

Precalculus 12th edition Sullivan Solutions Manual

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INSTRUCTOR SOLUTIONS MANUAL

TIM BRITT

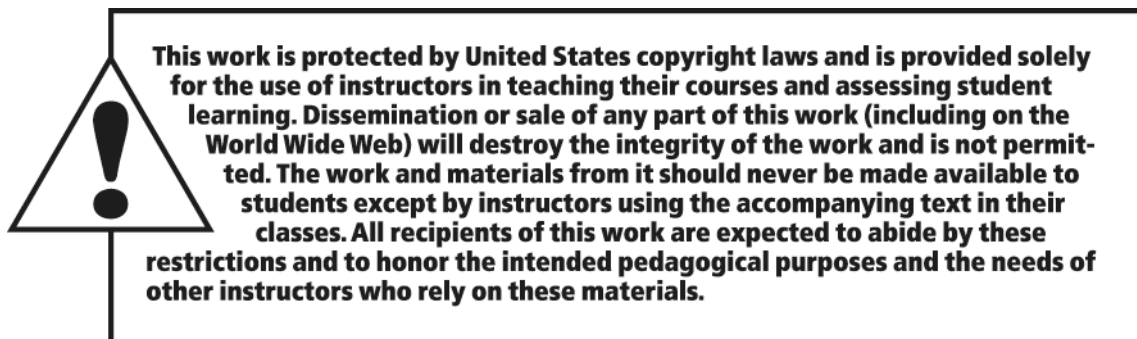
Jackson State Community College

PRECALCULUS TWELFTH EDITION

Michael Sullivan

Chicago State University





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Table of Contents

Chapter 1 Graphs

1.1 The Distance and Midpoint Formulas.....	1
1.2 Graphs of Equations in Two Variables; Intercepts; Symmetry	13
1.3 Lines	26
1.4 Circles	44
Chapter Review.....	57
Chapter Test.....	63
Chapter Projects	64

Chapter 2 Functions and Their Graphs

2.1 Functions.....	65
2.2 The Graph of a Function.....	83
2.3 Properties of Functions	92
2.4 Library of Functions; Piecewise-defined Functions	109
2.5 Graphing Techniques: Transformations	121
2.6 Mathematical Models: Building Functions.....	139
Chapter Review.....	147
Chapter Test.....	154
Cumulative Review	157
Chapter Projects	161

Chapter 3 Linear and Quadratic Functions

3.1 Properties of Linear Functions and Linear Models.....	163
3.2 Building Linear Functions from Data	174
3.3 Quadratic Functions and Their Properties	180
3.4 Build Quadratic Models from Verbal Descriptions and from Data	204
3.5 Inequalities Involving Quadratic Functions.....	211
Chapter Review.....	231
Chapter Test.....	239
Cumulative Review.....	241
Chapter Projects	244

Chapter 4 Polynomial and Rational Functions

4.1 Polynomial Functions	247
4.2 Graphing Polynomial Functions; Models	257
4.3 Properties of Rational Functions.....	273
4.4 The Graph of a Rational Function	283
4.5 Polynomial and Rational Inequalities	339
4.6 The Real Zeros of a Polynomial Function	360
4.7 Complex Zeros; Fundamental Theorem of Algebra	391
Chapter Review.....	400
Chapter Test.....	415
Cumulative Review.....	419
Chapter Projects	424

Chapter 5 Exponential and Logarithmic Functions

5.1 Composite Functions	426
5.2 One-to-One Functions; Inverse Functions	444
5.3 Exponential Functions	467
5.4 Logarithmic Functions	488
5.5 Properties of Logarithms	510
5.6 Logarithmic and Exponential Equations	519
5.7 Financial Models	540
5.8 Exponential Growth and Decay Models; Newton's Law; Logistic Growth and Decay Models	548
5.9 Building Exponential, Logarithmic, and Logistic Models from Data	558
Chapter Review	562
Chapter Test	575
Cumulative Review	579
Chapter Projects	582

Chapter 6 Trigonometric Functions

6.1 Angles, Arc Length, and Circular Motion	585
6.2 Trigonometric Functions: Unit Circle Approach	594
6.3 Properties of the Trigonometric Functions	612
6.4 Graphs of the Sine and Cosine Functions	626
6.5 Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions	648
6.6 Phase Shift; Sinusoidal Curve Fitting	658
Chapter Review	671
Chapter Test	679
Cumulative Review	683
Chapter Projects	687

Chapter 7 Analytic Trigonometry

7.1 The Inverse Sine, Cosine, and Tangent Functions	690
7.2 The Inverse Trigonometric Functions (Continued)	704
7.3 Trigonometric Equations	716
7.4 Trigonometric Identities	737
7.5 Sum and Difference Formulas	750
7.6 Double-angle and Half-angle Formulas	775
7.7 Product-to-Sum and Sum-to-Product Formulas	803
Chapter Review	816
Chapter Test	831
Cumulative Review	836
Chapter Projects	842

Chapter 8 Applications of Trigonometric Functions

8.1 Right Triangle Trigonometry; Applications	846
8.2 The Law of Sines	860
8.3 The Law of Cosines	875
8.4 Area of a Triangle	887
8.5 Simple Harmonic Motion; Damped Motion; Combining Waves	897
Chapter Review	907
Chapter Test	913
Cumulative Review	917
Chapter Projects	923

Chapter 9 Polar Coordinates; Vectors

9.1 Polar Coordinates.....	927
9.2 Polar Equations and Graphs.....	936
9.3 The Complex Plane; De Moivre's Theorem.....	965
9.4 Vectors.....	979
9.5 The Dot Product.....	992
9.6 Vectors in Space.....	998
9.7 The Cross Product.....	1005
Chapter Review.....	1016
Chapter Test.....	1025
Cumulative Review.....	1029
Chapter Projects.....	1031

Chapter 10 Analytic Geometry

10.2 The Parabola.....	1035
10.3 The Ellipse.....	1051
10.4 The Hyperbola.....	1068
10.5 Rotation of Axes; General Form of a Conic.....	1088
10.6 Polar Equations of Conics.....	1101
10.7 Plane Curves and Parametric Equations.....	1110
Chapter Review.....	1125
Chapter Test.....	1134
Cumulative Review.....	1139
Chapter Projects.....	1141

Chapter 11 Systems of Equations and Inequalities

11.1 Systems of Linear Equations: Substitution and Elimination.....	1145
11.2 Systems of Linear Equations: Matrices.....	1168
11.3 Systems of Linear Equations: Determinants.....	1192
11.4 Matrix Algebra.....	1206
11.5 Partial Fraction Decomposition.....	1225
11.6 Systems of Nonlinear Equations.....	1243
11.7 Systems of Inequalities.....	1271
11.8 Linear Programming.....	1286
Chapter Review.....	1300
Chapter Test.....	1315
Cumulative Review.....	1323
Chapter Projects.....	1327

Chapter 12 Sequences; Induction; the Binomial Theorem

12.1 Sequences.....	1329
12.2 Arithmetic Sequences.....	1339
12.3 Geometric Sequences; Geometric Series.....	1348
12.4 Mathematical Induction.....	1360
12.5 The Binomial Theorem.....	1369
Chapter Review.....	1376
Chapter Test.....	1380
Cumulative Review.....	1383
Chapter Projects.....	1386

Chapter 13 Counting and Probability

13.1 Counting.....	1389
13.2 Permutations and Combinations	1392
13.3 Probability.....	1397
Chapter Review.....	1404
Chapter Test.....	1406
Cumulative Review.....	1407
Chapter Projects.....	1410

Chapter 14 A Preview of Calculus: The Limit, Derivative, and Integral of a Function

14.1 Investigating Limits Using Tables and Graphs.....	1413
14.2 Algebraic Techniques for Finding Limits	1419
14.3 One-sided Limits; Continuity.....	1423
14.4 The Tangent Problem; The Derivative.....	1430
14.5 The Area Problem; The Integral	1439
Chapter Review.....	1453
Chapter Test.....	1460
Chapter Projects.....	1463

Appendix A Review

A.1 Algebra Essentials.....	1469
A.2 Geometry Essentials.....	1474
A.3 Polynomials.....	1480
A.4 Synthetic Division.....	1488
A.5 Rational Expressions.....	1490
A.6 Solving Equations	1495
A.7 Complex Numbers; Quadratic Equations in the Complex Number System	1509
A.8 Problem Solving: Interest, Mixture, Uniform Motion, Constant Rate Job Applications	1515
A.9 Interval Notation; Solving Inequalities	1522
A.10 n th Roots; Rational Exponents.....	1534

Appendix B Graphing Utilities

B.1 The Viewing Rectangle.....	1544
B.2 Using a Graphing Utility to Graph Equations	1545
B.3 Using a Graphing Utility to Locate Intercepts and Check for Symmetry	1550
B.5 Square Screens	1552

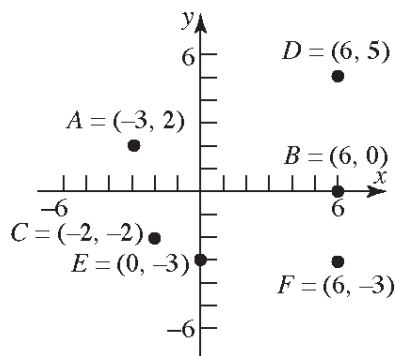
Chapter 1

Graphs

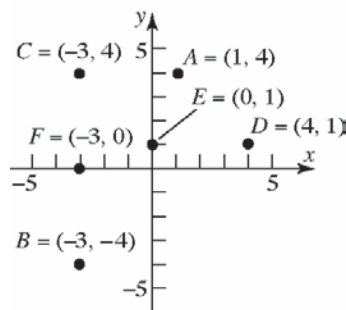
Section 1.1

1. 0
2. $|5 - (-3)| = |8| = 8$
3. $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$
4. $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$
Since the sum of the squares of two of the sides of the triangle equals the square of the third side, the triangle is a right triangle.
5. $\frac{1}{2}bh$
6. true
7. x -coordinate or abscissa; y -coordinate or ordinate
8. quadrants
9. midpoint
10. False; the distance between two points is never negative.
11. False; points that lie in Quadrant IV will have a positive x -coordinate and a negative y -coordinate. The point $(-1, 4)$ lies in Quadrant II.
12. True; $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
13. b
14. a
15. (a) Quadrant II
(b) x -axis
(c) Quadrant III
(d) Quadrant I
(e) y -axis

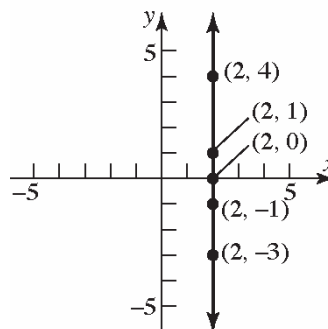
(f) Quadrant IV



16. (a) Quadrant I
(b) Quadrant III
(c) Quadrant II
(d) Quadrant I
(e) y -axis
(f) x -axis

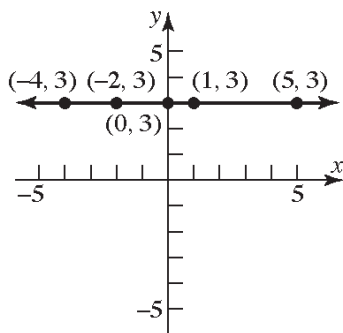


17. The points will be on a vertical line that is two units to the right of the y -axis.

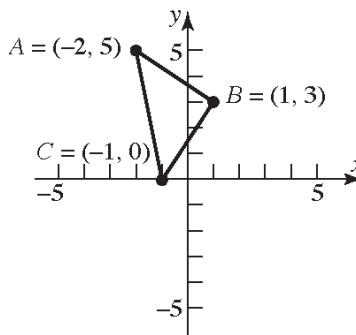


Chapter 1: Graphs

18. The points will be on a horizontal line that is three units above the x -axis.



19. $d(P_1, P_2) = \sqrt{(2-0)^2 + (1-0)^2}$
 $= \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$
20. $d(P_1, P_2) = \sqrt{(-2-0)^2 + (1-0)^2}$
 $= \sqrt{(-2)^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$
21. $d(P_1, P_2) = \sqrt{(-2-1)^2 + (2-1)^2}$
 $= \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$
22. $d(P_1, P_2) = \sqrt{(2-(-1))^2 + (2-1)^2}$
 $= \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$
23. $d(P_1, P_2) = \sqrt{(5-3)^2 + (4-(-4))^2}$
 $= \sqrt{2^2 + (8)^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$
24. $d(P_1, P_2) = \sqrt{(2-(-1))^2 + (4-0)^2}$
 $= \sqrt{(3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$
25. $d(P_1, P_2) = \sqrt{(4-(-7))^2 + (0-3)^2}$
 $= \sqrt{11^2 + (-3)^2} = \sqrt{121+9} = \sqrt{130}$
26. $d(P_1, P_2) = \sqrt{(4-2)^2 + (2-(-3))^2}$
 $= \sqrt{2^2 + 5^2} = \sqrt{4+25} = \sqrt{29}$
27. $d(P_1, P_2) = \sqrt{(6-5)^2 + (1-(-2))^2}$
 $= \sqrt{1^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$
28. $d(P_1, P_2) = \sqrt{(6-(-4))^2 + (2-(-3))^2}$
 $= \sqrt{10^2 + 5^2} = \sqrt{100+25}$
 $= \sqrt{125} = 5\sqrt{5}$
29. $d(P_1, P_2) = \sqrt{(2.3-(-0.2))^2 + (1.1-(-0.3))^2}$
 $= \sqrt{2.5^2 + 0.8^2} = \sqrt{6.25+0.64}$
 $= \sqrt{6.89} \approx 2.62$
30. $d(P_1, P_2) = \sqrt{(-0.3-1.2)^2 + (1.1-2.3)^2}$
 $= \sqrt{(-1.5)^2 + (-1.2)^2} = \sqrt{2.25+1.44}$
 $= \sqrt{3.69} \approx 1.92$
31. $d(P_1, P_2) = \sqrt{(0-a)^2 + (0-b)^2}$
 $= \sqrt{(-a)^2 + (-b)^2} = \sqrt{a^2 + b^2}$
32. $d(P_1, P_2) = \sqrt{(0-a)^2 + (0-a)^2}$
 $= \sqrt{(-a)^2 + (-a)^2}$
 $= \sqrt{a^2 + a^2} = \sqrt{2a^2} = |a|\sqrt{2}$
33. $A = (-2, 5), B = (1, 3), C = (-1, 0)$
 $d(A, B) = \sqrt{(1-(-2))^2 + (3-5)^2}$
 $= \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$
 $d(B, C) = \sqrt{(-1-1)^2 + (0-3)^2}$
 $= \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$
 $d(A, C) = \sqrt{(-1-(-2))^2 + (0-5)^2}$
 $= \sqrt{1^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}$



Section 1.1: The Distance and Midpoint Formulas

Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

$$(\sqrt{13})^2 + (\sqrt{13})^2 = (\sqrt{26})^2$$

$$13 + 13 = 26$$

$$26 = 26$$

The area of a triangle is $A = \frac{1}{2} \cdot bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A, B)] \cdot [d(B, C)]$$

$$= \frac{1}{2} \cdot \sqrt{13} \cdot \sqrt{13} = \frac{1}{2} \cdot 13$$

$$= \frac{13}{2} \text{ square units}$$

34. $A = (-2, 5)$, $B = (12, 3)$, $C = (10, -11)$

$$d(A, B) = \sqrt{(12 - (-2))^2 + (3 - 5)^2}$$

$$= \sqrt{14^2 + (-2)^2}$$

$$= \sqrt{196 + 4} = \sqrt{200}$$

$$= 10\sqrt{2}$$

$$d(B, C) = \sqrt{(10 - 12)^2 + (-11 - 3)^2}$$

$$= \sqrt{(-2)^2 + (-14)^2}$$

$$= \sqrt{4 + 196} = \sqrt{200}$$

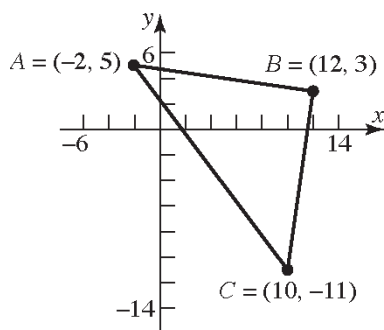
$$= 10\sqrt{2}$$

$$d(A, C) = \sqrt{(10 - (-2))^2 + (-11 - 5)^2}$$

$$= \sqrt{12^2 + (-16)^2}$$

$$= \sqrt{144 + 256} = \sqrt{400}$$

$$= 20$$



Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

$$(10\sqrt{2})^2 + (10\sqrt{2})^2 = (20)^2$$

$$200 + 200 = 400$$

$$400 = 400$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A, B)] \cdot [d(B, C)]$$

$$= \frac{1}{2} \cdot 10\sqrt{2} \cdot 10\sqrt{2}$$

$$= \frac{1}{2} \cdot 100 \cdot 2 = 100 \text{ square units}$$

35. $A = (-5, 3)$, $B = (6, 0)$, $C = (5, 5)$

$$d(A, B) = \sqrt{(6 - (-5))^2 + (0 - 3)^2}$$

$$= \sqrt{11^2 + (-3)^2} = \sqrt{121 + 9}$$

$$= \sqrt{130}$$

$$d(B, C) = \sqrt{(5 - 6)^2 + (5 - 0)^2}$$

$$= \sqrt{(-1)^2 + 5^2} = \sqrt{1 + 25}$$

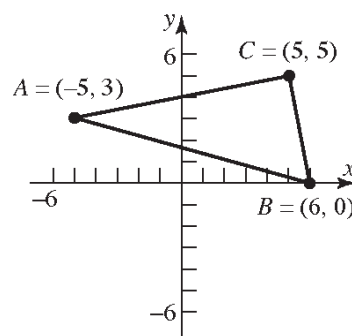
$$= \sqrt{26}$$

$$d(A, C) = \sqrt{(5 - (-5))^2 + (5 - 3)^2}$$

$$= \sqrt{10^2 + 2^2} = \sqrt{100 + 4}$$

$$= \sqrt{104}$$

$$= 2\sqrt{26}$$



Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

Chapter 1: Graphs

$$[d(A,C)]^2 + [d(B,C)]^2 = [d(A,B)]^2$$

$$(\sqrt{104})^2 + (\sqrt{26})^2 = (\sqrt{130})^2$$

$$104 + 26 = 130$$

$$130 = 130$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A,C)] \cdot [d(B,C)]$$

$$= \frac{1}{2} \cdot \sqrt{104} \cdot \sqrt{26}$$

$$= \frac{1}{2} \cdot 2\sqrt{26} \cdot \sqrt{26}$$

$$= \frac{1}{2} \cdot 2 \cdot 26$$

$$= 26 \text{ square units}$$

36. $A = (-6, 3)$, $B = (3, -5)$, $C = (-1, 5)$

$$d(A,B) = \sqrt{(3 - (-6))^2 + (-5 - 3)^2}$$

$$= \sqrt{9^2 + (-8)^2} = \sqrt{81 + 64}$$

$$= \sqrt{145}$$

$$d(B,C) = \sqrt{(-1 - 3)^2 + (5 - (-5))^2}$$

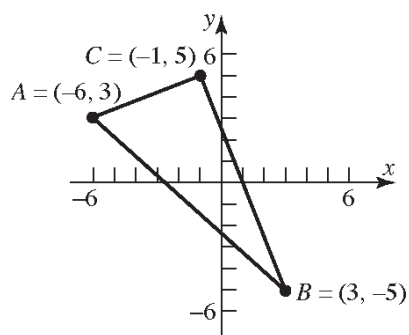
$$= \sqrt{(-4)^2 + 10^2} = \sqrt{16 + 100}$$

$$= \sqrt{116} = 2\sqrt{29}$$

$$d(A,C) = \sqrt{(-1 - (-6))^2 + (5 - 3)^2}$$

$$= \sqrt{5^2 + 2^2} = \sqrt{25 + 4}$$

$$= \sqrt{29}$$



Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

$$[d(A,C)]^2 + [d(B,C)]^2 = [d(A,B)]^2$$

$$(\sqrt{29})^2 + (2\sqrt{29})^2 = (\sqrt{145})^2$$

$$29 + 4 \cdot 29 = 145$$

$$29 + 116 = 145$$

$$145 = 145$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A,C)] \cdot [d(B,C)]$$

$$= \frac{1}{2} \cdot \sqrt{29} \cdot 2\sqrt{29}$$

$$= \frac{1}{2} \cdot 2 \cdot 29$$

$$= 29 \text{ square units}$$

37. $A = (4, -3)$, $B = (0, -3)$, $C = (4, 2)$

$$d(A,B) = \sqrt{(0 - 4)^2 + (-3 - (-3))^2}$$

$$= \sqrt{(-4)^2 + 0^2} = \sqrt{16 + 0}$$

$$= \sqrt{16}$$

$$= 4$$

$$d(B,C) = \sqrt{(4 - 0)^2 + (2 - (-3))^2}$$

$$= \sqrt{4^2 + 5^2} = \sqrt{16 + 25}$$

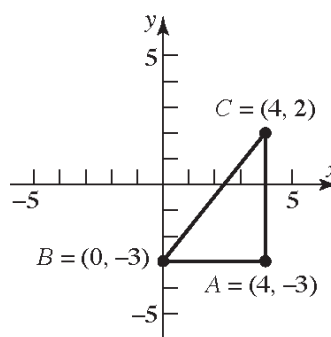
$$= \sqrt{41}$$

$$d(A,C) = \sqrt{(4 - 4)^2 + (2 - (-3))^2}$$

$$= \sqrt{0^2 + 5^2} = \sqrt{0 + 25}$$

$$= \sqrt{25}$$

$$= 5$$



Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

Section 1.1: The Distance and Midpoint Formulas

$$[d(A, B)]^2 + [d(A, C)]^2 = [d(B, C)]^2$$

$$4^2 + 5^2 = (\sqrt{41})^2$$

$$16 + 25 = 41$$

$$41 = 41$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A, B)] \cdot [d(A, C)]$$

$$= \frac{1}{2} \cdot 4 \cdot 5$$

$$= 10 \text{ square units}$$

38. $A = (4, -3), B = (4, 1), C = (2, 1)$

$$d(A, B) = \sqrt{(4-4)^2 + (1-(-3))^2}$$

$$= \sqrt{0^2 + 4^2}$$

$$= \sqrt{0+16}$$

$$= \sqrt{16}$$

$$= 4$$

$$d(B, C) = \sqrt{(2-4)^2 + (1-1)^2}$$

$$= \sqrt{(-2)^2 + 0^2} = \sqrt{4+0}$$

$$= \sqrt{4}$$

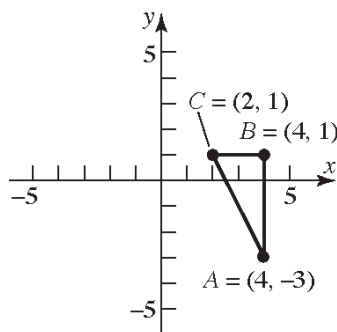
$$= 2$$

$$d(A, C) = \sqrt{(2-4)^2 + (1-(-3))^2}$$

$$= \sqrt{(-2)^2 + 4^2} = \sqrt{4+16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$



Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

$$4^2 + 2^2 = (2\sqrt{5})^2$$

$$16 + 4 = 20$$

$$20 = 20$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A, B)] \cdot [d(B, C)]$$

$$= \frac{1}{2} \cdot 4 \cdot 2$$

$$= 4 \text{ square units}$$

39. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{3+5}{2}, \frac{-4+4}{2} \right)$$

$$= \left(\frac{8}{2}, \frac{0}{2} \right)$$

$$= (4, 0)$$

40. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-2+2}{2}, \frac{0+4}{2} \right)$$

$$= \left(\frac{0}{2}, \frac{4}{2} \right)$$

$$= (0, 2)$$

41. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-1+8}{2}, \frac{4+0}{2} \right)$$

$$= \left(\frac{7}{2}, \frac{4}{2} \right)$$

$$= \left(\frac{7}{2}, 2 \right)$$

Chapter 1: Graphs

42. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{2+4}{2}, \frac{-3+2}{2} \right) \\&= \left(\frac{6}{2}, \frac{-1}{2} \right) \\&= \left(3, -\frac{1}{2} \right)\end{aligned}$$

43. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{7+9}{2}, \frac{-5+1}{2} \right) \\&= \left(\frac{16}{2}, \frac{-4}{2} \right) \\&= (8, -2)\end{aligned}$$

44. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{-4+2}{2}, \frac{-3+2}{2} \right) \\&= \left(\frac{-2}{2}, \frac{-1}{2} \right) \\&= \left(-1, -\frac{1}{2} \right)\end{aligned}$$

45. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{a+0}{2}, \frac{b+0}{2} \right) \\&= \left(\frac{a}{2}, \frac{b}{2} \right)\end{aligned}$$

46. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{a+0}{2}, \frac{a+0}{2} \right) \\&= \left(\frac{a}{2}, \frac{a}{2} \right)\end{aligned}$$

47. The x coordinate would be $2+3=5$ and the y coordinate would be $5-2=3$. Thus the new point would be $(5,3)$.

48. The new x coordinate would be $-1-2=-3$ and the new y coordinate would be $6+4=10$. Thus the new point would be $(-3,10)$.

49. a. If we use a right triangle to solve the problem, we know the hypotenuse is 13 units in length. One of the legs of the triangle will be $2+3=5$. Thus the other leg will be:

$$5^2 + b^2 = 13^2$$

$$25 + b^2 = 169$$

$$b^2 = 144$$

$$b = 12$$

Thus the coordinates will have an y value of $-1-12=-13$ and $-1+12=11$. So the points are $(3,11)$ and $(3,-13)$.

- b. Consider points of the form $(3, y)$ that are a distance of 13 units from the point $(-2, -1)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3 - (-2))^2 + (-1 - y)^2}$$

$$= \sqrt{(5)^2 + (-1 - y)^2}$$

$$= \sqrt{25 + 1 + 2y + y^2}$$

$$= \sqrt{y^2 + 2y + 26}$$

$$13 = \sqrt{y^2 + 2y + 26}$$

$$13^2 = \left(\sqrt{y^2 + 2y + 26} \right)^2$$

$$169 = y^2 + 2y + 26$$

$$0 = y^2 + 2y - 143$$

$$0 = (y - 11)(y + 13)$$

$$y - 11 = 0 \quad \text{or} \quad y + 13 = 0$$

$$y = 11 \quad \quad y = -13$$

Thus, the points $(3,11)$ and $(3,-13)$ are a distance of 13 units from the point $(-2,-1)$.

50. a. If we use a right triangle to solve the problem, we know the hypotenuse is 17 units in length. One of the legs of the triangle will be $2+6=8$. Thus the other leg will be:

$$8^2 + b^2 = 17^2$$

$$64 + b^2 = 289$$

$$b^2 = 225$$

$$b = 15$$

Thus the coordinates will have an x value of $1-15=-14$ and $1+15=16$. So the points are $(-14, -6)$ and $(16, -6)$.

- b. Consider points of the form $(x, -6)$ that are a distance of 17 units from the point $(1, 2)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(1 - x)^2 + (2 - (-6))^2}$$

$$= \sqrt{x^2 - 2x + 1 + (8)^2}$$

$$= \sqrt{x^2 - 2x + 1 + 64}$$

$$= \sqrt{x^2 - 2x + 65}$$

$$17 = \sqrt{x^2 - 2x + 65}$$

$$17^2 = (\sqrt{x^2 - 2x + 65})^2$$

$$289 = x^2 - 2x + 65$$

$$0 = x^2 - 2x - 224$$

$$0 = (x + 14)(x - 16)$$

$$x + 14 = 0 \quad \text{or} \quad x - 16 = 0$$

$$x = -14 \quad \quad \quad x = 16$$

Thus, the points $(-14, -6)$ and $(16, -6)$ are a distance of 17 units from the point $(1, 2)$.

51. Points on the x -axis have a y -coordinate of 0. Thus, we consider points of the form $(x, 0)$ that are a distance of 6 units from the point $(4, -3)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4 - x)^2 + (-3 - 0)^2}$$

$$= \sqrt{16 - 8x + x^2 + (-3)^2}$$

$$= \sqrt{16 - 8x + x^2 + 9}$$

$$= \sqrt{x^2 - 8x + 25}$$

$$6 = \sqrt{x^2 - 8x + 25}$$

$$6^2 = (\sqrt{x^2 - 8x + 25})^2$$

$$36 = x^2 - 8x + 25$$

$$0 = x^2 - 8x - 11$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-11)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{64 + 44}}{2} = \frac{8 \pm \sqrt{108}}{2}$$

$$= \frac{8 \pm 6\sqrt{3}}{2} = 4 \pm 3\sqrt{3}$$

$$x = 4 + 3\sqrt{3} \quad \text{or} \quad x = 4 - 3\sqrt{3}$$

Thus, the points $(4 + 3\sqrt{3}, 0)$ and $(4 - 3\sqrt{3}, 0)$ are on the x -axis and a distance of 6 units from the point $(4, -3)$.

52. Points on the y -axis have an x -coordinate of 0. Thus, we consider points of the form $(0, y)$ that are a distance of 6 units from the point $(4, -3)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4 - 0)^2 + (-3 - y)^2}$$

$$= \sqrt{4^2 + 9 + 6y + y^2}$$

$$= \sqrt{16 + 9 + 6y + y^2}$$

$$= \sqrt{y^2 + 6y + 25}$$

Chapter 1: Graphs

$$6 = \sqrt{y^2 + 6y + 25}$$

$$6^2 = \left(\sqrt{y^2 + 6y + 25}\right)^2$$

$$36 = y^2 + 6y + 25$$

$$0 = y^2 + 6y - 11$$

$$y = \frac{(-6) \pm \sqrt{(6)^2 - 4(1)(-11)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 + 44}}{2} = \frac{-6 \pm \sqrt{80}}{2}$$

$$= \frac{-6 \pm 4\sqrt{5}}{2} = -3 \pm 2\sqrt{5}$$

$$y = -3 + 2\sqrt{5} \quad \text{or} \quad y = -3 - 2\sqrt{5}$$

Thus, the points $(0, -3 + 2\sqrt{5})$ and $(0, -3 - 2\sqrt{5})$

are on the y -axis and a distance of 6 units from the point $(4, -3)$.

- 53. a.** To shift 3 units left and 4 units down, we subtract 3 from the x -coordinate and subtract 4 from the y -coordinate.

$$(2 - 3, 5 - 4) = (-1, 1)$$

- b.** To shift left 2 units and up 8 units, we subtract 2 from the x -coordinate and add 8 to the y -coordinate.

$$(2 - 2, 5 + 8) = (0, 13)$$

- 54.** Let the coordinates of point B be (x, y) . Using the midpoint formula, we can write

$$(2, 3) = \left(\frac{-1 + x}{2}, \frac{8 + y}{2}\right).$$

This leads to two equations we can solve.

$$\frac{-1 + x}{2} = 2 \quad \frac{8 + y}{2} = 3$$

$$-1 + x = 4 \quad 8 + y = 6$$

$$x = 5 \quad y = -2$$

Point B has coordinates $(5, -2)$.

- 55.** $M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$

$$P_1 = (x_1, y_1) = (-3, 6) \quad \text{and} \quad (x, y) = (-1, 4), \text{ so}$$

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$

$$-1 = \frac{-3 + x_2}{2} \quad 4 = \frac{6 + y_2}{2}$$

$$-2 = -3 + x_2 \quad 8 = 6 + y_2$$

$$1 = x_2 \quad 2 = y_2$$

Thus, $P_2 = (1, 2)$.

- 56.** $M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$

$P_2 = (x_2, y_2) = (7, -2)$ and $(x, y) = (5, -4)$, so

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$

$$5 = \frac{x_1 + 7}{2} \quad -4 = \frac{y_1 + (-2)}{2}$$

$$10 = x_1 + 7 \quad -8 = y_1 + (-2)$$

$$3 = x_1 \quad -6 = y_1$$

Thus, $P_1 = (3, -6)$.

- 57.** The midpoint of AB is: $D = \left(\frac{0+6}{2}, \frac{0+0}{2}\right)$
 $= (3, 0)$

The midpoint of AC is: $E = \left(\frac{0+4}{2}, \frac{0+4}{2}\right)$
 $= (2, 2)$

The midpoint of BC is: $F = \left(\frac{6+4}{2}, \frac{0+4}{2}\right)$
 $= (5, 2)$

$$d(C, D) = \sqrt{(0-4)^2 + (3-4)^2}$$

$$= \sqrt{(-4)^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}$$

$$d(B, E) = \sqrt{(2-6)^2 + (2-0)^2}$$

$$= \sqrt{(-4)^2 + 2^2} = \sqrt{16+4}$$

$$= \sqrt{20} = 2\sqrt{5}$$

$$d(A, F) = \sqrt{(2-0)^2 + (5-0)^2}$$

$$= \sqrt{2^2 + 5^2} = \sqrt{4+25}$$

$$= \sqrt{29}$$

Section 1.1: The Distance and Midpoint Formulas

58. Let $P_1 = (0, 0)$, $P_2 = (0, 4)$, $P = (x, y)$

$$\begin{aligned} d(P_1, P_2) &= \sqrt{(0-0)^2 + (4-0)^2} \\ &= \sqrt{16} = 4 \end{aligned}$$

$$\begin{aligned} d(P_1, P) &= \sqrt{(x-0)^2 + (y-0)^2} \\ &= \sqrt{x^2 + y^2} = 4 \\ \rightarrow x^2 + y^2 &= 16 \end{aligned}$$

$$\begin{aligned} d(P_2, P) &= \sqrt{(x-0)^2 + (y-4)^2} \\ &= \sqrt{x^2 + (y-4)^2} = 4 \\ \rightarrow x^2 + (y-4)^2 &= 16 \end{aligned}$$

Therefore,

$$y^2 = (y-4)^2$$

$$y^2 = y^2 - 8y + 16$$

$$8y = 16$$

$$y = 2$$

which gives

$$x^2 + 2^2 = 16$$

$$x^2 = 12$$

$$x = \pm 2\sqrt{3}$$

Two triangles are possible. The third vertex is $(-2\sqrt{3}, 2)$ or $(2\sqrt{3}, 2)$.

$$\begin{aligned} 59. \quad d(P_1, P_2) &= \sqrt{(-4-2)^2 + (1-1)^2} \\ &= \sqrt{(-6)^2 + 0^2} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

$$\begin{aligned} d(P_2, P_3) &= \sqrt{(-4-(-4))^2 + (-3-1)^2} \\ &= \sqrt{0^2 + (-4)^2} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

$$\begin{aligned} d(P_1, P_3) &= \sqrt{(-4-2)^2 + (-3-1)^2} \\ &= \sqrt{(-6)^2 + (-4)^2} \\ &= \sqrt{36+16} \\ &= \sqrt{52} \\ &= 2\sqrt{13} \end{aligned}$$

Since $[d(P_1, P_2)]^2 + [d(P_2, P_3)]^2 = [d(P_1, P_3)]^2$, the triangle is a right triangle.

$$\begin{aligned} 60. \quad d(P_1, P_2) &= \sqrt{(6-(-1))^2 + (2-4)^2} \\ &= \sqrt{7^2 + (-2)^2} \\ &= \sqrt{49+4} \\ &= \sqrt{53} \end{aligned}$$

$$\begin{aligned} d(P_2, P_3) &= \sqrt{(4-6)^2 + (-5-2)^2} \\ &= \sqrt{(-2)^2 + (-7)^2} \\ &= \sqrt{4+49} \\ &= \sqrt{53} \end{aligned}$$

$$\begin{aligned} d(P_1, P_3) &= \sqrt{(4-(-1))^2 + (-5-4)^2} \\ &= \sqrt{5^2 + (-9)^2} \\ &= \sqrt{25+81} \\ &= \sqrt{106} \end{aligned}$$

Since $[d(P_1, P_2)]^2 + [d(P_2, P_3)]^2 = [d(P_1, P_3)]^2$, the triangle is a right triangle.

Since $d(P_1, P_2) = d(P_2, P_3)$, the triangle is isosceles.

Therefore, the triangle is an isosceles right triangle.

$$\begin{aligned} 61. \quad d(P_1, P_2) &= \sqrt{(0-(-2))^2 + (7-(-1))^2} \\ &= \sqrt{2^2 + 8^2} = \sqrt{4+64} = \sqrt{68} \\ &= 2\sqrt{17} \end{aligned}$$

$$\begin{aligned} d(P_2, P_3) &= \sqrt{(3-0)^2 + (2-7)^2} \\ &= \sqrt{3^2 + (-5)^2} = \sqrt{9+25} \\ &= \sqrt{34} \end{aligned}$$

$$\begin{aligned} d(P_1, P_3) &= \sqrt{(3-(-2))^2 + (2-(-1))^2} \\ &= \sqrt{5^2 + 3^2} = \sqrt{25+9} \\ &= \sqrt{34} \end{aligned}$$

Since $d(P_2, P_3) = d(P_1, P_3)$, the triangle is isosceles.

Since $[d(P_1, P_3)]^2 + [d(P_2, P_3)]^2 = [d(P_1, P_2)]^2$, the triangle is also a right triangle.

Therefore, the triangle is an isosceles right triangle.

Chapter 1: Graphs

$$\begin{aligned}
 62. \quad d(P_1, P_2) &= \sqrt{(-4-7)^2 + (0-2)^2} \\
 &= \sqrt{(-11)^2 + (-2)^2} \\
 &= \sqrt{121+4} = \sqrt{125} \\
 &= 5\sqrt{5} \\
 d(P_2, P_3) &= \sqrt{(4-(-4))^2 + (6-0)^2} \\
 &= \sqrt{8^2 + 6^2} = \sqrt{64+36} \\
 &= \sqrt{100} \\
 &= 10 \\
 d(P_1, P_3) &= \sqrt{(4-7)^2 + (6-2)^2} \\
 &= \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} \\
 &= \sqrt{25} \\
 &= 5
 \end{aligned}$$

Since $[d(P_1, P_3)]^2 + [d(P_2, P_3)]^2 = [d(P_1, P_2)]^2$, the triangle is a right triangle.

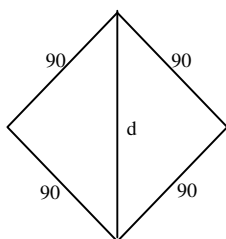
63. Using the Pythagorean Theorem:

$$90^2 + 90^2 = d^2$$

$$8100 + 8100 = d^2$$

$$16200 = d^2$$

$$d = \sqrt{16200} = 90\sqrt{2} \approx 127.28 \text{ feet}$$

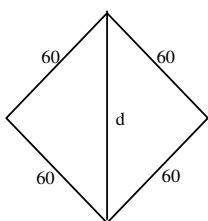


64. Using the Pythagorean Theorem:

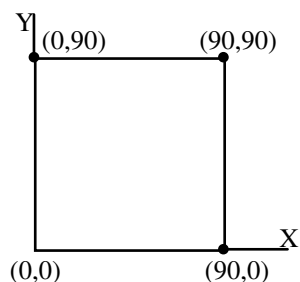
$$60^2 + 60^2 = d^2$$

$$3600 + 3600 = d^2 \rightarrow 7200 = d^2$$

$$d = \sqrt{7200} = 60\sqrt{2} \approx 84.85 \text{ feet}$$



65. a. First: (90, 0), Second: (90, 90), Third: (0, 90)



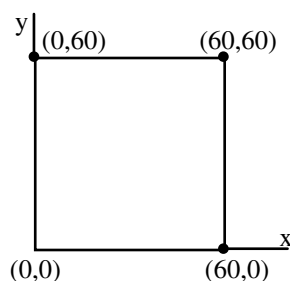
b. Using the distance formula:

$$\begin{aligned}
 d &= \sqrt{(310-90)^2 + (15-90)^2} \\
 &= \sqrt{220^2 + (-75)^2} = \sqrt{54025} \\
 &= 5\sqrt{2161} \approx 232.43 \text{ feet}
 \end{aligned}$$

c. Using the distance formula:

$$\begin{aligned}
 d &= \sqrt{(300-0)^2 + (300-90)^2} \\
 &= \sqrt{300^2 + 210^2} = \sqrt{134100} \\
 &= 30\sqrt{149} \approx 366.20 \text{ feet}
 \end{aligned}$$

66. a. First: (60, 0), Second: (60, 60), Third: (0, 60)



b. Using the distance formula:

$$\begin{aligned}
 d &= \sqrt{(180-60)^2 + (20-60)^2} \\
 &= \sqrt{120^2 + (-40)^2} = \sqrt{16000} \\
 &= 40\sqrt{10} \approx 126.49 \text{ feet}
 \end{aligned}$$

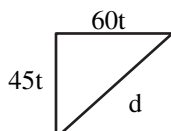
c. Using the distance formula:

$$\begin{aligned}
 d &= \sqrt{(220-0)^2 + (220-60)^2} \\
 &= \sqrt{220^2 + 160^2} = \sqrt{74000} \\
 &= 20\sqrt{185} \approx 272.03 \text{ feet}
 \end{aligned}$$

Section 1.1: The Distance and Midpoint Formulas

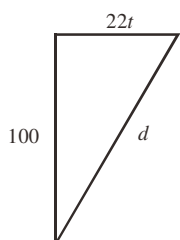
67. The Focus heading east moves a distance $60t$ after t hours. The Tesla heading south moves a distance $40t$ after t hours. Their distance apart after t hours is:

$$\begin{aligned} d &= \sqrt{(60t)^2 + (45t)^2} \\ &= \sqrt{3600t^2 + 2025t^2} \\ &= \sqrt{5625t^2} \\ &= 75t \text{ miles} \end{aligned}$$



68. $\frac{15 \text{ miles}}{1 \text{ hr}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} = 22 \text{ ft/sec}$

$$\begin{aligned} d &= \sqrt{100^2 + (22t)^2} \\ &= \sqrt{10000 + 484t^2} \text{ feet} \end{aligned}$$



69. a. The shortest side is between $P_1 = (2.6, 1.5)$ and $P_2 = (2.7, 1.7)$. The estimate for the desired intersection point is:

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) &= \left(\frac{2.6 + 2.7}{2}, \frac{1.5 + 1.7}{2} \right) \\ &= \left(\frac{5.3}{2}, \frac{3.2}{2} \right) \\ &= (2.65, 1.6) \end{aligned}$$

- b. Using the distance formula:

$$\begin{aligned} d &= \sqrt{(2.65 - 1.4)^2 + (1.6 - 1.3)^2} \\ &= \sqrt{(1.25)^2 + (0.3)^2} \\ &= \sqrt{1.5625 + 0.09} \\ &= \sqrt{1.6525} \\ &\approx 1.285 \text{ units} \end{aligned}$$

70. Let $P_1 = (2018, 232.89)$ and $P_2 = (2022, 513.98)$. The midpoint is:

$$\begin{aligned} (x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{2018 + 2022}{2}, \frac{232.89 + 513.98}{2} \right) \\ &= \left(\frac{4040}{2}, \frac{746.87}{2} \right) \\ &= (2020, 373.44) \end{aligned}$$

The estimate for 2020 is \$373.44 billion. The estimate net sales of Amazon.com in 2020 is \$12.62 billion off from the reported value of \$386.44 billion.

71. For 2014 we have the ordered pair $(2014, 5645)$ and for 2022 we have the ordered pair $(2022, 7951)$. The midpoint is

$$\begin{aligned} (\text{year}, \$) &= \left(\frac{2014 + 2022}{2}, \frac{5645 + 7951}{2} \right) \\ &= \left(\frac{4036}{2}, \frac{13596}{2} \right) \\ &= (2018, 6798) \end{aligned}$$

Using the midpoint, we estimate the average credit card debt in 2018 to be \$6,798. This is underestimate of the actual value.

72. Let $P_1 = (0, 0)$, $P_2 = (a, 0)$, and

$$P_3 = \left(\frac{a}{2}, \frac{\sqrt{3}a}{2} \right). \text{ Then}$$

$$\begin{aligned} d(P_1, P_2) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(a - 0)^2 + (0 - 0)^2} = \sqrt{a^2} = |a| \end{aligned}$$

$$\begin{aligned} d(P_2, P_3) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(\frac{a}{2} - a \right)^2 + \left(\frac{\sqrt{3}a}{2} - 0 \right)^2} \\ &= \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{4a^2}{4}} = \sqrt{a^2} = |a| \end{aligned}$$

Chapter 1: Graphs

$$\begin{aligned} d(P_1, P_3) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{\sqrt{3}a}{2} - 0\right)^2} \\ &= \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{4a^2}{4}} = \sqrt{a^2} = |a| \end{aligned}$$

Since the lengths of the three sides are all equal, the triangle is an equilateral triangle.

The midpoints of the sides are

$$M_{P_1P_2} = \left(\frac{0+a}{2}, \frac{0+0}{2}\right) = \left(\frac{a}{2}, 0\right)$$

$$M_{P_2P_3} = \left(\frac{a+\frac{a}{2}}{2}, \frac{0+\frac{\sqrt{3}a}{2}}{2}\right) = \left(\frac{3a}{4}, \frac{\sqrt{3}a}{4}\right)$$

$$M_{P_1P_3} = \left(\frac{0+\frac{a}{2}}{2}, \frac{0+\frac{\sqrt{3}a}{2}}{2}\right) = \left(\frac{a}{4}, \frac{\sqrt{3}a}{4}\right)$$

Then,

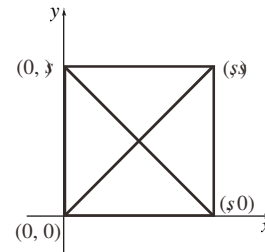
$$\begin{aligned} d(M_{P_1P_2}, M_{P_2P_3}) &= \sqrt{\left(\frac{3a}{4} - \frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}a}{4} - 0\right)^2} \\ &= \sqrt{\left(\frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}a}{4}\right)^2} \\ &= \sqrt{\frac{a^2}{16} + \frac{3a^2}{16}} = \frac{|a|}{2} \end{aligned}$$

$$\begin{aligned} d(M_{P_2P_3}, M_{P_1P_3}) &= \sqrt{\left(\frac{3a}{4} - \frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}a}{4} - \frac{\sqrt{3}a}{4}\right)^2} \\ &= \sqrt{\left(\frac{a}{2}\right)^2 + 0^2} \\ &= \sqrt{\frac{a^2}{4}} = \frac{|a|}{2} \end{aligned}$$

$$\begin{aligned} d(M_{P_1P_2}, M_{P_1P_3}) &= \sqrt{\left(\frac{a}{4} - \frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}a}{4} - 0\right)^2} \\ &= \sqrt{\left(-\frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}a}{4}\right)^2} \\ &= \sqrt{\frac{a^2}{16} + \frac{3a^2}{16}} = \frac{|a|}{2} \end{aligned}$$

Since the lengths of the sides of the triangle formed by the midpoints are all equal, the triangle is equilateral.

73. Let $P_1 = (0, 0)$, $P_2 = (0, s)$, $P_3 = (s, 0)$, and $P_4 = (s, s)$ be the vertices of the square.



The points P_1 and P_4 are endpoints of one diagonal and the points P_2 and P_3 are the endpoints of the other diagonal.

$$M_{P_1P_4} = \left(\frac{0+s}{2}, \frac{0+s}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$$

$$M_{P_2P_3} = \left(\frac{0+s}{2}, \frac{s+0}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$$

The midpoints of the diagonals are the same. Therefore, the diagonals of a square intersect at their midpoints.

74. Let $P = (a, 2a)$. Then

$$\begin{aligned} \sqrt{(a+5)^2 + (2a-1)^2} &= \sqrt{(a-4)^2 + (2a+4)^2} \\ (a+5)^2 + (2a-1)^2 &= (a-4)^2 + (2a+4)^2 \\ 5a^2 + 6a + 26 &= 5a^2 + 8a + 32 \\ 6a + 26 &= 8a + 32 \\ -2a &= 6 \\ a &= -3 \end{aligned}$$

Then $P = (-3, -6)$.

75. Arrange the parallelogram on the coordinate plane so that the vertices are

$$P_1 = (0, 0), P_2 = (a, 0), P_3 = (a+b, c) \text{ and } P_4 = (b, c)$$

Then the lengths of the sides are:

$$\begin{aligned} d(P_1, P_2) &= \sqrt{(a-0)^2 + (0-0)^2} \\ &= \sqrt{a^2} = |a| \end{aligned}$$

$$\begin{aligned} d(P_2, P_3) &= \sqrt{[(a+b)-a]^2 + (c-0)^2} \\ &= \sqrt{b^2 + c^2} \end{aligned}$$

Section 1.2: Graphs of Equations in Two Variables; Intercepts; Symmetry

$$\begin{aligned} d(P_3, P_4) &= \sqrt{[b - (a + b)]^2 + (c - c)^2} \\ &= \sqrt{a^2} = |a| \end{aligned}$$

and

$$\begin{aligned} d(P_1, P_4) &= \sqrt{(b - 0)^2 + (c - 0)^2} \\ &= \sqrt{b^2 + c^2} \end{aligned}$$

P_1 and P_3 are the endpoints of one diagonal, and P_2 and P_4 are the endpoints of the other diagonal. The lengths of the diagonals are

$$\begin{aligned} d(P_1, P_3) &= \sqrt{[(a + b) - 0]^2 + (c - 0)^2} \\ &= \sqrt{a^2 + 2ab + b^2 + c^2} \end{aligned}$$

and

$$\begin{aligned} d(P_2, P_4) &= \sqrt{(b - a)^2 + (c - 0)^2} \\ &= \sqrt{a^2 - 2ab + b^2 + c^2} \end{aligned}$$

Sum of the squares of the sides:

$$\begin{aligned} a^2 + (\sqrt{b^2 + c^2})^2 + a^2 + (\sqrt{b^2 + c^2})^2 \\ = 2a^2 + 2b^2 + 2c^2 \end{aligned}$$

Sum of the squares of the diagonals:

$$\begin{aligned} \left(\sqrt{a^2 + 2ab + b^2 + c^2} \right)^2 + \left(\sqrt{a^2 - 2ab + b^2 + c^2} \right)^2 \\ = 2a^2 + 2b^2 + 2c^2 \end{aligned}$$

76. Answers will vary.

Section 1.2

1. $2(x + 3) - 1 = -7$

$$2(x + 3) = -6$$

$$x + 3 = -3$$

$$x = -6$$

The solution set is $\{-6\}$.

2. $x^2 - 9 = 0$

$$x^2 = 9$$

$$x = \pm\sqrt{9} = \pm 3$$

The solution set is $\{-3, 3\}$.

3. intercepts

4. $y = 0$

5. y -axis

6. 4

7. $(-3, 4)$

8. True

9. False; the y -coordinate of a point at which the graph crosses or touches the x -axis is always 0. The x -coordinate of such a point is an x -intercept.

10. False; a graph can be symmetric with respect to both coordinate axes (in such cases it will also be symmetric with respect to the origin).

For example: $x^2 + y^2 = 1$

11. d

12. c

13. $y = x^4 - \sqrt{x}$

$$0 = 0^4 - \sqrt{0} \quad 1 = 1^4 - \sqrt{1} \quad 4 = (2)^4 - \sqrt{2}$$

$$0 = 0 \quad 1 \neq 0 \quad 4 \neq 16 - \sqrt{2}$$

The point $(0, 0)$ is on the graph of the equation.

14. $y = x^3 - 2\sqrt{x}$

$$0 = 0^3 - 2\sqrt{0} \quad 1 = 1^3 - 2\sqrt{1} \quad -1 = 1^3 - 2\sqrt{1}$$

$$0 = 0 \quad 1 \neq -1 \quad -1 = -1$$

The points $(0, 0)$ and $(1, -1)$ are on the graph of the equation.

15. $y^2 = x^2 + 9$

$$3^2 = 0^2 + 9 \quad 0^2 = 3^2 + 9 \quad 0^2 = (-3)^2 + 9$$

$$9 = 9 \quad 0 \neq 18 \quad 0 \neq 18$$

The point $(0, 3)$ is on the graph of the equation.

Chapter 1: Graphs

16. $y^3 = x + 1$

$$2^3 = 1 + 1 \quad 1^3 = 0 + 1 \quad 0^3 = -1 + 1$$

$$8 \neq 2 \quad 1 = 1 \quad 0 = 0$$

The points (0, 1) and (-1, 0) are on the graph of the equation.

17. $x^2 + y^2 = 4$

$$0^2 + 2^2 = 4 \quad (-2)^2 + 2^2 = 4 \quad (\sqrt{2})^2 + (\sqrt{2})^2 = 4$$

$$4 = 4 \quad 8 \neq 4 \quad 4 = 4$$

(0, 2) and $(\sqrt{2}, \sqrt{2})$ are on the graph of the equation.

18. $x^2 + 4y^2 = 4$

$$0^2 + 4 \cdot 1^2 = 4 \quad 2^2 + 4 \cdot 0^2 = 4 \quad 2^2 + 4 \left(\frac{1}{2}\right)^2 = 4$$

$$4 = 4 \quad 4 = 4 \quad 5 \neq 4$$

The points (0, 1) and (2, 0) are on the graph of the equation.

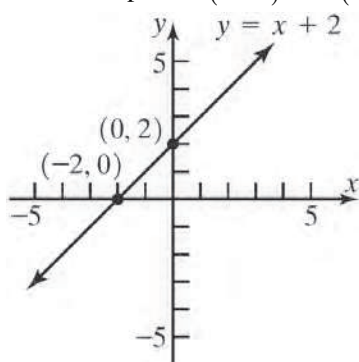
19. $y = x + 2$

x-intercept: $0 = x + 2$ y-intercept: $y = 0 + 2$

$$-2 = x$$

$$y = 2$$

The intercepts are (-2, 0) and (0, 2).



20. $y = x - 6$

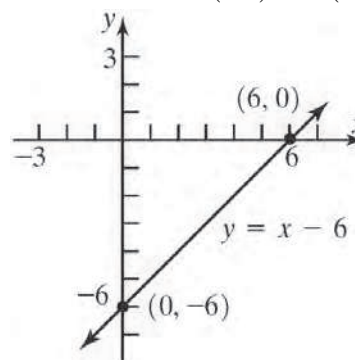
x-intercept: $0 = x - 6$

$$6 = x$$

y-intercept: $y = 0 - 6$

$$y = -6$$

The intercepts are (6, 0) and (0, -6).



21. $y = 2x + 8$

x-intercept: $0 = 2x + 8$

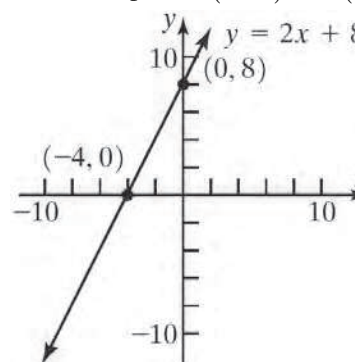
$$2x = -8$$

$$x = -4$$

y-intercept: $y = 2(0) + 8$

$$y = 8$$

The intercepts are (-4, 0) and (0, 8).



22. $y = 3x - 9$

x-intercept: $0 = 3x - 9$

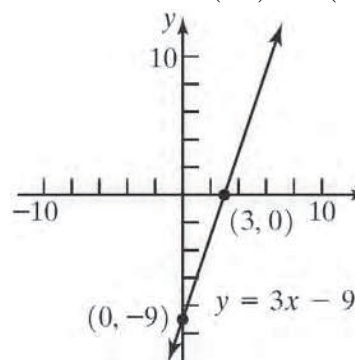
$$3x = 9$$

$$x = 3$$

y-intercept: $y = 3(0) - 9$

$$y = -9$$

The intercepts are (3, 0) and (0, -9).



Section 1.2: Graphs of Equations in Two Variables; Intercepts; Symmetry

23. $y = x^2 - 1$

x -intercepts:

$$0 = x^2 - 1$$

$$x^2 = 1$$

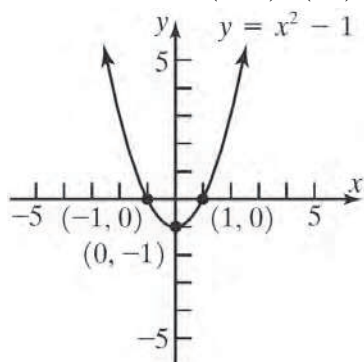
$$x = \pm 1$$

y -intercept:

$$y = 0^2 - 1$$

$$y = -1$$

The intercepts are $(-1, 0)$, $(1, 0)$, and $(0, -1)$.



24. $y = x^2 - 9$

x -intercepts:

$$0 = x^2 - 9$$

$$x^2 = 9$$

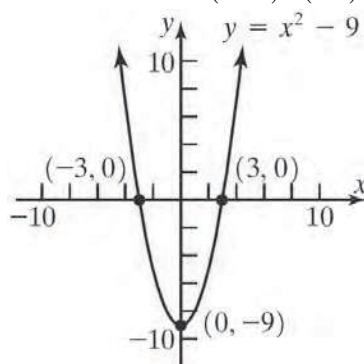
$$x = \pm 3$$

y -intercept:

$$y = 0^2 - 9$$

$$y = -9$$

The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, -9)$.



25. $y = -x^2 + 4$

x -intercepts:

$$0 = -x^2 + 4$$

$$x^2 = 4$$

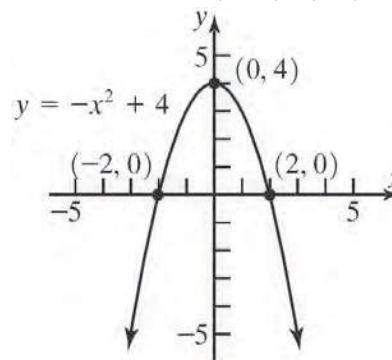
$$x = \pm 2$$

y -intercepts:

$$y = -(0)^2 + 4$$

$$y = 4$$

The intercepts are $(-2, 0)$, $(2, 0)$, and $(0, 4)$.



26. $y = -x^2 + 1$

x -intercepts:

$$0 = -x^2 + 1$$

$$x^2 = 1$$

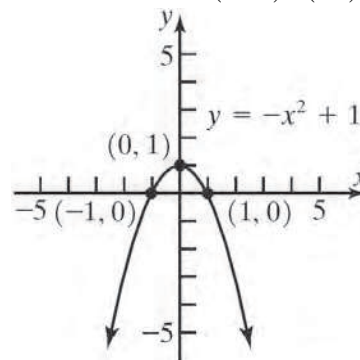
$$x = \pm 1$$

y -intercept:

$$y = -(0)^2 + 1$$

$$y = 1$$

The intercepts are $(-1, 0)$, $(1, 0)$, and $(0, 1)$.



27. $2x + 3y = 6$

x -intercepts:

$$2x + 3(0) = 6$$

$$2x = 6$$

$$x = 3$$

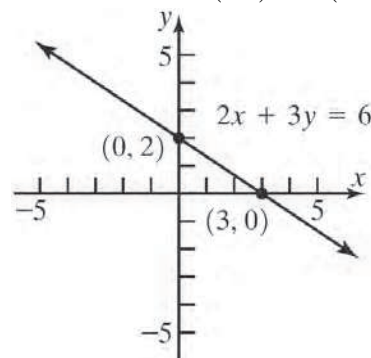
y -intercept:

$$2(0) + 3y = 6$$

$$3y = 6$$

$$y = 2$$

The intercepts are $(3, 0)$ and $(0, 2)$.



Chapter 1: Graphs

28. $5x + 2y = 10$

x -intercepts:

$$5x + 2(0) = 10$$

$$5x = 10$$

$$x = 2$$

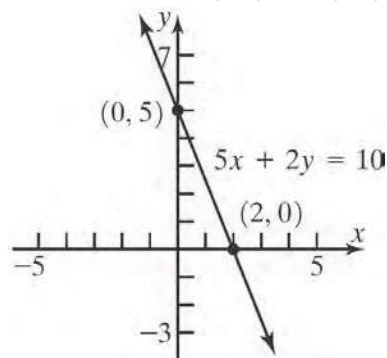
y -intercept:

$$5(0) + 2y = 10$$

$$2y = 10$$

$$y = 5$$

The intercepts are $(2, 0)$ and $(0, 5)$.



29. $9x^2 + 4y = 36$

x -intercepts:

$$9x^2 + 4(0) = 36$$

$$9x^2 = 36$$

$$x^2 = 4$$

$$x = \pm 2$$

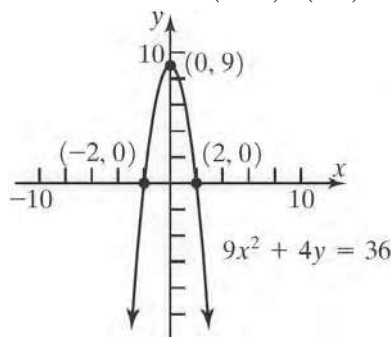
y -intercept:

$$9(0)^2 + 4y = 36$$

$$4y = 36$$

$$y = 9$$

The intercepts are $(-2, 0)$, $(2, 0)$, and $(0, 9)$.



30. $4x^2 + y = 4$

x -intercepts:

$$4x^2 + 0 = 4$$

$$4x^2 = 4$$

$$x^2 = 1$$

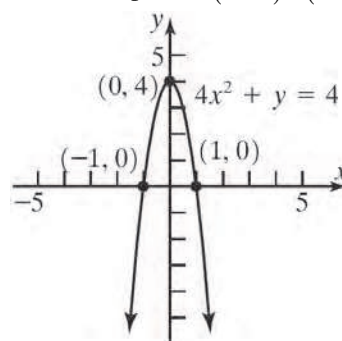
$$x = \pm 1$$

y -intercept:

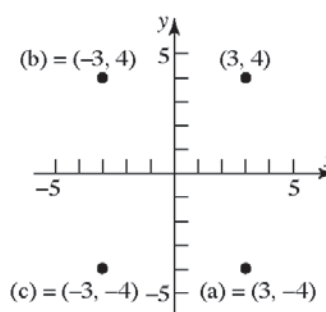
$$4(0)^2 + y = 4$$

$$y = 4$$

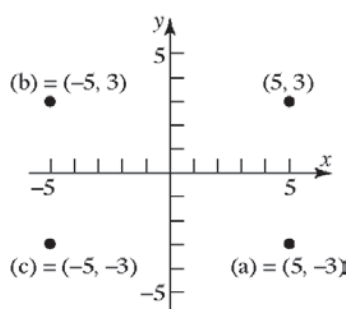
The intercepts are $(-1, 0)$, $(1, 0)$, and $(0, 4)$.



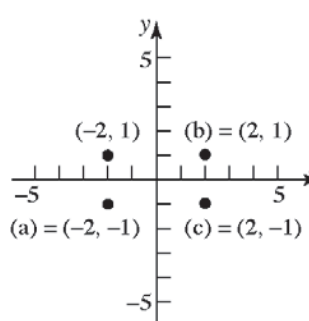
31.



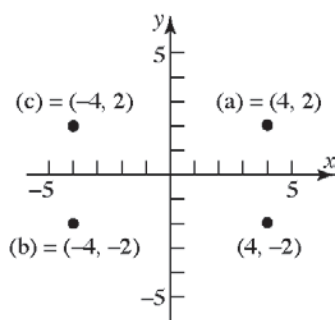
32.



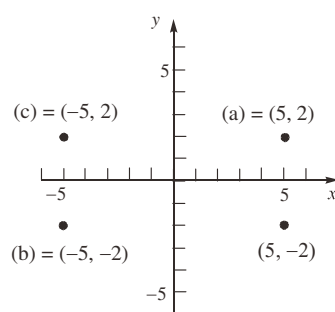
33.



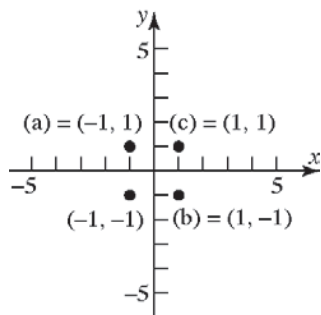
34.



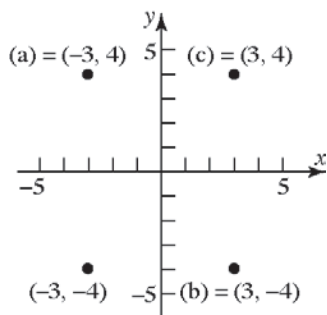
35.



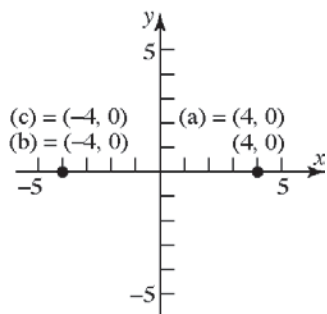
36.



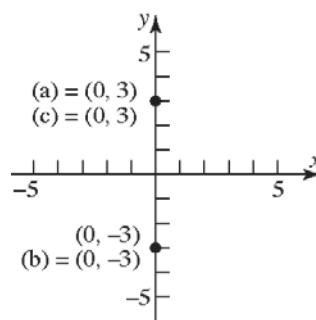
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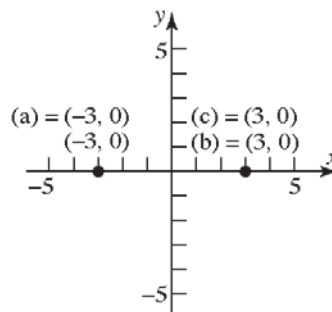
38.



39.



40.

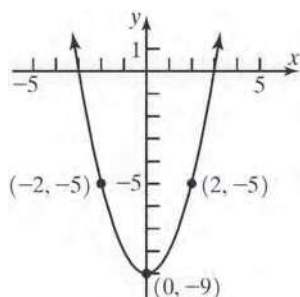


41. a. Intercepts: $(-1, 0)$ and $(1, 0)$
b. Symmetric with respect to the x -axis, y -axis, and the origin.
42. a. Intercepts: $(0, 1)$
b. Not symmetric to the x -axis, the y -axis, nor the origin
43. a. Intercepts: $(-\frac{\pi}{2}, 0)$, $(0, 1)$, and $(\frac{\pi}{2}, 0)$
b. Symmetric with respect to the y -axis.
44. a. Intercepts: $(-2, 0)$, $(0, -3)$, and $(2, 0)$
b. Symmetric with respect to the y -axis.
45. a. Intercepts: $(0, 0)$
b. Symmetric with respect to the x -axis.
46. a. Intercepts: $(-2, 0)$, $(0, 2)$, $(0, -2)$, and $(2, 0)$
b. Symmetric with respect to the x -axis, y -axis, and the origin.
47. a. Intercepts: $(-2, 0)$, $(0, 0)$, and $(2, 0)$
b. Symmetric with respect to the origin.
48. a. Intercepts: $(-4, 0)$, $(0, 0)$, and $(4, 0)$
b. Symmetric with respect to the origin.

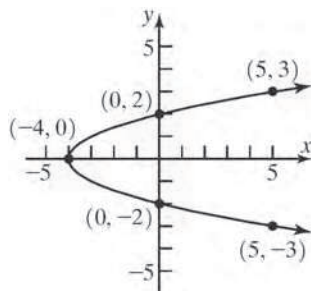
Chapter 1: Graphs

49. a. x -intercepts: $[-2, 1]$, y -intercept 0
 b. Not symmetric to x -axis, y -axis, or origin.
50. a. x -intercepts: $[-1, 2]$, y -intercept 0
 b. Not symmetric to x -axis, y -axis, or origin.
51. a. Intercepts: none
 b. Symmetric with respect to the origin.
52. a. Intercepts: none
 b. Symmetric with respect to the x -axis.

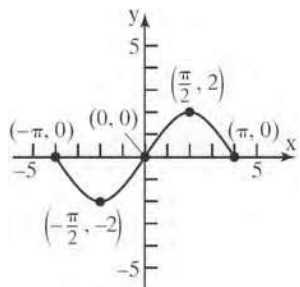
53.



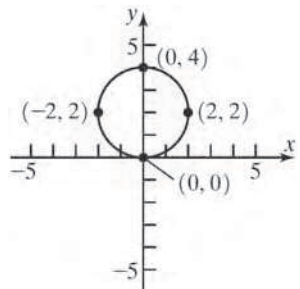
54.



55.



56.



57. $y^2 = x + 16$

x -intercepts:

$$0^2 = x + 16$$

$$-16 = x$$

y -intercepts:

$$y^2 = 0 + 16$$

$$y^2 = 16$$

$$y = \pm 4$$

The intercepts are $(-16, 0)$, $(0, -4)$ and $(0, 4)$.

Test x -axis symmetry: Let $y = -y$

$$(-y)^2 = x + 16$$

$$y^2 = x + 16 \text{ same}$$

Test y -axis symmetry: Let $x = -x$

$$y^2 = -x + 16 \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$(-y)^2 = -x + 16$$

$$y^2 = -x + 16 \text{ different}$$

Therefore, the graph will have x -axis symmetry.

58. $y^2 = x + 9$

x -intercepts:

$$(0)^2 = -x + 9$$

$$0 = -x + 9$$

$$x = 9$$

y -intercepts:

$$y^2 = 0 + 9$$

$$y^2 = 9$$

$$y = \pm 3$$

The intercepts are $(-9, 0)$, $(0, -3)$ and $(0, 3)$.

Test x -axis symmetry: Let $y = -y$

$$(-y)^2 = x + 9$$

$$y^2 = x + 9 \text{ same}$$

Test y -axis symmetry: Let $x = -x$

$$y^2 = -x + 9 \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$(-y)^2 = -x + 9$$

$$y^2 = -x + 9 \text{ different}$$

Therefore, the graph will have x -axis symmetry.

59. $y = \sqrt[3]{x}$

x -intercepts:

$$0 = \sqrt[3]{x}$$

$$0 = x$$

y -intercepts:

$$y = \sqrt[3]{0} = 0$$

The only intercept is $(0, 0)$.

Test x -axis symmetry: Let $y = -y$

$$-y = \sqrt[3]{x} \text{ different}$$

Section 1.2: Graphs of Equations in Two Variables; Intercepts; Symmetry

Test y-axis symmetry: Let $x = -x$

$$y = \sqrt[3]{-x} = -\sqrt[3]{x} \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \sqrt[3]{-x} = -\sqrt[3]{x}$$

$$y = \sqrt[3]{x} \text{ same}$$

Therefore, the graph will have origin symmetry.

60. $y = \sqrt[5]{x}$

x -intercepts: y -intercepts:

$$0 = \sqrt[5]{x} \quad y = \sqrt[5]{0} = 0$$

$$0 = x$$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = \sqrt[5]{x} \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \sqrt[5]{-x} = -\sqrt[5]{x} \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \sqrt[5]{-x} = -\sqrt[5]{x}$$

$$y = \sqrt[5]{x} \text{ same}$$

Therefore, the is symmetric with respect to the origin.

61. $x^2 + y - 9 = 0$

x -intercepts: y -intercepts:

$$x^2 - 9 = 0 \quad 0^2 + y - 9 = 0$$

$$x^2 = 9 \quad y = 9$$

$$x = \pm 3$$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,9)$.

Test x-axis symmetry: Let $y = -y$

$$x^2 - y - 9 = 0 \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$(-x)^2 + y - 9 = 0$$

$$x^2 + y - 9 = 0 \text{ same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$(-x)^2 - y - 9 = 0$$

$$x^2 - y - 9 = 0 \text{ different}$$

Therefore, the graph has y-axis symmetry.

62. $x^2 - y - 4 = 0$

x -intercepts: y -intercept:

$$x^2 - 0 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$0^2 - y - 4 = 0$$

$$-y = 4$$

$$y = -4$$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,-4)$.

Test x-axis symmetry: Let $y = -y$

$$x^2 - (-y) - 4 = 0$$

$$x^2 + y - 4 = 0 \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$(-x)^2 - y - 4 = 0$$

$$x^2 - y - 4 = 0 \text{ same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$(-x)^2 - (-y) - 4 = 0$$

$$x^2 + y - 4 = 0 \text{ different}$$

Therefore, the graph has y-axis symmetry.

63. $25x^2 + 4y^2 = 100$

x -intercepts: y -intercepts:

$$25x^2 + 4(0)^2 = 100 \quad 25(0)^2 + 4y^2 = 100$$

$$25x^2 = 100$$

$$4y^2 = 25$$

$$x^2 = 4$$

$$y^2 = 5$$

$$x = \pm 2$$

$$y = \pm 5$$

The intercepts are $(-2,0)$, $(2,0)$, $(0,-5)$, and $(0,5)$.

Test x-axis symmetry: Let $y = -y$

$$25x^2 + 4(-y)^2 = 100$$

$$25x^2 + 4y^2 = 100 \text{ same}$$

Test y-axis symmetry: Let $x = -x$

$$25(-x)^2 + 4y^2 = 100$$

$$25x^2 + 4y^2 = 100 \text{ same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$25(-x)^2 + 4(-y)^2 = 100$$

$$25x^2 + 4y^2 = 100 \text{ same}$$

Therefore, the graph has x-axis, y-axis, and origin symmetry.

64. $4x^2 + y^2 = 4$

x -intercepts: y -intercepts:

Chapter 1: Graphs

$$\begin{aligned} 4x^2 + 0^2 &= 4 & 4(0)^2 + y^2 &= 4 \\ 4x^2 &= 4 & y^2 &= 4 \\ x^2 &= 1 & y &= \pm 2 \\ x &= \pm 1 \end{aligned}$$

The intercepts are $(-1, 0)$, $(1, 0)$, $(0, -2)$, and $(0, 2)$.

Test x-axis symmetry: Let $y = -y$

$$\begin{aligned} 4x^2 + (-y)^2 &= 4 \\ 4x^2 + y^2 &= 4 \text{ same} \end{aligned}$$

Test y-axis symmetry: Let $x = -x$

$$\begin{aligned} 4(-x)^2 + y^2 &= 4 \\ 4x^2 + y^2 &= 4 \text{ same} \end{aligned}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$\begin{aligned} 4(-x)^2 + (-y)^2 &= 4 \\ 4x^2 + y^2 &= 4 \text{ same} \end{aligned}$$

Therefore, the graph has x-axis, y-axis, and origin symmetry.

65. $y = x^3 - 64$

x -intercepts:	y -intercepts:
$0 = x^3 - 64$	$y = 0^3 - 64$
$x^3 = 64$	$y = -64$
$x = 4$	

The intercepts are $(4, 0)$ and $(0, -64)$.

Test x-axis symmetry: Let $y = -y$

$$-y = x^3 - 64 \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$\begin{aligned} y &= (-x)^3 - 64 \\ y &= -x^3 - 64 \text{ different} \end{aligned}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$\begin{aligned} -y &= (-x)^3 - 64 \\ y &= x^3 + 64 \text{ different} \end{aligned}$$

Therefore, the graph has no symmetry.

66. $y = x^4 - 1$

x -intercepts:	y -intercepts:
------------------	------------------

$$\begin{aligned} 0 &= x^4 - 1 & y &= 0^4 - 1 \\ x^4 &= 1 & y &= -1 \\ x &= \pm 1 \end{aligned}$$

The intercepts are $(-1, 0)$, $(1, 0)$, and $(0, -1)$.

Test x-axis symmetry: Let $y = -y$

$$-y = x^4 - 1 \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$\begin{aligned} y &= (-x)^4 - 1 \\ y &= x^4 - 1 \text{ same} \end{aligned}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$\begin{aligned} -y &= (-x)^4 - 1 \\ -y &= x^4 - 1 \text{ different} \end{aligned}$$

Therefore, the graph has y-axis symmetry.

67. $y = x^2 - 2x - 8$

x -intercepts:	y -intercepts:
$0 = x^2 - 2x - 8$	$y = 0^2 - 2(0) - 8$
$0 = (x - 4)(x + 2)$	$y = -8$
$x = 4$ or $x = -2$	

The intercepts are $(4, 0)$, $(-2, 0)$, and $(0, -8)$.

Test x-axis symmetry: Let $y = -y$

$$-y = x^2 - 2x - 8 \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$\begin{aligned} y &= (-x)^2 - 2(-x) - 8 \\ y &= x^2 + 2x - 8 \text{ different} \end{aligned}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$\begin{aligned} -y &= (-x)^2 - 2(-x) - 8 \\ -y &= x^2 + 2x - 8 \text{ different} \end{aligned}$$

Therefore, the graph has no symmetry.

68. $y = x^2 + 4$

x -intercepts:	y -intercepts:
$0 = x^2 + 4$	$y = 0^2 + 4$
$x^2 = -4$	$y = 4$

no real solution

The only intercept is $(0, 4)$.

Test x-axis symmetry: Let $y = -y$

$$-y = x^2 + 4 \text{ different}$$

Section 1.2: Graphs of Equations in Two Variables; Intercepts; Symmetry

Test y-axis symmetry: Let $x = -x$

$$y = (-x)^2 + 4$$

$$y = x^2 + 4 \text{ same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = (-x)^2 + 4$$

$$-y = x^2 + 4 \text{ different}$$

Therefore, the graph has y-axis symmetry.

69. $y = \frac{4x}{x^2 + 16}$

x-intercepts:

$$0 = \frac{4x}{x^2 + 16}$$

$$4x = 0$$

$$x = 0$$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{4x}{x^2 + 16} \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{4(-x)}{(-x)^2 + 16}$$

$$y = -\frac{4x}{x^2 + 16} \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{4(-x)}{(-x)^2 + 16}$$

$$-y = -\frac{4x}{x^2 + 16}$$

$$y = \frac{4x}{x^2 + 16} \text{ same}$$

Therefore, the graph has origin symmetry.

70. $y = \frac{x^2 - 4}{2x}$

x-intercepts:

$$0 = \frac{x^2 - 4}{2x}$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

The intercepts are $(-2, 0)$ and $(2, 0)$.

y-intercepts:

$$y = \frac{0^2 - 4}{2(0)} = \frac{-4}{0}$$

undefined

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{x^2 - 4}{2x} \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{(-x)^2 - 4}{2(-x)}$$

$$y = -\frac{x^2 - 4}{2x} \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{(-x)^2 - 4}{2(-x)}$$

$$-y = \frac{x^2 - 4}{-2x}$$

$$y = \frac{x^2 - 4}{2x} \text{ same}$$

Therefore, the graph has origin symmetry.

71. $y = \frac{-x^3}{x^2 - 9}$

x-intercepts:

$$0 = \frac{-x^3}{x^2 - 9}$$

$$-x^3 = 0$$

$$x = 0$$

y-intercepts:

$$y = \frac{-0^3}{0^2 - 9} = \frac{0}{-9} = 0$$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{-x^3}{x^2 - 9}$$

$$y = \frac{x^3}{x^2 - 9} \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{-(-x)^3}{(-x)^2 - 9}$$

$$y = \frac{x^3}{x^2 - 9} \text{ different}$$

Chapter 1: Graphs

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{-(-x)^3}{(-x)^2 - 9}$$

$$-y = \frac{x^3}{x^2 - 9}$$

$$y = \frac{-x^3}{x^2 - 9} \text{ same}$$

Therefore, the graph has origin symmetry.

72. $y = \frac{x^4 + 1}{2x^5}$

x -intercepts:

$$0 = \frac{x^4 + 1}{2x^5}$$

$$x^4 = -1$$

no real solution

There are no intercepts for the graph of this equation.

Test x -axis symmetry: Let $y = -y$

$$-y = \frac{x^4 + 1}{2x^5} \text{ different}$$

Test y -axis symmetry: Let $x = -x$

$$y = \frac{(-x)^4 + 1}{2(-x)^5}$$

$$y = \frac{x^4 + 1}{-2x^5} \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

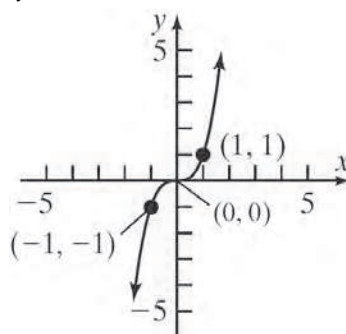
$$-y = \frac{(-x)^4 + 1}{2(-x)^5}$$

$$-y = \frac{x^4 + 1}{-2x^5}$$

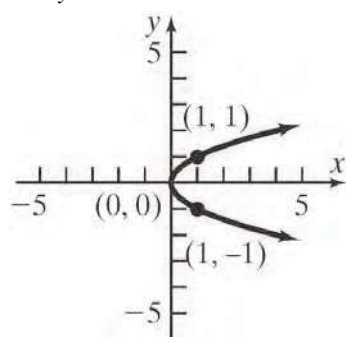
$$y = \frac{x^4 + 1}{2x^5} \text{ same}$$

Therefore, the graph has origin symmetry.

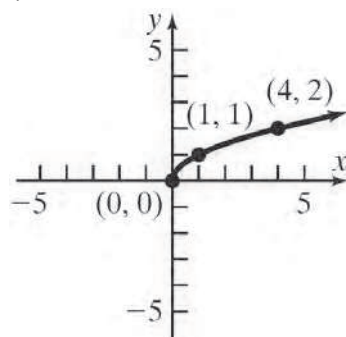
73. $y = x^3$



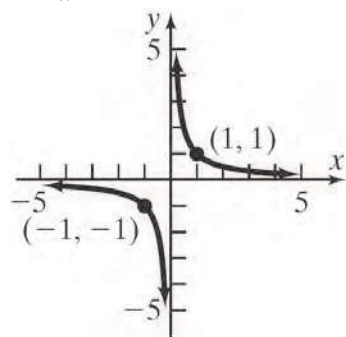
74. $x = y^2$



75. $y = \sqrt{x}$



76. $y = \frac{1}{x}$



Section 1.2: Graphs of Equations in Two Variables; Intercepts; Symmetry

77. If the point $(a, 4)$ is on the graph of

$$y = x^2 + 3x, \text{ then we have}$$

$$4 = a^2 + 3a$$

$$0 = a^2 + 3a - 4$$

$$0 = (a + 4)(a - 1)$$

$$a + 4 = 0 \quad \text{or} \quad a - 1 = 0$$

$$a = -4 \quad a = 1$$

Thus, $a = -4$ or $a = 1$.

78. If the point $(a, -5)$ is on the graph of

$$y = x^2 + 6x, \text{ then we have}$$

$$-5 = a^2 + 6a$$

$$0 = a^2 + 6a + 5$$

$$0 = (a + 5)(a + 1)$$

$$a + 5 = 0 \quad \text{or} \quad a + 1 = 0$$

$$a = -5 \quad a = -1$$

Thus, $a = -5$ or $a = -1$.

79. For a graph with origin symmetry, if the point (a, b) is on the graph, then so is the point $(-a, -b)$. Since the point $(1, 2)$ is on the graph of an equation with origin symmetry, the point $(-1, -2)$ must also be on the graph.

80. For a graph with y-axis symmetry, if the point (a, b) is on the graph, then so is the point $(-a, b)$. Since 6 is an x-intercept in this case, the point $(6, 0)$ is on the graph of the equation. Due to the y-axis symmetry, the point $(-6, 0)$ must also be on the graph. Therefore, -6 is another x-intercept.

81. For a graph with origin symmetry, if the point (a, b) is on the graph, then so is the point $(-a, -b)$. Since -4 is an x-intercept in this case, the point $(-4, 0)$ is on the graph of the equation. Due to the origin symmetry, the point $(4, 0)$ must also be on the graph. Therefore, 4 is another x-intercept.

82. For a graph with x-axis symmetry, if the point (a, b) is on the graph, then so is the point

$(a, -b)$. Since 2 is a y-intercept in this case, the point $(0, 2)$ is on the graph of the equation. Due to the x-axis symmetry, the point $(0, -2)$ must also be on the graph. Therefore, -2 is another y-intercept.

83. a. $(x^2 + y^2 - x)^2 = x^2 + y^2$

x-intercepts:

$$(x^2 + (0)^2 - x)^2 = x^2 + (0)^2$$

$$(x^2 - x)^2 = x^2$$

$$x^4 - 2x^3 + x^2 = x^2$$

$$x^4 - 2x^3 = 0$$

$$x^3(x - 2) = 0$$

$$x^3 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = 0 \quad x = 2$$

y-intercepts:

$$((0)^2 + y^2 - 0)^2 = (0)^2 + y^2$$

$$(y^2)^2 = y^2$$

$$y^4 = y^2$$

$$y^4 - y^2 = 0$$

$$y^2(y^2 - 1) = 0$$

$$y^2 = 0 \quad \text{or} \quad y^2 - 1 = 0$$

$$y = 0 \quad y^2 = 1$$

$$y = \pm 1$$

The intercepts are $(0, 0)$, $(2, 0)$, $(0, -1)$, and $(0, 1)$.

- b. Test x-axis symmetry: Let $y = -y$

$$(x^2 + (-y)^2 - x)^2 = x^2 + (-y)^2$$

$$(x^2 + y^2 - x)^2 = x^2 + y^2 \quad \text{same}$$

Test y-axis symmetry: Let $x = -x$

$$((-x)^2 + y^2 - (-x))^2 = (-x)^2 + y^2$$

$$(x^2 + y^2 + x)^2 = x^2 + y^2 \quad \text{different}$$

Chapter 1: Graphs

Test origin symmetry: Let $x = -x$ and $y = -y$

$$\begin{aligned} ((-x)^2 + (-y)^2 - (-x))^2 &= (-x)^2 + (-y)^2 \\ (x^2 + y^2 + x)^2 &= x^2 + y^2 \quad \text{different} \end{aligned}$$

Thus, the graph will have x -axis symmetry.

84. a. $16y^2 = 120x - 225$

y -intercepts:

$$16y^2 = 120(0) - 225$$

$$16y^2 = -225$$

$$y^2 = -\frac{225}{16}$$

no real solution

x -intercepts:

$$16(0)^2 = 120x - 225$$

$$0 = 120x - 225$$

$$-120x = -225$$

$$x = \frac{-225}{-120} = \frac{15}{8}$$

The only intercept is $\left(\frac{15}{8}, 0\right)$.

b. Test x -axis symmetry: Let $y = -y$

$$16(-y)^2 = 120x - 225$$

$$16y^2 = 120x - 225 \quad \text{same}$$

Test y -axis symmetry: Let $x = -x$

$$16y^2 = 120(-x) - 225$$

$$16y^2 = -120x - 225 \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$16(-y)^2 = 120(-x) - 225$$

$$16y^2 = -120x - 225 \quad \text{different}$$

Thus, the graph has x -axis symmetry.

85. Let $y = 0$.

$$(x^2 + 0^2)^2 = a^2(x^2 - 0^2)$$

$$x^4 = a^2(x^2)$$

$$x^4 - a^2x^2 = 0$$

$$x^2(x^2 - a^2) = 0$$

$$x^2 = 0 \quad \text{or} \quad (x^2 - a^2) = 0$$

$$x = 0 \quad \text{or} \quad x^2 = a^2$$

$$x = -a, a$$

Let $x = 0$.

$$(0^2 + y^2)^2 = a^2(0^2 - y^2)$$

$$y^4 = a^2(-y^2)$$

$$y^4 + a^2y^2 = 0$$

$$y^2(y^2 + a^2) = 0$$

$$y = 0$$

(Note that the solutions to $y^2 + a^2 = 0$ are not real)

So the intercepts are $(0,0)$, $(a,0)$ and $(-a,0)$.

Test x -axis symmetry: Replace y by $-y$

$$(x^2 + (-y)^2)^2 = a^2(x^2 - (-y)^2)$$

$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \quad \text{equivalent}$$

Test y -axis symmetry: replace x by $-x$

$$((-x)^2 + y^2)^2 = a^2((-x)^2 - y^2)$$

$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \quad \text{equivalent}$$

Test origin symmetry: replace x by $-x$ and y by $-y$

$$((-x)^2 + (-y)^2)^2 = a^2((-x)^2 - (-y)^2)$$

$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \quad \text{equivalent}$$

The graph is symmetric by respect to the x -axis, the y -axis, and the origin.

86. Let $y = 0$.

$$(x^2 + 0^2 - ax)^2 = b^2(x^2 + 0^2)$$

$$x^4 - 2ax^3 + a^2x^2 - b^2x^2 = 0$$

$$x^2[(x - (a + b))(x - (a - b))] = 0$$

$$x = 0 \quad \text{or} \quad x = a + b$$

$$\text{or} \quad x = a - b$$

Let $x = 0$.

$$(0^2 + y^2 - a \cdot 0)^2 = b^2(0^2 + y^2)$$

$$y^4 - b^2y^2 = 0$$

$$y^2(y + b)(y - b) = 0$$

$$y = 0, y = -b, y = b$$

So the intercepts are $(0,0)$, $(a-b,0)$, $(a+b,0)$, $(0,-b)$, $(0,b)$.

Test x -axis symmetry: replace y by $-y$

$$[x^2 + (-y)^2 - ax]^2 = b^2[x^2 + (-y)^2]$$

$$(x^2 + y^2 - ax)^2 = b^2(x^2 + y^2) \quad \text{Equivalent}$$

Section 1.2: Graphs of Equations in Two Variables; Intercepts; Symmetry

Test y-axis symmetry: replace x by $-x$

$$\left[(-x)^2 + y^2 - a(-x)\right]^2 = b^2 \left[(-x)^2 + y^2\right]$$

$$(x^2 + y^2 + ax)^2 = b^2(x^2 + y^2) \text{ Not equivalent}$$

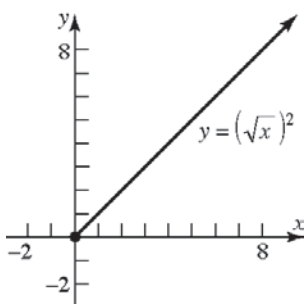
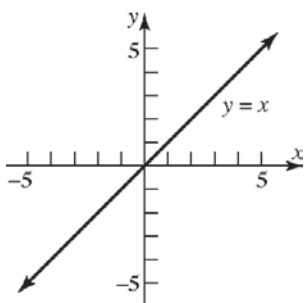
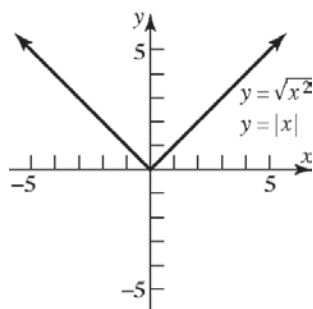
Test origin symmetry: replace x by $-x$ and y by $-y$

$$\left[(-x)^2 + (-y)^2 - a(-x)\right]^2 = b^2 \left[(-x)^2 + (-y)^2\right]$$

$$(x^2 + y^2 + ax)^2 = b^2(x^2 + y^2) \text{ No equivalent}$$

The graph is symmetric with respect to the x -axis only.

87. a.



b. Since $\sqrt{x^2} = |x|$ for all x , the graphs of $y = \sqrt{x^2}$ and $y = |x|$ are the same.

c. For $y = (\sqrt{x})^2$, the domain of the variable x is $x \geq 0$; for $y = x$, the domain of the

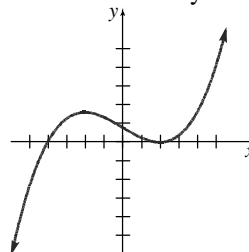
variable x is all real numbers. Thus,

$$(\sqrt{x})^2 = x \text{ only for } x \geq 0.$$

d. For $y = \sqrt{x^2}$, the range of the variable y is $y \geq 0$; for $y = x$, the range of the variable y is all real numbers. Also, $\sqrt{x^2} = x$ only if $x \geq 0$. Otherwise, $\sqrt{x^2} = -x$.

88. Answers will vary. A complete graph presents enough of the graph to the viewer so they can “see” the rest of the graph as an obvious continuation of what is shown.

89. Answers will vary. One example:



90. Answers will vary

91. Answers will vary

92. Answers will vary.

Case 1: Graph has x -axis and y -axis symmetry, show origin symmetry.

(x, y) on graph $\rightarrow (x, -y)$ on graph

(from x -axis symmetry)

$(x, -y)$ on graph $\rightarrow (-x, -y)$ on graph

(from y -axis symmetry)

Since the point $(-x, -y)$ is also on the graph, the graph has origin symmetry.

Case 2: Graph has x -axis and origin symmetry, show y -axis symmetry.

(x, y) on graph $\rightarrow (x, -y)$ on graph

(from x -axis symmetry)

$(x, -y)$ on graph $\rightarrow (-x, y)$ on graph

(from origin symmetry)

Since the point $(-x, y)$ is also on the graph, the graph has y -axis symmetry.

Case 3: Graph has y -axis and origin symmetry, show x -axis symmetry.

Chapter 1: Graphs

(x, y) on graph $\rightarrow (-x, y)$ on graph

(from y -axis symmetry)

$(-x, y)$ on graph $\rightarrow (x, -y)$ on graph

(from origin symmetry)

Since the point $(x, -y)$ is also on the graph, the graph has x -axis symmetry.

- 93.** Answers may vary. The graph must contain the points $(-2, 5)$, $(-1, 3)$, and $(0, 2)$. For the graph to be symmetric about the y -axis, the graph must also contain the points $(2, 5)$ and $(1, 3)$ (note that $(0, 2)$ is on the y -axis).

For the graph to also be symmetric with respect to the x -axis, the graph must also contain the points $(-2, -5)$, $(-1, -3)$, $(0, -2)$, $(2, -5)$, and $(1, -3)$. Recall that a graph with two of the symmetries (x -axis, y -axis, origin) will necessarily have the third. Therefore, if the original graph with y -axis symmetry also has x -axis symmetry, then it will also have origin symmetry.

Section 1.3

- 1.** undefined; 0

- 2.** 3; 2

$$x\text{-intercept: } 2x + 3(0) = 6$$

$$2x = 6$$

$$x = 3$$

$$y\text{-intercept: } 2(0) + 3y = 6$$

$$3y = 6$$

$$y = 2$$

- 3.** True

- 4.** False; the slope is $\frac{3}{2}$.

$$2y = 3x + 5$$

$$y = \frac{3}{2}x + \frac{5}{2}$$

$$\text{5. True; } 2(1) + (2) = 4$$

$$2 + 2 = 4$$

$$4 = 4 \quad \text{True}$$

$$\text{6. } m_1 = m_2; y\text{-intercepts; } m_1 \cdot m_2 = -1$$

$$\text{7. } 2$$

$$\text{8. } -\frac{1}{2}$$

$$\text{9. } c$$

$$\text{10. } d$$

$$\text{11. } b$$

$$\text{12. } d$$

$$\text{13. a. } \text{Slope} = \frac{1-0}{2-0} = \frac{1}{2}$$

- b.** If x increases by 2 units, y will increase by 1 unit.

$$\text{14. a. } \text{Slope} = \frac{1-0}{-2-0} = -\frac{1}{2}$$

- b.** If x increases by 2 units, y will decrease by 1 unit.

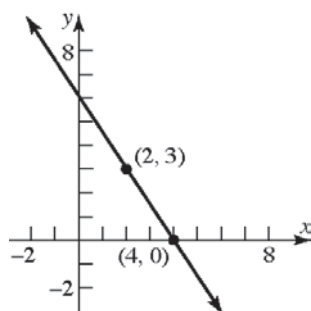
$$\text{15. a. } \text{Slope} = \frac{1-2}{1-(-2)} = -\frac{1}{3}$$

- b.** If x increases by 3 units, y will decrease by 1 unit.

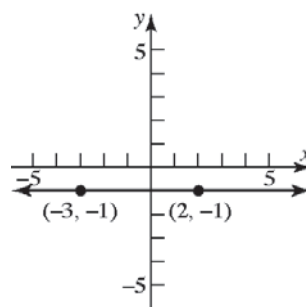
$$\text{16. a. } \text{Slope} = \frac{2-1}{2-(-1)} = \frac{1}{3}$$

- b.** If x increases by 3 units, y will increase by 1 unit.

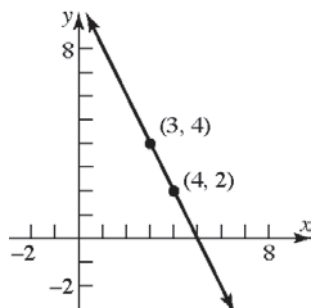
17. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{4 - 2} = -\frac{3}{2}$



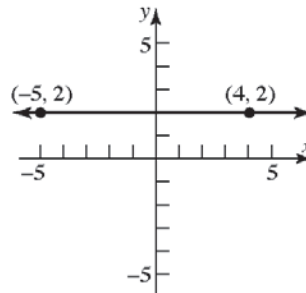
21. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-1)}{2 - (-3)} = \frac{0}{5} = 0$



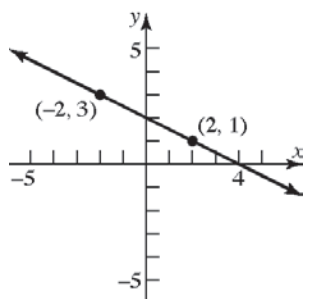
18. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{3 - 4} = \frac{2}{-1} = -2$



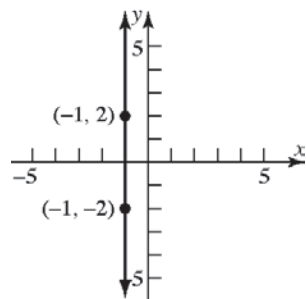
22. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{-5 - 4} = \frac{0}{-9} = 0$



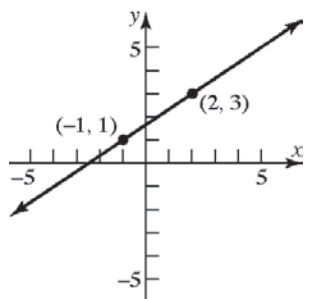
19. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{2 - (-2)} = \frac{-2}{4} = -\frac{1}{2}$



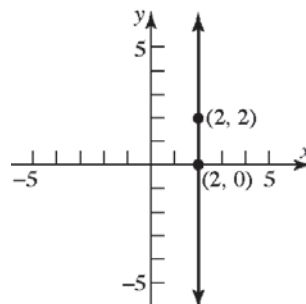
23. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 2}{-1 - (-1)} = \frac{-4}{0}$ undefined.



20. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{2 - (-1)} = \frac{2}{3}$

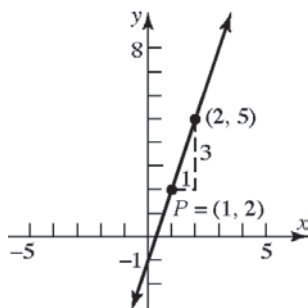


24. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{2 - 2} = \frac{2}{0}$ undefined.

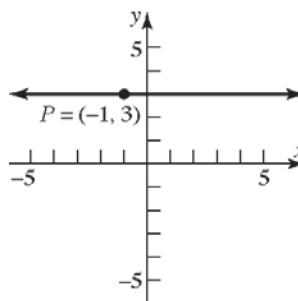


Chapter 1: Graphs

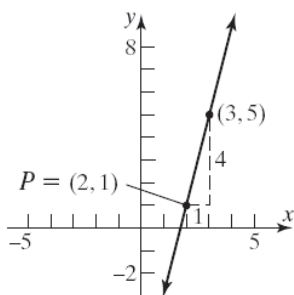
25. $P = (1, 2); m = 3$



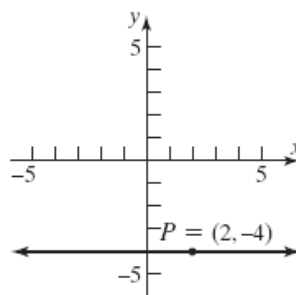
29. $P = (-1, 3); m = 0$



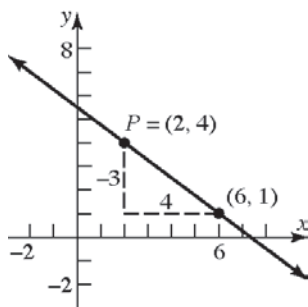
26. $P = (2, 1); m = 4$



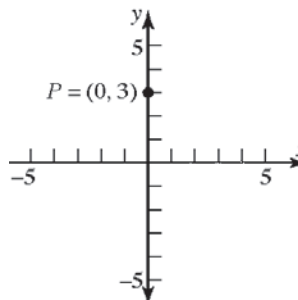
30. $P = (2, -4); m = 0$



27. $P = (2, 4); m = -\frac{3}{4}$

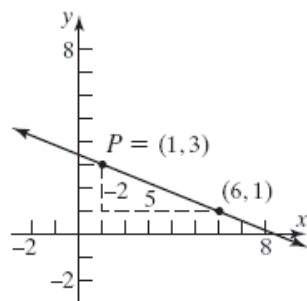


31. $P = (0, 3)$; slope undefined

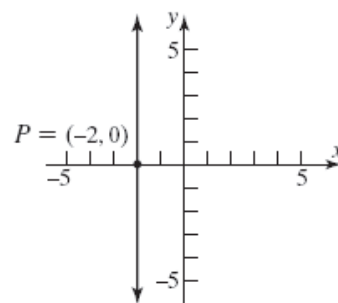


(note: the line is the y-axis)

28. $P = (1, 3); m = -\frac{2}{5}$



32. $P = (-2, 0)$; slope undefined



33. $P = (1, 2); m = 3; y - 2 = 3(x - 1)$

34. $P = (2, 1); m = 4; y - 1 = 4(x - 2)$

35. $P = (2, 4); m = -\frac{3}{4}; y - 4 = -\frac{3}{4}(x - 2)$

36. $P = (1, 3); m = -\frac{2}{5}; y - 3 = -\frac{2}{5}(x - 1)$

37. $P = (-1, 3); m = 0; y - 3 = 0$

38. $P = (2, -4); m = 0; y + 4 = 0$

39. Slope $= 4 = \frac{4}{1}$; point: $(1, 2)$

If x increases by 1 unit, then y increases by 4 units.

Answers will vary. Three possible points are:

$$x = 1 + 1 = 2 \text{ and } y = 2 + 4 = 6$$

$$(2, 6)$$

$$x = 2 + 1 = 3 \text{ and } y = 6 + 4 = 10$$

$$(3, 10)$$

$$x = 3 + 1 = 4 \text{ and } y = 10 + 4 = 14$$

$$(4, 14)$$

40. Slope $= 2 = \frac{2}{1}$; point: $(-2, 3)$

If x increases by 1 unit, then y increases by 2 units.

Answers will vary. Three possible points are:

$$x = -2 + 1 = -1 \text{ and } y = 3 + 2 = 5$$

$$(-1, 5)$$

$$x = -1 + 1 = 0 \text{ and } y = 5 + 2 = 7$$

$$(0, 7)$$

$$x = 0 + 1 = 1 \text{ and } y = 7 + 2 = 9$$

$$(1, 9)$$

41. Slope $= -\frac{3}{2} = \frac{-3}{2}$; point: $(2, -4)$

If x increases by 2 units, then y decreases by 3 units.

Answers will vary. Three possible points are:

$$x = 2 + 2 = 4 \text{ and } y = -4 - 3 = -7$$

$$(4, -7)$$

$$x = 4 + 2 = 6 \text{ and } y = -7 - 3 = -10$$

$$(6, -10)$$

$$x = 6 + 2 = 8 \text{ and } y = -10 - 3 = -13$$

$$(8, -13)$$

42. Slope $= \frac{4}{3}$; point: $(-3, 2)$

If x increases by 3 units, then y increases by 4 units.

Answers will vary. Three possible points are:

$$x = -3 + 3 = 0 \text{ and } y = 2 + 4 = 6$$

$$(0, 6)$$

$$x = 0 + 3 = 3 \text{ and } y = 6 + 4 = 10$$

$$(3, 10)$$

$$x = 3 + 3 = 6 \text{ and } y = 10 + 4 = 14$$

$$(6, 14)$$

43. Slope $= -2 = \frac{-2}{1}$; point: $(-2, -3)$

If x increases by 1 unit, then y decreases by 2 units.

Answers will vary. Three possible points are:

$$x = -2 + 1 = -1 \text{ and } y = -3 - 2 = -5$$

$$(-1, -5)$$

$$x = -1 + 1 = 0 \text{ and } y = -5 - 2 = -7$$

$$(0, -7)$$

$$x = 0 + 1 = 1 \text{ and } y = -7 - 2 = -9$$

$$(1, -9)$$

44. Slope $= -1 = \frac{-1}{1}$; point: $(4, 1)$

If x increases by 1 unit, then y decreases by 1 unit.

Answers will vary. Three possible points are:

$$x = 4 + 1 = 5 \text{ and } y = 1 - 1 = 0$$

$$(5, 0)$$

$$x = 5 + 1 = 6 \text{ and } y = 0 - 1 = -1$$

$$(6, -1)$$

$$x = 6 + 1 = 7 \text{ and } y = -1 - 1 = -2$$

$$(7, -2)$$

Chapter 1: Graphs

45. (0, 0) and (2, 1) are points on the line.

$$\text{Slope} = \frac{1-0}{2-0} = \frac{1}{2}$$

y-intercept is 0; using $y = mx + b$:

$$y = \frac{1}{2}x + 0$$

$$2y = x$$

$$0 = x - 2y$$

$$x - 2y = 0 \text{ or } y = \frac{1}{2}x$$

46. (0, 0) and (-2, 1) are points on the line.

$$\text{Slope} = \frac{1-0}{-2-0} = \frac{1}{-2} = -\frac{1}{2}$$

y-intercept is 0; using $y = mx + b$:

$$y = -\frac{1}{2}x + 0$$

$$2y = -x$$

$$x + 2y = 0$$

$$x + 2y = 0 \text{ or } y = -\frac{1}{2}x$$

47. (-1, 3) and (1, 1) are points on the line.

$$\text{Slope} = \frac{1-3}{1-(-1)} = \frac{-2}{2} = -1$$

Using $y - y_1 = m(x - x_1)$

$$y - 1 = -1(x - 1)$$

$$y - 1 = -x + 1$$

$$y = -x + 2$$

$$x + y = 2 \text{ or } y = -x + 2$$

48. (-1, 1) and (2, 2) are points on the line.

$$\text{Slope} = \frac{2-1}{2-(-1)} = \frac{1}{3}$$

Using $y - y_1 = m(x - x_1)$

$$y - 1 = \frac{1}{3}(x - (-1))$$

$$y - 1 = \frac{1}{3}(x + 1)$$

$$y - 1 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{4}{3}$$

$$x - 3y = -4 \text{ or } y = \frac{1}{3}x + \frac{4}{3}$$

49. $y - y_1 = m(x - x_1)$, $m = 2$

$$y - 3 = 2(x - 3)$$

$$y - 3 = 2x - 6$$

$$y = 2x - 3$$

$$2x - y = 3 \text{ or } y = 2x - 3$$

50. $y - y_1 = m(x - x_1)$, $m = -1$

$$y - 2 = -1(x - 1)$$

$$y - 2 = -x + 1$$

$$y = -x + 3$$

$$x + y = 3 \text{ or } y = -x + 3$$

51. $y - y_1 = m(x - x_1)$, $m = -\frac{1}{2}$

$$y - 2 = -\frac{1}{2}(x - 1)$$

$$y - 2 = -\frac{1}{2}x + \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

$$x + 2y = 5 \text{ or } y = -\frac{1}{2}x + \frac{5}{2}$$

52. $y - y_1 = m(x - x_1)$, $m = 1$

$$y - 1 = 1(x - (-1))$$

$$y - 1 = x + 1$$

$$y = x + 2$$

$$x - y = -2 \text{ or } y = x + 2$$

53. Slope = 3; containing (-2, 3)

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 3(x - (-2))$$

$$y - 3 = 3x + 6$$

$$y = 3x + 9$$

$$3x - y = -9 \text{ or } y = 3x + 9$$

54. Slope = 2; containing the point (4, -3)

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = 2(x - 4)$$

$$y + 3 = 2x - 8$$

$$y = 2x - 11$$

$$2x - y = 11 \text{ or } y = 2x - 11$$

55. Slope = $\frac{1}{2}$; containing the point (3, 1)

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{2}(x - 3)$$

$$y - 1 = \frac{1}{2}x - \frac{3}{2}$$

$$y = \frac{1}{2}x - \frac{1}{2}$$

$$x - 2y = 1 \text{ or } y = \frac{1}{2}x - \frac{1}{2}$$

56. Slope = $-\frac{2}{3}$; containing (1, -1)

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -\frac{2}{3}(x - 1)$$

$$y + 1 = -\frac{2}{3}x + \frac{2}{3}$$

$$y = -\frac{2}{3}x - \frac{1}{3}$$

$$2x + 3y = -1 \text{ or } y = -\frac{2}{3}x - \frac{1}{3}$$

57. Containing (1, 3) and (-1, 2)

$$m = \frac{2 - 3}{-1 - 1} = \frac{-1}{-2} = \frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{1}{2}(x - 1)$$

$$y - 3 = \frac{1}{2}x - \frac{1}{2}$$

$$y = \frac{1}{2}x + \frac{5}{2}$$

$$x - 2y = -5 \text{ or } y = \frac{1}{2}x + \frac{5}{2}$$

58. Containing the points (-3, 4) and (2, 5)

$$m = \frac{5 - 4}{2 - (-3)} = \frac{1}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{1}{5}(x - 2)$$

$$y - 5 = \frac{1}{5}x - \frac{2}{5}$$

$$y = \frac{1}{5}x + \frac{23}{5}$$

$$x - 5y = -23 \text{ or } y = \frac{1}{5}x + \frac{23}{5}$$

59. Slope = -3; y-intercept = 3

$$y = mx + b$$

$$y = -3x + 3$$

$$3x + y = 3 \text{ or } y = -3x + 3$$

60. Slope = -2; y-intercept = -2

$$y = mx + b$$

$$y = -2x + (-2)$$

$$2x + y = -2 \text{ or } y = -2x - 2$$

61. x-intercept = -4; y-intercept = 4

Points are (-4, 0) and (0, 4)

$$m = \frac{4 - 0}{0 - (-4)} = \frac{4}{4} = 1$$

$$y = mx + b$$

$$y = 1x + 4$$

$$y = x + 4$$

$$x - y = -4 \text{ or } y = x + 4$$

62. x-intercept = 2; y-intercept = -1

Points are (2, 0) and (0, -1)

$$m = \frac{-1 - 0}{0 - 2} = \frac{-1}{-2} = \frac{1}{2}$$

$$y = mx + b$$

$$y = \frac{1}{2}x - 1$$

$$x - 2y = 2 \text{ or } y = \frac{1}{2}x - 1$$

63. Slope undefined; containing the point (2, 4)
This is a vertical line.

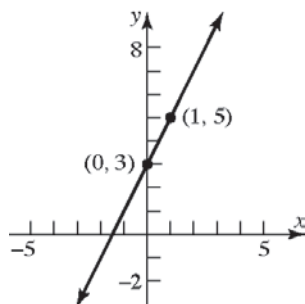
$$x = 2 \quad \text{No slope-intercept form.}$$

Chapter 1: Graphs

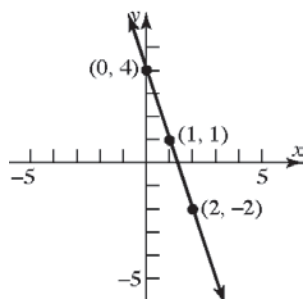
- 64.** Slope undefined; containing the point (3, 8)
This is a vertical line.
 $x = 3$ No slope-intercept form.
- 65.** Horizontal lines have slope $m = 0$ and take the form $y = b$. Therefore, the horizontal line passing through the point $(-3, 2)$ is $y = 2$.
- 66.** Vertical lines have an undefined slope and take the form $x = a$. Therefore, the vertical line passing through the point $(4, -5)$ is $x = 4$.
- 67.** Parallel to $y = 2x$; Slope = 2
Containing $(-1, 2)$
 $y - y_1 = m(x - x_1)$
 $y - 2 = 2(x - (-1))$
 $y - 2 = 2x + 2 \rightarrow y = 2x + 4$
 $2x - y = -4$ or $y = 2x + 4$
- 68.** Parallel to $y = -3x$; Slope = -3; Containing the point $(-1, 2)$
 $y - y_1 = m(x - x_1)$
 $y - 2 = -3(x - (-1))$
 $y - 2 = -3x - 3 \rightarrow y = -3x - 1$
 $3x + y = -1$ or $y = -3x - 1$
- 69.** Parallel to $x - 2y = -5$;
Slope = $\frac{1}{2}$; Containing the point $(0, 0)$
 $y - y_1 = m(x - x_1)$
 $y - 0 = \frac{1}{2}(x - 0) \rightarrow y = \frac{1}{2}x$
 $x - 2y = 0$ or $y = \frac{1}{2}x$
- 70.** Parallel to $2x - y = -2$; Slope = 2
Containing the point $(0, 0)$
 $y - y_1 = m(x - x_1)$
 $y - 0 = 2(x - 0)$
 $y = 2x$
 $2x - y = 0$ or $y = 2x$
- 71.** Parallel to $x = 5$; Containing $(4, 2)$
This is a vertical line.
 $x = 4$ No slope-intercept form.
- 72.** Parallel to $y = 5$; Containing the point $(4, 2)$
This is a horizontal line. Slope = 0
 $y = 2$
- 73.** Perpendicular to $y = \frac{1}{2}x + 4$; Containing $(1, -2)$
Slope of perpendicular = -2
 $y - y_1 = m(x - x_1)$
 $y - (-2) = -2(x - 1)$
 $y + 2 = -2x + 2 \rightarrow y = -2x$
 $2x + y = 0$ or $y = -2x$
- 74.** Perpendicular to $y = 2x - 3$; Containing the point $(1, -2)$
Slope of perpendicular = $-\frac{1}{2}$
 $y - y_1 = m(x - x_1)$
 $y - (-2) = -\frac{1}{2}(x - 1)$
 $y + 2 = -\frac{1}{2}x + \frac{1}{2} \rightarrow y = -\frac{1}{2}x - \frac{3}{2}$
 $x + 2y = -3$ or $y = -\frac{1}{2}x - \frac{3}{2}$
- 75.** Perpendicular to $x - 2y = -5$; Containing the point $(0, 4)$
Slope of perpendicular = -2
 $y = mx + b$
 $y = -2x + 4$
 $2x + y = 4$ or $y = -2x + 4$
- 76.** Perpendicular to $2x + y = 2$; Containing the point $(-3, 0)$
Slope of perpendicular = $\frac{1}{2}$
 $y - y_1 = m(x - x_1)$
 $y - 0 = \frac{1}{2}(x - (-3)) \rightarrow y = \frac{1}{2}x + \frac{3}{2}$
 $x - 2y = -3$ or $y = \frac{1}{2}x + \frac{3}{2}$
- 77.** Perpendicular to $x = 8$; Containing $(3, 4)$
Slope of perpendicular = 0 (horizontal line)
 $y = 4$

78. Perpendicular to $y = 8$;
Containing the point $(3, 4)$
Slope of perpendicular is undefined (vertical line). $x = 3$ No slope-intercept form.

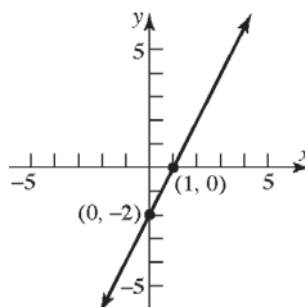
79. $y = 2x + 3$; Slope = 2; y-intercept = 3



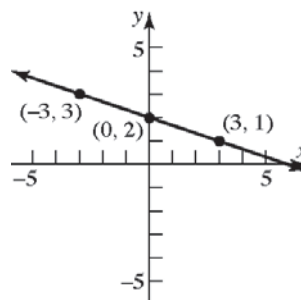
80. $y = -3x + 4$; Slope = -3; y-intercept = 4



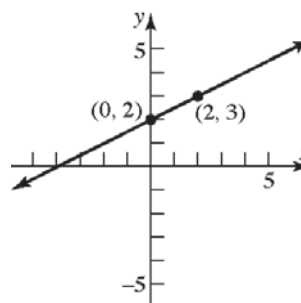
81. $\frac{1}{2}y = x - 1$; $y = 2x - 2$
Slope = 2; y-intercept = -2



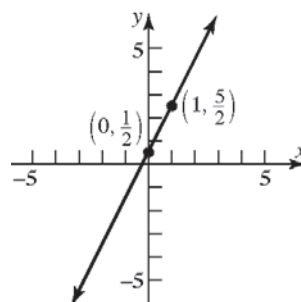
82. $\frac{1}{3}x + y = 2$; $y = -\frac{1}{3}x + 2$
Slope = $-\frac{1}{3}$; y-intercept = 2



83. $y = \frac{1}{2}x + 2$; Slope = $\frac{1}{2}$; y-intercept = 2



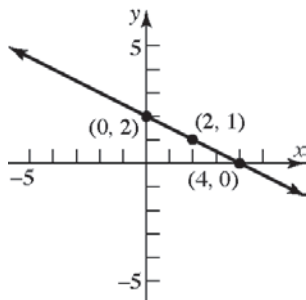
84. $y = 2x + \frac{1}{2}$; Slope = 2; y-intercept = $\frac{1}{2}$



Chapter 1: Graphs

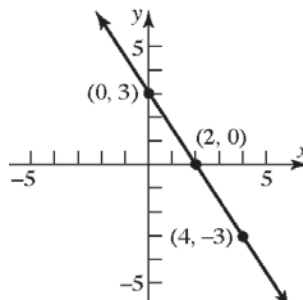
85. $x + 2y = 4$; $2y = -x + 4 \rightarrow y = -\frac{1}{2}x + 2$

Slope = $-\frac{1}{2}$; y-intercept = 2



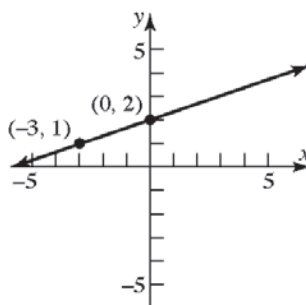
88. $3x + 2y = 6$; $2y = -3x + 6 \rightarrow y = -\frac{3}{2}x + 3$

Slope = $-\frac{3}{2}$; y-intercept = 3



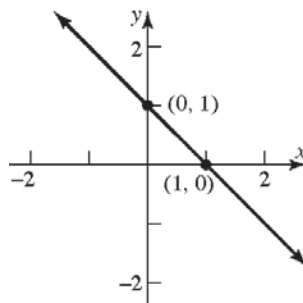
86. $-x + 3y = 6$; $3y = x + 6 \rightarrow y = \frac{1}{3}x + 2$

Slope = $\frac{1}{3}$; y-intercept = 2



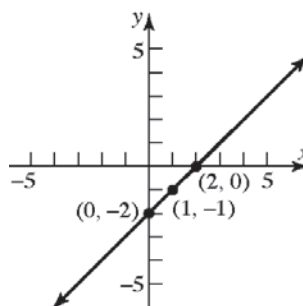
89. $x + y = 1$; $y = -x + 1$

Slope = -1; y-intercept = 1



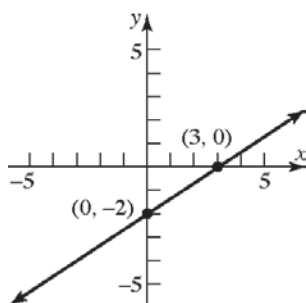
90. $x - y = 2$; $y = x - 2$

Slope = 1; y-intercept = -2

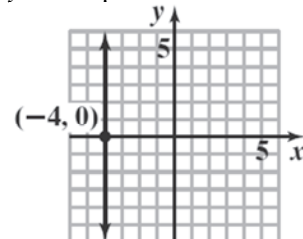


87. $2x - 3y = 6$; $-3y = -2x + 6 \rightarrow y = \frac{2}{3}x - 2$

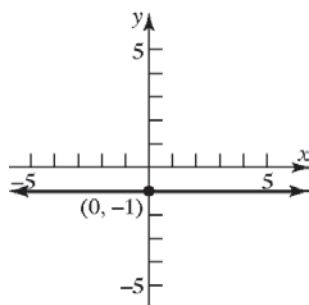
Slope = $\frac{2}{3}$; y-intercept = -2



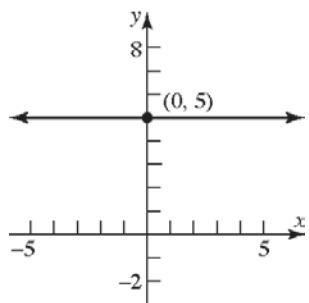
91. $x = -4$; Slope is undefined
y-intercept - none



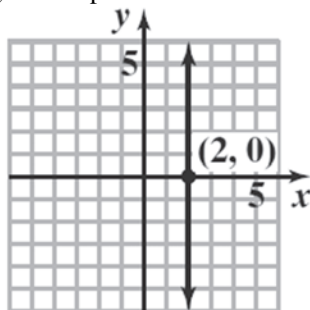
92. $y = -1$; Slope = 0; y-intercept = -1



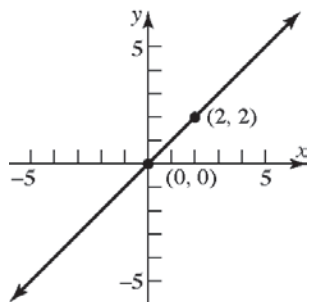
93. $y = 5$; Slope = 0; y-intercept = 5



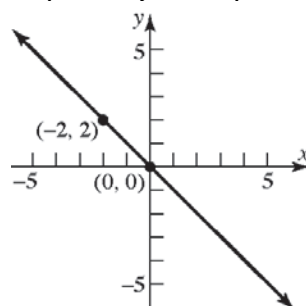
94. $x = 2$; Slope is undefined
y-intercept - none



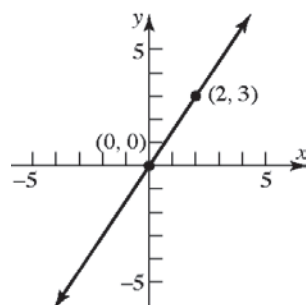
95. $y - x = 0$; $y = x$
Slope = 1; y-intercept = 0



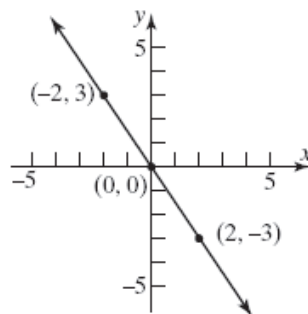
96. $x + y = 0$; $y = -x$
Slope = -1; y-intercept = 0



97. $2y - 3x = 0$; $2y = 3x \rightarrow y = \frac{3}{2}x$
Slope = $\frac{3}{2}$; y-intercept = 0



98. $3x + 2y = 0$; $2y = -3x \rightarrow y = -\frac{3}{2}x$
Slope = $-\frac{3}{2}$; y-intercept = 0



99. a. x-intercept: $2x + 3(0) = 6$
 $2x = 6$
 $x = 3$
The point (3, 0) is on the graph.

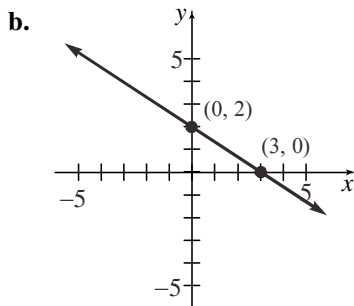
Chapter 1: Graphs

y-intercept: $2(0) + 3y = 6$

$$3y = 6$$

$$y = 2$$

The point $(0, 2)$ is on the graph.



100. a. x-intercept: $3x - 2(0) = 6$

$$3x = 6$$

$$x = 2$$

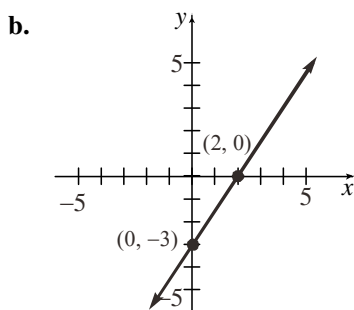
The point $(2, 0)$ is on the graph.

y-intercept: $3(0) - 2y = 6$

$$-2y = 6$$

$$y = -3$$

The point $(0, -3)$ is on the graph.



101. a. x-intercept: $-4x + 5(0) = 40$

$$-4x = 40$$

$$x = -10$$

The point $(-10, 0)$ is on the graph.

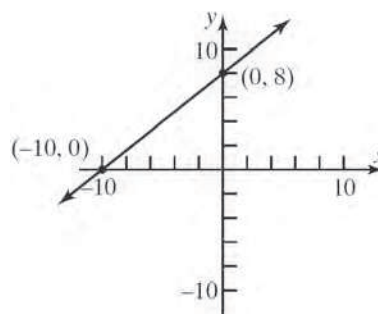
y-intercept: $-4(0) + 5y = 40$

$$5y = 40$$

$$y = 8$$

The point $(0, 8)$ is on the graph.

b.



102. a. x-intercept: $6x - 4(0) = 24$

$$6x = 24$$

$$x = 4$$

The point $(4, 0)$ is on the graph.

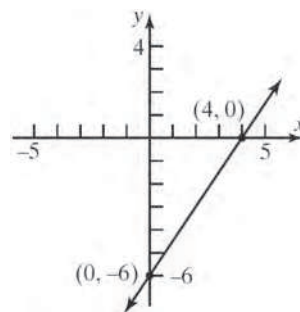
y-intercept: $6(0) - 4y = 24$

$$-4y = 24$$

$$y = -6$$

The point $(0, -6)$ is on the graph.

b.



103. a. x-intercept: $7x + 2(0) = 21$

$$7x = 21$$

$$x = 3$$

The point $(3, 0)$ is on the graph.

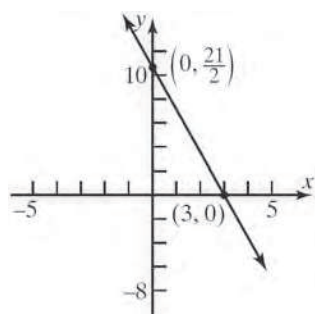
y-intercept: $7(0) + 2y = 21$

$$2y = 21$$

$$y = \frac{21}{2}$$

The point $\left(0, \frac{21}{2}\right)$ is on the graph.

b.



104. a. x-intercept: $5x + 3(0) = 18$

$$5x = 18$$

$$x = \frac{18}{5}$$

The point $(\frac{18}{5}, 0)$ is on the graph.

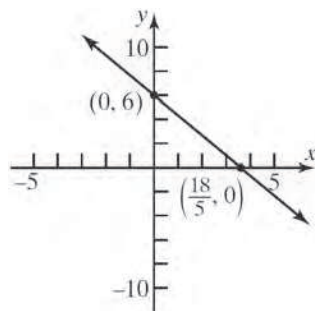
y-intercept: $5(0) + 3y = 18$

$$3y = 18$$

$$y = 6$$

The point $(0, 6)$ is on the graph.

b.



105. a. x-intercept: $\frac{1}{2}x + \frac{1}{3}(0) = 1$

$$\frac{1}{2}x = 1$$

$$x = 2$$

The point $(2, 0)$ is on the graph.

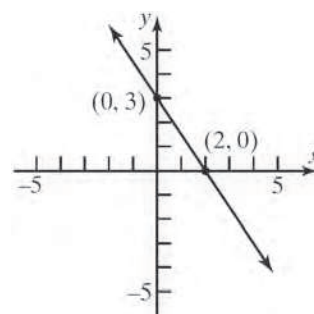
y-intercept: $\frac{1}{2}(0) + \frac{1}{3}y = 1$

$$\frac{1}{3}y = 1$$

$$y = 3$$

The point $(0, 3)$ is on the graph.

b.



106. a. x-intercept: $x - \frac{2}{3}(0) = 4$

$$x = 4$$

The point $(4, 0)$ is on the graph.

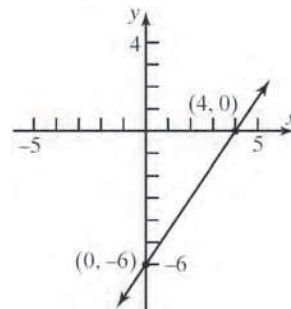
y-intercept: $(0) - \frac{2}{3}y = 4$

$$-\frac{2}{3}y = 4$$

$$y = -6$$

The point $(0, -6)$ is on the graph.

b.



107. a. x-intercept: $0.2x - 0.5(0) = 1$

$$0.2x = 1$$

$$x = 5$$

The point $(5, 0)$ is on the graph.

y-intercept: $0.2(0) - 0.5y = 1$

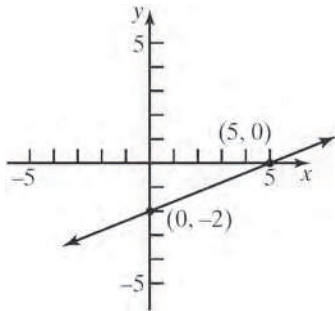
$$-0.5y = 1$$

$$y = -2$$

The point $(0, -2)$ is on the graph.

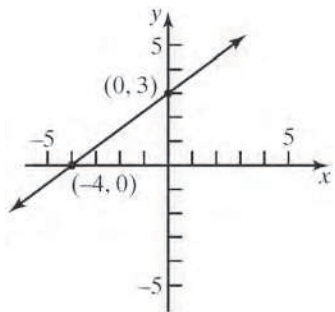
Chapter 1: Graphs

b.



108. a. x -intercept: $-0.3x + 0.4(0) = 1.2$
 $-0.3x = 1.2$
 $x = -4$
 The point $(-4, 0)$ is on the graph.
 y -intercept: $-0.3(0) + 0.4y = 1.2$
 $0.4y = 1.2$
 $y = 3$
 The point $(0, 3)$ is on the graph.

b.



109. The equation of the x -axis is $y = 0$. (The slope is 0 and the y -intercept is 0.)
 110. The equation of the y -axis is $x = 0$. (The slope is undefined.)
 111. The slopes are the same but the y -intercepts are different. Therefore, the two lines are parallel.
 112. The slopes are opposite-reciprocals. That is, their product is -1 . Therefore, the lines are perpendicular.
 113. The slopes are different and their product does not equal -1 . Therefore, the lines are neither parallel nor perpendicular.
 114. The slopes are different and their product does not equal -1 (in fact, the signs are the same so the product is positive). Therefore, the lines are neither parallel nor perpendicular.

115. Intercepts: $(0, 2)$ and $(-2, 0)$. Thus, slope = 1.
 $y = x + 2$ or $x - y = -2$

116. Intercepts: $(0, 1)$ and $(1, 0)$. Thus, slope = -1 .
 $y = -x + 1$ or $x + y = 1$

117. Intercepts: $(3, 0)$ and $(0, 1)$. Thus, slope = $-\frac{1}{3}$.
 $y = -\frac{1}{3}x + 1$ or $x + 3y = 3$

118. Intercepts: $(0, -1)$ and $(-2, 0)$. Thus,
 slope = $-\frac{1}{2}$.
 $y = -\frac{1}{2}x - 1$ or $x + 2y = -2$

119. $P_1 = (-2, 5)$, $P_2 = (1, 3)$: $m_1 = \frac{5-3}{-2-1} = \frac{2}{-3} = -\frac{2}{3}$
 $P_2 = (1, 3)$, $P_3 = (-1, 0)$: $m_2 = \frac{3-0}{1-(-1)} = \frac{3}{2}$

Since $m_1 \cdot m_2 = -1$, the line segments $\overline{P_1P_2}$ and $\overline{P_2P_3}$ are perpendicular. Thus, the points P_1 , P_2 , and P_3 are vertices of a right triangle.

120. $P_1 = (1, -1)$, $P_2 = (4, 1)$, $P_3 = (2, 2)$, $P_4 = (5, 4)$
 $m_{12} = \frac{1-(-1)}{4-1} = \frac{2}{3}$; $m_{24} = \frac{4-1}{5-4} = 3$;
 $m_{34} = \frac{4-2}{5-2} = \frac{2}{3}$; $m_{13} = \frac{2-(-1)}{2-1} = 3$

Each pair of opposite sides are parallel (same slope) and adjacent sides are not perpendicular. Therefore, the vertices are for a parallelogram.

121. $P_1 = (-1, 0)$, $P_2 = (2, 3)$, $P_3 = (1, -2)$, $P_4 = (4, 1)$
 $m_{12} = \frac{3-0}{2-(-1)} = \frac{3}{3} = 1$; $m_{24} = \frac{1-3}{4-2} = -1$;
 $m_{34} = \frac{1-(-2)}{4-1} = \frac{3}{3} = 1$; $m_{13} = \frac{-2-0}{1-(-1)} = -1$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). Therefore, the vertices are for a rectangle.

122. $P_1 = (0, 0)$, $P_2 = (1, 3)$, $P_3 = (4, 2)$, $P_4 = (3, -1)$

$$m_{12} = \frac{3-0}{1-0} = 3; \quad m_{23} = \frac{2-3}{4-1} = -\frac{1}{3};$$

$$m_{34} = \frac{-1-2}{3-4} = 3; \quad m_{14} = \frac{-1-0}{3-0} = -\frac{1}{3}$$

$$d_{12} = \sqrt{(1-0)^2 + (3-0)^2} = \sqrt{1+9} = \sqrt{10}$$

$$d_{23} = \sqrt{(4-1)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$$

$$d_{34} = \sqrt{(3-4)^2 + (-1-2)^2} = \sqrt{1+9} = \sqrt{10}$$

$$d_{14} = \sqrt{(3-0)^2 + (-1-0)^2} = \sqrt{9+1} = \sqrt{10}$$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). In addition, the length of all four sides is the same. Therefore, the vertices are for a square.

123. Let x = number of miles driven, and let C = cost in dollars.

Total cost = (cost per mile)(number of miles) + fixed cost

$$C = 0.60x + 39$$

When $x = 110$, $C = (0.60)(110) + 39 = \$105.00$.

When $x = 230$, $C = (0.60)(230) + 39 = \$177.00$.

124. Let x = number of pairs of jeans manufactured, and let C = cost in dollars.

Total cost = (cost per pair)(number of pairs) + fixed cost

$$C = 20x + 1200$$

When $x = 400$, $C = (20)(400) + 1200 = \$9200$.

When $x = 740$, $C = (20)(740) + 1200 = \$16,000$.

125. Let x = number of miles driven annually, and let C = cost in dollars.

Total cost = (approx cost per mile)(number of miles) + fixed cost

$$C = 0.28x + 6578$$

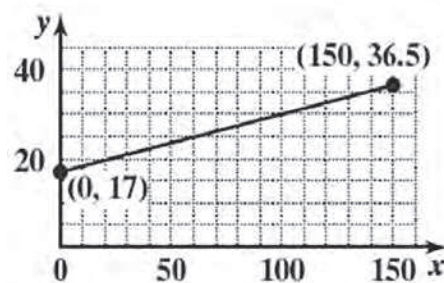
126. Let x = profit in dollars, and let S = salary in dollars.

Weekly salary = (% share of profit)(profit) + weekly pay

$$S = 0.05x + 525$$

127. a. $C = 0.13x + 17$; $0 \leq x \leq 1000$

b.



c. For 50 miles,

$$C = 0.13(50) + 17 = \$23.50$$

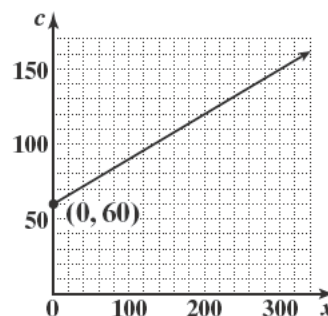
d. For 120 miles,

$$C = 0.13(120) + 17 = \$32.60$$

e. For every 1-mile in distance traveled, the cost will increase 13 cents.

128. a. $C = 60 + 0.25x$

b.



c. For 20 minutes,

$$C = 60 + 0.25(20) = \$65.00$$

d. For 60 minutes,

$$C = 60 + 0.25(60) = \$75.00$$

e. For every 1-minute increase in international calls the cost will increase by \$0.25 (that is, 25 cents).

Chapter 1: Graphs

129. $(^{\circ}C, ^{\circ}F) = (0, 32); (^{\circ}C, ^{\circ}F) = (100, 212)$

$$\text{slope} = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$$

$$^{\circ}F - 32 = \frac{9}{5}(^{\circ}C - 0)$$

$$^{\circ}F - 32 = \frac{9}{5}(^{\circ}C)$$

$$^{\circ}C = \frac{5}{9}(^{\circ}F - 32)$$

If $^{\circ}F = 70$, then

$$^{\circ}C = \frac{5}{9}(70 - 32) = \frac{5}{9}(38)$$

$$^{\circ}C \approx 21.1^{\circ}$$

130. a. $K = ^{\circ}C + 273$

b. $^{\circ}C = \frac{5}{9}(^{\circ}F - 32)$

$$K = \frac{5}{9}(^{\circ}F - 32) + 273$$

$$K = \frac{5}{9}^{\circ}F - \frac{160}{9} + 273$$

$$K = \frac{5}{9}^{\circ}F + \frac{2297}{9}$$

131. a. The y-intercept is $(0, 30)$, so $b = 30$. Since the ramp drops 2 inches for every 25 inches of run, the slope is $m = \frac{-2}{25} = -\frac{2}{25}$. Thus,

$$\text{the equation is } y = -\frac{2}{25}x + 30.$$

- b. Let $y = 0$.

$$0 = -\frac{2}{25}x + 30$$

$$\frac{2}{25}x = 30$$

$$\frac{25}{2}\left(\frac{2}{25}x\right) = \frac{25}{2}(30)$$

$$x = 375$$

The x-intercept is $(375, 0)$. This means that the ramp meets the floor 375 inches (or 31.25 feet) from the base of the platform.

- c. No. From part (b), the run is 31.25 feet which exceeds the required maximum of 30 feet.
- d. First, design requirements state that the maximum slope is a drop of 1 inch for each 12 inches of run. This means $|m| \leq \frac{1}{12}$.

Second, the run is restricted to be no more than 30 feet = 360 inches. For a rise of 30 inches, this means the minimum slope is

$$\frac{30}{360} = \frac{1}{12}. \text{ That is, } |m| \geq \frac{1}{12}. \text{ Thus, the}$$

only possible slope is $|m| = \frac{1}{12}$. The

diagram indicates that the slope is negative. Therefore, the only slope that can be used to obtain the 30-inch rise and still meet design

requirements is $m = -\frac{1}{12}$. In words, for every 12 inches of run, the ramp must drop *exactly* 1 inch.

132. a. Let x represent the percent of internet ad spending. Let y represent the percent of print ad spending. Then the points $(0.19, 0.26)$ and $(0.35, 0.16)$ are on the line.

$$\text{Thus, } m = \frac{16 - 26}{35 - 19} = -\frac{10}{16}$$

the point-slope formula we have

$$y - 26 = -0.625(x - 19)$$

$$y - 26 = -0.625x + 11.875$$

$$y = -0.625x + 37.875$$

- b. x-intercept: $0 = -0.625x + 37.875$
 $-37.875 = -0.625x$

$$60.6 = x$$

$$\text{y-intercept: } y = -0.625(0) + 37.875$$

$$= 37.875$$

The intercepts are $(60.6, 0)$ and $(0, 37.875)$.

- c. y-intercept: When Internet ads account for 0% of U.S. advertisement spending, print ads account for 37.875% of the spending.
x-intercept: When Internet ads account for 60.6% of U.S. advertisement spending, print ads account for 0% of the spending.

- d. Let $x = 39.2$.
 $y = -0.625(39.2) + 37.875 = 13.4\%$

133. a. Let x = number of boxes to be sold, and A = money, in dollars, spent on advertising. We have the points $(x_1, A_1) = (100,000, 40,000)$;

$$(x_2, A_2) = (200,000, 60,000)$$

$$\text{slope} = \frac{60,000 - 40,000}{200,000 - 100,000}$$

$$= \frac{20,000}{100,000} = \frac{1}{5}$$

$$A - 40,000 = \frac{1}{5}(x - 100,000)$$

$$A - 40,000 = \frac{1}{5}x - 20,000$$

$$A = \frac{1}{5}x + 20,000$$

- b. If $x = 300,000$, then

$$A = \frac{1}{5}(300,000) + 20,000 = \$80,000$$

- c. Each additional box sold requires an additional \$0.20 in advertising.

134. $2x - y = C$

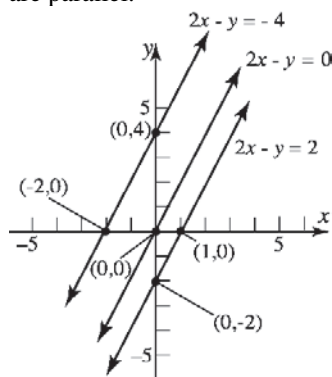
Graph the lines:

$$2x - y = -4$$

$$2x - y = 0$$

$$2x - y = 2$$

All the lines have the same slope, 2. The lines are parallel.



135. Put each linear equation in slope/intercept form.

$$x + 2y = 5 \quad 2x - 3y + 4 = 0 \quad ax + y = 0$$

$$2y = -x + 5 \quad -3y = -2x - 4 \quad y = -ax$$

$$y = -\frac{1}{2}x + \frac{5}{2} \quad y = \frac{2}{3}x + \frac{4}{3}$$

If the slope of $y = -ax$ equals the slope of either of the other two lines, then no triangle is formed.

$$\text{So, } -a = -\frac{1}{2} \Rightarrow a = \frac{1}{2} \text{ and } -a = \frac{2}{3} \Rightarrow a = -\frac{2}{3}.$$

Also if all three lines intersect at a single point,

then no triangle is formed. So, we find where

$$y = -\frac{1}{2}x + \frac{5}{2} \text{ and } y = \frac{2}{3}x + \frac{4}{3} \text{ intersect.}$$

$$-\frac{1}{2}x + \frac{5}{2} = \frac{2}{3}x + \frac{4}{3}$$

$$-\frac{7}{6}x = -\frac{7}{6}$$

$$x = 1$$

$$-\frac{1}{2}(1) + \frac{5}{2} = 2$$

The two lines intersect at $(1, 2)$. If $y = -ax$ also contains the point $(1, 2)$, then

$$2 = -a \cdot 1 \Rightarrow a = -2.$$

The three numbers are $\frac{1}{2}$, $-\frac{2}{3}$, and -2 .

136. The slope of the line containing (a, b) and (b, a) is

$$\frac{a-b}{b-a} = -1$$

The slope of the line $y = x$ is 1.

The two lines are perpendicular.

The midpoint of (a, b) and (b, a) is

$$M = \left(\frac{a+b}{2}, \frac{b+a}{2} \right).$$

Since the x and y coordinates of M are equal, M lies on the line $y = x$.

$$\text{Note: } \frac{a+b}{2} = \frac{b+a}{2}$$

137. The three midpoints are

$$\left(\frac{0+a}{2}, \frac{0+0}{2} \right) = \left(\frac{a}{2}, 0 \right), \left(\frac{a+b}{2}, \frac{0+c}{2} \right) = \left(\frac{a+b}{2}, \frac{c}{2} \right)$$

$$\text{and } \left(\frac{0+b}{2}, \frac{0+c}{2} \right) = \left(\frac{b}{2}, \frac{c}{2} \right).$$

Chapter 1: Graphs

Line 1 from $(0,0)$ to $\left(\frac{a+b}{2}, \frac{c}{2}\right)$

$$m = \frac{\frac{c}{2} - 0}{\frac{a+b}{2} - 0} = \frac{c}{a+b};$$

$$y - 0 = \frac{c}{a+b}(x - 0)$$

$$y = \frac{c}{a+b}x_1$$

Line 2 from $(a, 0)$ to $\left(\frac{b}{2}, \frac{c}{2}\right)$

$$m = \frac{\frac{c}{2} - 0}{\frac{b}{2} - a} = \frac{\frac{c}{2}}{\frac{b-2a}{2}} = \frac{c}{b-2a}$$

$$y - 0 = \frac{c}{b-2a}(x - a)$$

$$y = \frac{c}{b-2a}(x - a)$$

Line 3 from $\left(\frac{a}{2}, 0\right)$ to (b, c)

$$m = \frac{c - 0}{b - \frac{a}{2}} = \frac{2c}{2b - a}$$

$$y - 0 = \frac{2c}{2b - a}\left(x - \frac{a}{2}\right)$$

$$y = \frac{2c}{2b - a}\left(x - \frac{a}{2}\right)$$

Find where line 1 and line 2 intersect:

$$\frac{c}{a+b}x = \frac{c}{b-2a}(x - a)$$

$$\frac{b-2a}{a+b}x = x - a$$

$$\frac{b-2a-a-b}{a+b}x = -a$$

$$\frac{-3a}{a+b}x = -a$$

$$x = \frac{a+b}{3};$$

Substitute into line 1:

$$y = \frac{c}{a+b} \cdot \frac{a+b}{3} = \frac{c}{3}.$$

So, line 1 and line 2 intersect at $\left(\frac{a+b}{3}, \frac{c}{3}\right)$.

Show that line 3 contains the point $\left(\frac{a+b}{3}, \frac{c}{3}\right)$:

$$y = \frac{2c}{2b-a}\left(\frac{a+b}{3} - \frac{a}{2}\right) = \frac{2c}{2b-a} \cdot \frac{2b-a}{6} = \frac{c}{3} \quad \text{So}$$

the three lines intersect at $\left(\frac{a+b}{3}, \frac{c}{3}\right)$.

138. Refer to Figure 47. Assume $m_1 m_2 = -1$. Then

$$\begin{aligned} [d(A, B)]^2 &= (1-1)^2 + (m_1 - m_2)^2 \\ &= (m_1 - m_2)^2 \\ &= m_1^2 - 2m_1 m_2 + m_2^2 \\ &= m_1^2 - 2(-1) + m_2^2 \\ &= m_1^2 + m_2^2 + 2 \end{aligned}$$

Now,

$$\begin{aligned} [d(O, B)]^2 &= (1-0)^2 + (m_1 - 0)^2 = 1 + m_1^2 \\ [d(O, A)]^2 &= (1-0)^2 + (m_2 - 0)^2 = 1 + m_2^2 \end{aligned}$$

So

$$\begin{aligned} [d(O, B)]^2 + [d(O, A)]^2 &= 1 + m_1^2 + 1 + m_2^2 \\ &= m_1^2 + m_2^2 + 2 = [d(A, B)]^2 \end{aligned}$$

By the converse of the Pythagorean Theorem, $\triangle AOB$ is a right triangle with right angle at vertex O . Thus lines OA and OB are perpendicular.

139. (b), (c), (e) and (g)

The line has positive slope and positive y-intercept.

140. (a), (c), and (g)

The line has negative slope and positive y-intercept.

141. (c)

The equation $x - y = -2$ has slope 1 and y-intercept $(0, 2)$. The equation $x - y = 1$ has slope 1 and y-intercept $(0, -1)$. Thus, the lines are parallel with positive slopes. One line has a positive y-intercept and the other with a negative y-intercept.

142. (d)

The equation $y - 2x = 2$ has slope 2 and y-intercept $(0, 2)$. The equation $x + 2y = -1$ has slope $-\frac{1}{2}$ and y-intercept $\left(0, -\frac{1}{2}\right)$. The lines are perpendicular since $2\left(-\frac{1}{2}\right) = -1$. One line has a positive y-intercept and the other with a negative y-intercept.

143 – 145. Answers will vary.

146. No, the equation of a vertical line cannot be written in slope-intercept form because the slope is undefined.

147. No, a line does not need to have both an x-intercept and a y-intercept. Vertical and horizontal lines have only one intercept (unless they are a coordinate axis). Every line must have at least one intercept.

148. Two lines with equal slopes and equal y-intercepts are coinciding lines (i.e. the same).

149. Two lines that have the same x-intercept and y-intercept (assuming the x-intercept is not 0) are the same line since a line is uniquely defined by two distinct points.

150. No. Two lines with the same slope and different x-intercepts are distinct parallel lines and have no points in common.

Assume Line 1 has equation $y = mx + b_1$ and Line 2 has equation $y = mx + b_2$,

Line 1 has x-intercept $-\frac{b_1}{m}$ and y-intercept b_1 .

Line 2 has x-intercept $-\frac{b_2}{m}$ and y-intercept b_2 .

Assume also that Line 1 and Line 2 have unequal x-intercepts.

If the lines have the same y-intercept, then $b_1 = b_2$.

$$b_1 = b_2 \Rightarrow \frac{b_1}{m} = \frac{b_2}{m} \Rightarrow -\frac{b_1}{m} = -\frac{b_2}{m}$$

But $-\frac{b_1}{m} = -\frac{b_2}{m} \Rightarrow$ Line 1 and Line 2 have the same x-intercept, which contradicts the original assumption that the lines have unequal x-intercepts. Therefore, Line 1 and Line 2 cannot have the same y-intercept.

151. Yes. Two distinct lines with the same y-intercept, but different slopes, can have the same x-intercept if the x-intercept is $x = 0$.

Assume Line 1 has equation $y = m_1x + b$ and Line 2 has equation $y = m_2x + b$,

Line 1 has x-intercept $-\frac{b}{m_1}$ and y-intercept b .

Line 2 has x-intercept $-\frac{b}{m_2}$ and y-intercept b .

Assume also that Line 1 and Line 2 have unequal slopes, that is $m_1 \neq m_2$.

If the lines have the same x-intercept, then

$$-\frac{b}{m_1} = -\frac{b}{m_2}.$$

$$-\frac{b}{m_1} = -\frac{b}{m_2}$$

$$-m_2b = -m_1b$$

$$-m_2b + m_1b = 0$$

$$\text{But } -m_2b + m_1b = 0 \Rightarrow b(m_1 - m_2) = 0$$

$$\Rightarrow b = 0$$

$$\text{or } m_1 - m_2 = 0 \Rightarrow m_1 = m_2$$

Since we are assuming that $m_1 \neq m_2$, the only way that the two lines can have the same x-intercept is if $b = 0$.

152. Answers will vary.

$$153. \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 2}{1 - (-3)} = \frac{-6}{4} = -\frac{3}{2}$$

It appears that the student incorrectly found the slope by switching the direction of one of the subtractions.

Chapter 1: Graphs

Section 1.4

1. add; $\left(\frac{1}{2} \cdot 10\right)^2 = 25$

2. $(x-2)^2 = 9$

$$x-2 = \pm\sqrt{9}$$

$$x-2 = \pm 3$$

$$x = 2 \pm 3$$

$$x = 5 \text{ or } x = -1$$

The solution set is $\{-1, 5\}$.

3. False. For example, $x^2 + y^2 + 2x + 2y + 8 = 0$ is not a circle. It has no real solutions.

4. radius

5. True; $r^2 = 9 \rightarrow r = 3$

6. False; the center of the circle $(x+3)^2 + (y-2)^2 = 13$ is $(-3, 2)$.

7. d

8. a

9. Center = $(2, 1)$

Radius = distance from $(0, 1)$ to $(2, 1)$

$$= \sqrt{(2-0)^2 + (1-1)^2} = \sqrt{4} = 2$$

Equation: $(x-2)^2 + (y-1)^2 = 4$

10. Center = $(1, 2)$

Radius = distance from $(1, 0)$ to $(1, 2)$

$$= \sqrt{(1-1)^2 + (2-0)^2} = \sqrt{4} = 2$$

Equation: $(x-1)^2 + (y-2)^2 = 4$

11. Center = midpoint of $(1, 2)$ and $(4, 2)$

$$= \left(\frac{1+4}{2}, \frac{2+2}{2}\right) = \left(\frac{5}{2}, 2\right)$$

Radius = distance from $\left(\frac{5}{2}, 2\right)$ to $(4, 2)$

$$= \sqrt{\left(4 - \frac{5}{2}\right)^2 + (2-2)^2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

Equation: $\left(x - \frac{5}{2}\right)^2 + (y-2)^2 = \frac{9}{4}$

12. Center = midpoint of $(0, 1)$ and $(2, 3)$

$$= \left(\frac{0+2}{2}, \frac{1+3}{2}\right) = (1, 2)$$

Radius = distance from $(1, 2)$ to $(2, 3)$

$$= \sqrt{(2-1)^2 + (3-2)^2} = \sqrt{2}$$

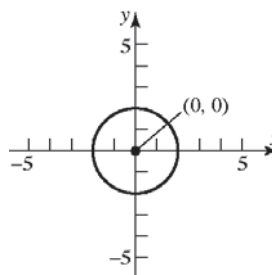
Equation: $(x-1)^2 + (y-2)^2 = 2$

13. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + (y-0)^2 = 2^2$$

$$x^2 + y^2 = 4$$

General form: $x^2 + y^2 - 4 = 0$

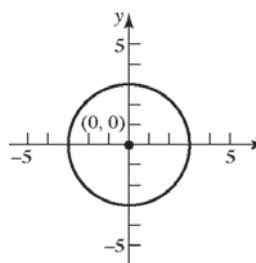


14. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + (y-0)^2 = 3^2$$

$$x^2 + y^2 = 9$$

General form: $x^2 + y^2 - 9 = 0$



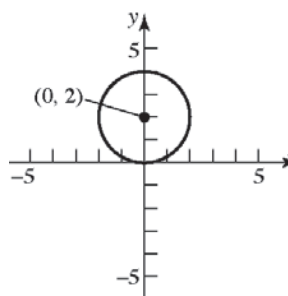
15. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + (y-2)^2 = 2^2$$

$$x^2 + (y-2)^2 = 4$$

General form: $x^2 + y^2 - 4y + 4 = 4$

$$x^2 + y^2 - 4y = 0$$



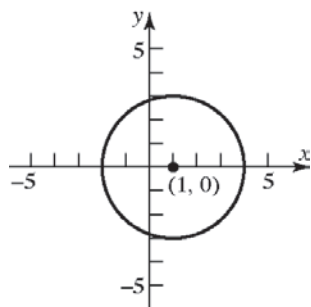
16. $(x-h)^2 + (y-k)^2 = r^2$

$(x-1)^2 + (y-0)^2 = 3^2$

$(x-1)^2 + y^2 = 9$

General form: $x^2 - 2x + 1 + y^2 = 9$

$x^2 + y^2 - 2x - 8 = 0$



17. $(x-h)^2 + (y-k)^2 = r^2$

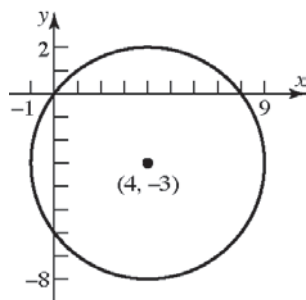
$(x-4)^2 + (y-(-3))^2 = 5^2$

$(x-4)^2 + (y+3)^2 = 25$

General form:

$x^2 - 8x + 16 + y^2 + 6y + 9 = 25$

$x^2 + y^2 - 8x + 6y = 0$



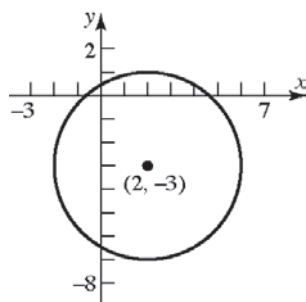
18. $(x-h)^2 + (y-k)^2 = r^2$

$(x-2)^2 + (y-(-3))^2 = 4^2$

$(x-2)^2 + (y+3)^2 = 16$

General form: $x^2 - 4x + 4 + y^2 + 6y + 9 = 16$

$x^2 + y^2 - 4x + 6y - 3 = 0$



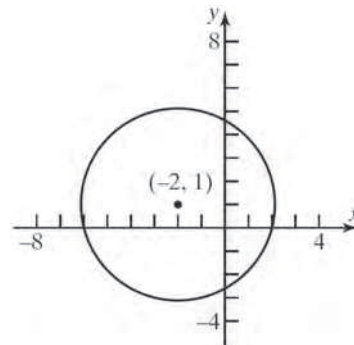
19. $(x-h)^2 + (y-k)^2 = r^2$

$(x-(-2))^2 + (y-1)^2 = 4^2$

$(x+2)^2 + (y-1)^2 = 16$

General form: $x^2 + 4x + 4 + y^2 - 2y + 1 = 16$

$x^2 + y^2 + 4x - 2y - 11 = 0$



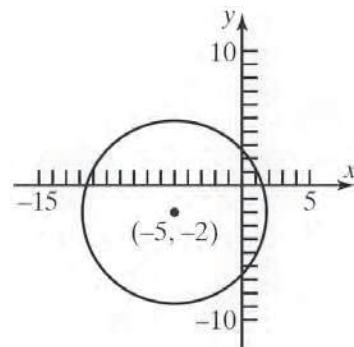
20. $(x-h)^2 + (y-k)^2 = r^2$

$(x-(-5))^2 + (y-(-2))^2 = 7^2$

$(x+5)^2 + (y+2)^2 = 49$

General form: $x^2 + 10x + 25 + y^2 + 4y + 4 = 49$

$x^2 + y^2 + 10x + 4y - 20 = 0$



21. $(x-h)^2 + (y-k)^2 = r^2$

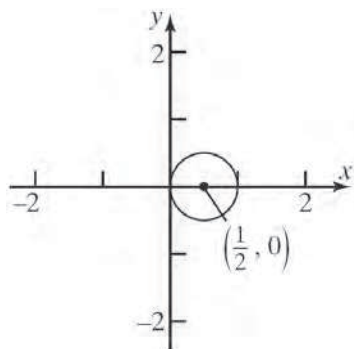
$\left(x - \frac{1}{2}\right)^2 + (y-0)^2 = \left(\frac{1}{2}\right)^2$

$\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$

General form: $x^2 - x + \frac{1}{4} + y^2 = \frac{1}{4}$

$x^2 + y^2 - x = 0$

Chapter 1: Graphs



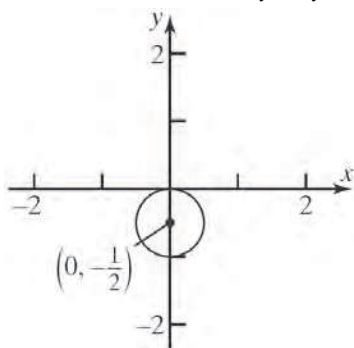
$$22. (x-h)^2 + (y-k)^2 = r^2$$

$$(x-0)^2 + \left(y - \left(-\frac{1}{2}\right)\right)^2 = \left(\frac{1}{2}\right)^2$$

$$x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{4}$$

General form: $x^2 + y^2 + y + \frac{1}{4} = \frac{1}{4}$

$$x^2 + y^2 + y = 0$$



$$23. (x-h)^2 + (y-k)^2 = r^2$$

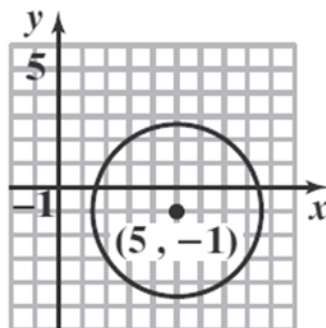
$$(x-5)^2 + (y-(-1))^2 = (\sqrt{13})^2$$

$$(x-5)^2 + (y+1)^2 = 13$$

General form:

$$x^2 - 10x + 25 + y^2 + 2y + 1 = 13$$

$$x^2 + y^2 - 10x + 2y + 13 = 0$$



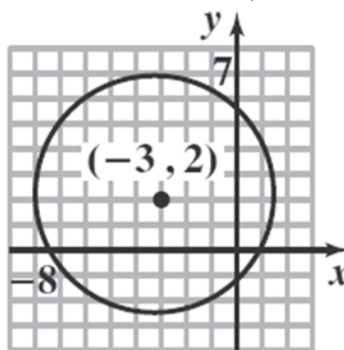
$$24. (x-h)^2 + (y-k)^2 = r^2$$

$$(x-(-3))^2 + (y-2)^2 = (2\sqrt{5})^2$$

$$(x+3)^2 + (y-2)^2 = 20$$

General form: $x^2 + 6x + 9 + y^2 - 4y + 4 = 20$

$$x^2 + y^2 + 6x - 4y - 7 = 0$$

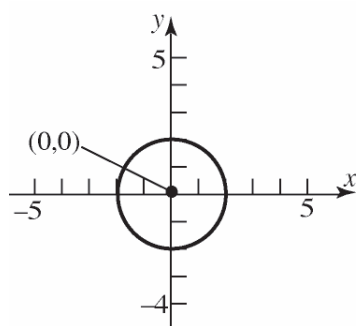


$$25. x^2 + y^2 = 4$$

$$x^2 + y^2 = 2^2$$

a. Center: (0,0); Radius = 2

b.



c. x -intercepts: $x^2 + (0)^2 = 4$

$$x^2 = 4$$

$$x = \pm\sqrt{4} = \pm 2$$

$$y$$
-intercepts: $(0)^2 + y^2 = 4$

$$y^2 = 4$$

$$y = \pm\sqrt{4} = \pm 2$$

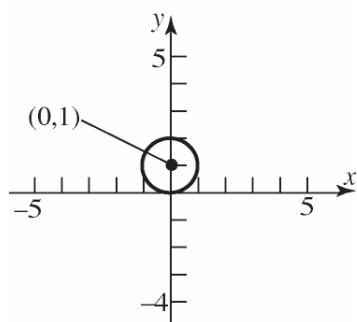
The intercepts are $(-2, 0)$, $(2, 0)$, $(0, -2)$, and $(0, 2)$.

26. $x^2 + (y-1)^2 = 1$

$$x^2 + (y-1)^2 = 1^2$$

 a. Center: $(0, 1)$; Radius = 1

b.



c. x -intercepts: $x^2 + (0-1)^2 = 1$

$$x^2 + 1 = 1$$

$$x^2 = 0$$

$$x = \pm\sqrt{0} = 0$$

$$y$$
-intercepts: $(0)^2 + (y-1)^2 = 1$

$$(y-1)^2 = 1$$

$$y-1 = \pm\sqrt{1}$$

$$y-1 = \pm 1$$

$$y = 1 \pm 1$$

$$y = 2 \text{ or } y = 0$$

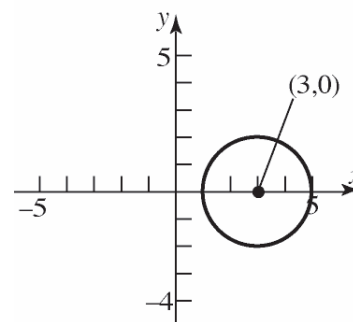
The intercepts are $(0, 0)$ and $(0, 2)$.

27. $2(x-3)^2 + 2y^2 = 8$

$$(x-3)^2 + y^2 = 4$$

 a. Center: $(3, 0)$; Radius = 2

b.



c. x -intercepts: $(x-3)^2 + (0)^2 = 4$

$$(x-3)^2 = 4$$

$$x-3 = \pm\sqrt{4}$$

$$x-3 = \pm 2$$

$$x = 3 \pm 2$$

$$x = 5 \text{ or } x = 1$$

$$y$$
-intercepts: $(0-3)^2 + y^2 = 4$

$$(-3)^2 + y^2 = 4$$

$$9 + y^2 = 4$$

$$y^2 = -5$$

No real solution.

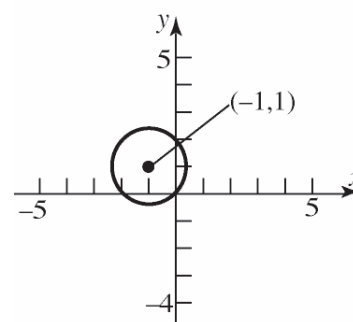
The intercepts are $(1, 0)$ and $(5, 0)$.

28. $3(x+1)^2 + 3(y-1)^2 = 6$

$$(x+1)^2 + (y-1)^2 = 2$$

 a. Center: $(-1, 1)$; Radius = $\sqrt{2}$

b.



Chapter 1: Graphs

c. x -intercepts: $(x+1)^2 + (0-1)^2 = 2$
 $(x+1)^2 + (-1)^2 = 2$
 $(x+1)^2 + 1 = 2$
 $(x+1)^2 = 1$
 $x+1 = \pm\sqrt{1}$
 $x+1 = \pm 1$
 $x = -1 \pm 1$
 $x = 0$ or $x = -2$

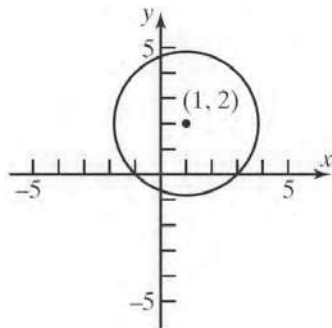
y -intercepts: $(0+1)^2 + (y-1)^2 = 2$
 $(1)^2 + (y-1)^2 = 2$
 $1 + (y-1)^2 = 2$
 $(y-1)^2 = 1$
 $y-1 = \pm\sqrt{1}$
 $y-1 = \pm 1$
 $y = 1 \pm 1$
 $y = 2$ or $y = 0$

The intercepts are $(-2, 0)$, $(0, 0)$, and $(0, 2)$.

29. $x^2 + y^2 - 2x - 4y - 4 = 0$
 $x^2 - 2x + y^2 - 4y = 4$
 $(x^2 - 2x + 1) + (y^2 - 4y + 4) = 4 + 1 + 4$
 $(x-1)^2 + (y-2)^2 = 3^2$

a. Center: $(1, 2)$; Radius = 3

b.



c. x -intercepts: $(x-1)^2 + (0-2)^2 = 3^2$
 $(x-1)^2 + (-2)^2 = 3^2$
 $(x-1)^2 + 4 = 9$
 $(x-1)^2 = 5$
 $x-1 = \pm\sqrt{5}$
 $x = 1 \pm \sqrt{5}$

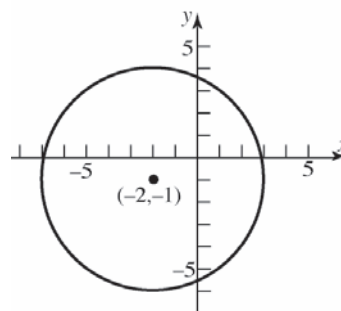
y -intercepts: $(0-1)^2 + (y-2)^2 = 3^2$
 $(-1)^2 + (y-2)^2 = 3^2$
 $1 + (y-2)^2 = 9$
 $(y-2)^2 = 8$
 $y-2 = \pm\sqrt{8}$
 $y-2 = \pm 2\sqrt{2}$
 $y = 2 \pm 2\sqrt{2}$

The intercepts are $(1-\sqrt{5}, 0)$, $(1+\sqrt{5}, 0)$, $(0, 2-2\sqrt{2})$, and $(0, 2+2\sqrt{2})$.

30. $x^2 + y^2 + 4x + 2y - 20 = 0$
 $x^2 + 4x + y^2 + 2y = 20$
 $(x^2 + 4x + 4) + (y^2 + 2y + 1) = 20 + 4 + 1$
 $(x+2)^2 + (y+1)^2 = 5^2$

a. Center: $(-2, -1)$; Radius = 5

b.



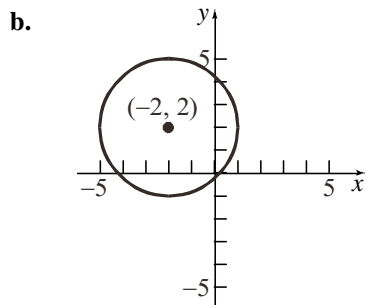
c. x -intercepts: $(x+2)^2 + (0+1)^2 = 5^2$
 $(x+2)^2 + 1 = 25$
 $(x+2)^2 = 24$
 $x+2 = \pm\sqrt{24}$
 $x+2 = \pm 2\sqrt{6}$
 $x = -2 \pm 2\sqrt{6}$

y -intercepts: $(0+2)^2 + (y+1)^2 = 5^2$
 $4 + (y+1)^2 = 25$
 $(y+1)^2 = 21$
 $y+1 = \pm\sqrt{21}$
 $y = -1 \pm \sqrt{21}$

The intercepts are $(-2-2\sqrt{6}, 0)$, $(-2+2\sqrt{6}, 0)$, $(0, -1-\sqrt{21})$, and $(0, -1+\sqrt{21})$.

$$\begin{aligned}
 31. \quad & x^2 + y^2 + 4x - 4y - 1 = 0 \\
 & x^2 + 4x + y^2 - 4y = 1 \\
 & (x^2 + 4x + 4) + (y^2 - 4y + 4) = 1 + 4 + 4 \\
 & (x+2)^2 + (y-2)^2 = 3^2
 \end{aligned}$$

a. Center: $(-2, 2)$; Radius = 3

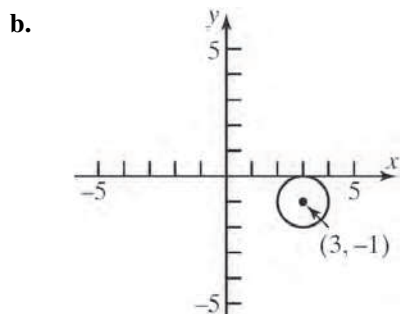


$$\begin{aligned}
 c. \quad & x\text{-intercepts: } (x+2)^2 + (0-2)^2 = 3^2 \\
 & (x+2)^2 + 4 = 9 \\
 & (x+2)^2 = 5 \\
 & x+2 = \pm\sqrt{5} \\
 & x = -2 \pm \sqrt{5} \\
 & y\text{-intercepts: } (0+2)^2 + (y-2)^2 = 3^2 \\
 & 4 + (y-2)^2 = 9 \\
 & (y-2)^2 = 5 \\
 & y-2 = \pm\sqrt{5} \\
 & y = 2 \pm \sqrt{5}
 \end{aligned}$$

The intercepts are $(-2 - \sqrt{5}, 0)$, $(-2 + \sqrt{5}, 0)$, $(0, 2 - \sqrt{5})$, and $(0, 2 + \sqrt{5})$.

$$\begin{aligned}
 32. \quad & x^2 + y^2 - 6x + 2y + 9 = 0 \\
 & x^2 - 6x + y^2 + 2y = -9 \\
 & (x^2 - 6x + 9) + (y^2 + 2y + 1) = -9 + 9 + 1 \\
 & (x-3)^2 + (y+1)^2 = 1^2
 \end{aligned}$$

a. Center: $(3, -1)$; Radius = 1



$$\begin{aligned}
 c. \quad & x\text{-intercepts: } (x-3)^2 + (0+1)^2 = 1^2 \\
 & (x-3)^2 + 1 = 1 \\
 & (x-3)^2 = 0 \\
 & x-3 = 0 \\
 & x = 3
 \end{aligned}$$

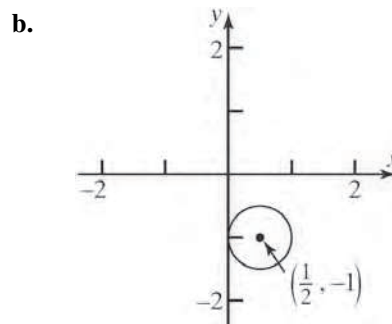
$$\begin{aligned}
 & y\text{-intercepts: } (0-3)^2 + (y+1)^2 = 1^2 \\
 & 9 + (y+1)^2 = 1 \\
 & (y+1)^2 = -8
 \end{aligned}$$

No real solution.

The intercept only intercept is $(3, 0)$.

$$\begin{aligned}
 33. \quad & x^2 + y^2 - x + 2y + 1 = 0 \\
 & x^2 - x + y^2 + 2y = -1 \\
 & \left(x^2 - x + \frac{1}{4}\right) + (y^2 + 2y + 1) = -1 + \frac{1}{4} + 1 \\
 & \left(x - \frac{1}{2}\right)^2 + (y+1)^2 = \left(\frac{1}{2}\right)^2
 \end{aligned}$$

a. Center: $\left(\frac{1}{2}, -1\right)$; Radius = $\frac{1}{2}$



$$\begin{aligned}
 c. \quad & x\text{-intercepts: } \left(x - \frac{1}{2}\right)^2 + (0+1)^2 = \left(\frac{1}{2}\right)^2 \\
 & \left(x - \frac{1}{2}\right)^2 + 1 = \frac{1}{4} \\
 & \left(x - \frac{1}{2}\right)^2 = -\frac{3}{4}
 \end{aligned}$$

No real solutions

$$\begin{aligned}
 & y\text{-intercepts: } \left(0 - \frac{1}{2}\right)^2 + (y+1)^2 = \left(\frac{1}{2}\right)^2 \\
 & \frac{1}{4} + (y+1)^2 = \frac{1}{4} \\
 & (y+1)^2 = 0 \\
 & y+1 = 0 \\
 & y = -1
 \end{aligned}$$

The only intercept is $(0, -1)$.

Chapter 1: Graphs

34. $x^2 + y^2 + x + y - \frac{1}{2} = 0$

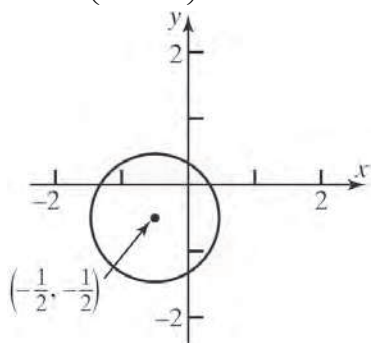
$$x^2 + x + y^2 + y = \frac{1}{2}$$

$$\left(x^2 + x + \frac{1}{4}\right) + \left(y^2 + y + \frac{1}{4}\right) = \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = 1^2$$

a. Center: $\left(-\frac{1}{2}, -\frac{1}{2}\right)$; Radius = 1

b.



c. x-intercepts: $\left(x + \frac{1}{2}\right)^2 + \left(0 + \frac{1}{2}\right)^2 = 1^2$

$$\left(x + \frac{1}{2}\right)^2 + \frac{1}{4} = 1$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{3}{4}$$

$$x + \frac{1}{2} = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{-1 \pm \sqrt{3}}{2}$$

y-intercepts: $\left(0 + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = 1^2$

$$\frac{1}{4} + \left(y + \frac{1}{2}\right)^2 = 1$$

$$\left(y + \frac{1}{2}\right)^2 = \frac{3}{4}$$

$$y + \frac{1}{2} = \pm \frac{\sqrt{3}}{2}$$

$$y = \frac{-1 \pm \sqrt{3}}{2}$$

The intercepts are $\left(\frac{-1-\sqrt{3}}{2}, 0\right)$, $\left(\frac{-1+\sqrt{3}}{2}, 0\right)$,

$\left(0, \frac{-1-\sqrt{3}}{2}\right)$, and $\left(0, \frac{-1+\sqrt{3}}{2}\right)$.

35. $2x^2 + 2y^2 - 12x + 8y - 24 = 0$

$$x^2 + y^2 - 6x + 4y = 12$$

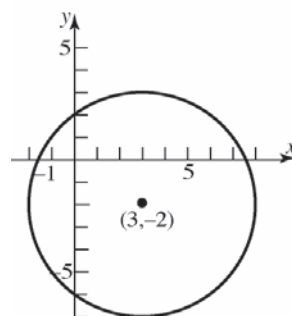
$$x^2 - 6x + y^2 + 4y = 12$$

$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$$

$$(x-3)^2 + (y+2)^2 = 5^2$$

a. Center: $(3, -2)$; Radius = 5

b.



c. x-intercepts: $(x-3)^2 + (0+2)^2 = 5^2$

$$(x-3)^2 + 4 = 25$$

$$(x-3)^2 = 21$$

$$x-3 = \pm\sqrt{21}$$

$$x = 3 \pm \sqrt{21}$$

y-intercepts: $(0-3)^2 + (y+2)^2 = 5^2$

$$9 + (y+2)^2 = 25$$

$$(y+2)^2 = 16$$

$$y+2 = \pm 4$$

$$y = -2 \pm 4$$

$$y = 2 \text{ or } y = -6$$

The intercepts are $(3-\sqrt{21}, 0)$, $(3+\sqrt{21}, 0)$, $(0, -6)$, and $(0, 2)$.

36. a. $2x^2 + 2y^2 + 8x + 7 = 0$

$$2x^2 + 8x + 2y^2 = -7$$

$$x^2 + 4x + y^2 = -\frac{7}{2}$$

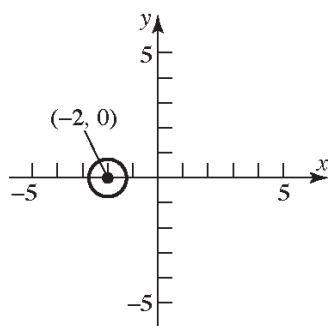
$$(x^2 + 4x + 4) + y^2 = -\frac{7}{2} + 4$$

$$(x+2)^2 + y^2 = \frac{1}{2}$$

$$(x+2)^2 + y^2 = \left(\frac{\sqrt{2}}{2}\right)^2$$

Center: $(-2, 0)$; Radius = $\frac{\sqrt{2}}{2}$

b.



$$\begin{aligned} \text{c. } x\text{-intercepts: } (x+2)^2 + (0)^2 &= \frac{1}{2} \\ (x+2)^2 &= \frac{1}{2} \\ x+2 &= \pm\sqrt{\frac{1}{2}} \\ x+2 &= \pm\frac{\sqrt{2}}{2} \\ x &= -2 \pm \frac{\sqrt{2}}{2} \end{aligned}$$

$$\begin{aligned} \text{y-intercepts: } (0+2)^2 + y^2 &= \frac{1}{2} \\ 4 + y^2 &= \frac{1}{2} \\ y^2 &= -\frac{7}{2} \end{aligned}$$

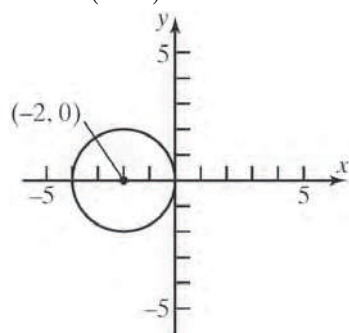
No real solutions.

The intercepts are $\left(-2 - \frac{\sqrt{2}}{2}, 0\right)$ and $\left(-2 + \frac{\sqrt{2}}{2}, 0\right)$.

$$\begin{aligned} 37. \quad 2x^2 + 8x + 2y^2 &= 0 \\ x^2 + 4x + y^2 &= 0 \\ x^2 + 4x + 4 + y^2 &= 0 + 4 \\ (x+2)^2 + y^2 &= 2^2 \end{aligned}$$

a. Center: $(-2, 0)$; Radius: $r = 2$

b.



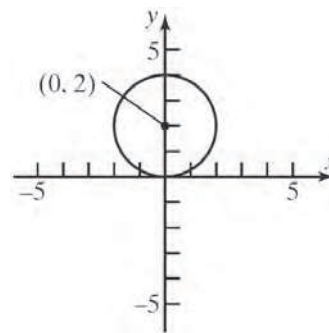
$$\begin{aligned} \text{c. } x\text{-intercepts: } (x+2)^2 + (0)^2 &= 2^2 \\ (x+2)^2 &= 4 \\ (x+2)^2 &= \pm\sqrt{4} \\ x+2 &= \pm 2 \\ x &= -2 \pm 2 \\ x &= 0 \quad \text{or} \quad x = -4 \\ \text{y-intercepts: } (0+2)^2 + y^2 &= 2^2 \\ 4 + y^2 &= 4 \\ y^2 &= 0 \\ y &= 0 \end{aligned}$$

The intercepts are $(-4, 0)$ and $(0, 0)$.

$$\begin{aligned} 38. \quad 3x^2 + 3y^2 - 12y &= 0 \\ x^2 + y^2 - 4y &= 0 \\ x^2 + y^2 - 4y + 4 &= 0 + 4 \\ x^2 + (y-2)^2 &= 4 \end{aligned}$$

a. Center: $(0, 2)$; Radius: $r = 2$

b.



$$\begin{aligned} \text{c. } x\text{-intercepts: } x^2 + (0-2)^2 &= 4 \\ x^2 + 4 &= 4 \\ x^2 &= 0 \\ x &= 0 \\ \text{y-intercepts: } 0^2 + (y-2)^2 &= 4 \\ (y-2)^2 &= 4 \\ y-2 &= \pm\sqrt{4} \\ y-2 &= \pm 2 \\ y &= 2 \pm 2 \\ y &= 4 \quad \text{or} \quad y = 0 \end{aligned}$$

The intercepts are $(0, 0)$ and $(0, 4)$.

Chapter 1: Graphs

39. Center at (0, 0); containing point (-2, 3).

$$r = \sqrt{(-2-0)^2 + (3-0)^2} = \sqrt{4+9} = \sqrt{13}$$

$$\text{Equation: } (x-0)^2 + (y-0)^2 = (\sqrt{13})^2$$

$$x^2 + y^2 = 13$$

40. Center at (1, 0); containing point (-3, 2).

$$r = \sqrt{(-3-1)^2 + (2-0)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$\text{Equation: } (x-1)^2 + (y-0)^2 = (\sqrt{20})^2$$

$$(x-1)^2 + y^2 = 20$$

41. Endpoints of a diameter are (1, 4) and (-3, 2).
The center is at the midpoint of that diameter:

$$\text{Center: } \left(\frac{1+(-3)}{2}, \frac{4+2}{2} \right) = (-1, 3)$$

$$\text{Radius: } r = \sqrt{(1-(-1))^2 + (4-3)^2} = \sqrt{4+1} = \sqrt{5}$$

$$\text{Equation: } (x-(-1))^2 + (y-3)^2 = (\sqrt{5})^2$$

$$(x+1)^2 + (y-3)^2 = 5$$

42. Endpoints of a diameter are (4, 3) and (0, 1).
The center is at the midpoint of that diameter:

$$\text{Center: } \left(\frac{4+0}{2}, \frac{3+1}{2} \right) = (2, 2)$$

$$\text{Radius: } r = \sqrt{(4-2)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$$

$$\text{Equation: } (x-2)^2 + (y-2)^2 = (\sqrt{5})^2$$

$$(x-2)^2 + (y-2)^2 = 5$$

43. $C = 2\pi r$

$$16\pi = 2\pi r$$

$$r = 8$$

$$(x-2)^2 + (y-(-4))^2 = (8)^2$$

$$(x-2)^2 + (y+4)^2 = 64$$

44. $A = \pi r^2$

$$49\pi = \pi r^2$$

$$r = 7$$

$$(x-(-5))^2 + (y-6)^2 = (7)^2$$

$$(x+5)^2 + (y-6)^2 = 49$$

45. (c); Center: 1; Radius = 2

46. (d); Center: (-3, 3); Radius = 3

47. (b); Center: (-1, 2); Radius = 2

48. (a); Center: (-3, 3); Radius = 3

49. The centers of the circles are: (4, -2) and (-1, 5).

The slope is $m = \frac{5-(-2)}{-1-4} = \frac{7}{-5} = -\frac{7}{5}$. Use the slope and one point to find the equation of the line.

$$y-(-2) = -\frac{7}{5}(x-4)$$

$$y+2 = -\frac{7}{5}x + \frac{28}{5}$$

$$5y+10 = -7x+28$$

$$7x+5y = 18$$

50. Find the centers of the two circles:

$$x^2 + y^2 - 4x + 6y + 4 = 0$$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -4 + 4 + 9$$

$$(x-2)^2 + (y+3)^2 = 9$$

$$\text{Center: } (2, -3)$$

$$x^2 + y^2 + 6x + 4y + 9 = 0$$

$$(x^2 + 6x + 9) + (y^2 + 4y + 4) = -9 + 9 + 4$$

$$(x+3)^2 + (y+2)^2 = 4$$

$$\text{Center: } (-3, -2)$$

Find the slope of the line containing the centers:

$$m = \frac{-2-(-3)}{-3-2} = -\frac{1}{5}$$

Find the equation of the line containing the centers:

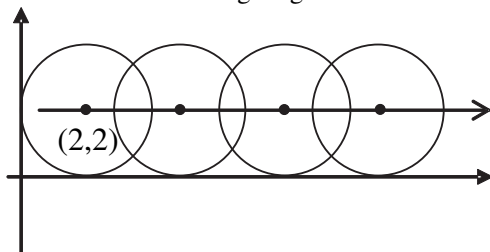
$$y+3 = -\frac{1}{5}(x-2)$$

$$5y+15 = -x+2$$

$$x+5y = -13$$

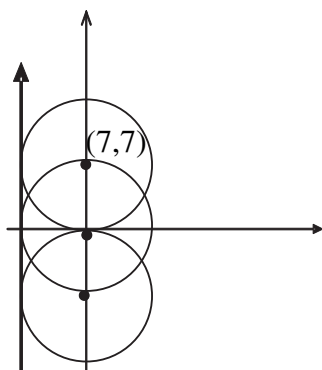
$$x+5y+13 = 0$$

51. Consider the following diagram:



Therefore, the path of the center of the circle has the equation $y = 2$.

52. Consider the following diagram:



Therefore the path of the center of the circle has the equation $x = 7$.

53. Let the upper-right corner of the square be the point (x, y) . The circle and the square are both centered about the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square. Therefore, we get

$$x^2 + y^2 = 9$$

$$x^2 + x^2 = 9$$

$$2x^2 = 9$$

$$x^2 = \frac{9}{2}$$

$$x = \sqrt{\frac{9}{2}} = \frac{3\sqrt{2}}{2}$$

The length of one side of the square is $2x$. Thus, the area is

$$A = s^2 = \left(2 \cdot \frac{3\sqrt{2}}{2}\right)^2 = (3\sqrt{2})^2 = 18 \text{ square units.}$$

54. The area of the shaded region is the area of the circle, less the area of the square. Let the upper-right corner of the square be the point (x, y) . The circle and the square are both centered about

the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square.

Therefore, we get

$$x^2 + y^2 = 36$$

$$x^2 + x^2 = 36$$

$$2x^2 = 36$$

$$x^2 = 18$$

$$x = 3\sqrt{2}$$

The length of one side of the square is $2x$. Thus,

the area of the square is $(2 \cdot 3\sqrt{2})^2 = 72$ square

units. From the equation of the circle, we have $r = 6$. The area of the circle is

$$\pi r^2 = \pi (6)^2 = 36\pi \text{ square units.}$$

Therefore, the area of the shaded region is

$$A = 36\pi - 72 \text{ square units.}$$

55. The diameter of the Ferris wheel was 250 feet, so the radius was 125 feet. The maximum height was 264 feet, so the center was at a height of $264 - 125 = 139$ feet above the ground. Since the center of the wheel is on the y -axis, it is the point $(0, 139)$. Thus, an equation for the wheel is:

$$(x - 0)^2 + (y - 139)^2 = 125^2$$

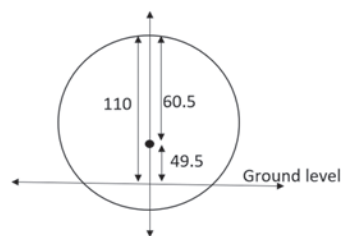
$$x^2 + (y - 139)^2 = 15,625$$

56. The diameter of the wheel is 520 feet, so the radius is 260 feet. The maximum height is 550 feet, so the center of the wheel is at a height of $550 - 260 = 290$ feet above the ground. Since the center of the wheel is on the y -axis, it is the point $(0, 290)$. Thus, an equation for the wheel is:

$$(x - 0)^2 + (y - 290)^2 = 260^2$$

$$x^2 + (y - 290)^2 = 67,600$$

- 57.



Refer to figure. Since the radius of the building is 60.5 m and the height of the building is 110 m,

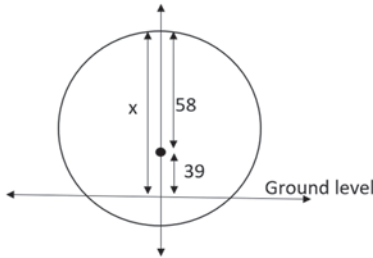
Chapter 1: Graphs

then the center of the building is 49.5 m above the ground, so the y-coordinate of the center is 49.5. The equation of the circle is given by $x^2 + (y - 49.5)^2 = 60.5^2 = 3660.25$

- 58.** Complete the square to find the equation of the circle representing the formula for the building.

$$x^2 + y^2 - 78y + 1521 = 1843 + 1521 = 3364$$

$$x^2 + (y - 39)^2 = 58^2$$



Refer to figure. The y coordinate of the center is 39. The radius is 58. Thus the height of the building is $58 + 39 = 97$ m.

- 59.** Center at (2, 3); tangent to the x-axis.
 $r = 3$

$$\begin{aligned}\text{Equation: } (x - 2)^2 + (y - 3)^2 &= 3^2 \\ (x - 2)^2 + (y - 3)^2 &= 9\end{aligned}$$

- 60.** Center at (-3, 1); tangent to the y-axis.
 $r = 3$

$$\begin{aligned}\text{Equation: } (x + 3)^2 + (y - 1)^2 &= 3^2 \\ (x + 3)^2 + (y - 1)^2 &= 9\end{aligned}$$

- 61.** Center at (-1, 3); tangent to the line $y = 2$.
This means that the circle contains the point (-1, 2), so the radius is $r = 1$.

$$\begin{aligned}\text{Equation: } (x + 1)^2 + (y - 3)^2 &= (1)^2 \\ (x + 1)^2 + (y - 3)^2 &= 1\end{aligned}$$

- 62.** Center at (4, -2); tangent to the line $x = 1$.
This means that the circle contains the point (1, -2), so the radius is $r = 3$.

$$\begin{aligned}\text{Equation: } (x - 4)^2 + (y + 2)^2 &= (3)^2 \\ (x - 4)^2 + (y + 2)^2 &= 9\end{aligned}$$

- 63. a.** Substitute $y = mx + b$ into $x^2 + y^2 = r^2$:

$$x^2 + (mx + b)^2 = r^2$$

$$x^2 + m^2x^2 + 2bmx + b^2 = r^2$$

$$(1 + m^2)x^2 + 2bmx + b^2 - r^2 = 0$$

This equation has one solution if and only if the discriminant is zero.

$$(2bm)^2 - 4(1 + m^2)(b^2 - r^2) = 0$$

$$4b^2m^2 - 4b^2 + 4r^2 - 4b^2m^2 + 4m^2r^2 = 0$$

$$-4b^2 + 4r^2 + 4m^2r^2 = 0$$

$$-b^2 + r^2 + m^2r^2 = 0$$

$$r^2(1 + m^2) = b^2$$

- b.** From part (a) we know

$(1 + m^2)x^2 + 2bmx + b^2 - r^2 = 0$. Using the quadratic formula, since the discriminant is zero, we get:

$$x = \frac{-2bm}{2(1 + m^2)} = \frac{-bm}{\left(\frac{b^2}{r^2}\right)} = \frac{-bmr^2}{b^2} = \frac{-mr^2}{b}$$

$$\begin{aligned}y &= m\left(\frac{-mr^2}{b}\right) + b \\ &= \frac{-m^2r^2}{b} + b = \frac{-m^2r^2 + b^2}{b} = \frac{r^2}{b}\end{aligned}$$

The point of tangency is $\left(\frac{-mr^2}{b}, \frac{r^2}{b}\right)$.

- c.** The slope of the tangent line is m .
The slope of the line joining the point of tangency and the center (0,0) is:

$$\frac{\left(\frac{r^2}{b} - 0\right)}{\left(\frac{-mr^2}{b} - 0\right)} = \frac{\frac{r^2}{b}}{\frac{-mr^2}{b}} = \frac{r^2}{b} \cdot \frac{b}{-mr^2} = -\frac{1}{m}$$

The two lines are perpendicular.

- 64.** Let (h, k) be the center of the circle.

$$x - 2y + 4 = 0$$

$$2y = x + 4$$

$$y = \frac{1}{2}x + 2$$

The slope of the tangent line is $\frac{1}{2}$. The slope from (h, k) to $(0, 2)$ is -2 .

$$\frac{2-k}{0-h} = -2$$

$$2-k = 2h$$

The other tangent line is $y = 2x - 7$, and it has slope 2.

The slope from (h, k) to $(3, -1)$ is $-\frac{1}{2}$.

$$\frac{-1-k}{3-h} = -\frac{1}{2}$$

$$2+2k = 3-h$$

$$2k = 1-h$$

$$h = 1-2k$$

Solve the two equations in h and k :

$$2-k = 2(1-2k)$$

$$2-k = 2-4k$$

$$3k = 0$$

$$k = 0$$

$$h = 1-2(0) = 1$$

The center of the circle is $(1, 0)$.

- 65.** The slope of the line containing the center $(0,0)$ and $(1, 2\sqrt{2})$ is

$$\frac{2\sqrt{2}-0}{1-0} = 2\sqrt{2}. \text{ Then the slope of the tangent}$$

$$\text{line is } \frac{-1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}.$$

So the equation of the tangent line is:

$$y - 2\sqrt{2} = -\frac{\sqrt{2}}{4}(x-1)$$

$$y - 2\sqrt{2} = -\frac{\sqrt{2}}{4}x + \frac{\sqrt{2}}{4}$$

$$4y - 8\sqrt{2} = -\sqrt{2}x + \sqrt{2}$$

$$\sqrt{2}x + 4y = 9\sqrt{2}$$

$$\mathbf{66.} \quad x^2 + y^2 - 4x + 6y + 4 = 0$$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -4 + 4 + 9$$

$$(x-2)^2 + (y+3)^2 = 9$$

Center: $(2, -3)$

The slope of the line containing the center and

$$(3, 2\sqrt{2} - 3) \text{ is } \frac{2\sqrt{2} - 3 - (-3)}{3 - 2} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$$

Then the slope of the tangent line is:

$$\frac{-1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

So, the equation of the tangent line is

$$y - (2\sqrt{2} - 3) = -\frac{\sqrt{2}}{4}(x - 3)$$

$$y - 2\sqrt{2} + 3 = -\frac{\sqrt{2}}{4}x + \frac{3\sqrt{2}}{4}$$

$$4y - 8\sqrt{2} + 12 = -\sqrt{2}x + 3\sqrt{2}$$

$$\sqrt{2}x + 4y = 11\sqrt{2} - 12$$

Chapter 1: Graphs

67. The center of the circle is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ and the radius is $\frac{1}{2}\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Then the equation of

the circle is $\left(x - \frac{x_1 + x_2}{2}\right)^2 + \left(y - \frac{y_1 + y_2}{2}\right)^2 = \frac{1}{4}[(x_1 - x_2)^2 + (y_1 - y_2)^2]$. Expanding, gives

$$\begin{aligned} x^2 - x(x_1 + x_2) + \frac{(x_1 + x_2)^2}{4} + y^2 - y(y_1 + y_2) + \frac{(y_1 + y_2)^2}{4} &= \frac{1}{4}[x_1^2 - 2x_1x_2 + x_2^2 + y_1^2 - 2y_1y_2 + y_2^2] \\ 4x^2 - 4x_1x - 4x_2x + x_1^2 + 2x_1x_2 + x_2^2 + 4y^2 - 4y_1y - 4y_2y + y_1^2 + 2y_1y_2 + y_2^2 &= x_1^2 - 2x_1x_2 + x_2^2 + y_1^2 - 2y_1y_2 + y_2^2 \\ 4x^2 - 4x_1x - 4x_2x + 4x_1x_2 + 4y^2 - 4y_1y - 4y_2y + 4y_1y_2 &= 0 \\ x^2 - x_1x - x_2x + x_1x_2 + y^2 - y_1y - y_2y + y_1y_2 &= 0 \\ x(x - x_1) - x_2(x + x_1) + y(y - y_1) - y_2(y + y_1) &= 0 \\ (x - x_1)(x - x_2) + (y - y_1)(y - y_2) &= 0 \end{aligned}$$

68. Complete the square to get $\left(x + \frac{d}{2}\right)^2 + \left(y + \frac{e}{2}\right)^2 = \frac{d^2 + e^2 - 4f}{4}$. The slope of the line between the center

$\left(-\frac{d}{2}, -\frac{e}{2}\right)$ and the point of tangency (x_0, y_0) is $m = \frac{y_0 + \frac{e}{2}}{x_0 + \frac{d}{2}}$. So the slope of the tangent line is $m_{\tan} = -\frac{x_0 + \frac{d}{2}}{y_0 + \frac{e}{2}}$.

Therefore, the equation of the tangent line is $y - y_0 = -\frac{x_0 + \frac{d}{2}}{y_0 + \frac{e}{2}}(x - x_0)$ which is equivalent to

$$\begin{aligned} (x - x_0)\left(x_0 + \frac{d}{2}\right) + (y - y_0)\left(y_0 + \frac{e}{2}\right) &= 0 \\ x_0x + \frac{d}{2}x - x_0^2 - \frac{d}{2}x_0 + y_0y + \frac{e}{2}y - y_0^2 - \frac{e}{2}y_0 &= 0 \\ x_0x + y_0y + \frac{d}{2}x + \frac{e}{2}y - \left(x_0^2 + y_0^2 + \frac{d}{2}x_0 + \frac{e}{2}y_0\right) &= 0 \end{aligned}$$

Because (x_0, y_0) is on the circle, $x_0^2 + y_0^2 + dx_0 + ey_0 + f = 0$, and

$$x_0^2 + y_0^2 + \frac{d}{2}x_0 + \frac{e}{2}y_0 = -\frac{d}{2}x_0 - \frac{e}{2}y_0 - f \quad \text{Substituting this result gives}$$

$$\begin{aligned} x_0^2 + y_0^2 + \frac{d}{2}x_0 + \frac{e}{2}y_0 - \left(-\frac{d}{2}x_0 - \frac{e}{2}y_0 - f\right) &= 0 \\ x_0^2 + y_0^2 + \frac{d}{2}x_0 + \frac{e}{2}y_0 + \frac{d}{2}x_0 + \frac{e}{2}y_0 + f &= 0 \\ x_0^2 + y_0^2 + d\left(\frac{x + x_0}{2}\right) + e\left(\frac{y + y_0}{2}\right) + f &= 0 \end{aligned}$$

69. (b), (c), (e) and (g)

We need $h, k > 0$ and $(0, 0)$ on the graph.

70. (b), (e) and (g)

We need $h < 0$, $k = 0$, and $|h| > r$.

71. Answers will vary.

72. The student has the correct radius, but the signs of the coordinates of the center are incorrect. The student needs to write the equation in the standard form $(x - h)^2 + (y - k)^2 = r^2$.

$$(x+3)^2 + (y-2)^2 = 16$$

$$(x-(-3))^2 + (y-2)^2 = 4^2$$

Thus, $(h, k) = (-3, 2)$ and $r = 4$.

Chapter 1 Review Exercises

1. $P_1 = (0, 0)$ and $P_2 = (4, 2)$

a. $d(P_1, P_2) = \sqrt{(4-0)^2 + (2-0)^2}$
 $= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

b. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{0+4}{2}, \frac{0+2}{2} \right) = \left(\frac{4}{2}, \frac{2}{2} \right) = (2, 1)$$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$

d. For each run of 2, there is a rise of 1.

2. $P_1 = (1, -1)$ and $P_2 = (-2, 3)$

a. $d(P_1, P_2) = \sqrt{(-2-1)^2 + (3-(-1))^2}$
 $= \sqrt{9+16} = \sqrt{25} = 5$

b. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{1+(-2)}{2}, \frac{-1+3}{2} \right)$$

$$= \left(\frac{-1}{2}, \frac{2}{2} \right) = \left(-\frac{1}{2}, 1 \right)$$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{3-(-1)}{-2-1} = \frac{4}{-3} = -\frac{4}{3}$

d. For each run of 3, there is a rise of -4 .

3. $P_1 = (4, -4)$ and $P_2 = (4, 8)$

a. $d(P_1, P_2) = \sqrt{(4-4)^2 + (8-(-4))^2}$
 $= \sqrt{0+144} = \sqrt{144} = 12$

b. The coordinates of the midpoint are:

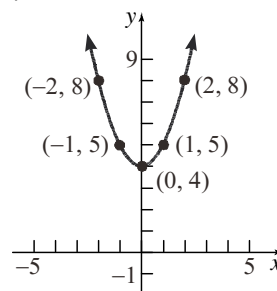
$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{4+4}{2}, \frac{-4+8}{2} \right) = \left(\frac{8}{2}, \frac{4}{2} \right) = (4, 2)$$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{8-(-4)}{4-4} = \frac{12}{0}$, undefined

d. An undefined slope means the points lie on a vertical line. There is no change in x .

4. $y = x^2 + 4$



5. x -intercepts: $-4, 0, 2$; y -intercepts: $-2, 0, 2$
Intercepts: $(-4, 0)$, $(0, 0)$, $(2, 0)$, $(0, -2)$, $(0, 2)$

6. $2x = 3y^2$

x -intercepts: y -intercepts:

$$2x = 3(0)^2 \quad 2(0) = 3y^2$$

$$2x = 0 \quad 0 = y^2$$

$$x = 0 \quad y = 0$$

The only intercept is $(0, 0)$.

Test x -axis symmetry: Let $y = -y$

$$2x = 3(-y)^2$$

$$2x = 3y^2 \text{ same}$$

Test y -axis symmetry: Let $x = -x$

$$2(-x) = 3y^2$$

$$-2x = 3y^2 \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$2(-x) = 3(-y)^2$$

$$-2x = 3y^2 \text{ different}$$

Therefore, the graph will have x -axis symmetry.

7. $x^2 + 4y^2 = 16$

Chapter 1: Graphs

x-intercepts:

$$x^2 + 4(0)^2 = 16$$

$$x^2 = 16$$

$$x = \pm 4$$

y-intercepts:

$$(0)^2 + 4y^2 = 16$$

$$4y^2 = 16$$

$$y^2 = 4$$

$$y = \pm 2$$

The intercepts are $(-4, 0)$, $(4, 0)$, $(0, -2)$, and $(0, 2)$.

Test x-axis symmetry: Let $y = -y$

$$x^2 + 4(-y)^2 = 16$$

$$x^2 + 4y^2 = 16 \quad \text{same}$$

Test y-axis symmetry: Let $x = -x$

$$(-x)^2 + 4y^2 = 16$$

$$x^2 + 4y^2 = 16 \quad \text{same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$(-x)^2 + 4(-y)^2 = 16$$

$$x^2 + 4y^2 = 16 \quad \text{same}$$

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

8. $y = x^4 + 2x^2 + 1$

x-intercepts:

$$0 = x^4 + 2x^2 + 1$$

$$0 = (x^2 + 1)(x^2 + 1)$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

no real solutions

The only intercept is $(0, 1)$.

Test x-axis symmetry: Let $y = -y$

$$-y = x^4 + 2x^2 + 1$$

$$y = -x^4 - 2x^2 - 1 \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$y = (-x)^4 + 2(-x)^2 + 1$$

$$y = x^4 + 2x^2 + 1 \quad \text{same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$-y = (-x)^4 + 2(-x)^2 + 1$$

$$-y = x^4 + 2x^2 + 1$$

$$y = -x^4 - 2x^2 - 1 \quad \text{different}$$

Therefore, the graph will have y-axis symmetry.

9. $y = x^3 - x$

x-intercepts:

$$0 = x^3 - x$$

$$0 = x(x^2 - 1)$$

$$0 = x(x+1)(x-1)$$

$$x = 0, x = -1, x = 1$$

y-intercepts:

$$y = (0)^3 - 0$$

$$= 0$$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(1, 0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = x^3 - x$$

$$y = -x^3 + x \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$y = (-x)^3 - (-x)$$

$$y = -x^3 + x \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$-y = (-x)^3 - (-x)$$

$$-y = -x^3 + x$$

$$y = x^3 - x \quad \text{same}$$

Therefore, the graph will have origin symmetry.

10. $x^2 + x + y^2 + 2y = 0$

x-intercepts: $x^2 + x + (0)^2 + 2(0) = 0$

$$x^2 + x = 0$$

$$x(x+1) = 0$$

$$x = 0, x = -1$$

y-intercepts: $(0)^2 + 0 + y^2 + 2y = 0$

$$y^2 + 2y = 0$$

$$y(y+2) = 0$$

$$y = 0, y = -2$$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(0, -2)$.

Test x-axis symmetry: Let $y = -y$

$$x^2 + x + (-y)^2 + 2(-y) = 0$$

$$x^2 + x + y^2 - 2y = 0 \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$(-x)^2 + (-x) + y^2 + 2y = 0$$

$$x^2 - x + y^2 + 2y = 0 \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$(-x)^2 + (-x) + (-y)^2 + 2(-y) = 0$$

$$x^2 - x + y^2 - 2y = 0 \quad \text{different}$$

The graph has none of the indicated symmetries.

11. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-(-2))^2 + (y-3)^2 = 4^2$$

$$(x+2)^2 + (y-3)^2 = 16$$

12. $(x-h)^2 + (y-k)^2 = r^2$

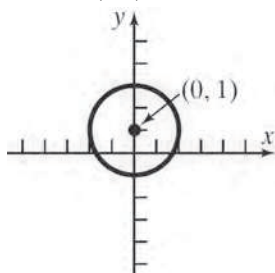
$$(x-(-1))^2 + (y-(-2))^2 = 1^2$$

$$(x+1)^2 + (y+2)^2 = 1$$

13. $x^2 + (y-1)^2 = 4$

$$x^2 + (y-1)^2 = 2^2$$

Center: (0,1); Radius = 2



x-intercepts: $x^2 + (0-1)^2 = 4$

$$x^2 + 1 = 4$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

y-intercepts: $0^2 + (y-1)^2 = 4$

$$(y-1)^2 = 4$$

$$y-1 = \pm 2$$

$$y = 1 \pm 2$$

$$y = 3 \text{ or } y = -1$$

The intercepts are $(-\sqrt{3}, 0)$, $(\sqrt{3}, 0)$, $(0, -1)$,

and $(0, 3)$.

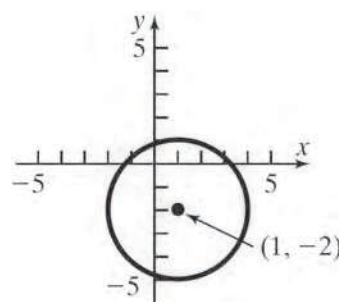
14. $x^2 + y^2 - 2x + 4y - 4 = 0$

$$x^2 - 2x + y^2 + 4y = 4$$

$$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 4 + 1 + 4$$

$$(x-1)^2 + (y+2)^2 = 3^2$$

Center: (1, -2) Radius = 3



x-intercepts: $(x-1)^2 + (0+2)^2 = 3^2$

$$(x-1)^2 + 4 = 9$$

$$(x-1)^2 = 5$$

$$x-1 = \pm\sqrt{5}$$

$$x = 1 \pm \sqrt{5}$$

y-intercepts: $(0-1)^2 + (y+2)^2 = 3^2$

$$1 + (y+2)^2 = 9$$

$$(y+2)^2 = 8$$

$$y+2 = \pm\sqrt{8}$$

$$y+2 = \pm 2\sqrt{2}$$

$$y = -2 \pm 2\sqrt{2}$$

The intercepts are $(1-\sqrt{5}, 0)$, $(1+\sqrt{5}, 0)$,

$(0, -2-2\sqrt{2})$, and $(0, -2+2\sqrt{2})$.

15. $3x^2 + 3y^2 - 6x + 12y = 0$

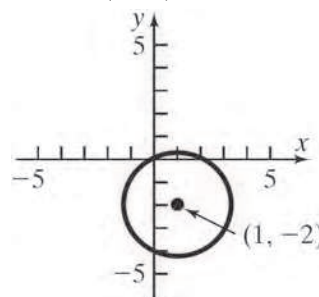
$$x^2 + y^2 - 2x + 4y = 0$$

$$x^2 - 2x + y^2 + 4y = 0$$

$$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 1 + 4$$

$$(x-1)^2 + (y+2)^2 = (\sqrt{5})^2$$

Center: (1, -2) Radius = $\sqrt{5}$



Chapter 1: Graphs

$$x\text{-intercepts: } (x-1)^2 + (0+2)^2 = (\sqrt{5})^2$$

$$(x-1)^2 + 4 = 5$$

$$(x-1)^2 = 1$$

$$x-1 = \pm 1$$

$$x = 1 \pm 1$$

$$x = 2 \quad \text{or} \quad x = 0$$

$$y\text{-intercepts: } (0-1)^2 + (y+2)^2 = (\sqrt{5})^2$$

$$1 + (y+2)^2 = 5$$

$$(y+2)^2 = 4$$

$$y+2 = \pm 2$$

$$y = -2 \pm 2$$

$$y = 0 \quad \text{or} \quad y = -4$$

The intercepts are $(0, 0)$, $(2, 0)$, and $(0, -4)$.

16. Slope = -2 ; containing $(3, -1)$

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -2(x - 3)$$

$$y + 1 = -2x + 6$$

$$y = -2x + 5 \quad \text{or} \quad 2x + y = 5$$

17. vertical; containing $(-3, 4)$

Vertical lines have equations of the form $x = a$, where a is the x -intercept. Now, a vertical line containing the point $(-3, 4)$ must have an x -intercept of -3 , so the equation of the line is $x = -3$. The equation does not have a slope-intercept form.

18. y -intercept = -2 ; containing $(5, -3)$

Points are $(5, -3)$ and $(0, -2)$

$$m = \frac{-2 - (-3)}{0 - 5} = \frac{1}{-5} = -\frac{1}{5}$$

$$y = mx + b$$

$$y = -\frac{1}{5}x - 2 \quad \text{or} \quad x + 5y = -10$$

19. Containing the points $(3, -4)$ and $(2, 1)$

$$m = \frac{1 - (-4)}{2 - 3} = \frac{5}{-1} = -5$$

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = -5(x - 3)$$

$$y + 4 = -5x + 15$$

$$y = -5x + 11 \quad \text{or} \quad 5x + y = 11$$

20. Parallel to $2x - 3y = -4$

$$2x - 3y = -4$$

$$-3y = -2x - 4$$

$$\frac{-3y}{-3} = \frac{-2x - 4}{-3}$$

$$y = \frac{2}{3}x + \frac{4}{3}$$

$$\text{Slope} = \frac{2}{3}; \text{ containing } (-5, 3)$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{2}{3}(x - (-5))$$

$$y - 3 = \frac{2}{3}(x + 5)$$

$$y - 3 = \frac{2}{3}x + \frac{10}{3}$$

$$y = \frac{2}{3}x + \frac{19}{3} \quad \text{or} \quad 2x - 3y = -19$$

21. Perpendicular to $x + y = 2$

$$x + y = 2$$

$$y = -x + 2$$

The slope of this line is -1 , so the slope of a line perpendicular to it is 1 .

Slope = 1 ; containing $(4, -3)$

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = 1(x - 4)$$

$$y + 3 = x - 4$$

$$y = x - 7 \quad \text{or} \quad x - y = 7$$

22. $4x - 5y = -20$

$$-5y = -4x - 20$$

$$y = \frac{4}{5}x + 4$$

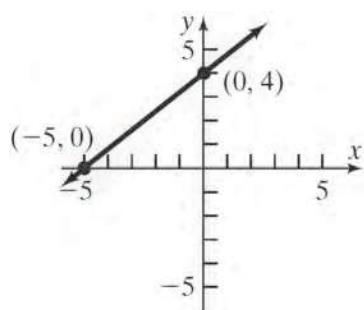
$$\text{slope} = \frac{4}{5}; y\text{-intercept} = 4$$

x -intercept: Let $y = 0$.

$$4x - 5(0) = -20$$

$$4x = -20$$

$$x = -5$$

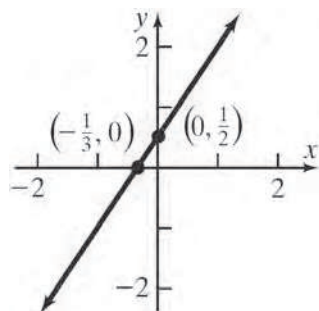


$$\begin{aligned} 23. \quad \frac{1}{2}x - \frac{1}{3}y &= -\frac{1}{6} \\ -\frac{1}{3}y &= -\frac{1}{2}x - \frac{1}{6} \\ y &= \frac{3}{2}x + \frac{1}{2} \end{aligned}$$

$$\text{slope} = \frac{3}{2}; \text{ y-intercept} = \frac{1}{2}$$

x-intercept: Let $y = 0$.

$$\begin{aligned} \frac{1}{2}x - \frac{1}{3}(0) &= -\frac{1}{6} \\ \frac{1}{2}x &= -\frac{1}{6} \\ x &= -\frac{1}{3} \end{aligned}$$



$$24. \quad 2x - 3y = 12$$

x-intercept:

$$2x - 3(0) = 12$$

$$2x = 12$$

$$x = 6$$

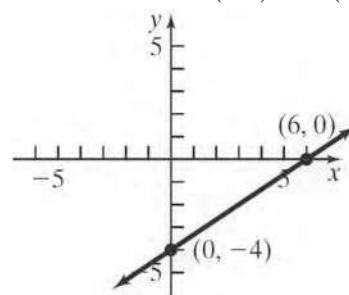
y-intercept:

$$2(0) - 3y = 12$$

$$-3y = 12$$

$$y = -4$$

The intercepts are $(6, 0)$ and $(0, -4)$.



$$25. \quad \frac{1}{2}x + \frac{1}{3}y = 2$$

x-intercept:

$$\frac{1}{2}x + \frac{1}{3}(0) = 2$$

$$\frac{1}{2}x = 2$$

$$x = 4$$

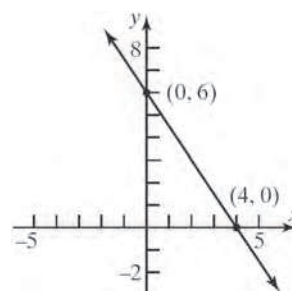
y-intercept:

$$\frac{1}{2}(0) + \frac{1}{3}y = 2$$

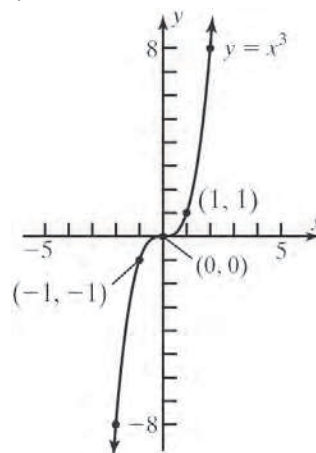
$$\frac{1}{3}y = 2$$

$$y = 6$$

The intercepts are $(4, 0)$ and $(0, 6)$.

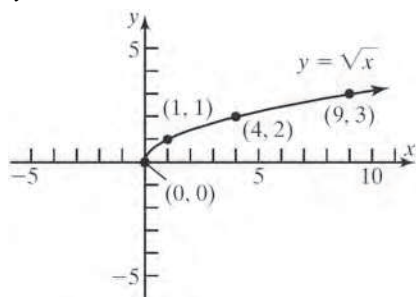


$$26. \quad y = x^3$$

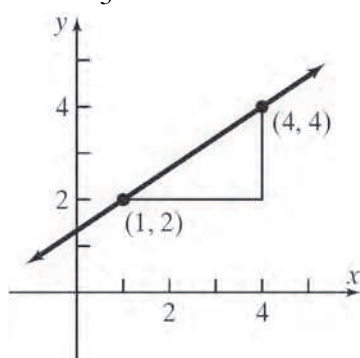


Chapter 1: Graphs

27. $y = \sqrt{x}$



28. slope = $\frac{2}{3}$, containing the point (1, 2)



29. Find the distance between each pair of points.

$$d_{A,B} = \sqrt{(1-3)^2 + (1-4)^2} = \sqrt{4+9} = \sqrt{13}$$

$$d_{B,C} = \sqrt{(-2-1)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$$

$$d_{A,C} = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{25+1} = \sqrt{26}$$

Since $AB = BC$, triangle ABC is isosceles.

30. Given the points $A = (-2, 0)$, $B = (-4, 4)$, and $C = (8, 5)$.

- a. Find the distance between each pair of points.

$$\begin{aligned} d(A, B) &= \sqrt{(-4 - (-2))^2 + (4 - 0)^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} = 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} d(B, C) &= \sqrt{(8 - (-4))^2 + (5 - 4)^2} \\ &= \sqrt{144 + 1} \\ &= \sqrt{145} \end{aligned}$$

$$\begin{aligned} d(A, C) &= \sqrt{(8 - (-2))^2 + (5 - 0)^2} \\ &= \sqrt{100 + 25} \\ &= \sqrt{125} = 5\sqrt{5} \end{aligned}$$

$$[d(A, B)]^2 + [d(A, C)]^2 = [d(B, C)]^2$$

$$(\sqrt{20})^2 + (\sqrt{125})^2 = (\sqrt{145})^2$$

$$20 + 125 = 145$$

$$145 = 145$$

The Pythagorean Theorem is satisfied, so this is a right triangle.

- b. Find the slopes:

$$m_{AB} = \frac{4-0}{-4-(-2)} = \frac{4}{-2} = -2$$

$$m_{BC} = \frac{5-4}{8-(-4)} = \frac{1}{12}$$

$$m_{AC} = \frac{5-0}{8-(-2)} = \frac{5}{10} = \frac{1}{2}$$

Since $m_{AB} \cdot m_{AC} = -2 \cdot \frac{1}{2} = -1$, the sides AB and AC are perpendicular and the triangle is a right triangle.

31. Endpoints of the diameter are $(-3, 2)$ and $(5, -6)$. The center is at the midpoint of the diameter:

$$\text{Center: } \left(\frac{-3+5}{2}, \frac{2+(-6)}{2} \right) = (1, -2)$$

$$\begin{aligned} \text{Radius: } r &= \sqrt{(1-(-3))^2 + (-2-2)^2} \\ &= \sqrt{16+16} \\ &= \sqrt{32} = 4\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{Equation: } (x-1)^2 + (y+2)^2 &= (4\sqrt{2})^2 \\ (x-1)^2 + (y+2)^2 &= 32 \end{aligned}$$

32. slope of $\overline{AB} = \frac{1-5}{6-2} = -1$

$$\text{slope of } \overline{AC} = \frac{-1-5}{8-2} = -1$$

Therefore, the points lie on a line.

Chapter 1 Test

$$\begin{aligned}
 1. \quad d(P_1, P_2) &= \sqrt{(5 - (-1))^2 + (-1 - 3)^2} \\
 &= \sqrt{6^2 + (-4)^2} \\
 &= \sqrt{36 + 16} \\
 &= \sqrt{52} = 2\sqrt{13}
 \end{aligned}$$

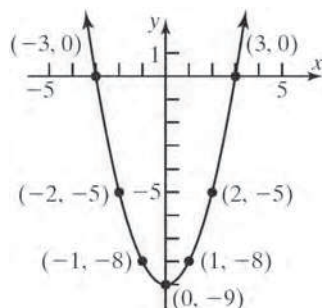
2. The coordinates of the midpoint are:

$$\begin{aligned}
 (x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{-1 + 5}{2}, \frac{3 + (-1)}{2} \right) \\
 &= \left(\frac{4}{2}, \frac{2}{2} \right) \\
 &= (2, 1)
 \end{aligned}$$

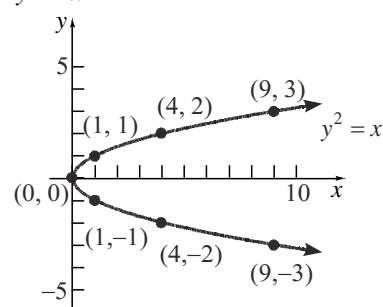
$$3. \quad a. \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{5 - (-1)} = \frac{-4}{6} = -\frac{2}{3}$$

b. If x increases by 3 units, y will decrease by 2 units.

$$4. \quad y = x^2 - 9$$



$$5. \quad y^2 = x$$



$$6. \quad x^2 + y = 9$$

x -intercepts:

y -intercept:

$$x^2 + 0 = 9 \quad (0)^2 + y = 9$$

$$x^2 = 9 \quad y = 9$$

$$x = \pm 3$$

The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, 9)$.

Test x -axis symmetry: Let $y = -y$

$$x^2 + (-y) = 9$$

$$x^2 - y = 9 \quad \text{different}$$

Test y -axis symmetry: Let $x = -x$

$$(-x)^2 + y = 9$$

$$x^2 + y = 9 \quad \text{same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$(-x)^2 + (-y) = 9$$

$$x^2 - y = 9 \quad \text{different}$$

Therefore, the graph will have y -axis symmetry.

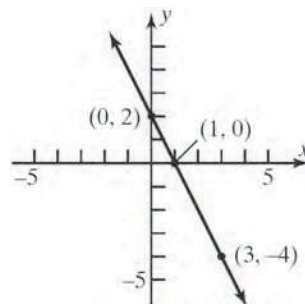
7. Slope = -2 ; containing $(3, -4)$

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = -2(x - 3)$$

$$y + 4 = -2x + 6$$

$$y = -2x + 2$$



$$8. \quad (x - h)^2 + (y - k)^2 = r^2$$

$$(x - 4)^2 + (y - (-3))^2 = 5^2$$

$$(x - 4)^2 + (y + 3)^2 = 25$$

$$\text{General form: } (x - 4)^2 + (y + 3)^2 = 25$$

$$x^2 - 8x + 16 + y^2 + 6y + 9 = 25$$

$$x^2 + y^2 - 8x + 6y = 0$$

$$9. \quad x^2 + y^2 + 4x - 2y - 4 = 0$$

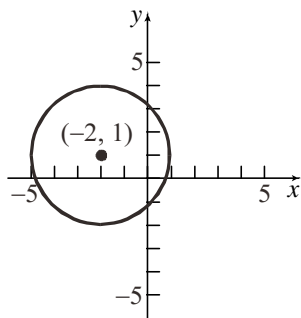
$$x^2 + 4x + y^2 - 2y = 4$$

$$(x^2 + 4x + 4) + (y^2 - 2y + 1) = 4 + 4 + 1$$

$$(x + 2)^2 + (y - 1)^2 = 3^2$$

Center: $(-2, 1)$; Radius = 3

Chapter 1: Graphs



10. $2x + 3y = 6$
 $3y = -2x + 6$
 $y = -\frac{2}{3}x + 2$

Parallel line

Any line parallel to $2x + 3y = 6$ has slope

$m = -\frac{2}{3}$. The line contains $(1, -1)$:

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -\frac{2}{3}(x - 1)$$
$$y + 1 = -\frac{2}{3}x + \frac{2}{3}$$
$$y = -\frac{2}{3}x - \frac{1}{3}$$

Perpendicular line

Any line perpendicular to $2x + 3y = 6$ has slope

$m = \frac{3}{2}$. The line contains $(0, 3)$:

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{3}{2}(x - 0)$$
$$y - 3 = \frac{3}{2}x$$
$$y = \frac{3}{2}x + 3$$

Chapter 1 Project

Internet-based Project

Chapter 2

Functions and Their Graphs

Section 2.1

1. $(-1, 3)$

$$\begin{aligned} 2. \quad 3(-2)^2 - 5(-2) + \frac{1}{(-2)} &= 3(4) - 5(-2) - \frac{1}{2} \\ &= 12 + 10 - \frac{1}{2} \\ &= \frac{43}{2} \text{ or } 21\frac{1}{2} \text{ or } 21.5 \end{aligned}$$

3. We must not allow the denominator to be 0.
 $x + 4 \neq 0 \Rightarrow x \neq -4$; Domain: $\{x \mid x \neq -4\}$.

4. $3 - 2x > 5$

$$-2x > 2$$

$$x < -1$$

Solution set: $\{x \mid x < -1\}$ or $(-\infty, -1)$



5. $\sqrt{5} + 2$

6. radicals

7. independent; dependent

8. a

9. c

10. False; $g \neq 0$

11. False; every function is a relation, but not every relation is a function. For example, the relation $x^2 + y^2 = 1$ is not a function.

12. verbally, numerically, graphically, algebraically

13. False; if the domain is not specified, we assume it is the largest set of real numbers for which the value of f is a real number.

14. False; if x is in the domain of a function f , we say that f is defined at x , or $f(x)$ exists.

15. difference quotient

16. explicitly

17. a. Domain: $\{0, 22, 40, 70, 100\}$ in $^{\circ}\text{C}$
 Range: $\{1.031, 1.121, 1.229, 1.305, 1.411\}$ in kg/m^3

b.

Temperature, $^{\circ}\text{C}$	Density, kg/m^3
0	1.411
22	1.305
40	1.229
70	1.121
100	1.031

c. $\{(0, 1.411), (22, 1.305), (40, 1.229), (70, 1.121), (100, 1.031)\}$

18. a. Domain: $\{1.80, 1.78, 1.77\}$
 Range: $\{87.1, 86.9, 92.0, 84.1, 86.4\}$

b.

Height, meters	Weight, kg
1.77	83.0
	84.1
1.78	86.9
	86.4
1.80	87.1

c. $\{(1.80, 87.1), (1.78, 86.9), (1.77, 83.0), (1.77, 84.1), (1.80, 86.4)\}$

19. Domain: $\{\text{Elvis, Colleen, Kaleigh, Marissa}\}$
 Range: $\{\text{Jan. 8, Mar. 15, Sept. 17}\}$
 Function

20. Domain: $\{\text{Bob, John, Chuck}\}$
 Range: $\{\text{Beth, Diane, Linda, Marcia}\}$
 Not a function

21. Domain: $\{20, 30, 40\}$
 Range: $\{200, 300, 350, 425\}$
 Not a function

22. Domain: $\{\text{Less than 9}^{\text{th}} \text{ grade, 9}^{\text{th}}\text{-12}^{\text{th}} \text{ grade, High School Graduate, Some College, College Graduate}\}$
 Range: $\{\$18,120, \$23,251, \$36,055, \$45,810, \$67,165\}$
 Function

23. Domain: $\{-3, 2, 4\}$
 Range: $\{6, 9, 10\}$
 Not a function

Chapter 2: Functions and Their Graphs

24. Domain: $\{-2, -1, 3, 4\}$
Range: $\{3, 5, 7, 12\}$
Function

25. Domain: $\{1, 2, 3, 4\}$
Range: $\{3\}$
Function

26. Domain: $\{0, 1, 2, 3\}$
Range: $\{-2, 3, 7\}$
Function

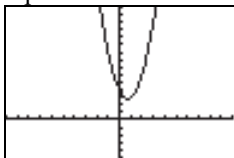
27. Domain: $\{-4, 0, 3\}$
Range: $\{1, 3, 5, 6\}$
Not a function

28. Domain: $\{-4, -3, -2, -1\}$
Range: $\{0, 1, 2, 3, 4\}$
Not a function

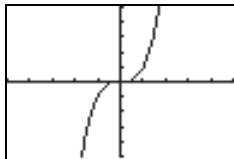
29. Domain: $\{-1, 0, 2, 4\}$
Range: $\{-1, 3, 8\}$
Function

30. Domain: $\{-2, -1, 0, 1\}$
Range: $\{3, 4, 16\}$
Function

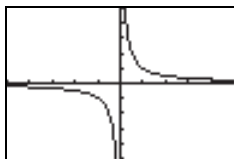
31. Graph $y = 2x^2 - 3x + 4$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



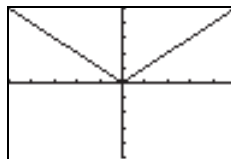
32. Graph $y = x^3$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



33. Graph $y = \frac{1}{x}$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



34. Graph $y = |x|$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



35. $x^2 = 8 - y^2$

Solve for y : $y = \pm\sqrt{8 - x^2}$

For $x = 0$, $y = \pm 2\sqrt{2}$. Thus, $(0, 2\sqrt{2})$ and $(0, -2\sqrt{2})$ are on the graph. This is not a function, since a distinct x -value corresponds to two different y -values.

36. $y = \pm\sqrt{1 - 2x}$

For $x = 0$, $y = \pm 1$. Thus, $(0, 1)$ and $(0, -1)$ are on the graph. This is not a function, since a distinct x -value corresponds to two different y -values.

37. $x = y^2$

Solve for y : $y = \pm\sqrt{x}$

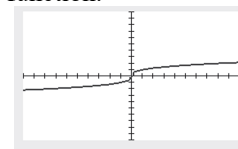
For $x = 1$, $y = \pm 1$. Thus, $(1, 1)$ and $(1, -1)$ are on the graph. This is not a function, since a distinct x -value corresponds to two different y -values.

38. $x + y^2 = 1$

Solve for y : $y = \pm\sqrt{1 - x}$

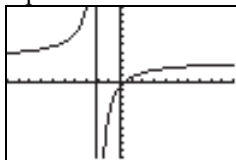
For $x = 0$, $y = \pm 1$. Thus, $(0, 1)$ and $(0, -1)$ are on the graph. This is not a function, since a distinct x -value corresponds to two different y -values.

39. Graph $y = \sqrt[3]{x}$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



40. Graph $y = \frac{3x-1}{x+2}$. The graph passes the

Vertical-Line Test. Thus, the equation represents a function.



41. $|y| = 2x + 3$

Solve for y : $y = 2x + 3$ or $y = -(2x + 3)$

For $x = 1$, $y = 5$ or $y = -5$. Thus, $(1, 5)$ and $(1, -5)$ are on the graph. This is not a function, since a distinct x -value corresponds to two different y -values.

42. $x^2 - 4y^2 = 1$

Solve for y : $x^2 - 4y^2 = 1$

$$4y^2 = x^2 - 1$$

$$y^2 = \frac{x^2 - 1}{4}$$

$$y = \frac{\pm\sqrt{x^2 - 1}}{2}$$

For $x = \sqrt{2}$, $y = \pm\frac{1}{2}$. Thus, $\left(\sqrt{2}, \frac{1}{2}\right)$ and

$\left(\sqrt{2}, -\frac{1}{2}\right)$ are on the graph. This is not a

function, since a distinct x -value corresponds to two different y -values.

43. $f(x) = 3x^2 + 2x - 4$

a. $f(0) = 3(0)^2 + 2(0) - 4 = -4$

b. $f(1) = 3(1)^2 + 2(1) - 4 = 3 + 2 - 4 = 1$

c. $f(-1) = 3(-1)^2 + 2(-1) - 4 = 3 - 2 - 4 = -3$

d. $f(-x) = 3(-x)^2 + 2(-x) - 4 = 3x^2 - 2x - 4$

e. $-f(x) = -(3x^2 + 2x - 4) = -3x^2 - 2x + 4$

f. $f(x+1) = 3(x+1)^2 + 2(x+1) - 4$
 $= 3(x^2 + 2x + 1) + 2x + 2 - 4$
 $= 3x^2 + 6x + 3 + 2x + 2 - 4$
 $= 3x^2 + 8x + 1$

g. $f(2x) = 3(2x)^2 + 2(2x) - 4 = 12x^2 + 4x - 4$

h. $f(x+h) = 3(x+h)^2 + 2(x+h) - 4$
 $= 3(x^2 + 2xh + h^2) + 2x + 2h - 4$
 $= 3x^2 + 6xh + 3h^2 + 2x + 2h - 4$

44. $f(x) = -2x^2 + x - 1$

a. $f(0) = -2(0)^2 + 0 - 1 = -1$

b. $f(1) = -2(1)^2 + 1 - 1 = -2$

c. $f(-1) = -2(-1)^2 + (-1) - 1 = -4$

d. $f(-x) = -2(-x)^2 + (-x) - 1 = -2x^2 - x - 1$

e. $-f(x) = -(-2x^2 + x - 1) = 2x^2 - x + 1$

f. $f(x+1) = -2(x+1)^2 + (x+1) - 1$
 $= -2(x^2 + 2x + 1) + x + 1 - 1$
 $= -2x^2 - 4x - 2 + x$
 $= -2x^2 - 3x - 2$

g. $f(2x) = -2(2x)^2 + (2x) - 1 = -8x^2 + 2x - 1$

h. $f(x+h) = -2(x+h)^2 + (x+h) - 1$
 $= -2(x^2 + 2xh + h^2) + x + h - 1$
 $= -2x^2 - 4xh - 2h^2 + x + h - 1$

45. $f(x) = \frac{x}{x^2 + 1}$

a. $f(0) = \frac{0}{0^2 + 1} = \frac{0}{1} = 0$

b. $f(1) = \frac{1}{1^2 + 1} = \frac{1}{2}$

c. $f(-1) = \frac{-1}{(-1)^2 + 1} = \frac{-1}{1 + 1} = -\frac{1}{2}$

d. $f(-x) = \frac{-x}{(-x)^2 + 1} = \frac{-x}{x^2 + 1}$

e. $-f(x) = -\left(\frac{x}{x^2 + 1}\right) = \frac{-x}{x^2 + 1}$

Chapter 2: Functions and Their Graphs

$$\begin{aligned} \text{f. } f(x+1) &= \frac{x+1}{(x+1)^2+1} \\ &= \frac{x+1}{x^2+2x+1+1} \\ &= \frac{x+1}{x^2+2x+2} \end{aligned}$$

$$\text{g. } f(2x) = \frac{2x}{(2x)^2+1} = \frac{2x}{4x^2+1}$$

$$\text{h. } f(x+h) = \frac{x+h}{(x+h)^2+1} = \frac{x+h}{x^2+2xh+h^2+1}$$

$$46. f(x) = \frac{x^2-1}{x+4}$$

$$\text{a. } f(0) = \frac{0^2-1}{0+4} = \frac{-1}{4} = -\frac{1}{4}$$

$$\text{b. } f(1) = \frac{1^2-1}{1+4} = \frac{0}{5} = 0$$

$$\text{c. } f(-1) = \frac{(-1)^2-1}{-1+4} = \frac{0}{3} = 0$$

$$\text{d. } f(-x) = \frac{(-x)^2-1}{-x+4} = \frac{x^2-1}{-x+4}$$

$$\text{e. } -f(x) = -\left(\frac{x^2-1}{x+4}\right) = \frac{-x^2+1}{x+4}$$

$$\begin{aligned} \text{f. } f(x+1) &= \frac{(x+1)^2-1}{(x+1)+4} \\ &= \frac{x^2+2x+1-1}{x+5} = \frac{x^2+2x}{x+5} \end{aligned}$$

$$\text{g. } f(2x) = \frac{(2x)^2-1}{2x+4} = \frac{4x^2-1}{2x+4}$$

$$\text{h. } f(x+h) = \frac{(x+h)^2-1}{(x+h)+4} = \frac{x^2+2xh+h^2-1}{x+h+4}$$

$$47. f(x) = |x| + 4$$

$$\text{a. } f(0) = |0| + 4 = 0 + 4 = 4$$

$$\text{b. } f(1) = |1| + 4 = 1 + 4 = 5$$

$$\text{c. } f(-1) = |-1| + 4 = 1 + 4 = 5$$

$$\text{d. } f(-x) = |-x| + 4 = |x| + 4$$

$$\text{e. } -f(x) = -(|x| + 4) = -|x| - 4$$

$$\text{f. } f(x+1) = |x+1| + 4$$

$$\text{g. } f(2x) = |2x| + 4 = 2|x| + 4$$

$$\text{h. } f(x+h) = |x+h| + 4$$

$$48. f(x) = \sqrt{x^2+x}$$

$$\text{a. } f(0) = \sqrt{0^2+0} = \sqrt{0} = 0$$

$$\text{b. } f(1) = \sqrt{1^2+1} = \sqrt{2}$$

$$\text{c. } f(-1) = \sqrt{(-1)^2+(-1)} = \sqrt{1-1} = \sqrt{0} = 0$$

$$\text{d. } f(-x) = \sqrt{(-x)^2+(-x)} = \sqrt{x^2-x}$$

$$\text{e. } -f(x) = -(\sqrt{x^2+x}) = -\sqrt{x^2+x}$$

$$\begin{aligned} \text{f. } f(x+1) &= \sqrt{(x+1)^2+(x+1)} \\ &= \sqrt{x^2+2x+1+x+1} \\ &= \sqrt{x^2+3x+2} \end{aligned}$$

$$\text{g. } f(2x) = \sqrt{(2x)^2+2x} = \sqrt{4x^2+2x}$$

$$\begin{aligned} \text{h. } f(x+h) &= \sqrt{(x+h)^2+(x+h)} \\ &= \sqrt{x^2+2xh+h^2+x+h} \end{aligned}$$

$$49. f(x) = \frac{2x+1}{3x-5}$$

$$\text{a. } f(0) = \frac{2(0)+1}{3(0)-5} = \frac{0+1}{0-5} = -\frac{1}{5}$$

$$\text{b. } f(1) = \frac{2(1)+1}{3(1)-5} = \frac{2+1}{3-5} = \frac{3}{-2} = -\frac{3}{2}$$

$$\text{c. } f(-1) = \frac{2(-1)+1}{3(-1)-5} = \frac{-2+1}{-3-5} = \frac{-1}{-8} = \frac{1}{8}$$

$$\text{d. } f(-x) = \frac{2(-x)+1}{3(-x)-5} = \frac{-2x+1}{-3x-5} = \frac{2x-1}{3x+5}$$

$$\text{e. } -f(x) = -\left(\frac{2x+1}{3x-5}\right) = \frac{-2x-1}{3x-5}$$

- f. $f(x+1) = \frac{2(x+1)+1}{3(x+1)-5} = \frac{2x+2+1}{3x+3-5} = \frac{2x+3}{3x-2}$
- g. $f(2x) = \frac{2(2x)+1}{3(2x)-5} = \frac{4x+1}{6x-5}$
- h. $f(x+h) = \frac{2(x+h)+1}{3(x+h)-5} = \frac{2x+2h+1}{3x+3h-5}$
50. $f(x) = 1 - \frac{1}{(x+2)^2}$
- a. $f(0) = 1 - \frac{1}{(0+2)^2} = 1 - \frac{1}{4} = \frac{3}{4}$
- b. $f(1) = 1 - \frac{1}{(1+2)^2} = 1 - \frac{1}{9} = \frac{8}{9}$
- c. $f(-1) = 1 - \frac{1}{(-1+2)^2} = 1 - \frac{1}{1} = 0$
- d. $f(-x) = 1 - \frac{1}{(-x+2)^2}$
- e. $-f(x) = -\left(1 - \frac{1}{(x+2)^2}\right) = \frac{1}{(x+2)^2} - 1$
- f. $f(x+1) = 1 - \frac{1}{(x+1+2)^2} = 1 - \frac{1}{(x+3)^2}$
- g. $f(2x) = 1 - \frac{1}{(2x+2)^2} = 1 - \frac{1}{4(x+1)^2}$
- h. $f(x+h) = 1 - \frac{1}{(x+h+2)^2}$
51. $f(x) = -5x + 4$
Domain: $\{x \mid x \text{ is any real number}\}$
52. $f(x) = x^2 + 2$
Domain: $\{x \mid x \text{ is any real number}\}$
53. $f(x) = \frac{x+1}{2x^2+8}$
Domain: $\{x \mid x \text{ is any real number}\}$
54. $f(x) = \frac{x^2}{x^2+1}$
Domain: $\{x \mid x \text{ is any real number}\}$
55. $g(x) = \frac{x}{x^2-16}$
 $x^2-16 \neq 0$
 $x^2 \neq 16 \Rightarrow x \neq \pm 4$
Domain: $\{x \mid x \neq -4, x \neq 4\}$
56. $h(x) = \frac{2x}{x^2-4}$
 $x^2-4 \neq 0$
 $x^2 \neq 4 \Rightarrow x \neq \pm 2$
Domain: $\{x \mid x \neq -2, x \neq 2\}$
57. $F(x) = \frac{x-2}{x^3+x}$
 $x^3+x \neq 0$
 $x(x^2+1) \neq 0$
 $x \neq 0, x^2 \neq -1$
Domain: $\{x \mid x \neq 0\}$
58. $G(x) = \frac{x+4}{x^3-4x}$
 $x^3-4x \neq 0$
 $x(x^2-4) \neq 0$
 $x \neq 0, x^2 \neq 4$
 $x \neq 0, x \neq \pm 2$
Domain: $\{x \mid x \neq -2, x \neq 0, x \neq 2\}$
59. $h(x) = \sqrt{3x-12}$
 $3x-12 \geq 0$
 $3x \geq 12$
 $x \geq 4$
Domain: $\{x \mid x \geq 4\}$

Chapter 2: Functions and Their Graphs

60. $G(x) = \sqrt{1-x}$

$$1-x \geq 0$$

$$-x \geq -1$$

$$x \leq 1$$

$$\text{Domain: } \{x \mid x \leq 1\}$$

61. $p(x) = \frac{x}{|2x+3|-1}$

$$|2x+3|-1 = 0$$

$$|2x+3| = 1$$

$$2x+3 = -1 \text{ or } 2x+3 = 1$$

$$2x = -4 \quad 2x = -2$$

$$x = -2 \quad x = -1$$

$$\text{Domain: } \{x \mid x \neq -2, x \neq -1\}$$

62. $p(x) = \frac{x-1}{|3x-1|-4}$

$$|3x-1|-4 = 0$$

$$|3x-1| = 4$$

$$3x-1 = -4 \text{ or } 3x-1 = 4$$

$$3x = -3 \quad 3x = 5$$

$$x = -1 \quad x = \frac{5}{3}$$

$$\text{Domain: } \left\{x \mid x \neq -1, x \neq \frac{5}{3}\right\}$$

63. $f(x) = \frac{x}{\sqrt{x-4}}$

$$x-4 > 0$$

$$x > 4$$

$$\text{Domain: } \{x \mid x > 4\}$$

64. $q(x) = \frac{-x}{\sqrt{-x-2}}$

$$-x-2 > 0$$

$$-x > 2$$

$$x < -2$$

$$\text{Domain: } \{x \mid x < -2\}$$

65. $P(t) = \frac{\sqrt{t-4}}{3t-21}$

$$t-4 \geq 0$$

$$t \geq 4$$

$$\text{Also } 3t-21 \neq 0$$

$$3t-21 \neq 0$$

$$3t \neq 21$$

$$t \neq 7$$

$$\text{Domain: } \{t \mid t \geq 4, t \neq 7\}$$

66. $h(z) = \frac{\sqrt{z+3}}{z-2}$

$$z+3 \geq 0$$

$$z \geq -3$$

$$\text{Also } z-2 \neq 0$$

$$z \neq 2$$

$$\text{Domain: } \{z \mid z \geq -3, z \neq 2\}$$

67. $f(x) = \sqrt[3]{5x-4}$

$$\text{Domain: } \{x \mid x \text{ is any real number}\}.$$

68. $g(t) = -t^2 + \sqrt[3]{t^2+7t}$

$$\text{Domain: } \{t \mid t \text{ is any real number}\}.$$

69. $M(t) = \sqrt[5]{\frac{t+1}{t^2-5t-14}}$

$$t^2-5t-14 = 0$$

$$(t+2)(t-7) = 0$$

$$t+2 = 0 \text{ or } t-7 = 0$$

$$t = -2 \quad t = 7$$

$$\text{Domain: } \{t \mid t \neq -2, t \neq 7\}$$

70. $N(p) = \sqrt[5]{\frac{p}{2p^2-98}}$

$$2p^2-98 = 0$$

$$2(p^2-49) = 0$$

$$2(p+7)(p-7) = 0$$

$$p+7 = 0 \text{ or } p-7 = 0$$

$$p = -7 \quad p = 7$$

$$\text{Domain: } \{p \mid p \neq -7, p \neq 7\}$$

71. $f(x) = 3x+4 \quad g(x) = 2x-3$

a. $(f+g)(x) = 3x+4+2x-3 = 5x+1$

$$\text{Domain: } \{x \mid x \text{ is any real number}\}.$$

Section 2.1: Functions

- b.** $(f - g)(x) = (3x + 4) - (2x - 3)$
 $= 3x + 4 - 2x + 3$
 $= x + 7$
Domain: $\{x \mid x \text{ is any real number}\}$.
- c.** $(f \cdot g)(x) = (3x + 4)(2x - 3)$
 $= 6x^2 - 9x + 8x - 12$
 $= 6x^2 - x - 12$
Domain: $\{x \mid x \text{ is any real number}\}$.
- d.** $\left(\frac{f}{g}\right)(x) = \frac{3x + 4}{2x - 3}$
 $2x - 3 \neq 0 \Rightarrow 2x \neq 3 \Rightarrow x \neq \frac{3}{2}$
Domain: $\left\{x \mid x \neq \frac{3}{2}\right\}$.
- e.** $(f + g)(3) = 5(3) + 1 = 15 + 1 = 16$
- f.** $(f - g)(4) = 4 + 7 = 11$
- g.** $(f \cdot g)(2) = 6(2)^2 - 2 - 12 = 24 - 2 - 12 = 10$
- h.** $\left(\frac{f}{g}\right)(1) = \frac{3(1) + 4}{2(1) - 3} = \frac{3 + 4}{2 - 3} = \frac{7}{-1} = -7$
- 72.** $f(x) = 2x + 1$ $g(x) = 3x - 2$
- a.** $(f + g)(x) = 2x + 1 + 3x - 2 = 5x - 1$
Domain: $\{x \mid x \text{ is any real number}\}$.
- b.** $(f - g)(x) = (2x + 1) - (3x - 2)$
 $= 2x + 1 - 3x + 2$
 $= -x + 3$
Domain: $\{x \mid x \text{ is any real number}\}$.
- c.** $(f \cdot g)(x) = (2x + 1)(3x - 2)$
 $= 6x^2 - 4x + 3x - 2$
 $= 6x^2 - x - 2$
Domain: $\{x \mid x \text{ is any real number}\}$.
- d.** $\left(\frac{f}{g}\right)(x) = \frac{2x + 1}{3x - 2}$
 $3x - 2 \neq 0$
 $3x \neq 2 \Rightarrow x \neq \frac{2}{3}$
Domain: $\left\{x \mid x \neq \frac{2}{3}\right\}$.
- e.** $(f + g)(3) = 5(3) - 1 = 15 - 1 = 14$
- f.** $(f - g)(4) = -4 + 3 = -1$
- g.** $(f \cdot g)(2) = 6(2)^2 - 2 - 2$
 $= 6(4) - 2 - 2$
 $= 24 - 2 - 2 = 20$
- h.** $\left(\frac{f}{g}\right)(1) = \frac{2(1) + 1}{3(1) - 2} = \frac{2 + 1}{3 - 2} = \frac{3}{1} = 3$
- 73.** $f(x) = x - 1$ $g(x) = 2x^2$
- a.** $(f + g)(x) = x - 1 + 2x^2 = 2x^2 + x - 1$
Domain: $\{x \mid x \text{ is any real number}\}$.
- b.** $(f - g)(x) = (x - 1) - (2x^2)$
 $= x - 1 - 2x^2$
 $= -2x^2 + x - 1$
Domain: $\{x \mid x \text{ is any real number}\}$.
- c.** $(f \cdot g)(x) = (x - 1)(2x^2) = 2x^3 - 2x^2$
Domain: $\{x \mid x \text{ is any real number}\}$.
- d.** $\left(\frac{f}{g}\right)(x) = \frac{x - 1}{2x^2}$
Domain: $\{x \mid x \neq 0\}$.
- e.** $(f + g)(3) = 2(3)^2 + 3 - 1$
 $= 2(9) + 3 - 1$
 $= 18 + 3 - 1 = 20$
- f.** $(f - g)(4) = -2(4)^2 + 4 - 1$
 $= -2(16) + 4 - 1$
 $= -32 + 4 - 1 = -29$
- g.** $(f \cdot g)(2) = 2(2)^3 - 2(2)^2$
 $= 2(8) - 2(4)$
 $= 16 - 8 = 8$
- h.** $\left(\frac{f}{g}\right)(1) = \frac{1 - 1}{2(1)^2} = \frac{0}{2(1)} = \frac{0}{2} = 0$
- 74.** $f(x) = 2x^2 + 3$ $g(x) = 4x^3 + 1$

Chapter 2: Functions and Their Graphs

$$\begin{aligned}\text{a. } (f+g)(x) &= 2x^2 + 3 + 4x^3 + 1 \\ &= 4x^3 + 2x^2 + 4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}\text{b. } (f-g)(x) &= (2x^2 + 3) - (4x^3 + 1) \\ &= 2x^2 + 3 - 4x^3 - 1 \\ &= -4x^3 + 2x^2 + 2\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}\text{c. } (f \cdot g)(x) &= (2x^2 + 3)(4x^3 + 1) \\ &= 8x^5 + 12x^3 + 2x^2 + 3\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\text{d. } \left(\frac{f}{g}\right)(x) = \frac{2x^2 + 3}{4x^3 + 1}$$

$$4x^3 + 1 \neq 0$$

$$4x^3 \neq -1$$

$$x^3 \neq -\frac{1}{4} \Rightarrow x \neq \sqrt[3]{-\frac{1}{4}} = -\frac{\sqrt[3]{2}}{2}$$

Domain: $\left\{x \mid x \neq -\frac{\sqrt[3]{2}}{2}\right\}$.

$$\begin{aligned}\text{e. } (f+g)(3) &= 4(3)^3 + 2(3)^2 + 4 \\ &= 4(27) + 2(9) + 4 \\ &= 108 + 18 + 4 = 130\end{aligned}$$

$$\begin{aligned}\text{f. } (f-g)(4) &= -4(4)^3 + 2(4)^2 + 2 \\ &= -4(64) + 2(16) + 2 \\ &= -256 + 32 + 2 = -222\end{aligned}$$

$$\begin{aligned}\text{g. } (f \cdot g)(2) &= 8(2)^5 + 12(2)^3 + 2(2)^2 + 3 \\ &= 8(32) + 12(8) + 2(4) + 3 \\ &= 256 + 96 + 8 + 3 = 363\end{aligned}$$

$$\text{h. } \left(\frac{f}{g}\right)(1) = \frac{2(1)^2 + 3}{4(1)^3 + 1} = \frac{2(1) + 3}{4(1) + 1} = \frac{2 + 3}{4 + 1} = \frac{5}{5} = 1$$

$$75. \quad f(x) = \sqrt{x} \quad g(x) = 3x - 5$$

$$\begin{aligned}\text{a. } (f+g)(x) &= \sqrt{x} + 3x - 5 \\ \text{Domain: } &\{x \mid x \geq 0\}.\end{aligned}$$

$$\begin{aligned}\text{b. } (f-g)(x) &= \sqrt{x} - (3x - 5) = \sqrt{x} - 3x + 5 \\ \text{Domain: } &\{x \mid x \geq 0\}.\end{aligned}$$

$$\begin{aligned}\text{c. } (f \cdot g)(x) &= \sqrt{x}(3x - 5) = 3x\sqrt{x} - 5\sqrt{x} \\ \text{Domain: } &\{x \mid x \geq 0\}.\end{aligned}$$

$$\begin{aligned}\text{d. } \left(\frac{f}{g}\right)(x) &= \frac{\sqrt{x}}{3x - 5} \\ &x \geq 0 \text{ and } 3x - 5 \neq 0\end{aligned}$$

$$3x \neq 5 \Rightarrow x \neq \frac{5}{3}$$

Domain: $\left\{x \mid x \geq 0 \text{ and } x \neq \frac{5}{3}\right\}$.

$$\begin{aligned}\text{e. } (f+g)(3) &= \sqrt{3} + 3(3) - 5 \\ &= \sqrt{3} + 9 - 5 = \sqrt{3} + 4\end{aligned}$$

$$\begin{aligned}\text{f. } (f-g)(4) &= \sqrt{4} - 3(4) + 5 \\ &= 2 - 12 + 5 = -5\end{aligned}$$

$$\begin{aligned}\text{g. } (f \cdot g)(2) &= 3(2)\sqrt{2} - 5\sqrt{2} \\ &= 6\sqrt{2} - 5\sqrt{2} = \sqrt{2}\end{aligned}$$

$$\text{h. } \left(\frac{f}{g}\right)(1) = \frac{\sqrt{1}}{3(1) - 5} = \frac{1}{3 - 5} = \frac{1}{-2} = -\frac{1}{2}$$

$$76. \quad f(x) = |x| \quad g(x) = x$$

$$\begin{aligned}\text{a. } (f+g)(x) &= |x| + x \\ \text{Domain: } &\{x \mid x \text{ is any real number}\}.\end{aligned}$$

$$\begin{aligned}\text{b. } (f-g)(x) &= |x| - x \\ \text{Domain: } &\{x \mid x \text{ is any real number}\}.\end{aligned}$$

$$\begin{aligned}\text{c. } (f \cdot g)(x) &= |x| \cdot x = x|x| \\ \text{Domain: } &\{x \mid x \text{ is any real number}\}.\end{aligned}$$

$$\begin{aligned}\text{d. } \left(\frac{f}{g}\right)(x) &= \frac{|x|}{x} \\ \text{Domain: } &\{x \mid x \neq 0\}.\end{aligned}$$

$$\text{e. } (f+g)(3) = |3| + 3 = 3 + 3 = 6$$

$$\text{f. } (f-g)(4) = |4| - 4 = 4 - 4 = 0$$

$$\text{g. } (f \cdot g)(2) = 2|2| = 2 \cdot 2 = 4$$

$$\text{h. } \left(\frac{f}{g}\right)(1) = \frac{|1|}{1} = \frac{1}{1} = 1$$

$$77. f(x) = 1 + \frac{1}{x} \quad g(x) = \frac{1}{x}$$

$$\text{a. } (f+g)(x) = 1 + \frac{1}{x} + \frac{1}{x} = 1 + \frac{2}{x}$$

$$\text{Domain: } \{x \mid x \neq 0\}.$$

$$\text{b. } (f-g)(x) = 1 + \frac{1}{x} - \frac{1}{x} = 1$$

$$\text{Domain: } \{x \mid x \neq 0\}.$$

$$\text{c. } (f \cdot g)(x) = \left(1 + \frac{1}{x}\right) \frac{1}{x} = \frac{1}{x} + \frac{1}{x^2}$$

$$\text{Domain: } \{x \mid x \neq 0\}.$$

$$\text{d. } \left(\frac{f}{g}\right)(x) = \frac{1 + \frac{1}{x}}{\frac{1}{x}} = \frac{\frac{x+1}{x}}{\frac{1}{x}} = \frac{x+1}{x} \cdot \frac{x}{1} = x+1$$

$$\text{Domain: } \{x \mid x \neq 0\}.$$

$$\text{e. } (f+g)(3) = 1 + \frac{2}{3} = \frac{5}{3}$$

$$\text{f. } (f-g)(4) = 1$$

$$\text{g. } (f \cdot g)(2) = \frac{1}{2} + \frac{1}{(2)^2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$\text{h. } \left(\frac{f}{g}\right)(1) = 1 + 1 = 2$$

$$78. f(x) = \sqrt{x-1} \quad g(x) = \sqrt{4-x}$$

$$\text{a. } (f+g)(x) = \sqrt{x-1} + \sqrt{4-x}$$

$$x-1 \geq 0 \text{ and } 4-x \geq 0$$

$$x \geq 1 \text{ and } -x \geq -4$$

$$x \leq 4$$

$$\text{Domain: } \{x \mid 1 \leq x \leq 4\}.$$

$$\text{b. } (f-g)(x) = \sqrt{x-1} - \sqrt{4-x}$$

$$x-1 \geq 0 \text{ and } 4-x \geq 0$$

$$x \geq 1 \text{ and } -x \geq -4$$

$$x \leq 4$$

$$\text{Domain: } \{x \mid 1 \leq x \leq 4\}.$$

$$\text{c. } (f \cdot g)(x) = (\sqrt{x-1})(\sqrt{4-x})$$

$$= \sqrt{-x^2 + 5x - 4}$$

$$x-1 \geq 0 \text{ and } 4-x \geq 0$$

$$x \geq 1 \text{ and } -x \geq -4$$

$$x \leq 4$$

$$\text{Domain: } \{x \mid 1 \leq x \leq 4\}.$$

$$\text{d. } \left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-1}}{\sqrt{4-x}} = \sqrt{\frac{x-1}{4-x}}$$

$$x-1 \geq 0 \text{ and } 4-x > 0$$

$$x \geq 1 \text{ and } -x > -4$$

$$x < 4$$

$$\text{Domain: } \{x \mid 1 \leq x < 4\}.$$

$$\text{e. } (f+g)(3) = \sqrt{3-1} + \sqrt{4-3}$$

$$= \sqrt{2} + \sqrt{1} = \sqrt{2} + 1$$

$$\text{f. } (f-g)(4) = \sqrt{4-1} - \sqrt{4-4}$$

$$= \sqrt{3} - \sqrt{0} = \sqrt{3} - 0 = \sqrt{3}$$

$$\text{g. } (f \cdot g)(2) = \sqrt{-(2)^2 + 5(2) - 4}$$

$$= \sqrt{-4 + 10 - 4} = \sqrt{2}$$

$$\text{h. } \left(\frac{f}{g}\right)(1) = \sqrt{\frac{1-1}{4-1}} = \sqrt{\frac{0}{3}} = \sqrt{0} = 0$$

$$79. f(x) = \frac{2x+3}{3x-2} \quad g(x) = \frac{4x}{3x-2}$$

$$\text{a. } (f+g)(x) = \frac{2x+3}{3x-2} + \frac{4x}{3x-2}$$

$$= \frac{2x+3+4x}{3x-2} = \frac{6x+3}{3x-2}$$

$$3x-2 \neq 0$$

$$3x \neq 2 \Rightarrow x \neq \frac{2}{3}$$

$$\text{Domain: } \left\{x \mid x \neq \frac{2}{3}\right\}.$$

Chapter 2: Functions and Their Graphs

$$\begin{aligned} \text{b. } (f-g)(x) &= \frac{2x+3}{3x-2} - \frac{4x}{3x-2} \\ &= \frac{2x+3-4x}{3x-2} = \frac{-2x+3}{3x-2} \end{aligned}$$

$$3x-2 \neq 0$$

$$3x \neq 2 \Rightarrow x \neq \frac{2}{3}$$

$$\text{Domain: } \left\{ x \mid x \neq \frac{2}{3} \right\}.$$

$$\text{c. } (f \cdot g)(x) = \left(\frac{2x+3}{3x-2} \right) \left(\frac{4x}{3x-2} \right) = \frac{8x^2+12x}{(3x-2)^2}$$

$$3x-2 \neq 0$$

$$3x \neq 2 \Rightarrow x \neq \frac{2}{3}$$

$$\text{Domain: } \left\{ x \mid x \neq \frac{2}{3} \right\}.$$

$$\text{d. } \left(\frac{f}{g} \right)(x) = \frac{\frac{2x+3}{3x-2}}{\frac{4x}{3x-2}} = \frac{2x+3}{3x-2} \cdot \frac{3x-2}{4x} = \frac{2x+3}{4x}$$

$$3x-2 \neq 0 \quad \text{and} \quad x \neq 0$$

$$3x \neq 2$$

$$x \neq \frac{2}{3}$$

$$\text{Domain: } \left\{ x \mid x \neq \frac{2}{3} \text{ and } x \neq 0 \right\}.$$

$$\text{e. } (f+g)(3) = \frac{6(3)+3}{3(3)-2} = \frac{18+3}{9-2} = \frac{21}{7} = 3$$

$$\text{f. } (f-g)(4) = \frac{-2(4)+3}{3(4)-2} = \frac{-8+3}{12-2} = \frac{-5}{10} = -\frac{1}{2}$$

$$\begin{aligned} \text{g. } (f \cdot g)(2) &= \frac{8(2)^2+12(2)}{(3(2)-2)^2} \\ &= \frac{8(4)+24}{(6-2)^2} = \frac{32+24}{(4)^2} = \frac{56}{16} = \frac{7}{2} \end{aligned}$$

$$\text{h. } \left(\frac{f}{g} \right)(1) = \frac{2(1)+3}{4(1)} = \frac{2+3}{4} = \frac{5}{4}$$

$$80. \quad f(x) = \sqrt{x+1} \quad g(x) = \frac{2}{x}$$

$$\text{a. } (f+g)(x) = \sqrt{x+1} + \frac{2}{x}$$

$$x+1 \geq 0 \quad \text{and} \quad x \neq 0$$

$$x \geq -1$$

$$\text{Domain: } \{x \mid x \geq -1, \text{ and } x \neq 0\}.$$

$$\text{b. } (f-g)(x) = \sqrt{x+1} - \frac{2}{x}$$

$$x+1 \geq 0 \quad \text{and} \quad x \neq 0$$

$$x \geq -1$$

$$\text{Domain: } \{x \mid x \geq -1, \text{ and } x \neq 0\}.$$

$$\text{c. } (f \cdot g)(x) = \sqrt{x+1} \cdot \frac{2}{x} = \frac{2\sqrt{x+1}}{x}$$

$$x+1 \geq 0 \quad \text{and} \quad x \neq 0$$

$$x \geq -1$$

$$\text{Domain: } \{x \mid x \geq -1, \text{ and } x \neq 0\}.$$

$$\text{d. } \left(\frac{f}{g} \right)(x) = \frac{\sqrt{x+1}}{\frac{2}{x}} = \frac{x\sqrt{x+1}}{2}$$

$$x+1 \geq 0 \quad \text{and} \quad x \neq 0$$

$$x \geq -1$$

$$\text{Domain: } \{x \mid x \geq -1, \text{ and } x \neq 0\}.$$

$$\text{e. } (f+g)(3) = \sqrt{3+1} + \frac{2}{3} = \sqrt{4} + \frac{2}{3} = 2 + \frac{2}{3} = \frac{8}{3}$$

$$\text{f. } (f-g)(4) = \sqrt{4+1} - \frac{2}{4} = \sqrt{5} - \frac{1}{2}$$

$$\text{g. } (f \cdot g)(2) = \frac{2\sqrt{2+1}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\text{h. } \left(\frac{f}{g} \right)(1) = \frac{1\sqrt{1+1}}{2} = \frac{\sqrt{2}}{2}$$

$$81. \quad f(x) = 3x+1 \quad (f+g)(x) = 6 - \frac{1}{2}x$$

$$6 - \frac{1}{2}x = 3x+1 + g(x)$$

$$5 - \frac{7}{2}x = g(x)$$

$$g(x) = 5 - \frac{7}{2}x$$

$$82. f(x) = \frac{1}{x} \quad \left(\frac{f}{g} \right)(x) = \frac{x+1}{x^2-x}$$

$$\frac{x+1}{x^2-x} = \frac{\frac{1}{x}}{g(x)}$$

$$g(x) = \frac{\frac{1}{x}}{\frac{x+1}{x^2-x}} = \frac{1}{x} \cdot \frac{x^2-x}{x+1}$$

$$= \frac{1}{x} \cdot \frac{x(x-1)}{x+1} = \frac{x-1}{x+1}$$

$$83. f(x) = 4x+3$$

$$\frac{f(x+h)-f(x)}{h} = \frac{4(x+h)+3-(4x+3)}{h}$$

$$= \frac{4x+4h+3-4x-3}{h}$$

$$= \frac{4h}{h} = 4$$

$$84. f(x) = -3x+1$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-3(x+h)+1-(-3x+1)}{h}$$

$$= \frac{-3x-3h+1+3x-1}{h}$$

$$= \frac{-3h}{h} = -3$$

$$85. f(x) = x^2 - 4$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-4-(x^2-4)}{h}$$

$$= \frac{x^2+2xh+h^2-4-x^2+4}{h}$$

$$= \frac{2xh+h^2}{h}$$

$$= 2x+h$$

$$86. f(x) = 3x^2 + 2$$

$$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)^2+2-(3x^2+2)}{h}$$

$$= \frac{3x^2+6xh+3h^2+2-3x^2-2}{h}$$

$$= \frac{6xh+3h^2}{h}$$

$$= 6x+3h$$

$$87. f(x) = x^2 - x + 4$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-(x+h)+4-(x^2-x+4)}{h}$$

$$= \frac{x^2+2xh+h^2-x-h+4-x^2+x-4}{h}$$

$$= \frac{2xh+h^2-h}{h}$$

$$= 2x+h-1$$

$$88. f(x) = 3x^2 - 2x + 6$$

$$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)^2-2(x+h)+6-[3x^2-2x+6]}{h}$$

$$= \frac{3(x^2+2xh+h^2)-2x-2h+6-3x^2+2x-6}{h}$$

$$= \frac{3x^2+6xh+3h^2-2h-3x^2}{h} = \frac{6xh+3h^2-2h}{h}$$

$$= 6x+3h-2$$

Chapter 2: Functions and Their Graphs

$$89. f(x) = \frac{5}{4x-3}$$

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{\frac{5}{4(x+h)-3} - \frac{5}{4x-3}}{h} \\ &= \frac{\frac{5(4x-3) - 5(4(x+h)-3)}{(4(x+h)-3)(4x-3)}}{h} \\ &= \left(\frac{20x+15-20x-15-20h}{(4(x+h)-3)(4x-3)} \right) \left(\frac{1}{h} \right) \\ &= \left(\frac{-20h}{(4(x+h)-3)(4x-3)} \right) \left(\frac{1}{h} \right) \\ &= \frac{-20}{(4(x+h)-3)(4x-3)} \end{aligned}$$

$$90. f(x) = \frac{1}{x+3}$$

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{\frac{1}{x+h+3} - \frac{1}{x+3}}{h} \\ &= \frac{\frac{x+3-(x+3+h)}{(x+h+3)(x+3)}}{h} \\ &= \left(\frac{x+3-x-3-h}{(x+h+3)(x+3)} \right) \left(\frac{1}{h} \right) \\ &= \left(\frac{-h}{(x+h+3)(x+3)} \right) \left(\frac{1}{h} \right) \\ &= \frac{-1}{(x+h+3)(x+3)} \end{aligned}$$

$$91. f(x) = \frac{2x}{x+3}$$

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{\frac{2(x+h)}{x+h+3} - \frac{2x}{x+3}}{h} \\ &= \frac{\frac{2(x+h)(x+3) - 2x(x+3+h)}{(x+h+3)(x+3)}}{h} \\ &= \frac{2x^2+6x+2hx+6h-2x^2-6x-2xh}{(x+h+3)(x+3)} \\ &= \left(\frac{6h}{(x+h+3)(x+3)} \right) \left(\frac{1}{h} \right) \\ &= \frac{6}{(x+h+3)(x+3)} \end{aligned}$$

$$92. f(x) = \frac{5x}{x-4}$$

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{\frac{5(x+h)}{x+h-4} - \frac{5x}{x-4}}{h} \\ &= \frac{\frac{5(x+h)(x-4) - 5x(x-4+h)}{(x+h-4)(x-4)}}{h} \\ &= \frac{5x^2-20x+5hx-20h-5x^2+20x-5xh}{(x+h-4)(x-4)} \\ &= \left(\frac{-20h}{(x+h-4)(x-4)} \right) \left(\frac{1}{h} \right) \\ &= \frac{-20}{(x+h-4)(x-4)} \end{aligned}$$

$$93. f(x) = \sqrt{x-2}$$

$$\begin{aligned} & \frac{f(x+h) - f(x)}{h} \\ &= \frac{\sqrt{x+h-2} - \sqrt{x-2}}{h} \\ &= \frac{\sqrt{x+h-2} - \sqrt{x-2}}{h} \cdot \frac{\sqrt{x+h-2} + \sqrt{x-2}}{\sqrt{x+h-2} + \sqrt{x-2}} \\ &= \frac{x+h-2 - x+2}{h(\sqrt{x+h-2} + \sqrt{x-2})} \\ &= \frac{h}{h(\sqrt{x+h-2} + \sqrt{x-2})} \\ &= \frac{1}{\sqrt{x+h-2} + \sqrt{x-2}} \end{aligned}$$

$$94. f(x) = \sqrt{x+1}$$

$$\begin{aligned} & \frac{f(x+h) - f(x)}{h} \\ &= \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\ &= \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}} \\ &= \frac{x+h+1 - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} = \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\ &= \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} \end{aligned}$$

$$95. f(x) = \frac{1}{x^2}$$

$$\begin{aligned} & \frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \\ &= \frac{\frac{x^2 - (x+h)^2}{x^2(x+h)^2}}{h} \\ &= \frac{x - (x^2 + 2xh + h^2)}{x^2(x+h)^2} \\ &= \left(\frac{1}{h}\right) \frac{-2xh - h^2}{x^2(x+h)^2} \\ &= \left(\frac{1}{h}\right) \frac{h(-2x-h)}{x^2(x+h)^2} \\ &= \frac{-2x-h}{x^2(x+h)^2} = \frac{-(2x+h)}{x^2(x+h)^2} \end{aligned}$$

$$96. f(x) = \frac{1}{x^2 + 1}$$

$$\begin{aligned} & \frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{(x+h)^2 + 1} - \frac{1}{x^2 + 1}}{h} \\ &= \frac{\frac{x^2 + 1 - ((x+h)^2 + 1)}{(x^2 + 1)((x+h)^2 + 1)}}{h} \\ &= \frac{x^2 + 1 - (x^2 + 2xh + h^2) - 1}{(x^2 + 1)((x+h)^2 + 1)} \\ &= \left(\frac{1}{h}\right) \frac{-2xh - h^2}{(x^2 + 1)((x+h)^2 + 1)} \\ &= \left(\frac{1}{h}\right) \frac{h(-2x-h)}{(x^2 + 1)((x+h)^2 + 1)} \\ &= \frac{-2x-h}{(x^2 + 1)((x+h)^2 + 1)} \\ &= \frac{-(2x+h)}{(x^2 + 1)((x+h)^2 + 1)} \end{aligned}$$

Chapter 2: Functions and Their Graphs

$$\begin{aligned}
 97. \quad f(x) &= \sqrt{4-x^2} \\
 \frac{f(x+h) - f(x)}{h} &= \frac{\sqrt{4-(x+h)^2} - \sqrt{4-x^2}}{h} \\
 &= \frac{\sqrt{4-(x+h)^2} - \sqrt{4-x^2}}{h} \cdot \frac{\sqrt{4-(x+h)^2} + \sqrt{4-x^2}}{\sqrt{4-(x+h)^2} + \sqrt{4-x^2}} \\
 &= \frac{4-(x+h)^2 - (4-x^2)}{h(\sqrt{4-(x+h)^2} + \sqrt{4-x^2})} \\
 &= \frac{4-(x^2+2xh+h^2)-(4-x^2)}{h(\sqrt{4-(x+h)^2} + \sqrt{4-x^2})} \\
 &= \frac{-2xh-h^2}{h(\sqrt{4-(x+h)^2} + \sqrt{4-x^2})} \\
 &= \frac{-2x-h}{\sqrt{4-(x+h)^2} + \sqrt{4-x^2}} \\
 &= \frac{-(2x+h)}{\sqrt{4-(x+h)^2} + \sqrt{4-x^2}}
 \end{aligned}$$

$$\begin{aligned}
 98. \quad f(x) &= \frac{1}{\sqrt{x+2}} \\
 \frac{f(x+h) - f(x)}{h} &= \frac{\frac{1}{\sqrt{x+h+2}} - \frac{1}{\sqrt{x+2}}}{h} \\
 &= \frac{\frac{\sqrt{x+2} - \sqrt{x+h+2}}{\sqrt{x+h+2}\sqrt{x+2}}}{h} \\
 &= \frac{\sqrt{x+2} - \sqrt{x+h+2}}{h\sqrt{x+h+2}\sqrt{x+2}} \cdot \frac{\sqrt{x+2} + \sqrt{x+h+2}}{\sqrt{x+2} + \sqrt{x+h+2}} \\
 &= \frac{x+2 - (x+h+2)}{h(x+2)\sqrt{x+h+2} + (x+h+2)\sqrt{x+2}} \\
 &= \frac{x+2 - x - h - 2}{h(x+2)\sqrt{x+h+2} + (x+h+2)\sqrt{x+2}} \\
 &= \frac{-h}{h(x+2)\sqrt{x+h+2} + (x+h+2)\sqrt{x+2}} \\
 &= -\frac{1}{(x+2)\sqrt{x+h+2} + (x+h+2)\sqrt{x+2}}
 \end{aligned}$$

$$\begin{aligned}
 99. \quad 11 &= x^2 - 2x + 3 \\
 0 &= x^2 - 2x - 8 \\
 0 &= (x-4)(x+2) \\
 x-4 &= 0 \quad \text{or} \quad x+2 = 0 \\
 x &= 4 \quad \text{or} \quad x = -2
 \end{aligned}$$

The solution set is: $\{-2, 4\}$

$$\begin{aligned}
 100. \quad -\frac{7}{16} &= \frac{5}{6}x - \frac{3}{4} \\
 -\frac{7}{16} + \frac{3}{4} &= \frac{5}{6}x \\
 \frac{5}{6}x &= -\frac{7}{16} + \frac{12}{16} \\
 \frac{5}{6}x &= \frac{5}{16} \\
 x &= \frac{5}{16} \cdot \frac{6}{5} = \frac{3}{8}
 \end{aligned}$$

The solution set is: $\left\{\frac{3}{8}\right\}$

$$\begin{aligned}
 101. \quad f(x) &= 2x^3 + Ax^2 + 4x - 5 \quad \text{and} \quad f(2) = 5 \\
 f(2) &= 2(2)^3 + A(2)^2 + 4(2) - 5 \\
 5 &= 16 + 4A + 8 - 5 \\
 5 &= 4A + 19 \\
 -14 &= 4A \\
 A &= \frac{-14}{4} = -\frac{7}{2}
 \end{aligned}$$

$$\begin{aligned}
 102. \quad f(x) &= 3x^2 - Bx + 4 \quad \text{and} \quad f(-1) = 12 : \\
 f(-1) &= 3(-1)^2 - B(-1) + 4 \\
 12 &= 3 + B + 4 \\
 B &= 5
 \end{aligned}$$

$$\begin{aligned}
 103. \quad f(x) &= \frac{3x+8}{2x-A} \quad \text{and} \quad f(0) = 2 \\
 f(0) &= \frac{3(0)+8}{2(0)-A} \\
 2 &= \frac{8}{-A} \\
 -2A &= 8 \\
 A &= -4
 \end{aligned}$$

104. $f(x) = \frac{2x-B}{3x+4}$ and $f(2) = \frac{1}{2}$

$$f(2) = \frac{2(2)-B}{3(2)+4}$$

$$\frac{1}{2} = \frac{4-B}{10}$$

$$5 = 4 - B$$

$$B = -1$$

105. Let x represent the length of the rectangle.

Then, $\frac{x}{2}$ represents the width of the rectangle

since the length is twice the width. The function

for the area is: $A(x) = x \cdot \frac{x}{2} = \frac{x^2}{2} = \frac{1}{2}x^2$

106. Let x represent the length of one of the two equal sides. The function for the area is:

$$A(x) = \frac{1}{2} \cdot x \cdot x = \frac{1}{2}x^2$$

107. Let x represent the number of hours worked.

The function for the gross salary is:

$$G(x) = 16x$$

108. Let x represent the number of items sold.

The function for the gross salary is:

$$G(x) = 10x + 100$$

109. a. $H(1) = 20 - 4.9(1)^2$
 $= 20 - 4.9 = 15.1$ meters

$$\begin{aligned} H(1.1) &= 20 - 4.9(1.1)^2 \\ &= 20 - 4.9(1.21) \\ &= 20 - 5.929 = 14.071 \text{ meters} \end{aligned}$$

$$\begin{aligned} H(1.2) &= 20 - 4.9(1.2)^2 \\ &= 20 - 4.9(1.44) \\ &= 20 - 7.056 = 12.944 \text{ meters} \end{aligned}$$

b. $H(x) = 15$:

$$15 = 20 - 4.9x^2$$

$$-5 = -4.9x^2$$

$$x^2 \approx 1.0204$$

$$x \approx 1.01 \text{ seconds}$$

$H(x) = 10$:

$$10 = 20 - 4.9x^2$$

$$-10 = -4.9x^2$$

$$x^2 \approx 2.0408$$

$$x \approx 1.43 \text{ seconds}$$

$H(x) = 5$:

$$5 = 20 - 4.9x^2$$

$$-15 = -4.9x^2$$

$$x^2 \approx 3.0612$$

$$x \approx 1.75 \text{ seconds}$$

c. $H(x) = 0$

$$0 = 20 - 4.9x^2$$

$$-20 = -4.9x^2$$

$$x^2 \approx 4.0816$$

$$x \approx 2.02 \text{ seconds}$$

110. a. $H(1) = 20 - 13(1)^2 = 20 - 13 = 7$ meters

$$\begin{aligned} H(1.1) &= 20 - 13(1.1)^2 = 20 - 13(1.21) \\ &= 20 - 15.73 = 4.27 \text{ meters} \end{aligned}$$

$$\begin{aligned} H(1.2) &= 20 - 13(1.2)^2 = 20 - 13(1.44) \\ &= 20 - 18.72 = 1.28 \text{ meters} \end{aligned}$$

b. $H(x) = 15$

$$15 = 20 - 13x^2$$

$$-5 = -13x^2$$

$$x^2 \approx 0.3846$$

$$x \approx 0.62 \text{ seconds}$$

$H(x) = 10$

$$10 = 20 - 13x^2$$

$$-10 = -13x^2$$

$$x^2 \approx 0.7692$$

$$x \approx 0.88 \text{ seconds}$$

Chapter 2: Functions and Their Graphs

$$H(x) = 5$$

$$5 = 20 - 13x^2$$

$$-15 = -13x^2$$

$$x^2 \approx 1.1538$$

$$x \approx 1.07 \text{ seconds}$$

$$\text{c. } H(x) = 0$$

$$0 = 20 - 13x^2$$

$$-20 = -13x^2$$

$$x^2 \approx 1.5385$$

$$x \approx 1.24 \text{ seconds}$$

$$111. \quad C(x) = 100 + \frac{x}{10} + \frac{36,000}{x}$$

$$\begin{aligned} \text{a. } C(500) &= 100 + \frac{500}{10} + \frac{36,000}{500} \\ &= 100 + 50 + 72 \\ &= \$222 \end{aligned}$$

$$\begin{aligned} \text{b. } C(450) &= 100 + \frac{450}{10} + \frac{36,000}{450} \\ &= 100 + 45 + 80 \\ &= \$225 \end{aligned}$$

$$\begin{aligned} \text{c. } C(600) &= 100 + \frac{600}{10} + \frac{36,000}{600} \\ &= 100 + 60 + 60 \\ &= \$220 \end{aligned}$$

$$\begin{aligned} \text{d. } C(400) &= 100 + \frac{400}{10} + \frac{36,000}{400} \\ &= 100 + 40 + 90 \\ &= \$230 \end{aligned}$$

$$112. \quad A(x) = 4x\sqrt{1-x^2}$$

$$\begin{aligned} \text{a. } A\left(\frac{1}{3}\right) &= 4 \cdot \frac{1}{3} \sqrt{1 - \left(\frac{1}{3}\right)^2} = \frac{4}{3} \sqrt{\frac{8}{9}} = \frac{4}{3} \cdot \frac{2\sqrt{2}}{3} \\ &= \frac{8\sqrt{2}}{9} \approx 1.26 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} \text{b. } A\left(\frac{1}{2}\right) &= 4 \cdot \frac{1}{2} \sqrt{1 - \left(\frac{1}{2}\right)^2} = 2\sqrt{\frac{3}{4}} = 2 \cdot \frac{\sqrt{3}}{2} \\ &= \sqrt{3} \approx 1.73 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} \text{c. } A\left(\frac{2}{3}\right) &= 4 \cdot \frac{2}{3} \sqrt{1 - \left(\frac{2}{3}\right)^2} = \frac{8}{3} \sqrt{\frac{5}{9}} = \frac{8}{3} \cdot \frac{\sqrt{5}}{3} \\ &= \frac{8\sqrt{5}}{9} \approx 1.99 \text{ ft}^2 \end{aligned}$$

$$113. \quad R(x) = \left(\frac{L}{P}\right)(x) = \frac{L(x)}{P(x)}$$

$$114. \quad T(x) = (V + P)(x) = V(x) + P(x)$$

$$115. \quad H(x) = (P \cdot I)(x) = P(x) \cdot I(x)$$

$$116. \quad N(x) = (I - T)(x) = I(x) - T(x)$$

$$\begin{aligned} 117. \quad \text{a. } P(x) &= R(x) - C(x) \\ &= (-1.2x^2 + 220x) - (0.05x^3 - 2x^2 + 65x + 500) \\ &= -1.2x^2 + 220x - 0.05x^3 + 2x^2 - 65x - 500 \\ &= -0.05x^3 + 0.8x^2 + 155x - 500 \end{aligned}$$

$$\begin{aligned} \text{b. } P(15) &= -0.05(15)^3 + 0.8(15)^2 + 155(15) - 500 \\ &= -168.75 + 180 + 2325 - 500 \\ &= \$1836.25 \end{aligned}$$

c. When 15 hundred smartphones are sold, the profit is \$1836.25.

118. a. P is the dependent variable; a is the independent variable

$$\begin{aligned} \text{b. } P(20) &= 0.027(20)^2 - 6.530(20) + 363.804 \\ &= 10.8 - 130.6 + 363.804 \\ &= 244.004 \end{aligned}$$

In 2015 there are 244.004 million people who are 20 years of age or older.

$$\begin{aligned} \text{c. } P(0) &= 0.027(0)^2 - 6.530(0) + 363.804 \\ &= 363.804 \end{aligned}$$

In 2015 there are 363.804 million people.

$$\begin{aligned} 119. \quad \text{a. } R(v) &= 2.2v; B(v) = 0.05v^2 + 0.4v - 15 \\ D(v) &= R(v) + B(v) \\ &= 2.2v + 0.05v^2 + 0.4v - 15 \\ &= 0.05v^2 + 2.6v - 15 \end{aligned}$$

$$\begin{aligned} \text{b. } D(60) &= 0.05(60)^2 + 2.6(60) - 15 \\ &= 180 + 156 - 15 \\ &= 321 \end{aligned}$$

- c. The car will need 321 feet to stop once the impediment is observed.

120. a. $h(x) = 2x$

$$h(a+b) = 2(a+b) = 2a+2b \\ = h(a) + h(b)$$

$$h(x) = 2x \text{ has the property.}$$

b. $g(x) = x^2$

$$g(a+b) = (a+b)^2 = a^2 + 2ab + b^2$$

Since

$$a^2 + 2ab + b^2 \neq a^2 + b^2 = g(a) + g(b),$$

$$g(x) = x^2 \text{ does not have the property.}$$

c. $F(x) = 5x - 2$

$$F(a+b) = 5(a+b) - 2 = 5a + 5b - 2$$

Since

$$5a + 5b - 2 \neq 5a - 2 + 5b - 2 = F(a) + F(b),$$

$$F(x) = 5x - 2 \text{ does not have the property.}$$

d. $G(x) = \frac{1}{x}$

$$G(a+b) = \frac{1}{a+b} \neq \frac{1}{a} + \frac{1}{b} = G(a) + G(b)$$

$$G(x) = \frac{1}{x} \text{ does not have the property.}$$

$$\begin{aligned} 121. \quad \frac{f(x+h) - f(x)}{h} &= \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} \\ &= \frac{(x+h)^{1/3} - x^{1/3}}{h} \\ &= \frac{(x+h)^{1/3} - x^{1/3}}{h} \cdot \frac{(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}}{(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}} \\ &= \frac{h}{(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}} \\ &= \frac{x+h-x}{h \left[(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3} \right]} = \frac{h}{h \left[(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3} \right]} \\ &= \frac{1}{(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}} \end{aligned}$$

122. $f\left(\frac{x+4}{5x-4}\right) = 3x^2 - 2$

Solve $\frac{x+4}{5x-4} = 1$.

$$\frac{x+4}{5x-4} = 1$$

$$x+4 = 5x-4$$

$$x = 2$$

$$\text{Therefore, } f(1) = 3(2)^2 - 2 = 10$$

123. We need $\frac{x^2+1}{7-|3x-1|} \geq 0$. Since $x^2+1 > 0$ for all

real numbers x , we need $7 - |3x-1| > 0$.

$$7 - |3x-1| > 0$$

$$|3x-1| < 7$$

$$-7 < 3x-1 < 7$$

$$-2 < x < \frac{8}{3}$$

The domain of f is $\left\{x \mid -2 < x < \frac{8}{3}\right\}$, or $\left(-2, \frac{8}{3}\right)$ in interval notation.

Chapter 2: Functions and Their Graphs

124. No. The domain of f is $\{x \mid x \text{ is any real number}\}$, but the domain of g is $\{x \mid x \neq -1\}$.

125. $\frac{3x - x^3}{(\text{your age})}$

126. Answers will vary.

127. $(x+12)^2 + y^2 = 16$

x-intercept ($y=0$):

$$(x+12)^2 + 0^2 = 16$$

$$(x+12)^2 = 16$$

$$(x+12) = \pm 4$$

$$x = -12 \pm 4$$

$$x = -16, x = -8$$

$$(-16, 0), (-8, 0)$$

y-intercept ($x=0$):

$$(0+12)^2 + y^2 = 16$$

$$(12)^2 + y^2 = 16$$

$$y^2 = 16 - 144 = -128$$

There are no real solutions so there are no y-intercepts.

Symmetry: $(x+12)^2 + (-y)^2 = 16$

$$(x+12)^2 + y^2 = 16$$

This shows x-axis symmetry.

128. $y = 3x^2 - 8\sqrt{x}$

$$y = 3(-1)^2 - 8\sqrt{-1}$$

There is no solution so $(-1, -5)$ is NOT a solution.

$$y = 3x^2 - 8\sqrt{x}$$

$$y = 3(4)^2 - 8\sqrt{4}$$

$$= 48 - 16 = 32$$

So $(4, 32)$ is a solution.

$$y = 3x^2 - 8\sqrt{x}$$

$$y = 3(9)^2 - 8\sqrt{9}$$

$$= 243 - 24 = 219 \neq 171$$

So $(9, 171)$ is NOT a solution.

129. Let x represent the amount of the 7% fat hamburger added.

% fat	tot. amt.	amt. of fat
20%	12	$(0.20)(12)$
7%	x	$(0.07)(x)$
15%	$12 + x$	$(0.15)(12 + x)$

$$(0.20)(12) + (0.07)(x) = (0.15)(12 + x)$$

$$2.4 + 0.07x = 1.8 + 0.15x$$

$$0.6 = .08x$$

$$x = 7.5$$

7.5 lbs. of the 7% fat hamburger must be added, producing 19.5 lbs. of the 15% fat hamburger.

130. $x^3 - 9x = 2x^2 - 18$

$$x^3 - 2x^2 - 9x + 18 = 0$$

$$(x^3 - 2x^2) - (9x - 18) = 0$$

$$x^2(x - 2) - 9(x - 2) = 0$$

$$(x^2 - 9)(x - 2) = 0$$

$$(x - 3)(x + 3)(x - 2) = 0$$

$$(x - 3) = 0 \text{ or } (x + 3) = 0 \text{ or } (x - 2) = 0$$

$$x = 3, x = -3, x = 2$$

The solution set is: $\{3, -3, 2\}$

131. $a + bx = ac + d$

$$a - ac = d - bx$$

$$a(1 - c) = d - bx$$

$$a = \frac{d - bx}{1 - c}$$

132. $r = kd^2$

$$0.4 = k(0.6)^2$$

$$\frac{10}{9} = k$$

Thus,

$$r = \frac{10}{9}(1.5)^2$$

$$= 2.5 \text{ kg} \cdot m^2$$

133. $3x - 10y = 12$

$$-10y = -3x + 12$$

$$y = \frac{3}{10}x - \frac{6}{5}$$

The slope of the line is $\frac{3}{10}$. The slope of a

perpendicular line would be $-\frac{10}{3}$.

$$\begin{aligned} 134. \frac{(4x^2 - 7) \cdot 3 - (3x + 5) \cdot 8x}{(4x^2 - 7)^2} &= \\ \frac{12x^2 - 21 - (24x^2 + 40x)}{(4x^2 - 7)^2} &= \\ \frac{12x^2 - 21 - 24x^2 - 40x}{(4x^2 - 7)^2} &= \frac{-12x^2 - 40x - 21}{(4x^2 - 7)^2} \\ &= -\frac{12x^2 + 40x + 21}{(4x^2 - 7)^2} \end{aligned}$$

135. Add the powers of x to obtain a degree of 7.

Section 2.2

1. $x^2 + 4y^2 = 16$

x -intercepts:

$$x^2 + 4(0)^2 = 16$$

$$x^2 = 16$$

$$x = \pm 4 \Rightarrow (-4, 0), (4, 0)$$

y -intercepts:

$$(0)^2 + 4y^2 = 16$$

$$4y^2 = 16$$

$$y^2 = 4$$

$$y = \pm 2 \Rightarrow (0, -2), (0, 2)$$

2. False; $x = 2y - 2$

$$-2 = 2y - 2$$

$$0 = 2y$$

$$0 = y$$

The point $(-2, 0)$ is on the graph.

3. vertical

4. $f(5) = -3$

5. $f(x) = ax^2 + 4$

$$a(-1)^2 + 4 = 2 \Rightarrow a = -2$$

6. False. The graph must pass the Vertical-Line Test in order to be the graph of a function.

7. False; e.g. $y = \frac{1}{x}$.

8. True

9. c

10. a

11. a. $f(0) = 3$ since $(0, 3)$ is on the graph.
 $f(-6) = -3$ since $(-6, -3)$ is on the graph.

b. $f(6) = 0$ since $(6, 0)$ is on the graph.
 $f(11) = 1$ since $(11, 1)$ is on the graph.

c. $f(3)$ is positive since $f(3) \approx 3.7$.

d. $f(-4)$ is negative since $f(-4) \approx -1$.

e. $f(x) = 0$ when $x = -3$, $x = 6$, and $x = 10$.

f. $f(x) > 0$ when $-3 < x < 6$, and $10 < x \leq 11$.

g. The domain of f is $\{x \mid -6 \leq x \leq 11\}$ or $[-6, 11]$.

h. The range of f is $\{y \mid -3 \leq y \leq 4\}$ or $[-3, 4]$.

i. The x -intercepts are -3 , 6 , and 10 .

j. The y -intercept is 3 .

k. The line $y = \frac{1}{2}$ intersects the graph 3 times.

l. The line $x = 5$ intersects the graph 1 time.

m. $f(x) = 3$ when $x = 0$ and $x = 4$.

n. $f(x) = -2$ when $x = -5$ and $x = 8$.

12. a. $f(0) = 0$ since $(0, 0)$ is on the graph.
 $f(6) = 0$ since $(6, 0)$ is on the graph.

Chapter 2: Functions and Their Graphs

- b. $f(2) = -2$ since $(2, -2)$ is on the graph.
 $f(-2) = 1$ since $(-2, 1)$ is on the graph.
- c. $f(3)$ is negative since $f(3) \approx -1$.
- d. $f(-1)$ is positive since $f(-1) \approx 1.0$.
- e. $f(x) = 0$ when $x = 0, x = 4$, and $x = 6$.
- f. $f(x) < 0$ when $0 < x < 4$.
- g. The domain of f is $\{x \mid -4 \leq x \leq 6\}$ or $[-4, 6]$.
- h. The range of f is $\{y \mid -2 \leq y \leq 3\}$ or $[-2, 3]$.
- i. The x -intercepts are 0, 4, and 6.
- j. The y -intercept is 0.
- k. The line $y = -1$ intersects the graph 2 times.
- l. The line $x = 1$ intersects the graph 1 time.
- m. $f(x) = 3$ when $x = 5$.
- n. $f(x) = -2$ when $x = 2$.
13. Not a function since vertical lines will intersect the graph in more than one point.
- a. Domain: $\{x \mid x \leq -1 \text{ or } x \geq 1\}$;
Range: $\{y \mid y \text{ is any real number}\}$
- b. Intercepts: $(-1, 0), (1, 0)$
- c. Symmetry about the x -axis, y -axis and the origin
14. Function
- a. Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y > 0\}$
- b. Intercepts: $(0, 1)$
- c. None
15. Function
- a. Domain: $\{x \mid -\pi \leq x \leq \pi\}$;
Range: $\{y \mid -1 \leq y \leq 1\}$
- b. Intercepts: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right), (0, 1)$
- c. Symmetry about y -axis.
16. Function
- a. Domain: $\{x \mid -\pi \leq x \leq \pi\}$;
Range: $\{y \mid -1 \leq y \leq 1\}$
- b. Intercepts: $(-\pi, 0), (\pi, 0), (0, 0)$
- c. Symmetry about the origin.
17. Not a function since vertical lines will intersect the graph in more than one point.
- a. Domain: $\{x \mid x \leq 0\}$;
Range: $\{y \mid y \text{ is any real number}\}$
- b. Intercepts: $(0, 0)$
- c. Symmetry about the x -axis
18. Not a function since vertical lines will intersect the graph in more than one point.
- a. Domain: $\{x \mid -2 \leq x \leq 2\}$;
Range: $\{y \mid -2 \leq y \leq 2\}$
- b. Intercepts: $(-2, 0), (2, 0), (0, -2), (0, 2)$
- c. Symmetry about the x -axis, y -axis and the origin
19. Function
- a. Domain: $\{x \mid 0 < x < 3\}$;
Range: $\{y \mid y < 2\}$
- b. Intercepts: $(1, 0)$
- c. None
20. Function
- a. Domain: $\{x \mid 0 \leq x < 4\}$;
Range: $\{y \mid 0 \leq y < 3\}$
- b. Intercepts: $(0, 0)$
- c. None
21. Function
- a. Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y \leq 2\}$
- b. Intercepts: $(-3, 0), (3, 0), (0, 2)$

- c. Symmetry about y -axis.
22. Function
- Domain: $\{x \mid x \geq -3\}$;
Range: $\{y \mid y \geq 0\}$
 - Intercepts: $(-3, 0)$, $(2, 0)$, $(0, 2)$
 - None
23. Function
- Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y \geq -3\}$
 - Intercepts: $(1, 0)$, $(3, 0)$, $(0, 9)$
 - None
24. Function
- Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y \leq 5\}$
 - Intercepts: $(-1, 0)$, $(2, 0)$, $(0, 4)$
 - None
25. $f(x) = 3x^2 + x - 2$
- $f(1) = 3(1)^2 + (1) - 2 = 2$
The point $(-1, 2)$ is on the graph of f .
 - $f(-2) = 3(-2)^2 + (-2) - 2 = 8$
The point $(-2, 8)$ is on the graph of f .
 - Solve for x :
 $-2 = 3x^2 + x - 2$
 $0 = 3x^2 + x$
 $0 = x(3x + 1) \Rightarrow x = 0, x = -\frac{1}{3}$
 $(0, -2)$ and $(-\frac{1}{3}, -2)$ are on the graph of f .
 - The domain of f is $\{x \mid x \text{ is any real number}\}$.
 - x -intercepts:
 $f(x) = 0 \Rightarrow 3x^2 + x - 2 = 0$
 $(3x - 2)(x + 1) = 0 \Rightarrow x = \frac{2}{3}, x = -1$
 - y -intercept: $f(0) = 3(0)^2 + 0 - 2 = -2$
26. $f(x) = -3x^2 + 5x$
- $f(-1) = -3(-1)^2 + 5(-1) = -8 \neq 2$
The point $(-1, 2)$ is not on the graph of f .
 - $f(-2) = -3(-2)^2 + 5(-2) = -22$
The point $(-2, -22)$ is on the graph of f .
 - Solve for x :
 $-2 = -3x^2 + 5x \Rightarrow 3x^2 - 5x - 2 = 0$
 $(3x + 1)(x - 2) = 0 \Rightarrow x = -\frac{1}{3}, x = 2$
 $(2, -2)$ and $(-\frac{1}{3}, -2)$ on the graph of f .
 - The domain of f is $\{x \mid x \text{ is any real number}\}$.
 - x -intercepts:
 $f(x) = 0 \Rightarrow -3x^2 + 5x = 0$
 $x(-3x + 5) = 0 \Rightarrow x = 0, x = \frac{5}{3}$
 - y -intercept:
 $f(0) = -3(0)^2 + 5(0) = 0$
27. $f(x) = \frac{x+2}{x-6}$
- $f(3) = \frac{3+2}{3-6} = -\frac{5}{3} \neq 14$
The point $(3, 14)$ is not on the graph of f .
 - $f(4) = \frac{4+2}{4-6} = \frac{6}{-2} = -3$
The point $(4, -3)$ is on the graph of f .
 - Solve for x :
 $2 = \frac{x+2}{x-6}$
 $2x - 12 = x + 2$
 $x = 14$
 $(14, 2)$ is a point on the graph of f .
 - The domain of f is $\{x \mid x \neq 6\}$.
 - x -intercepts:
 $f(x) = 0 \Rightarrow \frac{x+2}{x-6} = 0$
 $x + 2 = 0 \Rightarrow x = -2$

Chapter 2: Functions and Their Graphs

f. y-intercept: $f(0) = \frac{0+2}{0-6} = -\frac{1}{3}$

28. $f(x) = \frac{x^2+2}{x+4}$

a. $f(1) = \frac{1^2+2}{1+4} = \frac{3}{5}$

The point $\left(1, \frac{3}{5}\right)$ is on the graph of f .

b. $f(0) = \frac{0^2+2}{0+4} = \frac{2}{4} = \frac{1}{2}$

The point $\left(0, \frac{1}{2}\right)$ is on the graph of f .

c. Solve for x :

$$\frac{1}{2} = \frac{x^2+2}{x+4} \Rightarrow x+4 = 2x^2+4$$

$$0 = 2x^2 - x$$

$$x(2x-1) = 0 \Rightarrow x = 0 \text{ or } x = \frac{1}{2}$$

$\left(0, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \frac{1}{2}\right)$ are on the graph of f .

d. The domain of f is $\{x \mid x \neq -4\}$.

e. x-intercepts:

$$f(x) = 0 \Rightarrow \frac{x^2+2}{x+4} = 0 \Rightarrow x^2+2 = 0$$

This is impossible, so there are no x-intercepts.

f. y-intercept:

$$f(0) = \frac{0^2+2}{0+4} = \frac{2}{4} = \frac{1}{2}$$

29. $f(x) = \frac{12x^4}{x^2+1}$

a. $f(-1) = \frac{12(-1)^4}{(-1)^2+1} = \frac{12}{2} = 6$

The point $(-1, 6)$ is on the graph of f .

b. $f(3) = \frac{12(3)^4}{(3)^2+1} = \frac{972}{10} = \frac{486}{5}$

The point $\left(3, \frac{486}{5}\right)$ is on the graph of f .

c. Solve for x :

$$1 = \frac{12x^4}{x^2+1}$$

$$x^2+1 = 12x^4$$

$$12x^4 - x^2 - 1 = 0$$

$$(3x^2-1)(4x^2+1) = 0$$

$$3x^2-1 = 0 \Rightarrow x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$$

$\left(-\frac{\sqrt{3}}{3}, 1\right), \left(\frac{\sqrt{3}}{3}, 1\right)$ are on the graph of f .

d. The domain of f is $\{x \mid x \text{ is any real number}\}$.

e. x-intercept:

$$f(x) = 0 \Rightarrow \frac{12x^4}{x^2+1} = 0$$

$$12x^4 = 0 \Rightarrow x = 0$$

f. y-intercept:

$$f(0) = \frac{12(0)^4}{0^2+1} = \frac{0}{0+1} = 0$$

30. $f(x) = \frac{2x}{x-2}$

a. $f\left(\frac{1}{2}\right) = \frac{2\left(\frac{1}{2}\right)}{\frac{1}{2}-2} = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}$

The point $\left(\frac{1}{2}, -\frac{2}{3}\right)$ is on the graph of f .

b. $f(4) = \frac{2(4)}{4-2} = \frac{8}{2} = 4$

The point $(4, 4)$ is on the graph of f .

c. Solve for x :

$$1 = \frac{2x}{x-2} \Rightarrow x-2 = 2x \Rightarrow -2 = x$$

$(-2, 1)$ is a point on the graph of f .

d. The domain of f is $\{x \mid x \neq 2\}$.

e. x-intercept:

$$f(x) = 0 \Rightarrow \frac{2x}{x-2} = 0 \Rightarrow 2x = 0 \Rightarrow x = 0$$

f. y-intercept: $f(0) = \frac{0}{0-2} = 0$

Section 2.2: The Graph of a Function

31. a. $(f + g)(2) = f(2) + g(2) = 2 + 1 = 3$
 b. $(f + g)(4) = f(4) + g(4) = 1 + (-3) = -2$
 c. $(f - g)(6) = f(6) - g(6) = 0 - 1 = -1$
 d. $(g - f)(6) = g(6) - f(6) = 1 - 0 = 1$
 e. $(f \cdot g)(2) = f(2) \cdot g(2) = 2(1) = 2$
 f. $\left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{1}{-3} = -\frac{1}{3}$

32. $h(x) = -\frac{136x^2}{v^2} + 2.7x + 3.5$

- a. We want $h(15) = 10$.

$$\begin{aligned} -\frac{136(15)^2}{v^2} + 2.7(15) + 3.5 &= 10 \\ -\frac{30,600}{v^2} &= -34 \\ v^2 &= 900 \\ v &= 30 \text{ ft/sec} \end{aligned}$$

The ball needs to be thrown with an initial velocity of 30 feet per second.

b. $h(x) = -\frac{126x^2}{30^2} + 2.7x + 3.5$
 which simplifies to
 $h(x) = -\frac{34}{225}x^2 + 2.7x + 3.5$

- c. Using the velocity from part (b),

$$h(9) = -\frac{34}{225}(9)^2 + 2.7(9) + 3.5 = 15.56 \text{ ft}$$

The ball will be 15.56 feet above the floor when it has traveled 9 feet in front of the foul line.

- d. Select several values for x and use these to find the corresponding values for h . Use the results to form ordered pairs (x, h) . Plot the points and connect with a smooth curve.

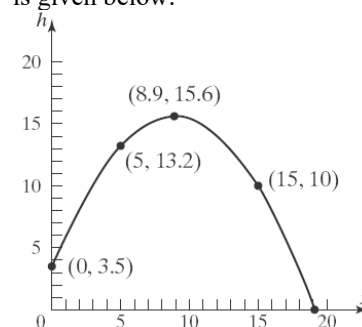
$$h(0) = -\frac{34}{225}(0)^2 + 2.7(0) + 3.5 = 3.5 \text{ ft}$$

$$h(5) = -\frac{34}{225}(5)^2 + 2.7(5) + 3.5 \approx 13.2 \text{ ft}$$

$$h(15) = -\frac{34}{225}(15)^2 + 2.7(15) + 3.5 \approx 10 \text{ ft}$$

Thus, some points on the graph are $(0, 3.5)$,

$(5, 13.2)$, and $(15, 10)$. The complete graph is given below.



33. $h(x) = -\frac{44x^2}{v^2} + x + 6$

a.
$$\begin{aligned} h(8) &= -\frac{44(8)^2}{28^2} + (8) + 6 \\ &= -\frac{2816}{784} + 14 \\ &\approx 10.4 \text{ feet} \end{aligned}$$

b.
$$\begin{aligned} h(12) &= -\frac{44(12)^2}{28^2} + (12) + 6 \\ &= -\frac{6336}{784} + 18 \\ &\approx 9.9 \text{ feet} \end{aligned}$$

- c. From part (a) we know the point $(8, 10.4)$ is on the graph and from part (b) we know the point $(12, 9.9)$ is on the graph. We could evaluate the function at several more values of x (e.g. $x = 0$, $x = 15$, and $x = 20$) to obtain additional points.

$$h(0) = -\frac{44(0)^2}{28^2} + (0) + 6 = 6$$

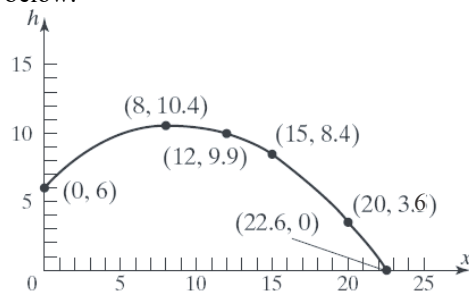
$$h(15) = -\frac{44(15)^2}{28^2} + (15) + 6 \approx 8.4$$

$$h(20) = -\frac{44(20)^2}{28^2} + (20) + 6 \approx 3.6$$

Some additional points are $(0, 6)$, $(15, 8.4)$ and $(20, 3.6)$. The complete graph is given

Chapter 2: Functions and Their Graphs

below.



d. $h(15) = -\frac{44(15)^2}{28^2} + (15) + 6 \approx 8.4$ feet

No; when the ball is 15 feet in front of the foul line, it will be below the hoop. Therefore it cannot go through the hoop.

In order for the ball to pass through the hoop, we need to have $h(15) = 10$.

$$10 = -\frac{44(15)^2}{v^2} + (15) + 6$$

$$-11 = -\frac{44(15)^2}{v^2}$$

$$v^2 = 4(225)$$

$$v^2 = 900$$

$$v = 30 \text{ ft/sec}$$

The ball must be shot with an initial velocity of 30 feet per second in order to go through the hoop.

34. $A(x) = 4x\sqrt{1-x^2}$

- a. Domain of $A(x) = 4x\sqrt{1-x^2}$; we know that x must be greater than or equal to zero, since x represents a length. We also need $1-x^2 \geq 0$, since this expression occurs under a square root. In fact, to avoid Area = 0, we require $x > 0$ and $1-x^2 > 0$.

Solve: $1-x^2 > 0$

$$(1+x)(1-x) > 0$$

Case1: $1+x > 0$ and $1-x > 0$

$$x > -1 \text{ and } x < 1$$

$$\text{(i.e. } -1 < x < 1\text{)}$$

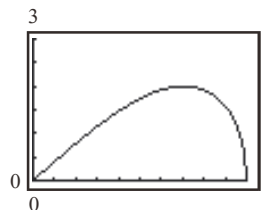
Case2: $1+x < 0$ and $1-x < 0$

$$x < -1 \text{ and } x > 1$$

(which is impossible)

Therefore the domain of A is $\{x \mid 0 < x < 1\}$.

b. Graphing $A(x) = 4x\sqrt{1-x^2}$



- c. When $x = 0.7$ feet, the cross-sectional area is maximized at approximately 1.9996 square feet. Therefore, the length of the base of the beam should be 1.4 feet in order to maximize the cross-sectional area.

X	Y1
0.3	1.1447
0.4	1.4664
0.5	1.7321
0.6	1.92
0.7	1.9996
0.8	1.92
0.9	1.5692

35. $h(x) = \frac{-32x^2}{130^2} + x$

a. $h(100) = \frac{-32(100)^2}{130^2} + 100$
 $= \frac{-320,000}{16,900} + 100 \approx 81.07$ feet

b. $h(300) = \frac{-32(300)^2}{130^2} + 300$
 $= \frac{-2,880,000}{16,900} + 300 \approx 129.59$ feet

c. $h(500) = \frac{-32(500)^2}{130^2} + 500$
 $= \frac{-8,000,000}{16,900} + 500 \approx 26.63$ feet

Section 2.2: The Graph of a Function

The ball is about 26.63 feet high after it has traveled 500 feet.

d. Solving $h(x) = \frac{-32x^2}{130^2} + x = 0$

$$\frac{-32x^2}{130^2} + x = 0$$

$$x \left(\frac{-32x}{130^2} + 1 \right) = 0$$

$$x = 0 \quad \text{or} \quad \frac{-32x}{130^2} + 1 = 0$$

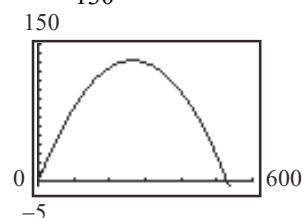
$$1 = \frac{32x}{130^2}$$

$$130^2 = 32x$$

$$x = \frac{130^2}{32} = 528.13 \text{ feet}$$

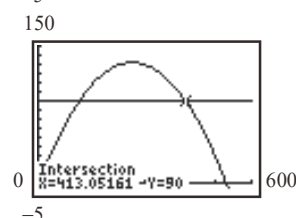
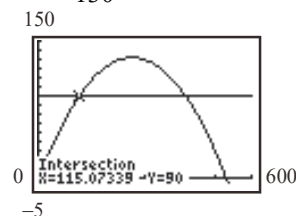
Therefore, the golf ball travels 528.13 feet.

e. $y_1 = \frac{-32x^2}{130^2} + x$



f. Use INTERSECT on the graphs of

$$y_1 = \frac{-32x^2}{130^2} + x \quad \text{and} \quad y_2 = 90.$$



The ball reaches a height of 90 feet twice. The first time is when the ball has traveled approximately 115.07 feet, and the second time is when the ball has traveled about 413.05 feet.

- g. The ball travels approximately 275 feet before it reaches its maximum height of approximately 131.8 feet.

X	Y1
200	124.26
225	129.14
250	131.66
275	131.8
300	129.59
325	125
350	118.05

- h. The ball travels approximately 264 feet before it reaches its maximum height of approximately 132.03 feet.

X	Y1
260	132
261	132.01
262	132.02
263	132.03
264	132.03
265	132.03
266	132.02

X	Y1
260	132
261	132.01
262	132.02
263	132.03
264	132.03
265	132.03
266	132.02

X	Y1
260	132
261	132.01
262	132.02
263	132.03
264	132.03
265	132.03
266	132.02

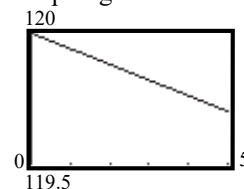
36. $W(h) = m \left(\frac{4000}{4000 + h} \right)^2$

- a. $h = 14110 \text{ feet} \approx 2.67 \text{ miles};$

$$W(2.67) = 120 \left(\frac{4000}{4000 + 2.67} \right)^2 \approx 119.84$$

On Pike's Peak, Amy will weigh about 119.84 pounds.

- b. Graphing:



- c. Create a TABLE:

X	Y1
0	120
.5	119.97
1	119.94
1.5	119.91
2	119.88
2.5	119.85
3	119.82

X	Y1
3.5	119.8
4	119.78
4.5	119.76
5	119.74

The weight W will vary from 120 pounds to about 119.7 pounds.

- d. By refining the table, Amy will weigh 119.95 lbs at a height of about 0.83 miles

Chapter 2: Functions and Their Graphs

(4382 feet).

X	Y1	
.5	119.97	
.6	119.96	
.7	119.95	
.8	119.95	
.9	119.95	
1	119.94	
1.1	119.93	
X=.8		

X	Y1	
.8	119.95	
.81	119.95	
.82	119.95	
.83	119.95	
.84	119.95	
.85	119.95	
.86	119.95	
Y1=119.950215496		

- e. Yes, 4382 feet is reasonable.

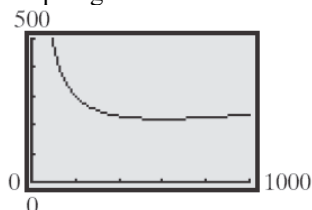
37. $C(x) = 100 + \frac{x}{10} + \frac{36000}{x}$

a. $C(480) = 100 + \frac{480}{10} + \frac{36000}{480}$
 $= \$223$

$C(600) = 100 + \frac{600}{10} + \frac{36000}{600}$
 $= \$220$

- b. $\{x \mid x > 0\}$

- c. Graphing:



- d. TblStart = 0; ΔTbl = 50

X	Y1	
0	ERROR	
50	825	
100	470	
150	355	
200	300	
250	269	
300	250	
Y1=100+X/10+36000/X		

- e. The cost per passenger is minimized to about \$220 when the ground speed is roughly 600 miles per hour.

X	Y1	
450	225	
500	222	
550	220.45	
600	220	
650	220.38	
700	221.43	
750	223	
X=600		

38. a. $C(0) = 5000$

This represents the fixed overhead costs. That is, the company will incur costs of \$5000 per day even if no computers are manufactured.

b. $C(10) = 19,000$

It costs the company \$19,000 to produce 10 computers in a day.

c. $C(50) = 51,000$

It costs the company \$51,000 to produce 50 computers in a day.

- d. The domain is $\{q \mid 0 \leq q \leq 80\}$. This indicates that production capacity is limited to 80 computers in a day.

- e. The graph is curved down and rises slowly at first. As production increases, the graph rises more quickly and changes to being curved up.

- f. The inflection point is where the graph changes from being curved down to being curved up.

39. a. $C(0) = \$50$

It costs \$50 if you use 0 gigabytes.

b. $C(5) = \$50$

It costs \$50 if you use 5 gigabytes.

c. $C(15) = \$150$

It costs \$90 if you use 15 gigabytes.

- d. The domain is $\{g \mid 0 \leq g \leq 30\}$. This indicates that there are at most 30 gigabytes in a month.

- e. The graph is flat at first and then rises in a straight line.

40. $g(-2) = 5 = f(-2) = 4$

Since $f(-2) = (-2)^2 - 4(-2) + c$
 $= 12 + c$

we have

$$\frac{12+c}{3} - 4 = 5$$

$$\frac{12+c}{3} = 9$$

$$12+c = 27$$

$$c = 15$$

$$f(3) = 3^2 - 4 \cdot 3 + 15 = 12$$

41. $g(5) = 5^2 + n = 25 + n$
 $f(g(5)) = f(25 + n) = \sqrt{25 + n} + 2 = 4$
 so, $\sqrt{25 + n} = 2$
 $25 + n = 4$
 $n = -21$
 $g(n) = n^2 + n = (-21)^2 + (-21) = 420$.

42. Answers will vary. From a graph, the domain can be found by visually locating the x -values for which the graph is defined. The range can be found in a similar fashion by visually locating the y -values for which the function is defined.

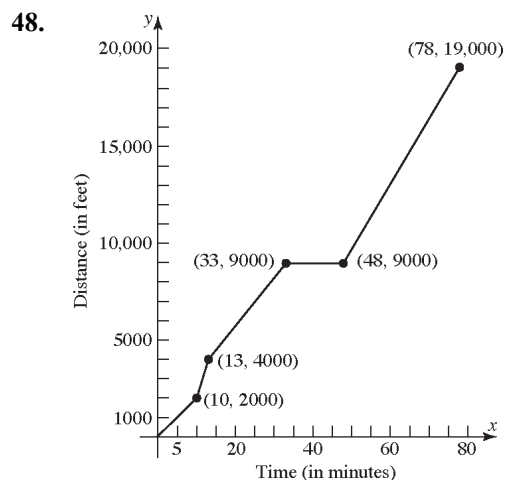
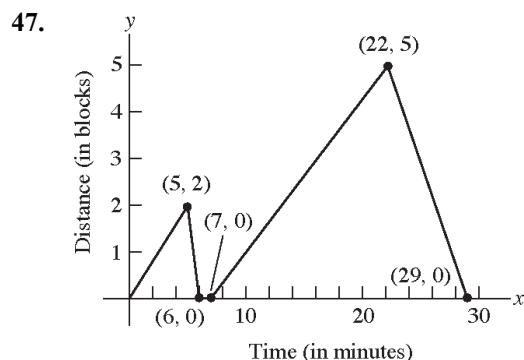
If an equation is given, the domain can be found by locating any restricted values and removing them from the set of real numbers. The range can be found by using known properties of the graph of the equation, or estimated by means of a table of values.

43. The graph of a function can have any number of x -intercepts. The graph of a function can have at most one y -intercept (otherwise the graph would fail the Vertical-Line Test).

44. Yes, the graph of a single point is the graph of a function since it would pass the Vertical-Line Test. The equation of such a function would be something like the following: $f(x) = 2$, where $x = 7$.

45. (a) III; (b) IV; (c) I; (d) V; (e) II

46. (a) II; (b) V; (c) IV; (d) III; (e) I



49. a. 2 hours elapsed; Sobia was between 0 and 3 miles from home.
 b. 0.5 hours elapsed; Sobia was 3 miles from home.
 c. 0.3 hours elapsed; Sobia was between 0 and 3 miles from home.
 d. 0.2 hours elapsed; Sobia was at home.
 e. 0.9 hours elapsed; Sobia was between 0 and 2.8 miles from home.
 f. 0.3 hours elapsed; Sobia was 2.8 miles from home.
 g. 1.1 hours elapsed; Sobia was between 0 and 2.8 miles from home.
 h. The farthest distance Sobia is from home is 3 miles.
 i. Sobia returned home 2 times.
50. a. Michael travels fastest between 7 and 7.4 minutes. That is, $(7, 7.4)$.
 b. Michael's speed is zero between 4.2 and 6 minutes. That is, $(4.2, 6)$.
 c. Between 0 and 2 minutes, Michael's speed increased from 0 to 30 miles/hour.
 d. Between 4.2 and 6 minutes, Michael was stopped (i.e., his speed was 0 miles/hour).
 e. Between 7 and 7.4 minutes, Michael was traveling at a steady rate of 50 miles/hour.
 f. Michael's speed is constant between 2 and 4 minutes, between 4.2 and 6 minutes, between 7 and 7.4 minutes, and between 7.6 and 8 minutes. That is, on the intervals $(2, 4)$, $(4.2, 6)$, $(7, 7.4)$, and $(7.6, 8)$.

Chapter 2: Functions and Their Graphs

51. Answers (graphs) will vary. Points of the form $(5, y)$ and of the form $(x, 0)$ cannot be on the graph of the function.
52. The only such function is $f(x) = 0$ because it is the only function for which $f(x) = -f(x)$. Any other such graph would fail the Vertical-Line Test.
53. Answers may vary.
54.
$$\begin{aligned} f(x-2) &= -(x-2)^2 + (x-2) - 3 \\ &= -(x^2 - 4x + 4) + x - 2 - 3 \\ &= -x^2 + 5x - 9 \end{aligned}$$
55.
$$\begin{aligned} d &= \sqrt{(1-3)^2 + (0-(-6))^2} \\ &= \sqrt{(-2)^2 + (-6)^2} \\ &= \sqrt{4+36} = \sqrt{40} = 2\sqrt{10} \end{aligned}$$
56.
$$\begin{aligned} y-4 &= \frac{2}{3}(x-(-6)) \\ y-4 &= \frac{2}{3}x+4 \\ y &= \frac{2}{3}x+8 \end{aligned}$$
57. Since the function contains a cube root then the domain is:
 $(-\infty, \infty)$
58.
$$\left[\frac{1}{2}(-12) \right]^2 = 36$$
59.
$$\frac{\frac{\sqrt{x}-\sqrt{6}}{x-6} \cdot \frac{\sqrt{x}+\sqrt{6}}{\sqrt{x}+\sqrt{6}}}{\frac{x-6}{(x-6)(\sqrt{x}+\sqrt{6})}} = \frac{1}{\sqrt{x}+\sqrt{6}}$$
60. The car traveling north travels a distance of $25t$ and the car traveling west travels a distance of $35t$ where t is the time of travel. Using the Pythagorean we have:
$$\begin{aligned} 40^2 &= (35t)^2 + (25t)^2 \\ 1600 &= 1225t^2 + 625t^2 \\ 1600 &= 1850t^2 \\ t^2 &= 0.8649 \\ t &= 0.93 \text{ hours} \end{aligned}$$

Converting to minutes we have
 $0.93(60) = 55.8 \text{ minutes}$
61.
$$\begin{aligned} 3x+4 &\leq 7 \quad \text{and} \quad 5-2x < 13 \\ 3x &\leq 3 \quad \quad -2x < 8 \\ x &\leq 1 \quad \quad x > -4 \end{aligned}$$

The solution set is $(-4, 1]$.
62.
$$\begin{aligned} (5x^2 - 7x + 2) - (8x - 10) \\ &= 5x^2 - 7x + 2 - 8x + 10 \\ &= 5x^2 - 15x + 12 \end{aligned}$$
63. $[-3, 10]$

Section 2.3

1. $2 < x < 5$
2.
$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{8-3}{3-(-2)} = \frac{5}{5} = 1$$
3. x -axis: $y \rightarrow -y$
$$\begin{aligned} (-y) &= 5x^2 - 1 \\ -y &= 5x^2 - 1 \\ y &= -5x^2 + 1 \quad \text{different} \end{aligned}$$

 y -axis: $x \rightarrow -x$
$$\begin{aligned} y &= 5(-x)^2 - 1 \\ y &= 5x^2 - 1 \quad \text{same} \end{aligned}$$

Section 2.3: Properties of Functions

origin: $x \rightarrow -x$ and $y \rightarrow -y$

$$(-y) = 5(-x)^2 - 1$$

$$-y = 5x^2 - 1$$

$$y = -5x^2 + 1 \text{ different}$$

The equation has symmetry with respect to the y -axis only.

4. $y - y_1 = m(x - x_1)$

$$y - (-2) = 5(x - 3)$$

$$y + 2 = 5(x - 3)$$

5. $y = x^2 - 9$

x -intercepts:

$$0 = x^2 - 9$$

$$x^2 = 9 \rightarrow x = \pm 3$$

y -intercept:

$$y = (0)^2 - 9 = -9$$

The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, -9)$.

6. increasing

7. even; odd

8. True

9. True

10. False; odd functions are symmetric with respect to the origin. Even functions are symmetric with respect to the y -axis.

11. c

12. d

13. Yes

14. No, it is increasing.

15. No

16. Yes

17. f is increasing on the intervals $[-8, -2]$, $[0, 2]$, $[5, 7]$.

18. f is decreasing on the intervals: $[-10, -8]$, $[-2, 0]$, $[2, 5]$.

19. Yes. The local maximum at $x = 2$ is 10.

20. No. There is a local minimum at $x = 5$; the local minimum is 0.

21. f has local maxima at $x = -2$ and $x = 2$. The local maxima are 6 and 10, respectively.

22. f has local minima at $x = -8$, $x = 0$ and $x = 5$. The local minima are -4 , 0, and 0, respectively.

23. f has absolute minimum of -4 at $x = -8$.

24. f has absolute maximum of 10 at $x = 2$.

25. a. Intercepts: $(-2, 0)$, $(2, 0)$, and $(0, 3)$.

b. Domain: $\{x \mid -4 \leq x \leq 4\}$ or $[-4, 4]$;

Range: $\{y \mid 0 \leq y \leq 3\}$ or $[0, 3]$.

c. Increasing: $[-2, 0]$ and $[2, 4]$;
Decreasing: $[-4, -2]$ and $[0, 2]$.

d. Since the graph is symmetric with respect to the y -axis, the function is even.

26. a. Intercepts: $(-1, 0)$, $(1, 0)$, and $(0, 2)$.

b. Domain: $\{x \mid -3 \leq x \leq 3\}$ or $[-3, 3]$;

Range: $\{y \mid 0 \leq y \leq 3\}$ or $[0, 3]$.

c. Increasing: $[-1, 0]$ and $[1, 3]$;
Decreasing: $[-3, -1]$ and $[0, 1]$.

d. Since the graph is symmetric with respect to the y -axis, the function is even.

27. a. Intercepts: $(0, 1)$.

b. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y > 0\}$ or $(0, \infty)$.

c. Increasing: $(-\infty, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the y -axis or the origin, the function is neither even nor odd.

28. a. Intercepts: $(1, 0)$.

b. Domain: $\{x \mid x > 0\}$ or $(0, \infty)$;

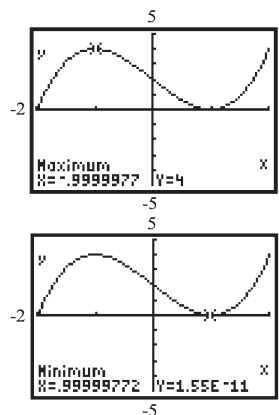
Range: $\{y \mid y \text{ is any real number}\}$.

c. Increasing: $[0, \infty)$; Decreasing: never.

Chapter 2: Functions and Their Graphs

- d. Since the graph is not symmetric with respect to the y -axis or the origin, the function is neither even nor odd.
29. a. Intercepts: $(-\pi, 0)$, $(\pi, 0)$, and $(0, 0)$.
 b. Domain: $\{x | -\pi \leq x \leq \pi\}$ or $[-\pi, \pi]$;
 Range: $\{y | -1 \leq y \leq 1\}$ or $[-1, 1]$.
 c. Increasing: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$;
 Decreasing: $\left[-\pi, -\frac{\pi}{2}\right]$ and $\left[\frac{\pi}{2}, \pi\right]$.
 d. Since the graph is symmetric with respect to the origin, the function is odd.
30. a. Intercepts: $\left(-\frac{\pi}{2}, 0\right)$, $\left(\frac{\pi}{2}, 0\right)$, and $(0, 1)$.
 b. Domain: $\{x | -\pi \leq x \leq \pi\}$ or $[-\pi, \pi]$;
 Range: $\{y | -1 \leq y \leq 1\}$ or $[-1, 1]$.
 c. Increasing: $[-\pi, 0]$; Decreasing: $[0, \pi]$.
 d. Since the graph is symmetric with respect to the y -axis, the function is even.
31. a. Intercepts: $\left(\frac{1}{3}, 0\right)$, $\left(\frac{5}{2}, 0\right)$, and $\left(0, \frac{1}{2}\right)$.
 b. Domain: $\{x | -3 \leq x \leq 3\}$ or $[-3, 3]$;
 Range: $\{y | -1 \leq y \leq 2\}$ or $[-1, 2]$.
 c. Increasing: $[2, 3]$; Decreasing: $[-1, 1]$;
 Constant: $[-3, -1]$ and $[1, 2]$.
 d. Since the graph is not symmetric with respect to the y -axis or the origin, the function is neither even nor odd.
32. a. Intercepts: $(-2.3, 0)$, $(3, 0)$, and $(0, 1)$.
 b. Domain: $\{x | -3 \leq x \leq 3\}$ or $[-3, 3]$;
 Range: $\{y | -2 \leq y \leq 2\}$ or $[-2, 2]$.
 c. Increasing: $[-3, -2]$ and $[0, 2]$;
 Decreasing: $[2, 3]$; Constant: $[-2, 0]$.
- d. Since the graph is not symmetric with respect to the y -axis or the origin, the function is neither even nor odd.
33. a. f has a local maximum value of 3 at $x = 0$.
 b. f has a local minimum value of 0 at both $x = -2$ and $x = 2$.
34. a. f has a local maximum value of 2 at $x = 0$.
 b. f has a local minimum value of 0 at both $x = -1$ and $x = 1$.
35. a. f has a local maximum value of 1 at $x = \frac{\pi}{2}$.
 b. f has a local minimum value of -1 at $x = -\frac{\pi}{2}$.
36. a. f has a local maximum value of 1 at $x = 0$.
 b. f has a local minimum value of -1 both at $x = -\pi$ and $x = \pi$.
37. $f(x) = 4x^3$
 $f(-x) = 4(-x)^3 = -4x^3 = -f(x)$
 Therefore, f is odd.
38. $f(x) = 2x^4 - x^2$
 $f(-x) = 2(-x)^4 - (-x)^2 = 2x^4 - x^2 = f(x)$
 Therefore, f is even.
39. $g(x) = 10 - x^2$
 $g(-x) = 10 - (-x)^2 = 10 - x^2 = g(x)$
 Therefore, g is even.
40. $h(x) = 3x^3 + 5$
 $h(-x) = 3(-x)^3 + 5 = -3x^3 + 5$
 h is neither even nor odd.
41. $F(x) = \sqrt[3]{4x}$
 $F(-x) = \sqrt[3]{-4x} = -\sqrt[3]{4x} = -F(x)$
 Therefore, F is odd.

42. $G(x) = \sqrt{x}$
 $G(-x) = \sqrt{-x}$
 G is neither even nor odd.
43. $f(x) = x + |x|$
 $f(-x) = -x + |-x| = -x + |x|$
 f is neither even nor odd.
44. $f(x) = \sqrt[3]{2x^2 + 1}$
 $f(-x) = \sqrt[3]{2(-x)^2 + 1} = \sqrt[3]{2x^2 + 1} = f(x)$
 Therefore, f is even.
45. $g(x) = \frac{1}{x^2 + 8}$
 $g(-x) = \frac{1}{(-x)^2 + 8} = \frac{1}{x^2 + 8} = g(x)$
 Therefore, g is even.
46. $h(x) = \frac{x}{x^2 - 1}$
 $h(-x) = \frac{-x}{(-x)^2 - 1} = \frac{-x}{x^2 - 1} = -h(x)$
 Therefore, h is odd.
47. $h(x) = \frac{-x^3}{3x^2 - 9}$
 $h(-x) = \frac{-(-x)^3}{3(-x)^2 - 9} = \frac{x^3}{3x^2 - 9} = -h(x)$
 Therefore, h is odd.
48. $F(x) = \frac{2x}{|x|}$
 $F(-x) = \frac{2(-x)}{|-x|} = \frac{-2x}{|x|} = -F(x)$
 Therefore, F is odd.
49. f has an absolute maximum of 4 at $x = 1$.
 f has an absolute minimum of 1 at $x = 5$.
 f has a local maximum value of 3 at $x = 3$.
 f has a local minimum value of 2 at $x = 2$.
50. f has an absolute maximum of 4 at $x = 4$.
 f has an absolute minimum of 0 at $x = 5$.
 f has a local maximum value of 4 at $x = 4$.
 f has a local minimum value of 1 at $x = 1$.
51. f has an absolute minimum of 1 at $x = 1$.
 f has an absolute maximum of 4 at $x = 3$.
 f has a local minimum value of 1 at $x = 1$.
 f has a local maximum value of 4 at $x = 3$.
52. f has an absolute minimum of 1 at $x = 0$.
 f has no absolute maximum.
 f has no local minimum.
 f has no local maximum.
53. f has an absolute minimum of 0 at $x = 0$.
 f has no absolute maximum.
 f has a local minimum value of 0 at $x = 0$.
 f has a local minimum value of 2 at $x = 3$.
 f has a local maximum value of 3 at $x = 2$.
54. f has an absolute maximum of 4 at $x = 2$.
 f has no absolute minimum.
 f has a local maximum value of 4 at $x = 2$.
 f has a local minimum value of 2 at $x = 0$.
55. f has no absolute maximum or minimum.
 f has no local maximum or minimum.
56. f has no absolute maximum or minimum.
 f has no local maximum or minimum.
57. $f(x) = x^3 - 3x + 2$ on the interval $(-2, 2)$
 Use MAXIMUM and MINIMUM on the graph of $y_1 = x^3 - 3x + 2$.



local maximum: $f(-1) = 4$

local minimum: $f(1) = 0$

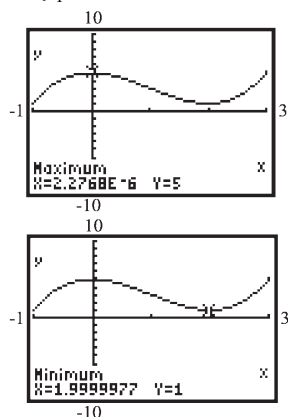
f is increasing on: $[-2, -1]$ and $[1, 2]$;

f is decreasing on: $[-1, 1]$

Chapter 2: Functions and Their Graphs

58. $f(x) = x^3 - 3x^2 + 5$ on the interval $(-1, 3)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = x^3 - 3x^2 + 5$.



local maximum: $f(0) = 5$

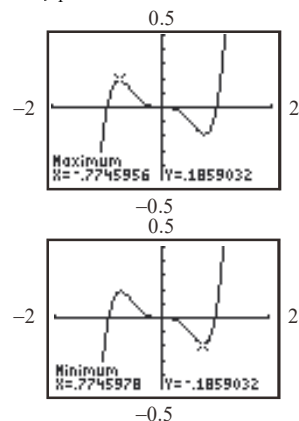
local minimum: $f(2) = 1$

f is increasing on: $[-1, 0]$ and $[2, 3]$;

f is decreasing on: $[0, 2]$

59. $f(x) = x^5 - x^3$ on the interval $(-2, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = x^5 - x^3$.



local maximum: $f(-0.77) = 0.19$

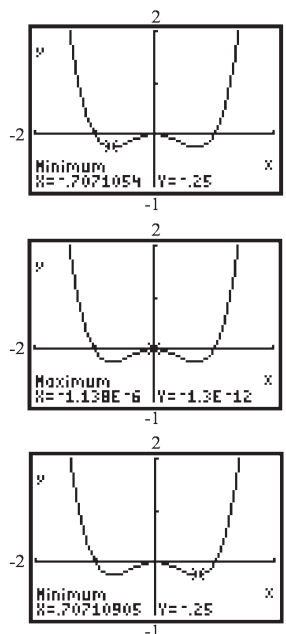
local minimum: $f(0.77) = -0.19$

f is increasing on: $[-2, -0.77]$ and $[0.77, 2]$;

f is decreasing on: $[-0.77, 0.77]$

60. $f(x) = x^4 - x^2$ on the interval $(-2, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = x^4 - x^2$.



local maximum: $f(0) = 0$

local minimum:

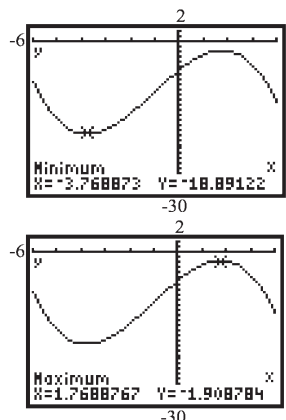
$f(-0.71) = -0.25$; $f(0.71) = -0.25$

f is increasing on: $[-0.71, 0]$ and $[0.71, 2]$;

f is decreasing on: $[-2, -0.71]$ and $[0, 0.71]$

61. $f(x) = -0.2x^3 - 0.6x^2 + 4x - 6$ on the interval $(-6, 4)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.2x^3 - 0.6x^2 + 4x - 6$.



local maximum: $f(1.77) = -1.91$

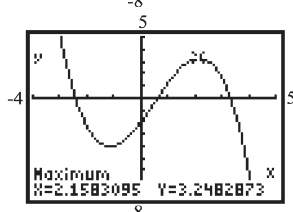
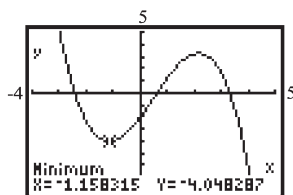
local minimum: $f(-3.77) = -18.89$

f is increasing on: $[-3.77, 1.77]$;

f is decreasing on: $[-6, -3.77]$ and $[1.77, 4]$

62. $f(x) = -0.4x^3 + 0.6x^2 + 3x - 2$ on the interval $(-4, 5)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^3 + 0.6x^2 + 3x - 2$.



local maximum: $f(2.16) = 3.25$

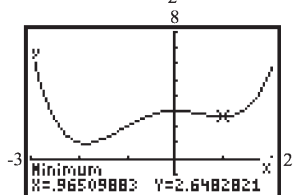
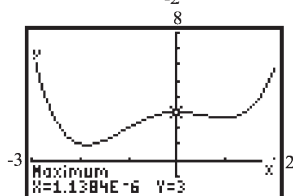
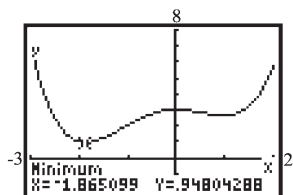
local minimum: $f(-1.16) = -4.05$

f is increasing on: $[-1.16, 2.16]$;

f is decreasing on: $[-4, -1.16]$ and $[2.16, 5]$

63. $f(x) = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$ on the interval $(-3, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$.



local maximum: $f(0) = 3$

local minimum:

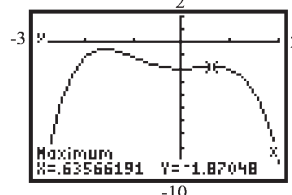
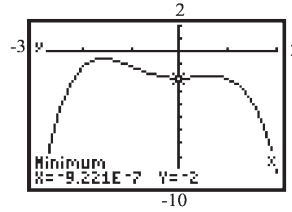
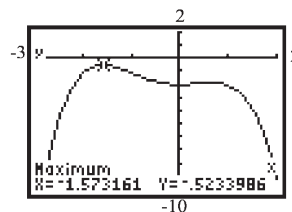
$$f(-1.87) = 0.95, f(0.97) = 2.65$$

f is increasing on: $[-1.87, 0]$ and $[0.97, 2]$;

f is decreasing on: $[-3, -1.87]$ and $[0, 0.97]$

64. $f(x) = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2$ on the interval $(-3, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2$.



local maxima: $f(-1.57) = -0.52$,

$$f(0.64) = -1.87$$

local minimum: $(0, -2)$ $f(0) = -2$

f is increasing on: $[-3, -1.57]$ and $[0, 0.64]$;

f is decreasing on: $[-1.57, 0]$ and $[0.64, 2]$

65. $f(x) = -2x^2 + 4$

- a. Average rate of change of f from $x = 0$ to $x = 2$

$$\begin{aligned} \frac{f(2) - f(0)}{2 - 0} &= \frac{(-2(2)^2 + 4) - (-2(0)^2 + 4)}{2} \\ &= \frac{(-4) - (4)}{2} = \frac{-8}{2} = -4 \end{aligned}$$

Chapter 2: Functions and Their Graphs

- b. Average rate of change of f from $x = 1$ to $x = 3$:

$$\begin{aligned}\frac{f(3) - f(1)}{3 - 1} &= \frac{(-2(3)^2 + 4) - (-2(1)^2 + 4)}{2} \\ &= \frac{(-14) - (-2)}{2} = \frac{-16}{2} = -8\end{aligned}$$

- c. Average rate of change of f from $x = 1$ to $x = 4$:

$$\begin{aligned}\frac{f(4) - f(1)}{4 - 1} &= \frac{(-2(4)^2 + 4) - (-2(1)^2 + 4)}{3} \\ &= \frac{(-28) - (-2)}{3} = \frac{-30}{3} = -10\end{aligned}$$

66. $f(x) = -x^3 + 1$

- a. Average rate of change of f from $x = 0$ to $x = 2$:

$$\begin{aligned}\frac{f(2) - f(0)}{2 - 0} &= \frac{(-(2)^3 + 1) - (-(0)^3 + 1)}{2} \\ &= \frac{-7 - 1}{2} = \frac{-8}{2} = -4\end{aligned}$$

- b. Average rate of change of f from $x = 1$ to $x = 3$:

$$\begin{aligned}\frac{f(3) - f(1)}{3 - 1} &= \frac{(-(3)^3 + 1) - (-(1)^3 + 1)}{2} \\ &= \frac{-26 - (0)}{2} = \frac{-26}{2} = -13\end{aligned}$$

- c. Average rate of change of f from $x = -1$ to $x = 1$:

$$\begin{aligned}\frac{f(1) - f(-1)}{1 - (-1)} &= \frac{(-(1)^3 + 1) - (-(-1)^3 + 1)}{2} \\ &= \frac{0 - 2}{2} = \frac{-2}{2} = -1\end{aligned}$$

67. $g(x) = x^3 - 4x + 7$

- a. Average rate of change of g from $x = -3$ to $x = -2$:

$$\begin{aligned}\frac{g(-2) - g(-3)}{-2 - (-3)} &= \frac{[(-2)^3 - 4(-2) + 7] - [(-3)^3 - 4(-3) + 7]}{1} \\ &= \frac{(7) - (-8)}{1} = \frac{15}{1} = 15\end{aligned}$$

- b. Average rate of change of g from $x = -1$ to $x = 1$:

$$\begin{aligned}\frac{g(1) - g(-1)}{1 - (-1)} &= \frac{[(1)^3 - 4(1) + 7] - [(-1)^3 - 4(-1) + 7]}{2} \\ &= \frac{(4) - (10)}{2} = \frac{-6}{2} = -3\end{aligned}$$

- c. Average rate of change of g from $x = 1$ to $x = 3$:

$$\begin{aligned}\frac{g(3) - g(1)}{3 - 1} &= \frac{[(3)^3 - 4(3) + 7] - [(1)^3 - 4(1) + 7]}{2} \\ &= \frac{(22) - (4)}{2} = \frac{18}{2} = 9\end{aligned}$$

68. $h(x) = x^2 - 2x + 3$

- a. Average rate of change of h from $x = -1$ to $x = 1$:

$$\begin{aligned}\frac{h(1) - h(-1)}{1 - (-1)} &= \frac{[(1)^2 - 2(1) + 3] - [(-1)^2 - 2(-1) + 3]}{2} \\ &= \frac{(2) - (6)}{2} = \frac{-4}{2} = -2\end{aligned}$$

- b. Average rate of change of h from $x = 0$ to $x = 2$:

$$\begin{aligned}\frac{h(2) - h(0)}{2 - 0} &= \frac{[(2)^2 - 2(2) + 3] - [(0)^2 - 2(0) + 3]}{2} \\ &= \frac{(3) - (3)}{2} = \frac{0}{2} = 0\end{aligned}$$

- c. Average rate of change of h from $x = 2$ to $x = 5$:

$$\begin{aligned} & \frac{h(5) - h(2)}{5 - 2} \\ &= \frac{[(5)^2 - 2(5) + 3] - [(2)^2 - 2(2) + 3]}{3} \\ &= \frac{(18) - (3)}{3} = \frac{15}{3} = 5 \end{aligned}$$

69. $f(x) = 5x - 2$

- a. Average rate of change of f from 1 to 3:

$$\frac{\Delta y}{\Delta x} = \frac{f(3) - f(1)}{3 - 1} = \frac{13 - 3}{3 - 1} = \frac{10}{2} = 5$$

Thus, the average rate of change of f from 1 to 3 is 5.

- b. From (a), the slope of the secant line joining $(1, f(1))$ and $(3, f(3))$ is 5. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$

$$y - 3 = 5(x - 1)$$

$$y - 3 = 5x - 5$$

$$y = 5x - 2$$

70. $f(x) = -4x + 1$

- a. Average rate of change of f from 2 to 5:

$$\begin{aligned} \frac{\Delta y}{\Delta x} &= \frac{f(5) - f(2)}{5 - 2} = \frac{-19 - (-7)}{5 - 2} \\ &= \frac{-12}{3} = -4 \end{aligned}$$

Therefore, the average rate of change of f from 2 to 5 is -4 .

- b. From (a), the slope of the secant line joining $(2, f(2))$ and $(5, f(5))$ is -4 . We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$

$$y - (-7) = -4(x - 2)$$

$$y + 7 = -4x + 8$$

$$y = -4x + 1$$

71. $g(x) = x^2 - 2$

- a. Average rate of change of g from -2 to 1 :

$$\frac{\Delta y}{\Delta x} = \frac{g(1) - g(-2)}{1 - (-2)} = \frac{-1 - 2}{1 - (-2)} = \frac{-3}{3} = -1$$

Therefore, the average rate of change of g from -2 to 1 is -1 .

- b. From (a), the slope of the secant line joining $(-2, g(-2))$ and $(1, g(1))$ is -1 . We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$

$$y - 2 = -1(x - (-2))$$

$$y - 2 = -x - 2$$

$$y = -x$$

72. $g(x) = x^2 + 1$

- a. Average rate of change of g from -1 to 2 :

$$\frac{\Delta y}{\Delta x} = \frac{g(2) - g(-1)}{2 - (-1)} = \frac{5 - 2}{2 - (-1)} = \frac{3}{3} = 1$$

Therefore, the average rate of change of g from -1 to 2 is 1 .

- b. From (a), the slope of the secant line joining $(-1, g(-1))$ and $(2, g(2))$ is 1 . We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$

$$y - 2 = 1(x - (-1))$$

$$y - 2 = x + 1$$

$$y = x + 3$$

73. $h(x) = x^2 - 2x$

- a. Average rate of change of h from 2 to 4 :

$$\frac{\Delta y}{\Delta x} = \frac{h(4) - h(2)}{4 - 2} = \frac{8 - 0}{4 - 2} = \frac{8}{2} = 4$$

Therefore, the average rate of change of h from 2 to 4 is 4 .

- b. From (a), the slope of the secant line joining $(2, h(2))$ and $(4, h(4))$ is 4 . We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$

$$y - 0 = 4(x - 2)$$

$$y = 4x - 8$$

Chapter 2: Functions and Their Graphs

74. $h(x) = -2x^2 + x$

- a. Average rate of change from 0 to 3:

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{h(3) - h(0)}{3 - 0} = \frac{-15 - 0}{3 - 0} \\ &= \frac{-15}{3} = -5\end{aligned}$$

Therefore, the average rate of change of h from 0 to 3 is -5 .

- b. From (a), the slope of the secant line joining $(0, h(0))$ and $(3, h(3))$ is -5 . We use the point-slope form to find the equation of the secant line:

$$\begin{aligned}y - y_1 &= m_{\text{sec}}(x - x_1) \\ y - 0 &= -5(x - 0) \\ y &= -5x\end{aligned}$$

75. a. $g(x) = x^3 - 27x$

$$\begin{aligned}g(-x) &= (-x)^3 - 27(-x) \\ &= -x^3 + 27x \\ &= -(x^3 - 27x) \\ &= -g(x)\end{aligned}$$

Since $g(-x) = -g(x)$, the function is odd.

- b. Since $g(x)$ is odd then it is symmetric about the origin so there exist a local maximum at $x = -3$.

$$g(-3) = (-3)^3 - 27(-3) = -27 + 81 = 54$$

So there is a local maximum of 54 at $x = -3$.

76. $f(x) = -x^3 + 12x$

a.
$$\begin{aligned}f(-x) &= -(-x)^3 + 12(-x) \\ &= x^3 - 12x \\ &= -(-x^3 + 12x) \\ &= -f(x)\end{aligned}$$

Since $f(-x) = -f(x)$, the function is odd.

- b. Since $f(x)$ is odd then it is symmetric about the origin so there exist a local maximum at $x = -3$.

$$f(-2) = -(-2)^3 + 12(-2) = 8 - 24 = -16$$

So there is a local minimum value of -16 at $x = -2$.

77. $F(x) = -x^4 + 8x^2 + 9$

a.
$$\begin{aligned}F(-x) &= -(-x)^4 + 8(-x)^2 + 9 \\ &= -x^4 + 8x^2 + 9 \\ &= F(x)\end{aligned}$$

Since $F(-x) = F(x)$, the function is even.

- b. Since the function is even, its graph has y -axis symmetry. The second local maximum value is 25 and occurs at $x = -2$.

- c. Because the graph has y -axis symmetry, the area under the graph between $x = 0$ and $x = 3$ bounded below by the x -axis is the same as the area under the graph between $x = -3$ and $x = 0$ bounded below the x -axis. Thus, the area is 50.4 square units.

78. $G(x) = -x^4 + 32x^2 + 144$

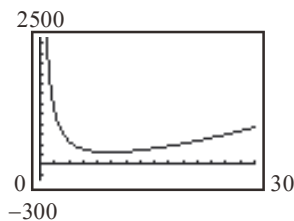
a.
$$\begin{aligned}G(-x) &= -(-x)^4 + 32(-x)^2 + 144 \\ &= -x^4 + 32x^2 + 144 \\ &= G(x)\end{aligned}$$

Since $G(-x) = G(x)$, the function is even.

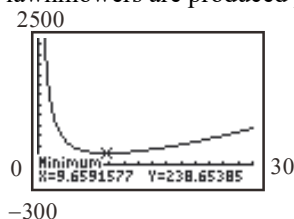
- b. Since the function is even, its graph has y -axis symmetry. The second local maximum is in quadrant II and is 400 and occurs at $x = -4$.
- c. Because the graph has y -axis symmetry, the area under the graph between $x = 0$ and $x = 6$ bounded below by the x -axis is the same as the area under the graph between $x = -6$ and $x = 0$ bounded below the x -axis. Thus, the area is 1612.8 square units.

79. $\bar{C}(x) = 0.3x^2 + 21x - 251 + \frac{2500}{x}$

a. $y_1 = 0.3x^2 + 21x - 251 + \frac{2500}{x}$

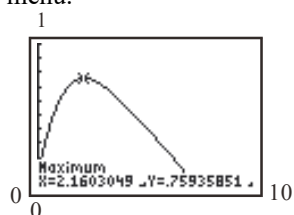


- b. Use MINIMUM. Rounding to the nearest whole number, the average cost is minimized when approximately 10 lawnmowers are produced per hour.



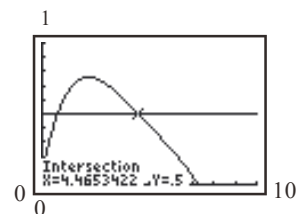
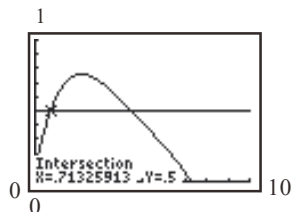
- c. The minimum average cost is approximately \$239 per mower.

80. a.
- $C(t) = -.002t^4 + .039t^3 - .285t^2 + .766t + .085$
-
- Graph the function on a graphing utility and use the Maximum option from the CALC menu.



The concentration will be highest after about 2.16 hours.

- b. Enter the function in Y1 and 0.5 in Y2. Graph the two equations in the same window and use the Intersect option from the CALC menu.



After taking the medication, the woman can feed her child within the first 0.71 hours (about 42 minutes) or after 4.47 hours (about 4 hours 28 minutes) have elapsed.

81. a.
$$\begin{aligned} \text{avg. rate of change} &= \frac{P(2.5) - P(0)}{2.5 - 0} \\ &= \frac{0.18 - 0.09}{2.5 - 0} \\ &= \frac{0.09}{2.5} \\ &= 0.036 \text{ gram per hour} \end{aligned}$$

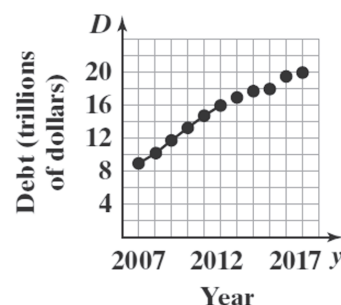
On average, the population is increasing at a rate of 0.036 gram per hour from 0 to 2.5 hours.

b.
$$\begin{aligned} \text{avg. rate of change} &= \frac{P(6) - P(4.5)}{6 - 4.5} \\ &= \frac{0.50 - 0.35}{6 - 4.5} \\ &= \frac{0.15}{1.5} \\ &= 0.1 \text{ gram per hour} \end{aligned}$$

On average, the population is increasing at a rate of 0.1 gram per hour from 4.5 to 6 hours.

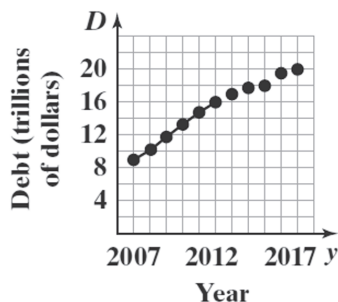
- c. The average rate of change is increasing as time passes. This indicates that the population is increasing at an increasing rate.

82. a.



Chapter 2: Functions and Their Graphs

b.



$$\begin{aligned} \text{c. avg. rate of change} &= \frac{P(2014) - P(2012)}{2014 - 2012} \\ &= \frac{9512 - 9273}{2} \\ &= \frac{239}{2} \\ &= \$119.50 \text{ dollars/yr} \end{aligned}$$

$$\begin{aligned} \text{d. avg. rate of change} &= \frac{P(2016) - P(2014)}{2016 - 2014} \\ &= \frac{9938 - 9512}{2} \\ &= \frac{426}{2} \\ &= \$213.00 \text{ dollars/yr} \end{aligned}$$

$$\begin{aligned} \text{e. avg. rate of change} &= \frac{P(2018) - P(2016)}{2018 - 2016} \\ &= \frac{10409 - 9938}{2} \\ &= \frac{471}{2} \\ &= \$235.50 \text{ dollars/yr} \end{aligned}$$

f. The average rate of change is increasing.

$$\begin{aligned} \text{g. avg. rate of change} &= \frac{P(2021) - P(2019)}{2021 - 2019} \\ &= \frac{9177 - 10507}{2} \\ &= -\frac{1330}{2} \\ &= -\$665.00 \text{ dollars/yr} \end{aligned}$$

The debt decreased.

h. Quarantine from Covid19 most likely had a effect on spending habits.

$$83. f(x) = x^2$$

a. Average rate of change of f from $x = 0$ to $x = 1$:

$$\frac{f(1) - f(0)}{1 - 0} = \frac{1^2 - 0^2}{1} = \frac{1}{1} = 1$$

b. Average rate of change of f from $x = 0$ to $x = 0.5$:

$$\frac{f(0.5) - f(0)}{0.5 - 0} = \frac{(0.5)^2 - 0^2}{0.5} = \frac{0.25}{0.5} = 0.5$$

c. Average rate of change of f from $x = 0$ to $x = 0.1$:

$$\frac{f(0.1) - f(0)}{0.1 - 0} = \frac{(0.1)^2 - 0^2}{0.1} = \frac{0.01}{0.1} = 0.1$$

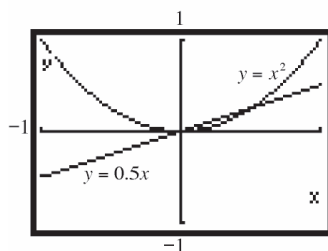
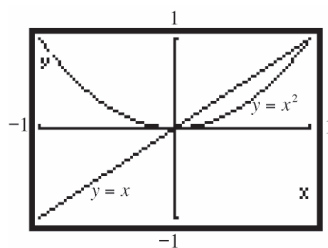
d. Average rate of change of f from $x = 0$ to $x = 0.01$:

$$\begin{aligned} \frac{f(0.01) - f(0)}{0.01 - 0} &= \frac{(0.01)^2 - 0^2}{0.01} \\ &= \frac{0.0001}{0.01} = 0.01 \end{aligned}$$

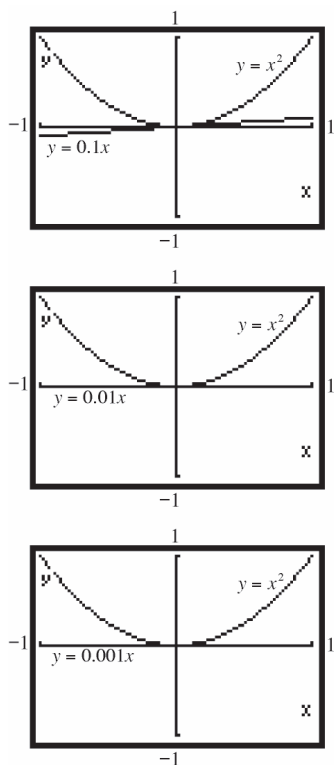
e. Average rate of change of f from $x = 0$ to $x = 0.001$:

$$\begin{aligned} \frac{f(0.001) - f(0)}{0.001 - 0} &= \frac{(0.001)^2 - 0^2}{0.001} \\ &= \frac{0.000001}{0.001} = 0.001 \end{aligned}$$

f. Graphing the secant lines:



Section 2.3: Properties of Functions



- g. The secant lines are beginning to look more and more like the tangent line to the graph of f at the point where $x = 0$.
- h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number zero.

84. $f(x) = x^2$

- a. Average rate of change of f from $x = 1$ to $x = 2$:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{2^2 - 1^2}{1} = \frac{3}{1} = 3$$

- b. Average rate of change of f from $x = 1$ to $x = 1.5$:

$$\frac{f(1.5) - f(1)}{1.5 - 1} = \frac{(1.5)^2 - 1^2}{0.5} = \frac{1.25}{0.5} = 2.5$$

- c. Average rate of change of f from $x = 1$ to $x = 1.1$:

$$\frac{f(1.1) - f(1)}{1.1 - 1} = \frac{(1.1)^2 - 1^2}{0.1} = \frac{0.21}{0.1} = 2.1$$

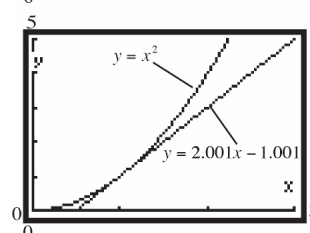
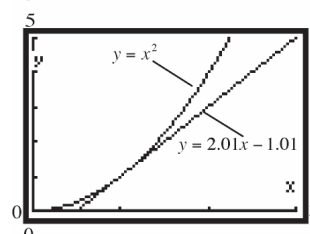
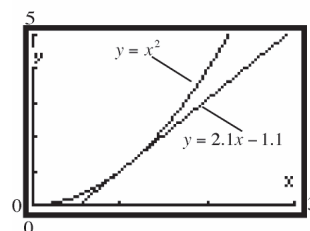
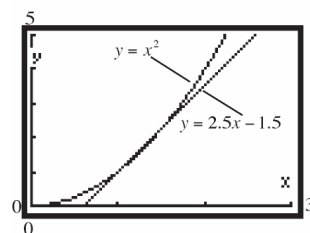
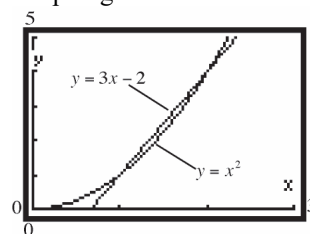
- d. Average rate of change of f from $x = 1$ to $x = 1.01$:

$$\frac{f(1.01) - f(1)}{1.01 - 1} = \frac{(1.01)^2 - 1^2}{0.01} = \frac{0.0201}{0.01} = 2.01$$

- e. Average rate of change of f from $x = 1$ to $x = 1.001$:

$$\begin{aligned} \frac{f(1.001) - f(1)}{1.001 - 1} &= \frac{(1.001)^2 - 1^2}{0.001} \\ &= \frac{0.002001}{0.001} = 2.001 \end{aligned}$$

- f. Graphing the secant lines:



Chapter 2: Functions and Their Graphs

- g. The secant lines are beginning to look more and more like the tangent line to the graph of f at the point where $x = 1$.
- h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number 2.

85. $f(x) = 2x + 5$

a.
$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{2(x+h) + 5 - 2x - 5}{h} = \frac{2h}{h} = 2$$

b. When $x = 1$:

$$h = 0.5 \Rightarrow m_{\text{sec}} = 2$$

$$h = 0.1 \Rightarrow m_{\text{sec}} = 2$$

$$h = 0.01 \Rightarrow m_{\text{sec}} = 2$$

$$\text{as } h \rightarrow 0, m_{\text{sec}} \rightarrow 2$$

c. Using the point $(1, f(1)) = (1, 7)$ and slope,

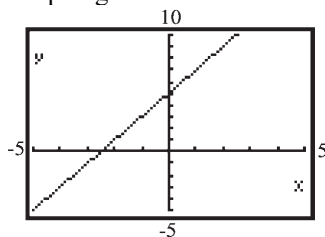
$m = 2$, we get the secant line:

$$y - 7 = 2(x - 1)$$

$$y - 7 = 2x - 2$$

$$y = 2x + 5$$

d. Graphing:



The graph and the secant line coincide.

86. $f(x) = -3x + 2$

a.
$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{-3(x+h) + 2 - (-3x + 2)}{h} = \frac{-3h}{h} = -3$$

b. When $x = 1$,

$$h = 0.5 \Rightarrow m_{\text{sec}} = -3$$

$$h = 0.1 \Rightarrow m_{\text{sec}} = -3$$

$$h = 0.01 \Rightarrow m_{\text{sec}} = -3$$

$$\text{as } h \rightarrow 0, m_{\text{sec}} \rightarrow -3$$

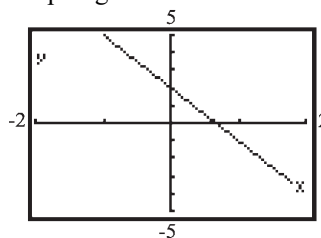
c. Using point $(1, f(1)) = (1, -1)$ and slope $= -3$, we get the secant line:

$$y - (-1) = -3(x - 1)$$

$$y + 1 = -3x + 3$$

$$y = -3x + 2$$

d. Graphing:



The graph and the secant line coincide.

87. $f(x) = x^2 + 2x$

a.
$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$$

$$= \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$$

$$= \frac{2xh + h^2 + 2h}{h}$$

$$= 2x + h + 2$$

b. When $x = 1$,

$$h = 0.5 \Rightarrow m_{\text{sec}} = 2 \cdot 1 + 0.5 + 2 = 4.5$$

$$h = 0.1 \Rightarrow m_{\text{sec}} = 2 \cdot 1 + 0.1 + 2 = 4.1$$

$$h = 0.01 \Rightarrow m_{\text{sec}} = 2 \cdot 1 + 0.01 + 2 = 4.01$$

$$\text{as } h \rightarrow 0, m_{\text{sec}} \rightarrow 2 \cdot 1 + 0 + 2 = 4$$

c. Using point $(1, f(1)) = (1, 3)$ and

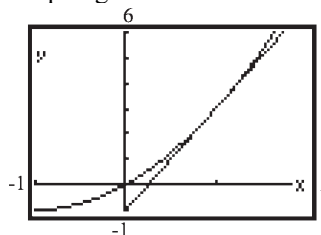
slope $= 4.01$, we get the secant line:

$$y - 3 = 4.01(x - 1)$$

$$y - 3 = 4.01x - 4.01$$

$$y = 4.01x - 1.01$$

d. Graphing:



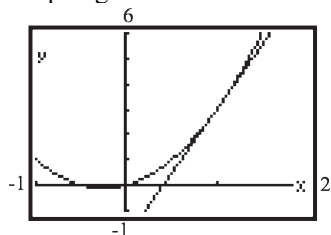
88. $f(x) = 2x^2 + x$

$$\begin{aligned} \text{a. } m_{\text{sec}} &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{2(x+h)^2 + (x+h) - (2x^2 + x)}{h} \\ &= \frac{2(x^2 + 2xh + h^2) + x + h - 2x^2 - x}{h} \\ &= \frac{2x^2 + 4xh + 2h^2 + x + h - 2x^2 - x}{h} \\ &= \frac{4xh + 2h^2 + h}{h} \\ &= 4x + 2h + 1 \end{aligned}$$

b. When $x = 1$,
 $h = 0.5 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.5) + 1 = 6$
 $h = 0.1 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.1) + 1 = 5.2$
 $h = 0.01 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.01) + 1 = 5.02$
 as $h \rightarrow 0$, $m_{\text{sec}} \rightarrow 4 \cdot 1 + 2(0) + 1 = 5$

c. Using point $(1, f(1)) = (1, 3)$ and
 slope = 5.02, we get the secant line:
 $y - 3 = 5.02(x - 1)$
 $y - 3 = 5.02x - 5.02$
 $y = 5.02x - 2.02$

d. Graphing:



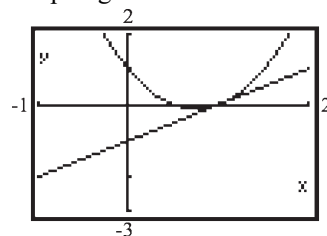
89. $f(x) = 2x^2 - 3x + 1$

$$\begin{aligned} \text{a. } m_{\text{sec}} &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h} \\ &= \frac{2(x^2 + 2xh + h^2) - 3x - 3h + 1 - 2x^2 + 3x - 1}{h} \\ &= \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1}{h} \\ &= \frac{4xh + 2h^2 - 3h}{h} \\ &= 4x + 2h - 3 \end{aligned}$$

b. When $x = 1$,
 $h = 0.5 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.5) - 3 = 2$
 $h = 0.1 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.1) - 3 = 1.2$
 $h = 0.01 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.01) - 3 = 1.02$
 as $h \rightarrow 0$, $m_{\text{sec}} \rightarrow 4 \cdot 1 + 2(0) - 3 = 1$

c. Using point $(1, f(1)) = (1, 0)$ and
 slope = 1.02, we get the secant line:
 $y - 0 = 1.02(x - 1)$
 $y = 1.02x - 1.02$

d. Graphing:



90. $f(x) = -x^2 + 3x - 2$

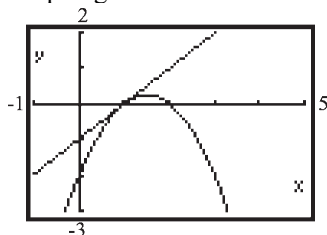
$$\begin{aligned} \text{a. } m_{\text{sec}} &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{-(x+h)^2 + 3(x+h) - 2 - (-x^2 + 3x - 2)}{h} \\ &= \frac{-(x^2 + 2xh + h^2) + 3x + 3h - 2 + x^2 - 3x + 2}{h} \\ &= \frac{-x^2 - 2xh - h^2 + 3x + 3h - 2 + x^2 - 3x + 2}{h} \\ &= \frac{-2xh - h^2 + 3h}{h} \\ &= -2x - h + 3 \end{aligned}$$

b. When $x = 1$,
 $h = 0.5 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.5 + 3 = 0.5$
 $h = 0.1 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.1 + 3 = 0.9$
 $h = 0.01 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.01 + 3 = 0.99$
 as $h \rightarrow 0$, $m_{\text{sec}} \rightarrow -2 \cdot 1 - 0 + 3 = 1$

c. Using point $(1, f(1)) = (1, 0)$ and
 slope = 0.99, we get the secant line:
 $y - 0 = 0.99(x - 1)$
 $y = 0.99x - 0.99$

Chapter 2: Functions and Their Graphs

d. Graphing:



91. $f(x) = \frac{1}{x}$

a.
$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right)}{h} = \frac{\left(\frac{x - (x+h)}{(x+h)x}\right)}{h}$$

$$= \left(\frac{x - x - h}{(x+h)x}\right)\left(\frac{1}{h}\right) = \left(\frac{-h}{(x+h)x}\right)\left(\frac{1}{h}\right)$$

$$= -\frac{1}{(x+h)x}$$

b. When $x = 1$,

$$h = 0.5 \Rightarrow m_{\text{sec}} = -\frac{1}{(1+0.5)(1)}$$

$$= -\frac{1}{1.5} = -\frac{2}{3} \approx -0.667$$

$$h = 0.1 \Rightarrow m_{\text{sec}} = -\frac{1}{(1+0.1)(1)}$$

$$= -\frac{1}{1.1} = -\frac{10}{11} \approx -0.909$$

$$h = 0.01 \Rightarrow m_{\text{sec}} = -\frac{1}{(1+0.01)(1)}$$

$$= -\frac{1}{1.01} = -\frac{100}{101} \approx -0.990$$

$$\text{as } h \rightarrow 0, m_{\text{sec}} \rightarrow -\frac{1}{(1+0)(1)} = -\frac{1}{1} = -1$$

c. Using point $(1, f(1)) = (1, 1)$ and

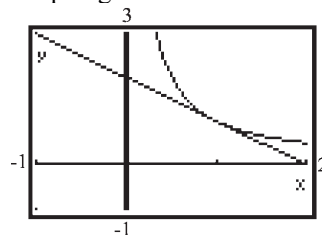
slope $= -\frac{100}{101}$, we get the secant line:

$$y - 1 = -\frac{100}{101}(x - 1)$$

$$y - 1 = -\frac{100}{101}x + \frac{100}{101}$$

$$y = -\frac{100}{101}x + \frac{201}{101}$$

d. Graphing:



92. $f(x) = \frac{1}{x^2}$

a.
$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\left(\frac{1}{(x+h)^2} - \frac{1}{x^2}\right)}{h}$$

$$= \frac{\left(\frac{x^2 - (x+h)^2}{(x+h)^2 x^2}\right)}{h}$$

$$= \left(\frac{x^2 - (x^2 + 2xh + h^2)}{(x+h)^2 x^2}\right)\left(\frac{1}{h}\right)$$

$$= \left(\frac{-2xh - h^2}{(x+h)^2 x^2}\right)\left(\frac{1}{h}\right)$$

$$= \frac{-2x - h}{(x+h)^2 x^2} = \frac{-2x - h}{(x^2 + 2xh + h^2)x^2}$$

b. When $x = 1$,

$$h = 0.5 \Rightarrow m_{\text{sec}} = \frac{-2 \cdot 1 - 0.5}{(1+0.5)^2 1^2} = -\frac{10}{9} \approx -1.1111$$

$$h = 0.1 \Rightarrow m_{\text{sec}} = \frac{-2 \cdot 1 - 0.1}{(1+0.1)^2 1^2} = -\frac{210}{121} \approx -1.7355$$

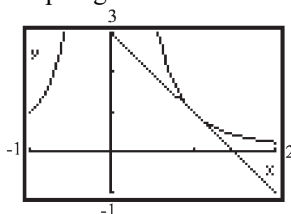
$$h = 0.01 \Rightarrow m_{\text{sec}} = \frac{-2 \cdot 1 - 0.01}{(1+0.01)^2 1^2}$$

$$= -\frac{20,100}{10,201} \approx -1.9704$$

$$\text{as } h \rightarrow 0, m_{\text{sec}} \rightarrow \frac{-2 \cdot 1 - 0}{(1+0)^2 1^2} = -2$$

- c. Using point $(1, f(1)) = (1, 1)$ and slope $= -1.9704$, we get the secant line:
 $y - 1 = -1.9704(x - 1)$
 $y - 1 = -1.9704x + 1.9704$
 $y = -1.9704x + 2.9704$

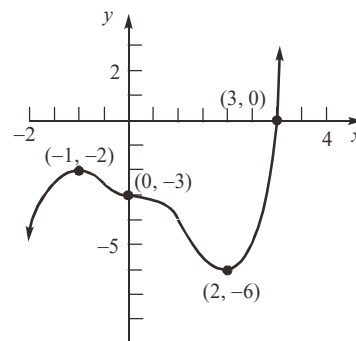
d. Graphing:



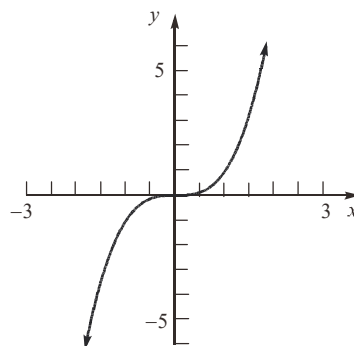
93. $f(2) = 12$ and $f(-1) = 8$, so
 $\frac{f(2) - f(-1)}{2 - (-1)} = \frac{12}{3} = 4$
 $f'(x) = 4 \rightarrow$
 $3x^2 + 4x - 1 = 4$
 $3x^2 + 4x - 5 = 0$
 $x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-5)}}{2(3)} = \frac{-4 \pm \sqrt{76}}{6}$
 $= \frac{-4 \pm 2\sqrt{19}}{6} = \frac{-2 \pm \sqrt{19}}{3}$
 $x = \frac{-2 - \sqrt{19}}{3} \approx -2.1$ is outside the interval
 $x = \frac{-2 + \sqrt{19}}{3} \approx .079$ is in the interval.
 The only such number is $\frac{-2 + \sqrt{19}}{3}$.

94. $g(-x) = -2f\left(-\frac{x}{3}\right) = -2f\left(\frac{x}{3}\right)$. Since f is odd,
 $f\left(-\frac{x}{3}\right) = -f\left(\frac{x}{3}\right)$
 $g(-x) = -2f\left(\frac{x}{3}\right) = 2f\left(-\frac{x}{3}\right) = -g(x)$
 so g is odd.
 So g is odd.

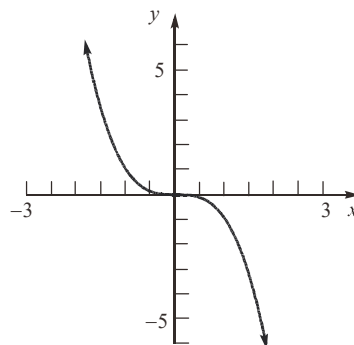
95. Answers will vary. One possibility follows:



96. Answers will vary. See solution to Problem 89 for one possibility.
97. A function that is increasing on an interval can have at most one x -intercept on the interval. The graph of f could not "turn" and cross it again or it would start to decrease.
98. An increasing function is a function whose graph goes up as you read from left to right.



A decreasing function is a function whose graph goes down as you read from left to right.



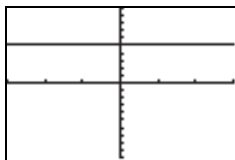
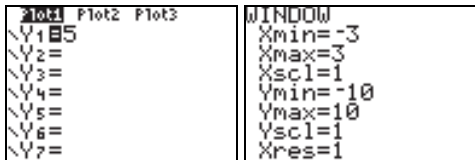
99. To be an even function we need $f(-x) = f(x)$ and to be an odd function we need $f(-x) = -f(x)$. In order for a function to be both

Chapter 2: Functions and Their Graphs

even and odd, we would need $f(x) = -f(x)$.

This is only possible if $f(x) = 0$.

100. The graph of $y = 5$ is a horizontal line.



The local maximum is $y = 5$ and it occurs at each x -value in the interval.

101. Not necessarily. It just means $f(5) > f(2)$.

The function could have both increasing and decreasing intervals.

$$102. \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{b - b}{x_2 - x_1} = 0$$

$$\frac{f(2) - f(-2)}{2 - (-2)} = \frac{0 - 0}{4} = 0$$

$$103. \sqrt{540} = \sqrt{36(15)} = 6\sqrt{15}$$

$$\begin{aligned} 104. (4a - b)^2 &= (4a - b)(4a - b) \\ &= 16a^2 - 4ab - 4ab + b^2 \\ &= 16a^2 - 8ab + b^2 \end{aligned}$$

$$105. C(x) = 0.80x + 40$$

$$106. S = \frac{k}{T}$$

$$33 = \frac{k}{10}$$

$$k = 330$$

$$S = \frac{330}{T}$$

$$S = \frac{330}{10}$$

$$S = 8.25 \text{ days}$$

$$107. (x - h)^2 + (y - k)^2 = r^2$$

$$(x - 3)^2 + (y - (-2))^2 = \left(\frac{\sqrt{6}}{2}\right)^2$$

$$(x - 3)^2 + (y + 2)^2 = \frac{3}{2}$$

- 108.

$$\begin{aligned} &(x^2 - 2)^3 2(3x^5 + 1)15x^4 + (3x^5 + 1)^2 3(x^2 - 2)^3 2x \\ &= 6x(x^2 - 2)^2(3x^5 + 1)[(x^2 - 2)5x^3 + (3x^5 + 1)] \\ &= 6x(x^2 - 2)^2(3x^5 + 1)[5x^5 - 10x^3 + 3x^5 + 1] \\ &= 6x(x^2 - 2)^2(3x^5 + 1)[8x^5 - 10x^3 + 1] \end{aligned}$$

$$109. \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) =$$

$$\left(\frac{-2 + \frac{3}{5}}{2}, \frac{1 + (-4)}{2}\right) =$$

$$\left(\frac{-\frac{7}{5}}{2}, \frac{-3}{2}\right) = \left(-\frac{7}{10}, -\frac{3}{2}\right)$$

$$110. |3x + 7| - 3 = 5$$

$$|3x + 7| = 8$$

$$3x + 7 = -8 \quad \text{or} \quad 3x + 7 = 8$$

$$3x = -15 \quad 3x = 1$$

$$x = -5 \quad x = \frac{1}{3}$$

The solution set is $\left\{-5, \frac{1}{3}\right\}$.

$$111. x^6 + 7x^3 = 8$$

$$x^6 + 7x^3 - 8 = 0$$

$$(x^3 + 8)(x^3 - 1) = 0$$

$$x^3 + 8 = 0 \quad \text{or} \quad x^3 - 1 = 0$$

$$x^3 = -8 \quad x^3 = 1$$

$$x = -2 \quad x = 1$$

The solution set is $\{-2, 1\}$.

112. $3y^2 \cdot D + 3x^2 - 3xy^2 - 3x^2y \cdot D = 0$

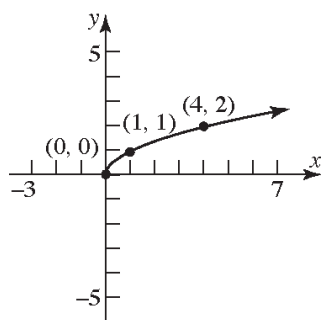
$$3y^2 \cdot D - 3x^2y \cdot D = -3x^2 + 3xy^2$$

$$D(3y^2 - 3x^2y) = -3x^2 + 3xy^2$$

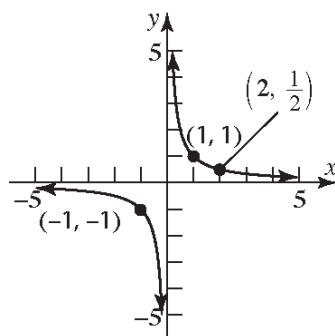
$$D = \frac{-3x^2 + 3xy^2}{3y^2 - 3x^2y} = \frac{xy^2 - x^2}{y^2 - x^2y} = \frac{x(y^2 - x)}{y(y - x^2)}$$

Section 2.4

1. $y = \sqrt{x}$



2. $y = \frac{1}{x}$



3. $y = x^3 - 8$

y-intercept:

Let $x = 0$, then $y = (0)^3 - 8 = -8$.

x-intercept:

Let $y = 0$, then $0 = x^3 - 8$

$$x^3 = 8$$

$$x = 2$$

The intercepts are $(0, -8)$ and $(2, 0)$.

4. $(-\infty, 0)$

5. piecewise-defined

6. True

7. False; the cube root function is odd and increasing on the interval $(-\infty, \infty)$.

8. False; the domain and range of the reciprocal function are both the set of real numbers except for 0.

9. b

10. a

11. C

12. A

13. E

14. G

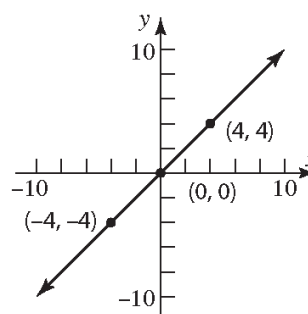
15. B

16. D

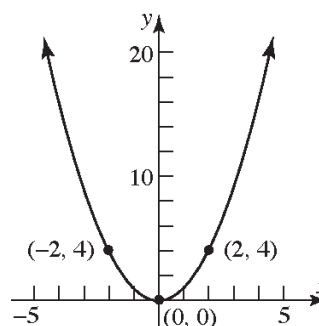
17. F

18. H

19. $f(x) = x$

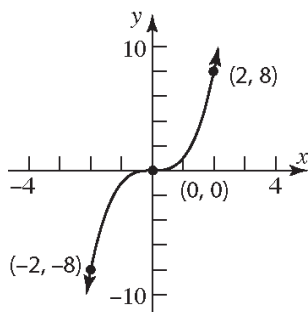


20. $f(x) = x^2$

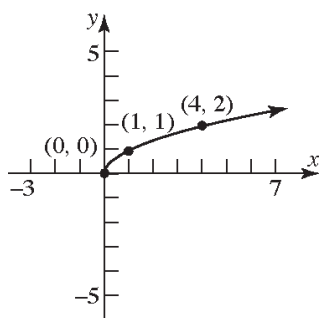


Chapter 2: Functions and Their Graphs

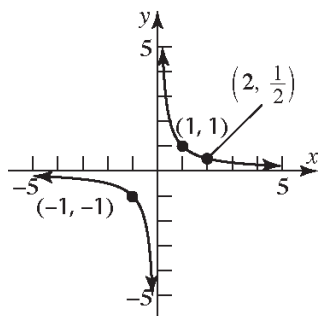
21. $f(x) = x^3$



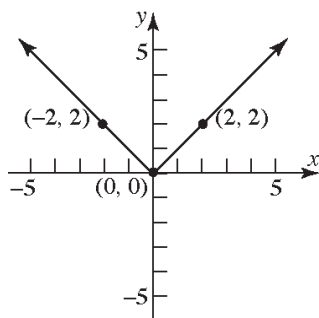
22. $f(x) = \sqrt{x}$



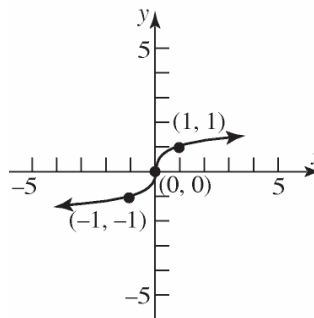
23. $f(x) = \frac{1}{x}$



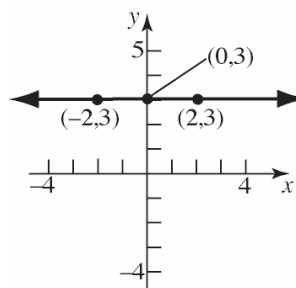
24. $f(x) = |x|$



25. $f(x) = \sqrt[3]{x}$



26. $f(x) = 3$



27. a. $f(-3) = -(-3)^2 = -9$

b. $f(0) = 4$

c. $f(2) = 3(3) - 2 = 7$

28. a. $f(-2) = -3(-2) = 6$

b. $f(-1) = 0$

c. $f(0) = 2(0)^2 + 1 = 1$

29. a. $f(-2) = 2(-2) + 4 = 0$

b. $f(0) = 2(0) + 4 = 4$

c. $f(1) = 2(1) + 4 = 6$

d. $f(3) = (3)^3 - 1 = 26$

30. a. $f(-1) = (-1)^3 = -1$

b. $f(0) = (0)^3 = 0$

c. $f(1) = 3(1) + 2 = 5$

Section 2.4: Library of Functions; Piecewise-defined Functions

d. $f(3) = 3(3) + 2 = 11$

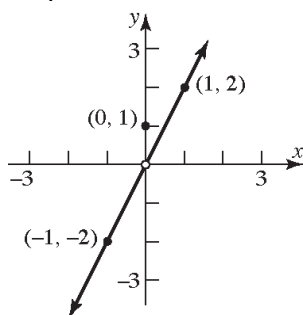
31. $f(x) = \begin{cases} 2x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x -intercept: none
 y -intercept:
 $f(0) = 1$

The only intercept is $(0, 1)$.

c. Graph:



d. Range: $\{y \mid y \neq 0\}; (-\infty, 0) \cup (0, \infty)$

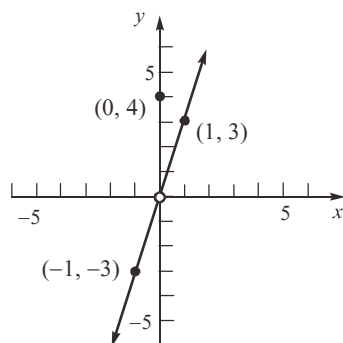
32. $f(x) = \begin{cases} 3x & \text{if } x \neq 0 \\ 4 & \text{if } x = 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x -intercept: none
 y -intercept: $f(0) = 4$

The only intercept is $(0, 4)$.

c. Graph:



d. Range: $\{y \mid y \neq 0\}; (-\infty, 0) \cup (0, \infty)$

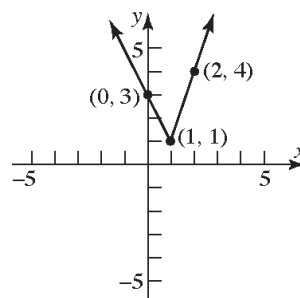
33. $f(x) = \begin{cases} -2x + 3 & \text{if } x < 1 \\ 3x - 2 & \text{if } x \geq 1 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x -intercept: none
 y -intercept: $f(0) = -2(0) + 3 = 3$

The only intercept is $(0, 3)$.

c. Graph:



d. Range: $\{y \mid y \geq 1\}; [1, \infty)$

34. $f(x) = \begin{cases} x + 3 & \text{if } x < -2 \\ -2x - 3 & \text{if } x \geq -2 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. $x + 3 = 0$ $-2x - 3 = 0$
 $x = -3$ $-2x = 3$

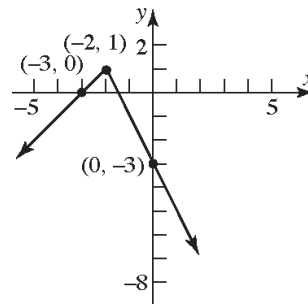
$$x = -\frac{3}{2}$$

x -intercepts: $-3, -\frac{3}{2}$

y -intercept: $f(0) = -2(0) - 3 = -3$

The intercepts are $(-3, 0)$, $(-\frac{3}{2}, 0)$, and $(0, -3)$.

c. Graph:



Chapter 2: Functions and Their Graphs

d. Range: $\{y \mid y \leq 1\}; (-\infty, 1]$

$$35. f(x) = \begin{cases} x+3 & \text{if } -2 \leq x < 1 \\ 5 & \text{if } x = 1 \\ -x+2 & \text{if } x > 1 \end{cases}$$

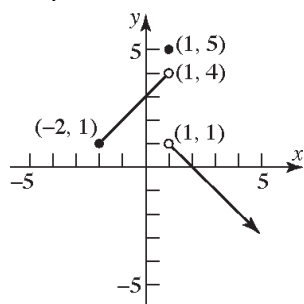
a. Domain: $\{x \mid x \geq -2\}; [-2, \infty)$

b. $x+3=0$ $-x+2=0$
 $x=-3$ $-x=-2$
(not in domain) $x=2$
x-intercept: 2

y-intercept: $f(0) = 0+3=3$

The intercepts are $(2, 0)$ and $(0, 3)$.

c. Graph:



d. Range: $\{y \mid y < 4, y = 5\}; (-\infty, 4) \cup \{5\}$

$$36. f(x) = \begin{cases} 2x+5 & \text{if } -3 \leq x < 0 \\ -3 & \text{if } x = 0 \\ -5x & \text{if } x > 0 \end{cases}$$

a. Domain: $\{x \mid x \geq -3\}; [-3, \infty)$

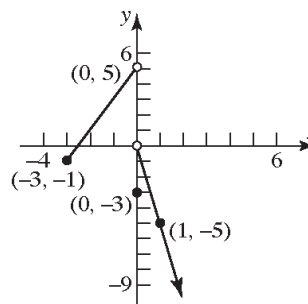
b. $2x+5=0$ $-5x=0$
 $2x=-5$ $x=0$
 $x=-\frac{5}{2}$ (not in domain of piece)

x-intercept: $-\frac{5}{2}$

y-intercept: $f(0) = -3$

The intercepts are $(-\frac{5}{2}, 0)$ and $(0, -3)$.

c. Graph:



d. Range: $\{y \mid y < 5\}; (-\infty, 5)$

$$37. f(x) = \begin{cases} 1+x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

a. Domain: $\{x \mid x \text{ is any real number}\}$

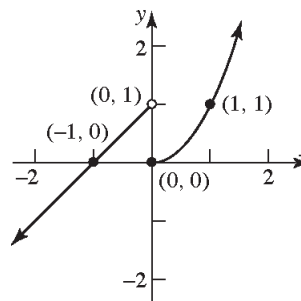
b. $1+x=0$ $x^2=0$
 $x=-1$ $x=0$

x-intercepts: $-1, 0$

y-intercept: $f(0) = 0^2 = 0$

The intercepts are $(-1, 0)$ and $(0, 0)$.

c. Graph:



d. Range: $\{y \mid y \text{ is any real number}\}$

$$38. f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ \sqrt[3]{x} & \text{if } x \geq 0 \end{cases}$$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. $\frac{1}{x}=0$ $\sqrt[3]{x}=0$
(no solution) $x=0$

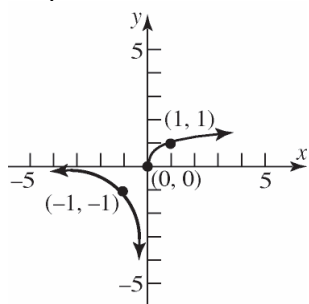
x-intercept: 0

y-intercept: $f(0) = \sqrt[3]{0} = 0$

The only intercept is $(0, 0)$.

Section 2.4: Library of Functions; Piecewise-defined Functions

c. Graph:



d. Range: $\{y \mid y \text{ is any real number}\}$

$$39. f(x) = \begin{cases} |x| & \text{if } -2 \leq x < 0 \\ x^3 & \text{if } x > 0 \end{cases}$$

a. Domain: $\{x \mid -2 \leq x < 0 \text{ and } x > 0\}$ or $\{x \mid x \geq -2, x \neq 0\}; [-2, 0) \cup (0, \infty)$.

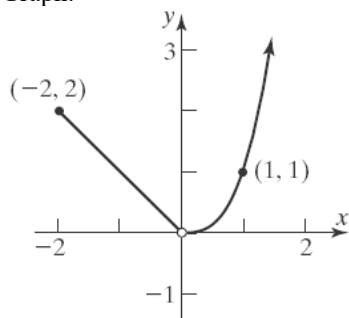
b. x -intercept: none

There are no x -intercepts since there are no values for x such that $f(x) = 0$.

y -intercept:

There is no y -intercept since $x = 0$ is not in the domain.

c. Graph:



d. Range: $\{y \mid y > 0\}; (0, \infty)$

$$40. f(x) = \begin{cases} 2-x & \text{if } -3 \leq x < 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$$

a. Domain: $\{x \mid -3 \leq x < 1 \text{ and } x > 1\}$ or $\{x \mid x \geq -3, x \neq 1\}; [-3, 1) \cup (1, \infty)$.

$$\begin{aligned} \text{b. } 2-x &= 0 & \sqrt{x} &= 0 \\ x &= 2 & x &= 0 \end{aligned}$$

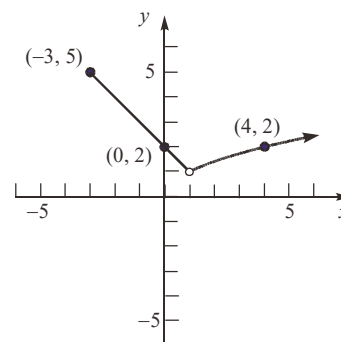
(not in domain of piece)

no x -intercepts

$$y\text{-intercept: } f(0) = 2 - 0 = 2$$

The intercept is $(0, 2)$.

c. Graph:



d. Range: $\{y \mid y > 1\}; (1, \infty)$

$$41. f(x) = \begin{cases} x^2 & \text{if } 0 < x \leq 2 \\ x+2 & \text{if } 2 < x < 5 \\ 7 & \text{if } \geq 5 \end{cases}$$

a. Domain: $\{x \mid x > 0\}; (0, \infty)$.

$$\begin{aligned} \text{b. } x^2 &= 0 \\ x &= 0 \end{aligned}$$

(not in domain
of piece)

$$x+2=0 \quad 7=0$$

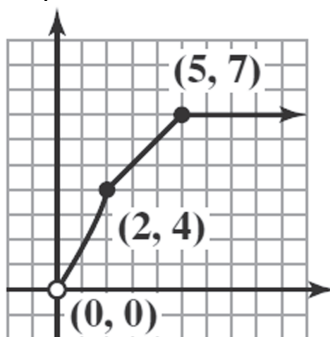
$$x=-2 \quad (\text{not possible})$$

(not in domain
of piece)

No intercepts.

Chapter 2: Functions and Their Graphs

c. Graph:



d. Range: $\{y \mid 0 < y \leq 7\}; (0, 7]$

$$42. f(x) = \begin{cases} 3x+5 & \text{if } -3 \leq x < 0 \\ 5 & \text{if } 0 \leq x \leq 2 \\ x^2+1 & \text{if } x > 2 \end{cases}$$

a. Domain: $\{x \mid x \geq -3\}; [3, \infty)$.

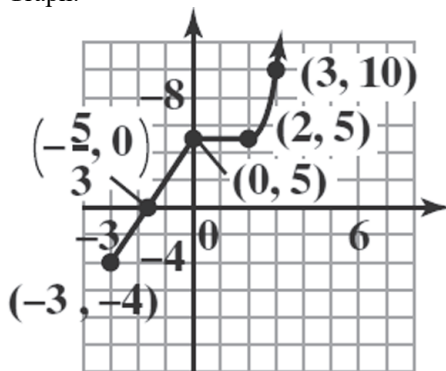
$$b. \quad \begin{array}{lll} 3x+5=0 & 5=0 & x^2+1=0 \\ x=-\frac{5}{3} & \text{(not possible)} & x^2=-1 \\ & & \text{(not possible)} \end{array}$$

$$\text{x-intercept: } -\frac{5}{3}$$

$$\text{y-intercept: } f(0) = 5$$

$$\text{The intercepts are } (0, 5) \text{ and } \left(-\frac{5}{3}, 0\right).$$

c. Graph:



d. Range: $[-4, \infty)$

43. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} -x & \text{if } -1 \leq x \leq 0 \\ \frac{1}{2}x & \text{if } 0 < x \leq 2 \end{cases}$$

44. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} x & \text{if } -1 \leq x \leq 0 \\ 1 & \text{if } 0 < x \leq 2 \end{cases}$$

45. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} -x & \text{if } x \leq 0 \\ -x+2 & \text{if } 0 < x \leq 2 \end{cases}$$

46. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} 2x+2 & \text{if } -1 \leq x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$

$$47. a. f(1.7) = \text{int}(2(1.7)) = \text{int}(3.4) = 3$$

$$b. f(2.8) = \text{int}(2(2.8)) = \text{int}(5.6) = 5$$

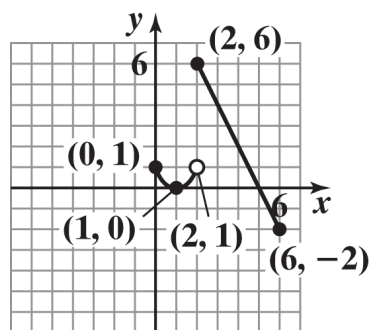
$$c. f(-3.6) = \text{int}(2(-3.6)) = \text{int}(-7.2) = -8$$

$$48. a. f(1.2) = \text{int}\left(\frac{1.2}{2}\right) = \text{int}(0.6) = 0$$

$$b. f(1.6) = \text{int}\left(\frac{1.6}{2}\right) = \text{int}(0.8) = 0$$

$$c. f(-1.8) = \text{int}\left(\frac{-1.8}{2}\right) = \text{int}(-0.9) = -1$$

49. a.

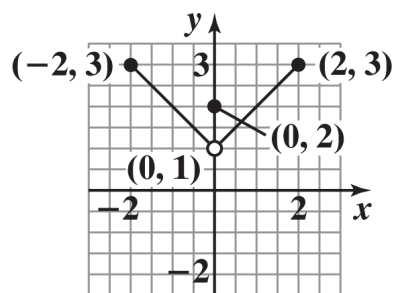


b. The domain is $[0, 6]$.

c. Absolute max: $f(2) = 6$

Absolute min: $f(6) = -2$

50. a.



- b. The domain is $[-2, 2]$.
 c. Absolute max: $f(-2) = f(2) = 3$
 Absolute min: none

51.
$$C = \begin{cases} 0.13x + 17 & \text{if } 0 \leq x \leq 3 \\ 0.4x - 23.5 & \text{if } x > 150 \end{cases}$$

- a. $C(76) = 0.13(76) + 17 = \26.88
 b. $C(133) = 0.13(133) + 17 = \34.29
 c. $C(189) = 0.4(189) - 23.5 = \52.10

52.
$$F(x) = \begin{cases} 3 - 3\text{int}(1 - x) & 0 < x \leq 2 \\ 6 - 4\text{int}(2 - x) & 2 < x \leq 4 \\ 36 & 4 < x \leq 8 \\ 77 & 8 < x \leq 24 \end{cases}$$

- a. $F(0.5) = 3 - 3\text{int}(1 - 0.5) = 3$
 Parking for 0.5 hours costs \$3.
 b. $F(2) = 3 - 3\text{int}(1 - 2) = 6$
 Parking for 2 hours costs \$6.
 c. $F(3.25) = 6 - 4\text{int}(2 - 3.25) = 14$
 Parking for 3.5 hours costs \$14.
 d. $F(8) = 36$
 Parking for 8 hours costs \$36.

53. a. Charge for 2 dozen: $C = 2.00(24)$
 $= \$48.00$

b. Charge for 120 cupcakes:
 $C = 100 + 1.5(x - 50)$
 $= 100 + 1.5(120 - 50)$
 $= \$205$

c. For $0 \leq x \leq 50$:
 $C = 2x$

 For $x \leq 200$:

$$C = 100 + 1.5(x - 50)$$

 For $x > 200$:

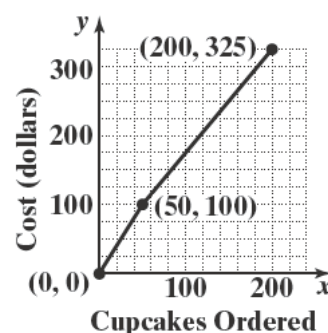
$$C = 100 + 1.5(150) + 1.15(x - 200)$$

$$= 325 + 1.15(x - 200)$$

The cost function:

$$C = \begin{cases} 2x & \text{if } 0 \leq x \leq 50 \\ 100 + 1.5(x - 50) & \text{if } 50 < x \leq 200 \\ 325 + 1.15(x - 200) & \text{if } x > 200 \end{cases}$$

d. Graph:



54. a. Taxes for \$48,000:

$$T = 0.08(48000)$$

$$= \$3840$$

b. Taxes for \$132,000:

$$T = 0.08(75000) + 0.16(57000)$$

$$= \$15,120$$

c. Taxes for \$231,000:

$$T = 0.08(75000) + 0.16(125000) + 0.22(31000)$$

$$= \$32,820$$

 d. For $0 \leq x \leq 75,000$:

$$T = 0.08x$$

 For $75,000 < x \leq 200,000$:

$$T = 0.08(75,000) + 0.16(x - 75,000)$$

$$= 6000 + 0.16(x - 75,000)$$

 For $200,000 < x \leq 350,000$:

$$T = 0.08(75,000) + 0.16(125,000)$$

$$+ 0.22(x - 200,000)$$

$$= 26000 + 0.22(x - 200000)$$

 For $x > 350,000$:

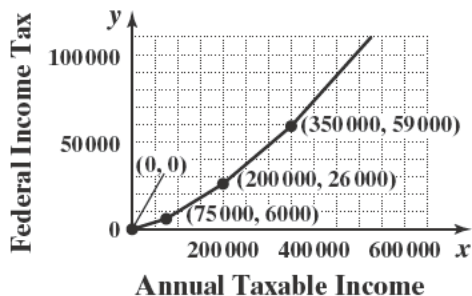
$$T = 59000 + 0.3(x - 350000)$$

Chapter 2: Functions and Their Graphs

The tax function:

$$T(x) = \begin{cases} 0.08x & \text{if } 0 \leq x \leq 75,000 \\ 6000 + 0.16(x - 75,000) & \text{if } 75,000 < x \leq 200,000 \\ 26,000 + 0.22(x - 200,000) & \text{if } 200,000 < x \leq 350,000 \\ 59,000 + 0.3(x - 350,000) & \text{if } x > 350,000 \end{cases}$$

e.



55. For schedule X:

$$f(x) = \begin{cases} 0.10x & \text{if } 0 < x \leq 11,000 \\ 1100 + 0.12(x - 11,000) & \text{if } 11,000 < x \leq 44,725 \\ 5,175 + 0.22(x - 44,725) & \text{if } 44,725 < x \leq 95,375 \\ 16,290 + 0.24(x - 95,375) & \text{if } 95,375 < x \leq 182,100 \\ 37,104 + 0.32(x - 182,100) & \text{if } 182,100 < x \leq 231,250 \\ 52,832 + 0.35(x - 231,250) & \text{if } 231,250 < x \leq 578,126 \\ 174,238 + 0.37(x - 578,126) & \text{if } x > 578,126 \end{cases}$$

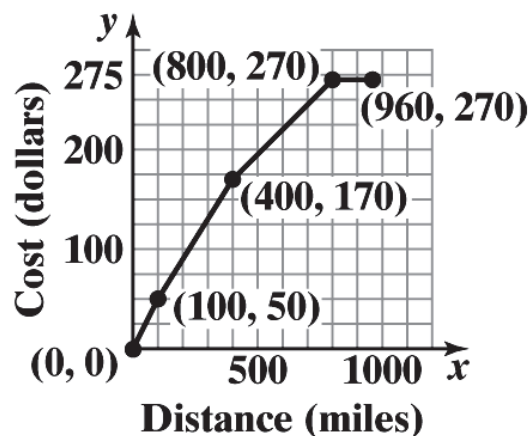
56. For Schedule Y-1:

$$f(x) = \begin{cases} 0.10x & \text{if } 0 < x \leq 22,000 \\ 2200 + 0.12(x - 22,000) & \text{if } 22,000 < x \leq 89,450 \\ 10,294 + 0.22(x - 89,450) & \text{if } 89,450 < x \leq 190,750 \\ 32,580 + 0.24(x - 190,750) & \text{if } 190,750 < x \leq 364,200 \\ 74,208 + 0.32(x - 364,200) & \text{if } 364,200 < x \leq 462,500 \\ 105,664 + 0.35(x - 462,500) & \text{if } 462,500 < x \leq 693,750 \\ 186,601 + 0.37(x - 693,750) & \text{if } x > 693,750 \end{cases}$$

57. a. Let x represent the number of miles and C be the cost of transportation.

$$C(x) = \begin{cases} 0.50x & \text{if } 0 \leq x \leq 100 \\ 0.50(100) + 0.40(x - 100) & \text{if } 100 < x \leq 400 \\ 0.50(100) + 0.40(300) + 0.25(x - 400) & \text{if } 400 < x \leq 800 \\ 0.50(100) + 0.40(300) + 0.25(400) + 0(x - 800) & \text{if } 800 < x \leq 960 \end{cases}$$

$$C(x) = \begin{cases} 0.50x & \text{if } 0 \leq x \leq 100 \\ 10 + 0.40x & \text{if } 100 < x \leq 400 \\ 70 + 0.25x & \text{if } 400 < x \leq 800 \\ 270 & \text{if } 800 < x \leq 960 \end{cases}$$

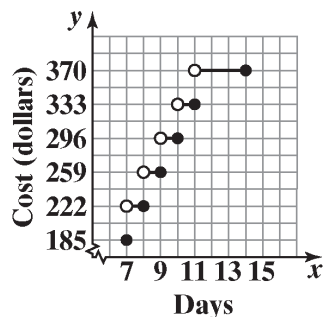


b. For hauls between 100 and 400 miles the cost is: $C(x) = 10 + 0.40x$.

c. For hauls between 400 and 800 miles the cost is: $C(x) = 70 + 0.25x$.

58. Let x = number of days car is used. The cost of renting is given by

$$C(x) = \begin{cases} 185 & \text{if } x = 7 \\ 222 & \text{if } 7 < x \leq 8 \\ 259 & \text{if } 8 < x \leq 9 \\ 296 & \text{if } 9 < x \leq 10 \\ 333 & \text{if } 10 < x \leq 11 \\ 370 & \text{if } 11 < x \leq 14 \end{cases}$$



59. a. Let s = the credit score of an individual who wishes to borrow \$300,000 with an 80% LTV ratio. The adverse market delivery charge is

given by

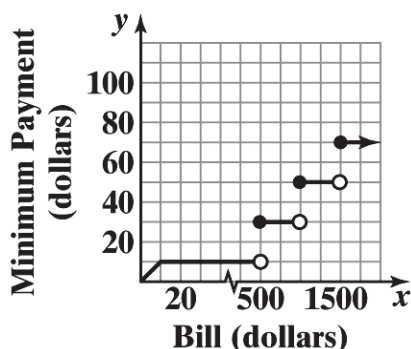
$$C(s) = \begin{cases} 9000 & \text{if } s \leq 659 \\ 8250 & \text{if } 660 \leq s \leq 679 \\ 5250 & \text{if } 680 \leq s \leq 699 \\ 3750 & \text{if } 700 \leq s \leq 719 \\ 2250 & \text{if } 720 \leq s \leq 739 \\ 1500 & \text{if } s \geq 740 \end{cases}$$

b. 725 is between 720 and 739 so the charge would be \$2250.

c. 670 is between 660 and 679 so the charge would be \$8250.

60. Let x = the amount of the bill in dollars. The minimum payment due is given by

$$f(x) = \begin{cases} x & \text{if } 0 \leq x < 10 \\ 10 & \text{if } 10 \leq x < 500 \\ 30 & \text{if } 500 \leq x < 1000 \\ 50 & \text{if } 1000 \leq x < 1500 \\ 70 & \text{if } x \geq 1500 \end{cases}$$



61. a. $W = 10^\circ C$
- b. $W = 33 - \frac{(10.45 + 10\sqrt{5} - 5)(33 - 10)}{22.04} \approx 4^\circ C$
- c. $W = 33 - \frac{(10.45 + 10\sqrt{15} - 15)(33 - 10)}{22.04} \approx -3^\circ C$
- d. $W = 33 - 1.5958(33 - 10) = -4^\circ C$
- e. When $0 \leq v < 1.79$, the wind speed is so small that there is no effect on the temperature.
- f. When the wind speed exceeds 20, the wind chill depends only on the air temperature.
62. a. $W = -10^\circ C$
- b. $W = 33 - \frac{(10.45 + 10\sqrt{5} - 5)(33 - (-10))}{22.04} \approx -21^\circ C$
- c. $W = 33 - \frac{(10.45 + 10\sqrt{15} - 15)(33 - (-10))}{22.04} \approx -34^\circ C$
- d. $W = 33 - 1.5958(33 - (-10)) = -36^\circ C$

63. Let x = the sales made and $S(x)$ = the total salary.
- For $0 \leq x \leq 250,000$: $S(x) = \$45,000 + 0.04x$
- For $250,000 < x \leq 500,000$:
- $$S(x) = 55,000 + 0.06(x - 250,000)$$
- For $x > 500,000$:
- $$S(x) = 70,000 + 0.09(x - 500,000)$$

64. Use intervals $(0, 8)$, $[8, 16)$, $[16, 32)$, $[32, 38)$ (exclude 0 and 38 since those would be the walls). Depth for the intervals $[8, 16)$ and $[32, 38)$ are constant (8 ft and 3 ft respectively). The other two are linear functions. On $(0, 8)$ the endpoint coordinates can be thought of as $(0, 3)$ and $(8, 8)$.

$$m = \frac{8-3}{8-0} = \frac{5}{8} \quad y = \frac{5}{8}x + 3$$

On $[16, 32)$ the endpoint coordinates can be thought of as $(16, 8)$ and $(32, 3)$.

$$m = \frac{3-8}{32-16} = -\frac{5}{16}$$

$$8 = -\frac{5}{16}(16) + b$$

$$13 = b$$

$$y = -\frac{5}{16}x + 13$$

Therefore,

$$d(x) = \begin{cases} \frac{5}{8}x + 3 & \text{if } 0 < x < 8 \\ 8 & \text{if } 8 \leq x < 16 \\ -\frac{5}{16}x + 13 & \text{if } 16 \leq x < 32 \\ 3 & \text{if } 32 \leq x < 38 \end{cases}$$

65. The function f changes definition at 2 and the function g changes definition at 0. Combining these together, the sum function will change definitions at 0 and 2.

On the interval $(-\infty, 0]$:

$$(f+g)(x) = f(x) + g(x) = (2x+3) + (-4x+1) = -2x+4$$

On the interval $(0, 2)$:

$$(f+g)(x) = f(x) + g(x) = (2x+3) + (x-7) = 3x-4$$

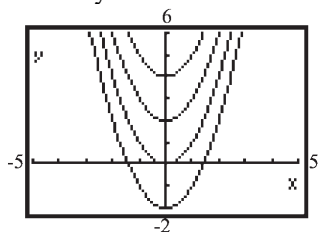
On the interval $[2, \infty)$:

$$(f+g)(x) = f(x) + g(x) = (x^2+5x) + (x-7) = x^2+6x-7$$

$$\text{So, } (f+g)(x) = \begin{cases} -2x+4 & \text{if } x \leq 0 \\ 3x-4 & \text{if } 0 < x < 2 \\ x^2+6x-7 & \text{if } x \geq 2 \end{cases}$$

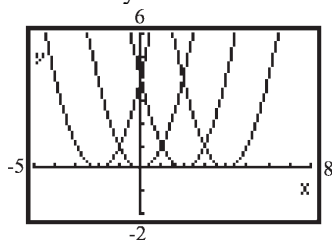
Section 2.4: Library of Functions; Piecewise-defined Functions

66. Each graph is that of $y = x^2$, but shifted vertically.



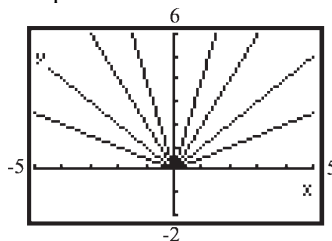
If $y = x^2 + k$, $k > 0$, the shift is up k units; if $y = x^2 - k$, $k > 0$, the shift is down k units. The graph of $y = x^2 - 4$ is the same as the graph of $y = x^2$, but shifted down 4 units. The graph of $y = x^2 + 5$ is the graph of $y = x^2$, but shifted up 5 units.

67. Each graph is that of $y = x^2$, but shifted horizontally.



If $y = (x - k)^2$, $k > 0$, the shift is to the right k units; if $y = (x + k)^2$, $k > 0$, the shift is to the left k units. The graph of $y = (x + 4)^2$ is the same as the graph of $y = x^2$, but shifted to the left 4 units. The graph of $y = (x - 5)^2$ is the graph of $y = x^2$, but shifted to the right 5 units.

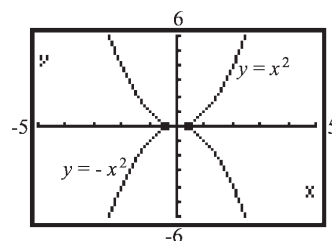
68. Each graph is that of $y = |x|$, but either compressed or stretched vertically.



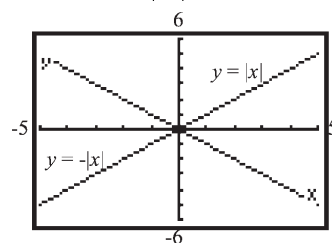
If $y = k|x|$ and $k > 1$, the graph is stretched vertically; if $y = k|x|$ and $0 < k < 1$, the graph is

compressed vertically. The graph of $y = \frac{1}{4}|x|$ is the same as the graph of $y = |x|$, but compressed vertically. The graph of $y = 5|x|$ is the same as the graph of $y = |x|$, but stretched vertically.

69. The graph of $y = -x^2$ is the reflection of the graph of $y = x^2$ about the x -axis.

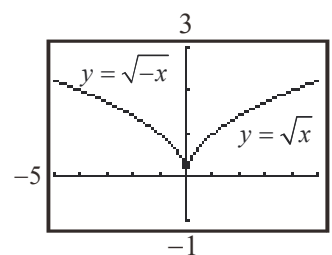


The graph of $y = -|x|$ is the reflection of the graph of $y = |x|$ about the x -axis.



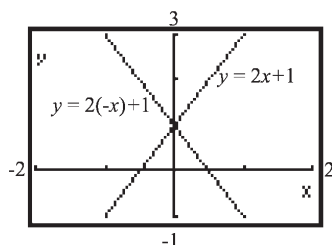
Multiplying a function by -1 causes the graph to be a reflection about the x -axis of the original function's graph.

70. The graph of $y = \sqrt{-x}$ is the reflection about the y -axis of the graph of $y = \sqrt{x}$.



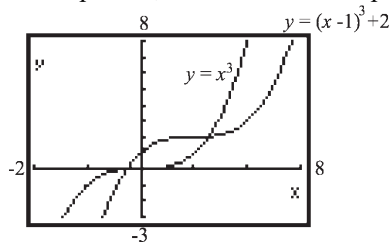
The same type of reflection occurs when graphing $y = 2x + 1$ and $y = 2(-x) + 1$.

Chapter 2: Functions and Their Graphs

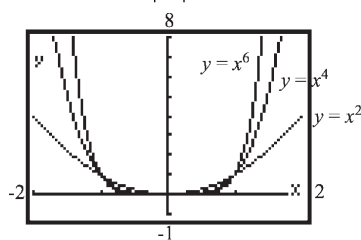


The graph of $y = f(-x)$ is the reflection about the y -axis of the graph of $y = f(x)$.

71. The graph of $y = (x-1)^3 + 2$ is a shifting of the graph of $y = x^3$ one unit to the right and two units up. Yes, the result could be predicted.

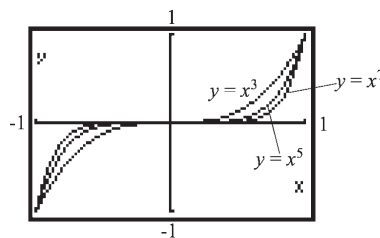


72. The graphs of $y = x^n$, n a positive even integer, are all U-shaped and open upward. All go through the points $(-1, 1)$, $(0, 0)$, and $(1, 1)$. As n increases, the graph of the function is narrower for $|x| > 1$ and flatter for $|x| < 1$.



73. The graphs of $y = x^n$, n a positive odd integer, all have the same general shape. All go through

the points $(-1, -1)$, $(0, 0)$, and $(1, 1)$. As n increases, the graph of the function increases at a greater rate for $|x| > 1$ and is flatter around 0 for $|x| < 1$.



74.
$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Yes, it is a function.

Domain = $\{x \mid x \text{ is any real number}\}$ or $(-\infty, \infty)$

Range = $\{0, 1\}$ or $\{y \mid y = 0 \text{ or } y = 1\}$

y -intercept: $x = 0 \Rightarrow x$ is rational $\Rightarrow y = 1$

So the y -intercept is $y = 1$.

x -intercept: $y = 0 \Rightarrow x$ is irrational

So the graph has infinitely many x -intercepts, namely, there is an x -intercept at each irrational value of x .

$f(-x) = 1 = f(x)$ when x is rational;

$f(-x) = 0 = f(x)$ when x is irrational.

Thus, f is even.

The graph of f consists of 2 infinite clusters of distinct points, extending horizontally in both directions. One cluster is located 1 unit above the x -axis, and the other is located along the x -axis.

75. Answers will vary.

76.
$$(x^{-3}y^5)^{-2} = x^6y^{-10} = \frac{x^6}{y^{10}}$$

Section 2.5: Graphing Techniques: Transformations

$$\begin{aligned}
 77. \quad & x^2 + y^2 = 6y + 16 \\
 & x^2 + y^2 - 6y = 16 \\
 & x^2 + (y^2 - 6y + 9) = 16 + 9 \\
 & x^2 + (y - 3)^2 = 5^2
 \end{aligned}$$

Center (h,k): (0, 3); Radius = 5

$$\begin{aligned}
 78. \quad & 4x - 5(2x - 1) = 4 - 7(x + 1) \\
 & 4x - 10x + 5 = 4 - 7x - 7 \\
 & -6x + 5 = -7x - 3 \\
 & x = -8
 \end{aligned}$$

The solution set is: $\{-8\}$

79. Let x represent the amount of money invested in a mutual fund. Then $60,000 - x$ represents the amount of money invested in CD's. Since the total interest is to be \$3700, we have:

$$\begin{aligned}
 & 0.08x + 0.03(60,000 - x) = 3700 \\
 & (100)(0.08x + 0.03(60,000 - x)) = (3700)(100) \\
 & 8x + 3(60,000 - x) = 370,000 \\
 & 8x + 180,000 - 3x = 370,000 \\
 & 5x + 180,000 = 370,000 \\
 & 5x = 190,000 \\
 & x = 38,000
 \end{aligned}$$

\$38,000 should be invested in a mutual fund at 8% and \$22,000 should be invested in CD's at 3%.

$$\begin{array}{r}
 80. \quad -2 \overline{) 1 \ 3 \ 0 \ -6} \\
 \underline{-2 \ -2 \ 4} \\
 1 \ 1 \ 6 \ -2 \\
 \text{Quotient: } x^2 + x - 2 \\
 \text{Remainder: } -2
 \end{array}$$

$$81. \quad \frac{3}{2} + 2i$$

$$82. \quad -2x^7$$

$$\begin{aligned}
 83. \quad & \sqrt{(5t^2)^2 + (25t^{7/2})^2} = \sqrt{25t^4 + 625t^7} \\
 & = \sqrt{25t^4(1 + 25t^3)} \\
 & = 5t^2\sqrt{1 + 25t^3}
 \end{aligned}$$

84. The radicand cannot be negative so:
 $x + 7 \geq 0$
 $x \geq -7$
The domain is $\{x \mid x \geq -7\}$.

$$\begin{aligned}
 85. \quad & 3x^3y - 2x^2y^2 + 18x - 12y = \\
 & (3x^3y - 2x^2y^2) + (18x - 12y) = \\
 & x^2y(3x - 2y) + 6(3x - 4y) = \\
 & (3x - 2y)(x^2y + 6)
 \end{aligned}$$

Section 2.5

- horizontal; right
- y
- False
- True; the graph of $y = -f(x)$ is the reflection about the x -axis of the graph of $y = f(x)$.
- d
- b
- B
- E
- H
- D
- I
- A
- L
- C
- F
- J
- G
- K
- $y = (x - 4)^3$
- $y = (x + 4)^3$
- $y = x^3 + 4$
- $y = x^3 - 4$

Chapter 2: Functions and Their Graphs

23. $y = (-x)^3 = -x^3$

24. $y = -x^3$

25. $y = 5x^3$

26. $y = \left(\frac{1}{4}x\right)^3 = \frac{1}{64}x^3$

27. $y = (2x)^3 = 8x^3$

28. $y = \frac{1}{4}x^3$

29. (1) $y = \sqrt{x} + 2$

(2) $y = -(\sqrt{x} + 2)$

(3) $y = -(\sqrt{-x} + 2) = -\sqrt{-x} - 2$

30. (1) $y = -\sqrt{x}$

(2) $y = -\sqrt{x-3}$

(3) $y = -\sqrt{x-3} - 2$

31. (1) $y = 3\sqrt{x}$

(2) $y = 3\sqrt{x} + 4$

(3) $y = 3\sqrt{x+5} + 4$

32. (1) $y = \sqrt{x} + 2$

(2) $y = \sqrt{-x} + 2$

(3) $y = \sqrt{-(x+3)} + 2 = \sqrt{-x-3} + 2$

33. (c); To go from $y = f(x)$ to $y = -f(x)$ we reflect about the x -axis. This means we change the sign of the y -coordinate for each point on the graph of $y = f(x)$. Thus, the point $(3, 6)$ would become $(3, -6)$.

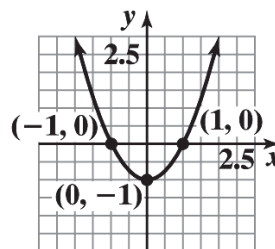
34. (d); To go from $y = f(x)$ to $y = f(-x)$, we reflect each point on the graph of $y = f(x)$ about the y -axis. This means we change the sign of the x -coordinate for each point on the graph of $y = f(x)$. Thus, the point $(3, 6)$ would become $(-3, 6)$.

35. (c); To go from $y = f(x)$ to $y = 2f(x)$, we stretch vertically by a factor of 2. Multiply the y -coordinate of each point on the graph of $y = f(x)$ by 2. Thus, the point $(1, 3)$ would become $(1, 6)$.

36. (c); To go from $y = f(x)$ to $y = f(2x)$, we compress horizontally by a factor of 2. Divide the x -coordinate of each point on the graph of $y = f(x)$ by 2. Thus, the point $(4, 2)$ would become $(2, 2)$.

37. $f(x) = x^2 - 1$

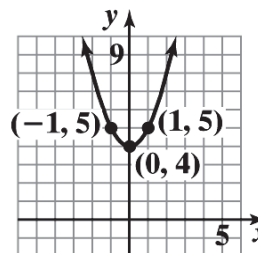
Using the graph of $y = x^2$, vertically shift downward 1 unit.



The domain is $(-\infty, \infty)$ and the range is $[-1, \infty)$.

38. $f(x) = x^2 + 4$

Using the graph of $y = x^2$, vertically shift upward 4 units.

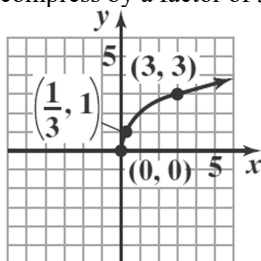


The domain is $(-\infty, \infty)$ and the range is $[4, \infty)$.

Section 2.5: Graphing Techniques: Transformations

39. $g(x) = \sqrt{3x}$

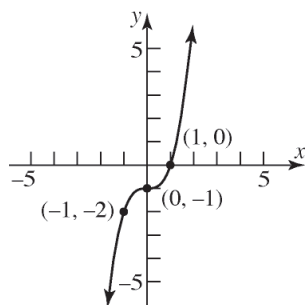
Using the graph of $y = \sqrt{x}$, horizontally compress by a factor of 3.



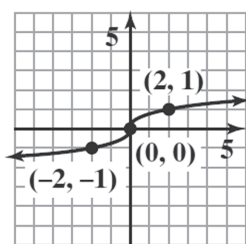
The domain is $[0, \infty)$ and the range is $[0, \infty)$.

40. $g(x) = \sqrt[3]{\frac{1}{2}x}$

Using the graph of $y = \sqrt[3]{x}$, horizontally stretch by a factor of $\frac{1}{2}$.

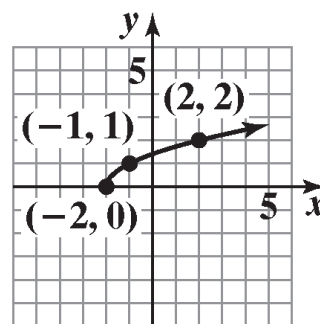


The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.



41. $h(x) = \sqrt{x+2}$

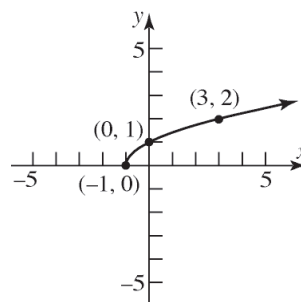
Using the graph of $y = \sqrt{x}$, horizontally shift to the left 2 units.



The domain is $[-2, \infty)$ and the range is $[0, \infty)$.

42. $h(x) = \sqrt{x+1}$

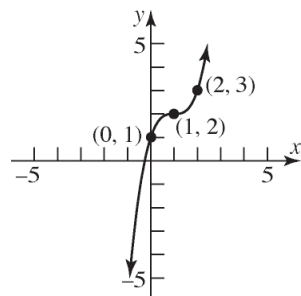
Using the graph of $y = \sqrt{x}$, horizontally shift to the left 1 unit.



The domain is $[-1, \infty)$ and the range is $[0, \infty)$.

43. $f(x) = (x-1)^3 + 2$

Using the graph of $y = x^3$, horizontally shift to the right 1 unit $[y = (x-1)^3]$, then vertically shift up 2 units $[y = (x-1)^3 + 2]$.

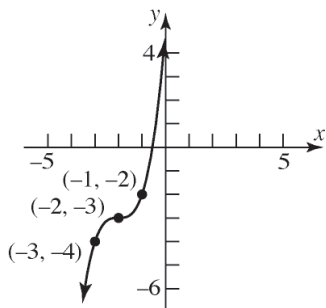


The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

Chapter 2: Functions and Their Graphs

44. $f(x) = (x+2)^3 - 3$

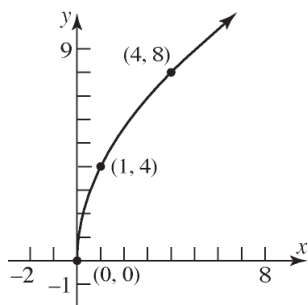
Using the graph of $y = x^3$, horizontally shift to the left 2 units $\left[y = (x+2)^3 \right]$, then vertically shift down 3 units $\left[y = (x+2)^3 - 3 \right]$.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

45. $g(x) = 4\sqrt{x}$

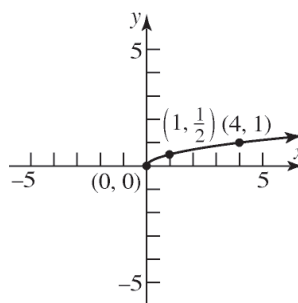
Using the graph of $y = \sqrt{x}$, vertically stretch by a factor of 4.



The domain is $[0, \infty)$ and the range is $[0, \infty)$.

46. $g(x) = \frac{1}{2}\sqrt{x}$

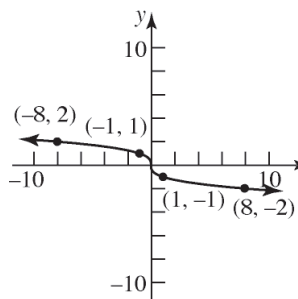
Using the graph of $y = \sqrt{x}$, vertically compress by a factor of $\frac{1}{2}$.



The domain is $[0, \infty)$ and the range is $[0, \infty)$.

47. $f(x) = -\sqrt[3]{x}$

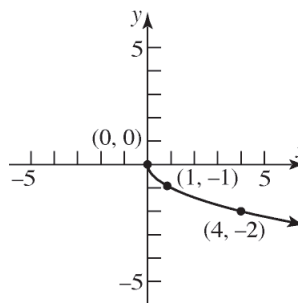
Using the graph of $y = \sqrt[3]{x}$, reflect the graph about the x -axis.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

48. $f(x) = -\sqrt{x}$

Using the graph of $y = \sqrt{x}$, reflect the graph about the x -axis.

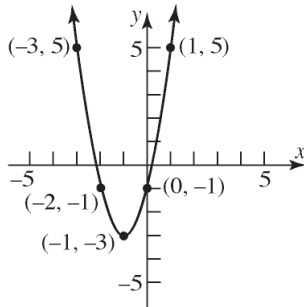


The domain is $[0, \infty)$ and the range is $(-\infty, 0]$.

Section 2.5: Graphing Techniques: Transformations

49. $f(x) = 2(x+1)^2 - 3$

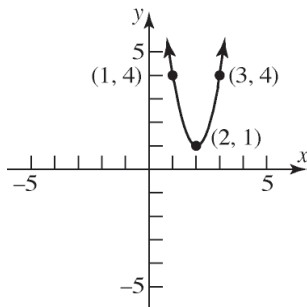
Using the graph of $y = x^2$, horizontally shift to the left 1 unit $\left[y = (x+1)^2 \right]$, vertically stretch by a factor of 2 $\left[y = 2(x+1)^2 \right]$, and then vertically shift downward 3 units $\left[y = 2(x+1)^2 - 3 \right]$.



The domain is $(-\infty, \infty)$ and the range is $[-3, \infty)$.

50. $f(x) = 3(x-2)^2 + 1$

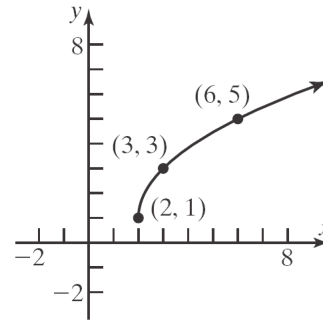
Using the graph of $y = x^2$, horizontally shift to the right 2 units $\left[y = (x-2)^2 \right]$, vertically stretch by a factor of 3 $\left[y = 3(x-2)^2 \right]$, and then vertically shift upward 1 unit $\left[y = 3(x-2)^2 + 1 \right]$.



The domain is $(-\infty, \infty)$ and the range is $[1, \infty)$.

51. $g(x) = 2\sqrt{x-2} + 1$

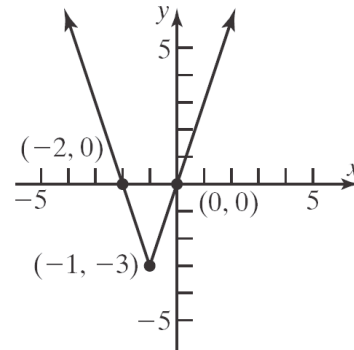
Using the graph of $y = \sqrt{x}$, horizontally shift to the right 2 units $\left[y = \sqrt{x-2} \right]$, vertically stretch by a factor of 2 $\left[y = 2\sqrt{x-2} \right]$, and vertically shift upward 1 unit $\left[y = 2\sqrt{x-2} + 1 \right]$.



The domain is $[2, \infty)$ and the range is $[1, \infty)$.

52. $g(x) = 3|x+1| - 3$

Using the graph of $y = |x|$, horizontally shift to the left 1 unit $\left[y = |x+1| \right]$, vertically stretch by a factor of 3 $\left[y = 3|x+1| \right]$, and vertically shift downward 3 units $\left[y = 3|x+1| - 3 \right]$.

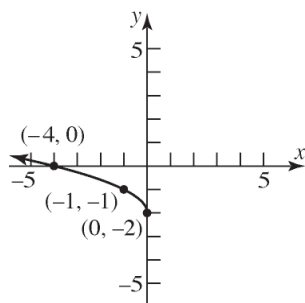


The domain is $(-\infty, \infty)$ and the range is $[-3, \infty)$.

Chapter 2: Functions and Their Graphs

53. $h(x) = \sqrt{-x} - 2$

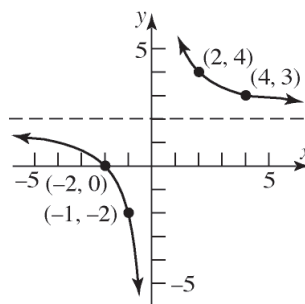
Using the graph of $y = \sqrt{x}$, reflect the graph about the y -axis $[y = \sqrt{-x}]$ and vertically shift downward 2 units $[y = \sqrt{-x} - 2]$.



The domain is $(-\infty, 0]$ and the range is $[-2, \infty)$.

54. $h(x) = \frac{4}{x} + 2 = 4\left(\frac{1}{x}\right) + 2$

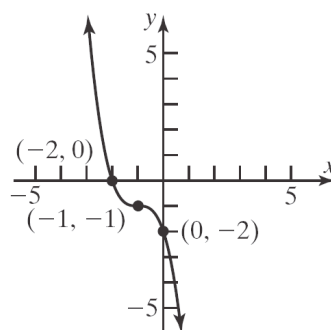
Stretch the graph of $y = \frac{1}{x}$ vertically by a factor of 4 $[y = 4 \cdot \frac{1}{x} = \frac{4}{x}]$ and vertically shift upward 2 units $[y = \frac{4}{x} + 2]$.



The domain is $(-\infty, 0) \cup (0, \infty)$ and the range is $(-\infty, 2) \cup (2, \infty)$.

55. $f(x) = -(x+1)^3 - 1$

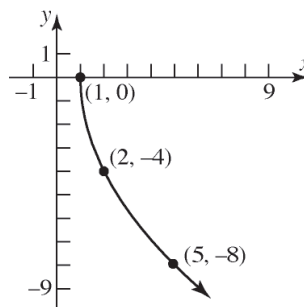
Using the graph of $y = x^3$, horizontally shift to the left 1 unit $[y = (x+1)^3]$, reflect the graph about the x -axis $[y = -(x+1)^3]$, and vertically shift downward 1 unit $[y = -(x+1)^3 - 1]$.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

56. $f(x) = -4\sqrt{x-1}$

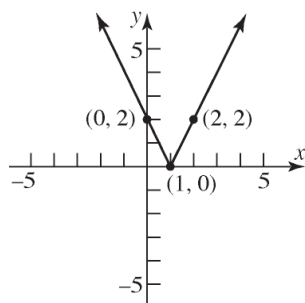
Using the graph of $y = \sqrt{x}$, horizontally shift to the right 1 unit $[y = \sqrt{x-1}]$, reflect the graph about the x -axis $[y = -\sqrt{x-1}]$, and stretch vertically by a factor of 4 $[y = -4\sqrt{x-1}]$.



The domain is $[1, \infty)$ and the range is $(-\infty, 0]$.

57. $g(x) = 2|1-x| = 2| -(-1+x) | = 2|x-1|$

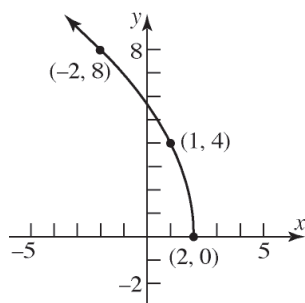
Using the graph of $y = |x|$, horizontally shift to the right 1 unit $[y = |x-1|]$, and vertically stretch by a factor of 2 $[y = 2|x-1|]$.



The domain is $(-\infty, \infty)$ and the range is $[0, \infty)$.

58. $g(x) = 4\sqrt{2-x} = 4\sqrt{-(x-2)}$

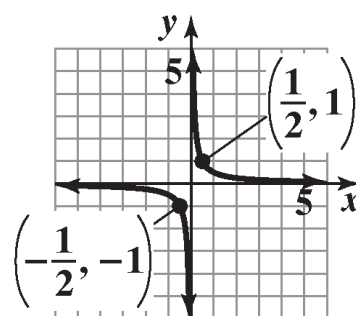
Using the graph of $y = \sqrt{x}$, reflect the graph about the y -axis $[y = \sqrt{-x}]$, horizontally shift to the right 2 units $[y = \sqrt{-(x-2)}]$, and vertically stretch by a factor of 4 $[y = 4\sqrt{-(x-2)}]$.



The domain is $(-\infty, 2]$ and the range is $[0, \infty)$.

59. $h(x) = \frac{1}{2x}$

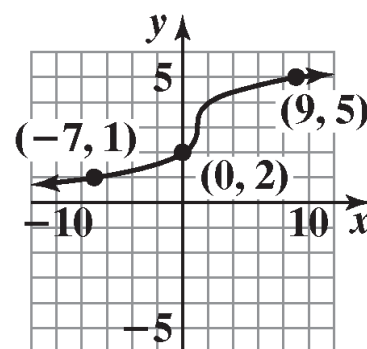
Using the graph of $y = \frac{1}{x}$, vertically compress by a factor of $\frac{1}{2}$.



The domain is $(-\infty, 0) \cup (0, \infty)$ and the range is $(-\infty, 0) \cup (0, \infty)$.

60. $f(x) = \sqrt[3]{x-1} + 3$

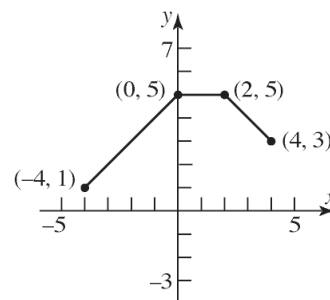
Using the graph of $f(x) = \sqrt[3]{x}$, horizontally shift to the right 1 unit $[y = \sqrt[3]{x-1}]$, then vertically shift up 3 units $[y = \sqrt[3]{x-1} + 3]$.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

61. a. $F(x) = f(x) + 3$

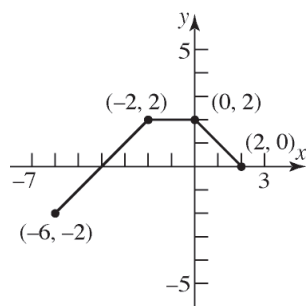
Shift up 3 units.



Chapter 2: Functions and Their Graphs

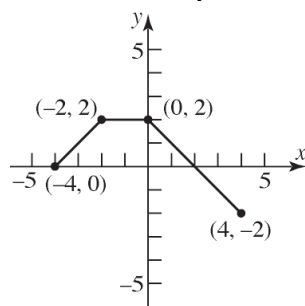
b. $G(x) = f(x+2)$

Shift left 2 units.



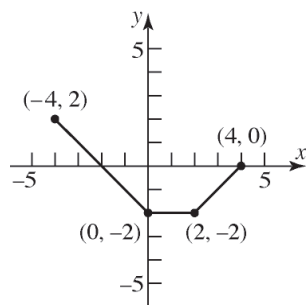
f. $g(x) = f(-x)$

Reflect about the y-axis.



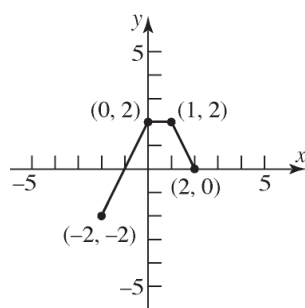
c. $P(x) = -f(x)$

Reflect about the x-axis.



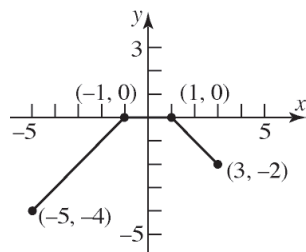
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



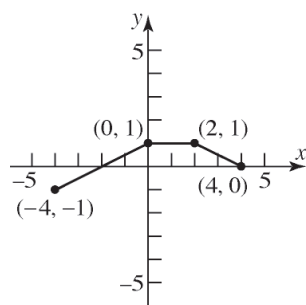
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



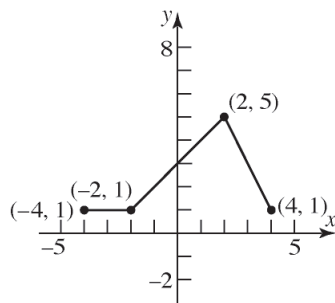
e. $Q(x) = \frac{1}{2}f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



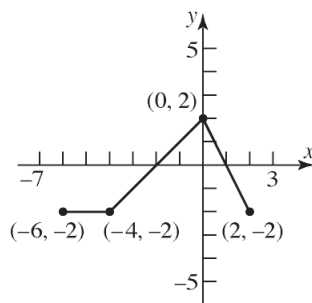
62. a. $F(x) = f(x) + 3$

Shift up 3 units.



b. $G(x) = f(x+2)$

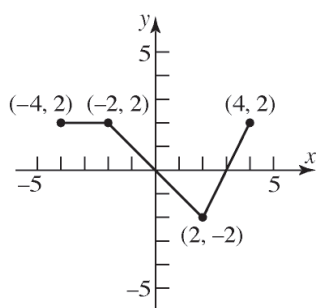
Shift left 2 units.



Section 2.5: Graphing Techniques: Transformations

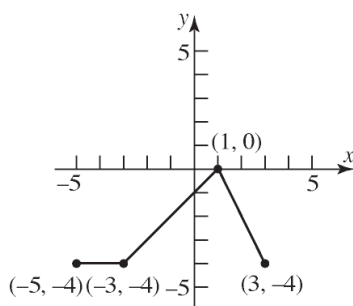
c. $P(x) = -f(x)$

Reflect about the x -axis.



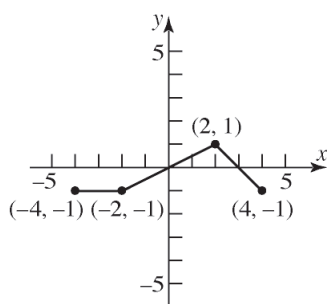
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



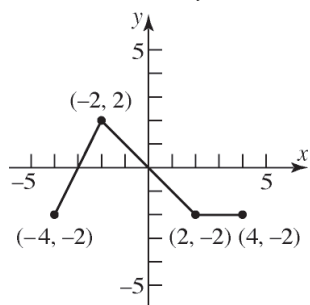
e. $Q(x) = \frac{1}{2}f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



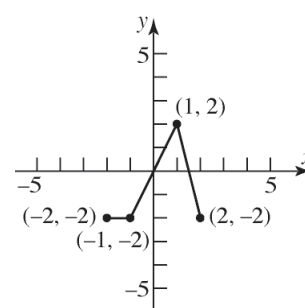
f. $g(x) = f(-x)$

Reflect about the y -axis.



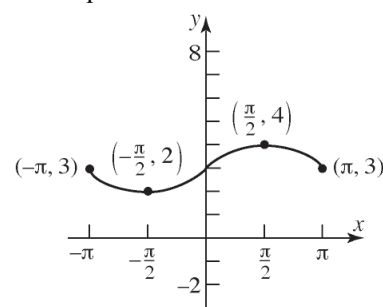
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



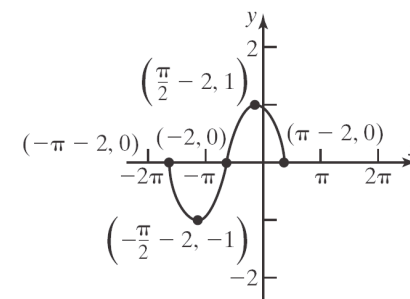
63. a. $F(x) = f(x) + 3$

Shift up 3 units.



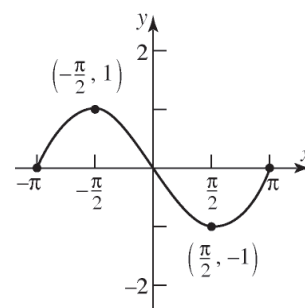
b. $G(x) = f(x+2)$

Shift left 2 units.



c. $P(x) = -f(x)$

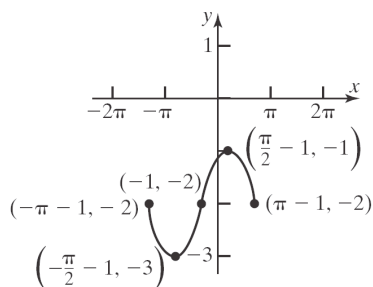
Reflect about the x -axis.



Chapter 2: Functions and Their Graphs

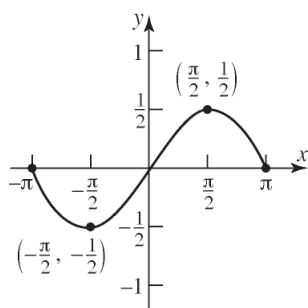
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



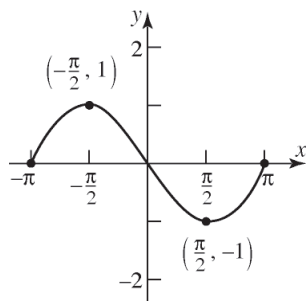
e. $Q(x) = \frac{1}{2}f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



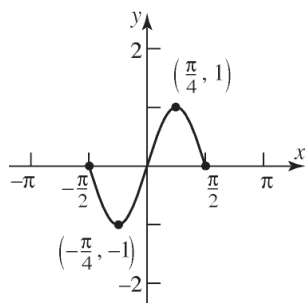
f. $g(x) = f(-x)$

Reflect about the y-axis.



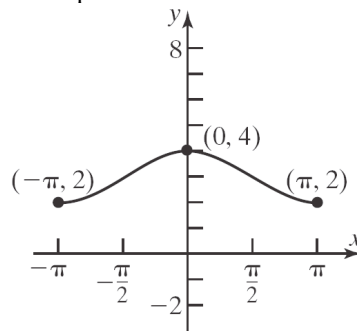
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



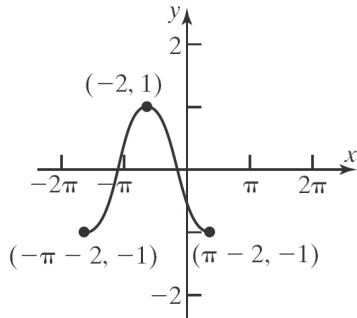
64. a. $F(x) = f(x) + 3$

Shift up 3 units.



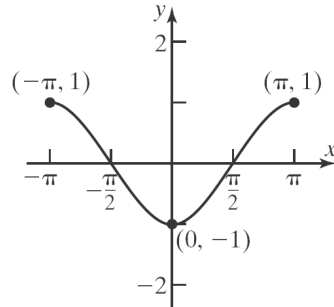
b. $G(x) = f(x+2)$

Shift left 2 units.



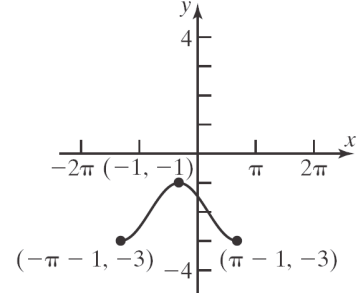
c. $P(x) = -f(x)$

Reflect about the x-axis.



d. $H(x) = f(x+1) - 2$

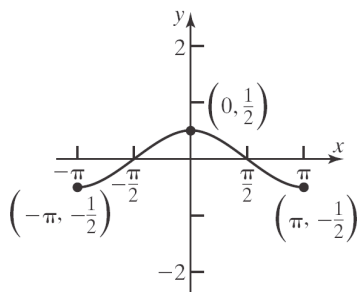
Shift left 1 unit and shift down 2 units.



Section 2.5: Graphing Techniques: Transformations

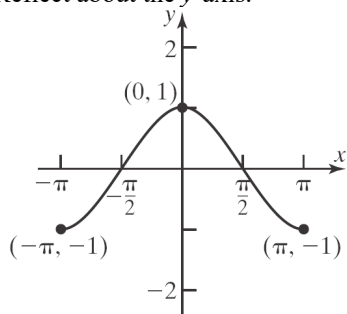
e. $Q(x) = \frac{1}{2}f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



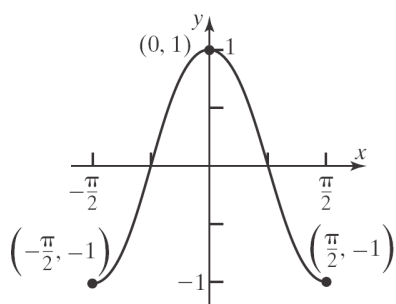
f. $g(x) = f(-x)$

Reflect about the y-axis.



g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



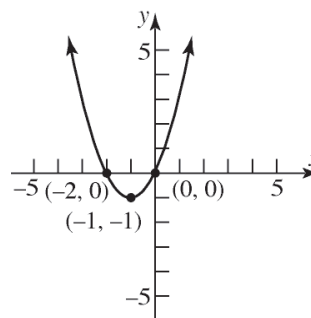
65. $f(x) = x^2 + 2x$

$$f(x) = (x^2 + 2x + 1) - 1$$

$$f(x) = (x + 1)^2 - 1$$

Using $f(x) = x^2$, shift left 1 unit and shift down

1 unit.

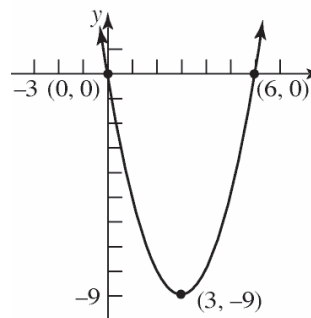


66. $f(x) = x^2 - 6x$

$$f(x) = (x^2 - 6x + 9) - 9$$

$$f(x) = (x - 3)^2 - 9$$

Using $f(x) = x^2$, shift right 3 units and shift down 9 units.

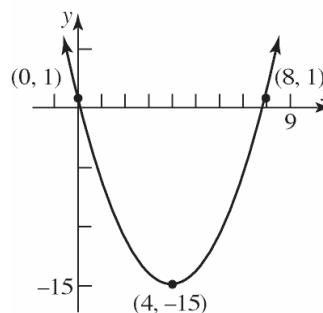


67. $f(x) = x^2 - 8x + 1$

$$f(x) = (x^2 - 8x + 16) + 1 - 16$$

$$f(x) = (x - 4)^2 - 15$$

Using $f(x) = x^2$, shift right 4 units and shift down 15 units.



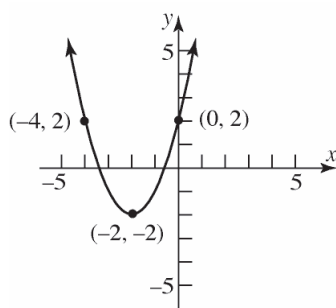
68. $f(x) = x^2 + 4x + 2$

$$f(x) = (x^2 + 4x + 4) + 2 - 4$$

$$f(x) = (x + 2)^2 - 2$$

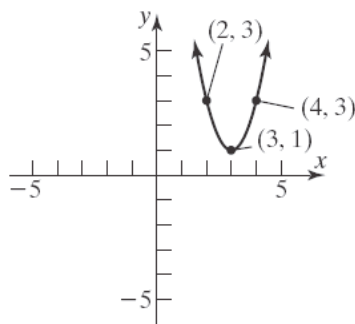
Chapter 2: Functions and Their Graphs

Using $f(x) = x^2$, shift left 2 units and shift down 2 units.



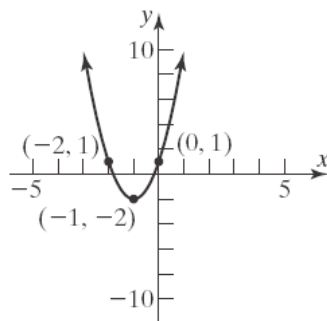
$$\begin{aligned} 69. \quad f(x) &= 2x^2 - 12x + 19 \\ &= 2(x^2 - 6x) + 19 \\ &= 2(x^2 - 6x + 9) + 19 - 18 \\ &= 2(x - 3)^2 + 1 \end{aligned}$$

Using $f(x) = x^2$, shift right 3 units, vertically stretch by a factor of 2, and then shift up 1 unit.



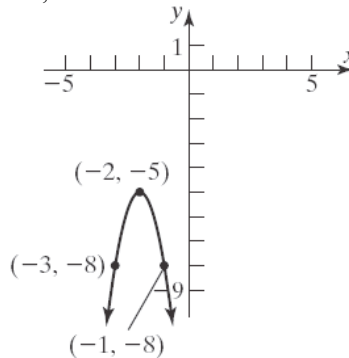
$$\begin{aligned} 70. \quad f(x) &= 3x^2 + 6x + 1 \\ &= 3(x^2 + 2x) + 1 \\ &= 3(x^2 + 2x + 1) + 1 - 3 \\ &= 3(x + 1)^2 - 2 \end{aligned}$$

Using $f(x) = x^2$, shift left 1 unit, vertically stretch by a factor of 3, and shift down 2 units.



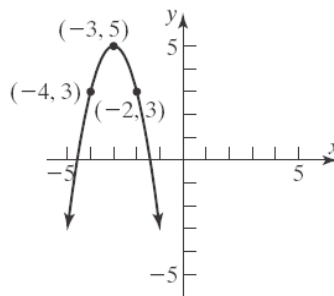
$$\begin{aligned} 71. \quad f(x) &= -3x^2 - 12x - 17 \\ &= -3(x^2 + 4x) - 17 \\ &= -3(x^2 + 4x + 4) - 17 + 12 \\ &= -3(x + 2)^2 - 5 \end{aligned}$$

Using $f(x) = x^2$, shift left 2 units, stretch vertically by a factor of 3, reflect about the x -axis, and shift down 5 units.



$$\begin{aligned} 72. \quad f(x) &= -2x^2 - 12x - 13 \\ &= -2(x^2 + 6x) - 13 \\ &= -2(x^2 + 6x + 9) - 13 + 18 \\ &= -2(x + 3)^2 + 5 \end{aligned}$$

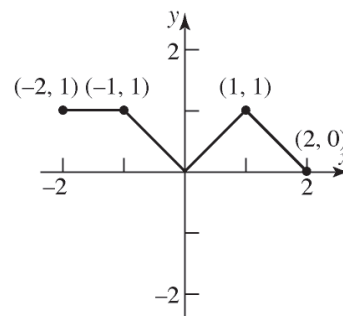
Using $f(x) = x^2$, shift left 3 units, stretch vertically by a factor of 2, reflect about the x -axis, and shift up 5 units.



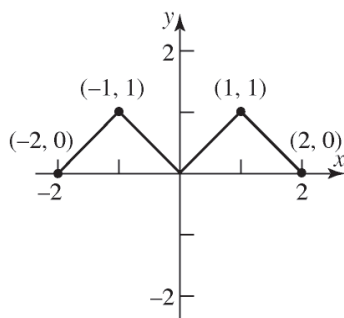
73. a. The graph of $y = f(x + 2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the x -intercepts are -7 and 1 .
- b. The graph of $y = f(x - 2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the right.

Section 2.5: Graphing Techniques: Transformations

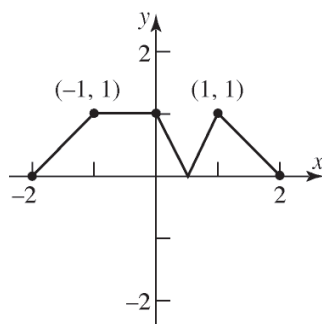
- the right. Therefore, the x -intercepts are -3 and 5 .
- c. The graph of $y = 4f(x)$ is the same as the graph of $y = f(x)$, but stretched vertically by a factor of 4. Therefore, the x -intercepts are still -5 and 3 since the y -coordinate of each is 0.
- d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y -axis. Therefore, the x -intercepts are 5 and -3 .
74. a. The graph of $y = f(x+4)$ is the same as the graph of $y = f(x)$, but shifted 4 units to the left. Therefore, the x -intercepts are -12 and -3 .
- b. The graph of $y = f(x-3)$ is the same as the graph of $y = f(x)$, but shifted 3 units to the right. Therefore, the x -intercepts are -5 and 4 .
- c. The graph of $y = 2f(x)$ is the same as the graph of $y = f(x)$, but stretched vertically by a factor of 2. Therefore, the x -intercepts are still -8 and 1 since the y -coordinate of each is 0.
- d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y -axis. Therefore, the x -intercepts are 8 and -1 .
75. a. The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is increasing on the interval $[-3, 3]$.
- b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is increasing on the interval $[4, 10]$.
- c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x -axis. Therefore, we can say that the graph of $y = -f(x)$ must be *decreasing* on the interval $[-1, 5]$.
- d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y -axis. Therefore, we can say that the graph of $y = f(-x)$ must be *decreasing* on the interval $[-5, 1]$.
76. a. The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is decreasing on the interval $[-4, 5]$.
- b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is decreasing on the interval $[3, 12]$.
- c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x -axis. Therefore, we can say that the graph of $y = -f(x)$ must be *increasing* on the interval $[-2, 7]$.
- d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y -axis. Therefore, we can say that the graph of $y = f(-x)$ must be *increasing* on the interval $[-7, 2]$.
77. a. $y = |f(x)|$



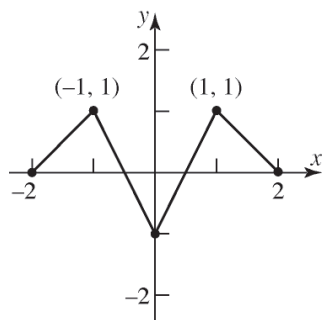
b. $y = f(|x|)$



78. a. To graph $y = |f(x)|$, the part of the graph for f that lies in quadrants III or IV is reflected about the x -axis.



- b. To graph $y = f(|x|)$, the part of the graph for f that lies in quadrants II or III is replaced by the reflection of the part in quadrants I and IV reflected about the y -axis.



79. a. The graph of $y = f(x+3)-5$ is the graph of $y = f(x)$ but shifted left 3 units and down 5 units. Thus, the point $(1,3)$ becomes the point $(-2,-2)$.

- b. The graph of $y = -2f(x-2)+1$ is the graph of $y = f(x)$ but shifted right 2 units, stretched vertically by a factor of 2, reflected about the x -axis, and shifted up 1 unit. Thus, the point $(1,3)$ becomes the point $(3,-5)$.

- c. The graph of $y = f(2x+3)$ is the graph of $y = f(x)$ but shifted left 3 units and horizontally compressed by a factor of 2. Thus, the point $(1,3)$ becomes the point $(-1,3)$.

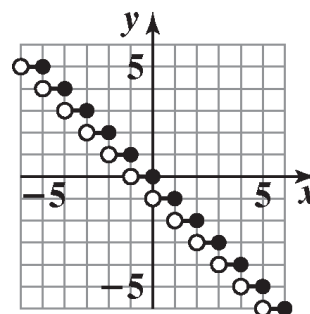
80. a. The graph of $y = g(x+1)-3$ is the graph of $y = g(x)$ but shifted left 1 unit and down 3 units. Thus, the point $(-3,5)$ becomes the point $(-4,2)$.

- b. The graph of $y = -3g(x-4)+3$ is the graph of $y = g(x)$ but shifted right 4 units, stretched vertically by a factor of 3, reflected about the x -axis, and shifted up 3 units. Thus, the point $(-3,5)$ becomes the point $(1,-12)$.

- c. The graph of $y = g(3x+9)$ is the graph of $y = g(x)$ but shifted left 9 units and horizontally compressed by a factor of 3. Thus, the point $(-3,5)$ becomes the point $(-4,5)$.

81. a. $f(x) = \text{int}(-x)$

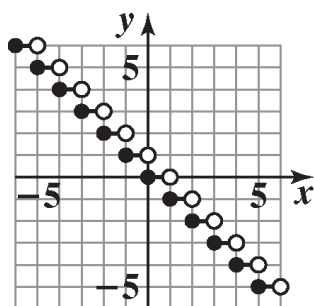
Reflect the graph of $y = \text{int}(x)$ about the y -axis.



- b. $g(x) = -\text{int}(x)$

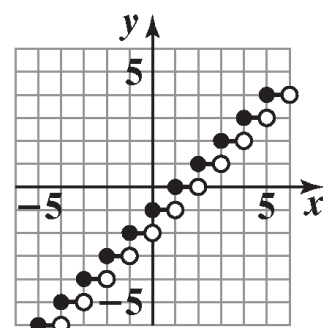
Section 2.5: Graphing Techniques: Transformations

Reflect the graph of $y = \text{int}(x)$ about the x -axis.



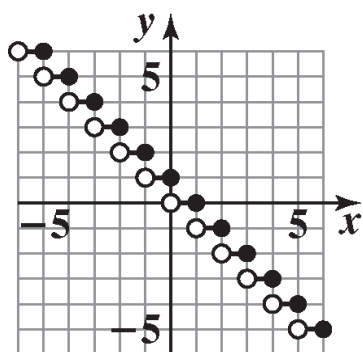
82. a. $f(x) = \text{int}(x-1)$

Shift the graph of $y = \text{int}(x)$ right 1 unit.



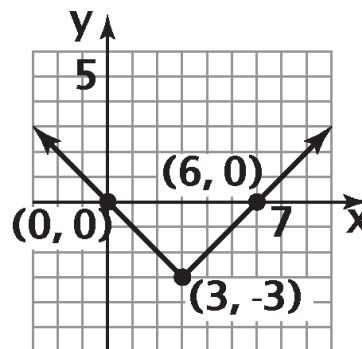
b. $g(x) = \text{int}(1-x) = \text{int}(-(x-1))$

Using the graph of $y = \text{int}(x)$, reflect the graph about the y -axis [$y = \text{int}(-x)$], horizontally shift to the right 1 unit [$y = \text{int}(-(x-1))$].



83. a. $f(x) = |x-3|-3$

Using the graph of $y = |x|$, horizontally shift to the right 3 units [$y = |x-3|$] and vertically shift downward 3 units [$y = |x-3|-3$].



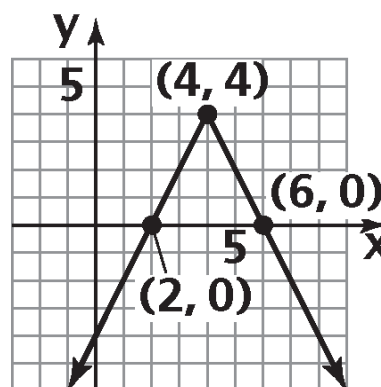
b. $A = \frac{1}{2}bh$

$$= \frac{1}{2}(6)(3) = 9$$

The area is 9 square units.

84. a. $f(x) = -2|x-4|+4$

Using the graph of $y = |x|$, horizontally shift to the right 4 units [$y = |x-4|$], vertically stretch by a factor of 2 and flip on the x -axis [$y = -2|x-4|$], and vertically shift upward 4 units [$y = -2|x-4|+4$].



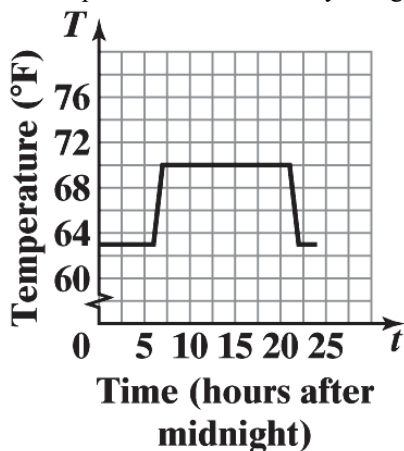
b. $A = \frac{1}{2}bh$

$$= \frac{1}{2}(4)(4) = 8$$

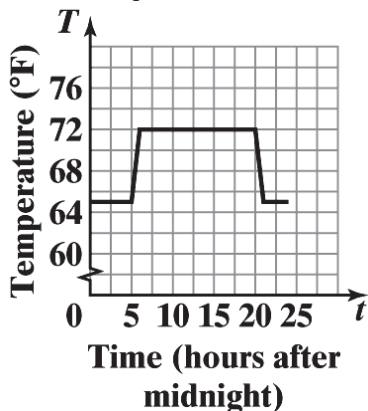
The area is 8 square units.

Chapter 2: Functions and Their Graphs

85. a. From the graph, the thermostat is set at 72°F during the daytime hours. The thermostat appears to be set at 65°F overnight.
- b. To graph $y = T(t) - 2$, the graph of $T(t)$ is shifted down 2 units. This change will lower the temperature in the house by 2 degrees.



- c. To graph $y = T(t+1)$, the graph of $T(t)$ should be shifted left one unit. This change will cause the program to switch between the daytime temperature and overnight temperature one hour sooner. The home will begin warming up at 5 a.m. instead of 6 a.m. and will begin cooling down at 8 p.m. instead of 9 p.m.



86. a. $f(0) = -0.12(0)^2 + 1.29(0) + 9.96 = 9.96$
The estimated hours of daylight in January is 9.96.

$$f(6) = -0.12(6)^2 + 1.29(6) + 9.96 = 13.38$$

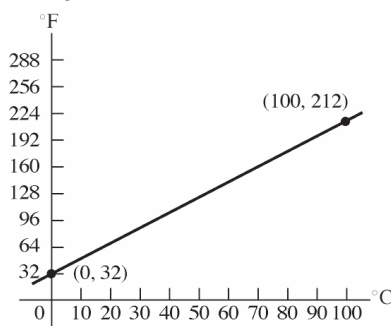
The estimated hours of daylight in July is 13.38.

$$f(11) = -0.12(11)^2 + 1.29(11) + 9.96 = 9.63$$

The estimated hours of daylight in December is 13.38.

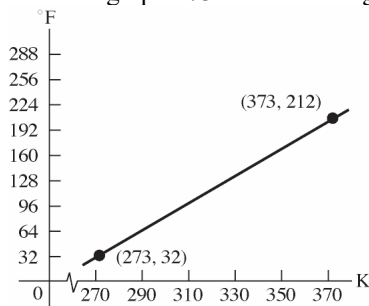
- b. x represents the number of months after July.
- c. January corresponds to:
- $$F(6) = f(6-6) = f(0)$$
- $$f(0) = 9.96$$
- d. Orlando is in the Northern Hemisphere and Alice Springs is in the Southern Hemisphere; therefore, they have "opposite" seasons in each month. For example, it is winter in Orlando in January while it is summer in Alice Spring.

$$87. F = \frac{9}{5}C + 32$$



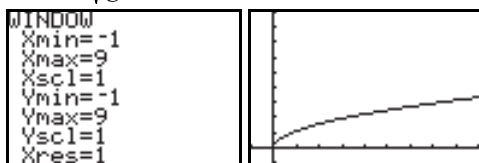
$$F = \frac{9}{5}(K - 273) + 32$$

Shift the graph 273 units to the right.



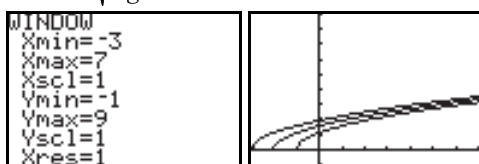
Section 2.5: Graphing Techniques: Transformations

88. a. $T = 2\pi\sqrt{\frac{l}{g}}$



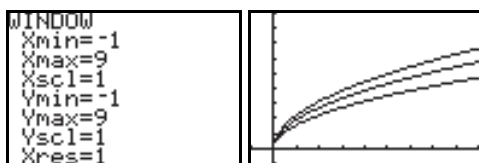
b. $T_1 = 2\pi\sqrt{\frac{l+1}{g}}; T_2 = 2\pi\sqrt{\frac{l+2}{g}};$

$T_3 = 2\pi\sqrt{\frac{l+3}{g}}$



- c. As the length of the pendulum increases, the period increases.

d. $T_1 = 2\pi\sqrt{\frac{2l}{g}}; T_2 = 2\pi\sqrt{\frac{3l}{g}}; T_3 = 2\pi\sqrt{\frac{4l}{g}}$



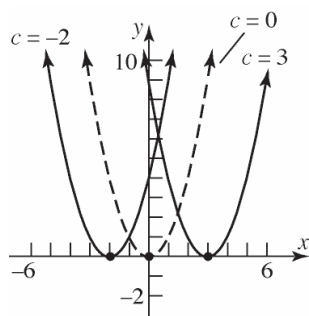
- e. If the length of the pendulum is multiplied by k , the period is multiplied by $\sqrt{k}$.

89. $y = (x-c)^2$

If $c = 0$, $y = x^2$.

If $c = 3$, $y = (x-3)^2$; shift right 3 units.

If $c = -2$, $y = (x+2)^2$; shift left 2 units.

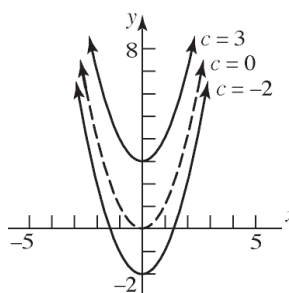


90. $y = x^2 + c$

If $c = 0$, $y = x^2$.

If $c = 3$, $y = x^2 + 3$; shift up 3 units.

If $c = -2$, $y = x^2 - 2$; shift down 2 units.



91. $f(x-5)$ is a shift right 5 units; increasing on $[2, 8]$ and $[16, 24]$; decreasing on $[8, 16]$.

$f(2x-5)$ compresses horizontally by a factor of $\frac{1}{2}$; increasing on $[1, 4]$ and $[8, 12]$; decreasing on $[4, 8]$. $-f(2x-5)$ reflects about the x -axis;

increasing on $[4, 8]$; decreasing on

$[1, 4]$ and $[8, 12]$. $-3f(2x-5)$ stretches

vertically by a factor of 3 but does not affect increasing/decreasing. Therefore $-3f(2x-5)$ is increasing on $[4, 8]$.

92. Write the general normal density as

$$f(x) = \frac{1}{\sigma} \cdot \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{\left(\frac{x-\mu}{\sigma} \right)^2}{2} \right]. \text{ Starting}$$

with the standard normal density,

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{x^2}{2} \right], \text{ stretch/compress}$$

horizontally by a factor of σ to get

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{\left(\frac{x}{\sigma} \right)^2}{2} \right]$$

$\left[\text{multiply all the } x\text{-coordinates by } \frac{1}{\sigma} \right], \text{ then shift}$

the graph horizontally $|\mu|$ units (left if $\mu < 0$ and right if $\mu > 0$) to get

Chapter 2: Functions and Their Graphs

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{\left(\frac{x-\mu}{\sigma} \right)^2}{2} \right]. \text{ Then}$$

stretch/compress vertically by a factor of $\frac{1}{\sigma}$ to get

$$f(x) = \frac{1}{\sigma} \cdot \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{\left(\frac{x-\mu}{\sigma} \right)^2}{2} \right]$$

[multiply all the y -coordinates by $\frac{1}{\sigma}$].

1. Stretch/compress horizontally by a factor of σ (stretch if $\sigma > 1$)
2. Shift horizontally $|\mu|$ units (left if $\mu < 0$ and right if $\mu > 0$).
3. Stretch/compress vertically by a factor of $\frac{1}{\sigma}$ (compress if $\sigma > 1$)

93. The graph of $y = 4f(x)$ is a vertical stretch of the graph of f by a factor of 4, while the graph of $y = f(4x)$ is a horizontal compression of the graph of f by a factor of $\frac{1}{4}$.

94. The graph of $y = f(x) - 2$ will shift the graph of $y = f(x)$ down by 2 units. The graph of $y = f(x - 2)$ will shift the graph of $y = f(x)$ to the right by 2 units.

95. The graph of $y = \sqrt{-x}$ is the graph of $y = \sqrt{x}$ but reflected about the y -axis. Therefore, our region is simply rotated about the y -axis and does not change shape. Instead of the region being bounded on the right by $x = 4$, it is bounded on the left by $x = -4$. Thus, the area of

the second region would also be $\frac{16}{3}$ square units.

96. The range of $f(x) = x^2$ is $[0, \infty)$. The graph of $g(x) = f(x) + k$ is the graph of f shifted up k units if $k > 0$ and shifted down $|k|$ units if $k < 0$, so the range of g is $[k, \infty)$.

97. The domain of $g(x) = \sqrt{x}$ is $[0, \infty)$. The graph of $g(x - k)$ is the graph of g shifted k units to the right, so the domain of g is $[k, \infty)$.

98. $3x - 5y = 30$
 $-5y = -3x + 30$
 $y = \frac{3}{5}x - 6$

The slope is $\frac{3}{5}$ and the y -intercept is -6 .

99. The total time run is $\frac{13.1}{7} + \frac{13.1}{2} = 8.4214$. The total distance is 26.2 mile. Thus the average speed is $\frac{26.2}{8.4214} = 3.11$ mph.

100. $W = kT$
 $7 = k4$
 $k = \frac{7}{4}$
 $W = \frac{7}{4}T = \frac{7}{4}(9) = 15.75$ gal

101. $y^2 = x + 4$

x-intercepts:

$(0)^2 = x + 4$

$0 = x + 4$

$x = -4$

y-intercepts:

$y^2 = 0 + 4$

$y^2 = 4$

$y = \pm 2$

 The intercepts are $(-4, 0)$, $(0, -2)$ and $(0, 2)$.

Test x-axis symmetry: Let $y = -y$

$(-y)^2 = x + 4$

$y^2 = x + 4$ same

Test y-axis symmetry: Let $x = -x$

$y^2 = -x + 4$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-y)^2 = -x + 4$

$y^2 = -x + 4$ different

Therefore, the graph will have x-axis symmetry.

102. The denominator must not be zero.

$x^2 - 5x - 14 = 0$

$(x - 7)(x + 2) = 0$

$x = 7, x = -2$

 So the domain is: $\{x \mid x \neq 7, x \neq -2\}$

103. $-16t^2 + 96t + 200 = 88$

$-16t^2 + 96t + 112 = 0$

$-16(t^2 - 6t - 7) = 0$

$-16(t - 7)(t + 1) = 0$

$t = 7, t = -1$

 Since t represents time the only answer that is reasonable is 7 seconds.

104. $\sqrt[3]{16x^5y^6z} = \sqrt[3]{8 \cdot 2x^3x^2y^6z} = 2xy^2\sqrt[3]{2x^2z}$

105. $\frac{f(x+h) - f(x)}{h} =$

$$\frac{3(x+h)^2 + 2(x+h) - 1 - (3x^2 + 2x - 1)}{h} =$$

$$\frac{3(x^2 + 2xh + h^2) + 2x + 2h - 1 - 3x^2 - 2x + 1}{h} =$$

$$\frac{3x^2 + 6xh + 3h^2 + 2x + 2h - 1 - 3x^2 - 2x + 1}{h} =$$

$$\frac{6xh + h^2 + 2h}{h} = \frac{h(6x + h + 2)}{h} = 6x + h + 2$$

106. $z^3 + 216 = (z)^3 + (6)^3$
 $= (z + 6)(z^2 - 6z + 36)$

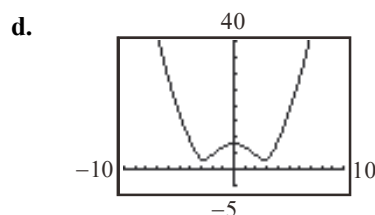
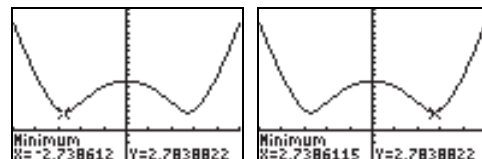
Section 2.6

 1. a. The distance d from P to the origin is
 $d = \sqrt{x^2 + y^2}$. Since P is a point on the graph of $y = x^2 - 8$, we have:

$$d(x) = \sqrt{x^2 + (x^2 - 8)^2} = \sqrt{x^4 - 15x^2 + 64}$$

b. $d(0) = \sqrt{0^4 - 15(0)^2 + 64} = \sqrt{64} = 8$

c. $d(1) = \sqrt{(1)^4 - 15(1)^2 + 64}$
 $= \sqrt{1 - 15 + 64} = \sqrt{50} = 5\sqrt{2} \approx 7.07$


 e. d is smallest when $x \approx -2.74$ or when $x \approx 2.74$.

 2. a. The distance d from P to $(0, -1)$ is

 $d = \sqrt{x^2 + (y + 1)^2}$. Since P is a point on the graph of $y = x^2 - 8$, we have:

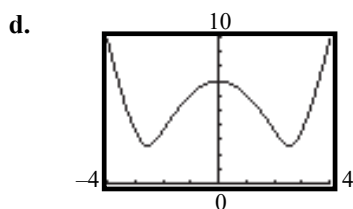
$$d(x) = \sqrt{x^2 + (x^2 - 8 + 1)^2}$$

$$= \sqrt{x^2 + (x^2 - 7)^2} = \sqrt{x^4 - 13x^2 + 49}$$

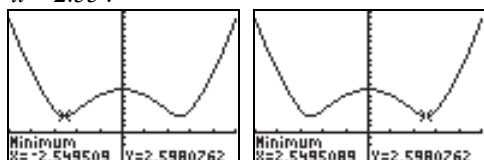
b. $d(0) = \sqrt{0^4 - 13(0)^2 + 49} = \sqrt{49} = 7$

c. $d(-1) = \sqrt{(-1)^4 - 13(-1)^2 + 49} = \sqrt{37} \approx 6.08$

Chapter 2: Functions and Their Graphs



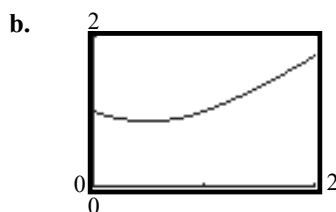
- e. d is smallest when $x \approx -2.55$ or when $x \approx 2.55$.



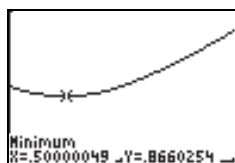
3. a. The distance d from P to the point $(1, 0)$ is $d = \sqrt{(x-1)^2 + y^2}$. Since P is a point on the graph of $y = \sqrt{x}$, we have:

$$d(x) = \sqrt{(x-1)^2 + (\sqrt{x})^2} = \sqrt{x^2 - x + 1}$$

where $x \geq 0$.



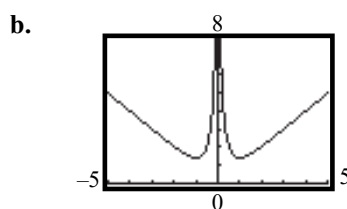
- c. d is smallest when $x = \frac{1}{2}$.



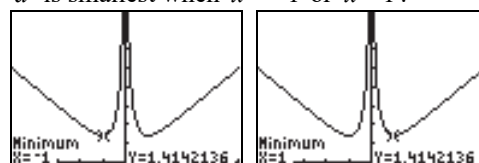
d. $d(x) = \sqrt{\frac{1}{2} - \frac{1}{2} + 1} = \frac{\sqrt{3}}{2}$

4. a. The distance d from P to the origin is $d = \sqrt{x^2 + y^2}$. Since P is a point on the graph of $y = \frac{1}{x}$, we have:

$$d(x) = \sqrt{x^2 + \left(\frac{1}{x}\right)^2} = \sqrt{x^2 + \frac{1}{x^2}}$$



- c. d is smallest when $x = -1$ or $x = 1$.



- d.

$$d(1) = \frac{\sqrt{1^2 + 1}}{|1|} = \sqrt{2}; d(-1) = \frac{\sqrt{(-1)^2 + 1}}{|-1|} = \sqrt{2}$$

5. By definition, a triangle has area

$$A = \frac{1}{2}bh, b = \text{base}, h = \text{height. From the figure,}$$

we know that $b = x$ and $h = y$. Expressing the area of the triangle as a function of x , we have:

$$A(x) = \frac{1}{2}xy = \frac{1}{2}x(x^3) = \frac{1}{2}x^4.$$

6. By definition, a triangle has area

$$A = \frac{1}{2}bh, b = \text{base}, h = \text{height. Because one}$$

vertex of the triangle is at the origin and the other is on the x -axis, we know that

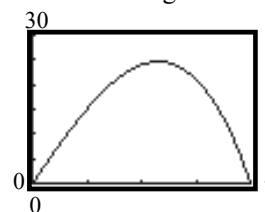
$b = x$ and $h = y$. Expressing the area of the triangle as a function of x , we have:

$$A(x) = \frac{1}{2}xy = \frac{1}{2}x(9 - x^2) = \frac{9}{2}x - \frac{1}{2}x^3.$$

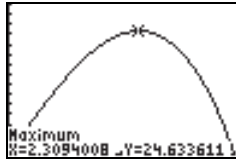
7. a. $A(x) = xy = x(16 - x^2)$

- b. Domain: $\{x \mid 0 < x < 4\}$

- c. The area is largest when $x \approx 2.31$.



Section 2.6: Mathematical Models: Building Functions

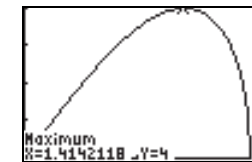
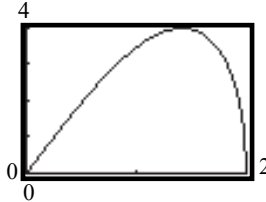


- d. The largest area is
 $A(2.31) = 2.31(16 - 2.31^2) \approx 24.63$ square units.

8. a. $A(x) = 2xy = 2x\sqrt{4 - x^2}$

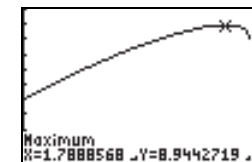
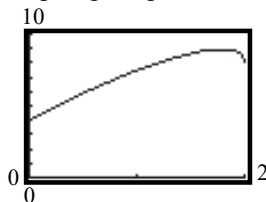
b. $p(x) = 2(2x) + 2(y) = 4x + 2\sqrt{4 - x^2}$

- c. Graphing the area equation:



The area is largest when $x \approx 1.41$.

- d. Graphing the perimeter equation:



The perimeter is largest when $x \approx 1.79$.

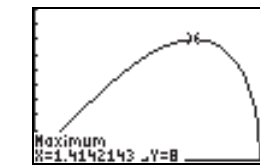
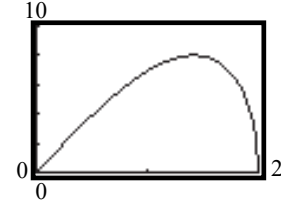
- e. The largest area is
 $A(1.41) = 2(1.41)\sqrt{4 - 1.41^2} \approx 4$ square units.
 The largest perimeter is
 $p(1.79) = 4(1.79) + 2\sqrt{4 - 1.79^2} \approx 8.94$ units.

9. a. In Quadrant I, $x^2 + y^2 = 4 \rightarrow y = \sqrt{4 - x^2}$

$$A(x) = (2x)(2y) = 4x\sqrt{4 - x^2}$$

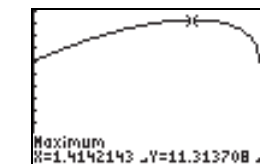
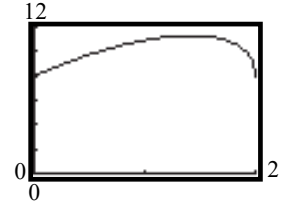
b. $p(x) = 2(2x) + 2(2y) = 4x + 4\sqrt{4 - x^2}$

- c. Graphing the area equation:



The area is largest when $x \approx 1.41$.

- d. Graphing the perimeter equation:



The perimeter is largest when $x \approx 1.41$.

10. a. $A(r) = (2r)(2r) = 4r^2$

b. $p(r) = 4(2r) = 8r$

11. a. C = circumference, A = total area,
 r = radius, x = side of square

$$C = 2\pi r = 10 - 4x \Rightarrow r = \frac{5 - 2x}{\pi}$$

$$\text{Total Area} = \text{area}_{\text{square}} + \text{area}_{\text{circle}} = x^2 + \pi r^2$$

$$A(x) = x^2 + \pi \left(\frac{5 - 2x}{\pi} \right)^2 = x^2 + \frac{25 - 20x + 4x^2}{\pi}$$

Chapter 2: Functions and Their Graphs

- b. Since the lengths must be positive, we have:

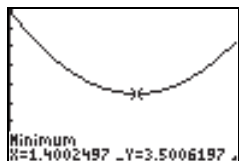
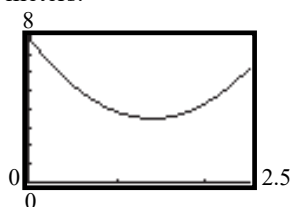
$$10 - 4x > 0 \quad \text{and} \quad x > 0$$

$$-4x > -10 \quad \text{and} \quad x > 0$$

$$x < 2.5 \quad \text{and} \quad x > 0$$

$$\text{Domain: } \{x \mid 0 < x < 2.5\}$$

- c. The total area is smallest when $x \approx 1.40$ meters.



12. a. C = circumference, A = total area,
 r = radius, x = side of equilateral triangle

$$C = 2\pi r = 10 - 3x \Rightarrow r = \frac{10 - 3x}{2\pi}$$

The height of the equilateral triangle is $\frac{\sqrt{3}}{2}x$.

Total Area = area_{triangle} + area_{circle}

$$= \frac{1}{2}x \left(\frac{\sqrt{3}}{2}x \right) + \pi r^2$$

$$\begin{aligned} A(x) &= \frac{\sqrt{3}}{4}x^2 + \pi \left(\frac{10 - 3x}{2\pi} \right)^2 \\ &= \frac{\sqrt{3}}{4}x^2 + \frac{100 - 60x + 9x^2}{4\pi} \end{aligned}$$

- b. Since the lengths must be positive, we have:

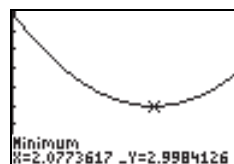
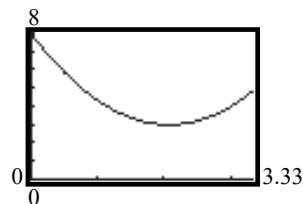
$$10 - 3x > 0 \quad \text{and} \quad x > 0$$

$$-3x > -10 \quad \text{and} \quad x > 0$$

$$x < \frac{10}{3} \quad \text{and} \quad x > 0$$

$$\text{Domain: } \left\{ x \mid 0 < x < \frac{10}{3} \right\}$$

- c. The area is smallest when $x \approx 2.08$ meters.



13. a. Since the wire of length x is bent into a circle, the circumference is x . Therefore,
 $C(x) = x$.

- b. Since $C = x = 2\pi r$, $r = \frac{x}{2\pi}$.

$$A(x) = \pi r^2 = \pi \left(\frac{x}{2\pi} \right)^2 = \frac{x^2}{4\pi}$$

14. a. Since the wire of length x is bent into a square, the perimeter is x . Therefore,
 $p(x) = x$.

- b. Since $P = x = 4s$, $s = \frac{1}{4}x$, we have

$$A(x) = s^2 = \left(\frac{1}{4}x \right)^2 = \frac{1}{16}x^2$$

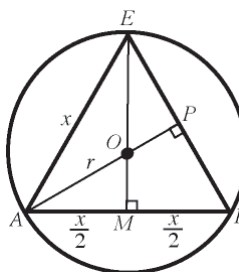
15. a. A = area, r = radius; diameter = $2r$

$$A(r) = (2r)(r) = 2r^2$$

- b. p = perimeter

$$p(r) = 2(2r) + 2r = 6r$$

16. C = circumference, r = radius;
 x = length of a side of the triangle



Since $\triangle ABC$ is equilateral, $EM = \frac{\sqrt{3}x}{2}$.

Section 2.6: Mathematical Models: Building Functions

Therefore, $OM = \frac{\sqrt{3}x}{2} - OE = \frac{\sqrt{3}x}{2} - r$

In $\triangle OAM$, $r^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2} - r\right)^2$

$$r^2 = \frac{x^2}{4} + \frac{3}{4}x^2 - \sqrt{3}rx + r^2$$

$$\sqrt{3}rx = x^2$$

$$r = \frac{x}{\sqrt{3}}$$

Therefore, the circumference of the circle is

$$C(x) = 2\pi r = 2\pi \left(\frac{x}{\sqrt{3}}\right) = \frac{2\pi\sqrt{3}}{3}x$$

17. Area of the equilateral triangle

$$A = \frac{1}{2}x \cdot \frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2$$

From problem 16, we have $r^2 = \frac{x^2}{3}$.

Area inside the circle, but outside the triangle:

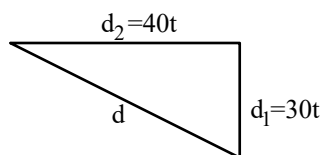
$$A(x) = \pi r^2 - \frac{\sqrt{3}}{4}x^2$$

$$= \pi \frac{x^2}{3} - \frac{\sqrt{3}}{4}x^2 = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)x^2$$

18. $d^2 = d_1^2 + d_2^2$

$$d^2 = (30t)^2 + (40t)^2$$

$$d(t) = \sqrt{900t^2 + 1600t^2} = \sqrt{2500t^2} = 50t$$



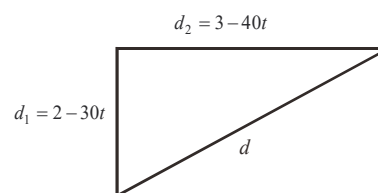
19. a. $d^2 = d_1^2 + d_2^2$

$$d^2 = (2 - 30t)^2 + (3 - 40t)^2$$

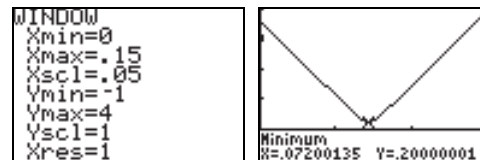
$$d(t) = \sqrt{(2 - 30t)^2 + (3 - 40t)^2}$$

$$= \sqrt{4 - 120t + 900t^2 + 9 - 240t + 1600t^2}$$

$$= \sqrt{2500t^2 - 360t + 13}$$



- b. The distance is smallest at $t \approx 0.07$ hours.



20. r = radius of cylinder, h = height of cylinder,
 V = volume of cylinder

$$r^2 + \left(\frac{h}{2}\right)^2 = R^2 \Rightarrow r^2 + \frac{h^2}{4} = R^2 \Rightarrow r^2 = R^2 - \frac{h^2}{4}$$

$$V = \pi r^2 h$$

$$V(h) = \pi \left(R^2 - \frac{h^2}{4}\right) h = \pi h \left(R^2 - \frac{h^2}{4}\right)$$

21. r = radius of cylinder, h = height of cylinder,
 V = volume of cylinder

By similar triangles: $\frac{H}{R} = \frac{H-h}{r}$

$$Hr = R(H-h)$$

$$Hr = RH - Rh$$

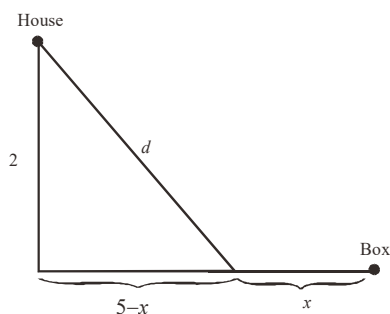
$$Rh = RH - Hr$$

$$h = \frac{RH - Hr}{R} = \frac{H(R-r)}{R}$$

$$V(r) = \pi r^2 h = \pi r^2 \left(\frac{H(R-r)}{R}\right) = \frac{\pi H(R-r)r^2}{R}$$

22. a. The total cost of installing the cable along the road is $500x$. If cable is installed x miles along the road, there are $5-x$ miles between the road to the house and where the cable ends along the road.

Chapter 2: Functions and Their Graphs



$$d = \sqrt{(5-x)^2 + 2^2}$$

$$= \sqrt{25 - 10x + x^2 + 4} = \sqrt{x^2 - 10x + 29}$$

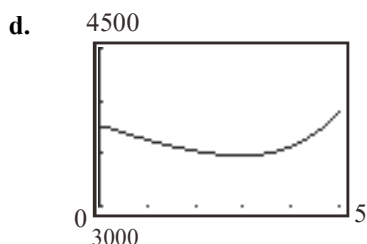
The total cost of installing the cable is:

$$C(x) = 500x + 700\sqrt{x^2 - 10x + 29}$$

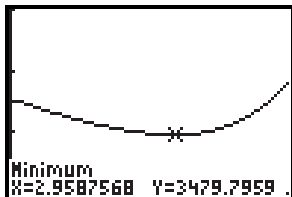
$$\text{Domain: } \{x \mid 0 \leq x \leq 5\}$$

b. $C(1) = 500(1) + 700\sqrt{1^2 - 10(1) + 29}$
 $= 500 + 700\sqrt{20} = \$3630.50$

c. $C(3) = 500(3) + 700\sqrt{3^2 - 10(3) + 29}$
 $= 1500 + 700\sqrt{8} = \$3479.90$

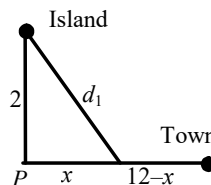


e. Using MINIMUM, the graph indicates that $x \approx 2.96$ miles results in the least cost.



23. a. The time on the boat is given by $\frac{d_1}{3}$. The

time on land is given by $\frac{12-x}{5}$.



$$d_1 = \sqrt{x^2 + 2^2} = \sqrt{x^2 + 4}$$

The total time for the trip is:

$$T(x) = \frac{12-x}{5} + \frac{d_1}{3} = \frac{12-x}{5} + \frac{\sqrt{x^2 + 4}}{3}$$

b. Domain: $\{x \mid 0 \leq x \leq 12\}$

c. $T(4) = \frac{12-4}{5} + \frac{\sqrt{4^2 + 4}}{3}$
 $= \frac{8}{5} + \frac{\sqrt{20}}{3} \approx 3.09 \text{ hours}$

d. $T(8) = \frac{12-8}{5} + \frac{\sqrt{8^2 + 4}}{3}$
 $= \frac{4}{5} + \frac{\sqrt{68}}{3} \approx 3.55 \text{ hours}$

24. a. Let A = amount of material,
 x = length of the base, h = height, and
 V = volume.

$$V = x^2 h = 10 \Rightarrow h = \frac{10}{x^2}$$

$$\text{Total Area } A = (\text{Area}_{\text{base}}) + (4)(\text{Area}_{\text{side}})$$

$$= x^2 + 4xh$$

$$= x^2 + 4x\left(\frac{10}{x^2}\right)$$

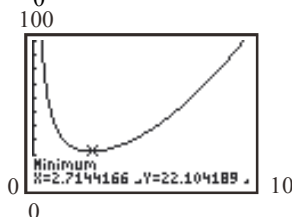
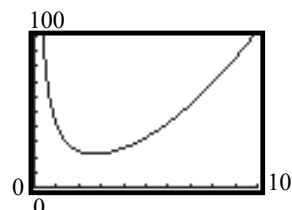
$$= x^2 + \frac{40}{x}$$

$$A(x) = x^2 + \frac{40}{x}$$

b. $A(1) = 1^2 + \frac{40}{1} = 1 + 40 = 41 \text{ ft}^2$

c. $A(2) = 2^2 + \frac{40}{2} = 4 + 20 = 24 \text{ ft}^2$

d. $y_1 = x^2 + \frac{40}{x}$



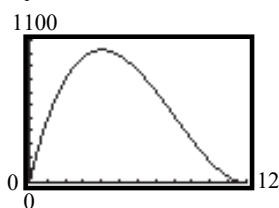
The amount of material is least when $x = 2.71$ ft.

e. The largest area is

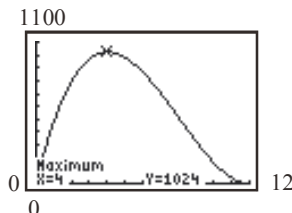
$$A(2.71) = 2.71^2 + \frac{40}{2.71} = 22.1 \text{ ft}^2$$

25. a. length = $24 - 2x$; width = $24 - 2x$; height = x
 $V(x) = x(24 - 2x)(24 - 2x) = x(24 - 2x)^2$
- b. $V(3) = 3(24 - 2(3))^2 = 3(18)^2$
 $= 3(324) = 972 \text{ in}^3$.
- c. $V(10) = 10(24 - 2(10))^2 = 10(4)^2$
 $= 10(16) = 160 \text{ in}^3$.

d. $y_1 = x(24 - 2x)^2$



Use MAXIMUM.

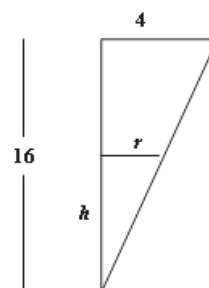
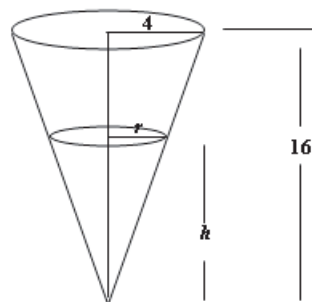


The volume is largest when $x = 4$ inches.

e. The largest volume is

$$V(4) = 4(24 - 2(4))^2 = 1024 \text{ in}^3$$

26. Consider the diagrams shown below.



There is a pair of similar triangles in the diagram. This allows us to write

$$\frac{r}{h} = \frac{4}{16} \Rightarrow \frac{r}{h} = \frac{1}{4} \Rightarrow r = \frac{1}{4}h$$

Substituting into the volume formula for the conical portion of water gives

$$V(h) = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{1}{4}h\right)^2 h = \frac{\pi}{48}h^3.$$

27. a. The total cost is the sum of the shipment cost, storage cost, and product cost. Since each shipment will contain x units, there are $600/x$ shipments per year, each costing \$15.

$$\text{So the shipment cost is } 15\left(\frac{600}{x}\right) = \left(\frac{9000}{x}\right).$$

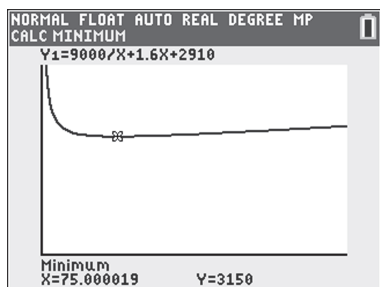
The storage cost for the year is given as $1.60x$. The product costs is

$$600(4.85) = 2910. \text{ So, the total cost is}$$

$$C(x) = \frac{9000}{x} + 1.60x + 2910.$$

Chapter 2: Functions and Their Graphs

b.



The retailer should order 75 drives per order for a minimum yearly cost of \$3150.

$$\begin{aligned}
 28. \quad |2x - 3| - 5 &= -2 \\
 |2x - 3| &= 3 \\
 2x - 3 &= -3 \text{ or } 2x - 3 = 3 \\
 2x &= 0 \quad \text{or} \quad 2x = 6 \\
 x &= 0 \quad \text{or} \quad x = 3
 \end{aligned}$$

The solution set is $\{0, 3\}$.

29. In order for the 16-foot long Ford Fusion to pass the 50-foot truck, the Ford Fusion must travel the length of the truck and the length of itself in the time frame of 5 seconds. Thus the Fusion must travel an additional 66 feet in 5 seconds.

Convert this to miles-per-hour.

$$5 \text{ sec} = \frac{5}{60} \text{ min} = \frac{5}{3600} \text{ hr} = \frac{1}{720} \text{ hr.}$$

$$66 \text{ ft} = \frac{66}{5280} \text{ mi}$$

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{\frac{66}{5280}}{\frac{1}{720}} = 9 \text{ mph}$$

Since the truck is traveling 55 mph, the Fusion must travel $55 + 9 = 64$ mph.

$$30. \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-2)}{1 - 3} = \frac{8}{-2} = -4$$

$$31. \quad \frac{10}{4} = \frac{14}{x}$$

$$10x = 4(14)$$

$$10x = 56$$

$$x = 5.6$$

$$32. \quad x = u - 1$$

$$y = \frac{u-1}{u-1+1} = \frac{u-1}{u}$$

$$\begin{aligned}
 33. \quad \frac{x+5}{3x^{2/3}} + x^{1/3} &= \frac{x+5}{3x^{2/3}} + \frac{3x}{3x^{2/3}} \\
 &= \frac{x+5+3x}{3x^{2/3}} \\
 &= \frac{4x+5}{3x^{2/3}}
 \end{aligned}$$

$$\begin{aligned}
 34. \quad -\sqrt{3x-2} &\geq 4 \\
 \sqrt{3x-2} &\leq -4
 \end{aligned}$$

No solution since a square root cannot be negative.

35. Since the graph is symmetric about the origin then $(3, -2)$ is symmetric to $(-3, 2)$.

$$\begin{aligned}
 36. \quad v &= \frac{2.6t}{d^2} \sqrt{\frac{E}{P}} \\
 \frac{vd^2}{2.6t} &= \sqrt{\frac{E}{P}} \\
 \left(\frac{vd^2}{2.6t} \right)^2 &= \frac{E}{P} \\
 \frac{v^2 d^4}{6.76t^2} &= \frac{E}{P} \\
 \frac{Pv^2 d^4}{6.76t^2} &= E \\
 P &= \frac{6.76t^2 E}{v^2 d^4}
 \end{aligned}$$

37. $3x^2 - 7x = 4x - 2$

$$3x^2 - 11x + 2 = 0$$

$$b^2 - 4ac = (-11)^2 - 4(3)(2) \\ = 121 - 24 = 97$$

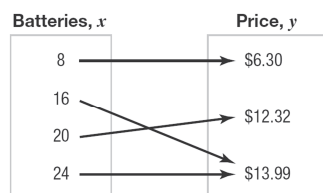
Chapter 2 Review Exercises

1. a. Domain $\{8, 16, 20, 24\}$

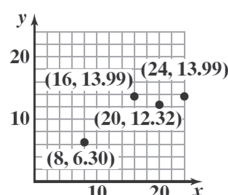
Range $\{\$6.30, \$12.32, \$13.99\}$

- b. $\{(8, \$6.30), (16, \$13.99), (20, \$12.32), (24, \$13.99)\}$

c.



d.



2. This relation represents a function.
Domain = $\{-1, 2, 4\}$; Range = $\{0, 3\}$.
3. Domain $\{2, 4\}$; Range $\{-1, 1, 2\}$
Not a function
4. not a function; domain $[-1, 3]$; range $[-2, 2]$
5. function; domain: all real numbers; range $[-3, \infty)$

6. $f(x) = \frac{3x}{x^2 - 1}$

a. $f(2) = \frac{3(2)}{(2)^2 - 1} = \frac{6}{4 - 1} = \frac{6}{3} = 2$

b. $f(-2) = \frac{3(-2)}{(-2)^2 - 1} = \frac{-6}{4 - 1} = \frac{-6}{3} = -2$

c. $f(-x) = \frac{3(-x)}{(-x)^2 - 1} = \frac{-3x}{x^2 - 1}$

d. $-f(x) = -\left(\frac{3x}{x^2 - 1}\right) = \frac{-3x}{x^2 - 1}$

e. $f(x-2) = \frac{3(x-2)}{(x-2)^2 - 1} \\ = \frac{3x-6}{x^2-4x+4-1} = \frac{3(x-2)}{x^2-4x+3}$

f. $f(2x) = \frac{3(2x)}{(2x)^2 - 1} = \frac{6x}{4x^2 - 1}$

7. $f(x) = \sqrt{x^2 - 4}$

a. $f(2) = \sqrt{2^2 - 4} = \sqrt{4 - 4} = \sqrt{0} = 0$

b. $f(-2) = \sqrt{(-2)^2 - 4} = \sqrt{4 - 4} = \sqrt{0} = 0$

c. $f(-x) = \sqrt{(-x)^2 - 4} = \sqrt{x^2 - 4}$

d. $-f(x) = -\sqrt{x^2 - 4}$

e. $f(x-2) = \sqrt{(x-2)^2 - 4} \\ = \sqrt{x^2 - 4x + 4 - 4} \\ = \sqrt{x^2 - 4x}$

f. $f(2x) = \sqrt{(2x)^2 - 4} = \sqrt{4x^2 - 4} \\ = \sqrt{4(x^2 - 1)} = 2\sqrt{x^2 - 1}$

8. $f(x) = \frac{x^2 - 4}{x^2}$

a. $f(2) = \frac{2^2 - 4}{2^2} = \frac{4 - 4}{4} = \frac{0}{4} = 0$

b. $f(-2) = \frac{(-2)^2 - 4}{(-2)^2} = \frac{4 - 4}{4} = \frac{0}{4} = 0$

c. $f(-x) = \frac{(-x)^2 - 4}{(-x)^2} = \frac{x^2 - 4}{x^2}$

d. $-f(x) = -\left(\frac{x^2 - 4}{x^2}\right) = \frac{4 - x^2}{x^2} = -\frac{x^2 - 4}{x^2}$

e. $f(x-2) = \frac{(x-2)^2 - 4}{(x-2)^2} = \frac{x^2 - 4x + 4 - 4}{(x-2)^2} \\ = \frac{x^2 - 4x}{(x-2)^2} = \frac{x(x-4)}{(x-2)^2}$

Chapter 2: Functions and Their Graphs

$$\begin{aligned} \text{f. } f(2x) &= \frac{(2x)^2 - 4}{(2x)^2} = \frac{4x^2 - 4}{4x^2} \\ &= \frac{4(x^2 - 1)}{4x^2} = \frac{x^2 - 1}{x^2} \end{aligned}$$

$$9. f(x) = \frac{x}{x^2 - 9}$$

The denominator cannot be zero:

$$x^2 - 9 \neq 0$$

$$(x+3)(x-3) \neq 0$$

$$x \neq -3 \text{ or } 3$$

$$\text{Domain: } \{x \mid x \neq -3, x \neq 3\}$$

$$10. f(x) = \sqrt{2-x}$$

The radicand must be non-negative:

$$2-x \geq 0$$

$$x \leq 2$$

$$\text{Domain: } \{x \mid x \leq 2\} \text{ or } (-\infty, 2]$$

$$11. g(x) = \frac{|x|}{x}$$

The denominator cannot be zero:

$$x \neq 0$$

$$\text{Domain: } \{x \mid x \neq 0\}$$

$$12. f(x) = \frac{x}{x^2 + 2x - 3}$$

The denominator cannot be zero:

$$x^2 + 2x - 3 \neq 0$$

$$(x+3)(x-1) \neq 0$$

$$x \neq -3 \text{ or } 1$$

$$\text{Domain: } \{x \mid x \neq -3, x \neq 1\}$$

$$13. f(x) = \frac{\sqrt{x+1}}{x^2 - 4}$$

The denominator cannot be zero:

$$x^2 - 4 \neq 0$$

$$(x+2)(x-2) \neq 0$$

$$x \neq -2 \text{ or } 2$$

Also, the radicand must be non-negative:

$$x+1 \geq 0$$

$$x \geq -1$$

$$\text{Domain: } [-1, 2) \cup (2, \infty)$$

$$14. f(x) = \frac{x}{\sqrt{x+8}}$$

The radicand must be non-negative and not zero:

$$x+8 > 0$$

$$x > -8$$

$$\text{Domain: } \{x \mid x > -8\}$$

$$15. f(x) = 2-x \quad g(x) = 3x+1$$

$$(f+g)(x) = f(x) + g(x)$$

$$= 2-x+3x+1 = 2x+3$$

$$\text{Domain: } \{x \mid x \text{ is any real number}\}$$

$$(f-g)(x) = f(x) - g(x)$$

$$= 2-x-(3x+1)$$

$$= 2-x-3x-1$$

$$= -4x+1$$

$$\text{Domain: } \{x \mid x \text{ is any real number}\}$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$= (2-x)(3x+1)$$

$$= 6x+2-3x^2-x$$

$$= -3x^2+5x+2$$

$$\text{Domain: } \{x \mid x \text{ is any real number}\}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2-x}{3x+1}$$

$$3x+1 \neq 0$$

$$3x \neq -1 \Rightarrow x \neq -\frac{1}{3}$$

$$\text{Domain: } \left\{x \mid x \neq -\frac{1}{3}\right\}$$

$$16. f(x) = 3x^2 + x + 1 \quad g(x) = 3x$$

$$(f+g)(x) = f(x) + g(x)$$

$$= 3x^2 + x + 1 + 3x$$

$$= 3x^2 + 4x + 1$$

$$\text{Domain: } \{x \mid x \text{ is any real number}\}$$

$$(f-g)(x) = f(x) - g(x)$$

$$= 3x^2 + x + 1 - 3x$$

$$= 3x^2 - 2x + 1$$

$$\text{Domain: } \{x \mid x \text{ is any real number}\}$$

$$\begin{aligned}
 (f \cdot g)(x) &= f(x) \cdot g(x) \\
 &= (3x^2 + x + 1)(3x) \\
 &= 9x^3 + 3x^2 + 3x
 \end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x^2 + x + 1}{3x}$$

$$3x \neq 0 \Rightarrow x \neq 0$$

Domain: $\{x \mid x \neq 0\}$

$$17. f(x) = \frac{x+1}{x-1} \quad g(x) = \frac{1}{x}$$

$$\begin{aligned}
 (f+g)(x) &= f(x) + g(x) \\
 &= \frac{x+1}{x-1} + \frac{1}{x} = \frac{x(x+1) + 1(x-1)}{x(x-1)} \\
 &= \frac{x^2 + x + x - 1}{x(x-1)} = \frac{x^2 + 2x - 1}{x(x-1)}
 \end{aligned}$$

Domain: $\{x \mid x \neq 0, x \neq 1\}$

$$\begin{aligned}
 (f-g)(x) &= f(x) - g(x) \\
 &= \frac{x+1}{x-1} - \frac{1}{x} = \frac{x(x+1) - 1(x-1)}{x(x-1)} \\
 &= \frac{x^2 + x - x + 1}{x(x-1)} = \frac{x^2 + 1}{x(x-1)}
 \end{aligned}$$

Domain: $\{x \mid x \neq 0, x \neq 1\}$

$$(f \cdot g)(x) = f(x) \cdot g(x) = \left(\frac{x+1}{x-1}\right)\left(\frac{1}{x}\right) = \frac{x+1}{x(x-1)}$$

Domain: $\{x \mid x \neq 0, x \neq 1\}$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{x+1}{x-1}}{\frac{1}{x}} = \left(\frac{x+1}{x-1}\right)\left(\frac{x}{1}\right) = \frac{x(x+1)}{x-1}$$

Domain: $\{x \mid x \neq 0, x \neq 1\}$

$$\begin{aligned}
 18. f(x) &= -2x^2 + x + 1 \\
 \frac{f(x+h) - f(x)}{h} &= \frac{-2(x+h)^2 + (x+h) + 1 - (-2x^2 + x + 1)}{h} \\
 &= \frac{-2(x^2 + 2xh + h^2) + x + h + 1 + 2x^2 - x - 1}{h} \\
 &= \frac{-2x^2 - 4xh - 2h^2 + x + h + 1 + 2x^2 - x - 1}{h} \\
 &= \frac{-4xh - 2h^2 + h}{h} = \frac{h(-4x - 2h + 1)}{h} \\
 &= -4x - 2h + 1
 \end{aligned}$$

$$19. \text{ a. Domain: } \{x \mid -4 \leq x \leq 3\}; [-4, 3]$$

$$\text{Range: } \{y \mid -3 \leq y \leq 3\}; [-3, 3]$$

$$\text{ b. Intercept: } (0, 0)$$

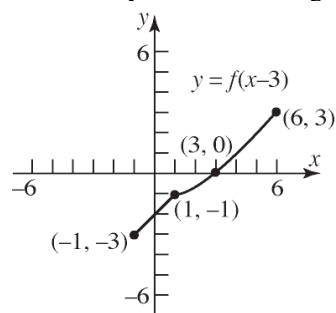
$$\text{ c. } f(-2) = -1$$

$$\text{ d. } f(x) = -3 \text{ when } x = -4$$

$$\text{ e. } f(x) > 0 \text{ when } 0 < x \leq 3$$

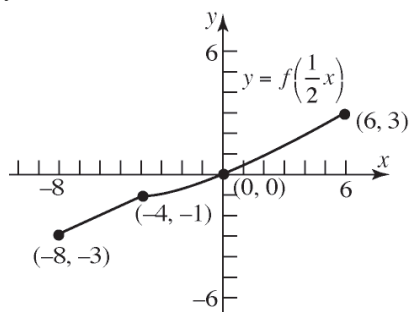
$$\{x \mid 0 < x \leq 3\}$$

f. To graph $y = f(x-3)$, shift the graph of f horizontally 3 units to the right.

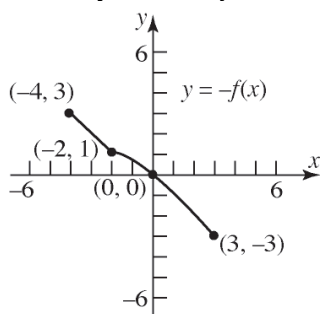


Chapter 2: Functions and Their Graphs

- g. To graph $y = f\left(\frac{1}{2}x\right)$, stretch the graph of f horizontally by a factor of 2.



- h. To graph $y = -f(x)$, reflect the graph of f vertically about the y -axis.



20. a. Domain: $(-\infty, 4]$
Range: $(-\infty, 3]$
- b. Increasing: $(-\infty, -2]$ and $[2, 4]$;
Decreasing: $[-2, 2]$
- c. Local minimum is -1 at $x = 2$;
Local maximum is 1 at $x = -2$
- d. No absolute minimum;
Absolute maximum is 3 at $x = 4$
- e. The graph has no symmetry.
- f. The function is neither.
- g. x -intercepts: $(-3, 0), (0, 0), (3, 0)$;
 y -intercept: $(0, 0)$

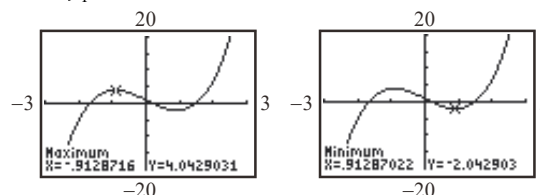
21. $f(x) = x^3 - 4x$
 $f(-x) = (-x)^3 - 4(-x) = -x^3 + 4x$
 $= -(x^3 - 4x) = -f(x)$
 f is odd.

22. $g(x) = \frac{4+x^2}{1+x^4}$
 $g(-x) = \frac{4+(-x)^2}{1+(-x)^4} = \frac{4+x^2}{1+x^4} = g(x)$
 g is even.

23. $G(x) = 1 - x + x^3$
 $G(-x) = 1 - (-x) + (-x)^3$
 $= 1 + x - x^3 \neq -G(x)$ or $G(x)$
 G is neither even nor odd.

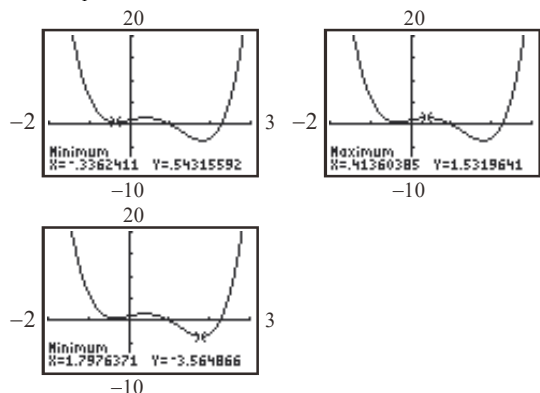
24. $f(x) = \frac{x}{1+x^2}$
 $f(-x) = \frac{-x}{1+(-x)^2} = \frac{-x}{1+x^2} = -f(x)$
 f is odd.

25. $f(x) = 2x^3 - 5x + 1$ on the interval $(-3, 3)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = 2x^3 - 5x + 1$.



local maximum value: 4.04 when $x \approx -0.91$
local minimum value: -2.04 when $x = 0.91$
 f is increasing on: $[-3, -0.91]$ and $[0.91, 3]$;
 f is decreasing on: $[-0.91, 0.91]$.

26. $f(x) = 2x^4 - 5x^3 + 2x + 1$ on the interval $(-2, 3)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = 2x^4 - 5x^3 + 2x + 1$.



local maximum: 1.53 when $x = 0.41$