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INSTRUCTOR'S SOLUTIONS MANUAL FOR
Microelectronic Circuits

EIGHTH EDITION

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Contents

Preface

This Instructor's Manual (ISM) contains complete solutions for about 450 in-chapter exercises and 1400 end-of-chapter problems included in the book ***Microelectronic Circuits, Eighth Edition***.

This manual has benefited from the work of the accuracy checkers, listed below. We are grateful to all of them. Dr. Amir Yazdani of Ryerson University deserves special mention as he has done a truly outstanding job in ensuring that this manual is as free of errors as possible. However, despite all of our combined efforts, there is little doubt that some errors remain. We will be most grateful to instructors who discover errors and point them out to us. Please send corrections and comments by email to: sedra@uwaterloo.ca.

As she has done for a number of editions of the book and the ISM, Jennifer Rodrigues most ably typed the majority of the solutions in this manual. We are very grateful for her excellent work.

At OUP, we wish to thank Senior Production Editor Keith Faivre, Senior Development Editor Eric Sinkins, and Assistant Editor Megan Carlson.

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Chapter 1

Solutions to Exercises within the Chapter

Ex: 1.1 When output terminals are open-circuited, as in Fig. 1.1a:

For circuit a. $v_{oc} = v_s(t)$

For circuit b. $v_{oc} = i_s(t) \times R_s$

When output terminals are short-circuited, as in Fig. 1.1b:

For circuit a. $i_{sc} = \dfrac{v_s(t)}{R_s}$

For circuit b. $i_{sc} = i_s(t)$

For equivalency

$R_s i_s(t) = v_s(t)$

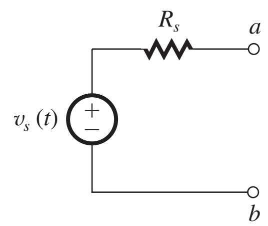


Figure 1.1a

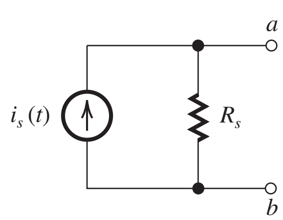
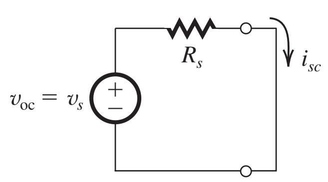


Figure 1.1b

Ex: 1.2



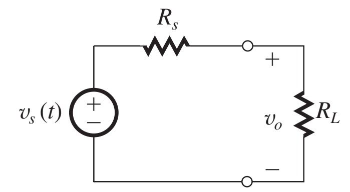
$v_{oc} = 10$ mV

$i_{sc} = 10\ \mu$A

$$R_s = \frac{v_{oc}}{i_{sc}} = \frac{10 \text{ mV}}{10\ \mu\text{A}} = 1 \text{ k}\Omega$$

Ex: 1.3 Using voltage divider:

$$v_o(t) = v_s(t) \times \frac{R_L}{R_s + R_L}$$



Given $v_s(t) = 10$ mV and $R_s = 1$ kΩ.

If $R_L = 100$ kΩ

$$v_o = 10 \text{ mV} \times \frac{100}{100 + 1} = 9.9 \text{ mV}$$

If $R_L = 10$ kΩ

$$v_o = 10 \text{ mV} \times \frac{10}{10 + 1} \simeq 9.1 \text{ mV}$$

If $R_L = 1$ kΩ

$$v_o = 10 \text{ mV} \times \frac{1}{1 + 1} = 5 \text{ mV}$$

If $R_L = 100\ \Omega$

$$v_o = 10 \text{ mV} \times \frac{100}{100 + 1 \text{ K}} \simeq 0.91 \text{ mV}$$

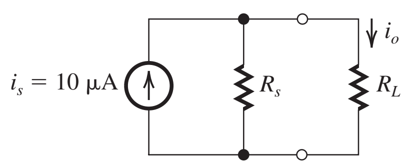
For $v_o = 0.8 v_s$,

$$\frac{R_L}{R_L + R_s} = 0.8$$

Since $R_s = 1$ kΩ,

$R_L = 4$ kΩ

Ex: 1.4 Using current divider:



$$i_o = i_s \times \frac{R_s}{R_s + R_L}$$

Given $i_s = 10\ \mu$A, $R_s = 100$ kΩ.

For

$$R_L = 1 \text{ k}\Omega,\ i_o = 10\ \mu\text{A} \times \frac{100}{100 + 1} = 9.9\ \mu\text{A}$$

For

$$R_L = 10 \text{ k}\Omega,\ i_o = 10\ \mu\text{A} \times \frac{100}{100 + 10} \simeq 9.1\ \mu\text{A}$$

For

$$R_L = 100 \text{ k}\Omega,\ i_o = 10\ \mu\text{A} \times \frac{100}{100 + 100} = 5\ \mu\text{A}$$

For $R_L = 1$ MΩ, $i_o = 10\ \mu\text{A} \times \dfrac{100 \text{ K}}{100 \text{ K} + 1 \text{ M}}$

$\simeq 0.9\ \mu$A

For $i_o = 0.8 i_s$, $\dfrac{100}{100 + R_L} = 0.8$

$\Rightarrow R_L = 25$ kΩ

Ex: 1.5 $f = \frac{1}{T} = \frac{1}{10^{-3}} = 1000 \text{ Hz}$

$\omega = 2\pi f = 2\pi \times 10^3 \text{ rad/s}$

Ex: 1.6 (a) $T = \frac{1}{f} = \frac{1}{60} \text{ s} = 16.7 \text{ ms}$

(b) $T = \frac{1}{f} = \frac{1}{10^{-3}} = 1000 \text{ s}$

(c) $T = \frac{1}{f} = \frac{1}{10^6} \text{ s} = 1\ \mu\text{s}$

Ex: 1.7 If 6 MHz is allocated for each channel, then 470 MHz to 806 MHz will accommodate

$\frac{806 - 470}{6} = 56$ channels

Since the broadcast band starts with channel 14, it will go from channel 14 to channel 69.

Ex: 1.8 $P = \frac{1}{T}\int_0^T \frac{v^2}{R}\,dt$

$= \frac{1}{T} \times \frac{V^2}{R} \times T = \frac{V^2}{R}$

Alternatively,

$P = P_1 + P_3 + P_5 + \cdots$

$= \left(\frac{4V}{\sqrt{2}\pi}\right)^2 \frac{1}{R} + \left(\frac{4V}{3\sqrt{2}\pi}\right)^2 \frac{1}{R}$

$+ \left(\frac{4V}{5\sqrt{2}\pi}\right)^2 \frac{1}{R} + \cdots$

$= \frac{V^2}{R} \times \frac{8}{\pi^2} \times \left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots\right)$

It can be shown by direct calculation that the infinite series in the parentheses has a sum that approaches $\pi^2/8$; thus P becomes V^2/R as found from direct calculation.

Fraction of energy in fundamental

$= 8/\pi^2 = 0.81$

Fraction of energy in first five harmonics

$= \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25}\right) = 0.93$

Fraction of energy in first seven harmonics

$= \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49}\right) = 0.95$

Fraction of energy in first nine harmonics

$= \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81}\right) = 0.96$

Note that 90% of the energy of the square wave is in the first three harmonics, that is, in the fundamental and the third harmonic.

Ex: 1.9 (a) D can represent 15 equally-spaced values between 0 and 3.75 V. Thus, the values are spaced 0.25 V apart.

$v_A = 0 \text{ V} \Rightarrow D = 0000$

$v_A = 0.25 \text{ V} \Rightarrow D = 0000$

$v_A = 1 \text{ V} \Rightarrow D = 0000$

$v_A = 3.75 \text{ V} \Rightarrow D = 0000$

(b) (i) 1 level spacing: $2^0 \times +0.25 = +0.25$ V

(ii) 2 level spacings: $2^1 \times +0.25 = +0.5$ V

(iii) 4 level spacings: $2^2 \times +0.25 = +1.0$ V

(iv) 8 level spacings: $2^3 \times +0.25 = +2.0$ V

(c) The closest discrete value represented by D is +1.25 V; thus $D = 0101$. The error is -0.05 V, or $-0.05/1.3 \times 100 = -4\%$.

Ex: 1.10 Voltage gain $= 20 \log 100 = 40$ dB

Current gain $= 20 \log 1000 = 60$ dB

Power gain $= 10 \log A_p = 10 \log (A_v A_i)$

$= 10 \log 10^5 = 50$ dB

Ex: 1.11 $P_{dc} = 15 \times 8 = 120$ mW

$P_L = \frac{(6/\sqrt{2})^2}{1} = 18 \text{ mW}$

$P_{\text{dissipated}} = 120 - 18 = 102 \text{ mW}$

$\eta = \frac{P_L}{P_{dc}} \times 100 = \frac{18}{120} \times 100 = 15\%$

Ex: 1.12 $v_o = 1 \times \frac{10}{10^6 + 10} \simeq 10^{-5} \text{ V} = 10\ \mu\text{V}$

$P_L = v_o^2/R_L = \frac{(10 \times 10^{-6})^2}{10} = 10^{-11} \text{ W}$

With the buffer amplifier:

$v_o = 1 \times \frac{R_i}{R_i + R_s} \times A_{vo} \times \frac{R_L}{R_L + R_o}$

$= 1 \times \frac{1}{1+1} \times 1 \times \frac{10}{10 + 10} = 0.25 \text{ V}$

$P_L = \frac{v_o^2}{R_L} = \frac{0.25^2}{10} = 6.25 \text{ mW}$

Voltage gain $= \frac{v_o}{v_s} = \frac{0.25 \text{ V}}{1 \text{ V}} = 0.25$ V/V

$= -12$ dB

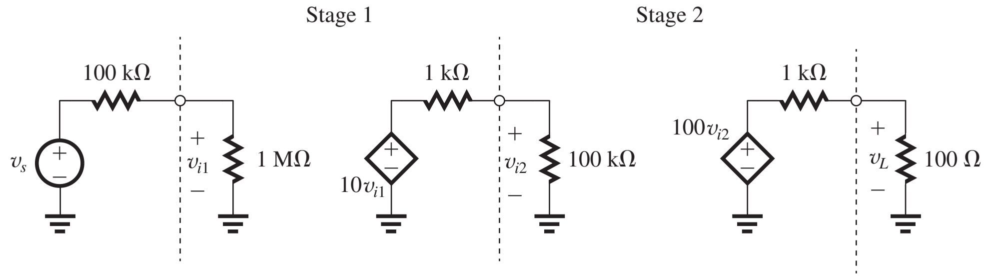
Power gain $(A_p) \equiv \frac{P_L}{P_i}$

where $P_L = 6.25$ mW and $P_i = v_i i_1$,

$v_i = 0.5$ V and

$i_i = \frac{1 \text{ V}}{1 \text{ M}\Omega + 1 \text{ M}\Omega} = 0.5\ \mu\text{A}$

This figure belongs to Exercise 1.15.



Thus,

$P_i = 0.5 \times 0.5 = 0.25\ \mu W$

and

$$A_p = \frac{6.25 \times 10^{-3}}{0.25 \times 10^{-6}} = 25 \times 10^3$$

$10 \log A_p = 44$ dB

Ex: 1.13 Open-circuit (no load) output voltage = $A_{vo} v_i$

Output voltage with load connected

$$= A_{vo} v_i \frac{R_L}{R_L + R_o}$$

$$0.8 = \frac{1}{R_o + 1} \Rightarrow R_o = 0.25\ \text{k}\Omega = 250\ \Omega$$

Ex: 1.14 $A_{vo} = 40\ \text{dB} = 100\ \text{V/V}$

$$P_L = \frac{v_o^2}{R_L} = \left(A_{vo} v_i \frac{R_L}{R_L + R_o}\right)^2 \Big/ R_L$$

$$= v_i^2 \times \left(100 \times \frac{1}{1+1}\right)^2 \Big/ 1000 = 2.5\ v_i^2$$

$$P_i = \frac{v_i^2}{R_i} = \frac{v_i^2}{10{,}000}$$

$$A_p \equiv \frac{P_L}{P_i} = \frac{2.5 v_i^2}{10^{-4} v_i^2} = 2.5 \times 10^4\ \text{W/W}$$

$10 \log A_p = 44$ dB

Ex: 1.15 Without stage 3 (see figure above)

$$\frac{v_L}{v_s} =$$

$$\left(\frac{1\ \text{M}\Omega}{100\ \text{k}\Omega + 1\ \text{M}\Omega}\right)(10)\left(\frac{100\ \text{k}\Omega}{100\ \text{k}\Omega + 1\ \text{k}\Omega}\right)$$

$$\times (100)\left(\frac{100}{100 + 1\ \text{k}\Omega}\right)$$

$$\frac{v_L}{v_s} = (0.909)(10)(0.9901)(100)(0.0909)$$

$= 81.8$ V/V

Ex: 1.16 Refer the solution to Example 1.3 in the text.

$$\frac{v_{i1}}{v_s} = 0.909\ \text{V/V}$$

$v_{i1} = 0.909\ v_s = 0.909 \times 1 = 0.909$ mV

$$\frac{v_{i2}}{v_s} = \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s} = 9.9 \times 0.909 = 9\ \text{V/V}$$

$v_{i2} = 9 \times v_S = 9 \times 1 = 9$ mV

$$\frac{v_{i3}}{v_s} = \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s} = 90.9 \times 9.9 \times 0.909$$

$= 818$ V/V

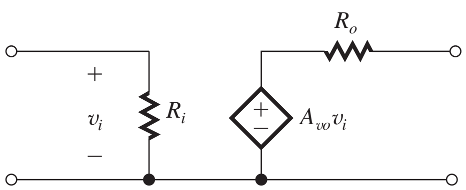
$v_{i3} = 818\ v_s = 818 \times 1 = 818$ mV

$$\frac{v_L}{v_s} = \frac{v_L}{v_{i3}} \times \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s}$$

$= 0.909 \times 90.9 \times 9.9 \times 0.909 \simeq 744$ V/V

$v_L = 744 \times 1\ \text{mV} = 744$ mV

Ex: 1.17 Using voltage amplifier model, the three-stage amplifier can be represented as



$R_i = 1\ \text{M}\Omega$

$R_o = 10\ \Omega$

$A_{vo} = A_{v1} \times A_{v2} \times A_{v3} = 9.9 \times 90.9 \times 1 = 900$ V/V

The overall voltage gain

$$\frac{v_o}{v_s} = \frac{R_i}{R_i + R_s} \times A_{vo} \times \frac{R_L}{R_L + R_o}$$

For $R_L = 10\ \Omega$

Overall voltage gain

$$= \frac{1\text{ M}}{1\text{ M} + 100\text{ K}} \times 900 \times \frac{10}{10 + 10} = 409\text{ V/V}$$

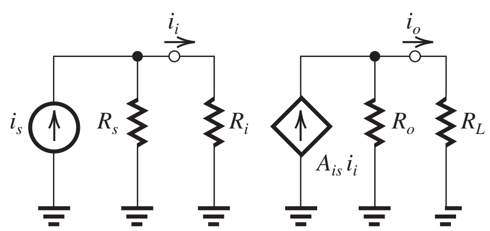
For $R_L = 1000\ \Omega$

Overall voltage gain

$$= \frac{1\text{ M}}{1\text{ M} + 100\text{ K}} \times 900 \times \frac{1000}{1000 + 10} = 810\text{ V/V}$$

$\therefore$ Range of voltage gain is from 409 V/V to 810 V/V.

Ex: 1.18



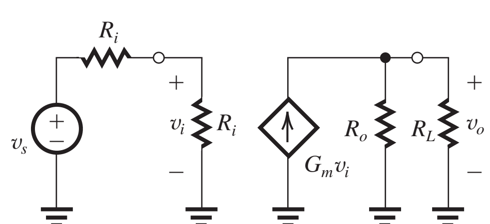
$$i_i = i_s \frac{R_s}{R_s + R_i}$$

$$i_o = A_{is} i_i \frac{R_o}{R_o + R_L} = A_{is} i_s \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L}$$

Thus,

$$\frac{i_o}{i_s} = A_{is} \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L}$$

Ex: 1.19



$$v_i = v_s \frac{R_i}{R_i + R_s}$$

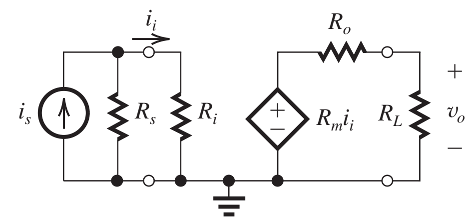
$$v_o = G_m v_i (R_o \parallel R_L)$$

$$= G_m v_s \frac{R_i}{R_i + R_s} (R_o \parallel R_L)$$

Thus,

$$\frac{v_o}{v_s} = G_m \frac{R_i}{R_i + R_s} (R_o \parallel R_L)$$

Ex: 1.20 Using the transresistance circuit model, the circuit will be



$$\frac{i_i}{i_s} = \frac{R_s}{R_i + R_s}$$

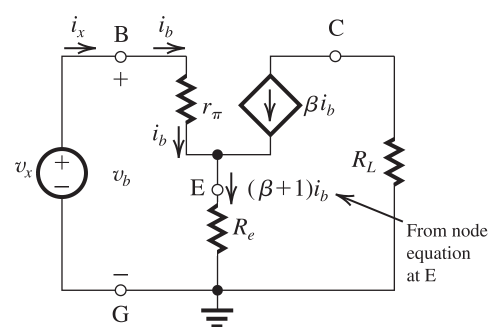
$$v_o = R_m i_i \times \frac{R_L}{R_L + R_o}$$

$$\frac{v_o}{i_i} = R_m \frac{R_L}{R_L + R_o}$$

$$\text{Now } \frac{v_o}{i_s} = \frac{v_o}{i_i} \times \frac{i_i}{i_s} = R_m \frac{R_L}{R_L + R_o} \times \frac{R_s}{R_i + R_s}$$

$$= R_m \frac{R_s}{R_s + R_i} \times \frac{R_L}{R_L + R_o}$$

Ex: 1.21



$$v_b = i_b r_\pi + (\beta + 1) i_b R_e$$

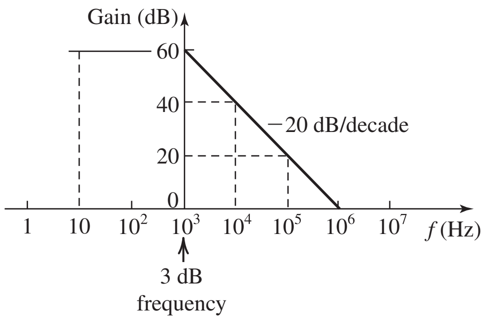
$$= i_b r_\pi + (\beta + 1) R_e$$

But $v_b = v_x$ and $i_b = i_x$, thus

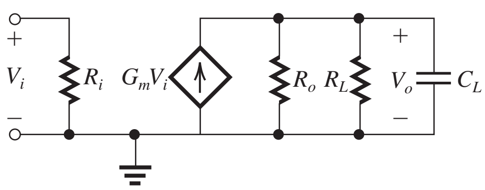
$$R_{\text{in}} \equiv \frac{v_x}{i_x} = \frac{v_b}{i_b} = r_\pi + (\beta + 1) R_e$$

Ex: 1.22

f	**Gain**
10 Hz	60 dB
10 kHz	40 dB
100 kHz	20 dB
1 MHz	0 dB



Ex: 1.23



$$V_o = G_m V_i R_o \parallel R_L \parallel C_L$$

$$= \frac{G_m V_i}{\dfrac{1}{R_o} + \dfrac{1}{R_L} + sC_L}$$

Thus, $\dfrac{V_o}{V_i} = \dfrac{G_m}{\dfrac{1}{R_o} + \dfrac{1}{R_L}} \times \dfrac{1}{1 + \dfrac{sC_L}{\dfrac{1}{R_o} + \dfrac{1}{R_L}}}$

$$\frac{V_o}{V_i} = \frac{G_m(R_L \parallel R_o)}{1 + sC_L(R_L \parallel R_o)}$$

which is of the STC LP type.

$$\omega_0 = \frac{1}{C_L(R_L \parallel R_o)}$$

$$= \frac{1}{4.5 \times 10^{-9}(10^3 \parallel R_o)}$$

For ω_0 to be at least $w\pi \times 40 \times 10^3$, the highest value allowed for R_o is

$$R_o = \frac{10^3}{2\pi \times 40 \times 10^3 \times 10^3 \times 4.5 \times 10^{-9} - 1}$$

$$= \frac{10^3}{1.131 - 1} = 7.64\ \text{k}\Omega$$

The dc gain is

$$G_m(R_L \parallel R_o)$$

To ensure a dc gain of at least 40 dB (i.e., 100), the minimum value of G_m is

$$\Rightarrow R_L \geq 100/(10^3 \parallel 7.64 \times 10^3) = 113.1\ \text{mA/V}$$

Ex: 1.24 Refer to Fig. E1.24

$$\frac{V_2}{V_s} = \frac{R_i}{R_s + \dfrac{1}{sC} + R_i} = \frac{R_i}{R_s + R_i} \frac{s}{s + \dfrac{1}{C(R_s + R_i)}}$$

which is an HP STC function.

$$f_{3\text{dB}} = \frac{1}{2\pi C(R_s + R_i)} \leq 100\ \text{Hz}$$

$$C \geq \frac{1}{2\pi(1 + 9)10^3 \times 100} = 0.16\ \mu\text{F}$$

Chapter 2

Solutions to Exercises within the Chapter

Ex: 2.1 The minimum number of terminals required by a single op amp is 5: two input terminals, one output terminal, one terminal for positive power supply, and one terminal for negative power supply.

The minimum number of terminals required by a quad op amp is 14: each op amp requires two input terminals and one output terminal (accounting for 12 terminals for the four op amps). In addition, the four op amps can all share one terminal for positive power supply and one terminal for negative power supply.

Ex: 2.2 Relevant equations are:

$v_3 = A(v_2 - v_1)$; $v_{Id} = v_2 - v_1$,

$v_{Icm} = \frac{1}{2}(v_1 + v_2)$

(a)

$$v_1 = v_2 - \frac{v_3}{A} = 0 - \frac{4}{10^3} = -0.004 \text{ V} = -4 \text{ mV}$$

$$v_{Id} = v_2 - v_1 = 0 - (-0.004) = +0.004 \text{ V}$$

$$= 4 \text{ mV}$$

$$v_{Icm} = \frac{1}{2}(-4 \text{ mV} + 0) = -2 \text{ mV}$$

(b) $-10 = 10^3(2 - v_1) \Rightarrow v_1 = 2.01$ V

$$v_{Id} = v_2 - v_1 = 2 - 2.01 = -0.01 \text{ V} = -10 \text{ mV}$$

$$v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(2.01 + 2) = 2.005 \text{ V}$$

$$\simeq 2 \text{ V}$$

(c)

$$v_3 = A(v_2 - v_1) = 10^3(1.998 - 2.002) = -4 \text{ V}$$

$$v_{Id} = v_2 - v_1 = 1.998 - 2.002 = -4 \text{ mV}$$

$$v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(2.002 + 1.998) = 2 \text{ V}$$

(d)

$$-1.2 = 10^3 v_2 - (-1.2) = 10^3(v_2 + 1.2)$$

$$\Rightarrow v_2 = -1.2012 \text{ V}$$

$$v_{Id} = v_2 - v_1 = -1.2012 - (-1.2)$$

$$= -0.0012 \text{ V} = -1.2 \text{ mV}$$

$$v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}[-1.2 + (-1.2012)]$$

$$\simeq -1.2 \text{ V}$$

Ex: 2.3 From Fig. E2.3 we have: $v_3 = \mu v_d$ and

$$v_d = (G_m v_2 - G_m v_1)R = G_m R(v_2 - v_1)$$

Therefore:

$$v_3 = \mu G_m R(v_2 - v_1)$$

That is, the open-loop gain of the op amp is $A = \mu G_m R$. For $G_m = 20$ mA/V and $\mu = 50$, we have:

$A = 50 \times 20 \times 5 = 5000$ V/V, or equivalently, 74 dB.

Ex: 2.4 The gain and input resistance of the inverting amplifier circuit shown in Fig. 2.5 are $-\frac{R_2}{R_1}$ and R_1, respectively. Therefore, we have:

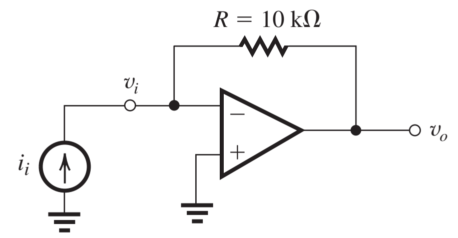
$R_1 = 100$ kΩ and

$$-\frac{R_2}{R_1} = -10 \Rightarrow R_2 = 10\ R_1$$

Thus:

$$R_2 = 10 \times 100 \text{ k}\Omega = 1 \text{ M}\Omega$$

Ex: 2.5



From Table 1.1 we have:

$$R_m = \left.\frac{v_o}{i_i}\right|_{i_o=0}$$; that is, output is open circuit

The negative input terminal of the op amp (i.e., v_i) is a virtual ground, thus $v_i = 0$:

$$v_o = v_i - Ri_i = 0 - Ri_i = -Ri_i$$

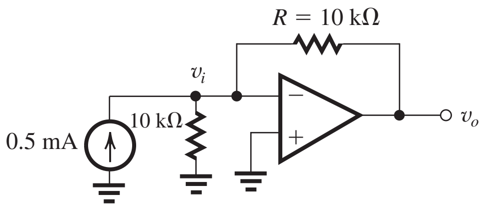
$$R_m = \left.\frac{v_o}{i_i}\right|_{i_o=0} = -\frac{Ri_i}{i_i} = -R \Rightarrow R_m = -R$$

$$= -10 \text{ k}\Omega$$

$R_i = \frac{v_i}{i_i}$ and v_i is a virtual ground ($v_i = 0$),

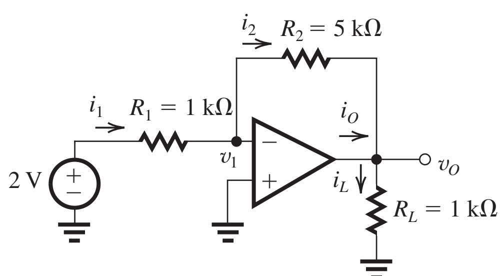
thus $R_i = \frac{0}{i_i} = 0 \Rightarrow R_i = 0\ \Omega$

Since we are assuming that the op amp in this transresistance amplifier is ideal, the op amp has zero output resistance and therefore the output resistance of this transresistance amplifier is also zero. That is $R_o = 0\ \Omega$.



Connecting the signal source shown in Fig. E2.5 to the input of this amplifier, we have:
v_i is a virtual ground that is $v_i = 0$, thus the current flowing through the 10-kΩ resistor connected between v_i and ground is zero. Therefore,
$v_o = v_i - R \times 0.5\ \text{mA} = 0 - 10\ \text{k}\Omega \times 0.5\ \text{mA}$
$= -5$ V.

Ex: 2.6



v_1 is a virtual ground, thus $v_1 = 0$ V

$$i_1 = \frac{2\ \text{V} - v_1}{R_1} = \frac{2 - 0}{1\ \text{k}\Omega} = 2\ \text{mA}$$

Assuming an ideal op amp, the current flowing into the negative input terminal of the op amp is zero. Therefore, $i_2 = i_1 \Rightarrow i_2 = 2$ mA

$$v_O = v_1 - i_2 R_2 = 0 - 2\ \text{mA} \times 5\ \text{k}\Omega = -10\ \text{V}$$

$$i_L = \frac{v_o}{R_L} = \frac{-10\ \text{V}}{1\ \text{k}\Omega} = -10\ \text{mA}$$

$$i_O = i_L - i_2 = -10\ \text{mA} - 2\ \text{mA} = -12\ \text{mA}$$

Voltage gain $= \dfrac{v_O}{2\ \text{V}} = \dfrac{-10\ \text{V}}{1\ \text{V}} = -5$ V/V or 14 dB

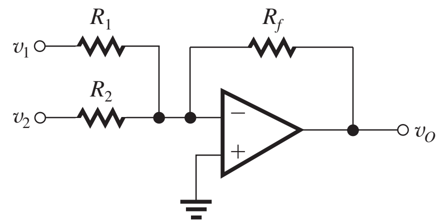
Current gain $= \dfrac{i_L}{i_1} = \dfrac{-10\ \text{mA}}{2\ \text{mA}} = -5$ A/A or 14 dB

Power gain

$$= \frac{P_L}{P_i} = \frac{-10(-10\ \text{mA})}{2\ \text{V} \times 2\ \text{mA}} = 25\ \text{W/W or 14 dB}$$

Note that power gain in dB is $10 \log_{10} \left| \dfrac{P_L}{P_i} \right|$.

Ex: 2.7



For the circuit shown above we have:

$$v_O = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2\right)$$

Since it is required that $v_O = -(v_1 + 4v_2)$,

we want to have:

$$\frac{R_f}{R_1} = 1 \quad \text{and} \quad \frac{R_f}{R_2} = 4$$

It is also desired that for a maximum output voltage of 4 V, the current in the feedback resistor not exceed 1 mA.

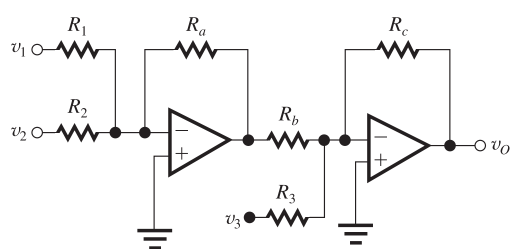
Therefore

$$\frac{4\ \text{V}}{R_f} \le 1\ \text{mA} \Rightarrow R_f \ge \frac{4\ \text{V}}{1\ \text{mA}} \Rightarrow R_f \ge 4\ \text{k}\Omega$$

Let us choose R_f to be 4 kΩ, then

$$R_1 = R_f = 4\ \text{k}\Omega \text{ and } R_2 = \frac{R_f}{4} = 1\ \text{k}\Omega$$

Ex: 2.8



$$v_O = \left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right)v_1 + \left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right)v_2 - \left(\frac{R_c}{R_3}\right)v_3$$

We want to design the circuit such that

$$v_O = 2v_1 + v_2 - 4v_3$$

Thus we need to have

$$\left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right) = 2, \left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) = 1, \text{ and } \frac{R_c}{R_3} = 4$$

From the above three equations, we have to find six unknown resistors; therefore, we can arbitrarily choose three of these resistors. Let us choose $R_a = R_b = R_c = 10\ \text{k}\Omega$.

Then we have

$$R_3 = \frac{R_c}{4} = \frac{10}{4} = 2.5\ \text{k}\Omega$$

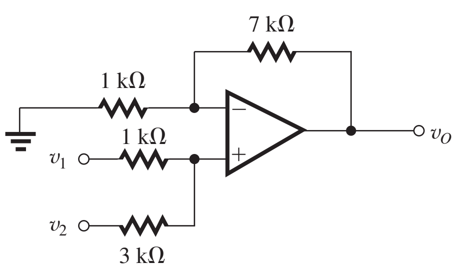
$$\left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right) = 2, \Rightarrow \frac{10}{R_1} \times \frac{10}{10} = 2$$

$$\Rightarrow R_1 = 5\ \text{k}\Omega$$

$$\left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) = 1 \Rightarrow \frac{10}{R_2} \times \frac{10}{10} = 1$$

$$\Rightarrow R_2 = 10\ \text{k}\Omega$$

Ex: 2.9 Using the superposition principle to find the contribution of v_1 to the output voltage v_O, we set $v_2 = 0$



v_+ (the voltage at the positive input of the op amp is: $v_+ = \dfrac{3}{1+3}v_1 = 0.75v_1$

Thus $v_O = \left(1 + \dfrac{7\ \text{k}\Omega}{1\ \text{k}\Omega}\right)v_+ = 8 \times 0.75v_1 = 6v_1$

To find the contribution of v_2 to the output voltage v_O we set $v_1 = 0$.

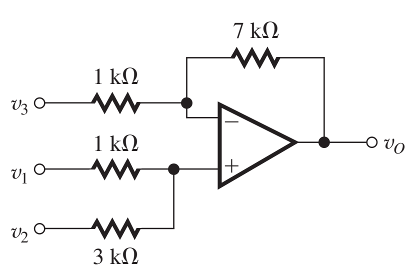
Then $v_+ = \dfrac{1}{1+3}v_2 = 0.25v_2$

Hence

$$v_O = \left(1 + \frac{7\ \text{k}\Omega}{1\ \text{k}\Omega}\right)v_+ = 8 \times 0.25v_2 = 2v_2$$

Combining the contributions of v_1 and v_2 to v_O, we have $v_O = 6v_1 + 2v_2$

Ex: 2.10



Using the superposition principle to find the contribution of v_1 to v_O, we set $v_2 = v_3 = 0$. Then we have (refer to the solution of Exercise 2.9): $v_O = 6v_1$

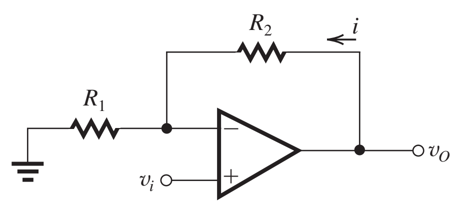
To find the contribution of v_2 to v_O, we set $v_1 = v_3 = 0$, then: $v_O = 2v_2$

To find the contribution of v_3 to v_O we set $v_1 = v_2 = 0$, then

$$v_o = -\frac{7\ \text{k}\Omega}{1\ \text{k}\Omega}v_3 = -7v_3$$

Combining the contributions of v_1, v_2, and v_3 to v_O we have: $v_O = 6v_1 + 4v_2 - 7v_3$.

Ex: 2.11



$$\frac{v_O}{v_i} = 1 + \frac{R_2}{R_1} = 2 \Rightarrow \frac{R_2}{R_1} = 1 \Rightarrow R_1 = R_2$$

If $v_O = 10$ V, then it is desired that $i = 10\ \mu$A.

Thus,

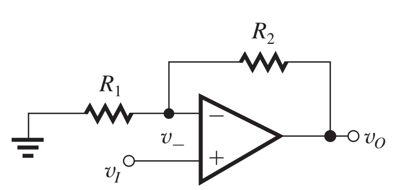
$$i = \frac{10\ \text{V}}{R_1 + R_2} = 10\ \mu\text{A} \Rightarrow R_1 + R_2 = \frac{10\ \text{V}}{10\ \mu\text{A}}$$

$R_1 + R_2 = 1\ \text{M}\Omega$ and

$R_1 = R_2 \Rightarrow R_1 = R_2 = 0.5\ \text{M}\Omega$

Ex: 2.12

(a)



$$v_I - v_- = v_O/A \Rightarrow v_- = v_I - v_O/A \qquad (1)$$

But from the voltage divider across v_O,

$$v_- = v_O\frac{R_1}{R_1 + R_2} \qquad (2)$$

Equating Eq. (1) and Eq. (2) gives

$$v_O\frac{R_1}{R_1 + R_2} = v_I - \frac{v_O}{A}$$

which can be manipulated to the form

$$\frac{v_O}{v_I} = \frac{1 + (R_2/R_1)}{1 + \dfrac{1 + (R_2/R_1)}{A}}$$

(b) For $R_1 = 1\ \text{k}\Omega$ and $R_2 = 9\ \text{k}\Omega$ the ideal value for the closed-loop gain is $1 + \frac{9}{1}$, that is, 10. The actual closed-loop gain is $G = \dfrac{10}{1 + 10/A}$.

If $A = 10^3$, then $G = 9.901$ and

$$\epsilon = \frac{G - 10}{10} \times 100 = -0.99\% \simeq -1\%$$

For $v_I = 1$ V, $v_O = G \times v_I = 9.901$ V and

$$v_O = A(v_+ - v_-) \Rightarrow v_+ - v_- = \frac{v_O}{A} = \frac{9.901}{1000}$$

$\simeq 9.9$ mV

If $A = 10^4$, then $G = 9.99$ and $\epsilon = -0.1\%$.

For $v_I = 1$ V, $v_O = G \times v_I = 9.99$ V,

therefore,

$$v_+ - v_- = \frac{v_O}{A} = \frac{9.99}{10^4} = 0.999\ \text{mV} \simeq 1\ \text{mV}$$

If $A = 10^5$, then $G = 9.999$ and $\epsilon = -0.01\%$

For $v_I = 1$ V, $v_O = G \times v_I = 9.999$ thus,

$$v_+ - v_- = \frac{v_O}{A} = \frac{9.999}{10^5} = 0.09999\ \text{mV}$$

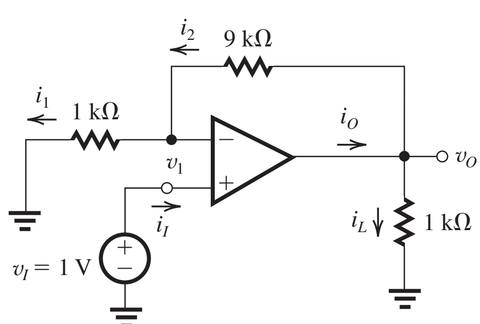
$\simeq 0.1$ mV

Ex: 2.13

$i_I = 0$ A, $v_1 = v_I = 1$ V

$$i_1 = \frac{v_1}{1\ \text{k}\Omega} = \frac{1\ \text{V}}{1\ \text{k}\Omega} = 1\ \text{mA}$$

$i_2 = i_1 = 1$ mA



$$v_O = v_1 + i_2 \times 9\ \text{k}\Omega = 1 + 1 \times 9 = 10\ \text{V}$$

$$i_L = \frac{v_O}{1\ \text{k}\Omega} = \frac{10\ \text{V}}{1\ \text{k}\Omega} = 10\ \text{mA}$$

$$i_O = i_L + i_2 = 11\ \text{mA}$$

$$\frac{v_O}{v_i} = \frac{10\ \text{V}}{1\ \text{V}} = 10\ \text{V/V or } 20\ \text{dB}$$

$$\frac{i_L}{i_I} = \frac{10\ \text{mA}}{0} = \infty$$

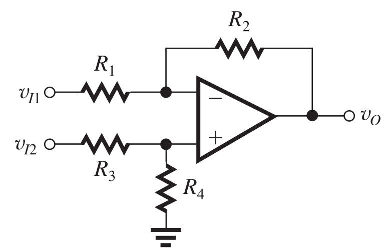
$$\frac{P_L}{P_I} = \frac{v_O \times i_L}{v_I \times i_I} = \frac{10 \times 10}{1 \times 0} = \infty$$

Ex: 2.14

(a) Load voltage $= \dfrac{1\ \text{k}\Omega}{1\ \text{k}\Omega + 1\ \text{M}\Omega} \times 1\ \text{V} \simeq 1\ \text{mV}$

(b) Load voltage = 1 V

Ex: 2.15



(a) $R_1 = R_3 = 2\ \text{k}\Omega$, $R_2 = R_4 = 200\ \text{k}\Omega$

Since $R_4/R_3 = R_2/R_1$ we have:

$$A_d = \frac{v_O}{v_{I2} - v_{I1}} = \frac{R_2}{R_1} = \frac{200}{2} = 100\ \text{V/V}$$

(b) $R_{id} = 2R_1 = 2 \times 2\ \text{k}\Omega = 4\ \text{k}\Omega$

Since we are assuming the op amp is ideal,

$R_o = 0\ \Omega$

(c)

$$A_{cm} \equiv \frac{v_O}{v_{Icm}} = \frac{R_4}{R_3 + R_4}\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

The worst-case common-mode gain (i.e., the largest A_{cm}) occurs when the resistor tolerances are such that the quantity in parentheses is maximum. This in turn occurs when R_2 and R_3 are at their highest possible values (each one percent above nominal) and R_1 and R_4 are at their lowest possible values (each one percent below nominal), resulting in

$$A_{cm} = \frac{R_4}{R_3 + R_4}\left(1 - \frac{1.01 \times 1.01}{0.99 \times 0.99}\right)$$

$$|A_{cm}| \simeq \frac{R_4}{R_3 + R_4} \times 0.04 \simeq \frac{200}{202} \times 0.04 \simeq 0.04\ \text{V/V}$$

The corresponding CMRR is

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{0.04} = 2500$$

or 68 dB.

Ex: 2.16 We choose $R_3 = R_1$ and $R_4 = R_2$. Then for the circuit to behave as a difference amplifier with a gain of 10 and an input resistance of 20 kΩ, we require

$A_d = \frac{R_2}{R_1} = 10$ and

$R_{Id} = 2R_1 = 20\text{ k}\Omega \Rightarrow R_1 = 10\text{ k}\Omega$ and

$R_2 = A_d R_1 = 10 \times 10\text{ k}\Omega = 100\text{ k}\Omega$

Therefore, $R_1 = R_3 = 10\text{ k}\Omega$ and

$R_2 = R_4 = 100\text{ k}\Omega$.

Ex: 2.17 Given $v_{Icm} = +5$ V

$v_{Id} = 10 \sin \omega t$ mV

$2R_1 = 1\text{ k}\Omega,\ R_2 = 0.5\text{ M}\Omega$

$R_3 = R_4 = 10\text{ k}\Omega$

$v_{I1} = v_{Icm} - \frac{1}{2}v_{Id} = 5 - \frac{1}{2} \times 0.01 \sin \omega t$

$= 5 - 0.005 \sin \omega t$ V

$v_{I2} = v_{Icm} + \frac{1}{2}v_{Id}$

$= 5 + 0.005 \sin \omega t$ V

$v_-(\text{op amp } A_1) = v_{I1} = 5 - 0.005 \sin \omega t$ V

$v_-(\text{op amp } A_2) = v_{I2} = 5 + 0.005 \sin \omega t$ V

$v_{Id} = v_{I2} - v_{I1} = 0.01 \sin \omega t$

$v_{O1} = v_{I1} - R_2 \times \frac{v_{Id}}{2R_1}$

$= 5 - 0.005 \sin \omega t - 500\text{ k}\Omega \times \frac{0.01 \sin \omega t}{1\text{ k}\Omega}$

$= (5 - 5.005 \sin \omega t)$ V

$v_{O2} = v_{I2} + R_2 \times \frac{v_{Id}}{2R_1}$

$= (5 + 5.005 \sin \omega t)$ V

$v_+(\text{op amp } A_3) = v_{O2} \times \frac{R_4}{R_3 + R_4} = v_{O2}\frac{10}{10 + 10}$

$= \frac{1}{2}v_{O2} = \frac{1}{2}(5 + 5.005 \sin \omega t)$

$= (2.5 + 2.5025 \sin \omega t)$V

$v_-(\text{op amp } A_3) = v_+(\text{op amp } A_3)$

$= (2.5 + 2.5025 \sin \omega t)$ V

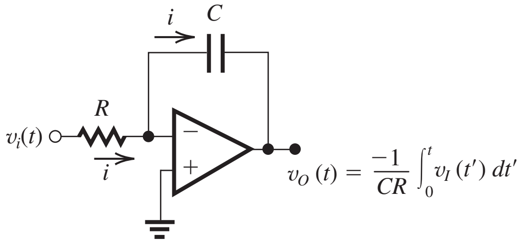
$v_O = \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right)v_{Id}$

$\frac{10\text{ k}\Omega}{10\text{ k}\Omega}\left(1 + \frac{0.5\text{ M}\Omega}{0.5\text{ k}\Omega}\right) \times 0.01 \sin \omega t$

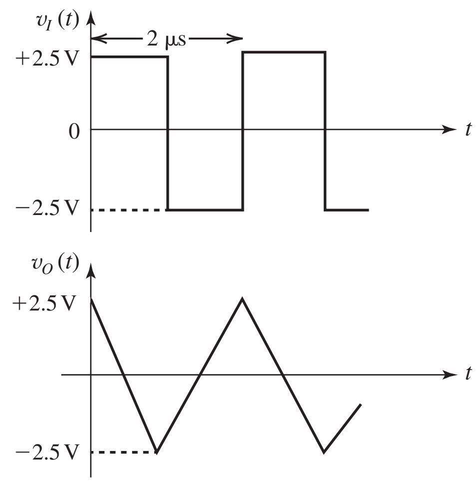
$= 1(1 + 1000) \times 0.01 \sin \omega t$

$= 10.01 \sin \omega t$ V

Ex: 2.18



The signal waveforms will be as shown.



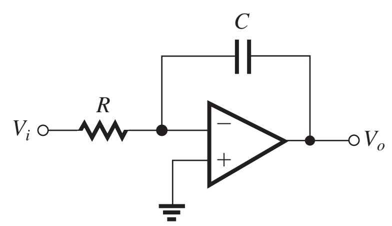
When $v_I = +2.5$ V, the current through the capacitor will be in the direction indicated, $i = 2.5\text{ V}/R$, and the output voltage will decrease linearly from +2.5 V to −2.5 V. Thus in $(T/2)$ seconds, the capacitor voltage changes by 5 V. The charge equilibrium equation can be expressed as

$i(T/2) = C \times 5$ V

$\frac{2.5}{R}\frac{T}{2} = 5C \Rightarrow CR = \frac{2.5T}{10} = \frac{1}{4} \times 2 \times 10^{-6}$

$= 0.5\ \mu\text{s}$

Ex: 2.19



The input resistance of this inverting integrator is R; therefore, $R = 10\text{ k}\Omega$.

Since the desired integration time constant is 10^{-3} s, we have: $CR = 10^{-3}$ s $\Rightarrow$

$$C = \frac{10^{-3}\text{ s}}{10\text{ k}\Omega} = 0.1\ \mu\text{F}$$

From Eq. (2.27) the transfer function of this integrator is:

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{1}{j\omega CR}$$

For $\omega = 10$ rad/s, the integrator transfer function has magnitude

$$\left|\frac{V_o}{V_i}\right| = \frac{1}{1 \times 10^{-3}} = 100\text{ V/V}$$

and phase $\phi = 90^\circ$.

For $\omega = 1$ rad/s, the integrator transfer function has magnitude

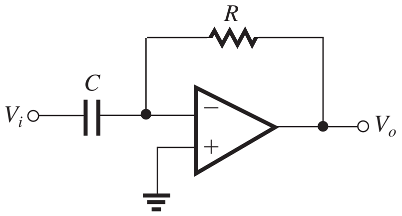
$$\left|\frac{V_o}{V_i}\right| = \frac{1}{1 \times 10^{-3}} = 1000\text{ V/V}$$

and phase $\phi = 90^\circ$.

The frequency at which the integrator gain magnitude is unity is

$$\omega_{\text{int}} = \frac{1}{CR} = \frac{1}{10^{-3}} = 1000\text{ rad/s}$$

Ex: 2.20



$C = 0.01\ \mu$F is the input capacitance of this differentiator. We want $CR = 10^{-2}$ s (the time constant of the differentiator); thus,

$$R = \frac{10^{-2}}{0.01\ \mu\text{F}} = 1\text{ M}\Omega$$

From Eq. (2.33), the transfer function of the differentiator is

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -j\omega CR$$

Thus, for $\omega = 10$ rad/s the differentiator transfer function has magnitude

$$\left|\frac{V_o}{V_i}\right| = 10 \times 10^{-2} = 0.1\text{ V/V}$$

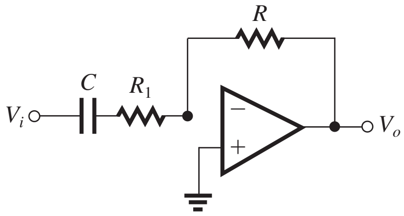
and phase $\phi = -90^\circ$.

For $\omega = 10^3$ rad/s, the differentiator transfer function has magnitude

$$\left|\frac{V_o}{V_i}\right| = 10^3 \times 10^{-2} = 10\text{ V/V}$$

and phase $\phi = -90^\circ$.

If we add a resistor in series with the capacitor to limit the high-frequency gain of the differentiator to 100, the circuit would be:



At high frequencies the capacitor C acts like a short circuit. Therefore, the high-frequency gain of this circuit is: $\frac{R}{R_1}$. To limit the magnitude of this high-frequency gain to 100, we should have:

$$\frac{R}{R_1} = 100 \Rightarrow R_1 = \frac{R}{100} = \frac{1\text{ M}\Omega}{100} = 10\text{ k}\Omega$$

Ex: 2.21

Refer to the model in Fig. 2.27 and observe that

$$v_+ - v_- = V_{OS} + v_2 - v_1 = V_{OS} + v_{Id}$$

and since $v_O = v_3 = A(v_+ - v_-)$, then

$$v_O = A(v_{Id} + V_{OS}) \qquad (1)$$

where $A = 10^4$ V/V and $V_{OS} = 5$ mV. From Eq. (1) we see that $v_{id} = 0$ results in $v_O = 50$ V, which is impossible; thus the op amp saturates and $v_O = +10$ V. This situation pertains for $v_{Id} \geq -4$ mV. If v_{Id} decreases below -4 mV, the op-amp output decreases correspondingly. For instance, $v_{Id} = -4.5$ mV results in $v_O = +5$ V; $v_{Id} = -5$ mV results in $v_O = 0$ V; $v_{Id} = -5.5$ mV results in $v_O = -5$ V; and $v_{Id} = -6$ mV results in $v_O = -10$ V, at which point the op amp saturates at the negative level of -10 V. Further decreases in v_{Id} have no effect on the output voltage. The result is the transfer characteristic sketched in Fig. E2.21. Observe that the linear range of the characteristic is now centered around $v_{Id} = -5$ mV rather than the ideal situation of $v_{Id} = 0$; this shift is obviously a result of the input offset voltage V_{OS}.

Ex: 2.22 (a) The inverting amplifier of -1000 V/V gain will exhibit an output dc offset voltage of $\pm V_{OS}(1 + R_2/R_1) =$ ± 3 mV $\times$ $(1 + 1000) = \pm 3.03$ V. Now, since the op-amp saturation levels are ± 10 V, the room left for output signal swing is approximately ± 7 V. Thus to avoid op-amp saturation and the attendant clipping of the peak of the output sinusoid, we must limit the peak amplitude of the input sine wave to approximately 7 V/1000 = 7 mV.

(b) If at room temperature (25°C), V_{OS} is trimmed to zero and (i) the circuit is operated at a constant temperature, the peak of the input sine wave can be increased to 10 mV. (ii) However, if the circuit is to operate in the temperature range of 0°C to 75°C (i.e., at a temperature that deviates from room temperature by a maximum of 50°C), the input offset voltage will drift from by a maximum of 10 μV/°C × 50°C = 500 μV or 0.5 mV. This will reduce the allowed peak amplitude of the input sinusoid to 9.5 mV.

Ex: 2.23

(a) If the amplifier is capacitively coupled in the manner of Fig. 2.32(a), then the input offset voltage V_{OS} will see a unity-gain amplifier [Fig. 2.32(b)] and the dc offset voltage at the output will be equal to V_{OS}, that is, 3 mV. Thus, almost the entire output range of ± 10 V will be available for signal swing, allowing a sine-wave input of approximately 10-mV peak without the risk of output clipping. Obviously, in this case there is no need for output trimming.

(b) We need to select a value of the coupling capacitor C that will place the 3-dB frequency of the resulting high-pass STC circuit at 1000 Hz, thus

$$1000 = \frac{1}{2\pi C R_1}$$

$$\Rightarrow C = \frac{1}{2\pi \times 1000 \times 1 \times 10^3} = 0.16\ \mu\text{F}$$

Ex: 2.24 From Eq. (2.35) we have:

$$V_O = I_{B1} R_2 \simeq I_B R_2$$

$$= 100\ \text{nA} \times 1\ \text{M}\Omega = 0.1\ \text{V}$$

From Eq. (2.37) the value of resistor R_3 (placed in series with positive input to minimize the output offset voltage) is

$$R_3 = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{10\ \text{k}\Omega \times 1\ \text{M}\Omega}{10\ \text{k}\Omega + 1\ \text{M}\Omega}$$

$$= 9.9\ \text{k}\Omega$$

$$R_3 = 9.9\ \text{k}\Omega \simeq 10\ \text{k}\Omega$$

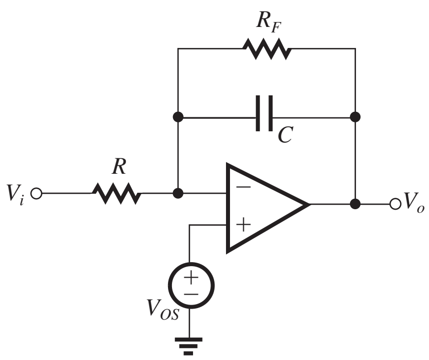
With this value of R_3, the new value of the output dc voltage [using Eq. (2.38)] is:

$$V_O = I_{OS} R_2 = 10\ \text{nA} \times 10\ \text{k}\Omega = 0.01\ \text{V}$$

Ex: 2.25 Using Eq. (2.39) we have:

$$v_O = V_{OS} + \frac{V_{OS}}{CR} t \Rightarrow 12 = 2\ \text{mV} + \frac{2\ \text{mV}}{1\ \text{ms}} t$$

$$\Rightarrow t = \frac{12\ \text{V} - 2\ \text{mV}}{2\ \text{mV}} \times 1\ \text{ms} \simeq 6\ \text{s}$$



With the feedback resistor R_F, to have at least ± 10 V of output signal swing available, we have to make sure that the output voltage due to V_{OS} has a magnitude of at most 2 V. From Eq. (2.34), we know that the output dc voltage due to V_{OS} is

$$V_O = V_{OS}\left(1 + \frac{R_F}{R}\right) \Rightarrow 2\ \text{V} = 2\ \text{mV}\left(1 + \frac{R_F}{10\ \text{k}\Omega}\right)$$

$$1 + \frac{R_F}{10\ \text{k}\Omega} = 1000 \Rightarrow R_F \simeq 10\ \text{M}\Omega$$

The corner frequency of the resulting STC network is $\omega_0 = \dfrac{1}{CR_F}$

We know $RC = 1$ ms and

$$R = 10\ \text{k}\Omega \Rightarrow C = 0.1\ \mu\text{F}$$

Thus $\omega_0 = \dfrac{1}{0.1\ \mu\text{F} \times 10\ \text{M}\Omega} = 1$ rad/s

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} = 0.16\ \text{Hz}$$

Ex: 2.26

$$20 \log A_0 = 106\ \text{dB} \Rightarrow A_0 = 200{,}000\ \text{V/V}$$

$$f_t = 3\ \text{MHz}$$

$$f_b = f_t / A_0 = \frac{3\ \text{MHz}}{200{,}000} = 15\ \text{Hz}$$

At f_b, the open-loop gain drops by 3 dB below its value at dc; thus it becomes 103 dB.

For $f \gg f_b$, $|A| \simeq f_t/f$; thus

At $f = 300$ Hz, $|A| = \dfrac{3\text{ MHz}}{300\text{ Hz}} = 10^4$ V/V

or 80 dB

At $f = 3$ kHz, $|A| = \dfrac{3\text{ MHz}}{3\text{ kHz}} = 10^3$ V/V

or 60 dB

At $f = 12$ kHz, which is two octaves higher than 3 kHz, the gain will be $2 \times 6 = 12$ dB below its value at 3 kHz; that is, $60 - 12 = 48$ dB.

At $f = 60$ kHz, $|A| = \dfrac{3\text{ MHz}}{60\text{ kHz}} = 50$ V/V

or 34 dB

Ex: 2.27

$A_0 = 10^6$ V/V or 120 dB

The gain falls off at the rate of 20 dB/decade. Thus, it reaches 40 dB at a frequency four decades higher than f_b,

$10^4 f_b = 100\text{ kHz} \Rightarrow f_b = 10$ Hz

The unity-gain frequency f_t will be two decades higher than 100 kHz, that is,

$f_t = 100 \times 100\text{ kHz} = 10$ MHz

Alternatively, we could have found f_t from the gain-bandwidth product

$f_t = A_0 f_b = 10^6 \times 10\text{ Hz} = 10$ MHz

At a frequency $f \gg f_b$,

$|A| = f_t/f$

For $f = 10$ kHz, $|A| = \dfrac{10\text{ MHz}}{10\text{ kHz}} = 10^3$ V/V or 60 dB

Ex: 2.28

$20 \log A_0 = 106\text{ dB} \Rightarrow A_0 = 200{,}000$ V/V

$f_t = 20$ MHz

For a noninverting amplifier with a nominal dc gain of 100,

$1 + \dfrac{R_2}{R_1} = 100$

Since the nominal dc gain is much lower than A_0,

$$f_{3\text{dB}} \simeq f_t \Big/ \left(1 + \frac{R_2}{R_1}\right)$$

$$= \frac{20\text{ MHz}}{100} = 200\text{ kHz}$$

Ex: 2.29 For the input voltage step of magnitude V the output waveform will still be given by the exponential waveform of Eq. (2.56) if

$$\omega_t V \le SR$$

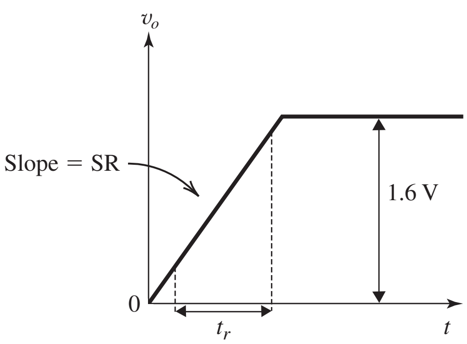
that is, $V \le \dfrac{SR}{\omega_t} \Rightarrow V \le \dfrac{SR}{2\pi f_t}$ resulting in

$$V \le 0.16\text{ V}$$

From Appendix F we know that the 10% to 90% rise time of the output waveform of the form of Eq. (2.56) is $t_r \simeq 2.2 \times$ time constant $= \dfrac{2.2}{\omega_t}$.

Thus, $t_r \simeq 0.35\ \mu$s

If an input step of amplitude 1.6 V (10 times as large compared to the previous case) is applied, the output will be slew-rate limited and thus linearly rising with a slope equal to the slew rate, as shown in the following figure.



$$t_r = \frac{0.9 \times 1.6 - 0.1 \times 1.6}{1\text{ V}/\mu\text{s}}$$

$\Rightarrow t_r = 1.28\ \mu$s

Ex: 2.30 From Eq. (2.57) we have:

$$f_M = \frac{SR}{2\pi V_{O\ \max}} = 318\text{ kHz}$$

Using Eq. (2.58), for an input sinusoid with frequency $f = 5 f_M$, the maximum possible amplitude that can be accommodated at the output without incurring SR distortion is:

$$V_O = V_{O\ \max}\left(\frac{f_M}{5 f_M}\right) = 5 \times \frac{1}{5} = 1\text{ V (peak)}$$

Chapter 3

Solutions to Exercises within the Chapter

Ex: 3.1 $T = 50$ K

$n_i = BT^{3/2}e^{-Eg/(2kT)}$

$= 7.3 \times 10^{15}(50)^{3/2}\ e^{-1.12/(2\times 8.62\times 10^{-5}\times 50)}$

$\simeq 9.6 \times 10^{-39}/\text{cm}^3$

$T = 350$ K

$n_i = BT^{3/2}e^{-Eg/(2kT)}$

$= 7.3 \times 10^{15}(350)^{3/2}\ e^{-1.12/(2\times 8.62\times 10^{-5}\times 350)}$

$= 4.15 \times 10^{11}/\text{cm}^3$

Ex: 3.2 $N_D = 10^{17}/\text{cm}^3$

From Exercise 3.1, n_i at

$T = 350\text{ K} = 4.15 \times 10^{11}/\text{cm}^3$

$n_n = N_D = 10^{17}/\text{cm}^3$

$p_n \cong \dfrac{ni^2}{N_D}$

$= \dfrac{(4.15 \times 10^{11})^2}{10^{17}}$

$= 1.72 \times 10^6/\text{cm}^3$

Ex: 3.3 At 300 K, $n_i = 1.5 \times 10^{10}/\text{cm}^3$

$p_p = N_A$

Want electron concentration

$= n_p = \dfrac{1.5 \times 10^{10}}{10^6} = 1.5 \times 10^4/\text{cm}^3$

$\therefore N_A = p_p = \dfrac{ni^2}{n_p}$

$= \dfrac{(1.5 \times 10^{10})^2}{1.5 \times 10^4}$

$= 1.5 \times 10^{16}/\text{cm}^3$

Ex: 3.4 (a) $v_{n\text{-drift}} = -\mu_n E$

Here negative sign indicates that electrons move in a direction opposite to E.

We use

$v_{n\text{-drift}} = 1350 \times \dfrac{1}{2 \times 10^{-4}}\ \because 1\ \mu\text{m} = 10^{-4}$ cm

$= 6.75 \times 10^6$ cm/s $= 6.75 \times 10^4$ m/s

(b) Time taken to cross 2-µm

length $= \dfrac{2 \times 10^{-6}}{6.75 \times 10^4} \simeq 30$ ps

(c) In n-type silicon, drift current density J_n is

$J_n = qn\mu_n E$

$= 1.6 \times 10^{-19} \times 10^{16} \times 1350 \times \dfrac{1\text{ V}}{2 \times 10^{-4}}$

$= 1.08 \times 10^4\ \text{A/cm}^2$

(d) Drift current $I_n = AJ_n$

$= 0.25 \times 10^{-8} \times 1.08 \times 10^4$

$= 27\ \mu$A

The resistance of the bar is

$R = \rho \times \dfrac{L}{A}$

$= qn\mu_n \times \dfrac{L}{A}$

$= 1.6 \times 10^{-19} \times 10^{16} \times 1350 \times \dfrac{2 \times 10^{-4}}{0.25 \times 10^{-8}}$

$= 37.0\ \text{k}\Omega$

Alternatively, we may simply use the preceding result for current and write

$R = V/I_n = 1\text{ V}/27\ \mu\text{A} = 37.0\ \text{k}\Omega$

Note that $0.25\ \mu\text{m}^2 = 0.25 \times 10^{-8}\ \text{cm}^2$.

Ex: 3.5 $J_n = qD_n \dfrac{dn(x)}{dx}$

From Fig. E3.5,

$n_0 = 10^{17}/\text{cm}^3 = 10^5/(\mu\text{m})^3$

$D_n = 35\ \text{cm}^2/\text{s} = 35 \times (10^4)^2\ (\mu\text{m})^2/\text{s}$

$= 35 \times 10^8\ (\mu\text{m})^2/\text{s}$

$\dfrac{dn}{dx} = \dfrac{10^5 - 0}{0.5} = 2 \times 10^5 \mu\text{m}^{-4}$

$J_n = qD_n \dfrac{dn(x)}{dx}$

$= 1.6 \times 10^{-19} \times 35 \times 10^8 \times 2 \times 10^5$

$= 112 \times 10^{-6}\ \text{A}/\mu\text{m}^2$

$= 112\ \mu\text{A}/\mu\text{m}^2$

For $I_n = 1\text{ mA} = J_n \times A$

$\Rightarrow A = \dfrac{1\text{ mA}}{J_n} = \dfrac{10^3\ \mu\text{A}}{112\ \mu\text{A}/(\mu\text{m})^2} \simeq 9\ \mu\text{m}^2$

Ex: 3.6 Using Eq. (3.20),

$\dfrac{D_n}{\mu_n} = \dfrac{D_p}{\mu_p} = V_T$

$D_n = \mu_n V_T = 1350 \times 25.9 \times 10^{-3}$

$\cong 35\ \text{cm}^2/\text{s}$

$D_p = \mu_p V_T = 480 \times 25.9 \times 10^{-3}$

$\cong 12.4\ \text{cm}^2/\text{s}$

Ex: 3.7 Equation (3.25)

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_0}$$

$$= \sqrt{\frac{2\epsilon_s}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)V_0}$$

$$W^2 = \frac{2\epsilon_s}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)V_0$$

$$V_0 = \frac{1}{2}\left(\frac{q}{\epsilon_s}\right)\left(\frac{N_A N_D}{N_A+N_D}\right)W^2$$

Ex: 3.8 In a p^+n diode $N_A \gg N_D$

Equation (3.26) $W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_0}$

We can neglect the term $\frac{1}{N_A}$ as compared to $\frac{1}{N_D}$, thus

$$W \simeq \sqrt{\frac{2\epsilon_s}{qN_D}\cdot V_0}$$

Equation (3.26) $x_n = W\frac{N_A}{N_A+N_D}$

$\simeq W\,\frac{N_A}{N_A}$

$= W$

Equation (3.28), $x_p = W\frac{N_D}{N_A+N_D}$

since $N_A \gg N_D$

$\simeq W\,\frac{N_D}{N_A} = W\Big/\left(\frac{N_A}{N_D}\right)$

Equation (3.28), $Q_J = Aq\left(\frac{N_A N_D}{N_A+N_D}\right)W$

$\simeq Aq\frac{N_A N_D}{N_A}W$

$= AqN_D W$

Equation (3.29), $Q_J = A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A+N_D}\right)V_0}$

$\simeq A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A}\right)V_0}$ since $N_A \gg N_D$

$= A\sqrt{2\epsilon_s q N_D V_0}$

Ex: 3.9 In Example 3.5, $N_A = 10^{18}/\text{cm}^3$ and $N_D = 10^{16}/\text{cm}^3$

In the n-region of this pn junction

$n_n = N_D = 10^{16}/\text{cm}^3$

$$p_n = \frac{n_i^2}{n_n} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4/\text{cm}^3$$

As one can see from above equation, to increase minority-carrier concentration (p_n) by a factor of 2, one must lower N_D ($= n_n$) by a factor of 2.

Ex: 3.10

Equation (3.40) $I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D}+\frac{D_n}{L_n N_A}\right)$

since $\frac{D_p}{L_p}$ and $\frac{D_n}{L_n}$ have approximately similar values, if $N_A \gg N_D$, then the term $\frac{D_n}{L_n N_A}$ can be neglected as compared to $\frac{D_p}{L_p N_D}$

$\therefore I_S \cong Aqn_i^2\frac{D_p}{L_p N_D}$

Ex: 3.11 $I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D}+\frac{D_n}{L_n N_A}\right)$

$= 10^{-4}\times1.6\times10^{-19}\times(1.5\times10^{10})^2$

$$\times\left(\frac{10}{5\times10^{-4}\times\frac{10^{16}}{2}}+\frac{18}{10\times10^{-4}\times10^{18}}\right)$$

$= 1.46\times10^{-14}$ A

$I = I_S(e^{V/V_T}-1)$

$\simeq I_S e^{V/V_T} = 1.45\times10^{-14}e^{0.605/(25.9\times10^{-3})}$

$= 0.2$ mA

Ex: 3.12 $W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_0-V_F)}$

$= \sqrt{\frac{2\times1.04\times10^{-12}}{1.6\times10^{-19}}\left(\frac{1}{10^{18}}+\frac{1}{10^{16}}\right)(0.814-0.605)}$

$= 1.66\times10^{-5}$ cm $= 0.166$ μm

Ex: 3.13 $W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_0+V_R)}$

$= \sqrt{\frac{2\times1.04\times10^{-12}}{1.6\times10^{-19}}\left(\frac{1}{10^{18}}+\frac{1}{10^{16}}\right)(0.814+2)}$

$= 6.08\times10^{-5}$ cm $= 0.608$ μm

Using Eq. (3.28),

$Q_J = Aq\left(\frac{N_A N_D}{N_A+N_D}\right)W$

$= 10^{-4}\times1.6\times10^{-19}\left(\frac{10^{18}\times10^{16}}{10^{18}+10^{16}}\right)\times6.08\times10^{-5}$ cm

$= 9.63$ pC

Reverse current

$$I = I_S = Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)$$

$$= 10^{-14} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2$$

$$\times \left(\frac{10}{5 \times 10^{-4} \times 10^{16}} + \frac{18}{10 \times 10^{-4} \times 10^{18}}\right)$$

$$= 7.3 \times 10^{-15} \text{ A}$$

Ex: 3.14 Equation (3.47),

$$C_{j0} = A\sqrt{\left(\frac{\epsilon_s q}{2}\right)\left(\frac{N_A N_D}{N_A + N_D}\right)\left(\frac{1}{V_0}\right)}$$

$$= 10^{-4}\sqrt{\left(\frac{1.04 \times 10^{-12} \times 1.6 \times 10^{-19}}{2}\right)}$$

$$\sqrt{\left(\frac{10^{18} \times 10^{16}}{10^{18} + 10^{16}}\right)\left(\frac{1}{0.814}\right)}$$

$$= 3.2 \text{ pF}$$

Equation (3.46),

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}}$$

$$= \frac{3.2 \times 10^{-12}}{\sqrt{1 + \frac{2}{0.814}}}$$

$$= 1.72 \text{ pF}$$

Ex: 3.15 $C_d = \frac{dQ}{dV} = \frac{d}{dV}(\tau_T I)$

$$= \frac{d}{dV}\tau_T \times I_S(e^{V/V_T} - 1)$$

$$= \tau_T I_S \frac{d}{dV}(e^{V/V_T} - 1)$$

$$= \tau_T I_S \frac{1}{V_T} e^{V/V_T}$$

$$= \frac{\tau_T}{V_T} \times I_S e^{V/V_T}$$

$$\cong \left(\frac{\tau_T}{V_T}\right) I$$

Ex: 3.16 Equation (3.49),

$$\tau_p = \frac{L_p^2}{D_p}$$

$$= \frac{(5 \times 10^{-4})^2}{10}$$

$$= 25 \text{ ns}$$

Equation (3.57),

$$C_d = \left(\frac{\tau_T}{V_T}\right) I$$

In Example 3.6, $N_A = 10^{18}/\text{cm}^3$,

$N_D = 10^{16}/\text{cm}^3$

Assuming $N_A \gg N_D$,

$\tau_T \simeq \tau_p = 25$ ns

$$\therefore C_d = \left(\frac{25 \times 10^{-9}}{25.9 \times 10^{-3}}\right) 0.1 \times 10^{-3}$$

$$= 96.5 \text{ pF}$$

Chapter 4

Solutions to Exercises within the Chapter

Ex: 4.1 Refer to Fig. 4.3(a). For $v_I \geq 0$, the diode conducts and presents a zero voltage drop. Thus $v_O = v_I$. For $v_I < 0$, the diode is cut off, zero current flows through R, and $v_O = 0$. The result is the transfer characteristic in Fig. E4.1.

Ex: 4.2 See Fig. 4.3a and 4.3b. During the positive half of the sinusoid, the diode is forward biased, so it conducts resulting in $v_D = 0$. During the negative half cycle of the input signal v_I, the diode is reverse biased. The diode does not conduct, resulting in no current flowing in the circuit. So $v_O = 0$ and $v_D = v_I - v_O = v_I$. This results in the waveform shown in Fig. E4.2.

Ex: 4.3 $\hat{i}_D = \dfrac{\hat{v}_I}{R} = \dfrac{10\text{ V}}{1\text{ k}\Omega} = 10\text{ mA}$

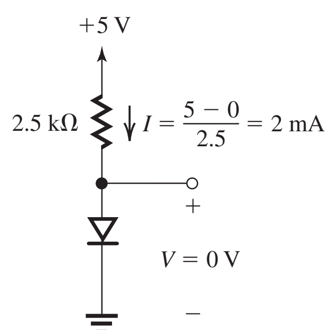
dc component of $v_O = \dfrac{1}{\pi}\hat{v}_O$

$= \dfrac{1}{\pi}\hat{v}_I = \dfrac{10}{\pi}$

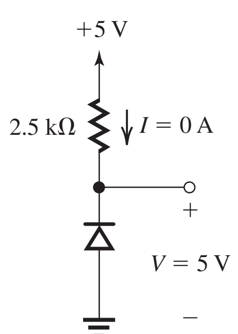
$= 3.18$ V

Ex: 4.4

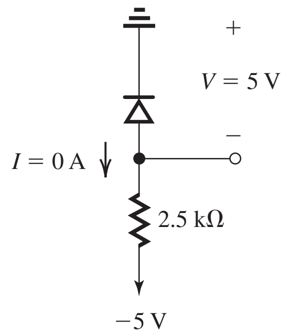
(a)



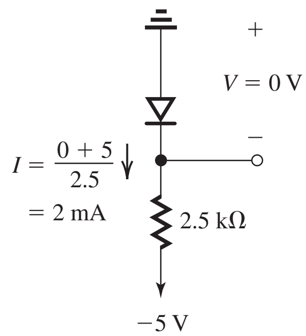
(b)



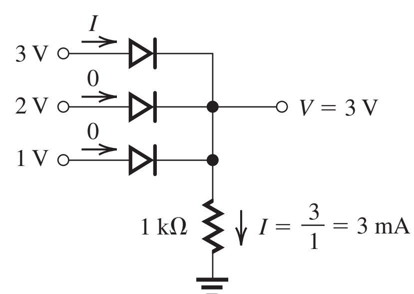
(c)



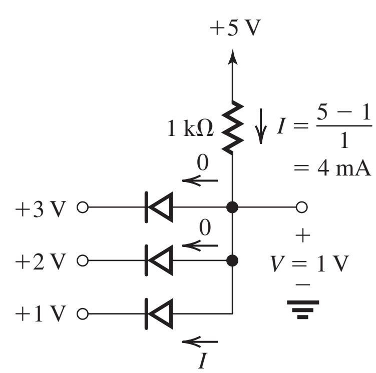
(d)



(e)



(f)



Ex: 4.5 $V_{\text{avg}} = \dfrac{10}{\pi}$

$$50 + R = \frac{\frac{10}{\pi}}{1\text{ mA}} = \frac{10}{\pi}\text{ k}\Omega$$

$\therefore\ R = 3.133\text{ k}\Omega$

Ex: 4.6 The maximum current arises when $|v_I| = 20$ V. In this case,

$$i_D = \frac{20-5}{R}$$

To ensure this is 50 mA,

$$R = \frac{20-5}{50} = 0.3\text{ k}\Omega = 300\Omega$$

Ex: 4.7 Equation (4.5)

$$V_2 - V_1 = 2.3\, V_T \log\left(\frac{I_2}{I_1}\right)$$

At room temperature $V_T = 25$ mV

$$V_2 - V_1 = 2.3 \times 25 \times 10^{-3} \times \log\left(\frac{10}{0.1}\right)$$

$= 115$ mV

Ex: 4.8 $i = I_S e^{v/V_T}$ (1)

$1\text{ (mA)} = I_S e^{0.7/V_T}$ (2)

Dividing (1) by (2), we obtain

$i\text{ (mA)} = e^{(v-0.7)/V_T}$

$\Rightarrow v = 0.7 + 0.025\ \ln(i)$

where i is in mA. Thus,

for $i = 0.1$ mA,

$v = 0.7 + 0.025\ \ln(0.1) = 0.64$ V

and for $i = 10$ mA,

$v = 0.7 + 0.025\ \ln(10) = 0.76$ V

Ex: 4.9 $i_D = I_S e^{v/V_T}$

$\Rightarrow I_S = i_D e^{-v/V_T} = 0.25 \times e^{-300/25}$

$= 1.23 \times 10^{6} mA = 1.23 \times 10^{-9}$A

Ex: 4.10 $\Delta T = 125 - 25 = 100°$C

$I_S = 10^{-14} \times 1.15^{\Delta T}$

$= 1.17 \times 10^{-8}$A

Ex: 4.11 At 20°C $I = \dfrac{1\text{ V}}{1\text{ M}\Omega} = 1\ \mu$A

Since the reverse leakage current doubles for every 10°C increase, at 40°C

$I = 4 \times 1\ \mu\text{A} = 4\ \mu$A

$\Rightarrow V = 4\ \mu\text{A} \times 1\text{ M}\Omega = 4.0$ V

@ 0°C $\quad I = \dfrac{1}{4}\ \mu$A

$\Rightarrow V = \dfrac{1}{4} \times 1 = 0.25$ V

Ex: 4.12 a. Use iteration:

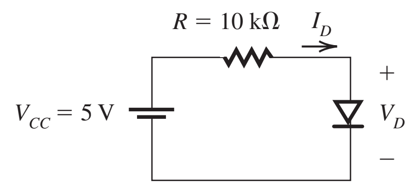
Diode has 0.7 V drop at 1 mA current.

Assume $V_D = 0.7$ V

$$I_D = \frac{5-0.7}{10\text{ k}\Omega} = 0.43\text{ mA}$$

Use Eq. (4.5) and note that

$V_1 = 0.7$ V, $\quad I_1 = 1$ mA



$$V_2 - V_1 = 2.3 \times V_T \log\left(\frac{I_2}{I_1}\right)$$

$$V_2 = V_1 + 2.3 \times V_T \log\left(\frac{I_2}{I_1}\right)$$

First iteration

$$V_2 = 0.7 + 2.3 \times 25 \times 10^{-3} \log\left(\frac{0.43}{1}\right)$$

$= 0.679$ V

Second iteration

$$I_2 = \frac{5-0.679}{10\text{ k}\Omega} = 0.432\text{ mA}$$

$$V_2 = 0.7 + 2.3 \times 25.3 \times 10^{-3} \log\left(\frac{0.432}{1}\right)$$

$= 0.679\text{ V} \simeq 0.68$ V

we get almost the same voltage.

∴ The iteration yields

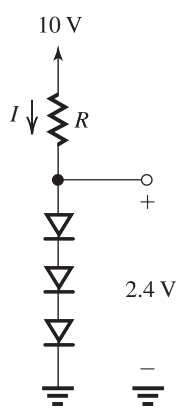
$I_D = 0.43$ mA, $V_D = 0.68$ V

b. Use constant voltage drop model:

$V_D = 0.7$ V constant voltage drop

$$I_D = \frac{5-0.7}{10\text{ k}\Omega} = 0.43\text{ mA}$$

Ex: 4.13



Diodes have 0.7 V drop at 1 mA

$\therefore 1\text{ mA} = I_S e^{0.7/V_T}$ (1)

At a current I (mA),

$I = I_S e^{V_D/V_T}$ (2)

Using (1) and (2), we obtain

$I = e^{(V_D - 0.7)/V_T}$

For an output voltage of 2.4 V, the voltage drop across each diode $= \frac{2.4}{3} = 0.8$ V

Now I, the current through each diode, is

$I = e^{(0.8-0.7)/0.025}$

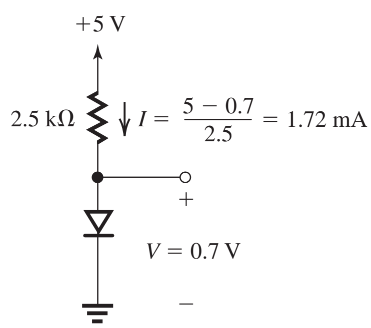
$= 54.6$ mA

$R = \frac{10 - 2.4}{54.6 \times 10^{-3}}$

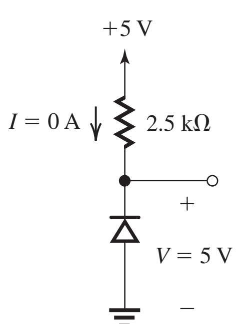
$= 139\ \Omega$

Ex: 4.14

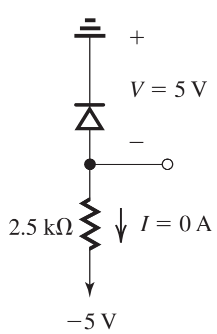
(a)



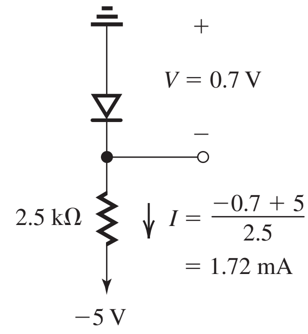
(b)



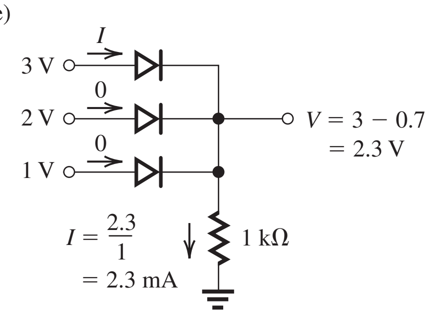
(c)



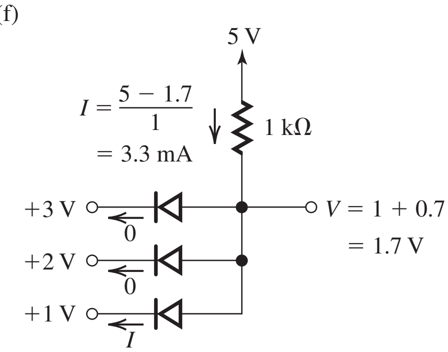
(d)



(e)



(f)



Ex: 4.15 With a reverse voltage, the diode in Fig. 4.10 will not conduct. Thus, the voltage drop on R will be zero, and the reverse voltage on the diode is $-V_D = -V_{DD}$. To ensure we respect the peak inverse voltage, we require $V_{DD} = V_D > -30$ V. Hence, the minimum voltage on V_{DD} is -30 V.

Ex: 4.16 When conducting a reverse current of 20 mA, the reverse voltage is

$V_Z = V_{ZT} + \Delta I_Z r_z =$
$3.5\text{ V} + (20\text{ mA} - 10\text{ mA})10\Omega$

$= 3.6$ V

The maximum current, I_{max}, is the current at 200 mW power dissipation.

$200\text{ mW} = I_{max}(3.5\text{ V} + 10\Omega(I_{max} - 10\text{ mA}))$

$0.2 = 3.5I_{max} + 10I_{max}^2 - 0.1I_{max}$

$\Rightarrow I_{max}^2 + 0.34I_{max} - 0.02 = 0$

$\Rightarrow I_{max} = \frac{1}{2}\left(-0.34 + \sqrt{0.34^2 + 4 \times 0.02}\right)$

$= 51$ mA

Ex: 4.17 $r_d = \frac{V_T}{I_D}$

$I_D = 0.1$ mA $\quad r_d = \frac{25 \times 10^{-3}}{0.1 \times 10^{-3}} = 250\ \Omega$

$I_D = 1$ mA $\quad r_d = \frac{25 \times 10^{-3}}{1 \times 10^{-3}} = 25\ \Omega$

$I_D = 10$ mA $\quad r_d = \frac{25 \times 10^{-3}}{10 \times 10^{-3}} = 2.5\ \Omega$

Ex: 4.18 For small signal model,

$\Delta i_D = \Delta v_D / r_d \qquad (1)$

where $r_d = \frac{V_T}{I_D}$

For exponential model,

$i_D = I_S e^{V/V_T}$

$\frac{i_{D2}}{i_{D1}} = e^{(V_2 - V_1)/V_T} = e^{\Delta v_D / V_T}$

$\Delta i_D = i_{D2} - i_{D1} = i_{D1} e^{\Delta v_D / V_T} - i_{D1}$

$= i_{D1}\left(e^{\Delta v_D / V_T} - 1\right) \qquad (2)$

In this problem, $i_{D1} = I_D = 1$ mA.

Using Eqs. (1) and (2) with $V_T = 25$ mV, we obtain

	Δv_D (mV)	Δi_D (mA) small signal	Δi_D (mA) exponential model
a	− 10	− 0.4	− 0.33
b	− 5	− 0.2	− 0.18
c	+ 5	+ 0.2	+ 0.22
d	+ 10	+ 0.4	+ 0.49

Ex: 4.19

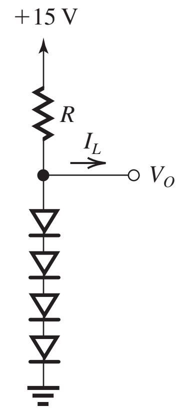
a. In this problem, $\frac{\Delta V_O}{\Delta i_L} = \frac{20 \text{ mV}}{1 \text{ mA}} = 20\ \Omega$.

$\therefore$ Total small-signal resistance of the four diodes $= 20\ \Omega$

$\therefore$ For each diode, $r_d = \frac{20}{4} = 5\ \Omega$.

But $r_d = \frac{V_T}{I_D} \Rightarrow 5 = \frac{25 \text{ mV}}{I_D}$.

$\therefore I_D = 5$ mA



and $R = \frac{15 - 3}{5 \text{ mA}} = 2.4\ \text{k}\Omega$.

b. For $V_O = 3$ V, voltage drop across each diode $= \frac{3}{4} = 0.75$ V

$i_D = I_S e^{V/V_T}$

$I_S = \frac{i_D}{e^{V/V_T}} = \frac{5 \times 10^{-3}}{e^{0.75/0.025}} = 4.7 \times 10^{-16}$ A

c. If $i_D = 5 - i_L = 5 - 1 = 4$ mA.

Across each diode the voltage drop is

$V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$

$= 25 \times 10^{-3} \times \ln\left(\frac{4 \times 10^{-3}}{4.7 \times 10^{-16}}\right)$

$= 0.7443$ V

Voltage drop across 4 diodes

$= 4 \times 0.7443 = 2.977$ V

so change in $V_O = 3 - 2.977 = 23$ mV.

Ex: 4.20 When the diode current is halved, the voltage changes by

$\Delta V_Z = r_z \Delta I_Z = 80\Omega \times \frac{-5 \text{ mA}}{2} = -200$ mV

$\Rightarrow V_Z = 6 - 0.2 = 5.8$ V

When the diode current is doubled,

$\Delta V_Z = r_z \Delta I_Z = 80\Omega \times 5 \text{ mA} = 400$ mV

$\Rightarrow V_Z = 6 + 0.4 = 6.4$ V

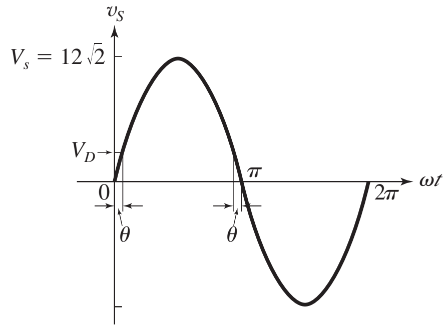
Finally, the value of V_{Z0} is that obtained by using the model at zero current.

$V_Z = V_{Z0} + r_z I_Z$

$\Rightarrow V_{Z0} = V_Z - r_z I_Z = 6 \text{ V} - 80\Omega \times 5 \text{ mA}$

$= 5.6$ V

Ex: 4.21



a. The diode starts conduction at

$v_S = V_D = 0.7$ V

$v_S = V_s \sin \omega t$, here $V_s = 12\sqrt{2}$

At $\omega t = \theta$,

$v_S = V_s \sin\theta = V_D = 0.7$ V

$12\sqrt{2}\sin\theta = 0.7$

$$\theta = \sin^{-1}\left(\frac{0.7}{12\sqrt{2}}\right) \simeq 2.4^\circ$$

Conduction starts at θ and stops at $180 - \theta$.

$\therefore$ Total conduction angle $= 180 - 2\theta = 175.2^\circ$

b. $$v_{O,\text{avg}} = \frac{1}{2\pi}\int_{\theta}^{(\pi-\theta)} (V_s \sin\phi - V_D)\, d\phi$$

$$= \frac{1}{2\pi}[-V_s \cos\phi - V_D\phi]_{\phi-\theta}^{\phi=\pi-\theta}$$

$$= \frac{1}{2\pi}[V_s\cos\theta - V_s\cos(\pi-\theta) - V_D(\pi - 2\theta)]$$

But $\cos\theta \simeq 1$, $\cos(\pi-\theta) \simeq -1$, and

$\pi - 2\theta \simeq \pi$

$$v_{O,\text{avg}} = \frac{2V_s}{2\pi} - \frac{V_D}{2}$$

$$= \frac{V_s}{\pi} - \frac{V_D}{2}$$

For $V_s = 12\sqrt{2}$ and $V_D = 0.7$ V

$$v_{O,\text{avg}} = \frac{12\sqrt{2}}{\pi} - \frac{0.7}{2} = 5.05 \text{ V}$$

c. The peak diode current occurs at the peak diode voltage.

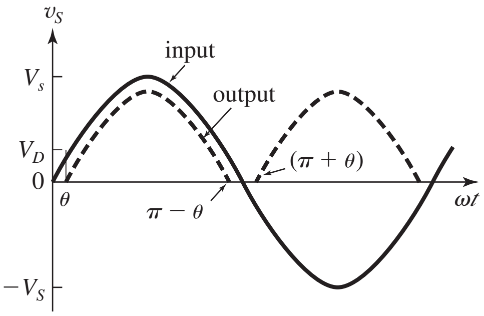
$$\therefore \hat{i}_D = \frac{V_s - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100}$$

$= 163$ mA

PIV $= +V_S = 12\sqrt{2}$

$\simeq 17$ V

Ex: 4.22



a. As shown in the diagram, the output is zero between $(\pi - \theta)$ to $(\pi + \theta)$

$= 2\theta$

Here θ is the angle at which the input signal reaches V_D.

$\therefore V_s \sin\theta = V_D$

$$\theta = \sin^{-1}\left(\frac{V_D}{V_s}\right)$$

$$2\theta = 2\sin^{-1}\left(\frac{V_D}{V_s}\right)$$

b. Average value of the output signal is given by

$$V_O = \frac{1}{2\pi}\left[2 \times \int_{\theta}^{(\pi-\theta)} (V_s \sin\phi - V_D)\, d\phi\right]$$

$$= \frac{1}{\pi}[-V_s\cos\phi - V_D\phi]_{\phi=\theta}^{\pi-\theta}$$

$$\simeq 2\frac{V_s}{\pi} - V_D, \qquad \text{for } \theta \text{ small.}$$

c. Peak current occurs when $\phi = \dfrac{\pi}{2}$.

Peak current

$$= \frac{V_s \sin(\pi/2) - V_D}{R} = \frac{V_s - V_D}{R}$$

If v_S is 12 V(rms),

then $V_s = \sqrt{2} \times 12 = 12\sqrt{2}$

$$\text{Peak current} = \frac{12\sqrt{2} - 0.7}{100} \simeq 163 \text{ mA}$$

Nonzero output occurs for angle $= 2(\pi - 2\theta)$

The fraction of the cycle for which $v_O > 0$ is

$$= \frac{2(\pi - 2\theta)}{2\pi} \times 100$$

$$= \frac{2\left[\pi - 2\sin^{-1}\left(\frac{0.7}{12\sqrt{2}}\right)\right]}{2\pi} \times 100$$

$\simeq 97.4\ \%$

Average output voltage V_O is

$$V_O = 2\frac{V_s}{\pi} - V_D = \frac{2 \times 12\sqrt{2}}{\pi} - 0.7 = 10.1 \text{ V}$$

Peak diode current $\hat{i}_D$ is

$$\hat{i}_D = \frac{V_s - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100}$$

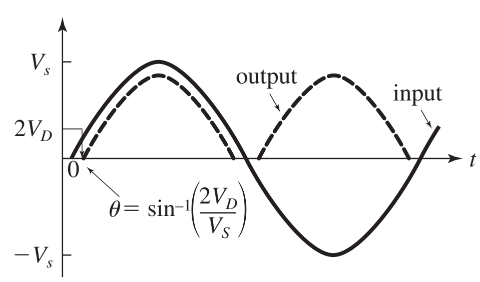
$= 163$ mA

$\text{PIV} = V_s - V_D + V_S$

$= 12\sqrt{2} - 0.7 + 12\sqrt{2}$

$= 33.2$ V

Ex: 4.23



(a) $V_{O,\text{avg}} = \dfrac{1}{2\pi}\displaystyle\int (V_s \sin\phi - 2V_D)\, d\phi$

$$= \frac{2}{2\pi}[-V_s\cos\phi - 2V_D\phi]_{\phi=\theta}^{\pi-\theta}$$

$$= \frac{1}{\pi}[V_s\cos\phi - V_s\cos(\pi-\theta) - 2V_D(\pi - 2\theta)]$$

But $\cos\theta \approx 1$,

$\cos(\pi - \theta) \approx -1$

$\pi - 2\theta \approx \pi$. Thus

$$\Rightarrow V_{O,\text{avg}} \simeq \frac{2V_s}{\pi} - 2V_D$$

$$= \frac{2 \times 12\sqrt{2}}{\pi} - 1.4 = 9.4 \text{ V}$$

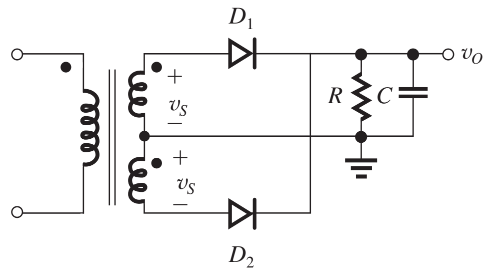
(b) Peak diode current $= \dfrac{\text{Peak voltage}}{R}$

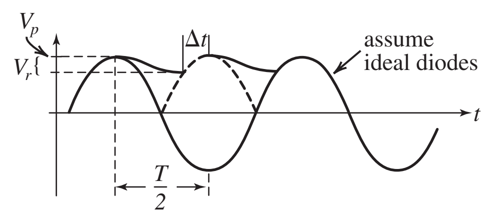
$$= \frac{V_s - 2V_D}{R} = \frac{12\sqrt{2} - 1.4}{100}$$

$= 156$ mA

$\text{PIV} = V_s - V_D = 12\sqrt{2} - 0.7 = 16.3$ V

Ex: 4.24 Full-wave peak rectifier:





The ripple voltage is the amount of voltage reduction during capacitor discharge that occurs when the diodes are not conducting. The output voltage is given by

$$v_O = V_p e^{-t/RC}$$

$V_p - V_r = V_p e^{-\frac{T/2}{RC}}$ $\leftarrow$ discharge is only half the period. We also assumed $\Delta t \ll \dfrac{T}{2}$.

$$V_r = V_p\left(1 - e^{-\frac{T/2}{RC}}\right)$$

$$e^{-\frac{T/2}{RC}} \simeq 1 - \frac{T/2}{RC}, \qquad \text{for } CR \gg \text{T}/2$$

Thus $V_r \simeq V_p\left(1 - 1 + \dfrac{T/2}{RC}\right)$

$$V_r = \frac{V_p}{2fRC} \qquad \text{(a)} \qquad \text{Q.E.D.}$$

To find the average diode current, note that the charge supplied to C during conduction is equal to the charge lost during discharge.

$Q_{\text{SUPPLIED}} = Q_{\text{LOST}}$

$i_{C\text{av}}\Delta t = CV_r$ SUB (a)

$$\left(i_{D,\text{av}} - I_L\right)\Delta t = C\frac{V_p}{2fRC} = \frac{V_p}{2fR}$$

$$= \frac{V_p\pi}{\omega R}$$

$$i_{D,\text{av}} = \frac{V_p\pi}{\omega\Delta t R} + I_L$$

where $\omega\Delta t$ is the conduction angle.

Note that the conduction angle has the same expression as for the half-wave rectifier and is given by Eq. (4.30),

$$\omega \Delta t \cong \sqrt{\frac{2V_r}{V_p}} \quad \text{(b)}$$

Substituting for $\omega \Delta t$, we get

$$\Rightarrow i_{D,\text{av}} = \frac{\pi V_p}{\sqrt{\frac{2V_r}{V_p}} \cdot R} + I_L$$

Since the output is approximately held at V_p, $\frac{V_p}{R} \approx I_L \cdot$ Thus

$$\Rightarrow i_{D,\text{av}} \cong \pi I_L \sqrt{\frac{V_p}{2V_r}} + I_L$$

$$= I_L \left[1 + \pi \sqrt{\frac{V_p}{2V_r}}\right] \qquad \text{Q.E.D.}$$

If $t = 0$ is at the peak, the maximum diode current occurs at the onset of conduction or at $t = -\omega \Delta t$.

During conduction, the diode current is given by

$$i_D = i_C + i_L$$

$$i_{D,\max} = C \left.\frac{dv_S}{dt}\right|_{t=-\omega\Delta t} + i_L$$

assuming i_L is const. $i_L \simeq \frac{V_p}{R} = I_L$

$$= C\frac{d}{dt}\left(V_p \cos \omega \text{t}\right) + I_L$$

$$= -C \sin \omega \text{ t} \times \omega V_p + I_L$$

$$= -C \sin(-\omega \Delta t) \times \omega V_p + I_L$$

For a small conduction angle

$\sin(-\omega \Delta t) \approx -\omega \Delta t$. Thus

$$\Rightarrow i_{D,\max} = C\omega \Delta t \times \omega V_p + I_L$$

Sub (b) to get

$$i_{D,\max} = C\sqrt{\frac{2V_r}{V_p}} \omega V_p + I_L$$

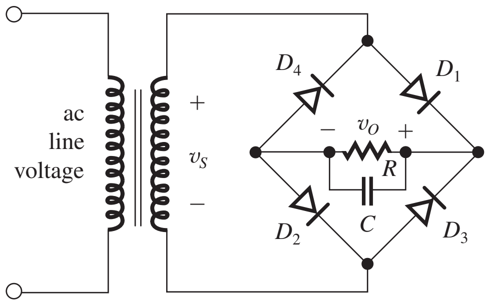
Substituting $\omega = 2\pi f$ and using (a) together with $V_p/R \simeq I_L$ results in

$$i_{D\max} = I_L \left[1 + 2\pi \sqrt{\frac{V_p}{2V_r}}\right] \qquad \text{Q.E.D.}$$

Ex: 4.25

The output voltage, v_O, can be expressed as

$$v_O = \left(V_p - 2V_D\right) e^{-t/RC}$$



At the end of the discharge interval

$$v_O = V_p - 2V_D - V_r$$

The discharge occurs almost over half of the time period $\simeq T/2$.

For time constant $RC \gg \frac{T}{2}$

$$e^{-t/RC} \simeq 1 - \frac{T}{2} \times \frac{1}{RC}$$

$$\therefore V_P - 2V_D - V_r = \left(V_p - 2V_D\right)\left(1 - \frac{T}{2} \times \frac{1}{RC}\right)$$

$$\Rightarrow V_r = \left(V_p - 2V_D\right) \times \frac{T}{2RC}$$

Here $V_p = 12\sqrt{2}$ and $V_r = 1$ V

$$V_D = 0.8 \text{ V}$$

$$T = \frac{1}{f} = \frac{1}{60} \text{ s}$$

$$1 = (12\sqrt{2} - 2 \times 0.8) \times \frac{1}{2 \times 60 \times 100 \times C}$$

$$C = \frac{(12\sqrt{2} - 1.6)}{2 \times 60 \times 100} = 1281 \ \mu\text{F}$$

Without considering the ripple voltage, the dc output voltage

$$= 12\sqrt{2} - 2 \times 0.8 = 15.4 \text{ V}$$

If ripple voltage is included, the output voltage is

$$= 12\sqrt{2} - 2 \times 0.8 - \frac{V_r}{2} = 14.9 \text{ V}$$

$$I_L = \frac{14.9}{100 \ \Omega} \simeq 0.15 \text{ A}$$

The conduction angle $\omega \Delta t$ can be obtained using Eq. (4.30) but substituting $V_p = 12\sqrt{2} - 2 \times 0.8$:

$$\omega \Delta t = \sqrt{\frac{2V_r}{V_p}} = \sqrt{\frac{2 \times 1}{12\sqrt{2} - 2 \times 0.8}}$$

$$= 0.36 \text{ rad} = 20.7^\circ$$

The average and peak diode currents can be calculated using Eqs. (4.34) and (4.35):

$$i_{Dav} = I_L\left(1 + \pi\sqrt{\frac{V_p}{2V_r}}\right), \quad \text{where } I_L = \frac{14.9\ \text{V}}{100\ \Omega},$$

$V_p = 12\sqrt{2} - 2 \times 0.8$, and $V_r = 1$ V; thus

$i_{Dav} = 1.45$ A

$$i_{D\max} = I\left(1 + 2\pi\sqrt{\frac{V_p}{2V_r}}\right)$$

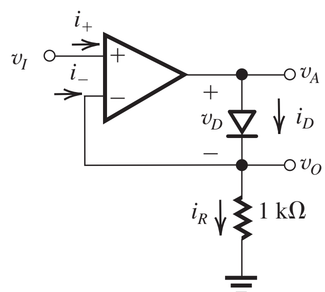
$= 2.76$ A

PIV of the diodes

$= V_S - V_{DO} = 12\sqrt{2} - 0.8 = 16.2$ V

To provide a safety margin, select a diode capable of a peak current of 3.5 to 4A and having a PIV rating of 20 V.

Ex: 4.26



The diode has 0.7 V drop at 1 mA current.

$i_D = I_S e^{v_D/V_T}$

$$\frac{i_D}{1\ \text{mA}} = e^{(v_D - 0.7)/V_T}$$

$$\Rightarrow v_D = V_T \ln\left(\frac{i_D}{1\ \text{mA}}\right) + 0.7\ \text{V}$$

For $v_I = 10$ mV, $\quad v_O = v_I = 10$ mV

It is an ideal op amp, so $i_+ = i_- = 0$.

$$\therefore i_D = i_R = \frac{10\ \text{mV}}{1\ \text{k}\Omega} = 10\ \mu\text{A}$$

$$v_D = 25 \times 10^{-3} \ln\left(\frac{10\ \mu\text{A}}{1\ \text{mA}}\right) + 0.7 = 0.58\ \text{V}$$

$v_A = v_D + 10$ mV

$= 0.58 + 0.01$

$= 0.59$ V

For $v_I = 1$ V

$v_O = v_I = 1$ V

$$i_D = \frac{v_O}{1\ \text{k}\Omega} = \frac{1}{1\ \text{k}\Omega} = 1\ \text{mA}$$

$v_D = 0.7$ V

$V_A = 0.7\ \text{V} + 1\ \text{k}\Omega \times 1\ \text{mA}$

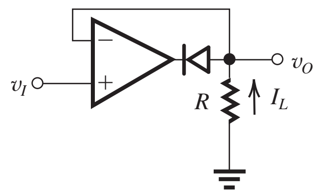
$= 1.7$ V

For $v_I = -1$ V, the diode is cut off.

$\therefore v_O = 0$ V

$v_A = -12$ V

Ex: 4.27



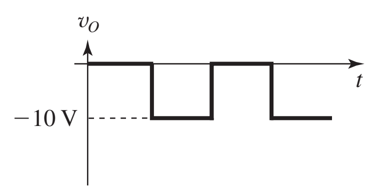
$v_I > 0 \sim$ diode is cut off, loop is open, and the opamp is saturated:

$v_O = 0$ V

$v_I < 0 \sim$ diode conducts and closes the negative feedback loop:

$v_O = v_I$

Ex: 4.28 Reversing the diode results in the peak output voltage being clamped at 0 V:



Here the dc component of $v_O = V_O = -5$ V

Ex: 4.29 The capacitor voltage accounts for the shift of the voltage waveforms from v_I to v_O. Thus, from Fig. 4.30, we see the capacitor voltage is $V_p + V_{CC} = 8$ V. The diode's peak inverse voltage arises at the peaks of v_O,

$\text{PIV} = v_{O\max} - V_{CC}$

$= V_{CC} + 2V_p - V_{CC}$

$= 2V_p = 6$ V

Ex: 4.30 $C_{j0} = 100$ fF, $V_0 = 3$ V, $m = 3$. Using Equation (3.47),

$$\text{at } V_R = 1\ \text{V: } C_j = \frac{100\ \text{fF}}{\left(1 + \frac{1}{3}\right)^3} = 42.2\ \text{fF, and}$$

$$\text{at } V_R = 3\ \text{V: } C_j = \frac{100\ \text{fF}}{\left(1 + \frac{3}{3}\right)^3} = 12.5\ \text{fF.}$$

Ex: 4.31 The reverse current is

$-i_D = I_D + i_P = I_D + R \times P$

At an incident light power of $P = 1$ mW,

$-i_D = 10^{-4} + 0.5 \times 1 = 0.5$ mA

At an incident light power of $P = 1$ μW,

$-i_D = 10^{-4} + 0.5 \times 10^{-3} = 6 \times 10^{-4}$ mA $=$ 0.6 μA

Ex: 4.32 Neglecting dark current,

$i_P = R \times P$

t

$10^{-6} = 0.3 \times 0.01 \times A$

$\Rightarrow A = \dfrac{10^{-6}}{0.3 \times 10^{-2}} = 3.33 \times 10^{-4}\ \text{m}^2 =$ 3.33 cm^2

The capacitance is 10 pF per mm^2 or, equivalently, 1 nF per cm^2. Thus,

$C_j = 1 \times 3.33 = 3.33$ nF

Ex: 4.33

$$R = \frac{9 - 3 \times 1.8}{20} = 0.18\ \text{k}\Omega = 180\Omega$$

Ex: 4.34

$$R = \frac{9 - 3 \times 2.2}{20} = 0.12\ \text{k}\Omega = 120\Omega$$

Chapter 5

Solutions to Exercises within the Chapter

Ex: 5.1

$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{34.5 \text{ pF/m}}{4 \text{ nm}} = 8.625 \text{ fF/}\mu\text{m}^2$

$\mu_n = 450 \text{ cm}^2/\text{V} \cdot \text{S}$

$k'_n = \mu_n C_{ox} = 388 \ \mu\text{A/V}^2$

$v_{OV} = v_{GS} - V_t = 0.5 \text{ V}$

$g_{DS} = \frac{1}{1 \text{ k}\Omega} = k'_n \frac{W}{L} v_{OV} \Rightarrow \frac{W}{L} = 5.15$

$L = 0.18 \ \mu\text{m}$, so $W = 0.93 \ \mu\text{m}$

Ex: 5.2

$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{34.5 \text{ pF/m}}{1.4 \text{ nm}} = 24.6 \text{ fF/}\mu\text{m}^2$

$\mu_n = 216 \text{ cm}^2/\text{V} \cdot \text{s}$

$k'_n = \mu_n C_{ox} = 531 \ \mu\text{A/V}^2$

$i_D = \frac{1}{2} k'_n \frac{W}{L} v_{OV}^2$

$50 = \frac{1}{2} \times 531 \times 10 \times v_{OV}^2$

$\therefore \ v_{OV} = 0.14 \text{ V}$

$v_{GS} = V_t + v_{OV} = 0.49 \text{ V}$

$v_{DS,\ \text{min}} = v_{OV} = 0.14 \text{ V},$

Ex: 5.3 $i_D = \frac{1}{2} k'_n \frac{W}{L} v_{OV}^2$ in saturation

Change in i_D is:

(a) double L, 0.5

(b) double W, 2

(c) double v_{OV}, $2^2 = 4$

(d) double v_{DS}, no change (ignoring length modulation)

(e) changes (a)–(d), 4

Case (c) would cause leaving saturation if

$v_{DS} < 2v_{OV}$

Ex: 5.4 For saturation $v_{DS} \geq v_{OV}$, so v_{DS} must be changed to $2v_{OV}$

$i_D = \frac{1}{2} k'_n \frac{W}{L} v_{OV}^2$, so i_D increases by a factor of 4.

Ex: 5.5 $v_{OV} = 0.5 \text{ V}$

$g_{DS} = k'_n \frac{W}{L} v_{OV} = \frac{1}{1 \text{ k}\Omega}$

$\therefore k_n = k'_n \frac{W}{L} = \frac{1}{1 \times 0.5} = 2 \text{ mA/V}^2$

At $v_{DS} = 0.2$ V, $v_{DS} < v_{OV}$, thus the transistor is operating in the triode region,

$$i_D = k_n (v_{OV} \ v_{DS} - \frac{1}{2} v_{DS}^2)$$
$$= 2(0.5 \times 0.2 - \frac{1}{2} \times 0.2^2) = 0.16 \text{ mA}$$

At $v_{DS} = 0.5$ V, $v_{DS} = v_{OV}$, thus the transistor is operating in saturation,

$i_D = \frac{1}{2} k_n v_{OV}^2 = \frac{1}{2} \times 2 \times 0.5^2 = 0.25 \text{ mA}$

At $v_{DS} = 1$ V, $v_{DS} > v_{OV}$ and the transistor is operating in saturation with $i_D = 0.25$ mA.

Ex: 5.6 $V_A = V'_A L = 5 \times 0.8 = 4 \text{ V}$

$\lambda = \frac{1}{V_A} = 0.25 \text{ V}^{-1}$

$v_{DS} = 0.8 \text{ V} > v_{OV} = 0.2 \text{ V}$

$\Rightarrow$ Saturation: $i_D = \frac{1}{2} k'_n \frac{W}{L} v_{OV}^2 \ (1 + \lambda v_{DS})$

$i_D = \frac{1}{2} \times 400 \times \frac{16}{0.8} \times 0.2^2 \ (1 + 0.25 \times 0.8)$

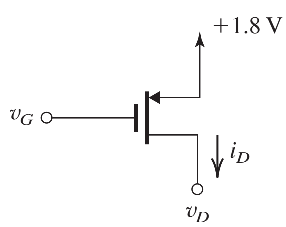
$= 0.192 \text{ mA}$

$r_o = \frac{V_A}{i_D} = \frac{4}{0.16} = 25 \text{ k}\Omega$

where i_D is the value of i_D without channel-length modulation taken into account.

$r_o = \frac{\Delta v_{DS}}{\Delta i_D} \Rightarrow \Delta i_D = \frac{1 \text{ V}}{25 \text{ k}\Omega} = 0.04 \text{ mA} =$ 40 μA

Ex: 5.7



$V_{tp} = -0.5 \text{ V}$

$k'_p = 100 \ \mu\text{A/V}^2$

$\frac{W}{L} = 10 \Rightarrow k_p = 1 \text{ mA/V}^2$

(a) Conduction occurs for $V_{SG} \geq |V_{tp}| = 0.5 \text{ V}$

$\Rightarrow v_G \leq 1.8 - 0.5 = +1.3 \text{ V}$

(b) Triode region occurs for $v_{DG} \geq |V_{tp}| = 0.5 \text{ V}$

$\Rightarrow v_D \geq v_G + 0.5$

(c) Conversely, for saturation

$v_{DG} \leq |V_{tp}| = 0.5$ V

$\Rightarrow v_D \leq v_G + 0.5$

(d) Given $\lambda \cong 0$,

$$i_D = \frac{1}{2} k'_p \frac{W}{L} |v_{OV}|^2 = 50\ \mu\text{A}$$

$\therefore |v_{OV}| = 0.32\text{ V} = v_{SG} - |V_{tp}|$

$\Rightarrow v_{SG} = |v_{OV}| + |V_{tp}| = 0.82$ V

$v_G = 1.8 - v_{SG} = 0.98$ V

$v_D \leq v_G + 0.5 = 1.48$ V

(e) For $\lambda = -0.2\text{ V}^{-1}$ and $|v_{OV}| = 0.32$ V,

$$i_D = 50\ \mu\text{A and } r_o = \frac{1}{|\lambda| i_D} = 100\ \text{k}\Omega$$

(f) At $v_D = +1$ V, $v_{SD} = 0.8$ V,

$$i_D = \frac{1}{2} k'_n \frac{W}{L} |v_{OV}|^2 (1 + |\lambda||v_{SD}|)$$

$= 50\ \mu\text{A}\ (1.16) = 58\ \mu\text{A}$

At $v_D = 0$ V, $v_{SD} = 1.8$ V,

$i_D = 50\ \mu\text{A}\ (1.36) = 68\ \mu\text{A}$

$$r_o = \frac{\Delta v_{DS}}{\Delta i_D} = \frac{1\text{ V}}{10\ \mu\text{A}} = 100\ \text{k}\Omega$$

which is the same value found in (c).

Ex: 5.8

$$R_D = \frac{V_{DD} - v_D}{I_D} = \frac{1 - 0.2}{0.1} = 8\ \text{k}\Omega$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2 \Rightarrow$$

$$100 = \frac{1}{2} \times 400 \times \frac{5}{0.4} V_{OV}^2 \Rightarrow$$

$V_{OV} = 0.2\text{ V} \Rightarrow V_{GS} = V_{OV} + V_t = 0.2 + 0.4$

$= 0.6$ V

$$V_S = -0.6\text{ V} \Rightarrow R_S = \frac{V_S - V_{SS}}{I_D}$$

$$= \frac{-0.6 - (-1)}{0.1}$$

$R_S = 4\ \text{k}\Omega$

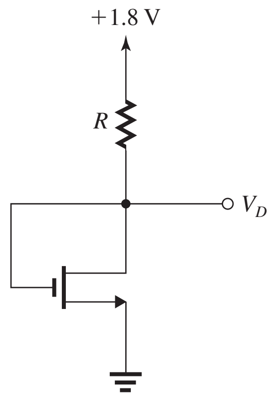
Ex: 5.9

$V_{tn} = 0.5$ V

$\mu_n C_{ox} = 0.4\text{ mA/V}^2$

$$\frac{W}{L} = \frac{0.72\ \mu\text{m}}{0.18\ \mu\text{m}} = 4.0$$

$\lambda = 0$



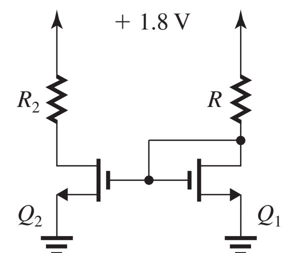
Saturation mode ($v_{GD} = 0 < V_{tn}$):

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_D - V_{tn})^2 = 0.032\ \text{mA}$$

$V_D = 0.7\text{ V} = 1.8 - I_D R$

$$\therefore R = \frac{1.8 - 0.7}{0.032\ \text{mA}} = 34.4\ \text{k}\Omega$$

Ex: 5.10



Since Q_2 is identical to Q_1 and their V_{GS} values are the same,

$I_{D2} = I_{D1} = 0.032$ mA

For Q_2 to operate at the triode–saturation boundary, we must have

$V_{D2} = V_{OV} = 0.7 - 0.5 = 0.2$ V

$$\therefore R_2 = \frac{1.8\text{ V} - 0.2\text{ V}}{0.032\ \text{mA}} = 50\ \text{k}\Omega$$

Ex: 5.11 $R_D = 6.55 \times 2 = 13.1\ \text{k}\Omega$

$V_{GS} = 2$ V, assume triode region:

$$\left.\begin{aligned} I_D &= k'_n \frac{W}{L} \left[(V_{GS} - V_{\text{tn}}) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \\ I_D &= \frac{V_{DD} - V_{DS}}{R} \end{aligned}\right\} \Rightarrow$$

$$\frac{2 - V_{DS}}{13.1} = 2 \times \left[(2 - 0.5) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$\Rightarrow V_{DS}^2 - 3.076 V_{DS} + 0.15 = 0$

$\Rightarrow V_{DS} = 0.05\text{ V} < V_{OV} \Rightarrow$ triode region

$$I_D = \frac{2 - 0.05}{13.1} = 0.15\text{ mA}$$

Ex: 5.12 As indicated in Example 5.6,

$V_D \geq V_G - V_t$ for the transistor to be in the saturation region.

$$V_{D\min} = V_G - V_t = 5 - 1 = 4\text{ V}$$

$$I_D = 0.5\text{ mA} \Rightarrow R_{D\max} = \frac{V_{DD} - V_{D\min}}{I_D}$$

$$= \frac{10 - 4}{0.5} = 12\text{ k}\Omega$$

Ex: 5.13

$$I_D = 0.32\text{ mA} = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2 = \frac{1}{2} \times 1 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.8\text{ V}$$

$$V_{GS} = 0.8 + 1 = 1.8\text{ V}$$

$$V_G = V_S + V_{GS} = 1.6 + 1.8 = 3.4\text{ V}$$

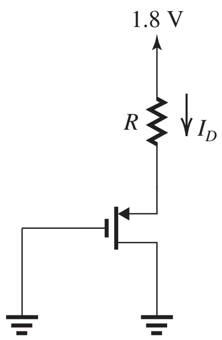
$$R_{G2} = \frac{V_G}{I} = \frac{3.4}{1\ \mu\text{A}} = 3.4\text{ M}\Omega$$

$$R_{G1} = \frac{5 - 3.4}{1\ \mu\text{A}} = 1.6\text{ M}\Omega$$

$$R_S = \frac{V_S}{0.32} = 5\text{ k}\Omega$$

$$V_D = 3.4\text{ V, then } R_D = \frac{5 - 3.4}{0.32} = 5\text{ k}\Omega$$

Ex: 5.14



$$V_{tp} = -0.4\text{ V}$$

$$k_p' = 0.1\text{ mA/V}^2$$

$$\frac{W}{L} = \frac{10\ \mu\text{m}}{0.18\ \mu\text{m}} \Rightarrow k_p = 5.56\text{ mA/V}^2$$

$$V_{SG} = |V_{tp}| + |V_{OV}|$$

$$= 0.4 + 0.6 = 1\text{ V}$$

$$V_S = +1\text{ V}$$

Since $V_{DG} = 0$, the transistor is operating in saturation, and

$$I_D = \frac{1}{2}k_pV_{OV}^2 = 1\text{ mA}$$

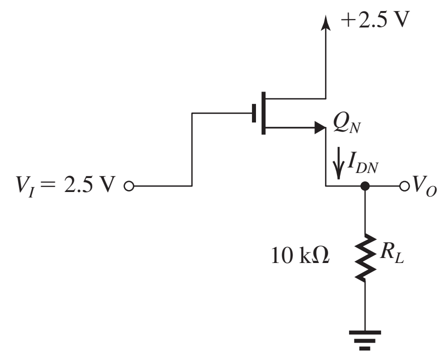
$$\therefore\ R = \frac{1.8 - V_S}{I_D} = \frac{1.8 - 1}{1} = 0.8\text{ k}\Omega$$

$$= 800\ \Omega$$

Ex: 5.15 $V_I = 0$: since the circuit is perfectly symmetrical, $V_O = 0$ and therefore $V_{GS} = 0$, which implies that the transistors are turned off and $I_{DN} = I_{DP} = 0$.

$V_I = 2.5$ V: if we assume that the NMOS is turned on, then V_O would be less than 2.5 V, and this implies that PMOS is off ($V_{SGP} < 0$).

$$I_{DN} = \frac{1}{2}k_n'\frac{W}{L}(V_{GS} - V_t)^2$$



$$I_{DN} = \frac{1}{2} \times 1(2.5 - V_O - 1)^2$$

$$I_{DN} = 0.5(1.5 - V_O)^2$$

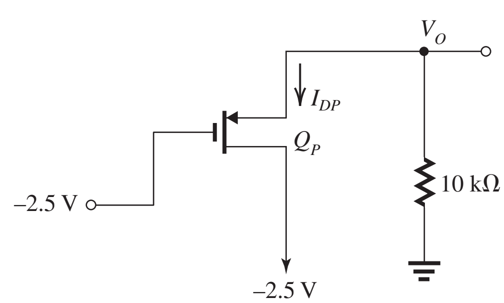
$$\text{Also: } V_O = R_L I_{DN} = 10I_{DN}$$

$$I_{DN} = 0.5(1.5 - 10I_{DN})^2$$

$$\Rightarrow 100I_{DN}^2 - 32I_{DN} + 2.25 = 0 \Rightarrow I_{DN}$$

$$= 0.104\text{ mA} \qquad I_{DP} = 0$$

$$V_O = 10 \times 0.104 = 1.04\text{V}$$



$V_I = -2.5$ V: Again if we assume that Q_P is turned on, then $V_O > -2.5$ V and $V_{GSN} < 0$, which implies that the NMOS Q_N is turned off.

$I_{DN} = 0$

Because of the symmetry,

$I_{DP} = 0.104$ mA,

$V_O = -I_{DP} \times 10$ kΩ

$= -1.04$ V

Ex: 5.16 $V_t = 0.8 + 0.4\left[\sqrt{0.7+3} - \sqrt{0.7}\right]$

$= 1.23$ V

Ex: 5.17 $v_{DS\min} = v_{GS} + |V_t|$

$= 1 + 2 = 3$ V

$I_D = \frac{1}{2} \times 2\ [1 - (-2)]^2$

$= 9$ mA

Chapter 6

Solutions to Exercises within the Chapter

Ex: 6.1 $i_C = I_S\, e^{v_{BE}/V_T}$

$v_{BE2} - v_{BE1} = V_T \ln\left[\frac{i_{C2}}{i_{C1}}\right]$

$v_{BE2} = 700 + 25 \ln\left[\frac{0.1}{1}\right]$

$= 642$ mV

$v_{BE3} = 700 + 25 \ln\left[\frac{10}{1}\right]$

$= 758$ mV

Ex: 6.2 $\therefore \alpha = \frac{\beta}{\beta + 1}$

$\frac{50}{50+1} < \alpha < \frac{150}{150+1}$

$0.980 < \alpha < 0.993$

Ex: 6.3 $I_C = I_E - I_B$

$= 1.460$ mA $-$ 0.01446 mA

$= 1.446$ mA

$\alpha = \frac{I_C}{I_E} = \frac{1.446}{1.460} = 0.99$

$\beta = \frac{I_C}{I_B} = \frac{1.446}{0.01446} = 100$

$I_C = I_S e^{v_{BE}/V_T}$

$I_S = \frac{I_C}{e^{v_{BE}/V_T}} = \frac{1.446}{e^{700/25}}$

$= \frac{1.446}{e^{28}}$ mA $= 10^{-15}$ A

Ex: 6.4 $\beta = \frac{\alpha}{1-\alpha}$ and $I_C = 10$ mA

For $\alpha = 0.99$, $\beta = \frac{0.99}{1-0.99} = 99$

$I_B = \frac{I_C}{\beta} = \frac{10}{99} = 0.1$ mA

For $\alpha = 0.98$, $\beta = \frac{0.98}{1-0.98} = 49$

$I_B = \frac{I_C}{\beta} = \frac{10}{49} = 0.2$ mA

Ex: 6.5 Given:

$I_S = 10^{-16}$ A, $\beta = 100$, $I_C = 1$ mA

We write

$I_{SE} = I_S/\alpha = I_S \times \left(1 + \frac{1}{\beta}\right)$

$= 10^{-16} \times 1.01 = 1.01 \times 10^{-16}$ A

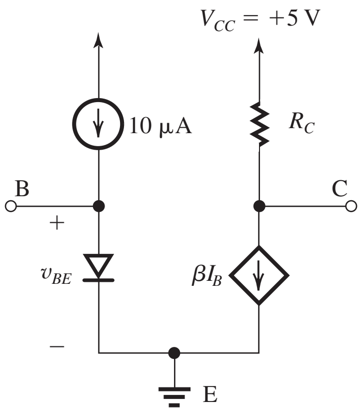
$I_{SB} = \frac{I_S}{\beta} = \frac{10^{-16}}{100} = 10^{-18}$ A

$V_{BE} = V_T \ln\left[\frac{I_C}{I_S}\right] = 25 \ln\left[\frac{1\text{ mA}}{10^{-16}\text{ A}}\right]$

$= 25 \times 29.9336$

$= 748$ mV

Ex: 6.6



$v_{BE} = 690$ mV

$I_C = 1$ mA

For active range $V_C \geq V_B$,

$R_{C\max} = \frac{V_{CC} - 0.690}{I_C}$

$= \frac{5 - 0.69}{1}$

$= 4.31$ kΩ

Ex: 6.7 $I_S = 10^{-15}$ A

$\text{Area}_C = 100 \times \text{Area}_E$

$I_{SC} = 100 \times I_S = 10^{-13}$ A

Ex: 6.8 $i_C = I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T}$

for $i_C = 0$

$I_S e^{v_{BE}/V_T} = I_{SC} e^{v_{BC}/V_T}$

$\frac{I_{SC}}{I_S} = \frac{e^{v_{BE}/V_T}}{e^{v_{BC}/V_T}}$

$= e^{(v_{BE} - v_{BC})/V_T}$

$\therefore V_{CE} = V_{BE} - V_{BC} = V_T \ln\left[\frac{I_{SC}}{I_S}\right]$

For collector Area = 100 × Emitter area

$V_{CE} = 25 \ln\left[\frac{100}{1}\right] = 115$ mV

Ex: 6.9 $I_C = I_S e^{V_{BE}/V_T} - I_{SC} e^{V_{BC}/V_T}$

$$I_B = \frac{I_S}{\beta} e^{V_{BE}/V_T} + I_{SC} e^{V_{BC}/V_T}$$

$$\beta_{\text{forced}} = \left.\frac{I_C}{I_B}\right|_{\text{sat}} < \beta$$

$$= \beta \frac{I_S e^{V_{BE}/V_T} - I_{SC} e^{V_{BC}/V_T}}{I_S e^{V_{BE}/V_T} + \beta I_{SC} e^{V_{BC}/V_T}}$$

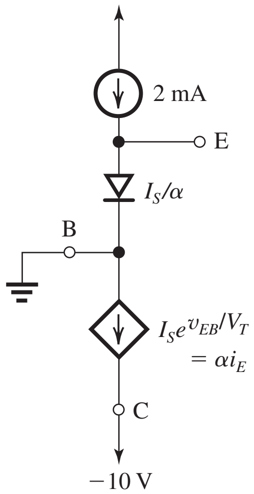
$$= \beta \frac{I_S e^{(V_{BE}-V_{BC})/V_T} - I_{SC}}{I_S e^{(V_{BE}-V_{BC})/V_T} + \beta I_{SC}}$$

$$= \beta \frac{e^{V_{CE\text{sat}}/V_T} - I_{SC}/I_S}{e^{V_{CE\text{sat}}/V_T} + \beta I_{SC}/I_S} \qquad \text{Q.E.D.}$$

$$\beta_{\text{forced}} = 100 \frac{e^{200/25} - 100}{e^{200/25} + 100 \times 100}$$

$$= 100 \times 0.2219 \approx 22.2$$

Ex: 6.10



$$I_E = \frac{I_S}{\alpha} e^{V_{BE}/V_T}$$

$$2 \text{ mA} = \frac{51}{50} 10^{-14} e^{V_{BE}/V_T}$$

$$V_{BE} = 25 \ln\left[\frac{2}{10^3} \times \frac{50}{51} \times 10^{14}\right]$$

$$= 650 \text{ mV}$$

$$I_C = \frac{\beta}{\beta+1} I_E = \frac{50}{51} \times 2$$

$$= 1.96 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{1.96}{50} \Rightarrow 39.2\ \mu\text{A}$$

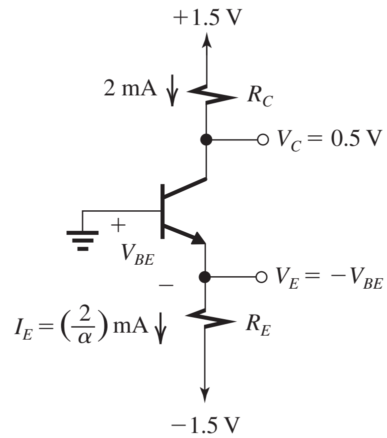
Ex: 6.11 $I_C = I_S e^{V_{BE}/V_T} = 1.5$ A

$$\therefore V_{BE} = V_T \ln\left[1.5/10^{-11}\right]$$

$$= 25 \times 25.734$$

$$= 643 \text{ mV}$$

Ex: 6.12



$$R_C = \frac{1.5 - V_C}{I_C} = \frac{1.5 - 0.5}{2}$$

$$= 0.5 \text{ k}\Omega = 500\ \Omega$$

Since at $I_C = 1$ mA, $V_{BE} = 0.8$ V, then at $I_C = 2$ mA,

$$V_{BE} = 0.8 + 0.025 \ln\left(\frac{2}{1}\right)$$

$$= 0.8 + 0.017$$

$$= 0.817 \text{ V}$$

$$V_E = -V_{BE} = -0.817 \text{ V}$$

$$I_E = \frac{2 \text{ mA}}{\alpha} = \frac{2}{0.99} = 2.02 \text{ mA}$$

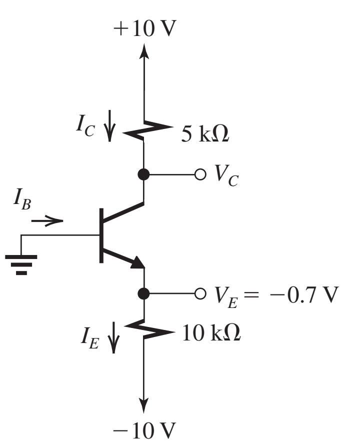
$$I_E = \frac{V_E - (-1.5)}{R_E}$$

Thus,

$$R_E = \frac{-0.817 + 1.5}{2.02} = 0.338 \text{ k}\Omega$$

$$= 338\ \Omega$$

Ex: 6.13



$$I_E = \frac{V_E - (-10)}{10} = \frac{-0.7 + 10}{10}$$

$= 0.93$ mA

Assuming active-mode operation,

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.93}{50 + 1} = 0.0182 \text{ mA}$$

$= 18.2\ \mu$A

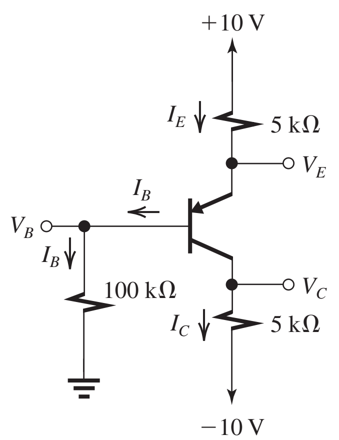
$I_C = I_E - I_B = 0.93 - 0.0182 = 0.91$ mA

$V_C = 10 - I_C \times 5$

$= 10 - 0.91 \times 5 = 5.45$ V

Since $V_C > V_B$, the transistor is operating in the active mode, as assumed.

Ex: 6.14



$V_B = 1.0$ V

Thus,

$$I_B = \frac{V_B}{100 \text{ k}\Omega} = 0.01 \text{ mA}$$

$V_E = +1.7$ V

Thus,

$$I_E = \frac{10 - V_E}{5 \text{ k}\Omega} = \frac{10 - 1.7}{5} = 1.66 \text{ mA}$$

and

$$\beta + 1 = \frac{I_E}{I_B} = \frac{1.66}{0.01} = 166$$

$\Rightarrow \beta = 165$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{165}{165 + 1} = 0.994$$

Assuming active-mode operation,

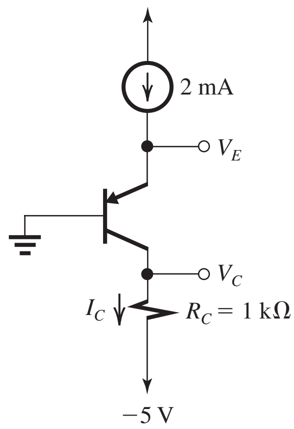
$I_C = \alpha I_E = 0.994 \times 1.66 = 1.65$ mA

and

$V_C = -10 + 1.65 \times 5 = -1.75$ V

Since $V_C < V_B$, the transistor is indeed operating in the active mode.

Ex: 6.15



The transistor is operating at a constant emitter current. Thus, a change in temperature of +30°C results in a change in V_{EB} by

$\triangle V_{EB} = -2 \text{ mV} \times 30 = -60$ mV

Thus,

$\triangle V_E = -60$ mV

Since the collector current remains unchanged at αI_E, the collector voltage does not change:

$\triangle V_C = 0$ V

Ex: 6.16 Refer to Fig. 6.19(a):

$$i_C = I_S e^{v_{BE}/V_T} + \frac{v_{CE}}{r_o} \qquad (1)$$

Now using Eqs. (6.21) and (6.22), we can express r_o as

$$r_o = \frac{V_A}{I_S e^{v_{BE}/V_T}}$$

Substituting in Eq. (1), we have

$$i_C = I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$$

which is Eq. (6.18). Q.E.D.

Ex: 6.17 $r_o = \dfrac{V_A}{I_C} = \dfrac{100}{I_C}$

At $I_C = 0.1$ mA, $r_o = 1$ MΩ

At $I_C = 1$ mA, $r_o = 100$ kΩ

At $I_C = 10$ mA, $r_o = 10$ kΩ

Ex: 6.18 $\triangle I_C = \dfrac{\triangle V_{CE}}{r_o}$

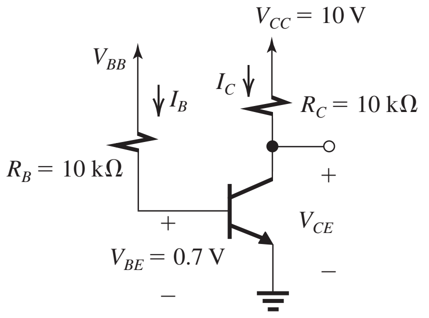
where

$r_o = \dfrac{V_A}{I_C} = \dfrac{100}{1} = 100\ \text{k}\Omega$

$\triangle I_C = \dfrac{11-1}{100} = 0.1$ mA

Thus, I_C becomes 1.1 mA.

Ex: 6.19



(a) For operation in the active mode with $V_{CE} = 5$ V,

$I_C = \dfrac{V_{CC} - V_C}{R_C} = \dfrac{10-5}{10} = 0.5$ mA

$I_B = \dfrac{I_C}{\beta} = \dfrac{0.5}{50} = 0.01$ mA

$V_{BB} = V_{BE} + I_B R_B$

$= 0.7 + 0.01 \times 10 = 0.8$ V

(b) For operation at the edge of saturation,

$V_{CE} = 0.3$ V

$I_C = \dfrac{V_{CC} - V_{CE}}{R_C} = \dfrac{10-0.3}{10} = 0.97$ mA

$I_B = \dfrac{I_C}{\beta} = \dfrac{0.97}{50} = 0.0194$ mA

$V_{BB} = V_B + I_B R_B$

$= 0.7 + 0.0194 \times 10 = 0.894$ V

(c) For operation deep in saturation with $\beta_{\text{forced}} = 10$, we have

$V_{CE} \simeq 0.2$ V

$I_C = \dfrac{10-0.2}{10} = 0.98$ mA

$I_B = \dfrac{I_C}{\beta_{\text{forced}}} = \dfrac{0.98}{10} = 0.098$ mA

$V_{BB} = V_B + I_B R_B$

$= 0.7 + 0.098 \times 10 = 1.68$ V

Ex: 6.20 For $V_{BB} = 0$ V, $I_B = 0$ and the transistor is cut off. Thus,

$I_C = 0$

and

$V_C = V_{CC} = +10$ V

Ex: 6.21 Refer to the circuit in Fig. 6.22 and let $V_{BB} = 1.7$ V. The current I_B can be found from

$I_B = \dfrac{V_{BB} - V_B}{R_B} = \dfrac{1.7-0.7}{10} = 0.1$ mA

Assuming operation in the active mode,

$I_C = \beta\ I_B = 50 \times 0.1 = 5$ mA

Thus,

$V_C = V_{CC} - R_C I_C$

$= 10 - 1 \times 5 = 5$ V

which is greater than V_B, verifying that the transistor is operating in the active mode, as assumed.

(a) To obtain operation at the edge of saturation, R_C must be increased to the value that results in $V_{CE} = 0.3$ V:

$R_C = \dfrac{V_{CC} - 0.3}{I_C}$

$= \dfrac{10-0.3}{5} = 1.94\ \text{k}\Omega$

(b) Further increasing R_C results in the transistor operating in saturation. To obtain saturation-mode operation with $V_{CE} = 0.2$ V and $\beta_{\text{forced}} = 10$, we use

$I_C = \beta_{\text{forced}} \times I_B$

$= 10 \times 0.1 = 1$ mA

The value of R_C required can be found from

$R_C = \dfrac{V_{CC} - V_{CE}}{I_C}$

$= \dfrac{10-0.2}{1} = 9.8\ \text{k}\Omega$

Ex: 6.22 Refer to the circuit in Fig. 6.23(a) with the base voltage raised from 4 V to V_B. If at this value of V_B, the transistor is at the edge of saturation then,

$V_C = V_B - 0.4$ V

Since $I_C \simeq I_E$, we can write

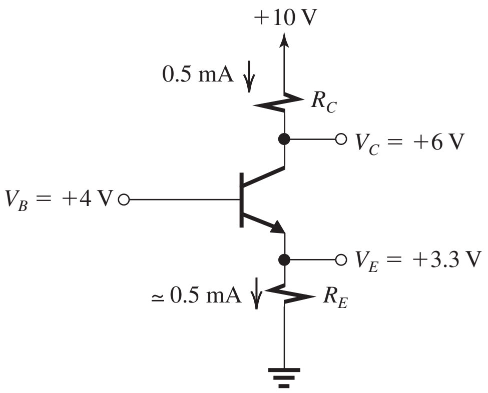
$\dfrac{10 - V_C}{R_C} = \dfrac{V_E}{R_E} = \dfrac{V_B - 0.7}{R_E}$

Thus,

$$\frac{10 - (V_B - 0.4)}{4.7} = \frac{V_B - 0.7}{3.3}$$

$$\Rightarrow V_B = +4.7 \text{ V}$$

Ex: 6.23



To establish a reverse-bias voltage of 2 V across the CBJ,

$$V_C = +6 \text{ V}$$

From the figure we see that

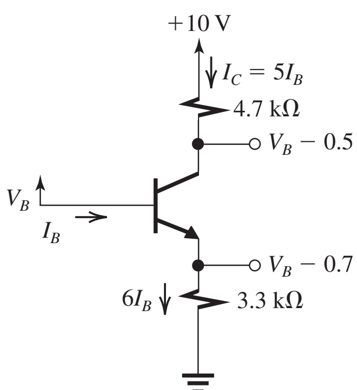
$$R_C = \frac{10 - 6}{0.5} = 8 \text{ k}\Omega$$

and

$$R_E = \frac{3.3}{0.5} = 6.6 \text{ k}\Omega$$

where we have assumed $\alpha \simeq 1$.

Ex: 6.24



The figure shows the circuit with the base voltage at V_B and the BJT operating in saturation with $V_{CE} = 0.2$ V and $\beta_{\text{forced}} = 5$.

$$I_C = 5I_B = \frac{10 - (V_B - 0.5)}{4.7} \quad (1)$$

$$I_E = 6I_B = \frac{V_B - 0.7}{3.3} \quad (2)$$

Dividing Eq. (1) by Eq. (2), we have

$$\frac{5}{6} = \frac{10.5 - V_B}{V_B - 0.7} \times \frac{3.3}{4.7}$$

$$\Rightarrow V_B = +5.18 \text{ V}$$

Ex: 6.25 Refer to the circuit in Fig. 6.26(a). The largest value for R_C while the BJT remains in the active mode corresponds to

$$V_C = +0.4 \text{ V}$$

Since the emitter and collector currents remain unchanged, then from Fig. 6.26(b) we obtain

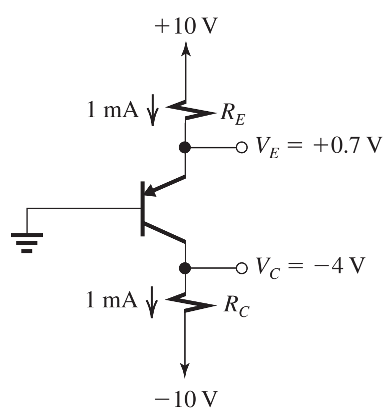
$$I_C = 4.65 \text{ mA}$$

Thus,

$$R_C = \frac{V_C - (-10)}{I_C}$$

$$= \frac{+0.4 + 10}{4.65} = 2.26 \text{ k}\Omega$$

Ex: 6.26



For a 4-V reverse-biased voltage across the CBJ,

$$V_C = -4 \text{ V}$$

Refer to the figure.

$$I_C = 1 \text{ mA} = \frac{V_C - (-10)}{R_C}$$

$$\Rightarrow R_C = \frac{-4 + 10}{1} = 6 \text{ k}\Omega$$

$$R_E = \frac{10 - V_E}{I_E}$$

Assuming $\alpha = 1$,

$$R_E = \frac{10 - 0.7}{1} = 9.3 \text{ k}\Omega$$

Ex: 6.27 Refer to the circuit in Fig. 6.27:

$$I_B = \frac{5 - 0.7}{100} = 0.043 \text{ mA}$$

To ensure that the transistor remains in the active mode for β in the range 50 to 150, we need to select R_C so that for the highest collector current possible, the BJT reaches the edge of saturation, that is, $V_{CE} = 0.3$ V. Thus,

$$V_{CE} = 0.3 = 10 - R_C I_{C\max}$$

where

$$I_{C\max} = \beta_{\max} I_B$$

$$= 150 \times 0.043 = 6.45 \text{ mA}$$

Thus,

$$R_C = \frac{10 - 0.3}{6.45} = 1.5 \text{ k}\Omega$$

For the lowest β,

$$I_C = \beta_{\min} I_B$$

$$= 50 \times 0.043 = 2.15 \text{ mA}$$

and the corresponding V_{CE} is

$$V_{CE} = 10 - R_C I_C = 10 - 1.5 \times 2.15$$

$$= 6.775 \text{ V}$$

Thus, V_{CE} will range from 0.3 V to 6.8 V.

Ex: 6.28 Refer to the solution of Example 6.10.

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + R_{BB}/(\beta + 1)}$$

$$= \frac{5 - 0.7}{3 + (33.3/51)} = 1.177 \text{ mA}$$

$$I_C = \alpha I_E = 0.98 \times 1.177 = 1.15 \text{ mA}$$

Thus the current is reduced by

$$\triangle I_C = 1.28 - 1.15 = 0.13 \text{ mA}$$

which is a -10% change.

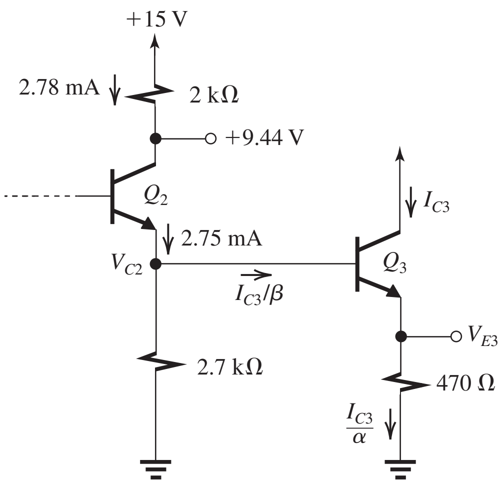
Ex: 6.29 Refer to the circuit in Fig. 6.30(b). The total current drawn from the power supply is

$$I = 0.103 + 1.252 + 2.78 = 4.135 \text{ mA}$$

Thus, the power dissipated in the circuit is

$$P = 15 \text{ V} \times 4.135 \text{ mA} = 62 \text{ mW}$$

Ex: 6.30



From the figure we see that

$$V_{E3} = \frac{I_{C3}}{\alpha} \times 0.47$$

$$V_{C2} = V_{E3} + 0.7 = \frac{I_{C3}}{\alpha} \times 0.47 + 0.7 \qquad (1)$$

A node equation at the collector of Q_2 yields

$$2.75 = \frac{V_{C2}}{2.7} + \frac{I_{C3}}{\beta}$$

Substituting for V_{C2} from Eq. (1), we obtain

$$2.75 = \frac{(0.47\, I_{C3}/\alpha) + 0.7}{2.7} + \frac{I_{C3}}{\beta}$$

Substituting $\alpha = 0.99$ and $\beta = 100$ and solving for I_{C3} results in

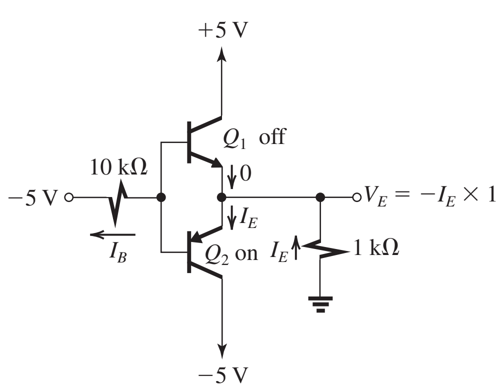
$$I_{C3} = 13.4 \text{ mA}$$

Now, V_{E3} and V_{C2} can be determined:

$$V_{E3} = \frac{I_{C3}}{\alpha} \times 0.47 = \frac{13.4}{0.99} \times 0.47 = +6.36 \text{ V}$$

$$V_{C2} = V_{E3} + 0.7 = +7.06 \text{ V}$$

Ex: 6.31



From the figure we see that Q_1 will be off and Q_2 will be on. Since the base of Q_2 will be at a voltage higher than −5 V, transistor Q_2 will be operating in the active mode. We can write a loop equation for the loop containing the 10-kΩ resistor, the EBJ of Q_2 and the 1-kΩ resistor:

$$-I_E \times 1 - 0.7 - I_B \times 10 = -5$$

Substituting $I_B = I_E/(\beta + 1) = I_E/101$ and rearranging gives

$$I_E = \frac{5 - 0.7}{\frac{10}{101} + 1} = 3.9 \text{ mA}$$

Thus,

$$V_E = -3.9 \text{ V}$$

$$V_{B2} = -4.6 \text{ V}$$

$$I_B = 0.039 \text{ mA}$$

Ex: 6.32 With the input at + 10 V, there is a strong possibility that the conducting transistor Q_1 will be saturated. Assuming this to be the case, the analysis steps will be as follows:

$$V_{CE\text{sat}}|_{Q_1} = 0.2 \text{ V}$$

$$V_E = 5 \text{ V} - V_{CE\text{sat}} = +4.8 \text{ V}$$

$$I_{E1} = \frac{4.8 \text{ V}}{1 \text{ k}\Omega} = 4.8 \text{ mA}$$

$$V_{B1} = V_E + V_{BE1} = 4.8 + 0.7 = +5.5 \text{ V}$$

$$I_{B1} = \frac{10 - 5.5}{10} = 0.45 \text{ mA}$$

$$I_{C1} = I_{E1} - I_{B1} = 4.8 - 0.45 = 4.35 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{4.35}{0.45} = 9.7$$

which is lower than $\beta_{\min}$, verifying that Q_1 is indeed saturated.

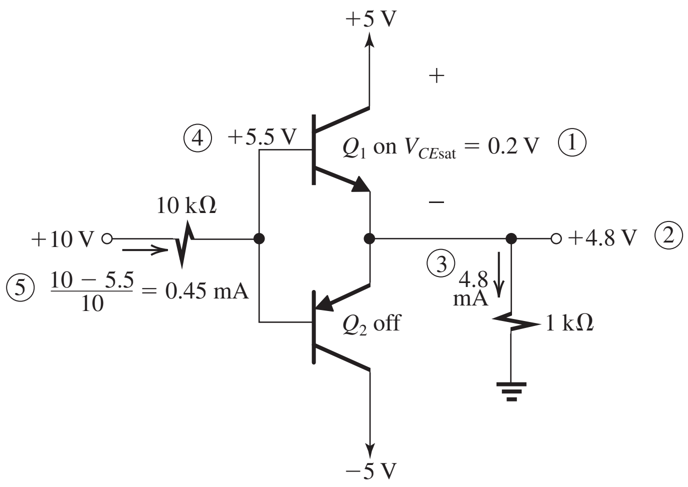
Finally, since Q_2 is off,

$$I_{C2} = 0$$

Ex: 6.33 $V_O = +10 - BV_{BCO} = 10 - 70$

$= -60$ V

This figure belongs to Exercise 6.32.



Chapter 7

Solutions to Exercises within the Chapter

Ex: 7.1 Refer to Fig. 7.2(a) and 7.2(b).

Coordinates of point A: V_t and V_{DD}; thus 0.4 V and 1.8 V. To determine the coordinates of point B, we use Eqs. (7.7) and (7.8) as follows:

$$V_{OV}\big|_{\rm B} = \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D}$$

$$= \frac{\sqrt{2 \times 4 \times 17.5 \times 1.8 + 1} - 1}{4 \times 17.5}$$

$$= 0.213 \text{ V}$$

Thus,

$$V_{GS}\big|_{\rm B} = V_t + V_{OV}\big|_{\rm B} = 0.4 + 0.213 = 0.613 \text{ V}$$

and

$$V_{DS}\big|_{\rm B} = V_{OV}\big|_{\rm B} = 0.213 \text{ V}$$

Thus, coordinates of B are 0.613 V and 0.213 V. At point C, the MOSFET is operating in the triode region, thus

$$i_D = k_n\left[(v_{GS}\big|_{\rm C} - V_t)v_{DS}\big|_{\rm C} - \frac{1}{2}v_{DS}^2\big|_{\rm C}\right]$$

If $v_{DS}\big|_{\rm C}$ is very small,

$$i_D \simeq k_n(v_{GS}\big|_{\rm C} - V_t)v_{DS}\big|_{\rm C}$$

$$= 4(1.8 - 0.4)v_{DS}\big|_{\rm C}$$

$$= 5.6v_{DS}\big|_{\rm C}, \text{ mA}$$

But

$$i_D = \frac{V_{DD} - v_{DS}\big|_{\rm C}}{R_D} \simeq \frac{V_{DD}}{R_D} = \frac{1.8}{17.5} = 0.1 \text{ mA}$$

Thus, $v_{DS}\big|_{\rm C} = \dfrac{0.1}{5.6} = 0.018 \text{ V} = 18 \text{ mV}$, which is indeed very small, as assumed.

Ex: 7.2 Refer to Example 7.1 and Fig. 7.4(a).

Design 1:

$$V_{OV} = 0.2 \text{ V}, \ V_{GS} = 0.6 \text{ V}$$

$$I_D = 0.08 \text{ mA}$$

Now,

$$A_v = -k_n V_{OV} R_D$$

Thus,

$$-10 = -0.4 \times 10 \times 0.2 \times R_D$$

$$\Rightarrow R_D = 12.5 \text{ k}\Omega$$

$$V_{DS} = V_{DD} - R_D I_D$$

$$= 1.8 - 12.5 \times 0.08 = 0.8 \text{ V}$$

Design 2:

$$R_D = 17.5 \text{ k}\Omega$$

$$A_v = -k_n V_{OV} R_D$$

$$-10 = -0.4 \times 10 \times V_{OV} \times 17.5$$

Thus,

$$V_{OV} = 0.14 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.4 + 0.14 = 0.54 \text{ V}$$

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2$$

$$= \frac{1}{2} \times 0.4 \times 10 \times 0.14^2 = 0.04 \text{ mA}$$

$$R_D = 17.5 \text{ k}\Omega$$

$$V_{DS} = V_{DD} - R_D I_D$$

$$= 1.8 - 17.5 \times 0.04 = 1.1 \text{ V}$$

Ex: 7.3

$$A_v = -\frac{I_C R_C}{V_T}$$

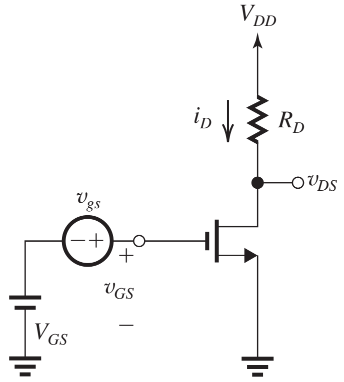
$$-320 = -\frac{1 \times R_C}{0.025} \Rightarrow R_C = 8 \text{ k}\Omega$$

$$V_C = V_{CC} - I_C R_C$$

$$= 10 - 1 \times 8 = 2 \text{ V}$$

Since the collector voltage is allowed to decrease to +0.3 V, the largest negative swing allowed at the output is 2 − 0.3 = 1.7 V. The corresponding input signal amplitude can be found by dividing 1.7 V by the gain magnitude (320 V/V), resulting in 5.3 mV.

Ex: 7.4



$$V_{DD} = 5 \text{ V}$$

$$V_{GS} = 2 \text{ V}$$

$$V_t = 1 \text{ V}$$

$$\lambda = 0$$

$$k_n' = 20 \text{ μA/V}^2$$

$$R_D = 10 \text{ k}\Omega$$

$\frac{W}{L} = 20$

(a) $V_{GS} = 2\text{ V} \Rightarrow V_{OV} = 1\text{ V}$

$I_D = \frac{1}{2}k_n' \frac{W}{L} V_{OV}^2 = 200\ \mu\text{A} = 0.2\text{ mA}$

$V_{DS} = V_{DD} - I_D R_D = +3\text{ V}$

(b) $g_m = k_n' \frac{W}{L} V_{OV} = 400\ \mu\text{A/V} = 0.4\text{ mA/V}$

(c) $A_v = \frac{v_{ds}}{v_{gs}} = -g_m R_D = -4\text{ V/V}$

(d) $v_{gs} = 0.2\sin\omega t\text{ V}$

$v_{ds} = -0.8\sin\omega t\text{ V}$

$v_{DS} = V_{DS} + v_{ds} \Rightarrow 2.2\text{ V} \le v_{DS} \le 3.8\text{ V}$

(e) Using Eq. (7.28), we obtain

$i_D = \frac{1}{2}k_n(V_{GS} - V_t)^2$

$+ k_n(V_{GS} - V_t)v_{gs} + \frac{1}{2}k_n v_{gs}^2$

$i_D = 200 + 80\sin\omega t$

$+ 8\sin^2\omega t,\ \mu\text{A}$

$= [200 + 80\sin\omega t + (4 - 4\ \cos 2\omega t)]$

$= 204 + 80\sin\omega t - 4\cos 2\omega t,\ \mu\text{A}$

Thus, I_D shifts by 4 μA and

$2\text{HD} = \frac{\hat{i}_{2\omega}}{\hat{i}_\omega} = \frac{4\ \mu\text{A}}{80\ \mu\text{A}} = 0.05\ (5\%)$

Ex: 7.5

(a) $V_{GS} = 1.5\text{V} \Rightarrow V_{OV} =$
$1.5 - 1 = 0.5\text{ V}$

$g_m = \frac{2I_D}{V_{OV}}$

$I_D = \frac{1}{2}k_n' \frac{W}{L} V_{OV}^2 = \frac{1}{2} \times 60 \times 40 \times 0.5^2$

$I_D = 300\ \mu\text{A} = 0.3\text{ mA}$

$g_m = \frac{2 \times 0.3}{0.5} = 1.2\text{ mA/V}$

$r_o = \frac{V_A}{I_D} = \frac{15}{0.3} = 50\text{ k}\Omega$

(b) $I_D = 0.5\text{ mA} \Rightarrow g_m = \sqrt{2\ \mu_n C_{ox} \frac{W}{L} I_D}$

$= \sqrt{2 \times 60 \times 40 \times 0.5 \times 10^3}$

$g_m = 1.55\text{ mA/V}$

$r_o = \frac{V_A}{I_D} = \frac{15}{0.5} = 30\text{ k}\Omega$

Ex: 7.6

$I_D = 0.1\text{ mA},\ g_m = 1\text{ mA/V},\ k_n' = 500\ \mu\text{A/V}^2$

$g_m = \frac{2I_D}{V_{OV}} \Rightarrow V_{OV} = \frac{2 \times 0.1}{1} = 0.2\text{ V}$

$I_D = \frac{1}{2}k_n' \frac{W}{L} V_{OV}^2 \Rightarrow \frac{W}{L} = \frac{2I_D}{k_n' V_{OV}^2}$

$= \frac{2 \times 0.1}{\frac{500}{1000} \times 0.2^2} = 10$

Ex: 7.7

$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV}$

Same bias conditions, so same V_{OV} and also same L and g_m for both PMOS and NMOS.

$\mu_n C_{ox} W_n = \mu_p C_{ox} W_p \Rightarrow \frac{\mu_p}{\mu_n} = 0.4 = \frac{W_n}{W_p}$

$\Rightarrow \frac{W_p}{W_n} = 2.5$

Ex: 7.8

$I_D = \frac{1}{2}k_p' \frac{W}{L}(V_{SG} - |V_t|)^2$

$= \frac{1}{2} \times 60 \times \frac{16}{0.8} \times\ (1.6 - 1)^2$

$I_D = 216\ \mu\text{A}$

$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 216}{1.6 - 1} = 720\ \mu\text{A/V}$

$= 0.72\text{ mA/V}$

$|\lambda| = 0.04 \Rightarrow |V_A'| = \frac{1}{|\lambda|} = \frac{1}{0.04} = 25\text{ V}/\mu\text{m}$

$r_o = \frac{|V_A'| \times L}{I_D} = \frac{25 \times 0.8}{0.216} = 92.6\text{ k}\Omega$

Ex: 7.9

$g_m r_o = \frac{2I_D}{V_{OV}} \times \frac{V_A}{I_D} = \frac{2V_A}{V_{OV}}$

$V_A = V_A' \times L = 6 \times 3 \times 0.18 = 3.24\text{ V}$

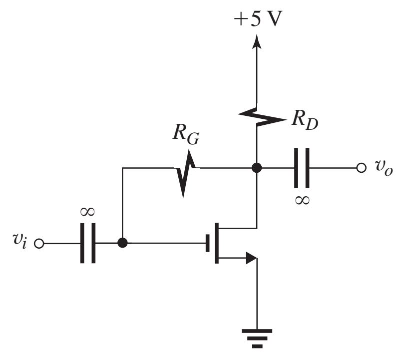
$g_m r_o = \frac{2 \times 3.24}{0.2} = 32.4\text{ V/V}$

Ex: 7.10

Refer to the solution of Example 7.3. From Eq. (7.47), $A_v \equiv \frac{v_o}{v_i} = -g_m R_D$ (note that R_L is absent).

Thus,

$g_m R_D = 25$



Substituting for $g_m = k_n V_{OV}$, we have

$k_n V_{OV} R_D = 25$

where $k_n = 1\ \text{mA/V}^2$, thus

$$V_{OV} R_D = 25 \qquad (1)$$

Next, consider the bias equation

$V_{GS} = V_{DS} = V_{DD} - R_D I_D$

Thus,

$V_t + V_{OV} = V_{DD} - R_D I_D$

Substituting $V_t = 0.7$ V, $V_{DD} = 5$ V, and

$$I_D = \frac{1}{2} k_n V_{OV}^2 = \frac{1}{2} \times 1 \times V_{OV}^2 = \frac{1}{2} V_{OV}^2$$

we obtain

$$0.7 + V_{OV} = 5 - \frac{1}{2} V_{OV}^2 R_D \qquad (2)$$

Equations (1) and (2) can be solved to obtain

$V_{OV} = 0.319$ V

and

$R_D = 78.5\ \text{k}\Omega$

The dc current I_D can be now found as

$$I_D = \frac{1}{2} k_n V_{OV}^2 = 50.9\ \mu\text{A}$$

To determine the required value of R_G we use Eq. (7.48), again noting that R_L is absent:

$$R_{\text{in}} = \frac{R_G}{1 + g_m R_D}$$

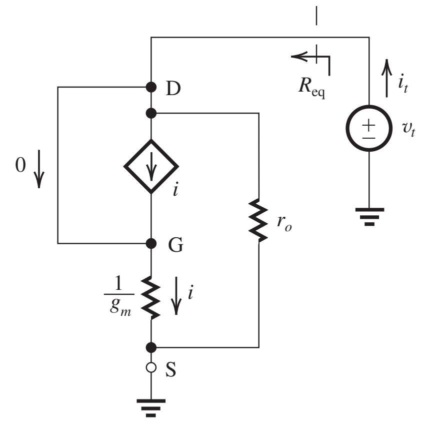
$$0.5\ \text{M}\Omega = \frac{R_G}{1 + 25}$$

$\Rightarrow R_G = 13\ \text{M}\Omega$

Finally, the maximum allowable input signal $\hat{v}_i$ can be found as follows:

$$\hat{v}_i = \frac{V_t}{|A_v| + 1} = \frac{0.7\ \text{V}}{25 + 1} = 27\ \text{mV}$$

Ex: 7.11



$$i_t = \frac{v_t}{r_o} + i = \frac{v_t}{r_o} + g_m v_t$$

$$\therefore R_{\text{eq}} = \frac{v_t}{i_t} = r_o \| \frac{1}{g_m}$$

Ex: 7.12

Given: $g_m = \left.\dfrac{\partial i_C}{\partial v_{BE}}\right|_{i_C = I_C}$

where $I_C = I_S e^{v_{BE}/V_T}$

$$\left.\frac{\partial i_C}{\partial v_{BE}}\right|_{i_C = I_C} = \frac{I_S e^{v_{BE}/V_T}}{V_T} = \frac{I_C}{V_T}$$

Thus,

$$g_m = \frac{I_C}{V_T}$$

Ex: 7.13

$$g_m = \frac{I_C}{V_T} = \frac{0.5\ \text{mA}}{25\ \text{mV}} = 20\ \text{mA/V}$$

Ex: 7.14

$I_C = 0.5$ mA (constant)

For $\beta = 50$:

$$g_m = \frac{I_C}{V_T} = \frac{0.5\ \text{mA}}{25\ \text{mV}} = 20\ \text{mA/V}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.5}{50} = 10\ \mu\text{A}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{50}{20} = 2.5\ \text{k}\Omega$$

For $\beta = 200$,

$$g_m = \frac{I_C}{V_T} = 20\ \text{mA/V}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.5\ \text{mA}}{200} = 2.5\ \mu\text{A}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{20} = 10\ \text{k}\Omega$$

Ex: 7.15

$\beta = 100 \quad I_C = 1\text{ mA}$

$g_m = \dfrac{1\text{ mA}}{25\text{ mV}} = 40\text{ mA/V}$

$r_e = \dfrac{V_T}{I_E} = \dfrac{\alpha V_T}{I_C} \simeq \dfrac{25\text{ mV}}{1\text{ mA}} = 25\ \Omega$

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{40} = 2.5\text{ k}\Omega$

Ex: 7.16

$g_m = \dfrac{I_C}{V_T} = \dfrac{1\text{ mA}}{25\text{ mV}} = 40\text{ mA/V}$

$A_v = \dfrac{v_{ce}}{v_{be}} = -g_m R_C$

$= -40 \times 10$

$= -400\text{ V/V}$

$V_C = V_{CC} - I_C R_C$

$= 15 - 1 \times 10 = 5\text{ V}$

$v_C(t) = V_C + v_c(t)$

$= V_C + A_v v_{be}(t)$

$= 5 - 400 \times 0.005 \sin\omega t$

$= 5 - 2 \sin\omega t$

$i_B(t) = I_B + i_b(t)$

where

$I_B = \dfrac{I_C}{\beta} = \dfrac{1\text{ mA}}{100} = 10\ \mu\text{A}$

and $i_b(t) = \dfrac{g_m v_{be}(t)}{\beta}$

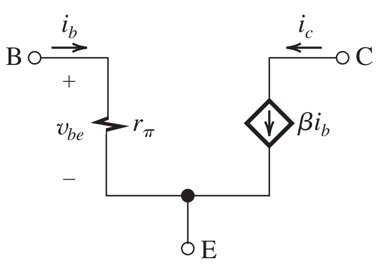
$= \dfrac{40 \times 0.005 \sin\omega t}{100}$

$= 2 \sin\omega t,\ \mu\text{A}$

Thus,

$i_B(t) = 10 + 2 \sin\omega t,\ \mu\text{A}$

Ex: 7.17



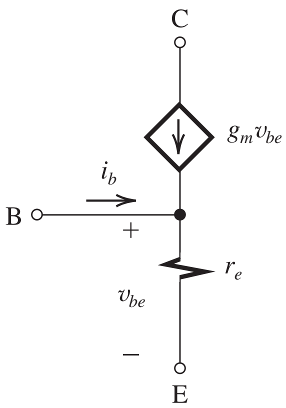
$i_c = \beta i_b = \beta \dfrac{v_{be}}{r_\pi}$

$= \left(\dfrac{\beta}{r_\pi}\right) v_{be} = g_m v_{be}$

$i_e = i_b + \beta i_b = (\beta + 1) i_b = (\beta + 1) \dfrac{v_{be}}{r_\pi}$

$= \dfrac{v_{be}}{r_\pi/(\beta + 1)} = \dfrac{v_{be}}{r_e}$

Ex: 7.18



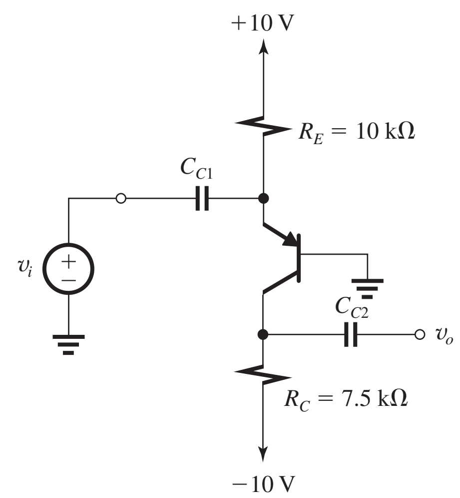
$i_b = \dfrac{v_{be}}{r_e} - g_m v_{be}$

$= v_{be}\left(\dfrac{1}{r_e} - g_m\right)$

$= v_{be}\left(\dfrac{1}{r_{\pi/\beta+1}} - \dfrac{\beta}{r_\pi}\right)$

$= v_{be}\left(\dfrac{\beta + 1}{r_\pi} - \dfrac{\beta}{r_\pi}\right) = \dfrac{v_{be}}{r_\pi}$

Ex: 7.19



$I_E = \dfrac{10 - 0.7}{10} = 0.93\text{ mA}$

$I_C = \alpha I_E = 0.99 \times 0.93$

$= 0.92\text{ mA}$

$V_C = -10 + I_C R_C$

$= -10 + 0.92 \times 7.5 = -3.1\text{ V}$

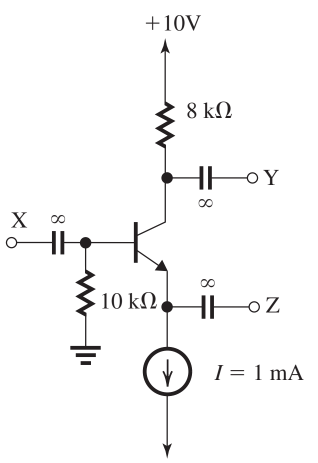
$A_v = \dfrac{v_o}{v_i} = \dfrac{\alpha R_C}{r_e}$

where $r_e = \dfrac{25\text{ mV}}{0.93\text{ mA}} = 26.9\ \Omega$

$A_v = \dfrac{0.99 \times 7.5 \times 10^3}{26.9} = 276.2\text{ V/V}$

For $\hat{v}_i = 10\text{ mV}$, $\hat{v}_o = 276.2 \times 10 = 2.76\text{ V}$

Ex: 7.20



$I_E = 1\text{ mA}$

$I_C = \dfrac{100}{101} \times 1 = 0.99\text{ mA}$

$I_B = \dfrac{1}{101} \times 1 = 0.0099\text{ mA}$

(a) $V_C = 10 - 8 \times 0.99 = 2.08 \simeq 2.1\text{ V}$

$V_B = -10 \times 0.0099 = -0.099 \simeq -0.1\text{ V}$

$V_E = -0.1 - 0.7 = -0.8\text{ V}$

(b) $g_m = \dfrac{I_C}{V_T} = \dfrac{0.99}{0.025} \simeq 40\text{ mA/V}$

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{40} \simeq 2.5\text{ k}\Omega$

$r_o = \dfrac{V_A}{I_C} = \dfrac{100}{0.99} = 101 \simeq 100\text{ k}\Omega$

(c) See figure below.
$R_{\text{sig}} = 2\text{ k}\Omega \quad R_B = 10\text{ k}\Omega \quad r_\pi = 2.5\text{ k}\Omega$

$g_m = 40\text{ mA/V}$

$R_C = 8\text{ k}\Omega \quad R_L = 8\text{ k}\Omega \quad r_o = 100\text{ k}\Omega$

$\dfrac{V_y}{V_{\text{sig}}} = \dfrac{V_\pi}{V_{\text{sig}}} \times \dfrac{V_y}{V_\pi}$

$= \dfrac{R_B \| r_\pi}{(R_B \| r_\pi) + R_{\text{sig}}} \times -g_m(R_C \| R_L \| r_o)$

$= \dfrac{10 \,\|\, 2.5}{(10 \,\|\, 2.5) + 2} \times -40(8 \,\|\, 8 \,\|\, 100)$

$= -0.5 \times 40 \times 3.846 = -77\text{ V/V}$

If r_o is negelected, $\dfrac{V_y}{V_{\text{sig}}} = -80$, for an error of 3.9%.

Ex: 7.21

$g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.25}{0.25} = 2\text{ mA/V}$

$R_{\text{in}} = \infty$

$A_{vo} = -g_m R_D = -2 \times 20 = -40\text{ V/V}$

$R_o = R_D = 20\text{ k}\Omega$

$A_v = A_{vo}\dfrac{R_L}{R_L + R_o} = -40 \times \dfrac{20}{20 + 20}$

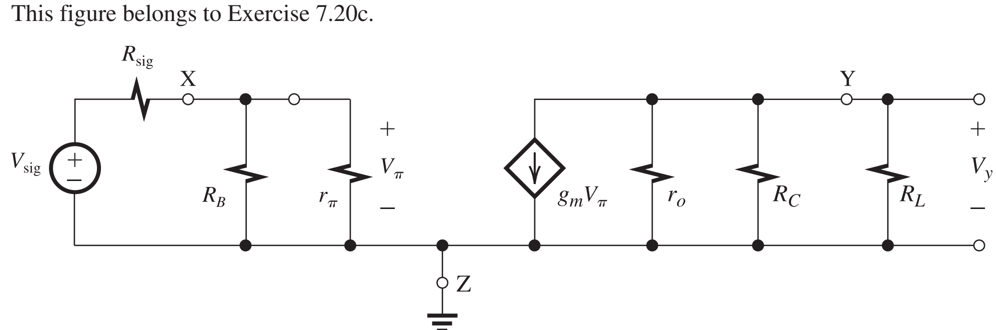
$= -20\text{ V/V}$

$G_v = A_v = -20\text{ V/V}$

$\hat{v}_i = 0.1 \times 2V_{OV} = 0.1 \times 2 \times 0.25 = 0.05\text{ V}$

$\hat{v}_o = 0.05 \times 20 = 1\text{ V}$

This figure belongs to Exercise 7.20c.



Ex: 7.22

$I_C = 0.5$ mA

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$R_{in} = r_\pi = 5 \text{ k}\Omega$

$A_{vo} = -g_m R_C = -20 \times 10 = -200$ V/V

$R_o = R_C = 10 \text{ k}\Omega$

$$A_v = A_{vo} \frac{R_L}{R_L + R_o} = -200 \times \frac{5}{5+10}$$

$= -66.7$ V/V

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_v = \frac{5}{5+5} \times -66.7$$

$= -33.3$ V/V

$\hat{v}_\pi = 5 \text{ mV} \Rightarrow \hat{v}_{sig} = 2 \times 5 = 10$ mV

$\hat{v}_o = 10 \times 33.3 = 0.33$ V

Although a larger fraction of the input signal reaches the amplifier input, linearity considerations cause the output signal to be in fact smaller than in the original design!

Ex: 7.23 Refer to the solution to Exercise 7.21. If $\hat{v}_{sig} = 0.2$ V and we wish to keep $\hat{v}_{gs} = 50$ mV, then we need to connect a resistance $R_s = \dfrac{3}{g_m}$ in the source lead. Thus,

$$R_s = \frac{3}{2 \text{ mA/V}} = 1.5 \text{ k}\Omega$$

$$G_v = A_v = -\frac{R_D \| R_L}{\dfrac{1}{g_m} + R_s}$$

$$= -\frac{20\|20}{0.5 + 1.5} = -5 \text{ V/V}$$

$\hat{v}_o = G_v \hat{v}_{sig} = 5 \times 0.2 = 1$ V (unchanged)

Ex: 7.24

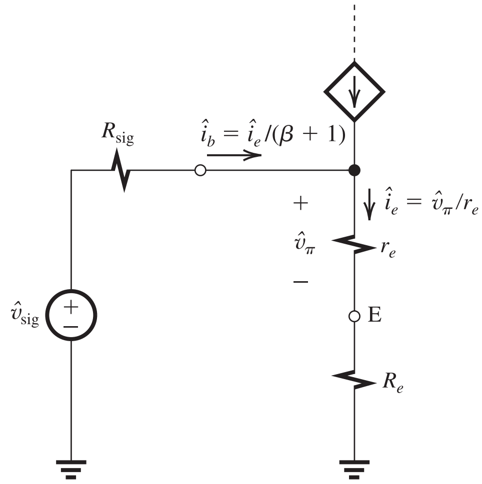
From the following figure we see that

$$\hat{v}_{sig} = \hat{i}_b R_{sig} + \hat{v}_\pi + \hat{i}_e R_e$$

$$= \frac{\hat{i}_e}{\beta + 1} R_{sig} + \hat{v}_\pi + \hat{i}_e R_e$$

$$= \frac{\hat{v}_\pi}{(\beta+1) r_e} R_{sig} + \hat{v}_\pi + \frac{\hat{v}_\pi}{r_e} R_e$$

$$\hat{v}_{sig} = \hat{v}_\pi \left(1 + \frac{R_e}{r_e} + \frac{R_{sig}}{r_\pi}\right) \qquad \text{Q.E.D}$$



For $I_C = 0.5$ mA and $\beta = 100$,

$$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} = \frac{0.99 \times 25}{0.5} \simeq 50\ \Omega$$

$r_\pi = (\beta + 1) r_e \simeq 5 \text{ k}\Omega$

For $\hat{v}_{sig} = 100$ mV, $R_{sig} = 10 \text{ k}\Omega$ and with $\hat{v}_\pi$ limited to 10 mV, the value of R_e required can be found from

$$100 = 10\left(1 + \frac{R_e}{50} + \frac{10}{5}\right)$$

$\Rightarrow R_e = 350\ \Omega$

$R_{in} = (\beta + 1)(r_e + R_e) = 101 \times (50 + 350)$

$= 40.4 \text{ k}\Omega$

$$G_v = -\beta \frac{R_C \| R_L}{R_{sig} + (\beta+1)(r_e + R_e)}$$

$$= -100 \frac{10}{10 + 101 \times 0.4} = -19.8 \text{ V/V}$$

Ex: 7.25

$$\frac{1}{g_m} = R_{sig} = 100\ \Omega$$

$$\Rightarrow g_m = \frac{1}{0.1 \text{ k}\Omega} = 10 \text{ mA/V}$$

But

$$g_m = \frac{2I_D}{V_{OV}}$$

Thus,

$$10 = \frac{2I_D}{0.2}$$

$\Rightarrow I_D = 1$ mA

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times g_m R_D$$

$= 0.5 \times 10 \times 2 = 10$ V/V

Ex: 7.26

$I_C = 1$ mA

$$r_e = \frac{V_T}{I_E} \simeq \frac{V_T}{I_C} = \frac{25\text{ mV}}{1\text{ mA}} = 25\ \Omega$$

$R_{\text{in}} = r_e = 25\ \Omega$

$A_{vo} = g_m R_C = 40 \times 5 = 200$ V/V

$R_o = R_C = 5$ kΩ

$$A_v = A_{vo}\frac{R_L}{R_L + R_o} = 200 \times \frac{5}{5+5} = 100\text{ V/V}$$

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \times A_v$$

$$= \frac{25}{25+5000} \times 100 = 0.5\text{ V/V}$$

Ex: 7.27

$R_{\text{in}} = r_e = 50\ \Omega$

$$\Rightarrow I_E = \frac{V_T}{r_e} = \frac{25\text{ mV}}{50\ \Omega} = 0.5\text{ mA}$$

$I_C \simeq I_E = 0.5$ mA

$$G_v = \frac{R_C \| R_L}{r_e + R_{\text{sig}}}$$

$$40 = \frac{R_C \| R_L}{(50+50)\Omega}$$

$R_C \| R_L = 4$ kΩ

Ex: 7.28 Refer to Fig. 7.42(c).

$R_o = 100\ \Omega$

Thus,

$$\frac{1}{g_m} = 100\ \Omega \Rightarrow g_m = 10\text{ mA/V}$$

But

$$g_m = \frac{2I_D}{V_{OV}}$$

Thus,

$$I_D = \frac{10 \times 0.25}{2} = 1.25\text{ mA}$$

$$\hat{v}_o = \hat{v}_i \times \frac{R_L}{R_L + R_o} = 1 \times \frac{1}{1+0.1} = 0.91\text{ V}$$

$$\hat{v}_{gs} = \hat{v}_i \frac{\frac{1}{g_m}}{\frac{1}{g_m} + R_L} = 1 \times \frac{0.1}{0.1+1} = 91\text{ mV}$$

Ex: 7.29

$R_o = 200\ \Omega$

$$\frac{1}{g_m} = 200\ \Omega$$

$\Rightarrow g_m = 5$ mA/V

But

$$gm = k_n'\left(\frac{W}{L}\right)V_{OV}$$

Thus,

$$5 = 0.4 \times \frac{W}{L} \times 0.25$$

$$\Rightarrow \frac{W}{L} = 50$$

$$I_D = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2$$

$$= \frac{1}{2} \times 0.4 \times 50 \times 0.25^2$$

$= 0.625$ mA

$R_L = 1$ kΩ to 10 kΩ

Correspondingly,

$$G_v = \frac{R_L}{R_L + R_o} = \frac{R_L}{R_L + 0.2}$$

will range from

$$G_v = \frac{1}{1+0.2} = 0.83\text{ V/V}$$

to

$$G_v = \frac{10}{10+0.2} = 0.98\text{ V/V}$$

Ex: 7.30

$I_C = 5$ mA

$$r_e = \frac{V_T}{I_E} \simeq \frac{V_T}{I_C} = \frac{25\text{ mV}}{5\text{ mA}} = 5\ \Omega$$

$R_{\text{sig}} = 10$ kΩ $\quad R_L = 1$ kΩ

$R_{\text{in}} = (\beta+1)\,(r_e + R_L)$

$= 101 \times (0.005 + 1)$

$= 101.5$ kΩ

$G_{vo} = 1$ V/V

$$R_{\text{out}} = r_e + \frac{R_{\text{sig}}}{\beta+1}$$

$$= 5 + \frac{10{,}000}{101} = 104\ \Omega$$

$$G_v = \frac{R_L}{R_L + r_e + \frac{R_{\text{sig}}}{\beta+1}} = \frac{R_L}{R_L + R_{\text{out}}}$$

$$= \frac{1}{1+0.104} = 0.91\text{ V/V}$$

$$v_\pi = v_{\text{sig}}\frac{r_e}{r_e + R_L + \frac{R_{\text{sig}}}{\beta+1}}$$

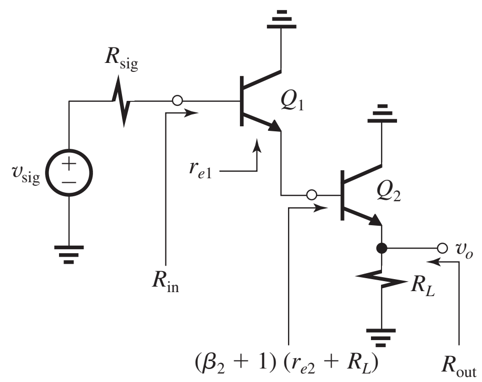
$$\hat{v}_{sig} = \hat{v}_{\pi}\left[1 + \frac{R_L}{r_e} + \frac{R_{sig}}{(\beta+1)\,r_e}\right]$$

$$\hat{v}_{sig} = 5\left[1 + \frac{1000}{5} + \frac{10{,}000}{101 \times 5}\right] = 1.1 \text{ V/V}$$

Correspondingly,

$$\hat{v}_o = G_v \times 1.1 = 0.91 \times 1.1 = 1 \text{ V}$$

Ex: 7.31



From the figure we can write

$$R_{in} = (\beta_1 + 1)\,[r_{e1} + (\beta_2 + 1)(r_{e2} + R_L)]$$

$$R_{out} = R_L \parallel \left[r_{e2} + \frac{r_{e1} + R_{sig}/(\beta_1 + 1)}{\beta_2 + 1}\right]$$

$$\frac{v_o}{v_{sig}} = \frac{R_L}{R_L + r_{e2} + \dfrac{r_{e1} + R_{sig}/(\beta_1 + 1)}{\beta_2 + 1}}$$

For $I_{E2} = 5$ mA, $\beta_1 = \beta_2 = 100$, $R_L = 1$ kΩ, and $R_{sig} = 100$ kΩ, we obtain

$$r_{e2} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5\ \Omega$$

$$I_{E1} = \frac{5}{\beta_2 + 1} = \frac{5}{101} \simeq 0.05 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{0.05 \text{ mA}} = 500\ \Omega$$

$$R_{in} = 101 \times (0.5 + 101 \times 1.005) = 10.3 \text{ M}\Omega$$

$$R_{out} = 1 \parallel \left[0.005 + \frac{0.5 + (100/101)}{101}\right] \simeq 20\ \Omega$$

$$\frac{v_o}{v_{sig}} = \frac{1}{1 + 0.005 + \dfrac{0.5 + (100/101)}{101}}$$

$$= 0.98 \text{ V/V}$$

Ex: 7.32

$$I_D = \frac{1}{2}k_n'\frac{W}{L}(V_{GS} - V_t)^2$$

$$0.5 = \frac{1}{2} \times 1(V_{GS} - 1)^2$$

$$\Rightarrow V_{GS} = 2 \text{ V}$$

If $V_t = 1.5$ V, then

$$I_D = \frac{1}{2} \times 1 \times (2 - 1.5)^2 = 0.125 \text{ mA}$$

$$\Rightarrow \frac{\Delta I_D}{I_D} = \frac{0.125 - 0.5}{0.5} = -0.75 = -75\%$$

Ex: 7.33

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{5 - 2}{0.5} = 6 \text{ k}\Omega$$

$$\rightarrow R_D = 6.2 \text{ k}\Omega$$

$$I_D = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2 \Rightarrow 0.5 = \frac{1}{2} \times 1 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 1 \text{ V}$$

$$\Rightarrow V_{GS} = V_{OV} + V_t = 1 + 1 = 2 \text{ V}$$

$$\Rightarrow V_S = -2 \text{ V}$$

$$R_S = \frac{V_S - V_{SS}}{I_D} = \frac{-2 - (-5)}{0.5} = 6 \text{ k}\Omega$$

$$\rightarrow R_S = 6.2 \text{ k}\Omega$$

If we choose $R_D = R_S = 6.2$ kΩ, then I_D will change slightly:

$$I_D = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2.$$

Also,

$$V_{GS} = -V_S = 5 - R_S I_D$$

Thus,

$$2I_D = (4 - 6.2I_D)^2$$

$$\Rightarrow 38.44I_D^2 - 51.6I_D^2 + 16 = 0$$

$$\Rightarrow I_D = 0.49 \text{ mA},\ 0.86 \text{ mA}$$

$I_D = 0.86$ results in $V_S > 0$ or $V_S > V_G$, which is not acceptable. Therefore $I_D = 0.49$ mA and

$$V_S = -5 + 6.2 \times 0.49 = -1.96 \text{ V}$$

$$V_D = 5 - 6.2 \times 0.49 = +1.96 \text{ V}$$

R_G should be selected in the range of 1 MΩ to 10 MΩ.

Ex: 7.34

$$I_D = 0.5 \text{ mA} = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2$$

$$\Rightarrow V_{OV}^2 = \frac{0.5 \times 2}{1} = 1$$

$$\Rightarrow V_{OV} = 1 \text{ V} \Rightarrow V_{GS} = 1 + 1 = 2 \text{ V}$$

$= V_D \Rightarrow R_D = \dfrac{5-2}{0.5} = 6\ \text{k}\Omega$

$\Rightarrow R_D = 6.2\ \text{k}\Omega$ (standard value). For this R_D we have to recalculate I_D:

$$I_D = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2$$

$$= \frac{1}{2}(V_{DD} - R_D I_D - 1)^2$$

$$(V_{GS} = V_D = V_{DD} - R_D I_D)$$

$$I_D = \frac{1}{2}(4 - 6.2 I_D)^2 \Rightarrow I_D \cong 0.49\text{ mA}$$

$$V_D = 5 - 6.2 \times 0.49 = 1.96\text{ V}$$

Ex: 7.35 Refer to Example 7.12.

(a) For design 1, $R_E = 3\ \text{k}\Omega$, $R_1 = 80\ \text{k}\Omega$, and $R_2 = 40\ \text{k}\Omega$. Thus, $V_{BB} = 4$ V.

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \dfrac{R_1 \| R_2}{\beta + 1}}$$

For the nominal case, $\beta = 100$ and

$$I_E = \frac{4 - 0.7}{3 + \dfrac{40\|80}{101}} = 1.01 \simeq 1\text{ mA}$$

For $\beta = 50$,

$$I_E = \frac{4 - 0.7}{3 + \dfrac{40\|80}{51}} = 0.94\text{ mA}$$

For $\beta = 150$,

$$I_E = \frac{4 - 0.7}{3 + \dfrac{40\|80}{151}} = 1.04\text{ mA}$$

Thus, I_E varies over a range approximately 10% of the nominal value of 1 mA.

(b) For design 2, $R_E = 3.3\ \text{k}\Omega$, $R_1 = 8\ \text{k}\Omega$, and $R_2 = 4\ \text{k}\Omega$. Thus, $V_{BB} = 4$ V. For the nominal case, $\beta = 100$ and

$$I_E = \frac{4 - 0.7}{3.3 + \dfrac{4\|8}{101}} = 0.99 \simeq 1\text{ mA}$$

For $\beta = 50$,

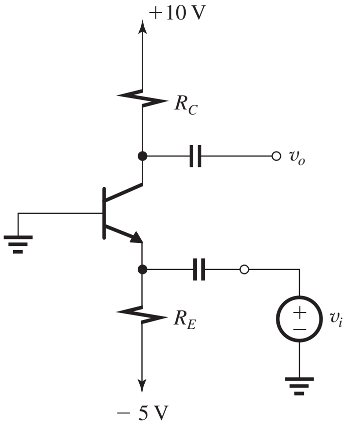
$$I_E = \frac{4 - 0.7}{3.3 + \dfrac{4\|8}{51}} = 0.984\text{ mA}$$

For $\beta = 150$,

$$I_E = \frac{4 - 0.7}{3.3 + \dfrac{4\|8}{151}} = 0.995\text{ mA}$$

Thus, I_E varies over a range of 1.1% of the nominal value of 1 mA. Note that lowering the resistances of the voltage divider considerably decreases the dependence on the value of β, a highly desirable result obtained at the expense of increased current and hence power dissipation.

Ex: 7.36 Refer to Fig. 7.55. Since the circuit is to be used as a common-base amplifier, we can dispense with R_B altogether and ground the base; thus $R_B = 0$. The circuit takes the form shown in the figure below.



To establish $I_E = 1$mA,

$$I_E = \frac{5 - V_{BE}}{R_E}$$

$$1\text{ mA} = \frac{5 - 0.7}{R_E}$$

$$\Rightarrow R_E = 4.3\ \text{k}\Omega$$

The voltage gain $\dfrac{v_o}{v_i} = g_m R_C$, where

$$g_m = \frac{I_C}{V_T} = 40\text{ mA/V}.$$

To maximize the voltage gain, we select R_C as large as possible, consistent with obtaining a $\pm$2-V signal swing at the collector. To maintain active-mode operation at all times, the collector voltage should not be allowed to fall below the value that causes the CBJ to become forward biased, namely, -0.4 V. Thus, the lowest possible dc voltage at the collector is $-0.4\text{ V} + 2\text{V} = +1.6$ V. Correspondingly,

$$R_C = \frac{10 - 1.6}{I_C} \simeq \frac{10 - 1.6}{1\text{ mA}} = 8.4\ \text{k}\Omega$$

Ex: 7.37 Refer to Fig. 7.56. For $I_E = 1$ mA and $V_C = 2.3$ V,

$$I_E = \frac{V_{CC} - V_C}{R_C}$$

$$1 = \frac{10 - 2.3}{R_C}$$

$$\Rightarrow R_C = 7.7\ \text{k}\Omega$$

Now, using Eq. (7.147), we obtain

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \dfrac{R_B}{\beta + 1}}$$

$$1 = \frac{10 - 0.7}{7.7 + \dfrac{R_B}{101}}$$

$$\Rightarrow R_B = 162 \text{ k}\Omega$$

Selecting standard 5% resistors (Appendix J), we use

$$R_B = 160 \text{ k}\Omega \qquad \text{and} \qquad R_C = 7.5 \text{ k}\Omega$$

The resulting value of I_E is found as

$$I_E = \frac{10 - 0.7}{7.5 + \dfrac{160}{101}} = 1.02 \text{ mA}$$

and the collector voltage will be

$$V_C = V_{CC} - I_E R_C = 2.3 \text{ V}$$

Ex: 7.38 Refer to Fig. 7.57(b).

$V_S = 3.5$ V and $I_D = 0.5$ mA; thus

$$R_S = \frac{V_S}{I_D} = \frac{3.5}{0.5} = 7 \text{ k}\Omega$$

$V_{DD} = 15$ V and $V_D = 6$ V; thus

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{15 - 6}{0.5 \text{ mA}} = 18 \text{ k}\Omega$$

To obtain V_{OV}, we use

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.5 = \frac{1}{2} \times 4 V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.5 \text{ V}$$

Thus,

$$V_{GS} = V_t + V_{OV} = 1 + 0.5 = 1.5 \text{ V}$$

We now can obtain the dc voltage required at the gate,

$$V_G = V_S + V_{GS} = 3.5 + 1.5 = 5 \text{ V}$$

Using a current of 2 µA in the voltage divider, we have

$$R_{G2} = \frac{5 \text{ V}}{2 \text{ µA}} = 2.5 \text{ M}\Omega$$

The voltage drop across R_{G1} is 10 V, thus

$$R_{G1} = \frac{10 \text{ V}}{2 \text{ µA}} = 5 \text{ M}\Omega$$

This completes the bias design. To obtain g_m and r_o, we use

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5}{0.5} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

Ex: 7.39 Refer to Fig. 7.57(a) and (c) and to the values found in the solution to Exercise 7.38 above.

$$R_{\text{in}} = R_{G1} \| R_{G2} = 5 \| 2.5 = 1.67 \text{ M}\Omega$$

$$R_o = R_D \| r_o = 18 \| 200 = 16.5 \text{ k}\Omega$$

$$G_v = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m (r_o \| R_D \| R_L)$$

$$= -\frac{1.67}{1.67 + 0.1} \times 2 \times (200 \| 18 \| 20)$$

$$= -17.1 \text{ V/V}$$

Ex: 7.40 To reduce v_{gs} to half its value, the unbypassed R_s is given by

$$R_s = \frac{1}{g_m}$$

From the solution to Exercise 7.38 above, $g_m = 2$ mA/V. Thus

$$R_s = \frac{1}{2} = 0.5 \text{ k}\Omega$$

Neglecting r_o, G_v is given by

$$G_v = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \times -\frac{R_D \| R_L}{\dfrac{1}{g_m} + R_s}$$

$$= -\frac{1.67}{1.67 + 0.1} \times \frac{18 \| 20}{0.5 + 0.5}$$

$$= -8.9 \text{ V/V}$$

Ex: 7.41 Refer to Fig. 7.58(a). For $V_B = 5$ V and 50-µA current through R_{B2}, we have

$$R_{B2} = \frac{5 \text{ V}}{0.05 \text{ mA}} = 100 \text{ k}\Omega$$

The base current is

$$I_B = \frac{I_E}{\beta + 1} \simeq \frac{0.5 \text{ mA}}{100} = 5 \text{ µA}$$

The current through R_{B1} is

$$I_{R_{B1}} = I_B + I_{R_{B2}} = 5 + 50 = 55 \text{ µA}$$

Since the voltage drop across R_{B1} is $V_{CC} - V_B = 10$ V, the value of R_{B1} can be found from

$$R_{B1} = \frac{10 \text{ V}}{0.055 \text{ µA}} = 182 \text{ k}\Omega$$

The value of R_E can be found from

$$I_E = \frac{V_B - V_{BE}}{R_E}$$

$$\Rightarrow R_E = \frac{5 - 0.7}{0.5} = 8.6 \text{ k}\Omega$$

The value of R_C can be found from

$$V_C = V_{CC} - I_C R_C$$

$6 = 15 - 0.99 \times 0.5 \times R_C$

$R_C \simeq 18\ \text{k}\Omega$

This completes the bias design. The values of g_m, r_π, and r_o can be found as follows:

$$g_m = \frac{I_C}{V_T} \simeq \frac{0.5\ \text{mA}}{0.025\ \text{V}} = 20\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5\ \text{k}\Omega$$

$$r_o = \frac{V_A}{I_C} \simeq \frac{100}{0.5} = 200\ \text{k}\Omega$$

Ex: 7.42 Refer to Fig. 7.58(b) and to the solution of Exercise 7.41 above.

$R_{\text{in}} = R_{B1} \| R_{B2} \| r_\pi$

$= 182 \| 100 \| 5 = 4.64\ \text{k}\Omega$

$R_o = R_C \| r_o = 18 \| 200 = 16.51\ \text{k}\Omega$

$$G_v = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m (R_C \| R_L \| r_o)$$

$$G_v = -\frac{4.64}{4.64 + 10} \times 20 \times (18 \| 20 \| 200)$$

$= -57.3\ \text{V/V}$

Ex: 7.43 Refer to the solutions of Exercises 7.41 and 7.42 above. With R_e included (i.e., left unbypassed), the input resistance becomes [refer to Fig. 7.59(b)]

$R_{\text{in}} = R_{B1} \| R_{B2} \| [(\beta + 1)(r_e + R_e)]$

Thus,

$10 = 182 \| 100 \| [101(0.05 + R_e)]$

where we have substituted $r_e = \dfrac{V_T}{I_E} =$ $\dfrac{25}{0.5} = 50\ \Omega$. The value of R_e is found from the equation above to be

$R_e = 67.2\ \Omega$

The overall voltage gain can be found from

$$G_v = -\alpha \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \frac{R_C \| R_L}{r_e + R_e}$$

$$G_v = -0.99 \times \frac{10}{10 + 10} \frac{18 \| 20}{0.05 + 0.0672}$$

$= -40\ \text{V/V}$

Ex: 7.44 Refer to Fig. 7.60(a).

$R_{\text{in}} = 50\ \Omega = r_e \| R_E \simeq r_e$

$$r_e = 50\ \Omega = \frac{V_T}{I_E}$$

$\Rightarrow I_E = 0.5\ \text{mA}$

$I_C = \alpha I_E \simeq I_E = 0.5\ \text{mA}$

$V_C = V_{CC} - R_C I_C$

For $V_C = 1$ V and $V_{CC} = 5$ V, we have

$1 = 5 - R_C \times 0.5$

$\Rightarrow R_C = 8\ \text{k}\Omega$

To obtain the required value of R_E, we note that the voltage drop across it is $(V_{EE} - V_{BE}) = 4.3$ V. Thus,

$$R_E = \frac{4.3}{0.5} = 8.6\ \text{k}\Omega$$

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m (R_C \| R_L)$$

$$= \frac{50\ \Omega}{50\ \Omega + 50\ \Omega} \times 20(8 \| 8)$$

$= 40\ \text{V/V}$

$\hat{v}_o = 40 \hat{v}_{\text{sig}} = 40 \times 10\ \text{mV} = 0.4\ \text{V}$

Ex: 7.45 Refer to Fig. 7.61. Consider first the bias design of the circuit in Fig. 7.61(a). Since the required $I_E = 1$ mA, the base current

$I_B = \dfrac{I_E}{\beta + 1} = \dfrac{1}{101} \simeq 0.01$ mA. For a dc voltage drop across R_B of 1 V, we obtain

$$R_B = \frac{1\ \text{V}}{0.01\ \text{mA}} = 100\ \text{k}\Omega$$

The result is a base voltage of –1 V and an emitter voltage of –1.7 V. The required value of R_E can now be determined as

$$R_E = \frac{-1.7 - (-5)}{I_E} = \frac{3.3}{1\ \text{mA}} = 3.3\ \text{k}\Omega$$

$R_{\text{in}} = R_B \| (\beta + 1) r_e + (R_E \| r_o \| R_L)$

where $r_o = \dfrac{V_A}{I_C} = \dfrac{100\ \text{V}}{1\ \text{mA}} = 100\ \text{k}\Omega$

$R_{\text{in}} = 100 \| (100 + 1) 0.025 + (3.3 \| 100 \| 1)$

$= 44.3\ \text{k}\Omega$

$$\frac{v_i}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{44.3}{44.3 + 50} = 0.469\ \text{V/V}$$

$$\frac{v_o}{v_i} = \frac{R_E \| r_o \| R_L}{r_e + (R_E \| r_o \| R_L)} = 0.968\ \text{V/V}$$

$$G_v \equiv \frac{v_o}{v_{\text{sig}}} = 0.469 \times 0.968 = 0.454\ \text{V/V}$$

$$R_{\text{out}} = r_o \| R_E \| \left[r_e + \frac{R_B \| R_{\text{sig}}}{\beta + 1} \right]$$

$$= 100 \| 3.3 \| \left[0.025 + \frac{100 \| 50}{101} \right]$$

$= 320\ \Omega$

Chapter 8

Solutions to Exercises within the Chapter

Ex: 8.1 In the current source of Example 8.1 (Fig. 8.1) we have $I_O = 100\ \mu\text{A}$ and we want to reduce the change in output current, ΔI_O, corresponding to a 1-V change in output voltage, ΔV_O, to 1% of I_O.

That is, $\Delta I_O = \dfrac{\Delta V_O}{r_{o2}} = 0.01 I_O \Rightarrow \dfrac{1\text{ V}}{r_{o2}}$

$= 0.01 \times 100\ \mu\text{A} = 1\ \mu\text{A}$

$$r_{o2} = \frac{1\text{ V}}{1\ \mu\text{A}} = 1\text{ M}\Omega$$

$$r_{o2} = \frac{V_A' \times L}{I_O} \Rightarrow 1\text{ M}\Omega = \frac{20 \times L}{100\ \mu\text{A}}$$

$$\Rightarrow L = \frac{100\text{ V}}{20\text{ V}/\ \mu\text{m}} = 5\ \mu\text{m}$$

To keep V_{OV} of the matched transistors the same as that in Example 8.1, $\dfrac{W}{L}$ of the transistor should remain the same. Therefore,

$$\frac{W}{5\ \mu\text{m}} = \frac{10\ \mu\text{m}}{1\ \mu\text{m}} \Rightarrow W = 50\ \mu\text{m}$$

So the dimensions of the matched transistors Q_1 and Q_2 should be changed to

$W = 50\ \mu\text{m}$ and $L = 5\ \mu\text{m}$

Ex: 8.2 For the circuit of Fig. 8.4 we have

$$I_2 = I_{\text{REF}}\frac{(W/L)_2}{(W/L)_1},\ I_3 = I_{\text{REF}}\frac{(W/L)_3}{(W/L)_1}$$

and $\quad I_5 = I_4\dfrac{(W/L)_5}{(W/L)_4}$

Since all channel lengths are equal, that is,

$L_1 = L_2 = \cdots = L_5 = 1\ \mu\text{m}$

and

$I_{\text{REF}} = 10\ \mu\text{A},\ I_2 = 60\ \mu\text{A},\ I_3 = 20\ \mu\text{A},\ I_4 = I_3 = 20\ \mu\text{A},$ and $I_5 = 80\ \mu\text{A},$

we have

$$I_2 = I_{\text{REF}}\frac{W_2}{W_1} \Rightarrow \frac{W_2}{W_1} = \frac{I_2}{I_{\text{REF}}} = \frac{60}{10} = 6$$

$$I_3 = I_{\text{REF}}\frac{W_3}{W_1} \Rightarrow \frac{W_3}{W_1} = \frac{I_3}{I_{\text{REF}}} = \frac{20}{10} = 2$$

$$I_5 = I_4\frac{W_5}{W_4} \Rightarrow \frac{W_5}{W_4} = \frac{I_5}{I_4} = \frac{80}{20} = 4$$

To allow the voltage at the drain of Q_2 to go down to within 0.2 V of the negative supply voltage, we need $V_{OV2} = 0.2$ V:

$$I_2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 V_{OV2}^2 = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_2 V_{OV2}^2$$

$$60\ \mu\text{A} = \frac{1}{2}200\frac{\mu\text{A}}{\text{V}^2}\left(\frac{W}{L}\right)_2 (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = \frac{120}{200 \times (0.2)^2} = 15 \Rightarrow W_2$$

$= 15 \times L_2$

$$W_2 = 15\ \mu\text{m},\ \frac{W_2}{W_1} = 6 \Rightarrow W_1 = \frac{W_2}{6} = 2.5\ \mu\text{m}$$

$$\frac{W_3}{W_1} = 2 \Rightarrow W_3 = 2 \times W_1 = 5\ \mu\text{m}$$

To allow the voltage at the drain of Q_5 to go up to within 0.2 V of positive supply, we need $V_{OV5} = 0.2$ V:

$$I_5 = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_5 V_{OV5}^2$$

$$80\ \mu\text{A} = \frac{1}{2}80\ \frac{\mu\text{A}}{\text{V}^2}\left(\frac{W}{L}\right)_5 (0.2)^2 \Rightarrow$$

$$\left(\frac{W}{L}\right)_5 = \frac{2 \times 80}{80 \times (0.2)^2} = 50 \Rightarrow W_5 = 50\ L_5$$

$W_5 = 50\ \mu\text{m}$

$$\frac{W_5}{W_4} = 4 \Rightarrow W_4 = \frac{50\ \mu\text{m}}{4} = 12.5\ \mu\text{m}$$

Thus:

$W_1 = 2.5\ \mu\text{m},\ W_2 = 15\ \mu\text{m},\ W_3 = 5\ \mu\text{m}$
$W_4 = 12.5\ \mu\text{m},$ and $W_5 = 50\ \mu\text{m}$

Ex: 8.3 $V_{BE} = V_T \ln(I_{REF}/I_S)$

$= 0.025 \ln(10^{-3}/10^{-15})$

$= 0.691$ V

From Eq. (8.21) we have

$$I_O = I_{\text{REF}}\left(\frac{m}{1 + \dfrac{m+1}{\beta}}\right)\left(1 + \frac{V_O - V_{BE}}{V_{A2}}\right)$$

$$I_O = 1\text{ mA}\left(\frac{1}{1 + \dfrac{1+1}{100}}\right)\left(1 + \frac{5 - 0.691}{100}\right)$$

$= 1.02$ mA

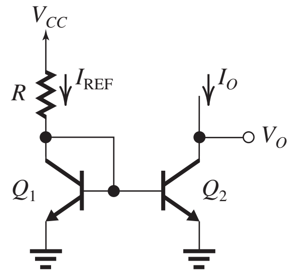
$I_O = 1.02$ mA

$$R_o = r_{o2} = \frac{V_A}{I_O} = \frac{100\text{ V}}{1.02\text{ mA}} = 98\text{ k}\Omega \simeq 100\text{ k}\Omega$$

Ex: 8.4

From Eq. (8.23), we have

$$I_O = \frac{I_{\text{REF}}}{1 + (2/\beta)}\left(1 + \frac{V_O - V_{BE}}{V_A}\right)$$



where $V_{BE} = V_T \ln\left(\frac{I_O}{I_S}\right)$

$= 0.025 \ln\left(\frac{0.5 \times 10^{-3}}{10^{-15}}\right) = 0.673 \text{ V}$

$$0.5 \text{ mA} = \frac{I_{REF}}{1 + (2/100)}\left(1 + \frac{2 - 0.673}{50}\right) \Rightarrow$$

$$I_{REF} = 0.5 \text{ mA}\frac{1.02}{1.0265} = 0.497 \text{ mA}$$

$$I_{REF} = \frac{V_{CC} - V_{BE}}{R} \Rightarrow R = \frac{V_{CC} - V_{BE}}{I_{REF}}$$

$$R = \frac{5 - 0.673}{0.497 \text{ mA}} = 8.71 \text{ k}\Omega$$

$V_{O\min} = V_{CE\text{sat}} = 0.3 \text{ V}$

For $V_O = 5$ V, From Eq. (8.23) we have

$$I_O = \frac{I_{REF}}{1 + (2/\beta)}\left(1 + \frac{V_O - V_{BE}}{V_A}\right)$$

$$I_O = \frac{0.497}{1 + (2/100)}\left(1 + \frac{5 - 0.673 \text{ V}}{50}\right) = 0.53 \text{ mA}$$

Ex: 8.5 $I_1 = I_2 = \cdots = I_N = I_C|_{Q_{REF}}$

At the input node,

$$I_{REF} = I_C|_{Q_{REF}} + I_B|_{Q_{REF}} + I_{B1} + \cdots + I_{BN}$$

$$= I_C|_{Q_{REF}} + (N + 1)\, I_B|_{Q_{REF}}$$

$$= I_C|_{Q_{REF}} + \frac{(N + 1)}{\beta}\, I_C|_{Q_{REF}}$$

$$\Rightarrow I_C|_{Q_{REF}} = \frac{I_{REF}}{1 + \frac{N + 1}{\beta}}$$

Thus,

$$I_1 = I_2 = \cdots = I_N = \frac{I_{REF}}{1 + \frac{N + 1}{\beta}} \qquad \text{Q.E.D}$$

For $\beta = 100$, to limit the error to 10%,

$$0.1 = \frac{N + 1}{\beta} = \frac{N + 1}{100}$$

$\Rightarrow N = 9$

Ex: 8.6

$$R_{in} \simeq \frac{1}{g_{m1}}$$

Now, $R_{in} = 1$ kΩ, thus

$g_{m1} = 1$ mA/V

But

$$g_{m1} = \sqrt{2(\mu_n C_{ox})\left(\frac{W}{L}\right)_1 I_{D1}}$$

$$1 = \sqrt{2 \times 0.4 \times \left(\frac{W}{L}\right)_1 \times 0.1}$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 12.5$$

To obtain

$$A_{is}\Big|_{\text{ideal}} = 5$$

$$5 = \frac{(W/L)_2}{(W/L)_1}$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 5 \times 12.5 = 62.5$$

$$R_o = r_{o2} = \frac{V_{A2}}{I_{D2}} = \frac{V_{A2}}{5I_{D1}}$$

Thus,

$$40 \text{ k}\Omega = \frac{V_{A2}}{5 \times 0.1}$$

$\Rightarrow V_{A2} = 20$ V

But

$V_{A2} = V'_{A2} L_2$

$20 = 20 \times L_2$

$\Rightarrow L_2 = 1\ \mu$m

Selecting $L_1 = L_2$, then

$L_1 = L_2 = 1\ \mu$m

$W_1 = 12.5\ \mu$m

$W_2 = 62.5\ \mu$m

The actual short-circuit current-transfer ratio is given by Eq. (8.31),

$$A_{is} = \frac{5}{1 + \frac{1}{g_{m1} r_{o1}}}$$

Thus the percentage error is

$$\text{Error} = \frac{-1/g_{m1} r_{o1}}{1 + (1/g_{m1} r_{o1})} \times 100\%$$

Substituting,

$g_{m1} = 1$ mA/V

$$r_{o1} = \frac{V_{A1}}{I_{D1}} = \frac{V'_A L_1}{I_{D1}} = \frac{20 \times 1}{0.1} = 200 \text{ k}\Omega$$

$$\text{Error} = \frac{-1/200}{1 + (1/200)} \times 100 = -0.5\%$$

Ex: 8.7

Using Eq. (8.42):

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)} \cdot \sqrt{I_D}$$

For $I_D = 10\ \mu A$, we have

$$g_m = \sqrt{2(387\ \mu A/V^2)(10)(10\ \mu A)}$$

$= 0.28$ mA/V

Using Eq. (8.46):

$$A_0 = V'_A \frac{\sqrt{2\mu_n C_{ox}(W/L)}}{\sqrt{I_D}}$$

$$= \frac{5\ V/\mu m\sqrt{2(387\ \mu A/V^2)(10)(0.36)^2}}{\sqrt{10\ \mu A}}$$

$A_0 = 50$ V/V

Since g_m varies with $\sqrt{I_D}$ and A_0 with $\dfrac{1}{\sqrt{I_D}}$,

for

$$I_D = 100\ \mu A \Rightarrow g_m = 0.28\ mA/V\left(\frac{100}{10}\right)^{1/2}$$

$= 0.88$ mA/V

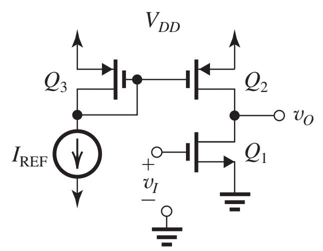
$$A_0 = 50\left(\frac{10}{100}\right)^{1/2} = 15.8\ V/V$$

For $I_D = 1$ mA, we have

$$g_m = 0.28\ mA/V\left(\frac{1}{0.010}\right)^{1/2} = 2.8\ mA/V$$

$$A_0 = 50\left(\frac{0.010}{1}\right)^{1/2} = 5\ V/V$$

Ex: 8.8



Since all transistors have the same

$$\frac{W}{L} = \frac{7.2\ \mu m}{0.36\ \mu m},$$

we have

$$I_{REF} = I_{D3} = I_{D2} = I_{D1} = 100\ \mu A$$

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\sqrt{I_{D1}}$$

$$= \sqrt{2(387\ \mu A/V^2)\left(\frac{7.2}{0.36}\right)(100\ \mu A)}$$

$= 1.24$ mA/V

$$r_{o1} = \frac{V'_{An}L_1}{I_{D1}} = \frac{5\ V/\mu m\,(0.36\ \mu m)}{0.1\ mA} = 18\ k\Omega$$

$$r_{o2} = \frac{|V'_{Ap}|L_2}{I_{D2}} = \frac{6\ V/\mu m\,(0.36\ \mu m)}{0.1\ mA} = 21.6\ k\Omega$$

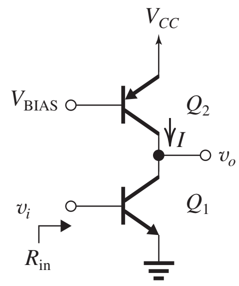
Voltage gain is

$$A_v = -g_{m1}(r_{o1} \parallel r_{o2})$$

$$A_v = -(1.24\ mA/V)(18\ k\Omega \parallel 21.6\ k\Omega)$$

$= -12.2$ V/V

Ex: 8.9



$$I_{C1} = I = 100\ \mu A = 0.1\ mA$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.1\ mA}{25\ mV} = 4\ mA/V$$

$$R_{in} = r_{\pi 1} = \frac{\beta_1}{g_{m1}} = \frac{100}{4\ mA/V} = 25\ k\Omega$$

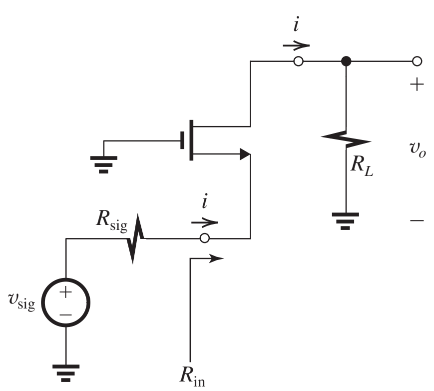
$$r_{o1} = \frac{V_A}{I} = \frac{50\ V}{0.1\ mA} = 500\ k\Omega$$

$$r_{o2} = \frac{|V_A|}{I} = \frac{50\ V}{0.1\ mA} = 500\ k\Omega$$

$A_0 = g_{m1}\, r_{o1} = (4\ mA/V)(500\ k\Omega) =$ 2000 V/V

$A_v = -g_{m1}(r_{o1} \parallel r_{o2}) = -(4\ mA/V) \times$ $(500\ k\Omega \parallel 500\ k\Omega) = -1000$ V/V

Ex: 8.10 Refer to Fig. 1,



$$v_o = i R_L$$

$$v_{sig} = i(R_s + R_{in})$$

Thus,

$$\frac{v_o}{v_{sig}} = \frac{R_L}{R_s + R_{in}} \qquad \text{Q.E.D}$$

Ex: 8.11 Since $g_m r_o \gg 1$, we use Eq. (8.52),

$$R_{in} \simeq \frac{1}{g_m} + \frac{R_L}{g_m r_o}$$

R_L	0	r_o	$(g_m r_o) r_o$	∞
R_{in}	$\frac{1}{g_m}$	$\frac{2}{g_m}$	r_o	∞

Ex: 8.12 For $g_m r_o \gg 1$, we use Eq. (8.57),

$$R_{out} \simeq r_o + (g_m r_o) R_s$$

to obtain

R_s	0	r_o	$(g_m r_o) r_o$	∞
R_{out}	r_o	$(g_m r_o) r_o$	$(g_m r_o)^2 r_o$	∞

Ex: 8.13 Substituting $R_s = r_o$ in Eq. (8.53) gives

$$A_{is} = \frac{1 + \dfrac{1}{g_m r_o}}{1 + \dfrac{2}{g_m r_o}}$$

Substituting the given values of $(g_m r_o)$ we obtain

$g_m r_o$	20	50	100
A_{is} (A/A)	0.95	0.98	0.99

Ex: 8.14 A_{vo} remains unchanged at $g_m r_o$. With a load resistance R_L connected,

$$A_v = A_{vo} \frac{R_L}{R_L + R_o}$$

$$= (g_m r_o) \frac{R_L}{R_L + (1 + g_m R_s) r_o}$$

Ex: 8.15 Since $g_m r_o \gg 1$ we use Eq. (8.62),

$$R_{in} \simeq r_\pi \| \left[\frac{1}{g_m} + \frac{R_L}{g_m r_o} \right]$$

For $R_L = 0$,

$$R_{in} \simeq r_\pi \| \frac{1}{g_m} = r_e$$

For $R_L = r_o$,

$$R_{in} = r_\pi \| (\frac{1}{g_m} + \frac{1}{g_m}) = r_\pi \| \frac{2}{g_m} \simeq 2r_e$$

For $R_L = \beta r_o$,

$$R_{in} = r_\pi \| (\frac{1}{g_m} + \frac{\beta r_o}{g_m r_o}) = r_\pi \| (\frac{1}{g_m} + r_\pi)$$

$$\simeq r_\pi \| r_\pi = \frac{1}{2} r_\pi$$

For $R_L = \infty$,

$$R_{in} = r_\pi$$

Summary:

R_L	0	r_o	βr_o	∞
R_{in}	r_e	$2r_e$	$\frac{1}{2} r_\pi$	r_π

Ex: 8.16 Using Eq. (8.69),

$$R_{out} \simeq r_o + (g_m r_o)(R_e \| r_\pi)$$

we obtain

R_e	0	r_e	r_π	r_o	∞
R_{out}	r_o	$2r_o$	$\left(\frac{\beta}{2} + 1\right) r_o$	$(\beta + 1) r_o$	$(\beta + 1) r_o$

Ex: 8.17 Using Eq. (8.65) with $R_e = r_o$,

$$A_{is} = \frac{1 + \dfrac{1}{g_m r_o}}{1 + \dfrac{1}{g_m r_o} + \dfrac{1}{g_m (r_o \| r_\pi)}}$$

$$= \frac{1 + \dfrac{1}{g_m r_o}}{1 + \dfrac{2}{g_m r_o} + \dfrac{1}{g_m r_\pi}} = \frac{1 + \dfrac{1}{g_m r_o}}{1 + \dfrac{2}{g_m r_o} + \dfrac{1}{\beta}}$$

For $\beta = 100$ and $g_m r_o = 100$,

$$A_{is} = \frac{1 + \dfrac{1}{100}}{1 + \dfrac{2}{100} + \dfrac{1}{100}} = 0.98 \text{ A/A}$$

For $\beta = 100$ and $g_m r_o = 1000$,

$$A_{is} = \frac{1 + \dfrac{1}{1000}}{1 + \dfrac{2}{1000} + \dfrac{1}{100}} = 0.99 \text{ A/A}$$

Ex: 8.18 $R_o = 1 + g_m (R_e \| r_\pi) r_o$

where

$$g_m = 4 \text{ mA/V}, \; r_\pi = \frac{\beta}{g_m} = 25 \text{ k}\Omega,$$

$$R_e = 1 \text{ k}\Omega, \text{ and } r_o = \frac{V_A}{I_C} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

Thus,

$R_o = [1 + 4(1 \parallel 25)] \times 200$

$\simeq 1\ \text{M}\Omega$

Without emitter degeneration,

$R_o = r_o = 200\ \text{k}\Omega$

Ex: 8.19 If L is halved $\left(L = \dfrac{0.55\ \mu\text{m}}{2}\right)$ and $|V_A| = |V_A'| \cdot L$, we obtain

$|V_A| = 5\ \text{V}/\mu\text{m} \left(\dfrac{0.55\ \mu\text{m}}{2}\right) = 1.375\ \text{V}$

$$R_o = \frac{|V_A|}{|V_{OV}|/2} \cdot \frac{|V_A|}{I_D} = \frac{2\,(1.375\ \text{V})^2}{(0.3\ \text{V})\ \ (100\ \mu\text{A})}$$

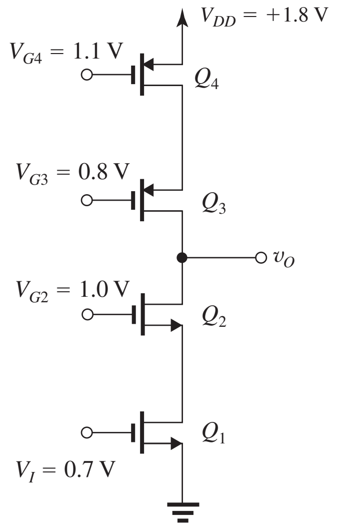
$= 126\ \text{k}\Omega$

Since $I_D = \dfrac{1}{2}(\mu_p C_{ox})\left(\dfrac{W}{L}\right)|V_{OV}|^2\left(1 + \dfrac{V_{SD}}{|V_A|}\right)$

$$\frac{W}{L} = \frac{2\,(100\ \mu\text{A})}{90\ \mu\text{A/V}^2\,(0.3\ \text{V})^2\left(1 + \dfrac{0.3\ \text{V}}{1.375\ \text{V}}\right)}$$

$$\frac{W}{L} = 20.3$$

Ex: 8.20



If all transistors are matched and are obviously operating at the same I_D, then all $|V_{OV}|$ will be equal and equal to that of Q_1, namely, $|V_{OV}| = 0.7 - 0.5 = 0.2$ V

To keep Q_2 in saturation, $v_{O\min} = V_{G2} - |V_t| = 0.5$ V

To keep Q_3 in saturation, $v_{O\max} = V_{G3} + |V_t| = 1.3$ V

Thus, the allowable range of v_O is from 0.5 V to 1.3 V.

Ex: 8.21 Refer to Fig. 8.31.

$g_{m1} = g_{m2} = g_{m3} = g_{m4} = \dfrac{2\ I_D}{|V_{OV}|} = \dfrac{2 \times 0.2}{0.2}$

$= 2\ \text{mA/V}$

$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \dfrac{|V_A|}{I_D} = \dfrac{2}{0.2} = 10\ \text{k}\Omega$

$R_{on} = (g_{m2} r_{o2}) r_{o1} = (2 \times 10) \times 10 = 200\ \text{k}\Omega$

$R_{op} = (g_{m3} r_{o3}) r_{o4} = (2 \times 10) \times 10 = 200\ \text{k}\Omega$

$R_o = R_{on} \parallel R_{op} = 200 \parallel 200 = 100\ \text{k}\Omega$

$A_v = -g_{m1} R_o = -2 \times 100 = -200\ \text{V/V}$

Ex: 8.22 $g_{m1} = g_{m2} = g_m$

$= \dfrac{I_D}{\dfrac{V_{OV}}{2}} = \dfrac{0.1\ \text{mA}}{(0.2/2)\ \text{V}} = 1\ \text{mA/V}$

$r_{o1} = r_{o2} = r_o$

$= \dfrac{V_A}{I_D} = \dfrac{2\ \text{V}}{0.1\ \text{mA}} = 20\ \text{k}\Omega$

so, $g_m r_o = 1\ \text{mA/V}\ (20\ \text{k}\Omega) = 20$

(a) For $R_L = 20\ \text{k}\Omega$,

$R_{\text{in}2} = \dfrac{R_L + r_{o2}}{1 + g_{m2} r_{o2}} = \dfrac{20\ \text{k}\Omega + 20\ \text{k}\Omega}{1 + 20} = 1.9\ \text{k}\Omega$

$\therefore A_{v1} = -g_{m1}\,(r_{o1} \parallel R_{\text{in}2})$

$= -1\ \text{mA/V}\ (20 \parallel 1.9) = -1.74\ \text{V/V}$

or

If we use the approximation of Eq. (8.84),

$R_{\text{in}2} \approx \dfrac{R_L}{g_{m2} r_{o2}} + \dfrac{1}{g_{m2}} = \dfrac{20\ \text{k}\Omega}{20} + \dfrac{1}{1\ \text{mA/V}} = 2\ \text{k}\Omega$

then

$A_{v1} = -1\ \text{mA/V}\ (20\ \text{k}\Omega \parallel 2\ \text{k}\Omega) = -1.82\ \text{V/V}$

Continuing, from Eq. (8.81),

$A_v = -g_{m1}\left[(g_{m2} r_{o2} r_{o1}) \parallel R_L\right]$

$A_v = -1\ \text{mA/V}\ \{[(20)\,(20\ \text{k}\Omega)] \parallel 20\ \text{k}\Omega\}$

$= -19.0\ \text{V/V}$

$A_{v2} = \dfrac{A_v}{A_{v1}} = \dfrac{-19.0}{-1.82} = 10.5\ \text{V/V}$

(b) Now, for $R_L = 400\ \text{k}\Omega$,

$R_{\text{in}2} \simeq \dfrac{R_L}{g_{m2} r_{o2}} + \dfrac{1}{g_{m2}} = \dfrac{400\ \text{k}\Omega}{20} + \dfrac{1}{1\ \text{mA/V}}$

$=\ 21\ \text{k}\Omega$

$A_{v1} = -1\ \text{mA/V}\ (20\ \text{k}\Omega \parallel 21\ \text{k}\Omega) = -10.2\ \text{V/V}$

$A_v = -1 \text{ mA/V } [(20)(20 \text{ k}\Omega)] \parallel 400 \text{ k}\Omega$

$= -200 \text{ V/V}$

$A_{v2} = \dfrac{A_v}{A_{v1}} = \dfrac{-200}{-10.2} = 19.6 \text{ V/V}$

Ex: 8.23 Referring to Fig. 8.34,

$R_{op} = (g_{m3}r_{o3})(r_{o4} \parallel r_{\pi 3})$ and

$R_{on} = (g_{m2}r_{o2})(r_{o1} \parallel r_{\pi 2})$

The maximum values of these resistances are obtained when $r_o \gg r_\pi$ and are given by

$R_{on}\Big|_{\max} = (g_{m2}r_{o2})\, r_{\pi 2}$

$R_{op}\Big|_{\max} = (g_{m3}r_{o3})\, r_{\pi 3}$

Since $g_m r_\pi = \beta$,

$R_{on}\Big|_{\max} = \beta_2 r_{o2}$

$R_{op}\Big|_{\max} = \beta_3 r_{o3}$

Since $A_v = -g_{m1}\left(R_{on} \parallel R_{op}\right)$,

$|A_{v\max}| = g_{m1}\left(\beta_2 r_{o2} \parallel \beta_3 r_{o3}\right)$

Ex: 8.24 For the *npn* transistors,

$g_{m1} = g_{m2} = \dfrac{|I_C|}{|V_T|} = \dfrac{0.2 \text{ mA}}{25 \text{ mV}} = 8 \text{ mA/V}$

$r_{\pi 1} = r_{\pi 2} = \dfrac{\beta}{g_m} = \dfrac{100}{8 \text{ mA/V}} = 12.5 \text{ k}\Omega$

$r_{o1} = r_{o2} = \dfrac{|V_A|}{|I_C|} = \dfrac{5 \text{ V}}{0.2 \text{ mA}} = 25 \text{ k}\Omega$

From Fig. 8.34,

$R_{on} = (g_{m2}r_{o2})(r_{o1} \parallel r_{\pi 2})$

$= (8 \text{ mA/V})(25 \text{ k}\Omega)(25 \text{ k}\Omega \parallel 12.5 \text{ k}\Omega)$

$R_{on} = 1.67 \text{ M}\Omega$

For the *pnp* transistors,

$g_{m3} = g_{m4} = \dfrac{|I_C|}{V_T} = \dfrac{0.2 \text{ mA}}{25 \text{ mV}} = 8 \text{ mA/V}$

$r_{\pi 3} = r_{\pi 4} = \dfrac{\beta}{g_m} = \dfrac{50}{8 \text{ mA/V}} = 6.25 \text{ k}\Omega$

$r_{o3} = r_{o4} = \dfrac{|V_A|}{|I_C|} = \dfrac{4 \text{ V}}{0.2 \text{ mA}} = 20 \text{ k}\Omega$

$R_{op} = (g_{m3}r_{o3})(r_{o4} \parallel r_{\pi 3})$

$= (8 \text{ mA/V})(20 \text{ k}\Omega)(20 \text{ k}\Omega \parallel 6.25 \text{ k}\Omega)$

$R_{op} = 762 \text{ k}\Omega$

$A_v = -g_{m1}\left(R_{on} \parallel R_{op}\right)$

$= -(8 \text{ mA/V})\ (1.67 \text{ M}\Omega \parallel 762 \text{ k}\Omega)$

$A_v = -4186 \text{ V/V}$

$A_{v\max}$ occurs when r_{o1} and r_{o4} are $\gg r_\pi$.

Then

$R_{on} = (g_{m2}r_{o2})\, r_{\pi 2} = \beta_2 r_{o2}$

$R_{on} = 100\,(25 \text{ k}\Omega) = 2.5 \text{ M}\Omega$

$R_{op} = (g_{m3}r_{o3})\, r_{\pi 3} = \beta_3 r_{o3}$

$R_{op} = 50\,(20 \text{ k}\Omega) = 1 \text{ M}\Omega$

Finally,

$A_{v\max} = -(8 \text{ mA/V})(2.5 \text{ M}\Omega \parallel 1.0 \text{ M}\Omega)$

$A_{v\max} = -5714 \text{ V/V}$

Ex: 8.25

$g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$

$g_{mb} = \chi g_m = 0.2 \times 2 = 0.4 \text{ mA/V}$

$r_{o1} = r_{o3} = \dfrac{V_A}{I_D} = \dfrac{5}{0.2} = 25 \text{ k}\Omega$

$R_L = r_{o1} \parallel r_{o3} \parallel \dfrac{1}{g_{mb}} = 25 \parallel 25 \parallel 2.5 \text{ k}\Omega$

$= 2.083 \text{ k}\Omega$

$\dfrac{v_o}{v_i} = \dfrac{R_L}{R_L + \dfrac{1}{g_m}} = \dfrac{2.083}{2.083 + \dfrac{1}{2}} = 0.81 \text{ V/V}$

To obtain R_{out} we use Eq. (8.95),

$R_{\text{out}} = \dfrac{1}{g_m} \| \dfrac{1}{g_{mb}} \| r_{o1} \| r_{o3}$

$= 0.5 \| 2.5 \| 25 \| 25$

$= 0.403 \text{ k}\Omega = 403\ \Omega$

Ex: 8.26 Refer to the circuit in Fig. 8.36. All transistors are operating at $I_D = I_{\text{REF}} = 100\ \mu\text{A}$ and equal V_{OV}, found from

$I_D = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{W}{L}\right)V_{OV}^2$

$100 = \dfrac{1}{2} \times 387 \times \dfrac{3.6}{0.36} \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.227 \text{ V}$

$V_{GS} = 0.227 + 0.5 = 0.727 \text{ V}$

$V_{O\min} = V_{G3} - V_{t3}$

$= V_{GS4} + V_{GS1} - V_{t3}$

Thus,

$V_{O\min} = 2V_{GS} - V_t$

$= V_t + 2\, V_{OV}$

$= 0.5 + 2 \times 0.227 = 0.95 \text{ V}$

$$g_m = \frac{2\,I_D}{V_{OV}} = \frac{2 \times 0.1}{0.227} = 0.88\text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{5 \times 0.36}{0.1} = 18\text{ k}\Omega$$

$$R_o = (g_{m3} r_{o3}) r_{o2} = (0.88 \times 18) \times 18$$
$$= 285\text{ k}\Omega$$

Ex: 8.27 For the Wilson mirror from Eq. (8.97), we have

$$\frac{I_O}{I_{REF}} \simeq \frac{1}{1 + \dfrac{2}{\beta^2}} = 0.9998$$

Thus $\dfrac{|I_O - I_{REF}|}{I_{REF}} \times 100 = 0.02\%$

whereas for the simple mirror from Eq. (8.18) we have

$$\frac{I_O}{I_{REF}} = \frac{1}{1 + \dfrac{2}{\beta}} = 0.98$$

Hence $\dfrac{|I_O - I_{REF}|}{I_{REF}} \times 100 = 2\%$

For the Wilson current mirror, we have

$$R_o = \frac{\beta r_o}{2} = \frac{100 \times 100\text{ k}\Omega}{2} = 5\text{ M}\Omega$$

and for the simple mirror, $R_o = r_o$
$= 100\text{ k}\Omega$.

Ex: 8.28 For the two current sources designed in Example 8.5, we have

$$g_m = \frac{I_C}{V_T} = \frac{10\ \mu\text{A}}{25\text{ mV}} = 0.4\,\frac{\text{mA}}{\text{V}}$$

$$r_o = \frac{V_A}{I_C} = \frac{100\text{ V}}{10\ \mu\text{A}} = 10\text{ M}\Omega,$$

$$r_\pi = \frac{\beta}{g_m} = 250\text{ k}\Omega$$

For the current source in Fig. 8.40(a), we have

$$R_o = r_{o2} = r_o = 10\text{ M}\Omega$$

For the current source in Fig. 8.40(b), from Eq. (8.105), we have

$$R_{out} \simeq \left[1 + g_m (R_E \parallel r_\pi)\right] r_o$$

From Example 8.5, $R_E = R_3 = 11.5\text{ k}\Omega$;

therefore,

$$R_{out} \simeq \left[1 + 0.4\frac{\text{mA}}{\text{V}}\,(11.5\text{ k}\Omega \parallel 250\text{ k}\Omega)\right] 10\text{ M}\Omega$$

$$\therefore R_{out} = 54\text{ M}\Omega$$

Chapter 9

Solutions to Exercises within the Chapter

Ex: 9.1 Referring to Fig. 9.3,

If R_D is doubled to 5 kΩ,

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} R_D$$

$$= 1.5 - \frac{0.4 \text{ mA}}{2} (5 \text{ k}\Omega) = 0.5 \text{ V}$$

$$V_{CM_{\max}} = V_t + V_D = 0.5 + 0.5 = +1.0 \text{ V}$$

Since the currents I_{D1}, and I_{D2} are still 0.2 mA each,

$$V_{GS} = 0.82 \text{ V}$$

So, $V_{CM_{\min}} = V_{SS} + V_{CS} + V_{GS}$

$$= -1.5 \text{ V} + 0.4 \text{ V} + 0.82 \text{ V} = -0.28 \text{ V}$$

So, the common-mode range is

-0.28 V to $+1.0$ V

Ex: 9.2 (a) The value of v_{id} that causes Q_1 to conduct the entire current is $\sqrt{2}\, V_{OV}$

$$\rightarrow \sqrt{2} \times 0.316 = 0.45 \text{ V}$$

then, $V_{D1} = V_{DD} - I \times R_D$

$$= 1.5 - 0.4 \times 2.5 = 0.5 \text{ V}$$

$$V_{D2} = V_{DD} = +1.5 \text{ V}$$

$$v_O = V_{D2} - V_{D1} = +1 \text{ V}$$

(b) For Q_2 to conduct the entire current:

$$v_{id} = -\sqrt{2}\, V_{OV} = -0.45 \text{ V}$$

then,

$$V_{D1} = V_{DD} = +1.5 \text{ V}$$

$$V_{D2} = 1.5 - 0.4 \times 2.5 = 0.5 \text{ V}$$

$$v_O = V_{D2} - V_{D1} = -1 \text{ V}$$

(c) Thus the differential output range is

$+1$ V to -1 V.

Ex: 9.3 Refer to answer table for Exercise 9.3 where values were obtained in the following way:

$$\frac{I}{2} = \frac{1}{2} k_n' \left(\frac{W}{L}\right) V_{OV}^2 \Rightarrow \frac{W}{L} = \frac{I}{k_n' V_{OV}^2} = \frac{2}{V_{OV}^2}$$

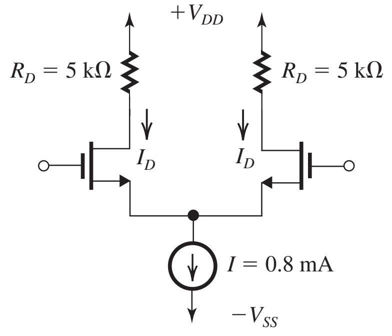
$$g_m = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}} = \frac{0.4}{V_{OV}} \text{ mA/V}$$

$$\left(\frac{v_{id}/2}{V_{OV}}\right)^2 = 0.1 \rightarrow v_{id} = 2\, V_{OV}\sqrt{0.1} =$$

$0.632\, V_{OV}$

Ex: 9.4 $I_D = \dfrac{I}{2} = \dfrac{0.8 \text{ mA}}{2} = 0.4 \text{ mA}$

$$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (V_{OV})^2$$



Thus,

$$V_{OV} = \sqrt{\frac{2 I_D}{k_n' \left(\frac{W}{L}\right)}} = \sqrt{\frac{2\,(0.4 \text{ mA})}{0.2\,(\text{mA/V}^2)\,(100)}}$$

$$= 0.2 \text{ V}$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{0.4 \text{ mA} \times 2}{0.2 \text{ V}} = 4 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20 \text{ V}}{0.4 \text{ mA}} = 50 \text{ k}\Omega$$

$$A_d = g_m\,(R_D \parallel r_o)$$

$$A_d = (4 \text{ mA/V})\,(5 \text{ k}\Omega \parallel 50 \text{ k}\Omega) = 18.2 \text{ V/V}$$

Ex: 9.5 With $I = 200$ μA, for all transistors,

$$I_D = \frac{I}{2} = \frac{200\ \mu\text{A}}{2} = 100\ \mu\text{A}$$

$$L = 2(0.18\ \mu\text{m}) = 0.36\ \mu\text{m}$$

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \frac{|V_A'|\, L}{I_D}$$

$$= \frac{(10 \text{ V}/\ \mu\text{m})\,(0.36\ \mu\text{m})}{0.1 \text{ mA}} = 36 \text{ k}\Omega$$

Since $I_{D1} = I_{D2} = \dfrac{1}{2}\, \mu_n C_{ox} \left(\dfrac{W}{L}\right) V_{OV}^2$,

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2 I_D}{\mu_n C_{ox} V_{OV}^2}$$

$$\frac{2(100\ \mu\text{A})}{(400\ \mu\text{A/V}^2)\,(0.2 \text{ V})^2} = 12.5$$

$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{2 I_D}{\mu_p C_{ox} |V_{OV}|^2}$$

$$\frac{2(100\ \mu\text{A})}{(100\ \mu\text{A/V}^2)\,(0.2)^2} = 50$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{(100\ \mu\text{A})\,(2)}{0.2 \text{ V}} = 1 \text{ mA/V},$$

so

$$A_d = g_{m1}(r_{o1} \parallel r_{o3}) = 1(\text{mA/V})\,(36 \text{ k}\Omega \parallel 36 \text{ k}\Omega)$$

$$= 18 \text{ V/V}$$

Ex: 9.6 $L = 2\,(0.18\ \mu\text{m}) = 0.36\ \mu\text{m}$

All $r_o = \dfrac{|V_A'| \cdot L}{I_D}$

The drain current for all transistors is

$$I_D = \frac{I}{2} = \frac{200\ \mu\text{A}}{2} = 100\ \mu\text{A}$$

$$r_o = \frac{(10\ \text{V}/\ \mu\text{m})\,(0.36\ \mu\text{m})}{0.1\ \text{mA}} = 36\ \text{k}\Omega$$

Referring to Fig. 9.13(a),

Since $I_D = \dfrac{1}{2}\,\mu_n C_{ox}\left(\dfrac{W}{L}\right)V_{OV}^2$ for all NMOS transistors,

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4$$

$$= \frac{2I_D}{\mu_n C_{ox} V_{OV}^2} = \frac{2(100\ \mu\text{A})}{400\ \mu\text{A}/\text{V}^2(0.2\ \text{V})^2} = 12.5$$

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8$$

$$= \frac{2I_D}{\mu_p C_{ox} V_{OV}^2} = \frac{2(100\ \mu\text{A})}{100\ \mu\text{A}/\text{V}^2(0.2\ \text{V})^2} = 50$$

For all transistors,

$$g_m = \frac{|I_D|}{|V_{OV}|/2} = \frac{(0.1\ \text{mA})\,(2)}{(0.2\ \text{V})} = 1\ \text{mA/V}$$

From Fig. 9.13(b),

$R_{on} = (g_{m3} r_{o3})\,r_{o1} = (1 \times 36) \times 36$

$= 1.296\ \text{M}\Omega$

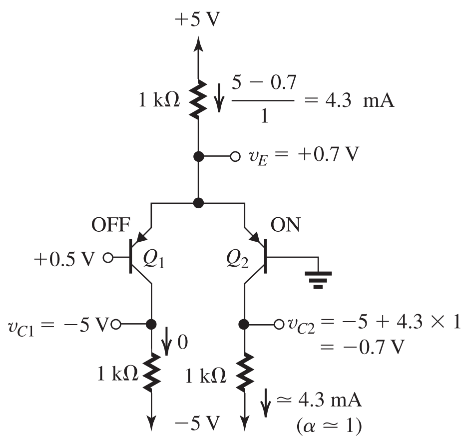
$R_{op} = (g_{m5} r_{o5}) r_{o7} = (1 \times 36) \times 36$

$= 1.296\ \text{M}\Omega$

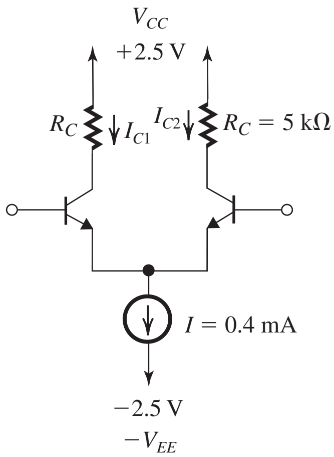
$A_d = g_{m1}\left(R_{on} \parallel R_{op}\right)$

$= (1\ \text{mA/V})\,(1.296\ \text{M}\Omega \parallel 1.296\ \text{M}\Omega)$
$= 648\ \text{V/V}$

Ex: 9.7



Ex: 9.8



$$I_{C1} = I_{C2} \simeq I_{E1} = I_{E2} = \frac{I}{2} = \frac{0.4\ \text{mA}}{2}$$

$= 0.2\ \text{mA}$

$V_{CM\max} \simeq V_C + 0.4\ \text{V}$

$= V_{CC} - I_C R_C + 0.4\ \text{V}$

$= 2.5 - 0.2\ \text{mA}\ (5\ \text{k}\Omega) + 0.4\ \text{V} = +1.9\ \text{V}$

$V_{CM\min} = -V_{EE} + V_{CS} + V_{BE}$

$V_{CM\min} = -2.5\ \text{V} + 0.3\ \text{V} + 0.7\ \text{V} = -1.5\ \text{V}$

Input common-mode range is -1.5 V to $+1.9$ V

Ex: 9.9 Substituting $i_{E1} + i_{E2} = I$ in Eq. (9.45) yields

$$i_{E1} = \frac{I}{1 + e^{(v_{B2} - v_{B1})/V_T}}$$

$$0.99\ I = \frac{I}{1 + e^{(v_{B2} - v_{B1})/V_T}}$$

$$v_{B1} - v_{B2} = -V_T \ln\left(\frac{1}{0.99} - 1\right)$$

$= -25 \ln(1/99)$

$= 25 \ln(99) = 115\ \text{mV}$

Ex: 9.10 (a) The DC current in each transister is 0.5 mA. Thus V_{BE} for each will be

$$V_{BE} = 0.7 + 0.025 \ln\left(\frac{0.5}{1}\right)$$

$= 0.683\ \text{V}$

$\Rightarrow v_E = 5 - 0.683 = +4.317\ \text{V}$

(b) $g_m = \dfrac{I_C}{V_T} = \dfrac{0.5}{0.025} = 20\ \dfrac{\text{mA}}{\text{V}}$

(c) $i_{C1} = 0.5 + g_{m1}\,\Delta v_{BE1}$

$= 0.5 + 20 \times 0.005 \sin(2\pi \times 1000t)$

$= 0.5 + 0.1 \sin(2\pi \times 1000t)$, mA

$i_{C2} = 0.5 - 0.1 \sin(2\pi \times 1000t)$, mA

(d) $v_{C1} = (V_{CC} - I_C R_C) - 0.1 \times R_C \sin(2\pi \times 1000t)$

$= (15 - 0.5 \times 10) - 0.1 \times 10 \sin(2\pi \times 1000t)$

$= 10 - 1 \sin(2\pi \times 1000t)$, V

$v_{C2} = 10 + 1 \sin(2\pi \times 1000t)$, V

(e) $v_{C2} - v_{C1} = 2 \cdot \sin(2\pi \times 1000t)$, V

(f) Voltage gain $\equiv \dfrac{v_{C2} - v_{C1}}{v_{B1} - v_{B2}}$

$= \dfrac{2 \text{ V peak}}{0.01 \text{ V peak}} = 200$ V/V

Ex: 9.11 The transconductance for each transistor is

$g_m = \sqrt{2\mu_n C_{ox} (W/L) I_D}$

$I_D = \dfrac{I}{2} = \dfrac{0.8 \text{ mA}}{2} = 0.4$ mA

Thus,

$g_m = \sqrt{2 \times 0.2 \times 100 \times 0.4} = 4$ mA/V

The differential gain for matched

R_D values is $A_d = \dfrac{v_{O2} - v_{O1}}{v_{id}} = g_m R_D$

If we ignore the 1% here, then we obtain

$A_d = g_m R_D = (4 \text{ mA/V})\,(5 \text{ k}\Omega) = 20$ V/V

$|A_{cm}| = \left(\dfrac{R_D}{2R_{SS}}\right)\left(\dfrac{\Delta R_D}{R_D}\right)$

$= \left(\dfrac{5}{2 \times 25}\right)(0.01) = 0.001$ V/V

CMRR (dB) $= 20 \log \dfrac{|A_d|}{|A_{CM}|} = 20 \log\left(\dfrac{20}{0.001}\right)$

$= 86$ dB

Ex: 9.12 From Exercise 9.11,

$g_m = 4$ mA/V

Using Eq. (9.85) and the fact that $R_{SS} = 25$ kΩ, we obtain

$\text{CMRR} = \dfrac{(2\, g_m R_{SS})}{\left(\dfrac{\Delta g_m}{g_m}\right)} = \dfrac{2(4 \text{ mA/V})\,(25 \text{ k}\Omega)}{0.01}$

$= 20,000$

CMRR (dB) $= 20 \log_{10} (20,000) = 86$ dB

Ex: 9.13 If the output of a MOS differential amplifier is taken single-endedly, then

$|A_d| = \dfrac{1}{2} g_m R_D$

(that is, half the gain obtained with the output taken differentially), and from Eq. (9.74) we have

$|A_{cm}| \simeq \dfrac{R_D}{2R_{SS}}$

Thus,

$\text{CMRR} \equiv \dfrac{|A_d|}{|A_{cm}|} = g_m R_{SS}$

Substituting $g_m = 4$ mA/V,

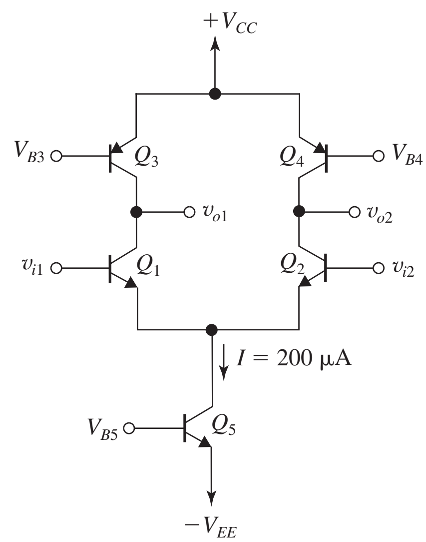
$R_D = 5$ kΩ, and $R_{SS} = 25$ kΩ,

$|A_d| = 10$ V/V

$|A_{cm}| = 0.1$ V/V

CMRR $= 100$ or 40 dB

Ex: 9.14



$I = 200\ \mu$A

Since $\beta \gg 1$,

$I_{C1} \approx I_{C2} \approx \dfrac{I}{2} = \dfrac{200\ \mu\text{A}}{2} = 100\ \mu$A

$g_{m1} = g_{m2} = g_m = \dfrac{I_C}{V_T} = \dfrac{100\ \mu\text{A}}{25 \text{ mV}} = 4$ mA/V

$R_{C1} = R_{C2} = R_C = r_o = \dfrac{|V_A|}{I_C}$

$= \dfrac{10 \text{ V}}{100\ \mu\text{A}} = 100$ kΩ

$r_{o1} = r_{o2} = \dfrac{V_A}{I/2} = \dfrac{10}{0.1} = 100$ kΩ

$r_{e1} = r_{e2} = r_e = \dfrac{V_T}{I_E} = \dfrac{25 \text{ mV}}{0.1 \text{ mA}} = 0.25$ kΩ

$|A_d| = \dfrac{R_C \parallel r_o}{r_e} = \dfrac{100 \text{ k}\Omega \parallel 100 \text{ k}\Omega}{0.25 \text{ k}\Omega}$

$= 200$ V/V

$R_{id} = 2r_\pi, \quad r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{4 \text{ mA/V}} = 25 \text{ k}\Omega$

$R_{id} = 2(25 \text{ k}\Omega) = 50 \text{ k}\Omega$

$R_{EE} = \dfrac{V_A}{I} = \dfrac{10 \text{ V}}{200 \ \mu\text{A}} = 50 \text{ k}\Omega$

If the total load resistance is assumed to be mismatched by 1%, then we have

$|A_{cm}| = \dfrac{R_C}{2R_{EE}} \dfrac{\Delta R_C}{R_C}$

$= \dfrac{100}{2 \times 50} \times 0.01 = 0.01 \text{ V/V}$

$\text{CMRR (dB)} = 20 \log_{10}\left|\dfrac{A_d}{A_{cm}}\right| = 20 \log_{10}\left|\dfrac{200}{0.01}\right|$

$= 86 \text{ dB}$

Using Eq. (9.96), we obtain

$$R_{icm} = \beta R_{EE} \cdot \frac{1 + \dfrac{R_C}{\beta r_o}}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

$$= 100 \times 50 \times \frac{1 + \dfrac{100}{100 \times 100}}{1 + \dfrac{100 + 2 \times 50}{100}}$$

$R_{icm} \simeq 1.68 \text{ M}\Omega$

Ex: 9.15 From Exercise 9.4:

$V_{OV} = 0.2 \text{ V}$

Using Eq. (9.97) we obtain V_{OS} due to $\Delta R_D / R_D$ as:

$V_{OS} = \left(\dfrac{V_{OV}}{2}\right) \cdot \left(\dfrac{\Delta R_D}{R_D}\right)$

$= \dfrac{0.2}{2} \times 0.02 = 0.002 \text{ V} \quad \text{i.e } 2 \text{ mV}$

To obtain V_{OS} due to Δk_n,

use Eq. (9.98),

$V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\left(\dfrac{\Delta k_n}{k_n}\right)$

$\Rightarrow V_{OS} = \left(\dfrac{0.2}{2}\right) \times 0.02 = 0.002$

$\Rightarrow 2 \text{ mV}$

The offset voltage arising from ΔV_t is obtained from Eq. (9.99):

$V_{OS} = \Delta V_t = 2 \text{ mV}$

Finally, from Eq. (9.100) the total input offset is

$V_{OS} =$

$$\left[\left(\frac{V_{OV}}{2} \frac{\Delta R_D}{R_D}\right)^2 + \left(\frac{V_{OV}}{2} \frac{\Delta k_n}{k_n}\right)^2 + (\Delta V_t)^2\right]^{1/2}$$

$= \sqrt{\left(2 \times 10^{-3}\right)^2 + \left(2 \times 10^{-3}\right)^2 + \left(2 \times 10^{-3}\right)^2}$

$= \sqrt{3 \times \left(2 \times 10^{-3}\right)^2}$

$= 3.5 \text{ mV}$

Ex: 9.16 From Eq. (9.103), we get

$V_{OS} = V_T \sqrt{\left(\dfrac{\Delta R_C}{R_C}\right)^2 + \left(\dfrac{\Delta I_S}{I_S}\right)^2}$

$= 25\sqrt{(0.02)^2 + (0.1)^2}$

$= 2.55 \text{ mV}$

$I_B = \dfrac{100}{2(\beta + 1)} = \dfrac{100}{2 \times 101} \cong 0.5 \ \mu\text{A}$

$I_{OS} = I_B \left(\dfrac{\Delta \beta}{\beta}\right)$

$= 0.5 \times 0.1 \ \mu\text{A} = 50 \text{ nA}$

Ex: 9.17 $I_D = \dfrac{1}{2} I = 0.4 \text{ mA}$

$I_D = \dfrac{1}{2} \mu_n C_{ox} \left(\dfrac{W}{L}\right)_n V_{OV}^2$

$0.4 = \dfrac{1}{2} \times 0.2 \times 100 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.2 \text{ V}$

$g_{m1,2} = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.4}{0.2} = 4 \text{ mA/V}$

$G_{md} = g_{m1,2} = 4 \text{ mA/V}$

$r_{o2} = \dfrac{V_{An}}{I_D} = \dfrac{20}{0.4} = 50 \text{ k}\Omega$

$r_{o4} = \dfrac{|V_{Ap}|}{I_D} = \dfrac{20}{0.4} = 50 \text{ k}\Omega$

$R_o = r_{o2} \parallel r_{o4} = 50 \parallel 50 = 25 \text{ k}\Omega$

$A_d = G_m R_o = 4 \times 25 = 100 \text{ V/V}$

Ex: 9.18 $G_{md} = g_{m1,2} \simeq \dfrac{I/2}{V_T} = \dfrac{0.4 \text{ mA}}{0.025 \text{ V}}$

$= 16 \text{ mA/V}$

$r_{o2} = r_{o4} = \dfrac{V_A}{I_C} = \dfrac{V_A}{I/2} = \dfrac{100}{0.4} = 250 \text{ k}\Omega$

$R_o = r_{o2} \parallel r_{o4} = 250 \parallel 250 = 125 \text{ k}\Omega$

$A_d = G_m R_o = 16 \times 125 = 2000 \text{ V/V}$

$R_{id} = 2r_\pi = 2 \times \dfrac{\beta}{g_{m1,2}} = 2 \times \dfrac{160}{16} = 20 \text{ k}\Omega$

Ex: 9.19 From Exercise 9.17, we get

$I_D = 0.4 \text{ mA}$

$V_{OV} = 0.2 \text{ V} \quad g_{m1,2} = 4 \text{ mA/V}$

$G_{md} = 4$ mA/V $\quad A_d = 100$ V/V

Now,

$R_{SS} = 25\ \text{k}\Omega$

$$g_{m3} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_p I_D}$$

$$= \sqrt{2 \times 0.1 \times 200 \times 0.4} = 4\ \text{mA/V}$$

$$|A_{cm}| = \frac{1}{2g_{m3}R_{SS}} = \frac{1}{2 \times 4 \times 25} = 0.005\ \text{V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{0.005}$$

$= 20{,}000$ or $20 \log 20{,}000 = 86$ dB

Ex: 9.20 Refer to Fig. (9.37).

(a) Using Eq. (9.142), we obtain

$$I_6 = \frac{(W/L)_6}{(W/L)_4}(I/2)$$

$$\Rightarrow 128 = \frac{(W/L)_6}{11.1} \times 64$$

thus, $(W/L)_6 = 22.2$

Using Eq. (9.143), we get

$$I_7 = \frac{(W/L)_7}{(W/L)_5} I$$

$$\Rightarrow 128 = \frac{(W/L)_7}{88.8} \times 128$$

thus, $(W/L)_7 = 88.8$

(b) For Q_1,

$$\frac{I}{2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 V_{OV_1}^2$$

$$\Rightarrow V_{OV1} = \sqrt{\frac{64}{\frac{1}{2} \times 128 \times 44.4}} = 0.15\ \text{V}$$

Similarly for Q_2, $V_{OV2} = 0.15$ V

For Q_6,

$$128 = \frac{1}{2} \times 512 \times 22.2 V_{OV6}^2$$

$\Rightarrow V_{OV6} = 0.15$ V

(c) $g_m = \dfrac{2I_D}{V_{OV}}$

	I_D	V_{OV}	g_m
Q_1	64 μA	0.15 V	0.853 mA/V
Q_2	64 μA	0.15 V	0.853 mA/V
Q_6	128 μA	0.15 V	1.71 mA/V

(d) $r_{o2} = 3.2/0.064 = 50\ \text{k}\Omega$

$r_{o4} = 3.2/0.064 = 50\ \text{k}\Omega$

$r_{o6} = 3.2/0.128 = 25\ \text{k}\Omega$

$r_{o7} = 3.2/0.128 = 25\ \text{k}\Omega$

(e) Eq. (9.140):

$$A_1 = -g_{m1}(r_{o2} \parallel r_{o4})$$

$$= -0.853(50 \parallel 50) = -21.3\ \frac{\text{V}}{\text{V}}$$

Eq. (9.141):

$$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$$

$= -21.3$ V/V

Overall voltage gain is

$$A_1 \times A_2 = -21.3 \times -21.3 = 454\ \text{V/V}$$

Ex: 9.21 $R_{id} = 20.2\ \text{k}\Omega$

$A_{vo} = 8513$ V/V

$R_o = 152\ \Omega$

With $R_S = 10\ \text{k}\Omega$ and $R_L = 1\ \text{k}\Omega$,

$$G_v = \frac{20.2}{20.2 + 10} \times 8513 \times \frac{1}{(1 + 0.152)}$$

$= 4943$ V/V

Ex: 9.22 $\dfrac{i_{e8}}{i_{b8}} = \beta_8 + 1 = 101$ A/A

$$\frac{i_{b8}}{i_{c7}} = \frac{R_5}{R_5 + R_{i4}} = \frac{15.7}{15.7 + 303.5} = 0.0492\ \text{A/A}$$

$$\frac{i_{c7}}{i_{b7}} = \beta_7 = 100\ \text{A/A}$$

$$\frac{i_{b7}}{i_{c5}} = \frac{R_3}{R_3 + R_{i3}} = \frac{3}{3 + 234.8} = 0.0126\ \text{A/A}$$

$$\frac{i_{c5}}{i_{b5}} = \beta_5 = 100\ \text{A/A}$$

$$\frac{i_{b5}}{i_{c2}} = \frac{R_1 + R_2}{R_1 + R_2 + R_{i2}} = \frac{40}{40 + 5.05} =$$

0.8879 A/A

$$\frac{i_{c2}}{i_1} = \beta_2 = 100\ \text{A/A}$$

Thus the overall current gain is

$$\frac{i_{e8}}{i_1} = 101 \times 0.0492 \times 100 \times 0.0126 \times 100$$

$\times\ 0.8879 \times 100$

$= 55{,}599$ A/A

and the overall voltage gain is

$$\frac{v_o}{v_{id}} = \frac{R_6}{R_{i1}} \cdot \frac{i_{e8}}{i_1}$$

$$= \frac{3}{20.2} \times 55599 = 8257\ \text{V/V}$$

Chapter 10

Solutions to Exercises within the Chapter

Ex: 10.1 $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11} \text{ F/m}}{3 \times 10^{-9} \text{ m}}$

$= 11.5 \times 10^{-3} \text{ F/m}^2$

$= 11.5 \text{ fF}/\mu\text{m}^2$

$C_{ov} = W L_{ov} C_{ox}$

$= 1.5 \times 0.03 \times 11.5 = 0.52 \text{ fF}$

$C_{gs} = \frac{2}{3} W L C_{ox} + C_{ov}$

$= \frac{2}{3} \times 1.5 \times 0.15 \times 11.5 + 0.52$

$= 2.25 \text{ fF}$

$C_{gd} = C_{ov} = 0.52 \text{ fF}$

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \frac{V_{SB}}{V_0}}} = \frac{1}{\sqrt{1 + \frac{0.2}{0.9}}} = 0.9 \text{ fF}$$

$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{V_{DB}}{V_0}}} = \frac{1}{\sqrt{1 + \frac{0.8 + 0.2}{0.9}}} = 0.7 \text{ fF}$$

Ex: 10.2 $g_m = \sqrt{2k_n'(W/L)I_D}$

$= \sqrt{2 \times 0.5 \times (1.5/0.15) \times 0.1} = 1 \text{ mA/V}$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$= \frac{1 \times 10^{-3}}{2\pi(2.25 + 0.52) \times 10^{-15}}$$

$f_T = 57.5 \text{ GHz}$

Ex: 10.3 $C_{de} = \tau_F g_m$

where

$\tau_F = 20 \text{ ps}$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

Thus,

$C_{de} = 20 \times 10^{-12} \times 40 \times 10^{-3} = 0.8 \text{ pF}$

$C_{je} \simeq 2C_{je0}$

$= 2 \times 20 = 40 \text{ fF}$

$C_\pi = C_{de} + C_{je}$

$= 0.8 + 0.04 = 0.84 \text{ pF}$

$$C_\mu = \frac{C_{\mu 0}}{\left(1 + \frac{V_{CB}}{V_{0c}}\right)^m}$$

$$= \frac{20}{\left(1 + \frac{2}{0.5}\right)^{0.33}} = 12 \text{ fF}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$= \frac{40 \times 10^{-3}}{2\pi(0.84 + 0.012) \times 10^{-12}} = 7.47 \text{ GHz}$$

Ex: 10.4 $|\beta| = 10$ at $f = 50$ MHz

Thus,

$f_T = 10 \times 50 = 500 \text{ MHz}$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 10^6}$$

$= 12.7 \text{ pF}$

$C_\pi = 12.7 - 2 = 10.7 \text{ pF}$

Ex: 10.5 $C_\pi = C_{de} + C_{je}$

$10.7 = C_{de} + 2$

$\Rightarrow C_{de} = 8.7 \text{ pF}$

Since C_{de} is proportional to g_m and hence I_C, at $I_C = 0.1$ mA,

$C_{de} = 0.87 \text{ pF}$

and

$C_\pi = 0.87 + 2 = 2.87 \text{ pF}$

$$f_T = \frac{4 \times 10^{-3}}{2\pi(2.87 + 2) \times 10^{-12}} = 130.7 \text{ MHz}$$

Ex: 10.6 $A_M = -g_m R_L'$

where

$g_m = 2 \text{ mA/V}$

$R_L' = R_L \| r_o = 10\|20 = 6.67 \text{ k}\Omega$

Thus,

$A_M = -2 \times 6.67 = -13.3 \text{ V/V}$

$C_{eq} = (1 + g_m R_L')C_{gd}$

$= (1 + 13.3) \times 5 = 71.5 \text{ fF}$

$C_{\text{in}} = C_{gs} + C_{eq} = 20 + 71.5 = 91.5$

$$f_H = \frac{1}{2\pi \; C_{\text{in}} R_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 91.5 \times 10^{-15} \times 20 \times 10^3}$$

$= 86.9 \text{ MHz}$

Ex: 10.7 $f_H = \frac{1}{2\pi \; C_{\text{in}} R_{\text{sig}}}$

For $f_H \geq 100$ MHz,

$$C_{\text{in}} \leq \frac{1}{2\pi \times 100 \times 10^6 \times 20 \times 10^3} = 79.6 \text{ fF}$$

But,

$C_{\text{in}} = C_{gs} + C_{eq} = 20 + C_{eq}$

Thus,

$C_{eq} \leq 59.6$ fF

$C_{gd}(1 + g_m R'_L) \leq 59.6$

$C_{gd} \leq \dfrac{59.6}{1+20} = 2.8$ fF

Ex: 10.8 Refer to Example 10.2.

To move f_t from 10.6 GHz to 3 GHz, f_H must be moved from 530.5 MHz to

$f_H = 530.5 \times \dfrac{3}{10.6} = 150.1$ MHz

Since,

$f_H = \dfrac{1}{2\pi(C_L + C_{gd})R'_L}$

then

$C_L + C_{gd} = \dfrac{1}{2\pi \times 150.1 \times 10^6 \times 10 \times 10^3} =$ 106 fF

$C_L = 106 - 5 = 101$ fF

Ex: 10.9 To reduce the midband gain to half the value found, we reduce R'_L by the same factor, thus

$R'_L = \dfrac{4.76}{2} = 2.38$ kΩ

But,

$R'_L = R_L \parallel r_o$

$2.38 = R_L \parallel 100$

$\Rightarrow R_L = 2.44$ kΩ

$C_{\text{in}} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 7 + 1(1 + 40 \times 2.38)$

$= 103.2$ pF

$f_H = \dfrac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$

$= \dfrac{1}{2\pi \times 103.2 \times 10^{-12} \times 1.67 \times 10^3}$

$= 923$ MHz

Thus, by accepting a reduction in gain by a factor of 2, the bandwidth is increased by a factor of 923/480 = 1.9, approximathely the same factor as the reduction in gain.

Ex: 10.10 $T(s) = \dfrac{1000}{1 + \dfrac{s}{2\pi \times 10^5}}$

GB $= 1000 \times 100 \times 10^3 = 10^8$ Hz

Ex: 10.11 $|A_M| = g_m R'_L = 2 \times 10 = 20$ V/V

GB $= |A_M| f_H$

$= 20 \times 56.8 = 1.14$ GHz

Ex: 10.12 $|A_M| = \dfrac{1}{2} \times 20 = 10$ V/V

$R_{gs} = 20$ kΩ

$R_{gd} = R_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= 20(1 + 10) + 5 = 225$ kΩ

$R_{C_L} = R'_L = 5$ kΩ

$\tau_{gs} = C_{gs}R_{gs} = 20 \times 10^{-15} \times 20 \times 10^3 = 400$ ps

$\tau_{gd} = C_{gd}R_{gd} = 5 \times 10^{-15} \times 225 \times 10^3 = 1125$ ps

$\tau_{C_L} = C_L R_{C_L} = 25 \times 10^{-15} \times 5 \times 10^3 = 125$ ps

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 400 + 1125 + 125$

$= 1650$ ps

$f_H = \dfrac{1}{2\pi \tau_H}$

$= \dfrac{1}{2\pi \times 1650 \times 10^{-12}} = 96.5$ MHz

GB $= 10 \times 96.5 = 0.965$ GHz

Ex: 10.13 $g_m = \sqrt{2\mu_n C_{ox} \dfrac{W}{L} I_D}$

Since I_D is increased by a factor of 4, g_m doubles:

$g_m = 2 \times 2 = 4$ mA/V

Since R'_L is $r_o/2$, increasing I_D by a factor of four results in r_o and hence R'_L decreasing by a factor of 4, thus

$R'_L = \dfrac{1}{4} \times 10 = 2.5$ kΩ

$|A_M| = g_m R'_L = 4 \times 2.5 = 10$ V/V

$R_{gs} = R_{\text{sig}} = 20$ kΩ

$R_{gd} = R_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= 20(1 + 10) + 2.5$

$= 222.5$ kΩ

$R_{C_L} = R'_L = 2.5$ kΩ

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= C_{gs}R_{\text{sig}} + C_{gd}R_{gd} + C_L R_{C_L}$

$= 20 \times 10^{-15} \times 20 \times 10^3 + 5 \times 10^{-15} \times 222.5$

$\quad \times 10^3 + 25 \times 10^{-15} \times 2.5 \times 10^3$

$= 400 + 1112.5 + 62.5$

$= 1575$ ps

$f_H = \dfrac{1}{2\pi \times 1575 \times 10^{-12}} = 101$ MHz

GB $= |A_M| f_H$

$= 10 \times 101 = 1.01$ GHz

Ex: 10.14 (a) $g_m = 40$ mA/V

$r_\pi = \frac{200}{40} = 5\ \text{k}\Omega$

$r_{on} = \frac{V_{An}}{I} = \frac{130}{1} = 130\ \text{k}\Omega$

$r_{op} = \frac{|V_{Ap}|}{I} = \frac{50}{1} = 50\ \text{k}\Omega$

$R'_L = r_{on} \parallel r_{op} = 130 \parallel 50 = 36.1\ \text{k}\Omega$

$A_M = -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m R'_L$

$= -\frac{5}{5+36} \times 40 \times 36.1$

$= -176$ V/V

(b) $C_{\text{in}} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 16 + 0.3(1 + 40 \times 36.1)$

$= 450$ pF

$R'_{\text{sig}} = r_\pi \parallel R_{\text{sig}}$

$= 5 \parallel 36 = 4.39\ \text{k}\Omega$

$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$

$= \frac{1}{2\pi \times 450 \times 10^{-12} \times 4.39 \times 10^3}$

$= 80.6$ kHz

(c) $R_\pi = R'_{\text{sig}} = 4.39\ \text{k}\Omega$

$R_\mu = R'_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= 4.39(1 + 40 \times 36.1) + 36.1$

$= 6.38\ \text{M}\Omega$

$R_{CL} = R'_L = 36.1\ \text{k}\Omega$

$\tau_H = C_\pi + C_\mu R_\mu + C_L R_{CL}$

$= 16 \times 4.39 + 0.3 \times 6.38 \times 10^3 + 5 \times 36.1$

$= 70.2 + 1914 + 180.5$

$= 2164.7$ ns

$f_H = \frac{1}{2\pi \times 2164.7 \times 10^{-9}}$

$= 73.5$ kHz

(d) $f_Z = \frac{g_m}{2\pi C_\mu}$

$= \frac{40 \times 10^{-3}}{2\pi \times 0.3 \times 10^{-12}} = 21.2$ GHz

(e) $\text{GB} = 175 \times 73.5 = 12.9$ MHz

Ex: 10.15 $R_{\text{in}} = \frac{R_L + r_o}{1 + g_m r_o}$

$= \frac{800 + 20}{1 + 40} \simeq 20\ \text{k}\Omega$

$G_v = \frac{R_L}{R_{\text{sig}} + R_{\text{in}}} = \frac{800}{20 + 20} = 20$ V/V

$R_{gs} = R_{\text{sig}} \parallel R_{\text{in}} = 20 \parallel 20 = 10\ \text{k}\Omega$

$R_{gd} = R_L \parallel R_o$

$= 800 \parallel 840 = 410\ \text{k}\Omega$

$\tau_H = C_{gs} R_{gs} + (C_{gd} + C_L) R_{gd}$

$= 20 \times 10^{-15} \times 10 \times 10^3 + (5 + 25) \times 10^{-15}$

$\times 410 \times 10^3$

$= 200 + 12{,}300$

$= 12{,}500$ ps

$f_H = \frac{1}{2\pi \times 12{,}500 \times 10^{-12}} = 12.7$ MHz

Thus, while the midband gain has been increased substantially (by a factor of 21), the bandwidth has been substantially lowered (by a factor of 20.7). Thus, the high-frequency advantage of the CG amplifier is completely lost!

Ex: 10.16 (a) $A_{\text{CS}} = -g_m(R_L \parallel r_o)$

$= -g_m(r_o \parallel r_o) = -\frac{1}{2} g_m r_o$

$= -\frac{1}{2} \times 40 = -20$ V/V

$A_{\text{cascode}} = -g_m(R_L \parallel R_o)$

$= -g_m(r_o \parallel g_m r_o r_o)$

$\simeq -g_m r_o = -40$ V/V

Thus,

$\frac{A_{\text{cascode}}}{A_{\text{CS}}} = 2$

(b) For the CS amplifier,

$\tau_H = C_{gs} R_{gs} + C_{gd} R_{gd}$

where

$R_{gs} = R_{\text{sig}}$

$R_{gd} = R_{\text{sig}}(1 + g_m R'_L) + R'_L$

$\simeq R_{\text{sig}}(1 + g_m R'_L)$

$= R_{\text{sig}}\left(1 + \frac{1}{2} g_m r_o\right)$

$= R_{\text{sig}}\left(1 + \frac{1}{2} \times 40\right) = 21 R_{\text{sig}}$

$\tau_H = C_{gs}R_{\text{sig}} + C_{gd} \times 21R_{\text{sig}}$

$= C_{gs}R_{\text{sig}} + 0.25C_{gs} \times 21R_{\text{sig}}$

$= 6.25C_{gs}R_{\text{sig}}$

$$f_H = \frac{1}{2\pi \times 6.25C_{gs}R_{\text{sig}}}$$

For the cascode amplifier,

$\tau_H \simeq R_{\text{sig}}C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})$

where

$$R_{d1} = r_{o1} \parallel R_{\text{in2}} = r_o \parallel \frac{r_o + r_o}{g_m r_o}$$

$$= r_o \parallel \frac{2}{g_m} = \frac{\frac{2}{g_m} r_o}{\frac{2}{g_m} + r_o}$$

$$= \frac{2r_o}{2 + g_m r_o} = \frac{2r_o}{2 + 40} = \frac{r_o}{21}$$

$$\tau_H = C_{gs}R_{\text{sig}}\left[1 + 0.25\left(1 + \frac{g_m r_o}{21}\right)\right]$$

$$= C_{gs}R_{\text{sig}}\left[1 + 0.25\left(1 + \frac{40}{21}\right)\right]$$

$= 1.73C_{gs}R_{\text{sig}}$

$$f_H = \frac{1}{2\pi \times 1.73\, C_{gs}R_{\text{sig}}}$$

Thus,

$$\frac{f_H(\text{cascode})}{f_H(\text{CS})} = \frac{6.25}{1.73} = 3.6$$

(c) $\dfrac{f_t(\text{cascode})}{f_t(\text{CS})} = 2 \times 3.6 = 7.2$

Ex: 10.17 $g_m = 40$ mA/V

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{40} = 5\ \text{k}\Omega$$

$R_{\text{in}} = r_\pi = 5\ \text{k}\Omega$

$A_0 = g_m r_o$

$= 40 \times 130 = 5200$

$R_{o1} = r_{o1} = 130\ \text{k}\Omega$

$$R_{\text{in2}} = r_{\pi 2} \parallel \frac{r_{o2} + R_L}{g_{m2}r_{o2}}$$

$$= 5 \parallel \frac{130 + 50}{5200}$$

$= 35\ \Omega$

$R_o \simeq \beta_2 r_{o2} = 200 \times 130 = 26\ \text{M}\Omega$

$$A_M = -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m(R_o \parallel R_L)$$

$$= -\frac{5}{5 + 36} 40(26{,}000 \parallel 50)$$

$A_M = -244$ V/V

$R'_{\text{sig}} = r_{\pi 1} \parallel R_{\text{sig}}$

$= 5 \parallel 36 = 4.39\ \text{k}\Omega$

$R_{\pi 1} = R'_{\text{sig}} = 4.39\ \text{k}\Omega$

$R_{c1} = r_{o1} \parallel R_{\text{in2}}$

$= 130\ \text{k}\Omega \parallel 35\ \Omega \simeq 35\ \Omega$

$R_{\mu 1} = R'_{\text{sig}}(1 + g_{m1}R_{c1}) + R_{c1}$

$= 4.39(1 + 40 \times 0.035) + 0.035$

$= 10.6\ \text{k}\Omega$

$\tau_H = C_{\pi 1}R_{\pi 1} + C_{\mu 1}R_{\mu 1} + C_{\pi 2}R_{c1}$

$\quad + (C_L + C_{\mu 2})(R_L \parallel R_o)$

$= 16 \times 4.39 + 0.3 \times 10.6 + 16 \times 0.035$

$\quad + (5 + 0.3)(50 \parallel 26{,}000)$

$= 70.24 + 3.18 + 0.56 + 264.5$

$= 338.5$ ns

$$f_H = \frac{1}{2\pi \times 338.5 \times 10^{-9}} = 470\ \text{kHz}$$

$f_t = |A_M| f_H = 244 \times 470 = 113.8$ MHz

Thus, in comparison to the CE amplifier of Exercise 10.19, we see that $|A_M|$ has increased from 175 V/V to 242 V/V, f_H has increased from 73.5 kHz to 470 kHz, and f_t has increased from 12.9 MHz to 113.8 MHz.

To have f_H equal to 1 MHz,

$$\tau_H = \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 1 \times 10^6} = 159.2\ \text{ns}$$

Thus,

$159.2 = 70.24 + 3.18 + 0.56$

$\quad + (C_L + C_\mu)(50 \parallel 26{,}000)$

$\Rightarrow C_L + C_\mu = 1.71$ pF

Thus, C_L must be reduced to 1.41 pF.

Ex: 10.18 From Eq. (10.103), we obtain

$$R_{gs} = \frac{R_{\text{sig}}}{g_m R'_L + 1} + \frac{R'_L}{g_m R'_L + 1} = \frac{R_{\text{sig}} + R'_L}{g_m R'_L + 1}$$

$R_{gd} = R_{\text{sig}}$

$$R_{C_L} = \frac{R'_L}{g_m R'_L + 1}$$

Ex: 10.19 From Example 10.8, we get

$\tau_H = b_1 = 104$ ps

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 104 \times 10^{-12}} = 1.53 \text{ GHz}$$

This is lower than the exact value found in Example 10.8 (i.e., 1.86 GHz) by about 18%, still not a bad estimate!

Ex: 10.20 $g_m = 40$ mA/V

$r_e = 25\ \Omega$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$R_{\text{sig}} = 1\text{ k}\Omega$

$R'_L = R_L \parallel r_o = 1 \parallel 100 = 0.99\text{ k}\Omega$

$$A_M = \frac{R'_L}{R'_L + r_e + \dfrac{R'_{\text{sig}}}{\beta + 1}}$$

$$= \frac{0.99}{0.99 + 0.025 + (1/101)} = 0.97 \text{ V/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 400 \times 10^6}$$

$= 15.9$ pF

$C_\mu = 2$ pF

$C_\pi = 13.9$ pF

$$f_Z = \frac{1}{2\pi C_\pi r_e} = \frac{1}{2\pi \times 13.9 \times 10^{-12} \times 25}$$

$= 458$ MHz

$$b_1 = \frac{\left[C_\pi + C_\mu\left(1 + \dfrac{R'_L}{r_e}\right)\right]R_{\text{sig}} + \left[C_\pi + C_L\left(1 + \dfrac{R_{\text{sig}}}{r_\pi}\right)\right]R'_L}{1 + \dfrac{R'_L}{r_e} + \dfrac{R_{\text{sig}}}{r_\pi}}$$

$$= \frac{\left[13.9 + 2\left(1 + \dfrac{0.99}{0.025}\right)\right] \times 1 + (13.9 + 0)0.99}{1 + \dfrac{0.99}{0.025} + \dfrac{1}{2.5}}$$

$= 2.66 \times 10^{-9}$ s

$$b_2 = \frac{C_\pi C_\mu R_L R_{\text{sig}}}{1 + \dfrac{R'_L}{r_e} + \dfrac{R_{\text{sig}}}{r_\pi}} = \frac{13.9 \times 2 \times 0.99 \times 1}{1 + \dfrac{0.99}{0.025} + \dfrac{1}{2.5}}$$

$= 0.671 \times 10^{-18}$

ω_{P1} and ω_{P2} are the roots of the equation

$1 + b_1 s + b_2 s^2 = 0$

Solving we obtain,

$f_{P1} = 67.2$ MHz

$f_{P2} = 562$ MHz

Since $f_{P1} \ll f_{P2}$,

$f_H \simeq f_{P1} = 67.2$ MHz

Ex: 10.21 (a) $I_{D1,2} = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{W}{L}\right)_{1,2} V_{OV}^2$

$$0.4 = \frac{1}{2} \times 0.2 \times 100 V_{OV}^2$$

$\Rightarrow V_{OV} = 0.2$ V

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.4}{0.2} = 4 \text{ mA/V}$$

(b) $A_d = g_m(R_D \parallel r_o)$

where

$$r_o = \frac{V_A}{I_D} = \frac{20}{0.4} = 50 \text{ k}\Omega$$

$A_d = 4(5 \parallel 50) = 4 \times 4.545$

$= 18.2$ V/V

(c) $f_H = \dfrac{1}{2\pi(C_L + C_{gd} + C_{db})(R_D \parallel r_o)}$

$$= \frac{1}{2\pi(100 + 10 + 10) \times 10^{-15} \times 4.545 \times 10^3}$$

$= 292$ MHz

(d) $\tau_{gs} = C_{gs}R_{\text{sig}} = 50 \times 10 = 500$ ps

$\tau_{gd} = C_{gd}R_{gd} = C_{gd}\left[R_{\text{sig}}(1 + g_m R'_L) + R'_L\right]$

$= 10[10(1 + 18.2) + 4.545]$

$= 1965.5$ ps

$\tau_{C_L} = (C_L + C_{db})R'_L = 110 \times 4.545 = 500$ ps

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 500 + 1965.5 + 500 = 2965.5$ ps

$$f_H = \frac{1}{2\pi \times 2965.5 \times 10^{-12}}$$

$= 53.7$ MHz

Ex: 10.22 $f_Z = \dfrac{1}{2\pi R_{SS}C_{SS}}$

$$= \frac{1}{2\pi \times 75 \times 10^3 \times 0.4 \times 10^{-12}}$$

$= 5.3$ MHz

Thus, the 3-dB frequency of the CMRR is 5.3 MHz.

Ex: 10.23 $A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

where

$g_{m1,2} = \dfrac{0.5}{0.025} = 20$ mA/V

$r_{o2} = r_{o4} = \dfrac{100}{0.5} = 200$ kΩ

$A_d = 20(200 \parallel 200) = 2000$ V/V

The dominant high-frequency pole is that introduced at the output node,

$$f_H = \frac{1}{2\pi C_L(r_{o2} \parallel r_{o4})}$$

$$= \frac{1}{2\pi \times 2 \times 10^{-12} \times 100 \times 10^3}$$

$= 0.8$ MHz

Ex: 10.24 (a) $A_M = -g_m R'_L$

where

$R'_L = R_L \parallel r_o = 20 \parallel 20 = 10$ kΩ

$A_M = -2 \times 10 = -20$ V/V

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R'_L$

$= C_{gs}R_{\text{sig}} + C_{gd}\left[R_{\text{sig}}(1 + g_m R'_L) + R'_L\right] + C_L R'_L$

$= 20 \times 20 + 5\,[20(1 + 20) + 10] + 5 \times 10$

$= 400 + 2150 + 50$

$= 2600$ ps

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 2600 \times 10^{-12}}$$

$= 61.2$ MHz

$\text{GB} = |A_M|\, f_H$

$= 20 \times 61.2$

$= 1.22$ GHz

(b) $G_m = \dfrac{g_m}{1 + g_m R_s} = \dfrac{2}{1+2} = 0.67$ mA/V

$R_o \simeq r_o(1 + g_m R_s)$

$= 20 \times 3 = 60$ kΩ

$R'_L = R_L \parallel R_o = 20 \parallel 60 = 15$ kΩ

$A_M = -G_m R'_L$

$= -0.67 \times 15 = -10$ V/V

$R_{gd} = R_{\text{sig}}(1 + G_m R'_L) + R'_L$

$= 20(1 + 10) + 15$

$= 235$ kΩ

$R_{C_L} = R'_L = 15$ kΩ

$$R_{gs} = \frac{R_{\text{sig}} + R_s + R_{\text{sig}}R_s/(r_o + R_L)}{1 + g_m R_s\left(\dfrac{r_o}{r_o + R_L}\right)}$$

where

$R_s = \dfrac{2}{g_m} = 1$ kΩ

$$R_{gs} = \frac{20 + 1 + \dfrac{20 \times 1}{20 + 20}}{1 + 2 \times \dfrac{20}{20 + 20}}$$

$= 10.75$ kΩ

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L}$

$= 20 \times 10.75 + 5 \times 235 + 5 \times 15$

$= 215 + 1175 + 75 = 1465$ ps

$$f_H = \frac{1}{2\pi \times 1465 \times 10^{-12}} = 109 \text{ MHz}$$

$\text{GB} = 10 \times 109 = 1.1$ GHz

Ex: 10.25 Refer to Fig. 10.35(b).

$$A_M = \frac{2r_\pi}{2r_\pi + R_{\text{sig}}} \times \frac{1}{2} \times g_m R_L$$

where

$g_m = 20$ mA/V

$r_\pi = \dfrac{100}{20} = 5$ kΩ

$$A_M = \frac{10}{10 + 10} \times \frac{1}{2} \times 20 \times 10 = 50 \text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi\left(\dfrac{C_\pi}{2} + C_\mu\right)(2r_\pi \parallel R_{\text{sig}})}$$

$$= \frac{1}{2\pi\left(\dfrac{6}{2} + 2\right) \times 10^{-12}(10 \parallel 10) \times 10^3}$$

$= 6.4$ MHz

$$f_{P2} = \frac{1}{2\pi C_\mu R_L}$$

$$= \frac{1}{2\pi \times 2 \times 10^{-12} \times 10 \times 10^3}$$

$= 8$ MHz

$$T(s) = \frac{50}{\left(1 + \dfrac{s}{\omega_{P1}}\right)\left(1 + \dfrac{s}{\omega_{P2}}\right)}$$

$$|T(j\omega)| = \frac{50}{\sqrt{\left[1 + \left(\dfrac{\omega}{\omega_{P1}}\right)^2\right]\left[1 + \left(\dfrac{\omega}{\omega_{P2}}\right)^2\right]}}$$

At $\omega = \omega_H$, $|T| = 50/\sqrt{2}$, thus

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{\omega_{P2}}\right)^2\right]$$

$$= 1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2 + \left(\frac{\omega_H}{\omega_{P2}}\right)^2 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\left(\frac{\omega_H}{\omega_{P2}}\right)^2$$

$$\frac{\omega_H^4}{\omega_{P1}^2\omega_{P2}^2} + \omega_H^2\left(\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2}\right) - 1 = 0$$

$$\frac{f_H^4}{f_{P1}^2 f_{P2}^2} + f_H^2\left(\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}\right) - 1 = 0$$

$$\frac{f_H^4}{6.4^2 \times 8^2} + f_H^2\left(\frac{1}{6.4^2} + \frac{1}{8^2}\right) - 1 = 0$$

$\Rightarrow f_H = 4.6$ MHz (Exact value)

Using Eq. (10.150), an approximathe value for f_H can be obtained:

$$f_H \simeq 1/\sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}}$$

$$= 1/\sqrt{\frac{1}{6.4^2} + \frac{1}{8^2}} = 5 \text{ MHz}$$

Ex: 10.26 $A_M = -\dfrac{R_G}{R_G + R_{\text{sig}}} g_m(R_D \parallel R_L)$

$$= -\frac{10}{10 + 0.1} \times 2(10 \parallel 10)$$

$$= -9.9 \text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_{\text{sig}} + R_G)}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6}(0.1 + 10) \times 10^6}$$

$$= 0.016 \text{ Hz}$$

$$f_{P2} = \frac{g_m + 1/R_S}{2\pi C_S}$$

$$= \frac{(2 + 0.1) \times 10^{-3}}{2\pi \times 1 \times 10^{-6}} = 334.2 \text{ Hz}$$

$$f_{P3} = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6}(10 + 10) \times 10^3}$$

$$= 8 \text{ Hz}$$

$$f_Z = \frac{1}{2\pi C_S R_S}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6} \times 10 \times 10^3}$$

$$= 15.9 \text{ Hz}$$

Since the highest-frequency pole is $f_{P2} = 334.2$ and the next highest-frequency singularity is f_Z at 15.9 Hz, the lower 3-dB frequency f_L will be

$$f_L \simeq f_{P2} = 334.2 \text{ Hz}$$

Ex: 10.27 Refer to Fig. 10.42.

$$\tau_{C1} = C_{C1}\left[R_{\text{sig}} + (R_B \parallel r_\pi)\right]$$

$$= 1 \times 10^{-6}\,[5 + (100 \parallel 2.5)] \times 10^3$$

$$= 7.44 \text{ ms}$$

$$\tau_{CE} = C_E\left[R_E \parallel \left(r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1}\right)\right]$$

$$\beta = g_m r_\pi = 40 \times 2.5 = 100$$

$$r_e \simeq 1/g_m = 25\ \Omega$$

$$\tau_{CE} = 1 \times 10^{-6}\left[5 \parallel \left(0.025 + \frac{100 \parallel 5}{101}\right)\right] \times 10^3$$

$$\tau_{CE} = 0.071 \text{ ms}$$

$$\tau_{C2} = C_{C2}(R_C + R_L)$$

$$= 1 \times 10^{-6}(8 + 5) \times 10^3$$

$$= 13 \text{ ms}$$

$$f_L = \frac{1}{2\pi}\left[\frac{1}{\tau_{C1}} + \frac{1}{\tau_{CE}} + \frac{1}{\tau_{C2}}\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{7.44} + \frac{1}{0.071} + \frac{1}{13}\right] \times 10^3$$

$$= 2.28 \text{ kHz}$$

$$f_Z = \frac{1}{2\pi C_E R_E}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6} \times 5 \times 10^3} = 31.8 \text{ Hz}$$

Since f_Z is much lower than f_L it will have a negligible effect on f_L.

Chapter 11

Solutions to Exercises within the Chapter

Ex: 11.1 (b)

$\beta = 1/ \left. A_f \right|_{\text{ideal}} = \dfrac{1}{10} = 0.1$ V/V

$0.1 = \dfrac{R_1}{R_1 + R_2}$

$\Rightarrow \dfrac{R_2}{R_1} = 9$

(c)

$A = 100 \quad \beta = 0.1$

$A\beta = 10$

$1 + A\beta = 11$ or 20.8 dB

$A_f = \dfrac{A}{1 + A\beta} = \dfrac{100}{11} = 9.091$ V/V

A_f differs from the ideal value of 10 V/V by -9.1%.

(d)

$10 = \dfrac{A}{1 + A\beta} = \dfrac{100}{1 + 100\beta}$

$\Rightarrow \beta = 0.09$ V/V

$0.09 = \dfrac{R_1}{R_1 + R_2}$

$\Rightarrow \dfrac{R_2}{R_1} = 10.11$

(e)

$V_o = A_f V_s = 10 \times 1 = 10$ V

$V_f = \beta V_o = 0.09 \times 10 = 0.9$ V

$V_i = V_s - V_f = 1 - 0.9 = 0.1$ V

(f)

$A = 0.8 \times 100 = 80$

$A_f = \dfrac{80}{1 + 80 \times 0.09}$

$= 9.76$ V/V

which is a 2.44% decrease.

Ex. 11.2 (b)

$\beta = 1/ \left. A_f \right|_{\text{ideal}} = 1/10^3$

$\beta = 0.001$ V/V

$0.001 = \dfrac{R_1}{R_1 + R_2}$

$\Rightarrow \dfrac{R_2}{R_1} = 999$

(c)

$A\beta = 10^4 \times 10^{-3} = 10$

$1 + A\beta = 11$ or 20.8 dB

$A_f = \dfrac{A}{1 + A\beta} = \dfrac{10^4}{11} = 909.1$ V/V

(d)

$1000 = \dfrac{10^4}{1 + 10^4\beta}$

$\Rightarrow \beta = 0.0009$

$0.0009 = \dfrac{R_1}{R_1 + R_2}$

$\Rightarrow \dfrac{R_2}{R_1} = 1110.1$

(e)

$V_o = A_f V_s = 1000 \times 0.01 = 10$ V

$V_f = \beta V_o = 0.0009 \times 10 = 0.009$ V

$V_i = V_s - V_f = 0.01 - 0.009 = 0.001$ V

(f)

$A = 0.8 \times 10^4$

$A_f = \dfrac{0.8 \times 10^4}{1 + 0.8 \times 10^4 \times 0.0009}$

$= 975.6$ V/V

which is a 2.44% decrease.

Ex. 11.3 To constrain the corresponding change in A_f to 0.1%, we need an amount-of-feedback of at least

$1 + A\beta = \dfrac{10\%}{0.1\%} = 100$

Thus the largest obtainable closed-loop gain will be

$A_f = \dfrac{A}{1 + A\beta} = \dfrac{1000}{100} = 10$ V/V

Each amplifier in the cascade will have a nominal gain of 10 V/V and a maximum variability of 0.1%; thus the overall voltage gain will be $(10)^3 = 1000$ V/V and the maximum variability will be 0.3%.

Ex. 11.4

$\beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{1}{1 + 9} = 0.1$

$A\beta = 10^4 \times 0.1 = 1000$

$1 + A\beta = 1001$

$A_f = \dfrac{A}{1 + A\beta}$

$$A_f = \frac{10^4}{1 + 10^4 \times 0.1} = 9.99 \text{ V/V}$$

$$f_{Hf} = f_H(1 + A\beta)$$

$$= 100 \times 1001 = 100.1 \text{ kHz}$$

Ex. 11.5 (a) Refer to Fig. 11.8(c).

$$\beta = \frac{R_1}{R_1 + R_2}$$

(b)

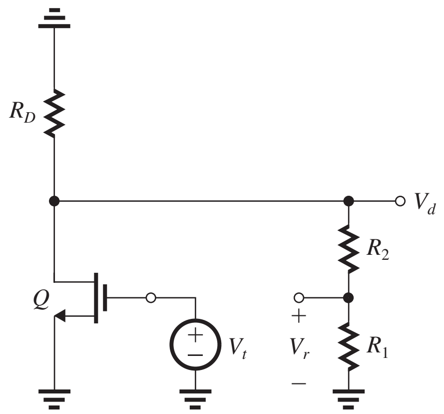


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain $A\beta$. Observe that we have eliminated the input signal V_s, and opened the loop at the gate of Q where the input impedance is infinite obviating the need for a termination resistance at the right-hand side of the break. Now we need to analyze the circuit to determine

$$A\beta \equiv -\frac{V_r}{V_t}$$

First, we write for the gain of the CS amplifier Q,

$$\frac{V_d}{V_t} = -g_m R_D \parallel (R_1 + R_2) \tag{1}$$

then we use the voltage-divider rule to find V_r,

$$\frac{V_r}{V_d} = \frac{R_1}{R_1 + R_2} \tag{2}$$

Combining Eqs. (1) and (2) gives

$$A\beta \equiv -\frac{V_r}{V_t} = g_m R_D \parallel (R_1 + R_2)\frac{R_1}{R_1 + R_2}$$

which can be simplified to

$$A\beta = g_m \frac{R_D R_1}{R_D + R_1 + R_2}$$

(c) $A = \dfrac{A\beta}{\beta}$

$$= g_m \frac{R_D(R_1 + R_2)}{R_D + R_1 + R_2}$$

(d) $\beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{20}{20 + 80} = 0.2$ V/V

$$A\beta = 4\frac{10 \times 20}{10 + 20 + 80} = 7.27$$

$$A = \frac{7.27}{0.2} = 36.36 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{36.36}{1 + 36.36 \times 0.2} = 4.4 \text{ V/V}$$

If $A\beta$ were $\gg 1$, then

$$A_f \simeq \frac{1}{\beta} = \frac{1}{0.2} = 5 \text{ V/V}$$

Ex. 11.6 From the solution of Example 11.5,

$$1 + A\beta = 60.1$$

Thus,

$$f_{Hf} = (1 + A\beta) f_H$$

$$= 60.1 \times 1$$

$$= 60.1 \text{ kHz}$$

Ex. 11.7 Refer to Fig. E11.7. The 1-mA bias current will split equally between the emitters of Q_1 and Q_2, thus

$$I_{E1} = I_{E2} = 0.5 \text{ mA}$$

Transistor Q_3 will be operating at an emitter current

$$I_{E3} = 5 \text{ mA}$$

determined by the 5-mA current source. Since the dc component of $V_s = 0$, the negative feedback will force the dc voltage at the output to be approximately zero.

The β circuit is shown in Fig. 1 (on next page) together with the determination of β and of the loading effects of the β circuit on the A circuit,

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{1}{1 + 9} = 0.1 \text{ V/V}$$

$$R_{11} = R_1 \parallel R_2 = 1 \parallel 9 = 0.9 \text{ k}\Omega$$

$$R_{22} = R_1 + R_2 = 1 + 9 = 10 \text{ k}\Omega$$

The A circuit is shown in Fig. 2 (on next page).

$$r_{e1} = r_{e2} = \frac{V_T}{I_{E1,2}} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \ \Omega$$

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5 \ \Omega$$

$$i_e = \frac{V_i}{r_{e1} + r_{e2} + \dfrac{R_s + R_{11}}{\beta + 1}}$$

These figures belong to Exercise 11.7.

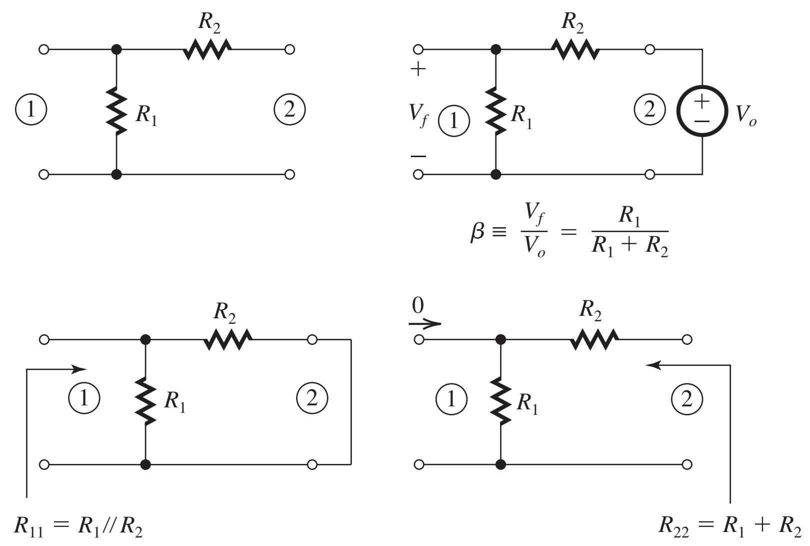


Figure 1

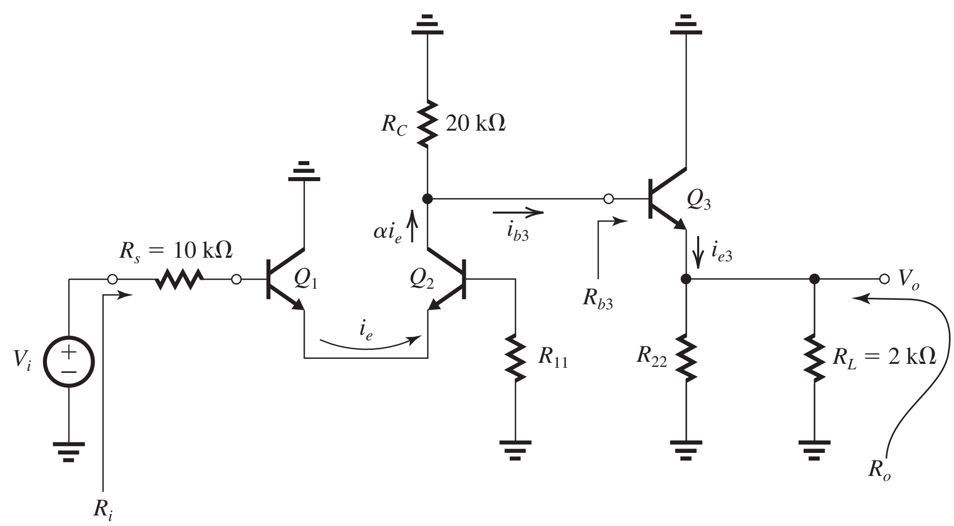


Figure 2

$$i_e = \frac{V_i}{0.05 + 0.05 + \dfrac{10 + 0.9}{101}}$$

$$\Rightarrow i_e = 4.81 V_i \qquad (1)$$

$$R_{b3} = (\beta + 1)[r_{e3} + (R_{22} \parallel R_L)]$$

$$= 101[0.005 + (10 \parallel 2)]$$

$$= 168.84 \text{ k}\Omega$$

$$i_{b3} = \alpha i_e \frac{R_C}{R_C + R_{b3}}$$

$$= 0.99\, i_e \frac{20}{20 + 168.84}$$

$$\Rightarrow i_{b3} = 0.105 i_e \qquad (2)$$

$$V_o = i_{e3}(R_{22} \parallel R_L)$$

$$= (\beta + 1) i_{b3}(R_{22} \parallel R_L)$$

$$= i_{b3} \times 101(10 \parallel 2)$$

$$\Rightarrow V_o = 168.33 i_{b3} \qquad (3)$$

Combining (1)–(3), we obtain

$$A \equiv \frac{V_o}{V_i} = 85 \text{ V/V}$$

$$\beta = 0.1 \text{ V/V}$$

$A\beta = 8.5$

$1 + A\beta = 9.5$

$A_f = \frac{85}{9.5} = 8.95$ V/V

From the A circuit, we have

$R_i = R_s + R_{11} + (\beta + 1)(r_{e1} + r_{e2})$

$= 10 + 0.9 + 101 \times 0.1$

$= 21$ kΩ

$R_{if} = R_i(1 + A\beta)$

$= 21 \times 9.5 = 199.5$ kΩ

$R_{\text{in}} = R_{if} - R_s = 199.5 - 10 = 189.5$ kΩ

From the A circuit, we have

$R_o = R_L \parallel R_{22} \parallel \left(r_{e5} + \frac{R_C}{\beta + 1}\right)$

$= 2 \parallel 10 \parallel \left(0.005 + \frac{20}{101}\right)$

$= 181\ \Omega$

$R_{of} = \frac{R_o}{1 + A\beta}$

$= \frac{181}{9.5} = 19.1\ \Omega$

$R_{of} = R_L \parallel R_{\text{out}}$

$19.1 = 2\text{ k}\Omega \parallel R_{\text{out}}$

$\Rightarrow R_{\text{out}} = 19.2\ \Omega$

Ex. 11.8 Figure 1 shows the β circuit together with the determination of β, R_{11} and R_{22}.

$\beta = \frac{R_1}{R_1 + R_2}$

$R_{11} = R_1 \parallel R_2$

$R_{22} = R_1 + R_2$

Figure 2 shows the A circuit. We can write

$V_o = g_m(R_D \parallel R_{22})V_i$

Thus,

$A \equiv \frac{V_o}{V_i} = g_m R_D \parallel (R_1 + R_2)$

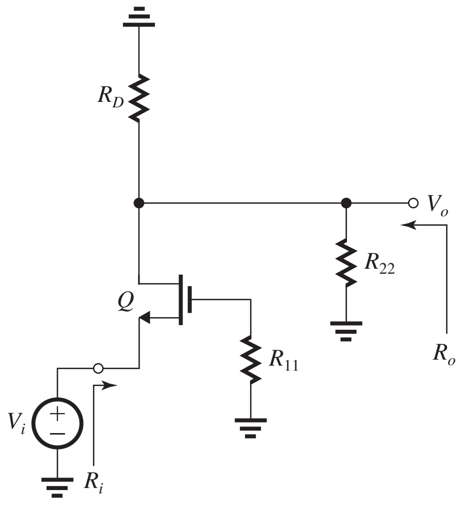


Figure 2

$\beta = \frac{R_1}{R_1 + R_2}$

$A_f = \frac{A}{1 + A\beta}$

From A circuit, we have

$R_i = \frac{1}{g_m}$

This figure belongs to Exercise 11.8.

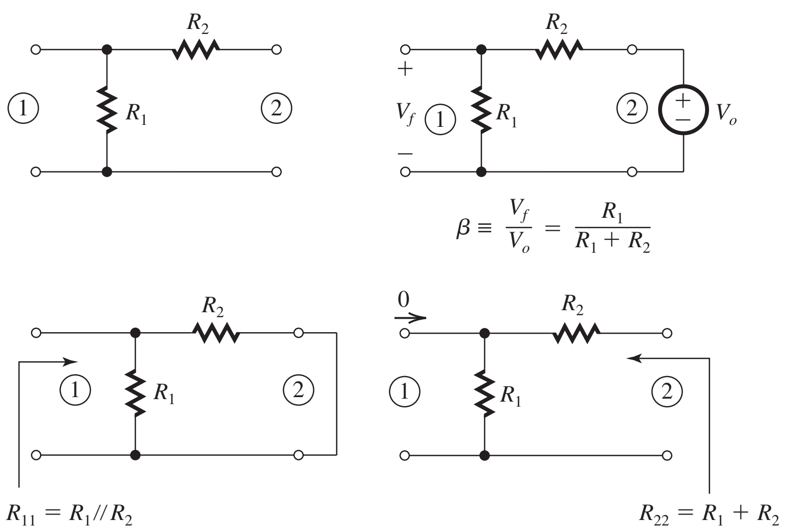


Figure 1

$R_o = R_D \parallel R_{22}$

$R_{\text{in}} = R_{if} = R_i(1 + A\beta)$

$$R_{\text{in}} = \frac{1}{g_m}(1 + A\beta)$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$R_{\text{out}} = \frac{R_D \parallel (R_1 + R_2)}{1 + A\beta}$$

Comparison with the results of Exercise 11.6 shows that the expressions for A and β are identical. However, R_{in} and R_{out} cannot be determined using the method of Exercise 11.6.

Ex. 11.9 $A_f\big|_{\text{ideal}} = 10$ A/A

$$\beta = \frac{1}{A_f\big|_{\text{ideal}}} = 0.1 \text{ A/A}$$

$A = 1000$ A/A

$1 + A\beta = 1 + 1000 \times 0.1 = 101$

$$A_f = \frac{A}{1 + A\beta} = \frac{1000}{101} = 9.9 \text{ A/A}$$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{1000\ \Omega}{101} = 9.9\ \Omega$$

$R_{of} = R_o(1 + A\beta) = 100\ \text{k}\Omega \times 101$

$= 10.1\ \text{M}\Omega$

Ex. 11.10 Refer to the solution of Example 11.7. If μ is reduced from 1000 V/V to 100 V/V, A will be reduced by the same factor (10) to become

$A = 64.9$ mA/V

and the loop gain will be correspondingly reduced,

$A\beta = 64.9$

Thus, the amount of feedback becomes

$1 + A\beta = 65.9$

and the closed-loop gain A_f becomes

$$A_f = \frac{A}{1 + A\beta} = \frac{64.9}{65.9} = 0.985 \text{ mA/V}$$

The input resistance becomes

$R_{\text{in}} = R_{if} = (1 + A\beta)R_i$

$= 65.9 \times 101 = 6.7\ \text{M}\Omega$

The output resistance becomes

$R_{\text{out}} = R_{of} = (1 + A\beta)R_o$

$= 65.9 \times 61 = 4\ \text{M}\Omega$

Ex. 11.11 Refer to the solution of Example 11.7. To double $A_f\big|_{\text{ideal}}$ we need to reduce β to half its value. Since $\beta = R_F$, the feedback resistance R_F becomes

$R_F = 500\ \Omega$

Now, since A is given by

$$A = \frac{R_{id}}{R_{id} + R_F} \frac{\mu}{\frac{1}{g_m} + (R_F \| r_{o2})} \frac{r_{o2}}{R_F + r_{o2}}$$

the value of A becomes

$$A = \frac{100}{100 + 0.5} \frac{1000}{0.5 + (0.5 \| 20)} \frac{20}{0.5 + 20}$$

$= 982.7$ mA/V

Thus, A_f becomes

$$A_f = \frac{A}{1 + A\beta} = \frac{982.7}{1 + 982.7 \times 0.5}$$

$= 1.996$ mA/V

Ex. 11.12 To obtain $A_f\big|_{\text{ideal}} = 100$ mA/V, the feedback factor β must be

$$\beta = \frac{1}{100} = 0.01 \text{ V/mA} = 10\ \Omega$$

But,

$$\beta = \frac{R_{E1}R_{E2}}{R_{E1} + R_{E2} + R_F}$$

For $R_{E1} = R_{E2} = 100\ \Omega$,

$$10 = \frac{100 \times 100}{100 + 100 + R_F}$$

$\Rightarrow R_F = 800\ \Omega$

Since the ideal value of (I_o/V_s) is 100 mA/V, the ideal value of the voltage gain is,

$$\frac{V_o}{V_s} = -\frac{I_oR_C}{V_s} = -100 \times 0.6$$

$= -60$ V/V

Ex. 11.13 See figure on next page. Figure 1 shows the circuit prepared for the determination of the loop gain,

$$A\beta \equiv -\frac{V_r}{V_t}$$

We will trace the signal around the loop as follows:

$$I_{c2} = g_{m2}V_t \tag{1}$$

$$I_{b3} = I_{c2}\frac{R_{C2}}{R_{C2} + R_{i3}} \tag{2}$$

where

$R_{i3} =$
$$(\beta + 1)\{r_{e3} + R_{E2} \parallel (R_F + (R_{E1} \parallel r_{e1}))\} \tag{3}$$

$$I_{e3} = (\beta + 1)I_{b3} \tag{4}$$

This figure belongs to Exercise 11.13.

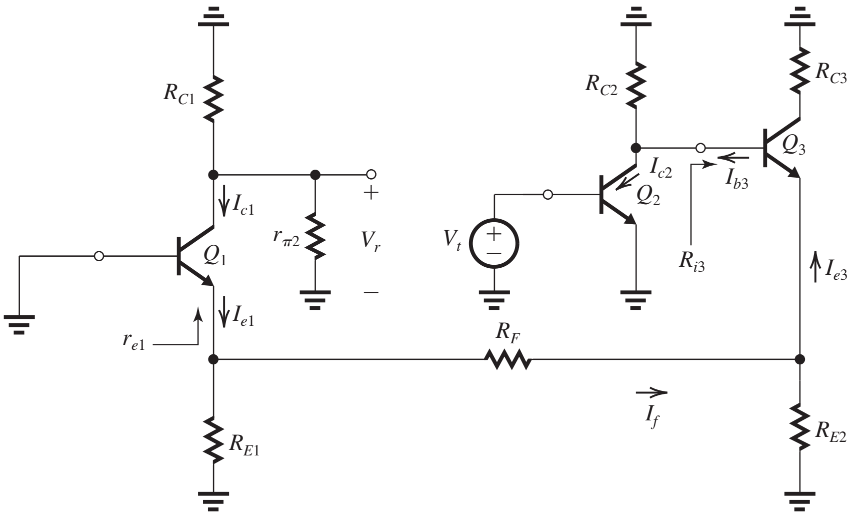


Figure 1

$$I_f = I_{e3}\frac{R_{E2}}{R_{E2}+R_F+(R_{E1}\parallel r_{e1})} \qquad (5)$$

$$I_{e1} = I_f\frac{R_{E1}}{R_{E1}+r_{e1}} \qquad (6)$$

$$I_{c1} = \alpha I_{e1} \qquad (7)$$

$$V_r = -I_{c1}(R_{C1}\parallel r_{\pi 2}) \qquad (8)$$

Combining (1)–(8) gives V_r in terms of V_t and hence $A\beta \equiv -V_r/V_t$. We shall do this numerically using the values in Example 11.8:

$g_{m2} = 40$ mA/V, $R_{C2} = 5$ kΩ, $\beta = 100$,

$r_{e3} = 6.25$ Ω, $R_{E1} = R_{E2} = 100$ Ω, $R_F = 640$ Ω,

$r_{e1} = 41.7$ Ω, $\alpha_1 = 0.99$, $R_{C1} = 9$ kΩ, and $r_{\pi 2} = 2.5$ kΩ

$$R_{i3} = 101\{0.00625+0.1\parallel(0.64 + (0.1\parallel 0.0417))\}$$

$= 9.42$ kΩ

$$I_{c2} = 40V_t \qquad (9)$$

$$I_{b3} = 0.347I_{c2} \qquad (10)$$

$$I_{e3} = 101I_{b3} \qquad (11)$$

$$I_f = 0.13I_{e3} \qquad (12)$$

$$I_{e1} = 0.706I_f \qquad (13)$$

$$I_{c1} = 0.99I_{e1} \qquad (14)$$

$$V_r = -1.957I_{c1} \qquad (15)$$

Combining (9)–(15), we obtain

$$A\beta = -\frac{V_r}{V_t} = 249.2$$

Ex. 11.14

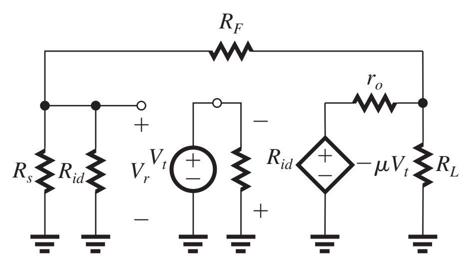


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain

$$A\beta \equiv -\frac{V_r}{V_t}$$

Using the voltage-divider rule, we can write by inspection

$$V_r = -\mu V_t\frac{R_L\parallel[R_F+(R_s\parallel R_{id})]}{r_o+\{R_L\parallel[R_F+(R_s\parallel R_{id})]\}}\,\frac{(R_s\parallel R_{id})}{R_F+(R_s\parallel R_{id})}$$

$$V_r = -\mu V_t\frac{R_L(R_s\parallel R_{id})}{r_o[R_L+R_F+(R_s\parallel R_{id})]+R_L[R_F+(R_s\parallel R_{id})]}$$

Thus,

$$A\beta = -\frac{V_r}{V_t} = \frac{\mu R_L(R_{id}\parallel R_s)}{r_o[R_L+R_F+(R_{id}\parallel R_s)]+R_L[R_F+(R_{id}\parallel R_s)]}$$

Q.E.D.

Using the numerical values in Example 11.9, we get

$$A\beta = 1035.2$$

This figure belongs to Exercise 11.15.

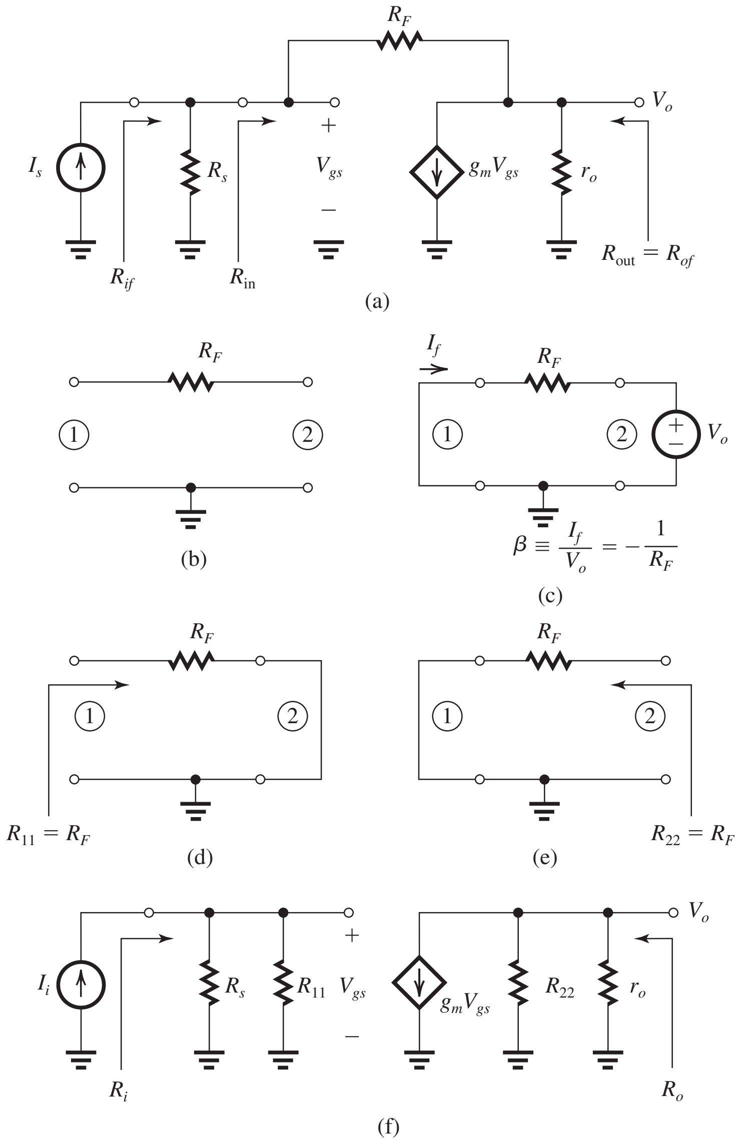


Figure 1

Ex. 11.15 See figure on next page. Figure 1(a) shows the feedback amplifier circuit. The β circuit is shown in Fig. 1(b), and the determination of β is shown in Fig. 1(c),

$$\beta = -\frac{1}{R_F}$$

(a)

$$A_f\Big|_{ideal} = \frac{1}{\beta} = -R_F$$

(b) The determination of R_{11} and R_{22} is illustrated in Figs. 1(d) and (e), respectively:

$$R_{11} = R_{22} = R_F$$

Finally, the A circuit is shown in Fig. 1(f). We can write by inspection

$$R_i = R_s \parallel R_{11} = R_s \parallel R_F$$

$$R_o = r_o \parallel R_{22} = r_o \parallel R_F$$

$$V_{gs} = I_i R_i$$

$$V_o = -g_m V_{gs}(r_o \parallel R_{22})$$

Thus,

$$A \equiv \frac{V_o}{I_i} = -g_m(R_s \parallel R_F)(r_o \parallel R_F)$$

$$A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$A_f =$

$$-\frac{g_m(R_s \parallel R_F)(r_o \parallel R_F)}{1 + g_m(R_s \parallel R_F)(r_o \parallel R_F)/R_F} \qquad \text{Q.E.D.}$$

(c) $R_{if} = \dfrac{R_i}{1 + A\beta}$

$$R_{if} = \frac{R_s \parallel R_F}{1 + g_m(R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_F} + \frac{g_m(r_o \parallel R_F)}{R_F}$$

But,

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_{\text{in}}}$$

thus,

$$\frac{1}{R_{\text{in}}} = \frac{1}{R_F} 1 + g_m(r_o \parallel R_F)$$

$$\Rightarrow R_{\text{in}} = \frac{R_F}{1 + g_m(r_o \parallel R_F)} \qquad \text{Q.E.D.}$$

(d) $R_{\text{out}} = R_{of} = \dfrac{R_o}{1 + A\beta}$

$$= \frac{r_o \parallel R_F}{1 + g_m(R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

$$\frac{1}{R_{\text{out}}} = \frac{1}{r_o} + \frac{1}{R_F} + \frac{g_m(R_s \parallel R_F)}{R_F}$$

$$\Rightarrow R_{\text{out}} = r_o \parallel \frac{R_F}{1 + g_m(R_s \parallel R_F)} \qquad \text{Q.E.D.}$$

(e) $A = -5(1 \parallel 10)(20 \parallel 10)$

$A = -30.3 \text{ k}\Omega$

$$\beta = -\frac{1}{R_F} = -\frac{1}{10} = -0.1 \text{ mA/V}$$

$A\beta = 3.03$

$1 + A\beta = 4.03$

$$A_f = \frac{A}{1 + A\beta} = -\frac{30.3}{4.03} = -7.52 \text{ k}\Omega$$

(Compare to the ideal value of -10 kΩ).

$R_i = R_s \parallel R_F = 1 \parallel 10 = 909\ \Omega$

$R_o = r_o \parallel R_F = 20 \parallel 10 = 6.67 \text{ k}\Omega$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{909}{4.03} = 226\ \Omega$$

$$R_{\text{in}} = 1 \Big/ \left[\frac{1}{R_{if}} - \frac{1}{R_s}\right] = 291\ \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{6.67}{4.03} = 1.66 \text{ k}\Omega$$

$R_{\text{out}} = R_{of} = 1.66 \text{ k}\Omega$

Ex. 11.16 Refer to the solution of Example 11.10.

$$A = -\mu \frac{R_i}{1/g_m + (R_1 \| R_2 \| r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \| R_2)}$$

$$= -100 \frac{100}{\frac{1}{5} + (10\|90\|20)} \frac{20}{20 + (10\|90)}$$

$= -1076.4$ A/A

$\beta = -0.1$

$$A_f = \frac{A}{1 + A\beta} = -\frac{1076.4}{1 + 1076.4 \times 0.1}$$

$$= -\frac{1076.4}{1 + 107.64} = -\frac{1076.4}{108.64}$$

$= -9.91$ A/A

$R_i = 100 \text{ k}\Omega$

$$R_{\text{in}} = R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{100}{108.64}$$

$= 921\ \Omega$

$R_{\text{out}} = R_{of} = R_o(1 + A\beta)$

$= 929 \times 108.64$

$= 101 \text{ M}\Omega$

Ex. 11.17 Refer to the solution of Example 11.10. With $R_2 = 0$, $\beta = -\dfrac{1}{1 + 0} = -1$

$$A_f\Big|_{\text{ideal}} = \frac{1}{\beta} = -1 \text{ A/A}$$

$R_i = R_s \| R_{id} \| (R_1 + R_2)$

$\Rightarrow R_i = R_1$

$R_o = r_{o2} + (R_1 \| R_2) + (g_m r_{o2})(R_1 \| R_2)$

$\Rightarrow R_o = r_{o2}$

$$A = -\mu \frac{R_i}{1/g_m + (R_1 \| R_2 \| r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \| R_2)}$$

$\Rightarrow A = -\mu g_m R_1$

$A\beta = \mu g_m R_1$

$$A_f = \frac{A}{1 + A\beta}$$

$$= -\frac{\mu g_m R_1}{1 + \mu g_m R_1}$$

$$R_{\text{in}} = R_{if} = \frac{R_i}{1 + A\beta}$$

$$R_{\text{in}} = \frac{R_1}{1 + \mu g_m R_1} \simeq \frac{1}{\mu g_m}$$

$R_{\text{out}} = R_{of} = R_o(1 + A\beta)$

$= r_{o2}(1 + \mu g_m R_1)$

$\Rightarrow R_{\text{out}} \simeq \mu(g_m r_{o2}) R_1$

Ex. 11.18 The feedback shifts the pole by a factor equal to the amount of feedback:

$1 + A_0\beta = 1 + 10^5 \times 0.01 = 1001$

The pole will be shifted to a frequency

$f_{Pf} = f_P(1 + A_0\beta)$

$= 100 \times 1001 = 100.1$ kHz

If β is changed to a value that results in a nominal closed-loop gain of 1, then we obtain

$\beta = 1$

and

$1 + A_0\beta = 1 + 10^5 \times 1 \simeq 10^5$

then the pole will be shifted to a frequency

$f_{Pf} = 10^5 \times 100 = 10$ MHz

Ex. 11.19 From Eq. (11.40), we see that the poles coincide when

$(\omega_{P1} + \omega_{P2})^2 = 4(1 + A_0\beta)\omega_{P1}\omega_{P2}$

$(10^4 + 10^6)^2 = 4(1 + 100\beta) \times 10^4 \times 10^6$

$\Rightarrow 1 + 100\beta = 25.5$

$\Rightarrow \beta = 0.245$

The corresponding value of $Q = 0.5$. This can also be verified by substituting in Eq. (11.42).

A maximally flat response is obtained when $Q = 1/\sqrt{2}$. Substituting in Eq. (11.42), we obtain

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{(1 + 100\beta) \times 10^4 \times 10^6}}{10^4 + 10^6}$$

$\Rightarrow \beta = 0.5$

In this case, the low-frequency closed-loop gain is

$$A_f(0) = \frac{A_0}{1 + A_0\beta}$$

$$= \frac{100}{1 + 100 \times 0.5} = 1.96 \text{ V/V}$$

Ex. 11.20 The closed-loop poles are the roots of the characteristic equation

$1 + A(s)\beta = 0$

$$1 + \left(\frac{10}{1 + \frac{s}{10^4}}\right)^3 \beta = 0$$

To simplify matters, we normalize s by the factor 10^4, thus obtaining the normalized complex-frequency variable $S = s/10^4$, and the characteristic equation becomes

$$(S + 1)^3 + 10^3\beta = 0 \quad (1)$$

This equation has three roots, a real one and a pair that can be complex conjugate. The real pole can be found from

$(S + 1)^3 = -10^3\beta$

$$\Rightarrow S = -1 - 10\beta^{1/3} = -\left(1 + 10\beta^{1/3}\right) \quad (2)$$

Dividing the characteristic polynomial in (1) by $\left(S + 1 + 10\beta^{1/3}\right)$ gives a quadratic whose two roots are the remaining poles of the feedback amplifier. After some straightforward but somewhat tedious algebra, we obtain

$$S^2 + \left(10\beta^{1/3} - 2\right) S + \left(1 + 100\beta^{2/3} - 10\beta^{1/3}\right)$$

$$= 0 \quad (3)$$

The pair of poles can now be obtained as

$$S = \left(-1 + 5\beta^{1/3}\right) \pm j5\sqrt{3}\,\beta^{1/3} \quad (4)$$

Equations (1) and (3) describe the three poles shown in Fig. E11.20.

From Eq. (2) we see that the pair of complex poles lie on the $j\omega$ axis for the value of β that makes the coefficient of S equal to zero, thus

$$\beta_{cr} = \left(\frac{2}{10}\right)^3 = 0.008$$

Ex. 11.21

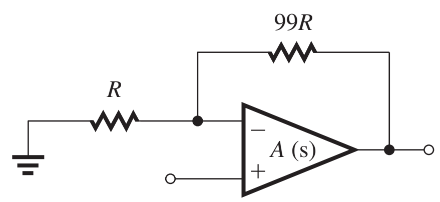


Figure 1

From Fig. 1, we can easily obtain the loop gain as

$A\beta = A(s) \times 0.01$

$$= \frac{10^5}{1 + \frac{s}{2\pi \times 10}} \times 0.01$$

$$= \frac{1000}{1 + \frac{s}{2\pi \times 10}}$$

From this single-pole response (low-pass STC response) we can find the unity-gain frequency by inspection as

$f_1 = f_P \times 1000$

$= 10^4$ Hz

The phase angle at f_1 will be -90° and thus the phase margin is 90°.

Ex. 11.22 From Eq. (11.48), we obtain

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + e^{-j\theta}|$$

$= 1/|1 + \cos\theta - j\sin\theta|$

(a) For PM $= 30^\circ$, $\theta = 180 - 30 = 150^\circ$, thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 150^\circ - j\sin 150^\circ|$$

$= 1.93$

(b) For PM $= 60^\circ$, $\theta = 180 - 60 = 120^\circ$, thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 120^\circ - j\sin 120^\circ|$$

$= 1$

(c) For PM $= 90^\circ$, $\theta = 180 - 90 = 90^\circ$, thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 90^\circ - j\sin 90^\circ|$$

$= 1/\sqrt{2} = 0.707$

Ex. 11.23 For a phase margin of 45°, $\phi = -135^\circ$. Locating the point $\phi = -135^\circ$ on the phase plot in Fig. 11.38, we find the corresponding point on the gain plot at 80 dB, that is,

$20\log(1/\beta) = 80$ dB

$\Rightarrow \beta = 10^{-4}$

$$A_f = \frac{A}{1 + A\beta} = \frac{10^5}{1 + 10^5 \times 10^{-4}}$$

$= 9.09 \times 10^3$

$= 79.2$ dB $\simeq 80$ dB

Ex. 11.24 To obtain stable performance for closed-loop gains as low as 20 dB (which is 80 dB below A_0, or equivalently 10^4 below A_0), we must place the new dominant pole at 1 MHz/10^4 = 100 Hz.

Ex. 11.25 The frequency of the first pole must be lowered from 1 MHz to a new frequency

$$f'_D = \frac{10\text{ MHz}}{10^4} = 1000\text{ Hz}$$

that is, by a factor of 1000. Thus, the capacitance at the controlling node must be increased by a factor of 1000.

Chapter 12

Solutions to Exercises within the Chapter

Ex: 12.1 To allow for v_O to reach $-V_{CC} + V_{CE\text{sat}} = -15 + 0.2 = -14.8$ V, with Q_1 just cutting off (i.e. $i_{E1} = 0$),

$$I = \frac{14.8 \text{ V}}{R_L} = \frac{14.8}{1 \text{ k}\Omega} = 14.8 \text{ mA}$$

The value of R can now be found from

$$I = \frac{V_R}{R} = \frac{V_{CC} - V_D}{R}$$

$$14.8 = \frac{15 - 0.7}{R}$$

$$\Rightarrow R = \frac{14.3}{14.8} = 0.97 \text{ k}\Omega$$

The resulting output signal swing will be -14.8 V to $+14.8$ V. The minimum current in $Q_1 = 0$. The maximum current in $Q_1 = 14.8 + 14.8 = 29.6$ mA

Ex: 12.2 At $v_O = -10$ V, we have

$$i_L = \frac{-10}{1} = -10 \text{ mA}$$

$$i_{E1} = I + i_L = 14.8 - 10 = 4.8 \text{ mA}$$

$$v_{BE1} = 0.6 + 0.025 \ln\left(\frac{4.8}{1}\right)$$

$$= 0.64 \text{ V}$$

$$v_I = v_O + v_{BE1}$$

$$= -10 + 0.64 = -9.36 \text{ V}$$

At $v_O = 0$ V, we have

$$i_L = 0 \text{ mA}$$

$$i_{E1} = I = 14.8 \text{ mA}$$

$$v_{BE1} = 0.6 + 0.025 \ln\left(\frac{14.8}{1}\right)$$

$$= 0.67 \text{ V}$$

$$v_I = v_O + v_{BE1}$$

$$= 0 + 0.67 = 0.67 \text{ V}$$

At $v_O = 10$ V

$$i_L = \frac{10}{1} = 10 \text{ mA}$$

$$i_{E1} = I + i_L = 14.8 + 10 = 24.8 \text{ mA}$$

$$v_{BE1} = 0.6 + 0.025 \ln\left(\frac{24.8}{1}\right)$$

$$= 0.68 \text{ V}$$

$$v_I = v_O + v_{BE1}$$

$$= 10 + 0.68 = 10.68 \text{ V}$$

At $v_O = -10$ V, we have

$$i_{E1} = 4.8 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{4.8 \text{ mA}} = 5.2 \ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}} = \frac{1}{1 + 0.0052} = 0.995 \text{ V/V}$$

At $v_O = 0$ V,

$$i_{E1} = 14.8 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{14.8 \text{ mA}} = 1.7 \ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}} = \frac{1}{1 + 0.0017} = 0.998 \text{ V/V}$$

At $v_O = +10$ V,

$$i_{E1} = 24.8 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{24.8 \text{ mA}} = 1.0 \ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}} = \frac{1}{1 + 0.001} = 0.999 \text{ V/V}$$

Ex: 12.3

a. $$P_L = \frac{\left(\hat{V}_o/\sqrt{2}\right)^2}{R_L} = \frac{\left(8/\sqrt{2}\right)^2}{100} = 0.32 \text{ W}$$

$$P_S = 2V_{CC} \times I = 2 \times 10 \times 100 \times 10^{-3}$$

$$= 2 \text{ W}$$

$$\text{Efficiency } \eta = \frac{P_L}{P_S} \times 100$$

$$= \frac{0.32}{2} \times 100$$

$$= 16\%$$

Ex: 12.4 (a) $P_L = \dfrac{1}{2}\dfrac{\hat{V}_o^2}{R_L}$

$$= \frac{1}{2}\frac{(4.5)^2}{4} = 2.53 \text{ W}$$

(b) $P_{S+} = P_{S-} = V_{CC} \times \dfrac{1}{\pi}\dfrac{\hat{V}_o}{R_L}$

$$= 6 \times \frac{1}{\pi} \times \frac{4.5}{4} = 2.15 \text{ W}$$

(c) $\eta = \dfrac{P_L}{P_S} \times 100 = \dfrac{2.53}{2 \times 2.15} \times 100$

$$= 59\%$$

(d) Peak input currents $= \dfrac{1}{\beta + 1}\dfrac{\hat{V}_o}{R_L}$

$= \frac{1}{51} \times \frac{4.5}{4}$

$= 22.1$ mA

(e) Using Eq. (12.22), we obtain

$$P_{DN\max} = P_{DP\max} = \frac{V_{CC}^2}{\pi^2 R_L}$$

$$= \frac{6^2}{\pi^2 \times 4} = 0.91 \text{ W}$$

Ex: 12.5 (a) The quiescent power dissipated in each transistor $= I_Q \times V_{CC}$

Total power dissipated in the two transistors

$= 2I_Q \times V_{CC}$

$= 2 \times 2 \times 10^{-3} \times 15$

$= 60$ mW

(b) I_Q is increased to 10 mA

At $v_O = 0$, we have $i_N = i_P = 10$ mA

From Eq. (12.31), we obtain

$$R_{\text{out}} = \frac{V_T}{i_P + i_N} = \frac{25}{10 + 10} = 1.25\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 1.25}$$

$$\frac{v_o}{v_i} = 0.988 \text{ at } v_O = 0 \text{ V}$$

At $v_O = 10$ V, we have

$$i_L = \frac{10 \text{ V}}{100\ \Omega} = 0.1 \text{ A} = 100 \text{ mA}$$

Use Eq. (12.27) to calculate i_N:

$$i_N^2 - i_N i_L - I_Q^2 = 0$$

$$i_N^2 - 100\, i_N - 10^2 = 0$$

$$\Rightarrow i_N = 101.0 \text{ mA}$$

Using Eq. (12.26), we obtain

$$i_P = \frac{I_Q^2}{i_N} \simeq 1 \text{ mA}$$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{101.0 + 1} \simeq 0.2451\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 0.2451} \simeq 1$$

$$\% \text{ change} = \frac{1 - 0.988}{1} \times 100 = 1.2\%$$

In Example 12.3, $I_Q = 2$ mA, and for $v_O = 0$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{2 + 2} = 6.25\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 6.25} = 0.94$$

$v_O = 10$ V

$$i_L = \frac{10 \text{ V}}{100\ \Omega} = 100 \text{ mA}$$

Again calculate i_N (for $I_Q = 2$ mA) using Eq. (12.27) ($i_N = 100.04$ mA):

$$i_P = \frac{I_Q^2}{I_N} = \frac{2^2}{100.04} = 0.04 \text{ mA}$$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{100.04 + 0.04} = 0.25\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} \simeq 1$$

$$\% \text{ Change} = \frac{1 - 0.94}{1} \times 100 = 6\%$$

For $I_Q = 10$ mA, change is 1.2%

For $I_Q = 2$ mA, change is 6%

(c) The quiesent power dissipated in each transistor $= I_Q \times V_{CC}$

Total power dissipated $= 2 \times 10 \times 10^{-3} \times 15$

$= 300$ mW

Ex: 12.6 From Example 12.4, we have $V_{CC} = 15$ V,
$R_L = 100\ \Omega$

Q_N and Q_P matched and $I_S = 10^{-13}$ A and $\beta = 50$, $I_{\text{Bias}} = 3$ mA

For $v_O = 10$ V, we have $i_L = \frac{10}{100} = 0.1$ A

As a first approximation, $i_N \simeq 0.1$ A,

$$i_P = 0,\ i_{BN} \simeq \frac{0.1 \text{ A}}{50 + 1} \simeq 2 \text{ mA}$$

$$i_D = I_{\text{Bias}} - i_{BN} = 3 - 2 = 1 \text{ mA}$$

$$V_{BB} = 2V_T \ln\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) \tag{1}$$

This $\frac{1}{3}$ is because biasing diodes have $\frac{1}{3}$ area of the output devices.

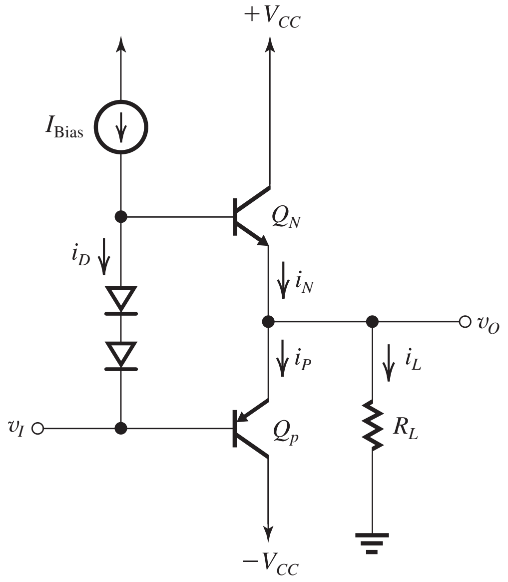
But $V_{BB} = V_{BEN} + V_{BEP}$

$$= V_T \ln\left(\frac{i_N}{I_S}\right) + V_T \ln\left(\frac{i_N - i_L}{I_S}\right)$$

$$= V_T \ln\left[\frac{i_N\,(i_N - i_L)}{I_S^2}\right] \tag{2}$$

Equating Eqs. 1 and 2, we obtain

$$2V_T \ln\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) = V_T \ln\left[\frac{i_N(i_N - i_L)}{I_S^2}\right]$$



$$\left(\frac{10^{-3}}{\frac{1}{3}\times 10^{-13}}\right)^2 = \frac{i_N\,(i_N-0.1)}{(10^{-13})^2}$$

$i_N(i_N-0.1) = 9\times 10^{-6}$

If i_N is in mA, then

$i_N(i_N-100) = 9$

$i_N^2 - 100\,i_N - 9 = 0$

$\Rightarrow i_N = 100.1$ mA

$i_P = i_N - i_L = 0.1$ mA

For $v_O = -10$ V and $i_L = \dfrac{-10}{100} = -0.1$ A

$= -100$ mA:

As a first approximation assume $i_P \cong 100$ mA,

$i_N \simeq 0$. Since $i_N = 0$, current through diodes $= 3$ mA

$$\therefore V_{BB} = 2V_T \ln\left(\frac{3\times 10^{-3}}{\frac{1}{3}\times 10^{-13}}\right) \qquad (3)$$

But $V_{BB} = V_T \ln\left(\dfrac{i_N}{10^{-13}}\right) + V_T \ln\left(\dfrac{i_P}{10^{-13}}\right)$

$$= V_T \ln\left(\frac{i_P + i_L}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right) \qquad (4)$$

Here $i_L = -0.1$ A

Equating Eqs. (3) and (4), we obtain

$$2V_T \ln\left(\frac{3\times 10^{-3}}{\frac{1}{3}\times 10^{-13}}\right) =$$

$$V_T \ln\left(\frac{i_P - 0.1}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right)$$

$$\left(\frac{3\times 10^{-3}}{\frac{1}{3}\times 10^{-13}}\right)^2 = \frac{i_P\,(i_P-0.1)}{(10^{-13})^2}$$

$i_P(i_P-0.1) = 81\times 10^{-6}$

Expressing currents in mA, we have

$i_P\,(i_P-100) = 81$

$i_P^2 - 100\,i_P - 81 = 0$

$\Rightarrow i_P = 100.8$ mA

$i_N = i_P + i_L = 0.8$ mA

Ex: 12.7 $\Delta I_C = g_m \times 2\text{ mV}/^\circ\text{C} \times 5\,^\circ\text{C}$, mA

where g_m is in mA / mV

$$g_m = \frac{10\text{ mA}}{25\text{ mV}} = 0.4\text{ mA/mV}$$

Thus, $\Delta I_C = 0.4 \times 2 \times 5 = 4$ mA

Ex: 12.8 Refer to Fig. 12.14.

(a) To obtain a terminal voltage of 1.2 V, and since β_1 is very large, it follows that $V_{R1} = V_{R2} = 0.6$ V.

Thus $I_{C1} = 1$ mA

$$I_R = \frac{1.2\text{ V}}{R_1 + R_2} = \frac{1.2}{2.4} = 0.5\text{ mA}$$

Thus, $I = I_{C1} + I_R = 1.5$ mA

(b) For $\Delta V_{BB} = +50$ mV:

$V_{BB} = 1.25$ V $\quad I_R = \dfrac{1.25}{2.4} = 0.52$ mA

$V_{BE} = \dfrac{1.25}{2} = 0.625$ V

$I_{C1} = 1 \times e^{\Delta V_{BE}/V_T} = e^{0.025/0.025}$

$= 2.72$ mA

$I = 2.72 + 0.52 = 3.24$ mA

For $\Delta V_{BB} = +100$ mV, we have

$V_{BB} = 1.3$ V, $\quad I_R = \dfrac{1.3}{2.4} = 0.54$ mA

$V_{BE} = \dfrac{1.3}{2} = 0.65$ V

$I_{C1} = 1 \times e^{\Delta V_{BE}/V_T} = 1 \times e^{0.05/0.025}$

$= 7.39$ mA

$I = 7.39 + 0.54 = 7.93$ mA

For $\Delta V_{BB} = +200$ mV:

$V_{BB} = 1.4\text{ V}, \quad I_R = \frac{1.4}{2.4} = 0.58\text{ mA}$

$V_{BE} = 0.7\text{ V}$

$I_{C1} = 1 \times e^{0.1/0.025} = 54.60\text{ mA}$

$I = 54.60 + 0.58 = 55.18\text{ mA}$

For $\Delta V_{BB} = -50\text{ mV}$:

$V_{BB} = 1.15\text{ V}, \quad I_R = \frac{1.15}{2.4} = 0.48\text{ mA}$

$V_{BE} = \frac{1.15}{2}$

$= 0.575$

$I_{C1} = 1 \times e^{-0.025/0.025} = 0.37\text{ mA}$

$I = 0.48 + 0.37 = 0.85\text{ mA}$

For $\Delta V_{BB} = -100\text{ mV}$:

$V_{BB} = 1.1\text{ V} \quad I_R = \frac{1.1}{2.4} = 0.46\text{ mA}$

$V_{BE} = 0.55\text{ V}$

$I_{C1} = 1 \times e^{-0.05/0.025} = 0.13\text{ mA}$

$I = 0.46 + 0.13 = 0.59\text{ mA}$

For $\Delta V_{BB} = -200\text{ mV}$:

$V_{BB} = 1.0\text{ V} \quad I_R = \frac{1}{2.4} = 0.417\text{ mA}$

$V_{BE} = 0.5\text{ V}$

$I_{C1} = 1 \times e^{-0.1/0.025} = 0.018\text{ mA}$

$I = 0.43\text{ mA}$

Ex: 12.9 (a) From symmetry we see that all transistors will conduct equal currents and have equal V_{BE}'s. Thus,

$v_O = 0\text{ V}$

If $V_{BE} \simeq 0.7\text{ V}$, then

$V_{E1} = 0.7\text{ V}$ and $I_1 = \frac{15 - 0.7}{5} = 2.86\text{ mA}$

If we neglect I_{B3}, then

$I_{C1} \simeq 2.86\text{ mA}$

At this current, $|V_{BE}|$ is given by

$|V_{BE}| = 0.025 \ln\left(\frac{2.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right) \simeq 0.63\text{ V}$

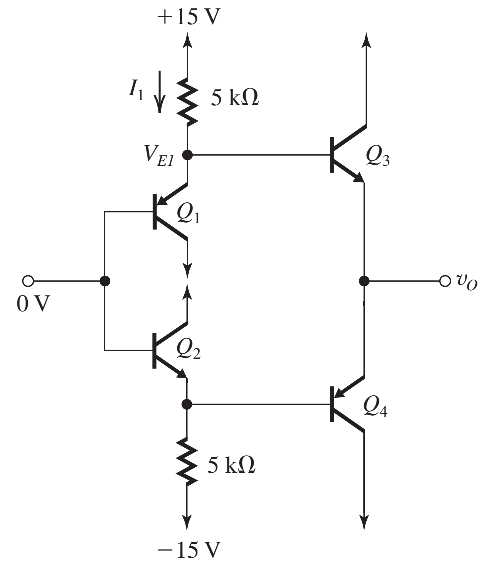
Thus $V_{E1} = 0.63\text{ V}$ and $I_1 = 2.87\text{ mA}$

No more iterations are required and

$i_{C1} = i_{C2} = i_{C3} = i_{C4} \simeq 2.87\text{ mA}$

(b) For $v_I = +10\text{ V}$

To start the iterations, let $V_{BE1} \simeq 0.7\text{ V}$

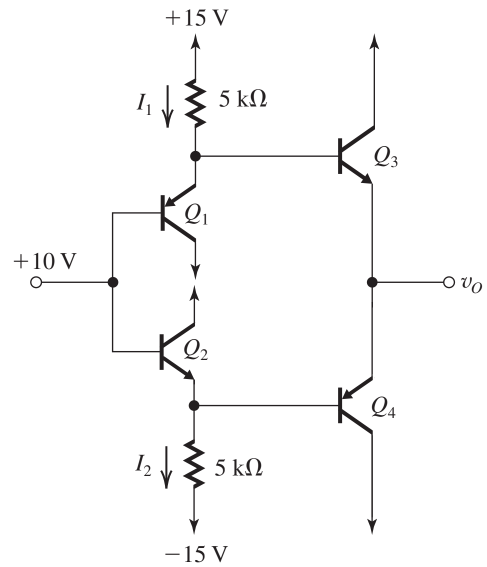


Thus,

$V_{E1} = 10.7\text{ V}$

and

$I_1 = \frac{15 - 10.7}{5} = 0.86\text{ mA}$



Neglecting I_{B3}, we obtain

$I_{C1} \simeq I_{E1} \simeq I_1 = 0.86\text{ mA}$

But at this current

$|V_{BE1}| = V_T \ln\left(\frac{I_{C1}}{I_S}\right)$

$= 0.025 \ln\left(\frac{0.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$

$= 0.6\text{ V}$

Thus, $V_{E1} = +10.6$ V and $I_1 = 0.88$ mA. No further iterations are required and $I_{C1} \simeq 0.88$ mA.

To find I_{C2}, we use an identical procedure:

$V_{BE2} \simeq 0.7$ V

$V_{E2} = 10 - 0.7 = +9.3$ V

$$I_2 = \frac{9.3 - (-15)}{5} = 4.86 \text{ mA}$$

$$V_{BE2} = 0.025 \ln\left(\frac{4.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$= 0.643$ V

$V_{E2} = 10 - 0.643 = +9.357$

$I_2 = 4.87$ mA

$I_{C2} \simeq 4.87$ mA

Finally,

$I_{C3} = I_{C4} = 3.3 \times 10^{-14}\ e^{V_{BE}/V_T}$

where

$$V_{BE} = \frac{V_{E1} - V_{E2}}{2} = 0.62 \text{ V}$$

Thus, $I_{C3} = I_{C4} = 1.95$ mA

The symmetry of the circuit enables us to find the values for $v_I = -10$ V as follows:

$I_{C1} = 4.87$ mA $I_{C2} = 0.88$ mA

$I_{C3} = I_{C4} = 1.95$ mA

For $v_I = +10$ V, we have $v_O = V_{E1} - V_{BE3}$

$= 10.6 - 0.62 = +9.98$ V

For $v_I = -10$ V, we have $v_O = V_{E1} - V_{BE3}$

$= -9.357 - 0.62 = -9.98$ V

(c) For $v_I = +10$ V, we have

$v_O \simeq 10$ V

$I_L \simeq 100$ mA

$I_{C3} \simeq 100$ mA

$$I_{B3} = \frac{100}{201} \simeq 0.5 \text{ mA}$$

Assuming that $|V_{BE1}|$ has not changed much from 0.6 V, then

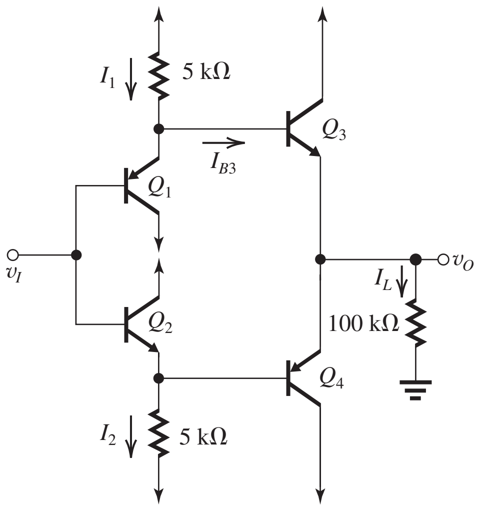
$V_{E1} \simeq 10.6$ V

$$I_1 = \frac{15 - 10.6}{5} = 0.88 \text{ mA}$$

$I_{E1} = I_1 - I_{B3} = 0.88 - 0.5 = 0.38$ mA

$I_{C1} \simeq 0.38$ mA

$$|V_{BE1}| = 0.025 \ln\left(\frac{0.38 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$



$= 0.58$ V

$V_{E1} = 10.58$ V

$$I_1 = \frac{15 - 10.58}{5} = 0.88 \text{ mA}$$

Thus, $I_{C1} \simeq 0.38$ mA

Now for Q_2 we have

$V_{BE2} = 0.643$ V

$V_{E2} = 10 - 0.643 = 9.357$

$I_2 = 4.87$ mA

$I_{B4} \simeq 0$

$I_{C2} \simeq 4.87$ mA (as in (b))

Assuming that $I_{C3} \simeq 100$ mA, we have

$$V_{BE3} = 0.025 \ln\left(\frac{100 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$= 0.72$ V

Thus, $v_O = V_{E1} - V_{BE3}$

$= 10.58 - 0.72 = +9.86$ V

$|V_{BE4}| = v_O - V_{E2}$

$9.86 - 9.36 = 0.5$ V

Thus, $I_{C4} = 3.3 \times 10^{-14} e^{0.5/0.025}$

$\simeq 0.02$ mA

From symmetry we find the values for the case

$v_I = -10$ V as:

$I_{C1} = 4.87$ mA, $\quad I_{C2} = 0.38$ mA

$I_{C3} = 0.02$ mA, $\quad I_{C4} = 100$ mA

$v_O = -9.86$ V.

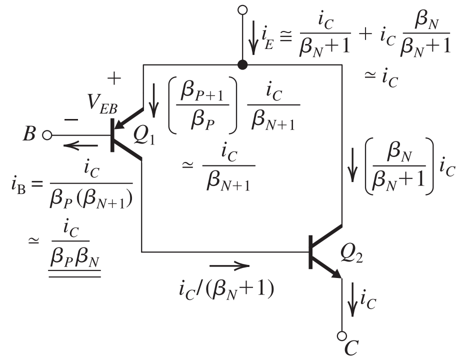
Ex: 12.10 For Q_1:

$i_{C1} = I_{SP} e^{v_{EB}/V_T}$

$\dfrac{i_C}{\beta_N + 1} = I_{SP} e^{v_{EB}/V_T}$

$i_C \simeq \beta_N I_{SP} e^{v_{EB}/V_T}$

Thus, effective scale current $= \beta_N I_{SP}$



(b) Effective current gain $\equiv \dfrac{i_C}{i_B} = \beta_P \beta_N$

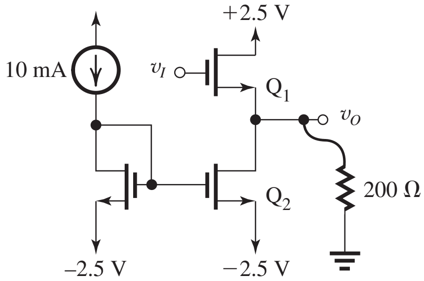
$= 20 \times 50 = 1000$

$100 \times 10^{-3} = 50 \times 10^{-14}\ e^{v_{EB}/0.025}$

$v_{EB} = 0.025 \ln\left(2 \times 10^{11}\right)$

$= 0.651$ V

Ex: 12.11



$v_O = 0\text{ V} \Rightarrow I_{D1} = 10$ mA

$\Rightarrow V_{GS1} = V_{tn} + V_{OV1}$

$= v_{tn} + \sqrt{\dfrac{2I_{D1}}{k_n}}$

$= 0.5\text{ V} + \sqrt{\dfrac{20\text{ mA}}{50\text{ mA/V}^2}}$

$= 1.13$ V

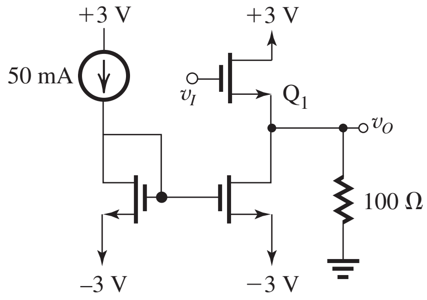
$\Rightarrow v_I = 1.13$ V

From Eq. (12.38),

$R_L \geq \dfrac{V_{SS} - V_{OV2}}{I} = \dfrac{2.5\text{ V} - 0.63\text{ V}}{10\text{ mA}}$

$= 187\Omega$

Ex: 12.12



When $v_O = 0$ V,

$I_{D1} = 50$ mA

$g_{m1} = \sqrt{2k_n I_{D1}} = \sqrt{2 \times 80 \times 50} = 89$ mA/V

$A_v = \dfrac{R_L}{R_L + 1/g_{m1}}$

$= \dfrac{100}{100 + 1/0.089}$

$= 0.899$ V/V

$V_{OV2} = \sqrt{\dfrac{2I}{k_n}}$

$= \sqrt{\dfrac{2 \times 50}{80}}$

$= 1.12$ V

$\Rightarrow v_{O\min} = -2.5 + 1.12$

$= -1.38$ V

We must also ensure Equation (12.38) is satisfied.

$I \geq \dfrac{V_{SS} - V_{OV2}}{R_L}$

$50\text{ mA} \geq \dfrac{1.38\text{ V}}{100\Omega} = 13.8$ mA

Thus, when $v_O = v_{O\min}$:

$I_{D1} = 50\text{ mA} - \dfrac{1.38\text{ V}}{100\Omega}$

$= 36.2$ mA

$\Rightarrow g_{m1} = \sqrt{2k_n I_{D1}}$

$= \sqrt{2 \times 80 \times 36.2}$

$= 76.1$ mA/V

$\Rightarrow A_v = \dfrac{R_L}{R_L + 1/g_{m1}}$

$= \dfrac{100}{100 + 1/0.0761}$

$= 0.884$ V/V

The percentage change is therefore,

$\left|\dfrac{0.884 - 0.899}{0.899}\right| \times 100\% = 1.7\%$

Ex: 12.13 According to Equation (12.44),

$$R_{\text{out}} \cong \frac{1}{\mu g_{mp}}$$

$$\Rightarrow \mu \cong \frac{1}{g_{mp} R_{\text{out}}}$$

$$= \frac{1}{0.05 \times 1}$$

$$= 20 \text{ V/V}$$

$$g_{mp} = \frac{2I}{V_{OV1}}$$

$$\Rightarrow I = \frac{g_{mp} V_{OV1}}{2}$$

$$= \frac{50 \times 0.3}{2}$$

$$= 7.5 \text{ mA}$$

Ex: 12.14 $I_Q = 0.1 \times \dfrac{2 \text{ V}}{100\Omega} = 2 \text{ mA}$

$$g_{mn} = g_{mp} = \frac{2I_Q}{V_{OV1,2}} = \frac{2 \times 2}{0.3} = 13.33 \text{ mA/V}$$

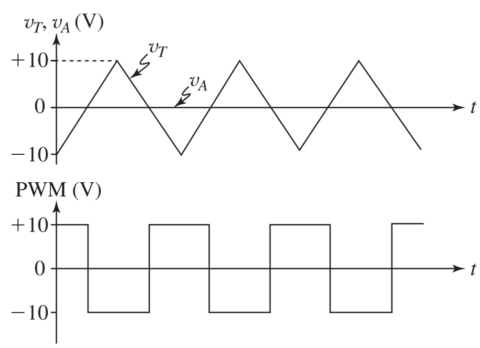
According to Equation (12.45),

$$R_{\text{out}} = \frac{1}{\mu(g_{mp} + g_{mn})}$$

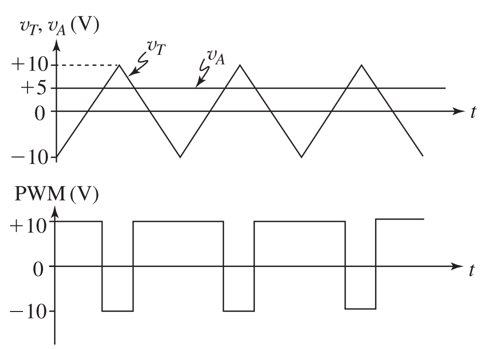
$$= \frac{1}{10 \times (13.3 + 13.3)}$$

$$= 3.75\Omega$$

Ex: 12.15 See Fig. 1.

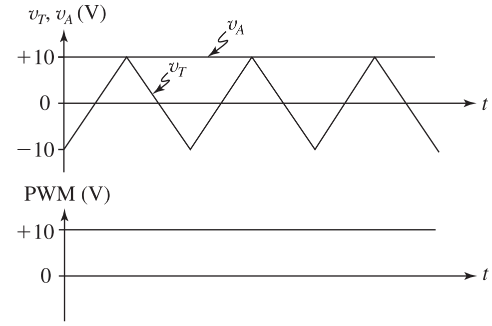


(a) $v_A = 0$: $D = 50\%$; Average $= 0$

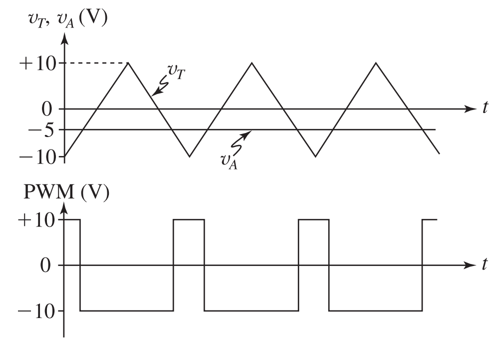


(b) $v_A = +5$ V: $D = 75\%$; Average $= +5$ V

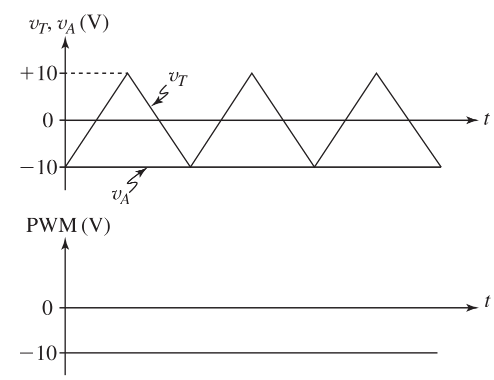
Figure 1 *continued*



(c) $v_A = +10$ V: $D = 100\%$; Average $= +10$ V



(d) $v_A = -5$ V: $D = 25\%$; Average $= -5$ V



(e) $v_A = -10$ V: $D = 0\%$; Average $= -10$ V

Figure 1

Ex: 12.16

$f_s = 10 \times$ highest frequency in audio signal

$= 10 \times 20 = 200$ kHz

Since f_s is a decade higher than f_P, the gain will have fallen by 40 dB. Thus the PWM component at f_s will be attenuated by 40 dB.

Ex: 12.17 Maximum peak amplitude $= V_{DD}$

Maximum power delivered to $R_L = \dfrac{(V_{DD}/\sqrt{2})^2}{R_L}$

$$= \frac{V_{DD}^2}{2R_L}$$

For $V_{DD} = 35$ V and $R_L = 8\ \Omega$:

Peak amplitude $= 35$ V

Maximum power $= \dfrac{35^2}{2 \times 8} = 76.6$ W

Power delivered by power supplies

$= \dfrac{P_L}{\eta} = \dfrac{76.6}{0.9} = 85.1$ W

Chapter 13

Solutions to Exercises within the Chapter

Ex: 13.1 Using Eq. (13.2), we obtain

$V_{ICM\min} = -V_{SS} + V_{tn} + V_{OV3} - |V_{tp}|$

$= -1.65 + 0.5 + 0.3 - 0.5$

$= -1.35$ V

Using Eq. (13.3), we get

$V_{ICM\max} = V_{DD} - |V_{OV3}| - |V_{tp}| - |V_{OV1}|$

$= 1.65 - 0.3 - 0.5 - 0.3$

$= +0.55$ V

Thus,

$-1.35 \text{ V} \leq V_{ICM} \leq +0.55 \text{ V}$

Using Eq. (13.5), we obtain

$-V_{SS} + V_{OV6} \leq v_O \leq V_{DD} - |V_{OV7}|$

Thus

$-1.65 + 0.5 \leq v_O \leq 1.65 - 0.5$

$\Rightarrow -1.15 \text{ V} \leq v_O \leq +1.15 \text{ V}$

In a unity-gain configuration, the output is connected to the gate of Q_1 and must, therefore, respect both limits. The overlap is

$-1.15 \text{ V} \leq v_O \leq +0.55 \text{ V}$

Ex: 13.2 For all devices, we have

$|V_{An}| = 5 \text{ V/ µm} \times 0.5 \text{ µm} = 2.5 \text{ V}$

$|V_{Ap}| = 6 \text{ V/ µm} \times 0.5 \text{ µm} = 3 \text{ V}$

Using Eq. (13.11), we get

$$A_1 = -\frac{2}{|V_{OV1}|} \Big/ \left[\frac{1}{|V_{A2}|} + \frac{1}{V_{A4}}\right]$$

$$= -\frac{2}{0.2 \text{ V}} \Big/ \left[\frac{1}{3 \text{ V}} + \frac{1}{2.5 \text{ V}}\right] = -13.6 \text{ V/V}$$

Using Eq. (13.16), we obtain

$$A_2 = -\frac{2}{|V_{OV6}|} \Big/ \left[\frac{1}{V_{A6}} + \frac{1}{|V_{A4}|}\right]$$

$$= -\frac{2}{0.3 \text{ V}} \Big/ \left[\frac{1}{2.5 \text{ V}} + \frac{1}{3 \text{ V}}\right] = -9.1 \text{ V/V}$$

The total gain is

$A_v = A_1 A_2$

$= -13.6 \times -9.1$

$= 123.8$ V/V

We find the op-amp output resistance as follows,

$$r_{o6} = \frac{V_{An}}{I_{D6}} = \frac{2.5 \text{ V}}{0.1 \text{ mA}} = 25 \text{ k}\Omega$$

$$r_{o7} = \frac{|V_{Ap}|}{I_{D6}} = \frac{3 \text{ V}}{0.1 \text{ mA}} = 30 \text{ k}\Omega$$

$R_o = r_{o6} \parallel r_{o7} = 25 \parallel 30 = 13.6 \text{ k}\Omega$

Ex: 13.3 The feedback is of the voltage sampling type (i.e., the connection at the output is a shunt one), thus

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

where

$R_o = r_{o6} \parallel r_{o7}$

$A = g_{m1}(r_{o2} \parallel r_{o4}) g_{m6}(r_{o6} \parallel r_{o7})$

$\beta = 1$

Thus,

$$R_{\text{out}} = \frac{r_{o6} \parallel r_{o7}}{1 + g_{m1}(r_{o2} \parallel r_{o4}) g_{m6}(r_{o6} \parallel r_{o7})}$$

Usually,

$A \gg 1$

Thus,

$$R_{\text{out}} \simeq \frac{1}{g_{m6}[g_{m1}(r_{o2} \parallel r_{o4})]}$$

Ex: 13.4 Using Eq. (13.31), we get

$$f_t = \frac{G_{m1}}{2\pi C_C}$$

$$\Rightarrow C_C = \frac{G_{m1}}{2\pi f_t}$$

$$= \frac{0.35 \times 10^{-3}}{2\pi \times 20 \times 10^6}$$

$= 2.8$ pF

From Eq. (13.26), we have

$$f_Z = \frac{G_{m2}}{2\pi C_C}$$

$$= \frac{1.0 \times 10^{-3}}{2\pi \times 2.8 \times 10^{-12}}$$

$\simeq 57$ MHz

From Eq. (13.30), we have

$$f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

$$= \frac{1.0 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 80 \text{ MHz}$$

Thus, f_t is lower than f_Z and f_{P2}.

Ex: 13.5 (a) Using Eq. (13.36), we have

$$f_t = \frac{G_{m1}}{2\pi C_C}$$

$$\Rightarrow C_C = \frac{G_{m1}}{2\pi f_t}$$

$$= \frac{1 \times 10^{-3}}{2\pi \times 100 \times 10^6}$$

$= 1.6$ pF

$$A_0 = G_{m1}(r_{o2} \parallel r_{o4})G_{m2}(r_{o6} \parallel r_{o7})$$

$$= 1(100 \parallel 100) \times 2(40 \parallel 40)$$

$= 50 \times 2 \times 20 = 2000$ V/V

$$f_{3dB} = \frac{f_t}{A_0} = \frac{100 \times 10^6}{2000} = 50 \text{ kHz}$$

(b) From Eq. (13.34), we have

$$R = \frac{1}{G_{m2}} = \frac{1}{2 \times 10^{-3}} = 500\ \Omega$$

(c) From Eq. (13.35), we have

$$f_{P2} = \frac{G_{m2}}{2\pi C_2}$$

$$= \frac{2 \times 10^{-3}}{2\pi \times 1 \times 10^{-12}} = 318 \text{ MHz}$$

$$\phi_{P2} = -\tan^{-1}\frac{f_t}{f_{P2}}$$

$$= -\tan^{-1}\frac{100}{318}$$

$= -17.4^\circ$

PM $= 90 - 17.4 = 72.6^\circ$

Ex: 13.6 Using Eq. (13.47), we obtain

SR $= V_{OV1}\omega_t$

$= 0.2 \times 2\pi \times 100 \times 10^6$

$= 126$ V/µs

Using Eq. (13.45),

$$\text{SR} = \frac{I}{C_C}$$

$$\Rightarrow I = 126 \times 10^6 \times 1.6 \times 10^{-12}$$

$= 200$ µA

Ex: 13.7

$$G_{m2} = g_{m6} = \frac{2I_{D6}}{V_{OV}} = \frac{2 \times 0.5}{0.25} = 4 \text{ mA/V}$$

$$f_{P2} = \frac{G_{m2}}{2\pi \times C_2} = \frac{4}{2\pi \times 15 \times 10^{-12}} = 42 \text{ MHz}$$

$$\tan^{-1}\frac{f_t}{f_{P2}} = 20^\circ$$

$$\Rightarrow f_t = f_{P2} \times \tan 20^\circ = 15.3 \text{ MHz}$$

$$G_{m1} = g_{m1} = \frac{2(I/2)}{V_{OV}} = \frac{2 \times 0.3}{0.25} = 1.2 \text{ mA/V}$$

$$f_t = \frac{G_{m1}}{2\pi \times C_C} \Rightarrow C_C = \frac{G_{m1}}{2\pi \times f_t}$$

$$= \frac{1.2 \times 10^{-3}}{2\pi \times 15.3 \times 10^6} = 12.5 \text{ pF}$$

Ex: 13.8

$$I_{REF} = \frac{0.2 \text{ mA}}{10} = 20\ \mu\text{A} = I_{D8}$$

$$I_{D8} = \frac{1}{2}k'_p\left(\frac{W}{L}\right)_8 V_{OV}^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_8 = \frac{2I_{D8}}{k'_p \times V_{OV}^2}$$

$$= \frac{2 \times 20 \times 10^{-6}}{86 \times 10^{-6} \times 0.5^2} = 1.86$$

$W_8 = 1.86 \times L_8 = 1.86 \times 0.6 = 1.1$ µm

$$\frac{W_5}{W_8} = \frac{I_{D5}}{I_{D8}} \Rightarrow W_5 = \frac{200}{20} \times 1.1 = 11\ \mu\text{m}$$

$$\frac{W_6}{W_8} = \frac{I_{D6}}{I_{D8}} \Rightarrow W_6 = \frac{400}{20} \times 1.1 = 22\ \mu\text{m}$$

$|V_{GS8}| = |V_{tp}| + |V_{OV}| = 0.5 + 0.5 = 1.0$ V

The required resistor value is

$$R = \frac{1\text{ V} - 1\text{ V} - (-1\text{ V})}{I_{REF}} = \frac{1\text{ V}}{20\mu\text{A}} = 50\text{ k}\Omega$$

Ex: 13.9 Total bias current $= 300\ \mu\text{A} = 2I_B$

$\Rightarrow I_B = 150$ µA

$I_B = I_{D1} + I_{D3}$

$150 = I_{D1} + 0.25I_{D1}$

$\Rightarrow I_{D1} = 120$ µA

$I = I_{D1} + I_{D2}$

$= 120 + 120 = 240$ µA

$I_{D3,4} = 0.25I_{D1,2} = 0.25 \times 120$

$= 30$ µA

Ex: 13.10 Using Eq. (13.50), we get

$V_{ICM\max} = V_{DD} - |V_{OV9}| + V_{tn}$

$= 1.65 - 0.3 + 0.5 = +1.85$ V

Using Eq. (13.51), we obtain

$V_{ICM\min} = -V_{SS} + V_{OV11} + V_{OV1} + V_{tn}$

$V_{ICM\min} = -1.65 + 0.3 + 0.3 + 0.5$

$= -0.55$ V

Thus,

$-0.55 \text{ V} \leq V_{ICM} \leq +1.85 \text{ V}$

Using Eq. (13.54), we get

$V_{O\max} = V_{DD} - |V_{OV10}| - |V_{OV4}|$

$= 1.65 - 0.3 - 0.3 = +1.05$ V

Using Eq. (13.55), we obtain

$V_{O\min} = -V_{SS} + V_{OV7} + V_{OV5} + V_{tn}$

$= -1.65 + 0.3 + 0.3 + 0.5$

$= -0.55$ V

Thus,

$-0.55 \text{ V} \leq v_O \leq +1.05 \text{ V}$

Ex: 13.11 $G_m = g_{m1} = g_{m2}$

$$G_m = \frac{2(I/2)}{V_{OV1}} = \frac{I}{V_{OV1}}$$

$$= \frac{0.24}{0.2} = 1.2 \text{ mA/V}$$

$$r_{o2} = \frac{|V_A|}{I/2} = \frac{20}{0.12} = 166.7 \text{ k}\Omega$$

$$r_{o4} = \frac{|V_A|}{I_{D4}} = \frac{|V_A|}{I_B - \dfrac{I}{2}} = \frac{20}{0.150 - 0.120}$$

$$= \frac{20}{0.03} = 666.7 \text{ k}\Omega$$

$$r_{o10} = \frac{|V_A|}{I_B}$$

$$= \frac{20}{0.15} = 133.3 \text{ k}\Omega$$

$$g_{m4} = \frac{2I_{D4}}{|V_{OV}|} = \frac{2 \times 0.03}{0.2} = 0.3 \text{ mA/V}$$

$R_{o4} = (g_{m4}r_{o4})(r_{o2} \parallel r_{o10})$

$= (0.3 \times 666.7)(166.7 \parallel 133.3)$

$= 14.8$ MΩ

$$g_{m6} = \frac{2I_{D6}}{V_{OV}} = \frac{2 \times 0.03}{0.2} = 0.3 \text{ mA/V}$$

$$r_{o6} = r_{o8} = \frac{|V_A|}{I_{D6,8}} = \frac{20}{0.03} = 666.7 \text{ k}\Omega$$

$R_{o6} = g_{m6}r_{o6}r_{o8}$

$= 0.3 \times 666.7 \times 666.7 = 133.3$ MΩ

$R_o = R_{o4} \parallel R_{o6}$

$= 14.8 \parallel 133.3 = 13.3$ MΩ

$A_v = G_m R_o = 1.2 \times 13.3 \times 10^3 = 16{,}000$ V/V

Ex: 13.12 (a) The NMOS input stage operates over the following input common-mode range:

$-V_{SS} + 2V_{OV} + V_{tn} \leq V_{ICM} \leq V_{DD} - |V_{OV}| + V_{tn}$

that is,

$(-2.5 + 0.6 + 0.7) \leq V_{ICM} \leq (2.5 - 0.3 + 0.7)$

$-1.2 \text{ V} \leq V_{ICM} \leq +2.9 \text{ V}$

(b) The PMOS input stage operates over the following input common-mode range:

$-V_{SS} + V_{OV} - |V_{tp}| \leq V_{ICM} \leq$
$V_{DD} - 2|V_{OV}| - |V_{tp}|$

that is,

$(-2.5 + 0.3 - 0.7) \leq V_{ICM} \leq (2.5 - 0.6 - 0.7)$

$-2.9 \text{ V} \leq V_{ICM} \leq +1.2 \text{ V}$

(c) The overlap range is

$-1.2 \text{ V} \leq V_{ICM} \leq +1.2 \text{ V}$

(d) The input common-mode range is

$-2.9 \text{ V} \leq V_{ICM} \leq +2.9 \text{ V}$

Ex: 13.13 Denote the (W/L) of the transistors in the wide-swing mirror by $(W/L)_M$. Transistor Q_4 has

$$(W/L)_5 = \frac{1}{4}(W/L)_M$$

$$I_{\text{REF}} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_5 V_{OV5}^2$$

$$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_M \times \frac{1}{4}V_{OV5}^2$$

$$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_M (V_{OV5}/2)^2$$

Thus,

$$\frac{V_{OV5}}{2} = V_{OV}$$

where V_{OV} is the overdrive voltage for each of the mirror transistors. Thus,

$V_5 = V_{tn} + 2V_{OV}$

which is the value of V_{BIAS} needed in the circuit of Fig. 13.13(b).

Ex: 13.14 $I_{S2} = 2I_{S1}$

Using Eq. (13.72), we obtain

$$I = \frac{V_T}{R_2}\ln\frac{I_{S2}}{I_{S1}}$$

$$0.01 = \frac{0.025}{R_2}\ln 2$$

$\Rightarrow R_2 = 1.73$ kΩ

$R_3 = R_4 = \dfrac{0.2 \text{ V}}{0.01 \text{ mA}} = 20 \text{ k}\Omega$

Ex: 13.15 To obtain $I_5 = 10$ μA, transistor Q_5 must have the same emitter area as Q_2 and

$R_5 = R_2 = 1.73 \text{ k}\Omega$

To obtain $I_6 = 5$ μA, transistor Q_6 must have one-half the emitter area of Q_2 and

$R_6 = 2R_2 = 3.46 \text{ k}\Omega$

To obtain $I_7 = 20$ μA, transistor Q_7 must have double the emitter area of Q_3 and

$R_7 = 0.5R_3 = 10 \text{ k}\Omega$

To obtain $I_8 = 10$ μA, transistor Q_8 must have the same emitter area as Q_3 and

$R_8 = R_3 = 20 \text{ k}\Omega$

Ex: 13.16 Refer to Fig. 13.23. At node X,

$$I_{REF} = \frac{2I}{1 + \dfrac{2}{\beta_P}} + \frac{2I}{\beta_P}$$

$$I_{REF} = 2I\left[\frac{\beta_P + 1 + \dfrac{2}{\beta_P}}{\beta_P\left(1 + \dfrac{2}{\beta_P}\right)}\right]$$

$$\simeq 2I\frac{\beta_P + 1}{\beta_P + 2}$$

$$\simeq 2I$$

Thus,

$$I = \frac{I_{REF}}{2} = 9.5 \text{ μA}$$

resulting in

$$I_{C1} = I_{C2} = I_{C3} = I_{C4} = 9.5 \text{ μA}$$

Ex: 13.17 Figure 1 on next page shows the determination of the loop gain of the feedback circuit that stabilizes the bias currents of the first stage of the 741 op amp. Note that since I_{REF} is assumed to be constant, we have shown its incremental value at node X to be zero. Observe that this circuit shows only incremental quantities. The analysis shown provides the returned current signal as

$$I_r = -I_t \frac{\beta_P}{1 + \dfrac{2}{\beta_P}}$$

For $\beta_P \gg 1$, we have

$$I_r \simeq -\beta_P I_t$$

and the loop gain $A\beta$ is

$$A\beta \equiv -\frac{I_r}{I_t} = \beta_P$$

Ex: 13.18 $I_B = \dfrac{1}{2}(I_{B1} + I_{B2})$

$$= \frac{1}{2}\left(\frac{I}{\beta_N + 1} + \frac{I}{\beta_N + 1}\right)$$

$$= \frac{I}{\beta_N + 1} \simeq \frac{I}{\beta_N}$$

$$= \frac{9.5}{200} = 47.5 \text{ nA}$$

$$I_{OS} = 0.1 \times I_B = 4.75 \text{ nA}$$

Ex: 13.19
$V_{C1} = V_{CC} - V_{EB8} = 15 - 0.6 = 14.4$ V

Q_1 and Q_2 saturate when V_{ICM} exceeds V_{C1} by 0.3 V. Thus,

$$V_{ICM\max} = +14.7 \text{ V}$$

Ex: 13.20 $r_e = \dfrac{V_T}{I_E} \simeq \dfrac{25 \text{ mV}}{9.5 \text{ μA}} = 2.63 \text{ k}\Omega$

$$g_{m1} \simeq \frac{1}{r_e} = 0.38 \text{ mA/V}$$

$$G_{m1} = \frac{1}{2}g_{m1} = 0.19 \text{ mA/V}$$

$$R_{id} = (\beta_N + 1) \times 4r_e$$

$$= 201 \times 4 \times 2.63$$

$$= 2.1 \text{ M}\Omega$$

Ex: 13.21 $r_{o4} = \dfrac{|V_{Ap}|}{I} = \dfrac{50 \text{ V}}{9.5 \text{ μA}} = 5.26 \text{ M}\Omega$

$$g_{m4} = 0.38 \text{ mA/V}$$

$$r_{e2} = 2.63 \text{ k}\Omega$$

$$r_{\pi 4} = \frac{\beta_P}{g_{m4}} = \frac{50}{0.38} = 131.6 \text{ k}\Omega$$

$$R_{o4} = r_{o4}1 + g_{m4}(r_{e2} \parallel r_{\pi 4})$$

$$= 5.261 + 0.38(2.63 \parallel 131.6)$$

$$= 10.4 \text{ M}\Omega$$

10.5 MΩ is obtained by neglecting $r_{\pi 4}$.

$$r_{o6} = \frac{V_{An}}{I} = \frac{125 \text{ V}}{9.5 \text{ μA}} = 13.16 \text{ M}\Omega$$

$$g_{m6} = 0.38 \text{ mA/V}$$

$$R_6 = 1 \text{ k}\Omega$$

$$r_{\pi 6} = \frac{200}{0.38} = 526.3 \text{ k}\Omega$$

$$R_{o6} = r_{o6}1 + g_{m6}(R_2 \parallel r_{\pi 6})$$

$$= 13.161 + 0.38(1 \parallel 526.3)$$

This figure belongs to Exercise 13.17.

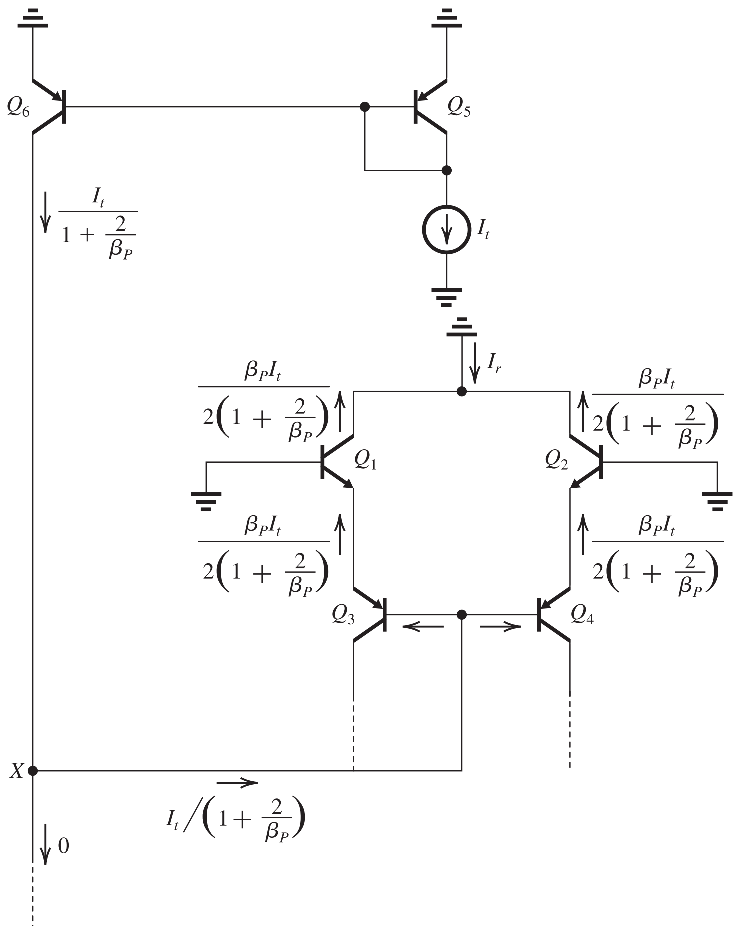


Figure 1

$= 18.2\ \text{M}\Omega$

$R_{o1} = 0.5 \times R_{o4}$

$= 5.3\ \text{M}\Omega$

Ex: 13.22 $|A_{vo}| = G_{m1}R_{o1}$

Using G_{m1} given in the answer to Exercise 13.20,

$G_{m1} = 0.19$ mA/V

and R_{o1} given in the answer to Exercise 13.21,

$R_{o1} = 5.3\ \text{M}\Omega$

we obtain

$|A_{vo}| = G_{m1}R_{o1}$

$= 0.19 \times 5.3 = 1007$ V/V

Ex: 13.23

$$R_{o6} = r_{o6} = \frac{|V_{Ap}|}{I_{C6}} = \frac{50\ \text{V}}{19\ \mu\text{A}} = 2.63\ \text{M}\Omega$$

$$R_{o7} = r_{o7}[1 + g_{m7}(R_4 \parallel r_{\pi 7})]$$

where

$$r_{o7} = \frac{V_{An}}{I_{C7}} = \frac{125\ \text{V}}{19\ \mu\text{A}} = 6.58\ \text{M}\Omega$$

$$g_{m7} = \frac{I_{C7}}{V_T} = \frac{19\ \mu\text{A}}{0.025\ \text{V}} = 0.76\ \text{mA/V}$$

$R_4 = 5\ \text{k}\Omega$

$$r_{\pi 7} = \frac{\beta_N}{g_{m7}} = \frac{200}{0.76} = 263.2\ \text{k}\Omega$$

$R_{o7} = 6.58[1 + 0.76(5 \parallel 263.2)]$

$= 31.1\ \text{M}\Omega$

$R_o = R_{o6} \parallel R_{o7}$

$= 2.63 \parallel 31.1 = 2.43\ \text{M}\Omega$

Ex: 13.24 Using Eq. (13.84), we obtain

$$G_{mcm} = \frac{\epsilon_m}{2R_o}$$

$$= \frac{5.5 \times 10^{-3}}{2 \times 2.43 \times 10^{6}} = 1.13 \times 10^{-6}\ \text{mA/V}$$

$$\text{CMRR} = \frac{G_{m1}}{G_{mcm}} = \frac{0.19}{1.13 \times 10^{-6}} = 1.68 \times 10^5$$

or 104.5 dB

Without common-mode feedback, the CMRR is reduced by a factor equal to β_P. Equivalently,

$\text{CMRR} = 104.5 - 20 \log \beta_P$

$= 104.5 - 20 \log 50$

$= 70.5$ dB

Ex: 13.25 Refer to the circuit in Fig. 13.29.

(a) Current gain from v_{IP} to output
$= (\beta_1 + 1)(\beta_2 + 1)\beta_P$

$\simeq \beta_1\beta_2\beta_P = \beta_N\beta_P^2$

Current gain from v_{IN} to output $= (\beta_3 + 1)\beta_N$

$\simeq \beta_3\beta_N = \beta_N^2$

(b) For $i_L = +10$ mA,

current needed at v_{IP} input

$$= \frac{10}{\beta_N\beta_P^2} = \frac{10}{40 \times 10^2} = 2.5\ \mu\text{A}$$

For $i_L = -10$ mA,

current needed at v_{IN} input $= \dfrac{10}{\beta_N^2} = \dfrac{10}{40^2}$

$= 6.25\ \mu$A

Ex: 13.26 $I_Q = 0.4$ mA, $I = 10\ \mu$A,

$$\frac{I_{SN}}{I_{S10}} = 10, \quad \frac{I_{S7}}{I_{S11}} = 2$$

Using Eq. (13.93), we obtain

$$0.4 \times 10^3 = 2\left(\frac{I_{\text{REF}}^2}{10}\right) \times 10 \times 2$$

where I_{REF} is in μA. Thus,

$I_{\text{REF}} = 10\ \mu$A

For $i_L = -10$ mA, then

$i_P = 10 + i_N$

Using Eq. (13.94), we get

$$\frac{i_N(10 + i_N)}{i_N + 10 + i_N} = 0.2$$

$\Rightarrow i_N^2 - 9.6i_N + 2 = 0$

$\Rightarrow i_N = 0.2$ mA

$i_P = 10.2$ mA

Chapter 14

Solutions to Exercises within the Chapter

Ex: 14.1 $A = -20\ \log|T|\,[\text{dB}]$

$\lvert T\rvert =$	1	0.99	0.9	0.8	0.7	0.5	0.1	0
$A \simeq$	0	0.1	1	2	3	6	20	∞

Ex: 14.2 $A_{\max} = -20 \log 0.9 \simeq 0.9$ dB

$$A_{\min} = 20 \log\left[\frac{1}{0.01}\right] = 40\ \text{dB}$$

Ex: 14.3 (a)

$$T(s) = \frac{K}{s^3 + b_2 s^2 + b_1 s + b_0}$$

This is a low-pass filter with 3 transmission zeros at $s = \infty$.

(b)

$$T(s) = \frac{Ks}{s^3 + b_2 s^2 + b_1 s + b_0}$$

There is one transmission zero at $s = 0$, and two transmission zeros at $s = \infty$. $\Rightarrow$ Bandpass filter.

(c)

$$T(s) = \frac{Ks^2}{s^3 + b_2 s^2 + b_1 s + b_0}$$

There are two transmission zeros at $s = 0$, and one transmission zero at $s = \infty$. $\Rightarrow$ Bandpass.

(d)

$$T(s) = \frac{Ks^3}{s^3 + b_2 s^2 + b_1 s + b_0}$$

Three transmission zeros at $s = 0$. $\Rightarrow$ Highpass.

Ex: 14.4

$$T(s) = k\frac{(s + j2)(s - j2)}{\left(s + \frac{1}{2} + j\sqrt{\frac{3}{2}}\right)\left(s + \frac{1}{2} - j\sqrt{\frac{3}{2}}\right)}$$

$$= k\frac{\left(s^2 + 4\right)}{s^2 + s + \frac{1}{4} + \frac{3}{4}}$$

$$= k\frac{\left(s^2 + 4\right)}{s^2 + s + 1}$$

$$T(0) = k\frac{4}{1} = 1$$

$$k = \frac{1}{4}$$

$$\therefore T(s) = \frac{1}{4}\frac{\left(s^2 + 4\right)}{s^2 + s + 1}$$

Ex: 14.5

$$T(s) = a_3\frac{s\left(s^2 + 4\right)}{(s + 0.1 + j0.8)(s + 0.1 - j0.8)} \times \frac{1}{(s + 0.1 + j1.2)(s + 0.1 - j1.2)}$$

$$= a_3\frac{s\left(s^2 + 4\right)}{\left(s^2 + 0.2s + 0.65\right)\left(s^2 + 0.2s + 1.45\right)}$$

Ex: 14.6

$$T(s) = -\frac{R_2}{R_1}\frac{1}{1 + \frac{s}{\omega_0}}$$

where $R_2/R_1 = 10$ and $\omega_0 = 10^5$ rad/s. The $|\ T\ |$ will be reduced by 3 dB from the value at dc, at $\omega = \omega_0 = 10^5$ rad/s. Thus,

$\omega_P = 10^5$ rad/s

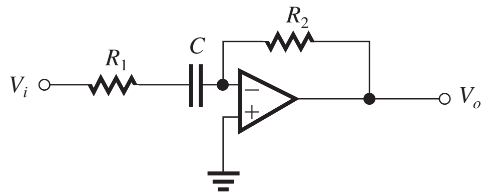
The $|\ T\ |$ at $\omega = \omega_s = 3\omega_0$ is

$$|\ T(j\omega_s)\ | = \frac{10}{\sqrt{1 + \left(\frac{3\omega_0}{\omega_0}\right)^2}}$$

Thus, relative to $|\ T(0)\ |$, the transmission will be lower by $20 \log\sqrt{1 + 3^2}$, which is 10 dB. Thus,

$A(\omega_s) = 10$ dB

Ex: 14.7



$$10^4 = \frac{1}{CR_1},\quad R_1 = 10\ \text{k}\Omega$$

$C = 0.01\ \mu$F = 10 nF

$$\text{H.F. gain} = \frac{-R_2}{R_1} = -10$$

$R_2 = 100\ \text{k}\Omega$

Ex: 14.8 $T(s) = \dfrac{\omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$

For maximally flat response, $Q = 1/\sqrt{2}$, thus

$$T(s) = \frac{\omega_0^2}{s^2 + s\sqrt{2}\,\omega_0 + \omega_0^2}$$

$$T(j\omega) = \frac{\omega_0^2}{(\omega_0^2 - \omega^2) + j\sqrt{2}\,\omega\omega_0}$$

$$|T(j\omega)| = \frac{\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2+2\omega^2\omega_0^2}}$$

$$= \frac{\omega_0^2}{\sqrt{\omega_0^4+\omega^4}}$$

$$= \frac{1}{\sqrt{1+\left(\dfrac{\omega}{\omega_0}\right)^4}}$$

At $\omega = \omega_0$,

$$|T(j\omega_0)| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

which is 3 dB below the value at $\omega = 0$ (0 dB). Q.E.D.

Ex: 14.9 $T(s) = \dfrac{sK\left(\dfrac{\omega_0}{Q}\right)}{s^2+s\dfrac{\omega_0}{Q}+\omega_0^2}$

where K is the center-frequency gain. For $\omega_0 = 10^5$ rad/s and 3-dB bandwidth $= 10^3$ rad/s, we have

$$\text{3-dB } BW = \frac{\omega_0}{Q}$$

$$10^3 = \frac{10^5}{Q}$$

$$\Rightarrow Q = 100$$

Also, for a center-frequency gain of 10, we have

$$K = 10$$

Thus,

$$T(s) = \frac{10^4 s}{s^2+10^3 s+10^{10}}$$

Ex: 14.10

$$T(s) = k\,\frac{s^2+\omega_n^2}{s^2+s\,\frac{\omega_0}{Q}+\omega_0^2}$$

For $\omega_n = 1.2$ rad/s, $\omega_0 = 1$ rad/s, $Q = 5$, and dc gain $= k\,\dfrac{\omega_n^2}{\omega_0^2} = 1$, i.e., $k = 0.694$,

$$T(s) = 0.694\,\frac{s^2+1.44}{s^2+0.2\,s+1}$$

$$T(\infty) = 0.694$$

Ex: 14.11 $|\,T(j\omega)\,| = 1/\sqrt{1+\epsilon^2\left(\dfrac{\omega}{\omega_P}\right)^{2N}}$

$\omega_P = \omega_{3\text{dB}}$, $\epsilon = 1$, and $N = 5$,

$$|\,T(j\omega)\,| = 1/\sqrt{1+\left(\frac{\omega}{\omega_{3\text{dB}}}\right)^{10}}$$

$$= 1/\sqrt{1+\left(\frac{f}{f_{3\text{dB}}}\right)^{10}}$$

$$|\,T(j\omega_s)\,| = 1/\sqrt{1+\left(\frac{30}{10}\right)^{10}}$$

$$A(\omega_s) = -20\log|\,T(j\omega_s)\,|$$

$$= 47.7 \text{ dB}$$

Ex: 14.12

$$\epsilon = \sqrt{10^{\frac{A_{\max}}{10}}-1} = \sqrt{10^{\frac{1}{10}}-1} = 0.5088$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}}}$$

$$A(\omega_s) = -20\log|T(j\omega_s)|$$

$$= 10\log\left[1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$

Thus,

$$10\log\left[1+0.5088^2\times 1.5^{2N}\right] \geq 30$$

$N = 10$: LHS $= 29.35$ dB

$N = 11$: LHS $= 32.87$ dB

$\therefore$ Use $N = 11$ and obtain

$$A_{\min} = 32.87 \text{ dB}$$

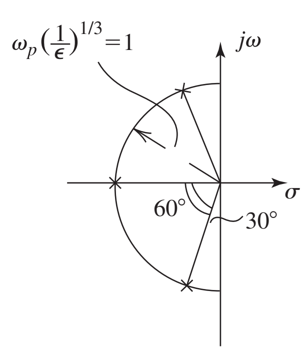
Ex: 14.13 The real pole is at $s = -1$

The complex conjugate poles are at

$$s = -\cos 60^\circ \pm j\sin 60^\circ$$

$$= -0.5 \pm j\sqrt{\frac{3}{2}}$$

Note that $\omega_0 = 1$ rad/s and $Q = 1$.



$$T(s) = \frac{1}{(s+1)\left(s+0.5+j\sqrt{\dfrac{3}{2}}\right)\left(s+0.5-j\sqrt{\dfrac{3}{2}}\right)}$$

$$= \frac{1}{(s+1)\left(s^2+s+1\right)}$$

DC gain $= 1$

Ex: 14.14

$$\epsilon = \sqrt{10^{\frac{A_{\max}}{10}} - 1} = \sqrt{10^{\frac{0.5}{10}} - 1} = 0.3493$$

$$A(\omega_s) = 10 \log\left[1 + \epsilon^2 \cosh^2\left(N \cosh^{-1}\frac{\omega_s}{\omega_p}\right)\right]$$

$$= 10 \log\left[1 + 0.3493^2 \cosh^2\left(7 \cosh^{-1} 2\right)\right]$$

$$= 64.9 \text{ dB}$$

For $A_{\max} = 1$ dB, $\epsilon = \sqrt{10^{0.1} - 1} = 0.5088$

$$A(\omega_s) = 10 \log\left[1 + 0.5088^2 \cosh^2\left(7 \cosh^{-1} 2\right)\right]$$

$$= 68.2 \text{ dB}$$

This is an increase of 3.3 dB

Ex: 14.15 $\epsilon = \sqrt{10^{\frac{1}{10}} - 1} = 0.5088$

(a) For the Chebyshev filter:

$$A(\omega_s)$$

$$= 10 \log\left[1 + 0.5088^2 \cosh^2\left(N \cosh^{-1} 1.5\right)\right]$$

$$\geq 50 \text{ dB}$$

$N = 7.4 \ \therefore$ choose $N = 8$

Excess attenuation $=$

$$10 \log\left[1 + 0.5088^2 \cosh^2\left(8 \cosh^{-1} 1.5\right)\right] - 50$$

$$= 55 - 50 = 5 \text{ dB}$$

(b) For a Butterworth filter

$$\epsilon = 0.5088$$

$$A(\omega_s) = 10 \log\left[1 + \epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$

$$= 10 \log\left(1 + 0.5088^2 (1.5)^{2N}\right) \geq 50$$

$N = 15.9 \ \therefore$ choose $N = 16$

Excess attenuation $=$

$$10 \log\left(\left[1 + 0.5088^2 (1.5)^{32}\right] - 50\right) = 0.5 \text{ dB}$$

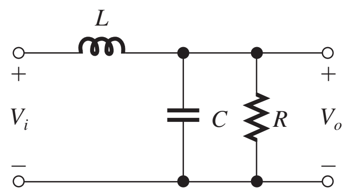
Ex: 14.16 Maximally flat $\Rightarrow Q = \dfrac{1}{\sqrt{2}}$

$$\omega_0 = 2\pi \times 100 \times 10^3$$

Arbitrarily selecting $R = 1$ kΩ, we get

$$Q = \omega_0 CR \Rightarrow C = \frac{1}{\sqrt{2} \times 2\pi 10^5 \times 10^3}$$

$$= 1125 \text{ pF}$$



Also $Q = \dfrac{R}{\omega_0 L}$

$$\therefore L = \frac{R}{\omega_0 Q} = \frac{10^3}{2\pi 10^5 \times \frac{1}{\sqrt{2}}} = 2.25 \text{ mH}$$

Ex: 14.17 Refer to Fig. 14.19(c).

$$\omega_0 = 1/\sqrt{LC} = 10^5 \text{ rad/s}$$

$$\text{3-dB } BW = \frac{\omega_0}{Q} = \frac{1}{CR} = 10^3 \text{ rad/s}$$

Selecting $R = 100$ kΩ,

$$C = \frac{1}{10^3 R} = \frac{1}{10^3 \times 10^5} = 10^{-8} \text{ F} = 10 \text{ nF}$$

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{10^{10} \times 10^{-8}} = 0.01 \text{ H} = 10 \text{ mH}$$

Ex: 14.18 Refer to Fig. 14.20(a). Selecting,

$$R_1 = R_2 = R_3 = R_5 = R$$

and

$$C_4 = C$$

gives

$$Z_{\text{in}} = sCR^2$$

Thus,

$$L = CR^2$$

To obtain $L = 100$ mH, we select $C = 1$ nF. Thus,

$$100 \times 10^{-3} = 1 \times 10^{-9} \ R^2$$

$$\Rightarrow R = 10^4 = 10 \text{ k}\Omega$$

The values of C and R are practically convenient.

Ex: 14.19

$f_0 = 10$ kHz, $\Delta f_{3\text{dB}} = 500$ Hz

$$Q = \frac{f}{\Delta f_{3\text{dB}}} = \frac{10^4}{500} = 20$$

Using Eqs. (14.53) and (14.54) and selecting,

$C = 1.2$ nF,

we get,

$$R = \frac{1}{\omega_0 C}$$

$$= \frac{1}{2\pi 10^4 \times 1.2 \times 10^{-9}} = 13.26 \text{ k}\Omega$$

$R_6 = QR = 20 \times 13.26 = 265 \text{ k}\Omega$

Now,

$K = \text{center-frequency gain} = 10$

Thus,

$1 + r_2/r_1 = 10$

Selecting $r_1 = 10 \text{ k}\Omega$, we obtain $r_2 = 90 \text{ k}\Omega$.

Ex: 14.20 Refer to the KHN circuit in Fig. 14.24. Choosing $C = 1$ nF, we obtain

$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi 10^4 \times 10^{-9}} = 15.9 \text{ k}\Omega$$

Using Eq. (14.65) and selecting $R_1 = 10 \text{ k}\Omega$, we get

$R_f = R_1 = 10 \text{ k}\Omega$

Using Eq. (14.66) and setting $R_2 = 10 \text{ k}\Omega$, we obtain

$R_3 = R_2(2Q - 1) = 10(2 \times 2 - 1) = 30 \text{ k}\Omega$

$$\text{High-frequency gain} = K = 2 - \frac{1}{Q} = 1.5 \text{ V/V}$$

The transfer function to the output of the first integrator is

$$\frac{V_{bp}}{V_i} = -\frac{1}{sCR} \times \frac{V_{hp}}{V_i} = \frac{sK/(CR)}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

Thus the center-frequency gain is given by

$$\frac{K}{CR}\frac{Q}{\omega_0} = KQ = 1.5 \times 2 = 3 \text{ V/V}$$

Ex: 14.21 Refer to Fig. 14.25 (b)

$$CR = \frac{1}{\omega_0} \Rightarrow C = \frac{1}{2\pi 10^4 \times 10^4} = 1.59 \text{ nF}$$

$R_d = QR = 20 \times 10 = 200 \text{ k}\Omega$

Center frequency gain $= KQ = 1$

$$\therefore K = \frac{1}{Q} = \frac{1}{20}$$

$R_g = R/K = 20R = 200 \text{ k}\Omega$

Ex: 14.22 Refer to Fig. 14.27(a) and Eqns. (14.80)-(14.85). Selecting

$C_1 = C_2 = C = 1$ nF,

and,

$R_3 = R$

then,

$$R_4 = \frac{R}{4Q^2} = \frac{R}{4 \times 25} = \frac{R}{100}$$

and

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 5}{10^5} = 10^{-4} \text{ s}$$

$$R = \frac{10^{-4}}{10^{-9}} = 10^5 \ \Omega = 100 \text{ k}\Omega$$

$R_3 = R = 100 \text{ k}\Omega$

$$R_4 = \frac{R}{100} = 1 \text{ k}\Omega$$

The center-frequency gain is given by Eq. (14.85),

$K = -2Q^2 = -2 \times 25 = -50 \text{ V/V}$

Ex: 14.23 Refer to Fig. 14.29 and Eqns. (14.94)-(14.101). Selecting

$R_1 = R_2 = R$

$C_4 = C$

then

$$C_3 = \frac{C}{4Q^2} = \frac{C}{4 \times \frac{1}{2}} = \frac{C}{2}$$

Thus, if we select $C = 1$ nF then

$C_4 = 1$ nF

$C_3 = 0.5$ nF

Now,

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 1/\sqrt{2}}{10^5} = 1.414 \times 10^{-5} \text{ s}$$

Thus,

$$R = \frac{1.414 \times 10^{-5}}{1 \times 10^{-9}} = 1.414 \times 10^4 = 14.14 \text{ k}\Omega$$

$\Rightarrow R_1 = R_2 = 14.14 \text{ k}\Omega$

Finally, from the transfer function in Eq. (14.94)

DC gain $= 1$ V/V

Ex: 14.24 From Eq. (14.108),

$C_3 = C_4 = \omega_0 T_c C$

$$= 2\pi 10^4 \times \frac{1}{200 \times 10^3} \times 20$$

$= 6.283$ pF

From Eq. (14.110),

$$C_5 = \frac{C_4}{Q} = \frac{6.283}{20} = 0.314 \text{ pF}$$

From Eq. (14.111),

$$\text{Centre-frequency gain} = \frac{C_6}{C_5} = 1$$

$C_6 = C_5 = 0.314$ pF

Chapter 15

Solutions to Exercises within the Chapter

Ex: 15.1 Pole frequency $f_0 = 1$ kHz

Center-frequency gain $= \dfrac{1}{\text{Amplifier gain}}$

$= \dfrac{1}{2}$ V/V

Ex: 15.2 Refer to Fig. 15.2(a).

$\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.1 \times 10^{-3} \times 10 \times 10^{-9}}} =$
10^6 rad/s

The loop gain at $\omega_0 = (1 + \dfrac{r_2}{r_1}) \times 1 = 1.01.$

Thus, the condition of oscillation is satisfied.

Ex: 15.3 $\omega_0 = \dfrac{1}{\sqrt{LC}}$

(a)

$L \longrightarrow 1.01L$

$\omega_0 + \triangle\omega_0 = \dfrac{1}{\sqrt{1.01LC}} = \dfrac{0.995}{\sqrt{LC}}$

Thus,

$\dfrac{\triangle\omega_0}{\omega_0} \times 100 = -0.5\%$

(b)

$C \longrightarrow 1.01C$

$\omega_0 + \triangle\omega_0 = \dfrac{1}{\sqrt{L \times 1.01C}} = \dfrac{0.995}{\sqrt{LC}}$

$\dfrac{\triangle\omega_0}{\omega_0} \times 100 = -0.5\%$

(c)

$R \longrightarrow 1.01R$

Since ω_0 is *not* a function of R, changing R has no effect on ω_0, that is, $\triangle\omega_0 = 0$.

Ex: 15.4 Refer to Fig. 15.5. Noting that the sinusoid at the positive input terminal of the op amp has a peak amplitude of 0.9 V, to obtain a 5-V peak amplitude at the output, the closed-loop gain of the op amp must be

$1 + \dfrac{r_2}{r_1} = \dfrac{5}{0.9} = 5.6$

$\Rightarrow \dfrac{r_2}{r_1} = 4.6$

Ex: 15.5 (a) $L(s) = \left(1 + \dfrac{R_2}{R_1}\right)\dfrac{Z_P}{Z_P + Z_S}$

$= \left(1 + \dfrac{R_2}{R_1}\right)\dfrac{1}{1 + Z_S Y_P}$

$= \left(1 + \dfrac{20.3}{10}\right)\dfrac{1}{1 + \left(R + \dfrac{1}{sC}\right)\left(\dfrac{1}{R} + sC\right)}$

$= \dfrac{3.03}{3 + sCR + \dfrac{1}{sCR}}$

where $R = 10$ kΩ and $C = 16$ nF

Thus,

$L(s) = \dfrac{3.03}{3 + s16 \times 10^{-5} + \dfrac{1}{s \times 16 \times 10^{-5}}}$

The closed-loop poles are found by setting $L(s) = 1$; that is, they are the values of s, satisfying

$3 + s \times 16 \times 10^{-5} + \dfrac{1}{s \times 16 \times 10^{-5}} = 3.03$

$\Rightarrow s = \dfrac{10^5}{16}(0.015 \pm j)$

(b) The frequency of oscillation is $\left(10^5/16\right)$ rad/s or approximately 1 kHz.

(c) Using Eq. (15.11), the positive peak amplitude can be found as

$\hat{v}_{o+} =$
$\left[\dfrac{1}{3} \times 15 + \left(1 + \dfrac{1}{3}\right) \times 0.7\right] / \left(\dfrac{2}{3} - \dfrac{1}{3} \times \dfrac{1}{3}\right)$

$= 10.68$ V

Since the limiter circuit is symmetric, the negative peak amplitude also will be 10.68 V. Thus, the peak-to-peak amplitude of the output sinusoid will be

$\hat{v}_{op-p} = 2 \times 10.68 = 21.36$ V

Ex: 15.6 (a) For oscillations to start, we need from Eq. (15.10), $R_2/R_1 = 2$; thus the potentiometer should be set so that its resistance to ground is 20 kΩ.

(b) From Eq. (15.9),

$f_0 = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi 10 \times 10^3 \times 16 \times 10^{-9}}$

$= 1$ kHz

Ex: 15.7 Figure 1 shows the circuit together with the analysis details. The current I can be found as

$I = \dfrac{V_o}{R_f}\left(1 + \dfrac{1}{sCR}\right) + \dfrac{V_o}{sCRR_f}\left(2 + \dfrac{1}{sCR}\right)$

Finally, V_x can be found from

$V_x = -\dfrac{1}{sCR_f}V_o\left(2 + \dfrac{1}{sCR}\right) - \dfrac{I}{sC}$

$= -\dfrac{1}{sCR_f}V_o\left(2 + \dfrac{1}{sCR}\right) - \dfrac{V_o}{sCR_f}\left(1 + \dfrac{1}{sCR}\right)$

$-\dfrac{V_o}{s^2C^2RR_f}\left(2 + \dfrac{1}{sCR}\right)$

This figure belongs to Exercise 15.7.

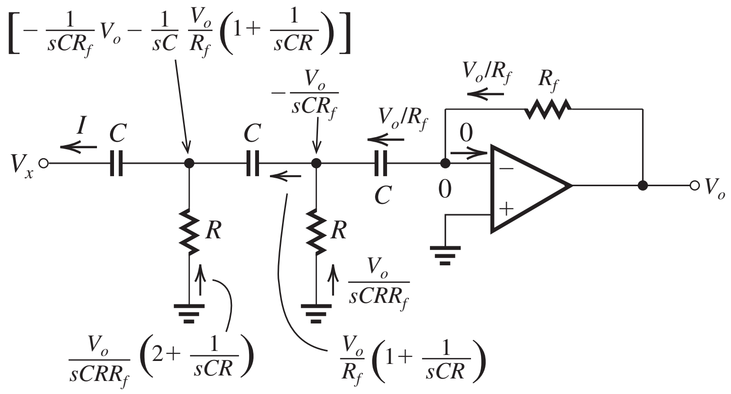


Figure 1

$$-\frac{V_x}{V_o}=\frac{3}{sCR_f}+\frac{4}{s^2C^2RR_f}+\frac{1}{s^3C^3R^2R_f}$$

$$\frac{V_o}{V_x}=-\frac{s^2C^2RR_f}{4+s3CR+\frac{1}{sCR}}$$

For $s=j\omega$,

$$\frac{V_o}{V_x}=\frac{\omega^2C^2RR_f}{4+j\left(3\omega CR-\frac{1}{\omega CR}\right)}$$

Ex: 15.8 From the loop-gain expression

$$\frac{V_o}{V_x}=\frac{\omega^2C^2RR_f}{4+j\left(3\omega CR-\frac{1}{\omega CR}\right)}$$

we see that the phase will be zero at

$$3\omega_0CR=\frac{1}{\omega_0CR}$$

$$\Rightarrow \omega_0=\frac{1}{\sqrt{3}CR}$$

At this frequency, we have

$$\frac{V_o}{V_i}=\frac{\omega_0^2C^2RR_f}{4}$$

$$=\frac{1}{12}\frac{R_f}{R}$$

Thus the minimum value that R_f must have for oscillations to start is

$$R_f=12R$$

Using the values in Fig. 15.10, we obtain

$$\omega_0=\frac{1}{\sqrt{3}}\frac{1}{16\times10^{-9}\times10\times10^3}=3.608\text{ krad/s}$$

$$f_0=\frac{\omega_0}{2\pi}=\frac{3.608\times10^3}{2\pi}=574.3\text{ Hz}$$

$$R_f\geq12\times10=120\text{ k}\Omega$$

Ex: 15.9 $\omega_0=\frac{1}{CR}\Rightarrow CR=\frac{1}{2\pi10^3}$

For $C=16$ nF, we have $R=10$ kΩ.

The peak-to-peak amplitude of the square wave is 1.4 V, which has a fundamental-frequency component of $\frac{4\times1.4}{\pi}=1.8$ V peak-to-peak. Since the output is twice as large as the voltage across the resonator, its peak-to-peak amplitude is 3.6 V.

Ex: 15.10 Refer to Fig. 15.14(a). The admittance seen by the transistor between drain and source is

$$Y=sC_1+\frac{1}{sL+\frac{1}{sC_2}}$$

$$=\frac{s^2LC_1+\frac{C_1}{C_2}+1}{sL+\frac{1}{sC_2}}$$

$$=\frac{s^2+1/\left[L\frac{C_1C_2}{C_1+C_2}\right]}{s\frac{1}{C_1}+\frac{1}{sLC_1C_2}}$$

$$Y(j\omega)=\frac{-\omega^2+1/\left[L\frac{C_1C_2}{C_1+C_2}\right]}{j\frac{\omega}{C_1}\left(1-\frac{1}{\omega^2LC_2}\right)}$$

from which we see that at

$\omega=\omega_0=1/\sqrt{L\frac{C_1C_2}{C_1+C_2}}$ the numerator is zero and $Y(j\omega_0)=0$. Q.E.D.

Ex: 15.11 Refer to the circuit in Fig. 15.15(a).

$$R=R_{\text{coil}}\parallel r_o\parallel R_L$$

$$=\frac{Q}{\omega_0C_1}\parallel100\times10^3\parallel100\times10^3$$

$$=\frac{100}{10^6\times0.01\times10^{-6}}\parallel10^5\parallel10^5$$