k_1^2 - 4k_2}\right)t} - \frac{(-gk_1 + 2k_2 v_0 + g\sqrt{k_1^2 - 4k_2})\left(\frac{-k_1}{2} - \frac{1}{2}\sqrt{k_1^2 - 4k_2}\right)}{2k_2\sqrt{k_1^2 - 4k_2}} e^{\left(\frac{-k_1}{2} - \frac{1}{2}\sqrt{k_1^2 - 4k_2}\right)t}$$

WITH NUMBERS... $y'(t) = 46.6 e^{-2.54t} - 21.6 e^{-9.46t}$ in./sec

Lim $y(t) = \frac{g}{k_2}$ as $t \rightarrow \infty$ SO... $y_f = \frac{g}{k_2} = \frac{32.2(12)}{24} \rightarrow y_f = 16.10$ in.

PLOTTING $y(t)$ AND $y'(t)$...



• FIND TIME:

$$0.95 y_f = y(t) = 16.10 - 18.38 e^{-2.54t} + 2.28 e^{-9.46t} = 0.95(16.10)$$

$$t = 1.234 \text{ sec}$$

$$2/33 \quad \sigma = 0.7 \text{ m/s}^2, \quad \eta = 0.2 \text{ m}^{-1}, \quad v_0 = 4 \text{ m/s}$$

$$a = \sigma - \eta v^2 = \frac{v dv}{dy} \longrightarrow \int_{y=0}^y dy = \int_{v_0}^v \frac{v dv}{\sigma - \eta v^2} \quad \begin{cases} u = \sigma - \eta v^2 \\ du = -2\eta v dv \end{cases}$$

$$y = -\frac{1}{2\eta} \int_{v_0}^v \frac{du}{u} = -\frac{1}{2\eta} \ln u \Big|_{v_0}^v = -\frac{1}{2\eta} \ln(\sigma - \eta v^2) \Big|_{v_0}^v = \frac{1}{2\eta} \ln \left(\frac{\sigma - \eta v_0^2}{\sigma - \eta v^2} \right)$$

$$\text{If } v = \frac{v_0}{2} = 2 \text{ m/s} \dots y = \frac{1}{2(0.2)} \ln \left(\frac{0.7 - 0.2(4)^2}{0.7 - 0.2(2)^2} \right) \longrightarrow \underline{y = 8.05 \text{ m}}$$

$$\text{FOR TERMINAL VELOCITY... } a = 0 = \sigma - \eta v_T^2 \longrightarrow v_T = \sqrt{\frac{\sigma}{\eta}}$$

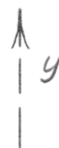
$$\text{WITH NUMBERS... } v_T = \sqrt{\frac{0.7}{0.2}} \longrightarrow \underline{v_T = 1.871 \text{ m/s}}$$

$$\begin{aligned} 2/34 \quad a &= g - cy^2 = v \frac{dv}{dy} \\ \int_0^{y_m} (g - cy^2) dy &= \int_{v_0}^0 v dv \\ \left(gy - c \frac{y^3}{3} \right) \Big|_0^{y_m} &= \frac{v^2}{2} \Big|_{v_0}^0 \\ gy_m - c \frac{y_m^3}{3} &= -\frac{v_0^2}{2} \Rightarrow c = \frac{3v_0^2 + 6gy_m}{2y_m^3} \end{aligned}$$

WILEY

$$\frac{2}{35} \quad \text{Up: } a_u = -g - kv^2 = v \frac{dv}{dy}$$

$$\int_0^h dy = - \int_{v_0}^0 \frac{v dv}{g + kv^2}$$



$$h = -\frac{1}{2k} \ln[g + kv^2]_{v_0}^0 = \frac{1}{2k} \ln\left[\frac{g + kv_0^2}{g}\right]$$

$$h = \frac{1}{2(0.006)} \ln\left[\frac{9.81 + 0.006(30)^2}{9.81}\right] = \underline{36.5 \text{ m}}$$

$$\text{Down: } a_d = -g + kv^2 = v \frac{dv}{dy}$$

$$\int_h^0 dy = \int_{v_f}^0 \frac{v dv}{-g + kv^2}$$

$$-h = \frac{1}{2k} \ln[-g + kv^2]_0^{v_f} = \frac{1}{2k} \ln\left[\frac{g - kv_f^2}{g}\right]$$

$$\Rightarrow v_f = \sqrt{\frac{g}{k}(1 - e^{-2kh})}$$

$$= \sqrt{\frac{9.81}{0.006}(1 - e^{-2(0.006)(36.5)})} = \underline{24.1 \text{ m/s}}$$

WILEY

2/36

$$\text{Up: } a_u = -g - kv^2 = \frac{dv}{dt}$$

$$\int_0^{t_u} dt = - \int_{v_0}^0 \frac{dv}{g + kv^2}$$

$$t_u = \frac{1}{\sqrt{gk}} \tan^{-1} \left(\frac{v \sqrt{gk}}{g} \right) \Big|_0^{v_0} = \frac{1}{\sqrt{gk}} \tan^{-1} \left(v_0 \sqrt{\frac{k}{g}} \right)$$

$$t_u = \frac{1}{\sqrt{9.81(0.006)}} \tan^{-1} \left(30 \sqrt{\frac{0.006}{9.81}} \right) = \underline{2.63 \text{ s}}$$

$$\text{Down: } a_d = -g + kv^2 = \frac{dv}{dt}$$

$$\int_0^{t_d} dt = \int_0^{v_f} \frac{dv}{-g + kv^2}$$

$$t_d = \frac{1}{\sqrt{gk}} \tanh^{-1} \left(\frac{v \sqrt{gk}}{g} \right) \Big|_0^{v_f} = \frac{1}{\sqrt{gk}} \tanh^{-1} \left(v_f \sqrt{\frac{k}{g}} \right)$$

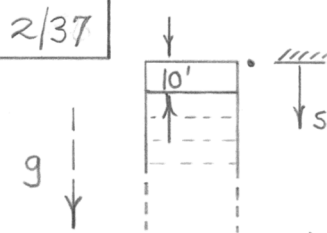
$$= \frac{1}{\sqrt{9.81(0.006)}} \tanh^{-1} \left(24.1 \sqrt{\frac{0.006}{9.81}} \right)$$

$$= \underline{2.83 \text{ s}}$$

(Refer to solution of Prob. 2/35)

WILEY

2/37



$$s = s_0' + v_0' t + \frac{1}{2} g t^2$$

When $s = 10 \text{ ft}$,

$$10 = \frac{1}{2} (32.2) t_{10'}^2, \quad t_{10'} = 0.788 \text{ sec}$$

Time to pass first story from the top is $t_1 = t_{10'} - t_0' = 0.788 - 0 = \underline{0.788 \text{ sec}}$

10th story: $90 = \frac{1}{2} (32.2) t_{90'}^2, \quad t_{90'} = 2.36 \text{ sec}$

$$100 = \frac{1}{2} (32.2) t_{100'}^2, \quad t_{100'} = 2.49 \text{ sec}$$

$$t_{10} = t_{100'} - t_{90'} = 2.49 - 2.36 = \underline{0.1279 \text{ sec}}$$

100th story: $990 = \frac{1}{2} (32.2) t_{990'}^2,$

$$1000 = \frac{1}{2} (32.2) t_{1000'}^2,$$

$$t_{100} = t_{1000'} - t_{990'} = 7.88 - 7.84 = \underline{0.0395 \text{ sec}}$$

WILEY

2/38

$$a = g - kv^2 = \frac{dv}{dt}$$



$$\int_0^t dt = \int_0^v \frac{dv}{g - kv^2}$$

$$\begin{aligned} \text{(see Art. C/10): } t &= \frac{1}{\sqrt{gk}} \tanh^{-1} \sqrt{\frac{k}{g}} v \Big|_0^v \\ &= \frac{1}{\sqrt{gk}} \tanh^{-1} \sqrt{\frac{k}{g}} v \end{aligned}$$

$$\Rightarrow v = \frac{ds}{dt} = \sqrt{\frac{g}{k}} \tanh(\sqrt{gk} t)$$

$$\int_0^s ds = \sqrt{\frac{g}{k}} \int_0^t \tanh(\sqrt{gk} t) dt$$

$$s = \frac{1}{k} \ln \cosh \sqrt{gk} t$$

$$\text{or } t = \frac{\cosh^{-1}(e^{sk})}{\sqrt{gk}} = \frac{\cosh^{-1}(e^{0.0055s})}{0.401}$$

s, ft	t, sec
0	0
10	0.795
90	2.54
100	2.70
990	14.06
1000	14.19

The time t_1 to pass first story

$$\text{is } t_1 = t_{10} - t_0 = 0.795 - 0 = \underline{0.795 \text{ sec}}$$

Similarly,

$$t_{10} = \underline{0.1592 \text{ sec}}$$

$$t_{100} = \underline{0.1246 \text{ sec}}$$

$$\underline{2/39} \quad (a) \quad a = 2 \text{ m/s}^2 = \text{constant}$$

With $v = 250/3.6 = 69.4 \text{ m/s}$, we have

$$v^2 - v_0^2 = 2a(s - s_0) : 69.4^2 - 0^2 = 2(2)s$$

$$\underline{s = 1206 \text{ m}}$$

$$(b) \quad a = a_0 - kv^2 = v \frac{dv}{ds}$$

$$\int_0^s ds = \int_0^v \frac{v dv}{a_0 - kv^2}$$

$$s = -\frac{1}{2k} \ln(a_0 - kv^2) \Big|_0^v$$

$$= -\frac{1}{2k} \ln \left[\frac{a_0 - kv^2}{a_0} \right]$$

$$s = -\frac{1}{2(4)(10^{-5})} \ln \left[\frac{2 - 4(10^{-5})(69.4)^2}{2} \right]$$

$$= \underline{1268 \text{ m}}$$

WILEY

$$\begin{aligned}
 & \boxed{2/40} \quad a = -kv^2, \quad v \\
 & v dv = a dx, \quad \int_{v_0}^v \frac{v dv}{-kv^2} = \int_0^x dx, \quad x = \left. \frac{-1}{k} \ln v \right|_{v_0}^v \\
 & \quad \quad \quad x = \frac{1}{k} \ln v_0/v \\
 & \text{when } v = v_0/2, \quad x = D = \frac{1}{k} \ln 2 = \underline{0.693/k} \\
 & v = \frac{dx}{dt} \quad \text{where } kx = \ln v_0/v, \quad v = v_0 e^{-kx} \\
 & \text{so } \frac{dx}{v_0 e^{-kx}} = dt \quad \text{or } \int_0^t dt = \frac{1}{v_0} \int_0^x e^{kx} dx \\
 & \quad \quad \quad t = \left. \frac{1}{v_0} \frac{1}{k} e^{kx} \right|_0^x = \frac{1}{kv_0} [e^{kx} - 1] \\
 & \text{For } x = D, \quad e^{kx} = 2 \quad \text{so } t = \frac{1}{kv_0} [2 - 1], \quad \underline{t = \frac{1}{kv_0}}
 \end{aligned}$$

WILEY

2/41

$$\int v dv = \int a dx \quad \text{or} \quad \int_{(40)^2}^{0^2} d(v^2) = 2 \int_0^x a dx$$

$$= 2 \left[\text{area under } a \text{ vs } x \text{ curve} \right]$$

$$-1600 = -2 \left[\frac{1}{2} 1000(0.5) + 1000(x-0.5) + \frac{1}{2} (x-0.5) \frac{2000}{0.5} (x-0.5) \right]$$

$$1600 = 500 + 2000(x-0.5) + 4000(x-0.5)^2, \quad x > 0.5 \text{ m}$$

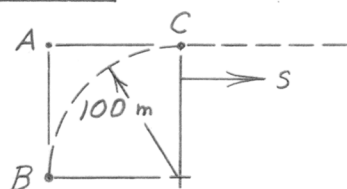
$$x^2 - 0.5x - 0.275 = 0, \quad x = \frac{0.5 \pm \sqrt{(0.5)^2 + 4(0.275)}}{2}$$

$$= 0.25 \pm 0.581$$

$$\underline{x = 0.831 \text{ m}} \quad (\text{or } -0.331)$$

WILEY

2/42



First, determine B's
acceleration time:

$$v = v_0 + at$$

$$\frac{100(10^3)}{3600} = \frac{40(10^3)}{3600} + 0.981 t$$

$$t = 16.99 \text{ s}$$

Distance traveled by A in that time:

$$d_A = \frac{100(10^3)}{3600} (16.99) = 472 \text{ m}$$

Displacement beyond C: $s_A = 472 - 100 = 372 \text{ m}$

Distance traveled by B in 16.99 s:

$$d_B = v_0 t + \frac{1}{2} a t^2 = \frac{40(10^3)}{3600} (16.99) + \frac{1}{2} (0.981) (16.99)^2$$

$$= 330 \text{ m}$$

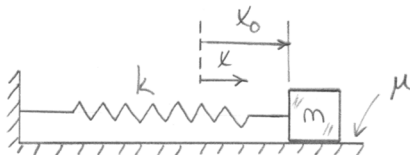
Displacement beyond C: $s_B = 330 - \frac{\pi(100)}{2} = 173.3 \text{ m}$

So A is ahead of B by $s_A - s_B = 372 - 173.3$

$= 198.7 \text{ m}$ (in the steady state)

WILEY

2/43



$$\begin{cases} m = 5 \text{ kg} \\ \mu = 0.40 \\ k = 150 \text{ N/m} \\ x_0 = 200 \text{ mm} \end{cases}$$

MINIMUM STRETCH TO OVERCOME STATIC FRICTION.

$$a = 0 = \mu g - \frac{k}{m} x_{\min} \rightarrow x_{\min} = \frac{\mu mg}{k} = \frac{0.40(5)(9.81)}{150}$$

$$x_{\min} = 0.1308 \text{ m OR } 130.8 \text{ mm}$$

(MASS WILL MOVE LEFT)

$$a = \mu g - \frac{k}{m} x = \frac{v dv}{dx} \rightarrow \int_{x_0}^x \left(\mu g - \frac{k}{m} x \right) dx = \int_{v_0=0}^v v dv$$

$$\text{SO... } \frac{1}{2} v^2 = \mu g (x - x_0) - \frac{k}{2m} (x^2 - x_0^2)$$

FOR COMPLETE STOP... $v = 0$, AND SOLVE FOR $x = x_f$

$$x_f = x_0 \text{ OR } x_f = \frac{2mg\mu - kx_0}{k} = \frac{2(5)(9.81)(0.40) - 150(0.2)}{150}$$

$$x_f = 0.0616 \text{ m OR } 61.6 \text{ mm}$$

► 2/44 $m = 5 \text{ kg}, \mu = 0.40, k = 150 \text{ N/m}, x_0 = 500 \text{ mm}$

From 2/43, $x_{\min} = 130.8 \text{ mm}$ TO CAUSE MOTION

FOR AN INITIAL STRETCH (MOTION TO THE LEFT), $a = \mu g - \frac{k}{m} x$

FOR STOP... $x_f = \frac{2mg\mu - kx_0}{k} = \frac{2(5)(9.81)(0.40) - 150(0.5)}{150}$

$x_f = -0.238 \text{ m OR } -238 \text{ mm}$

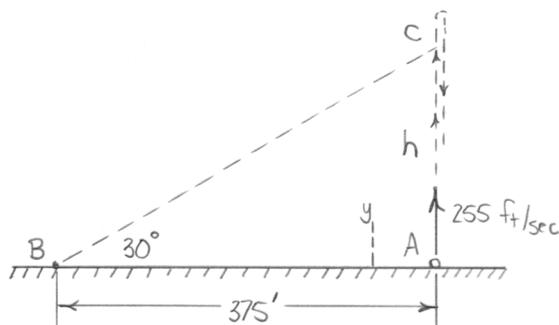
SINCE $238 > 130.8$, THE BLOCK WILL SLIDE PAST THE MINIMUM STRETCH DISTANCE ON THE COMPRESSION SIDE OF THE SPRING'S UNDEFORMED POSITION, AND WILL THEN SLIDE BACK TO THE RIGHT.

NOW... $a = -\mu g - \frac{k}{m} x$ (SINCE FRICTION TERM WILL POINT LEFT)

AS BEFORE... $x_f = \frac{-2mg\mu - kx_0}{k}$ WITH $x_0 = -0.238 \text{ m}$

$x_f = \frac{-2(5)(9.81)(0.40) - 150(-0.238)}{150} \rightarrow x_f = -0.0232 \text{ m OR } -23.2 \text{ mm}$

2/45



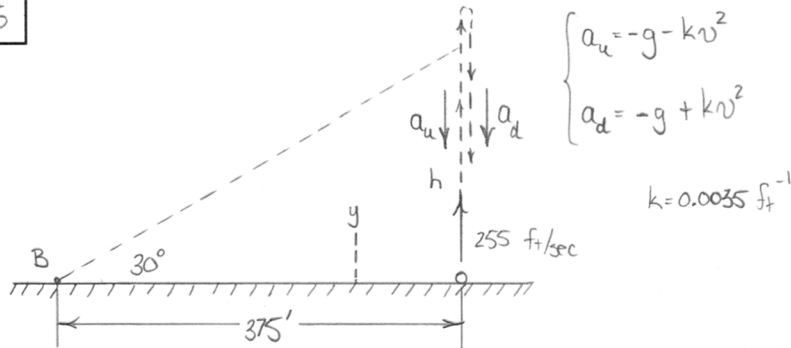
$$h = 375 \tan 30^\circ = 217'$$

$$\begin{cases} h = v_{y0}t - \frac{1}{2}gt^2 \\ 217 = 255t - \frac{1}{2}(32.2)t^2 \end{cases} \rightarrow \underline{t = 0.900 \text{ sec AND } 14.94 \text{ sec}}$$

$$v = v_y - gt = 255 - 32.2(0.900) \rightarrow \underline{v = 226 \text{ ft/sec}}$$

(BOTH CASES ARE THE SAME)

► 2/46



$$h = 375 \tan 30^\circ = 217'$$

•UPWARD MOTIONS:

$$a_u = -g - kv^2 = \frac{v dv}{dy} \rightarrow -\int_0^y dy = \int_{v_0}^v \frac{v}{g + kv^2} dv$$

$$\begin{cases} u = g + kv^2 \\ du = 2kv dv \end{cases} \text{ so... } y = -\frac{1}{2k} \int \frac{1}{u} du = -\frac{\ln u}{2k} = -\frac{\ln(g + kv^2)}{2k} \Big|_{v_0}^v$$

$$y = -\frac{1}{2k} \ln \left(\frac{g + kv^2}{g + kv_0^2} \right) \rightarrow 217 = -\frac{1}{2(0.0035)} \ln \left(\frac{32.2 + 0.0035 v^2}{32.2 + 0.0035 (255)^2} \right)$$

$$v = 84.3 \text{ ft/sec} \quad (\text{USE ARTICLE C/10})$$

$$a_u = -g - kv^2 = \frac{dv}{dt} \rightarrow \int_0^t dt = -\int_{v_0}^v \frac{dv}{g + kv^2} = -\frac{1}{\sqrt{gk}} \tanh^{-1} \left(\frac{\sqrt{k} v}{\sqrt{g}} \right) \Big|_{v_0}^v$$

$$t = \frac{-1}{\sqrt{32.2(0.0035)}} \left[\tanh^{-1} \left(\frac{\sqrt{0.0035} 84.3}{\sqrt{32.2}} \right) - \tanh^{-1} \left(\frac{\sqrt{0.0035} 255}{\sqrt{32.2}} \right) \right] \rightarrow t = 1.460 \text{ sec}$$

$$\begin{cases} y_{\max} = y \text{ As } v \rightarrow 0 \\ y_{\max} = \frac{-1}{2(0.0035)} \ln \left(\frac{32.2}{32.2 + 0.0035 (255)^2} \right) = 298 \text{ ft} \\ t_{y_{\max}} = \frac{1}{\sqrt{gk}} \tanh^{-1} \left(\frac{\sqrt{k} v_0}{\sqrt{g}} \right) = \frac{1}{\sqrt{32.2(0.0035)}} \tanh^{-1} \left(\frac{\sqrt{0.0035} 255}{\sqrt{32.2}} \right) = 3.61 \text{ sec} \end{cases}$$

•DOWNWARD MOTION: (USE PREVIOUS RESULT)

$$a_d = -g + kv^2 = \frac{v dv}{dy} \rightarrow \int_{y_{\max}}^y dy = \int_0^v \frac{v dv}{-g + kv^2} \rightarrow y - y_{\max} = \frac{1}{2k} \ln \left(\frac{kv^2 - g}{-g} \right)$$

$$\text{AND } y = 217' \quad 217 - 298 = \frac{1}{2(0.0035)} \ln \left(\frac{0.0035 v^2 - 32.2}{-32.2} \right)$$

$$\text{SOLVING... } v = 63.3 \text{ ft/sec} \quad \downarrow (\text{MINUS})$$

$$a_d = -g + kv^2 = \frac{dv}{dt} \rightarrow \int_{t_{y_{\max}}}^t dt = \int_0^v \frac{dv}{-g + kv^2} = -\frac{1}{g} \int \frac{dv}{1 - \frac{kv^2}{g}}$$

$$u = \frac{\sqrt{k} v}{\sqrt{g}} \text{ so... } t - t_{y_{\max}} = \frac{-1}{\sqrt{gk}} \int \frac{du}{1 - u^2} = \frac{-1}{\sqrt{gk}} \tanh^{-1}(u) = \frac{-1}{\sqrt{gk}} \tanh^{-1} \left(\frac{\sqrt{k} v}{\sqrt{g}} \right) \Big|_0^v$$

$$du = \frac{\sqrt{k} dv}{\sqrt{g}} \quad t - 3.61 = \frac{-1}{\sqrt{32.2(0.0035)}} \tanh^{-1} \left(\frac{\sqrt{0.0035} (-63.3)}{\sqrt{32.2}} \right)$$

$$t = 5.97 \text{ sec}$$

$$\begin{aligned}\boxed{2/47} \quad \underline{v}_{av} &= \frac{\Delta \underline{r}}{\Delta t} = \frac{(5.1 \underline{i} + 0.4 \underline{j}) - 5 \underline{i}}{0.02} \\ &= 5 \underline{i} + 20 \underline{j} \text{ m/s} \\ v_{av} &= \sqrt{5^2 + 20^2} = \underline{20.6 \text{ m/s}} \\ \theta &= \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{20}{5}\right) = \underline{76.0^\circ}\end{aligned}$$

WILEY

$$\boxed{2/48} \quad \underline{a}_{Av} = \frac{\Delta \underline{v}}{\Delta t} = \frac{0.3\hat{i} + 0.4\hat{j}}{0.1} = \underline{3\hat{i} + 4\hat{j} \frac{m}{s^2}}$$

$$a = a_{av} = \sqrt{3^2 + 4^2} = \underline{5 \text{ m/s}^2}$$

$$\tan \theta = \frac{a_y}{a_x} = \frac{4}{3} \quad , \quad \underline{\theta = 53.1^\circ}$$

WILEY

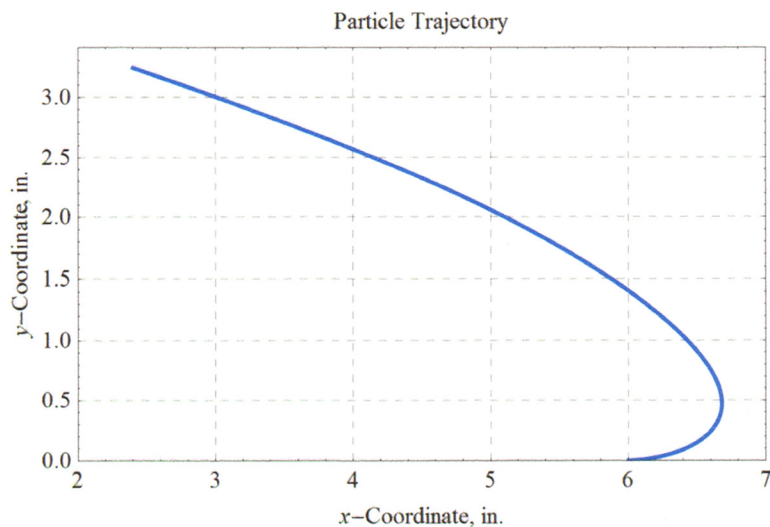
$$\boxed{2/49} \quad (x_0, y_0) = (6, 0) \text{ in.} \quad \text{AND} \quad (v_{x_0}, v_{y_0}) = (0, 0)$$

$$\begin{cases} a_x = 0.5 - 0.35t \text{ in./sec}^2 \\ v_x = v_{x_0} + \int a_x(t) dt = 0 + \int (0.5 - 0.35t) dt = 0.5t - 0.175t^2 \text{ in./sec} \\ x = x_0 + \int v_x(t) dt = 6 + \int (0.5t - 0.175t^2) dt = \underline{6 + 0.25t^2 - 0.0583t^3 \text{ in.}} \end{cases}$$

$$\begin{cases} a_y = 0.15t - 0.02t^2 \text{ in./sec}^2 \\ v_y = v_{y_0} + \int a_y(t) dt = 0 + \int (0.15t - 0.02t^2) dt = 0.075t^2 - 0.667(10^{-3})t^3 \text{ in./sec} \\ y = y_0 + \int v_y(t) dt = 0 + \int (0.075t^2 - 0.667(10^{-3})t^3) dt = \underline{0.025t^3 - 1.667(10^{-3})t^4 \text{ in.}} \end{cases}$$

$$\text{At } t = 6 \text{ sec} \quad \begin{cases} x = 6 + 0.25(6)^2 - 0.0583(6)^3 \longrightarrow x = 2.4 \text{ in.} \\ y = 0.025(6)^3 - 1.667(10^{-3})(6)^4 \longrightarrow y = 3.24 \text{ in.} \end{cases}$$

$$\text{so... } \underline{(x, y) = (2.4, 3.24) \text{ in.}}$$



2/50

$$x = 20 + \frac{1}{4}t^2, \dot{x} = \frac{1}{2}t, \ddot{x} = \frac{1}{2} \text{ mm/s}^2$$

$$y = 15 - \frac{1}{6}t^3, \dot{y} = -\frac{1}{2}t^2, \ddot{y} = -t \text{ mm/s}^2$$

For $t = 2 \text{ s}$,

$$\dot{x} = 1 \text{ mm/s}$$

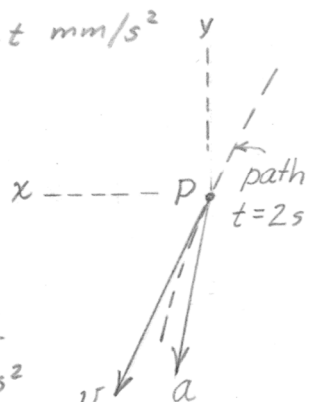
$$\dot{y} = -2 \text{ mm/s}$$

$$\ddot{x} = \frac{1}{2} \text{ mm/s}^2$$

$$\ddot{y} = -2 \text{ mm/s}^2$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{1^2 + (-2)^2} = \underline{2.24 \text{ mm/s}}$$

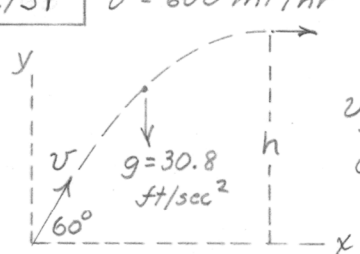
$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(\frac{1}{2})^2 + (-2)^2} = \underline{2.06 \text{ mm/s}^2}$$



The diagram shows a 2D Cartesian coordinate system with x and y axes. A dashed line represents the path of a particle. At a point P on this path, which is labeled 't=2s', two vectors originate: a velocity vector v pointing downwards and to the left, and an acceleration vector a pointing downwards and to the right. The path is indicated by a dashed line with an arrow pointing towards P.

WILEY

2/51 $v = 600 \text{ mi/hr} \approx 880 \text{ ft/sec}$



$a_y = -30.8 \text{ ft/sec}^2$

$v_y = v_{y_0} + a_y t$

$0 = 880 \sin 60^\circ - 30.8 t$

$t = \frac{880(0.866)}{30.8} = \underline{24.7 \text{ sec}}$

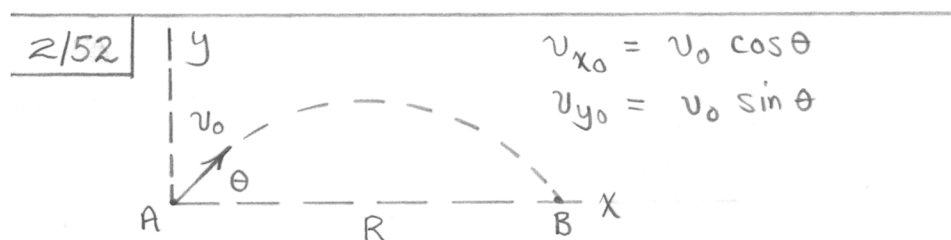
$v_y^2 = v_{y_0}^2 + 2a_y y$

$0 = (880 \sin 60^\circ)^2 - 2(30.8)h$

$h = \frac{[880(0.866)]^2}{2(30.8)} = 9429 \text{ ft}$

or $h = \frac{9429}{5280} = \underline{1.786 \text{ mi}}$

WILEY



$$x = x_0 + v_{x0} t \text{ @ B: } R = 0 + (v_0 \cos \theta) t_f \quad (1)$$

$$y = y_0 + v_{y0} t - \frac{1}{2} g t^2 \text{ @ B: } 0 = 0 + (v_0 \sin \theta) t_f - \frac{g}{2} t_f^2$$

$$(2): t_f = 0, \frac{2 v_0 \sin \theta}{g} \quad (t = 0 \text{ is launch time})^{(2)}$$

$$(1): R = (v_0 \cos \theta) \left(\frac{2 v_0 \sin \theta}{g} \right) = \frac{v_0^2 \sin 2\theta}{g}$$

$$\frac{dR}{d\theta} = 0: \frac{v_0^2}{g} 2 \cos 2\theta = 0 \Rightarrow \theta = 45^\circ$$

$$R_{\max} = \frac{v_0^2 \sin (2 \cdot 45^\circ)}{g} = \frac{v_0^2}{g}$$

WILEY

2/53

Angle θ for maximum range and, hence, minimum u is 45°

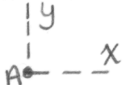
From Sample Prob. 2/6 range is

$$2s = \frac{u^2}{g} \sin 2\theta, \text{ so } 12(10^3) = \frac{u^2}{9.81} (1)$$

$$u = \sqrt{9.81(12000)} = \underline{343 \text{ m/s}}$$

WILEY

2/54



$$v_{x_0} = v_0 \cos \theta = 45 \cos 40^\circ = 34.5 \text{ ft/sec}$$

$$v_{y_0} = v_0 \sin \theta = 45 \sin 40^\circ = 28.9 \text{ ft/sec}$$

$$x = x_0 + v_{x_0} t \text{ at wall: } 60 = 0 + 34.5 t$$

$$t = 1.741 \text{ sec}$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \text{ at wall:}$$

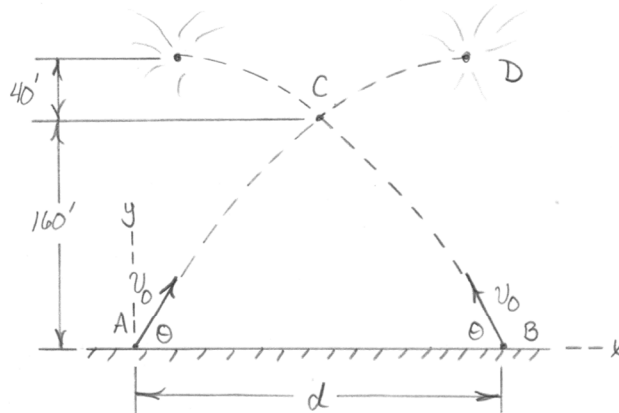
$$y = 1 + 28.9 (1.741) - 16.1 (1.741)^2 = 2.57 \text{ ft}$$

So water strikes wall 2.57 ft above B.

WILEY

2/55

$$\theta = 60^\circ$$



$$v_y^2 = v_{y0}^2 - 2g\Delta y \rightarrow 0 = (v_0 \sin 60^\circ)^2 - 2(32.2)(200) \rightarrow v_0 = 131.0 \text{ ft/sec}$$

$$y = y_A + v_{y0} t_{AC} - \frac{1}{2} g t_{AC}^2 \rightarrow 160 = 131.0 \sin 60^\circ t_{AC} - \frac{1}{2} (32.2) t_{AC}^2$$

$$\therefore t_{AC} = 1.948 \text{ sec} \quad (\text{OR } 5.10 \text{ sec})^{\text{TOO LATE!}}$$

$$x_C = x_A + v_{x0} t_{AC} \rightarrow \frac{d}{2} = 131.0 \cos 60^\circ (1.948) \rightarrow d = 255 \text{ ft}$$

$$y_D = y_A + v_{y0} t_{AD} - \frac{1}{2} g t_{AD}^2 \rightarrow 200 = 131.0 \sin 60^\circ t_{AD} - \frac{1}{2} (32.2) t_{AD}^2$$

$$\therefore t_{AD} = 3.52 \text{ sec}$$

2/56 Set up x-y axes at the initial location of G.

$$\left. \begin{aligned} x &= x_0 + v_{x_0} t : 1 = (v_0 \cos \theta) t \\ y &= y_0 + v_{y_0} t - \frac{1}{2} g t^2 : 1.06 = (v_0 \sin \theta) t - \frac{9.81}{2} t^2 \\ v_y &= v_{y_0} - g t : 0 = v_0 \sin \theta - 9.81 t \end{aligned} \right\}$$

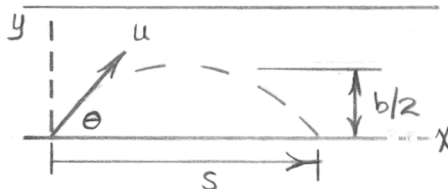
$$\text{Solve simultaneously: } \left\{ \begin{aligned} t &= 0.465 \text{ s} \\ v_0 &= 5.04 \text{ m/s} \\ \theta &= 64.7^\circ \end{aligned} \right.$$

WILEY

2/57

$$a_y = -\frac{eE}{m}, \text{ constant}$$

$$a_x = 0$$



$$v_y^2 - v_{y_0}^2 = 2a_y y : \text{ At top, } 0 - (u \sin \theta)^2 = 2 \left(-\frac{eE}{m} \right) \frac{b}{2}$$

$$E = \frac{mu^2 \sin^2 \theta}{eb}$$

$$v_y = v_{y_0} + a_y t : \text{ At top, } 0 = u \sin \theta - \frac{eE}{m} t$$

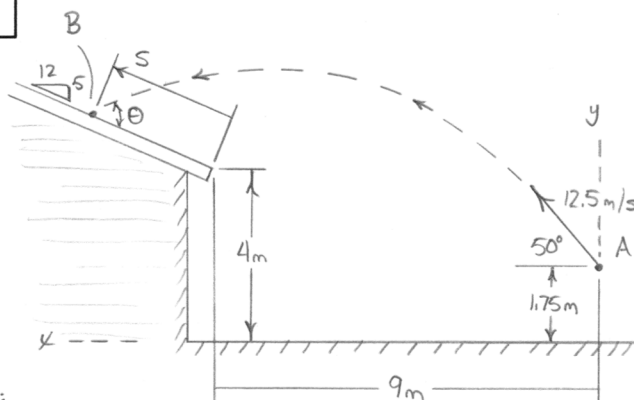
$$t = \frac{mu \sin \theta}{eE}$$

$$x = v_{x_0} t : s = (u \cos \theta)(2t) = u \cos \theta \left(\frac{2mu \sin \theta}{eE} \right)$$

$$= u \cos \theta \left(\frac{2mu \sin \theta}{e \frac{mu^2 \sin^2 \theta}{eb}} \right) = \underline{2b \cot \theta}$$

WILEY

2/58



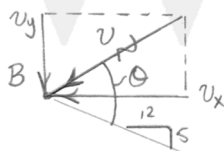
• IMPACT:

$$\begin{cases} x_B = x_A + v_{x0}t \rightarrow 9 + \frac{12}{13}S = 12.5 \cos 50^\circ t \\ y_B = y_A + v_{y0}t - \frac{1}{2}gt^2 \rightarrow 4 + \frac{5}{13}S = 1.75 + 12.5 \sin 50^\circ t - \frac{1}{2}(9.81)t^2 \end{cases}$$

SOLVING... $t = 1.477 \text{ s}$ AND $S = 3.10 \text{ m}$

• VELOCITY AT B

$$\begin{cases} v_x = v_{x0} = 12.5 \cos 50^\circ = 8.03 \text{ m/s} \\ v_y = v_{y0} - gt = 12.5 \sin 50^\circ - 9.81(1.477) = -4.91 \text{ m/s} \end{cases}$$



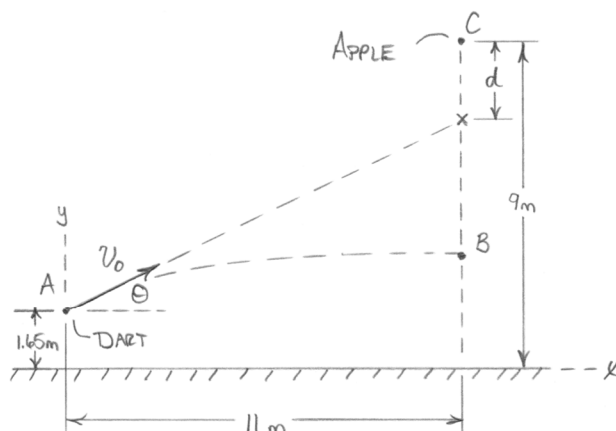
$$\theta = \tan^{-1}\left(\frac{|v_y|}{v_x}\right) + \tan^{-1}\left(\frac{5}{12}\right) = \tan^{-1}\left(\frac{4.91}{8.03}\right) + \tan^{-1}\left(\frac{5}{12}\right)$$

$\theta = 54.1^\circ$

2/59

$$v_0 = 14 \text{ m/s}$$

THERE IS A 215 ms DELAY BEFORE THE THROW.



• DART:

$$x_B = x_A + v_{x_0} t \rightarrow 11 = 14 \cos \theta t$$

$$y_B = y_A + v_{y_0} t - \frac{1}{2} g t^2 \rightarrow y_B = 1.65 + 14 \sin \theta t - \frac{1}{2} (9.81) t^2$$

• APPLE:

$$y_B = y_C + v_{y_0} t' - \frac{1}{2} g t'^2 \rightarrow y_B = 9 - \frac{1}{2} (9.81) (t + 0.215)^2$$

SOLVING... $y_B = 3.21 \text{ m}$, $t = 0.872 \text{ s}$, $\theta = 25.7^\circ$ (OTHER SOLUTION HAS NEGATIVE TIME)

$$1.65 + 11 \tan \theta + d = 9 \rightarrow d = 2.07 \text{ m}$$

2/60 For y-motion $a_y = g = 9.81 \text{ m/s}^2$
 $y = \frac{1}{2}gt^2$

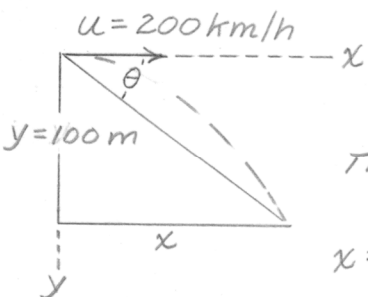
$u = 200 \text{ km/h}$

For x-motion $a_x = 0$ so
 $x = ut$

Thus $y = \frac{gx^2}{2u^2}$, $x = u\sqrt{\frac{2y}{g}}$

$x = \frac{200}{3.6} \sqrt{\frac{2(100)}{9.81}} = 251 \text{ m}$

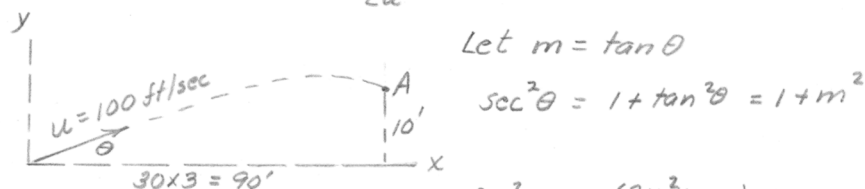
$\theta = \tan^{-1} \frac{y}{x} = \tan^{-1} \frac{100}{251} = 21.7^\circ$



WILEY

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$$y = x \tan \theta - \frac{gx^2}{2u^2} \sec^2 \theta$$



$$y = xm - \frac{gx^2}{2u^2} (1+m^2), \quad m^2 - \frac{2u^2}{gx} m + \left(\frac{2u^2 y}{gx^2} + 1 \right) = 0$$

$$\text{At A, } m^2 - \frac{2(10^2)^2}{32.2(90)} m + \left(\frac{2(10^2)^2 10}{32.2(90)^2} + 1 \right) = 0$$

$$m^2 - 6.901m + 1.7668 = 0$$

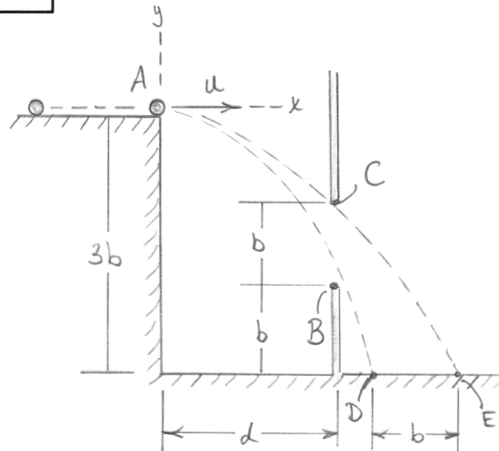
$$m = \frac{6.901}{2} \pm \frac{1}{2} \sqrt{(6.901)^2 - 4(1.7668)}$$

$$= \frac{6.901 \pm \sqrt{40.56}}{2} = 0.266 \text{ or } 6.635$$

$$\theta = \tan^{-1} m = \underline{14.91^\circ} \quad (\text{or } 81.4^\circ)$$

WILEY

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• MISS CORNER B:

$$\begin{cases} y_B = y_A + v_{yA} t_B - \frac{1}{2} g t_B^2 \rightarrow -2b = -\frac{1}{2} g t_B^2 \rightarrow t_B = 2\sqrt{\frac{b}{g}} \\ x_B = x_A + v_{xA} t_B \rightarrow d = 2u\sqrt{\frac{b}{g}} \quad \text{so...} \quad u = \frac{d}{2}\sqrt{\frac{g}{b}} \end{cases}$$

$$\begin{cases} y_D = y_A + v_{yA} t_f - \frac{1}{2} g t_f^2 \rightarrow -3b = -\frac{1}{2} g t_f^2 \rightarrow t_f = \sqrt{\frac{6b}{g}} \quad (\text{BOTH CASES}) \\ x_D = x_A + v_{xA} t_f = u\sqrt{\frac{6b}{g}} = \sqrt{\frac{3}{2}} d \end{cases}$$

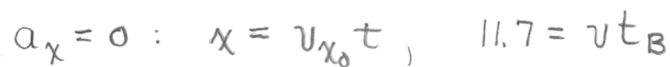
• MISS CORNER C:

$$\begin{cases} y_C = y_A + v_{yA} t_c - \frac{1}{2} g t_c^2 \rightarrow -b = -\frac{1}{2} g t_c^2 \rightarrow t_c = \sqrt{\frac{2b}{g}} \\ x_C = x_A + v_{xA} t_c \rightarrow d = u\sqrt{\frac{2b}{g}} \quad \text{so...} \quad u = d\sqrt{\frac{g}{2b}} \end{cases}$$

$$x_E = x_A + v_{xA} t_f = u\sqrt{\frac{6b}{g}} = \sqrt{3}d$$

$$x_E - x_D = b \rightarrow \sqrt{3}d - \sqrt{\frac{3}{2}}d = b \rightarrow \underline{d = 1.971 b}$$

$$\text{SUBSTITUTE FOR } u \dots \underline{0.986\sqrt{bg} < u < 1.394\sqrt{bg}}$$

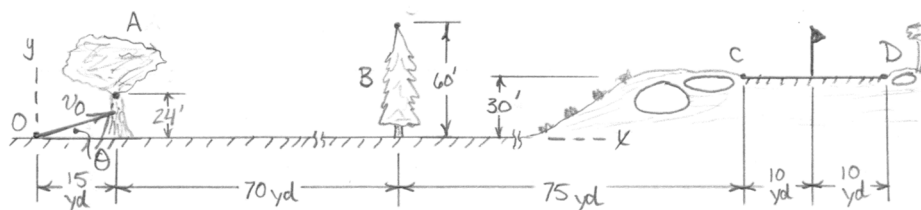


$$\text{At B: } (2.55 - 1.05) = 0 + \frac{1}{2} 9.81 t_B^2, \quad t_B = 0.553 \text{ s}$$

$$\text{At C: } 2.55 = \frac{1}{2}(9.81)t_c^2, \quad t_c = 0.721 \text{ s}$$

$$S + 11.7 = 21.2(0.721) \quad , \quad \underline{S = 3.55 \text{ m}}$$

$$2/64 \quad v_0 = 115 \frac{\text{mi}}{\text{hr}} = 168.7 \text{ ft/sec}, \quad \theta = 18^\circ$$



•CHECK A:

$$\begin{cases} x_A = x_0 + v_{x0} t_A \rightarrow 15(3) = 168.7 \cos 18^\circ t_A \rightarrow t_A = 0.281 \text{ sec} \\ y_A = y_0 + v_{y0} t_A - \frac{1}{2} g t_A^2 = 168.7 \sin 18^\circ (0.281) - \frac{1}{2} (32.2) (0.281)^2 = 13.35 \text{ ft} \quad \text{OK} \end{cases}$$

•CHECK B:

$$\begin{cases} x_B = x_0 + v_{x0} t_B \rightarrow 85(3) = 168.7 \cos 18^\circ t_B \rightarrow t_B = 1.590 \text{ sec} \\ y_B = y_0 + v_{y0} t_B - \frac{1}{2} g t_B^2 = 168.7 \sin 18^\circ (1.590) - \frac{1}{2} (32.2) (1.590)^2 = 42.2 \text{ ft} \end{cases}$$

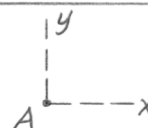
FIRST IMPACT POINT IS 42.2 ft UP IN THE TREE AT B.

2/65

(a) $v_o = 42 \text{ m/s}$ and $\theta = 8^\circ$:

$$x = x_o + v_{x_o} t \text{ at B: } 60 = 0 + (42 \cos 8^\circ) t$$

$$t = 1.443 \text{ s}$$



$$y = y_o + v_{y_o} t - \frac{1}{2} g t^2 \text{ at B:}$$

$$-(2.3 - h) = 0 + 42 \sin 8^\circ (1.443) - \frac{1}{2} (9.81) (1.443)^2$$

$$h = 0.525 \text{ m}$$

(b) $v_o = 36 \text{ m/s}$ and $\theta = 12^\circ$:

$$x = x_o + v_{x_o} t \text{ at B: } 60 = 0 + (36 \cos 12^\circ) t$$

$$t = 1.704 \text{ s}$$

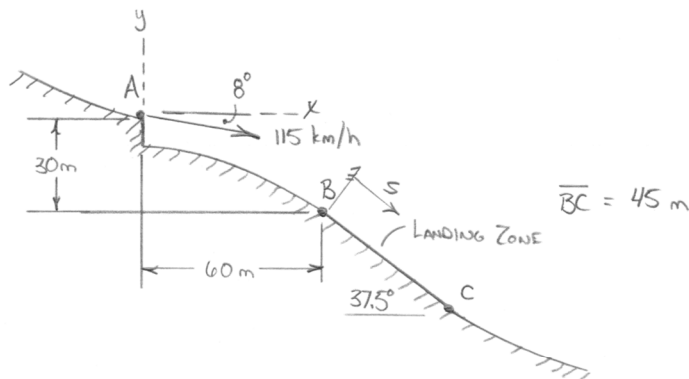
$$y = y_o + v_{y_o} t - \frac{1}{2} g t^2 \text{ at B:}$$

$$-(2.3 - h) = 0 + 36 \sin 12^\circ (1.704) - \frac{1}{2} (9.81) (1.704)^2$$

$$h = 0.813 \text{ m}$$

(In baseball, the time of flight is critical;
low trajectories, even with one hop, are better.)

2/66



$$\begin{cases} x_f = x_A + v_{x_0} t_f \rightarrow 60 + S \cos 37.5^\circ = \frac{115}{3.6} \cos 8^\circ t_f \\ y_f = y_A + v_{y_0} t_f \rightarrow -30 - S \sin 37.5^\circ = -\frac{115}{3.6} \sin 8^\circ t_f - \frac{1}{2}(9.81) t_f^2 \end{cases}$$

Solving... $t_f = 2.92 \text{ s}$

$S = 41.0 \text{ m}$

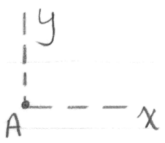
(OTHER SOLUTION HAS NEGATIVE S)

$$x_f = 60 + 41.0 \cos 37.5^\circ = 92.5 \text{ m}$$

$$y_f = -30 - 41.0 \sin 37.5^\circ = -54.9 \text{ m}$$

$$d = \sqrt{x_f^2 + y_f^2} = \sqrt{92.5^2 + 54.9^2} \rightarrow d = 107.6 \text{ m}$$

2/67 Set up x-y coordinates with origin at A.



$v_{x0} = 25 \cos \theta$
 $v_{y0} = 25 \sin \theta$

Evaluate $v_y^2 = v_{y0}^2 - 2g(y - y_0)$ at ceiling:

$$0^2 = (25 \sin \theta)^2 - 2(9.81)(5 - 0)$$

$$\theta = 23.3^\circ$$

Range $R = \frac{v_0^2 \sin 2\theta}{g} = \frac{25^2 \sin(2 \cdot 23.3^\circ)}{9.81}$

$R = 46.4 \text{ m}$

WILEY

2/68

x-y coordinates with origin at release

point : 

$$v_{x_0} = v_0 \sin \theta = 12 \sin \theta$$

$$v_{y_0} = v_0 \cos \theta = 12 \cos \theta \quad (\text{m/s})$$

$v_y = v_{y_0} - gt$ applied at end of flight:

$$-12 \cos \theta = 12 \cos \theta - 9.81 t_f, \quad t_f = 2.45 \cos \theta$$

$v_x = v_{x_0} - 0.4 t$ applied at end of flight:

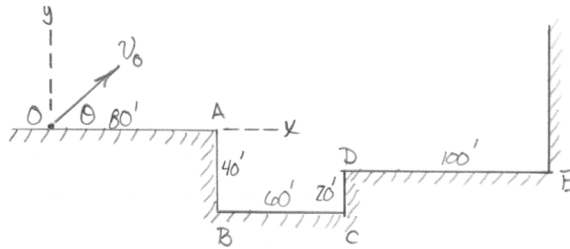
$$-12 \sin \theta = 12 \sin \theta - 0.4 (2.45 \cos \theta)$$

$$24 \sin \theta = 0.979 \cos \theta, \quad \tan \theta = 0.041$$

$$\theta = 2.33^\circ$$

WILEY

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• CASE A: $v_0 = 60 \text{ ft/sec}$ AND $\theta = 40^\circ$ (START WITH CORNER A)

$$\begin{cases} x_A = x_0 + v_{x0} t_A \rightarrow 80 = 60 \cos 40^\circ t_A \rightarrow t_A = 1.741 \text{ sec} \\ y_A = y_0 + v_{y0} t_A - \frac{1}{2} g t_A^2 = 60 \sin 40^\circ (1.741) - \frac{1}{2} (32.2) (1.741)^2 = 18.35 \text{ ft} \quad (\text{OK}) \end{cases}$$

$$\begin{cases} x_D = x_0 + v_{x0} t_D \rightarrow 140 = 60 \cos 40^\circ t_D \rightarrow t_D = 3.05 \text{ sec} \\ y_D = y_0 + v_{y0} t_D - \frac{1}{2} g t_D^2 = 60 \sin 40^\circ (3.05) - \frac{1}{2} (32.2) (3.05)^2 = -31.9 \text{ ft} \quad (\text{HITS } \overline{CD}) \end{cases}$$

$$(x_f, y_f) = (140, -31.9) \text{ ft}$$

• CASE B: $v_0 = 85 \text{ ft/sec}$ AND $\theta = 15^\circ$

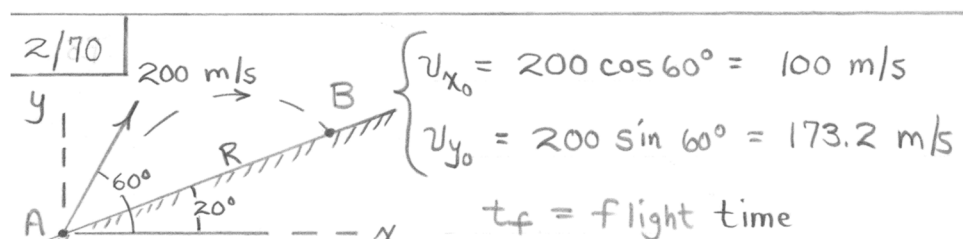
$$\begin{cases} x_A = x_0 + v_{x0} t_A \rightarrow 80 = 85 \cos 15^\circ t_A \rightarrow t_A = 0.974 \text{ sec} \\ y_A = y_0 + v_{y0} t_A - \frac{1}{2} g t_A^2 = 85 \sin 15^\circ (0.974) - \frac{1}{2} (32.2) (0.974)^2 = 6.15 \text{ ft} \quad (\text{OK}) \end{cases}$$

$$\begin{cases} x_D = x_0 + v_{x0} t_D \rightarrow 140 = 85 \cos 15^\circ t_D \rightarrow t_D = 1.705 \text{ sec} \\ y_D = y_0 + v_{y0} t_D - \frac{1}{2} g t_D^2 = 85 \sin 15^\circ (1.705) - \frac{1}{2} (32.2) (1.705)^2 = -9.30 \text{ ft} \quad (\text{OK}) \end{cases}$$

$$y_f = y_0 + v_{y0} t_f - \frac{1}{2} g t_f^2 \rightarrow -20 = 85 \sin 15^\circ t_f - \frac{1}{2} (32.2) t_f^2 \rightarrow t_f = 1.991 \text{ sec}$$

$$x_f = x_0 + v_{x0} t_f = 85 \cos 15^\circ (1.991) = 163.4 \text{ ft} \quad (x_f, y_f) = (163.4, -20) \text{ ft}$$

2/70



$u_{x_0} = 200 \cos 60^\circ = 100 \text{ m/s}$
 $u_{y_0} = 200 \sin 60^\circ = 173.2 \text{ m/s}$
 $t_f = \text{flight time}$

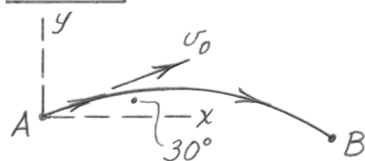
$x = x_0 + u_{x_0} t \text{ @ B: } R \cos 20^\circ = 100 t_f \quad (1)$
 $y = y_0 + u_{y_0} t - \frac{1}{2} g t^2 \text{ @ B: } R \sin 20^\circ = 173.2 t_f - \frac{9.81}{2} t_f^2 \quad (2)$

$(1): t_f = 0.00940 R$
 $(2): R \sin 20^\circ = 173.2 (0.00940 R) - \frac{9.81}{2} (0.00940 R)^2$
 $\underline{R = 2970 \text{ m}}$

WILEY

2/71

Set up x - y axes at A , target at B :



$$\left. \begin{aligned} x\text{-eq.} : x_B &= (v_0 \cos 30^\circ)t \\ y\text{-eq.} : y_B &= (v_0 \sin 30^\circ)t - \frac{1}{2}gt^2 \end{aligned} \right\}$$

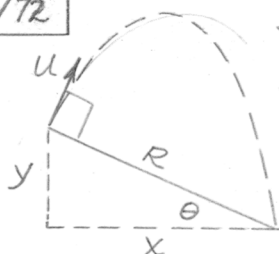
$$\text{For } x_B = 3.5 \text{ m}, y_B = -0.100 \text{ m} : \begin{cases} v_0 = 6.15 \text{ m/s} \\ t = 0.658 \text{ s} \end{cases}$$

$$\text{For } x_B = 4.1 \text{ m}, y_B = -0.100 \text{ m} : \begin{cases} v_0 = 6.68 \text{ m/s} \\ t = 0.709 \text{ s} \end{cases}$$

So the range is $6.15 \leq v_0 \leq 6.68 \text{ m/s}$

WILEY

2/72



$$y = R \sin \theta = -u t \cos \theta + \frac{1}{2} g t^2$$

$$x = R \cos \theta = u t \sin \theta$$

Eliminate t and get

$$R \sin \theta = -R \frac{\cos \theta}{\sin \theta} \cos \theta + \frac{1}{2} g \left(\frac{R \cos \theta}{u \sin \theta} \right)^2$$

$$\frac{1}{\sin \theta} = \frac{g}{2} \frac{R}{u^2 \tan^2 \theta}$$

$$R = \frac{2u^2 \tan^2 \theta}{g \sin \theta} = \frac{2u^2}{g} \tan \theta \sec \theta$$

WILEY

2/73

With x - y coordinates, origin at A: 

$$x = x_0 + v_{x_0} t \quad @ \quad B: \quad 360 = 0 + (100 \cos \alpha) t_f \quad (1)$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \quad @ \quad B: \quad -80 = 0 + (100 \sin \alpha) t_f - \frac{1}{2} (32.2) t_f^2 \quad (2)$$

Simultaneous solutions of (1) & (2):

$$\begin{cases} t_f = 4.03 \text{ sec}, & \alpha = 26.8^\circ \end{cases} \quad (a)$$

$$\begin{cases} t_f = 5.68 \text{ sec}, & \alpha = 50.7^\circ \end{cases} \quad (b)$$

Check at corner $[(x, y) = (280, 0)]$:

$$(a) \quad t_c = \frac{280}{100 \cos 26.8^\circ} = 3.14 \text{ sec}$$

$$y_c = 100 \sin 26.8^\circ (3.14) - \frac{32.2}{2} (3.14)^2 = -16.94 \text{ ft}$$

So conditions (a) are not possible.

$$(b) \quad t_c = \frac{280}{100 \cos 50.7^\circ} = 4.42 \text{ sec}$$

$$y_c = 100 \sin 50.7^\circ (4.42) - \frac{32.2}{2} (4.42)^2 = 27.5 \text{ ft}$$

Conditions (b) result in clearance at corner

$$\text{Ans.} \quad \underline{\alpha = 50.7^\circ}$$

2/74

$t \leq 2 \text{ sec}$

$\begin{cases} a_x = 2g \cos 60^\circ = g \\ a_y = 2g \sin 60^\circ - g = 0.732g \end{cases}$

$\begin{cases} v_x = v_{x0} + a_x t = gt, & v_x(2) = 64.4 \frac{\text{ft}}{\text{sec}} \\ v_y = v_{y0} + a_y t = 100 + 0.732gt, & v_y(2) = 147.1 \frac{\text{ft}}{\text{sec}} \end{cases}$

$\begin{cases} x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 = \frac{1}{2}gt^2, & x(2) = 64.4 \text{ ft} \\ y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 = 100t + \frac{1}{2}(0.732g)t^2 \end{cases}$

$t > 2 \text{ sec}$

$\begin{cases} a_x = 0 \\ a_y = -g \end{cases}$

$y(2) = 247 \text{ ft}$

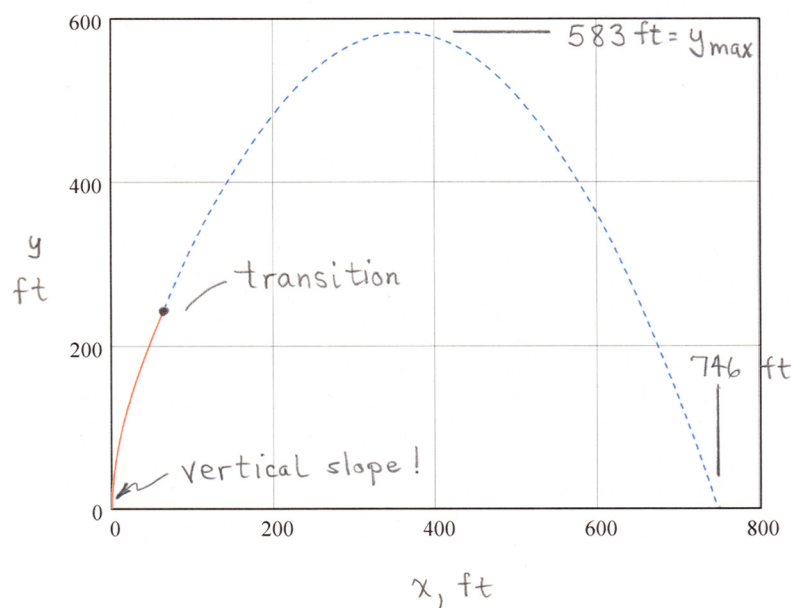
$\begin{cases} v_x = v_x(2) = 64.4 \text{ ft/sec} \\ v_y = v_y(2) - g(t-2) = 147.1 - g(t-2) = 212 - gt \end{cases}$

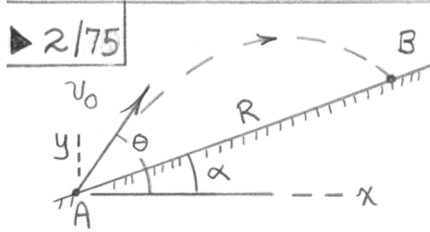
$\begin{cases} x = x(2) + v_x(2)(t-2) = 64.4 + 64.4(t-2) \\ \quad \quad \quad = -64.4 + 64.4t \\ y = y(2) + v_y(2)(t-2) - \frac{1}{2}g(t-2)^2 \\ \quad \quad \quad = 247 + 147.1(t-2) - 16.1(t-2)^2 \\ \quad \quad \quad = -111.5 + 212t - 16.1t^2 \end{cases}$

$v_y \rightarrow 0 @ t = 6.57 \text{ sec} \Rightarrow y = 583 \text{ ft} = h$

Set $y = 0$ to obtain $t_f = 12.59 \text{ sec}$

$x(12.59) = d = 746 \text{ ft}$





$$x = x_0 + v_{x_0} t \quad @ \quad B: \quad R \cos \alpha = (v_0 \cos \theta) t_f$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \quad @ \quad B: \quad R \sin \alpha = (v_0 \sin \theta) t_f - \frac{1}{2} g t_f^2$$

$$x\text{-eq:} \quad t_f = \frac{R \cos \alpha}{v_0 \cos \theta}$$

$$y\text{-eq:} \quad R \sin \alpha = (v_0 \sin \theta) \left(\frac{R \cos \alpha}{v_0 \cos \theta} \right) - \frac{1}{2} g \left(\frac{R \cos \alpha}{v_0 \cos \theta} \right)^2$$

$$\Rightarrow R = \frac{2 v_0^2 \cos^2 \theta}{g \cos \alpha} (\tan \theta - \tan \alpha)$$

$$\frac{dR}{d\theta} = 0: \quad \frac{4 v_0^2 \cos \theta (-\sin \theta)}{g \cos \alpha} (\tan \theta - \tan \alpha) + \frac{2 v_0^2 \cos^2 \theta}{g \cos \alpha} \frac{1}{\cos^2 \theta} = 0$$

$$\frac{2 v_0^2}{g \cos \alpha} \left[2 \cos \theta \sin \theta (\tan \alpha - \tan \theta) + 1 \right] = 0$$

$$\Rightarrow 2 \cos \theta \sin \theta \left(\tan \alpha - \frac{\sin \theta}{\cos \theta} \right) + 1 = 0$$

$$(2 \cos \theta \sin \theta) \tan \alpha - 2 \sin^2 \theta + 1 = 0$$

$$\sin 2\theta \tan \alpha - 2 \left(\frac{1}{2} - \frac{1}{2} \cos 2\theta \right) + 1 = 0$$

$$\sin 2\theta \tan \alpha + \cos 2\theta = 0$$

$$\tan 2\theta = -\frac{1}{\tan \alpha}$$

$$2\theta = \tan^{-1} \left(-\frac{1}{\tan \alpha} \right) = 180^\circ - \tan^{-1} \left(\frac{1}{\tan \alpha} \right)$$

$$= 180^\circ - (90^\circ - \alpha) = 90^\circ + \alpha$$

$$\therefore \theta = \frac{90^\circ + \alpha}{2}$$

Specific results :

$$\begin{cases} \alpha = 0^\circ, & \theta = 45^\circ \\ \alpha = 30^\circ, & \theta = 60^\circ \\ \alpha = 45^\circ, & \theta = 67.5^\circ \end{cases}$$

$$\begin{aligned} \boxed{2/76} \quad \underline{a} &= -k\underline{v} - g\underline{j} \quad \left. \begin{aligned} \therefore a_x &= -kv_x \\ a_x \underline{i} + a_y \underline{j} &= -k(v_x \underline{i} + v_y \underline{j}) - g\underline{j} \end{aligned} \right\} \begin{aligned} a_y &= -kv_y - g \end{aligned} \\ \underline{x}: \quad a_x &= \frac{dv_x}{dt} = -kv_x \\ \int_{v_{x0}}^{v_x} \frac{dv_x}{v_x} &= -\int_0^t k dt \Rightarrow v_x = v_{x0} e^{-kt} \\ &\text{or } \underline{v_x = (v_0 \cos \theta) e^{-kt}} \end{aligned}$$

$$\begin{aligned} v_x &= \frac{dx}{dt} = v_{x0} e^{-kt} \\ \int_0^x dx &= \int_0^t v_{x0} e^{-kt} dt \\ x &= \frac{v_{x0}}{k} [1 - e^{-kt}] = \underline{\underline{\frac{v_0 \cos \theta}{k} [1 - e^{-kt}]}} \end{aligned}$$

$$\begin{aligned} \underline{y}: \quad a_y &= \frac{dv_y}{dt} = -kv_y - g \\ \int_{v_{y0}}^{v_y} \frac{dv_y}{kv_y + g} &= -\int_0^t dt \\ \Rightarrow v_y &= \left[v_{y0} + \frac{g}{k} \right] e^{-kt} - \frac{g}{k} = \underline{\underline{\left[v_0 \sin \theta + \frac{g}{k} \right] e^{-kt} - \frac{g}{k}}} \end{aligned}$$

$$\begin{aligned} v_y &= \frac{dy}{dt} = \left[v_{y0} + \frac{g}{k} \right] e^{-kt} - \frac{g}{k} \\ \int_0^y dy &= \int_0^t \left\{ \left[v_{y0} + \frac{g}{k} \right] e^{-kt} - \frac{g}{k} \right\} dt \\ y &= \underline{\underline{\frac{1}{k} \left[v_0 \sin \theta + \frac{g}{k} \right] [1 - e^{-kt}] - \frac{g}{k} t}} \end{aligned}$$

Terminal velocity ($t \rightarrow \infty$): $v_x \rightarrow 0$

$$\underline{\underline{v_y \rightarrow -\frac{g}{k}}}$$

$$\begin{aligned} \underline{2/77} \quad v &= r\dot{\theta} = \frac{13.5}{12} \left[45 \frac{2\pi}{60} \right] = \underline{5.30 \text{ ft/sec}} \\ a_n &= r\dot{\theta}^2 = \frac{13.5}{12} \left[45 \frac{2\pi}{60} \right]^2 = \underline{25.0 \text{ ft/sec}^2} \\ &(\text{Could have used } a_n = \frac{v^2}{r}) \end{aligned}$$

WILEY

$$\begin{aligned} \boxed{2/78} \quad & \text{For } a_t = \text{constant}, \quad v = v_0 + a_t t \\ \text{So } a_t &= \frac{v}{t} = \frac{100 (1000)}{3600} / 10 = 2.78 \text{ m/s}^2 \\ v_8 &= 2.78(8) = 22.2 \text{ m/s} \\ a_n &= \frac{v^2}{r} = \frac{22.2^2}{80} = 6.17 \text{ m/s}^2 \\ a &= \sqrt{a_t^2 + a_n^2} = \sqrt{2.78^2 + 6.17^2} = \underline{6.77 \text{ m/s}^2} \end{aligned}$$

WILEY

- | | |
|------|--|
| 2/79 | a_1 : speed is increasing , no path curvature. |
| | a_2 : speed increasing , car turning to left. |
| | a_3 : speed stationary , car turning to left. |
| | a_4 : speed decreasing , car turning to left. |
| | a_5 : speed decreasing , no path curvature . |
| | a_6 : speed decreasing , car turning to right. |

WILEY

$$\boxed{2/80} \quad a_n = \frac{v^2}{r}$$
$$\textcircled{A} \quad 0.88(9.81) = \frac{v_A^2}{16} \quad , \quad \underline{v_A = 11.75 \text{ m/s}}$$
$$\textcircled{B} \quad 0.88(9.81) = \frac{v_B^2}{21} \quad , \quad \underline{v_B = 13.46 \text{ m/s}}$$

WILEY

2/81

$$\begin{cases} v = 215 \text{ km/h or } 59.7 \text{ m/s} \\ a_t = 18 \text{ km/h per second or } 5 \text{ m/s}^2 \text{ (SLOWING)} \\ a = 3.5g = 3.5(9.81) = 34.3 \text{ m/s}^2 \end{cases}$$

$$a_n = \sqrt{a^2 - a_t^2} = \sqrt{34.3^2 - 5^2} \rightarrow a_n = 34.0 \text{ m/s}^2$$

$$a_n = \frac{v^2}{r_c} \rightarrow 34.0 = \frac{59.7^2}{r_c} \rightarrow r_c = 105 \text{ m}$$

$$\text{so... } r_{\text{TRACK}} = r_c + 0.8 = \underline{105.8 \text{ m}}$$

WILEY

$$\begin{aligned} 2/82 \quad a = a_n &= v^2/\rho, \quad v^2 = \rho a_n \\ \rho &= 98 + 2 = 100 \text{ m}, \quad v^2 = 100(0.4)(9.81) \\ v &= 19.81 \frac{\text{m}}{\text{s}} \quad \text{or} \quad 71.3 \frac{\text{km}}{\text{h}} \end{aligned}$$

WILEY

2/83

$$\Delta\theta = (25-15)\frac{\pi}{180} = 0.1745 \text{ rad}$$

$$v_B = 14 \text{ ft/sec}$$

$$v_A = 12 \text{ ft/sec}$$

$$v_{av} = \frac{12+14}{2} = 13 \text{ ft/sec}$$

$$a_n = \frac{\Delta v_n}{\Delta t} = \frac{v_{av}(\Delta\theta)}{\Delta t} = \frac{13(0.1745)}{2.62-2.4} = \underline{10.31 \frac{\text{ft}}{\text{sec}^2}}$$

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{14-12}{0.22} = \underline{9.09 \frac{\text{ft}}{\text{sec}^2}}$$

WILEY

2/84

$$v_{\max} = 40 \text{ km/h At } s = 60 \text{ m}$$

THE MAXIMUM ACCELERATION FOR THE SPRINTER WILL OCCUR THE INSTANT HE REACHES HIS TOP SPEED AT THE 60-m MARK. AT THIS LOCATION HE WILL EXPERIENCE HIS MAXIMUM TANGENTIAL COMPONENT OF ACCELERATION AS WELL AS HIS MAXIMUM NORMAL COMPONENT OF ACCELERATION.

$$v_f^2 = v_o^2 + 2a_t \Delta s \rightarrow \left(\frac{40}{3.6}\right)^2 = 2a_t(60) \rightarrow a_t = 1.029 \text{ m/s}^2$$

$$a_n = \frac{v_f^2}{\rho} = \frac{\left(\frac{40}{3.6}\right)^2}{39.9} \rightarrow a_n = 3.09 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{1.029^2 + 3.09^2} \rightarrow a = 3.26 \text{ m/s}^2$$

$$\begin{aligned}
 \underline{2/85} \quad v &= v_0 + a_t t : \quad \frac{50}{3.6} = \frac{100}{3.6} + 12 a_t \\
 a_t &= -1.157 \text{ m/s}^2 \\
 a &= \sqrt{a_t^2 + a_n^2} : \quad 2 = \sqrt{1.157^2 + a_n^2} \\
 a_n &= 1.631 \text{ m/s}^2 \\
 v_6 &= v_0 + a_t t = \frac{100}{3.6} - 1.157(6) = 20.8 \text{ m/s} \\
 \text{From } a_n &= \frac{v^2}{r}, \quad r = \frac{v^2}{a_n} = \frac{20.8^2}{1.631} \\
 &= \underline{266 \text{ m}}
 \end{aligned}$$

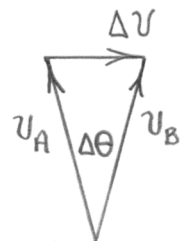
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$$2/86 \quad v_A = v_B = v = 2 \text{ m/s}$$

$$\Delta v = 2v \sin \frac{\Delta \theta}{2} = 4 \sin \frac{\Delta \theta}{2} \text{ m/s}$$

$$\Delta t = \frac{r \Delta \theta}{v} = \frac{0.8 \Delta \theta}{2} = 0.4 \Delta \theta \text{ s}$$

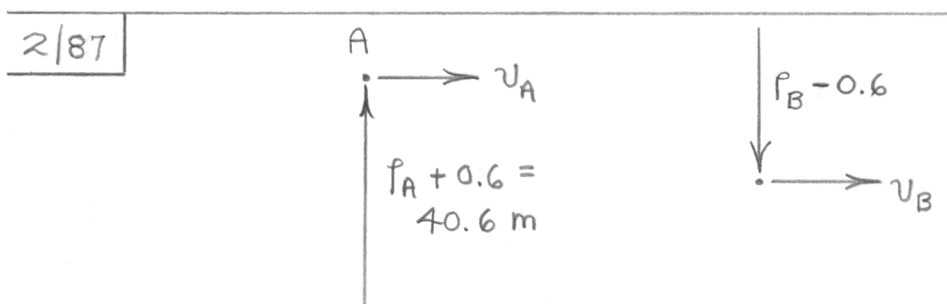
$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{4 \sin \frac{\Delta \theta}{2}}{0.4 \Delta \theta} = 5 \frac{\sin \frac{\Delta \theta}{2}}{\Delta \theta/2}$$



	$\Delta \theta^\circ$	$\frac{\Delta \theta}{2}^\circ$	$\frac{\Delta \theta}{2} \text{ rad}$	$\sin \frac{\Delta \theta}{2}$	$a_{av}, \text{ m/s}^2$	% diff.
(a)	30°	15°	0.262	0.259	<u>4.94</u>	1.138
(b)	15°	7.5°	0.1309	0.1305	<u>4.99</u>	0.285
(c)	5°	2.5°	0.0436	0.0436	<u>4.998</u>	0.0317

$$a_n = \frac{v^2}{r} = \frac{2^2}{0.8} = \underline{5 \text{ m/s}^2}$$

WILEY



$$v_A = \frac{50}{3.6} = 13.89 \text{ m/s}, \quad v_B = \frac{100}{3.6} = 27.8 \text{ m/s}$$

$$v_B = v_A + a_t t, \quad a_t = \frac{27.8 - 13.89}{10} = 1.389 \text{ m/s}^2$$

$$\text{At A: } a_n = \frac{v^2}{r} = \frac{13.89^2}{40.6} = 4.75 \text{ m/s}^2$$

$$\text{Total: } a_A = \sqrt{4.75^2 + 1.389^2} = 4.95 \text{ m/s}^2$$

$$\text{At B: For } a_A = a_B = 4.95 \text{ m/s}^2,$$

$$4.95 = \sqrt{1.389^2 + a_n^2}$$

$$a_n = 4.75 = \frac{27.8^2}{r_B - 0.6}, \quad r_B = 163.0 \text{ m}$$

WILEY

$$\underline{2/88} \quad a_t = \frac{v_f - v_i}{\Delta t} = \frac{6 - 3}{2} = 1.5 \text{ m/s}^2$$

Halfway through time interval, $v = 4.5 \text{ m/s}$

$$a_{p_1} = \sqrt{a_t^2 + a_n^2} = \sqrt{1.5^2 + \left(\frac{4.5^2}{0.060}\right)^2}$$

$$= \underline{338 \text{ m/s}^2} \quad (34.4g!)$$

$$a_{p_2} = a_t = \underline{1.5 \text{ m/s}^2}$$

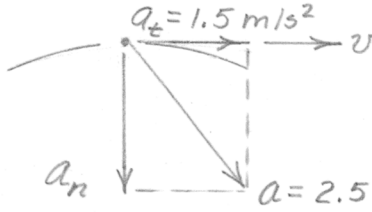
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$$\begin{aligned} \underline{2/89} \quad v &= r\dot{\theta} = (R \cos \gamma) \dot{\theta} \\ &= 6371(10^3) \cos 40^\circ (0.7292 \cdot 10^{-4}) \\ &= \underline{356 \text{ m/s}} \end{aligned}$$

$$\begin{aligned} a &= a_n = r\dot{\theta}^2 = R \cos \gamma \dot{\theta}^2 \\ &= 6371(10^3) \cos 40^\circ (0.7292 \cdot 10^{-4})^2 \\ &= \underline{0.0260 \text{ m/s}^2} \end{aligned}$$

WILEY

2/90



$a_t = 1.5 \text{ m/s}^2$
 $a^2 = a_n^2 + a_t^2$
 $a_n = \frac{v^2}{\rho} = \sqrt{a^2 - a_t^2}$
 $a = 2.5 \text{ m/s}^2$

$$v^2 = 200 \sqrt{2.5^2 - 1.5^2} = 200(2) = 400 \text{ m}^2/\text{s}^2$$

$$v = 20 \text{ m/s} \text{ or } 20(3.6) = \underline{72 \text{ km/h}}$$

WILEY

2/91

$$a_t = 20/3.6 = 5.56 \text{ m/s}^2$$

$$a^2 = a_n^2 + a_t^2, \quad a_n^2 = 3(9.81)^2 - 5.56^2 = 835.2$$

$$a_n = 28.90 \text{ m/s}^2$$

$$a_n = v^2/\rho, \quad \rho = \frac{(800/3.6)^2}{28.90} = 1709 \text{ m}$$

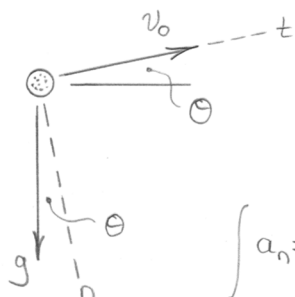
WILEY

2/92

$$v_0 = 165 \text{ mph or } 242 \text{ ft/sec}$$

$$\theta = 12^\circ$$

At LAUNCH:

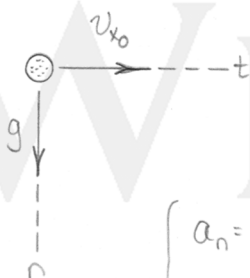


$$\begin{cases} a_t = -g \sin \theta = -32.2 \sin 12^\circ \\ a_t = -6.69 \text{ ft/sec}^2 \end{cases}$$

$$\begin{cases} a_n = \frac{v_0^2}{\rho} = g \cos \theta \rightarrow \frac{242^2}{\rho} = 32.2 \cos 12^\circ \\ \rho = 1859 \text{ ft} \end{cases}$$

At APEX:

$$v_{x_0} = v_0 \cos \theta = 242 \cos 12^\circ = 237 \text{ ft/sec}$$

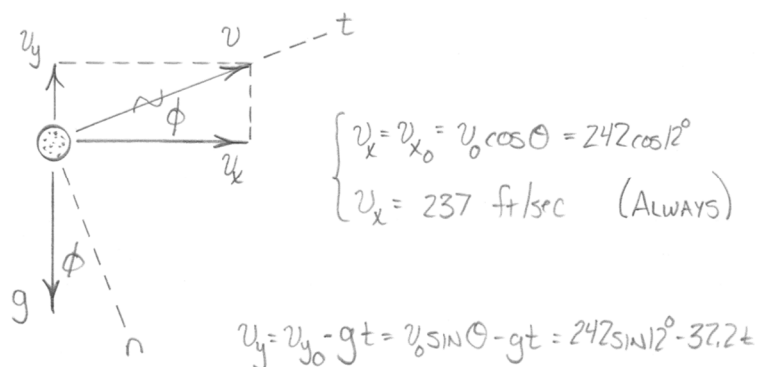


$$a_t = 0$$

$$\begin{cases} a_n = \frac{v_{x_0}^2}{\rho} = g \rightarrow \frac{237^2}{\rho} = 32.2 \\ \rho = 1740 \text{ ft} \end{cases}$$

*2/93 $v_0 = 165 \text{ mph OR } 242 \text{ ft/sec}$ $\Theta = 12^\circ$ $\rho = 1800 \text{ ft}$

ARBITRARY LOCATION ALONG THE FLIGHT OF THE BALL:



$$\tan \phi = \frac{u_y}{u_x} = \frac{242 \sin 12^\circ - 32.2t}{237} = 0.213 - 0.1360t$$

$$a_n = g \cos \phi = \frac{v^2}{\rho} \rightarrow 32.2 \cos \phi = \frac{v_x^2 + v_y^2}{1800}$$

SUBSTITUTE FOR u_x AND u_y INTO a_n AND SOLVE NUMERICALLY...

$$\begin{cases} \phi = 8.59^\circ \\ u_y = 35.7 \text{ ft/sec} \\ t = 0.452 \text{ sec} \end{cases} \quad \text{AND} \quad \begin{cases} \phi = -8.59^\circ \\ u_y = -35.7 \text{ ft/sec} \\ t = 2.67 \text{ sec} \end{cases}$$

2/94 The radius of Jupiter is

$$R = \frac{139\,822}{2} (10^3) = 6.99 (10^7) \text{ m}$$

$$\text{So } r = R+h = 6.99 (10^7) + 10^6 \text{ m} = 7.09 (10^7) \text{ m}$$

From the gravitational law

$$a_n = g = g_0 \frac{R^2}{(R+h)^2} = g_0 \frac{R^2}{r^2} = 24.79 \frac{[6.99 (10^7)]^2}{[7.09 (10^7)]^2}$$

$$= 24.1 \text{ m/s}^2$$

$$\text{From } a_n = \frac{v^2}{r}, \quad v^2 = a_n r = (24.1)(7.09 \cdot 10^7)$$

$$v = 41300 \text{ m/s} \quad \text{or} \quad v = 148800 \text{ km/h}$$

WILEY

$$\boxed{2/95} \quad \left\{ \begin{array}{l} \rho_A = 300' \\ a_A = 0.60g \end{array} \right. \quad \left\{ \begin{array}{l} \rho_B = 320' \\ a_B = 0.76g \end{array} \right. \quad \begin{array}{l} \text{BOTH TRAVEL AT CONSTANT SPEED.} \\ \theta = 60^\circ = \frac{\pi}{3} \text{ rad} \end{array}$$

$$\left\{ \begin{array}{l} a_{A_n} = a_A = \frac{v_A^2}{\rho_A} \rightarrow 0.60(32.2) = \frac{v_A^2}{300} \rightarrow v_A = 76.1 \text{ ft/sec} \\ a_{B_n} = a_B = \frac{v_B^2}{\rho_B} \rightarrow 0.76(32.2) = \frac{v_B^2}{320} \rightarrow v_B = 88.5 \text{ ft/sec} \end{array} \right.$$

$$\left\{ \begin{array}{l} t_A = \frac{s_A}{v_A} = \frac{\rho_A \theta}{v_A} = \frac{300 \frac{\pi}{3}}{76.1} \rightarrow t_A = 4.13 \text{ sec} \\ t_B = \frac{s_B}{v_B} = \frac{\rho_B \theta}{v_B} = \frac{320 \frac{\pi}{3}}{88.5} \rightarrow t_B = 3.79 \text{ sec} \end{array} \right.$$

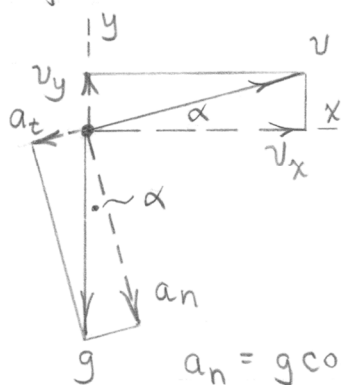
CAR B CROSSES DD FIRST.

$$\delta = v_A(t_A - t_B) = 76.1(4.13 - 3.79) \rightarrow \underline{\delta = 25.9 \text{ ft}}$$

$$\begin{aligned} \boxed{2/96} \quad v^2 &= v_0^2 + 2a_t \Delta s, \quad 18^2 = 2^2 + 2a_t(8) \\ a_t &= 20 \text{ m/s}^2 \\ a_n &= v^2/r = 3^2/0.150 = 60 \text{ m/s}^2 \\ a &= \sqrt{a_t^2 + a_n^2} = \sqrt{20^2 + 60^2} = \underline{63.2 \text{ m/s}^2} \end{aligned}$$

WILEY

2/97 The time t_{up} to apex is found from
 $v_y = v_{y0} - gt: 0 = 80 \sin 35^\circ - 32.2 t_{up}$,
 $t_{up} = 1.425 \text{ sec}$. So $t = 1 \text{ sec}$ is before
apex and $t = 2 \text{ sec}$ is after.



(a) $t = 1 \text{ sec}$

$$v_x = 80 \cos 35^\circ = 65.5 \text{ ft/sec}$$

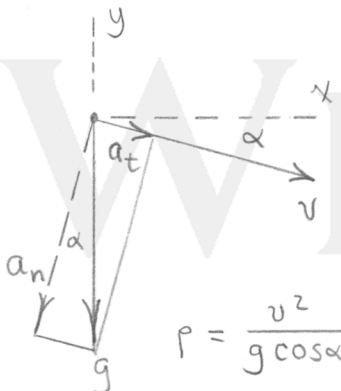
$$v_y = 80 \sin 35^\circ - 32.2(1) = 13.69 \text{ ft/sec}$$

$$v = \sqrt{65.5^2 + 13.69^2} = 66.9 \text{ ft/sec}$$

$$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = 11.80^\circ$$

$$a_n = g \cos \alpha = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{g \cos \alpha}$$

$$\text{or } r = \frac{66.9^2}{32.2 \cos 11.80^\circ} = 142.2 \text{ ft}; a_t = -g \sin \alpha = -6.58 \text{ ft/sec}^2$$



(b) $t = 2 \text{ sec}$

$$v_x \text{ remains at } 65.5 \text{ ft/sec}$$

$$v_y = 80 \sin 35^\circ - 32.2(2) = -18.51 \text{ ft/sec}$$

$$v = \sqrt{65.5^2 + 18.51^2} = 68.1 \text{ ft/sec}$$

$$\alpha = \tan^{-1} \frac{18.51}{65.5} = 15.78^\circ$$

$$r = \frac{v^2}{g \cos \alpha} = \frac{68.1^2}{32.2 \cos 15.78^\circ} = 149.7 \text{ ft}$$

$$a_t = g \sin \alpha = 32.2 \sin 15.78^\circ = 8.75 \text{ ft/sec}^2$$

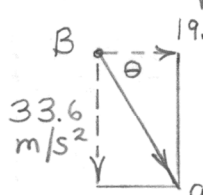
2/98 (a) $a = 2g \rightarrow (19.62 \text{ m/s}^2) \quad \underline{\theta_x = 0}$

(b) $v_B^2 = v_A^2 + 2a_t(s_B - s_A)$
 $= 0^2 + 2(2 \cdot 9.81)(3), \quad v_B = 10.85 \text{ m/s}$

$a_n = \frac{v_B^2}{\rho} = \frac{10.85^2}{3.5} = 33.6 \text{ m/s}^2$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{19.62^2 + 33.6^2} = 38.9 \text{ m/s}^2$

$\theta = \tan^{-1}\left(\frac{33.6}{19.62}\right) = 59.7^\circ$
 $\underline{\theta_x = -59.7^\circ}$

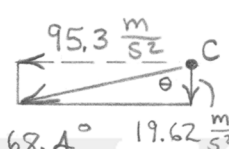


(c) $v_c^2 = v_A^2 + 2a_t(s_c - s_A)$
 $= 0^2 + 2(2 \cdot 9.81)\left[3 + 3.5\frac{\pi}{2}\right], \quad v_c = 18.26 \text{ m/s}$

$a_n = \frac{v_c^2}{\rho} = \frac{18.26^2}{3.5} = 95.3 \text{ m/s}^2$

$a = \sqrt{19.62^2 + 95.3^2} = 97.3 \text{ m/s}^2$

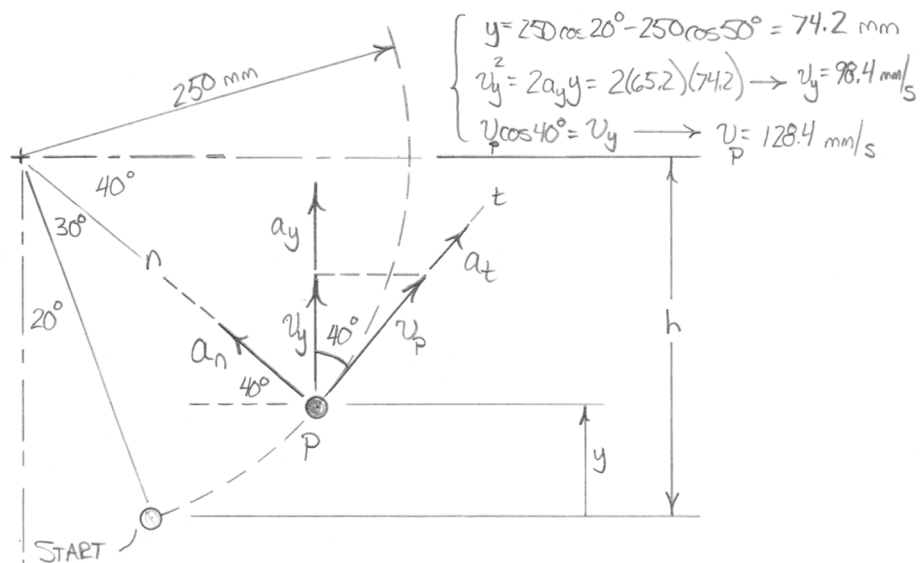
$\theta = \tan^{-1}\left(\frac{95.3}{19.62}\right) = 78.4^\circ \Rightarrow \underline{\theta_x = -168.4^\circ}$



2/99 $v_{\max} = 175 \text{ mm/s}$ VERTICAL MOTION OF P MUST MATCH THE GUIDE.

$$\begin{cases} h = 250 \cos 20^\circ \\ h = 235 \text{ mm} \end{cases} \quad v_{\max}^2 = v_o^2 + 2a_y h \rightarrow 175^2 = 2a_y(235)$$

$$a_y = 65.2 \text{ mm/s}^2$$



$$a_n = \frac{v_p^2}{r} = \frac{178.4^2}{250} \rightarrow a_n = 66.0 \text{ mm/s}^2$$

a_t AND a_n MUST COMBINE TO GIVE a_y .

$$a_t \cos 40^\circ + a_n \sin 40^\circ = a_y \rightarrow a_t \cos 40^\circ + 66.0 \sin 40^\circ = 65.2$$

$$a_t = 29.7 \text{ mm/s}^2$$

2/100

Accel. directed from A to N is

$$a = g = g_0 \left(\frac{R}{R+h} \right)^2$$

$$= 9.821 \left(\frac{6371}{16000} \right)^2 = 1.557 \text{ m/s}^2$$

$$a_n = 1.557 \cos \theta = 1.557 (0.866)$$

$$= 1.348 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho}, \rho = \frac{(17970)^2}{1.348(10^{-3})(3600)^2} = 18480 \text{ km}$$

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2/101 $y - 10 = kx^2$ $k = -\frac{1}{10} \text{ in.}^{-1}$

So $y = 10(1 - \frac{x^2}{100}) \text{ in.}$

$\dot{x} = 15 \text{ in./sec}$, $\ddot{x} = 0$

$\dot{y} = -x\dot{x}/5$, $\ddot{y} = -\dot{x}^2/5$

So $a = a_y = \ddot{y} = -\frac{15^2}{5} = -45 \text{ in./sec}^2$

$\frac{dy}{dx} = -\frac{x}{5} = -\frac{6}{5}$ @ $x = 6 \text{ in.}$

$\theta = \tan^{-1} \frac{6}{5} = 50.2^\circ$

$a_n = a \cos \theta = 45 \cos 50.2^\circ = 28.8 \text{ in./sec}^2$

$a_t = a \sin \theta = 45 \sin 50.2^\circ = 34.6 \text{ in./sec}^2$

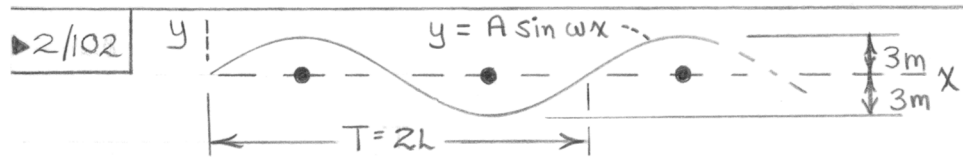
For $x = 6 \text{ in.}$, $\dot{y} = -6(15)/5 = -18 \text{ in./sec}$

$v = \sqrt{15^2 + 18^2} = 23.4 \text{ in./sec}$

$a_n = \frac{v^2}{\rho}$, $\rho = \frac{23.4^2}{28.8} = 19.06 \text{ in.}$

Check: $\rho_{xy} = \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2} = \frac{[1 + (-6/5)^2]^{3/2}}{-1/5}$

$= -19.06 \text{ in.} \checkmark$



$$y = A \sin wx, \text{ where } A = 3m \text{ \& } w = \frac{2\pi}{T}$$

$$\text{Radius of curvature } \rho = \frac{[1 + (\frac{dy}{dx})^2]^{3/2}}{d^2y/dx^2}$$

$$\frac{dy}{dx} = Aw \cos wx, \quad \frac{d^2y}{dx^2} = -Aw^2 \sin wx$$

$$\text{Set } \frac{d\rho}{dx} = 0 \text{ to show that } |\rho| \text{ is a min @ } x = \frac{T}{4}$$

$$\neq x = \frac{3T}{4}.$$

$$\rho_{\min} = \frac{[1 + \{A \frac{2\pi}{T} \cos(\frac{2\pi}{T} \cdot \frac{T}{4})\}^2]^{3/2}}{+ A (\frac{2\pi}{T})^2 \sin(\frac{2\pi}{T} \cdot \frac{T}{4})} = \frac{T^2}{4\pi^2 A}$$

$$a_n = \frac{v^2}{\rho} : 0.7(9.81) = \frac{(80/3.6)^2}{T^2/(4\pi^2 \cdot 3)}$$

$$T = 92.3 \text{ m} = 2L, \quad L = 46.1 \text{ m}$$

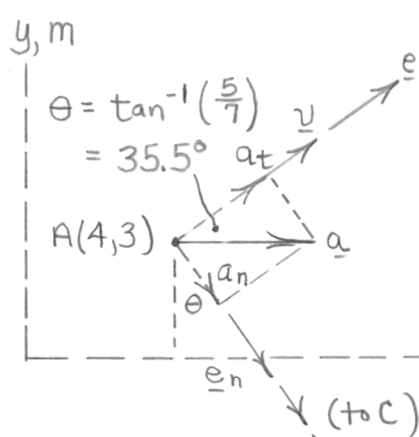
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$$\begin{cases} x = 2t^2 + 3t - 1; & \dot{x} = 4t + 3; & \ddot{x} = 4 \\ y = 5t - 2; & \dot{y} = 5; & \ddot{y} = 0 \end{cases}$$

At $t = 1\text{ s}$:

$$\begin{cases} x = 4\text{ m}; & \dot{x} = 7\text{ m/s}; & \ddot{x} = 4\text{ m/s}^2 \\ y = 3\text{ m}; & \dot{y} = 5\text{ m/s}; & \ddot{y} = 0 \end{cases}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{7^2 + 5^2} = 8.60\text{ m/s}$$



$\theta = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$

$\underline{e}_t = \cos 35.5^\circ \underline{i} + \sin 35.5^\circ \underline{j} = 0.814 \underline{i} + 0.581 \underline{j}$

$\underline{e}_n$ is $\perp$ $\underline{e}_t$, so

$\underline{e}_n = 0.581 \underline{i} - 0.814 \underline{j}$

$a_n = \underline{a} \cdot \underline{e}_n = 4(0.581) = 2.32\text{ m/s}^2$

From $a_n = v^2/\rho$, $\rho = \frac{v^2}{a_n}$

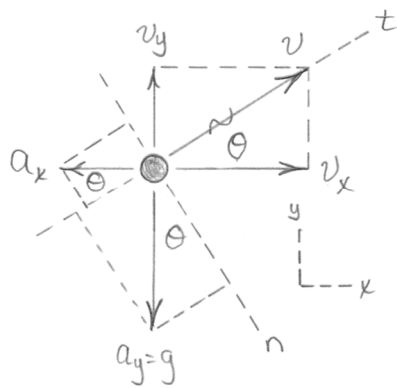
or $\rho = \frac{8.60^2}{2.32} = 31.8\text{ m}$

$$x_c = x_A + \rho \sin \theta = 4 + 31.8 \sin 35.5^\circ = \underline{22.5\text{ m}}$$

$$y_c = y_A - \rho \cos \theta = 3 - 31.8 \cos 35.5^\circ = \underline{-22.9\text{ m}}$$

*2/104 $v_0 = 120 \text{ m/s}$ $\theta_0 = 35^\circ$ $a_x = 5 \text{ m/s}^2$ (CONSTANT)

• ARBITRARY POSITION IN FLIGHT:



$$\begin{cases} v_x = v_0 \cos \theta_0 - a_x t \\ x = v_0 \cos \theta_0 t - \frac{1}{2} a_x t^2 \end{cases}$$

$$\begin{cases} v_y = v_0 \sin \theta_0 - g t \\ y = v_0 \sin \theta_0 t - \frac{1}{2} g t^2 \end{cases}$$

$$\theta = \tan^{-1} \left(\frac{v_y}{v_x} \right) \quad \& \quad a_t = -g \sin \theta - a_x \cos \theta$$

$$a_n = g \cos \theta - a_x \sin \theta \quad \& \quad a_n = \frac{v^2}{\rho} \rightarrow \rho = \frac{v_x^2 + v_y^2}{g \cos \theta - a_x \sin \theta}$$

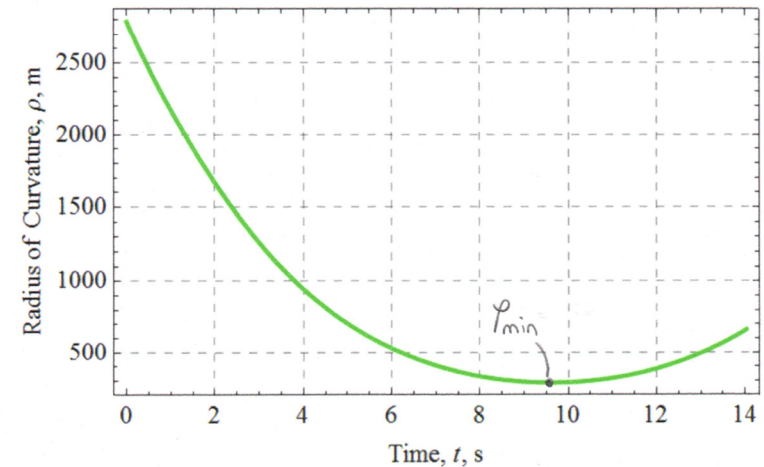
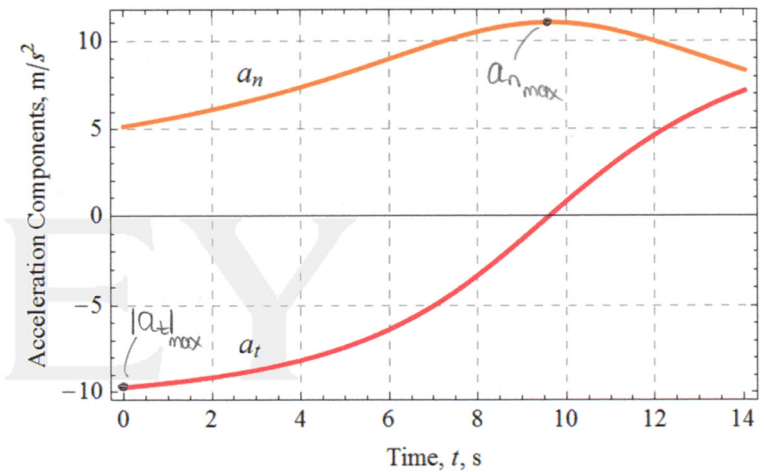
• FLIGHT TIME:

$$y_f' = y_0' + v_{y0} t_f - \frac{1}{2} g t_f^2 \rightarrow 0 = 120 \sin 35^\circ t_f - \frac{1}{2} (9.81) t_f^2$$

$$t_f = 14.03 \text{ s}$$

EVALUATE v_x AND v_y , COMPUTE θ , a_t , a_n , AND ρ AT EACH TIME INCREMENT FROM $t=0$ TO $t_f = 14.03 \text{ s}$, AND PLOT THE VALUES.

$$\left. \begin{aligned} &\text{FOR } \varphi_{\min}, \text{ SET } \frac{d\varphi}{dt} = 0 \text{ SO... } \underline{\varphi_{\min} = 288^\circ \text{ @ } t = 9.62 \text{ s}} \\ &\text{FOR } a_{n\max}, \text{ SET } \frac{da_n}{dt} = 0 \text{ SO... } \underline{a_{n\max} = 11.01 \text{ m/s}^2 \text{ @ } t = 9.62 \text{ s}} \\ &\text{MAX } a_t \text{ OCCURS AT } t=0 \dots \underline{|a_t|_{\max} = 9.72 \text{ m/s}^2 \text{ @ } t=0} \\ &\text{(ACTUALLY A NEGATIVE \#)} \end{aligned} \right\} \text{ SAME TIME}$$



2/105

$v = 65 \text{ mi/hr}$
 $\begin{cases} v = 65 \left(\frac{5280}{3600} \right) = 95.3 \text{ ft/sec} \\ r = \frac{100}{\sin 60^\circ} = 115.5 \text{ ft} \end{cases}$

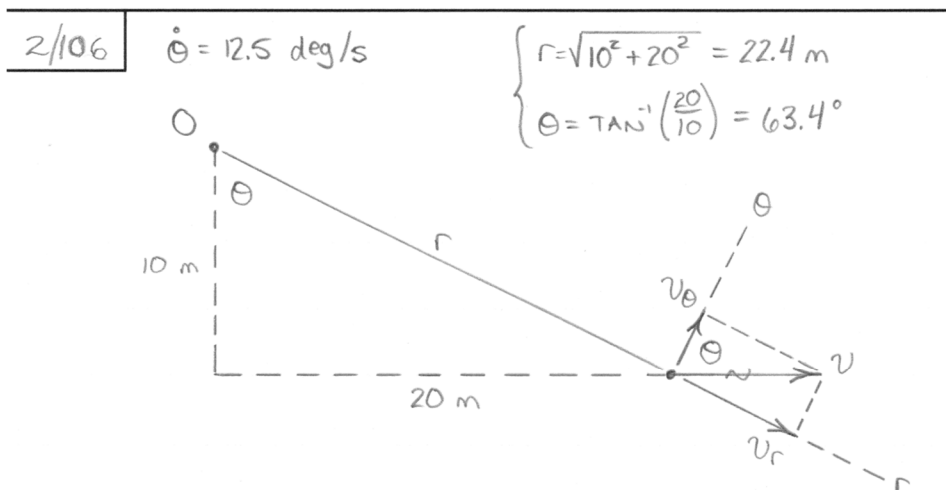
$v_r = \dot{r} = v \cos \theta = 95.3 \cos 60^\circ = \underline{47.7 \text{ ft/sec}}$

$v_\theta = r\dot{\theta} = -v \sin \theta$

$\dot{\theta} = -\frac{v \sin \theta}{r} = -\frac{95.3 \sin 60^\circ}{115.5} = -0.715 \text{ rad/sec}$

$\text{or } \dot{\theta} = -0.715 \left(\frac{180}{\pi} \right) = \underline{-41.0 \text{ deg/sec}}$

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$$v_\theta = r\dot{\theta} = v \cos \theta = v \cos 63.4^\circ = 22.4 \left(12.5 \frac{\pi}{180} \right) \rightarrow \underline{v = 10.91 \text{ m/s}}$$

$$v_r = \dot{r} = v \sin \theta = 10.91 \sin 63.4^\circ \rightarrow \underline{\dot{r} = 9.76 \text{ m/s}}$$

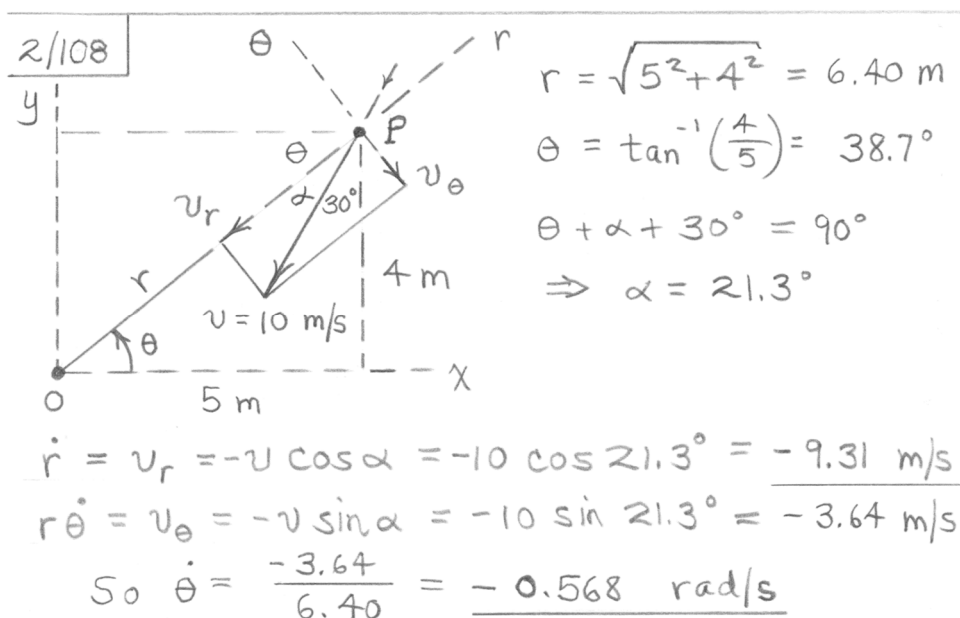
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Position	r	$\dot{r}$	$\ddot{r}$	θ	$\dot{\theta}$	$\ddot{\theta}$
A	+	-	+	+	+	+
B	+	0	+	+	+	0
C	+	+	+	+	+	-

- Notes :
- (1) $r \geq 0$, always, by definition
 - (2) $\dot{r}$ determined by inspection
 - (3) $\ddot{r}$ found from $a_r = \ddot{r} - r\dot{\theta}^2 = 0$
 - (4) $\theta \geq 0$, by definition in figure
 - (5) $\dot{\theta} > 0$ here, by inspection
 - (6) $\ddot{\theta}$ found from $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

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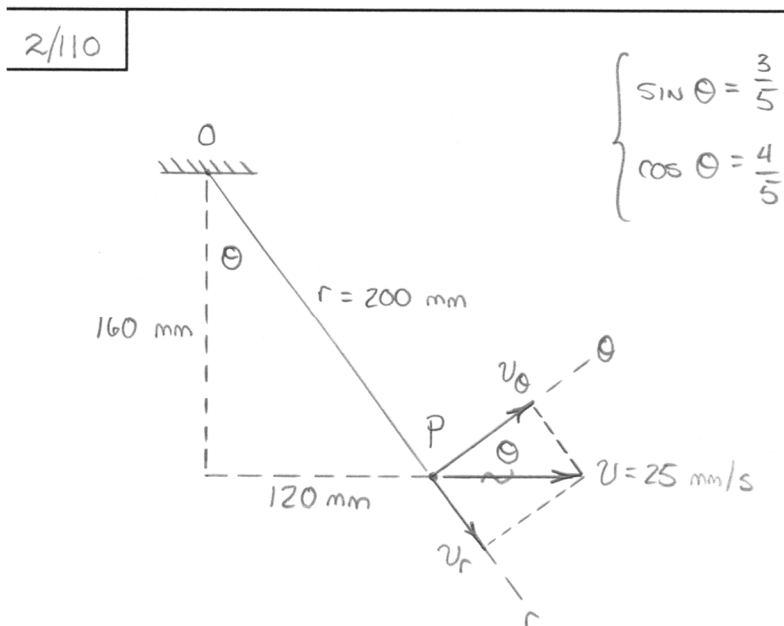
2/109 | Since $\underline{a} = \underline{0}$, any and all components of $\underline{a}$ are zero.

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 \Rightarrow \ddot{r} = 6.40(-0.568)^2$$
$$= \underline{2.07 \text{ m/s}^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \Rightarrow \ddot{\theta} = \frac{-2(-9.31)(-0.568)}{6.40}$$
$$= \underline{-1.653 \text{ rad/s}^2}$$

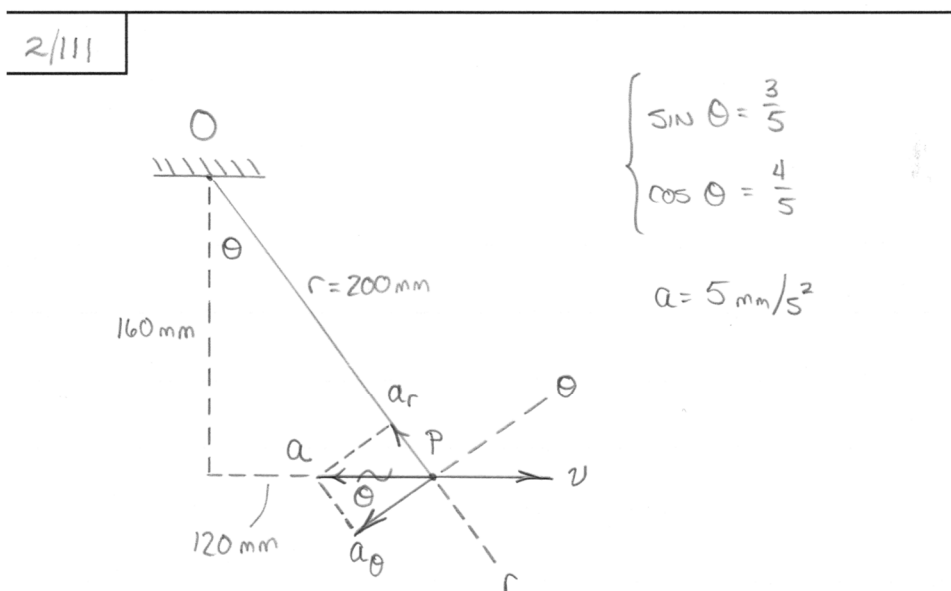
(Values of r , $\dot{r}$, and $\dot{\theta}$ from the solution to the previous problem.)

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$$v_r = \dot{r} = v \sin \theta = \frac{3}{5}(25) \rightarrow \underline{\dot{r} = 15 \text{ mm/s}}$$

$$v_\theta = r\dot{\theta} = v \cos \theta = \frac{4}{5}(25) = 200\dot{\theta} \rightarrow \underline{\dot{\theta} = 0.1 \text{ rad/s}}$$



From Prob. 2/110 ... $\dot{r} = 15 \text{ mm/s}$ and $\dot{\theta} = 0.1 \text{ rad/s}$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -a \sin \theta = -\frac{3}{5}(5) = \ddot{r} - 200(0.1)^2$$

$$\ddot{r} = -1 \text{ mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -a \cos \theta = -\frac{4}{5}(5) = 200\ddot{\theta} + 2(15)(0.1)$$

$$\ddot{\theta} = -0.035 \text{ rad/s}^2$$

$$\begin{aligned}
 \underline{2/112} \quad \underline{\underline{v}} &= \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta = 1.5 \underline{e}_r + (24+7)(5 \frac{\pi}{180}) \underline{e}_\theta \\
 &= 1.5 \underline{e}_r + 2.71 \underline{e}_\theta \quad \text{ft/sec} \\
 \underline{a} &= (\ddot{r} - r \dot{\theta}^2) \underline{e}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \underline{e}_\theta \\
 &= [-4 - 31(5 \frac{\pi}{180})^2] \underline{e}_r + [31(2 \frac{\pi}{180}) + 2(1.5)(5 \frac{\pi}{180})] \underline{e}_\theta \\
 &= -4.24 \underline{e}_r + 1.344 \underline{e}_\theta \quad \text{ft/sec}^2
 \end{aligned}$$

WILEY

2/113 $\underline{\dot{r}} \neq \underline{v}$, $\dot{r} \neq \underline{v}$, $\underline{\ddot{r}} \neq \underline{a}$, and $\ddot{r} \neq \underline{a}$ because
scalars are NEVER equal to vectors.

$\dot{r} \neq v$ because $v \neq (v_r = \dot{r})$.

$\ddot{r} \neq a$ because $\ddot{r}$ is only one part of the magnitude
of a_r (and therefore of a).

$\underline{\dot{r}} \neq \dot{r}\underline{e}_r$ because $\underline{v} = \underline{\dot{r}}$ also contains an $\underline{e}_\theta$ -Component.

$\underline{\ddot{r}} \neq \ddot{r}\underline{e}_r$ because $\underline{a} = \underline{\ddot{r}}$ contains another $\underline{e}_r$ -
Component and also an $\underline{e}_\theta$ -Component.

$\underline{\dot{r}} \neq r\dot{\theta}\underline{e}_\theta$ because $\underline{\dot{r}} = \underline{v}$ also contains an $\underline{e}_r$ -
Component.

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$$\boxed{2/114} \quad \dot{r} = 6 \text{ in./sec}, \quad \ddot{r} = -2 \text{ in./sec}^2, \quad \dot{\theta} = 10 \frac{\pi}{180} \text{ rad/s}, \quad \ddot{\theta} = 0$$

$$r = 6 \text{ ft} = 72 \text{ in.}$$

$$v_r = \dot{r} = 6 \text{ in./sec} \quad v_\theta = r\dot{\theta} = 72 \left(\frac{\pi}{18} \right) = 12.57 \text{ in./sec}$$

$$\begin{cases} a_r = \ddot{r} - r\dot{\theta}^2 = -2 - 72 \left(\frac{\pi}{18} \right)^2 = -4.19 \text{ in./sec}^2 \\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(6) \left(\frac{\pi}{18} \right) = 2.09 \text{ in./sec}^2 \end{cases}$$

$$\underline{v} = v_r \underline{e}_r + v_\theta \underline{e}_\theta \rightarrow \underline{v} = 6 \underline{e}_r + 12.57 \underline{e}_\theta \text{ in./sec}$$

$$\underline{a} = a_r \underline{e}_r + a_\theta \underline{e}_\theta \rightarrow \underline{a} = -4.19 \underline{e}_r + 2.09 \underline{e}_\theta \text{ in./sec}^2$$

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$$\textcircled{A} \quad \underline{v}_A = \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta = \underline{v} \underline{e}_r + l \Omega \underline{e}_\theta$$

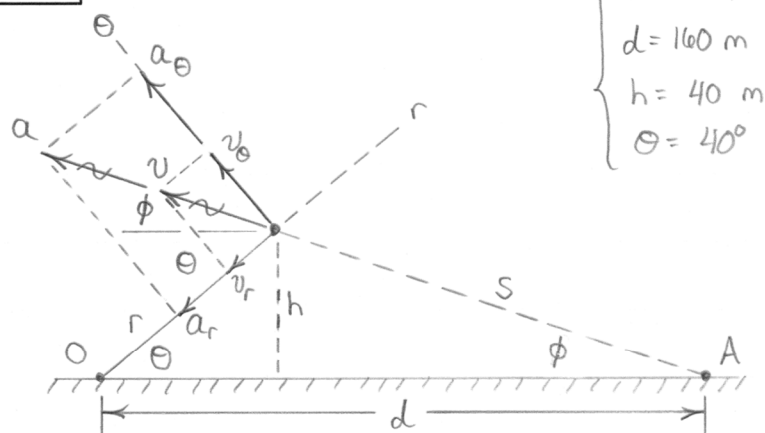
$$\underline{a}_A = (\ddot{r} - r \dot{\theta}^2) \underline{e}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \underline{e}_\theta$$

$$= -l \Omega^2 \underline{e}_r + 2v \Omega \underline{e}_\theta$$

$$\textcircled{B} \quad \underline{v}_B = 4v \underline{e}_r + 2l \Omega \underline{e}_\theta$$

$$\underline{a}_B = -2l \Omega^2 \underline{e}_r + 8v \Omega \underline{e}_\theta$$

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$$\left\{ \begin{array}{l} v = 28 \text{ m/s} \\ d = 160 \text{ m} \\ h = 40 \text{ m} \\ \theta = 40^\circ \end{array} \right.$$

$$\left\{ \begin{array}{l} r \sin \theta = h \rightarrow r \sin 40^\circ = 40 \rightarrow r = 62.2 \text{ m} \\ s^2 = r^2 + d^2 - 2rd \cos \theta = 62.2^2 + 160^2 - 2(62.2)(160) \cos 40^\circ \rightarrow s = 119.2 \text{ m} \\ \phi = \sin^{-1} \left(\frac{h}{s} \right) = \sin^{-1} \left(\frac{40}{119.2} \right) \rightarrow \phi = 19.60^\circ \quad \& \quad \theta + \phi = 59.6^\circ \\ v^2 = v_0^2 + 2as \rightarrow 28^2 = 0 + 2a(119.2) \rightarrow a = 3.29 \text{ m/s}^2 \end{array} \right.$$

$$\begin{cases} a_r = \ddot{r} - r\dot{\theta}^2 = -a \cos(\theta + \phi) = -3.29 \cos 59.6^\circ = \ddot{r} - (6.22(0.388))^2 \\ \ddot{r} = 7.71 \text{ m/s}^2 \\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = a \sin(\theta + \phi) = 3.29 \sin 59.6^\circ = 6.22\ddot{\theta} + 2(-14.17)(0.388) \\ \ddot{\theta} = 0.222 \text{ rad/s}^2 \end{cases}$$

$$\begin{aligned} 2/117 \quad \theta &= 0.8t - \frac{t^2}{20} \text{ rad} & r &= 1.6 - 0.2t \text{ m} \\ \dot{\theta} &= 0.8 - 0.1t \text{ rad/s} & \dot{r} &= -0.2 \text{ m/s} \\ \ddot{\theta} &= -0.1 \text{ rad/s}^2 & \ddot{r} &= 0 \end{aligned}$$

$$\begin{aligned} \text{At } t = 4 \text{ s : } \theta &= 2.4 \text{ rad } (137.5^\circ), \quad \dot{\theta} = 0.4 \text{ rad/s} \\ r &= 0.8 \text{ m} \end{aligned}$$

$$\begin{aligned} \underline{v} &= \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = -0.2\underline{e}_r + 0.8(0.4)\underline{e}_\theta \\ &= -0.2\underline{e}_r + 0.32\underline{e}_\theta \text{ m/s} \end{aligned}$$

$$v = 0.377 \text{ m/s}$$

$$\beta = \tan^{-1}\left(\frac{0.2}{0.32}\right) = 32.0^\circ \quad \underline{e}_\theta$$

$$\alpha = \theta + 90^\circ + \beta = 260^\circ$$

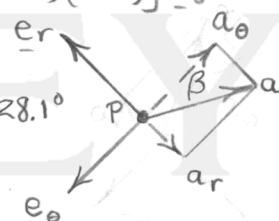
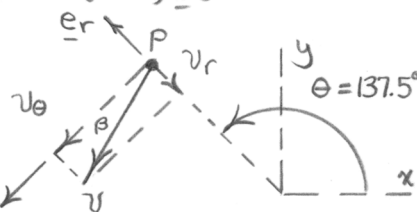
$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

$$= [0 - 0.8(0.4)^2]\underline{e}_r + [0.8(-0.1) + 2(-0.2)(0.4)]\underline{e}_\theta$$

$$= -0.1280\underline{e}_r - 0.240\underline{e}_\theta \text{ m/s}^2$$

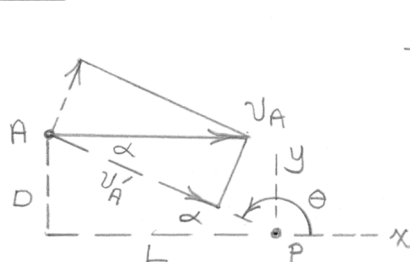
$$a = 0.272 \text{ m/s}^2, \quad \beta = \tan^{-1} \frac{0.1280}{0.240} = 28.1^\circ$$

$$\alpha = \theta - 90^\circ + \beta = 19.44^\circ$$



2/118

Consider Car A :



$$\tan \alpha = \frac{D}{L} = \frac{7}{60}$$

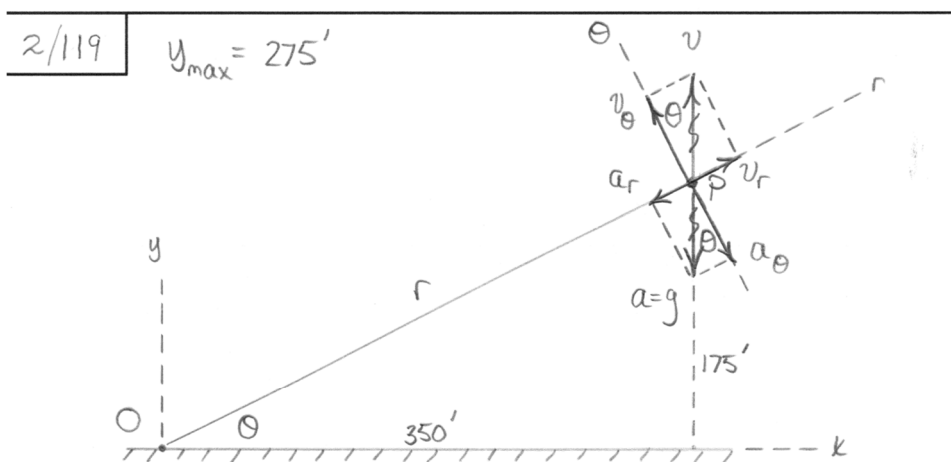
$$\alpha = 6.65^\circ$$

$$v'_A = v_A \cos \alpha = 110 \cos 6.65^\circ = \underline{109.3 \text{ km/h}}$$

$$\text{For Car B, } \alpha = \tan^{-1} \left(\frac{2D}{L} \right) = \tan^{-1} \left(\frac{2 \cdot 7}{60} \right) \\ = 13.13^\circ$$

$$v'_B = v_A \cos \alpha = 110 \cos 13.13^\circ = \underline{107.1 \text{ km/h}}$$

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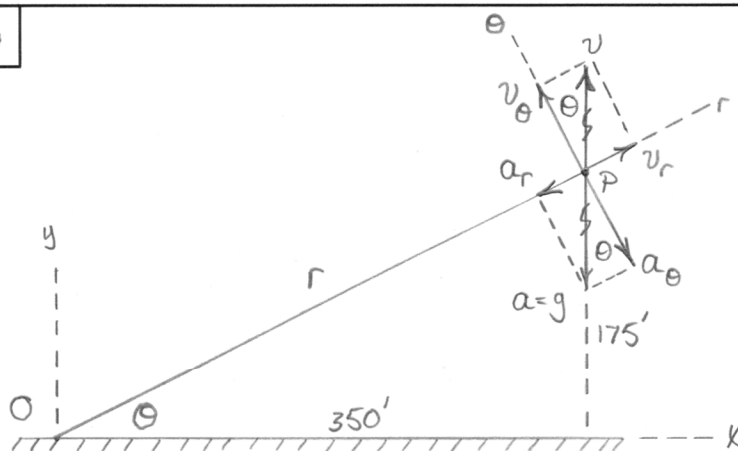


$$\left\{ \begin{array}{l} \text{At } y_{\max}: v_f^2 = v_o^2 - 2gy_{\max} \rightarrow 0 = v_o^2 - 2(32.2)(275) \rightarrow v_o = 133.1 \frac{\text{ft}}{\text{sec}} \\ \text{At } y=175': v^2 = v_o^2 - 2gy = 133.1^2 - 2(32.2)(175) \rightarrow v = 80.2 \frac{\text{ft}}{\text{sec}} \end{array} \right.$$

$$r = \sqrt{175^2 + 350^2} = 391 \text{ ft} \quad \theta = \tan^{-1}\left(\frac{175}{350}\right) = 26.7^\circ$$

$$\left\{ \begin{array}{l} v_r = \dot{r} = v \sin \theta = 80.2 \sin 26.7^\circ \rightarrow \dot{r} = 35.9 \frac{\text{ft}}{\text{sec}} \\ v_\theta = r \dot{\theta} = v \cos \theta = 80.2 \cos 26.7^\circ = 391 \dot{\theta} \rightarrow \dot{\theta} = 0.1834 \text{ rad/sec} \end{array} \right.$$

2/120



From Prob. 2/119 ... $\begin{cases} r = 391 \text{ ft} & \text{and} & \dot{r} = 35.9 \text{ ft/sec} \\ \theta = 26.7^\circ & \text{and} & \dot{\theta} = 0.1834 \text{ rad/sec} \end{cases}$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -a \sin \theta = -32.2 \sin 26.7^\circ = \ddot{r} - 391(0.1834)^2$$

$$\ddot{r} = -1.234 \text{ ft/sec}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -a \cos \theta = -32.2 \cos 26.7^\circ = 391\ddot{\theta} + 2(35.9)(0.1834)$$

$$\ddot{\theta} = -0.1072 \text{ rad/sec}^2$$

2/121

$$v_\theta = r\dot{\theta} = 9000(0.02) = 180 \text{ m/s}$$

$$v = v_\theta / \cos 60^\circ = 180 / 0.5 = \underline{360 \text{ m/s}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

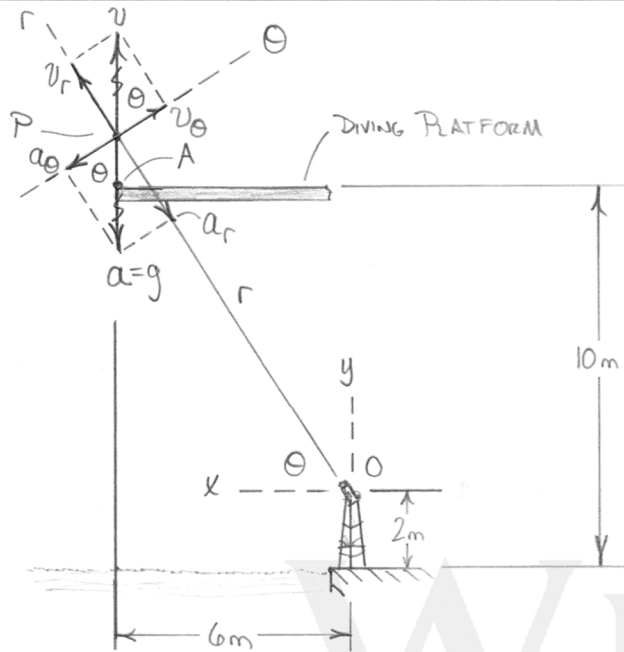
$$= 21 - 9000(0.02)^2 = 17.4 \text{ m/s}^2$$

$$a = a_r / \sin 60^\circ$$

$$= 17.4 / 0.866 = \underline{20.1 \text{ m/s}^2}$$

WILEY

*2/122



$$\begin{cases} v_0 = 2.5 \text{ m/s} \\ x_0 = x_f = 6 \text{ m} \\ y_0 = 8 \text{ m} \\ y_f = -2 \text{ m} \end{cases}$$

• ARBITRARY POSITION:

$$\begin{cases} y = y_0 + v_0 t - \frac{1}{2} g t^2 = 8 + 2.5t - \frac{1}{2}(9.81)t^2 & \textcircled{1} \\ x = x_0 = 6 \text{ m} & \textcircled{2} \end{cases}$$

$$v = v_0 - g t = 2.5 - 9.81 t \quad \textcircled{3}$$

$$r = \sqrt{x^2 + y^2} \quad \textcircled{4} \quad \text{AND} \quad \theta = \tan^{-1}\left(\frac{y}{x}\right) \quad \textcircled{5}$$

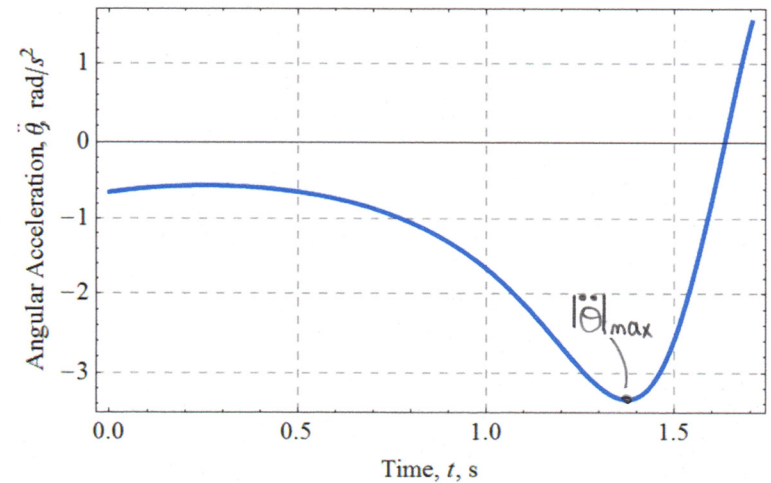
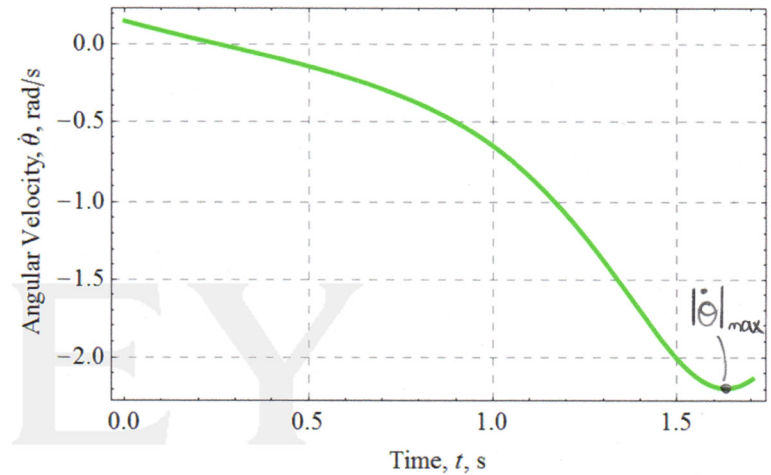
$$\begin{cases} v_r = v \sin \theta = \dot{r} & \textcircled{6} \\ v_\theta = v \cos \theta = r \dot{\theta} & \textcircled{7} \end{cases} \quad \begin{cases} a_r = -g \sin \theta = \ddot{r} - r \dot{\theta}^2 & \textcircled{8} \\ a_\theta = -g \cos \theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta} & \textcircled{9} \end{cases}$$

$$y_f = y_0 + v_0 t_f - \frac{1}{2} g t_f^2 \rightarrow -2 = 8 + 2.5 t_f - \frac{1}{2}(9.81) t_f^2 \quad \text{so...} \quad t_f = 1.705 \text{ s}$$

EVALUATE ① THROUGH ③ AND SOLVE ④ AND ⑤. WITH ① THROUGH ⑤, ⑥ ⑦ AND ⑧ CAN BE SOLVED FOR $\dot{r}$, $\dot{\theta}$, AND $\ddot{\theta}$.

FROM PLOTS...

$$\begin{cases} |\dot{\theta}|_{\max} = 2.19 \text{ rad/s} @ t = 1.633 \text{ s} \\ |\ddot{\theta}|_{\max} = 3.34 \text{ rad/s}^2 @ t = 1.373 \text{ s} \end{cases} \quad \text{(BOTH VALUES ARE NEGATIVE)}$$



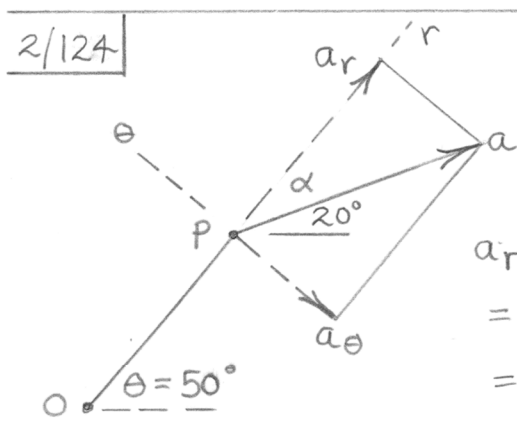
AT $t = 1.705 \text{ s}$, $\dot{\theta} = -2.13 \text{ rad/s}$ AND $\ddot{\theta} = 1.565 \text{ rad/s}^2$

2/123

$\alpha = 50^\circ - 20^\circ = 30^\circ$
 $r = 500 / \cos 60^\circ = 1000 \text{ ft}$
 $v_r = \dot{r} = v \cos \alpha = 140$
 $v \cos 30^\circ = 140, \quad v = 161.7 \frac{\text{ft}}{\text{sec}}$
 $v_\theta = r \dot{\theta} = -v \sin \alpha$
 $\dot{\theta} = -\frac{v \sin \alpha}{r} = -\frac{161.7 \sin 30^\circ}{1000}$
 $= -0.0808 \text{ rad/sec}$

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Values of α , r , $\dot{\theta}$ from solution to previous problem.

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$= 14 - 1000(-0.0808)^2$$

$$= 7.47 \text{ ft/sec}^2$$

Then $a = \frac{a_r}{\cos \alpha} = \frac{7.47}{\cos 30^\circ} = \underline{8.62 \text{ ft/sec}^2}$

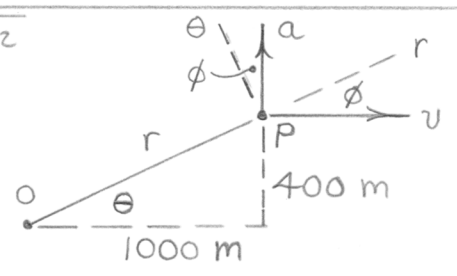
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$$-8.62 \sin 30^\circ = 1000\ddot{\theta} + 2(140)(-0.0808)$$

$$\ddot{\theta} = \underline{0.01832 \text{ rad/sec}^2}$$

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2/125 $r = \sqrt{1000^2 + 400^2}$
 $= 1077 \text{ m}$
 $\theta = \tan^{-1} \frac{400}{1200} = 21.8^\circ$
 $v = \frac{600}{3.6} = 166.7 \text{ m/s}$
 $a = a_n = \frac{v^2}{r} = \frac{166.7^2}{1200} = 23.1 \text{ m/s}^2$



$v_r = \dot{r} = v \cos \theta = 166.7 \cos 21.8^\circ = 154.7 \text{ m/s}$
 $v_\theta = r\dot{\theta} : -166.7 \sin 21.8^\circ = 1077\dot{\theta}, \dot{\theta} = -0.0575 \frac{\text{rad}}{\text{s}}$
 $a_r = \ddot{r} - r\dot{\theta}^2 : 23.1 \sin 21.8^\circ = \ddot{r} - 1077(-0.0575)^2$
 $\ddot{r} = 12.15 \text{ m/s}^2$
 $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 23.1 \cos 21.8^\circ = 1077\ddot{\theta} + 2(154.7)(-0.0575)$
 $\ddot{\theta} = 0.0365 \text{ rad/s}^2$

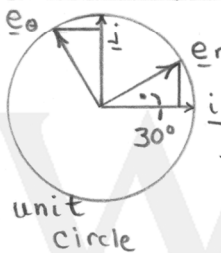
$$\begin{aligned} 2/126 \quad \left\{ \begin{array}{ll} r = 0.75 + 0.5 = 1.25 \text{ m} & \theta = 30^\circ \\ \dot{r} = 0.2 \text{ m/s} & \dot{\theta} = 0.1745 \frac{\text{rad}}{\text{s}} \\ \ddot{r} = -0.3 \text{ m/s}^2 & \ddot{\theta} = 0 \end{array} \right. \end{aligned}$$

$$\begin{aligned} \underline{v} &= v_r \underline{e}_r + v_\theta \underline{e}_\theta = \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta \\ &= 0.2 \underline{e}_r + 1.25(0.1745) \underline{e}_\theta = 0.2 \underline{e}_r + 0.218 \underline{e}_\theta \frac{\text{m}}{\text{s}} \end{aligned}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \underline{0.296 \text{ m/s}}$$

$$\begin{aligned} \underline{a} &= a_r \underline{e}_r + a_\theta \underline{e}_\theta = (\ddot{r} - r \dot{\theta}^2) \underline{e}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \underline{e}_\theta \\ &= [-0.3 - 1.25(0.1745)^2] \underline{e}_r + [1.25(0) + 2(0.2)(0.1745)] \underline{e}_\theta \\ &= -0.338 \underline{e}_r + 0.0698 \underline{e}_\theta \text{ m/s}^2 \end{aligned}$$

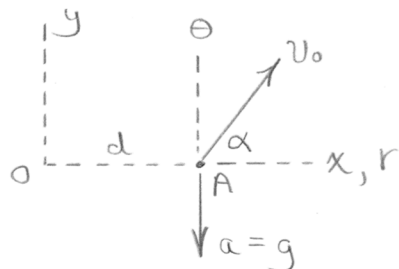
$$a = \sqrt{a_r^2 + a_\theta^2} = \underline{0.345 \text{ m/s}^2}$$



$$\begin{aligned} \underline{e}_r &= \underline{i} \cos 30^\circ + \underline{j} \sin 30^\circ \\ \underline{e}_\theta &= -\underline{i} \sin 30^\circ + \underline{j} \cos 30^\circ \\ \underline{v} &= 0.2 [\underline{i} \cos 30^\circ + \underline{j} \sin 30^\circ] + 0.218 [-\underline{i} \sin 30^\circ + \underline{j} \cos 30^\circ] \\ &= 0.064 \underline{i} + 0.289 \underline{j} \text{ m/s} \end{aligned}$$

$$\begin{aligned} \underline{a} &= -0.338 [\underline{i} \cos 30^\circ + \underline{j} \sin 30^\circ] + 0.0698 [-\underline{i} \sin 30^\circ + \underline{j} \cos 30^\circ] \\ &= -0.328 \underline{i} - 0.1086 \underline{j} \text{ m/s}^2 \end{aligned}$$

2/127



$$r = d, \quad \theta = 0$$

$$v_r = \dot{r} = v_0 \cos \alpha$$

$$v_\theta = r\dot{\theta} = v_0 \sin \alpha,$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0,$$

$$\dot{\theta} = \frac{v_0 \sin \alpha}{d}$$

$$\ddot{r} = r\dot{\theta}^2 = \frac{v_0^2 \sin^2 \alpha}{d}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -g, \quad \ddot{\theta} = -\frac{2\dot{r}\dot{\theta} + g}{r}$$

$$\text{or } \ddot{\theta} = -\frac{2(v_0 \cos \alpha)\left(\frac{v_0 \sin \alpha}{d}\right) + g}{d}$$

$$= -\frac{1}{d} \left(\frac{2v_0^2}{d} \cos \alpha \sin \alpha + g \right)$$

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$$v = 12,149 \left(\frac{5280}{3600} \right) = 17,819 \frac{\text{ft}}{\text{sec}}$$

$$v_{\theta} = r\dot{\theta} : 17,819 \cos 30^{\circ} = 8400(5280)\dot{\theta}$$

$$\dot{\theta} = 3.48(10^{-4}) \text{ rad/sec}$$

$$v_r = \dot{r} : 17,819 \sin 30^{\circ} = \dot{r}$$

$$\dot{r} = 8910 \frac{\text{ft}}{\text{sec}}$$

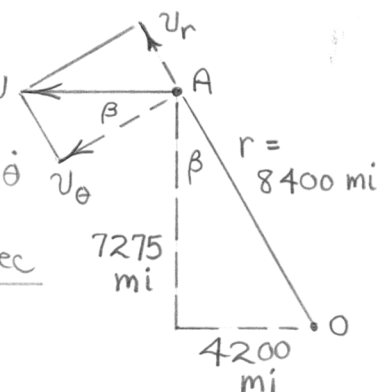
$$\beta = \tan^{-1} \left(\frac{4200}{7275} \right) = 30^{\circ}$$

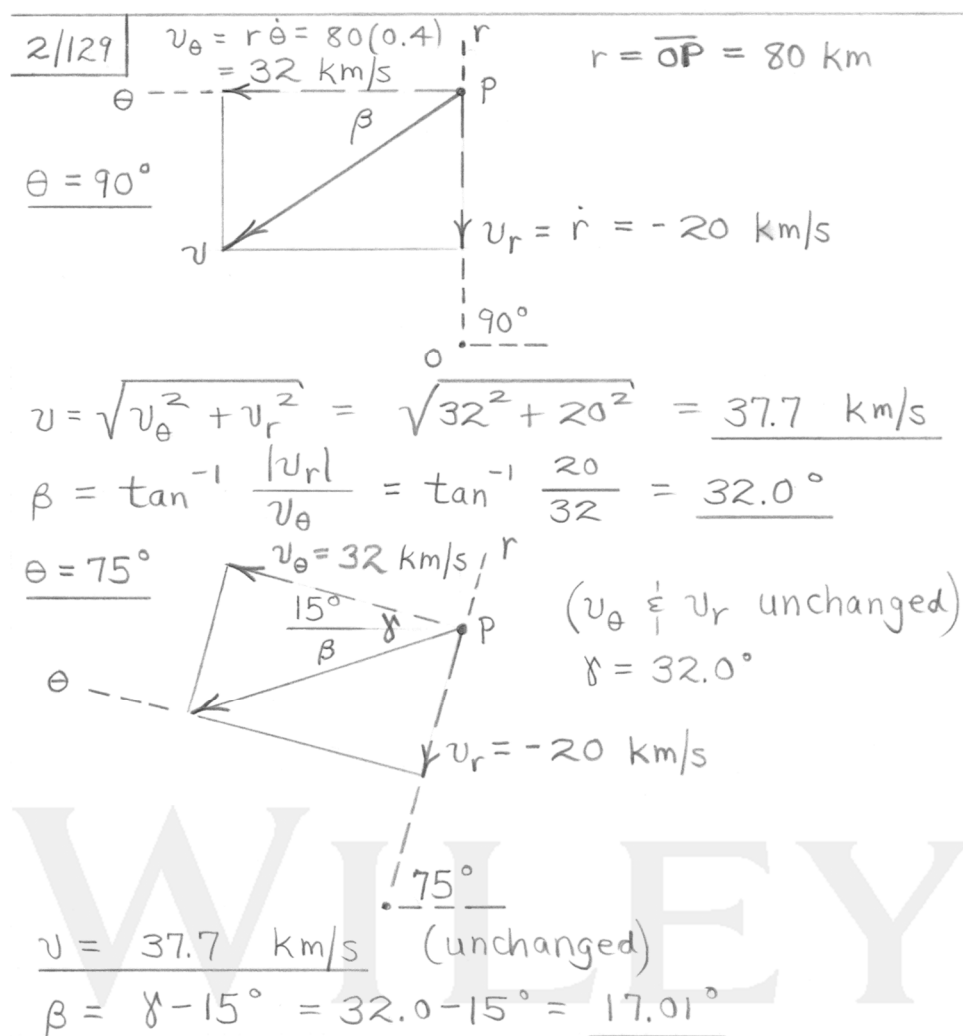
$$a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 0 = 8400(5280)\ddot{\theta} + 2(8910)(3.48)(10^{-4})$$

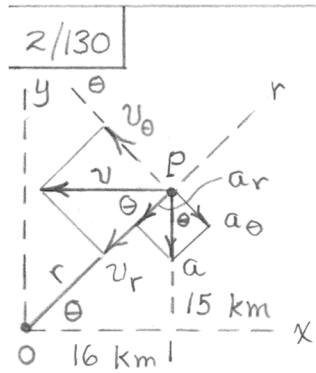
$$\ddot{\theta} = -1.398(10^{-7}) \text{ rad/sec}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 : -7.159 = \ddot{r} - 8400(5280)(3.48 \times 10^{-4})^2$$

$$\ddot{r} = -1.790 \text{ ft/sec}^2$$







$$v = \frac{360}{3.6} = 100 \text{ m/s}$$

$$a = \frac{100^2}{3000} = 3.33 \text{ m/s}^2$$

$$r = \sqrt{16000^2 + 15000^2} = 21900 \text{ m}$$

$$\theta = \tan^{-1} \frac{15}{16} = 43.2^\circ$$

$$\begin{aligned} \dot{r} &= -v \cos \theta = -100 \cos 43.2^\circ \\ &= -73.0 \text{ m/s} \end{aligned}$$

$$\begin{aligned} v_\theta &= v \sin \theta = 100 \sin 43.2^\circ = 68.4 \text{ m/s} = r\dot{\theta} \\ \Rightarrow \dot{\theta} &= 0.00312 \text{ rad/s} \end{aligned}$$

$$\begin{aligned} a_r &= -a \sin \theta = -3.33 \sin 43.2^\circ = -2.28 \text{ m/s}^2 \\ \text{So } -2.28 &= \ddot{r} - r\dot{\theta}^2 \Rightarrow \ddot{r} = -2.07 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} a_\theta &= -a \cos \theta = -3.33 \cos 43.2^\circ = -2.43 \text{ m/s}^2 \\ \text{So } -2.43 &= r\ddot{\theta} + 2\dot{r}\dot{\theta} \Rightarrow \ddot{\theta} = -9.01(10^{-5}) \text{ rad/s}^2 \end{aligned}$$

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$x = x_0 + v_{x_0} t$
 $= 0 + 100 \cos 30^\circ (0.5)$
 $= 43.3 \text{ ft}$

$\dot{x} = v_{x_0} = 100 \cos 30^\circ = 86.6 \frac{\text{ft}}{\text{sec}}$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 = 6 + 100 \sin 30^\circ (0.5) - 16.1 (0.5)^2 = 27.0 \text{ ft}$

$\dot{y} = v_{y_0} - g t = 100 \sin 30^\circ - 32.2 (0.5) = 33.9 \text{ ft/sec}$

$r = \sqrt{x^2 + y^2} = \sqrt{43.3^2 + 27.0^2} = 51.0 \text{ ft}$

$\theta = \tan^{-1} \left(\frac{y}{x} \right) = \tan^{-1} (27.0/43.3) = 31.9^\circ$

$\alpha = \tan^{-1} (v_y/v_x) = \tan^{-1} (33.9/86.6) = 21.4^\circ$

$\beta = \theta - \alpha = 10.54^\circ, v = \sqrt{\dot{x}^2 + \dot{y}^2} = 93.0 \text{ ft/sec}$

$v_r = v \cos \beta = 93.0 \cos 10.54^\circ = 91.4 \text{ ft/sec} = \dot{r}$

$v_\theta = -v \sin \beta = -93.0 \sin 10.54^\circ = -17.02 \text{ ft/sec} = \dot{\theta} r$

$-17.02 = r \dot{\theta} = 51.0 \dot{\theta}, \quad \dot{\theta} = -0.334 \text{ rad/sec}$

$a = \ddot{y} = -32.2 \text{ ft/sec}^2$

$a_r = -g \sin \theta = -32.2 \sin 31.9^\circ = -17.03 \text{ ft/sec}^2$

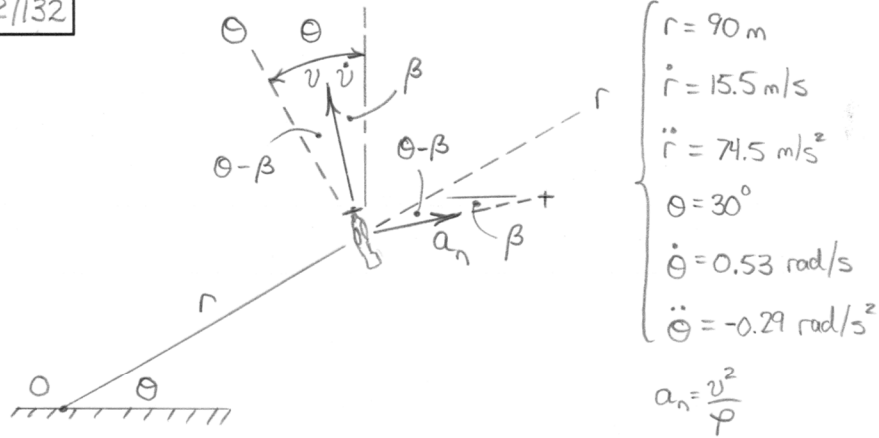
$-17.03 = \ddot{r} - r \dot{\theta}^2 = \ddot{r} - 51.0 (-0.334)^2, \quad \ddot{r} = -11.35 \text{ ft/sec}^2$

$a_\theta = -g \cos \theta = -32.2 \cos 31.9^\circ = -27.3 \text{ ft/sec}^2$

$-27.3 = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 51.0 \ddot{\theta} + 2(91.4)(-0.334)$

$\ddot{\theta} = 0.660 \text{ rad/sec}^2$

► 2/132



$$v_r = \dot{r} = v \sin(\theta - \beta) = v \sin(30^\circ - \beta) = 15.5 \text{ (m/s)} \quad (1)$$

$$v_\theta = r\dot{\theta} = v \cos(\theta - \beta) = v \cos(30^\circ - \beta) = 90(0.53) \text{ (m/s)} \quad (2)$$

$$a_r = a_n \cos(\theta - \beta) + \dot{v} \sin(\theta - \beta) = \ddot{r} - r\dot{\theta}^2$$

$$\frac{v^2}{r} \cos(30^\circ - \beta) + \dot{v} \sin(30^\circ - \beta) = 74.5 - 90(0.53)^2 \quad (3)$$

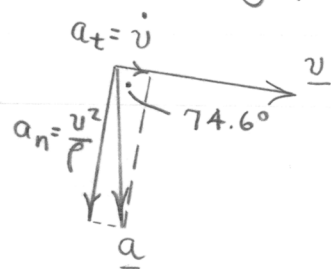
$$a_\theta = \dot{v} \cos(\theta - \beta) - a_n \sin(\theta - \beta) = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\dot{v} \cos(30^\circ - \beta) - \frac{v^2}{r} \sin(30^\circ - \beta) = 90(-0.29) + 2(15.5)(0.53) \quad (4)$$

Solving... $v = 50.2 \text{ m/s}$, $\dot{v} = 6.01 \text{ m/s}^2$, $r = 50.5 \text{ m}$, $\beta = 12.00^\circ$

$$\begin{aligned} \underline{v} &= 6\underline{i} - 3\underline{j} + 2\underline{k} \text{ m/s}, \quad v = \sqrt{6^2 + 3^2 + 2^2} = 7 \frac{\text{m}}{\text{s}} \\ \underline{a} &= 3\underline{i} - \underline{j} - 5\underline{k} \text{ m/s}^2, \quad a = \sqrt{3^2 + 1^2 + 5^2} = \sqrt{35} \frac{\text{m}}{\text{s}^2} \\ \theta &= \cos^{-1} \left[\frac{\underline{v} \cdot \underline{a}}{va} \right] = \cos^{-1} \left[\frac{6(3) - 3(-1) - 2(5)}{7\sqrt{35}} \right] \\ &= \underline{74.6^\circ} \end{aligned}$$

In osculating plane:



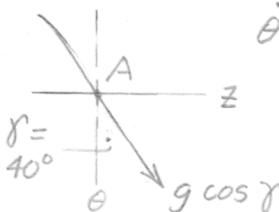
$$\begin{aligned} a_t &= \dot{v} = a \cos 74.6^\circ = \underline{1.571 \text{ m/s}^2} \\ a_n &= a \sin 74.6^\circ = 5.70 \text{ m/s}^2 \\ &= \frac{v^2}{\rho} \\ \therefore \rho &= \frac{v^2}{a_n} = \frac{7^2}{5.70} = \underline{8.59 \text{ m}} \end{aligned}$$

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$$\begin{aligned}
 2/134 \quad v_{z_0} &= 600 \sin 60^\circ = 520 \text{ ft/sec} \\
 v_{xy_0} &= 600 \cos 60^\circ = 300 \text{ ft/sec} \\
 v_z &= v_{z_0} - gt = 520 - 32.2(20) = \underline{-124.4 \text{ ft/sec}} \\
 v_{xy} &= v_{xy_0} = 300 \text{ ft/sec} = \text{constant} \\
 v_x &= -v_{xy} \sin 20^\circ = -300 \sin 20^\circ = \underline{-102.6 \text{ ft/sec}} \\
 v_y &= v_{xy} \cos 20^\circ = 300 \cos 20^\circ = \underline{282 \text{ ft/sec}} \\
 d_{xy} &= v_{xy} t = 300(20) = 6000 \text{ ft} \\
 x &= -d_{xy} \sin 20^\circ = -6000 \sin 20^\circ = \underline{-2050 \text{ ft}} \\
 y &= d_{xy} \cos 20^\circ = 6000 \cos 20^\circ = \underline{5640 \text{ ft}} \\
 z &= v_{z_0} t - \frac{1}{2} g t^2 = 520(20) - 16.1(20)^2 = \underline{3950 \text{ ft}} \\
 a_x &= a_y = 0, \quad a_z = -g = -32.2 \text{ ft/sec}^2
 \end{aligned}$$

2/135 $v_\theta = r\dot{\theta}$ & $v_\theta = v \cos \gamma$ so $\dot{\theta} = \frac{v}{r} \cos \gamma$

$\dot{\theta} = \frac{15}{5} (0.7660) = 2.298 \text{ rad/s}$



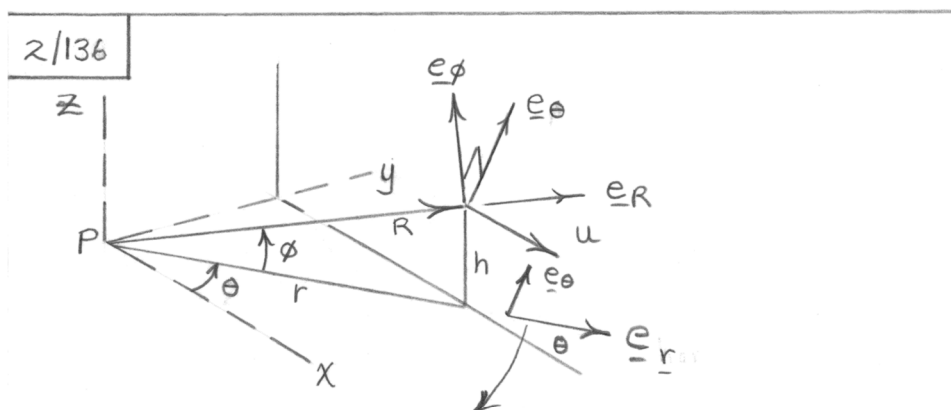
$a_\theta = g \cos^2 \gamma = 9.81 (0.7660)^2 = 5.76 \text{ m/s}^2$

$a_z = g \cos \gamma \sin \gamma = 9.81 (0.7660)(0.6428) = 4.83 \text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(2.298)^2 = -26.41 \text{ m/s}^2$

$a = \sqrt{26.41^2 + 5.76^2 + 4.83^2} = \underline{27.5 \text{ m/s}^2}$

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Resolve u along $\underline{e}_\theta$ & $\underline{e}_r$ directions :

$$\underline{e}_\theta : \underline{u \sin \theta} = v_\theta$$

$$\underline{e}_r : u \cos \theta = v_r$$

Now resolve v_r along $\underline{e}_R$ & $\underline{e}_\phi$ directions :

$$\underline{e}_R : v_r \cos \phi = \underline{u \cos \theta \cos \phi} = v_R$$

$$\underline{e}_\phi : -v_r \sin \phi = \underline{-u \cos \theta \sin \phi} = v_\phi$$

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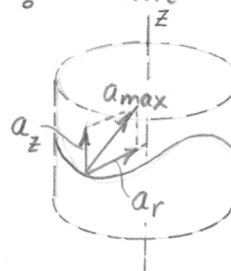
$$2/137 \quad a_r = \ddot{r} - r\dot{\theta}^2 = 0 - r\omega^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$$

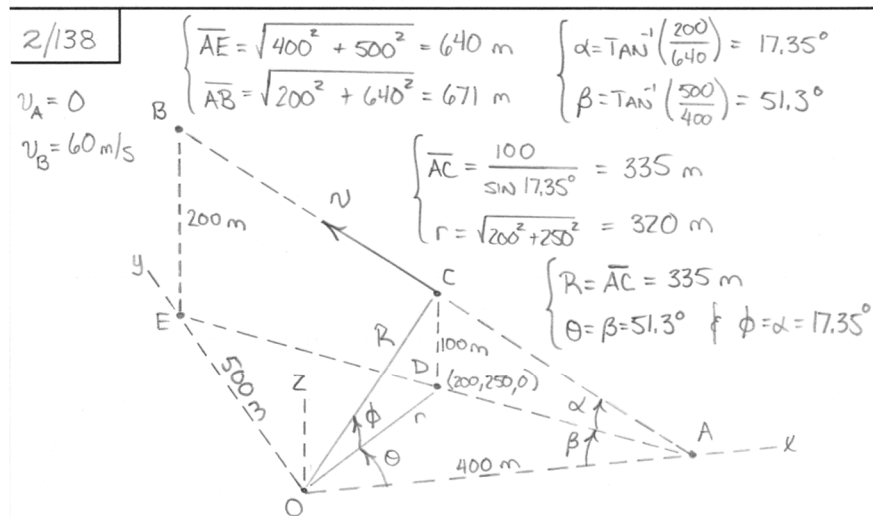
$$a_z = \frac{d^2}{dt^2}(z_0 \sin 2\pi nt) = -4n^2\pi^2 z_0 \sin 2\pi nt$$

$$a = \sqrt{(-r\omega^2)^2 + (-4n^2\pi^2 z_0 \sin 2\pi nt)^2}$$

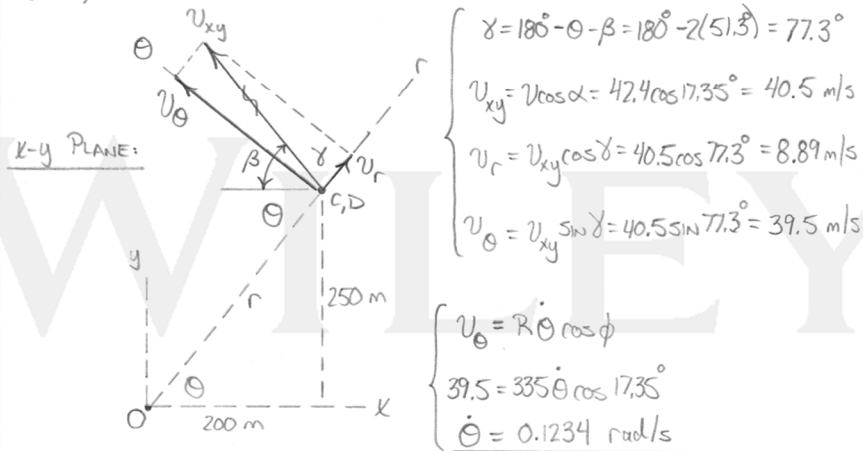
$$a_{\max} = \sqrt{r^2\omega^4 + 16n^4\pi^4 z_0^2}$$



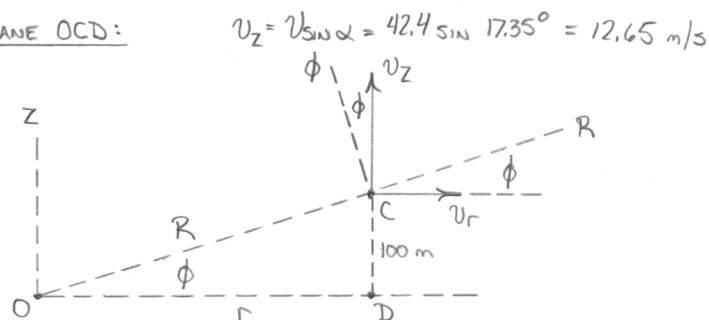
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$$\begin{cases} v_B^2 = v_A^2 + 2a\overline{AB} \rightarrow 60^2 = 2a(671) \rightarrow a = 2.68 \text{ m/s}^2 \\ v^2 = v_A^2 + 2a\overline{AC} = 2(2.68)(335) \rightarrow v = 42.4 \text{ m/s} \end{cases}$$



PLANE OCD:

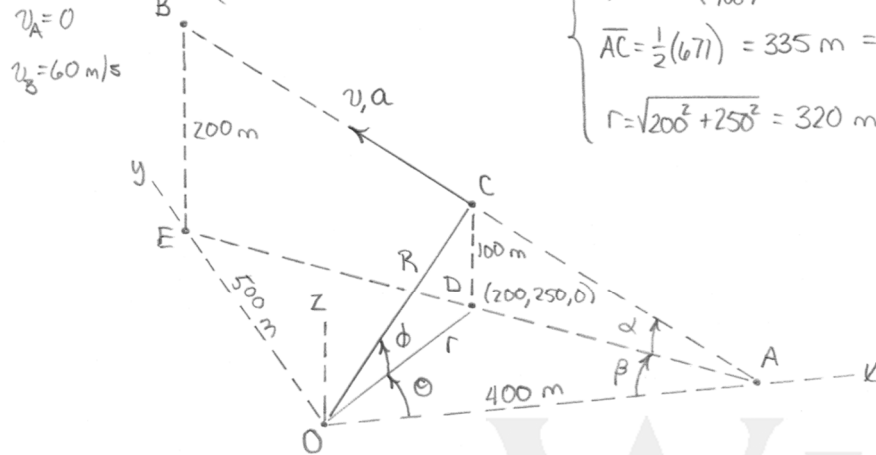


$$\begin{cases} v_R = v_r \cos \phi + v_z \sin \phi = 8.89 \cos 17.35^\circ + 12.65 \sin 17.35^\circ = 12.26 \text{ m/s} \\ v_R = \dot{R} = 12.26 \text{ m/s} \end{cases}$$

$$\begin{cases} v_\phi = v_z \cos \phi - v_r \sin \phi = 12.65 \cos 17.35^\circ - 8.89 \sin 17.35^\circ = 9.42 \text{ m/s} \\ v_\phi = R \dot{\phi} \rightarrow 9.42 = 335 \dot{\phi} \rightarrow \dot{\phi} = 0.0281 \text{ rad/s} \end{cases}$$

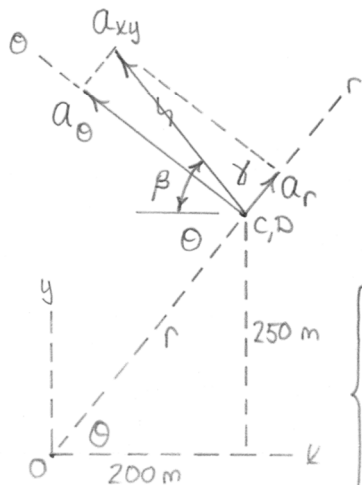
2/139

$$\begin{cases} \overline{AE} = \sqrt{400^2 + 500^2} = 640 \text{ m} \\ \overline{AB} = \sqrt{200^2 + 640^2} = 671 \text{ m} \\ \alpha = \tan^{-1}\left(\frac{200}{640}\right) = 17.35^\circ = \phi \\ \beta = \tan^{-1}\left(\frac{500}{400}\right) = 51.3^\circ = \theta \\ \overline{AC} = \frac{1}{2}(671) = 335 \text{ m} = R \\ r = \sqrt{200^2 + 250^2} = 320 \text{ m} \end{cases}$$



$$v_B^2 = v_A^2 + 2a\overline{AB} \rightarrow 60^2 = 2a(671) \rightarrow a = 2.68 \text{ m/s}^2$$

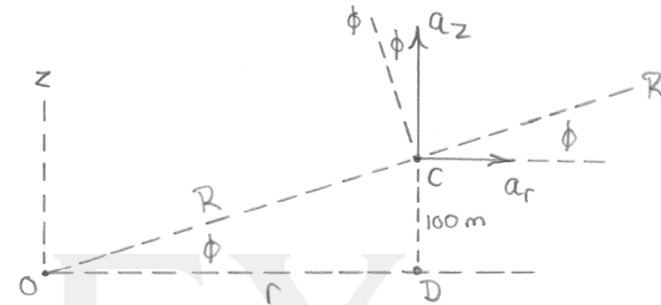
$$x-y \text{ PLANE: } \dot{R} = 12.26 \text{ m/s}, \dot{\theta} = 0.1234 \text{ rad/s}, \dot{\phi} = 0.0281 \text{ rad/s}$$



$$\begin{cases} \gamma = 180^\circ - \theta - \beta = 180^\circ - 2(51.3^\circ) = 77.3^\circ \\ a_{xy} = a \cos \alpha = 2.68 \cos 17.35^\circ = 2.56 \text{ m/s}^2 \\ a_r = a_{xy} \cos \gamma = 2.56 \cos 77.3^\circ = 0.562 \text{ m/s}^2 \\ a_\theta = a_{xy} \sin \gamma = 2.56 \sin 77.3^\circ = 2.50 \text{ m/s}^2 \end{cases}$$

$$\begin{cases} a_\theta = 2\dot{R}\dot{\theta}\cos\phi - 2R\dot{\theta}\dot{\phi}\sin\phi + R\ddot{\theta}\cos\phi \\ = 2(12.26)(0.1234)\cos 17.35^\circ + 335\ddot{\theta}\cos 17.35^\circ \\ - 2(335)(0.1234)(0.0281)\sin 17.35^\circ = 2.50 \\ \ddot{\theta} = 9.52(10^{-4}) \text{ rad/s}^2 \end{cases}$$

$$\text{PLANE OCD: } a_z = a \sin \alpha = 2.68 \sin 17.35^\circ = 0.8 \text{ m/s}^2$$

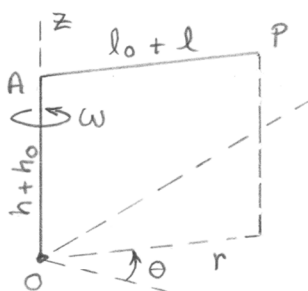


$$\begin{cases} a_R = a_r \cos \phi + a_z \sin \phi = 0.562 \cos 17.35^\circ + 0.8 \sin 17.35^\circ = 0.775 \text{ m/s}^2 \\ a_R = \ddot{R} - R\dot{\theta}^2 \cos^2 \phi - R\dot{\phi}^2 = \ddot{R} - 335(0.1234)^2 \cos^2 17.35^\circ - 335(0.0281)^2 = 0.775 \\ \ddot{R} = 5.69 \text{ m/s}^2 \end{cases}$$

$$a_\phi = a_z \cos \phi - a_r \sin \phi = 0.8 \cos 17.35^\circ - 0.562 \sin 17.35^\circ = 0.596 \text{ m/s}^2$$

$$\begin{aligned} a_\phi &= 2\dot{R}\dot{\phi} + R\ddot{\phi} + R\dot{\theta}^2 \sin \phi \cos \phi = 2(12.26)(0.0281) + 335\ddot{\phi} \dots \\ &\quad + 335(0.1234)^2 \sin 17.35^\circ \cos 17.35^\circ = 0.596 \\ \ddot{\phi} &= -4.61(10^{-3}) \text{ rad/s}^2 \end{aligned}$$

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$$r = l_0 + l, \quad \dot{r} = \dot{l}, \quad \ddot{r} = \ddot{l}$$

$$z = h_0 + h, \quad \dot{z} = \dot{h}, \quad \ddot{z} = \ddot{h}$$

$$\omega = \dot{\theta} = \text{constant}$$

$$v_r = \dot{r} = \dot{l}$$

$$v_\theta = r\dot{\theta} = (l_0 + l)\omega$$

$$v_z = \dot{z} = \dot{h}$$

$$v_p = \sqrt{\dot{l}^2 + (l_0 + l)^2 \omega^2 + \dot{h}^2}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = \ddot{l} - (l_0 + l)\omega^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2\dot{l}\omega$$

$$a_z = \ddot{z} = \ddot{h}$$

$$a_p = \sqrt{(\ddot{l} - (l_0 + l)\omega^2)^2 + (2\dot{l}\omega)^2 + \ddot{h}^2}$$

$$= \sqrt{(\ddot{l} - (l_0 + l)\omega^2)^2 + 4\dot{l}^2\omega^2 + \ddot{h}^2}$$

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$\dot{\theta} = \omega = \frac{40}{180} \pi = 0.698 \text{ rad/s}$
 $\dot{\beta} = \frac{10}{180} \pi = 0.1745 \text{ rad/s}$
 $\ddot{\beta} = \frac{20}{180} \pi = 0.349 \text{ rad/s}^2$
 Use cylindrical coordinates

$r = b \cos \beta + c \sin \beta, \quad \dot{r} = (-b \sin \beta + c \cos \beta) \dot{\beta}$
 $\ddot{r} = (-b \cos \beta - c \sin \beta) \dot{\beta}^2 + (-b \sin \beta + c \cos \beta) \ddot{\beta}$
 $z = h + b \sin \beta - c \cos \beta, \quad \dot{z} = (b \cos \beta + c \sin \beta) \dot{\beta}$
 $\ddot{z} = (-b \sin \beta + c \cos \beta) \dot{\beta}^2 + (b \cos \beta + c \sin \beta) \ddot{\beta}$

For $\beta = 30^\circ$, $\dot{r} = (-300 \times 0.5 + 200 \times 0.866)(0.1745) = 4.050 \text{ mm/s}$
 $\ddot{r} = (-300 \times 0.866 - 200 \times 0.5)(0.1745)^2$
 $\quad + (-300 \times 0.5 + 200 \times 0.866)(0.349) = -2.860 \text{ mm/s}^2$
 $\ddot{z} = (-300 \times 0.5 + 200 \times 0.866)(0.1745)^2$
 $\quad + (300 \times 0.866 + 200 \times 0.5)(0.349) = 126.30 \text{ mm/s}^2$

$a_r = \ddot{r} - r \dot{\theta}^2 = -2.860 - 359.8(0.698)^2 = -178.23 \text{ mm/s}^2$
 $a_\theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 0 + 2(4.049)(0.698) = 5.65 \text{ mm/s}^2$
 $a_z = \ddot{z} = 126.30 \text{ mm/s}^2$

$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2} = 219.5 \text{ mm/s}^2$

2/142 Helix angle is $\gamma = \tan^{-1} \frac{10}{24\pi} = 7.55^\circ$

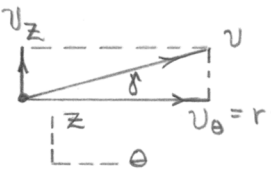


Diagram showing velocity vector v at an angle γ to the horizontal. The horizontal component is v_z and the vertical component is $v_\theta = r\dot{\theta}$. The angle between the horizontal axis and the velocity vector is γ .

$$v = 15 \left(\frac{5280}{3600} \right) = 22 \text{ ft/sec}$$

$$v_z = v \sin \gamma = 22 \sin 7.55^\circ = 2.89 \text{ ft/sec}$$

$$v_\theta = v \cos \gamma = 22 \cos 7.55^\circ = 21.8 \text{ ft/sec}$$

From $v_\theta = r\dot{\theta}$, $\dot{\theta} = \frac{v_\theta}{r} = \frac{21.8}{24} = 0.909 \text{ rad/sec}$

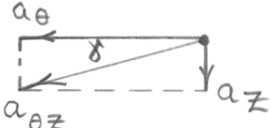


Diagram showing acceleration vector a at an angle γ to the horizontal. The horizontal component is a_z and the vertical component is a_θ . The angle between the horizontal axis and the acceleration vector is γ .

$$a_\theta = 2 \left(\frac{5280}{3600} \right) = 2.93 \text{ ft/sec}^2$$

$$a_\theta = -2.93 \cos 7.55^\circ = -2.91 \text{ ft/sec}^2$$

$$a_z = -2.93 \sin 7.55^\circ = -0.386 \text{ ft/sec}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 24(0.909)^2 = -19.82 \text{ ft/sec}^2$$

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2/143 Use Eq. 2/19 where $\dot{\varphi} = -\dot{\beta}$, $R = L$, $\dot{\theta} = \omega$

$$a_R = 0 - 1.2\left(-\frac{3}{2}\right)^2 - 1.2(2)^2\frac{1}{2} = \underline{-5.10 \text{ m/s}^2}$$

$$\begin{aligned} a_\theta &= \frac{\sin\beta}{L}(2L\dot{L}\omega + 0) + 2L\omega\dot{\beta}\cos\beta = 2\omega(L\sin\beta + L\dot{\beta}\cos\beta) \\ &= 2(2)\left(0.9\frac{1}{\sqrt{2}} + 1.2\left(\frac{3}{2}\right)\frac{1}{\sqrt{2}}\right) = \frac{10.8}{\sqrt{2}} = \underline{7.64 \text{ m/s}^2} \end{aligned}$$

$$\begin{aligned} a_\phi &= -2\dot{L}\dot{\beta} + L\omega^2\cos\beta\sin\beta = -2(0.9)\frac{3}{2} + 1.2(2^2)\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} \\ &= -2.7 + 2.4 = \underline{-0.3 \text{ m/s}^2} \end{aligned}$$

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$$R = 200 + 50 \sin 4\pi t \quad \dot{\theta} = 120 \left(\frac{2\pi}{60} \right) = 4\pi \text{ rad/s}$$

$$\dot{R} = 200\pi \cos 4\pi t \quad \ddot{\theta} = 0$$

$$\ddot{R} = -800\pi^2 \sin 4\pi t \quad \phi = \frac{\pi}{2} - \beta = 60^\circ, \dot{\phi} = \ddot{\phi} = 0$$

For $\dot{R}$ maximum, $\cos 4\pi t = 1$ and $\sin 4\pi t = 0$

$$\text{Eq. 2/19: } a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2 \cos^2 \phi$$

$$= 0 + (200 + 0)0 - (200 + 0)(4\pi)^2 \cos^2 60^\circ = -800\pi^2 \frac{\text{mm}}{\text{s}^2}$$

$$a_\theta = \frac{\cos \phi}{R} \frac{d}{dt} (R^2 \dot{\theta}) - 2R\dot{\theta}\dot{\phi} \sin \phi$$

$$= 2\dot{R}\dot{\theta} \cos \phi - 2R\dot{\theta}\dot{\phi} \sin \phi$$

$$= 2(200\pi \cdot 1)(4\pi) \cos 60^\circ - 2(200 + 0)(4\pi)(0) \sin 60^\circ$$

$$= 800\pi^2 \text{ mm/s}^2$$

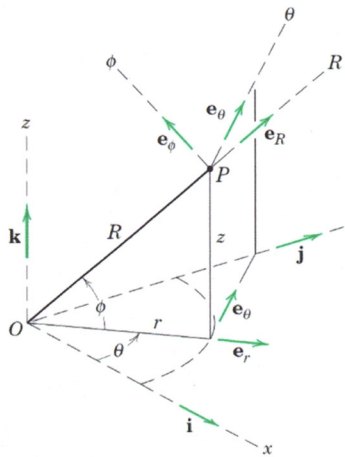
$$a_\phi = \frac{1}{R} \frac{d}{dt} (R^2 \dot{\phi}) + R\dot{\theta}^2 \sin \phi \cos \phi$$

$$= 0 + (200 + 0)(4\pi)^2 \sin 60^\circ \cos 60^\circ = 800\sqrt{3} \pi^2 \text{ mm/s}^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = 17660 \text{ mm/s}^2 \text{ or } \underline{17.66 \text{ m/s}^2}$$

► 2/145

SINCE THE VECTOR-DIFFERENTIATION THEOREM IS NOT USED UNTIL CHAPTER 5, WE WILL UTILIZE GEOMETRY AND FIXED UNIT VECTORS.



$$\underline{v} = \dot{R}\underline{e}_R + R\dot{\theta}\cos\phi\underline{e}_\theta + R\dot{\phi}\underline{e}_\phi$$

$$\underline{\dot{v}} = \underline{a} = \ddot{R}\underline{e}_R + \dot{R}\dot{\underline{e}}_R + \dot{R}\dot{\theta}\cos\phi\underline{e}_\theta + R\ddot{\theta}\cos\phi\underline{e}_\theta - R\dot{\theta}\dot{\phi}\sin\phi\underline{e}_\theta + R\dot{\theta}\cos\phi\underline{\dot{e}}_\theta + \dot{R}\dot{\phi}\underline{e}_\phi + R\ddot{\phi}\underline{e}_\phi + R\dot{\phi}\dot{\underline{e}}_\phi$$

NOW, WE WRITE $\underline{e}_R$, $\underline{e}_\theta$, AND $\underline{e}_\phi$ IN TERMS OF $\underline{i}$, $\underline{j}$, AND $\underline{k}$. THEN, WE WRITE $\underline{i}$, $\underline{j}$, AND $\underline{k}$ IN TERMS OF $\underline{e}_R$, $\underline{e}_\theta$, AND $\underline{e}_\phi$, SUBSTITUTE AND SIMPLIFY TO GET THE FINAL RESULT.

$$\underline{e}_R = \cos\phi\cos\theta\underline{i} + \cos\phi\sin\theta\underline{j} + \sin\phi\underline{k}$$

$$\underline{\dot{e}}_R = (-\dot{\phi}\sin\phi\cos\theta - \dot{\theta}\cos\phi\sin\theta)\underline{i} + (\dot{\theta}\cos\phi\cos\theta - \dot{\phi}\sin\phi\sin\theta)\underline{j} + \dot{\phi}\cos\phi\underline{k}$$

$$\underline{e}_\theta = -\sin\theta\underline{i} + \cos\theta\underline{j}$$

$$\underline{\dot{e}}_\theta = -\dot{\theta}\cos\theta\underline{i} - \dot{\theta}\sin\theta\underline{j}$$

$$\underline{e}_\phi = -\sin\phi\cos\theta\underline{i} - \sin\phi\sin\theta\underline{j} + \cos\phi\underline{k}$$

$$\underline{\dot{e}}_\phi = (-\dot{\phi}\cos\phi\cos\theta + \dot{\theta}\sin\phi\sin\theta)\underline{i} - (\dot{\phi}\cos\phi\sin\theta + \dot{\theta}\sin\phi\cos\theta)\underline{j} - \dot{\phi}\sin\phi\underline{k}$$

$$\begin{cases} \underline{i} = \cos\theta\cos\phi\underline{e}_R - \sin\theta\underline{e}_\theta - \cos\theta\sin\phi\underline{e}_\phi \\ \underline{j} = \sin\theta\cos\phi\underline{e}_R + \cos\theta\underline{e}_\theta - \sin\theta\sin\phi\underline{e}_\phi \\ \underline{k} = \sin\phi\underline{e}_R + \cos\phi\underline{e}_\phi \end{cases}$$

$$\begin{aligned} \underline{\dot{e}}_R &= (-\dot{\phi}\sin\phi\cos\theta - \dot{\theta}\cos\phi\sin\theta)(\cos\theta\cos\phi\underline{e}_R - \sin\theta\underline{e}_\theta - \sin\phi\cos\theta\underline{e}_\phi) \\ &\quad + (\dot{\theta}\cos\phi\cos\theta - \dot{\phi}\sin\phi\sin\theta)(\sin\theta\cos\phi\underline{e}_R + \cos\theta\underline{e}_\theta - \sin\phi\sin\theta\underline{e}_\phi) \\ &\quad + \dot{\phi}\cos\phi(\sin\phi\underline{e}_R + \cos\phi\underline{e}_\phi) = 0\underline{e}_R + \dot{\theta}\cos\phi\underline{e}_\theta + \dot{\phi}\underline{e}_\phi \end{aligned}$$

$$\begin{aligned} \underline{\dot{e}}_\theta &= -\dot{\theta}\cos\theta(\cos\theta\cos\phi\underline{e}_R - \sin\theta\underline{e}_\theta - \cos\theta\sin\phi\underline{e}_\phi) \\ &\quad - \dot{\theta}\sin\theta(\sin\theta\cos\phi\underline{e}_R + \cos\theta\underline{e}_\theta - \sin\theta\sin\phi\underline{e}_\phi) \\ &= -\dot{\theta}\cos\phi\underline{e}_R + 0\underline{e}_\theta + \dot{\theta}\sin\phi\underline{e}_\phi \end{aligned}$$

$$\begin{aligned} \underline{\dot{e}}_\phi &= (-\dot{\phi}\cos\phi\cos\theta + \dot{\theta}\sin\phi\sin\theta)(\cos\phi\cos\theta\underline{e}_R - \sin\theta\underline{e}_\theta - \sin\phi\cos\theta\underline{e}_\phi) \\ &\quad - (\dot{\phi}\cos\phi\sin\theta + \dot{\theta}\sin\phi\cos\theta)(\cos\phi\sin\theta\underline{e}_R + \cos\theta\underline{e}_\theta - \sin\phi\sin\theta\underline{e}_\phi) \\ &\quad - \dot{\phi}\sin\phi(\sin\phi\underline{e}_R + \cos\phi\underline{e}_\phi) = -\dot{\phi}\underline{e}_R - \dot{\theta}\sin\phi\underline{e}_\theta + 0\underline{e}_\phi \end{aligned}$$

NOW, WE SUBSTITUTE FOR $\underline{\dot{e}}_R$, $\underline{\dot{e}}_\theta$, AND $\underline{\dot{e}}_\phi$ IN $\underline{a}$ AND COLLECT TERMS.

$$\begin{aligned} \underline{a} &= \ddot{R}\underline{e}_R + \dot{R}\dot{\underline{e}}_R + \dot{R}\dot{\theta}\cos\phi\underline{e}_\theta + \dot{R}\dot{\phi}\underline{e}_\phi + (\dot{R}\ddot{\theta}\cos\phi + R\ddot{\theta}\cos\phi - R\dot{\theta}\dot{\phi}\sin\phi)\underline{e}_\theta \\ &\quad - R\dot{\theta}^2\cos^2\phi\underline{e}_R + R\dot{\theta}^2\sin\phi\cos\phi\underline{e}_\phi + (\dot{R}\dot{\phi} + R\ddot{\phi})\underline{e}_\phi - R\dot{\phi}^2\underline{e}_R \\ &\quad - R\dot{\theta}\dot{\phi}\sin\phi\underline{e}_\theta \end{aligned}$$

$$\begin{aligned} \underline{a} &= (\ddot{R} - R\dot{\theta}^2\cos^2\phi - R\dot{\phi}^2)\underline{e}_R + (2\dot{R}\dot{\theta}\cos\phi - 2R\dot{\theta}\dot{\phi}\sin\phi + R\ddot{\theta}\cos\phi)\underline{e}_\theta \\ &\quad + (2\dot{R}\dot{\phi} + R\ddot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi)\underline{e}_\phi \quad \left(\text{THIS IS EQUIVALENT TO THE BOOK EXPRESSION!} \right) \end{aligned}$$

► 2/146 $R = \text{const}$ $\theta = \omega t$ $\sin \phi = z/R$
 $z = \frac{h}{2}(1 - \cos 2\theta)$, $\dot{z} = \omega h \sin 2\theta$ where $\dot{\theta} = \omega$
 $(\cos \phi) \dot{\phi} = \frac{1}{R} \dot{z}$, $\dot{\phi} = \frac{\omega h \sin 2\theta}{R \cos \phi}$

$$v_R = \dot{R} = 0$$

$$v_\theta = R \dot{\phi} \cos \phi = R \omega \sqrt{1 - \sin^2 \phi} = R \omega \sqrt{1 - \left(\frac{h}{2R}[1 - \cos 2\theta]\right)^2}$$

$$v_\phi = R \dot{\phi} = \frac{\omega h \sin 2\theta}{\cos \phi} = h \omega \frac{\sin 2\theta}{\sqrt{1 - \left(\frac{h}{2R}[1 - \cos 2\theta]\right)^2}}$$

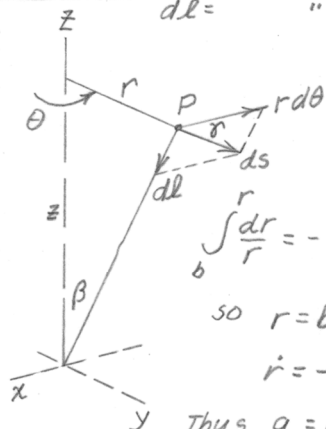
When $\theta = \omega t = \pi/4$, $1 - \cos 2\theta = 1$ so that

$$\underline{v_\theta = R \omega \sqrt{1 - (h/2R)^2}}, \quad \underline{v_\phi = \frac{h \omega}{\sqrt{1 - (h/2R)^2}}}, \quad \underline{v_R = 0}$$

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► 2/14.7

ds = differential distance along curve
 $d\ell$ = " " " in direction of cone element



$$r = \frac{b}{h} z, \tan \beta = b/h$$

$$d\ell = r d\theta \tan \gamma$$

$$dr = -d\ell \sin \beta = -r d\theta \tan \gamma \sin \beta$$

$$\int_b^r \frac{dr}{r} = -\tan \gamma \sin \beta \int_0^\theta d\theta, \ln r \Big|_b^r = -\tan \gamma \sin \beta \theta$$

$$\text{so } r = b e^{-\tan \gamma \sin \beta \theta}, r = b e^{-K\theta} \text{ where } K = \tan \gamma \sin \beta$$

$$\dot{r} = -bK\dot{\theta} e^{-K\theta}, \ddot{r} = bK^2\dot{\theta}^2 e^{-K\theta}, \ddot{\theta} = 0$$

$$\text{Thus } a_r = \ddot{r} - r\dot{\theta}^2 = b\dot{\theta}^2 e^{-K\theta} (K^2 - 1)$$

$$\text{or } a_r = b\dot{\theta}^2 (\tan^2 \gamma \sin^2 \beta - 1) e^{-\theta \tan \gamma \sin \beta}$$

$$\text{where } \beta = \tan^{-1} \frac{b}{h}$$

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2/148 The terms appearing in Eq. 2/19 are

$$R = 50 + 200\left(\frac{1}{2}\right)^2 = 100 \text{ mm}, \quad \dot{R} = 400t = 400\left(\frac{1}{2}\right) = 200 \text{ mm/s}$$

$$\ddot{R} = 400 \text{ mm/s}^2$$

$$\theta = \omega t = \frac{\pi}{3}\left(\frac{1}{2}\right) = \pi/6 \text{ rad}, \quad \dot{\theta} = \pi/3 \text{ rad/s}, \quad \ddot{\theta} = 0$$

$$\phi = \dot{\phi} t = \frac{2\pi}{3}\left(\frac{1}{2}\right) = \pi/3 \text{ rad}, \quad \dot{\phi} = 2\pi/3 \text{ rad/s}, \quad \ddot{\phi} = 0$$

$$\sin \theta = 1/2, \quad \cos \theta = \sqrt{3}/2, \quad \sin \phi = \sqrt{3}/2, \quad \cos \phi = 1/2$$

$$\frac{d}{dt}(R^2\dot{\theta}) = 2R\dot{R}\dot{\theta} + R^2\ddot{\theta} = 2(0.1)(0.2)\pi/3 + 0 = \frac{0.04\pi}{3} \text{ (m/s)}^2$$

$$\frac{d}{dt}(R^2\dot{\phi}) = 2R\dot{R}\dot{\phi} + R^2\ddot{\phi} = 2(0.1)(0.2)2\pi/3 + 0 = \frac{0.08\pi}{3} \text{ (m/s)}^2$$

Thus the components of $\underline{a}$ from Eq. 2/19 become

$$a_R = 0.40 - 0.10(2\pi/3)^2 - 0.10(\pi/3)^2(1/2)^2 = -0.0661 \text{ m/s}^2$$

$$a_\theta = \frac{1/2}{0.10} \frac{0.04\pi}{3} - 2(0.10)(\pi/3)(2\pi/3)(\sqrt{3}/2) = -0.1704 \text{ m/s}^2$$

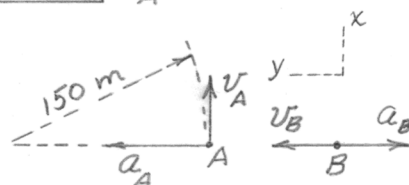
$$a_\phi = \frac{1}{0.10} \frac{0.08\pi}{3} + 0.10(\pi/3)^2(\sqrt{3}/2)(1/2) = 0.885 \text{ m/s}^2$$

The magnitude of the acceleration is, then,

$$a = \sqrt{(-0.0661)^2 + (-0.1704)^2 + (0.885)^2} = \underline{0.904 \text{ m/s}^2}$$

WILEY

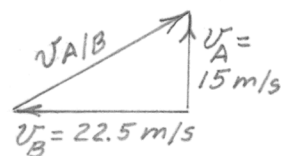
$$2/149 \quad v_A = 54/3.6 = 15 \text{ m/s}, \quad v_B = 81/3.6 = 22.5 \text{ m/s}$$



$$a_A = \frac{v_A^2}{\rho} = \frac{15^2}{150} = 1.5 \text{ m/s}^2$$

$$a_B = 3 \text{ m/s}^2$$

$$v_A = v_B + v_{A/B}$$

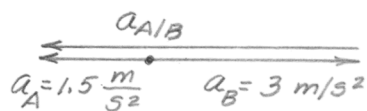


$$v_{A/B} = \sqrt{(22.5)^2 + (15)^2}$$

$$= 27.0 \text{ m/s}$$

$$\underline{v_{A/B}} = 15\hat{i} - 22.5\hat{j} \text{ m/s}$$

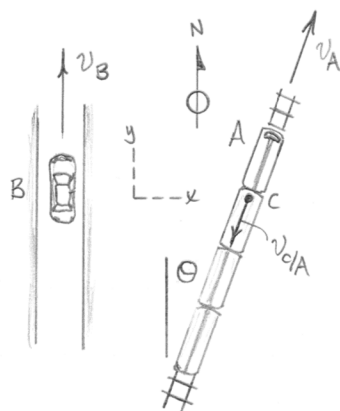
$$a_A = a_B + a_{A/B}$$



$$\underline{a_{A/B}} = 4.5\hat{j} \text{ m/s}^2$$

WILEY

2/150



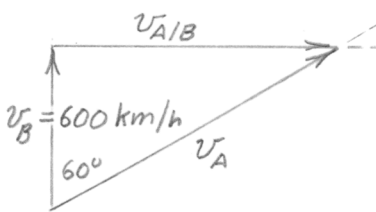
$$\begin{cases} v_A = 35 \frac{\text{mi}}{\text{hr}} = 51.3 \text{ ft/sec} \\ v_{C/A} = 4 \text{ ft/sec} \end{cases}$$

$$v_{B/C} = 16 \text{ ft/sec WEST}$$

$$\underline{v}_C = \underline{v}_A + \underline{v}_{C/A} = (51.3 - 4)(\sin \theta \underline{i} + \cos \theta \underline{j})$$

$$\begin{cases} \underline{v}_B = \underline{v}_C + \underline{v}_{B/C} \\ v_B \underline{j} = (51.3 - 4)(\sin \theta \underline{i} + \cos \theta \underline{j}) - v_{B/C} \underline{i} \end{cases}$$

$$\begin{cases} \underline{i}: 0 = (51.3 - 4) \sin \theta - 16 \longrightarrow \theta = 19.76^\circ \\ \underline{j}: v_B = (51.3 - 4) \cos \theta \longrightarrow \underline{v}_B = 44.5 \text{ ft/sec OR } 30.4 \text{ mi/hr} \end{cases}$$

$$\begin{aligned} 2/151 \quad \vec{v}_A &= \vec{v}_B + \vec{v}_{A/B} \\ \vec{v}_A &= \frac{600}{\cos 60^\circ} = 1200 \text{ km/h} \\ \vec{v}_{A/B} &= 600 \tan 60^\circ \\ &= 1039 \text{ km/h} \end{aligned}$$


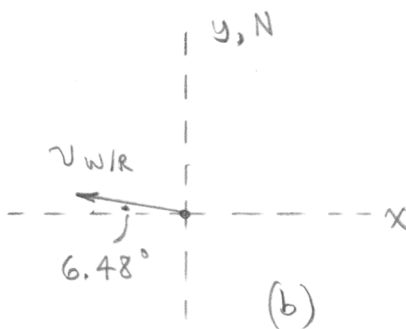
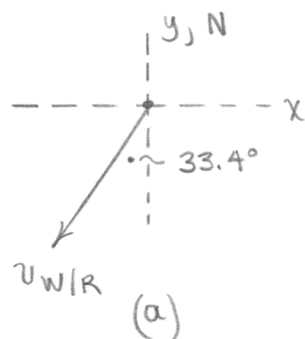
A velocity triangle diagram showing the relationship between three vectors. Vector $\vec{v}_B$ is a vertical arrow pointing upwards, labeled $v_B = 600 \text{ km/h}$. Vector $\vec{v}_{A/B}$ is a horizontal arrow pointing to the right, starting from the tip of $\vec{v}_B$. Vector $\vec{v}_A$ is the hypotenuse of the triangle, starting from the tail of $\vec{v}_B$ and ending at the tip of $\vec{v}_{A/B}$. The angle between $\vec{v}_B$ and $\vec{v}_A$ is labeled 60° . A dashed line extends from the tip of $\vec{v}_{A/B}$ to the right.

WILEY

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$$\begin{aligned}\underline{v}_{W/R} &= \underline{v}_W - \underline{v}_R \\ &= 15(-\cos 35^\circ \underline{i} - \sin 35^\circ \underline{j}) - 10 \underline{j} \\ &= -12.29 \underline{i} - 18.60 \underline{j} \text{ mi/hr}\end{aligned}$$

$$v_{W/R} = 22.3 \text{ mi/hr}, 33.4^\circ \text{ West of due South}$$



$$\begin{aligned}(b) \quad \underline{v}_{W/R} &= \underline{v}_W - \underline{v}_R \\ &= 15(-\cos 35^\circ \underline{i} - \sin 35^\circ \underline{j}) - (-10 \underline{j}) \\ &= -12.29 \underline{i} + 1.396 \underline{j} \text{ mi/hr}\end{aligned}$$

$$v_{W/R} = 12.37 \text{ mi/hr}, 6.48^\circ \text{ north of due west}$$

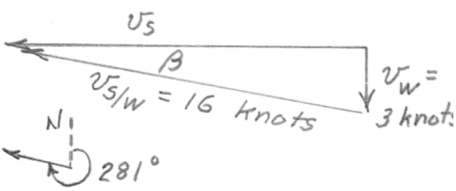
Runner has a headwind in both cases!

2/153 $w = \text{water}, s = \text{ship}$

$$\underline{v}_s = \underline{v}_w + \underline{v}_{s/w}$$

$$\beta = \sin^{-1} 3/16 = 10.8^\circ$$

$$\text{Heading} = 270 + 10.8 = 281^\circ$$

$$\text{time } t = \frac{\text{Dist.}}{\text{vel.}} = \frac{24}{\sqrt{(16)^2 - 3^2}} = 1.527 \text{ hr}$$


WILEY

$$\begin{aligned}
 \underline{2/154} \quad \underline{\underline{v_{A/B} = v_A - v_B}}, \quad \underline{\underline{\Omega = 3(2\pi/60) = 0.314 \frac{\text{rad}}{\text{s}}}} \\
 = \underline{\underline{\frac{18}{3.6} \underline{i} - 9(0.314)(\cos 45^\circ \underline{i} - \sin 45^\circ \underline{j})}} \\
 = \underline{\underline{3.00 \underline{i} + 1.999 \underline{j} \text{ m/s}}}
 \end{aligned}$$

$$\begin{aligned}
 \underline{\underline{a_{A/B} = a_A - a_B = 3 \underline{i} - 9(0.314)^2(-\cos 45^\circ \underline{i} - \sin 45^\circ \underline{j})}} \\
 = \underline{\underline{3.63 \underline{i} + 0.628 \underline{j} \text{ m/s}^2}}
 \end{aligned}$$

WILEY

$$\begin{array}{l}
 \boxed{2/155} \\
 \begin{array}{c} y \\ | \\ \text{---} x \end{array}
 \end{array}
 \quad
 \begin{aligned}
 \underline{v}_{W/B} &= \underline{v}_W - \underline{v}_B \\
 &= 10(\sin 40^\circ \underline{i} + \cos 40^\circ \underline{j}) - v_B \underline{i} \\
 &= (6.43 - v_B) \underline{i} + 7.66 \underline{j}
 \end{aligned}$$

The captain can reduce the head/tailwind to zero : $6.43 - v_B = 0 \Rightarrow v_B = 6.43 \text{ m/s}$

The passengers will still feel a side wind of 7.66 m/s from the south (starboard side).

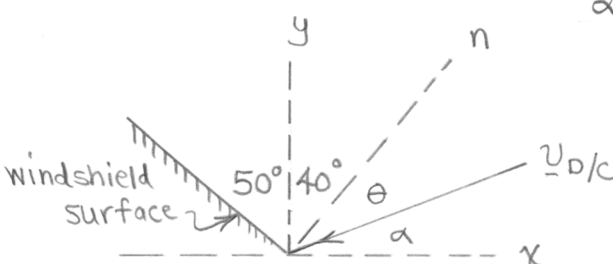
WILEY

2/156 Drop: $v_D = \sqrt{2gh} = \sqrt{2(9.81)(6)} = 10.85 \text{ m/s}$

Car: $v_C = 100/3.6 = 27.8 \text{ m/s}$

$\underline{v}_{D/C} = \underline{v}_D - \underline{v}_C = -10.85\hat{j} - 27.8\hat{i} \text{ m/s}$

$\alpha = \tan^{-1} \frac{10.85}{27.8} = 21.3^\circ$



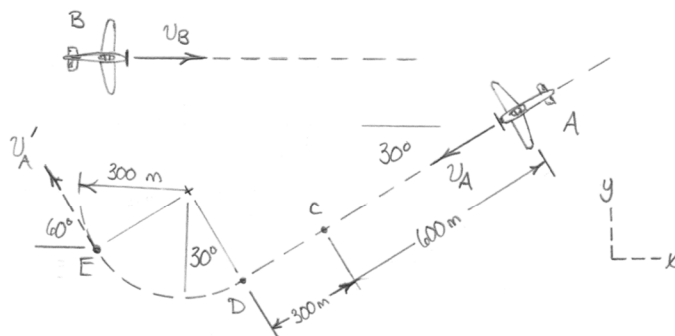
$40^\circ + \theta + \alpha = 90^\circ \Rightarrow \underline{\theta = 28.7^\circ \text{ below normal}}$

WILEY

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$$v_A = 285 \text{ km/h (CONSTANT)}$$

$$v_B = 350 \text{ km/h (CONSTANT)}$$



A_T C :

$$v_{A/B} = v_A - v_B = 285(-\cos 30^\circ \underline{i} - \sin 30^\circ \underline{j}) - 350 \underline{i}$$

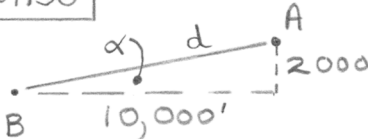
$$v_{A/B} = -597 \underline{i} - 142.5 \underline{j} \text{ km/h}$$

A_T E :

$$v_{A/B} = v_A - v_B = 285(-\sin 30^\circ \underline{i} + \cos 30^\circ \underline{j}) - 350 \underline{i}$$

$$v_{A/B} = -492 \underline{i} + 247 \underline{j} \text{ km/h}$$

2/158



$$\alpha = \tan^{-1} \frac{2}{10} = 11.31^\circ$$

$$d = \sqrt{2000^2 + 10,000^2} = 10,200 \text{ ft}$$

$$v_{B/A} = \frac{d}{t} = \frac{10,200}{120} = 85.0 \text{ ft/sec}$$

must lie along BA.

$$\begin{aligned} \underline{v}_B &= \underline{v}_A + \underline{v}_{B/A} \\ &= 300 \left(\frac{88}{60} \right) \underline{i} + 85.0 (\cos 11.31^\circ \underline{i} + \sin 11.31^\circ \underline{j}) \\ &= 523 \underline{i} + 16.67 \underline{j} \text{ ft/sec} \\ &\text{(or } 524 \text{ ft/sec @ } 1.824^\circ \rightarrow) \end{aligned}$$

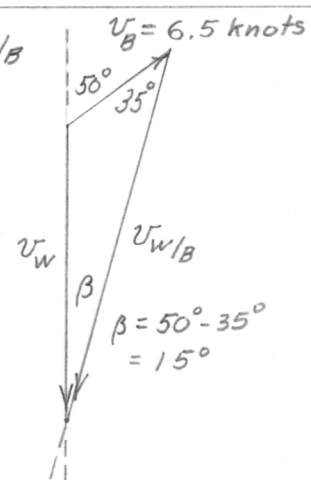
WILEY

2/159 $w = \text{wind}$ $\vec{v}_w = \vec{v}_B + \vec{v}_{w/B}$
 $B = \text{boat}$

Law of sines,

$$\frac{v_w}{\sin 35^\circ} = \frac{6.5}{\sin 15^\circ}$$

$$v_w = \frac{6.5(0.5736)}{0.2588} = \underline{14.40 \text{ knots}}$$



$v_B = 6.5 \text{ knots}$
 50°
 35°
 β
 $\beta = 50^\circ - 35^\circ = 15^\circ$
 v_w
 $v_{w/B}$

WILEY

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$v_A = 40 \text{ km/h} = 11.11 \text{ m/s}$
 $v_B = 30 \text{ km/h} = 8.33 \text{ m/s}$

$r = \sqrt{105^2 + 75^2 - 2(105)(75)\cos(45^\circ + 30^\circ)}$
 $\quad \quad \quad = 112.1 \text{ m}$

$\phi = \cos^{-1} \left[\frac{105^2 + 112.1^2 - 75^2}{2(105)(112.1)} \right] = 40.2^\circ$

$\theta = 90^\circ - \phi = 49.8^\circ$
 $\beta = \theta - 30^\circ = 19.75^\circ$

$\begin{cases} \underline{e}_r = -\sin \beta \underline{i} + \cos \beta \underline{j} \\ \underline{e}_\theta = -\cos \beta \underline{i} - \sin \beta \underline{j} \end{cases}$

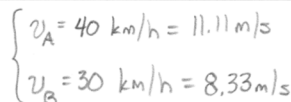
$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = 11.11(\cos 45^\circ \underline{i} - \sin 45^\circ \underline{j}) - 8.33(\sin 30^\circ \underline{i} + \cos 30^\circ \underline{j})$

$\underline{v}_{A/B} = 3.69 \underline{i} - 15.07 \underline{j} \text{ m/s}$

$\underline{v}_r = \underline{v}_{A/B} \cdot \underline{e}_r = (3.69 \underline{i} - 15.07 \underline{j}) \cdot (-\sin 19.75^\circ \underline{i} + \cos 19.75^\circ \underline{j}) \rightarrow \underline{v}_r = \dot{r} = -15.43 \text{ m/s}$

$\underline{v}_\theta = \underline{v}_{A/B} \cdot \underline{e}_\theta = (3.69 \underline{i} - 15.07 \underline{j}) \cdot (-\cos 19.75^\circ \underline{i} - \sin 19.75^\circ \underline{j}) \rightarrow \underline{v}_\theta = 1.622 \text{ m/s}$

$\underline{v}_\theta = r \dot{\theta} \rightarrow 1.622 = 112.1 \dot{\theta} \rightarrow \underline{\dot{\theta}} = 0.01446 \text{ rad/s}$



$$\begin{cases} a_{At} = 1.25 \text{ m/s}^2 \\ a_{An} = \frac{v_A^2}{\rho_A} = \frac{11.1^2}{75} = 1.646 \text{ m/s}^2 \end{cases}$$

$$\begin{cases} a_{Bt} = 2,5 \text{ m/s}^2 \\ a_{Bn} = \frac{v_B^2}{r_B} = \frac{8,33^2}{105} = 0,661 \text{ m/s}^2 \end{cases}$$

$$\begin{cases} r = 112,1 \text{ m}, & \phi = 40,2^\circ \\ \theta = 49,8^\circ, & \beta = 19,75^\circ \\ \dot{r} = -15,43 \text{ m/s}, & \dot{\theta} = 0,01446 \end{cases}$$

$$\begin{cases} \underline{e}_r = -\sin\beta \underline{i} + \cos\beta \underline{j} \\ \underline{e}_\theta = -\cos\beta \underline{i} - \sin\beta \underline{j} \end{cases}$$

$$\begin{aligned} a_{A/B} &= a_A - a_B = 1.25(-\cos 45^\circ \underline{i} + \sin 45^\circ \underline{j}) + 1.646(-\cos 45^\circ \underline{i} - \sin 45^\circ \underline{j}) \\ &\quad - [2.5(\sin 30^\circ \underline{i} + \cos 30^\circ \underline{j}) + 0.661(-\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j})] \\ a_{A/B} &= -2.73 \underline{i} - 2.78 \underline{j} \text{ m/s}^2 \end{aligned}$$

$$\begin{cases} \vec{a}_r = \vec{a}_{A/B} \cdot \vec{e}_r = (-2.73 \vec{i} - 2.78 \vec{j}) \cdot (-\sin 19.75^\circ \vec{i} + \cos 19.75^\circ \vec{j}) = -1.691 \text{ m/s}^2 \\ \vec{a}_r = \ddot{r} - r \dot{\theta}^2 \rightarrow -1.691 = \ddot{r} - 112.1 (0.01446)^2 \rightarrow \ddot{r} = -1.668 \text{ m/s}^2 \end{cases}$$

$$a_{\theta} = a_{A/B} \cdot e_{\theta} = (-2.73 \underline{i} - 2.78 \underline{j}) \cdot (-\cos 19.75^{\circ} \underline{i} - \sin 19.75^{\circ} \underline{j}) = 3.50 \text{ m/s}^2$$

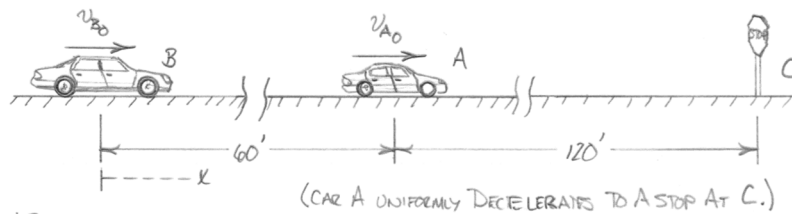
$$a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} \rightarrow 3,50 = 112,1\ddot{\theta} + 2(-15,43)(0,01446) \rightarrow \ddot{\theta} = 0,0352 \text{ rad/s}^2$$

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$$v_{A_0} = 25 \frac{\text{mi}}{\text{hr}} \text{ or } 36.7 \text{ ft/sec}$$

$$v_{B_0} = 40 \frac{\text{mi}}{\text{hr}} \text{ or } 58.7 \text{ ft/sec}$$

CAR B WAITS 1.3 SEC BEFORE DECELERATING AT 18 ft/sec^2



• FIRST 1.3 SECONDS:

$$\begin{cases} x_A' = x_{A_0} + v_{A_0}t - \frac{1}{2}a_A t^2 = 60 + 36.7(1.3) - \frac{1}{2}\left(\frac{36.7^2}{2 \times 120}\right)(1.3)^2 = 102.9 \text{ ft} \\ v_A = v_{A_0} - a_A t = 36.7 - \left(\frac{36.7^2}{2 \times 120}\right)(1.3) = 29.4 \text{ ft/sec} \\ x_B = x_{B_0} + v_{B_0}t = 0 + 58.7(1.3) = 76.3 \text{ ft} \quad \text{AND} \quad v_B = v_{B_0} = 58.7 \text{ ft/sec} \end{cases}$$

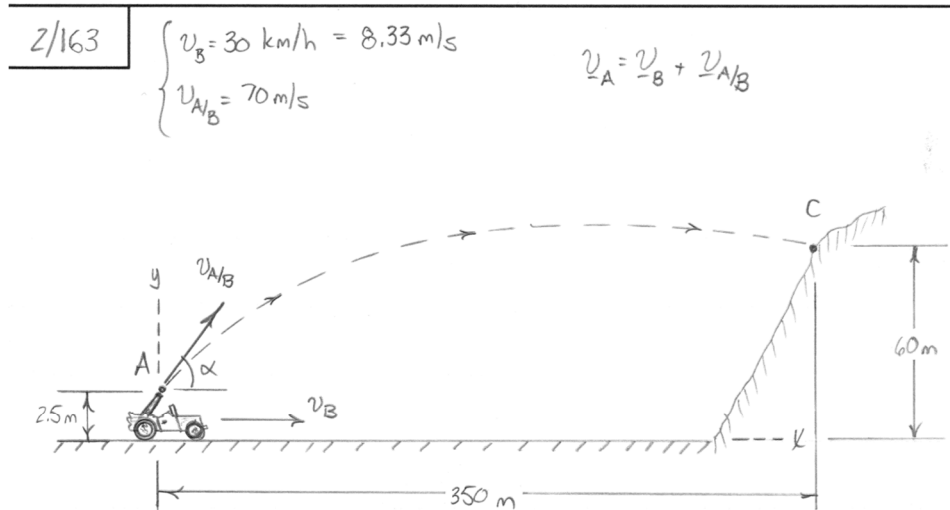
• COLLISION POINT: MEASURE TIME FROM END OF 1.3 SECONDS

$$\begin{aligned} x_A' &= x_B' \rightarrow x_A + v_A \Delta t - \frac{1}{2}a_A \Delta t^2 = x_B + v_B \Delta t - \frac{1}{2}a_B \Delta t^2 \\ 102.9 + 29.4 \Delta t - \frac{1}{2}\left(\frac{36.7^2}{2 \times 120}\right) \Delta t^2 &= 76.3 + 58.7 \Delta t - \frac{1}{2}(18) \Delta t^2 \\ \therefore \Delta t &= 1.232 \text{ sec} \quad \text{OR} \quad \Delta t = 3.49 \text{ sec} \quad (\text{CHOOSE THE FIRST}) \end{aligned}$$

$$x_B' = 76.3 + 58.7(1.232) - \frac{1}{2}(18)(1.232)^2 \rightarrow x_A' = x_B' = 134.9 \text{ ft} \quad (\text{BEFORE C})$$

$$\begin{cases} v_A' = v_A - a_A \Delta t = 29.4 - \left(\frac{36.7^2}{2 \times 120}\right)(1.232) = 22.5 \text{ ft/sec} \\ v_B' = v_B - a_B \Delta t = 58.7 - 18(1.232) = 36.5 \text{ ft/sec} \end{cases}$$

$$v_{B/A} = v_B - v_A = 36.5 \text{ ft/sec} - 22.5 \text{ ft/sec} = 14.01 \text{ ft/sec} \quad \text{SO...} \quad v_{B/A} = 14.01 \text{ ft/sec} = 9.55 \frac{\text{mi}}{\text{hr}}$$



$$\begin{cases} x_C = x_A + v_{x0} t_{AC} \rightarrow 350 = (8.33 + 70 \cos \alpha) t_{AC} \\ y_C = y_A + v_{y0} t_{AC} - \frac{1}{2} g t_{AC}^2 \rightarrow 60 = 2.5 + 70 \sin \alpha t_{AC} - \frac{1}{2} (9.81) t_{AC}^2 \end{cases}$$

Solving...

$$\begin{cases} t_{AC} = 5.14 \text{ s} \\ \alpha = 31.3^\circ \end{cases} \quad \text{OR} \quad \begin{cases} t_{AC} = 12.82 \text{ s} \\ \alpha = 74.3^\circ \end{cases}$$

IF SITTING STILL...

$$\begin{cases} x_C = x_A + v_{x0} t_{AC} \rightarrow 350 = 70 \cos \alpha t_{AC} \\ y_C = y_A + v_{y0} t_{AC} - \frac{1}{2} g t_{AC}^2 \rightarrow 60 = 2.5 + 70 \sin \alpha t_{AC} - \frac{1}{2} (9.81) t_{AC}^2 \end{cases}$$

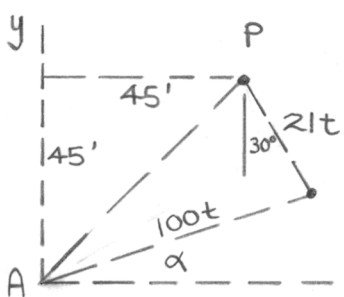
Solving...

$$\begin{cases} t_{AC} = 6.03 \text{ s} \\ \alpha = 34.0^\circ \end{cases} \quad \text{OR} \quad \begin{cases} t_{AC} = 11.99 \text{ s} \\ \alpha = 65.4^\circ \end{cases}$$

$$\begin{aligned} 2/164 \quad \text{Use } g &= g_0 \left(\frac{R}{R+h} \right)^2 \\ \text{For A, } g_A &= 32.23 \left(\frac{3959}{3959+200} \right)^2 = 29.2 \text{ ft/sec}^2 \\ \text{For B, } g_B &= 32.23 \left(\frac{3959}{3959+22,300} \right)^2 = 0.733 \text{ ft/sec}^2 \\ a_{B/A} &= a_B - a_A = +0.733 \underline{i} - (-29.2) \underline{j} \\ &= +0.733 \underline{i} + 29.2 \underline{j} \text{ ft/sec}^2 \end{aligned}$$

WILEY

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$$\begin{cases} 100t \cos \alpha = 45 + 21t \sin 30^\circ \\ 100t \sin \alpha = 45 - 21t \cos 30^\circ \end{cases}$$

or

$$\begin{cases} 100 \cos \alpha = \frac{45}{t} + 21 \sin 30^\circ \\ 100 \sin \alpha = \frac{45}{t} - 21 \cos 30^\circ \end{cases}$$

Subtract 2nd from 1st:

$$100 (\cos \alpha - \sin \alpha) = 21 \left(\frac{1}{t} + \frac{\sqrt{3}}{2} \right) \text{ or } \cos \alpha - \sin \alpha = 0.287$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = 0.287 + \sin \alpha$$

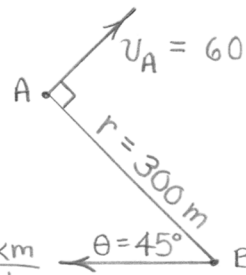
SQBS & rearrange : $2 \sin^2 \alpha + 0.574 \sin \alpha - 0.918 = 0$

Positive solution : $\alpha = 33.3^\circ$; Then $t = 0.616 \text{ sec}$

$$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = 100 [\cos \alpha \underline{i} + \sin \alpha \underline{j}] - 21 [\sin 30^\circ \underline{i} - \cos 30^\circ \underline{j}]$$

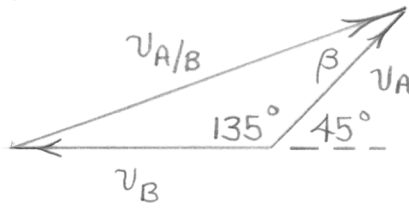
$$= \underline{73.1 \underline{i} + 73.1 \underline{j} \text{ ft/sec}}$$

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$v_A = 60 \text{ km/h}$ $v_B = 80 \frac{\text{km}}{\text{h}}$ $\theta = 45^\circ$ $r = 300 \text{ m}$

$\underline{v_A} = \underline{v_B} + \underline{v_{A/B}}$



$v_{A/B}$ β v_A 135° 45° v_B

$$v_{A/B}^2 = 60^2 + 80^2 - 2(60)(80)\cos 135^\circ$$

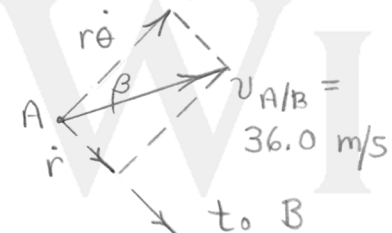
$$v_{A/B} = 129.6 \text{ km/h} \quad \text{or} \quad \underline{36.0 \text{ m/s}}$$

$$\frac{36.0}{\sin 135^\circ} = \frac{80/3.6}{\sin \beta}, \quad \beta = 25.9^\circ$$

$$r\dot{\theta} = v_{A/B} \cos \beta : 300 \dot{\theta} = 36.0 \cos 25.9^\circ$$

$$\underline{\dot{\theta} = 0.1079 \text{ rad/s}}$$

$$\dot{r} = -v_{A/B} \sin \beta : \dot{r} = -36.0 \sin 25.9^\circ$$

$$\underline{\dot{r} = -15.71 \text{ m/s}}$$


$\dot{r}$ $r\dot{\theta}$ β $v_{A/B} = 36.0 \text{ m/s}$ $t \rightarrow B$

2/167 From Prob. 2/166,

$\dot{r} = -15.71 \text{ m/s}, \quad \dot{\theta} = 0.1079 \text{ rad/s}$

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$

$a_A = \frac{v_A^2}{\rho} = \frac{(60/3.6)^2}{300}$

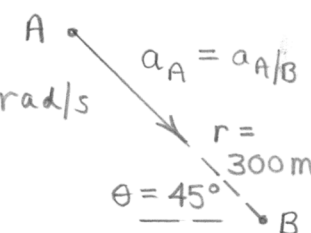
$= 0.926 \text{ m/s}^2 = a_{A/B}$

$(a_{A/B})_r = \ddot{r} - r\dot{\theta}^2 : -0.926 = \ddot{r} - 300(0.1079)^2$

$\ddot{r} = 2.57 \text{ m/s}^2$

$(a_{A/B})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 0 = 300\ddot{\theta} + 2(-15.71)(0.1079)$

$\ddot{\theta} = 0.01131 \text{ rad/s}^2$



WILEY

2/168 Find flight time t :

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2 : 7 = 3 + 100 \sin 30^\circ t - 16.1t^2$$

Solve to obtain 0.0822 sec (discard) & $t = 3.02$ sec

$$\text{Range } R = x_0 + v_{x0}t = 0 + 100 \cos 30^\circ (3.02) \\ = 262 \text{ ft}$$

$$\text{Fielder must run } 262 - 220 = 41.8 \text{ ft}$$

$$\text{in } (3.02 - 0.25) \text{ sec} \Rightarrow v_B = \frac{41.8}{2.77} = 15.08 \text{ ft/sec}$$

Velocity components of ball when caught :

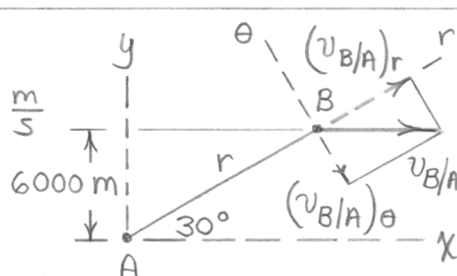
$$v_x = v_{x0} = 100 \cos 30^\circ = 86.6 \text{ ft/sec}$$

$$v_y = v_{y0} - gt = 100 \sin 30^\circ - 32.2(3.02) = -47.4 \frac{\text{ft}}{\text{sec}}$$

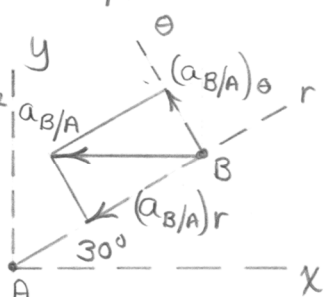
$$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = (86.6\hat{i} - 47.4\hat{j}) - 15.08\hat{i} \\ = \underline{71.5\hat{i} - 47.4\hat{j} \text{ ft/sec}}$$

WILEY

► 2/169 $v_{B/A} = v_B - v_A$
 $= (1500 - 1000) / 3.6 = 138.9 \frac{\text{m}}{\text{s}}$
 $(v_{B/A})_r = \dot{r} = 138.9 \cos 30^\circ$
 $= 120.3 \text{ m/s}$
 $(v_{B/A})_\theta = r\dot{\theta} = -138.9 \sin 30^\circ = \frac{6000}{\sin 30^\circ} \dot{\theta}$
 $\dot{\theta} = -0.00579 \text{ rad/s}$



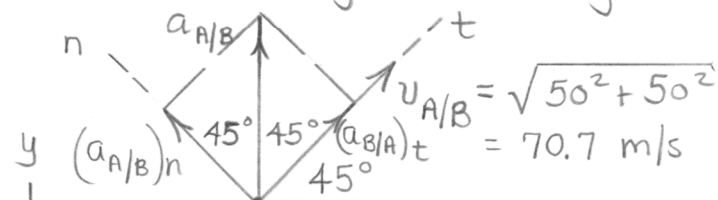
$a_{B/A} = a_B - a_A = 0 - 1.2 = -1.2 \text{ m/s}^2$
 $(a_{B/A})_r = \ddot{r} - r\dot{\theta}^2$
 $-1.2 \cos 30^\circ = \ddot{r} - 12000(-0.00579)^2$
 $\ddot{r} = -0.637 \text{ m/s}^2$
 $(a_{B/A})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
 $1.2 \sin 30^\circ = 12000\ddot{\theta} + 2(120.3)(-0.00579)$
 $\ddot{\theta} = 0.1660(10^{-3}) \text{ rad/s}^2$



►2/170 (a) $\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = 50\hat{i} - (-50\hat{j}) = 50\hat{i} + 50\hat{j} \text{ m/s}$

$$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = \frac{v_A^2}{r_A} \hat{j} - 0 = \frac{50^2}{2000} \hat{j} = 1.25 \hat{j} \text{ m/s}^2$$

(b) Use the results of part (a) for a normal-tangential analysis:



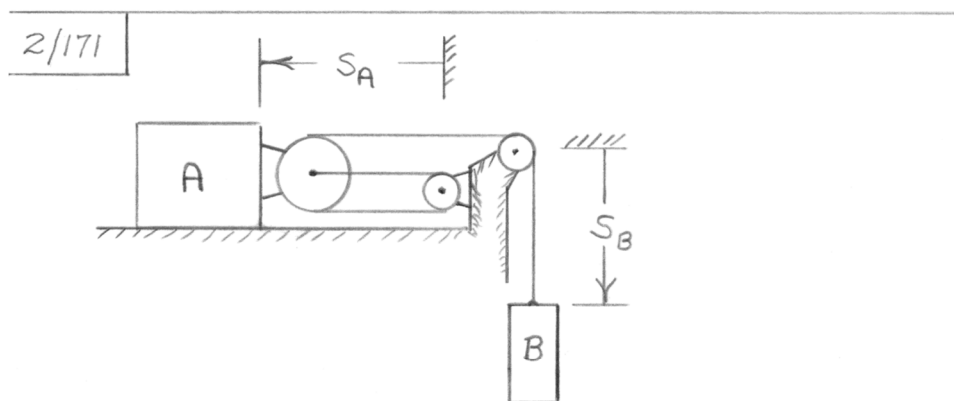
$$v_{A/B} = \sqrt{50^2 + 50^2} = 70.7 \text{ m/s}$$

$$\dot{r} = (a_{B/A})_t = a_{A/B} \cos 45^\circ = 1.250 \frac{\sqrt{2}}{2} = 0.884 \text{ m/s}^2$$

$$(a_{B/A})_n = a_{B/A} \sin 45^\circ = \frac{v_{A/B}^2}{r}$$

$$1.250 \frac{\sqrt{2}}{2} = \frac{70.7^2}{r}, r = 5660 \text{ m}$$

(in n-direction from A)



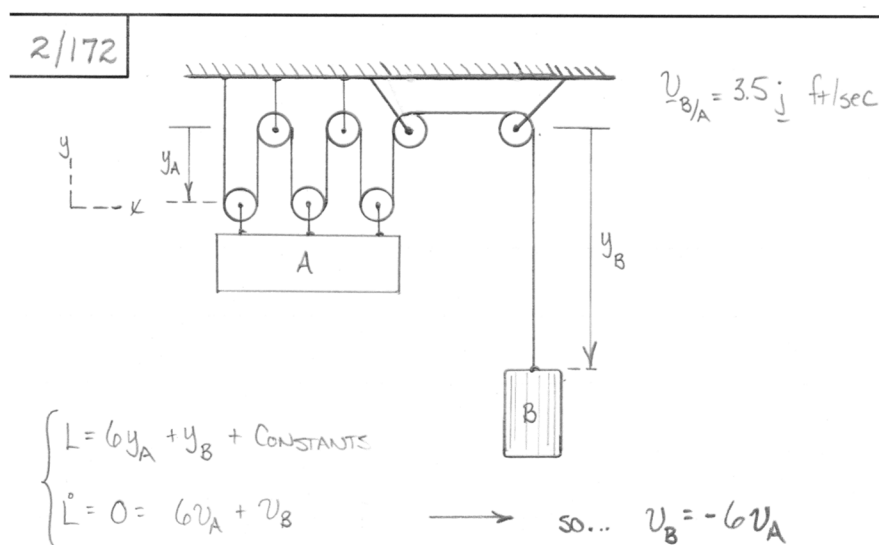
Length of cable $L = 3s_A + s_B + \text{constant}$

Differentiate: $0 = 3v_A + v_B$

$$v_B = -3v_A = -3(-3.6)$$

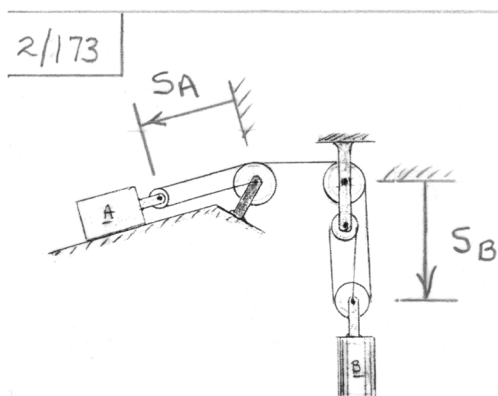
$$= \underline{10.8 \text{ ft/sec (down)}}$$

WILEY



$$\begin{cases} \underline{v}_B = \underline{v}_A + \underline{v}_{B/A} \longrightarrow \underline{v}_{B/A} = \underline{v}_B - \underline{v}_A \\ \underline{j}: 3.5 = \underline{v}_B - \underline{v}_A = -7\underline{v}_A \longrightarrow \underline{v}_A = -0.5 \underline{j} \text{ m/s} \end{cases}$$

$$\underline{v}_B = -6\underline{v}_A = -6(-0.5) \longrightarrow \underline{v}_B = 3 \underline{j} \text{ m/s}$$



Cable length $L = 2s_A + 3s_B + \text{constants}$

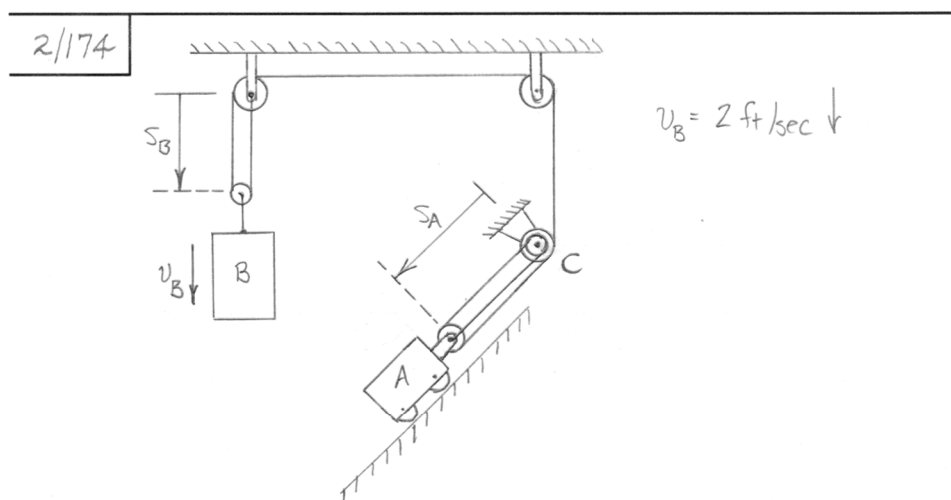
$$0 = 2v_A + 3v_B \quad (1)$$

$$0 = 2a_A + 3a_B \quad (2)$$

$$(1): v_A = -\frac{3}{2}v_B = -\frac{3}{2}(1.2) = \underline{-1.8 \text{ m/s (up)}}$$

$$(2): a_A = -\frac{3}{2}a_B = -\frac{3}{2}(-2) = \underline{3 \text{ m/s}^2 \text{ (down)}}$$

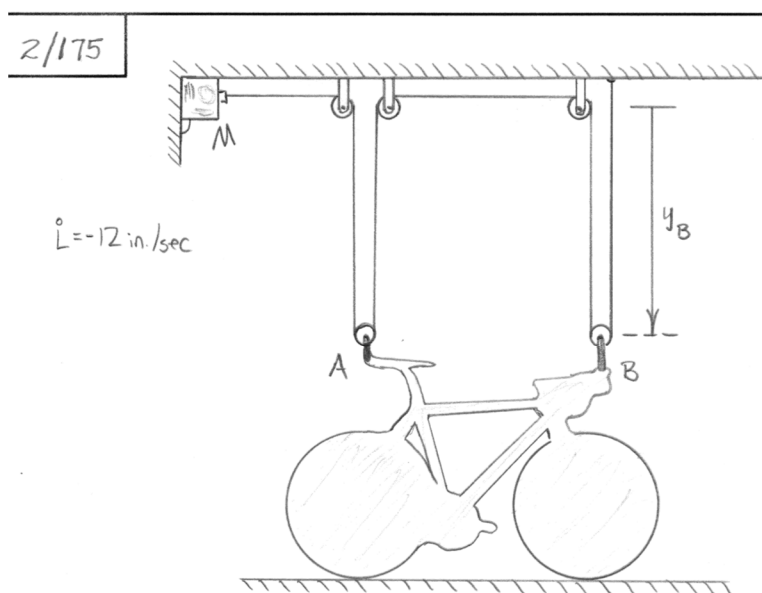
WILEY



$$\begin{cases} L = 2s_B + 3s_A + \text{CONSTANTS} \\ \dot{L} = 0 = 2v_B + 3v_A \end{cases}$$

So... $v_A = -\frac{2}{3}v_B = -\frac{2}{3}(2) = -\frac{4}{3} \rightarrow \underline{v_A = 1.333 \text{ ft/sec} \nearrow}$

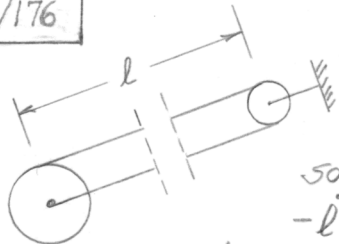
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$$\begin{cases} L = 4y_B + \text{CONSTANTS} \\ \dot{L} = -12 = 4\dot{y}_B = 4v_B \end{cases} \rightarrow v_B = -3 \text{ in./sec or } 3 \text{ in./sec} \uparrow$$

$$d = v_B t \rightarrow 5(12) = 3t \rightarrow \underline{t = 20 \text{ sec}}$$

2/176



Length of cable is

$$L = 2l$$

$$\dot{L} = 2\dot{l}$$

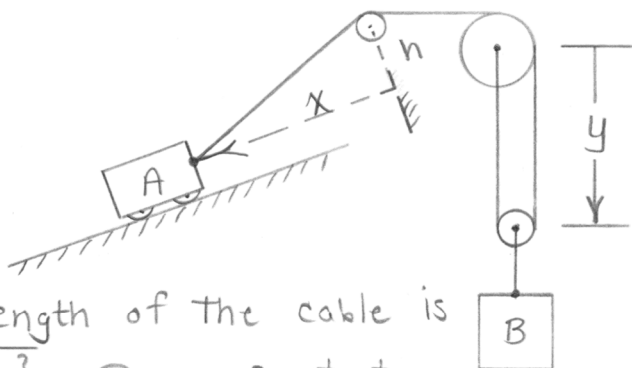
So velocity of truck is

$$-\dot{l} = \frac{1}{2}(-\dot{L}) = \frac{1}{2}(40) = 20 \text{ mm/s}$$

time $t = \frac{\text{distance}}{\text{velocity}} = \frac{4(10^3)}{20} = 200 \text{ s or } \underline{3 \text{ min } 20 \text{ s}}$

WILEY

2/177



The total length of the cable is

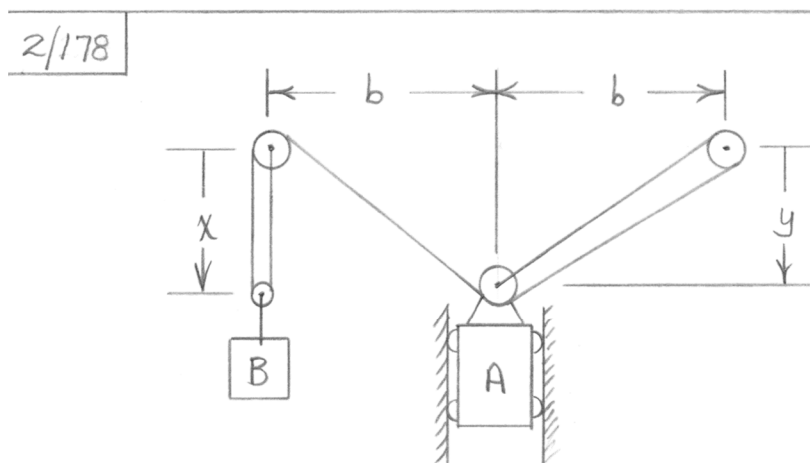
$$L = \sqrt{x^2 + h^2} + 2y + \text{constant}$$

$$\dot{L} = 0 = \frac{1}{2} \frac{2x\dot{x}}{\sqrt{x^2 + h^2}} + 2\dot{y}$$

With $v_A = \dot{x}$ & $v_B = -\dot{y}$:

$$v_A = + \frac{2\sqrt{x^2 + h^2}}{x} v_B$$

WILEY



The total length of the cable is

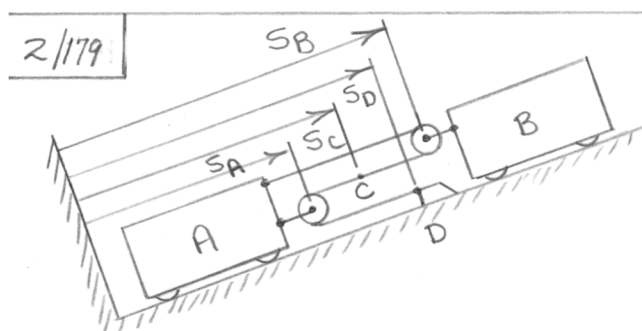
$$L = 2x + 3\sqrt{y^2 + b^2} + \text{constant}$$

Differentiate to obtain

$$\dot{L} = 0 = 2\dot{x} + 3\frac{y\dot{y}}{\sqrt{y^2 + b^2}}$$

With $\dot{x} = v_B$ and $\dot{y} = v_A$, we have

$$v_B = -\frac{3y}{2\sqrt{y^2 + b^2}}v_A$$



The cable length is $L = 2(s_B - s_A) + s_D - s_A$
 Differentiating:
 + Constants

$$0 = 2v_B - 3v_A \quad ; \quad 0 = 2a_B - 3a_A$$

$$\text{So } v_A = \frac{2}{3}v_B = \frac{2}{3}(3) = 2 \text{ ft/sec}$$

$$a_A = \frac{2}{3}a_B = \frac{2}{3}(6) = 4 \text{ ft/sec}^2$$

$$v_{B/A} = v_B - v_A = 3 - 2 = \underline{1 \text{ ft/sec}}$$

$$a_{B/A} = a_B - a_A = 6 - 4 = \underline{2 \text{ ft/sec}^2}$$

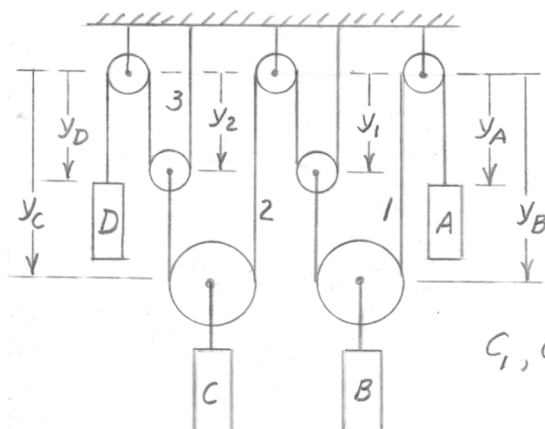
The length of cable between A and C is

$$L' = (s_B - s_A) + (s_B - s_C) = 2s_B - s_A - s_C + \text{constants}$$

$$\Rightarrow 0 = 2v_B - v_A - v_C \quad ; \quad v_C = 2v_B - v_A = 2(3) - 2 = \underline{4 \text{ ft/sec}}$$

(All answers are quantities directed up in cline.)

2/180



3 degrees of freedom

C_1, C_2, C_3 are constant lengths

$$L_1 = y_B + y_A + (y_B - y_1) + C_1; \quad 0 = 2\dot{y}_B + \dot{y}_A - \dot{y}_1$$

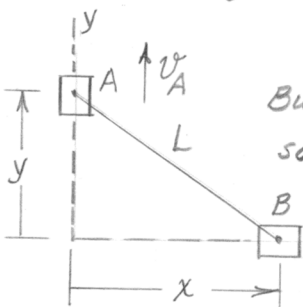
$$L_2 = y_C + 2y_1 + (y_C - y_2) + C_2 \quad 0 = 2\dot{y}_C + 2\dot{y}_1 - \dot{y}_2$$

$$L_3 = 2y_2 + y_D + C_3, \quad 0 = 2\dot{y}_2 + \dot{y}_D$$

Eliminate $\dot{y}_1$ & $\dot{y}_2$ & get $4\ddot{y}_A + 8\ddot{y}_B + 4\ddot{y}_C + \ddot{y}_D = 0$

WILEY

2/181



$$x^2 + y^2 = L^2; \quad x\dot{x} + y\dot{y} = 0$$

$$\neq \dot{x}^2 + x\ddot{x} + \dot{y}^2 + y\ddot{y} = 0$$

But $\dot{y} = v_A, \ddot{y} = 0$

so $a_x = \ddot{x} = -\frac{\dot{x}^2 + \dot{y}^2}{x} = -\frac{\dot{y}^2 \frac{y^2}{x^2} + \dot{y}^2}{x}$

$$a_x = -\frac{L^2 \dot{y}^2}{x^3} = -\frac{L^2 v_A^2}{(L^2 - y^2)^{3/2}}$$

WILEY

2/182

$$x_A^2 + y_A^2 = R^2 \quad (1)$$

$$2x_A \dot{x}_A + 2y_A \dot{y}_A = 0 \quad (2)$$

or $x_A \dot{x}_A + y_A \dot{y}_A = 0$

$$x_B \text{ varies, } y_B = -R \quad (3)$$

$$(x_B - x_A)^2 + (y_B - y_A)^2 = (2R)^2 \quad (4)$$

$$\Rightarrow [x_B - x_A][\dot{x}_B - \dot{x}_A] + [y_B - y_A][\dot{y}_B - \dot{y}_A] = 0$$

Retain $x_B, \dot{x}_B, y_A, \dot{y}_A$. With (1), (2), and (3):

$$[x_B - \sqrt{R^2 - y_A^2}][\dot{x}_B + \frac{y_A \dot{y}_A}{x_A}] + [-R - y_A][0 - \dot{y}_A] = 0 \quad (5)$$

At position shown, $\begin{cases} x_A = -R \sin 60^\circ \\ y_A = -R \cos 60^\circ \end{cases}$

With $\dot{x}_B = 2 \text{ m/s}$, $x_B = 1.070R$ (from (4)), then

from (5) we obtain $\dot{y}_A = -2.39 \text{ m/s}$

$$\begin{aligned} \text{From (2)} \quad \dot{x}_A &= -\frac{y_A \dot{y}_A}{x_A} = -\frac{(-R \cos 60^\circ)(-2.39)}{-R \sin 60^\circ} \\ &= 1.382 \text{ m/s} \end{aligned}$$

$$\text{Then } v_A = \sqrt{\dot{x}_A^2 + \dot{y}_A^2} = \underline{2.76 \text{ m/s}}$$

2/183

Thus $v_A = \frac{2y}{\sqrt{y^2 + b^2}} v_B$

Cable length is

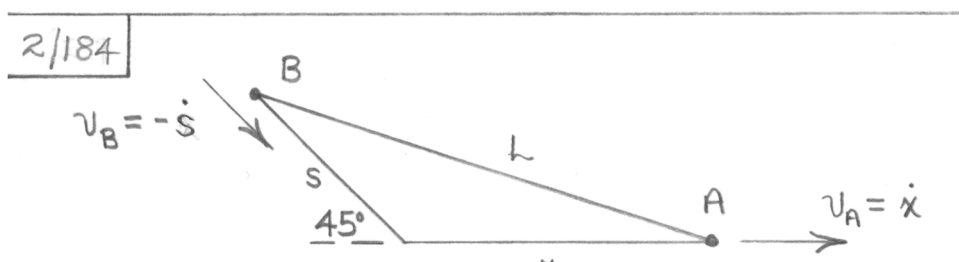
$$L = y_A + 2\sqrt{y_B^2 + b^2}$$

$$0 = \dot{y}_A + \frac{2y\dot{y}}{\sqrt{y^2 + b^2}}$$

But $v_A = -\dot{y}_A$, $v_B = \dot{y}$

WILEY

2/184



$$L^2 = \left(x + \frac{s}{\sqrt{2}}\right)^2 + \left(\frac{s}{\sqrt{2}}\right)^2 = x^2 + \frac{2xs}{\sqrt{2}} + s^2$$

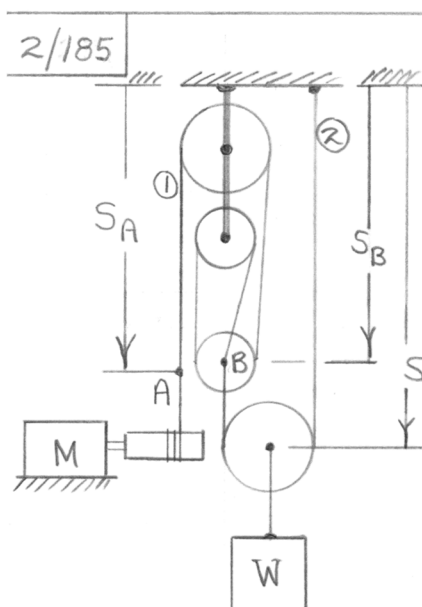
$$0 = 2x\dot{x} + \frac{2\dot{x}s}{\sqrt{2}} + \frac{2x\dot{s}}{\sqrt{2}} + 2s\dot{s} = 0$$

$$0 = x v_A + \frac{s}{\sqrt{2}} v_A - \frac{x}{\sqrt{2}} v_B - s v_B$$

$$v_B \left(s + \frac{x}{\sqrt{2}}\right) = v_A \left(x + \frac{s}{\sqrt{2}}\right) \text{ so } v_B = \frac{x + s/\sqrt{2}}{s + x/\sqrt{2}} v_A$$

$$\text{or } \underline{v_B = \frac{s + \sqrt{2}x}{x + \sqrt{2}s} v_A}$$

WILEY



Let A be a point on cable 1, and let B be the indicated pulley.

$$L_1 = s_A + 3s_B + \text{Constant}$$

$$0 = v_A + 3v_B \quad (1)$$

$$L_2 = s_W - s_B + s_W$$

$$= 2s_W - s_B$$

$$0 = 2v_W - v_B \quad (2)$$

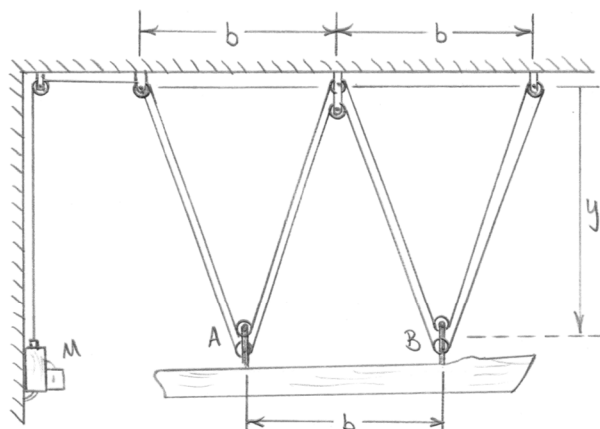
Combine (1) & (2) : $v_A + 6v_W = 0$

So $v_W = -\frac{1}{6}v_A$

Hence W rises $\frac{1}{6}(180)(10) = \underline{300 \text{ mm} = h}$

2/186

CABLE IS REELED IN AT A CONSTANT RATE $\dot{l}$.



$$L = 8\sqrt{y^2 + \left(\frac{b}{2}\right)^2} + \text{CONSTANTS}$$

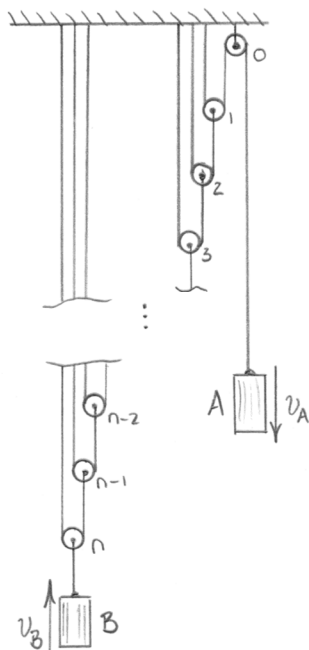
$$v = -\dot{y}$$

$$\frac{dL}{dt} = -\dot{l} = 8 \left[\frac{1}{2} \left(y^2 + \left(\frac{b}{2} \right)^2 \right)^{-1/2} (2y\dot{y}) \right] \rightarrow \dot{y} = \frac{-\dot{l} \sqrt{y^2 + \left(\frac{b}{2} \right)^2}}{8y}$$

$$\text{So... } v = \frac{\dot{l} \sqrt{4y^2 + b^2}}{16y} \quad \text{AND} \quad a = \frac{dv}{dt} = \frac{\dot{l} (8y\dot{y}) \left(\frac{1}{2} \right) (4y^2 + b^2)^{-1/2}}{16y} - \frac{\dot{l} \sqrt{4y^2 + b^2} \dot{y}}{16y^2}$$

$$a = \frac{\dot{l} \dot{y}}{4\sqrt{4y^2 + b^2}} - \frac{\dot{l} \dot{y} \sqrt{4y^2 + b^2}}{16y^2} = \frac{-\dot{l}^2}{64y} + \frac{\dot{l}^2 (4y^2 + b^2)}{256y^3} \rightarrow a = \frac{b^2 \dot{l}^2}{256y^3}$$

2/187



By inspection, the center of pulley 1 will move $\frac{1}{2}$ as fast as mass A. The center of pulley 2 will move $\frac{1}{2}$ as fast as pulley 1 or $\frac{1}{4}$ as fast as mass A. The center of pulley 3 will move $\frac{1}{2}$ as fast as pulley 2 or $\frac{1}{8}$ as fast as mass A.

$$\therefore \underline{v_B = \frac{1}{2^n} v_A \uparrow}$$

2/188

$$s^2 = 2b^2 + 2b^2 \cos \theta$$

$$= 2b^2(1 + \cos \theta)$$

$$2s\dot{s} = 2b^2(-\dot{\theta} \sin \theta)$$

$$v_B = -\dot{s} = \frac{b^2}{s} \dot{\theta} \sin \theta$$

$$y = l \sin \theta$$

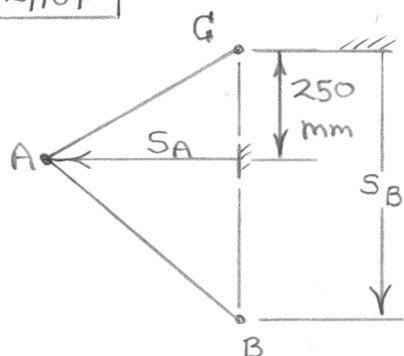
$$(v_A)_y = \dot{y} = l \dot{\theta} \cos \theta$$

Eliminate $\dot{\theta}$ & get

$$(v_A)_y = l \cos \theta \frac{s v_B}{b^2 \sin \theta} = \frac{l \sqrt{2(1 + \cos \theta)}}{b \tan \theta} v_B$$

WILEY

2/189



$$S_A = 425 \text{ mm}$$

Length of cord

$$L = 1050 \text{ mm}$$

$$\sqrt{S_A^2 + 250^2} + \sqrt{S_A^2 + (S_B - 250)^2} = L \quad (1)$$

Differentiate with respect to time :

$$\frac{1}{2}(S_A^2 + 250^2)^{-\frac{1}{2}}(2S_A\dot{S}_A) + \frac{1}{2}[S_A^2 + (S_B - 250)^2]^{-\frac{1}{2}} \times [2S_A\dot{S}_A + 2(S_B - 250)\dot{S}_B] = 0$$

or

$$\frac{S_A v_A}{\sqrt{S_A^2 + 250^2}} + \frac{S_A v_A + (S_B - 250)v_B}{\sqrt{S_A^2 + (S_B - 250)^2}} = 0 \quad (2)$$

With $S_A = 425 \text{ mm}$, $v_A = 25 \text{ mm/s}$ & $L = 1050 \text{ mm}$,

Eq. (1) gives $S_B = 610 \text{ mm}$

Then Eq. (2) gives $v_B = -62.9 \text{ mm/s}$
(up)

► 2/190 From the solution to Prob. 2/189, the velocity constraint equation is

$$s_A v_A (s_A^2 + 250^2)^{-1/2} + [s_A v_A + (s_B - 250) v_B] \times [s_A^2 + (s_B - 250)^2]^{-1/2} = 0$$

The next derivative is

$$\begin{aligned} & \dot{s}_A v_A (s_A^2 + 250^2)^{-1/2} + s_A \dot{v}_A (s_A^2 + 250^2)^{-1/2} \\ & + s_A v_A (-\frac{1}{2})(s_A^2 + 250^2)^{-3/2}(2s_A \dot{s}_A) \\ & + [\dot{s}_A v_A + s_A \dot{v}_A + \dot{s}_B v_B + (s_B - 250) \dot{v}_B] \times \\ & [s_A^2 + (s_B - 250)^2]^{-1/2} + [s_A v_A + (s_B - 250) v_B] \times \\ & (-\frac{1}{2})[s_A^2 + (s_B - 250)^2]^{-3/2} [2s_A \dot{s}_A + 2(s_B - 250) \dot{s}_B] = 0 \end{aligned}$$

Slight simplification:

$$\begin{aligned} & v_A^2 [s_A^2 + 250^2]^{-1/2} + s_A a_A [s_A^2 + 250^2]^{-1/2} \\ & - s_A^2 v_A^2 [s_A^2 + 250^2]^{-3/2} + [s_A a_A + v_A^2 + v_B^2 + (s_B - 250) a_B] \times \\ & [s_A^2 + (s_B - 250)^2]^{-1/2} + [s_A v_A + (s_B - 250) v_B] \times \\ & [s_A^2 + (s_B - 250)^2]^{-3/2} [s_A v_A + (s_B - 250) v_B] = 0 \end{aligned}$$

With $s_A = 425 \text{ mm}$, $s_B = 610 \text{ mm}$, $v_A = 25 \text{ mm/s}$, $v_B = -62.9 \text{ mm/s}$ (Prob. 2/189), and $a_A = 0$:

$$\underline{a_B = -11.93 \text{ mm/s}^2 \text{ (up)}}$$

$$\begin{aligned} 2/191 \quad s &= 8e^{-0.4t} - 6t + t^2 \\ v &= \frac{ds}{dt} = -3.2e^{-0.4t} - 6 + 2t \\ a &= \frac{dv}{dt} = 1.28e^{-0.4t} + 2 \\ a &= 3 \text{ m/s}^2 \text{ when } 1.28e^{-0.4t} + 2 = 3 \\ 1.28e^{-0.4t} &= 1, \quad e^{-0.4t} = 0.781 \\ -0.4t &= -0.247, \quad t = 0.617 \text{ s} \\ v &= -3.2e^{-0.4(0.617)} - 6 + 2(0.617) \\ &= \underline{-7.27 \text{ m/s}} \end{aligned}$$

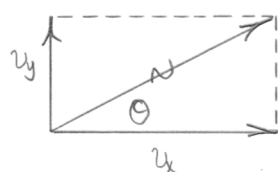
WILEY

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$$\underline{v} = 7.25 \underline{i} + 3.48 \underline{j} \text{ m/s}$$

$$\underline{a} = 0.85 \underline{j} \text{ m/s}^2$$

(ONLY in y-DIRECTION)



$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{3.48}{7.25}\right) = 25.6^\circ$$

$$\text{so... } \theta' = \theta + 30^\circ = 55.6^\circ$$

$$\tan \theta' = \frac{v_y'}{v_x'} \rightarrow \tan 55.6^\circ = \frac{v_y'}{7.25} \rightarrow v_y' = 10.60 \text{ m/s}$$

$$\begin{cases} v_y' = v_{y_0} + a_y t \\ 10.60 = 3.48 + 0.85 t \\ t = 8.38 \text{ s} \end{cases}$$

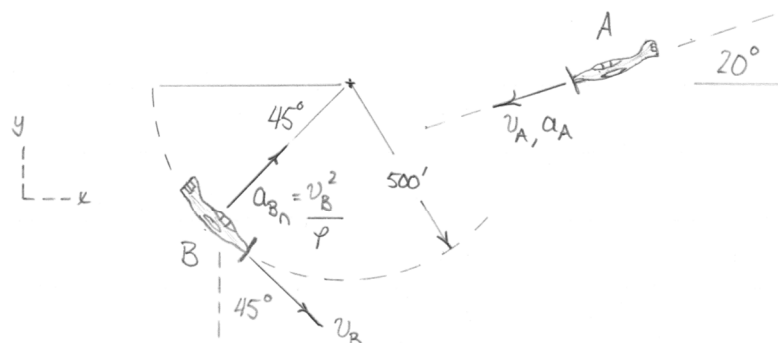
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$$v_A = 265 \text{ mi/hr}$$

$$a_A = 4 \text{ mi/hr/sec}$$

$$v_B = 150 \text{ mi/hr (CONSTANT)}$$



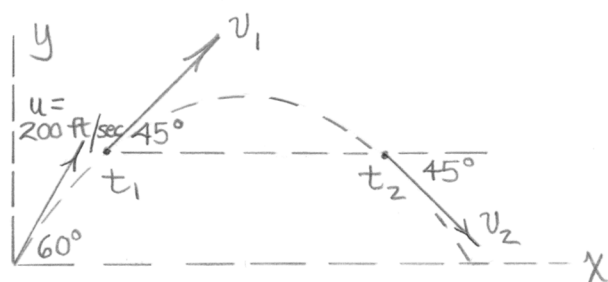
$$\underline{v}_{B/A} = \underline{v}_B - \underline{v}_A = 150(\sin 45^\circ \underline{i} - \cos 45^\circ \underline{j}) - 265(-\cos 20^\circ \underline{i} - \sin 20^\circ \underline{j})$$

$$\underline{v}_{B/A} = 355 \underline{i} - 15.43 \underline{j} \text{ mi/hr}$$

$$\underline{a}_{B/A} = \underline{a}_B - \underline{a}_A = \frac{\left[150 \left(\frac{5280}{3600}\right)\right]^2}{500} (\cos 45^\circ \underline{i} + \sin 45^\circ \underline{j}) - 4 \left(\frac{5280}{3600}\right) (-\cos 20^\circ \underline{i} - \sin 20^\circ \underline{j})$$

$$\underline{a}_{B/A} = 74.0 \underline{i} + 70.5 \underline{j} \text{ ft/sec}^2$$

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$$\dot{x} = u \cos \theta = 200 \cos 60^\circ = 100 \text{ ft/sec}$$

$$\dot{y} = u \sin \theta - gt = 200 \sin 60^\circ - 32.2t = 173.2 - 32.2t$$

$$\text{At } t_1: \dot{x} = \dot{y}: 100 = 173.2 - 32.2t_1, \quad \underline{t_1 = 2.27 \text{ sec}}$$

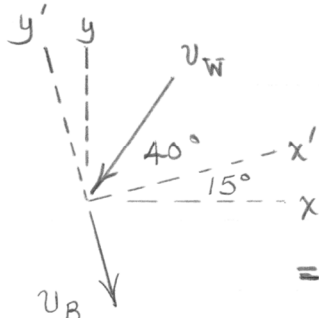
$$\text{At } t_2: \dot{x} = -\dot{y}: 100 = -173.2 + 32.2t_2, \quad \underline{t_2 = 8.48 \text{ sec}}$$

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2/195 (a) $\underline{v}_{W/B} = \underline{v}_W - \underline{v}_B$ $\begin{matrix} y \\ \vdots \\ x \end{matrix}$

$$= 20(-\sin 35^\circ \underline{i} - \cos 35^\circ \underline{j}) - 16(\sin 15^\circ \underline{i} - \cos 15^\circ \underline{j})$$

$$= -15.61 \underline{i} - 0.928 \underline{j} \text{ mi/hr}$$

(b) 

$$\underline{v}_{W/B} = \underline{v}_W - \underline{v}_B$$

$$= 20(-\cos 40^\circ \underline{i}' - \sin 40^\circ \underline{j}') - v_B(-\underline{j}')$$

$$= -15.32 \underline{i}' + (v_B - 12.86) \underline{j}' \text{ mi/hr}$$

For no $\underline{j}'$ -component: $\underline{v}_B = 12.86 \text{ mi/hr}$

and $\underline{v}_{W/B} = 15.32 \text{ mi/hr}$

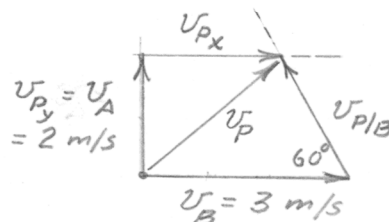
2/196 Let B = point on slot B coincident with P .

$$\underline{v}_P = \underline{v}_B + \underline{v}_{P/B} \quad \text{But } \underline{v}_P = \underline{v}_{Px} + \underline{v}_{Py} \\ = \underline{v}_{Px} + \underline{v}_A$$



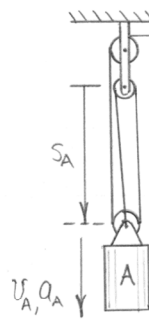
$$\text{So } \underline{v}_{Px} + \underline{v}_A = \underline{v}_B + \underline{v}_{P/B}$$

$$v_P = \sqrt{2^2 + \left(3 - \frac{2}{\sqrt{3}}\right)^2} \\ = 2.72 \text{ m/s}$$



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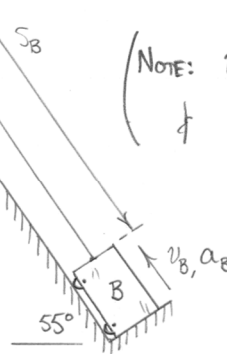
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$$a_{B/A} = 2.4 \text{ m/s}^2$$

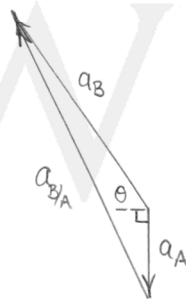
$$\theta = 55^\circ$$

(NOTE: $v_B = -\dot{s}_B$, $a_B = -\ddot{s}_B$)
 $\uparrow$ $v_A = \dot{s}_A$, $a_A = \ddot{s}_A$



$$\begin{cases} L = 3s_A + s_B + \text{CONSTANTS} \\ \dot{L} = 0 = 3\dot{s}_A + \dot{s}_B \\ \ddot{L} = 0 = 3\ddot{s}_A + \ddot{s}_B \end{cases} \rightarrow$$

$$\begin{cases} \text{IN MAGNITUDE... } v_A = \frac{1}{3} v_B \\ \text{IN MAGNITUDE... } a_A = \frac{1}{3} a_B \end{cases} \quad (1)$$



$$a_{B/A}^2 = a_B^2 + a_A^2 - 2a_A a_B \cos(\theta + 90^\circ) \quad (2)$$

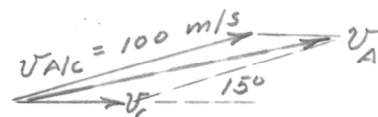
SOLVING (1) AND (2)... $\begin{cases} a_A = 0.621 \text{ m/s}^2 \downarrow \\ a_B = 1.864 \text{ m/s}^2 \swarrow \end{cases}$

$$d = \frac{1}{2} a_B t^2 \rightarrow 5 = \frac{1}{2} (1.864) t^2 \rightarrow \underline{t = 2.32 \text{ s}}$$

$$v_A = \frac{1}{3} v_B = \frac{1}{3} (a_B t) = \frac{1}{3} (1.864)(2.32) \rightarrow \underline{v_A = 1.439 \text{ m/s} \downarrow}$$

2/198 Carrier deck has a constant velocity, so may be used as an inertial coordinate base.
Velocity of aircraft relative to carrier is

$$v_{A/C}^2 = 2as = 2(50)(100) = 10\,000 \text{ (m/s)}^2, v_{A/C} = 100 \text{ m/s}$$

$$\vec{v}_A = \vec{v}_C + \vec{v}_{A/C}$$


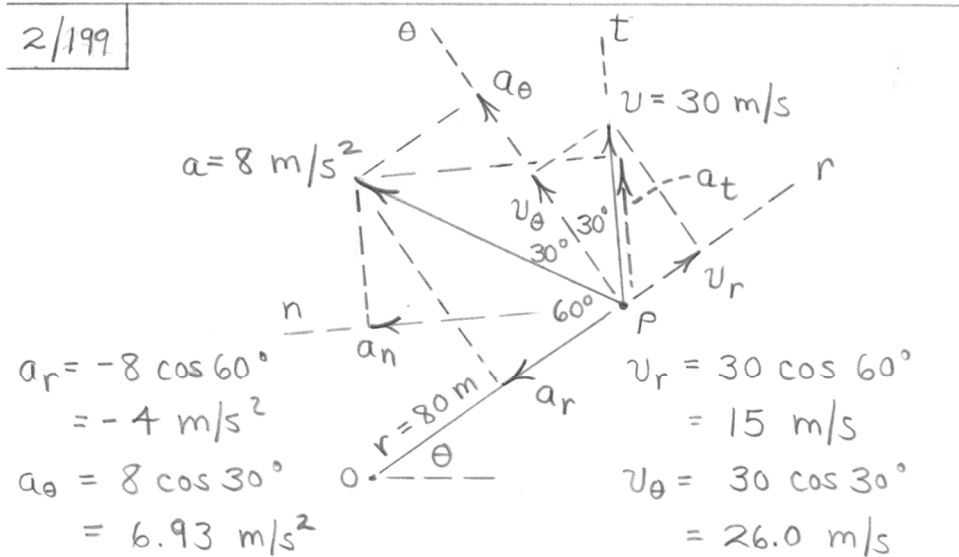
$v_{A/C} = 100 \text{ m/s}$
 15°

$$v_C = 30(1.852)/3.6 = 15.43 \text{ m/s}$$

$$v_A^2 = (100)^2 + (15.43)^2 + 2(100)(15.43)\cos 15^\circ = 13\,220 \text{ (m/s)}^2$$

$$v_A = 115.0 \text{ m/s or } v_A = v = 115.0(3.6) = \underline{414 \text{ km/h}}$$

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$[r-\theta] \quad v_r = \dot{r} = 15 \text{ m/s}$
 $v_\theta = r\dot{\theta} : 26.0 = 80\dot{\theta}, \quad \dot{\theta} = 0.325 \text{ rad/s}$
 $a_r = \ddot{r} - r\dot{\theta}^2 : -4 = \ddot{r} - 80(0.325)^2, \quad \ddot{r} = 4.44 \text{ m/s}^2$
 $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 6.93 = 80\ddot{\theta} + 2(15)(0.325), \quad \ddot{\theta} = -0.0352 \text{ rad/s}^2$

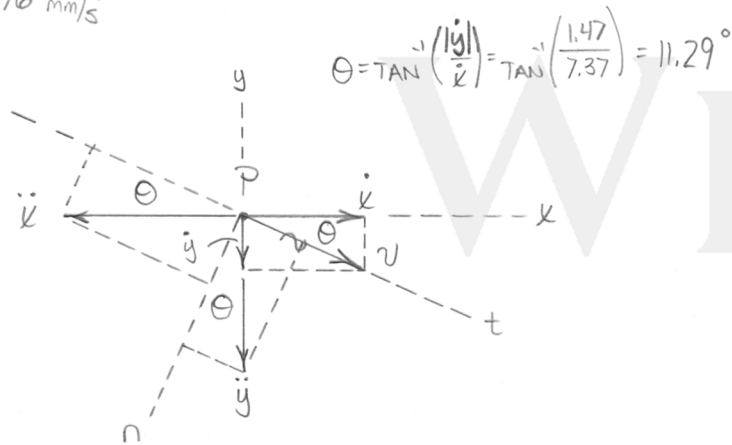
$[n-t] : a_n = 8 \cos 30^\circ = 6.93 \text{ m/s}^2$
 $a_t = 8 \cos 60^\circ = 4 \text{ m/s}^2$
 $a_n = \frac{v^2}{\rho}, \quad \rho = \frac{v^2}{a_n} = \frac{30^2}{6.93} = 129.9 \text{ m}$

2/200

$t = 3.25 \text{ s}$

$$\begin{cases} x = 10.25t + 1.75t^2 - 0.45t^3 = 10.25(3.25) + 1.75(3.25)^2 - 0.45(3.25)^3 = 36.3 \text{ mm} \\ \dot{x} = 10.25 + 3.5t - 1.35t^2 = 10.25 + 3.5(3.25) - 1.35(3.25)^2 = 7.37 \text{ mm/s} \\ \ddot{x} = 3.5 - 2.7t = 3.5 - 2.7(3.25) = -5.28 \text{ mm/s}^2 \end{cases}$$

$$\begin{cases} y = 6.32 + 14.65t - 2.48t^2 = 6.32 + 14.65(3.25) - 2.48(3.25)^2 = 27.7 \text{ mm} \\ \dot{y} = 14.65 - 4.96t = 14.65 - 4.96(3.25) = -1.47 \text{ mm/s} \\ \ddot{y} = -4.96 \text{ mm/s}^2 \end{cases}$$



$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{7.37^2 + 1.47^2} \rightarrow \underline{v = 7.51 \text{ mm/s}}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{5.28^2 + 4.96^2} \rightarrow \underline{a = 7.24 \text{ mm/s}^2}$$

$$\underline{v} = 7.37 \underline{i} - 1.47 \underline{j} \text{ mm/s}$$

$$\underline{a} = -5.28 \underline{i} - 4.96 \underline{j} \text{ mm/s}^2$$

$$\underline{e}_t = \frac{\underline{v}}{v} = \frac{7.37 \underline{i} - 1.47 \underline{j}}{7.51} \rightarrow \underline{e}_t = 0.981 \underline{i} - 0.1957 \underline{j}$$

$$\text{Thus... } \underline{e}_n = -0.1957 \underline{i} - 0.981 \underline{j}$$

$$a_t = \underline{a} \cdot \underline{e}_t = (-5.28 \underline{i} - 4.96 \underline{j}) \cdot (0.981 \underline{i} - 0.1957 \underline{j}) \rightarrow \underline{a_t = -4.20 \text{ mm/s}^2}$$

$$\underline{a}_t = a_t \underline{e}_t = -4.20 (0.981 \underline{i} - 0.1957 \underline{j}) \rightarrow \underline{a_t = -4.12 \underline{i} + 0.822 \underline{j} \text{ mm/s}^2}$$

$$a_n = \underline{a} \cdot \underline{e}_n = (-5.28 \underline{i} - 4.96 \underline{j}) \cdot (-0.1957 \underline{i} - 0.981 \underline{j}) \rightarrow \underline{a_n = 5.90 \text{ mm/s}^2}$$

$$\underline{a}_n = a_n \underline{e}_n = 5.90 (-0.1957 \underline{i} - 0.981 \underline{j}) \rightarrow \underline{a_n = -1.154 \underline{i} - 5.78 \underline{j} \text{ mm/s}^2}$$

$$a_n = \frac{v^2}{\rho} \rightarrow \rho = \frac{v^2}{a_n} = \frac{7.51^2}{5.90} \rightarrow \underline{\rho = 9.57 \text{ mm}}$$

$$v = \rho \dot{\beta} \rightarrow \dot{\beta} = \frac{v}{\rho} = \frac{7.51}{9.57} \rightarrow \underline{\dot{\beta} = 0.785 \text{ rad/s}}$$

2/201

$$K = 10.25t + 1.75t^2 - 0.45t^3 \text{ mm} \quad \& \quad y = 6.32 + 14.65t - 2.48t^2 \text{ mm}$$

At $t = 3.25 \text{ s}$:

$$\begin{cases} K = 10.25(3.25) + 1.75(3.25)^2 - 0.45(3.25)^3 = 36.3 \text{ mm} \\ \dot{K} = 10.25 + 3.5(3.25) - 1.35(3.25)^2 = 7.37 \text{ mm/s} \\ \ddot{K} = 3.5 - 2.70(3.25) = -5.28 \text{ mm/s}^2 \end{cases}$$

$$\begin{cases} y = 6.32 + 14.65(3.25) - 2.48(3.25)^2 = 27.7 \text{ mm} \\ \dot{y} = 14.65 - 4.96(3.25) = -1.47 \text{ mm/s} \\ \ddot{y} = -4.96 \text{ mm/s}^2 \end{cases}$$

$$\begin{cases} v = \sqrt{\dot{K}^2 + \dot{y}^2} = \sqrt{7.37^2 + 1.47^2} \rightarrow v = 7.51 \text{ mm/s} \\ \underline{v} = \dot{K}\underline{i} + \dot{y}\underline{j} \rightarrow \underline{v} = 7.37\underline{i} - 1.47\underline{j} \text{ mm/s} \end{cases}$$

$$\begin{cases} a = \sqrt{\ddot{K}^2 + \ddot{y}^2} = \sqrt{5.28^2 + 4.96^2} \rightarrow a = 7.24 \text{ mm/s}^2 \\ \underline{a} = \ddot{K}\underline{i} + \ddot{y}\underline{j} \rightarrow \underline{a} = -5.28\underline{i} - 4.96\underline{j} \text{ mm/s}^2 \end{cases}$$

$$\begin{cases} r = \sqrt{K^2 + y^2} = \sqrt{36.3^2 + 27.7^2} \rightarrow r = 45.7 \text{ mm} \\ \Theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{27.7}{36.3}\right) \rightarrow \Theta = 37.3^\circ \end{cases}$$

$$\begin{cases} \underline{e}_r = \cos\Theta\underline{i} + \sin\Theta\underline{j} = \cos 37.3^\circ\underline{i} + \sin 37.3^\circ\underline{j} \rightarrow \underline{e}_r = 0.795\underline{i} + 0.607\underline{j} \\ \underline{e}_\theta = \cos(\Theta+90^\circ)\underline{i} + \sin(\Theta+90^\circ)\underline{j} = \cos(127.3^\circ)\underline{i} + \sin(127.3^\circ)\underline{j} \rightarrow \underline{e}_\theta = -0.607\underline{i} + 0.795\underline{j} \end{cases}$$

$$\underline{v}_r = \underline{v} \cdot \underline{e}_r = (7.37\underline{i} - 1.47\underline{j}) \cdot (0.795\underline{i} + 0.607\underline{j}) \rightarrow \underline{v}_r = 4.96 \text{ mm/s}$$

$$\underline{v}_\theta = \underline{v} \cdot \underline{e}_\theta = (7.37\underline{i} - 1.47\underline{j}) \cdot (-0.607\underline{i} + 0.795\underline{j}) \rightarrow \underline{v}_\theta = -5.64 \text{ mm/s}$$

$$\underline{a}_r = \underline{a} \cdot \underline{e}_r = (-5.28\underline{i} - 4.96\underline{j}) \cdot (0.795\underline{i} + 0.607\underline{j}) \rightarrow \underline{a}_r = -7.20 \text{ mm/s}^2$$

$$\underline{a}_\theta = \underline{a} \cdot \underline{e}_\theta = (-5.28\underline{i} - 4.96\underline{j}) \cdot (-0.607\underline{i} + 0.795\underline{j}) \rightarrow \underline{a}_\theta = -0.743 \text{ mm/s}^2$$

$$\underline{a}_r = \underline{a} \cdot \underline{e}_r = (-5.28\underline{i} - 4.96\underline{j}) \cdot (0.795\underline{i} + 0.607\underline{j}) \rightarrow \underline{a}_r = -7.20 \text{ mm/s}^2$$

$$\underline{a}_\theta = \underline{a} \cdot \underline{e}_\theta = (-5.28\underline{i} - 4.96\underline{j}) \cdot (-0.607\underline{i} + 0.795\underline{j}) \rightarrow \underline{a}_\theta = -0.743 \text{ mm/s}^2$$

$$\underline{a}_r = \underline{a} \cdot \underline{e}_r = (-5.28\underline{i} - 4.96\underline{j}) \cdot (0.795\underline{i} + 0.607\underline{j}) \rightarrow \underline{a}_r = -7.20 \text{ mm/s}^2$$

$$\underline{a}_\theta = \underline{a} \cdot \underline{e}_\theta = (-5.28\underline{i} - 4.96\underline{j}) \cdot (-0.607\underline{i} + 0.795\underline{j}) \rightarrow \underline{a}_\theta = -0.743 \text{ mm/s}^2$$

$$\underline{v}_r = \dot{r} = 4.96 \text{ mm/s} \quad \underline{v}_\theta = r\dot{\Theta} = -5.64 = 45.7\dot{\Theta} \rightarrow \dot{\Theta} = -0.1233 \text{ rad/s}$$

$$\underline{a}_r = \ddot{r} - r\dot{\Theta}^2 = -7.20 = \ddot{r} - 45.7(-0.1233)^2 \rightarrow \ddot{r} = -6.51 \text{ mm/s}^2$$

$$\underline{a}_\theta = r\ddot{\Theta} + 2\dot{r}\dot{\Theta} = -0.743 = 45.7\ddot{\Theta} + 2(4.96)(-0.1233) \rightarrow \ddot{\Theta} = 0.01052 \text{ rad/s}^2$$

2/202 Vertical motion :

$$z = z_0 + v_{z0}t - \frac{1}{2}gt^2 \text{ at impact : } 0 = 1500 - 16.1t^2$$
$$t = 9.65 \text{ sec}$$

Horizontal motion :

$$x = x_0 + v_{x0}t : x = 0 - 130(9.65) = \underline{-1255 \text{ ft}}$$

$$y = y_0 + v_{y0}t : y = 1000 + 20(9.65) = \underline{1193 \text{ ft}}$$

$$\underline{z = 0}$$

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2/203 $v_0 = 27000/3.6 = 7500 \text{ m/s}$

$H = 35(10^4) \text{ m}$

$g_0 = 9.832 \text{ m/s}^2$ (Fig. 1/1)

$R = 6371 \text{ km}$ or $6.371(10^6) \text{ m}$

$g = g_0 \left(\frac{R}{R+H+y} \right)^2$

From $a = v \frac{dv}{dy}$, we have

$$\int_{v_0}^0 v dv = \int_0^h -g_0 \left(\frac{R}{R+H+y} \right)^2 dy$$

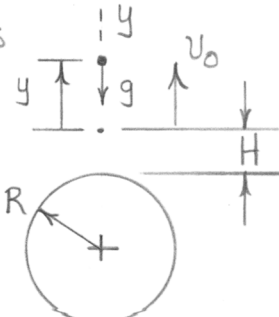
$$-\frac{1}{2}v_0^2 = g_0 R^2 \frac{1}{R+H+y} \Big|_0^h$$

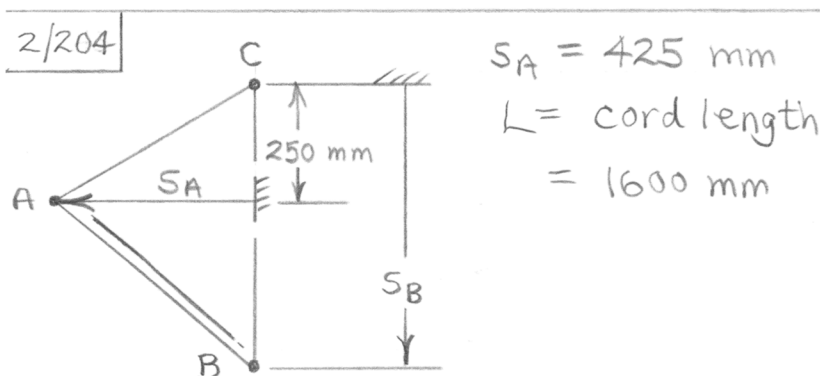
$$v_0^2 = 2g_0 R^2 \frac{h}{(R+H)(R+H+h)}$$

Solve for h : $h = \frac{(R+H)^2 v_0^2}{2g_0 R^2 - (R+H)v_0^2}$

Substitute numerical values and obtain

$h = 6048(10^3) \text{ m}$ or $\underline{h = 6048 \text{ km}}$





$$\sqrt{s_A^2 + 250^2} + 2\sqrt{s_A^2 + (s_B - 250)^2} = L \quad (1)$$

Differentiate WRT time :

$$\frac{1}{2}(s_A^2 + 250^2)^{-\frac{1}{2}}(2s_A \dot{s}_A) + 2\left(\frac{1}{2}\right)[s_A^2 + (s_B - 250)^2]^{-\frac{1}{2}} \times [2s_A \dot{s}_A + 2(s_B - 250)\dot{s}_B] = 0$$

or

$$\frac{s_A v_A}{\sqrt{s_A^2 + 250^2}} + \frac{2[s_A v_A + (s_B - 250)v_B]}{\sqrt{s_A^2 + (s_B - 250)^2}} = 0 \quad (2)$$

With $s_A = 425$, $v_A = 25 \text{ mm/s}$, & $L = 1600 \text{ mm}$,
 Eq. (1) gives $s_B = 605 \text{ mm}$

Then Eq. (2) gives $v_B = -46.8 \text{ mm/s (up)}$

*2/205 From the solution to Prob. 2/204, the velocity constraint equation is

$$s_A v_A (s_A^2 + 250^2)^{-1/2} + 2 [s_A v_A + (s_B - 250) v_B] \times [s_A^2 + (s_B - 250)^2]^{-1/2} = 0$$

The next derivative is

$$\begin{aligned} & (\dot{s}_A v_A + s_A \dot{v}_A) (s_A^2 + 250^2)^{-1/2} + s_A v_A \left(-\frac{1}{2}\right) (s_A^2 + 250^2)^{-3/2} (2s_A \dot{s}_A) \\ & + 2 [\dot{s}_A v_A + s_A \dot{v}_A + \dot{s}_B v_B + (s_B - 250) \dot{v}_B] \times \\ & [s_A^2 + (s_B - 250)^2]^{-1/2} + 2 [s_A v_A + (s_B - 250) v_B] \times \\ & \left(-\frac{1}{2}\right) [s_A^2 + (s_B - 250)^2]^{-3/2} [2s_A \dot{s}_A + 2(s_B - 250) \dot{s}_B] = 0 \end{aligned}$$

Slight simplification:

$$\begin{aligned} & (v_A^2 + s_A a_A) [s_A^2 + 250^2]^{-1/2} - s_A^2 v_A^2 [s_A^2 + 250^2]^{-3/2} \\ & + 2 [s_A a_A + v_A^2 + v_B^2 + (s_B - 250) a_B] \times [s_A^2 + (s_B - 250)^2]^{-1/2} \\ & - 2 [s_A v_A + (s_B - 250) v_B] [s_A^2 + (s_B - 250)^2]^{-3/2} \times \\ & [s_A v_A + (s_B - 250) v_B] = 0 \end{aligned}$$

With $s_A = 425 \text{ mm}$, $v_A = 25 \text{ mm/s}$, $s_B = 605 \text{ mm}$,

and $v_B = -46.8 \text{ mm/s}$ (Prob. 2/204), along with

$a_A = 0$: $a_B = -7.86 \text{ mm/s}^2$ (up)

$$\triangleright 2/206 \quad \theta = \theta_0 \cos \omega t, \quad \dot{\theta} = -\theta_0 \omega \sin \omega t, \quad \ddot{\theta} = -\theta_0 \omega^2 \cos \omega t$$

$$\dot{\phi} = K, \quad \ddot{\phi} = 0, \quad R = b, \quad \dot{R} = \ddot{R} = 0$$

From Eq. 2/19

$$a_R = 0 - bK^2 - b\theta_0^2 \omega^2 \sin^2 \omega t \cos^2 \phi$$

$$a_\theta = b \cos \phi (-\theta_0 \omega^2 \cos \omega t) - 2b(-\theta_0 \omega \sin \omega t)K \sin \phi$$

$$a_\phi = 0 + b(\theta_0 \omega \sin \omega t)^2 \sin \phi \cos \phi$$

At A $\cos \omega t = -1, \sin \omega t = 0$

$$a_R = -bK^2, \quad a_\theta = b\omega^2 \theta_0 \cos \phi, \quad a_\phi = 0$$

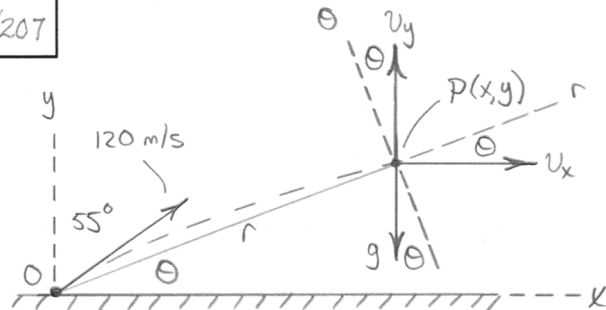
$$\text{So } a = b\sqrt{K^4 + \omega^4 \theta_0^2 \cos^2 \phi}$$

At B $\cos \omega t = 0, \sin \omega t = 1, \phi = \pi/2$

$$\text{So } a = bK\sqrt{K^2 + 4\omega^2 \theta_0^2}$$

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* 2/207



$$\begin{aligned}\theta_0 &= 35^\circ \\ v_0 &= 120 \text{ m/s} \\ x_0 &= 0 \\ y_0 &= 0\end{aligned}$$

$$\text{At } t = 9_s: \begin{cases} x = 885 \text{ m}, y = 222 \text{ m}, r = 912 \text{ m}, \theta = 14.10^\circ \\ v_x = 98.3 \text{ m/s}, v_y = -19.46 \text{ m/s (Down)} \end{cases}$$

$$\begin{cases} v_r = 90.6 \text{ m/s}, v_\theta = -42.8 \text{ m/s} \\ a_r = -2.39 \text{ m/s}^2, a_\theta = -9.51 \text{ m/s}^2 \end{cases}$$

ARBITRARY POSITION:

$$x = x_0 + v_{x0} t = 120 \sin 55^\circ t \quad (1)$$

$$y = y_0 + v_{y0} t - \frac{1}{2} g t^2 = 120 \cos 55^\circ t - \frac{1}{2} (9.81) t^2 \quad (2)$$

$$v_x = v_{x0} = 120 \sin 55^\circ \quad (3)$$

$$v_y = v_{y0} - g t = 120 \cos 55^\circ - 9.81 t \quad (4)$$

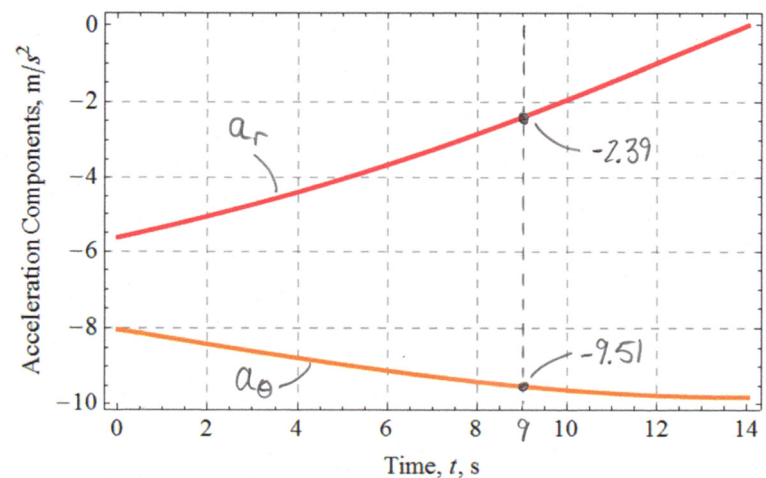
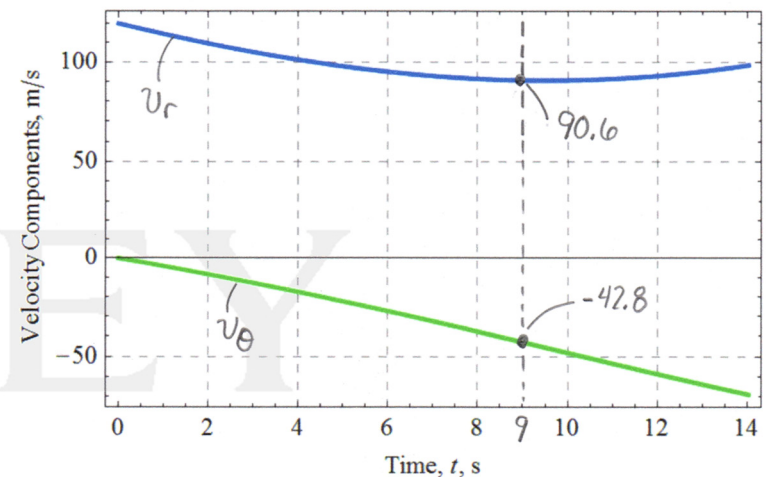
$$\text{AND} \quad \begin{cases} a_x = 0 \\ a_y = -g = -9.81 \text{ m/s}^2 \end{cases}$$

$$r = \sqrt{x^2 + y^2} \quad \text{AND} \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\begin{cases} v_r = v_x \cos \theta + v_y \sin \theta \quad \text{AND} \quad v_\theta = v_y \cos \theta - v_x \sin \theta \\ a_r = -g \sin \theta \quad \text{AND} \quad a_\theta = -g \cos \theta \end{cases}$$

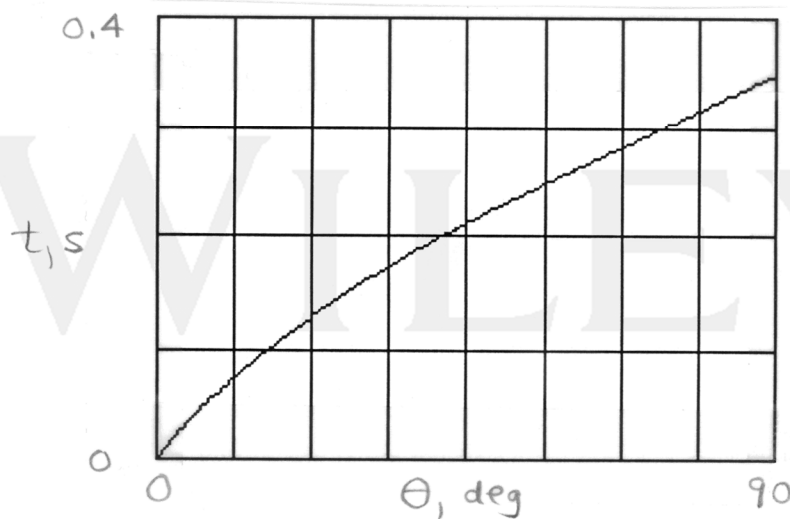
EVALUATE (1) THROUGH (4) AT EACH TIME STEP AND SOLVE FOR $\theta, v_r, v_\theta, a_r$, AND a_θ . PLOT EACH FROM $t=0$ TO $t=t_f$.

FLIGHT TIME: $y_f' = y_0' + v_{y0} t_f - \frac{1}{2} g t_f^2 \rightarrow 0 = 120 \cos 55^\circ t_f - \frac{1}{2} (9.81) t_f^2$
 $t_f = 14.03 \text{ s}$



$$\begin{aligned} \ddot{\theta} &= \dot{\theta} \frac{d\dot{\theta}}{d\theta} = \frac{g}{l} \cos \theta \\ \int_{\dot{\theta}_0}^{\dot{\theta}} \dot{\theta} d\dot{\theta} &= \frac{g}{l} \int_0^{\theta} \cos \theta d\theta \\ \dot{\theta} &= \left[\dot{\theta}_0^2 + \frac{2g}{l} \sin \theta \right]^{1/2} \\ \text{Then } \dot{\theta} = \frac{d\theta}{dt} &= \left[\dot{\theta}_0^2 + \frac{2g}{l} \sin \theta \right]^{1/2} \\ t &= \int_0^{\theta} \frac{d\theta}{\sqrt{\dot{\theta}_0^2 + \frac{2g}{l} \sin \theta}} \end{aligned}$$

With $\dot{\theta}_0 = 2 \text{ rad/s}$, $l = 0.6 \text{ m}$, $g = 9.81$, at $\theta = \frac{\pi}{2}$,
a numerical integration yields $t' = 0.349 \text{ s}$.

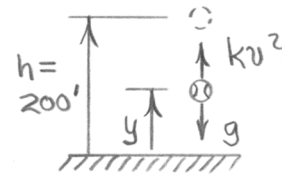


*2/209 $a = v \frac{dv}{dy} = -g + kv^2$

$$\int_0^v \frac{v dv}{-g + kv^2} = \int_h^y dy$$

$$\frac{1}{2k} \ln[-g + kv^2]_0^v = y|_h^y$$

$$\frac{1}{2k} \ln \left[\frac{-g + kv^2}{-g} \right] = y - h \Rightarrow v = \sqrt{\frac{g}{k} [1 - e^{2k(y-h)}]}$$

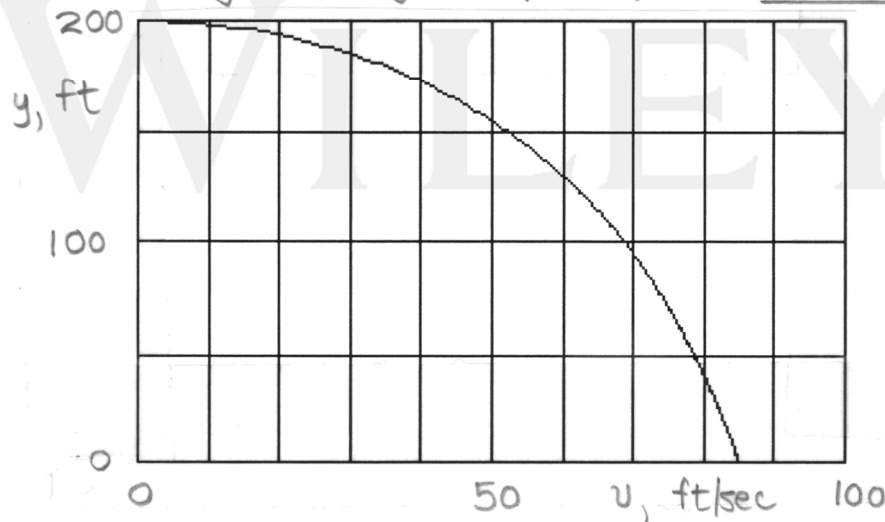


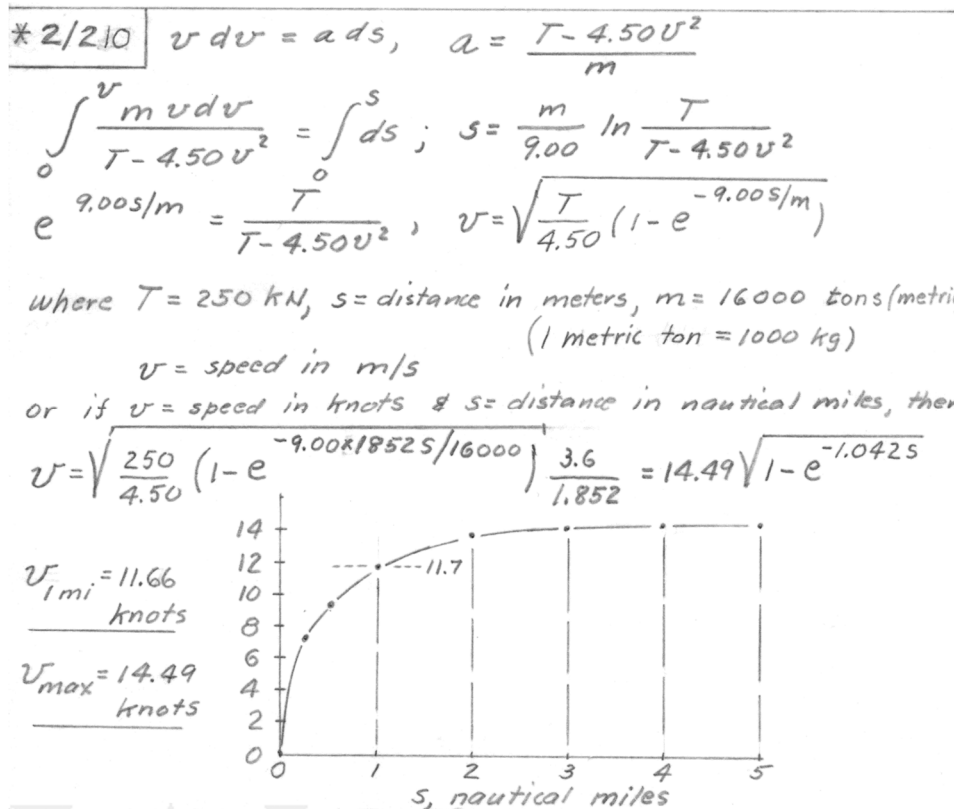
Given numbers : $85 = \sqrt{\frac{32.2}{k} [1 - e^{2k(0-200)}]}$

Numerical solution : $k = 0.00323 \text{ ft}^{-1}$

Terminal speed : $g = kv^2 : 32.2 = 0.00323 v_t^2$
 $v_t = 99.8 \text{ ft/sec}$

Without drag : $v' = \sqrt{2gh} = \sqrt{2(32.2)(200)} = 113.5 \text{ ft/sec}$





$$*2/211 \quad a_t = \frac{dv}{dt} = g \cos \theta - \frac{k}{m} v$$

$$\text{With } v = r\dot{\theta}: \quad \frac{d}{dt}(r\dot{\theta}) = g \cos \theta - \frac{k}{m} (r\dot{\theta})$$

$$\text{or } \frac{d^2\theta}{dt^2} + \frac{k}{m} \frac{d\theta}{dt} - \frac{g}{r} \cos \theta = 0$$

This is a nonlinear, second-order differential equation, so a numerical integration is in order.

To switch to first order form, we let

$$x_1 = \theta \quad \dot{x}_2 = \dot{\theta}$$

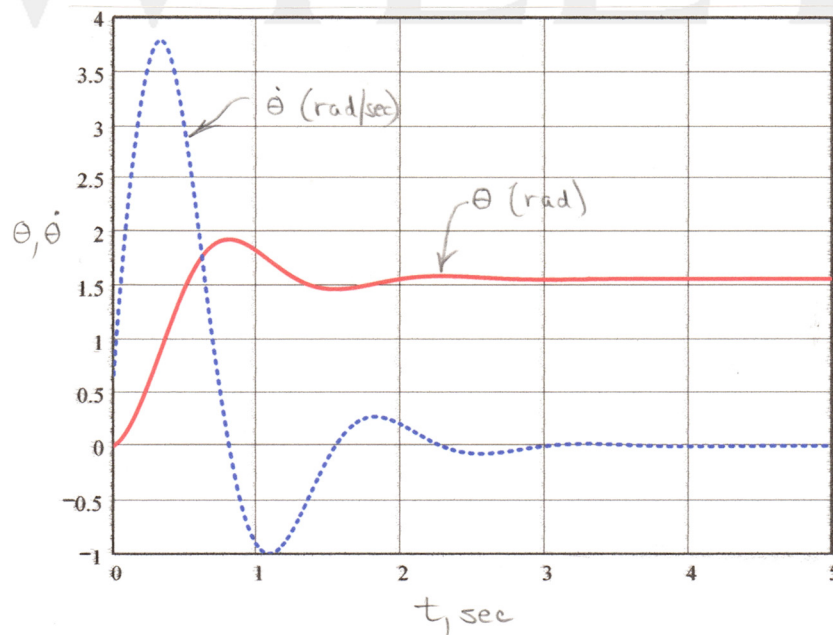
$$\text{So } \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m} x_2 + \frac{g}{r} \cos x_1 \end{cases} \quad \left\| \begin{array}{l} x_{10} = \theta_0 = 0 \\ x_{20} = \dot{\theta}_0 = \frac{v_0}{r} \end{array} \right.$$

The plots below show θ and $\dot{\theta}$ as functions of t .

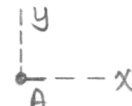
$$\theta_{\max} = 110.4^\circ \text{ @ } t = 0.802 \text{ sec}$$

$$\dot{\theta}_{\max} = 3.79 \text{ rad/sec @ } t = 0.324 \text{ sec}$$

$$\theta = 90^\circ \text{ @ } t = 0.526 \text{ sec}$$



$$\begin{aligned} \text{Values used: } m &= \frac{1.8}{32.2} \text{ slugs}, \quad g = 32.2 \frac{\text{ft}}{\text{sec}^2} \\ k &= 0.2 \text{ lb-sec/ft}, \quad \theta_0 = 0, \quad \dot{\theta}_0 = \frac{v_0}{r} = \frac{1}{1.5} \\ &= 0.667 \text{ rad/s} \end{aligned}$$

*2/212 Set up x-y coordinates @ A: 

Let coordinates of B be $(R, -h)$.

$$x = x_0 + v_{x0}t \text{ @ B: } R = 0 + (v_0 \cos \alpha) t_f \quad (1)$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2 \text{ @ B: } -h = 0 + (v_0 \sin \alpha) t_f - \frac{1}{2}gt_f^2 \quad (2)$$

Solve (1) & (2) for R and t_f :

$$\left\{ R = \frac{v_0 \cos \alpha}{g} \left[v_0 \sin \alpha + \sqrt{v_0^2 \sin^2 \alpha + 2gh} \right] \right.$$

$$\left. t_f = \frac{1}{g} \left[v_0 \sin \alpha + \sqrt{v_0^2 \sin^2 \alpha + 2gh} \right] \right.$$

(Plus sign in quadratic formula has been taken)

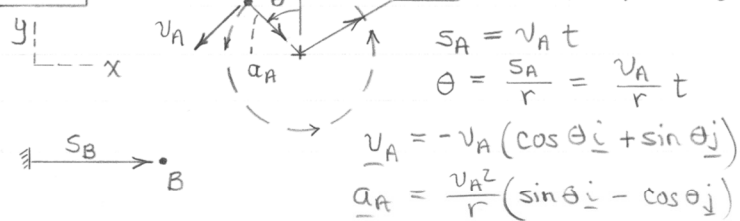
Set $\frac{dR}{d\alpha} = 0$ (done by computer) to find

$$\underline{\alpha = 42.2^\circ} \quad \left\{ \begin{array}{l} \text{for } h = 10 \text{ m, } v_0 = 30 \text{ m/s,} \\ \text{and } g = 9.81 \text{ m/s}^2 \end{array} \right.$$

The corresponding value of R is

$$\underline{R = 101.3 \text{ m}}$$

*2/213 For all time t :



$0 \leq t \leq 10 \text{ s}$:

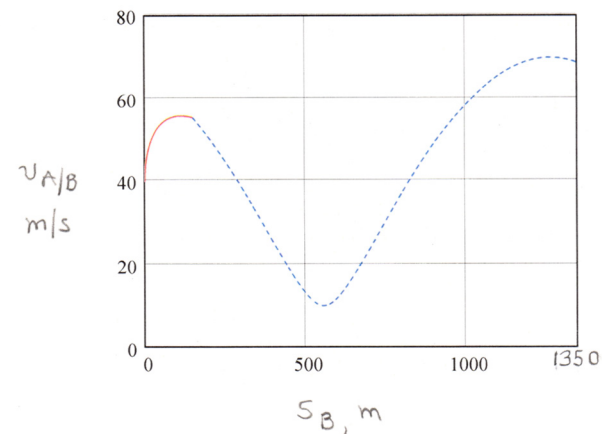
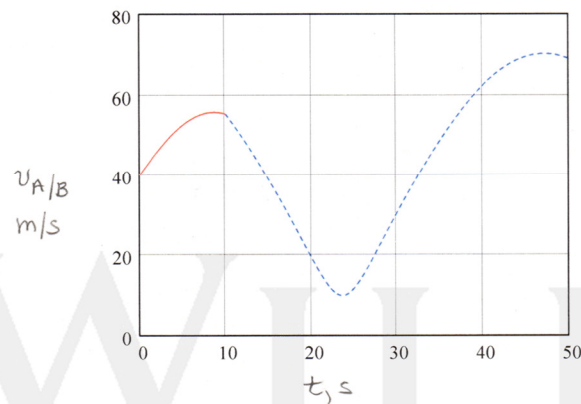
$$\underline{v}_B = 3 \underline{i}, \quad \underline{v}_B = 3t \underline{i}$$

$10 \leq t \leq 20 \text{ s}$:

$$\underline{v}_B = 0, \quad \underline{v}_B = 30(t-10) \underline{i}$$

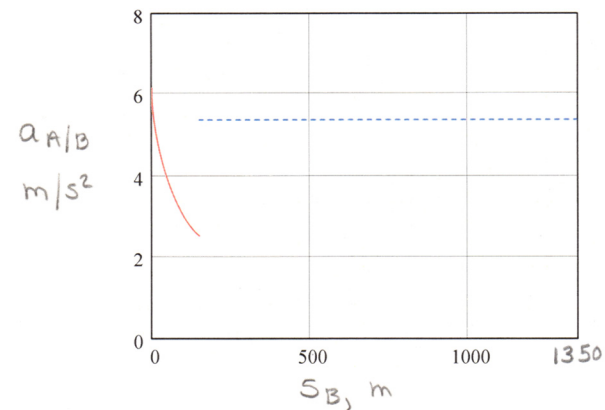
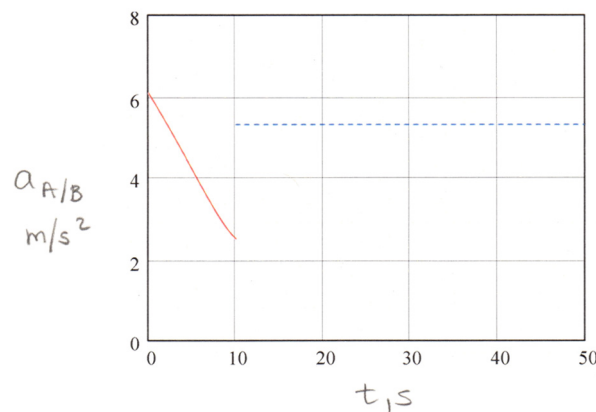
$$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B$$

$$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B$$



Maximum $v_{A/B} = 70 \text{ m/s}$ at $\begin{cases} t = 47.1 \text{ s} \\ s_B = 1264 \text{ m} \end{cases}$

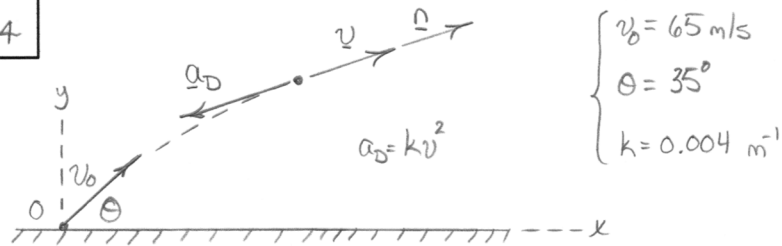
Minimum $v_{A/B} = 10 \text{ m/s}$ at $\begin{cases} t = 23.6 \text{ s} \\ s_B = 557 \text{ m} \end{cases}$



Maximum $a_{A/B} = 6.12 \text{ m/s}^2$ at $\begin{cases} t = 0 \\ s_B = 0 \end{cases}$

Minimum $a_{A/B} = 2.52 \text{ m/s}^2$ at $\begin{cases} t = 10 \text{ s} \\ s_B = 150 \text{ m} \end{cases}$

*2/214



• WITHOUT DRAG:

$$\begin{cases} y_f = y_0 + v_{y0} t_f - \frac{1}{2} g t_f^2 \rightarrow 0 = 65 \sin 35^\circ t_f - \frac{1}{2} (9.81) t_f^2 \\ x_f = x_0 + v_{x0} t_f = 65 \cos 35^\circ (7.60) \rightarrow x_f = 405 \text{ m} \end{cases} \therefore t_f = 7.60 \text{ s}$$

• WITH DRAG: $v_x = \dot{x}$ AND $v_y = \dot{y}$; $a_x = \ddot{x}$ AND $a_y = \ddot{y}$

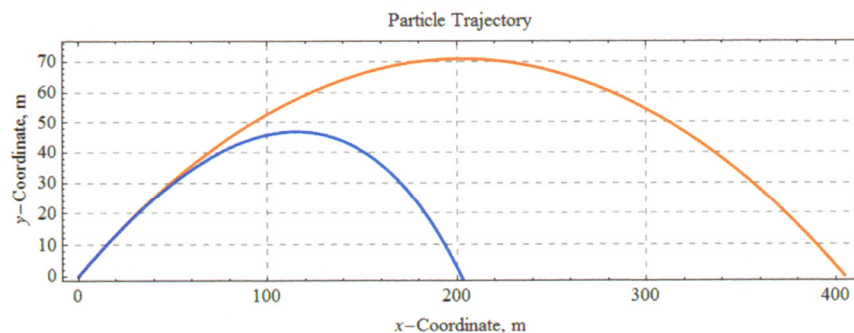
$$\underline{a_D} = -k v^2 \underline{n} = -k (v_x^2 + v_y^2) \left[\frac{v_x \underline{i} + v_y \underline{j}}{\sqrt{v_x^2 + v_y^2}} \right] = -k \sqrt{v_x^2 + v_y^2} (v_x \underline{i} + v_y \underline{j})$$

So...

$$\begin{cases} a_x = \ddot{x} = -k \dot{x} \sqrt{\dot{x}^2 + \dot{y}^2} & \textcircled{1} \\ a_y = \ddot{y} = -g - k \dot{y} \sqrt{\dot{x}^2 + \dot{y}^2} & \textcircled{2} \end{cases} \quad \begin{cases} x(0) = 0 \text{ AND } \dot{x}(0) = v_0 \cos \theta \\ y(0) = 0, \dot{y}(0) = v_0 \sin \theta \end{cases}$$

THESE TWO 2ND ORDER DIFFERENTIAL EQUATIONS MUST BE NUMERICALLY INTEGRATED SINCE THEY ARE NONLINEAR AND COUPLED.

PLOTTING THE NUMERICAL SOLUTION FOR THE TRAJECTORY WITH DRAG (BLUE) AGAINST THE TRAJECTORY FOR NO DRAG (ORANGE) YIELDS THE FOLLOWING PLOT.



• FOR DRAG:

ROOT FINDING TECHNIQUES YIELD...

$$t_f = 6.14 \text{ s} \quad \text{AND} \quad x_f = 202 \text{ m}$$

Chapter 2 Reserve Problem Solutions (Part 1)

Question Title: Chapter 2, Reserve Problem 2/001

The velocity of a particle which moves along the s axis is given by $v = 2 - 4t + 5t^{3/2}$, where t is in seconds and v is in meters per second. Evaluate the position s , velocity v , and acceleration a when $t = 3$ s. The particle is at the position $s_0 = 3$ m when $t = 0$.

SOLUTION

$$v = 2 - 4t + 5t^{3/2}$$

$$a = \frac{dv}{dt} = -4 + \frac{15}{2}t^{1/2}$$

$$\frac{ds}{dt} = 2 - 4t + 5t^{3/2}$$

$$\int_{s_0=3}^s ds = \int_0^t (2 - 4t + 5t^{3/2}) dt$$

$$s = 3 + 2t - 2t^2 + 2t^{5/2}$$

$$\text{At } t = 3 \text{ s} : \begin{cases} s = 22.2 \text{ m} \\ v = 15.98 \text{ m/s} \\ a = 8.99 \text{ m/s}^2 \end{cases}$$

Question Title: Chapter 2, Reserve Problem 2/002

The displacement of a particle which moves along the s -axis is given by $s = (-2 + 3t)e^{-0.5t}$, where s is in meters and t is in seconds. Plot the displacement, velocity, and acceleration versus time for the first 20 seconds of motion. Determine the time at which the acceleration is zero.

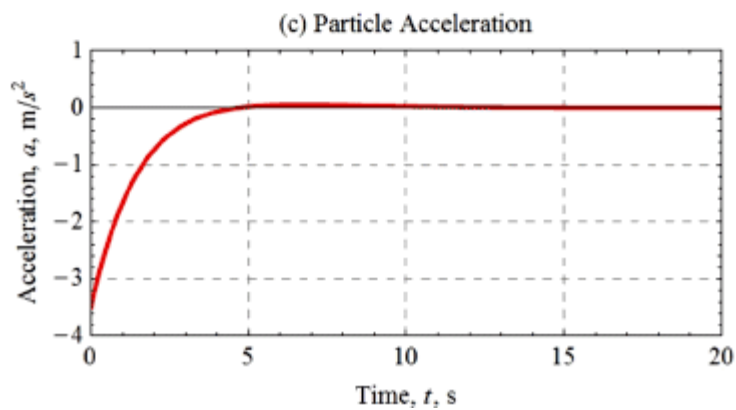
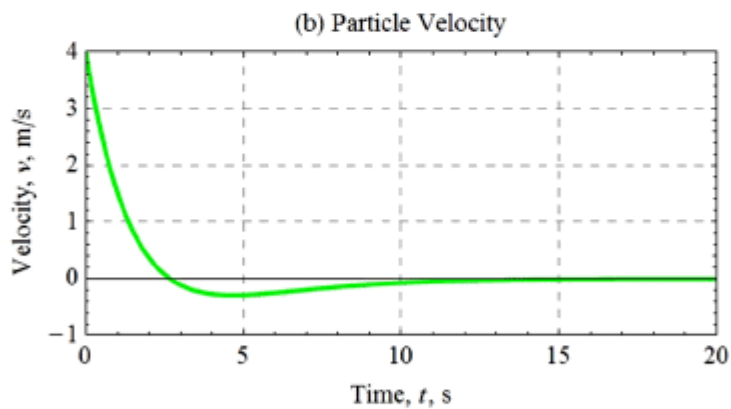
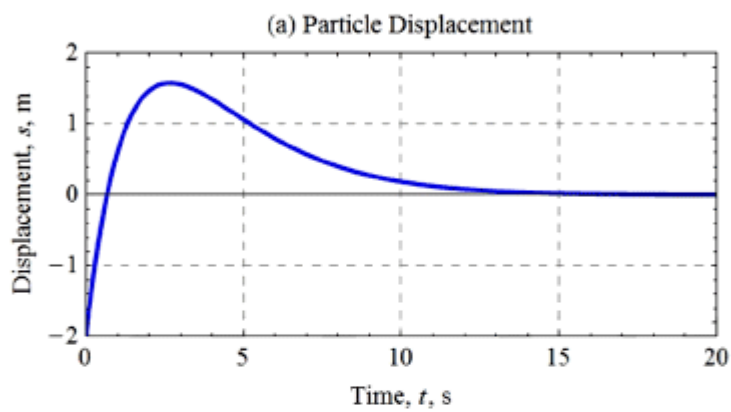
SOLUTION

$$s = (-2 + 3t)e^{-0.5t}$$

$$v = \frac{ds}{dt} = 3e^{-0.5t} + (-2 + 3t)(-0.5)e^{-0.5t}$$
$$= \underline{(4 - 1.5t)e^{-0.5t}}$$

$$a = \frac{dv}{dt} = -1.5e^{-0.5t} + (4 - 1.5t)(-0.5)e^{-0.5t}$$
$$= \underline{(-3.5 + 0.75t)e^{-0.5t}}$$

$$a = 0 : (-3.5 + 0.75t)e^{-0.5t} = 0, \underline{t = 4.67 \text{ s}}$$



Question Title: Chapter 2, Reserve Problem 2/003

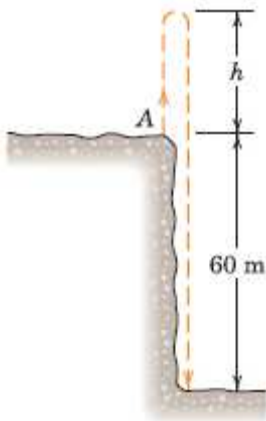
Calculate the constant acceleration a in g 's which the catapult of an aircraft carrier must provide to produce a launch velocity of 180 mi/hr in a distance of 300 ft. Assume that the carrier is at anchor.

SOLUTION

$$\begin{aligned}\text{For } a = \text{constant, } v^2 &= v_0^2 + 2as \\ \left[\frac{180 (5280)}{3600} \right]^2 &= 0^2 + 2a(300) \\ a &= 116.2 \text{ ft/sec}^2 \\ a &= 116.2 / 32.2 = \underline{3.61g}\end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/004 (Multistep version available)

A ball is thrown vertically up with a velocity of 30 m/s at the edge of a 60-m cliff. Calculate the height h to which the ball rises and the total time t after release for the ball to reach the bottom of the cliff. Neglect air resistance and take the downward acceleration to be 9.81 m/s^2 .



SOLUTION

$$\begin{aligned} \uparrow +y \quad y &= v_0 t + \frac{1}{2} a t^2, \quad y = 30t - \frac{1}{2} 9.81 t^2 \\ \perp \quad \text{for } y &= -60 \text{ m} \\ -60 &= 30t - \frac{9.81}{2} t^2 \\ \text{or } 9.81 t^2 - 60t - 120 &= 0 \\ t &= \frac{60 \pm \sqrt{60^2 + 4(9.81)(120)}}{2(9.81)} = \underline{7.70 \text{ s}} \quad (\text{or } -1.588 \text{ s}) \\ \text{For } v &= 0, \quad v^2 = v_0^2 + 2ay, \quad y = h = \frac{0 - 30^2}{-2(9.81)} = \underline{45.9 \text{ m}} \end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/005 (GO Tutorial version available)

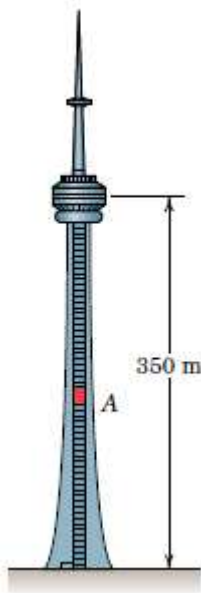
A car comes to a complete stop from an initial speed of 50 mi/hr in a distance of 100 ft. With the same constant acceleration, what would be the stopping distance s from an initial speed of 70 mi/hr?

SOLUTION

$$\begin{aligned} v^2 - v_0^2 &= 2a(s - s_0) \\ 0 - \left[50 \frac{5280}{3600} \right]^2 &= 2a(100), a = -26.9 \frac{\text{ft}}{\text{sec}^2} \\ \text{Then } 0 - \left[70 \frac{5280}{3600} \right]^2 &= 2(-26.9)s \\ \underline{s = 196.0 \text{ ft}} \end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/006

The main elevator A of the tower (modeled after the CN Tower in Toronto) rises about 350 m and for most of its run has a constant speed of 22 km/h. Assume that both the acceleration and deceleration have a constant magnitude of $0.25g$ and determine the time duration t of the elevator run. Note that your numbers on this problem do not represent the actual CN tower in Toronto.



SOLUTION

Acceleration period :

$$v = v_0 + at : \frac{22}{3.6} = 0 + \frac{9.81}{4} t_a, t_a = 2.49 \text{ s}$$

Note that The deceleration time $t_d = t_a$

$$v^2 = v_0^2 + 2a \Delta s : \left(\frac{22}{3.6}\right)^2 = 0^2 + 2 \frac{9.81}{4} \Delta s_a$$

$$\Delta s_a = 7.61 \text{ m} = \Delta s_d$$

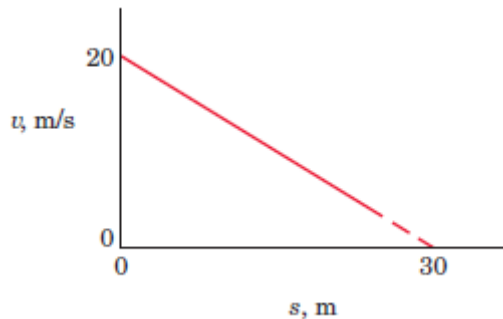
$$\text{Cruise period : } \Delta s_c = 350 - \Delta s_a - \Delta s_d = 335 \text{ m}$$

$$\Delta s = v_c t_c : 335 = \frac{22}{3.6} t_c, t_c = 54.8 \text{ s}$$

$$\text{Total run time } t = t_c + t_a + t_d = \underline{59.8 \text{ s}}$$

Question Title: Chapter 2, Reserve Problem 2/007

If the velocity v of a particle moving along a straight line decreases linearly with its displacement s from 20 m/s to a value approaching zero at $s = 30$ m, determine the acceleration a of the particle when $s = 15$ m and show that the particle never reaches the 30-m displacement.



SOLUTION

Concise Solution

$$v = 20 - ks \quad \text{where } k = \frac{2}{3} \text{ s}^{-1}$$

$$\text{So } v = 20 - \frac{2}{3}s, \quad \frac{dv}{ds} = -\frac{2}{3} \text{ s}^{-1}$$

$$\text{Note that } v = 10 \text{ m/s when } s = 15 \text{ m}$$

$$\therefore a = v \frac{dv}{ds} = 10 \left(-\frac{2}{3} \right) = \underline{-6.67 \text{ m/s}^2}$$

$$v = \frac{ds}{dt} \Rightarrow \int_0^t dt = \int_0^{30} \frac{ds}{v} = \int_0^{30} \frac{ds}{20 - \frac{2}{3}s}$$

$$t = -\frac{3}{2} \ln \left(20 - \frac{2}{3}s \right) \Big|_0^{30} = -\frac{3}{2} \ln \frac{0}{20} \rightarrow \underline{\underline{\infty}}$$

Detailed Solution

Solution Strategy:

- 1) Calculate the velocity v as a function of displacement s .
- 2) Calculate the acceleration a as a function of displacement s .
- 3) Calculate the acceleration a when displacement $s = 15$ m.
- 4) Prove that the particle can never reach a displacement $s = 30$ m.

Learning Outcomes:

- 1) Evaluate rectilinear kinematics of a particle via integration.
- 2) Evaluate rectilinear kinematics of a particle via differentiation.

1) Calculate the velocity v as a function of displacement s .

The general equation of the line, shown in the problem statement, is:

$$v = As + B \quad (1)$$

Where A and B are constants.

The boundary conditions are:

$$s = 0 \text{ m}, v = 20 \text{ m/s}$$

$$s = 30 \text{ m}, v = 0 \text{ m/s}$$

Apply the 1st boundary condition to Eq. 1:

$$20 = A(0) + B \quad (2)$$

So:

$$\boxed{B = 20 \text{ m/s}} \leftarrow \text{intermediate ans}$$

Apply the 2nd boundary condition to Eq. 1:

$$0 = A(30) + B \quad (3)$$

Substitute the calculated value into Eq. 3:

$$0 = A(30) + 20$$

So:

$$\boxed{A = -\frac{2}{3}} \leftarrow \text{intermediate ans}$$

Substitute the calculated values to Eq. 1:

$$\boxed{v = -\frac{2}{3}s + 20} \quad (4) \leftarrow \text{intermediate equation}$$

2) Calculate the acceleration a as a function of displacement s .

The differential equation that relates displacement s , velocity v , and acceleration a is:

$$a \, ds = v \, dv \quad (5)$$

Rearrange Eq. 5:

$$a = v \frac{dv}{ds} \quad (6)$$

Substitute Eq. 4 into Eq. 6:

$$a = \left(-\frac{2}{3}s + 20 \right) \frac{d\left(-\frac{2}{3}s + 20 \right)}{ds} \quad (7)$$

Evaluate the derivative in Eq. 7:

$$a = \left(-\frac{2}{3}s + 20 \right) \left(-\frac{2}{3} \right) \quad (8)$$

Simplify Eq. 8:

$$a = \left(\frac{4}{9}s - \frac{40}{3} \right) \quad (9)$$

3) Calculate the acceleration a when displacement $s = 15$ m.

Substitute the known value into Eq. 9:

$$a = \left(\frac{4}{9}(15) - \frac{40}{3} \right) \text{ m/s}^2$$

Or:

$$\underline{\underline{a = -6.67 \text{ m/s}^2 \leftarrow \text{ans}}}$$

4) Prove that the particle can never reach a displacement $s = 30$ m.

The instantaneous velocity of a particle is given by:

$$v = \frac{ds}{dt} \quad (10)$$

Substitute Eq. 4 into Eq. 10:

$$-\frac{2}{3}s + 20 = \frac{ds}{dt} \quad (11)$$

Separate the variables of Eq. 11:

$$dt = \frac{ds}{-\frac{2}{3}s + 20} \quad (12)$$

Apply limits of integration to Eq. 12:

$$\int_0^t dt = \int_0^s \frac{ds}{-\frac{2}{3}s + 20} \quad (13)$$

Rearrange Eq. 13:

$$\int_0^t dt = -\frac{3}{2} \int_0^s \frac{ds}{(s-30)} \quad (14)$$

Evaluate the integral of Eq. 14 and simplify:

$$t = -\frac{3}{2} [\ln(s-30) - \ln(-30)] \quad (15)$$

Or:

$$t = -\frac{3}{2} \ln\left(\frac{s-30}{-30}\right) \quad (16)$$

Substitute the known value of $s = 30$ m into Eq. 16:

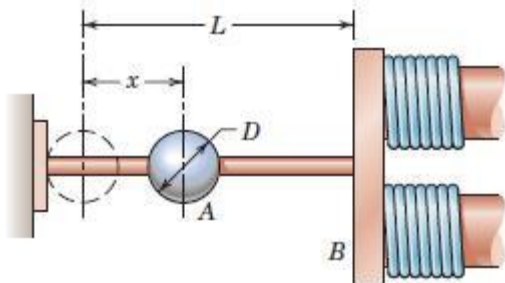
$$t = -\frac{3}{2} \ln\left(\frac{30-30}{-30}\right) \text{ sec}$$

So:

$$t = -\frac{3}{2} \ln(0) = \infty, \text{ which is impossible} \leftarrow \text{ans}$$

Question Title: Chapter 2, Reserve Problem 2/008

The steel ball *A* of diameter *D* slides freely on the horizontal rod which leads to the pole face of the electromagnet. The force of attraction obeys an inverse-square law, and the resulting acceleration of the ball is $a = K/(L - x)^2$, where *K* is a measure of the strength of the magnetic field. If the ball is released from rest at $x = 0$, determine the velocity *v* with which it strikes the pole face.



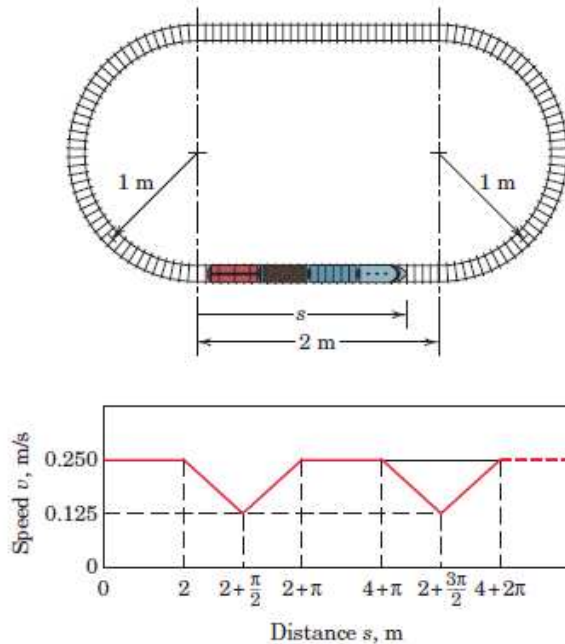
SOLUTION

This solution provides a guideline to solving the problem, not the numerical answer.

$$v dv = a dx, \quad \int_0^v v dv = \int_0^x \frac{K}{(L-x)^2} dx$$
$$\frac{v^2}{2} = \frac{K}{L-x} \Big|_0^x; \quad v^2 = \frac{2Kx}{L(L-x)}$$
$$\text{For } x = L - D/2, \quad v = 2 \sqrt{\frac{K(L - D/2)}{LD}}$$

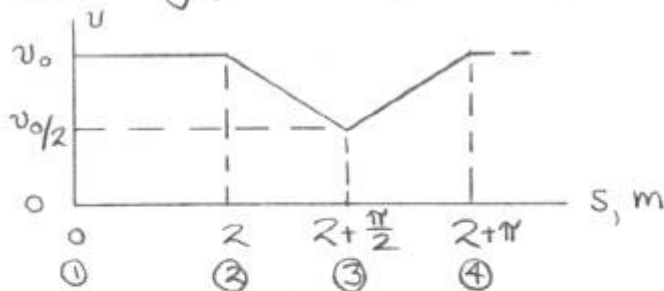
Question Title: Chapter 2, Reserve Problem 2/009

The electronic throttle control of a model train is programmed so that the train speed varies with position as shown in the plot. Determine the time t required for the train to complete one lap.



SOLUTION

Compute the time $t_{1/2}$ required for $\frac{1}{2}$ lap.
 Temporarily, let v_0 be the straightaway speed:



Time t_{1-2} to go from ① to ②: $t_{1-2} = \frac{2}{v_0}$

For t_{2-3} : Note that $\frac{dv}{ds} = -c$ ($c = \text{pos. const.}$)

$$\text{so that } a_t = v \frac{dv}{ds} = -cv = \frac{dv}{dt}$$

$$-c \int_{t_2}^{t_3} dt = \int_{v_2}^{v_3} \frac{dv}{v} : -ct_{2-3} = \ln\left(\frac{v_0/2}{v_0}\right)$$

$$t_{2-3} = \frac{1}{c} \ln 2$$

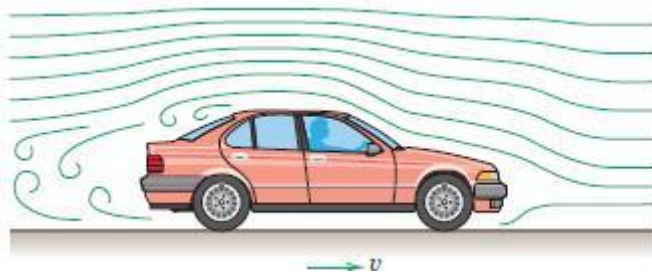
likewise, $t_{3-4} = \frac{1}{c} \ln 2$

$$c = \frac{v_0/2}{\pi/2} = \frac{v_0}{\pi}, \quad t_{1/2} = t_{1-2} + t_{2-3} + t_{3-4} = \frac{6.36}{v_0}$$

$$\text{lap time } t = 2t_{1/2} = \frac{12.71}{v_0} = \frac{12.71}{0.25} = \underline{50.8\text{ s}}$$

Question Title: Chapter 2, Reserve Problem 2/010

The aerodynamic resistance to motion of a car is nearly proportional to the square of its velocity. Additional frictional resistance is constant, so that the acceleration of the car when coasting may be written $a = -C_1 - C_2v^2$, where C_1 and C_2 are constants which depend on the mechanical configuration of the car. If the car has an initial velocity v_0 when the engine is disengaged, derive an expression for the distance D required for the car to coast to a stop.



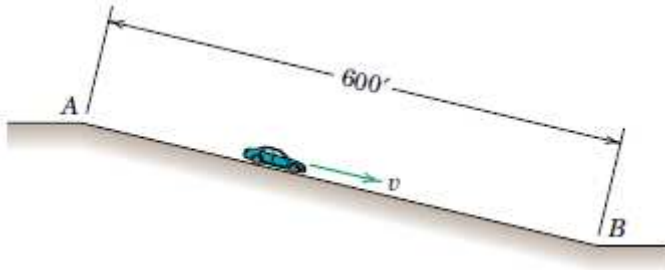
SOLUTION

This solution provides a guideline to solving the problem, not the numerical answer.

$$v dv = a dx, \quad \int_0^x dx = \int_{v_0}^v \frac{v dv}{-C_1 - C_2 v^2}$$
$$x = \left. \frac{-1}{2C_2} \ln(C_1 + C_2 v^2) \right|_{v_0}^v = \frac{1}{2C_2} \ln \frac{C_1 + C_2 v_0^2}{C_1 + C_2 v^2}$$
$$\text{when } v=0, \quad x=D = \frac{1}{2C_2} \ln \left(1 + \frac{C_2}{C_1} v_0^2 \right)$$

Question Title: Chapter 2, Reserve Problem 2/011

The driver of a car, which is initially at rest at the top A of the grade, releases the brakes and coasts down the grade with an acceleration in feet per second squared given by $a = 3.22 - 0.04v^2$, where v is the velocity in feet per second. Determine the velocity v_B at the bottom B of the grade.



SOLUTION

$$a = v \frac{dv}{ds} = 3.22 - 0.004v^2$$

$$\int_0^{v_B} \frac{v dv}{3.22 - 0.004v^2} = \int_0^{600} ds$$

$$\frac{1}{2(-0.004)} \ln(3.22 - 0.004v^2) \Big|_0^{v_B} = 600$$

$$\ln \left[\frac{3.22 - 0.004v_B^2}{3.22} \right] = 600(2)(-0.004)$$

$$\frac{3.22 - 0.004v_B^2}{3.22} = 0.00823$$

$$\underline{v_B = 28.3 \text{ ft/sec}}$$

Question Title: Chapter 2, Reserve Problem 2/012

The vertical acceleration of a certain solid-fuel rocket is given by $a = ke^{-bt} - cv - g$ where k , b , and c are constants, v is the vertical velocity acquired, and g is the gravitational acceleration, essentially constant for atmospheric flight. The exponential term represents the effect of a decaying thrust as fuel is burned, and the term $-cv$ approximates the retardation due to atmospheric resistance. Determine the expression for the vertical velocity v of the rocket t seconds after firing.

SOLUTION

This solution provides a guideline to solving the problem, not the numerical answer.

$$\frac{dv}{dt} = ke^{-bt} - cv - g, \quad \frac{dv}{dt} + cv = ke^{-bt} - g$$

For the standard form for the solution of the first order linear differential equation,

$$e^{-\int c dt} = e^{-ct} \quad \&$$

$$v = Ae^{-ct} + e^{-ct} \int (ke^{-bt} - g)e^{ct} dt$$

$$= Ae^{-ct} + \frac{k}{c-b} e^{-bt} - \frac{g}{c}$$

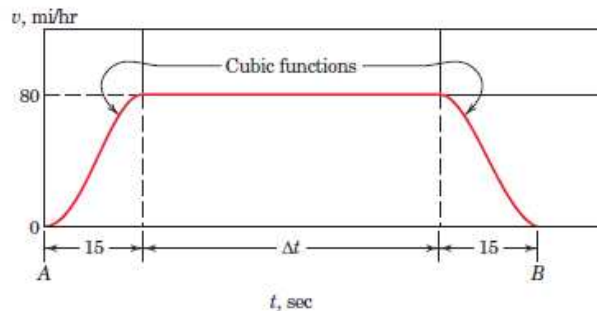
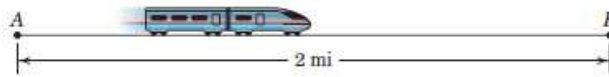
$$\text{when } t=0, v=0 \quad \text{so } 0 = A + \frac{k}{c-b} - \frac{g}{c}, \quad A = \frac{g}{c} - \frac{k}{c-b}$$

$$\text{Thus } v = \left(\frac{g}{c} - \frac{k}{c-b} \right) e^{-ct} + \frac{k}{c-b} e^{-bt} - \frac{g}{c}$$

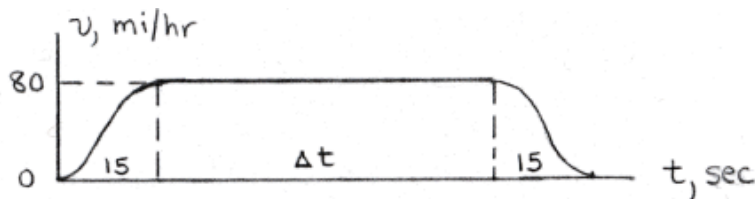
$$\text{or } \underline{v = \frac{g}{c}(e^{-ct} - 1) + \frac{k}{c-b}(e^{-bt} - e^{-ct})}$$

Question Title: Chapter 2, Reserve Problem 2/013

The preliminary design for a rapid-transit system calls for the train velocity to vary with time as shown in the plot as the train runs the two miles between stations A and B. The slopes of the cubic transition curves (which are of form $a + bt + ct^2 + dt^3$) are zero at the end points. Determine the total run time t between the stations and the magnitude of the maximum acceleration.



SOLUTION



$$\text{Cubic form: } v = a + bt + ct^2 + dt^3$$

$$v(0) = a = 0, \quad \frac{dv}{dt} = b + 2ct + 3dt^2$$

$$\frac{dv}{dt}\bigg|_{t=0} = 0 \Rightarrow b = 0; \quad \frac{dv}{dt}\bigg|_{t=15} = 30c + 675d = 0$$

$$v(15) = c(15)^2 + d(15)^3 = 80 \left(\frac{5280}{3600} \right)$$

$$\text{Solve to obtain } c = 1.564 \text{ ft/sec}^3, \quad d = -6.95(10^{-2}) \frac{\text{ft}}{\text{sec}^4}$$

$$\therefore v = 1.564t^2 - 0.0695t^3 \quad \text{for } 0 \leq t \leq 15 \text{ sec}$$

$$s = \int_0^t (1.564t^2 - 0.0695t^3) dt = 0.521t^3 - 0.01738t^4$$

$$s(15) = 880 \text{ ft}; \quad 880 \text{ ft also covered during decel.}$$

$$\text{Distance during cruise: } (2)(5280) - 2(880) = 8800 \text{ ft}$$

$$\Delta s = v \Delta t: \quad 8800 = 80 \left(\frac{5280}{3600} \right) \Delta t, \quad \Delta t = 75 \text{ sec}$$

$$\therefore \text{Run time } t = 75 + 2(15) = \underline{105 \text{ sec}}$$

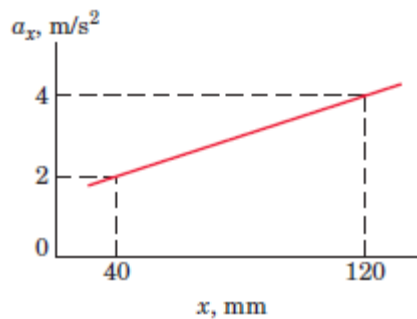
$$a = \frac{dv}{dt} = 3.13t - 0.209t^2$$

$$\frac{da}{dt} = 3.13 - 0.417t = 0, \quad t = 7.5 \text{ sec}$$

$$a_{\max} = 3.13(7.5) - 0.209(7.5)^2 = \underline{11.73 \text{ ft/sec}^2}$$

Question Title: Chapter 2, Reserve Problem 2/014

A particle moves along the positive x -axis with an acceleration in meters per second squared which increases linearly with x expressed in millimeters, as shown on the graph for an interval of its motion. If the velocity of the particle at $x = 40$ mm is 0.4 m/s, determine the velocity at $x = 120$ mm.



SOLUTION

$$\int_{0.4}^v v \, dv = \int a_x \, dx ; \quad \frac{1}{2} (v^2 - 0.4^2) = \text{area under } a_x - x \text{ curve}$$

$$\text{Area} = \int a_x \, dx = (a_x)_{av} \Delta x = 3(120 - 40)10^{-3} = 0.240 (\text{m/s})^2$$

$$\text{Thus } v^2 = 0.4^2 + 2(0.240) = 0.16 + 0.48 = 0.64$$

$$v = \sqrt{0.64} = \underline{0.8 \text{ m/s}}$$

Question Title: Chapter 2, Reserve Problem 2/015

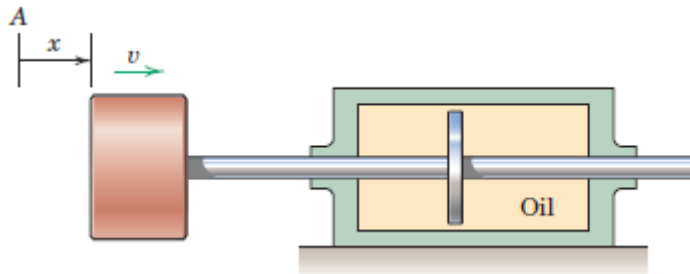
A retarding force is applied to a body moving in a straight line so that, during an interval of its motion, its speed v decreases with increased position coordinate s according to the relation $v^2 = k/s$, where k is a constant. If the body has a forward speed of 2 in./sec and its position coordinate is 9 in. at time $t = 0$, determine the speed v at $t = 3$ sec.

SOLUTION

$$\begin{aligned} v^2 &= \frac{k}{s}, \quad v = \dot{s} = \sqrt{k/s} \text{ where } k = 2^2(9) = 36 \text{ in}^3/\text{sec}^2 \\ \int_9^s \sqrt{s} \, ds &= 6 \int_0^t dt, \quad \left[\frac{2}{3} s^{3/2} \right]_9^s = 6(t-0), \\ s^{3/2} &= 27 + 9t, \text{ but } v = \sqrt{k} s^{-1/2} \text{ so} \\ v &= 6(27 + 9t)^{-1/3} \quad \& \text{ at } t = 3 \text{ sec, } v = 6(27 + 27)^{-1/3} \\ &= 2^{2/3} = \underline{\underline{1.587 \frac{\text{in.}}{\text{sec}}}} \end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/016

The horizontal motion of the plunger and shaft is arrested by the resistance of the attached disk which moves through the oil bath. If the velocity of the plunger is v_0 in the position A where $x = 0$ and $t = 0$, and if the deceleration is proportional to v so that $a = -kv$, derive expressions for the velocity v and position coordinate x in terms of the time t . Also express v in terms of x .



SOLUTION

This solution provides a guideline to solving the problem, not the numerical answer.

$$a = \frac{dv}{dt} = -kv, \quad \int_{v_0}^v \frac{dv}{v} = -k \int_0^t dt$$

$$\ln \frac{v}{v_0} = -kt, \quad \underline{v = v_0 e^{-kt}}$$

$$v = \frac{dx}{dt} = v_0 e^{-kt}, \quad \int_0^x dx = \int_0^t v_0 e^{-kt} dt$$

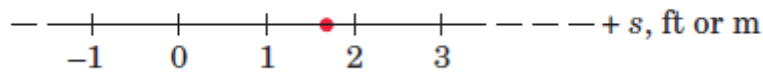
$$\underline{x = \frac{v_0}{k} [1 - e^{-kt}]}$$

$$v dv = a dx, \quad \frac{v dv}{v} = -k dx$$

$$\int_{v_0}^v dv = -k \int_0^x dx, \quad \underline{v = v_0 - kx}$$

Question Title: Chapter 2, Reserve Problem 2/017

The velocity of a particle is given by $v = 25t^2 - 80t - 200$, where v is in feet per second and t is in seconds. Plot the velocity v and acceleration a versus time for the first 6 seconds of motion and evaluate the velocity when a is zero.



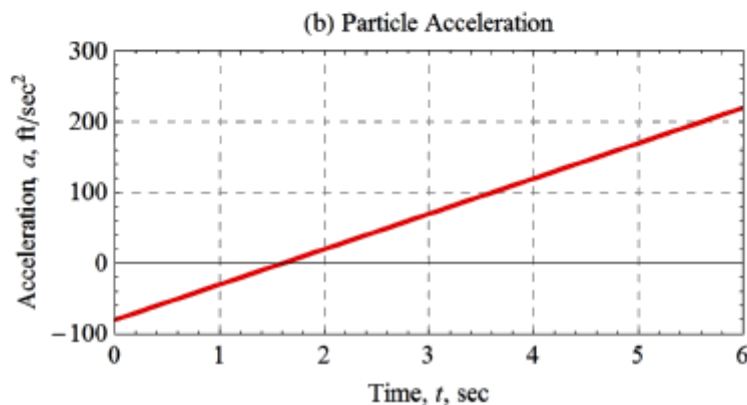
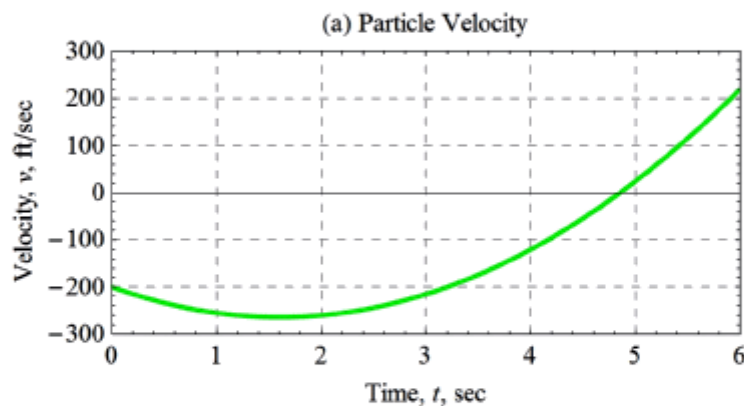
SOLUTION

This solution provides a guideline to solving the problem and might not match the values in your text.

$$\left. \begin{aligned} v &= 25t^2 - 80t - 200 \\ a &= \frac{dv}{dt} = 50t - 80 \end{aligned} \right\} \text{See plots}$$

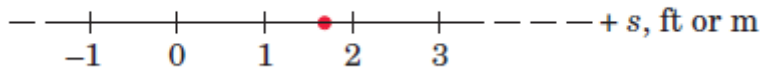
$$a = 0 : 50t - 80 = 0, \quad t = 1.6 \text{ sec}$$

$$\text{At } t = 1.6 \text{ sec, } v = 25(1.6)^2 - 80(1.6) - 200 = -264 \frac{\text{ft}}{\text{sec}}$$



Question Title: Chapter 2, Reserve Problem 2/018

The acceleration of a particle is given by $a = -kt^2$, where a is in meters per second squared and the time t is in seconds. If the initial velocity of the particle at $t = 0$ is $v_0 = 12$ m/s and the particle takes 6 seconds to reverse direction, determine the constant k . What is the net displacement Δs of the particle over the same 6-second interval of motion?



SOLUTION

At $t=0$, $v_0 = 12$ m/s

$$a = -kt^2 = \frac{dv}{dt} \rightarrow \int_0^t -kt^2 dt = \int_{v_0}^v dv$$

$$v = v_0 - \frac{1}{3}kt^3 \quad \text{AND} \quad \text{At } t=6, v=0 = 12 - \frac{1}{3}k(6)^3$$

$$\text{So... } \underline{k = \frac{1}{6}}$$

For UNITS...

$$[a] = -[k][t^2] \rightarrow \frac{L}{T^2} = [k]T^2 \rightarrow [k] = \frac{L}{T^4} \propto \underline{\frac{m}{s^4}}$$

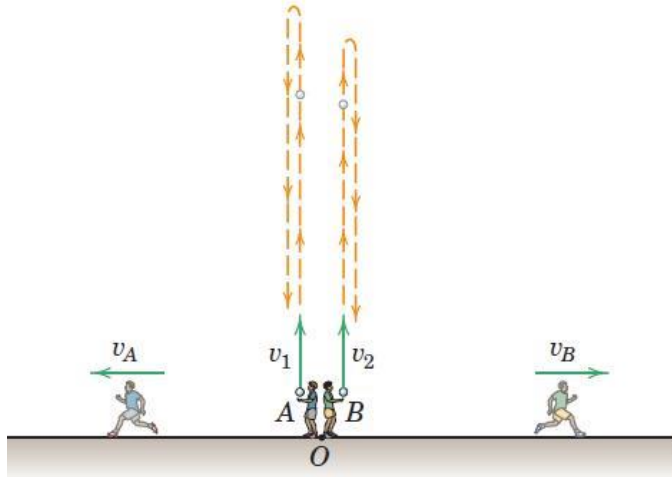
$$\text{So... } v = 12 - \frac{1}{18}t^3 = \frac{ds}{dt}$$

$$\int_{s_0}^{s_6} ds = \int_{t_0=0}^{t=6} (12 - \frac{1}{18}t^3) dt \rightarrow s_6 - s_0 = \Delta s = 12t - \frac{1}{72}t^4 \Big|_0^6$$

$$\underline{\Delta s = 54 \text{ m}}$$

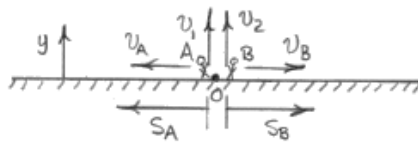
Question Title: Chapter 2, Reserve Problem 2/019

A game requires that two children each throw a ball upward as high as possible from point O and then run horizontally in opposite directions away from O . The child who travels the greater distance before their thrown ball impacts the ground wins. If child A throws a ball upward with a speed of $v_1 = 70$ ft/sec and immediately runs leftward at a constant speed of $v_A = 16$ ft/sec while child B throws the ball upward with a speed of $v_2 = 64$ ft/sec and immediately runs rightward with a constant speed of $v_B = 18$ ft/sec, which child will win the game? Calculate the distances s_A and s_B that each child travels while the respective ball is in the air.



SOLUTION

$$v_A = 16 \text{ ft/sec}, v_B = 18 \text{ ft/sec}, v_1 = 70 \text{ ft/sec}, v_2 = 64 \text{ ft/sec}$$



FLIGHT TIME:

$$y_f = y_o + v_{y_o} t_f - \frac{1}{2} g t_f^2 \rightarrow t_f = 0 \text{ (TRIVIAL)} \text{ OR } t_f = \frac{2v_{y_o}}{g}$$

$$\begin{cases} \text{A: } t_A = \frac{2v_1}{g} = \frac{2(70)}{32.2} \rightarrow t_A = 4.35 \text{ sec} \\ \text{B: } t_B = \frac{2v_2}{g} = \frac{2(64)}{32.2} \rightarrow t_B = 3.98 \text{ sec} \end{cases}$$

DISTANCE:

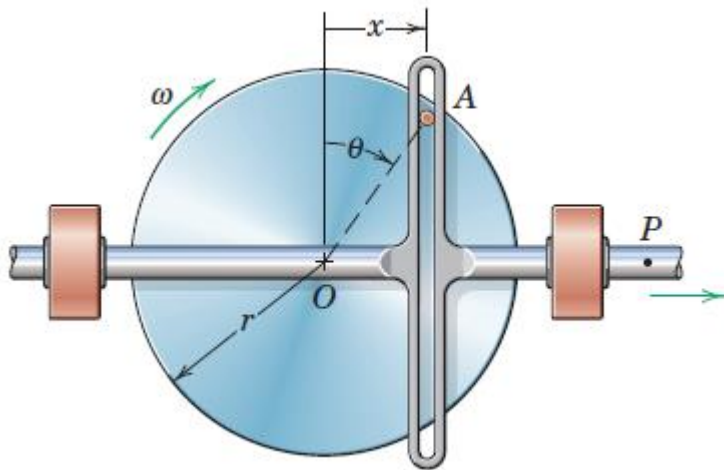
$$s_f = s_o + v_{s_o} t_f + \frac{1}{2} a_s t_f^2 \rightarrow s_f = v_{s_o} t_f$$

$$\begin{cases} \text{A: } s_A = v_A t_A = 16(4.35) \rightarrow s_A = 69.6 \text{ ft} \\ \text{B: } s_B = v_B t_B = 18(3.98) \rightarrow s_B = 71.6 \text{ ft} \end{cases}$$

So, CHILD B WINS BY 1.988 ft

Question Title: Chapter 2, Reserve Problem 2/020 (Multistep version available)

A Scotch-yoke mechanism is used to convert rotary motion into reciprocating motion. As the disk rotates at the constant angular rate ω , a pin A slides in a vertical slot causing the slotted member to displace horizontally according to $x = r \sin(\omega t)$ relative to the fixed disk center O . Determine the expressions for the velocity v_P and acceleration a_P of a point P on the output shaft of the mechanism as functions of time. Use $r = 75 \text{ mm}$ and $\omega = \pi \text{ rad/s}$.



SOLUTION

$$r = 75 \text{ mm} \quad \omega = \pi \text{ rad/s}$$

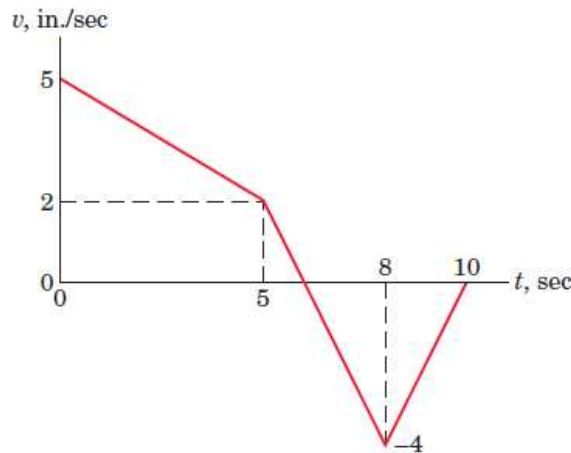
$$\begin{cases} x = r \sin(\omega t) \\ \dot{x} = v = r\omega \cos(\omega t) & \text{so... } v = 75\pi \cos(\pi t) \text{ mm/s} \\ \ddot{x} = a = -r\omega^2 \sin(\omega t) & \text{so... } a = -75\pi^2 \sin(\pi t) \text{ mm/s}^2 \end{cases}$$

$$v_{\max} = 75\pi \text{ OR } 236 \text{ mm/s}$$

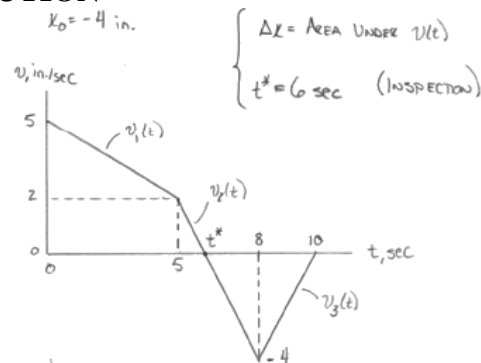
$$a_{\max} = 75\pi^2 \text{ OR } 740 \text{ mm/s}^2$$

Question Title: Chapter 2, Reserve Problem 2/021

A particle moves along the x -axis with the velocity history shown. If the particle is at the position $x = -4$ in, at time $t = 0$, plot the corresponding displacement history for the time interval $0 \leq t \leq 10$ sec. Additionally, find the net displacement and total distance traveled by the particle for this interval.



SOLUTION



FIND ALL v & x FUNCTIONS.

$$\begin{cases} v_1(t) = 5 - \frac{3}{5}t \text{ in./sec} \\ x_1(t) = x_0 + \int v_1(t) dt = -4 + \int (5 - \frac{3}{5}t) dt \rightarrow x_1(t) = -4 + 5t - \frac{3}{10}t^2 \text{ in.} \end{cases}$$

At $t = 5$ sec... $x_1(5) = -4 + 5(5) - \frac{3}{10}(5)^2 = 13.5$ in./sec

$$\begin{cases} v_2(t) = 2 - \frac{6}{3}(t-5) = 12 - 2t \text{ in./sec} \\ x_2(t) = \int v_2(t) dt + C_1 = \int (12 - 2t) dt + C_1 = C_1 + 12t - t^2 \end{cases}$$

At $t = 5$, $x_2(5) = 13.5 = C_1 + 12(5) - 5^2$

$C_1 = -21.5$ $x_2(t) = -21.5 + 12t - t^2$ in.

$$\begin{cases} v_3(t) = -4 + \frac{4}{2}(t-8) = -20 + 2t \text{ in./sec} \\ x_3(t) = \int v_3(t) dt + C_2 = \int (-20 + 2t) dt + C_2 = C_2 - 20t + t^2 \end{cases}$$

At $t = 8$ sec... $x_2(8) = -21.5 + 12(8) - 8^2 = 10.5$ in.

So... $x_3(8) = 10.5 = C_2 - 20(8) + 8^2 \rightarrow C_2 = 106.5$

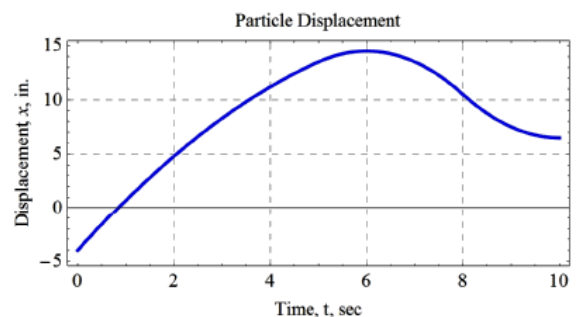
$x_3(t) = 106.5 - 20t + t^2$ in. { THE THREE FUNCTIONS CAN BE PLOTTED OVER THE APPROPRIATE TIME INTERVALS. }

$$\begin{cases} \text{At } t^* = 6 \text{ sec... } x_2(6) = -21.5 + 12(6) - 6^2 = 14.5 \text{ in.} \\ \text{At } t = 10 \text{ sec... } x_3(10) = 106.5 - 20(10) + 10^2 = 6.5 \text{ in.} \end{cases}$$

DISPLACEMENT... $\Delta x = x_f - x_0 = 6.5 - (-4) \rightarrow \Delta x = 10.5$ in.

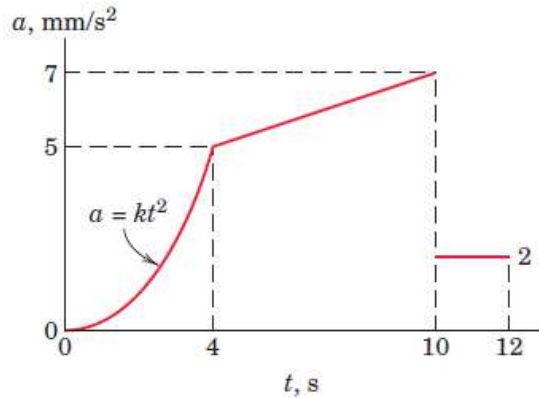
DISTANCE TRAVELED... $x_{\text{TOTAL}} = (14.5 - (-4)) + (14.5 - 6.5)$

$x_{\text{TOTAL}} = 26.5$ in.



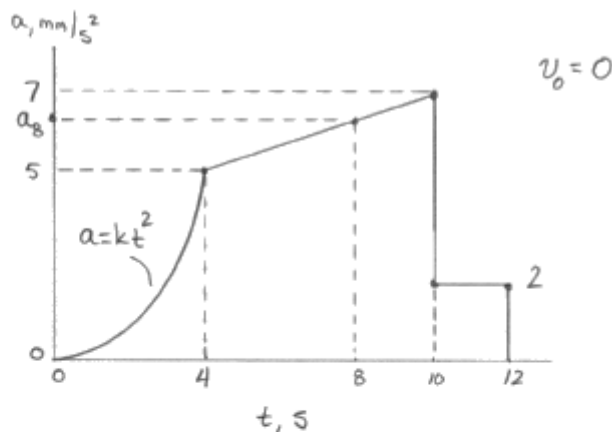
Question Title: Chapter 2, Reserve Problem 2/022 (Multistep version available)

The graph shows the rectilinear acceleration of a particle as a function of time over a 12-second interval. If the particle is at rest at the position $s_0 = 0$ at time $t = 0$, determine the velocity of the particle when (a) $t = 4$ s, (b) $t = 8$ s, and (c) $t = 12$ s.



SOLUTION

At $t = 4$ s, $a = 5 \text{ mm/s}^2 \dots \quad 5 = k(4)^2 \rightarrow k = \frac{5}{16} \text{ mm} \cdot \text{s}^{-4}$



$$v_4 = \int_0^4 \frac{5}{16} t^2 dt = \frac{5}{48} t^3 \Big|_0^4 \rightarrow \underline{v_4 = 6.67 \text{ mm/s}}$$

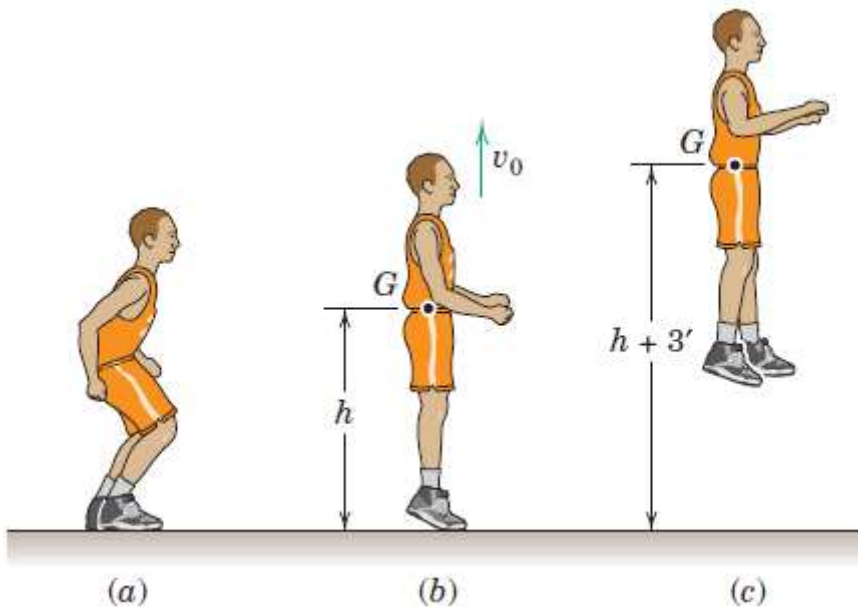
At $t = 8$ s, $a_8 = 5 + \frac{2}{3}(7-5) \rightarrow a_8 = 6.33 \text{ mm/s}^2$

$$v_8 = 6.67 + \frac{5 + 6.33}{2}(4) \rightarrow \underline{v_8 = 29.3 \text{ mm/s}}$$

$$v_{12} = 6.67 + \frac{5+7}{2}(6) + 2(2) \rightarrow \underline{v_{12} = 46.7 \text{ mm/s}}$$

Question Title: Chapter 2, Reserve Problem 2/023

In a test of vertical leaping ability, a basketball player (*a*) crouches just before jumping, (*b*) has given his mass center *G* a vertical velocity v_0 at the instant his feet leave the surface, and (*c*) reaches the maximum height. If the player can raise his mass center 3 feet as shown, estimate the initial velocity v_0 of his mass center in position (*b*).

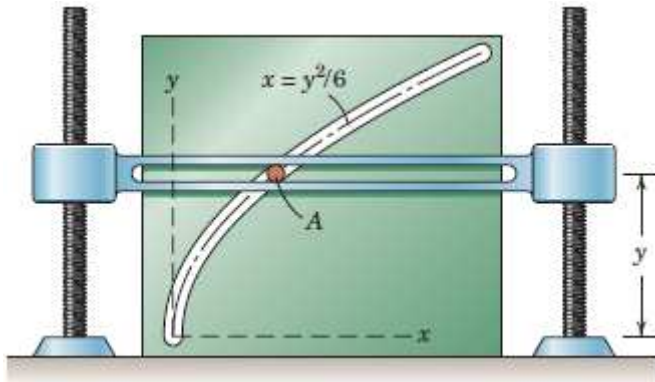


SOLUTION

$$\begin{aligned} +\uparrow \quad v_y^2 &= v_{y_0}^2 - 2g(y - y_0) \\ 0^2 &= v_0^2 - 2(32.2)(3) \\ v_0 &= 13.90 \text{ ft/sec} \end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/024 (Multistep version available)

For a certain interval of motion the pin A is forced to move in the fixed parabolic slot by the horizontal slotted arm which is elevated in the y-direction at the constant rate of 3 in./sec. All measurements are in inches and seconds. Calculate the velocity v and acceleration a of pin A when $x = 6$ in.



SOLUTION

$$x = y^2/6 \quad \& \quad \dot{y} = 3 \text{ in./sec}$$

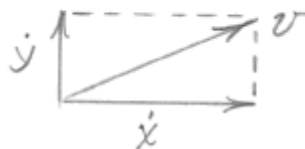
$$\dot{x} = \frac{y}{3} \dot{y} \quad , \quad \ddot{x} = \frac{\dot{y}^2}{3} + \frac{y}{3} \ddot{y} \quad \text{but } \ddot{y} = 0 \quad \& \quad \dot{y} = 3 \text{ in./sec}$$

$$\text{Also when } x = 6 \text{ in.}, \quad y = \sqrt{36} = 6 \text{ in.}$$

$$\text{So } \dot{x} = \frac{6}{3} (3) = 6 \text{ in./sec,}$$

$$\text{Hence } v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{6^2 + 3^2} = 3\sqrt{5} \text{ in./sec}$$

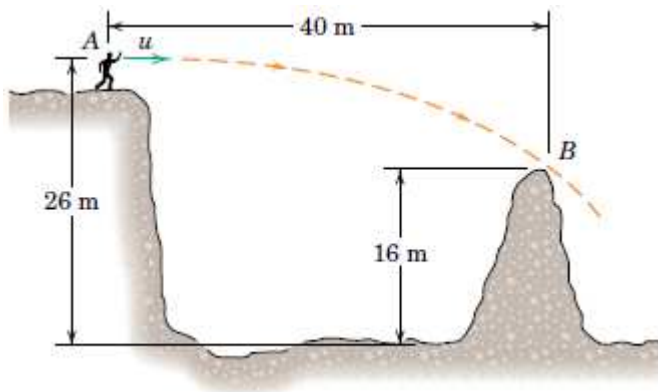
$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(3^2/3)^2 + 0} = 3 \text{ in./sec}^2$$



$$a = \ddot{x}$$

Question Title: Chapter 2, Reserve Problem 2/025

With what minimum horizontal velocity u can a boy throw a rock at A and have it just clear the obstruction at B?



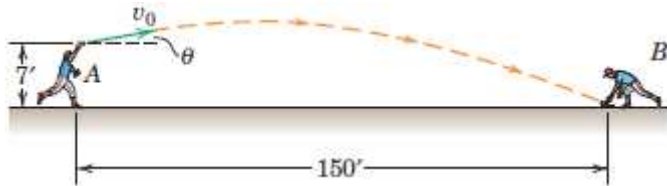
SOLUTION

$a_y = -g$ so $y = 0 - \frac{1}{2}gt^2$, $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(26-16)}{9.81}}$
 $= 1.428 \text{ s}$
 $x = ut$; $u = 40/1.428$
 $= \underline{28.0 \text{ m/s}}$

A hand-drawn diagram showing the rock's trajectory. It is a coordinate system with a vertical y-axis and a horizontal x-axis. The rock starts at point A, which is 26 m above the x-axis. It follows a parabolic path, clearing an obstruction at point B, which is 16 m above the x-axis. The horizontal distance from A to B is 40 m. The initial velocity u is shown as a horizontal arrow from A.

Question Title: Chapter 2, Reserve Problem 2/026

During a baseball practice session, the cutoff man *A* executes a throw to the third baseman *B*. If the initial speed of the baseball is $v_0 = 130$ ft/sec, what angle θ is best if the ball is to arrive at third base at essentially ground level?



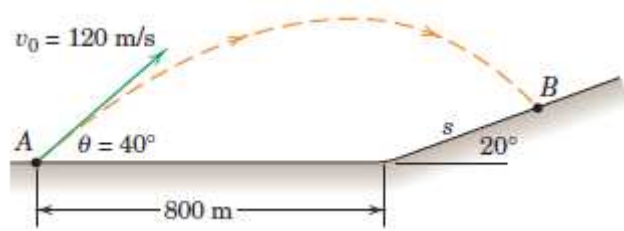
SOLUTION

$$\begin{aligned}
 & x = x_0 + v_{x_0} t \text{ at } B: \\
 & 150 = 0 + (130 \cos \theta) t_f \quad (1) \\
 & y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \text{ at } B: \\
 & 0 = 7 + (130 \sin \theta) t_f - 16.1 t_f^2 \quad (2) \\
 & \text{Solve (1) \& (2): } t_f = 8.05 \text{ sec}, \theta = 81.8^\circ \\
 & \text{or } t_f = 1.159 \text{ sec}, \theta = 5.57^\circ
 \end{aligned}$$

Clearly, the second solution is better in baseball!

Question Title: Chapter 2, Reserve Problem 2/027

A projectile is launched from point A with the initial conditions shown in the figure. Determine the slant distance s which locates the point B of impact. Calculate the time of flight t .



SOLUTION

Set up x - y coordinates with origin at A.

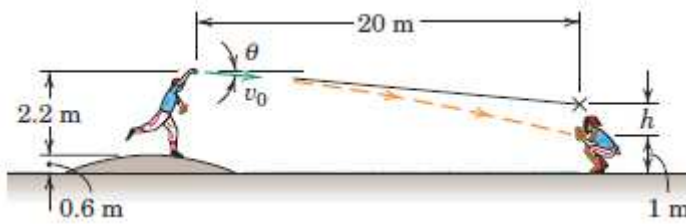
$$x = x_0 + v_{x_0}t \quad \text{@ B: } 800 + s \cos 20^\circ = (120 \cos 40^\circ)t \quad (1)$$
$$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2 \quad \text{@ B:}$$
$$s \sin 20^\circ = (120 \sin 40^\circ)t - \frac{1}{2}9.81t^2 \quad (2)$$

Solve (1) & (2):

$$\underline{s = 455 \text{ m}}, \quad \underline{t = 13.35 \text{ s}}$$

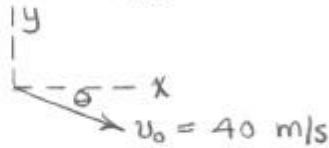
Question Title: Chapter 2, Reserve Problem 2/028

Determine the location h of the spot toward which the pitcher must throw if the ball is to hit the catcher's mitt. The ball is released with a speed of 40 m/s.



SOLUTION

Set up x - y coordinates with origin at release point:



$$x = x_0 + v_{x_0} t \quad \text{at mitt:} \quad 20 = (40 \cos \theta) t_f \quad (1)$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \quad \text{at mitt:}$$

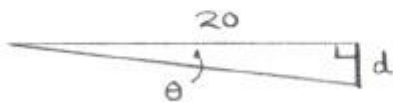
$$-1.8 = 0 + (-40 \sin \theta) t_f - \frac{9.81}{2} t_f^2 \quad (2)$$

$$(1): \quad t_f = \frac{1}{2 \cos \theta}$$

$$(2): \quad -1.8 = -40 \sin \theta \left(\frac{1}{2 \cos \theta} \right) - \frac{9.81}{2} \left(\frac{1}{2 \cos \theta} \right)^2$$

$$\text{Use } \frac{1}{\cos^2 \theta} = \tan^2 \theta + 1: \quad 1.226 \tan^2 \theta + 20 \tan \theta - 0.574 = 0$$

$$\Rightarrow \theta = 1.640^\circ$$

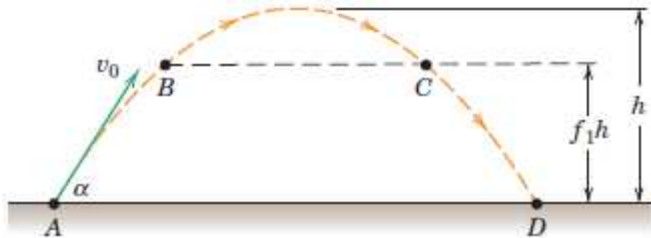


$$d = 20 \tan 1.640^\circ = 0.573'$$

$$h = (2.2 + 0.6) - (0.573 + 1) = \underline{1.227 \text{ m}}$$

Question Title: Chapter 2, Reserve Problem 2/029

A projectile is launched from point A and lands on the same level at D . Its maximum altitude is h . Determine and plot the fraction of the total flight time f_2 that the projectile is above the level $f_1 h$ where f_1 is a fraction which can vary from zero to 1. State the value of f_2 for $f_1 = 3/4$.

**SOLUTION**

Set up x - y coordinates at A :

From $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ evaluated at D ,
 we have $0 = 0 + v_0 \sin \alpha t_D - \frac{1}{2}gt_D^2$ or $t_D = \frac{2v_0 \sin \alpha}{g}$

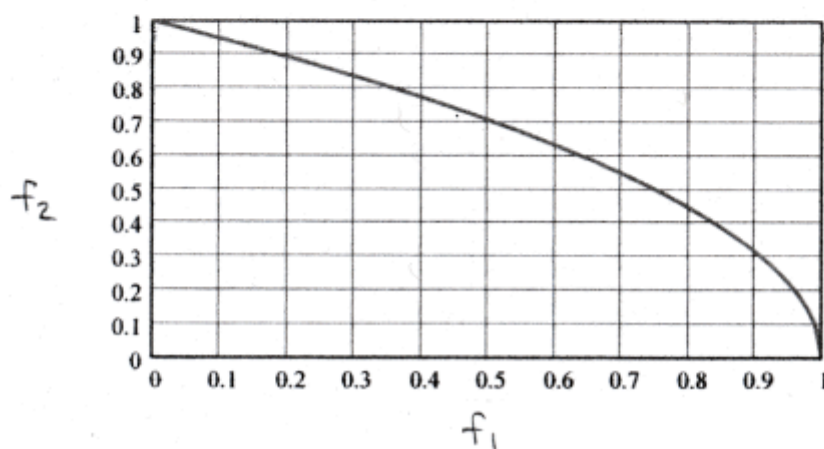
From $v_y^2 = v_{y0}^2 - 2g(y - y_0)$ evaluated at apex, we
 have $0 = v_0^2 \sin^2 \alpha - 2gh$ or $h = \frac{v_0^2 \sin^2 \alpha}{2g}$

Time to $f_1 h$: $f_1 h = f_1 \frac{v_0^2 \sin^2 \alpha}{2g} = (v_0 \sin \alpha)t - \frac{1}{2}gt^2$

Solve quadratic to obtain $\begin{cases} t_B = \frac{v_0 \sin \alpha (1 - \sqrt{1 - f_1})}{g} \\ t_C = \frac{v_0 \sin \alpha (1 + \sqrt{1 - f_1})}{g} \end{cases}$

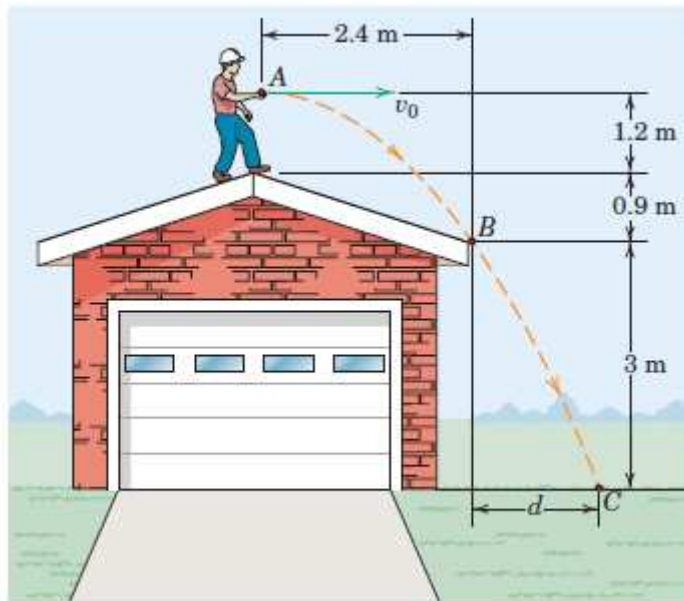
So $t_{BC} = t_C - t_B = \frac{2\sqrt{1 - f_1} v_0 \sin \alpha}{g}$

$f_2 = \frac{t_{BC}}{t_D} = \sqrt{1 - f_1}$; $f_2 = \frac{1}{2}$ for $f_1 = \frac{3}{4}$.



Question Title: Chapter 2, Reserve Problem 2/030

A roofer tosses a small tool to the ground. What minimum magnitude v_0 of horizontal velocity is required to just miss the roof corner B ? Also determine the distance d .

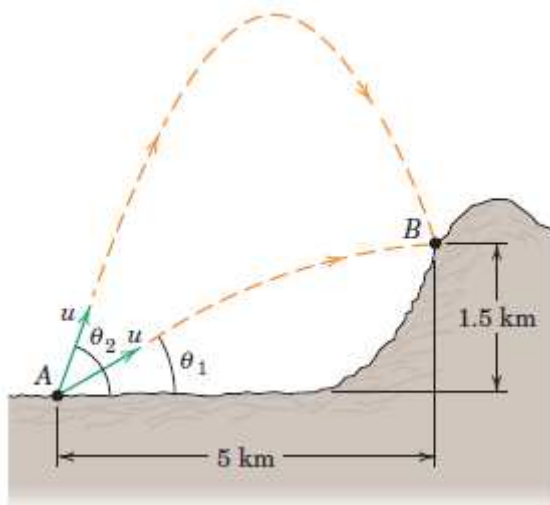


SOLUTION

$$\begin{aligned}
 & y = y_0 + v_{y0}t - \frac{1}{2}gt^2 \quad \text{at } B: \\
 & -2.1 = 0 + 0 - \frac{9.81}{2}t_B^2, \quad t_B = 0.654 \text{ s} \\
 & x = x_0 + v_{x0}t \quad \text{at } B: \\
 & 2.4 = 0 + v_0(0.654), \quad \underline{v_0 = 3.67 \frac{\text{m}}{\text{s}}} \\
 & \text{y-displacement equation at C:} \\
 & -5.1 = -\frac{9.81}{2}t_C^2, \quad t_C = 1.020 \text{ s} \\
 & \text{x-displacement equation at C:} \\
 & 2.4 + d = 3.67(1.020), \quad \underline{d = 1.340 \text{ m}}
 \end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/031

The muzzle velocity of a long-range rifle at A is $u = 400$ m/s. Determine the two angles of elevation which will permit the projectile to hit the mountain target B.



SOLUTION

Concise Solution

$$y = x \tan \theta - \frac{g x^2}{2 u^2} \sec^2 \theta$$

$$= x \tan \theta - \frac{g x^2}{2 u^2} (1 + \tan^2 \theta)$$

Substitute values & get

$$1500 = 5000 \tan \theta - \frac{9.81 (5000)^2}{2 (400)^2} (1 + \tan^2 \theta)$$

$$\text{or } \tan^2 \theta - 6.524 \tan \theta + 2.957 = 0$$

solution gives roots $\theta_1 = 26.1^\circ$ & $\theta_2 = 80.6^\circ$

Detailed Solution

Solution Strategy:

- 1) Clearly identify all known and unknown displacement, velocity, and acceleration values in the x and y -directions.
- 2) Apply kinematic equations in the x and y -directions.
- 3) Calculate angles θ_1 and θ_2 .

Learning Outcomes:

- 1) Apply kinematic equations in the Cartesian coordinate system.
- 2) Solve a system of non-linear algebraic equations in closed form.

Draw a representative sketch (note that θ is shown generally; two values of θ will ultimately be found: θ_1 and θ_2):

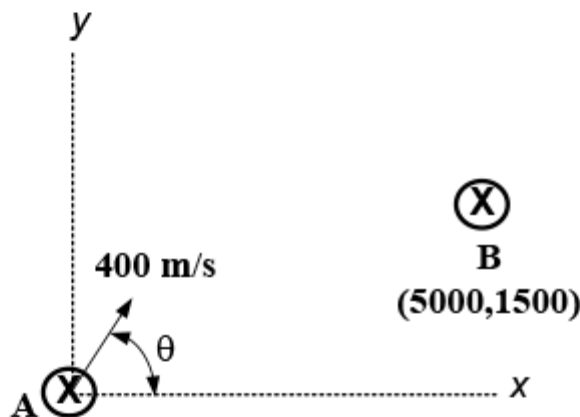


Figure 1

- 1) Clearly identify all known and unknown displacement, velocity, and acceleration values in the x and y -directions.

Table 1

x	y
$x_0 = 0 \text{ m}$	$y_0 = 0 \text{ m}$
$x = 5000 \text{ m}$	$y = 1500 \text{ m}$
$v_x = v_{x0} = 400 \cos \theta \text{ m/s}$	$v_{y0} = 400 \sin \theta \text{ m/s}$
$a_x = 0 \text{ m/s}^2$	$a_y = -9.81 \text{ m/s}^2$

- 2) Apply kinematic equations in the x and y -directions.

Apply the following kinematic equation in the x -direction:

$$x = x_0 + v_x t \quad (v_x \text{ is constant}) \quad (1)$$

Substitute the known values into Eq. 1:

$$5000 = 0 + 400(\cos \theta)t \quad (2)$$

Simplify and rearrange Eq. 2:

$$t = \frac{5000}{400 \cos(\theta)} \leftarrow \text{intermediate equation} \quad (3)$$

Apply the following kinematic equation in the y-direction:

$$y = y_0 + v_{y0} \cdot t + \frac{1}{2} a_y t^2 \quad (a_y \text{ is constant}) \quad (4)$$

Substitute the known values into Eq. 4 and simplify:

$$1500 = 400(\sin \theta)t + \frac{1}{2}(-9.81)t^2 \quad \leftarrow \text{intermediate equation} \quad (5)$$

3) Calculate angles θ_1 and θ_2 .

SPECIAL NOTE: Eqs. 3 and 5 contain two unknowns. There are two equations and two unknowns, so it is feasible to solve for t and θ . However, Eqs. 3 and 5 form a set of non-linear algebraic equations, so the solution strategy is not conventional.

One solution that yields the correct solution is provided below. There are other strategies that would also produce the same solution.

Substitute Eq. 3 into Eq. 5:

$$1500 = [400 \sin \theta] \cdot \left[\frac{5000}{400 \cos \theta} \right] - 4.905 \cdot \left[\frac{5000}{400 \cos \theta} \right]^2 \quad (6)$$

Substitute $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\frac{1}{\cos^2 \theta} = \sec^2 \theta$ into Eq. 6:

$$1500 = 5000 \tan \theta - 766.4 \sec^2 \theta \quad (7)$$

Substitute the trigonometric identity $\sec^2 \theta = 1 + \tan^2 \theta$ into Eq. 7 and simplify:

$$0 = -2266.4 + 5000 \tan \theta - 766.4 \tan^2 \theta \quad (8)$$

Substitute $m = \tan \theta$ into Eq. 8:

$$0 = -2266.4 + 5000m - 766.4m^2 \quad (9)$$

Solve Eq. 9 for the two values of m :

$$m_1 = 0.490$$

$$m_2 = 6.034$$

Substitute $\tan \theta_1 = m_1$ and $\tan \theta_2 = m_2$ into the above calculated values:

$$\tan \theta_1 = 0.490$$

$$\tan \theta_2 = 6.034$$

So:

$$\underline{\underline{\theta_1 = 26.1^\circ \leftarrow \text{ans}}}$$

$$\underline{\underline{\theta_2 = 80.6^\circ \leftarrow \text{ans}}}$$

Question Title: Chapter 2, Reserve Problem 2/032

At time $t = 10$ s, the velocity of a particle moving in the x - y plane is $\mathbf{v} = 0.1\mathbf{i} + 2\mathbf{j}$ m/s. By time $t = 10.1$ s, its velocity has become $-0.1\mathbf{i} + 1.8\mathbf{j}$ m/s. Determine the magnitude a_{av} of its average acceleration during this interval and the angle θ made by the average acceleration with the positive x -axis.

SOLUTION

$$\underline{a}_{av} = \frac{\Delta \underline{v}}{\Delta t} = \frac{(-0.1\mathbf{i} + 1.8\mathbf{j}) - (0.1\mathbf{i} + 2\mathbf{j})}{0.1}$$

$$= -2\mathbf{i} - 2\mathbf{j} \text{ m/s}$$

$$a_{av} = \sqrt{2^2 + 2^2} = \underline{2.83 \text{ m/s}^2}$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-2}{-2}\right) = \underline{225^\circ}$$

Question Title: Chapter 2, Reserve Problem 2/033

The rectangular coordinates of a particle which moves with curvilinear motion are given by $x = 10.25t + 1.75t^2 - 0.45t^3$ and $y = 6.32 + 14.65t - 2.48t^2$, where x and y are in millimeters and the time t is in seconds, beginning from $t = 0$. Determine the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ of the particle when $t = 5$ s. Also, determine the first time that the velocity of the particle makes an angle of 45° with the positive x -axis.

SOLUTION

$$\begin{cases} x = 10.25t + 1.75t^2 - 0.45t^3 \text{ mm} \\ v_x = \frac{dx}{dt} = 10.25 + 3.5t - 1.35t^2 \text{ mm/s} \\ a_x = \frac{dv_x}{dt} = 3.5 - 2.7t \text{ mm/s}^2 \end{cases}$$

$$\begin{cases} y = 6.32 + 14.65t - 2.48t^2 \text{ mm} \\ v_y = \frac{dy}{dt} = 14.65 - 4.96t \text{ mm/s} \\ a_y = \frac{dv_y}{dt} = -4.96 \text{ mm/s}^2 \end{cases}$$

$$\text{At } t = 5 \text{ s} \dots \begin{cases} v_x = 10.25 + 3.5(5) - 1.35(5)^2 = -6 \text{ mm/s} \\ v_y = 14.65 - 4.96(5) = -10.15 \text{ mm/s} \end{cases}$$

$$\underline{\underline{\mathbf{v} = -6\mathbf{i} - 10.15\mathbf{j} \text{ mm/s}}}$$

$$\begin{cases} a_x = 3.5 - 2.7(5) = -10 \text{ mm/s}^2 \\ a_y = -4.96 \text{ mm/s}^2 \end{cases}$$

$$\underline{\underline{\mathbf{a} = -10\mathbf{i} - 4.96\mathbf{j} \text{ mm/s}^2}}$$

FOR TIME WHEN $\mathbf{v}$ IS ORIENTED AT 45° WITH THE HORIZONTAL,

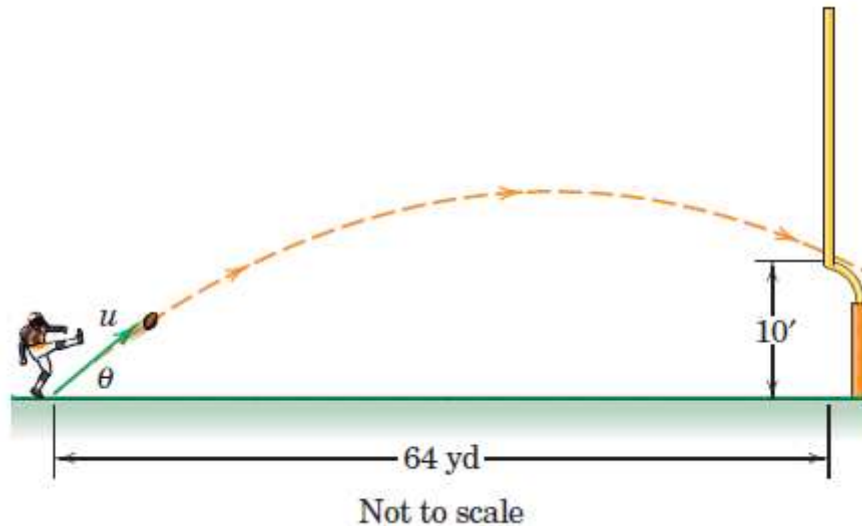
v_x IS EQUAL TO v_y

$$v_x = v_y \quad 10.25 + 3.5t - 1.35t^2 = 14.65 - 4.96t$$

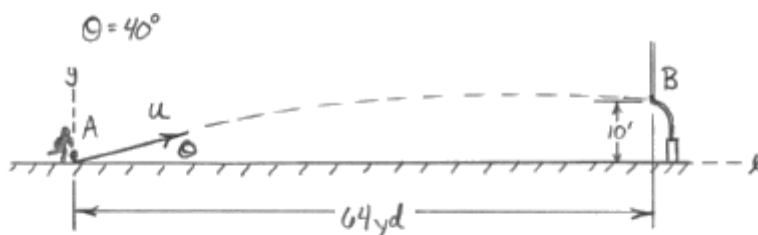
$$\text{Solving... } \underline{\underline{t = 0.572 \text{ AND } 5.69 \text{ s}}}$$

Question Title: Chapter 2, Reserve Problem 2/034

A placekicker is attempting to make a 64-yard field goal. If the launch angle of the football is 40° , what is the minimum initial speed u which will allow the kicker to succeed?



SOLUTION

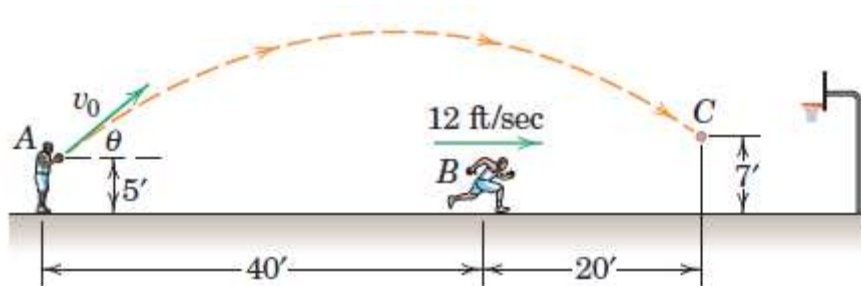


$$\begin{cases} x_B = x_A + v_{x_0} t \rightarrow 64(3) = u \cos 40^\circ t \\ y_B = y_A + v_{y_0} t - \frac{1}{2} g t^2 \rightarrow 10 = u \sin 40^\circ t - \frac{1}{2} (32.2) t^2 \end{cases}$$

Solving... $t = 3.06 \text{ sec}$ AND $\underline{u = 81.8 \text{ ft/sec}}$

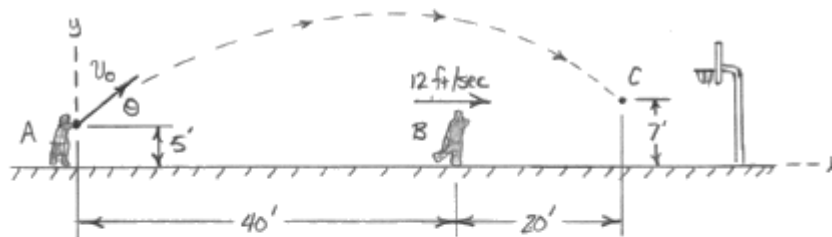
Question Title: Chapter 2, Reserve Problem 2/035 (GO Tutorial version available)

In a basketball game, the point guard A intends to throw a pass to the shooting guard B, who is breaking toward the basket at a constant speed of 12 ft/sec. If the shooting guard is to catch the ball at a height of 7 ft at C while in full stride to execute a layup, determine the speed v_0 and launch angle θ with which the point guard should throw the ball.



SOLUTION

This solution provides a guideline to solving the problem and might not match the values in your text.



BASKETBALL FLIGHT:

$$x_c = x_A + v_{x_0} t \rightarrow 60 = v_0 \cos \theta t$$

$$y_c = y_A + v_{y_0} t - \frac{1}{2} g t^2 \rightarrow 7 = 5 + v_0 \sin \theta t - \frac{1}{2} (32.2) t^2$$

PLAYER B MOTION:

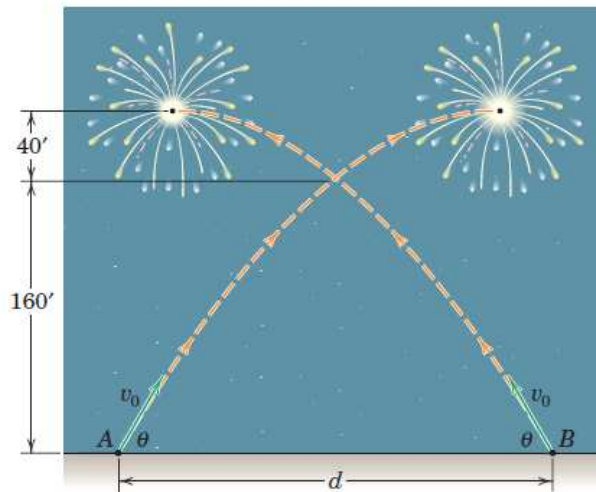
$$x_c = x_B + v_{x_0} t \rightarrow 60 = 40 + 12 t \rightarrow t = 1.667 \text{ sec}$$

Solving... $v_0 = 45.6 \text{ ft/sec}$ AND $\theta = 37.9^\circ$

(THE OTHER SOLUTION HAS NEGATIVE VALUES)

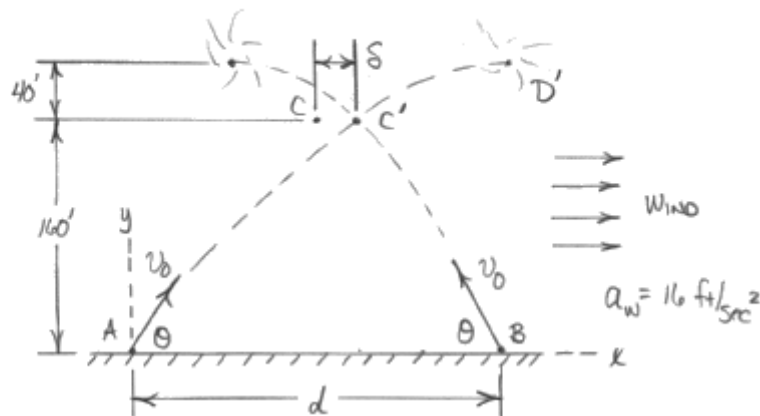
Question Title: Chapter 2, Reserve Problem 2/036

A fireworks show is choreographed to have two shells cross paths at a height of 160 feet and explode at an apex of 200 feet under normal weather conditions. The shells have a launch angle $\theta = 60^\circ$ above the horizontal. If a strong wind induces a constant rightward acceleration of 16 ft/sec^2 for the fireworks shells, determine the horizontal shift Δx of the crossing point of the shells.



SOLUTION

$\theta = 60^\circ$ From Prob. 2/03 ... $v_0 = 131.0 \text{ ft/sec}$, $d = 255 \text{ ft}$



SINCE THE ONLY CHANGE IS a_w WHICH IS HORIZONTAL, NONE OF THE FLIGHT TIMES WILL CHANGE.
 $\therefore t_{AC} = 1.948 \text{ sec}$

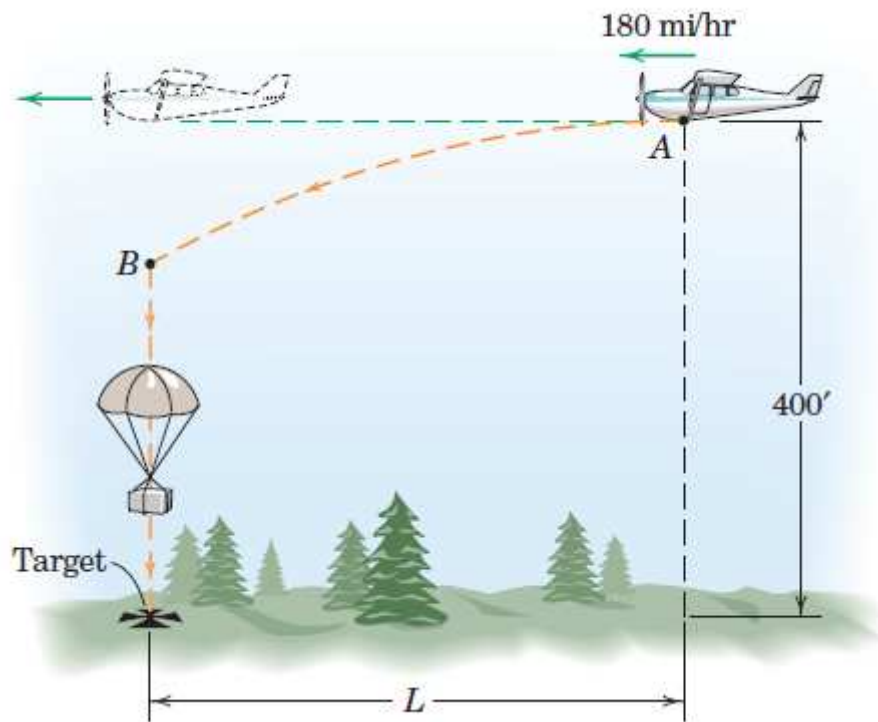
$$x_c' = x_A + v_{x_0} t_{AC} + \frac{1}{2} a_w t_{AC}^2 = 131.0 \cos 60^\circ (1.948) + \frac{1}{2} (16) (1.948)^2$$

$$x_c' = 158.0 \text{ ft}$$

$$\delta = x_c' - \frac{d}{2} = 158.0 - \frac{255}{2} \rightarrow \delta = 30.4 \text{ ft TO THE RIGHT}$$

Question Title: Chapter 2, Reserve Problem 2/037 (Multistep version available)

A small airplane flying horizontally with a speed of 180 mi/hr at an altitude of 400 ft above a remote valley drops an emergency medical package at A. The package has a parachute which deploys at B and allows the package to descend vertically at the constant rate of 6 ft/sec. If the drop is designed so that the package is to reach the ground 37 seconds after release at A, determine the horizontal lead L so that the package hits the target. Neglect atmospheric resistance from A to B.



SOLUTION

$t = 0$: package dropped at A
 t_B, t_C : times package at point B, C

From A to B: $y = \frac{1}{2}gt_B^2$ (1)

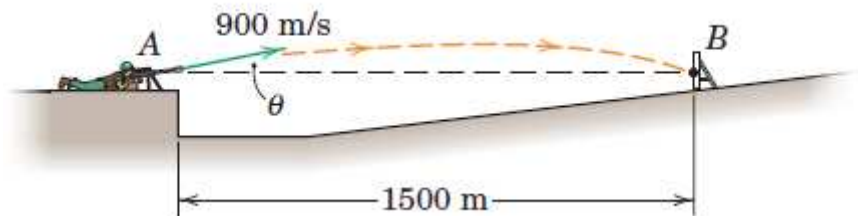
From B to C: $(400 - y) = 6(t_C - t_B)$ (2)

Also, $t_C = 37$ sec. Solve (1) & (2) to obtain $t_B = 3.52$ sec, $y = 199.1$ ft

$L = 180 \left(\frac{5280}{3600} \right) (3.52) = \underline{928 \text{ ft}}$

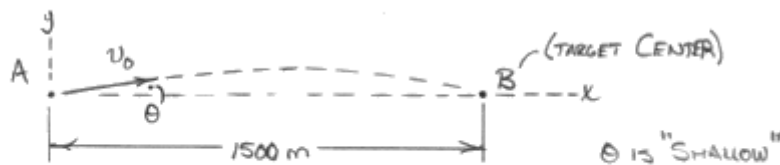
Question Title: Chapter 2, Reserve Problem 2/038

A marksman fires a practice round from A toward a target B. If the target diameter is 160 mm and the target center is at the same altitude as the end of the rifle barrel, determine the range of “shallow” launch angles θ for which the round will strike the target. Neglect aerodynamic drag and assume that the round is directed along the vertical centerline of the target. (Note: The word “shallow” indicates a low-flying trajectory for the round.)



SOLUTION

$v_0 = 900 \text{ m/s}$ $d = 160 \text{ mm}$ (TARGET DIAMETER)



BOTTOM OF TARGET:

$$\begin{cases} x_B = x_A + v_0 t \rightarrow 1500 = 900 \cos \theta t \\ y_B = y_A + v_0 t - \frac{1}{2} g t^2 \rightarrow -\frac{0.160}{2} = 900 \sin \theta t - \frac{1}{2} (9.81) t^2 \end{cases}$$

Solving... $t = 1.667 \text{ s}$ AND $\theta = 0.517^\circ$

(ALSO A "STEEP" ANGLE SOLUTION WITH $t = 183.5 \text{ s}$ AND $\theta = 89.5^\circ$)

TOP OF TARGET:

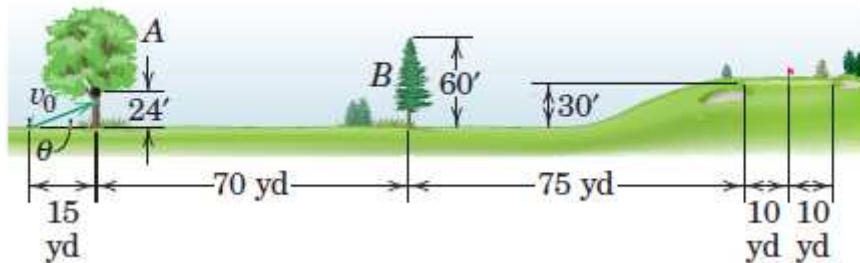
$$\begin{cases} x_B = x_A + v_0 t \rightarrow 1500 = 900 \cos \theta t \\ y_B = y_A + v_0 t - \frac{1}{2} g t^2 \rightarrow \frac{0.160}{2} = 900 \sin \theta t - \frac{1}{2} (9.81) t^2 \end{cases}$$

Solving... $t = 1.667 \text{ s}$ AND $\theta = 0.524^\circ$ (STEEP SOLUTION IS THE SAME WITH S.F.)

SO... $\underline{0.517^\circ \leq \theta \leq 0.524^\circ}$

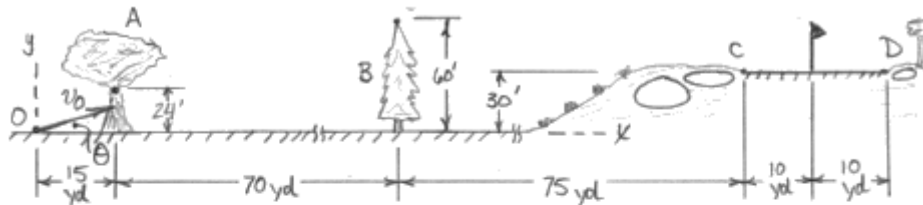
Question Title: Chapter 2, Reserve Problem 2/039 (Multistep version available)

A golfer is attempting to reach the elevated green by hitting his ball under a low-hanging branch in one tree A, but over the top of a second tree B. If the launch speed of the golf ball is $v_0 = 115$ mi/hr, what launch angle θ will put the first impact point of the ball closest to the pin? How far from the pin (distance d) is this impact point?



SOLUTION

$$v_0 = 115 \frac{\text{mi}}{\text{hr}} = 168.7 \frac{\text{ft}}{\text{sec}}, \quad \theta = 18^\circ$$



•CHECK A:

$$\begin{cases} x_A = x_0 + v_{x0} t_A \rightarrow 15(3) = 168.7 \cos 18^\circ t_A \rightarrow t_A = 0.281 \text{ sec} \\ y_A = y_0 + v_{y0} t_A - \frac{1}{2} g t_A^2 = 168.7 \sin 18^\circ (0.281) - \frac{1}{2} (32.2) (0.281)^2 = 13.35 \text{ ft} \quad \text{OK} \end{cases}$$

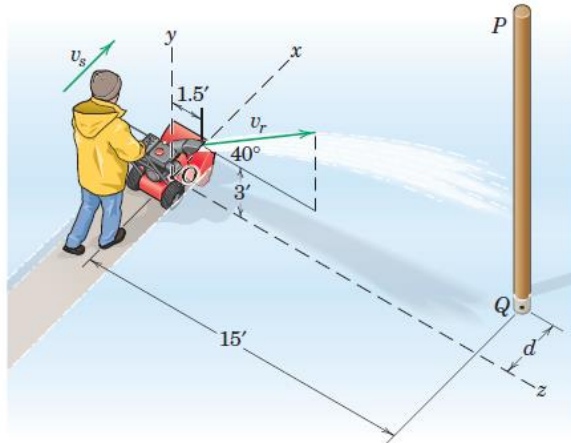
•CHECK B:

$$\begin{cases} x_B = x_0 + v_{x0} t_B \rightarrow 85(3) = 168.7 \cos 18^\circ t_B \rightarrow t_B = 1.590 \text{ sec} \\ y_B = y_0 + v_{y0} t_B - \frac{1}{2} g t_B^2 = 168.7 \sin 18^\circ (1.590) - \frac{1}{2} (32.2) (1.590)^2 = 42.2 \text{ ft} \end{cases}$$

FIRST IMPACT POINT IS 42.2 ft UP IN THE TREE AT B.

Question Title: Chapter 2, Reserve Problem 2/040 (GO Tutorial version available)

A snow blower travels forward at a constant speed $v_s = 1.4$ ft/sec along the straight and level path shown. The snow is ejected with a speed $v_r = 30$ ft/sec relative to the machine at the 40° angle indicated. Determine the distance d which locates the snowblower position from which an ejected snow particle ultimately strikes the slender pole P . At what height h above the ground does the snow strike the pole? Points O and Q are at ground level.



SOLUTION

This solution provides a guideline to solving the problem and might not match the values in your text.

USE THE $Oxyz$ COORDINATE SYSTEM OF THE GIVEN FIGURE.

$$\underline{v}_0 = \underline{v}_s + \underline{v}_r = 1.4 \underline{i} + 30 \sin 40^\circ \underline{j} + 30 \cos 40^\circ \underline{k} \text{ ft/sec}$$

FIND TIME FOR SNOW TO TRAVEL DOWN Z -AXIS TO THE POLE.

$$z_f = z_0 + v_{z_0} t \rightarrow 15 = 1.5 + 30 \cos 40^\circ t \rightarrow t = 0.587 \text{ sec}$$

FIND DISTANCE THE SNOW BLOWER WILL BE DOWN THE X -AXIS FROM THE POLE FOR THE X -COMPONENT OF VELOCITY TO REACH THE POLE IN 0.587 sec.
LET THE ORIGIN MOVE WITH THE SNOW BLOWER.

$$x_f = x_0 + v_{x_0} t \rightarrow d = 1.4(0.587) \rightarrow \underline{d = 0.822 \text{ ft}}$$

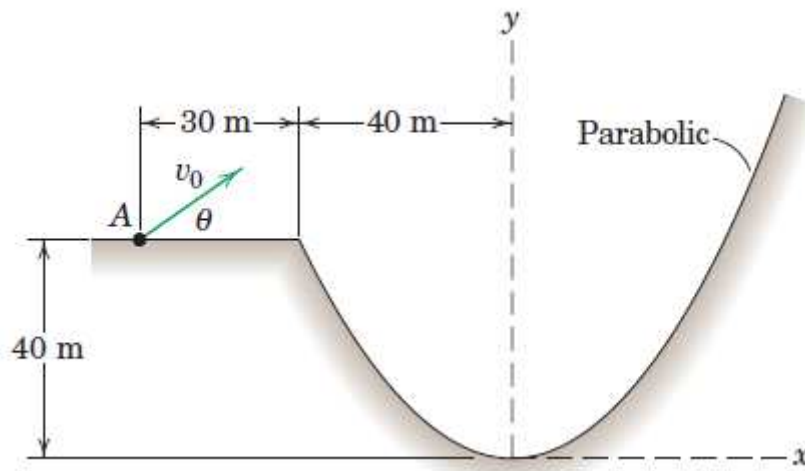
FIND HEIGHT OF SNOW AT TIME OF IMPACT.

$$y_f = y_0 + v_{y_0} t - \frac{1}{2} g t^2 = 3 + 30 \sin 40^\circ (0.587) - \frac{1}{2} (32.2) (0.587)^2 = h$$

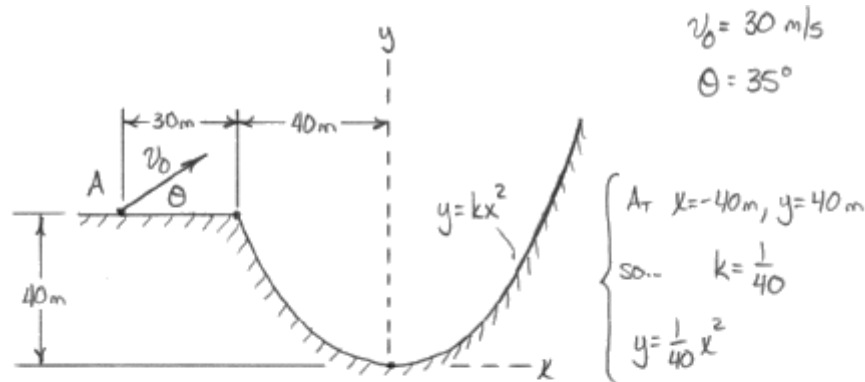
$$\underline{h = 8.77 \text{ ft}}$$

Question Title: Chapter 2, Reserve Problem 2/041

A projectile is launched from point A with $v_0 = 30$ m/s and $\theta = 35^\circ$. Determine the x - and y -coordinates of the point of impact.



SOLUTION



$$\begin{cases} x_f = x_0 + v_{x0} t_f = -70 + 30 \cos 35^\circ t_f \\ y_f = y_0 + v_{y0} t_f - \frac{1}{2} g t_f^2 = 40 + 30 \sin 35^\circ t_f - \frac{1}{2} (9.81) t_f^2 \\ y_f = \frac{1}{40} x_f^2 \end{cases}$$

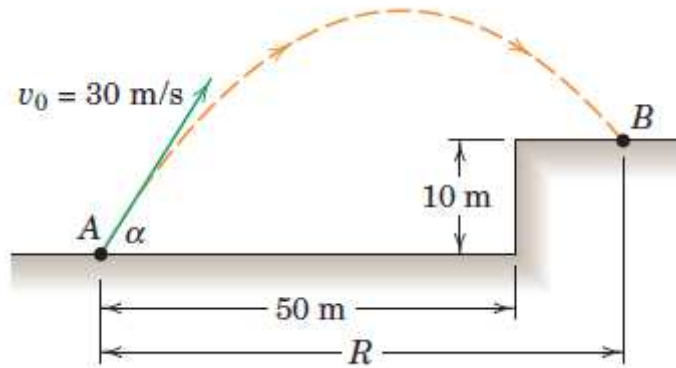
Solving...

$$\begin{cases} t_f = 4.17 \text{ s} \\ x_f = 32.5 \text{ m} \\ y_f = 26.4 \text{ m} \end{cases}$$

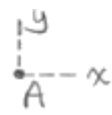
(Other solution has $t_f = 0.989$ s BUT THIS IS WHERE IT WOULD CROSS THE LEFT SIDE OF THE PARABOLA.)

Question Title: Chapter 2, Reserve Problem 2/042

A projectile is launched from point A with speed $v_0 = 30$ m/s. Determine the value of the launch angle α which maximizes the range R indicated in the figure. Determine the corresponding value of R .



SOLUTION

Set up x-y coordinates @ A : 

Let coordinates of B be (R, h) .

$$x = x_0 + v_{x0}t \text{ @ B: } R = 0 + (v_0 \cos \alpha) t_f \quad (1)$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2 \text{ @ B: } h = 0 + (v_0 \sin \alpha) t_f - \frac{1}{2}gt_f^2 \quad (2)$$

Solve (1) & (2) for R & t_f :

$$\left. \begin{aligned} R &= \frac{v_0 \cos \alpha}{g} \left[v_0 \sin \alpha + \sqrt{v_0^2 \sin^2 \alpha - 2gh} \right] \\ t_f &= \frac{1}{g} \left[v_0 \sin \alpha + \sqrt{v_0^2 \sin^2 \alpha - 2gh} \right] \end{aligned} \right\}$$

where the + sign has been taken to maximize R .

Set $\frac{dR}{d\alpha} = 0$ (done by computer) to find

$$\underline{\alpha = 48.5^\circ} \quad (\text{for } h = 10 \text{ m, } v_0 = 30 \text{ m/s, and } g = 9.81 \text{ m/s}^2)$$

The corresponding value of R is

$$\underline{R = 81.1 \text{ m}}$$

So the 10-m plateau is indeed achieved, as assumed (because $R > 50$ m).

Question Title: Chapter 2, Reserve Problem 2/043

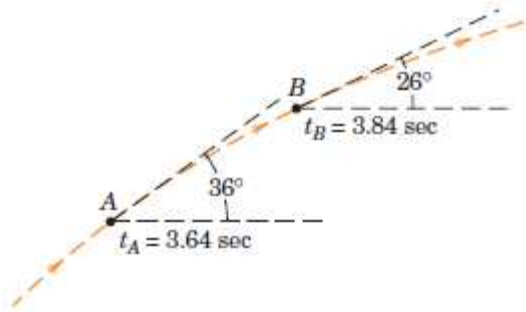
A ship which moves at a steady 20-knot speed (1 knot = 1.852 km/h) executes a turn to port by changing its compass heading at a constant counterclockwise rate. If it requires 60 s to alter course 90° , calculate the magnitude of the acceleration a of the ship during the turn.

SOLUTION

$$a = a_n = v\dot{\beta} = \frac{20(1.852)}{3.6} \frac{\pi}{2} \frac{1}{60} = \underline{0.269 \text{ m/s}^2}$$

Question Title: Chapter 2, Reserve Problem 2/044

A particle moves along the curved path shown. If the particle has a speed of 40 ft/sec at A at time t_A and a speed of 44 ft/sec at B at time t_B , determine the average values of the acceleration of the particle between A and B, both normal and tangent to the path.



SOLUTION

$$\Delta \theta = \frac{36 - 26}{180} \pi$$

$$= 0.1745 \text{ rad}$$

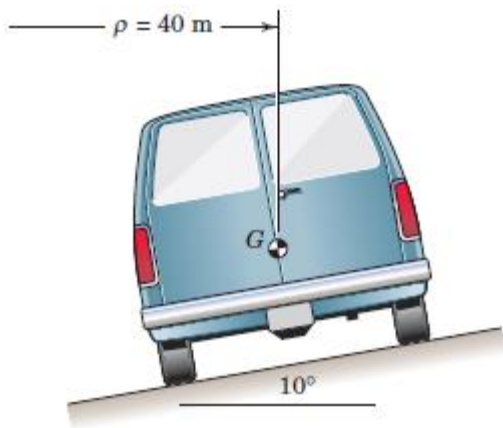
$$v_{AV} = 42 \text{ ft/sec}$$

$$a_n = \frac{\Delta v_n}{\Delta t} = \frac{v \Delta \theta}{\Delta t} = \frac{42 (0.1745)}{3.84 - 3.64} = \underline{36.7 \frac{\text{ft}}{\text{sec}^2}}$$

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{44 - 40}{0.2} = \underline{20 \text{ ft/sec}^2}$$

Question Title: Chapter 2, Reserve Problem 2/045

A minivan starts from rest on the road whose constant radius of curvature is 40 m and whose bank angle is 10° . The motion occurs in a horizontal plane. If the constant forward acceleration of the minivan is 1.8 m/s^2 , determine the magnitude a of its total acceleration 5 seconds after starting.

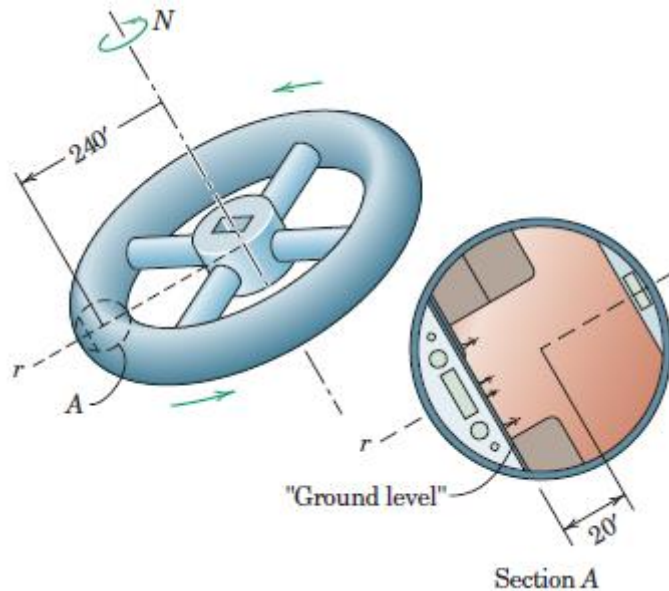


SOLUTION

$$\begin{aligned}v &= v_0 + a_t t = 0 + 1.8(5) = 9 \text{ m/s} \\a_n &= \frac{v^2}{\rho} = \frac{9^2}{40} = 2.025 \text{ m/s}^2 \\a &= \sqrt{a_t^2 + a_n^2} = \sqrt{(1.8)^2 + (2.025)^2} = \underline{\underline{2.71 \frac{\text{m}}{\text{s}^2}}}\end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/046

The preliminary design for a “small” space station to orbit the earth in a circular path consists of a ring (torus) with a circular cross section as shown. The living space within the torus is shown in section A, where the “ground level” is 20 ft from the center of the section. Calculate the angular speed N in revolutions per minute required to simulate standard gravity at the surface of the earth (32.17 ft/sec^2). Recall that you would be unaware of a gravitational field if you were in a non-rotating spacecraft in a circular orbit around the earth.



SOLUTION

Relative to space station, $a_n = r\dot{\theta}^2$

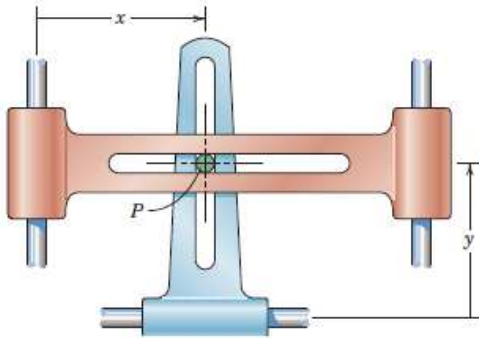
where $a_n = 32.17 \text{ ft/sec}^2$.

$$\text{Thus } 32.17 = (240 + 20) \dot{\theta}^2, \quad \dot{\theta} = 0.352 \frac{\text{rad}}{\text{sec}}$$

$$N = 0.352 \left(\frac{60}{2\pi} \right) = \underline{3.36 \text{ rev/min}}$$

Question Title: Chapter 2, Reserve Problem 2/047 (GO Tutorial version available)

During a short interval the slotted guides are designed to move according to $x = 16 - 12t + 4t^2$ and $y = 2 + 15t - 3t^2$, where x and y are in millimeters and t is in seconds. At the instant when $t = 2$ s, determine the radius of curvature ρ of the path of the constrained pin P .



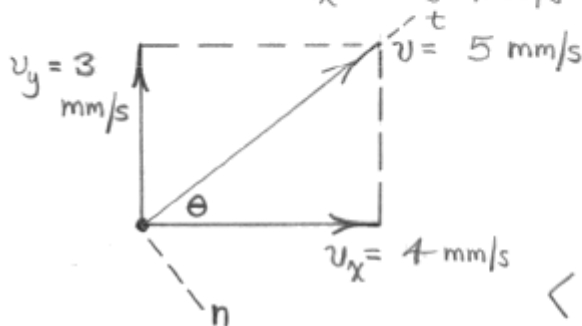
SOLUTION

$$\begin{aligned} x &= 16 - 12t + 4t^2 \\ \dot{x} &= 8t - 12 \\ \ddot{x} &= 8 \end{aligned}$$

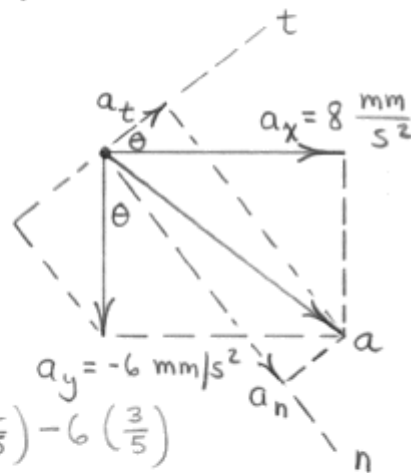
$$\begin{aligned} y &= 2 + 15t - 3t^2 \\ \dot{y} &= 15 - 6t \\ \ddot{y} &= -6 \end{aligned}$$

At $t = 2$ s : $\dot{x} = 4$ mm/s $\dot{y} = 15 - 12 = 3$ mm/s

$\ddot{x} = 8$ mm/s² $\ddot{y} = -6$ mm/s²



$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$



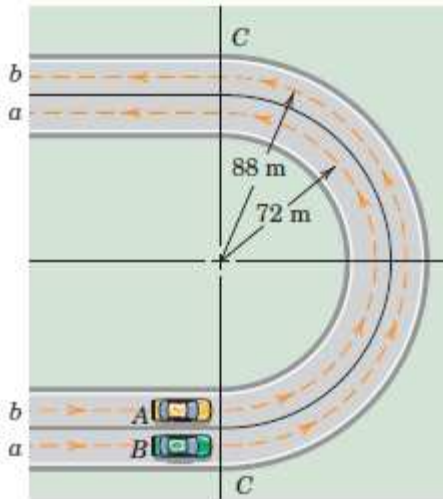
$$\begin{aligned} a_t &= 8 \cos \theta - 6 \sin \theta = 8\left(\frac{4}{5}\right) - 6\left(\frac{3}{5}\right) \\ &= 2.8 \text{ mm/s}^2 \end{aligned}$$

$$\begin{aligned} a_n &= 6 \cos \theta + 8 \sin \theta = 6\left(\frac{4}{5}\right) + 8\left(\frac{3}{5}\right) \\ &= 9.6 \text{ mm/s}^2 \end{aligned}$$

$$a_n = \frac{v^2}{\rho}, \quad \rho = \frac{v^2}{a_n} = \frac{5^2}{9.6} = \underline{2.60 \text{ mm}}$$

Question Title: Chapter 2, Reserve Problem 2/048

The two cars A and B enter an unbanked and level turn. They cross line C-C simultaneously, and each car has the speed corresponding to a maximum normal acceleration of $0.9g$ in the turn. Determine the elapsed time for each car between its two crossings of line C-C. What is the relative position of the two cars as the second car exits the turn? Assume no speed changes throughout.



SOLUTION

$$a_n = \frac{v^2}{r}, \quad t = \frac{\pi r}{v}$$

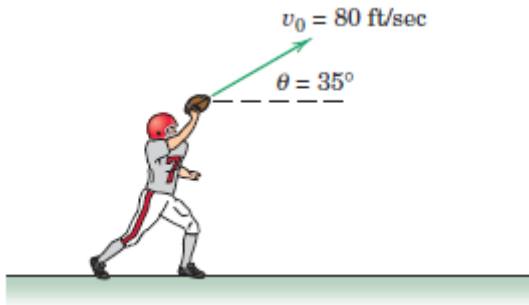
$$\textcircled{A}: 0.9(9.81) = \frac{v_A^2}{72}, \quad v_A = 25.2 \frac{\text{m}}{\text{s}}, \quad t_A = \frac{\pi 72}{25.2} = 8.97 \text{ s}$$

$$\textcircled{B}: 0.9(9.81) = \frac{v_B^2}{88}, \quad v_B = 27.9 \frac{\text{m}}{\text{s}}, \quad t_B = \frac{\pi 88}{27.9} = 9.92 \text{ s}$$

So A exits the turn $(9.92 - 8.97) = 0.947 \text{ s}$
 ahead of B, which translates to
 $25.2(0.947) = \underline{23.9 \text{ m}}.$

Question Title: Chapter 2, Reserve Problem 2/049

A football player releases a ball with the initial conditions shown in the figure. Determine the radius of curvature of the trajectory (*a*) just after release and (*b*) at the apex. For each case, compute the time rate of change of the speed.



SOLUTION

$v_0 = 80 \text{ ft/sec}$
 $\theta = 35^\circ$
 $g = 32.2 \text{ ft/sec}^2$

(a) $a_n = g \cos 35^\circ = \frac{v^2}{\rho}$

$$\rho = \frac{80^2}{g \cos 35^\circ} = \underline{243 \text{ ft}}$$

$\dot{v} = -g \sin 35^\circ = \underline{-18.47 \frac{\text{ft}}{\text{sec}^2}}$

(b) $a_n = g = \frac{v^2}{\rho} = \frac{v_{x_0}^2}{\rho}$

$$\rho = \frac{(80 \cos 35^\circ)^2}{32.2} = \underline{133.4 \text{ ft}}$$

$\dot{v} = a_t = \underline{0}$

Question Title: Chapter 2, Reserve Problem 2/050

A particle which moves in two-dimensional curvilinear motion has coordinates in millimeters which vary with time t in seconds according to $x = 5t^2 + 4$ and $y = 2t^3 + 6$. For time $t = 3$ s, determine the radius of curvature ρ of the particle path and the magnitudes of the normal and tangential accelerations.

SOLUTION

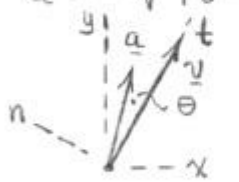
$$\begin{array}{l}
 x = 5t^2 + 4 \quad \left| \quad \begin{array}{l} \text{--- } t = 3 \text{ s:} \\ x = 49 \text{ mm} \\ \dot{x} = 30 \text{ mm/s} \\ \ddot{x} = 10 \text{ mm/s}^2 \end{array} \right. \quad \left| \quad \begin{array}{l} y = 2t^3 + 6 \\ \dot{y} = 6t^2 \\ \ddot{y} = 12t \end{array} \right. \quad \left| \quad \begin{array}{l} \text{--- } t = 3 \text{ s:} \\ y = 60 \text{ mm} \\ \dot{y} = 54 \frac{\text{mm}}{\text{s}} \\ \ddot{y} = 36 \frac{\text{mm}}{\text{s}^2} \end{array}
 \end{array}$$

(Position is of no consequence here)

$$\underline{v} = \dot{x} \underline{i} + \dot{y} \underline{j} = 30 \underline{i} + 54 \underline{j} \text{ mm/s}$$

$$v = \sqrt{30^2 + 54^2} = 61.8 \text{ mm/s}$$

$$\underline{a} = \ddot{x} \underline{i} + \ddot{y} \underline{j} = 10 \underline{i} + 36 \underline{j} \text{ mm/s}^2$$

$$a = \sqrt{10^2 + 36^2} = 37.4 \text{ mm/s}^2$$


$$\cos \theta = \frac{\underline{v} \cdot \underline{a}}{va}$$

$$\cos \theta = \frac{30(10) + 54(36)}{61.8(37.4)} = 0.972$$

$$\theta = 13.53^\circ$$

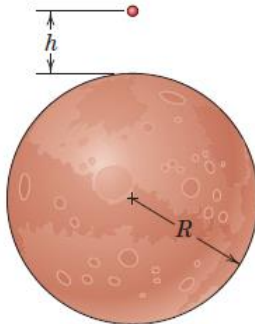
$$a_n = a \sin \theta = 37.4 \sin 13.53^\circ = 8.74 \text{ mm/s}^2$$

$$\text{Then } \rho = \frac{v^2}{a_n} = \frac{61.8^2}{8.74} = 437 \text{ mm}$$

$$a_t = a \cos \theta = 37.4 \cos 13.53^\circ = 36.3 \text{ mm/s}^2$$

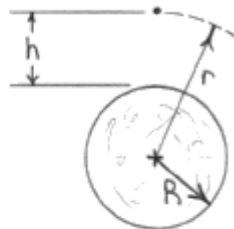
Question Title: Chapter 2, Reserve Problem 2/051 (GO Tutorial version available)

Compute the impact speed of a body released from rest at an altitude $h = 650$ miles above the surface of Mars. (a) First assume a constant gravitational acceleration $g_{m0} = 12.3 \text{ ft/sec}^2$ (equal to that at the surface) and (b) then account for the variation of g with altitude (refer to Art. 1/5). Neglect any effects of atmospheric drag.



SOLUTION

$$\begin{aligned} h &= 650 \text{ mi} \\ g_{m0} &= 12.3 \text{ ft/sec}^2 \\ R &= 2109 \text{ mi} \end{aligned}$$



• CONSTANT g :

$$v^2 = v_0^2 + 2g_{m0}h = 2(12.3)(650 \times 5280)$$

$$\underline{v = 9190 \text{ ft/sec}}$$

• Now - CONSTANT g :

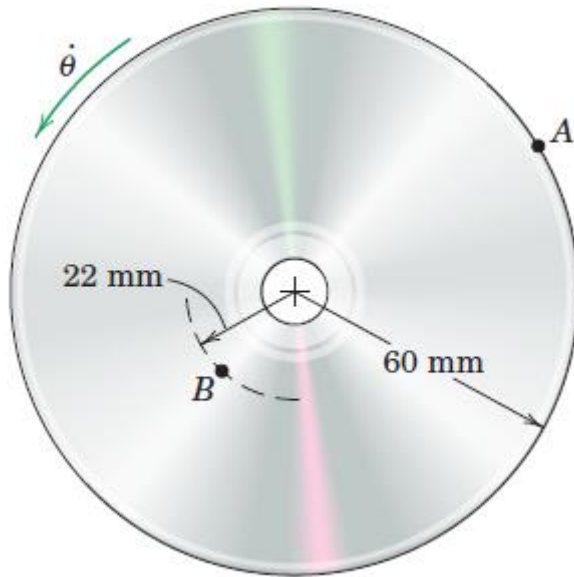
$$a = -\frac{g_{m0}R^2}{r^2} = \frac{v dv}{dr} \rightarrow -g_{m0}R^2 \int_{r_0}^r \frac{dr}{r^2} = \int_{v_0}^v v dv$$

$$-g_{m0}R^2 \left(-\frac{1}{r} \right) \bigg|_{r_0=R+h}^{r=R} = \frac{1}{2} v^2 \bigg|_{v_0=0}^v \rightarrow v = \sqrt{2g_{m0}R^2 \left(\frac{1}{R} - \frac{1}{R+h} \right)}$$

$$\underline{v = \sqrt{2(12.3)(2109 \times 5280)^2 \left[\frac{1}{2109 \times 5280} - \frac{1}{(2109+650) \times 5280} \right]} = 8033 \text{ ft/sec}}$$

Question Title: Chapter 2, Reserve Problem 2/052 (Multistep version available)

If the compact disc is spinning at a constant angular rate $\dot{\theta} = 360 \text{ rev/min}$, determine the magnitudes of the accelerations of points A and B at the instant shown.



SOLUTION

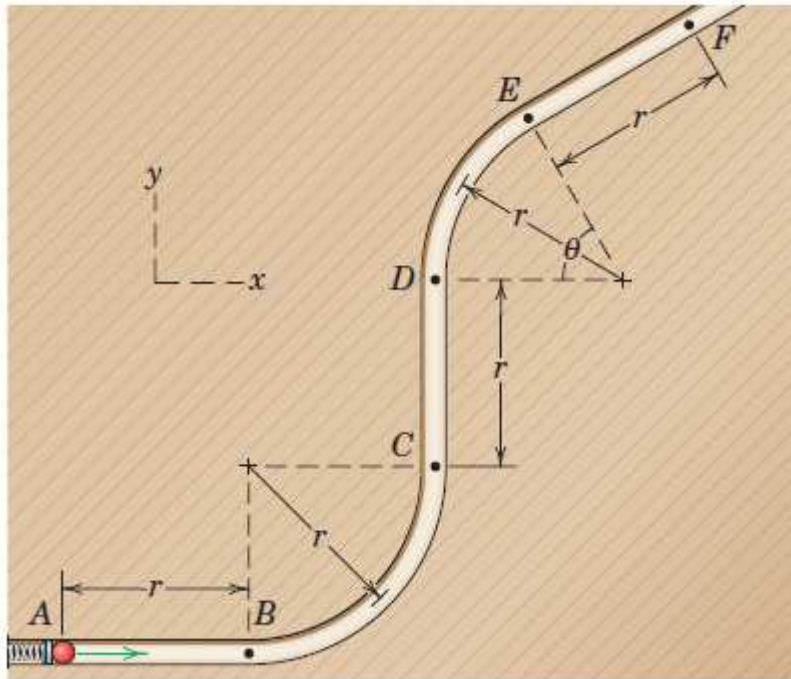
$$r_A = 60 \text{ mm} \quad r_B = 22 \text{ mm} \quad \dot{\theta} = 360 \text{ rpm}$$

$$\text{POINT A: } a_{A_n} = r_A \dot{\theta}^2 = 0.060 \left(360 \frac{2\pi}{60} \right)^2 = \underline{85.3 \text{ m/s}^2}$$

$$\text{POINT B: } a_{B_n} = r_B \dot{\theta}^2 = 0.022 \left(360 \frac{2\pi}{60} \right)^2 = \underline{31.3 \text{ m/s}^2}$$

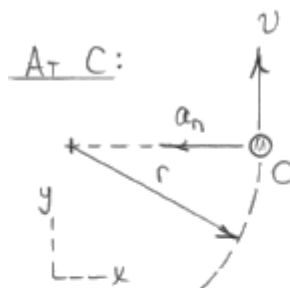
Question Title: Chapter 2, Reserve Problem 2/053

An overhead view of part of a pinball game is shown. If the plunger imparts a speed of 3 m/s to the ball which travels in the smooth horizontal slot, determine the acceleration $\mathbf{a}$ of the ball (a) just before it exits the curve at C and (b) when it is halfway between points D and E . Use the values $r = 150$ mm and $\theta = 60^\circ$.



SOLUTION

$$v = 3 \text{ m/s} \quad r = 150 \text{ mm} \quad a_t = 0$$



$$a_n = \frac{v^2}{r} = \frac{3^2}{0.150} = 60 \text{ m/s}^2$$

$$\underline{\underline{\mathbf{a} = -60 \mathbf{i} \text{ m/s}^2}}$$

HALFWAY BETWEEN D AND E:



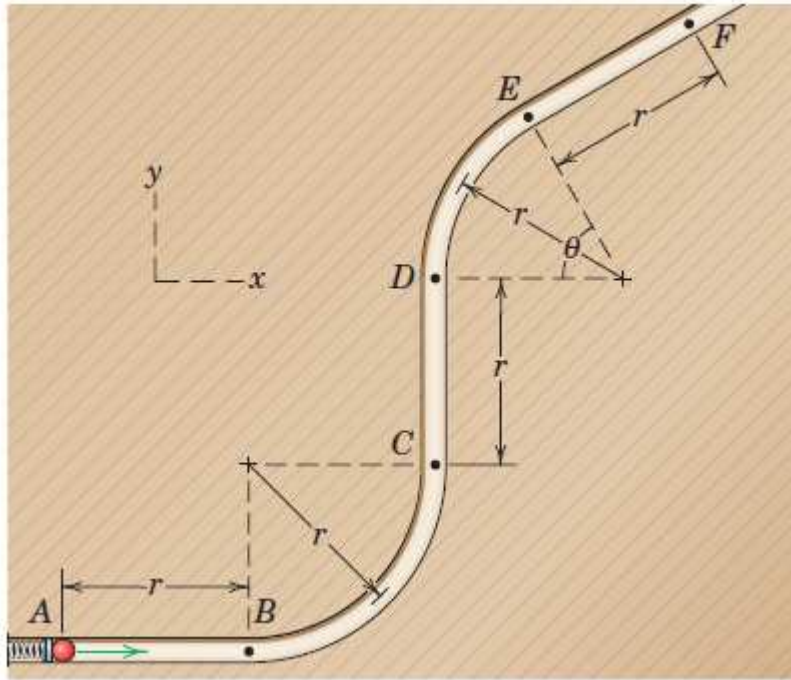
$$a_n = \frac{v^2}{r} = \frac{3^2}{0.150} = 60 \text{ m/s}^2$$

$$\underline{\underline{\mathbf{a} = 60(\cos 30^\circ \mathbf{i} - \sin 30^\circ \mathbf{j})}}$$

$$\underline{\underline{\mathbf{a} = 52.0 \mathbf{i} - 30 \mathbf{j} \text{ m/s}^2}}$$

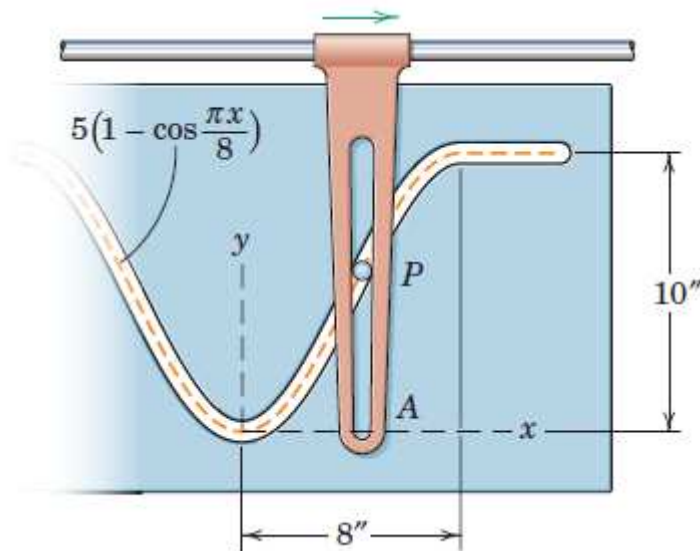
Question Title: Chapter 2, Reserve Problem 2/054

An overhead view of part of a pinball game is shown. If the plunger imparts an initial speed of 3 m/s to the ball at time $t = 0$, determine the acceleration $\mathbf{a}$ of the ball (*a*) at time $t = 0.08$ s and (*b*) at time $t = 0.20$ s. At point F , the speed of the pinball has decreased by 10% from the initial value, and this decrease may be assumed to occur uniformly over the total distance traveled by the pinball. Use the values $r = 150$ mm and $\theta = 60^\circ$.



Question Title: Chapter 2, Reserve Problem 2/055

The figure shows a portion of a plate cam used in the design of a control mechanism. The motion of pin P in the fixed slot of the plate cam is controlled by the vertical guide A , which travels horizontally at a constant speed of 6 in./sec over the central sinusoidal portion of the slot. Determine the normal component of acceleration when the pin is at the position $x = 2$ in.



SOLUTION

$$k = 2 \text{ in.} \quad \dot{x} = 6 \text{ in./sec (CONSTANT)}$$

$$y = 5\left(1 - \cos \frac{\pi x}{8}\right) = 5\left(1 - \cos \frac{2\pi}{8}\right) \rightarrow y = 1.464 \text{ in.}$$

$$\dot{y} = \frac{5\pi}{8} \sin \frac{\pi x}{8} \dot{x} = \frac{5\pi}{8} \sin \frac{2\pi}{8} (6) \rightarrow \dot{y} = 8.33 \text{ in./sec}$$

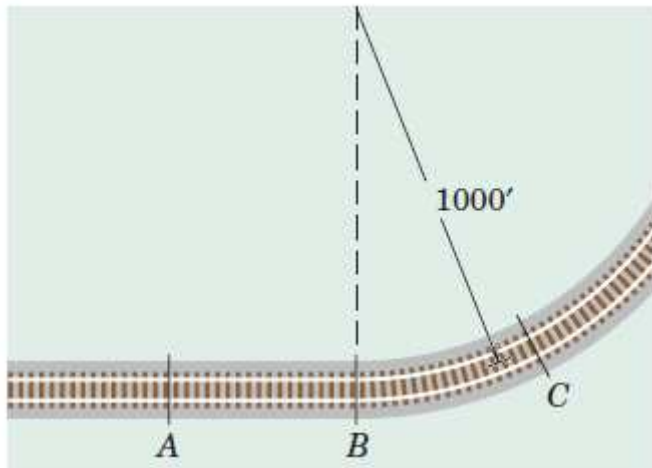
$$\rho_{xy} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + \left(\frac{5\pi}{8} \sin \frac{\pi x}{8}\right)^2\right]^{3/2}}{\frac{5\pi^2}{64} \cos \frac{\pi x}{8}} = \frac{\left[1 + \left(\frac{5\pi}{8} \sin \frac{2\pi}{8}\right)^2\right]^{3/2}}{\frac{5\pi^2}{64} \cos \frac{2\pi}{8}}$$

$$\rho_{xy} = 9.19 \text{ in.}$$

$$a_n = \frac{v^2}{\rho_{xy}} = \frac{\dot{x}^2 + \dot{y}^2}{\rho_{xy}} = \frac{6^2 + 8.33^2}{9.19} \rightarrow a_n = 11.47 \text{ in./sec}^2$$

Question Title: Chapter 2, Reserve Problem 2/056

An inexperienced designer of a roadbed for a new high-speed train proposes to join a straight section of track to a circular section of 1000-ft radius as shown. For a train that would travel at a constant speed of 90 mi/hr, plot the magnitude of its acceleration as a function of distance along the track between points A and C and explain why this design is unacceptable.

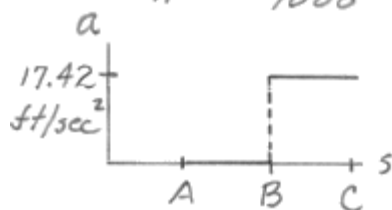


SOLUTION

From A to B $a = 0$

From B to C $a = a_n = v^2/\rho$

$$a_n = \frac{(90 \times 44/30)^2}{1000} = 17.42 \text{ ft/sec}^2$$

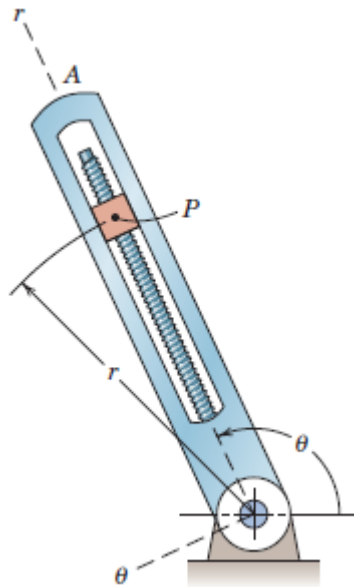


Abrupt acceleration would cause abrupt forces which would be uncomfortable for passengers.

A transition section to change curvature gradually over an interval of track would be required.

Question Title: Chapter 2, Reserve Problem 2/057 (Multistep version available)

Motion of the sliding block P in the rotating radial slot is controlled by the power screw as shown. For the instant represented, $\dot{\theta} = 0.1 \text{ rad/s}$, $\ddot{\theta} = -0.04 \text{ rad/s}^2$, and $r = 300 \text{ mm}$. Also, the screw turns at a constant speed giving $\dot{r} = 40 \text{ mm/s}$. For this instant, determine the magnitudes of the velocity v and acceleration a of P . Sketch v and a if $\theta = 120^\circ$.

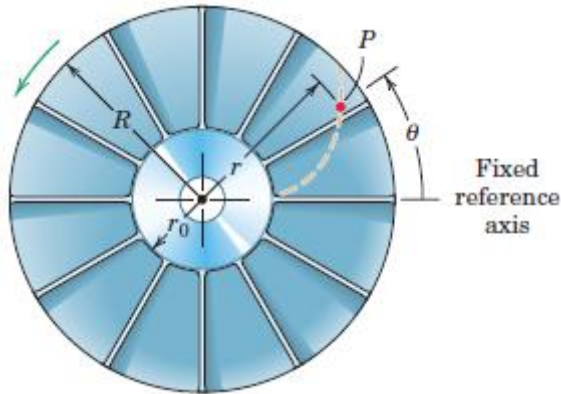


SOLUTION

$$\begin{aligned}
 v_r &= \dot{r} = 40 \text{ mm/s} \\
 v_\theta &= r\dot{\theta} = 300(0.1) = 30 \text{ mm/s} \\
 v &= \sqrt{40^2 + 30^2} = 50 \text{ mm/s} \\
 a_r &= \ddot{r} - r\dot{\theta}^2 = 0 - 300(0.1)^2 = -3 \text{ mm/s}^2 \\
 a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300(-0.04) + 2(40)(0.1) = -4 \text{ mm/s}^2 \\
 a &= \sqrt{3^2 + 4^2} = 5 \text{ mm/s}^2
 \end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/058

The radial position of a fluid particle P in a certain centrifugal pump with radial vanes is approximated by $r = r_0 \cosh Kt$, where t is time and $K = \dot{\theta}$ is the constant angular rate at which the impeller turns. Determine the expression for the magnitude of the total acceleration of the particle just prior to leaving the vane in terms of r_0 , R , and K .



SOLUTION

This solution provides a guideline to solving the problem and might not match the values in your text.

$$\begin{cases} r = r_0 \cosh Kt \\ \dot{r} = r_0 K \sinh Kt \\ \ddot{r} = r_0 K^2 \cosh Kt \end{cases}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = r_0 K^2 \cosh Kt - (r_0 \cosh Kt) K^2 = 0$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2r_0 K \sinh Kt (K) \\ = 2r_0 K^2 \sinh Kt$$

$$\text{But } \cosh^2 Kt - \sinh^2 Kt = 1, \sinh Kt = \sqrt{\cosh^2 Kt - 1} \\ = \sqrt{\left(\frac{r}{r_0}\right)^2 - 1}$$

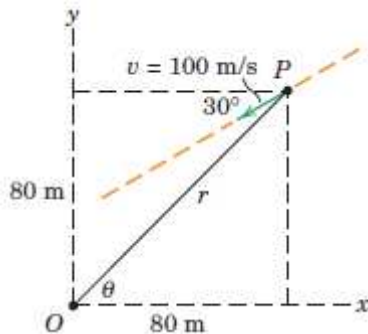
$$\text{When } r = R, \sinh Kt = \sqrt{\left(\frac{R}{r_0}\right)^2 - 1}$$

So at the instant of leaving the vane,

$$a = a_\theta = 2r_0 K^2 \sqrt{\left(\frac{R}{r_0}\right)^2 - 1} = \underline{2K^2 \sqrt{R^2 - r_0^2}}$$

Question Title: Chapter 2, Reserve Problem 2/059

As it passes the position shown, the particle P has a constant speed $v = 100$ m/s along the straight line shown. Determine the corresponding values of $\dot{r}$, $\dot{\theta}$, $\ddot{r}$, and $\ddot{\theta}$.

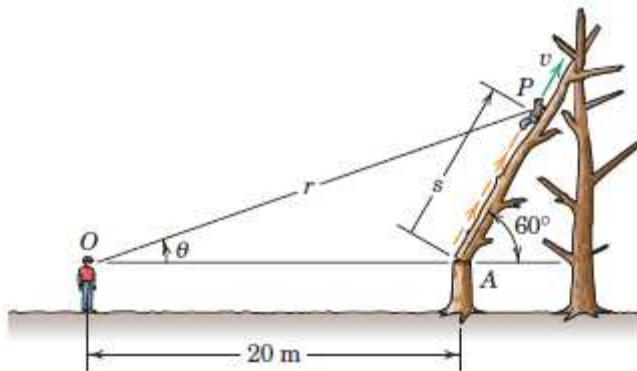


SOLUTION

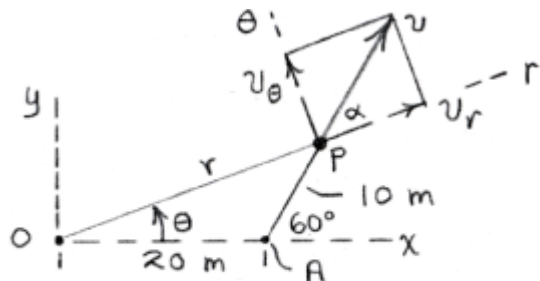
$$\begin{aligned} & \begin{cases} v = 100 \text{ m/s} \\ r = 80\sqrt{2} = 113.1 \text{ m} \end{cases} \\ & v_r = \dot{r} = -v \cos 15^\circ = -100 \cos 15^\circ \\ & \quad = \underline{-96.6 \text{ m/s}} \\ & v_\theta = r \dot{\theta} = v \sin 15^\circ \\ & \dot{\theta} = \frac{100 \sin 15^\circ}{113.1} \\ & \quad = \underline{0.229 \text{ rad/s}} \\ & a_r = \ddot{r} - r \dot{\theta}^2 = 0, \quad \ddot{r} = r \dot{\theta}^2 = 113.1 (0.229)^2 = \underline{5.92 \frac{\text{m}}{\text{s}^2}} \\ & a_\theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 0, \quad \ddot{\theta} = -\frac{2 \dot{r} \dot{\theta}}{r} \\ & \quad = -\frac{2(-96.6)(0.229)}{113.1} = \underline{0.391 \frac{\text{rad}}{\text{s}^2}} \end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/060

A hiker pauses to watch a squirrel P run up a partially downed tree trunk. If the squirrel's speed is $v = 2$ m/s when the position $s = 10$ m, determine the corresponding values of $\dot{r}$ and $\dot{\theta}$.



SOLUTION



$$r^2 = 20^2 + 10^2 - 2(20)(10)\cos 120^\circ, \quad r = 26.5 \text{ m}$$

$$\frac{\sin \theta}{10} = \frac{\sin 120^\circ}{26.5} \quad \theta = 19.11^\circ$$

$$\alpha = 60^\circ - 19.11^\circ = 40.9^\circ$$

$$\dot{r} = v_r = v \cos \alpha$$

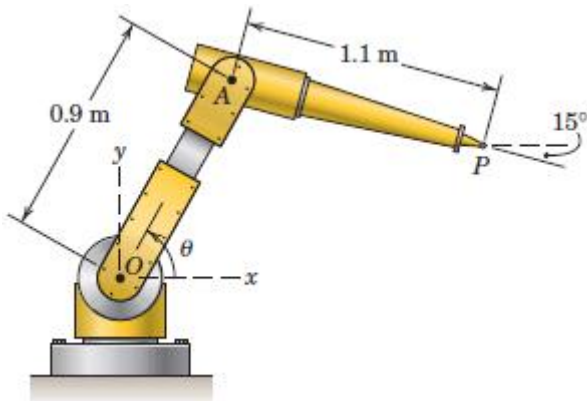
$$= 2 \cos 40.9^\circ = \underline{1.512 \text{ m/s}}$$

$$r\dot{\theta} = v_\theta = v \sin \alpha = 2 \sin 40.9^\circ = 1.309 \text{ m/s}$$

$$\Rightarrow \underline{\dot{\theta} = 0.0495 \text{ rad/s}}$$

Question Title: Chapter 2, Reserve Problem 2/061

The member OA of the industrial robot telescopes and pivots about the fixed axis at point O . At the instant shown, $\theta = 60^\circ$, $\dot{\theta} = 1.2 \text{ rad/s}$, $\ddot{\theta} = 0.8 \text{ rad/s}^2$, $\overline{OA} = 0.9 \text{ m}$, $\dot{\overline{OA}} = 0.5 \text{ m/s}$, and $\ddot{\overline{OA}} = -6 \text{ m/s}^2$. Determine the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ of joint A of the robot. The base of the robot does not revolve about a vertical axis.

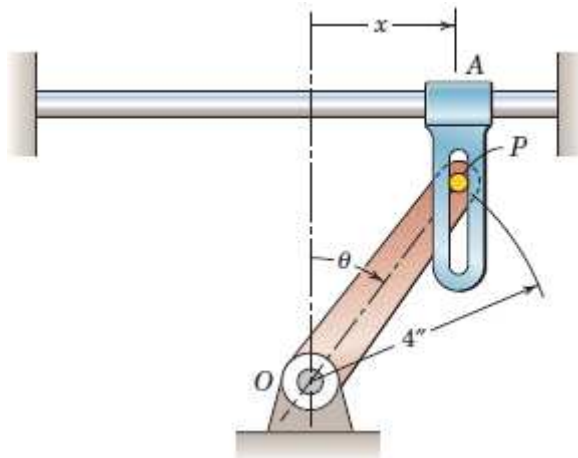


SOLUTION

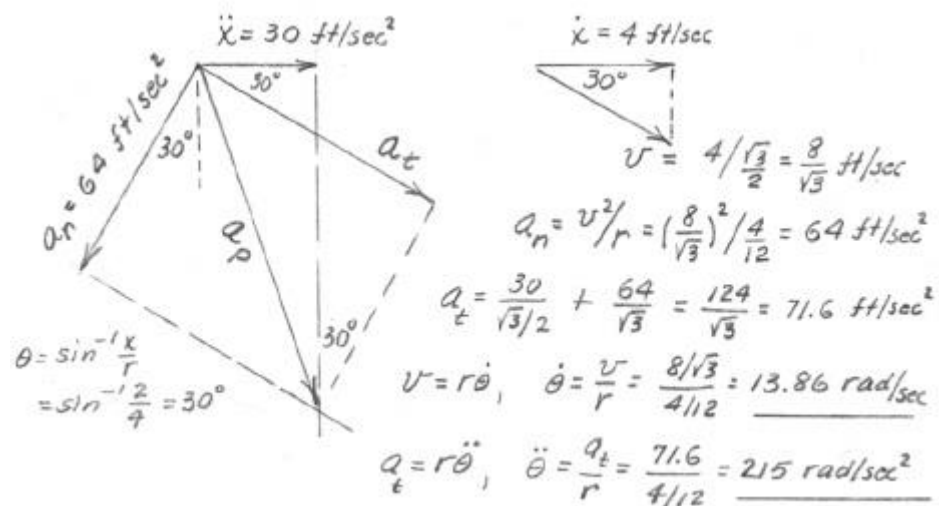
$$\begin{aligned}
 v_r &= \dot{r} = 0.5 \text{ m/s} \\
 v_\theta &= r\dot{\theta} = 0.9(1.2) = 1.08 \text{ m/s} \\
 v_A &= \sqrt{0.5^2 + 1.08^2} = 1.190 \text{ m/s} \\
 a_r &= \ddot{r} - r\dot{\theta}^2 = -6 - 0.9(1.2)^2 \\
 &= -7.30 \text{ m/s}^2 \\
 a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} \\
 &= 0.9(+0.8) + 2(0.5)(1.2) \\
 &= 1.92 \text{ m/s}^2 \\
 a_A &= \sqrt{7.30^2 + 1.92^2} = 7.54 \text{ m/s}^2 \\
 \alpha &= \tan^{-1}\left(\frac{1.08}{0.5}\right) = 65.2^\circ \\
 \text{So for } v_A, \theta_x &= 60^\circ + 65.2^\circ = 125.2^\circ \\
 \beta &= \tan^{-1}\left(\frac{7.30}{1.92}\right) = 75.3^\circ \\
 \text{For } a_A, \theta_x &= 60^\circ + 90^\circ + 75.3^\circ = 225^\circ
 \end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/062

Rotation of the arm PO is controlled by the horizontal motion of the vertical slotted link. If $\dot{x} = 4$ ft/sec and $\ddot{x} = 30$ ft/sec² when $x = 2$ in., determine $\dot{\theta}$ and $\ddot{\theta}$ for this instant.

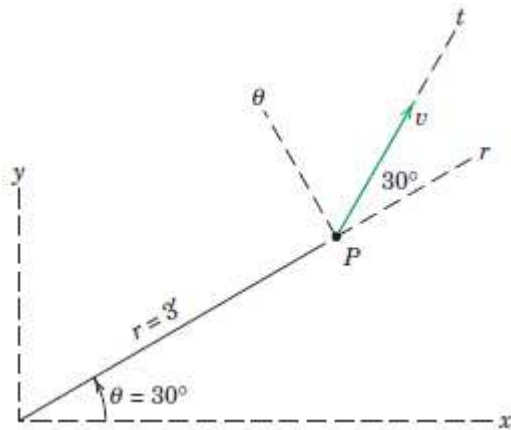


SOLUTION



Question Title: Chapter 2, Reserve Problem 2/063

For the instant represented the particle P has a velocity $v = 6 \text{ ft/sec}$ in the direction shown and has acceleration components $a_x = 15 \text{ ft/sec}^2$ and $a_\theta = -15 \text{ ft/sec}^2$. Determine a_r , a_y , a_t , a_n , and the radius of curvature ρ of the path for this position. (Hint: Draw the related acceleration components of the total acceleration of the particle and take advantage of the simplified geometry for your calculations.)



SOLUTION

$v = 6 \text{ ft/sec}$
 $a_x = 15 \text{ ft/sec}^2$
 $a_\theta = -15 \text{ ft/sec}^2$

Total acceleration must lie on dotted lines ① & ②

From 30-60 triangles,

$$a_n = a = 15 / \cos 30^\circ = 10\sqrt{3} \frac{\text{ft}}{\text{sec}^2}$$

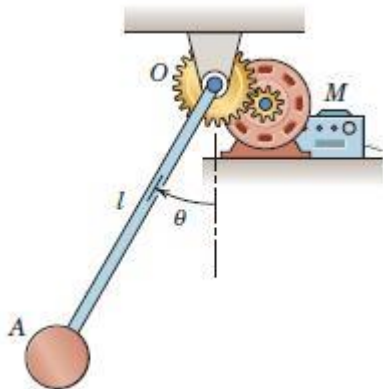
$$a_t = 0$$

$$a_y = -15 \tan 30^\circ = -5\sqrt{3} \text{ ft/sec}^2, \quad a_r = 10\sqrt{3} \sin 30^\circ = 5\sqrt{3} \text{ ft/sec}^2$$

$$a_n = v^2 / \rho, \quad \rho = \frac{v^2}{a_n} = \frac{6^2}{10\sqrt{3}} = \frac{6\sqrt{3}}{5} \text{ ft}$$

Question Title: Chapter 2, Reserve Problem 2/064

By means of the control unit M , the pendulum OA is given an oscillatory motion about the vertical given by $\theta = \theta_0 \sin \sqrt{\frac{g}{l}} t$, where θ_0 is the maximum angular displacement in radians, g is the acceleration of gravity, l is the pendulum length, and t is the time in seconds measured from an instant when OA is vertical. Determine and plot the magnitude a of the acceleration of A as a function of time and as a function of θ over the first quarter cycle of motion. Determine the minimum and maximum values of a and the corresponding values of t and θ . Use the values $\theta_0 = \pi/3$ radians, $l = 0.8$ m, and $g = 9.81$ m/s². (Note: The prescribed motion is not precisely that of a freely swinging pendulum for large amplitudes.)



SOLUTION

$$\text{Let } \omega_n = \sqrt{g/l} : \begin{cases} \theta = \theta_0 \sin \omega_n t \\ \dot{\theta} = \theta_0 \omega_n \cos \omega_n t \\ \ddot{\theta} = -\theta_0 \omega_n^2 \sin \omega_n t \end{cases}$$

$$a_t = l\ddot{\theta} = -l\theta_0 \omega_n^2 \sin \omega_n t = -g\theta_0 \sin \omega_n t$$

$$a_n = l\dot{\theta}^2 = l\theta_0^2 \omega_n^2 \cos^2 \omega_n t = g\theta_0^2 \cos^2 \omega_n t$$

$$a = \sqrt{a_t^2 + a_n^2} = g\theta_0 \sqrt{\sin^2 \omega_n t + \theta_0^2 \cos^2 \omega_n t}$$

a^2 (and therefore a) is an extreme when

$$\frac{da^2}{d\theta} = 0 = g^2 \theta_0^2 [2 \sin \omega_n t (\cos \omega_n t) + \theta_0^2 4 \cos^3 \omega_n t (-\sin \omega_n t)]$$

$$\Rightarrow [1 - 2\theta_0^2 \cos^2 \omega_n t] = 0$$

$$[1 - 2(\frac{\pi}{3})^2 \cos^2 \omega_n t] = 0, \quad \omega_n t = 0.830 \text{ rad}$$

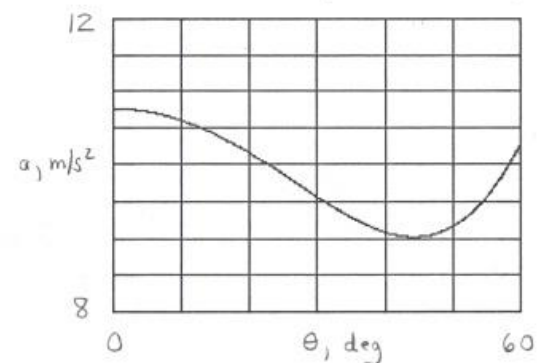
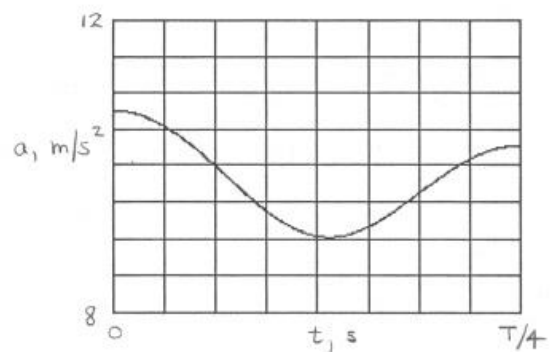
With $\omega_n = \sqrt{g/l} = \sqrt{\frac{9.81}{0.8}} = 3.50 \text{ s}^{-1}$, $t = 0.237 \text{ s}$

$$\theta = \frac{\pi}{3} \sin (0.830) = 0.772 \text{ rad } (44.3^\circ)$$

As can be seen from the plots below, the above represents a minimum:

$$a_{\min} = 9.03 \text{ m/s}^2 \text{ @ } \theta = 44.3^\circ \text{ \& } t = 0.237 \text{ s}$$

$$a_{\max} = 10.76 \text{ m/s}^2 \text{ @ } \theta = 0 \text{ \& } t = 0$$

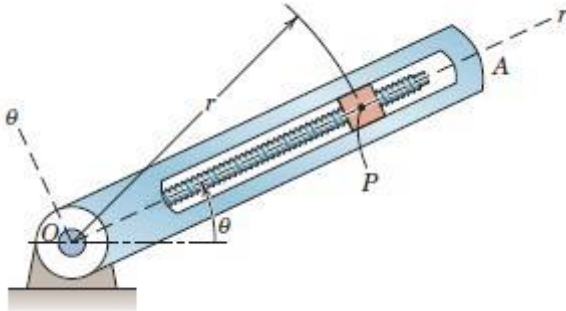


Note: Period $T = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.50} = 1.794 \text{ s}$

$$\frac{T}{4} = 0.449 \text{ s}$$

Question Title: Chapter 2, Reserve Problem 2/065

The position of the slider P in the rotating slotted arm OA is controlled by a power screw as shown. At the instant represented, $\dot{\theta} = 8 \text{ rad/s}$ and $\ddot{\theta} = -20 \text{ rad/s}^2$. Also at this same instant, $r = 200 \text{ mm}$, $\dot{r} = -300 \text{ mm/s}$, and $\ddot{r} = 0$. For this instant determine the r - and θ -components of the acceleration of P .



SOLUTION

Concise Solution

$$\begin{aligned}a_r &= \ddot{r} - r\dot{\theta}^2 = 0 - 200(8^2) = -12800 \text{ mm/s}^2 \\ &\text{or } a_r = \underline{-12.80 \text{ m/s}^2} \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} = 200(-20) + 2(-300)(8) = -8800 \text{ m/s}^2 \\ &\text{or } a_\theta = \underline{-8.80 \text{ m/s}^2}\end{aligned}$$

Detailed Solution

Solution Strategy:

- 1) Clearly identify the known values of the problem statement.
- 2) Calculate the r - component a_r of the acceleration at point P .
- 3) Calculate the θ - component a_θ of the acceleration at point P .

Learning Outcome:

- Calculate the r - and θ - components of acceleration of a particle.

1) Clearly identify the known values of the problem statement.

According to the problem statement:

$$\dot{\theta} = 8 \text{ rad/s (CW, according to the problem statement)}$$

$$\ddot{\theta} = -20 \text{ rad/s}^2 \text{ (CCW, according to the problem statement)}$$

$$r = 200 \text{ mm}$$

$$\dot{r} = -300 \text{ mm/s}$$

$$\ddot{r} = 0$$

2) Calculate the r - component a_r of the acceleration of point P .

The r - component of the acceleration a_r is given by:

$$a_r = \ddot{r} - r\dot{\theta}^2 \quad (1)$$

Substitute the known values into Eq. 1:

$$a_r = 0 - (200)(-8)^2 \text{ mm/s}^2$$

So:

$$\underline{\underline{a_r = -12.8 \text{ m/s}^2 \leftarrow \text{ans}}}$$

3) Calculate the θ - component a_θ of the acceleration at point P .

The θ - component of the acceleration a_θ is given by:

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \quad (2)$$

Substitute the known values into Eq. 2:

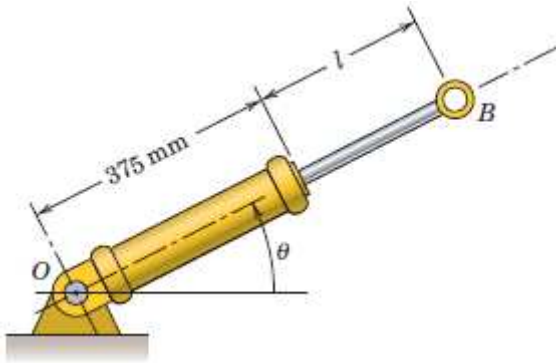
$$a_\theta = (200)(-20) + 2(-300)(8) \text{ mm/s}^2$$

So:

$$\underline{\underline{a_\theta = -8.80 \text{ m/s}^2 \leftarrow \text{ans}}}$$

Question Title: Chapter 2, Reserve Problem 2/066

As the hydraulic cylinder rotates around O , the exposed length l of the piston rod P is controlled by the action of oil pressure in the cylinder. If the cylinder rotates at the constant rate $\dot{\theta} = 60$ deg/s and l is decreasing at the constant rate of 150 mm/s, calculate the magnitudes of the velocity v and acceleration a of end B when $l = 125$ mm.



SOLUTION

$$r = 375 + 125 = 500 \text{ mm}, \quad \dot{r} = \dot{l} = -150 \frac{\text{mm}}{\text{s}}$$

$$\ddot{r} = 0, \quad \dot{\theta} = 60 \left(\frac{\pi}{180} \right) = \frac{\pi}{3} \text{ rad/s}, \quad \ddot{\theta} = 0$$

$$v_r = \dot{r} = -150 \frac{\text{mm}}{\text{s}}, \quad v_\theta = r\dot{\theta} = 500 \left(\frac{\pi}{3} \right) = 524 \frac{\text{mm}}{\text{s}}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-150)^2 + (524)^2} = \underline{545 \frac{\text{mm}}{\text{s}}}$$

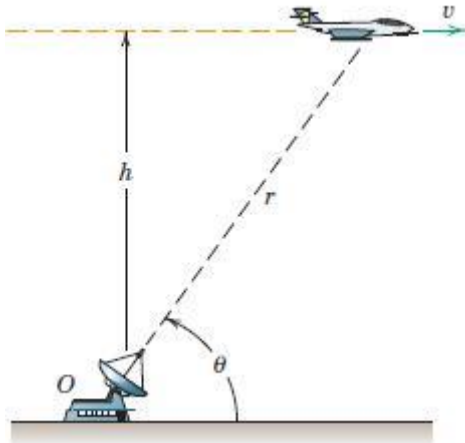
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 500 \left(\frac{\pi}{3} \right)^2 = -548 \text{ mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-150) \left(\frac{\pi}{3} \right) = -314 \text{ mm/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-548)^2 + (-314)^2} = \underline{632 \text{ mm/s}^2}$$

Question Title: Chapter 2, Reserve Problem 2/067

A jet plane flying at a constant speed v at an altitude $h = 10$ km is being tracked by radar located at O directly below the line of flight. If the angle θ is decreasing at the rate of 0.020 rad/s when $\theta = 60^\circ$, determine the value of $\dot{r}$ at this instant and the magnitude of the velocity v of the plane.



SOLUTION

Acceleration in all directions is zero, so

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0, \quad \ddot{r} = r\dot{\theta}^2$$

$$r = h/\sin\theta, \quad r = \frac{10}{\sqrt{3}/2} = 11.55 \text{ km}$$

$$\ddot{r} = 11.55 (-0.020)^2 = 0.00462 \text{ km/s}^2 = 4.62 \text{ m/s}^2$$

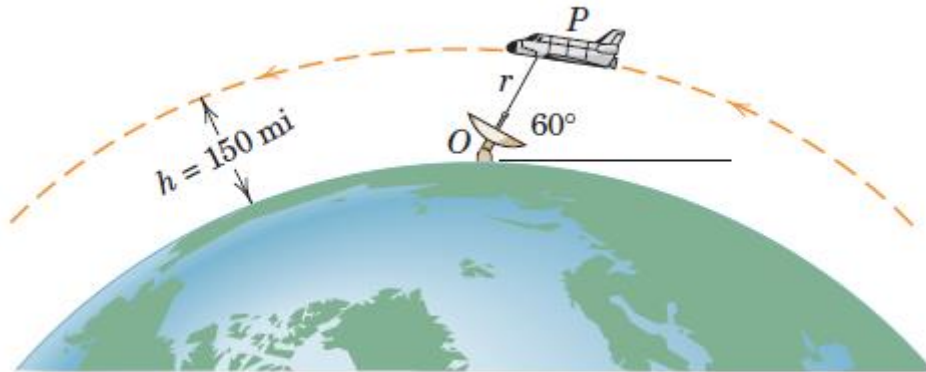
$$v = |r\dot{\theta}|/\sin\theta = \frac{h\dot{\theta}}{\sin^2\theta}$$

$$= \frac{10(-0.020)}{(\sqrt{3}/2)^2} = 0.267 \text{ km/s}$$

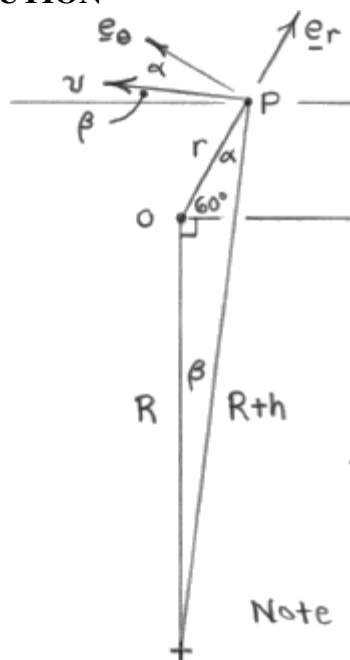
or $v = 0.267 (3600) = 960 \text{ km/h}$

Question Title: Chapter 2, Reserve Problem 2/068

At the instant depicted in the figure, the radar station at O measures the range rate of the space shuttle P to be $\dot{r} = -12,272$ ft/sec, with O considered fixed. If it is known that the shuttle is in a circular orbit at an altitude $h = 150$ mi, determine the orbital speed v of the shuttle from this information.



SOLUTION



$$\frac{\sin 150^\circ}{R+h} = \frac{\sin \alpha}{R}$$

$$\frac{\sin 150^\circ}{3959+150} = \frac{\sin \alpha}{3959}$$

$$\alpha = 28.8^\circ$$

$$\alpha + \beta + 150^\circ = 180^\circ \Rightarrow \beta = 1.200^\circ$$

$$v_r = \dot{r} = -12,272 = -v \sin \alpha$$

$$-12,272 = -v \sin 28.8^\circ$$

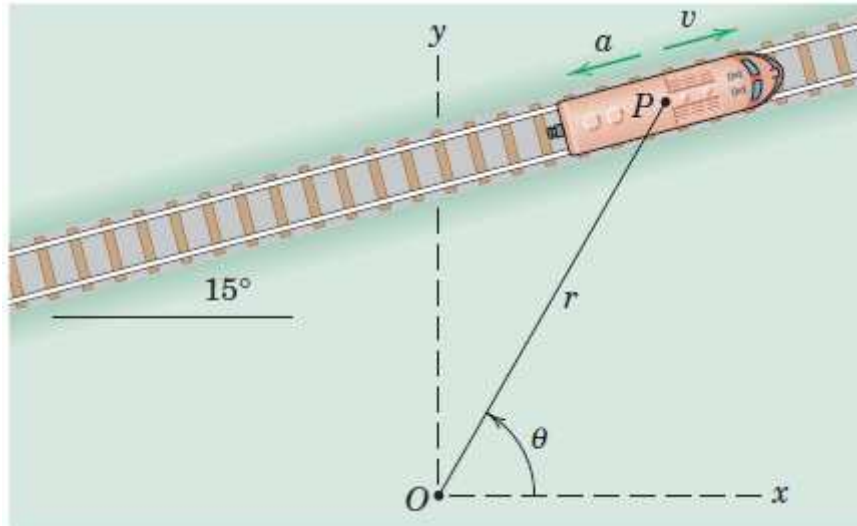
$$v = \underline{25,470 \text{ ft/sec}}$$

Note: Because v is nearly parallel to the horizontal at O ($\beta = 1.200^\circ$), one obtains a close approximation ($v = 24,544 \frac{\text{ft}}{\text{sec}}$) to the correct answer by neglecting β (assuming a flat earth).

Question Title: Chapter 2, Reserve Problem 2/069 (GO Tutorial version available)

A locomotive is traveling on the straight and level track with a speed $v = 90 \text{ km/h}$ and a deceleration $a = 0.5 \text{ m/s}^2$ as shown. Relative to the fixed observer at O , determine the quantities

$\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$ at the instant when $\theta = 60^\circ$ and $r = 400 \text{ m}$.



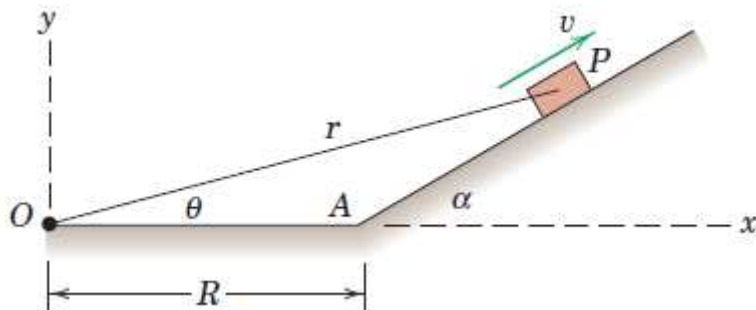
SOLUTION

$v = \frac{90}{3.6} = 25 \text{ m/s}$
 $a = 0.5 \text{ m/s}^2$

$v_r = \dot{r} = v \sin 45^\circ = 25 \frac{\sqrt{2}}{2} = 17.68 \text{ m/s}$
 $v_\theta = r\dot{\theta} : -25 \cos 45^\circ = 400\dot{\theta}, \quad \dot{\theta} = -0.0442 \text{ rad/s}$
 $a_r = \ddot{r} - r\dot{\theta}^2 : -0.5 \cos 45^\circ = \ddot{r} - 400(-0.0442)^2$
 $\ddot{r} = 0.428 \text{ m/s}^2$
 $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 0.5 \sin 45^\circ = 400\ddot{\theta} + 2(17.68)(-0.0442)$
 $\ddot{\theta} = 0.00479 \text{ rad/s}^2$

Question Title: Chapter 2, Reserve Problem 2/070

The small block P starts from rest at time $t = 0$ at point A and moves up the incline with constant acceleration a . Determine $\dot{r}$ as a function of time.



SOLUTION

This solution provides a guideline to solving the problem, not the numerical answer.

$$x = R + s \cos \alpha = R + (s_0 + v_0 t + \frac{1}{2} a t^2) \cos \alpha$$

$$= R + \frac{1}{2} a t^2 \cos \alpha$$

$$y = s \sin \alpha$$

$$= \frac{1}{2} a t^2 \sin \alpha$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(R + \frac{1}{2} a t^2 \cos \alpha)^2 + (\frac{1}{2} a t^2 \sin \alpha)^2}$$

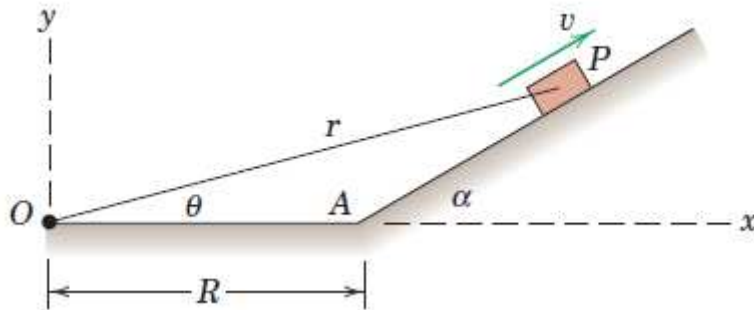
$$= \sqrt{R^2 + R a t^2 \cos \alpha + \frac{1}{4} a^2 t^4}$$

$$\dot{r} = \frac{1}{2} (R^2 + R a t^2 \cos \alpha + \frac{1}{4} a^2 t^4)^{-1/2} [2 R a t \cos \alpha + a^2 t^3]$$

$$= \frac{\frac{1}{2} a t (2 R \cos \alpha + a t^2)}{\sqrt{R^2 + R a t^2 \cos \alpha + \frac{1}{4} a^2 t^4}}$$

Question Title: Chapter 2, Reserve Problem 2/071

The small block P starts from rest at time $t = 0$ at point A and moves up the incline with constant acceleration a . Determine $\dot{\theta}$ as a function of time.



SOLUTION

This solution provides a guideline to solving the problem, not the numerical answer.

From the solution to Prob. 2/157 :

$$\begin{cases} x = R + \frac{1}{2}at^2 \cos \alpha \\ y = \frac{1}{2}at^2 \sin \alpha \end{cases}$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right) = \tan^{-1} \left[\frac{\frac{1}{2}at^2 \sin \alpha}{R + \frac{1}{2}at^2 \cos \alpha} \right]$$

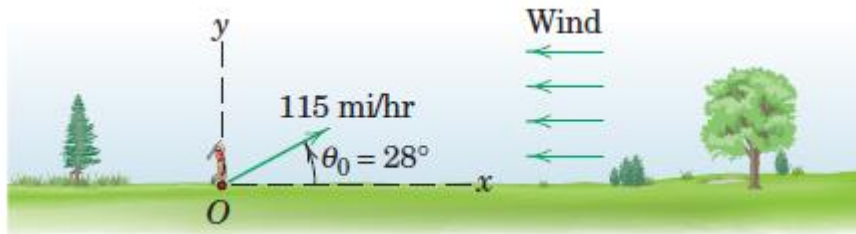
$$\dot{\theta} = \frac{\frac{(R + \frac{1}{2}at^2 \cos \alpha)(at \sin \alpha) - (\frac{1}{2}at^2 \sin \alpha)(at \cos \alpha)}{(R + \frac{1}{2}at^2 \cos \alpha)^2}}{1 + \left[\frac{\frac{1}{2}at^2 \sin \alpha}{R + \frac{1}{2}at^2 \cos \alpha} \right]^2}$$

Simplify to

$$\dot{\theta} = \frac{Rat \sin \alpha}{R^2 + Rat^2 \cos \alpha + \frac{1}{4}a^2 t^4}$$

Question Title: Chapter 2, Reserve Problem 2/072 (GO Tutorial version available)

A golf ball is driven with the initial conditions shown in the figure. If the wind imparts a constant horizontal deceleration of 4 ft/sec^2 , determine the values of r , $\dot{r}$, $\ddot{r}$, θ , $\dot{\theta}$, and $\ddot{\theta}$ when $t = 1.05$ sec. Take the r -coordinate to be measured from the origin.



SOLUTION

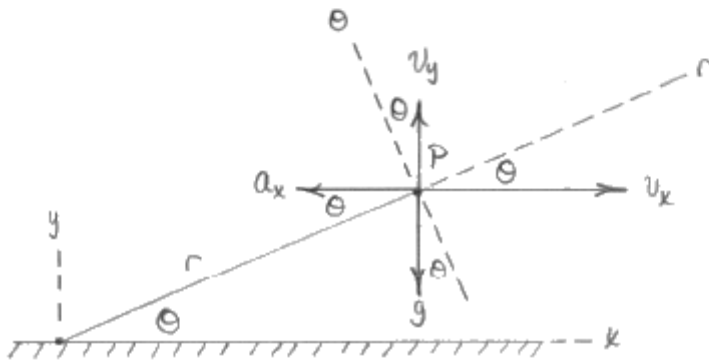
$$v_0 = 115 \text{ mph} = 168.7 \text{ ft/sec}, \quad \theta_0 = 28^\circ, \quad a_x = -4 \text{ ft/sec}$$

At $t = 1.05 \text{ sec}$:

$$\begin{cases} x = x_0 + v_{x_0}t + \frac{1}{2}a_x t^2 = 168.7 \cos 28^\circ (1.05) - \frac{1}{2}(4)(1.05)^2 = 154.2 \text{ ft} \\ y = y_0 + v_{y_0}t - \frac{1}{2}g t^2 = 168.7 \sin 28^\circ (1.05) - \frac{1}{2}(32.2)(1.05)^2 = 65.4 \text{ ft} \end{cases}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{154.2^2 + 65.4^2} = 167.5 \text{ ft} \quad \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{65.4}{154.2}\right) = 23.0^\circ$$

$$\begin{cases} v_x = v_{x_0} + a_x t = 168.7 \cos 28^\circ - 4(1.05) = 144.7 \text{ ft/sec} \\ v_y = v_{y_0} - g t = 168.7 \sin 28^\circ - 32.2(1.05) = 45.4 \text{ ft/sec} \end{cases}$$



$$v_r = v_x \cos \theta + v_y \sin \theta = \dot{r} = 144.7 \cos 23.0^\circ + 45.4 \sin 23.0^\circ$$

$$\dot{r} = 151.0 \text{ ft/sec}$$

$$v_\theta = r\dot{\theta} = v_y \cos \theta - v_x \sin \theta = 45.4 \cos 23.0^\circ - 144.7 \sin 23.0^\circ = 16.75 \dot{\theta}$$

$$\dot{\theta} = -0.0880 \text{ rad/sec}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -a_x \cos \theta - g \sin \theta = -4 \cos 23.0^\circ - 32.2 \sin 23.0^\circ = \ddot{r} - 16.75(-0.0880)^2$$

$$\ddot{r} = -14.96 \text{ ft/sec}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = a_x \sin \theta - g \cos \theta = 4 \sin 23.0^\circ - 32.2 \cos 23.0^\circ = 16.75\ddot{\theta} + 2(151.0)(-0.0880)$$

$$\ddot{\theta} = -0.00897 \text{ rad/sec}^2$$

Question Title: Chapter 2, Reserve Problem 2/073 (Multistep version available)

The rectangular coordinates of a particle are given in millimeters as functions of time t in seconds by $x = 30 \cos 2t$, $y = 40 \sin 2t$, and $z = 20t + 3t^2$. Determine the angle θ_1 between the position vector $\mathbf{r}$ and the velocity $\mathbf{v}$ and the angle θ_2 between the position vector $\mathbf{r}$ and the acceleration $\mathbf{a}$, both at time $t = 2$ s. Note that the arguments of the trigonometric functions are in radians (not degrees).

SOLUTION

$$\begin{cases} x = 30 \cos 2t; & y = 40 \sin 2t; & z = 20t + 3t^2 \\ \dot{x} = -60 \sin 2t; & \dot{y} = 80 \cos 2t; & \dot{z} = 20 + 6t \\ \ddot{x} = -120 \cos 2t; & \ddot{y} = -160 \sin 2t; & \ddot{z} = 6 \end{cases}$$

At $t = 2$ s :

$$\begin{cases} x = -19.61 \text{ mm} & ; & y = -30.3 \text{ mm} & ; & z = 52 \text{ mm} \\ \dot{x} = 45.4 \text{ mm/s} & ; & \dot{y} = -52.3 \text{ mm/s} & ; & \dot{z} = 32 \text{ mm/s} \\ \ddot{x} = 78.4 \text{ mm/s}^2 & ; & \ddot{y} = 121.1 \text{ mm/s}^2 & ; & \ddot{z} = 6 \text{ mm/s}^2 \end{cases}$$

$$r = (x^2 + y^2 + z^2)^{1/2} = 63.3 \text{ mm} \quad \left\| \quad a = (\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2)^{1/2} \right.$$

$$v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} = 76.3 \text{ mm/s} \quad \left\| \quad = 144.4 \text{ mm/s}^2 \right.$$

$$\theta_1 = \cos^{-1} \left[\frac{\mathbf{r} \cdot \mathbf{v}}{rv} \right] = \cos^{-1} \left[\frac{-19.61(45.4) - 30.3(-52.3) + 52(32)}{(63.3)(76.3)} \right]$$

$$= 60.8^\circ$$

$$\theta_2 = \cos^{-1} \left[\frac{\mathbf{r} \cdot \mathbf{a}}{ra} \right] = \cos^{-1} \left[\frac{-19.61(78.4) - 30.3(121.1) - 52(6)}{(63.3)(144.4)} \right]$$

$$= 122.4^\circ$$

Question Title: Chapter 2, Reserve Problem 2/074

The base structure of the firetruck ladder rotates about a vertical axis through O with a constant angular velocity $\Omega = 10$ deg/s. At the same time, the ladder unit OB elevates at a constant rate $\dot{\phi} = 7$ deg/s, and section AB of the ladder extends from within section OA at the constant rate of 0.5 m/s. At the instant under consideration, $\phi = 30^\circ$, $\overline{OA} = 9$ m, and $\overline{AB} = 6$ m. Determine the magnitudes of the velocity v and acceleration a of the end B of the ladder.



SOLUTION

Spherical coordinates

$$v_R = \dot{R} = 0.5 \text{ m/s}$$

$$v_\theta = R\dot{\theta} \cos \phi = 15 \left(10 \frac{\pi}{180} \right) \cos 30^\circ = 2.27 \text{ m/s}$$

$$v_\phi = R\dot{\phi} = 15 \left(7 \frac{\pi}{180} \right) = 1.833 \text{ m/s}$$

$$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = \underline{2.96 \text{ m/s}}$$

$$a_R = \ddot{R} - R\dot{\theta}^2 - R\dot{\phi}^2 \cos^2 \phi$$

$$= 0 - 15 \left(7 \frac{\pi}{180} \right)^2 - 15 \left(10 \frac{\pi}{180} \right)^2 \cos^2 30^\circ = -0.567 \text{ m/s}^2$$

$$a_\theta = \frac{\cos \phi}{R} [R^2 \ddot{\theta} + 2R\dot{R}\dot{\theta}] - 2R\dot{\theta}\dot{\phi} \sin \phi$$

$$= \frac{\cos 30^\circ}{15} [0 + 2(15)(0.5) \left(10 \frac{\pi}{180} \right)] - 2(15) \left(10 \frac{\pi}{180} \right) \left(7 \frac{\pi}{180} \right) \sin 30^\circ$$

$$= -0.1687 \text{ m/s}^2$$

$$a_\phi = \frac{1}{R} [R^2 \ddot{\phi} + 2R\dot{R}\dot{\phi}] + R\dot{\theta}^2 \sin \phi \cos \phi$$

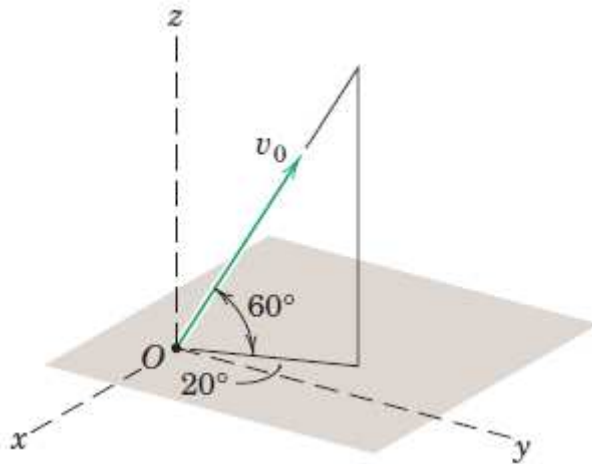
$$= \frac{1}{15} [0 + 2(15)(0.5) \left(7 \frac{\pi}{180} \right)] + 15 \left(10 \frac{\pi}{180} \right)^2 \sin 30^\circ \cos 30^\circ$$

$$= 0.320 \text{ m/s}^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = \underline{0.672 \text{ m/s}^2}$$

Question Title: Chapter 2, Reserve Problem 2/075

A projectile is launched from point O with an initial velocity of magnitude $v_0 = 600$ ft/sec, directed as shown in the figure. Compute the x -, y -, and z -components of position, velocity, and acceleration 20 seconds after launch. Neglect aerodynamic drag.



SOLUTION

$$v_{z_0} = 600 \sin 60^\circ = 520 \text{ ft/sec}$$

$$v_{xy_0} = 600 \cos 60^\circ = 300 \text{ ft/sec}$$

$$v_z = v_{z_0} - gt = 520 - 32.2(20) = \underline{-124.4 \text{ ft/sec}}$$

$$v_{xy} = v_{xy_0} = 300 \text{ ft/sec} = \text{constant}$$

$$v_x = -v_{xy} \sin 20^\circ = -300 \sin 20^\circ = \underline{-102.6 \text{ ft/sec}}$$

$$v_y = v_{xy} \cos 20^\circ = 300 \cos 20^\circ = \underline{282 \text{ ft/sec}}$$

$$d_{xy} = v_{xy} t = 300(20) = 6000 \text{ ft}$$

$$x = -d_{xy} \sin 20^\circ = -6000 \sin 20^\circ = \underline{-2050 \text{ ft}}$$

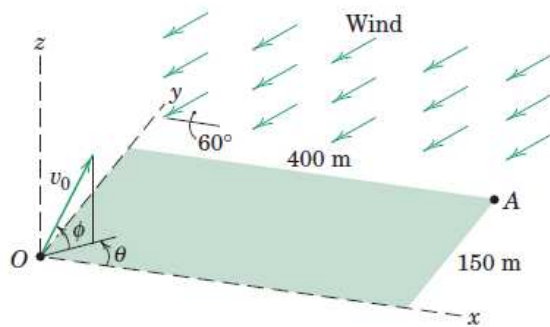
$$y = d_{xy} \cos 20^\circ = 6000 \cos 20^\circ = \underline{5640 \text{ ft}}$$

$$z = v_{z_0} t - \frac{1}{2} g t^2 = 520(20) - 16.1(20)^2 = \underline{3950 \text{ ft}}$$

$$\underline{a_x = a_y = 0, \quad a_z = -g = -32.2 \text{ ft/sec}^2}$$

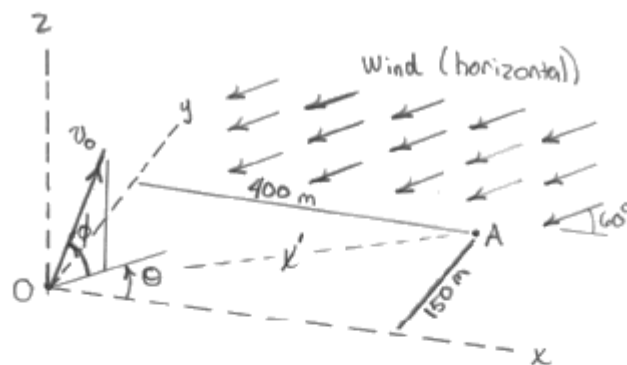
Question Title: Chapter 2, Reserve Problem 2/076 (Multistep version available)

A projectile is launched from point O at a speed $v_0 = 80$ m/s with the goal of hitting the target A . At the launch instant, a strong horizontal wind begins blowing and imparts a constant acceleration of 1.25 m/s² to the projectile in the same direction as the wind. If the launch conditions are chosen so that the particle would impact the target in the absence of wind effects, determine the impact coordinates of the projectile. Note: There is more than one solution to this problem. Choose the solution that corresponds to the lower trajectory.

**SOLUTION**

$$a_w = 1.25 \text{ m/s}^2$$

$$v_0 = 80 \text{ m/s}$$

**• WITHOUT WIND:**

WITHOUT WIND, THE PROJECTILE IS AIMED DIRECTLY AT A , AND WE WILL EXAMINE MOTION ALONG LINE OA , WITHIN THE $x'-z$ PLANE.

$$\theta = \tan^{-1}\left(\frac{150}{400}\right) = 20.6^\circ \quad \overline{OA} = \sqrt{150^2 + 400^2} = 427 \text{ m}$$

$$\begin{cases} x'_A = x'_{/0} + v'_{x_0} t \\ 427 = 80 \cos \phi t \end{cases} \quad \begin{cases} z'_A = z'_{/0} + v'_{z_0} t - \frac{1}{2} g t^2 \\ 0 = 80 \sin \phi t - \frac{1}{2} (9.81) t^2 \end{cases}$$

$$\text{Solving...} \quad \begin{cases} \phi = 20.5^\circ \\ t = 5.70 \text{ s} \end{cases} \quad \text{OR} \quad \begin{cases} \phi = 69.5^\circ \\ t = 15.28 \text{ s} \end{cases} \quad \left(\begin{array}{l} \text{THE LONGER TIME} \\ \text{WILL ALLOW THE} \\ \text{WIND TO MAKE} \\ \text{A BIGGER EFFECT} \end{array} \right)$$

• WITH WIND: USE $t = 5.70$ s.

$$x_f = x_{/0} + v_{x_0} t_f + \frac{1}{2} a_x t_f^2 = 80 \cos(20.5^\circ) \cos(20.6^\circ) (5.70) - \frac{1}{2} (1.25 \cos 60^\circ) (5.70)^2$$

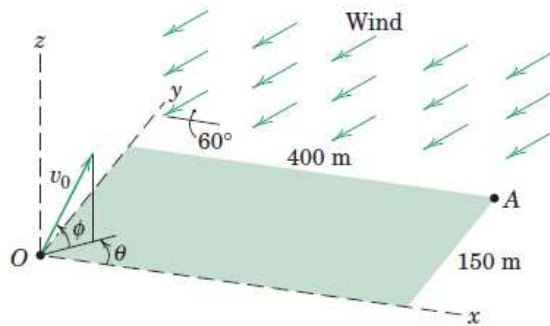
$$\underline{x_f = 390 \text{ m}}$$

$$y_f = y_{/0} + v_{y_0} t_f + \frac{1}{2} a_y t_f^2 = 80 \cos(20.5^\circ) \sin(20.6^\circ) (5.70) - \frac{1}{2} (1.25 \sin 60^\circ) (5.70)^2$$

$$\underline{y_f = 132.4 \text{ m}}$$

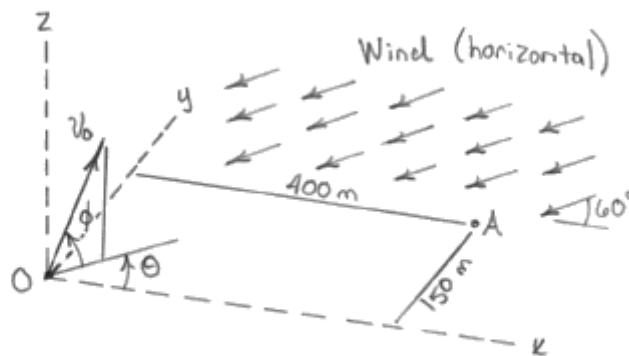
Question Title: Chapter 2, Reserve Problem 2/077

A projectile is launched from point O at a speed $v_0 = 80$ m/s with the goal of hitting the target A . At the launch instant, a strong horizontal wind begins blowing and imparts a constant acceleration of 1.25 m/s² to the projectile in the same direction as the wind. What values of θ and ϕ (positive and less than 90°) will ensure that the projectile impacts the target at A if the wind conditions are considered. Please list both possible combinations of the angles.



SOLUTION

$$a_w = 1.25 \text{ m/s}^2 \quad v_0 = 80 \text{ m/s}$$



$$\begin{cases} x_A = x_0 + v_{x0} t_A + \frac{1}{2} a_x t_A^2 \\ 400 = 80 \cos \phi \cos \theta t_A - \frac{1}{2} (1.25 \cos 60^\circ) t_A^2 \end{cases}$$

$$\begin{cases} y_A = y_0 + v_{y0} t_A + \frac{1}{2} a_y t_A^2 \\ 150 = 80 \cos \phi \sin \theta t_A - \frac{1}{2} (1.25 \sin 60^\circ) t_A^2 \end{cases}$$

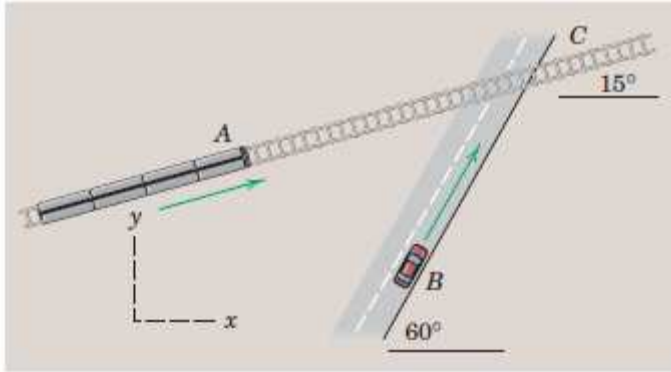
$$\begin{cases} z_A = z_0 + v_{z0} t_A - \frac{1}{2} g t_A^2 \\ 0 = 80 \sin \phi t_A - \frac{1}{2} (9.81) t_A^2 \end{cases}$$

Solving Numerically...

$$\begin{cases} \theta = 22.4^\circ \\ \phi = 21.5^\circ \\ t_A = 5.97 \text{ s} \end{cases} \quad \text{OR} \quad \begin{cases} \theta = 29.5^\circ \\ \phi = 62.5^\circ \\ t_A = 14.46 \text{ s} \end{cases}$$

Question Title: Chapter 2, Reserve Problem 2/078 (GO Tutorial version available)

Train A travels with a constant speed $v_A = 120$ km/h along the straight and level track. The driver of car B , anticipating the railway grade crossing C , decreases the car speed of 90 km/h at the rate of 3 m/s^2 . Determine the velocity $\mathbf{v}_{A/B}$ and acceleration $\mathbf{a}_{A/B}$ of the train relative to the car.



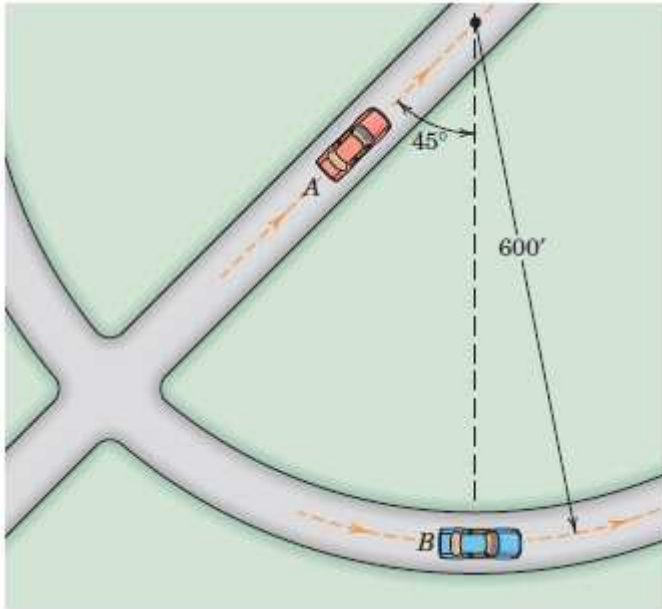
SOLUTION

This solution provides a guideline to solving the problem and might not match the values in your text.

$$\begin{aligned}
 \mathbf{v}_{A/B} &= \mathbf{v}_A - \mathbf{v}_B \\
 &= 120 [\cos 15^\circ \mathbf{i} + \sin 15^\circ \mathbf{j}] - 90 [\cos 60^\circ \mathbf{i} + \sin 60^\circ \mathbf{j}] \\
 &= 70.9 \mathbf{i} - 46.9 \mathbf{j} \text{ km/h} \\
 \mathbf{a}_{A/B} &= \mathbf{a}_A - \mathbf{a}_B = \mathbf{0} - 3 (-\cos 60^\circ \mathbf{i} - \sin 60^\circ \mathbf{j}) \\
 &= 1.5 \mathbf{i} + 2.60 \mathbf{j} \text{ m/s}^2
 \end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/079

For the instant represented, car A has an acceleration in the direction of its motion, and car B has a speed of 45 mi/hr which is increasing. If the acceleration of B as observed from A is zero for this instant, determine the magnitude of the acceleration of A and the rate at which the speed of B is changing.



SOLUTION

With $\underline{a}_{B/A} = 0$, $\underline{a}_A = \underline{a}_B$

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(45 \frac{44}{30})^2}{600}$$

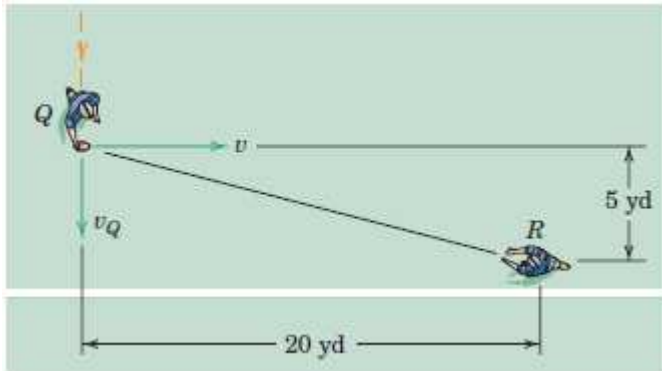
$$= 7.26 \text{ ft/sec}^2$$

$$\dot{v}_B = (a_B)_t = 7.26 \text{ ft/sec}^2$$

$$a_A = a_B = 7.26\sqrt{2} = \underline{10.27 \frac{\text{ft}}{\text{sec}^2}}$$

Question Title: Chapter 2, Reserve Problem 2/080

While scrambling directly toward the sideline, the football quarterback Q throws a pass toward the stationary receiver R . At what speed v_Q should the quarterback run if the direction of the velocity of the ball relative to the quarterback is to be directly down the field as indicated? The speed of the ball relative to the quarterback is 60 ft/sec. What is the absolute speed v_b of the ball? Treat the problem as two-dimensional.



SOLUTION

$$\begin{aligned}
 & \alpha = \tan^{-1} \left(\frac{5}{20} \right) = 14.04^\circ \\
 & \frac{v_Q}{v_{B/Q}} = 0.25 \Rightarrow \underline{v_Q = 15 \text{ ft/sec}} \\
 & v_B = \sqrt{60^2 + 15^2} = \underline{61.8 \text{ ft/sec}}
 \end{aligned}$$

Handwritten notes: $v_{B/Q} \leftarrow 60 \text{ ft/sec}$ (B = ball)

Question Title: Chapter 2, Reserve Problem 2/081

As part of a training exercise, the pilot of aircraft A adjusts her airspeed (speed relative to the wind) to 220 km/h while in the level portion of the approach path and thereafter holds her absolute speed constant as she negotiates the 10° glide path. The absolute speed of the aircraft carrier is 30 km/h and that of the wind is 48 km/h. What will be the angle β of the glide path with respect to the horizontal as seen by an observer on the ship?



SOLUTION

$$\underline{v}_A = \underline{v}_W + \underline{v}_{A/W} = -48\hat{i} + 220\hat{i} = 172\hat{i} \frac{\text{km}}{\text{h}}$$

On descent: $\underline{v}_A = 172(\cos 10^\circ\hat{i} - \sin 10^\circ\hat{j})$ km/h

A coordinate system is shown with the x-axis horizontal and the y-axis vertical.

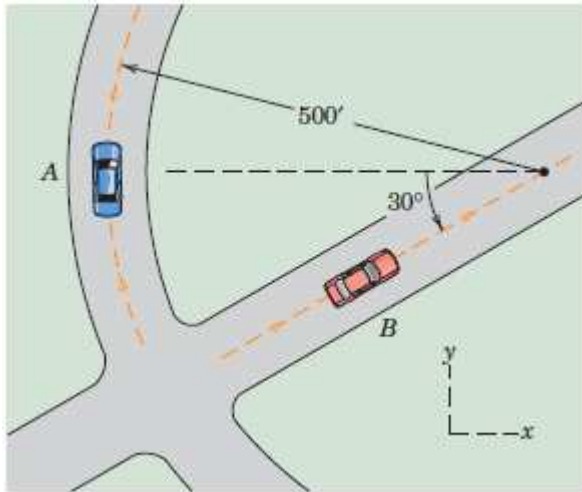
$$\underline{v}_{A/C} = \underline{v}_A - \underline{v}_C = 172(\cos 10^\circ\hat{i} - \sin 10^\circ\hat{j}) - 30\hat{i}$$

$$= 139.4\hat{i} - 29.9\hat{j} \text{ km/h}$$

$$\beta = \tan^{-1} \left(\frac{29.9}{139.4} \right) = \underline{12.09^\circ}$$

Question Title: Chapter 2, Reserve Problem 2/082

For the instant represented, car A is rounding the circular curve at a constant speed of 30 mi/hr, while car B is slowing down at the rate of 5 mi/hr per second. Determine the magnitude of the acceleration that car A appears to have to an observer in car B.



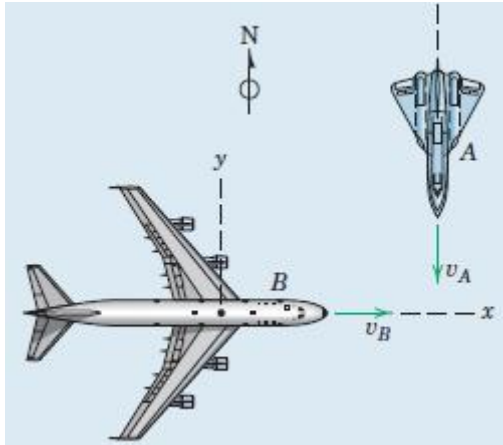
SOLUTION

$$\begin{aligned} \underline{a}_A &= \underline{a}_B + \underline{a}_{A/B} \\ a_A &= v^2/\rho = (44)^2/500 = 3.87 \text{ ft/sec}^2 \\ a_B &= \frac{5}{30} 44 = 7.33 \text{ ft/sec}^2 \end{aligned}$$

$$\begin{aligned} \underline{a}_{A/B} &= (7.33 \cos 30^\circ + 3.87) \underline{i} + 7.33 \sin 30^\circ \underline{j} \\ &= 10.22 \underline{i} + 3.67 \underline{j} \text{ ft/sec}^2 \\ a_{A/B} &= \sqrt{10.22^2 + 3.67^2} = \underline{10.86 \text{ ft/sec}^2} \end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/083

The passenger aircraft B is flying east with a velocity $v_B = 800$ km/h. A military jet traveling south with a velocity $v_A = 1200$ km/h passes under B at a slightly lower altitude. What velocity does A appear to have to a passenger in B , and what is the direction of that apparent velocity?



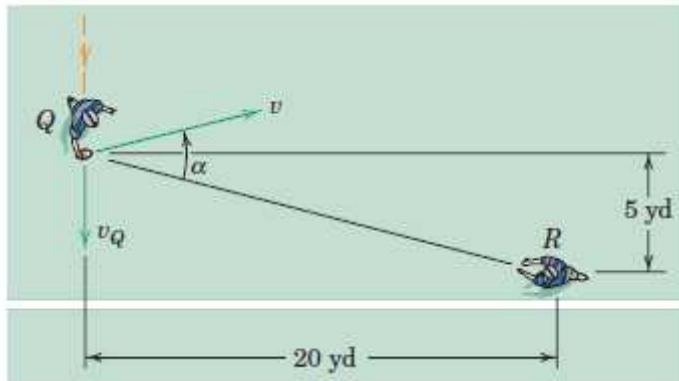
SOLUTION

$$\begin{aligned} \vec{v}_A &= \vec{v}_B + \vec{v}_{A/B} \\ v_{A/B} &= \sqrt{(1200)^2 + (800)^2} = 1442 \text{ km/h} \\ \beta &= \tan^{-1} \frac{800}{1200} = 33.7^\circ \text{ west of south} \end{aligned}$$

The velocity triangle shows the relationship between the velocities. The horizontal vector $v_B = 800$ km/h points to the right. The vertical vector $v_A = 1200$ km/h points downwards. The resultant vector $v_{A/B}$ is the hypotenuse of the right triangle. The angle β is measured from the vertical vector v_A towards the horizontal vector v_B , indicating it is west of south.

Question Title: Chapter 2, Reserve Problem 2/084

While scrambling directly toward the sideline at a speed $v_Q = 20$ ft/sec, the football quarterback Q throws a pass toward the stationary receiver R . At what angle α should the quarterback release the ball? The speed of the ball relative to the quarterback is 60 ft/sec. Treat the problem as two-dimensional.



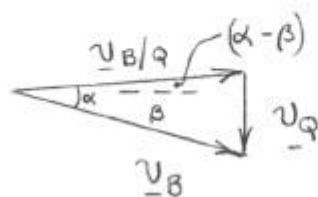
SOLUTION

$$\begin{aligned} \underline{v}_B &= \underline{v}_Q + \underline{v}_{B/Q} \quad (B = \text{ball}) \\ \underline{v}_B (\cos \beta \underline{i} - \sin \beta \underline{j}) &= -20 \underline{j} + 60 (\cos(\alpha - \beta) \underline{i} + \sin(\alpha - \beta) \underline{j}) \end{aligned}$$

Solution: $\alpha = 18.87^\circ$

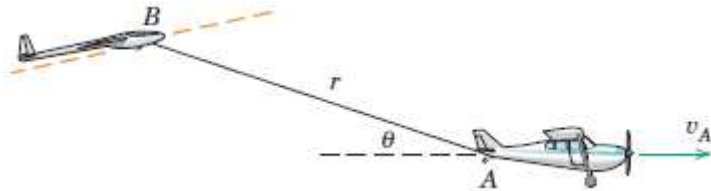
$v_B = 61.6 \text{ ft/sec}$

$\beta = \tan^{-1} \left(\frac{5}{20} \right) = 14.04^\circ$



Question Title: Chapter 2, Reserve Problem 2/085

Airplane A is flying horizontally with a constant speed of 200 km/h and is towing the glider B, which is gaining altitude. If the tow cable has a length $r = 60$ m and θ is increasing at the constant rate of 5 degrees per second, determine the magnitudes of the velocity v and acceleration a of the glider for the instant when $\theta = 15^\circ$.



SOLUTION

$$\begin{aligned} \underline{v}_B &= \underline{v}_A + \underline{v}_{B/A}, \quad v_{B/A} = r\dot{\theta} = 60\left(\frac{5}{180}\pi\right) = 5.24 \text{ m/s} \\ \underline{v}_B &= \underline{v}_A + \underline{v}_{B/A} \end{aligned}$$

$$v_B^2 = (5.24)^2 + (55.6)^2 + 2(5.24)(55.6)\cos 75^\circ = 3264 \text{ (m/s)}^2$$

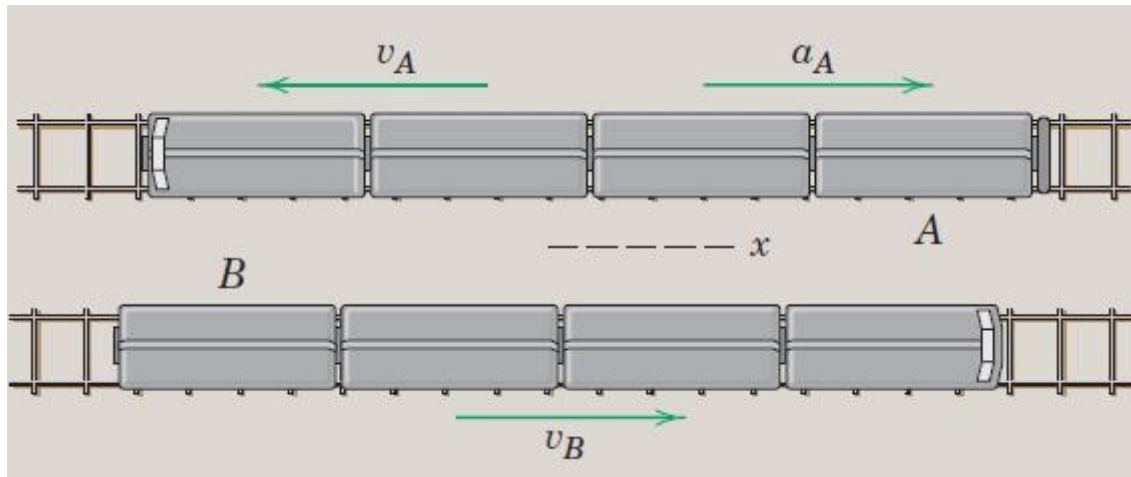
$$v_B = 57.1 \text{ m/s} \quad \text{or} \quad v_B = 57.1(3.6) = \underline{206 \text{ km/h}}$$

$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}, \quad a_A = 0, \quad a_{B/A} = r\dot{\theta}^2 = 60\left(\frac{5\pi}{180}\right)^2 = 0.457 \text{ m/s}^2$$

Thus $a_B = a_{B/A} = \underline{0.457 \text{ m/s}^2}$ from B to A

Question Title: Chapter 2, Reserve Problem 2/086

Rapid-transit trains *A* and *B* travel on parallel tracks. Train *A* has a speed of 80 km/h and is slowing at the rate of 2 m/s², while train *B* has a constant speed of 40 km/h. Determine the velocity and acceleration of train *B* relative to train *A*.



SOLUTION

$$\underline{v}_{B/A} = \underline{v}_B - \underline{v}_A = 40\hat{i} - (-80\hat{i}) = \underline{120\hat{i} \frac{\text{km}}{\text{h}}}$$

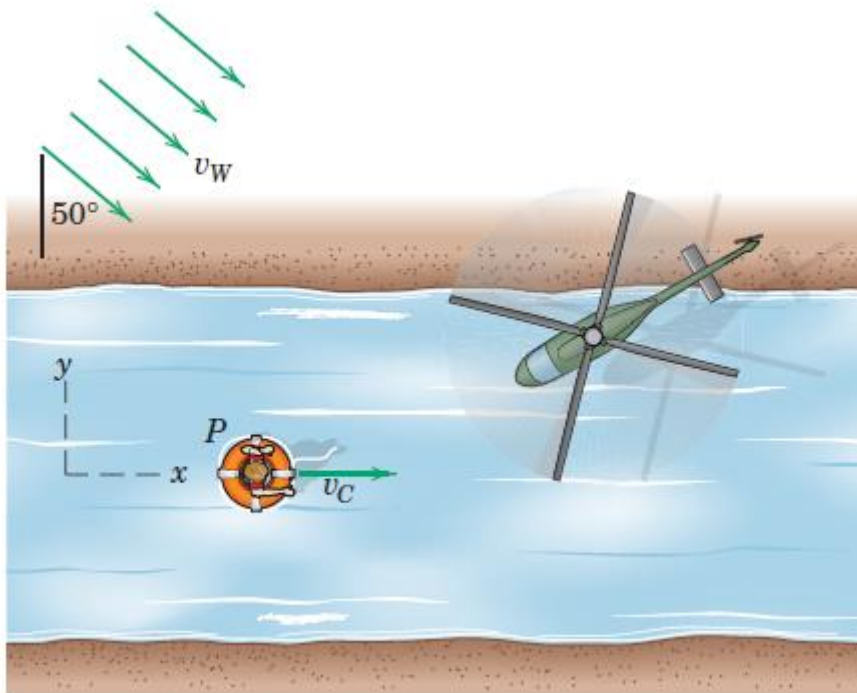
$$\underline{a}_{B/A} = \underline{a}_B - \underline{a}_A = 0 - 2\hat{i} = \underline{-2\hat{i} \text{ m/s}^2}$$

— — — — x

Chapter 2 Reserve Problem Solutions (Part 2)

Question Title: Chapter 2, Reserve Problem 2/087

A helicopter approaches a rescue scene. A victim P is drifting along with the river current of speed $v_C = 2$ m/s. The wind is blowing at a speed $v_W = 3$ m/s as indicated. Determine the velocity $\mathbf{v}_{H/W}$ relative to the wind which the helicopter must acquire so that it maintains a steady overhead position relative to the victim.

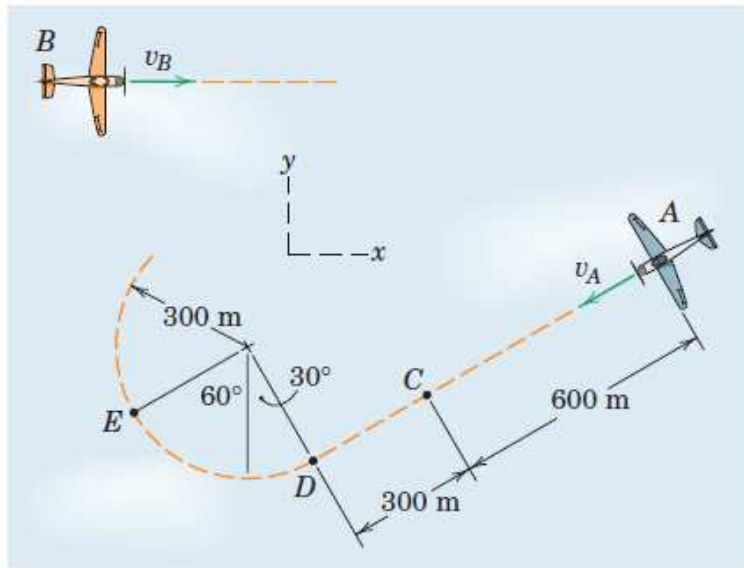


SOLUTION

$$\begin{aligned}
 \underline{v}_H &= \underline{v}_C = 2\hat{i} \text{ m/s} \\
 \underline{v}_{H/W} &= \underline{v}_H - \underline{v}_W \\
 &= 2\hat{i} - 3(\sin 50^\circ \hat{i} - \cos 50^\circ \hat{j}) \\
 &= \underline{-0.298\hat{i} + 1.928\hat{j} \text{ m/s}}
 \end{aligned}$$

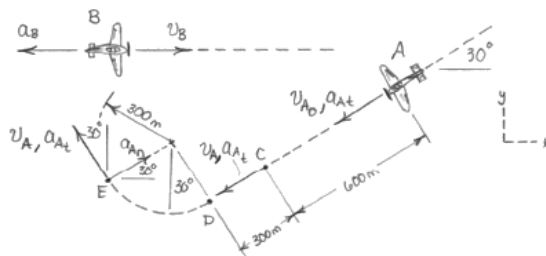
Question Title: Chapter 2, Reserve Problem 2/088 (Multistep version available)

Plane A has speed $v_A = 285 \text{ km/h}$ at the position shown. Plane A increases its speed at a constant rate and acquires a speed of 415 km/h by the time it reaches position E. Plane B has the speed $v_B = 350 \text{ km/h}$ in the position shown. Plane B experiences a steady deceleration of 1.5 m/s^2 . Relative to the pilot in plane B, what are the velocities and accelerations which plane A appears to have when it is at positions C and E?



SOLUTION

$$v_{A_0} = 285 \text{ km/h}, v_{B_0} = 350 \text{ km/h}, v_A = 415 \text{ km/h}, a_B = 1.5 \text{ m/s}^2$$



$$v_A = v_{A_0} + 2a_{At} s_{AE} \rightarrow \left(\frac{415}{3.6}\right)^2 = \left(\frac{285}{3.6}\right)^2 + 2a_{At} \left(300 + \frac{300\pi}{2}\right)$$

$$\therefore a_{At} = 2.56 \text{ m/s}^2$$

At C:

$$v_A = \sqrt{v_{A_0}^2 + 2a_{At} s_{AC}} = \sqrt{\left(\frac{285}{3.6}\right)^2 + 2(2.56)(600)} = 96.6 \text{ m/s OR } 348 \text{ km/h}$$

$$v_A = v_{A_0} + a_{At} t_{AC} \rightarrow 96.6 = \frac{285}{3.6} + 2.56 t_{AC} \rightarrow t_{AC} = 6.83 \text{ s}$$

$$v_B = v_{B_0} - a_B t_{AC} = \frac{350}{3.6} - 1.5(6.83) = 87.0 \text{ m/s OR } 313 \text{ km/h}$$

At E:

$$v_A = v_{A_0} + a_{At} t_{AE} \rightarrow \frac{415}{3.6} = \frac{285}{3.6} + 2.56 t_{AE} \rightarrow t_{AE} = 14.10 \text{ s}$$

$$v_B = v_{B_0} - a_B t_{AE} = \frac{350}{3.6} - 1.5(14.10) = 76.1 \text{ m/s OR } 274 \text{ km/h}$$

At C:

$$v_{A/B} = v_A - v_B = 348(-\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}) - 313 \hat{i}$$

$$v_{A/B} = -614 \hat{i} - 174.0 \hat{j} \text{ km/h}$$

$$a_{A/B} = a_A - a_B = 2.56(-\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}) - (-1.5 \hat{i})$$

$$a_{A/B} = -0.717 \hat{i} - 1.280 \hat{j} \text{ m/s}^2$$

At E:

$$v_{A/B} = v_A - v_B = 415(-\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j}) - 274 \hat{i}$$

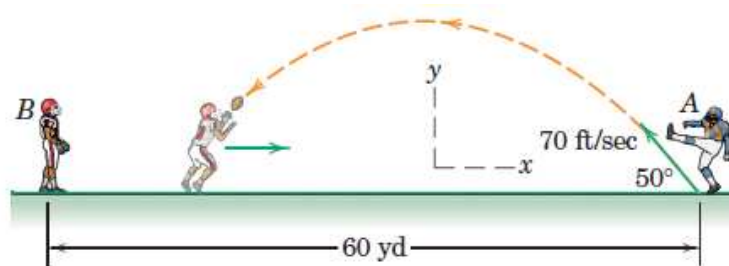
$$v_{A/B} = -481 \hat{i} + 359 \hat{j} \text{ km/h}$$

$$a_{A/B} = a_A - a_B = 2.56(-\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j}) + \frac{(415/3.6)^2}{300}(\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}) - (-1.5 \hat{i})$$

$$a_{A/B} = 38.6 \hat{i} + 24.4 \hat{j} \text{ m/s}^2$$

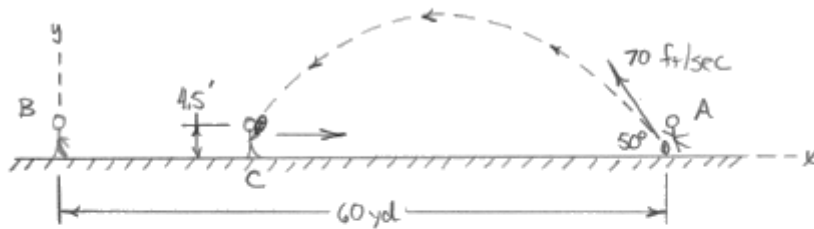
Question Title: Chapter 2, Reserve Problem 2/089 (Multistep version available)

A place kicker A executes a “pooch” kick, which is designed to eliminate a potential return by the receiving team. The “pooch” kick features a high trajectory and short range, thereby preventing the deep kick returner B from reaching his maximum speed before encountering coverage. To offset this, player B hesitates for 1.15 sec after the ball is kicked, and then accelerates at a uniform rate, reaching his maximum speed at the instant he catches the ball. Determine the velocity of the football relative to the receiver at the instant the catch is made if the football is caught when it is 4.5 ft above the playing surface.



SOLUTION

PLAYER B HESITATES FOR 1.15 SEC BEFORE RUNNING.



• FLIGHT TIME OF BALL:

$$\begin{cases} x_c = x_A + v_{x_0} t_{AC} = 60(3) - 70 \cos 50^\circ t_{AC} \\ y_c = y_A + v_{y_0} t_{AC} - \frac{1}{2} g t_{AC}^2 \rightarrow 4.5 = 0 + 70 \sin 50^\circ t_{AC} - \frac{1}{2} (32.2) t_{AC}^2 \end{cases}$$

Solving... $t_{AC} = 0.0861 \text{ sec}$ OR $t_{AC} = 3.24 \text{ sec}$ (CHOOSE LATER TIME)

SO... $x_{AC} = 34.0 \text{ ft}$

• FOOTBALL VELOCITY AT C:

$$\begin{cases} v_x = v_{x_0} = -70 \cos 50^\circ = -45.0 \text{ ft/sec} \\ v_y = v_{y_0} - g t_{AC} = 70 \sin 50^\circ - 32.2(3.24) = -50.8 \text{ ft/sec} \end{cases}$$

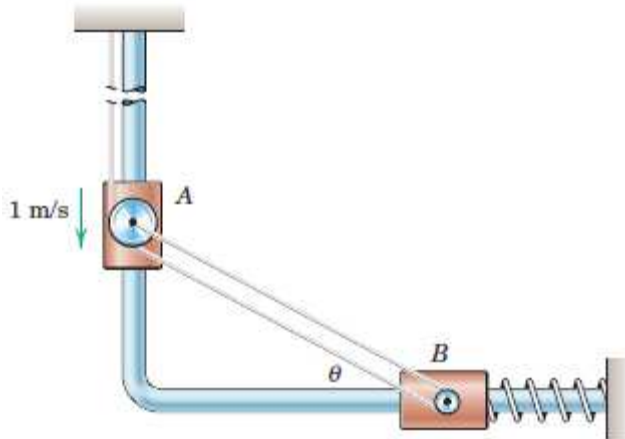
• RECEIVER FROM B TO C: (HE HAS $3.24 - 1.15 = 2.09 \text{ sec}$ TO REACH POINT C.)

$$\begin{cases} x_c = x_B + v_{x_0} t_{BC} + \frac{1}{2} a t_{BC}^2 \rightarrow 34.0 = \frac{1}{2} a (2.09)^2 \rightarrow a = 15.51 \text{ ft/sec}^2 \\ v_c = v_{x_0} + a t_{BC} = 15.51 (2.09) = 32.5 \text{ ft/sec} \end{cases}$$

$$v_{\text{BALL/B}} = v_{\text{BALL}} - v_B = (-45.0 \underline{i} - 50.8 \underline{j}) - 32.5 \underline{i} \rightarrow v_{\text{BALL/B}} = -77.5 \underline{i} - 50.8 \underline{j} \text{ ft/sec}$$

Question Title: Chapter 2, Reserve Problem 2/090

Cylinder A has a constant downward speed of 1 m/s. Compute the velocity of cylinder B for (a) $\theta = 45^\circ$, (b) $\theta = 30^\circ$, and (c) $\theta = 15^\circ$. The spring is in tension throughout the motion range of interest, and the pulleys are connected by the cable of fixed length.



SOLUTION

$$\begin{aligned}
 \theta = 45^\circ : \quad s_B/s_A &= 1 \\
 \theta = 30^\circ : \quad s_B/s_A &= \sqrt{3} \\
 \theta = 15^\circ : \quad s_B/s_A &= 3.73
 \end{aligned}$$

$$L = (s_P - s_A) + 2\sqrt{s_A^2 + s_B^2}$$

$$0 = -v_A + 2(s_A v_A + s_B v_B) / \sqrt{s_A^2 + s_B^2}$$

$$\Rightarrow v_B = \left[\sqrt{1 + (s_B/s_A)^2} - 2 \right] \frac{v_A}{2(s_B/s_A)}$$

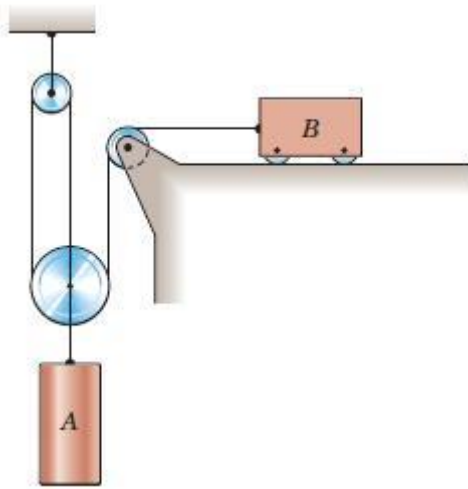
$$\theta = 45^\circ : v_B = -0.293 v_A = -0.293(-1) = \underline{0.293 \frac{m}{s}}$$

$$\theta = 30^\circ : v_B = \underline{0}$$

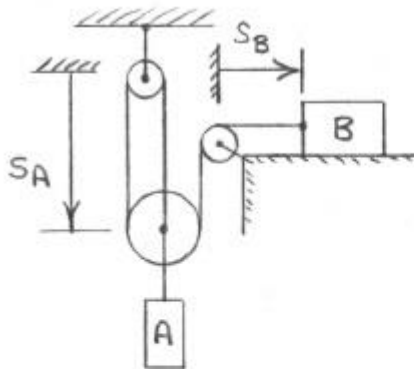
$$\theta = 15^\circ : v_B = 0.250 v_A = 0.250(-1) = \underline{-0.250 \frac{m}{s}}$$

Question Title: Chapter 2, Reserve Problem 2/091

If block *B* has a leftward velocity of 1.2 m/s, determine the velocity of cylinder *A* (positive if down, negative if up).



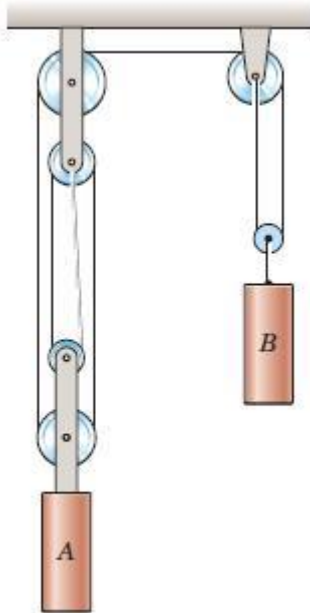
SOLUTION



$$\begin{aligned} \text{Length of cable } L &= s_B + 3s_A + \text{constants} \\ 0 &= v_B + 3v_A, \quad v_A = -\frac{v_B}{3} = -\left(\frac{-1.2}{3}\right) = 0.4 \frac{\text{m}}{\text{s}} \\ &\quad \text{(down)} \end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/092

Cylinder B has a downward velocity in feet per second given by $v_B = t^2/2 + t^3/6$, where t is in seconds. Calculate the acceleration of A when $t = 2$ sec. The acceleration of A is positive if downward, negative if upward.



SOLUTION

Total length of cable to within constants is

$$L = 4y_A + 2y_B$$

$$0 = 4\dot{y}_A + 2\dot{y}_B$$

$$0 = 4\ddot{y}_A + 2\ddot{y}_B$$

Upward accel. of A is

$$a_A = -\ddot{y}_A = \frac{1}{2}\ddot{y}_B = \frac{1}{2}a_B$$

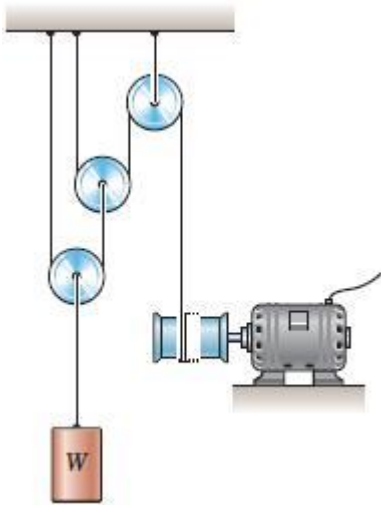
$$v_B = t^2/2 + t^3/6, \quad a_B = \dot{v}_B = t + t^2/2$$

For $t = 2$ sec, $a_B = 2 + 4/2 = 4 \text{ ft/sec}^2$

Thus $a_A = \frac{1}{2}a_B = \underline{2 \text{ ft/sec}^2}$

Question Title: Chapter 2, Reserve Problem 2/093

Determine the vertical rise h of the load W during 5 seconds if the hoisting drum wraps cable around it at the constant rate of 320 mm/s.



SOLUTION

Let A be a point on cable 1:

$$L_1 = s_A + 2s_B$$

$$0 = v_A + 2v_B \quad (1)$$

$$L_2 = s_C + (s_C - s_B)$$

$$0 = 2v_C - v_B \quad (2)$$

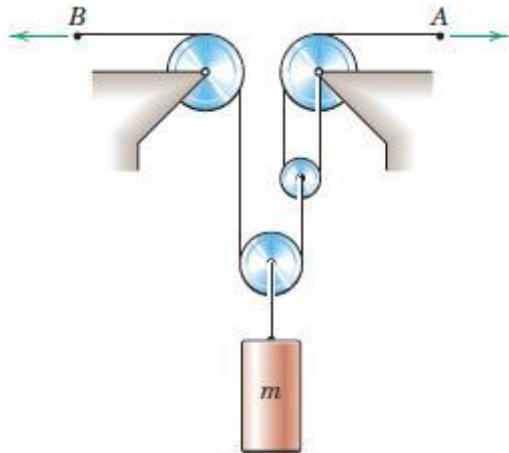
Combine (1) & (2) to obtain

$$v_C = -\frac{1}{4}v_A = -\frac{1}{4}(320) = -80 \text{ mm/s}$$

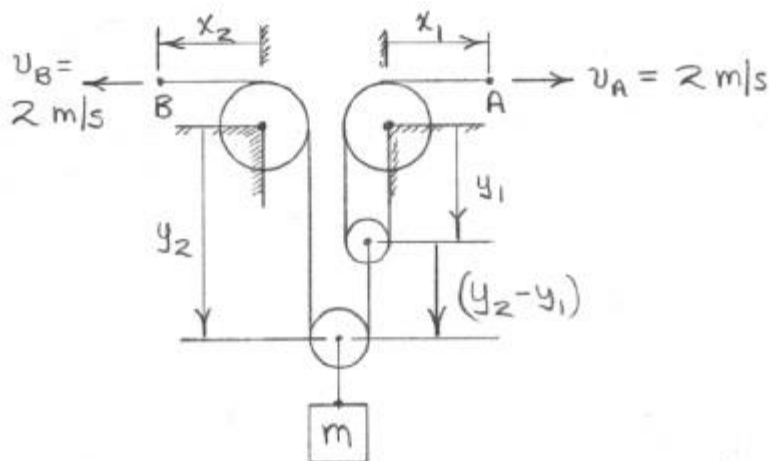
So Q (and W) rises $h = 80(5) = \underline{400 \text{ mm}}$

Question Title: Chapter 2, Reserve Problem 2/094

For the pulley system shown, each of the cables at A and B is given a velocity of 2 m/s in the direction of the arrow. Determine the upward velocity v of the load m .



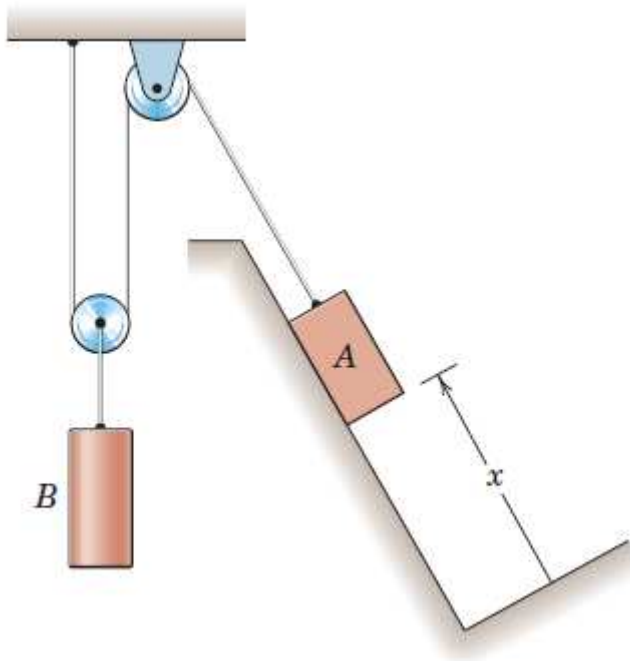
SOLUTION



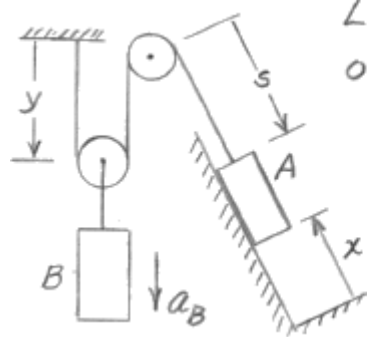
$$\begin{aligned}
 x_1 + 2y_1 &= \text{constant}; \quad \dot{x}_1 + 2\dot{y}_1 = 0, \quad \dot{y}_1 = -\frac{\dot{x}_1}{2} \\
 x_2 + y_2 + (y_2 - y_1) &= \text{constant}; \quad \dot{x}_2 + 2\dot{y}_2 - \dot{y}_1 = 0 \\
 \dot{x}_2 + 2\dot{y}_2 - \left(-\frac{\dot{x}_1}{2}\right) &= 0 \\
 \dot{x}_2 + 2\dot{y}_2 + \frac{\dot{x}_1}{2} &= 0, \quad \dot{y}_2 = -\frac{\dot{x}_2}{2} - \frac{\dot{x}_1}{4} = -\frac{2}{2} - \frac{2}{4} = -1.5 \text{ m/s} \\
 &\text{or } \underline{1.5 \text{ m/s up}}
 \end{aligned}$$

Question Title: Chapter 2, Reserve Problem 2/095

If the velocity $\dot{x}$ of block A up the incline is increasing at the rate of 0.044 m/s each second, determine the acceleration of B. The acceleration of B is positive if up and negative if down.



SOLUTION



$$L = 2y + s + \text{const.}$$

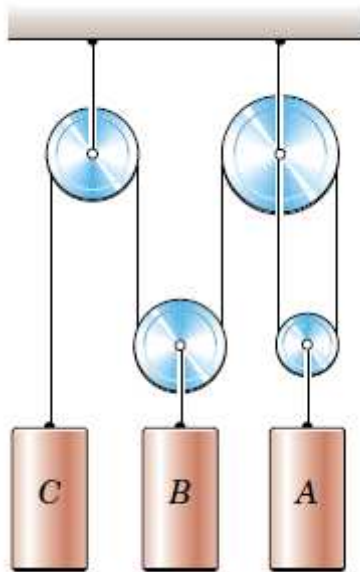
$$0 = 2\dot{y} + \dot{s}, \quad 0 = 2\ddot{y} + \ddot{s}$$

$$a_B = \ddot{y} = -\frac{\ddot{s}}{2} = +\frac{\ddot{x}}{2}$$

$$a_B = \frac{0.044}{2} = \underline{0.022 \text{ m/s}^2}$$

Question Title: Chapter 2, Reserve Problem 2/096 (Multistep version available)

Determine the relation which governs the accelerations of A, B, and C, all measured positive down. Identify the number of degrees of freedom.



SOLUTION

Length of cable $L = 2y_A + 2y_B + y_C + \text{const.}$

$$0 = 2\dot{y}_A + 2\dot{y}_B + \dot{y}_C$$

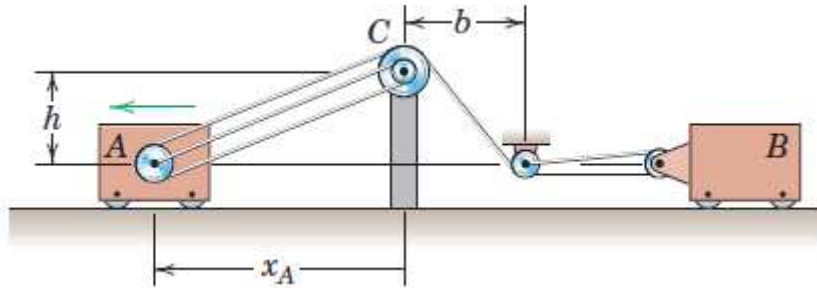
$$0 = 2\ddot{y}_A + 2\ddot{y}_B + \ddot{y}_C$$

so $\underline{2a_A + 2a_B + a_C = 0}$

2 degrees of freedom

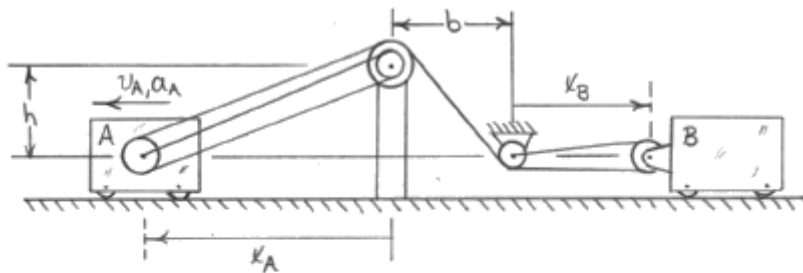
Question Title: Chapter 2, Reserve Problem 2/097

Cart A has a leftward velocity v_A and a leftward acceleration a_A at the instant represented. Find the velocity and acceleration of cart B (both positive if to the right, negative if to the left). The position of cart A is x_A , the distance h , and the distance b . Neglect the diameters of the pulleys and assume that there is no mechanical interference. The two pulleys at C are pivoted independently.



SOLUTION

This solution provides a guideline to solving the problem, not the numerical answer.



$$L = 3\sqrt{x_A^2 + h^2} + 2x_B + \text{CONSTANTS}$$

$$\dot{L} = 0 = 3 \left[\frac{1}{2} (x_A^2 + h^2)^{-1/2} (2x_A \dot{x}_A) \right] + 2\dot{x}_B \rightarrow v_B = \frac{-3x_A v_A}{2\sqrt{x_A^2 + h^2}}$$

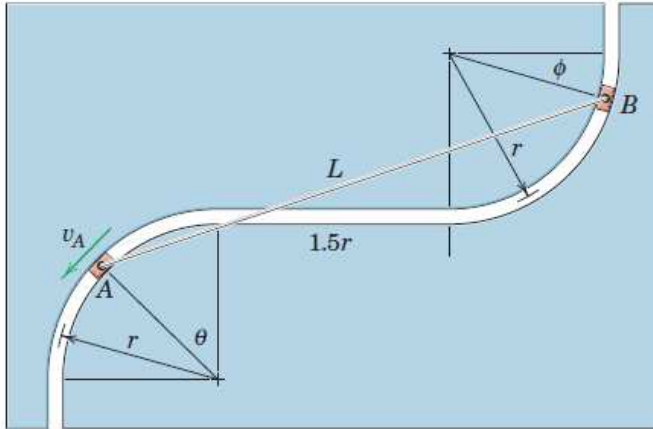
$$a_B = \frac{dv_B}{dt} = -\frac{3\dot{x}_A v_A + 3x_A \dot{v}_A}{2\sqrt{x_A^2 + h^2}} - \frac{3x_A v_A}{2} \left[-\frac{1}{2} (x_A^2 + h^2)^{-3/2} (2x_A \dot{x}_A) \right]$$

$$a_B = -\frac{3(v_A^2 + x_A a_A)}{2\sqrt{x_A^2 + h^2}} + \frac{3x_A^2 v_A^2}{2(x_A^2 + h^2)^{3/2}}$$

A MINUS VALUE INDICATES A LEFTWARD DIRECTION.

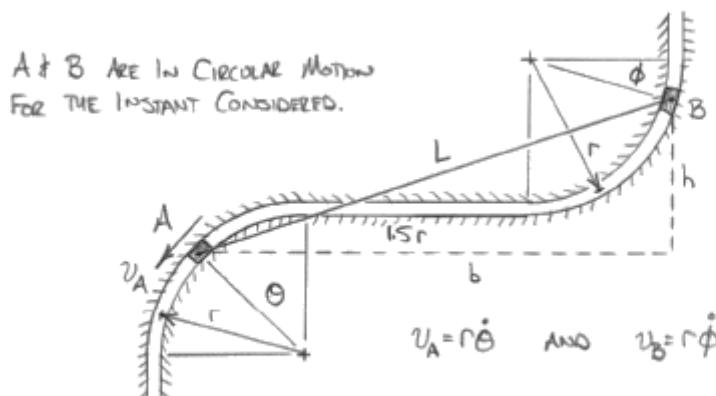
Question Title: Chapter 2, Reserve Problem 2/098

The two sliders are connected by the light rigid bar and move in the smooth vertical-plane guide. At the instant illustrated, the speed of slider A is 25 mm/s, $\theta = 45^\circ$, and $\phi = 15^\circ$. Determine the speed of slider B for this instant if $r = 175$ mm.



SOLUTION

$$v_A = 25 \text{ mm/s} \quad r = 175 \text{ mm} \quad \theta = 45^\circ \quad \phi = 15^\circ$$



$$L^2 = b^2 + h^2 = (r\cos\phi + r\sin\theta + 1.5r)^2 + [(r - r\sin\phi) + (r - r\cos\theta)]^2$$

DIFFERENTIATE BOTH SIDES WRT TIME t

$$2L\frac{dL}{dt} = 2(r\cos\phi + r\sin\theta + 1.5r)(r\dot{\theta}\cos\theta - r\dot{\phi}\sin\phi) + 2(2r - r\sin\phi - r\cos\theta)(-r\dot{\phi}\cos\phi + r\dot{\theta}\sin\theta)$$

$$0 = (r\cos\phi + r\sin\theta + 1.5r)(v_A\cos\theta - v_B\sin\phi) + (2r - r\sin\phi - r\cos\theta)(v_A\sin\theta - v_B\cos\phi)$$

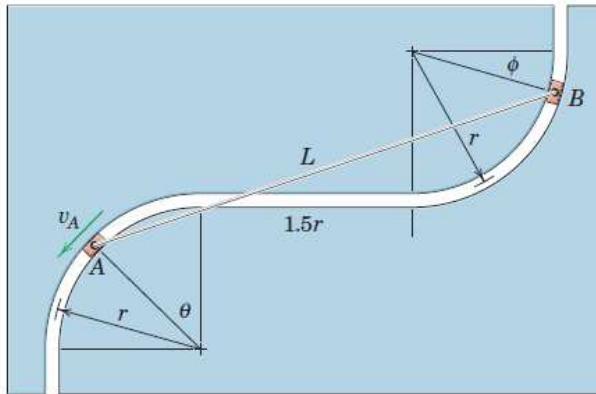
WITH NUMBERS...

$$0 = (175\cos 15^\circ + 175\sin 45^\circ + 1.5(175))(25\cos 45^\circ - v_B\sin 15^\circ) + (2(175) - 175\sin 15^\circ - 175\cos 45^\circ)(25\sin 45^\circ - v_B\cos 15^\circ)$$

$$\therefore \underline{v_B = 40.9 \text{ mm/s}}$$

Question Title: Chapter 2, Reserve Problem 2/099

The two sliders are connected by the light rigid bar and move in the smooth vertical-plane guide. At the instant illustrated, the speed of slider A is 25 mm/s, $\theta = 45^\circ$, and $\phi = 15^\circ$. Determine the time rate of change of speed for slider B if the speed of slider A is constant over a short interval which includes the position shown. The distance $r = 175$ mm.



SOLUTION

$v_A = 25 \text{ mm/s (CONST)}$ $r = 175 \text{ mm}$ $\theta = 45^\circ$ $\phi = 15^\circ$

From Prob. 2/227

$$\begin{cases} v_A = r\dot{\theta} \\ v_B = r\dot{\phi} \end{cases} \quad \begin{cases} a_{A_t} = 0 = r\ddot{\theta} \\ a_{B_t} = r\ddot{\phi} \end{cases} \quad \begin{cases} a_{A_n} = r\dot{\theta}^2 = \frac{v_A^2}{r} \\ a_{B_n} = r\dot{\phi}^2 = \frac{v_B^2}{r} \end{cases}$$

$$0 = (r\cos\phi + r\sin\theta + 1.5r)(r\dot{\theta}\cos\theta - r\dot{\phi}\sin\phi) + (2r - r\sin\phi - r\cos\theta)(r\dot{\theta}\sin\theta - r\dot{\phi}\cos\phi)$$

so... $\ddot{\theta} = 0$

$$\begin{aligned} 0 &= r^2\ddot{\theta}\cos\theta\cos\phi - r^2\dot{\phi}\sin\phi\cos\phi + r^2\ddot{\theta}\sin\theta\cos\theta - r^2\dot{\phi}\sin\theta\sin\phi + 1.5r^2\ddot{\theta}\cos\theta \\ &\quad - 1.5r^2\dot{\phi}\sin\phi + 2r^2\ddot{\theta}\sin\theta - 2r^2\dot{\phi}\cos\phi - r^2\ddot{\theta}\sin\theta\sin\phi + r^2\dot{\phi}\sin\phi\cos\phi \\ &\quad - r^2\ddot{\theta}\sin\theta\cos\theta + r^2\dot{\phi}\cos\theta\cos\phi \\ 0 &= r^2\ddot{\theta}\cos\theta\cos\phi - r^2\dot{\phi}\sin\theta\sin\phi + 1.5r^2\ddot{\theta}\cos\theta - 1.5r^2\dot{\phi}\sin\phi + 2r^2\ddot{\theta}\sin\theta - 2r^2\dot{\phi}\cos\phi \\ &\quad - r^2\ddot{\theta}\sin\theta\sin\phi + r^2\dot{\phi}\cos\theta\cos\phi \end{aligned}$$

DIFFERENTIATE WRT TIME, t

$$\begin{aligned} 0 &= r^2\ddot{\theta}'\cos\theta\cos\phi - r^2\ddot{\theta}^2\sin\theta\cos\phi - r^2\dot{\theta}\dot{\phi}\cos\theta\sin\phi - r^2\dot{\phi}^2\sin\theta\sin\phi \\ &\quad - r^2\ddot{\theta}\dot{\phi}\cos\theta\sin\phi - r^2\dot{\phi}^2\sin\theta\cos\phi + 1.5r^2\ddot{\theta}'\cos\theta - 1.5r^2\ddot{\theta}^2\sin\theta - 1.5r^2\dot{\theta}\dot{\phi}\sin\phi \\ &\quad - 1.5r^2\dot{\phi}^2\cos\phi + 2r^2\ddot{\theta}'\sin\theta + 2r^2\ddot{\theta}^2\cos\theta - 2r^2\dot{\theta}\dot{\phi}\cos\phi + 2r^2\dot{\phi}^2\sin\phi \\ &\quad - r^2\ddot{\theta}'\sin\theta\sin\phi - r^2\ddot{\theta}^2\cos\theta\sin\phi - r^2\dot{\theta}\dot{\phi}\sin\theta\cos\phi + r^2\dot{\phi}^2\cos\theta\cos\phi \\ &\quad - r^2\ddot{\theta}\dot{\phi}\sin\theta\cos\phi - r^2\dot{\phi}^2\cos\theta\sin\phi \end{aligned}$$

$$\begin{aligned}
 0 = & -r^2 \dot{\theta}^2 (\sin \theta \cos \phi + \cos \theta \sin \phi + 1.5 \sin \theta - 2 \cos \theta) \\
 & + r^2 \ddot{\phi} (\cos \theta \cos \phi - \sin \theta \sin \phi - 1.5 \sin \phi - 2 \cos \phi) \\
 & - 2r^2 \dot{\theta} \dot{\phi} (\cos \theta \sin \phi + \sin \theta \cos \phi) \\
 & - r^2 \dot{\phi}^2 (\sin \theta \cos \phi + \cos \theta \sin \phi + 1.5 \cos \phi - 2 \sin \phi)
 \end{aligned}$$

$$\begin{aligned}
 0 = & -v_A^2 (\sin \theta \cos \phi + \cos \theta \sin \phi + 1.5 \sin \theta - 2 \cos \theta) - 2v_A v_B (\cos \theta \sin \phi + \sin \theta \cos \phi) \\
 & + r a_{B_t} (\cos \theta \cos \phi - \sin \theta \sin \phi - 1.5 \sin \phi - 2 \cos \phi) - v_B^2 (\sin \theta \cos \phi + \cos \theta \sin \phi + 1.5 \cos \phi - 2 \sin \phi)
 \end{aligned}$$

$$\begin{aligned}
 0 = & -25^2 (\sin 45^\circ \cos 15^\circ + \cos 45^\circ \sin 15^\circ + 1.5 \sin 45^\circ - 2 \cos 45^\circ) \\
 & - 2(25)(40.9) (\cos 45^\circ \sin 15^\circ + \sin 45^\circ \cos 15^\circ) \\
 & + 175 a_{B_t} (\cos 45^\circ \cos 15^\circ - \sin 45^\circ \sin 15^\circ - 1.5 \sin 15^\circ - 2 \cos 15^\circ) \\
 & - 40.9^2 (\sin 45^\circ \cos 15^\circ + \cos 45^\circ \sin 15^\circ + 1.5 \cos 15^\circ - 2 \sin 15^\circ)
 \end{aligned}$$

$$\therefore \underline{a_{B_t} = -15.98 \text{ mm/s}^2 = \dot{v}_B}$$