

Trigonometry 12th edition Sullivan Solutions Manual

SKU: 9780138234645-SOLUTIONS

INSTRUCTOR'S SOLUTIONS MANUAL

TIM BRITT

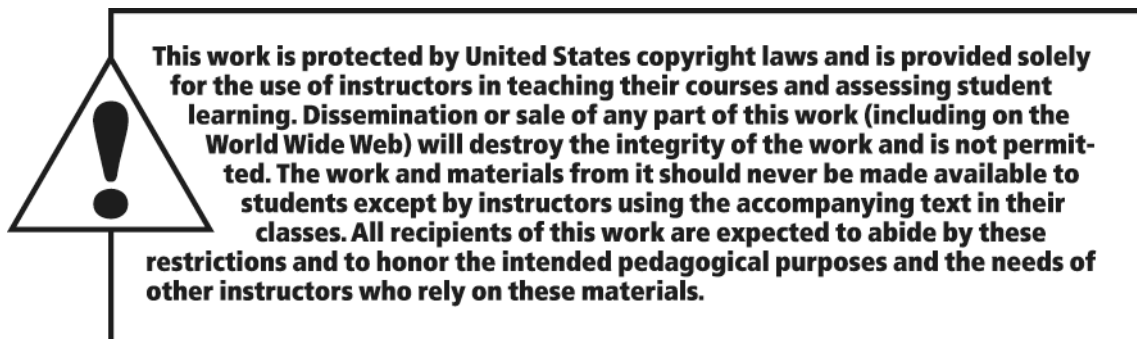
Jackson State Community College

TRIGONOMETRY: A UNIT CIRCLE APPROACH TWELFTH EDITION

Michael Sullivan

Chicago State University





Copyright © 2025, 2021, 2017 by Pearson Education, Inc. or its affiliates. All Rights Reserved. Manufactured in the United States of America. This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise. For information regarding permissions, request forms, and the appropriate contacts within the Pearson Education Global Rights and Permissions department, please visit www.pearsoned.com/permissions/.

PEARSON and MYLAB are exclusive trademarks owned by Pearson Education, Inc. or its affiliates in the U.S. and/or other countries.

Unless otherwise indicated herein, any third-party trademarks, logos, or icons that may appear in this work are the property of their respective owners, and any references to third-party trademarks, logos, icons, or other trade dress are for demonstrative or descriptive purposes only. Such references are not intended to imply any sponsorship, endorsement, authorization, or promotion of Pearson's products by the owners of such marks, or any relationship between the owner and Pearson Education, Inc., or its affiliates, authors, licensees, or distributors.



PPID: A103000347144

Table of Contents

Chapter 1 Preliminaries

1.1 The Distance and Midpoint Formulas.....	1
1.2 Graphs of Equations in Two Variables; Intercepts; Symmetry	13
1.3 Lines	26
1.4 Circles	44
Chapter Review.....	57
Chapter Test.....	63
Chapter Projects	64

Chapter 2 Functions and Their Graphs

2.1 Functions.....	65
2.2 The Graph of a Function.....	83
2.3 Properties of Functions	92
2.4 Library of Functions; Piecewise-defined Functions	109
2.5 Graphing Techniques: Transformations	121
2.6 Mathematical Models: Building Functions.....	139
2.7 One-to-One Functions; Inverse Functions.....	146
Chapter Review.....	169
Chapter Test.....	177
Cumulative Review	181
Chapter Projects	184

Chapter 3 Trigonometric Functions

3.1 Angles, Arc Length, and Circular Motion	186
3.2 Trigonometric Functions: Unit Circle Approach	195
3.3 Properties of the Trigonometric Functions	213
3.4 Graphs of the Sine and Cosine Functions	227
3.5 Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions.....	248
3.6 Phase Shift; Sinusoidal Curve Fitting	258
Chapter Review.....	271
Chapter Test.....	279
Cumulative Review.....	282
Chapter Projects	284

Chapter 4 Analytic Trigonometry

4.1 The Inverse Sine, Cosine, and Tangent Functions.....	288
4.2 The Inverse Trigonometric Functions (Continued)	302
4.3 Trigonometric Equations	314
4.4 Trigonometric Identities	335
4.5 Sum and Difference Formulas	348
4.6 Double-angle and Half-angle Formulas.....	373
4.7 Product-to-Sum and Sum-to-Product Formulas.....	401
Chapter Review.....	414
Chapter Test.....	429
Cumulative Review.....	434
Chapter Projects	438

Chapter 5 Applications of Trigonometric Functions

5.1 Right Triangle Trigonometry; Applications	442
5.2 The Law of Sines	456
5.3 The Law of Cosines	470
5.4 Area of a Triangle	483
5.5 Simple Harmonic Motion; Damped Motion; Combining Waves	492
Chapter Review.....	502
Chapter Test.....	509
Cumulative Review.....	512
Chapter Projects.....	516

Chapter 6 Polar Coordinates; Vectors

6.1 Polar Coordinates.....	520
6.2 Polar Equations and Graphs.....	529
6.3 The Complex Plane; De Moivre's Theorem	558
6.4 Vectors	572
6.5 The Dot Product.....	585
6.6 Vectors in Space	591
6.7 The Cross Product.....	598
Chapter Review.....	608
Chapter Test.....	618
Cumulative Review.....	621
Chapter Projects.....	623

Chapter 7 Analytic Geometry

7.2 The Parabola	627
7.3 The Ellipse	643
7.4 The Hyperbola	660
7.5 Rotation of Axes; General Form of a Conic	680
7.6 Polar Equations of Conics.....	693
7.7 Plane Curves and Parametric Equations	702
Chapter Review.....	716
Chapter Test.....	726
Cumulative Review.....	730
Chapter Projects.....	732

Chapter 8 Exponential and Logarithmic Functions

8.1 Exponential Functions	736
8.2 Logarithmic Functions.....	756
8.3 Properties of Logarithms	778
8.4 Logarithmic and Exponential Equations.....	787
8.5 Financial Models	807
8.6 Exponential Growth and Decay Models; Newton's Law; Logistic Growth and Decay Models	815
8.7 Building Exponential, Logarithmic, and Logistic Models from Data.....	825
Chapter Review.....	829
Chapter Test.....	838
Chapter Projects.....	841

Appendix A Review

A.1 Algebra Essentials.....	844
A.2 Geometry Essentials.....	849
A.3 Polynomials.....	855
A.4 Solving Equations	863
A.5 Complex Numbers; Quadratic Equations in the Complex Number System	877
A.6 Problem Solving: Interest, Mixture, Uniform Motion, Constant Rate Job Applications	883
A.7 Interval Notation; Solving Inequalities	890
A.8 n th Roots; Rational Exponents.....	902

Appendix B Graphing Utilities

B.1 The Viewing Rectangle.....	912
B.2 Using a Graphing Utility to Graph Equations	913
B.3 Using a Graphing Utility to Locate Intercepts and Check for Symmetry	918
B.5 Square Screens	920

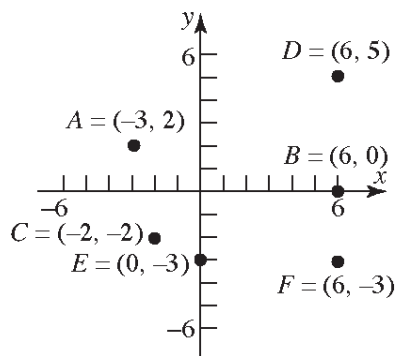
Chapter 1

Preliminaries

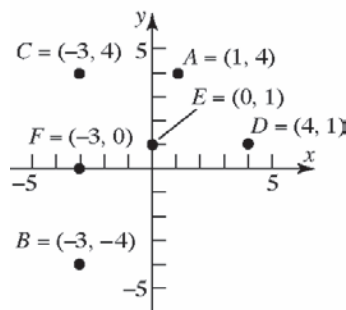
Section 1.1

1. 0
2. $|5 - (-3)| = |8| = 8$
3. $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$
4. $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$
Since the sum of the squares of two of the sides of the triangle equals the square of the third side, the triangle is a right triangle.
5. $\frac{1}{2}bh$
6. true
7. x -coordinate or abscissa; y -coordinate or ordinate
8. quadrants
9. midpoint
10. False; the distance between two points is never negative.
11. False; points that lie in Quadrant IV will have a positive x -coordinate and a negative y -coordinate. The point $(-1, 4)$ lies in Quadrant II.
12. True; $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
13. b
14. a
15. (a) Quadrant II
(b) x -axis
(c) Quadrant III
(d) Quadrant I
(e) y -axis

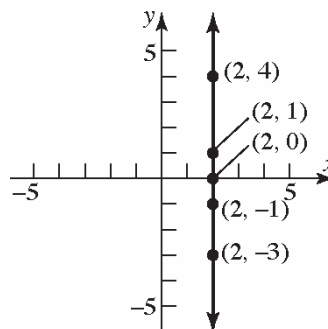
(f) Quadrant IV



16. (a) Quadrant I
(b) Quadrant III
(c) Quadrant II
(d) Quadrant I
(e) y -axis
(f) x -axis

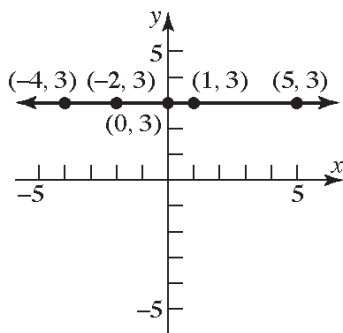


17. The points will be on a vertical line that is two units to the right of the y -axis.



Chapter 1: Preliminaries

18. The points will be on a horizontal line that is three units above the x -axis.



19. $d(P_1, P_2) = \sqrt{(2-0)^2 + (1-0)^2}$
 $= \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$
20. $d(P_1, P_2) = \sqrt{(-2-0)^2 + (1-0)^2}$
 $= \sqrt{(-2)^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$
21. $d(P_1, P_2) = \sqrt{(-2-1)^2 + (2-1)^2}$
 $= \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$
22. $d(P_1, P_2) = \sqrt{(2-(-1))^2 + (2-1)^2}$
 $= \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$
23. $d(P_1, P_2) = \sqrt{(5-3)^2 + (4-(-4))^2}$
 $= \sqrt{2^2 + (8)^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$
24. $d(P_1, P_2) = \sqrt{(2-(-1))^2 + (4-0)^2}$
 $= \sqrt{(3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$
25. $d(P_1, P_2) = \sqrt{(4-(-7))^2 + (0-3)^2}$
 $= \sqrt{11^2 + (-3)^2} = \sqrt{121+9} = \sqrt{130}$
26. $d(P_1, P_2) = \sqrt{(4-2)^2 + (2-(-3))^2}$
 $= \sqrt{2^2 + 5^2} = \sqrt{4+25} = \sqrt{29}$
27. $d(P_1, P_2) = \sqrt{(6-5)^2 + (1-(-2))^2}$
 $= \sqrt{1^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$

$$\begin{aligned} 28. \quad d(P_1, P_2) &= \sqrt{(6-(-4))^2 + (2-(-3))^2} \\ &= \sqrt{10^2 + 5^2} = \sqrt{100+25} \\ &= \sqrt{125} = 5\sqrt{5} \end{aligned}$$

$$\begin{aligned} 29. \quad d(P_1, P_2) &= \sqrt{(2.3-(-0.2))^2 + (1.1-(0.3))^2} \\ &= \sqrt{2.5^2 + 0.8^2} = \sqrt{6.25+0.64} \\ &= \sqrt{6.89} \approx 2.62 \end{aligned}$$

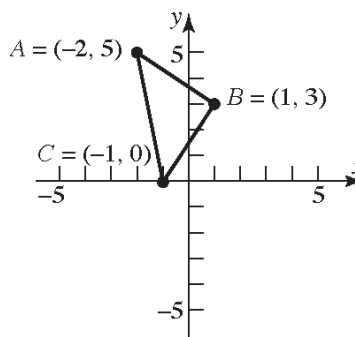
$$\begin{aligned} 30. \quad d(P_1, P_2) &= \sqrt{(-0.3-1.2)^2 + (1.1-2.3)^2} \\ &= \sqrt{(-1.5)^2 + (-1.2)^2} = \sqrt{2.25+1.44} \\ &= \sqrt{3.69} \approx 1.92 \end{aligned}$$

$$\begin{aligned} 31. \quad d(P_1, P_2) &= \sqrt{(0-a)^2 + (0-b)^2} \\ &= \sqrt{(-a)^2 + (-b)^2} = \sqrt{a^2 + b^2} \end{aligned}$$

$$\begin{aligned} 32. \quad d(P_1, P_2) &= \sqrt{(0-a)^2 + (0-a)^2} \\ &= \sqrt{(-a)^2 + (-a)^2} \\ &= \sqrt{a^2 + a^2} = \sqrt{2a^2} = |a|\sqrt{2} \end{aligned}$$

33. $A = (-2, 5), B = (1, 3), C = (-1, 0)$

$$\begin{aligned} d(A, B) &= \sqrt{(1-(-2))^2 + (3-5)^2} \\ &= \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13} \\ d(B, C) &= \sqrt{(-1-1)^2 + (0-3)^2} \\ &= \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13} \\ d(A, C) &= \sqrt{(-1-(-2))^2 + (0-5)^2} \\ &= \sqrt{1^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26} \end{aligned}$$



Section 1.1: The Distance and Midpoint Formulas

Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

$$(\sqrt{13})^2 + (\sqrt{13})^2 = (\sqrt{26})^2$$

$$13 + 13 = 26$$

$$26 = 26$$

The area of a triangle is $A = \frac{1}{2} \cdot bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A, B)] \cdot [d(B, C)]$$

$$= \frac{1}{2} \cdot \sqrt{13} \cdot \sqrt{13} = \frac{1}{2} \cdot 13$$

$$= \frac{13}{2} \text{ square units}$$

34. $A = (-2, 5)$, $B = (12, 3)$, $C = (10, -11)$

$$d(A, B) = \sqrt{(12 - (-2))^2 + (3 - 5)^2}$$

$$= \sqrt{14^2 + (-2)^2}$$

$$= \sqrt{196 + 4} = \sqrt{200}$$

$$= 10\sqrt{2}$$

$$d(B, C) = \sqrt{(10 - 12)^2 + (-11 - 3)^2}$$

$$= \sqrt{(-2)^2 + (-14)^2}$$

$$= \sqrt{4 + 196} = \sqrt{200}$$

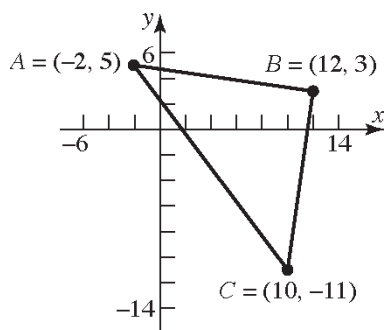
$$= 10\sqrt{2}$$

$$d(A, C) = \sqrt{(10 - (-2))^2 + (-11 - 5)^2}$$

$$= \sqrt{12^2 + (-16)^2}$$

$$= \sqrt{144 + 256} = \sqrt{400}$$

$$= 20$$



Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

$$(10\sqrt{2})^2 + (10\sqrt{2})^2 = (20)^2$$

$$200 + 200 = 400$$

$$400 = 400$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A, B)] \cdot [d(B, C)]$$

$$= \frac{1}{2} \cdot 10\sqrt{2} \cdot 10\sqrt{2}$$

$$= \frac{1}{2} \cdot 100 \cdot 2 = 100 \text{ square units}$$

35. $A = (-5, 3)$, $B = (6, 0)$, $C = (5, 5)$

$$d(A, B) = \sqrt{(6 - (-5))^2 + (0 - 3)^2}$$

$$= \sqrt{11^2 + (-3)^2} = \sqrt{121 + 9}$$

$$= \sqrt{130}$$

$$d(B, C) = \sqrt{(5 - 6)^2 + (5 - 0)^2}$$

$$= \sqrt{(-1)^2 + 5^2} = \sqrt{1 + 25}$$

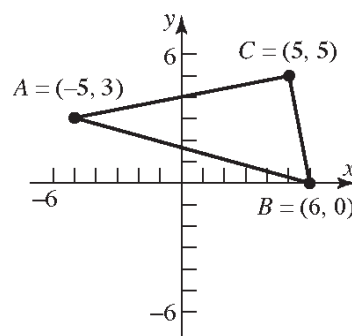
$$= \sqrt{26}$$

$$d(A, C) = \sqrt{(5 - (-5))^2 + (5 - 3)^2}$$

$$= \sqrt{10^2 + 2^2} = \sqrt{100 + 4}$$

$$= \sqrt{104}$$

$$= 2\sqrt{26}$$



Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

Chapter 1: Preliminaries

$$[d(A, C)]^2 + [d(B, C)]^2 = [d(A, B)]^2$$

$$(\sqrt{104})^2 + (\sqrt{26})^2 = (\sqrt{130})^2$$

$$104 + 26 = 130$$

$$130 = 130$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A, C)] \cdot [d(B, C)]$$

$$= \frac{1}{2} \cdot \sqrt{104} \cdot \sqrt{26}$$

$$= \frac{1}{2} \cdot 2\sqrt{26} \cdot \sqrt{26}$$

$$= \frac{1}{2} \cdot 2 \cdot 26$$

$$= 26 \text{ square units}$$

36. $A = (-6, 3)$, $B = (3, -5)$, $C = (-1, 5)$

$$d(A, B) = \sqrt{(3 - (-6))^2 + (-5 - 3)^2}$$

$$= \sqrt{9^2 + (-8)^2} = \sqrt{81 + 64}$$

$$= \sqrt{145}$$

$$d(B, C) = \sqrt{(-1 - 3)^2 + (5 - (-5))^2}$$

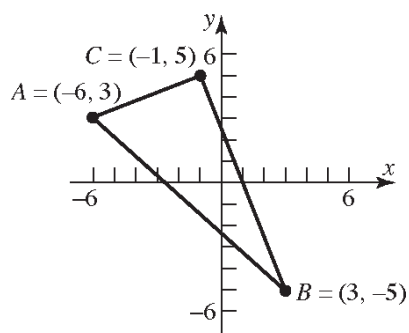
$$= \sqrt{(-4)^2 + 10^2} = \sqrt{16 + 100}$$

$$= \sqrt{116} = 2\sqrt{29}$$

$$d(A, C) = \sqrt{(-1 - (-6))^2 + (5 - 3)^2}$$

$$= \sqrt{5^2 + 2^2} = \sqrt{25 + 4}$$

$$= \sqrt{29}$$



Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

$$[d(A, C)]^2 + [d(B, C)]^2 = [d(A, B)]^2$$

$$(\sqrt{29})^2 + (2\sqrt{29})^2 = (\sqrt{145})^2$$

$$29 + 4 \cdot 29 = 145$$

$$29 + 116 = 145$$

$$145 = 145$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A, C)] \cdot [d(B, C)]$$

$$= \frac{1}{2} \cdot \sqrt{29} \cdot 2\sqrt{29}$$

$$= \frac{1}{2} \cdot 2 \cdot 29$$

$$= 29 \text{ square units}$$

37. $A = (4, -3)$, $B = (0, -3)$, $C = (4, 2)$

$$d(A, B) = \sqrt{(0 - 4)^2 + (-3 - (-3))^2}$$

$$= \sqrt{(-4)^2 + 0^2} = \sqrt{16 + 0}$$

$$= \sqrt{16}$$

$$= 4$$

$$d(B, C) = \sqrt{(4 - 0)^2 + (2 - (-3))^2}$$

$$= \sqrt{4^2 + 5^2} = \sqrt{16 + 25}$$

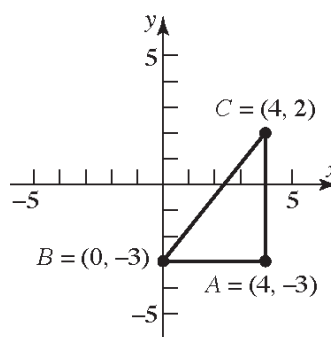
$$= \sqrt{41}$$

$$d(A, C) = \sqrt{(4 - 4)^2 + (2 - (-3))^2}$$

$$= \sqrt{0^2 + 5^2} = \sqrt{0 + 25}$$

$$= \sqrt{25}$$

$$= 5$$



Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

Section 1.1: The Distance and Midpoint Formulas

$$[d(A, B)]^2 + [d(A, C)]^2 = [d(B, C)]^2$$

$$4^2 + 5^2 = (\sqrt{41})^2$$

$$16 + 25 = 41$$

$$41 = 41$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A, B)] \cdot [d(A, C)]$$

$$= \frac{1}{2} \cdot 4 \cdot 5$$

$$= 10 \text{ square units}$$

38. $A = (4, -3), B = (4, 1), C = (2, 1)$

$$d(A, B) = \sqrt{(4-4)^2 + (1-(-3))^2}$$

$$= \sqrt{0^2 + 4^2}$$

$$= \sqrt{0+16}$$

$$= \sqrt{16}$$

$$= 4$$

$$d(B, C) = \sqrt{(2-4)^2 + (1-1)^2}$$

$$= \sqrt{(-2)^2 + 0^2} = \sqrt{4+0}$$

$$= \sqrt{4}$$

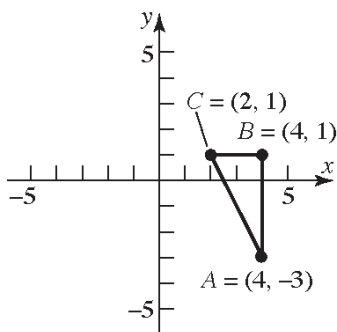
$$= 2$$

$$d(A, C) = \sqrt{(2-4)^2 + (1-(-3))^2}$$

$$= \sqrt{(-2)^2 + 4^2} = \sqrt{4+16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$



Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

$$4^2 + 2^2 = (2\sqrt{5})^2$$

$$16 + 4 = 20$$

$$20 = 20$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A, B)] \cdot [d(B, C)]$$

$$= \frac{1}{2} \cdot 4 \cdot 2$$

$$= 4 \text{ square units}$$

39. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{3+5}{2}, \frac{-4+4}{2} \right)$$

$$= \left(\frac{8}{2}, \frac{0}{2} \right)$$

$$= (4, 0)$$

40. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-2+2}{2}, \frac{0+4}{2} \right)$$

$$= \left(\frac{0}{2}, \frac{4}{2} \right)$$

$$= (0, 2)$$

41. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-1+8}{2}, \frac{4+0}{2} \right)$$

$$= \left(\frac{7}{2}, \frac{4}{2} \right)$$

$$= \left(\frac{7}{2}, 2 \right)$$

Chapter 1: Preliminaries

42. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{2+4}{2}, \frac{-3+2}{2} \right) \\&= \left(\frac{6}{2}, \frac{-1}{2} \right) \\&= \left(3, -\frac{1}{2} \right)\end{aligned}$$

43. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{7+9}{2}, \frac{-5+1}{2} \right) \\&= \left(\frac{16}{2}, \frac{-4}{2} \right) \\&= (8, -2)\end{aligned}$$

44. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{-4+2}{2}, \frac{-3+2}{2} \right) \\&= \left(\frac{-2}{2}, \frac{-1}{2} \right) \\&= \left(-1, -\frac{1}{2} \right)\end{aligned}$$

45. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{a+0}{2}, \frac{b+0}{2} \right) \\&= \left(\frac{a}{2}, \frac{b}{2} \right)\end{aligned}$$

46. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{a+0}{2}, \frac{a+0}{2} \right) \\&= \left(\frac{a}{2}, \frac{a}{2} \right)\end{aligned}$$

47. The x coordinate would be $2+3=5$ and the y coordinate would be $5-2=3$. Thus the new point would be $(5,3)$.

48. The new x coordinate would be $-1-2=-3$ and the new y coordinate would be $6+4=10$. Thus the new point would be $(-3,10)$

49. a. If we use a right triangle to solve the problem, we know the hypotenuse is 13 units in length. One of the legs of the triangle will be $2+3=5$. Thus the other leg will be:

$$5^2 + b^2 = 13^2$$

$$25 + b^2 = 169$$

$$b^2 = 144$$

$$b = 12$$

Thus the coordinates will have an y value of $-1-12=-13$ and $-1+12=11$. So the points are $(3,11)$ and $(3,-13)$.

- b. Consider points of the form $(3, y)$ that are a distance of 13 units from the point $(-2, -1)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3 - (-2))^2 + (-1 - y)^2}$$

$$= \sqrt{(5)^2 + (-1 - y)^2}$$

$$= \sqrt{25 + 1 + 2y + y^2}$$

$$= \sqrt{y^2 + 2y + 26}$$

$$13 = \sqrt{y^2 + 2y + 26}$$

$$13^2 = \left(\sqrt{y^2 + 2y + 26} \right)^2$$

$$169 = y^2 + 2y + 26$$

$$0 = y^2 + 2y - 143$$

$$0 = (y - 11)(y + 13)$$

$$y - 11 = 0 \quad \text{or} \quad y + 13 = 0$$

$$y = 11 \qquad y = -13$$

Thus, the points $(3,11)$ and $(3,-13)$ are a distance of 13 units from the point $(-2,-1)$.

Section 1.1: The Distance and Midpoint Formulas

- 50. a.** If we use a right triangle to solve the problem, we know the hypotenuse is 17 units in length. One of the legs of the triangle will be $2+6=8$. Thus the other leg will be:

$$8^2 + b^2 = 17^2$$

$$64 + b^2 = 289$$

$$b^2 = 225$$

$$b = 15$$

Thus the coordinates will have an x value of $1-15=-14$ and $1+15=16$. So the points are $(-14, -6)$ and $(16, -6)$.

- b.** Consider points of the form $(x, -6)$ that are a distance of 17 units from the point $(1, 2)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(1-x)^2 + (2-(-6))^2}$$

$$= \sqrt{x^2 - 2x + 1 + (8)^2}$$

$$= \sqrt{x^2 - 2x + 1 + 64}$$

$$= \sqrt{x^2 - 2x + 65}$$

$$17 = \sqrt{x^2 - 2x + 65}$$

$$17^2 = (\sqrt{x^2 - 2x + 65})^2$$

$$289 = x^2 - 2x + 65$$

$$0 = x^2 - 2x - 224$$

$$0 = (x+14)(x-16)$$

$$x+14=0 \quad \text{or} \quad x-16=0$$

$$x=-14 \quad \quad \quad x=16$$

Thus, the points $(-14, -6)$ and $(16, -6)$ are a distance of 17 units from the point $(1, 2)$.

- 51.** Points on the x -axis have a y -coordinate of 0. Thus, we consider points of the form $(x, 0)$ that are a distance of 6 units from the point $(4, -3)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4-x)^2 + (-3-0)^2}$$

$$= \sqrt{16-8x+x^2+(-3)^2}$$

$$= \sqrt{16-8x+x^2+9}$$

$$= \sqrt{x^2-8x+25}$$

$$6 = \sqrt{x^2-8x+25}$$

$$6^2 = (\sqrt{x^2-8x+25})^2$$

$$36 = x^2 - 8x + 25$$

$$0 = x^2 - 8x - 11$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-11)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{64+44}}{2} = \frac{8 \pm \sqrt{108}}{2}$$

$$= \frac{8 \pm 6\sqrt{3}}{2} = 4 \pm 3\sqrt{3}$$

$$x = 4 + 3\sqrt{3} \quad \text{or} \quad x = 4 - 3\sqrt{3}$$

Thus, the points $(4+3\sqrt{3}, 0)$ and $(4-3\sqrt{3}, 0)$ are on the x -axis and a distance of 6 units from the point $(4, -3)$.

- 52.** Points on the y -axis have an x -coordinate of 0. Thus, we consider points of the form $(0, y)$ that are a distance of 6 units from the point $(4, -3)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4-0)^2 + (-3-y)^2}$$

$$= \sqrt{4^2 + 9 + 6y + y^2}$$

$$= \sqrt{16 + 9 + 6y + y^2}$$

$$= \sqrt{y^2 + 6y + 25}$$

Chapter 1: Preliminaries

$$6 = \sqrt{y^2 + 6y + 25}$$

$$6^2 = \left(\sqrt{y^2 + 6y + 25}\right)^2$$

$$36 = y^2 + 6y + 25$$

$$0 = y^2 + 6y - 11$$

$$y = \frac{(-6) \pm \sqrt{(6)^2 - 4(1)(-11)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 + 44}}{2} = \frac{-6 \pm \sqrt{80}}{2}$$

$$= \frac{-6 \pm 4\sqrt{5}}{2} = -3 \pm 2\sqrt{5}$$

$$y = -3 + 2\sqrt{5} \quad \text{or} \quad y = -3 - 2\sqrt{5}$$

Thus, the points $(0, -3 + 2\sqrt{5})$ and $(0, -3 - 2\sqrt{5})$

are on the y -axis and a distance of 6 units from the point $(4, -3)$.

- 53. a.** To shift 3 units left and 4 units down, we subtract 3 from the x -coordinate and subtract 4 from the y -coordinate.

$$(2 - 3, 5 - 4) = (-1, 1)$$

- b.** To shift left 2 units and up 8 units, we subtract 2 from the x -coordinate and add 8 to the y -coordinate.

$$(2 - 2, 5 + 8) = (0, 13)$$

- 54.** Let the coordinates of point B be (x, y) . Using the midpoint formula, we can write

$$(2, 3) = \left(\frac{-1 + x}{2}, \frac{8 + y}{2}\right).$$

This leads to two equations we can solve.

$$\frac{-1 + x}{2} = 2$$

$$\frac{8 + y}{2} = 3$$

$$-1 + x = 4$$

$$8 + y = 6$$

$$x = 5$$

$$y = -2$$

Point B has coordinates $(5, -2)$.

- 55.** $M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$

$$P_1 = (x_1, y_1) = (-3, 6) \quad \text{and} \quad (x, y) = (-1, 4), \text{ so}$$

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$

$$-1 = \frac{-3 + x_2}{2} \quad 4 = \frac{6 + y_2}{2}$$

$$-2 = -3 + x_2 \quad 8 = 6 + y_2$$

$$1 = x_2 \quad 2 = y_2$$

Thus, $P_2 = (1, 2)$.

- 56.** $M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$

$$P_2 = (x_2, y_2) = (7, -2) \quad \text{and} \quad (x, y) = (5, -4), \text{ so}$$

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$

$$5 = \frac{x_1 + 7}{2} \quad -4 = \frac{y_1 + (-2)}{2}$$

$$10 = x_1 + 7 \quad -8 = y_1 + (-2)$$

$$3 = x_1 \quad -6 = y_1$$

Thus, $P_1 = (3, -6)$.

- 57.** The midpoint of AB is: $D = \left(\frac{0+6}{2}, \frac{0+0}{2}\right)$
 $= (3, 0)$

The midpoint of AC is: $E = \left(\frac{0+4}{2}, \frac{0+4}{2}\right)$
 $= (2, 2)$

The midpoint of BC is: $F = \left(\frac{6+4}{2}, \frac{0+4}{2}\right)$
 $= (5, 2)$

$$d(C, D) = \sqrt{(0-4)^2 + (3-4)^2}$$

$$= \sqrt{(-4)^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}$$

$$d(B, E) = \sqrt{(2-6)^2 + (2-0)^2}$$

$$= \sqrt{(-4)^2 + 2^2} = \sqrt{16+4}$$

$$= \sqrt{20} = 2\sqrt{5}$$

$$d(A, F) = \sqrt{(2-0)^2 + (5-0)^2}$$

$$= \sqrt{2^2 + 5^2} = \sqrt{4+25}$$

$$= \sqrt{29}$$

Section 1.1: The Distance and Midpoint Formulas

58. Let $P_1 = (0, 0)$, $P_2 = (0, 4)$, $P = (x, y)$

$$\begin{aligned} d(P_1, P_2) &= \sqrt{(0-0)^2 + (4-0)^2} \\ &= \sqrt{16} = 4 \end{aligned}$$

$$\begin{aligned} d(P_1, P) &= \sqrt{(x-0)^2 + (y-0)^2} \\ &= \sqrt{x^2 + y^2} = 4 \\ \rightarrow x^2 + y^2 &= 16 \end{aligned}$$

$$\begin{aligned} d(P_2, P) &= \sqrt{(x-0)^2 + (y-4)^2} \\ &= \sqrt{x^2 + (y-4)^2} = 4 \\ \rightarrow x^2 + (y-4)^2 &= 16 \end{aligned}$$

Therefore,

$$y^2 = (y-4)^2$$

$$y^2 = y^2 - 8y + 16$$

$$8y = 16$$

$$y = 2$$

which gives

$$x^2 + 2^2 = 16$$

$$x^2 = 12$$

$$x = \pm 2\sqrt{3}$$

Two triangles are possible. The third vertex is $(-2\sqrt{3}, 2)$ or $(2\sqrt{3}, 2)$.

$$\begin{aligned} 59. \quad d(P_1, P_2) &= \sqrt{(-4-2)^2 + (1-1)^2} \\ &= \sqrt{(-6)^2 + 0^2} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

$$\begin{aligned} d(P_2, P_3) &= \sqrt{(-4-(-4))^2 + (-3-1)^2} \\ &= \sqrt{0^2 + (-4)^2} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

$$\begin{aligned} d(P_1, P_3) &= \sqrt{(-4-2)^2 + (-3-1)^2} \\ &= \sqrt{(-6)^2 + (-4)^2} \\ &= \sqrt{36+16} \\ &= \sqrt{52} \\ &= 2\sqrt{13} \end{aligned}$$

Since $[d(P_1, P_2)]^2 + [d(P_2, P_3)]^2 = [d(P_1, P_3)]^2$, the triangle is a right triangle.

$$\begin{aligned} 60. \quad d(P_1, P_2) &= \sqrt{(6-(-1))^2 + (2-4)^2} \\ &= \sqrt{7^2 + (-2)^2} \\ &= \sqrt{49+4} \\ &= \sqrt{53} \end{aligned}$$

$$\begin{aligned} d(P_2, P_3) &= \sqrt{(4-6)^2 + (-5-2)^2} \\ &= \sqrt{(-2)^2 + (-7)^2} \\ &= \sqrt{4+49} \\ &= \sqrt{53} \end{aligned}$$

$$\begin{aligned} d(P_1, P_3) &= \sqrt{(4-(-1))^2 + (-5-4)^2} \\ &= \sqrt{5^2 + (-9)^2} \\ &= \sqrt{25+81} \\ &= \sqrt{106} \end{aligned}$$

Since $[d(P_1, P_2)]^2 + [d(P_2, P_3)]^2 = [d(P_1, P_3)]^2$, the triangle is a right triangle.

Since $d(P_1, P_2) = d(P_2, P_3)$, the triangle is isosceles.

Therefore, the triangle is an isosceles right triangle.

$$\begin{aligned} 61. \quad d(P_1, P_2) &= \sqrt{(0-(-2))^2 + (7-(-1))^2} \\ &= \sqrt{2^2 + 8^2} = \sqrt{4+64} = \sqrt{68} \\ &= 2\sqrt{17} \end{aligned}$$

$$\begin{aligned} d(P_2, P_3) &= \sqrt{(3-0)^2 + (2-7)^2} \\ &= \sqrt{3^2 + (-5)^2} = \sqrt{9+25} \\ &= \sqrt{34} \end{aligned}$$

$$\begin{aligned} d(P_1, P_3) &= \sqrt{(3-(-2))^2 + (2-(-1))^2} \\ &= \sqrt{5^2 + 3^2} = \sqrt{25+9} \\ &= \sqrt{34} \end{aligned}$$

Since $d(P_2, P_3) = d(P_1, P_3)$, the triangle is isosceles.

Since $[d(P_1, P_3)]^2 + [d(P_2, P_3)]^2 = [d(P_1, P_2)]^2$, the triangle is also a right triangle.

Therefore, the triangle is an isosceles right triangle.

Chapter 1: Preliminaries

$$\begin{aligned}
 62. \quad d(P_1, P_2) &= \sqrt{(-4-7)^2 + (0-2)^2} \\
 &= \sqrt{(-11)^2 + (-2)^2} \\
 &= \sqrt{121+4} = \sqrt{125} \\
 &= 5\sqrt{5} \\
 d(P_2, P_3) &= \sqrt{(4-(-4))^2 + (6-0)^2} \\
 &= \sqrt{8^2 + 6^2} = \sqrt{64+36} \\
 &= \sqrt{100} \\
 &= 10 \\
 d(P_1, P_3) &= \sqrt{(4-7)^2 + (6-2)^2} \\
 &= \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} \\
 &= \sqrt{25} \\
 &= 5
 \end{aligned}$$

Since $[d(P_1, P_3)]^2 + [d(P_2, P_3)]^2 = [d(P_1, P_2)]^2$, the triangle is a right triangle.

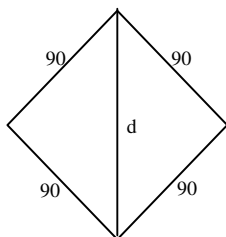
63. Using the Pythagorean Theorem:

$$90^2 + 90^2 = d^2$$

$$8100 + 8100 = d^2$$

$$16200 = d^2$$

$$d = \sqrt{16200} = 90\sqrt{2} \approx 127.28 \text{ feet}$$

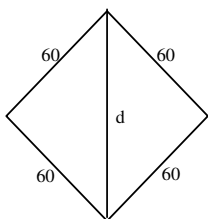


64. Using the Pythagorean Theorem:

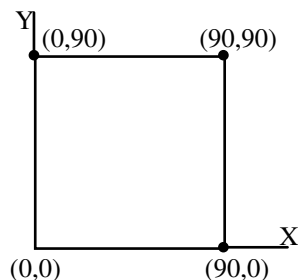
$$60^2 + 60^2 = d^2$$

$$3600 + 3600 = d^2 \rightarrow 7200 = d^2$$

$$d = \sqrt{7200} = 60\sqrt{2} \approx 84.85 \text{ feet}$$



65. a. First: (90, 0), Second: (90, 90), Third: (0, 90)



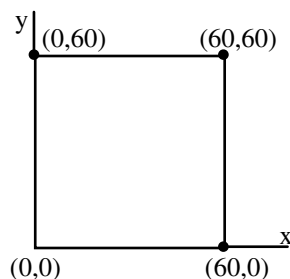
b. Using the distance formula:

$$\begin{aligned}
 d &= \sqrt{(310-90)^2 + (15-90)^2} \\
 &= \sqrt{220^2 + (-75)^2} = \sqrt{54025} \\
 &= 5\sqrt{2161} \approx 232.43 \text{ feet}
 \end{aligned}$$

c. Using the distance formula:

$$\begin{aligned}
 d &= \sqrt{(300-0)^2 + (300-90)^2} \\
 &= \sqrt{300^2 + 210^2} = \sqrt{134100} \\
 &= 30\sqrt{149} \approx 366.20 \text{ feet}
 \end{aligned}$$

66. a. First: (60, 0), Second: (60, 60), Third: (0, 60)



b. Using the distance formula:

$$\begin{aligned}
 d &= \sqrt{(180-60)^2 + (20-60)^2} \\
 &= \sqrt{120^2 + (-40)^2} = \sqrt{16000} \\
 &= 40\sqrt{10} \approx 126.49 \text{ feet}
 \end{aligned}$$

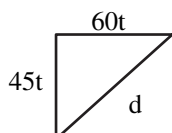
c. Using the distance formula:

$$\begin{aligned}
 d &= \sqrt{(220-0)^2 + (220-60)^2} \\
 &= \sqrt{220^2 + 160^2} = \sqrt{74000} \\
 &= 20\sqrt{185} \approx 272.03 \text{ feet}
 \end{aligned}$$

Section 1.1: The Distance and Midpoint Formulas

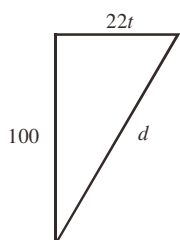
67. The Focus heading east moves a distance $60t$ after t hours. The Tesla heading south moves a distance $40t$ after t hours. Their distance apart after t hours is:

$$\begin{aligned} d &= \sqrt{(60t)^2 + (45t)^2} \\ &= \sqrt{3600t^2 + 2025t^2} \\ &= \sqrt{5625t^2} \\ &= 75t \text{ miles} \end{aligned}$$



68. $\frac{15 \text{ miles}}{1 \text{ hr}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} = 22 \text{ ft/sec}$

$$\begin{aligned} d &= \sqrt{100^2 + (22t)^2} \\ &= \sqrt{10000 + 484t^2} \text{ feet} \end{aligned}$$



69. a. The shortest side is between $P_1 = (2.6, 1.5)$ and $P_2 = (2.7, 1.7)$. The estimate for the desired intersection point is:

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) &= \left(\frac{2.6 + 2.7}{2}, \frac{1.5 + 1.7}{2} \right) \\ &= \left(\frac{5.3}{2}, \frac{3.2}{2} \right) \\ &= (2.65, 1.6) \end{aligned}$$

- b. Using the distance formula:

$$\begin{aligned} d &= \sqrt{(2.65 - 1.4)^2 + (1.6 - 1.3)^2} \\ &= \sqrt{(1.25)^2 + (0.3)^2} \\ &= \sqrt{1.5625 + 0.09} \\ &= \sqrt{1.6525} \\ &\approx 1.285 \text{ units} \end{aligned}$$

70. Let $P_1 = (2018, 232.89)$ and $P_2 = (2022, 513.98)$. The midpoint is:

$$\begin{aligned} (x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{2018 + 2022}{2}, \frac{232.89 + 513.98}{2} \right) \\ &= \left(\frac{4040}{2}, \frac{746.87}{2} \right) \\ &= (2020, 373.44) \end{aligned}$$

The estimate for 2020 is \$373.44 billion. The estimate net sales of Amazon.com in 2020 is \$12.62 billion off from the reported value of \$386.44 billion.

71. For 2014 we have the ordered pair $(2014, 5645)$ and for 2022 we have the ordered pair $(2022, 7951)$. The midpoint is

$$\begin{aligned} (\text{year}, \$) &= \left(\frac{2014 + 2022}{2}, \frac{5645 + 7951}{2} \right) \\ &= \left(\frac{4036}{2}, \frac{13596}{2} \right) \\ &= (2018, 6798) \end{aligned}$$

Using the midpoint, we estimate the average credit card debt in 2018 to be \$6,798. This is underestimate of the actual value.

72. Let $P_1 = (0, 0)$, $P_2 = (a, 0)$, and

$$P_3 = \left(\frac{a}{2}, \frac{\sqrt{3}a}{2} \right). \text{ Then}$$

$$\begin{aligned} d(P_1, P_2) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(a - 0)^2 + (0 - 0)^2} = \sqrt{a^2} = |a| \end{aligned}$$

$$\begin{aligned} d(P_2, P_3) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(\frac{a}{2} - a \right)^2 + \left(\frac{\sqrt{3}a}{2} - 0 \right)^2} \\ &= \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{4a^2}{4}} = \sqrt{a^2} = |a| \end{aligned}$$

Chapter 1: Preliminaries

$$\begin{aligned} d(P_1, P_3) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{\sqrt{3}a}{2} - 0\right)^2} \\ &= \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{4a^2}{4}} = \sqrt{a^2} = |a| \end{aligned}$$

Since the lengths of the three sides are all equal, the triangle is an equilateral triangle.

The midpoints of the sides are

$$M_{P_1P_2} = \left(\frac{0+a}{2}, \frac{0+0}{2}\right) = \left(\frac{a}{2}, 0\right)$$

$$M_{P_2P_3} = \left(\frac{a+\frac{a}{2}}{2}, \frac{0+\frac{\sqrt{3}a}{2}}{2}\right) = \left(\frac{3a}{4}, \frac{\sqrt{3}a}{4}\right)$$

$$M_{P_1P_3} = \left(\frac{0+\frac{a}{2}}{2}, \frac{0+\frac{\sqrt{3}a}{2}}{2}\right) = \left(\frac{a}{4}, \frac{\sqrt{3}a}{4}\right)$$

Then,

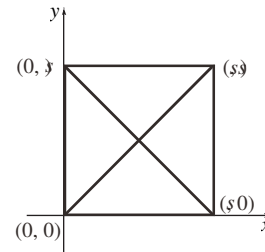
$$\begin{aligned} d(M_{P_1P_2}, M_{P_2P_3}) &= \sqrt{\left(\frac{3a}{4} - \frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}a}{4} - 0\right)^2} \\ &= \sqrt{\left(\frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}a}{4}\right)^2} \\ &= \sqrt{\frac{a^2}{16} + \frac{3a^2}{16}} = \frac{|a|}{2} \end{aligned}$$

$$\begin{aligned} d(M_{P_2P_3}, M_{P_1P_3}) &= \sqrt{\left(\frac{3a}{4} - \frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}a}{4} - \frac{\sqrt{3}a}{4}\right)^2} \\ &= \sqrt{\left(\frac{a}{2}\right)^2 + 0^2} \\ &= \sqrt{\frac{a^2}{4}} = \frac{|a|}{2} \end{aligned}$$

$$\begin{aligned} d(M_{P_1P_2}, M_{P_1P_3}) &= \sqrt{\left(\frac{a}{4} - \frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}a}{4} - 0\right)^2} \\ &= \sqrt{\left(\frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}a}{4}\right)^2} \\ &= \sqrt{\frac{a^2}{16} + \frac{3a^2}{16}} = \frac{|a|}{2} \end{aligned}$$

Since the lengths of the sides of the triangle formed by the midpoints are all equal, the triangle is equilateral.

73. Let $P_1 = (0, 0)$, $P_2 = (0, s)$, $P_3 = (s, 0)$, and $P_4 = (s, s)$ be the vertices of the square.



The points P_1 and P_4 are endpoints of one diagonal and the points P_2 and P_3 are the endpoints of the other diagonal.

$$M_{P_1P_4} = \left(\frac{0+s}{2}, \frac{0+s}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$$

$$M_{P_2P_3} = \left(\frac{0+s}{2}, \frac{s+0}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$$

The midpoints of the diagonals are the same. Therefore, the diagonals of a square intersect at their midpoints.

74. Let $P = (a, 2a)$. Then

$$\begin{aligned} \sqrt{(a+5)^2 + (2a-1)^2} &= \sqrt{(a-4)^2 + (2a+4)^2} \\ (a+5)^2 + (2a-1)^2 &= (a-4)^2 + (2a+4)^2 \\ 5a^2 + 6a + 26 &= 5a^2 + 8a + 32 \\ 6a + 26 &= 8a + 32 \\ -2a &= 6 \\ a &= -3 \end{aligned}$$

Then $P = (-3, -6)$.

75. Arrange the parallelogram on the coordinate plane so that the vertices are

$$P_1 = (0, 0), P_2 = (a, 0), P_3 = (a+b, c) \text{ and } P_4 = (b, c)$$

Then the lengths of the sides are:

$$\begin{aligned} d(P_1, P_2) &= \sqrt{(a-0)^2 + (0-0)^2} \\ &= \sqrt{a^2} = |a| \end{aligned}$$

$$\begin{aligned} d(P_2, P_3) &= \sqrt{[(a+b)-a]^2 + (c-0)^2} \\ &= \sqrt{b^2 + c^2} \end{aligned}$$

Section 1.2: Graphs of Equations in Two Variables; Intercepts; Symmetry

$$\begin{aligned} d(P_3, P_4) &= \sqrt{[b - (a + b)]^2 + (c - c)^2} \\ &= \sqrt{a^2} = |a| \end{aligned}$$

and

$$\begin{aligned} d(P_1, P_4) &= \sqrt{(b - 0)^2 + (c - 0)^2} \\ &= \sqrt{b^2 + c^2} \end{aligned}$$

P_1 and P_3 are the endpoints of one diagonal, and P_2 and P_4 are the endpoints of the other diagonal. The lengths of the diagonals are

$$\begin{aligned} d(P_1, P_3) &= \sqrt{[(a + b) - 0]^2 + (c - 0)^2} \\ &= \sqrt{a^2 + 2ab + b^2 + c^2} \end{aligned}$$

and

$$\begin{aligned} d(P_2, P_4) &= \sqrt{(b - a)^2 + (c - 0)^2} \\ &= \sqrt{a^2 - 2ab + b^2 + c^2} \end{aligned}$$

Sum of the squares of the sides:

$$\begin{aligned} a^2 + (\sqrt{b^2 + c^2})^2 + a^2 + (\sqrt{b^2 + c^2})^2 \\ = 2a^2 + 2b^2 + 2c^2 \end{aligned}$$

Sum of the squares of the diagonals:

$$\begin{aligned} \left(\sqrt{a^2 + 2ab + b^2 + c^2} \right)^2 + \left(\sqrt{a^2 - 2ab + b^2 + c^2} \right)^2 \\ = 2a^2 + 2b^2 + 2c^2 \end{aligned}$$

76. Answers will vary.

Section 1.2

1. $2(x + 3) - 1 = -7$

$$2(x + 3) = -6$$

$$x + 3 = -3$$

$$x = -6$$

The solution set is $\{-6\}$.

2. $x^2 - 9 = 0$

$$x^2 = 9$$

$$x = \pm\sqrt{9} = \pm 3$$

The solution set is $\{-3, 3\}$.

3. intercepts

4. $y = 0$

5. y -axis

6. 4

7. $(-3, 4)$

8. True

9. False; the y -coordinate of a point at which the graph crosses or touches the x -axis is always 0. The x -coordinate of such a point is an x -intercept.

10. False; a graph can be symmetric with respect to both coordinate axes (in such cases it will also be symmetric with respect to the origin).

For example: $x^2 + y^2 = 1$

11. d

12. c

13. $y = x^4 - \sqrt{x}$

$$0 = 0^4 - \sqrt{0} \quad 1 = 1^4 - \sqrt{1} \quad 4 = (2)^4 - \sqrt{2}$$

$$0 = 0 \quad 1 \neq 0 \quad 4 \neq 16 - \sqrt{2}$$

The point $(0, 0)$ is on the graph of the equation.

14. $y = x^3 - 2\sqrt{x}$

$$0 = 0^3 - 2\sqrt{0} \quad 1 = 1^3 - 2\sqrt{1} \quad -1 = 1^3 - 2\sqrt{1}$$

$$0 = 0 \quad 1 \neq -1 \quad -1 = -1$$

The points $(0, 0)$ and $(1, -1)$ are on the graph of the equation.

15. $y^2 = x^2 + 9$

$$3^2 = 0^2 + 9 \quad 0^2 = 3^2 + 9 \quad 0^2 = (-3)^2 + 9$$

$$9 = 9 \quad 0 \neq 18 \quad 0 \neq 18$$

The point $(0, 3)$ is on the graph of the equation.

Chapter 1: Graphs

16. $y^3 = x + 1$

$$2^3 = 1 + 1 \quad 1^3 = 0 + 1 \quad 0^3 = -1 + 1$$

$$8 \neq 2 \quad 1 = 1 \quad 0 = 0$$

The points (0, 1) and (-1, 0) are on the graph of the equation.

17. $x^2 + y^2 = 4$

$$0^2 + 2^2 = 4 \quad (-2)^2 + 2^2 = 4 \quad (\sqrt{2})^2 + (\sqrt{2})^2 = 4$$

$$4 = 4 \quad 8 \neq 4 \quad 4 = 4$$

(0, 2) and $(\sqrt{2}, \sqrt{2})$ are on the graph of the equation.

18. $x^2 + 4y^2 = 4$

$$0^2 + 4 \cdot 1^2 = 4 \quad 2^2 + 4 \cdot 0^2 = 4 \quad 2^2 + 4 \left(\frac{1}{2}\right)^2 = 4$$

$$4 = 4 \quad 4 = 4 \quad 5 \neq 4$$

The points (0, 1) and (2, 0) are on the graph of the equation.

19. $y = x + 2$

x-intercept:

$$0 = x + 2$$

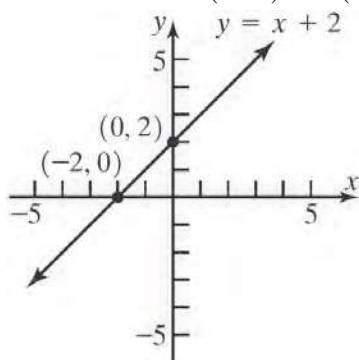
$$-2 = x$$

y-intercept:

$$y = 0 + 2$$

$$y = 2$$

The intercepts are (-2, 0) and (0, 2).



20. $y = x - 6$

x-intercept:

$$0 = x - 6$$

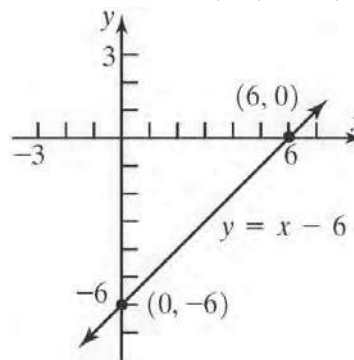
$$6 = x$$

y-intercept:

$$y = 0 - 6$$

$$y = -6$$

The intercepts are (6, 0) and (0, -6).



21. $y = 2x + 8$

x-intercept:

$$0 = 2x + 8$$

$$2x = -8$$

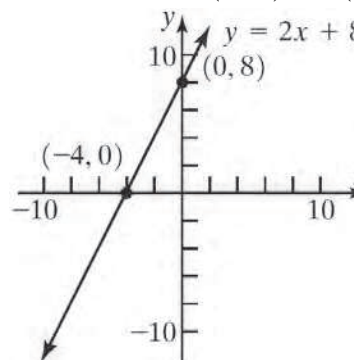
$$x = -4$$

y-intercept:

$$y = 2(0) + 8$$

$$y = 8$$

The intercepts are (-4, 0) and (0, 8).



22. $y = 3x - 9$

x-intercept:

$$0 = 3x - 9$$

$$3x = 9$$

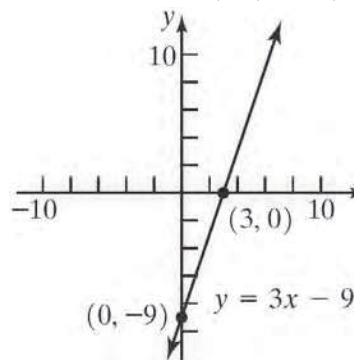
$$x = 3$$

y-intercept:

$$y = 3(0) - 9$$

$$y = -9$$

The intercepts are (3, 0) and (0, -9).



Section 1.2: Graphs of Equations in Two Variables; Intercepts; Symmetry

23. $y = x^2 - 1$

x -intercepts:

$$0 = x^2 - 1$$

$$x^2 = 1$$

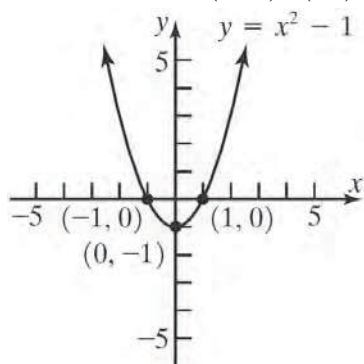
$$x = \pm 1$$

y -intercept:

$$y = 0^2 - 1$$

$$y = -1$$

The intercepts are $(-1, 0)$, $(1, 0)$, and $(0, -1)$.



24. $y = x^2 - 9$

x -intercepts:

$$0 = x^2 - 9$$

$$x^2 = 9$$

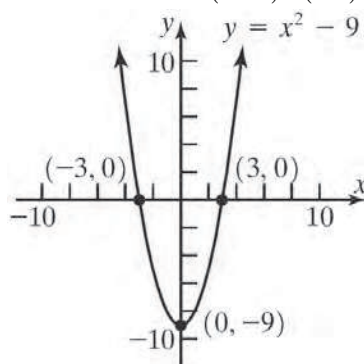
$$x = \pm 3$$

y -intercept:

$$y = 0^2 - 9$$

$$y = -9$$

The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, -9)$.



25. $y = -x^2 + 4$

x -intercepts:

$$0 = -x^2 + 4$$

$$x^2 = 4$$

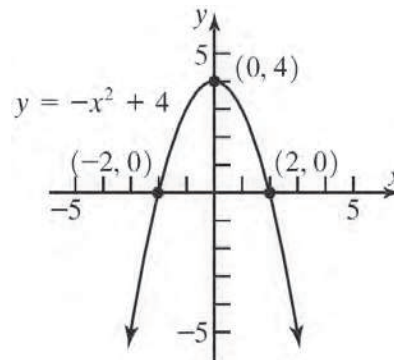
$$x = \pm 2$$

y -intercepts:

$$y = -(0)^2 + 4$$

$$y = 4$$

The intercepts are $(-2, 0)$, $(2, 0)$, and $(0, 4)$.



26. $y = -x^2 + 1$

x -intercepts:

$$0 = -x^2 + 1$$

$$x^2 = 1$$

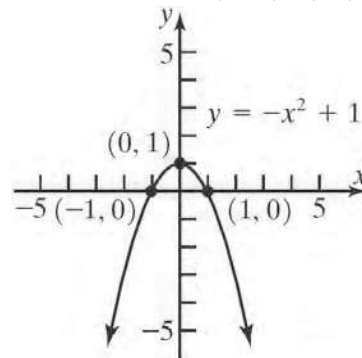
$$x = \pm 1$$

y -intercept:

$$y = -(0)^2 + 1$$

$$y = 1$$

The intercepts are $(-1, 0)$, $(1, 0)$, and $(0, 1)$.



27. $2x + 3y = 6$

x -intercepts:

$$2x + 3(0) = 6$$

$$2x = 6$$

$$x = 3$$

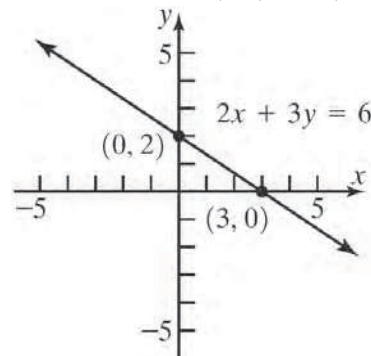
y -intercept:

$$2(0) + 3y = 6$$

$$3y = 6$$

$$y = 2$$

The intercepts are $(3, 0)$ and $(0, 2)$.



Chapter 1: Graphs

28. $5x + 2y = 10$

x -intercepts:

$$5x + 2(0) = 10$$

$$5x = 10$$

$$x = 2$$

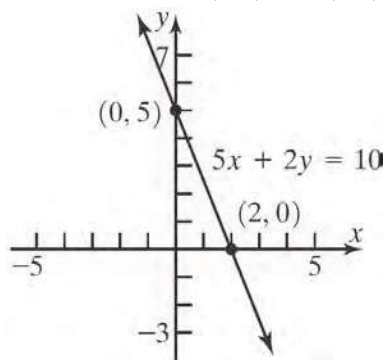
y -intercept:

$$5(0) + 2y = 10$$

$$2y = 10$$

$$y = 5$$

The intercepts are $(2, 0)$ and $(0, 5)$.



29. $9x^2 + 4y = 36$

x -intercepts:

$$9x^2 + 4(0) = 36$$

$$9x^2 = 36$$

$$x^2 = 4$$

$$x = \pm 2$$

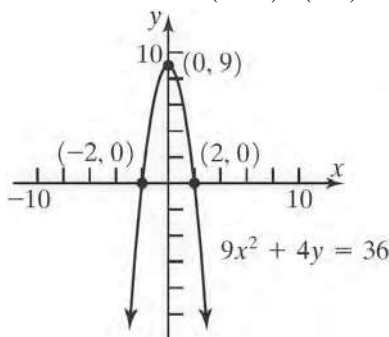
y -intercept:

$$9(0)^2 + 4y = 36$$

$$4y = 36$$

$$y = 9$$

The intercepts are $(-2, 0)$, $(2, 0)$, and $(0, 9)$.



30. $4x^2 + y = 4$

x -intercepts:

$$4x^2 + 0 = 4$$

$$4x^2 = 4$$

$$x^2 = 1$$

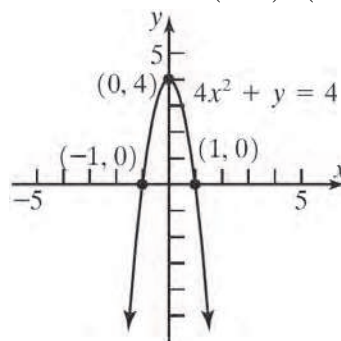
$$x = \pm 1$$

y -intercept:

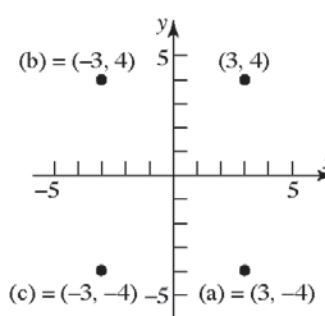
$$4(0)^2 + y = 4$$

$$y = 4$$

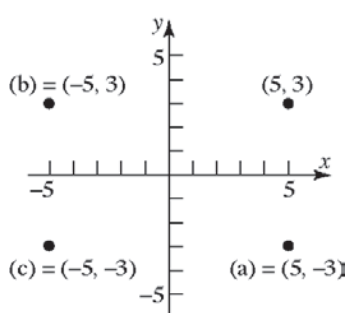
The intercepts are $(-1, 0)$, $(1, 0)$, and $(0, 4)$.



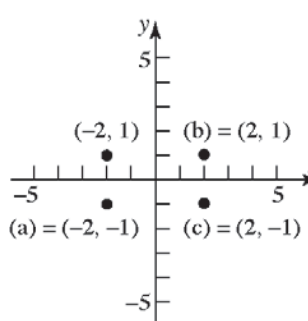
31.



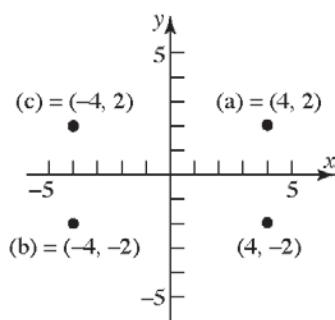
32.



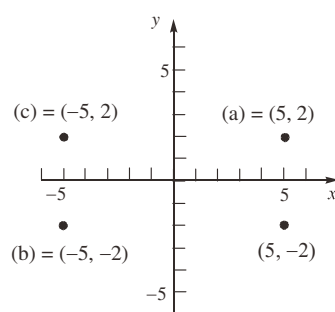
33.



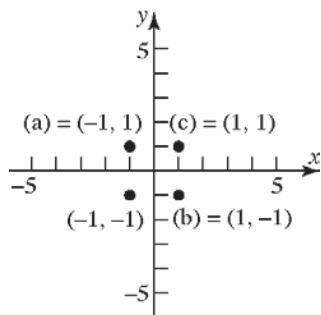
34.



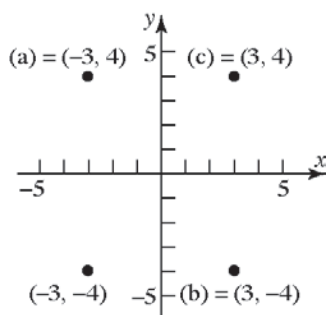
35.



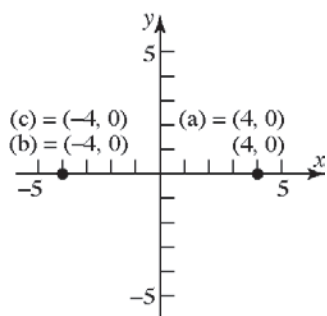
36.



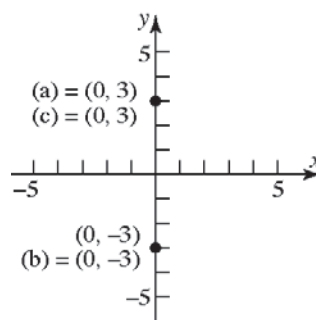
37.



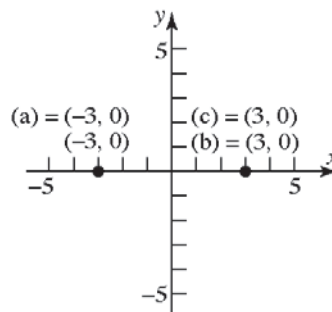
38.



39.



40.

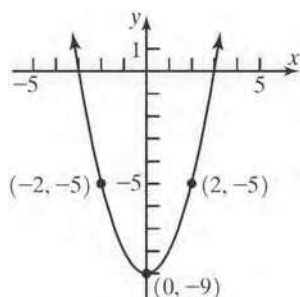


41. a. Intercepts: $(-1, 0)$ and $(1, 0)$
b. Symmetric with respect to the x -axis, y -axis, and the origin.
42. a. Intercepts: $(0, 1)$
b. Not symmetric to the x -axis, the y -axis, nor the origin
43. a. Intercepts: $(-\frac{\pi}{2}, 0)$, $(0, 1)$, and $(\frac{\pi}{2}, 0)$
b. Symmetric with respect to the y -axis.
44. a. Intercepts: $(-2, 0)$, $(0, -3)$, and $(2, 0)$
b. Symmetric with respect to the y -axis.
45. a. Intercepts: $(0, 0)$
b. Symmetric with respect to the x -axis.
46. a. Intercepts: $(-2, 0)$, $(0, 2)$, $(0, -2)$, and $(2, 0)$
b. Symmetric with respect to the x -axis, y -axis, and the origin.
47. a. Intercepts: $(-2, 0)$, $(0, 0)$, and $(2, 0)$
b. Symmetric with respect to the origin.
48. a. Intercepts: $(-4, 0)$, $(0, 0)$, and $(4, 0)$
b. Symmetric with respect to the origin.

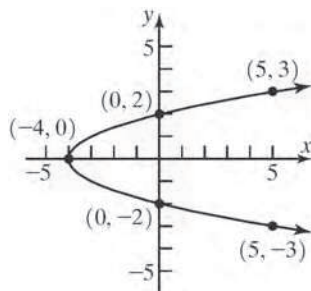
Chapter 1: Graphs

49. a. x -intercepts: $[-2, 1]$, y -intercept 0
 b. Not symmetric to x -axis, y -axis, or origin.
50. a. x -intercepts: $[-1, 2]$, y -intercept 0
 b. Not symmetric to x -axis, y -axis, or origin.
51. a. Intercepts: none
 b. Symmetric with respect to the origin.
52. a. Intercepts: none
 b. Symmetric with respect to the x -axis.

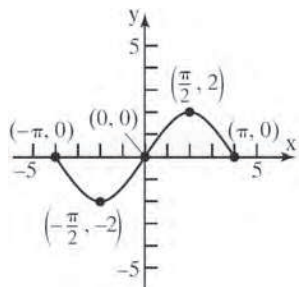
53.



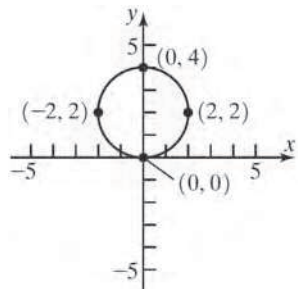
54.



55.



56.



57. $y^2 = x + 16$

x -intercepts:

$$0^2 = x + 16$$

$$-16 = x$$

y -intercepts:

$$y^2 = 0 + 16$$

$$y^2 = 16$$

$$y = \pm 4$$

The intercepts are $(-16, 0)$, $(0, -4)$ and $(0, 4)$.

Test x -axis symmetry: Let $y = -y$

$$(-y)^2 = x + 16$$

$$y^2 = x + 16 \text{ same}$$

Test y -axis symmetry: Let $x = -x$

$$y^2 = -x + 16 \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$(-y)^2 = -x + 16$$

$$y^2 = -x + 16 \text{ different}$$

Therefore, the graph will have x -axis symmetry.

58. $y^2 = x + 9$

x -intercepts:

$$(0)^2 = -x + 9$$

$$0 = -x + 9$$

$$x = 9$$

y -intercepts:

$$y^2 = 0 + 9$$

$$y^2 = 9$$

$$y = \pm 3$$

The intercepts are $(-9, 0)$, $(0, -3)$ and $(0, 3)$.

Test x -axis symmetry: Let $y = -y$

$$(-y)^2 = x + 9$$

$$y^2 = x + 9 \text{ same}$$

Test y -axis symmetry: Let $x = -x$

$$y^2 = -x + 9 \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$(-y)^2 = -x + 9$$

$$y^2 = -x + 9 \text{ different}$$

Therefore, the graph will have x -axis symmetry.

59. $y = \sqrt[3]{x}$

x -intercepts:

$$0 = \sqrt[3]{x}$$

$$0 = x$$

y -intercepts:

$$y = \sqrt[3]{0} = 0$$

The only intercept is $(0, 0)$.

Test x -axis symmetry: Let $y = -y$

$$-y = \sqrt[3]{x} \text{ different}$$

Section 1.2: Graphs of Equations in Two Variables; Intercepts; Symmetry

Test y-axis symmetry: Let $x = -x$

$$y = \sqrt[3]{-x} = -\sqrt[3]{x} \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \sqrt[3]{-x} = -\sqrt[3]{x}$$

$$y = \sqrt[3]{x} \text{ same}$$

Therefore, the graph will have origin symmetry.

60. $y = \sqrt[5]{x}$

x -intercepts: y -intercepts:

$$0 = \sqrt[5]{x} \quad y = \sqrt[5]{0} = 0$$

$$0 = x$$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = \sqrt[5]{x} \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \sqrt[5]{-x} = -\sqrt[5]{x} \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \sqrt[5]{-x} = -\sqrt[5]{x}$$

$$y = \sqrt[5]{x} \text{ same}$$

Therefore, the is symmetric with respect to the origin.

61. $x^2 + y - 9 = 0$

x -intercepts: y -intercepts:

$$x^2 - 9 = 0 \quad 0^2 + y - 9 = 0$$

$$x^2 = 9 \quad y = 9$$

$$x = \pm 3$$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,9)$.

Test x-axis symmetry: Let $y = -y$

$$x^2 - y - 9 = 0 \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$(-x)^2 + y - 9 = 0$$

$$x^2 + y - 9 = 0 \text{ same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$(-x)^2 - y - 9 = 0$$

$$x^2 - y - 9 = 0 \text{ different}$$

Therefore, the graph has y-axis symmetry.

62. $x^2 - y - 4 = 0$

x -intercepts: y -intercept:

$$x^2 - 0 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$0^2 - y - 4 = 0$$

$$-y = 4$$

$$y = -4$$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,-4)$.

Test x-axis symmetry: Let $y = -y$

$$x^2 - (-y) - 4 = 0$$

$$x^2 + y - 4 = 0 \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$(-x)^2 - y - 4 = 0$$

$$x^2 - y - 4 = 0 \text{ same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$(-x)^2 - (-y) - 4 = 0$$

$$x^2 + y - 4 = 0 \text{ different}$$

Therefore, the graph has y-axis symmetry.

63. $25x^2 + 4y^2 = 100$

x -intercepts: y -intercepts:

$$25x^2 + 4(0)^2 = 100 \quad 25(0)^2 + 4y^2 = 100$$

$$25x^2 = 100$$

$$4y^2 = 25$$

$$x^2 = 4$$

$$y^2 = 5$$

$$x = \pm 2$$

$$y = \pm 5$$

The intercepts are $(-2,0)$, $(2,0)$, $(0,-5)$, and $(0,5)$.

Test x-axis symmetry: Let $y = -y$

$$25x^2 + 4(-y)^2 = 100$$

$$25x^2 + 4y^2 = 100 \text{ same}$$

Test y-axis symmetry: Let $x = -x$

$$25(-x)^2 + 4y^2 = 100$$

$$25x^2 + 4y^2 = 100 \text{ same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$25(-x)^2 + 4(-y)^2 = 100$$

$$25x^2 + 4y^2 = 100 \text{ same}$$

Therefore, the graph has x-axis, y-axis, and origin symmetry.

64. $4x^2 + y^2 = 4$

x -intercepts: y -intercepts:

Chapter 1: Graphs

$$\begin{aligned} 4x^2 + 0^2 &= 4 & 4(0)^2 + y^2 &= 4 \\ 4x^2 &= 4 & y^2 &= 4 \\ x^2 &= 1 & y &= \pm 2 \\ x &= \pm 1 \end{aligned}$$

The intercepts are $(-1, 0)$, $(1, 0)$, $(0, -2)$, and $(0, 2)$.

Test x-axis symmetry: Let $y = -y$

$$\begin{aligned} 4x^2 + (-y)^2 &= 4 \\ 4x^2 + y^2 &= 4 \text{ same} \end{aligned}$$

Test y-axis symmetry: Let $x = -x$

$$\begin{aligned} 4(-x)^2 + y^2 &= 4 \\ 4x^2 + y^2 &= 4 \text{ same} \end{aligned}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$\begin{aligned} 4(-x)^2 + (-y)^2 &= 4 \\ 4x^2 + y^2 &= 4 \text{ same} \end{aligned}$$

Therefore, the graph has x-axis, y-axis, and origin symmetry.

65. $y = x^3 - 64$

x -intercepts:	y -intercepts:
$0 = x^3 - 64$	$y = 0^3 - 64$
$x^3 = 64$	$y = -64$
$x = 4$	

The intercepts are $(4, 0)$ and $(0, -64)$.

Test x-axis symmetry: Let $y = -y$

$$-y = x^3 - 64 \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$\begin{aligned} y &= (-x)^3 - 64 \\ y &= -x^3 - 64 \text{ different} \end{aligned}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$\begin{aligned} -y &= (-x)^3 - 64 \\ y &= x^3 + 64 \text{ different} \end{aligned}$$

Therefore, the graph has no symmetry.

66. $y = x^4 - 1$

x -intercepts:	y -intercepts:
------------------	------------------

$$\begin{aligned} 0 &= x^4 - 1 & y &= 0^4 - 1 \\ x^4 &= 1 & y &= -1 \\ x &= \pm 1 \end{aligned}$$

The intercepts are $(-1, 0)$, $(1, 0)$, and $(0, -1)$.

Test x-axis symmetry: Let $y = -y$

$$-y = x^4 - 1 \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$\begin{aligned} y &= (-x)^4 - 1 \\ y &= x^4 - 1 \text{ same} \end{aligned}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$\begin{aligned} -y &= (-x)^4 - 1 \\ -y &= x^4 - 1 \text{ different} \end{aligned}$$

Therefore, the graph has y-axis symmetry.

67. $y = x^2 - 2x - 8$

x -intercepts:	y -intercepts:
$0 = x^2 - 2x - 8$	$y = 0^2 - 2(0) - 8$
$0 = (x - 4)(x + 2)$	$y = -8$
$x = 4$ or $x = -2$	

The intercepts are $(4, 0)$, $(-2, 0)$, and $(0, -8)$.

Test x-axis symmetry: Let $y = -y$

$$-y = x^2 - 2x - 8 \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$\begin{aligned} y &= (-x)^2 - 2(-x) - 8 \\ y &= x^2 + 2x - 8 \text{ different} \end{aligned}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$\begin{aligned} -y &= (-x)^2 - 2(-x) - 8 \\ -y &= x^2 + 2x - 8 \text{ different} \end{aligned}$$

Therefore, the graph has no symmetry.

68. $y = x^2 + 4$

x -intercepts:	y -intercepts:
$0 = x^2 + 4$	$y = 0^2 + 4$
$x^2 = -4$	$y = 4$

no real solution

The only intercept is $(0, 4)$.

Test x-axis symmetry: Let $y = -y$

$$-y = x^2 + 4 \text{ different}$$

Section 1.2: Graphs of Equations in Two Variables; Intercepts; Symmetry

Test y-axis symmetry: Let $x = -x$

$$y = (-x)^2 + 4$$

$$y = x^2 + 4 \text{ same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = (-x)^2 + 4$$

$$-y = x^2 + 4 \text{ different}$$

Therefore, the graph has y-axis symmetry.

69. $y = \frac{4x}{x^2 + 16}$

x-intercepts:

$$0 = \frac{4x}{x^2 + 16}$$

$$4x = 0$$

$$x = 0$$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{4x}{x^2 + 16} \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{4(-x)}{(-x)^2 + 16}$$

$$y = -\frac{4x}{x^2 + 16} \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{4(-x)}{(-x)^2 + 16}$$

$$-y = -\frac{4x}{x^2 + 16}$$

$$y = \frac{4x}{x^2 + 16} \text{ same}$$

Therefore, the graph has origin symmetry.

70. $y = \frac{x^2 - 4}{2x}$

x-intercepts:

$$0 = \frac{x^2 - 4}{2x}$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

The intercepts are $(-2, 0)$ and $(2, 0)$.

y-intercepts:

$$y = \frac{0^2 - 4}{2(0)} = \frac{-4}{0}$$

undefined

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{x^2 - 4}{2x} \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{(-x)^2 - 4}{2(-x)}$$

$$y = -\frac{x^2 - 4}{2x} \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{(-x)^2 - 4}{2(-x)}$$

$$-y = \frac{x^2 - 4}{-2x}$$

$$y = \frac{x^2 - 4}{2x} \text{ same}$$

Therefore, the graph has origin symmetry.

71. $y = \frac{-x^3}{x^2 - 9}$

x-intercepts:

$$0 = \frac{-x^3}{x^2 - 9}$$

$$-x^3 = 0$$

$$x = 0$$

y-intercepts:

$$y = \frac{-0^3}{0^2 - 9} = \frac{0}{-9} = 0$$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{-x^3}{x^2 - 9}$$

$$y = \frac{x^3}{x^2 - 9} \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{-(-x)^3}{(-x)^2 - 9}$$

$$y = \frac{x^3}{x^2 - 9} \text{ different}$$

Chapter 1: Graphs

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{-(-x)^3}{(-x)^2 - 9}$$

$$-y = \frac{x^3}{x^2 - 9}$$

$$y = \frac{-x^3}{x^2 - 9} \text{ same}$$

Therefore, the graph has origin symmetry.

72. $y = \frac{x^4 + 1}{2x^5}$

x -intercepts:

$$0 = \frac{x^4 + 1}{2x^5}$$

$$x^4 = -1$$

no real solution

There are no intercepts for the graph of this equation.

Test x -axis symmetry: Let $y = -y$

$$-y = \frac{x^4 + 1}{2x^5} \text{ different}$$

Test y -axis symmetry: Let $x = -x$

$$y = \frac{(-x)^4 + 1}{2(-x)^5}$$

$$y = \frac{x^4 + 1}{-2x^5} \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

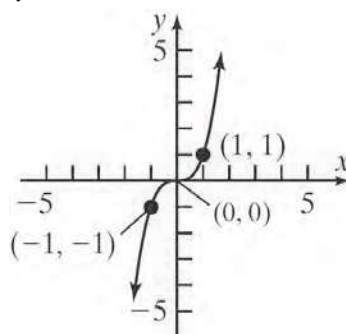
$$-y = \frac{(-x)^4 + 1}{2(-x)^5}$$

$$-y = \frac{x^4 + 1}{-2x^5}$$

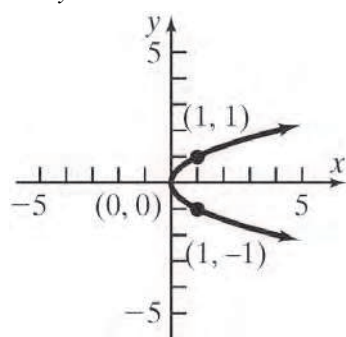
$$y = \frac{x^4 + 1}{2x^5} \text{ same}$$

Therefore, the graph has origin symmetry.

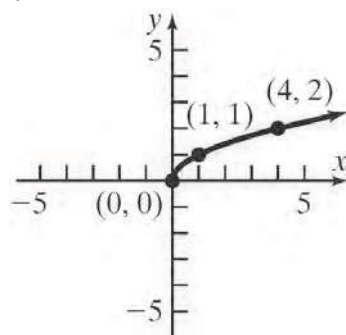
73. $y = x^3$



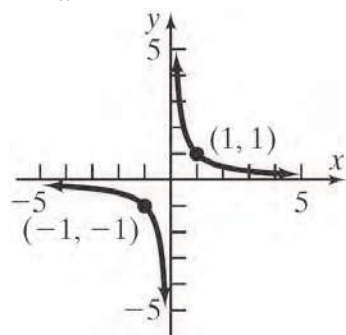
74. $x = y^2$



75. $y = \sqrt{x}$



76. $y = \frac{1}{x}$



Section 1.2: Graphs of Equations in Two Variables; Intercepts; Symmetry

77. If the point $(a, 4)$ is on the graph of

$$y = x^2 + 3x, \text{ then we have}$$

$$4 = a^2 + 3a$$

$$0 = a^2 + 3a - 4$$

$$0 = (a + 4)(a - 1)$$

$$a + 4 = 0 \quad \text{or} \quad a - 1 = 0$$

$$a = -4 \quad a = 1$$

Thus, $a = -4$ or $a = 1$.

78. If the point $(a, -5)$ is on the graph of

$$y = x^2 + 6x, \text{ then we have}$$

$$-5 = a^2 + 6a$$

$$0 = a^2 + 6a + 5$$

$$0 = (a + 5)(a + 1)$$

$$a + 5 = 0 \quad \text{or} \quad a + 1 = 0$$

$$a = -5 \quad a = -1$$

Thus, $a = -5$ or $a = -1$.

79. For a graph with origin symmetry, if the point (a, b) is on the graph, then so is the point $(-a, -b)$. Since the point $(1, 2)$ is on the graph of an equation with origin symmetry, the point $(-1, -2)$ must also be on the graph.

80. For a graph with y-axis symmetry, if the point (a, b) is on the graph, then so is the point $(-a, b)$. Since 6 is an x-intercept in this case, the point $(6, 0)$ is on the graph of the equation. Due to the y-axis symmetry, the point $(-6, 0)$ must also be on the graph. Therefore, -6 is another x-intercept.

81. For a graph with origin symmetry, if the point (a, b) is on the graph, then so is the point $(-a, -b)$. Since -4 is an x-intercept in this case, the point $(-4, 0)$ is on the graph of the equation. Due to the origin symmetry, the point $(4, 0)$ must also be on the graph. Therefore, 4 is another x-intercept.

82. For a graph with x-axis symmetry, if the point (a, b) is on the graph, then so is the point

$(a, -b)$. Since 2 is a y-intercept in this case, the point $(0, 2)$ is on the graph of the equation. Due to the x-axis symmetry, the point $(0, -2)$ must also be on the graph. Therefore, -2 is another y-intercept.

83. a. $(x^2 + y^2 - x)^2 = x^2 + y^2$

x-intercepts:

$$(x^2 + (0)^2 - x)^2 = x^2 + (0)^2$$

$$(x^2 - x)^2 = x^2$$

$$x^4 - 2x^3 + x^2 = x^2$$

$$x^4 - 2x^3 = 0$$

$$x^3(x - 2) = 0$$

$$x^3 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = 0 \quad x = 2$$

y-intercepts:

$$((0)^2 + y^2 - 0)^2 = (0)^2 + y^2$$

$$(y^2)^2 = y^2$$

$$y^4 = y^2$$

$$y^4 - y^2 = 0$$

$$y^2(y^2 - 1) = 0$$

$$y^2 = 0 \quad \text{or} \quad y^2 - 1 = 0$$

$$y = 0 \quad y^2 = 1$$

$$y = \pm 1$$

The intercepts are $(0, 0)$, $(2, 0)$, $(0, -1)$, and $(0, 1)$.

- b. Test x-axis symmetry: Let $y = -y$

$$(x^2 + (-y)^2 - x)^2 = x^2 + (-y)^2$$

$$(x^2 + y^2 - x)^2 = x^2 + y^2 \quad \text{same}$$

Test y-axis symmetry: Let $x = -x$

$$((-x)^2 + y^2 - (-x))^2 = (-x)^2 + y^2$$

$$(x^2 + y^2 + x)^2 = x^2 + y^2 \quad \text{different}$$

Chapter 1: Graphs

Test origin symmetry: Let $x = -x$ and $y = -y$

$$\begin{aligned} ((-x)^2 + (-y)^2 - (-x))^2 &= (-x)^2 + (-y)^2 \\ (x^2 + y^2 + x)^2 &= x^2 + y^2 \quad \text{different} \end{aligned}$$

Thus, the graph will have x -axis symmetry.

84. a. $16y^2 = 120x - 225$

y -intercepts:

$$16y^2 = 120(0) - 225$$

$$16y^2 = -225$$

$$y^2 = -\frac{225}{16}$$

no real solution

x -intercepts:

$$16(0)^2 = 120x - 225$$

$$0 = 120x - 225$$

$$-120x = -225$$

$$x = \frac{-225}{-120} = \frac{15}{8}$$

The only intercept is $\left(\frac{15}{8}, 0\right)$.

b. Test x -axis symmetry: Let $y = -y$

$$16(-y)^2 = 120x - 225$$

$$16y^2 = 120x - 225 \quad \text{same}$$

Test y -axis symmetry: Let $x = -x$

$$16y^2 = 120(-x) - 225$$

$$16y^2 = -120x - 225 \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$16(-y)^2 = 120(-x) - 225$$

$$16y^2 = -120x - 225 \quad \text{different}$$

Thus, the graph has x -axis symmetry.

85. Let $y = 0$.

$$(x^2 + 0^2)^2 = a^2(x^2 - 0^2)$$

$$x^4 = a^2(x^2)$$

$$x^4 - a^2x^2 = 0$$

$$x^2(x^2 - a^2) = 0$$

$$x^2 = 0 \quad \text{or} \quad (x^2 - a^2) = 0$$

$$x = 0 \quad \text{or} \quad x^2 = a^2$$

$$x = -a, a$$

Let $x = 0$.

$$(0^2 + y^2)^2 = a^2(0^2 - y^2)$$

$$y^4 = a^2(-y^2)$$

$$y^4 + a^2y^2 = 0$$

$$y^2(y^2 + a^2) = 0$$

$$y = 0$$

(Note that the solutions to $y^2 + a^2 = 0$ are not real)

So the intercepts are $(0,0)$, $(a,0)$ and $(-a,0)$.

Test x -axis symmetry: Replace y by $-y$

$$(x^2 + (-y)^2)^2 = a^2(x^2 - (-y)^2)$$

$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \quad \text{equivalent}$$

Test y -axis symmetry: replace x by $-x$

$$((-x)^2 + y^2)^2 = a^2((-x)^2 - y^2)$$

$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \quad \text{equivalent}$$

Test origin symmetry: replace x by $-x$ and y by $-y$

$$((-x)^2 + (-y)^2)^2 = a^2((-x)^2 - (-y)^2)$$

$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \quad \text{equivalent}$$

The graph is symmetric by respect to the x -axis, the y -axis, and the origin.

86. Let $y = 0$.

$$(x^2 + 0^2 - ax)^2 = b^2(x^2 + 0^2)$$

$$x^4 - 2ax^3 + a^2x^2 - b^2x^2 = 0$$

$$x^2[(x - (a + b))(x - (a - b))] = 0$$

$$x = 0 \quad \text{or} \quad x = a + b$$

$$\text{or} \quad x = a - b$$

Let $x = 0$.

$$(0^2 + y^2 - a \cdot 0)^2 = b^2(0^2 + y^2)$$

$$y^4 - b^2y^2 = 0$$

$$y^2(y + b)(y - b) = 0$$

$$y = 0, y = -b, y = b$$

So the intercepts are $(0,0)$, $(a-b,0)$, $(a+b,0)$, $(0,-b)$, $(0,b)$.

Test x -axis symmetry: replace y by $-y$

$$[x^2 + (-y)^2 - ax]^2 = b^2[x^2 + (-y)^2]$$

$$(x^2 + y^2 - ax)^2 = b^2(x^2 + y^2) \quad \text{Equivalent}$$

Section 1.2: Graphs of Equations in Two Variables; Intercepts; Symmetry

Test y-axis symmetry: replace x by $-x$

$$\left[(-x)^2 + y^2 - a(-x)\right]^2 = b^2 \left[(-x)^2 + y^2\right]$$

$$(x^2 + y^2 + ax)^2 = b^2(x^2 + y^2) \text{ Not equivalent}$$

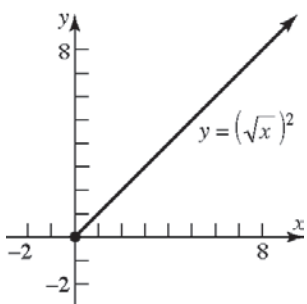
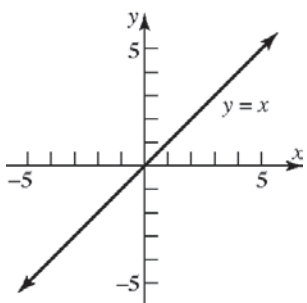
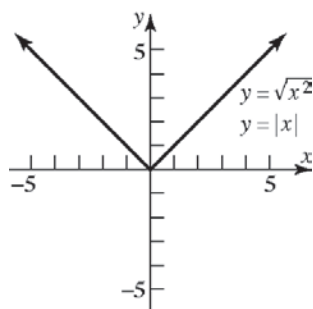
Test origin symmetry: replace x by $-x$ and y by $-y$

$$\left[(-x)^2 + (-y)^2 - a(-x)\right]^2 = b^2 \left[(-x)^2 + (-y)^2\right]$$

$$(x^2 + y^2 + ax)^2 = b^2(x^2 + y^2) \text{ No equivalent}$$

The graph is symmetric with respect to the x -axis only.

87. a.



b. Since $\sqrt{x^2} = |x|$ for all x , the graphs of $y = \sqrt{x^2}$ and $y = |x|$ are the same.

c. For $y = (\sqrt{x})^2$, the domain of the variable x is $x \geq 0$; for $y = x$, the domain of the

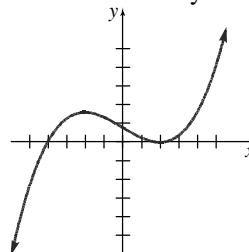
variable x is all real numbers. Thus,

$$(\sqrt{x})^2 = x \text{ only for } x \geq 0.$$

d. For $y = \sqrt{x^2}$, the range of the variable y is $y \geq 0$; for $y = x$, the range of the variable y is all real numbers. Also, $\sqrt{x^2} = x$ only if $x \geq 0$. Otherwise, $\sqrt{x^2} = -x$.

88. Answers will vary. A complete graph presents enough of the graph to the viewer so they can "see" the rest of the graph as an obvious continuation of what is shown.

89. Answers will vary. One example:



90. Answers will vary

91. Answers will vary

92. Answers will vary.

Case 1: Graph has x -axis and y -axis symmetry, show origin symmetry.

(x, y) on graph $\rightarrow (x, -y)$ on graph

(from x -axis symmetry)

$(x, -y)$ on graph $\rightarrow (-x, -y)$ on graph

(from y -axis symmetry)

Since the point $(-x, -y)$ is also on the graph, the graph has origin symmetry.

Case 2: Graph has x -axis and origin symmetry, show y -axis symmetry.

(x, y) on graph $\rightarrow (x, -y)$ on graph

(from x -axis symmetry)

$(x, -y)$ on graph $\rightarrow (-x, y)$ on graph

(from origin symmetry)

Since the point $(-x, y)$ is also on the graph, the graph has y -axis symmetry.

Case 3: Graph has y -axis and origin symmetry, show x -axis symmetry.

Chapter 1: Graphs

(x, y) on graph $\rightarrow (-x, y)$ on graph

(from y -axis symmetry)

$(-x, y)$ on graph $\rightarrow (x, -y)$ on graph

(from origin symmetry)

Since the point $(x, -y)$ is also on the graph, the graph has x -axis symmetry.

93. Answers may vary. The graph must contain the points $(-2, 5)$, $(-1, 3)$, and $(0, 2)$. For the graph to be symmetric about the y -axis, the graph must also contain the points $(2, 5)$ and $(1, 3)$ (note that $(0, 2)$ is on the y -axis).

For the graph to also be symmetric with respect to the x -axis, the graph must also contain the points $(-2, -5)$, $(-1, -3)$, $(0, -2)$, $(2, -5)$, and $(1, -3)$. Recall that a graph with two of the symmetries (x -axis, y -axis, origin) will necessarily have the third. Therefore, if the original graph with y -axis symmetry also has x -axis symmetry, then it will also have origin symmetry.

Section 1.3

1. undefined; 0

2. 3; 2

x -intercept: $2x + 3(0) = 6$

$$2x = 6$$

$$x = 3$$

y -intercept: $2(0) + 3y = 6$

$$3y = 6$$

$$y = 2$$

3. True

4. False; the slope is $\frac{3}{2}$.

$$2y = 3x + 5$$

$$y = \frac{3}{2}x + \frac{5}{2}$$

5. True; $2(1) + (2) = 4$

$$2 + 2 = 4$$

$$4 = 4 \quad \text{True}$$

6. $m_1 = m_2$; y -intercepts; $m_1 \cdot m_2 = -1$

7. 2

8. $-\frac{1}{2}$

9. c

10. d

11. b

12. d

13. a. Slope = $\frac{1-0}{2-0} = \frac{1}{2}$

- b. If x increases by 2 units, y will increase by 1 unit.

14. a. Slope = $\frac{1-0}{-2-0} = -\frac{1}{2}$

- b. If x increases by 2 units, y will decrease by 1 unit.

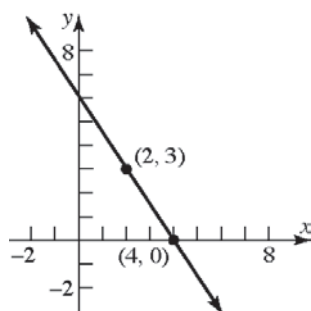
15. a. Slope = $\frac{1-2}{1-(-2)} = -\frac{1}{3}$

- b. If x increases by 3 units, y will decrease by 1 unit.

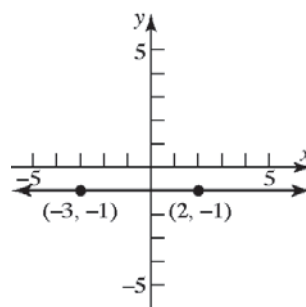
16. a. Slope = $\frac{2-1}{2-(-1)} = \frac{1}{3}$

- b. If x increases by 3 units, y will increase by 1 unit.

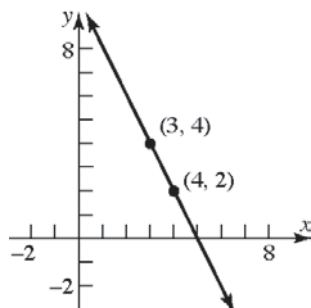
17. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{4 - 2} = -\frac{3}{2}$



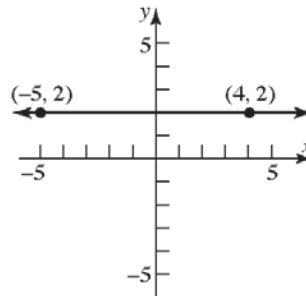
21. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-1)}{2 - (-3)} = \frac{0}{5} = 0$



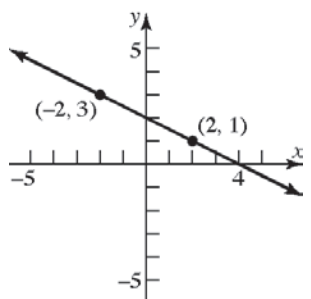
18. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{3 - 4} = \frac{2}{-1} = -2$



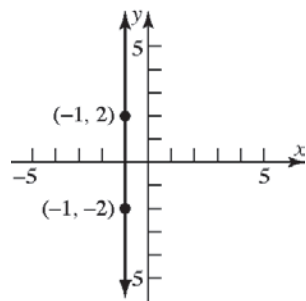
22. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{-5 - 4} = \frac{0}{-9} = 0$



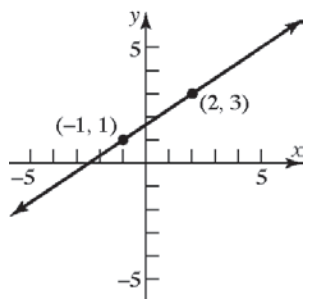
19. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{2 - (-2)} = \frac{-2}{4} = -\frac{1}{2}$



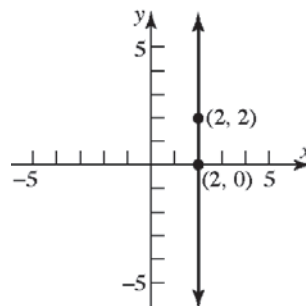
23. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 2}{-1 - (-1)} = \frac{-4}{0}$ undefined.



20. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{2 - (-1)} = \frac{2}{3}$

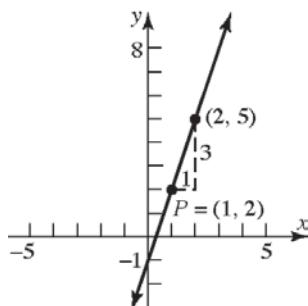


24. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{2 - 2} = \frac{2}{0}$ undefined.

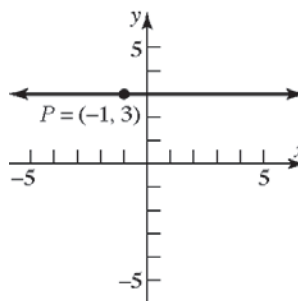


Chapter 1: Graphs

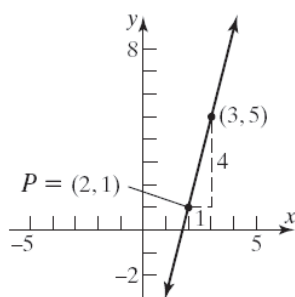
25. $P = (1, 2); m = 3$



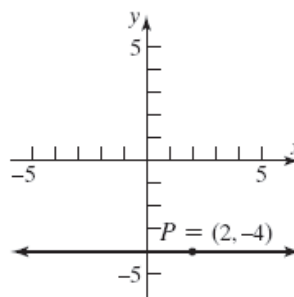
29. $P = (-1, 3); m = 0$



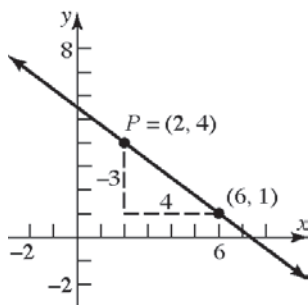
26. $P = (2, 1); m = 4$



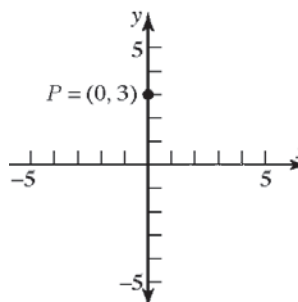
30. $P = (2, -4); m = 0$



27. $P = (2, 4); m = -\frac{3}{4}$

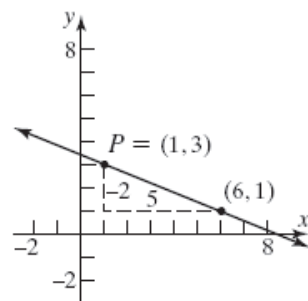


31. $P = (0, 3)$; slope undefined

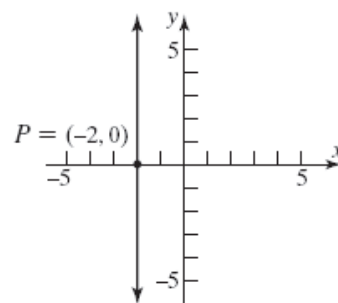


(note: the line is the y-axis)

28. $P = (1, 3); m = -\frac{2}{5}$



32. $P = (-2, 0)$; slope undefined



33. $P = (1, 2); m = 3; y - 2 = 3(x - 1)$

34. $P = (2, 1); m = 4; y - 1 = 4(x - 2)$

35. $P = (2, 4); m = -\frac{3}{4}; y - 4 = -\frac{3}{4}(x - 2)$

36. $P = (1, 3); m = -\frac{2}{5}; y - 3 = -\frac{2}{5}(x - 1)$

37. $P = (-1, 3); m = 0; y - 3 = 0$

38. $P = (2, -4); m = 0; y + 4 = 0$

39. Slope $= 4 = \frac{4}{1}$; point: $(1, 2)$

If x increases by 1 unit, then y increases by 4 units.

Answers will vary. Three possible points are:

$$x = 1 + 1 = 2 \text{ and } y = 2 + 4 = 6$$

$$(2, 6)$$

$$x = 2 + 1 = 3 \text{ and } y = 6 + 4 = 10$$

$$(3, 10)$$

$$x = 3 + 1 = 4 \text{ and } y = 10 + 4 = 14$$

$$(4, 14)$$

40. Slope $= 2 = \frac{2}{1}$; point: $(-2, 3)$

If x increases by 1 unit, then y increases by 2 units.

Answers will vary. Three possible points are:

$$x = -2 + 1 = -1 \text{ and } y = 3 + 2 = 5$$

$$(-1, 5)$$

$$x = -1 + 1 = 0 \text{ and } y = 5 + 2 = 7$$

$$(0, 7)$$

$$x = 0 + 1 = 1 \text{ and } y = 7 + 2 = 9$$

$$(1, 9)$$

41. Slope $= -\frac{3}{2} = \frac{-3}{2}$; point: $(2, -4)$

If x increases by 2 units, then y decreases by 3 units.

Answers will vary. Three possible points are:

$$x = 2 + 2 = 4 \text{ and } y = -4 - 3 = -7$$

$$(4, -7)$$

$$x = 4 + 2 = 6 \text{ and } y = -7 - 3 = -10$$

$$(6, -10)$$

$$x = 6 + 2 = 8 \text{ and } y = -10 - 3 = -13$$

$$(8, -13)$$

42. Slope $= \frac{4}{3}$; point: $(-3, 2)$

If x increases by 3 units, then y increases by 4 units.

Answers will vary. Three possible points are:

$$x = -3 + 3 = 0 \text{ and } y = 2 + 4 = 6$$

$$(0, 6)$$

$$x = 0 + 3 = 3 \text{ and } y = 6 + 4 = 10$$

$$(3, 10)$$

$$x = 3 + 3 = 6 \text{ and } y = 10 + 4 = 14$$

$$(6, 14)$$

43. Slope $= -2 = \frac{-2}{1}$; point: $(-2, -3)$

If x increases by 1 unit, then y decreases by 2 units.

Answers will vary. Three possible points are:

$$x = -2 + 1 = -1 \text{ and } y = -3 - 2 = -5$$

$$(-1, -5)$$

$$x = -1 + 1 = 0 \text{ and } y = -5 - 2 = -7$$

$$(0, -7)$$

$$x = 0 + 1 = 1 \text{ and } y = -7 - 2 = -9$$

$$(1, -9)$$

44. Slope $= -1 = \frac{-1}{1}$; point: $(4, 1)$

If x increases by 1 unit, then y decreases by 1 unit.

Answers will vary. Three possible points are:

$$x = 4 + 1 = 5 \text{ and } y = 1 - 1 = 0$$

$$(5, 0)$$

$$x = 5 + 1 = 6 \text{ and } y = 0 - 1 = -1$$

$$(6, -1)$$

$$x = 6 + 1 = 7 \text{ and } y = -1 - 1 = -2$$

$$(7, -2)$$

Chapter 1: Graphs

45. (0, 0) and (2, 1) are points on the line.

$$\text{Slope} = \frac{1-0}{2-0} = \frac{1}{2}$$

y-intercept is 0; using $y = mx + b$:

$$y = \frac{1}{2}x + 0$$

$$2y = x$$

$$0 = x - 2y$$

$$x - 2y = 0 \text{ or } y = \frac{1}{2}x$$

46. (0, 0) and (-2, 1) are points on the line.

$$\text{Slope} = \frac{1-0}{-2-0} = -\frac{1}{2}$$

y-intercept is 0; using $y = mx + b$:

$$y = -\frac{1}{2}x + 0$$

$$2y = -x$$

$$x + 2y = 0$$

$$x + 2y = 0 \text{ or } y = -\frac{1}{2}x$$

47. (-1, 3) and (1, 1) are points on the line.

$$\text{Slope} = \frac{1-3}{1-(-1)} = -\frac{2}{2} = -1$$

Using $y - y_1 = m(x - x_1)$

$$y - 1 = -1(x - 1)$$

$$y - 1 = -x + 1$$

$$y = -x + 2$$

$$x + y = 2 \text{ or } y = -x + 2$$

48. (-1, 1) and (2, 2) are points on the line.

$$\text{Slope} = \frac{2-1}{2-(-1)} = \frac{1}{3}$$

Using $y - y_1 = m(x - x_1)$

$$y - 1 = \frac{1}{3}(x - (-1))$$

$$y - 1 = \frac{1}{3}(x + 1)$$

$$y - 1 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{4}{3}$$

$$x - 3y = -4 \text{ or } y = \frac{1}{3}x + \frac{4}{3}$$

49. $y - y_1 = m(x - x_1)$, $m = 2$

$$y - 3 = 2(x - 3)$$

$$y - 3 = 2x - 6$$

$$y = 2x - 3$$

$$2x - y = 3 \text{ or } y = 2x - 3$$

50. $y - y_1 = m(x - x_1)$, $m = -1$

$$y - 2 = -1(x - 1)$$

$$y - 2 = -x + 1$$

$$y = -x + 3$$

$$x + y = 3 \text{ or } y = -x + 3$$

51. $y - y_1 = m(x - x_1)$, $m = -\frac{1}{2}$

$$y - 2 = -\frac{1}{2}(x - 1)$$

$$y - 2 = -\frac{1}{2}x + \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

$$x + 2y = 5 \text{ or } y = -\frac{1}{2}x + \frac{5}{2}$$

52. $y - y_1 = m(x - x_1)$, $m = 1$

$$y - 1 = 1(x - (-1))$$

$$y - 1 = x + 1$$

$$y = x + 2$$

$$x - y = -2 \text{ or } y = x + 2$$

53. Slope = 3; containing (-2, 3)

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 3(x - (-2))$$

$$y - 3 = 3x + 6$$

$$y = 3x + 9$$

$$3x - y = -9 \text{ or } y = 3x + 9$$

54. Slope = 2; containing the point (4, -3)

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = 2(x - 4)$$

$$y + 3 = 2x - 8$$

$$y = 2x - 11$$

$$2x - y = 11 \text{ or } y = 2x - 11$$

55. Slope = $\frac{1}{2}$; containing the point (3, 1)

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{2}(x - 3)$$

$$y - 1 = \frac{1}{2}x - \frac{3}{2}$$

$$y = \frac{1}{2}x - \frac{1}{2}$$

$$x - 2y = 1 \text{ or } y = \frac{1}{2}x - \frac{1}{2}$$

56. Slope = $-\frac{2}{3}$; containing (1, -1)

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -\frac{2}{3}(x - 1)$$

$$y + 1 = -\frac{2}{3}x + \frac{2}{3}$$

$$y = -\frac{2}{3}x - \frac{1}{3}$$

$$2x + 3y = -1 \text{ or } y = -\frac{2}{3}x - \frac{1}{3}$$

57. Containing (1, 3) and (-1, 2)

$$m = \frac{2 - 3}{-1 - 1} = \frac{-1}{-2} = \frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{1}{2}(x - 1)$$

$$y - 3 = \frac{1}{2}x - \frac{1}{2}$$

$$y = \frac{1}{2}x + \frac{5}{2}$$

$$x - 2y = -5 \text{ or } y = \frac{1}{2}x + \frac{5}{2}$$

58. Containing the points (-3, 4) and (2, 5)

$$m = \frac{5 - 4}{2 - (-3)} = \frac{1}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{1}{5}(x - 2)$$

$$y - 5 = \frac{1}{5}x - \frac{2}{5}$$

$$y = \frac{1}{5}x + \frac{23}{5}$$

$$x - 5y = -23 \text{ or } y = \frac{1}{5}x + \frac{23}{5}$$

59. Slope = -3; y-intercept = 3

$$y = mx + b$$

$$y = -3x + 3$$

$$3x + y = 3 \text{ or } y = -3x + 3$$

60. Slope = -2; y-intercept = -2

$$y = mx + b$$

$$y = -2x + (-2)$$

$$2x + y = -2 \text{ or } y = -2x - 2$$

61. x-intercept = -4; y-intercept = 4

Points are (-4, 0) and (0, 4)

$$m = \frac{4 - 0}{0 - (-4)} = \frac{4}{4} = 1$$

$$y = mx + b$$

$$y = 1x + 4$$

$$y = x + 4$$

$$x - y = -4 \text{ or } y = x + 4$$

62. x-intercept = 2; y-intercept = -1

Points are (2, 0) and (0, -1)

$$m = \frac{-1 - 0}{0 - 2} = \frac{-1}{-2} = \frac{1}{2}$$

$$y = mx + b$$

$$y = \frac{1}{2}x - 1$$

$$x - 2y = 2 \text{ or } y = \frac{1}{2}x - 1$$

63. Slope undefined; containing the point (2, 4)

This is a vertical line.

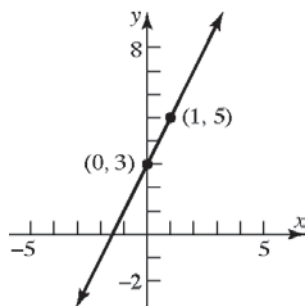
$x = 2$ No slope-intercept form.

Chapter 1: Graphs

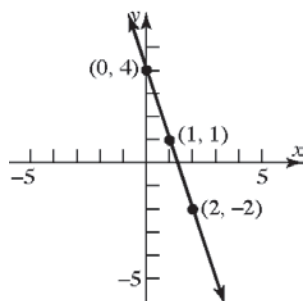
- 64.** Slope undefined; containing the point (3, 8)
This is a vertical line.
 $x = 3$ No slope-intercept form.
- 65.** Horizontal lines have slope $m = 0$ and take the form $y = b$. Therefore, the horizontal line passing through the point $(-3, 2)$ is $y = 2$.
- 66.** Vertical lines have an undefined slope and take the form $x = a$. Therefore, the vertical line passing through the point $(4, -5)$ is $x = 4$.
- 67.** Parallel to $y = 2x$; Slope = 2
Containing $(-1, 2)$
 $y - y_1 = m(x - x_1)$
 $y - 2 = 2(x - (-1))$
 $y - 2 = 2x + 2 \rightarrow y = 2x + 4$
 $2x - y = -4$ or $y = 2x + 4$
- 68.** Parallel to $y = -3x$; Slope = -3; Containing the point $(-1, 2)$
 $y - y_1 = m(x - x_1)$
 $y - 2 = -3(x - (-1))$
 $y - 2 = -3x - 3 \rightarrow y = -3x - 1$
 $3x + y = -1$ or $y = -3x - 1$
- 69.** Parallel to $x - 2y = -5$;
Slope = $\frac{1}{2}$; Containing the point $(0, 0)$
 $y - y_1 = m(x - x_1)$
 $y - 0 = \frac{1}{2}(x - 0) \rightarrow y = \frac{1}{2}x$
 $x - 2y = 0$ or $y = \frac{1}{2}x$
- 70.** Parallel to $2x - y = -2$; Slope = 2
Containing the point $(0, 0)$
 $y - y_1 = m(x - x_1)$
 $y - 0 = 2(x - 0)$
 $y = 2x$
 $2x - y = 0$ or $y = 2x$
- 71.** Parallel to $x = 5$; Containing $(4, 2)$
This is a vertical line.
 $x = 4$ No slope-intercept form.
- 72.** Parallel to $y = 5$; Containing the point $(4, 2)$
This is a horizontal line. Slope = 0
 $y = 2$
- 73.** Perpendicular to $y = \frac{1}{2}x + 4$; Containing $(1, -2)$
Slope of perpendicular = -2
 $y - y_1 = m(x - x_1)$
 $y - (-2) = -2(x - 1)$
 $y + 2 = -2x + 2 \rightarrow y = -2x$
 $2x + y = 0$ or $y = -2x$
- 74.** Perpendicular to $y = 2x - 3$; Containing the point $(1, -2)$
Slope of perpendicular = $-\frac{1}{2}$
 $y - y_1 = m(x - x_1)$
 $y - (-2) = -\frac{1}{2}(x - 1)$
 $y + 2 = -\frac{1}{2}x + \frac{1}{2} \rightarrow y = -\frac{1}{2}x - \frac{3}{2}$
 $x + 2y = -3$ or $y = -\frac{1}{2}x - \frac{3}{2}$
- 75.** Perpendicular to $x - 2y = -5$; Containing the point $(0, 4)$
Slope of perpendicular = -2
 $y = mx + b$
 $y = -2x + 4$
 $2x + y = 4$ or $y = -2x + 4$
- 76.** Perpendicular to $2x + y = 2$; Containing the point $(-3, 0)$
Slope of perpendicular = $\frac{1}{2}$
 $y - y_1 = m(x - x_1)$
 $y - 0 = \frac{1}{2}(x - (-3)) \rightarrow y = \frac{1}{2}x + \frac{3}{2}$
 $x - 2y = -3$ or $y = \frac{1}{2}x + \frac{3}{2}$
- 77.** Perpendicular to $x = 8$; Containing $(3, 4)$
Slope of perpendicular = 0 (horizontal line)
 $y = 4$

78. Perpendicular to $y = 8$;
Containing the point $(3, 4)$
Slope of perpendicular is undefined (vertical line). $x = 3$ No slope-intercept form.

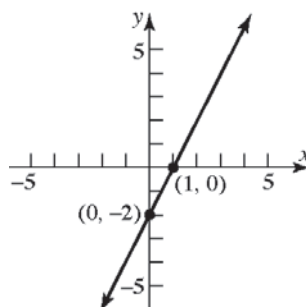
79. $y = 2x + 3$; Slope = 2; y-intercept = 3



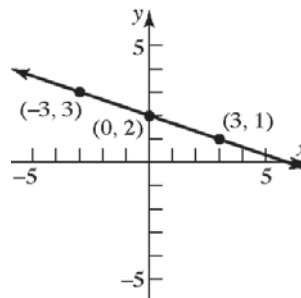
80. $y = -3x + 4$; Slope = -3; y-intercept = 4



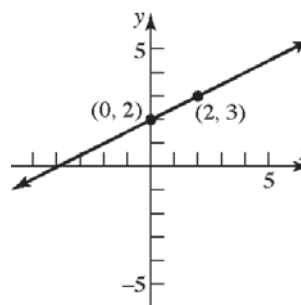
81. $\frac{1}{2}y = x - 1$; $y = 2x - 2$
Slope = 2; y-intercept = -2



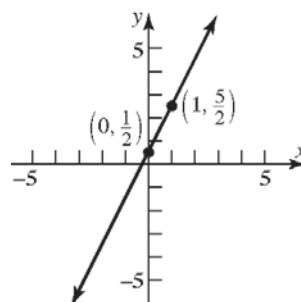
82. $\frac{1}{3}x + y = 2$; $y = -\frac{1}{3}x + 2$
Slope = $-\frac{1}{3}$; y-intercept = 2



83. $y = \frac{1}{2}x + 2$; Slope = $\frac{1}{2}$; y-intercept = 2



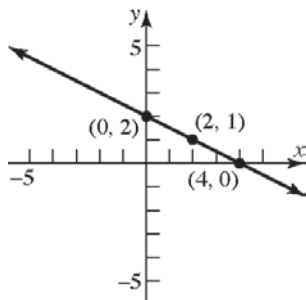
84. $y = 2x + \frac{1}{2}$; Slope = 2; y-intercept = $\frac{1}{2}$



Chapter 1: Graphs

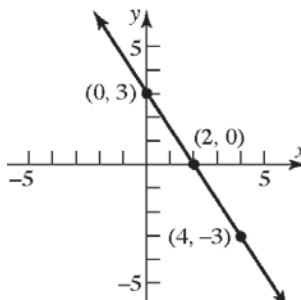
85. $x + 2y = 4$; $2y = -x + 4 \rightarrow y = -\frac{1}{2}x + 2$

Slope = $-\frac{1}{2}$; y-intercept = 2



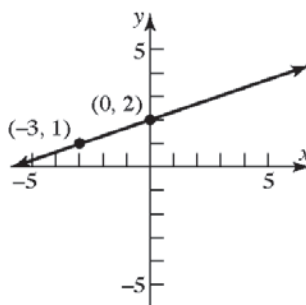
88. $3x + 2y = 6$; $2y = -3x + 6 \rightarrow y = -\frac{3}{2}x + 3$

Slope = $-\frac{3}{2}$; y-intercept = 3



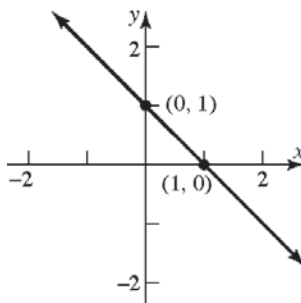
86. $-x + 3y = 6$; $3y = x + 6 \rightarrow y = \frac{1}{3}x + 2$

Slope = $\frac{1}{3}$; y-intercept = 2



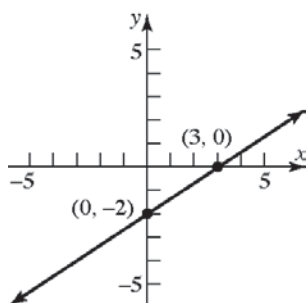
89. $x + y = 1$; $y = -x + 1$

Slope = -1; y-intercept = 1



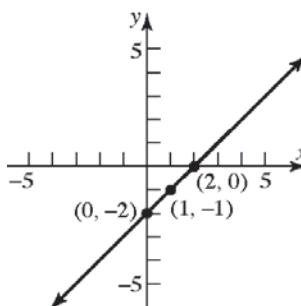
87. $2x - 3y = 6$; $-3y = -2x + 6 \rightarrow y = \frac{2}{3}x - 2$

Slope = $\frac{2}{3}$; y-intercept = -2

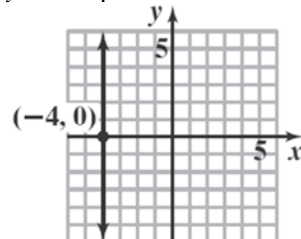


90. $x - y = 2$; $y = x - 2$

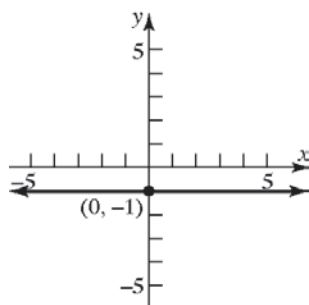
Slope = 1; y-intercept = -2



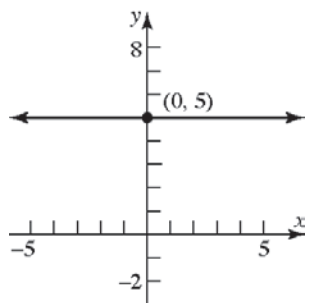
91. $x = -4$; Slope is undefined
y-intercept - none



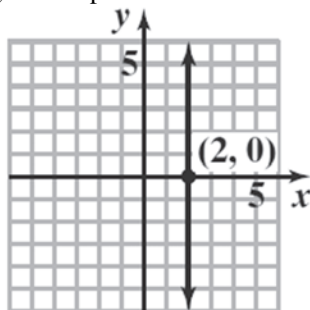
92. $y = -1$; Slope = 0; y-intercept = -1



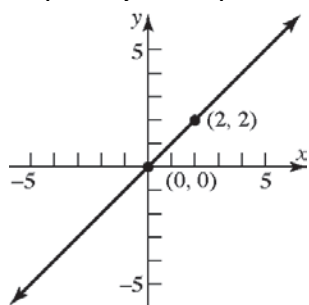
93. $y = 5$; Slope = 0; y-intercept = 5



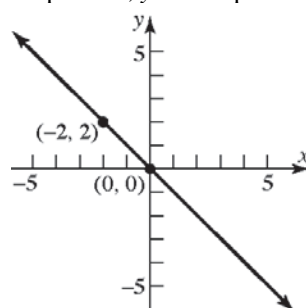
94. $x = 2$; Slope is undefined
y-intercept - none



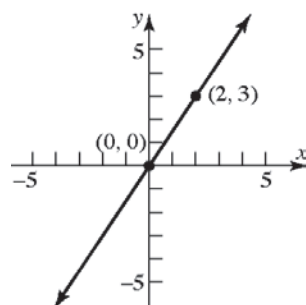
95. $y - x = 0$; $y = x$
Slope = 1; y-intercept = 0



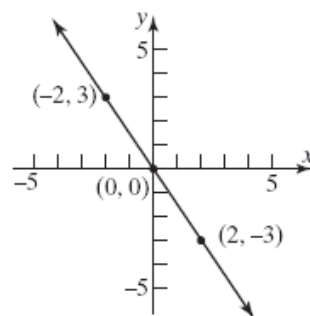
96. $x + y = 0$; $y = -x$
Slope = -1; y-intercept = 0



97. $2y - 3x = 0$; $2y = 3x \rightarrow y = \frac{3}{2}x$
Slope = $\frac{3}{2}$; y-intercept = 0



98. $3x + 2y = 0$; $2y = -3x \rightarrow y = -\frac{3}{2}x$
Slope = $-\frac{3}{2}$; y-intercept = 0



99. a. x-intercept: $2x + 3(0) = 6$
 $2x = 6$
 $x = 3$
The point (3, 0) is on the graph.

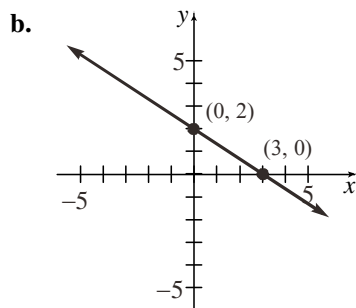
Chapter 1: Graphs

y-intercept: $2(0) + 3y = 6$

$$3y = 6$$

$$y = 2$$

The point $(0, 2)$ is on the graph.



100. a. x-intercept: $3x - 2(0) = 6$

$$3x = 6$$

$$x = 2$$

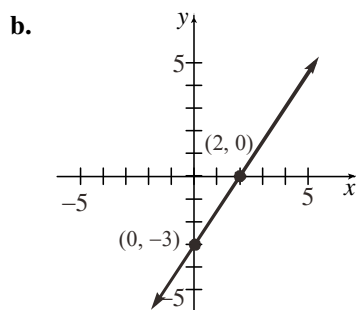
The point $(2, 0)$ is on the graph.

y-intercept: $3(0) - 2y = 6$

$$-2y = 6$$

$$y = -3$$

The point $(0, -3)$ is on the graph.



101. a. x-intercept: $-4x + 5(0) = 40$

$$-4x = 40$$

$$x = -10$$

The point $(-10, 0)$ is on the graph.

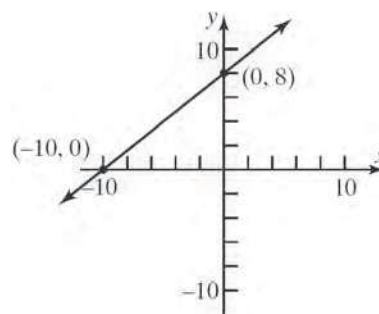
y-intercept: $-4(0) + 5y = 40$

$$5y = 40$$

$$y = 8$$

The point $(0, 8)$ is on the graph.

b.



102. a. x-intercept: $6x - 4(0) = 24$

$$6x = 24$$

$$x = 4$$

The point $(4, 0)$ is on the graph.

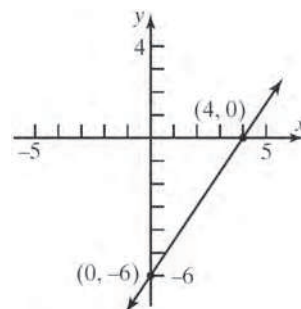
y-intercept: $6(0) - 4y = 24$

$$-4y = 24$$

$$y = -6$$

The point $(0, -6)$ is on the graph.

b.



103. a. x-intercept: $7x + 2(0) = 21$

$$7x = 21$$

$$x = 3$$

The point $(3, 0)$ is on the graph.

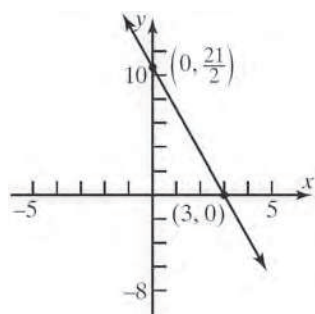
y-intercept: $7(0) + 2y = 21$

$$2y = 21$$

$$y = \frac{21}{2}$$

The point $\left(0, \frac{21}{2}\right)$ is on the graph.

b.



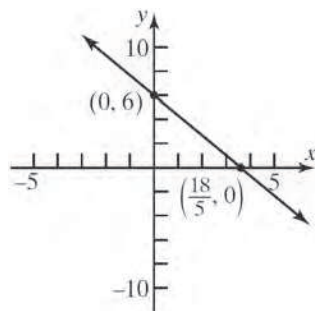
104. a. x-intercept: $5x + 3(0) = 18$
 $5x = 18$
 $x = \frac{18}{5}$

The point $(\frac{18}{5}, 0)$ is on the graph.

y-intercept: $5(0) + 3y = 18$
 $3y = 18$
 $y = 6$

The point $(0, 6)$ is on the graph.

b.



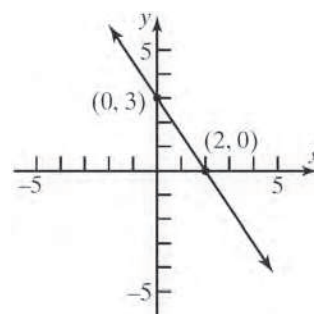
105. a. x-intercept: $\frac{1}{2}x + \frac{1}{3}(0) = 1$
 $\frac{1}{2}x = 1$
 $x = 2$

The point $(2, 0)$ is on the graph.

y-intercept: $\frac{1}{2}(0) + \frac{1}{3}y = 1$
 $\frac{1}{3}y = 1$
 $y = 3$

The point $(0, 3)$ is on the graph.

b.



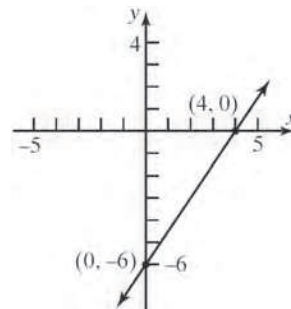
106. a. x-intercept: $x - \frac{2}{3}(0) = 4$
 $x = 4$

The point $(4, 0)$ is on the graph.

y-intercept: $(0) - \frac{2}{3}y = 4$
 $-\frac{2}{3}y = 4$
 $y = -6$

The point $(0, -6)$ is on the graph.

b.



107. a. x-intercept: $0.2x - 0.5(0) = 1$
 $0.2x = 1$
 $x = 5$

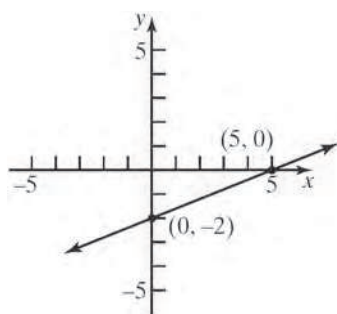
The point $(5, 0)$ is on the graph.

y-intercept: $0.2(0) - 0.5y = 1$
 $-0.5y = 1$
 $y = -2$

The point $(0, -2)$ is on the graph.

Chapter 1: Graphs

b.



108. a. x -intercept: $-0.3x + 0.4(0) = 1.2$

$$-0.3x = 1.2$$

$$x = -4$$

The point $(-4, 0)$ is on the graph.

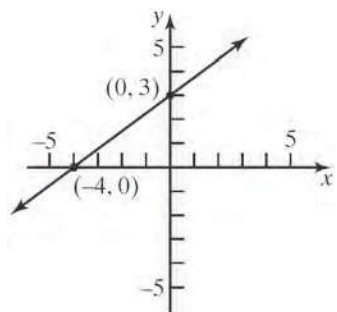
y -intercept: $-0.3(0) + 0.4y = 1.2$

$$0.4y = 1.2$$

$$y = 3$$

The point $(0, 3)$ is on the graph.

b.



109. The equation of the x -axis is $y = 0$. (The slope is 0 and the y -intercept is 0.)

110. The equation of the y -axis is $x = 0$. (The slope is undefined.)

111. The slopes are the same but the y -intercepts are different. Therefore, the two lines are parallel.

112. The slopes are opposite-reciprocals. That is, their product is -1 . Therefore, the lines are perpendicular.

113. The slopes are different and their product does not equal -1 . Therefore, the lines are neither parallel nor perpendicular.

114. The slopes are different and their product does not equal -1 (in fact, the signs are the same so the product is positive). Therefore, the lines are neither parallel nor perpendicular.

115. Intercepts: $(0, 2)$ and $(-2, 0)$. Thus, slope = 1.
 $y = x + 2$ or $x - y = -2$

116. Intercepts: $(0, 1)$ and $(1, 0)$. Thus, slope = -1 .
 $y = -x + 1$ or $x + y = 1$

117. Intercepts: $(3, 0)$ and $(0, 1)$. Thus, slope = $-\frac{1}{3}$.
 $y = -\frac{1}{3}x + 1$ or $x + 3y = 3$

118. Intercepts: $(0, -1)$ and $(-2, 0)$. Thus,
slope = $-\frac{1}{2}$.

$$y = -\frac{1}{2}x - 1 \text{ or } x + 2y = -2$$

119. $P_1 = (-2, 5)$, $P_2 = (1, 3)$: $m_1 = \frac{5-3}{-2-1} = \frac{2}{-3} = -\frac{2}{3}$

$$P_2 = (1, 3), P_3 = (-1, 0): m_2 = \frac{3-0}{1-(-1)} = \frac{3}{2}$$

Since $m_1 \cdot m_2 = -1$, the line segments $\overline{P_1P_2}$ and $\overline{P_2P_3}$ are perpendicular. Thus, the points P_1 , P_2 , and P_3 are vertices of a right triangle.

120. $P_1 = (1, -1)$, $P_2 = (4, 1)$, $P_3 = (2, 2)$, $P_4 = (5, 4)$

$$m_{12} = \frac{1-(-1)}{4-1} = \frac{2}{3}; m_{24} = \frac{4-1}{5-4} = 3;$$

$$m_{34} = \frac{4-2}{5-2} = \frac{2}{3}; m_{13} = \frac{2-(-1)}{2-1} = 3$$

Each pair of opposite sides are parallel (same slope) and adjacent sides are not perpendicular. Therefore, the vertices are for a parallelogram.

121. $P_1 = (-1, 0)$, $P_2 = (2, 3)$, $P_3 = (1, -2)$, $P_4 = (4, 1)$

$$m_{12} = \frac{3-0}{2-(-1)} = \frac{3}{3} = 1; m_{24} = \frac{1-3}{4-2} = -1;$$

$$m_{34} = \frac{1-(-2)}{4-1} = \frac{3}{3} = 1; m_{13} = \frac{-2-0}{1-(-1)} = -1$$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). Therefore, the vertices are for a rectangle.

122. $P_1 = (0, 0)$, $P_2 = (1, 3)$, $P_3 = (4, 2)$, $P_4 = (3, -1)$

$$m_{12} = \frac{3-0}{1-0} = 3; \quad m_{23} = \frac{2-3}{4-1} = -\frac{1}{3};$$

$$m_{34} = \frac{-1-2}{3-4} = 3; \quad m_{14} = \frac{-1-0}{3-0} = -\frac{1}{3}$$

$$d_{12} = \sqrt{(1-0)^2 + (3-0)^2} = \sqrt{1+9} = \sqrt{10}$$

$$d_{23} = \sqrt{(4-1)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$$

$$d_{34} = \sqrt{(3-4)^2 + (-1-2)^2} = \sqrt{1+9} = \sqrt{10}$$

$$d_{14} = \sqrt{(3-0)^2 + (-1-0)^2} = \sqrt{9+1} = \sqrt{10}$$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). In addition, the length of all four sides is the same. Therefore, the vertices are for a square.

123. Let x = number of miles driven, and let C = cost in dollars.

Total cost = (cost per mile)(number of miles) + fixed cost

$$C = 0.60x + 39$$

When $x = 110$, $C = (0.60)(110) + 39 = \$105.00$.

When $x = 230$, $C = (0.60)(230) + 39 = \$177.00$.

124. Let x = number of pairs of jeans manufactured, and let C = cost in dollars.

Total cost = (cost per pair)(number of pairs) + fixed cost

$$C = 20x + 1200$$

When $x = 400$, $C = (20)(400) + 1200 = \$9200$.

When $x = 740$, $C = (20)(740) + 1200 = \$16,000$.

125. Let x = number of miles driven annually, and let C = cost in dollars.

Total cost = (approx cost per mile)(number of miles) + fixed cost

$$C = 0.28x + 6578$$

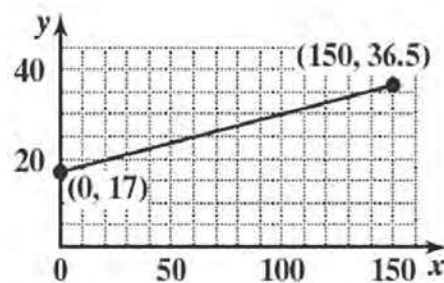
126. Let x = profit in dollars, and let S = salary in dollars.

Weekly salary = (% share of profit)(profit) + weekly pay

$$S = 0.05x + 525$$

127. a. $C = 0.13x + 17$; $0 \leq x \leq 1000$

b.



c. For 50 miles,

$$C = 0.13(50) + 17 = \$23.50$$

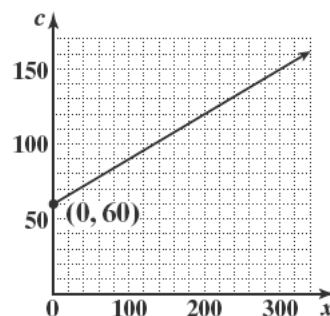
d. For 120 miles,

$$C = 0.13(120) + 17 = \$32.60$$

e. For every 1-mile in distance traveled, the cost will increase 13 cents.

128. a. $C = 60 + 0.25x$

b.



c. For 20 minutes,

$$C = 60 + 0.25(20) = \$65.00$$

d. For 60 minutes,

$$C = 60 + 0.25(60) = \$75.00$$

e. For every 1-minute increase in international calls the cost will increase by \$0.25 (that is, 25 cents).

Chapter 1: Graphs

129. $(^{\circ}C, ^{\circ}F) = (0, 32); (^{\circ}C, ^{\circ}F) = (100, 212)$

$$\text{slope} = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$$

$$^{\circ}F - 32 = \frac{9}{5}(^{\circ}C - 0)$$

$$^{\circ}F - 32 = \frac{9}{5}(^{\circ}C)$$

$$^{\circ}C = \frac{5}{9}(^{\circ}F - 32)$$

If $^{\circ}F = 70$, then

$$^{\circ}C = \frac{5}{9}(70 - 32) = \frac{5}{9}(38)$$

$$^{\circ}C \approx 21.1^{\circ}$$

130. a. $K = ^{\circ}C + 273$

b. $^{\circ}C = \frac{5}{9}(^{\circ}F - 32)$

$$K = \frac{5}{9}(^{\circ}F - 32) + 273$$

$$K = \frac{5}{9}^{\circ}F - \frac{160}{9} + 273$$

$$K = \frac{5}{9}^{\circ}F + \frac{2297}{9}$$

131. a. The y-intercept is $(0, 30)$, so $b = 30$. Since the ramp drops 2 inches for every 25 inches of run, the slope is $m = \frac{-2}{25} = -\frac{2}{25}$. Thus,

$$\text{the equation is } y = -\frac{2}{25}x + 30.$$

- b. Let $y = 0$.

$$0 = -\frac{2}{25}x + 30$$

$$\frac{2}{25}x = 30$$

$$\frac{25}{2}\left(\frac{2}{25}x\right) = \frac{25}{2}(30)$$

$$x = 375$$

The x-intercept is $(375, 0)$. This means that the ramp meets the floor 375 inches (or 31.25 feet) from the base of the platform.

- c. No. From part (b), the run is 31.25 feet which exceeds the required maximum of 30 feet.
- d. First, design requirements state that the maximum slope is a drop of 1 inch for each 12 inches of run. This means $|m| \leq \frac{1}{12}$.

Second, the run is restricted to be no more than 30 feet = 360 inches. For a rise of 30 inches, this means the minimum slope is

$$\frac{30}{360} = \frac{1}{12}. \text{ That is, } |m| \geq \frac{1}{12}. \text{ Thus, the}$$

only possible slope is $|m| = \frac{1}{12}$. The

diagram indicates that the slope is negative. Therefore, the only slope that can be used to obtain the 30-inch rise and still meet design

requirements is $m = -\frac{1}{12}$. In words, for every 12 inches of run, the ramp must drop *exactly* 1 inch.

132. a. Let x represent the percent of internet ad spending. Let y represent the percent of print ad spending. Then the points $(0.19, 0.26)$ and $(0.35, 0.16)$ are on the line.

$$\text{Thus, } m = \frac{16 - 26}{35 - 19} = -\frac{10}{16}$$

the point-slope formula we have

$$y - 26 = -0.625(x - 19)$$

$$y - 26 = -0.625x + 11.875$$

$$y = -0.625x + 37.875$$

- b. x-intercept: $0 = -0.625x + 37.875$
 $-37.875 = -0.625x$

$$60.6 = x$$

$$\text{y-intercept: } y = -0.625(0) + 37.875$$

$$= 37.875$$

The intercepts are $(60.6, 0)$ and $(0, 37.875)$.

- c. y-intercept: When Internet ads account for 0% of U.S. advertisement spending, print ads account for 37.875% of the spending.
x-intercept: When Internet ads account for 60.6% of U.S. advertisement spending, print ads account for 0% of the spending.

- d. Let $x = 39.2$.
 $y = -0.625(39.2) + 37.875 = 13.4\%$

133. a. Let x = number of boxes to be sold, and A = money, in dollars, spent on advertising. We have the points $(x_1, A_1) = (100,000, 40,000)$;

$$(x_2, A_2) = (200,000, 60,000)$$

$$\text{slope} = \frac{60,000 - 40,000}{200,000 - 100,000}$$

$$= \frac{20,000}{100,000} = \frac{1}{5}$$

$$A - 40,000 = \frac{1}{5}(x - 100,000)$$

$$A - 40,000 = \frac{1}{5}x - 20,000$$

$$A = \frac{1}{5}x + 20,000$$

- b. If $x = 300,000$, then

$$A = \frac{1}{5}(300,000) + 20,000 = \$80,000$$

- c. Each additional box sold requires an additional \$0.20 in advertising.

134. $2x - y = C$

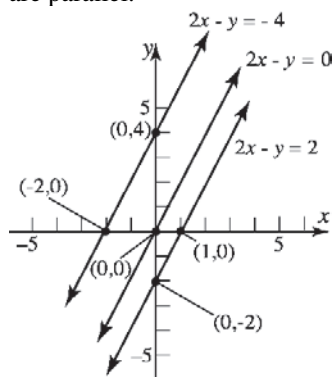
Graph the lines:

$$2x - y = -4$$

$$2x - y = 0$$

$$2x - y = 2$$

All the lines have the same slope, 2. The lines are parallel.



135. Put each linear equation in slope/intercept form.

$$x + 2y = 5 \quad 2x - 3y + 4 = 0 \quad ax + y = 0$$

$$2y = -x + 5 \quad -3y = -2x - 4 \quad y = -ax$$

$$y = -\frac{1}{2}x + \frac{5}{2} \quad y = \frac{2}{3}x + \frac{4}{3}$$

If the slope of $y = -ax$ equals the slope of either of the other two lines, then no triangle is formed.

$$\text{So, } -a = -\frac{1}{2} \Rightarrow a = \frac{1}{2} \text{ and } -a = \frac{2}{3} \Rightarrow a = -\frac{2}{3}.$$

Also if all three lines intersect at a single point,

then no triangle is formed. So, we find where

$$y = -\frac{1}{2}x + \frac{5}{2} \text{ and } y = \frac{2}{3}x + \frac{4}{3} \text{ intersect.}$$

$$-\frac{1}{2}x + \frac{5}{2} = \frac{2}{3}x + \frac{4}{3}$$

$$-\frac{7}{6}x = -\frac{7}{6}$$

$$x = 1$$

$$-\frac{1}{2}(1) + \frac{5}{2} = 2$$

The two lines intersect at $(1, 2)$. If $y = -ax$ also contains the point $(1, 2)$, then

$$2 = -a \cdot 1 \Rightarrow a = -2.$$

The three numbers are $\frac{1}{2}$, $-\frac{2}{3}$, and -2 .

136. The slope of the line containing (a, b) and

(b, a) is

$$\frac{a-b}{b-a} = -1$$

The slope of the line $y = x$ is 1.

The two lines are perpendicular.

The midpoint of (a, b) and (b, a) is

$$M = \left(\frac{a+b}{2}, \frac{b+a}{2} \right).$$

Since the x and y coordinates of M are equal, M lies on the line $y = x$.

$$\text{Note: } \frac{a+b}{2} = \frac{b+a}{2}$$

137. The three midpoints are

$$\left(\frac{0+a}{2}, \frac{0+0}{2} \right) = \left(\frac{a}{2}, 0 \right), \left(\frac{a+b}{2}, \frac{0+c}{2} \right) = \left(\frac{a+b}{2}, \frac{c}{2} \right)$$

$$\text{and } \left(\frac{0+b}{2}, \frac{0+c}{2} \right) = \left(\frac{b}{2}, \frac{c}{2} \right).$$

Chapter 1: Graphs

Line 1 from $(0,0)$ to $\left(\frac{a+b}{2}, \frac{c}{2}\right)$

$$m = \frac{\frac{c}{2} - 0}{\frac{a+b}{2} - 0} = \frac{c}{a+b};$$

$$y - 0 = \frac{c}{a+b}(x - 0)$$

$$y = \frac{c}{a+b}x_1$$

Line 2 from $(a, 0)$ to $\left(\frac{b}{2}, \frac{c}{2}\right)$

$$m = \frac{\frac{c}{2} - 0}{\frac{b}{2} - a} = \frac{\frac{c}{2}}{\frac{b-2a}{2}} = \frac{c}{b-2a}$$

$$y - 0 = \frac{c}{b-2a}(x - a)$$

$$y = \frac{c}{b-2a}(x - a)$$

Line 3 from $\left(\frac{a}{2}, 0\right)$ to (b, c)

$$m = \frac{c - 0}{b - \frac{a}{2}} = \frac{2c}{2b - a}$$

$$y - 0 = \frac{2c}{2b - a}\left(x - \frac{a}{2}\right)$$

$$y = \frac{2c}{2b - a}\left(x - \frac{a}{2}\right)$$

Find where line 1 and line 2 intersect:

$$\frac{c}{a+b}x = \frac{c}{b-2a}(x - a)$$

$$\frac{b-2a}{a+b}x = x - a$$

$$\frac{b-2a-a-b}{a+b}x = -a$$

$$\frac{-3a}{a+b}x = -a$$

$$x = \frac{a+b}{3};$$

Substitute into line 1:

$$y = \frac{c}{a+b} \cdot \frac{a+b}{3} = \frac{c}{3}.$$

So, line 1 and line 2 intersect at $\left(\frac{a+b}{3}, \frac{c}{3}\right)$.

Show that line 3 contains the point $\left(\frac{a+b}{3}, \frac{c}{3}\right)$:

$$y = \frac{2c}{2b-a}\left(\frac{a+b}{3} - \frac{a}{2}\right) = \frac{2c}{2b-a} \cdot \frac{2b-a}{6} = \frac{c}{3} \quad \text{So}$$

the three lines intersect at $\left(\frac{a+b}{3}, \frac{c}{3}\right)$.

138. Refer to Figure 47. Assume $m_1 m_2 = -1$. Then

$$\begin{aligned} [d(A, B)]^2 &= (1-1)^2 + (m_1 - m_2)^2 \\ &= (m_1 - m_2)^2 \\ &= m_1^2 - 2m_1 m_2 + m_2^2 \\ &= m_1^2 - 2(-1) + m_2^2 \\ &= m_1^2 + m_2^2 + 2 \end{aligned}$$

Now,

$$\begin{aligned} [d(O, B)]^2 &= (1-0)^2 + (m_1 - 0)^2 = 1 + m_1^2 \\ [d(O, A)]^2 &= (1-0)^2 + (m_2 - 0)^2 = 1 + m_2^2 \end{aligned}$$

So

$$\begin{aligned} [d(O, B)]^2 + [d(O, A)]^2 &= 1 + m_1^2 + 1 + m_2^2 \\ &= m_1^2 + m_2^2 + 2 = [d(A, B)]^2 \end{aligned}$$

By the converse of the Pythagorean Theorem, $\triangle AOB$ is a right triangle with right angle at vertex O . Thus lines OA and OB are perpendicular.

139. (b), (c), (e) and (g)

The line has positive slope and positive y-intercept.

140. (a), (c), and (g)

The line has negative slope and positive y-intercept.

141. (c)

The equation $x - y = -2$ has slope 1 and y-intercept $(0, 2)$. The equation $x - y = 1$ has slope 1 and y-intercept $(0, -1)$. Thus, the lines are parallel with positive slopes. One line has a positive y-intercept and the other with a negative y-intercept.

142. (d)

The equation $y - 2x = 2$ has slope 2 and y-intercept $(0, 2)$. The equation $x + 2y = -1$ has slope $-\frac{1}{2}$ and y-intercept $\left(0, -\frac{1}{2}\right)$. The lines are perpendicular since $2\left(-\frac{1}{2}\right) = -1$. One line has a positive y-intercept and the other with a negative y-intercept.

143 – 145. Answers will vary.

146. No, the equation of a vertical line cannot be written in slope-intercept form because the slope is undefined.

147. No, a line does not need to have both an x-intercept and a y-intercept. Vertical and horizontal lines have only one intercept (unless they are a coordinate axis). Every line must have at least one intercept.

148. Two lines with equal slopes and equal y-intercepts are coinciding lines (i.e. the same).

149. Two lines that have the same x-intercept and y-intercept (assuming the x-intercept is not 0) are the same line since a line is uniquely defined by two distinct points.

150. No. Two lines with the same slope and different x-intercepts are distinct parallel lines and have no points in common.

Assume Line 1 has equation $y = mx + b_1$ and Line 2 has equation $y = mx + b_2$,

Line 1 has x-intercept $-\frac{b_1}{m}$ and y-intercept b_1 .

Line 2 has x-intercept $-\frac{b_2}{m}$ and y-intercept b_2 .

Assume also that Line 1 and Line 2 have unequal x-intercepts.

If the lines have the same y-intercept, then $b_1 = b_2$.

$$b_1 = b_2 \Rightarrow \frac{b_1}{m} = \frac{b_2}{m} \Rightarrow -\frac{b_1}{m} = -\frac{b_2}{m}$$

But $-\frac{b_1}{m} = -\frac{b_2}{m} \Rightarrow$ Line 1 and Line 2 have the same x-intercept, which contradicts the original assumption that the lines have unequal x-intercepts. Therefore, Line 1 and Line 2 cannot have the same y-intercept.

151. Yes. Two distinct lines with the same y-intercept, but different slopes, can have the same x-intercept if the x-intercept is $x = 0$.

Assume Line 1 has equation $y = m_1x + b$ and Line 2 has equation $y = m_2x + b$,

Line 1 has x-intercept $-\frac{b}{m_1}$ and y-intercept b .

Line 2 has x-intercept $-\frac{b}{m_2}$ and y-intercept b .

Assume also that Line 1 and Line 2 have unequal slopes, that is $m_1 \neq m_2$.

If the lines have the same x-intercept, then

$$-\frac{b}{m_1} = -\frac{b}{m_2}.$$

$$-\frac{b}{m_1} = -\frac{b}{m_2}$$

$$-m_2b = -m_1b$$

$$-m_2b + m_1b = 0$$

$$\text{But } -m_2b + m_1b = 0 \Rightarrow b(m_1 - m_2) = 0$$

$$\Rightarrow b = 0$$

$$\text{or } m_1 - m_2 = 0 \Rightarrow m_1 = m_2$$

Since we are assuming that $m_1 \neq m_2$, the only way that the two lines can have the same x-intercept is if $b = 0$.

152. Answers will vary.

$$153. \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 2}{1 - (-3)} = \frac{-6}{4} = -\frac{3}{2}$$

It appears that the student incorrectly found the slope by switching the direction of one of the subtractions.

Chapter 1: Graphs

Section 1.4

1. add; $\left(\frac{1}{2} \cdot 10\right)^2 = 25$

2. $(x-2)^2 = 9$

$$x-2 = \pm\sqrt{9}$$

$$x-2 = \pm 3$$

$$x = 2 \pm 3$$

$$x = 5 \text{ or } x = -1$$

The solution set is $\{-1, 5\}$.

3. False. For example, $x^2 + y^2 + 2x + 2y + 8 = 0$ is not a circle. It has no real solutions.

4. radius

5. True; $r^2 = 9 \rightarrow r = 3$

6. False; the center of the circle $(x+3)^2 + (y-2)^2 = 13$ is $(-3, 2)$.

7. d

8. a

9. Center = $(2, 1)$

Radius = distance from $(0, 1)$ to $(2, 1)$

$$= \sqrt{(2-0)^2 + (1-1)^2} = \sqrt{4} = 2$$

Equation: $(x-2)^2 + (y-1)^2 = 4$

10. Center = $(1, 2)$

Radius = distance from $(1, 0)$ to $(1, 2)$

$$= \sqrt{(1-1)^2 + (2-0)^2} = \sqrt{4} = 2$$

Equation: $(x-1)^2 + (y-2)^2 = 4$

11. Center = midpoint of $(1, 2)$ and $(4, 2)$

$$= \left(\frac{1+4}{2}, \frac{2+2}{2}\right) = \left(\frac{5}{2}, 2\right)$$

Radius = distance from $\left(\frac{5}{2}, 2\right)$ to $(4, 2)$

$$= \sqrt{\left(4 - \frac{5}{2}\right)^2 + (2-2)^2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

Equation: $\left(x - \frac{5}{2}\right)^2 + (y-2)^2 = \frac{9}{4}$

12. Center = midpoint of $(0, 1)$ and $(2, 3)$

$$= \left(\frac{0+2}{2}, \frac{1+3}{2}\right) = (1, 2)$$

Radius = distance from $(1, 2)$ to $(2, 3)$

$$= \sqrt{(2-1)^2 + (3-2)^2} = \sqrt{2}$$

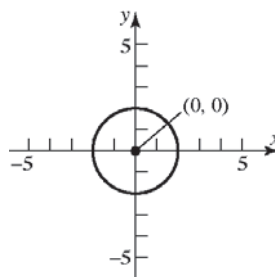
Equation: $(x-1)^2 + (y-2)^2 = 2$

13. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + (y-0)^2 = 2^2$$

$$x^2 + y^2 = 4$$

General form: $x^2 + y^2 - 4 = 0$

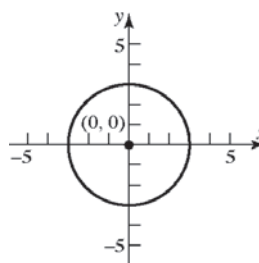


14. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + (y-0)^2 = 3^2$$

$$x^2 + y^2 = 9$$

General form: $x^2 + y^2 - 9 = 0$



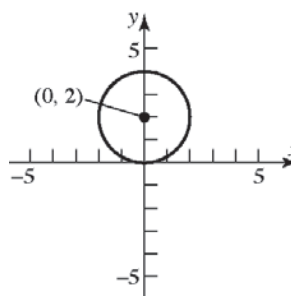
15. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + (y-2)^2 = 2^2$$

$$x^2 + (y-2)^2 = 4$$

General form: $x^2 + y^2 - 4y + 4 = 4$

$$x^2 + y^2 - 4y = 0$$



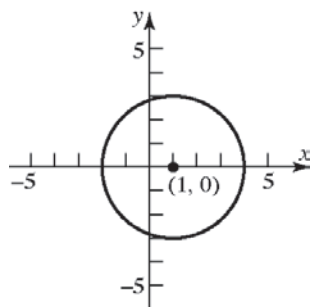
16. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-1)^2 + (y-0)^2 = 3^2$$

$$(x-1)^2 + y^2 = 9$$

General form: $x^2 - 2x + 1 + y^2 = 9$

$$x^2 + y^2 - 2x - 8 = 0$$



17. $(x-h)^2 + (y-k)^2 = r^2$

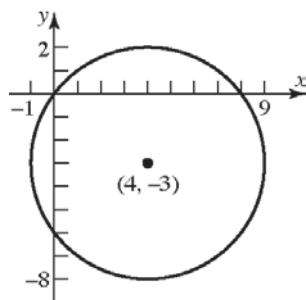
$$(x-4)^2 + (y-(-3))^2 = 5^2$$

$$(x-4)^2 + (y+3)^2 = 25$$

General form:

$$x^2 - 8x + 16 + y^2 + 6y + 9 = 25$$

$$x^2 + y^2 - 8x + 6y = 0$$



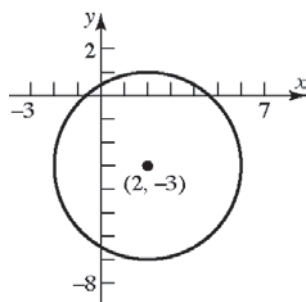
18. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-2)^2 + (y-(-3))^2 = 4^2$$

$$(x-2)^2 + (y+3)^2 = 16$$

General form: $x^2 - 4x + 4 + y^2 + 6y + 9 = 16$

$$x^2 + y^2 - 4x + 6y - 3 = 0$$



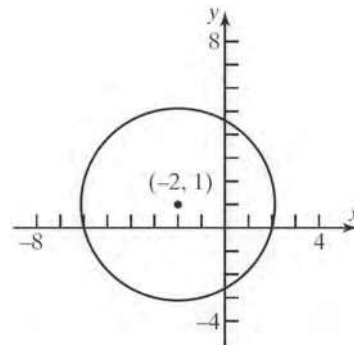
19. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-(-2))^2 + (y-1)^2 = 4^2$$

$$(x+2)^2 + (y-1)^2 = 16$$

General form: $x^2 + 4x + 4 + y^2 - 2y + 1 = 16$

$$x^2 + y^2 + 4x - 2y - 11 = 0$$



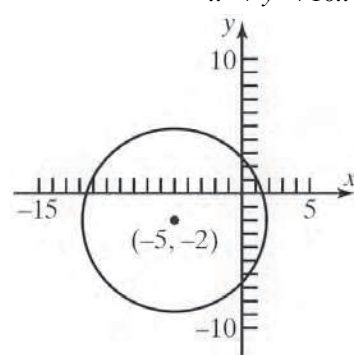
20. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-(-5))^2 + (y-(-2))^2 = 7^2$$

$$(x+5)^2 + (y+2)^2 = 49$$

General form: $x^2 + 10x + 25 + y^2 + 4y + 4 = 49$

$$x^2 + y^2 + 10x + 4y - 20 = 0$$



21. $(x-h)^2 + (y-k)^2 = r^2$

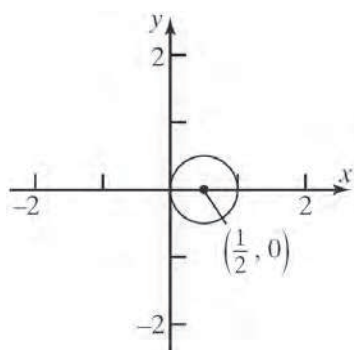
$$\left(x - \frac{1}{2}\right)^2 + (y-0)^2 = \left(\frac{1}{2}\right)^2$$

$$\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$$

General form: $x^2 - x + \frac{1}{4} + y^2 = \frac{1}{4}$

$$x^2 + y^2 - x = 0$$

Chapter 1: Graphs



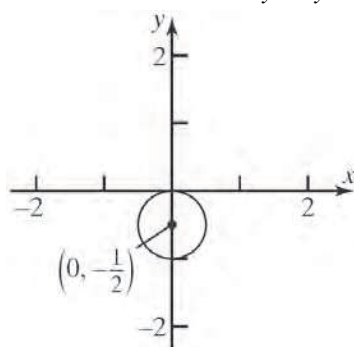
22. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + \left(y - \left(-\frac{1}{2}\right)\right)^2 = \left(\frac{1}{2}\right)^2$$

$$x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{4}$$

General form: $x^2 + y^2 + y + \frac{1}{4} = \frac{1}{4}$

$$x^2 + y^2 + y = 0$$



23. $(x-h)^2 + (y-k)^2 = r^2$

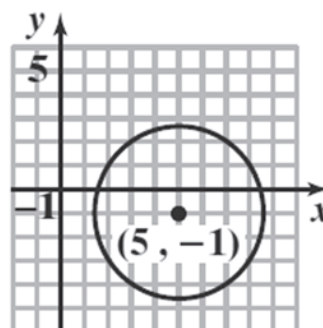
$$(x-5)^2 + (y-(-1))^2 = (\sqrt{13})^2$$

$$(x-5)^2 + (y+1)^2 = 13$$

General form:

$$x^2 - 10x + 25 + y^2 + 2y + 1 = 13$$

$$x^2 + y^2 - 10x + 2y + 13 = 0$$



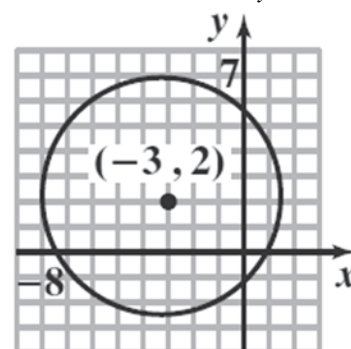
24. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-(-3))^2 + (y-2)^2 = (2\sqrt{5})^2$$

$$(x+3)^2 + (y-2)^2 = 20$$

General form: $x^2 + 6x + 9 + y^2 - 4y + 4 = 20$

$$x^2 + y^2 + 6x - 4y - 7 = 0$$

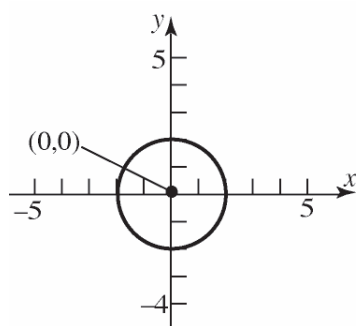


25. $x^2 + y^2 = 4$

$$x^2 + y^2 = 2^2$$

a. Center: $(0, 0)$; Radius = 2

b.



c. x -intercepts: $x^2 + (0)^2 = 4$

$$x^2 = 4$$

$$x = \pm\sqrt{4} = \pm 2$$

y -intercepts: $(0)^2 + y^2 = 4$

$$y^2 = 4$$

$$y = \pm\sqrt{4} = \pm 2$$

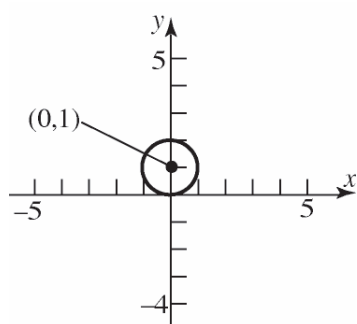
The intercepts are $(-2, 0)$, $(2, 0)$, $(0, -2)$, and $(0, 2)$.

26. $x^2 + (y-1)^2 = 1$

$$x^2 + (y-1)^2 = 1^2$$

 a. Center: $(0, 1)$; Radius = 1

b.



c. x -intercepts: $x^2 + (0-1)^2 = 1$

$$x^2 + 1 = 1$$

$$x^2 = 0$$

$$x = \pm\sqrt{0} = 0$$

y -intercepts: $(0)^2 + (y-1)^2 = 1$

$$(y-1)^2 = 1$$

$$y-1 = \pm\sqrt{1}$$

$$y-1 = \pm 1$$

$$y = 1 \pm 1$$

$$y = 2 \text{ or } y = 0$$

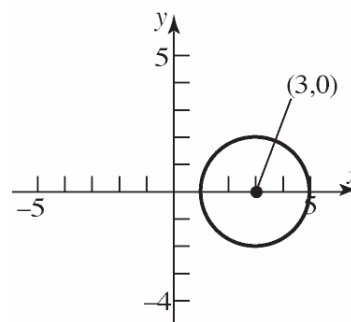
The intercepts are $(0, 0)$ and $(0, 2)$.

27. $2(x-3)^2 + 2y^2 = 8$

$$(x-3)^2 + y^2 = 4$$

 a. Center: $(3, 0)$; Radius = 2

b.



c. x -intercepts: $(x-3)^2 + (0)^2 = 4$

$$(x-3)^2 = 4$$

$$x-3 = \pm\sqrt{4}$$

$$x-3 = \pm 2$$

$$x = 3 \pm 2$$

$$x = 5 \text{ or } x = 1$$

y -intercepts: $(0-3)^2 + y^2 = 4$

$$(-3)^2 + y^2 = 4$$

$$9 + y^2 = 4$$

$$y^2 = -5$$

No real solution.

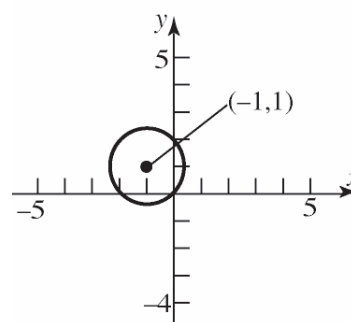
The intercepts are $(1, 0)$ and $(5, 0)$.

28. $3(x+1)^2 + 3(y-1)^2 = 6$

$$(x+1)^2 + (y-1)^2 = 2$$

 a. Center: $(-1, 1)$; Radius = $\sqrt{2}$

b.



Chapter 1: Graphs

c. x -intercepts: $(x+1)^2 + (0-1)^2 = 2$
 $(x+1)^2 + (-1)^2 = 2$
 $(x+1)^2 + 1 = 2$
 $(x+1)^2 = 1$
 $x+1 = \pm\sqrt{1}$
 $x+1 = \pm 1$
 $x = -1 \pm 1$
 $x = 0$ or $x = -2$

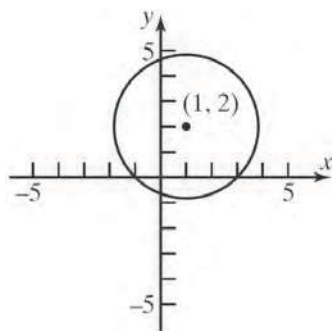
y -intercepts: $(0+1)^2 + (y-1)^2 = 2$
 $(1)^2 + (y-1)^2 = 2$
 $1 + (y-1)^2 = 2$
 $(y-1)^2 = 1$
 $y-1 = \pm\sqrt{1}$
 $y-1 = \pm 1$
 $y = 1 \pm 1$
 $y = 2$ or $y = 0$

The intercepts are $(-2, 0)$, $(0, 0)$, and $(0, 2)$.

29. $x^2 + y^2 - 2x - 4y - 4 = 0$
 $x^2 - 2x + y^2 - 4y = 4$
 $(x^2 - 2x + 1) + (y^2 - 4y + 4) = 4 + 1 + 4$
 $(x-1)^2 + (y-2)^2 = 3^2$

a. Center: $(1, 2)$; Radius = 3

b.



c. x -intercepts: $(x-1)^2 + (0-2)^2 = 3^2$
 $(x-1)^2 + (-2)^2 = 3^2$
 $(x-1)^2 + 4 = 9$
 $(x-1)^2 = 5$
 $x-1 = \pm\sqrt{5}$
 $x = 1 \pm \sqrt{5}$

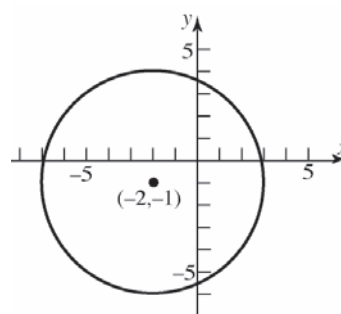
y -intercepts: $(0-1)^2 + (y-2)^2 = 3^2$
 $(-1)^2 + (y-2)^2 = 3^2$
 $1 + (y-2)^2 = 9$
 $(y-2)^2 = 8$
 $y-2 = \pm\sqrt{8}$
 $y-2 = \pm 2\sqrt{2}$
 $y = 2 \pm 2\sqrt{2}$

The intercepts are $(1-\sqrt{5}, 0)$, $(1+\sqrt{5}, 0)$, $(0, 2-2\sqrt{2})$, and $(0, 2+2\sqrt{2})$.

30. $x^2 + y^2 + 4x + 2y - 20 = 0$
 $x^2 + 4x + y^2 + 2y = 20$
 $(x^2 + 4x + 4) + (y^2 + 2y + 1) = 20 + 4 + 1$
 $(x+2)^2 + (y+1)^2 = 5^2$

a. Center: $(-2, -1)$; Radius = 5

b.



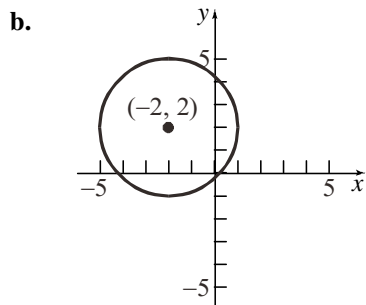
c. x -intercepts: $(x+2)^2 + (0+1)^2 = 5^2$
 $(x+2)^2 + 1 = 25$
 $(x+2)^2 = 24$
 $x+2 = \pm\sqrt{24}$
 $x+2 = \pm 2\sqrt{6}$
 $x = -2 \pm 2\sqrt{6}$

y -intercepts: $(0+2)^2 + (y+1)^2 = 5^2$
 $4 + (y+1)^2 = 25$
 $(y+1)^2 = 21$
 $y+1 = \pm\sqrt{21}$
 $y = -1 \pm \sqrt{21}$

The intercepts are $(-2-2\sqrt{6}, 0)$, $(-2+2\sqrt{6}, 0)$, $(0, -1-\sqrt{21})$, and $(0, -1+\sqrt{21})$.

$$\begin{aligned}
 31. \quad & x^2 + y^2 + 4x - 4y - 1 = 0 \\
 & x^2 + 4x + y^2 - 4y = 1 \\
 & (x^2 + 4x + 4) + (y^2 - 4y + 4) = 1 + 4 + 4 \\
 & (x+2)^2 + (y-2)^2 = 3^2
 \end{aligned}$$

a. Center: $(-2, 2)$; Radius = 3

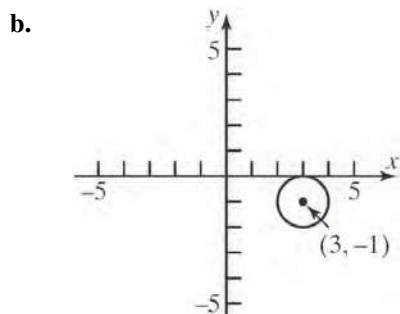


$$\begin{aligned}
 c. \quad & x\text{-intercepts: } (x+2)^2 + (0-2)^2 = 3^2 \\
 & (x+2)^2 + 4 = 9 \\
 & (x+2)^2 = 5 \\
 & x+2 = \pm\sqrt{5} \\
 & x = -2 \pm \sqrt{5} \\
 & y\text{-intercepts: } (0+2)^2 + (y-2)^2 = 3^2 \\
 & 4 + (y-2)^2 = 9 \\
 & (y-2)^2 = 5 \\
 & y-2 = \pm\sqrt{5} \\
 & y = 2 \pm \sqrt{5}
 \end{aligned}$$

The intercepts are $(-2 - \sqrt{5}, 0)$, $(-2 + \sqrt{5}, 0)$, $(0, 2 - \sqrt{5})$, and $(0, 2 + \sqrt{5})$.

$$\begin{aligned}
 32. \quad & x^2 + y^2 - 6x + 2y + 9 = 0 \\
 & x^2 - 6x + y^2 + 2y = -9 \\
 & (x^2 - 6x + 9) + (y^2 + 2y + 1) = -9 + 9 + 1 \\
 & (x-3)^2 + (y+1)^2 = 1^2
 \end{aligned}$$

a. Center: $(3, -1)$; Radius = 1



$$\begin{aligned}
 c. \quad & x\text{-intercepts: } (x-3)^2 + (0+1)^2 = 1^2 \\
 & (x-3)^2 + 1 = 1 \\
 & (x-3)^2 = 0 \\
 & x-3 = 0 \\
 & x = 3
 \end{aligned}$$

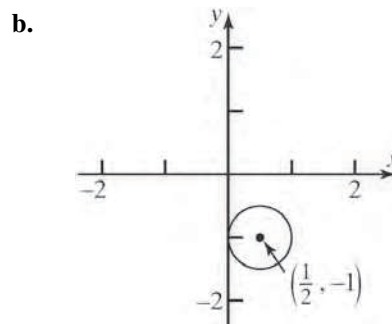
$$\begin{aligned}
 & y\text{-intercepts: } (0-3)^2 + (y+1)^2 = 1^2 \\
 & 9 + (y+1)^2 = 1 \\
 & (y+1)^2 = -8
 \end{aligned}$$

No real solution.

The intercept only intercept is $(3, 0)$.

$$\begin{aligned}
 33. \quad & x^2 + y^2 - x + 2y + 1 = 0 \\
 & x^2 - x + y^2 + 2y = -1 \\
 & \left(x^2 - x + \frac{1}{4}\right) + (y^2 + 2y + 1) = -1 + \frac{1}{4} + 1 \\
 & \left(x - \frac{1}{2}\right)^2 + (y+1)^2 = \left(\frac{1}{2}\right)^2
 \end{aligned}$$

a. Center: $\left(\frac{1}{2}, -1\right)$; Radius = $\frac{1}{2}$



$$\begin{aligned}
 c. \quad & x\text{-intercepts: } \left(x - \frac{1}{2}\right)^2 + (0+1)^2 = \left(\frac{1}{2}\right)^2 \\
 & \left(x - \frac{1}{2}\right)^2 + 1 = \frac{1}{4} \\
 & \left(x - \frac{1}{2}\right)^2 = -\frac{3}{4}
 \end{aligned}$$

No real solutions

$$\begin{aligned}
 & y\text{-intercepts: } \left(0 - \frac{1}{2}\right)^2 + (y+1)^2 = \left(\frac{1}{2}\right)^2 \\
 & \frac{1}{4} + (y+1)^2 = \frac{1}{4} \\
 & (y+1)^2 = 0 \\
 & y+1 = 0 \\
 & y = -1
 \end{aligned}$$

The only intercept is $(0, -1)$.

Chapter 1: Graphs

34. $x^2 + y^2 + x + y - \frac{1}{2} = 0$

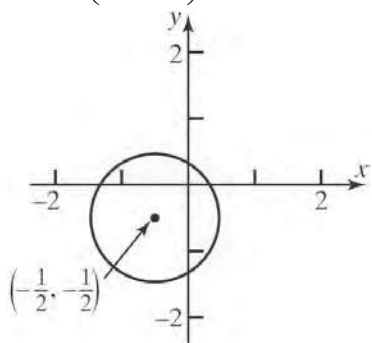
$$x^2 + x + y^2 + y = \frac{1}{2}$$

$$\left(x^2 + x + \frac{1}{4}\right) + \left(y^2 + y + \frac{1}{4}\right) = \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = 1^2$$

a. Center: $\left(-\frac{1}{2}, -\frac{1}{2}\right)$; Radius = 1

b.



c. x-intercepts: $\left(x + \frac{1}{2}\right)^2 + \left(0 + \frac{1}{2}\right)^2 = 1^2$

$$\left(x + \frac{1}{2}\right)^2 + \frac{1}{4} = 1$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{3}{4}$$

$$x + \frac{1}{2} = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{-1 \pm \sqrt{3}}{2}$$

y-intercepts: $\left(0 + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = 1^2$

$$\frac{1}{4} + \left(y + \frac{1}{2}\right)^2 = 1$$

$$\left(y + \frac{1}{2}\right)^2 = \frac{3}{4}$$

$$y + \frac{1}{2} = \pm \frac{\sqrt{3}}{2}$$

$$y = \frac{-1 \pm \sqrt{3}}{2}$$

The intercepts are $\left(\frac{-1-\sqrt{3}}{2}, 0\right)$, $\left(\frac{-1+\sqrt{3}}{2}, 0\right)$,

$\left(0, \frac{-1-\sqrt{3}}{2}\right)$, and $\left(0, \frac{-1+\sqrt{3}}{2}\right)$.

35. $2x^2 + 2y^2 - 12x + 8y - 24 = 0$

$$x^2 + y^2 - 6x + 4y = 12$$

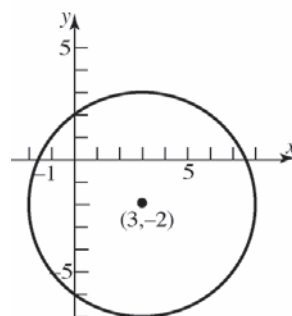
$$x^2 - 6x + y^2 + 4y = 12$$

$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$$

$$(x-3)^2 + (y+2)^2 = 5^2$$

a. Center: $(3, -2)$; Radius = 5

b.



c. x-intercepts: $(x-3)^2 + (0+2)^2 = 5^2$

$$(x-3)^2 + 4 = 25$$

$$(x-3)^2 = 21$$

$$x-3 = \pm\sqrt{21}$$

$$x = 3 \pm \sqrt{21}$$

y-intercepts: $(0-3)^2 + (y+2)^2 = 5^2$

$$9 + (y+2)^2 = 25$$

$$(y+2)^2 = 16$$

$$y+2 = \pm 4$$

$$y = -2 \pm 4$$

$$y = 2 \text{ or } y = -6$$

The intercepts are $(3-\sqrt{21}, 0)$, $(3+\sqrt{21}, 0)$, $(0, -6)$, and $(0, 2)$.

36. a. $2x^2 + 2y^2 + 8x + 7 = 0$

$$2x^2 + 8x + 2y^2 = -7$$

$$x^2 + 4x + y^2 = -\frac{7}{2}$$

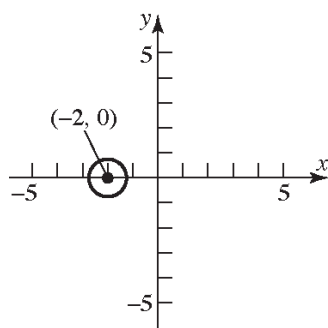
$$(x^2 + 4x + 4) + y^2 = -\frac{7}{2} + 4$$

$$(x+2)^2 + y^2 = \frac{1}{2}$$

$$(x+2)^2 + y^2 = \left(\frac{\sqrt{2}}{2}\right)^2$$

Center: $(-2, 0)$; Radius = $\frac{\sqrt{2}}{2}$

b.



$$\begin{aligned} \text{c. } x\text{-intercepts: } (x+2)^2 + (0)^2 &= \frac{1}{2} \\ (x+2)^2 &= \frac{1}{2} \\ x+2 &= \pm\sqrt{\frac{1}{2}} \\ x+2 &= \pm\frac{\sqrt{2}}{2} \\ x &= -2 \pm \frac{\sqrt{2}}{2} \end{aligned}$$

$$\begin{aligned} \text{y-intercepts: } (0+2)^2 + y^2 &= \frac{1}{2} \\ 4 + y^2 &= \frac{1}{2} \\ y^2 &= -\frac{7}{2} \end{aligned}$$

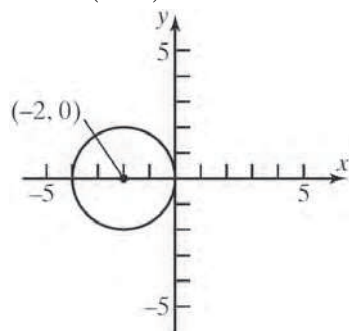
No real solutions.

The intercepts are $\left(-2 - \frac{\sqrt{2}}{2}, 0\right)$ and $\left(-2 + \frac{\sqrt{2}}{2}, 0\right)$.

$$\begin{aligned} 37. \quad 2x^2 + 8x + 2y^2 &= 0 \\ x^2 + 4x + y^2 &= 0 \\ x^2 + 4x + 4 + y^2 &= 0 + 4 \\ (x+2)^2 + y^2 &= 2^2 \end{aligned}$$

a. Center: $(-2, 0)$; Radius: $r = 2$

b.



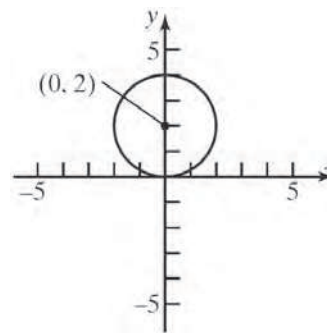
$$\begin{aligned} \text{c. } x\text{-intercepts: } (x+2)^2 + (0)^2 &= 2^2 \\ (x+2)^2 &= 4 \\ (x+2)^2 &= \pm\sqrt{4} \\ x+2 &= \pm 2 \\ x &= -2 \pm 2 \\ x &= 0 \quad \text{or} \quad x = -4 \\ \text{y-intercepts: } (0+2)^2 + y^2 &= 2^2 \\ 4 + y^2 &= 4 \\ y^2 &= 0 \\ y &= 0 \end{aligned}$$

The intercepts are $(-4, 0)$ and $(0, 0)$.

$$\begin{aligned} 38. \quad 3x^2 + 3y^2 - 12y &= 0 \\ x^2 + y^2 - 4y &= 0 \\ x^2 + y^2 - 4y + 4 &= 0 + 4 \\ x^2 + (y-2)^2 &= 4 \end{aligned}$$

a. Center: $(0, 2)$; Radius: $r = 2$

b.



$$\begin{aligned} \text{c. } x\text{-intercepts: } x^2 + (0-2)^2 &= 4 \\ x^2 + 4 &= 4 \\ x^2 &= 0 \\ x &= 0 \\ \text{y-intercepts: } 0^2 + (y-2)^2 &= 4 \\ (y-2)^2 &= 4 \\ y-2 &= \pm\sqrt{4} \\ y-2 &= \pm 2 \\ y &= 2 \pm 2 \\ y &= 4 \quad \text{or} \quad y = 0 \end{aligned}$$

The intercepts are $(0, 0)$ and $(0, 4)$.

Chapter 1: Graphs

39. Center at (0, 0); containing point (-2, 3).

$$r = \sqrt{(-2-0)^2 + (3-0)^2} = \sqrt{4+9} = \sqrt{13}$$

$$\text{Equation: } (x-0)^2 + (y-0)^2 = (\sqrt{13})^2$$

$$x^2 + y^2 = 13$$

40. Center at (1, 0); containing point (-3, 2).

$$r = \sqrt{(-3-1)^2 + (2-0)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$\text{Equation: } (x-1)^2 + (y-0)^2 = (\sqrt{20})^2$$

$$(x-1)^2 + y^2 = 20$$

41. Endpoints of a diameter are (1, 4) and (-3, 2).
The center is at the midpoint of that diameter:

$$\text{Center: } \left(\frac{1+(-3)}{2}, \frac{4+2}{2} \right) = (-1, 3)$$

$$\text{Radius: } r = \sqrt{(1-(-1))^2 + (4-3)^2} = \sqrt{4+1} = \sqrt{5}$$

$$\text{Equation: } (x-(-1))^2 + (y-3)^2 = (\sqrt{5})^2$$

$$(x+1)^2 + (y-3)^2 = 5$$

42. Endpoints of a diameter are (4, 3) and (0, 1).
The center is at the midpoint of that diameter:

$$\text{Center: } \left(\frac{4+0}{2}, \frac{3+1}{2} \right) = (2, 2)$$

$$\text{Radius: } r = \sqrt{(4-2)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$$

$$\text{Equation: } (x-2)^2 + (y-2)^2 = (\sqrt{5})^2$$

$$(x-2)^2 + (y-2)^2 = 5$$

43. $C = 2\pi r$

$$16\pi = 2\pi r$$

$$r = 8$$

$$(x-2)^2 + (y-(-4))^2 = (8)^2$$

$$(x-2)^2 + (y+4)^2 = 64$$

44. $A = \pi r^2$

$$49\pi = \pi r^2$$

$$r = 7$$

$$(x-(-5))^2 + (y-6)^2 = (7)^2$$

$$(x+5)^2 + (y-6)^2 = 49$$

45. (c); Center: 1; Radius = 2

46. (d); Center: (-3, 3); Radius = 3

47. (b); Center: (-1, 2); Radius = 2

48. (a); Center: (-3, 3); Radius = 3

49. The centers of the circles are: (4, -2) and (-1, 5).

The slope is $m = \frac{5-(-2)}{-1-4} = \frac{7}{-5} = -\frac{7}{5}$. Use the slope and one point to find the equation of the line.

$$y-(-2) = -\frac{7}{5}(x-4)$$

$$y+2 = -\frac{7}{5}x + \frac{28}{5}$$

$$5y+10 = -7x+28$$

$$7x+5y = 18$$

50. Find the centers of the two circles:

$$x^2 + y^2 - 4x + 6y + 4 = 0$$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -4 + 4 + 9$$

$$(x-2)^2 + (y+3)^2 = 9$$

$$\text{Center: } (2, -3)$$

$$x^2 + y^2 + 6x + 4y + 9 = 0$$

$$(x^2 + 6x + 9) + (y^2 + 4y + 4) = -9 + 9 + 4$$

$$(x+3)^2 + (y+2)^2 = 4$$

$$\text{Center: } (-3, -2)$$

Find the slope of the line containing the centers:

$$m = \frac{-2-(-3)}{-3-2} = -\frac{1}{5}$$

Find the equation of the line containing the centers:

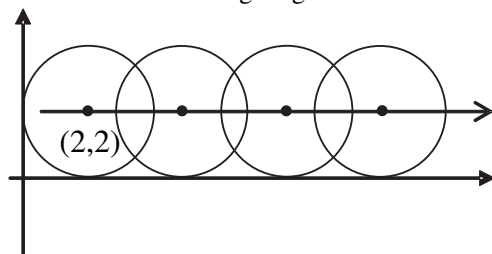
$$y+3 = -\frac{1}{5}(x-2)$$

$$5y+15 = -x+2$$

$$x+5y = -13$$

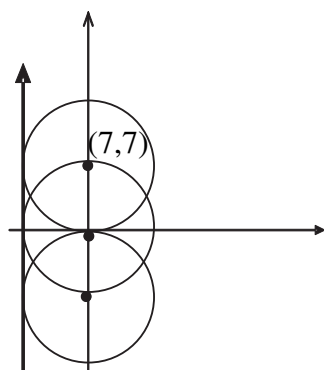
$$x+5y+13 = 0$$

51. Consider the following diagram:



Therefore, the path of the center of the circle has the equation $y = 2$.

52. Consider the following diagram:



Therefore the path of the center of the circle has the equation $x = 7$.

53. Let the upper-right corner of the square be the point (x, y) . The circle and the square are both centered about the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square. Therefore, we get

$$x^2 + y^2 = 9$$

$$x^2 + x^2 = 9$$

$$2x^2 = 9$$

$$x^2 = \frac{9}{2}$$

$$x = \sqrt{\frac{9}{2}} = \frac{3\sqrt{2}}{2}$$

The length of one side of the square is $2x$. Thus, the area is

$$A = s^2 = \left(2 \cdot \frac{3\sqrt{2}}{2}\right)^2 = (3\sqrt{2})^2 = 18 \text{ square units.}$$

54. The area of the shaded region is the area of the circle, less the area of the square. Let the upper-right corner of the square be the point (x, y) . The circle and the square are both centered about

the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square.

Therefore, we get

$$x^2 + y^2 = 36$$

$$x^2 + x^2 = 36$$

$$2x^2 = 36$$

$$x^2 = 18$$

$$x = 3\sqrt{2}$$

The length of one side of the square is $2x$. Thus,

the area of the square is $(2 \cdot 3\sqrt{2})^2 = 72$ square

units. From the equation of the circle, we have $r = 6$. The area of the circle is

$$\pi r^2 = \pi (6)^2 = 36\pi \text{ square units.}$$

Therefore, the area of the shaded region is

$$A = 36\pi - 72 \text{ square units.}$$

55. The diameter of the Ferris wheel was 250 feet, so the radius was 125 feet. The maximum height was 264 feet, so the center was at a height of $264 - 125 = 139$ feet above the ground. Since the center of the wheel is on the y -axis, it is the point $(0, 139)$. Thus, an equation for the wheel is:

$$(x - 0)^2 + (y - 139)^2 = 125^2$$

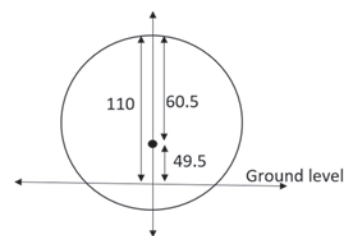
$$x^2 + (y - 139)^2 = 15,625$$

56. The diameter of the wheel is 520 feet, so the radius is 260 feet. The maximum height is 550 feet, so the center of the wheel is at a height of $550 - 260 = 290$ feet above the ground. Since the center of the wheel is on the y -axis, it is the point $(0, 290)$. Thus, an equation for the wheel is:

$$(x - 0)^2 + (y - 290)^2 = 260^2$$

$$x^2 + (y - 290)^2 = 67,600$$

- 57.



Refer to figure. Since the radius of the building is 60.5 m and the height of the building is 110 m,

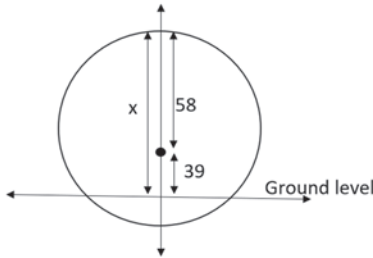
Chapter 1: Graphs

then the center of the building is 49.5 m above the ground, so the y-coordinate of the center is 49.5. The equation of the circle is given by $x^2 + (y - 49.5)^2 = 60.5^2 = 3660.25$

- 58.** Complete the square to find the equation of the circle representing the formula for the building.

$$x^2 + y^2 - 78y + 1521 = 1843 + 1521 = 3364$$

$$x^2 + (y - 39)^2 = 58^2$$



Refer to figure. The y coordinate of the center is 39. The radius is 58. Thus the height of the building is $58 + 39 = 97$ m.

- 59.** Center at (2, 3); tangent to the x-axis.
 $r = 3$

$$\begin{aligned} \text{Equation: } (x-2)^2 + (y-3)^2 &= 3^2 \\ (x-2)^2 + (y-3)^2 &= 9 \end{aligned}$$

- 60.** Center at (-3, 1); tangent to the y-axis.
 $r = 3$

$$\begin{aligned} \text{Equation: } (x+3)^2 + (y-1)^2 &= 3^2 \\ (x+3)^2 + (y-1)^2 &= 9 \end{aligned}$$

- 61.** Center at (-1, 3); tangent to the line $y = 2$.
This means that the circle contains the point (-1, 2), so the radius is $r = 1$.

$$\begin{aligned} \text{Equation: } (x+1)^2 + (y-3)^2 &= (1)^2 \\ (x+1)^2 + (y-3)^2 &= 1 \end{aligned}$$

- 62.** Center at (4, -2); tangent to the line $x = 1$.
This means that the circle contains the point (1, -2), so the radius is $r = 3$.

$$\begin{aligned} \text{Equation: } (x-4)^2 + (y+2)^2 &= (3)^2 \\ (x-4)^2 + (y+2)^2 &= 9 \end{aligned}$$

- 63. a.** Substitute $y = mx + b$ into $x^2 + y^2 = r^2$:

$$x^2 + (mx + b)^2 = r^2$$

$$x^2 + m^2x^2 + 2bmx + b^2 = r^2$$

$$(1 + m^2)x^2 + 2bmx + b^2 - r^2 = 0$$

This equation has one solution if and only if the discriminant is zero.

$$(2bm)^2 - 4(1 + m^2)(b^2 - r^2) = 0$$

$$4b^2m^2 - 4b^2 + 4r^2 - 4b^2m^2 + 4m^2r^2 = 0$$

$$-4b^2 + 4r^2 + 4m^2r^2 = 0$$

$$-b^2 + r^2 + m^2r^2 = 0$$

$$r^2(1 + m^2) = b^2$$

- b.** From part (a) we know

$(1 + m^2)x^2 + 2bmx + b^2 - r^2 = 0$. Using the quadratic formula, since the discriminant is zero, we get:

$$x = \frac{-2bm}{2(1 + m^2)} = \frac{-bm}{\left(\frac{b^2}{r^2}\right)} = \frac{-bmr^2}{b^2} = \frac{-mr^2}{b}$$

$$\begin{aligned} y &= m\left(\frac{-mr^2}{b}\right) + b \\ &= \frac{-m^2r^2}{b} + b = \frac{-m^2r^2 + b^2}{b} = \frac{r^2}{b} \end{aligned}$$

The point of tangency is $\left(\frac{-mr^2}{b}, \frac{r^2}{b}\right)$.

- c.** The slope of the tangent line is m .
The slope of the line joining the point of tangency and the center (0,0) is:

$$\frac{\left(\frac{r^2}{b} - 0\right)}{\left(\frac{-mr^2}{b} - 0\right)} = \frac{\frac{r^2}{b}}{\frac{-mr^2}{b}} = \frac{r^2}{b} \cdot \frac{b}{-mr^2} = -\frac{1}{m}$$

The two lines are perpendicular.

- 64.** Let (h, k) be the center of the circle.

$$x - 2y + 4 = 0$$

$$2y = x + 4$$

$$y = \frac{1}{2}x + 2$$

The slope of the tangent line is $\frac{1}{2}$. The slope from (h, k) to $(0, 2)$ is -2 .

$$\frac{2-k}{0-h} = -2$$

$$2-k = 2h$$

The other tangent line is $y = 2x - 7$, and it has slope 2.

The slope from (h, k) to $(3, -1)$ is $-\frac{1}{2}$.

$$\frac{-1-k}{3-h} = -\frac{1}{2}$$

$$2+2k = 3-h$$

$$2k = 1-h$$

$$h = 1-2k$$

Solve the two equations in h and k :

$$2-k = 2(1-2k)$$

$$2-k = 2-4k$$

$$3k = 0$$

$$k = 0$$

$$h = 1-2(0) = 1$$

The center of the circle is $(1, 0)$.

- 65.** The slope of the line containing the center $(0,0)$ and $(1, 2\sqrt{2})$ is

$$\frac{2\sqrt{2}-0}{1-0} = 2\sqrt{2}. \text{ Then the slope of the tangent}$$

$$\text{line is } \frac{-1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}.$$

So the equation of the tangent line is:

$$y - 2\sqrt{2} = -\frac{\sqrt{2}}{4}(x-1)$$

$$y - 2\sqrt{2} = -\frac{\sqrt{2}}{4}x + \frac{\sqrt{2}}{4}$$

$$4y - 8\sqrt{2} = -\sqrt{2}x + \sqrt{2}$$

$$\sqrt{2}x + 4y = 9\sqrt{2}$$

$$\mathbf{66.} \quad x^2 + y^2 - 4x + 6y + 4 = 0$$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -4 + 4 + 9$$

$$(x-2)^2 + (y+3)^2 = 9$$

Center: $(2, -3)$

The slope of the line containing the center and

$$(3, 2\sqrt{2} - 3) \text{ is } \frac{2\sqrt{2} - 3 - (-3)}{3 - 2} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$$

Then the slope of the tangent line is:

$$\frac{-1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

So, the equation of the tangent line is

$$y - (2\sqrt{2} - 3) = -\frac{\sqrt{2}}{4}(x - 3)$$

$$y - 2\sqrt{2} + 3 = -\frac{\sqrt{2}}{4}x + \frac{3\sqrt{2}}{4}$$

$$4y - 8\sqrt{2} + 12 = -\sqrt{2}x + 3\sqrt{2}$$

$$\sqrt{2}x + 4y = 11\sqrt{2} - 12$$

Chapter 1: Graphs

67. The center of the circle is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ and the radius is $\frac{1}{2}\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Then the equation of

the circle is $\left(x - \frac{x_1 + x_2}{2}\right)^2 + \left(y - \frac{y_1 + y_2}{2}\right)^2 = \frac{1}{4}[(x_1 - x_2)^2 + (y_1 - y_2)^2]$. Expanding, gives

$$\begin{aligned} x^2 - x(x_1 + x_2) + \frac{(x_1 + x_2)^2}{4} + y^2 - y(y_1 + y_2) + \frac{(y_1 + y_2)^2}{4} &= \frac{1}{4}[x_1^2 - 2x_1x_2 + x_2^2 + y_1^2 - 2y_1y_2 + y_2^2] \\ 4x^2 - 4x_1x - 4x_2x + x_1^2 + 2x_1x_2 + x_2^2 + 4y^2 - 4y_1y - 4y_2y + y_1^2 + 2y_1y_2 + y_2^2 &= x_1^2 - 2x_1x_2 + x_2^2 + y_1^2 - 2y_1y_2 + y_2^2 \\ 4x^2 - 4x_1x - 4x_2x + 4x_1x_2 + 4y^2 - 4y_1y - 4y_2y + 4y_1y_2 &= 0 \\ x^2 - x_1x - x_2x + x_1x_2 + y^2 - y_1y - y_2y + y_1y_2 &= 0 \\ x(x - x_1) - x_2(x + x_1) + y(y - y_1) - y_2(y + y_1) &= 0 \\ (x - x_1)(x - x_2) + (y - y_1)(y - y_2) &= 0 \end{aligned}$$

68. Complete the square to get $\left(x + \frac{d}{2}\right)^2 + \left(y + \frac{e}{2}\right)^2 = \frac{d^2 + e^2 - 4f}{4}$. The slope of the line between the center

$\left(-\frac{d}{2}, -\frac{e}{2}\right)$ and the point of tangency (x_0, y_0) is $m = \frac{y_0 + \frac{e}{2}}{x_0 + \frac{d}{2}}$. So the slope of the tangent line is $m_{\tan} = -\frac{x_0 + \frac{d}{2}}{y_0 + \frac{e}{2}}$.

Therefore, the equation of the tangent line is $y - y_0 = -\frac{x_0 + \frac{d}{2}}{y_0 + \frac{e}{2}}(x - x_0)$ which is equivalent to

$$\begin{aligned} (x - x_0)\left(x_0 + \frac{d}{2}\right) + (y - y_0)\left(y_0 + \frac{e}{2}\right) &= 0 \\ x_0x + \frac{d}{2}x - x_0^2 - \frac{d}{2}x_0 + y_0y + \frac{e}{2}y - y_0^2 - \frac{e}{2}y_0 &= 0 \\ x_0x + y_0y + \frac{d}{2}x + \frac{e}{2}y - \left(x_0^2 + y_0^2 + \frac{d}{2}x_0 + \frac{e}{2}y_0\right) &= 0 \end{aligned}$$

Because (x_0, y_0) is on the circle, $x_0^2 + y_0^2 + dx_0 + ey_0 + f = 0$, and

$$x_0^2 + y_0^2 + \frac{d}{2}x_0 + \frac{e}{2}y_0 = -\frac{d}{2}x_0 - \frac{e}{2}y_0 - f \quad \text{Substituting this result gives}$$

$$\begin{aligned} x_0^2 + y_0^2 + \frac{d}{2}x_0 + \frac{e}{2}y_0 - \left(-\frac{d}{2}x_0 - \frac{e}{2}y_0 - f\right) &= 0 \\ x_0^2 + y_0^2 + \frac{d}{2}x_0 + \frac{e}{2}y_0 + \frac{d}{2}x_0 + \frac{e}{2}y_0 + f &= 0 \\ x_0^2 + y_0^2 + d\left(\frac{x + x_0}{2}\right) + e\left(\frac{y + y_0}{2}\right) + f &= 0 \end{aligned}$$

69. (b), (c), (e) and (g)

We need $h, k > 0$ and $(0, 0)$ on the graph.

70. (b), (e) and (g)

We need $h < 0$, $k = 0$, and $|h| > r$.

71. Answers will vary.

72. The student has the correct radius, but the signs of the coordinates of the center are incorrect. The student needs to write the equation in the standard form $(x - h)^2 + (y - k)^2 = r^2$.

$$(x+3)^2 + (y-2)^2 = 16$$

$$(x-(-3))^2 + (y-2)^2 = 4^2$$

Thus, $(h, k) = (-3, 2)$ and $r = 4$.

Chapter 1 Review Exercises

1. $P_1 = (0, 0)$ and $P_2 = (4, 2)$

a. $d(P_1, P_2) = \sqrt{(4-0)^2 + (2-0)^2}$
 $= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

b. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{0+4}{2}, \frac{0+2}{2} \right) = \left(\frac{4}{2}, \frac{2}{2} \right) = (2, 1)$$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$

d. For each run of 2, there is a rise of 1.

2. $P_1 = (1, -1)$ and $P_2 = (-2, 3)$

a. $d(P_1, P_2) = \sqrt{(-2-1)^2 + (3-(-1))^2}$
 $= \sqrt{9+16} = \sqrt{25} = 5$

b. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{1+(-2)}{2}, \frac{-1+3}{2} \right)$$

$$= \left(\frac{-1}{2}, \frac{2}{2} \right) = \left(-\frac{1}{2}, 1 \right)$$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{3-(-1)}{-2-1} = \frac{4}{-3} = -\frac{4}{3}$

d. For each run of 3, there is a rise of -4 .

3. $P_1 = (4, -4)$ and $P_2 = (4, 8)$

a. $d(P_1, P_2) = \sqrt{(4-4)^2 + (8-(-4))^2}$
 $= \sqrt{0+144} = \sqrt{144} = 12$

b. The coordinates of the midpoint are:

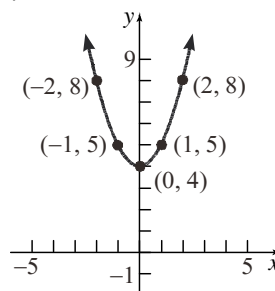
$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{4+4}{2}, \frac{-4+8}{2} \right) = \left(\frac{8}{2}, \frac{4}{2} \right) = (4, 2)$$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{8-(-4)}{4-4} = \frac{12}{0}$, undefined

d. An undefined slope means the points lie on a vertical line. There is no change in x .

4. $y = x^2 + 4$



5. x -intercepts: $-4, 0, 2$; y -intercepts: $-2, 0, 2$
Intercepts: $(-4, 0)$, $(0, 0)$, $(2, 0)$, $(0, -2)$, $(0, 2)$

6. $2x = 3y^2$

x -intercepts:	y -intercepts:
$2x = 3(0)^2$	$2(0) = 3y^2$
$2x = 0$	$0 = y^2$
$x = 0$	$y = 0$

The only intercept is $(0, 0)$.

Test x -axis symmetry: Let $y = -y$

$$2x = 3(-y)^2$$

$$2x = 3y^2 \text{ same}$$

Test y -axis symmetry: Let $x = -x$

$$2(-x) = 3y^2$$

$$-2x = 3y^2 \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$2(-x) = 3(-y)^2$$

$$-2x = 3y^2 \text{ different}$$

Therefore, the graph will have x -axis symmetry.

7. $x^2 + 4y^2 = 16$

Chapter 1: Graphs

x-intercepts:

$$x^2 + 4(0)^2 = 16$$

$$x^2 = 16$$

$$x = \pm 4$$

y-intercepts:

$$(0)^2 + 4y^2 = 16$$

$$4y^2 = 16$$

$$y^2 = 4$$

$$y = \pm 2$$

The intercepts are $(-4, 0)$, $(4, 0)$, $(0, -2)$, and $(0, 2)$.

Test x-axis symmetry: Let $y = -y$

$$x^2 + 4(-y)^2 = 16$$

$$x^2 + 4y^2 = 16 \quad \text{same}$$

Test y-axis symmetry: Let $x = -x$

$$(-x)^2 + 4y^2 = 16$$

$$x^2 + 4y^2 = 16 \quad \text{same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$(-x)^2 + 4(-y)^2 = 16$$

$$x^2 + 4y^2 = 16 \quad \text{same}$$

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

8. $y = x^4 + 2x^2 + 1$

x-intercepts:

$$0 = x^4 + 2x^2 + 1$$

$$0 = (x^2 + 1)(x^2 + 1)$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

no real solutions

The only intercept is $(0, 1)$.

Test x-axis symmetry: Let $y = -y$

$$-y = x^4 + 2x^2 + 1$$

$$y = -x^4 - 2x^2 - 1 \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$y = (-x)^4 + 2(-x)^2 + 1$$

$$y = x^4 + 2x^2 + 1 \quad \text{same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$-y = (-x)^4 + 2(-x)^2 + 1$$

$$-y = x^4 + 2x^2 + 1$$

$$y = -x^4 - 2x^2 - 1 \quad \text{different}$$

Therefore, the graph will have y-axis symmetry.

9. $y = x^3 - x$

x-intercepts:

$$0 = x^3 - x$$

$$0 = x(x^2 - 1)$$

$$0 = x(x+1)(x-1)$$

$$x = 0, x = -1, x = 1$$

y-intercepts:

$$y = (0)^3 - 0$$

$$= 0$$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(1, 0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = x^3 - x$$

$$y = -x^3 + x \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$y = (-x)^3 - (-x)$$

$$y = -x^3 + x \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$-y = (-x)^3 - (-x)$$

$$-y = -x^3 + x$$

$$y = x^3 - x \quad \text{same}$$

Therefore, the graph will have origin symmetry.

10. $x^2 + x + y^2 + 2y = 0$

x-intercepts: $x^2 + x + (0)^2 + 2(0) = 0$

$$x^2 + x = 0$$

$$x(x+1) = 0$$

$$x = 0, x = -1$$

y-intercepts: $(0)^2 + 0 + y^2 + 2y = 0$

$$y^2 + 2y = 0$$

$$y(y+2) = 0$$

$$y = 0, y = -2$$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(0, -2)$.

Test x-axis symmetry: Let $y = -y$

$$x^2 + x + (-y)^2 + 2(-y) = 0$$

$$x^2 + x + y^2 - 2y = 0 \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$(-x)^2 + (-x) + y^2 + 2y = 0$$

$$x^2 - x + y^2 + 2y = 0 \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$(-x)^2 + (-x) + (-y)^2 + 2(-y) = 0$$

$$x^2 - x + y^2 - 2y = 0 \quad \text{different}$$

The graph has none of the indicated symmetries.

11. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-(-2))^2 + (y-3)^2 = 4^2$$

$$(x+2)^2 + (y-3)^2 = 16$$

12. $(x-h)^2 + (y-k)^2 = r^2$

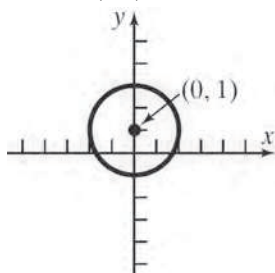
$$(x-(-1))^2 + (y-(-2))^2 = 1^2$$

$$(x+1)^2 + (y+2)^2 = 1$$

13. $x^2 + (y-1)^2 = 4$

$$x^2 + (y-1)^2 = 2^2$$

Center: (0,1); Radius = 2



x-intercepts: $x^2 + (0-1)^2 = 4$

$$x^2 + 1 = 4$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

y-intercepts: $0^2 + (y-1)^2 = 4$

$$(y-1)^2 = 4$$

$$y-1 = \pm 2$$

$$y = 1 \pm 2$$

$$y = 3 \text{ or } y = -1$$

The intercepts are $(-\sqrt{3}, 0)$, $(\sqrt{3}, 0)$, $(0, -1)$,

and $(0, 3)$.

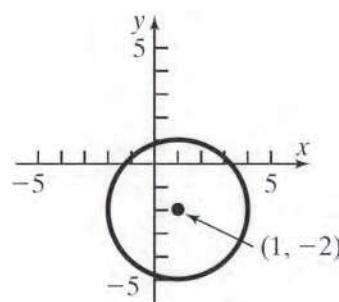
14. $x^2 + y^2 - 2x + 4y - 4 = 0$

$$x^2 - 2x + y^2 + 4y = 4$$

$$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 4 + 1 + 4$$

$$(x-1)^2 + (y+2)^2 = 3^2$$

Center: (1, -2) Radius = 3



x-intercepts: $(x-1)^2 + (0+2)^2 = 3^2$

$$(x-1)^2 + 4 = 9$$

$$(x-1)^2 = 5$$

$$x-1 = \pm\sqrt{5}$$

$$x = 1 \pm \sqrt{5}$$

y-intercepts: $(0-1)^2 + (y+2)^2 = 3^2$

$$1 + (y+2)^2 = 9$$

$$(y+2)^2 = 8$$

$$y+2 = \pm\sqrt{8}$$

$$y+2 = \pm 2\sqrt{2}$$

$$y = -2 \pm 2\sqrt{2}$$

The intercepts are $(1-\sqrt{5}, 0)$, $(1+\sqrt{5}, 0)$,

$(0, -2-2\sqrt{2})$, and $(0, -2+2\sqrt{2})$.

15. $3x^2 + 3y^2 - 6x + 12y = 0$

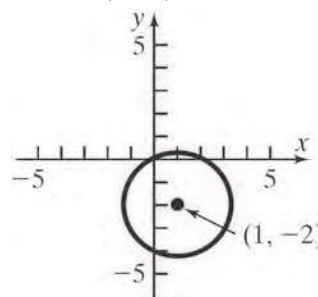
$$x^2 + y^2 - 2x + 4y = 0$$

$$x^2 - 2x + y^2 + 4y = 0$$

$$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 1 + 4$$

$$(x-1)^2 + (y+2)^2 = (\sqrt{5})^2$$

Center: (1, -2) Radius = $\sqrt{5}$



Chapter 1: Graphs

$$x\text{-intercepts: } (x-1)^2 + (0+2)^2 = (\sqrt{5})^2$$

$$(x-1)^2 + 4 = 5$$

$$(x-1)^2 = 1$$

$$x-1 = \pm 1$$

$$x = 1 \pm 1$$

$$x = 2 \quad \text{or} \quad x = 0$$

$$y\text{-intercepts: } (0-1)^2 + (y+2)^2 = (\sqrt{5})^2$$

$$1 + (y+2)^2 = 5$$

$$(y+2)^2 = 4$$

$$y+2 = \pm 2$$

$$y = -2 \pm 2$$

$$y = 0 \quad \text{or} \quad y = -4$$

The intercepts are $(0, 0)$, $(2, 0)$, and $(0, -4)$.

16. Slope = -2 ; containing $(3, -1)$

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -2(x - 3)$$

$$y + 1 = -2x + 6$$

$$y = -2x + 5 \quad \text{or} \quad 2x + y = 5$$

17. vertical; containing $(-3, 4)$

Vertical lines have equations of the form $x = a$, where a is the x -intercept. Now, a vertical line containing the point $(-3, 4)$ must have an x -intercept of -3 , so the equation of the line is $x = -3$. The equation does not have a slope-intercept form.

18. y -intercept = -2 ; containing $(5, -3)$

Points are $(5, -3)$ and $(0, -2)$

$$m = \frac{-2 - (-3)}{0 - 5} = \frac{1}{-5} = -\frac{1}{5}$$

$$y = mx + b$$

$$y = -\frac{1}{5}x - 2 \quad \text{or} \quad x + 5y = -10$$

19. Containing the points $(3, -4)$ and $(2, 1)$

$$m = \frac{1 - (-4)}{2 - 3} = \frac{5}{-1} = -5$$

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = -5(x - 3)$$

$$y + 4 = -5x + 15$$

$$y = -5x + 11 \quad \text{or} \quad 5x + y = 11$$

20. Parallel to $2x - 3y = -4$

$$2x - 3y = -4$$

$$-3y = -2x - 4$$

$$\frac{-3y}{-3} = \frac{-2x - 4}{-3}$$

$$y = \frac{2}{3}x + \frac{4}{3}$$

Slope = $\frac{2}{3}$; containing $(-5, 3)$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{2}{3}(x - (-5))$$

$$y - 3 = \frac{2}{3}(x + 5)$$

$$y - 3 = \frac{2}{3}x + \frac{10}{3}$$

$$y = \frac{2}{3}x + \frac{19}{3} \quad \text{or} \quad 2x - 3y = -19$$

21. Perpendicular to $x + y = 2$

$$x + y = 2$$

$$y = -x + 2$$

The slope of this line is -1 , so the slope of a line perpendicular to it is 1 .

Slope = 1 ; containing $(4, -3)$

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = 1(x - 4)$$

$$y + 3 = x - 4$$

$$y = x - 7 \quad \text{or} \quad x - y = 7$$

22. $4x - 5y = -20$

$$-5y = -4x - 20$$

$$y = \frac{4}{5}x + 4$$

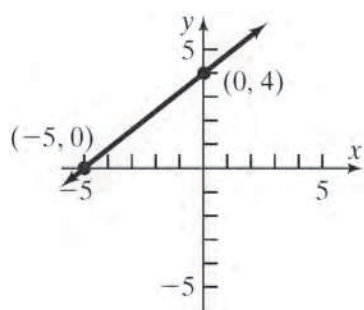
slope = $\frac{4}{5}$; y -intercept = 4

x -intercept: Let $y = 0$.

$$4x - 5(0) = -20$$

$$4x = -20$$

$$x = -5$$

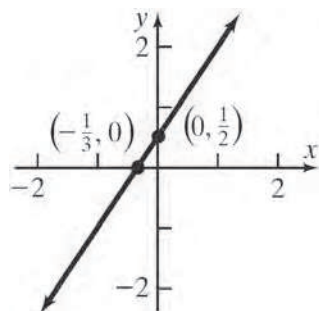


$$\begin{aligned} 23. \quad \frac{1}{2}x - \frac{1}{3}y &= -\frac{1}{6} \\ -\frac{1}{3}y &= -\frac{1}{2}x - \frac{1}{6} \\ y &= \frac{3}{2}x + \frac{1}{2} \end{aligned}$$

$$\text{slope} = \frac{3}{2}; \text{ y-intercept} = \frac{1}{2}$$

x-intercept: Let $y = 0$.

$$\begin{aligned} \frac{1}{2}x - \frac{1}{3}(0) &= -\frac{1}{6} \\ \frac{1}{2}x &= -\frac{1}{6} \\ x &= -\frac{1}{3} \end{aligned}$$



$$24. \quad 2x - 3y = 12$$

x-intercept:

$$2x - 3(0) = 12$$

$$2x = 12$$

$$x = 6$$

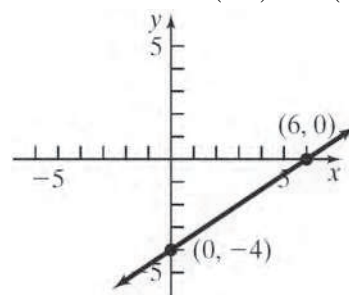
y-intercept:

$$2(0) - 3y = 12$$

$$-3y = 12$$

$$y = -4$$

The intercepts are $(6, 0)$ and $(0, -4)$.



$$25. \quad \frac{1}{2}x + \frac{1}{3}y = 2$$

x-intercept:

$$\frac{1}{2}x + \frac{1}{3}(0) = 2$$

$$\frac{1}{2}x = 2$$

$$x = 4$$

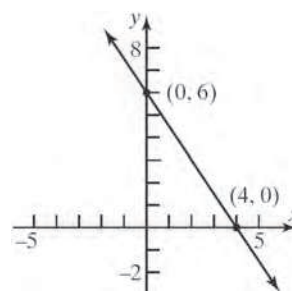
y-intercept:

$$\frac{1}{2}(0) + \frac{1}{3}y = 2$$

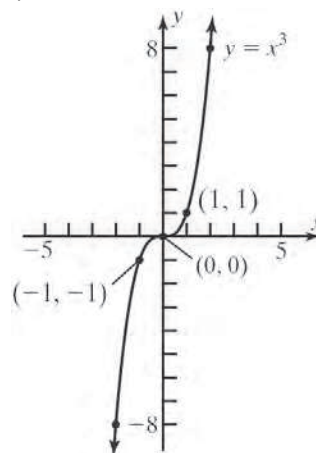
$$\frac{1}{3}y = 2$$

$$y = 6$$

The intercepts are $(4, 0)$ and $(0, 6)$.

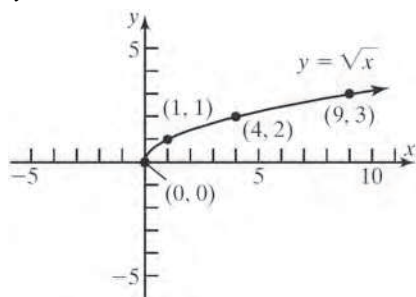


$$26. \quad y = x^3$$

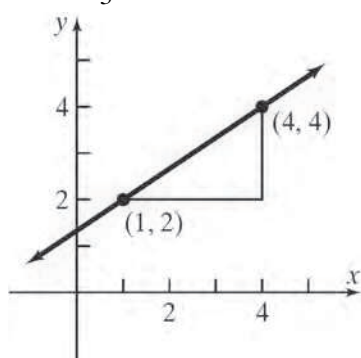


Chapter 1: Graphs

27. $y = \sqrt{x}$



28. slope = $\frac{2}{3}$, containing the point (1, 2)



29. Find the distance between each pair of points.

$$d_{A,B} = \sqrt{(1-3)^2 + (1-4)^2} = \sqrt{4+9} = \sqrt{13}$$

$$d_{B,C} = \sqrt{(-2-1)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$$

$$d_{A,C} = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{25+1} = \sqrt{26}$$

Since $AB = BC$, triangle ABC is isosceles.

30. Given the points $A = (-2, 0)$, $B = (-4, 4)$, and $C = (8, 5)$.

- a. Find the distance between each pair of points.

$$\begin{aligned} d(A, B) &= \sqrt{(-4 - (-2))^2 + (4 - 0)^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} = 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} d(B, C) &= \sqrt{(8 - (-4))^2 + (5 - 4)^2} \\ &= \sqrt{144 + 1} \\ &= \sqrt{145} \end{aligned}$$

$$\begin{aligned} d(A, C) &= \sqrt{(8 - (-2))^2 + (5 - 0)^2} \\ &= \sqrt{100 + 25} \\ &= \sqrt{125} = 5\sqrt{5} \end{aligned}$$

$$[d(A, B)]^2 + [d(A, C)]^2 = [d(B, C)]^2$$

$$(\sqrt{20})^2 + (\sqrt{125})^2 = (\sqrt{145})^2$$

$$20 + 125 = 145$$

$$145 = 145$$

The Pythagorean Theorem is satisfied, so this is a right triangle.

- b. Find the slopes:

$$m_{AB} = \frac{4-0}{-4-(-2)} = \frac{4}{-2} = -2$$

$$m_{BC} = \frac{5-4}{8-(-4)} = \frac{1}{12}$$

$$m_{AC} = \frac{5-0}{8-(-2)} = \frac{5}{10} = \frac{1}{2}$$

Since $m_{AB} \cdot m_{AC} = -2 \cdot \frac{1}{2} = -1$, the sides AB and AC are perpendicular and the triangle is a right triangle.

31. Endpoints of the diameter are $(-3, 2)$ and $(5, -6)$. The center is at the midpoint of the diameter:

$$\text{Center: } \left(\frac{-3+5}{2}, \frac{2+(-6)}{2} \right) = (1, -2)$$

$$\begin{aligned} \text{Radius: } r &= \sqrt{(1-(-3))^2 + (-2-2)^2} \\ &= \sqrt{16+16} \\ &= \sqrt{32} = 4\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{Equation: } (x-1)^2 + (y+2)^2 &= (4\sqrt{2})^2 \\ (x-1)^2 + (y+2)^2 &= 32 \end{aligned}$$

32. slope of $\overline{AB} = \frac{1-5}{6-2} = -1$

$$\text{slope of } \overline{AC} = \frac{-1-5}{8-2} = -1$$

Therefore, the points lie on a line.

Chapter 1 Test

$$\begin{aligned}
 1. \quad d(P_1, P_2) &= \sqrt{(5 - (-1))^2 + (-1 - 3)^2} \\
 &= \sqrt{6^2 + (-4)^2} \\
 &= \sqrt{36 + 16} \\
 &= \sqrt{52} = 2\sqrt{13}
 \end{aligned}$$

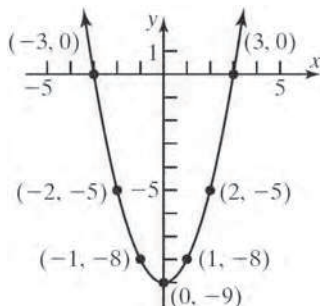
2. The coordinates of the midpoint are:

$$\begin{aligned}
 (x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{-1 + 5}{2}, \frac{3 + (-1)}{2} \right) \\
 &= \left(\frac{4}{2}, \frac{2}{2} \right) \\
 &= (2, 1)
 \end{aligned}$$

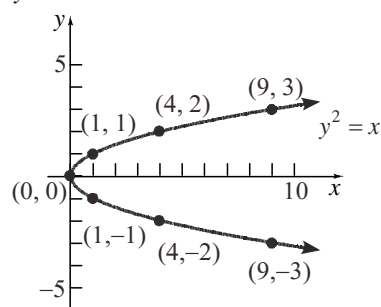
$$3. \quad a. \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{5 - (-1)} = \frac{-4}{6} = -\frac{2}{3}$$

b. If x increases by 3 units, y will decrease by 2 units.

$$4. \quad y = x^2 - 9$$



$$5. \quad y^2 = x$$



$$6. \quad x^2 + y = 9$$

x -intercepts: y -intercept:

$$x^2 + 0 = 9 \qquad (0)^2 + y = 9$$

$$x^2 = 9 \qquad y = 9$$

$$x = \pm 3$$

The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, 9)$.

Test x -axis symmetry: Let $y = -y$

$$x^2 + (-y) = 9$$

$$x^2 - y = 9 \quad \text{different}$$

Test y -axis symmetry: Let $x = -x$

$$(-x)^2 + y = 9$$

$$x^2 + y = 9 \quad \text{same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$(-x)^2 + (-y) = 9$$

$$x^2 - y = 9 \quad \text{different}$$

Therefore, the graph will have y -axis symmetry.

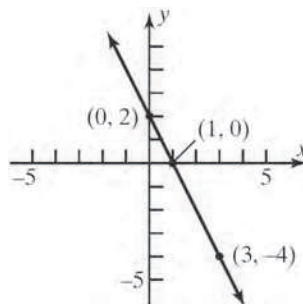
7. Slope = -2 ; containing $(3, -4)$

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = -2(x - 3)$$

$$y + 4 = -2x + 6$$

$$y = -2x + 2$$



$$8. \quad (x - h)^2 + (y - k)^2 = r^2$$

$$(x - 4)^2 + (y - (-3))^2 = 5^2$$

$$(x - 4)^2 + (y + 3)^2 = 25$$

$$\text{General form: } (x - 4)^2 + (y + 3)^2 = 25$$

$$x^2 - 8x + 16 + y^2 + 6y + 9 = 25$$

$$x^2 + y^2 - 8x + 6y = 0$$

$$9. \quad x^2 + y^2 + 4x - 2y - 4 = 0$$

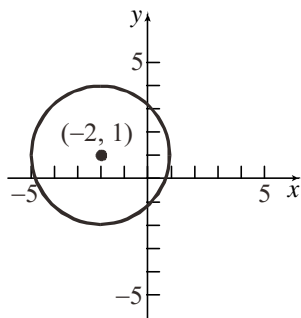
$$x^2 + 4x + y^2 - 2y = 4$$

$$(x^2 + 4x + 4) + (y^2 - 2y + 1) = 4 + 4 + 1$$

$$(x + 2)^2 + (y - 1)^2 = 3^2$$

Center: $(-2, 1)$; Radius = 3

Chapter 1: Graphs



10. $2x + 3y = 6$
 $3y = -2x + 6$
 $y = -\frac{2}{3}x + 2$

Parallel line

Any line parallel to $2x + 3y = 6$ has slope

$m = -\frac{2}{3}$. The line contains $(1, -1)$:

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -\frac{2}{3}(x - 1)$$
$$y + 1 = -\frac{2}{3}x + \frac{2}{3}$$
$$y = -\frac{2}{3}x - \frac{1}{3}$$

Perpendicular line

Any line perpendicular to $2x + 3y = 6$ has slope

$m = \frac{3}{2}$. The line contains $(0, 3)$:

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{3}{2}(x - 0)$$
$$y - 3 = \frac{3}{2}x$$
$$y = \frac{3}{2}x + 3$$

Chapter 1 Project

Internet-based Project

Chapter 2

Functions and Their Graphs

Section 2.1

1. $(-1, 3)$

$$\begin{aligned} 2. \quad 3(-2)^2 - 5(-2) + \frac{1}{(-2)} &= 3(4) - 5(-2) - \frac{1}{2} \\ &= 12 + 10 - \frac{1}{2} \\ &= \frac{43}{2} \text{ or } 21\frac{1}{2} \text{ or } 21.5 \end{aligned}$$

3. We must not allow the denominator to be 0.
 $x + 4 \neq 0 \Rightarrow x \neq -4$; Domain: $\{x \mid x \neq -4\}$.

4. $3 - 2x > 5$

$$-2x > 2$$

$$x < -1$$

Solution set: $\{x \mid x < -1\}$ or $(-\infty, -1)$



5. $\sqrt{5} + 2$

6. radicals

7. independent; dependent

8. a

9. c

10. False; $g \neq 0$

11. False; every function is a relation, but not every relation is a function. For example, the relation $x^2 + y^2 = 1$ is not a function.

12. verbally, numerically, graphically, algebraically

13. False; if the domain is not specified, we assume it is the largest set of real numbers for which the value of f is a real number.

14. False; if x is in the domain of a function f , we say that f is defined at x , or $f(x)$ exists.

15. difference quotient

16. explicitly

17. a. Domain: $\{0, 22, 40, 70, 100\}$ in $^{\circ}\text{C}$
 Range: $\{1.031, 1.121, 1.229, 1.305, 1.411\}$ in kg/m^3

b.

Temperature, $^{\circ}\text{C}$	Density, kg/m^3
0	1.411
22	1.305
40	1.229
70	1.121
100	1.031

c. $\{(0, 1.411), (22, 1.305), (40, 1.229), (70, 1.121), (100, 1.031)\}$

18. a. Domain: $\{1.80, 1.78, 1.77\}$
 Range: $\{87.1, 86.9, 92.0, 84.1, 86.4\}$

b.

Height, meters	Weight, kg
1.77	83.0
	84.1
1.78	86.9
	86.4
1.80	87.1

c. $\{(1.80, 87.1), (1.78, 86.9), (1.77, 83.0), (1.77, 84.1), (1.80, 86.4)\}$

19. Domain: $\{\text{Elvis, Colleen, Kaleigh, Marissa}\}$
 Range: $\{\text{Jan. 8, Mar. 15, Sept. 17}\}$
 Function

20. Domain: $\{\text{Bob, John, Chuck}\}$
 Range: $\{\text{Beth, Diane, Linda, Marcia}\}$
 Not a function

21. Domain: $\{20, 30, 40\}$
 Range: $\{200, 300, 350, 425\}$
 Not a function

22. Domain: $\{\text{Less than 9}^{\text{th}} \text{ grade, 9}^{\text{th}}\text{-12}^{\text{th}} \text{ grade, High School Graduate, Some College, College Graduate}\}$
 Range: $\{\$18,120, \$23,251, \$36,055, \$45,810, \$67,165\}$
 Function

23. Domain: $\{-3, 2, 4\}$
 Range: $\{6, 9, 10\}$
 Not a function

Chapter 2: Functions and Their Graphs

24. Domain: $\{-2, -1, 3, 4\}$
Range: $\{3, 5, 7, 12\}$
Function

25. Domain: $\{1, 2, 3, 4\}$
Range: $\{3\}$
Function

26. Domain: $\{0, 1, 2, 3\}$
Range: $\{-2, 3, 7\}$
Function

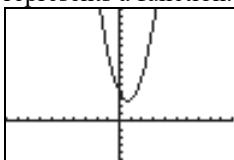
27. Domain: $\{-4, 0, 3\}$
Range: $\{1, 3, 5, 6\}$
Not a function

28. Domain: $\{-4, -3, -2, -1\}$
Range: $\{0, 1, 2, 3, 4\}$
Not a function

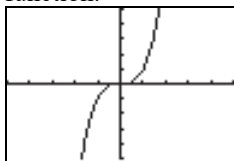
29. Domain: $\{-1, 0, 2, 4\}$
Range: $\{-1, 3, 8\}$
Function

30. Domain: $\{-2, -1, 0, 1\}$
Range: $\{3, 4, 16\}$
Function

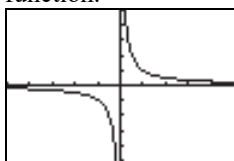
31. Graph $y = 2x^2 - 3x + 4$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



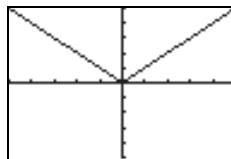
32. Graph $y = x^3$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



33. Graph $y = \frac{1}{x}$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



34. Graph $y = |x|$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



35. $x^2 = 8 - y^2$

Solve for y : $y = \pm\sqrt{8 - x^2}$

For $x = 0$, $y = \pm 2\sqrt{2}$. Thus, $(0, 2\sqrt{2})$ and $(0, -2\sqrt{2})$ are on the graph. This is not a function, since a distinct x -value corresponds to two different y -values.

36. $y = \pm\sqrt{1 - 2x}$

For $x = 0$, $y = \pm 1$. Thus, $(0, 1)$ and $(0, -1)$ are on the graph. This is not a function, since a distinct x -value corresponds to two different y -values.

37. $x = y^2$

Solve for y : $y = \pm\sqrt{x}$

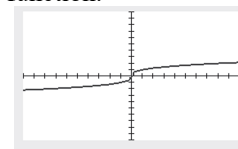
For $x = 1$, $y = \pm 1$. Thus, $(1, 1)$ and $(1, -1)$ are on the graph. This is not a function, since a distinct x -value corresponds to two different y -values.

38. $x + y^2 = 1$

Solve for y : $y = \pm\sqrt{1 - x}$

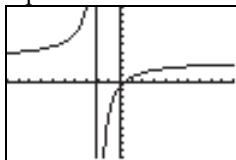
For $x = 0$, $y = \pm 1$. Thus, $(0, 1)$ and $(0, -1)$ are on the graph. This is not a function, since a distinct x -value corresponds to two different y -values.

39. Graph $y = \sqrt[3]{x}$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



40. Graph $y = \frac{3x-1}{x+2}$. The graph passes the

Vertical-Line Test. Thus, the equation represents a function.



41. $|y| = 2x + 3$

Solve for y : $y = 2x + 3$ or $y = -(2x + 3)$

For $x = 1$, $y = 5$ or $y = -5$. Thus, $(1, 5)$ and $(1, -5)$ are on the graph. This is not a function, since a distinct x -value corresponds to two different y -values.

42. $x^2 - 4y^2 = 1$

Solve for y : $x^2 - 4y^2 = 1$

$$4y^2 = x^2 - 1$$

$$y^2 = \frac{x^2 - 1}{4}$$

$$y = \frac{\pm\sqrt{x^2 - 1}}{2}$$

For $x = \sqrt{2}$, $y = \pm\frac{1}{2}$. Thus, $\left(\sqrt{2}, \frac{1}{2}\right)$ and

$\left(\sqrt{2}, -\frac{1}{2}\right)$ are on the graph. This is not a

function, since a distinct x -value corresponds to two different y -values.

43. $f(x) = 3x^2 + 2x - 4$

- a. $f(0) = 3(0)^2 + 2(0) - 4 = -4$
- b. $f(1) = 3(1)^2 + 2(1) - 4 = 3 + 2 - 4 = 1$
- c. $f(-1) = 3(-1)^2 + 2(-1) - 4 = 3 - 2 - 4 = -3$
- d. $f(-x) = 3(-x)^2 + 2(-x) - 4 = 3x^2 - 2x - 4$
- e. $-f(x) = -(3x^2 + 2x - 4) = -3x^2 - 2x + 4$

$$\begin{aligned} \text{f. } f(x+1) &= 3(x+1)^2 + 2(x+1) - 4 \\ &= 3(x^2 + 2x + 1) + 2x + 2 - 4 \\ &= 3x^2 + 6x + 3 + 2x + 2 - 4 \\ &= 3x^2 + 8x + 1 \end{aligned}$$

$$\text{g. } f(2x) = 3(2x)^2 + 2(2x) - 4 = 12x^2 + 4x - 4$$

$$\begin{aligned} \text{h. } f(x+h) &= 3(x+h)^2 + 2(x+h) - 4 \\ &= 3(x^2 + 2xh + h^2) + 2x + 2h - 4 \\ &= 3x^2 + 6xh + 3h^2 + 2x + 2h - 4 \end{aligned}$$

$$44. f(x) = -2x^2 + x - 1$$

$$\text{a. } f(0) = -2(0)^2 + 0 - 1 = -1$$

$$\text{b. } f(1) = -2(1)^2 + 1 - 1 = -2$$

$$\text{c. } f(-1) = -2(-1)^2 + (-1) - 1 = -4$$

$$\text{d. } f(-x) = -2(-x)^2 + (-x) - 1 = -2x^2 - x - 1$$

$$\text{e. } -f(x) = -(-2x^2 + x - 1) = 2x^2 - x + 1$$

$$\begin{aligned} \text{f. } f(x+1) &= -2(x+1)^2 + (x+1) - 1 \\ &= -2(x^2 + 2x + 1) + x + 1 - 1 \\ &= -2x^2 - 4x - 2 + x \\ &= -2x^2 - 3x - 2 \end{aligned}$$

$$\text{g. } f(2x) = -2(2x)^2 + (2x) - 1 = -8x^2 + 2x - 1$$

$$\begin{aligned} \text{h. } f(x+h) &= -2(x+h)^2 + (x+h) - 1 \\ &= -2(x^2 + 2xh + h^2) + x + h - 1 \\ &= -2x^2 - 4xh - 2h^2 + x + h - 1 \end{aligned}$$

$$45. f(x) = \frac{x}{x^2 + 1}$$

$$\text{a. } f(0) = \frac{0}{0^2 + 1} = \frac{0}{1} = 0$$

$$\text{b. } f(1) = \frac{1}{1^2 + 1} = \frac{1}{2}$$

$$\text{c. } f(-1) = \frac{-1}{(-1)^2 + 1} = \frac{-1}{1 + 1} = -\frac{1}{2}$$

$$\text{d. } f(-x) = \frac{-x}{(-x)^2 + 1} = \frac{-x}{x^2 + 1}$$

$$\text{e. } -f(x) = -\left(\frac{x}{x^2 + 1}\right) = \frac{-x}{x^2 + 1}$$

Chapter 2: Functions and Their Graphs

$$\begin{aligned} \text{f. } f(x+1) &= \frac{x+1}{(x+1)^2+1} \\ &= \frac{x+1}{x^2+2x+1+1} \\ &= \frac{x+1}{x^2+2x+2} \end{aligned}$$

$$\text{g. } f(2x) = \frac{2x}{(2x)^2+1} = \frac{2x}{4x^2+1}$$

$$\text{h. } f(x+h) = \frac{x+h}{(x+h)^2+1} = \frac{x+h}{x^2+2xh+h^2+1}$$

$$46. f(x) = \frac{x^2-1}{x+4}$$

$$\text{a. } f(0) = \frac{0^2-1}{0+4} = \frac{-1}{4} = -\frac{1}{4}$$

$$\text{b. } f(1) = \frac{1^2-1}{1+4} = \frac{0}{5} = 0$$

$$\text{c. } f(-1) = \frac{(-1)^2-1}{-1+4} = \frac{0}{3} = 0$$

$$\text{d. } f(-x) = \frac{(-x)^2-1}{-x+4} = \frac{x^2-1}{-x+4}$$

$$\text{e. } -f(x) = -\left(\frac{x^2-1}{x+4}\right) = \frac{-x^2+1}{x+4}$$

$$\begin{aligned} \text{f. } f(x+1) &= \frac{(x+1)^2-1}{(x+1)+4} \\ &= \frac{x^2+2x+1-1}{x+5} = \frac{x^2+2x}{x+5} \end{aligned}$$

$$\text{g. } f(2x) = \frac{(2x)^2-1}{2x+4} = \frac{4x^2-1}{2x+4}$$

$$\text{h. } f(x+h) = \frac{(x+h)^2-1}{(x+h)+4} = \frac{x^2+2xh+h^2-1}{x+h+4}$$

$$47. f(x) = |x| + 4$$

$$\text{a. } f(0) = |0| + 4 = 0 + 4 = 4$$

$$\text{b. } f(1) = |1| + 4 = 1 + 4 = 5$$

$$\text{c. } f(-1) = |-1| + 4 = 1 + 4 = 5$$

$$\text{d. } f(-x) = |-x| + 4 = |x| + 4$$

$$\text{e. } -f(x) = -(|x| + 4) = -|x| - 4$$

$$\text{f. } f(x+1) = |x+1| + 4$$

$$\text{g. } f(2x) = |2x| + 4 = 2|x| + 4$$

$$\text{h. } f(x+h) = |x+h| + 4$$

$$48. f(x) = \sqrt{x^2+x}$$

$$\text{a. } f(0) = \sqrt{0^2+0} = \sqrt{0} = 0$$

$$\text{b. } f(1) = \sqrt{1^2+1} = \sqrt{2}$$

$$\text{c. } f(-1) = \sqrt{(-1)^2+(-1)} = \sqrt{1-1} = \sqrt{0} = 0$$

$$\text{d. } f(-x) = \sqrt{(-x)^2+(-x)} = \sqrt{x^2-x}$$

$$\text{e. } -f(x) = -(\sqrt{x^2+x}) = -\sqrt{x^2+x}$$

$$\begin{aligned} \text{f. } f(x+1) &= \sqrt{(x+1)^2+(x+1)} \\ &= \sqrt{x^2+2x+1+x+1} \\ &= \sqrt{x^2+3x+2} \end{aligned}$$

$$\text{g. } f(2x) = \sqrt{(2x)^2+2x} = \sqrt{4x^2+2x}$$

$$\begin{aligned} \text{h. } f(x+h) &= \sqrt{(x+h)^2+(x+h)} \\ &= \sqrt{x^2+2xh+h^2+x+h} \end{aligned}$$

$$49. f(x) = \frac{2x+1}{3x-5}$$

$$\text{a. } f(0) = \frac{2(0)+1}{3(0)-5} = \frac{0+1}{0-5} = -\frac{1}{5}$$

$$\text{b. } f(1) = \frac{2(1)+1}{3(1)-5} = \frac{2+1}{3-5} = \frac{3}{-2} = -\frac{3}{2}$$

$$\text{c. } f(-1) = \frac{2(-1)+1}{3(-1)-5} = \frac{-2+1}{-3-5} = \frac{-1}{-8} = \frac{1}{8}$$

$$\text{d. } f(-x) = \frac{2(-x)+1}{3(-x)-5} = \frac{-2x+1}{-3x-5} = \frac{2x-1}{3x+5}$$

$$\text{e. } -f(x) = -\left(\frac{2x+1}{3x-5}\right) = \frac{-2x-1}{3x-5}$$

$$\text{f. } f(x+1) = \frac{2(x+1)+1}{3(x+1)-5} = \frac{2x+2+1}{3x+3-5} = \frac{2x+3}{3x-2}$$

$$\text{g. } f(2x) = \frac{2(2x)+1}{3(2x)-5} = \frac{4x+1}{6x-5}$$

$$\text{h. } f(x+h) = \frac{2(x+h)+1}{3(x+h)-5} = \frac{2x+2h+1}{3x+3h-5}$$

$$50. f(x) = 1 - \frac{1}{(x+2)^2}$$

$$\text{a. } f(0) = 1 - \frac{1}{(0+2)^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\text{b. } f(1) = 1 - \frac{1}{(1+2)^2} = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\text{c. } f(-1) = 1 - \frac{1}{(-1+2)^2} = 1 - \frac{1}{1} = 0$$

$$\text{d. } f(-x) = 1 - \frac{1}{(-x+2)^2}$$

$$\text{e. } -f(x) = -\left(1 - \frac{1}{(x+2)^2}\right) = \frac{1}{(x+2)^2} - 1$$

$$\text{f. } f(x+1) = 1 - \frac{1}{(x+1+2)^2} = 1 - \frac{1}{(x+3)^2}$$

$$\text{g. } f(2x) = 1 - \frac{1}{(2x+2)^2} = 1 - \frac{1}{4(x+1)^2}$$

$$\text{h. } f(x+h) = 1 - \frac{1}{(x+h+2)^2}$$

$$51. f(x) = -5x + 4$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$52. f(x) = x^2 + 2$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$53. f(x) = \frac{x+1}{2x^2+8}$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$54. f(x) = \frac{x^2}{x^2+1}$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$55. g(x) = \frac{x}{x^2-16}$$

$$x^2 - 16 \neq 0$$

$$x^2 \neq 16 \Rightarrow x \neq \pm 4$$

Domain: $\{x \mid x \neq -4, x \neq 4\}$

$$56. h(x) = \frac{2x}{x^2-4}$$

$$x^2 - 4 \neq 0$$

$$x^2 \neq 4 \Rightarrow x \neq \pm 2$$

Domain: $\{x \mid x \neq -2, x \neq 2\}$

$$57. F(x) = \frac{x-2}{x^3+x}$$

$$x^3 + x \neq 0$$

$$x(x^2+1) \neq 0$$

$$x \neq 0, \quad x^2 \neq -1$$

Domain: $\{x \mid x \neq 0\}$

$$58. G(x) = \frac{x+4}{x^3-4x}$$

$$x^3 - 4x \neq 0$$

$$x(x^2-4) \neq 0$$

$$x \neq 0, \quad x^2 \neq 4$$

$$x \neq 0, \quad x \neq \pm 2$$

Domain: $\{x \mid x \neq -2, x \neq 0, x \neq 2\}$

$$59. h(x) = \sqrt{3x-12}$$

$$3x - 12 \geq 0$$

$$3x \geq 12$$

$$x \geq 4$$

Domain: $\{x \mid x \geq 4\}$

Chapter 2: Functions and Their Graphs

60. $G(x) = \sqrt{1-x}$

$$1-x \geq 0$$

$$-x \geq -1$$

$$x \leq 1$$

$$\text{Domain: } \{x \mid x \leq 1\}$$

61. $p(x) = \frac{x}{|2x+3|-1}$

$$|2x+3|-1 = 0$$

$$|2x+3| = 1$$

$$2x+3 = -1 \text{ or } 2x+3 = 1$$

$$2x = -4 \quad 2x = -2$$

$$x = -2 \quad x = -1$$

$$\text{Domain: } \{x \mid x \neq -2, x \neq -1\}$$

62. $p(x) = \frac{x-1}{|3x-1|-4}$

$$|3x-1|-4 = 0$$

$$|3x-1| = 4$$

$$3x-1 = -4 \text{ or } 3x-1 = 4$$

$$3x = -3 \quad 3x = 5$$

$$x = -1 \quad x = \frac{5}{3}$$

$$\text{Domain: } \left\{x \mid x \neq -1, x \neq \frac{5}{3}\right\}$$

63. $f(x) = \frac{x}{\sqrt{x-4}}$

$$x-4 > 0$$

$$x > 4$$

$$\text{Domain: } \{x \mid x > 4\}$$

64. $q(x) = \frac{-x}{\sqrt{-x-2}}$

$$-x-2 > 0$$

$$-x > 2$$

$$x < -2$$

$$\text{Domain: } \{x \mid x < -2\}$$

65. $P(t) = \frac{\sqrt{t-4}}{3t-21}$

$$t-4 \geq 0$$

$$t \geq 4$$

$$\text{Also } 3t-21 \neq 0$$

$$3t-21 \neq 0$$

$$3t \neq 21$$

$$t \neq 7$$

$$\text{Domain: } \{t \mid t \geq 4, t \neq 7\}$$

66. $h(z) = \frac{\sqrt{z+3}}{z-2}$

$$z+3 \geq 0$$

$$z \geq -3$$

$$\text{Also } z-2 \neq 0$$

$$z \neq 2$$

$$\text{Domain: } \{z \mid z \geq -3, z \neq 2\}$$

67. $f(x) = \sqrt[3]{5x-4}$

$$\text{Domain: } \{x \mid x \text{ is any real number}\}.$$

68. $g(t) = -t^2 + \sqrt[3]{t^2+7t}$

$$\text{Domain: } \{t \mid t \text{ is any real number}\}.$$

69. $M(t) = \sqrt[5]{\frac{t+1}{t^2-5t-14}}$

$$t^2-5t-14 = 0$$

$$(t+2)(t-7) = 0$$

$$t+2 = 0 \text{ or } t-7 = 0$$

$$t = -2 \quad t = 7$$

$$\text{Domain: } \{t \mid t \neq -2, t \neq 7\}$$

70. $N(p) = \sqrt[5]{\frac{p}{2p^2-98}}$

$$2p^2-98 = 0$$

$$2(p^2-49) = 0$$

$$2(p+7)(p-7) = 0$$

$$p+7 = 0 \text{ or } p-7 = 0$$

$$p = -7 \quad p = 7$$

$$\text{Domain: } \{p \mid p \neq -7, p \neq 7\}$$

71. $f(x) = 3x+4 \quad g(x) = 2x-3$

a. $(f+g)(x) = 3x+4+2x-3 = 5x+1$

$$\text{Domain: } \{x \mid x \text{ is any real number}\}.$$

Section 2.1: Functions

- b.** $(f - g)(x) = (3x + 4) - (2x - 3)$
 $= 3x + 4 - 2x + 3$
 $= x + 7$
 Domain: $\{x \mid x \text{ is any real number}\}$.
- c.** $(f \cdot g)(x) = (3x + 4)(2x - 3)$
 $= 6x^2 - 9x + 8x - 12$
 $= 6x^2 - x - 12$
 Domain: $\{x \mid x \text{ is any real number}\}$.
- d.** $\left(\frac{f}{g}\right)(x) = \frac{3x + 4}{2x - 3}$
 $2x - 3 \neq 0 \Rightarrow 2x \neq 3 \Rightarrow x \neq \frac{3}{2}$
 Domain: $\left\{x \mid x \neq \frac{3}{2}\right\}$.
- e.** $(f + g)(3) = 5(3) + 1 = 15 + 1 = 16$
- f.** $(f - g)(4) = 4 + 7 = 11$
- g.** $(f \cdot g)(2) = 6(2)^2 - 2 - 12 = 24 - 2 - 12 = 10$
- h.** $\left(\frac{f}{g}\right)(1) = \frac{3(1) + 4}{2(1) - 3} = \frac{3 + 4}{2 - 3} = \frac{7}{-1} = -7$
- 72.** $f(x) = 2x + 1$ $g(x) = 3x - 2$
- a.** $(f + g)(x) = 2x + 1 + 3x - 2 = 5x - 1$
 Domain: $\{x \mid x \text{ is any real number}\}$.
- b.** $(f - g)(x) = (2x + 1) - (3x - 2)$
 $= 2x + 1 - 3x + 2$
 $= -x + 3$
 Domain: $\{x \mid x \text{ is any real number}\}$.
- c.** $(f \cdot g)(x) = (2x + 1)(3x - 2)$
 $= 6x^2 - 4x + 3x - 2$
 $= 6x^2 - x - 2$
 Domain: $\{x \mid x \text{ is any real number}\}$.
- d.** $\left(\frac{f}{g}\right)(x) = \frac{2x + 1}{3x - 2}$
 $3x - 2 \neq 0$
 $3x \neq 2 \Rightarrow x \neq \frac{2}{3}$
 Domain: $\left\{x \mid x \neq \frac{2}{3}\right\}$.
- e.** $(f + g)(3) = 5(3) - 1 = 15 - 1 = 14$
- f.** $(f - g)(4) = -4 + 3 = -1$
- g.** $(f \cdot g)(2) = 6(2)^2 - 2 - 2$
 $= 6(4) - 2 - 2$
 $= 24 - 2 - 2 = 20$
- h.** $\left(\frac{f}{g}\right)(1) = \frac{2(1) + 1}{3(1) - 2} = \frac{2 + 1}{3 - 2} = \frac{3}{1} = 3$
- 73.** $f(x) = x - 1$ $g(x) = 2x^2$
- a.** $(f + g)(x) = x - 1 + 2x^2 = 2x^2 + x - 1$
 Domain: $\{x \mid x \text{ is any real number}\}$.
- b.** $(f - g)(x) = (x - 1) - (2x^2)$
 $= x - 1 - 2x^2$
 $= -2x^2 + x - 1$
 Domain: $\{x \mid x \text{ is any real number}\}$.
- c.** $(f \cdot g)(x) = (x - 1)(2x^2) = 2x^3 - 2x^2$
 Domain: $\{x \mid x \text{ is any real number}\}$.
- d.** $\left(\frac{f}{g}\right)(x) = \frac{x - 1}{2x^2}$
 Domain: $\{x \mid x \neq 0\}$.
- e.** $(f + g)(3) = 2(3)^2 + 3 - 1$
 $= 2(9) + 3 - 1$
 $= 18 + 3 - 1 = 20$
- f.** $(f - g)(4) = -2(4)^2 + 4 - 1$
 $= -2(16) + 4 - 1$
 $= -32 + 4 - 1 = -29$
- g.** $(f \cdot g)(2) = 2(2)^3 - 2(2)^2$
 $= 2(8) - 2(4)$
 $= 16 - 8 = 8$
- h.** $\left(\frac{f}{g}\right)(1) = \frac{1 - 1}{2(1)^2} = \frac{0}{2(1)} = \frac{0}{2} = 0$
- 74.** $f(x) = 2x^2 + 3$ $g(x) = 4x^3 + 1$

Chapter 2: Functions and Their Graphs

$$\begin{aligned}\text{a. } (f+g)(x) &= 2x^2 + 3 + 4x^3 + 1 \\ &= 4x^3 + 2x^2 + 4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}\text{b. } (f-g)(x) &= (2x^2 + 3) - (4x^3 + 1) \\ &= 2x^2 + 3 - 4x^3 - 1 \\ &= -4x^3 + 2x^2 + 2\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}\text{c. } (f \cdot g)(x) &= (2x^2 + 3)(4x^3 + 1) \\ &= 8x^5 + 12x^3 + 2x^2 + 3\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\text{d. } \left(\frac{f}{g}\right)(x) = \frac{2x^2 + 3}{4x^3 + 1}$$

$$4x^3 + 1 \neq 0$$

$$4x^3 \neq -1$$

$$x^3 \neq -\frac{1}{4} \Rightarrow x \neq \sqrt[3]{-\frac{1}{4}} = -\frac{\sqrt[3]{2}}{2}$$

Domain: $\left\{x \mid x \neq -\frac{\sqrt[3]{2}}{2}\right\}$.

$$\begin{aligned}\text{e. } (f+g)(3) &= 4(3)^3 + 2(3)^2 + 4 \\ &= 4(27) + 2(9) + 4 \\ &= 108 + 18 + 4 = 130\end{aligned}$$

$$\begin{aligned}\text{f. } (f-g)(4) &= -4(4)^3 + 2(4)^2 + 2 \\ &= -4(64) + 2(16) + 2 \\ &= -256 + 32 + 2 = -222\end{aligned}$$

$$\begin{aligned}\text{g. } (f \cdot g)(2) &= 8(2)^5 + 12(2)^3 + 2(2)^2 + 3 \\ &= 8(32) + 12(8) + 2(4) + 3 \\ &= 256 + 96 + 8 + 3 = 363\end{aligned}$$

$$\text{h. } \left(\frac{f}{g}\right)(1) = \frac{2(1)^2 + 3}{4(1)^3 + 1} = \frac{2(1) + 3}{4(1) + 1} = \frac{2 + 3}{4 + 1} = \frac{5}{5} = 1$$

$$75. f(x) = \sqrt{x} \quad g(x) = 3x - 5$$

$$\begin{aligned}\text{a. } (f+g)(x) &= \sqrt{x} + 3x - 5 \\ \text{Domain: } &\{x \mid x \geq 0\}.\end{aligned}$$

$$\begin{aligned}\text{b. } (f-g)(x) &= \sqrt{x} - (3x - 5) = \sqrt{x} - 3x + 5 \\ \text{Domain: } &\{x \mid x \geq 0\}.\end{aligned}$$

$$\begin{aligned}\text{c. } (f \cdot g)(x) &= \sqrt{x}(3x - 5) = 3x\sqrt{x} - 5\sqrt{x} \\ \text{Domain: } &\{x \mid x \geq 0\}.\end{aligned}$$

$$\begin{aligned}\text{d. } \left(\frac{f}{g}\right)(x) &= \frac{\sqrt{x}}{3x - 5} \\ &x \geq 0 \text{ and } 3x - 5 \neq 0\end{aligned}$$

$$3x \neq 5 \Rightarrow x \neq \frac{5}{3}$$

Domain: $\left\{x \mid x \geq 0 \text{ and } x \neq \frac{5}{3}\right\}$.

$$\begin{aligned}\text{e. } (f+g)(3) &= \sqrt{3} + 3(3) - 5 \\ &= \sqrt{3} + 9 - 5 = \sqrt{3} + 4\end{aligned}$$

$$\begin{aligned}\text{f. } (f-g)(4) &= \sqrt{4} - 3(4) + 5 \\ &= 2 - 12 + 5 = -5\end{aligned}$$

$$\begin{aligned}\text{g. } (f \cdot g)(2) &= 3(2)\sqrt{2} - 5\sqrt{2} \\ &= 6\sqrt{2} - 5\sqrt{2} = \sqrt{2}\end{aligned}$$

$$\text{h. } \left(\frac{f}{g}\right)(1) = \frac{\sqrt{1}}{3(1) - 5} = \frac{1}{3 - 5} = \frac{1}{-2} = -\frac{1}{2}$$

$$76. f(x) = |x| \quad g(x) = x$$

$$\begin{aligned}\text{a. } (f+g)(x) &= |x| + x \\ \text{Domain: } &\{x \mid x \text{ is any real number}\}.\end{aligned}$$

$$\begin{aligned}\text{b. } (f-g)(x) &= |x| - x \\ \text{Domain: } &\{x \mid x \text{ is any real number}\}.\end{aligned}$$

$$\begin{aligned}\text{c. } (f \cdot g)(x) &= |x| \cdot x = x|x| \\ \text{Domain: } &\{x \mid x \text{ is any real number}\}.\end{aligned}$$

$$\begin{aligned}\text{d. } \left(\frac{f}{g}\right)(x) &= \frac{|x|}{x} \\ \text{Domain: } &\{x \mid x \neq 0\}.\end{aligned}$$

$$\text{e. } (f+g)(3) = |3| + 3 = 3 + 3 = 6$$

$$\text{f. } (f-g)(4) = |4| - 4 = 4 - 4 = 0$$

$$\text{g. } (f \cdot g)(2) = 2|2| = 2 \cdot 2 = 4$$

$$\text{h. } \left(\frac{f}{g}\right)(1) = \frac{|1|}{1} = \frac{1}{1} = 1$$

$$77. f(x) = 1 + \frac{1}{x} \quad g(x) = \frac{1}{x}$$

$$\text{a. } (f+g)(x) = 1 + \frac{1}{x} + \frac{1}{x} = 1 + \frac{2}{x}$$

$$\text{Domain: } \{x \mid x \neq 0\}.$$

$$\text{b. } (f-g)(x) = 1 + \frac{1}{x} - \frac{1}{x} = 1$$

$$\text{Domain: } \{x \mid x \neq 0\}.$$

$$\text{c. } (f \cdot g)(x) = \left(1 + \frac{1}{x}\right) \frac{1}{x} = \frac{1}{x} + \frac{1}{x^2}$$

$$\text{Domain: } \{x \mid x \neq 0\}.$$

$$\text{d. } \left(\frac{f}{g}\right)(x) = \frac{1 + \frac{1}{x}}{\frac{1}{x}} = \frac{x+1}{1} = x+1$$

$$\text{Domain: } \{x \mid x \neq 0\}.$$

$$\text{e. } (f+g)(3) = 1 + \frac{2}{3} = \frac{5}{3}$$

$$\text{f. } (f-g)(4) = 1$$

$$\text{g. } (f \cdot g)(2) = \frac{1}{2} + \frac{1}{(2)^2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$\text{h. } \left(\frac{f}{g}\right)(1) = 1 + 1 = 2$$

$$78. f(x) = \sqrt{x-1} \quad g(x) = \sqrt{4-x}$$

$$\text{a. } (f+g)(x) = \sqrt{x-1} + \sqrt{4-x}$$

$$x-1 \geq 0 \text{ and } 4-x \geq 0$$

$$x \geq 1 \text{ and } -x \geq -4$$

$$x \leq 4$$

$$\text{Domain: } \{x \mid 1 \leq x \leq 4\}.$$

$$\text{b. } (f-g)(x) = \sqrt{x-1} - \sqrt{4-x}$$

$$x-1 \geq 0 \text{ and } 4-x \geq 0$$

$$x \geq 1 \text{ and } -x \geq -4$$

$$x \leq 4$$

$$\text{Domain: } \{x \mid 1 \leq x \leq 4\}.$$

$$\text{c. } (f \cdot g)(x) = (\sqrt{x-1})(\sqrt{4-x})$$

$$= \sqrt{-x^2 + 5x - 4}$$

$$x-1 \geq 0 \text{ and } 4-x \geq 0$$

$$x \geq 1 \text{ and } -x \geq -4$$

$$x \leq 4$$

$$\text{Domain: } \{x \mid 1 \leq x \leq 4\}.$$

$$\text{d. } \left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-1}}{\sqrt{4-x}} = \sqrt{\frac{x-1}{4-x}}$$

$$x-1 \geq 0 \text{ and } 4-x > 0$$

$$x \geq 1 \text{ and } -x > -4$$

$$x < 4$$

$$\text{Domain: } \{x \mid 1 \leq x < 4\}.$$

$$\text{e. } (f+g)(3) = \sqrt{3-1} + \sqrt{4-3}$$

$$= \sqrt{2} + \sqrt{1} = \sqrt{2} + 1$$

$$\text{f. } (f-g)(4) = \sqrt{4-1} - \sqrt{4-4}$$

$$= \sqrt{3} - \sqrt{0} = \sqrt{3} - 0 = \sqrt{3}$$

$$\text{g. } (f \cdot g)(2) = \sqrt{-(2)^2 + 5(2) - 4}$$

$$= \sqrt{-4 + 10 - 4} = \sqrt{2}$$

$$\text{h. } \left(\frac{f}{g}\right)(1) = \sqrt{\frac{1-1}{4-1}} = \sqrt{\frac{0}{3}} = \sqrt{0} = 0$$

$$79. f(x) = \frac{2x+3}{3x-2} \quad g(x) = \frac{4x}{3x-2}$$

$$\text{a. } (f+g)(x) = \frac{2x+3}{3x-2} + \frac{4x}{3x-2}$$

$$= \frac{2x+3+4x}{3x-2} = \frac{6x+3}{3x-2}$$

$$3x-2 \neq 0$$

$$3x \neq 2 \Rightarrow x \neq \frac{2}{3}$$

$$\text{Domain: } \{x \mid x \neq \frac{2}{3}\}.$$

Chapter 2: Functions and Their Graphs

$$\begin{aligned} \text{b. } (f-g)(x) &= \frac{2x+3}{3x-2} - \frac{4x}{3x-2} \\ &= \frac{2x+3-4x}{3x-2} = \frac{-2x+3}{3x-2} \end{aligned}$$

$$3x-2 \neq 0$$

$$3x \neq 2 \Rightarrow x \neq \frac{2}{3}$$

$$\text{Domain: } \left\{ x \mid x \neq \frac{2}{3} \right\}.$$

$$\text{c. } (f \cdot g)(x) = \left(\frac{2x+3}{3x-2} \right) \left(\frac{4x}{3x-2} \right) = \frac{8x^2+12x}{(3x-2)^2}$$

$$3x-2 \neq 0$$

$$3x \neq 2 \Rightarrow x \neq \frac{2}{3}$$

$$\text{Domain: } \left\{ x \mid x \neq \frac{2}{3} \right\}.$$

$$\text{d. } \left(\frac{f}{g} \right)(x) = \frac{\frac{2x+3}{3x-2}}{\frac{4x}{3x-2}} = \frac{2x+3}{3x-2} \cdot \frac{3x-2}{4x} = \frac{2x+3}{4x}$$

$$3x-2 \neq 0 \quad \text{and} \quad x \neq 0$$

$$3x \neq 2$$

$$x \neq \frac{2}{3}$$

$$\text{Domain: } \left\{ x \mid x \neq \frac{2}{3} \text{ and } x \neq 0 \right\}.$$

$$\text{e. } (f+g)(3) = \frac{6(3)+3}{3(3)-2} = \frac{18+3}{9-2} = \frac{21}{7} = 3$$

$$\text{f. } (f-g)(4) = \frac{-2(4)+3}{3(4)-2} = \frac{-8+3}{12-2} = \frac{-5}{10} = -\frac{1}{2}$$

$$\begin{aligned} \text{g. } (f \cdot g)(2) &= \frac{8(2)^2+12(2)}{(3(2)-2)^2} \\ &= \frac{8(4)+24}{(6-2)^2} = \frac{32+24}{(4)^2} = \frac{56}{16} = \frac{7}{2} \end{aligned}$$

$$\text{h. } \left(\frac{f}{g} \right)(1) = \frac{2(1)+3}{4(1)} = \frac{2+3}{4} = \frac{5}{4}$$

$$80. \quad f(x) = \sqrt{x+1} \quad g(x) = \frac{2}{x}$$

$$\text{a. } (f+g)(x) = \sqrt{x+1} + \frac{2}{x}$$

$$x+1 \geq 0 \quad \text{and} \quad x \neq 0$$

$$x \geq -1$$

$$\text{Domain: } \{x \mid x \geq -1, \text{ and } x \neq 0\}.$$

$$\text{b. } (f-g)(x) = \sqrt{x+1} - \frac{2}{x}$$

$$x+1 \geq 0 \quad \text{and} \quad x \neq 0$$

$$x \geq -1$$

$$\text{Domain: } \{x \mid x \geq -1, \text{ and } x \neq 0\}.$$

$$\text{c. } (f \cdot g)(x) = \sqrt{x+1} \cdot \frac{2}{x} = \frac{2\sqrt{x+1}}{x}$$

$$x+1 \geq 0 \quad \text{and} \quad x \neq 0$$

$$x \geq -1$$

$$\text{Domain: } \{x \mid x \geq -1, \text{ and } x \neq 0\}.$$

$$\text{d. } \left(\frac{f}{g} \right)(x) = \frac{\sqrt{x+1}}{\frac{2}{x}} = \frac{x\sqrt{x+1}}{2}$$

$$x+1 \geq 0 \quad \text{and} \quad x \neq 0$$

$$x \geq -1$$

$$\text{Domain: } \{x \mid x \geq -1, \text{ and } x \neq 0\}.$$

$$\text{e. } (f+g)(3) = \sqrt{3+1} + \frac{2}{3} = \sqrt{4} + \frac{2}{3} = 2 + \frac{2}{3} = \frac{8}{3}$$

$$\text{f. } (f-g)(4) = \sqrt{4+1} - \frac{2}{4} = \sqrt{5} - \frac{1}{2}$$

$$\text{g. } (f \cdot g)(2) = \frac{2\sqrt{2+1}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\text{h. } \left(\frac{f}{g} \right)(1) = \frac{1\sqrt{1+1}}{2} = \frac{\sqrt{2}}{2}$$

$$81. \quad f(x) = 3x+1 \quad (f+g)(x) = 6 - \frac{1}{2}x$$

$$6 - \frac{1}{2}x = 3x+1 + g(x)$$

$$5 - \frac{7}{2}x = g(x)$$

$$g(x) = 5 - \frac{7}{2}x$$

$$82. \quad f(x) = \frac{1}{x} \quad \left(\frac{f}{g} \right)(x) = \frac{x+1}{x^2-x}$$

$$\frac{x+1}{x^2-x} = \frac{\frac{1}{x}}{g(x)}$$

$$g(x) = \frac{\frac{1}{x}}{\frac{x+1}{x^2-x}} = \frac{1}{x} \cdot \frac{x^2-x}{x+1}$$

$$= \frac{1}{x} \cdot \frac{x(x-1)}{x+1} = \frac{x-1}{x+1}$$

$$83. \quad f(x) = 4x+3$$

$$\frac{f(x+h)-f(x)}{h} = \frac{4(x+h)+3-(4x+3)}{h}$$

$$= \frac{4x+4h+3-4x-3}{h}$$

$$= \frac{4h}{h} = 4$$

$$84. \quad f(x) = -3x+1$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-3(x+h)+1-(-3x+1)}{h}$$

$$= \frac{-3x-3h+1+3x-1}{h}$$

$$= \frac{-3h}{h} = -3$$

$$85. \quad f(x) = x^2 - 4$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-4-(x^2-4)}{h}$$

$$= \frac{x^2+2xh+h^2-4-x^2+4}{h}$$

$$= \frac{2xh+h^2}{h}$$

$$= 2x+h$$

$$86. \quad f(x) = 3x^2 + 2$$

$$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)^2+2-(3x^2+2)}{h}$$

$$= \frac{3x^2+6xh+3h^2+2-3x^2-2}{h}$$

$$= \frac{6xh+3h^2}{h}$$

$$= 6x+3h$$

$$87. \quad f(x) = x^2 - x + 4$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-(x+h)+4-(x^2-x+4)}{h}$$

$$= \frac{x^2+2xh+h^2-x-h+4-x^2+x-4}{h}$$

$$= \frac{2xh+h^2-h}{h}$$

$$= 2x+h-1$$

$$88. \quad f(x) = 3x^2 - 2x + 6$$

$$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)^2-2(x+h)+6-[3x^2-2x+6]}{h}$$

$$= \frac{3(x^2+2xh+h^2)-2x-2h+6-3x^2+2x-6}{h}$$

$$= \frac{3x^2+6xh+3h^2-2h-3x^2}{h} = \frac{6xh+3h^2-2h}{h}$$

$$= 6x+3h-2$$

Chapter 2: Functions and Their Graphs

$$89. f(x) = \frac{5}{4x-3}$$

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{\frac{5}{4(x+h)-3} - \frac{5}{4x-3}}{h} \\ &= \frac{\frac{5(4x-3) - 5(4(x+h)-3)}{(4(x+h)-3)(4x-3)}}{h} \\ &= \left(\frac{20x+15-20x-15-20h}{(4(x+h)-3)(4x-3)} \right) \left(\frac{1}{h} \right) \\ &= \left(\frac{-20h}{(4(x+h)-3)(4x-3)} \right) \left(\frac{1}{h} \right) \\ &= \frac{-20}{(4(x+h)-3)(4x-3)} \end{aligned}$$

$$90. f(x) = \frac{1}{x+3}$$

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{\frac{1}{x+h+3} - \frac{1}{x+3}}{h} \\ &= \frac{\frac{x+3-(x+3+h)}{(x+h+3)(x+3)}}{h} \\ &= \left(\frac{x+3-x-3-h}{(x+h+3)(x+3)} \right) \left(\frac{1}{h} \right) \\ &= \left(\frac{-h}{(x+h+3)(x+3)} \right) \left(\frac{1}{h} \right) \\ &= \frac{-1}{(x+h+3)(x+3)} \end{aligned}$$

$$91. f(x) = \frac{2x}{x+3}$$

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{\frac{2(x+h)}{x+h+3} - \frac{2x}{x+3}}{h} \\ &= \frac{\frac{2(x+h)(x+3) - 2x(x+3+h)}{(x+h+3)(x+3)}}{h} \\ &= \frac{\frac{2x^2+6x+2hx+6h-2x^2-6x-2xh}{(x+h+3)(x+3)}}{h} \\ &= \left(\frac{6h}{(x+h+3)(x+3)} \right) \left(\frac{1}{h} \right) \\ &= \frac{6}{(x+h+3)(x+3)} \end{aligned}$$

$$92. f(x) = \frac{5x}{x-4}$$

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{\frac{5(x+h)}{x+h-4} - \frac{5x}{x-4}}{h} \\ &= \frac{\frac{5(x+h)(x-4) - 5x(x-4+h)}{(x+h-4)(x-4)}}{h} \\ &= \frac{\frac{5x^2-20x+5hx-20h-5x^2+20x-5xh}{(x+h-4)(x-4)}}{h} \\ &= \left(\frac{-20h}{(x+h-4)(x-4)} \right) \left(\frac{1}{h} \right) \\ &= \frac{-20}{(x+h-4)(x-4)} \end{aligned}$$

$$93. f(x) = \sqrt{x-2}$$

$$\begin{aligned} & \frac{f(x+h) - f(x)}{h} \\ &= \frac{\sqrt{x+h-2} - \sqrt{x-2}}{h} \\ &= \frac{\sqrt{x+h-2} - \sqrt{x-2}}{h} \cdot \frac{\sqrt{x+h-2} + \sqrt{x-2}}{\sqrt{x+h-2} + \sqrt{x-2}} \\ &= \frac{x+h-2 - x+2}{h(\sqrt{x+h-2} + \sqrt{x-2})} \\ &= \frac{h}{h(\sqrt{x+h-2} + \sqrt{x-2})} \\ &= \frac{1}{\sqrt{x+h-2} + \sqrt{x-2}} \end{aligned}$$

$$94. f(x) = \sqrt{x+1}$$

$$\begin{aligned} & \frac{f(x+h) - f(x)}{h} \\ &= \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\ &= \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}} \\ &= \frac{x+h+1 - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} = \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\ &= \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} \end{aligned}$$

$$95. f(x) = \frac{1}{x^2}$$

$$\begin{aligned} & \frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \\ &= \frac{\frac{x^2 - (x+h)^2}{x^2(x+h)^2}}{h} \\ &= \frac{x - (x^2 + 2xh + h^2)}{x^2(x+h)^2} \\ &= \left(\frac{1}{h}\right) \frac{-2xh - h^2}{x^2(x+h)^2} \\ &= \left(\frac{1}{h}\right) \frac{h(-2x - h)}{x^2(x+h)^2} \\ &= \frac{-2x - h}{x^2(x+h)^2} = \frac{-(2x+h)}{x^2(x+h)^2} \end{aligned}$$

$$96. f(x) = \frac{1}{x^2 + 1}$$

$$\begin{aligned} & \frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{(x+h)^2 + 1} - \frac{1}{x^2 + 1}}{h} \\ &= \frac{\frac{x^2 + 1 - ((x+h)^2 + 1)}{(x^2 + 1)((x+h)^2 + 1)}}{h} \\ &= \frac{x^2 + 1 - (x^2 + 2xh + h^2) - 1}{(x^2 + 1)((x+h)^2 + 1)} \\ &= \left(\frac{1}{h}\right) \frac{-2xh - h^2}{(x^2 + 1)((x+h)^2 + 1)} \\ &= \left(\frac{1}{h}\right) \frac{h(-2x - h)}{(x^2 + 1)((x+h)^2 + 1)} \\ &= \frac{-2x - h}{(x^2 + 1)((x+h)^2 + 1)} \\ &= \frac{-(2x+h)}{(x^2 + 1)((x+h)^2 + 1)} \end{aligned}$$

Chapter 2: Functions and Their Graphs

$$\begin{aligned}
 97. \quad f(x) &= \sqrt{4-x^2} \\
 \frac{f(x+h) - f(x)}{h} &= \frac{\sqrt{4-(x+h)^2} - \sqrt{4-x^2}}{h} \\
 &= \frac{\sqrt{4-(x+h)^2} - \sqrt{4-x^2}}{h} \cdot \frac{\sqrt{4-(x+h)^2} + \sqrt{4-x^2}}{\sqrt{4-(x+h)^2} + \sqrt{4-x^2}} \\
 &= \frac{4-(x+h)^2 - (4-x^2)}{h(\sqrt{4-(x+h)^2} + \sqrt{4-x^2})} \\
 &= \frac{4-(x^2+2xh+h^2)-(4-x^2)}{h(\sqrt{4-(x+h)^2} + \sqrt{4-x^2})} \\
 &= \frac{-2xh-h^2}{h(\sqrt{4-(x+h)^2} + \sqrt{4-x^2})} \\
 &= \frac{-2x-h}{\sqrt{4-(x+h)^2} + \sqrt{4-x^2}} \\
 &= \frac{-(2x+h)}{\sqrt{4-(x+h)^2} + \sqrt{4-x^2}}
 \end{aligned}$$

$$\begin{aligned}
 98. \quad f(x) &= \frac{1}{\sqrt{x+2}} \\
 \frac{f(x+h) - f(x)}{h} &= \frac{\frac{1}{\sqrt{x+h+2}} - \frac{1}{\sqrt{x+2}}}{h} \\
 &= \frac{\frac{\sqrt{x+2} - \sqrt{x+h+2}}{\sqrt{x+h+2}\sqrt{x+2}}}{h} \\
 &= \frac{\sqrt{x+2} - \sqrt{x+h+2}}{h\sqrt{x+h+2}\sqrt{x+2}} \cdot \frac{\sqrt{x+2} + \sqrt{x+h+2}}{\sqrt{x+2} + \sqrt{x+h+2}} \\
 &= \frac{x+2 - (x+h+2)}{h(x+2)\sqrt{x+h+2} + (x+h+2)\sqrt{x+2}} \\
 &= \frac{x+2 - x - h - 2}{h(x+2)\sqrt{x+h+2} + (x+h+2)\sqrt{x+2}} \\
 &= \frac{-h}{h(x+2)\sqrt{x+h+2} + (x+h+2)\sqrt{x+2}} \\
 &= -\frac{1}{(x+2)\sqrt{x+h+2} + (x+h+2)\sqrt{x+2}}
 \end{aligned}$$

$$\begin{aligned}
 99. \quad 11 &= x^2 - 2x + 3 \\
 0 &= x^2 - 2x - 8 \\
 0 &= (x-4)(x+2) \\
 x-4 &= 0 \quad \text{or} \quad x+2 = 0 \\
 x &= 4 \quad \text{or} \quad x = -2
 \end{aligned}$$

The solution set is: $\{-2, 4\}$

$$\begin{aligned}
 100. \quad -\frac{7}{16} &= \frac{5}{6}x - \frac{3}{4} \\
 -\frac{7}{16} + \frac{3}{4} &= \frac{5}{6}x \\
 \frac{5}{6}x &= -\frac{7}{16} + \frac{12}{16} \\
 \frac{5}{6}x &= \frac{5}{16} \\
 x &= \frac{5}{16} \cdot \frac{6}{5} = \frac{3}{8}
 \end{aligned}$$

The solution set is: $\left\{\frac{3}{8}\right\}$

$$\begin{aligned}
 101. \quad f(x) &= 2x^3 + Ax^2 + 4x - 5 \quad \text{and} \quad f(2) = 5 \\
 f(2) &= 2(2)^3 + A(2)^2 + 4(2) - 5 \\
 5 &= 16 + 4A + 8 - 5 \\
 5 &= 4A + 19 \\
 -14 &= 4A \\
 A &= \frac{-14}{4} = -\frac{7}{2}
 \end{aligned}$$

$$\begin{aligned}
 102. \quad f(x) &= 3x^2 - Bx + 4 \quad \text{and} \quad f(-1) = 12 : \\
 f(-1) &= 3(-1)^2 - B(-1) + 4 \\
 12 &= 3 + B + 4 \\
 B &= 5
 \end{aligned}$$

$$\begin{aligned}
 103. \quad f(x) &= \frac{3x+8}{2x-A} \quad \text{and} \quad f(0) = 2 \\
 f(0) &= \frac{3(0)+8}{2(0)-A} \\
 2 &= \frac{8}{-A} \\
 -2A &= 8 \\
 A &= -4
 \end{aligned}$$

104. $f(x) = \frac{2x-B}{3x+4}$ and $f(2) = \frac{1}{2}$

$$f(2) = \frac{2(2)-B}{3(2)+4}$$

$$\frac{1}{2} = \frac{4-B}{10}$$

$$5 = 4 - B$$

$$B = -1$$

105. Let x represent the length of the rectangle.

Then, $\frac{x}{2}$ represents the width of the rectangle

since the length is twice the width. The function

for the area is: $A(x) = x \cdot \frac{x}{2} = \frac{x^2}{2} = \frac{1}{2}x^2$

106. Let x represent the length of one of the two equal sides. The function for the area is:

$$A(x) = \frac{1}{2} \cdot x \cdot x = \frac{1}{2}x^2$$

107. Let x represent the number of hours worked.

The function for the gross salary is:

$$G(x) = 16x$$

108. Let x represent the number of items sold.

The function for the gross salary is:

$$G(x) = 10x + 100$$

109. a. $H(1) = 20 - 4.9(1)^2$
 $= 20 - 4.9 = 15.1$ meters

$$\begin{aligned} H(1.1) &= 20 - 4.9(1.1)^2 \\ &= 20 - 4.9(1.21) \\ &= 20 - 5.929 = 14.071 \text{ meters} \end{aligned}$$

$$\begin{aligned} H(1.2) &= 20 - 4.9(1.2)^2 \\ &= 20 - 4.9(1.44) \\ &= 20 - 7.056 = 12.944 \text{ meters} \end{aligned}$$

b. $H(x) = 15$:

$$15 = 20 - 4.9x^2$$

$$-5 = -4.9x^2$$

$$x^2 \approx 1.0204$$

$$x \approx 1.01 \text{ seconds}$$

$H(x) = 10$:

$$10 = 20 - 4.9x^2$$

$$-10 = -4.9x^2$$

$$x^2 \approx 2.0408$$

$$x \approx 1.43 \text{ seconds}$$

$H(x) = 5$:

$$5 = 20 - 4.9x^2$$

$$-15 = -4.9x^2$$

$$x^2 \approx 3.0612$$

$$x \approx 1.75 \text{ seconds}$$

c. $H(x) = 0$

$$0 = 20 - 4.9x^2$$

$$-20 = -4.9x^2$$

$$x^2 \approx 4.0816$$

$$x \approx 2.02 \text{ seconds}$$

110. a. $H(1) = 20 - 13(1)^2 = 20 - 13 = 7$ meters

$$\begin{aligned} H(1.1) &= 20 - 13(1.1)^2 = 20 - 13(1.21) \\ &= 20 - 15.73 = 4.27 \text{ meters} \end{aligned}$$

$$\begin{aligned} H(1.2) &= 20 - 13(1.2)^2 = 20 - 13(1.44) \\ &= 20 - 18.72 = 1.28 \text{ meters} \end{aligned}$$

b. $H(x) = 15$

$$15 = 20 - 13x^2$$

$$-5 = -13x^2$$

$$x^2 \approx 0.3846$$

$$x \approx 0.62 \text{ seconds}$$

$H(x) = 10$

$$10 = 20 - 13x^2$$

$$-10 = -13x^2$$

$$x^2 \approx 0.7692$$

$$x \approx 0.88 \text{ seconds}$$

Chapter 2: Functions and Their Graphs

$$H(x) = 5$$

$$5 = 20 - 13x^2$$

$$-15 = -13x^2$$

$$x^2 \approx 1.1538$$

$$x \approx 1.07 \text{ seconds}$$

$$\text{c. } H(x) = 0$$

$$0 = 20 - 13x^2$$

$$-20 = -13x^2$$

$$x^2 \approx 1.5385$$

$$x \approx 1.24 \text{ seconds}$$

$$111. \quad C(x) = 100 + \frac{x}{10} + \frac{36,000}{x}$$

$$\begin{aligned} \text{a. } C(500) &= 100 + \frac{500}{10} + \frac{36,000}{500} \\ &= 100 + 50 + 72 \\ &= \$222 \end{aligned}$$

$$\begin{aligned} \text{b. } C(450) &= 100 + \frac{450}{10} + \frac{36,000}{450} \\ &= 100 + 45 + 80 \\ &= \$225 \end{aligned}$$

$$\begin{aligned} \text{c. } C(600) &= 100 + \frac{600}{10} + \frac{36,000}{600} \\ &= 100 + 60 + 60 \\ &= \$220 \end{aligned}$$

$$\begin{aligned} \text{d. } C(400) &= 100 + \frac{400}{10} + \frac{36,000}{400} \\ &= 100 + 40 + 90 \\ &= \$230 \end{aligned}$$

$$112. \quad A(x) = 4x\sqrt{1-x^2}$$

$$\begin{aligned} \text{a. } A\left(\frac{1}{3}\right) &= 4 \cdot \frac{1}{3} \sqrt{1 - \left(\frac{1}{3}\right)^2} = \frac{4}{3} \sqrt{\frac{8}{9}} = \frac{4}{3} \cdot \frac{2\sqrt{2}}{3} \\ &= \frac{8\sqrt{2}}{9} \approx 1.26 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} \text{b. } A\left(\frac{1}{2}\right) &= 4 \cdot \frac{1}{2} \sqrt{1 - \left(\frac{1}{2}\right)^2} = 2\sqrt{\frac{3}{4}} = 2 \cdot \frac{\sqrt{3}}{2} \\ &= \sqrt{3} \approx 1.73 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} \text{c. } A\left(\frac{2}{3}\right) &= 4 \cdot \frac{2}{3} \sqrt{1 - \left(\frac{2}{3}\right)^2} = \frac{8}{3} \sqrt{\frac{5}{9}} = \frac{8}{3} \cdot \frac{\sqrt{5}}{3} \\ &= \frac{8\sqrt{5}}{9} \approx 1.99 \text{ ft}^2 \end{aligned}$$

$$113. \quad R(x) = \left(\frac{L}{P}\right)(x) = \frac{L(x)}{P(x)}$$

$$114. \quad T(x) = (V + P)(x) = V(x) + P(x)$$

$$115. \quad H(x) = (P \cdot I)(x) = P(x) \cdot I(x)$$

$$116. \quad N(x) = (I - T)(x) = I(x) - T(x)$$

$$\begin{aligned} 117. \quad \text{a. } P(x) &= R(x) - C(x) \\ &= (-1.2x^2 + 220x) - (0.05x^3 - 2x^2 + 65x + 500) \\ &= -1.2x^2 + 220x - 0.05x^3 + 2x^2 - 65x - 500 \\ &= -0.05x^3 + 0.8x^2 + 155x - 500 \end{aligned}$$

$$\begin{aligned} \text{b. } P(15) &= -0.05(15)^3 + 0.8(15)^2 + 155(15) - 500 \\ &= -168.75 + 180 + 2325 - 500 \\ &= \$1836.25 \end{aligned}$$

c. When 15 hundred smartphones are sold, the profit is \$1836.25.

118. a. P is the dependent variable; a is the independent variable

$$\begin{aligned} \text{b. } P(20) &= 0.027(20)^2 - 6.530(20) + 363.804 \\ &= 10.8 - 130.6 + 363.804 \\ &= 244.004 \end{aligned}$$

In 2015 there are 244.004 million people who are 20 years of age or older.

$$\begin{aligned} \text{c. } P(0) &= 0.027(0)^2 - 6.530(0) + 363.804 \\ &= 363.804 \end{aligned}$$

In 2015 there are 363.804 million people.

$$\begin{aligned} 119. \quad \text{a. } R(v) &= 2.2v; B(v) = 0.05v^2 + 0.4v - 15 \\ D(v) &= R(v) + B(v) \\ &= 2.2v + 0.05v^2 + 0.4v - 15 \\ &= 0.05v^2 + 2.6v - 15 \end{aligned}$$

$$\begin{aligned} \text{b. } D(60) &= 0.05(60)^2 + 2.6(60) - 15 \\ &= 180 + 156 - 15 \\ &= 321 \end{aligned}$$

- c. The car will need 321 feet to stop once the impediment is observed.

120. a. $h(x) = 2x$

$$h(a+b) = 2(a+b) = 2a+2b \\ = h(a) + h(b)$$

$$h(x) = 2x \text{ has the property.}$$

b. $g(x) = x^2$

$$g(a+b) = (a+b)^2 = a^2 + 2ab + b^2$$

Since

$$a^2 + 2ab + b^2 \neq a^2 + b^2 = g(a) + g(b),$$

$$g(x) = x^2 \text{ does not have the property.}$$

c. $F(x) = 5x - 2$

$$F(a+b) = 5(a+b) - 2 = 5a + 5b - 2$$

Since

$$5a + 5b - 2 \neq 5a - 2 + 5b - 2 = F(a) + F(b),$$

$$F(x) = 5x - 2 \text{ does not have the property.}$$

d. $G(x) = \frac{1}{x}$

$$G(a+b) = \frac{1}{a+b} \neq \frac{1}{a} + \frac{1}{b} = G(a) + G(b)$$

$$G(x) = \frac{1}{x} \text{ does not have the property.}$$

$$\begin{aligned} 121. \quad \frac{f(x+h) - f(x)}{h} &= \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} \\ &= \frac{(x+h)^{1/3} - x^{1/3}}{h} \\ &= \frac{(x+h)^{1/3} - x^{1/3}}{h} \cdot \frac{(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}}{(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}} \\ &= \frac{h}{(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}} \\ &= \frac{x+h-x}{h \left[(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3} \right]} = \frac{h}{h \left[(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3} \right]} \\ &= \frac{1}{(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}} \end{aligned}$$

122. $f\left(\frac{x+4}{5x-4}\right) = 3x^2 - 2$

Solve $\frac{x+4}{5x-4} = 1$.

$$\frac{x+4}{5x-4} = 1$$

$$x+4 = 5x-4$$

$$x = 2$$

$$\text{Therefore, } f(1) = 3(2)^2 - 2 = 10$$

123. We need $\frac{x^2+1}{7-|3x-1|} \geq 0$. Since $x^2+1 > 0$ for all

real numbers x , we need $7 - |3x-1| > 0$.

$$7 - |3x-1| > 0$$

$$|3x-1| < 7$$

$$-7 < 3x-1 < 7$$

$$-2 < x < \frac{8}{3}$$

The domain of f is $\left\{x \mid -2 < x < \frac{8}{3}\right\}$, or $\left(-2, \frac{8}{3}\right)$ in interval notation.

Chapter 2: Functions and Their Graphs

124. No. The domain of f is $\{x \mid x \text{ is any real number}\}$, but the domain of g is $\{x \mid x \neq -1\}$.

125. $\frac{3x - x^3}{(\text{your age})}$

126. Answers will vary.

127. $(x+12)^2 + y^2 = 16$

x-intercept ($y=0$):

$$(x+12)^2 + 0^2 = 16$$

$$(x+12)^2 = 16$$

$$(x+12) = \pm 4$$

$$x = -12 \pm 4$$

$$x = -16, x = -8$$

$$(-16, 0), (-8, 0)$$

y-intercept ($x=0$):

$$(0+12)^2 + y^2 = 16$$

$$(12)^2 + y^2 = 16$$

$$y^2 = 16 - 144 = -128$$

There are no real solutions so there are no y-intercepts.

Symmetry: $(x+12)^2 + (-y)^2 = 16$

$$(x+12)^2 + y^2 = 16$$

This shows x-axis symmetry.

128. $y = 3x^2 - 8\sqrt{x}$

$$y = 3(-1)^2 - 8\sqrt{-1}$$

There is no solution so $(-1, -5)$ is NOT a solution.

$$y = 3x^2 - 8\sqrt{x}$$

$$y = 3(4)^2 - 8\sqrt{4}$$

$$= 48 - 16 = 32$$

So $(4, 32)$ is a solution.

$$y = 3x^2 - 8\sqrt{x}$$

$$y = 3(9)^2 - 8\sqrt{9}$$

$$= 243 - 24 = 219 \neq 171$$

So $(9, 171)$ is NOT a solution.

129. $3x - 10y = 12$

$$-10y = -3x + 12$$

$$y = \frac{3}{10}x - \frac{6}{5}$$

The slope of the line is $\frac{3}{10}$. The slope of a

perpendicular line would be $-\frac{10}{3}$.

130. $x^3 - 9x = 2x^2 - 18$

$$x^3 - 2x^2 - 9x + 18 = 0$$

$$(x^3 - 2x^2) - (9x - 18) = 0$$

$$x^2(x - 2) - 9(x - 2) = 0$$

$$(x^2 - 9)(x - 2) = 0$$

$$(x - 3)(x + 3)(x - 2) = 0$$

$$(x - 3) = 0 \text{ or } (x + 3) = 0 \text{ or } (x - 2) = 0$$

$$x = 3, x = -3, x = 2$$

The solution set is: $\{3, -3, 2\}$

131. $a + bx = ac + d$

$$a - ac = d - bx$$

$$a(1 - c) = d - bx$$

$$a = \frac{d - bx}{1 - c}$$

132. $\frac{(4x^2 - 7) \cdot 3 - (3x + 5) \cdot 8x}{(4x^2 - 7)^2} =$

$$\frac{12x^2 - 21 - (24x^2 + 40x)}{(4x^2 - 7)^2} =$$

$$\frac{12x^2 - 21 - 24x^2 - 40x}{(4x^2 - 7)^2} = \frac{-12x^2 - 40x - 21}{(4x^2 - 7)^2}$$

$$= -\frac{12x^2 + 40x + 21}{(4x^2 - 7)^2}$$

Section 2.2

1. $x^2 + 4y^2 = 16$

x -intercepts:

$$x^2 + 4(0)^2 = 16$$

$$x^2 = 16$$

$$x = \pm 4 \Rightarrow (-4, 0), (4, 0)$$

y -intercepts:

$$(0)^2 + 4y^2 = 16$$

$$4y^2 = 16$$

$$y^2 = 4$$

$$y = \pm 2 \Rightarrow (0, -2), (0, 2)$$

2. False; $x = 2y - 2$

$$-2 = 2y - 2$$

$$0 = 2y$$

$$0 = y$$

The point $(-2, 0)$ is on the graph.

3. vertical

4. $f(5) = -3$

5. $f(x) = ax^2 + 4$

$$a(-1)^2 + 4 = 2 \Rightarrow a = -2$$

6. False. The graph must pass the Vertical-Line Test in order to be the graph of a function.

7. False; e.g. $y = \frac{1}{x}$.

8. True

9. c

10. a

11. a. $f(0) = 3$ since $(0, 3)$ is on the graph.
 $f(-6) = -3$ since $(-6, -3)$ is on the graph.

b. $f(6) = 0$ since $(6, 0)$ is on the graph.
 $f(11) = 1$ since $(11, 1)$ is on the graph.

c. $f(3)$ is positive since $f(3) \approx 3.7$.

d. $f(-4)$ is negative since $f(-4) \approx -1$.

e. $f(x) = 0$ when $x = -3$, $x = 6$, and $x = 10$.

f. $f(x) > 0$ when $-3 < x < 6$, and $10 < x \leq 11$.

g. The domain of f is $\{x \mid -6 \leq x \leq 11\}$ or $[-6, 11]$.

h. The range of f is $\{y \mid -3 \leq y \leq 4\}$ or $[-3, 4]$.

i. The x -intercepts are -3 , 6 , and 10 .

j. The y -intercept is 3 .

k. The line $y = \frac{1}{2}$ intersects the graph 3 times.

l. The line $x = 5$ intersects the graph 1 time.

m. $f(x) = 3$ when $x = 0$ and $x = 4$.

n. $f(x) = -2$ when $x = -5$ and $x = 8$.

12. a. $f(0) = 0$ since $(0, 0)$ is on the graph.
 $f(6) = 0$ since $(6, 0)$ is on the graph.

b. $f(2) = -2$ since $(2, -2)$ is on the graph.
 $f(-2) = 1$ since $(-2, 1)$ is on the graph.

c. $f(3)$ is negative since $f(3) \approx -1$.

d. $f(-1)$ is positive since $f(-1) \approx 1.0$.

e. $f(x) = 0$ when $x = 0$, $x = 4$, and $x = 6$.

f. $f(x) < 0$ when $0 < x < 4$.

g. The domain of f is $\{x \mid -4 \leq x \leq 6\}$ or $[-4, 6]$.

h. The range of f is $\{y \mid -2 \leq y \leq 3\}$ or $[-2, 3]$.

i. The x -intercepts are 0 , 4 , and 6 .

j. The y -intercept is 0 .

k. The line $y = -1$ intersects the graph 2 times.

l. The line $x = 1$ intersects the graph 1 time.

m. $f(x) = 3$ when $x = 5$.

n. $f(x) = -2$ when $x = 2$.

13. Not a function since vertical lines will intersect the graph in more than one point.

Chapter 2: Functions and Their Graphs

- a. Domain: $\{x \mid x \leq -1 \text{ or } x \geq 1\}$;
Range: $\{y \mid y \text{ is any real number}\}$
- b. Intercepts: $(-1, 0), (1, 0)$
- c. Symmetry about the x-axis, y-axis and the origin
- 14. Function**
- a. Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y > 0\}$
- b. Intercepts: $(0, 1)$
- c. None
- 15. Function**
- a. Domain: $\{x \mid -\pi \leq x \leq \pi\}$;
Range: $\{y \mid -1 \leq y \leq 1\}$
- b. Intercepts: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right), (0, 1)$
- c. Symmetry about y-axis.
- 16. Function**
- a. Domain: $\{x \mid -\pi \leq x \leq \pi\}$;
Range: $\{y \mid -1 \leq y \leq 1\}$
- b. Intercepts: $(-\pi, 0), (\pi, 0), (0, 0)$
- c. Symmetry about the origin.
- 17. Not a function since vertical lines will intersect the graph in more than one point.**
- a. Domain: $\{x \mid x \leq 0\}$;
Range: $\{y \mid y \text{ is any real number}\}$
- b. Intercepts: $(0, 0)$
- c. Symmetry about the x-axis
- 18. Not a function since vertical lines will intersect the graph in more than one point.**
- a. Domain: $\{x \mid -2 \leq x \leq 2\}$;
Range: $\{y \mid -2 \leq y \leq 2\}$
- b. Intercepts: $(-2, 0), (2, 0), (0, -2), (0, 2)$
- c. Symmetry about the x-axis, y-axis and the origin
- 19. Function**
- a. Domain: $\{x \mid 0 < x < 3\}$;
Range: $\{y \mid y < 2\}$
- b. Intercepts: $(1, 0)$
- c. None
- 20. Function**
- a. Domain: $\{x \mid 0 \leq x < 4\}$;
Range: $\{y \mid 0 \leq y < 3\}$
- b. Intercepts: $(0, 0)$
- c. None
- 21. Function**
- a. Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y \leq 2\}$
- b. Intercepts: $(-3, 0), (3, 0), (0, 2)$
- c. Symmetry about y-axis.
- 22. Function**
- a. Domain: $\{x \mid x \geq -3\}$;
Range: $\{y \mid y \geq 0\}$
- b. Intercepts: $(-3, 0), (2, 0), (0, 2)$
- c. None
- 23. Function**
- a. Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y \geq -3\}$
- b. Intercepts: $(1, 0), (3, 0), (0, 9)$
- c. None
- 24. Function**
- a. Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y \leq 5\}$
- b. Intercepts: $(-1, 0), (2, 0), (0, 4)$
- c. None
- 25. $f(x) = 3x^2 + x - 2$**

Section 2.2: The Graph of a Function

- a.** $f(1) = 3(1)^2 + (1) - 2 = 2$
The point $(-1, 2)$ is on the graph of f .
- b.** $f(-2) = 3(-2)^2 + (-2) - 2 = 8$
The point $(-2, 8)$ is on the graph of f .
- c.** Solve for x :

$$-2 = 3x^2 + x - 2$$

$$0 = 3x^2 + x$$

$$0 = x(3x + 1) \Rightarrow x = 0, x = -\frac{1}{3}$$
 $(0, -2)$ and $(-\frac{1}{3}, -2)$ are on the graph of f .
- d.** The domain of f is $\{x \mid x \text{ is any real number}\}$.
- e.** x -intercepts:

$$f(x) = 0 \Rightarrow 3x^2 + x - 2 = 0$$

$$(3x - 2)(x + 1) = 0 \Rightarrow x = \frac{2}{3}, x = -1$$
- f.** y -intercept: $f(0) = 3(0)^2 + 0 - 2 = -2$
- 26.** $f(x) = -3x^2 + 5x$
- a.** $f(-1) = -3(-1)^2 + 5(-1) = -8 \neq 2$
The point $(-1, 2)$ is not on the graph of f .
- b.** $f(-2) = -3(-2)^2 + 5(-2) = -22$
The point $(-2, -22)$ is on the graph of f .
- c.** Solve for x :

$$-2 = -3x^2 + 5x \Rightarrow 3x^2 - 5x - 2 = 0$$

$$(3x + 1)(x - 2) = 0 \Rightarrow x = -\frac{1}{3}, x = 2$$
 $(2, -2)$ and $(-\frac{1}{3}, -2)$ on the graph of f .
- d.** The domain of f is $\{x \mid x \text{ is any real number}\}$.
- e.** x -intercepts:

$$f(x) = 0 \Rightarrow -3x^2 + 5x = 0$$

$$x(-3x + 5) = 0 \Rightarrow x = 0, x = \frac{5}{3}$$
- f.** y -intercept:

$$f(0) = -3(0)^2 + 5(0) = 0$$
- 27.** $f(x) = \frac{x+2}{x-6}$
- a.** $f(3) = \frac{3+2}{3-6} = -\frac{5}{3} \neq 14$
The point $(3, 14)$ is not on the graph of f .
- b.** $f(4) = \frac{4+2}{4-6} = \frac{6}{-2} = -3$
The point $(4, -3)$ is on the graph of f .
- c.** Solve for x :

$$2 = \frac{x+2}{x-6}$$

$$2x - 12 = x + 2$$

$$x = 14$$
 $(14, 2)$ is a point on the graph of f .
- d.** The domain of f is $\{x \mid x \neq 6\}$.
- e.** x -intercepts:

$$f(x) = 0 \Rightarrow \frac{x+2}{x-6} = 0$$

$$x + 2 = 0 \Rightarrow x = -2$$
- f.** y -intercept: $f(0) = \frac{0+2}{0-6} = -\frac{1}{3}$
- 28.** $f(x) = \frac{x^2 + 2}{x + 4}$
- a.** $f(1) = \frac{1^2 + 2}{1 + 4} = \frac{3}{5}$
The point $(1, \frac{3}{5})$ is on the graph of f .
- b.** $f(0) = \frac{0^2 + 2}{0 + 4} = \frac{2}{4} = \frac{1}{2}$
The point $(0, \frac{1}{2})$ is on the graph of f .
- c.** Solve for x :

$$\frac{1}{2} = \frac{x^2 + 2}{x + 4} \Rightarrow x + 4 = 2x^2 + 4$$

$$0 = 2x^2 - x$$

$$x(2x - 1) = 0 \Rightarrow x = 0 \text{ or } x = \frac{1}{2}$$
 $(0, \frac{1}{2})$ and $(\frac{1}{2}, \frac{1}{2})$ are on the graph of f .

Chapter 2: Functions and Their Graphs

d. The domain of f is $\{x \mid x \neq -4\}$.

e. x -intercepts:

$$f(x)=0 \Rightarrow \frac{x^2+2}{x+4}=0 \Rightarrow x^2+2=0$$

This is impossible, so there are no x -intercepts.

f. y -intercept:

$$f(0)=\frac{0^2+2}{0+4}=\frac{2}{4}=\frac{1}{2}$$

29. $f(x)=\frac{12x^4}{x^2+1}$

a. $f(-1)=\frac{12(-1)^4}{(-1)^2+1}=\frac{12}{2}=6$

The point $(-1, 6)$ is on the graph of f .

b. $f(3)=\frac{12(3)^4}{(3)^2+1}=\frac{972}{10}=\frac{486}{5}$

The point $\left(3, \frac{486}{5}\right)$ is on the graph of f .

c. Solve for x :

$$\begin{aligned} 1 &= \frac{12x^4}{x^2+1} \\ x^2+1 &= 12x^4 \\ 12x^4 - x^2 - 1 &= 0 \\ (3x^2-1)(4x^2+1) &= 0 \\ 3x^2-1 &= 0 \Rightarrow x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3} \\ \left(-\frac{\sqrt{3}}{3}, 1\right), \left(\frac{\sqrt{3}}{3}, 1\right) &\text{ are on the graph of } f. \end{aligned}$$

d. The domain of f is $\{x \mid x \text{ is any real number}\}$.

e. x -intercept:

$$f(x)=0 \Rightarrow \frac{12x^4}{x^2+1}=0$$

$$12x^4=0 \Rightarrow x=0$$

f. y -intercept:

$$f(0)=\frac{12(0)^4}{0^2+1}=\frac{0}{0+1}=0$$

30. $f(x)=\frac{2x}{x-2}$

a. $f\left(\frac{1}{2}\right)=\frac{2\left(\frac{1}{2}\right)}{\frac{1}{2}-2}=\frac{1}{-\frac{3}{2}}=-\frac{2}{3}$

The point $\left(\frac{1}{2}, -\frac{2}{3}\right)$ is on the graph of f .

b. $f(4)=\frac{2(4)}{4-2}=\frac{8}{2}=4$

The point $(4, 4)$ is on the graph of f .

c. Solve for x :

$$1=\frac{2x}{x-2} \Rightarrow x-2=2x \Rightarrow -2=x$$

$(-2, 1)$ is a point on the graph of f .

d. The domain of f is $\{x \mid x \neq 2\}$.

e. x -intercept:

$$\begin{aligned} f(x)=0 &\Rightarrow \frac{2x}{x-2}=0 \Rightarrow 2x=0 \\ &\Rightarrow x=0 \end{aligned}$$

f. y -intercept: $f(0)=\frac{0}{0-2}=0$

31. a. $(f+g)(2)=f(2)+g(2)=2+1=3$

b. $(f+g)(4)=f(4)+g(4)=1+(-3)=-2$

c. $(f-g)(6)=f(6)-g(6)=0-1=-1$

d. $(g-f)(6)=g(6)-f(6)=1-0=1$

e. $(f \cdot g)(2)=f(2) \cdot g(2)=2(1)=2$

f. $\left(\frac{f}{g}\right)(4)=\frac{f(4)}{g(4)}=\frac{1}{-3}=-\frac{1}{3}$

32. $h(x)=-\frac{136x^2}{v^2}+2.7x+3.5$

a. We want $h(15)=10$.

$$-\frac{136(15)^2}{v^2}+2.7(15)+3.5=10$$

$$-\frac{30,600}{v^2}=-34$$

$$v^2=900$$

$$v=30 \text{ ft/sec}$$

The ball needs to be thrown with an initial velocity of 30 feet per second.

Section 2.2: The Graph of a Function

b. $h(x) = -\frac{126x^2}{30^2} + 2.7x + 3.5$

which simplifies to

$$h(x) = -\frac{34}{225}x^2 + 2.7x + 3.5$$

- c. Using the velocity from part (b),

$$h(9) = -\frac{34}{225}(9)^2 + 2.7(9) + 3.5 = 15.56 \text{ ft}$$

The ball will be 15.56 feet above the floor when it has traveled 9 feet in front of the foul line.

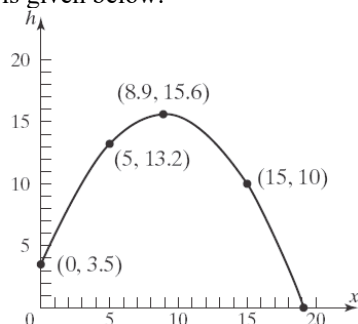
- d. Select several values for x and use these to find the corresponding values for h . Use the results to form ordered pairs (x, h) . Plot the points and connect with a smooth curve.

$$h(0) = -\frac{34}{225}(0)^2 + 2.7(0) + 3.5 = 3.5 \text{ ft}$$

$$h(5) = -\frac{34}{225}(5)^2 + 2.7(5) + 3.5 \approx 13.2 \text{ ft}$$

$$h(15) = -\frac{24}{225}(15)^2 + 2.7(15) + 3.5 \approx 10 \text{ ft}$$

Thus, some points on the graph are $(0, 3.5)$, $(5, 13.2)$, and $(15, 10)$. The complete graph is given below.



33. $h(x) = -\frac{44x^2}{v^2} + x + 6$

a.
$$\begin{aligned} h(8) &= -\frac{44(8)^2}{28^2} + (8) + 6 \\ &= -\frac{2816}{784} + 14 \\ &\approx 10.4 \text{ feet} \end{aligned}$$

b.
$$\begin{aligned} h(12) &= -\frac{44(12)^2}{28^2} + (12) + 6 \\ &= -\frac{6336}{784} + 18 \\ &\approx 9.9 \text{ feet} \end{aligned}$$

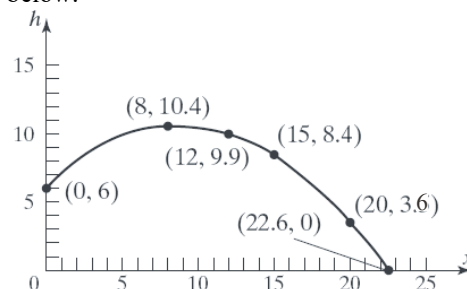
- c. From part (a) we know the point $(8, 10.4)$ is on the graph and from part (b) we know the point $(12, 9.9)$ is on the graph. We could evaluate the function at several more values of x (e.g. $x = 0$, $x = 15$, and $x = 20$) to obtain additional points.

$$h(0) = -\frac{44(0)^2}{28^2} + (0) + 6 = 6$$

$$h(15) = -\frac{44(15)^2}{28^2} + (15) + 6 \approx 8.4$$

$$h(20) = -\frac{44(20)^2}{28^2} + (20) + 6 \approx 3.6$$

Some additional points are $(0, 6)$, $(15, 8.4)$ and $(20, 3.6)$. The complete graph is given below.



d.
$$h(15) = -\frac{44(15)^2}{28^2} + (15) + 6 \approx 8.4 \text{ feet}$$

No; when the ball is 15 feet in front of the foul line, it will be below the hoop. Therefore it cannot go through the hoop.

In order for the ball to pass through the hoop, we need to have $h(15) = 10$.