

***Elements of Electromagnetics 7th edition Sadiku
Solutions Manual***

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CHAPTER 10

P. E. 10.1 (a)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2 \times 10^8} = \underline{31.42 \text{ ns}},$$

$$\lambda = uT = 3 \times 10^8 \times 31.42 \times 10^{-9} = \underline{9.425 \text{ m}}$$

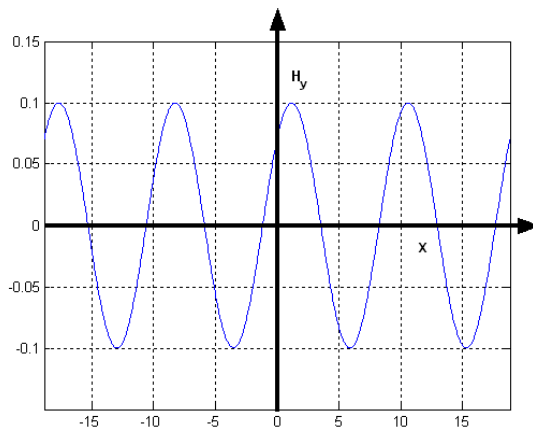
$$k = \beta = 2\pi / \lambda = \underline{0.6667 \text{ rad/m}}$$

$$(b) \quad t_1 = T/8 = \underline{3.927 \text{ ns}}$$

(c)

$$\mathbf{H}(t = t_1) = 0.1 \cos(2 \times 10^8 \frac{\pi}{8 \times 10^8} - 2x/3) \mathbf{a}_y = 0.1 \cos(2x/3 - \pi/4) \mathbf{a}_y$$

as sketched below.

P. E. 10.2 Let $x_o = \sqrt{1 + (\sigma / \omega \epsilon)^2}$, then

$$\alpha = \omega \sqrt{\frac{\mu_o \epsilon_o}{2} \mu_r \epsilon_r (x_o - 1)} = \frac{\omega}{c} \sqrt{\frac{16}{2}} \sqrt{x_o - 1}$$

$$\text{or } \sqrt{x_o - 1} = \frac{\alpha c}{\omega \sqrt{8}} = \frac{1/3 \times 3 \times 10^8}{10^8 \sqrt{8}} = \frac{1}{\sqrt{8}} \longrightarrow x_o = 9/8$$

$$x_o^2 = \frac{81}{64} = 1 + (\sigma / \omega \epsilon)^2 \longrightarrow \frac{\sigma}{\omega \epsilon} = 0.5154$$

$$\tan 2\theta_{\eta} = 0.5154 \longrightarrow \theta_{\eta} = 13.63^{\circ}$$

$$\frac{\beta}{\alpha} = \sqrt{\frac{x_o + 1}{x_o - 1}} = \sqrt{17}$$

$$(a) \quad \beta = \alpha \sqrt{17} = \frac{\sqrt{17}}{3} = \underline{1.374 \text{ rad/m}}$$

$$(b) \quad \frac{\sigma}{\omega \epsilon} = \underline{0.5154}$$

$$(c) \quad |\eta| = \frac{\sqrt{\mu / \epsilon}}{\sqrt{x_o}} = \frac{120\pi \sqrt{2/8}}{\sqrt{9/8}} = 177.72$$

$$\eta = \underline{177.72 \angle 13.63^{\circ} \Omega}$$

$$(d) \quad u = \frac{\omega}{\beta} = \frac{10^8}{1.374} = \underline{7.278 \times 10^7 \text{ m/s}}$$

$$(e) \quad \mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E \longrightarrow \mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_H \longrightarrow \mathbf{a}_H = \mathbf{a}_y$$

$$\mathbf{H} = \frac{0.5}{177.5} e^{-z/3} \sin(10^8 t - \beta z - 13.63^{\circ}) \mathbf{a}_y = \underline{2.817 e^{-z/3} \sin(10^8 t - \beta z - 13.63^{\circ}) \mathbf{a}_y \text{ mA/m}}$$

P. E. 10.3 (a) Along -z direction

$$(b) \quad \lambda = \frac{2\pi}{\beta} = 2\pi / 2 = \underline{3.142 \text{ m}}$$

$$f = \frac{\omega}{2\pi} = \frac{10^8}{2\pi} = \underline{15.92 \text{ MHz}}$$

$$\beta = \omega \sqrt{\mu \epsilon} = \omega \sqrt{\mu_o \epsilon_o} \sqrt{\mu_r \epsilon_r} = \frac{\omega}{c} \sqrt{(I) \epsilon_r}$$

$$\text{or } \sqrt{\epsilon_r} = \beta c / \omega = \frac{3 \times 10^8 \times 2}{10^8} = 6 \longrightarrow \underline{\underline{\epsilon_r = 36}}$$

$$(c) \quad \theta_{\eta} = 0, |\eta| = \sqrt{\mu / \epsilon} = \sqrt{\mu_o / \epsilon_o} \sqrt{I / \epsilon_r} = \frac{120\pi}{6} = 20\pi$$

$$\mathbf{a}_k = \mathbf{a}_E \times \mathbf{a}_H \longrightarrow -\mathbf{a}_z = \mathbf{a}_y \times \mathbf{a}_H \longrightarrow \mathbf{a}_H = \mathbf{a}_x$$

$$\mathbf{H} = \frac{50}{20\pi} \sin(\omega t + \beta z) \mathbf{a}_x = \underline{\underline{795.8 \sin(10^8 t + 2z) \mathbf{a}_x}} \text{ mA/m}$$

P. E. 10.4 (a)

$$\frac{\sigma}{\omega \epsilon} = \frac{10^{-2}}{10^9 \pi \times 4 \times \frac{10^{-9}}{36\pi}} = 0.09$$

$$\alpha \cong \omega \sqrt{\frac{\mu \epsilon}{2} \left[1 + \frac{1}{2} \left(\frac{\sigma}{\omega \epsilon} \right)^2 - 1 \right]} = \frac{\omega}{2c} \sqrt{\mu_r \epsilon_r} \frac{\sigma}{\omega \epsilon} = \frac{10^9 \pi}{2 \times 3 \times 10^8} (2)(0.09) = 0.9425 \text{ Np/m}$$

$$\beta \cong \omega \sqrt{\frac{\mu \epsilon}{2} \left[1 + \frac{1}{2} \left(\frac{\sigma}{\omega \epsilon} \right)^2 + 1 \right]} = \frac{10^9 \pi}{3 \times 10^8} \sqrt{2[2 + 0.5(0.09)^2]} = 20.965 \text{ rad/m}$$

$$\mathbf{E} = 30e^{-0.9425y} \cos(10^9 \pi t - 20.96y + \pi/4) \mathbf{a}_z$$

At $t = 2\text{ns}$, $y = 1\text{m}$,

$$\mathbf{E} = 30e^{-0.9425} \cos(2\pi - 20.96 + \pi/4) \mathbf{a}_z = \underline{\underline{2.844 \mathbf{a}_z}} \text{ V/m}$$

$$(b) \quad \beta y = 10^\circ = \frac{10\pi}{180} \text{ rad}$$

or

$$y = \frac{\pi}{18} \frac{1}{\beta} = \frac{\pi}{18 \times 20.965} = \underline{\underline{8.325 \text{ mm}}}$$

$$(c) \quad 30(0.6) = 30 e^{-\alpha y}$$

$$y = \frac{1}{\alpha} \ln(1/0.6) = \frac{1}{0.9425} \ln \frac{1}{0.6} = \underline{\underline{542 \text{ mm}}}$$

(d)

$$|\eta| \cong \frac{\sqrt{\mu/\epsilon}}{[1 + \frac{1}{4}(0.09)^2]} = \frac{60\pi}{1.002} = 188.11 \Omega$$

$$2\theta_{\eta} = \tan^{-1} 0.09 \longrightarrow \theta_{\eta} = 2.571^{\circ}$$

$$\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x$$

$$\mathbf{H} = \frac{30}{188.11} e^{-0.9425y} \cos(10^9 \pi t - 20.96y + \pi/4 - 2.571^{\circ}) \mathbf{a}_x$$

At $y = 2\text{m}$, $t = 2\text{ns}$,

$$\mathbf{H} = (0.1595)(0.1518) \cos(-34.8963\text{rad}) \mathbf{a}_x = \underline{\underline{-22.83 \mathbf{a}_x \text{ mA/m}}}$$

P. E. 10.5

$$I_s = \int_0^w \int_0^{\infty} J_{xs} dy dz = J_{xs}(0) \int_0^w dy \int_0^{\infty} e^{-z(1+j)/\delta} dz = \frac{J_{xs}(0)w\delta}{1+j}$$

$$\underline{\underline{|I_s| = \frac{J_{xs}(0)w\delta}{\sqrt{2}}}}$$

P. E. 10.6 (a)

$$\frac{R_{ac}}{R_{dc}} = \frac{a}{2\delta} = \frac{a}{2} \sqrt{\pi f \mu \sigma} = \frac{1.3 \times 10^{-3}}{2} \sqrt{\pi \times 10^7 \times 4\pi \times 10^{-7} \times 3.5 \times 10^7} = \underline{\underline{24.16}}$$

(b)

$$\frac{R_{ac}}{R_{dc}} = \frac{1.3 \times 10^{-3}}{2} \sqrt{\pi \times 2 \times 10^9 \times 4\pi \times 10^{-7} \times 3.5 \times 10^7} = \underline{\underline{341.7}}$$

P. E. 10.7

$$E = \text{Re}[E_s e^{j\omega t}] = \text{Re}[E_o e^{j\omega t} e^{-j\beta z} \mathbf{a}_x + E_o e^{-j\pi/2} e^{j\omega t} e^{-j\beta z} \mathbf{a}_y]$$

$$= E_o \cos(\omega t - \beta z) \mathbf{a}_x + E_o \cos(\omega t - \beta z - \pi/2) \mathbf{a}_y$$

$$= E_o \cos(\omega t - \beta z) \mathbf{a}_x + E_o \sin(\omega t - \beta z) \mathbf{a}_y$$

At $z = 0$, $E_x = E_o \cos \omega t$, $E_y = E_o \sin \omega t$

$$\cos^2 \omega t + \sin^2 \omega t = 1 \longrightarrow \left(\frac{E_x}{E_o} \right)^2 + \left(\frac{E_y}{E_o} \right)^2 = 1$$

which describes a circle. Hence the polarization is circular.

P. E. 10.8

$$\mathcal{P}_{\text{ave}} = \frac{1}{2} \eta H_o^2 \mathbf{a}_x$$

(a) Let $f(x, z) = x + y - l = 0$

$$\mathbf{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{\mathbf{a}_x + \mathbf{a}_y}{\sqrt{2}}, \quad d\mathbf{S} = dS \mathbf{a}_n$$

$$P_t = \int \mathbf{P} \cdot d\mathbf{S} = \mathbf{P} \cdot \mathbf{S} \mathbf{a}_n = \frac{1}{2} \eta H_o^2 \mathbf{a}_x \cdot \frac{\mathbf{a}_x + \mathbf{a}_y}{\sqrt{2}}$$

$$= \frac{l}{2\sqrt{2}} (120\pi)(0.2)^2 (0.1)^2 = \underline{53.31 \text{ mW}}$$

(b) $d\mathbf{S} = dydz \mathbf{a}_x$, $P_t = \int \mathcal{P} \cdot d\mathbf{S} = \frac{1}{2} \eta H_o^2 S$

$$P_t = \frac{l}{2} (120\pi)(0.2)^2 \pi (0.05)^2 = \underline{59.22 \text{ mW}}$$

P. E. 10.9 $\eta_1 = \eta_o = 120\pi, \eta_2 = \sqrt{\frac{\mu}{\epsilon}} = \frac{\eta_o}{2}$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 2/3, \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1/3$$

$$E_{ro} = \Gamma E_{io} = -\frac{10}{3}$$

$$\underline{\underline{E_{rs} = -\frac{10}{3} e^{j\beta_1 z} \mathbf{a}_x \text{ V/m}}}$$

where $\beta_1 = \omega / c = 100\pi / 3$.

$$E_{io} = \tau E_{is} = \frac{20}{3}$$

$$\underline{\underline{E_{is} = \frac{20}{3} e^{-j\beta_2 z} \mathbf{a}_x \text{ V/m}}}$$

where $\beta_2 = \omega \sqrt{\epsilon_r} / c = 2\beta_1 = 200\pi / 3$.

P. E. 10.10

$$\alpha_1 = 0, \quad \beta_1 = \frac{\omega}{c} \sqrt{\mu_r \epsilon_r} = \frac{2\omega}{c} = 5 \longrightarrow \omega = 5c/2 = 7.5 \times 10^8$$

$$\frac{\sigma_2}{\omega \epsilon_2} = \frac{0.1}{7.5 \times 10^8 \times 4 \times \frac{10^{-9}}{36\pi}} = 1.2\pi$$

$$\alpha_2 = \frac{\omega}{c} \sqrt{\frac{4}{2} \left[\sqrt{1 + 1.44\pi^2} - 1 \right]} = 6.021$$

$$\beta_2 = \frac{\omega}{c} \sqrt{\frac{4}{2} \left[\sqrt{1 + 1.44\pi^2} + 1 \right]} = 7.826$$

$$|\eta_2| = \frac{60\pi}{\sqrt[4]{1 + 1.44\pi^2}} = 95.445, \quad \eta_1 = 120\pi \sqrt{\epsilon_{r1}} = 754$$

$$\tan 2\theta_{\eta_2} = 1.2\pi \longrightarrow \theta_{\eta_2} = 37.57^\circ$$

$$\eta_2 = 95.445 \angle 37.57^\circ$$

(a)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{95.445 \angle 37.57^\circ - 754}{95.445 \angle 37.57^\circ + 754} = \underline{\underline{0.8186 \angle 171.08^\circ}}$$

$$\tau = 1 + \Gamma = \underline{\underline{0.2295 \angle 33.56^\circ}}$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.8186}{1 - 0.8186} = \underline{\underline{10.025}}$$

$$(b) \quad \mathbf{E}_i = 50 \sin(\omega t - 5x) \mathbf{a}_y = \text{Im}(\mathbf{E}_{is} e^{j\omega t}), \text{ where } \mathbf{E}_{is} = 50 e^{-j5x} \mathbf{a}_y.$$

$$\mathbf{E}_{ro} = \Gamma \mathbf{E}_{io} = 0.8186 e^{j171.08^\circ} (50) = 40.93 e^{j171.08^\circ}$$

$$\mathbf{E}_{rs} = 40.93 e^{j5x + j171.08^\circ} \mathbf{a}_y$$

$$\mathbf{E}_r = \text{Im}(\mathbf{E}_{rs} e^{j\omega t}) = \underline{\underline{40.93 \sin(\omega t + 5x + 171.1^\circ) \mathbf{a}_y}} \text{ V/m}$$

$$\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E = -\mathbf{a}_x \times \mathbf{a}_y = -\mathbf{a}_z$$

$$\mathbf{H}_r = -\frac{40.93}{754} \sin(\omega t + 5x + 171.1^\circ) \mathbf{a}_z = \underline{\underline{-0.0543 \sin(\omega t + 5x + 171.1^\circ) \mathbf{a}_z}} \quad \text{A/m}$$

(c)

$$E_{io} = \tau E_{io} = 0.229 e^{j33.56^\circ} (50) = 11.475 e^{j33.56^\circ}$$

$$\mathbf{E}_{ts} = 11.475 e^{-j\beta_2 x + j33.56^\circ} e^{-\alpha_2 x} \mathbf{a}_y$$

$$\mathbf{E}_t = \text{Im}(\mathbf{E}_{ts} e^{j\omega t}) = \underline{\underline{11.475 e^{-6.021x} \sin(\omega t - 7.826x + 33.56^\circ) \mathbf{a}_y}} \quad \text{V/m}$$

$$\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$

$$\begin{aligned} \mathbf{H}_t &= \frac{11.495}{95.445} e^{-6.021x} \sin(\omega t - 7.826x + 33.56^\circ - 37.57^\circ) \mathbf{a}_z \\ &= \underline{\underline{0.1202 e^{-6.021x} \sin(\omega t - 7.826x - 4.01^\circ) \mathbf{a}_z}} \quad \text{A/m} \end{aligned}$$

(d)

$$\mathcal{P}_{\text{ave}} = \frac{E_{io}^2}{2\eta_1} \mathbf{a}_x + \frac{E_{ro}^2}{2\eta_1} (-\mathbf{a}_x) = \frac{1}{2(240\pi)} [50^2 \mathbf{a}_x - 40.93^2 \mathbf{a}_x] = \underline{\underline{0.5469 \mathbf{a}_x}} \quad \text{W/m}^2$$

$$\mathbf{P}_{\text{ave}} = \frac{E_{to}^2}{2|\eta_2|} e^{-2\alpha_2 x} \cos \theta_{\eta_2} \mathbf{a}_x = \frac{(11.475)^2}{2(95.445)} \cos 37.57^\circ e^{-2(6.021)x} \mathbf{a}_x = \underline{\underline{0.5469 e^{-12.04x} \mathbf{a}_x}} \quad \text{W/m}^2$$

P. E. 10.11 (a)

$$\mathbf{k} = -2\mathbf{a}_y + 4\mathbf{a}_z \longrightarrow k = \sqrt{2^2 + 4^2} = \sqrt{20}$$

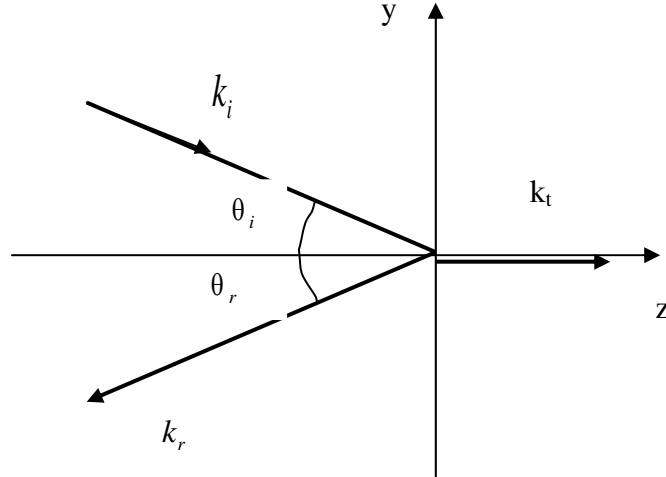
$$\omega = kc = 3 \times 10^8 \sqrt{20} = \underline{\underline{1.342 \times 10^9 \text{ rad/s}}},$$

$$\lambda = 2\pi / k = \underline{\underline{1.405 \text{ m}}}$$

$$\begin{aligned} \text{(b)} \quad \mathbf{H} &= \frac{\mathbf{a}_k \times \mathbf{E}}{\eta_o} = \frac{(-2\mathbf{a}_y + 4\mathbf{a}_z)}{\sqrt{20}(120\pi)} \times (10\mathbf{a}_y + 5\mathbf{a}_z) \cos(\omega t - \mathbf{k} \cdot \mathbf{r}) \\ &= \underline{\underline{-29.66 \cos(1.342 \times 10^9 t + 2y - 4z) \mathbf{a}_x}} \quad \text{mA/m} \end{aligned}$$

$$(c) \quad \mathcal{P}_{\text{ave}} = \frac{|E_o|^2}{2\eta_o} \mathbf{a}_k = \frac{125}{2(120\pi)} \frac{(-2\mathbf{a}_y + 4\mathbf{a}_z)}{\sqrt{20}} = \underline{\underline{-74.15\mathbf{a}_y + 148.9\mathbf{a}_z}} \quad \text{mW/m}^2$$

P. E. 10.12 (a)



$$\tan \theta_i = \frac{k_{iy}}{k_{iz}} = \frac{2}{4} \longrightarrow \underline{\underline{\theta_i = 26.56 = \theta_r}}$$

$$\sin \theta_t = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}} \sin \theta_i = \frac{1}{2} \sin 26.56^\circ \longrightarrow \underline{\underline{\theta_t = 12.92^\circ}}$$

(b) $\eta_1 = \eta_o, \eta_2 = \eta_o / 2$ $\mathbf{E}$ is parallel to the plane of incidence. Since $\mu_1 = \mu_2 = \mu_o$, we may use the result of Prob. 10.42, i.e.

$$\Gamma_{\parallel} = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} = \frac{\tan(-13.64^\circ)}{\tan(39.48^\circ)} = \underline{\underline{-0.2946}}$$

$$\tau_{\parallel} = \frac{2 \cos 26.56^\circ \sin 12.92^\circ}{\sin 39.48^\circ \cos(-13.64^\circ)} = \underline{\underline{0.6474}}$$

(c) $\mathbf{k}_r = -\beta_1 \sin \theta_r \mathbf{a}_y - \beta_1 \cos \theta_r \mathbf{a}_z$. Once $\mathbf{k}_r$ is known, $\mathbf{E}_r$ is chosen such that

$\mathbf{k}_r \cdot \mathbf{E}_r = 0$ or $\nabla \cdot \mathbf{E}_r = 0$. Let

$$\mathbf{E}_r = \pm E_{or} (-\cos \theta_r \mathbf{a}_y + \sin \theta_r \mathbf{a}_z) \cos(\omega t + \beta_1 \sin \theta_r y + \beta_1 \cos \theta_r z)$$

Only the positive sign will satisfy the boundary conditions. It is evident that

$$\mathbf{E}_i = E_{oi}(\cos \theta_i \mathbf{a}_y + \sin \theta_i \mathbf{a}_z) \cos(\omega t + 2y - 4z)$$

Since $\theta_r = \theta_i$,

$$E_{or} \cos \theta_r = \Gamma_{//} E_{oi} \cos \theta_i = 10 \Gamma_{//} = -2.946$$

$$E_{or} \sin \theta_r = \Gamma_{//} E_{oi} \sin \theta_i = 5 \Gamma_{//} = -1.473$$

$$\beta_1 \sin \theta_r = 2, \beta_1 \cos \theta_r = 4$$

i.e.

$$\mathbf{E}_r = -(2.946 \mathbf{a}_y - 1.473 \mathbf{a}_z) \cos(\omega t + 2y + 4z)$$

$$\mathbf{E}_1 = \mathbf{E}_i + \mathbf{E}_r = \underline{\underline{(10 \mathbf{a}_y + 5 \mathbf{a}_z) \cos(\omega t + 2y - 4z) + (-2.946 \mathbf{a}_y + 1.473 \mathbf{a}_z) \cos(\omega t + 2y + 4z)}}$$

V/m

(d) $\mathbf{k}_t = -\beta_2 \sin \theta_t \mathbf{a}_y + \beta_2 \cos \theta_t \mathbf{a}_z$. Since $\mathbf{k}_r \bullet \mathbf{E}_r = 0$, let

$$\mathbf{E}_t = E_{ot}(\cos \theta_t \mathbf{a}_y + \sin \theta_t \mathbf{a}_z) \cos(\omega t + \beta_2 y \sin \theta_t - \beta_2 z \cos \theta_t)$$

$$\beta_2 = \omega \sqrt{\mu_2 \epsilon_2} = \beta_1 \sqrt{\epsilon_{r2}} = 2\sqrt{20}$$

$$\sin \theta_t = \frac{1}{2} \sin \theta_i = \frac{1}{2\sqrt{5}}, \quad \cos \theta_t = \frac{\sqrt{9}}{\sqrt{20}}$$

$$\beta_2 \cos \theta_t = 2\sqrt{20} \sqrt{\frac{19}{20}} = 8.718$$

$$E_{ot} \cos \theta_t = \tau_{//} E_{oi} \cos \theta_i = 0.6474 \sqrt{125} \sqrt{\frac{19}{20}} = 7.055$$

$$E_{ot} \sin \theta_t = \tau_{//} E_{oi} \sin \theta_i = 0.6474 \sqrt{125} \sqrt{\frac{1}{20}} = 1.6185$$

Hence

$$\mathbf{E}_2 = \mathbf{E}_t = \underline{\underline{(7.055 \mathbf{a}_y + 1.6185 \mathbf{a}_z) \cos(\omega t + 2y - 8.718z)}} \quad \text{V/m}$$

$$(d) \tan \theta_{B//} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = 2 \longrightarrow \underline{\underline{\theta_{B//} = 63.43^\circ}}$$

P.E. 10.13

$$S_i = \frac{1 + 0.4}{1 - 0.4} = \frac{1.4}{0.6} = \underline{\underline{2.333}}$$

$$S_o = \frac{1 + 0.2}{1 - 0.2} = \frac{1.2}{0.8} = \underline{\underline{1.5}}$$

Prob. 10.1 (a) Wave propagates along $+\underline{\underline{a_x}}$.

(b)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi \times 10^6} = \underline{\underline{1\mu s}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6} = \underline{\underline{1.047\text{m}}}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi \times 10^6}{6} = \underline{\underline{1.047 \times 10^6 \text{ m/s}}}$$

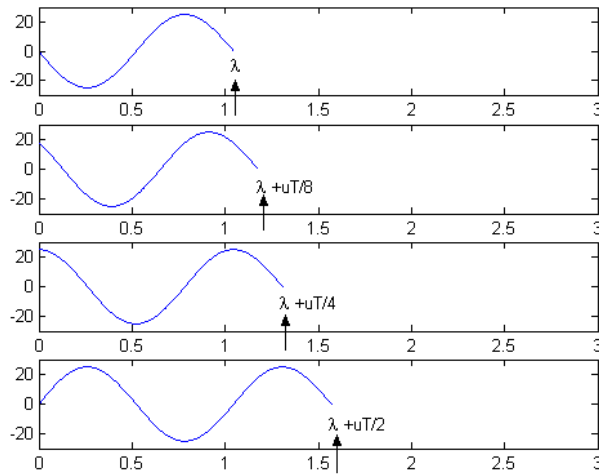
(c) At $t=0$, $E_z = 25 \sin(-6x) = -25 \sin 6x$

$$\text{At } t=T/8, E_z = 25 \sin\left(\frac{2\pi}{T} \frac{T}{8} - 6x\right) = 25 \sin\left(\frac{\pi}{4} - 6x\right)$$

$$\text{At } t=T/4, E_z = 25 \sin\left(\frac{2\pi}{T} \frac{T}{4} - 6x\right) = 25 \sin(-6x + 90^\circ) = 25 \cos 6x$$

$$\text{At } t=T/2, E_z = 25 \sin\left(\frac{2\pi}{T} \frac{T}{2} - 6x\right) = 25 \sin(-6x + \pi) = 25 \sin 6x$$

These are sketched below.

**Prob. 10.2**

$$(a) \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{60} = \underline{\underline{5 \times 10^6 \text{ m}}}$$

$$(b) \quad \lambda = \frac{3 \times 10^8}{2 \times 10^6} = \underline{\underline{150 \text{ m}}}$$

$$(c) \quad \lambda = \frac{3 \times 10^8}{120 \times 10^6} = \underline{\underline{2.5 \text{ m}}}$$

$$(d) \quad \lambda = \frac{3 \times 10^8}{2.4 \times 10^9} = \underline{\underline{0.125 \text{ m}}}$$

Prob. 10.3

$$(a) \quad \omega = \underline{\underline{10^8 \text{ rad/s}}}$$

$$(b) \quad \beta = \frac{\omega}{c} = \frac{10^8}{3 \times 10^8} = \underline{\underline{0.333 \text{ rad/m}}}$$

$$(c) \quad \lambda = \frac{2\pi}{\beta} = 6\pi = \underline{\underline{18.85 \text{ m}}}$$

(d) Along $-\mathbf{a}_y$

At $y=1, t=10\text{ms}$,

$$(e) \quad H = 0.5 \cos(10^8 t \times 10 \times 10^{-9} + \frac{1}{3} \times 3) = 0.5 \cos(1+1) \\ = \underline{\underline{-0.1665 \text{ A/m}}}$$

Prob. 10.4

(a)

$$\frac{\partial E}{\partial x} = -\sin(x + \omega t) - \sin(x - \omega t)$$

$$\frac{\partial^2 E}{\partial x^2} = -\cos(x + \omega t) - \cos(x - \omega t) = -E$$

$$\frac{\partial E}{\partial t} = -\omega \sin(x + \omega t) - \omega \sin(x - \omega t)$$

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 \cos(x + \omega t) - \omega^2 \cos(x - \omega t) = -\omega^2 E$$

$$\frac{\partial^2 E}{\partial t^2} - u^2 \frac{\partial^2 E}{\partial x^2} = -\omega^2 E + u^2 E = 0$$

if $\omega^2 = u^2$ and hence, eq. (10.1) is satisfied.

(b) $u = \omega$ **Prob. 10.5** If

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) = -\omega^2\mu\varepsilon + j\omega\mu\sigma \quad \text{and } \gamma = \alpha + j\beta, \text{ then}$$

$$|\gamma^2| = \sqrt{(\alpha^2 - \beta^2)^2 + 4\alpha^2\beta^2} = \sqrt{(\alpha^2 + \beta^2)^2} = \alpha^2 + \beta^2$$

i.e.

$$\alpha^2 + \beta^2 = \omega\mu\sqrt{(\sigma^2 + \omega^2\varepsilon^2)} \quad (1)$$

$$\text{Re}(\gamma^2) = \alpha^2 - \beta^2 = -\omega^2\mu\varepsilon$$

$$\beta^2 - \alpha^2 = \omega^2\mu\varepsilon \quad (2)$$

Subtracting and adding (1) and (2) lead respectively to

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2} \left[\sqrt{I + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - I \right]}$$

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2} \left[\sqrt{I + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + I \right]}$$

(b) From eq. (10.25), $E_s(z) = E_o e^{-\gamma z} \mathbf{a}_x$.

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}_s \longrightarrow \mathbf{H}_s = \frac{j}{\omega\mu} \nabla \times \mathbf{E}_s = \frac{j}{\omega\mu} (-\gamma E_o e^{-\gamma z} \mathbf{a}_y)$$

But $\mathbf{H}_s(z) = H_o e^{-\gamma z} \mathbf{a}_y$, hence $H_o = \frac{E_o}{\eta} = -\frac{j\gamma}{\omega\mu} E_o$

$$\eta = \frac{j\omega\mu}{\gamma}$$

(c) From (b),

$$\eta = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1 - j\frac{\sigma}{\omega\epsilon}}}$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\sqrt[4]{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}}, \tan 2\theta_\eta = \left(\frac{\omega\epsilon}{\sigma}\right)^{-1} = \frac{\sigma}{\omega\epsilon}$$

Prob. 10.6 (a)

From eq. (10.18),

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{j\omega\mu j\omega\epsilon \left(1 - \frac{j\sigma}{\omega\epsilon}\right)} = j\omega\sqrt{\mu\epsilon} \left(1 - \frac{j\sigma}{\omega\epsilon}\right)^{1/2}$$

Assuming that $\frac{\sigma}{\omega\epsilon} \ll 1$, we include up to the second power in $\frac{\sigma}{\omega\epsilon}$ and neglect higher-order terms.

$$\gamma = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} \left(1 - \frac{\sigma^2}{\omega^2\epsilon^2}\right) + j\omega\sqrt{\mu\epsilon} \left(1 + \frac{\sigma^2}{8\omega^2\epsilon^2}\right) = \alpha + j\beta$$

Thus,

$$\beta = \omega\sqrt{\mu\epsilon} \left(1 + \frac{\sigma^2}{8\omega^2\epsilon^2}\right)$$

Prob. 10.7

(a)

$$\frac{\sigma}{\omega\epsilon} = \frac{8 \times 10^{-2}}{2\pi \times 50 \times 10^6 \times 3.6 \times \frac{10^{-9}}{36\pi}} = 8$$

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2} - 1 \right]} = \frac{2\pi \times 50 \times 10^6}{3 \times 10^8} \sqrt{\frac{2.1 \times 3.6}{2} [\sqrt{65} - 1]} = 5.41$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2} + 1 \right]} = 6.129$$

$$\gamma = \alpha + j\beta = \underline{\underline{5.41 + j6.129 \text{ /m}}}$$

$$(b) \quad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6.129} = \underline{\underline{1.025 \text{ m}}}$$

$$(c) \quad u = \frac{\omega}{\beta} = \frac{2\pi \times 50 \times 10^6}{6.129} = \underline{\underline{5.125 \times 10^7 \text{ m/s}}}$$

$$(d) \quad |\eta| = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\sqrt[4]{1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2}} = \frac{120\pi \sqrt{\frac{2.1}{3.6}}}{\sqrt[4]{65}} = 101.4$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon} = 8 \longrightarrow \theta_\eta = 41.44^\circ$$

$$\eta = \underline{\underline{101.41 \angle 41.44^\circ \Omega}}$$

$$(e) \quad \mathbf{H}_s = \mathbf{a}_k \times \frac{\mathbf{E}_s}{\eta} = \mathbf{a}_x \times \frac{6}{\eta} e^{-\gamma z} \mathbf{a}_z = -\frac{6}{\eta} e^{-\gamma z} \mathbf{a}_y = \underline{\underline{-59.16 e^{-j41.44^\circ} e^{-\gamma z} \mathbf{a}_y \text{ mA/m}}}$$

Prob. 10.8

$$(a) \quad \tan \theta = \frac{\sigma}{\omega\epsilon} = \frac{10^{-2}}{2\pi \times 12 \times 10^6 \times 10 \times \frac{10^{-9}}{36\pi}} = \underline{\underline{1.5}}$$

$$(b) \quad \tan \theta = \frac{10^{-4}}{2\pi \times 12 \times 10^6 \times 4 \times \frac{10^{-9}}{36\pi}} = \underline{\underline{3.75 \times 10^{-2}}}$$

$$(c) \tan \theta = \frac{4}{2\pi \times 12 \times 10^6 \times 81 \times \frac{10^{-9}}{36\pi}} = \underline{\underline{74.07}}$$

Prob. 10.9

(a)

$$\begin{aligned} \alpha &= \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2} - 1 \right]} = \frac{2\pi \times 15 \times 10^9}{3 \times 10^8} \sqrt{\frac{1 \times 9.6}{2} \left[\sqrt{1 + 9 \times 10^{-8}} - 1 \right]} \\ &= 100\pi \sqrt{4.8 \left(\frac{1}{2} \times 9 \times 10^{-8} \right)} = 0.146 \end{aligned}$$

$$\delta = \frac{1}{\alpha} = \underline{\underline{6.85 \text{ m}}}$$

$$(b) A = \alpha \ell = 0.146 \times 5 \times 10^{-3} = \underline{\underline{0.73 \times 10^{-3} \text{ Np}}}$$

Prob. 10.10

If $\sigma = \epsilon\omega$, the loss tangent is $\frac{\sigma}{\epsilon\omega} = 1$

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2} - 1 \right]} = \omega \sqrt{\frac{\mu\epsilon}{2} [\sqrt{1+1^2} - 1]} = \omega \sqrt{\frac{\mu\epsilon}{2} [\sqrt{2} - 1]}$$

$$\text{But } \lambda_o = \frac{2\pi}{\beta} = \frac{2\pi c}{\omega} \rightarrow \omega = \frac{2\pi c}{\lambda_o}$$

$$\alpha = \frac{2\pi c}{\lambda_o} \sqrt{\frac{\mu\epsilon}{2} \sqrt{2} - 1} = \frac{2\pi \times 3 \times 10^8}{12 \times 10^{-2}} \sqrt{\frac{1}{2} \times 4\pi \times 10^{-7} \times 4 \times \frac{10^{-9}}{36\pi}} (0.6436) = \underline{\underline{47.66 \text{ Np/m}}}$$

$$\begin{aligned} \beta &= \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2} + 1 \right]} = \frac{2\pi c}{\lambda_o} \sqrt{\frac{\mu\epsilon}{2} \sqrt{2} + 1} \\ &= \frac{2\pi \times 3 \times 10^8}{12 \times 10^{-2}} \sqrt{\frac{1}{2} \times 4\pi \times 10^{-7} \times 4 \times \frac{10^{-9}}{36\pi}} (1.5538) \\ &= \underline{\underline{115.06 \text{ rad/m}}} \end{aligned}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi c}{\lambda_o \beta} = \frac{2\pi \times 3 \times 10^8}{12 \times 10^{-2} \times 115.06} = \underline{\underline{1.3652 \times 10^8 \text{ m/s}}}$$

Prob. 10.11

For silver, the loss tangent is

$$\frac{\sigma}{\omega\epsilon} = \frac{6.1 \times 10^7}{2\pi \times 10^8 \times \frac{10^{-9}}{36\pi}} = 6.1 \times 18 \times 10^8 \gg 1$$

Hence, silver is a good conductor

For rubber,

$$\frac{\sigma}{\omega\epsilon} = \frac{10^{-15}}{2\pi \times 10^8 \times 3.1 \times \frac{10^{-9}}{36\pi}} = \frac{18}{3.1} \times 10^{-14} \ll 1$$

Hence, rubber is a poor conductor or a good insulator.

Prob. 10.12

$$\frac{\sigma}{\omega\epsilon} = \frac{4}{2\pi \times 10^5 \times 80 \times 10^{-9} / 36\pi} = 9,000 \gg 1$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi \times 10^5}{2} \times 4\pi \times 10^{-7} \times 4} = 0.4\pi$$

$$(a) \quad u = \omega / \beta = \frac{2\pi \times 10^5}{0.4\pi} = \underline{\underline{5 \times 10^5}} \text{ m/s}$$

$$(b) \quad \lambda = 2\pi / \beta = \frac{2\pi}{0.4\pi} = \underline{\underline{5}} \text{ m}$$

$$(c) \quad \delta = 1 / \alpha = \frac{1}{0.4\pi} = \underline{\underline{0.796}} \text{ m}$$

$$(d) \quad \eta = |\eta| \angle \theta_\eta, \theta_\eta = 45^\circ$$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\sqrt[4]{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}} \cong \sqrt{\frac{\mu}{\epsilon} \frac{\omega\epsilon}{\sigma}} = \sqrt{\frac{4\pi \times 10^{-7} \times 2\pi \times 10^5}{4}} = 0.4443$$

$$\eta = \underline{\underline{0.4443 \angle 45^\circ}} \quad \Omega$$

Prob. 10.13 (a)

$$T = 1/f = 2\pi/\omega = \frac{2\pi}{\pi \times 10^8} = \underline{\underline{20 \text{ ns}}}$$

$$(b) \text{ Let } x = \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$$

$$\frac{\alpha}{\beta} = \left(\frac{x-1}{x+1}\right)^{1/2}$$

$$\text{But } \alpha = \frac{\omega}{c} \sqrt{\frac{\mu_r \epsilon_r}{2}} \sqrt{x-1}$$

$$\sqrt{x-1} = \frac{\alpha c}{\omega \sqrt{\frac{\mu_r \epsilon_r}{2}}} = \frac{0.1 \times 3 \times 10^8}{\pi \times 10^8 \sqrt{2}} = 0.06752 \longrightarrow x = 1.0046$$

$$\beta = \left(\frac{x+1}{x-1}\right)^{1/2} \quad \alpha = \left(\frac{2.0046}{0.0046}\right)^{1/2} \quad 0.1 = 2.088$$

$$\lambda = 2\pi/\beta = \frac{2\pi}{2.088} = \underline{\underline{3 \text{ m}}}$$

$$(c) \quad |\eta| = \frac{\sqrt{\mu/\epsilon}}{\sqrt{x}} = \frac{377}{2\sqrt{1.0046}} = 188.1$$

$$x = \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} = 1.0046$$

$$\frac{\sigma}{\omega \epsilon} = 0.096 = \tan 2\theta_\eta \longrightarrow \theta_\eta = 2.74^\circ$$

$$\eta = 188.1 \angle 2.74^\circ \quad \Omega$$

$$E_o = \eta H_o = 12 \times 188.1 = 2257.2$$

$$\mathbf{a}_E \times \mathbf{a}_H = \mathbf{a}_k \longrightarrow \mathbf{a}_E \times \mathbf{a}_x = \mathbf{a}_y \longrightarrow \mathbf{a}_E = \mathbf{a}_z$$

$$\underline{\underline{\mathbf{E} = 2.257e^{-0.1y} \sin(\pi \times 10^8 t - 2.088y + 2.74^\circ) \mathbf{a}_z \text{ kV/m}}}$$

(d) The phase difference is 2.74° .

Prob. 10.14

This is a lossy medium in which $\mu = \mu_0$.

$$\text{Let } x = \left(\frac{\sigma}{\omega \epsilon} \right)^2$$

$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j2\pi \times 10^9 \times 4\pi}{100 + j200} = 35.31 \angle 26.57^\circ$$

$$E_o = 0.05 \times 35.31 = 1.765$$

$$\mathbf{a}_E = \mathbf{a}_H \times \mathbf{a}_k = -\mathbf{a}_z$$

Thus, we obtain

$$\underline{\underline{E = -1.765 \cos(2\pi \times 10^9 t - 200x + 26.57^\circ) \mathbf{a}_z \text{ V/m}}}$$

$$\sqrt{\epsilon_r(1/3)} = \frac{\alpha c}{\omega} = \frac{100 \times 3 \times 10^8}{2\pi \times 10^9} = \frac{15}{\pi}$$

$$\epsilon_r \frac{1}{3} = 4.776 \longrightarrow \epsilon_r = 14.32$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega \epsilon} = \frac{4}{3} \longrightarrow \theta_\eta = 26.57^\circ$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\sqrt[4]{1+x}} = \frac{\frac{377}{\sqrt{14.32}}}{\sqrt[4]{5/3}} = 77.175$$

$$E_o = |\eta| H_o = 77.175 \times 50 \times 10^{-3} = 3.858$$

$$\mathbf{a}_E = -(\mathbf{a}_k \times \mathbf{a}_H) = -(\mathbf{a}_x \times \mathbf{a}_y) = -\mathbf{a}_z$$

$$\underline{\underline{E = -3.858 e^{-100x} \cos(2\pi \times 10^9 t - 200x + 26.57^\circ) \mathbf{a}_z \text{ V/m}}}$$

Prob. 10.15

$$\frac{\sigma}{\omega\epsilon} = \frac{1}{2\pi \times 10^9 \times 4 \times \frac{10^{-9}}{36\pi}} = 4.5$$

$$\begin{aligned}\alpha &= \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1 \right]} \\ &= 2\pi \times 10^9 \sqrt{\frac{4\pi}{2} \times 10^{-7} \times 4 \times 9 \times \frac{10^{-9}}{36\pi} \left[\sqrt{1 + 4.5^2} - 1 \right]} \\ &= 20\pi \sqrt{2[\sqrt{21.25} - 1]} = \underline{\underline{168.8 \text{ Np/m}}}\end{aligned}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1 \right]} = 20\pi \sqrt{2[\sqrt{21.25} + 1]} = \underline{\underline{210.5 \text{ rad/m}}}$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon} = 4.5 \quad \longrightarrow \quad \theta_\eta = 38.73^\circ$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\sqrt[4]{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}} = \frac{120\pi\sqrt{9/4}}{\sqrt[4]{1 + 4.5^2}} = 263.38$$

$$\eta = \underline{\underline{263.38 \angle 38.73^\circ \ \Omega}}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi \times 10^9}{210.5} = \underline{\underline{2.985 \times 10^7 \text{ m/s}}}$$

Prob. 10.16

$$(a) \quad \beta = 6.5 = \omega \sqrt{\mu_o \epsilon_o} = \frac{\omega}{c}$$

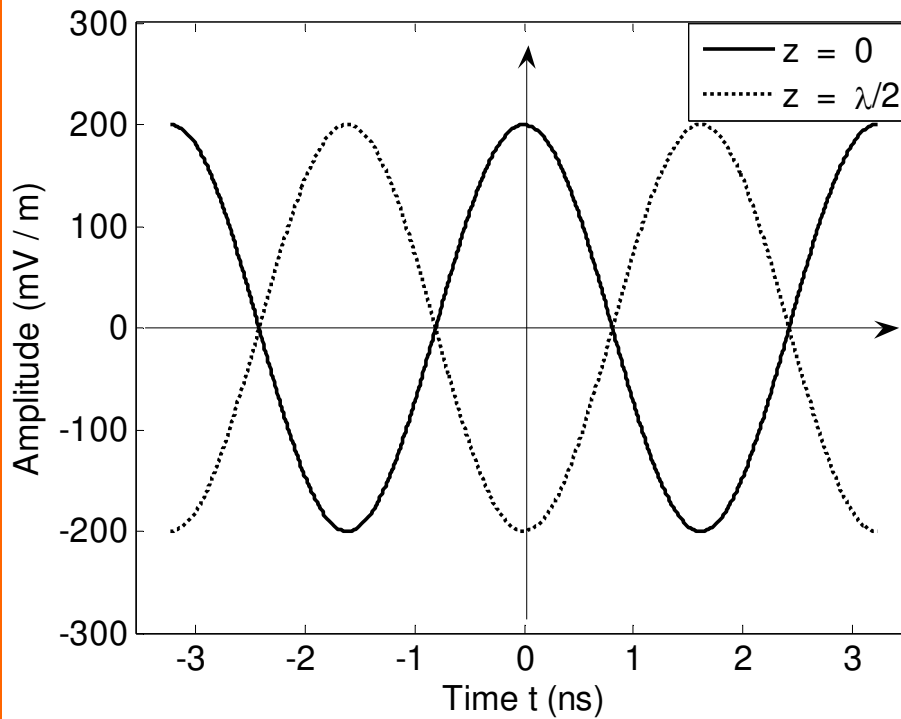
$$\omega = \beta c = 6.5 \times 3 \times 10^8 = \underline{\underline{1.95 \times 10^9 \text{ rad/s}}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6.5} = \underline{\underline{0.9666 \text{ m}}}$$

$$(b) \quad \text{For } z=0, \quad E_z = 0.2 \cos \omega t$$

$$\text{For } z=\lambda/2, \quad E_z = 0.2 \cos\left(\omega t - \frac{2\pi}{\lambda} \frac{\lambda}{2}\right) = -0.2 \cos \omega t$$

The two waves are sketched below.



(c) $\mathbf{H} = H_o \cos(\omega t - 6.5z)\mathbf{a}_H$

$$H_o = \frac{E_o}{\eta_o} = \frac{0.2}{377} = 5.305 \times 10^{-4}$$

$$\mathbf{a}_E \times \mathbf{a}_H = \mathbf{a}_k \longrightarrow \mathbf{a}_x \times \mathbf{a}_H = \mathbf{a}_z \longrightarrow \mathbf{a}_H = \mathbf{a}_y$$

$$\mathbf{H} = 0.5305 \cos(\omega t - 6.5z)\mathbf{a}_y \text{ mA/m}$$

Prob. 10.17

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\epsilon}}$$

$$\frac{\lambda_o}{\lambda} = \frac{\sqrt{\mu_o\epsilon_o\epsilon_r}}{\sqrt{\mu_o\epsilon_o}} = \sqrt{\epsilon_r}$$

$$\sqrt{\epsilon_r} = \frac{\lambda_o}{\lambda} = \frac{6.4}{2.8} = 2.286 \longrightarrow \underline{\underline{\epsilon_r = 5.224}}$$

Prob. 10.18 (a) Along -x direction.

(b) $\beta = 6, \quad \omega = 2 \times 10^8,$

$$\beta = \omega \sqrt{\mu \epsilon} = \frac{\omega}{c} \sqrt{\mu_r \epsilon_r}$$

$$\sqrt{\epsilon_r} = \beta c / \omega = \frac{6 \times 3 \times 10^8}{2 \times 10^8} = 9 \quad \longrightarrow \quad \epsilon_r = 81$$

$$\epsilon = \epsilon_o \epsilon_r = \frac{10^{-9}}{36\pi} \times 81 = \underline{\underline{7.162 \times 10^{-10} \text{ F/m}}}$$

(c) $\eta = \sqrt{\mu / \epsilon} = \sqrt{\mu_o / \epsilon_o} \sqrt{\mu_r / \epsilon_r} = \frac{120\pi}{9} = 41.89 \Omega$

$$E_o = H_o \eta = 25 \times 10^{-3} \times 41.88 = 1.047$$

$$\mathbf{a}_E \times \mathbf{a}_H = \mathbf{a}_k \longrightarrow \mathbf{a}_E \times \mathbf{a}_y = -\mathbf{a}_x \longrightarrow \mathbf{a}_E = \mathbf{a}_z$$

$$\underline{\underline{E = 1.047 \sin(2 \times 10^8 t + 6x) \mathbf{a}_z \text{ V/m}}}$$

Prob. 10.19 (a) $\frac{\sigma}{\omega \epsilon} = \frac{10^{-6}}{2\pi \times 10^7 \times 5 \times \frac{10^{-9}}{36\pi}} = 3.6 \times 10^{-4} \ll 1$

Thus, the material is lossless at this frequency.

(b) $\beta = \omega \sqrt{\mu \epsilon} = \frac{2\pi \times 10^7}{3 \times 10^8} \sqrt{5 \times 750} = \underline{\underline{12.83 \text{ rad/m}}}$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{12.83} = \underline{\underline{0.49 \text{ m}}}$$

(c) Phase difference = $\beta l = \underline{\underline{25.66 \text{ rad}}}$

(d) $\eta = \sqrt{\mu / \epsilon} = 120\pi \sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi \sqrt{\frac{750}{5}} = \underline{\underline{4.62 \text{ k}\Omega}}$

Prob. 10.20

(a)

$$\beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c} = \frac{10^8\pi}{3 \times 10^8} = \frac{\pi}{3} = \underline{\underline{1.0472 \text{ rad/m}}}$$

(b)

$$E = 0 \longrightarrow \sin(10^8\pi t_o - \beta x_o) = 0 = \sin(n\pi), n = 1, 2, 3, \dots$$

$$10^8\pi t_o - \beta x_o = \pi$$

$$10^8\pi \times 5 \times 10^{-3} - \frac{\pi}{3} x_o = \pi \longrightarrow x_o = \underline{\underline{5 \times 10^5 \text{ m}}}$$

(c)

$$\mathbf{H} = H_o \sin(10^8\pi t - \beta x) \mathbf{a}_H$$

$$H_o = \frac{E_o}{\eta} = \frac{50 \times 10^{-3}}{120\pi} = 132.63 \text{ } \mu\text{A/m}$$

$$\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_x \times \mathbf{a}_z = -\mathbf{a}_y$$

$$\mathbf{H} = \underline{\underline{-132.63 \sin(10^8\pi t - 1.0472x) \mathbf{a}_y \text{ } \mu\text{A/m}}}$$

Prob. 10.21

This is a lossless material.

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = 377 \sqrt{\frac{\mu_r}{\epsilon_r}} = 105 \quad (1)$$

$$u = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}} = 7.6 \times 10^7 \quad (2)$$

From (1),

$$\sqrt{\frac{\mu_r}{\epsilon_r}} = \frac{105}{377} = 0.2785 \quad (1)a$$

From (2),

$$\frac{1}{\sqrt{\mu_r\epsilon_r}} = \frac{7.6 \times 10^7}{3 \times 10^8} = 0.2533 \quad (2)a$$

Multiplying (1)a by (2)a,

$$\frac{1}{\epsilon_r} = 0.2785 \times 0.2533 = 0.07054 \longrightarrow \epsilon_r = \underline{\underline{14.175}}$$

Dividing (1)a by (2)a,

$$\mu_r = \frac{0.2785}{0.2533} = \underline{\underline{1.0995}}$$

Prob. 10.22

$$\begin{aligned}
 \text{(a)} \quad \nabla \times \mathbf{E} &= \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(z,t) & E_y(z,t) & 0 \end{vmatrix} = -\frac{\partial E_y}{\partial z} \mathbf{a}_x + \frac{\partial E_x}{\partial z} \mathbf{a}_y \\
 &= -6\beta \cos(\omega t - \beta z) \mathbf{a}_x + 8\beta \sin(\omega t - \beta z) \mathbf{a}_y
 \end{aligned}$$

$$\text{But } \nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \longrightarrow \mathbf{H} = -\frac{1}{\mu} \int \nabla \times \mathbf{E} dt$$

$$\mathbf{H} = \underline{\underline{\frac{6\beta}{\mu\omega} \sin(\omega t - \beta z) \mathbf{a}_x + \frac{8\beta}{\mu\omega} \cos(\omega t - \beta z) \mathbf{a}_y}}}$$

$$\text{(b)} \quad \beta = \omega \sqrt{\mu\epsilon} = \frac{2\pi f}{c} \sqrt{4.5} = \frac{2\pi \times 40 \times 10^6}{3 \times 10^8} \sqrt{4.5} = \underline{\underline{1.777 \text{ rad/m}}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{1.777} = \underline{\underline{3.536 \text{ m}}}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{4.5}} = \underline{\underline{177.72 \Omega}}$$

$$u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{4.5}} = \frac{3 \times 10^8}{\sqrt{4.5}} = \underline{\underline{1.4142 \times 10^8 \text{ m/s}}}$$

Prob. 10.23

$$\text{(a)} \quad \mathbf{E} = \text{Re}[\mathbf{E}_s e^{j\omega t}] = (5\mathbf{a}_x + 12\mathbf{a}_y) e^{-0.2z} \cos(\omega t - 3.4z)$$

$$\text{At } z = 4\text{m}, \quad t = T/8, \quad \omega t = \frac{2\pi}{T} \frac{T}{8} = \frac{\pi}{4}$$

$$\mathbf{E} = (5\mathbf{a}_x + 12\mathbf{a}_y) e^{-0.8} \cos(\pi/4 - 13.6)$$

$$|\mathbf{E}| = 13e^{-0.8} |\cos(\pi/4 - 13.6)| = \underline{\underline{5.662 \text{ V/m}}}$$

$$\text{(b)} \quad \text{loss} = \alpha \Delta z = 0.2(3) = 0.6 \text{ Np. Since } 1 \text{ Np} = 8.686 \text{ dB,}$$

$$\text{loss} = 0.6 \times 8.686 = \underline{\underline{5.212 \text{ dB}}}$$

$$(c) \text{ Let } x = \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$$

$$\frac{\alpha}{\beta} = \left(\frac{x-1}{x+1}\right)^{1/2} = 0.2/3.4 = \frac{1}{17}$$

$$\frac{x-1}{x+1} = 1/289 \quad \longrightarrow \quad x = 1.00694$$

$$\alpha = \omega \sqrt{\mu \epsilon / 2} \sqrt{x-1} = \frac{\omega}{c} \sqrt{\epsilon_r / 2} \sqrt{x-1}$$

$$\sqrt{\frac{\epsilon_r}{2}} = \frac{\alpha c}{\omega \sqrt{x-1}} = \frac{0.2 \times 3 \times 10^8}{10^8 \sqrt{0.00694}} = 7.2 \quad \longrightarrow \quad \epsilon_r = 103.68$$

$$|\eta| = \frac{\sqrt{\frac{\mu_o}{\epsilon_o}} \cdot \frac{1}{\sqrt{\epsilon_r}}}{\sqrt{x}} = \frac{120\pi}{\sqrt{103.68 \times 1.00694}} = 36.896$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega \epsilon} = \sqrt{x^2 - 1} = 0.118 \quad \longrightarrow \quad \theta_\eta = 3.365^\circ$$

$$\eta = \underline{\underline{36.896 \angle 3.365^\circ \, \Omega}}$$

Prob. 10.24

$$\alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}} \quad \longrightarrow \quad \sigma = \frac{2\alpha^2}{\omega \mu} = \frac{2 \times 12^2}{2\pi \times 10^6 \times 4\pi \times 10^{-7}} = \underline{\underline{36.48}}$$

$$\eta = |\eta| \angle \theta_\eta = \sqrt{\frac{\omega \mu}{\sigma}} \angle 45^\circ$$

$$|\eta| = \sqrt{\frac{\omega \mu}{\sigma}} = \sqrt{\frac{2\pi \times 10^6 \times 4\pi \times 10^{-7}}{36.48}} = 0.4652$$

$$E_o = |\eta| H_o = 0.4652 \times 20 \times 10^{-3} = 9.305 \times 10^{-3}$$

$$\mathbf{a}_E = \mathbf{a}_H \times \mathbf{a}_k = \mathbf{a}_y \times (-\mathbf{a}_z) = -\mathbf{a}_x$$

$$\mathbf{E} = E_o e^{-\alpha z} \sin(\omega t + \beta z) \mathbf{a}_E$$

$$= \underline{\underline{-9.305 e^{-12z} \sin(2\pi \times 10^6 t + 12z + 45^\circ) \mathbf{a}_x \text{ mV/m}}}$$

Prob. 10.25 For a good conductor, $\frac{\sigma}{\omega\epsilon} \gg 1$, say $\frac{\sigma}{\omega\epsilon} > 100$

$$(a) \quad \frac{\sigma}{\omega\epsilon} = \frac{10^{-2}}{2\pi \times 8 \times 10^6 \times 15 \times \frac{10^{-9}}{36\pi}} = 1.5 \longrightarrow \text{lossy}$$

No, not conducting.

$$(b) \quad \frac{\sigma}{\omega\epsilon} = \frac{0.025}{2\pi \times 8 \times 10^6 \times 16 \times \frac{10^{-9}}{36\pi}} = 3.515 \longrightarrow \text{lossy}$$

No, not conducting.

$$(c) \quad \frac{\sigma}{\omega\epsilon} = \frac{25}{2\pi \times 8 \times 10^6 \times 81 \times \frac{10^{-9}}{36\pi}} = 694.4 \longrightarrow \text{conducting}$$

Yes, conducting.

Prob. 10.26

$$\alpha = \beta = \frac{1}{\delta}$$

$$\text{But } u = \frac{\omega}{\beta} = \delta\omega = 0.02 \times 10^{-3} \times 2\pi \times 100 \times 10^6 = 4\pi \times 10^3 = \underline{\underline{1.256 \times 10^4 \text{ m/s}}}$$

Prob. 10.27 (a)

$$R_{dc} = \frac{l}{\sigma S} = \frac{l}{\sigma \pi a^2} = \frac{600}{5.8 \times 10^7 \times \pi \times (1.2)^2 \times 10^{-6}} = \underline{\underline{2.287 \Omega}}$$

$$(b) \quad R_{ac} = \frac{l}{\sigma 2\pi a \delta}. \quad \text{At 100 MHz, } \delta = 6.6 \times 10^{-3} \text{ mm} = 6.6 \times 10^{-6} \text{ m mm for copper}$$

(see Table 10.2).

$$R_{ac} = \frac{600}{5.8 \times 10^7 \times 2\pi \times (1.2 \times 10^{-3}) \times 6.6 \times 10^{-6}} = \underline{\underline{207.61 \Omega}}$$

$$(c) \quad \frac{R_{ac}}{R_{dc}} = \frac{a}{2\delta} = 1 \longrightarrow \delta = a/2 = \frac{66.1 \times 10^{-3}}{\sqrt{f}}$$

$$\sqrt{f} = \frac{66.1 \times 2 \times 10^{-3}}{a} = \frac{66.1 \times 2}{1.2} \longrightarrow f = \underline{\underline{12.137 \text{ kHz}}}$$

Prob. 10.28

(a) Copper is a good conductor.

$$\alpha = \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 10^{10} \times 4\pi \times 10^{-7} \times 5.8 \times 10^7} = 2\pi \times 10^5 \sqrt{5.8}$$

$$\underline{\underline{\alpha = 1.513 \times 10^6 \text{ Np/m}}}$$

$$(b) \quad \delta = \frac{1}{\alpha} = \frac{1}{1.513 \times 10^6} = \underline{\underline{6.609 \times 10^{-7} \text{ m}}}$$

$$(c) \quad \eta = \frac{1+j}{\sigma \delta} = \frac{1+j}{5.8 \times 10^7 \times 6.609 \times 10^{-7}} = \underline{\underline{26.09(1+j) \times 10^{-3} \Omega}}$$

Prob. 10.29

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \longrightarrow f = \frac{1}{\delta^2 \pi \mu \sigma}$$

$$f = \frac{1}{4 \times 10^{-6} \times \pi \times 4\pi \times 10^{-7} \times 6.1 \times 10^7} = \underline{\underline{1.038 \text{ kHz}}}$$

Prob. 10.30

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \times 5.8 \times 10^7 (60) 4\pi \times 10^{-7}}} = \frac{1}{2\pi \sqrt{5.8(60)}} = \underline{\underline{8.531 \text{ mm}}}$$

Prob. 10.31

This is a good conductor.

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

$$24.6 = \sqrt{\frac{2\pi \times 12 \times 10^6 \times 4\pi \times 10^{-7}}{\sigma}} = \sqrt{\frac{16\pi^2 (0.6)}{\sigma}}$$

$$\sigma = \frac{16\pi^2 (0.6)}{24.6^2} = \underline{\underline{0.1566 \text{ S/m}}}$$

$$\beta = \alpha = \frac{1}{\delta} = \frac{1}{0.12} = \underline{\underline{8.333 \text{ rad/m}}}$$

$$\lambda = \frac{2\pi}{\beta} = 2\pi(0.12) = \underline{\underline{0.754 \text{ m}}}$$

$$u = \frac{\omega}{\beta} = \omega\delta = 2\pi \times 12 \times 10^6 \times 0.12 = \underline{\underline{9.05 \times 10^6 \text{ m/s}}}$$

Prob. 10.32

$$\frac{\sigma}{\omega\epsilon} = \frac{0.12}{2\pi \times 2.42 \times 10^9 \times 5.5 \times \frac{10^{-9}}{36\pi}} = 0.1623$$

This is a lossy material.

$$\begin{aligned} \alpha &= \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2} - 1 \right]} \\ &= 2\pi \times 2.42 \times 10^9 \sqrt{\frac{4\pi}{2} \times 10^{-7} \times 5.5 \times \frac{10^{-9}}{36\pi} \left[\sqrt{1 + 0.1623^2} - 1 \right]} \\ &= 15.21(10^9)(10^{-8})\sqrt{0.01308} \\ &= 17.39 \end{aligned}$$

$$\delta = \frac{1}{\alpha} = \underline{\underline{57.5 \text{ mm}}}$$

Prob. 10.33

$$t = 5\delta = \frac{5}{\sqrt{\pi f \mu \sigma}} = \frac{5}{\sqrt{\pi \times 12 \times 10^9 \times 4\pi \times 10^{-7} \times 6.1 \times 10^7}} = \underline{\underline{2.94 \times 10^{-6} \text{ m}}}$$

Prob. 10.34

$$\frac{\sigma}{\omega\epsilon} = \frac{4}{2\pi \times 2 \times 10^9 \times 24 \times \frac{10^{-9}}{36\pi}} = 1.5$$

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2} - 1 \right]} = 2\pi \times 2 \times 10^9 \sqrt{\frac{4\pi \times 10^{-7} \times 24 \times \frac{10^{-9}}{36\pi}}{2} \left[\sqrt{1 + (1.5)^2} - 1 \right]}$$

$$= 130.01 \text{ Np/m}$$

$$10^{-5} E_o = E_o e^{-\alpha d}$$

Taking the log of both sides gives

$$-5 \ln 10 = -\alpha d \quad \longrightarrow \quad d = \frac{5 \ln 10}{\alpha} = \frac{5 \ln 10}{130.01} = \underline{\underline{0.0886 \text{ m}}}$$

Prob. 10.35

(a) Linearly polarized along $\mathbf{a}_z$

$$(b) \quad \omega = 2\pi f = 2\pi \times 10^7 \quad \longrightarrow \quad f = 10^7 = \underline{\underline{10 \text{ MHz}}}$$

$$\beta = \omega \sqrt{\mu\epsilon} = \omega \sqrt{\mu_o \epsilon_o} \sqrt{\epsilon_r} = \frac{\omega}{c} \sqrt{\epsilon_r}$$

$$(c) \quad \sqrt{\epsilon_r} = \frac{\beta c}{\omega} = \frac{3 \times 3 \times 10^8}{2\pi \times 10^7} = 14.32 \quad \longrightarrow \quad \epsilon_r = \underline{\underline{205.18}}$$

$$\text{Let } \mathbf{H} = H_o \sin(\omega t - 3y) \mathbf{a}_H$$

$$H_o = \frac{E_o}{\eta}, \quad \eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{\eta_o}{\sqrt{\epsilon_r}} = \frac{120\pi}{14.32} = 26.33$$

$$(d) \quad H_o = \frac{12}{26.33} = 0.456$$

$$\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x$$

$$\underline{\underline{\mathbf{H} = 0.456 \sin(2\pi \times 10^7 t - 3y) \mathbf{a}_x \text{ A/m}}}$$

Prob. 10.36

$$\mathbf{E} = (2\mathbf{a}_y - 5\mathbf{a}_z) \sin(\omega t - \beta x)$$

The ratio E_y / E_z remains the same as t changes. Hence the wave is linearly polarized

Prob. 10.37

(a)

$$E_x = E_o \cos(\omega t + \beta y), \quad E_y = E_o \sin(\omega t + \beta y)$$

$$E_x(0, t) = E_o \cos \omega t \quad \longrightarrow \quad \cos \omega t = \frac{E_x(0, t)}{E_o}$$

$$E_y(0, t) = E_o \sin \omega t \quad \longrightarrow \quad \sin \omega t = \frac{E_y(0, t)}{E_o}$$

$$\cos^2 \omega t + \sin^2 \omega t = 1 \quad \longrightarrow \quad \left(\frac{E_x}{E_o} \right)^2 + \left(\frac{E_y}{E_o} \right)^2 = 1$$

Hence, we have circular polarization.

(b)

$$E_x = E_o \cos(\omega t - \beta y), \quad E_y = -3E_o \sin(\omega t - \beta y)$$

In the $y=0$ plane,

$$E_x(0, t) = E_o \cos \omega t \quad \longrightarrow \quad \cos \omega t = \frac{E_x(0, t)}{E_o}$$

$$E_y(0, t) = E_o \sin \omega t \quad \longrightarrow \quad \sin \omega t = \frac{-E_y(0, t)}{3E_o}$$

$$\cos^2 \omega t + \sin^2 \omega t = 1 \quad \longrightarrow \quad \left(\frac{E_x}{E_o} \right)^2 + \frac{1}{9} \left(\frac{E_y}{E_o} \right)^2 = 1$$

Hence, we have elliptical polarization.

Prob. 10.38

(a) We can write

$$\mathbf{E} = \text{Re}(\mathbf{E}_s e^{j\omega t}) = (40\mathbf{a}_x + 60\mathbf{a}_y) \cos(\omega t - 10z)$$

Since E_x / E_y does not change with time, the wave is linearly polarized.

(b) This is elliptically polarized.

Prob. 10.39(a) The wave is elliptically polarized.

(b)

$$\text{Let } \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2,$$

$$\text{where } \mathbf{E}_1 = 40 \cos(\omega t - \beta z) \mathbf{a}_x, \quad \mathbf{E}_2 = 60 \sin(\omega t - \beta z) \mathbf{a}_y$$

$$\mathbf{H}_1 = H_{o1} \cos(\omega t - \beta z) \mathbf{a}_{H1}$$

$$H_{o1} = \frac{40}{\eta_o} = \frac{40}{120\pi} = 0.106$$

$$\mathbf{a}_{H1} = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y$$

$$\mathbf{H}_1 = 0.106 \cos(\omega t - \beta z) \mathbf{a}_y$$

$$\mathbf{H}_2 = H_{o2} \sin(\omega t - \beta z) \mathbf{a}_{H2}$$

$$H_{o2} = \frac{60}{\eta_o} = \frac{60}{120\pi} = 0.1592$$

$$\mathbf{a}_{H2} = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_z \times \mathbf{a}_y = -\mathbf{a}_x$$

$$\mathbf{H}_2 = -0.1592 \sin(\omega t - \beta z) \mathbf{a}_x$$

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 = \underline{\underline{-159.2 \sin(\omega t - \beta z) \mathbf{a}_x + 106 \cos(\omega t - \beta z) \mathbf{a}_y \text{ mA/m}}}$$

Prob. 10.40We can write $\mathbf{E}_s$ as

$$\mathbf{E}_s = \mathbf{E}_1(z) + \mathbf{E}_2(z)$$

where

$$\mathbf{E}_1(z) = \frac{1}{2} E_o (\mathbf{a}_x - j \mathbf{a}_y) e^{-j\beta z}$$

$$\mathbf{E}_2(z) = \frac{1}{2} E_o (\mathbf{a}_x + j \mathbf{a}_y) e^{-j\beta z}$$

We recognize that $\mathbf{E}_1$ and $\mathbf{E}_2$ are circularly polarized waves. The problem is therefore proved.**Prob. 10.41**

(a)

When $\phi = 0$,

$$\mathbf{E}(y, t) = (E_{o1} \mathbf{a}_x + E_{o2} \mathbf{a}_z) \cos(\omega t - \beta y)$$

The two components are in phase and the wave is linearly polarized.

(b)

When $\phi = \pi/2$,

$$\mathbf{E}_z = E_{o2} \cos(\omega t - \beta y + \pi/2) = -E_{o2} \sin(\omega t - \beta y)$$

We can combine $\mathbf{E}_x$ and $\mathbf{E}_z$ to show that the wave is elliptically polarized.

(c)

When $\phi = \pi$,

$$\begin{aligned} \mathbf{E}(y, t) &= E_{o1} \cos(\omega t - \beta y) \mathbf{a}_x + E_{o2} \cos(\omega t - \beta y + \pi) \mathbf{a}_z \\ &= (E_{o1} \mathbf{a}_x - E_{o2} \mathbf{a}_z) \cos(\omega t - \beta y) \end{aligned}$$

Thus, the wave is linearly polarized.**Prob. 10.42**

$$\text{Let } \mathbf{E}_s = \mathbf{E}_r + j\mathbf{E}_i \quad \text{and} \quad \mathbf{H}_s = \mathbf{H}_r + j\mathbf{H}_i$$

$$\mathbf{E} = \text{Re}(\mathbf{E}_s e^{j\omega t}) = \mathbf{E}_r \cos \omega t - \mathbf{E}_i \sin \omega t$$

Similarly,

$$\mathbf{H} = \mathbf{H}_r \cos \omega t - \mathbf{H}_i \sin \omega t$$

$$\mathcal{P} = \mathbf{E} \times \mathbf{H} = \mathbf{E}_r \times \mathbf{H}_r \cos^2 \omega t + \mathbf{E}_i \times \mathbf{H}_i \sin^2 \omega t - \frac{1}{2}(\mathbf{E}_r \times \mathbf{H}_i + \mathbf{E}_i \times \mathbf{H}_r) \sin 2\omega t$$

$$\mathcal{P}_{\text{ave}} = \frac{1}{T} \int_0^T \mathcal{P} dt = \frac{1}{T} \int_0^T \cos^2 \omega t dt (\mathbf{E}_r \times \mathbf{H}_r) + \frac{1}{T} \int_0^T \sin^2 \omega t dt (\mathbf{E}_i \times \mathbf{H}_i) - \frac{1}{2T} \int_0^T \sin 2\omega t dt (\mathbf{E}_i \times \mathbf{H}_i + \mathbf{E}_i \times \mathbf{H}_r)$$

$$= \frac{1}{2}(\mathbf{E}_r \times \mathbf{H}_r + \mathbf{E}_i \times \mathbf{H}_i) = \frac{1}{2} \text{Re}[(\mathbf{E}_r + j\mathbf{E}_i) \times (\mathbf{H}_r - j\mathbf{H}_i)]$$

$$\mathcal{P}_{\text{ave}} = \frac{1}{2} \text{Re}(\mathbf{E}_s \times \mathbf{H}_s^*)$$

as required.

Prob. 10.43

(a)

$$\beta = \omega \sqrt{\mu \epsilon} = \omega \sqrt{\mu_o \epsilon_o} \sqrt{\epsilon_r} = \frac{\omega}{c} \sqrt{\epsilon_r}$$

$$\sqrt{\epsilon_r} = \frac{\beta c}{\omega} = \frac{8 \times 3 \times 10^8}{10^9} = 2.4$$

$$\epsilon_r = \underline{\underline{5.76}}$$

$$(b) \quad \eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_o}{\epsilon_o}} \frac{1}{\sqrt{\epsilon_r}} = \frac{377}{2.4} = \underline{\underline{157.1 \Omega}}$$

$$(c) \quad u = \frac{\omega}{\beta} = \frac{10^9}{8} = \underline{\underline{1.25 \times 10^8 \text{ m/s}}}$$

(d)

Let $\mathbf{H} = H_o \cos(10^9 t + 8x) \mathbf{a}_H$

$$H_o = \frac{E_o}{\eta} = \frac{150}{157.1} = 0.955$$

$$\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E = -\mathbf{a}_x \times \mathbf{a}_z = \mathbf{a}_y$$

$$\mathbf{H} = \underline{\underline{0.955 \cos(10^9 t + 8x) \mathbf{a}_y \text{ A/m}}}$$

(e)

$$\begin{aligned} \mathcal{P} &= \mathbf{E} \times \mathbf{H} = -150(0.955) \cos^2(10^9 t + 8x) \mathbf{a}_x \\ &= \underline{\underline{-143.25 \cos^2(10^9 t + 8x) \mathbf{a}_x \text{ W/m}^2}} \end{aligned}$$

Prob. 10.44

$$\mathcal{P} = \mathbf{E} \times \mathbf{H} = \frac{E_o^2}{\eta} \cos^2(\omega t - 10z) \mathbf{a}_z$$

$$\mathcal{P}_{ave} = \frac{1}{T} \int_0^T \mathcal{P} dt = \frac{E_o^2}{2\eta} \mathbf{a}_z$$

$$P = \int_S \mathcal{P}_{ave} \cdot d\mathbf{S} = \frac{E_o^2 S}{2\eta} = \frac{(40)^2 \times \pi (1.5)^2}{2 \times 120\pi} = \frac{(60)^2}{240} = \underline{\underline{15 \text{ W}}}$$

Prob. 10.45

(a)

Let $\mathbf{H}_s = \frac{H_o}{r} \sin \theta e^{-j3r} \mathbf{a}_H$

$$H_o = \frac{E_o}{\eta_o} = \frac{10}{120\pi} = \frac{1}{12\pi}$$

$$\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_r \times \mathbf{a}_\theta = \mathbf{a}_\phi$$

$$\mathbf{H}_s = \underline{\underline{\frac{1}{12\pi r} \sin \theta e^{-j3r} \mathbf{a}_\phi \text{ A/m}}}$$

(b)

$$\mathcal{P}_{ave} = \frac{1}{2} \text{Re}(\mathbf{E}_s \times \mathbf{H}_s) = \frac{10}{2 \times 12\pi r^2} \sin^2 \theta \mathbf{a}_r$$

$$P_{ave} = \int_S \mathcal{P}_{ave} \cdot d\mathbf{S}, \quad d\mathbf{S} = r^2 \sin \theta d\theta d\phi \mathbf{a}_r$$

$$\begin{aligned} P_{ave} &= \frac{10}{24\pi} \int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi/6} r^2 \sin^3 \theta d\theta \bigg|_{r=2} = \frac{5}{8} - \frac{5\sqrt{3}}{32} = 0.007145 \\ &= \underline{\underline{7.145 \text{ mW}}} \end{aligned}$$

Prob. 10.46

$$(a) \quad P_{ave} = \frac{1}{2} \operatorname{Re}(E_s H_s^*) = \frac{1}{2} \operatorname{Re}\left(\frac{|E_s|}{|\eta|}\right) = \frac{8^2}{2|\eta|} e^{-0.2z}$$

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1 \right]}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1 \right]}$$

$$\text{Let } x = \sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}$$

$$\frac{\alpha}{\beta} = \frac{\sqrt{x-1}}{\sqrt{x+1}} = 0.1/0.3 = 1/3$$

$$\frac{x-1}{x+1} = \frac{1}{9} \quad \longrightarrow \quad x = 5/4$$

$$\frac{5}{4} = \sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} \quad \longrightarrow \quad \frac{\sigma}{\omega\epsilon} = 3/4$$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\sqrt[4]{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}} = \frac{120\pi / \sqrt{81}}{\sqrt[4]{\frac{5}{4}}} = 37.4657$$

$$P_{ave} = \frac{64}{2(37.4657)} e^{-0.2z} = \underline{\underline{0.8541 e^{-0.2z} \text{ W/m}^2}}$$

$$(b) \quad 20\text{dB} = 10 \log \frac{P_1}{P_2} \quad \longrightarrow \quad \frac{P_1}{P_2} = 100$$

$$\frac{P_2}{P_1} = e^{-0.2z} = \frac{1}{100} \quad \longrightarrow \quad e^{0.2z} = 100$$

$$z = 5 \log 100 = \underline{\underline{23 \text{ m}}}$$

Prob. 10.47

$$(a) \quad u = \omega / \beta \quad \longrightarrow \quad \omega = u\beta = \frac{\beta c}{\sqrt{4.5}} = \frac{2 \times 3 \times 10^8}{\sqrt{4.5}} = \underline{\underline{2.828 \times 10^8}} \text{ rad/s}$$

$$\eta = \frac{120\pi}{\sqrt{4.5}} = 177.7 \Omega$$

$$\mathbf{H} = \mathbf{a}_k \times \frac{\mathbf{E}}{\eta} = \frac{\mathbf{a}_z}{\eta} \times \frac{40}{\rho} \sin(\omega t - 2z) \mathbf{a}_\rho = \frac{0.225}{\rho} \sin(\omega t - 2z) \mathbf{a}_\phi \text{ A/m}$$

$$(b) \quad \mathbf{P} = \mathbf{E} \times \mathbf{H} = \frac{9}{\rho^2} \sin^2(\omega t - 2z) \mathbf{a}_z \text{ W/m}^2$$

$$(c) \quad \mathcal{P}_{\text{ave}} = \frac{4.5}{\rho^2} \mathbf{a}_z, \quad d\mathbf{S} = \rho d\phi d\rho \mathbf{a}_z$$

$$\mathbf{P}_{\text{ave}} = \int \mathbf{P}_{\text{ave}} \cdot d\mathbf{S} = 4.5 \int_{2\text{mm}}^{3\text{mm}} \frac{d\rho}{\rho} \int_0^{2\pi} d\phi = 4.5 \ln(3/2) (2\pi) = \underline{\underline{11.46 \text{ W}}}$$

Prob. 10.48

$$\mathcal{P} = \mathbf{E} \times \mathbf{H} = \frac{E_o^2}{\eta r^2} \sin^2 \theta \sin^2 \omega(t - r/c) \mathbf{a}_r$$

$$\mathbf{P}_{\text{ave}} = \frac{1}{T} \int_0^T \mathcal{P} dt = \frac{E_o^2}{2\eta r^2} \sin^2 \theta \mathbf{a}_r$$

Prob. 10.49

$$\beta = \frac{\omega}{c} \quad \longrightarrow \quad \omega = \beta c = 40(3 \times 10^8) = \underline{\underline{12 \times 10^9}} \text{ rad/s}$$

$$\mathbf{E} = \frac{1}{\epsilon} \int \nabla \times \mathbf{H} dt$$

$$\begin{aligned} \nabla \times \mathbf{H} &= \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 10 \sin(\omega t - 40x) & -20 \sin(\omega t - 40x) \end{vmatrix} \\ &= -800 \cos(\omega t - 40x) \mathbf{a}_y - 400 \cos(\omega t - 40x) \mathbf{a}_z \end{aligned}$$

$$\begin{aligned}
 \mathbf{E} &= \frac{1}{\epsilon} \int \nabla \times \mathbf{H} dt = -\frac{800}{\omega \epsilon} \sin(\omega t - 40x) \mathbf{a}_y - \frac{400}{\omega \epsilon} \sin(\omega t - 40x) \mathbf{a}_z \\
 &= -\frac{800}{12 \times 10^9 \times \frac{10^{-9}}{36\pi}} \sin(\omega t - 40x) \mathbf{a}_y - \frac{400}{12 \times 10^9 \times \frac{10^{-9}}{36\pi}} \sin(\omega t - 40x) \mathbf{a}_z \\
 &= -7.539 \sin(\omega t - 40x) \mathbf{a}_y - 3.77 \sin(\omega t - 40x) \mathbf{a}_z \text{ kV/m}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{P} &= \mathbf{E} \times \mathbf{H} = \begin{vmatrix} 0 & E_y & E_z \\ 0 & H_y & H_z \end{vmatrix} = (E_y H_z - E_z H_y) \mathbf{a}_x \\
 &= [20(7.537) \sin^2(\omega t - 40x) + 37.7 \sin^2(\omega t - 40x)] \mathbf{a}_x 10^3 \\
 \mathbf{P}_{\text{ave}} &= \frac{1}{2} [20(7.537) + 37.7] \mathbf{a}_x 10^3 = \underline{\underline{94.23 \mathbf{a}_x \text{ kW/m}^2}}
 \end{aligned}$$

Prob. 10.50

$$\begin{aligned}
 P &= \frac{E_o^2}{2\eta_o} \longrightarrow E_o^2 = 2\eta_o P = 2(120\pi)10 \times 10^{-3} = 7.539 \\
 E_o &= \underline{\underline{2.746 \text{ V/m}}}
 \end{aligned}$$

Prob. 10.51

Let $T = \omega t - \beta z$.

$$-\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos T & \sin T & 0 \end{vmatrix}$$

$$-\mu \frac{\partial \mathbf{H}}{\partial t} = \beta \cos T \mathbf{a}_x + \beta \sin T \mathbf{a}_y$$

$$\mathbf{H} = -\frac{\beta}{\mu} \int [\cos T \mathbf{a}_x + \sin T \mathbf{a}_y] dt = -\frac{\beta}{\mu \omega} \sin T \mathbf{a}_x + \frac{\beta}{\mu \omega} \cos T \mathbf{a}_y$$

$$\mathcal{P} = \mathbf{E} \times \mathbf{H} = \begin{vmatrix} \cos T & \sin T & 0 \\ -\frac{\beta}{\mu \omega} \sin T & \frac{\beta}{\mu \omega} \cos T & 0 \end{vmatrix} = \frac{\beta}{\mu \omega} (\cos^2 T + \sin^2 T) \mathbf{a}_z$$

$$= \frac{\beta}{\mu \omega} \mathbf{a}_z = \sqrt{\frac{\epsilon}{\mu}} \mathbf{a}_z$$

which is constant everywhere.

Prob. 10.52

$$\phi = \frac{E_o^2}{2\eta_o}$$

$$P = \phi S = \frac{E_o^2 S}{2\eta_o} = \frac{(2.4 \times 10^3)^2 \times 450 \times 10^{-4}}{2 \times 377} = \underline{\underline{343.8 \text{ W}}}$$

Prob. 10.53

$$\mathcal{P} = \mathbf{E} \times \mathbf{H} = \frac{V_o I_o}{2\pi\rho^2 \ln(b/a)} \sin^2(\omega t - \beta z) \mathbf{a}_z$$

$$\begin{aligned} \text{(a)} \quad \mathcal{P}_{ave} &= \frac{1}{T} \int_0^T \mathcal{P} dt = \frac{V_o I_o}{2\pi\rho^2 \ln(b/a)} \frac{1}{T} \int_0^T \sin^2(\omega t - \beta z) dt \mathbf{a}_z = \frac{V_o I_o}{2\pi\rho^2 \ln(b/a)} \frac{1}{2} \mathbf{a}_z \\ &= \underline{\underline{\frac{V_o I_o}{4\pi\rho^2 \ln(b/a)} \mathbf{a}_z}} \end{aligned}$$

(b)

$$\begin{aligned} P_{ave} &= \int_S \mathcal{P}_{ave} \cdot d\mathbf{S}, \quad d\mathbf{S} = \rho d\rho d\phi \mathbf{a}_z \\ &= \frac{V_o I_o}{4\pi \ln(b/a)} \int_{\phi=0}^{2\pi} \int_{\rho=a}^b \frac{1}{\rho^2} \rho d\rho d\phi = \frac{V_o I_o}{4\pi \ln(b/a)} (2\pi) \ln(b/a) \\ &= \underline{\underline{\frac{1}{2} V_o I_o}} \end{aligned}$$

Prob. 10.54

$$\text{(a)} \quad P_{i,ave} = \frac{E_{io}^2}{2\eta_1}, \quad P_{r,ave} = \frac{E_{ro}^2}{2\eta_1}, \quad P_{t,ave} = \frac{E_{to}^2}{2\eta_2}$$

$$R = \frac{P_{r,ave}}{P_{i,ave}} = \frac{E_{ro}^2}{E_{io}^2} = \Gamma^2 = \underline{\underline{\left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right)^2}}$$

$$R = \left(\frac{\sqrt{\frac{\mu_o}{\epsilon_2}} - \sqrt{\frac{\mu_o}{\epsilon_1}}}{\sqrt{\frac{\mu_o}{\epsilon_2}} + \sqrt{\frac{\mu_o}{\epsilon_1}}} \right)^2 = \left(\frac{\sqrt{\mu_o \epsilon_1} - \sqrt{\mu_o \epsilon_2}}{\sqrt{\mu_o \epsilon_1} + \sqrt{\mu_o \epsilon_2}} \right)^2$$

$$\text{Since } n_1 = c\sqrt{\mu_1 \epsilon_1} = c\sqrt{\mu_o \epsilon_1}, \quad n_2 = c\sqrt{\mu_o \epsilon_2},$$

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

$$T = \frac{P_{t,ave}}{P_{i,ave}} = \frac{\eta_1}{\eta_2} \frac{E_{to}^2}{E_{io}^2} = \frac{\eta_1}{\eta_2} \tau^2 = \frac{\eta_1}{\eta_2} (1 + \Gamma)^2 = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

$$(b) \text{ If } P_{r,ave} = P_{t,ave} \longrightarrow RP_{i,ave} = TP_{i,ave} \longrightarrow R = T$$

$$\text{i.e. } (n_1 - n_2)^2 = 4n_1 n_2 \longrightarrow n_1^2 - 6n_1 n_2 + n_2^2 = 0$$

$$\text{or } \left(\frac{n_1}{n_2} \right)^2 - 6 \left(\frac{n_1}{n_2} \right) + 1 = 0, \text{ so}$$

$$\frac{n_1}{n_2} = 3 \pm \sqrt{8} = \underline{\underline{5.828}} \quad \text{or} \quad \underline{\underline{0.1716}}$$

(Note that these values are mutual reciprocals, reflecting the inherent symmetry of the problem.)

Prob. 10.55

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \sqrt{\frac{2\mu_o}{8\epsilon_o}} = \frac{\eta_o}{2}, \quad \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \sqrt{\frac{\mu_o}{16\epsilon_o}} = \frac{\eta_o}{4}$$

$$\Gamma = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \frac{\eta_o/4 - \eta_o/2}{\eta_o/4 + \eta_o/2} = -\frac{1}{3}, \quad \tau = \Gamma + 1 = \frac{2}{3}$$

$$\begin{aligned} \underline{\underline{E_r}} &= -\frac{1}{3}(60)\sin(\omega t + 10z)\mathbf{a}_x - \frac{1}{3}(30)\sin(\omega t + 10z + \pi/6)\mathbf{a}_y \\ &= -20\sin(\omega t + 10z)\mathbf{a}_x - 10\sin(\omega t + 10z + \pi/6)\mathbf{a}_y \text{ V/m} \end{aligned}$$

$$\begin{aligned} \underline{\underline{E_t}} &= \frac{2}{3}(60)\sin(\omega t - 10z)\mathbf{a}_x + \frac{2}{3}(30)\sin(\omega t - 10z + \pi/6)\mathbf{a}_y \\ &= 40\sin(\omega t - 10z)\mathbf{a}_x + 20\sin(\omega t - 10z + \pi/6)\mathbf{a}_y \text{ V/m} \end{aligned}$$

Prob. 10.56

$$\eta_1 = \eta_o = 120\pi, \quad \eta_2 = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_o}{9\epsilon_o}} = \frac{\eta_o}{3} = 40\pi$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_o/3 - \eta_o}{\eta_o/3 + \eta_o} = -\frac{1}{2}, \quad \tau = 1 + \Gamma = \frac{1}{2}$$

$$E_r = \Gamma E_o = -E_o/2, \quad E_t = \tau E_o = E_o/2$$

$$P_{iave} = \frac{|E_o|^2}{2\eta_1} = \frac{E_o^2}{2\eta_o}$$

$$P_{tave} = \frac{|E_t|^2}{2\eta_2} = \frac{\frac{1}{4}E_o^2}{2(\eta_o/3)} = \frac{E_o^2}{2\eta_o} \frac{3}{4}$$

$$\frac{P_{tave}}{P_{iave}} = \frac{3}{4} = \underline{\underline{0.75}}$$

Prob. 10.57

(a) $\eta_1 = \eta_o$

$$E_i = E_{io} \sin(\omega t - 5x) \mathbf{a}_E$$

$$E_{io} = H_{io} \eta_o = 120\pi \times 4 = 480\pi$$

$$\mathbf{a}_E \times \mathbf{a}_H = \mathbf{a}_k \longrightarrow \mathbf{a}_E \times \mathbf{a}_y = \mathbf{a}_x \longrightarrow \mathbf{a}_E = -\mathbf{a}_z$$

$$E_i = -480\pi \sin(\omega t - 5x) \mathbf{a}_z$$

$$\eta_2 = \sqrt{\frac{\mu_o}{4\epsilon_o}} = \frac{120\pi}{\sqrt{4}} = 60\pi$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = -1/3, \quad \tau = 1 + \Gamma = 2/3$$

$$E_{ro} = \Gamma E_{io} = (-1/3)(-480\pi) = 160\pi$$

$$E_r = 160\pi \sin(\omega t + 5x) \mathbf{a}_z$$

$$E_1 = E_i + E_r = \underline{\underline{-1.508 \sin(\omega t - 5x) \mathbf{a}_z + 0.503 \sin(\omega t + 5x) \mathbf{a}_z}} \quad \text{kV/m}$$

$$(b) \quad E_{io} = \tau E_{io} = (2/3)(480\pi) = 320\pi$$

$$\mathcal{P} = \frac{E_{io}^2}{2\eta_2} \mathbf{a}_x = \frac{(320\pi)^2}{2(60\pi)} \mathbf{a}_x = \underline{\underline{2.68 \mathbf{a}_x \text{ kW/m}^2}}$$

$$(c) \quad s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/3}{1-1/3} = 2$$

$$\text{Prob. 10.58} \quad \eta_I = \sqrt{\frac{\mu_I}{\epsilon_I}} = \eta_o / 2, \quad \eta_2 = \eta_o$$

$$\Gamma = \frac{\eta_2 - \eta_I}{\eta_2 + \eta_I} = 1/3, \quad \tau = 1 + \Gamma = 4/3$$

$$E_{or} = \Gamma E_{io} = (1/3)(5) = 5/3, \quad E_{ot} = \tau E_{io} = 20/3$$

$$\beta = \frac{\omega}{c} \sqrt{\mu_r \epsilon_r} = \frac{10^8}{3 \times 10^8} \sqrt{4} = 2/3$$

$$(a) \quad \mathbf{E}_r = \frac{5}{3} \cos(10^8 t - 2y/3) \mathbf{a}_z$$

$$\underline{\underline{\mathbf{E}_1 = \mathbf{E}_i + \mathbf{E}_r = 5 \cos(10^8 t + \frac{2}{3} y) \mathbf{a}_z + \frac{5}{3} \cos(10^8 t - \frac{2}{3} y) \mathbf{a}_z \text{ V/m}}}$$

$$(b) \quad \mathcal{P}_{\text{ave1}} = \frac{E_{io}^2}{2\eta_1} (-\mathbf{a}_y) + \frac{E_{ro}^2}{2\eta_1} (+\mathbf{a}_y) = \frac{25}{2(60\pi)} (1 - \frac{1}{9}) (-\mathbf{a}_y) = \underline{\underline{-0.0589 \mathbf{a}_y \text{ W/m}^2}}$$

$$(c) \quad \mathcal{P}_{\text{ave2}} = \frac{E_{io}^2}{2\eta_2} (-\mathbf{a}_y) = \frac{400}{9(2)(120\pi)} (-\mathbf{a}_y) = \underline{\underline{-0.0589 \mathbf{a}_y \text{ W/m}^2}}$$

Prob. 10.59

$$\beta_1 = \omega \sqrt{\mu_1 \epsilon_1} = \omega \sqrt{\mu_o \epsilon_o} \sqrt{\mu_{r1} \epsilon_{r1}} = \frac{\omega}{c} \sqrt{16} = \frac{90 \times 10^9 (4)}{3 \times 10^8} = 300(4) = 1200$$

$$\beta_2 = \omega \sqrt{\mu_o \epsilon_o} = \frac{90 \times 10^9}{3 \times 10^8} = 300$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \sqrt{\frac{\mu_o}{\epsilon_o}} \sqrt{\frac{\mu_{r1}}{\epsilon_{r1}}} = \frac{\eta_o}{4} = 30\pi, \quad \eta_2 = \eta_o = 120\pi$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_o - \eta_o/4}{\eta_o + \eta_o/4} = \frac{3}{5}, \quad \tau = 1 + \Gamma = \frac{8}{5}$$

$$\mathcal{P} = \mathbf{E} \times \mathbf{H}$$

$$\mathcal{P}_1 = \mathcal{P}_i + \mathcal{P}_r = \frac{60^2}{\eta_1} \cos^2(\omega t - \beta_1 z) \mathbf{a}_x + \frac{(60 \times \frac{3}{5})^2}{\eta_1} \cos^2(\omega t + \beta_1 z) \mathbf{a}_x$$

$$\frac{60^2}{\eta_1} = \frac{60^2}{30\pi} = 38.197, \quad \frac{(60 \times \frac{3}{5})^2}{\eta_1} = 13.75$$

$$\underline{\underline{\mathcal{P}_1 = 38.197 \cos^2(\omega t - \beta_1 z) \mathbf{a}_x + 13.75 \cos^2(\omega t + \beta_1 z) \mathbf{a}_x \text{ W/m}^2, \text{ where } \beta_1 = 1200}}$$

$$\mathcal{P}_2 = \mathcal{P}_t = \frac{(60 \times \frac{8}{5})^2}{\eta_o} \cos^2(\omega t - \beta_2 z) \mathbf{a}_x$$

$$\frac{(60 \times \frac{8}{5})^2}{\eta_o} = \frac{96^2}{120\pi} = 24.46$$

$$\underline{\underline{\mathcal{P}_2 = 24.46 \cos^2(\omega t - \beta_2 z) \mathbf{a}_x \text{ W/m}^2, \text{ where } \beta_2 = 300}}$$

Prob. 10.60

(a) In air, $\beta_I = 1, \lambda_I = 2\pi / \beta_I = 2\pi = \underline{\underline{6.283 \text{ m}}}$

$$\omega = \beta_I c = \underline{\underline{3 \times 10^8 \text{ rad/s}}}$$

In the dielectric medium, ω is the same .

$$\omega = \underline{\underline{3 \times 10^8 \text{ rad/s}}}$$

$$\beta_2 = \frac{\omega}{c} \sqrt{\epsilon_{r2}} = \beta_I \sqrt{\epsilon_{r2}} = \sqrt{3}$$

$$\lambda_2 = \frac{2\pi}{\beta_2} = \frac{2\pi}{\sqrt{3}} = \underline{\underline{3.6276 \text{ m}}}$$

$$(b) \quad H_o = \frac{E_o}{\eta_o} = \frac{10}{120\pi} = 0.0265$$

$$\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_z \times \mathbf{a}_y = -\mathbf{a}_x$$

$$\underline{\underline{\mathbf{H}_i = -26.5 \cos(\omega t - z) \mathbf{a}_x \text{ mA/m}}}$$

$$(c) \quad \eta_1 = \eta_o, \quad \eta_2 = \eta_o / \sqrt{3}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{(1/\sqrt{3}) - 1}{(1/\sqrt{3}) + 1} = \underline{\underline{-0.268}}, \quad \tau = 1 + \Gamma = \underline{\underline{0.732}}$$

$$(d) \quad E_{io} = \tau E_{i0} = 7.32, \quad E_{ro} = \Gamma E_{i0} = -2.68$$

$$\underline{\underline{\mathbf{E}_1 = \mathbf{E}_i + \mathbf{E}_r = 10 \cos(\omega t - z) \mathbf{a}_y - 2.68 \cos(\omega t + z) \mathbf{a}_y \text{ V/m}}}$$

$$\underline{\underline{\mathbf{E}_2 = \mathbf{E}_t = 7.32 \cos(\omega t - z) \mathbf{a}_y \text{ V/m}}}$$

$$\mathcal{P}_{\text{ave1}} = \frac{1}{2\eta_1} (\mathbf{a}_z) [E_{io}^2 - E_{ro}^2] = \frac{1}{2(120\pi)} (\mathbf{a}_z) (10^2 - 2.68^2) = \underline{\underline{0.1231 \mathbf{a}_z \text{ W/m}^2}}$$

$$\mathcal{P}_{\text{ave2}} = \frac{E_{to}^2}{2\eta_2} (\mathbf{a}_z) = \frac{\sqrt{3}}{2 \times 120\pi} (7.32)^2 (\mathbf{a}_z) = \underline{\underline{0.1231 \mathbf{a}_z \text{ W/m}^2}}$$

Prob. 10.61

$$\eta_1 = \eta_o = 120\pi$$

For seawater (lossy medium),

$$\eta_2 = \sqrt{\frac{j\omega\mu_o}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j2\pi \times 10^8 \times 4}{4 + j2\pi \times 10^8 \times 81 \times \frac{10^{-9}}{36\pi}}} = 10.44 + j9.333$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.9461 \angle 177.16$$

$$|\Gamma|^2 = 0.8952, \quad 1 - |\Gamma| = 0.1084$$

$$\frac{P_r}{P_i} = \underline{\underline{89.51\%}}, \quad \frac{P_t}{P_i} = \underline{\underline{10.84\%}},$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{7.924 \angle 43.975 - 377}{7.924 \angle 43.975 + 377} = 0.9702 \angle 178.2^\circ$$

The fraction of the incident power reflected is

$$\frac{P_r}{P_i} = |\Gamma|^2 = 0.9702^2 = \underline{\underline{0.9413}}$$

The transmitted fraction is

$$\frac{P_t}{P_i} = 1 - |\Gamma|^2 = 1 - 0.9702^2 = \underline{\underline{0.0587}}$$

Prob. 10.62

(a)

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \frac{120\pi}{\sqrt{4}} = 188.5, \quad \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \frac{120\pi}{\sqrt{3.2}} = 210.75$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{210.75 - 188.5}{210.75 + 188.5} = 0.0557, \quad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times 210.75}{210.75 + 188.5} = 1.0557$$

$$E_{ro} = \Gamma E_{io} = (0.0557)(12) = 0.6684 \quad E_{to} = \tau E_{io} = 1.0557(12) = 12.668$$

$$\beta_1 = \omega \sqrt{\mu_1 \epsilon_1} = \frac{\omega}{c} \sqrt{4} \quad \longrightarrow \quad \omega = \frac{\beta_1 c}{2} = \frac{40\pi(3 \times 10^8)}{2} = \underline{\underline{6\pi \times 10^9 \text{ rad/s}}}$$

(b)

$$\beta_2 = \omega \sqrt{\mu_2 \epsilon_2} = \frac{\omega}{c} \sqrt{3.2} = \frac{6\pi \times 10^9 \sqrt{3.2}}{3 \times 10^8} = 112.4$$

$$E_r = E_{ro} \cos(\omega t + 40\pi x) \mathbf{a}_z = \underline{\underline{0.6684 \cos(6\pi \times 10^9 t + 40\pi x) \mathbf{a}_z \text{ V/m}}}$$

$$E_t = E_{to} \cos(\omega t - \beta_2 x) \mathbf{a}_z = \underline{\underline{12.668 \cos(6\pi \times 10^9 t - 112.4x) \mathbf{a}_z \text{ V/m}}}$$

Prob. 10.63 (a) $\omega = \beta c = 3 \times 3 \times 10^8 = \underline{\underline{9 \times 10^8 \text{ rad/s}}}$

(b) $\lambda = 2\pi / \beta = 2\pi / 3 = \underline{\underline{2.094 \text{ m}}}$

(c) $\frac{\sigma}{\omega \epsilon} = \frac{4}{9 \times 10^8 \times 80 \times 10^{-9} / 36\pi} = 2\pi = \underline{\underline{6.288}}$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega \epsilon} = 6.288 \quad \longrightarrow \quad \theta_\eta = 40.47^\circ$$

$$|\eta_2| = \frac{\sqrt{\mu_2 / \epsilon_2}}{\sqrt[4]{1 + \left(\frac{\sigma_2}{\omega \epsilon_2}\right)^2}} = \frac{377 / \sqrt{80}}{\sqrt[4]{1 + 4\pi^2}} = 16.71$$

$$\eta_2 = \underline{16.71 \angle 40.47^\circ \Omega}$$

$$(d) \quad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{16.71 \angle 40.47^\circ - 377}{16.71 \angle 40.47^\circ + 377} = 0.935 \angle 179.7^\circ$$

$$E_{or} = \Gamma E_{oi} = 9.35 \angle 179.7^\circ$$

$$\underline{E_r = 9.35 \sin(\omega t - 3z + 179.7) \mathbf{a}_x \text{ V/m}}$$

$$\alpha_2 = \frac{\omega}{c} \sqrt{\frac{\mu_{r2} \epsilon_{r2}}{2} \left[\sqrt{1 + \left(\frac{\sigma_2}{\omega \epsilon_2} \right)^2} - 1 \right]} = \frac{9 \times 10^8}{3 \times 10^8} \sqrt{\frac{80}{2} \left[\sqrt{1 + 4\pi^2} - 1 \right]} = 43.94 \text{ Np/m}$$

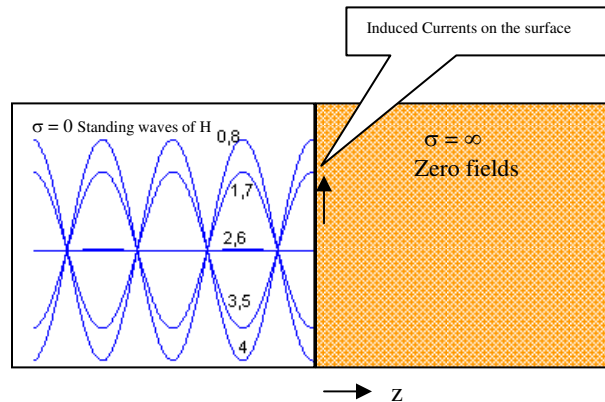
$$\beta_2 = \frac{9 \times 10^8}{3 \times 10^8} \sqrt{\frac{80}{2} \left[\sqrt{1 + 4\pi^2} + 1 \right]} = 51.48 \text{ rad/m}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times 16.71 \angle 40.47^\circ}{16.71 \angle 40.47^\circ + 377} = 0.0857 \angle 38.89^\circ$$

$$E_{ot} = \tau E_o = 0.857 \angle 38.89^\circ$$

$$\underline{E_t = 0.857 e^{43.94z} \sin(9 \times 10^8 t + 51.48z + 38.89^\circ) \mathbf{a}_x \text{ V/m}}$$

Prob. 10.64



Curve 0 is at $t = 0$; curve 1 is at $t = T/8$; curve 2 is at $t = T/4$; curve 3 is at $t = 3T/8$, etc.

Prob. 10.65 Since $\mu_o = \mu_l = \mu_2$,

$$\sin \theta_{t1} = \sin \theta_i \sqrt{\frac{\epsilon_o}{\epsilon_l}} = \frac{\sin 45^\circ}{\sqrt{4.5}} = 0.3333 \quad \longrightarrow \quad \underline{\underline{\theta_{t1} = 19.47^\circ}}$$

$$\sin \theta_{t2} = \sin \theta_{t1} \sqrt{\frac{\epsilon_l}{\epsilon_2}} = \frac{1}{3} \sqrt{\frac{4.5}{2.25}} = 0.4714 \quad \longrightarrow \quad \underline{\underline{\theta_{t2} = 28.13^\circ}}$$

Prob. 10.66

$$\begin{aligned} \mathbf{E}_s &= \frac{20(e^{jk_x x} - e^{-jk_x x})}{j2} \frac{(e^{jk_y y} + e^{-jk_y y})}{2} \mathbf{a}_z \\ &= -j5 \left[e^{j(k_x x + k_y y)} + e^{j(k_x x - k_y y)} - e^{-j(k_x x - k_y y)} - e^{-j(k_x x + k_y y)} \right] \mathbf{a}_z \end{aligned}$$

which consists of four plane waves.

$$\nabla \times \mathbf{E}_s = -j\omega\mu_o \mathbf{H}_s \quad \longrightarrow \quad \mathbf{H}_s = \frac{j}{\omega\mu_o} \nabla \times \mathbf{E}_s = \frac{j}{\omega\mu_o} \left(\frac{\partial E_z}{\partial y} \mathbf{a}_x - \frac{\partial E_z}{\partial x} \mathbf{a}_y \right)$$

$$\underline{\underline{\mathbf{H}_s = -\frac{j20}{\omega\mu_o} \left[k_y \sin(k_x x) \sin(k_y y) \mathbf{a}_x + k_x \cos(k_x x) \cos(k_y y) \mathbf{a}_y \right]}}$$

Prob. 10.67

$$\eta_1 = \eta_o = 377\Omega$$

For η_2 ,

$$\frac{\sigma_2}{\omega\epsilon_2} = \frac{4}{2\pi \times 1.2 \times 10^9 \times 50 \times \frac{10^{-9}}{36\pi}} = 1.2$$

$$\tan 2\theta_{\eta_2} = \frac{\sigma_2}{\omega\epsilon_2} = 1.2 \quad \longrightarrow \quad \theta_{\eta_2} = 25.1^\circ$$

$$|\eta_2| = \frac{\sqrt{\mu/\epsilon}}{\sqrt[4]{1 + \left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2}} = \frac{120\pi\sqrt{1/50}}{\sqrt[4]{1 + 1.2^2}} = 42.658$$

$$\eta_2 = 42.658 \angle 25.1^\circ$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{42.658 \angle 25.1^\circ - 377}{42.658 \angle 25.1^\circ + 377} = \underline{\underline{0.8146 \angle 174.4^\circ}}$$

Prob. 10.68

(a)

$$P_t = (1 - |\Gamma|^2) P_i$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} \longrightarrow |\Gamma| = \frac{s - 1}{s + 1}$$

$$\frac{P_t}{P_i} = 1 - \left(\frac{s - 1}{s + 1} \right)^2 = \frac{4s}{(s + 1)^2}$$

$$(b) \quad P_i = P_r + P_t \longrightarrow \frac{P_r}{P_i} = 1 - \frac{P_t}{P_i} = \left(\frac{s - 1}{s + 1} \right)^2$$

Prob. 10.69

If $\mathbf{A}$ is a uniform vector and $\Phi(r)$ is a scalar,

$$\nabla \times (\Phi \mathbf{A}) = \nabla \Phi \times \mathbf{A} + \Phi (\nabla \times \mathbf{A}) = \nabla \Phi \times \mathbf{A}$$

since $\nabla \times \mathbf{A} = \mathbf{0}$.

$$\begin{aligned} \nabla \times \mathbf{E} &= \left(\frac{\partial}{\partial x} \mathbf{a}_x + \frac{\partial}{\partial y} \mathbf{a}_y + \frac{\partial}{\partial z} \mathbf{a}_z \right) \times \mathbf{E}_o e^{j(k_x x + k_y y + k_z z - \omega t)} = j(k_x \mathbf{a}_x + k_y \mathbf{a}_y + k_z \mathbf{a}_z) e^{j(\mathbf{k} \cdot \mathbf{r} - \omega t)} \times \mathbf{E}_o \\ &= j\mathbf{k} \times \mathbf{E}_o e^{j(\mathbf{k} \cdot \mathbf{r} - \omega t)} = j\mathbf{k} \times \mathbf{E} \end{aligned}$$

$$\text{Also, } -\frac{\partial \mathbf{B}}{\partial t} = j\omega\mu\mathbf{H}. \quad \text{Hence } \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \text{ becomes } \mathbf{k} \times \mathbf{E} = \omega\mu\mathbf{H}$$

$$\text{From this, } \underline{\underline{\mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_H}}$$

Prob. 10.70

$$k = |\mathbf{k}| = \sqrt{124^2 + 124^2 + 263^2} = 316.1$$

$$\lambda = \frac{2\pi}{k} = \underline{\underline{19.88 \text{ mm}}}$$

$$k = \omega\sqrt{\mu\epsilon} = \frac{2\pi f}{c} \longrightarrow f = \frac{kc}{2\pi} = \frac{316.1 \times 3 \times 10^8}{2\pi} = \underline{\underline{15.093 \text{ GHz}}}$$

$$\mathbf{k} \cdot \mathbf{a}_x = k \cos \theta_x \longrightarrow \cos \theta_x = \frac{124}{316.1} \longrightarrow \theta_x = 66.9^\circ = \theta_y$$

$$\theta_z = \cos^{-1} \frac{263}{316.1} = 33.69^\circ$$

Thus,

$$\underline{\underline{\theta_x = \theta_y = 66.9^\circ, \theta_z = 33.69^\circ}}$$

Prob. 10.71

$$\mathbf{k} = -3.4\mathbf{a}_x + 4.2\mathbf{a}_y$$

$$\mathbf{k} \cdot \mathbf{E} = 0 \quad \longrightarrow \quad 0 = -3.4E_o + 4.2$$

$$E_o = \frac{4.2}{3.4} = \underline{\underline{1.235}}$$

$$k = |\mathbf{k}| = \beta = \sqrt{(-3.4)^2 + (4.2)^2} = 5.403$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5.403} = 1.162$$

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{1.162} = \underline{\underline{258 \text{ MHz}}}$$

$$\mathbf{H}_s = \frac{1}{\mu\omega} \mathbf{k} \times \mathbf{E}_s = \frac{1}{\mu k c} \mathbf{k} \times \mathbf{E}_s$$

$$= \frac{1}{4\pi \times 10^{-7} \times 5.403 \times 3 \times 10^8} \begin{vmatrix} -3.4 & 4.2 & 0 \\ E_o & 1 & 3 + j4 \end{vmatrix} A_o$$

$$\text{where } A_o = e^{-j3.4x + 4.2y}$$

$$\begin{aligned} \mathbf{H}_s &= 4.91 A_o \times 10^{-4} [4.2(3 + j4)\mathbf{a}_x + 3.4(3 + j4)\mathbf{a}_y + (-3.4 - 4.2E_o)\mathbf{a}_z] \\ &= 0.491 [(12.6 + j16.8)\mathbf{a}_x + (10.2 + j13.6)\mathbf{a}_y - 8.59\mathbf{a}_z] e^{-j3.4x + 4.2y} \text{ mA/m} \end{aligned}$$

Prob. 10.72

$$\nabla \cdot \mathbf{E} = \left(\frac{\partial}{\partial x} \mathbf{a}_x + \frac{\partial}{\partial y} \mathbf{a}_y + \frac{\partial}{\partial z} \mathbf{a}_z \right) \cdot \mathbf{E}_o e^{j(k_x x + k_y y + k_z z - \omega t)} = j(k_x \mathbf{a}_x + k_y \mathbf{a}_y + k_z \mathbf{a}_z) e^{j(\mathbf{k} \cdot \mathbf{r} - \omega t)} \cdot \mathbf{E}_o$$

$$= j\mathbf{k} \cdot \mathbf{E}_o e^{j(\mathbf{k} \cdot \mathbf{r} - \omega t)} = j\mathbf{k} \cdot \mathbf{E} = 0 \quad \longrightarrow \quad \mathbf{k} \cdot \mathbf{E} = 0$$

Similarly,

$$\nabla \cdot \mathbf{H} = j\mathbf{k} \cdot \mathbf{H} = 0 \quad \longrightarrow \quad \mathbf{k} \cdot \mathbf{H} = 0$$

It has been shown in the previous problem that

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \longrightarrow \quad \mathbf{k} \times \mathbf{E} = \omega \mu \mathbf{H}$$

Similarly,

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \quad \longrightarrow \quad \mathbf{k} \times \mathbf{H} = -\epsilon \omega \mathbf{E}$$

From $\mathbf{k} \times \mathbf{E} = \omega\mu\mathbf{H}$, $\mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_H$ and

From $\mathbf{k} \times \mathbf{H} = -\varepsilon\omega\mathbf{E}$, $\mathbf{a}_k \times \mathbf{a}_H = -\mathbf{a}_E$

Prob. 10.73

$$\text{If } \mu_o = \mu_l = \mu_2, \quad \eta_l = \frac{\eta_o}{\sqrt{\varepsilon_{r1}}}, \eta_2 = \frac{\eta_o}{\sqrt{\varepsilon_{r2}}}$$

$$\Gamma_{\parallel} = \frac{\frac{I}{\sqrt{\varepsilon_{r2}}} \cos\theta_t - \frac{I}{\sqrt{\varepsilon_{r1}}} \cos\theta_i}{\frac{I}{\sqrt{\varepsilon_{r2}}} \cos\theta_t + \frac{I}{\sqrt{\varepsilon_{r1}}} \cos\theta_i}$$

$$\sqrt{\varepsilon_{r1}} \sin\theta_i = \sqrt{\varepsilon_{r2}} \sin\theta_t \quad \longrightarrow \quad \frac{\sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}}} = \frac{\sin\theta_i}{\sin\theta_t}$$

$$\begin{aligned} \Gamma_{\parallel} &= \frac{\cos\theta_t - \frac{\sin\theta_i}{\sin\theta_t} \cos\theta_i}{\cos\theta_t + \frac{\sin\theta_i}{\sin\theta_t} \cos\theta_i} = \frac{\sin\theta_t \cos\theta_t - \sin\theta_i \cos\theta_i}{\sin\theta_t \cos\theta_t + \sin\theta_i \cos\theta_i} \\ &= \frac{\sin 2\theta_t - \sin 2\theta_i}{\sin 2\theta_t + \sin 2\theta_i} = \frac{\sin(\theta_t - \theta_i) \cos(\theta_t + \theta_i)}{\cos(\theta_t - \theta_i) \sin(\theta_t + \theta_i)} = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} \end{aligned}$$

Similarly,

$$\begin{aligned} \tau_{\parallel} &= \frac{\frac{2}{\sqrt{\varepsilon_{r2}}} \cos\theta_i}{\frac{I}{\sqrt{\varepsilon_{r2}}} \cos\theta_t + \frac{I}{\sqrt{\varepsilon_{r1}}} \cos\theta_i} = \frac{2 \cos\theta_i}{\cos\theta_t + \frac{\sin\theta_i}{\sin\theta_t} \cos\theta_i} \\ &= \frac{2 \cos\theta_i \sin\theta_t}{\sin\theta_t \cos\theta_t (\sin^2\theta_i + \cos^2\theta_i) + \sin\theta_i \cos\theta_i (\sin^2\theta_t + \cos^2\theta_t)} \\ &= \frac{2 \cos\theta_i \sin\theta_t}{(\sin\theta_i \cos\theta_t + \sin\theta_t \cos\theta_i)(\cos\theta_i \cos\theta_t + \sin\theta_i \sin\theta_t)} \\ &= \frac{2 \cos\theta_i \sin\theta_t}{\sin(\theta_i + \theta_t) \cos(\theta_i - \theta_t)} \end{aligned}$$

$$\Gamma_{\perp} = \frac{\frac{I}{\sqrt{\epsilon_{r2}}} \cos \theta_i - \frac{I}{\sqrt{\epsilon_{r1}}} \cos \theta_t}{\frac{I}{\sqrt{\epsilon_{r2}}} \cos \theta_i + \frac{I}{\sqrt{\epsilon_{r1}}} \cos \theta_t} = \frac{\cos \theta_i - \frac{\sin \theta_i}{\sin \theta_t} \cos \theta_t}{\cos \theta_i + \frac{\sin \theta_i}{\sin \theta_t} \cos \theta_t} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}$$

$$\tau_{\perp} = \frac{\frac{2}{\sqrt{\epsilon_{r2}}} \cos \theta_i}{\frac{I}{\sqrt{\epsilon_{r2}}} \cos \theta_i + \frac{I}{\sqrt{\epsilon_{r1}}} \cos \theta_t} = \frac{2 \cos \theta_i}{\cos \theta_i + \frac{\sin \theta_i}{\sin \theta_t} \cos \theta_t} = \frac{2 \cos \theta_i \sin \theta_t}{\sin(\theta_t + \theta_i)}$$

Prob. 10.74

(a) $n_1 = 1, \quad n_2 = c\sqrt{\mu_2 \epsilon_2} = c\sqrt{6.4 \epsilon_o \times \mu_o} = \sqrt{6.4} = 2.5298$

$$\sin \theta_t = \frac{n_1}{n_2} \sin \theta_i = \frac{1}{2.5298} \sin 12^\circ = 0.082185 \quad \longrightarrow \quad \theta_t = 4.714^\circ$$

$$\eta_1 = 120\pi, \quad \eta_2 = 120\pi \sqrt{\frac{1}{6.4}} = 47.43\pi$$

$$\begin{aligned} \frac{E_{ro}}{E_{io}} = \Gamma &= \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} = \frac{47.43\pi \cos 4.714^\circ - 120\pi \cos 12^\circ}{47.43\pi \cos 4.714^\circ + 120\pi \cos 12^\circ} \\ &= \frac{47.27 - 117.38}{47.27 + 117.38} = \underline{\underline{-0.4258}} \end{aligned}$$

$$\frac{E_{to}}{E_{io}} = \tau = \frac{2\eta_2 \cos \theta_t}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} = \frac{2 \times 47.43 \cos 12^\circ}{47.27 + 117.33} = \frac{92.787}{164.65} = \underline{\underline{0.5635}}$$

Prob. 10.75

(a) $\mathbf{k}_i = 4\mathbf{a}_y + 3\mathbf{a}_z$

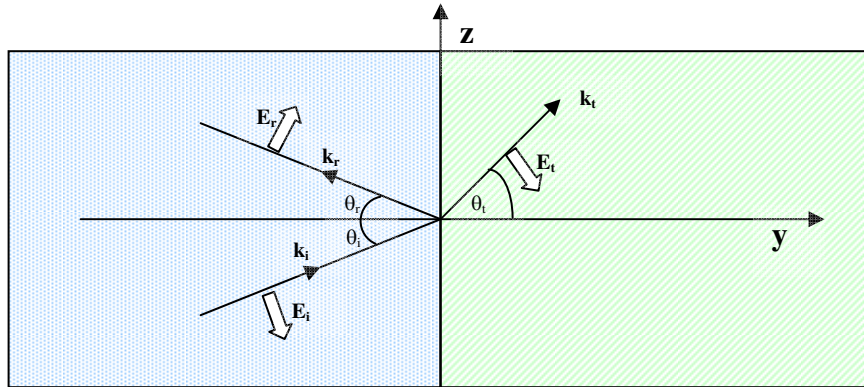
$$\mathbf{k}_i \bullet \mathbf{a}_n = k_i \cos \theta_i \quad \longrightarrow \quad \cos \theta_i = 4/5 \quad \longrightarrow \quad \underline{\underline{\theta_i = 36.87^\circ}}$$

(b)

$$\mathbf{P}_{\text{ave}} = \frac{1}{2} \operatorname{Re}(\mathbf{E}_s \times \mathbf{H}_s^*) = \frac{E_o^2}{2\eta} \mathbf{a}_k = \frac{(\sqrt{8^2 + 6^2})^2}{2 \times 120\pi} \frac{(4\mathbf{a}_y + 3\mathbf{a}_z)}{5} = \underline{\underline{106.1\mathbf{a}_y + 79.58\mathbf{a}_z \text{ mW/m}^2}}$$

(c) $\theta_r = \theta_i = 36.87^\circ$. Let

$$\mathbf{E}_r = (E_{ry}\mathbf{a}_y + E_{rz}\mathbf{a}_z) \sin(\omega t - \mathbf{k}_r \bullet \mathbf{r})$$



From the figure, $\mathbf{k}_r = k_{rz}\mathbf{a}_z - k_{ry}\mathbf{a}_y$. But $k_r = k_i = 5$

$$k_{rz} = k_r \sin \theta_r = 5(3/5) = 3, \quad k_{ry} = k_r \cos \theta_r = 5(4/5) = 4,$$

Hence, $\mathbf{k}_r = -4\mathbf{a}_y + 3\mathbf{a}_z$

$$\sin \theta_t = \frac{n_1}{n_2} \sin \theta_i = \frac{c\sqrt{\mu_1 \epsilon_1}}{c\sqrt{\mu_2 \epsilon_2}} \sin \theta_i = \frac{3/5}{\sqrt{4}} = 0.3$$

$$\theta_t = 17.46, \quad \cos \theta_t = 0.9539, \quad \eta_1 = \eta_o = 120\pi, \eta_2 = \eta_o / 2 = 60\pi$$

$$\Gamma_{||} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} = \frac{\frac{\eta_o}{2}(0.9539) - \eta_o(0.8)}{\frac{\eta_o}{2}(0.9539) + \eta_o(0.8)} = -0.253$$

$$E_{ro} = \Gamma_{||} E_{io} = -0.253(10) = -2.53$$

$$\text{But } (E_{ry}\mathbf{a}_y + E_{rz}\mathbf{a}_z) = E_{ro}(\sin \theta_r \mathbf{a}_y + \cos \theta_r \mathbf{a}_z) = -2.53\left(\frac{3}{5}\mathbf{a}_y + \frac{4}{5}\mathbf{a}_z\right)$$

$$\underline{\underline{\mathbf{E}_r = -(1.518\mathbf{a}_y + 2.024\mathbf{a}_z) \sin(\omega t + 4y - 3z) \text{ V/m}}}$$

Similarly, let

$$\mathbf{E}_t = (E_{ty}\mathbf{a}_y + E_{tz}\mathbf{a}_z) \sin(\omega t - \mathbf{k}_t \bullet \mathbf{r})$$

$$k_t = \beta_2 = \omega \sqrt{\mu_2 \epsilon_2} = \omega \sqrt{4\mu_o \epsilon_o}$$

But $k_i = \beta_1 = \omega \sqrt{\mu_o \epsilon_o}$

$$\frac{k_t}{k_i} = 2 \quad \longrightarrow \quad k_t = 2k_i = 10$$

$$k_{ty} = k_t \cos \theta_t = 9.539, \quad k_{tz} = k_t \sin \theta_t = 3,$$

$$k_t = 9.539a_y + 3a_z$$

Note that $k_{iz} = k_{rz} = k_{tz} = 3$

$$\tau_{\parallel} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} = \frac{\eta_o(0.8)}{\frac{\eta_o}{2}(0.9539) + \eta_o(0.8)} = 0.6265$$

$$E_{to} = \tau_{\parallel} E_{io} = 6.265$$

But

$$(E_{ty}a_y + E_{tz}a_z) = E_{to}(\sin \theta_t a_y - \cos \theta_t a_z) = 6.256(0.3a_y - 0.9539a_z)$$

Hence,

$$\underline{\underline{E_t = (1.879a_y - 5.968a_z) \sin(\omega t - 9.539y - 3z) \text{ V/m}}}$$

Prob. 10.76

(a)

$$\tan \theta_i = \frac{k_{ix}}{k_{iz}} = \frac{1}{\sqrt{8}} \quad \longrightarrow \quad \underline{\underline{\theta_i = \theta_r = 19.47^\circ}}$$

$$\sin \theta_t = \sin \theta_i \sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}} = \frac{1}{3}(3) = 1 \quad \longrightarrow \quad \underline{\underline{\theta_t = 90^\circ}}$$

$$(b) \quad \beta_1 = \frac{\omega}{c} \sqrt{\epsilon_{r1}} = \frac{10^9}{3 \times 10^8} \times 3 = 10 = k \sqrt{1+8} = 3k \quad \longrightarrow \quad \underline{\underline{k = 3.333}}$$

$$(c) \quad \lambda = 2\pi / \beta, \quad \lambda_1 = 2\pi / \beta_1 = 2\pi / 10 = \underline{\underline{0.6283 \text{ m}}}$$

$$\beta_2 = \omega / c = 10/3, \quad \lambda_2 = 2\pi / \beta_2 = 2\pi \times 3/10 = \underline{\underline{1.885 \text{ m}}}$$

$$\begin{aligned}
 \text{(d)} \quad \underline{\underline{E_i}} &= \eta_1 \mathbf{H}_x \times \mathbf{a}_k = 40\pi(0.2) \cos(\omega t - \mathbf{k} \cdot \mathbf{r}) \mathbf{a}_y \times \frac{(\mathbf{a}_x + \sqrt{8}\mathbf{a}_z)}{3} \\
 &= \underline{\underline{(23.6954\mathbf{a}_x - 8.3776\mathbf{a}_z) \cos(10^9 t - kx - k\sqrt{8}z) \text{ V/m}}}
 \end{aligned}$$

$$\text{(e)} \quad \tau_{//} = \frac{2 \cos \theta_i \sin \theta_t}{\sin(\theta_i + \theta_t) \cos(\theta_t - \theta_i)} = \frac{2 \cos 19.47^\circ \sin 90^\circ}{\sin 19.47^\circ \cos 19.47^\circ} = 6$$

$$\Gamma_{//} = -\frac{\cot 19.47^\circ}{\cot 19.47^\circ} = -1$$

$$\text{Let } \underline{\underline{E_t}} = -E_{io} (\cos \theta_t \mathbf{a}_x - \sin \theta_t \mathbf{a}_z) \cos(10^9 t - \beta_2 x \sin \theta_t - \beta_2 z \cos \theta_t)$$

where

$$\underline{\underline{E_t}} = -E_{io} (\cos \theta_t \mathbf{a}_x - \sin \theta_t \mathbf{a}_z) \cos(10^9 t - \beta_1 x \sin \theta_i - \beta_1 z \cos \theta_i)$$

$$\sin \theta_t = 1, \quad \cos \theta_t = 0, \quad \beta_2 \sin \theta_t = 10/3$$

$$E_{io} \sin \theta_t = \tau_{//} E_{io} = 6(24\pi)(3)(1) = 1357.2$$

Hence,

$$\underline{\underline{E_t}} = 1357 \cos(10^9 t - 3.333x) \mathbf{a}_z \text{ V/m}$$

$$\text{Since } \Gamma = -1, \quad \theta_r = \theta_i$$

$$\underline{\underline{E_r}} = (213.3\mathbf{a}_x + 75.4\mathbf{a}_z) \cos(10^9 t - kx + k\sqrt{8}z) \text{ V/m}$$

$$\text{(f)} \quad \tan \theta_{B//} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \sqrt{\frac{\epsilon_o}{9\epsilon_o}} = 1/3 \quad \longrightarrow \quad \underline{\underline{\theta_{B//} = 18.43^\circ}}$$

Prob. 10.77

$$\text{(a)} \quad \underline{\underline{E_{i\perp}}} = 5 \cos(\omega t - 0.5\pi x - 0.866\pi z) \mathbf{a}_y$$

$$\underline{\underline{E_{i\parallel}}} = (4\mathbf{a}_x - 3\mathbf{a}_z) \cos(\omega t - 0.5\pi x - 0.866\pi z)$$

(b) Comparing $\underline{\underline{E_{i\parallel}}}$ with eq. (10.115a),

$$4\mathbf{a}_x - 3\mathbf{a}_z = (\cos \theta_i \mathbf{a}_x - \sin \theta_i \mathbf{a}_z) E_{io}$$

$$\tan \theta_i = \frac{\sin \theta_i}{\cos \theta_i} = \frac{3}{4} \quad \longrightarrow \quad \underline{\underline{\theta_i = 36.87^\circ}}$$

Prob. 10.78

$$\tan \theta_B = \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \sqrt{\frac{\epsilon_o \epsilon_r}{\epsilon_o}} = \sqrt{\epsilon_r}$$

$$\epsilon_r = \tan^2 \theta_B = \tan^2 68 = \underline{\underline{6.126}}$$

Prob. 10.79

$$(a) \quad n = \frac{c}{u} = \sqrt{\mu_r \epsilon_r} = \sqrt{2.1 \times 1} = \underline{\underline{1.45}}$$

$$(b) \quad n = \sqrt{\mu_r \epsilon_r} = \sqrt{1 \times 81} = \underline{\underline{9}}$$

$$(c) \quad n = \sqrt{\epsilon_r} = \sqrt{2.7} = \underline{\underline{1.643}}$$

Prob. 10.80

Microwave is used:

- (1) For surveying land with a piece of equipment called the *tellurometer*. This radar system can precisely measure the distance between two points.
- (2) For guidance. The guidance of missiles, the launching and homing guidance of space vehicles, and the control of ships are performed with the aid of microwaves.
- (3) In semiconductor devices. A large number of new microwave semiconductor devices have been developed for the purpose of microwave oscillator, amplification, mixing/detection, frequency multiplication, and switching. Without such achievement, the majority of today's microwave systems could not exist.

Prob. 10.81

(a) In terms of the S-parameters, the T-parameters are given by

$$T_{11} = 1/S_{21}, \quad T_{12} = -S_{22}/S_{21}, \quad T_{21} = S_{11}/S_{21}, \quad T_{22} = S_{12} - S_{11} S_{22}/S_{21}$$

$$(b) \quad T_{11} = 1/0.4 = 2.5, \quad T_{12} = -0.2/0.4,$$

$$T_{21} = 0.2/0.4, \quad T_{22} = 0.4 - 0.2 \times 0.2/0.4 = 0.3$$

Hence,

$$T = \underline{\underline{\begin{bmatrix} 2.5 & -0.5 \\ 0.5 & 0.3 \end{bmatrix}}}$$

Prob. 10.82

Since $Z_L = Z_o$, $\Gamma_L = 0$.

$$\Gamma_i = S_{11} = \underline{0.33 - j0.15}$$

$$\Gamma_g = (Z_g - Z_0) / (Z_g + Z_0) = (2 - 1) / (2 + 1) = 1/3$$

$$\begin{aligned}\Gamma_o &= S_{22} + S_{12}S_{21}\Gamma_g / (1 - S_{11}\Gamma_g) \\ &= 0.44 - j0.62 + 0.56 \times 0.56 \times (1/3) / [1 - (0.11 - j0.05)] \\ &= \underline{0.5571 - j0.6266}\end{aligned}$$

Prob. 10.83 The microwave wavelengths are of the same magnitude as the circuit components. The wavelength in air at a microwave frequency of 300 GHz, for example, is 1 mm. The physical dimension of the lumped element must be in this range to avoid interference. Also, the leads connecting the lumped element probably have much more inductance and capacitance than is needed.

Prob. 10.84

$$\lambda = c/f = \frac{3 \times 10^8}{8.4 \times 10^9} = \underline{35.71 \text{ mm}}$$

CHAPTER 12

P. E. 12.1 (a) For TE_{10} , $f_c = 3$ GHz,

$$\sqrt{1 - (f_c / f)^2} = \sqrt{1 - (3 / 15)^2} = \sqrt{0.96}, \quad \beta_o = \omega / u_o = 4\pi f / c$$

$$\beta = \frac{4\pi f}{c} \sqrt{0.96} = \frac{4\pi \times 15 \times 10^9}{3 \times 10^8} \sqrt{0.96} = \underline{\underline{615.6}} \text{ rad/m}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi \times 15 \times 10^9}{615.6} = \underline{\underline{1.531 \times 10^8}} \text{ m/s}$$

$$\eta' = \sqrt{\frac{\mu}{\epsilon}} = 60\pi, \quad \eta_{TE} = \frac{60\pi}{\sqrt{0.96}} = \underline{\underline{192.4\Omega}}$$

(b) For TM_{11} , $f_c = 3\sqrt{7.25}$ GHz, $\sqrt{1 - (f_c / f)^2} = 0.8426$

$$\beta = \frac{4\pi f}{c} (0.8426) = \frac{4\pi \times 15 \times 10^9 (0.8426)}{3 \times 10^8} = \underline{\underline{529.4}} \text{ rad/m}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi \times 15 \times 10^9}{529.4} = \underline{\underline{1.78 \times 10^8}} \text{ m/s}$$

$$\eta_{TM} = 60\pi (0.8426) = \underline{\underline{158.8\Omega}}$$

P. E. 12.2 (a) Since $E_z \neq 0$, this is a TM mode

$$E_{zs} = E_o \sin(m\pi x / a) \sin(n\pi y / b) e^{-j\beta z}$$

$$E_o = 20, \quad \frac{m\pi}{a} = 40\pi \quad \longrightarrow \quad m=2, \quad \frac{n\pi}{b} = 50\pi \quad \longrightarrow \quad n=1$$

i.e. TM₂₁ mode.

$$(b) \quad f_c = \frac{u'}{2} \sqrt{(m/a)^2 + (n/b)^2} = \frac{3 \times 10^8}{2} \sqrt{40^2 + 50^2} = 1.5\sqrt{41} \text{ GHz}$$

$$\beta = \omega \sqrt{\mu\epsilon} \sqrt{1 - (f_c / f)^2} = \frac{2\pi f}{c} \sqrt{f^2 - f_c^2} = \frac{2\pi \times 10^9}{3 \times 10^8} \sqrt{225 - 92.25} = \underline{\underline{241.3 \text{ rad/m}}}$$

(c)

$$E_{xs} = \frac{-j\beta}{h^2} (40\pi) 20 \cos 40\pi x \sin 50\pi y e^{-j\beta z}$$

$$E_{ys} = \frac{-j\beta}{h^2} (50\pi) 20 \sin 40\pi x \cos 50\pi y e^{-j\beta z}$$

$$\frac{E_y}{E_x} = \underline{\underline{1.25 \tan 40\pi x \cot 50\pi y}}$$

P. E. 12.3 If TE₁₃ mode is assumed, f_c and β remain the same.

$$\underline{\underline{f_c = 28.57 \text{ GHz}, \beta = 1718.81 \text{ rad/m}, \gamma = j\beta}}$$

$$\eta_{TE13} = \frac{377 / 2}{\sqrt{1 - (28.57 / 50)^2}} = \underline{\underline{229.69 \Omega}}$$

For $m=1, n=3$, the field components are:

$$E_z = 0$$

$$H_z = H_o \cos(\pi x / a) \cos(3\pi y / b) \cos(\omega t - \beta z)$$

$$E_x = -\frac{\omega\mu}{h^2} \left(\frac{3\pi}{b} \right) H_o \cos(\pi x / a) \sin(3\pi y / b) \sin(\omega t - \beta z)$$

$$E_y = \frac{\omega\mu}{h^2} \left(\frac{\pi}{a} \right) H_o \sin(\pi x / a) \cos(3\pi y / b) \sin(\omega t - \beta z)$$

$$H_x = -\frac{\beta}{h^2} \left(\frac{\pi}{a} \right) H_o \sin(\pi x / a) \cos(3\pi y / b) \sin(\omega t - \beta z)$$

$$H_y = -\frac{\beta}{h^2} \left(\frac{3\pi}{b} \right) H_o \cos(\pi x / a) \sin(3\pi y / b) \sin(\omega t - \beta z)$$

$$\text{Given that } H_{ox} = 2 = -\frac{\beta}{h^2} (\pi / a) H_o,$$

$$H_{oy} = -\frac{\beta}{h^2} (3\pi / b) H_o = 6a / b = 6(1.5) / 8 = 11.25$$

$$H_{oz} = H_o = -\frac{2h^2 a}{\beta\pi} = \frac{-2 \times 14.51\pi^2 \times 10^4 \times 1.5 \times 10^{-2}}{1718.81\pi} = -7.96$$

$$E_{oy} = \frac{\omega\mu}{h^2} \left(\frac{\pi}{a} \right) H_o = -\frac{2\omega\mu}{\beta} = 2\eta_{TE} = -459.4$$

$$E_{ox} = -E_{oy} \frac{3a}{b} = 459.4(4.5 / 0.8) = 2584.1$$

$$E_x = 2584.1 \cos(\pi x / a) \sin(3\pi y / b) \sin(\omega t - \beta z) \text{ V/m,}$$

$$E_y = -459.4 \sin(\pi x / a) \cos(3\pi y / b) \sin(\omega t - \beta z) \text{ V/m,}$$

$$E_z = 0,$$

$$H_y = 11.25 \cos(\pi x / a) \sin(3\pi y / b) \sin(\omega t - \beta z) \text{ A/m},$$

$$H_z = -7.96 \cos(\pi x / a) \cos(3\pi y / b) \cos(\omega t - \beta z) \text{ A/m}$$

P. E. 12.4

$$f_{c11} = \frac{u'}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = \frac{3 \times 10^8 \times 10^2}{2} \sqrt{1/8.636^2 + 1/4.318^2} = 3.883 \text{ GHz}$$

$$u_p = \frac{3 \times 10^8}{\sqrt{1 - (3.883/4)^2}} = \underline{\underline{12.5 \times 10^8}} \text{ m/s},$$

$$u_s = \frac{9 \times 10^{16}}{12.5 \times 10^8} = \underline{\underline{7.2 \times 10^7}} \text{ m/s}$$

P. E. 12.5 The dominant mode becomes TE₀₁ mode

$$f_{c01} = \frac{c}{2b} = 3.75 \text{ GHz}, \quad \eta_{TE} = 406.7\Omega$$

From Example 12.2,

$$E_x = -E_o \sin(3\pi y / b) \sin(\omega t - \beta z), \quad \text{where } E_o = \frac{\omega \mu b}{\pi} H_o.$$

$$\mathcal{P}_{ave} = \int_{x=0}^a \int_{y=0}^b \frac{|E_{xs}|^2}{2\eta} dx dy = \frac{E_o^2 ab}{4\eta}$$

Hence $E_o = 63.77 \text{ V/m}$ as in Example 12.5.

$$H_o = \frac{\pi E_o}{\omega \mu b} = \frac{\pi \times 63.77}{2\pi \times 10^{10} \times 4\pi \times 10^{-7} \times 4 \times 10^{-2}} = \underline{\underline{63.34}} \text{ mA/m}$$

P. E. 12.6 (a) For $m=1, n=0$, $f_c = u'/(2a)$

$$\frac{\sigma}{\omega \epsilon} = \frac{10^{-15}}{2\pi \times 9 \times 10^9 \times 2.6 \times 10^{-9} / (36\pi)} = \frac{10^{-15}}{1.3} \ll 1$$

Hence,

$$u' \cong \frac{1}{\sqrt{\mu\epsilon}} = c / \sqrt{2.6}, \quad f_c = \frac{3 \times 10^8}{2 \times 2.4 \times 10^{-2} \sqrt{2.6}} = 2.2149 \text{ GHz}$$

$$\alpha_d = \frac{\sigma \eta'}{2 \sqrt{1 - (f_c / f)^2}} = \frac{10^{-15} \times 377 / \sqrt{2.6}}{2 \sqrt{1 - (2.2149 / 9)^2}} = 1.205 \times 10^{-13} \text{ Np/m}$$

For $n = 0, m = 1$,

$$\begin{aligned} \alpha_c &= \frac{2R_s}{b\eta' \sqrt{1 - (f_c / f)^2}} \left[\frac{1}{2} + \frac{b}{a} (f_c / f)^2 \right] \\ &= \frac{2\sqrt{2.6} \sqrt{\pi \times 9 \times 10^9 \times 1.1 \times 10^7 \times 4\pi \times 10^{-7}}}{377 \times 1.5 \times 10^{-2} \times 1.1 \times 10^7 \sqrt{1 - (2.2149 / 9)^2}} [0.5 + (2.4 / 1.5)(2.2148 / 9)^2] = \underline{\underline{2 \times 10^{-2} \text{ Np/m}}} \end{aligned}$$

(b) Since $\alpha_c \gg \alpha_d, \alpha = \alpha_c + \alpha_d \cong \alpha_c = 2 \times 10^{-2}$

$$\text{loss} = \alpha l = 2 \times 10^{-2} \times 0.4 = 0.8 \times 10^{-2} \text{ Np} = \underline{\underline{0.06945 \text{ dB}}}$$

P. E. 12.7 For TE_{11} , $m = 1 = n$,

$$H_{zs} = H_o \cos(\pi x / a) \cos(\pi y / b) e^{-\gamma z}$$

$$E_{xs} = \frac{j\omega}{h^2} (\pi / b) H_o \cos(\pi x / a) \sin(\pi y / b) e^{-\gamma z}$$

$$E_{ys} = -\frac{j\omega\mu}{h^2} (\pi / a) H_o \sin(\pi x / a) \cos(\pi y / b) e^{-\gamma z}$$

$$H_{xs} = \frac{j\beta}{h^2} (\pi / a) H_o \sin(\pi x / a) \cos(\pi y / b) e^{-\gamma z}$$

$$H_{ys} = \frac{j\beta}{h^2} (\pi / b) H_o \cos(\pi x / a) \sin(\pi y / b) e^{-\gamma z}$$

$$E_{zs} = 0$$

For the electric field lines,

$$\frac{dy}{dx} = \frac{E_y}{E_x} = (a / b) \tan(\pi x / a) \cot(\pi y / b)$$

For the magnetic field lines

$$\frac{dy}{dx} = \frac{H_y}{H_x} = -(a/b) \cot(\pi x/a) \tan(\pi y/b)$$

Notice that $\left(\frac{E_y}{E_x}\right)\left(\frac{H_y}{H_x}\right) = -1$

showing that the electric and magnetic field lines are mutually orthogonal. The field lines are as shown in Fig. 12.14.

P. E. 12.8

$$u' = \frac{l}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}}$$

$$f_{TE101} = \frac{1.5 \times 10^{10}}{\sqrt{3}} \sqrt{1/25 + 0 + 1/100} = \underline{\underline{1.936 \text{ GHz}}}$$

$$Q_{TE101} = \frac{1}{61\delta}, \text{ where}$$

$$\delta = \frac{1}{\sqrt{\pi f_{101} \mu \sigma_c}} = \frac{1}{\sqrt{\pi \times 1.936 \times 10^9 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7}} = 1.5 \times 10^{-6}$$

$$Q_{TE101} = \frac{10^6}{61 \times 1.5} = \underline{\underline{10,929}}$$

P. E. 12.9

(a) By Snell's law, $n_1 \sin \theta_1 = n_2 \sin \theta_2$. Thus
 $\theta_2 = 90^\circ \longrightarrow \sin \theta_2 = 1$

$$\sin \theta_1 = n_2/n_1, \quad \theta_1 = \sin^{-1} n_2/n_1 = \sin^{-1} 1.465/1.48 = \underline{\underline{81.83^\circ}}$$

$$(b) \text{ NA} = \sqrt{n_1^2 - n_2^2} = \sqrt{1.48^2 - 1.465^2} = \underline{\underline{0.21}}$$

P. E. 12.10

$$\alpha_1 = 10 \log P(0)/P(1) = 0.2 \times 10 = 2$$

$$P(0)/P(1) = 10^{0.2}, \text{ i.e. } P(1) = P(0) 10^{-0.2} = 0.631 P(0)$$

$$\text{i.e. } \underline{\underline{63.1 \%}}$$

Prob. 12.1

(a) For TE_{10} mode, $f_c = \frac{u'}{2a} = \frac{3 \times 10^8}{2 \times 6 \times 10^{-2}} = \underline{\underline{2.5 \text{ GHz}}}$

(b) $f = 3f_c = 7.5 \text{ GHz}$

$$f_{cmn} = \frac{u'}{2} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}} = \frac{3 \times 10^8 \times 10^2}{2} \sqrt{\left(\frac{m}{6}\right)^2 + \left(\frac{n}{4}\right)^2}$$

$$= 15 \sqrt{\left(\frac{m}{6}\right)^2 + \left(\frac{n}{4}\right)^2} \text{ GHz}$$

$$f_{c20} = 15 \times \frac{2}{6} = 5 \text{ GHz}$$

$$f_{c01} = 3.75 \text{ GHz}, \quad f_{c02} = 7.5 \text{ GHz}$$

$$f_{c10} = 2.5 \text{ GHz}, \quad f_{c20} = 5.0 \text{ GHz}$$

$$f_{c21} = 6.25 \text{ GHz}, \quad f_{c30} = 7.5 \text{ GHz}$$

$$f_{12} = 15 \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{2}{4}\right)^2} = 7.91 \text{ GHz}$$

$$f_{11} = 15 \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{4}\right)^2} = 4.507 \text{ GHz}$$

The following modes are transmitted

$TE_{01}, TE_{02}, TE_{10}, TE_{11}, TE_{20}, TE_{21}, TE_{30}$

TM_{11}, TM_{21}

i.e. 7 TE modes and 2 TM modes

Prob. 12.2

$$f_{c10} = \frac{u'}{2a}, f_{c11} = \frac{u'}{2} \sqrt{\frac{1}{a^2} + \frac{1}{a^2}} = \frac{u'}{2a} \sqrt{2}$$

Since the guide can only propagate TE_{10} mode,

$$f_{c10} < f < f_{c11} \rightarrow \frac{u'}{2a} < f < \frac{u'}{2a} \sqrt{2} \rightarrow u' < 2af < u' \sqrt{2}$$

$$\frac{u'}{f} < 2a < \frac{u'}{f} \sqrt{2} \rightarrow \lambda < 2a < \lambda \sqrt{2}$$

$$\underline{\underline{\frac{\lambda}{2} < a < \frac{\lambda}{\sqrt{2}}}}}$$

Prob. 12.3

$$f_c = \frac{u'}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3 \times 10^8}{2 \sqrt{2.25} \times 10^{-2}} \left[\left(\frac{m}{2.28}\right)^2 + \left(\frac{n}{1.01}\right)^2 \right]^{1/2}$$

$$= \frac{15}{\sqrt{2.25}} \left[\left(\frac{m}{2.28}\right)^2 + \left(\frac{n}{1.01}\right)^2 \right]^{1/2} \text{ GHz}$$

Using this formula, we obtain the cutoff frequencies for the given modes as shown below.

Mode	f_c (GHz)
TE ₀₁	9.901
TE ₁₀	4.386
TE ₁₁	10.829
TE ₀₂	19.802
TE ₂₂	21.658
TM ₁₁	10.829
TM ₁₂	20.282
TM ₂₁	13.228

Prob. 12.4

(a)

For TE₁₀ mode,

$$f_c = \frac{u'}{2} \sqrt{\left(\frac{1}{a}\right)^2} = \frac{3 \times 10^8}{2 \times 2.4 \times 10^{-2}} = \underline{\underline{6.25 \text{ GHz}}}$$

For TE₀₁ mode,

$$f_c = \frac{u'}{2} \sqrt{\left(\frac{1}{b}\right)^2} = \frac{3 \times 10^8}{2 \times 1.2 \times 10^{-2}} = \underline{\underline{12.5 \text{ GHz}}}$$

For TE₂₀ mode,

$$f_c = \frac{u'}{2} \sqrt{\left(\frac{2}{a}\right)^2} = 2 \times 6.25 = \underline{\underline{12.5 \text{ GHz}}}$$

For TE₀₂ mode,

$$f_c = \frac{u'}{2} \sqrt{\left(\frac{2}{b}\right)^2} = 2 \times 12.5 = \underline{\underline{25 \text{ GHz}}}$$

(b) Since $f = 12 \text{ GHz}$, only TE₁₀ mode will propagate.

Prob. 12.5 $a/b = 3 \longrightarrow a = 3b$

$$f_{c10} = \frac{u'}{2a} \longrightarrow a = \frac{u'}{2f_{c10}} = \frac{3 \times 10^8}{2 \times 18 \times 10^9} \text{ m} = 0.833 \text{ cm}$$

A design could be $a = 9 \text{ mm}$, $b = 3 \text{ mm}$.

Prob. 12.6 For the dominant mode,

$$f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 8} = 18.75 \text{ MHz}$$

(a) It will not pass the AM signal, (b) it will pass the FM signal.

Prob. 12.7 (a) For TE_{10} mode, $f_c = \frac{u'}{2a}$

$$\text{Or } a = \frac{u'}{2f_c} = \frac{3 \times 10^8}{2 \times 5 \times 10^9} = \underline{\underline{3 \text{ cm}}}$$

For TE_{01} mode, $f_c = \frac{u'}{2b}$

$$\text{Or } b = \frac{u'}{2f_c} = \frac{3 \times 10^8}{2 \times 12 \times 10^9} = \underline{\underline{1.25 \text{ cm}}}$$

(b) Since $a > b$, $1/a < 1/b$, the next higher modes are calculated as shown below.

Mode	f_c (GHz)
TE_{10}	5
* TE_{20}	10
TE_{30}	15
TE_{40}	20
* TE_{01}	12
TE_{02}	24
* TE_{11}	13
TE_{21}	15.62

The next three higher modes are starred ones, i.e. TE_{20} , TE_{01} , TE_{11}

$$(c) \ u' = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{2.25}} = 2 \times 10^8 \text{ m/s}$$

For TE_{11} modes,

$$f_c = \frac{3 \times 10^8}{2 \times 10^{-2} \sqrt{2.25}} \sqrt{\frac{1}{3^2} + \frac{1}{1.25^2}} = \underline{\underline{8.67 \text{ GHz}}}$$

Prob. 12.8

$$\text{Let } F_{12} = \sqrt{1 - \left(\frac{f_{c12}}{f}\right)^2} = \sqrt{1 - \left(\frac{25}{40}\right)^2} = 0.7806$$

$$\lambda' = \frac{c}{f} = \frac{3 \times 10^8}{40 \times 10^9} = 0.0075 \text{ m} = 7.5 \times 10^{-3} \text{ m}$$

$$\lambda_{12} = \frac{\lambda'}{F_{12}} = \frac{7.5 \times 10^{-3} \text{ m}}{0.7806} = \underline{\underline{9.608 \times 10^{-3} \text{ m}}}$$

$$u_{12} = \frac{u'}{F_{12}} = \frac{3 \times 10^8}{0.7806} = \underline{\underline{3.843 \times 10^8 \text{ m/s}}}$$

$$\beta_{12} = \frac{2\pi}{\lambda_{12}} = \frac{2\pi}{9.608 \times 10^{-3}} = \underline{\underline{653.95 \text{ rad/m}}}$$

$$\eta_{TE12} = \frac{\eta'}{F_{12}} = \frac{120\pi}{0.7806} = \underline{\underline{482.95 \Omega}}$$

Prob. 12.9

$$f_c = \frac{u'}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

For TE₁₀ mode, m=1, n=0,

$$f_c = \frac{u'}{2a} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 5 \times 10^{-2}} = 3 \text{ GHz}$$

$$\beta = \omega \sqrt{\mu \epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 2\pi \times 12.5 \times 10^9 \sqrt{4\pi \times 10^{-7} \times \frac{10^{-9}}{36\pi}} \sqrt{1 - \left(\frac{3}{12.5}\right)^2} = \frac{785.4}{3} (0.9708)$$

$$\underline{\underline{\beta = 254.15 \text{ rad/m}}}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi \times 12.5 \times 10^9}{254.15} = \underline{\underline{3.09 \times 10^8 \text{ m/s}}}$$

$$\eta_{TE} = \frac{\eta'}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{120\pi}{0.9708} = \underline{\underline{388.3 \Omega}}$$

Prob. 12.10

$$f_c = \frac{u'}{2a} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 4 \times 10^{-2}} = \underline{\underline{3.75 \text{ GHz}}}$$

$$\beta = \omega \sqrt{\mu \epsilon} \sqrt{1 - \left(\frac{f_c}{f} \right)^2} = \frac{2\pi \times 24 \times 10^9}{3 \times 10^8} \sqrt{1 - \left(\frac{3.75}{24} \right)^2} = \frac{480\pi}{3} (0.9877)$$

$$\underline{\underline{\beta = 496.48 \text{ rad/m}}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{496.48} = \underline{\underline{0.0127 \text{ m}}}$$

Prob. 12.11

$$u = \frac{\omega}{\beta} = \frac{u'}{\sqrt{1 - (f_c / f)^2}} = \frac{3 \times 10^8}{\sqrt{1 - (6.5 / 7.2)^2}} = 6.975 \times 10^8 \text{ m/s}$$

$$u_g = \frac{9 \times 10^{16}}{u} = 1.2903 \times 10^8 \text{ m/s}$$

$$t = \frac{2l}{u_g} = \frac{300}{1.2903 \times 10^8} = \underline{\underline{2.325 \mu\text{s}}}$$

Prob. 12.12

$$f_c = \frac{u'}{2a} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 2.4 \times 10^{-2}} = \underline{\underline{6.25 \text{ GHz}}}$$

$$f = 1.25 f_c = \underline{\underline{7.813 \text{ GHz}}}$$

$$\eta = \frac{\eta'}{\sqrt{1 - \left(\frac{f_c}{f} \right)^2}} = \frac{377}{\sqrt{1 - \left(\frac{1}{1.25} \right)^2}} = \frac{377}{0.6} = \underline{\underline{628.32 \Omega}}$$

Prob. 12.13

$$\begin{aligned} (a) f_{c10} &= \frac{u'}{2a} = \frac{c}{2a\sqrt{\epsilon_r}} \\ &= \frac{3 \times 10^8}{2 \times 1.067 \times 10^{-2} \sqrt{6.8}} \\ &= \frac{30}{2 \times 1.067 \sqrt{6.8}} \text{ GHz} \\ &= \underline{\underline{5.391 \text{ GHz}}} \end{aligned}$$

(b)

$$\begin{aligned}
 F &= \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \\
 &= \sqrt{1 - \left(\frac{5.391}{6}\right)^2} \\
 &= 0.439
 \end{aligned}$$

$$u_\rho = \frac{u'}{F} = \frac{c}{F\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{0.439 \times \sqrt{6.8}} = \underline{\underline{2.62 \times 10^8 \text{ m/s}}}$$

(c)

$$\begin{aligned}
 \lambda &= \frac{\lambda'}{F} = \frac{u'/f}{F} = \frac{c}{fF\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{0.439 \times 6 \times 10^9 \times \sqrt{6.8}} \\
 &= \frac{10^{-1}}{2 \times 0.439 \sqrt{6.8}} = 0.04368 \text{ m} = \underline{\underline{4.368 \text{ cm}}}
 \end{aligned}$$

Prob.12.14

In evanescent mode,

$$\begin{aligned}
 k^2 &= \omega^2 \mu \epsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \\
 \beta &= 0, \quad \gamma = \alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2} = \sqrt{4\pi^2 \mu \epsilon f_c^2 - \omega^2 \mu \epsilon} \\
 \alpha &= \sqrt{\mu \epsilon} \sqrt{4\pi^2 f_c^2 - 4\pi^2 f^2} = \underline{\underline{2\pi \sqrt{\mu \epsilon} f_c \sqrt{1 - \left(\frac{f}{f_c}\right)^2}}}
 \end{aligned}$$

Prob. 12.15 $E_z \neq 0$. This must be TM₂₃ mode (m=2, n=3). Since a=2b,

$$f_c = \frac{c}{4b} \sqrt{m^2 + 4n^2} = \frac{3 \times 10^8}{4 \times 3 \times 10^{-2}} \sqrt{4 + 36} = 15.81 \text{ GHz}, \quad f = \frac{\omega}{2\pi} = \frac{10^{12}}{2\pi} = 159.2 \text{ GHz}$$

$$\eta_{TM} = 377 \sqrt{1 - (15.81/159.2)^2} = \underline{\underline{375.1 \Omega}}$$

$$\mathcal{P}_{\text{ave}} = \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\eta_{TM}} \mathbf{a}_z$$

$$= \frac{\beta^2 E_o^2}{2h^4 \eta_{TM}} \left[(2\pi/a)^2 \cos^2(2\pi x/a) \sin^2(3\pi y/b) + (3\pi/b)^2 \sin^2(2\pi x/a) \cos^2(3\pi y/b) \right] \mathbf{a}_z$$

$$P_{ave} = \int \mathcal{P}_{ave} \cdot d\mathbf{S} = \int_{x=0}^a \int_{y=0}^b \mathcal{P}_{ave} \cdot dxdy \mathbf{a}_z$$

$$= \frac{\beta^2 E_o^2}{2h^4 \eta_{TM}} \frac{ab}{4} \left[\frac{4\pi^2}{a^2} + \frac{9\pi^2}{b^2} \right] = \frac{\beta^2 E_o^2 ab}{8h^2 \eta_{TM}}$$

But

$$\beta = \frac{\omega}{c} \sqrt{1 - (f_c/f)^2} = \frac{10^{12}}{3 \times 10^8} \sqrt{1 - (15.81/159.2)^2} = 3.317 \times 10^3$$

$$h^2 = \frac{4\pi^2}{a^2} + \frac{9\pi^2}{b^2} = \frac{10\pi^2}{b^2} = 1.097 \times 10^5$$

$$P_{ave} = \frac{(3.317)^2 \times 10^6 \times 5^2 \times 18 \times 10^{-4}}{8 \times (1.098 \times 10^5) \times 375.1} = \underline{\underline{1.5 \text{ mW}}}$$

Prob. 12.16 (a) Since $m=2$ and $n=1$, we have TE₂₁ mode

$$(b) \beta = \beta' \sqrt{1 - (f_c/f)^2} = \omega \sqrt{\mu_o \epsilon_o} \sqrt{1 - (\omega_c/\omega)^2}$$

$$\beta c = \sqrt{\omega^2 - \omega_c^2} \longrightarrow \omega_c^2 = \sqrt{\omega^2 - \beta^2 c^2}$$

$$f_c = \frac{\omega_c}{2\pi} = \sqrt{f^2 - \frac{\beta^2 c^2}{4\pi^2}} = \sqrt{36 \times 10^{18} - \frac{144 \times 9 \times 10^{16}}{4\pi^2}} = \underline{\underline{5.973 \text{ GHz}}}$$

$$(c) \eta_{TE} = \frac{\eta}{\sqrt{1 - (f_c/f)^2}} = \frac{377}{\sqrt{1 - (5.973/6)^2}} = \underline{\underline{3978 \Omega}}$$

(d) For TE mode,

$$E_y = \frac{\omega \mu}{h^2} (m\pi/a) H_o \sin(m\pi x/a) \cos(n\pi y/b) \sin(\omega t - \beta z)$$

$$H_x = \frac{-\beta}{h^2} (m\pi/a) H_o \sin(m\pi x/a) \cos(n\pi y/b) \sin(\omega t - \beta z)$$

$$\beta = 12, m = 2, n = 1$$

$$E_{oy} = \frac{\omega \mu}{h^2} (m\pi / a) H_o, \quad H_{ox} = \frac{\beta}{h^2} (m\pi / a) H_o$$

$$\eta_{TE} = \frac{E_{oy}}{H_{ox}} = \frac{\omega \mu}{\beta} = \frac{2\pi \times 6 \times 10^9 \times 4\pi \times 10^{-7}}{12} = 4\pi^2 \times 100$$

$$H_{ox} = \frac{E_{oy}}{\eta_{TE}} = \frac{5}{4\pi^2 \times 100} = 1.267 \text{ mA/m}$$

$$\underline{H_x = -1.267 \sin(m\pi x / a) \cos(n\pi y / b) \sin(\omega t - \beta z) \text{ mA/m}}$$

Prob. 12.17 (a) Since $m=2, n=3$, the mode is TE₂₃.

$$(b) \quad \beta = \beta' \sqrt{1 - (f_c / f)^2} = \frac{2\pi f}{c} \sqrt{1 - (f_c / f)^2}$$

But

$$f_c = \frac{u'}{2} \sqrt{(m/a)^2 + (n/b)^2} = \frac{3 \times 10^8}{2 \times 10^{-2}} \sqrt{(2/2.86)^2 + (3/1.016)^2} = 46.19 \text{ GHz}, \quad f = 50 \text{ GHz}$$

$$\beta = \frac{2\pi \times 50 \times 10^9}{3 \times 10^8} \sqrt{1 - (46.19/50)^2} = 400.68 \text{ rad/m}$$

$$\gamma = j\beta = \underline{j400.7 \text{ /m}}$$

$$(c) \quad \eta = \frac{\eta'}{\sqrt{1 - (f_c / f)^2}} = \frac{377}{\sqrt{1 - (46.19/50)^2}} = \underline{985.3 \Omega}$$

Prob. 12.18 In free space,

$$\eta_l = \frac{\eta_o}{\sqrt{1 - (f_c / f)^2}}, \quad f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 5 \times 10^{-2}} = 3 \text{ GHz}$$

$$\eta_l = \frac{377}{\sqrt{1 - (3/8)^2}} = 406.7 \Omega$$

$$\eta_2 = \frac{\eta'_l}{\sqrt{1 - (f_c / f)^2}}, \quad \eta'_l = \frac{120\pi}{\sqrt{2.25}} = 80\pi, \quad f_c = \frac{u'}{2a}, \quad u' = \frac{c}{\sqrt{\epsilon_r}}$$

$$f_c = \frac{3 \times 10^8}{2 \times 5 \times 10^{-2} \sqrt{2.25}} = 2 \text{ GHz}, \quad \eta_2 = \frac{80\pi}{\sqrt{1 - (2/8)^2}} = 259.57\Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.2208$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \underline{\underline{1.5667}}$$

Prob. 12.19 Substituting $E_z = R\Phi Z$ into the wave equation,

$$\frac{\Phi Z}{\rho} \frac{d}{d\rho}(\rho R') + \frac{RZ}{\rho^2} \Phi'' + R\Phi Z'' + k^2 R\Phi Z = 0$$

Dividing by $R\Phi Z$,

$$\frac{1}{R\rho} \frac{d}{d\rho}(\rho R') + \frac{\Phi''}{\Phi\rho^2} + k^2 = -\frac{Z''}{Z} = -k_z^2$$

$$\text{i.e.} \quad \underline{\underline{Z'' - k_z^2 Z = 0}}$$

$$\frac{1}{R\rho} \frac{d}{d\rho}(\rho R') + \frac{\Phi''}{\Phi\rho^2} + (k^2 + k_z^2) = 0$$

$$\frac{\rho}{R} \frac{d}{d\rho}(\rho R') + (k^2 + k_z^2)\rho^2 = -\frac{\Phi''}{\Phi} = k_\phi^2$$

or

$$\underline{\underline{\Phi'' + k_\phi^2 \Phi = 0}}$$

$$\rho \frac{d}{d\rho}(\rho R') + (k_\rho^2 \rho^2 - k_\phi^2)R = 0, \text{ where } k_\rho^2 = k^2 + k_z^2. \text{ Hence}$$

$$\underline{\underline{\rho^2 R'' + \rho R' + (k_\rho^2 \rho^2 - k_\phi^2)R = 0}}$$

Prob. 12.20

The MATLAB code and the plot of the phase and group velocities are presented below.

```
% Plot U_p and U_g versus frequency f (10<f<100) in GHz
```

```
c=3*10^8;
```

```
for k=1:91
```

```
    f(k)=9+k
```

```
    fac = sqrt( 1 - (8/f(k))^2 );
```

```
    up(k) = c/fac;
```

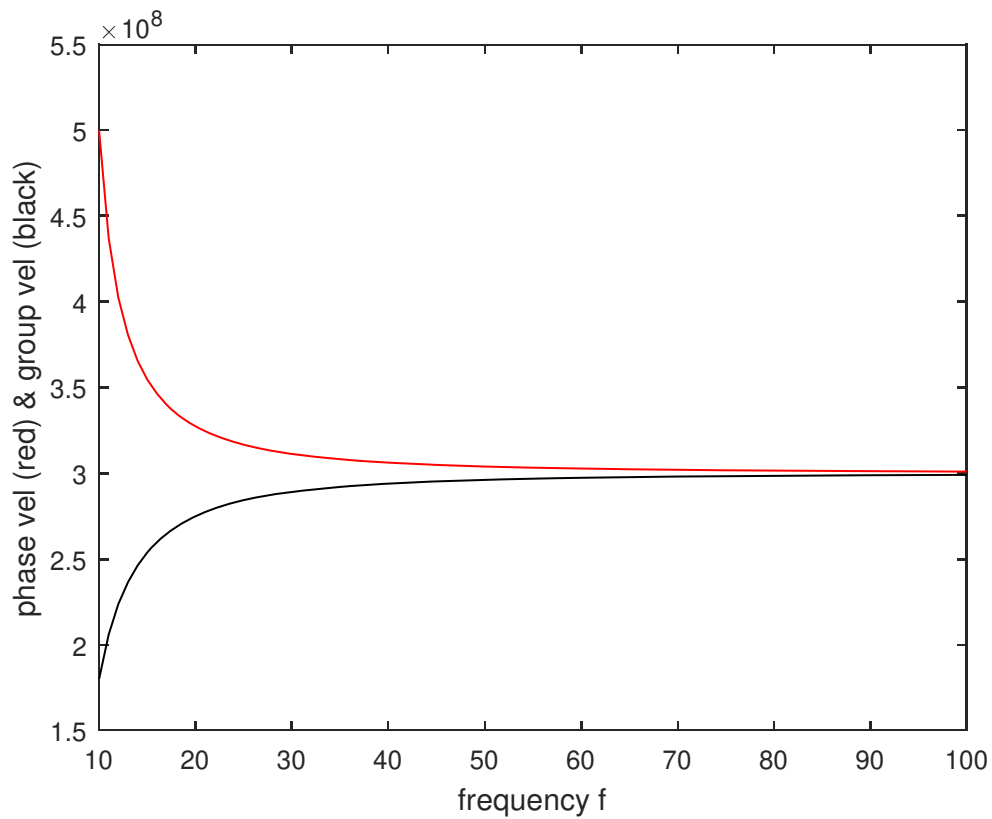
```
    ug(k) = c*fac;
```

```
end
```

```
plot(f,up, 'r', f, ug, 'k')
```

```
xlabel('frequency f')
```

```
ylabel('phase vel (red) & group vel (black)')
```



Prob. 12.21

(a)

For TE_{10} mode,

$$f_c = \frac{u'}{2a} = \frac{3 \times 10^8}{2 \times 7.2 \times 10^{-2}} = 2.083 \text{ GHz}$$

$$\text{Let } F = \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \sqrt{1 - \left(\frac{2.083}{6.2}\right)^2} = 0.942$$

$$\beta = \omega \sqrt{\mu \epsilon} F = \frac{\omega F}{c} = \frac{2\pi \times 6.2 \times 10^9 \times 0.942}{3 \times 10^8} = \underline{\underline{122.32 \text{ rad/m}}}$$

$$u_p = \frac{\omega}{\beta} = \frac{c}{F} = \frac{3 \times 10^8}{0.942} = \underline{\underline{3.185 \times 10^8 \text{ m/s}}}$$

$$u_g = u' F = 3 \times 10^8 (0.942) = \underline{\underline{2.826 \times 10^8 \text{ m/s}}}$$

$$\eta_{TE} = \frac{\eta'}{F} = \frac{377}{0.942} = \underline{\underline{400.21 \Omega}}$$

(b)

$$u' = \frac{1}{\sqrt{\mu \epsilon}} = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8$$

$$f_c = \frac{u'}{2a} = \frac{2 \times 10^8}{2 \times 7.2 \times 10^{-2}} = 1.389 \text{ GHz}$$

$$\text{Let } F = \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \sqrt{1 - \left(\frac{1.389}{6.2}\right)^2} = 0.9746$$

$$\beta = \omega \sqrt{\mu \epsilon} F = \frac{\omega F \sqrt{\epsilon_r}}{c} = \frac{2\pi \times 6.2 \times 10^9 \times 0.9746 \times 1.5}{3 \times 10^8} = \underline{\underline{189.83 \text{ rad/m}}}$$

$$u_p = \frac{\omega}{\beta} = \frac{2\pi \times 6.2 \times 10^9}{189.83} = \underline{\underline{2.052 \times 10^8 \text{ m/s}}}$$

$$u_g = u' F = 2 \times 10^8 (0.9746) = \underline{\underline{1.949 \times 10^8 \text{ m/s}}}$$

$$\eta_{TE} = \frac{\eta'}{F} = \frac{377}{1.5 \times 0.9746} = \underline{\underline{257.88 \Omega}}$$

Prob. 12.22

$$f_{c10} = \frac{u'}{2a} = \frac{1}{2a\sqrt{\mu\epsilon}} = \frac{c}{2a\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{2 \times 7.214 \times 10^{-2} \sqrt{2.5}} = 1.315 \text{ GHz}$$

$$\text{Let } F = \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \sqrt{1 - \left(\frac{1.315}{4}\right)^2} = 0.9444$$

$$\beta = \omega\sqrt{\mu\epsilon}F = \frac{\omega\sqrt{\epsilon_r}F}{c}$$

$$u_p = \frac{\omega}{\beta} = \frac{c}{F\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{0.9444 \times \sqrt{2.5}} = \underline{\underline{2.009 \times 10^8 \text{ m/s}}}$$

$$u_g = u'F = \frac{cF}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8 \times 0.9444}{\sqrt{2.5}} = \underline{\underline{1.792 \times 10^8 \text{ m/s}}}$$

Prob. 12.23

$$u_p = \frac{\omega}{\beta} = \frac{\omega\lambda}{2\pi} \rightarrow \omega = \beta u_p = \beta c \frac{\lambda_o^2}{\lambda^2} = \frac{c\lambda_o^2\beta^3}{4\pi^2}, \quad (\lambda = 2\pi / \beta)$$

$$u_g = \frac{d\omega}{d\beta} = 3 \left(\frac{c\lambda_o^2}{4\pi^2} \right) \beta^2 = 3c \left(\frac{\lambda_o}{\lambda} \right)^2 = \underline{\underline{3u_p}}$$

Prob. 12.24

$$u_g = \frac{1}{4}c = u' \sqrt{1 - \left(\frac{f_c}{f_1}\right)^2} \quad (1)$$

$$u_g = \frac{1}{3}c = u' \sqrt{1 - \left(\frac{f_c}{f_2}\right)^2} \quad (2)$$

Dividing (1) by (2),

$$\frac{1/4}{1/3} = \frac{\sqrt{1 - \left(\frac{f_c}{f_1}\right)^2}}{\sqrt{1 - \left(\frac{f_c}{f_2}\right)^2}} \rightarrow \left(\frac{3}{4}\right)^2 = 0.5625 = \frac{1 - \left(\frac{f_c}{f_1}\right)^2}{1 - \left(\frac{f_c}{f_2}\right)^2}$$

$$1 - \left(\frac{f_c}{f_1}\right)^2 = 0.5625 \left[1 - \left(\frac{f_c}{f_2}\right)^2 \right]$$

Assuming f_c is in GHz,

$$1 - \frac{f_c^2}{144} = 0.5625 - \frac{0.5625 f_c^2}{225} \rightarrow f_c^2 = 98.44 \rightarrow \underline{\underline{f_c = 9.9216 \text{ GHz}}}$$

From (1),

$$u' = \frac{0.25c}{\sqrt{1 - \left(\frac{f_c}{f_1}\right)^2}} = \frac{0.25c}{\sqrt{1 - \left(\frac{9.9216}{12}\right)^2}} = 0.4444c$$

$$\text{But } u' = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}}$$

$$\frac{c}{\sqrt{\epsilon_r}} = 0.4444c \rightarrow \epsilon_r = \left(\frac{1}{0.4444}\right)^2 = \underline{\underline{5.0625}}$$

Prob. 12.25

$$f_c = \frac{u'}{2a}$$

$$u_g = u' \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \longrightarrow \left(\frac{f_c}{f}\right)^2 = 1 - \left(\frac{u_g}{u'}\right)^2 = 1 - \left(\frac{1.8 \times 10^8}{\frac{3 \times 10^8}{\sqrt{2.2}}}\right)^2 = 0.208$$

$$f_c = \sqrt{0.208} f = 2.0523 \text{ GHz}$$

$$a = \frac{u'}{2f_c} = \frac{3 \times 10^8}{2\sqrt{2.2} \times 2.053 \times 10^9} = \underline{\underline{4.927 \text{ cm}}}$$

Prob. 12.26

$$\text{Let } F = \sqrt{1 - (f_c / f)^2} = \sqrt{1 - (16 / 24)^2} = 0.7453$$

$$u' = \frac{1}{\sqrt{\mu\epsilon}} = \frac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8, \quad u_p = \frac{u'}{F}, \quad u_g = u' F = 2 \times 10^8 \times 0.7453 = \underline{\underline{1.491 \times 10^8}} \\ \text{m/s}$$

$$\eta_{TE} = \eta' / F = \frac{377}{1.5 \times 0.7453} = \underline{\underline{337.2 \Omega}}$$

Prob. 12.27

For the TE_{10} mode,

$$H_{zs} = H_o \cos\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

$$H_{xs} = \frac{j\beta a}{\pi} H_o \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

$$E_{ys} = -\frac{j\omega\mu a}{\pi} H_o \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

$$E_{xs} = 0 = E_{zs} = H_{ys}$$

$$\begin{aligned} \mathbf{E}_s \times \mathbf{H}_s^* &= \begin{vmatrix} 0 & E_{ys} & 0 \\ H_{xs}^* & 0 & H_{zs}^* \end{vmatrix} = E_{ys} H_{zs}^* \mathbf{a}_x - E_{ys} H_{xs}^* \mathbf{a}_z \\ &= -\frac{j\omega\mu a}{\pi} H_o^2 \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi x}{a}\right) \mathbf{a}_x + \frac{\omega\mu\beta a^2}{\pi^2} H_o^2 \sin^2\left(\frac{\pi x}{a}\right) \mathbf{a}_z \\ \mathcal{P}_{ave} &= \frac{1}{2} \text{Re}[\mathbf{E}_s \times \mathbf{H}_s^*] = \underline{\underline{\frac{\omega\mu\beta a^2}{2\pi^2} H_o^2 \sin^2\left(\frac{\pi x}{a}\right) \mathbf{a}_z}} \end{aligned}$$

Prob. 12.28

$$\mathbf{P}_{ave} = \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\eta} \mathbf{a}_z = \underline{\underline{\frac{\omega^2 \mu^2 \pi^2}{2\eta b^2 h^4} H_o^2 \sin^2 \pi y / b \mathbf{a}_z}}$$

where $\eta = \eta_{TE10}$.

$$\mathbf{P}_{ave} = \int \mathbf{P}_{ave} dS = \frac{\omega^2 \mu^2 \pi^2}{2\eta b^2 h^4} H_o^2 \int_{x=0}^a \int_{y=0}^b \sin^2 \pi y / b dx dy$$

$$P_{\text{ave}} = \frac{\omega^2 \mu^2 \pi^2}{2\eta b^2 h^4} H_o^2 ab / 2$$

But $h^2 = (m\pi / a)^2 + (n\pi / b)^2 = \frac{\pi^2}{b^2}$,

$$P_{\text{ave}} = \frac{\omega^2 \mu^2 ab^3 H_o^2}{4\pi^2 \eta}$$

Prob. 12.29

$$R_s = \sqrt{\frac{\pi \mu f}{\sigma_c}} = \sqrt{\frac{\pi \times 12 \times 10^9 \times 4\pi \times 10^{-7}}{5.8 \times 10^7}} = 2.858 \times 10^{-2}$$

$$f_{c10} = \frac{u'}{2a} = \frac{3 \times 10^8}{2\sqrt{2.6} \times 2 \times 10^{-2}} = 4.651 \text{ GHz}$$

$$f_{c11} = \frac{u'}{2} \left[\frac{1}{a^2} + \frac{1}{b^2} \right]^{1/2} = 10.4 \text{ GHz}$$

$$\eta' = \sqrt{\frac{\mu}{\epsilon}} = \frac{377}{\sqrt{2.6}} = 233.81 \Omega$$

(a) For TE₁₀ mode, eq.(12.57) gives

$$\begin{aligned} \alpha_d + j\beta_d &= \sqrt{-\omega^2 \mu \epsilon + k_x^2 + k_y^2 + j\omega \mu \sigma_d} \\ &= \sqrt{-\omega^2 / u^2 + \frac{\pi^2}{a^2} + j\omega \mu \sigma_d} \\ &= \sqrt{-\left(\frac{2\pi \times 12 \times 10^9}{3 \times 10^8} \right)^2 (2.6) + \frac{\pi^2}{(2 \times 10^{-2})^2} + j2\pi \times 12 \times 10^9 \times 4\pi \times 10^{-7} \times 10^{-4}} \\ &= 0.012682 + j373.57 \end{aligned}$$

$$\underline{\alpha_d = 0.012682 \text{ Np/m}}$$

$$\alpha_c = \frac{2R_s}{b\eta' \sqrt{1 - (f_c / f)^2}} \left[\frac{1}{2} + \frac{b}{a} \left(\frac{f_c}{f} \right)^2 \right]$$

$$= \frac{2 \times 2.858 \times 10^{-2}}{10^{-2}(233.81)\sqrt{1-(4.651/12)^2}} \left[\frac{1}{2} + \frac{1}{2} \left(\frac{4.651}{12} \right)^2 \right] = \underline{\underline{0.0153}} \text{ Np/m}$$

(b) For TE₁₁ mode,

$$\begin{aligned} \alpha_d + j\beta_d &= \sqrt{-\omega^2 / u^2 + 1/a^2 + 1/b^2 + j\omega\mu\sigma_d} \\ &= \sqrt{-139556.21 + \frac{\pi^2}{(10^{-2})^2} + j9.4748} = 0.02344 + j202.14 \end{aligned}$$

$$\underline{\underline{\alpha_d = 0.02344 \text{ Np/m}}}$$

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1-(f_c/f)^2}} \left[\frac{(b/a)^3 + 1}{(b/a)^2 + 1} \right] = \frac{2 \times 2.858 \times 10^{-2}}{10^{-2}(233.81)\sqrt{1-(10.4/12)^2}} \left[\frac{(1/8) + 1}{(1/4) + 1} \right]$$

$$\underline{\underline{\alpha_c = 0.0441 \text{ Np/m}}}$$

Prob. 12.30 $\epsilon_c = \epsilon' - j\epsilon'' = \epsilon - j\frac{\sigma}{\omega}$

Comparing this with

$$\epsilon_c = 16\epsilon_o(1 - j10^{-4}) = 16\epsilon_o - j16\epsilon_o \times 10^{-4}$$

$$\epsilon = 16\epsilon_o, \quad \frac{\sigma}{\omega} = 16\epsilon_o \times 10^{-4}$$

For TM₂₁ mode,

$$f_c = \frac{u'}{2} \left[\frac{m^2}{a^2} + \frac{n^2}{b^2} \right]^{1/2} = 2.0963 \text{ GHz}, \quad f = 1.1f_c = 2.3059 \text{ GHz}$$

$$\sigma = 16\epsilon_o\omega \times 10^{-4} = 16 \times 2\pi \times 2.3059 \times 10^9 \times \frac{10^{-9}}{36\pi} \times 10^{-4} = 2.0525 \times 10^{-4}$$

$$\eta' = \sqrt{\frac{\mu}{\epsilon}} = 30\pi \Omega$$

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1-(f_c/f)^2}} = \frac{4.1 \times 10^{-4} \times 30\pi}{2\sqrt{1-1/1.12}} = \underline{\underline{0.0231}} \text{ Np/m}$$

$$E_o e^{-\alpha_d z} = 0.8E_o \quad \longrightarrow \quad z = \frac{1}{\alpha_d} \ln(1/0.8) = \underline{\underline{9.66 \text{ m}}}$$

Prob. 12.31

For TM₂₁ mode,

$$\alpha_c = \frac{2R_s}{b\eta' \sqrt{1 - (f_c/f)^2}}$$

$$R_s = \frac{1}{\sigma_c \delta} = \sqrt{\frac{\pi f \mu}{\sigma_c}} = \sqrt{\frac{\pi \times 2.3059 \times 10^9 \times 4\pi \times 10^{-7}}{1.5 \times 10^7}} = 0.0246 \, \Omega$$

$$\alpha_c = \frac{2 \times 0.0246}{4\pi \times 10^{-2} \times 30\pi \times 0.4166} = 0.0314 \, \text{Np/m}$$

$$E_o e^{-(\alpha_c + \alpha_d)z} = 0.7 E_o \quad \longrightarrow \quad z = \frac{1}{\alpha_c + \alpha_d} \ln(1/0.7) = \underline{\underline{6.5445 \, \text{m}}}$$

Prob. 12.32

$$f_{c10} = \frac{u'}{2a} = \frac{3 \times 10^8}{2 \times 6 \times 10^{-2}} = 2.5 \, \text{GHz}$$

$$\eta_{TE} = \frac{\eta'}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1 - \left(\frac{2.5}{4}\right)^2}} = 483 \, \Omega$$

From Example 12.5,

$$P_{ave} = \frac{E_o^2 ab}{2\eta} = \frac{(2.2)^2 \times 10^6 \times 6 \times 3 \times 10^{-4}}{2 \times 483} = \underline{\underline{9.0196 \, \text{mW}}}$$

Prob. 12.33

For TE₁₀ mode,

$$f_c = \frac{u'}{2a} = \frac{3 \times 10^8}{2\sqrt{2.11} \times 4.8 \times 10^{-2}} = 2.151 \, \text{GHz}$$

$$(a) \text{ loss tangent} = \frac{\sigma}{\omega \epsilon} = d$$

$$\sigma = d\omega\epsilon = 3 \times 10^{-4} \times 2\pi \times 4 \times 10^9 \times 2.11 \times \frac{10^{-9}}{36\pi} = 1.4086 \times 10^{-4}$$

$$\eta' = \frac{120\pi}{\sqrt{2.11}} = 259.53 \, \Omega$$

$$\alpha_d = \frac{\sigma \eta'}{2\sqrt{1-(f_c/f)^2}} = \frac{1.4067 \times 10^{-4} \times 259.53}{2\sqrt{1-(2.151/4)^2}} = \underline{\underline{2.165 \times 10^{-2} \text{ Np/m}}}$$

$$(b) R_s = \sqrt{\frac{\mu f \pi}{\sigma_c}} = \sqrt{\frac{\pi \times 4 \times 10^9 \times 4\pi \times 10^{-7}}{4.1 \times 10^7}} = 1.9625 \times 10^{-2} \Omega$$

$$\alpha_c = \frac{2R_s}{b\eta' \sqrt{1-(f_c/f)^2}} \left[\frac{1}{2} + \frac{b}{a} (f_c/f)^2 \right] = \frac{3.925 \times 10^{-2} (0.5 + 0.5 \times 0.2892)}{2.4 \times 10^{-2} \times 259.53 \times 0.8431}$$

$$= \underline{\underline{4.818 \times 10^{-3} \text{ Np/m}}}$$

Prob.12.34

$$\alpha_c = \frac{2R_s}{b\eta' \sqrt{1-\left(\frac{f_c}{f}\right)^2}} \left[\frac{1}{2} + \frac{b}{a} \left(\frac{f_c}{f}\right)^2 \right] = \frac{2\sqrt{\frac{\pi f \mu}{\sigma_c}}}{b\eta' \sqrt{1-\left(\frac{f_c}{f}\right)^2}} \left[\frac{1}{2} + \frac{1}{2} \left(\frac{f_c}{f}\right)^2 \right]$$

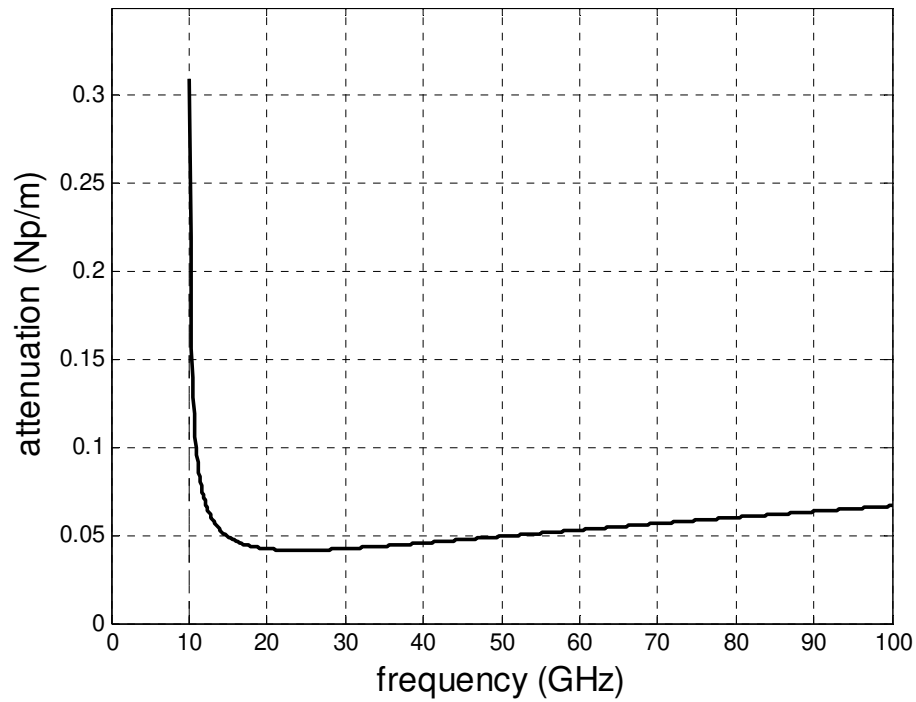
$$= \frac{2\sqrt{4\pi \times 10^{-7} \times \pi} \sqrt{f} \times \frac{1}{2}}{0.5 \times 10^{-2} \times (120\pi / \sqrt{2.25}) \sqrt{5.8 \times 10^7} \sqrt{1-\left(\frac{f_c}{f}\right)^2}} \left[1 + \left(\frac{f_c}{f}\right)^2 \right]$$

$$= \frac{10^{-5} \sqrt{f}}{30\sqrt{(5.8/2.25)} \sqrt{1-\left(\frac{f_c}{f}\right)^2}} \left[1 + \left(\frac{f_c}{f}\right)^2 \right]$$

The MATLAB code is shown below

```
k=10^(-5)/(30*sqrt(5.8/2.25));
fc=10^10;
for n=1:1000
    f(n)=fc*(n/100+1);
    fn=f(n);
    num=sqrt(fn)*(1+(fc/fn)^2);
    den=sqrt(1-(fc/fn)^2);
    alpha(n)=k*num/den;
end
plot(f/10^9,alpha)
xlabel('frequency (GHz)')
ylabel('attenuation')
grid
```

The plot of attenuation versus frequency is shown below.



Prob. 12.35

The cutoff frequency of the dominant mode is

$$f_{c10} = \frac{u}{2a} = \frac{3 \times 10^8}{4.576 \times 10^{-2}} = 6.56 \text{ GHz}$$

The surface resistance is

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma_c}} = \sqrt{\frac{\pi \times 8.4 \times 10^9 \times 4\pi \times 10^{-7}}{5.8 \times 10^7}} = 23.91 \times 10^{-3} \Omega$$

For TE₁₀ mode,

$$\alpha_c = \frac{2R_s}{b\eta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \left[0.5 + \frac{b}{a} \left(\frac{f_c}{f}\right)^2 \right]$$

$$\frac{f_c}{f} = \frac{6.56}{8.4} = 0.781, \quad \eta' = \eta_o = 377$$

$$\begin{aligned}\alpha_c &= \frac{2 \times 23.91 \times 10^{-3}}{1.016 \times 10^{-2} \times 377 \sqrt{1 - 0.781^2}} \left[0.5 + \frac{1.016}{2.286} (0.781)^2 \right] \\ &= \frac{47.82 \times 10^{-3} (0.5 + 0.2711)}{3.83 \times 0.6245} = 15.42 \times 10^{-3} \text{ Np/m} \\ &= 15.42 \times 10^{-3} \times 8.686 \text{ dB/m} = \underline{\underline{0.1339 \text{ dB/m}}}\end{aligned}$$

Prob. 12.36

$$(a) \quad f_{c10} = \frac{u'}{2a} = \frac{3 \times 10^8}{2 \times 3.8 \times 10^{-2}} = 3.947 \text{ GHz}$$

$$u_g = u' \sqrt{1 - \left(\frac{f_c}{f} \right)^2} = 3 \times 10^8 \sqrt{1 - (0.3947)^2} = \underline{\underline{2.756 \times 10^8 \text{ m/s}}}$$

$$(b) \quad \alpha = \alpha_d + \alpha_c$$

$\alpha_d = 0$ since the guide is air-filled.

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma_c}} = \sqrt{\frac{\pi \times 10^{10} \times 4\pi \times 10^{-7}}{5.8 \times 10^7}} = 2.609 \times 10^{-2} \Omega$$

$$\begin{aligned}\alpha_c &= \frac{2R_s}{b\eta' \sqrt{1 - \left(\frac{f_c}{f} \right)^2}} \left[0.5 + \frac{b}{a} \left(\frac{f_c}{f} \right)^2 \right] \\ &= \frac{2 \times 2.609 \times 10^{-2}}{1.6 \times 10^{-2} (377) \sqrt{1 - (0.3947)^2}} \left[0.5 + \frac{1.6}{3.8} (0.3947)^2 \right] = \frac{5.218 \times 0.5656}{554.23} \\ &= 5.325 \times 10^{-3} \text{ Np/m}\end{aligned}$$

$$\alpha_c (\text{dB}) = 8.686 \times 5.325 \times 10^{-3} = 0.04626 \text{ dB/m}$$

Prob. 12.37

$$f_{c10} = \frac{u'}{2a} = \frac{c}{2a\sqrt{\epsilon_r \mu_r}} = \frac{3 \times 10^8}{2 \times 2.5 \times 10^{-2} \sqrt{2.26}} = 3.991 \text{ GHz}$$

$$\beta' = \frac{\omega}{u'} = \frac{2\pi f \sqrt{\epsilon_r}}{c}$$

$$F = \sqrt{1 - \left(\frac{f_c}{f} \right)^2} = \sqrt{1 - \left(\frac{3.991}{7.5} \right)^2} = 0.8467$$

$$\beta = \beta' F = \frac{2\pi \times 7.5 \times 10^9 \sqrt{2.26}}{3 \times 10^8} \times 0.8467 = \underline{\underline{199.94 \text{ rad/m}}}$$

$$\alpha_d = \frac{\sigma \eta'}{2F} = \frac{\sigma \eta_o}{2F \sqrt{\epsilon_r}} = \frac{10^{-4}(377)}{2 \times 0.8467 \sqrt{2.26}} = \underline{\underline{1.481 \times 10^{-2} \text{ Np/m}}}$$

$$\alpha_c = \frac{2R_s}{b\eta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \left[0.5 + \frac{b}{a} \left(\frac{f_c}{f}\right)^2 \right]$$

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma_c}} = \sqrt{\frac{\pi \times 7.5 \times 10^9 \times 4\pi \times 10^{-7}}{1.1 \times 10^7}} = 0.0519 \Omega$$

$$\alpha_c = \frac{2 \times 0.0519 \left[0.5 + \frac{1.5}{2.5} \left(\frac{3.991}{7.5}\right)^2 \right]}{1.5 \times 10^{-2} \times \frac{377}{\sqrt{2.66}} \times 0.8467} = \frac{0.1038 \times 0.6698}{3.1848}$$

$$= \underline{\underline{0.02183 \text{ Np/m}}}$$

$$u_p = \frac{u'}{F} = \frac{c}{F \sqrt{\epsilon_r}} = \frac{3 \times 10^8}{0.8467 \sqrt{2.26}} = \underline{\underline{2.357 \times 10^8 \text{ m/s}}}$$

$$u_g = u' F = \frac{3 \times 10^8 \times 0.8467}{\sqrt{2.26}} = \underline{\underline{1.689 \times 10^8 \text{ m/s}}}$$

$$\lambda_c = \frac{u'}{f_c} = \frac{c}{f_c \sqrt{\epsilon_r}} = \frac{3 \times 10^8}{3.991 \times 10^9 \sqrt{2.26}} = 0.05 \text{ m} = \underline{\underline{5 \text{ cm}}} (= 2a, \text{ as expected})$$

Prob. 12.38 (a) For TE₁₀ mode,

$$f_c = \frac{u'}{2a}, \quad u' = \frac{c}{\sqrt{2.11}}$$

$$f_c = \frac{3 \times 10^8}{\sqrt{2.11}(2 \times 2.25 \times 10^{-2})} = \underline{\underline{4.589 \text{ GHz}}}$$

$$(b) \quad \alpha_{cTE10} = \frac{2R_s}{b\eta' \sqrt{1 - (f_c/f)^2}} \left[\frac{1}{2} + \frac{b}{a} (f_c/f)^2 \right]$$

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma_c}} = \sqrt{\frac{\pi \times 5 \times 10^9 \times 4\pi \times 10^{-7}}{1.37 \times 10^7}} = 3.796 \times 10^{-2} \Omega$$

$$\eta' = \frac{377}{\sqrt{2.11}} = 259.54 \Omega$$

$$\alpha_c = \frac{2 \times 3.796 \times 10^{-2} [0.5 + \frac{1.5}{2.25} (4.589/5)^2]}{1.5 \times 10^{-4} (259.54) \sqrt{1 - (4.589/5)^2}} = \underline{\underline{0.05217}} \text{ Np/m}$$

Prob. 12.39 For TE₁₀ mode,

$$\alpha_c = \frac{2R_s}{b\eta' \sqrt{1 - (f_c/f)^2}} \left[\frac{1}{2} + \frac{b}{a} \left(\frac{f_c}{f} \right)^2 \right]$$

But $a = b$, $R_s = \frac{I}{\sigma_c \delta} = \sqrt{\frac{\pi f \mu}{\sigma_c}}$

$$\alpha_c = \frac{2 \sqrt{\frac{\pi f \mu}{\sigma_c}}}{a\eta' \sqrt{1 - (f_c/f)^2}} \left[\frac{1}{2} + \left(\frac{f_c}{f} \right)^2 \right] = \frac{k \sqrt{f} \left[\frac{1}{2} + \left(\frac{f_c}{f} \right)^2 \right]}{\sqrt{1 - (f_c/f)^2}}$$

where k is a constant.

$$\frac{d\alpha_c}{df} = \frac{k [1 - (f_c/f)^2]^{-1/2} [\frac{1}{4} f^{-1/2} - \frac{3}{2} f_c^2 f^{-5/2}] - \frac{k}{2} [\frac{1}{2} f^{1/2} + f_c^2 f^{-3/2}] (2 f_c^2 f^{-3}) [1 - (f_c/f)^2]^{-1/2}}{1 - (f_c/f)^2}$$

For minimum value, $\frac{d\alpha_c}{df} = 0$. This leads to $\underline{\underline{f = 2.962 f_c}}$.

Prob. 12.40 For the TE mode to z ,

$$E_{zs} = 0, H_{zs} = H_o \cos(m\pi x/a) \cos(n\pi y/b) \sin(p\pi z/c)$$

$$E_{ys} = -\frac{\gamma}{h^2} \frac{\partial E_{zs}}{\partial y} + \frac{j\omega\mu}{h^2} \frac{\partial H_{zs}}{\partial x} = -\frac{j\omega\mu}{h^2} (m\pi/a) H_o \sin(m\pi x/a) \cos(n\pi y/b) \sin(p\pi z/c)$$

as required.

$$E_{xs} = -\frac{\gamma}{h^2} \frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_{zs}}{\partial y} = \frac{j\omega\mu}{h^2} (n\pi/b) H_o \cos(m\pi x/a) \sin(n\pi y/b) \sin(p\pi z/c)$$

From Maxwell's equation,

$$-j\omega\mu\mathbf{H}_s = \nabla \times \mathbf{E}_s = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{xs} & E_{ys} & 0 \end{vmatrix}$$

$$H_{xs} = \frac{1}{j\omega\mu} \frac{\partial E_{ys}}{\partial z} = -\frac{1}{h^2} (m\pi/a)(p\pi/c) H_o \sin(m\pi x/a) \cos(n\pi y/b) \cos(p\pi z/c)$$

Prob. 12.41 Maxwell's equation can be written as

$$H_{xs} = \frac{j\omega\epsilon}{h^2} \frac{\partial E_{zs}}{\partial y} - \frac{\gamma}{h^2} \frac{\partial H_{zs}}{\partial x}$$

For a rectangular cavity,

$$h^2 = k_x^2 + k_y^2 = (m\pi/a)^2 + (n\pi/b)^2$$

For TM mode, $H_{zs} = 0$ and

$$E_{zs} = E_o \sin(m\pi x/a) \sin(n\pi y/b) \cos(p\pi z/c)$$

Thus

$$H_{xs} = \frac{j\omega\epsilon}{h^2} \frac{\partial E_{zs}}{\partial y} = \frac{j\omega\epsilon}{h^2} (n\pi/b) E_o \sin(m\pi x/a) \cos(n\pi y/b) \cos(p\pi z/c)$$

as required.

$$\begin{aligned} H_{xs} &= -\frac{j\omega\epsilon}{h^2} \frac{\partial E_{zs}}{\partial x} - \frac{\gamma}{h^2} \frac{\partial H_{zs}}{\partial y} \\ &= -\frac{j\omega\epsilon}{h^2} (m\pi/a) E_o \cos(m\pi x/a) \sin(n\pi y/b) \cos(p\pi z/c) \end{aligned}$$

From Maxwell's equation,

$$j\omega\epsilon\mathbf{E}_s = \nabla \times \mathbf{H}_s = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_{xs} & H_{ys} & 0 \end{vmatrix}$$

$$E_{ys} = \frac{1}{j\omega\epsilon} \frac{\partial H_{xs}}{\partial z} = \frac{1}{h^2} (n\pi/b)(p\pi/c) E_o \sin(m\pi x/a) \cos(n\pi y/b) \cos(p\pi z/c)$$

Prob. 12.42

$$f_r = \frac{u'}{2} \sqrt{(m/a)^2 + (n/b)^2 + (p/c)^2}$$

where for TM mode to z, $m = 1, 2, 3, \dots$, $n = 1, 2, 3, \dots$, $p = 0, 1, 2, \dots$

and for TE mode to z, $m = 0, 1, 2, 3, \dots$, $n = 0, 1, 2, 3, \dots$, $p = 1, 2, 3, \dots$, $(m+n) \neq 0$.

(a) If $a < b < c$, $1/a > 1/b > 1/c$,

The lowest TM mode is TM_{110} with $f_r = \frac{u'}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

The lowest TE mode is TE_{011} with $f_r = \frac{u'}{2} \sqrt{\frac{1}{b^2} + \frac{1}{c^2}} < \frac{u'}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

Hence the dominant mode is TE_{011} .

(b) If $a > b > c$, $1/a < 1/b < 1/c$,

The lowest TM mode is TM_{110} with $f_r = \frac{u'}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

The lowest TE mode is TE_{101} with $f_r = \frac{u'}{2} \sqrt{\frac{1}{a^2} + \frac{1}{c^2}} > \frac{u'}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

Hence the dominant mode is TM_{110} .

(c) If $a = c > b$, $1/a = 1/c < 1/b$,

The lowest TM mode is TM_{110} with $f_r = \frac{u'}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

The lowest TE mode is TE_{101} with $f_r = \frac{u'}{2} \sqrt{\frac{1}{a^2} + \frac{1}{c^2}} < \frac{u'}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

Hence the dominant mode is TE_{101} .

Prob. 12.43

$$(a) \quad u' = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{4.6}}$$

$$f_c = \frac{u'}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

For the dominant mode, $m = 1, n=0, p=1$

$$f_c = \frac{u'}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2} = \frac{3 \times 10^8}{2\sqrt{4.6}} \sqrt{\frac{1}{9 \times 10^{-4}} + \frac{1}{36 \times 10^{-4}}} = \frac{3 \times 10^{10}}{2(2.1447)} (0.37267) = \underline{\underline{2.606 \text{ GHz}}}$$

$$(b) \quad Q = \frac{(a^2 + c^2)abc}{\delta [2b(a^3 + c^3) + ac(a^2 + c^2)]}$$

$$\delta = \frac{1}{\sqrt{\pi f_{r101} \sigma \mu_o}} = \frac{1}{\sqrt{\pi \times 2.606 \times 10^9 \times 1.57 \times 10^7 \times 4\pi \times 10^{-7}}} = 2.49 \times 10^{-6} \text{ m}$$

$$Q = \frac{(9+36)(72) \times 10^{-2}}{\delta [8(27+216) + 18(9+36)]} = \frac{32.42}{2.49 \times 10^{-6} (2754)} = \underline{\underline{4727.7}}$$

Prob. 12.44

(a)

$$f_r = \frac{u'}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$f_{rTE101} = 1.5 \times 10^{10} \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2} = 1.5 \times 10^{10} \sqrt{0.1736} = \underline{\underline{6.25 \text{ GHz}}}$$

$$f_{rTE011} = 1.5 \times 10^{10} \sqrt{\frac{1}{6.25} + \frac{1}{16}} = 1.5 \times 10^{10} \sqrt{0.2225} = \underline{\underline{7.075 \text{ GHz}}}$$

$$f_{rTE110} = 1.5 \times 10^{10} \sqrt{\frac{1}{9} + \frac{1}{6.25}} = 1.5 \times 10^{10} \sqrt{0.2711} = \underline{\underline{7.81 \text{ GHz}}}$$

Prob. 12.45

$$u' = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{2.5}} = \frac{3 \times 10^8}{\sqrt{2.5}} = 1.897 \times 10^8$$

$$f_c = \frac{u'}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2} = \frac{1.897 \times 10^8 \times 10^2}{2} \sqrt{\left(\frac{m}{1}\right)^2 + \left(\frac{n}{2}\right)^2 + \left(\frac{p}{3}\right)^2}$$

$$= 9.485 \sqrt{m^2 + 0.25n^2 + 0.111p^2} \text{ GHz}$$

$$f_{r101} = 9.485 \sqrt{1 + 0 + 0.111} = 10 \text{ GHz}$$

$$f_{r011} = 9.485 \sqrt{0 + 0.25 + 0.111} = 5.701 \text{ GHz}$$

$$f_{r012} = 9.485 \sqrt{0 + 0.25 + 0.444} = 7.906 \text{ GHz}$$

$$f_{r013} = 9.485 \sqrt{0 + 0.25 + 0.999} = 10.61 \text{ GHz}$$

$$f_{r021} = 9.485 \sqrt{0 + 1 + 0.111} = 10 \text{ GHz}$$

Thus, the first five resonant frequencies are:

$$5.701 \text{ GHz (TE}_{011}\text{)}$$

$$7.906 \text{ GHz (TE}_{012}\text{)}$$

$$10 \text{ GHz (TE}_{101}\text{ and TE}_{021}\text{)}$$

$$10.61 \text{ GHz (TE}_{013}\text{ or TM}_{110}\text{)}$$

$$\underline{\underline{11.07 \text{ GHz (TE}_{111}\text{ or TM}_{111}\text{)}}}$$

Prob. 12.46

$$Q = \frac{(a^2 + c^2)abc}{\delta [2b(a^3 + c^3) + ac(a^2 + c^2)]}$$

When $a = b = c$,

$$Q = \frac{2a^2a^3}{\delta [2a \times 2a^3 + a^2 \times 2a^2]} = \frac{2a^5}{6\delta a^4} = \underline{\underline{\frac{a}{3\delta}}}$$

Prob. 12.47

(a) Since $a > b < c$, the dominant mode is TE_{101}

$$f_{r101} = \frac{u'}{2} \sqrt{\frac{1}{a^2} + 0 + \frac{1}{c^2}} = \frac{3 \times 10^8 \times 10^2}{2} \sqrt{\frac{1}{2^2} + \frac{1}{1^2}} = \underline{\underline{16.77 \text{ GHz}}}$$

$$\begin{aligned}
 \text{(b)} \quad Q_{TE101} &= \frac{(a^2 + c^2)abc}{\delta [2b(a^3 + c^3) + ac(a^2 + c^2)]} \\
 &= \frac{(400 + 100)20 \times 8 \times 10 \times 10^{-3}}{\delta [16(8000 + 1000) + 200(400 + 100)]} = \frac{3.279 \times 10^{-3}}{\delta}
 \end{aligned}$$

$$\text{But } \delta = \frac{1}{\sqrt{\pi f_{r101} \mu_o \sigma}} = \frac{1}{\sqrt{\pi 16.77 \times 10^9 \times 4\pi \times 10^{-7} \times 6.1 \times 10^7}} = \frac{10^{-4}}{200.961} \text{ m}$$

$$Q_{TE101} = 3.279 \times 10^{-3} \frac{200.961}{10^{-4}} = \underline{\underline{6589.51}}$$

Prob. 12.48

$$f_r = \frac{c}{2a} \sqrt{m^2 + n^2 + p^2}$$

The lowest possible modes are TE_{101} , TE_{011} , and TM_{110} . Hence

$$f_r = \frac{c}{2a} \sqrt{2} \longrightarrow a = \frac{c}{f_r \sqrt{2}} = \frac{3 \times 10^8}{\sqrt{2} \times 3 \times 10^9} = 7.071 \text{ cm}$$

$$\underline{\underline{a = b = c = 7.071 \text{ cm}}}$$

Prob. 12.49

$$\text{(a)} \quad a = b = c$$

$$f_r = \frac{u'}{2a} \sqrt{m^2 + n^2 + p^2}$$

For the dominant mode TE_{101} ,

$$f_r = \frac{u'}{2a} \sqrt{1+1} = \frac{c}{2a} \sqrt{2}$$

$$a = \frac{c\sqrt{2}}{2f_r} = \frac{3 \times 10^8 \sqrt{2}}{2 \times 5.6 \times 10^9} = 0.03788 \text{ m}$$

$$\underline{\underline{a = b = c = 3.788 \text{ cm}}}$$

(b)

$$\text{For } \epsilon_r = 2.05, \quad u' = \frac{c}{\sqrt{\epsilon_r}}$$

$$a = \frac{c\sqrt{2}}{2f_r \sqrt{\epsilon_r}} = \frac{0.03788}{\sqrt{2.05}} = 0.02646$$

$$\underline{\underline{a = b = c = 2.646 \text{ cm}}}$$

Prob. 12.50

(a)

This is a TM mode to z. From Maxwell's equations,

$$\nabla \times \mathbf{E}_s = -j\omega\mu\mathbf{H}_s$$

$$\mathbf{H}_s = -\frac{1}{j\omega\mu} \nabla \times \mathbf{E}_s = \frac{j}{\omega\mu} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_{zs}(x, y) \end{vmatrix} = \frac{j}{\omega\mu} \left(\frac{\partial E_{zs}}{\partial y} \mathbf{a}_x - \frac{\partial E_{zs}}{\partial x} \mathbf{a}_y \right)$$

But

$$E_{zs} = 200 \sin 30\pi x \sin 30\pi y, \quad \frac{1}{\omega\mu} = \frac{1}{6 \times 10^9 \times 4\pi \times 10^{-7}} = \frac{10^{-2}}{24\pi}$$

$$\mathbf{H}_s = \frac{j10^{-2}}{24\pi} \times 200 \times 30\pi \left\{ \sin 30\pi x \cos 30\pi y \mathbf{a}_x - \cos 30\pi x \sin 30\pi y \mathbf{a}_y \right\}$$

$$\mathbf{H} = \text{Re} (\mathbf{H}_s e^{j\omega t})$$

$$\mathbf{H} = 2.5 \left\{ -\sin 30\pi x \cos 30\pi y \mathbf{a}_x + \cos 30\pi x \sin 30\pi y \mathbf{a}_y \right\} \sin 6 \times 10^9 \pi t \quad \text{A/m}$$

(b)

$$\mathbf{E} = E_z \mathbf{a}_z, \quad \mathbf{H} = H_x \mathbf{a}_x + H_y \mathbf{a}_y$$

$$\mathbf{E} \cdot \mathbf{H} = 0$$

Prob. 12.51

$$(a) \quad a = b = c \longrightarrow f_{r101} = \frac{3 \times 10^8}{a\sqrt{2}} = 12 \times 10^9$$

$$a = \frac{3 \times 10^8}{\sqrt{2} \times 12 \times 10^9} = \underline{\underline{1.77 \text{ cm}}}$$

$$(b) \quad Q_{TE101} = \frac{a}{3\delta} = \frac{a\sqrt{\pi f_{r101}\mu\sigma}}{3}$$

$$= \frac{1.77 \times 10^{-2} \sqrt{\pi \times 12 \times 10^9 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7}}{3} = \underline{\underline{9767.61}}$$

Prob. 12.52

$$f_r = \frac{u'}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$f_{r101} = \frac{3 \times 10^8}{2} \sqrt{\frac{1}{(10.2)^2} + \frac{1}{(3.6)^2}} = 44.186 \text{ MHz}$$

$$f_{r011} = 150 \sqrt{\frac{1}{(8.7)^2} + \frac{1}{(3.6)^2}} \text{ MHz} = 45.093 \text{ MHz}$$

$$f_{r111} = 150 \sqrt{\frac{1}{(10.2)^2} + \frac{1}{(8.7)^2} + \frac{1}{(3.6)^2}} \text{ MHz} = 47.43 \text{ MHz}$$

$$f_{r110} = 150 \sqrt{\frac{1}{(10.2)^2} + \frac{1}{(8.7)^2}} \text{ MHz} = 22.66 \text{ MHz}$$

$$f_{r102} = 150 \sqrt{\frac{1}{(10.2)^2} + \frac{4}{(3.6)^2}} \text{ MHz} = 84.62 \text{ MHz}$$

$$f_{r201} = 150 \sqrt{\frac{4}{(10.2)^2} + \frac{1}{(3.6)^2}} \text{ MHz} = 51 \text{ MHz}$$

Thus, the resonant frequencies below 50 MHz are

$$\underline{\underline{f_{r110}, f_{r101}, f_{r011}, \text{ and } f_{r111}}}$$

Prob. 12.53

$$n = c/u_m = \frac{3 \times 10^8}{2.1 \times 10^8} = \underline{\underline{1.4286}}$$

Prob. 12.54

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.51^2 - 1.45^2} = \sqrt{0.1776} = 0.421$$

Prob. 12.55

$$(a) \quad NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.62^2 - 1.604^2} = \underline{\underline{0.2271}}$$

$$(b) \quad NA = \sin \theta_a = 0.2271 \quad \text{or} \quad \theta_a = \sin^{-1} 0.2271 = \underline{\underline{13.13^\circ}}$$

$$(c) \quad V = \frac{\pi d}{\lambda} NA = \frac{\pi \times 50 \times 10^{-6} \times 0.2271}{1300 \times 10^{-9}} = 27.441$$

$$N = V^2/2 \quad \underline{\underline{6 \text{ modes}}}$$

Prob. 12.56

$$V = \frac{\pi d}{\lambda} \sqrt{n_1^2 - n_2^2} = \frac{\pi \times 2 \times 5 \times 10^{-6}}{1300 \times 10^{-9}} \sqrt{1.48^2 - 1.46^2} = 5.86$$

$$N = \frac{V^2}{2} = 17.17 \text{ or } \underline{\underline{17 \text{ modes}}}$$

Prob. 12.57

$$(a) \text{ NA} = \sin \theta_a = \sqrt{n_1^2 - n_2^2} = \sqrt{1.53^2 - 1.45^2} = 0.4883$$

$$\theta_a = \sin^{-1} 0.4883 = \underline{\underline{29.23^\circ}}$$

$$(b) P(1)/P(0) = 10^{-\alpha \ell / 10} = 10^{-0.4 \times 5 / 10} = 0.631$$

$$\text{i.e. } \underline{\underline{63.1 \%}}$$

Prob. 12.58

$$P(\ell) = P(0) 10^{-\alpha \ell / 10} = 10 \times 10^{-0.5 \times 0.85 / 10} = \underline{\underline{9.0678 \text{ mW}}}$$

Prob. 12.59

As shown in Eq. (10.35), $\log_{10} P_1/P_2 = 0.434 \ln P_1/P_2$,

$$1 \text{ Np} = 20 \log_{10} e = 8.686 \text{ dB} \text{ or } 1 \text{ Np/km} = 8.686 \text{ dB/km},$$

or $1 \text{ Np/m} = 8686 \text{ dB/km}$. Thus,

$$\alpha_{12} = \underline{\underline{8686 \alpha_{10}}}$$

Prob. 12.60

$$\alpha \ell = 10 \log_{10} \frac{P_{in}}{P_{out}} = 10 \log_{10} \frac{1.2 \times 10^{-3}}{1 \times 10^{-6}} = 30.792$$

$$\alpha = 0.4 \text{ dB/km} = \frac{0.4}{8.686} \text{ Np/km}$$

$$\ell = \frac{30.792}{\alpha} = \frac{30.392 \text{ dB}}{0.4 \text{ dB/km}} = \underline{\underline{76.98 \text{ km}}}$$

Prob. 12.61

$$P(0) = P(1) 10^{\alpha \ell / 10} = 0.2 \times 10^{0.4 \times 30 / 10} \text{ mW} = \underline{\underline{3.1698 \text{ mW}}}$$

Prob. 12.62 See text.

CHAPTER 2

P. E. 2.1

(a) At $P(1,3,5)$, $x = 1$, $y = 3$, $z = 5$,
 $\rho = \sqrt{x^2 + y^2} = \sqrt{10}$, $z = 5$, $\phi = \tan^{-1} y/x = \tan^{-1} 3 = 71.6^\circ$
 $P(\rho, \phi, z) = P(\sqrt{10}, \tan^{-1} 3, 5) = \underline{\underline{P(3.162, 71.6^\circ, 5)}}$

Spherical system:

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{35} = 5.916$$

$$\theta = \tan^{-1} \sqrt{x^2 + y^2} / z = \tan^{-1} \sqrt{10} / 5 = \tan^{-1} 0.6325 = 32.31^\circ$$

$$P(r, \theta, \phi) = \underline{\underline{P(5.916, 32.31^\circ, 71.57^\circ)}}$$

At $T(0,-4,3)$, $x = 0$, $y = -4$, $z = 3$;
 $\rho = \sqrt{x^2 + y^2} = 4$, $z = 3$, $\phi = \tan^{-1} y/x = \tan^{-1} -4/0 = 270^\circ$
 $T(\rho, \phi, z) = \underline{\underline{T(4, 270^\circ, 3)}}$.

Spherical system:

$$r = \sqrt{x^2 + y^2 + z^2} = 5$$
, $\theta = \tan^{-1} \rho / z = \tan^{-1} 4/3 = 53.13^\circ$.
 $T(r, \theta, \phi) = \underline{\underline{T(5, 53.13^\circ, 270^\circ)}}$.

At $S(-3,-4,-10)$, $x = -3$, $y = -4$, $z = -10$;
 $\rho = \sqrt{x^2 + y^2} = 5$, $\phi = \tan^{-1} \left(\frac{-4}{-3} \right) = 233.1^\circ$
 $S(\rho, \phi, z) = \underline{\underline{S(5, 233.1^\circ, -10)}}$.

Spherical system:

$$r = \sqrt{x^2 + y^2 + z^2} = 5\sqrt{5} = 11.18.$$

$$\theta = \tan^{-1} \rho / z = \tan^{-1} \frac{5}{-10} = 153.43^\circ;$$

$$S(r, \theta, \phi) = \underline{\underline{S(11.18, 153.43^\circ, 233.1^\circ)}}$$
.

(b) In Cylindrical system, $\rho = \sqrt{x^2 + y^2}$; $yz = z\rho \sin \phi$,
 $Q_x = \frac{\rho}{\sqrt{\rho^2 + z^2}}$; $Q_y = 0$; $Q_z = -\frac{z\rho \sin \phi}{\sqrt{\rho^2 + z^2}}$;

$$\begin{bmatrix} Q_\rho \\ Q_\phi \\ Q_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_x \\ 0 \\ Q_z \end{bmatrix};$$

$$Q_\rho = Q_x \cos\phi = \frac{\rho \cos\phi}{\sqrt{\rho^2 + z^2}}, \quad Q_\phi = -Q_x \sin\phi = \frac{-\rho \sin\phi}{\sqrt{\rho^2 + z^2}}$$

Hence,

$$\underline{\underline{Q = \frac{\rho}{\sqrt{\rho^2 + z^2}} (\cos\phi \mathbf{a}_\rho - \sin\phi \mathbf{a}_\phi - z \sin\phi \mathbf{a}_z).}}$$

In Spherical coordinates:

$$Q_x = \frac{r \sin\theta}{r} = \sin\theta;$$

$$Q_z = -r \sin\phi \sin\theta r \cos\theta \frac{1}{r} = -r \sin\theta \cos\theta \sin\phi.$$

$$\begin{bmatrix} Q_r \\ Q_\theta \\ Q_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta \cos\phi & \sin\theta \sin\phi & \cos\theta \\ \cos\theta \cos\phi & \cos\theta \sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} Q_x \\ 0 \\ Q_z \end{bmatrix};$$

$$Q_r = Q_x \sin\theta \cos\phi + Q_z \cos\theta = \sin^2\theta \cos\phi - r \sin\theta \cos^2\theta \sin\phi.$$

$$Q_\theta = Q_x \cos\theta \cos\phi - Q_z \sin\theta = \sin\theta \cos\theta \cos\phi + r \sin^2\theta \cos\theta \sin\phi.$$

$$Q_\phi = -Q_x \sin\phi = -\sin\theta \sin\phi.$$

$$\underline{\underline{\therefore Q = \sin\theta (\sin\theta \cos\phi - r \cos^2\theta \sin\phi) \mathbf{a}_r + \sin\theta \cos\theta (\cos\phi + r \sin\theta \sin\phi) \mathbf{a}_\theta - \sin\theta \sin\phi \mathbf{a}_\phi.}}$$

At T :

$$Q(x, y, z) = \frac{4}{5} \mathbf{a}_x + \frac{12}{5} \mathbf{a}_z = 0.8 \mathbf{a}_x + 2.4 \mathbf{a}_z;$$

$$\begin{aligned} Q(\rho, \phi, z) &= \frac{4}{5} (\cos 270^\circ \mathbf{a}_\rho - \sin 270^\circ \mathbf{a}_\phi - 3 \sin 270^\circ \mathbf{a}_z) \\ &= 0.8 \mathbf{a}_\phi + 2.4 \mathbf{a}_z; \end{aligned}$$

$$\begin{aligned} Q(r, \theta, \phi) &= \frac{4}{5} \left(0 - \frac{45}{25} (-1) \right) \mathbf{a}_r + \frac{4}{5} \left(\frac{3}{5} \right) \left(0 + \frac{20}{5} (-1) \right) \mathbf{a}_\theta - \frac{4}{5} (-1) \mathbf{a}_\phi \\ &= \frac{36}{25} \mathbf{a}_r - \frac{48}{25} \mathbf{a}_\theta + \frac{4}{5} \mathbf{a}_\phi = \underline{\underline{1.44 \mathbf{a}_r - 1.92 \mathbf{a}_\theta + 0.8 \mathbf{a}_\phi;}} \end{aligned}$$

Note, that the magnitude of vector $Q = 2.53$ in all 3 cases above.

P.E. 2.2 (a)

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho z \sin \phi \\ 3\rho \cos \phi \\ \rho \cos \phi \sin \phi \end{bmatrix}$$

$$A = (\rho z \cos \phi \sin \phi - 3\rho \cos \phi \sin \phi) \mathbf{a}_x + (\rho z \sin^2 \phi + 3\rho \cos^2 \phi) \mathbf{a}_y + \rho \cos \phi \sin \phi \mathbf{a}_z.$$

$$\text{But } \rho = \sqrt{x^2 + y^2}, \quad \tan \phi = \frac{y}{x}, \quad \cos \phi = \frac{x}{\sqrt{x^2 + y^2}}, \quad \sin \phi = \frac{y}{\sqrt{x^2 + y^2}};$$

Substituting all this yields:

$$A = \frac{1}{\sqrt{x^2 + y^2}} [(xyz - 3xy) \mathbf{a}_x + (zy^2 + 3x^2) \mathbf{a}_y + xy \mathbf{a}_z].$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\ \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} r^2 \\ 0 \\ \sin \theta \end{bmatrix}$$

$$\text{Since } r = \sqrt{x^2 + y^2 + z^2}, \quad \tan \theta = \frac{\sqrt{x^2 + y^2}}{z}, \quad \tan \phi = \frac{y}{x};$$

$$\text{and } \sin \theta = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}, \quad \cos \theta = \frac{z}{\sqrt{x^2 + y^2 + z^2}};$$

$$\text{and } \sin \phi = \frac{y}{\sqrt{x^2 + y^2}}, \quad \cos \phi = \frac{x}{\sqrt{x^2 + y^2}};$$

$$B_x = r^2 \sin \theta \cos \phi - \sin \theta \sin \phi = rx - \frac{y}{r} = \frac{1}{r}(r^2 x - y).$$

$$B_y = r^2 \sin \theta \sin \phi + \sin \theta \cos \phi = ry + \frac{x}{r} = \frac{1}{r}(r^2 y + x).$$

$$B_z = r^2 \cos \theta = rz = \frac{1}{r}(r^2 z).$$

Hence,

$$\mathbf{B} = \frac{1}{\sqrt{x^2 + y^2 + z^2}} [\{x(x^2 + y^2 + z^2) - y\} \mathbf{a}_x + \{y(x^2 + y^2 + z^2) + x\} \mathbf{a}_y + z(x^2 + y^2 + z^2) \mathbf{a}_z].$$

P.E.2.3 (a) At:

$$(1, \pi/3, 0), \quad \mathbf{H} = (0, 0.06767, 1)$$

$$\mathbf{a}_x = \cos \phi \mathbf{a}_\rho - \sin \phi \mathbf{a}_\phi = \frac{1}{2}(\mathbf{a}_\rho - \sqrt{3} \mathbf{a}_\phi)$$

$$\mathbf{H} \bullet \mathbf{a}_x = \underline{\underline{-0.0586.}}$$

(b) At:

$$(1, \pi/3, 0), \quad \mathbf{a}_\theta = \cos \theta \mathbf{a}_\rho - \sin \theta \mathbf{a}_z = -\mathbf{a}_z.$$

$$\mathbf{H} \times \mathbf{a}_z = \begin{vmatrix} \mathbf{a}_\rho & \mathbf{a}_\phi & \mathbf{a}_z \\ 0 & 0.06767 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \underline{\underline{-0.06767 \mathbf{a}_\rho.}}$$

$$(c) \quad (\mathbf{H} \bullet \mathbf{a}_\rho) \mathbf{a}_\rho = \underline{\underline{0 \mathbf{a}_\rho.}}$$

$$(d) \quad \mathbf{H} \times \mathbf{a}_z = \begin{vmatrix} \mathbf{a}_\rho & \mathbf{a}_\phi & \mathbf{a}_z \\ 0 & 0.06767 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0.06767 \mathbf{a}_\rho.$$

$$|\mathbf{H} \times \mathbf{a}_z| = \underline{\underline{0.06767}}$$

P.E. 2.4

(a)

$$\mathbf{A} \bullet \mathbf{B} = (3, 2, -6) \bullet (4, 0, 3) = \underline{\underline{-6.}}$$

$$(b) \quad |\mathbf{A} \times \mathbf{B}| = \begin{vmatrix} 3 & 2 & -6 \\ 4 & 0 & 3 \end{vmatrix} = |6\mathbf{a}_r - 33\mathbf{a}_\theta - 8\mathbf{a}_\phi|.$$

$$\text{Thus the magnitude of } \mathbf{A} \times \mathbf{B} = \underline{\underline{34.48.}}$$

(c)

$$\text{At } (1, \pi/3, 5\pi/4), \quad \theta = \pi/3,$$

$$\mathbf{a}_z = \cos \theta \mathbf{a}_r - \sin \theta \mathbf{a}_\theta = \frac{1}{2} \mathbf{a}_r - \frac{\sqrt{3}}{2} \mathbf{a}_\theta.$$

$$(\mathbf{A} \bullet \mathbf{a}_z) \mathbf{a}_z = \left(\frac{3}{2} - \sqrt{3} \right) \left(\frac{1}{2} \mathbf{a}_r - \frac{\sqrt{3}}{2} \mathbf{a}_\theta \right) = \underline{\underline{-0.116 \mathbf{a}_r + 0.201 \mathbf{a}_\theta}}$$

Prob. 2.1

(a)

$$\rho = \sqrt{x^2 + y^2} = \sqrt{4 + 25} = 5.3852, \quad \phi = \tan^{-1} \frac{y}{x} = \tan^{-1} 2.5 = 68.2^\circ$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{4 + 25 + 1} = 5.477, \quad \theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1} \frac{5.3852}{1} = 79.48^\circ$$

$$P(\rho, \phi, z) = \underline{\underline{P(5.3852, 68.2^\circ, 1)}}, \quad P(r, \theta, \phi) = \underline{\underline{P(5.477, 79.48^\circ, 68.2^\circ)}}$$

(b)

$$\rho = \sqrt{x^2 + y^2} = \sqrt{9 + 16} = 5, \quad \phi = \tan^{-1} \frac{y}{x} = \tan^{-1} \frac{4}{-3} = 360^\circ - 53.123^\circ = 306.88^\circ$$

$$r = \sqrt{x^2 + y^2 + z^2} = 5, \quad \theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1} \infty = 90^\circ$$

$$Q(\rho, \phi, z) = \underline{\underline{Q(5, 306.88^\circ, 0)}}, \quad P(r, \theta, \phi) = \underline{\underline{P(5, 90^\circ, 306.88^\circ)}}$$

(c)

$$\rho = \sqrt{x^2 + y^2} = \sqrt{36 + 4} = 6.325, \quad \phi = \tan^{-1} \frac{y}{x} = \tan^{-1} \frac{2}{6} = 18.43^\circ$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{36 + 4 + 16} = 7.483,$$

$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1} \frac{6.325}{-4} = 180^\circ - 57.69^\circ = 122.31^\circ$$

$$R(\rho, \phi, z) = \underline{\underline{R(6.325, 18.43^\circ, -4)}}, \quad R(r, \theta, \phi) = \underline{\underline{R(7.483, 122.31^\circ, 18.43^\circ)}}$$

Prob. 2.2

(a)

$$x = \rho \cos \phi = 2 \cos 30^\circ = 1.732;$$

$$y = \rho \sin \phi = 2 \sin 30^\circ = 1;$$

$$z = 5;$$

$$P_1(x, y, z) = \underline{\underline{P_1(1.732, 1, 5)}}.$$

(b)

$$x = 1 \cos 90^\circ = 0; \quad y = 1 \sin 90^\circ = 1; \quad z = -3.$$

$$P_2(x, y, z) = \underline{\underline{P_2(0, 1, -3)}}.$$

(c)

$$x = r \sin \theta \cos \phi = 10 \sin(\pi/4) \cos(\pi/3) = 3.535;$$

$$y = r \sin \theta \sin \phi = 10 \sin(\pi/4) \sin(\pi/3) = 6.124;$$

$$z = r \cos \theta = 10 \cos(\pi/4) = 7.0711$$

$$P_3(x, y, z) = \underline{\underline{P_3(3.535, 6.124, 7.0711)}}.$$

(d)

$$x = 4 \sin 30^\circ \cos 60^\circ = 1$$

$$y = 4 \sin 30^\circ \sin 60^\circ = 1.7321$$

$$z = r \cos \theta = 4 \cos 30^\circ = 3.464$$

$$P_4(x, y, z) = \underline{\underline{P_4(1, 1.7321, 3.464)}}.$$

Prob. 2.3

$$\rho = \sqrt{x^2 + y^2} = \sqrt{4 + 36} = 6.324$$

$$(a) \quad \phi = \tan^{-1} \frac{y}{x} = \tan^{-1} \frac{6}{2} = 71.56^\circ$$

$$P \text{ is } \underline{\underline{(6.324, 71.56^\circ, -4)}}$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{4 + 36 + 16} = 7.485$$

$$(b) \quad \theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1} \frac{6.324}{-4} = 90^\circ + \tan^{-1} \frac{4}{6.324} = 122.3^\circ$$

$$P \text{ is } \underline{\underline{(7.483, 122.3^\circ, 71.56^\circ)}}$$

Prob. 2.4

(a)

$$x = \rho \cos \phi = 5 \cos 120^\circ = -2.5$$

$$y = \rho \sin \phi = 5 \sin 120^\circ = 4.33$$

$$z = 1$$

$$\text{Hence } Q = \underline{\underline{(-2.5, 4.33, 1)}}$$

(b)

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{\rho^2 + z^2} = \sqrt{25 + 1} = 5.099$$

$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1} \frac{\rho}{z} = \tan^{-1} \frac{5}{1} = 78.69^\circ$$

$$\phi = 120^\circ$$

$$\text{Hence } Q = \underline{\underline{(5.099, 78.69^\circ, 120^\circ)}}$$

Prob. 2.5

$$T(r, \theta, \phi) \longrightarrow r = 10, \theta = 60^\circ, \phi = 30^\circ$$

$$x = r \sin \theta \cos \phi = 10 \sin 60^\circ \cos 30^\circ = 7.5$$

$$y = r \sin \theta \sin \phi = 10 \sin 60^\circ \sin 30^\circ = 4.33$$

$$z = r \cos \theta = 10 \cos 60^\circ = 5$$

$$T(x, y, z) = \underline{\underline{(7.5, 4.33, 5)}}$$

$$\rho = r \sin \theta = 10 \sin 60^\circ = 8.66$$

$$T(\rho, \phi, z) = \underline{\underline{(8.66, 30^\circ, 5)}}$$

Prob. 2.6

(a)

$$x = \rho \cos \phi, \quad y = \rho \sin \phi,$$

$$V = \underline{\underline{\rho z \cos \phi - \rho^2 \sin \phi \cos \phi + \rho z \sin \phi}}$$

(b)

$$U = x^2 + y^2 + z^2 + y^2 + 2z^2$$

$$= r^2 + r^2 \sin^2 \theta \sin^2 \phi + 2r^2 \cos^2 \theta$$

$$= \underline{\underline{r^2 [1 + \sin^2 \theta \sin^2 \phi + 2 \cos^2 \theta]}}$$

Prob. 2.7

(a)

$$\begin{bmatrix} F_\rho \\ F_\phi \\ F_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{x}{\sqrt{\rho^2 + z^2}} \\ \frac{y}{\sqrt{\rho^2 + z^2}} \\ \frac{4}{\sqrt{\rho^2 + z^2}} \end{bmatrix}$$

$$F_\rho = \frac{1}{\sqrt{\rho^2 + z^2}} [\rho \cos^2 \phi + \rho \sin^2 \phi] = \frac{\rho}{\sqrt{\rho^2 + z^2}};$$

$$F_\phi = \frac{1}{\sqrt{\rho^2 + z^2}} [-\rho \cos \phi \sin \phi + \rho \cos \phi \sin \phi] = 0;$$

$$F_z = \frac{4}{\sqrt{\rho^2 + z^2}};$$

$$\underline{\underline{\bar{F} = \frac{1}{\sqrt{\rho^2 + z^2}} (\rho \mathbf{a}_\rho + 4 \mathbf{a}_z)}}$$

In Spherical:

$$\begin{bmatrix} F_r \\ F_\theta \\ F_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{bmatrix} \begin{bmatrix} \frac{x}{r} \\ \frac{y}{r} \\ \frac{4}{r} \end{bmatrix}$$

$$F_r = \frac{r}{r} \sin^2 \theta \cos^2 \phi + \frac{r}{r} \sin^2 \theta \sin^2 \phi + \frac{4}{r} \cos \theta = \sin^2 \theta + \frac{4}{r} \cos \theta;$$

$$F_\theta = \sin \theta \cos \theta \cos^2 \phi + \sin \theta \cos \theta \sin^2 \phi - \frac{4}{r} \sin \theta = \sin \theta \cos \theta - \frac{4}{r} \sin \theta;$$

$$F_\phi = -\sin \theta \cos \phi \sin \phi + \sin \theta \sin \phi \cos \phi = 0;$$

$$\underline{\underline{\therefore \bar{F} = (\sin^2 \theta + \frac{4}{r} \cos \theta) \mathbf{a}_r + \sin \theta (\cos \theta - \frac{4}{r}) \mathbf{a}_\theta}}$$

(b)

$$\begin{bmatrix} G_\rho \\ G_\phi \\ G_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{x\rho^2}{\sqrt{\rho^2 + z^2}} \\ \frac{y\rho^2}{\sqrt{\rho^2 + z^2}} \\ \frac{z\rho^2}{\sqrt{\rho^2 + z^2}} \end{bmatrix}$$

$$G_\rho = \frac{\rho^2}{\sqrt{\rho^2 + z^2}} [\rho \cos^2 \phi + \rho \sin^2 \phi] = \frac{\rho^3}{\sqrt{\rho^2 + z^2}};$$

$$G_\phi = 0;$$

$$G_z = \frac{z\rho^2}{\sqrt{\rho^2 + z^2}};$$

$$\underline{\underline{\mathbf{G} = \frac{\rho^2}{\sqrt{\rho^2 + z^2}} (\rho \mathbf{a}_\rho + z \mathbf{a}_z)}}$$

Spherical :

$$\mathbf{G} = \frac{\rho^2}{r} (x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z) = \frac{r^2 \sin^2 \theta}{r} r\mathbf{a}_r = \underline{\underline{r^2 \sin^2 \theta \mathbf{a}_r}}$$

Prob. 2.8

$$\mathbf{B} = \rho \mathbf{a}_x + \frac{y}{\rho} \mathbf{a}_y + z \mathbf{a}_z$$

$$\begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho \\ y/\rho \\ z \end{bmatrix}$$

$$B_\rho = \rho \cos \phi + \frac{y}{\rho} \sin \phi$$

$$B_\phi = -\rho \sin \phi + \frac{y}{\rho} \cos \phi$$

$$B_z = z$$

$$\text{But } y = \rho \sin \phi$$

$$B_\rho = \rho \cos \phi + \sin^2 \phi, B_\phi = -\rho \sin \phi + \sin \phi \cos \phi$$

Hence,

$$\underline{\underline{\mathbf{B} = (\rho \cos \phi + \sin^2 \phi) \mathbf{a}_\rho + \sin \phi (\cos \phi - \rho) \mathbf{a}_\phi + z \mathbf{a}_z}}$$

Prob. 2.9

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\text{At } P, \rho = 2, \quad \phi = \pi/2, \quad z = -1$$

$$A_x = 2 \cos \phi - 3 \sin \phi = 2 \cos 90^\circ - 3 \sin 90^\circ = -3$$

$$A_y = 2 \sin \phi + 3 \cos \phi = 2 \sin 90^\circ + 3 \cos 90^\circ = 2$$

$$A_z = 4$$

$$\text{Hence, } \underline{\underline{A = -3\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z}}$$

Prob. 2.10

(a)

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho \sin \phi \\ \rho \cos \phi \\ -2z \end{bmatrix}$$

$$A_x = \rho \sin \phi \cos \phi - \rho \cos \phi \sin \phi = 0$$

$$A_y = \rho \sin^2 \phi + \rho \cos^2 \phi = \rho = \sqrt{x^2 + y^2}$$

$$A_z = -2z$$

Hence,

$$\underline{\underline{A = \sqrt{x^2 + y^2} \mathbf{a}_y - 2z \mathbf{a}_z}}$$

(b)

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\ \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} 4r \cos \phi \\ r \\ 0 \end{bmatrix}$$

$$B_x = 4r \sin \theta \cos^2 \phi + r \cos \theta \cos \phi$$

$$B_y = 4r \sin \theta \sin \phi \cos \phi + r \cos \theta \sin \phi$$

$$B_z = 4r \cos \theta \cos \phi - r \sin \theta$$

$$\text{But } r = \sqrt{x^2 + y^2 + z^2}, \quad \sin \theta = \frac{\sqrt{x^2 + y^2}}{r}, \quad \cos \theta = \frac{z}{r}$$

$$\sin \phi = \frac{y}{\sqrt{x^2 + y^2}}, \quad \cos \phi = \frac{x}{\sqrt{x^2 + y^2}}$$

$$B_x = 4\sqrt{x^2 + y^2} \frac{x^2}{x^2 + y^2} + \frac{zx}{\sqrt{x^2 + y^2}}$$

$$B_y = 4\sqrt{x^2 + y^2} \frac{xy}{x^2 + y^2} + \frac{zy}{\sqrt{x^2 + y^2}}$$

$$B_z = 4z \frac{x}{\sqrt{x^2 + y^2}} - \sqrt{x^2 + y^2}$$

$$\mathbf{B} = \frac{1}{\sqrt{x^2 + y^2}} \left[x(4x + z)\mathbf{a}_x + y(4x + z)\mathbf{a}_y + (4xz - x^2 - y^2)\mathbf{a}_z \right]$$

Prob. 2.11

Method 1:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\ \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} 4/r^2 \\ 0 \\ 0 \end{bmatrix}$$

$$F_x = \frac{4}{r^2} \sin \theta \cos \phi, \quad F_y = \frac{4}{r^2} \sin \theta \sin \phi, \quad F_z = \frac{4}{r^2} \cos \theta$$

$$r^2 = x^2 + y^2 + z^2, \quad \sin \theta = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}, \quad \cos \theta = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\sin \phi = \frac{y}{\sqrt{x^2 + y^2}}, \quad \cos \phi = \frac{x}{\sqrt{x^2 + y^2}}$$

$$F_x = \frac{4}{x^2 + y^2 + z^2} \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}} \frac{x}{\sqrt{x^2 + y^2}} = \frac{4x}{(x^2 + y^2 + z^2)^{3/2}}$$

$$F_y = \frac{4}{x^2 + y^2 + z^2} \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}} \frac{y}{\sqrt{x^2 + y^2}} = \frac{4y}{(x^2 + y^2 + z^2)^{3/2}}$$

$$F_z = \frac{4}{x^2 + y^2 + z^2} \frac{z}{(x^2 + y^2 + z^2)} = \frac{4z}{(x^2 + y^2 + z^2)^{3/2}}$$

Thus,

$$\mathbf{F} = \frac{4}{(x^2 + y^2 + z^2)^{3/2}} \left[x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z \right]$$

Method 2:

$$\mathbf{F} = \frac{4\mathbf{a}_r}{r^2} \cdot \frac{\mathbf{r}}{r} = \frac{4r\mathbf{a}_r}{r^3}$$

$$\mathbf{F} = \frac{4}{(x^2 + y^2 + z^2)^{3/2}} [x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z]$$

Prob. 2.12

$$r = 2, \quad \theta = \pi/2, \quad \phi = 3\pi/2$$

$$(a) \quad \mathbf{B} = 2 \sin(\pi/2)\mathbf{a}_r - 4 \cos(3\pi/2)\mathbf{a}_\phi = \underline{\underline{2\mathbf{a}_r}}$$

(b)

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\ \cos \theta \cos \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} r \sin \theta \\ 0 \\ -r^2 \cos \phi \end{bmatrix}$$

$$B_x = r \sin^2 \theta \cos \phi - r^2 \sin \phi \cos \phi, \quad B_y = r \sin \theta \cos \theta \cos \phi - r^2 \cos^2 \phi$$

$$B_z = r \sin \theta \cos \theta$$

$$\text{But } r = \sqrt{x^2 + y^2 + z^2}, \cos \theta = \frac{z}{r}, \sin \theta = \frac{\rho}{r} = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos \phi = \frac{x}{\rho} = \frac{x}{\sqrt{x^2 + y^2}}, \quad \sin \phi = \frac{y}{\rho} = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\begin{aligned} B_x &= \sqrt{x^2 + y^2 + z^2} \frac{x^2 + y^2}{x^2 + y^2 + z^2} \frac{x}{\sqrt{x^2 + y^2}} - (x^2 + y^2 + z^2) \frac{xy}{x^2 + y^2} \\ &= \frac{x\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}} - \frac{xy(x^2 + y^2 + z^2)}{x^2 + y^2} \end{aligned}$$

$$\begin{aligned} B_y &= \sqrt{x^2 + y^2 + z^2} \frac{z\sqrt{x^2 + y^2}}{x^2 + y^2 + z^2} \frac{x}{\sqrt{x^2 + y^2}} - (x^2 + y^2 + z^2) \frac{x^2}{x^2 + y^2} \\ &= \frac{xz}{\sqrt{x^2 + y^2 + z^2}} - \frac{x^2(x^2 + y^2 + z^2)}{x^2 + y^2} \end{aligned}$$

$$B_z = \sqrt{x^2 + y^2 + z^2} \frac{z\sqrt{x^2 + y^2}}{x^2 + y^2 + z^2} = \frac{z\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}$$

$$\mathbf{B} = B_x \mathbf{a}_x + B_y \mathbf{a}_y + B_z \mathbf{a}_z$$

Prob. 2.13

$$(a) \quad x = \rho \cos \phi$$

$$\underline{\underline{B = \rho \cos \phi \mathbf{a}_z}}$$

$$x = r \sin \theta \cos \phi$$

$$(b) \quad \mathbf{B} = r \sin \theta \cos \phi \mathbf{a}_z, \quad B_x = 0 = B_y, B_z = r \sin \theta \cos \phi$$

$$\begin{bmatrix} B_r \\ B_\theta \\ B_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ r \sin \theta \cos \phi \end{bmatrix}$$

$$B_r = r \sin \theta \cos \theta \cos \phi = 0.5r \sin(2\theta) \cos \phi$$

$$B_\theta = -r \sin^2 \theta \cos \phi, \quad B_\phi = 0$$

$$\underline{\underline{\mathbf{B} = 0.5r \sin(2\theta) \cos \phi \mathbf{a}_r - r \sin^2 \theta \cos \phi \mathbf{a}_\theta}}$$

Prob. 2.14

(a)

$$\mathbf{a}_x \times \mathbf{a}_\rho = (\cos \phi \mathbf{a}_\rho - \sin \phi \mathbf{a}_\phi) \times \mathbf{a}_\rho = \cos \phi$$

$$\mathbf{a}_x \times \mathbf{a}_\phi = (\cos \phi \mathbf{a}_\rho - \sin \phi \mathbf{a}_\phi) \times \mathbf{a}_\phi = -\sin \phi$$

$$\mathbf{a}_y \times \mathbf{a}_\rho = (\sin \phi \mathbf{a}_\rho + \cos \phi \mathbf{a}_\phi) \times \mathbf{a}_\rho = \sin \phi$$

$$\bar{\mathbf{a}}_y \times \bar{\mathbf{a}}_\phi = (\sin \phi \mathbf{a}_\rho + \sin \phi \mathbf{a}_\phi) \times \mathbf{a}_\phi = \cos \phi$$

(b) and (c)

In spherical system :

$$\mathbf{a}_x = \sin \theta \cos \phi \mathbf{a}_r + \cos \theta \cos \phi \mathbf{a}_\theta - \sin \phi \mathbf{a}_\phi.$$

$$\mathbf{a}_y = \sin \theta \sin \phi \mathbf{a}_r + \cos \theta \sin \phi \mathbf{a}_\theta - \cos \phi \mathbf{a}_\phi.$$

$$\mathbf{a}_z = \cos \theta \mathbf{a}_r - \sin \theta \mathbf{a}_\theta.$$

Hence,

$$\mathbf{a}_x \times \mathbf{a}_r = \sin \theta \cos \phi;$$

$$\mathbf{a}_x \times \mathbf{a}_\theta = \cos \theta \cos \phi;$$

$$\mathbf{a}_y \times \mathbf{a}_r = \sin \theta \sin \phi;$$

$$\mathbf{a}_y \times \mathbf{a}_\theta = \cos \theta \sin \phi;$$

$$\bar{\mathbf{a}}_z \times \bar{\mathbf{a}}_r = \cos \theta;$$

$$\bar{\mathbf{a}}_z \times \bar{\mathbf{a}}_\theta = -\sin \theta;$$

Prob. 2.15

(a)

$$\mathbf{a}_\rho = \cos \phi \mathbf{a}_x + \sin \phi \mathbf{a}_y, \quad \mathbf{a}_\phi = -\sin \phi \mathbf{a}_x + \cos \phi \mathbf{a}_y$$

$$\mathbf{a}_\rho \times \mathbf{a}_\phi = \begin{vmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \end{vmatrix} = (\cos^2 \phi + \sin^2 \phi) \mathbf{a}_z = \mathbf{a}_z$$

$$\mathbf{a}_z \times \mathbf{a}_\rho = \begin{vmatrix} 0 & 0 & 1 \\ \cos \phi & \sin \phi & 0 \end{vmatrix} = -\sin \phi \mathbf{a}_x + \cos \phi \mathbf{a}_y = \mathbf{a}_\phi$$

$$\mathbf{a}_\phi \times \mathbf{a}_z = \begin{vmatrix} -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos \phi \mathbf{a}_x + \sin \phi \mathbf{a}_y = \mathbf{a}_\rho$$

(b)

$$\mathbf{a}_r = \sin \theta \cos \phi \mathbf{a}_x + \sin \theta \sin \phi \mathbf{a}_y + \cos \theta \mathbf{a}_z$$

$$\mathbf{a}_\theta = \cos \theta \cos \phi \mathbf{a}_x + \cos \theta \sin \phi \mathbf{a}_y - \sin \theta \mathbf{a}_z$$

$$\mathbf{a}_\phi = -\sin \phi \mathbf{a}_x + \cos \phi \mathbf{a}_y$$

$$\mathbf{a}_r \times \mathbf{a}_\theta = \begin{vmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \end{vmatrix}$$

$$= (-\sin^2 \theta \sin \phi - \cos^2 \theta \sin \phi) \mathbf{a}_x + (\cos^2 \theta \cos \phi + \sin^2 \theta \cos \phi) \mathbf{a}_y$$

$$+ (\sin \theta \cos \theta \sin \phi \cos \phi - \sin \theta \cos \theta \sin \phi \cos \phi) \mathbf{a}_z$$

$$= -\sin \phi \mathbf{a}_x + \cos \phi \mathbf{a}_y = \mathbf{a}_\phi$$

$$\begin{aligned}
 \mathbf{a}_\phi \times \mathbf{a}_r &= \begin{vmatrix} -\sin \phi & \cos \phi & 0 \\ \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \end{vmatrix} \\
 &= \cos \theta \cos \phi \mathbf{a}_x + \cos \theta \sin \phi \mathbf{a}_y + (-\sin \theta \sin^2 \phi - \sin \theta \cos^2 \phi) \mathbf{a}_z \\
 &= \cos \theta \cos \phi \mathbf{a}_x + \cos \theta \sin \phi \mathbf{a}_y - \sin \theta \mathbf{a}_z = \mathbf{a}_\theta \\
 \mathbf{a}_\theta \times \mathbf{a}_\phi &= \begin{vmatrix} \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{vmatrix} \\
 &= \sin \theta \cos \phi \mathbf{a}_x + \sin \theta \sin \phi \mathbf{a}_y + (\cos \theta \cos^2 \phi + \cos \theta \sin^2 \phi) \mathbf{a}_z \\
 &= \sin \theta \cos \phi \mathbf{a}_x + \sin \theta \sin \phi \mathbf{a}_y + \cos \theta \mathbf{a}_z = \mathbf{a}_r
 \end{aligned}$$

Prob. 2.16

(a)

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{\rho^2 + z^2}.$$

$$\theta = \tan^{-1} \frac{\rho}{z}; \quad \phi = \phi.$$

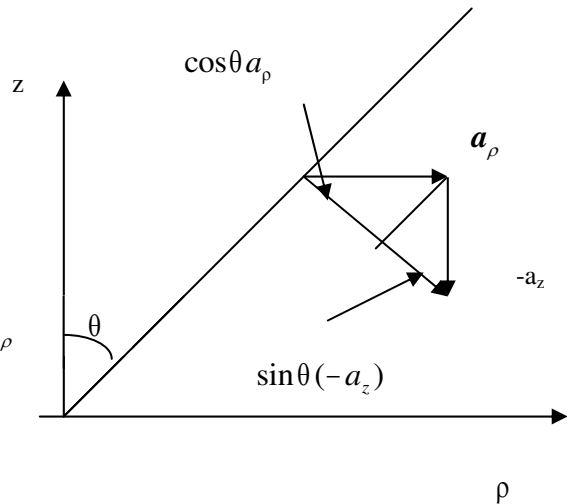
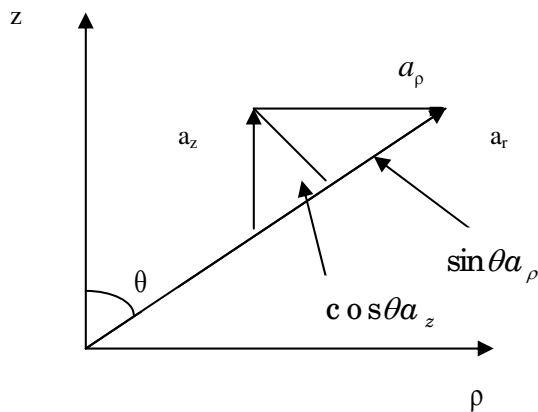
or

$$\rho = \sqrt{x^2 + y^2} = \sqrt{r^2 \sin^2 \theta \cos^2 \phi + r^2 \sin^2 \theta \sin^2 \phi}.$$

$$= r \sin \theta;$$

$$z = r \cos \theta; \quad \phi = \phi.$$

(b) From the figures below,



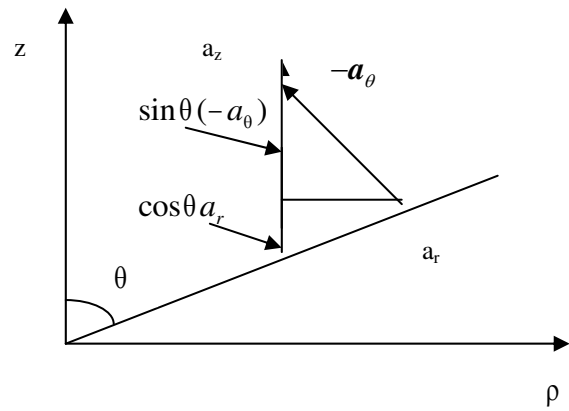
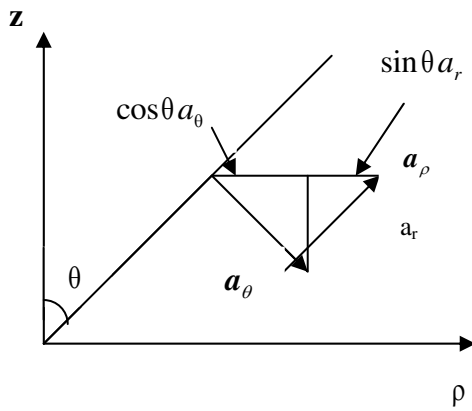
$$\mathbf{a}_r = \sin\theta \mathbf{a}_\rho + \cos\theta \mathbf{a}_z; \quad \mathbf{a}_\theta = \cos\theta \mathbf{a}_\rho - \sin\theta \mathbf{a}_z; \quad \mathbf{a}_\phi = \mathbf{a}_\phi.$$

Hence,

$$\begin{bmatrix} \mathbf{a}_r \\ \mathbf{a}_\theta \\ \mathbf{a}_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{a}_\rho \\ \mathbf{a}_\phi \\ \mathbf{a}_z \end{bmatrix}$$

From the figures below,

$$\mathbf{a}_\rho = \cos\theta \mathbf{a}_\theta + \sin\theta \mathbf{a}_r; \quad \mathbf{a}_z = \cos\theta \mathbf{a}_r - \sin\theta \mathbf{a}_\theta; \quad \mathbf{a}_\phi = \mathbf{a}_\phi.$$



$$\begin{bmatrix} \mathbf{a}_\rho \\ \mathbf{a}_\phi \\ \mathbf{a}_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \mathbf{a}_r \\ \mathbf{a}_\theta \\ \mathbf{a}_z \end{bmatrix}$$

Prob. 2.17

$$\text{At } P(2, 0, -1), \quad \phi = 0, \quad \theta = \cos^{-1} \left(\frac{z}{\sqrt{x^2 + y^2 + z^2}} \right) = \cos^{-1} \left(\frac{-1}{\sqrt{5}} \right) = 116.56^\circ$$

- (a) $\mathbf{a}_\rho \bullet \mathbf{a}_x = \cos\phi = \underline{\underline{1}}$
- (b) $\mathbf{a}_\phi \bullet \mathbf{a}_y = \cos\phi = \underline{\underline{1}}$
- (c) $\mathbf{a}_r \bullet \mathbf{a}_z = \cos\theta = \underline{\underline{-0.4472}}$

Prob. 2.18

If **A** and **B** are perpendicular to each other, $\mathbf{A} \cdot \mathbf{B} = 0$

$$\begin{aligned}\mathbf{A} \cdot \mathbf{B} &= \rho^2 \sin^2 \phi + \rho^2 \cos^2 \phi - \rho^2 \\ &= \rho^2 (\sin^2 \phi + \cos^2 \phi) - \rho^2 \\ &= \rho^2 - \rho^2 \\ &= 0\end{aligned}$$

As expected.

Prob. 2.19

$$(a) \mathbf{A} + \mathbf{B} = \underline{\underline{8\mathbf{a}_\rho + 2\mathbf{a}_\phi - 7\mathbf{a}_z}}$$

$$(b) \mathbf{A} \cdot \mathbf{B} = 15 + 0 - 8 = \underline{\underline{7}}$$

$$\begin{aligned}(c) \mathbf{A} \times \mathbf{B} &= \begin{vmatrix} 3 & 2 & 1 \\ 5 & 0 & -8 \end{vmatrix} \\ &= -16\mathbf{a}_\rho + (5 + 24)\mathbf{a}_\phi - 10\mathbf{a}_z \\ &= \underline{\underline{-16\mathbf{a}_\rho + 29\mathbf{a}_\phi - 10\mathbf{a}_z}}\end{aligned}$$

$$\begin{aligned}(d) \cos \theta_{AB} &= \frac{\mathbf{A} \cdot \mathbf{B}}{AB} = \frac{7}{\sqrt{9+4+1}\sqrt{25+64}} = \frac{7}{\sqrt{14}\sqrt{89}} \\ &= 0.19831\end{aligned}$$

$$\underline{\underline{\theta_{AB} = 78.56^\circ}}$$

Prob. 2.20

$$\begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} G_\rho \\ G_\phi \\ G_z \end{bmatrix}$$

$$\begin{aligned}G_x &= G_\rho \cos \phi - G_\phi \sin \phi = 3\rho \cos \phi - \rho \cos \phi \sin \phi \\ &= 3x - x \sin \phi = 3(3) - (3) \sin(306.87^\circ) = 11.4\end{aligned}$$

$$\mathbf{G}_x = G_x \mathbf{a}_x = \underline{\underline{11.4\mathbf{a}_x}}$$

Prob. 2.21

$$\begin{bmatrix} G_\rho \\ G_\phi \\ G_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} yz \\ xz \\ xy \end{bmatrix}$$

$$G_\rho = yz \cos \phi + xz \sin \phi$$

$$x = \rho \cos \phi, y = \rho \sin \phi, \quad yz = \rho z \sin \phi, xz = \rho z \cos \phi$$

$$G_\rho = \rho z \sin \phi \cos \phi + \rho z \cos \phi \sin \phi = 2\rho z \sin \phi \cos \phi = \rho z \sin 2\phi$$

$$G_\phi = -yz \sin \phi + xz \cos \phi = \rho z (\cos^2 \phi - \sin^2 \phi) = \rho z \cos 2\phi$$

$$G_z = xy = \rho^2 \cos \phi \sin \phi = 0.5\rho^2 \sin 2\phi$$

$$\underline{\underline{G = \rho z \sin 2\phi \mathbf{a}_y + \rho z \cos 2\phi \mathbf{a}_\phi + 0.5\rho^2 \sin 2\phi \mathbf{a}_z}}$$

Prob. 2.22

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$= \begin{bmatrix} \frac{x}{\sqrt{x^2 + y^2}} & -\frac{y}{\sqrt{x^2 + y^2}} & 0 \\ \frac{y}{\sqrt{x^2 + y^2}} & \frac{x}{\sqrt{x^2 + y^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\ \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$= \begin{bmatrix} \frac{x}{\sqrt{x^2 + y^2 + z^2}} & \frac{xz}{\sqrt{x^2 + y^2} \sqrt{x^2 + y^2 + z^2}} & \frac{-y}{\sqrt{x^2 + y^2}} \\ \frac{y}{\sqrt{x^2 + y^2 + z^2}} & \frac{yz}{\sqrt{x^2 + y^2} \sqrt{x^2 + y^2 + z^2}} & \frac{x}{\sqrt{x^2 + y^2}} \\ \frac{z}{\sqrt{x^2 + y^2 + z^2}} & -\frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}} & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Prob. 2.23 (a) Using the results in Prob.2.14,

$$A_\rho = \rho z \sin \phi = r^2 \sin \theta \cos \theta \sin \phi$$

$$A_\phi = 3\rho \cos \phi = 3r \sin \theta \cos \phi$$

$$A_z = \rho \cos \phi \sin \phi = r \sin \theta \cos \phi \sin \phi$$

Hence,

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta & 0 & \cos \theta \\ \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r^2 \sin \theta \cos \theta \sin \phi \\ 3r \sin \theta \cos \phi \\ r \sin \theta \cos \phi \sin \phi \end{bmatrix}$$

$$\underline{\underline{A(r, \theta, \phi) = r \sin \theta \left[\sin \phi \cos \theta (r \sin \theta + \cos \phi) \mathbf{a}_r + \sin \phi (r \cos^2 \theta - \sin \theta \cos \phi) \mathbf{a}_\theta + 3 \cos \phi \mathbf{a}_\phi \right]}}$$

At $(10, \pi/2, 3\pi/4)$, $r = 10, \theta = \pi/2, \phi = 3\pi/4$

$$\underline{\underline{A = 10(0\mathbf{a}_r + 0.5\mathbf{a}_\theta - \frac{3}{\sqrt{2}}\mathbf{a}_\phi) = 5\mathbf{a}_\theta - 21.21\mathbf{a}_\phi}}$$

(b) $B_r = r^2 = (\rho^2 + z^2), \quad B_\theta = 0, \quad B_\phi = \sin \theta = \frac{\rho}{\sqrt{\rho^2 + z^2}}$

$$\begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} = \begin{bmatrix} \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} B_r \\ B_\theta \\ B_\phi \end{bmatrix}$$

$$\underline{\underline{B(\rho, \phi, z) = \sqrt{\rho^2 + z^2} \left(\rho \mathbf{a}_\rho + \frac{\rho}{\rho^2 + z^2} \mathbf{a}_\phi + z \mathbf{a}_z \right)}}$$

At $(2, \pi/6, 1)$, $\rho = 2, \phi = \pi/6, z = 1$

$$\underline{\underline{B = \sqrt{5}(2\mathbf{a}_\rho + 0.4\mathbf{a}_\phi + \mathbf{a}_z) = 4.472\mathbf{a}_\rho + 0.8944\mathbf{a}_\phi + 2.236\mathbf{a}_z}}$$

Prob. 2.24

(a) $d = \sqrt{(6-2)^2 + (-1-1)^2 + (2-5)^2} = \sqrt{29} = \underline{\underline{5.385}}$

(b) $d^2 = 3^2 + 5^2 - 2(3)(5) \cos \pi + (-1-5)^2 = 100$
 $d = \sqrt{100} = \underline{\underline{10}}$

(c)

$$\begin{aligned}
 d^2 &= 10^2 + 5^2 - 2(10)(5) \cos \frac{\pi}{4} \cos \frac{\pi}{6} - 2(10)(5) \sin \frac{\pi}{4} \sin \frac{\pi}{6} \cos \left(7\frac{\pi}{4} - \frac{3\pi}{4} \right) \\
 &= 125 - 100(0.7071)(0.866) - 100(0.7071)(0.5)(-0.2334) \\
 &= 125 - 61.23 + 35.33 = 99.118 \\
 d &= \sqrt{99.118} = \underline{\underline{9.956}}.
 \end{aligned}$$

Prob. 2.25

Using eq. (2.33),

$$\begin{aligned}
 d^2 &= r_1^2 + r_2^2 - 2r_1r_2 \cos \theta_1 \cos \theta_2 - 2r_1r_2 \sin \theta_1 \sin \theta_2 \cos(\phi_2 - \phi_1) \\
 &= 16 + 36 - 2(4)(6) \cos 30^\circ \cos 90^\circ - 2(4)(6) \sin 30^\circ \sin 90^\circ \cos(180^\circ) \\
 &= 16 + 36 - 0 - 48(0.5)(1)(-1) = 52 + 24 = 76 \\
 d &= 8.718
 \end{aligned}$$

Prob. 2.26

$$\begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$a_\rho = \cos \phi a_x + \sin \phi a_y, \quad a_\phi = -\sin \phi a_x + \cos \phi a_y$$

$$\text{At } (0, 4, -1), \quad \phi = 90^\circ$$

$$a_\rho = \sin 90^\circ a_y = \underline{\underline{a_y}}$$

$$a_\phi = -\sin 90^\circ a_x = \underline{\underline{-a_x}}$$

Prob. 2.27

$$\text{At } (1, 60^\circ, -1), \quad \rho = 1, \phi = 60^\circ, z = -1,$$

$$\begin{aligned}
 \text{(a) } \mathbf{A} &= (-2 - \sin 60^\circ) \mathbf{a}_\rho + (4 + 2 \cos 60^\circ) \mathbf{a}_\phi - 3(1)(-1) \mathbf{a}_z \\
 &= -2.866 \mathbf{a}_\rho + 5 \mathbf{a}_\phi + 3 \mathbf{a}_z
 \end{aligned}$$

$$\mathbf{B} = 1 \cos 60^\circ \mathbf{a}_\rho + \sin 60^\circ \mathbf{a}_\phi + \mathbf{a}_z = 0.5 \mathbf{a}_\rho + 0.866 \mathbf{a}_\phi + \mathbf{a}_z$$

$$\mathbf{A} \cdot \mathbf{B} = -1.433 + 4.33 + 3 = 5.897$$

$$AB = \sqrt{2.866^2 + 26 + 9} \sqrt{0.25 + 1 + 0.866^2} = 9.1885$$

$$\cos \theta_{AB} = \frac{\mathbf{A} \cdot \mathbf{B}}{AB} = \frac{5.897}{9.1885} = 0.6419 \quad \longrightarrow \quad \theta_{AB} = \underline{\underline{50.07^\circ}}$$

Let $\mathbf{D} = \mathbf{A} \times \mathbf{B}$. At $(1, 90^\circ, 0)$, $\rho = 1, \phi = 90^\circ, z = 0$

$$(b) \mathbf{A} = -\sin 90^\circ \mathbf{a}_\rho + 4\mathbf{a}_\phi = -\mathbf{a}_\rho + 4\mathbf{a}_\phi$$

$$\mathbf{B} = 1 \cos 90^\circ \mathbf{a}_\rho + \sin 90^\circ \mathbf{a}_\phi + \mathbf{a}_z = \mathbf{a}_\phi + \mathbf{a}_z$$

$$\mathbf{D} = \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_\rho & \mathbf{a}_\phi & \mathbf{a}_z \\ -1 & 4 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 4\mathbf{a}_\rho + \mathbf{a}_\phi - \mathbf{a}_z$$

$$\mathbf{a}_D = \frac{\mathbf{D}}{D} = \frac{(4, 1, -1)}{\sqrt{16+1+1}} = \underline{\underline{0.9428\mathbf{a}_\rho + 0.2357\mathbf{a}_\phi - 0.2357\mathbf{a}_z}}$$

Prob.2.28

At $P(0, 2, -5)$, $\phi = 90^\circ$;

$$\begin{aligned} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} &= \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 1 \\ -3 \end{bmatrix} \end{aligned}$$

$$\mathbf{B} = -\mathbf{a}_x - 5\mathbf{a}_y - 3\mathbf{a}_z$$

$$(a) \mathbf{A} + \mathbf{B} = (2, 4, 10) + (-1, -5, -3)$$

$$= \underline{\underline{\mathbf{a}_x - \mathbf{a}_y + 7\mathbf{a}_z}}$$

$$(b) \cos \theta_{AB} = \frac{\mathbf{A} \cdot \mathbf{B}}{AB} = \frac{-52}{\sqrt{4200}}$$

$$\theta_{AB} = \cos^{-1}\left(\frac{-52}{\sqrt{4200}}\right) = \underline{\underline{143.36^\circ}}$$

$$(c) A_B = \mathbf{A} \cdot \mathbf{a}_B = \frac{\mathbf{A} \cdot \mathbf{B}}{B} = -\frac{52}{\sqrt{35}} = \underline{\underline{-8.789}}$$

Prob. 2.29

$$\mathbf{B} \cdot \mathbf{a}_x = B_x$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix}$$

$$\begin{aligned} B_x &= B_\rho \cos \phi - B_\phi \sin \phi = \rho^2 \sin \phi \cos \phi - (z-1) \cos \phi \sin \phi \\ &= 16(0.5) - (-2)(0.5) = 8 + 1 = \underline{\underline{9}} \end{aligned}$$

Prob. 2.30

$$\bar{\mathbf{G}} = \cos^2 \phi \mathbf{a}_x + \frac{2r \cos \theta \sin \phi}{r \sin \theta} \bar{\mathbf{a}}_y + (1 - \cos^2 \phi) \bar{\mathbf{a}}_z$$

$$= \cos^2 \phi \bar{\mathbf{a}}_x + 2 \cot \theta \sin \phi \bar{\mathbf{a}}_y + \sin^2 \phi \bar{\mathbf{a}}_z$$

$$\begin{pmatrix} G_r \\ G_\theta \\ G_\phi \end{pmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{bmatrix} \begin{bmatrix} \cos^2 \phi \\ 2 \cot \theta \sin \phi \\ \sin^2 \phi \end{bmatrix}$$

$$\begin{aligned} G_r &= \sin \theta \cos^3 \phi + 2 \cos \theta \sin^2 \phi + \cos \theta \sin^2 \phi \\ &= \sin \theta \cos^3 \phi + 3 \cos \theta \sin^2 \phi \end{aligned}$$

$$G_\theta = \cos \theta \cos^3 \phi + 2 \cot \theta \cos \theta \sin^2 \phi - \sin \theta \sin^2 \phi$$

$$G_\phi = -\sin \phi \cos^2 \phi + 2 \cot \theta \sin \phi \cos \phi$$

$$\begin{aligned} \bar{\mathbf{G}} &= [\sin \theta \cos^3 \phi + 3 \cos \theta \sin^2 \phi] \mathbf{a}_r \\ &\quad + [\cos \theta \cos^3 \phi + 2 \cot \theta \cos \theta \sin^2 \phi - \sin \theta \sin^2 \phi] \mathbf{a}_\theta \\ &\quad + \underline{\underline{\sin \phi \cos \phi (2 \cot \theta - \cos \phi) \mathbf{a}_\phi}} \end{aligned}$$

Prob. 2.31

- (a) An infinite line parallel to the z-axis.
- (b) Point (2,-1,10).
- (c) A circle of radius $r \sin \theta = 5$, i.e. the intersection of a cone and a sphere.
- (d) An infinite line parallel to the z-axis.
- (e) A semi-infinite line parallel to the x-y plane.
- (f) A semi-circle of radius 5 in the y-z plane.

Prob. 2.32

$$(a) \mathbf{J}_z = (\mathbf{J} \cdot \mathbf{a}_z) \mathbf{a}_z.$$

$$\text{At } (2, \pi/2, 3\pi/2), \mathbf{a}_z = \cos \theta \mathbf{a}_r - \sin \theta \mathbf{a}_\theta = -\mathbf{a}_\theta.$$

$$\mathbf{J}_z = -\cos 2\theta \sin \phi \mathbf{a}_\theta = -\cos \pi \sin(3\pi/2) \mathbf{a}_\theta = -\mathbf{a}_\theta.$$

$$(b) \mathbf{J}_\phi = \tan \frac{\theta}{2} \ln r \mathbf{a}_\phi = \tan \frac{\pi}{4} \ln 2 \mathbf{a}_\phi = \ln 2 \mathbf{a}_\phi = 0.6931 \mathbf{a}_\phi.$$

$$(c) \mathbf{J}_t = \mathbf{J} - \mathbf{J}_n = \mathbf{J} - \mathbf{J}_r = -\mathbf{a}_\theta + \ln 2 \mathbf{a}_\phi = \underline{\underline{-\mathbf{a}_\theta + 0.6931 \mathbf{a}_\phi}}$$

$$(d) \mathbf{J}_p = (\mathbf{J} \cdot \mathbf{a}_x) \mathbf{a}_x$$

$$\mathbf{a}_x = \sin \theta \cos \phi \mathbf{a}_r + \cos \theta \cos \phi \mathbf{a}_\theta - \sin \phi \mathbf{a}_\phi = \mathbf{a}_\phi.$$

$$\text{At } (2, \pi/2, 3\pi/2),$$

$$\mathbf{J}_p = \underline{\underline{\ln 2 \mathbf{a}_\phi.}}$$

Prob. 2.33

$$\mathbf{H} \cdot \mathbf{a}_x = H_x$$

$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho^2 \cos \phi \\ -\rho \sin \phi \\ 0 \end{bmatrix}$$

$$H_x = \rho^2 \cos^2 \phi + \rho \sin^2 \phi$$

$$\text{At P, } \rho = 2, \phi = 60^\circ, z = -1$$

$$H_x = 4(1/4) + 2(3/4) = 1 + 1.5 = \underline{\underline{2.5}}$$

Prob. 2.34

$$(a) 5 = \mathbf{r} \cdot \mathbf{a}_x + \mathbf{r} \cdot \mathbf{a}_y = x + y \quad \underline{\underline{\text{a plane}}}$$

$$(b) 10 = |\mathbf{r} \times \mathbf{a}_z| = \begin{vmatrix} x & y & z \\ 0 & 0 & 1 \end{vmatrix} = |y \mathbf{a}_x - x \mathbf{a}_y| = \sqrt{x^2 + y^2} = \rho$$

a cylinder of infinite length