Precalculus with Limits 5th edition Larson Solutions Manual

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Complete Solutions Manual

Precalculus with Limits

FIFTH EDITION

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CHAPTER 1
Functions and Their Graphs

CHAPTER 1
Functions and Their Graphs

Section 1.1 Rectangular Coordinates

1. Cartesian

2. Midpoint Formula

3. The x-axis is the horizontal real number line.
Matches (c).

4. The y-axis is the vertical real number line.
Matches (f).

5. The origin is the point of intersection of the vertical and horizontal axes.
Matches (a).

6. The quadrants are four regions of the coordinate plane.
Matches (d).

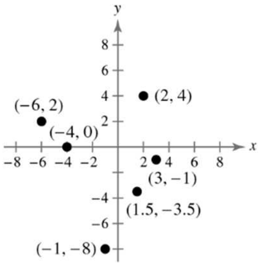
7. An x-coordinate is the directed distance from the y-axis. Matches (e).

8. A y-coordinate is the directed distance from the x-axis.
Matches (b).

9. $A: (2, 6)$, $B: (-6, -2)$, $C: (4, -4)$, $D: (-3, 2)$

10. $A: \left(\frac{3}{2}, -4\right)$; $B: (0, -2)$; $C: \left(-3, \frac{5}{2}\right)$; $D: (-6, 0)$

11.



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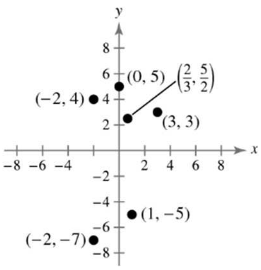


13. $(-3, 4)$

14. $(-12, 0)$

15. $x > 0$ and $y < 0$ in Quadrant IV.

16. $x < 0$ and $y < 0$ in Quadrant III.

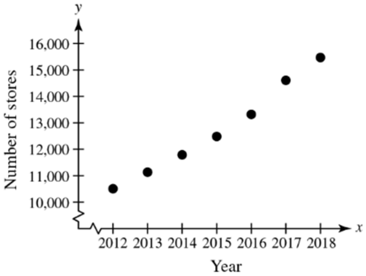
17. $x = -4$ and $y > 0$ in Quadrant II.

18. $x < 0$ and $y = 7$ in Quadrant II.

19. $x + y = 0, x \neq 0, y \neq 0$ means
$x = -y$ or $y = -x$. This occurs in Quadrant II or IV.

20. $(x, y), xy > 0$ means x and y have the same signs.
This occurs in Quadrant I or III.

21.



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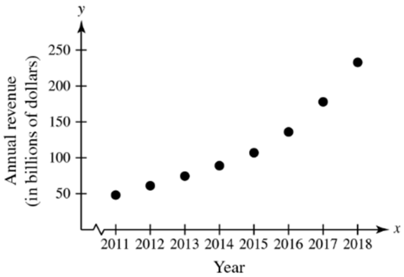


23.
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - (-2))^2 + (-6 - 6)^2} \\ &= \sqrt{(5)^2 + (-12)^2} \\ &= \sqrt{25 + 144} \\ &= 13 \text{ units} \end{aligned}$$

24.
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0 - 8)^2 + (20 - 5)^2} \\ &= \sqrt{(-8)^2 + (15)^2} \\ &= \sqrt{64 + 225} \\ &= \sqrt{289} \\ &= 17 \text{ units} \end{aligned}$$

25. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-5 - 1)^2 + (-1 - 4)^2}$
$= \sqrt{(-6)^2 + (-5)^2}$
$= \sqrt{36 + 25}$
$= \sqrt{61}$ units

26. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(3 - 1)^2 + (-2 - 3)^2}$
$= \sqrt{(2)^2 + (-5)^2}$
$= \sqrt{4 + 25}$
$= \sqrt{29}$ units

27. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{\left(2 - \frac{1}{2}\right)^2 + \left(-1 - \frac{4}{3}\right)^2}$
$= \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{7}{3}\right)^2}$
$= \sqrt{\frac{9}{4} + \frac{49}{9}}$
$= \sqrt{\frac{277}{36}}$
$= \frac{\sqrt{277}}{6}$ units

28. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-3.9 - 9.5)^2 + (8.2 - (-2.6))^2}$
$= \sqrt{(-13.4)^2 + (10.8)^2}$
$= \sqrt{179.56 + 116.64}$
$= \sqrt{296.2}$

29. (a) $(1, 0), (13, 5)$

Distance $= \sqrt{(13 - 1)^2 + (5 - 0)^2}$
$= \sqrt{12^2 + 5^2} = \sqrt{169} = 13$

$(13, 5), (13, 0)$

Distance $= |5 - 0| = |5| = 5$

$(1, 0), (13, 0)$

Distance $= |1 - 13| = |-12| = 12$

(b) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

30. (a) The distance between $(-1, 1)$ and $(9, 1)$ is 10.

The distance between $(9, 1)$ and $(9, 4)$ is 3.

The distance between $(-1, 1)$ and $(9, 4)$ is

$\sqrt{(9 - (-1))^2 + (4 - 1)^2} = \sqrt{100 + 9} = \sqrt{109}.$

(b) $10^2 + 3^2 = 109 = \left(\sqrt{109}\right)^2$

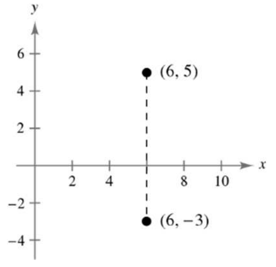
31. $d_1 = \sqrt{(4 - 2)^2 + (0 - 1)^2} = \sqrt{4 + 1} = \sqrt{5}$
$d_2 = \sqrt{(4 + 1)^2 + (0 + 5)^2} = \sqrt{25 + 25} = \sqrt{50}$
$d_3 = \sqrt{(2 + 1)^2 + (1 + 5)^2} = \sqrt{9 + 36} = \sqrt{45}$
$\left(\sqrt{5}\right)^2 + \left(\sqrt{45}\right)^2 = \left(\sqrt{50}\right)^2$

32. $d_1 = \sqrt{(3 - (-1))^2 + (5 - 3)^2} = \sqrt{16 + 4} = \sqrt{20}$
$d_2 = \sqrt{(5 - 3)^2 + (1 - 5)^2} = \sqrt{4 + 16} = \sqrt{20}$
$d_3 = \sqrt{(5 - (-1))^2 + (1 - 3)^2} = \sqrt{36 + 4} = \sqrt{40}$
$\left(\sqrt{20}\right)^2 + \left(\sqrt{20}\right)^2 = \left(\sqrt{40}\right)^2$

33. $d_1 = \sqrt{(1 - 3)^2 + (-3 - 2)^2} = \sqrt{4 + 25} = \sqrt{29}$
$d_2 = \sqrt{(3 + 2)^2 + (2 - 4)^2} = \sqrt{25 + 4} = \sqrt{29}$
$d_3 = \sqrt{(1 + 2)^2 + (-3 - 4)^2} = \sqrt{9 + 49} = \sqrt{58}$
$d_1 = d_2$

34. $d_1 = \sqrt{(4 - 2)^2 + (9 - 3)^2} = \sqrt{4 + 36} = \sqrt{40}$
$d_2 = \sqrt{(-2 - 4)^2 + (7 - 9)^2} = \sqrt{36 + 4} = \sqrt{40}$
$d_3 = \sqrt{(2 - (-2))^2 + (3 - 7)^2} = \sqrt{16 + 16} = \sqrt{32}$
$d_1 = d_2$

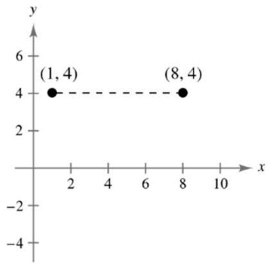
35. (a)



(b) $d = \sqrt{(5 - (-3))^2 + (6 - 6)^2} = \sqrt{64} = 8$

(c) $\left(\frac{6 + 6}{2}, \frac{5 + (-3)}{2}\right) = (6, 1)$

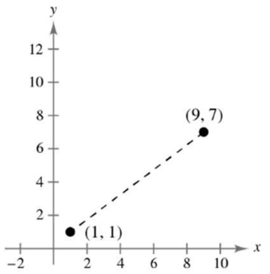
36. (a)



(b) $d = \sqrt{(4-4)^2 + (8-1)^2} = \sqrt{49} = 7$

(c) $\left(\dfrac{1+8}{2}, \dfrac{4+4}{2}\right) = \left(\dfrac{9}{2}, 4\right)$

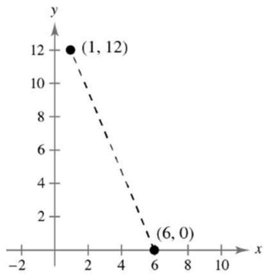
37. (a)



(b) $d = \sqrt{(9-1)^2 + (7-1)^2} = \sqrt{64+36} = 10$

(c) $\left(\dfrac{9+1}{2}, \dfrac{7+1}{2}\right) = (5, 4)$

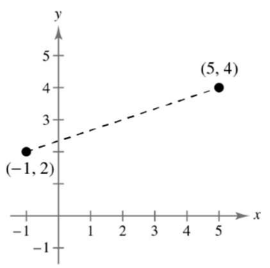
38. (a)



(b) $d = \sqrt{(1-6)^2 + (12-0)^2} = \sqrt{25+144} = 13$

(c) $\left(\dfrac{1+6}{2}, \dfrac{12+0}{2}\right) = \left(\dfrac{7}{2}, 6\right)$

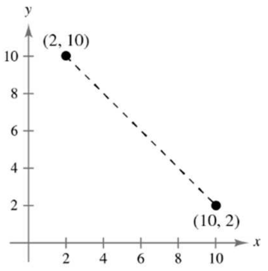
39. (a)



(b) $d = \sqrt{(5+1)^2 + (4-2)^2} + = \sqrt{36+4} = 2\sqrt{10}$

(c) $\left(\dfrac{-1+5}{2}, \dfrac{2+4}{2}\right) = (2, 3)$

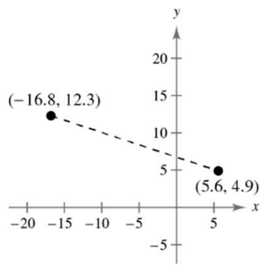
40. (a)



(b) $d = \sqrt{(2-10)^2 + (10-2)^2}$
$= \sqrt{64+64} = 8\sqrt{2}$

(c) $\left(\dfrac{2+10}{2}, \dfrac{10+2}{2}\right) = (6, 6)$

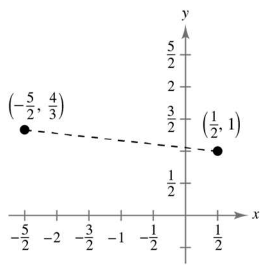
41. (a)



(b) $d = \sqrt{(-16.8-5.6)^2 + (12.3-4.9)^2}$
$= \sqrt{501.76 + 54.76} = \sqrt{556.52}$

(c) $\left(\dfrac{-16.8+5.6}{2}, \dfrac{12.3+4.9}{2}\right) = (-5.6, 8.6)$

42. (a)



(b) $d = \sqrt{\left(\dfrac{1}{2} + \dfrac{5}{2}\right)^2 + \left(1 - \dfrac{4}{3}\right)^2}$
$= \sqrt{9 + \dfrac{1}{9}} = \dfrac{\sqrt{82}}{3}$

(c) $\left(\dfrac{-(5/2)+(1/2)}{2}, \dfrac{(4/3)+1}{2}\right) = \left(-1, \dfrac{7}{6}\right)$

43. $d = \sqrt{(42 - 18)^2 + (50 - 12)^2}$
$= \sqrt{24^2 + 38^2}$
$= \sqrt{2020}$
$= 2\sqrt{505}$
≈ 45

The pass is about 45 yards.

44. $d = \sqrt{120^2 + 150^2}$
$= \sqrt{36,900}$
$= 30\sqrt{41}$
≈ 192.09

The plane flies about 192 kilometers.

45. $\text{midpoint} = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
$= \left(\dfrac{2016 + 2018}{2}, \dfrac{485.9 + 514.4}{2}\right)$
$= (2017, 500.15)$

In 2017, the sales were about \$500.15 billion.

46. $\text{midpoint} = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$. In this problem, you have $\left(\dfrac{2017 + 2019}{2}, \dfrac{6.16 + y_2}{2}\right) = (2018, 7.57)$.

So, $\dfrac{6.16 + y_2}{2} = 7.57 \Rightarrow y_2 = 8.98$.

In 2019, the earnings per share were about \$8.98.

47. $(-2 + 2, \; -4 + 5) = (0, 1)$
$(2 + 2, \; -3 + 5) = (4, 2)$
$(-1 + 2, \; -1 + 5) = (1, 4)$

48. $(-3 + 6, 6 - 3) = (3, 3)$
$(-5 + 6, 3 - 3) = (1, 0)$
$(-3 + 6, 0 - 3) = (3, -3)$
$(-1 + 6, 3 - 3) = (5, 0)$

49. $(-7 + 4, -2 + 8) = (-3, 6)$
$(-2 + 4, 2 + 8) = (2, 10)$
$(-2 + 4, -4 + 8) = (2, 4)$
$(-7 + 4, -4 + 8) = (-3, 4)$

50. $(5 - 10, 8 - 6) = (-5, 2)$
$(3 - 10, 6 - 6) = (-7, 0)$
$(7 - 10, 6 - 6) = (-3, 0)$

51. True. Because $x < 0$ and $y > 0$, $2x < 0$ and $-3y < 0$, which is located in Quadrant III.

52. False. The Midpoint Formula would be used 15 times.

53. True. Two sides of the triangle have lengths $\sqrt{149}$ and the third side has a length of $\sqrt{18}$.

54. False. The polygon could be a rhombus. For example, consider the points $(4, 0)$, $(0, 6)$, $(-4, 0)$, and $(0, -6)$.

55. The y-coordinate of a point on the x-axis is 0. The x-coordinates of a point on the y-axis is 0.

56. The x- and y-coordinates are switched. For example, $(3, 2)$ should be $(2, 3)$.

57. Because $x_m = \dfrac{x_1 + x_2}{2}$ and $y_m = \dfrac{y_1 + y_2}{2}$ we have:

$$\begin{aligned} 2x_m &= x_1 + x_2 & \qquad 2y_m &= y_1 + y_2 \\ 2x_m - x_1 &= x_2 & \qquad 2y_m - y_1 &= y_2 \end{aligned}$$

So, $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$.

58. The midpoint of the given line segment is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

The midpoint between (x_1, y_1) and $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is $\left(\dfrac{x_1 + \frac{x_1 + x_2}{2}}{2}, \dfrac{y_1 + \frac{y_1 + y_2}{2}}{2}\right) = \left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right)$.

The midpoint between $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ and (x_2, y_2) is $\left(\dfrac{\frac{x_1 + x_2}{2} + x_2}{2}, \dfrac{\frac{y_1 + y_2}{2} + y_2}{2}\right) = \left(\dfrac{x_1 + 3x_2}{4}, \dfrac{y_1 + 3y_2}{4}\right)$.

So, the three points are $\left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right)$, $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$, and $\left(\dfrac{x_1 + 3x_2}{4}, \dfrac{y_1 + 3y_2}{4}\right)$.

59. Use the Midpoint Formula to prove the diagonals of the parallelogram bisect each other.

$$\left(\frac{b+a}{2}, \frac{c+0}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$$

$$\left(\frac{a+b+0}{2}, \frac{c+0}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$$

60. (a) Because (x_0, y_0) lies in Quadrant II, $(x_0, -y_0)$ must lie in Quadrant III. Matches (ii).

(b) Because (x_0, y_0) lies in Quadrant II, $(-2x_0, y_0)$ must lie in Quadrant I. Matches (iii).

(c) Because (x_0, y_0) lies in Quadrant II, $\left(x_0, \frac{1}{2}y_0\right)$ must lie in Quadrant II. Matches (iv).

(d) Because (x_0, y_0) lies in Quadrant II, $(-x_0, -y_0)$ must lie in Quadrant IV. Matches (i).

61. (a) **First Set**

$$d(A, B) = \sqrt{(2-2)^2 + (3-6)^2} = \sqrt{9} = 3$$

$$d(B, C) = \sqrt{(2-6)^2 + (6-3)^2} = \sqrt{16+9} = 5$$

$$d(A, C) = \sqrt{(2-6)^2 + (3-3)^2} = \sqrt{16} = 4$$

Because $3^2 + 4^2 = 5^2$, A, B, and C are the vertices of a right triangle.

Second Set

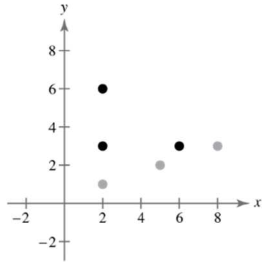
$$d(A, B) = \sqrt{(8-5)^2 + (3-2)^2} = \sqrt{10}$$

$$d(B, C) = \sqrt{(5-2)^2 + (2-1)^2} = \sqrt{10}$$

$$d(A, C) = \sqrt{(8-2)^2 + (3-1)^2} = \sqrt{40}$$

A, B, and C are the vertices of an isosceles triangle or are collinear: $\sqrt{10} + \sqrt{10} = 2\sqrt{10} = \sqrt{40}$.

(b)



First set: Not collinear

Second set: Collinear.

(c) A set of three points is collinear when the sum of two distances among the points is exactly equal to the third distance.

62. (a) distance $= \sqrt{(x_1 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(y_2 - y_1)^2} = |y_2 - y_1|$

(b) distance $= \sqrt{(x_2 - x_1)^2 + (y_1 - y_1)^2}$
$= \sqrt{(x_2 - x_1)^2} = |x_2 - x_1|$

Answers will vary.

63. $4x - 6$

(a) $4(-1) - 6 = -4 - 6 = -10$

(b) $4(0) - 6 = 0 - 6 = -6$

64. $9 - 7x$

(a) $9 - 7(-3) = 9 + 21 = 30$

(b) $9 - 7(3) = 9 - 21 = -12$

65. (a) When $x = -3$, $2x^3 = 2(-3)^3 = 2(-27) = -54$.

(b) When $x = 0$, $2x^3 = 2(0)^3 = 0$.

66. (a) When $x = 0$, $-3x^{-4} = \frac{-3}{x^4} = -\frac{3}{0}$. Division by 0 is undefined.

(b) When $x = -2$, $-3x^{-4} = \frac{-3}{x^4} = \frac{-3}{(-2)^4} = -\frac{3}{16}$.

67. $(-x)^2 - 2 = x^2 - 2$

68. $(-x)^3 - (-x)^2 + 2 = -x^3 - x^2 + 2$

69. $-x^2 + (-x)^2 = -x^2 + x^2 = 0$

70. $-x^3 + (-x)^3 + (-x)^4 - x^4 = -x^3 - x^3 + x^4 - x^4$
$= -2x^3$

71. (a) $P = R - C$
$= 135x - (93x + 35{,}000)$
$= 42x - 35{,}000$

(b) $P = 42(5000) - 35{,}000 = \$175{,}000$

72. (a) Time to copy one page: $\frac{1}{50}$ min

(b) Time to copy x pages: $\frac{x}{50}$ min

(c) Time to copy 120 pages: $\frac{120}{50} = \frac{12}{5} = 2.4$ min

(d) Time to copy 10,000 pages: $\frac{10{,}000}{50} = 200$ min

Section 1.2 Graphs of Equations

1. solution or solution point

2. graph

3. intercepts

4. y-axis

5. origin

6. numerical

7. Two other approaches to solve problems mathematically are algebraic and graphical.

8. Let (x, y) be any point on the circle. The distance between the center (h, k) and (x, y) is the radius r.

So, $\sqrt{(x-h)^2 + (y-k)^2} = r$.

9. (a) $(2, 0)$: $(2)^2 - 3(2) + 2 \stackrel{?}{=} 0$

$$4 - 6 + 2 \stackrel{?}{=} 0$$
$$0 = 0$$

Yes, the point *is* on the graph.

(b) $(-2, 8)$: $(-2)^2 - 3(-2) + 2 \stackrel{?}{=} 8$

$$4 + 6 + 2 \stackrel{?}{=} 8$$
$$12 \neq 8$$

No, the point *is not* on the graph.

10. (a) $(0, 2)$: $2 \stackrel{?}{=} \sqrt{0 + 4}$

$$2 = 2$$

Yes, the point *is* on the graph.

(b) $(5, 3)$: $3 \stackrel{?}{=} \sqrt{5 + 4}$

$$3 \stackrel{?}{=} \sqrt{9}$$
$$3 = 3$$

Yes, the point *is* on the graph.

11. (a) $(1, 5)$: $5 \stackrel{?}{=} 4 - |1 - 2|$

$$5 \stackrel{?}{=} 4 - 1$$
$$5 \neq 3$$

No, the point *is not* on the graph.

(b) $(6, 0)$: $0 \stackrel{?}{=} 4 - |6 - 2|$

$$0 \stackrel{?}{=} 4 - 4$$
$$0 = 0$$

Yes, the point *is* on the graph.

12. (a) $(6, 0)$: $2(6)^2 + 5(0)^2 \stackrel{?}{=} 8$

$$2(36) + 5(0) \stackrel{?}{=} 8$$
$$72 + 0 \stackrel{?}{=} 8$$
$$72 \neq 8$$

No, the point *is not* on the graph.

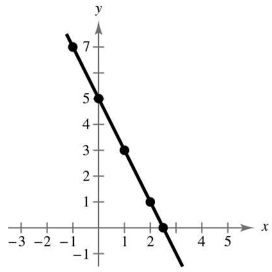
(b) $(0, 4)$: $2(0)^2 + 5(4)^2 \stackrel{?}{=} 8$

$$2(0) + 5(16) \stackrel{?}{=} 8$$
$$0 + 80 \stackrel{?}{=} 8$$
$$80 \neq 8$$

No, the point *is not* on the graph.

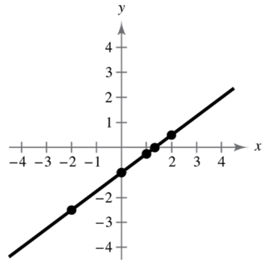
13. $y = -2x + 5$

x	-1	0	1	2	$\frac{5}{2}$
y	7	5	3	1	0
(x, y)	$(-1, 7)$	$(0, 5)$	$(1, 3)$	$(2, 1)$	$\left(\frac{5}{2}, 0\right)$



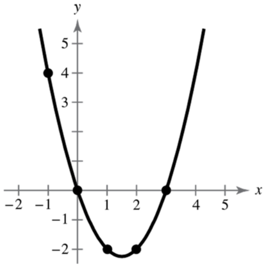
14. $y = \frac{3}{4}x - 1$

x	-2	0	1	$\frac{4}{3}$	2
y	$-\frac{5}{2}$	-1	$-\frac{1}{4}$	0	$\frac{1}{2}$
(x, y)	$\left(-2, -\frac{5}{2}\right)$	$(0, -1)$	$\left(1, -\frac{1}{4}\right)$	$\left(\frac{4}{3}, 0\right)$	$\left(2, \frac{1}{2}\right)$



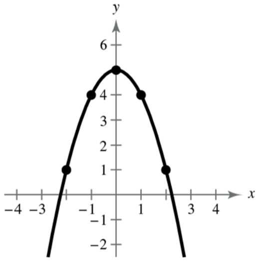
15. $y = x^2 - 3x$

x	-1	0	1	2	3
y	4	0	-2	-2	0
(x, y)	$(-1, 4)$	$(0, 0)$	$(1, -2)$	$(2, -2)$	$(3, 0)$



16. $y = 5 - x^2$

x	-2	-1	0	1	2
y	1	4	5	4	1
x, y	$(-2, 1)$	$(-1, 4)$	$(0, 5)$	$(1, 4)$	$(2, 1)$



17. x-intercept: $(-2, 0)$

y-intercept: $(0, 2)$

18. x-intercept: $(4, 0)$

y-intercepts: $(0, \pm 2)$

19. x-intercept: $(3, 0)$

y-intercept: $(0, 9)$

20. x-intercepts: $(\pm 2, 0)$

y-intercept: $(0, 16)$

21. $y = 5x - 6$

Let $y = 0$.

$0 = 5x - 6$

$6 = 5x$

$\frac{6}{5} = x$.

x-intercept: $\left(\frac{6}{5}, 0\right)$

Let $x = 0$.

$y = 5(0) - 6$

$y = 0 - 6$

$y = -6$

y-intercept: $(0, -6)$

22. $y = 8 - 3x$

Let $y = 0$.

$0 = 8 - 3x$

$3x = 8$

$x = \frac{8}{3}$

x-intercept: $\left(\frac{8}{3}, 0\right)$

Let $x = 0$.

$y = 8 - 3(0)$

$y = 8 - 0$

$y = 8$

y-intercept: $(0, 8)$

23. $y = \sqrt{x + 4}$

Let $y = 0$.

$0 = \sqrt{x + 4}$

$0 = x + 4$

$-4 = x$

x-intercept: $(-4, 0)$

Let $x = 0$.

$y = \sqrt{0 + 4}$

$y = \sqrt{4}$

$y = 2$

y-intercept: $(0, 2)$

24. $y = \sqrt{2x - 1}$

Let $y = 0$.

$0 = \sqrt{2x - 1}$

$0 = 2x - 1$

$1 = 2x$

$x = \frac{1}{2}$

x-intercept: $\left(\frac{1}{2}, 0\right)$

Let $x = 0$.

$y = \sqrt{2(0) - 1}$

$y = \sqrt{0 - 1}$

$y = \sqrt{-1}$

y-intercept: None

25. $y = |3x - 7|$

Let $y = 0$.

$0 = |3x - 7|$

$0 = 3x - 7$

$7 = 3x$

$\frac{7}{3} = x$

x-intercept: $\left(\frac{7}{3}, 0\right)$

Let $x = 0$.

$y = |3(0) - 7|$

$y = |-7|$

$y = 7$

y-intercept: $(0, 7)$

26. $y = -|x + 10|$

Let $y = 0$.

$0 = -|x + 10|$

$0 = x + 10$

$-10 = x$

x-intercept: $(-10, 0)$

Let $x = 0$.

$y = -|(0) + 10|$

$y = -|10|$

$y = -10$

y-intercept: $(0, -10)$

27. $y = 2x^3 - 4x^2$

Let $y = 0$.

$0 = 2x^3 - 4x^2$

$0 = 2x^2(x - 2)$

$x = 0 \quad \text{or} \quad x = 2$

x-intercepts: $(0, 0), (2, 0)$

Let $x = 0$.

$y = 2(0)^3 - 4(0)^2$

$y = 0 - 0$

$y = 0$

y-intercept: $(0, 0)$

28. $y^2 = x + 1$

Let $y = 0$.

$0 = x + 1$

$-1 = x$

x-intercept: $(-1, 0)$

Let $x = 0$.

$y^2 = (0) + 1$

$y^2 = 1$

$y = \pm\sqrt{1}$

$y = \pm 1$

y-intercepts: $(0, \pm 1)$

29. $x^2 - y = 0$

$(-x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow$ y-axis symmetry

$x^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No x-axis symmetry

$(-x)^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No origin symmetry

30. $x - y^2 = 0$

$(-x) - y^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No y-axis symmetry

$x - (-y)^2 = 0 \Rightarrow x - y^2 = 0 \Rightarrow$ x-axis symmetry

$(-x) - (-y)^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No origin symmetry

31. $y = x^3$

$y = (-x)^3 \Rightarrow y = -x^3 \Rightarrow$ No y-axis symmetry

$-y = x^3 \Rightarrow y = -x^3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow y = x^3 \Rightarrow$ Origin symmetry

32. $y = x^4 - x^2 + 3$

$y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = x^4 - x^2 + 3 \Rightarrow$ y-axis symmetry

$-y = x^4 - x^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No origin symmetry

33. $y = \dfrac{x}{x^2 + 1}$

$y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No y-axis symmetry

$-y = \dfrac{x}{x^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow -y = \dfrac{-x}{x^2 + 1} \Rightarrow y = \dfrac{x}{x^2 + 1} \Rightarrow$ Origin symmetry

34. $y = \dfrac{1}{1 + x^2}$

$y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{1}{1 + x^2} \Rightarrow$ y-axis symmetry

$-y = \dfrac{1}{1 + x^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No origin symmetry

35. $xy^2 + 10 = 0$

$(-x)y^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No y-axis symmetry

$x(-y)^2 + 10 = 0 \Rightarrow xy^2 + 10 = 0 \Rightarrow$ x-axis symmetry

$(-x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No origin symmetry

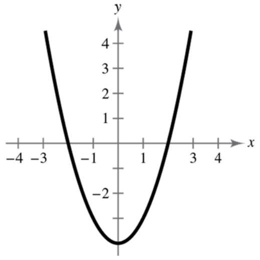
36. $xy = 4$

$(-x)y = 4 \Rightarrow xy = -4 \Rightarrow$ No y-axis symmetry

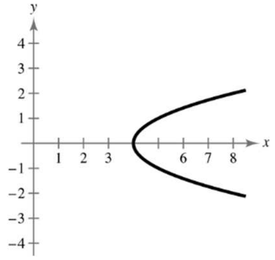
$x(-y) = 4 \Rightarrow xy = -4 \Rightarrow$ No x-axis symmetry

$(-x)(-y) = 4 \Rightarrow xy = 4 \Rightarrow$ Origin symmetry

37.



38.



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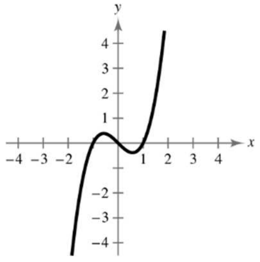

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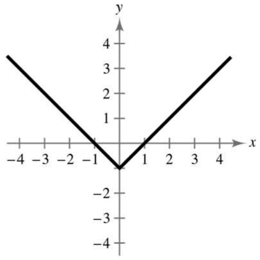
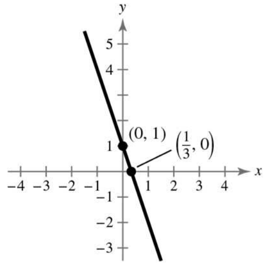


41. $y = -3x + 1$

x-intercept: $\left(\frac{1}{3}, 0\right)$

y-intercept: $(0, 1)$

No symmetry

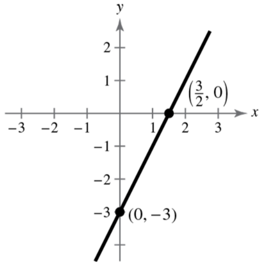


42. $y = 2x - 3$

x-intercept: $\left(\frac{3}{2}, 0\right)$

y-intercept: $(0, -3)$

No symmetry



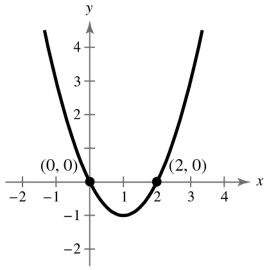
43. $y = x^2 - 2x$

x-intercepts: $(0, 0), (2, 0)$

y-intercept: $(0, 0)$

No symmetry

x	-1	0	1	2	3
y	3	0	-1	0	3

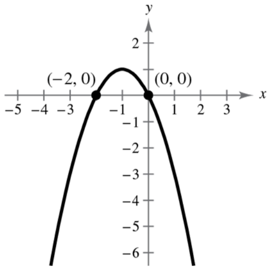


44. $y = -x^2 - 2x$

x-intercepts: $(-2, 0), (0, 0)$

y-intercept: $(0, 0)$

No symmetry



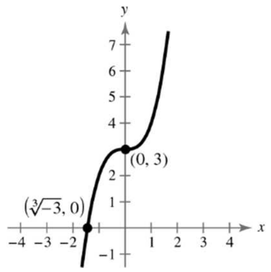
45. $y = x^3 + 3$

x-intercept: $\left(\sqrt[3]{-3}, 0\right)$

y-intercept: $(0, 3)$

No symmetry

x	-2	-1	0	1	2
y	-5	2	3	4	11

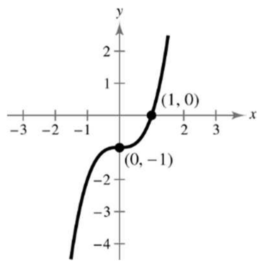


46. $y = x^3 - 1$

x-intercept: $(1, 0)$

y-intercept: $(0, -1)$

No symmetry



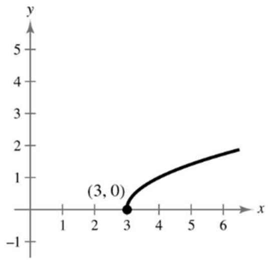
47. $y = \sqrt{x - 3}$

x-intercept: $(3, 0)$

y-intercept: none

No symmetry

x	3	4	7	12
y	0	1	2	3

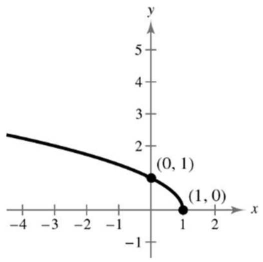


48. $y = \sqrt{1 - x}$

x-intercept: $(1, 0)$

y-intercept: $(0, 1)$

No symmetry



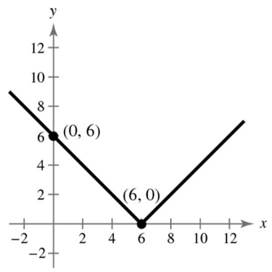
49. $y = |x - 6|$

x-intercept: $(6, 0)$

y-intercept: $(0, 6)$

No symmetry

x	-2	0	2	4	6	8	10
y	8	6	4	2	0	2	4

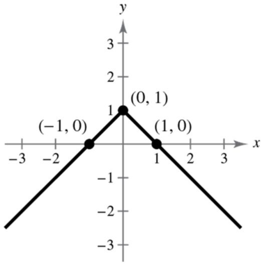


50. $y = 1 - |x|$

x-intercepts: $(1, 0), (-1, 0)$

y-intercept: $(0, 1)$

y-axis symmetry



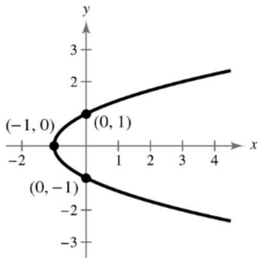
51. $x = y^2 - 1$

x-intercept: $(-1, 0)$

y-intercepts: $(0, -1), (0, 1)$

x-axis symmetry

x	-1	0	3
y	0	± 1	± 2

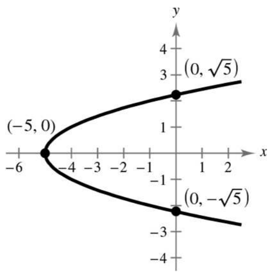


52. $x = y^2 - 5$

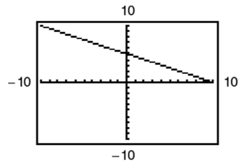
x-intercept: $(-5, 0)$

y-intercepts: $(0, \sqrt{5}), (0, -\sqrt{5})$

x-axis symmetry

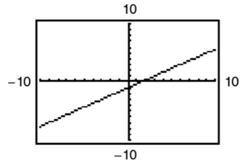


53. $y = 5 - \frac{1}{2}x$



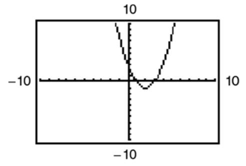
Intercepts: $(10, 0), (0, 5)$

54. $y = \frac{2}{3}x - 1$



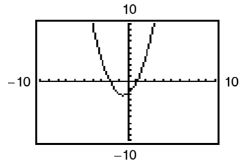
Intercepts: $(0, -1), \left(\frac{3}{2}, 0\right)$

55. $y = x^2 - 4x + 3$



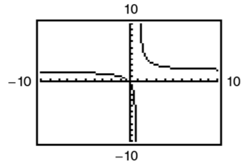
Intercepts: $(3, 0), (1, 0), (0, 3)$

56. $y = x^2 + x - 2$



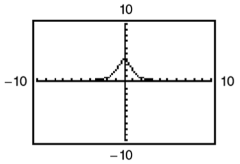
Intercepts: $(-2, 0), (1, 0), (0, -2)$

57. $y = \dfrac{2x}{x - 1}$



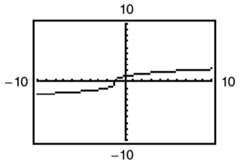
Intercept: $(0, 0)$

58. $y = \dfrac{4}{x^2 + 1}$



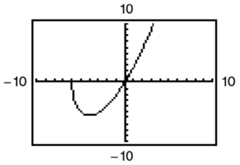
Intercept: $(0, 4)$

59. $y = \sqrt[3]{x + 1}$



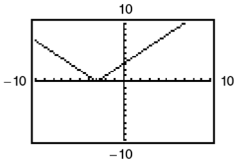
Intercepts: $(-1, 0), (0, 1)$

60. $y = x\sqrt{x + 6}$



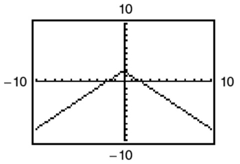
Intercepts: $(0, 0), (-6, 0)$

61. $y = |x + 3|$



Intercepts: $(-3, 0), (0, 3)$

62. $y = 2 - |x|$



Intercepts: $(\pm 2, 0), (0, 2)$

63. Center: $(0, 0)$; Radius: 3

$$\begin{aligned}(x - 0)^2 + (y - 0)^2 &= 3^2\\ x^2 + y^2 &= 9\end{aligned}$$

64. Center: $(0, 0)$; Radius: 7

$$\begin{aligned}(x - 0)^2 + (y - 0)^2 &= 7^2\\ x^2 + y^2 &= 49\end{aligned}$$

65. Center: $(-4, 5)$; Radius: 2

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2\\ \left[x - (-4)\right]^2 + \left[y - 5\right]^2 &= 2^2\\ (x + 4)^2 + (y - 5)^2 &= 4\end{aligned}$$

66. Center: $(1, -3)$; Radius: $\sqrt{11}$

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2\\ (x - 1)^2 + \left[y - (-3)\right]^2 &= \sqrt{11}^2\\ (x - 1)^2 + (y + 3)^2 &= 11\end{aligned}$$

67. Center: $(3, 8)$; Solution point: $(-9, 13)$

$$\begin{aligned}r &= \sqrt{(x - h)^2 + (y - k)^2}\\ &= \sqrt{(-9 - 3)^2 + (13 - 8)^2}\\ &= \sqrt{(-12)^2 + (5)^2}\\ &= \sqrt{144 + 25}\\ &= \sqrt{169}\\ &= 13\end{aligned}$$

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2\\ (x - 3)^2 + (y - 8)^2 &= 13^2\\ (x - 3)^2 + (y - 8)^2 &= 169\end{aligned}$$

68. Center: $(-2, -6)$; Solution point: $(1, -10)$

$$\begin{aligned}r &= \sqrt{(x - h)^2 + (y - k)^2}\\ &= \sqrt{\left[1 - (-2)\right]^2 + \left[-10 - (-6)\right]^2}\\ &= \sqrt{(3)^2 + (-4)^2}\\ &= \sqrt{9 + 16}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2\\ \left[x - (-2)\right]^2 + \left[y - (-6)\right]^2 &= 5^2\\ (x + 2)^2 + (y + 6)^2 &= 25\end{aligned}$$

69. Endpoints of a diameter: $(3, 2), (-9, -8)$

$$\begin{aligned} r &= \frac{1}{2}\sqrt{(-9-3)^2 + (-8-2)^2} \\ &= \frac{1}{2}\sqrt{(-12)^2 + (-10)^2} \\ &= \frac{1}{2}\sqrt{144 + 100} \\ &= \frac{1}{2}\sqrt{244} = \frac{1}{2}(2)\sqrt{61} = \sqrt{61} \end{aligned}$$

$$(h, k): \left(\frac{3 + (-9)}{2}, \frac{2 + (-8)}{2}\right) = \left(\frac{-6}{2}, \frac{-6}{2}\right) = (-3, -3)$$

$$\begin{aligned} (x-h)^2 + (y-k)^2 &= r^2 \\ [x - (-3)]^2 + [y - (-3)]^2 &= \left(\sqrt{61}\right)^2 \\ (x+3)^2 + (y+3)^2 &= 61 \end{aligned}$$

70. Endpoints of a diameter: $(11, -5), (3, 15)$

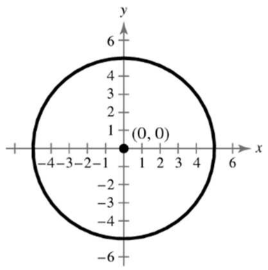
$$\begin{aligned} r &= \frac{1}{2}\sqrt{(3-11)^2 + [15 - (-5)]^2} \\ &= \frac{1}{2}\sqrt{(-8)^2 + (20)^2} \\ &= \frac{1}{2}\sqrt{64 + 400} \\ &= \frac{1}{2}\sqrt{464} = \frac{1}{2}(4)\sqrt{29} = 2\sqrt{29} \end{aligned}$$

$$(h, k): \left(\frac{11 + 3}{2}, \frac{-5 + 15}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$$

$$\begin{aligned} (x-h)^2 + (y-k)^2 &= r^2 \\ (x-7)^2 + (y-5)^2 &= \left(2\sqrt{29}\right)^2 \\ (x-7)^2 + (y-5)^2 &= 116 \end{aligned}$$

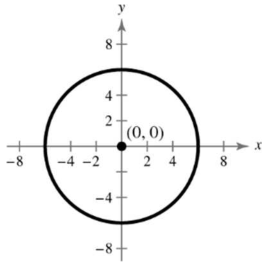
71. $x^2 + y^2 = 25$

Center: $(0, 0)$, Radius: 5



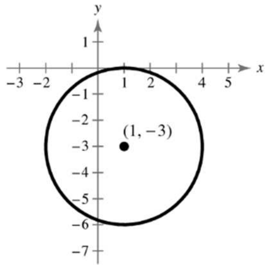
72. $x^2 + y^2 = 36$

Center: $(0, 0)$, Radius: 6



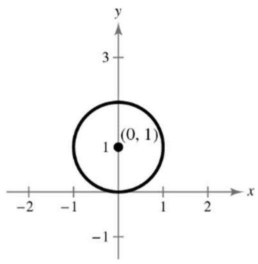
73. $(x-1)^2 + (y+3)^2 = 9$

Center: $(1, -3)$, Radius: 3



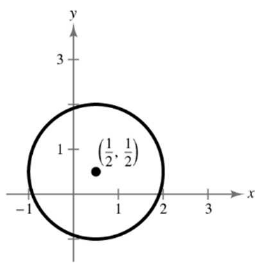
74. $x^2 + (y-1)^2 = 1$

Center: $(0, 1)$, Radius: 1



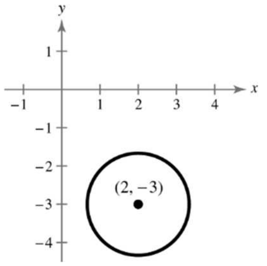
75. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

Center: $\left(\frac{1}{2}, \frac{1}{2}\right)$, Radius: $\frac{3}{2}$

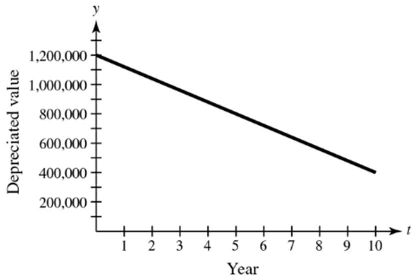


76. $(x - 2)^2 + (y + 3)^2 = \frac{16}{9}$

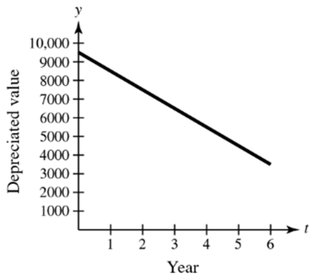
Center: $(2, -3)$, Radius: $\frac{4}{3}$



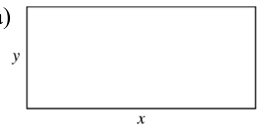
77. $y = 1{,}200{,}000 - 80{,}000t,\ 0 \le t \le 10$



78. $y = 9500 - 1000t,\ 0 \le t \le 6$



79. (a)



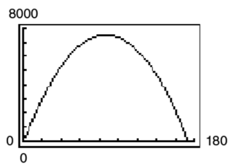
(b) $2x + 2y = \frac{1040}{3}$

$2y = \frac{1040}{3} - 2x$

$y = \frac{520}{3} - x$

$A = xy = x\left(\frac{520}{3} - x\right)$

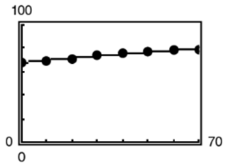
(c)



(d) When $x = y = 86\frac{2}{3}$ yards, the area is a maximum of $7511\frac{1}{9}$ square yards.

(e) A regulation NFL playing field is 120 yards long and $53\frac{1}{3}$ yards wide. The actual area is 6400 square yards.

80. (a)



The model fits the data well.

Each data value is close to the graph of the model.

(b) Using the graph or the model (with $t = 40$),

The life expectancy is approximately 75.2 years.

(c) The model and the line $y = 70.1$ intersect at $t = 11.1$, corresponding to the year 1961.

Algebraically, $\dfrac{68.0 + 0.33(11.1)}{1 + 0.002(11.1)} \approx 70.1.$

(d) The y-intercept is $(0, 68.0)$. In 1950 $(t = 0)$, the life expectancy of a child (at birth) was 68.0 years.

(e) Answers will vary.

81. False. The line $y = x$ is symmetric with respect to the origin.

82. False. The line $y = 0$ has infinitely many x-intercepts.

83. The test is for symmetry with respect to the x-axis. The statement should read: The graph of $x = 3y^2$ is symmetric with respect to the x-axis because

84. x-axis symmetry:
$$x^2 + y^2 = 1$$
$$x^2 + (-y)^2 = 1$$
$$x^2 + y^2 = 1$$

y-axis symmetry:
$$x^2 + y^2 = 1$$
$$(-x)^2 + y^2 = 1$$
$$x^2 + y^2 = 1$$

Origin symmetry:
$$x^2 + y^2 = 1$$
$$(-x)^2 + (-y)^2 = 1$$
$$x^2 + y^2 = 1$$

So, the graph of the equation is symmetric with respect to x-axis, y-axis, and origin.

85. $y = ax^2 + bx^3$

(a) $y = a(-x)^2 + b(-x)^3$

$= ax^2 - bx^3$

To be symmetric with respect to the y-axis; a can be any non-zero real number, b must be zero.

Sample answer: $a = 1, b = 0$

(b) $-y = a(-x)^2 + b(-x)^3$

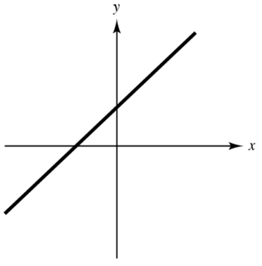
$-y = ax^2 - bx^3$

$y = -ax^2 + bx^3$

To be symmetric with respect to the origin; a must be zero, b can be any non-zero real number.

Sample answer: $a = 0, b = 1$

86. The line would rise from left to right, passing through quadrants I, II, and III.



87. $3(7x + 1) = 3(7x) + 3(1) = 21x + 3$

88. $5(x - 6) = 5(x) - 5(6) = 5x - 30$

89. $6(x - 1) + 4 = 6(x) - 6(1) = 6x - 6 + 4 = 6x - 2$

90. $4(x + 2) - 12 = 4(x) + 4(2) = 4x + 8 - 12$

$= 4x - 4$

91. The least common denominator is $3(4) = 12$.

92. The least common denominator is 9.

93. The least common denominator is $x - 4$.

94. The least common denominator is $x^2 - 4 = (x + 2)(x - 2)$.

95. $$\begin{aligned} 7\sqrt{72} - 5\sqrt{18} &= 7\sqrt{2 \cdot 36} - 5\sqrt{2 \cdot 9} \\ &= 7(6)\sqrt{2} - 5(3)\sqrt{2} \\ &= (42 - 15)\sqrt{2} \\ &= 27\sqrt{2} \end{aligned}$$

96. $$\begin{aligned} -10\sqrt{25y} - \sqrt{y} &= (-10)(5)\left(\sqrt{y} - \sqrt{y}\right) \\ &= -50\left(\sqrt{y} - \sqrt{y}\right) \\ &= -51\sqrt{y} \end{aligned}$$

97. $7^{3/2} \cdot 7^{11/2} = 7^{3/2+11/2} = 7^7 = 823{,}543$

98. $\dfrac{10^{17/4}}{10^{5/4}} = 10^{17/4-5/4} = 10^{12/4} = 10^3 = 1000$

99. $(9x - 4) + (2x^2 - x + 15) = 2x^2 + 8x + 11$

100. $$\begin{aligned} 4x(11 - x + 3x^2) &= 44x - 4x^2 + 12x^3 \\ &= 12x^3 - 4x^2 + 44x \end{aligned}$$

101. $$\begin{aligned} (2x + 9)(x - 7) &= 2x^2 + 9x - 14x - 63 \\ &= 2x^2 - 5x - 63 \end{aligned}$$

102. $$\begin{aligned} (3x^2 - 5)(-x^2 + 1) &= -3x^4 + 5x^2 + 3x^2 - 5 \\ &= -3x^4 + 8x^2 - 5 \end{aligned}$$

Section 1.3 Linear Equations in Two Variables

1. linear

2. slope

3. point-slope

4. parallel

5. rate or rate of change

6. linear extrapolation

7. They are perpendicular to each other.

8. $$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - y_1 &= mx - mx_1 \\ mx_1 - y_1 &= mx - y \\ 0 &= mx - y - mx_1 + y_1 \\ mx - y - (mx_1 - y_1) &= 0 \end{aligned}$$

9. (a) $m = \frac{2}{3}$. Because the slope is positive, the line rises. Matches L_2.

(b) m is undefined. The line is vertical. Matches L_3.

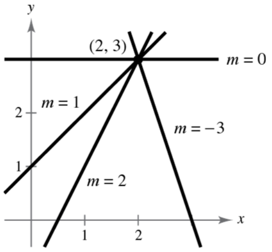
(c) $m = -2$. The line falls. Matches L_1.

10. (a) $m = 0$. The line is horizontal. Matches L_2.

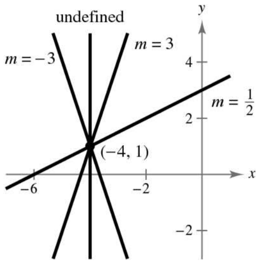
(b) $m = -\frac{3}{4}$. Because the slope is negative, the line falls. Matches L_1.

(c) $m = 1$. Because the slope is positive, the line rises. Matches L_3.

11.



12.



13. Two points on the line: $(0, 0)$ and $(4, 6)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6}{4} = \frac{3}{2}$$

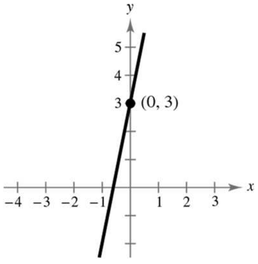
14. The line appears to go through $(0, 7)$ and $(7, 0)$.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 7}{7 - 0} = -1$$

15. $y = 5x + 3$

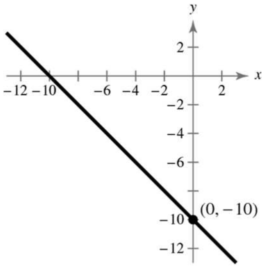
Slope: $m = 5$

y-intercept: $(0, 3)$



16. Slope: $m = -1$

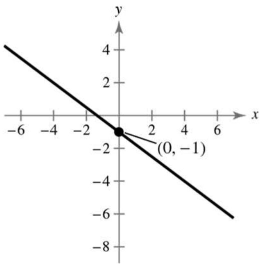
y-intercept: $(0, -10)$



17. $y = -\frac{3}{4}x - 1$

Slope: $m = -\frac{3}{4}$

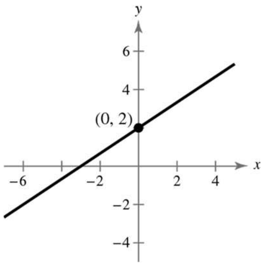
y-intercept: $(0, -1)$



18. $y = \frac{2}{3}x + 2$

Slope: $m = \frac{2}{3}$

y-intercept: $(0, 2)$

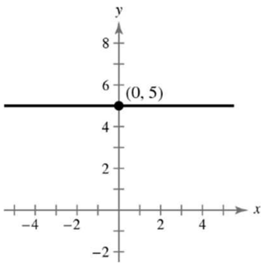


19. $y - 5 = 0$

$y = 5$

Slope: $m = 0$

y-intercept: $(0, 5)$

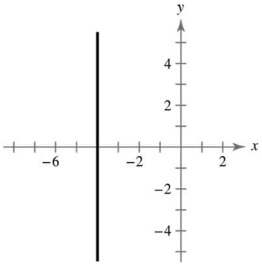


20. $x + 4 = 0$

$$x = -4$$

Slope: undefined (vertical line)

y-intercept: none

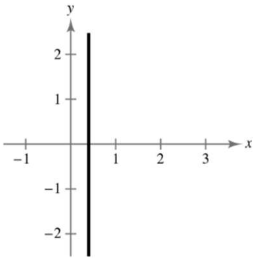


21. $5x - 2 = 0$

$$x = \tfrac{2}{5}, \text{ vertical line}$$

Slope: undefined

y-intercept: none



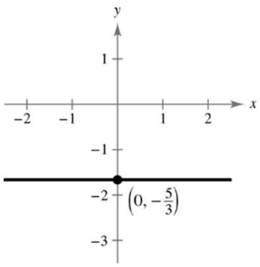
22. $3y + 5 = 0$

$$3y = -5$$

$$y = -\tfrac{5}{3}$$

Slope: $m = 0$

y-intercept: $\left(0, -\tfrac{5}{3}\right)$



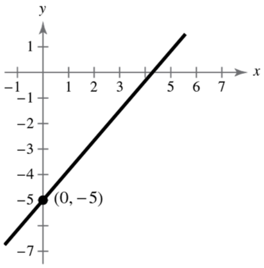
23. $7x - 6y = 30$

$$-6y = -7x + 30$$

$$y = \tfrac{7}{6}x - 5$$

Slope: $m = \tfrac{7}{6}$

y-intercept: $(0, -5)$



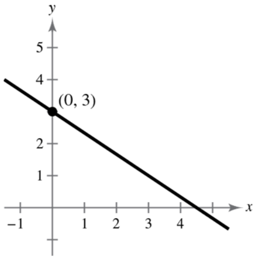
24. $2x + 3y = 9$

$$3y = -2x + 9$$

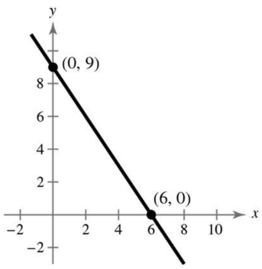
$$y = -\tfrac{2}{3}x + 3$$

Slope: $m = -\tfrac{2}{3}$

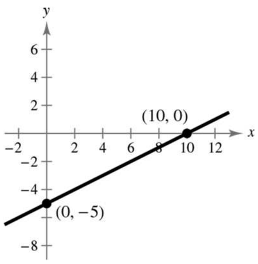
y-intercept: $(0, 3)$



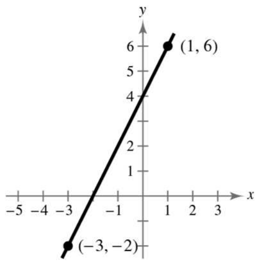
25. $m = \dfrac{0 - 9}{6 - 0} = \dfrac{-9}{6} = -\dfrac{3}{2}$



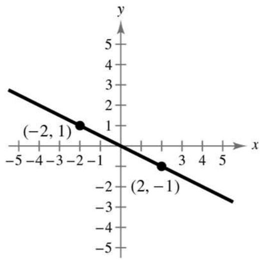
26. $m = \dfrac{-5 - 0}{0 - 10} = \dfrac{-5}{-10} = \dfrac{1}{2}$



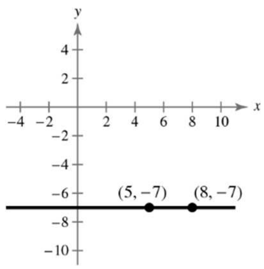
27. $m = \dfrac{6 - (-2)}{1 - (-3)} = \dfrac{8}{4} = 2$



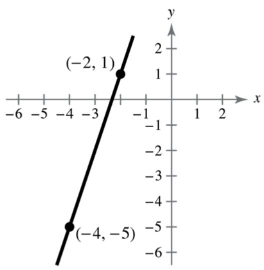
28. $m = \dfrac{1 - (-1)}{-2 - 2} = \dfrac{2}{-4} = -\dfrac{1}{2}$



29. $m = \dfrac{-7 - (-7)}{8 - 5} = \dfrac{0}{3} = 0$

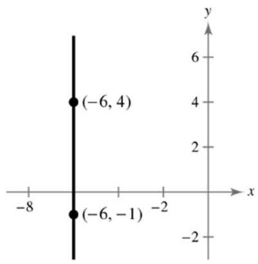


30. $m = \dfrac{-5 - 1}{-4 - (-2)} = 3$

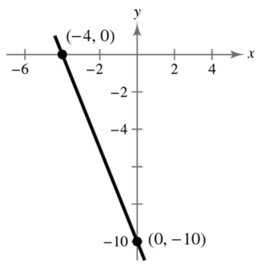


31. $m = \dfrac{4 - (-1)}{-6 - (-6)} = \dfrac{5}{0}$

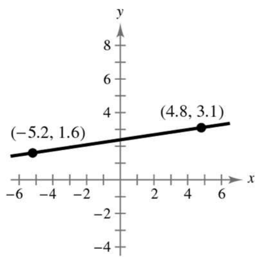
m is undefined.



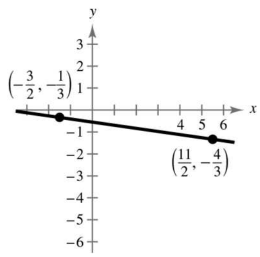
32. $m = \dfrac{0 - (-10)}{-4 - 0} = -\dfrac{5}{2}$



33. $m = \dfrac{1.6 - 3.1}{-5.2 - 4.8} = \dfrac{-1.5}{-10} = 0.15$



34. $m = \dfrac{-\dfrac{1}{3} - \left(-\dfrac{4}{3}\right)}{-\dfrac{3}{2} - \dfrac{11}{2}} = -\dfrac{1}{7}$



35. Point: $(5, 7)$, Slope: $m = 0$

Because $m = 0$, y does not change. Three other points are $(-1, 7)$, $(0, 7)$, and $(4, 7)$.

36. Point: $(3, -2)$, Slope: $m = 0$

Because $m = 0$, y does not change. Three other points are $(1, -2)$, $(10, -2)$, and $(-6, -2)$.

37. Point: $(-5, 4)$, Slope: $m = 2$

Because $m = 2 = \frac{2}{1}$, y increases by 2 for every one unit increase in x. Three additional points are $(-4, 6)$, $(-3, 8)$, and $(-2, 10)$.

38. Point: $(0, -9)$, Slope: $m = -2$

Because $m = -2$, y decreases by 2 for every one unit increase in x. Three other points are $(-2, -5)$, $(1, -11)$, and $(3, -15)$.

39. Point: $(4, 5)$, Slope: $m = -\frac{1}{3}$

Because $m = -\frac{1}{3}$, y decreases by 1 unit for every three units increase in x. Three additional points are $(-2, 7)$, $\left(0, -\frac{19}{4}\right)$, and $(1, 6)$.

40. Point: $(3, -4)$, Slope: $m = \frac{1}{4}$

Because $m = \frac{1}{4}$, y increases by 1 unit for every four unit increase in x. Three additional points are $(-1, -5)$, $(1, -11)$, and $(3, -15)$.

41. Point: $(-4, 3)$, Slope is undefined.

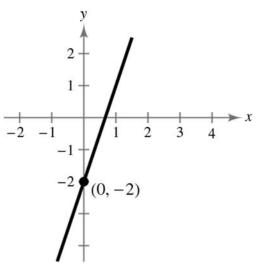
Because m is undefined, x does not change. Three points are $(-4, 0)$, $(-4, 5)$, and $(-4, 2)$.

42. Point: $(2, 14)$, Slope is undefined.

Because m is undefined, x does not change. Three other points are $(2, -3)$, $(2, 0)$, and $(2, 4)$.

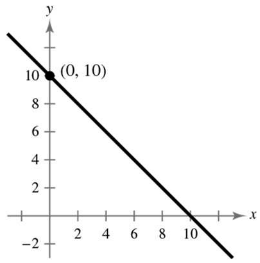
43. Point: $(0, -2)$; $m = 3$

$$y + 2 = 3(x - 0)$$
$$y = 3x - 2$$



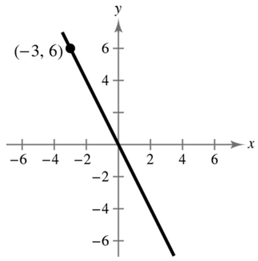
44. Point: $(0, 10)$; $m = -1$

$$y - 10 = -1(x - 0)$$
$$y - 10 = -x$$
$$y = -x + 10$$



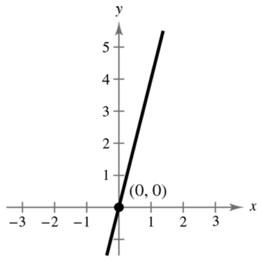
45. Point: $(-3, 6)$; $m = -2$

$$y - 6 = -2(x + 3)$$
$$y = -2x$$



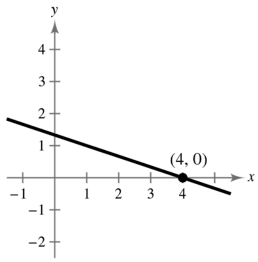
46. Point: $(0, 0)$; $m = 4$

$$y - 0 = 4(x - 0)$$
$$y = 4x$$



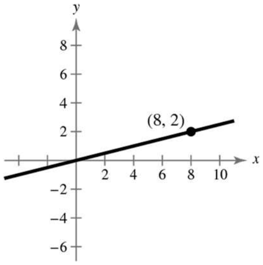
47. Point: $(4, 0)$; $m = -\frac{1}{3}$

$$y - 0 = -\frac{1}{3}(x - 4)$$
$$y = -\frac{1}{3}x + \frac{4}{3}$$



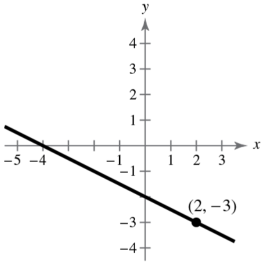
48. Point: $(8, 2)$; $m = \frac{1}{4}$

$$y - 2 = \tfrac{1}{4}(x - 8)$$
$$y - 2 = \tfrac{1}{4}x - 2$$
$$y = \tfrac{1}{4}x$$



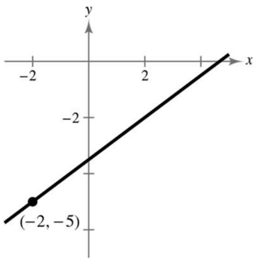
49. Point: $(2, -3)$; $m = -\dfrac{1}{2}$

$$y - (-3) = -\frac{1}{2}(x - 2)$$
$$y + 3 = -\frac{1}{2}x + 1$$
$$y = -\frac{1}{2}x - 2$$



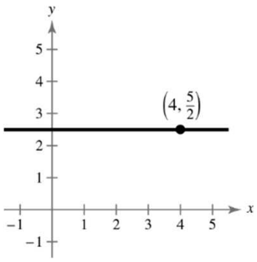
50. Point: $(-2, -5)$; $m = \frac{3}{4}$

$$y + 5 = \tfrac{3}{4}(x + 2)$$
$$4y + 20 = 3x + 6$$
$$4y = 3x - 14$$
$$y = \tfrac{3}{4}x - \tfrac{7}{2}$$



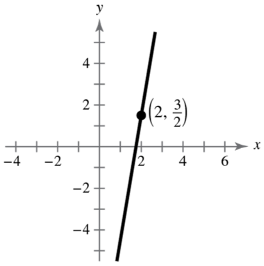
51. Point: $\left(4, \frac{5}{2}\right)$; $m = 0$

$$y - \tfrac{5}{2} = 0(x - 4)$$
$$y - \tfrac{5}{2} = 0$$
$$y = \tfrac{5}{2}$$



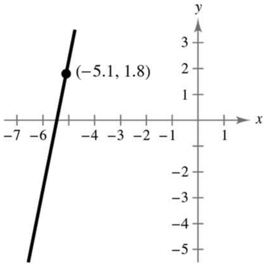
52. Point: $\left(2, \frac{3}{2}\right)$; $m = 6$

$$y - \tfrac{3}{2} = 6(x - 2)$$
$$y - \tfrac{3}{2} = 6x - 12$$
$$y = 6x - \tfrac{21}{2}$$



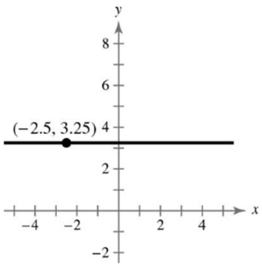
53. Point: $(-5.1, 1.8)$; $m = 5$

$$y - 1.8 = 5(x - (5.1))$$
$$y = 5x + 27.3$$



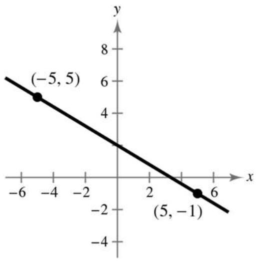
54. Point: $(-2.5, 3.25)$; $m = 0$

$$y - 3.25 = 0(x - (-2.5))$$
$$y - 3.25 = 0$$
$$y = 3.25$$



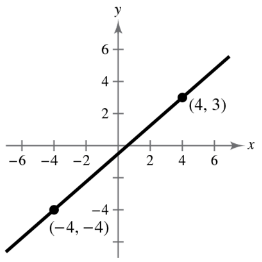
55. $(5, -1)$, $(-5, 5)$

$$y + 1 = \frac{5 + 1}{-5 - 5}(x - 5)$$
$$y = -\frac{3}{5}(x - 5) - 1$$
$$y = -\frac{3}{5}x + 2$$



56. $(4, 3), (-4, -4)$

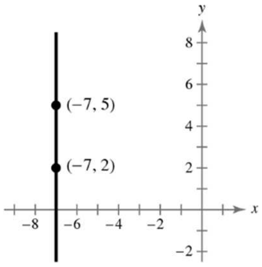
$$y - 3 = \frac{-4 - 3}{-4 - 4}(x - 4)$$
$$y - 3 = \frac{7}{8}(x - 4)$$
$$y - 3 = \frac{7}{8}x - \frac{7}{2}$$
$$y = \frac{7}{8}x - \frac{1}{2}$$



57. $(-7, 2), (-7, 5)$

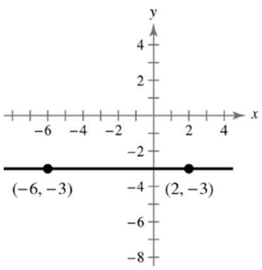
$$m = \frac{5 - 2}{-7 - (-7)} = \frac{3}{0}$$

m is undefined.



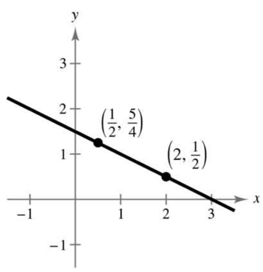
58. $(-6, -3), (2, -3)$

$$y - 4 = \frac{-3 - (-3)}{2 - (-6)}(x + 6)$$
$$y + 3 = 0(x + 6)$$
$$y + 3 = 0$$
$$y = -3$$



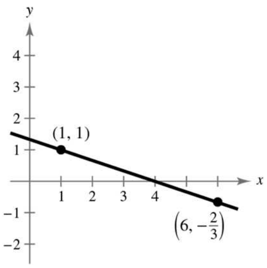
59. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$

$$y - \frac{1}{2} = \frac{\frac{5}{4} - \frac{1}{2}}{\frac{1}{2} - 2}(x - 2)$$
$$y = -\frac{1}{2}(x - 2) + \frac{1}{2}$$
$$y = -\frac{1}{2}x + \frac{3}{2}$$



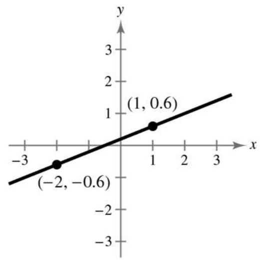
60. $(1, 1), \left(6, -\frac{2}{3}\right)$

$$y - 1 = \frac{-\frac{2}{3} - 1}{6 - 1}(x - 1)$$
$$y - 1 = -\frac{1}{3}(x - 1)$$
$$y - 1 = -\frac{1}{3}x + \frac{1}{3}$$
$$y = -\frac{1}{3}x + \frac{4}{3}$$



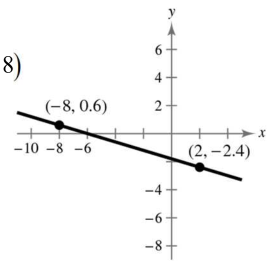
61. $(1, 0.6), (-2, -0.6)$

$$y - 0.6 = \frac{-0.6 - 0.6}{-2 - 1}(x - 1)$$
$$y = 0.4(x - 1) + 0.6$$
$$y = 0.4x + 0.2$$



62. $(-8, 0.6), (2, -2.4)$

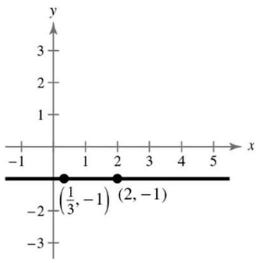
$$y - 0.6 = \frac{-2.4 - 0.6}{2 - (-8)}(x + 8)$$
$$y - 0.6 = -\frac{3}{10}(x + 8)$$
$$10y - 6 = -3(x + 8)$$
$$10y - 6 = -3x - 24$$
$$10y = -3x - 18$$
$$y = -\frac{3}{10}x - \frac{9}{5} \text{ or } y = -0.3x - 1.8$$



63. $(2, -1), \left(\frac{1}{3}, -1\right)$

$$y + 1 = \frac{-1 - (-1)}{\frac{1}{3} - 2}(x - 2)$$
$$y + 1 = 0$$
$$y = -1$$

The line is horizontal.

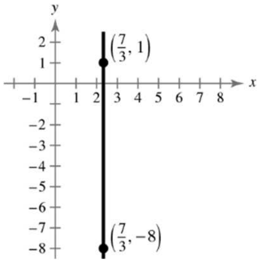


64. $\left(\frac{7}{3}, -8\right), \left(\frac{7}{3}, 1\right)$

$$m = \frac{1-(-8)}{\frac{7}{3}-\frac{7}{3}} = \frac{9}{0} \text{ and is undefined.}$$

$$x = \frac{7}{3}$$

The line is vertical.



65. $L_1: y = -\frac{2}{3}x - 3$

$m_1 = -\frac{2}{3}$

$L_2: y = -\frac{2}{3}x - 1$

$m_2 = -\frac{2}{3}$

The slopes are equal, so the lines are parallel.

66. $L_1: y = \frac{1}{4}x - 1$

$m_1 = \frac{1}{4}$

$L_2: y = 4x + 7$

$m_2 = 4$

The lines are neither parallel nor perpendicular.

67. $L_1: y = \frac{1}{2}x - 3$

$m_1 = \frac{1}{2}$

$L_2: y = -\frac{1}{2}x + 1$

$m_2 = -\frac{1}{2}$

The lines are neither parallel nor perpendicular.

68. $L_1: y = -\frac{4}{5}x - 5$

$m_1 = -\frac{4}{5}$

$L_2: y = \frac{5}{4}x + 1$

$m_2 = \frac{5}{4}$

The slopes are negative reciprocals, so the lines are perpendicular.

69. $L_1: (0, -1), (5, 9)$

$$m_1 = \frac{9+1}{5-0} = 2$$

$L_2: (0, 3), (4, 1)$

$$m_2 = \frac{1-3}{4-0} = -\frac{1}{2}$$

The slopes are negative reciprocals, so the lines are perpendicular.

70. $L_1: (-2, -1), (1, 5)$

$$m_1 = \frac{5-(-1)}{1-(-2)} = \frac{6}{3} = 2$$

$L_2: (1, 3), (5, -5)$

$$m_2 = \frac{-5-3}{5-1} = \frac{-8}{4} = -2$$

The lines are neither parallel nor perpendicular.

71. $L_1: (-6, -3), (2, -3)$

$$m_1 = \frac{-3-(-3)}{2-(-6)} = \frac{0}{8} = 0$$

$L_2: \left(3, -\frac{1}{2}\right), \left(6, -\frac{1}{2}\right)$

$$m_2 = \frac{-\frac{1}{2}-\left(-\frac{1}{2}\right)}{6-3} = \frac{0}{3} = 0$$

L_1 and L_2 are both horizontal lines, so they are parallel.

72. $L_1: (4, 8), (-4, 2)$

$$m_1 = \frac{2-8}{-4-4} = \frac{-6}{-8} = \frac{3}{4}$$

$L_2: (3, -5), \left(-1, \frac{1}{3}\right)$

$$m_2 = \frac{\frac{1}{3}-(-5)}{-1-3} = \frac{\frac{16}{3}}{-4} = -\frac{4}{3}$$

The slopes are negative reciprocals, so the lines are perpendicular.

73. $4x - 2y = 3$

$y = 2x - \frac{3}{2}$

Slope: $m = 2$

(a) $(2, 1), m = 2$

$y - 1 = 2(x - 2)$

$y = 2x - 3$

(b) $(2, 1), m = -\frac{1}{2}$

$y - 1 = -\frac{1}{2}(x - 2)$

$y = -\frac{1}{2}x + 2$

74. $x + y = 7$

$y = -x + 7$

Slope: $m = -1$

(a) $m = -1, (-3, 2)$

$y - 2 = -1(x + 3)$

$y - 2 = -x - 3$

$y = -x - 1$

(b) $m = 1, (-3, 2)$

$y - 2 = 1(x + 3)$

$y = x + 5$

75. $3x + 4y = 7$

$y = -\frac{3}{4}x + \frac{7}{4}$

Slope: $m = -\frac{3}{4}$

(a) $\left(-\frac{2}{3}, \frac{7}{8}\right), m = -\frac{3}{4}$

$y - \frac{7}{8} = -\frac{3}{4}\left(x - \left(-\frac{2}{3}\right)\right)$

$y = -\frac{3}{4}x + \frac{3}{8}$

(b) $\left(-\frac{2}{3}, \frac{7}{8}\right), m = \frac{4}{3}$

$y - \frac{7}{8} = \frac{4}{3}\left(x - \left(-\frac{2}{3}\right)\right)$

$y = \frac{4}{3}x + \frac{127}{72}$

76. $5x + 3y = 0$

$3y = -5x$

$y = -\frac{5}{3}x$

Slope: $m = -\frac{5}{3}$

(a) $m = -\frac{5}{3}, \left(\frac{7}{8}, \frac{3}{4}\right)$

$y - \frac{3}{4} = -\frac{5}{3}\left(x - \frac{7}{8}\right)$

$24y - 18 = -40\left(x - \frac{7}{8}\right)$

$24y - 18 = -40x + 35$

$24y = -40x + 53$

$y = -\frac{5}{3}x + \frac{53}{24}$

(b) $m = \frac{3}{5}, \left(\frac{7}{8}, \frac{3}{4}\right)$

$y - \frac{3}{4} = \frac{3}{5}\left(x - \frac{7}{8}\right)$

$40y - 30 = 24\left(x - \frac{7}{8}\right)$

$40y - 30 = 24x - 21$

$40y = 24x + 9$

$y = \frac{3}{5}x + \frac{9}{40}$

77. $y + 5 = 0$

$y = -5$

Slope: $m = 0$

(a) $(-2, 4), m = 0$

$y = 4$

(b) $(-2, 4)$, m is undefined.

$x = -2$

78. $x - 4 = 0$

$x = 4$

Slope: m is undefined.

(a) $(3, -2)$, m is undefined.

$x = 3$

(b) $(3, -2), m = 0$

$y = -2$

79. $x - y = 4$

$y = x - 4$

Slope: $m = 1$

(a) $(2.5, 6.8), m = 1$

$y - 6.8 = 1(x - 2.5)$

$y = x + 4.3$

(b) $(2.5, 6.8), m = -1$

$y - 6.8 = (-1)(x - 2.5)$

$y = -x + 9.3$

80. $6x + 2y = 9$

$2y = -6x + 9$

$y = -3x + \frac{9}{2}$

Slope: $m = -3$

(a) $(-3.9, -1.4), m = -3$

$y - (-1.4) = -3(x - (-3.9))$

$y + 1.4 = -3x - 11.7$

$y = -3x - 13.1$

(b) $(-3.9, -1.4), m = \frac{1}{3}$

$y - (-1.4) = \frac{1}{3}(x - (-3.9))$

$y + 1.4 = \frac{1}{3}x + 1.3$

$y = \frac{1}{3}x - 0.1$

81. $\frac{x}{3} + \frac{y}{5} = 1$

$(15)\left(\frac{x}{3} + \frac{y}{5}\right) = 1(15)$

$5x + 3y - 15 = 0$

82. $(-3, 0), (0, 4)$

$$\frac{x}{-3} + \frac{y}{4} = 1$$

$$(-12)\frac{x}{-3} + (-12)\frac{y}{4} = (-12)\cdot 1$$

$$4x - 3y + 12 = 0$$

83. $$\frac{x}{-1/6} + \frac{y}{-2/3} = 1$$

$$6x + \frac{3}{2}y = -1$$

$$12x + 3y + 2 = 0$$

84. $\left(\frac{2}{3}, 0\right), (0, -2)$

$$\frac{x}{2/3} + \frac{y}{-2} = 1$$

$$\frac{3x}{2} - \frac{y}{2} = 1$$

$$3x - y - 2 = 0$$

85. $$\frac{x}{c} + \frac{y}{c} = 1, c \neq 0$$

$$x + y = c$$

$$1 + 2 = c$$

$$3 = c$$

$$x + y = 3$$

$$x + y - 3 = 0$$

86. $(d, 0), (0, d), (-3, 4)$

$$\frac{x}{d} + \frac{y}{d} = 1$$

$$x + y = d$$

$$-3 + 4 = d$$

$$1 = d$$

$$x + y = 1$$

$$x + y - 1 = 0$$

87. (a) $m = 135$. The sales are increasing 135 units per year.

(b) $m = 0$. There is no change in sales during the year.

(c) $m = -40$. The sales are decreasing 40 units per year.

88. (a) The slopes of each line segment are:

$$\frac{182.80 - 170.91}{14 - 13} = 11.89 \qquad \frac{229.23 - 215.64}{17 - 16} = 13.59$$

$$\frac{233.72 - 182.80}{15 - 14} = 50.92 \text{ (greatest)} \qquad \frac{265.60 - 229.23}{18 - 17} = 36.37$$

$$\frac{215.64 - 233.72}{16 - 15} = -18.08 \text{ (least)} \qquad \frac{260.17 - 265.60}{19 - 18} = -5.43$$

The sales increased the greatest between 2014 and 2015. The sales increased the least between 2015 to 2016.

(b) $$m = \frac{260.17 - 170.91}{19 - 13} = \frac{89.26}{6} \approx 14.88$$

The slope of the line is about 14.88.

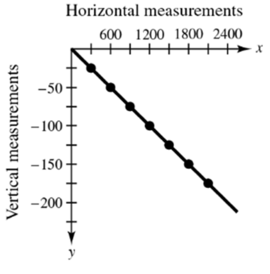
(c) Sales increased an average of about \$14.88 billion per year between 2013 and 2019.

89. $y = \frac{6}{100}x$

$y = \frac{6}{100}(200) = 12$ feet

90. (a) and (b)

x	300	600	900	1200	1500	1800	2100
y	–25	–50	–75	–100	–125	–150	–175



(c) $$m = \frac{-50 - (-25)}{600 - 300} = \frac{-25}{300} = -\frac{1}{12}$$

$$y - (-50) = -\frac{1}{12}(x - 600)$$

$$y + 50 = -\frac{1}{12}x + 50$$

$$y = -\frac{1}{12}x$$

(d) Because $m = -\frac{1}{12}$, for every change in the horizontal measurement of 12 feet, the vertical measurement decreases by 1 foot.

(e) $\frac{1}{12} \approx 0.083 = 8.3\%$ grade

91. Using the points $(0, 32)$ and $(100, 212)$, where the first coordinate represents a temperature in degrees Celsius and the second coordinate represents a temperature in degrees Fahrenheit, you have

$$m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}.$$

Since the point $(0, 32)$ is the F- intercept, $b = 32$, the equation is $F = 1.8C + 32$ or $C = \frac{5}{9}F - \frac{160}{9}$.

92. (a) Using the points $(1, 970)$ and $(3, 1270)$, you have

$$m = \frac{1270 - 970}{3 - 1} = \frac{300}{2} = 150.$$

Using the point-slope form with $m = 150$ and the point $(1, 970)$, you have

$$y - y_1 = m(t - t_1)$$
$$y - 970 = 150(t - 1)$$
$$y - 970 = 150t - 150$$
$$y = 150t + 820.$$

(b) The slope is $m = 150$. The slope tells you the amount of increase in the weight of an average male child's brain each year.

(c) Let $t = 2$:

$$y = 150(2) + 820$$
$$y = 300 + 820$$
$$y = 1120$$

The average brain weight at age 2 is 1120 grams.

(d) Answers will vary.

(e) Answers will vary. *Sample answer:* No. The brain stops growing after reaching a certain age.

93. Using the points $(0, 830)$ and $(5, 0)$, where the first coordinate represents the year t and the second coordinate represents the value V, you have

$$V = \frac{0 - 830}{5 - 0}t + 830 = -166t + 830, \ 0 \le t \le 5.$$

94. Using the points $(0, 24{,}000)$ and $(10, 2000)$, where the first coordinate represents the year t and the second coordinate represents the value V, you have

$$m = \frac{2{,}000 - 24{,}000}{10 - 0} = \frac{-22{,}000}{10} = -2200.$$

Since the point $(0, 24{,}000)$ is the V-intercept, $b = 24{,}000$, the equation is $V = -2200t + 24{,}000, \ 0 \le t \le 10.$

95. (a) Total Cost = cost for fuel and maintainance + cost for operator + purchase cost

$$C = 9.5t + 11.5t + 42{,}000$$
$$C = 21t + 42{,}000$$

(b) Revenue = Rate per hour · Hours

$$R = 45t$$

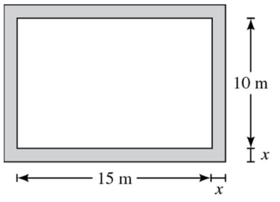
(c)
$$P = R - C$$
$$P = 45t - (21t + 42{,}000)$$
$$P = 24t - 42{,}000$$

(d) Let $P = 0$, and solve for t.

$$0 = 24t - 42{,}000$$
$$42{,}000 = 24t$$
$$1750 = t$$

The equipment must be used 1750 hours to yield a profit of 0 dollars.

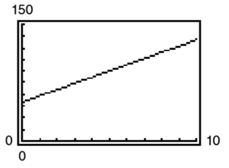
96. (a)



(b) $y = 2(15 + 2x) + 2(10 + 2x) = 8x + 50$

Because $m = 8$, each 1-meter increase in x will increase y by 8 meters.

(c)



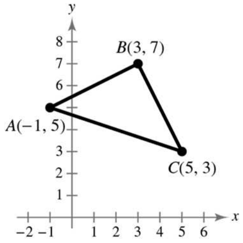
97. False. The slope with the greatest magnitude corresponds to the steepest line.

98. $(-8, 2)$ and $(-1, 4)$: $m_1 = \dfrac{4 - 2}{-1 - (-8)} = \dfrac{2}{7}$

$(0, -4)$ and $(-7, 7)$: $m_2 = \dfrac{7 - (-4)}{-7 - 0} = \dfrac{11}{-7}$

False. The lines are not parallel.

99. Find the slope of the line segments between the points A and B, and B and C.



$$m_{AB} = \frac{7-5}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$$

$$m_{BC} = \frac{3-7}{5-3} = \frac{-4}{2} = -2$$

Since the slopes are negative reciprocals, the line segments are perpendicular and therefore intersect to form a right angle. So, the triangle is a right triangle.

100.
$$\begin{aligned} d_1 &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-0)^2 + (m_1 - 0)^2} \\ &= \sqrt{1 + (m_1)^2} \end{aligned}$$

$$\begin{aligned} d_2 &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-0)^2 + (m_2 - 0)^2} \\ &= \sqrt{1 + (m_2)^2} \end{aligned}$$

Using the Pythagorean Theorem:

$$\begin{aligned} (d_1)^2 + (d_2)^2 &= \left(\text{distance between } (1, m_1), \text{ and } (1, m_2)\right)^2 \\ \left(\sqrt{1+(m_1)^2}\right)^2 + \left(\sqrt{1+(m_2)^2}\right)^2 &= \left(\sqrt{(1-1)^2 + (m_2 - m_1)^2}\right)^2 \\ 1 + (m_1)^2 + 1 + (m_2)^2 &= (m_2 - m_1)^2 \\ (m_1)^2 + (m_2)^2 + 2 &= (m_2)^2 - 2m_1m_2 + (m_1)^2 \\ 2 &= -2m_1m_2 \\ -\frac{1}{m_2} &= m_1 \end{aligned}$$

101. (a) The slope is $\frac{0-(-1)}{2-0} = \frac{1}{2}$, not 2.

(b) The y-intercept is $(0, -3)$, not $(0, 4)$.

102. (a) Matches graph (ii).

The slope is –20, which represents the decrease in the amount of the loan each week. The y-intercept is $(0, 200)$, which represents the original amount of the loan.

(b) Matches graph (iii).

The slope is 2, which represents the increase in the hourly wage for each unit produced. The y-intercept is $(0, 12.5)$, which represents the hourly rate if the employee produces no units.

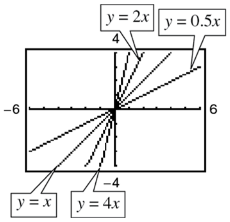
(c) Matches graph (i).

The slope is 0.32, which represents the increase in travel cost for each mile driven. The y-intercept is $(0, 32)$, which represents the fixed cost of \$30 per day for meals. This amount does not depend on the number of miles driven.

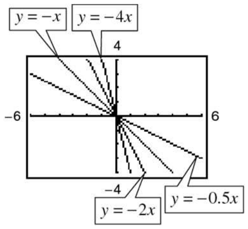
(d) Matches graph (iv).

The slope is –100, which represents the amount by which the computer depreciates each year. The y-intercept is $(0, 750)$, which represents the original purchase price.

103. The line $y = 4x$ rises most quickly.



The line $y = -4x$ falls most quickly.



The greater the magnitude of the slope (the absolute value of the slope), the faster the line rises or falls.

104. Because $\left|-4\right| > \left|\frac{5}{2}\right|$, the steeper line is the one with a slope of – 4. The slope with the greatest magnitude corresponds to the steepest line.

105. $y = x^2 - x$

When $x = -2$: $y = (-2)^2 - (-2) = 4 + 2 = 6$

When $x = 0$: $y = 0^2 - 0 = 0$

When $x = 3$: $y = 3^2 - 3 = 9 - 3 = 6$

When $x = 6$: $y = 6^2 - 6 = 36 - 6 = 30$

106. $y = x + 5y \Rightarrow 4y = -x \Rightarrow y = \dfrac{-x}{4}$

(a) When $x = -2$: $y = \dfrac{-(-2)}{4} = \dfrac{1}{2}$

(b) When $x = 0$: $y = \dfrac{-0}{4} = 0$

(c) When $x = 3$: $y = \dfrac{-3}{4}$

(d) When $x = 6$: $y = \dfrac{-6}{4} = \dfrac{-3}{2}$

107. $2f = \dfrac{(7 - x^3)}{5} \Rightarrow f = \dfrac{(7 - x^3)}{10}$

(a) When $x = -2$: $f = \dfrac{\left(7 - (-2)^3\right)}{10} = \dfrac{(7 + 8)}{10} = \dfrac{15}{10} = \dfrac{3}{2}$

(b) When $x = 0$: $f = \dfrac{(7 - 0^3)}{10} = \dfrac{7}{10}$

(c) When $x = 3$: $f = \dfrac{(7 - 3^3)}{10} = \dfrac{(7 - 27)}{10} = \dfrac{-20}{10} = -2$

(d) When $x = 6$: $f = \dfrac{(7 - 6^3)}{10} = \dfrac{(7 - 216)}{10} = \dfrac{-209}{10}$

108. $\dfrac{g}{4} = \dfrac{\sqrt{2 + x}}{3} \Rightarrow g = \dfrac{4\sqrt{2 + x}}{3}$

(a) When $x = -2$: $g = \dfrac{4\sqrt{2 - 2}}{3} = \dfrac{4\sqrt{0}}{3} = 0$

(b) When $x = 0$: $g = \dfrac{4\sqrt{2 + 0}}{3} = \dfrac{4\sqrt{2}}{3}$

(c) When $x = 3$: $g = \dfrac{4\sqrt{2 + 3}}{3} = \dfrac{4\sqrt{5}}{3}$

(d) When $x = 6$: $g = \dfrac{4\sqrt{2 + 6}}{3} = \dfrac{4\sqrt{8}}{3} = \dfrac{8\sqrt{2}}{3}$

109.
$$\begin{aligned}
2x^3 - 5x^2 - 2x + 5 &= 0\\
x^2(2x - 5) - (2x - 5) &= 0\\
(x^2 - 1)(2x - 5) &= 0\\
x &= \pm 1, \frac{5}{2}
\end{aligned}$$

110.
$$\begin{aligned}
x^4 - 3x^3 - x - 2 &= 1 - 2x\\
x^4 - 3x^3 + x - 3 &= 0\\
x^3(x - 3) + (x - 3) &= 0\\
(x^3 + 1)(x - 3) &= 0\\
(x + 1)(x^2 - x + 1)(x - 3) &= 0\\
x &= -1, 3
\end{aligned}$$

111.
$$x^6 + 4x^3 - 5 = 0$$
$$(x^3 + 5)(x^3 - 1) = 0$$
$$x = -\sqrt[3]{5},\ 1$$

112.
$$x^4 - 5x^2 + 4 = 0$$
$$(x^2 - 4)(x^2 - 1) = 0$$
$$(x + 2)(x - 2)(x + 1)(x - 1) = 0$$
$$x = \pm 2,\ \pm 1$$

113. $3\sqrt{x} - 5\sqrt{18} = 0$
$$3\sqrt{x} = 5(3)\sqrt{2}$$
$$\sqrt{x} = 5\sqrt{2}$$
$$x = 25(2) = 50$$

114. $(x + 1)^{3/5} - 8 = 0$
$$(x + 1)^{3/5} = 8$$
$$(x + 1) = 8^{5/3} = 2^5 = 32$$
$$x = 31$$

115.
$$x = \frac{2}{x} + 1$$
$$x^2 = 2 + x$$
$$x^2 - x - 2 = 0$$
$$(x - 2)(x + 1) = 0$$
$$x = 2,\ -1$$

116.
$$\frac{3}{x} - \frac{1}{2} = \frac{x}{4}$$
$$3 - \frac{1}{2}x = \frac{x^2}{4}$$
$$12 - 2x = x^2$$
$$x^2 + 2x - 12 = 0$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4(-12)}}{2}$$
$$= \frac{-2 \pm \sqrt{52}}{2}$$
$$= -1 \pm \sqrt{13}$$

117. $x + |x - 9| = 7$
$$|x - 9| = 7 - x$$
If $x - 9 \geq 0$, then $x - 9 = 7 - x \Rightarrow 2x = 16 \Rightarrow$ $x = 8$, which is extraneous.

If $x - 9 < 0$, then $-(x - 9) = 7 - x$, which is extraneous.

No real solution.

118. $|3x - 6| = x^2 - 4$

If $3x - 6 \geq 0$, then $3x - 6 = x^2 - 4 \Rightarrow x^2 - 3x - 2 = 0$
$\Rightarrow (x - 2)(x - 1) = 0 \Rightarrow x = 1, 2$
$(x = 1$ is extraneous.$)$

If $3x - 6 < 0$, then $6 - 3x = x^2 - 4 \Rightarrow x^2 + 3x - 10 = 0$
$\Rightarrow (x + 5)(x - 2) = 0 \Rightarrow x = 2, -5$

Solutions: $x = 2, -5$

119.
$$\frac{\left[(x - 1)^2 + 1\right] - (x^2 + 1)}{x} = \frac{(x^2 - 2x + 1 + 1) - (x^2 + 1)}{x}$$
$$= \frac{-2x + 1}{x}$$

120.
$$\frac{\left[(t + 1)^2 - (t + 1) - 2\right] - (t^2 - t - 2)}{t} = \frac{(t^2 + 2t + 1 - t - 1 - 2) - (t^2 - t - 2)}{t}$$
$$= \frac{t^2 + t - 2 - t^2 + t + 2}{t}$$
$$= \frac{2t}{t} = 2,\ t \neq 0$$

121. $$\frac{4-x^2-3}{x-1}+x^2+x+4 = \frac{4-x^2-3+(x-1)(x^2+x+4)}{x-1}$$
$$= \frac{4-x^2-3+(x^3+x^2+4x-x^2-x-4)}{x-1}$$
$$= \frac{4-x^2-3+x^3+3x-4}{x-1}$$
$$= \frac{x^3-x^2+3x-3}{x-1}$$
$$= \frac{x^2(x-1)+3(x-1)}{x-1}$$
$$= \frac{(x-1)(x^2+3)}{x-1}$$
$$= x^2+3,\ x \neq 1$$

Alternate solution:

$$\frac{4-x^2-3}{x-1}+x^2+x+4 = \frac{(1-x)(1+x)}{x-1}+x^2+x+4$$
$$= -1-x+x^2+x+4,\ x \neq 1$$
$$= x^2+3,\ x \neq 1$$

122. $$\frac{x^3+x}{x^2-4}-\frac{5}{2(x+2)}+\frac{5}{2(x-2)}+2 = \frac{2(x^3+x)-5(x-2)+5(x+2)+4(x^2-4)}{2(x^2-4)}$$
$$= \frac{2x^3+2x-5x+10+5x+10+4x^2-16}{2(x^2-4)}$$
$$= \frac{2x^3+4x^2+2x+4}{2(x^2-4)}$$
$$= \frac{x^3+2x^2+x+2}{x^2-4}$$
$$= \frac{x^2(x+2)+(x+2)}{x^2-4}$$
$$= \frac{(x+2)(x^2+1)}{(x+2)(x-2)}$$
$$= \frac{x^2+1}{x-2},\ x \neq -2$$

Section 1.4 Functions

1. independent; dependent

2. difference quotient

3. A relation is any set of ordered pairs. A function is a relation in which no two ordered pairs have the same first component and different second components.

4. For $g(x) = 4x - 5$, to find $g(x+2)$, substitute $x+2$ for x, and simplify: $g(x+2) = 4(x+2) - 5$
$= 4x + 3.$

5. The domain of a piece-wise function must be explicitly described, so that it can determine which equation is used to evaluate the function.

6. The domain of $y = \sqrt{4-x^2}$ is determined by solving the inequality $4 - x^2 \geq 0$.

7. Yes, the relationship is a function. Each domain value is matched with exactly one range value.

8. No, the relationship is not a function. The domain value of –1 is matched with two output values.

9. No, it does not represent a function. The input values of 10 and 7 are each matched with two output values.

10. Yes, the table does represent a function. Each input value is matched with exactly one output value.

Input, x	-2	0	2	4	6
Output, y	1	1	1	1	1

11. (a) Each element of A is matched with exactly one element of B, so it does represent a function.

(b) Each element of A is matched with exactly one element of B, so it does represent a function.

(c) The element 2 in A is not matched with an element of B, so the relation does not represent a function.

12. (a) The element c in A is matched with two elements, 2 and 3 of B, so it is not a function.

(b) Each element of A is matched with exactly one element of B, so it does represent a function.

(c) This is not a function from A to B (it represents a function from B to A instead).

13. $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$

No, y *is not* a function of x.

14. $x^2 - y = 9 \Rightarrow y = x^2 - 9$

Yes, y *is* a function of x.

15. $y = \sqrt{16 - x^2}$

Yes, y *is* a function of x.

16. $y = \sqrt{x + 5}$

Yes, y *is* a function of x.

17. $y = 4 - |x|$

Yes, y *is* a function of x.

18. $|y| = 4 - x \Rightarrow y = 4 - x$ or $y = -(4 - x)$

No, y *is not* a function of x.

19. $y = -75$ or $y = -75 + 0x$

Yes, y *is* a function of x.

20. $x - 1 = 0$

$x = 1$

No, this *is not* a function of x.

21. $g(t) = 4t^2 - 3t + 5$

(a) $g(2) = 4(2)^2 - 3(2) + 5 = 16 - 6 + 5 = 15$

(b) $g(-1) = 4(-1)^2 - 3(-1) + 5 = 4 + 3 + 5 = 12$

(c)
$$\begin{aligned} g(t + 2) &= 4(t + 2)^2 - 3(t + 2) + 5 \\ &= 4(t^2 + 4t + 4) - 3t - 6 + 5 \\ &= 4t^2 + 13t + 15 \end{aligned}$$

22. $V(r) = \frac{4}{3}\pi r^3$

(a) $V(3) = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi$

(b) $V\left(\frac{3}{2}\right) = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{27}{8}\right) = \frac{9}{2}\pi$

(c) $V(2r) = \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi(8r^3) = \frac{32}{3}\pi r^3$

23. $f(y) = 3 - \sqrt{y}$

(a) $f(4) = 3 - \sqrt{4} = 1$

(b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$

(c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

24. $f(x) = \sqrt{x + 8} + 2$

(a) $f(-8) = \sqrt{(-8) + 8} + 2 = 2$

(b) $f(1) = \sqrt{(1) + 8} + 2 = 5$

(c) $f(x - 8) = \sqrt{(x - 8) + 8} + 2 = \sqrt{x} + 2$

25. $q(x) = \dfrac{1}{x^2 - 9}$

(a) $q(0) = \dfrac{1}{0^2 - 9} = -\dfrac{1}{9}$

(b) $q(3) = \dfrac{1}{3^2 - 9}$ is undefined.

(c) $q(y + 3) = \dfrac{1}{(y + 3)^2 - 9} = \dfrac{1}{y^2 + 6y}$

26. $q(t) = \dfrac{2t^2 + 3}{t^2}$

(a) $q(2) = \dfrac{2(2)^2 + 3}{(2)^2} = \dfrac{8 + 3}{4} = \dfrac{11}{4}$

(b) $q(0) = \dfrac{2(0)^2 + 3}{(0)^2}$

Division by zero is undefined.

(c) $q(-x) = \dfrac{2(-x)^2 + 3}{(-x)^2} = \dfrac{2x^2 + 3}{x^2}$

27. $f(x) = \dfrac{|x|}{x}$

(a) $f(2) = \dfrac{|2|}{2} = 1$

(b) $f(-2) = \dfrac{|-2|}{-2} = -1$

(c) $f(x-1) = \dfrac{|x-1|}{x-1} = \begin{cases} -1, & \text{if } x < 1 \\ 1, & \text{if } x > 1 \end{cases}$

28. $f(x) = |x| + 4$

(a) $f(2) = |2| + 4 = 6$

(b) $f(-2) = |-2| + 4 = 6$

(c) $f(x^2) = |x^2| + 4 = x^2 + 4$

29. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

30. $f(x) = \begin{cases} -3x - 3, & x < -1 \\ x^2 + 2x - 1, & x \geq -1 \end{cases}$

(a) $f(-2) = -3(-2) - 3 = 3$

(b) $f(-1) = (-1)^2 + 2(-1) - 1 = -2$

(c) $f(1) = (1)^2 + 2(1) - 1 = 2$

31. $f(x) = -x^2 + 5$

$f(-2) = -(-2)^2 + 5 = 1$

$f(-1) = -(-1)^2 + 5 = 4$

$f(0) = -(0)^2 + 5 = 5$

$f(1) = -(1)^2 + 5 = 4$

$f(2) = -(2)^2 + 5 = 1$

x	-2	-1	0	1	2
$f(x)$	1	4	5	-4	1

32. $h(t) = \frac{1}{2}|t + 3|$

$h(-5) = \frac{1}{2}|-5 + 3| = 1$

$h(-4) = \frac{1}{2}|-4 + 3| = \frac{1}{2}$

$h(-3) = \frac{1}{2}|-3 + 3| = 0$

$h(-2) = \frac{1}{2}|-2 + 3| = \frac{1}{2}$

$h(-1) = \frac{1}{2}|-1 + 3| = 1$

t	-5	-4	-3	-2	-1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

33. $f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \leq 0 \\ (x - 2)^2, & x > 0 \end{cases}$

$f(-2) = -\frac{1}{2}(-2) + 4 = 5$

$f(-1) = -\frac{1}{2}(-1) + 4 = 4\frac{1}{2} = \frac{9}{2}$

$f(0) = -\frac{1}{2}(0) + 4 = 4$

$f(1) = (1 - 2)^2 = 1$

$f(2) = (2 - 2)^2 = 0$

x	-2	-1	0	1	2
$f(x)$	5	$\frac{9}{2}$	4	1	0

34. $f(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \geq 3 \end{cases}$

$f(1) = 9 - (1)^2 = 8$

$f(2) = 9 - (2)^2 = 5$

$f(3) = (3) - 3 = 0$

$f(4) = (4) - 3 = 1$

$f(5) = (5) - 3 = 2$

x	1	2	3	4	5
$f(x)$	8	5	0	1	2

35. $15 - 3x = 0$

$3x = 15$

$x = 5$

36. $f(x) = 4x + 6$

$4x + 6 = 0$

$4x = -6$

$x = -\frac{3}{2}$

37. $\dfrac{3x - 4}{5} = 0$

$3x - 4 = 0$

$x = \dfrac{4}{3}$

38. $f(x) = \dfrac{12 - x^2}{8}$

$\dfrac{12 - x^2}{8} = 0$

$x^2 = 12$

$x = \pm\sqrt{12} = \pm 2\sqrt{3}$

39. $f(x) = x^2 - 81$

$x^2 - 81 = 0$

$x^2 = 81$

$x = \pm 9$

40. $f(x) = x^2 - 6x - 16$

$x^2 - 6x - 16 = 0$

$(x - 8)(x + 2) = 0$

$x - 8 = 0 \Rightarrow x = 8$

$x + 2 = 0 \Rightarrow x = -2$

41. $x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x + 1)(x - 1) = 0$

$x = 0$, $x = -1$, or $x = 1$

42. $f(x) = x^3 - x^2 - 3x + 3$

$x^3 - x^2 - 3x + 3 = 0$

$x^2(x - 1) - 3(x - 1) = 0$

$(x - 1)(x^2 - 3) = 0$

$x - 1 = 0 \Rightarrow x = 1$

$x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}$

43. $f(x) = g(x)$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x - 2 = 0 \quad x + 1 = 0$

$x = 2 \quad x = -1$

44. $f(x) = g(x)$

$x^2 + 2x + 1 = 5x + 19$

$x^2 - 3x - 18 = 0$

$(x - 6)(x + 3) = 0$

$x - 6 = 0 \quad x + 3 = 0$

$x = 6 \quad x = -3$

45. $f(x) = g(x)$

$x^4 - 2x^2 = 2x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x^2(x + 2)(x - 2) = 0$

$x^2 = 0 \Rightarrow x = 0$

$x + 2 = 0 \Rightarrow x = -2$

$x - 2 = 0 \Rightarrow x = 2$

46. $f(x) = g(x)$

$\sqrt{x} - 4 = 2 - x$

$x + \sqrt{x} - 6 = 0$

$(\sqrt{x} + 3)(\sqrt{x} - 2) = 0$

$\sqrt{x} + 3 = 0 \Rightarrow \sqrt{x} = -3$, which is a contradiction, since $\sqrt{x}$ represents the principal square root.

$\sqrt{x} - 2 = 0 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$

47. $f(x) = 5x^2 + x - 1$

Because $f(x)$ is a polynomial, the domain is all real numbers x.

48. $f(x) = 1 - 2x^2$

Because $f(x)$ is a polynomial, the domain is all real numbers x.

49. $g(y) = \sqrt{y + 6}$

Domain: $y + 6 \geq 0$

$y \geq -6$

The domain is all real numbers y such that $y \geq -6$.

50. $f(t) = \sqrt[3]{t + 4}$

Because $f(t)$ is a cube root, the domain is all real numbers t.

51. $g(x) = \dfrac{1}{x} - \dfrac{3}{x+2}$

The domain is all real numbers x except $x = 0, x = -2$.

52. $h(x) = \dfrac{6}{x^2 - 4x}$

$x^2 - 4x \neq 0$

$x(x - 4) \neq 0$

$x \neq 0$

$x - 4 \neq 0 \Rightarrow x \neq 4$

The domain is all real numbers x except $x = 0, x = 4$.

53. $f(s) = \dfrac{\sqrt{s-1}}{s-4}$

Domain: $s - 1 \geq 0 \Rightarrow s \geq 1$ and $s \neq 4$

The domain consists of all real numbers s, such that $s \geq 1$ and $s \neq 4$.

54. $f(x) = \dfrac{x+2}{\sqrt{x-10}}$

$x - 10 > 0$

$x > 10$

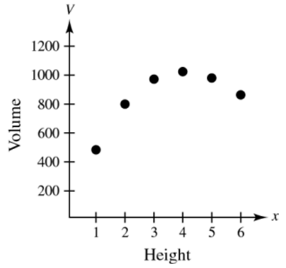
The domain is all real numbers x such that $x > 10$.

55. (a)

Height, x	Volume, V
1	484
2	800
3	972
4	1024
5	980
6	864

The volume is maximum when $x = 4$ and $V = 1024$ cubic centimeters.

(b)



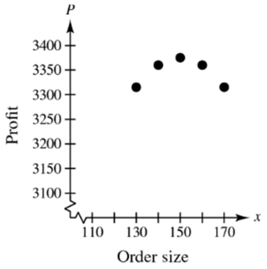
V is a function of x.

(c) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

56. (a) The maximum profit is \$3375.

(b)



Yes, P is a function of x.

(c) Profit = Revenue − Cost

$$= \begin{pmatrix}\text{price}\\ \text{per unit}\end{pmatrix}\begin{pmatrix}\text{number}\\ \text{of units}\end{pmatrix} - (\text{cost})\begin{pmatrix}\text{number}\\ \text{of units}\end{pmatrix}$$

$$= [90 - (x - 100)(0.15)]x - 60x, x > 100$$

$$= (90 - 0.15x + 15)x - 60x$$

$$= (105 - 0.15x)x - 60x$$

$$= 105x - 0.15x^2 - 60x$$

$$= 45x - 0.15x^2, x > 100$$

57. $y = -\frac{1}{10}x^2 + 3x + 6$

$y(25) = -\frac{1}{10}(25)^2 + 3(25) + 6 = 18.5$ feet

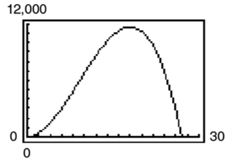
If the child holds a glove at a height of 5 feet, then the ball *will* be over the child's head because it will be at a height of 18.5 feet.

58. (a) $V = l \cdot w \cdot h = x \cdot y \cdot x = x^2y$ where $4x + y = 108$. So, $y = 108 - 4x$ and

$V = x^2(108 - 4x) = 108x^2 - 4x^3$.

Domain: $0 < x < 27$

(b)



(c) The dimensions that will maximize the volume of the package are $18 \times 18 \times 36$. From the graph, the maximum volume occurs when $x = 18$. To find the dimension for y, use the equation $y = 108 - 4x$.

$y = 108 - 4x = 108 - 4(18) - 72 = 36$

59. $A = \frac{1}{2}bh = \frac{1}{2}xy$

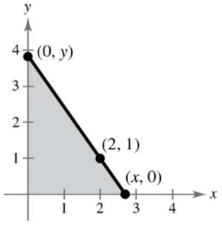
Because $(0, y), (2, 1)$, and $(x, 0)$ all lie on the same line, the slopes between any pair are equal.

$$\frac{1-y}{2-0} = \frac{0-1}{x-2}$$

$$\frac{1-y}{2} = \frac{-1}{x-2}$$

$$y = \frac{2}{x-2} + 1$$

$$y = \frac{x}{x-2}$$



So, $A = \frac{1}{2}x\left(\frac{x}{x-2}\right) = \frac{x^2}{2(x-2)}$.

The domain of A includes x-values such that $x^2/[2(x-2)] > 0$. By solving this inequality, the domain is $x > 2$.

60. $A = l \cdot w = (2x)y = 2xy$

But $y = \sqrt{36 - x^2}$, so $A = 2x\sqrt{36 - x^2}$.

The domain is $0 < x < 6$.

61. $p(t) = \begin{cases} 1.76t + 58.3, & 12 \le t < 16 \\ 0.90t + 71.5, & 16 \le t \le 18 \end{cases}$

For 2012 through 2015, use the first equation.

2012: $p(12) = 1.76(12) + 58.3 = 79.42\%$

2013: $p(13) = 1.76(13) + 58.3 = 81.18\%$

2014: $p(14) = 1.76(14) + 58.3 = 82.94\%$

2015: $p(15) = 1.76(15) + 58.3 = 84.70\%$

For 2016 through 2019, use the second equation.

2016: $p(16) = 0.90(16) + 71.5 = 85.90\%$

2017: $p(17) = 0.90(17) + 71.5 = 86.80\%$

2018: $p(18) = 0.90(18) + 71.5 = 87.70\%$

62. $p(t) = \begin{cases} 4.011t^2 - 76.89t + 586.7, & 7 \le t < 12 \\ 14.94t + 70.0, & 12 \le t < 17 \\ -4.263t^2 + 152.04t - 1030.4, & 17 \le t < 19 \end{cases}$

For 2007 through 2011, use the first equation.

2007: $p(7) = 4.011(7)^2 - 76.89(7) + 586.7 = 245.009 \Rightarrow \$245,009$

2008: $p(8) = 4.011(8)^2 - 76.89(8) + 586.7 = 228.284 \Rightarrow \$228,284$

2009: $p(9) = 4.011(9)^2 - 76.89(9) + 586.7 = 219.581 \Rightarrow \$219,581$

2010: $p(10) = 4.011(10)^2 - 76.89(10) + 586.7 = 218.900 \Rightarrow \$218,900$

2011: $p(11) = 4.011(11)^2 - 76.89(11) + 586.7 = 226.241 \Rightarrow \$226,241$

For 2012 through 2016, use the second equation.

2012: $p(12) = 14.94(12) + 70.0 = 249.28 \Rightarrow \$249,280$

2013: $p(13) = 14.94(13) + 70.0 = 264.22 \Rightarrow \$264,220$

2014: $p(14) = 14.94(14) + 70.0 = 279.16 \Rightarrow \$279,160$

2015: $p(15) = 14.94(15) + 70.0 = 294.1 \Rightarrow \$294,100$

2016: $p(16) = 14.94(16) + 70.0 = 309.04 \Rightarrow \$309,040$

For 2017 through 2019, use the third equation.

2017: $p(17) = -4.263(17)^2 + 152.04(17) - 1030.4 = 322.273 \Rightarrow \$322,273$

2018: $p(18) = -4.263(18)^2 + 152.04(18) - 1030.4 = 325.108 \Rightarrow \$325,108$

2019: $p(19) = -4.263(19)^2 + 152.04(19) - 1030.4 = 319.417 \Rightarrow \$319,417$