

***Principles of Modern Communication Systems 1st
edition Agbo Solutions Manual***

SKU: 9781107107922-SOLUTIONS

CHAPTER 2

P. P 2.1

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T y^2(t) dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T 4 \cos^2 \pi t dt = \lim_{T \rightarrow \infty} \frac{4}{2T} \int_{-T}^T \frac{1}{2} (1 + \cos 2\pi t) dt$$

$$= \lim_{T \rightarrow \infty} \frac{4}{2T} \frac{2T}{2} = 2$$

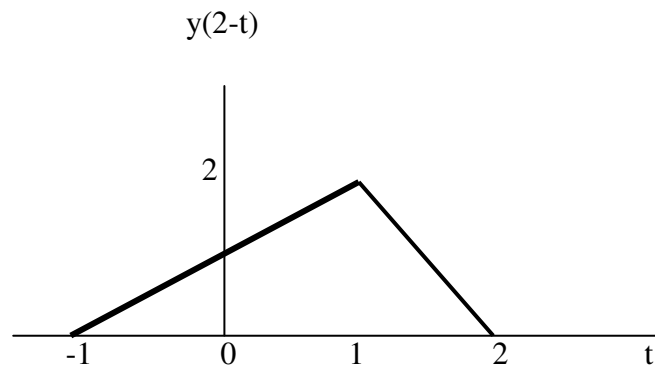
Since $0 < P < \infty$, i.e. the signal has finite power, it is a power signal.

P.P. 2.2

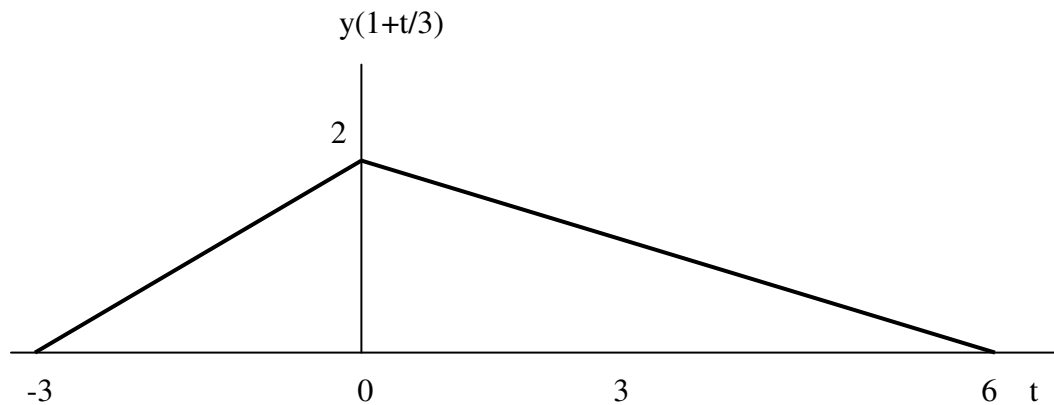
$$(a) \quad t(2-t) = \begin{cases} 2(2-t), & 0 < 2-t < 1 \\ 3-(2-t), & 1 < 2-t < 3 \end{cases} = \begin{cases} 4-2t, & 1 < t < 2 \\ 1+t, & -1 < t < 1 \end{cases} = \begin{cases} 1+t, & -1 < t < 1 \\ 4-2t, & 1 < t < 2 \end{cases}$$

$$(b) \quad y(1+t/3) = \begin{cases} 2(1+t/3), & 0 \leq 1+t/3 < 1 \\ 3-(1+t/3), & 1 \leq 1+t/3 \leq 3 \end{cases} = \begin{cases} 2+2t/3, & -3 \leq t \leq 0 \\ 2-t/3, & 0 < t < 6 \end{cases}$$

These are sketched below.



(a)



(b)

P.P. 2.3

Comparing the given signal with the square wave in Table 2.2, we notice that

$A = 1, T = 2, \omega_o = 2\pi / T = \pi$. Hence,

$$f(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)\pi t = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{1}{n} \sin \pi n t, \quad n = 2k-1$$

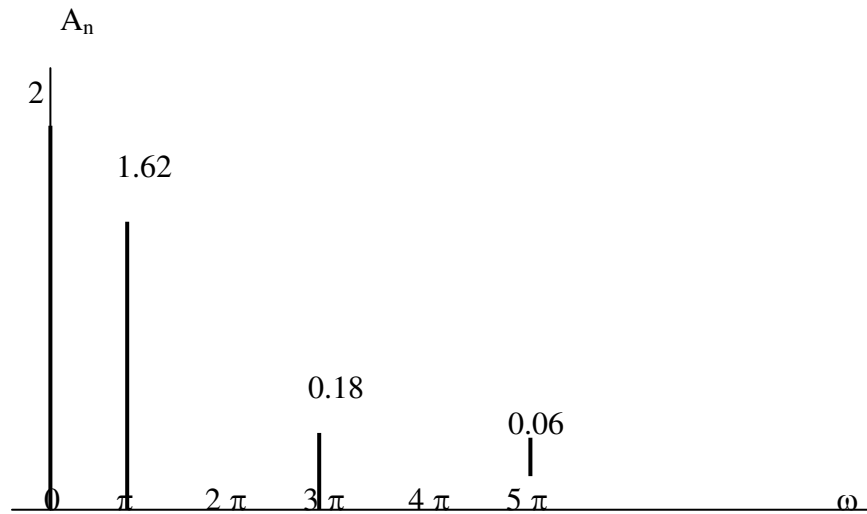
P.P. 2.4

We comparing the given signal with the triangular wave in Table 2.2, we notice that

$A = 4, T = 2, \omega_o = 2\pi / T = \pi$. Hence,

$$h(t) = \frac{4}{2} - \frac{16}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos(2n-1)\pi t}{(2n-1)^2} = 2 - \frac{16}{\pi^2} \sum_{k=1}^{\infty} \frac{\cos n\pi t}{n^2}, \quad n = 2k-1$$

The phase spectrum is zero, while the amplitude spectrum is shown below.

**P.P. 2.5**

$T = 10, \omega_o = 2\pi / T = \pi / 5$

$$\begin{aligned} c_n &= \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_o t} dt = \frac{1}{10} \int_{-1}^1 10 e^{-jn\omega_o t} dt = \frac{1}{-jn\omega_o} e^{-jn\omega_o t} \Big|_{-1}^1 = \frac{1}{-jn\omega_o} (e^{-jn\omega_o} - e^{jn\omega_o}) \\ &= \frac{2}{n\omega_o} \left(\frac{e^{jn\omega_o} - e^{-jn\omega_o}}{2j} \right) = 2 \frac{\sin n\omega_o}{n\omega_o} = 2 \frac{\sin n\pi / 5}{n\pi / 5} \end{aligned}$$

$$f(t) = 2 \sum_{n=-\infty}^{\infty} \frac{\sin \lambda}{\lambda} e^{j\lambda t}, \quad \lambda = \frac{n\pi}{5}$$

P.P. 2.6

$$C_n = \frac{1}{T} \int_0^T f(t) e^{-j\omega_o n t} dt, \quad \omega_o = \frac{2\pi}{2} = \pi$$

$$C_n = \frac{1}{2} \int_{-1}^1 t e^{-jn\pi t} dt$$

Using integration by parts,

$$u = t \text{ and } du = dt$$

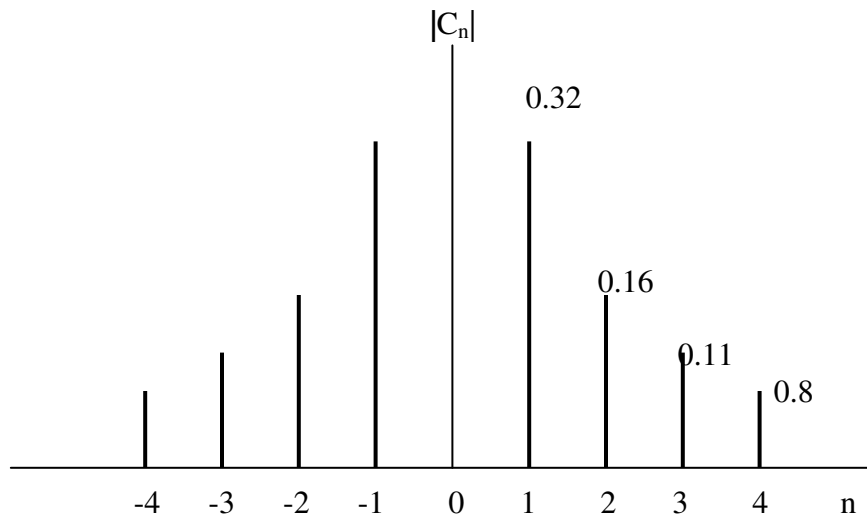
$$dv = e^{-jn\pi t} dt \text{ which leads to } v = -[1/(2jn\pi)] e^{-jn\pi t}$$

$$\begin{aligned} C_n &= -\frac{t}{2jn\pi} e^{-jn\pi t} \Big|_{-1}^1 - \frac{1}{2jn\pi} \int_{-1}^1 e^{-jn\pi t} dt \\ &= \frac{j}{2n\pi} [e^{-jn\pi} + e^{jn\pi}] + \frac{1}{2n^2\pi^2(-j)^2} e^{-jn\pi t} \Big|_{-1}^1 \\ &= [j/(n\pi)] \cos(n\pi) - [1/(2n^2\pi^2)] (e^{-jn\pi} - e^{jn\pi}) \\ C_n &= \frac{j(-1)^n}{n\pi} - \frac{2j}{2n^2\pi^2} \sin(n\pi) = \frac{j(-1)^n}{n\pi} \end{aligned}$$

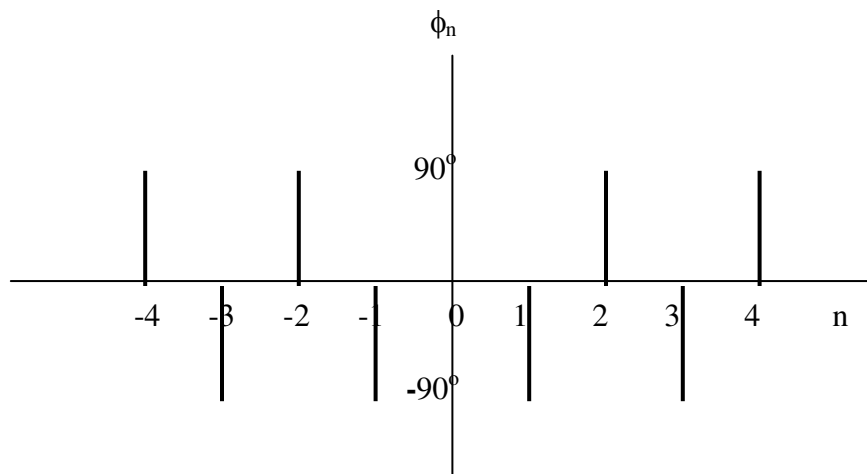
Thus

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_o t} = \sum_{n=-\infty}^{\infty} (-1)^n \frac{j}{n\pi} e^{jn\pi t}$$

The amplitude and phase spectra are shown next.



(a)



(b)

P.P. 2.7

$$P = \frac{1}{T} \int_0^T f^2(t) dt = \frac{1}{2\pi} \int_0^\pi (1)^2 dt = \frac{\pi}{2\pi} = 0.5$$

$$C_o = \frac{1}{T} \int_0^T f(t) dt = \frac{1 \times \pi}{2\pi} = 0.5$$

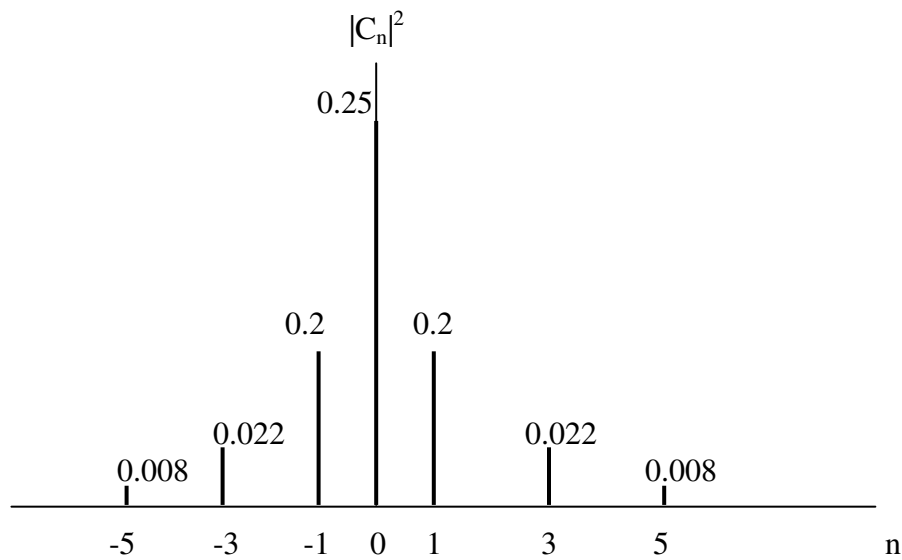
$$\omega_o = 2\pi / T = 1$$

$$C_n = \frac{1}{T} \int_0^T f(t) e^{-jn\omega_o t} dt = \frac{1}{2\pi} \int_0^\pi (1) e^{-jnt} dt = \frac{1}{2\pi(-jn)} (e^{-jn\pi} - 1)$$

$$= \frac{j}{2n\pi} (\cos n\pi - j \sin n\pi - 1) = \begin{cases} \frac{-j}{n\pi}, & n = 1, 3, 5, 7, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$|C_n|^2 = \begin{cases} \frac{1}{n^2 \pi^2}, & n = 1, 3, 5, 7, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

The power spectrum is shown below.



P.P. 2.8

$$(a) \quad X(\omega) = \int_0^1 (1) e^{-j\omega t} dt = -\frac{1}{j\omega} [e^{-j\omega} - 1] = \frac{1}{j\omega} [1 - e^{-j\omega}]$$

$$(b) \quad \mathcal{F}[\delta(t - t_o)] = e^{-j\omega t_o}$$

Let $t_o = -3$.

$$X(\omega) = e^{j3\omega}$$

$$(c) \quad X(\omega) = 2\mathcal{F}\left[\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right]$$

$$= 2\pi [\delta(\omega + \omega_o) + \delta(\omega - \omega_o)]$$

P.P. 2.9

$$X(\omega) = \int_{-\tau}^0 \left(1 + \frac{t}{\tau}\right) e^{-j\omega t} dt + \int_0^{\tau} \left(1 - \frac{t}{\tau}\right) e^{-j\omega t} dt$$

$$= \int_0^{\tau} \left(1 - \frac{t}{\tau}\right) e^{+j\omega t} dt + \int_0^{\tau} \left(1 - \frac{t}{\tau}\right) e^{-j\omega t} dt$$

$$= 2 \int_0^{\tau} \left(1 - \frac{t}{\tau}\right) \cos \omega t dt$$

$$= \frac{2 \sin \omega \tau}{\omega} - \frac{2}{\tau \omega^2} (\cos \omega \tau + \omega \tau \sin \omega \tau - 1)$$

$$= \frac{2}{\tau \omega^2} (1 - \cos \omega \tau)$$

But $\cos 2u = 1 - 2 \sin^2 u$

$$1 - \cos \omega \tau = 2 \sin^2 \omega \tau / 2$$

$$X(\omega) = \frac{4}{\tau \omega^2} \sin^2 \omega \tau / 2 = \tau \frac{\sin^2 \omega \tau / 2}{(\omega \tau / 2)^2}$$

$$= \tau \sin^2 \frac{\omega \tau}{2}$$

P.P. 2.10

$$f(t) = \begin{cases} e^{at}, & t < 0 \\ 0, & t > 0 \end{cases}$$

$$\begin{aligned} F(\omega) &= \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt = \int_{-\infty}^0 e^{at} e^{-j\omega t} dt = \int_{-\infty}^0 e^{(a-j\omega)t} dt \\ &= \frac{1}{a-j\omega} e^{(a-j\omega)t} \bigg|_{-\infty}^0 = \frac{1}{a-j\omega} \end{aligned}$$

The energy spectral density is

$$G(\omega) = |F(\omega)|^2 = \frac{1}{a^2 + \omega^2}, \quad \omega = 2\pi f \quad (2.10.2)$$

Thus, the energy contained in the frequency range of $-f_o < f < f_o$ is

$$E = \int_{-f_o}^{f_o} \frac{df}{a^2 + (2\pi f)^2} \quad (2.10.3)$$

The integrand is even so that we can integrate from 0 to f_o and multiply by 2.

If we let $u = 2\pi f/a$ so that $df = adu/2\pi$, eq. (2.10.3) becomes

$$E = \frac{2a}{2\pi a^2} \int_0^{2\pi f_o/a} \frac{du}{1+u^2} = \frac{1}{\pi a} \tan^{-1} \frac{2\pi f_o}{a}$$

P.P. 2.11

$$(a) \quad X(\omega) = 1(e^{-j\omega^2} + e^{-j\omega} + e^{j\omega} + e^{j\omega^2}) = 2(\cos \omega + \cos 2\omega)$$

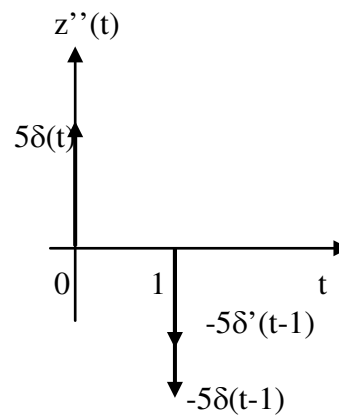
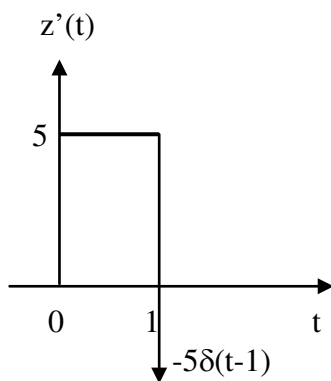
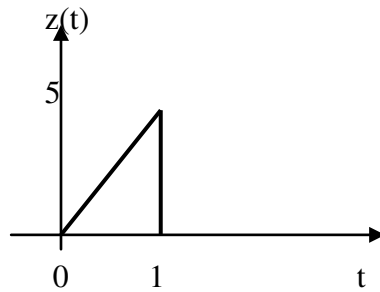
$$(b) \quad y(t) = 2te^{-t}u(t)$$

$$\text{Let } g(t) = 2e^{-t}u(t) \longrightarrow G(\omega) = \frac{2}{1+j\omega}$$

$$y(t) = tg(t) \longrightarrow j \frac{dG(\omega)}{d\omega} = j2(-1)j(1+j\omega)^{-2}$$

$$Y(\omega) = \frac{2}{(1+j\omega)^2}$$

(c) $z(t)$ and its derivatives are shown below.



$$z''(t) = 5\delta(t) - 5\delta(t-1) - 5\delta'(t-1)$$

$$(j\omega)^2 Z(\omega) = 5 - 5e^{-j\omega} - 5j\omega e^{-j\omega}$$

$$Z(\omega) = \frac{-5}{\omega^2} (1 - e^{-j\omega} - j\omega e^{-j\omega})$$

P.P. 2.12

Taking the first derivative gives $y'(t)$ in Fig. A. Taking the second derivative gives $y''(t)$ as in Fig. B.

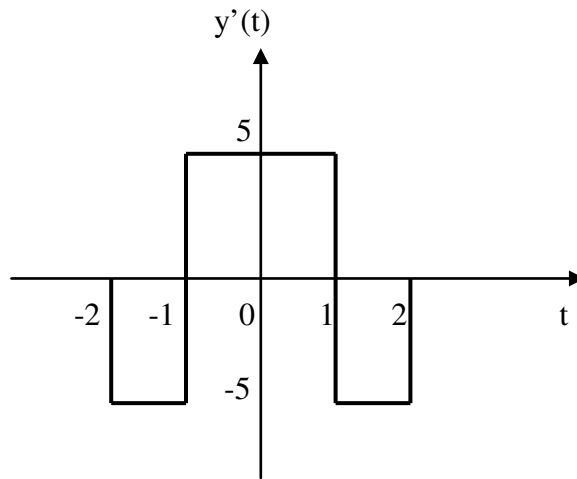


Fig. A

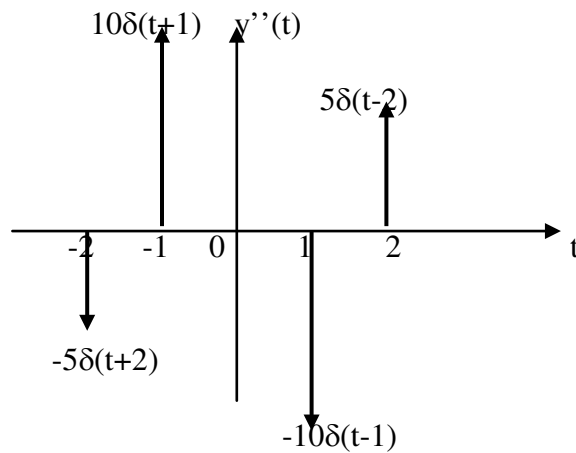


Fig. B

From Fig. B,

$$y''(t) = -5\delta(t+2) + 10\delta(t+1) - 10\delta(t-1) + 5\delta(t-2)$$

Taking the Fourier transform of each term,

$$\begin{aligned}(j\omega)^2 Y(\omega) &= -5e^{j2\omega} + 10e^{j\omega} - 10e^{-j\omega} + 5e^{-j2\omega} \\ &= -5(e^{j2\omega} - e^{-j2\omega}) + 10(e^{j\omega} - e^{-j\omega})\end{aligned}$$

$$-\omega^2 Y(\omega) = 20j \sin \omega - 10j \sin 2\omega$$

or

$$Y(\omega) = \frac{j10}{\omega^2} (\sin 2\omega - \sin \omega)$$

P.P. 2.13

(a) $f(t) = e^{-(t-2)}u(t-2)$

$$\begin{aligned}
 \text{(b) } g(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\pi \delta(\omega) e^{j\omega t}}{(5+j\omega)(2+j\omega)} d\omega \\
 &= \frac{1}{2} \frac{1}{(5)(2)} = \frac{1}{20} = 0.05
 \end{aligned}$$

P.P. 2.14

$$-20 = 20 \log |H| \longrightarrow H = 10^{-20/20} = 0.1$$

$$1 + \left(\frac{\omega}{\omega_c} \right)^2 = \frac{1}{0.1^2} = 100$$

$$\text{Let } X = \frac{\omega}{\omega_c} = \frac{2\pi f}{2\pi f_c} = \frac{f}{f_c}$$

$$X^n = 99 \longrightarrow X = 99^{1/n} = 99^{1/3} = 4.6261$$

$$f_c = \frac{f}{X} = \frac{50}{4.6261} = 10.81 \text{ MHz}$$

$$H(s) = \frac{\omega_c^3}{s^3 + 2\omega_c s^2 + 2\omega_c^2 s + \omega_c^3}$$

$$\text{where } \omega_c = 2\pi f_c = 2\pi \times 10.81 \times 10^6 \text{ rad/s} = 6.791 \times 10^7 \text{ rad/s}$$

P.P. 2.15

$$\begin{aligned}
 H(\omega) &= \frac{V_o}{V_i} = \frac{R \parallel \frac{1}{j\omega C}}{j\omega L + R \parallel \frac{1}{j\omega C}} = \frac{\frac{R}{1+j\omega C}}{j\omega L + R \parallel \frac{1}{j\omega C}} = \frac{\frac{R}{1+j\omega RC}}{j\omega L + \frac{R}{1+j\omega RC}} \\
 &= \frac{R}{R(1-\omega^2 LC) + j\omega L}
 \end{aligned}$$

Since $R = 1$,

$$H(s) = \frac{1}{s^2 LC + sL + 1} \quad (s = j\omega)$$

$$H(s) = \frac{1/LC}{s^2 + s/C + 1/LC}$$

We compare this with

$$H(s) = \frac{k}{s^2 + 1.4142\omega_c s + \omega_c^2}$$

$$H(0) = \frac{k}{\omega_c^2} = 1$$

$$\omega_c^2 = \frac{1}{LC}$$

$$\frac{1}{C} = 1.4142 \quad \longrightarrow \quad C = 0.7071 = 707.1 \text{ mF}$$

$$\text{If } \omega_c = 1, \quad L = 1/C = 1.4142 \text{ H}$$

Prob.2.1

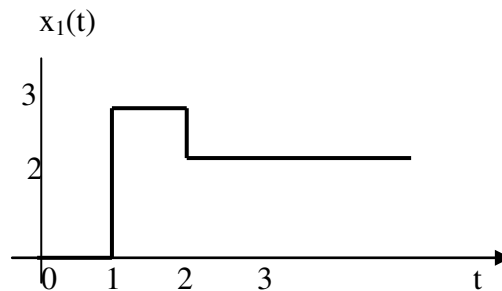
- (a) An analog signal is a continuous-time signal in which the variation with time is analogous to some physical phenomenon.
- (b) A digital signal is a discrete-time signal that can have a finite number of values (usually binary).
- (c) A continuous-time signal takes a value at every instant of time.
- (d) A discrete-time signal is defined only at particular instants of time.

Prob. 2.2

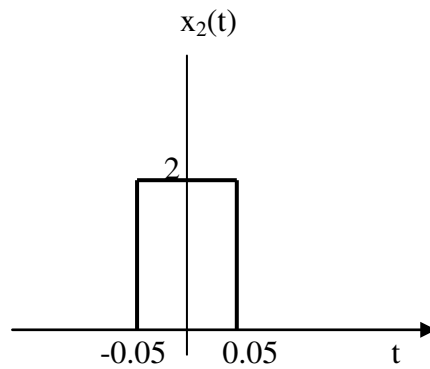
- (a) A periodic signal is one that repeats itself every T seconds.
- (b) An aperiodic signal is a nonperiodic signal, i.e. it does not repeat itself.
- (c) A signal is said to be an energy signal when the total energy E of the signal satisfies the condition $0 < E < \infty$
- (d) A signal is called a power signal when its average power P satisfies the condition $0 < P < \infty$

Prob. 2.3

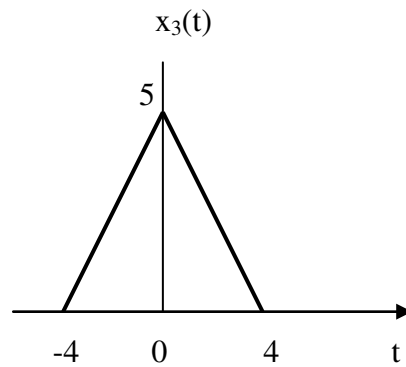
- (a) $x_1(t)$ is sketched below.



(b) $x_2(t) = 2\Pi(10t) = 2\Pi(t/0.1)$. It is sketched below.



(c) $x_3(t)$ is shown below.

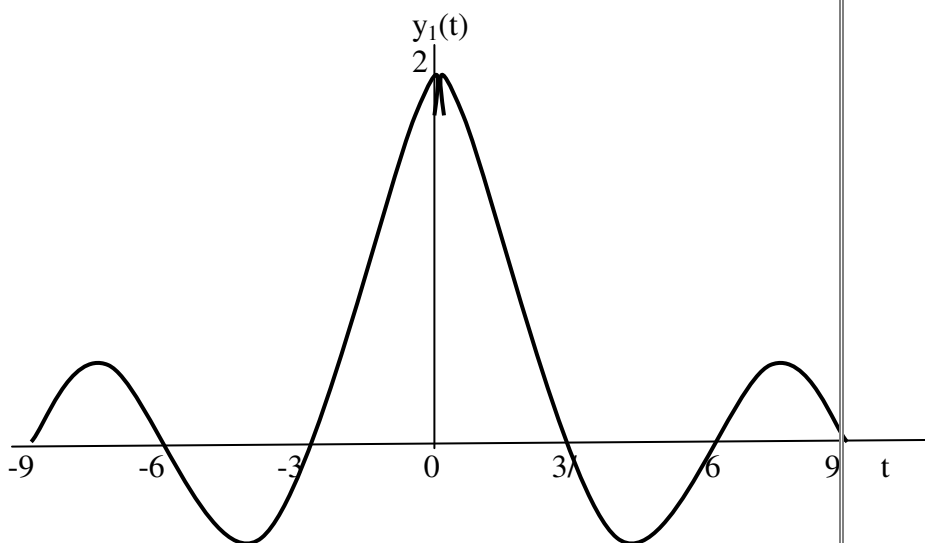


Prob. 2.4

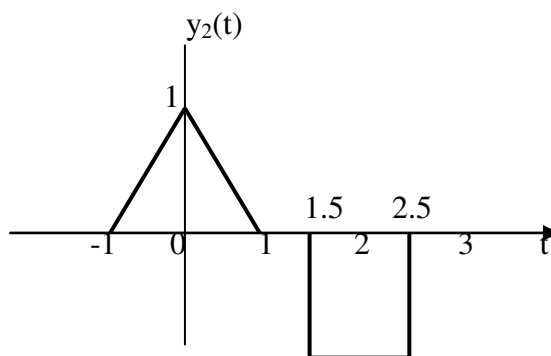
(a) Since $\text{sinc}(x) = \sin(x)/x$,

$$\sin(\pi t_o / 3) = \sin(n\pi) \longrightarrow t_o = 3n$$

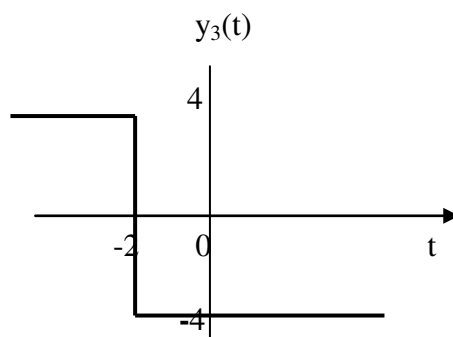
$y_1(t)$ is sketched below.



(b) $y_2(t)$ is sketched below.

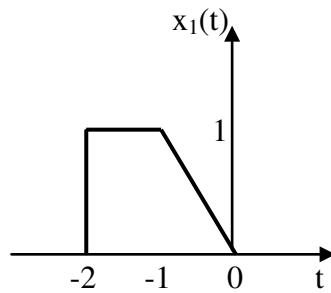


(c) $y_3(t)$ is shown below.

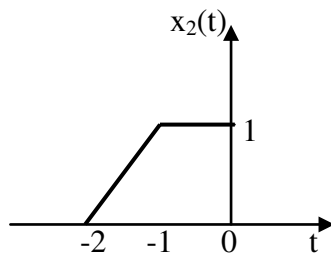


Prob. 2.5

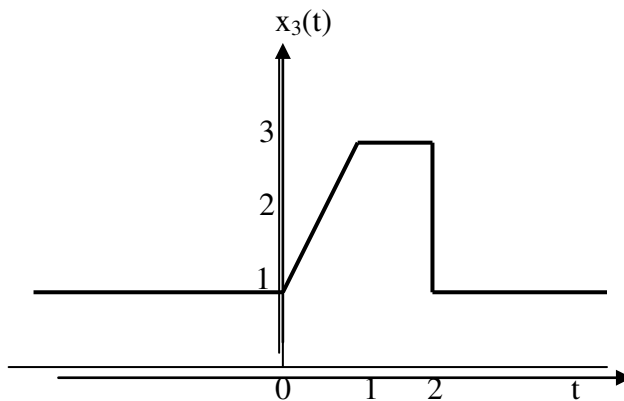
(a) $x_1(t) = x(-t)$ is sketched below.



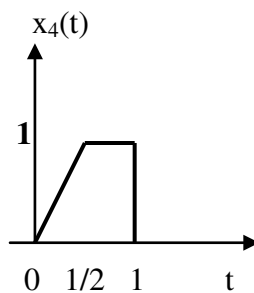
(b) $x_2(t) = x(t+2)$ is sketched below.



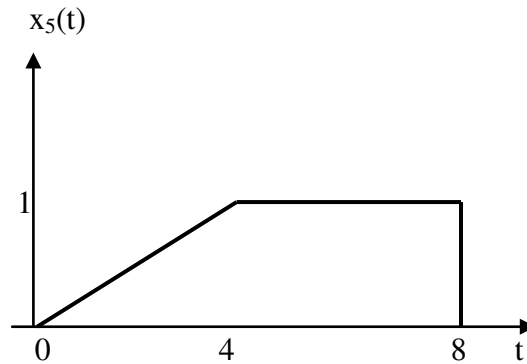
(c) $x_3(t) = 1 + 2x(t)$ is shown below.



(d) $x_4(t) = x(2t)$ is shown below.



(e) $x_5(t) = x(t/4)$ is sketched below.



Prob. 2.6

$$(a) \quad E = \int_{-\infty}^{\infty} (x(t))^2 dt = \int_{-\infty}^{\infty} e^{-2|t|} dt = 2 \int_0^{\infty} e^{-2t} dt = \frac{2}{-2} e^{-2t} \Big|_0^{\infty} = 1 < \infty$$

i.e. $x(t)$ is an energy signal.

$$(b) \quad E = \lim_{T \rightarrow \infty} \int_0^T (y(t))^2 dt = \lim_{T \rightarrow \infty} \int_0^T t^2 dt = \lim_{T \rightarrow \infty} \frac{T^3}{3} = \infty$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (y(t))^2 dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T t^2 dt = \lim_{T \rightarrow \infty} \frac{1}{T} \frac{T^3}{3} = \infty$$

i.e. $y(t)$ is neither energy nor power signal.

$$(c) \quad E = \int_{-\tau}^{\tau} (z(t))^2 dt = 2 \int_0^{\tau} \left(1 - \frac{t}{\tau}\right)^2 dt = 2 \left(t - \frac{t^2}{\tau} + \frac{t^3}{3\tau^2} \right) \Big|_0^{\tau} = 2 \left(\tau - \tau + \frac{1}{3} \tau \right) = \frac{2}{3} \tau$$

Hence, $z(t)$ is an energy signal.

Prob. 2.7

It is not possible to generate power signal in a lab because such a signal would have infinite duration and infinite energy. Signals generated in the lab have finite energy and are energy signals.

Prob. 2.8

$$P = \frac{1}{T} \int_0^T f^2(t) dt$$

$$P_g = \frac{1}{T} \int_0^T g^2(t) dt = \frac{1}{T} \int_0^T a^2 f^2(bt + c) dt$$

$$\text{Let } bt + c = \lambda, \quad dt = d\lambda / b$$

$$P_g = \frac{a^2}{b} \frac{1}{T} \int_c^{bT+c} f^2(\lambda) d\lambda = \frac{a^2}{b} P$$

Prob. 2.9

- (a) A system is linear when its output is linearly related to its input.
- (b) A nonlinear system is one in which the output is not linearly related to its input.
- (c) A continuous-time system has input and output signals that are continuous-time.
- (d) A discrete-time system has input and output signals that are discrete-time.

Prob. 2.10

- (a) A time-varying system is one in which the input-output relationship varies with time.
- (b) A time-invariant system is one in which the input-output relationship does not vary with time.
- (c) A casual system is one whose output signal (response) does not start before the input signal (excitation) is applied.
- (d) A noncausal system is one in which the response depends on the future values of the input.
- (e) An analog system is one whose input signal is analog.
- (f) A digital system is one whose input is in the form of a sequence of digits.

Prob. 2.11

- (a) Linear
- (b) Nonlinear
- (c) Linear

Prob. 2.12

- (a) Linear
- (b) Nonlinear
- (c) Nonlinear

Prob. 2.13

$$T = 4, \omega_o = 2\pi / T = \pi / 2$$

$$f(t) = \begin{cases} 5, & 0 < t < 1 \\ 10, & 1 < t < 2 \\ 0, & 2 < t < 4 \end{cases}$$

$$a_0 = \frac{1}{T} \int_0^T f(t) dt = \frac{1}{4} \left[\int_0^1 5 dt + \int_1^2 10 dt \right] = 3.75$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos n\omega_o t dt = \frac{2}{4} \left[\int_0^1 5 \cos \frac{n\pi t}{2} dt + \int_1^2 10 \cos \frac{n\pi t}{2} dt \right] = \frac{-5}{n\pi} \sin \frac{n\pi}{2}$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin n\omega_o t dt = \frac{2}{4} \left[\int_0^1 5 \sin \frac{n\pi t}{2} dt + \int_1^2 10 \sin \frac{n\pi t}{2} dt \right] = \frac{5}{n\pi} \left(1 - 2 \cos n\pi + \cos \frac{n\pi}{2} \right)$$

Prob. 2.14

$$T = 4, \omega_o = 2\pi / T = \pi / 2, \quad b_n = 0 \text{ since } f(t) \text{ is an even function.}$$

$$f(t) = \begin{cases} 0, & -2 < t < -1 \\ \cos \frac{\pi}{2} t, & -1 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

$$a_0 = \frac{2}{T} \int_0^{T/2} f(t) dt = \frac{2}{4} \int_0^1 \cos \frac{\pi}{2} t dt = \frac{1}{2} \frac{2}{\pi} \sin \frac{\pi}{2} t \Big|_0^1 = \frac{1}{\pi}$$

$$a_n = \frac{4}{T} \int_0^T f(t) \cos n\omega_o t dt = \int_0^1 \cos \frac{\pi}{2} t \cos \frac{n\pi t}{2} dt$$

We apply the trig identity: $\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$

$$a_n = \frac{1}{2} \int_0^1 \left[\cos \frac{\pi}{2} (n+1)t + \cos \frac{\pi}{2} (n-1)t \right] dt$$

For $n = 1$,

$$a_1 = \frac{1}{2} \int_0^1 [\cos \pi t + 1] dt = \frac{1}{2} \left[\frac{\sin \pi t}{\pi} + t \right] \Big|_0^1 = \frac{1}{2}$$

For $n > 1$,

$$a_n = \frac{1}{\pi(n+1)} \sin \frac{\pi}{2} (n+1) + \frac{1}{\pi(n-1)} \sin \frac{\pi}{2} (n-1)$$

When $n = \text{odd}$, $(n+1)$ and $(n-1)$ are even so that $a_n = 0$.

When $n = \text{even}$, $(n+1)$ and $(n-1)$ are odd so that

$$\sin \frac{\pi}{2}(n+1) = -\sin \frac{\pi}{2}(n-1) = \cos \frac{\pi}{2}n = (-1)^{n/2}$$

$$a_n = \frac{(-1)^{n/2}}{\pi(n+1)} + \frac{-(-1)^{n/2}}{\pi(n-1)} = \frac{-2(-1)^{n/2}}{\pi(n^2-1)}$$

Thus,

$$f(t) = \frac{1}{\pi} + \frac{1}{2} \cos \frac{\pi}{2}t - \frac{2}{\pi} \sum_{n=\text{even}} \frac{(-1)^{n/2}}{(n^2-1)} \cos \frac{n\pi}{2}t$$

Prob. 2.15

$$T = 4, \omega_0 = 2\pi/T = \pi/2, \quad a_0 = 0, \quad f(t) = 2(t-1)$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos n\omega_0 t dt = \frac{2}{4} \int_0^2 2(t-1) \cos \frac{n\pi}{2}t dt = \frac{1}{a^2} \cos at + \frac{1}{a}t \sin at - \frac{1}{a} \sin at \Big|_0^2, \quad a = \frac{n\pi}{2}$$

$$a_n = \frac{1}{a^2}(\cos n\pi - 1) + \frac{2}{a} \sin n\pi - 0 - \frac{1}{a} \sin n\pi - 0 = \frac{4}{n^2\pi^2}(\cos n\pi - 1)$$

$$b_n = \int_0^2 (t-1) \sin at dt = \frac{1}{a^2} \sin at - \frac{1}{a}t \cos at + \frac{1}{a} \cos at \Big|_0^2$$

$$= 0 - \frac{1}{a}(2 \cos n\pi - 1) + \frac{1}{a}(\cos n\pi - 1) = \frac{-2}{n\pi}(2 + \cos n\pi)$$

$$f(t) = \sum_{n=1}^{\infty} \left[\frac{4}{n^2\pi^2}(\cos n\pi - 1) \cos \frac{n\pi}{2}t - \frac{2}{n\pi}(2 + \cos n\pi) \sin \frac{n\pi}{2}t \right]$$

Prob. 2.16

$b_n = 0$ since this is an even function

$$T = 6, \quad \omega = 2\pi/6 = \pi/3$$

$$a_0 = \frac{2}{T} \int_0^{T/2} f(t) dt = \frac{2}{6} \left[\int_1^2 (4t-4) dt + \int_2^3 4 dt \right]$$

$$= \frac{1}{3} \left[(2t^2 - 4t) \Big|_1^2 + 4(3-2) \right] = 2$$

$$a_n = \frac{4}{T} \int_0^{T/4} f(t) \cos(n\pi t/3) dt$$

$$= (4/6) \left[\int_1^2 (4t-4) \cos(n\pi t/3) dt + \int_2^3 4 \cos(n\pi t/3) dt \right]$$

$$= \frac{16}{6} \left[\frac{9}{n^2 \pi^2} \cos\left(\frac{n\pi t}{3}\right) + \frac{3t}{n\pi} \sin\left(\frac{n\pi t}{3}\right) - \frac{3}{n\pi} \sin\left(\frac{n\pi t}{3}\right) \right]_1^2 + \frac{16}{6} \left[\frac{3}{n\pi} \sin\left(\frac{n\pi t}{3}\right) \right]_2^3$$

$$= [24/(n^2 \pi^2)] [\cos(2n\pi/3) - \cos(n\pi/3)]$$

Thus
$$f(t) = 2 + \frac{24}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left[\cos\left(\frac{2n\pi t}{3}\right) - \cos\left(\frac{n\pi t}{3}\right) \right] \cos\left(\frac{n\pi t}{3}\right)$$

At $t = 2$,

$$f(2) = 2 + (24/\pi^2) [(\cos(2\pi/3) - \cos(\pi/3))\cos(2\pi/3)$$

$$+ (1/4)(\cos(4\pi/3) - \cos(2\pi/3))\cos(4\pi/3)$$

$$+ (1/9)(\cos(2\pi) - \cos(\pi))\cos(2\pi) + \dots]$$

$$= 2 + 2.432(0.5 + 0 + 0.2222 + \dots)$$

$$f(2) = 3.756$$

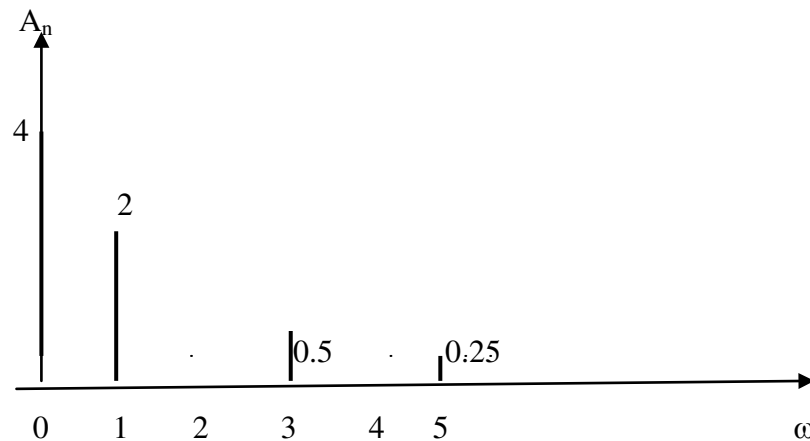
Prob. 2.17

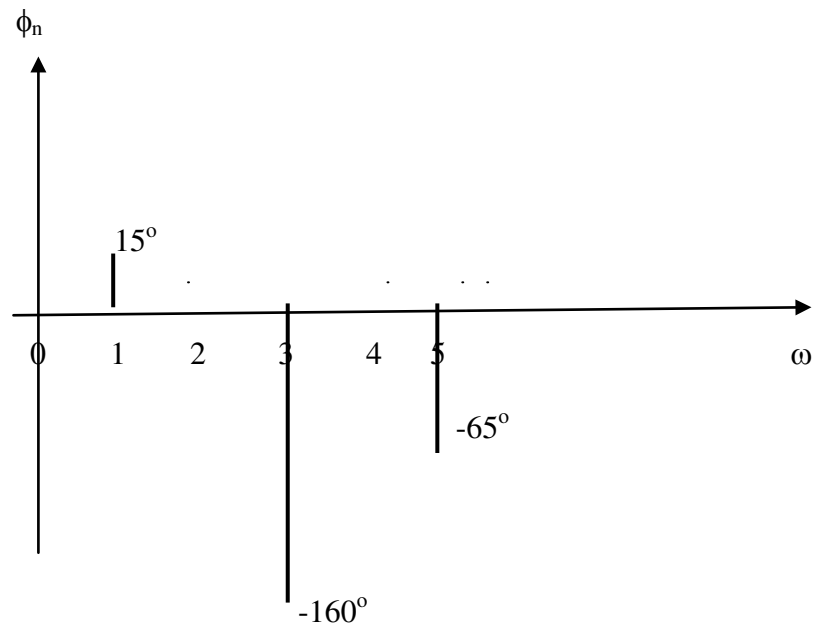
Express each term in the standard form as $a_n \cos(\omega_n t + \theta_n)$.

$$\frac{1}{4} \sin(5t + 25^\circ) = \frac{1}{4} \cos(5t + 25^\circ - 90^\circ) = \frac{1}{4} \cos(5t - 65^\circ)$$

$$-\frac{1}{2} \cos(3t + 20^\circ) = \frac{1}{2} \cos(3t - 160^\circ)$$

Thus, the magnitude and phase spectra are shown below.



**Prob. 2.18**

$$a_n = 20/(n^2\pi^2), \quad b_n = -3/(n\pi), \quad \omega_n = 2n$$

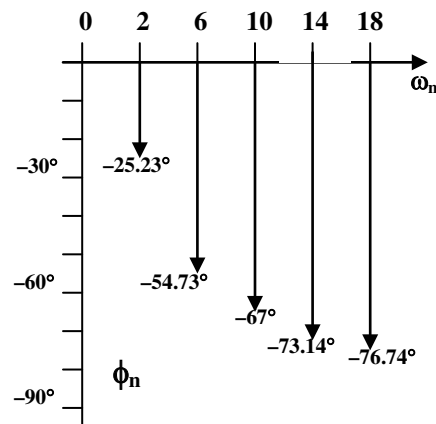
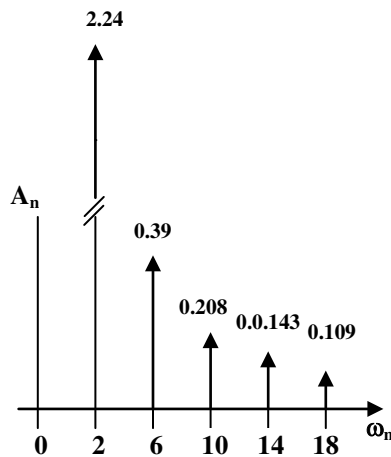
$$A_n = \sqrt{a_n^2 + b_n^2} = \sqrt{\frac{400}{n^4\pi^4} + \frac{9}{n^2\pi^2}}$$

$$= \frac{3}{n\pi} \sqrt{1 + \frac{44.44}{n^2\pi^2}}, \quad n = 1, 3, 5, 7, 9, \text{ etc.}$$

n	A_n
1	2.24
3	0.39
5	0.208
7	0.143
9	0.109

$$\phi_n = \tan^{-1}(b_n/a_n) = \tan^{-1}\{[-3/(n\pi)][n^2\pi^2/20]\} = \tan^{-1}(-n \times 0.4712)$$

n	ϕ_n
1	-25.23°
3	-54.73°
5	-67°
7	-73.14°
9	-76.74°
∞	-90°

**Prob. 2.19**

$$f(t) = 40 \cos 5\pi t - 20 \sin(2\pi t + \pi/6) \cos 5\pi t$$

But $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

$$f(t) = 40 \cos 5\pi t - 10 \sin(7\pi t + \pi/6) - 10 \sin(-3\pi t + \pi/6)$$

Also, $\sin \alpha = \cos(\alpha - 90^\circ)$

$$f(t) = 40 \cos 5\pi t - 10 \cos(7\pi t - 60^\circ) + 10 \cos(3\pi t - 120^\circ)$$

Thus,

$$a_1 = 40, a_2 = -10, a_3 = 10, \omega_1 = 5\pi, \omega_2 = 7\pi, \omega_3 = 3\pi, \theta_1 = 0, \theta_2 = -60^\circ, \theta_3 = -120^\circ$$

Prob. 2.20

This is an even function.

$$b_n = 0, T = 4, \omega_o = 2\pi / T = \pi / 2$$

$$a_0 = \frac{1}{2} \frac{(2+4)4}{4} = 3$$

$$f(t) = \begin{cases} 4t, & 0 < t < 1 \\ 4, & 1 < t < 3 \\ 8-4t, & 3 < t < 4 \end{cases}$$

$$a_n = \frac{4}{T} \int_0^{T/2} f(t) \cos n\omega_o t dt, \quad \alpha = n\omega_o = n\pi / 2$$

$$\begin{aligned} a_n &= \frac{4}{4} \left[\int_0^1 4t \cos \alpha t dt + \int_1^2 4 \cos \alpha t dt \right] = 4 \left[\frac{1}{\alpha^2} \cos \alpha t + \frac{t}{\alpha} \sin \alpha t \right] \bigg|_0^1 - \frac{4}{\alpha} \cos \alpha t \bigg|_1^2 \\ &= 4 \left[\frac{1}{\alpha^2} (\cos \alpha - 1) + \frac{1}{\alpha} \sin \alpha - 0 \right] - \frac{4}{\alpha} (\cos 2\alpha - \cos \alpha) \\ &= \frac{16}{n^2 \pi^2} (\cos \frac{n\pi}{2} - 1) + \frac{8}{n\pi} (\sin \frac{n\pi}{2} + \cos \frac{n\pi}{2} - \cos n\pi) \end{aligned}$$

$$f(t) = 3 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{2} t$$

Prob. 2.21

(a) Since $f(t)$ is an odd function, $a_0 = 0 = a_n$. $T = \pi, \omega_o = 2\pi / T = 2$

$$\begin{aligned} b_n &= \frac{4}{T} \int_0^{T/2} f(t) \sin n\omega_o t dt = \frac{4}{\pi} \int_0^{\pi/2} t \sin 2nt dt = \frac{4}{\pi} \left[\frac{1}{4n^2} \sin 2nt - \frac{t}{n\pi} \cos 2nt \right] \bigg|_0^{\pi/2} \\ &= \frac{4}{\pi} \left[-\frac{\pi}{2n\pi} \cos n\pi \right] = \frac{2}{n\pi} (-1)^{n+1} \end{aligned}$$

$$f(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin 2nt$$

(b) The total average power is

$$P_T = \frac{1}{T} \int_0^T f^2(t) dt = \frac{2}{\pi} \int_0^{\pi/2} \frac{4t^2}{\pi^2} dt = \frac{8}{\pi^3} \frac{t^3}{3} \bigg|_0^{\pi/2} = \frac{1}{3}$$

$$\begin{aligned} P_4 &= a_0^2 + \frac{1}{2} \sum A_n^2 = \frac{1}{2} \sum b_n^2 = \frac{1}{2} \left(\frac{4}{\pi^2} + \frac{4}{4\pi^2} + \frac{4}{9\pi^2} + \frac{4}{25\pi^2} \right) = \frac{2}{\pi^2} \left(1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{25} \right) \\ &= 0.2026(1.4011) = 0.2839 \end{aligned}$$

Prob. 2.22

$$T = 2\pi, \omega_0 = 2\pi / T = 1$$

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} t^2 e^{-jnt} dt$$

We integrate by parts twice and obtain

$$C_n = \frac{2}{n^2} \cos n\pi = \frac{2}{n^2} (-1)^n, \quad n \neq 0$$

For $n = 0$,

$$C_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} t^2 dt = \frac{\pi^2}{3}$$

Hence,

$$f(t) = \frac{\pi^2}{3} + \sum_{n=-\infty, n \neq 0}^{\infty} \frac{2(-1)^n}{n^2} e^{jnt}$$

Prob. 2.23

$$\begin{aligned} C_n &= \frac{1}{T} \int_0^T f(t) e^{-j\omega_0 nt} dt = \frac{1}{T} \int_{-\tau}^0 (1) e^{-j\omega_0 nt} dt + \frac{1}{T} \int_0^{\tau} (-1) e^{-j\omega_0 nt} dt \\ &= \frac{1}{T} \frac{e^{-j\omega_0 nt}}{-j\omega_0 n} \Big|_{-\tau}^0 - \frac{1}{T} \frac{e^{-j\omega_0 nt}}{-j\omega_0 n} \Big|_0^{\tau} = \frac{1}{-j\omega_0 nT} (1 - e^{-j\omega_0 n\tau}) + \frac{1}{j\omega_0 nT} (e^{-j\omega_0 n\tau} - 1) = \frac{2}{j\omega_0 nT} (e^{-j\omega_0 n\tau} - 1) \end{aligned}$$

$$\text{But } \omega_0 = \frac{2\pi}{T},$$

$$C_n = \frac{1}{jn\pi} (e^{-j2n\pi\tau/T} - 1), n \neq 0$$

For $n = 0$,

$$C_0 = \frac{1}{T} \int_{-\tau}^0 e^{-j\omega_0 t} dt + \frac{1}{T} \int_0^{\tau} (-1) e^{-j\omega_0 t} dt = \frac{1}{T} \frac{e^{-j\omega_0 t}}{-j\omega_0} \Big|_{-\tau}^0 - \frac{1}{T} \frac{e^{-j\omega_0 t}}{-j\omega_0} \Big|_0^{\tau}$$

$$\text{But } \omega_0 = \frac{2\pi}{T} \longrightarrow \omega_0 T = 2\pi$$

$$C_0 = \frac{-1}{2\pi} (1 - e^{j\omega_0 \tau}) + \frac{1}{2\pi} (e^{-j\omega_0 \tau} - 1) = \frac{1}{\pi} (\cos \omega_0 \tau - 1)$$

$$f(t) = \frac{1}{\pi} (\cos 2\pi\tau/T - 1) + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{1}{jn\pi} (e^{-j2n\pi\tau/T} - 1) e^{jn\pi t/T}$$

Prob. 2.24

$$T = 2, \omega_0 = 2\pi / T = \pi$$

$$\begin{aligned} C_n &= \frac{1}{2} \int_{-1}^0 4(1+t)e^{-jn\pi t} dt = 2 \left[\frac{e^{-jn\pi t}}{-jn\pi} + \frac{e^{-jn\pi t}}{-n^2\pi^2} (-jn\pi t - 1) \right] \Bigg|_{-1}^0 \\ &= 2 \left[\frac{(1 - e^{jn\pi})}{-jn\pi} - \frac{1}{n^2\pi^2} + \frac{e^{jn\pi}}{n^2\pi^2} (jn\pi - 1) \right] = 2 \left[\frac{(1 - \cos n\pi - j \sin n\pi)}{-jn\pi} - \frac{1}{n^2\pi^2} + \frac{(\cos n\pi + j \sin n\pi)}{n^2\pi^2} (jn\pi - 1) \right] \\ &= 2 \left[\frac{j}{n\pi} (1 - \cos n\pi) - \frac{1}{n^2\pi^2} + \frac{(jn\pi - 1)}{n^2\pi^2} \cos n\pi \right] \end{aligned}$$

Prob. 2.25

$$(a) \quad g(t) = f(t-2) = \sum C_n e^{jn\omega_0(t-2)} = \sum C_n e^{-j2n\omega_0} e^{jn\omega_0 t}$$

$$C'_n = C_n e^{-j2n\omega_0}$$

$$(b) \quad h(t) = 2 \frac{df}{dt} = \sum 2C_n (jn\omega_0) e^{jn\omega_0 t}$$

$$C'_n = 2jn\omega_0 C_n$$

$$(c) \quad y(t) = \frac{d^2 f}{dt^2} - \frac{df}{dt} = \sum C_n (j^2 n^2 \omega_0^2 - jn\omega_0) e^{jn\omega_0 t}$$

$$C'_n = -C_n (n^2 \omega_0^2 + jn\omega_0)$$

Prob. 2.26

$$(a) \quad X(\omega) = \frac{\pi}{(1+j\omega)^2 + \pi^2}$$

$$(b) \quad Y(\omega) = \frac{1}{3} [e^{-j\omega/3} + e^{j\omega/3}]$$

$$(c) \quad \text{sgn}(t) \Leftrightarrow \frac{2}{j\omega}$$

$$Z(\omega) = \frac{2}{j(\omega+1)}$$

Prob. 2.27

$$\begin{aligned}
 \text{(a)} \quad X(\omega) &= \int_{-1}^1 4e^{-j\omega t} dt = \frac{4}{-j\omega} e^{-j\omega t} \bigg|_{-1}^1 = \frac{4}{-j\omega} (e^{-j\omega} - e^{j\omega}) \\
 &= \frac{8}{\omega} \frac{(e^{j\omega} - e^{-j\omega})}{j2} = \frac{8}{\omega} \sin \omega = 8 \sin c \omega
 \end{aligned}$$

$$\text{Or } X(\omega) = \mathcal{F}[4\Pi(t/\tau)] = 4\tau \sin c(\omega\tau/2) = 8 \sin c \omega, \quad (\tau = 2)$$

$$\begin{aligned}
 \text{(b)} \quad Y(\omega) &= \int_0^1 2e^{-j\omega t} dt + \int_1^2 (1)e^{-j\omega t} dt = \frac{2}{-j\omega} e^{-j\omega t} \bigg|_0^1 + \frac{1}{-j\omega} e^{-j\omega t} \bigg|_1^2 \\
 &= \frac{1}{j\omega} (2 - e^{-j\omega} - e^{-j2\omega})
 \end{aligned}$$

$$\text{Or } y'(t) = 2\delta(t) - \delta(t-1) - \delta(t-2)$$

$$j\omega Y(\omega) = (2 - e^{-j\omega} - e^{-j2\omega})$$

$$Y(\omega) = \frac{1}{j\omega} (2 - e^{-j\omega} - e^{-j2\omega})$$

$$\text{(c)} \quad z''(t) = \delta(t+1) - 2\delta(t) + \delta(t-1)$$

$$-\omega^2 Z(\omega) = e^{j\omega} - 2 + e^{-j\omega} = 2 \cos \omega - 2$$

$$Z(\omega) = \frac{-2}{\omega^2} (\cos \omega - 1)$$

$$\text{But } \cos 2A = 1 - 2\sin^2 A$$

$$Z(\omega) = \frac{4 \sin^2 \omega/2}{\omega^2} = \sin^2 c(\omega/2)$$

$$\text{Or } z(t) = \Delta(t) \longrightarrow Z(\omega) = \sin^2 c(\omega/2)$$

Prob. 2.28

(a) Taking the first and second derivatives gives

$$f''(t) = \delta(t+2) - 4\delta(t-1) + 3\delta(t-2)$$

Taking the Fourier transform of each term:

$$-\omega^2 F(\omega) = e^{j2\omega} - 4e^{-j\omega} + 3e^{-j2\omega}$$

$$F(\omega) = \frac{1}{\omega^2} (4e^{-j\omega} - 3e^{-j2\omega} - e^{j2\omega})$$

$$\text{(b)} \quad g(t) = 2\Delta(t-1) - 2\Pi[(t-3)/2]$$

$$G(\omega) = 2e^{-j\omega} \sin^2 c(\omega/2) - 2e^{-j3} 2 \sin c \omega$$

$$G(\omega) = 2e^{-j\omega} \sin^2 c\left(\frac{\omega}{2}\right) - 4e^{-j3} \sin c \omega$$

Prob. 2.29

Taking second derivative of the signal gives

$$h''(t) = A\delta'(t) - \frac{A}{\tau}\delta(t) + \frac{A}{\tau}\delta(t - \tau)$$

$$-\omega^2 H(\omega) = Aj\omega - \frac{A}{\tau}(1) + \frac{A}{\tau}e^{-j\omega\tau}$$

$$H(\omega) = \frac{A}{\omega^2\tau} [1 - j\omega\tau - e^{-j\omega\tau}]$$

Prob. 2.30

(a) Let $h(t) = 1 + m \cos \alpha t$

$$H(\omega) = 2\pi\delta(\omega) + m\pi[\delta(\omega + \alpha) + \delta(\omega - \alpha)]$$

$$F(\omega) = \frac{1}{2}[H(\omega + \beta) + H(\omega - \beta)]$$

$$= \pi[\delta(\omega + \beta) + \delta(\omega - \beta)] + \frac{m\pi}{2}[\delta(\omega + \alpha + \beta) + \delta(\omega + \alpha - \beta) + \delta(\omega - \alpha + \beta) + \delta(\omega - \alpha - \beta)]$$

(b) $g(t) = \sin(t) = \frac{e^{jt} - e^{-jt}}{2j}, \quad 0 < t < 2\pi$

$$\begin{aligned} G(\omega) &= \int_0^{2\pi} \frac{e^{jt} - e^{-jt}}{2j} e^{-j\omega t} dt = \frac{1}{2j} \int_0^{2\pi} (e^{jt(1-\omega)} - e^{-jt(1+\omega)}) dt \\ &= \frac{1}{2j} \left[\frac{e^{jt(1-\omega)}}{j(1-\omega)} + \frac{e^{-jt(1+\omega)}}{j(1+\omega)} \right] \Bigg|_0^{2\pi} = \frac{-1}{2} \left[\frac{e^{j2\pi(1-\omega)} - 1}{(1-\omega)} + \frac{e^{-j2\pi(1+\omega)} - 1}{(1+\omega)} \right] \end{aligned}$$

But $e^{j2\pi} = \cos(2\pi) + j\sin(2\pi) = 1$

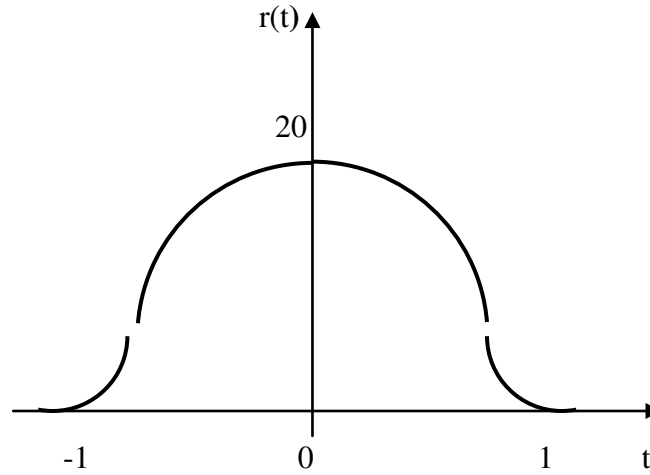
$$G(\omega) = \frac{-1}{2} \left[\frac{e^{-j2\pi\omega} - 1}{(1-\omega)} + \frac{e^{-j2\pi\omega} - 1}{(1+\omega)} \right] = \frac{-1}{2} \frac{(e^{-j2\pi\omega} - 1)(2)}{(1-\omega^2)} = \frac{1 - e^{-j2\pi\omega}}{1 - \omega^2}$$

Prob. 2.31

$$P(\omega) = \int_{-\tau/2}^{\tau/2} \cos \frac{\pi t}{2} e^{-j\omega t} dt = \frac{2\tau \cos(\omega\tau/2)}{\pi \left(1 - \left(\frac{\omega\tau}{\pi} \right)^2 \right)}$$

Prob. 2.32

(a) $r(t)$ is sketched below.



$$\begin{aligned}
 \text{(b)} \quad R(\omega) &= \int_{-\infty}^{\infty} r(t)e^{-j\omega t} dt = \int_{-1}^1 10(1 + \cos \pi t)e^{-j\omega t} dt \\
 &= 10 \frac{e^{-j\omega t}}{-j\omega} \Big|_{-1}^1 + 5 \int_{-1}^1 (e^{j\pi t} + e^{-j\pi t})e^{-j\omega t} dt \\
 &= \frac{10}{-j\omega} (e^{-j\omega} - e^{j\omega}) + \frac{5e^{j(\pi-\omega)t}}{j(\pi-\omega)} + \frac{5e^{-j(\pi+\omega)t}}{-j(\pi+\omega)} \Big|_{-1}^1 \\
 &= \frac{20}{\omega} \sin \omega + 5 \frac{(e^{j(\pi-\omega)} - e^{-j(\pi-\omega)})}{j(\pi-\omega)} - 5 \frac{(e^{-j(\pi+\omega)} - e^{j(\pi+\omega)})}{j(\pi+\omega)} \\
 &= 20 \sin \omega + 10 \sin(\pi - \omega) + 10 \sin(\pi + \omega)
 \end{aligned}$$

Prob. 2.33

$$\text{(a)} \quad F_1(\omega) = \frac{1}{2}(e^{j\omega a} + e^{-j\omega a}), \quad a = \pi/4$$

$$f_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(\omega)e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{2} [e^{j\omega(t+a)} + e^{j\omega(t-a)}] d\omega$$

$$\text{But} \quad \int_{-\infty}^{\infty} e^{j(t \pm a)\omega} d\omega = 2\pi\delta(t \pm a)$$

$$f_1(t) = \frac{1}{4\pi} 2\pi[\delta(t+a) - \delta(t-a)] = \frac{1}{2}[\delta(t+\pi/4) - \delta(t-\pi/4)]$$

(b) $f_2(t) = e^{-4(t-2)}u(t-2)$

(c) Let $\omega^2 = x$

$$F_3 = \frac{x+2}{x^2+3x+2} = \frac{x+2}{(x+2)(x+1)} = \frac{1}{x+1} = \frac{1}{\omega^2+1}$$

$$f_3(t) = \frac{1}{2}e^{-|t|}$$

Prob. 2.34

$$2 \sin \pi \omega = \frac{2}{2j} [e^{j\pi\omega} - e^{-j\pi\omega}]$$

$$\begin{aligned} f(t) &= \frac{1}{2\pi j} \int_{-1}^1 [e^{j\pi\omega} - e^{-j\pi\omega}] e^{j\omega t} d\omega = \frac{1}{2\pi j} \int_{-1}^1 [e^{j\omega(\pi+t)} - e^{-j\omega(\pi-t)}] d\omega \\ &= \frac{1}{2\pi j} \left[\frac{e^{j\omega(\pi+t)}}{j(\pi+t)} + \frac{e^{-j\omega(\pi-t)}}{j(\pi-t)} \right] \Big|_0^1 = \frac{2j \sin t}{t^2 - \pi^2} \end{aligned}$$

Prob. 2.35

$$\begin{aligned} f(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega \\ &= \frac{1}{2\pi} \int_{-\omega_o-\tau}^{-\omega_o+\tau} \frac{A}{2} e^{j\omega t} d\omega + \frac{1}{2\pi} \int_{-\tau}^{\tau} A e^{j\omega t} d\omega + \frac{1}{2\pi} \int_{\omega_o-\tau}^{\omega_o+\tau} \frac{A}{2} e^{j\omega t} d\omega \\ &= \frac{A}{4\pi} \frac{e^{j\omega t}}{jt} \Big|_{-\omega_o-\tau}^{-\omega_o+\tau} + \frac{A}{2\pi} \frac{e^{j\omega t}}{jt} \Big|_{-\tau}^{\tau} + \frac{A}{4\pi} \frac{e^{j\omega t}}{jt} \Big|_{\omega_o-\tau}^{\omega_o+\tau} \\ &= A \frac{e^{-j\omega_o t}}{2\pi t} \sin \tau t + \frac{A\tau}{\pi} \sin c(\tau t) + A \frac{e^{j\omega_o t}}{2\pi t} \sin \tau t \\ f(t) &= \frac{A\tau}{\pi} \sin(\tau t) [1 + \cos(\omega_o t)] \end{aligned}$$

Prob. 2.36

$$\begin{aligned}
 f(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \left[\int_{-1}^0 (-2\omega) e^{j\omega t} d\omega + \int_0^1 (2\omega) e^{j\omega t} d\omega \right] \\
 &= \frac{1}{2\pi} \left[-\frac{2e^{j\omega t} (j\omega t - 1)}{\omega^2} \Big|_{-1}^0 + \frac{2e^{j\omega t} (j\omega t - 1)}{\omega^2} \Big|_0^1 \right] \\
 &= \frac{1}{\pi\omega^2} \left[1 - jte^{-jt} - e^{-jt} + jte^{jt} - e^{jt+1} \right] \\
 &= \frac{1}{\pi\omega^2} [2 - 2t \sin t - 2 \cos t]
 \end{aligned}$$

Prob. 2.37

(a) Let $h(t) = g(-2t)$

$$H(\omega) = \frac{1}{2} \frac{20}{(1 - \frac{j\omega}{2})} = \frac{20}{2 - j\omega}$$

(b) Let $f(t) = tg(t)$

$$F(\omega) = j \frac{dG(\omega)}{d\omega} = 20j(-j)(1+j\omega)^{-2} = \frac{20}{(1+j\omega)^2}$$

Let $h(t) = (t+1)g(t+1)u(t+1) = f(t+1)u(t+1)$

$$H(\omega) = e^{j\omega} F(\omega) = \frac{20e^{j\omega}}{(1+j\omega)^2}$$

$$(c) \quad \frac{dg}{dt} \Leftrightarrow \frac{20j\omega}{1+j\omega}$$

$$t \frac{dg}{dt} = j \frac{d}{d\omega} \left(\frac{20j\omega}{1+j\omega} \right) = j20 \frac{[(1+j\omega)j - j\omega(j)]}{(1+j\omega)^2} = -\frac{20}{(1+j\omega)^2}$$

(d) Let $y(t) = g(t) \cos \pi t$

$$Y(\omega) = \frac{1}{2} [G(\omega + \pi) + G(\omega - \pi)] = \left[\frac{10}{1 + j(\omega + \pi)} + \frac{10}{1 + j(\omega - \pi)} \right]$$

Prob. 3.38

(a) Let $f(t) = x(t)e^{-j2t}$

$$F(\omega) = X(\omega + 2) = \frac{4 + j(\omega + 2)}{-(\omega + 2)^2 + j2(\omega + 2) + 2}$$

(b) Let $g(t) = x(t) \sin \pi(t-1) = x(t)[\sin \pi t \cos \pi - \cos \pi t \sin \pi]$

Since $\cos \pi = -1$ and $\sin \pi = 0$,

$$g(t) = -x(t) \sin \pi t$$

$$\begin{aligned} G(\omega) &= -\frac{1}{2}[X(\omega + \pi) - X(\omega - \pi)] \\ &= \frac{\frac{1}{2}[4 + j(\omega - \pi)]}{-(\omega - \pi)^2 + j2(\omega - \pi) + 3} - \frac{\frac{1}{2}[4 + j(\omega + \pi)]}{-(\omega + \pi)^2 + j2(\omega + \pi) + 3} \end{aligned}$$

(c) Let $h(t) = x(t) * \delta(t-2) = x(t-2)$

$$H(\omega) = e^{-j2\omega} X(\omega) = \frac{(4 + j\omega)e^{-j2\omega}}{-\omega^2 + j2\omega + 3}$$

(d) Let $y(t) = \int_{-\infty}^t x(\tau) d\tau$

$$Y(\omega) = \frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega) = \frac{4 + j\omega}{j\omega(-\omega^2 + j2\omega + 3)} + \frac{4}{3}\pi\delta(\omega)$$

Prob. 2.39

$$(a) \quad u(t+\tau) - u(t-\tau) \quad \Leftrightarrow \quad \frac{2 \sin \omega \tau}{\omega}$$

$$\frac{\sin \omega \tau}{\omega \tau} \quad \Leftrightarrow \quad \frac{1}{2\tau} [u(t+\tau) - u(t-\tau)]$$

Let $\tau=1$

$$\frac{\sin \omega}{\omega} \quad \Leftrightarrow \quad \frac{1}{2} [u(t+1) - u(t-1)]$$

$$\mathcal{F} \left[\frac{\sin t}{t} \right] = \frac{1}{2} 2\pi [u(-\omega+1) - u(-\omega-1)]$$

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{\sin^2 t}{t^2} dt &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \pi^2 [u(-\omega+1) - u(-\omega-1)]^2 d\omega \\ &= \frac{\pi}{2} \Big|_{-\omega-1}^{-\omega+1} = \frac{\pi}{2} (2) = \pi \end{aligned}$$

(b) Let $f(t) = \frac{4}{t^2 + 4}$

$$\frac{2a}{t^2 + a^2} \Leftrightarrow 2\pi e^{-a|\omega|}, \quad a = 2$$

$$f^2(t) = \frac{16}{(t^2 + 4)^2}$$

$$\int_{-\infty}^{\infty} f^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{dt}{(t^2 + 4)^2} &= \frac{1}{16} \frac{1}{2\pi} \int_{-\infty}^{\infty} 4\pi^2 e^{-4|\omega|} d\omega = \frac{\pi}{4} \int_0^{\infty} e^{-4\omega} d\omega \\ &= \frac{\pi}{4} \frac{e^{-4\omega}}{-4} = \frac{\pi}{16} \end{aligned}$$

Prob. 2.40

(a) $X(\omega) = \frac{1}{a + j\omega}, \quad a = 3$

$$|X(\omega)|^2 = \frac{1}{a^2 + \omega^2}$$

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{a^2 + \omega^2} = \frac{2}{2\pi} \frac{1}{3} \tan^{-1} \frac{\omega}{3} \Big|_0^{\infty} = \frac{2}{2\pi} \frac{1}{3} \frac{\pi}{2} = \frac{1}{6} = 0.1667 \text{ W}$$

(b) $Y(\omega) = 4 \sin c\left(\frac{\omega 4}{2}\right) = 4 \sin c(2\omega)$

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} 16 \sin^2(2\omega) d\omega$$

But $\int \frac{\sin^2 x}{x^2} dx = \int \sin^2 x dx = \frac{\pi}{2}$

Let $2\omega = x \longrightarrow d\omega = \frac{dx}{2}$

$$E = \frac{16}{2\pi} \int \sin^2 x \frac{dx}{2} = \frac{8}{2\pi} \frac{\pi}{2} = 2$$

Prob. 2.41

$$F(\omega) = 5(2) \sin c\left(\frac{\omega 2}{2}\right) = 10 \sin c \omega$$

Applying the time shifting property,

$$G(\omega) = F(\omega)e^{j\omega 2} + F(\omega)e^{-j\omega 2} = 10 \sin c \omega (e^{j\omega 2} + e^{-j\omega 2})$$

$$G(\omega) = 20 \cos(2\omega) \sin c \omega$$

Prob. 2.42

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{j\omega RC}{1 + j\omega RC - \omega^2 LC}$$

$$H(0) = 0, \quad H(\infty) = 0 \quad \longrightarrow \quad \text{a bandpass filter}$$

Prob. 2.43

$$E_x = \int_{-\infty}^{\infty} x^2(t) dt = \int_0^{\infty} e^{-200\pi t} dt = \frac{1}{200\pi}$$

$$x(t) = e^{-100\pi t} \quad \longrightarrow \quad X(\omega) = \frac{1}{100\pi + j\omega}$$

$$Y(\omega) = H(\omega)X(\omega) = \frac{1}{100\pi + j\omega}, \quad -B < \omega < B$$

$$E_{out} = \frac{1}{3} E_x = \frac{1}{600\pi} = \frac{1}{2\pi} \int_{-B}^B |Y(\omega)|^2 d\omega = \frac{1}{\pi} \int_0^B \frac{d\omega}{\pi^2 10^4 + \omega^2}$$

$$\frac{1}{600\pi} = \frac{1}{\pi} \left(\frac{1}{100\pi} \tan^{-1} \frac{B}{100\pi} \right) \quad \longrightarrow \quad \frac{B}{100\pi} = \tan \frac{\pi}{6}$$

$$B = 100\pi \tan \frac{\pi}{6} = 5,441.4 \text{ rad/s}$$

Prob. 5.44

$$H(s) = \frac{\omega_c^3}{s^3 + 2\omega_c s + 2\omega_c^2 + \omega_c^3} = \frac{1}{s^3 + 2s^2 + 2s + 1} = \frac{1}{(s+1)(s^2 + s + 1)}$$

$$H(s) = \frac{A}{s+1} + \frac{Bs+C}{s^2+s+1}$$

$$1 = A(s^2 + s + 1) + B(s^2 + s) + C(s + 1)$$

Equating coefficients,

$$s^2: \quad 0 = A + B \quad \longrightarrow \quad B = -A$$

$$s: \quad 0 = A + B + C = 0 + C \quad \longrightarrow \quad C = 0$$

$$\text{constant: } 1 = A + C \quad \longrightarrow \quad A = 1, B = -1$$

$$\begin{aligned} H(s) &= \frac{1}{s+1} - \frac{s}{s^2 + s + 1} = \frac{1}{s+1} - \frac{s}{(s+1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\ &= \frac{1}{s+1} - \frac{(s+1/2)}{(s+1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1/2}{(s+1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\ &= \frac{1}{s+1} - \frac{(s+1/2)}{(s+1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1/2}{\frac{\sqrt{3}}{2} \left[(s+1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 \right]} \end{aligned}$$

$$h(t) = e^{-t} - e^{-t/2} \cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{1}{\sqrt{3}} e^{-t/2} \sin\left(\frac{\sqrt{3}}{2}t\right)$$

Prob. 2.45

This is $H(s)$ for third-order Butterworth filter. Hence,

$$H(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

Prob. 2.46

$$Z_1 = 1 + sL, Z_2 = \frac{1}{sC} \parallel 1 = \frac{\frac{1}{sC}}{1 + \frac{1}{sC}} = \frac{1}{1 + sC}$$

$$V_o = \frac{Z_2}{Z_1 + Z_2} V_s$$

$$H = \frac{V_o}{V_s} = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{1}{1 + sC}}{1 + sL + \frac{1}{1 + sC}} = \frac{1}{s^2 LC + s(L + C) + 2}$$

$$= \frac{1/LC}{s^2 + s(1/C + 1/L) + 2/LC}$$

Comparing this with

$$H(s) = \frac{k}{s^2 + 1.414\omega_c s + \omega_c^2}$$

we get

$$k = 1/LC, \quad \omega_c^2 = \frac{2}{LC}, \quad 1.414\omega_c = \frac{1}{C} + \frac{1}{L},$$

$$\frac{1}{C} = \frac{L\omega_c^2}{2} = 50L$$

$$14.14 = 50L + \frac{1}{L} \quad \longrightarrow \quad 50L^2 - 14.14L + 1 = 0$$

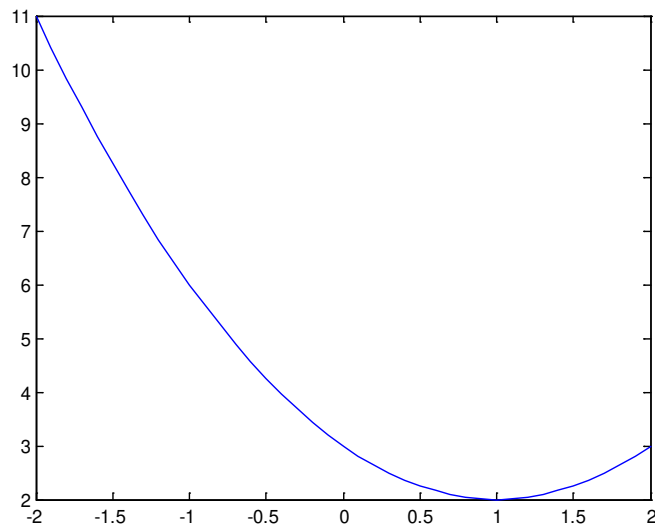
$$L = \frac{14.14 \pm \sqrt{14.14^2 - 4(50)(1)}}{100} \approx 0.1414$$

$$L = 141.4 \text{ mH}, \quad C = \frac{1}{50L} = 0.1414 = 141.4 \text{ mF}$$

Prob. 2.47

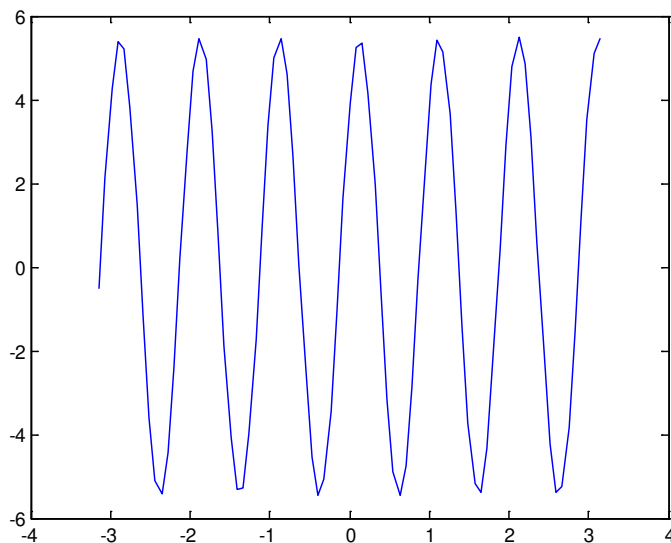
(a) The MATLAB code with the plot is presented below.

```
t=-2:0.1:2;
x = t.*t -2*t +3;
plot(t,x)
```



(b) The MATLAB codes with the plot is shown below.

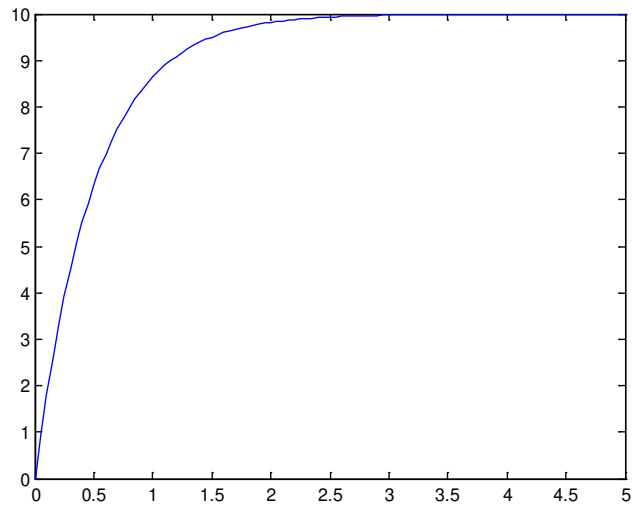
```
t=-pi:pi/40:pi;
y = 4*cos(2*pi*t - 12*pi/180) + 3*sin(2*pi*t);
plot(t,y)
```



(c) The MATLAB code with the plot is shown below.

```
t= 0:0.05:5;
z = 10*( 1 - exp(-2*t));
```

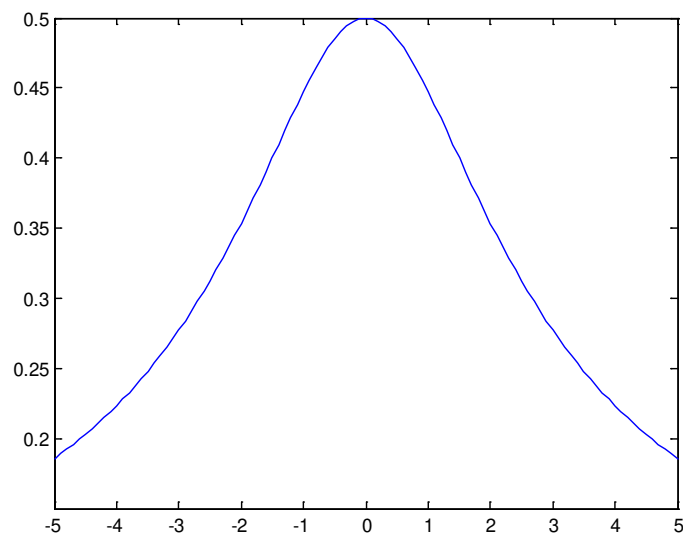
```
plot(t,z)
```



Prob. 2.48

(a) The MATLAB with the plot is shown below.

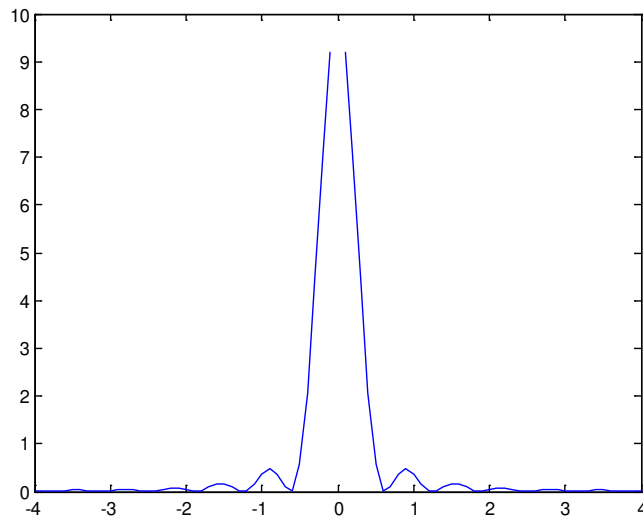
```
w= -5:0.1:5;
F = 1./sqrt( 4 + w.*w);
plot(w,F)
```



(b) The MATLAB with the plot is shown below.

```
w= -4:0.1:4;
G = 10*( sin(5*w).*sin(5*w) )./(25*w.*w);
```

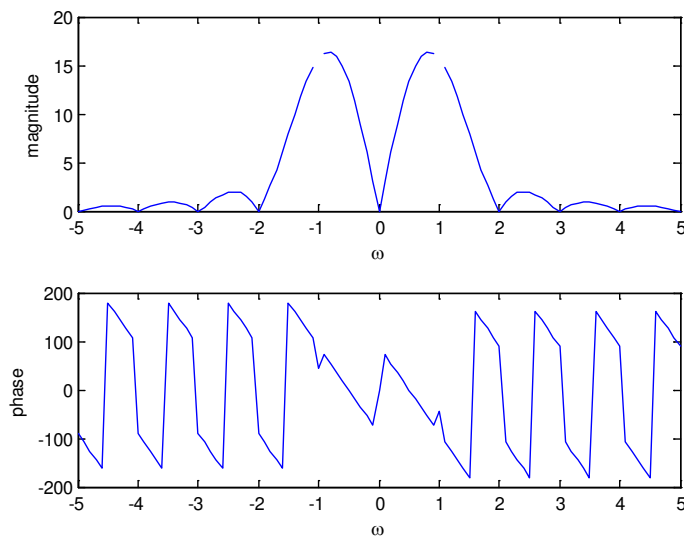
plot(w,G)



Prob. 4.49

The MATLAB code with the plots is presented below.

```
w= -5:0.1:5;
a=pi*w
F = 10*j.*exp(-j*a).*sin(a) ./ (1-w.*w);
FM = abs(F);
FP = angle(F)*180/pi;
subplot(2,1,1); plot(w,FM);
xlabel('\omega'); ylabel('magnitude');
subplot(2,1,2); plot(w,FP);
xlabel('\omega'); ylabel('phase');
```

**Prob. 2.50**

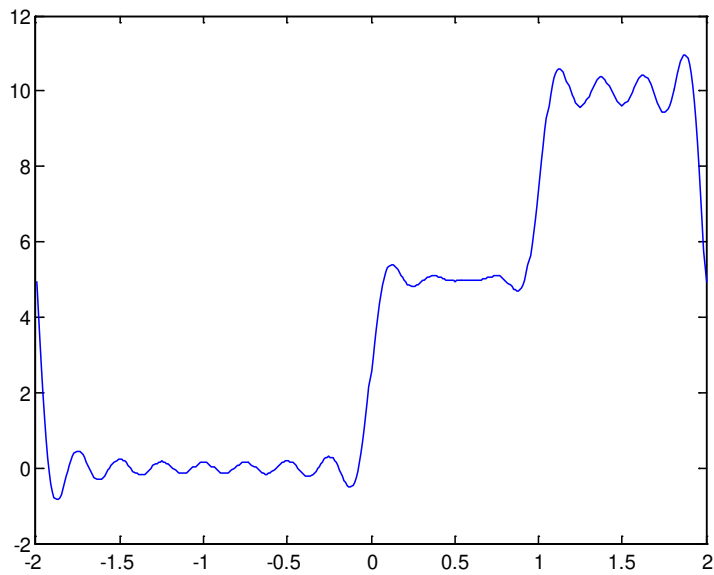
From Prob. 2.13,

$$a_0 = 3.75, a_n = \frac{-5}{n\pi} \sin \frac{n\pi}{2}, b_n = \frac{5}{n\pi} \left(1 - 2 \cos n\pi + \cos \frac{n\pi}{2} \right)$$

$$fN = 3.75 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t), \quad \omega_0 = \frac{\pi}{2}$$

The MATLAB code with the plot of partial sum is shown below.

```
N=15;
t = -2:0.01:2;
w = pi/2
f0=3.75;
fN = f0*ones(size(t));
for n=1:N
    fac=5/(pi*n);
    an= -fac*sin(w*n);
    bn=fac*(1 - 2*cos(n*pi) + cos(w*n));
    fN = fN +an*cos(n*w*t) + bn*sin(n*w*t);
end
plot(t,fN)
```

**Prob. 2.51**

From Problem 2.14,

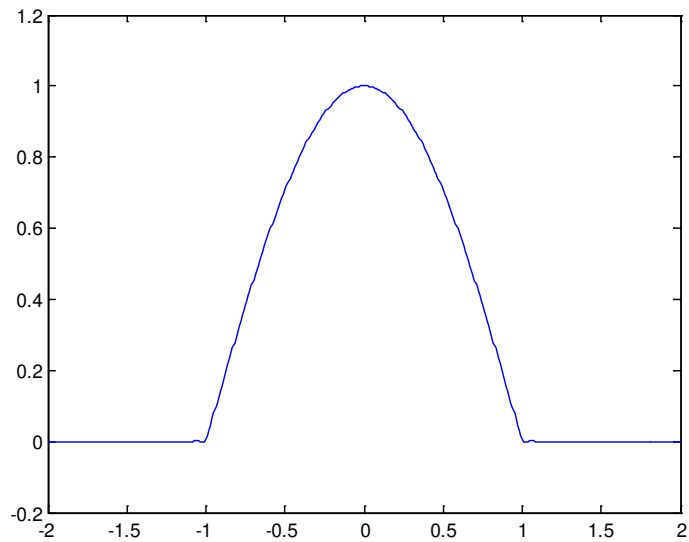
$$f(t) = \frac{1}{\pi} + \frac{1}{2} \cos \frac{\pi}{2} t - \frac{2}{\pi} \sum_{n=\text{even}}^{\infty} \frac{(-1)^{n/2}}{(n^2 - 1)} \cos \frac{n\pi}{2} t$$

The MATLAB code with the plot is shown below.

```

N = 25;
t = -2:0.01:2;
w = pi/2;
f0 = 1/pi;
fN = f0*ones(size(t));
fN = fN + 0.5*cos(w*t)
fac=-2/pi;
for k=1:N
    n=2*k;
    fac1= (-1)^(n/2);
    fac2 = n^2 -1;
    fN = fN +fac*fac1*cos(n*w*t)/fac2;
end
plot(t,fN)

```

**Prob. 2.52**

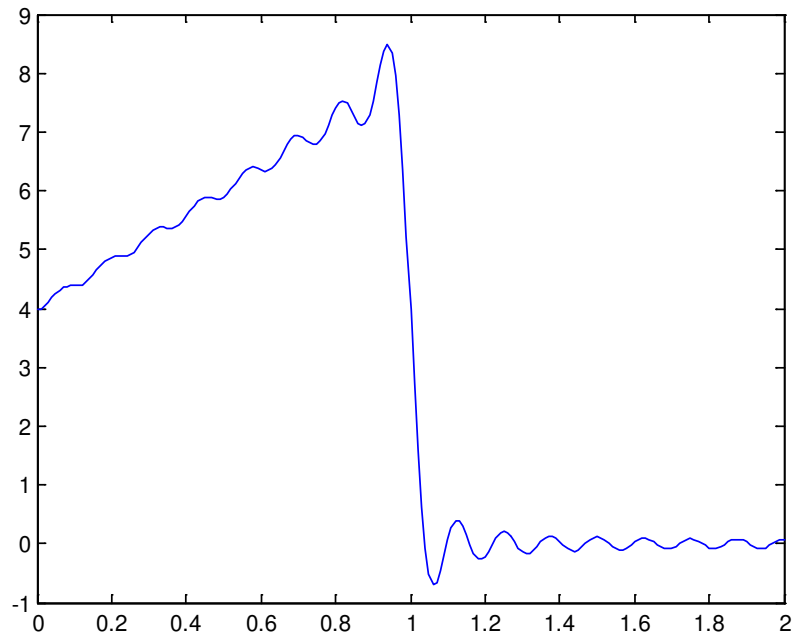
From Problem 2.20,

$$a_n = \frac{16}{n^2 \pi^2} \left(\cos \frac{n\pi}{2} - 1 \right) + \frac{16}{n\pi} \sin \frac{n\pi}{2}$$

$$f(t) = 3 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{2} t$$

The MATLAB code with the plot of partial sum is shown below.

```
N = 31;
t = 0:0.01:2;
w = pi/2;
f0 = 3;
fN = f0*ones(size(t));
for n=1:N
    fac1= 16*( cos(w*n)-1)/(n*n*pi*pi);
    fac2 = 16*sin(w*n)/(n*pi);
    an=fac1 + fac2;
    fN = fN + an*cos(n*w*t);
end
plot(t,fN)
```



Prob. 2.53

The MATLAB code is shown below.

```
den = [ 1  3.236  5.236  5.236  3.236  1];
roots(den)

ans =

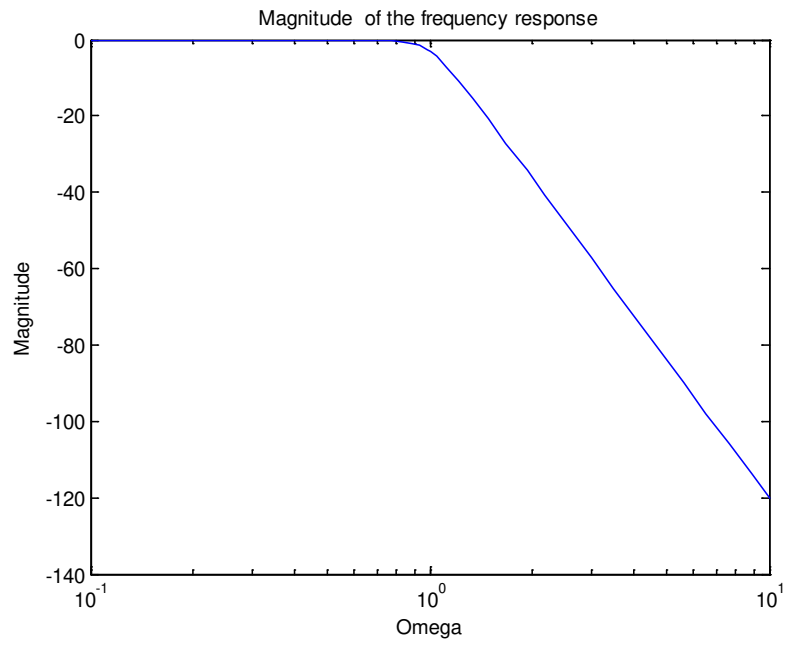
-0.3090 + 0.9510i
-0.3090 - 0.9510i
-1.0000 + 0.0000i
-0.8090 + 0.5879i
-0.8090 - 0.5879i
```

Prob. 2.54

The MATLAB code with the plot is shown below.

```
[z,p,k]=buttap(6); % returns the zeros, poles, and constant k of the
% 6th-order Butterworth filter
num=k*poly(z); % forms the numerator
den=poly(p); % forms the denominator
[mag,phase,w]=bode(num,den); % returns magnitude, phase (in
%degrees), and frequency vector w (automatically)
semilogx(w,20*log10(abs(mag)))
% plot(w,mag) % plots the magnitude verse w
title('Magnitude of the frequency response')
```

```
xlabel('Omega')  
ylabel('Magnitude')
```



Chapter 3

$$3.1 \text{ (a) } L_{\min} = 0.1\lambda = 0.1 \frac{c}{f_m} = 0.1 \frac{3 \times 10^8 \text{ m/s}}{30 \times 10^3 \text{ Hz}}$$

$$L_{\min} = 10^3 \text{ m or 1 km}$$

$$(b) f_c = 100f_m = 100 \times 30 \times 10^3 \text{ Hz} = 3 \times 10^6 \text{ Hz}$$

$$L_{\min} = 0.1\lambda = 0.1 \frac{c}{f_c} = 0.1 \frac{3 \times 10^8 \text{ m/s}}{3 \times 10^6 \text{ Hz}} = 10 \text{ m}$$

$$3.2 \text{ (a) For } f_c = 550 \text{ kHz,}$$

$$H_{\min} = 0.1\lambda = 0.1 \frac{c}{f_c} = 0.1 \frac{3 \times 10^8 \text{ m/s}}{550 \times 10^3 \text{ Hz}} = 54.54 \text{ m}$$

$$(b) \text{ For } f_c = 1600 \text{ kHz,}$$

$$H_{\min} = 0.1\lambda = 0.1 \frac{c}{f_c} = 0.1 \frac{3 \times 10^8 \text{ m/s}}{1600 \times 10^3 \text{ Hz}} = 18.75 \text{ m}$$

$$3.3 \quad A_c = 10; \text{ Modulation index } = \mu = \frac{M_p}{A_c} = \frac{A_m}{A_c}$$

$$\text{For waveform sketches, } A_{\max} = A_c + A_m; \quad A_{\min} = A_c - A_m$$

$$\text{For From Figure 3.35 (a), } A_{\max} = A_c + A_m = 8; \quad A_{\min} = A_c - A_m = 2$$

$$\therefore A_c = \frac{1}{2}(A_{\max} + A_{\min}) = \frac{1}{2}(8 + 2) = 5; \quad A_m = \frac{1}{2}(A_{\max} - A_{\min}) = \frac{1}{2}(8 - 2) = 3$$

$$\mu = \frac{A_m}{A_c} = \frac{3}{5} = 0.6 \text{ or } 60\%$$

$$(a) \quad \mu = \frac{4}{10} = 0.4$$

$$\text{Sketch is similar to Fig. 3.2 (a) but } A_{\max} = 10 + 4 = 14 \text{ and } A_{\min} = 10 - 4 = 6.$$

$$(b) \text{ For } A_m = 10, \quad \mu = \frac{10}{10} = 1.0 \text{ (100\% modulation, not recommended!)}$$

$$\text{Sketch is similar to Fig. 3.2 (b) but } A_{\max} = 10 + 10 = 20 \text{ and } A_{\min} = 10 - 10 = 0$$