Renewable Energy Engineering 1st edition Jenkins Solutions Manual

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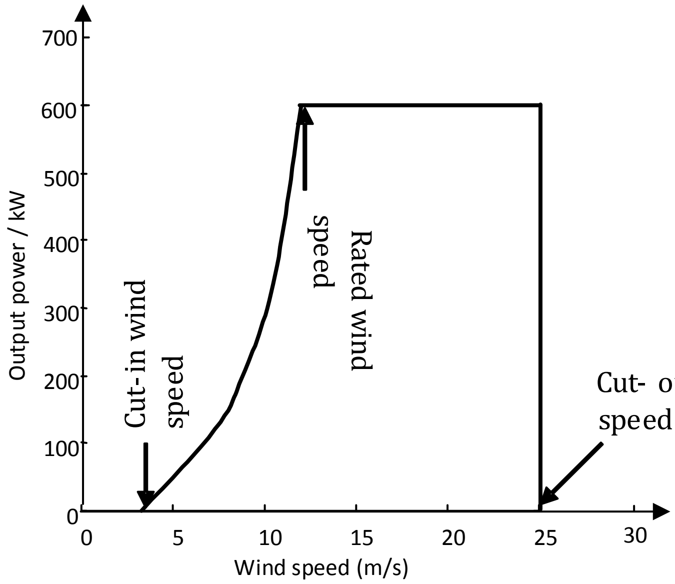
Chapter 2

WIND ENERGY

Take the density of air to be 1.25 kg/m^3

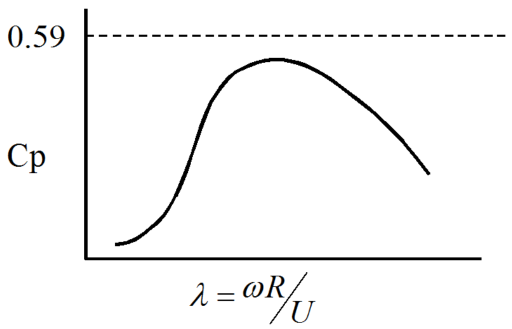
1. Sketch and explain the Power Curve of a wind turbine.

   Solution:



2. Sketch and explain the C_P/λ curve of a wind turbine rotor. Define C_P and λ. What is the Betz Limit?

   Solution:



- Power extracted by the aerodynamic rotor = C_p x Power available where C_p is the power coefficient
- Tip speed ratio is defined as $\lambda = \dfrac{\omega R}{V} = \dfrac{\text{Speeds of rotor tip}}{\text{Free wind speeds}}$
- The maximum value of the power coefficient C_p is 59% and it is called the Betz Limit.

3. Explain using appropriate diagrams and sketches how power may be regulated in a wind turbine by pitch regulation and stall regulation. Why is stall regulation only used for fixed speed wind turbines?

<u>Solution:</u> See Section 2.6.1

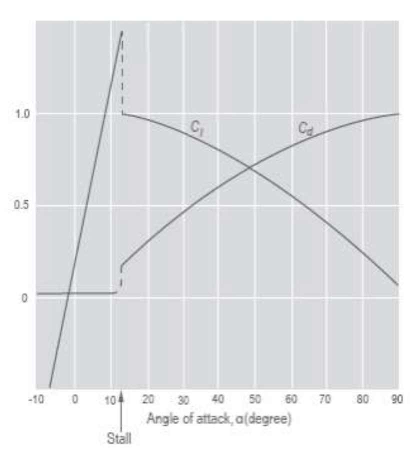


Figure 2.11

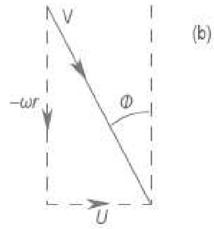


Fig 2.13b

There are two ways of reducing the rotational force F_r and hence the output power of the wind turbine, *pitch and stall regulation.* Consider the airfoil characteristics (Figure 2.11) with the blade operating at an initial angle of attack of 10^0. C_d is very small and so can be ignored but the C_l characteristic is a steeply rising almost straight line. With *pitch regulation* the angle of incidence (α) is reduced by mechanically turning the blade about its axis. This reduces C_l , F_r, and hence the power generated by the wind turbine.

For a fixed speed wind turbine, the angular velocity ω of the rotor is held constant by the electrical generator, which is locked on to the 50 or 60 Hz frequency of the power network. Considering the triangle of velocities *V*, *U* and $-\omega r$ The velocity of the blade element ωr is constant and so an increase of the free wind speed *U*, will increase Φ. Hence α increases as *V* swings round. Consider Figure 2.11 with an initial operating angle of attack of 10^0. Once the free wind speed U has increased sufficiently for the angle of attack to exceed around 13^0 the blade stalls and the lift coefficient and hence the torque decreases. This is *stall regulation* that does not require any physical change in the pitch angle of the blades.

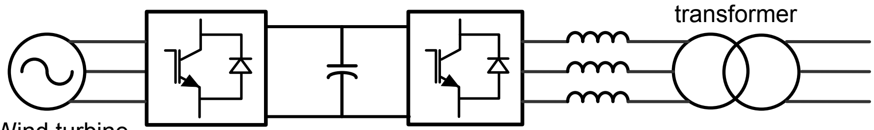
For stall regulation it is necessary to keep the rotor speed of rotation ω constant so that as U increases the angle of attack α also increases and the blade goes into stall.

4. Why are large wind turbines operated at variable speed? Explain the principle of operation of a Full Power Converter variable speed wind turbine.

<u>Solution:</u>

- For maximum power extraction, the turbine should operate at a particular value of λ. In order to achieve this the generator speed should change with the wind speed.
- Under variable speed operation, the mechanical loading on the wind turbine is reduced.
- As variable speed wind turbines often have a power electronic interface, turbines will be able to meet the grid code requirements with less additional equipment.

In variable speed wind turbines, back-to-back converters are used to extract maximum power from wind. The following figure shows a converter system typically used to control a large full power converter variable speed wind turbine. The generator may be synchronous (wound rotor or permanent magnet) or an induction machine. Operation is possible over a wide speed range.



Full power converter (FPC) variable-speed generator

5. Describe briefly what is meant by the measure-correlate-predict technique for establishing the long-term wind resource at a wind farm site

   Solution: See Section 2.9.1

   The measure-correlate-predict technique takes a series of measurements of wind speed and direction at the mast erected at the wind farm site and correlates them with simultaneous wind speed measurements made at a local meteorological station, which may be at a local airport. The averaging period of the site-measured data is synchronised with that of the meteorological station data. This gives a scatter plot with the mast measurements on one axis and the simultaneous meteorological station measurements on the other. Linear regression is used to establish a relationship between the measured site wind speed and the long-term meteorological wind speed data.

   The coefficients relating the two sets of measurements, sometimes known as speed-up factors, are calculated for the twelve 30^0 directional sectors. Then these coefficients are applied to transfer the long term data record of the meteorological station to the site. The speed up factors allow the long term wind speed record held by the meteorological station to be used as an estimate of what the wind speed at the wind farm site would have been over that period. It is then assumed that this long term wind speed record at the site is representative of the wind speed over the projected life of the wind farm. It is common to use a long term record of 20 years.

   The drawbacks of using the measure-correlate-predict technique include that high site masts are necessary if large wind turbines are planned. There may not be a suitable meteorological station nearby (within say 50 km) or with a similar exposure and wind climate. The data obtained from the meteorological station may not always be of good quality and may include gaps. Therefore, it may be time consuming to ensure that it is properly correlated with the site data. Critically the technique assumes that the previous long-term record provides a good estimate of the future wind resource over the lifetime of the wind farm.

6. Discuss how Weibull parameters may be used to describe hourly mean wind speed data at a site.

   Solution: See Section 2.8.1

It has been found from experience that either the Weibull or the simpler Raleigh distribution (when k=2) can be used to describe the shape of the probabilities of long term records of wind speeds.
The probability density function of a Weibull distribution is given by

$$f(U) = \frac{k}{c}\left(\frac{U}{c}\right)^{k-1} \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

where
c: scale parameter [m/s]
k: shape parameter

and the cumulative distribution function (that the wind speed will be less than or equal to *U*) is given by

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

7. A wind turbine has a rotor diameter of 33 m. At a wind speed of 13 m/s,
   a) What is the power in the wind?
   b) What is the power in the shaft of the turbine if the rotor has a power coefficient C_P of 0.35?
   c) If the speed of rotation of the shaft is 35 r.p.m. calculate the tip speed and torque on the shaft.

   [Answer: a) 1.17 MW; b) 411 kW; c) 60.5 m/s, 112 kNm]

   Solution:

   a) $P_{wind} = \frac{1}{2}\rho A U^3 = \frac{1}{2} \times 1.25 \times \pi \times 16.5^2 \times 13^3 = 1174$ kW

   b) $P_{shaft} = P_{wind} \times C_P = 1174 \times 0.35 = 411$ kW

   c) $\omega = \frac{35}{60} \times 2\pi = 3.67$ rad/sec

   $V_{tip} = \omega R = 3.67 \times 16.5 = 60.5$ m/s

   Torque $= \frac{P}{\omega} = \frac{411}{3.67} = 112$ kNm

8. A wind turbine has a rotor of 55 metres in diameter, mounted at a hub height of 60 metres. The gearbox ratio is 1:28. If it operates at a tip speed ratio (λ) of 8 in a wind speed of 10 m/s, with a power coefficient of 0.35 and a thrust coefficient of 0.55 calculate:
   a) The torque on the generator shaft
   b) The overturning moment at the base of the tower

   [Answer: a) 6.38 kNm; b) 4.9 MNm]

Solution:

a) From $\lambda = \frac{\omega R}{U}$, $\omega = \frac{U\lambda}{R} = \frac{10 \times 8}{27.5} = 2.91$ rad/sec

$$P_{wind} = \frac{1}{2}\rho A U^3 = \frac{1}{2} \times 1.25 \times \pi \times 27.5^2 \times 10^3 = 1.485 \text{ MW}$$

$$P_{shaft} = C_p P_{wind} = 0.35 \times 1.485 = 520 \text{ kW}$$

$$Q_{ls\ shaft} = \frac{P}{\omega} = \frac{520}{2.91} = 178.7 \text{ kNm}$$

$$Q_{gen\ shaft} = \frac{178.7}{28} = 6.38 \text{ kNm}$$

b) $Thrust = C_t \frac{1}{2}\rho A U^2 = 0.55 \times 0.5 \times 1.25 \times \pi \times 27.5^2 \times 10^2 = 81.67$ kN

$$Moment = 81.67 \times 60 = 4.9 \text{ MNm}$$

9. A large horizontal axis wind turbine has a rotor of diameter of 80 m at a hub height of 90 m. It operates at a rotational speed of 15 r.p.m. with a combined efficiency of the gearbox and generator of 95% to produce 2 MW of electrical output power.
   a) If the hub height wind speed is 12 m/s calculate C_p (the Power Coefficient) of the rotor.
   b) Write down the tip speed ratio and hence C_Q (the Torque Coefficient)
   c) Estimate the wind speed at a 10 m high met mast adjacent to the wind turbine

[Answer: a) 0.39; b) 5.23, 0.075; c) 8.8 m/s]

Solution:

a) $P_{wind} = \frac{1}{2}\rho A U^3 = \frac{1}{2} \times 1.25 \times \pi \times 40^2 \times 12^3 = 5.43$ MW

$$P_{shaft} = \frac{2}{0.95} = 2.1 \text{ MW}$$

$$C_p = \frac{P_{shaft}}{P_{wind}} = \frac{2.1}{5.43} = 0.39$$

b) $\lambda = \frac{\omega R}{U} = \frac{\left(15 \times 2\pi/60\right) \times 40}{12} = 5.23$

$$C_Q = \frac{C_p}{\lambda} = \frac{0.39}{5.23} = 0.075$$

c) $$U_{hub} = U_{ref}\left(\frac{z_{hub}}{z_{ref}}\right)^{1/7}$$

$$U_{ref} = U_{hub} 1/\left(\frac{z_{hub}}{z_{ref}}\right)^{0.1428} = 12/\left(\frac{90}{10}\right)^{0.1428} = 8.8 \text{ m/s}$$

10. A horizontal axis wind turbine has a rotor diameter of 20 m, and reaches its full load output at the generator terminals of 150 kW at a wind speed of 13 m/s. The overall efficiency of the gearbox and generator is 94%.

    a) Calculate the power in the air passing through the rotor disk and hence the Power Coefficient (C_p) of the aerodynamic rotor.

    b) The turbine operates at a Tip Speed Ratio (λ) of 4. Calculate the rotational speed of the aerodynamic rotor and hence the torque on the rotor shaft.

    c) The inertia of the complete rotor, gearbox and generator is 50x10^3 kgm^2. Calculate the speed of the rotor 0.5 seconds after the connection to the electrical power network is broken and write down the energy that is absorbed by the brake when it brings the rotor rapidly to rest.

    [Answer: a) 431 kW, 0.37; b) 5.2 rad/sec, 30.7 kNm, c) 5.5 rad/sec, 756 kJ]

    Solution:

    a) $$P_{wind} = \tfrac{1}{2}\rho A U^3 = \frac{1}{2}\times 1.25\times\pi\times 10^2\times 13^3 = 431 \text{ kW}$$

    $$C_p = \frac{P_{shaft}}{P_{wind}} = \frac{150/0.94}{431} = 0.37$$

    b) $$\omega = \frac{U\lambda}{R} = \frac{13\times 4}{10} = 5.2 \text{ rad/sec}$$

    $$Q_{ls\ shaft} = \frac{P}{\omega} = \frac{150/0.94}{5.2} = 30.7 \text{ kNm}$$

    c) KE during operation $= \frac{1}{2}I\omega^2 = \frac{1}{2}\times 50,000\times 5.2^2 = 676$ kJ

    Additional KE = $= 150/0.94 \times 0.5 = 79.8$ kJ

    Therefore energy to be dissipated in brake = 755.8 kJ

Maximum speed

$$\omega = \sqrt{\frac{2KE}{I}} = \sqrt{\frac{2 \times 755.8}{50}} = 5.5 \text{ rad/sec}$$

11. A wind turbine is rated at 400 kW at 12.5 m/s. If the annual distribution of hourly mean wind speeds at the site may be represented by a Weibull distribution with
    Scale parameter of 8.5
    Shape parameter of 2.5

    Calculate the annual energy production in kWh per year that the wind turbine will produce at rated power.

    [Answer: 254 MWh]

    Solution:

    The time that the wind speed is below 12.5 m/s is calculated from

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^k\right] = 1 - \exp\left[-\left(\frac{12.5}{8.5}\right)^{2.5}\right]$$

$$F(U) = 0.927, \text{ or } 8124 \text{ hours/year}$$

Assuming that the number of hours the wind turbine operates above cut-out wind speed is negligible, the number of hours the wind turbine operates at rated power
= 8760 – 8124 = 636 hrs
Total energy wind turbine will produce = 400 kW x 636 = 254,400 kWh

12. A wind farm consists of 10 wind turbines each with:

Hub height	50 m
Rated power	2 MW
Cut in wind speed	5 m/s
Rated wind speed	13 m/s
Cut-out wind speed	20 m/s

Long term monitoring wind speeds has been undertaken using an anemometer at 20 m height and the site wind speeds can be represented by a Weibull distribution with:

Scale parameter	8
Shape parameter	2 (Raleigh Distribution)

Assume that the Scale parameter increases with height according to the 1/7 power law.

Calculate:
a) The number of hours each year the turbines operate
b) The annual energy production (in MWh per year) which the wind turbines will produce when operating above rated power.

[Answer: a) 6414 hours; b) 21.5 GWh]

Note:

The following are likely to be useful

$$\frac{U(z)}{U(h)} = \left(\frac{z}{h}\right)^{\alpha}$$

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

<u>Solution:</u>

a) When the hub height wind speed is 5 m/s, then wind speed at 20 m height

$$= \left(\frac{20}{50}\right)^{1/7} \times 5 = 4.3865 \text{ m/s}$$

The time that the wind speed is below 4.3865 m/s is calculated from

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right] = 1 - \exp\left[-\left(\frac{4.3865}{8.0}\right)^{2}\right]$$

$F(U) = 0.26,$ or 2274.6 hours/year

When the hub height wind speed is 20 m/s, then wind speed at 20 m height

$$= \left(\frac{20}{50}\right)^{1/7} \times 20 = 17.546 \text{ m/s}$$

The time that the wind speed is below 17.546 m/s is calculated from

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right] = 1 - \exp\left[-\left(\frac{17.546}{8.0}\right)^{2}\right]$$

$F(U) = 0.9919,$ or 8688.65 hours/year

The number of hours each year the turbines operate = 8688.65 – 2274.6 = 6414 hrs

b) When the hub height wind speed is 13 m/s, then wind speed at 20 m height

$$= \left(\frac{20}{50}\right)^{1/7} \times 13 = 11.405 \text{ m/s}$$

The time that the wind speed is below 11.405 m/s is calculated from

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right] = 1 - \exp\left[-\left(\frac{11.405}{8.0}\right)^{2}\right]$$

$F(U) = 0.869,$ or 7612.2 hours/year

No of hours wind turbine operates above rated speed = 8688.65 – 7612.2 = 1076.45 hrs

Total energy 10 wind turbines will produce = 10 x 2 MW x 1076.45 = 21.5 GWh

13. Estimate the Weibull parameters of the following set of hourly mean wind speed measured over one year.

Wind speed (m/s)	Hours
0-4	2050
4-8	4000
8-12	2200
12-16	460
16-20	50

[Answer: c 7.2, k 2]

Note: Graph Paper is provided and the following may be useful:

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^k\right]$$

Solution:

$$\ln\left[-\ln\left(1-F\left(U\right)\right)\right] = k\ln U - k\ln c$$

Wind speed (m/s)	Hours	ln (U)	ln(-ln(1-F(U))
0-4	2050	1.386	-1.24
4-8	4000	2.079	0.11
8-12	2200	2.485	0.99
12-16	460	2.772	1.73
16-20	50	2.995	10

Plotting ln(U) versus ln[-ln(1-F(U))] gives a graph similar to that of Figure 2.22 and so allows estimation of c and k.

U	Hours	F(U)	1-F(U)	ln(1-F(U))	ln(U)	ln(-ln(1-F(U)))
4	2200	0.251	0.749	-1.382	1.386	-1.241
8	3700	0.674	0.326	-0.395	2.079	0.113
12	2280	0.934	0.066	-0.069	2.485	0.999
16	550	0.997	0.003	-0.003	2.773	1.736
20	30	1.000	0.000	0.000	2.996	

14. The annual distribution of hub height hourly mean wind speeds at a site may be described by a Weibull distribution with:
    Scale parameter, c, of 8.5
    Shape parameter, k, of 2

    Calculate the gross annual energy yield of a wind turbine with a Power Curve shown in the Table, installed at this site.

    [Answer: 2191 MWh]

Wind speed m/s	Output kW
0-5	0
6	50
7	120
8	280
9	440
10-25	600

The following are likely to be useful

$$f(U) = \frac{k}{c}\left(\frac{U}{c}\right)^{k-1} \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

Solution:

	8.5	2		
U	P	f(U)/F(U)	Hours/year	kWh
6	50	0.101	884	44200
7	120	0.098	862	103380
8	280	0.091	800	223996
9	440	0.081	711	312969
10+	600	0.287	2512	1507172
				2191717

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Simplification of variance.

Show that

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(U_i - \bar{U}\right)^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}U_i{}^2 - n\bar{U}^2\right]$$

$$\begin{aligned}
\sum_{i=1}^{n}\left(U_i - \bar{U}\right)^2 &= \sum_{i=1}^{n}\left(U_i^2 - 2U_i\bar{U} + \bar{U}^2\right) \\
&= \sum_{i=1}^{n}U_i^2 - 2\bar{U}\sum_{i=1}^{n}U_i + n\bar{U}^2 \\
&= \sum_{i=1}^{n}U_i^2 - 2\bar{U}n\bar{U} + n\bar{U}^2 \\
&= \sum_{i=1}^{n}U_i^2 - n\bar{U}^2
\end{aligned}$$

and so

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}U_i^2 - n\bar{U}^2$$

Renewable Energy Engineering

Extended exercise 2.1

Use of wind speed data and demonstration of Probability Density Functions, Cumulative Density Functions and a Wind Rose

Using the set of 10-minute mean wind data, plot and explain a:

a. Binned Histogram of wind speed at each height
b. Probability Density Function of each anemometer
c. Cumulative Density Function of each anemometer
d. Wind Rose

Data and Resources to support the Exercise

- Data: FD1t50_students.xlsx
- Explanation of data: Example 2.1 Wind resource explanation.ppt
- Example solution: Example 2.1 Example solution.ppt
- Matlab m. files to give example solution

Renewable Energy Engineering

Extended exercise 2.2

Use of wind speed data to demonstrate wind shear and determination of Weibull parameters

Using the set of 10-minute mean wind data provided for Exercise 2.1:

a. Estimate the wind shear and compare the data with $1/7^{th}$ power law.

b. Estimate the Weibull parameters of the data.

Data and resources for extended exercise

- Data: Ex 2_1 Data.xlsx
- Explanation of data: Example 2.2 Wind resource explanation.ppt
- Example solution: Example 2.2 Example solution.ppt
- MATLAB files to give example solution

Renewable Energy Engineering

Extended exercise 2.3

Estimation of the power production of a wind turbine

Using the set of 1 hour mean wind data used and the power curves of the 600 kW (50m diameter), 1.3 MW (55m diameter), MW and 2 MW (80m diameter) wind turbines estimate the annual energy produced by each wind turbine. First move the wind speed data to the correct hub height of each turbine using the 1/7th power law.

Data and Resources to support the Exercise

Data and resources for extended exercise

- Data: Exercise 2.1 data.xlsx and Turbine Power Data.xlsx
- Explanation of data: Example 2.3 Explanation of Data.ppt
- Power curves of 3 wind turbines given in: Data on 3 wind turbines included in the “Turbine Power Data.xlsx” excel sheet.
- Example solution: Exercise 2.3 Example solution.ppt
- MATLAB file to give example solution

Renewable Energy Engineering

Extended exercise 2.4

Wind turbine performance using BLADED

Use models of 2 wind turbines: 1.3 MW (stall regulated, 55m diameter) and 2 MW (pitch regulated, 80m diameter). Open each model in BLADED, run the appropriate calculation and plot the following:

a. Run a Steady Power Curve calculation.
   - View the turbine power curve, by plotting electrical power v. hub wind speed. Mark on the power curves the cut-in, rated and cut-out wind speeds. Compare performance of a stall and pitch wind turbine.
   - Plot pitch angle v. hub wind speed. Show the rated wind speed.
   - Plot power coefficient v. hub wind speed
   - Plot thrust coefficient v. hub wind speed

b. Run a Performance Coefficients calculation.
   - Plot power coefficient v tip speed ratio

c. Run a Steady Operational Loads calculation.
   - Plot Tower Fx load at any tower height v. Hub wind speed. This is the force acting on the tower in the direction of the turbine axis. This force is also referred to as the 'Thrust' force. At what wind speed is this force a maximum? Can you explain why this is?
   - Plot Hub Mx load v. hub wind speed. This is the useful torque load generated by the rotor blades that acts to turn the turbine drive shaft. What do you notice about the variation of this load above rated wind speed? Why do we see this?

d. Run a Steady Power curve calculation.
   Plot Rotor speed (rad/s) v. hub wind speed.

A plot of generator torque (or generator power) against generator speed can be obtained by running a Steady Power Curve calculation and then selecting both of the variables individually in separate channels. The x-axis can then be set as generator speed.

Data and Resources to support the exercise

1. Introduction to BLADED
2. To obtain an Educational copy of BLADED contact DNVGL through:

Email: bladed@dnvgl.com

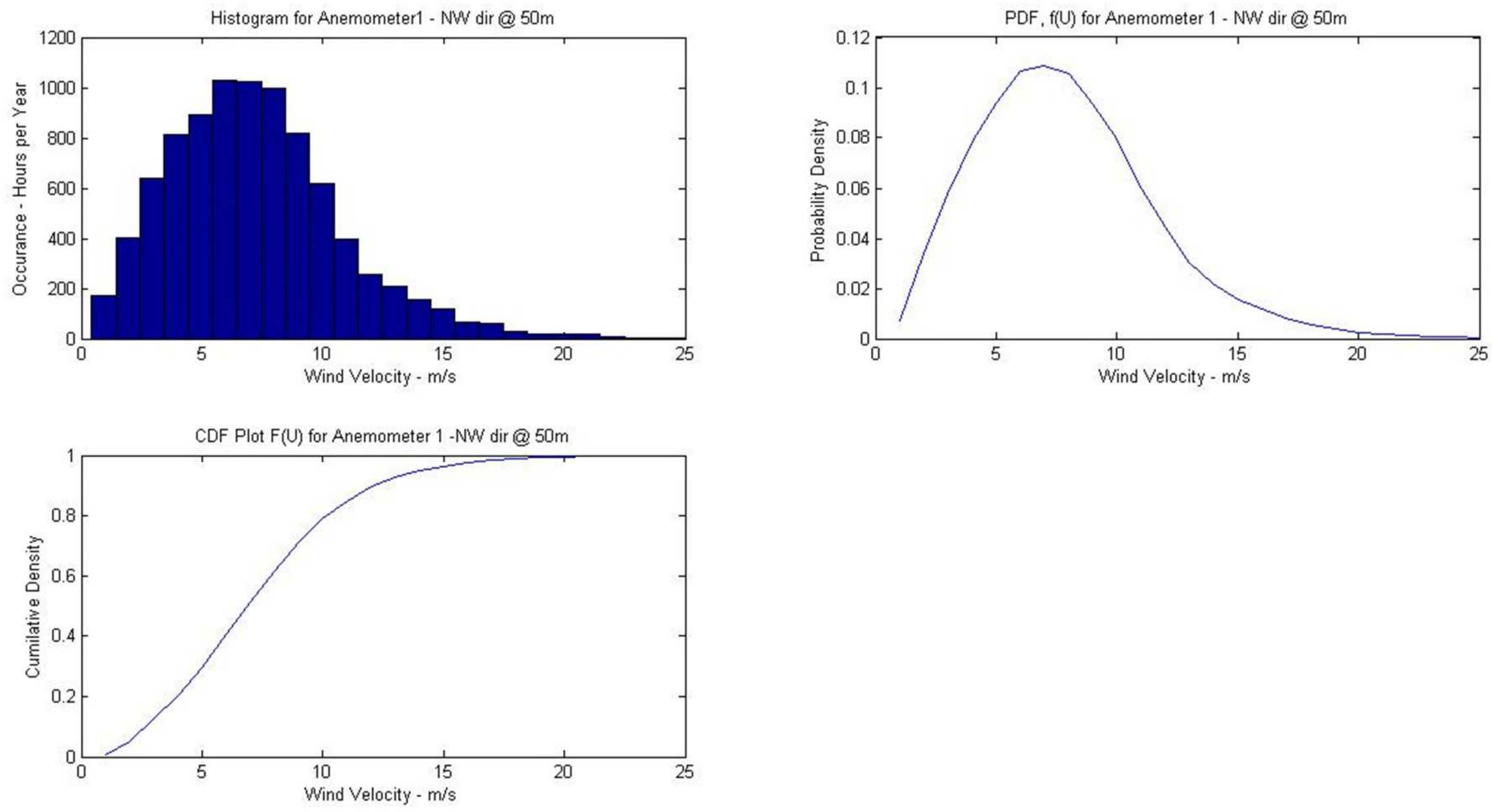
Website: https://www.dnvgl.com/services/bladed-3775

3. BLADED models of 80m diameter and 55m diameter wind turbines

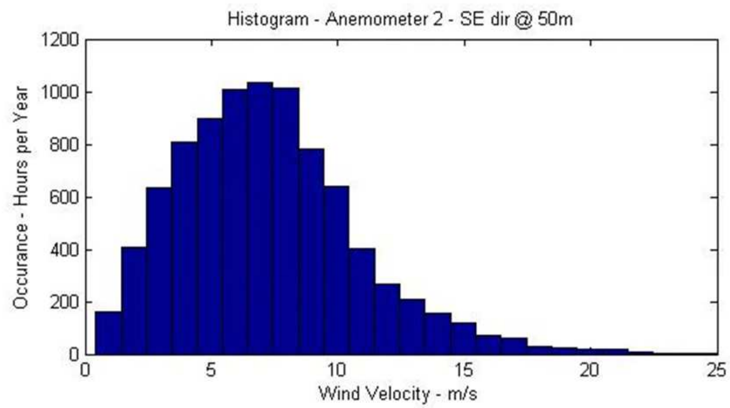
Exercise 2.1
Example solutions

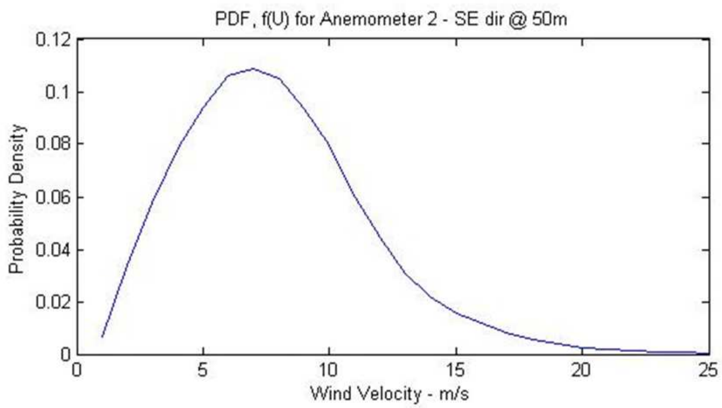
Solutions prepared by Lahiru Thedavikum
Cardiff PhD student

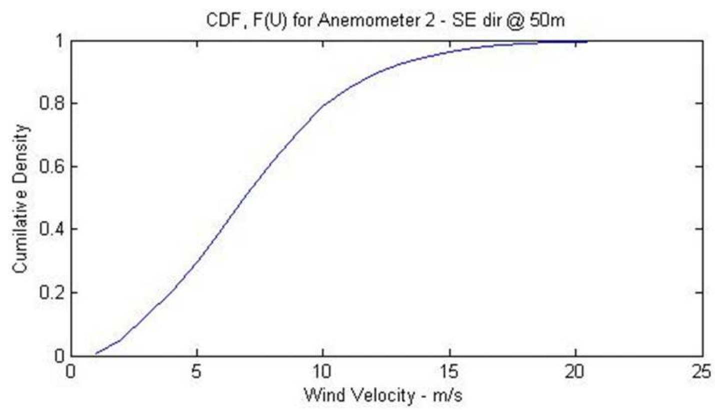
Anemometer 1: 50m, NW cross arm (1 year of data)



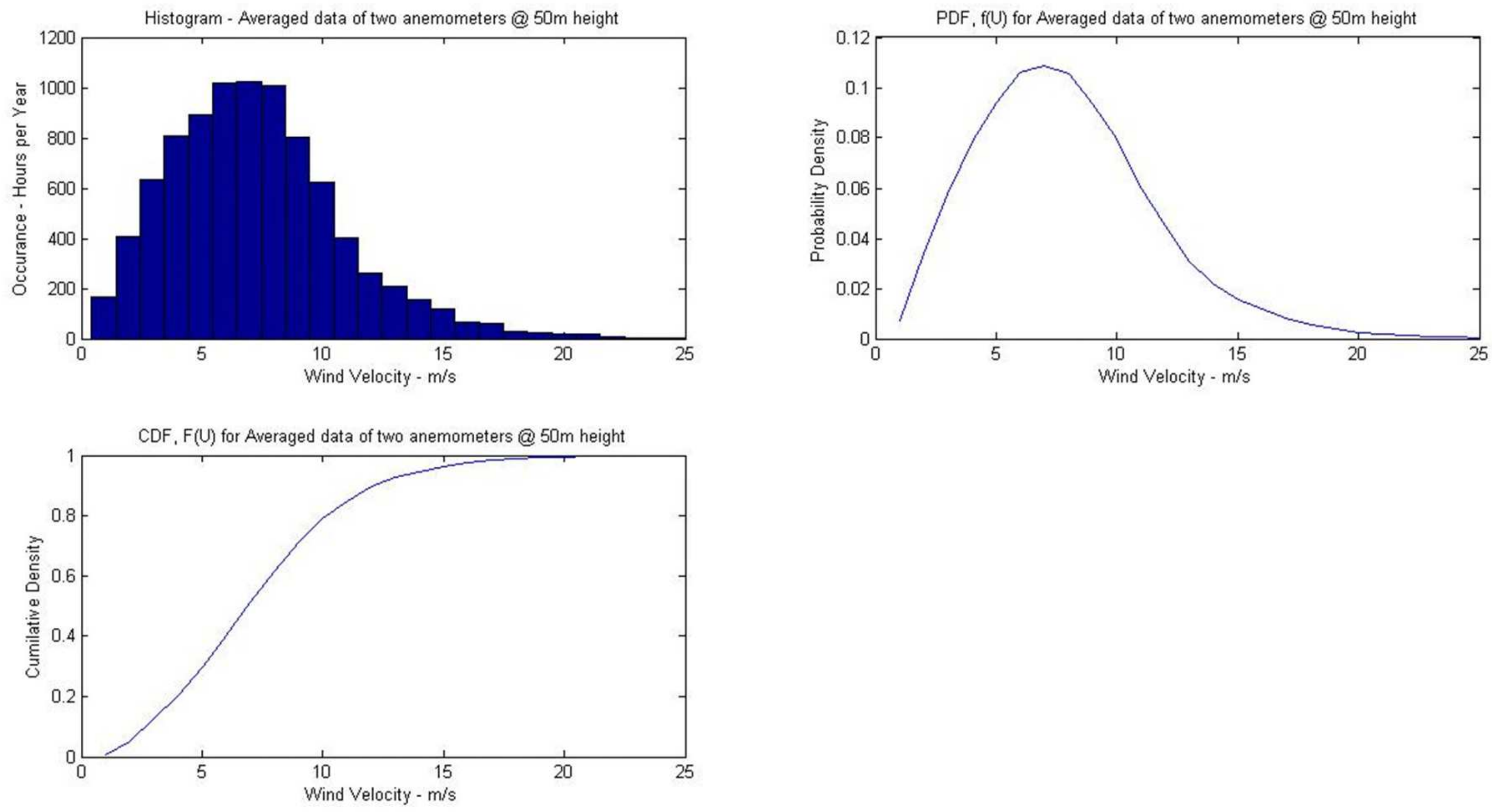
Anemometer 2: 50m, SE cross arm (1 year of data)



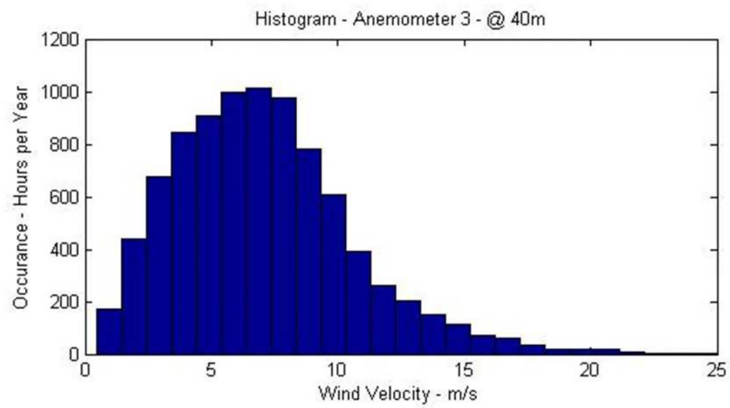


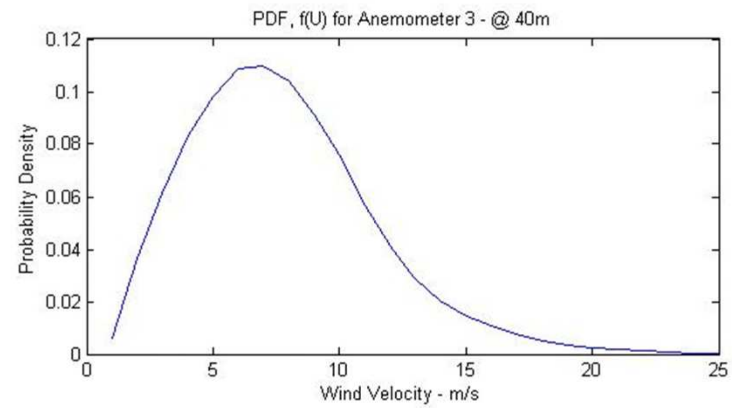


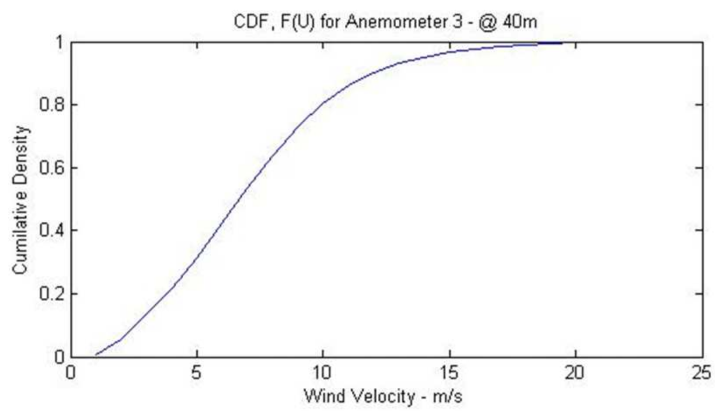
Average 50m
(1 year of data)



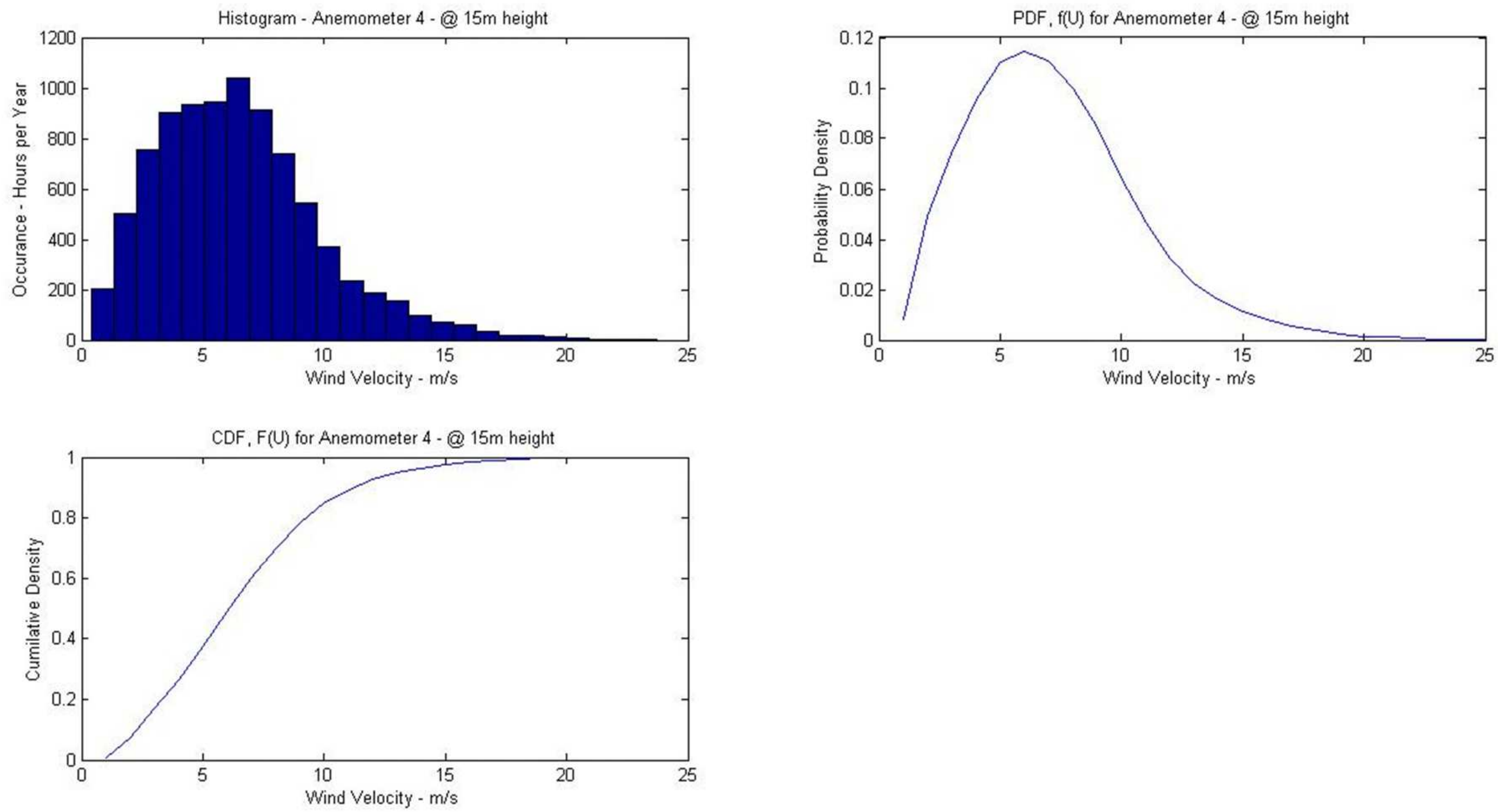
Anemometer 3: 40m (1 year of data)



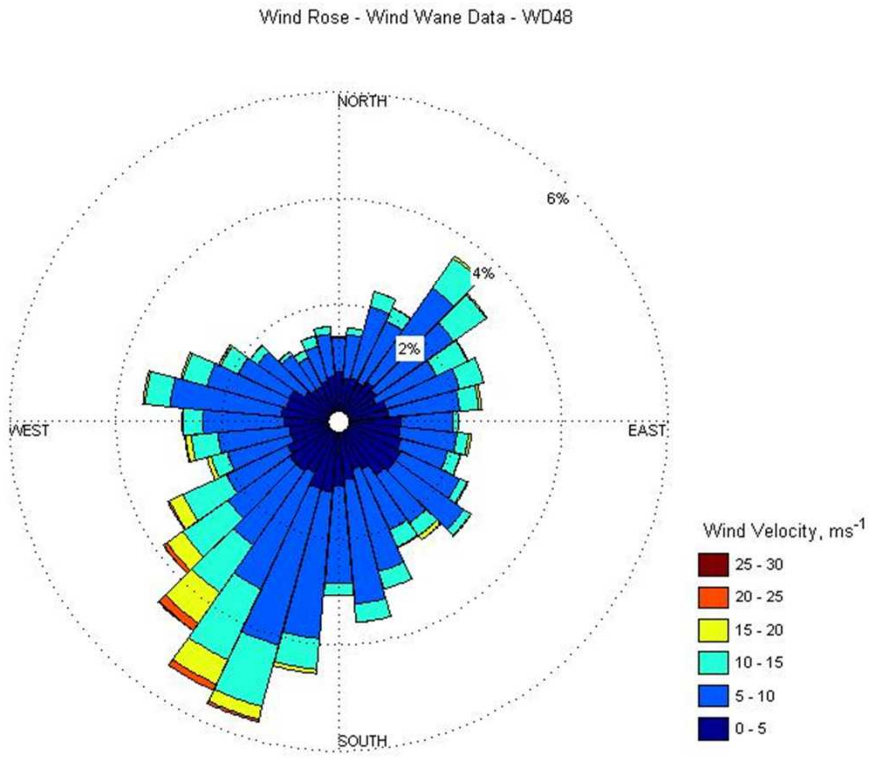




Anemometer 4: 15m
(1 year of data)



The Wind Rose (50m anemometers, 48 m wind direction)



Exercise 2.2
Example solutions

Solutions prepared by Lahiru Thedavikum
Cardiff PhD student

Exercise 2.2a Wind Shear

Estimate the wind shear and compare the data with the 1/7th power law. Estimate best power law coefficient to fit the data

Data – Hourly mean wind velocity data at 3 heights – 15m, 40m and 50m (***Ex 2_1 Data.xlsx***)

Then calculate yearly mean velocities for the heights -

For 50m height, the yearly mean wind speed AN50y,

$$AN50y = \frac{\sum_{i=1}^{8760}(AN50h_i)}{8760} = 7.2278\, m/s$$

- $AN50h_i$ represent the wind speed value for the hour i, since one year is considered and therefore we have (24 x 365) 8760 values.
- Similarly, for other heights the yearly average wind speed is calculated. (AN15y, AN40y).
- The four points [0, AN15y, AN40y, AN50y] = [0, 6.4543, 7.0467, 7.2278] are plotted against the height, [0,15,40,50].

Then calculate the wind velocities at different heights,

using the 1/7th power law.

$$u = u_r \left(\frac{z}{z_r}\right)^{1/7}$$

Taking the reference as, $z_r = 15m, u_r = 6.4543\ m/s$

Plot the calculated wind velocities against the height.

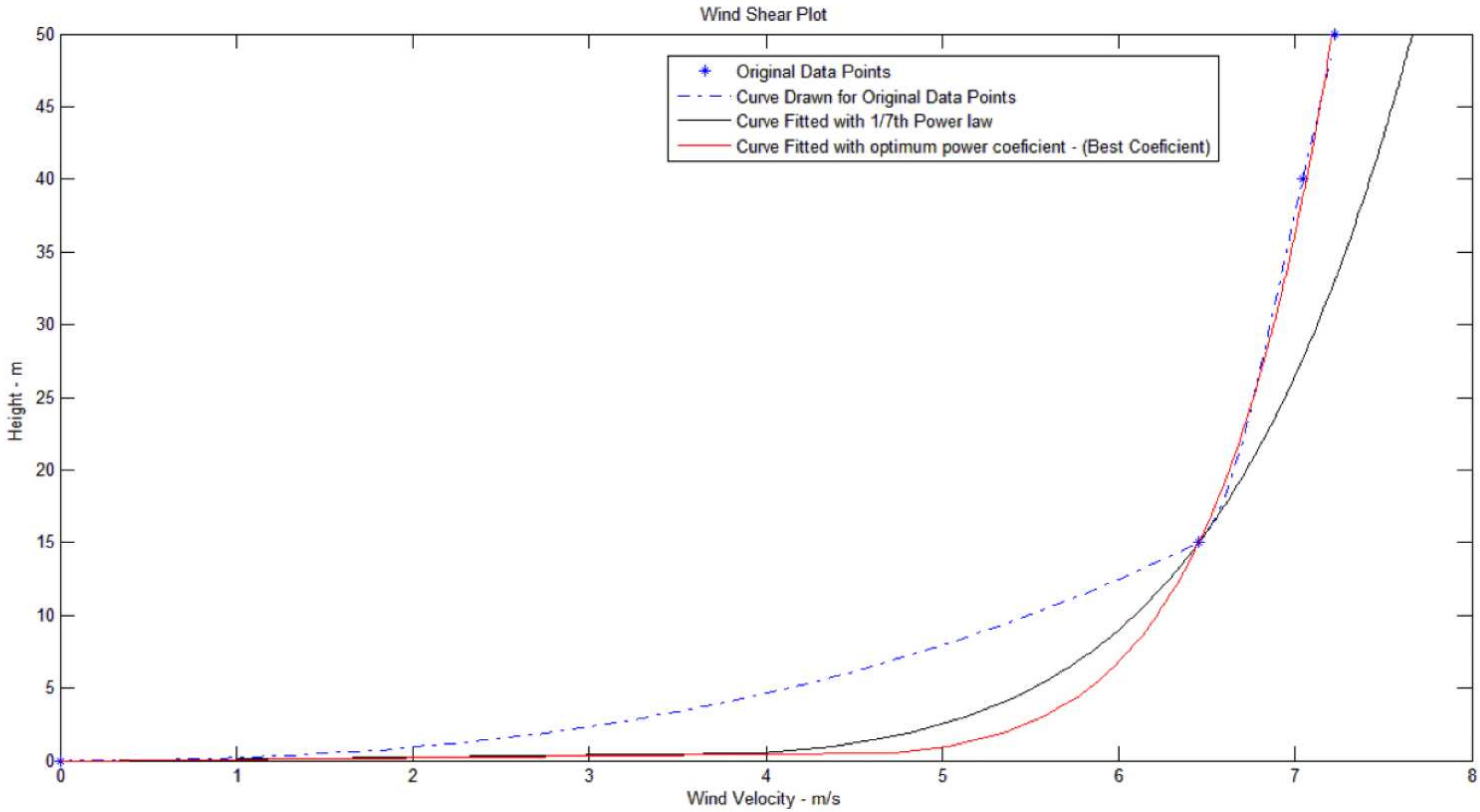
Estimate the best power coefficient to fit the data.

- Consider the general power law, $u = u_r \left(\frac{z}{z_r}\right)^a$
- Take the reference as, $z_r = 15m$, $u_r = 6.4543\ m/s$ and by taking the log in the base 10, the equation becomes,

$$\log10(\mathrm{u}) = \mathrm{y} = 0.81 + \mathrm{a} \times \log10(\mathrm{z}) - 1.176 \times \mathrm{a} \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (A)$$

- Original points, $\mathrm{y1} = \log10(7.0467)$ and $\mathrm{y2} = \log10(7.2278)$.
- Error function, $\mathrm{S} = (\mathrm{y1} - \mathrm{y11})^2 + (\mathrm{y2} - \mathrm{y22})^2$
- Calculate $\mathrm{y11}$ and $\mathrm{y22}$ from equation A, for $\mathrm{z} = 40$and $\mathrm{z} = 50$ respectively.
- Find the minimum of S for a range of power coefficients, a .
  ($\boldsymbol{S\_\mathrm{min}} = \mathbf{2.2656 \times 10^{-6}}$)
- At the minimum S, by back calculating the corresponding a value is found.

$$\boldsymbol{Optimum\ power\ Coefficient = 0.092 \cong \frac{1}{10}}$$



Wind Shear Plot
Original Data Points
Curve Drawn for Original Data Points
Curve Fitted with 1/7th Power law
Curve Fitted with optimum power coeficient - (Best Coeficient)
Height - m
Wind Velocity - m/s
50
45
40
35
30
25
20
15
10
5
0
0
1
2
3
4
5
6
7
8

Ex.2.2b Weibull parameters

Estimate the Weibull Parameters of the wind speed data.

Approach –

- Ex. For 3 m/s bin, calculating Probability Density and Cumulative Density.
- f (3) $= \frac{Occurance\ in\ considered\ year}{8760}$
- F (3) = f (1) + f (2) +f (3)
- Similarly, for bins 1 m/s to 25 m/s Probability Density and Cumulative Density are calculated.

Manual calculation of Weibull parameters for the hourly averaged wind speed data.

- Consider equation, $F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^k\right]$ for calculating Cumulative Density.
- Taking natural log twice, the equation becomes,
$$\ln[-\ln(Q(U))] = k \times \ln(U) - k \times \ln(c), \quad \text{here } Q(U) = 1 - F(U)$$
- Calculate $\ln(U)$ and $\ln[-\ln(Q(U))]$ separately for each wind speed bin. (1 – 25 m/s)
- Plot $\ln[-\ln(Q(U))]$ vs. $\ln(U)$ and a linear function ($y = mx + n$) is fitted to the points.
- When plotted the coefficients of the linear equation becomes, $m = k$ and $n = -k \times \ln(c)$

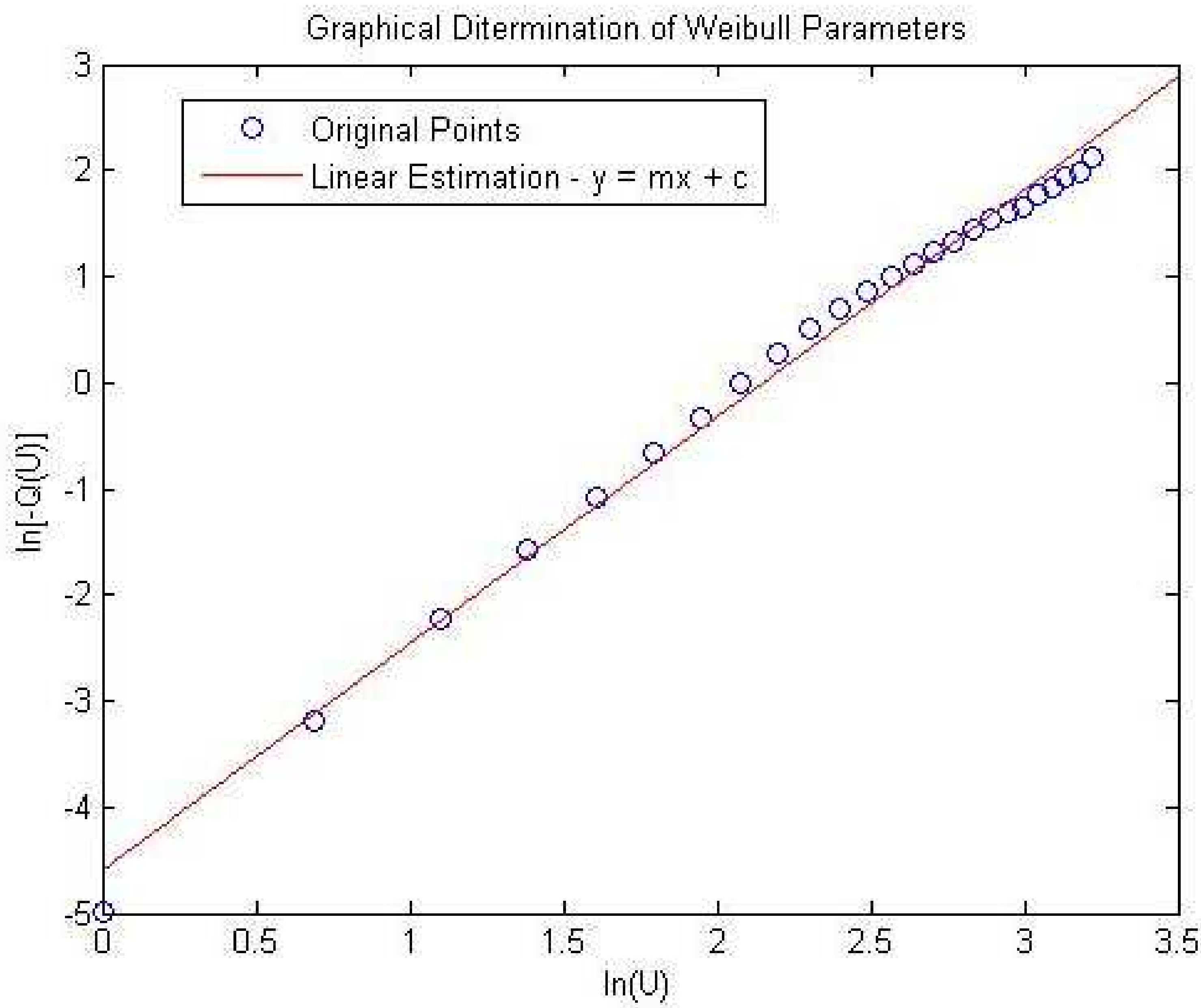


Figure 1 - Graphical Determination of Weibull Parameters for 50m data

From the graph,

m = **k = 2.1364** and

n = $-\mathrm{k} \times \ln(\mathrm{c})$ = - 4.6006 which will give **c = 8.6142**

Using the calculated parameters and considering equations,

$$f(U) = \frac{k}{c}\left(\frac{U}{c}\right)^{k-1} \times exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

and

$$\mathrm{F(U)} = 1 - \exp\left[-\left(\frac{\mathrm{U}}{\mathrm{c}}\right)^{\mathrm{k}}\right]$$

the Probability Density and Cumulative Density are calculated and plotted.

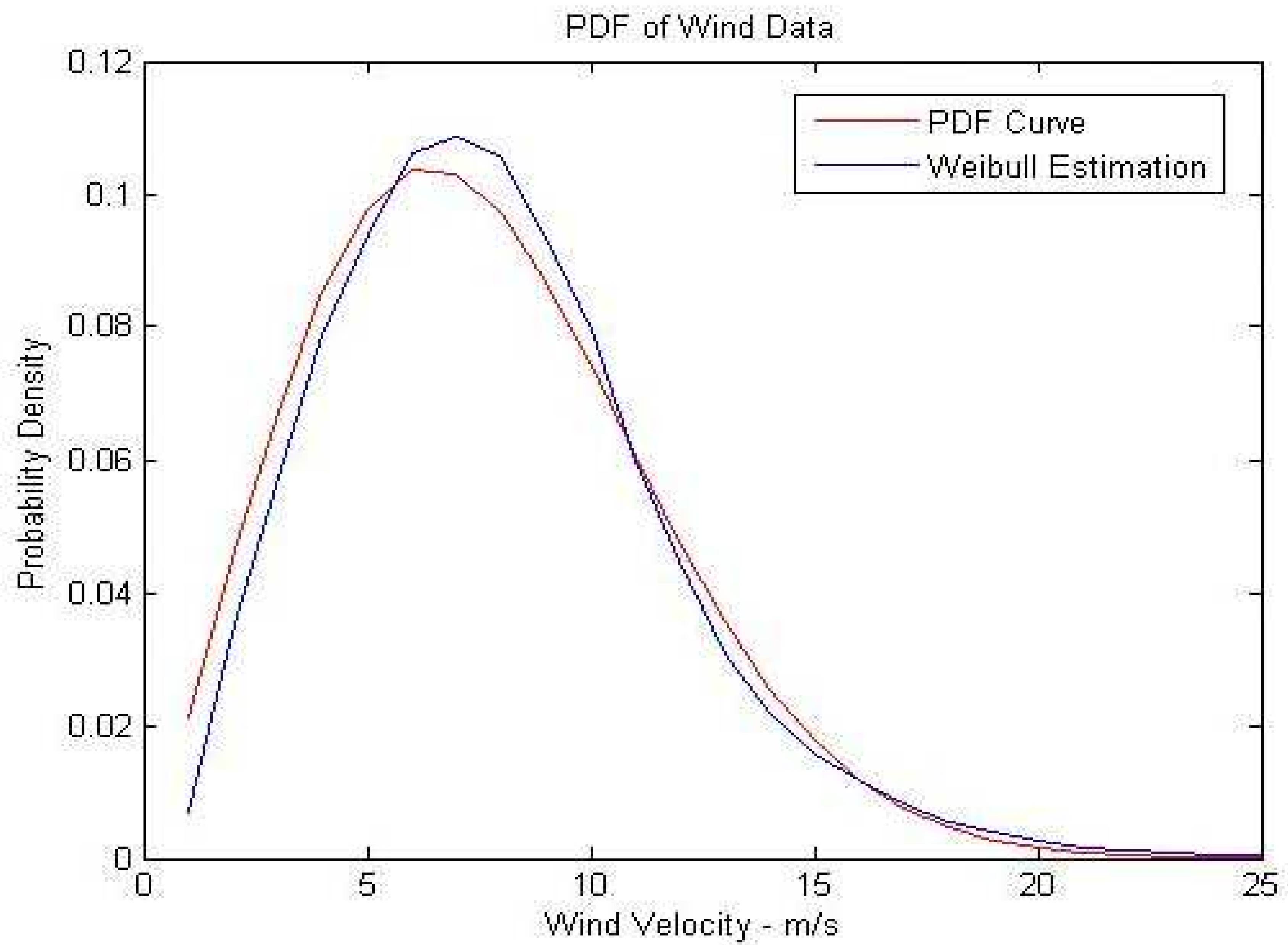


Figure 2 – PDF plot with Weibull Estimation for 50m Data

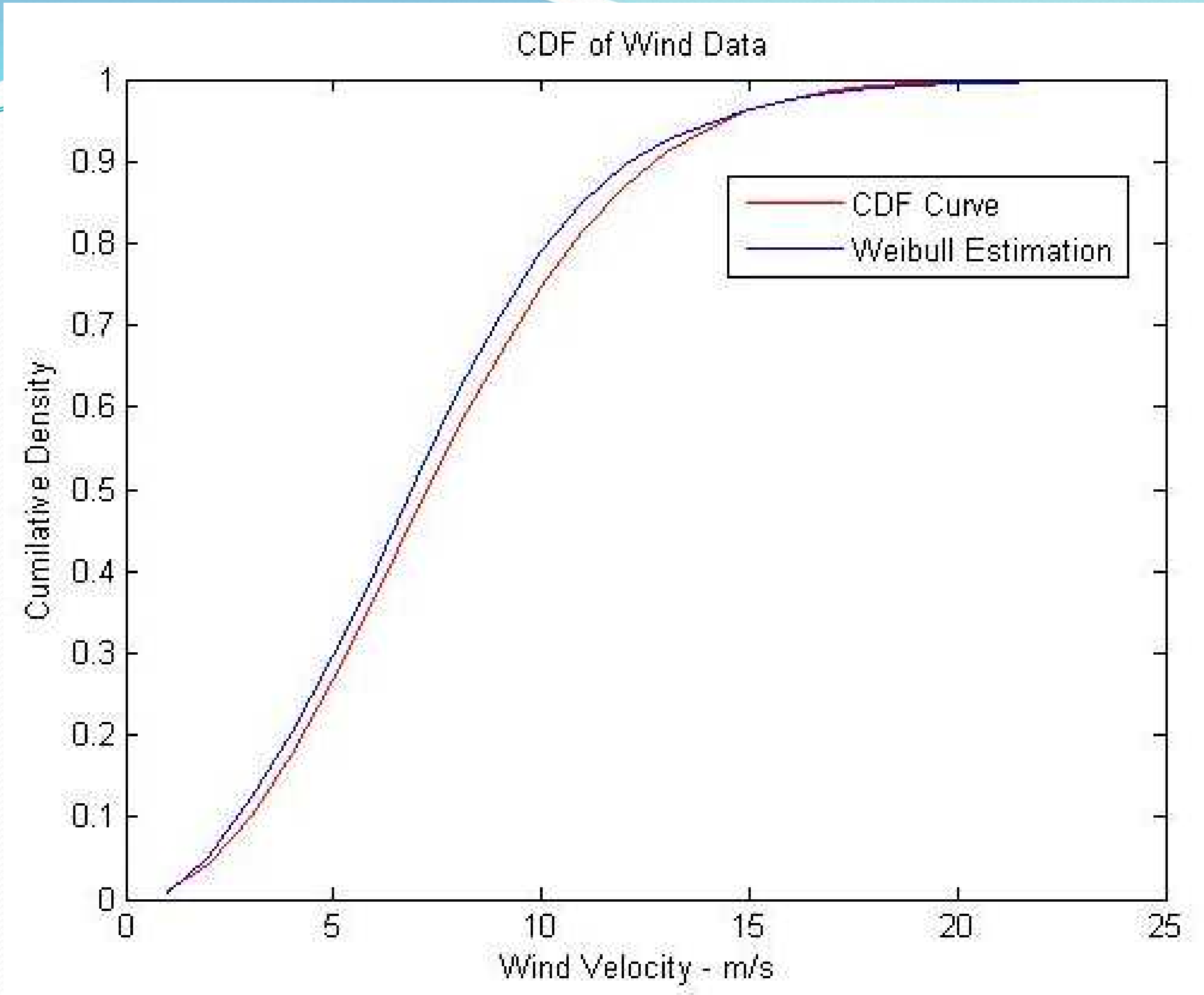


Figure 3 - CDF plot with Weibull Estimation for 50m Data

Exercise 2.3
Example solutions

Solutions prepared by Lahiru Thedavikum
Cardiff PhD student

Using the set of mean wind speed data provided and the power curves of the 600 kW (50m diameter), 1.3 MW (55m diameter), MW and 2 MW (80m diameter) wind turbines estimate the annual energy produced by each wind turbine.

Data – Hourly mean wind velocity data at 3 heights – 15m, 40m and 50m (***Ex 2_1 Data.xlsx***)

Power Data for the three wind turbine data – (***Turbines power data.xlsx***)

Approach –

- Since the three turbines have the hub heights to be 61.5m, 55m and 50m, the average wind velocity data at 50m height is selected as reference data u_r.
- The power law is used to move the wind speed data into the corresponding hub heights.

$$\boldsymbol{u} = \boldsymbol{u_r}\left(\frac{\boldsymbol{z}}{\boldsymbol{z_r}}\right)^{\boldsymbol{1/7}}$$

- Histograms created with 1m/s bins to calculate the occurrence of each wind velocity bin; 1m/s to 25m/s.

$$\boldsymbol{Annual\ Energy\ Produced(\ at\ 6m/s)\ = Yearly\ Occurance \times Power\ Produced(\ at\ 6m/s)}$$ **(kWh/year)**

- Yearly occurrence is referred from the histogram data and the Power Produced at each velocity bin is refereed from the relevant Turbine Power data.

Therefore,

$$\boldsymbol{Total\ Annual\ Energy\ Produced = \frac{\sum_{i=1}^{25} Annual\ Energy\ Produced_i}{1000}}$$ **MWh.**

For 2MW Wind Turbine,

$$\boldsymbol{Total\ Annual\ Energy\ Produced = 5653.5\ \mathbf{MWh}}$$

For 1.3MW Wind Turbine,

$$\boldsymbol{Total\ Annual\ Energy\ Produced = 2347.6\ \mathbf{MWh}}$$

For 600kW Wind Turbine,

$$\boldsymbol{Total\ Annual\ Energy\ Produced = 1990.7\ \mathbf{MWh}}$$

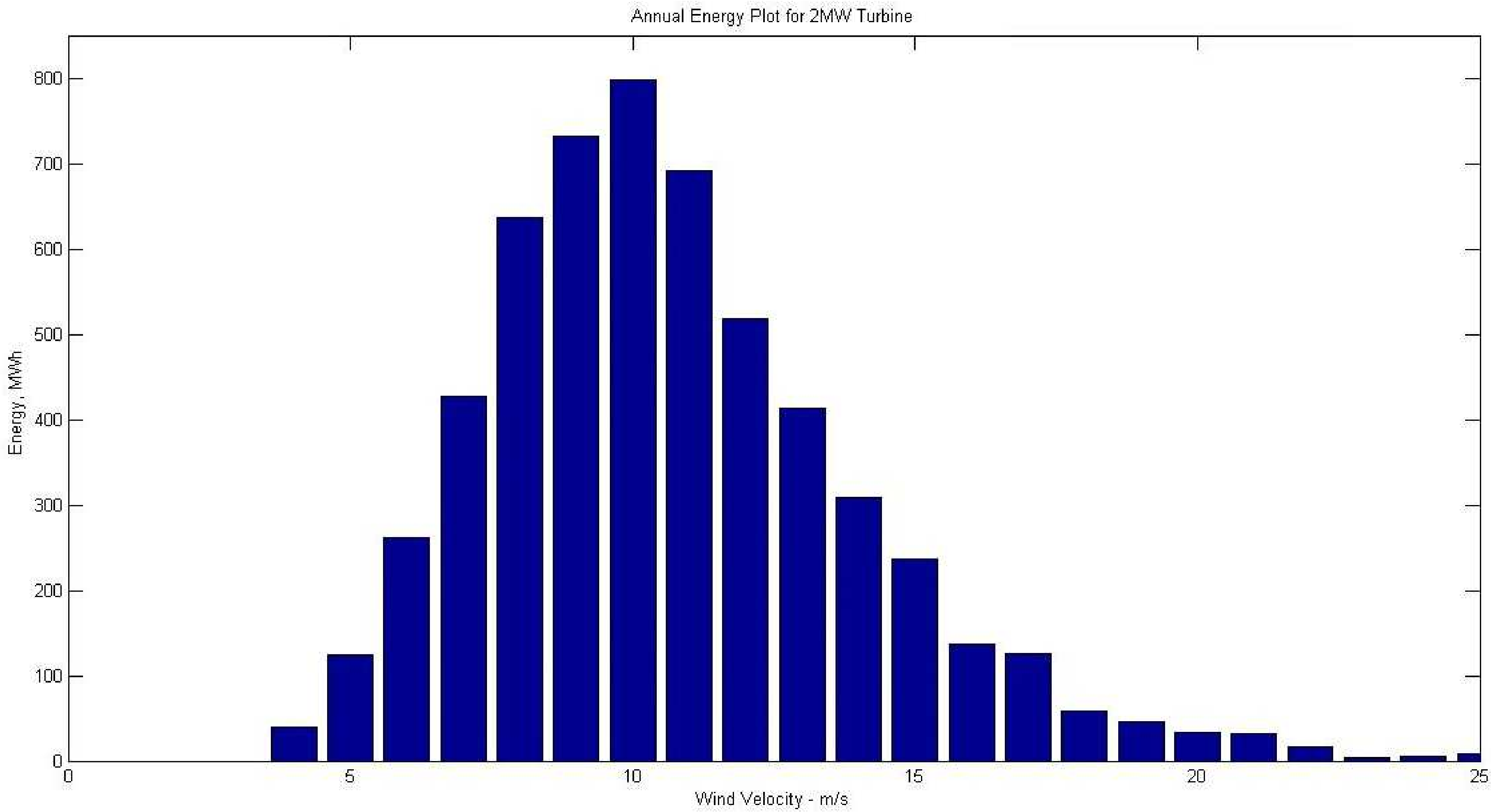


Figure 1 - Annual Energy Plot for 2MW Turbine

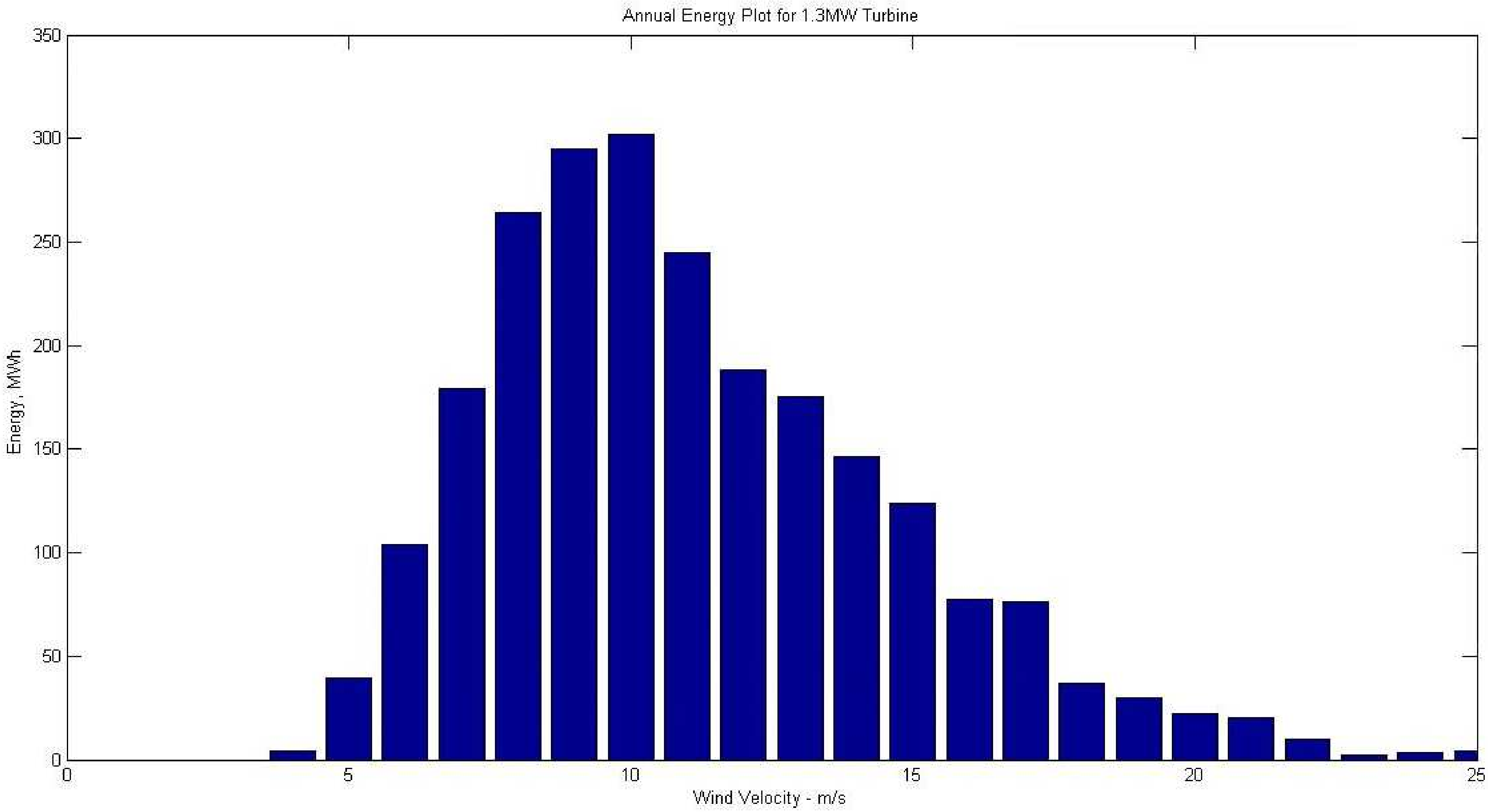


Figure 2 - Annual Energy Plot for 1.3 MW Turbine

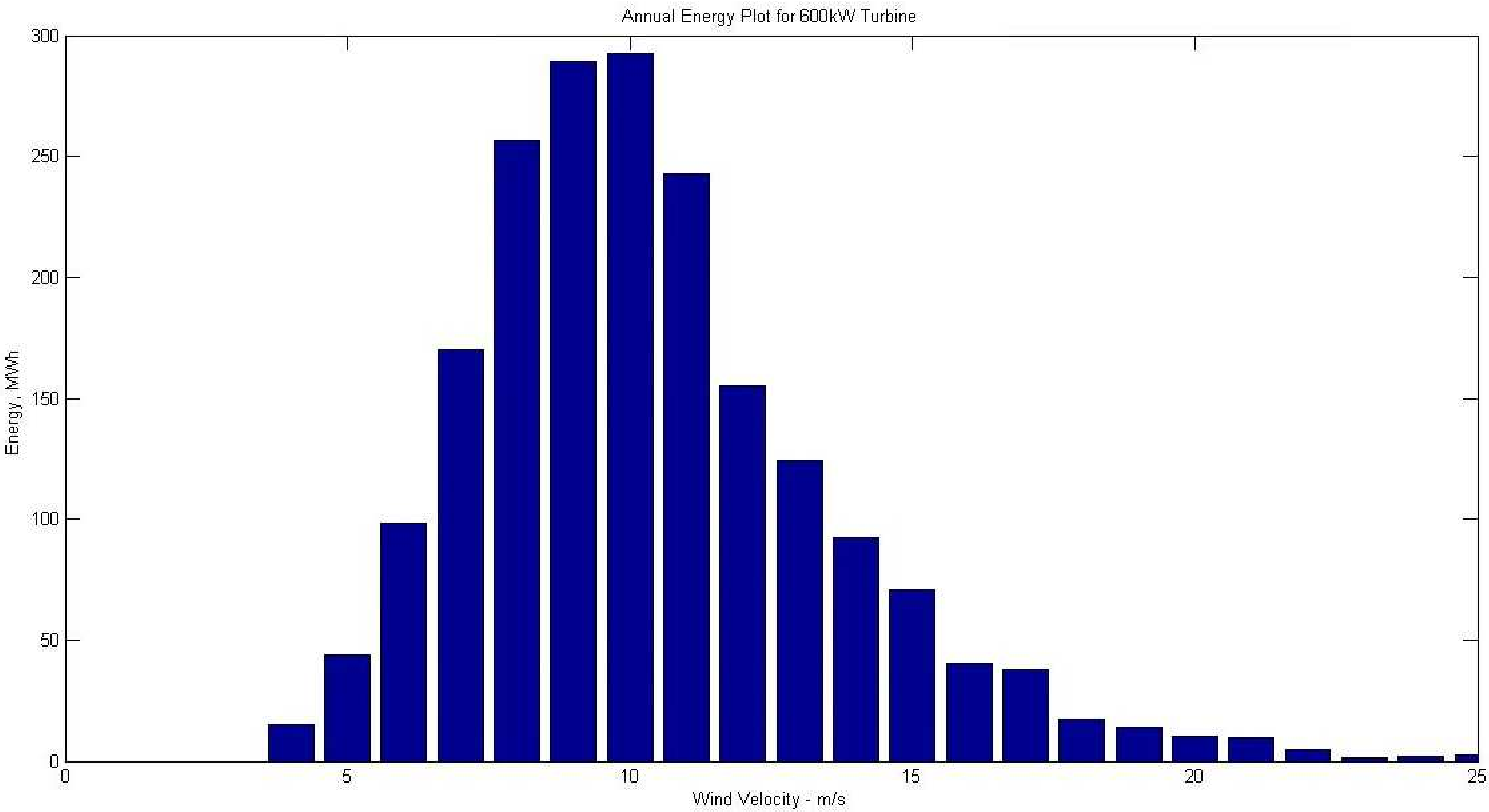
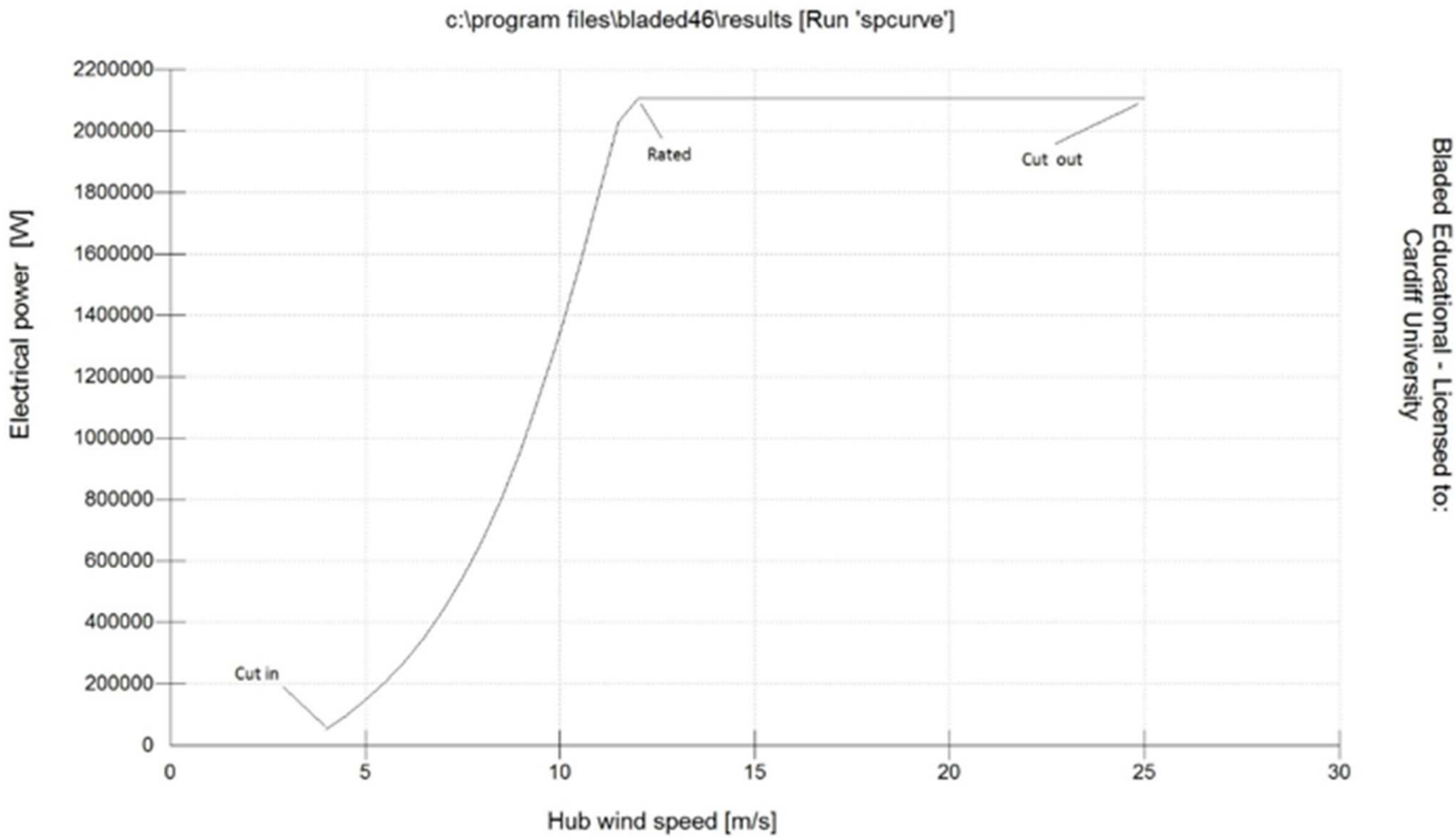


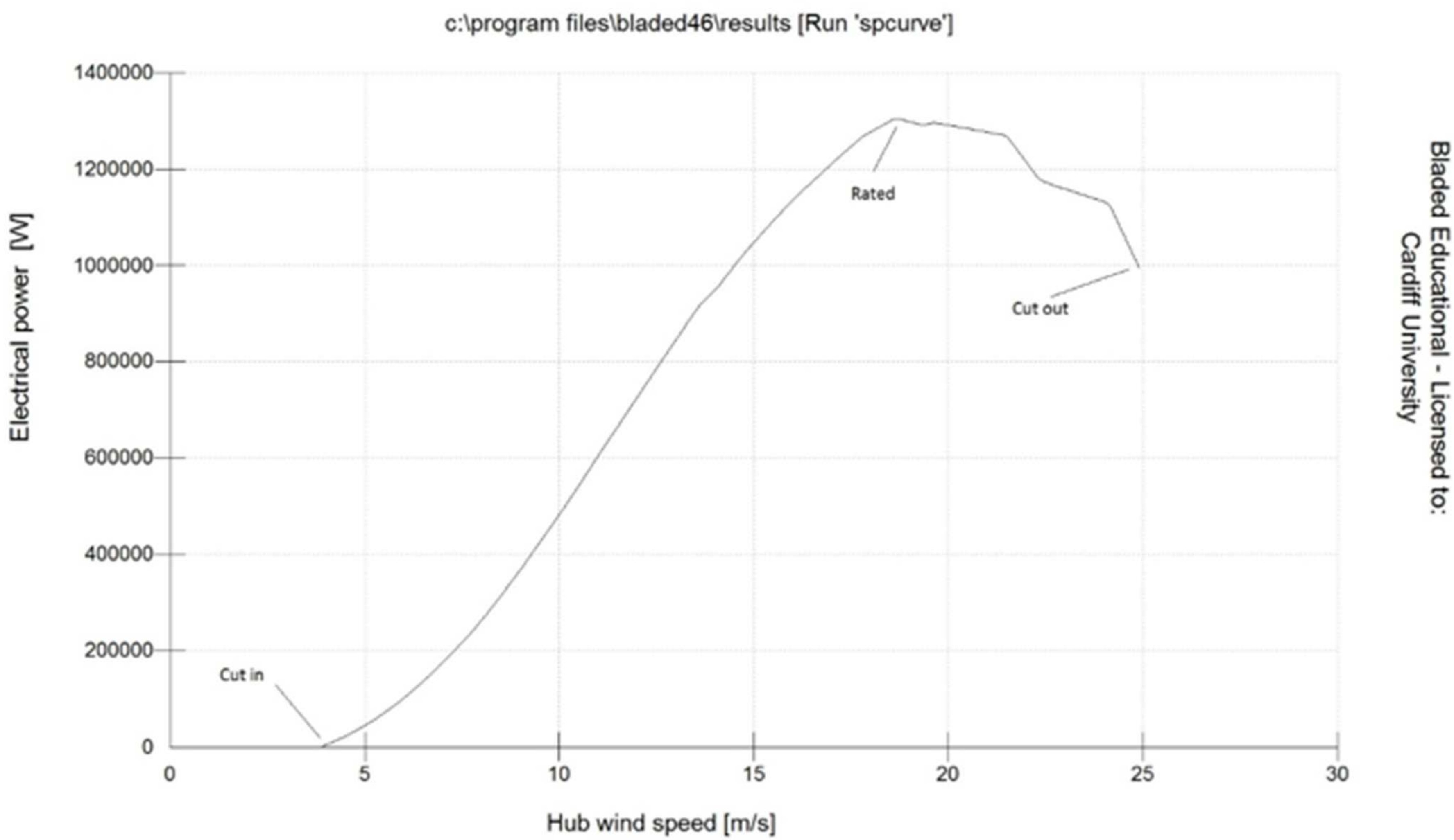
Figure 3 - Annual Energy Plot for 600kW Turbine

Exercise 2.4
Example solutions

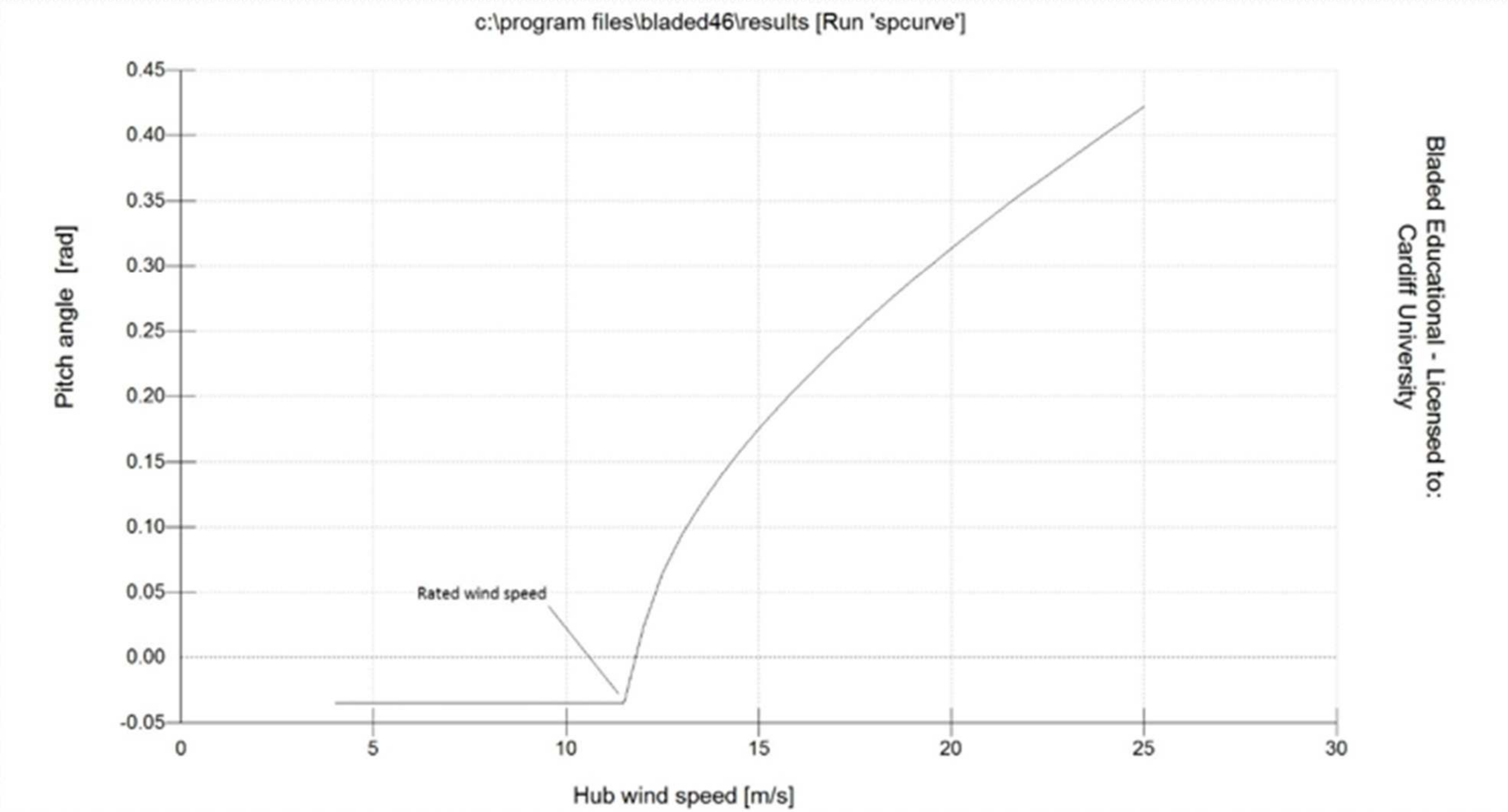
Power curve of 80m diameter, variable speed, pitch regulated wind turbine



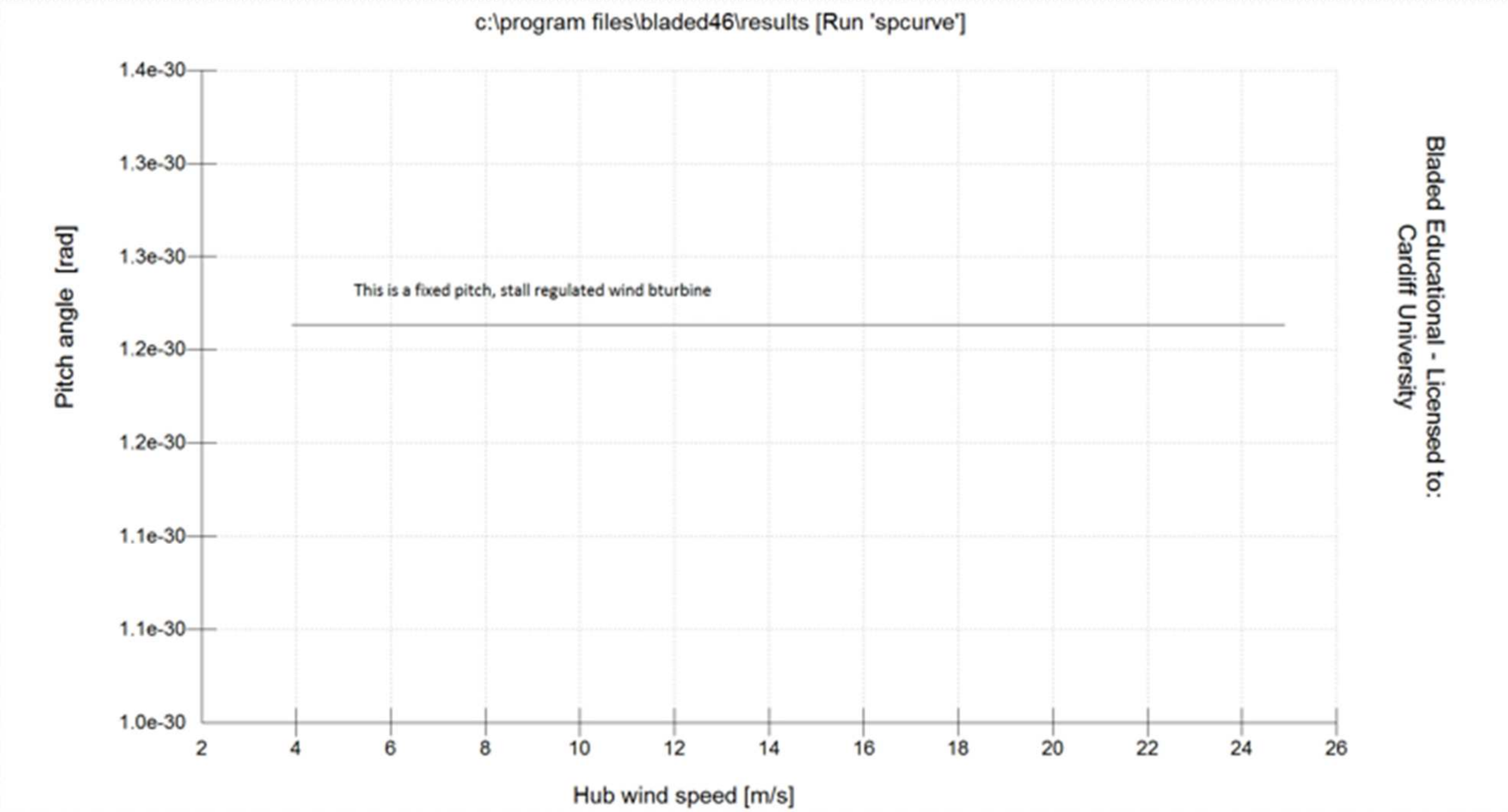
Power curve of 55m diameter, stall regulated wind turbine



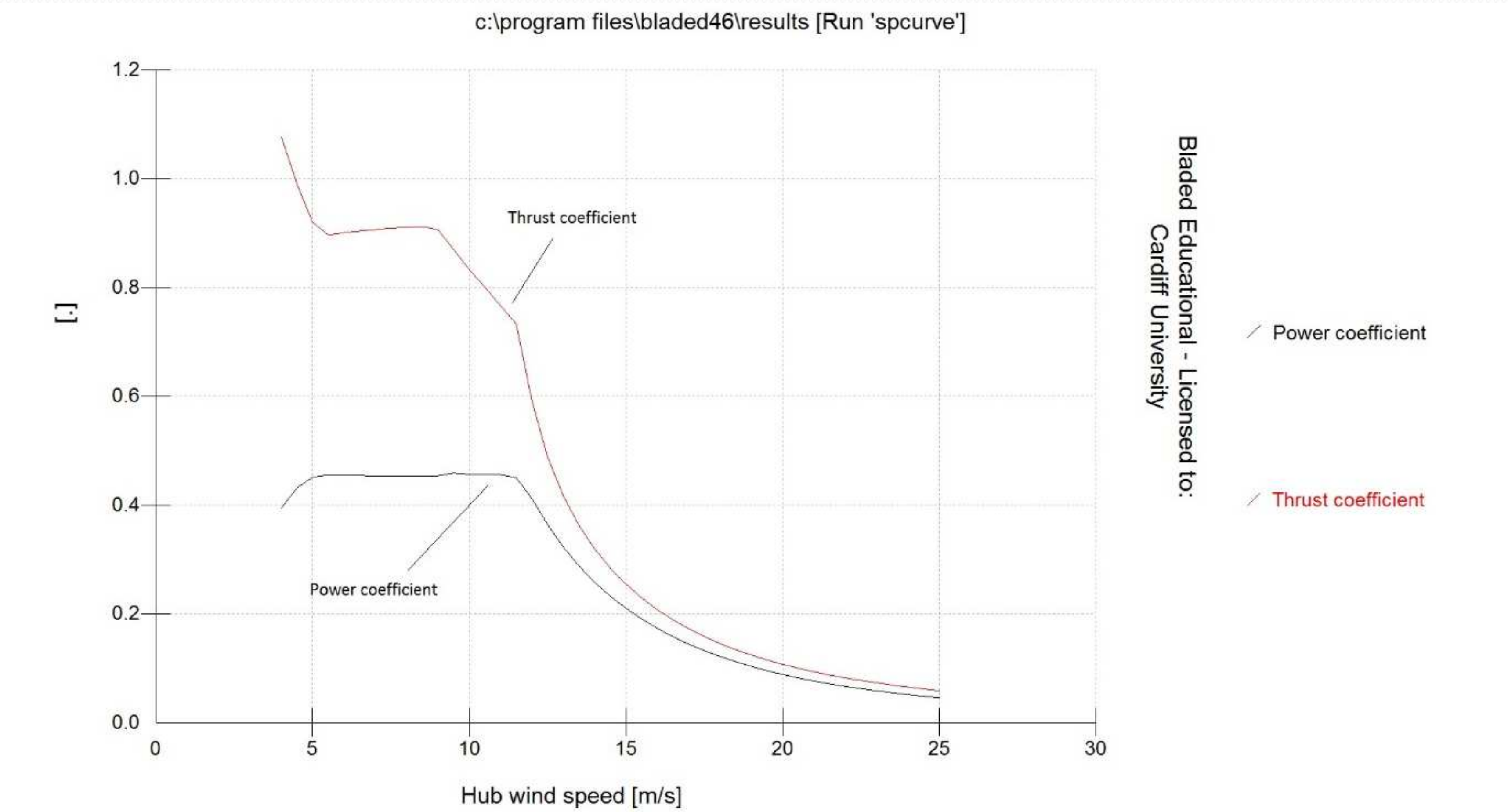
Pitch angle of 80m diameter, variable speed, pitch regulated wind turbine



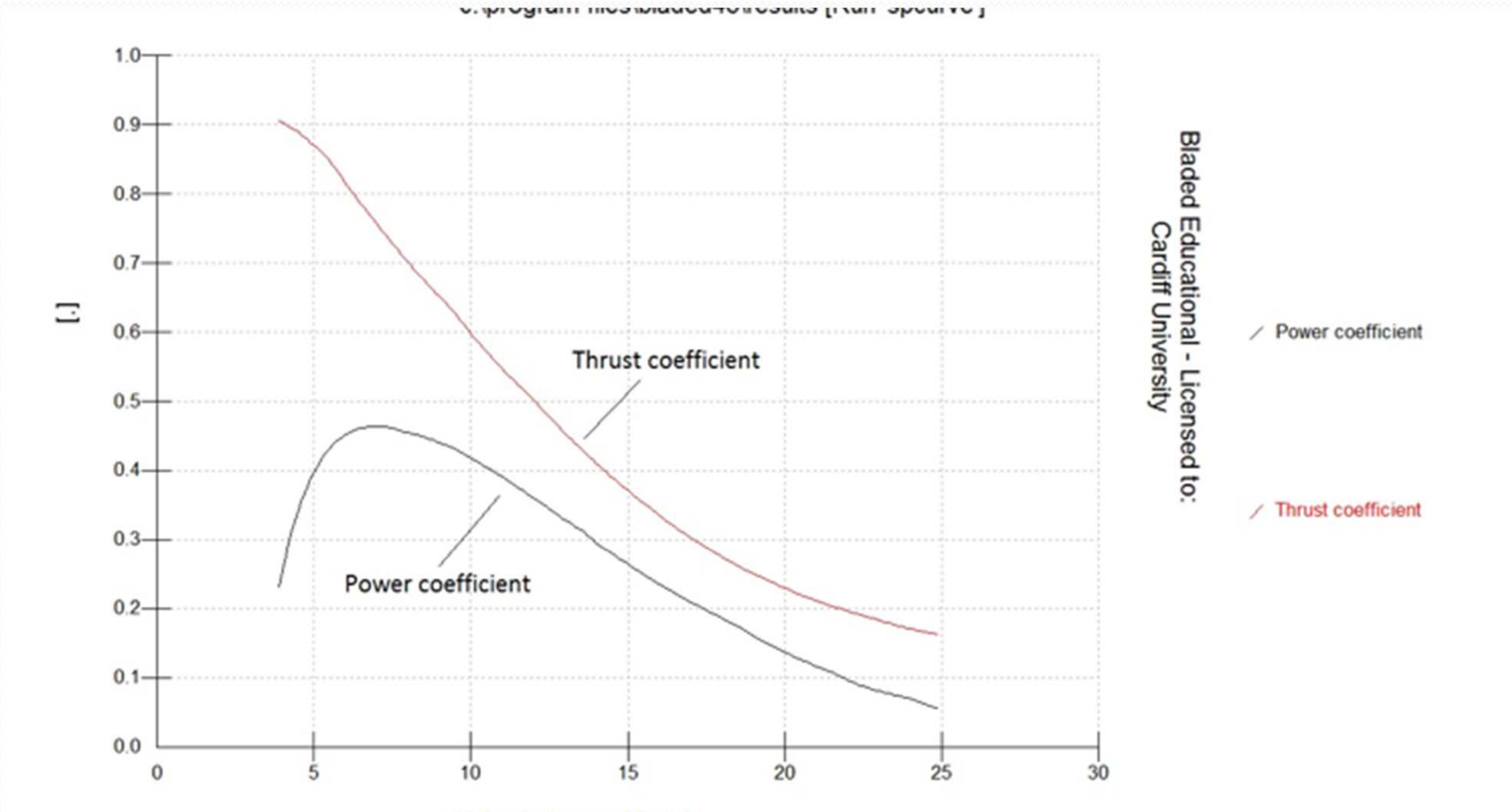
Pitch angle of 55m diameter, fixed speed, stall regulated wind turbine. Note pitch angle is fixed.



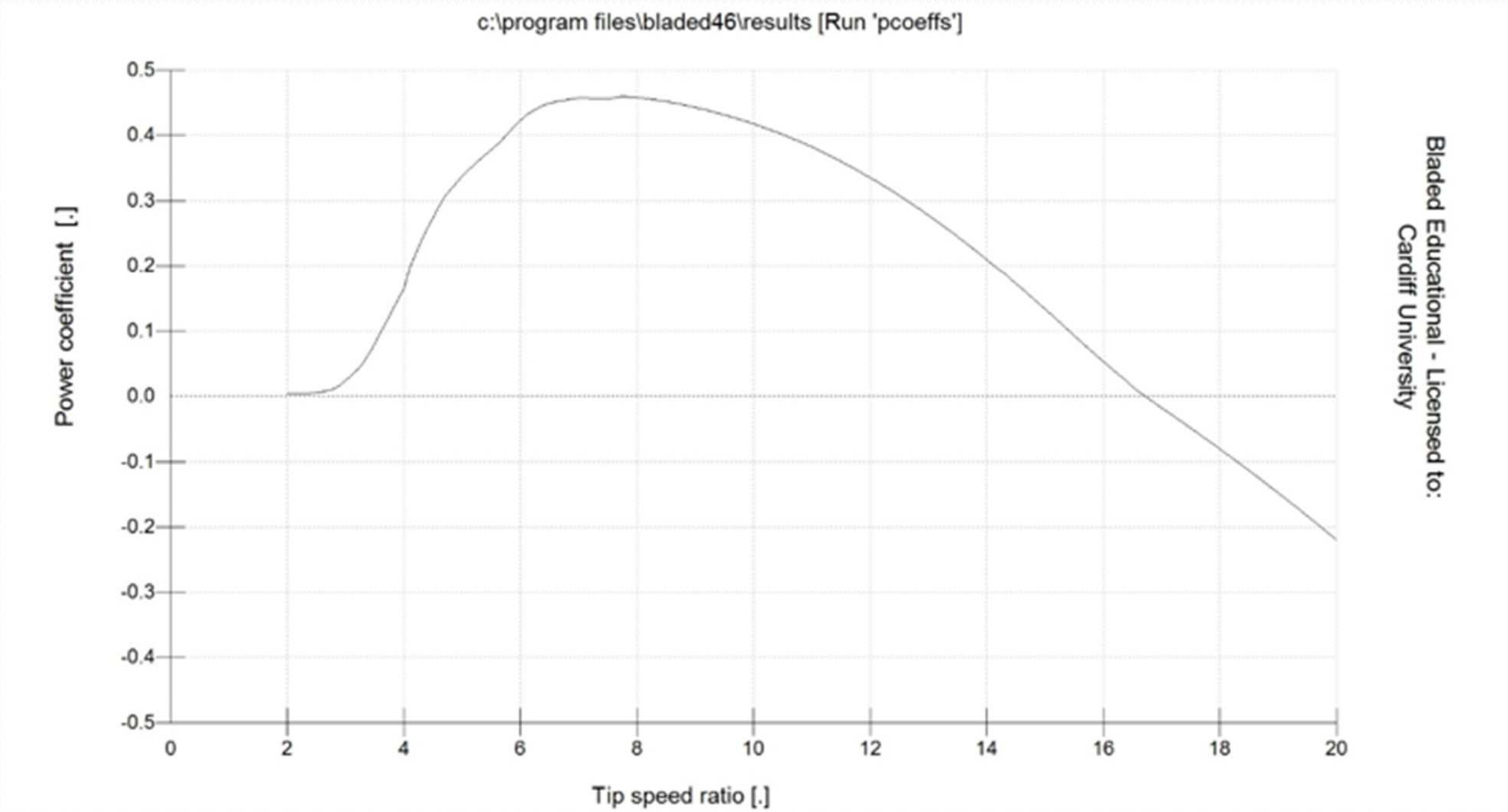
Power and Thrust coefficients of 80m diameter variable speed, pitch regulated wind turbine



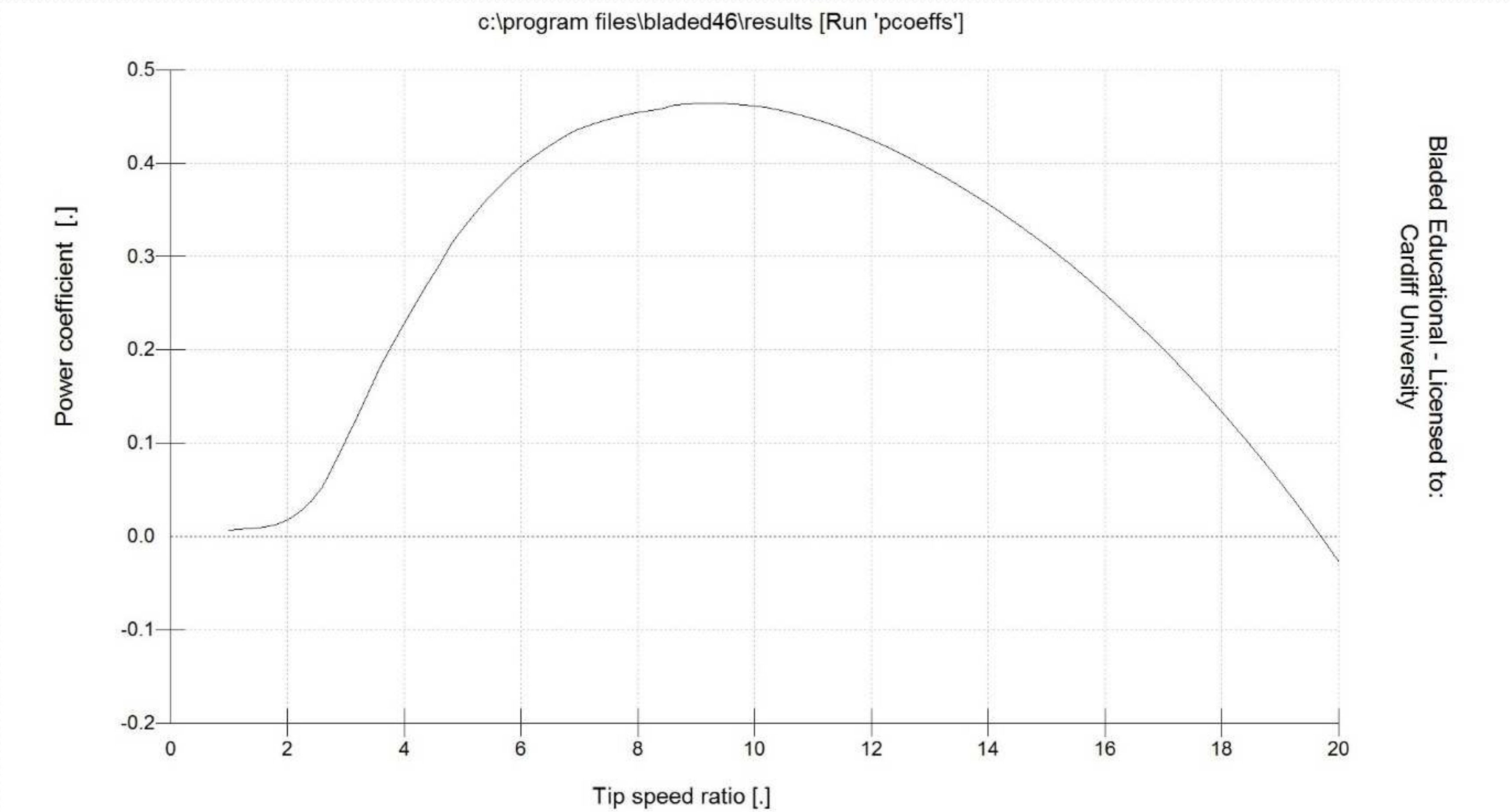
Power and Thrust coefficients of 55m diameter fixed speed, stall regulated wind turbine



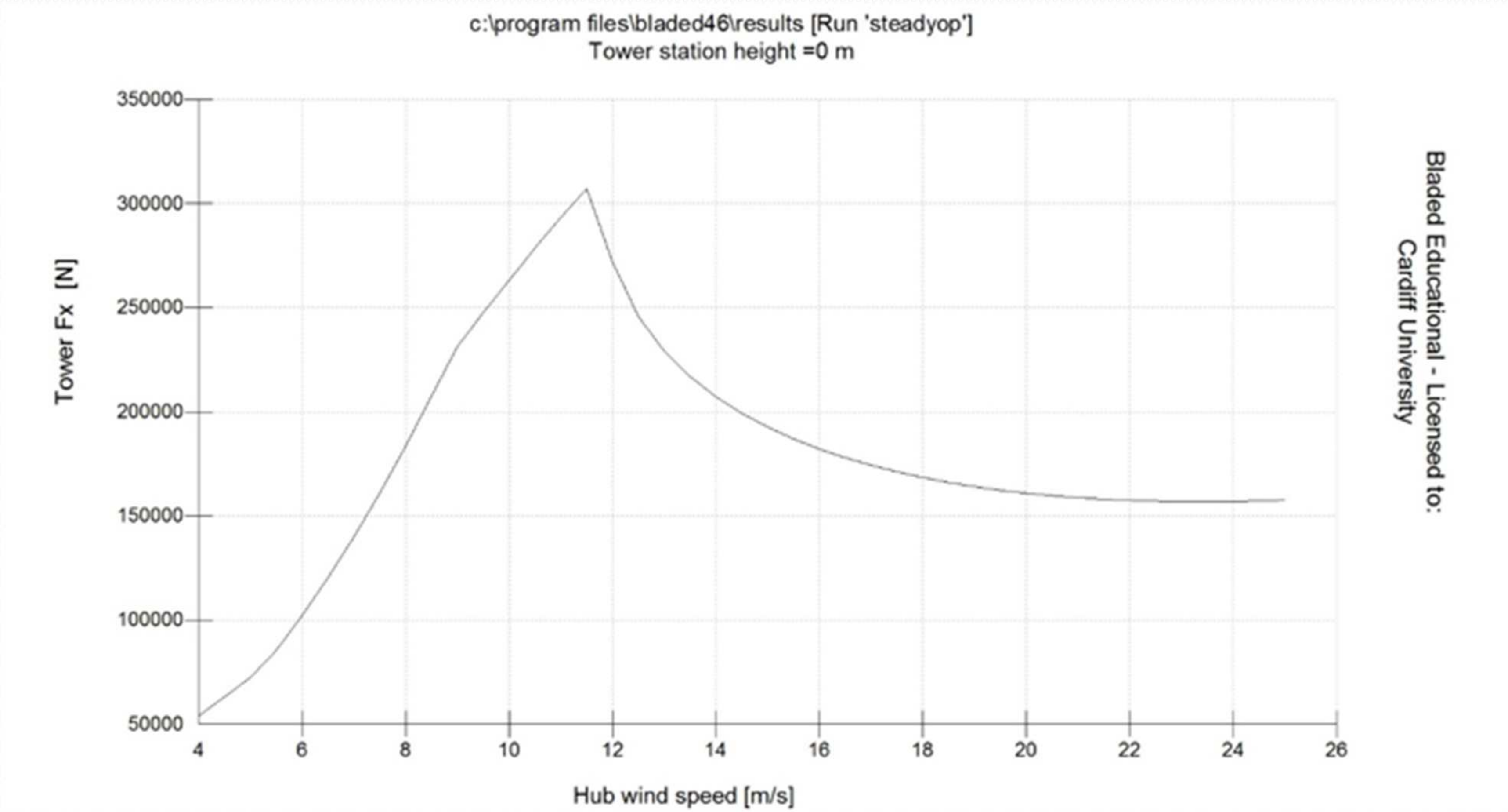
Power coefficient versus tip speed ratio of 80m diameter variable speed, wind turbine



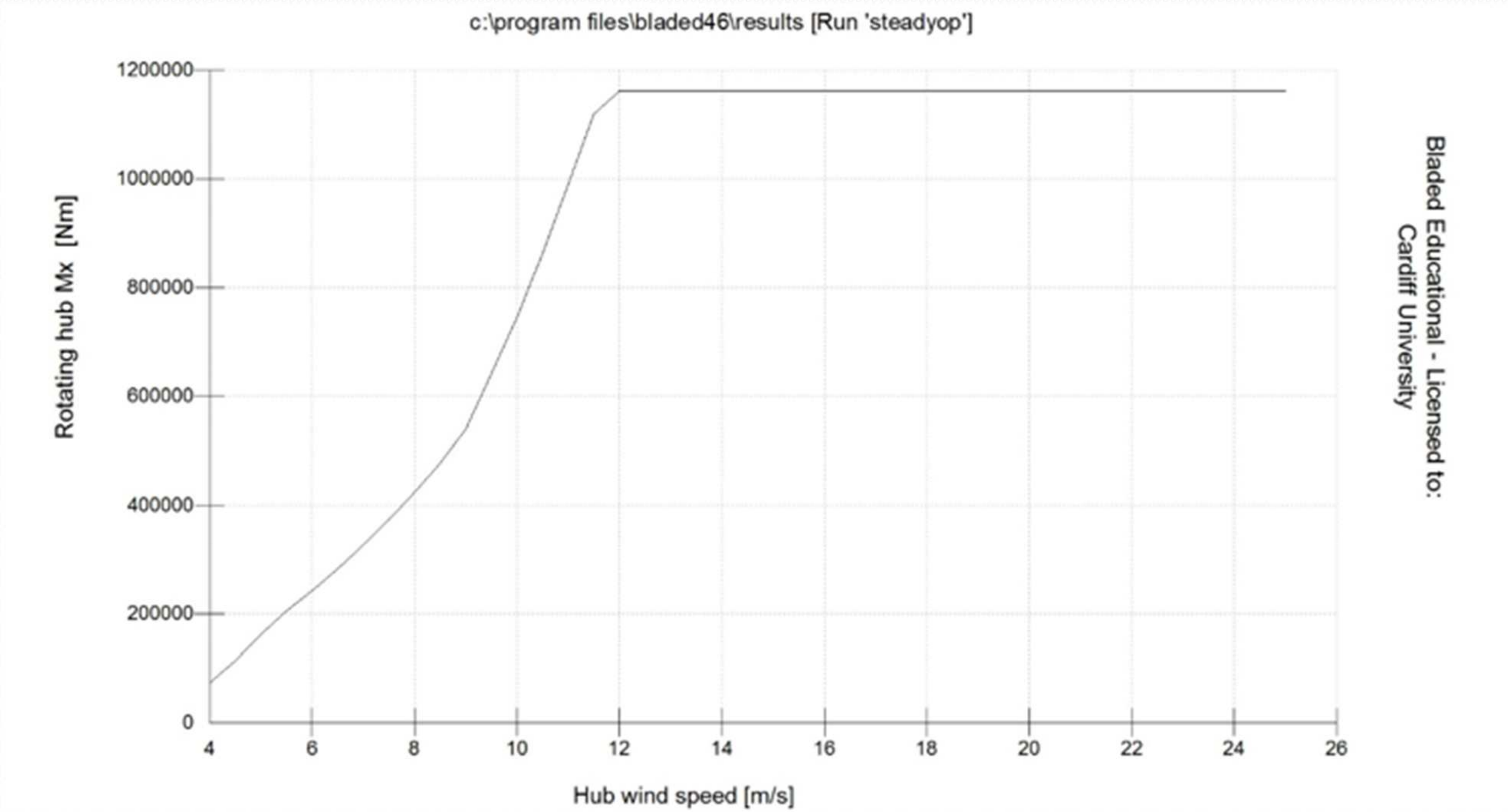
Power coefficient versus tip speed ratio of 55 m diameter, stall regulated, wind turbine



Tower thrust (Fx) loads of 80m diameter wind turbine



Rotating (Mx) loads giving useful torque of 80m diameter turbine.



Generator speed v. generator torque for 80m diameter wind turbine.

This characteristic is the basis of the variable speed control system.