Introduction to Thermal and Fluids Engineering 1st edition Kaminski Solutions Manual

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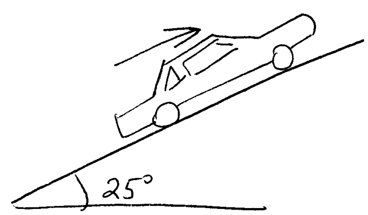
2-1 A 2000-kg car accelerates from 20 to 60 km/h on an uphill road. The car travels 120 m and the slope of the road from the horizontal is 25°. Determine the work done by the engine.

Approach:

Apply the first law choosing the car to be the system under consideration. Eliminate all terms except kinetic energy, potential energy, and work.



Assumptions:

1. The car is isothermal.
2. No heat is transferred to or from the car.

Solution:

Consider the car to be the system. From the first law

$$\Delta E = Q - W \qquad \text{or} \qquad \Delta KE + \Delta PE + \Delta U = Q - W$$

The car does not change temperature, so $\Delta U = 0$. No heat is transferred, so $Q = 0$ and the first law becomes

$$\Delta KE + \Delta PE = -W$$

$$(KE_2 - KE_1) + (PE_2 - PE_1) = -W$$

Using expressions for kinetic and potential energy

$$W = \frac{1}{2}m\left(V_1^{\,2} - V_2^{\,2}\right) + mg(z_1 - z_2)$$

Let $z_1 = 0$ $\quad\therefore z_2 = (120\text{m})(\sin 25^0) = 50.7\text{m}$

$$W = \frac{1}{2}(2000\,\text{kg})\left\{20^2 - 60^2\right\}\left(\frac{\text{km}}{\text{h}}\right)^2\left(\frac{1000\,\text{m}}{1\,\text{km}}\right)^2\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)^2 + (2000\,\text{kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)(50.7\,\text{m})$$

$$= -1.24\times10^6\ \text{J} = -1240\ \text{kJ}$$

The system is the car. Work done <u>on</u> a system is negative. The work done <u>by</u> the engine is

$W = 1240\ \text{kJ}$ ⟵ **Answer**

2-2 A missile is launched vertically upward from the surface of the earth with an initial velocity of 350 m/s. If the missile mass is 1200 kg, calculate the maximum height the missile will attain. Assume no aerodynamic drag or other work during the flight and no heat transfer.

Approach:

Write the first law for a closed system and eliminate all terms except kinetic and potential energy changes.

Assumptions:

1. The missile does not exchange heat with the surroundings.
2. Aerodynamic drag is negligible.
3. The missile is isothermal.

Solution:

From the first law

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

With no work, heat transfer, or change in internal energy, this becomes

$$\Delta KE + \Delta PE = 0$$

$$\frac{1}{2}m\left(V_2^{\,2} - V_1^{\,2}\right) + mg(z_2 - z_1) = 0$$

Setting $z_1 = 0$ and solving for z_2

$$z_2 = \frac{V_1^{\,2} - V_2^{\,2}}{2g} = \frac{\left(350\,\frac{\text{m}}{\text{s}}\right)^2 - 0}{2\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)} = 6244\,\text{m}$$

Answer

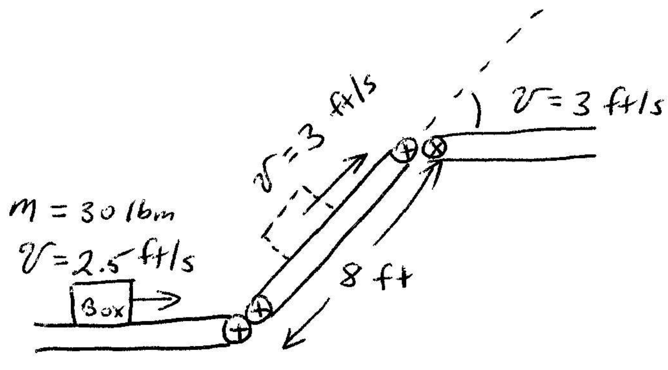
Comment:

Mass is not needed in the calculation.

2-3 A system of conveyor belts is used to transport a box of 30 lbm, as shown in the figure. Note that the inclined belt and the upper belt travel at a faster speed than the lower belt. Calculate the work done by the motor which drives the inclined belt. Neglect all friction.

Approach:

Apply the first law choosing the box to be the system under consideration. Eliminate all terms except kinetic energy, potential energy, and work.



Assumptions:

1. The box is isothermal.
2. No heat is transferred to or from the box.
3. The conveyor belts operate in steady state.
4. Frictional effects are negligible

Solution:

Consider the box to be the system. From the first law

$$\Delta E = Q - W$$

or

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

The box does not change temperature, thus $\Delta U = 0$. No heat is transferred, so $Q = 0$, and the first law reduces to

$$\Delta KE + \Delta PE = -W$$

$$(KE_2 - KE_1) + (PE_2 - PE_1) = -W$$

$$W = \frac{1}{2} m \left(V_1^{\,2} - V_2^{\,2} \right) + mg(z_1 - z_2)$$

where 1 is the bottom of the belt and 2 is the top. Let $z_1 = 0$. Then

$$z_2 = (8\,\text{ft})\sin(20^\circ) = 2.74\,\text{ft}$$

$$W = \frac{1}{2}(30\,\text{lbm})\left\{(2.5)^2 - (3)^2\right\}\left(\frac{\text{ft}}{\text{s}}\right)^2 \left(\frac{1\,\text{lbf}}{32.17\,\dfrac{\text{lbm}\cdot\text{ft}}{\text{s}^2}} \right) + (30\,\text{lbm})\left(32.17\,\frac{\text{ft}}{\text{s}^2}\right)(-2.74\text{ft})\left(\frac{1\,\text{lbf}}{32.17\,\dfrac{\text{lbm}\cdot\text{ft}}{\text{s}^2}} \right)$$

$$W = -83.4\ \text{ft}\cdot\text{lbf}$$

The system is the box. Work done on a system is negative. The work done by the motor is

$W = 83.4$ ft·lbf ⟵ **Answer**

2-4 In a front-wheel-drive car, 60% of the braking energy is dissipated in the front wheels and 40% is dissipated in the rear. If a car with a mass of 2650 lbm is decelerated from 60 mph to 15 mph on level ground by braking, calculate the energy dissipated in each front wheel (in Btu). Neglect aerodynamic drag and rolling resistance.

Approach:

Write the first law for a closed system and eliminate all terms except kinetic and internal energy changes. The energy dissipated in the brakes may be expressed as a rise in internal energy.

Assumptions:

1. The car does not exchange heat with the surroundings.
2. Aerodynamic drag is negligible.
3. Rolling resistance is negligible

Solution:

The system is the car. From the first law

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

The car moves on level ground, so $\Delta PE = 0$. There is no drag or rolling resistance, so $W = 0$. The work of the brake pads on the wheels is <u>internal</u> to the system and is not included. Assume that in the short time frame, no heat, Q, leaves the vehicle. (This is a conservative assumption, which will result in the highest estimate of brake pad temperature). Therefore

$$\Delta KE + \Delta U = 0$$

$$\Delta U = -\Delta KE = \frac{1}{2} m\left(V_1^{\,2} - V_2^{\,2}\right)$$

$$\Delta U = \frac{1}{2}(2650\,\text{lbm})\left\{(60)^2 - (15)^2\right\}\left(\frac{\text{mi}}{\text{h}}\right)^2 \left(\frac{1.47\,\dfrac{\text{ft}}{\text{s}}}{1\,\dfrac{\text{mi}}{\text{h}}}\right)^2 \left(\frac{1\ \text{lbf}}{32.17\ \dfrac{\text{lbm}\cdot\text{ft}}{\text{s}^2}}\right)\left(\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right)$$

$$\Delta U = 386\ \text{Btu}$$

Of this internal energy, 60% is dissipated in the front wheels; therefore, each front wheel receives 30%. The final solution is

$$\Delta U_{\text{wheel}} = (0.3)(386)\,\text{Btu} = 116\,\text{Btu} \longleftarrow \textbf{Answer}$$

2-5 A mass of 1200 kg of fish at 20 °C is to be frozen solid at –20 °C. The freezing point of the fish is –2.2 °C and the specific heats above and below the freezing point are 3.2 and 1.7 kJ/kg·K, respectively. The heat of fusion (the amount of heat needed to freeze 1 kg of fish) is 235 kJ/kg. Find the heat transferred.

Approach:

Write the first law for a closed system and eliminate all terms except heat and internal energy. Calculate the rise in internal energy as the sensible heat needed to reach the freezing point, the latent heat needed to freeze the fish and the sensible heat needed to lower the fish to the final temperature.

Assumptions:

1. Kinetic and potential energy are negligible.
2. No work is done on the fish.
3. The specific heats are constant.

Solution:

The first law is

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

In this case, there is no kinetic energy, potential energy, or work. Therefore

$$\Delta U = Q$$

The change in internal energy is calculated in 3 parts: from 20°C to -2.2°C, from unfrozen to frozen, and from -2.2°C to -20°C. Thus

$$mc_1\Delta T_1 + mh_{fs} + mc_2\Delta T_2 = Q$$

where h_{fs} is the latent heat of fusion of the fish. Substituting values

$$Q = (1200\,\text{kg})\left\{\left(3.2\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left[20\text{-(-2.2)}\right]{}^\circ\text{C} + 235\frac{\text{kJ}}{\text{kg}} + \left(1.7\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left[-2.2-(-20)\right]{}^\circ\text{C}\right\}$$

$$= 4.04\times10^5\ \text{kJ} \longleftarrow \textbf{Answer}$$

Comments:

This would take a big freezer.

2-6 A steel bar initially at 1000°F is quenched by immersion in a bath of liquid water initially at 70°F. The mass of the bar is 2.5 lbm and the volume of the water is 7 ft^3. Heat is transferred from the bath to the surroundings, which are at 70°F. After some time, the bar and water reach an equilibrium temperature of 70°F. Find the heat transferred. (For the steel, use c = 0.106 Btu/lbm·R.)

Approach:

Select the combination of the bar and the liquid as the closed system and apply the first law. All terms except heat and internal energy may be eliminated.

Assumptions:

1. Kinetic and potential energy are negligible.
2. No work is done on the bar or water.
3. The specific heats are constant.

Solution:

Take the bar and water together to be the system. From the first law

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

In this process, there is no change in kinetic energy or potential energy and no work is done. Thus

$$\Delta U = Q$$

$$m_b c_b (T_{b2} - T_{b1}) + m_w c_w (T_{w2} - T_{w1}) = Q$$

where b refers to the bar and w refers to the water. The water begins at 70°F, rises in temperature due to heat transfer from the hot bar, and then cools back to 70°F. As a result there is no <u>net</u> change in the water temperature and $T_{w1} = T_{w2}$. Then

$$Q = m_b c_b (T_{b2} - T_{b1})$$

$$Q = (2.5\,\text{lbm})\left(0.106\frac{\text{Btu}}{\text{lbm}\cdot\text{R}}\right)(70-1000)^\circ\text{F}$$

$$Q = -247\,\text{Btu}$$ ⟵ **Answer**

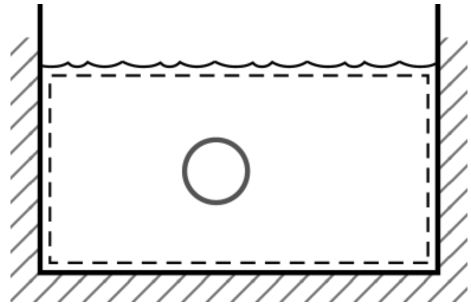
Comments:

Q is negative because the system (water and bar) is being cooled.

2-7 A 0.14 lbm aluminum ball at 400°F is dropped into a water bath at 70°F. The bath contains 0.52 ft^3 of water and is well-insulated. What is the final temperature of the ball after the ball and water reach equilibrium?

Approach:

Apply the first law choosing the ball and water to be the system under consideration. Eliminate all terms except enthalpy change and heat.



Assumptions:

1. The process occurs at constant pressure.
2. The container is well-insulated.
3. Specific heats are constant
4. Kinetic and potential energy changes are negligible.
5. No work is done on or by the system.

Solution:

Take the ball and water together to be the system. The process occurs at constant pressure; therefore, from the first law

$$Q = \Delta H$$

Since the bath is well-insulated on the outside,

$$0 = \Delta H$$

The total enthalpy change is the sum of the enthalpy changes of the ball and water:

$$O = \Delta H_b + \Delta H_w = m_b \Delta h_b + m_w \Delta h_w$$

Assuming constant specific heat

$$O = m_b c_{pb} \Delta T_b + m_w c_{pw} \Delta T_w = m_b c_{pb} \left(T_{b_2} - T_{b_1}\right) + m_w c_{pw} \left(T_{2w} - T_{1w}\right)$$

Both the ball and the water reach the same final temperature, T_2, at equilibrium, so

$$O = m_b c_{pb} \left(T_2 - T_{b1}\right) + m_w c_{pw} \left(T_2 - T_{1w}\right)$$

The mass of water is calculated from the density and volume so that

$$O = m_b c_{pb} \left(T_2 - T_{b1}\right) + \rho_w V_w c_{pw} \left(T_2 - T_{1w}\right)$$

Rearranging

$$T_2 = \frac{m_b c_{pb} T_{b1} + \rho_w V_w c_{pw} T_{1w}}{m_b c_{pb} + \rho_w V_w c_{pw}}$$

Using specific heat data in Tables B-2 and B-6,

$$T_2 = \frac{(0.14\,\text{lbm})\left(0.216\dfrac{\text{Btu}}{\text{lbm}\cdot\text{R}}\right)(400^\circ\text{F}) + \left(62.2\dfrac{\text{lbm}}{\text{ft}^3}\right)(0.52\,\text{ft}^3)\left(0.998\dfrac{\text{Btu}}{\text{lbm}\cdot{}^\circ\text{F}}\right)(70^\circ\text{F})}{(0.14\,\text{lbm})\left(0.216\dfrac{\text{Btu}}{\text{lbm}\cdot\text{R}}\right) + \left(62.2\dfrac{\text{lbm}}{\text{ft}^3}\right)(0.52\,\text{ft}^3)\left(0.998\dfrac{\text{Btu}}{\text{lbm}\cdot{}^\circ\text{F}}\right)}$$

$T_2 = 70.3^\circ\text{F}$ ⟵ **Answer**

2-8 In a new process, a thin metal film is produced when very high velocity particles strike a surface, melt, and adhere to the surface. Imagine an aluminum particle with a diameter of 40 µm (1 µm = 10^{-6} m) at a temperature of 20°C. The particle strikes a cold aluminum surface, also at 20°C. The particle energy is just high enough so that the particle and a portion of the surface with the same mass as the particle completely melt. What is the velocity of the particle? Assume pure aluminum with a constant specific heat of 1146 J/kg·K. The latent heat of fusion (the amount of heat needed to melt 1 kg of aluminum) is 404 kJ/kg.

Approach:

Write the first law for a constant pressure process of a closed system and eliminate all terms except the kinetic energy change and the enthalpy change. Include both sensible and latent heat of fusion in evaluating the enthalpy term.

Assumptions:

1. There is no potential energy change.
2. There is no heat transfer from the surface during the process.
3. Specific heat is constant.

Solution:

Choose the particle and the part of the surface which melts as the control volume. The process begins just before the particle hits the surface and ends as the particle is brought to a complete halt and the aluminum has melted. The process occurs at constant pressure, therefore the first law may be written in the form

$$\Delta KE + \Delta PE + \Delta H = Q$$

We assume no change in potential energy and no heat transfer to the air from the surface, therefore:

$$\Delta KE + \Delta H = 0$$

The enthalpy change has two components: the enthalpy rise as the particle temperature changes from its initial value to the melting point of aluminum and the enthalpy rise as the solid aluminum and the particle melt. Thus

$$\frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2 + 2mc_p\left(T_2 - T_1\right) + 2m\,h_{fs} = 0$$

where h_{fs} is the latent heat of fusion of aluminum. The final velocity, V_2 is zero. Solving for V_1

$$V_1 = \sqrt{4c_p\left(T_2 - T_1\right) + 4h_{fs}} = 2\sqrt{c_p\left(T_2 - T_1\right) + h_{fs}}$$

Substituting the melting temperature of aluminum from Table A-2

$$V_1 = 2\sqrt{\left(1146\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(933 - 273 - 20\right)\text{K} + 404\frac{\text{kJ}}{\text{kg}}\left(\frac{1000\,\text{J}}{1\,\text{kJ}}\right)}$$

$$V_1 = 2133\,\text{m/s} \longleftarrow \textbf{Answer}$$

2-9 An object weights 40 N on a space station which has an artificial gravitational acceleration of 5 m/s^2. What is the weight of the object on Earth?

Approach:

Use Newton's second law to determine the mass of the object knowing its weight on the space station. Use this mass to find the weight on the surface of the earth.

Assumptions:

none

Solution:

On the space station, by Newton's second law,

$$F_1 = mg_1$$

Mass is the same on the space station and on earth. Therefore, on Earth

$$F_2 = mg_2$$

Eliminating mass between the last two equations,

$$\frac{F_1}{g_1} = \frac{F_2}{g_2}$$

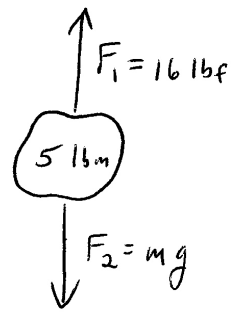
$$F_2 = \frac{g_2 F_1}{g_1} = \frac{\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(40\,\text{N})}{5\,\frac{\text{m}}{\text{s}^2}}$$

$$F_2 = 78.5\,\text{N} \longleftarrow$$ **Answer**

2-10 A mass of 5 lbm is acted on by an upward force of 16 lbf. The only additional force on the mass is the force of gravity. Find the acceleration in ft/s^2. Is this acceleration up or down?

Approach:

Draw a free body diagram of the mass included both upward and downward forces. Find the net force and use it in Newton's second law to determine the acceleration.



Assumptions:

None

Solution:

The weight force on the object is, by Newton's second law,

$$F_2 = mg \quad = (5 \text{ lbm})\left(32.17 \frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\,\text{lbf}}{32.17 \dfrac{\text{lbm}\cdot\text{ft}}{\text{s}^2}}\right) = 5 \text{ lbf}$$

The net force is

$$F_{net} = F_1 - F_2 = 16 - 5 = 11 \text{ lbf}$$

Since F_1 is greater than F_2, the net force is upward. To find acceleration, use Newton's second law

$$F = ma$$

Rearranging

$$a = \frac{F}{m} = \frac{11 \text{ lbf}}{(5 \text{ lbm})\left(\dfrac{1 \text{ lbf}}{32.17\,\text{lbm}\cdot\text{ft/s}^2}\right)}$$

$$a = 70.8 \frac{\text{ft}}{\text{s}^2} \quad \text{upward} \longleftarrow$$ **Answer**

2-11 An airplane of mass 18,300 kg travels at 500 mph through the atmosphere. Calculate the kinetic energy of the plane in kJ.

Approach:

Use the formula for the kinetic energy of a moving object and convert units.

Assumptions:

none

Solution:

Kinetic energy is given by

$$KE = \frac{1}{2} m V^2$$

Substitute values and convert miles per hour to m/s to obtain

$$KE = \frac{1}{2}(18300\,\text{kg})(500\,\text{mph})^2 \left(\frac{1\frac{\text{m}}{\text{s}}}{2.237\,\text{mph}} \right)^2$$

Since kg, m, and s are all SI units, the result will be in J.

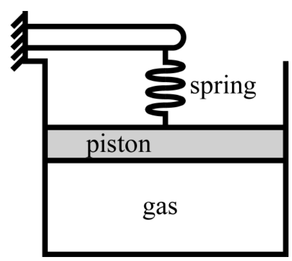
$$KE = 4.57 \times 10^8\ \text{J}$$

$$KE = 4.57 \times 10^5\ \text{kJ} \longleftarrow \textbf{Answer}$$

2-12 A gas is contained in a piston-cylinder assembly as shown in the figure below. A compressed spring exerts a force of 60 N on the top of the piston. The mass of the piston is 4 kg, and the surface area is 35 cm^2. If atmospheric pressure is 95 kPa, what is the pressure of the gas in the cylinder?

Approach:

Draw a free body diagram for the piston, including atmospheric pressure forces, the pressure force due to the gas in the cylinder, the force exerted by the spring and the weight of the piston. In equilibrium, these forces sum to zero.



Assumptions:

none

Solution:

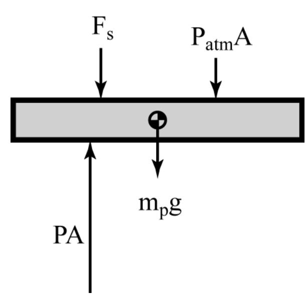
A force balance on the piston gives:

$$PA = P_{atm}A + F_s + m_p g$$

Solving for gas pressure

$$P = P_{atm} + \frac{F_s}{A} + \frac{m_p g}{A}$$

Substituting values



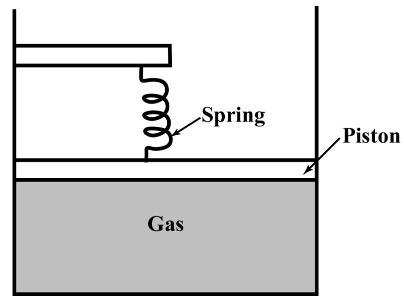
$$P = 95\text{kPa} + \left(\frac{60\text{N}}{35\text{cm}^2}\right)\left(\frac{100\text{cm}}{1\text{m}}\right)^2\left(\frac{1\text{kPa}}{1000\text{Pa}}\right) + \frac{(4\text{kg})(9.8\,\text{m/s}^2)}{3.5\,\text{cm}^2}\left(\frac{100\text{cm}}{1\text{m}}\right)^2\left(\frac{1\text{kPa}}{1000\text{Pa}}\right)$$

$$P = 123.3\,\text{kPa} \longleftarrow \textbf{Answer}$$

2-13 A gas is contained in a piston-cylinder assembly, as shown below. A compressed spring exerts a downward force on the piston. The spring is compressed 2 in. and the spring constant is 6.7 lbf/in. The piston is made of steel with a density of 490 lbm/ft^3 and a thickness of 0.5 in. The cylinder has a 7 in. diameter. Calculate the gage pressure of the gas in the tank.

Approach:

Perform a force balance on the piston, including atmospheric pressure forces, the pressure force due to the gas in the cylinder, the force exerted by the spring and the weight of the piston. In equilibrium, these forces sum to zero.



Assumptions:

none

Solution:

A force balance on the piston gives:

$$P_{abs}A = P_{atm}A + kx + mg$$

where P_{abs} is the absolute gas pressure, x is the length of compression of the spring, and k is the spring constant. The gage pressure is the difference between absolute and atmospheric pressure, so rearrange the above equation in the form

$$P_{abs} - P_{atm} = \frac{kx + mg}{A}$$

Using the fact that the mass of the piston is the density times the volume,

$$P_g = \frac{kx + mg}{A} = \frac{kx + \rho hAg}{A}$$

where h is the thickness of the piston and P_g is gage pressure. Simplifying

$$P_g = \frac{kx}{A} + \rho hg$$

Substituting values

$$P_g = \frac{\left(6.7\frac{\text{lbf}}{\text{in.}}\right)(2\text{in.})}{\pi(3.5\text{in.})^2} + \left(490\frac{\text{lbm}}{\text{ft}^3}\right)(0.5\text{in.})\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\frac{1\,\text{lbf}}{32.2\frac{\text{lbm}}{(\text{ft/s}^2)}}\left(\frac{1\ \text{ft}}{(12)^3\ \text{in.}^3}\right)$$

$P_g = 0.49$ psig ← **Answer**

2-14 Find the density of hydrogen at a pressure of 150 kPa and a temperature of 50°C.

Approach:
Use the ideal gas law.

Assumptions:
1. Hydrogen behaves like an ideal gas under these conditions.

Solution:
From the ideal gas law:

$$\rho = \frac{PM}{\bar{R}T}$$

The value of the molecular weight, M, is found in Table A-1. Substituting values and making unit conversions

$$\rho = \frac{(150\,\text{kPa})(2.016\,\text{kg/kmol})\left(\frac{1000\text{Pa}}{1\,\text{kPa}}\right)}{(8.314\,\text{kJ/kmol}\cdot\text{K})(50+273)\,\text{K}\left(\frac{1000\text{J}}{1\,\text{kJ}}\right)}$$

$$\rho = 0.113\,\text{kg/m}^3 \quad \longleftarrow \quad \textbf{Answer}$$

2-15 A pressurized nitrogen tank used on a paintball gun has a volume of 88 in^3. If the pressure of nitrogen is 4500 psia, calculate the mass of nitrogen in the tank. Assume a temperature of 70°F.

Approach:
Use the ideal gas law.

Assumptions:
1. Nitrogen behaves like an ideal gas under these conditions.

Solution:
From the ideal gas law:

$$m = \frac{MPV}{\bar{R}T}$$

The value of the molecular weight, M, is found in Table B-1. Substituting values and making unit conversions

$$m = \frac{\left(28.0\,\frac{\text{lbm}}{\text{lbmol}}\right)(4500\,\text{psia})(88\,\text{in.}^3)\frac{1\ \text{ft}^3}{(12)^3\ \text{in.}^3}}{\left(10.73\,\frac{(\text{psia})(\text{ft}^3)}{(\text{lbmol})(\text{R})}\right)(70+460)\,\text{R}}$$

$$m = 1.13\,\text{lbm} \quad \longleftarrow \quad \textbf{Answer}$$

2-16 Air is pumped from a vacuum chamber until the pressure drops to 3 torr. If the air temperature at the end of the pumping process is 5°C, calculate the air density. Eventually, the air temperature in the vacuum chamber rises to 20°C because of heat transfer with the surroundings. Assuming the volume is constant, find the final pressure, in torr.

Approach:

Use the ideal gas law to find the air density. Density remains constant during the heating process. The ideal gas law can be used again to find the final pressure.

Assumptions:

1. Air behaves like an ideal gas under these conditions.

Solution:

From the ideal gas law,

$$P_1 = \frac{\rho \bar{R} T_1}{M}$$

Solving for density

$$\rho = \frac{P_1 M}{\bar{R} T_1}$$

From Table A-1, $M = 28.97$ for air. Substituting values:

$$\rho = \frac{(3\,\text{torr})\left(\dfrac{0.133\,\text{kPa}}{1\text{ torr}}\right)\left(\dfrac{1000\,\text{Pa}}{1\,\text{kPa}}\right)\left(28.97\dfrac{\text{kg}}{\text{kmol}}\right)}{\left(8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)\left(\dfrac{1000\,\text{J}}{1\,\text{kJ}}\right)(5+273)\,\text{K}}$$

$$\rho = 0.005\frac{\text{kg}}{\text{m}^3}$$

Since all units have been converted to SI, the result for density is in the appropriate SI unit (kg/m^3). At the end of the heating

$$P_2 = \frac{\rho \bar{R} T_2}{M}$$

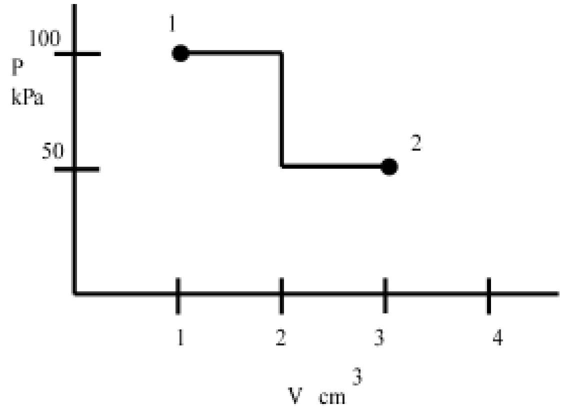
Density does not change during the heating, so

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$P_2 = P_1\frac{T_2}{T_1} = (3\,\text{torr})\left(\frac{20+237\,K}{5+237\,K}\right)$$

$$P_2 = 3.16\text{ torr} \longleftarrow \textbf{Answer}$$

2-17 Calculate the work, in joules, that is done in the quasi-equilibrium process from state 1 to state 2 shown in the figure.



Approach:

The work done is the area under the curve of *P* versus *V*. Use geometry to calculate this area.

Assumptions:

1. The process is quasi-equilibrium.

Solution:

The work for a quasi-equilibrium process is

$$W = \int P dV$$

The area under the curve from state 1 to state 2 is represented as

$$W = (100\text{kPa})(2\text{-}1)\text{cm}^3 + (50\text{kPa})(3\text{-}2)\text{cm}^3$$

$$= (150\ \text{kPa}\cdot\text{cm}^3)\left(\frac{1000\text{Pa}}{1\text{kPa}}\right)\left(\frac{1}{100}\right)^3\frac{\text{m}^3}{\text{cm}^3}$$

$W = 0.15$ J ← **Answer**

2-18 In a certain quasi-equilibrium process, pressure increases from 200 kPa to 350 kPa. The initial gas volume is 0.2 m^3. During the process, pressure varies with volume according to

$$(V-0.1)10^5 = (P-100)^2$$

where V is in m^3 and P is in kPa. Calculate the work done.

Approach:

Calculate the work as the integral of PdV. Use the initial and final volumes as the limits of the integral.

Assumptions:

1. The process proceeds through a set of equilibrium states.

Solution:

In a quasi-equilibrium process:

$$W = \int_{V_1}^{V_2} PdV$$

Solve the expression given in the problem statement for pressure to get

$$P = [(V-0.1)^{\frac{1}{2}}(10^5)^{\frac{1}{2}} + 100]\frac{1000\,\text{Pa}}{1\,\text{kPa}}$$

$$P = (V-0.1)^{\frac{1}{2}}10^{\frac{11}{2}} + 10^5$$

Substitute this expression into the equation for work

$$W = \int_{V_1}^{V_2}\left[(V-0.1)^{\frac{1}{2}}10^{\frac{11}{2}} + 10^5\right]dV$$

where V_1 and V_2 are the initial and final volumes. Performing the integration:

$$W = \left(10^{\frac{11}{2}}\right)\frac{2}{3}(V-0.1)^{\frac{3}{2}}\Big|_{V_1}^{V_2} + 10^5(V_2 - V_1)$$

$$W = \frac{2}{3}10^{\frac{11}{2}}\left[(V_2-0.1)^{\frac{3}{2}} - (V_1-0.1)^{\frac{3}{2}}\right] + 10^5(V_2 - V_1)$$

To find V_2, use the equation in the problem statement, bearing in mind that V is in m^3 and P is in kPa,

$$V = (P-100)^2 10^{-5} + 0.1$$

$$V_2 = (350-100)^2 10^{-5} + 0.1$$

$$= 0.725\,\text{m}^2$$

Work may now be evaluated as:

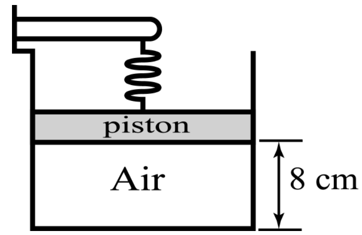
$$W = \frac{2}{3}10^{\frac{11}{2}}\left[(0.725-0.1)^{\frac{3}{2}} - (0.20-0.1)^{\frac{3}{2}}\right] + 10^5(0.725-0.20)$$

$$= 1.50\times10^5\,\text{J} = 150\,\text{kJ}$$ **Answer**

Comments:

In the evaluation of work, the SI units for V and P were used. Therefore the SI unit for work, J, will result.

2-19 Air is contained in a piston-cylinder assembly, as shown in the figure. The piston, which is assumed massless, is held in place by a spring. Initially, the spring is not compressed, and exerts no force on the piston. Then, the air is heated until the volume increases by 25%. The force exerted by the spring on the piston is $F = kx$, where k = 130 N/cm. The piston diameter is 6 cm. and the initial height of the piston is 8 cm. Calculate the amount of work done by the gas during this process. Assume atmospheric pressure is 101 kPa.



Approach:

Find an expression for the pressure of the gas as a function of x, the amount by which the spring is compressed. Use this expression to find work from

$$W = \int P dV$$

Assumptions:

1. The process is quasi-equilibrium.
2. The piston is massless
3. There is no friction between the piston and the cylinder.

Solution:

The pressure of the gas is

$$P(x) = P_{atm} + \frac{kx}{A}$$

Because the process is quasi-equilibrium,

$$W = \int P dV = \int PA\,dx = \int_{x_1}^{x_2}\left(P_{atm} + \frac{kx}{A}\right)A\,dx$$

Performing the integration and simplifying

$$W = P_{atm}A\left(x_2 - x_1\right) + k\left(\frac{x_2^{\,2} - x_1^{\,2}}{2}\right)$$

$$= (101\text{kPa})\left(\pi 3^2\right)\text{cm}^2\,(2\,\text{cm})\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)^3\left(\frac{1000\text{Pa}}{1\text{kPa}}\right) + \left(130\frac{\text{N}}{\text{cm}}\right)\left(\frac{2^2}{2}\right)\text{cm}^2\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)$$

$W = 8.31$ J ⟵ **Answer**

2-20 A propeller operating at 85 rpm applies a torque of 61 N·m. If the propeller has been rotating for 30 minutes, find the work done in kWh (kilowatt-hours).

Approach:

Use the expression for shaft work, integrating over time. Since both torque and speed are constant, they may be removed from the integral and the integration becomes trivial.

Assumptions:

1. Torque does not vary with time.
2. Rotational speed does not vary with time.

Solution:

For shaft work

$$W = \int \Im \omega dt = \Im \omega \int dt = \Im \omega \Delta t$$

Substituting values

$$W = (61\text{J})\left(85\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right)(30\,\text{min})\left(\frac{1\ \text{kWh}}{3.6\times10^3\ \text{kJ}}\right)\left(\frac{1\ \text{kJ}}{1000\text{J}}\right)$$

$$W = 0.2715\ \text{kWh}$$ ← **Answer**

2-21 A resistance heater is being used to heat a tank of nitrogen. If 3 amps are supplied to the resistor, which has a resistance of 60 Ω, how long will it take for 1200 J of work to be done?

Approach:

Use the expression for electrical work, integrating over time. Since both voltage and current are constant, they may be removed from the integral and the integration becomes trivial.

Assumptions:

1. Voltage does not vary with time.
2. Current does not vary with time.

Solution:

For electrical work

$$W = \int \xi i dt = \xi i \int dt = \xi i t$$

From Ohm's law

$$\xi = iR$$

Substituting gives

$$W = i^2 R t$$

Solving for time

$$t = \frac{W}{i^2 R} = \frac{1200\,\text{J}}{(3\,\text{A})^2(60\Omega)}$$

$$t = 2.22\ \text{s}$$ ← **Answer**

2-22 An electric motor operates in steady state at 1000 rpm for 45 minutes. The motor draws 8 amps at 110 volts and delivers a torque of 7.6 N·m. Find the total electrical energy input in kWh (kilowatt-hours) and the total shaft work produced in both kWh and Btu.

Approach:

Use the formula for electrical work to find the electrical energy input and the formula for shaft work to find shaft work.

Assumptions:

1. Current and voltage to the motor are constant.

Solution:

Electrical work is given by

$$W = \int \xi i \, \mathrm{d}t$$

Because this is a DC motor, current and voltage are constant and the integral is just

$$W = \xi i \, \Delta t = (110\,\mathrm{V})(8\,\mathrm{A})(45\,\mathrm{min})\left(\frac{1\,\mathrm{h}}{60\ \mathrm{min}}\right)\left(\frac{1\,\mathrm{kW}}{1000\ \mathrm{W}}\right)$$

$$= 0.66\,\mathrm{kWh}$$

Recall that volts times amps equals watts. For shaft work

$$W = \int \Im \omega \mathrm{d}t$$

Since speed and torque are constant,

$$W = \Im \omega \Delta t$$

$$= (7.6\,\mathrm{N}\cdot\mathrm{m})\left(1000\frac{\mathrm{rev}}{\mathrm{min}}\right)(45\,\mathrm{min})\left(\frac{2\pi\,\mathrm{rad}}{\mathrm{rev}}\right)\left(\frac{1\mathrm{J}}{1\,\mathrm{N}\cdot\mathrm{m}}\right)\left(\frac{1\,\mathrm{kWh}}{3.6\times10^{6}\,\mathrm{J}}\right)$$

$$= 0.597\,\mathrm{kWh}$$

$$= (0.597\,\mathrm{kWh})\left(\frac{3.6\times10^{3}\ \mathrm{kJ}}{1\,\mathrm{kWh}}\right)\left(\frac{1\,\mathrm{Btu}}{1.055\,\mathrm{kJ}}\right)$$

$$= 2037\ \ \mathrm{Btu}$$ **Answer**

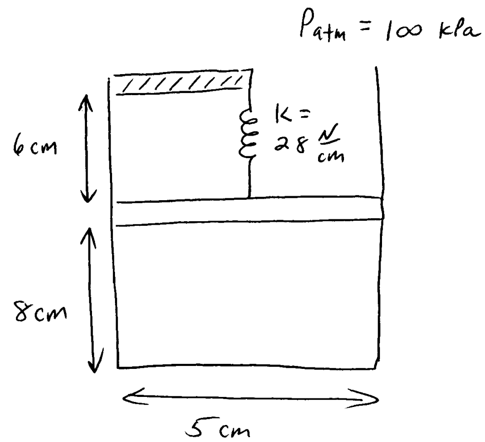
2-23 Nitrogen at 28 °C and 100 kPa is heated in a piston-cylinder assembly. Initially the spring shown is uncompressed and exerts no force on the piston, which is massless. If 4.5 J of work is done by the N_2,

a. how far does the piston rise?

b. what is the final temperature?

Approach:

The work done is the sum of the expansion work against the atmosphere and the work done on the spring. Knowing the total work done, one can calculate the amount of compression of the spring. Find the final temperature from the ideal gas law.



Assumptions:

1. The mass of the piston is negligible.
2. Nitrogen behaves like an ideal gas under these conditions.

Solution:

a) The piston does expansion work against the atmosphere and work against the spring. The sum is

$$W_{tot} = W_{exp} + W_{spring}$$

Defining x_1 as the initial piston location and x_2 as the final piston location, the expansion work is

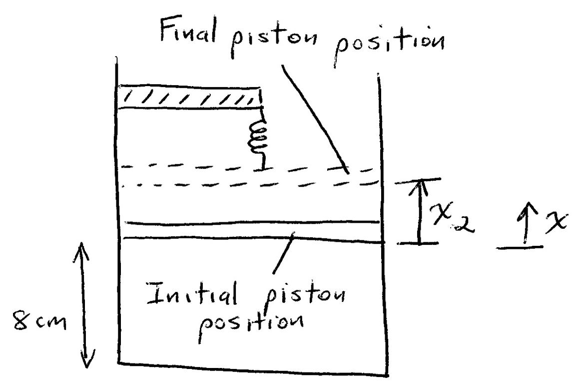
$$W_{exp} = P_{atm}\Delta V = P_{atm}\left(x_2 - x_1\right)A$$

Arbitrarily set $x_1 = 0$, so that

$$W_{exp} = P_{atm} x_2 A$$

The spring work is

$$W_{spring} = k\left(\frac{x_2^2 - x_1^2}{2}\right) = k\frac{x_2^2}{2}$$



The total work done is

$$W_{tot} = P_{atm} x_2 A + k\frac{x_2^2}{2}$$

Substituting values

$$(4.5\,\text{J}) = (100{,}000\,\text{Pa})\; x_2\, \pi (0.025\,\text{m})^2 + \left(28\frac{\text{N}}{\text{cm}}\right)\left(\frac{100\text{ cm}}{1\text{ m}}\right)\frac{x_2^2}{2}$$

Solving for x_2

$$x_2 = 0.02\text{ m} = 2\text{ cm}$$ ← **Answer**

b) From the ideal gas law

$$P_1 V_1 = \frac{m\bar{R}T_1}{M} \quad \text{and} \quad P_2 V_2 = \frac{m\bar{R}T_2}{M}$$

Dividing gives

$$\frac{P_1 V_1}{P_2 V_2} = \frac{T_1}{T_2}$$

The final pressure is

$$P_2 = P_{atm} + \frac{k x_2}{A}$$

Substituting values

$$P_2 = (100,000 \text{ Pa}) + \frac{\left(28 \frac{\text{N}}{\text{cm}}\right)(2 \text{ cm})}{\pi (0.025 \text{ m})^2} = 129 \times 10^3 \text{ Pa} = 129 \text{ kPa}$$

The initial and final volumes are

$$V_1 = (8 \text{ cm})(A)$$

$$V_2 = (10 \text{ cm})(A)$$

Substituting these expressions into the ideal gas law produces

$$\frac{(100 \text{ kPa})(8 \text{ cm})A}{(129 \text{ kPa})(10 \text{ cm})A} = \frac{(28+273)}{(T_2+273)}$$

$T_2 = 212\ °\text{C}$ ← **Answer**

2-24 A piston-cylinder assembly contains 0.49 g of air at a pressure of 150 kPa. The initial volume is 425 cm^3. The air is then compressed while 16.4 J of work are done and 3.2 J of heat are transferred to the surroundings. Calculate the final air temperature.

Approach:

Apply the first law. Write the change in internal energy in terms of the specific heat at constant volume and the temperature difference. Find the initial temperature from the ideal gas law.

Assumptions:

1. Specific heat is constant.
2. Air behaves like an ideal gas under these conditions.

Solution:

From the first law,

$$Q = \Delta U + W = m\Delta u + W$$

Writing internal energy in terms of specific heat,

$$Q = mc_v\Delta T + W = mc_v\left(T_2 - T_1\right) + W$$

What is T_1? From the ideal gas law,

$$T_1 = \frac{P_1V_1M}{m\bar{R}} = \frac{\left(150\,\text{kPa}\right)\left(425\,\text{cm}^3\right)\left(\dfrac{1000\ \text{Pa}}{1\,\text{kPa}}\right)\left(\dfrac{1\,\text{m}^3}{10^6\text{cm}^3}\right)\left(28.97\dfrac{\text{kg}}{\text{kmol}}\right)}{\left(0.49\,\text{g}\right)\left(\dfrac{1\ \text{kg}}{1000\,\text{g}}\right)\left(8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)\left(\dfrac{1000\,\text{J}}{1\,\text{kJ}}\right)}$$

$$T_1 = 453\text{ K} = 180^\circ\text{C}$$

Solving the first law for T_2,

$$T_2 = \frac{Q - W}{mc_v} + T_1$$

The preferred practice is to evaluate specific heat at the average of the initial and final temperatures; however, we do not know the final temperature. Instead, we will evaluate specific heat at the known initial temperature. We can make a correction later if more accuracy is desired. With values of c_v from Table A-8 at 450 K,

$$T_2 = \frac{\left\{(-3.2) - (-16)\right\}\text{J}\left(\dfrac{1\,\text{kJ}}{1000\,\text{J}}\right)}{\left(0.49\,\text{g}\right)\left(\dfrac{0.733\,\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left(\dfrac{1\,\text{kg}}{1000\,\text{g}}\right)} + 180^\circ\text{C}$$

$$T_2 = 215^\circ\text{C}$$ ◄——— **Answer**

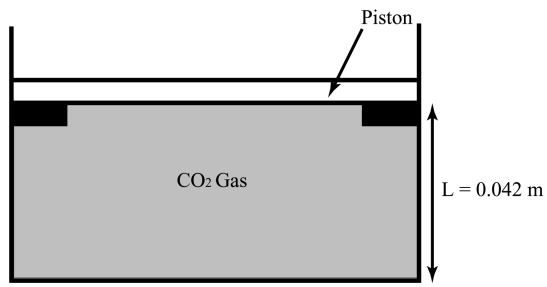
Comments:

For more accuracy, repeat the calculation evaluating specific heat at the average of 215ºC and 180ºC. Specific heat does not vary substantially between these two temperatures, so the estimate of final temperature will change very little.

2-25 In the figure below, a piston is resting on a set of stops. The cylinder contains CO_2 initially at –30 °C and 45 kPa. The mass of the piston is 1.2 kg and its diameter is 0.06 m. Assuming atmospheric pressure is 101 kPa, how much heat must be added to just lift the piston off the stops.

Approach:

A force balance on the piston can be used to calculate the gas pressure at piston lift-off. The ideal gas law is used to determine the final temperature. As a last step, apply the first law to find heat added.



Assumptions:

1. Specific heat is constant.
2. Carbon dioxide behaves like an ideal gas under these conditions.

Solution:

The piston will just lift off when the pressure inside equals the pressure outside plus the weight of the piston per unit area.

$$P_2 = P_{atm} + \frac{m_p g}{A_p} = \left(101\times10^3\ \text{Pa}\right) + \frac{(1.2\,\text{kg})\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)}{\pi(0.03\,\text{m})^2} = 105.1\ \text{kPa}$$

To find T_2, use the ideal gas law, noting that

$$P_1V_1 = \frac{m\bar{R}T_1}{M} \qquad \text{and} \qquad P_2V_2 = \frac{m\bar{R}T_2}{M}$$

Dividing the last two equations and realizing that $V_1 = V_2$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right) = (-30+273)\,\text{K}\left(\frac{105.1}{45}\right) = 568\,\text{K}$$

From the first law,

$$Q = \Delta U + W$$

Since there is no volume change, $W = 0$, and

$$Q = \Delta U = m\Delta u = mc_v\left(T_2 - T_1\right)$$

To evaluate c_v, use average temperature, i.e., $T_{ave} = \left(T_1 + T_2\right)/2 = \left(243+568\right)/2 = 405\,\text{K}$

From Table A-8, $c_v \approx 0.75\ \ \text{kJ/kg}\cdot\text{K}$. To find m, use the ideal gas law

$$m = \frac{P_1V_1M}{\bar{R}T_1} = \frac{\left(45\times10^3\ \text{Pa}\right)(0.042\,\text{m})(\pi)(0.03\,\text{m})^2\left(44.01\,\frac{\text{kg}}{\text{kmol}}\right)}{\left(8.314\,\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(243\,\text{K})\left(\frac{1000\,\text{J}}{1\,\text{kJ}}\right)} = 1.16\times10^{-4}\,\text{kg}$$

From the first law

$$Q = mc_v\left(T_2 - T_1\right) = \left(1.16\times10^{-4}\,\text{kg}\right)\left(0.75\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left(\frac{1000\,\text{J}}{1\,\text{kJ}}\right)(568\text{-}243)\,\text{K}$$

$$Q = 28.3\,\text{J} \longleftarrow \textbf{Answer}$$

2-26 A closed tank of volume 2.8 ft^3 contains oxygen at 70^{o}F and an absolute pressure of 14.3 lbf/in^2. The gas is then heated until the pressure becomes 45 lbf/in^2. Treating oxygen as an ideal gas,

a. find the final temperature.

b. find the total change in enthalpy, *H*, in Btu for this process.

Approach:

Use the ideal gas law to find the final temperature, T_2.
The change is enthalpy is found using the specific heat at constant pressure.

Assumptions:

1. Oxygen is an ideal gas at these conditions.
2. Specific heat is constant.

Solution:

a) From the ideal gas law

$$P_1V_1 = \frac{m_1\bar{R}T_1}{M} \qquad \text{and} \qquad P_2V_2 = \frac{m_2\bar{R}T_2}{M}$$

Neither the volume nor the mass changes during the heating process, so $V_1 = V_2$, and $m_1 = m_2$. Dividing the last two equations gives

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$T_2 = \frac{T_1P_2}{P_1} = \frac{(70+460)\,\mathrm{R}\,(45\,\mathrm{psia})}{14.3\,\mathrm{psia}} = 1668\,\mathrm{R} = 1208\ °\mathrm{F}$$

b) The total enthalpy change is related to specific enthalpy change by

$$\Delta H = m\Delta h = m(h_2 - h_1) = m\,c_{p,ave}\,(T_2 - T_1)$$

Evaluate specific heat at the average temperature, $T_{ave} = (70+1208)/2 = 639\ °\mathrm{F}$. From Table B-8 $c_p \approx 0.24\,\mathrm{Btu}/(\mathrm{lbm}\cdot\mathrm{R})$. To find mass, use the ideal gas law

$$m = \frac{P_1V_1M}{\bar{R}T_1} = \frac{\left(14.3\dfrac{\mathrm{lbf}}{\mathrm{in}^2}\right)\left(2.8\ \mathrm{ft}^3\right)\left(32\dfrac{\mathrm{lbm}}{\mathrm{lbmol}}\right)\left(\dfrac{144\ \mathrm{in}^2}{1\ \mathrm{ft}^2}\right)\left(\dfrac{1\ \mathrm{Btu}}{778.2\ \mathrm{lbf}\cdot\mathrm{ft}}\right)}{\left(1.986\dfrac{\mathrm{Btu}}{\mathrm{lbmol}\cdot\mathrm{R}}\right)(70+460)\,\mathrm{R}} = 0.225\,\mathrm{lbm}$$

Substituting values

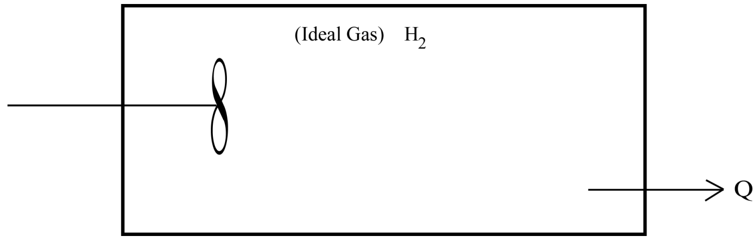
$$\Delta H = (0.225\,\mathrm{lbm})\left(0.24\frac{\mathrm{Btu}}{\mathrm{lbm}\cdot\mathrm{R}}\right)(1208-70)$$

$$\Delta H = 61.5\,\mathrm{Btu} \longleftarrow \textbf{Answer}$$

2-27 A rigid tank of volume 0.26 m^3 contains hydrogen at 15 °C and 101 kPa. A paddlewheel stirs the tank, adding 17.8 kJ of work. Over the same time period, the tank loses 9.3 kJ of heat to the environment. Assuming the specific heat of hydrogen does not vary with temperature, find the final temperature.

Approach:

Use the first law to relate heat, work, and internal energy. Apply the ideal gas law to find the total mass of hydrogen present. Express the internal energy is terms of temperature and solve for the final temperature.



Assumptions:

1. Hydrogen is an ideal gas at these conditions.
2. Specific heat is constant.

Solution:

From the first law

$$\Delta U = m\Delta u = Q - W$$

Writing internal energy in terms of specific heat,

$$mc_v\left(T_2 - T_1\right) = Q - W$$

Mass may be found from the ideal gas law

$$m = \frac{P_1 V_1 M}{\bar{R} T_1} = \frac{\left(101\,\text{kPa}\right)\left(0.26\,\text{m}^3\right)\left(2.016\,\frac{\text{kJ}}{\text{kmol}}\right)\left(\frac{1000\,\text{Pa}}{1\,\text{kPa}}\right)}{\left(8.314\,\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)\left(15{+}273\,\text{K}\right)\left(\frac{1000\,\text{J}}{1\,\text{kJ}}\right)} = 0.0221\,\text{kg}$$

where molecular weight was obtained from Table A-1. Solving the first law for final temperature

$$T_2 = \frac{Q - W}{mc_v} + T_1$$

The final temperature is unknown. As an approximation, evaluate specific heat at the initial temperature. If the temperature change is large, then reevaluate specific heat at the average of initial and final temperatures and perform the calculation again. Using c_v from Table A-8 at 15ºC,

$$T_2 = \frac{\left[-9.3 - \left(-17.8\right)\right]\text{kJ}}{\left(0.0221\,\text{kg}\right)\left(10.12\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)} + 15^\circ\text{C}$$

$$T_2 = 53.0\,^\circ\text{C} \longleftarrow \textbf{Answer}$$

Comments:

For more accuracy, repeat the calculation evaluating specific heat at the average of 15ºC and 53ºC. Specific heat does not vary substantially between these two temperatures, so the estimate of final temperature will change very little.

2-28 A chamber is divided equally in two parts by a membrane. One side contains H_2 at a pressure of 130 kPa and the other side is evacuated. The total chamber volume is 0.004 m^3. At time $t = 0$, the membrane ruptures and the hydrogen expands freely into the evacuated side. If the chamber is considered adiabatic, find the final pressure.

Approach:

Use the first law to relate heat, work, and internal energy. There is no work done and no heat transferred, therefore internal energy does not change and the process is isothermal. Apply the ideal gas law to find the final pressure.

Assumptions:

1. Hydrogen is an ideal gas at these conditions.
2. The chamber is perfectly insulated.

Solution:

From the first law

$$Q = \Delta U + W$$

No work is done in a free expansion and no heat is transferred since the chamber is adiabatic, so

$$\Delta U = 0$$

Writing internal energy in terms of temperature:

$$mc_v \Delta T = mc_v \left(T_2 - T_1\right) = 0$$

which implies

$$T_1 = T_2$$

From the ideal gas law

$$P_1 V_1 = \frac{m\bar{R}T_1}{M} \quad \text{and} \quad P_2 V_2 = \frac{m\bar{R}T_2}{M}$$

Dividing these two equations and remembering that $T_1 = T_2$

$$P_1 V_1 = P_2 V_2$$

Rearranging

$$P_2 = \frac{P_1 V_1}{V_2} = \left(130\,\text{kPa}\right)\left(\frac{0.002\,\text{m}^3}{0.004\,\text{m}^3}\right)$$

$$P_2 = 65\text{ kPa}$$ ← **Answer**

2-29 Air at 20°C, 250 kPa is contained in a piston-cylinder assembly. Initially, the piston is held in place by a pin. Then the pin is removed and the gas expands rapidly. During the expansion, there is no time for any heat transfer to occur. The final air temperature and pressure are –16 °C and 100 kPa. The mass of air in the cylinder is 0.4 kg. Find the work done on the atmosphere.

Approach:

Use the first law for a constant pressure process of a closed system. Calculate mass from the ideal gas law.

Assumptions:

1. Air behaves like an ideal gas under these conditions.
2. Specific heat is constant.
3. The process is adiabatic.
4. There is negligible friction between the piston and cylinder walls.
5. The mass of the piston is negligible.

Solution:

Choose the air to be the system under consideration. From the first law

$$\Delta U = Q - W$$

Since the process is adiabatic

$$\Delta U = -W$$

Assuming constant specific heat

$$W = mc_v(T_1 - T_2)$$

The specific heat may be found in Table A-8. Substituting values

$$W = (0.4\,\mathrm{kg})\left(0.717\frac{\mathrm{kJ}}{\mathrm{kg\ K}}\right)\left[20-(-16)\right]{}^\circ\mathrm{C}$$

$$W = 10.3\,\mathrm{kJ} \longleftarrow \textbf{Answer}$$

Comments:

The work is positive because work is done by the gas on the atmosphere.

2-30 Nitrogen at 50 psia and 650°F is contained in a piston-cylinder assembly. The initial volume is 25 ft^3. The nitrogen is cooled slowly while the pressure stays constant until the temperature drops to 150°F. Find the heat transferred.

Approach:

Use the first law for a constant pressure process of a closed system. Calculate mass from the ideal gas law.

Assumptions:

1. Nitrogen behaves like an ideal gas under these conditions.
2. Specific heat is constant.

Solution:

From the first law for a constant pressure process

$$Q = \Delta H = mc_P \Delta T$$

From the ideal gas law

$$m = \frac{P_1 V_1 M}{\bar{R} T_1}$$

$$m = \frac{(50\,\text{psia})(25\ \text{ft}^3)\left(28.0\dfrac{\text{lbm}}{\text{lbmol}}\right)}{\left(10.73\dfrac{\text{psia ft}^3}{\text{lbmol R}}\right)(650+460)R} = 2.94\ \text{lbm}$$

From Table B-8 at an average temperature of

$$T_{ave} = \frac{150+650}{2} = 400^\circ\text{F} \qquad c_P = 0.251\frac{\text{Btu}}{\text{lbm R}}$$

Using these values in the first law gives

$$Q = (2.94\ \text{lbm})\left(0.251\frac{\text{Btu}}{\text{lbm R}}\right)(150-650)^\circ\text{F}$$

$$= -369\,\text{Btu} \longleftarrow$$ **Answer**

Comments:

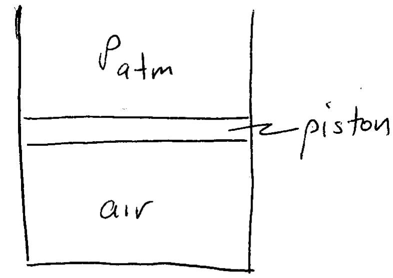
The heat transfer is negative because the system is being cooled.

2-31 Air at 30°C is contained in a piston-cylinder assembly, as shown in the figure. The piston has a weight of 15 N and a cross-sectional area of 0.12 m^2. The initial volume of air is 3.5 m^3. Heat is added until the volume of the air becomes 6.5 m^3. Atmospheric pressure is 100 kPa.

a. Find the final air temperature
b. Determine the work done by the air on both the piston and the atmosphere

Approach:

Use the ideal gas law to find the final temperature and $W = \int PdV$ to find the work.



Assumptions:

1. Air behaves like an ideal gas under these conditions.
2. There is no friction between the piston and cylinder walls.
3. The process proceeds through a succession of quasi-equilibrium states

Solution:

a) Select the air to be the system under consideration. This is a constant pressure process, therefore

$$P_1 = P_2$$

From the ideal gas law:

$$P_1V_1 = \frac{m\bar{R}T_1}{M} \qquad P_2V_2 = \frac{m\bar{R}T_2}{M}$$

The last two equations imply that

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

Solving for T_2,

$$T_2 = T_1\left(\frac{V_2}{V_1}\right) = (30+273)\text{K}\left(\frac{6.5\,\text{m}^3}{3.5\,\text{m}^3}\right)$$

$$T_2 = 563\,\text{K} = 290\,^\circ\text{C}$$ **Answer**

b) Since this is a quasi-equilibrium process

$$W = \int_{V_1}^{V_2} PdV = P\int_{V_1}^{V_2} dV = P(V_2 - V_1)$$

where pressure can be removed from the integral because, in this process, it is a constant. The pressure is due to atmospheric pressure plus the weight of the piston distributed over the piston area:

$$P = P_{\text{atm}} + \frac{\text{mg}}{A}$$

$$= 100\,\text{kPa} + \frac{15\,\text{N}}{0.12\,\text{m}^2}\left(\frac{1\,\text{kPa}}{1000\,\text{Pa}}\right)$$

$$= 100.1\,\text{kPa}$$

Work may now be calculated as

$$W = (100.1\,\text{kPa})\left(\frac{1000\,\text{Pa}}{1\,\text{kPa}}\right)(6.5-3.5)\text{m}^2$$

$$W = 300{,}300\,\text{J} = 300\,\text{kJ}$$ **Answer**

Comments:

Note that the weight of the piston is insignificant compared to atmospheric pressure in this case.

2-32 An ideal gas with $c_p = 0.7$ kJ/kg·K and a molecular weight of 25.6 is initially at 75 kPa and 40 °C. First the gas is expanded at constant pressure until its volume doubles. Then it is heated at constant volume until the pressure doubles. If the mass of gas is 4.5 kg, find

a. the total work for the entire process
b. the heat transferred for the entire process

Approach:

Consider this as a two-step process. Calculate work for each step and add to get total work. Also calculate heat for each step and add. Find the work from the integral of PdV and the heat from the first law, after work is known.

Assumptions:

1. Both processes proceed through a succession of quasi-equilibrium states.
2. Specific heat is constant.

Solution:

a. For a quasi-equilibrium process, work is

$$W = \int P\mathrm{d}V$$

Applying this to the first step of the process, which proceeds from state 1 to state 2

$$W_{12} = \int_{V_1}^{V_2} P\mathrm{d}V = P\int_{V_1}^{V_2} \mathrm{d}V$$

where P has been removed from the integral because pressure is constant in the first step. Performing the integral and noting that the final volume is twice the initial volume

$$W_{12} = P(V_2 - V_1) = P(2V_1 - V_1)$$

Using the ideal gas law:

$$W_{12} = PV_1 = P_1V_1 = \frac{m\bar{R}T_1}{M} = \frac{(4.5\,\text{kg})\left(8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(40+273)\text{K}}{25.6\,\dfrac{\text{kg}}{\text{kmol}}} = 457\,\text{kJ}$$

For the second step of the process, work done is zero, because volume does not change; therefore, the total work for the process is

$$W = W_{12} = 457\,\text{kJ}$$ **Answer**

b. To find the heat transferred, use the first law

$$\Delta U = Q - W$$

For the first step of the process

$$U_2 - U_1 = Q_{12} - W_{12}$$

$$Q_{12} = mc_v(T_2 - T_1) + W_{12}$$

From the ideal gas law

$$P_1V_1 = \frac{m\bar{R}T_1}{M} \qquad P_2V_2 = \frac{m\bar{R}T_2}{M}$$

but $P_1 = P_2$ and $V_2 = 2V_1$, so

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} = \frac{V_1}{2V_1} = \frac{1}{2}$$

The temperature at state 2 becomes

$$T_2 = 2T_1 = 2(40+273)\text{K} = 626\,\text{K}$$

To find c_v, use

$$c_v = c_p - \frac{\bar{R}}{M}$$

$$c_v = 0.7\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - \frac{8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{25.6\dfrac{\text{kg}}{\text{kmol}}} = 0.375\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Substituting values

$$Q_{12} = (4.5\,\text{kg})\left(0.375\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(626-313)\text{K}+457\text{ kJ} = 985\,\text{kJ}$$

For the second step of the process, apply the first law

$$\Delta U = Q - W$$

$$U_3 - U_2 = Q_{23} - W_{23}$$

Volume is constant during the second step; therefore, $W_{23} = 0.$ Using the relation for the specific heat of an ideal gas

$$Q_{23} = mc_v(T_3 - T_2)$$

From the ideal gas law

$$P_3V_3 = \frac{m\bar{R}T_3}{M} \qquad P_2V_2 = \frac{m\bar{R}T_2}{M}$$

Since $P_3 = 2P_2$ and $V_2 = V_3$,

$$\frac{P_3}{P_2} = \frac{T_3}{T_2} = \frac{2P_2}{P_2} = 2$$

$$T_3 = 2T_2 = 2(626)\text{K}=1252\,\text{K}$$

The heat transferred becomes

$$Q_{23} = (4.5\,\text{kg})\left(0.375\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(1252-626)\text{K}$$

$$= 1057\text{ kJ}$$

The total heat transferred is the sum of the heat in each step; that is,

$$Q_{Tot} = Q_{12} + Q_{23} = 985 + 1057 = 2043\,\text{kJ}$$ **Answer**

2-33 A piston-cylinder assembly contains 0.2 kg of argon at 200 K and 50 kPa. If the argon is expanded isothermally to 30 kPa, find the work done.

Approach:

Apply the formula for work done during an isothermal expansion of an ideal gas. Use the ideal gas law to rewrite the formula in terms of pressure rather than volume.

Assumptions:

1. Argon is an ideal gas under these conditions.
2. The process proceeds through a succession of equilibrium states.

Solution:

For an ideal gas undergoing an isothermal expansion

$$W = \frac{m\bar{R}T}{M}\ln\frac{V_2}{V_1}$$

From the ideal gas law

$$P_1V_1 = \frac{m\bar{R}T}{M} \qquad \text{and} \qquad P_2V_2 = \frac{m\bar{R}T}{M}$$

Solving for V_1 and V_2 and substituting

$$W = \frac{m\bar{R}T}{M}\ln\left(\frac{m\bar{R}T}{MP_2}\bigg/\frac{m\bar{R}T}{MP_1}\right) = \frac{m\bar{R}T}{M}\ln\left(\frac{P_1}{P_2}\right)$$

Using molecular weight from Table A-1,

$$W = \frac{(0.2\,\text{kg})\left(8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(200\,\text{K})\ln\left(\frac{50}{30}\right)}{39.9\frac{\text{kg}}{\text{kmol}}}$$

$W = 4.26\,\text{kJ}$ ⟵ **Answer**

2-34 An ideal gas with a molecular weight of 37.2 is contained in a piston-cylinder assembly. The gas is initially at 130 kPa, 25°C, and has a mass of 2.34 x 10^{-4} kg. The gas expands slowly and isothermally until the final pressure is 100 kPa. Calculate the work done.

Approach:

Apply the formula for work done during an isothermal expansion of an ideal gas. Use the ideal gas law to determine initial and final volumes.

Assumptions:

1. The process proceeds through a succession of equilibrium states.

Solution:

For an ideal gas undergoing an isothermal expansion

$$W = \frac{m\bar{R}T}{M}\ln\frac{V_2}{V_1}$$

From the ideal gas law

$$V_1 = \frac{m\bar{R}T_1}{MP_1} = \frac{(2.34\times10^{-4}\,\text{kg})\left(8.314\,\dfrac{\text{kPa}\cdot\text{m}^3}{\text{kmol}\cdot\text{K}}\right)(25+273)\text{K}}{\left(37.2\,\dfrac{\text{kg}}{\text{kmol}}\right)(130\ \text{kPa})} = 1.2\times10^{-4}\,\text{m}^3$$

$$V_2 = \frac{m\bar{R}T_2}{MP_2} = \frac{(2.34\times10^{-4}\,\text{kg})\left(8.314\,\dfrac{\text{kPa}\cdot\text{m}^3}{\text{kmol}\cdot\text{K}}\right)(25+273)\text{K}}{\left(37.2\,\dfrac{\text{kg}}{\text{kmol}}\right)(100\ \text{kPa})} = 1.56\times10^{-4}\,\text{m}^3$$

Finally, the work done is

$$W = \frac{(2.34\times10^{-4}\,\text{kg})\left(8.314\,\dfrac{\text{kPa}\cdot\text{m}^3}{\text{kmol}\cdot\text{K}}\right)(25+273)\text{K}}{\left(37.2\,\dfrac{\text{kg}}{\text{kmol}}\right)\left(\dfrac{1\,\text{kPa}}{1000\,\text{Pa}}\right)}\ln\left(\frac{1.56\times10^{-4}}{1.2\times10^{-4}}\right)$$

$$W = 4.09\,\text{J} \longleftarrow$$ **Answer**

2-35 An ideal gas with a volume of 0.5 ft^3 and an absolute pressure of 15 lbf/in.^2 is contained in a piston-cylinder assembly. The gas is compressed isothermally until the pressure doubles. Calculate the heat transferred in Btu. Is the heat moving from the gas to the surroundings or vice-versa?

Approach:

Apply the formula for work done during an isothermal compression of an ideal gas. Use the first law to determine heat transferred.

Assumptions:

1. The process proceeds through a succession of equilibrium states.

Solution:

From the ideal gas law

$$P_1V_1 = \frac{m\bar{R}T}{M} \qquad \text{and} \qquad P_2V_2 = \frac{m\bar{R}T}{M}$$

Since temperature is unchanged in this process,

$$P_1V_1 = P_2V_2$$

Rearranging

$$V_2 = \frac{P_1V_1}{P_2} = \frac{(15\,\text{psia})(0.5\,\text{ft}^3)}{(30\,\text{psia})} = 0.25\,\text{ft}^3$$

For an isothermal compression of an ideal gas, the work done is

$$W = P_1V_1 \ln\frac{V_2}{V_1} = \left(15\frac{\text{lbf}}{\text{in.}^2}\right)(0.5\text{ft}^3)\left(\frac{144\,\text{in.}^2}{\text{ft}^2}\right)\ln\left(\frac{0.25}{0.5}\right) = -748.6\,\text{ft}\cdot\text{lbf}$$

From the first law

$$\Delta U = Q - W$$

But since the process is isothermal, $\Delta U = 0$,

$$Q = W = (-748.6\,\text{ft}\cdot\text{lbf})\left(\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right) = -0.962\,\text{Btu}$$ **Answer**

Comment:

Since Q is negative, heat moves from the gas to the surroundings.

2-36 Air in a piston-cylinder assembly is compressed slowly and isothermally from an initial volume of 350 cm^3 to a final volume of 200 cm^3. The air is initially at 100 kPa.

a. Find the work done

b. Find the heat transferred

Approach:

Apply the formula for work done during an isothermal compression of an ideal gas.

Assumptions:

1. Air is an ideal gas under these conditions.
2. The process proceeds through a succession of equilibrium states.

Solution:

a) For an ideal gas undergoing an isothermal compression

$$W = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$W = (100\,\text{kPa})\left(\frac{1000\,\text{Pa}}{1\,\text{kPa}}\right)\left(350\,\text{cm}^3\right)\left(\frac{1\,\text{m}^3}{10^6\,\text{cm}^3}\right)\ln\left(\frac{200}{350}\right)$$

$$W = -19.6\ \text{J} \longleftarrow \textbf{Answer}$$

b) From the first law

$$Q = \Delta U + W = mc_v \Delta T + W$$

but the process is isothermal and $\Delta T = 0$, therefore,

$$Q = W$$

$$Q = -19.6\ \text{J} \longleftarrow \textbf{Answer}$$

Comments:

Work is negative because, during a compression, work is done on the system. Heat is negative because the system is cooled during the process. One must cool the system while compressing it so that temperature will remain constant.

2-37 A piston-cylinder assembly contains 0.4 kg of CO_2. The gas expands at constant temperature from an initial state of 250 kPa, 100ºC to a final pressure of 100 kPa. Calculate the heat transferred during the process.

Approach:

Apply the formula for work done during an isothermal expansion of an ideal gas. Use the ideal gas law to rewrite the formula in terms of pressure rather than volume. Finally, use the first law to find heat transferred.

Assumptions:

1. Carbon dioxide is an ideal gas under these conditions.
2. The process proceeds through a succession of equilibrium states.

Solution:

For an ideal gas undergoing an isothermal expansion

$$W = \frac{m\bar{R}T}{M}\ln\frac{V_2}{V_1}$$

From the ideal gas law

$$P_1V_1 = \frac{m\bar{R}T}{M} \qquad \text{and} \qquad P_2V_2 = \frac{m\bar{R}T}{M}$$

Solving for V_1 and V_2 and substituting

$$W = \frac{m\bar{R}T}{M}\ln\left(\frac{m\bar{R}T}{MP_2}\Big/\frac{m\bar{R}T}{MP_1}\right) = \frac{m\bar{R}T}{M}\ln\left(\frac{P_1}{P_2}\right)$$

Using molecular weight from Table A-1,

$$W = = (0.4\,\text{kg})\left(\frac{8.314\ \text{kJ}/(\text{kmol}\cdot\text{K})}{44.01\,\text{kg/kmol}}\right)(373.15\,\text{K})\ln\left(\frac{250}{100}\right)$$

$$W = 25.8\,\text{kJ}$$

From the first law, recognizing that temperature is constant,

$$\Delta U = Q - W = c_V\Delta T = O$$

Therefore

$$Q = W = 25.8\,\text{kJ}$$ **Answer**

Comments:

Work is positive because during an expansion, work is done by the system. Heat is positive because the system is heated during the process. One must heat the system during expansion so that temperature will remain constant.

2-38 Air at 180 °F and 25 psia is compressed slowly and isothermally to 86 psia. If the initial mass of air is 0.0043 lbm, find:

a. the work done.

b. the heat transferred.

.

Approach:

Apply the formula for work done during an isothermal compression of an ideal gas. Use the ideal gas law to rewrite the formula in terms of pressure rather than volume. Apply the first law to find heat transferred.

Assumptions:

1. Air is an ideal gas under these conditions.
2. The process proceeds through a succession of equilibrium states.

Solution:

a) For an ideal gas undergoing an isothermal compression

$$W = \frac{m\bar{R}T}{M}\ln\frac{V_2}{V_1}$$

From the ideal gas law

$$P_1V_1 = \frac{m\bar{R}T}{M} \qquad \text{and} \qquad P_2V_2 = \frac{m\bar{R}T}{M}$$

Solving for V_1 and V_2 and substituting

$$W = \frac{m\bar{R}T}{M}\ln\left(\frac{m\bar{R}T}{MP_2} \bigg/ \frac{m\bar{R}T}{MP_1}\right) = \frac{m\bar{R}T}{M}\ln\left(\frac{P_1}{P_2}\right)$$

Using molecular weight from Table A-1,

$$W = (0.0043\,\text{lbm})\left(\frac{1545\ \text{ft}\cdot\text{lbf}/(\text{lbmol}\cdot\text{R})}{28.97\,\text{lbm/lbmol}}\right)(180+460)\,\text{R}\ln\left(\frac{25}{86}\right)$$

$$W = -181\ \text{ft}\cdot\text{lbf} \longleftarrow$$ **Answer**

b) From the first law, recognizing that temperature is constant,

$$\Delta U = Q - W = c_V\Delta T = O$$

Therefore

$$Q = W = -181\,\text{ft}\cdot\text{lbf}$$

Converting to Btu,

$$Q = (-181\ \text{ft}\cdot\text{lbf})\left(\frac{1\ \text{Btu}}{778\ \text{lbf}\cdot\text{ft}}\right)$$

$$= -0.233\ \text{Btu} \longleftarrow$$ **Answer**

Comments:

Work is negative because, during a compression, work is done on the system. Heat is negative because the system is cooled during the process. One must cool the system while compressing it so that temperature will remain constant.

2-39 A piston cylinder assembly of initial volume 150 cm^3 contains 0.3 g of oxygen at 120 kPa. The oxygen is then compressed slowly, isothermally, and frictionlessly, while 5.9 J of heat is removed. Find the final pressure.

Approach:

Use the first law to determine the work done. From the formula for work done during an isothermal compression of an ideal gas, you can determine the final pressure. Use the ideal gas law to find properties.

Assumptions:

1. Oxygen is an ideal gas under these conditions.
2. The process proceeds through a succession of equilibrium states.

Solution:

From the first law, recognizing that temperature is constant,

$$\Delta U = Q - W = c_V \Delta T = O$$

Therefore

$$Q = W = -5.9\,\text{J}$$

For an ideal gas undergoing an isothermal compression

$$W = \frac{m\bar{R}T}{M}\ln\frac{V_2}{V_1}$$

To find temperature, use the ideal gas law (the molecular weight of oxygen is found in Table A-1)

$$T_1 = \frac{P_1 V_1 M}{m\bar{R}} = \frac{(120\,\text{kPa})(150\,\text{cm}^3)\left(31.99\dfrac{\text{kg}}{\text{kmol}}\right)\left(\dfrac{1\,\text{m}}{100\,\text{cm}}\right)^3\left(\dfrac{1000\,\text{Pa}}{1\,\text{kPa}}\right)}{(0.3\,\text{g})\left(8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)\left(\dfrac{1\,\text{kg}}{1000\,\text{g}}\right)\left(\dfrac{1000\,\text{J}}{1\,\text{kJ}}\right)} = 231\text{ K}$$

Rearranging the work equation

$$V_2 = V_1 \exp\left[\frac{MW}{m\bar{R}T_1}\right] = (150\,\text{cm}^3)\exp\left[\frac{\left(31.99\dfrac{\text{kg}}{\text{kmol}}\right)(-5.9\,\text{J})\left(\dfrac{1\,\text{kJ}}{1000\,\text{J}}\right)}{(0.3\,\text{g})\left(8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)\left(\dfrac{1\,\text{kg}}{1000\,\text{g}}\right)(231\,\text{K})}\right]$$

$$V_2 = 108\,\text{cm}^3$$

To find the final pressure, apply the ideal gas law

$$P_2 = \frac{m\bar{R}T_2}{MV_2} = \frac{(0.3\,\text{g})\left(8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(231\,\text{K})\left(\dfrac{1\,\text{kg}}{1000\,\text{g}}\right)\left(\dfrac{1000\,\text{J}}{1\,\text{kJ}}\right)}{\left(31.99\dfrac{\text{kg}}{\text{kmol}}\right)(108\,\text{cm}^3)\left(\dfrac{1\,\text{m}}{100\,\text{cm}}\right)^3}$$

$$P_2 = 166{,}700\,\text{Pa} = 166.7\,\text{kPa} \longleftarrow$$ **Answer**

2-40 Carbon dioxide is expanded slowly and isothermally in a piston-cylinder assembly from 33.7 psia to 14.7 psia. The initial volume is 39 in^3 and the temperature is 100°F. Calculate the work done.

Approach:

Apply the formula for work done during an isothermal expansion of an ideal gas. Use the ideal gas law to rewrite the formula in terms of pressure rather than volume.

Assumptions:

1. Carbon dioxide is an ideal gas under these conditions.
2. The process proceeds through a succession of equilibrium states.

Solution:

For an ideal gas undergoing an isothermal expansion

$$W = P_1 V_1 \ln\frac{V_2}{V_1}$$

From the ideal gas law

$$P_1 V_1 = \frac{m\bar{R}T}{M} \qquad \text{and} \qquad P_2 V_2 = \frac{m\bar{R}T}{M}$$

Solving for V_1 and V_2 and substituting

$$W = P_1 V_1 \ln\left(\frac{m\bar{R}T}{MP_2} \Big/ \frac{m\bar{R}T}{MP_1}\right) = P_1 V_1 \ln\left(\frac{P_1}{P_2}\right)$$

Substituting values

$$W = \left(33.7\,\frac{\text{lbf}}{\text{in.}^2}\right)\left(39\,\text{in.}^3\right)\ln\left(\frac{33.7}{14.7}\right)\left(\frac{1\,\text{ft}}{12\,\text{in.}}\right)$$

$$W = 90.9\,\text{ft}\cdot\text{lbf}$$ **Answer**

Comment:

The problem can be solved without knowing the temperature.

2-41 Fifteen grams of nitrogen in a piston-cylinder assembly is compressed slowly and isothermally from 100 kPa, 25° C, to 2500 kPa. Calculate the heat transferred and the work done.

Approach:

Apply the formula for work done during an isothermal expansion of an ideal gas. Use the ideal gas law to rewrite the formula in terms of pressure rather than volume. Use the first law to find heat transferred.

Assumptions:

1. Nitrogen is an ideal gas under these conditions.
2. The process proceeds through a succession of equilibrium states.

Solution:

For an ideal gas undergoing an isothermal expansion

$$W = \frac{m\bar{R}T}{M}\ln\frac{V_2}{V_1}$$

From the ideal gas law

$$P_1V_1 = \frac{m\bar{R}T}{M} \qquad \text{and} \qquad P_2V_2 = \frac{m\bar{R}T}{M}$$

Solving for V_1 and V_2 and substituting

$$W = \frac{m\bar{R}T}{M}\ln\left(\frac{m\bar{R}T}{MP_2}\Big/\frac{m\bar{R}T}{MP_1}\right) = \frac{m\bar{R}T}{M}\ln\left(\frac{P_1}{P_2}\right)$$

Using molecular weight from Table A-1,

$$W = \frac{(0.015\,\text{kg})\left(8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(25+273)\,\text{K}\ln\left(\dfrac{100}{2500}\right)}{28.01\dfrac{\text{kg}}{\text{kmol}}}$$

$$W = -4.27\,\text{kJ} \longleftarrow \textbf{Answer}$$

From the first law, recognizing that temperature is constant,

$$\Delta U = Q - W = c_V\Delta T = O$$

Therefore

$$Q = W = -4.27\,\text{kJ} \longleftarrow \textbf{Answer}$$

Comments:

Work is negative because, during a compression, work is done on the system. Heat is negative because the system is cooled during the process. One must cool the system while compressing it so that temperature will remain constant.

2-42 Air in a piston-cylinder assembly is slowly compressed from 100 kPa to 300 kPa. The mass of the air is 1.5×10^{-4} kg and its initial temperature is 20°C. During the entire process, pressure is related to volume as

$$PV^{1.4} = \text{a constant}$$

Calculate the work done.

Approach:

Apply the formula for work done during a polytropic process of an ideal gas. Use the ideal gas law to determine volume.

Assumptions:

1. Air is an ideal gas under these conditions.
2. The process proceeds through a succession of equilibrium states.
3. Specific heat is constant.

Solution:

For a polytropic process of an ideal gas, work done is

$$W = \frac{P_2V_2 - P_1V_1}{1-n}$$

where n = 1.4 in this case. To find V_1, apply the ideal gas law

$$V_1 = \frac{m\bar{R}T_1}{P_1M} = \frac{\left(1.5\times10^{-4}\text{ kg}\right)\left(8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)\left(20+273\right)\text{K}}{\left(100\,\text{kPa}\right)\left(28.97\dfrac{\text{kg}}{\text{kmol}}\right)} = 1.26\times10^{-4}\text{ m}^3$$

To find V_2, note that $P_1V_1^{1.4} = P_2V_2^{1.4}$; therefore

$$V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1.4}} V_1 = \left(\frac{100}{300}\right)^{\frac{1}{1.4}}\left(1.26\times10^{-4}\right) = 5.75\times10^{-5}\text{ m}^3$$

$$W = \left[\frac{\left(300\,\text{kPa}\right)\left(5.75\times10^{-5}\text{m}^3\right) - \left(100\,\text{kPa}\right)\left(1.26\times10^{-4}\text{m}^3\right)}{1-1.4}\right]\left(\frac{1000\,\text{Pa}}{1\,\text{kPa}}\right)$$

$$W = -11.6\text{ J}$$ ← **Answer**

Comments:

Work is negative because work is being done **on** the air in this compression process.

2-43 Air is compressed from 150 kPa to 600 kPa while the temperature rises from 20°C to 100°C. The process is polytropic with

$$PV^n = \text{constant}$$

The initial volume of air is 1 m^3. Find

a. the value of n

b. the work

c. the heat transfer

Approach:

Use the defining equation for a polytropic process and the ideal gas law to find n. After work is calculated using the expression for work done in a polytropic process, heat may be calculated from the first law.

Assumptions:

1. Air is an ideal gas under these conditions.
2. The process proceeds through a succession of equilibrium states.
3. Specific heat is constant.

Solution:

a) For a polytropic process, by definition:

$$P_1V_1^n = P_2V_2^n$$

To find V_2, apply the ideal gas law at each state:

$$P_1V_1 = \frac{m\bar{R}T_1}{M} \qquad P_2V_2 = \frac{m\bar{R}T_2}{M}$$

Dividing one equation by the other

$$\frac{P_1V_1}{P_2V_2} = \frac{\left(\dfrac{m\bar{R}T_1}{M}\right)}{\left(\dfrac{m\bar{R}T_2}{M}\right)}$$

Solving for V_2 yields:

$$V_2 = \frac{P_1V_1}{P_2}\left(\frac{T_2}{T_1}\right) = \frac{(150\,\text{kPa})(1\text{ m}^3)}{(600\,\text{kPa})}\left[\frac{(100+273)\,\text{K}}{(20+273)\,\text{K}}\right] = 0.318\,\text{m}^3$$

To find n, return to the defining equation for a polytropic process

$$P_1V_1^n = P_2V_2^n$$

Rearranging:

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n$$

Taking the natural logarithm of both sides

$$\ln\left(\frac{P_1}{P_2}\right) = n\ln\left(\frac{V_2}{V_1}\right)$$

Solving for n

$$n = \frac{\ln\left(\dfrac{P_1}{P_2}\right)}{\ln\left(\dfrac{V_2}{V_1}\right)} = \frac{\ln\left(\dfrac{150}{600}\right)}{\ln\left(\dfrac{0.318}{1}\right)} = 1.21 \longleftarrow \textbf{Answer}$$

b) As long as $n \neq 1$, the work done in a polytropic process is

$$W = \frac{P_2V_2 - P_1V_1}{1-n}$$

Substituting values

$$W = \frac{\left[(600\,\text{kPa})(0.318)\text{m}^3 - (150\,\text{kPa})(1\,\text{m}^3)\right]\left(\dfrac{1000\,\text{Pa}}{1\text{kPa}}\right)}{1\text{-}1.21} = -1.95\times10^5\,\text{J}$$

$$W = -195\,\text{kJ} \longleftarrow \textbf{Answer}$$

c) From the first law

$$\Delta U = Q - W$$

$$Q = W + \Delta U = W + mc_V(T_2 - T_1)$$

The average temperature is $(20+100)/2 = 60\,^\circ\text{C} = 333\text{K}$. From Table A-8 for air at 333K, $c_V \approx 0.72$ kJ/(kg K).
The molecular weight of air is available in Table A-1. Find the mass of air from the ideal gas law:

$$m = \frac{P_1V_1M}{\bar{R}T_1}$$

$$m = \frac{(150\,\text{kPa})(1\,\text{m}^3)\left(28.97\dfrac{\text{kg}}{\text{kmol}}\right)\left(\dfrac{1000\,\text{Pa}}{1\,\text{kPa}}\right)}{\left(8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(20+273)\text{K}\left(\dfrac{1000\,\text{J}}{1\text{kJ}}\right)}$$

$$m = 1.78\,\text{kg}$$

Substituting values in the first law:

$$Q = -195\,\text{kJ} + (1.78\,\text{kg})\left(0.72\frac{\text{kJ}}{\text{kg K}}\right)(100-20)^\circ\text{C}$$

$$Q = -92.3\text{kJ} \longleftarrow \textbf{Answer}$$

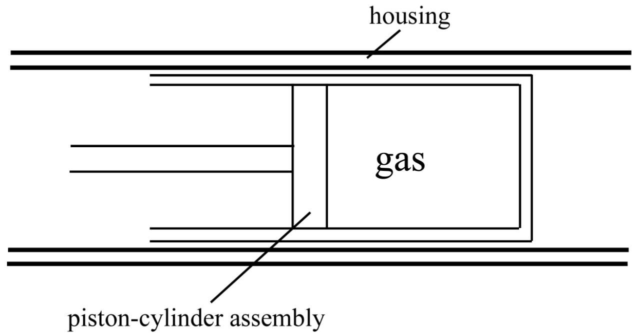
Comments:

The heat calculated is negative, and this indicates that the system is losing heat during this process.

2-44 A piston-cylinder assembly of total mass 16 lbm is free to move within a housing as shown in the figure. Initially the cylinder contains gas at an absolute pressure of 20 lbf/in.^{2} and 0.07 ft^3 and is at rest. The piston is then moved so that the entire assembly accelerates rightward and reaches a final velocity of 7.5 ft/s. During this process, the gas is compressed to a final pressure of 35 lbf/in.^{2} The process is adiabatic and the pressure is related to the volume by $PV^{1.4}$ = constant. Calculate the change in internal energy for this process in Btu.

Approach:

Apply the formula for work done during a polytropic process of an ideal gas. Use the first law including both kinetic energy change and compression work to determine the internal energy change.



Assumptions:

1. The gas is ideal.
2. Neglect the mass of the gas compared to the mass of the housing.
3. Specific heat is constant.
4. The process is adiabatic.
5. The compression is a quasi-equilibrium process.

Solution:

Choose the system to be the piston-cylinder assembly and the gas it contains. For a polytropic process of an ideal gas, work done is

$$W = \frac{P_2V_2 - P_1V_1}{1-n}$$

where n = 1.4 in this case. To find V_2, note that $P_1V_1^{1.4} = P_2V_2^{1.4}$; therefore

$$V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1.4}} V_1 = \left(\frac{20}{35}\right)^{\frac{1}{1.4}} (0.07) = 0.0469 \text{ ft}^2$$

$$W = \left[\frac{\left(35\,\frac{\text{lbf}}{\text{in.}^2}\right)\left(0.0469\,\text{ft}^3\right) - \left(20\,\frac{\text{lbf}}{\text{in.}^2}\right)\left(0.07\,\text{ft}^3\right)}{1-1.4}\right]\left(\frac{144\,\text{in.}^2}{1\,\text{ft}^2}\right)$$

$$W = -87.4 \text{ ft}\cdot\text{lbf}$$

Next find the change in kinetic energy

$$\Delta KE = \frac{1}{2}mV^2 = \left(\frac{1}{2}\right)(16\,\text{lbm})\left(7.5\,\frac{\text{ft}}{\text{s}}\right)^2\left(\frac{1\,\text{lbf}}{32.2\,\frac{\text{lbm}\cdot\text{ft}}{\text{s}^2}}\right) = 13.98\,\text{ft}\cdot\text{lbf}$$

From the first law:

$$\Delta U + \Delta KE + \Delta PE = Q - W$$

Since the process is adiabatic and there is no change in potential energy,

$$\Delta U = -\Delta KE - W = (-13.98 + 87.4)\,\text{lbf}\cdot\text{ft}\left(\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right)$$

$$\Delta U = 0.994 \text{ Btu}$$ ← **Answer**

2-45 Natural gas is a mixture of methane, ethane, propane and butane as well as other components. Composition varies by point of origin of the gas. Consider natural gas with an equivalent molecular weight of 23.6 and an equivalent specific heat, c_p, of 2.01 kJ/Kg K. The gas is slowly compressed in a frictionless, adiabatic process from an initial volume of 212 cm^3 to a final volume of 98 cm^3. If the initial pressure is 39 kPa, and the initial temperature is 15°C, find the final temperature and pressure. Assume the mixture can be modeled as an ideal gas.

Approach:

To find the final temperature use

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

The ratio of specific heat, k, can be determined after c_v is calculated with $c_p = c_v + \bar{R}/M$.

Assumptions:

1. Natural gas is ideal under these conditions.
2. The process is a quasi-equilibrium process.
3. Specific heat is constant.
4. The process is adiabatic.

Solution:

For an ideal adiabatic process of an ideal gas

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

where $k = \frac{c_p}{c_v}$. To find c_v, use

$$c_v = c_p - \frac{\bar{R}}{M} = 2.01 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} - \frac{8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}}{23.6 \frac{\text{kg}}{\text{kmol}}} = 1.66 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

The ratio of specific heats may now be determined from

$$k = \frac{c_p}{c_v} = \frac{2.01}{1.66} = 1.21$$

Solving for T_2 and substituting values,

$$T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{k-1} = (15 + 273)\left(\frac{212}{98}\right)^{(1.21-1)}$$

$$T_2 = 338.7 \text{ K} = 65.7^\circ\text{C} \longleftarrow \textbf{Answer}$$

For final pressure, use,

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{k}$$

Rearranging

$$P_2 = 39\left(\frac{212}{98}\right)^{1.21}$$

$$P_2 = 99.2 \text{ kPa} \longleftarrow \textbf{Answer}$$

2-46 Carbon monoxide is expanded slowly in a well-insulated, frictionless piston-cylinder assembly from 300 cm^3, 25ºC to 400 cm^3. Find the final temperature

Approach:

To find the final temperature use

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

Assumptions:

1. Carbon monoxide behaves like an ideal gas under these conditions.
2. The process is a quasi-equilibrium process.
3. Specific heat is constant.
4. The process is adiabatic.

Solution:

Since the process is slow and frictionless, it may be assumed quasi-static. Since it is also well-insulated, it maybe be assumed adiabatic. For an ideal adiabatic process of an ideal gas

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1}$$

Values for the specific heat ratio, k, are available in Table A-8. The final temperature is not known, but it is expected to be less than the initial temperature. As shown in the table, k is very insensitive to temperature, being almost the same at 250, 300, and 350 K. Therefore, k = 1.4 will do, and

$$T_2 = (25+273)\left(\frac{300}{400}\right)^{(1.4-1)} = 266 \text{ K} = -7^\circ\text{C}$$ **Answer**

2-47 Hydrogen with a mass of 1.1 kg is compressed slowly and adiabatically from 100 kPa, 25 °C to 450 kPa in a piston-cylinder assembly. Assuming constant specific heat, calculate the final temperature and the work done.

Approach:

Find the final temperature from $\frac{T_2}{T_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1-K}{K}}$ Then apply the first law to determine work.

Assumptions:

1. The process proceeds through a succession of quasi-equilibrium states.
2. Specific heat is constant.
3. Hydrogen behaves like an ideal gas under these conditions.
4. The process is adiabatic.

Solution:

For an adiabatic compression of an ideal gas with constant specific heat

$$\frac{T_2}{T_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1-K}{K}}$$

Solving for final temperature and substituting values

$$T_2 = (25+273)\text{K}\left(\frac{100\,\text{kPa}}{450\text{ kPa}}\right)^{\frac{1-1.405}{1.405}} = 460\,\text{K}$$

where k = 1.405 at 300K from Table A-8. From the first law

$$\Delta U = Q - W$$

Since the process is adiabatic Q = 0 and the first law becomes

$$W = -\Delta U = -mc_v(T_2 - T_1)$$

Use values for c_v from Table A-8 at 300 K to get

$$W = -(1.1\,\text{kg})\left(10.2\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left[460-(25+273)\right]\text{K}$$

$$W = -1818\text{ kJ}$$ **Answer**

2-48 Air at 14.7 psia and 100°F is contained in a well-insulated piston-cylinder assembly of initial volume 0.6 ft^3. The air is slowly expanded by applying 560 ft·lbf of work. What is the final pressure? Assume constant specific heats.

Approach:

Use the formula for work done in a polytropic process with $n = k$. Solve this simultaneously with

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^k$$

Assumptions:

1. The process proceeds through a succession of quasi-equilibrium states.
2. Specific heat is constant.
3. Air behaves like an ideal gas under these conditions.
4. The system is adiabatic.

Solution:

This is an adiabatic expansion of an ideal gas with constant specific heat, so it is polytropic with $n = k$. The work done is

$$W = \frac{P_2V_2 - P_1V_1}{1-k}$$

Also use:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^k$$

Rewriting the numerator of the work equation

$$W = \frac{P_1\left(\frac{P_2}{P_1}\right)V_2 - P_1V_1}{1-k}$$

Replacing the pressure ratio with a volume ratio

$$W = \frac{P_1\left(\frac{V_1}{V_2}\right)^k V_2 - P_1V_1}{1-k} = \frac{\left(P_1V_1^k\right)V_2^{1-k} - P_1V_1}{1-k}$$

Solving for V_2

$$V_2^{1-k} = \frac{W(1-k) + P_1V_1}{P_1V_1^k}$$

Simplifying and substituting the value of k for air at 100°F from Table B-8

$$V_2 = \left[\frac{W(1-k)+P_1V_1}{P_1V_1^k}\right]^{\frac{1}{1-k}} = \left[\frac{(560\,\text{ft}\cdot\text{lbf})(1\text{-}1.4)+\left(14.7\frac{\text{lbf}}{\text{in.}^2}\right)\left(\frac{144\,\text{in.}^2}{1\,\text{ft}^2}\right)(0.6\,\text{ft}^3)}{\left(14.7\frac{\text{lbf}}{\text{in.}^2}\right)\left(\frac{144\,\text{in.}^2}{1\,\text{ft}^2}\right)\left(0.6\,\text{ft}^3\right)^{1.4}}\right]^{\frac{1}{1-1.4}} = 0.975\,\text{ft}^3$$

Finally, the final pressure is

$$P_2 = \left(\frac{V_1}{V_2}\right)^k P_1 = \left(14.7\frac{\text{lbf}}{\text{in.}^2}\right)\left(\frac{0.6\,\text{ft}^2}{0.975\,\text{ft}^3}\right)^{1.4}$$

$$P_2 = 7.45\,\text{psia} \longleftarrow$$ **Answer**

2-49 Oxygen at 14.7 psia and 70°F is contained in a piston-cylinder assembly with an initial volume of 150 in.^{3} The oxygen is compressed slowly and adiabatically to a final volume of 50 in.^{3} Assume constant specific heat. Find

a. the final temperature
b. the final pressure
c. the work done (in ft·lbf)

Approach:

Find the final temperature from $\frac{T_2}{T_1}=\left(\frac{V_1}{V_2}\right)^{k-1}$ and the final pressure from $\frac{P_2}{P_1}=\left(\frac{V_1}{V_2}\right)^{k}$ Then apply the first law to determine work.

Assumptions:

1. The process proceeds through a succession of quasi-equilibrium states.
2. Specific heat is constant.
3. Oxygen behaves like an ideal gas under these conditions.
4. The process is adiabatic.

Solution:

a) For an adiabatic compression of an ideal gas with constant specific heat

$$\frac{T_2}{T_1}=\left(\frac{V_1}{V_2}\right)^{k-1}$$

Solving for T_2

$$T_2=T_1\left(\frac{V_1}{V_2}\right)^{k-1}$$

The final temperature is unknown. For simplicity, we evaluate the ratio of specific heats, k, at the given initial temperature. Once final temperature is estimated, we can correct the value of k if necessary. For oxygen at 70°F, k = 1.39 from Table B-8. Using this and given values

$$T_2=(70+460)\,\mathrm{R}\left(\frac{150\,\mathrm{in.}^3}{50\,\mathrm{in.}^3}\right)^{(1.39-1)}$$

$$T_2=813\,\mathrm{R}=353^\circ\mathrm{F} \longleftarrow \textbf{Answer}$$

b) To find the final pressure, use

$$\frac{P_2}{P_1}=\left(\frac{V_1}{V_2}\right)^{k}$$

Solving for P_2

$$P_2=P_1\left(\frac{V_1}{V_2}\right)^{k}=14.7\,\mathrm{psia}\left(\frac{150\,\mathrm{in.}^3}{50\,\mathrm{in.}^3}\right)^{1.39}$$

$$P_2=67.7\,\mathrm{psia} \longleftarrow \textbf{Answer}$$

c) From the first law

$$\Delta U=Q-W$$

Since the process is adiabatic, $Q=0$ and

$$\Delta U=-W$$

The oxygen is assumed to be an ideal gas, therefore

$$W = mc_v(T_1 - T_2)$$

To find the mass, use

$$m = \frac{P_1 V_1 M}{\bar{R} T_1}$$

Substituting values:

$$m = \frac{(14.7\ \text{psia})(150\ \text{in.}^3)\left(32\,\frac{\text{lbm}}{\text{lbmol}}\right)}{\left(10.73\,\frac{\text{psia}\cdot\text{ft}^3}{\text{lbmol}\cdot\text{R}}\right)\left(\frac{12\,\text{in.}}{1\,\text{ft}}\right)^3(70+460)\text{R}} = 0.00718\ \text{lbm}$$

The work may now be evaluated from

$$W = (0.00718\ \text{lbm})\left(0.157\,\frac{\text{Btu}}{\text{lbm}\cdot\text{R}}\right)(70-353)\,^{\circ}\text{F}\left(\frac{778\,\text{ft}\cdot\text{lbf}}{1\,\text{Btu}}\right)$$

$$W = -248\,\text{ft}\cdot\text{lbf}$$ **Answer**

Comments:

The final temperature is 353 °F; therefore, the average temperature for the process is $T_{ave} = (70+353)/2 = 211\,^{\circ}\text{F}$. The ratio of specific heats, *k*, at the average temperature is close to the value at the initial temperature of 70°F, as seen in Table B-8. As a result, there is no need to iterate.

2-50 Nitrogen at 850 K, 2 MPa expands slowly and adiabatically until the final temperature is 300 K. Assuming constant specific heat, find the final pressure and the ratio of final to initial volume.

Approach:

Find the final pressure from $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$ Then apply $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$ to find the percent increase in volume.

Assumptions:

1. The process proceeds through a succession of quasi-equilibrium states.
2. Specific heat is constant.
3. Nitrogen behaves like an ideal gas under these conditions.
4. The process is adiabatic.

Solution:

For an adiabatic expansion of an ideal gas with constant specific heat

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

Solving for P_2

$$P_2 = P_1\left(\frac{T_1}{T_2}\right)^{\frac{k}{1-k}}$$

Evaluate k for nitrogen at the average temperature of 575 K. From Table A-8, k = 1.38. Substituting values:

$$P_2 = (2000\,\text{kPa})\left(\frac{850}{300}\right)^{\frac{1.38}{1-1.38}}$$

$$P_2 = 45.6\,\text{kPa} \longleftarrow \textbf{Answer}$$

For percent increase in volume, use

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

Rearranging

$$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{k-1}}$$

Substituting values

$$\frac{V_2}{V_1} = \left(\frac{850}{300}\right)^{\frac{1}{1.38-1}} = 15.5 \longleftarrow \textbf{Answer}$$

Comments:

For this process to occur, the volume must increase by more than 15 times.

2-51 Air with a mass of 0.17 lbm is slowly compressed in a well-insulated, frictionless, piston-cylinder assembly from 14.7 psia to 68 psia. If the air is initially at 60°F,

a. find the final temperature.

b. find the work done (in ft·lbf).

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Approach:

Find the final pressure from $\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$ Then apply the first law to find the work done.

Assumptions:

1. The process proceeds through a succession of quasi-equilibrium states.
2. Specific heat is constant.
3. Air behaves like an ideal gas under these conditions.
4. The process is adiabatic.

Solution:

a. For an adiabatic compression of an ideal gas with constant specific heat

$$T_2=T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

The final temperature is unknown. As an approximation, we use the ratio of specific heats at the initial temperature. We will iterate later if necessary. From Table B-8, for air at 60ºF, k = 1.4. Therefore

$$T_2=(60+460)\left(\frac{68}{14.7}\right)^{\frac{1.4-1}{1.4}}=805\,\text{R}=345\,^{\circ}\text{F}$$ **Answer**

b. From the first law

$$Q=\Delta U+W$$

The process is adiabatic, the gas is ideal, and the specific heats are constant, so

$$W=-\Delta U=-m\Delta u=-mc_v\Delta T=-mc_v\left(T_2-T_1\right)$$

To evaluate specific heat, we take advantage of having a good estimate of the final temperature from part a. and evaluate specific heat at the average of initial and final temperatures, or

$$T_{ave}=\frac{345+60}{2}=202\,^{\circ}\text{F}\quad\therefore c_v\approx 0.173\frac{\text{Btu}}{\text{lbm}\cdot\text{R}}$$

where Table B-8 has again been used. Substituting values

$$W=-(0.17\,\text{lbm})\left(0.173\frac{\text{Btu}}{\text{lbm}\cdot\text{R}}\right)(345-60)\,\text{R}$$

$$=-8.38\,\text{Btu}\left(\frac{778\,\text{ft}\cdot\text{lbf}}{1\,\text{Btu}}\right)$$

$$W=6521\,\text{ft}\cdot\text{lbf}$$ **Answer**

2-52 Air is slowly expanded at constant pressure from an initial temperature of 300 K to a final temperature of 700 K in a piston-cylinder assembly. The initial volume of air is 250 cm^3 and the pressure is 150 kPa. Calculate the work done and the heat transferred

a. using variable specific heats.
b. using constant specific heats.

Approach:

Find work from $W = P\left(V_2 - V_1\right)$ and then use the first law to find heat. In part a., use values of internal energy from Table A-9, and in part b., rewrite internal energy in terms of temperature and specific heat from Table A-8.

Assumptions:

1. The process proceeds through a succession of quasi-equilibrium states.
2. Air behaves like an ideal gas under these conditions.

Solution:

a) From the first law

$$\Delta U = Q - W$$

Since the process is slow, and pressure is constant, the work is obtained from

$$W = \int P\,dV = P\int dV = P\left(V_2 - V_1\right)$$

The ideal gas law may be used to find the final volume, V_2. Start with

$$P_1V_1 = \frac{m\bar{R}T_1}{M} \qquad \text{and} \qquad P_2V_2 = \frac{m\bar{R}T_2}{M}$$

Dividing one equation by the other and noting that $P_1 = P_2$,

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

Solving for V_2,

$$V_2 = V_1\left(\frac{T_2}{T_1}\right) = \left(250\,\text{cm}^3\right)\left(\frac{700\,\text{K}}{300\,\text{K}}\right) = 583\,\text{cm}^3$$

Work may now be calculated as

$$W = (150\,\text{kPa})\left(\frac{1000\,\text{Pa}}{1\,\text{kPa}}\right)(583-250)\,\text{cm}^3\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)^3 = 50\,\text{J}$$

Find heat from the first law in the form

$$Q = W + \Delta U = W + m\left(u_2 - u_1\right)$$

To find mass, use the ideal gas law

$$m = \frac{P_1V_1M}{\bar{R}T_1} = \frac{(150\,\text{kPa})\left(250\,\text{cm}^3\right)\left(28.97\,\frac{\text{kg}}{\text{kmol}}\right)\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)^3}{\left(8.314\,\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(300\,\text{K})} = 4.35\times10^{-4}\,\text{kg}$$

In part a., variable specific heats are used. Taking values of u_1 and u_2 from Table A-9,

$$Q = 50\,\text{J} + \left(4.35\times10^{-4}\text{kg}\right)(512.3\text{-}214.1)\frac{\text{kJ}}{\text{kg}}\left(\frac{1000\,\text{J}}{1\,\text{kJ}}\right)$$

$$Q = 180\,\text{J}$$ **Answer**

b) If constant specific heats are used, the first law may be written

$$Q = W + \Delta U = W + m\left(u_2 - u_1\right) = W + mc_v\left(T_2 - T_1\right)$$

The value of c_v is chosen at the average of T_1 and T_2, or T_{ave} = 500 K. Using c_v for air from Table A-8, and noting that work, mass, and temperature are unchanged,

$$Q = 50\,\text{J} + \left(4.35\times10^{-4}\text{kg}\right)\left(0.742\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(700\text{-}300)\,\text{K}\left(\frac{1000\,\text{J}}{1\,\text{kJ}}\right)$$

$Q = 179.3\,\mathrm{J}$ ⟵ **Answer**

Comment:

The result for variable specific heat was very close to that for constant specific heat. This is often the case for gases when the temperature change is not too large.

2-53 A rigid tank of volume 4.2 ft^3 contains air initially at 100°F and 14.7 psia. Heat is added until the final pressure is 70.9 psia. Assuming variable specific heat, find the heat added.

Approach:

Find heat from the first law, noting that work is zero in a rigid tank. Use values of internal energy from Table B-9 and calculate mass with the ideal gas law.

Assumptions:

1. The process proceeds through a succession of quasi-equilibrium states.
2. Air behaves like an ideal gas under these conditions.

Solution:

From the first law

$$\Delta U = Q - W$$

The tank is rigid, so $W = 0$, and

$$m\left(u_2 - u_1\right) = Q$$

To find u_2, we need the final temperature, T_2. From the ideal gas law,

$$P_1V_1 = \frac{m\bar{R}T_1}{M} \qquad \text{and} \qquad P_2V_2 = \frac{m\bar{R}T_2}{M}$$

Dividing one equation by the other and noting that $V_1 = V_2$,

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

Rearranging

$$T_2 = T_1\left(\frac{P_2}{P_1}\right) = \left(100+460\right)\text{R}\left(\frac{70.9}{14.7}\right) = 2701\,\text{R}$$

We also need the mass, m, of air in the tank. From the ideal gas law

$$m = \frac{P_1V_1M}{\bar{R}T_1} = \frac{(14.7\text{ psia})(4.2\text{ ft}^3)\left(28.97\dfrac{\text{lbm}}{\text{lbmol}}\right)}{\left(10.73\dfrac{\text{psia}\cdot\text{ft}^3}{\text{lbmol}\cdot\text{R}}\right)(100+460)\text{R}} = 0.298\text{ lbm}$$

Taking values of u_1 and u_2 from Table B-9 (interpolating for u_2),

$$Q = \left(0.298\,\text{lbm}\right)\left(518.5-95.47\right)\frac{\text{Btu}}{\text{lbm}}$$

$Q = 126\,\text{Btu}$ ⟵ **Answer**

2-54 A rigid tank contains 0.05 kg of air at 800 K and 300 kPa. The tank is cooled while 6.35 kJ of heat are transferred. Find the final air temperature and pressure assuming variable specific heat.

Approach:

Find heat from the first law, noting that work is zero in a rigid tank. Use values of internal energy from Table B-9 to determine the final air temperature. The pressure can then be found from the ideal gas law.

Assumptions:

1. The process proceeds through a succession of quasi-equilibrium states.
2. Air behaves like an ideal gas under these conditions.

Solution:

From the first law

$$\Delta U = Q - W$$

The tank is rigid, so $W = 0$, and

$$m(u_2 - u_1) = Q$$

Using the value of u_2 at 800 K from Table A-9, and noting that Q is negative because the air is cooled,

$$u_2 = \frac{Q}{m} + u_1 = \frac{-6.35\,\text{kJ}}{0.05\,\text{kg}} + 592.3\frac{\text{kJ}}{\text{kg}} = 465.3\frac{\text{kJ}}{\text{kg}}$$

From Table A-9, the temperature corresponding to u_2 is

$$T_2 = 640\,\text{K}$$ **Answer**

To find pressure, use the ideal gas law

$$P_1V_1 = \frac{m\bar{R}T_1}{M} \quad \text{and} \quad P_2V_2 = \frac{m\bar{R}T_2}{M}$$

Dividing one equation by the other and noting that $V_1 = V_2$,

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

Rearranging

$$P_2 = P_1\left(\frac{T_2}{T_1}\right) = (300)\,\text{kPa}\left(\frac{640}{800}\right) = 240\,\text{kPa}$$ **Answer**