Trigonometry A Right Triangle Approach 1st edition Larson Solutions Manual

SKU: 9780357381809-SOLUTIONS

CHAPTER 1
Right Triangle Trigonometry

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CHAPTER 1
Right Triangle Trigonometry

Section 1.1 Angles, Degree Measure, and Special Triangles

1. angle

2. degree

3. quadrantal

4. acute; obtuse

5. hypotenuse; legs

6. You can add or subtract $360°$ or a multiple of $360°$.

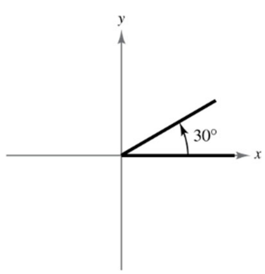
7. The sum of two complementary angles is $90°$ The sum of two supplementary angles is $180°$

8. A scalene triangle has no equal sides. An isosceles triangle has two equal sides. An equilateral triangle has three equal sides.

9. An acute triangle has three acute angles. Obtuse and right triangles have exactly two acute angles.

10. For a 30°-60°-90° triangle, the length of the shorter leg (opposite the 30°-angle) is one-half the length of the hypotenuse. The length of the longer leg (opposite the 60°-angle) is $\sqrt{3}/2$ times the length of the hypotenuse.

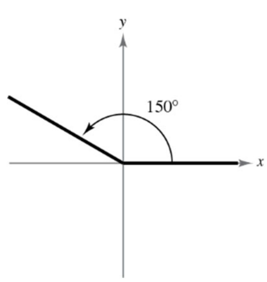
11. (a)



Not a quadrantal angle

Angle lies in Quadrant I.

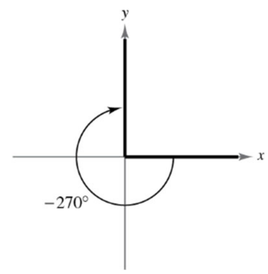
(b)



Not a quadrantal angle

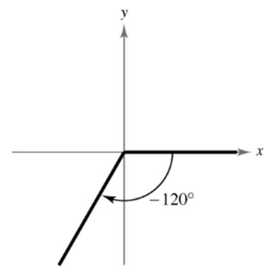
Angle lies in Quadrant II.

12. (a)



Yes, this is a quadrantal angle.

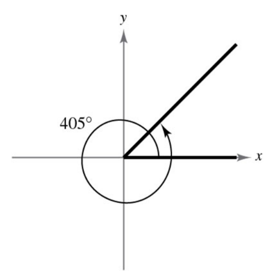
(b)



Not a quadrantal angle

Angle lies in Quadrant III.

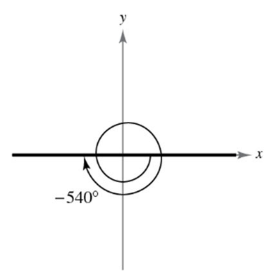
13. (a)



Not a quadrantal angle

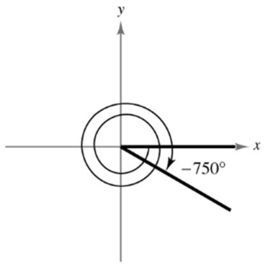
Angle lies in Quadrant I.

(b)



Yes, this is a quadrantal angle.

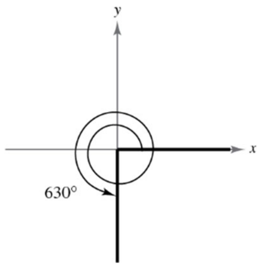
14. (a)



Not a quadrantal angle

Angle lies in Quadrant IV.

(b)



Yes, this is a quadrantal angle.

15. (a) Coterminal angles for $120°$

$120° + 360° = 480°$

$120° - 360° = -240°$

(b) Coterminal angles for $-210°$

$-210° + 360° = 150°$

$-210° - 360° = -570°$

16. (a) Coterminal angles for $45°$

$45° + 360° = 405°$

$45° - 360° = -315°$

(b) Coterminal angles for $-420°$

$-420° + 720° = 300°$

$-420° + 360° = -60°$

17. (a) Coterminal angles for $-300°$

$-300° + 360° = 60°$

$-300° - 360° = -660°$

(b) Coterminal angles for $740°$

$740° - 720° = 20°$

$740° - 1080° = -340°$

18. (a) Coterminal angles for $-520°$

$-520° + 720° = 200°$

$-520° + 360° = -160°$

(b) Coterminal angles for $230°$

$230° + 360° = 590°$

$230° - 360° = -130°$

19. Complement: $90° - 18° = 72°$

Supplement: $180° - 18° = 162°$

20. Complement: $90° - 85° = 5°$

Supplement: $180° - 85° = 95°$

21. Complement: $90° - 46° = 44°$

Supplement: $180° - 46° = 134°$

22. Complement: Not possible. $93°$ is greater than $90°$.

Supplement: $180° - 93° = 87°$

23. Complement: $90° - 24° = 66°$

Supplement: $180° - 24° = 156°$

24. Complement: Not possible. $126°$ is greater than $90°$.

Supplement: $180° - 126° = 54°$

25. Complement: Not possible. $109°$ is greater than $90°$

Supplement: $180° - 109° = 71°$

26. Complement: $90° - 74° = 16°$

Supplement: $180° - 74° = 106°$

27. Let θ be the unknown angle.

$\theta = 2(90° - \theta)$

$3\theta = 180°$

$\theta = 60°$

28. Let θ be the unknown angle.

$\theta = (90° - \theta) - 8°$

$2\theta = 82°$

$\theta = 41°$

29. Let θ be the unknown angle.

$\theta = 4(180° - \theta) - 15°$

$5\theta = 705°$

$\theta = 141°$

30. Let θ be the unknown angle.

$\theta = \frac{1}{3}(180° - \theta) + 4°$

$\frac{4}{3}\theta = 64°$

$\theta = 48°$

31. $100° + 55° + 35° = 190° \neq 180°$

The angle measures do not form a triangle.

32. $86° + 84° + 10° = 180°$

The angle measures form a triangle.

33. $91° + 45° + 44° = 180°$

The angle measures form a triangle.

34. $83° + 63° + 33° = 179° \neq 180°$
The angle measures do not form a triangle.

35. $112° + 41° + 27° = 180°$
The angle measures form a triangle.

36. $72° + 72° + 38° = 182° \neq 180°$
The angle measures do not form a triangle.

37. $$\begin{aligned} x° + 88° + 45° &= 180° \\ x &= 180 - 88 - 45 \\ x &= 47 \end{aligned}$$
$A = 47°$

38. $$\begin{aligned} x° + 2x° + 66° &= 180° \\ 3x &= 114 \\ x &= 38 \end{aligned}$$
$B = 38°$
$C = 2(38°) = 76°$

39. $$\begin{aligned} (x + 8)° + (x - 8)° &= 90° \\ 2x &= 90 \\ x &= 45 \end{aligned}$$
$A = 45° + 8° = 53°$
$B = 45° - 8° = 37°$

40. $$\begin{aligned} x° + (x + 22)° &= 90° \\ 2x &= 68 \\ x &= 34 \end{aligned}$$
$A = 34° + 22° = 56°$
$C = 34°$

41. $$\begin{aligned} 16x° + 18x° + 11x &= 180° \\ 45x &= 180 \\ x &= 4 \end{aligned}$$
$A = 16(4°) = 64°$
$B = 18(4°) = 72°$
$C = 11(4°) = 44°$

42. $$\begin{aligned} \tfrac{3}{4}x° + (3x + 14)° + 2x° &= 180° \\ \tfrac{23}{4}x &= 166 \\ x &= \left(\tfrac{664}{23}\right) \approx 28.87° \end{aligned}$$
$A \approx \frac{3}{4}(28.87°) = 21.65°$
$B \approx 3(28.87°) + 14° = 100.61°$
$C \approx 2(28.87°) = 57.74°$

43. (a) The triangle has two equal sides, so it is isosceles.
(b) The triangle has three acute angles, so it is acute.

44. (a) The triangle has no equal sides, so it is scalene.
(b) The triangle has one obtuse angle, so it is obtuse.

45. (a) The triangle has three equal sides so it is equilateral.
(b) The triangle has three acute angles, so it is acute.

46. (a) The triangle has no equal sides, so it is scalene.
(b) The triangle has three acute angles, so it is acute.

47. (a) The triangle has no equal sides, so it is scalene.
(b) The triangle has one right angle, so it is right.

48. (a) The triangle has two equal sides, so it is isosceles.
(b) The triangle has one obtuse angle, so it is obtuse.
$[180° - 2(20°) = 180° - 40° = 140°]$

49. (a) The triangle has two equal sides, so it is isosceles.
(b) The triangle has one obtuse angle, so it is obtuse.
$[180° - 2(43°) = 180° - 86° = 94°]$

50. (a) The triangle has no equal sides, so it is scalene.
(b) The triangle has one right angle, so it is right.
$(59° + 31° = 90°)$

51. Let A be an acute angle.
$$\begin{aligned} A + 9A &= 90° \\ 10A &= 90° \\ A &= 9° \end{aligned}$$
The unknown angles are $9°$ and $9(9°) = 81°$.

52. Let A be an acute angle.
$$\begin{aligned} A + (A - 36°) &= 90° \\ 2A &= 126° \\ A &= 63° \end{aligned}$$
The unknown angles are $63°$ and $63° - 36° = 27°$.

53. $$\begin{aligned} A + B + C &= 180° \\ 84° + 3(C + 8°) + C &= 180° \\ 4C &= 72° \\ C &= 18° \end{aligned}$$
$B = 3(18° + 8°) = 78°$

54. Let A, B, and C be the angle measures of the triangle.

$$\begin{aligned} A + B + C &= 180° \\ A + 4A + \tfrac{5}{4}(A + 4A) &= 180° \\ 5A + \tfrac{25}{4}A &= 180° \\ \left(\tfrac{45}{4}\right)A &= 180° \\ A &= 16° \end{aligned}$$

$A = 16°, B = 4(16°) = 64°,$

$C = 180° - 16° - 64° = 100°$

55. $$\begin{aligned} x^2 + 8^2 &= 12^2 \\ x^2 &= 144 - 64 = 80 \\ x &= \sqrt{80} = 4\sqrt{5} \end{aligned}$$

56. $$\begin{aligned} x^2 + 9^2 &= (x + 1)^2 \\ x^2 + 81 &= x^2 + 2x + 1 \\ 80 &= 2x \\ x &= 40 \end{aligned}$$

57. $$\begin{aligned} x^2 + (x + 2)^2 &= 10^2 \\ x^2 + x^2 + 4x + 4 &= 100 \\ 2x^2 + 4x - 96 &= 0 \\ x^2 + 2x - 48 &= 0 \\ (x - 6)(x + 8) &= 0 \end{aligned}$$

Choose the positive solution, so $x = 6$.

58. $$\begin{aligned} 7^2 + (x - 1)^2 &= x^2 \\ 49 + x^2 - 2x + 1 &= x^2 \\ 50 &= 2x \\ x &= 25 \end{aligned}$$

59. $$\begin{aligned} x^2 + (x + 1)^2 &= (2x - 11)^2 \\ x^2 + x^2 + 2x + 1 &= 4x^2 - 44x + 121 \\ 2x^2 - 46x + 120 &= 0 \\ x^2 - 23x + 60 &= 0 \\ (x - 20)(x - 3) &= 0 \\ x &= 20 \end{aligned}$$

(**Note:** If $x = 3$, then the hypotenuse is $2(3) - 11 = -5$ which is not possible.)

60. $$\begin{aligned} x^2 + (2x - 2)^2 &= (2x + 2)^2 \\ x^2 + 4x^2 - 8x + 4 &= 4x^2 + 8x + 4 \\ x^2 - 16x &= 0 \\ x(x - 16) &= 0 \end{aligned}$$

Choose the positive solution, so $x = 16$.

61. $7^2 + 24^2 = 49 + 576 = 625 = 25^2$

The triangle is a right triangle.

62. $6^2 + 1.1^2 = 36 + 1.21 = 37.21 = 6.1^2$

The triangle is a right triangle.

63. $12^2 + \left(2\sqrt{13}\right)^2 = 144 + 52 = 196 = 14^2$

The triangle is a right triangle.

64. $80^2 + 35^2 = 6400 + 1225 = 7625 \neq 89^2 = 7921$

The triangle is not a right triangle.

65. $1^2 + 2^2 = 1 + 4 = 5 \neq 3^2 = 9$

Not a Pythagorean triple.

66. $15^2 + 26^2 = 225 + 676 = 901 \neq 27^2 = 729$

Not a Pythagorean triple

67. $16^2 + 63^2 = 256 + 3969 = 4225 = 65^2$

Pythagorean triple

68. $36^2 + 77^2 = 1296 + 5929 = 7225 = 85^2$

Pythagorean triple

69. The triangle is an isosceles right triangle. Let x be the length of a leg.

$$\begin{aligned} x^2 + x^2 &= \left(16\sqrt{2}\right)^2 \\ 2x^2 &= 512 \\ x^2 &= 256 \\ x &= 16 \end{aligned}$$

$\text{Legs} = 16$

70. The triangle is an isosceles right triangle. Let x be the length of the hypotenuse.

$$\begin{aligned} 9^2 + 9^2 &= x^2 \\ 81 + 81 &= x^2 \\ x &= 9\sqrt{2} \end{aligned}$$

$\text{Legs} = 9$, hypotenuse $= 9\sqrt{2}$

71. The triangle is a 30°-60°-90° right triangle. The shortest side is $\sqrt{3}$, so the hypotenuse is $2\sqrt{3}$. The other leg is $\left(\sqrt{3}/2\right)\left(2\sqrt{3}\right) = 3$.

72. The triangle is a 30°-60°-90° right triangle. The shortest side is 12, so the hypotenuse is $2(12) = 24$. The other leg is $\left(\sqrt{3}/2\right)(24) = 12\sqrt{3}$.

73. The triangle is a 30°-60°-90° right triangle. The longest side is $6\sqrt{3}$, so the hypotenuse is $\left(\sqrt{3}/2\right)x = 6\sqrt{3} \Rightarrow x = 12$. The shortest side is 6.

74. The triangle is a 30°-60°-90° right triangle. The hypotenuse is $8\sqrt{3}$, so the shortest side is $4\sqrt{3}$. The longest side is $\left(8\sqrt{3}\right)\left(\sqrt{3}/2\right) = 12$.

75. The hypotenuse is $2\sqrt{2}$, so each leg measures $\left(2\sqrt{2}\right)/\sqrt{2} = 2$.

76. Each leg measures $\sqrt{2}$, so the hypotenuse is $\left(\sqrt{2}\right)\left(\sqrt{2}\right) = 2$.

77. The longest side is 3, so the hypotenuse x satisfies $\left(\sqrt{3}/2\right)x = 3 \Rightarrow x = 2\sqrt{3}$. The shortest side is $\sqrt{3}$.

78. The hypotenuse is 4, so the shortest side is 2 and the longest side is $\left(\sqrt{3}/2\right)4 = 2\sqrt{3}$.

79. The hypotenuse is 12, so the longest side is $\left(\sqrt{3}/2\right)(12) = 6\sqrt{3}$ feet. The bed extends about $6\sqrt{3} \approx 10.4$ feet from the frame.

80. (a) At 2:00, the angle is 60°.
(b) At 7:00, the angle is $180° - 30° = 150°$.
(c) *Sample answer:* The hands form a straight angle at 6:00.

81. True. If α and β are coterminal angles, then $\alpha = \beta + n(360°)$ or $\alpha = \beta + n(2\pi)$, where n is an integer. The difference between α and β is $\alpha - \beta = n(360°)$, or $\alpha - \beta = n(2\pi)$ if expressed in radians.

82. False. The terminal side of $-1260°$ lies on the negative x-axis.

83. False. The sum of the angle measures of a triangle is equal to $180°$, whereas the measure of two obtuse angles is greater than $180°$

84. True. An equilateral triangle has three acute angles that each measure $60°$.

85. True. A 45°-45°-90° triangle is isosceles.

86. False. The longest side of a 30°-60°-90° triangle is twice the length of the shortest side, whereas $9 = 3(3)$.

87. The first step is incorrect. The hypotenuse x is twice the length of the shorter leg.
So, $x = 2\left(5\sqrt{3}\right) = 10\sqrt{3}$.

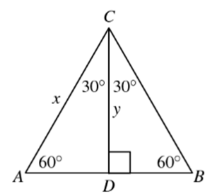
88. The Pythagorean is applied incorrectly.
The first step should be $x^2 + 10^2 = 26^2$.
Then $x^2 = 26^2 - 10^2 = 576 \Rightarrow x = 24$.

89. The side lengths are $3x$, $5x - 12$, and $x + 20$. If $3x = 5x - 12$, then $2x = 12 \Rightarrow x = 6$. If $3x = x + 20$, then $2x = 20 \Rightarrow x = 10$. If $5x - 12 = x + 20$, then $4x = 32 \Rightarrow x = 8$. So, $x = 6,\ 8$, or 10.

90. Angles B and C are coterminal with Angle A, since they have the same initial and terminal side as Angle A.

91. Let x be the length of the legs. Then the hypotenuse measures $\sqrt{x^2 + x^2} = x\sqrt{2}$.

92. Consider an equilateral triangle divided into two 30°-60°-90° triangles.



If the hypotenuse is x, then the shorter leg AD measures $\frac{1}{2}x$.

By the Pythagorean Theorem,

$$\left(\frac{1}{2}x\right)^2 + y^2 = x^2$$
$$y^2 = \frac{3}{4}x^2$$
$$y = \frac{\sqrt{3}}{2}x.$$

So, the longer leg is $\sqrt{3}$ times the length of the shorter leg $\frac{1}{2}x$.

93. $x + 5 = 13$
$x = 8$

Check: $8 + 5 = 13$

94. $4d = -28$
$d = -\frac{28}{4} = -7$

Check: $4(-7) = -28$

95. $-2a - 3 = 9$

$-2a = 12$

$a = -6$

Check: $-2(-6) - 3 = 12 - 3 = 9$

96. $16 = \frac{1}{2}y + 7$

$9 = \frac{1}{2}y$

$y = 18$

Check: $\frac{1}{2}(18) + 7 = 9 + 7 = 16$

97. $-1 = 5w - (2 + w)$

$-1 = 5w - 2 - w$

$1 = 4w$

$w = \frac{1}{4}$

Check: $5\left(\frac{1}{4}\right) - \left(2 + \frac{1}{4}\right) = \frac{5}{4} - \frac{9}{4} = -\frac{4}{4} = -1$

98. $13 - (3z - 4) = 22$

$-3z + 17 = 22$

$-3z = 5$

$z = -\frac{5}{3}$

Check: $13 - \left[3\left(-\frac{5}{3}\right) - 4\right] = 13 - (-9) = 22$

99. $\frac{n}{6} = \frac{20}{24}$

$24n = 20(6)$

$n = \frac{120}{24} = 5$

Check: $\frac{20}{24} = \frac{5}{6}$

100. $\frac{15}{3} = \frac{21}{y}$

$15y = 3(21)$

$5y = 21$

$y = \frac{21}{5}$

Check: $\frac{21}{(21/5)} = 5 = \frac{15}{3}$

101. $\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

102. $-\frac{\sqrt{6}}{\sqrt{7x}} = -\frac{\sqrt{6}}{\sqrt{7x}} \bullet \frac{\sqrt{7x}}{\sqrt{7x}}$

$= -\frac{\sqrt{42x}}{7x}$

103. $\sqrt{\frac{4w^2}{5}} = \frac{2w}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{2\sqrt{5}\,w}{5}$

104. $\sqrt{\frac{20}{3t^2}} = \frac{2\sqrt{5}}{\sqrt{3}\,t} \bullet \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{15}}{3t}$

Section 1.2 Similar Triangles

1. (a) alternate interior

(b) alternate exterior

(c) corresponding

(d) consecutive interior

2. Congruent; Similar

3. Vertical angles are formed by two lines that intersect at a point, forming two pairs of angles that lie opposite each other.

4. *Sample answer:* $\frac{AB}{XY} = \frac{BC}{YZ}$

$\frac{AB}{XY} = \frac{CA}{ZX}$

$\frac{BC}{YZ} = \frac{CA}{ZX}$

5. (a) Vertical angles: $\angle 1$ and $\angle 6$, $\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 8$, $\angle 4$ and $\angle 7$

(b) Alternate interior angles: $\angle 2$ and $\angle 7$, $\angle 3$ and $\angle 6$

(c) Alternate exterior angles: $\angle 1$ and $\angle 8$, $\angle 4$ and $\angle 5$

(d) Corresponding angles: $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$, $\angle 5$ and $\angle 7$, $\angle 6$ and $\angle 8$

(e) Consecutive interior angles: $\angle 2$ and $\angle 3$, $\angle 6$ and $\angle 7$

6. (a) $\angle 1$ and $\angle 14$: alternate exterior angles

(b) $\angle 5$ and $\angle 7$: corresponding angles

(c) $\angle 11$ and $\angle 8$: alternate interior angles

(d) $\angle 10$ and $\angle 13$: vertical angles

(e) $\angle 2$ and $\angle 3$: consecutive interior angles

(f) $\angle 9$ and $\angle 16$: alternate exterior angles

(g) $\angle 15$ and $\angle 12$: vertical angles

(h) $\angle 4$ and $\angle 6$: none of these

7. The angles are supplementary.

$$\begin{aligned}(6x - 15)^\circ + 57^\circ &= 180^\circ\\ (6x) &= 138\\ x &= 23\end{aligned}$$

8. The angles are supplementary.

$$\begin{aligned}(12x + 5)^\circ + 115^\circ &= 180^\circ\\ (12x) &= 60\\ x &= 5\end{aligned}$$

9. $\frac{1}{3}y^\circ + y^\circ = 180^\circ \Rightarrow \frac{4}{3}y = 180 \Rightarrow y = 135$

$x^\circ + 3x^\circ = 180^\circ \Rightarrow 4x = 180 \Rightarrow x = 45$

10. $\begin{cases}(x + 30)^\circ = (y - 30)^\circ\\ (x - 30)^\circ = 3(y - 130)^\circ\end{cases}$

$\begin{cases}x + 30 = y - 30\\ x - 30 = 3y - 390\end{cases}$

Subtract the two equations to get
$60 = -2y + 360 \Rightarrow y = 150.$

Finally, $x + 30 = 150 - 30 \Rightarrow x = 90.$

11. The triangles are not similar because none of the acute angles have the same measure.

12. The triangles are similar because corresponding angles have the same measure. (The angle at E measures $180^\circ - 50^\circ - 85^\circ = 45^\circ.$)

13. The triangles are similar because corresponding sides are proportional:

$$\frac{AC}{DF} = \frac{AB}{ED} = \frac{BC}{EF} = \frac{4}{5}.$$

14. The triangles are not similar because corresponding sides are not proportional. For example, $\frac{AC}{DF} = \frac{12}{11}$ and $\frac{AB}{DE} = \frac{7}{6}.$

15. $B = 90^\circ - 32^\circ = 58^\circ = E$

$C = 32^\circ = F$

16.
$$\begin{aligned}7x^\circ + 4(x + 1)^\circ + 4(x - 1)^\circ &= 180^\circ\\ 15x &= 180\\ x &= 12\end{aligned}$$

$D = 4(x - 1)^\circ = 44^\circ = A$

$B = 7x^\circ = 84^\circ = E$

$C = 4(x + 1)^\circ = 52^\circ = F$

17.
$$\begin{aligned}\frac{AC}{DF} &= \frac{AB}{DE}\\ \frac{18}{2x} &= \frac{x}{16}\\ 2x^2 &= 288\\ x^2 &= 144\\ x &= 12\\ \frac{BC}{EF} &= \frac{AB}{DE}\\ \frac{y}{20} &= \frac{12}{16}\\ y &= 15\end{aligned}$$

$AB = 12$ units, $BC = 15$ units, $DF = 24$ units

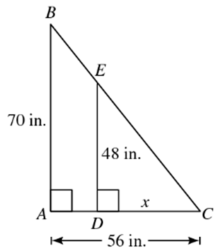
18.
$$\begin{aligned}\frac{AC}{DF} &= \frac{BC}{EF}\\ \frac{12}{16} &= \frac{4x - 3}{4x}\\ 48x &= 64x - 48\\ 48 &= 16x\\ x &= 3\\ \frac{AC}{DF} &= \frac{AB}{DE}\\ \frac{12}{16} &= \frac{y}{y + 5}\\ 12y + 60 &= 16y\\ 60 &= 4y\\ y &= 15\end{aligned}$$

$AB = 15$ units, $BC = 9$ units, $DE = 20$ units, $EF = 12$ units

19. The radius of the moon is DE. The triangles ABC and DEC are similar.

$$\begin{aligned}\frac{DE}{AB} &= \frac{DC}{AC}\\ \frac{DE}{432{,}300} &= \frac{239{,}000}{93{,}000{,}000}\\ DE &= \frac{239{,}000}{93{,}000{,}000}(432{,}300) \approx 1111 \text{ miles}\end{aligned}$$

20.



Let x be the length of the shorter shadow. Triangle ABC and DEC are similar.

$$\frac{DC}{56} = \frac{48}{70}$$

$$DC = \frac{48}{70}(56) = 38.4$$

The shadow is 38.4 inches.

21. True. Vertical angles that measure $45°$ are complementary.

22. False. If a pair of lines cut by a transversal are not parallel, then the alternate interior angles do not have equal measure.

23. False. Similar triangles can have different sizes.

24. True. Congruent triangles have the same shape.

25. It is true that all equilateral triangles are similar. All the angles measure $60°$. However, not all right triangles are similar because corresponding acute angles are not always equal.

26. Sample answer: Triangles BCD and ACE are similar, so you can use the proportion $\frac{x}{x + 24} = \frac{120}{150}$ to find the distance x between the shoreline and the buoy. (**Note:** $x = 96$ m)

27. (a) The angles form a straight angle, so $\angle 1 + \angle 2 + \angle 3 = 180°$.

(b) $\angle 1$ and $\angle 6$ are corresponding angles of parallel lines cut by a transversal, as are $\angle 3$ and $\angle 5$. So, $\angle 1 = \angle 6$ and $\angle 3 = \angle 5$. $\angle 2$ and $\angle 4$ are vertical angles, so $\angle 2 = \angle 4$.

(c) Because $\angle 1 + \angle 2 + \angle 3 = 180°$, by substitution, $\measuredangle 6 + \measuredangle 4 + \measuredangle 5 = 180°$.

28. Let x be the length of the hypotenuse. Then $\left(\sqrt{3}/2\right)x = 5\sqrt{3}$, which implies that $x = 10$. Finally, the shorter leg measures 5 units.

29. $f(t) = -4t + 10$

(a) $f(-1) = -4(-1) + 10 = 4 + 10 = 14$

(b) $f(2) = -4(2) + 10 = -8 + 10 = 2$

(c) $f(5) = -4(5) + 10 = -20 + 10 = -10$

30. $f(x) = 2x^2 - 3x + 5$

(a) $f(-2) = 2(-2)^2 - 3(-2) + 5$
$= 8 + 6 + 5$
$= 19$

(b) $f(0) = 2\left(0^2\right) - 3(0) + 5 = 5$

(c) $f(3.5) = 2(3.5)^2 - 3(3.5) + 5$
$= 24.5 - 10.5 + 5$
$= 19$

31. $f(x) = \sqrt{x - 9}$

(a) $f(5) = \sqrt{5 - 9} = \sqrt{-4} \Rightarrow$ Not possible

(b) $f(9) = \sqrt{9 - 9} = 0$

(c) $f(25) = \sqrt{25 - 9} = \sqrt{16} = 4$

32. $f(n) = \dfrac{3n - 6}{n}$

(a) $f(-2) = \dfrac{3(-2) - 6}{-2} = \dfrac{-12}{-2} = 6$

(b) $f(0)$ not possible (division by 0).

(c) $f(2) = \dfrac{3(2) - 6}{2} = \dfrac{0}{2} = 0$

Section 1.3 Trigonometric Functions: Right Triangle Ratios

1. sine, cotangent, secant

2. minute; second

3. (a) $\dfrac{\text{opposite}}{\text{hypotenuse}} = \sin\theta$ (v) (b) $\dfrac{\text{adjacent}}{\text{hypotenuse}} = \cos\theta$ (iv) (c) $\dfrac{\text{opposite}}{\text{adjacent}} = \tan\theta$ (vi)

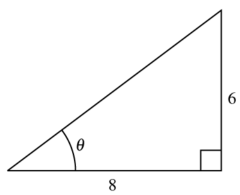
(d) $\dfrac{\text{hypotenuse}}{\text{opposite}} = \csc\theta$ (iii) (e) $\dfrac{\text{hypotenuse}}{\text{adjacent}} = \sec\theta$ (i) (f) $\dfrac{\text{adjacent}}{\text{opposite}} = \cot\theta$ (ii)

4. (a) $\sin 30°$, $\cos 60°$ matches (i) $\dfrac{1}{2}$. (b) $\cos 30°$, $\sin 60°$ matches (iv) $\dfrac{\sqrt{3}}{2}$.

(c) $\tan 30°$ matches (ii) $\dfrac{\sqrt{3}}{3}$. (d) $\sin 45°$, $\cos 45°$ matches (iii) $\dfrac{\sqrt{2}}{2}$.

(e) $\tan 45°$ matches (v) 1. (f) $\tan 60°$ matches (vi) $\sqrt{3}$.

5. $\text{hyp} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

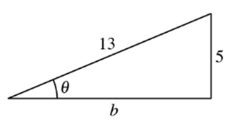


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{10}{6} = \frac{5}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{8}{10} = \frac{4}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{10}{8} = \frac{5}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{6}{8} = \frac{3}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{8}{6} = \frac{4}{3}$$

6. $\text{adj} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = 12$

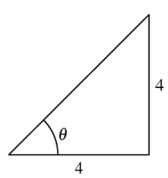


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{13}{5}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{13}{12}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{5}{12} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{12}{5}$$

7. $\text{hyp} = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$

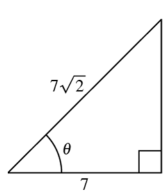


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{4\sqrt{2}}{4} = \sqrt{2}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{4\sqrt{2}}{4} = \sqrt{2}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{4} = 1 \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{4} = 1$$

8. $\text{opp} = \sqrt{\left(7\sqrt{2}\right)^2 - 7^2} = \sqrt{98 - 49} = \sqrt{49} = 7$

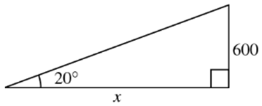


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{7}{7\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{7\sqrt{2}}{7} = \sqrt{2}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{7}{7\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{7\sqrt{2}}{7} = \sqrt{2}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{7}{7} = 1 \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{7}{7} = 1$$

9. $\text{adj} = \sqrt{41^2 - 9^2} = \sqrt{1681 - 81} = \sqrt{1600} = 40$

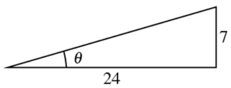


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{9}{41} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{41}{9}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{40}{41} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{41}{40}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{9}{40} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{40}{9}$$

10. $\text{hyp} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$

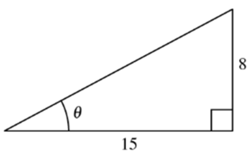


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{7}{25} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{25}{7}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{24}{25} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{25}{24}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{7}{24} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{24}{7}$$

11. $\text{hyp} = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$

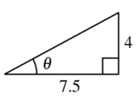


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{17} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{17}{8}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{15}{17} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{17}{15}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{15} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{15}{8}$$

$\text{hyp} = \sqrt{7.5^2 + 4^2} = \dfrac{17}{2}$



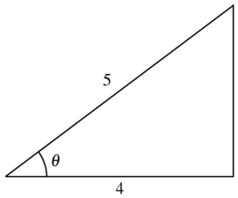
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{(17/2)} = \frac{8}{17} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{(17/2)}{4} = \frac{17}{8}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{7.5}{(17/2)} = \frac{15}{17} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{(17/2)}{7.5} = \frac{17}{15}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{7.5} = \frac{8}{15} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{7.5}{4} = \frac{15}{8}$$

The function values are the same because the triangles are similar, and corresponding sides are proportional.

12. opp $= \sqrt{5^2 - 4^2} = 3$

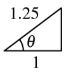


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{3}$$

opp $= \sqrt{1.25^2 - 1^2} = 0.75$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{0.75}{1.25} = \frac{3}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{1.25}{0.75} = \frac{5}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{1.25} = \frac{4}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{1.25}{1} = \frac{5}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{0.75}{1} = \frac{3}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{0.75} = \frac{4}{3}$$

The function values are the same since the triangles are similar and the corresponding sides are proportional.

13. Given: $\cos\theta = \frac{15}{17} = \frac{\text{adj}}{\text{hyp}}$

$$(\text{opp})^2 + 15^2 = 17^2$$

$$\text{opp} = \sqrt{289 - 225}$$

$$\text{opp} = \sqrt{64} = 8$$

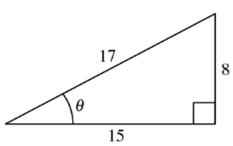
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{17}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{15}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{17}{8}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{17}{15}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{15}{8}$$



14. Given: $\sin\theta = \frac{3}{5} = \frac{\text{opp}}{\text{hyp}}$

$$(\text{adj})^2 + 3^2 = 5^2$$

$$\text{adj} = \sqrt{25 - 9} = 4$$

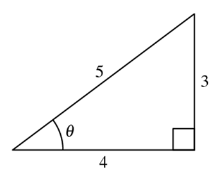
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$$

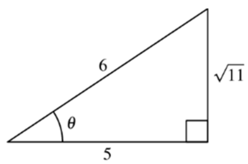
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{3}$$



15. Given: $\sec\theta = \frac{6}{5} = \frac{\text{hyp}}{\text{adj}}$

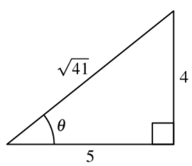
$$(\text{opp})^2 + 5^2 = 6^2$$
$$\text{opp} = \sqrt{36 - 25} = \sqrt{11}$$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{11}}{6}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{6}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{11}}{5}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{6\sqrt{11}}{11}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{5\sqrt{11}}{11}$$

16. Given: $\tan\theta = \frac{4}{5} = \frac{\text{opp}}{\text{adj}}$

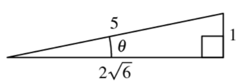
$$4^2 + 5^2 = (\text{hyp})^2$$
$$\text{hyp} = \sqrt{41}$$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4\sqrt{41}}{41}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5\sqrt{41}}{41}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{41}}{4}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{41}}{5}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{5}{4}$$

17. Given: $\sin\theta = \frac{1}{5} = \frac{\text{opp}}{\text{hyp}}$

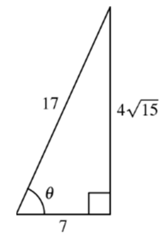
$$1^2 + (\text{adj})^2 = 5^2$$
$$\text{adj} = \sqrt{24} = 2\sqrt{6}$$



$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2\sqrt{6}}{5}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{6}}{12}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = 5$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5\sqrt{6}}{12}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = 2\sqrt{6}$$

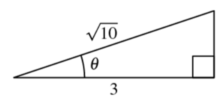
18. Given: $\sec\theta = \frac{17}{7} = \frac{\text{hyp}}{\text{adj}}$

$$(\text{opp})^2 + 7^2 = 17^2$$
$$\text{opp} = \sqrt{240} = 4\sqrt{15}$$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4\sqrt{15}}{17}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{7}{17}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4\sqrt{15}}{7}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{17\sqrt{15}}{60}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{7\sqrt{15}}{60}$$

19. Given: $\cot\theta = 3 = \frac{3}{1} = \frac{\text{adj}}{\text{opp}}$

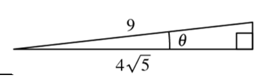


$$1^2 + 3^2 = (\text{hyp})^2$$
$$\text{hyp} = \sqrt{10}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{10}}{10}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3\sqrt{10}}{10}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{3}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \sqrt{10}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{10}}{3}$$

20. Given: $\csc\theta = 9 = \frac{9}{1} = \frac{\text{hyp}}{\text{opp}}$

$$1^2 + (\text{adj})^2 = 9^2$$
$$\text{adj} = \sqrt{80} = 4\sqrt{5}$$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{9}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4\sqrt{5}}{9}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{5}}{20}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{9\sqrt{5}}{20}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = 4\sqrt{5}$$

21. $\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

22. $\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

23. $\tan\theta = 1 \Rightarrow \theta = 45°$

24. $\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°$

25. $\csc\theta = \frac{2\sqrt{3}}{3} \Rightarrow \theta = 60°$

26. $\sec\theta = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \theta = 45°$

27. $\cot\theta = 1 \Rightarrow \theta = 45°$

28. $\cos\theta = \frac{2\sqrt{3}}{3}$ not possible because $\frac{2\sqrt{3}}{3} > 1$

29. $\cos 60° = \frac{x}{18}$

$x = 18\cos 60° = 18\left(\frac{1}{2}\right) = 9$

$\sin 60° = \frac{y}{18}$

$y = 18\sin 60° = 18\frac{\sqrt{3}}{2} = 9\sqrt{3}$

30. $\tan 30° = \frac{30}{x}$

$\frac{1}{\sqrt{3}} = \frac{30}{x}$

$x = 30\sqrt{3}$

$\sin 30° = \frac{30}{r}$

$r = \frac{30}{\sin 30°} = \frac{30}{\frac{1}{2}} = 60$

31. $\tan 45° = \frac{20}{x}$

$x = \frac{20}{\tan 45°}$

$x = \frac{20}{1} = 20$

$\sin 45° = \frac{20}{r}$

$r = \frac{20}{\sin 45°} = \frac{20}{\frac{\sqrt{2}}{2}} = 20\sqrt{2}$

32. $\tan 60° = \frac{32}{y}$

$\sqrt{3} = \frac{32}{y}$

$\sqrt{3}y = 32$

$y = \frac{32}{\sqrt{3}} = \frac{32\sqrt{3}}{3}$

$\sin 60° = \frac{32}{r}$

$r = \frac{32}{\sin 60°}$

$r = \frac{32}{\frac{\sqrt{3}}{2}} = \frac{64\sqrt{3}}{3}$

33. $\sin 71° = \cos(90° - 71°) = \cos 19°$

34. $\cos 22° = \sin(90° - 22°) = \sin 68°$

35. $\tan 8° = \cot(90° - 8°) = \cot 82°$

36. $\csc 56° = \sec(90° - 56°) = \sec 34°$

37.
$$\begin{aligned} \sin(2A + 3°) &= \cos(3A - 68°) \\ (2A + 3°) + (3A - 68°) &= 90° \\ 5A - 65° &= 90° \\ 5A &= 155° \\ A &= 31° \end{aligned}$$

38.
$$\begin{aligned} \cos 5A &= \sin(82° - A) \\ 5A + (82° - A) &= 90° \\ 4A &= 8° \\ A &= 2° \end{aligned}$$

39. $\csc\left(\frac{5}{2}A + 42°\right) = \sec\left(42° - \frac{3}{2}A\right)$

$$\left(\tfrac{5}{2}A + 42°\right) + \left(42° - \tfrac{3}{2}A\right) = 90°$$
$$A = 90° - 84°$$
$$A = 6°$$

40. $\tan\left(\frac{1}{2}A + 17°\right) = \cot(3A + 10°)$

$$\left(\tfrac{1}{2}A + 17°\right) + (3A + 10°) = 90°$$
$$\tfrac{7}{2}A + 27° = 90°$$
$$\tfrac{7}{2}A = 63°$$
$$A = 18°$$

41. $\cos\theta = \frac{1}{3}$

(a) $\sec\theta = \dfrac{1}{\cos\theta} = 3$

(b) $\sin(90° - \theta) = \cos\theta = \frac{1}{3}$

(c) $\csc(90° - \theta) = \sec\theta = 3$

42. $\cot\theta = \sqrt{3}$

(a) $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

(b) $\tan(90° - \theta) = \cot\theta = \sqrt{3}$

(c) $\cot(90° - \theta) = \tan\theta = \dfrac{\sqrt{3}}{3}$

43. $34.121° = 34° + 0.121°$

$$= 34° + 0.121°\left(\frac{60'}{1°}\right)$$
$$= 34° + 7.26'$$
$$= 34° + 7' + 0.26'$$
$$= 34° + 7' + 0.26'\left(\frac{60''}{1'}\right)$$
$$= 34° + 7' + 15.6''$$

So, to the nearest second, $34.121° = 34°\,7'\,16''$.

44. $0.457° = 0.457°\left(\dfrac{60'}{1°}\right)$

$$= 27.42'$$
$$= 27' + 0.42'\left(\frac{60''}{1°}\right)$$
$$= 27' + 25.2''$$

So, to the nearest second, $0.457 = 27'\,25''$.

45. $2.559° = 2° + 0.559°$

$$= 2° + 0.559°\left(\frac{60'}{1°}\right)$$
$$= 2° + 33.54'$$
$$= 2° + 33' + 0.54'\left(\frac{60''}{1°}\right)$$
$$= 2° + 33' + 32.4''$$

So, to the nearest second, $2.559° = 2°\,33'\,32''$.

46. $63.583° = 63° + 0.583°$

$$= 63° + 0.583°\left(\frac{60'}{1°}\right)$$
$$= 63° + 34.98'$$
$$= 63° + 34' + 0.98'\left(\frac{60''}{1°}\right)$$
$$= 63° + 34' + 58.8''$$

So, to the nearest second, $63.583° = 63°\,34'\,59''$.

47. $35°\,38' = 35° + \left(\dfrac{38}{60}\right)^{\circ} = 35.633°$

48. $30°\,32'' = 30° + \left(\dfrac{32}{3600}\right)^{\circ} = 30.009°$

49. $85°\,18'\,25'' = 85° + \left(\dfrac{18}{60}\right)^{\circ} + \left(\dfrac{25}{3600}\right)^{\circ}$

$$= 85.307°$$

50. $48°\,16'\,20'' = 48° + \left(\dfrac{16}{60}\right)^{\circ} + \left(\dfrac{20}{3600}\right)^{\circ}$

$$= 48.272°$$

51. $\cot(73°\,45') = \dfrac{1}{\tan(73°\,45')} \approx 0.291$

52. $\cos(37°\,30'\,36'') \approx 0.793$

53. $\sin(51°\,20'\,15'') \approx 0.781$

54. $\sec(61°\,42'\,32'') = \dfrac{1}{\cos(61°\,42'\,32'')}$

$$\approx 2.110$$

55. (a) No, the road is not steeper because $\tan 4.3° \approx 0.0752 < 0.08 = \frac{2}{25}$.

(b) $x = 100\tan\theta = 100(0.0752) = 7.52\%$

The maximum recommended percent grade is $100(0.08) = 8.0\%$. The results are the same.

56. (a) Yes, the road is steeper because $\tan 4.6° \approx 0.0804 > 0.08 = \frac{2}{25}$.

(b) $x = 100 \tan \theta = 100(0.0804) = 8.04\%$

The maximum recommended percent grade is $100(0.08) = 8\%$. The results are the same.

57. True. $1'' = 1 \text{ second} = \dfrac{1}{3600}(1°) \Rightarrow 3600'' = 1°$

58. False. $1'' = 1 \text{ second} = \dfrac{1}{60}(1') \Rightarrow 60'' = 1'$

59. False. $\sec 30° = \dfrac{1}{\cos 30°} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$ and

$\csc 30° = \dfrac{1}{\sin 30°} = 2.$

60. (a) The side y is opposite angle θ.

(b) The side y is adjacent to angle $90° - \theta$.

(c) $\sin \theta = y/r$ and $\cos(90° - \theta) = y/r$

61.
$$x^2 + x^2 = 72^2$$
$$2x^2 = 5184$$
$$x^2 = 2592$$
$$x = \sqrt{2592} = 36\sqrt{2}$$

62.
$$(1.5)^2 + (4x)^2 = (5x)^2$$
$$2.25 + 16x^2 = 25x^2$$
$$9x^2 = 2.25$$
$$x^2 = 0.25$$
$$x = 0.5$$

63.
$$x^2 + [(x + 1)/2]^2 = (x + 2)^2$$
$$x^2 + \frac{x^2 + 2x + 1}{4} = x^2 + 4x + 4$$
$$4x^2 + (x^2 + 2x + 1) = 4x^2 + 16x + 16$$
$$x^2 - 14x - 15 = 0$$
$$(x - 15)(x + 1) = 0 \Rightarrow x = 15$$

Check: $15^2 + 8^2 = 289 = 17^2$

(**Note:** The solution $x = -1$ does not make sense in the context of the problem.)

64.
$$(x + 2)^2 + \left(\frac{x}{2}\right)^2 = (x + 3)^2$$
$$(x^2 + 4x + 4) + \frac{x^2}{4} = x^2 + 6x + 9$$
$$(4x^2 + 16x + 16) + x^2 = 4x^2 + 24x + 36$$
$$x^2 - 8x - 20 = 0$$
$$(x - 10)(x + 2) = 0 \Rightarrow x = 10$$

(**Note:** The solution $x = -2$ does not make sense in the context of the problem.)

65.
$$\frac{AB}{XY} = \frac{BC}{YZ}$$
$$\frac{n + 2}{n} = \frac{7.5}{5} = 1.5$$
$$n + 2 = 1.5n$$
$$2 = 0.5n$$
$$n = 4$$
$$\frac{AC}{XZ} = \frac{AB}{XY}$$
$$\frac{m - 10}{\frac{1}{3}m} = \frac{6}{4} = \frac{3}{2}$$
$$m - 10 = \frac{1}{3}m\left(\frac{3}{2}\right) = \frac{m}{2}$$
$$m = 20$$

$AB = 6$ units, $AC = 10$ units, $XY = 4$ units, $\text{XZ} = \frac{20}{3}$ units

Section 1.4 Solving Right Triangles

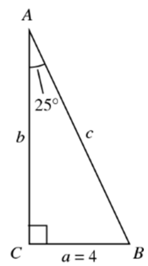
1. solving

2. elevation; depression

3. Given: $A = 25°, a = 4$

$B = 90° - A = 65°$

$$\sin A = \frac{a}{c} \Rightarrow c = \frac{a}{\sin A} = \frac{4}{\sin 25°} \approx 9.46$$



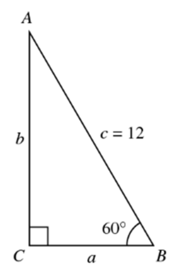
$$\tan A = \frac{a}{b} \Rightarrow b = \frac{a}{\tan A} = \frac{4}{\tan 25°} \approx 8.58$$

4. Given: $B = 60°, c = 12$

$A = 90° - 60° = 30°$

$$\sin B = \frac{b}{c} \Rightarrow b = c \bullet \sin B = 12 \sin 60° \approx 10.39$$

$$\cos B = \frac{a}{c} \Rightarrow a = c \bullet \cos B = 12 \cos 60° = 6$$

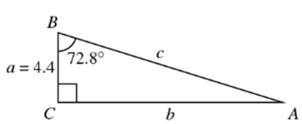


5. Given: $B = 72.8°, a = 4.4$

$$\cos B = \frac{a}{c} \Rightarrow c = \frac{a}{\cos B} = \frac{4.4}{\cos 72.8°} \approx 14.88$$

$$\tan B = \frac{b}{a} \Rightarrow b = a \tan B = 4.4 \tan 72.8° \approx 14.21$$

$A = 90° - 72.8° = 17.2°$

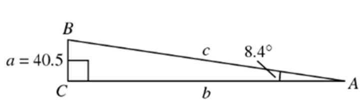


6. Given: $A = 8.4°, a = 40.5$

$$B = 90° - A = 90° - 8.4° = 81.6°$$

$$\tan A = \frac{a}{b} \Rightarrow b = \frac{a}{\tan A} = \frac{40.5}{\tan 8.4°} \approx 274.27$$

$$\sin A = \frac{a}{c} \Rightarrow c = \frac{a}{\sin A} = \frac{40.5}{\sin 8.4°} \approx 277.24$$

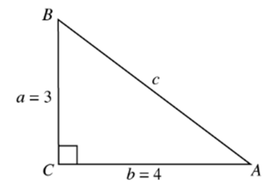


7. Given: $a = 3, b = 4$

$$a^2 + b^2 = c^2 \Rightarrow c^2 = (3)^2 + (4)^2 \Rightarrow c = 5$$

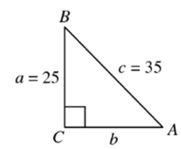
$$\tan A = \frac{a}{b} \Rightarrow A = \tan^{-1}\left(\frac{a}{b}\right) = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.87°$$

$B = 90° - 36.87° = 53.13°$



8. Given: $a = 25, c = 35$

$$b = \sqrt{c^2 - a^2} = \sqrt{35^2 - 25^2} = \sqrt{600} \approx 24.49$$



$$\sin A = \frac{a}{c} \Rightarrow A = \arcsin\frac{a}{c} = \arcsin\frac{25}{35} \approx 45.58°$$

$$\cos B = \frac{a}{c} \Rightarrow B = \arccos\frac{a}{c} = \arccos\frac{25}{35} \approx 44.42°$$

9. Given: $b = 15.70, c = 55.16$

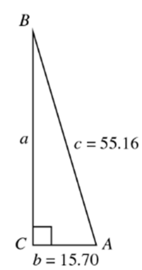
$$a = \sqrt{55.16^2 - 15.7^2} \approx 52.88$$

$$\cos A = \frac{b}{c}$$

$$\cos A = \frac{15.7}{55.15}$$

$$A = \arccos\frac{15.7}{55.15} \approx 73.46°$$

$B = 90° - 73.46° \approx 16.54°$

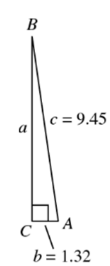


10. Given: $b = 1.32, c = 9.45$

$$a = \sqrt{c^2 - b^2} = \sqrt{87.5601} \approx 9.36$$

$$\cos A = \frac{b}{c} \Rightarrow A = \arccos\frac{b}{c} = \arccos\frac{1.32}{9.45} \approx 81.97°$$

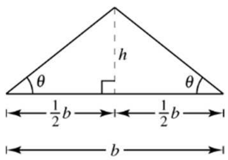
$$\sin B = \frac{b}{c} \Rightarrow B = \arcsin\frac{b}{c} = \arcsin\frac{1.32}{9.45} \approx 8.03°$$



11. $\theta = 45°, b = 6$

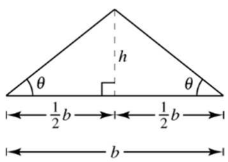
$$\tan\theta = \frac{h}{(1/2)b} \Rightarrow h = \frac{1}{2}b\tan\theta$$

$$h = \frac{1}{2}(6)\tan 45° = 3.00 \text{ units}$$



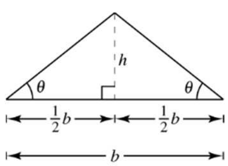
12. $\tan\theta = \frac{h}{(1/2)b} \Rightarrow h = \frac{1}{2}b\tan\theta$

$$h = \frac{1}{2}(14)\tan 22° \approx 2.83 \text{ units}$$



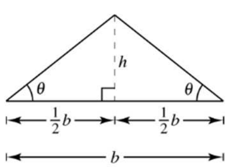
13. $\tan\theta = \frac{h}{(1/2)b} \Rightarrow h = \frac{1}{2}b\tan\theta$

$$h = \frac{1}{2}(8)\tan 32° \approx 2.50 \text{ units}$$



14. $\tan\theta = \frac{h}{(1/2)b} \Rightarrow h = \frac{1}{2}b\tan\theta$

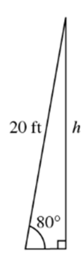
$$h = \frac{1}{2}(11)\tan 27° \approx 2.80 \text{ units}$$



15. $\sin 80° = \frac{h}{20}$

$$20\sin 80° = h$$

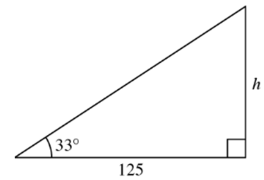
$$h \approx 19.7 \text{ feet}$$



16. $\tan 33° = \frac{h}{125}$

$$h = 125\tan 33$$

$$\approx 81.2 \text{ feet}$$



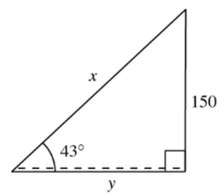
17. $\tan\theta = \frac{\text{opp}}{\text{adj}}$

$$\tan 54° = \frac{w}{100}$$

$$w = 100\tan 54° \approx 137.6 \text{ feet}$$

18. $\tan 43° = \frac{150}{y}$

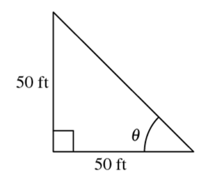
$$y = \frac{150}{\tan 43°} \approx 160.9 \text{ ft}$$



19. $\tan\theta = \frac{\text{opp.}}{\text{adj.}}$

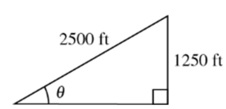
$$= \frac{50}{50} = 1$$

$$\theta = 45°$$



20. $\sin\theta = \frac{1250}{2500} = \frac{1}{2}$

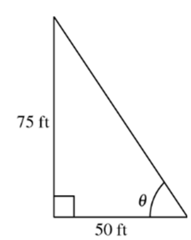
$$\theta = 30° = \frac{\pi}{6}$$



21. $\tan\theta = \frac{\text{opp.}}{\text{adj.}}$

$$= \frac{75}{50} = \frac{3}{2}$$

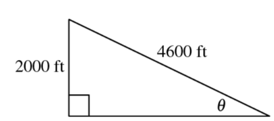
$$\theta = \tan^{-1}\left(\frac{3}{2}\right) \approx 56.3°$$



22. $\sin\theta = \frac{\text{opp.}}{\text{hyp.}}$

$$= \frac{2000}{4600}$$

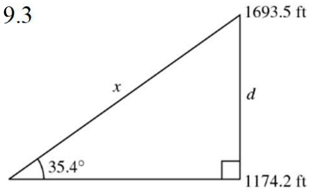
$$\theta = \sin^{-1}\left(\frac{2000}{4600}\right) \approx 25.8°$$



23. $d = 1693.5 - 1174.2 = 519.3$

$\sin 35.4° = \dfrac{\text{opp.}}{\text{hyp.}} = \dfrac{d}{x}$

$x = \dfrac{d}{\sin 35.4} \approx 896.5$ feet



24. (a) $\tan 82° = \dfrac{x}{45}$

$x = 45 \tan 82°$

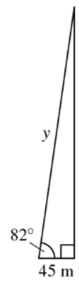
Height of the building:

$123 + 45 \tan 82° \approx 443.2$ meters

(b) Distance between friends:

$\cos 82° = \dfrac{45}{y} \Rightarrow y = \dfrac{45}{\cos 82°}$

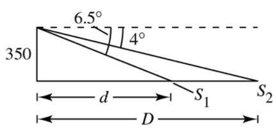
≈ 323.34 meters



25. $\tan 6.5° = \dfrac{350}{d} \Rightarrow d \approx 3071.91$ ft

$\tan 4° = \dfrac{350}{D} \Rightarrow D \approx 5005.23$ ft

Distance between ships: $D - d \approx 1933.3$ ft



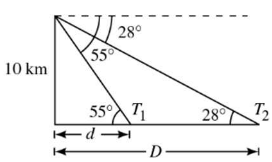
Not drawn to scale

26. $\cot 55° = \dfrac{d}{10} \Rightarrow d \approx 7$ kilometers

$\cot 28° = \dfrac{D}{10} \Rightarrow D \approx 18.8$ kilometers

Distance between towns:

$D - d = 18.8 - 7 = 11.8$ kilometers



27. Let the height of the castle $= x$ and the height of the castle and tower $= y$.

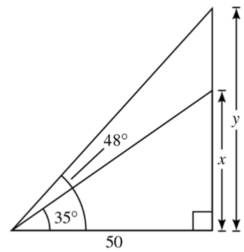
$\tan 35° = \dfrac{x}{50}$ and $\tan 48° = \dfrac{y}{50}$

$x = 50 \tan 35° \approx 35.01$ and

$y = 50 \tan 48° \approx 55.53$

$h = y - x = 55.53 - 35.01 = 20.52.$

$h \approx 20.5$ feet



28. Let $h =$ the height of the mountain.

Let $x =$ the horizontal distance from where the $9°$ angle of elevation is sighted to the point at that level directly below the mountain peak.

Then $\tan 3.5° = \dfrac{h}{x + 13}$ and $\tan 9° = \dfrac{h}{x}$.

$\tan 9° = \dfrac{h}{x} \Rightarrow x = \dfrac{h}{\tan 9°}$

Substitute $x = \dfrac{h}{\tan 9°}$ into the expression for $\tan 3.5°$.

$$\tan 3.5° = \frac{h}{\dfrac{h}{\tan 9°} + 13}$$

$$\tan 3.5° = \frac{h \tan 9°}{h + 13 \tan 9°}$$

$$h \tan 3.5° + 13 \tan 9° \tan 3.5° = h \tan 9°$$

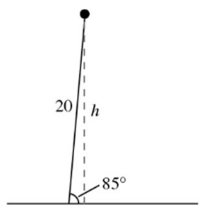
$$13 \tan 9° \tan 3.5° = h(\tan 9° - \tan 3.5°)$$

$$\frac{13 \tan 9° \tan 3.5°}{\tan 9° - \tan 3.5°} = h$$

$$1.2953 \approx h$$

The mountain is about 1.3 miles high.

29.



(a) $\sin 85° = \dfrac{h}{20}$

$h = 20 \sin 85° \approx 19.9$ meters

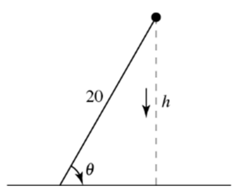
(b) The side of the triangle labeled h will become shorter.

(c)

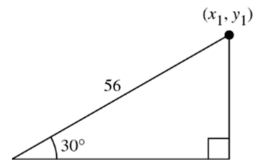
Angle, θ	Height (in meters)
80°	19.7
70°	18.8
60°	17.3
50°	15.3
40°	12.9
30°	10.0
20°	6.8
10°	3.5

(d) The height of the balloon decreases.

As $\theta \to 0°, h \to 0$.



30.



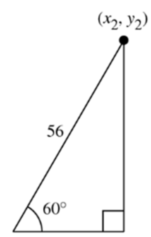
$$\sin 30° = \frac{y_1}{56}$$

$$y_1 = (\sin 30°)(56) = \left(\frac{1}{2}\right)(56) = 28$$

$$\cos 30° = \frac{x_1}{56}$$

$$x_1 = \cos 30°(56) = \frac{\sqrt{3}}{2}(56) = 28\sqrt{3}$$

$$(x_1, y_1) = \left(28\sqrt{3}, 28\right)$$



$$\sin 60° = \frac{y_2}{56}$$

$$y_2 = \sin 60°(56) = \left(\frac{\sqrt{3}}{2}\right)(56) = 28\sqrt{3}$$

$$\cos 60° = \frac{x_2}{56}$$

$$x_2 = (\cos 60°)(56) = \left(\frac{1}{2}\right)(56) = 28$$

$$(x_2, y_2) = \left(28, 28\sqrt{3}\right)$$

31. True. You can use the Pythagorean Theorem to find the third side and the tangent function to find one of the acute angles.

32. False. You also need to know at least one side to solve a right triangle.

33. False. An angle of elevation is an acute angle by definition.

34. $\tan B = \dfrac{15}{b} \Rightarrow b = \dfrac{15}{\tan B} = \dfrac{15}{\tan 40°}$

$\sin C = \dfrac{15}{c} \Rightarrow c = \dfrac{15}{\sin C} = \dfrac{15}{\sin 40°}$

35. The point $(1, 3)$ lies in Quadrant I.

36. The point $(9, -2)$ lies in Quadrant IV.

37. The point $(-5, 4)$ lies in Quadrant II.

38. The point $(-6, -7)$ lies in Quadrant III.

39. $y = -x$ lies in Quadrants II and IV.

40. $y = \frac{1}{3}x$ lies in Quadrants I and III.

41. $2x - y = 0 \Leftrightarrow y = 2x$ lies in Quadrant I and III.

42. $4x + 3y = 0 \Leftrightarrow y = -\dfrac{4x}{3}$ lies in Quadrant II and IV.

43. (a) Coterminal angles for $\theta = 20°$

$20° + 360° = 380°$

$20° - 360° = -340°$

(b) Coterminal angles for $\theta = -30°$

$-30° + 360° = 330°$

$-30° - 360° = -390°$

44. (a) Coterminal angles for $\theta = 150°$

$150° + 360° = 510°$

$150° - 360° = -210°$

(b) Coterminal angles for $\theta = -160°$

$-160° + 360° = 200°$

$-160° - 360° = -520°$

45. (a) Coterminal angles for $\theta = 300°$

$300° + 360° = 660°$

$300° - 360° = -60°$

(b) Coterminal angles for $\theta = -150°$

$-150° + 360° = 210°$

$-150° - 360° = -510°$

46. (a) Coterminal angles for $\theta = 15°$

$15° + 360° = 375°$

$15° - 360° = -345°$

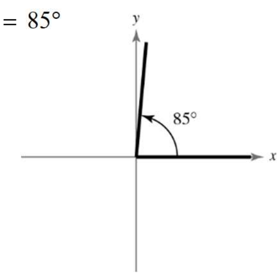
(b) Coterminal angles for $\theta = -20°$

$-20° + 360° = 340°$

$-20° - 360° = -380°$

Review Exercises for Chapter 1

1. (a) $\theta = 85°$

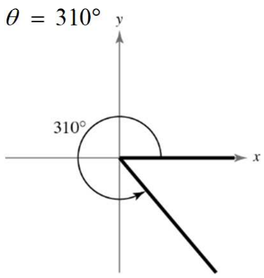


(b) The angle lies in Quadrant I.

(c) Coterminal angles: $85° + 360° = 445°$

$85° - 360° = -275°$

2. (a) $\theta = 310°$



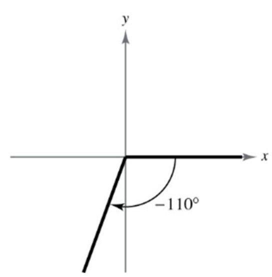
(b) The angle lies in Quadrant IV.

(c) Coterminal angles: $310° + 360° = 670°$

$310° - 360° = -50°$

3. $\theta = -110°$

(a)



(b) Quadrant III

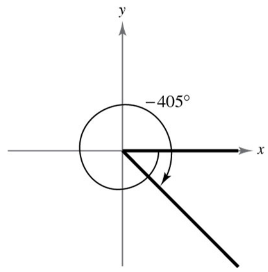
(c) Coterminal angles:

$-110° + 360° = 250°$

$-110° - 360° = -470°$

4. $\theta = -405°$

(a)



(b) Quadrant IV

(c) $-405° + 720° = 315°$

$-405° + 360° = -45°$

5. The complement of $55°$ is $90° - 55° = 35°$.

The supplement of $55°$ is $180° - 55° = 125°$.

6. The complement of $100°$ does not exist because $100°$ is greater than $90°$

The supplement of $100°$ is $180° - 100° = 80°$.

7. The complement of $89°$ is $90° - 89° = 1°$.

The supplement of $89°$ is $180° - 89° = 91°$.

8. The complement of $35°$ is $90° - 35° = 55°$.

The supplement of $35°$ is $180° - 35° = 145°$.

9. Let θ be the unknown angle.

$$\begin{aligned} \theta &= 17(90° - \theta) \\ 18\theta &= 17(90°) \\ \theta &= 85° \end{aligned}$$

10. Let θ be the unknown angle.

$$\begin{aligned} \theta &= 2(180° - \theta) + 30° \\ 3\theta &= 2(180°) + 30° \\ 3\theta &= 390° \\ \theta &= 130° \end{aligned}$$

11.
$$\begin{aligned} x° + 38° &= 90° \\ x &= 90 - 38 \\ x &= 52 \end{aligned}$$

12.
$$\begin{aligned} 6x° + 4x° &= 90° \\ 10x &= 90 \\ x &= 9 \end{aligned}$$
$$\begin{aligned} 6x° &= 54° \\ 4x° &= 36° \end{aligned}$$

13.
$$\begin{aligned} 7x° + (10x + 6)° &= 90° \\ 17x &= 84 \\ x &= \left(\tfrac{84}{17}\right) \approx 4.941 \end{aligned}$$
$$\begin{aligned} 7x° &\approx 34.6° \\ (10x + 6)° &\approx 55.4° \end{aligned}$$

14.
$$\begin{aligned} (x - 1)° + (x + 1)° &= 90° \\ 2x &= 90 \\ x &= 45 \end{aligned}$$
$$\begin{aligned} (x - 1) &= 44° \\ (x + 1) &= 46° \end{aligned}$$

15.
$$\begin{aligned} x° &= 180° - 67.5° - 22.5° \\ x &= 90 \end{aligned}$$

16.
$$\begin{aligned} x° + x° + 74° &= 180° \\ 2x &= 106 \\ x &= 53 \end{aligned}$$

17.
$$\begin{aligned} x° + (x - 7)° + 31° &= 180° \\ 2x &= 156 \\ x &= 78 \end{aligned}$$

18.
$$\begin{aligned} (4x - 2)° + (13x + 6)° + 5x° &= 180° \\ 22x &= 176 \\ x &= 8 \end{aligned}$$
$$\begin{aligned} 5x° &= 40° \\ (13x + 6)° &= 110° \\ (4x - 2)° &= 30° \end{aligned}$$

19. Let x be the unknown angle.

$$\begin{aligned} 3x° &= 180° \\ x° &= 60° \quad \text{(equilateral triangle)} \end{aligned}$$

The unknown angles are 60°, 60°, and 60°.

20. Let x be an unknown angle.

$$\begin{aligned} 4x° + x° &= 90° \\ 5x° &= 90° \\ x° &= 18° \end{aligned}$$

The unknown angles are 18° and 72°.

21.
$$\begin{aligned} x^2 + 10^2 &= 14^2 \\ x^2 &= 14^2 - 10^2 = 96 \\ x &= \sqrt{96} = 4\sqrt{6} \end{aligned}$$

22.
$$\begin{aligned} x^2 + (4x - 1)^2 &= (4x + 1)^2 \\ x^2 + 16x^2 - 8x + 1 &= 16x^2 + 8x + 1 \\ x^2 - 16x &= 0 \\ x(x - 16) &= 0 \\ x &= 16 \end{aligned}$$

(**Note:** The solution $x = 0$ does not make sense in the context of the problem.)

23. $33^2 + 56^2 = 1089 + 3136 = 4225 = 65^2$

The triangle is a right triangle.

24. $(\sqrt{6})^2 + (\sqrt{5})^2 = 6 + 5 = 11 = (\sqrt{11})^2$

The triangle is a right triangle.

25. The hypotenuse is $2(16) = 32$.

The longer leg is $\left(\dfrac{\sqrt{3}}{2}\right)(32) = 16\sqrt{3}$.

26.
$$\begin{aligned} 2x^2 &= 10^2 = 100 \\ x^2 &= 50 \\ x &= 5\sqrt{2} \end{aligned}$$

Each leg measures $5\sqrt{2}$.

27. $(5\sqrt{2})^2 + (5/\sqrt{2})^2 = 100 = 10^2$

The hypotenuse is 10.

28. The hypotenuse is $\left(\dfrac{2}{\sqrt{3}}\right)9 = \dfrac{18}{\sqrt{3}} = 6\sqrt{3}$.

The shorter leg is $3\sqrt{3}$.

29.
$$\begin{aligned} (13x - 39)^\circ + 102^\circ &= 180^\circ \\ 13x &= 117 \\ x &= 9 \end{aligned}$$

30.
$$\begin{aligned} 9x^\circ &= 63^\circ \\ x &= 7 \end{aligned}$$

31.
$$\begin{aligned} 30^\circ &= \tfrac{1}{3}(y - 30)^\circ \\ 90 &= (y - 30) \\ 120 &= y \end{aligned}$$

In the triangle between the parallel lines, the smallest angle is $\frac{1}{3}(y - 30)^\circ = 30^\circ$. So, the other equal angles are $\left(\dfrac{150}{2}\right)^\circ = 75^\circ$. So, $x = 180 - 75 = 105$.

32.
$$\begin{cases} y^\circ = 2(x + 17)^\circ \\ y^\circ + 4(x + 11)^\circ = 180^\circ \end{cases}$$

$$\begin{cases} y - 2x = 34 \\ y + 4x = 136 \end{cases}$$

Subtract the two equations to get $6x = 102 \Rightarrow x = 17$. Then, $y = 136 - 4x = 68$.

33. $A = 180^\circ - 53^\circ - 44^\circ = 83^\circ$

$E = 180^\circ - 44^\circ - 83^\circ = 53^\circ$

The triangles are similar because corresponding angles have the same measure.

34. $\dfrac{AC}{DF} = \dfrac{16}{18} = \dfrac{8}{9}$

$\dfrac{BC}{EF} = \dfrac{10}{12} = \dfrac{5}{6}$

The triangles are not similar because corresponding sides are not proportional.

35.
$$\begin{aligned} 2x^\circ + x^\circ + 3x^\circ &= 180^\circ \\ 6x &= 180 \\ x &= 30 \end{aligned}$$

$A = 2x^\circ = 60^\circ = D$

$B = 3x^\circ = 90^\circ = E$

$C = x^\circ = 30^\circ = F$

36.
$$\begin{aligned} (x + 30)^\circ + 3x^\circ &= 90^\circ \\ 4x &= 60 \\ x &= 15 \end{aligned}$$

$B = (x + 30)^\circ = E = 45^\circ$

$C = 3x^\circ = F = 45^\circ$

37.
$$\begin{aligned} \frac{AB}{DE} &= \frac{AC}{FD} \\ \frac{x - 7}{24} &= \frac{x}{45} \\ 45(x - 7) &= 24x \\ 21x &= 315 \\ x &= 15 \end{aligned}$$

$24^2 + 45^2 = y^2 \Rightarrow y = \sqrt{2601} = 51$

$AC = 15$ units, $AB = 8$ units, $EF = 51$ units

38. $\dfrac{AB}{ED} = \dfrac{AC}{FD}$

$\dfrac{28}{y} = \dfrac{24}{18} = \dfrac{4}{3}$

$4y = 3(28)$

$y = 21$

$\dfrac{BC}{EF} = \dfrac{AC}{FD}$

$\dfrac{36}{x} = \dfrac{24}{18} = \dfrac{4}{3}$

$4x = 3(36)$

$x = 27$

$DE = 21$ units, $EF = 27$ units

39. The triangles VWX and YZX are similar.

$\dfrac{WX}{ZX} = \dfrac{VW}{YZ}$

$\dfrac{W}{3} = \dfrac{57}{4}$

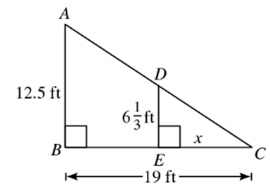
$W = \dfrac{3(57)}{4} = 42.75$ feet

40. Triangles ABC and DEC are similar.

$\dfrac{AB}{DE} = \dfrac{BC}{X}$

$\dfrac{12.5}{6.333} \approx \dfrac{19}{x}$

$x = \dfrac{19(6.333)}{12.5} \approx 9.63$



The shadow is about 9.63 feet, or 115.5 inches.

41. opp $= 4$, adj $= 5$, hyp $= \sqrt{4^2 + 5^2} = \sqrt{41}$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{4}{\sqrt{41}} = \dfrac{4\sqrt{41}}{41}$ $\quad \csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{\sqrt{41}}{4}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{5}{\sqrt{41}} = \dfrac{5\sqrt{41}}{41}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{\sqrt{41}}{5}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{4}{5}$ $\quad \cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{5}{4}$

42. adj $= 4$, hyp $= 8$, opp $= \sqrt{8^2 - 4^2} = \sqrt{48} = 4\sqrt{3}$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{4\sqrt{3}}{8} = \dfrac{\sqrt{3}}{2}$ $\quad \csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{8}{4\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{4}{8} = \dfrac{1}{2}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{8}{4} = 2$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{4\sqrt{3}}{4} = \sqrt{3}$ $\quad \cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{4}{4\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

43. (a) $\sin\theta = \dfrac{\sqrt{2}}{2} \Rightarrow \theta = 45°$

(b) $\cot\theta = \dfrac{\sqrt{3}}{3} \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

44. (a) $\csc\theta = 2 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

(b) $\cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

45. $\cos 63° = \sin(90° - 63°) = \sin 27°$

46. $\sec 48° = \csc 90° - 48° = \csc 42°$

47.
$$\begin{aligned} \cot(4A + 1°) &= \tan(3A + 5°) \\ (4A + 1°) + (3A + 5°) &= 90° \\ 7A + 6° &= 90° \\ 7A &= 84° \\ A &= 12° \end{aligned}$$

48.
$$\begin{aligned} \cos\left(\tfrac{5}{3}A - 58°\right) &= \sin\left(61° - \tfrac{2}{3}A\right) \\ \left(\tfrac{5}{3}A - 58°\right) + \left(61° - \tfrac{2}{3}A\right) &= 90° \\ A = 3° &= 90° \\ A &= 87° \end{aligned}$$

49. $\tan\theta = 4$

(a) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{4}$

(b) $\cot(90° - \theta) = \tan\theta = 4$

(c) $\tan(90° - \theta) = \cot\theta = \dfrac{1}{4}$

50. $\sec\theta = 3$

(a) $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{3}$

(b) $\csc(90° - \theta) = \sec\theta = 3$

(c) $\sin(90° - \theta) = \cos\theta = \dfrac{1}{3}$

51.
$$\begin{aligned} 18.345° &= 18° + 0.345° \\ &= 18° + 0.345°\left(\frac{60'}{1°}\right) \\ &= 18° + 20.7' \\ &= 18° + 20' + 0.7'\left(\frac{60''}{1'}\right) \\ &= 18° + 20' + 42'' \end{aligned}$$

So, to the nearest second, $18.345° = 18°\,20'\,42''$.

52.
$$\begin{aligned} 74.981° &= 74° + 0.981° \\ &= 74° + 0.981°\left(\frac{60'}{1°}\right) \\ &= 74° + 58.86' \\ &= 74° + 58' + 0.86'\left(\frac{60''}{1'}\right) \\ &= 74° + 58' + 51.6'' \end{aligned}$$

So, to the nearest second, $74.981° = 74°\,58'\,52''$.

53.
$$\begin{aligned} 63°\,52' &= 63° + \left(\tfrac{52}{60}\right)^{\circ} \\ &= 63° + 0.867° \\ &= 63.867° \end{aligned}$$

54.
$$\begin{aligned} 5°\,39'\,26'' &= 5° + \left(\tfrac{39}{60}\right)^{\circ} + \left(\tfrac{26}{3600}\right)^{\circ} \\ &= 5° + 0.65° + 0.07° \\ &= 5.657° \end{aligned}$$

55. $\sec(84.96°) = \dfrac{1}{\cos(84.96°)} \approx 11.383$

56. $\sin(19°\,45'\,18'') \approx 0.338$

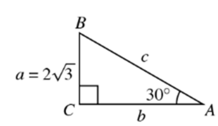
57. $\csc(19°\,45'\,18'') = \dfrac{1}{\sin(19°\,45'\,18'')} \approx 2.959$

58. $\tan(46°\,16'\,25'') \approx 1.045$

59. Yes. The roof has a conventional slope because $\tan(18.5°) \approx 0.3346$ lies within the interval $\frac{1}{3} \le m \le \frac{3}{4}$.

60. No. The roof does not have a steep slope because $\tan(55.2°) \approx 1.4388$, which is less than $\frac{7}{4} = 1.75$.

61. Given: $A = 30°, a = 2\sqrt{3}$



$B = 90° - A = 90° - 30° = 60°$

30°-60°-90° right triangle

$c = 2a = 4\sqrt{3} \approx 6.93$

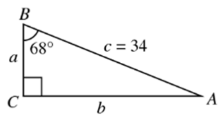
$b = \left(\dfrac{\sqrt{3}}{2}\right)(c) = \left(\dfrac{\sqrt{3}}{2}\right)\left(4\sqrt{3}\right) = 6$

62. Given: $B = 68°, c = 34$

$A = 90° - B = 90° - 68° = 22°$

$\sin B = \dfrac{b}{c} \Rightarrow b = c \sin B = 34 \sin(68°) \approx 31.52$

$\sin A = \dfrac{a}{c} \Rightarrow a = c \sin A = 34 \sin(22°) \approx 12.74$

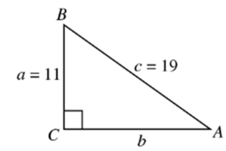


63. Given: $a = 11, c = 19$

$b = \sqrt{19^2 - 11^2} = \sqrt{240} = 4\sqrt{15} \approx 15.49$

$\sin \theta = \dfrac{a}{c} = \dfrac{11}{19} \Rightarrow A \approx 35.38°$

$\sin B = \dfrac{b}{c} = \dfrac{15.49}{19} \Rightarrow B \approx 54.62°$

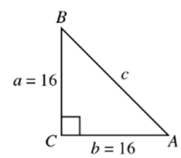


64. Given: $a = b = 16$

$c = \sqrt{16^2 + 16^2} = \sqrt{512} = 16\sqrt{2} \approx 22.63$

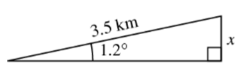
45°-45°-90° right triangle

$A = B = 45°$



65. $\sin 1.2° = \dfrac{x}{3.5}$

$x = 3.5 \sin 1.2° \approx 0.0733$ kilometer or 73.3 meters

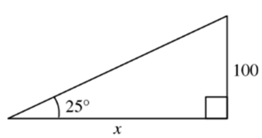


Not drawn to scale

66. $\tan 25° = \dfrac{100}{x}$

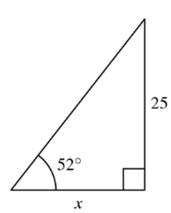
$x = \dfrac{100}{\tan 25°}$

≈ 214.45 feet

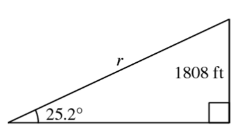


67. $\tan 52° = \dfrac{25}{x}$

$x = \dfrac{25}{\tan 52°} \approx 19.5$ feet



68.



$r = \dfrac{1808}{\sin 25.2°} \approx 4246.33$ ft

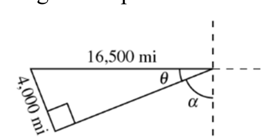
69. $12{,}500 + 4000 = 16{,}500$

$\sin \theta = \dfrac{4000}{16{,}500}$

$\theta = \arcsin\left(\dfrac{4000}{16{,}500}\right)$

$\theta \approx 14.03°$

Angle of depression $= \alpha \approx 90° - 14.03° = 75.97°$



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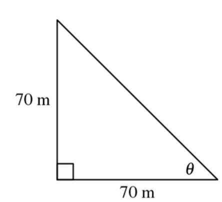
70.



This is a 45°-45°-90° right triangle, so, $\theta = 45°$.

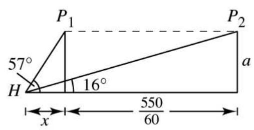
71. $\tan 57° = \dfrac{a}{x} \Rightarrow x = a \cot 57°$

$$\tan 16° = \frac{a}{x + (55/6)}$$

$$\tan 16° = \frac{a}{a \cot 57° + (55/6)}$$

$$\cot 16° = \frac{a \cot 57° + (55/6)}{a}$$

$$a \cot 16° - a \cot 57° = \frac{55}{6} \Rightarrow a \approx 3.23 \text{ miles}$$
$$\approx 17{,}054 \text{ feet}$$



72. $\tan 3° = \dfrac{x}{15}$

$$x = 15 \tan 3°$$
$$d = 5 + 2x = 5 + 2(15 \tan 3°) \approx 6.57 \text{ centimeters}$$

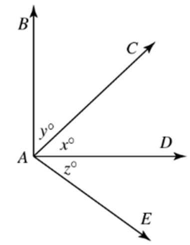
73. False. The complement is 90° less than the supplement.

74. False. When a transversal cuts two lines that are not parallel, consecutive interior angles are not supplementary.

75. True. $1' = \frac{1}{60}(1°)$, so $60' = 1°$.

76. False. An angle of depression is acute.

77. Label the figure as follows.



$\measuredangle BAC$ and $\measuredangle CAD$ are complementary, so $y° + x° = 90°$.

$\measuredangle DAE$ and $\measuredangle CAD$ are complementary, so $z° + x° = 90°$.

Subtracting the equations, $y° = z° \Rightarrow \measuredangle BAC = \measuredangle DAE$.

78. Yes. Each ratio of corresponding angles is 1.

79. Yes. Given $\tan \theta$, $\sec \theta$ can be found from the identity $1 + \tan^2 \theta = \sec^2 \theta$.

80. (a)

θ	6°	12°	18°	24°	30°
$\tan \theta$	0.105	0.213	0.325	0.445	0.577
$\dfrac{\sin \theta}{\cos \theta}$	0.105	0.213	0.325	0.445	0.577

(b) Conjecture: $\tan \theta = \dfrac{\sin \theta}{\cos \theta}$ (which is true!)

Problem Solving for Chapter 1

1. (a) $\sin 39° = \dfrac{3000}{d}$

$$d = \frac{3000}{\sin 39°} \approx 4767 \text{ feet}$$

(b) $\tan 39° = \dfrac{3000}{x}$

$$x = \frac{3000}{\tan 39°} \approx 3705 \text{ feet}$$

(c) $\tan 63° = \dfrac{w + 3705}{3000}$

$$3000 \tan 63° = w + 3705$$
$$w = 3000 \tan 63° - 3705 \approx 2183 \text{ feet}$$

2. Triangles ABC and DEF are similar right triangles.

The slope of the line through A and B is

$$\frac{BC}{AC} = \frac{y_2 - y_1}{x_2 - x_1}.$$

The slope of the line through D and E is

$$\frac{EF}{DF} = \frac{y_4 - y_3}{x_4 - x_3}.$$

Because the sides of the triangles are proportional

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_4 - y_3}{x_4 - x_3} = \text{slope}$$

So, any two points on a line can be used to calculate the slope of the line.

3. The area A of the large square is $(a + b)^2$. This area is also given by $4\left(\frac{1}{2}ab\right) + c^2$. So,

$$(a + b)^2 = 4\left(\tfrac{1}{2}ab\right) + c^2$$
$$a^2 + 2ab + b^2 = 2ab + c^2$$
$$a^2 + b^2 = c^2.$$

4. (a) $r = \sqrt{2}(1) = \sqrt{2}$ (45°-45°-90° right triangle)

$s = \sqrt{1 + r^2} = \sqrt{3}$ (Pythagorean Theorem)

$t = \sqrt{1 + s^2} = \sqrt{4} = 2$ (Pythagorean Theorem)

$u = \sqrt{1 + 2^2} = \sqrt{5}$ (Pythagorean Theorem)

$v = \sqrt{1 + u^2} = \sqrt{6}$ (Pythagorean Theorem)

$w = \sqrt{1 + v^2} = \sqrt{7}$ (Pythagorean Theorem)

(b) The hypotenuse is $\sqrt{n + 1}$.

(c) The right triangle with legs of length 1 and hypotenuse $r = \sqrt{2}$ is a 45°-45°-90° right triangle.

(d) The right triangle with legs of length 1 and $s = \sqrt{3}$, and hypotenuse $t = 2$ is a 30°-60°-90° triangle.

5. Gear 1: $\frac{24}{32}(360°) = 270° = \frac{3\pi}{2}$ radians

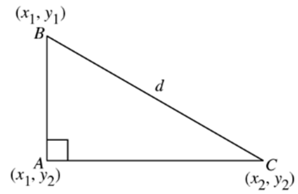
Gear 2: $\frac{24}{26}(360°) \approx 332.3° \approx \frac{24\pi}{13}$ radians

Gear 3: $\frac{24}{22}(360°) \approx 392.7° \approx \frac{24\pi}{11}$ radians

Gear 4: $\frac{40}{32}(360°) = 450° = \frac{5\pi}{2}$ radians

Gear 5: $\frac{24}{19}(360°) \approx 454.7° \approx \frac{48\pi}{19}$ radians

6. Consider the right triangle ABC below.



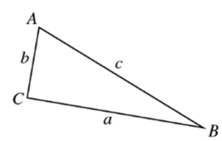
The coordinates of A are (x_1, y_2). Then by the Pythagorean Theorem,

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

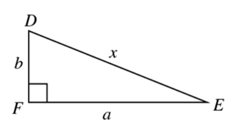
7. (a) Because the lines are parallel, triangles ABC and DEF are similar. So, $\angle BAC = \angle EDF$, showing that the slope of ℓ and m are equal.

(b) If ℓ and m have the same slope, then $\angle BAC = \angle EDF$. So, the line ℓ and m are parallel.

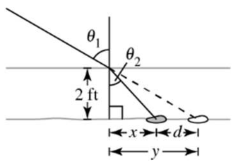
8. Consider $\triangle ABC$, in which $a^2 + b^2 = c^2$.



Now consider right $\triangle DEF$ with legs a and b, and hypotenuse x.



By the Pythagorean Theorem, $a^2 + b^2 = x^2$. So, $x^2 = c^2 \Rightarrow x = c$, and the top triangle is a right triangle.

9. 

(a) $\dfrac{\sin \theta_1}{\sin \theta_2} = 1.333$

$\sin \theta_2 = \dfrac{\sin \theta_1}{1.333} = \dfrac{\sin 60°}{1.333} \approx 0.6497$

$\theta_2 = 40.5°$

(b) $\tan \theta_2 = \dfrac{x}{2} \Rightarrow x = 2 \tan 40.52° \approx 1.71$ feet

$\tan \theta_1 = \dfrac{y}{2} \Rightarrow y = 2 \tan 60° \approx 3.46$ feet

(c) $d = y - x = 3.46 - 1.71 = 1.75$ feet

(d) As you move closer to the rock, θ_1 decreases, which causes y to decrease, which in turn causes d to decrease.

10. (a) Assume the nonvertical lines ℓ and n are perpendicular. Let k and j be parallel, and note that $\measuredangle BCA = \measuredangle DAC$. By complementary angles, $\measuredangle BAC = \measuredangle DAE$. So, $\triangle ABC$ is similar to $\triangle ADE$. Corresponding sides are proportional, so $\dfrac{ED}{AD} = \dfrac{BC}{AB} = \dfrac{1}{AB/BC}$.

The slope of line ℓ is $\dfrac{ED}{AD}$. The slope of line n is $-\dfrac{AB}{BC}$. So,

$\left(\dfrac{ED}{AD}\right)\left(-\dfrac{AB}{BC}\right) = \dfrac{1}{AB/BC}(-AB/BC) = -1.$

(b) Assume that the product of the slopes of ℓ and n is -1. You can also assume that $AE = AC$ in the figure. Then you have

$\left(\dfrac{ED}{AD}\right)\left(-\dfrac{AB}{BC}\right) = -1 \Rightarrow \dfrac{ED}{AD} = \dfrac{BC}{AB}.$

So, $\triangle ABC$ and $\triangle ADE$ are congruent (and similar). So $\measuredangle DAE = \measuredangle BAC$, which means $\measuredangle EAC$ is a right angle. Therefore, ℓ and n are perpendicular.

11. (a) *Sample answer:* 5, 12, 13; 8, 15, 17

(b) *Sample answer:*

$5 \bullet 12 \bullet 13 = 780$

$780 \div 3 = 260;\ 780 \div 4 = 195;\ 780 \div 5 = 156$

$8 \bullet 15 \bullet 17 = 2040$

$2040 \div 3 = 680;\ 2040 \div 4 = 510;\ 2040 \div 5 = 408$

Yes. The product of the three numbers in each Pythagorean triple is divisible by 3, by 4, and by 5.

(c) Conjecture: if a, b, and c form a Pythagorean triple, then abc is divisible by 60.