College Algebra 8th edition Stewart Solutions Manual

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P PREREQUISITES

P.1 MODELING THE REAL WORLD WITH ALGEBRA

1. Using this model, we find that 15 cars have $W = 4(15) = 60$ wheels. To find the number of cars that have a total of W wheels, we write $W = 4X \Leftrightarrow X = \dfrac{W}{4}$.If the cars in a parking lot have a total of 124 wheels, we find that there are $X = \frac{124}{4} = 31$ cars in the lot.

2. If each gallon of gas costs \$3.50, then x gallons of gas costs $\$3.5x$. Thus, $C = 3.5x$. We find that 12 gallons of gas would cost $C = 3.5(12) = \$42$.

3. If $x = \$120$ and $T = 0.06x$, then $T = 0.06(120) = 7.2$. The sales tax is \$7.20.

4. If $x = 62{,}000$ and $T = 0.005x$, then $T = 0.005(62{,}000) = 310$. The wage tax is \$310.

5. If $v = 70$, $t = 3.5$, and $d = vt$, then $d = 70 \cdot 3.5 = 245$. The car has traveled 245 miles.

6. $V = \pi r^2 h = \pi\left(3^2\right)(5) = 45\pi \approx 141.4\ \text{in}^3$

7. **(a)** $M = \dfrac{N}{G} = \dfrac{240}{8} = 30$ miles/gallon

(b) $25 = \dfrac{175}{G} \Leftrightarrow G = \dfrac{175}{25} = 7$ gallons

8. **(a)** $T = 70 - 0.003h = 70 - 0.003(1500) = 65.5°$ F

(b) $64 = 70 - 0.003h \Leftrightarrow 0.003h = 6 \Leftrightarrow h = 2000$ ft

9. **(a)** $V = 9.5S = 9.5\left(4\ \text{km}^3\right) = 38\ \text{km}^3$

(b) $19\ \text{km}^3 = 9.5S \Leftrightarrow S = 2\ \text{km}^3$

10. **(a)** $P = 0.06s^3 = 0.06\left(12^3\right) = 103.7$ hp

(b) $7.5 = 0.06s^3 \Leftrightarrow s^3 = 125$ so $s = 5$ knots

11. **(a)**

Depth (ft)	Pressure (lb/in^2)
0	$0.45(0) + 14.7 = 14.7$
10	$0.45(10) + 14.7 = 19.2$
20	$0.45(20) + 14.7 = 23.7$
30	$0.45(30) + 14.7 = 28.2$
40	$0.45(40) + 14.7 = 32.7$
50	$0.45(50) + 14.7 = 37.2$
60	$0.45(60) + 14.7 = 41.7$

(b) We know that $P = 30$ and we want to find d, so we solve the equation $30 = 14.7 + 0.45d \Leftrightarrow 15.3 = 0.45d \Leftrightarrow d = \dfrac{15.3}{0.45} = 34.0$. Thus, if the pressure is 30 lb/in^2, the depth is 34 ft.

12. **(a)**

Population	Water use (gal)
0	0
1000	$40(1000) = 40{,}000$
2000	$40(2000) = 80{,}000$
3000	$40(3000) = 120{,}000$
4000	$40(4000) = 160{,}000$
5000	$40(5000) = 200{,}000$

(b) We solve the equation $40x = 120{,}000 \Leftrightarrow x = \dfrac{120{,}000}{40} = 3000$. Thus, the population is about 3000.

13. The number N of cents in q quarters is $N = 25q$.

14. The average A of two numbers, a and b, is $A = \dfrac{a+b}{2}$.

15. The cost C of purchasing x gallons of gas at \$3.50 a gallon is $C = 3.5x$.

16. The amount T of a 15% tip on a restaurant bill of x dollars is $T = 0.15x$.

17. The distance d in miles that a car travels in t hours at 60 mi/h is $d = 60t$.

18. The speed r of a boat that travels d miles in 3 hours is $r = \dfrac{d}{3}$.

19. **(a)** $\$12 + 3\,(\$1) = \$12 + \$3 = \$15$

(b) The cost C, in dollars, of a pizza with n toppings is $C = 12 + n$.

(c) Using the model $C = 12 + n$ with $C = 16$, we get $16 = 12 + n \Leftrightarrow n = 4$. So the pizza has four toppings.

20. **(a)** $3\,(30) + 280\,(0.10) = 90 + 28 = \$118$

(b) The cost is $\left(\begin{matrix}\text{daily}\\ \text{rental}\end{matrix}\right) \times \left(\begin{matrix}\text{days}\\ \text{rented}\end{matrix}\right) + \left(\begin{matrix}\text{cost}\\ \text{per mile}\end{matrix}\right) \times \left(\begin{matrix}\text{miles}\\ \text{driven}\end{matrix}\right)$, so $C = 30n + 0.1m$.

(c) We have $C = 140$ and $n = 3$. Substituting, we get $140 = 30\,(3) + 0.1m \Leftrightarrow 140 = 90 + 0.1m \Leftrightarrow 50 = 0.1m \Leftrightarrow m = 500$. So the rental was driven 500 miles.

21. **(a)** **(i)** For an all-electric car, the energy cost of driving x miles is $C_e = 0.04x$.

(ii) For an average gasoline powered car, the energy cost of driving x miles is $C_g = 0.12x$.

(b) **(i)** The cost of driving 10,000 miles with an all-electric car is $C_e = 0.04\,(10{,}000) = \$400$.

(ii) The cost of driving 10,000 miles with a gasoline powered car is $C_g = 0.12\,(10{,}000) = \$1200$.

22. **(a)** If the width is 20, then the length is 40, so the volume is $20 \cdot 20 \cdot 40 = 16{,}000\text{ in}^3$.

(b) In terms of width, $V = x \cdot x \cdot 2x = 2x^3$.

23. **(a)** The GPA is $\dfrac{4a + 3b + 2c + 1d + 0f}{a + b + c + d + f} = \dfrac{4a + 3b + 2c + d}{a + b + c + d + f}$.

(b) Using $a = 2 \cdot 3 = 6$, $b = 4$, $c = 3 \cdot 3 = 9$, and $d = f = 0$ in the formula from part (a), we find the GPA to be $\dfrac{4 \cdot 6 + 3 \cdot 4 + 2 \cdot 9}{6 + 4 + 9} = \dfrac{54}{19} \approx 2.84$.

P.2 THE REAL NUMBERS

1. **(a)** The natural numbers are $\{1, 2, 3, \ldots\}$.

(b) The numbers $\{\ldots, -3, -2, -1, 0\}$ are integers but not natural numbers.

(c) Any irreducible fraction $\dfrac{p}{q}$ with $q \neq 1$ is rational but is not an integer. Examples: $\frac{3}{2}$, $-\frac{5}{12}$, $\frac{1729}{23}$.

(d) Any number which cannot be expressed as a ratio $\dfrac{p}{q}$ of two integers is irrational. Examples are $\sqrt{2}$, $\sqrt{3}$, π, and e.

2. **(a)** $ab = ba$; Commutative Property of Multiplication

(b) $a + (b + c) = (a + b) + c$; Associative Property of Addition

(c) $a\,(b + c) = ab + ac$; Distributive Property

3. **(a)** In set-builder notation: $\{x \mid -3 < x < 5\}$

(b) In interval notation: $(-3, 5)$

(c) As a graph:

4. The symbol $|x|$ stands for the *absolute value* of the number x. If x is not 0, then the sign of $|x|$ is always *positive*.

5. The distance between a and b on the real line is $d\,(a, b) = |b - a|$. So the distance between -5 and 2 is $|2 - (-5)| = 7$.

6. **(a)** If $a < b$, then any interval between a and b (whether or not it contains either endpoint) contains infinitely many numbers—including, for example $a + \dfrac{b - a}{2^n}$ for every positive n. (If an interval extends to infinity in either or both directions, then it obviously contains infinitely many numbers.)

(b) No, because $(5, 6)$ does not include 5.

7. **(a)** No: $a - b = -(b - a) \neq b - a$ in general.

(b) No; by the Distributive Property, $-2(a-5) = -2a + -2(-5) = -2a + 10 \neq -2a - 10$.

8. **(a)** Yes, absolute values (such as the distance between two different numbers) are always positive.

(b) Yes, $|b - a| = |a - b|$.

9. **(a)** Natural number: 100

(b) Integers: $0, 100, -8$

(c) Rational numbers: $-1.5, 0, \frac{5}{2}, 2.71, 3.1\overline{4}, 100, -8$

(d) Irrational numbers: $\sqrt{7}, -\pi$

10. **(a)** Natural numbers: $2, \sqrt{9}\,(=3), 10$

(b) Integers: $2, -\frac{100}{2}\,(=-50), \sqrt{9}\,(=3), 10$

(c) Rational numbers: $4.5\left(=\frac{9}{2}\right), \frac{1}{3}, 1.6666\ldots\left(=\frac{5}{3}\right),$ $2, -\frac{100}{2}, \sqrt{9}\,(=3), 10$

(d) Irrational numbers: $\sqrt{2}, \sqrt{3.14}$

11. Commutative Property of addition

12. Commutative Property of multiplication

13. Associative Property of addition

14. Distributive Property

15. Distributive Property

16. Distributive Property

17. Commutative Property of multiplication

18. Distributive Property

19. $x + 3 = 3 + x$

20. $7(3x) = (7 \cdot 3)x$

21. $4(A + B) = 4A + 4B$

22. $5x + 5y = 5(x + y)$

23. $-2(x + y) = -2x - 2y$

24. $(a - b)(-5) = -5a + 5b$

25. $5(2xy) = (5 \cdot 2)xy = 10xy$

26. $\frac{4}{3}(-6y) = \left[\frac{4}{3}(-6)\right]y = -8y$

27. $-\frac{5}{2}(2x - 4y) = -\frac{5}{2}(2x) + \frac{5}{2}(4y) = -5x + 10y$

28. $(3a)(b + c - 2d) = 3ab + 3ac - 6ad$

29. **(a)** $\frac{2}{3} + \frac{5}{7} = \frac{14}{21} + \frac{15}{21} = \frac{29}{21}$

(b) $\frac{5}{12} - \frac{3}{8} = \frac{10}{24} - \frac{9}{24} = \frac{1}{24}$

30. **(a)** $\frac{2}{5} - \frac{3}{8} = \frac{16}{40} - \frac{15}{40} = \frac{1}{40}$

(b) $\frac{3}{2} - \frac{5}{8} + \frac{1}{6} = \frac{36}{24} - \frac{15}{24} + \frac{4}{24} = \frac{25}{24}$

31. **(a)** $\frac{2}{3}\left(6 - \frac{3}{2}\right) = \frac{2}{3} \cdot 6 - \frac{2}{3} \cdot \frac{3}{2} = 4 - 1 = 3$

(b) $\left(3 + \frac{1}{4}\right)\left(1 - \frac{4}{5}\right) = \left(\frac{12}{4} + \frac{1}{4}\right)\left(\frac{5}{5} - \frac{4}{5}\right) = \frac{13}{4} \cdot \frac{1}{5} = \frac{13}{20}$

32. **(a)** $\dfrac{2}{\frac{2}{3}} - \dfrac{\frac{2}{3}}{2} = 2 \cdot \frac{3}{2} - \frac{2}{3} \cdot \frac{1}{2} = 3 - \frac{1}{3} = \frac{9}{3} - \frac{1}{3} = \frac{8}{3}$

(b) $\dfrac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{3}{15}} = \dfrac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{1}{5}} = \dfrac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{1}{5}} \cdot \frac{10}{10} = \frac{4+5}{1+2} = \frac{9}{3} = 3$

33. **(a)** $2 \cdot 3 = 6$ and $2 \cdot \frac{7}{2} = 7$, so $3 < \frac{7}{2}$

(b) $-6 > -7$

(c) $3.5 = \frac{7}{2}$

34. **(a)** $3 \cdot \frac{2}{3} = 2$ and $3 \cdot 0.67 = 2.01$, so $\frac{2}{3} < 0.67$

(b) $\frac{2}{3} > -0.67$

(c) $|0.6| = |-0.6|$

35. **(a)** False

(b) True

36. **(a)** False: $\sqrt{3} \approx 1.73205 < 1.7325$.

(b) False

37. **(a)** True **(b)** False

38. **(a)** True **(b)** True

39. **(a)** $x > 0$ **(b)** $t < 4$ **(c)** $a \geq \pi$ **(d)** $-5 < x < \frac{1}{3}$ **(e)** $|3 - p| \leq 5$

40. **(a)** $y < 0$ **(b)** $z \geq 3$ **(c)** $b \leq 8$ **(d)** $0 < w \leq 17$ **(e)** $|y - \pi| \geq 2$

41. **(a)** $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ **(b)** $A \cap B = \{2, 4, 6\}$

42. **(a)** $B \cup C = \{2, 4, 6, 7, 8, 9, 10\}$ **(b)** $B \cap C = \{8\}$

43. **(a)** $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ **(b)** $A \cap C = \{7\}$

44. **(a)** $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ **(b)** $A \cap B \cap C = \varnothing$

45. **(a)** $B \cup C = \{x \mid x \leq 5\}$ **(b)** $B \cap C = \{x \mid -1 < x < 4\}$

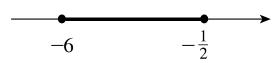
46. **(a)** $A \cap C = \{x \mid -1 < x \leq 5\}$ **(b)** $A \cap B = \{x \mid -2 \leq x < 4\}$

47. $(-3, 0) = \{x \mid -3 < x < 0\}$

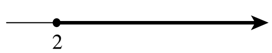
48. $(2, 8] = \{x \mid 2 < x \leq 8\}$

49. $[2, 8) = \{x \mid 2 \leq x < 8\}$

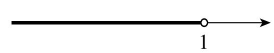
50. $\left[-6, -\frac{1}{2}\right] = \left\{x \mid -6 \leq x \leq -\frac{1}{2}\right\}$



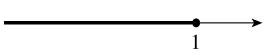
51. $[2, \infty) = \{x \mid x \geq 2\}$



52. $(-\infty, 1) = \{x \mid x < 1\}$

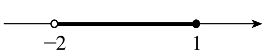


53. $x \leq 1 \Leftrightarrow x \in (-\infty, 1]$

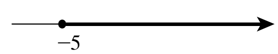


54. $1 \leq x \leq 2 \Leftrightarrow x \in [1, 2]$

55. $-2 < x \leq 1 \Leftrightarrow x \in (-2, 1]$



56. $x \geq -5 \Leftrightarrow x \in [-5, \infty)$



57. $x > -1 \Leftrightarrow x \in (-1, \infty)$

58. $-5 < x < 2 \Leftrightarrow x \in (-5, 2)$

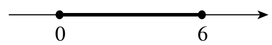
59. **(a)** $[-3, 5]$ **(b)** $(-3, 5]$ **(c)** $(-3, \infty)$

60. **(a)** $[0, 2)$ **(b)** $(-2, 0]$ **(c)** $(-\infty, 0]$

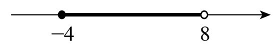
61. $(-2, 0) \cup (-1, 1) = (-2, 1)$

62. $(-2, 0) \cap (-1,) = (-1, 0)$

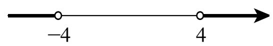
63. $[-4, 6] \cap [0, 8) = [0, 6]$



64. $[-4, 6] \cup [0, 8) = [-4, 8)$



65. $(-\infty, -4) \cup (4, \infty)$



66. $(-\infty, 6] \cap (2, 10) = (2, 6]$

67. **(a)** $|50| = 50$

(b) $|-13| = 13$

68. **(a)** $|2 - 8| = |-6| = 6$

(b) $|8 - |-2|| = |8 - 2| = |6| = 6$

69. **(a)** $||-6| - |-4|| = |6 - 4| = |2| = 2$

(b) $\frac{-1}{|-1|} = \frac{-1}{1} = -1$

70. **(a)** $|2 - |-12|| = |2 - 12| = |-10| = 10$

(b) $-1 - |1 - |-1|| = -1 - |1 - 1| = -1 - |0| = -1$

71. **(a)** $|(-2) \cdot 6| = |-12| = 12$

(b) $\left|\left(-\frac{1}{3}\right)(-15)\right| = |5| = 5$

72. **(a)** $\left|\frac{-6}{24}\right| = \left|-\frac{1}{4}\right| = \frac{1}{4}$

(b) $\left|\frac{7-12}{12-7}\right| = \left|-\frac{5}{5}\right| = |-1| = 1$

73. $|(-2) - 3| = |-5| = 5$

74. $|-2.5 - 1.5| = |-4| = 4$

75. **(a)** $|17 - 2| = 15$

(b) $|21 - (-3)| = |21 + 3| = |24| = 24$

(c) $\left|-\frac{3}{10} - \frac{11}{8}\right| = \left|-\frac{12}{40} - \frac{55}{40}\right| = \left|-\frac{67}{40}\right| = \frac{67}{40}$

76. **(a)** $\left|\frac{7}{15} - \left(-\frac{1}{21}\right)\right| = \left|\frac{49}{105} + \frac{5}{105}\right| = \left|\frac{54}{105}\right| = \left|\frac{18}{35}\right| = \frac{18}{35}$

(b) $|-38 - (-57)| = |-38 + 57| = |19| = 19.$

(c) $|-2.6 - (-1.8)| = |-2.6 + 1.8| = |-0.8| = 0.8.$

77. **(a)** Let $x = 0.777\ldots$. So $10x = 7.7777\ldots \Leftrightarrow x = 0.7777\ldots \Leftrightarrow 9x = 7$. Thus, $x = \frac{7}{9}$.

(b) Let $x = 0.2888\ldots$. So $100x = 28.8888\ldots \Leftrightarrow 10x = 2.8888\ldots \Leftrightarrow 90x = 26$. Thus, $x = \frac{26}{90} = \frac{13}{45}$.

(c) Let $x = 0.575757\ldots$. So $100x = 57.5757\ldots \Leftrightarrow x = 0.5757\ldots \Leftrightarrow 99x = 57$. Thus, $x = \frac{57}{99} = \frac{19}{33}$.

78. **(a)** Let $x = 5.2323\ldots$. So $100x = 523.2323\ldots \Leftrightarrow 1x = 5.2323\ldots \Leftrightarrow 99x = 518$. Thus, $x = \frac{518}{99}$.

(b) Let $x = 1.3777\ldots$. So $100x = 137.7777\ldots \Leftrightarrow 10x = 13.7777\ldots \Leftrightarrow 90x = 124$. Thus, $x = \frac{124}{90} = \frac{62}{45}$.

(c) Let $x = 2.13535\ldots$. So $1000x = 2135.3535\ldots \Leftrightarrow 10x = 21.3535\ldots \Leftrightarrow 990x = 2114$. Thus, $x = \frac{2114}{990} = \frac{1057}{495}$.

79. $\pi > 3$, so $|\pi - 3| = \pi - 3$.

80. $\sqrt{2} > 1$, so $\left|1 - \sqrt{2}\right| = \sqrt{2} - 1$.

81. $a < b$, so $|a - b| = -(a - b) = b - a$.

82. $a + b + |a - b| = a + b + b - a = 2b$

83. **(a)** $-a$ is negative because a is positive.

(b) bc is positive because the product of two negative numbers is positive.

(c) $a - ba + (-b)$ is positive because it is the sum of two positive numbers.

(d) $ab + ac$ is negative: each summand is the product of a positive number and a negative number, and the sum of two negative numbers is negative.

84. **(a)** $-b$ is positive because b is negative.

(b) $a + bc$ is positive because it is the sum of two positive numbers.

(c) $c - a = c + (-a)$ is negative because c and $-a$ are both negative.

(d) ab^2 is positive because both a and b^2 are positive.

85. Distributive Property

86. **(a)** When $L = 60$, $x = 8$, and $y = 6$, we have $L + 2(x + y) = 60 + 2(8 + 6) = 60 + 28 = 88$. Because $88 \leq 108$ the post office will accept this package.
When $L = 48$, $x = 24$, and $y = 24$, we have $L + 2(x + y) = 48 + 2(24 + 24) = 48 + 96 = 144$, and since $144 \not\leq 108$, the post office will *not* accept this package.
(b) If $x = y = 9$, then $L + 2(9 + 9) \leq 108 \Leftrightarrow L + 36 \leq 108 \Leftrightarrow L \leq 72$. So the length can be as long as 72 in. = 6 ft.

87. Let $x = \dfrac{m_1}{n_1}$ and $y = \dfrac{m_2}{n_2}$ be rational numbers. Then $x + y = \dfrac{m_1}{n_1} + \dfrac{m_2}{n_2} = \dfrac{m_1 n_2 + m_2 n_1}{n_1 n_2}$, $x - y = \dfrac{m_1}{n_1} - \dfrac{m_2}{n_2} = \dfrac{m_1 n_2 - m_2 n_1}{n_1 n_2}$, and $x \cdot y = \dfrac{m_1}{n_1} \cdot \dfrac{m_2}{n_2} = \dfrac{m_1 m_2}{n_1 n_2}$. This shows that the sum, difference, and product of two rational numbers are again rational numbers. However the product of two irrational numbers is not necessarily irrational; for example, $\sqrt{2} \cdot \sqrt{2} = 2$, which is rational. Also, the sum of two irrational numbers is not necessarily irrational; for example, $\sqrt{2} + \left(-\sqrt{2}\right) = 0$ which is rational.

88. $\frac{1}{2} + \sqrt{2}$ is irrational. If it were rational, then by Exercise 6(a), the sum $\left(\frac{1}{2} + \sqrt{2}\right) + \left(-\frac{1}{2}\right) = \sqrt{2}$ would be rational, but this is not the case.
Similarly, $\frac{1}{2} \cdot \sqrt{2}$ is irrational.
(a) Following the hint, suppose that $r + t = q$, a rational number. Then by Exercise 6(a), the sum of the two rational numbers $r + t$ and $-r$ is rational. But $(r + t) + (-r) = t$, which we know to be irrational. This is a contradiction, and hence our original premise—that $r + t$ is rational—was false.
(b) r is a nonzero rational number, so $r = \dfrac{a}{b}$ for some nonzero integers a and b. Let us assume that $rt = q$, a rational number. Then by definition, $q = \dfrac{c}{d}$ for some integers c and d. But then $rt = q \Leftrightarrow \dfrac{a}{b}t = \dfrac{c}{d}$, whence $t = \dfrac{bc}{ad}$, implying that t is rational. Once again we have arrived at a contradiction, and we conclude that the product of a rational number and an irrational number is irrational.

89.

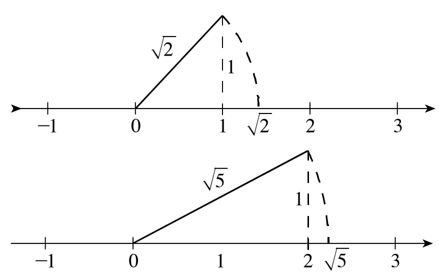
x	1	2	10	100	1000
$\frac{1}{x}$	1	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

As x gets large, the fraction $1/x$ gets small. Mathematically, we say that $1/x$ goes to zero.

x	1	0.5	0.1	0.01	0.001
$\frac{1}{x}$	1	$\frac{1}{0.5} = 2$	$\frac{1}{0.1} = 10$	$\frac{1}{0.01} = 100$	$\frac{1}{0.001} = 1000$

As x gets small, the fraction $1/x$ gets large. Mathematically, we say that $1/x$ goes to infinity.

90. We can construct the number $\sqrt{2}$ on the number line by transferring the length of the hypotenuse of a right triangle with legs of length 1 and 1.



Similarly, to locate $\sqrt{5}$, we construct a right triangle with legs of length 1 and 2. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{1^2 + 2^2} = \sqrt{5}$. Then transfer the length of the hypotenuse to the number line.
The square root of any rational number can be located on a number line in this fashion.
The circle in the second figure in the text has circumference π, so if we roll it along a number line one full rotation, we have found π on the number line. Similarly, any rational multiple of π can be found this way.

91. **(a)** Suppose that $a > b$, so $\max(a,b) = a$ and $|a-b| = a-b$. Then $\dfrac{a+b+|a-b|}{2} = \dfrac{a+b+a-b}{2} = a$.
On the other hand, if $b > a$, then $\max(a,b) = b$ and $|a-b| = -(a-b) = b-a$. In this case, $\dfrac{a+b+|a-b|}{2} = \dfrac{a+b+b-a}{2} = b$.
If $a = b$, then $|a-b| = 0$ and the result is trivial.

(b) If $a < b$, then $\min(a,b) = a$ and $|a-b| = b-a$. In this case $\dfrac{a+b-|a-b|}{2} = \dfrac{a+b-(b-a)}{2} = a$.
Similarly, if $b < a$, then $\dfrac{a+b-|a-b|}{2} = b$; and if $a = b$, the result is trivial.

92. Answers will vary.

93. **(a)** Subtraction is not commutative. For example, $5-1 \neq 1-5$.
(b) Division is not commutative. For example, $5 \div 1 \neq 1 \div 5$.
(c) Putting on your socks and putting on your shoes are not commutative. If you put on your socks first, then your shoes, the result is not the same as if you proceed the other way around.
(d) Putting on your hat and putting on your coat are commutative. They can be done in either order, with the same result.
(e) Washing laundry and drying it are not commutative.

94. **(a)** If $x = 2$ and $y = 3$, then $|x+y| = |2+3| = |5| = 5$ and $|x| + |y| = |2| + |3| = 5$.
If $x = -2$ and $y = -3$, then $|x+y| = |-5| = 5$ and $|x| + |y| = 5$.
If $x = -2$ and $y = 3$, then $|x+y| = |-2+3| = 1$ and $|x| + |y| = 5$.
In each case, $|x+y| \le |x| + |y|$ and the Triangle Inequality is satisfied.

(b) *Case 0:* If either x or y is 0, the result is equality, trivially.

Case 1: If x and y have the same sign, then $|x+y| = \left\{ \begin{array}{ll} x+y & \text{if } x \text{ and } y \text{ are positive} \\ -(x+y) & \text{if } x \text{ and } y \text{ are negative} \end{array} \right\} = |x| + |y|$.

Case 2: If x and y have opposite signs, then suppose without loss of generality that $x < 0$ and $y > 0$. Then $|x+y| < |-x+y| = |x| + |y|$.

P.3 INTEGER EXPONENTS AND SCIENTIFIC NOTATION

1. Using exponential notation we can write the product $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ as 5^6.

2. Yes, there is a difference: $(-5)^4 = (-5)(-5)(-5)(-5) = 625$, while $-5^4 = -(5 \cdot 5 \cdot 5 \cdot 5) = -625$.

3. In the expression 3^4, the number 3 is called the *base* and the number 4 is called the *exponent*.

4. **(a)** When we multiply two powers with the same base, we *add* the exponents. So $3^4 \cdot 3^5 = 3^9$.
(b) When we divide two powers with the same base, we *subtract* the exponents. So $\dfrac{3^5}{3^2} = 3^3$.

5. When we raise a power to a new power, we *multiply* the exponents. So $\left(3^4\right)^2 = 3^8$.

6. **(a)** $2^{-1} = \dfrac{1}{2}$ **(b)** $2^{-3} = \dfrac{1}{8}$ **(c)** $\left(\dfrac{1}{2}\right)^{-1} = 2$ **(d)** $\dfrac{1}{2^{-3}} = 2^3 = 8$

7. To move a number raised to a power from numerator to denominator or from denominator to numerator change the sign of the *exponent*. So $a^{-2} = \dfrac{1}{a^2}$, $\dfrac{1}{b^{-2}} = b^2$, $\dfrac{a^{-3}}{b^2} = \dfrac{1}{a^3b^2}$, and $\dfrac{6a^2}{b^{-3}} = 6a^2b^3$.

8. Scientists express very large or very small numbers using *scientific* notation. In scientific notation, 8,300,000 is 8.3×10^6 and 0.0000327 is 3.27×10^{-5}.

9. (a) No, $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2} = \frac{9}{4}$.

(b) No, $(-5)^3 = -125$ and $-5^3 = -\left(5^3\right) = -125$.

10. (a) No, $\left(x^2\right)^3 = x^{2\cdot 3} = x^6$.

(b) No, $\left(2x^4\right)^3 = 2^3 \cdot \left(x^4\right)^3 = 8x^{12}$.

11. (a) $-2^6 = -\left(2^6\right) = -64$

(b) $(-2)^6 = 64$

(c) $\left(\frac{1}{5}\right)^2 \cdot (-3)^3 = \frac{(-3)^3}{5^2} = -\frac{27}{25}$

12. (a) $(-5)^3 = -125$

(b) $-5^3 = -\left(5^3\right) = -125$

(c) $(-5)^2 \cdot \left(\frac{2}{5}\right)^2 = 4$

13. (a) $\left(\frac{5}{3}\right)^0 \cdot 2^{-1} = \frac{1}{2}$

(b) $\frac{2^{-3}}{3^0} = \frac{1}{2^3} = \frac{1}{8}$

(c) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

14. (a) $-2^3 \cdot (-2)^0 = -\left(2^3\right) \cdot 1 = -8$

(b) $-2^{-3} \cdot (-2)^0 = -\frac{1}{2^3} \cdot 1 = -\frac{1}{8}$

(c) $\left(\frac{-3}{5}\right)^{-3} = \frac{5^3}{(-3)^3} = -\frac{125}{27}$

15. (a) $5^3 \cdot 5 = 5^{3+1} = 625$

(b) $5^4 \cdot 5^{-2} = 5^{4-2} = 25$

(c) $\left(2^2\right)^3 = 2^{2\cdot 3} = 64$

16. (a) $3^8 \cdot 3^5 = 3^{8+5} = 3^{13}$

(b) $\frac{10^7}{10^4} = 10^{7-4} = 1000$

(c) $\left(3^5\right)^4 = 3^{5\cdot 4} = 3^{20}$

17. (a) $\frac{7^{10}}{7^8} = 7^{10-8} = 7^2 = 49$

(b) $\frac{6^2}{6^{-2}} = 6^{2-(-2)} = 6^4 = 1296$

(c) $4^{-1} \cdot 4^{-2} = 4^{-1+(-2)} = 4^{-3} = \frac{1}{64}$

18. (a) $\frac{8^3}{8} = \frac{8^3}{8^1} = 8^{3-1} = 8^2 = 64$

(b) $\frac{8^{-1}}{8} = \frac{8^{-1}}{8^1} = 8^{-1-1} = 8^{-2} = \frac{1}{64}$

(c) $5^{-3} \cdot 5^{-1} = 5^{-3+(-1)} = 5^{-4} = \frac{1}{625}$

19. (a) $t^5 t^2 = t^{5+2} = t^7$

(b) $\left(4z^3\right)^2 = 4^2 \left(z^3\right)^2 = 16z^{3\cdot 2} = 16z^6$

(c) $x^{-3}x^5 = x^{-3+5} = x^2$

20. (a) $a^4 a^6 = a^{4+6} = a^{10}$

(b) $\left(-2b^{-3}\right)^3 = (-2)^3 \left(b^{-3}\right)^3 = -8b^{-3\cdot 3} = -\frac{8}{b^9}$

(c) $-2y^{10}y^{-11} = -2y^{10+(-11)} = -2y^{-1} = -\frac{2}{y}$

21. (a) $x^{-5} \cdot x^3 = x^{-5+3} = x^{-2} = \frac{1}{x^2}$

(b) $w^{-2}w^{-4}w^5 = w^{-2-4+5} = w^{-1} = \frac{1}{w}$

(c) $\frac{x^{16}}{x^{10}} = x^{16-10} = x^6$

22. (a) $y^2 \cdot y^{-5} = y^{2-5} = y^{-3} = \frac{1}{y^3}$

(b) $z^5 z^{-3} z^{-4} = z^{5-3-4} = z^{-2} = \frac{1}{z^2}$

(c) $\frac{y^7 y^0}{y^{10}} = y^{7+0-10} = y^{-3} = \frac{1}{y^3}$

23. (a) $\frac{a^9 a^{-2}}{a} = a^{9-2-1} = a^6$

(b) $\left(a^2 a^4\right)^3 = \left(a^{2+4}\right)^3 = \left(a^6\right)^3 = a^{6\cdot 3} = a^{18}$

(c) $\left(\frac{x}{2}\right)^3 \left(5x^6\right) = \frac{x^3 \cdot 5x^6}{2^3} = \frac{5x^9}{8}$

24. (a) $\frac{z^2 z^4}{z^3 z^{-1}} = z^{2+4-3-(-1)} = z^4$

(b) $\left(2a^3a^2\right)^4 = 2^4 \left(a^{3+2}\right)^4 = 16\left(a^5\right)^4 = 16a^{20}$

(c) $\left(-3z^2\right)^3 \left(2z^3\right) = (-3)^3 \cdot z^{2\cdot 3} \cdot 2z^3 = -54z^9$

25. (a) $\left(3x^3y^2\right)\left(2y^3\right) = 3 \cdot 2x^3y^2y^3 = 6x^3y^5$

(b) $\left(5w^2z^{-2}\right)^2\left(z^3\right) = \left(\dfrac{5w^2}{z^2}\right)^2\left(z^3\right) = \dfrac{5^2\left(w^2\right)^2z^3}{\left(z^2\right)^2} = \dfrac{25w^4}{z}$

26. (a) $\left(8m^{-2}n^4\right)\left(\frac{1}{2}n^{-2}\right) = 8\cdot\frac{1}{2}m^{-2}n^{4-2} = \dfrac{4n^2}{m^2}$

(b) $\left(3a^4b^{-2}\right)^3\left(a^2b^{-1}\right) = \left(3^3a^{4\cdot3}b^{-2\cdot3}\right)\left(a^2b^{-1}\right) = 3^3a^{12+2}b^{-6-1} = \dfrac{27a^{14}}{b^7}$

27. (a) $\dfrac{x^2y^{-1}}{x^{-5}} = x^{2-(-5)}y^{-1} = \dfrac{x^7}{y}$

(b) $\left(\dfrac{a^3}{2b^2}\right)^3 = \dfrac{a^{3\cdot3}}{2^3\left(b^2\right)^3} = \dfrac{a^9}{8b^6}$

28. (a) $\dfrac{y^{-2}z^{-3}}{y^{-1}} = \dfrac{y}{y^2z^3} = \dfrac{1}{yz^3}$

(b) $\left(\dfrac{x^3y^{-2}}{x^{-3}y^2}\right)^{-2} = \left[x^{3-(-3)}y^{-2-2}\right]^{-2} = \left(x^6y^{-4}\right)^{-2} = \dfrac{y^8}{x^{12}}$

29. (a) $\left(\dfrac{a^2}{b}\right)^5\left(\dfrac{a^3b^2}{c^3}\right)^3 = a^{2\cdot5+3\cdot3}b^{-5+2\cdot3}c^{-3\cdot3} = \dfrac{a^{19}b}{c^9}$

(b) $\dfrac{\left(u^{-1}v^2\right)^2}{\left(u^3v^{-2}\right)^3} = u^{-1\cdot2-3\cdot3}v^{2\cdot2-(-2)3} = \dfrac{v^{10}}{u^{11}}$

30. (a) $\left(\dfrac{x^4z^2}{4y^5}\right)\left(\dfrac{2x^3y^2}{z^3}\right)^2 = \dfrac{2^2}{4}x^{4+3\cdot2}y^{-5+2\cdot2}z^{2+(-3)2} = \dfrac{x^{10}}{yz^4}$

(b) $\dfrac{\left(rs^2\right)^3}{\left(r^{-3}s^2\right)^2} = r^{3-(-3)2}s^{2\cdot3-2\cdot2} = r^9s^2$

31. (a) $\dfrac{8a^3b^{-4}}{2a^{-5}b^5} = 4a^{3-(-5)}b^{-4-5} = \dfrac{4a^8}{b^9}$

(b) $\left(\dfrac{y}{5x^{-2}}\right)^{-3} = 5^{-1(-3)}x^{-(-2)(-3)}y^{-3} = \dfrac{125}{x^6y^3}$

32. (a) $\dfrac{5xy^{-2}}{x^{-1}y^{-3}} = 5x^{1-(-1)}y^{-2-(-3)} = 5x^2y$

(b) $\left(\dfrac{2a^{-1}b}{a^2b^{-3}}\right)^{-3} = 2^{-3}a^{-1(-3)-2(-3)}b^{-3-(-3)(-3)} = \dfrac{a^9}{8b^{12}}$

33. (a) $69{,}300{,}000 = 6.93\times10^7$

(b) $7{,}200{,}000{,}000{,}000 = 7.2\times10^{12}$

(c) $0.000028536 = 2.8536\times10^{-5}$

(d) $0.0001213 = 1.213\times10^{-4}$

34. (a) $129{,}540{,}000 = 1.2954\times10^8$

(b) $7{,}259{,}000{,}000 = 7.259\times10^9$

(c) $0.000\,000\,001\,4 = 1.4\times10^{-9}$

(d) $0.0007029 = 7.029\times10^{-4}$

35. (a) $3.19\times10^5 = 319{,}000$

(b) $2.721\times10^8 = 272{,}100{,}000$

(c) $2.670\times10^{-8} = 0.000\,000\,026\,70$

(d) $9.999\times10^{-9} = 0.000\,000\,009\,999$

36. (a) $7.1\times10^{14} = 710{,}000{,}000{,}000{,}000$

(b) $6\times10^{12} = 6{,}000{,}000{,}000{,}000$

(c) $8.55\times10^{-3} = 0.00855$

(d) $6.257\times10^{-10} = 0.000\,000\,000\,625\,7$

37. (a) $5{,}900{,}000{,}000{,}000$ mi$= 5.9\times10^{12}$ mi

(b) $0.000\,000\,000\,000\,4 \text{ cm} = 4 \times 10^{-13} \text{ cm}$

(c) $33 \text{ billion billion molecules} = 33 \times 10^9 \times 10^9 = 3.3 \times 10^{19} \text{ molecules}$

38. (a) $93{,}000{,}000 \text{ mi} = 9.3 \times 10^7 \text{ mi}$

(b) $0.000\,000\,000\,000\,000\,000\,000\,053 \text{ g} = 5.3 \times 10^{-23} \text{ g}$

(c) $5{,}970{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ kg} = 5.97 \times 10^{24} \text{ kg}$

39. $\left(7.2 \times 10^{-9}\right)\left(1.806 \times 10^{-12}\right) = 7.2 \times 1.806 \times 10^{-9} \times 10^{-12} \approx 13.0 \times 10^{-21} = 1.3 \times 10^{-20}$

40. $\left(1.062 \times 10^{24}\right)\left(8.61 \times 10^{19}\right) = 1.062 \times 8.61 \times 10^{24} \times 10^{19} \approx 9.14 \times 10^{43}$

41. $\dfrac{1.295643 \times 10^9}{(3.610 \times 10^{-17})(2.511 \times 10^6)} = \dfrac{1.295643}{3.610 \times 2.511} \times 10^{9+17-6} \approx 0.1429 \times 10^{19} = 1.429 \times 10^{19}$

42. $\dfrac{(73.1)\left(1.6341 \times 10^{28}\right)}{0.0000000019} = \dfrac{(7.31 \times 10)\left(1.6341 \times 10^{28}\right)}{1.9 \times 10^{-9}} = \dfrac{7.31 \times 1.6341}{1.9} \times 10^{1+28-(-9)} \approx 6.3 \times 10^{38}$

43. $\dfrac{(0.0000162)(0.01582)}{(594621000)(0.0058)} = \dfrac{\left(1.62 \times 10^{-5}\right)\left(1.582 \times 10^{-2}\right)}{(5.94621 \times 10^8)(5.8 \times 10^{-3})} = \dfrac{1.62 \times 1.582}{5.94621 \times 5.8} \times 10^{-5-2-8+3} = 0.074 \times 10^{-12}$
$= 7.4 \times 10^{-14}$

44. $\dfrac{\left(3.542 \times 10^{-6}\right)^9}{(5.05 \times 10^4)^{12}} = \dfrac{(3.542)^9 \times 10^{-54}}{(5.05)^{12} \times 10^{48}} = \dfrac{87747.96}{275103767.10} \times 10^{-54-48} \approx 3.19 \times 10^{-4} \times 10^{-102} \approx 3.19 \times 10^{-106}$

45. $\left|10^{50} - 10^{10}\right| < 10^{50}$, whereas $\left|10^{101} - 10^{100}\right| = 10^{100}\,|10-1| = 9 \times 10^{100} > 10^{50}$. So 10^{10} is closer to 10^{50} than 10^{100} is to 10^{101}.

46. (a) b^5 is negative since a negative number raised to an odd power is negative.

(b) b^{10} is positive since a negative number raised to an even power is positive.

(c) ab^2c^3 we have (positive) (negative)^2 (negative)^3 = (positive) (positive) (negative) which is negative.

(d) Since $b - a$ is negative, $(b-a)^3 = (\text{negative})^3$ which is negative.

(e) Since $b - a$ is negative, $(b-a)^4 = (\text{negative})^4$ which is positive.

(f) $\dfrac{a^3c^3}{b^6c^6} = \dfrac{(\text{positive})^3\,(\text{negative})^3}{(\text{negative})^6\,(\text{negative})^6} = \dfrac{(\text{positive})\,(\text{negative})}{(\text{positive})\,(\text{positive})} = \dfrac{\text{negative}}{\text{positive}}$ which is negative.

47. Since one light year is 5.9×10^{12} miles, Centauri is about $4.3 \times 5.9 \times 10^{12} \approx 2.54 \times 10^{13}$, or 25,400,000,000,000 miles away.

48. $9.3 \times 10^7 \text{ mi} = 186{,}000 \dfrac{\text{mi}}{\text{s}} \times t \text{ s} \Leftrightarrow t = \dfrac{9.3 \times 10^7}{186{,}000} \text{ s} = 500 \text{ s} = 8\tfrac{1}{3} \text{ min.}$

49. $\text{Volume} = (\text{Average depth})\,(\text{Area}) = \left(3.7 \times 10^3 \text{ m}\right)\left(3.6 \times 10^{14} \text{ m}^2\right)\left(\dfrac{10^3 \text{ liters}}{\text{m}^3}\right) \approx 1.33 \times 10^{21} \text{ liters}$

50. Each person's share is equal to $\dfrac{\text{National debt}}{\text{Population}} = \dfrac{2.670 \times 10^{13}}{3.3145 \times 10^8} \approx 8.0555 \times 10^4 \approx \$80{,}555$.

51. First, we estimate the total mass of the stars in the observable universe:

$$\begin{aligned}\text{Total star mass} &\approx (\text{Number of galaxies})\left(\frac{\text{Number of stars}}{\text{Galaxy}}\right)(\text{Mass of typical star}) = (1.5 \cdot 2 \cdot 1.77)\left(10^{12} \cdot 10^{11} \cdot 10^{30}\right)\\ &\approx 5.31 \times 10^{53} \text{ kg}\end{aligned}$$

Thus, the number of hydrogen atoms in the observable universe is

$$\frac{\text{Total star mass}}{\text{Mass of a single hydrogen atom}} \approx \frac{5.31 \times 10^{53}}{1.67 \times 10^{-27}} \approx 3.18 \times 10^{80}$$

52. (a)

Patient	Weight	Height	BMI $= 703\dfrac{W}{H^2}$	Result
A	295 lb	5 ft 10 in. = 70 in.	42.32	obese
B	105 lb	5 ft 6 in. = 66 in.	16.95	underweight
C	220 lb	6 ft 4 in. = 76 in.	26.78	overweight
D	110 lb	5 ft 2 in. = 62 in.	20.12	normal

(b) Answers will vary.

53. $I = 15{,}000\left[(1.00375)^{12n} - 1\right]$

Year	Total interest
1	\$689.10
2	1409.85
3	2163.72
4	2952.22
5	3776.94

54. Since $10^6 = 10^3 \cdot 10^3$ it would take 1000 days ≈ 2.74 years to spend the million dollars.
Since $10^9 = 10^3 \cdot 10^6$ it would take $10^6 = 1{,}000{,}000$ days ≈ 2739.72 years to spend the billion dollars.

55. (a) $\dfrac{18^5}{9^5} = \left(\dfrac{18}{9}\right)^5 = 2^5 = 32$ **(b)** $20^6 \cdot (0.5)^6 = (20 \cdot 0.5)^6 = 10^6 = 1{,}000{,}000$

56. (a) $\dfrac{a^m}{a^n} = \dfrac{\overbrace{a \cdot a \cdot \cdots \cdot a}^{m \text{ factors}}}{\underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}}$. Because $m > n$, we can cancel n factors of a from numerator and denominator and are left with $m - n$ factors of a in the numerator. Thus, $\dfrac{a^m}{a^n} = a^{m-n}$.

(b) $\left(\dfrac{a}{b}\right)^n = \overbrace{\dfrac{a}{b} \cdot \dfrac{a}{b} \cdot \cdots \cdot \dfrac{a}{b}}^{n \text{ factors}} = \dfrac{\overbrace{a \cdot a \cdot \cdots \cdot a}^{n \text{ factors}}}{\underbrace{b \cdot b \cdot \cdots \cdot b}_{n \text{ factors}}} = \dfrac{a^n}{b^n}$

57. (a) $\left(\dfrac{a}{b}\right)^{-n} = \dfrac{1}{(a/b)^n} = \dfrac{1}{a^n/b^n} = \dfrac{b^n}{a^n}$ **(b)** $\dfrac{a^{-n}}{b^{-m}} = \dfrac{1/a^n}{1/b^m} = \dfrac{1}{a^n} \cdot b^m = \dfrac{b^m}{a^n}$

P.4 RATIONAL EXPONENTS AND RADICALS

1. (a) Using exponential notation we can write $\sqrt[3]{5}$ as $5^{1/3}$.

(b) Using radicals we can write $5^{1/2}$ as $\sqrt{5}$.

(c) No. $\sqrt{5^2} = \left(5^2\right)^{1/2} = 5^{2(1/2)} = 5$ and $\left(\sqrt{5}\right)^2 = \left(5^{1/2}\right)^2 = 5^{(1/2)2} = 5$.

2. $\left(4^{1/2}\right)^3 = 2^3 = 8$; $\left(4^3\right)^{1/2} = 64^{1/2} = 8$

3. Because the denominator is of the form $\sqrt{a}$, we multiply numerator and denominator by $\sqrt{a}$: $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

4. $5^{1/3} \cdot 5^{2/3} = 5^1 = 5$

5. No. If a is negative, then $\sqrt{4a^2} = -2a$.

6. No. For example, if $a = -2$, then $\sqrt{a^2 + 4} = \sqrt{8} = 2\sqrt{2}$, but $a + 2 = 0$.

7. (a) $\sqrt{144} = \sqrt{12^2} = 12$

(b) $\sqrt[3]{-125} = \sqrt[3]{(-5)^3} = -5$

(c) $\sqrt[4]{\frac{1}{81}} = \sqrt[4]{\left(\frac{1}{3}\right)^4} = \frac{1}{3}$

8. (a) $\sqrt{49} = \sqrt{7^2} = 7$

(b) $\sqrt[5]{32} = \sqrt[5]{2^5} = 2$

(c) $\sqrt[3]{-\frac{1}{27}} = \sqrt[3]{\left(-\frac{1}{3}\right)^3} = -\frac{1}{3}$

9. (a) $3\sqrt[3]{16} = 3\sqrt[3]{8 \cdot 2} = 3\sqrt[3]{8}\sqrt[3]{2} = 6\sqrt[3]{2}$

(b) $\dfrac{\sqrt{18}}{\sqrt{81}} = \sqrt{\dfrac{18}{81}} = \sqrt{\dfrac{2}{9}} = \dfrac{\sqrt{2}}{3}$

(c) $\sqrt{\dfrac{27}{4}} = \dfrac{3\sqrt{3}}{2} = \dfrac{\sqrt{9 \cdot 3}}{\sqrt{4}} = \dfrac{3\sqrt{3}}{2}$

10. (a) $2\sqrt[3]{81} = 2\sqrt[3]{27 \cdot 3} = 2\sqrt[3]{27}\sqrt[3]{3} = 6\sqrt[3]{3}$

(b) $\dfrac{\sqrt{18}}{\sqrt{25}} = \dfrac{\sqrt{9 \cdot 2}}{5} = \dfrac{3\sqrt{2}}{5}$

(c) $\sqrt{\dfrac{12}{49}} = \dfrac{\sqrt{4 \cdot 3}}{\sqrt{49}} = \dfrac{2\sqrt{3}}{7}$

11. (a) $\sqrt{3}\sqrt{15} = \sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$

(b) $\dfrac{\sqrt{48}}{\sqrt{3}} = \sqrt{\dfrac{48}{3}} = \sqrt{16} = 4$

(c) $\sqrt[3]{24}\sqrt[3]{18} = \sqrt[3]{24 \cdot 18} = \sqrt[3]{6^3 \cdot 2} = 6\sqrt[3]{2}$

12. (a) $\sqrt{10}\sqrt{32} = \sqrt{320} = \sqrt{64 \cdot 5} = 8\sqrt{5}$

(b) $\dfrac{\sqrt{54}}{\sqrt{6}} = \sqrt{\dfrac{54}{6}} = 3$

(c) $\sqrt[3]{15}\sqrt[3]{75} = \sqrt[3]{15 \cdot 75} = \sqrt[3]{5^3 \cdot 9} = 5\sqrt[3]{9}$

13. (a) $\dfrac{\sqrt{72}}{\sqrt{8}} = \sqrt{\dfrac{72}{8}} = \sqrt{9} = 3$

(b) $\sqrt[3]{9}\sqrt[3]{24} = \sqrt[3]{9 \cdot 24} = \sqrt[3]{216} = 6$

(c) $\sqrt[4]{\dfrac{1}{32}}\sqrt[4]{\dfrac{1}{8}} = \sqrt[4]{\dfrac{1}{32} \cdot \dfrac{1}{8}} = \sqrt[4]{\dfrac{1}{256}} = \dfrac{1}{4}$

14. (a) $\sqrt[3]{\dfrac{3}{4}}\sqrt[3]{\dfrac{9}{2}} = \sqrt[3]{\dfrac{3}{4} \cdot \dfrac{9}{2}} = \sqrt[3]{\dfrac{27}{8}} = \dfrac{3}{2}$

(b) $\sqrt[5]{\frac{1}{4}}\sqrt[5]{128} = \sqrt[5]{\frac{1}{4}(128)} = \sqrt[5]{32} = 2$

(c) $\dfrac{\sqrt[3]{12}}{\sqrt[3]{96}} = \sqrt[3]{\dfrac{12}{96}} = \sqrt[3]{\dfrac{1}{8}} = \dfrac{1}{2}$

15. (a) $\sqrt[4]{x^4} = |x|$

(b) $\sqrt[4]{16x^8} = \sqrt[4]{2^4x^8} = 2x^2$

16. (a) $\sqrt[5]{x^{10}} = \left(x^{10}\right)^{1/5} = x^2$

(b) $\sqrt[3]{x^3y^6} = \left(x^3y^6\right)^{1/3} = xy^2$

17. (a) $\sqrt[3]{27w^4} = \sqrt[3]{3^3w^3 \cdot w} = 3w\sqrt[3]{w}$

(b) $\sqrt[4]{64y^6} = \sqrt[4]{2^4y^4 \cdot 4y^2} = 2|y|\sqrt[4]{4y^2}$

18. (a) $\sqrt[4]{x^2z^5} = \sqrt[4]{z^4 \cdot x^2z} = z\sqrt[4]{x^2z}$, $z \geq 0$

(b) $\sqrt{x^6y^6} = \sqrt{\left(x^3y^3\right)^2} = |x|^3|y|^3$

19. (a) $\sqrt[3]{8x^9y^3} = \sqrt[3]{2^3\left(x^3\right)^3y^3} = 2x^3y$

(b) $\sqrt[4]{8x^6y^2}\sqrt[4]{2x^2y^2} = \sqrt[4]{8x^6y^2 \cdot 2x^2y^2} = \sqrt[4]{16x^8y^4}$

$= \sqrt[4]{2^4\left(x^2\right)^4y^4} = 2x^2|y|$

20. (a) $\sqrt{16x^4y^6z^2} = \sqrt{4^2\left(x^2\right)^2\left(y^3\right)^2z^2} = 4x^2|y|^3|z|$

(b) $\sqrt[3]{\sqrt[3]{512x^9}} = \left(512x^9\right)^{1/9} = 2x$

21. **(a)** $\sqrt{32}+\sqrt{18}=\sqrt{16\cdot 2}+\sqrt{9\cdot 2}=\sqrt{16}\sqrt{2}+\sqrt{9}\sqrt{2}=4\sqrt{2}+3\sqrt{2}=7\sqrt{2}$

(b) $\sqrt{75}+\sqrt{48}=\sqrt{25\cdot 3}+\sqrt{16\cdot 3}=\sqrt{25}\sqrt{3}+\sqrt{16}\sqrt{3}=5\sqrt{3}+4\sqrt{3}=9\sqrt{3}$

22. **(a)** $\sqrt{125}+\sqrt{45}=\sqrt{25\cdot 5}+\sqrt{9\cdot 5}=5\sqrt{5}+3\sqrt{3}=8\sqrt{5}$

(b) $\sqrt[3]{54}-\sqrt[3]{16}=\sqrt[3]{27\cdot 2}-\sqrt[3]{8\cdot 2}=\sqrt[3]{27}\sqrt[3]{2}-\sqrt[3]{8}\sqrt[3]{2}=\sqrt[3]{2}$

23. **(a)** $\sqrt{9a^3}+\sqrt{a}=\sqrt{9}\sqrt{a^2\cdot a}+\sqrt{a}=3a\sqrt{a}+\sqrt{a}=(3a+1)\sqrt{a}$

(b) $\sqrt{16x}+\sqrt{x^5}=\sqrt{16}\sqrt{x}+\sqrt{x^4}\sqrt{x}=\left(x^2+4\right)\sqrt{x}$

24. **(a)** $\sqrt[3]{x^4}+\sqrt[3]{8x}=\sqrt[3]{x^3\cdot x}+\sqrt[3]{2^3x}=(x+2)\sqrt[3]{x}$

(b) $4\sqrt{18rt^3}+5\sqrt{32r^3t^5}=4\sqrt{\left(3^2\cdot 2\right)r\left(t^2\cdot t\right)}+5\sqrt{\left(4^2\cdot 2\right)\left(r^2\cdot r\right)\left(t^4\cdot t\right)}=4(3)(t)\sqrt{2rt}+5(4)(r)\left(t^2\right)\sqrt{2rt}$

$=\left(20rt^2+12t\right)\sqrt{2rt}$

25. **(a)** $\sqrt{36x^2+36x^4}=\sqrt{6^2x^2\left(1+x^2\right)}=6x\sqrt{1+x^2}$

(b) $\sqrt{81x^2+81y^2}=\sqrt{9^2\left(x^2+y^2\right)}=9\sqrt{x^2+y^2}$

26. **(a)** $\sqrt{27a^3+63a^2}=\sqrt{3^2a^2\left(3a+7\right)}=3a\sqrt{3a+7}$

(b) $\sqrt{25t^2+100t^2}=\sqrt{125t^2}=5\sqrt{5}t,\ t\geq 0$

27. $\sqrt{10}=10^{1/2}$

28. $\sqrt[5]{6}=6^{1/5}$

29. $7^{3/5}=\left(7^3\right)^{1/5}=\sqrt[5]{7^3}$

30. $6^{-5/2}=\dfrac{1}{6^{5/2}}=\dfrac{1}{\left(6^5\right)^{1/2}}=\dfrac{1}{\sqrt{6^5}}$

31. $\dfrac{1}{\sqrt{5}}=\dfrac{1}{5^{1/2}}=5^{-1/2}$

32. $\dfrac{1}{\sqrt[3]{7}}=\dfrac{1}{7^{1/3}}=7^{-1/3}$

33. $5^{-3/4}=\dfrac{1}{5^{3/4}}=\dfrac{1}{\sqrt[4]{5^3}}=\dfrac{1}{\sqrt[4]{125}}$

34. $y^{-1.5}=y^{-3/2}=\dfrac{1}{\sqrt{y^3}}$

35. $\dfrac{1}{\sqrt[3]{x^2}}=\dfrac{1}{x^{2/3}}=x^{-2/3}$

36. $\dfrac{1}{\sqrt[4]{x^3}}=\dfrac{1}{x^{3/4}}=x^{-3/4}$

37. **(a)** $16^{1/4}=\sqrt[4]{16}=2$ **(b)** $-8^{1/3}=\sqrt[3]{-8}=-2$ **(c)** $9^{-1/2}=\dfrac{1}{\sqrt{9}}=\dfrac{1}{3}$

38. **(a)** $27^{1/3}=\sqrt[3]{27}=3$ **(b)** $(-8)^{1/3}=\sqrt[3]{-8}=-2$ **(c)** $-\left(\dfrac{1}{8}\right)^{1/3}=-\dfrac{1}{\sqrt[3]{8}}=-\dfrac{1}{2}$

39. **(a)** $32^{2/5}=\left(\sqrt[5]{32}\right)^2=2^2=4$ **(b)** $\left(\dfrac{4}{9}\right)^{-1/2}=\dfrac{1}{\sqrt{4/9}}=\sqrt{\dfrac{9}{4}}=\dfrac{3}{2}$ **(c)** $\left(\dfrac{16}{81}\right)^{3/4}=\left(\dfrac{\sqrt[4]{16}}{\sqrt[4]{81}}\right)^3=\dfrac{8}{27}$

40. **(a)** $125^{2/3}=\left(\sqrt[3]{125}\right)^2=5^2=25$

(b) $\left(\dfrac{25}{64}\right)^{3/2}=\left(\dfrac{\sqrt{25}}{\sqrt{64}}\right)^3=\left(\dfrac{5}{8}\right)^3=\dfrac{125}{512}$

(c) $27^{-4/3}=\dfrac{1}{\left(\sqrt[3]{27}\right)^4}=\dfrac{1}{3^4}=\dfrac{1}{81}$

41. **(a)** $5^{2/3}\cdot 5^{1/3}=5^{(2/3)+(1/3)}=5^1=5$

(b) $\dfrac{3^{3/5}}{3^{2/5}}=3^{(3/5)-(2/5)}=3^{1/5}=\sqrt[5]{3}$

(c) $\left(\sqrt[3]{4}\right)^3=\left(4^{1/3}\right)^3=4^{(1/3)\cdot 3}=4^1=4$

42. **(a)** $3^{2/7}\cdot 3^{12/7}=3^{2/7+12/7}=3^2=9$

(b) $\dfrac{7^{2/3}}{7^{5/3}} = 7^{(2/3)-(5/3)} = 7^{-1} = \dfrac{1}{7}$

(c) $\left(\sqrt[5]{6}\right)^{-10} = \left(6^{1/5}\right)^{-10} = 6^{(1/5)(-10)} = 6^{-2} = \dfrac{1}{6^2} = \dfrac{1}{36}$

43. When $x = 3$, $y = 4$, $z = -1$ we have $\sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.

44. When $x = 3$, $y = 4$, $z = -1$ we have $\sqrt[4]{x^3 + 14y + 2z} = \sqrt[4]{3^3 + 14\,(4) + 2\,(-1)} = \sqrt[4]{27 + 56 - 2} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$.

45. When $x = 3$, $y = 4$, $z = -1$ we have

$$\begin{aligned}(9x)^{2/3} + (2y)^{2/3} + z^{2/3} &= (9 \cdot 3)^{2/3} + (2 \cdot 4)^{2/3} + (-1)^{2/3} = \left(3^3\right)^{2/3} + \left(2^3\right)^{2/3} + (1)^{1/3} \\ &= 3^2 + 2^2 + 1 = 9 + 4 + 1 = 14.\end{aligned}$$

46. When $x = 3$, $y = 4$, $z = -1$ we have $(xy)^{2z} = (3 \cdot 4)^{2 \cdot (-1)} = 12^{-2} = \frac{1}{144}$.

47. (a) $x^{3/4}x^{5/4} = x^{3/4+5/4} = x^2$

(b) $y^{2/3}y^{4/3} = y^{2/3+4/3} = y^2$

48. (a) $(4b)^{1/2}\left(8b^{1/4}\right) = 4^{1/2} \cdot 8b^{1/2+1/4} = 16b^{3/4}$

(b) $\left(3a^{3/4}\right)^2\left(5a^{1/2}\right) = 3^2 \cdot 5a^{(3/4)\cdot 2+(1/2)} = 45a^2$

49. (a) $\dfrac{w^{4/3}w^{2/3}}{w^{1/3}} = w^{4/3+2/3-1/3} = w^{5/3}$

(b) $\left(3x^{1/2}y^{1/3}\right)^6 = 3^6x^{(1/2)6}y^{(1/3)6} = 3^6 = 729x^3y^2$

50. (a) $\left(8y^3\right)^{-2/3} = 8^{-2/3}y^{3(-2/3)} = \dfrac{1}{4y^2}$

(b) $\left(u^4v^6\right)^{-1/3} = u^{4(-1/3)}v^{6(-1/3)} = \dfrac{1}{u^{4/3}v^2}$

51. (a) $\left(8a^6b^{3/2}\right)^{2/3} = 8^{2/3}a^{6(2/3)}b^{(3/2)(2/3)} = 4a^4b$

(b) $\left(4a^{-4}b^6\right)^{3/2} = 4^{3/2}a^{-4(3/2)}b^{6(3/2)} = 8a^{-6}b^9 = \dfrac{8b^9}{a^6}$

52. (a) $\left(x^{-5}y^{1/3}\right)^{-3/5} = x^{-5(-3/5)}y^{(1/3)(-3/5)} = \dfrac{x^3}{y^{1/5}}$

(b) $\left(u^3v^2\right)^{1/3}\left(16u^6v^{2/3}\right)^{1/2} = u^{3(1/3)}v^{2(1/3)}16^{1/2}u^{6(1/2)}v^{(2/3)(1/2)} = 4uv^{2/3}u^3v^{1/3} = 4u^4v$

53. (a) $\left(\dfrac{3x^{1/4}}{y}\right)^2\left(\dfrac{x^{-1}}{y^4}\right)^{1/2} = \dfrac{3^2x^{(1/4)2}}{y^2}\dfrac{x^{-1(1/2)}}{y^{4(1/2)}} = \dfrac{9x^{1/2}}{y^2}\dfrac{x^{-1/2}}{y^2} = \dfrac{9}{y^4}$

(b) $\left(\dfrac{3w^{-1/3}}{z^{-1}}\right)^{-4}\left(\dfrac{2w^{1/3}}{z^{1/2}}\right)^2 = \dfrac{3^{-4}w^{(-1/3)(-4)}}{z^{-1(-4)}}\dfrac{2^2w^{(1/3)2}}{z^{(1/2)2}} = \dfrac{w^{4/3}}{81z^4}\dfrac{4w^{2/3}}{z} = \dfrac{4w^{(4/3)+(2/3)}}{81z^5} = \dfrac{4w^2}{81z^5}$

54. (a) $\left(\dfrac{y^{2/3}}{x^{-1}}\right)^3\left(\dfrac{x}{y^{-2}}\right)^{-1} = \dfrac{\left(y^{2/3}\right)^3}{\left(x^{-1}\right)^3}\dfrac{x^{-1}}{\left(y^{-2}\right)^{-1}} = \dfrac{y^2}{x^{-3}}\dfrac{x^{-1}}{y^2} = x^2$

(b) $\left(\dfrac{4y^3z^{2/3}}{x^{1/2}}\right)^2\left(\dfrac{x^{-3}y^6}{8z^4}\right)^{1/3} = \dfrac{4^2\left(y^3\right)^2\left(z^{2/3}\right)^2}{\left(x^{1/2}\right)^2}\dfrac{\left(x^{-3}\right)^{1/3}\left(y^6\right)^{1/3}}{8^{1/3}\left(z^4\right)^{1/3}} = \dfrac{16y^6z^{4/3}}{x}\dfrac{x^{-1}y^2}{2z^{4/3}} = \dfrac{8y^8}{x^2}$

55. (a) $\sqrt{x^3} = \left(x^3\right)^{1/2} = x^{3(1/2)} = x^{3/2}$

(b) $\sqrt[5]{x^6} = \left(x^6\right)^{1/5} = x^{6(1/5)} = x^{6/5}$

56. (a) $\sqrt{x^5} = \left(x^5\right)^{1/2} = x^{5(1/2)} = x^{5/2}$

(b) $\sqrt[4]{x^6} = \left(x^6\right)^{1/4} = x^{6(1/4)} = x^{3/2}$

57. (a) $\sqrt[6]{y^5}\,\sqrt[3]{y^2} = y^{5/6} \cdot y^{2/3} = y^{5/6+2/3} = y^{3/2}$

(b) $\left(5\sqrt[3]{x}\right)\left(2\sqrt[4]{x}\right) = 5 \cdot 2x^{1/3+1/4} = 10x^{7/12}$

58. (a) $\sqrt[4]{b^3}\sqrt{b} = b^{3/4+1/2} = b^{5/4}$

(b) $\left(2\sqrt{a}\right)\left(\sqrt[3]{a^2}\right) = 2a^{1/2+2/3} = 2a^{7/6}$

59. (a) $\sqrt{4st^3}\,\sqrt[6]{s^3t^2} = 4^{1/2}s^{1/2+3/6}t^{3/2+2/6} = 2st^{11/6}$

(b) $\dfrac{\sqrt[4]{x^7}}{\sqrt[4]{x^3}} = x^{7/4-3/4} = x$

60. (a) $\sqrt[5]{x^3y^2}\,\sqrt[10]{x^4y^{16}} = x^{3/5+4/10}y^{2/5+16/10} = xy^2$

(b) $\dfrac{\sqrt[3]{8x^2}}{\sqrt{x}} = 8^{1/3}x^{2/3-1/2} = 2x^{1/6}$

61. (a) $\sqrt[3]{y\sqrt{y}} = \left(y^{1+1/2}\right)^{1/3} = y^{(3/2)(1/3)} = y^{1/2}$

(b) $\sqrt{\dfrac{18u^5v}{2u^3v^3}} = \sqrt{\dfrac{9u^2}{v^2}} = \dfrac{3u}{v}$

62. (a) $\sqrt{s\sqrt{s^3}} = \left(s^{1+3/2}\right)^{1/2} = s^{5/4}$

(b) $\sqrt[3]{\dfrac{54x^2y^4}{2x^5y}} = \sqrt[3]{\dfrac{27y^3}{x^3}} = \dfrac{3y}{x}$

63. (a) $\dfrac{1}{\sqrt{6}} = \dfrac{1}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{\sqrt{6}}{6}$

(b) $\sqrt{\dfrac{3}{2}} = \sqrt{\dfrac{3}{2}}\sqrt{\dfrac{2}{2}} = \dfrac{\sqrt{6}}{2}$

(c) $\dfrac{9}{\sqrt[4]{2}} = \dfrac{9}{2^{1/4}} \cdot \dfrac{2^{3/4}}{2^{3/4}} = \dfrac{9\sqrt[4]{8}}{2}$

64. (a) $\dfrac{12}{\sqrt{3}} = \dfrac{12}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{12\sqrt{3}}{3} = 4\sqrt{3}$

(b) $\sqrt{\dfrac{12}{5}} = \sqrt{\dfrac{4 \cdot 3}{5}}\sqrt{\dfrac{5}{5}} = \dfrac{2\sqrt{15}}{5}$

(c) $\dfrac{8}{\sqrt[3]{5^2}} = \dfrac{8}{5^{2/3}} \cdot \dfrac{5^{1/3}}{5^{1/3}} = \dfrac{8\sqrt[3]{5}}{5}$

65. (a) $\dfrac{1}{\sqrt{5x}} = \dfrac{1}{\sqrt{5x}} \cdot \dfrac{\sqrt{5x}}{\sqrt{5x}} = \dfrac{\sqrt{5x}}{5x}$

(b) $\sqrt{\dfrac{x}{5}} = \sqrt{\dfrac{x}{5}}\sqrt{\dfrac{5}{5}} = \dfrac{\sqrt{5x}}{5}$

(c) $\sqrt[5]{\dfrac{1}{x^3}} = \dfrac{1}{x^{3/5}} \cdot \dfrac{x^{2/5}}{x^{2/5}} = \dfrac{\sqrt[5]{x^2}}{x}$

66. (a) $\sqrt{\dfrac{s}{3t}} = \sqrt{\dfrac{s}{3t}}\sqrt{\dfrac{3t}{3t}} = \dfrac{\sqrt{3st}}{3t}$

(b) $\dfrac{a}{\sqrt[6]{b^2}} = \dfrac{a}{\sqrt[3]{b}} \cdot \dfrac{\sqrt[3]{b^2}}{\sqrt[3]{b^2}} = \dfrac{a\sqrt[3]{b^2}}{b}$

(c) $\dfrac{1}{c^{3/5}} = \dfrac{1}{c^{3/5}} \cdot \dfrac{c^{2/5}}{c^{2/5}} = \dfrac{\sqrt[5]{c^2}}{c}$

67. (a) $\sqrt[4]{\dfrac{1}{4}}\sqrt[4]{\dfrac{1}{64}} = \sqrt[4]{\dfrac{1}{4 \cdot 4^3}} = \dfrac{1}{4}$

(b) $\dfrac{\sqrt{5}}{\sqrt{40}} = \sqrt{\dfrac{5}{40}} = \sqrt{\dfrac{1}{8}} = \sqrt{\dfrac{2}{16}} = \dfrac{\sqrt{2}}{4}$

68. (a) $\left(\dfrac{x^{3/2}}{y^{-1/2}}\right)^4 = \dfrac{x^{(3/2)4}}{y^{(-1/2)(4)}} = \dfrac{x^6}{y^{-2}} = x^6y^2$

(b) $\left(25u^2v^{-4}\right)^{1/2} = 25^{1/2}u^{2(1/2)}v^{-4(1/2)} = 5\,|u|\,v^{-2} = \dfrac{5\,|u|}{v^2}$. Since $u < 0$, this is equivalent to $-\dfrac{5u}{v^2}$.

69. (a) $\sqrt{y}\sqrt[4]{y^2} = y^{1/2}y^{2/4} = y$

(b) $\left(81\sqrt[4]{w^8z^8}\right)^{1/2} = 81^{1/2}w^{(8/4)(1/2)}z^{(8/4)(1/2)} = 9\,|w|\,|z|$. Since $w > 0$ and $z < 0$, this is equivalent to $-9wz$.

70. (a) $\left(\dfrac{x^{3/2}}{y^{-1/2}}\right)^4\left(\dfrac{x^{-2}}{y^3}\right) = \dfrac{\left(x^{3/2}\right)^4}{\left(y^{-1/2}\right)^4} \cdot \dfrac{x^{-2}}{y^3} = \dfrac{x^6 \cdot x^{-2}}{y^{-2} \cdot y^3} = \dfrac{x^4}{y}$

(b) $\left(\dfrac{w^4}{\sqrt[3]{w^3z^6}}\right)^{1/2} = \dfrac{w^{4(1/2)}}{w^{1/2}z^{2(1/2)}} = \dfrac{w^2}{\sqrt{wz}} = \dfrac{w^{3/2}}{z}$

71. (a) Since $\frac{1}{2} > \frac{1}{3}$, $2^{1/2} > 2^{1/3}$.

(b) $\left(\frac{1}{2}\right)^{1/2} = 2^{-1/2}$ and $\left(\frac{1}{2}\right)^{1/3} = 2^{-1/3}$. Since $-\frac{1}{2} < -\frac{1}{3}$, we have $\left(\frac{1}{2}\right)^{1/2} < \left(\frac{1}{2}\right)^{1/3}$.

72. (a) We find a common root: $7^{1/4} = 7^{3/12} = \left(7^3\right)^{1/12} = 343^{1/12}$; $4^{1/3} = 4^{4/12} = \left(4^4\right)^{1/12} = 256^{1/12}$. So $7^{1/4} > 4^{1/3}$.

(b) We find a common root: $\sqrt[3]{5} = 5^{1/3} = 5^{2/6} = \left(5^2\right)^{1/6} = 25^{1/6}$; $\sqrt{3} = 3^{1/2} = 3^{3/6} = \left(3^3\right)^{1/6} = 27^{1/6}$. So $\sqrt[3]{5} < \sqrt{3}$.

73. First convert 1135 feet to miles. This gives $1135 \text{ ft} = 1135 \cdot \dfrac{1 \text{ mile}}{5280 \text{ feet}} = 0.215$ mi. Thus the distance you can see is given by $D = \sqrt{2rh + h^2} = \sqrt{2\,(3960)\,(0.215) + (0.215)^2} \approx \sqrt{1702.8} \approx 41.3$ miles.

74. (a) Using $f = 0.4$ and substituting $d = 65$, we obtain $s = \sqrt{30fd} = \sqrt{30 \times 0.4 \times 65} \approx 28$ mi/h.

(b) Using $f = 0.5$ and substituting $s = 50$, we find d. This gives $s = \sqrt{30fd} \Leftrightarrow 50 = \sqrt{30 \cdot (0.5)\,d} \Leftrightarrow 50 = \sqrt{15d} \Leftrightarrow 2500 = 15d \Leftrightarrow d = \frac{500}{3} \approx 167$ feet.

75. Since 1 day $=$ 86,400 s, 365.25 days $=$ 31,557,600 s. Substituting, we obtain

$$d = \left(\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{4\pi^2}\right)^{1/3} \cdot \left(3.15576 \times 10^7\right)^{2/3} \approx 1.5 \times 10^{11} \text{ m} = 1.5 \times 10^8 \text{ km}.$$

76. **(a)** Substituting the given values we get $V = 1.486 \cdot \dfrac{75^{2/3} \cdot 0.050^{1/2}}{24.1^{2/3} \cdot 0.040} \approx 17.707$ ft/s.

(b) Since the volume of the flow is $V \cdot A$, the canal discharge is $17.707 \cdot 75 \approx 1328.0$ ft^3/s.

77. **(a)**

n	1	2	5	10	100
$2^{1/n}$	$2^{1/1} = 2$	$2^{1/2} = 1.414$	$2^{1/5} = 1.149$	$2^{1/10} = 1.072$	$2^{1/100} = 1.007$

So when n gets large, $2^{1/n}$ decreases to 1.

(b)

n	1	2	5	10	100
$\left(\frac{1}{2}\right)^{1/n}$	$\left(\frac{1}{2}\right)^{1/1} = 0.5$	$\left(\frac{1}{2}\right)^{1/2} = 0.707$	$\left(\frac{1}{2}\right)^{1/5} = 0.871$	$\left(\frac{1}{2}\right)^{1/10} = 0.933$	$\left(\frac{1}{2}\right)^{1/100} = 0.993$

So when n gets large, $\left(\frac{1}{2}\right)^{1/n}$ increases to 1.

P.5 ALGEBRAIC EXPRESSIONS

1. **(a)** $2x^3 - \frac{1}{2}x + \sqrt{3}$ is a polynomial. (The constant term is not an integer, but all exponents are integers.)

(b) $x^2 - \frac{1}{2} - 3\sqrt{x} = x^2 - \frac{1}{2} - 3x^{1/2}$ is not a polynomial because the exponent $\frac{1}{2}$ is not an integer.

(c) $\dfrac{1}{x^2 + 4x + 7}$ is not a polynomial. (It is the reciprocal of the polynomial $x^2 + 4x + 7$.)

(d) $x^5 + 7x^2 - x + 100$ is a polynomial.

(e) $\sqrt[3]{8x^6 - 5x^3 + 7x - 3}$ is not a polynomial. (It is the cube root of the polynomial $8x^6 - 5x^3 + 7x - 3$.)

(f) $\sqrt{3}x^4 + \sqrt{5}x^2 - 15x$ is a polynomial. (Some coefficients are not integers, but all exponents are integers.)

2. To add polynomials we add *like* terms. So
$\left(3x^2 + 2x + 4\right) + \left(8x^2 - x + 1\right) = (3 + 8)x^2 + (2 - 1)x + (4 + 1) = 11x^2 + x + 5.$

3. To subtract polynomials we subtract *like* terms. So
$\left(2x^3 + 9x^2 + x + 10\right) - \left(x^3 + x^2 + 6x + 8\right) = (2 - 1)x^3 + (9 - 1)x^2 + (1 - 6)x + (10 - 8) = x^3 + 8x^2 - 5x + 2.$

4. We use FOIL to multiply two polynomials: $(x + 2)(x + 3) = x \cdot x + x \cdot 3 + 2 \cdot x + 2 \cdot 3 = x^2 + 5x + 6.$

5. The Special Product Formula for the "square of a sum" is $(A + B)^2 = A^2 + 2AB + B^2$. So
$(2x + 3)^2 = (2x)^2 + 2(2x)(3) + 3^2 = 4x^2 + 12x + 9.$

6. The Special Product Formula for the "product of the sum and difference of terms" is $(A + B)(A - B) = A^2 - B^2$. So
$(6 + x)(6 - x) = 6^2 - x^2 = 36 - x^2.$

7. **(a)** No; $(x + 5)^2 = x^2 + 2(5x) + 25 \neq x^2 + 25$.

(b) Yes; $(x + a)^2 = x^2 + 2xa + a^2$.

8. **(a)** Yes; by a Special Product Formula, $(x + 5)(x - 5) = x^2 - 25$.

(b) No, $(x + a)(x - a) = x^2 - a^2$, by a Special Product Formula.

9. Type: binomial. Terms: $5x^3$ and 6. Degree: 3.

10. Type: trinomial. Terms: $-2x^2$, $5x$, and -3. Degree: 2.

11. Type: monomial. Term: -8. Degree: 0.

12. Type: monomial. Terms: $\frac{1}{2}x^7$. Degree: 7.

13. Type: quadrinomial. Terms: x, $-x^2$, x^3, and $-x^4$. Degree: 4.

14. Type: binomial. Terms: $\sqrt{2}x$ and $-\sqrt{3}$. Degree: 1.

15. $(12x-7)-(5x-12)=12x-7-5x+12=7x+5$

16. $(5-3x)+(2x-8)=-x-3$

17. $\left(-2x^2-3x+1\right)+\left(3x^2+5x-4\right)=-2x^2-3x+1+3x^2+5x-4=x^2+2x-3$

18. $\left(3x^2+x+1\right)-\left(2x^2-3x-5\right)=3x^2+x+1-2x^2+3x+5=x^2+4x+6$

19. $\left(5x^3+4x^2-3x\right)-\left(x^2+7x+2\right)=5x^3+4x^2-3x-x^2-7x-2=5x^3+3x^2-10x-2$

20. $\left(5x^2-3\right)-\left(1-4x-3x^2\right)=5x^2-3-1+4x+3x^2=8x^2+4x-4$

21. $3(x-1)+4(x+2)=3x-3+4x+8=7x+5$

22. $8(2x+5)-7(x-9)=16x+40-7x+63=9x+103$

23. $4\left(x^2-3x+5\right)-3\left(x^2-2x+1\right)=4x^2-12x+20-3x^2+6x-3=x^2-6x+17$

24. $\left(5x^3-3x^2+2x\right)-2\left(3+x-4x^3\right)=5x^3-3x^2+2x-2(3)-2(x)-2\left(-4x^3\right)=13x^3-3x^2-6$

25. $4w(w-5)=4w^2-20w$

26. $10x(1-2x)=10x-20x^2=-20x^2+10x$

27. $y^3\left(y^2-y\right)=y^3\cdot y^2-y^3\cdot y=y^5-y^4$

28. $4z\left(2-z^2\right)=-4z^3+8z$

29. $x^3\left(x^2+3x\right)-2x\left(x^4-3x^2\right)=x^5+3x^4-2x^5+6x^3=-x^5+3x^4+6x^3$

30. $4x\left(1-x^3\right)+3x^3\left(x^3-x\right)=4x-4x^4+3x^6-3x^4=3x^6-7x^4+4x$

31. $4x(x-2)-2\left(x^2-4x\right)+2x^2(x-1)=4x^2-8x-2x^2+8x+2x^3-2x^2=2x^3$

32. $6x^3\left(x^2-1\right)-2x\left(2+3x^2\right)+2(2x-4)=6x^5-6x^3-4x-6x^3+4x-8=6x^5-12x^3-8$

33. $(x+4)(x+10)=x^2+10x+4x+4\cdot 10=x^2+14x+40$

34. $(y-1)(y-7)=y^2-7y-y-1(-7)=y^2-8y+7$

35. $(z-1)(4z+5)=4z^2+5z-4z-1(5)=4z^2+z-5$

36. $(3x+2)(x-5)=3x^2-3\cdot 5x+2x+2(-5)=3x^2-13x-10$

37. $(3t-2)(7t-4)=21t^2-12t-14t+8=21t^2-26t+8$

38. $(4s-1)(2s+5)=8s^2+18s-5$

39. $(3x+5)(2x-1)=6x^2+10x-3x-5=6x^2+7x-5$

40. $(7y-3)(2y-1)=14y^2-13y+3$

41. $(x+3y)(2x-y)=2x^2+5xy-3y^2$

42. $(4x-5y)(3x-y)=12x^2-19xy+5y^2$

43. $(3u-4v)(4u-3v)=12u^2-25uv+12v^2$

44. $(2r-s)(r+2s)=2r^2+3rs-2s^2$

45. $(4x+3)^2=16x^2+24x+9$

46. $(2-7y)^2=49y^2-28y+4$

47. $(1-3z)^2=9z^2-6z+1$

48. $(2u+3)^2=4u^2+12u+9$

49. $(y-3x)^2=y^2-6xy+9x^2$

50. $(5x-y)^2=25x^2-10xy+y^2$

51. $(2x+3y)^2 = 4x^2 + 12xy + 9y^2$

52. $(r-2s)^2 = r^2 - 4rs + 4s^2$

53. $\left(5-y^3\right)^2 = 5^2 + \left(-y^3\right)^2 + 2(5)\left(-y^3\right) = y^6 - 10y^3 + 25$

54. $\left(x^4+3\right)^2 = x^8 + 6x^4 + 9$

55. $(w+7)(w-7) = w^2 - 49$

56. $(5-y)(5+y) = 25 - y^2$

57. $(3x-4)(3x+4) = (3x)^2 - 4^2 = 9x^2 - 16$

58. $(2y+5)(2y-5) = 4y^2 - 25$

59. $(\sqrt{x}+2)(\sqrt{x}-2) = x - 4$

60. $\left(\sqrt{y}+\sqrt{2}\right)\left(\sqrt{y}-\sqrt{2}\right) = y - 2$

61. $(y+2)^3 = y^3 + 3y^2(2) + 3y\left(2^2\right) + 2^3 = y^3 + 6y^2 + 12y + 8$

62. $(x-3)^3 = x^3 - 9x^2 + 27x - 27$

63. $(2-5u)^3 = -125u^3 + 150u^2 - 60u + 8$

64. $(3x-2)^3 = 27x^3 - 54x^2 + 36x - 8$

65. $(x+2)\left(x^2+2x+3\right) = x^3 + 2x^2 + 3x + 2x^2 + 4x + 6 = x^3 + 4x^2 + 7x + 6$

66. $(x+1)\left(2x^2-x+1\right) = 2x^3 + x^2 + 1$

67. $(2x-5)\left(x^2-x+1\right) = 2x^3 - 2x^2 + 2x - 5x^2 + 5x - 5 = 2x^3 - 7x^2 + 7x - 5$

68. $(1+2x)\left(x^2-3x+1\right) = 2x^3 - 5x^2 - x + 1$

69. $\sqrt{x}(x-\sqrt{x}) = x\sqrt{x} - (\sqrt{x})^2 = x\sqrt{x} - x$

70. $x^{3/2}(\sqrt{x} - 1/\sqrt{x}) = x^2 - x$

71. $y^{1/3}\left(y^{2/3}+y^{5/3}\right) = y^{1/3+2/3} + y^{1/3+5/3} = y^2 + y$

72. $x^{1/4}\left(2x^{3/4}-x^{1/4}\right) = 2x - \sqrt{x}$

73. $\left(x^{1/2}-y^{1/2}\right)^2 = x - 2\sqrt{xy} + y$

74. $\left(\sqrt{u}+\dfrac{1}{\sqrt{u}}\right)^2 = u + 2 + \dfrac{1}{u}$

75. $\left(x^2-a^2\right)\left(x^2+a^2\right) = x^4 - a^4$

76. $\left(x^{1/2}+y^{1/2}\right)\left(x^{1/2}-y^{1/2}\right) = x - y$

77. $(\sqrt{a}-b)(\sqrt{a}+b) = a - b^2$

78. $\left(\sqrt{h^2+1}+1\right)\left(\sqrt{h^2+1}-1\right) = h^2$

79. $\left((x-1)+x^2\right)\left((x-1)-x^2\right) = (x-1)^2 - \left(x^2\right)^2 = x^2 - 2x + 1 - x^4 = -x^4 + x^2 - 2x + 1$

80. $\left(x+\left(2+x^2\right)\right)\left(x-\left(2+x^2\right)\right) = -x^4 - 3x^2 - 4$

81. $(2x+y-3)(2x+y+3) = (2x+y)^2 - 3^2 = 4x^2 + 4xy + y^2 - 9$

82. $(x+y+z)(x-y-z) = x^2 - y^2 - z^2 - 2yz$

83. **(a)** $\frac{1}{2}\left[(a+b)^2 - \left(a^2+b^2\right)\right] = \frac{1}{2}\left[a^2 + 2ab + b^2 - a^2 - b^2\right] = \frac{1}{2}(2ab) = ab.$

(b)
$$\begin{aligned}\left(a^2+b^2\right)^2 - \left(a^2-b^2\right)^2 &= \left[\left(a^2+b^2\right) - \left(a^2-b^2\right)\right]\left[\left(a^2+b^2\right) + \left(a^2-b^2\right)\right] \\ &= \left(a^2+b^2-a^2+b^2\right)\left(a^2+b^2+a^2-b^2\right) = \left(2b^2\right)\left(2a^2\right) = 4a^2b^2\end{aligned}$$

84. LHS $= \left(a^2+b^2\right)\left(c^2+d^2\right) = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2.$

RHS $= (ac+bd)^2 + (ad-bc)^2 = a^2c^2 + 2abcd + b^2d^2 + a^2d^2 - 2abcd + b^2c^2 = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2.$

So LHS = RHS, that is, $\left(a^2+b^2\right)\left(c^2+d^2\right) = (ac+bd)^2 + (ad-bc)^2.$

85. **(a)** The height of the box is x, its width is $6-2x$, and its length is $10-2x$. Since Volume = height × width × length, we have $V = x(6-2x)(10-2x)$.

(b) $V = x\left(60 - 32x + 4x^2\right) = 60x - 32x^2 + 4x^3$, degree 3.

(c) When $x = 1$, the volume is $V = 60(1) - 32\left(1^2\right) + 4\left(1^3\right) = 32$, and when $x = 2$, the volume is $V = 60(2) - 32\left(2^2\right) + 4\left(2^3\right) = 24$.

86. (a) The width is the width of the lot minus the setbacks of 10 feet each. Thus width $= x - 20$ and length $= y - 20$. Since Area $=$ width $\times$ length, we get $A = (x - 20)(y - 20)$.

(b) $A = (x - 20)(y - 20) = xy - 20x - 20y + 400$

(c) For the 100×400 lot, the building envelope has $A = (100 - 20)(400 - 20) = 80(380) = 30{,}400$. For the 200×200, lot the building envelope has $A = (200 - 20)(200 - 20) = 180(180) = 32{,}400$. The 200×200 lot has a larger building envelope.

87. (a) $A = 4000(1 + r)^3 = 2000\left(1 + 3r + 3r^2 + r^3\right) = 2000 + 6000r + 6000r^2 + 2000r^3$, degree 3.

(b) Remember that % means divide by 100, so $2\% = 0.02$.

Interest rate r	2%	3.5%	5%	6%	10%
Amount A	\$4244.83	\$4434.87	\$4630.50	\$4764.06	\$5324.00

88. (a) When $x = 1$, $(x + 5)^2 = (1 + 5)^2 = 36$ and $x^2 + 25 = 1^2 + 25 = 26$.

(b) $(x + 5)^2 = x^2 + 10x + 25$

89. (a) The degree of the product is the sum of the degrees of the original polynomials.

(b) The degree of the sum could be lower than either of the degrees of the original polynomials, but is at most the largest of the degrees of the original polynomials.

(c) Product: $$\left(2x^3 + x - 3\right)\left(-2x^3 - x + 7\right) = -4x^6 - 2x^4 + 14x^3 - 2x^4 - x^2 + 7x + 6x^3 + 3x - 21$$
$$= -4x^6 - 4x^4 + 20x^3 - x^2 + 10x - 21$$

Sum: $\left(2x^3 + x - 3\right) + \left(-2x^3 - x + 7\right) = 4$.

P.6 FACTORING

1. (a) The polynomial $2x^3 + 3x^2 + 10x$ has three terms: $2x^3$, $3x^2$, and $10x$.

(b) The factor x is common to each term, so $2x^3 + 3x^2 + 10x = x\left(2x^2 + 3x + 10\right)$.

2. The greatest common factor in the expression $18x^3 + 30x$ is $6x$, and the expression factors as $6x\left(3x^2 + 5\right)$.

3. To factor the trinomial $x^2 + 8x + 12$ we look for two integers whose product is 12 and whose sum is 8. These integers are 6 and 2, so the trinomial factors as $(x + 6)(x + 2)$.

4. The Special Factoring Formula for the "difference of squares" is $A^2 - B^2 = (A - B)(A + B)$. So $49x^2 - 9 = (7x - 3)(7x + 3)$.

5. The Special Factoring Formula for a "perfect square" is $A^2 + 2AB + B^2 = (A + B)^2$. So $x^2 + 10x + 25 = (x + 5)^2$.

6. The greatest common factor in the expression $4(x + 1)^2 - x(x + 1)^2$ is $(x + 1)^2$, and the expression factors as $4(x + 1)^2 - x(x + 1)^2 = (x + 1)^2(4 - x)$.

7. $10x + 15 = 5(2x + 3)$

8. $12 - 3y = 3(4 - y)$

9. $4x^4 - x^2 = x^2\left(4x^2 - 1\right) = x^2(2x + 1)(2x - 1)$

10. $3x^4 - 6x^3 - x^2 = x^2\left(3x^2 - 6x - 1\right)$

11. $4x^3y^2 - 6xy^3 + 8x^2y^4 = 2xy^2\left(2x^2 - 3y + 4xy^2\right)$

12. $-7x^4y^2 + 14xy^3 + 21xy^4 = 7xy^2\left(-x^3 + 2y + 3y^2\right)$

13. $y(y - 6) + 9(y - 6) = (y - 6)(y + 9)$

14. $(z + 2)^2 - 5(z + 2) = (z + 2)[(z + 2) - 5] = (z + 2)(z - 3)$

15. $z^2 - 11z + 18 = (z - 9)(z - 2)$

16. $x^2 + 4x - 5 = (x + 5)(x - 1)$

17. $10x^2 - 19x + 6 = (5x - 2)(2x - 3)$

18. $6y^2 + 11y - 21 = (y + 3)(6y - 7)$

19. $3x^2 - 16x + 5 = (3x - 1)(x - 5)$

20. $5x^2 - 7x - 6 = (5x + 3)(x - 2)$

21. $(3x + 2)^2 + 8(3x + 2) + 12 = [(3x + 2) + 2][(3x + 2) + 6] = (3x + 4)(3x + 8)$

22. $2(a + b)^2 + 5(a + b) - 3 = [(a + b) + 3][2(a + b) - 1] = (a + b + 3)(2a + 2b - 1)$

23. $x^2 - 400 = (x + 20)(x - 20)$

24. $64 - y^2 = (8 + y)(8 - y)$

25. $36a^2 - 49 = (6a + 7)(6a - 7)$

26. $4z^2 - 81 = (2z + 9)(2z - 9)$

27. $x^2 - 49y^2 = (x + 7y)(x - 7y)$

28. $4u^2 - 25v^2 = (2u + 5v)(2u - 5v)$

29. $(z + 5)^2 - 9w^2 = (z + 5 + 3w)(z + 5 - 3w)$

30. $y^2 - (x - 1)^2 = (y + x - 1)(y - x + 1)$

31. $x^2 + 12x + 36 = x^2 + 2(6x) + 6^2 = (x + 6)^2$

32. $16 - 8y + y^2 = y^2 + 2(-4y) + (-4)^2 = (y - 4)^2$

33. $100 - 20z + z^2 = z^2 + 2(-10z) + (-10)^2 = (z - 10)^2$

34. $w^2 - 14w + 49 = w^2 + 2(-7w) + (-7)^2 = (w - 7)^2$

35. $25 + 30t + 9t^2 = (3t)^2 + 2(3t)(5) + 5^2 = (3t + 5)^2$

36. $16z^2 - 24z + 9 = (4z)^2 - 2(4z)(3) + 3^2 = (4z - 3)^2$

37. $4u^2 - 36uv + 81v^2 = (2u)^2 - 2(2u)(9v) + (9v)^2 = (2u - 9v)^2$

38. $4y^2 + 20yz + 25z^2 = (2y)^2 + 2(2y)(5z) + (5z)^2 = (2y + 5z)^2$

39. $x^3 - 125 = x^3 - 5^3 = (x - 5)\left(x^2 + 5x + 25\right)$

40. $y^3 + 64 = y^3 + 4^3 = (y + 4)\left(y^2 - 4y + 16\right)$

41. $8w^3 + 27 = (2w)^3 + 3^3 = (2w + 3)\left(4w^2 - 6w + 9\right)$

42. $1 - 27a^3 = 1^3 - (3a)^3 = (1 - 3a)\left(1 + 3a + 9a^2\right)$

43. $27x^3 + y^3 = (3x)^3 + y^3 = (3x + y)\left[(3x)^2 + 3xy + y^2\right] = (3x + y)\left(9x^2 - 3xy + y^2\right)$

44. $8x^3 - y^3 = (2x)^3 - y^3 = (2x - y)\left(4x^2 + 2xy + y^2\right)$

45. $a^3 - b^6 = a^3 - \left(b^2\right)^3 = \left(a - b^2\right)\left[a^2 + ab^2 + \left(b^2\right)^2\right] = \left(a - b^2\right)\left(a^2 + ab^2 + b^4\right)$

46. $64x^3y^3 - 27 = (4xy)^3 - 3^3 = (4xy - 3)\left(16x^2y^2 + 12xy + 9\right)$

47. $x^3 + 4x^2 + x + 4 = x^2(x + 4) + 1(x + 4) = (x + 4)\left(x^2 + 1\right)$

48. $3x^3 - x^2 + 6x - 2 = x^2(3x - 1) + 2(3x - 1) = (3x - 1)\left(x^2 + 2\right)$

49. $5x^3 + x^2 + 5x + 1 = x^2(5x + 1) + (5x + 1) = \left(x^2 + 1\right)(5x + 1)$

50. $18x^3 + 9x^2 + 2x + 1 = 9x^2(2x + 1) + (2x + 1) = \left(9x^2 + 1\right)(2x + 1)$

51. $x^3 + x^2 + x + 1 = x^2(x + 1) + 1(x + 1) = (x + 1)\left(x^2 + 1\right)$

52. $x^5 + x^4 + x + 1 = x^4(x + 1) + 1(x + 1) = (x + 1)\left(x^4 + 1\right)$

53. Start by factoring out the power of x with the smallest exponent, that is, $x^{2/3}$.
Thus, $x^{2/3} + 3x^{5/3} = x^{2/3}(1 + 3x)$

54. Start by factoring out the power of x with the smallest exponent, that is, $x^{-1/4}$.
Thus, $x^{3/4} - 5x^{-1/4} = x^{-1/4}(x - 5)$

55. Start by factoring out the power of x with the smallest exponent, that is, $x^{-3/2}$.
Thus, $x^{-3/2} - x^{-1/2} + x^{1/2} = x^{-3/2}\left(1 - x + x^2\right) = x^{-3/2}\left(x^2 - x + 1\right)$

56. Start by factoring out the power of x with the smallest exponent, that is, $x^{-1/3}$.
Thus, $x^{5/3}+x^{2/3}+2x^{-1/3}=x^{-1/3}\left(x^2+x+2\right)$

57. Start by factoring out the power of x^2+1 with the smallest exponent, that is, $\left(x^2+1\right)^{-1/2}$.
Thus, $\left(x^2+1\right)^{1/2}+2\left(x^2+1\right)^{-1/2}=\left(x^2+1\right)^{-1/2}\left[\left(x^2+1\right)+2\right]=\left(x^2+1\right)^{-1/2}\left(x^2+3\right)=\dfrac{x^2+3}{\sqrt{x^2+1}}$.

58. $x^{-1/2}(x+1)^{1/2}+x^{1/2}(x+1)^{-1/2}=x^{-1/2}(x+1)^{-1/2}[(x+1)+x]=x^{-1/2}(x+1)^{-1/2}(2x+1)=\dfrac{2x+1}{\sqrt{x}\sqrt{x+1}}$

59. $2x+12x^3=2x\left(1+6x^2\right)$

60. $12x^2+3x^3=3x^2(4+x)$

61. $15x^3-10x^2=5x^2(3x-2)$

62. $6xyz-3xz=3xz(2y-1)$

63. $x^2-2x-8=(x-4)(x+2)$

64. $x^2-14x+48=(x-8)(x-6)$

65. $y^2+4y-21=(y+7)(y-3)$

66. $t^2-10t+24=(t-4)(t-6)$

67. $2x^2+5x+3=(2x+3)(x+1)$

68. $2x^2+7x-4=(2x-1)(x+4)$

69. $9x^2-36x-45=9\left(x^2-4x-5\right)=9(x-5)(x+1)$

70. $8x^2+10x+3=(4x+3)(2x+1)$

71. $12x^2-x-20=(4x+5)(3x-4)$

72. $15-26t+8t^2=(4t-3)(2t-5)$

73. $x^2-121=(x+11)(x-11)$

74. $4t^2-900=(2t+30)(2t-30)$

75. $49-4y^2=(7-2y)(7+2y)$

76. $4t^2-9s^2=(2t-3s)(2t+3s)$

77. $t^2-6t+9=(t-3)^2$

78. $x^2+10x+25=(x+5)^2$

79. $y^2-10yz+25z^2=(y-5z)^2$

80. $r^2-6rs+9s^2=(r-3s)^2$

81. $1-8t^3=(1-2t)\left(1+2t+4t^2\right)$

82. $27x^3+1000=(3x+10)\left(9x^2-30x+100\right)$

83. $8x^3-125=(2x)^3-5^3=(2x-5)\left[(2x)^2+(2x)(5)+5^2\right]=(2x-5)\left(4x^2+10x+25\right)$

84. $x^6+64=x^6+2^6=\left(x^2\right)^3+(4)^3=\left(x^2+4\right)\left[\left(x^2\right)^2-4\left(x^2\right)+(4)^2\right]=\left(x^2+4\right)\left(x^4-4x^2+16\right)$

85. $x^3+2x^2+x=x\left(x^2+2x+1\right)=x(x+1)^2$

86. $3x^3-27x=3x\left(x^2-9\right)=3x(x-3)(x+3)$

87. $x^6+x^5-42x^4=x^4\left(x^2+x-42\right)=x^4(x+7)(x-6)$

88. $3t^4-2t^3-5t^2=t^2\left(3t^2-2t-5\right)=t^2(t+1)(3t-5)$

89. $x^4y^3-x^2y^5=x^2y^3\left(x^2-y^2\right)=x^2y^3(x+y)(x-y)$

90. $18y^3x^2-2xy^4=2xy^3(9x-y)$

91. $27x^6-y^3=\left(3x^2\right)^3-y^3=\left(3x^2-y\right)\left(9x^4+3x^2y+y^2\right)$

92. $a^3+64b^6=\left(a+4b^2\right)\left(a^2-4ab^2+16b^4\right)$

93. $y^3-5y^2-9y+45=y\left(y^2-9\right)-5\left(y^2-9\right)=(y-5)(y+3)(y-3)$

94. $y^3+y^2-y-1=y^2(y+1)-(y+1)=\left(y^2-1\right)(y+1)=(y+1)(y-1)(y+1)=(y+1)^2(y-1)$

95. $3x^3 - x^2 - 12x + 4 = 3x\left(x^2 - 4\right) - \left(x^2 - 4\right) = (3x - 1)\left(x^2 - 4\right) = (3x - 1)(x + 2)(x - 2)$

96. $9x^3 + 18x^2 - x - 2 = 9x^2(x + 2) - (x + 2) = \left(9x^2 - 1\right)(x + 2) = (3x + 1)(3x - 1)(x + 2)$

97. $(a + b)^2 - (a - b)^2 = [(a + b) - (a - b)][(a + b) + (a - b)] = (2b)(2a) = 4ab$

98. $$\left(1 + \frac{1}{x}\right)^2 - \left(1 - \frac{1}{x}\right)^2 = \left[\left(1 + \frac{1}{x}\right) - \left(1 - \frac{1}{x}\right)\right]\left[\left(1 + \frac{1}{x}\right) + \left(1 - \frac{1}{x}\right)\right]$$
$$= \left(1 + \frac{1}{x} - 1 + \frac{1}{x}\right)\left(1 + \frac{1}{x} + 1 - \frac{1}{x}\right) = \left(\frac{2}{x}\right)(2) = \frac{4}{x}$$

99. $x^2\left(x^2 - 1\right) - 9\left(x^2 - 1\right) = \left(x^2 - 1\right)\left(x^2 - 9\right) = (x - 1)(x + 1)(x - 3)(x + 3)$

100. $$\left(a^2 - 1\right)(b - 2)^2 - 4\left(a^2 - 1\right) = \left(a^2 - 1\right)\left[(b - 2)^2 - 4\right] = (a + 1)(a - 1)[(b - 2) + 2][(b - 2) - 2]$$
$$= (a + 1)(a - 1)\,b\,(b - 4)$$

101. Start by factoring out the power of x with the smallest exponent, that is, $x^{-3/2}$. So
$$x^{-3/2} + 2x^{-1/2} + x^{1/2} = x^{-3/2}\left(1 + 2x + x^2\right) = x^{-3/2}(1 + x)^2 = \frac{(1 + x)^2}{x^{3/2}}.$$

102. $$(x - 1)^{7/2} - (x - 1)^{3/2} = (x - 1)^{3/2}\left[(x - 1)^2 - 1\right] = (x - 1)^{3/2}[(x - 1) - 1][(x - 1) + 1]$$
$$= (x - 1)^{3/2}(x - 2)(x)$$

103. $(x - 1)(x + 2)^2 - (x - 1)^2(x + 2) = (x - 1)(x + 2)[(x + 2) - (x - 1)] = 3(x - 1)(x + 2)$

104. $y^4(y + 2)^3 + y^5(y + 2)^4 = y^4(y + 2)^3[(1) + y(y + 2)] = y^4(y + 2)^3\left(y^2 + 2y + 1\right) = y^4(y + 2)^3(y + 1)^2$

105. Start by factoring $y^2 - 7y + 10$, and then substitute $a^2 + 1$ for y. This gives
$$\left(a^2 + 1\right)^2 - 7\left(a^2 + 1\right) + 10 = \left[\left(a^2 + 1\right) - 2\right]\left[\left(a^2 + 1\right) - 5\right] = \left(a^2 - 1\right)\left(a^2 - 4\right) = (a - 1)(a + 1)(a - 2)(a + 2)$$

106. $$\left(a^2 + 2a\right)^2 - 2\left(a^2 + 2a\right) - 3 = \left[\left(a^2 + 2a\right) - 3\right]\left[\left(a^2 + 2a\right) + 1\right] = \left(a^2 + 2a - 3\right)\left(a^2 + 2a + 1\right)$$
$$= (a - 1)(a + 3)(a + 1)^2$$

107. $$\left(x^2 + 3\right)^2(x - 1)^3 - (4x + 1)^2(x - 1)^3 = (x - 1)^3\left[\left(x^2 + 3\right)^2 - (4x + 1)^2\right]$$
$$= (x - 1)^3\left[\left(x^2 + 3\right) + (4x + 1)\right]\left[\left(x^2 + 3\right) - (4x + 1)\right] = (x - 1)^3\left(x^2 + 4x + 4\right)\left(x^2 - 4x + 2\right)$$
$$= (x - 1)^3(x + 2)^2\left(x^2 - 4x + 2\right)$$

108. $$(x + 1)^2(x + 2) - 6(x + 1)(x + 2) + 9(x + 2) = (x + 2)\left[(x + 1)^2 - 6(x + 1) + 9\right]$$
$$= (x + 2)[(x + 1) - 3]^2 = (x + 2)(x - 2)^2$$

109. $$5\left(x^2 + 4\right)^4(2x)(x - 2)^4 + \left(x^2 + 4\right)^5(4)(x - 2)^3 = 2\left(x^2 + 4\right)^4(x - 2)^3\left[(5)(x)(x - 2) + \left(x^2 + 4\right)(2)\right]$$
$$= 2\left(x^2 + 4\right)^4(x - 2)^3\left(5x^2 - 10x + 2x^2 + 8\right) = 2\left(x^2 + 4\right)^4(x - 2)^3\left(7x^2 - 10x + 8\right)$$

110. $$3(2x - 1)^2(2)(x + 3)^{1/2} + (2x - 1)^3\left(\tfrac{1}{2}\right)(x + 3)^{-1/2} = (2x - 1)^2(x + 3)^{-1/2}\left[6(x + 3) + (2x - 1)\left(\tfrac{1}{2}\right)\right]$$
$$= (2x - 1)^2(x + 3)^{-1/2}\left(6x + 18 + x - \tfrac{1}{2}\right) = (2x - 1)^2(x + 3)^{-1/2}\left(7x + \tfrac{35}{2}\right)$$

111. $$\left(x^2 + 3\right)^{-1/3} - \tfrac{2}{3}x^2\left(x^2 + 3\right)^{-4/3} = \left(x^2 + 3\right)^{-4/3}\left[\left(x^2 + 3\right) - \tfrac{2}{3}x^2\right] = \left(x^2 + 3\right)^{-4/3}\left(\tfrac{1}{3}x^2 + 3\right) = \frac{\frac{1}{3}x^2 + 3}{\left(x^2 + 3\right)^{4/3}}$$

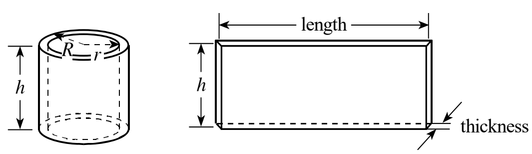
112. $$\tfrac{1}{2}x^{-1/2}(3x + 4)^{1/2} - \tfrac{3}{2}x^{1/2}(3x + 4)^{-1/2} = \tfrac{1}{2}x^{-1/2}(3x + 4)^{-1/2}[(3x + 4) - 3x] = \tfrac{1}{2}x^{-1/2}(3x + 4)^{-1/2}(4)$$
$$= 2x^{-1/2}(3x + 4)^{-1/2}$$

113. $$\begin{aligned} 4a^2c^2 - \left(c^2 - b^2 + a^2\right)^2 &= (2ac)^2 - \left(c^2 - b^2 + a^2\right) \\ &= \left[(2ac) - \left(c^2 - b^2 + a^2\right)\right]\left[(2ac) + \left(c^2 - b^2 + a^2\right)\right] \text{ (difference of squares)} \\ &= \left(2ac - c^2 + b^2 - a^2\right)\left(2ac + c^2 - b^2 + a^2\right) \\ &= \left[b^2 - \left(c^2 - 2ac + a^2\right)\right]\left[\left(c^2 + 2ac + a^2\right) - b^2\right] \text{ (regrouping)} \\ &= \left[b^2 - (c-a)^2\right]\left[(c+a)^2 - b^2\right] \text{ (perfect squares)} \\ &= [b - (c-a)][b + (c-a)][(c+a) - b][(c+a) + b] \text{ (each factor is a difference of squares)} \\ &= (b - c + a)(b + c - a)(c + a - b)(c + a + b) \\ &= (a + b - c)(-a + b + c)(a - b + c)(a + b + c) \end{aligned}$$

114. **(a)** Mowed portion $=$ field $-$ habitat

(b) Using the difference of squares, we get $b^2 - (b - 2x)^2 = [b - (b - 2x)][b + (b - x)] = 2x(2b - 2x) = 4x(b - x)$.

115. The volume of the shell is the difference between the volumes of the outside cylinder (with radius R) and the inside cylinder (with radius r). Thus $V = \pi R^2 h - \pi r^2 h = \pi\left(R^2 - r^2\right)h = \pi(R - r)(R + r)h = 2\pi \cdot \dfrac{R+r}{2} \cdot h \cdot (R - r)$. The average radius is $\dfrac{R+r}{2}$ and $2\pi \cdot \dfrac{R+r}{2}$ is the average circumference (length of the rectangular box), h is the height, and $R - r$ is the thickness of the rectangular box. Thus $V = \pi R^2 h - \pi r^2 h = 2\pi \cdot \dfrac{R+r}{2} \cdot h \cdot (R - r) = 2\pi \cdot \text{(average radius)} \cdot \text{(height)} \cdot \text{(thickness)}$



116. **(a)** $528^2 - 527^2 = (528 - 527)(528 + 527) = 1(1055) = 1055$

(b) $1020^2 - 1010^2 = (1020 - 1010)(1020 + 1010) = 10(2030) = 20{,}300$

(c) $501 \cdot 499 = (500 + 1)(500 - 1) = 500^2 - 1^2 = 250{,}000 - 1 = 249{,}999$

(d) $1002 \cdot 998 = (1000 + 2)(1000 - 2) = 1000^2 - 2^2 = 1{,}000{,}000 - 4 = 999{,}996$

117. **(a)**

$$\begin{array}{r} A + 1 \\ \times \quad A - 1 \\ \hline -A - 1 \\ A^2 + A \quad \\ \hline A^2 \quad\quad - 1 \end{array} \qquad \begin{array}{r} A^2 + A + 1 \\ \times \quad A - 1 \\ \hline -A^2 - A - 1 \\ A^3 + A^2 + A \quad \\ \hline A^3 \quad\quad\quad\quad - 1 \end{array} \qquad \begin{array}{r} A^3 + A^2 + A + 1 \\ \times \quad A - 1 \\ \hline -A^3 - A^2 - A - 1 \\ A^4 + A^3 + A^2 + A \quad \\ \hline A^4 \quad\quad\quad\quad\quad\quad - 1 \end{array}$$

(b) Based on the pattern in part (a), we suspect that $A^5 - 1 = (A - 1)\left(A^4 + A^3 + A^2 + A + 1\right)$. Check:

$$\begin{array}{r} A^4 + A^3 + A^2 + A + 1 \\ \times \quad A - 1 \\ \hline -A^4 - A^3 - A^2 - A - 1 \\ A^5 + A^4 + A^3 + A^2 + A \quad \\ \hline A^5 \quad\quad\quad\quad\quad\quad\quad\quad - 1 \end{array}$$

The general pattern is $A^n - 1 = (A - 1)\left(A^{n-1} + A^{n-2} + \cdots + A^2 + A + 1\right)$, where n is a positive integer.

118. **(a)** $(A-B)\left(A^2+AB+B^2\right)=A^3+A^2B+AB^2-A^2B-AB^2-B^3=A^3-B^3$

(b) $(A+B)\left(A^2-AB+B^2\right)=A^3-A^2B+AB^2+A^2B-AB^2+B^3=A^3+B^3$

P.7 RATIONAL EXPRESSIONS

1. A rational expression has the form $\dfrac{P(x)}{Q(x)}$, where P and Q are polynomials.

(a) $\dfrac{3x}{x^2-1}$ is a rational expression.

(b) $\dfrac{\sqrt{x+1}}{2x+3}$ is not a rational expression. A rational expression must be a polynomial divided by a polynomial, and the numerator of the expression is $\sqrt{x+1}$, which is not a polynomial.

(c) $\dfrac{x(x^2-1)}{x+3}=\dfrac{x^3-x}{x+3}$ is a rational expression.

2. To simplify a rational expression we cancel factors that are common to the *numerator* and *denominator*. So, the expression $\dfrac{(x+1)(x+2)}{(x+3)(x+2)}$ simplifies to $\dfrac{x+1}{x+3}$.

3. To multiply two rational expressions we multiply their *numerators* together and multiply their *denominators* together. So $\dfrac{2}{x+1}\cdot\dfrac{x}{x+3}$ is the same as $\dfrac{2\cdot x}{(x+1)\cdot(x+3)}=\dfrac{2x}{x^2+4x+3}$.

4. **(a)** $\dfrac{1}{x}-\dfrac{2}{(x+1)}-\dfrac{x}{(x+1)^2}$ has three terms.

(b) The least common denominator of all the terms is $x(x+1)^2$.

(c)
$$\frac{1}{x}-\frac{2}{(x+1)}-\frac{x}{(x+1)^2}=\frac{(x+1)^2}{x(x+1)^2}-\frac{2x(x+1)}{(x+1)}-\frac{x(x)}{(x+1)^2}=\frac{(x+1)^2-2x(x+1)-x^2}{x(x+1)^2}$$
$$=\frac{x^2+2x+1-2x^2-2x-x^2}{x(x+1)^2}=\frac{-2x^2+1}{x(x+1)^2}$$

5. **(a)** Yes. Cancelling $x+1$, we have $\dfrac{x(x+1)}{(x+1)^2}=\dfrac{x}{x+1}$.

(b) No; $(x+5)^2=x^2+10x+25\neq x^2+25$, so $x+5=\sqrt{x^2+10x+25}\neq\sqrt{x^2+25}$.

6. **(a)** Yes, $\dfrac{3+a}{3}=\dfrac{3}{3}+\dfrac{a}{3}=1+\dfrac{a}{3}$.

(b) No. We cannot "separate" the denominator in this way; only the numerator, as in part (a). (See also Exercise 101.)

7. The domain of $4x^2-10x+3$ is all real numbers.

8. The domain of $-x^4+x^3+9x$ is all real numbers.

9. Since $x-3\neq 0$ we have $x\neq 3$. Domain: $\{x\mid x\neq 3\}$

10. Since $3t+6\neq 0$ we have $t\neq -2$. Domain: $\{t\mid t\neq -2\}$

11. Since $x+3\geq 0$, $x\geq -3$. Domain; $\{x\mid x\geq -3\}$

12. Since $x-1>0$, $x>1$. Domain; $\{x\mid x>1\}$

13. $x^2-x-2=(x+1)(x-2)\neq 0\Leftrightarrow x\neq -1$ or 2, so the domain is $\{x\mid x\neq -1,2\}$.

14. $\dfrac{x}{x^2-4}=\dfrac{x}{(x+2)(x-2)}$. $(x+2)(x-2)\neq 0\Leftrightarrow x\neq\pm 2$, so the domain is $\{x\mid x\neq\pm 2\}$.

15. $\dfrac{\sqrt{x-2}}{x+3}$ is defined for $\{x\mid x\geq 2, x\neq -3\}$, so its domain is $\{x\mid x\geq 2\}$.

16. $\dfrac{\sqrt{x-2}}{x^2-9}$ is defined for $\{x\mid x\geq 2, x\neq\pm 3\}$, so its domain is $\{x\mid x\geq 2, x\neq 3\}$.

17. $\dfrac{(x-5)(x+5)}{2x-10}=\dfrac{(x-5)(x+5)}{2(x-5)}=\frac{1}{2}(x+5)$

18. $\dfrac{x^3-2x}{x^2+x}=\dfrac{x\left(x^2-2\right)}{x(x+1)}=\dfrac{x^2-2}{x+1}$

19. $\dfrac{x-2}{x^2-4}=\dfrac{x-2}{(x-2)(x+2)}=\dfrac{1}{x+2}$

20. $\dfrac{x^2-x-2}{x^2-1}=\dfrac{(x-2)(x+1)}{(x-1)(x+1)}=\dfrac{x-2}{x-1}$

21. $\dfrac{x^2-7x-8}{x^2-10x+16}=\dfrac{(x-8)(x+1)}{(x-8)(x-2)}=\dfrac{x+1}{x-2}$

22. $\dfrac{x^2-x-12}{x^2+5x+6}=\dfrac{(x-4)(x+3)}{(x+2)(x+3)}=\dfrac{x-4}{x+2}$

23. $\dfrac{y^2+y}{y^2-1}=\dfrac{y(y+1)}{(y-1)(y+1)}=\dfrac{y}{y-1}$

24. $\dfrac{y^3-5y^2-24y}{y^3-9y}=\dfrac{y\left(y^2-5y-24\right)}{y\left(y^2-9\right)}=\dfrac{y(y-8)(y+3)}{y(y+3)(y-3)}=\dfrac{y-8}{y-3}$

25. $\dfrac{2x^3-x^2-6x}{2x^2-7x+6}=\dfrac{x\left(2x^2-x-6\right)}{(2x-3)(x-2)}=\dfrac{x(2x+3)(x-2)}{(2x-3)(x-2)}=\dfrac{x(2x+3)}{2x-3}$

26. $\dfrac{1-x^2}{x^3-1}=\dfrac{(1-x)(1+x)}{(x-1)\left(x^2+x+1\right)}=\dfrac{-(x-1)(1+x)}{(x-1)\left(x^2+x+1\right)}=\dfrac{-(x+1)}{x^2+x+1}$

27. $\dfrac{4x}{x^2-4}\cdot\dfrac{x+2}{16x}=\dfrac{4x}{(x-2)(x+2)}\cdot\dfrac{x+2}{16x}=\dfrac{1}{4(x-2)}$

28. $\dfrac{x^2-25}{x^2-16}\cdot\dfrac{x+4}{x+5}=\dfrac{(x-5)(x+5)}{(x-4)(x+4)}\cdot\dfrac{x+4}{x+5}=\dfrac{x-5}{x-4}$

29. $\dfrac{x^2+2x-15}{x^2-25}\cdot\dfrac{x-5}{x+2}=\dfrac{(x+5)(x-3)(x-5)}{(x+5)(x-5)(x+2)}=\dfrac{x-3}{x+2}$

30. $\dfrac{x^2+2x-3}{x^2-2x-3}\cdot\dfrac{3-x}{3+x}=\dfrac{(x+3)(x-1)}{(x-3)(x+1)}\cdot\dfrac{-(x-3)}{x+3}=\dfrac{-(x-1)}{x+1}=\dfrac{-x+1}{x+1}=\dfrac{1-x}{1+x}$

31. $\dfrac{2t+3}{t^2+9}\cdot\dfrac{2t-3}{4t^2-9}=\dfrac{(2t+3)(2t-3)}{\left(t^2+9\right)(2t+3)(2t-3)}=\dfrac{1}{t^2+9}$

32. $\dfrac{3y^2+9y}{y^3-9y}\cdot\dfrac{y^2-9}{2y^2+3y-9}=\dfrac{3y(y+3)(y+3)(y-3)}{y\left(y^2-9\right)(y+3)(2y-3)}=\dfrac{3}{2y-3}$

33. $\dfrac{x^3-2x^2-8x}{x^2+8x+12}\cdot\dfrac{x^2+2x-24}{x^3-16x}=\dfrac{x(x+2)(x-4)(x-4)(x+6)}{(x+6)(x+2)x(x+4)(x-4)}=\dfrac{x-4}{x+4}$

34. $\dfrac{2x^2+xy-y^2}{x^2+xy-2y^2}\cdot\dfrac{x^2-2xy+y^2}{2x^2-3xy+y^2}=\dfrac{(2x-y)(x+y)(x-y)^2}{(x-y)(x+2y)(2x-y)(x-y)}=\dfrac{x+y}{x+2y}$

35. $\dfrac{x+3}{4x^2-9}\div\dfrac{x^2+7x+12}{2x^2+7x-15}=\dfrac{x+3}{4x^2-9}\cdot\dfrac{2x^2+7x-15}{x^2+7x+12}=\dfrac{x+3}{(2x-3)(2x+3)}\cdot\dfrac{(x+5)(2x-3)}{(x+3)(x+4)}=\dfrac{x+5}{(2x+3)(x+4)}$

36. $\dfrac{2x+1}{2x^2+x-15}\div\dfrac{6x^2-x-2}{x+3}=\dfrac{2x+1}{(x+3)(2x-5)}\cdot\dfrac{x+3}{(2x+1)(3x-2)}=\dfrac{1}{(2x-5)(3x-2)}$

37. $\dfrac{\dfrac{x^3}{x+1}}{\dfrac{x}{x^2+2x+1}}=\dfrac{x^3}{x+1}\cdot\dfrac{x^2+2x+1}{x}=\dfrac{x^3(x+1)(x+1)}{(x+1)x}=x^2(x+1)$

38. $\dfrac{\dfrac{2x^2-3x-2}{x^2-1}}{\dfrac{2x^2+5x+2}{x^2+x-2}}=\dfrac{2x^2-3x-2}{x^2-1}\cdot\dfrac{x^2+x-2}{2x^2+5x+2}=\dfrac{(x-2)(2x+1)}{(x-1)(x+1)}\cdot\dfrac{(x-1)(x+2)}{(x+2)(2x+1)}=\dfrac{x-2}{x+1}$

39. $\dfrac{x/y}{z} = \dfrac{x}{y} \cdot \dfrac{1}{z} = \dfrac{x}{yz}$

40. $\dfrac{x}{y/z} = x \div \dfrac{y}{z} = \dfrac{x}{1} \cdot \dfrac{z}{y} = \dfrac{xz}{y}$

41. $1 + \dfrac{1}{x+3} = \dfrac{x+3}{x+3} + \dfrac{1}{x+3} = \dfrac{x+4}{x+3}$

42. $\dfrac{3x-2}{x+1} - 2 = \dfrac{3x-2}{x+1} - \dfrac{2(x+1)}{x+1} = \dfrac{3x-2-2x-2}{x+1} = \dfrac{x-4}{x+1}$

43. $\dfrac{1}{x+5} + \dfrac{2}{x-3} = \dfrac{x-3}{(x+5)(x-3)} + \dfrac{2(x+5)}{(x+5)(x-3)} = \dfrac{x-3+2x+10}{(x+5)(x-3)} = \dfrac{3x+7}{(x+5)(x-3)}$

44. $\dfrac{1}{x+1} + \dfrac{1}{x-1} = \dfrac{x-1}{(x+1)(x-1)} + \dfrac{x+1}{(x+1)(x-1)} = \dfrac{x-1+x+1}{(x+1)(x-1)} = \dfrac{2x}{(x+1)(x-1)}$

45. $\dfrac{3}{x+1} - \dfrac{1}{x+2} = \dfrac{3(x+2)}{(x+1)(x+2)} - \dfrac{1(x+1)}{(x+1)(x+2)} = \dfrac{3x+6-x-1}{(x+1)(x+2)} = \dfrac{2x+5}{(x+1)(x+2)}$

46. $\dfrac{x}{x-4} - \dfrac{3}{x+6} = \dfrac{x(x+6)}{(x-4)(x+6)} + \dfrac{-3(x-4)}{(x-4)(x+6)} = \dfrac{x^2+6x-3x+12}{(x-4)(x+6)} = \dfrac{x^2+3x+12}{(x-4)(x+6)}$

47. $\dfrac{5}{2x-3} - \dfrac{3}{(2x-3)^2} = \dfrac{5(2x-3)}{(2x-3)^2} - \dfrac{3}{(2x-3)^2} = \dfrac{10x-15-3}{(2x-3)^2} = \dfrac{10x-18}{(2x-3)^2} = \dfrac{2(5x-9)}{(2x-3)^2}$

48. $\dfrac{x}{(x+1)^2} + \dfrac{2}{x+1} = \dfrac{x}{(x+1)^2} + \dfrac{2(x+1)}{(x+1)(x+1)} = \dfrac{x+2x+2}{(x+1)^2} = \dfrac{3x+2}{(x+1)^2}$

49. $u + 1 + \dfrac{u}{u+1} = \dfrac{(u+1)(u+1)}{u+1} + \dfrac{u}{u+1} = \dfrac{u^2+2u+1+u}{u+1} = \dfrac{u^2+3u+1}{u+1}$

50. $\dfrac{2}{a^2} - \dfrac{3}{ab} + \dfrac{4}{b^2} = \dfrac{2b^2}{a^2b^2} - \dfrac{3ab}{a^2b^2} + \dfrac{4a^2}{a^2b^2} = \dfrac{2b^2-3ab+4a^2}{a^2b^2}$

51. $\dfrac{1}{x^2} + \dfrac{1}{x^2+x} = \dfrac{1}{x^2} + \dfrac{1}{x(x+1)} = \dfrac{x+1}{x^2(x+1)} + \dfrac{x}{x^2(x+1)} = \dfrac{2x+1}{x^2(x+1)}$

52. $\dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3} = \dfrac{x^2}{x^3} + \dfrac{x}{x^3} + \dfrac{1}{x^3} = \dfrac{x^2+x+1}{x^3}$

53. $$\begin{aligned}\dfrac{2}{x+3} - \dfrac{1}{x^2+7x+12} &= \dfrac{2}{x+3} - \dfrac{1}{(x+3)(x+4)} = \dfrac{2(x+4)}{(x+3)(x+4)} + \dfrac{-1}{(x+3)(x+4)} \\ &= \dfrac{2x+8-1}{(x+3)(x+4)} = \dfrac{2x+7}{(x+3)(x+4)}\end{aligned}$$

54. $$\begin{aligned}\dfrac{x}{x^2-4} + \dfrac{1}{x-2} &= \dfrac{x}{(x-2)(x+2)} + \dfrac{1}{x-2} = \dfrac{x}{(x-2)(x+2)} + \dfrac{x+2}{(x-2)(x+2)} \\ &= \dfrac{2x+2}{(x-2)(x+2)} = \dfrac{2(x+1)}{(x-2)(x+2)}\end{aligned}$$

55. $\dfrac{1}{x+3} + \dfrac{1}{x^2-9} = \dfrac{1}{x+3} + \dfrac{1}{(x-3)(x+3)} = \dfrac{x-3}{(x-3)(x+3)} + \dfrac{1}{(x-3)(x+3)} = \dfrac{x-2}{(x-3)(x+3)}$

56. $$\begin{aligned}\dfrac{x}{x^2+x-2} - \dfrac{2}{x^2-5x+4} &= \dfrac{x}{(x-1)(x+2)} + \dfrac{-2}{(x-1)(x-4)} \\ &= \dfrac{x(x-4)}{(x-1)(x+2)(x-4)} + \dfrac{-2(x+2)}{(x-1)(x+2)(x-4)} = \dfrac{x^2-4x-2x-4}{(x-1)(x+2)(x-4)} = \dfrac{x^2-6x-4}{(x-1)(x+2)(x-4)}\end{aligned}$$

57. $\dfrac{2}{x} + \dfrac{3}{x-1} - \dfrac{4}{x^2-x} = \dfrac{2}{x} + \dfrac{3}{x-1} - \dfrac{4}{x(x-1)} = \dfrac{2(x-1)}{x(x-1)} + \dfrac{3x}{x(x-1)} + \dfrac{-4}{x(x-1)} = \dfrac{2x-2+3x-4}{x(x-1)} = \dfrac{5x-6}{x(x-1)}$

58. $\dfrac{1}{x^3y^3z} + \dfrac{1}{xy^3z^3} + \dfrac{1}{x^3yz^3} = \dfrac{z^2}{x^3y^3z^3} + \dfrac{x^2}{x^3y^3z^3} + \dfrac{y^2}{x^3y^3z^3} = \dfrac{x^2+y^2+z^2}{x^3y^3z^3}$

59. $$\frac{1}{x^2(x+1)}+\frac{1}{x^2(x+1)^2}+\frac{1}{x^3(x+1)^2}=\frac{x(x+1)}{x^3(x+1)^2}+\frac{x}{x^3(x+1)^2}+\frac{1}{x^3(x+1)^2}=\frac{x^2+2x+1}{x^3(x+1)^2}$$
$$=\frac{(x+1)^2}{x^3(x+1)^2}=\frac{1}{x^3}$$

60. $$\frac{1}{x+1}-\frac{2}{(x+1)^2}+\frac{3}{x^2-1}=\frac{1}{x+1}+\frac{-2}{(x+1)^2}+\frac{3}{(x-1)(x+1)}$$
$$=\frac{(x+1)(x-1)}{(x-1)(x+1)^2}+\frac{-2(x-1)}{(x-1)(x+1)^2}+\frac{3(x+1)}{(x-1)(x+1)^2}$$
$$=\frac{x^2-1}{(x-1)(x+1)^2}+\frac{-2x+2}{(x-1)(x+1)^2}+\frac{3x+3}{(x-1)(x+1)^2}=\frac{x^2-1-2x+2+3x+3}{(x-1)(x+1)^2}=\frac{x^2+x+4}{(x-1)(x+1)^2}$$

61. $$\frac{1+\frac{1}{x}}{\frac{1}{x}-2}=\frac{x\left(1+\frac{1}{x}\right)}{x\left(\frac{1}{x}-2\right)}=\frac{x+1}{1-2x}$$

62. $$\frac{1-\frac{2}{y}}{\frac{3}{y}-1}=\frac{y\left(1-\frac{2}{y}\right)}{y\left(\frac{3}{y}-1\right)}=\frac{y-2}{3-y}$$

63. $$\frac{1+\dfrac{1}{x+2}}{1-\dfrac{1}{x+2}}=\frac{(x+2)\left(1+\dfrac{1}{x+2}\right)}{(x+2)\left(1-\dfrac{1}{x+2}\right)}=\frac{(x+2)+1}{(x+2)-1}=\frac{x+3}{x+1}$$

64. $$\frac{1+\dfrac{1}{c-1}}{1-\dfrac{1}{c-1}}=\frac{c-1+1}{c-1-1}=\frac{c}{c-2}$$

65. $$\frac{\dfrac{1}{x-1}+\dfrac{1}{x+3}}{x+1}=\frac{(x-1)(x+3)\left(\dfrac{1}{x-1}+\dfrac{1}{x+3}\right)}{(x-1)(x+3)(x+1)}=\frac{(x+3)+(x-1)}{(x-1)(x+1)(x+3)}=\frac{2(x+1)}{(x-1)(x+1)(x+3)}$$
$$=\frac{2}{(x-1)(x+3)}$$

66. $$\frac{\dfrac{x-3}{x-4}-\dfrac{x+2}{x+1}}{x+3}=\frac{(x-3)(x+1)-(x+2)(x-4)}{(x-4)(x+3)(x+1)}=\frac{x^2-2x-3-\left(x^2-2x-8\right)}{(x-4)(x+3)(x+1)}=\frac{5}{(x-4)(x+3)(x+1)}$$

67. $$\frac{x-\dfrac{x}{y}}{y-\dfrac{y}{x}}=\frac{xy\left(x-\dfrac{x}{y}\right)}{xy\left(y-\dfrac{y}{x}\right)}=\frac{x^2y-x^2}{xy^2-y^2}=\frac{x^2(y-1)}{y^2(x-1)}$$

68. $$\frac{x+\dfrac{y}{x}}{y+\dfrac{x}{y}}=\frac{xy\left(x+\dfrac{y}{x}\right)}{xy\left(y+\dfrac{x}{y}\right)}=\frac{x^2y+y^2}{xy^2+x^2}=\frac{y\left(y+x^2\right)}{x\left(x+y^2\right)}$$

69. $\dfrac{\dfrac{x}{y}-\dfrac{y}{x}}{\dfrac{1}{x^2}-\dfrac{1}{y^2}} = \dfrac{\dfrac{x^2-y^2}{xy}}{\dfrac{y^2-x^2}{x^2y^2}} = \dfrac{x^2-y^2}{xy}\cdot\dfrac{x^2y^2}{y^2-x^2} = \dfrac{xy}{-1} = -xy$. An alternative method is to multiply the numerator and denominator by the common denominator of both the numerator and denominator, in this case x^2y^2:

$$\frac{\frac{x}{y}-\frac{y}{x}}{\frac{1}{x^2}-\frac{1}{y^2}} = \frac{\left(\frac{x}{y}-\frac{y}{x}\right)}{\left(\frac{1}{x^2}-\frac{1}{y^2}\right)}\cdot\frac{x^2y^2}{x^2y^2} = \frac{x^3y-xy^3}{y^2-x^2} = \frac{xy\left(x^2-y^2\right)}{y^2-x^2} = -xy.$$

70. $x-\dfrac{y}{\dfrac{x}{y}+\dfrac{y}{x}} = x-\dfrac{y}{\dfrac{x}{y}+\dfrac{y}{x}}\cdot\dfrac{xy}{xy} = x-\dfrac{xy^2}{x^2+y^2} = \dfrac{x\left(x^2+y^2\right)}{x^2+y^2}-\dfrac{xy^2}{x^2+y^2} = \dfrac{x^3+xy^2-xy^2}{x^2+y^2} = \dfrac{x^3}{x^2+y^2}$

71. $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} = \dfrac{\dfrac{1}{x^2}-\dfrac{1}{y^2}}{\dfrac{1}{x}+\dfrac{1}{y}} = \dfrac{\dfrac{y^2}{x^2y^2}-\dfrac{x^2}{x^2y^2}}{\dfrac{y}{xy}+\dfrac{x}{xy}} = \dfrac{y^2-x^2}{x^2y^2}\cdot\dfrac{xy}{y+x} = \dfrac{(y-x)(y+x)xy}{x^2y^2(y+x)} = \dfrac{y-x}{xy}$

Alternatively, $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} = \dfrac{\left(\dfrac{1}{x^2}-\dfrac{1}{y^2}\right)}{\left(\dfrac{1}{x}+\dfrac{1}{y}\right)}\cdot\dfrac{x^2y^2}{x^2y^2} = \dfrac{y^2-x^2}{xy^2+x^2y} = \dfrac{(y-x)(y+x)}{xy(y+x)} = \dfrac{y-x}{xy}$.

72. $\dfrac{1}{1+u}+\dfrac{1}{1+\dfrac{1}{u}} = \dfrac{1}{1+u}+\dfrac{u}{u+1} = 1$

73. $1-\dfrac{1}{1-\dfrac{1}{x}} = 1-\dfrac{x}{x-1} = \dfrac{x-1-x}{x-1} = \dfrac{1}{1-x}$

74. $1+\dfrac{1}{1+\dfrac{1}{1+x}} = 1+\dfrac{1+x}{(1+x)+1} = 1+\dfrac{x+1}{x+2} = \dfrac{x+2+x+1}{x+2} = \dfrac{2x+3}{x+2}$

75. $\dfrac{\dfrac{1}{1+x+h}-\dfrac{1}{1+x}}{h} = \dfrac{(1+x)-(1+x+h)}{h(1+x)(1+x+h)} = -\dfrac{1}{(1+x)(1+x+h)}$

76. In calculus it is necessary to eliminate the h in the denominator, and we do this by rationalizing the numerator:

$$\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h} = \frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}} = \frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)} = -\frac{1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}.$$

77. $\dfrac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h} = \dfrac{x^2-(x+h)^2}{hx^2(x+h)^2} = \dfrac{x^2-\left(x^2+2xh+h^2\right)}{hx^2(x+h)^2} = -\dfrac{2x+h}{x^2(x+h)^2}$

78. $\dfrac{(x+h)^2+3(x+h)-x^2-3x}{h} = \dfrac{x^2+h^2+2xh+3x+3h-x^2-3x}{h} = \dfrac{h(h+2x+3)}{h} = 2x+h+3$

79. $\sqrt{1+\left(\dfrac{x}{\sqrt{1-x^2}}\right)^2} = \sqrt{1+\dfrac{x^2}{1-x^2}} = \sqrt{\dfrac{1-x^2}{1-x^2}+\dfrac{x^2}{1-x^2}} = \sqrt{\dfrac{1}{1-x^2}} = \dfrac{1}{\sqrt{1-x^2}}$

80. $\sqrt{1+\left(x^3-\dfrac{1}{4x^3}\right)^2} = \sqrt{1+x^6-\dfrac{2x^3}{4x^3}+\dfrac{1}{16x^6}} = \sqrt{1+x^6-\frac{1}{2}+\dfrac{1}{16x^6}} = \sqrt{x^6+\frac{1}{2}+\dfrac{1}{16x^6}}$

$= \sqrt{\left(x^3+\dfrac{1}{4x^3}\right)^2} = \left|x^3+\dfrac{1}{4x^3}\right|$

81. $\dfrac{2(x-3)(x+5)^3-3(x+5)^2(x-3)^2}{(x+5)^6}=\dfrac{(x-3)(x+5)^2[2(x+5)-3(x-3)]}{(x+5)^6}=\dfrac{(x-3)(19-x)}{(x+5)^4}$

82. $\dfrac{4x^3(1-x)^3-3(1-x)^2(-1)\left(x^4\right)}{(1-x)^6}=\dfrac{x^3(1-x)^2[4(1-x)+3x]}{(1-x)^6}=\dfrac{x^3(4-x)}{(1-x)^4}$

83. $\dfrac{2(1+x)^{1/2}-x(1+x)^{-1/2}}{1+x}=\dfrac{(1+x)^{-1/2}[2(1+x)-x]}{1+x}=\dfrac{x+2}{(1+x)^{3/2}}$

84. $\dfrac{\left(1-x^2\right)^{1/2}+x^2\left(1-x^2\right)^{-1/2}}{1-x^2}=\dfrac{\left(1-x^2\right)^{-1/2}\left(1-x^2+x^2\right)}{1-x^2}=\dfrac{1}{\left(1-x^2\right)^{3/2}}$

85. $\dfrac{3(1+x)^{1/3}-x(1+x)^{-2/3}}{(1+x)^{2/3}}=\dfrac{(1+x)^{-2/3}[3(1+x)-x]}{(1+x)^{2/3}}=\dfrac{2x+3}{(1+x)^{4/3}}$

86. $\dfrac{(7-3x)^{1/2}+\frac{3}{2}x(7-3x)^{-1/2}}{7-3x}=\dfrac{(7-3x)^{-1/2}\left(7-3x+\frac{3}{2}x\right)}{7-3x}=\dfrac{7-\frac{3}{2}x}{(7-3x)^{3/2}}$

87. $\dfrac{1}{3+\sqrt{10}}=\dfrac{3-\sqrt{10}}{\left(3+\sqrt{10}\right)\left(3-\sqrt{10}\right)}=\dfrac{3-\sqrt{10}}{9-10}=\sqrt{10}-3$

88. $\dfrac{3}{2-\sqrt{5}}=\dfrac{\left(2+\sqrt{5}\right)3}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}=\dfrac{6+3\sqrt{5}}{4-5}=-6-3\sqrt{5}$

89. $\dfrac{2}{\sqrt{5}-\sqrt{3}}=\dfrac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}=\dfrac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}=\sqrt{5}+\sqrt{3}$

90. $\dfrac{1}{\sqrt{x}+1}=\dfrac{1}{\sqrt{x}+1}\cdot\dfrac{\sqrt{x}-1}{\sqrt{x}-1}=\dfrac{\sqrt{x}-1}{x-1}$

91. $\dfrac{y}{\sqrt{3}+\sqrt{y}}=\dfrac{y}{\sqrt{3}+\sqrt{y}}\cdot\dfrac{\sqrt{3}-\sqrt{y}}{\sqrt{3}-\sqrt{y}}=\dfrac{y\left(\sqrt{3}-\sqrt{y}\right)}{3-y}=\dfrac{y\sqrt{3}-y\sqrt{y}}{3-y}$

92. $\dfrac{2(x-y)}{\sqrt{x}-\sqrt{y}}=\dfrac{2(x-y)}{\sqrt{x}-\sqrt{y}}\cdot\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\dfrac{2(x-y)\left(\sqrt{x}+\sqrt{y}\right)}{x-y}=2\left(\sqrt{x}+\sqrt{y}\right)=2\sqrt{x}+2\sqrt{y}$

93. $\dfrac{2-\sqrt{5}}{5}=\dfrac{\left(2-\sqrt{5}\right)\left(2+\sqrt{5}\right)}{5\left(2+\sqrt{5}\right)}=\dfrac{4-5}{5\left(2+\sqrt{5}\right)}=\dfrac{-1}{5\left(2+\sqrt{5}\right)}$

94. $\dfrac{\sqrt{3}+\sqrt{5}}{2}=\dfrac{\sqrt{3}+\sqrt{5}}{2}\cdot\dfrac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}=\dfrac{3-5}{2\left(\sqrt{3}-\sqrt{5}\right)}=\dfrac{-2}{2\left(\sqrt{3}-\sqrt{5}\right)}=\dfrac{-1}{\sqrt{3}-\sqrt{5}}$

95. $\dfrac{\sqrt{r}+\sqrt{2}}{5}=\dfrac{\sqrt{r}+\sqrt{2}}{5}\cdot\dfrac{\sqrt{r}-\sqrt{2}}{\sqrt{r}-\sqrt{2}}=\dfrac{r-2}{5\left(\sqrt{r}-\sqrt{2}\right)}$

96. $\dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}=\dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\dfrac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}=\dfrac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$

$$=\dfrac{-h}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}=\dfrac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

97. $\sqrt{x^2+1}-x=\dfrac{\sqrt{x^2+1}-x}{1}\cdot\dfrac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}=\dfrac{x^2+1-x^2}{\sqrt{x^2+1}+x}=\dfrac{1}{\sqrt{x^2+1}+x}$

98. $\sqrt{x+1}-\sqrt{x}=\dfrac{\sqrt{x+1}-\sqrt{x}}{1}\cdot\dfrac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}=\dfrac{x+1-x}{\sqrt{x+1}+\sqrt{x}}=\dfrac{1}{\sqrt{x+1}+\sqrt{x}}$

99. **(a)** $R = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} \cdot \dfrac{R_1 R_2}{R_1 R_2} = \dfrac{R_1 R_2}{R_2 + R_1}$

(b) Substituting $R_1 = 10$ ohms and $R_2 = 20$ ohms gives $R = \dfrac{(10)(20)}{(20) + (10)} = \dfrac{200}{30} \approx 6.7$ ohms.

100. **(a)** The average cost $A = \dfrac{\text{Cost}}{\text{number of shirts}} = \dfrac{500 + 6x + 0.01x^2}{x}$.

(b)

x	10	20	50	100	200	500	1000
Average cost	\$56.10	\$31.20	\$16.50	\$12.00	\$10.50	\$12.00	\$16.50

101.

x	2.80	2.90	2.95	2.99	2.999	3	3.001	3.01	3.05	3.10	3.20
$\dfrac{x^2-9}{x-3}$	5.80	5.90	5.95	5.99	5.999	?	6.001	6.01	6.05	6.10	6.20

From the table, we see that the expression $\dfrac{x^2-9}{x-3}$ approaches 6 as x approaches 3. We simplify the expression: $\dfrac{x^2-9}{x-3} = \dfrac{(x-3)(x+3)}{x-3} = x + 3$, $x \neq 3$. Clearly as x approaches 3, $x + 3$ approaches 6. This explains the result in the table.

102. No, squaring $\dfrac{2}{\sqrt{x}}$ changes its value by a factor of $\dfrac{2}{\sqrt{x}}$.

103. Answers will vary.

Algebraic Error	Counterexample
$\dfrac{1}{a} + \dfrac{1}{b} \neq \dfrac{1}{a+b}$	$\dfrac{1}{2} + \dfrac{1}{2} \neq \dfrac{1}{2+2}$
$(a+b)^2 \neq a^2 + b^2$	$(1+3)^2 \neq 1^2 + 3^2$
$\sqrt{a^2+b^2} \neq a + b$	$\sqrt{5^2+12^2} \neq 5 + 12$
$\dfrac{a+b}{a} \neq b$	$\dfrac{2+6}{2} \neq 6$
$\dfrac{a}{a+b} \neq \dfrac{1}{b}$	$\dfrac{1}{1+1} \neq 1$
$\dfrac{a^m}{a^n} \neq a^{m/n}$	$\dfrac{3^5}{3^2} \neq 3^{5/2}$

104. **(a)** $\dfrac{5+a}{5} = \dfrac{5}{5} + \dfrac{a}{5} = 1 + \dfrac{a}{5}$, so the statement is true.

(b) This statement is false. For example, take $x = 5$ and $y = 2$. Then LHS $= \dfrac{x+1}{y+1} = \dfrac{5+1}{2+1} = \dfrac{6}{3} = 2$, while RHS $= \dfrac{x}{y} = \dfrac{5}{2}$, and $2 \neq \dfrac{5}{2}$.

(c) This statement is false. For example, take $x = 0$ and $y = 1$. Then LHS $= \dfrac{x}{x+y} = \dfrac{0}{0+1} = 0$, while RHS $= \dfrac{1}{1+y} = \dfrac{1}{1+1} = \dfrac{1}{2}$, and $0 \neq \dfrac{1}{2}$.

(d) This statement is false. For example, take $x = 1$ and $y = 1$. Then LHS $= 2\left(\dfrac{a}{b}\right) = 2\left(\dfrac{1}{1}\right) = 2$, while RHS $= \dfrac{2a}{2b} = \dfrac{2}{2} = 1$, and $2 \neq 1$.

(e) This statement is true: $\dfrac{-a}{b} = (-a)\left(\dfrac{1}{b}\right) = (-1)(a)\left(\dfrac{1}{b}\right) = (-1)\left(\dfrac{a}{b}\right) = -\dfrac{a}{b}$.

(f) This statement is true: $\dfrac{1+x+x^2}{x} = \dfrac{1}{x} + \dfrac{x}{x} + \dfrac{x^2}{x} = \dfrac{1}{x} + 1 + x$.

105. (a)

x	1	3	$\frac{1}{2}$	$\frac{9}{10}$	$\frac{99}{100}$	$\frac{999}{1000}$	$\frac{9999}{10{,}000}$
$x+\frac{1}{x}$	2	3.333	2.5	2.011	2.0001	2.000001	2.00000001

It appears that the smallest possible value of $x + \dfrac{1}{x}$ is 2.

(b) Because $x > 0$, we can multiply both sides by x and preserve the inequality: $x + \dfrac{1}{x} \geq 2 \Leftrightarrow x\left(x + \dfrac{1}{x}\right) \geq 2x \Leftrightarrow x^2 + 1 \geq 2x \Leftrightarrow x^2 - 2x + 1 \geq 0 \Leftrightarrow (x-1)^2 \geq 0$. The last statement is true for all $x > 0$, and because each step is reversible, we have shown that $x + \dfrac{1}{x} \geq 2$ for all $x > 0$.

P.8 SOLVING BASIC EQUATIONS

1. Substituting $x = 3$ in the equation $4x - 2 = 10$ makes the equation true, so the number 3 is a *solution* of the equation.

2. Subtracting 4 from both sides of the given equation, $3x + 4 = 10$, we obtain $3x + 4 - 4 = 10 - 4 \Leftrightarrow 3x = 6$. Multiplying by $\frac{1}{3}$, we have $\frac{1}{3}(3x) = \frac{1}{3}(6) \Leftrightarrow x = 2$, so the solution is $x = 2$.

3. (a) $\dfrac{x}{2} + 2x = 10$ is equivalent to $\frac{5}{2}x - 10 = 0$, so it is a linear equation.

(b) $\dfrac{2}{x} - 2x = 1$ is not linear because it contains the term $\dfrac{2}{x}$, a multiple of the reciprocal of the variable.

(c) $x + 7 = 5 - 3x \Leftrightarrow 4x - 2 = 0$, so it is linear.

4. (a) $x(x+1) = 6 \Leftrightarrow x^2 + x = 6$ is not linear because it contains the square of the variable.

(b) $\sqrt{x+2} = x$ is not linear because it contains the square root of $x + 2$.

(c) $3x^2 - 2x - 1 = 0$ is not linear because it contains a multiple of the square of the variable.

5. (a) This is true: If $a = b$, then $a + x = b + x$.

(b) This is false, because the number could be zero. However, it is true that multiplying each side of an equation by a *nonzero* number always gives an equivalent equation.

(c) This is false. For example, $-5 = 5$ is false, but $(-5)^2 = 5^2$ is true.

6. To solve the equation $x^3 = 125$ we take the *cube* root of each side. So the solution is $x = \sqrt[3]{125} = 5$.

7. (a) When $x = -2$, LHS $= 4(-2) + 7 = -8 + 7 = -1$ and RHS $= 9(-2) - 3 = -18 - 3 = -21$. Since LHS $\neq$ RHS, $x = -2$ is not a solution.

(b) When $x = 2$, LHS $= 4(2) + 7 = 8 + 7 = 15$ and RHS $= 9(2) - 3 = 18 - 3 = 15$. Since LHS $=$ RHS, $x = 2$ is a solution.

8. (a) When $x = -1$, LHS $= 2 - 5(-1) = 2 + 5 = 7$ and RHS $= 8 + (-1) = 7$. Since LHS $=$ RHS, $x = -1$ is a solution.

(b) When $x = 1$, LHS $= 2 - 5(1) = 2 - 5 = -3$ and RHS $= 8 + (1) = 9$. Since LHS $\neq$ RHS, $x = 1$ is not a solution.

9. (a) When $x = 2$, LHS $= 1 - [2 - (3 - (2))] = 1 - [2 - 1] = 1 - 1 = 0$ and RHS $= 4(2) - (6 + (2)) = 8 - 8 = 0$. Since LHS $=$ RHS, $x = 2$ is a solution.

(b) When $x = 4$ LHS $= 1 - [2 - (3 - (4))] = 1 - [2 - (-1)] = 1 - 3 = -2$ and RHS $= 4(4) - (6 + (4)) = 16 - 10 = 6$. Since LHS $\neq$ RHS, $x = 4$ is not a solution.

10. **(a)** When $x = 2$, LHS $= \frac{1}{2} - \dfrac{1}{2-4} = \frac{1}{2} - \dfrac{1}{-2} = \frac{1}{2} + \frac{1}{2} = 1$ and RHS $= 1$. Since LHS $=$ RHS, $x = 2$ is a solution.

(b) When $x = 4$ the expression $\dfrac{1}{4-4}$ is not defined, so $x = 4$ is not a solution.

11. **(a)** When $x = -1$, LHS $= 2(-1)^{1/3} - 3 = 2(-1) - 3 = -2 - 3 = -5$. Since LHS $\neq 1$, $x = -1$ is not a solution.

(b) When $x = 8$ LHS $= 2(8)^{1/3} - 3 = 2(2) - 3 = 4 - 3 = 1 =$ RHS. So $x = 8$ is a solution.

12. **(a)** When $x = 4$, LHS $= \dfrac{4^{3/2}}{4-6} = \dfrac{2^3}{-2} = \dfrac{8}{-2} = -4$ and RHS $= (4) - 8 = -4$. Since LHS $=$ RHS, $x = 4$ is a solution.

(b) When $x = 8$, LHS $= \dfrac{8^{3/2}}{8-6} = \dfrac{\left(2^3\right)^{3/2}}{2} = \dfrac{2^{9/2}}{2} = 2^{7/2}$ and RHS $= (8) - 8 = 0$. Since LHS $\neq$ RHS, $x = 8$ is not a solution.

13. **(a)** When $x = 0$, LHS $= \dfrac{0-a}{0-b} = \dfrac{-a}{-b} = \dfrac{a}{b} =$ RHS. So $x = 0$ is a solution.

(b) When $x = b$, LHS $= \dfrac{b-a}{b-b} = \dfrac{b-a}{0}$ is not defined, so $x = b$ is not a solution.

14. **(a)** When $x = \dfrac{b}{2}$, LHS $= \left(\dfrac{b}{2}\right)^2 - b\left(\dfrac{b}{2}\right) + \frac{1}{4}b^2 = \dfrac{b^2}{4} - \dfrac{b^2}{2} + \dfrac{b^2}{4} = 0 =$ RHS. So $x = \dfrac{b}{2}$ is a solution.

(b) When $x = \dfrac{1}{b}$, LHS $= \left(\dfrac{1}{b}\right)^2 - b\left(\dfrac{1}{b}\right) + \frac{1}{4}b^2 = \dfrac{1}{b^2} - 1 + \dfrac{b^2}{4}$, so $x = \dfrac{1}{b}$ is not a solution.

15. $5x - 6 = 14 \Leftrightarrow 5x = 20 \Leftrightarrow x = 4$

16. $3x + 4 = 7 \Leftrightarrow 3x = 3 \Leftrightarrow x = 1$

17. $7 - 2x = 15 \Leftrightarrow 2x = -8 \Leftrightarrow x = -4$

18. $4x - 95 = 1 \Leftrightarrow 4x = 96 \Leftrightarrow x = 24$

19. $\frac{1}{2}x + 7 = 3 \Leftrightarrow \frac{1}{2}x = -4 \Leftrightarrow x = -8$

20. $2 + \frac{1}{3}x = -4 \Leftrightarrow \frac{1}{3}x = -6 \Leftrightarrow x = -18$

21. $-3x - 3 = 5x - 3 \Leftrightarrow 0 = 8x \Leftrightarrow x = 0$

22. $2x + 3 = 5 - 2x \Leftrightarrow 4x = 2 \Leftrightarrow x = \frac{1}{2}$

23. $7x + 1 = 4 - 2x \Leftrightarrow 9x = 3 \Leftrightarrow x = \frac{1}{3}$

24. $1 - x = x + 4 \Leftrightarrow -3 = 2x \Leftrightarrow x = -\frac{3}{2}$

25. $-x + 3 = 4x \Leftrightarrow 3 = 5x \Leftrightarrow x = \frac{3}{5}$

26. $2x + 3 = 7 - 3x \Leftrightarrow 5x = 4 \Leftrightarrow x = \frac{4}{5}$

27. $\frac{x}{3} - 1 = \frac{5}{3}x + 7 \Leftrightarrow x - 3 = 5x + 21 \Leftrightarrow 4x = -24 \Leftrightarrow x = -6$

28. $\frac{2}{5}x - 1 = \frac{3}{10}x + 3 \Leftrightarrow 4x - 10 = 3x + 30 \Leftrightarrow x = 40$

29. $2(1-x) = 3(1+2x) + 5 \Leftrightarrow 2 - 2x = 3 + 6x + 5 \Leftrightarrow 2 - 2x = 8 + 6x \Leftrightarrow -6 = 8x \Leftrightarrow x = -\frac{3}{4}$

30. $5(x+3) + 9 = -2(x-2) - 1 \Leftrightarrow 5x + 15 + 9 = -2x + 4 - 1 \Leftrightarrow 5x + 24 = -2x + 3 \Leftrightarrow 7x = -21 \Leftrightarrow x = -3$

31. $4\left(y - \frac{1}{2}\right) - y = 6(5-y) \Leftrightarrow 4y - 2 - y = 30 - 6y \Leftrightarrow 3y - 2 = 30 - 6y \Leftrightarrow 9y = 32 \Leftrightarrow y = \frac{32}{9}$

32. $r - 2[1 - 3(2r+4)] = 61 \Leftrightarrow r - 2(1 - 6r - 12) = 61 \Leftrightarrow r - 2(-6r - 11) = 61 \Leftrightarrow r + 12r + 22 = 61 \Leftrightarrow 13r = 39 \Leftrightarrow r = 3$

33. $x - \frac{1}{3}x - \frac{1}{2}x - 5 = 0 \Leftrightarrow 6x - 2x - 3x - 30 = 0$ (multiply both sides by 6) $\Leftrightarrow x = 30$

34. $\frac{2}{3}y + \frac{1}{2}(y-3) = \dfrac{y+1}{4} \Leftrightarrow 8y + 6(y-3) = 3(y+1) \Leftrightarrow 8y + 6y - 18 = 3y + 3 \Leftrightarrow 14y - 18 = 3y + 3 \Leftrightarrow 11y = 21 \Leftrightarrow y = \frac{21}{11}$

35. $2x - \dfrac{x}{2} + \dfrac{x+1}{4} = 6x \Leftrightarrow 8x - 2x + x + 1 = 24x \Leftrightarrow 7x + 1 = 24x \Leftrightarrow 1 = 17x \Leftrightarrow x = \frac{1}{17}$

36. $3x - \dfrac{5x}{2} = \dfrac{x+1}{3} - \dfrac{1}{6} \Leftrightarrow 18x - 15x = 2(x+1) - 1 \Leftrightarrow 3x = 2x + 1 \Leftrightarrow x = 1$

37. $(x-1)(x+2) = (x-2)(x-3) \Leftrightarrow x^2 + x - 2 = x^2 - 5x + 6 \Leftrightarrow x - 2 = -5x + 6 \Leftrightarrow 6x = 8 \Leftrightarrow x = \frac{4}{3}$

38. $x(x+1)=(x+3)^2 \Leftrightarrow x^2+x=x^2+6x+9 \Leftrightarrow x=6x+9 \Leftrightarrow -5x=9 \Leftrightarrow x=-\frac{9}{5}$

39. $(x-1)(4x+5)=(2x-3)^2 \Leftrightarrow 4x^2+x-5=4x^2-12x+9 \Leftrightarrow x-5=-12x+9 \Leftrightarrow 13x=14 \Leftrightarrow x=\frac{14}{13}$

40. $(t-4)^2=(t+4)^2+32 \Leftrightarrow t^2-8t+16=t^2+8t+16+32 \Leftrightarrow -16t=32 \Leftrightarrow t=-2$

41. $\frac{1}{x}=\frac{4}{3x}+1 \Rightarrow 3=4+3x$ (multiply both sides by the LCD, $3x$) $\Leftrightarrow -1=3x \Leftrightarrow x=-\frac{1}{3}$

42. $\frac{2}{x}-5=\frac{6}{x}+4 \Rightarrow 2-5x=6+4x \Leftrightarrow -4=9x \Leftrightarrow -\frac{4}{9}=x$

43. $\frac{2x-1}{x+2}=\frac{4}{5} \Rightarrow 5(2x-1)=4(x+2) \Leftrightarrow 10x-5=4x+8 \Leftrightarrow 6x=13 \Leftrightarrow x=\frac{13}{6}$

44. $\frac{2x-7}{2x+4}=\frac{2}{3} \Rightarrow (2x-7)3=2(2x+4)$ (cross multiply) $\Leftrightarrow 6x-21=4x+8 \Leftrightarrow 2x=29 \Leftrightarrow x=\frac{29}{2}$

45. $\frac{2}{t+6}=\frac{3}{t-1} \Rightarrow 2(t-1)=3(t+6)$ [multiply both sides by the LCD, $(t-1)(t+6)$] $\Leftrightarrow 2t-2=3t+18 \Leftrightarrow -20=t$

46. $\frac{6}{x-3}=\frac{5}{x+4} \Rightarrow 6(x+4)=5(x-3) \Leftrightarrow 6x+24=5x-15 \Leftrightarrow x=-39$

47. $\frac{3}{x+1}-\frac{1}{2}=\frac{1}{3x+3} \Rightarrow 3(6)-(3x+3)=2$ [multiply both sides by $6(x+1)$] $\Leftrightarrow 18-3x-3=2 \Leftrightarrow -3x+15=2 \Leftrightarrow$ $-3x=-13 \Leftrightarrow x=\frac{13}{3}$

48. $\frac{12x-5}{6x+3}=2-\frac{5}{x} \Rightarrow (12x-5)x=2x(6x+3)-5(6x+3) \Leftrightarrow 12x^2-5x=12x^2+6x-30x-15 \Leftrightarrow$ $12x^2-5x=12x^2-24x-15 \Leftrightarrow 19x=-15 \Leftrightarrow x=-\frac{15}{19}$

49. $\frac{1}{z}-\frac{1}{2z}-\frac{1}{5z}=\frac{10}{z+1} \Rightarrow 10(z+1)-5(z+1)-2(z+1)=10(10z)$ [multiply both sides by $10z(z+1)$] $\Leftrightarrow$ $3(z+1)=100z \Leftrightarrow 3z+3=100z \Leftrightarrow 3=97z \Leftrightarrow \frac{3}{97}=z$

50. $\frac{1}{3-t}+\frac{4}{3+t}+\frac{15}{9-t^2}=0 \Rightarrow (3+t)+4(3-t)+15=0 \Leftrightarrow 3+t+12-4t+15=0 \Leftrightarrow -3t+30=0 \Leftrightarrow -3t=-30$ $\Leftrightarrow t=10$

51. $\frac{x}{2x-4}-2=\frac{1}{x-2} \Rightarrow x-2(2x-4)=2$ [multiply both sides by $2(x-2)$] $\Leftrightarrow x-4x+8=2 \Leftrightarrow -3x=-6 \Leftrightarrow x=2$. But substituting $x=2$ into the original equation does not work, since we cannot divide by 0. Thus there is no solution.

52. $\frac{1}{x+3}+\frac{5}{x^2-9}=\frac{2}{x-3} \Rightarrow (x-3)+5=2(x+3) \Leftrightarrow x+2=2x+6 \Leftrightarrow x=-4$

53. $\frac{3}{x+4}=\frac{1}{x}+\frac{6x+12}{x^2+4x} \Rightarrow 3(x)=(x+4)+6x+12$ (multiply both sides by $x(x+4)$] $\Leftrightarrow 3x=7x+16 \Leftrightarrow -4x=16$ $\Leftrightarrow x=-4$. But substituting $x=-4$ into the original equation does not work, since we cannot divide by 0. Thus, there is no solution.

54. $\frac{1}{x}-\frac{2}{2x+1}=\frac{1}{2x^2+x} \Rightarrow (2x+1)-2(x)=1 \Leftrightarrow 1=1$. This is an identity for $x\neq 0$ and $x\neq -\frac{1}{2}$, so the solutions are all real numbers except 0 and $-\frac{1}{2}$.

55. $x^2=25 \Leftrightarrow x=\pm 5$

56. $3x^2=48 \Leftrightarrow x^2=16 \Leftrightarrow x=\pm 4$

57. $5x^2=15 \Leftrightarrow x^2=3 \Leftrightarrow x=\pm\sqrt{3}$

58. $x^2=1000 \Leftrightarrow x=\pm\sqrt{1000}=\pm 10\sqrt{10}$

59. $8x^2-64=0 \Leftrightarrow x^2-8=0 \Leftrightarrow x^2=8 \Leftrightarrow x=\pm\sqrt{8}=\pm 2\sqrt{2}$

60. $5x^2-125=0 \Leftrightarrow 5\left(x^2-25\right)=0 \Leftrightarrow x^2=25 \Leftrightarrow x=\pm 5$

61. $x^2+16=0 \Leftrightarrow x^2=-16$ which has no real solution.

62. $6x^2 + 100 = 0 \Leftrightarrow 6x^2 = -100 \Leftrightarrow x^2 = -\frac{50}{3}$, which has no real solution.

63. $(x-3)^2 = 5 \Leftrightarrow x - 3 = \pm\sqrt{5} \Leftrightarrow x = 3 \pm \sqrt{5}$

64. $(3x-4)^2 = 7 \Leftrightarrow 3x - 4 = \pm\sqrt{7} \Leftrightarrow 3x = 4 \pm \sqrt{7} \Leftrightarrow x = \dfrac{4 \pm \sqrt{7}}{3}$

65. $x^3 = 27 \Leftrightarrow x = 27^{1/3} = 3$

66. $x^5 + 32 = 0 \Leftrightarrow x^5 = -32 \Leftrightarrow x = -32^{1/5} = -2$

67. $0 = x^4 - 16 = \left(x^2+4\right)\left(x^2-4\right) = \left(x^2+4\right)(x-2)(x+2)$. $x^2 + 4 = 0$ has no real solution. If $x - 2 = 0$, then $x = 2$. If $x + 2 = 0$, then $x = -2$. The solutions are ± 2.

68. $64x^6 = 27 \Leftrightarrow x^6 = \dfrac{27}{64} \Rightarrow x = \pm\left(\dfrac{27}{64}\right)^{1/6} = \pm\dfrac{27^{1/6}}{64^{1/6}} = \pm\dfrac{\sqrt{3}}{2}$

69. $x^4 + 64 = 0 \Leftrightarrow x^4 = -64$ which has no real solution.

70. $(x-1)^3 + 8 = 0 \Leftrightarrow (x-1)^3 = -8 \Leftrightarrow x - 1 = (-8)^{1/3} = -2 \Leftrightarrow x = -1$.

71. $(x+2)^4 - 81 = 0 \Leftrightarrow (x+2)^4 = 81 \Leftrightarrow \left[(x+2)^4\right]^{1/4} = \pm 81^{1/4} \Leftrightarrow x + 2 = \pm 3$. So $x + 2 = 3$, then $x = 1$. If $x + 2 = -3$, then $x = -5$. The solutions are -5 and 1.

72. $(x+1)^4 + 16 = 0 \Leftrightarrow (x+1)^4 = -16$, which has no real solution.

73. $3(x-3)^3 = 375 \Leftrightarrow (x-3)^3 = 125 \Leftrightarrow (x-3) = 125^{1/3} = 5 \Leftrightarrow x = 3 + 5 = 8$

74. $4(x+2)^5 = 1 \Leftrightarrow (x+2)^5 = \frac{1}{4} \Rightarrow x + 2 = \sqrt[5]{\frac{1}{4}} \Leftrightarrow x = -2 + \sqrt[5]{\frac{1}{4}}$

75. $\sqrt[3]{x} = 5 \Leftrightarrow x = 5^3 = 125$

76. $x^{4/3} - 16 = 0 \Leftrightarrow x^{4/3} = 16 = 2^4 \Leftrightarrow \left(x^{4/3}\right)^3 = \left(2^4\right)^3 = 2^{12} \Leftrightarrow x^4 = 2^{12} \Leftrightarrow x = \pm\left(2^{12}\right)^{1/4} = \pm 2^3 = \pm 8$

77. $2x^{5/3} + 64 = 0 \Leftrightarrow 2x^{5/3} = -64 \Leftrightarrow x^{5/3} = -32 \Leftrightarrow x = (-32)^{3/5} = \left(-2^5\right)^{1/5} = (-2)^3 = -8$

78. $6x^{2/3} - 216 = 0 \Leftrightarrow 6x^{2/3} = 216 \Leftrightarrow x^{2/3} = 36 = (\pm 6)^2 \Leftrightarrow \left(x^{2/3}\right)^{3/2} = \left[(\pm 6)^2\right]^{3/2} \Leftrightarrow x = (\pm 6)^3 = \pm 216$

79. $3.02x + 1.48 = 10.92 \Leftrightarrow 3.02x = 9.44 \Leftrightarrow x = \dfrac{9.44}{3.02} \approx 3.13$

80. $8.36 - 0.95x = 9.97 \Leftrightarrow -0.95x = 1.61 \Leftrightarrow x = \dfrac{1.61}{-0.95} \approx -1.69$

81. $2.15x - 4.63 = x + 1.19 \Leftrightarrow 1.15x = 5.82 \Leftrightarrow x = \dfrac{5.82}{1.19} \approx 5.06$

82. $3.95 - x = 2.32x + 2.00 \Leftrightarrow 1.95 = 3.32x \Leftrightarrow x = \dfrac{1.95}{3.32} \approx 0.59$

83. $3.16(x + 4.63) = 4.19(x - 7.24) \Leftrightarrow 3.16x + 14.63 = 4.19x - 30.34 \Leftrightarrow 44.97 = 1.03x \Leftrightarrow x = \dfrac{44.97}{1.03} \approx 43.66$

84. $2.14(x - 4.06) = 2.27 - 0.11x \Leftrightarrow 2.14x - 8.6684 = 2.27 - 0.11x \Leftrightarrow 2.25x = 10.9584 \Leftrightarrow x = 4.8704 \approx 4.87$

85. $\dfrac{0.26x - 1.94}{3.03 - 2.44x} = 1.76 \Rightarrow 0.26x - 1.94 = 1.76(3.03 - 2.44x) \Leftrightarrow 0.26x - 1.94 = 5.33 - 4.29x \Leftrightarrow 4.55x = 7.27 \Leftrightarrow x = \dfrac{7.27}{4.55} \approx 1.60$

86. $\dfrac{1.73x}{2.12 + x} = 1.51 \Leftrightarrow 1.73x = 1.51(2.12 + x) \Leftrightarrow 1.73x = 3.20 + 1.51x \Leftrightarrow 0.22x = 3.20 \Leftrightarrow x = \dfrac{3.20}{0.22} \approx 14.55$

87. $r = \dfrac{12}{M} \Leftrightarrow M = \dfrac{12}{r}$

88. $wd = rTH \Leftrightarrow T = \dfrac{wd}{rH}$

89. $PV = nRT \Leftrightarrow R = \dfrac{PV}{nT}$

90. $F = G\dfrac{mM}{r^2} \Leftrightarrow m = \dfrac{Fr^2}{GM}$

91. $P = 2l + 2w \Leftrightarrow 2w = P - 2l \Leftrightarrow w = \dfrac{P - 2l}{2}$

92. $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} \Leftrightarrow R_1 R_2 = R R_2 + R R_1$(multiply both sides by the LCD, $R R_1 R_2$). Thus $R_1 R_2 - R R_1 = R R_2 \Leftrightarrow$ $R_1 (R_2 - R) = R R_2 \Leftrightarrow R_1 = \dfrac{R R_2}{R_2 - R}$.

93. $V = \frac{1}{3}\pi r^2 h \Leftrightarrow r^2 = \dfrac{3V}{\pi h} \Rightarrow r = \pm\sqrt{\dfrac{3V}{\pi h}}$

94. $F = G\dfrac{mM}{r^2} \Leftrightarrow r^2 = G\dfrac{mM}{F} \Rightarrow r = \pm\sqrt{G\dfrac{mM}{F}}$

95. $V = \frac{4}{3}\pi r^3 \Leftrightarrow r^3 = \dfrac{3V}{4\pi} \Leftrightarrow r = \sqrt[3]{\dfrac{3V}{4\pi}}$

96. $a^2 + b^2 = c^2 \Leftrightarrow b^2 = c^2 - a^2 \Rightarrow b = \pm\sqrt{c^2 - a^2}$

97. $A = P\left(1 + \dfrac{i}{100}\right)^2 \Leftrightarrow \dfrac{A}{P} = \left(1 + \dfrac{i}{100}\right)^2 \Rightarrow 1 + \dfrac{i}{100} = \pm\sqrt{\dfrac{A}{P}} \Leftrightarrow \dfrac{i}{100} = -1 \pm \sqrt{\dfrac{A}{P}} \Leftrightarrow i = -100 \pm 100\sqrt{\dfrac{A}{P}}$

98. $a^2 x + (a - 1) = (a + 1)x \Leftrightarrow a^2 x - (a + 1)x = -(a - 1) \Leftrightarrow \left(a^2 - (a + 1)\right)x = -a + 1 \Leftrightarrow \left(a^2 - a - 1\right)x = -a + 1$ $\Leftrightarrow x = \dfrac{-a + 1}{a^2 - a - 1}$

99. $\dfrac{ax + b}{cx + d} = 2 \Leftrightarrow ax + b = 2(cx + d) \Leftrightarrow ax + b = 2cx + 2d \Leftrightarrow ax - 2cx = 2d - b \Leftrightarrow (a - 2c)x = 2d - b \Leftrightarrow x = \dfrac{2d - b}{a - 2c}$

100. $\dfrac{a + 1}{b} = \dfrac{a - 1}{b} + \dfrac{b + 1}{a} \Leftrightarrow a(a + 1) = a(a - 1) + b(b + 1) \Leftrightarrow a^2 + a = a^2 - a + b^2 + b \Leftrightarrow 2a = b^2 + b \Leftrightarrow$ $a = \frac{1}{2}\left(b^2 + b\right)$

101. **(a)** The shrinkage factor when $w = 250$ is $S = \dfrac{0.032(250) - 2.5}{10{,}000} = \dfrac{8 - 2.5}{10{,}000} = 0.00055$. So the beam shrinks $0.00055 \times 12.025 \approx 0.007$ m, so when it dries it will be $12.025 - 0.007 = 12.018$ m long.

(b) Substituting $S = 0.00050$ we get $0.00050 = \dfrac{0.032w - 2.5}{10{,}000} \Leftrightarrow 5 = 0.032w - 2.5 \Leftrightarrow 7.5 = 0.032w \Leftrightarrow$ $w = \dfrac{7.5}{0.032} \approx 234.375$. So the water content should be 234.375 kg/m^3.

102. Substituting $C = 3600$ we get $3600 = 450 + 3.75x \Leftrightarrow 3150 = 3.75x \Leftrightarrow x = \dfrac{3150}{3.75} = 840$. So the toy manufacturer can manufacture 840 toy trucks.

103. **(a)** Solving for v when $P = 10{,}000$ we get $10{,}000 = 15.6v^3 \Leftrightarrow v^3 \approx 641.02 \Leftrightarrow v \approx 8.6$ km/h.

(b) Solving for v when $P = 50{,}000$ we get $50{,}000 = 15.6v^3 \Leftrightarrow v^3 \approx 3205.13 \Leftrightarrow v \approx 14.7$ km/h.

104. Substituting $F = 300$ we get $300 = 0.3x^{3/4} \Leftrightarrow 1000 = 10^3 = x^{3/4} \Leftrightarrow x^{1/4} = 10 \Leftrightarrow x = 10^4 = 10{,}000$ lb.

105. **(a)** $C = \frac{1}{2}(1000)\left(2^2\right) + 1000(9.81)(1) + 200{,}000 = 211{,}810$

(b) $\frac{1}{2}\rho v^2 + \rho g h + P = C \Leftrightarrow P = C - \frac{1}{2}\rho v^2 - \rho g h$.

If the height increases to 5 m, then $P = 211{,}810 - \frac{1}{2}(1000)\left(2^2\right) - 1000(9.81)(5) = 160{,}760$ Pa.

If $h = 1$ m and $v = 4$ m/s, then $P = 211{,}810 - \frac{1}{2}(1000)\left(4^2\right) - 1000(9.81)(1) = 194{,}000$ Pa.

106. **(a)** $3(0) + k - 5 = k(0) - k + 1 \Leftrightarrow k - 5 = -k + 1 \Leftrightarrow 2k = 6 \Leftrightarrow k = 3$

(b) $3(1) + k - 5 = k(1) - k + 1 \Leftrightarrow 3 + k - 5 = k - k + 1 \Leftrightarrow k - 2 = 1 \Leftrightarrow k = 3$

(c) $3(2) + k - 5 = k(2) - k + 1 \Leftrightarrow 6 + k - 5 = 2k - k + 1 \Leftrightarrow k + 1 = k + 1$. $x = 2$ is a solution for every value of k. That is, $x = 2$ is a solution to every member of this family of equations.

107. When we multiplied by x, we introduced $x = 0$ as a solution. When we divided by $x - 1$, we are really dividing by 0, since $x = 1 \Leftrightarrow x - 1 = 0$.

P.9 MODELING WITH EQUATIONS

1. An equation modeling a real-world situation can be used to help us understand a real-world problem using mathematical methods. We translate real-world ideas into the language of algebra to construct our model, and translate our mathematical results back into real-world ideas in order to interpret our findings.

2. In the formula $I = Prt$ for simple interest, P stands for *principal*, r for *interest rate*, and t for *time (in years)*.

3. **(a)** A square of side x has area $A = x^2$.

(b) A rectangle of length l and width w has area $A = lw$.

(c) A circle of radius r has area $A = \pi r^2$.

4. Balsamic vinegar contains 5% acetic acid, so a 32 ounce bottle of balsamic vinegar contains $32 \cdot 5\% = 32 \cdot \frac{5}{100} = 1.6$ ounces of acetic acid.

5. A painter paints a wall in x hours, so the fraction of the wall she paints in one hour is $\dfrac{1 \text{ wall}}{x \text{ hours}} = \dfrac{1}{x}$.

6. Solving $d = rt$ for r, we find $\dfrac{d}{t} = \dfrac{rt}{t} \Rightarrow r = \dfrac{d}{t}$. Solving $d = rt$ for t, we find $\dfrac{d}{r} = \dfrac{rt}{r} \Rightarrow t = \dfrac{d}{r}$.

7. If n is the first integer, then $n + 1$ is the middle integer, and $n + 2$ is the third integer. So the sum of the three consecutive integers is $n + (n + 1) + (n + 2) = 3n + 3$.

8. If n is the middle integer, then $n - 1$ is the first integer, and $n + 1$ is the third integer. So the sum of the three consecutive integers is $(n - 1) + n + (n + 1) = 3n$.

9. If n is the first even integer, then $n + 2$ is the second even integer and $n + 4$ is the third. So the sum of three consecutive even integers is $n + (n + 2) + (n + 4) = 3n + 6$.

10. If n is the first integer, then the next integer is $n + 1$. The sum of their squares is $n^2 + (n + 1)^2 = n^2 + \left(n^2 + 2n + 1\right) = 2n^2 + 2n + 1$.

11. If s is the third test score, then since the other test scores are 78 and 82, the average of the three test scores is $\dfrac{78 + 82 + s}{3} = \dfrac{160 + s}{3}$.

12. If q is the fourth quiz score, then since the other quiz scores are 8, 8, and 8, the average of the four quiz scores is $\dfrac{8 + 8 + 8 + q}{4} = \dfrac{24 + q}{4}$.

13. If x dollars are invested at $2\frac{1}{2}\%$ simple interest, then the first year you will receive $0.025x$ dollars in interest.

14. If n is the number of months the apartment is rented, and each month the rent is \$945, then the total rent paid is $945n$.

15. Since w is the width of the rectangle, the length is three times the width, or $4w$. Then area $=$ length $\times$ width $= 4w \times w = 4w^2$ ft^2.

16. Since w is the width of the rectangle, the length is $w + 6$. The perimeter is

$$2 \times \text{length} + 2 \times \text{width} = 2\,(w + 6) + 2\,(w) = 4w + 12$$

17. If d is the given distance, in miles, and distance $=$ rate $\times$ time, we have time $= \dfrac{\text{distance}}{\text{rate}} = \dfrac{d}{55}$.

18. Since distance $=$ rate$\times$ time we have distance $= s \times (45 \text{ min}) \dfrac{1 \text{ h}}{60 \text{ min}} = \frac{3}{4}s$ mi.

19. If x is the quantity of pure water added, the mixture will contain 25 oz of salt and $3 + x$ gallons of water. Thus the concentration is $\dfrac{25}{3 + x}$.

20. If p is the number of pennies in the purse, then the number of nickels is $2p$, the number of dimes is $4 + 2p$, and the number of quarters is $(2p) + (4 + 2p) = 4p + 4$. Thus the value (in cents) of the change in the purse is $1 \cdot p + 5 \cdot 2p + 10 \cdot (4 + 2p) + 25 \cdot (4p + 4) = p + 10p + 40 + 20p + 100p + 100 = 131p + 140$.

21. If d is the number of days and m the number of miles, then the cost of a rental is $C = 65d + 0.40m$. In this case, $d = 3$ and $C = 283$, so we solve for m: $283 = 65 \cdot 3 + 0.40m \Leftrightarrow 283 = 195 + 0.4m \Leftrightarrow 0.4m = 88 \Leftrightarrow m = \dfrac{88}{0.4} = 220$. Thus, the truck was driven 220 miles.

22. The cost of speaking for m minutes on this plan is $C = \begin{cases} 100 & \text{if } m \le 250 \\ 100 + 0.25\,(m - 250) & \text{if } m > 250 \end{cases}$ In this case $m > 250$ and $C = 120.50$, so we solve $100 + 0.25\,(m - 250) = 120.5 \Leftrightarrow 0.25\,(m - 250) = 20.5 \Leftrightarrow m - 250 = 4\,(20.5) = 82 \Leftrightarrow m = 250 + 82 = 332$. Therefore, the tourist used 332 minutes of talk in the month of June.

23. If x is the student's score on their final exam, then because the final counts twice as much as each midterm, the average score is $\dfrac{82 + 75 + 71 + 2x}{3\,(100) + 200} = \dfrac{228 + 2x}{500} = \dfrac{114 + x}{250}$. For the average to be 80%, we must have $\dfrac{114 + x}{250} = 80\% = 0.8 \Leftrightarrow 114 + x = 250\,(0.8) = 200 \Leftrightarrow x = 86$. So the student scored 86% on their final exam.

24. Six students scored 100 and three students scored 60. Let x be the average score of the remaining $25 - 6 - 3 = 16$ students. Because the overall average is $84\% = 0.84$, we have $\dfrac{6\,(100) + 3\,(60) + 16x}{25\,(100)} = 0.84 \Leftrightarrow 780 + 16x = 0.84\,(2500) = 2100$ $\Leftrightarrow 16x = 1320 \Leftrightarrow x = \frac{1320}{16} = 82.5$. Thus, the remaining 16 students' average score was 82.5%.

25. Let m be the amount invested at $2\frac{1}{2}\%$. Then $12{,}000 - m$ is the amount invested at 3%.
Since the total interest is equal to the interest earned at 2.5% plus the interest earned at 3%, we have $318 = 0.025m + 0.03\,(12{,}000 - m) \Leftrightarrow 318 = 0.025m + 360 - 0.03m \Leftrightarrow 0.005m = 42 \Leftrightarrow m = \dfrac{42}{0.005} = 8400$. Thus, \$8400 is invested at $2\frac{1}{2}\%$ and $12{,}000 - 8400 = \$3600$ is invested at 3%.

26. Let m be the amount invested at 5%. Then $8000 + m$ is the total amount invested. Thus

$$4\% \text{ of the total investment} = \text{interest earned at } 3\tfrac{1}{2}\% + \text{interest earned at } 5\%$$

So $0.04\,(8000 + m) = 0.035\,(8000) + 0.05m \Leftrightarrow 320 + 0.04m = 280 + 0.05m \Leftrightarrow m = \dfrac{40}{0.01} = 4000$. Thus, \$4000 must be invested at 5%.

27. Using the formula $I = Prt$ and solving for r, we get $262.50 = 3500 \cdot r \cdot 1 \Leftrightarrow r = \dfrac{262.5}{3500} = 0.075$ or 7.5%.

28. If \$3000 is invested at an interest rate $a\%$, then \$5000 is invested at $\left(a + \frac{1}{2}\right)\%$, so, remembering that a is expressed as a percentage, the total interest is $I = 3000 \cdot \dfrac{a}{100} \cdot 1 + 5000 \cdot \dfrac{a + \frac{1}{2}}{100} \cdot 1 = 30a + 50a + 25 = 80a + 25$. Since the total interest is \$265, we have $265 = 80a + 25 \Leftrightarrow 80a = 240 \Leftrightarrow a = 3$. Thus, \$3000 is invested at 3% interest.

29. Let x be the monthly salary. Since annual salary $= 12 \times$ (monthly salary) $+$ (Christmas bonus), we have $180{,}100 = 12x + 7300 \Leftrightarrow 172{,}800 = 12x \Leftrightarrow x \approx 14{,}400$. The monthly salary is \$14,400.

30. Let s be the assistant's annual salary. Then the foreman's annual salary is $1.15s$. Their total income is the sum of their salaries, so $s + 1.15s = 113{,}305 \Leftrightarrow 2.15s = 113{,}305 \Leftrightarrow s = 52{,}700$. Thus, the assistant's annual salary is \$52,700.

31. Let x be the overtime hours worked. Since gross pay $=$ regular salary $+$ overtime pay, we obtain the equation $814 = 18.50 \cdot 35 + 18.50 \cdot 1.5 \cdot x \Leftrightarrow 814 = 647.50 + 27.75x \Leftrightarrow 166.50 = 27.75x \Leftrightarrow x = \dfrac{166.50}{27.75} = 6$. Thus, the lab technician worked 6 hours of overtime.

32. Let x be the number of hours worked by the plumber. Then the cabinet maker works for $9x$ hours. The total labor charge is the sum of their charges, so $2610 = 150x + 80\,(9x) \Leftrightarrow 2610 = 150x + 720x \Leftrightarrow x = \frac{2610}{870} = 3$. Thus, the plumber works for 3 hours and the cabinet maker works for $3 \cdot 9 = 27$ hours.

33. All ages are in terms of the daughter's age 7 years ago. Let y be age of the daughter 7 years ago. Then $11y$ is the age of the movie star 7 years ago. Today, the daughter is $y + 7$, and the movie star is $11y + 7$. But the movie star is also 4 times his daughter's age today. So $4(y+7) = 11y + 7 \Leftrightarrow 4y + 28 = 11y + 7 \Leftrightarrow 21 = 7y \Leftrightarrow y = 3$. Thus, today the movie star is $11(3) + 7 = 40$ years old.

34. Let h be number of home runs Babe Ruth hit. Then $h + 41$ is the number of home runs that Hank Aaron hit. So $1469 = h + h + 41 \Leftrightarrow 1428 = 2h \Leftrightarrow h = 714$. Thus Babe Ruth hit 714 home runs.

35. Let n be the number of nickels. Then there are also n dimes and n quarters. The total value of the coins in the purse is the sum of the values of nickels, dimes, and quarters, so $2.80 = 0.05n + 0.10n + 0.25n \Leftrightarrow 2.8 = 0.4p \Leftrightarrow p = \frac{2.8}{0.4} = 7$. So the purse contains 7 nickels, 7 dimes, and 7 quarters.

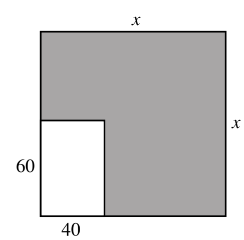
36. Let q be the number of quarters. Then $2q$ is the number of dimes, and $2q + 5$ is the number of nickels. Thus $3.00 =$ value of nickels+ value of dimes+ value of quarters, so
$3.00 = 0.05(2q+5) + 0.10(2q) + 0.25q \Leftrightarrow 3.00 = 0.10q + 0.25 + 0.20q + 0.25q \Leftrightarrow 2.75 = 0.55q \Leftrightarrow q = \frac{2.75}{0.55} = 5$.
Thus you have 5 quarters, $2(5) = 10$ dimes, and $2(5) + 5 = 15$ nickels.

37. Let l be the length of the garden. Since area $=$ width $\cdot$ length, we obtain the equation $1125 = 25l \Leftrightarrow l = \frac{1125}{25} = 45$ ft. So the garden is 45 feet long.

38. Let w be the width of the pasture. Then the length of the pasture is $3w$. Since area $=$ length $\times$ width we have $132{,}300 = w(3w) = 3w^2 \Leftrightarrow w^2 = \frac{132{,}300}{3} = 44{,}100 \Rightarrow w = \pm 210$. Thus the width of the pasture is 210 feet.

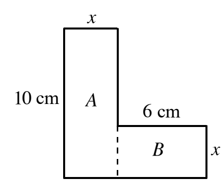
39. Let w be the width of the building lot. Then the length of the building lot is $5w$. Since a half-acre is $\frac{1}{2} \cdot 43{,}560 = 21{,}780$ and area is length times width, we have $21{,}780 = w(5w) = 5w^2 \Leftrightarrow w^2 = 4{,}356 \Rightarrow w = \pm 66$. Thus the width of the building lot is 66 feet and the length of the building lot is $5(66) = 330$ feet.

40. Let x be the length of a side of the square plot. As shown in the figure, area of the plot $=$ area of the building $+$ area of the parking lot. Thus, $x^2 = 60(40) + 12{,}000 = 2{,}400 + 12{,}000 = 14{,}400 \Rightarrow x = \pm 120$. So the plot of land measures 120 feet by 120 feet.



41. The shaded area is the sum of the area of a square and the area of a triangle. So $A = y^2 + \frac{1}{2}(y)(y) = \frac{3}{2}y^2$. We are given that the area is 120 in^2, so $120 = \frac{3}{2}y^2 \Leftrightarrow y^2 = 80 \Leftrightarrow y = \pm\sqrt{80} = \pm 4\sqrt{5}$. y is positive, so $y = 4\sqrt{5} \approx 8.94$ in.

42. First we write a formula for the area of the figure in terms of x. Region A has dimensions 10 cm and x cm and region B has dimensions 6 cm and x cm. So the shaded region has area $(10 \cdot x) + (6 \cdot x) = 16x$ cm^2. We are given that this is equal to 144 cm^2, so $144 = 16x \Leftrightarrow x = \frac{144}{16} = 9$ cm.

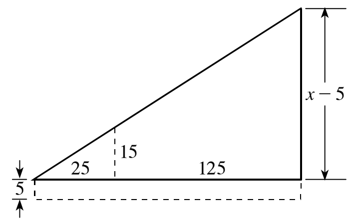


43. Let x be the width of the strip. Then the length of the mat is $20 + 2x$, and the width of the mat is $15 + 2x$. Now the perimeter is twice the length plus twice the width, so $102 = 2(20 + 2x) + 2(15 + 2x) \Leftrightarrow 102 = 40 + 4x + 30 + 4x \Leftrightarrow 102 = 70 + 8x \Leftrightarrow 32 = 8x \Leftrightarrow x = 4$. Thus the strip of mat is 4 inches wide.

44. Let x be the width of the strip. Then the width of the poster is $100 + 2x$ and its length is $140 + 2x$. The perimeter of the printed area is $2(100) + 2(140) = 480$, and the perimeter of the poster is $2(100 + 2x) + 2(140 + 2x)$. Now we use the fact that the perimeter of the poster is $1\frac{1}{2}$ times the perimeter of the printed area: $2(100 + 2x) + 2(140 + 2x) = \frac{3}{2} \cdot 480 \Leftrightarrow 480 + 8x = 720 \Leftrightarrow 8x = 240 \Leftrightarrow x = 30$. The blank strip is thus 30 cm wide.

45. Let x be the length of the person's shadow, in meters. Using similar triangles, $\dfrac{10 + x}{6} = \dfrac{x}{2} \Leftrightarrow 20 + 2x = 6x \Leftrightarrow 4x = 20 \Leftrightarrow x = 5$. Thus the person's shadow is 5 meters long.

46. Let x be the height of the tall tree. Here we use the property that corresponding sides in similar triangles are proportional. The base of the similar triangles starts at eye level of the woodcutter, 5 feet. Thus we obtain the proportion $\frac{x-5}{15}=\frac{150}{25} \Leftrightarrow 25(x-5)=15(150) \Leftrightarrow 25x-125=2250 \Leftrightarrow 25x=2375 \Leftrightarrow x=95$. Thus the tree is 95 feet tall.



47. Let x be the amount (in mL) of 60% acid solution to be used. Then $300-x$ mL of 30% solution would have to be used to yield a total of 300 mL of solution.

	60% acid	30% acid	Mixture
mL	x	$300-x$	300
Rate (% acid)	0.60	0.30	0.50
Value	$0.60x$	$0.30(300-x)$	$0.50(300)$

Thus the total amount of pure acid used is $0.60x+0.30(300-x)=0.50(300) \Leftrightarrow 0.3x+90=150 \Leftrightarrow x=\frac{60}{0.3}=200$. So 200 mL of 60% acid solution must be mixed with 100 mL of 30% solution to get 300 mL of 50% acid solution.

48. The amount of pure acid in the original solution is $300(50\%)=150$. Let x be the number of mL of pure acid added. Then the final volume of solution is $300+x$. Because its concentration is to be 60%, we must have $\frac{150+x}{300+x}=60\%=0.6 \Leftrightarrow 150+x=0.6(300+x) \Leftrightarrow 150+x=180+0.6x \Leftrightarrow 0.4x=30 \Leftrightarrow x=\frac{30}{0.4}=75$. Thus, 75 mL of pure acid must be added.

49. Let x be the number of grams of silver added. The weight of the rings is $5 \times 18\text{ g}=90\text{ g}$.

	5 rings	Pure silver	Mixture
Grams	90	x	$90+x$
Rate (% gold)	0.90	0	0.75
Value	$0.90(90)$	$0x$	$0.75(90+x)$

So $0.90(90)+0x=0.75(90+x) \Leftrightarrow 81=67.5+0.75x \Leftrightarrow 0.75x=13.5 \Leftrightarrow x=\frac{13.5}{0.75}=18$. Thus 18 grams of silver must be added to get the required mixture.

50. Let x be the number of liters of water to be boiled off. The result will contain $6-x$ liters.

	Original	Water	Final
Liters	6	$-x$	$6-x$
Concentration	120	0	200
Amount	$120(6)$	0	$200(6-x)$

So $120(6)+0=200(6-x) \Leftrightarrow 720=1200-200x \Leftrightarrow 200x=480 \Leftrightarrow x=2.4$. Thus 2.4 liters need to be boiled off.

51. Let x be the number of liters of coolant removed and replaced by water.

	60% antifreeze	60% antifreeze (removed)	Water	Mixture
Liters	3.6	x	x	3.6
Rate (% antifreeze)	0.60	0.60	0	0.50
Value	$0.60\,(3.6)$	$-0.60x$	$0x$	$0.50\,(3.6)$

So $0.60\,(3.6) - 0.60x + 0x = 0.50\,(3.6) \Leftrightarrow 2.16 - 0.6x = 1.8 \Leftrightarrow -0.6x = -0.36 \Leftrightarrow x = \dfrac{-0.36}{-0.6} = 0.6$. Thus 0.6 liters must be removed and replaced by water.

52. Let x be the number of gallons of 2% bleach removed from the tank. This is also the number of gallons of pure bleach added to make the 5% mixture.

	Original 2%	Pure bleach	5% mixture
Gallons	$100 - x$	x	100
Concentration	0.02	1	0.05
Bleach	$0.02\,(100 - x)$	$1x$	$0.05 \cdot 100$

So $0.02\,(100 - x) + x = 0.05 \cdot 100 \Leftrightarrow 2 - 0.02x + x = 5 \Leftrightarrow 0.98x = 3 \Leftrightarrow x = 3.06$. Thus 3.06 gallons need to removed and replaced with pure bleach.

53. Let c be the concentration of fruit juice in the cheaper brand. The new mixture consists of 650 mL of the original fruit punch and 100 mL of the cheaper fruit punch.

	Original Fruit Punch	Cheaper Fruit Punch	Mixture
mL	650	100	750
Concentration	0.50	c	0.48
Juice	$0.50 \cdot 650$	$100c$	$0.48 \cdot 750$

So $0.50 \cdot 650 + 100c = 0.48 \cdot 750 \Leftrightarrow 325 + 100c = 360 \Leftrightarrow 100c = 35 \Leftrightarrow c = 0.35$. Thus the cheaper brand is only 35% fruit juice.

54. Let x be the number of ounces of \$3.00/oz tea Then $80 - x$ is the number of ounces of \$2.75/oz tea.

	\$3.00 tea	\$2.75 tea	Mixture
Ounces	x	$80 - x$	80
Rate (cost per ounce)	3.00	2.75	2.90
Value	$3.00x$	$2.75\,(80 - x)$	$2.90\,(80)$

So $3.00x + 2.75\,(80 - x) = 2.90\,(80) \Leftrightarrow 3.00x + 220 - 2.75x = 232 \Leftrightarrow 0.25x = 12 \Leftrightarrow x = 48$. The mixture uses 48 ounces of \$3.00/oz tea and $80 - 48 = 32$ ounces of \$2.75/oz tea.

55. Let t be the time in minutes it would take to wash the car if the friends worked together. Friend 1 washes $\frac{1}{25}$ of the car per minute, while Friend 2 washes $\frac{1}{35}$ of the car per minute. The sum of these fractions is equal to the fraction of the job they can do working together, so we have $\dfrac{1}{t} = \dfrac{1}{25} + \dfrac{1}{35} \Leftrightarrow \dfrac{875t}{t} = \dfrac{875t}{25} + \dfrac{875t}{35} \Leftrightarrow 875 = 35t + 25t = 60t \Leftrightarrow t = \frac{875}{60} \Leftrightarrow$ $t = 14\frac{35}{60}$ minutes, or 14 minutes 35 seconds

56. Let t be the time, in minutes, it takes the landscaper to mow the lawn. Since the assistant is half as fast, it would take them $2t$ minutes to mow the lawn alone. Thus, $\dfrac{1}{10} = \dfrac{1}{t} + \dfrac{1}{2t} \Leftrightarrow \dfrac{10t}{10} = \dfrac{10t}{t} + \dfrac{10t}{2t} \Leftrightarrow t = 10 + 5 = 15$. Thus, it would the assistant $2\,(15) = 30$ minutes to mow the lawn alone.

57. Let t be the number of hours it would take your friend to paint a house alone. Then working together, it takes $\frac{2}{3}t$ hours. Because it takes you 7 hours, we have $\dfrac{1}{7}+\dfrac{1}{t}=\dfrac{1}{\frac{2}{3}t} \Leftrightarrow \dfrac{14t}{7}+\dfrac{14t}{t}=\dfrac{14t}{\frac{2}{3}t}=\dfrac{\frac{3}{2}(14t)}{t} \Leftrightarrow 2t+14=21 \Leftrightarrow t=\frac{7}{2}$. Thus, it would take your friend 3.5 h to paint a house alone.

58. Let h be the time, in hours, to fill the swimming pool using the smaller hose alone. Since the larger hose takes 20% less time, it takes $0.8h$ to fill the pool alone. Thus $16\cdot\dfrac{1}{h}+16\cdot\dfrac{1}{0.8h}=1 \Leftrightarrow 16\cdot 0.8+16=0.8h \Leftrightarrow 12.8+16=0.8h \Leftrightarrow h=\frac{28.8}{0.8}=36$. Thus, the smaller hose takes 36 hours to fill the pool alone, and the larger hose takes $0.8(36)=28.8$ hours.

59. Let t be the time in hours that the commuter spent on the train. Then $\frac{11}{2}-t$ is the time in hours that commuter spent on the bus. We construct a table:

	Rate	Time	Distance
By train	40	t	$40t$
By bus	60	$\frac{11}{2}-t$	$60\left(\frac{11}{2}-t\right)$

The total distance traveled is the sum of the distances traveled by bus and by train, so $300=40t+60\left(\frac{11}{2}-t\right) \Leftrightarrow$ $300=40t+330-60t \Leftrightarrow -30=-20t \Leftrightarrow t=\frac{-30}{-20}=1.5$ hours. So the time spent on the train is $5.5-1.5=4$ hours.

60. Let r be the speed of the slower cyclist, in mi/h. Then the speed of the faster cyclist is $2r$.

	Rate	Time	Distance
Slower cyclist	r	2	$2r$
Faster cyclist	$2r$	2	$4r$

When they meet, they will have traveled a combined total of 90 miles, so $2r+4r=90 \Leftrightarrow 6r=90 \Leftrightarrow r=15$. The speed of the slower cyclist is 15 mi/h, while the speed of the faster cyclist is $2(15)=30$ mi/h.

61. Let r be the speed of the plane from Montreal to Los Angeles. Then $r+0.20r=1.20r$ is the speed of the plane from Los Angeles to Montreal.

	Rate	Time	Distance
Montreal to L.A.	r	$\dfrac{2500}{r}$	2500
L.A. to Montreal	$1.2r$	$\dfrac{2500}{1.2r}$	2500

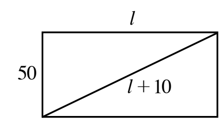
The total time is the sum of the times each way, so $9\frac{1}{6}=\dfrac{2500}{r}+\dfrac{2500}{1.2r} \Leftrightarrow \dfrac{55}{6}=\dfrac{2500}{r}+\dfrac{2500}{1.2r} \Leftrightarrow$ $55\cdot 1.2r=2500\cdot 6\cdot 1.2+2500\cdot 6 \Leftrightarrow 66r=18{,}000+15{,}000 \Leftrightarrow 66r=33{,}000 \Leftrightarrow r=\frac{33{,}000}{66}=500$. Thus the plane flew at a speed of 500 mi/h on the trip from Montreal to Los Angeles.

62. Let x be the speed of the car in mi/h. Since a mile contains 5280 ft and an hour contains 3600 s, 1 mi/h $=\frac{5280\text{ ft}}{3600\text{ s}}=\frac{22}{15}$ ft/s. The truck is traveling at $50\cdot\frac{22}{15}=\frac{220}{3}$ ft/s. So in 6 seconds, the truck travels $6\cdot\frac{220}{3}=440$ feet. Thus the back end of the car must travel the length of the car, the length of the truck, and the 440 feet in 6 seconds, so its speed must be $\frac{14+30+440}{6}=\frac{242}{3}$ ft/s. Converting to mi/h, we have that the speed of the car is $\frac{242}{3}\cdot\frac{15}{22}=55$ mi/h.

63. Let x be the distance from the fulcrum to where the 125-pound friend sits. Then substituting the known values into the formula given, we have $100(8)=125x \Leftrightarrow 800=125x \Leftrightarrow x=6.4$. So the 125-pound friend should sit 6.4 feet from the fulcrum.

64. Let w be the largest weight that can be hung. In this exercise, the edge of the building acts as the fulcrum, so the 240 lb man is sitting 25 feet from the fulcrum. Then substituting the known values into the formula given in Exercise 63, we have $240\,(25) = 5w \Leftrightarrow 6000 = 5w \Leftrightarrow w = 1200$. Therefore, 1200 pounds is the largest weight that can be hung.

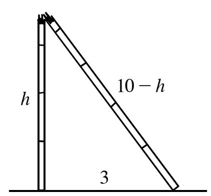
65. Let l be the length of the lot in feet. Then the length of the diagonal is $l + 10$. We apply the Pythagorean Theorem with the hypotenuse as the diagonal. So $l^2 + 50^2 = (l + 10)^2 \Leftrightarrow l^2 + 2500 = l^2 + 20l + 100 \Leftrightarrow 20l = 2400 \Leftrightarrow l = 120$. Thus the length of the lot is 120 feet.



66. Let r be the radius of the running track. The running track consists of two semicircles and two straight sections 110 yards long, so we get the equation $2\pi r + 220 = 440 \Leftrightarrow 2\pi r = 220 \Leftrightarrow r = \frac{110}{\pi} = 35.03$. Thus the radius of the semicircle is about 35 yards.

67. Let h be the height in feet of the structure. The structure is composed of a right cylinder with radius 10 and height $\frac{2}{3}h$ and a cone with base radius 10 and height $\frac{1}{3}h$. Using the formulas for the volume of a cylinder and that of a cone, we obtain the equation $1400\pi = \pi\,(10)^2\left(\frac{2}{3}h\right) + \frac{1}{3}\pi\,(10)^2\left(\frac{1}{3}h\right) \Leftrightarrow 1400\pi = \frac{200\pi}{3}h + \frac{100\pi}{9}h \Leftrightarrow 126 = 6h + h$ (multiply both sides by $\dfrac{9}{100\pi}$) $\Leftrightarrow 126 = 7h \Leftrightarrow h = 18$. Thus the height of the structure is 18 feet.

68. Let h be the height of the break, in feet. Then the portion of the bamboo above the break is $10 - h$. Applying the Pythagorean Theorem, we obtain $h^2 + 3^2 = (10 - h)^2 \Leftrightarrow h^2 + 9 = 100 - 20h + h^2 \Leftrightarrow -91 = -20h \Leftrightarrow h = \frac{91}{20} = 4.55$. Thus the break is 4.55 ft above the ground.



69. Answers will vary.

CHAPTER P REVIEW

1. **(a)** Since there are initially 250 tablets and the patient takes 2 tablets per day, the number of tablets T that are left in the bottle after x days is $T = 250 - 2x$.

(b) After 30 days, there are $250 - 2\,(30) = 190$ tablets left.

(c) We set $T = 0$ and solve: $T = 250 - 2x = 0 \Leftrightarrow 250 = 2x \Leftrightarrow x = 125$. The patient will run out after 125 days.

2. **(a)** The total cost is \$11 per calzone plus the \$7 delivery charge, so $C = 11x + 7$.

(b) Four calzones would cost $11\,(4) + 7 = \$51$.

(c) We solve $C = 11x + 7 = 40 \Leftrightarrow 11x = 33 \Leftrightarrow x = 3$. You can order three calzones.

3. **(a)** 16 is rational. It is an integer, and more precisely, a natural number.

(b) -16 is rational. It is an integer, but because it is negative, it is not a natural number.

(c) $\sqrt{16} = 4$ is rational. It is an integer, and more precisely, a natural number.

(d) $\sqrt{2}$ is irrational.

(e) $\frac{8}{3}$ is rational, but is neither a natural number nor an integer.

(f) $-\frac{8}{2} = -4$ is rational. It is an integer, but because it is negative, it is not a natural number.

4. **(a)** -5 is rational. It is an integer, but not a natural number.

(b) $-\frac{25}{6}$ is rational, but is neither an integer nor a natural number.

(c) $\sqrt{25} = 5$ is rational, a natural number, and an integer.

(d) 3π is irrational.

(e) $\frac{24}{16} = \frac{3}{2}$ is rational, but is neither a natural number nor an integer.

(f) 10^{20} is rational, a natural number, and an integer.

5. Commutative Property for addition.

6. Commutative Property for multiplication.

7. Distributive Property.

8. Distributive Property.

9. (a) $\frac{5}{6} + \frac{2}{3} = \frac{5}{6} + \frac{4}{6} = \frac{9}{6} = \frac{3}{2}$

(b) $\frac{5}{6} - \frac{2}{3} = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$

10. (a) $\frac{7}{10} - \frac{11}{15} = \frac{21}{30} - \frac{22}{30} = -\frac{1}{30}$

(b) $\frac{7}{10} + \frac{11}{15} = \frac{21}{30} + \frac{22}{30} = \frac{43}{30}$

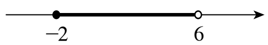
11. (a) $\frac{15}{8} \cdot \frac{12}{5} = \frac{15 \cdot 12}{8 \cdot 5} = \frac{3 \cdot 3}{2 \cdot 1} = \frac{9}{2}$

(b) $\frac{15}{8} \div \frac{12}{5} = \frac{15 \cdot 5}{8 \cdot 12} = \frac{5 \cdot 5}{8 \cdot 4} = \frac{25}{32}$

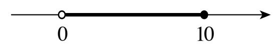
12. (a) $\frac{30}{7} \div \frac{12}{35} = \frac{30 \cdot 35}{7 \cdot 12} = \frac{5 \cdot 5}{1 \cdot 2} = \frac{25}{2}$

(b) $\frac{30}{7} \cdot \frac{12}{35} = \frac{30 \cdot 12}{7 \cdot 35} = \frac{6 \cdot 12}{7 \cdot 7} = \frac{72}{49}$

13. $x \in [-2, 6) \Leftrightarrow -2 \le x < 6$



14. $x \in (0, 10] \Leftrightarrow 0 < x \le 10$



15. $x \in (-\infty, 4] \Leftrightarrow x \le 4$

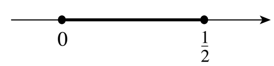
16. $x \in [-2, \infty) \Leftrightarrow -2 \le x$

17. $x \ge 5 \Leftrightarrow x \in [5, \infty)$

18. $x < -3 \Leftrightarrow x \in (-\infty, -3)$

19. $-1 < x \le 5 \Leftrightarrow x \in (-1, 5]$

20. $0 \le x \le \frac{1}{2} \Leftrightarrow x \in \left[0, \frac{1}{2}\right]$



21. (a) $A \cup B = \left\{-1, 0, \frac{1}{2}, 1, 2, 3, 4\right\}$

(b) $A \cap B = \{1\}$

22. (a) $C \cup D = (-1, 2]$

(b) $C \cap D = (0, 1]$

23. (a) $A \cap C = \{1, 2\}$

(b) $B \cap D = \left\{\frac{1}{2}, 1\right\}$

24. (a) $A \cap D = \{0, 1\}$

(b) $B \cap C = \left\{\frac{1}{2}, 1\right\}$

25. $|1 - |-4|| = |1 - 4| = |-3| = 3$

26. $5 - |10 - |-4|| = 5 - |10 - 4| = 5 - 6 = -1$

27. $2^{1/2} 8^{1/2} = \sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$

28. $2^{-3} - 3^{-2} = \frac{1}{8} - \frac{1}{9} = \frac{9}{72} - \frac{8}{72} = \frac{1}{72}$

29. $216^{-1/3} = \frac{1}{216^{1/3}} = \frac{1}{\sqrt[3]{216}} = \frac{1}{6}$

30. $64^{2/3} = \left(4^3\right)^{2/3} = 4^2 = 16$

31. $\frac{\sqrt{242}}{\sqrt{2}} = \sqrt{\frac{242}{2}} = \sqrt{121} = 11$

32. $\sqrt{2}\sqrt{50} = \sqrt{100} = 10$

33. (a) $|5 - 3| = |2| = 2$

(b) $|-5 - 3| = |-8| = 8$

34. (a) $|-4 - 0| = |-4| = 4$

(b) $|-4 - 4| = |-8| = 8$

35. **(a)** $\sqrt[3]{7} = 7^{1/3}$

(b) $\sqrt[5]{7^4} = 7^{4/5}$

36. **(a)** $\sqrt[3]{5^7} = 5^{7/3}$

(b) $\left(\sqrt[4]{5}\right)^3 = \left(5^{1/4}\right)^3 = 5^{3/4}$

37. **(a)** $\sqrt[6]{x^5} = x^{5/6}$

(b) $\left(\sqrt{x}\right)^9 = \left(x^{1/2}\right)^9 = x^{9/2}$

38. **(a)** $\sqrt{y^3} = \left(y^3\right)^{1/2} = y^{3/2}$

(b) $\left(\sqrt[8]{y}\right)^2 = \left(y^{1/8}\right)^2 = y^{1/4}$

39. **(a)** $\left(a^2\right)^{-3}\left(a^3b\right)^2\left(b^3\right)^4 = a^{-6} \cdot a^6b^2 \cdot b^{12} = a^{-6+6}b^{2+12} = b^{14}$

(b) $\left(3xy^2\right)^3\left(\frac{2}{3}x^{-1}y\right)^2 = 27x^3y^6 \cdot \frac{4}{9}x^{-2}y^2 = 27 \cdot \frac{4}{9}x^{3-2}y^{6+2} = 12xy^8$

40. **(a)** $\dfrac{x^4(3x)^2}{x^3} = \dfrac{x^4 \cdot 9x^2}{x^3} = 9x^{4+2-3} = 9x^3$

(b) $\left(\dfrac{r^2s^{4/3}}{r^{1/3}s}\right)^6 = \dfrac{r^{12}s^8}{r^2s^6} = r^{12-2}s^{8-6} = r^{10}s^2$

41. **(a)** $\sqrt[3]{\left(x^3y\right)^2y^4} = \sqrt[3]{x^6y^4y^2} = \sqrt[3]{x^6y^6} = x^2y^2$

(b) $\sqrt{w^8z^{10}} = \sqrt{w^8} \cdot \sqrt{z^{10}} = w^4\,|z|^5$

42. **(a)** $\dfrac{8r^{1/2}s^{-3}}{2r^{-2}s^4} = 4r^{(1/2)-(-2)}s^{-3-4} = 4r^{5/2}s^{-7} = \dfrac{4r^{5/2}}{s^7}$

(b) $\left(\dfrac{ab^2c^{-3}}{2a^3b^{-4}}\right)^{-2} = \dfrac{a^{-2}b^{-4}c^6}{2^{-2}a^{-6}b^8} = 2^2a^{-2-(-6)}b^{-4-8}c^6 = 4a^4b^{-12}c^6 = \dfrac{4a^4c^6}{b^{12}}$

43. $78{,}250{,}000{,}000 = 7.825 \times 10^{10}$

44. $2.08 \times 10^{-8} = 0.000\,000\,020\,8$

45. $\dfrac{ab}{c} \approx \dfrac{(0.00000293)\left(1.582 \times 10^{-14}\right)}{2.8064 \times 10^{12}} = \dfrac{\left(2.93 \times 10^{-6}\right)\left(1.582 \times 10^{-14}\right)}{2.8064 \times 10^{12}} = \dfrac{2.93 \cdot 1.582}{2.8064} \times 10^{-6-14-12}$
$\approx 1.65 \times 10^{-32}$

46. $80\,\dfrac{\text{times}}{\text{minute}} \cdot \dfrac{60\text{ minutes}}{\text{hour}} \cdot \dfrac{24\text{ hours}}{\text{day}} \cdot \dfrac{365\text{ days}}{\text{year}} \cdot 90\text{ years} \approx 3.8 \times 10^9\text{ times}$

47. $x^2 + 5x - 14 = (x+7)(x-2)$

48. $12x^2 + 10x - 8 = 2\left(6x^2 + 5x - 4\right) = 2(3x+4)(2x-1)$

49. $2x^2y - 6xy^2 = 2xy(x - 3y)$

50. $4y^2z^3 + 10y^3z - 12y^5z^2 = 2y^2z\left(2z^2 + 5y - 6y^3z\right)$

51. $3x^2 - 2x - 1 = (3x+1)(x-1)$

52. $x^4 + x^2 - 2 = \left(x^2\right)^2 + x^2 - 2 = \left(x^2+2\right)\left(x^2-1\right) = \left(x^2+2\right)(x+1)(x-1)$

53. $4t^2 - 13t - 12 = (4t+3)(t-4)$

54. $x^4 - 2x^2 + 1 = \left(x^2-1\right)^2 = [(x-1)(x+1)]^2 = (x-1)^2(x+1)^2$

55. $16 - 4t^2 = 4\left(4 - t^2\right) = 4(2-t)(2+t) = -4(t-2)(t+2)$

56. $2y^6 - 32y^2 = 2y^2\left(y^4 - 16\right) = 2y^2\left(y^2+4\right)\left(y^2-4\right) = 2y^2\left(y^2+4\right)(y+2)(y-2)$

57. $x^6 - 1 = \left(x^3-1\right)\left(x^3+1\right) = (x-1)\left(x^2+x+1\right)(x+1)\left(x^2-x+1\right)$

58. $16a^4b^2 + 2ab^5 = 2ab^2\left(8a^3 + b^3\right) = 2ab^2(2a + b)\left(4a^2 - 2ab + b^2\right)$

59. $x^3 - 27 = (x - 3)\left(x^2 + 3x + 9\right)$

60. $3y^3 - 81x^3 = 3\left(y^3 - 27x^3\right) = 3(y - 3x)\left(y^2 + 3xy + 9x^2\right)$

61. $-3x^{-1/2} + 2x^{1/2} + 5x^{3/2} = x^{-1/2}\left(5x^3 + 2x^2 - 3x\right) = x^{-1/2}(5x - 3)(x + 1)$

62. $7x^{-3/2} - 8x^{-1/2} + x^{1/2} = x^{-3/2}\left(x^2 - 8x^2 + 7\right) = x^{-3/2}(x - 1)(x - 7)$

63. $5x^3 + 15x^2 - x - 3 = 5x^2(x + 3) - (x + 3) = \left(5x^2 - 1\right)(x + 3)$

64. $w^3 - 3w^2 - 4w + 12 = w^2(w - 3) - 4(w - 3) = \left(w^2 - 4\right)(w - 3) = (w - 3)(w - 2)(w + 2)$

65. $(a + b)^2 - 3(a + b) - 10 = (a + b - 5)(a + b + 2)$

66. $(3x + 2)^2 - (3x + 2) - 6 = [(3x + 2) - 3][(3x + 2) + 2] = (3x - 1)(3x + 4)$

67. $(2y - 7)(2y + 7) = 4y^2 - 49$

68. $(1 + x)(2 - x) - (3 - x)(3 + x) = 2 + x - x^2 - \left(9 - x^2\right) = 2 + x - x^2 - 9 + x^2 = -7 + x$

69. $x^2(x - 2) + x(x - 2)^2 = x^3 - 2x^2 + x\left(x^2 - 4x + 4\right) = x^3 - 2x^2 + x^3 - 4x^2 + 4x = 2x^3 - 6x^2 + 4x$

70. $\dfrac{x^3 + 2x^2 + 3x}{x} = \dfrac{x\left(x^2 + 2x + 3\right)}{x} = x^2 + 2x + 3$

71. $\sqrt{x}\left(\sqrt{x} + 1\right)\left(2\sqrt{x} - 1\right) = \left(x + \sqrt{x}\right)\left(2\sqrt{x} - 1\right) = 2x\sqrt{x} - x + 2x - \sqrt{x} = 2x^{3/2} + x - x^{1/2}$

72. $(2x + 1)^3 = (2x)^3 + 3(2x)^2(1) + 3(2x)(1)^2 + (1)^3 = 8x^3 + 12x^2 + 6x + 1$

73. $\dfrac{x^2 - 2x - 3}{2x^2 + 5x + 3} = \dfrac{(x - 3)(x + 1)}{(2x + 3)(x + 1)} = \dfrac{x - 3}{2x + 3}$

74. $\dfrac{t^3 - 1}{t^2 - 1} = \dfrac{(t - 1)\left(t^2 + t + 1\right)}{(t - 1)(t + 1)} = \dfrac{t^2 + t + 1}{t + 1}$

75. $\dfrac{x^2 - 4x - 5}{x^2 - 25} \cdot \dfrac{x^2 + 12x + 36}{x^2 + 7x + 6} = \dfrac{(x - 5)(x + 1)}{(x + 5)(x - 5)} \cdot \dfrac{(x + 6)^2}{(x + 6)(x + 1)} = \dfrac{x + 1}{x + 5} \cdot \dfrac{x + 6}{x + 1} = \dfrac{x + 6}{x + 5}$

76. $\dfrac{x^2 - 2x - 15}{x^2 - 6x + 5} \div \dfrac{x^2 - x - 12}{x^2 - 1} = \dfrac{(x - 5)(x + 3)}{(x - 5)(x - 1)} \cdot \dfrac{(x - 1)(x + 1)}{(x - 4)(x + 3)} = \dfrac{x + 1}{x - 4}$

77. $x - \dfrac{1}{x + 1} = \dfrac{x(x + 1)}{x + 1} - \dfrac{1}{x + 1} = \dfrac{x^2 + x - 1}{x + 1}$

78. $\dfrac{1}{x - 1} - \dfrac{x}{x^2 + 1} = \dfrac{x^2 + 1}{(x - 1)\left(x^2 + 1\right)} - \dfrac{x(x - 1)}{(x - 1)\left(x^2 + 1\right)} = \dfrac{x^2 + 1 - x^2 + x}{(x - 1)\left(x^2 + 1\right)} = \dfrac{x + 1}{(x - 1)\left(x^2 + 1\right)}$

79.
$$\frac{2}{x} + \frac{1}{x - 2} + \frac{3}{(x - 2)^2} = \frac{2(x - 2)^2}{x(x - 2)^2} + \frac{x(x - 2)}{x(x - 2)^2} + \frac{3x}{x(x - 2)^2}$$
$$= \frac{2\left(x^2 - 4x + 4\right) + x^2 - 2x + 3x}{x(x - 2)^2} = \frac{2x^2 - 8x + 8 + x^2 - 2x + 3x}{x(x - 2)^2} = \frac{3x^2 - 7x + 8}{x(x - 2)^2}$$

80.
$$\frac{1}{x + 2} + \frac{1}{x^2 - 4} - \frac{2}{x^2 - x - 2} = \frac{1}{x + 2} + \frac{1}{(x - 2)(x + 2)} - \frac{2}{(x - 2)(x + 1)}$$
$$= \frac{(x - 2)(x + 1)}{(x - 2)(x + 1)(x + 2)} + \frac{x + 1}{(x - 2)(x + 1)(x + 2)} - \frac{2(x + 2)}{(x - 2)(x + 1)(x + 2)}$$
$$= \frac{x^2 - x - 2 + x + 1 - 2x - 4}{(x - 2)(x + 1)(x + 2)} = \frac{x^2 - 2x - 5}{(x - 2)(x + 1)(x + 2)}$$

81. $\dfrac{\dfrac{1}{x}-\dfrac{1}{2}}{x-2}=\dfrac{\dfrac{2}{2x}-\dfrac{x}{2x}}{x-2}=\dfrac{2-x}{2x}\cdot\dfrac{1}{x-2}=\dfrac{-1\,(x-2)}{2x}\cdot\dfrac{1}{x-2}=\dfrac{-1}{2x}$

82. $\dfrac{\dfrac{1}{x}-\dfrac{1}{x+1}}{\dfrac{1}{x}+\dfrac{1}{x+1}}=\dfrac{\dfrac{1}{x}-\dfrac{1}{x+1}}{\dfrac{1}{x}+\dfrac{1}{x+1}}\cdot\dfrac{x\,(x+1)}{x\,(x+1)}=\dfrac{(x+1)-x}{(x+1)+x}=\dfrac{1}{2x+1}$

83. $\dfrac{\sqrt{x}-2}{\sqrt{x}+2}=\dfrac{\sqrt{x}-2}{\sqrt{x}+2}\cdot\dfrac{\sqrt{x}-2}{\sqrt{x}-2}=\dfrac{x+4-4\sqrt{x}}{x-4}$

84. $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}=\dfrac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=\dfrac{(x+h)-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\dfrac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\dfrac{1}{\sqrt{x+h}+\sqrt{x}}$

85. $\dfrac{1}{\sqrt{11}}=\dfrac{1}{\sqrt{11}}\cdot\dfrac{\sqrt{11}}{\sqrt{11}}=\dfrac{\sqrt{11}}{11}$

86. $\dfrac{3}{\sqrt{6}}=\dfrac{3}{\sqrt{6}}\cdot\dfrac{\sqrt{6}}{\sqrt{6}}=\dfrac{3\sqrt{6}}{6}=\dfrac{\sqrt{6}}{2}$

87. $\dfrac{10}{\sqrt{2}-1}=\dfrac{10}{\sqrt{2}-1}\cdot\dfrac{\sqrt{2}+1}{\sqrt{2}+1}=\dfrac{10+10\sqrt{2}}{2-1}=10+10\sqrt{2}$

88. $\dfrac{14}{3-\sqrt{2}}=\dfrac{14}{3-\sqrt{2}}\cdot\dfrac{3+\sqrt{2}}{3+\sqrt{2}}=\dfrac{42+14\sqrt{2}}{3^2-\left(\sqrt{2}\right)^2}=\dfrac{42+14\sqrt{2}}{7}=6+2\sqrt{2}$

89. $\dfrac{x}{2+\sqrt{x}}=\dfrac{x}{2+\sqrt{x}}\cdot\dfrac{2-\sqrt{x}}{2-\sqrt{x}}=\dfrac{2x-x\sqrt{x}}{2^2-\left(\sqrt{x}\right)^2}=\dfrac{x\left(\sqrt{x}-2\right)}{x-4}$

90. $\dfrac{\sqrt{x}-2}{\sqrt{x}+2}=\dfrac{\sqrt{x}-2}{\sqrt{x}+2}\cdot\dfrac{\sqrt{x}-2}{\sqrt{x}-2}=\dfrac{x-4\sqrt{x}+4}{x-4}$

91. $\dfrac{x+5}{x+10}$ is defined whenever $x+10\neq 0\Leftrightarrow x\neq -10$, so its domain is $\{x\mid x\neq -10\}$.

92. $\dfrac{2x}{x^2-9}$ is defined whenever $x^2-9\neq 0\Leftrightarrow x^2\neq 9\Leftrightarrow x\neq\pm 3$, so its domain is $\{x\mid x\neq -3\text{ and }x\neq 3\}$.

93. $\dfrac{\sqrt{x}}{x^2-3x-4}$ is defined whenever $x\geq 0$ (so that $\sqrt{x}$ is defined) and $x^2-3x-4=(x+1)(x-4)\neq 0\Leftrightarrow x\neq -1$ and $x\neq 4$. Thus, its domain is $\{x\mid x\geq 0\text{ and }x\neq 4\}$.

94. $\dfrac{\sqrt{x-3}}{x^2-4x+4}$ is defined whenever $x-3\geq 0\Leftrightarrow x\geq 3$ and $x^2-4x+4=(x-2)^2\neq 0\Leftrightarrow x\neq\pm 2$. Thus, its domain is $\{x\mid x\geq 3\}$.

95. This statement is false. For example, take $x=1$ and $y=1$. Then LHS $=(x+y)^3=(1+1)^3=2^3=8$, while RHS $=x^3+y^3=1^3+1^3=1+1=2$, and $8\neq 2$.

96. This statement is true for $a\neq 1$: $\dfrac{1+\sqrt{a}}{1-a}=\dfrac{1+\sqrt{a}}{1-a}\cdot\dfrac{1-\sqrt{a}}{1-\sqrt{a}}=\dfrac{1-a}{(1-a)\left(1-\sqrt{a}\right)}=\dfrac{1}{1-\sqrt{a}}$.

97. This statement is true: $\dfrac{12+y}{y}=\dfrac{12}{y}+\dfrac{y}{y}=\dfrac{12}{y}+1$.

98. This statement is false. For example, take $a=1$ and $b=1$. Then LHS $=\sqrt[3]{a+b}=\sqrt[3]{1+1}=\sqrt[3]{2}$, while RHS $=\sqrt[3]{a}+\sqrt[3]{b}=\sqrt[3]{1}+\sqrt[3]{1}=1+1=2$, and $\sqrt[3]{2}\neq 2$.

99. This statement is false. For example, take $a=-1$. Then LHS $=\sqrt{a^2}=\sqrt{(-1)^2}=\sqrt{1}=1$, which does not equal $a=-1$. The true statement is $\sqrt{a^2}=|a|$.

100. This statement is false. For example, take $x = 1$. Then LHS $= \dfrac{1}{x+4} = \dfrac{1}{1+4} = \dfrac{1}{5}$, while RHS $= \dfrac{1}{x} + \dfrac{1}{4} = \dfrac{1}{1} + \dfrac{1}{4} = \dfrac{5}{4}$, and $\dfrac{1}{5} \neq \dfrac{5}{4}$.

101. $3x + 12 = 24 \Leftrightarrow 3x = 12 \Leftrightarrow x = 4$

102. $5x - 7 = 42 \Leftrightarrow 5x = 49 \Leftrightarrow x = \frac{49}{5}$

103. $7x - 6 = 4x + 9 \Leftrightarrow 3x = 15 \Leftrightarrow x = 5$

104. $8 - 2x = 14 + x \Leftrightarrow -3x = 6 \Leftrightarrow x = -2$

105. $\frac{1}{3}x - \frac{1}{2} = 2 \Leftrightarrow 2x - 3 = 12 \Leftrightarrow 2x = 15 \Leftrightarrow x = \frac{15}{2}$

106. $\frac{2}{3}x + \frac{3}{5} = \frac{1}{5} - 2x \Leftrightarrow 10x + 9 = 3 - 30x \Leftrightarrow 40x = -6 \Leftrightarrow x = -\frac{6}{40} = -\frac{3}{20}$

107. $2(x+3) - 4(x-5) = 8 - 5x \Leftrightarrow 2x + 6 - 4x + 20 = 8 - 5x \Leftrightarrow -2x + 26 = 8 - 5x \Leftrightarrow 3x = -18 \Leftrightarrow x = -6$

108. $\dfrac{x-5}{2} - \dfrac{2x+5}{3} = \dfrac{5}{6} \Leftrightarrow 3(x-5) - 2(2x+5) = 5 \Leftrightarrow 3x - 15 - 4x - 10 = 5 \Leftrightarrow -x = 30 \Leftrightarrow x = -30$

109. $\dfrac{x+1}{x-1} = \dfrac{2x-1}{2x+1} \Leftrightarrow (x+1)(2x+1) = (2x-1)(x-1) \Leftrightarrow 2x^2 + 3x + 1 = 2x^2 - 3x + 1 \Leftrightarrow 6x = 0 \Leftrightarrow x = 0$

110. $\dfrac{x}{x+2} - 3 = \dfrac{1}{x+2} \Leftrightarrow x - 3(x+2) = 1 \Leftrightarrow x - 3x - 6 = 1 \Leftrightarrow -2x = 7 \Leftrightarrow x = -\dfrac{7}{2}$

111. $\dfrac{x+1}{x-1} = \dfrac{3x}{3x-6} = \dfrac{3x}{3(x-2)} = \dfrac{x}{x-2} \Leftrightarrow (x+1)(x-2) = x(x-1) \Leftrightarrow x^2 - x - 2 = x^2 - x \Leftrightarrow -2 = 0$. Since this last equation is never true, there is no real solution to the original equation.

112. $(x+2)^2 = (x-4)^2 \Leftrightarrow (x+2)^2 - (x-4)^2 = 0 \Leftrightarrow [(x+2) - (x-4)][(x+2) + (x-4)] = 0 \Leftrightarrow$ $[x + 2 - x + 4][x + 2 + x - 4] = 6(2x - 2) = 0 \Leftrightarrow 2x - 2 = 0 \Leftrightarrow x = 1$.

113. $x^2 = 144 \Rightarrow x = \pm 12$

114. $4x^2 = 49 \Leftrightarrow x^2 = \frac{49}{4} \Rightarrow x = \pm\frac{7}{2}$

115. $x^3 - 27 = 0 \Leftrightarrow x^3 = 27 \Rightarrow x = 3$.

116. $6x^4 + 15 = 0 \Leftrightarrow 6x^4 = -15 \Leftrightarrow x^4 = -\frac{5}{2}$. Since x^4 must be nonnegative, there is no real solution.

117. $(x+1)^3 = -64 \Leftrightarrow x + 1 = -4 \Leftrightarrow x = -1 - 4 = -5$.

118. $(x+2)^2 - 2 = 0 \Leftrightarrow (x+2)^2 = 2 \Leftrightarrow x + 2 = \pm\sqrt{2} \Leftrightarrow x = -2 \pm \sqrt{2}$.

119. $\sqrt[3]{x} = -3 \Leftrightarrow x = (-3)^3 = -27$.

120. $x^{2/3} - 4 = 0 \Leftrightarrow \left(x^{1/3}\right)^2 = 4 \Rightarrow x^{1/3} = \pm 2 \Leftrightarrow x = \pm 8$.

121. $4x^{3/4} - 500 = 0 \Leftrightarrow 4x^{3/4} = 500 \Leftrightarrow x^{3/4} = 125 \Leftrightarrow x = 125^{4/3} = 5^4 = 625$.

122. $(x-2)^{1/5} = 2 \Leftrightarrow x - 2 = 2^5 = 32 \Leftrightarrow x = 2 + 32 = 34$.

123. $A = \dfrac{x+y}{2} \Leftrightarrow 2A = x + y \Leftrightarrow x = 2A - y$.

124. $V = xy + yz + xz \Leftrightarrow V = y(x+z) + xz \Leftrightarrow V - xz = y(x+z) \Leftrightarrow y = \dfrac{V - xz}{x+z}$.

125. Multiply through by t: $J = \dfrac{1}{t} + \dfrac{1}{2t} + \dfrac{1}{3t} \Leftrightarrow tJ = 1 + \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{11}{6} \Leftrightarrow t = \dfrac{11}{6J}$, $J \neq 0$.

126. $F = k\dfrac{q_1q_2}{r^2} \Leftrightarrow r^2 = k\dfrac{q_1q_2}{F} \Leftrightarrow r = \pm\sqrt{k\dfrac{q_1q_2}{F}}$. (In real-world applications, r represents distance, so we would take the positive root.)

127. Let x be the number of pounds of raisins. Then the number of pounds of nuts is $50 - x$.

	Raisins	Nuts	Mixture
Pounds	x	$50 - x$	50
Rate (cost per pound)	3.20	2.40	2.72

So $3.20x + 2.40(50 - x) = 2.72(50) \Leftrightarrow 3.20x + 120 - 2.40x = 136 \Leftrightarrow 0.8x = 16 \Leftrightarrow x = 20$. Thus the mixture uses 20 pounds of raisins and $50 - 20 = 30$ pounds of nuts.

128. Let t be the number of hours that the district supervisor drives. Then the store manager drives for $t - \frac{1}{4}$ hours.

	Rate	Time	Distance
Supervisor	45	t	$45t$
Manager	40	$t - \frac{1}{4}$	$40\left(t - \frac{1}{4}\right)$

When they pass each other, they will have traveled a total of 160 miles. So $45t + 40\left(t - \frac{1}{4}\right) = 160 \Leftrightarrow 45t + 40t - 10 = 160$ $\Leftrightarrow 85t = 170 \Leftrightarrow t = 2$. Since the supervisor leaves at 2:00 P.M. and travels for 2 hours, they pass each other at 4:00 P.M.

129. Let x be the amount invested in the account earning 1.5% interest. Then the amount invested in the account earning 2.5% is $7000 - x$.

	1.5% Account	2.5% Account	Total
Amount invested	x	$7000 - x$	7000
Interest earned	$0.015x$	$0.025(7000 - x)$	120.25

From the table, we see that $0.015x + 0.025(7000 - x) = 120.25 \Leftrightarrow 0.015x + 175 - 0.025x = 120.25 \Leftrightarrow 54.75 = 0.01x$ $\Leftrightarrow x = 5475$. Thus, \$5475 is invested in the account earning 1.5% interest and \$1525 is invested in the account earning 2.5% interest.

130. The amount of interest currently earned is $6000(0.03) = \$180$ per year. If a total of \$300 is desired, another \$120 in interest must be earned at a rate of 1.25% per year. If the additional amount invested is x, we have $0.0125x = 120 \Leftrightarrow x = 9600$. Thus, an additional \$9600 must be invested at 1.25% simple interest to earn a total of \$300 interest per year.

131. Let t be the time it would take the interior decorator to paint a living room if they work alone. It would take the assistant $2t$ hours alone, and it would take the apprentice $3t$ hours alone. Thus, the decorator does $\dfrac{1}{t}$ of the job per hour, the assistant does $\dfrac{1}{2t}$ of the job per hour, and the apprentice does $\dfrac{1}{3t}$ of the job per hour. So $\dfrac{1}{t} + \dfrac{1}{2t} + \dfrac{1}{3t} = 1 \Leftrightarrow 6 + 3 + 2 = 6t \Leftrightarrow$ $6t = 11 \Leftrightarrow t = \frac{11}{6}$. Thus, it would take the decorator 1 hour 50 minutes to paint the living room alone.

132. Let w be width of the pool. Then the length of the pool is $2w$, and its volume is $8(w)(2w) = 8464 \Leftrightarrow 16w^2 = 8464 \Leftrightarrow$ $w^2 = 529 \Rightarrow w = \pm 23$. Since $w > 0$, we reject the negative value. The pool is 23 feet wide, $2(23) = 46$ feet long, and 8 feet deep.

CHAPTER P TEST

1. (a) The cost is $C = 9 + 1.5x$.

(b) There are four extra toppings, so $x = 4$ and $C = 9 + 1.5(4) = \$15$.

(c) We have $C = 19.5 = 9 + 1.5x \Leftrightarrow 1.5x = 10.5 \Leftrightarrow x = 7$. Thus, a \$19.50 pizza has 7 toppings.

2. (a) 5 is rational. It is an integer, and more precisely, a natural number.

(b) $\sqrt{5}$ is irrational.

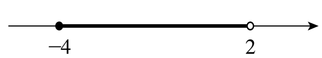
(c) $-\frac{9}{3} = -3$ is rational, and it is an integer.

(d) $-1{,}000{,}000$ is rational, and it is an integer.

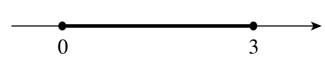
3. (a) $A \cap B = \{0, 1, 5\}$

(b) $A \cup B = \left\{-2, 0, \frac{1}{2}, 1, 3, 5, 7\right\}$

4. (a)



$[-4, 2)$



$[0, 3]$

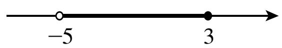
(b)

$[-4, 2) \cap [0, 3] = [0, 2)$

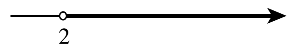
$[-4, 2) \cup [0, 3] = [-4, 3]$

(c) $|2 - (-4)| = |6| = 6$

5. (a)



$(-5, 3]$



$(2, \infty)$

(b) $x \le 3 \Leftrightarrow x \in (-\infty, 3]$; $-1 \le x < 4 \Leftrightarrow x \in [-1, 4)$

6. (a) $(-3)^4 = 81$

(b) $-3^4 = -81$

(c) $3^{-4} = \dfrac{1}{3^4} = \frac{1}{81}$

(d) $\dfrac{3^{75}}{3^{72}} = 3^{75-72} = 3^3 = 27$

(e) $\left(\dfrac{2}{3}\right)^{-2} = \dfrac{3^2}{2^2} = \dfrac{9}{4}$

(f) $\dfrac{\sqrt[5]{32}}{\sqrt{16}} = \dfrac{2}{4} = \dfrac{1}{2}$

(g) $\sqrt[4]{\dfrac{3^8}{2^{16}}} = \dfrac{3^2}{2^4} = \dfrac{9}{16}$

(h) $16^{-3/4} = \left(2^4\right)^{-3/4} = 2^{-3} = \frac{1}{8}$

7. (a) $186{,}000{,}000{,}000 = 1.86 \times 10^{11}$

(b) $0.000\,000\,396\,5 = 3.965 \times 10^{-7}$

8. (a) $\dfrac{a^3b^2}{ab^3} = \dfrac{a^2}{b}$

(b) $\sqrt{200} - \sqrt{32} = 10\sqrt{2} - 4\sqrt{2} = 6\sqrt{2}$

(c) $\left(2x^{1/2}y^2\right)\left(3x^{1/4}y^{-1}\right)^2 = 2 \cdot 3^2 x^{(1/2)+2(1/4)} y^{2+2(-1)} = 18x$

(d) $\left(3a^3b^3\right)\left(4ab^2\right)^2 = 3a^3b^3 \cdot 4^2a^2b^4 = 48a^5b^7$

(e) $\sqrt{9x^2y^4} = \sqrt{\left(3xy^2\right)^2} = 3\,|x|\,y^2$

(f) $\left(\dfrac{4x^9y^3}{xy^7}\right)^{-1/2} = \left(\dfrac{4x^8}{y^4}\right)^{-1/2} = \left(\dfrac{y^4}{4x^8}\right)^{1/2} = \dfrac{y^2}{2x^4}$

9. (a) $z\,(4z - 3) + 2z\,(3 - 2z) = 4z^2 - 3z + 6z - 4z^2 = 3z$

(b) $(x + 3)\,(4x - 5) = 4x^2 - 5x + 12x - 15 = 4x^2 + 7x - 15$

(c) $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2 = a - b$

(d) $(2x + 3)^2 = (2x)^2 + 2\,(2x)\,(3) + (3)^2 = 4x^2 + 12x + 9$

(e) $(x + 2)^3 = (x)^3 + 3\,(x)^2\,(2) + 3\,(x)\,(2)^2 + (2)^3 = x^3 + 6x^2 + 12x + 8$

(f) $x^2\,(x - 3)\,(x + 3) = x^2\left(x^2 - 9\right) = x^4 - 9x^2$

10. (a) $4x^2 - 25 = (2x - 5)\,(2x + 5)$

(b) $2x^2 + 5x - 12 = (2x - 3)\,(x + 4)$

(c) $x^3 - 3x^2 - 4x + 12 = x^2\,(x - 3) - 4\,(x - 3) = (x - 3)\left(x^2 - 4\right) = (x - 3)\,(x - 2)\,(x + 2)$

(d) $x^4 + 27x = x\left(x^3 + 27\right) = x\,(x + 3)\left(x^2 - 3x + 9\right)$

(e) $2x^{3/2} + 8x^{1/2} - 10x^{-1/2} = 2x^{-1/2}\left(x^2 + 4x - 5\right) = 2x^{-1/2}\,(x + 5)\,(x - 1)$

(f) $x^4y^2 - 9x^2y^2 = x^2y^2\left(x^2-9\right) = x^2y^2(x-3)(x+3)$

11. (a) $\dfrac{w^2+4w+3}{w^2-2w-3} = \dfrac{(w+3)(w+1)}{(w-3)(w+1)} = \dfrac{w+3}{w-3}$

(b) $\dfrac{2x^2-x-1}{x^2-9} \cdot \dfrac{x+3}{2x+1} = \dfrac{(2x+1)(x-1)(x+3)}{(x+3)(x-3)(2x+1)} = \dfrac{x-1}{x-3}$

(c) $\dfrac{x^2}{x^2-4} - \dfrac{x+1}{x+2} = \dfrac{x^2}{(x-2)(x+2)} - \dfrac{x+1}{x+2} = \dfrac{x^2}{(x-2)(x+2)} + \dfrac{-(x+1)(x-2)}{(x-2)(x+2)}$

$= \dfrac{x^2-\left(x^2-x-2\right)}{(x-2)(x+2)} = \dfrac{x+2}{(x-2)(x+2)} = \dfrac{1}{x-2}$

(d) $\dfrac{\frac{y}{x}-\frac{x}{y}}{\frac{1}{y}-\frac{1}{x}} = \dfrac{\frac{y}{x}-\frac{x}{y}}{\frac{1}{y}-\frac{1}{x}} \cdot \dfrac{xy}{xy} = \dfrac{y^2-x^2}{x-y} = \dfrac{(y-x)(y+x)}{x-y} = \dfrac{-(x-y)(y+x)}{x-y} = -(x+y)$

12. (a) $\dfrac{6}{\sqrt[3]{4}} = \dfrac{6}{\sqrt[3]{2^2}} = \dfrac{6}{\sqrt[3]{2^2}} \cdot \dfrac{\sqrt[3]{2}}{\sqrt[3]{2}} = \dfrac{6\sqrt[3]{2}}{2} = 3\sqrt[3]{2}$

(b) $\dfrac{\sqrt{10}}{\sqrt{5}-2} = \dfrac{\sqrt{10}}{\sqrt{5}-2} \cdot \dfrac{\sqrt{5}+2}{\sqrt{5}+2} = \dfrac{\sqrt{10}\left(\sqrt{5}+2\right)}{5-4} = \sqrt{50}+2\sqrt{10} = 5\sqrt{2}+2\sqrt{10}$

(c) $\dfrac{1}{1+\sqrt{x}} = \dfrac{1}{1+\sqrt{x}} \cdot \dfrac{1-\sqrt{x}}{1-\sqrt{x}} = \dfrac{1-\sqrt{x}}{1-x}$

13. (a) $4x-3 = 2x+7 \Leftrightarrow 4x-2x = 7-(-3) \Leftrightarrow 2x = 10 \Leftrightarrow x = 5$.

(b) $8x^3 = -125 \Leftrightarrow \sqrt[3]{8x^3} = \sqrt[3]{-125} \Leftrightarrow 2x = -5 \Leftrightarrow x = -\frac{5}{2}$.

(c) $x^{2/3} - 64 = 0 \Leftrightarrow x^{2/3} = 64 \Leftrightarrow \left(x^{2/3}\right)^{3/2} = 64^{3/2} \Leftrightarrow x = 8^3 = 512$.

(d) $\dfrac{x}{2x-5} = \dfrac{x+3}{2x-1} \Leftrightarrow x(2x-1) = (x+3)(2x-5) \Leftrightarrow 2x^2-x = 2x^2-5x+6x-15 \Leftrightarrow -x = x-15 \Leftrightarrow 2x = 15$
$\Leftrightarrow x = \frac{15}{2}$.

(e) $3(x+1)^2 - 18 = 0 \Leftrightarrow 3(x+1)^2 = 18 \Leftrightarrow (x+1)^2 = 6 \Leftrightarrow x+1 = \pm\sqrt{6} \Leftrightarrow x = \pm\sqrt{6}-1$

14. $E = mc^2 \Leftrightarrow \dfrac{E}{m} = c^2 \Leftrightarrow c = \sqrt{\dfrac{E}{m}}$. (We take the positive root because c represents the speed of light, which is positive.)

15. Let t be the time (in hours) it took the trucker to drive from Amity to Belleville. Then $4.4-t$ is the time it took the trucker to drive from Belleville to Amity. Since the distance from Amity to Belleville equals the distance from Belleville to Amity, we have $50t = 60(4.4-t) \Leftrightarrow 50t = 264-60t \Leftrightarrow 110t = 264 \Leftrightarrow t = 2.4$ hours. Thus the distance is $50(2.4) = 120$ mi.

FOCUS ON MODELING Making Optimal Decisions

1. (a) The total cost is $\begin{pmatrix}\text{cost of}\\\text{printer}\end{pmatrix}+\begin{pmatrix}\text{maintenance}\\\text{cost}\end{pmatrix}\begin{pmatrix}\text{number}\\\text{of months}\end{pmatrix}+\begin{pmatrix}\text{printing}\\\text{cost}\end{pmatrix}\begin{pmatrix}\text{number}\\\text{of months}\end{pmatrix}$. Each month the printing cost is $8000\cdot 0.03=240$. Thus we get $C_1=5800+25n+240n=5800+265n$.

(b) In this case the cost is $\begin{pmatrix}\text{rental}\\\text{cost}\end{pmatrix}\begin{pmatrix}\text{number}\\\text{of months}\end{pmatrix}+\begin{pmatrix}\text{printing}\\\text{cost}\end{pmatrix}\begin{pmatrix}\text{number}\\\text{of months}\end{pmatrix}$. Each month the printing cost is $8000\cdot 0.06=480$. Thus we get $C_2=95n+480n=575n$.

(c)

Years	n	Purchase	Rental
1	12	8,980	6,900
2	24	12,160	13,800
3	36	15,340	20,700
4	48	18,520	27,600
5	60	21,700	34,500
6	72	24,880	41,400

(d) The cost is the same when $C_1=C_2$ are equal. So $5800+265n=575n \Leftrightarrow 5800=310n \Leftrightarrow n\approx 18.71$ months.

2. (a) The cost of Plan 1 is $3\cdot\begin{pmatrix}\text{daily}\\\text{cost}\end{pmatrix}+\begin{pmatrix}\text{cost per}\\\text{mile}\end{pmatrix}\begin{pmatrix}\text{number}\\\text{of miles}\end{pmatrix}=3\cdot 65+0.15x=195+0.15x$.

The cost of Plan 2 is $3\cdot\begin{pmatrix}\text{daily}\\\text{cost}\end{pmatrix}=3\cdot 90=270$.

(b) When $x=400$, Plan 1 costs $195+0.15\,(400)=\$255$ and Plan 2 costs \$270, so Plan 1 is cheaper. When $x=800$, Plan 1 costs $195+0.15\,(800)=\$315$ and Plan 2 costs \$270, so Plan 2 is cheaper.

(c) The cost is the same when $195+0.15x=270 \Leftrightarrow 0.15=75x \Leftrightarrow x=500$. So both plans cost \$270 when you drive 500 miles.

3. (a) The total cost is $\begin{pmatrix}\text{setup}\\\text{cost}\end{pmatrix}+\begin{pmatrix}\text{cost per}\\\text{tire}\end{pmatrix}\begin{pmatrix}\text{number}\\\text{of tires}\end{pmatrix}$. So $C=8000+22x$.

(b) The revenue is $\begin{pmatrix}\text{price per}\\\text{tire}\end{pmatrix}\begin{pmatrix}\text{number}\\\text{of tires}\end{pmatrix}$. So $R=49x$.

(c) Profit $=$ Revenue $-$ Cost. So $P=R-C=49x-(8000+22x)=27x-8000$.

(d) Break even is when profit is zero. Thus $27x-8000=0 \Leftrightarrow 27x=8000 \Leftrightarrow x\approx 296.3$. So they need to sell at least 297 tires to break even.

4. (a) *Option 1:* In this option the width is constant at 100. Let x be the increase in length. Then the additional area is width $\times \begin{pmatrix} \text{increase} \\ \text{in length} \end{pmatrix} = 100x$. The cost is the sum of the costs of moving the old fence, and of installing the new one. The cost of moving is $\$6 \cdot 100 = \$600$ and the cost of installation is $2 \cdot 10 \cdot x = 20x$, so the total cost is $C = 20x + 600$. Solving for x, we get $C = 20x + 600 \Leftrightarrow 20x = C - 600 \Leftrightarrow x = \dfrac{C - 600}{20}$. Substituting in the area we have $A_1 = 100\left(\dfrac{C - 600}{20}\right) = 5(C - 600) = 5C - 3{,}000$.

Option 2: In this option the length is constant at 180. Let y be the increase in the width. Then the additional area is length $\times \begin{pmatrix} \text{increase} \\ \text{in width} \end{pmatrix} = 180y$. The cost of moving the old fence is $6 \cdot 180 = \$1080$ and the cost of installing the new one is $2 \cdot 10 \cdot y = 20x$, so the total cost is $C = 20y + 1080$. Solving for y, we get $C = 20y + 1080 \Leftrightarrow 20y = C - 1080$ $\Leftrightarrow y = \dfrac{C - 1080}{20}$. Substituting in the area we have $A_2 = 180\left(\dfrac{C - 1080}{20}\right) = 9(C - 1080) = 9C - 9{,}720$.

(b)

Cost C	Area gain A_1 from Option 1	Area gain A_2 from Option 2
\$1100	2,500 ft^2	180 ft^2
\$1200	3,000 ft^2	1,080 ft^2
\$1500	4,500 ft^2	3,780 ft^2
\$2000	7,000 ft^2	8,280 ft^2
\$2500	9,500 ft^2	12,780 ft^2
\$3000	12,000 ft^2	17,280 ft^2

(c) If the farmer has only \$1200, Option 1 gives the greatest gain. If the farmer has only \$2000, Option 2 gives the greatest gain.

5. (a) Design 1 is a square and the perimeter of a square is four times the length of a side. $24 = 4x$, so each side is $x = 6$ feet long. Thus the area is $6^2 = 36$ ft^2.

Design 2 is a circle with perimeter $2\pi r$ and area πr^2. Thus we must solve $2\pi r = 24 \Leftrightarrow r = \dfrac{12}{\pi}$. Thus, the area is $\pi\left(\dfrac{12}{\pi}\right)^2 = \dfrac{144}{\pi} \approx 45.8$ ft^2. Design 2 gives the largest area.

(b) In Design 1, the cost is \$3 times the perimeter p, so $120 = 3p$ and the perimeter is 40 feet. By part (a), each side is then $\frac{40}{4} = 10$ feet long. So the area is $10^2 = 100$ ft^2.

In Design 2, the cost is \$4 times the perimeter p. Because the perimeter is $2\pi r$, we get $120 = 4(2\pi r)$ so $r = \dfrac{120}{8\pi} = \dfrac{15}{\pi}$. The area is $\pi r^2 = \pi\left(\dfrac{15}{\pi}\right)^2 = \dfrac{225}{\pi} \approx 71.6$ ft^2. Design 1 gives the largest area.

6. (a) Plan 1: Tomatoes every year. Profit $=$ acres $\times$ (Revenue $-$ cost) $= 100(1600 - 300) = 130{,}000$. Then for n years the profit is $P_1 = 130{,}000n$.

(b) Plan 2: Soybeans followed by tomatoes. The profit for two years is Profit $=$ acres $\times$ $\left[\begin{pmatrix} \text{soybean} \\ \text{revenue} \end{pmatrix} + \begin{pmatrix} \text{tomato} \\ \text{revenue} \end{pmatrix}\right] = 100(1200 + 1600) = 280{,}000$. Remember that no fertilizer is needed in this plan. Then for $2k$ years, the profit is $P_2 = 280{,}000k$.

(c) When $n = 10$, $P_1 = 130{,}000(10) = 1{,}300{,}000$. Since $2k = 10$ when $k = 5$, $P_2 = 280{,}000(5) = 1{,}400{,}000$. So Plan B is more profitable.

7. (a)

Data (GB)	Plan A	Plan B	Plan C
1	\$25	\$40	\$60
1.5	$25 + 5\,(2.00) = \$35$	$40 + 5\,(1.50) = \$47.50$	$60 + 5\,(1.00) = \$65$
2	$25 + 10\,(2.00) = \$45$	$40 + 10\,(1.50) = \$55$	$60 + 10\,(1.00) = \$70$
2.5	$25 + 15\,(2.00) = \$55$	$40 + 15\,(1.50) = \$62.50$	$60 + 15\,(1.00) = \$75$
3	$25 + 20\,(2.00) = \$65$	$40 + 20\,(1.50) = \$70$	$60 + 20\,(1.00) = \$80$
3.5	$25 + 25\,(2.00) = \$75$	$40 + 25\,(1.50) = \$77.50$	$60 + 25\,(1.00) = \$85$
4	$25 + 30\,(2.00) = \$85$	$40 + 30\,(1.50) = \$85$	$60 + 30\,(1.00) = \$90$

(b) For Plan A: $C_A = 25 + 2\,(10x - 10) = 20x + 5$. For Plan B: $C_B = 40 + 1.5\,(10x - 10) = 15x + 25$. For Plan C: $C_C = 60 + 1\,(10x - 10) = 10x + 50$. Note that these equations are valid only for $x \geq 1$.

(c) If 2.2 GB are used, Plan A costs $25 + 12\,(2) = \$49$, Plan B costs $40 + 12\,(1.5) = \$58$, and Plan C costs $60 + 12\,(1) = \$72$.

If 3.7 GB are used, Plan A costs $25 + 27\,(2) = \$79$, Plan B costs $40 + 27\,(1.5) = \$80.50$, and Plan C costs $60 + 27\,(1) = \$87$.

If 4.9 GB are used, Plan A costs $25 + 39\,(2) = \$103$, Plan B costs $40 + 39\,(1.5) = \$98.50$, and Plan C costs $60 + 39\,(1) = \$99$.

(d) **(i)** We set $C_A = C_B \Leftrightarrow 20x + 5 = 15x + 25 \Leftrightarrow 5x = 20 \Leftrightarrow x = 4$. Plans A and B cost the same when 4 GB are used.

(ii) We set $C_A = C_C \Leftrightarrow 20x + 5 = 10x + 50 \Leftrightarrow 10x = 45 \Leftrightarrow x = 4.5$. Plans A and C cost the same when 4.5 GB are used.

(iii) We set $C_B = C_C \Leftrightarrow 15x + 25 = 10x + 50 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5$. Plans B and C cost the same when 5 GB are used.

8. (a) In this plan, Company A gets \$3.2 million and Company B gets \$3.2 million. Company A's investment is \$1.4 million, so they make a profit of $3.2 - 1.4 = \$1.8$ million. Company B's investment is \$2.6 million, so they make a profit of $3.2 - 2.6 = \$0.6$ million. So Company A makes three times the profit that Company B does, which is not fair.

(b) The original investment is $1.4 + 2.6 = \$4$ million. So after giving the original investment back, they then share the profit of \$2.4 million. So each gets an additional \$1.2 million. So Company A gets a total of $1.4 + 1.2 = \$2.6$ million and Company B gets $2.6 + 1.2 = \$3.8$ million. So even though Company B invests more, they make the same profit as Company A, which is not fair.

(c) The original investment is \$4 million, so Company A gets $\frac{1.4}{4} \cdot 6.4 = \$2.24$ million and Company B gets $\frac{2.6}{4} \cdot 6.4 = \$4.16$ million. This seems the fairest.