Algebra for College Students 10th edition Lial Solutions Manual

SKU: 9780138173241-SOLUTIONS

INSTRUCTOR'S SOLUTIONS MANUAL

ALGEBRA FOR COLLEGE STUDENTS

TENTH EDITION

Lial/Hornsby/McGinnis/Daniels

Pearson

Pearson

Pearson Product ID: A103000322314

Table of Contents

Chapter R
Review of the Real Number System

R.1 Fractions

Classroom Examples, Now Try Exercises

1. 90 is composite and can be written as

$$\begin{tikzcd}[column sep=small] & 90 \arrow[dl] \arrow[dr] & & & \\ \boxed{2} & & 45 \arrow[dl] \arrow[dr] & & \\ & \boxed{3} & & 15 \arrow[dl] \arrow[dr] & \\ & & \boxed{3} & & \boxed{5} \end{tikzcd}$$

Writing 90 as the product of primes gives us $90 = 2\cdot3\cdot3\cdot5$.

N1. 60 is composite and can be written as

$$\begin{tikzcd}[column sep=small] & 60 \arrow[dl] \arrow[dr] & & & \\ \boxed{2} & & 30 \arrow[dl] \arrow[dr] & & \\ & \boxed{3} & & 10 \arrow[dl] \arrow[dr] & \\ & & \boxed{2} & & \boxed{5} \end{tikzcd}$$

Writing 60 as the product of primes gives us $60 = 2\cdot2\cdot3\cdot5$.

2. (a) $\frac{12}{20}=\frac{3\cdot4}{5\cdot4}=\frac{3}{5}\cdot\frac{4}{4}=\frac{3}{5}\cdot1=\frac{3}{5}$

(b) $\frac{8}{48}=\frac{8}{6\cdot8}=\frac{1}{6\cdot1}=\frac{1}{6}$

(c) $\frac{90}{162}=\frac{5\cdot18}{9\cdot18}=\frac{5}{9}\cdot1=\frac{5}{9}$

N2. (a) $\frac{30}{42}=\frac{5\cdot6}{7\cdot6}=\frac{5}{7}\cdot\frac{6}{6}=\frac{5}{7}\cdot1=\frac{5}{7}$

(b) $\frac{10}{70}=\frac{10}{7\cdot10}=\frac{1}{7\cdot1}\cdot=\frac{1}{7}$

(c) $\frac{72}{120}=\frac{3\cdot24}{5\cdot24}=\frac{3}{5}\cdot1=\frac{3}{5}$

3. The fraction bar represents division. Divide the numerator of the improper fraction by the denominator.

$$\begin{array}{r} 3 \\ 10\overline{)37} \\ \underline{30} \\ 7 \end{array}$$

Thus, $\frac{37}{10}=3\frac{7}{10}$.

N3. The fraction bar represents division. Divide the numerator of the improper fraction by the denominator.

$$\begin{array}{r} 18 \\ 5\overline{)92} \\ \underline{5} \\ 42 \\ \underline{40} \\ 2 \end{array}$$

Thus, $\frac{92}{5}=18\frac{2}{5}$.

4. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$5\cdot3=15$ and $15+4=19$
The denominator of the improper fraction is the same as the denominator in the mixed number.

Thus, $3\frac{4}{5}=\frac{19}{5}$.

N4. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$3\cdot11=33$ and $33+2=35$
The denominator of the improper fraction is the same as the denominator in the mixed number.

Thus, $11\frac{2}{3}=\frac{35}{3}$.

5. (a) To multiply two fractions, multiply their numerators and then multiply their denominators. Then simplify and write the answer in lowest terms.

$$\begin{aligned}\frac{5}{9}\cdot\frac{18}{25}&=\frac{5\cdot18}{9\cdot25}\\&=\frac{90}{225}\\&=\frac{2\cdot45}{5\cdot45}\\&=\frac{2}{5}\end{aligned}$$

(b) To multiply two mixed numbers, first write them as improper fractions. Multiply their numerators and then multiply their denominators. Then simplify and write the answer as a mixed number in lowest terms.

$$3\frac{1}{3}\cdot 1\frac{3}{4}=\frac{10}{3}\cdot\frac{7}{4}$$
$$=\frac{10\cdot 7}{3\cdot 4}$$
$$=\frac{2\cdot 5\cdot 7}{3\cdot 2\cdot 2}$$
$$=\frac{35}{6},\text{ or }5\frac{5}{6}$$

N5. **(a)** To multiply two fractions, multiply their numerators and then multiply their denominators. Then simplify and write the answer in lowest terms.

$$\frac{4}{7}\cdot\frac{5}{8}=\frac{4\cdot 5}{7\cdot 8}$$
$$=\frac{20}{56}$$
$$=\frac{5\cdot 4}{14\cdot 4}$$
$$=\frac{5}{14}$$

(b) To multiply two mixed numbers, first write them as improper fractions. Multiply their numerators and then multiply their denominators. Then simplify and write the answer as a mixed number in lowest terms.

$$3\frac{2}{5}\cdot 6\frac{2}{3}=\frac{17}{5}\cdot\frac{20}{3}$$
$$=\frac{17\cdot 20}{5\cdot 3}$$
$$=\frac{17\cdot 5\cdot 4}{5\cdot 3}$$
$$=\frac{68}{3},\text{ or }22\frac{2}{3}$$

6. **(a)** To divide fractions, multiply by the reciprocal of the divisor.

$$\frac{9}{10}\div\frac{3}{5}=\frac{9}{10}\cdot\frac{5}{3}$$
$$=\frac{3\cdot 3\cdot 5}{2\cdot 5\cdot 3}$$
$$=\frac{3}{2},\text{ or }1\frac{1}{2}$$

(b) Change both mixed numbers to improper fractions. Then multiply by the reciprocal of the second fraction.

$$2\frac{3}{4}\div 3\frac{1}{3}=\frac{11}{4}\div\frac{10}{3}$$
$$=\frac{11}{4}\cdot\frac{3}{10}$$
$$=\frac{33}{40}$$

N6. **(a)** To divide fractions, multiply by the reciprocal of the divisor.

$$\frac{2}{7}\div\frac{8}{9}=\frac{2}{7}\cdot\frac{9}{8}$$
$$=\frac{2\cdot 3\cdot 3}{7\cdot 2\cdot 4}$$
$$=\frac{9}{28}$$

(b) To divide fractions, multiply by the reciprocal of the divisor.

$$3\frac{3}{4}\div 4\frac{2}{7}=\frac{15}{4}\div\frac{30}{7}$$
$$=\frac{15}{4}\cdot\frac{7}{30}$$
$$=\frac{15\cdot 7}{4\cdot 2\cdot 15}$$
$$=\frac{7}{8}$$

7. **(a)** To find the sum of two fractions having the same denominator, add the numerators and keep the same denominator.

$$\frac{1}{9}+\frac{5}{9}=\frac{1+5}{9}$$
$$=\frac{6}{9}$$
$$=\frac{2\cdot 3}{3\cdot 3}$$
$$=\frac{2}{3}$$

(b) To find the difference of two fractions having the same denominator, subtract the numerators and keep the same denominator.

$$\frac{6}{7}-\frac{2}{7}=\frac{6-2}{7}$$
$$=\frac{4}{7}$$

N7. **(a)** To find the sum of two fractions having the same denominator, add the numerators and keep the same denominator.

$$\frac{1}{8}+\frac{3}{8}=\frac{1+3}{8}=\frac{4}{8}=\frac{1\cdot 4}{2\cdot 4}=\frac{1}{2}$$

(b) To find the difference of two fractions having the same denominator, subtract the numerators and keep the same denominator.

$$\frac{7}{9}-\frac{2}{9}=\frac{7-2}{9}=\frac{5}{9}$$

8. **(a)** Since $30=2\cdot 3\cdot 5$ and $45=3\cdot 3\cdot 5$, the least common denominator must have one factor of 2 (from 30), two factors of 3 (from 45), and one factor of 5 (from either 30 or 45), so it is $2\cdot 3\cdot 3\cdot 5=90$.
Write each fraction with a denominator of 90.

$$\frac{7}{30}=\frac{7}{30}\cdot\frac{3}{3}=\frac{21}{90} \text{ and } \frac{2}{45}=\frac{2}{45}\cdot\frac{2}{2}=\frac{4}{90}$$

Now add.

$$\frac{7}{30}+\frac{2}{45}=\frac{21}{90}+\frac{4}{90}=\frac{21+4}{90}=\frac{25}{90}$$

Write $\frac{25}{90}$ in lowest terms.

$$\frac{25}{90}=\frac{5\cdot 5}{18\cdot 5}=\frac{5}{18}$$

(b) Write each mixed number as an improper fraction.

$$4\frac{5}{6}+2\frac{1}{3}=\frac{29}{6}+\frac{7}{3}$$

The least common denominator is 6, so write each fraction with a denominator of 6.

$$\frac{29}{6} \text{ and } \frac{7}{3}=\frac{7}{3}\cdot\frac{2}{2}=\frac{14}{6}$$

Now add.

$$\frac{29}{6}+\frac{7}{3}=\frac{29}{6}+\frac{14}{6}=\frac{29+14}{6}=\frac{43}{6}, \text{ or } 7\frac{1}{6}$$

N8. **(a)** Since $12=2\cdot 2\cdot 3$ and $8=2\cdot 2\cdot 2$, the least common denominator must have three factors of 2 (from 8) and one factor of 3 (from 12), so it is $2\cdot 2\cdot 2\cdot 3=24$.
Write each fraction with a denominator of 24.

$$\frac{5}{12}=\frac{5}{12}\cdot\frac{2}{2}=\frac{10}{24} \text{ and } \frac{3}{8}=\frac{3}{8}\cdot\frac{3}{3}=\frac{9}{24}$$

Now add.

$$\frac{5}{12}+\frac{3}{8}=\frac{10}{24}+\frac{9}{24}=\frac{10+9}{24}=\frac{19}{24}$$

(b) Write each mixed number as an improper fraction.

$$3\frac{1}{4}+5\frac{5}{8}=\frac{13}{4}+\frac{45}{8}$$

The least common denominator is 8, so write each fraction with a denominator of 8.

$$\frac{45}{8} \text{ and } \frac{13}{4}=\frac{13}{4}\cdot\frac{2}{2}=\frac{26}{8}$$

Now add.

$$\frac{13}{4}+\frac{45}{8}=\frac{26}{8}+\frac{45}{8}=\frac{26+45}{8}=\frac{71}{8}, \text{ or } 8\frac{7}{8}$$

9. **(a)** Since $10=2\cdot 5$ and $4=2\cdot 2$, the least common denominator is $2\cdot 2\cdot 5=20$. Write each fraction with a denominator of 20.

$$\frac{3}{10}=\frac{3}{10}\cdot\frac{2}{2}=\frac{6}{20} \text{ and } \frac{1}{4}=\frac{1}{4}\cdot\frac{5}{5}=\frac{5}{20}$$

Now subtract.

$$\frac{3}{10}-\frac{1}{4}=\frac{6}{20}-\frac{5}{20}=\frac{1}{20}$$

(b) Write each mixed number as an improper fraction.

$$3\frac{3}{8}-1\frac{1}{2}=\frac{27}{8}-\frac{3}{2}$$

The least common denominator is 8. Write each fraction with a denominator of 8. $\frac{27}{8}$ remains unchanged, and $\frac{3}{2}=\frac{3}{2}\cdot\frac{4}{4}=\frac{12}{8}$.

Now subtract.

$$\frac{27}{8}-\frac{3}{2}=\frac{27}{8}-\frac{12}{8}=\frac{27-12}{8}=\frac{15}{8}, \text{ or } 1\frac{7}{8}$$

N9. **(a)** Since $11=11$ and $9=3\cdot 3$, the least common denominator is $3\cdot 3\cdot 11=99$. Write each fraction with a denominator of 99.

$$\frac{5}{11}=\frac{5}{11}\cdot\frac{9}{9}=\frac{45}{99} \text{ and } \frac{2}{9}=\frac{2}{9}\cdot\frac{11}{11}=\frac{22}{99}$$

Now subtract.

$$\frac{5}{11}-\frac{2}{9}=\frac{45}{99}-\frac{22}{99}=\frac{23}{99}$$

(b) Write each mixed number as an improper fraction.

$$4\frac{1}{3}-2\frac{5}{6}=\frac{13}{3}-\frac{17}{6}$$

The least common denominator is 6. Write each fraction with a denominator of 6. $\frac{17}{6}$ remains unchanged, and $\frac{13}{3}=\frac{13}{3}\cdot\frac{2}{2}=\frac{26}{6}$.

Now subtract.

$$\frac{13}{3}-\frac{17}{6}=\frac{26}{6}-\frac{17}{6}=\frac{26-17}{6}=\frac{9}{6}$$

Now reduce.

$$\frac{9}{6}=\frac{3\cdot 3}{2\cdot 3}=\frac{3}{2}, \text{ or } 1\frac{1}{2}$$

Exercises

1. True; the number above the fraction bar is called the numerator and the number below the fraction bar is called the denominator.

2. True; 5 divides the 31 six times with a remainder of one, so $\frac{31}{5}=6\frac{1}{5}$.

3. False; this is an improper fraction. Its value is 1.

4. False; the number 1 is neither prime nor composite.

5. False; the fraction $\frac{13}{39}$ can be written in lowest terms as $\frac{1}{3}$ since $\frac{13}{39}=\frac{13\cdot 1}{13\cdot 3}=\frac{1}{3}$.

6. False; the reciprocal of $\frac{6}{2}=3$ is $\frac{2}{6}=\frac{1}{3}$.

7. False; Factors of 12 are 1, 2, 3, 4, 6, 12. Factors of 18 are 1, 2, 3, 6, 9, 18. The GCF is 6.

8. False; $\frac{2}{3}=\frac{2}{3}\cdot\frac{3}{3}=\frac{6}{9}$, so $\frac{2}{3}$ is equivalent to $\frac{6}{9}$.

9. $\frac{16}{24}=\frac{2\cdot 8}{3\cdot 8}=\frac{2}{3}$

Therefore, C is correct.

10. Simplify each fraction to find which are equal to $\frac{5}{9}$.

$$\frac{15}{27}=\frac{3\cdot 5}{3\cdot 9}=\frac{5}{9}$$
$$\frac{30}{54}=\frac{6\cdot 5}{6\cdot 9}=\frac{5}{9}$$
$$\frac{40}{74}=\frac{2\cdot 20}{2\cdot 37}=\frac{20}{37}$$
$$\frac{55}{99}=\frac{11\cdot 5}{11\cdot 9}=\frac{5}{9}$$

Therefore, C is correct.

11. We need to multiply 8 by 3 to get 24 in the denominator, so we must multiply 5 by 3 as well.

$$\frac{5}{8}=\frac{5\cdot 3}{8\cdot 3}=\frac{15}{24}$$

Therefore, B is correct.

12. A common denominator for $\frac{p}{q}$ and $\frac{r}{s}$ must be a multiple of both denominators, q and s. Such a number is $q\cdot s$. Therefore, A is correct.

13. Since 19 has only itself and 1 as factors, it is a prime number.

14. Since 31 has only itself and 1 as factors, it is a prime number.

15.
$$\begin{aligned}30&=2\cdot 15\\&=2\cdot 3\cdot 5\end{aligned}$$

Since 30 has factors other than itself and 1, it is a composite number.

16.
$$\begin{aligned}50&=2\cdot 25\\&=2\cdot 5\cdot 5,\end{aligned}$$

so 50 is a composite number.

17.
$$\begin{aligned}64&=2\cdot 32\\&=2\cdot 2\cdot 16\\&=2\cdot 2\cdot 2\cdot 8\\&=2\cdot 2\cdot 2\cdot 2\cdot 4\\&=2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\end{aligned}$$

Since 64 has factors other than itself and 1, it is a composite number.

18.
$$\begin{aligned}81&=3\cdot 27\\&=3\cdot 3\cdot 9\\&=3\cdot 3\cdot 3\cdot 3\end{aligned}$$

Since 81 has factors other than itself and 1, it is a composite number.

19. As stated in the text, the number 1 is neither prime nor composite, by agreement.

20. The number 0 is not a natural number, so it is neither prime nor composite.

21. $57 = 3 \cdot 19$, so 57 is a composite number.

22. $51 = 3 \cdot 17$, so 51 is a composite number.

23. Since 79 has only itself and 1 as factors, it is a prime number.

24. Since 83 has only itself and 1 as factors, it is a prime number.

25. $$\begin{aligned} 124 &= 2 \cdot 62 \\ &= 2 \cdot 2 \cdot 31, \end{aligned}$$
so 124 is a composite number.

26. $$\begin{aligned} 138 &= 2 \cdot 69 \\ &= 2 \cdot 3 \cdot 23, \end{aligned}$$
so 138 is a composite number.

27. $$\begin{aligned} 500 &= 2 \cdot 250 \\ &= 2 \cdot 2 \cdot 125 \\ &= 2 \cdot 2 \cdot 5 \cdot 25 \\ &= 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5, \end{aligned}$$
so 500 is a composite number.

28. $$\begin{aligned} 700 &= 2 \cdot 350 \\ &= 2 \cdot 2 \cdot 175 \\ &= 2 \cdot 2 \cdot 5 \cdot 35 \\ &= 2 \cdot 2 \cdot 5 \cdot 5 \cdot 7, \end{aligned}$$
so 700 is a composite number.

29. $$\begin{aligned} 3458 &= 2 \cdot 1729 \\ &= 2 \cdot 7 \cdot 247 \\ &= 2 \cdot 7 \cdot 13 \cdot 19 \end{aligned}$$
Since 3458 has factors other than itself and 1, it is a composite number.

30. $$\begin{aligned} 1025 &= 5 \cdot 205 \\ &= 5 \cdot 5 \cdot 41 \end{aligned}$$
Since 1025 has factors other than itself and 1, it is a composite number.

31. $\frac{8}{16} = \frac{1 \cdot 8}{2 \cdot 8} = \frac{1}{2} \cdot \frac{8}{8} = \frac{1}{2} \cdot 1 = \frac{1}{2}$

32. $\frac{4}{12} = \frac{1 \cdot 4}{3 \cdot 4} = \frac{1}{3} \cdot \frac{4}{4} = \frac{1}{3} \cdot 1 = \frac{1}{3}$

33. $\frac{15}{18} = \frac{3 \cdot 5}{3 \cdot 6} = \frac{3}{3} \cdot \frac{5}{6} = 1 \cdot \frac{5}{6} = \frac{5}{6}$

34. $\frac{16}{20} = \frac{4 \cdot 4}{5 \cdot 4} = \frac{4}{5} \cdot \frac{4}{4} = \frac{4}{5} \cdot 1 = \frac{4}{5}$

35. $\frac{90}{150} = \frac{3 \cdot 30}{5 \cdot 30} = \frac{3}{5} \cdot \frac{30}{30} = \frac{3}{5} \cdot 1 = \frac{3}{5}$

36. $\frac{100}{140} = \frac{5 \cdot 20}{7 \cdot 20} = \frac{5}{7} \cdot \frac{20}{20} = \frac{5}{7} \cdot 1 = \frac{5}{7}$

37. $\frac{18}{90} = \frac{1 \cdot 18}{5 \cdot 18} = \frac{1}{5} \cdot \frac{18}{18} = \frac{1}{5} \cdot 1 = \frac{1}{5}$

38. $\frac{16}{64} = \frac{1 \cdot 16}{4 \cdot 16} = \frac{1}{4} \cdot \frac{16}{16} = \frac{1}{4} \cdot 1 = \frac{1}{4}$

39. $\frac{144}{120} = \frac{6 \cdot 24}{5 \cdot 24} = \frac{6}{5} \cdot \frac{24}{24} = \frac{6}{5} \cdot 1 = \frac{6}{5}$

40. $\frac{132}{77} = \frac{12 \cdot 11}{7 \cdot 11} = \frac{12}{7} \cdot \frac{11}{11} = \frac{12}{7} \cdot 1 = \frac{12}{7}$

41. $$\begin{array}{r} 1 \\ 7\overline{)12} \\ \underline{7} \\ 5 \end{array}$$

Therefore, $\frac{12}{7} = 1\frac{5}{7}$.

42. $$\begin{array}{r} 1 \\ 9\overline{)16} \\ \underline{9} \\ 7 \end{array}$$

Therefore, $\frac{16}{9} = 1\frac{7}{9}$.

43. $$\begin{array}{r} 6 \\ 12\overline{)77} \\ \underline{72} \\ 5 \end{array}$$

Therefore, $\frac{77}{12} = 6\frac{5}{12}$.

44. $$\begin{array}{r} 6 \\ 15\overline{)101} \\ \underline{90} \\ 11 \end{array}$$

Therefore, $\frac{101}{15} = 6\frac{11}{15}$.

45. $$\begin{array}{r} 7 \\ 11\overline{)83} \\ \underline{77} \\ 6 \end{array}$$

Therefore, $\frac{83}{11} = 7\frac{6}{11}$.

46. $$\begin{array}{r} 5 \\ 13\overline{)67} \\ \underline{65} \\ 2 \end{array}$$

Therefore, $\frac{67}{13} = 5\frac{2}{13}$.

47. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$5 \cdot 2 = 10$ and $10 + 3 = 13$
The denominator of the improper fraction is the same as the denominator in the mixed number.
Thus, $2\frac{3}{5} = \frac{13}{5}$.

48. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$7 \cdot 5 = 35$ and $35 + 6 = 41$
The denominator of the improper fraction is the same as the denominator in the mixed number.
Thus, $5\frac{6}{7} = \frac{41}{7}$.

49. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$8 \cdot 10 = 80$ and $80 + 3 = 83$
The denominator of the improper fraction is the same as the denominator in the mixed number.
Thus, $10\frac{3}{8} = \frac{83}{8}$.

50. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$3 \cdot 12 = 36$ and $36 + 2 = 38$
The denominator of the improper fraction is the same as the denominator in the mixed number.
Thus, $12\frac{2}{3} = \frac{38}{3}$.

51. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$5 \cdot 10 = 50$ and $50 + 1 = 51$
The denominator of the improper fraction is the same as the denominator in the mixed number.
Thus, $10\frac{1}{5} = \frac{51}{5}$.

52. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$6 \cdot 18 = 108$ and $108 + 1 = 109$
The denominator of the improper fraction is the same as the denominator in the mixed number.
Thus, $18\frac{1}{6} = \frac{109}{6}$.

53. $\frac{4}{5} \cdot \frac{6}{7} = \frac{4 \cdot 6}{5 \cdot 7} = \frac{24}{35}$

54. $\frac{5}{9} \cdot \frac{2}{7} = \frac{5 \cdot 2}{9 \cdot 7} = \frac{10}{63}$

55. $\frac{2}{15} \cdot \frac{3}{8} = \frac{2 \cdot 3}{15 \cdot 8} = \frac{6}{120} = \frac{1 \cdot 6}{20 \cdot 6} = \frac{1}{20}$

56. $\frac{3}{20} \cdot \frac{5}{21} = \frac{3 \cdot 5}{20 \cdot 21} = \frac{15}{420} = \frac{1 \cdot 15}{28 \cdot 15} = \frac{1}{28}$

57. $\frac{1}{10} \cdot \frac{12}{5} = \frac{1 \cdot 12}{10 \cdot 5} = \frac{1 \cdot 2 \cdot 6}{2 \cdot 5 \cdot 5} = \frac{6}{25}$

58. $\frac{1}{8} \cdot \frac{10}{7} = \frac{1 \cdot 10}{8 \cdot 7} = \frac{1 \cdot 2 \cdot 5}{2 \cdot 4 \cdot 7} = \frac{5}{28}$

59. $$\begin{aligned} \frac{15}{4} \cdot \frac{8}{25} &= \frac{15 \cdot 8}{4 \cdot 25} \\ &= \frac{3 \cdot 5 \cdot 4 \cdot 2}{4 \cdot 5 \cdot 5} \\ &= \frac{3 \cdot 2}{5} \\ &= \frac{6}{5}, \text{ or } 1\frac{1}{5} \end{aligned}$$

60. $$\begin{aligned} \frac{21}{8} \cdot \frac{4}{7} &= \frac{21 \cdot 4}{8 \cdot 7} \\ &= \frac{3 \cdot 7 \cdot 4}{4 \cdot 2 \cdot 7} \\ &= \frac{3}{2}, \text{ or } 1\frac{1}{2} \end{aligned}$$

61. $21 \cdot \frac{3}{7} = \frac{21}{1} \cdot \frac{3}{7}$
$= \frac{21 \cdot 3}{1 \cdot 7}$
$= \frac{3 \cdot 7 \cdot 3}{1 \cdot 7}$
$= \frac{3 \cdot 3}{1} = 9$

62. $36 \cdot \frac{4}{9} = \frac{36}{1} \cdot \frac{4}{9}$
$= \frac{36 \cdot 4}{1 \cdot 9}$
$= \frac{4 \cdot 9 \cdot 4}{1 \cdot 9}$
$= \frac{4 \cdot 4}{1} = 16$

63. Change both mixed numbers to improper fractions.
$3\frac{1}{4} \cdot 1\frac{2}{3} = \frac{13}{4} \cdot \frac{5}{3}$
$= \frac{13 \cdot 5}{4 \cdot 3}$
$= \frac{65}{12}$, or $5\frac{5}{12}$

64. Change both mixed numbers to improper fractions.
$2\frac{2}{3} \cdot 1\frac{3}{5} = \frac{8}{3} \cdot \frac{8}{5}$
$= \frac{8 \cdot 8}{3 \cdot 5}$
$= \frac{64}{15}$, or $4\frac{4}{15}$

65. Change both mixed numbers to improper fractions.
$2\frac{3}{8} \cdot 3\frac{1}{5} = \frac{19}{8} \cdot \frac{16}{5}$
$= \frac{19 \cdot 16}{8 \cdot 5}$
$= \frac{19 \cdot 2 \cdot 8}{8 \cdot 5}$
$= \frac{38}{5}$, or $7\frac{3}{5}$

66. Change both mixed numbers to improper fractions.
$3\frac{3}{5} \cdot 7\frac{1}{6} = \frac{18}{5} \cdot \frac{43}{6}$
$= \frac{18 \cdot 43}{5 \cdot 6}$
$= \frac{3 \cdot 6 \cdot 43}{5 \cdot 6}$
$= \frac{3 \cdot 43}{5}$
$= \frac{129}{5}$, or $25\frac{4}{5}$

67. Change both numbers to improper fractions.
$5 \cdot 2\frac{1}{10} = \frac{5}{1} \cdot \frac{21}{10}$
$= \frac{5 \cdot 21}{1 \cdot 10}$
$= \frac{5 \cdot 21}{1 \cdot 2 \cdot 5}$
$= \frac{21}{1 \cdot 2}$
$= \frac{21}{2}$, or $10\frac{1}{2}$

68. Change both numbers to improper fractions.
$3 \cdot 4\frac{2}{9} = \frac{3}{1} \cdot \frac{38}{9}$
$= \frac{3 \cdot 38}{1 \cdot 9}$
$= \frac{3 \cdot 38}{1 \cdot 3 \cdot 3}$
$= \frac{38}{1 \cdot 3}$
$= \frac{38}{3}$, or $12\frac{2}{3}$

69. To divide fractions, multiply by the reciprocal of the divisor.
$\frac{7}{9} \div \frac{3}{2} = \frac{7}{9} \cdot \frac{2}{3}$
$= \frac{7 \cdot 2}{9 \cdot 3}$
$= \frac{14}{27}$

70. To divide fractions, multiply by the reciprocal of the divisor.

$$\frac{6}{11} \div \frac{5}{4} = \frac{6}{11} \cdot \frac{4}{5} = \frac{6 \cdot 4}{11 \cdot 5} = \frac{24}{55}$$

71. To divide fractions, multiply by the reciprocal of the divisor.

$$\frac{5}{4} \div \frac{3}{8} = \frac{5}{4} \cdot \frac{8}{3} = \frac{5 \cdot 8}{4 \cdot 3} = \frac{5 \cdot 4 \cdot 2}{4 \cdot 3} = \frac{5 \cdot 2}{3} = \frac{10}{3}, \text{ or } 3\frac{1}{3}$$

72. To divide fractions, multiply by the reciprocal of the divisor.

$$\frac{7}{5} \div \frac{3}{10} = \frac{7}{5} \cdot \frac{10}{3} = \frac{7 \cdot 10}{5 \cdot 3} = \frac{7 \cdot 2 \cdot 5}{5 \cdot 3} = \frac{14}{3}, \text{ or } 4\frac{2}{3}$$

73. To divide fractions, multiply by the reciprocal of the divisor.

$$\frac{32}{5} \div \frac{8}{15} = \frac{32}{5} \cdot \frac{15}{8} = \frac{32 \cdot 15}{5 \cdot 8} = \frac{8 \cdot 4 \cdot 3 \cdot 5}{1 \cdot 5 \cdot 8} = \frac{4 \cdot 3}{1} = 12$$

74. To divide fractions, multiply by the reciprocal of the divisor.

$$\frac{24}{7} \div \frac{6}{21} = \frac{24}{7} \cdot \frac{21}{6} = \frac{24 \cdot 21}{7 \cdot 6} = \frac{4 \cdot 6 \cdot 3 \cdot 7}{1 \cdot 7 \cdot 6} = \frac{4 \cdot 3}{1} = 12$$

75. To divide fractions, multiply by the reciprocal of the divisor.

$$\frac{3}{4} \div 12 = \frac{3}{4} \cdot \frac{1}{12} = \frac{3 \cdot 1}{4 \cdot 12} = \frac{3 \cdot 1}{4 \cdot 3 \cdot 4} = \frac{1}{4 \cdot 4} = \frac{1}{16}$$

76. To divide fractions, multiply by the reciprocal of the divisor.

$$\frac{2}{5} \div 30 = \frac{2}{5} \cdot \frac{1}{30} = \frac{2 \cdot 1}{5 \cdot 30} = \frac{2 \cdot 1}{5 \cdot 2 \cdot 15} = \frac{1}{5 \cdot 15} = \frac{1}{75}$$

77. To divide fractions, multiply by the reciprocal of the divisor.

$$6 \div \frac{3}{5} = \frac{6}{1} \cdot \frac{5}{3} = \frac{6 \cdot 5}{1 \cdot 3} = \frac{2 \cdot 3 \cdot 5}{1 \cdot 3} = \frac{2 \cdot 5}{1} = 10$$

78. To divide fractions, multiply by the reciprocal of the divisor.

$$\begin{aligned} 8 \div \frac{4}{9} &= \frac{8}{1} \cdot \frac{9}{4} \\ &= \frac{8 \cdot 9}{1 \cdot 4} \\ &= \frac{2 \cdot 4 \cdot 9}{1 \cdot 4} \\ &= \frac{2 \cdot 9}{1} = 18 \end{aligned}$$

79. Change the first number to an improper fraction, and then multiply by the reciprocal of the divisor.

$$\begin{aligned} 6\frac{3}{4} \div \frac{3}{8} &= \frac{27}{4} \div \frac{3}{8} \\ &= \frac{27}{4} \cdot \frac{8}{3} \\ &= \frac{27 \cdot 8}{4 \cdot 3} \\ &= \frac{3 \cdot 9 \cdot 2 \cdot 4}{4 \cdot 3} \\ &= \frac{9 \cdot 2}{1} = 18 \end{aligned}$$

80. Change the first number to an improper fraction, and then multiply by the reciprocal of the divisor.

$$\begin{aligned} 5\frac{3}{5} \div \frac{7}{10} &= \frac{28}{5} \div \frac{7}{10} \\ &= \frac{28}{5} \cdot \frac{10}{7} \\ &= \frac{28 \cdot 10}{5 \cdot 7} \\ &= \frac{4 \cdot 7 \cdot 2 \cdot 5}{5 \cdot 7} \\ &= \frac{4 \cdot 2}{1} = 8 \end{aligned}$$

81. Change both mixed numbers to improper fractions, and then multiply by the reciprocal of the divisor.

$$\begin{aligned} 2\frac{1}{2} \div 1\frac{5}{7} &= \frac{5}{2} \div \frac{12}{7} \\ &= \frac{5}{2} \cdot \frac{7}{12} \\ &= \frac{5 \cdot 7}{2 \cdot 12} \\ &= \frac{35}{24}, \text{ or } 1\frac{11}{24} \end{aligned}$$

82. Change both mixed numbers to improper fractions, and then multiply by the reciprocal of the divisor.

$$\begin{aligned} 2\frac{2}{9} \div 1\frac{2}{5} &= \frac{20}{9} \div \frac{7}{5} \\ &= \frac{20}{9} \cdot \frac{5}{7} \\ &= \frac{20 \cdot 5}{9 \cdot 7} \\ &= \frac{100}{63}, \text{ or } 1\frac{37}{63} \end{aligned}$$

83. Change both mixed numbers to improper fractions, and then multiply by the reciprocal of the divisor.

$$\begin{aligned} 2\frac{5}{8} \div 1\frac{15}{32} &= \frac{21}{8} \div \frac{47}{32} \\ &= \frac{21}{8} \cdot \frac{32}{47} \\ &= \frac{21 \cdot 32}{8 \cdot 47} \\ &= \frac{21 \cdot 8 \cdot 4}{8 \cdot 47} \\ &= \frac{21 \cdot 4}{47} \\ &= \frac{84}{47}, \text{ or } 1\frac{37}{47} \end{aligned}$$

84. Change both mixed numbers to improper fractions, and then multiply by the reciprocal of the divisor.

$$\begin{aligned} 2\frac{3}{10} \div 1\frac{4}{5} &= \frac{23}{10} \div \frac{9}{5} \\ &= \frac{23}{10} \cdot \frac{5}{9} \\ &= \frac{23 \cdot 5}{2 \cdot 5 \cdot 9} \\ &= \frac{23}{18}, \text{ or } 1\frac{5}{8} \end{aligned}$$

85. $\frac{7}{15} + \frac{4}{15} = \frac{7+4}{15} = \frac{11}{15}$

86. $\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9} = \frac{7}{9}$

87. $\dfrac{7}{12}+\dfrac{1}{12}=\dfrac{7+1}{12}$
$=\dfrac{8}{12}$
$=\dfrac{2\cdot 4}{3\cdot 4}$
$=\dfrac{2}{3}$

88. $\dfrac{3}{16}+\dfrac{5}{16}=\dfrac{3+5}{16}=\dfrac{8}{16}=\dfrac{1}{2}$

89. Since $9=3\cdot 3$, and 3 is prime, the LCD (least common denominator) is $3\cdot 3=9$.
$\dfrac{1}{3}=\dfrac{1}{3}\cdot\dfrac{3}{3}=\dfrac{3}{9}$
Now add the two fractions with the same denominator.
$\dfrac{5}{9}+\dfrac{1}{3}=\dfrac{5}{9}+\dfrac{3}{9}=\dfrac{8}{9}$

90. To add $\dfrac{4}{15}$ and $\dfrac{1}{5}$, first find the LCD. Since $15=3\cdot 5$ and 5 is prime, the LCD is 15.
$\dfrac{4}{15}+\dfrac{1}{5}=\dfrac{4}{15}+\dfrac{1}{5}\cdot\dfrac{3}{3}$
$=\dfrac{4}{15}+\dfrac{3}{15}$
$=\dfrac{4+3}{15}$
$=\dfrac{7}{15}$

91. Since $8=2\cdot 2\cdot 2$ and $6=2\cdot 3$, the LCD is $2\cdot 2\cdot 2\cdot 3=24$.
$\dfrac{3}{8}=\dfrac{3}{8}\cdot\dfrac{3}{3}=\dfrac{9}{24}$ and $\dfrac{5}{6}=\dfrac{5}{6}\cdot\dfrac{4}{4}=\dfrac{20}{24}$
Now add fractions with the same denominator.
$\dfrac{3}{8}+\dfrac{5}{6}=\dfrac{9}{24}+\dfrac{20}{24}=\dfrac{29}{24}$, or $1\dfrac{5}{24}$

92. Since $6=2\cdot 3$ and $9=3\cdot 3$, the LCD is $2\cdot 3\cdot 3=18$.
$\dfrac{5}{6}=\dfrac{5}{6}\cdot\dfrac{3}{3}=\dfrac{15}{18}$ and $\dfrac{2}{9}=\dfrac{2}{9}\cdot\dfrac{2}{2}=\dfrac{4}{18}$
Now add fractions with the same denominator.
$\dfrac{5}{6}+\dfrac{2}{9}=\dfrac{15}{18}+\dfrac{4}{18}=\dfrac{19}{18}$, or $1\dfrac{1}{18}$

93. Since $9=3\cdot 3$ and $16=4\cdot 4$, the LCD is $3\cdot 3\cdot 4\cdot 4=144$.
$\dfrac{5}{9}=\dfrac{5}{9}\cdot\dfrac{16}{16}=\dfrac{80}{144}$ and $\dfrac{3}{16}=\dfrac{3}{16}\cdot\dfrac{9}{9}=\dfrac{27}{144}$
Now add fractions with the same denominator.
$\dfrac{5}{9}+\dfrac{3}{16}=\dfrac{80}{144}+\dfrac{27}{144}=\dfrac{107}{144}$

94. Since $4=2\cdot 2$ and $25=5\cdot 5$, the LCD is $2\cdot 2\cdot 5\cdot 5=100$.
$\dfrac{3}{4}=\dfrac{3}{4}\cdot\dfrac{25}{25}=\dfrac{75}{100}$ and $\dfrac{6}{25}=\dfrac{6}{25}\cdot\dfrac{4}{4}=\dfrac{24}{100}$
Now add fractions with the same denominator.
$\dfrac{3}{4}+\dfrac{6}{25}=\dfrac{75}{100}+\dfrac{24}{100}=\dfrac{99}{100}$

95. $3\dfrac{1}{8}=3+\dfrac{1}{8}=\dfrac{24}{8}+\dfrac{1}{8}=\dfrac{25}{8}$
$2\dfrac{1}{4}=2+\dfrac{1}{4}=\dfrac{8}{4}+\dfrac{1}{4}=\dfrac{9}{4}$
$3\dfrac{1}{8}+2\dfrac{1}{4}=\dfrac{25}{8}+\dfrac{9}{4}$
Since $8=2\cdot 2\cdot 2$ and $4=2\cdot 2$, the LCD is $2\cdot 2\cdot 2$ or 8.
$3\dfrac{1}{8}+2\dfrac{1}{4}=\dfrac{25}{8}+\dfrac{9}{4}\cdot\dfrac{2}{2}$
$=\dfrac{25}{8}+\dfrac{18}{8}$
$=\dfrac{43}{8}$, or $5\dfrac{3}{8}$

96. $4\dfrac{2}{3}=4+\dfrac{2}{3}=\dfrac{12}{3}+\dfrac{2}{3}=\dfrac{14}{3}$
$2\dfrac{1}{6}=2+\dfrac{1}{6}=\dfrac{12}{6}+\dfrac{1}{6}=\dfrac{13}{6}$
Since $6=2\cdot 3$, the LCD is 6.
$4\dfrac{2}{3}+2\dfrac{1}{6}=\dfrac{14}{3}\cdot\dfrac{2}{2}+\dfrac{13}{6}$
$=\dfrac{28}{6}+\dfrac{13}{6}$
$=\dfrac{41}{6}$, or $6\dfrac{5}{6}$

97. $3\dfrac{1}{4}=3+\dfrac{1}{4}=\dfrac{12}{4}+\dfrac{1}{4}=\dfrac{13}{4}$
$1\dfrac{4}{5}=1+\dfrac{4}{5}=\dfrac{5}{5}+\dfrac{4}{5}=\dfrac{9}{5}$

Since $4 = 2 \cdot 2$, and 5 is prime, the LCD is $2 \cdot 2 \cdot 5 = 20$.

$$3\frac{1}{4} + 1\frac{4}{5} = \frac{13}{4} \cdot \frac{5}{5} + \frac{9}{5} \cdot \frac{4}{4}$$
$$= \frac{65}{20} + \frac{36}{20}$$
$$= \frac{101}{20}, \text{ or } 5\frac{1}{20}$$

98. To add $5\frac{3}{4}$ and $1\frac{1}{3}$, first change to improper fractions then find the LCD, which is 12.

$$5\frac{3}{4} + 1\frac{1}{3} = \frac{23}{4} + \frac{4}{3}$$
$$= \frac{23}{4} \cdot \frac{3}{3} + \frac{4}{3} \cdot \frac{4}{4}$$
$$= \frac{69}{12} + \frac{16}{12}$$
$$= \frac{85}{12}, \text{ or } 7\frac{1}{12}$$

99. $\frac{7}{9} - \frac{2}{9} = \frac{7-2}{9} = \frac{5}{9}$

100. $\frac{8}{11} - \frac{3}{11} = \frac{8-3}{11} = \frac{5}{11}$

101.
$$\frac{13}{15} - \frac{3}{15} = \frac{13-3}{15}$$
$$= \frac{10}{15}$$
$$= \frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3}$$

102.
$$\frac{11}{12} - \frac{3}{12} = \frac{11-3}{12}$$
$$= \frac{8}{12}$$
$$= \frac{2 \cdot 4}{3 \cdot 4} = \frac{2}{3}$$

103. Since $12 = 4 \cdot 3$ (12 is a multiple of 3), the LCD is 12.

$$\frac{1}{3} \cdot \frac{4}{4} = \frac{4}{12}$$

Now subtract fractions with the same denominator.

$$\frac{7}{12} - \frac{1}{3} = \frac{7}{12} - \frac{4}{12} = \frac{3}{12} = \frac{1 \cdot 3}{4 \cdot 3} = \frac{1}{4}$$

104. Since $6 = 3 \cdot 2$ (6 is a multiple of 2), the LCD is 6.

$$\frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6}$$

Now subtract fractions with the same denominator.

$$\frac{5}{6} - \frac{1}{2} = \frac{5}{6} - \frac{3}{6} = \frac{2}{6} = \frac{1 \cdot 2}{3 \cdot 2} = \frac{1}{3}$$

105. Since $12 = 2 \cdot 2 \cdot 3$ and $9 = 3 \cdot 3$, the LCD is $2 \cdot 2 \cdot 3 \cdot 3 = 36$.

$$\frac{7}{12} = \frac{7}{12} \cdot \frac{3}{3} = \frac{21}{36} \text{ and } \frac{1}{9} \cdot \frac{4}{4} = \frac{4}{36}$$

Now subtract fractions with the same denominator.

$$\frac{7}{12} - \frac{1}{9} = \frac{21}{36} - \frac{4}{36} = \frac{17}{36}$$

106. $\frac{11}{16} - \frac{1}{12} = \frac{11}{16} \cdot \frac{3}{3} - \frac{1}{12} \cdot \frac{4}{4}$ The LCD of 12 and 16 is 48.

$$= \frac{33}{48} - \frac{4}{48}$$
$$= \frac{29}{48}$$

107.
$$4\frac{3}{4} = 4 + \frac{3}{4} = \frac{16}{4} + \frac{3}{4} = \frac{19}{4}$$
$$1\frac{2}{5} = 1 + \frac{2}{5} = \frac{5}{5} + \frac{2}{5} = \frac{7}{5}$$

Since $4 = 2 \cdot 2$, and 5 is prime, the LCD is $2 \cdot 2 \cdot 5 = 20$.

$$4\frac{3}{4} - 1\frac{2}{5} = \frac{19}{4} \cdot \frac{5}{5} - \frac{7}{5} \cdot \frac{4}{4}$$
$$= \frac{95}{20} - \frac{28}{20}$$
$$= \frac{67}{20}, \text{ or } 3\frac{7}{20}$$

108. Change both numbers to improper fractions then add, using 45 as the common denominator.

$$3\frac{4}{5} - 1\frac{4}{9} = \frac{19}{5} - \frac{13}{9}$$
$$= \frac{19}{5} \cdot \frac{9}{9} - \frac{13}{9} \cdot \frac{5}{5}$$
$$= \frac{171}{45} - \frac{65}{45}$$
$$= \frac{106}{45}, \text{ or } 2\frac{16}{45}$$

109. $6\frac{1}{4}=6+\frac{1}{4}=\frac{24}{4}+\frac{1}{4}=\frac{25}{4}$

$5\frac{1}{3}=5+\frac{1}{3}=\frac{15}{3}+\frac{1}{3}=\frac{16}{3}$

Since $4=2\cdot 2$, and 3 is prime, the LCD is $2\cdot 2\cdot 3=12.$

$$\begin{aligned}6\frac{1}{4}-5\frac{1}{3}&=\frac{25}{4}-\frac{16}{3}\\&=\frac{25}{4}\cdot\frac{3}{3}-\frac{16}{3}\cdot\frac{4}{4}\\&=\frac{75}{12}-\frac{64}{12}\\&=\frac{11}{12}\end{aligned}$$

110. $5\frac{1}{3}=5+\frac{1}{3}=\frac{15}{3}+\frac{1}{3}=\frac{16}{3}$

$4\frac{1}{2}=4+\frac{1}{2}=\frac{8}{2}+\frac{1}{2}=\frac{9}{2}$

2 and 3 are prime, so the LCD is $2\cdot 3=6.$

$$\begin{aligned}5\frac{1}{3}-4\frac{1}{2}&=\frac{16}{3}\cdot\frac{2}{2}-\frac{9}{2}\cdot\frac{3}{3}\\&=\frac{32}{6}-\frac{27}{6}\\&=\frac{5}{6}\end{aligned}$$

111. $8\frac{2}{9}=8+\frac{2}{9}=\frac{72}{9}+\frac{2}{9}=\frac{74}{9}$

$4\frac{2}{3}=4+\frac{2}{3}=\frac{12}{3}+\frac{2}{3}=\frac{14}{3}$

Since $9=3\cdot 3$, and 3 is prime, the LCD is $3\cdot 3=9.$

$$\begin{aligned}8\frac{2}{9}-4\frac{2}{3}&=\frac{74}{9}-\frac{14}{3}\cdot\frac{3}{3}\\&=\frac{74}{9}-\frac{42}{9}\\&=\frac{32}{9}\text{, or }3\frac{5}{9}\end{aligned}$$

112. $7\frac{5}{12}=7+\frac{5}{12}=\frac{84}{12}+\frac{5}{12}=\frac{89}{12}$

$4\frac{5}{6}=4+\frac{5}{6}=\frac{24}{6}+\frac{5}{6}=\frac{29}{6}$

Since $12=2\cdot 2\cdot 3$ and $6=2\cdot 3$, the LCD is $2\cdot 2\cdot 3=12.$

$$\begin{aligned}7\frac{5}{12}-4\frac{5}{6}&=\frac{89}{12}-\frac{29}{6}\cdot\frac{2}{2}\\&=\frac{89}{12}-\frac{58}{12}\\&=\frac{31}{12}\text{, or }2\frac{7}{12}\end{aligned}$$

113. Observe that there are 24 dots in the entire figure, 6 dots in the triangle, 12 dots in the rectangle, and 2 dots in the overlapping region.

(a) $\frac{12}{24}=\frac{1}{2}$ of all the dots are in the rectangle.

(b) $\frac{6}{24}=\frac{1}{4}$ of all the dots are in the triangle.

(c) $\frac{2}{6}=\frac{1}{3}$ of the dots in the triangle are in the overlapping region.

(d) $\frac{2}{12}=\frac{1}{6}$ of the dots in the rectangle are in the overlapping region.

114. **(a)** 12 is $\frac{1}{3}$ of 36, so Maya got a hit in exactly $\frac{1}{3}$ of her at-bats.

(b) 1 is a little less than $\frac{1}{10}$ of 11, so Deion got a home run in just less than $\frac{1}{10}$ of his at-bats.

(c) 9 is a little less than $\frac{1}{4}$ of 40, so Christine got a hit in just less than $\frac{1}{4}$ of her at-bats.

(d) 8 is $\frac{1}{2}$ of 16, and 10 is $\frac{1}{2}$ of 20, so Joe and Greg each got hits $\frac{1}{2}$ of the time they were at bat.

R.2 Decimals and Percents

Classroom Examples, Now Try Exercises

1. **(a)** $0.15 = \frac{15}{100}$

(b) $0.009 = \frac{9}{1000}$

(c) $2.5 = 2\frac{5}{10} = \frac{25}{10}$

N1. **(a)** $0.8 = \frac{8}{10}$

(b) $0.431 = \frac{431}{1000}$

(c) $2.58 = 2\frac{58}{100} = \frac{258}{100}$

2. **(a)**
$$\begin{array}{r} 42.830 \\ 71.000 \\ +\ \ 3.074 \\ \hline 116.904 \end{array}$$

(b)
$$\begin{array}{r} 32.50 \\ -\,21.72 \\ \hline 10.78 \end{array}$$

(c)
$$\begin{array}{r} 10.000 \\ -\,0.125 \\ \hline 9.875 \end{array}$$

N2. **(a)**
$$\begin{array}{r} 68.900 \\ 42.720 \\ +\ \ 8.973 \\ \hline 120.593 \end{array}$$

(b)
$$\begin{array}{r} 351.800 \\ -\quad 2.706 \\ \hline 349.094 \end{array}$$

(c)
$$\begin{array}{r} 5.000 \\ -\,0.375 \\ \hline 4.625 \end{array}$$

3. **(a)**
$$\begin{array}{rr} 30.2 & 1 \text{ decimal place} \\ \times 0.052 & 3 \text{ decimal places} \\ \hline 604 & \downarrow \\ 1510\ & 1+3=4 \\ \hline 1.5704 & 4 \text{ decimal places} \end{array}$$

(b)
$$\begin{array}{rr} 0.06 & 2 \text{ decimal places} \\ \times 0.12 & 2 \text{ decimal places} \\ \hline 12 & \downarrow \\ 6\ & 2+2=4 \\ \hline 0.0072 & 4 \text{ decimal places} \end{array}$$

(c) To change the divisor 3.1 into a whole number, move each decimal point one place to the right. Move the decimal point straight up and divide as with whole numbers.

$$\begin{array}{r} 1.21 \\ 31\overline{)37.60} \\ \underline{31} \\ 66 \\ \underline{62} \\ 40 \\ \underline{31} \\ 9 \end{array}$$

We carried out the division to 2 decimal places so that we could round to 1 decimal place. Therefore, $3.76 \div 3.1 \approx 1.2$.

N3. **(a)**
$$\begin{array}{rr} 9.32 & 2 \text{ decimal places} \\ \times\ 1.4 & 1 \text{ decimal place} \\ \hline 3728 & \downarrow \\ 932\ & 2+1=3 \\ \hline 13.048 & 3 \text{ decimal places} \end{array}$$

(b)
$$\begin{array}{rr} 0.6 & 1 \text{ decimal place} \\ \times 0.004 & 3 \text{ decimal places} \\ \hline 24 & 1+3=4 \\ \hline 0.0024 & 4 \text{ decimal places} \end{array}$$

(c) To change the divisor 1.3 into a whole number, move each decimal point one place to the right. Move the decimal point straight up and divide as with whole numbers.

$$\begin{array}{r}
5.641 \\
13\overline{)73.340} \\
\underline{65} \\
83 \\
\underline{78} \\
54 \\
\underline{52} \\
20 \\
\underline{13} \\
7
\end{array}$$

We carried out the division to 3 decimal places so that we could round to 2 decimal places. Therefore, $7.334 \div 1.3 \approx 5.64$.

4. **(a)** Move the decimal point three places to the right.
$19.5 \times 1000 = 19,500$

(b) Move the decimal point one place to the left.
$960.1 \div 10 = 96.01$

N4. **(a)** Move the decimal point one place to the right.
$294.72 \times 10 = 2947.2$

(b) Move the decimal point two places to the left. Insert a 0 in front of the 4 to do this.
$4.793 \div 100 = 0.04793$

5. **(a)** Divide 3 by 50. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r}
0.06 \\
50\overline{)3.00} \\
\underline{3\ 00} \\
0
\end{array}$$

Therefore, $\frac{3}{50} = 0.06$.

(b) Divide 11 by 1. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r}
0.090909... \\
11\overline{)1.000000...} \\
\underline{99} \\
100 \\
\underline{99} \\
100 \\
\underline{99} \\
1
\end{array}$$

When rounded to the nearest thousandth, this is 0.091. When rounded to the nearest hundredth, this is 0.09. When rounded to the nearest tenth, this is 0.1. Note that the pattern repeats. Therefore, $\frac{1}{11} = 0.\overline{09}$,

N5. **(a)** Divide 20 by 17. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r}
0.85 \\
20\overline{)17.00} \\
\underline{160} \\
100 \\
\underline{100} \\
0
\end{array}$$

Therefore, $\frac{17}{20} = 0.85$.

(b) Divide 2 by 9. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r}
0.222... \\
9\overline{)2.000...} \\
\underline{18} \\
20 \\
\underline{18} \\
20 \\
\underline{18} \\
2
\end{array}$$

When rounded to the nearest thousandth, this is 0.222. When rounded to the nearest hundredth, this is 0.22. When rounded to the nearest tenth, this is 0.2. Note that the pattern repeats. Therefore, $\frac{2}{9} = 0.\overline{2}$.

6. **(a)** $5\frac{1}{4}\% = 5.25\%$
$$= \frac{5.25}{100}$$
$$= 0.0525$$

(b) $200\% = \frac{200}{100} = 2.00$, or 2

N6. **(a)** $23\% = \frac{23}{100} = 0.23$

(b) $350\% = \frac{350}{100} = 3.50$, or 3.5

7. **(a)** $0.06 = 0.06 \cdot 100\% = 6\%$

(b) $1.75 = 1.75 \cdot 100\% = 175\%$

N7. **(a)** $0.31 = 0.31 \cdot 100\% = 31\%$

(b) $1.32 = 1.32 \cdot 100\% = 132\%$

8. **(a)** $85\% = 0.85$

(b) $110\% = 1.10$, or 1.1

(c) $0.30 = 30\%$

(d) $0.165 = 16.5\%$

N8. **(a)** $52\% = 0.52$

(b) $2\% = 02\% = 0.02$

(c) $0.45 = 45\%$

(d) $3.5 = 3.50 = 350\%$

9. **(a)** $65\% = \frac{65}{100}$

In lowest terms,

$$\frac{65}{100} = \frac{13 \cdot 5}{20 \cdot 5} = \frac{13}{20}$$

(b) $1.5\% = \frac{1.5}{100} = \frac{1.5}{100} \cdot \frac{10}{10} = \frac{15}{1000} = \frac{3}{200}$

N9. **(a)** $20\% = \frac{20}{100}$

In lowest terms,

$$\frac{20}{100} = \frac{1 \cdot 20}{5 \cdot 20} = \frac{1}{5}$$

(b) $160\% = \frac{160}{100}$

In lowest terms,

$$\frac{160}{100} = \frac{8 \cdot 20}{5 \cdot 20} = \frac{8}{5}, \text{ or } 1\frac{3}{5}$$

10. **(a)**
$$\begin{aligned} \frac{3}{50} &= \frac{3}{50} \cdot 100\% \\ &= \frac{3}{50} \cdot \frac{100}{1}\% \\ &= \frac{3 \cdot 50 \cdot 2}{50}\% \\ &= 6\% \end{aligned}$$

(b)
$$\begin{aligned} \frac{1}{3} &= \frac{1}{3} \cdot 100\% \\ &= \frac{1}{3} \cdot \frac{100}{1}\% \\ &= \frac{100}{3}\% \\ &= 33\frac{1}{3}\%, \text{ or } 33.\overline{3}\% \end{aligned}$$

N10. **(a)**
$$\begin{aligned} \frac{6}{25} &= \frac{6}{25} \cdot 100\% \\ &= \frac{6}{25} \cdot \frac{100}{1}\% \\ &= \frac{6 \cdot 25 \cdot 4}{25}\% \\ &= 24\% \end{aligned}$$

(b)
$$\begin{aligned} \frac{7}{9} &= \frac{7}{9} \cdot 100\% \\ &= \frac{7}{9} \cdot \frac{100}{1}\% \\ &= \frac{700}{9}\% \\ &= 77\frac{7}{9}\%, \text{ or } 77.\overline{7}\% \end{aligned}$$

Exercises

1. 367.9412

(a) Tens: 6

(b) Tenths: 9

(c) Thousandths: 1

(d) Ones: 7

(e) Hundredths: 4

2. Answers will vary. One example is 5243.0164.

3. 46.249

(a) 46.25

(b) 46.2

(c) 46

(d) 50

4. **(a)** 0.976

(b) 0.865

(c) 0.889

(d) 0.444

5. $0.4 = \frac{4}{10}$

6. $0.6 = \frac{6}{10}$

7. $0.64 = \frac{64}{100}$

8. $0.82 = \dfrac{82}{100}$

9. $0.138 = \dfrac{138}{1000}$

10. $0.104 = \dfrac{104}{1000}$

11. $0.043 = \dfrac{43}{1000}$

12. $0.087 = \dfrac{87}{1000}$

13. $3.805 = 3\dfrac{805}{1000} = \dfrac{3805}{1000}$

14. $5.166 = 5\dfrac{166}{1000} = \dfrac{5166}{1000}$

15.
$$\begin{array}{r} 25.320 \\ 109.200 \\ +\ \ 8.574 \\ \hline 143.094 \end{array}$$

16.
$$\begin{array}{r} 90.527 \\ 32.430 \\ +589.800 \\ \hline 712.757 \end{array}$$

17.
$$\begin{array}{r} 28.73 \\ -\ 3.12 \\ \hline 25.61 \end{array}$$

18.
$$\begin{array}{r} 46.88 \\ -13.45 \\ \hline 33.43 \end{array}$$

19.
$$\begin{array}{r} 43.50 \\ -28.17 \\ \hline 15.33 \end{array}$$

20.
$$\begin{array}{r} 345.10 \\ -\ 56.31 \\ \hline 288.79 \end{array}$$

21.
$$\begin{array}{r} 3.87 \\ 15.00 \\ +\ 2.90 \\ \hline 21.77 \end{array}$$

22.
$$\begin{array}{r} 8.20 \\ 1.09 \\ +12.00 \\ \hline 21.29 \end{array}$$

23.
$$\begin{array}{r} 32.560 \\ 47.356 \\ +\ 1.800 \\ \hline 81.716 \end{array}$$

24.
$$\begin{array}{r} 75.200 \\ 123.960 \\ +\ \ 3.897 \\ \hline 203.057 \end{array}$$

25.
$$\begin{array}{r} 18.000 \\ -\ 2.789 \\ \hline 15.211 \end{array}$$

26.
$$\begin{array}{r} 29.000 \\ -\ 8.582 \\ \hline 20.418 \end{array}$$

27.
$$\begin{array}{rr} 12.8 & \text{1 decimal place} \\ \times\ 9.1 & \text{1 decimal place} \\ \hline 128 & \downarrow \\ 1152 & 1+1=2 \\ \hline 116.48 & \text{2 decimal places} \end{array}$$

28.
$$\begin{array}{rr} 34.04 & \text{2 decimal places} \\ \times\ 0.56 & \text{2 decimal places} \\ \hline 20424 & \downarrow \\ 17020 & 2+2=4 \\ \hline 19.0624 & \text{4 decimal places} \end{array}$$

29.
$$\begin{array}{rr} 22.41 & \text{2 decimal places} \\ \times\ 33 & \text{0 decimal places} \\ \hline 6723 & \downarrow \\ 6723 & 2+0=2 \\ \hline 739.53 & \text{2 decimal places} \end{array}$$

30.
$$\begin{array}{rr} 55.76 & \text{2 decimal places} \\ \times\ 72 & \text{0 decimal places} \\ \hline 11152 & \downarrow \\ 39032 & 2+0=2 \\ \hline 4014.72 & \text{2 decimal places} \end{array}$$

31.
$$\begin{array}{rr} 0.2 & \text{1 decimal place} \\ \underline{\times 0.03} & \text{2 decimal places} \\ \underline{6} & 1+2=3 \\ 0.006 & \text{3 decimal places} \end{array}$$

32.
$$\begin{array}{rr} 0.07 & \text{2 decimal places} \\ \underline{\times 0.004} & \text{3 decimal places} \\ \underline{28} & 2+3=5 \\ 0.00028 & \text{5 decimal places} \end{array}$$

33.
$$\begin{array}{r} 7.15 \\ 11\overline{)78.65} \\ \underline{77} \\ 16 \\ \underline{11} \\ 55 \\ \underline{55} \\ 0 \end{array}$$

34.
$$\begin{array}{r} 5.24 \\ 14\overline{)73.36} \\ \underline{70} \\ 33 \\ \underline{28} \\ 56 \\ \underline{56} \\ 0 \end{array}$$

35. To change the divisor 11.6 into a whole number, move each decimal point one place to the right. Move the decimal point straight up and divide as with whole numbers.
$$\begin{array}{r} 2.8 \\ 116\overline{)324.8} \\ \underline{232} \\ 928 \\ \underline{928} \\ 0 \end{array}$$
Therefore, $32.48 \div 11.6 = 2.8$.

36. To change the divisor 17.4 into a whole number, move each decimal point one place to the right. Move the decimal point straight up and divide as with whole numbers.
$$\begin{array}{r} 4.9 \\ 174\overline{)852.6} \\ \underline{696} \\ 1566 \\ \underline{1566} \\ 0 \end{array}$$
Therefore, $85.26 \div 17.4 = 4.9$.

37. To change the divisor 9.74 into a whole number, move each decimal point two places to the right. Move the decimal point straight up and divide as with whole numbers.
$$\begin{array}{r} 2.05 \\ 974\overline{)1996.70} \\ \underline{1948} \\ 4870 \\ \underline{4870} \\ 0 \end{array}$$
Therefore, $19.967 \div 9.74 = 2.05$.

38. To change the divisor 5.27 into a whole number, move each decimal point two places to the right. Move the decimal point straight up and divide as with whole numbers.
$$\begin{array}{r} 8.44 \\ 527\overline{)4447.88} \\ \underline{4216} \\ 2318 \\ \underline{2108} \\ 2108 \\ \underline{2108} \\ 0 \end{array}$$
Therefore, $44.4788 \div 5.27 = 8.44$.

39. Move the decimal point one place to the right.
$123.26 \times 10 = 1232.6$

40. Move the decimal point one place to the right.
$785.91 \times 10 = 7859.1$

41. Move the decimal point two places to the right.
$57.116 \times 100 = 5711.6$

42. Move the decimal point two places to the right.
$82.053 \times 100 = 8205.3$

43. Move the decimal point three places to the right.
$0.094 \times 1000 = 94$

44. Move the decimal point three places to the right.
$0.025 \times 1000 = 25$

45. Move the decimal point one place to the left.
$1.62 \div 10 = 0.162$

46. Move the decimal point one place to the left.
$8.04 \div 10 = 0.804$

47. Move the decimal point two places to the left.
$124.03 \div 100 = 1.2403$

48. Move the decimal point two places to the left.
$490.35 \div 100 = 4.9035$

49. Move the decimal point three places to the left.
$23.29 \div 1000 = 023.29 \div 1000 = 0.02329$

50. Move the decimal point three places to the left.
$59.8 \div 1000 = 059.8 \div 1000 = 0.0598$

51. Convert from a decimal to a percent.
$0.01 = 0.01 \cdot 100\% = 1\%$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{100}$	0.01	1%

52. Convert from a percent to a decimal.

$$2\% = \frac{2}{100} = 0.02$$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{50}$	0.02	2%

53. Convert from a percent to a fraction.

$$5\% = \frac{5}{100}$$

In lowest terms,

$$\frac{5}{100} = \frac{1 \cdot 5}{20 \cdot 5} = \frac{1}{20}$$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{20}$	0.05	5%

54. Convert to a decimal first. Divide 1 by 10. Move the decimal point one place to the left.
$1 \div 10 = 0.1$
Convert the decimal to a percent.
$0.1 = 0.1 \cdot 100\% = 10\%$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{10}$	0.1	10%

55. Convert the decimal to a percent.
$0.125 = 0.125 \cdot 100\% = 12.5\%$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{8}$	0.125	12.5%

56. Convert the percent to a decimal first.
$20\% = 0.20$, or 0.2
Convert from a percent to a fraction.

$$20\% = \frac{20}{100}$$

In lowest terms,

$$\frac{20}{100} = \frac{1 \cdot 20}{5 \cdot 20} = \frac{1}{5}$$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{5}$	0.2	20%

57. Convert to a decimal first. Divide 1 by 4. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Convert the decimal to a percent.
$0.25 = 0.25 \cdot 100\% = 25\%$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{4}$	0.25	25%

58. Convert to a decimal first. Divide 1 by 3. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.33\ldots \\ 3\overline{)1.00\ldots} \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

Note that the pattern repeats. Therefore,

$\frac{1}{3} = 0.\overline{3}.$

Convert the decimal to a percent.

$0.33\overline{3} = 0.33\overline{3} \cdot 100\% = 33.\overline{3}\%,$ or $33\frac{1}{3}\%$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{3}$	$0.\overline{3}$	$33.\overline{3}\%$ or $33\frac{1}{3}\%$

59. Convert the percent to a decimal first.

$50\% = 0.50,$ or 0.5

Convert from a percentl to a fraction.

$50\% = \frac{50}{100}$

In lowest terms,

$\frac{50}{100} = \frac{1 \cdot 50}{2 \cdot 50} = \frac{1}{2}$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{2}$	0.5	50%

60. Divide 2 by 3. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.66\ldots \\ 3\overline{)2.00\ldots} \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

Note that the pattern repeats. Therefore,

$\frac{2}{3} = 0.\overline{6}.$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{2}{3}$	$0.\overline{6}$	$66.\overline{6}\%$ or $66\frac{2}{3}\%$

61. Convert the decimal to a percent first.

$0.75 = 0.75 \cdot 100\% = 75\%$

Convert from a percent to a fraction.

$75\% = \frac{75}{100}$

In lowest terms,

$\frac{75}{100} = \frac{3 \cdot 25}{4 \cdot 25} = \frac{3}{4}$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{3}{4}$	0.75	75%

62. Convert the decimal to a percent.

$1.0 = 1.0 \cdot 100\% = 100\%$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
1	1.0	100%

63. Divide 2 by 5. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ \underline{2\,0} \\ 0 \end{array}$$

64. Divide 3 by 5. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.6 \\ 5\overline{)3.0} \\ \underline{3\,0} \\ 0 \end{array}$$

65. Divide 9 by 4. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 2.25 \\ 4\overline{)9.00} \\ \underline{8} \\ 1\,0 \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

66. Divide 15 by 4. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 3.75 \\ 4\overline{)15.00} \\ \underline{12} \\ 3\,0 \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

67. Divide 3 by 8. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

68. Divide 7 by 8. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ \underline{64} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

69. Divide 5 by 9. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.555\ldots \\ 9\overline{)5.000\ldots} \\ \underline{45} \\ 50 \\ \underline{45} \\ 50 \\ \underline{45} \\ 5 \end{array}$$

Note that the pattern repeats. Therefore, $\frac{5}{9} = 0.\overline{5},$ or 0.556 rounded to the nearest thousandth, 0.56 rounded to the nearest hundredth, and 0.6 rounded to the nearest tenth.

70. Divide 8 by 9. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.888\ldots \\ 9\overline{)8.000\ldots} \\ \underline{72} \\ 80 \\ \underline{72} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

Note that the pattern repeats. Therefore, $\frac{8}{9} = 0.\overline{8},$ or 0.889 rounded to the nearest thousandth, 0.89 rounded to the nearest hundredth, and 0.9 rounded to the nearest tenth.

71. Divide 5 by 6. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.833\ldots \\ 6\overline{)5.000\ldots} \\ \underline{48} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

Note that the pattern repeats. Therefore, $\frac{5}{6} = 0.8\overline{3},$ or 0.833 rounded to the nearest thousandth, 0.83 rounded to the nearest hundredth, and 0.8 rounded to the nearest tenth.

72. Divide 1 by 6. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.166\ldots \\ 6\overline{)1.000\ldots} \\ \underline{6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

Note that the pattern repeats. Therefore, $\frac{1}{6} = 0.1\bar{6}$, or 0.167 rounded to the nearest thousandth, 0.17 rounded to the nearest hundredth, and 0.2 rounded to the nearest tenth.

73. $54\% = 0.54$

74. $39\% = 0.39$

75. $7\% = 07\% = 0.07$

76. $4\% = 04\% = 0.04$

77. $117\% = 1.17$

78. $189\% = 1.89$

79. $2.4\% = 02.4\% = 0.024$

80. $3.1\% = 03.1\% = 0.031$

81. $6\frac{1}{4}\% = 6.25\% = 06.25\% = 0.0625$

82. $5\frac{1}{2}\% = 5.5\% = 05.5\% = 0.055$

83. $0.8\% = 00.8\% = 0.008$

84. $0.9\% = 00.9\% = 0.009$

85. $0.79 = 79\%$

86. $0.83 = 83\%$

87. $0.02 = 2\%$

88. $0.08 = 8\%$

89. $0.004 = 0.4\%$

90. $0.005 = 0.5\%$

91. $1.28 = 128\%$

92. $2.35 = 235\%$

93. $0.40 = 40\%$

94. $0.6 = 0.60 = 60\%$

95. $6 = 6.00 = 600\%$

96. $10 = 10.00 = 1000\%$

97. $51\% = \frac{51}{100}$

98. $47\% = \frac{47}{100}$

99. $15\% = \frac{15}{100}$

In lowest terms,

$$\frac{15}{100} = \frac{3 \cdot 5}{20 \cdot 5} = \frac{3}{20}$$

100. $35\% = \frac{35}{100}$

In lowest terms,

$$\frac{35}{100} = \frac{7 \cdot 5}{20 \cdot 5} = \frac{7}{20}$$

101. $2\% = \frac{2}{100}$

In lowest terms,

$$\frac{2}{100} = \frac{1 \cdot 2}{50 \cdot 2} = \frac{1}{50}$$

102. $8\% = \frac{8}{100}$

In lowest terms,

$$\frac{8}{100} = \frac{2 \cdot 4}{25 \cdot 4} = \frac{2}{25}$$

103. $140\% = \frac{140}{100}$

In lowest terms,

$$\frac{140}{100} = \frac{7 \cdot 20}{5 \cdot 20} = \frac{7}{5}, \text{ or } 1\frac{2}{5}$$

104. $180\% = \frac{180}{100}$

In lowest terms,

$$\frac{180}{100} = \frac{9 \cdot 20}{5 \cdot 20} = \frac{9}{5}, \text{ or } 1\frac{4}{5}$$

105. $7.5\% = \frac{7.5}{100} = \frac{7.5}{100} \cdot \frac{10}{10} = \frac{75}{1000}$

In lowest terms,

$$\frac{75}{1000} = \frac{3 \cdot 25}{40 \cdot 25} = \frac{3}{40}$$

106. $2.5\% = \frac{2.5}{100} = \frac{2.5}{100} \cdot \frac{10}{10} = \frac{25}{1000}$

In lowest terms,

$\frac{25}{1000} = \frac{1 \cdot 25}{40 \cdot 25} = \frac{1}{40}$

107. $\frac{4}{5} = \frac{4}{5} \cdot 100\% = \frac{4}{5} \cdot \frac{100}{1}\% = \frac{4 \cdot 5 \cdot 20}{5}\% = 80\%$

108. $\frac{3}{25} = \frac{3}{25} \cdot 100\%$

$= \frac{3}{25} \cdot \frac{100}{1}\%$

$= \frac{3 \cdot 4 \cdot 25}{25}\%$

$= 12\%$

109. $\frac{7}{50} = \frac{7}{50} \cdot 100\%$

$= \frac{7}{50} \cdot \frac{100}{1}\%$

$= \frac{7 \cdot 2 \cdot 50}{50}\%$

$= 14\%$

110. $\frac{9}{20} = \frac{9}{20} \cdot 100\%$

$= \frac{9}{20} \cdot \frac{100}{1}\%$

$= \frac{9 \cdot 5 \cdot 20}{20}\%$

$= 45\%$

111. $\frac{9}{4} = \frac{9}{4} \cdot 100\% = \frac{9}{4} \cdot \frac{100}{1}\% = \frac{9 \cdot 4 \cdot 25}{4}\% = 225\%$

112. $\frac{8}{5} = \frac{8}{5} \cdot 100\% = \frac{8}{5} \cdot \frac{100}{1}\% = \frac{8 \cdot 5 \cdot 20}{5}\% = 160\%$

113. $\frac{5}{6} = \frac{5}{6} \cdot 100\%$

$= \frac{5}{6} \cdot \frac{100}{1}\%$

$= \frac{5 \cdot 2 \cdot 50}{2 \cdot 3}\%$

$= \frac{250}{3}\%$

$= 83\frac{1}{3}\%, \text{ or } 83.\overline{3}\%$

114. $\frac{5}{12} = \frac{5}{12} \cdot 100\%$

$= \frac{5}{12} \cdot \frac{100}{1}\%$

$= \frac{5 \cdot 4 \cdot 25}{3 \cdot 4}\%$

$= \frac{125}{3}\%$

$= 41\frac{2}{3}\%, \text{ or } 41.\overline{6}\%$

115. $\frac{4}{9} = \frac{4}{9} \cdot 100\% = \frac{4}{9} \cdot \frac{100}{1}\% = \frac{400}{9}\% = 44.\overline{4}\%$

116. $\frac{2}{11} = \frac{2}{11} \cdot 100\%$

$= \frac{2}{11} \cdot \frac{100}{1}\%$

$= \frac{200}{11}\%$

$= 18.\overline{18}\%$

117. The tip is 20% of \$150. The word *of* here means multiply.

$$\begin{array}{ccccc} 20\% & \text{of} & \$150 & & \\ \downarrow & \downarrow & \downarrow & & \\ 0.20 & \cdot & \$150 & = & \$30 \end{array}$$

The tip is \$30, and the total bill is \$150 + \$30 = \$180.

118. The raise is 10% of \$15. The word *of* here means multiply.

$$\begin{array}{ccccc} 10\% & \text{of} & \$15 & & \\ \downarrow & \downarrow & \downarrow & & \\ 0.10 & \cdot & \$15 & = & \$1.50 \end{array}$$

The raise is \$1.50 per hour, and the new hourly rate is \$1.50 + \$15 = \$16.50.

119. The discount is 15% of \$1000. The word *of* here means multiply.

$$\begin{array}{ccccc} 15\% & \text{of} & \$1000 & & \\ \downarrow & \downarrow & \downarrow & & \\ 0.15 & \cdot & \$1000 & = & \$150 \end{array}$$

The amount of the discount is \$150. The sale price is found by subtracting.

$\$1000 - \$150 = \$850$

120. The discount is 25% of \$600. The word *of* here means multiply.

$$\begin{array}{ccccc} 25\% & \text{of} & \$600 & & \\ \downarrow & \downarrow & \downarrow & & \\ 0.25 & \cdot & \$600 & = & \$150 \end{array}$$

The amount of the discount is \$150. The sale price is found by subtracting.
$\$600 - \$160 = \$450$

R.3 Basic Concepts from Algebra

Classroom Examples, Now Try Exercises

1. $\{x \mid x \text{ is a natural number greater than } 12\}$
$\{13, 14, 15, \ldots\}$ is the set of natural numbers greater than 12.

N1. $\{p \mid p \text{ is a natural number less than } 6\}$
$\{1, 2, 3, 4, 5\}$ is the set of natural numbers less than 6.

2. One answer is $\{x \mid x \text{ is a whole number less than } 6\}$.

N2. One answer is $\{x \mid x \text{ is a natural number between 8 and } 13\}$.

3. **(a)** -7 is an integer, a rational number $\left(\text{since } -7 = \frac{-7}{1}\right)$, and a real number.

(b) 3.14, a terminating decimal, is a rational number and a real number.

(c) $\sqrt{4} = 2$, so it is a whole number, an integer, a rational number, and a real number.

N3. **(a)** The whole numbers in the given set are in the set $\{0, 5\}$.

(b) The rational numbers in the given set are in the set $\left\{-2.4, -\sqrt{1}, -\frac{1}{2}, 0, 0.\overline{3}, 5\right\}$.

(c) The irrational numbers in the given set are in the set $\left\{\sqrt{5}, \pi\right\}$.

4. **(a)** The statement "Some integers are whole numbers" is *true* since 0 and the positive integers are whole numbers. The negative integers are not whole numbers.

(b) The statement "Every real number is irrational" is *false.* Some real numbers such as $\frac{4}{9}$, $0.\overline{3}$, and $\sqrt{16}$ are rational. Others such as $\sqrt{2}$ and π are irrational.

N4. **(a)** The statement "All integers are irrational numbers" is *false.* In fact, all integers are rational.

(b) The statement "Every whole number is an integer" is *true* since the integers include the whole numbers, the negatives of the whole numbers, and 0.

5. **(a)** $|3| = 3$

(b) $|-3| = -(-3) = 3$

(c) Evaluate the absolute value. Then find the additive inverse.
$-|3| = -(3) = -3$

(d) $-|-3| = -(3) = -3$

(e) Evaluate each absolute value, and then add.
$|8| + |-1| = 8 + 1 = 9$

(f) $|8 - 1| = |7| = 7$

N5. **(a)** $|-7| = -(-7) = 7$

(b) Evaluate the absolute value. Then find the additive inverse.
$-|7| = -(7) = -7$

(c) Evaluate the absolute value. Then find the additive inverse.
$-|-7| = -(7) = -7$

(d) Evaluate each absolute value, and then subtract.
$|4| - |-4| = 4 - 4 = 0$

(e) $|4 - 4| = |0| = 0$

6. Information security analysts: $|34.7| = 34.7$
Word processors and typists: $|-38.2| = 38.2$
Since $38.2 > 34.7$, word processors and typists will show the greater change.

N6. Wind turbine service technicians: $|44.3| = 44.3$
Parking enforcement workers: $|-37.1| = 37.1$
Word processors and typists: $|-38.2| = 38.2$
Since 37.1 is less than 38.2 and 44.3, parking enforcement workers will show the least change.

7. (a) $-2 < 5$ is *true*, since -2 is to the left of 5 on a number line.

(b) $-8 > -4$ is *false*, since -8 is to the left of -4 on a number line, -8 is less than, not greater than, -4.

N7. (a) $-5 > -1$ is *false*, since -5 is to the left of -1 on a number line, -5 is less than, not greater than, -1.

(b) $-7 < -6$ is *true*, since -7 is to the left of -6 on a number line.

8. (a) $-2 \le -3$ is *false*, since -2 is to the right of -3 on a number line, it must be greater.

(b) $-1 \ge -9$ reads "-1 is greater than or equal to -9". This is *true*, since -1 is to the right of -9 on a number line.

(c) $8 \le 8$ is *true*. The statement is "8 is less than *or* equal to 8". Since 8 is equal to 8, the statement is true.

(d) Since $3(4) = 12$ and $2 \cdot 6 = 12$, the statement becomes $12 > 12$, which is *false*.

N8. (a) Since -6 is to the left of -5 on a number line, -6 is less than, not greater than -5. The statement is *false*.

(b) Since -10 is to the left of -2 on a number line, the statement is *true*.

(c) Since the statement is "0.5 is less than *or* equal to 0.5." and 0.5 is equal to 0.5, the statement is *true*.

(d) Since $10 \cdot 6 = 60$ and $8(7) = 56$, the statement becomes $60 > 56$, which is *true*.

Exercises

1. Yes, the choice of variables is not important. The sets have the same description so they represent the same set.

2. (a) The only integer between 6.75 and 7.75 is 7.

(b) There are an infinite number of rational numbers between $\frac{1}{4}$ and $\frac{3}{4}$; one example is $\frac{1}{2}$.

(c) The number 0 is the only whole number that is not a natural number.

(d) The opposite of any natural number is an integer that is not a whole number; one example is -1.

(e) There are an infinite number of irrational numbers between $\sqrt{4}$ and $\sqrt{9}$; one example is $\sqrt{5}$.

(f) One example is $-\pi$. Other answers are possible.

3. The set of natural numbers is $\{1, 2, 3, \ldots\}$, so the set of natural numbers less than 6 is $\{1, 2, 3, 4, 5\}$.

4. The set is $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

5. The set of integers is $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$, so the set of integers greater than 4 is $\{5, 6, 7, 8, \ldots\}$.

6. The set is $\{9, 10, 11, 12, \ldots\}$.

7. The set of integers is $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$, so the set of integers less than or equal to 4 is $\{\ldots, -1, 0, 1, 2, 3, 4\}$.

8. The set is $\{\ldots, -2, -1, 0, 1, 2\}$.

9. The set of even integers is $\{\ldots, -2, 0, 2, 4, \ldots\}$, so the set of even integers greater than 8 is $\{10, 12, 14, 16, \ldots\}$.

10. The set is $\{\ldots, -7, -5, -3, -1\}$.

11. This is the set of numbers that lie a distance of 4 units from 0 on the number line. Thus, the set of numbers whose absolute value is 4 is $\{-4, 4\}$.

12. The set is $\{-7, 7\}$.

13. $\{x \mid x$ is an irrational number that is also rational$\}$
Irrational numbers cannot also be rational numbers. The set of irrational numbers that are also rational is $\varnothing$.

14. The set is $\varnothing$, the empty set.

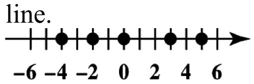
15. $\{2, 4, 6, 8\}$ can be described by $\{x \mid x$ is an even natural number less than or equal to $8\}$.

16. $\{11, 12, 13, 14\}$ can be described by $\{x \mid x$ is an integer between 10 and $15\}$.

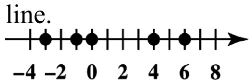
17. $\{4, 8, 12, 16, \ldots\}$ can be described by $\{x \mid x$ is a multiple of 4 greater than $0\}$.

18. $\{\ldots, -6, -3, 0, 3, 6, \ldots\}$ can be described by $\{x \mid x$ is an integer multiple of $3\}$.

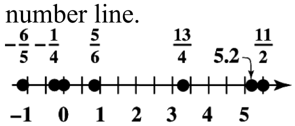
19. Place dots for $-4, -2, 0, 3,$ and 5 on a number line.



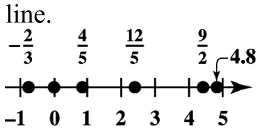
20. Place dots for $-3, -1, 0, 4,$ and 6 on a number line.



21. Place dots for $-\dfrac{6}{5} = -1.2, -\dfrac{1}{4} = -0.25, 0,$ $\dfrac{5}{6} = 0.8\overline{3}, \dfrac{13}{4} = 3.25, 5.2,$ and $\dfrac{11}{2} = 5.5,$ on a number line.



22. Place dots to indicate the points $-\dfrac{2}{3} = -0.\overline{6}, 0,$ $\dfrac{4}{5} = 0.8, \dfrac{12}{5} = 2.4, \dfrac{9}{2} = 4.5,$ and 4.8 on a number line.



23. **(a)** The elements 8, 13, and $\dfrac{75}{5}$ (or 15) are natural numbers.

(b) The elements 0, 8, 13, and $\dfrac{75}{5}$ are whole numbers.

(c) The elements -9, 0, 8, 13, and $\dfrac{75}{5}$ are integers.

(d) The elements $-9, -0.7, 0, \dfrac{6}{7}, 4.\overline{6},$ $8, \dfrac{21}{2}, 13,$ and $\dfrac{75}{5}$ are rational numbers.

(e) The elements $-\sqrt{6}$ and $\sqrt{7}$ are irrational numbers.

(f) All the elements are real numbers.

24. **(a)** The elements 5, 17, and $\dfrac{40}{2}$ (or 20) are natural numbers.

(b) The elements 0, 5, 17, and $\dfrac{40}{2}$ are whole numbers.

(c) The elements -8, 0, 5, 17, and $\dfrac{40}{2}$ are integers.

(d) The elements $-8, -0.6, 0, \frac{3}{4}, 5, \frac{13}{2},$ 17, and $\frac{40}{2}$ are rational numbers.

(e) The elements $-\sqrt{5}, \sqrt{3},$ and π are irrational numbers.

(f) All the elements are real numbers.

25. The statement "Every integer is a whole number" is *false.* Some integers are whole numbers, but the negative integers are not whole numbers.

26. The statement "Every natural number is an integer" is *true.* The integers include the natural numbers, their negatives, and zero.

27. The statement "Every irrational number is an integer" is *false.* Irrational numbers have decimal representations that neither terminate nor repeat, so no irrational numbers are integers.

28. The statement "Every integer is a rational number" is *true.* Every integer q can be written as $\dfrac{q}{1}$.

29. The statement "Every natural number is a whole number" is *true.* Whole numbers consist of the natural numbers and zero.

30. The statement "Some rational numbers are irrational" is *false.* No rational numbers are irrational numbers.

31. The statement "Some rational numbers are whole numbers" is *true.* Every whole number is rational.

32. The statement "Some real numbers are integers" is *true.* Every integer is a real number.

33. The statement "The absolute value of any number is the same as the absolute value of its additive inverse" is *true.* The distance on a number line from 0 to a number is the same as the distance from 0 to its additive inverse.

34. The statement "The absolute value of any nonzero number is positive" is *true.* Every nonzero number is a positive distance from 0.

35. **(a)** $-(-4)=4$ (Choice A)

(b) $|-4|=4$ (Choice A)

(c) $-|-4|=-(4)=-4$ (Choice B)

(d) $-|-(-4)|=-|4|=-4$ (Choice B)

36. In general $|x|=a$ (with $a \neq 0$), is true for two values of a; namely, a and $-a$. In this case, $|x|=4$ is true for $x=4$ and $x=-4$.

37. **(a)** The additive inverse of 6 is -6.

(b) $6>0$, so $|6|=6$.

38. **(a)** The additive inverse of 9 is -9.

(b) $9>0$, so $|9|=9$.

39. **(a)** The additive inverse of -12 is $-(-12)=12$.

(b) $-12<0$, so $|-12|=-(-12)=12$.

40. **(a)** The additive inverse of -14 is $-(-14)=14$.

(b) $-14<0$, so $|-14|=-(-14)=14$.

41. **(a)** The additive inverse of $\frac{6}{5}$ is $-\frac{6}{5}$.

(b) $\frac{6}{5}>0$, so $\left|\frac{6}{5}\right|=\frac{6}{5}$.

42. **(a)** The additive inverse of 0.16 is -0.16.

(b) $0.16>0$, so $|0.16|=0.16$.

43. Use the definition of absolute value.
$-8<0$, so $|-8|=-(-8)=8$.

44. $|-19|=-(-19)=19$

45. $\frac{3}{2}>0$, so $\left|\frac{3}{2}\right|=\frac{3}{2}$.

46. $\left|\frac{3}{4}\right|=\frac{3}{4}$

47. $-|5|=-(5)=-5$

48. $-|12|=-(12)=-12$

49. $-|-2|=-[-(-2)]=-(2)=-2$

50. $-|-6|=-[-(-6)]=-(6)=-6$

51. $-|4.5|=-(4.5)=-4.5$

52. $-|12.4|=-(12.4)=-12.4$

53. $|-2|+|3|=2+3=5$

54. $|-16|+|14|=16+14=30$

55. $-|10-9|=-(1)=-1$

56. $-|12-6|=-(6)=-6$

57. $|-9|-|-3|=9-3=6$

58. $|-10|-|-7|=10-7=3$

59.
$$\begin{aligned}|-1|+|-2|-|-3|&=1+2-3\\&=3-3\\&=0\end{aligned}$$

60. $|-7|+|-3|-|-10|=7+3-10=0$

61. **(a)** The greatest absolute value is $|18.4|=18.4$. Therefore, Utah had the greatest change in population. The population increased by 18.4%.

(b) The least absolute value is $|2.3|=2.3$. Therefore, Ohio had the least change in population. The population increased by 2.3%.

62. **(a)** The greatest absolute value is $|-2.9|=2.9$. Therefore, used cars/trucks showed the greatest change in CPI. It decreased by 2.9%.

(b) The least absolute value is $|0.5|=0.5$. Therefore, food showed the least change in CPI. It increased by 0.5%.

63. Compare the depths of the bodies of water. The deepest, that is, the body of water whose depth has the greatest absolute value, is the Pacific Ocean $(|-14,040|=14,040)$ followed by the Indian Ocean $(|-12,800|=12,800)$, the Caribbean Sea $(|-8448|=8448)$, the South China Sea $(|-4802|=4802)$, and the Gulf of California $(|-2375|=2375)$.

64. Compare the heights of the mountains. The shortest, that is, the smallest value, is Point Success $(|14,164|=14,164)$ followed by Rainier $(|14,410|=14,410)$, Matlalcueyetl

$(|14,636| = 14,636)$, Steele
$(|16,624| = 16,624)$, and Denali
$(|20,310| = 20,310)$.

65. *True*; the absolute value of the depth of the Pacific Ocean is $|-14,040| = 14,040$ which is greater than the absolute value of the depth of the Indian Ocean, $|-12,800| = 12,800$.

66. *False*; the absolute value of the depth of the Gulf of California is $|-2375| = 2375$ which is less than the absolute value of the depth of the Caribbean Sea, $|-8448| = 8448$.

67. *True*; since -6 is to the left of -1 on a number line, -6 is less than -1.

68. *True*; since -4 is to the left of -2 on a number line, -4 is less than -2.

69. *False*; since -4 is to the left of -3 on a number line, -4 is *less* than -3, not greater.

70. *False*; since -3 is to the left of -1 on a number line, -3 is *less* than -1, not greater.

71. *True*; since 3 is to the right of -2 on a number line, 3 is greater than -2.

72. *True*; since 6 is to the right of -3 on a number line, 6 is greater than -3.

73. *False*; since $-3 = -3, -3$ is *not* greater than -3.

74. *False*; since $-5 = -5, -5$ is *not* less than -5.

75. The inequality $6 > 2$ can also be written $2 < 6$. In each case, the inequality symbol points toward the smaller number so both inequalities are true.

76. $5 > 1$ is equivalent to $1 < 5$.

77. $-9 < 4$ is equivalent to $4 > -9$.

78. $-6 < 1$ is equivalent to $1 > -6$.

79. $-5 > -10$ is equivalent to $-10 < -5$.

80. $-7 > -12$ is equivalent to $-12 < -7$.

81. "7 is greater than -1" can be written as $7 > -1$.

82. "-4 is less than 10" can be written as $-4 < 10$.

83. "5 is greater than or equal to 5" can be written as $5 \geq 5$.

84. "-6 is less than or equal to -6" can be written as $-6 \leq -6$.

85. "$13-3$ is less than or equal to 10" can be written as $13-3 \leq 10$.

86. "$5+14$ is greater than or equal to 19" can be written as $5+14 \geq 19$.

87. "$5+0$ is not equal to 0" can be written as $5+0 \neq 0$.

88. "$6+7$ is not equal to -13" can be written as $6+7 \neq -13$.

89. $0 \leq -5$ False

90. $-11 \geq 0$ False

91. $7 \leq 7$ True

92. $10 \geq 10$ True

93. $-6 \overset{?}{<} 7+3$
$-6 < 10$ True
The last statement is true since -6 is to the left of 10 on a number line.

94. $-7 \overset{?}{<} 4+1$
$-7 < 5$ True

95. $-2 \cdot 5 \overset{?}{\geq} 4+6$
$10 \geq 10$ True

96. $-8+7 \overset{?}{\leq} 3 \cdot 5$
$15 \geq 15$ True

97. $-|-3| \overset{?}{\geq} -3$
$-3 \geq -3$ True

98. $-|-4| \overset{?}{\leq} -4$
$-4 \leq -4$ True

99. $-8 \overset{?}{>} -|-6|$
$-8 > -6$ False

100. $-10 \overset{?}{>} -|-4|$
$-10 > -4$ False

101. 2021: IA $= 15.0$, OH $= 10.5$, PA $= 8.1$
In 2021, Iowa (IA), Ohio (OH), and Pennsylvania (PA) had production greater than 8 billion eggs.

102. IA: $15.0 < 15.5$; PA: $8.1 < 9.0$
For Iowa (IA) and Pennsylvania (PA), egg production was less in 2021 than in 2020.

103. 2021: TX $= x = 6.4$, OH $= y = 10.5$
Since $6.4 < 10.5$, $x < y$ is true.

104. 2020: IA $= x = 15.5$, PA $= y = 9.0$
Two inequalities that compare the production in these two states are $x > y$ and $y < x$.

R.4 Operations on Real Numbers

Classroom Examples, Now Try Exercises

1. **(a)** $-6 + (-15) = -(6 + 15) = -21$

(b) $-1.1 + (-1.2) = -(1.1 + 1.2) = -2.3$

(c) $$-\frac{3}{4} + \left(-\frac{5}{8}\right) = -\left(\frac{3}{4} + \frac{5}{8}\right) = -\left(\frac{6}{8} + \frac{5}{8}\right) = -\frac{11}{8}$$

N1. **(a)** $-4 + (-9) = -(4 + 9) = -13$

(b) $-7.25 + (-3.57) = -(7.25 + 3.57) = -10.82$

(c) $$-\frac{2}{5} + \left(-\frac{3}{10}\right) = -\left(\frac{2}{5} + \frac{3}{10}\right) = -\left(\frac{4}{10} + \frac{3}{10}\right) = -\frac{7}{10}$$

2. **(a)** Subtract the lesser absolute value from the greater absolute value $(7 - 3)$, and take the sign of the larger.
$3 + (-7) = -(7 - 3) = -4$

(b) $-3 + 7 = 7 - 3 = 4$

(c) $-3.8 + 4.6 = 4.6 - 3.8 = 0.8$

(d) $$-\frac{3}{8} + \frac{1}{4} = -\frac{3}{8} + \frac{1 \cdot 2}{4 \cdot 2} = -\frac{3}{8} + \frac{2}{8} = -\left(\frac{3}{8} - \frac{2}{8}\right) = -\frac{1}{8}$$

N2. **(a)** Subtract the lesser absolute value from the greater absolute value $(15 - 7)$, and take the sign of the larger.
$-15 + 7 = -(15 - 7) = -8$

(b) $-5 + 12 = 12 - 5 = 7$

(c) $4.6 + (-2.8) = 4.6 - 2.8 = 1.8$

(d) $$-\frac{5}{9} + \frac{2}{7} = \frac{5 \cdot 7}{9 \cdot 7} + \frac{2 \cdot 9}{7 \cdot 9} = -\frac{35}{63} + \frac{18}{63} = -\left(\frac{35}{63} - \frac{18}{63}\right) = -\frac{17}{63}$$

3. **(a)** $5 - 12 = 5 + (-12) = -7$

(b) $12 - (-5) = 12 + 5 = 17$

(c) $$-11.5 - (-6.3) = -11.5 + 6.3 = -(11.5 - 6.3) = -5.2$$

(d) $$\frac{3}{4} - \left(-\frac{2}{3}\right) = \frac{3}{4} + \frac{2}{3} = \frac{3 \cdot 3}{4 \cdot 3} + \frac{2 \cdot 4}{3 \cdot 4} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12}$$

N3. **(a)** $9 - 15 = 9 + (-15) = -6$

(b) $-4 - 11 = -4 + (-11) = -15$

(c) $$-5.67 - (-2.34) = -5.67 + 2.34 = -(5.67 - 2.34) = -3.33$$

(d) $$\frac{4}{9} - \frac{3}{5} = \frac{4}{9} - \frac{3}{5} = \frac{4 \cdot 5}{9 \cdot 5} + \frac{3 \cdot 9}{5 \cdot 9} = \frac{20}{45} - \frac{27}{45} = -\frac{7}{45}$$

4. **(a)** $$-6 - (-2) - 8 - 1 = -6 + 2 - 8 - 1 = -4 + (-8) - 1 = -12 - 1 = -13$$

(b) $$-3 - [(-7) + 15] - 6 = -3 - 8 - 6 = -11 - 6 = -17$$

N4. $-4-(-2-7)-12 = -4-(-9)-12$
$$= -4+9-12$$
$$= 5-12$$
$$= -7$$

5. $|-1-(-8)| = |-1+8| = |7| = 7$

N5. $|10-(-7)| = |10+7| = |17| = 17$

6. **(a)** $7(-2) = -14$

The numbers have different signs, so the product is negative.

(b) $-0.9(-15) = 13.5$

(c) $-\frac{5}{8}(16) = -\frac{5\cdot 2\cdot 8}{8} = -10$

(d) $0(-3) = 0\cdot(-3) = 0$

The multiplication property of 0 states that any product of any real number and 0 is 0.

N6. **(a)** $-3(-10) = 30$

The numbers have the same sign, so the product is positive.

(b) $0.7(-1.2) = -0.84$

The numbers have different signs, so the product is negative.

(c) $-\frac{8}{11}(33) = \frac{8\cdot 3\cdot 11}{11} = -24$

(d) $14(0) = 14\cdot 0 = 0$

The multiplication property of 0 states that any product of any real number and 0 is 0.

7. **(a)** $\frac{-15}{-3} = -15\left(-\frac{1}{3}\right) = 5$

(b) $\frac{2.7}{-0.3} = -9$

(c) $\frac{10}{0}$ is undefined.

(d) $\frac{3}{4} \div \left(-\frac{7}{16}\right) = \frac{3}{4}\cdot\left(-\frac{16}{7}\right) = -\frac{3\cdot 4\cdot 4}{4\cdot 7}$
$$= -\frac{3\cdot 4}{7} = -\frac{12}{7}$$

(e) $\frac{-\frac{3}{8}}{\frac{11}{16}} = -\frac{3}{8} \div \frac{11}{16} = -\frac{3}{8}\cdot\frac{16}{11}$
$$= -\frac{3\cdot 2\cdot 8}{8\cdot 11} = -\frac{6}{11}$$

N7. **(a)** $\frac{-10}{-5} = -10\left(-\frac{1}{5}\right) = 2$

(b) $\frac{-1.5}{0.3} = -5$

(c) $\frac{0}{-4} = 0$

(d) $-\frac{6}{5} \div \left(-\frac{3}{7}\right) = -\frac{6}{5}\cdot\left(-\frac{7}{3}\right) = \frac{2\cdot 3\cdot 7}{5\cdot 3}$
$$= \frac{2\cdot 7}{5} = \frac{14}{5}$$

(e) $\frac{-\frac{10}{3}}{\frac{3}{8}} = -\frac{10}{3} \div \frac{3}{8} = -\frac{10}{3}\cdot\frac{8}{3}$
$$= -\frac{10\cdot 8}{3\cdot 3} = -\frac{80}{9}$$

Exercises

1. The sum of two positive numbers is a positive number. For example, $18+6=24$.

2. The sum of two negative numbers is a negative number. For example, $-7+(-21)=-28$.

3. The sum of a positive number and a negative number is negative if the negative number has the greater absolute value. For example, $-14+9=-5$.

4. The sum of a positive number and a negative number is positive if the positive number has the greater absolute value. For example, $15+(-2)=13$.

5. The difference of two positive numbers is negative if the number with the greater absolute value is subtracted from the one with the lesser absolute value. For example, $5-12=-7$.

6. The difference of two negative numbers is negative if the number with lesser absolute value is subtracted from the one with greater absolute value. For example, $-15-(-3)=-12$.

7. The product of two numbers with the same sign is positive. For example, $(-2)(-8) = 16$ and $2 \cdot 8 = 16$.

8. The product of two numbers with different signs is negative. For example, $-5(15) = -75$.

9. The quotient formed by any nonzero number divided by 0 is undefined, and the quotient formed by 0 divided by any nonzero number is 0. For example, $\frac{-17}{0}$ is undefined and $\frac{0}{42}$ is equal to 0.

10. The sum of a positive number and a negative number is 0 if the numbers are additive inverses. For example, $4 + (-4) = 0$.

11. $-6 + (-13) = -(6 + 13) = -19$

12. $-8 + (-16) = -(8 + 16) = -24$

13. $-15 + 6 = -(15 - 6) = -9$

14. $-17 + 9 = -(17 - 9) = -8$

15. $13 + (-4) = 13 - 4 = 9$

16. $19 + (-13) = 19 - 13 = 6$

17. $-17 + 22 = 22 - 17 = 5$

18. $-12 + 16 = 16 - 12 = 4$

19. $-\frac{7}{3} + \frac{3}{4} = -\frac{28}{12} + \frac{9}{12} = -\frac{19}{12}$

20. $-\frac{5}{6} + \frac{4}{9} = -\frac{15}{18} + \frac{8}{18} = -\frac{7}{18}$

21. The difference between 4.5 and 2.8 is 1.7. The number with the greater absolute value, 4.5 is positive, so the answer is positive. Thus, $-2.8 + 4.5 = 1.7$.

22. $$\begin{aligned} -3.8 + 6.2 &= 6.2 - 3.8 \\ &= 2.4 \end{aligned}$$

23. $4 - 9 = 4 + (-9) = -5$

24. $3 - 7 = 3 + (-7) = -4$

25. $-6 - 5 = -6 + (-5) = -(6 + 5) = -11$

26. $-8 - 17 = -8 + (-17) = -(8 + 17) = -25$

27. $8 - (-13) = 8 + 13 = 21$

28. $12 - (-22) = 12 + 22 = 34$

29. $-16 - (-3) = -16 + 3 = -13$

30. $-21 - (-6) = -21 + 6 = -15$

31. $$\begin{aligned} -12.31 - (-2.13) &= -12.31 + 2.13 \\ &= -(12.31 - 2.13) \\ &= -10.18 \end{aligned}$$

32. $$\begin{aligned} -15.88 - (-9.42) &= -15.88 + 9.42 \\ &= -(15.88 - 9.42) \\ &= -6.46 \end{aligned}$$

33. $\frac{9}{10} - \left(-\frac{4}{3}\right) = \frac{9}{10} + \frac{4}{3} = \frac{27}{30} + \frac{40}{30} = \frac{67}{30}$

34. $\frac{3}{14} - \left(-\frac{3}{4}\right) = \frac{3}{14} + \frac{3}{4} = \frac{6}{28} + \frac{21}{28} = \frac{27}{28}$

35. $|-8 - 6| = |-14| = -(-14) = 14$

36. $|-7 - 15| = |-22| = -(-22) = 22$

37. $-|-4 + 9| = -|5| = -5$

38. $-|-5 + 6| = -|1| = -1$

39. $-2 - |-4| = -2 - 4 = -2 + (-4) = -6$

40. $16 - |-13| = 16 - 13 = 16 + (-13) = 3$

41. $-7 + 5 - 9 = (-7 + 5) - 9 = -2 - 9 = -11$

42. $-12 + 14 - 18 = 2 - 18 = -16$

43. $6 - (-2) - 8 = 6 + 2 - 8 = 8 - 8 = 0$

44. $7 - (-4) - 11 = 7 + 4 - 11 = 11 - 11 = 0$

45. $$\begin{aligned} 8 - (-12) - 2 - 6 &= 8 + 12 - 2 - 6 \\ &= 20 - 2 - 6 \\ &= 18 - 6 \\ &= 12 \end{aligned}$$

46. $$\begin{aligned} 3 - (-14) - 6 - 4 &= 3 + 14 - 6 - 4 \\ &= 17 - 6 - 4 \\ &= 11 - 4 \\ &= 7 \end{aligned}$$

47. $$\begin{aligned} -9 - 4 - (-3) + 6 &= (-9 - 4) + 3 + 6 \\ &= -13 + 3 + 6 \\ &= -10 + 6 \\ &= -4 \end{aligned}$$

48. $-10-6-(-12)+9=-10+(-6)+12+9$
$=-16+12+9$
$=-4+9$
$=5$

49. $-0.38+4-0.62=3.62-0.62$
$=3$

50. $-2.95+8-0.05=5.05-0.05$
$=5$

51. $\left(-\frac{5}{4}-\frac{2}{3}\right)+\frac{1}{6}=\left(-\frac{15}{12}-\frac{8}{12}\right)+\left(\frac{2}{12}\right)$
$=-\frac{23}{12}+\frac{2}{12}$
$=-\frac{21}{12}$, or $-\frac{7}{4}$

52. $\left(-\frac{5}{9}-\frac{1}{6}\right)+\frac{1}{2}=\left(-\frac{10}{18}-\frac{3}{18}\right)+\frac{1}{2}$
$=-\frac{13}{18}+\frac{9}{18}$
$=-\frac{4}{18}$, or $-\frac{2}{9}$

53. $-\frac{3}{4}-\left(\frac{1}{2}-\frac{3}{5}\right)=-\frac{6}{8}-\left(\frac{4}{8}-\frac{3}{8}\right)$
$=-\frac{6}{8}-\frac{1}{8}$
$=-\frac{7}{8}$

54. $-\frac{1}{2}-\left(\frac{5}{6}-\frac{5}{12}\right)=-\frac{1}{2}-\left(\frac{10}{12}-\frac{5}{12}\right)$
$=-\frac{1}{2}-\left(\frac{5}{12}\right)$
$=-\frac{6}{12}-\frac{5}{12}$
$=-\frac{11}{12}$

55. $-4-[-3-(-6)]+9$
$=-4-[-3+6]+9$
$=-4-3+9$
$=-7+9$
$=2$

56. $-10-[-2-(-5)]+16$
$=-10-[-2+5]+16$
$=-10-3+16$
$=-13+16$
$=3$

57. $|-11|-|-5|-|7|+|-2|$
$=11-5-7+2$
$=6-7+2$
$=-1+2$
$=1$

58. $|-6|+|-3|-|4|-|-8|$
$=6+3-4-8$
$=9-4-8$
$=5-8$
$=-3$

59. $A=-4$ and $B=2$. The distance between A and B is the absolute value of their difference.
$|-4-2|=|-6|=6$

60. $A=-4$ and $C=5$. The distance between A and C is the absolute value of their difference.
$|-4-5|=|-9|=9$

61. $D=-6$ and $F=\frac{1}{2}$. The distance between D and F is the absolute value of their difference.
$\left|-6-\frac{1}{2}\right|=\left|-\frac{12}{2}-\frac{1}{2}\right|$
$=\left|-\frac{13}{2}\right|$
$=\frac{13}{2}$, or $6\frac{1}{2}$

62. $E=-\frac{3}{2}$ and $C=5$. The distance between E and C is the absolute value of their difference.
$\left|-\frac{3}{2}-5\right|=\left|-\frac{3}{2}-\frac{10}{2}\right|$
$=\left|-\frac{13}{2}\right|$
$=\frac{13}{2}$, or $6\frac{1}{2}$

63. It is true for multiplication (and division). It is false for addition and subtraction when the number to be subtracted has the lesser absolute value. A more precise statement is, "The product or quotient of two negative numbers is positive."

64. Because the reciprocal of a number is the quotient of 1 (a positive number) and the number, the reciprocal always has the same sign as the number.

65. The product of two numbers with the same sign is positive, so $-8(-5)=40$.

66. The product of two numbers with the same sign is positive, so $-20(-4)=80$.

67. The product of two numbers with different signs is negative, so $5(-7)=-35$.

68. The product of two numbers with different signs is negative, so $6(-9)=-54$.

69. $4(0)=4\cdot 0=0$

The multiplication property of 0 states that any product of any real number and 0 is 0.

70. $0(-8)=0\cdot -8=0$

The multiplication property of 0 states that any product of any real number and 0 is 0.

71. $\frac{1}{2}(0)=\frac{1}{2}\cdot 0=0$

The multiplication property of 0 states that any product of any real number and 0 is 0.

72. $0(-4.5)=0\cdot -4.5=0$

The multiplication property of 0 states that any product of any real number and 0 is 0.

73. The product of two numbers with different signs is negative, so $\frac{3}{4}(-16)=-\frac{3}{4}\cdot 4\cdot 4=-12$.

74. The product of two numbers with different signs is negative, so $\frac{3}{5}(-35)=-\frac{3}{5}\cdot 5\cdot 7=-21$.

75. The product of two numbers with the same sign is positive, so $-10\left(-\frac{1}{5}\right)=2\cdot 5\cdot\left(\frac{1}{5}\right)=2$.

76. The product of two numbers with the same sign is positive, so $-\frac{1}{2}(-18)=\frac{1}{2}\cdot 2\cdot 9=9$.

77. The product of two numbers with the same sign is positive, so $-\frac{5}{2}\left(-\frac{12}{25}\right)=\frac{5\cdot 2\cdot 6}{2\cdot 5\cdot 5}=\frac{6}{5}$.

78. The product of two numbers with the same sign is positive, so $-\frac{9}{7}\left(-\frac{21}{36}\right)=\frac{9\cdot 3\cdot 7}{7\cdot 4\cdot 9}=\frac{3}{4}$.

79. The product of two numbers with the same sign is positive, so $-\frac{3}{8}\left(-\frac{24}{9}\right)=\frac{3\cdot 3\cdot 8}{8\cdot 9}=1$.

80. The product of two numbers with the same sign is positive, so $-\frac{2}{11}\left(-\frac{22}{4}\right)=\frac{2\cdot 2\cdot 11}{11\cdot 2\cdot 2}=1$.

81. The product of two numbers with the same sign is positive, so $-0.8(-0.5)=0.4$.

82. The product of two numbers with the same sign is positive, so $-0.5(-0.6)=0.3$.

83. The product of two numbers with different signs is negative, so $-0.06(0.4)=-0.024$.

84. The product of two numbers with different signs is negative, so $-0.08(0.7)=-0.056$.

85. The quotient of two nonzero real numbers with different signs is negative, so
$\frac{-14}{2}=-14\cdot\frac{1}{2}=-2\cdot 7\cdot\frac{1}{2}=-7$.

86. The quotient of two nonzero real numbers with different signs is negative, so
$\frac{-39}{13}=-39\cdot\frac{1}{13}=-3\cdot 13\cdot\frac{1}{13}=-3$.

87. The quotient of two nonzero real numbers with the same sign is positive, so
$\frac{-24}{-4}=24\cdot\frac{1}{4}=4\cdot 6\cdot\frac{1}{4}=6$.

88. The quotient of two nonzero real numbers with the same sign is positive, so
$\frac{-45}{-9}=45\cdot\frac{1}{9}=5\cdot 9\cdot\frac{1}{9}=5$.

89. The quotient of two nonzero real numbers with different signs is negative, so
$\frac{100}{-25}=-100\cdot\frac{1}{25}=-4\cdot 25\cdot\frac{1}{25}=-4$.

90. The quotient of two nonzero real numbers with different signs is negative, so
$\frac{150}{-30}=-150\cdot\frac{1}{30}=-5\cdot 30\cdot\frac{1}{30}=-5$.

91. $\frac{0}{-8}=0\cdot\left(-\frac{1}{8}\right)=0$

92. $\frac{0}{-14}=0\cdot\left(-\frac{1}{14}\right)=0$

93. Division by 0 is undefined, so $\frac{5}{0}$ is undefined.

94. Division by 0 is undefined, so $\frac{13}{0}$ is undefined.

95. The quotient of two nonzero real numbers with the same sign is positive, so
$-\frac{10}{17}\div\left(-\frac{12}{5}\right)=\frac{10}{17}\cdot\frac{5}{12}=\frac{2\cdot5\cdot5}{17\cdot2\cdot6}=\frac{25}{102}$.

96. $-\frac{22}{23}\div\left(-\frac{33}{5}\right)=\frac{22}{23}\cdot\frac{5}{33}=\frac{2\cdot11\cdot5}{23\cdot3\cdot11}=\frac{10}{69}$

97. $\frac{\frac{12}{13}}{-\frac{4}{3}}=\frac{12}{13}\div\left(-\frac{4}{3}\right)=\frac{12}{13}\left(-\frac{3}{4}\right)$
$=-\frac{3\cdot4\cdot3}{13\cdot4}=-\frac{9}{13}$

98. $\frac{\frac{7}{6}}{-\frac{2}{3}}=\frac{7}{6}\div\left(-\frac{2}{3}\right)=\frac{7}{6}\left(-\frac{3}{2}\right)$
$=-\frac{7\cdot3}{2\cdot3\cdot2}=-\frac{7}{4}$

99. $\frac{-7.2}{0.8}=-9$

100. $\frac{-4.5}{0.9}=-5$

101. $\frac{-1.28}{-0.4}=3.2$

102. $\frac{-1.82}{-0.2}=9.1$

103. $\frac{1}{6}-\left(-\frac{7}{9}\right)=\frac{1}{6}+\frac{7}{9}=\frac{3}{18}+\frac{14}{18}=\frac{17}{18}$

104. $\frac{7}{10}-\left(-\frac{1}{6}\right)=\frac{7}{10}+\frac{1}{6}=\frac{21}{30}+\frac{5}{30}=\frac{26}{30}=\frac{13}{15}$

105. $-\frac{1}{9}+\frac{7}{12}=-\frac{4}{36}+\frac{21}{36}=\frac{17}{36}$

106. $-\frac{1}{12}+\frac{13}{16}=-\frac{4}{48}+\frac{39}{48}=\frac{35}{48}$

107. $-\frac{3}{8}-\frac{5}{12}=-\frac{9}{24}-\frac{10}{24}=-\frac{19}{24}$

108. $-\frac{11}{15}-\frac{4}{9}=-\frac{33}{45}-\frac{20}{45}=-\frac{53}{45}$

109. $-\frac{7}{30}+\frac{2}{45}-\frac{3}{10}=-\frac{21}{90}+\frac{4}{90}-\frac{27}{90}=-\frac{44}{90}=-\frac{22}{45}$

110. $-\frac{8}{15}+\frac{7}{6}-\frac{3}{20}=-\frac{32}{60}+\frac{70}{60}-\frac{9}{60}=\frac{29}{60}$

111. $\frac{8}{25}\left(-\frac{5}{12}\right)=-\frac{2\cdot4\cdot5}{5\cdot5\cdot3\cdot4}=-\frac{2}{5\cdot3}=-\frac{2}{15}$

112. $\frac{9}{20}\left(-\frac{7}{15}\right)=-\frac{3\cdot3\cdot7}{4\cdot5\cdot3\cdot5}=-\frac{3\cdot7}{4\cdot5\cdot5}=-\frac{21}{100}$

113. $\frac{5}{6}\left(-\frac{9}{10}\right)\left(-\frac{4}{5}\right)=\frac{5\cdot3\cdot3\cdot2\cdot2}{2\cdot3\cdot2\cdot5\cdot5}=\frac{3}{5}$

114. $\frac{4}{3}\left(-\frac{9}{20}\right)\left(-\frac{5}{12}\right)=\frac{2\cdot2\cdot3\cdot3\cdot5}{3\cdot2\cdot2\cdot5\cdot4\cdot3}=\frac{1}{4}$

115. $\frac{8}{3}\div\left(-\frac{14}{15}\right)=\frac{8}{3}\cdot\left(-\frac{15}{14}\right)$
$=-\frac{2\cdot4\cdot3\cdot5}{3\cdot2\cdot7}$
$=-\frac{4\cdot5}{7}$
$=-\frac{20}{7}$

116. $\frac{12}{5}\div\left(-\frac{18}{25}\right)=\frac{12}{5}\cdot\left(-\frac{25}{18}\right)$
$=-\frac{2\cdot2\cdot3\cdot5\cdot5}{5\cdot2\cdot3\cdot3}$
$=-\frac{2\cdot5}{3}$
$=-\frac{10}{3}$, or $-3\frac{1}{3}$

117. $\frac{\frac{2}{3}}{-2}=\frac{2}{3}\div\frac{-2}{1}=\frac{2}{3}\cdot\frac{1}{-2}=-\frac{2\cdot1}{3\cdot2}=-\frac{2}{6}=-\frac{1}{3}$

118. $\frac{\frac{3}{4}}{-6}=\frac{3}{4}\div\frac{-6}{1}=\frac{3}{4}\cdot\frac{1}{-6}=-\frac{3\cdot1}{4\cdot6}=-\frac{3}{24}=-\frac{1}{8}$

119. $\dfrac{-\frac{8}{9}}{\frac{2}{3}} = -\dfrac{8}{9} \div \dfrac{2}{3} = -\dfrac{8}{9} \cdot \dfrac{3}{2} = -\dfrac{8 \cdot 3}{9 \cdot 2} = -\dfrac{24}{18} = -\dfrac{4}{3}$

120. $\dfrac{-\frac{15}{16}}{\frac{3}{8}} = -\dfrac{15}{16} \div \dfrac{3}{8} = -\dfrac{15}{16} \cdot \dfrac{8}{3}$

$= -\dfrac{15 \cdot 8}{16 \cdot 3} = -\dfrac{120}{48} = -\dfrac{5}{2}$

121. $-8.6 - 23.751 = -(8.6 + 23.751) = -32.351$

122. $-37.8 - 13.582 = -(37.8 + 13.582) = -51.382$

123. $-2.5(0.8)(1.5) = (-2)(1.5) = -3$

124. $-1.6(0.5)(2.5) = (-0.8)(2.5)$
$= -2$

125. $-24.84 \div 6 = -4.14$

126. $-32.84 \div 8 = -4.105$

127. $-2496 \div (-0.52) = 4800$

128. $-1875 \div (-0.25) = 7500$

129. $\dfrac{-100}{-0.01} = \dfrac{100}{\frac{1}{100}} = 100 \div \dfrac{1}{100} = 100 \cdot 100$
$= 10{,}000$

130. $\dfrac{-60}{-0.06} = \dfrac{60}{\frac{6}{100}} = 60 \div \dfrac{6}{100} = 60 \cdot \dfrac{100}{6}$
$= 10 \cdot 100 = 1000$

131. $-14.2 + 9.81 = -4.39$

132. $-89.41 + 21.325 = -68.085$

133. To find the difference between these two temperatures, subtract the lowest temperature from the highest temperature.
$100° - (-80°) = 100° + 80°$
$= 180°$
The difference is 180°F.

134. $120° - (-29°) = 120° + 29°$
$= 149°$
The difference is 149°F.

135. $48.35 - 35.99 - 20.00 - 28.50 + 66.27$
$= 12.36 - 20.00 - 28.50 + 66.27$
$= -7.64 - 28.50 + 66.27$
$= -36.14 + 66.27$
$= 30.13$
His balance is \$30.13.

136. $37.60 - 25.99 - 19.34 - 25.00 + 58.66$
$= 11.61 - 19.34 - 25.00 + 58.66$
$= -7.73 - 25.00 + 58.66$
$= -32.73 + 58.66$
$= 25.93$
Her balance is \$25.93.

137. $-382.45 + 25.10 + 34.50 - 45.00 - 98.17$
$= -466.02$
His balance is $-\$466.02$.

(a) To pay off the balance, his payment should be \$466.02.

(b) $-466.02 + 300 - 24.66 = -190.68$
His balance is $-\$190.68$.

138. $-237.59 + 47.25 - 12.39 - 20.00 = -222.73$
Her balance is $-\$222.73$.

(a) To pay off her balance, her payment should be \$222.73.

(b) $-222.73 + 75.00 - 32.06 = -179.79$
Her balance is $-\$179.79$.

139. **(a)** $4.93 + 16.27 + 8.41 + (-28.46) + 30.18$
$= 31.33$
The sum for 2018–2022 was 31.33%.

(b) $-28.46 - (8.41) = -36.87$
The difference between the percents from 2021 and 2020 was -36.87%.

(c) $30.18 - (-28.46) = 58.64$
The difference between the percents from 2022 and 2021 was 58.64%.

140. **(a)** $-142 - 225 - 185 + 77 = -475$
The total is $-\$475$ thousand, which represents a loss of \$475 thousand.

(b) $77 - (-185) = 77 + 185$
$= 262$
The difference was \$262 thousand dollars.

(c) $-225-(-142)=-225+142$
$=-83$
The difference was $-\$83$ thousand dollars.

141.

Year	Difference (in billions)
2001	$\$1991-\$1863=\$128$
2006	$\$2407-\$2655=-\$248$
2011	$\$2303-\$3603=-\$1300$
2016	$\$3268-\$3853=-\$585$
2021	$\$4047-\$6822=-\$2775$

142. There was a surplus in the year 2001. There were defecits in the years 2006, 2011, 2016, and 2021. A positive difference between receipts and outlays indicates a surplus, while a negative difference indicates a deficit.

R.5 Exponents, Roots, and Order of Operations

Classroom Examples, Now Try Exercises

1. (a) $\frac{2}{7}\cdot\frac{2}{7}\cdot\frac{2}{7}\cdot\frac{2}{7}=\left(\frac{2}{7}\right)^4$

(b) $(-10)(-10)(-10)=(-10)^3$

(c) $y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y=y^8$

N1. (a) $(-3)(-3)(-3)=(-3)^3$

(b) $(0.5)(0.5)=(0.5)^2$

(c) $t\cdot t\cdot t\cdot t\cdot t=t^5$

2. (a) $2^5=2\cdot 2\cdot 2\cdot 2\cdot 2=32$

(b) $(0.5)^2=(0.5)(0.5)=0.25$

(c) $(-4)^3=(-4)(-4)(-4)=-64$

(d) $(-4)^4=(-4)(-4)(-4)(-4)=256$

N2. (a) $5^3=5\cdot 5\cdot 5=125$

(b) $\left(\frac{2}{5}\right)^4=\frac{2}{5}\cdot\frac{2}{5}\cdot\frac{2}{5}\cdot\frac{2}{5}=\frac{16}{625}$

(c) $(-3)^2=(-3)(-3)=9$

(d) $(-3)^3=(-3)(-3)(-3)=-27$

3. (a) $3^4=3\cdot 3\cdot 3\cdot 3$
$=9\cdot 3\cdot 3$
$=27\cdot 3=81$

(b) $(-3)^4=(-3)(-3)(-3)(-3)=81$

(c) $-3^4=-(3\cdot 3\cdot 3\cdot 3)=-81$

N3. (a) $7^2=7\cdot 7=49$

(b) $(-7)^2=(-7)(-7)=49$

(c) $-7^2=-(7\cdot 7)=-49$

4. (a) $\sqrt{49}=7$ because 7 is positive and $7^2=49$.

(b) $\sqrt{\frac{121}{81}}=\frac{11}{9}$ because $\left(\frac{11}{9}\right)^2=\frac{121}{81}$, so $-\sqrt{\frac{121}{81}}=-\frac{11}{9}$.

(c) $-\sqrt{49}=-7$ because the negative sign is outside the radical symbol.

(d) $\sqrt{-49}$ is not a real number.

N4. (a) $\sqrt{121}=11$ because $11^2=121$.

(b) $\sqrt{\frac{100}{9}}=\frac{10}{3}$ because $\left(\frac{10}{3}\right)^2=\frac{100}{9}$.

(c) $-\sqrt{121}=-11$ because the negative sign is outside the radical symbol.

(d) $\sqrt{-121}$ is not a real number.

5. (a) $5\cdot 9+2\cdot 4=45+8$ Multiply.
$=53$ Add.

(b) $4-12\div 4\cdot 2=4-3\cdot 2$ Divide.
$=4-6$ Multiply.
$=-2$ Subtract.

N5. $15-3\cdot 4+2=15-12+2$ Multiply.
$=3+2$ Subtract.
$=5$ Add.

6. (a) Add and subtract inside parentheses.
$(4+2)-4^2-(8-3)$
$=6-4^2-5$

Evaluate the power.

$= 6 - 16 - 5$

$= -10 - 5$ Subtract from left to right.

$= -15$

(b) $6 + \frac{5}{4}(-12) - \frac{2}{3} \cdot 18$

$= 6 + \frac{5}{4} \cdot \frac{-12}{1} - \frac{2}{3} \cdot \frac{18}{1}$

$= 6 - \frac{60}{4} - \frac{36}{3}$ Multiply.

Subtract left from right.

$= 6 - 15 - 12$

$= -21$

N6. **(a)** Work inside the absolute value bars.

$-5^2 + 10 \div 5 - |3 - 7|$

Subtract inside the absolute value bars.

$= -5^2 + 10 \div 5 - |-4|$

$= -5^2 + 10 \div 5 - 4$ Take absolute value.

$= -25 + 10 \div 5 - 4$ Evaluate the power.

$= -25 + 2 - 4$ Divide.

$= -23 - 4$ Add.

$= -27$ Subtract.

(b) $6 + \frac{2}{3}(-9) - \frac{5}{8} \cdot 16$

$= 6 + \frac{2}{3} \cdot \frac{-9}{1} - \frac{5}{8} \cdot \frac{16}{1}$

$= 6 - \frac{18}{3} - \frac{80}{8}$ Multiply

$= 6 - 6 - 10$ Subtract from left

$= -10$ to right.

7. $\dfrac{\frac{1}{2} \cdot 10 - 6 + \sqrt{9}}{\frac{5}{6} \cdot 12 - 3^2 - 1}$

$= \dfrac{\frac{1}{2} \cdot 10 - 6 + 3}{\frac{5}{6} \cdot 12 - 9 - 1}$ Evaluate powers and roots.

$= \dfrac{5 - 6 + 3}{10 - 9 - 1}$ Multiply.

$= \dfrac{-1 + 3}{1 - 1}$ Add and subtract.

$= \dfrac{2}{0}$ Undefined.

N7. $\dfrac{\sqrt{36} - 4 \cdot 3^2}{-2^2 - 8 \cdot 3 + 28}$

$= \dfrac{6 - 4 \cdot 9}{-4 - 8 \cdot 3 + 28}$ Evaluate powers and roots.

$= \dfrac{6 - 36}{-4 - 24 + 28}$ Multiply.

$= \dfrac{-30}{-28 + 28}$ Add and subtract.

$= \dfrac{-30}{0}$ Undefined.

8. **(a)** $\dfrac{5x + z\sqrt{y}}{x - 1} = \dfrac{5(-12) + (-3)\sqrt{64}}{-12 - 1}$

$= \dfrac{5(-12) - 3(8)}{-12 - 1}$

$= \dfrac{-60 - 24}{-12 - 1}$

$= \dfrac{-84}{-13} = \dfrac{84}{13}$

(b) $w^2 + 2z^3 = 4^2 + 2(-3)^3$

$= 16 + 2(-27)$

$= 16 + (-54)$

$= -38$

N8. **(a)** $3y - 2x = 3(7) - 2(-4)$

$= 21 - (-8)$

$= 21 + 8$

$= 29$

(b) $\dfrac{x^2 - \sqrt{z}}{-3xy} = \dfrac{(-4)^2 - \sqrt{36}}{-3(-4)(7)}$

$= \dfrac{16 - 6}{-3(-4)(7)}$

$= \dfrac{16 - 6}{12(7)}$

$= \dfrac{16 - 6}{84}$

$= \dfrac{10}{84} = \dfrac{5}{42}$

Exercises

1. $-7^6 = -(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7) = -117{,}649,$

$(-7)^6 = (-7)(-7)(-7)(-7)(-7)(-7)$

$= 117{,}649$

Thus, the original statement, $-7^6 = (-7)^6$, is *false*. Note that the statement $-7^6 = -(7^6)$ is *true*.

2. $-5^7 = (-5)^7$ is *true.* $(-5)^7 = -5^7$ since $(-5)^7$ has an odd number of negative factors.

3. The statement "$\sqrt{25}$ is a positive number" is *true.* The symbol $\sqrt{}$ always gives a positive square root provided the radicand is positive.

4. $3+5\cdot 8 = 3+(5\cdot 8)$ is *true.* The order of operations says that multiplication should be done before addition.

5. The statement "$(-6)^7$ is a negative number" is *true.* $(-6)^7$ gives an odd number of negative factors, so the product is negative.

6. The statement "$(-6)^8$ is a positive number" is *true.* $(-6)^8$ gives an even number of negative factors, so the product is positive.

7. The statement "The product of 10 positive factors and 10 negative factors is positive" is *true.* The product of an even number of negative factors is positive.

8. The statement "The product of 5 positive factors and 5 negative factors is positive" is *false.* The product of 5 positive factors is positive and the product of 5 negative factors is negative, so the product of these two products is negative.

9. The statement "In the exponential expression -2^5, the base is -2" is *false.* The base is 2, not -2. If the problem were written $(-2)^5$, then -2 would be the base.

10. The statement "$\sqrt{a}$ is positive for all positive numbers a" is *true.*

11. $10\cdot 10\cdot 10\cdot 10 = 10^4$

12. $8\cdot 8\cdot 8 = 8^3$

13. $\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4} = \left(\frac{3}{4}\right)^5$

14. $\frac{1}{2}\cdot\frac{1}{2} = \left(\frac{1}{2}\right)^2$

15. $(-9)(-9)(-9) = (-9)^3$

16. $(-4)(-4)(-4)(-4) = (-4)^4$

17. $(0.8)(0.8) = (0.8)^2$

18. $(0.1)(0.1)(0.1)(0.1)(0.1)(0.1) = (0.1)^6$

19. $z\cdot z\cdot z\cdot z\cdot z\cdot z\cdot z = z^7$

20. $a\cdot a\cdot a\cdot a\cdot a = a^5$

21. **(a)** $8^2 = 64$

(b) $-8^2 = -(8\cdot 8) = -64$

(c) $(-8)^2 = (-8)(-8) = 64$

(d) $-(-8)^2 = -(64) = -64$

22. **(a)** $4^3 = 64$

(b) $-4^3 = -(4\cdot 4\cdot 4) = -64$

(c) $(-4)^3 = (-4)(-4)(-4) = -64$

(d) $-(-4)^3 = -(-64) = 64$

23. $4^2 = 4\cdot 4 = 16$

24. $6^2 = 6\cdot 6 = 36$

25. $(0.3)^3 = (0.3)(0.3)(0.3) = 0.027$

26. $(0.1)^3 = (0.1)(0.1)(0.1) = 0.001$

27. $\left(\frac{1}{5}\right)^3 = \frac{1}{5}\cdot\frac{1}{5}\cdot\frac{1}{5} = \frac{1}{125}$

28. $\left(\frac{1}{6}\right)^4 = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$

$= \frac{1}{6\cdot 6\cdot 6\cdot 6} = \frac{1}{1296}$

29. $\left(\frac{4}{5}\right)^4 = \left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)$

$= \frac{4\cdot 4\cdot 4\cdot 4}{5\cdot 5\cdot 5\cdot 5} = \frac{256}{625}$

30. $\left(\frac{7}{10}\right)^3 = \left(\frac{7}{10}\right)\left(\frac{7}{10}\right)\left(\frac{7}{10}\right) = \frac{343}{1000}$

31. $(-5)^3 = (-5)(-5)(-5) = -125$

32. $(-3)^5 = (-3)(-3)(-3)(-3)(-3) = -243$

33. $(-2)^8 = (-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2)$
$= 256$

34. $(-3)^6 = (-3)(-3)(-3)(-3)(-3)(-3) = 729$

35. $-3^6 = -(3\cdot3\cdot3\cdot3\cdot3\cdot3) = -729$

36. $-4^6 = (4\cdot4\cdot4\cdot4\cdot4\cdot4) = -4096$

37. $-8^4 = -(8\cdot8\cdot8\cdot8) = -4096$

38. $-10^3 = -(10\cdot10\cdot10) = -1000$

39. **(a)** -7^2 is negative because the negative sign is not part of the base.

(b) $(-7)^2$ is positive because the negative sign is part of the base, and the base is squared, so there are an even number of factors.

(c) -7^3 is negative because the negative sign is not part of the base.

(d) $(-7)^3$ is negative because the negative sign is part of the base, and the base is cubed, so there are an odd number of factors.

(e) -7^4 is negative because the negative sign is not part of the base.

(f) $(-7)^4$ is positive because the negative sign is part of the base, and the base is raised to the fourth power, so there are an even number of factors.

40. **(a)** $\sqrt{144} = 12,$ which is choice B.

(b) $\sqrt{-144}$ is not a real number, which is choice C.

(c) $-\sqrt{144} = -12,$ which is choice A.

41. $\sqrt{81} = 9$ since 9 is positive and $9^2 = 81$.

42. $\sqrt{64} = 8$ since 8 is positive and $8^2 = 64$.

43. $\sqrt{169} = 13$ since 13 is positive and $13^2 = 169$.

44. $\sqrt{225} = 15$ since 15 is positive and $15^2 = 225$.

45. $-\sqrt{400} = -\left(\sqrt{400}\right) = -(20) = -20$

46. $-\sqrt{900} = -\left(\sqrt{900}\right) = -(30) = -30$

47. $\sqrt{\dfrac{100}{121}} = \dfrac{10}{11}$ since $\dfrac{10}{11}$ is positive and $\left(\dfrac{10}{11}\right)^2 = \dfrac{100}{121}$.

48. $\sqrt{\dfrac{225}{169}} = \dfrac{15}{13}$ since $\dfrac{15}{13}$ is positive and $\left(\dfrac{15}{13}\right)^2 = \dfrac{225}{169}$.

49. $-\sqrt{0.49} = -\sqrt{(0.7)^2} = -(0.7) = -0.7$

50. $-\sqrt{0.64} = -\sqrt{(0.8)^2} = -(0.8) = -0.8$

51. There is no real number whose square is negative, so $\sqrt{-36}$ is not a real number.

52. There is no real number whose square is negative, so $\sqrt{-121}$ is not a real number.

53. If a is a positive number, then $-a$ is a negative number. Therefore, $\sqrt{-a}$ is not a real number, and $-\sqrt{-a}$ *is not a real number*.

54. If a is a positive number, then $\sqrt{a}$ is also a positive number, and $-\sqrt{a}$ is a *negative* number.

55. **(a)** Since $9+15\div3 = 9+5 = 14,$ Student B was correct.

(b) The reasoning was incorrect. The operation of division must be done first, and then the addition follows. The "Order of Process rule" given by Student B is not correct. It just happens coincidentally in this problem that the correct answer was obtained in the wrong way.

56.
$$\begin{aligned} 7+7\div7+7\times7-7 &= 7+1+49-7 \\ &= 8+49-7 \\ &= 57-7 \\ &= 50 \end{aligned}$$

57. $12+3\cdot4 = 12+12$ Multiply.
$= 24$ Add.

58. $15+5\cdot2 = 15+10$ Multiply.
$= 25$ Add.

59. $6 \cdot 3 - 12 \div 4 = 18 - 12 \div 4$ Multiply.
$= 18 - 3$ Divide.
$= 15$ Subtract.

60. $9 \cdot 4 - 8 \div 2 = 36 - 8 \div 2$ Multiply.
$= 36 - 4$ Divide.
$= 32$ Subtract.

61. $10 + 30 \div 2 \cdot 3 = 10 + 15 \cdot 3$ Divide.
$= 10 + 45$ Multiply.
$= 55$ Add.

62. $12 + 24 \div 3 \cdot 2 = 12 + 8 \cdot 2$ Divide.
$= 12 + 16$ Multiply.
$= 28$ Add.

63. $-3(5)^2 - (-2)(-8)$
$= -3(25) - (-2)(-8)$ Evaluate power.
$= -75 - 16$ Multiply.
$= -91$ Subtract.

64. $-9(2)^2 - (-3)(-2)$
$= -9(4) - (-3)(-2)$ Evaluate power.
$= -36 - 6$ Multiply.
$= -42$ Subtract.

65. $5 - 7 \cdot 3 - (-2)^3$
$= 5 - 7 \cdot 3 - (-8)$ Evaluate power.
$= 5 - 21 + 8$ Multiply, change sign.
$= -16 + 8$ Subtract.
$= -8$ Add.

66. $-4 - 3 \cdot 5 - (-3)^3$
$= -4 - 3 \cdot 5 - (-27)$ Evaluate power.
$= -4 - 15 + 27$ Multiply.
$= -19 + 27$ Subtract.
$= 8$ Add.

67. $-7\left(\sqrt{36}\right) - (-2)(-3)$
$= -7(6) - (-2)(-3)$ Evaluate root.
$= -42 - 6$ Multiply.
$= -48$ Subtract.

68. $-8\left(\sqrt{64}\right) - (-3)(-7)$
$= -8(8) - (-3)(-7)$ Evaluate root.
$= -64 - 21$ Multiply.
$= -85$ Subtract.

69. Simplify within absolute value bars.
$6|4 - 5| - 24 \div 3$
$= 6|-1| - 24 \div 3$
$= 6(1) - 24 \div 3$ Take absolute value.
$= 6 - 8$ Multiply and divide.
$= -2$ Subtract.

70. Simplify within absolute value bars.
$-4|2 - 4| + 8 \cdot 2$
$= -4|-2| + 8 \cdot 2$
$= -4(2) + 8 \cdot 2$ Take absolute value.
$= -8 + 16$ Multiply.
$= 8$ Add.

71. Simplify within absolute value bars.
$|-6 - 5|(-8) - 3^2$
$= |-11|(-8) - 3^2$
$= 11(-8) - 3^2$ Take absolute value.
$= 11(-8) - 9$ Evaluate power.
$= -88 - 9$ Multiply.
$= -97$ Subtract.

72. Simplify within absolute value bars.
$|-2 - 3|(-9) - 4^2$
$= |-5|(-9) - 4^2$
$= 5(-9) - 4^2$ Take absolute value.
$= 5(-9) - 16$ Evaluate power.
$= -45 - 16$ Multiply.
$= -61$ Subtract.

73. Simplify within parentheses.
$18 - 4^2 + 5 - (3 - 7)$
$= 18 - 4^2 + 5 - (-4)$
$= 18 - 4^2 + 5 + 4$ Simplify parentheses.
$= 18 - 16 + 5 + 4$ Evaluate power.
$= 2 + 5 + 4$ Subtract.
$= 7 + 4$ Add.
$= 11$ Add.

74. Simplify within parentheses.

$10-2^2+9-(1-8)$

$=10-2^2+9-(-7)$

$=10-2^2+9+7$ Simplify parentheses.

$=10-4+9+7$ Evaluate power.

$=6+9+7$ Subtract.

$=15+7$ Add.

$=22$ Add.

75. $6+\frac{2}{3}(-9)-\frac{5}{8}\cdot 16=6+(-6)-10$ Multiply.

$=0-10$ Add.

$=-10$ Subtract.

76. $7-\frac{3}{4}(-8)+12\cdot\frac{5}{6}=7+6+10$ Multiply.

$=13+10$ Add.

$=23$ Add.

77. $-14\left(-\frac{2}{7}\right)\div(2\cdot 6-10)$

$=4\div(12-10)$ Multiply.

$=4\div 2$ Work inside parentheses.

$=2$ Divide.

78. $-12\left(-\frac{3}{4}\right)-(6\cdot 5\div 3)$

$=9-(30\div 3)$ Multiply.

$=9-10$ Work inside parentheses.

$=-1$ Subtract.

79. Evaluate root and power in numerator, and subtract in the denominator.

$\frac{(-5+\sqrt{4})-2^2}{-5-2}$

$=\frac{(-5+2)-4}{-7}$

$=\frac{(-3)-4}{-7}$ Work inside parentheses.

$=\frac{-7}{-7}$ Subtract.

$=1$ Divide.

80. Evaluate root and power in numerator, and subtract in the denominator.

$\frac{(-9+\sqrt{16})-3^2}{-6-1}$

$=\frac{(-9+4)-9}{-7}$

$=\frac{(-5)-9}{-7}$ Work inside parentheses.

$=\frac{-14}{-7}$ Subtract.

$=2$ Divide.

81. $\frac{2(-5)+(-3)(-2)}{-8+3^2-1}$

$=\frac{-10+6}{-8+9-1}$ Evaluate power, multiply.

$=\frac{-4}{1-1}$ Add.

$=\frac{-4}{0}$ Subtract.

Since division by 0 is undefined, the given expression is *undefined.*

82. $\frac{3(-4)+(-5)(-8)}{2^3-2-6}$

$=\frac{-12+40}{8-2-6}$ Evaluate power, multiply.

$=\frac{28}{6-6}$ Add, Subtract.

$=\frac{28}{0}$ Subtract.

Since division by 0 is undefined, the given expression is *undefined.*

83. $3a+\sqrt{b}=3(-3)+\sqrt{64}$

$=3(-3)+8$

$=-9+8$

$=-1$

84. $-2a-\sqrt{b}=-2(-3)-\sqrt{64}$

$=6-8$

$=-2$

85. $\sqrt{b}+c-a=\sqrt{64}+6-(-3)$

$=8+6+3$

$=14+3=17$

86. $\sqrt{b}-c+a=\sqrt{64}-6+(-3)$

$=8-6+(-3)$

$=2+(-3)=-1$

87. $4a^3+2c=4(-3)^3+2(6)$
$=4(-27)+12$
$=-108+12=-96$

88. $-3a^4-3c=-3(-3)^4-3(6)$
$=-3(81)-18$
$=-243-18=-261$

89. $2(a-c)^2-ac=2(-3-6)^2-(-3)(6)$
$=2(-9)^2-(-18)$
$=2(81)+18$
$=162+18$
$=180$

90. $-4ac+(c-a)^2=-4(-3)(6)+(6-(-3))^2$
$=-4(-3)(6)+(6+3)^2$
$=-4(-3)(6)+(9)^2$
$=-4(-3)(6)+81$
$=72+81$
$=153$

91. $\dfrac{\sqrt{b}-4a}{c^2}=\dfrac{\sqrt{64}-4(-3)}{(6)^2}$
$=\dfrac{8-4(-3)}{36}$
$=\dfrac{8+12}{36}$
$=\dfrac{20}{36}$
$=\dfrac{5}{9}$

92. $\dfrac{a^3+2c}{-\sqrt{b}}=\dfrac{(-3)^3+2(6)}{-\sqrt{64}}$
$=\dfrac{-27+2(6)}{-8}$
$=\dfrac{-27+12}{-8}$
$=\dfrac{-15}{-8}$
$=\dfrac{15}{8}$

93. $\dfrac{2c+a^3}{4b+6a}=\dfrac{2(6)+(-3)^3}{4(64)+6(-3)}$
$=\dfrac{12+(-27)}{256+(-18)}$
$=\dfrac{-15}{238}=-\dfrac{15}{238}$

94. $\dfrac{3c+a^2}{2b-6a}=\dfrac{3(6)+(-3)^2}{2(64)-6(6)}$
$=\dfrac{18+9}{128-36}$
$=\dfrac{27}{92}$

95. $wy-8x=4\left(\dfrac{1}{2}\right)-8\left(-\dfrac{3}{4}\right)$
$=2+6$
$=8$

96. $wz-12y=4(1.25)-12\left(\dfrac{1}{2}\right)$
$=5-6$
$=-1$

97. $xy+y^4=-\dfrac{3}{4}\left(\dfrac{1}{2}\right)+\left(\dfrac{1}{2}\right)^4$
$=-\dfrac{3}{8}+\dfrac{1}{16}$
$=-\dfrac{6}{16}+\dfrac{1}{16}=-\dfrac{5}{16}$

98. $xy-x^2=-\dfrac{3}{4}\left(\dfrac{1}{2}\right)-\left(-\dfrac{3}{4}\right)^2$
$=-\dfrac{3}{8}-\dfrac{9}{16}$
$=-\dfrac{6}{16}-\dfrac{9}{16}=-\dfrac{15}{16}$

99. $-w+2x+3y+z$
$=-4+2\left(-\dfrac{3}{4}\right)+3\left(\dfrac{1}{2}\right)+1.25$
$=-4-\dfrac{3}{2}+\dfrac{3}{2}+1.25$
$=-4+1.25$
$=-2.75$

100. $w-6x+5y-3z$
$=4-6\left(-\frac{3}{4}\right)+5\left(\frac{1}{2}\right)-3(1.25)$
$=4+4.5+2.5-3.75$
$=7.25$

101. $(v\times 0.5485-4850)\div 1000\times 31.44$
$=(150{,}000\times 0.5485-4850)\div 1000\times 31.44$
$=(82{,}275-4850)\div 1000\times 31.44$
$=77{,}425\div 1000\times 31.44$
$=77.425\times 31.44$
$=2434.242\approx 2434$
The owner would pay \$2434 in property taxes.

102. $(v\times 0.5485-4850)\div 1000\times 31.44$
$=(200{,}000\times 0.5485-4850)\div 1000\times 31.44$
$=(109{,}700-4850)\div 1000\times 31.44$
$=104{,}850\div 1000\times 31.44$
$=104.85\times 31.44$
$=3296.484\approx 3296$
The owner would pay \$3296 in property taxes.

103. **(a)** number of oz $\times$ % alcohol $\times 0.075\div$ body weight in lb $-$ hr of drinking $\times 0.015$
$=48\times 3.2\times 0.075\div 190-2\times 0.015$

(b) $48\times 3.2\times 0.075\div 190-2\times 0.015$
$=153.6\times 0.075\div 190-2\times 0.015$
$=11.52\div 190-2\times 0.015$
$\approx 0.061-2\times 0.015$
$=0.061-0.03$
$=0.031$

104. number of oz $\times$ % alcohol $\times 0.075\div$ body weight in lb $-$ hr of drinking $\times 0.015$
$=36\times 4.0\times 0.075\div 135-3\times 0.015$
$=144\times 0.075\div 135-3\times 0.015$
$=10.8\div 135-3\times 0.015$
$=0.08-3\times 0.015$
$=0.08-0.015$
$=0.035$

105. **(a)** BAC for the man
$=48\times 3.2\times 0.075\div(190+25)-2\times 0.015$
≈ 0.024
BAC for the woman
$=36\times 4.0\times 0.075\div(135+25)-3\times 0.015$
≈ 0.023
Increased weight results in lower BACs.

(b) Decreased weight will result in higher BACs.
BAC for the man
$=48\times 3.2\times 0.075\div(190-25)-2\times 0.015$
≈ 0.040
BAC for the woman
$=36\times 4.0\times 0.075\div(135-25)-3\times 0.015$
≈ 0.053

106. BAC for the man
$=48\times 3.2\times 0.075\div 190-1\times 0.015$
≈ 0.046
BAC for the woman
$=36\times 4.0\times 0.075\div 135-2\times 0.015$
$=0.05$
Decreased time results in higher BACs.

107. $10.396x-20{,}893$

(a) $10.396(2015)-20{,}893\approx \$54.9$ billion

(b) $10.396(2018)-20{,}893\approx \$86.1$ billion

(c) $10.396(2021)-20{,}893\approx \$117.3$ billion

(d) The amount spent on pets more than doubled from 2015 to 2021.

108. **(a)**

Year	Average Price (in dollars)
2000	5.50
2005	6.47
2010	$0.1930(2010)-380.5=7.43$
2015	$0.1930(2015)-380.5\approx 8.40$
2021	$0.1930(2021)-380.5\approx 9.55$

(b) The average price of a movie ticket in the United States increased by $\$9.55-\$5.50=\$4.05$ from 2000 to 2021.

R.6 Properties of Real Numbers

Classroom Examples, Now Try Exercises

1. **(a)** $-4(p-5)=-4[p+(-5)]$
$=-4(p)+(-4)(-5)$
$=-4p+20$

(b) Use the second form of the distributive property.
$-6m+2m=(-6+2)m$
$=-4m$

(c) Since there is no common number or variable here, the distributive property cannot be used.

(d) $5(4p-2q+r)=5(4p)+5(-2q)+5(r)$
$=20p-10q+5r$

N1. (a) $-2(3x-y)=-2[3x+(-y)]$
$=-2(3x)+(-2)(-y)$
$=-6x+2y$

(b) Use the second form of the distributive property.
$4k-12k=[4+(-12)]k$
$=-8k$

(c) Since there is no common number or variable here, the distributive property cannot be used.

2. (a) $x-3x=1x-3x$ Identity property
$=(1-3)x$ Distributive property
$=-2x$

(b) $-(3+4p)$
$=-1(3+4p)$ Identity property
$=-1(3)+(-1)(4p)$ Distributive property
$=-3+(-4p)$
$=-3-4p$

N2. (a) $7x+x=7x-1x$ Identity property
$=(7+1)x$ Distributive property
$=8x$

(b) $-(5p-3q)$
$=-1(5p-3q)$ Identity prop.
$=-1(5p)+(-1)(-3q)$ Distributive prop.
$=-5p+3q$

3. (See Example 3 for more detailed steps.)
$12b-9+4b-7b+1$
$=12b+4b-7b-9+1$
$=(12+4-7)b-9+1$
$=9b-8$

N3. (See Example 3 for more detailed steps.)
$-7x+10-3x-4+x$
$=-7x-3x+1x+10-4$
$=(-7-3+1)x+10-4$
$=-9x+6$

4. (a) $12b-9b+5b-7b$
$=(12-9+5-7)b$
$=1b$
$=b$

(b) $6-(2x+7)-3$
$=6-2x-7-3$ Distributive property
$=-2x+6-7-3$ Commutative prop.
$=-2x-4$ Combine like terms.

(c) $4m(2n)$
$=[4m(2)]n$ Associative property
$=[4(m\cdot 2)]n$ Associative property
$=[4(2m)]n$ Commutative property
$=[(4\cdot 2)m]n$ Associative property
$=(8m)n$ Multiply.
$=(8mn)$ Associative property
$=8mn$

(d) $2(x+3)-5(2x-1)$
$=2x+6-10x+5$ Distributive property
$=2x-10x+6+5$ Commutative prop.
$=-8x+11$ Combine like terms.

N4. (a) $-3(t-4)-t+15$
$=-3t+12-t+15$ Distributive prop.
$=-3t-t+12+15$ Commutative prop.
$=-4t+27$ Combine terms.

(b) $7x-(4x-2)$
$=7x-1(4x-2)$ Identity property
$=7x-4x+2$ Distributive property
$=3x+2$ Subtract.

(c) $5x(6y)$
$=[5x(6)]y$ Associative property
$=[5(x\cdot 6)]y$ Associative property
$=[5(6x)]y$ Commutative property
$=[(5\cdot 6)x]y$ Associative property
$=(30x)y$ Multiply.
$=30(xy)$ Associative property
$=30xy$

(d) $3(5x-7)-8(x+4)$
$=15x-21-8x-32$ Distributive property
$=15x-8x-21-32$ Commutative prop.
$=7x-53$ Combine like terms.

Exercises

1. The identity element for addition is 0 since, for any real number a, $a+0=0+a=a$. Choice B is correct.

2. The identity element for multiplication is 1 since, for any real number a, $a\cdot 1=1\cdot a=a$. Choice C is correct.

3. The additive inverse of a is $-a$ since, for any real number a, $a+(-a)=0$ and $-a+a=0$. Choice A is correct.

4. The multiplicative inverse of a is $\frac{1}{a}$ since, for any nonzero real number a, $a\cdot\frac{1}{a}=1$ and $\frac{1}{a}\cdot a=1$. Choice D is correct.

5. The distributive property provides a way to rewrite a product such as $a(b+c)$ as the sum $\underline{ab+ac}$.

6. The commutative property is used to change the <u>order</u> of two terms or factors.

7. The associative property is used to change the <u>grouping</u> of three terms or factors.

8. Like terms are terms with the <u>same</u> variables raised to the <u>same</u> powers.

9. When simplifying an expression, only <u>like</u> terms can be combined.

10. The numerical coefficient in the term $-7yz^2$ is $\underline{-7}$.

11. Using the distributive property, $2(m+p)=2m+2p$.

12. $3(a+b)=3a+3b$

13. Using the distributive property,
$$\begin{aligned}-12(x-y)&=-12[x+(-y)]\\&=-12(x)+(-12)(-y)\\&=-12x+12y.\end{aligned}$$

14. $$\begin{aligned}-10(p-q)&=-10[p+(-q)]\\&=-10p+10q\end{aligned}$$

15. Use the second form of the distributive property.
$$\begin{aligned}5k+3k&=(5+3)k\\&=8k\end{aligned}$$

16. $$\begin{aligned}6a+5a&=(6+5)a\\&=11a\end{aligned}$$

17. $$\begin{aligned}7r-9r&=7r+(-9r)\\&=[7+(-9)]r\\&=-2r\end{aligned}$$

18. $$\begin{aligned}4n-6n&=4n+(-6n)\\&=[4+(-6)]n\\&=-2n\end{aligned}$$

19. Since there is no common variable factor here, we cannot use the distributive property to simplify the expression.

20. Since there is no common variable factor here, we cannot use the distributive property to simplify the expression.

21. Use the identity property, then the distributive property.
$$\begin{aligned}a+7a&=1a+7a\\&=(1+7)a\\&=8a\end{aligned}$$

22. $$\begin{aligned}s+9s&=1\cdot s+9s\quad\text{Identity property}\\&=(1+9)s\\&=10s\end{aligned}$$

23. $$\begin{aligned}x+x&=1x+1x\\&=(1+1)x\\&=2x\end{aligned}$$

24. $$\begin{aligned}a+a&=1a+1a\\&=(1+1)a\\&=2a\end{aligned}$$

25. $$\begin{aligned}-(2d-f)&=-1(2d-f)\\&=-1(2d)+(-1)(-f)\\&=-2d+f\end{aligned}$$

26. $$\begin{aligned}-(3m-n)&=-1(3m-n)\\&=-1(3m)+(-1)(-n)\\&=-3m+n\end{aligned}$$

27. $$\begin{aligned}-(-x-y)&=-1(-x-y)\\&=-1(-x)+(-1)(-y)\\&=x+y\end{aligned}$$

28. $-(-3x-4y)=-1(-3x-4y)$
$=-1(-3x)+(-1)(-4y)$
$=3x+4y$

29. $2(x-3y+2z)=2x+2(-3y)+2(2z)$
$=2x-6y+4z$

30. $8(3x+y-5z)=8(3x)+8y+8(-5z)$
$=24x+8y-40z$

31. $-12y+4y+3y+2y$
$=(-12+4+3+2)y$
$=-3y$

32. $-5r-9r+8r-5r$
$=(-5-9+8-5)r$
$=-11r$

33. $-6p+5-4p+6+11p$
$=-6p-4p+11p+5+6$
$=(-6-4+11)p+11$
$=1p+11$ or $p+11$

34. $-8x-12+3x-5x+9$
$=-8x+3x-5x-12+9$
$=(-8+3-5)x-3$
$=-10x-3$

35. $3(k+2)-5k+6+3$
$=3k+6-5k+6+3$
$=3k-5k+6+6+3$
$=(3-5)k+6+6+3$
$=-2k+15$

36. $5(r-3)+6r-2r+4$
$=5r-15+6r-2r+4$
$=5r+6r-2r-15+4$
$=(5+6-2)r-11$
$=9r-11$

37. $10-(4y+8)$
$=10-1(4y+8)$
$=10-4y-8$
$=-4y+2$

38. $6-(9y+5)$
$=6-1(9y+5)$
$=6-9y-5$
$=-9y+1$

39. $10x(3)(y)=[10x(3)]y$
$=[30x]y$
$=30xy$

40. $8x(6)(y)=[8x(6)]y$
$=[48x]y$
$=48xy$

41. $-\frac{2}{3}(12w)(7z)=\left[-\frac{2}{3}(12w)\right](7z)$
$=[-8w](7z)$
$=-56wz$

42. $-\frac{5}{6}(18w)(5z)=\left[-\frac{5}{6}(18w)\right](5z)$
$=[-15w](5z)$
$=-75wz$

43. $3(m-4)-2(m+1)$
$=3m-12-2m-2$
$=3m-2m-12-2$
$=m-14$

44. $6(a-5)-4(a+6)$
$=6a-30-4a-24$
$=6a-4a-30-24$
$=2a-54$

45. $0.25(8+4p)-0.5(6+2p)$
$=0.25(8)+0.25(4p)+(-0.5)(6)+(-0.5)(2p)$
$=2+p-3-p$
$=p-p+2-3$
$=(1-1)p+2-3$
$=0p-1$
$=-1$

46. $0.4(10-5x)-0.8(5+10x)$
$=0.4(10)+0.4(-5x)+(-0.8)(5)+(-0.8)(10x)$
$=4-2x-4-8x$
$=-10x$

47. $-(2p+5)+3(2p+4)-2p$
$=-2p-5+6p+12-2p$
$=(-2+6-2)p+(-5+12)$
$=2p+7$

48. $-(7m-12)+2(4m+7)-6m$
$=-1(7m-12)+2(4m+7)-6m$
$=-7m+12+8m+14-6m$
$=-5m+26$

49. $2+3(2z-5)-3(4z+6)-8$
$=2+6z-15-12z-18-8$
$=6z-12z+2-15-18-8$
$=(6-12)z-13-18-8$
$=-6z-31-8$
$=-6z-39$

50. $-4+4(4k-3)-6(2k+8)+7$
$=-4+16k-12-12k-48+7$
$=16k-12k-4-12-48+7$
$=4k-57$

51. $5x+8x=(5+8)x=13x$ Distributive prop.

52. $9y-6y=(9-6)y=3y$ Distributive property

53. $5(9r)=(5\cdot 9)r=45r$ Associative prop.

54. $-4+(12+8)=(-4+12)+8$ Assoc. prop.
$=8+8=16$

55. $5x+9y=9y+5x$ Commutative prop.

56. $-5\cdot 7=7\cdot(-5)=-35$ Commutative prop.

57. $1\cdot 7=7$ Identity property

58. $-12x+0=-12x$ Identity property

59. $-\frac{1}{4}ty+\frac{1}{4}ty=0$ Inverse prop.
A number plus its opposite equals 0.

60. $-\frac{9}{8}\left(-\frac{8}{9}\right)=1$ Inverse prop.

61. $8(-4+x)=8(-4)+8x$ Distributive prop.
$=-32+8x$

62. $3(x-y+z)=3x-3y+3z$ Distributive prop.

63. Use the multiplication property of 0.
$0(0.875x+9y)=0$
Zero times any quantity equals 0.

64. Use the multiplication property of 0.
$0(0.35t+12u)=0$

65. Answers will vary. One example of commutativity is washing your face and brushing your teeth. The activities can be carried out in either order. An example of non-commutativity is putting on your socks and putting on your shoes.

66. Answers will vary. One example is waking up and going to sleep. Another example is tying and untying your shoes. These activities are inverses.

67. $96\cdot 19+4\cdot 19=(96+4)19$
$=(100)19$
$=1900$

68. $27\cdot 60+27\cdot 40=27(60+40)$
$=27(100)$
$=2700$

69. $58\cdot\frac{3}{2}-8\cdot\frac{3}{2}=(58-8)\frac{3}{2}$
$=(50)\frac{3}{2}=\frac{50}{1}\cdot\frac{3}{2}$
$=\frac{150}{2}=75$

70. $\frac{8}{5}(17)+\frac{8}{5}(13)=\frac{8}{5}(17+13)$
$=\frac{8}{5}(30)=8\cdot\frac{30}{5}$
$=8\cdot 6=48$

71. $8.75(15)-8.75(5)=8.75(15-5)$
$=8.75(10)$
$=87.5$

72. $4.31(69)+4.31(31)=4.31(69+31)$
$=4.31(100)$
$=431$

73. The terms have been grouped using the associative property of addition.

74. The terms have been regrouped using the associative property of addition.

75. The order of the terms inside the parentheses has been changed using the commutative property of addition.

76. The terms have been regrouped using the associative property of addition.

77. The common factor, x, has been factored out using the distributive property.

78. The numbers in parentheses have been added using arithmetic facts to simplify the expression.

Chapter R Test

1. $\frac{3}{4}+\frac{1}{6}=\frac{3}{4}\cdot\frac{3}{3}+\frac{1}{6}\cdot\frac{2}{2}$
$=\frac{9}{12}+\frac{2}{12}$
$=\frac{9+2}{12}$
$=\frac{11}{12}$

2. $\frac{3}{7}\div\frac{9}{14}=\frac{3}{7}\cdot\frac{14}{9}$
$=\frac{3\cdot 14}{7\cdot 9}$
$=\frac{3\cdot 2\cdot 7}{7\cdot 3\cdot 3}$
$=\frac{2}{3}$

3.
$$\begin{array}{r} 13.250 \\ -\ \ 6.417 \\ \hline 6.833 \end{array}$$

4.
$$\begin{array}{rl} 0.7 & \text{1 decimal place} \\ \times 0.04 & \text{2 decimal places} \\ \hline 28 & 1+2=3 \\ \hline 0.028 & \text{3 decimal places} \end{array}$$

5. Convert the percent to a decimal first.
$4\%=0.04$
Convert from a decimal to a fraction.
$0.04=\frac{4}{100}=\frac{1}{25}$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{25}$	0.04	4%

6. Convert the fraction to a decimal first.
$\frac{5}{6}=0.8\overline{3}$
Convert from a decimal to a percent.
$0.8\overline{3}=0.8\overline{3}\cdot 100\%=83.\overline{3}\%$

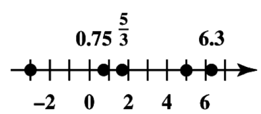
Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{5}{6}$	$0.8\overline{3}$	$83.\overline{3}\%$

7. Convert the decimal to a percent first.
$1.5=1.5\cdot 100\%=150\%$
Convert from a decimal to a fraction.
$1.5=1\frac{1}{2}=\frac{3}{2}$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{3}{2}$	1.5	150%

8. $\{-3, 0.75, \frac{5}{3}, 5, 6.3\}$

Place dots at $-3, 0.75, \frac{5}{3}=1.\overline{6}, 5,$ and 6.3.



9. The elements $0, 3, \sqrt{25}$ (or 5), and $\frac{24}{2}$ (or 12) are whole numbers.

10. The elements $-1, 0, 3, \sqrt{25}$ (or 5), and $\frac{24}{2}$ (or 12) are integers.

11. The elements $-1, -0.5, 0, 3, \sqrt{25}$ (or 5), 7.5, and $\frac{24}{2}$ (or 12) are rational numbers.

12. All the elements in the set are real numbers except $\sqrt{-4}$.

13. $-6+14+(-11)-(-3)$
$=8+(-11)+3$
$=-3+3=0$

14. $-\frac{5}{7}-\left(-\frac{10}{9}+\frac{2}{3}\right)=-\frac{5}{7}+\frac{10}{9}-\frac{2}{3}$

$=-\frac{5}{7}\cdot\frac{9}{9}+\frac{10}{9}\cdot\frac{7}{7}-\frac{2}{3}\cdot\frac{21}{21}$

$=-\frac{45}{63}+\frac{70}{63}-\frac{42}{63}$

$=\frac{-45+70-42}{63}$

$=-\frac{17}{63}$

15. $10-4\cdot 3+6(-4)$

$=10-12+(-24)$

$=-2+(-24)=-26$

16. $7-4^2+2(6)+(-4)^2$

$=7-16+12+16$

$=19$

17. $\frac{-2[3-(-1-2)+2]}{\sqrt{9}(-3)-(-2)}$

$=\frac{-2[3-(-3)+2]}{3(-3)-(-2)}$

$=\frac{-2[8]}{-9-(-2)}$

$=\frac{-16}{-7}=\frac{16}{7}$

18. $\frac{8\cdot 4-3^2\cdot 5-2(-1)}{-3\cdot 2^3+24}$

$=\frac{8\cdot 4-9\cdot 5-2(-1)}{-3\cdot 8+24}$

$=\frac{32-45+2}{-24+24}$

$=\frac{-11}{0}=$ Undefined

19. $\sqrt{196}=14,$ because 14 is positive and $14^2=196.$

20. $-\sqrt{225}=-(\sqrt{225})=-(15)=-15$

21. Since there is no real number whose square is $-16,\ \sqrt{-16}$ is not a real number.

22. Let $k=-3,\ m=-3,$ and $r=25$

$\frac{8k+2m^2}{r-2}$

$=\frac{8(-3)+2(-3)^2}{25-2}$

$=\frac{8(-3)+2(9)}{23}$

$=\frac{-24+18}{23}$

$=\frac{-6}{23}$ or $-\frac{6}{23}$

23. $-3(2k-4)+4(3k-5)-2+4k$

$=-3(2k)+(-3)(-4)+4(3k)$

$+4(-5)-2+4k$

$=-6k+12+12k-20-2+4k$

$=-6k+12k+4k+12-20-2$

$=10k-10$

24. When simplifying $(3r+8)-(-4r+6),$ the subtraction sign in front of $(-4r+6)$ changes the sign of the terms $-4r$ and 6.

$(3r+8)-(-4r+6)$

$=3r+8-(-4r)-6$

$=3r+8+4r-6$

$=3r+4r+8-6$

$=7r+2$

25. **(a)** The answer is B, Inverse property. The sum of 6 and its inverse, $-6,$ equals zero.

(b) The answer is D, Associative property. The order of the terms is the same, but the grouping has changed.

(c) The answer is A, Distributive property. This is the second form of the distributive property.

(d) The answer is F, Multiplication property of 0. Multiplication by 0 always equals 0.

(e) The answer is C, Identity property. The addition of 0 to any number does not change the number.

(f) The answer is C, Identity property. Multiplication of any number by 1 does not change the number.

(g) The answer is E, Commutative property. The order of the terms *a* and *b* is reversed.

Chapter 1
Linear Equations, Inequalities, and Applications

1.1 Linear Equations in One Variable

Classroom Examples, Now Try Exercises

1. (a) $9x+10=0$ is an *equation* because it contains an equality symbol.

(b) $9x+10$ is an *expression* because it does not contain an equality symbol.

N1. (a) $2x+17-3x$ is an *expression* because it does not contain an equality symbol.

(b) $2x+17=3x$ is an *equation* because it contains an equality symbol.

2.
$$\begin{aligned} 4x+8x &= -9+17x-1 \\ 12x &= 17x-10 && \text{Combine terms.} \\ 12x-17x &= 17x-10-17x && \text{Subtract } 17x. \\ -5x &= -10 && \text{Combine terms.} \\ \frac{-5x}{-5} &= \frac{-10}{-5} && \text{Divide by } -5. \\ x &= 2 \end{aligned}$$

Check by substituting 2 for x in the original equation.

$$\begin{aligned} 4x+8x &= -9+17x-1 \\ 4(2)+8(2) &\stackrel{?}{=} -9+17(2)-1 && \text{Let } x=2. \\ 8+16 &\stackrel{?}{=} -9+34-1 \\ 24 &\stackrel{?}{=} 25-1 \\ 24 &= 24 && \text{True.} \end{aligned}$$

The solution set is $\{2\}$.

N2.
$$\begin{aligned} 5x+11 &= 2x-13-3x \\ 5x+11 &= -x-13 && \text{Combine terms.} \\ 5x+11+x &= -x-13+x && \text{Add } x. \\ 6x+11 &= -13 && \text{Combine terms.} \\ 6x+11-11 &= -13-11 && \text{Subtract } 11. \\ 6x &= -24 && \text{Combine terms.} \\ \frac{6x}{6} &= \frac{-24}{6} && \text{Divide by } 6. \\ x &= -4 \end{aligned}$$

Check by substituting -4 for x in the original equation.

$$\begin{aligned} 5x+11 &= 2x-13-3x \\ 5(-4)+11 &\stackrel{?}{=} 2(-4)-13-3(-4) && \text{Let } x=-4. \\ -20+11 &\stackrel{?}{=} -8-13+12 \\ -9 &\stackrel{?}{=} -21+12 \\ -9 &= -9 && \text{True.} \end{aligned}$$

The solution set is $\{-4\}$.

3.
$$\begin{aligned} 3(4+x)-x &= 5x-3 \\ 12+3x-x &= 5x-3 && \text{Distributive prop.} \\ 12+2x &= 5x-3 && \text{Combine terms.} \\ 12+2x-2x &= 5x-3-2x && \text{Subtract } 2x. \\ 12 &= 3x-3 && \text{Combine terms.} \\ 12+3 &= 3x-3+3 && \text{Add } 3. \\ 15 &= 3x && \text{Combine terms.} \\ \frac{15}{3} &= \frac{3x}{3} && \text{Divide by } 3. \\ 5 &= x \end{aligned}$$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

Check $x=5$: $27-5=25-3$ True

The solution set is $\{5\}$.

N3.
$$\begin{aligned} 5(x-4)-12 &= 3-2x \\ 5x-20-12 &= 3-2x && \text{Distributive prop.} \\ 5x-32 &= 3-2x && \text{Combine terms.} \\ 5x-32+2x &= 3-2x+2x && \text{Add } 2x. \\ 7x-32 &= 3 && \text{Combine terms.} \\ 7x-32+32 &= 3+32 && \text{Add } 32. \\ 7x &= 35 && \text{Combine terms.} \\ \frac{7x}{7} &= \frac{35}{7} && \text{Divide by } 7. \\ x &= 5 \end{aligned}$$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

Check $x=5$: $5-12=3-10$ True

The solution set is $\{5\}$.

4. $6-(4+x)=8x-2(3x+5)$

$$\begin{aligned}
6-4-x&=8x-6x-10 && \text{Distributive prop.}\\
2-x&=2x-10 && \text{Combine terms.}\\
2-x+x&=2x-10+x && \text{Add } x.\\
2&=3x-10 && \text{Combine terms.}\\
2+10&=3x-10+10 && \text{Add 10.}\\
12&=3x && \text{Combine terms.}\\
\frac{12}{3}&=\frac{3x}{3} && \text{Divide by 3.}\\
4&=x
\end{aligned}$$

Check $x=4$: $-2=32-34$ True

The solution set is $\{4\}$.

N4. $2-3(2+6x)=4(x+1)+36$

$$\begin{aligned}
2-6-18x&=4x+4+36 && \text{Dist. prop.}\\
-4-18x&=4x+40 && \text{Combine terms.}\\
-4-18x+18x&=4x+40+18x && \text{Add } 18x.\\
-4&=22x+40 && \text{Combine terms.}\\
-4-40&=22x+40-40 && \text{Subtract 40.}\\
-44&=22x && \text{Combine terms.}\\
\frac{-44}{22}&=\frac{22x}{22} && \text{Divide by 22.}\\
-2&=x
\end{aligned}$$

Check $x=-2$: $2+30=-4+36$ True

The solution set is $\{-2\}$.

5. $\frac{x}{3}-5=-\frac{x}{2}$

$$\begin{aligned}
6\left(\frac{x}{3}-5\right)&=6\left(-\frac{x}{2}\right) && \text{LCD is 6.}\\
6\left(\frac{x}{3}\right)-6(5)&=6\left(-\frac{x}{2}\right) && \text{Dist. prop.}\\
2x-30&=-3x && \text{Multiply.}\\
2x-30-2x&=-3x-2x && \text{Subtract } 2x.\\
-30&=-5x && \text{Combine terms.}\\
\frac{-30}{-5}&=\frac{-5x}{-5} && \text{Divide by } -5.\\
6&=x
\end{aligned}$$

Check $x=6$: $2-5=-3$ True

The solution set is $\{6\}$.

N5. $\frac{x}{2}-7=-\frac{x}{5}$

$$\begin{aligned}
10\left(\frac{x}{2}-7\right)&=10\left(-\frac{x}{5}\right) && \text{LCD is 10.}\\
10\left(\frac{x}{2}\right)-10(7)&=10\left(-\frac{x}{5}\right) && \text{Dist. prop.}\\
5x-70&=-2x && \text{Multiply.}\\
5x-70-5x&=-2x-5x && \text{Subtract } 5x.\\
-70&=-7x && \text{Combine terms.}\\
\frac{-70}{-7}&=\frac{-7x}{-7} && \text{Divide by } -7.\\
10&=x
\end{aligned}$$

Check $x=10$: $5-7=-2$ True

The solution set is $\{10\}$.

6. Multiply each side by the LCD, 4, and use the distributive property

$$\begin{aligned}
\frac{x+1}{2}+\frac{x+3}{4}&=\frac{1}{2}\\
4\left(\frac{x+1}{2}\right)+4\left(\frac{x+3}{4}\right)&=4\left(\frac{1}{2}\right)\\
2(x+1)+1(x+3)&=2\\
2x+2+x+3&=2\\
3x+5&=2\\
3x&=-3 && \text{Subtract 5.}\\
x&=-1 && \text{Divide by 3.}
\end{aligned}$$

Check $x=-1$: $0+\frac{2}{4}=\frac{1}{2}$ True

The solution set is $\{-1\}$.

N6. Multiply each side by the LCD, 8, and use the distributive property.

$$\begin{aligned}
\frac{x-4}{4}+\frac{2x+4}{8}&=5\\
8\left(\frac{x-4}{4}\right)+8\left(\frac{2x+4}{8}\right)&=8(5)\\
2(x-4)+1(2x+4)&=40\\
2x-8+2x+4&=40\\
4x-4&=40\\
4x&=44 && \text{Add 4.}\\
x&=11 && \text{Divide by 4.}
\end{aligned}$$

Check $x=11$: $\frac{7}{4}+\frac{13}{4}=5$ True

The solution set is $\{11\}$.

7. Multiply each term by 100.

$$\begin{aligned} 0.02(60)+0.04x &= 0.03(50+x) \\ 2(60)+4x &= 3(50+x) \\ 120+4x &= 150+3x \\ 4x &= 30+3x \\ x &= 30 \end{aligned}$$

Check $x=30$: $1.2+1.2=2.4$ True

The solution set is $\{30\}$.

N7. Multiply each term by 100.

$$\begin{aligned} 0.08x-0.12(x-4) &= 0.03(x-5) \\ 8x-12(x-4) &= 3(x-5) \\ 8x-12x+48 &= 3x-15 \\ -4x+48 &= 3x-15 \\ -4x+63 &= 3x \\ 63 &= 7x \\ 9 &= x \end{aligned}$$

Check $x=9$: $0.72-0.60=0.12$ True

The solution set is $\{9\}$.

8. (a) $5(x+2)-2(x+1)=3x+1$

$$\begin{aligned} 5x+10-2x-2 &= 3x+1 \\ 3x+8 &= 3x+1 \\ 3x+8-3x &= 3x+1-3x \\ 8 &= 1 \qquad \text{False} \end{aligned}$$

Since the result, $8=1$ is *false*, the equation has no solution and is called a *contradiction*.
The solution set is $\varnothing$.

(b) Multiply each side by the LCD, 3, and use the distributive property.

$$\begin{aligned} \frac{x+1}{3}+\frac{2x}{3} &= x+\frac{1}{3} \\ 3\left(\frac{x+1}{3}\right)+3\left(\frac{2x}{3}\right) &= 3\left(x+\frac{1}{3}\right) \\ x+1+2x &= 3x+1 \\ 3x+1 &= 3x+1 \end{aligned}$$

This is an *identity*. Any real number will make the equation true.
The solution set is {all real numbers}.

(c) $5(3x+1)=x+5$

$$\begin{aligned} 15x+5 &= x+5 \\ 14x+5 &= 5 \qquad \text{Subtract } x. \\ 14x &= 0 \qquad \text{Subtract 5.} \\ x &= 0 \qquad \text{Divide by 14.} \end{aligned}$$

This is a *conditional equation*.

Check $x=0$: $5(1)=0+5$ True

The solution set is $\{0\}$.

N8. (a) $9x-3(x+4)=6(x-2)$

$$\begin{aligned} 9x-3x-12 &= 6x-12 \\ 6x-12 &= 6x-12 \end{aligned}$$

This is an *identity*. Any real number will make the equation true.
The solution set is $\{$all real numbers$\}$.

(b) $-3(2x-1)-2x=3+x$

$$\begin{aligned} -6x+3-2x &= 3+x \\ -8x+3 &= 3+x \\ -9x+3 &= 3 \qquad \text{Subtract } x. \\ -9x &= 0 \qquad \text{Subtract 3.} \\ x &= 0 \qquad \text{Divide by} -9. \end{aligned}$$

This is a *conditional equation*.
Check $x=0$: $-3(-1)=3$ True

The solution set is $\{0\}$.

(c)

$$\begin{aligned} 10x-21 &= 2(x-5)+8x \\ 10x-21 &= 2x-10+8x \\ 10x-21 &= 10x-10 \\ 10x-21-10x &= 10x-10-10x \\ -21 &= -10 \qquad \text{False} \end{aligned}$$

Since the result, $-21=-10$, is *false*, the equation has no solution and is called a *contradiction*.
The solution set is $\varnothing$.

Exercises

1. A collection of numbers, variables, operation symbols, and grouping symbols, such as $2(8x-15)$, is an algebraic expression. While an equation *does* include an equality symbol, there *is not* an equality symbol in an algebraic expression.

2. A linear equation in one variable (here x) can be written in the form $ax+b=0$, with $a\neq 0$. Another name for a linear equation is a first-degree equation, because the greatest power on the variable is *one*.

3. If we let $x=2$ in the linear equation $2x+5=9$, a *true* statement results. The number 2 is a solution of the equation, and $\{2\}$ is the solution set.

4. A linear equation with one solution in its solution set, such as $2x+5=9$, is a conditional equation.

5. A linear equation with an infinite number of solutions is an identity. Its solution set is $\{\text{all real numbers}\}$.

6. A linear equation with no solution is a contradiction. Its solution set is the empty set $\varnothing$.

7. $3x+x-2=0$ can be written as $4x=2$, so it is linear.
$9x-4=9$ is in linear form.
Choices A and C are linear.

8. $12=x^2$ in choice B is nonlinear because the variable is squared.
$3x+2y=6$ in choice D is nonlinear because there are two variables.
Choices B and D are nonlinear.

9. $-3x+2-4=x$ is an *equation* because it contains an equality symbol.

10. $-3x+2-4-x=4$ is an *equation* because it contains an equality symbol.

11. $4(x+3)-2(x+1)-10$ is an *expression* because it does not contain an equality symbol.

12. $4(x+3)-2(x+1)+10$ is an *expression* because it does not contain an equality symbol.

13. $-10x+12-4x=-3$ is an *equation* because it contains an equality symbol.

14. $-10x+12-4x+3=0$ is an *equation* because it contains an equality symbol.

15. A sign error was made when the distributive property was applied. The left side of the second line should be $8x-4x+6$.

$$
\begin{aligned}
8x-2(2x-3)&=3x+7 && \\
8x-4x+6&=3x+7 && \text{Distributive property}\\
4x+6&=3x+7 && \text{Combine like terms.}\\
x&=1 && \text{Subtract } 3x \text{ and } 6.
\end{aligned}
$$

The correct solution is 1.

16. The first step should have been to distribute -2 to $(3x+1)$.

$$
\begin{aligned}
12-2(3x+1)&=11 && \\
12-6x-2&=11 && \text{Distributive property}\\
-6x+10&=11 && \text{Combine terms.}\\
-6x+10-10&=11-10 && \text{Subtract 10.}\\
-6x&=1 && \\
x&=-\frac{1}{6} && \text{Divide by } -6.
\end{aligned}
$$

The correct solution is $-\frac{1}{6}$.

17. **(a)** The true statement $7=7$ indicates that the solution set is {all real numbers}, choice B.

(b) The final line is $x=0$, which is the form of solution for a conditional equation. The solution set is $\{0\}$, choice A.

(c) The false statement $7=0$ indicates that the equation has no solution. The solution set is the empty set $\varnothing$, choice C.

18. An equation will have the solution set {all real numbers} if it is an identity. The only equation listed here that is **not** an identity is choice C:

$$
\begin{aligned}
4x&=3x\\
4x-3x&=0\\
x&=0
\end{aligned}
$$

19.

$$
\begin{aligned}
7x+8&=1 && \\
7x+8-8&=1-8 && \text{Subtract 8.}\\
7x&=-7 && \\
\frac{7x}{7}&=\frac{-7}{7} && \text{Divide by 7.}\\
x&=-1 &&
\end{aligned}
$$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.
Check $x=-1$: $-7+8=1$ True
The solution set is $\{-1\}$.

20.

$$
\begin{aligned}
5x-4&=21 && \\
5x-4+4&=21+4 && \text{Add 4.}\\
5x&=25 && \\
\frac{5x}{5}&=\frac{25}{5} && \text{Divide by 5.}\\
x&=5 &&
\end{aligned}
$$

Check $x=5$: $25-4=21$ True
The solution set is $\{5\}$.

21.
$$\begin{aligned}5x+2&=3x-6\\5x+2-3x&=3x-6-3x &&\text{Subtract } 3x.\\2x+2&=-6\\2x+2-2&=-6-2 &&\text{Subtract 2.}\\2x&=-8\\\frac{2x}{2}&=\frac{-8}{2} &&\text{Divide by 2.}\\x&=-4\end{aligned}$$

Check $x=-4$: $-20+2=-12-6$ True

The solution set is $\{-4\}$.

22.
$$\begin{aligned}9x+1&=7x-9\\9x+1-7x&=7x-9-7x &&\text{Subtract } 7x.\\2x+1&=-9\\2x+1-1&=-9-1 &&\text{Subtract 1.}\\2x&=-10\\\frac{2x}{2}&=\frac{-10}{2} &&\text{Divide by 2.}\\x&=-5\end{aligned}$$

Check $x=-5$: $-45+1=-35-9$ True

The solution set is $\{-5\}$.

23.
$$\begin{aligned}7x-5x+15&=x+8\\2x+15&=x+8 &&\text{Combine terms.}\\2x+15-x&=x+8-x &&\text{Subtract } x.\\x+15&=8 &&\text{Combine terms.}\\x+15-15&=8-15 &&\text{Subtract 15.}\\x&=-7 &&\text{Combine terms.}\end{aligned}$$

Check $x=-7$: $-49+35+15=-7+8$ True

The solution set is $\{-7\}$.

24.
$$\begin{aligned}2x+4-x&=4x-5\\x+4&=4x-5 &&\text{Combine terms.}\\-3x+4&=-5 &&\text{Subtract } 4x.\\-3x&=-9 &&\text{Subtract 4.}\\x&=3 &&\text{Divide by } -3.\end{aligned}$$

Check $x=3$: $6+4-3=12-5$ True

The solution set is $\{3\}$.

25.
$$\begin{aligned}12w+15w-9+5&=-3w+5-9\\27w-4&=-3w-4 &&\text{Combine terms.}\\30w-4&=-4. &&\text{Add } 3w.\\30w&=0 &&\text{Add 4.}\\w&=0 &&\text{Divide by 30.}\end{aligned}$$

Check $w=0$: $-9+5=5-9$ True

The solution set is $\{0\}$.

26.
$$\begin{aligned}-4x+5x-8+4&=6x-4\\x-4&=6x-4 &&\text{Combine terms.}\\-5x-4&=-4 &&\text{Subtract } 6x.\\-5x&=0 &&\text{Add 4.}\\x&=0 &&\text{Divide by } -5.\end{aligned}$$

Check $x=0$: $-8+4=-4$ True

The solution set is $\{0\}$.

27.
$$\begin{aligned}3(2t-4)&=20-2t\\6t-12&=20-2t &&\text{Distributive property}\\8t-12&=20 &&\text{Add } 2t.\\8t&=32 &&\text{Add 12.}\\t&=4 &&\text{Divide by 8.}\end{aligned}$$

Check $t=4$: $3(4)=20-8$ True

The solution set is $\{4\}$.

28.
$$\begin{aligned}2(3-2x)&=x-4\\6-4x&=x-4 &&\text{Distributive property}\\6-5x&=-4 &&\text{Subtract } x.\\-5x&=-10 &&\text{Subtract 6.}\\x&=2 &&\text{Divide by } -5.\end{aligned}$$

Check $x=2$: $2(-1)=2-4$ True

The solution set is $\{2\}$.

29.
$$\begin{aligned}-5(x+1)+3x+2&=6x+4\\-5x-5+3x+2&=6x+4 &&\text{Distributive prop.}\\-2x-3&=6x+4 &&\text{Combine terms.}\\-3&=8x+4 &&\text{Add } 2x.\\-7&=8x &&\text{Subtract 4.}\\-\frac{7}{8}&=x &&\text{Divide by 8.}\end{aligned}$$

Check: Substitute $-\frac{7}{8}$ for x and show that both sides equal -1.25. The screen shows a typical Check on a calculator.

```
-7/8→X
                 -.875
-5(X+1)+3X+2
                 -1.25
6X+4
                 -1.25
```

The solution set is $\left\{-\frac{7}{8}\right\}$.

30. $$\begin{aligned} 5(x+3)+4x-5&=4-2x \\ 5x+15+4x-5&=4-2x && \text{Distributive prop.} \\ 9x+10&=4-2x && \text{Combine terms.} \\ 11x+10&=4 && \text{Add } 2x. \\ 11x&=-6 && \text{Subtract 10.} \\ x&=-\frac{6}{11} && \text{Divide by 11.} \end{aligned}$$

Check

$x=-\frac{6}{11}$: $\frac{135}{11}-\frac{24}{11}-\frac{55}{11}=\frac{44}{11}+\frac{12}{11}$ True

The solution set is $\left\{-\frac{6}{11}\right\}$.

31. $$\begin{aligned} -2x+5x-9&=3(x-4)-5 \\ 3x-9&=3x-12-5 \\ 3x-9&=3x-17 \\ -9&=-17 \quad \text{False} \end{aligned}$$

The equation is a *contradiction.*
The solution set is $\varnothing$.

32. $$\begin{aligned} -6x+2x-11&=-2(2x-3)+4 \\ -4x-11&=-4x+6+4 \\ -4x-11&=-4x+10 \\ -11&=10 \quad \text{False} \end{aligned}$$

The equation is a *contradiction.*
The solution set is $\varnothing$.

33. $$\begin{aligned} -2(x+3)&=-6(x+7) \\ -2x-6&=-6x-42 \\ 4x-6&=-42 \\ 4x&=-36 \\ x&=-9 \end{aligned}$$

Check $x=-9$: $-2(-6)=-6(-2)$ True

The solution set is $\{-9\}$.

34. $$\begin{aligned} -4(x-9)&=-8(x+3) \\ -4x+36&=-8x-24 \\ 4x+36&=-24 \\ 4x&=-60 \\ x&=-15 \end{aligned}$$

Check $x=-15$: $-4(-24)=-8(-12)$ True

The solution set is $\{-15\}$.

35. $$\begin{aligned} 3(2x+1)-2(x-2)&=5 \\ 6x+3-2x+4&=5 \\ 4x+7&=5 \\ 4x&=-2 \\ x&=\frac{-2}{4}=-\frac{1}{2} \end{aligned}$$

Check $x=-\frac{1}{2}$: $3(0)-2\left(-\frac{5}{2}\right)=5$ True

The solution set is $\left\{-\frac{1}{2}\right\}$.

36. $$\begin{aligned} 4(x-2)+2(x+3)&=6 \\ 4x-8+2x+6&=6 \\ 6x-2&=6 \\ 6x&=8 \\ x&=\frac{8}{6}=\frac{4}{3} \end{aligned}$$

Check $x=\frac{4}{3}$: $4\left(-\frac{2}{3}\right)+2\left(\frac{13}{3}\right)=6$ True

The solution set is $\left\{\frac{4}{3}\right\}$.

37. $$\begin{aligned} 2x+3(x-4)&=2(x-3) \\ 2x+3x-12&=2x-6 \\ 5x-12&=2x-6 \\ 3x&=6 \\ x&=\frac{6}{3}=2 \end{aligned}$$

Check $x=2$: $4+3(-2)=2(-1)$ True

The solution set is $\{2\}$.

38. $6x-3(5x+2)=4(1-x)$

$$\begin{aligned} 6x-15x-6&=4-4x\\ -9x-6&=4-4x\\ -5x&=10\\ x&=\frac{10}{-5}=-2 \end{aligned}$$

Check $x=-2$: $-12-3(-8)=4(3)$ True

The solution set is $\{-2\}$.

39. $6x-4(3-2x)=5(x-4)-10$

$$\begin{aligned} 6x-12+8x&=5x-20-10\\ 14x-12&=5x-30\\ 9x&=-18\\ x&=-2 \end{aligned}$$

Check $x=-2: -12-4(7)=5(-6)-10$ True

The solution set is $\{-2\}$.

40. $-2x-3(4-2x)=2(x-3)+2$

$$\begin{aligned} -2x-12+6x&=2x-6+2\\ 4x-12&=2x-4\\ 2x&=8\\ x&=4 \end{aligned}$$

Check $x=4$: $-8-3(-4)=2(1)+2$ True

The solution set is $\{4\}$.

41. $-2(x+3)-x-4=-3(x+4)+2$

$$\begin{aligned} -2x-6-x-4&=-3x-12+2\\ -3x-10&=-3x-10 \end{aligned}$$

The equation is an *identity.*
The solution set is {all real numbers}.

42. $4(2x+7)=2x+25+3(2x+1)$

$$\begin{aligned} 8x+28&=2x+25+6x+3\\ 8x+28&=8x+28 \end{aligned}$$

The equation is an *identity.*
The solution set is {all real numbers}.

43. $2[x-(2x+4)+3]=2(x+1)$

$$\begin{aligned} 2[x-2x-4+3]&=2(x+1)\\ 2[-x-1]&=2(x+1)\\ -x-1&=x+1 && \text{Divide by 2.}\\ -1&=2x+1 && \text{Add } x.\\ -2&=2x && \text{Subtract 1.}\\ -1&=x && \text{Divide by 2.} \end{aligned}$$

Check $x=-1$: $2[-1-2+3]=0$ True

The solution set is $\{-1\}$.

44. $4[2x-(3-x)+5]=-(2+7x)$

$$\begin{aligned} 4[2x-3+x+5]&=-(2+7x)\\ 4[3x+2]&=-(2+7x)\\ 12x+8&=-2-7x\\ 19x+8&=-2 && \text{Add } 7x.\\ 19x&=-10 && \text{Subtract 8.}\\ x&=-\frac{10}{19} && \text{Divide by 19.} \end{aligned}$$

Check

$x=-\frac{10}{19}$:

$$4\left[-\frac{20}{19}-\frac{67}{19}+\frac{95}{19}\right]=-\left(-\frac{32}{19}\right) \quad \text{True}$$

The solution set is $\left\{-\frac{10}{19}\right\}$.

45. $-[2x-(5x+2)]=2+(2x+7)$

$$\begin{aligned} -[2x-5x-2]&=2+2x+7\\ -[-3x-2]&=2+2x+7\\ 3x+2&=2x+9\\ x&=7 \end{aligned}$$

Check $x=7$: $-[14-37]=2+21$ True

The solution set is $\{7\}$.

46. $-[6x-(4x+8)]=9+(6x+3)$

$$\begin{aligned} -[6x-4x-8]&=9+6x+3\\ -(2x-8)&=6x+12\\ -2x+8&=6x+12\\ -8x&=4\\ x&=\frac{4}{-8}=-\frac{1}{2} \end{aligned}$$

Check $x=-\frac{1}{2}$: $-[-3-6]=9+0$ True

The solution set is $\left\{-\frac{1}{2}\right\}$.

47. $-3x+6-5(x-1)=-5x-(2x-4)+5$

$$\begin{aligned} -3x+6-5x+5&=-5x-2x+4+5\\ -8x+11&=-7x+9\\ -x&=-2\\ x&=2 \end{aligned}$$

Check
$x=2$: $-6+6-5=-10-0+5$ True

The solution set is $\{2\}$.

48. $4(x+2)-8x-5=-3x+9-2(x+6)$

$$4x+8-8x-5=-3x+9-2x-12$$
$$-4x+3=-5x-3$$
$$x=-6$$

Check
$x=-6$: $-16+48-5=18+9-0$ True

The solution set is $\{-6\}$.

49. $7[2-(3+4x)]-2x=-9+2(1-15x)$

$$7[2-3-4x]-2x=-9+2-30x$$
$$7[-1-4x]-2x=-7-30x$$
$$-7-28x-2x=-7-30x$$
$$-7-30x=-7-30x$$

The equation is an *identity.*
The solution set is {all real numbers}.

50. $4[6-(1+2x)]+10x=2(10-3x)+8x$

$$4[6-1-2x]+10x=20-6x+8x$$
$$4(5-2x)+10x=20+2x$$
$$20-8x+10x=20+2x$$
$$20+2x=20+2x$$

The equation is an *identity.*
The solution set is {all real numbers}.

51. The denominators of the fractions are 3, 4, 6, and 1. The least common denominator is 12 since it is the smallest number into which each denominator can divide without a remainder.

52. Yes, the coefficients will be larger, but you will get the correct solution. As long as you multiply both sides of the equation by the *same* nonzero number, the resulting equation is equivalent and the solution does not change.

53. **(a)** We need to make 0.05 and 0.12 integers. Multiplying by 10^2, or 100, will eliminate the decimal points so that all the coefficients are integers.

(b) We need to make 0.006, 0.007, and 0.0.009 integers. Multiplying by 10^3, or 1000, will eliminate the decimal points so that all the coefficients are integers.

54. $0.06(10-x)(100)$

$$=0.06(100)(10-x)$$
$$=6(10-x)$$
$$=60-6x \quad \text{Choice B is correct.}$$

55. $-\frac{5}{9}x=2$

$-5x=18$ Multiply by 9.

$x=\frac{18}{-5}=-\frac{18}{5}$ Divide by -5.

Check
$x=-\frac{18}{5}$: $\left(-\frac{5}{9}\right)\left(-\frac{18}{5}\right)=2$ True

The solution set is $\left\{-\frac{18}{5}\right\}$.

56. $-\frac{7}{8}x=6$

$-7x=48$ Multiply by 8.

$x=\frac{48}{-7}=-\frac{48}{7}$ Divide by -7.

Check $x=-\frac{48}{7}$: $\left(-\frac{7}{8}\right)\left(-\frac{48}{7}\right)=6$ True

The solution set is $\left\{-\frac{48}{7}\right\}$.

57. $\frac{6}{5}x=-1$

$6x=-5$ Multiply by 5.

$x=\frac{-5}{6}=-\frac{5}{6}$ Divide by 6.

Check
$x=-\frac{5}{6}$: $\left(\frac{6}{5}\right)\left(-\frac{5}{6}\right)=-1$ True

The solution set is $\left\{-\frac{5}{6}\right\}$.

58. $\frac{3}{11}x=-5$

$3x=-55$ Multiply by 11.

$x=\frac{-55}{3}=-\frac{55}{3}$ Divide by 3.

Check $x=-\frac{55}{3}$: $\left(\frac{3}{11}\right)\left(-\frac{55}{3}\right)=-5$ True

The solution set is $\left\{-\frac{55}{3}\right\}$.

59. Multiply both sides by the LCD, 6.

$$\frac{x}{2}-5=-\frac{x}{3}$$
$$6\left(\frac{x}{2}-5\right)=6\left(-\frac{x}{3}\right)$$
$$6\left(\frac{x}{2}\right)+6(-5)=-2x \quad \text{Distributive prop.}$$
$$3x-30=-2x$$
$$3x-30+2x=-2x+2x \quad \text{Add } 2x.$$
$$5x-30=0 \quad \text{Combine terms.}$$
$$5x-30+30=0+30 \quad \text{Add 30.}$$
$$5x=30 \quad \text{Combine terms.}$$
$$x=6 \quad \text{Divide by 5.}$$

Check $x=6$: $3-5=-2$ True

The solution set is $\{6\}$.

60. Multiply both sides by the LCD, 20.

$$\frac{x}{5}-1=\frac{x}{4}$$
$$20\left(\frac{x}{5}-1\right)=20\left(\frac{x}{4}\right)$$
$$20\left(\frac{x}{5}\right)-20(1)=5x \quad \text{Distributive prop.}$$
$$4x-20=5x$$
$$4x-20-4x=5x-4x \quad \text{Subtract } 4x.$$
$$-20=x \quad \text{Combine terms.}$$

Check $x=-20$: $-4-1=-5$ True

The solution set is $\{-20\}$.

61. Multiply both sides by the LCD, 4.

$$\frac{3x}{4}+\frac{5x}{2}=13$$
$$4\left(\frac{3x}{4}+\frac{5x}{2}\right)=4(13)$$
$$4\left(\frac{3x}{4}\right)+4\left(\frac{5x}{2}\right)=4(13) \quad \text{Distributive prop.}$$
$$3x+10x=52$$
$$13x=52 \quad \text{Combine terms.}$$
$$x=4 \quad \text{Divide by 13.}$$

Check $x=4$: $3+10=13$ True

The solution set is $\{4\}$.

62. Multiply both sides by the LCD, 6.

$$\frac{8x}{3}-\frac{x}{2}=-13$$
$$6\left(\frac{8x}{3}-\frac{x}{2}\right)=6(-13)$$
$$6\left(\frac{8x}{3}\right)-6\left(\frac{x}{2}\right)=6(-13) \quad \text{Distributive prop.}$$
$$16x-3x=-78$$
$$13x=-78$$
$$x=-6 \quad \text{Divide by 13.}$$

Check $x=-6$: $-16+3=-13$ True

The solution set is $\{-6\}$.

63. Multiply both sides by the LCD, 8.

$$\frac{3}{4}x-\frac{1}{2}x=-\frac{1}{8}x-12$$
$$8\left(\frac{3}{4}x-\frac{1}{2}x\right)=8\left(-\frac{1}{8}x-12\right)$$
$$8\left(\frac{3}{4}x\right)+8\left(-\frac{1}{2}x\right)=8\left(-\frac{1}{8}x\right)+8(-12)$$
$$6x-4x=-x-96$$
$$2x=-x-96$$
$$2x+x=-x-96+x$$
$$3x=-96$$
$$x=-32$$

Check $x=-32$: $-24+16=4-12$ True

The solution set is $\{-32\}$.

64. Multiply both sides by the LCD, 10.

$$\frac{3}{5}x-\frac{1}{2}x=\frac{7}{10}x+3$$
$$10\left(\frac{3}{5}x-\frac{1}{2}x\right)=10\left(\frac{7}{10}x+3\right)$$
$$10\left(\frac{3}{5}x\right)+10\left(-\frac{1}{2}x\right)=10\left(\frac{7}{10}x\right)+10(3)$$
$$6x-5x=7x+30$$
$$x=7x+30$$
$$x-x=7x+30-x$$
$$0=6x+30$$
$$0-30=6x+30-30$$
$$-30=6x$$
$$-5=x$$

Check $x=-5$: $-3+\frac{5}{2}=-\frac{7}{2}+3$ True

The solution set is $\{-5\}$.

65. Multiply both sides by the LCD, 35.

$$\frac{1}{7}(3x+2)-\frac{1}{5}(x+4)=2$$

$$35\left[\frac{1}{7}(3x+2)-\frac{1}{5}(x+4)\right]=35(2)$$

$$\begin{aligned} 5(3x+2)-7(x+4)&=70 && \text{Distrib.}\\ 15x+10-7x-28&=70 && \text{Distrib.}\\ 8x-18&=70 && \text{Combine.}\\ 8x-18+18&=70+18 && \text{Add 18.}\\ 8x&=88 && \text{Combine.}\\ x&=11 && \text{Div. by 8.} \end{aligned}$$

Check $x=11$: $5-3=2$ True

The solution set is $\{11\}$.

66. Multiply both sides by the LCD, 12.

$$\frac{1}{4}(3x-1)+\frac{1}{6}(x+3)=3$$

$$12\left[\frac{1}{4}(3x-1)+\frac{1}{6}(x+3)\right]=12(3)$$

$$\begin{aligned} 3(3x-1)+2(x+3)&=36 && \text{Distrib.}\\ 9x-3+2x+6&=36 && \text{Distrib.}\\ 11x+3&=36 && \text{Combine.}\\ 11x+3-3&=36-3 && \text{Subtract 3.}\\ 11x&=33 && \text{Combine.}\\ x&=3 && \text{Div. by 11.} \end{aligned}$$

Check $x=3$: $2+1=3$ True

The solution set is $\{3\}$.

67. Multiply both sides by the LCD, 6.

$$\frac{x-10}{5}+\frac{2}{5}=-\frac{x}{3}$$

$$15\left(\frac{x-10}{5}+\frac{2}{5}\right)=15\left(-\frac{x}{3}\right)$$

$$\begin{aligned} 3(x-10)+3(2)&=-5x\\ 3x-30+6&=-5x\\ 8x&=24\\ x&=\frac{24}{8}=3 \end{aligned}$$

Check $x=3$: $-\frac{7}{5}+\frac{2}{5}=-1$ True

The solution set is $\{3\}$.

68. Multiply both sides by the LCD, 54.

$$\frac{5-x}{6}+\frac{5}{6}=\frac{x}{54}$$

$$54\left(\frac{5-x}{6}+\frac{5}{6}\right)=54\left(\frac{x}{54}\right)$$

$$\begin{aligned} 9(5-x)+9(5)&=x\\ 45-9x+45&=x\\ -9x+90&=x\\ 90&=10x\\ x&=\frac{90}{10}=9 \end{aligned}$$

Check $x=9$: $\frac{-36}{54}+\frac{45}{54}=\frac{9}{54}$ True

The solution set is $\{9\}$.

69. Multiply both sides by the LCD, 12.

$$\frac{3x-1}{4}+\frac{x+3}{6}=3$$

$$12\left(\frac{3x-1}{4}+\frac{x+3}{6}\right)=12(3)$$

$$\begin{aligned} 3(3x-1)+2(x+3)&=36\\ 9x-3+2x+6&=36\\ 11x+3&=36\\ 11x&=33\\ x&=3 \end{aligned}$$

Check $x=3$: $2+1=3$ True

The solution set is $\{3\}$.

70. Multiply both sides by the LCD, 35.

$$\frac{3x+2}{7}-\frac{x+4}{5}=2$$

$$35\left(\frac{3x+2}{7}-\frac{x+4}{5}\right)=35(2)$$

$$\begin{aligned} 5(3x+2)-7(x+4)&=70\\ 15x+10-7x-28&=70\\ 8x-18&=70\\ 8x&=88\\ x&=11 \end{aligned}$$

Check $x=11$: $5-3=2$ True

The solution set is $\{11\}$.

71. Multiply both sides by the LCD, 6.

$$\frac{4x+1}{3}=\frac{x+5}{6}+\frac{x-3}{6}$$
$$6\left(\frac{4x+1}{3}\right)=6\left(\frac{x+5}{6}+\frac{x-3}{6}\right)$$
$$2(4x+1)=(x+5)+(x+3)$$
$$8x+2=2x+2$$
$$6x=0$$
$$x=0$$

Check $x=0$: $\frac{1}{3}=\frac{5}{6}-\frac{3}{6}$ True

The solution set is $\{0\}$.

72. Multiply both sides by the LCD, 80.

$$\frac{2x+5}{5}=\frac{3x+1}{2}+\frac{-x+8}{16}$$
$$80\left(\frac{2x+5}{5}\right)=80\left(\frac{3x+1}{2}+\frac{-x+8}{16}\right)$$
$$16(2x+5)=40(3x+1)+5(-x+8)$$
$$32x+80=120x+40-5x+40$$
$$32x+80=115x+80$$
$$0=83x$$
$$\frac{0}{83}=x$$
$$0=x$$

Check $x=0$: $1=\frac{1}{2}+\frac{1}{2}$ True

The solution set is $\{0\}$.

73. Multiply each term by 10.

$$2.7x-3=5.2x+2$$
$$27x-30=52x+20$$
$$27x-50=52x$$
$$-50=25x$$
$$-2=x$$

Check $x=-2$: $-5.4-3=-10.4+2$ True

The solution set is $\{-2\}$.

74. Multiply each term by 10.

$$3.6x-1=5.1x+5$$
$$36x-10=51x+50$$
$$36x-60=51x$$
$$-60=15x$$
$$-4=x$$

Check $x=-4$: $-14.4-1=-20.4+5$ True

The solution set is $\{-4\}$.

75. Multiply each term by 100.

$$0.25x-1=0.32x-0.3$$
$$25x-100=32x-30$$
$$25x-70=32x$$
$$-70=7x$$
$$-10=x$$

Check $x=-10$: $-2.5-1=-3.2-0.3$ True

The solution set is $\{-10\}$.

76. Multiply each term by 100.

$$0.45x-0.2=0.29x-1$$
$$45x-20=29x-100$$
$$45x=29x-80$$
$$16x=-80$$
$$x=-5$$

Check $x=-5$: $-2.25-0.2=-1.45-1$ True

The solution set is $\{-5\}$.

77. Multiply each term by 100.

$$0.04x+0.06+0.03x=0.03x+1.46$$
$$4x+6+3x=3x+146$$
$$4x+6=146$$
$$4x=140$$
$$x=35$$

Check
$x=35$: $1.4+0.06+1.05=1.05+1.46$ True

The solution set is $\{35\}$.

78. Multiply each term by 100.

$$0.05x+0.08+0.06x=0.07x+0.68$$
$$5x+8+6x=7x+68$$
$$11x+8=7x+68$$
$$4x=60$$
$$x=15$$

Check
$x=15$: $0.75+0.08+0.9=1.05+0.68$ True

The solution set is $\{15\}$.

79. Multiply each term by 1000.

$$0.006x-0.02x+0.03=0.008x+0.25$$
$$6x-20x+30=8x+250$$
$$-14x+30=8x+250$$
$$-22x=220$$
$$x=\frac{220}{-22}$$
$$x=-10$$

Check
$x=-10: -0.06+0.2+0.03=0.08+0.25$ True
The solution set is $\{-10\}$.

80. Multiply each term by 100.

$$\begin{aligned} 0.05x-0.1x+0.6&=0.04x+2.22\\ 5x-10x+60&=4x+222\\ -5x+60&=4x+222\\ -9x&=162\\ x&=\frac{162}{-9}\\ x&=-18 \end{aligned}$$

Check
$x=-18: -0.9+1.8+0.6=-0.72+2.22$ True
The solution set is $\{-18\}$.

81. Multiply each term by 100.

$$\begin{aligned} 0.05x+0.12(x+5000)&=940\\ 5x+12(x+5000)&=100(940)\\ 5x+12x+60,000&=94,000\\ 17x&=34,000\\ x&=2000 \end{aligned}$$

Check $x=2000$: $100+840=940$ True
The solution set is $\{2000\}$.

82. Multiply each term by 100.

$$\begin{aligned} 0.09x+0.13(x+300)&=61\\ 100[0.09x+0.13(x+300)]&=100(61)\\ 100(0.09x)+100(0.13)(x+300)&=6100\\ 9x+13(x+300)&=6100\\ 9x+13x+3900&=6100\\ 22x&=2200\\ x&=\frac{2200}{22}=100 \end{aligned}$$

Check $x=100$: $9+52=61$ True
The solution set is $\{100\}$.

83. Multiply each term by 100.

$$\begin{aligned} 0.02(50)+0.08x&=0.04(50+x)\\ 2(50)+8x&=4(50+x)\\ 100+8x&=200+4x\\ 4x&=100\\ x&=25 \end{aligned}$$

Check $x=25$: $1+2=3$ True
The solution set is $\{25\}$.

84. Multiply each term by 100.

$$\begin{aligned} 0.04(90)+0.12x&=0.06(460+x)\\ 100[0.04(90)+0.12x]&=100[0.06(460+x)]\\ 4(90)+12x&=6(460+x)\\ 360+12x&=2760+6x\\ 360+6x&=2760\\ 6x&=2400\\ x&=400 \end{aligned}$$

Check $x=400$: $3.6+48=51.6$ True
The solution set is $\{400\}$.

85. Multiply each term by 100.

$$\begin{aligned} 0.05x+0.10(200-x)&=0.45x\\ 5x+10(200-x)&=45x\\ 5x+2000-10x&=45x\\ 2000-5x&=45x\\ 2000&=50x\\ 40&=x \end{aligned}$$

Check $x=40$: $2+16=18$ True
The solution set is $\{40\}$.

86. Multiply each term by 100.

$$\begin{aligned} 0.08x+0.12(260-x)&=0.48x\\ 8x+12(260-x)&=48x\\ 8x+3120-12x&=48x\\ -4x+3120&=48x\\ 3120&=52x\\ x&=\frac{3120}{52}=60 \end{aligned}$$

Check $x=60$: $4.8+24=28.8$ True
The solution set is $\{60\}$.

87. Multiply each term by 1000.

$$\begin{aligned} 0.006(x+2)&=0.007x+0.009\\ 6(x+2)&=7x+9\\ 6x+12&=7x+9\\ 3&=x \end{aligned}$$

Check $x=3$: $0.03=0.021+0.009$ True
The solution set is $\{3\}$.

88. Multiply each term by 1000.

$$\begin{aligned} 0.006(50-x)&=0.272-0.004x\\ 6(50-x)&=272-4x\\ 300-6x&=272-4x\\ -2x&=-28\\ x&=14 \end{aligned}$$

Check $x = 14$: $0.216 = 0.272 - 0.056$ True

The solution set is $\{14\}$.

1.2 Formulas and Percent

Classroom Examples, Now Try Exercises

1. To solve $d = rt$ for r, treat r as the only variable.

$d = rt$

$\frac{d}{t} = \frac{rt}{t}$ Divide by t.

$\frac{d}{t} = r$, or $r = \frac{d}{t}$

N1. To solve $I = prt$ for p, treat p as the only variable.

$I = prt$

$I = p(rt)$ Associative property

$\frac{I}{rt} = \frac{p(rt)}{rt}$ Divide by rt.

$\frac{I}{rt} = p$, or $p = \frac{I}{rt}$

2. Solve $P = 2L + 2W$ for L.

$P = 2L + 2W$

$P - 2W = 2L$ Subtract $2W$.

$\frac{P - 2W}{2} = \frac{2L}{2}$ Divide by 2.

$\frac{P - 2W}{2} = L$, or $L = \frac{P}{2} - W$

N2. Solve $P = a + 2b + c$ for b.

$P - a = 2b + c$ Subtract a.

$P - a - c = 2b$ Subtract c.

$\frac{P - a - c}{2} = \frac{2b}{2}$ Divide by 2.

$\frac{P - a - c}{2} = b$, or $b = \frac{P - a - c}{2}$

3. Solve $y = \frac{1}{2}(x + 3)$ for x.

$2y = x + 3$ Multiply by 2.

$2y - 3 = x$, or $x = 2y - 3$ Subtract 3.

N3. Solve $P = 2(L + W)$ for L.

$\frac{P}{2} = \frac{2(L + W)}{2}$ Divide by 2.

$\frac{P}{2} = L + W$

$\frac{P}{2} - W = L$ Subtract W.

$L = \frac{P}{2} - W$, or $L = \frac{P - 2W}{2}$

4. Solve $2x + 7y = 7$ for y.

$2x + 7y - 2x = 7 - 2x$ Subtract $2x$.

$7y = 7 - 2x$

$\frac{7y}{7} = \frac{7 - 2x}{7}$ Divide by 7.

$y = \frac{7 - 2x}{7}$, or $y = -\frac{2}{7}x + 1$

N4. Solve $5x - 6y = 12$ for y.

$-6y = 12 - 5x$ Subtract $5x$.

$y = \frac{12 - 5x}{-6}$, Divide by -6.

or $y = \frac{5}{6}x - 2$

5. Use $d = rt$. Solve for r.

$\frac{d}{t} = \frac{rt}{t}$ Divide by t.

$\frac{d}{t} = r$, or $r = \frac{d}{t}$

Now substitute $d = 15$ and $t = \frac{1}{3}$.

$$r = \frac{15}{\frac{1}{3}} = 15 \times \frac{3}{1} = 45$$

The average rate is 45 miles per hour.

N5. Use $d = rt$. Solve for r.

$\frac{d}{t} = \frac{rt}{t}$ Divide by t.

$\frac{d}{t} = r$, or $r = \frac{d}{t}$

Now substitute $d = 21$ and $t = \frac{1}{2}$.

$$r = \frac{21}{\frac{1}{2}} = 21 \times \frac{2}{1} = 42$$

The average rate is 42 miles per hour.

6. (a) Let a represent the amount of commission earned.

$$\begin{aligned} p \cdot b &= a && \text{percent} \cdot \text{whole} = \text{partial} \\ 0.08 \cdot 12{,}000 &= a && \text{Let } p = 0.08, b = 12{,}000. \\ 960 &= a && \text{Multiply.} \end{aligned}$$

The salesperson earns \$960.

(b) The given amount of mixture is 20 oz. The part that is oil is 1 oz.

$$\begin{aligned} p \cdot b &= a && \text{percent} \cdot \text{whole} = \text{partial} \\ p \cdot 20 &= 1 && \text{Let } b = 20 \text{ and } a = 1. \\ \frac{20p}{20} &= \frac{1}{20} && \text{Comm. prop.; div. by 20.} \\ p &= 0.05 && 0.05 = \tfrac{5}{100} = 5\% \end{aligned}$$

The mixture is 5% oil.

(c) The partial amount \$22,000 is 80% of the down payment.

$$\begin{aligned} p \cdot b &= a && \text{percent} \cdot \text{whole} = \text{partial} \\ 0.8 \cdot b &= 22{,}000 && \text{Let } p = 0.8,\ a = 22{,}000. \\ \frac{0.8b}{0.8} &= \frac{22{,}000}{0.8} && \text{Divide by 0.8.} \\ b &= 27{,}500 \end{aligned}$$

The total amount needed for the down payment is \$27,500.

N6. (a) Let a represent the amount of interest earned.

$$\begin{aligned} p \cdot b &= a && \text{percent} \cdot \text{whole} = \text{partial} \\ 0.025 \cdot 7500 &= a && \text{Let } p = 0.025, b = 7500. \\ 187.5 &= a && \text{Multiply.} \end{aligned}$$

The interest earned is \$187.50.

(b) The given amount of mixture is 5 L. The part that is antifreeze is 2 L.

$$\begin{aligned} p \cdot b &= a && \text{percent} \cdot \text{whole} = \text{partial} \\ p \cdot 5 &= 2 && \text{Let } b = 5 \text{ and } a = 2. \\ \frac{5p}{5} &= \frac{2}{5} && \text{Comm. prop.; div. by 5.} \\ p &= 0.40 && 0.40 = \tfrac{40}{100} = 40\% \end{aligned}$$

The mixture is 40% antifreeze.

(c) The partial amount \$450 is 90% of the down payment.

$$\begin{aligned} p \cdot b &= a && \text{percent} \cdot \text{whole} = \text{partial} \\ 0.90 \cdot b &= 450 && \text{Let } p = 0.90,\ a = 450. \\ \frac{0.90b}{0.90} &= \frac{450}{0.90} && \text{Divide by 0.90.} \\ b &= 500 \end{aligned}$$

The total amount needed for the down payment is \$500.

7. Let a represent the amount spent on vet care/product sales.

$$\begin{aligned} p \cdot b &= a && \text{percent} \cdot \text{whole} = \text{partial} \\ 0.278 \cdot 123.6 &= a && \text{Let } p = 0.278,\ b = 123.6. \\ 34.4 &\approx \text{a} && \text{Multiply; nearest tenth} \end{aligned}$$

Therefore, about \$34.4 billion was spent on vet care/product sales.

N7. Let a represent the amount spent on supplies/live animals/medicine.

$$\begin{aligned} p \cdot b &= a && \text{percent} \cdot \text{whole} = \text{partial} \\ 0.241 \cdot 123.6 &= a && \text{Let } p = 0.241,\ b = 123.6. \\ 29.8 &\approx \text{a} && \text{Multiply; nearest tenth} \end{aligned}$$

Therefore, about \$29.8 billion was spent on supplies/live animals/medicine.

8. (a) Let x = the percent increase (as a decimal).

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

$$x = \frac{1352 - 1300}{1300}$$

$$x = \frac{52}{1300}$$

$$x = 0.04$$

The percent increase was 4%.

(b) Let x = the percent decrease (as a decimal).

$$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$

$$x = \frac{54.00 - 51.30}{54.00}$$

$$x = \frac{2.70}{54.00}$$

$$x = 0.05$$

The percent decrease was 5%.

N8. (a) Let x = the percent decrease (as a decimal).

$$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$

$$x = \frac{80 - 56}{80}$$

$$x = \frac{24}{80}$$

$$x = 0.3$$

The percent markdown was 30%.

(b) Let x = the percent increase (as a decimal).

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$
$$x = \frac{689-650}{650}$$
$$x = \frac{39}{650}$$
$$x = 0.06$$

The percent increase was 6%.

9. $S = \dfrac{4k+7}{k+90}$

$S = \dfrac{4(9.072)+7}{(9.072)+90}$ Substitute $k = 9.072$.

$S = 0.44$

The child has a body surface area of $0.44\ \text{m}^2$.

N9. $S = \dfrac{4k+7}{k+90}$

$S = \dfrac{4(14.515)+7}{(14.515)+90}$ Substitute $k = 14.515$.

$S = 0.62$

The child has a body surface area of $0.62\ \text{m}^2$.

10. The surface area of the child is $0.44\ \text{m}^2$. The adult dosage is 500 mg.

$$C = \frac{\text{body surface area in m}^2}{1.7} \times D$$
$$C = \frac{0.44}{1.7} \times 500$$
$$C = 129$$

The child dosage is 129 mg.

N10. The surface area of the child is $0.62\ \text{m}^2$. The adult dosage is 100 mg.

$$C = \frac{\text{body surface area in m}^2}{1.7} \times D$$
$$C = \frac{0.62}{1.7} \times 100$$
$$C = 36$$

The child dosage is 36 mg.

Exercises

1. A *formula* is an equation in which variables are used to describe a relationship.

2. To solve a formula for a specified variable, treat that variable as if it were the only one and treat all other variables like constants (numbers).

3. **(a)** $0.35 = \dfrac{35}{100} = 35\%$

(b) $0.18 = \dfrac{18}{100} = 18\%$

(c) $0.02 = \dfrac{2}{100} = 2\%$

(d) $0.075 = \dfrac{7.5}{100} = 7.5\%$

(e) $1.5 = \dfrac{150}{100} = 150\%$

4. **(a)** $60\% = \dfrac{60}{100} = 0.6$

(b) $37\% = \dfrac{37}{100} = 0.37$

(c) $8\% = \dfrac{8}{100} = 0.08$

(d) $3.5\% = \dfrac{3.5}{100} = 0.035$

(e) $210\% = \dfrac{210}{100} = 2.1$

5. Solve $I = prt$ for r.

$$I = prt$$
$$\frac{I}{pt} = \frac{prt}{pt}$$
$$\frac{I}{pt} = r, \quad \text{or} \quad r = \frac{I}{pt}$$

6. Solve $d = rt$ for t.

$\dfrac{d}{r} = \dfrac{rt}{r}$ Divide by r.

$\dfrac{d}{r} = t$, or $t = \dfrac{d}{r}$

7. **(a)** Solve $A = LW$ for W.

$\dfrac{A}{L} = W$ Divide by L.

(b) Solve $A = LW$ for L.

$\dfrac{A}{W} = L$ Divide by W.

8. **(a)** Solve $A = bh$ for b.

$\frac{A}{h} = b$ Divide by h.

(b) Solve $A = bh$ for h.

$\frac{A}{b} = h$ Divide by b.

9. Solve $P = 2L + 2W$ for L.

$$P = 2L + 2W$$
$$P - 2W = 2L$$
$$\frac{P - 2W}{2} = \frac{2L}{2}$$
$$\frac{P - 2W}{2} = L, \quad \text{or} \quad L = \frac{P}{2} - W$$

10. **(a)** Solve $P = a + b + c$ for b.

$$P - (a + c) = a + b + c - (a + c)$$
$$P - a - c = b$$

(b) Solve $P = a + b + c$ for c.

$$P - (a + b) = a + b + c - (a + b)$$
$$P - a - b = c$$

11. **(a)** Solve for $V = LWH$ for W.

$$V = LWH$$
$$\frac{V}{LH} = \frac{LWH}{LH}$$
$$\frac{V}{LH} = W, \quad \text{or} \quad W = \frac{V}{LH}$$

(b) Solve for $V = LWH$ for H.

$$V = LWH$$
$$\frac{V}{LW} = \frac{LWH}{LW}$$
$$\frac{V}{LW} = H, \quad \text{or} \quad H = \frac{V}{LW}$$

12. **(a)** Solve $A = \frac{1}{2}bh$ for h.

$$\frac{2A}{b} = \frac{2}{b}\left(\frac{1}{2}bh\right)$$
$$\frac{2A}{b} = h$$

(b) Solve $A = \frac{1}{2}bh$ for b.

$$\frac{2A}{h} = \frac{2}{h}\left(\frac{1}{2}bh\right)$$
$$\frac{2A}{h} = b$$

13. Solve $C = 2\pi r$ for r.

$$C = 2\pi r$$

$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$ Divide by 2π.

$$\frac{C}{2\pi} = r$$

14. Solve $V = \pi r^2 h$ for h.

$$V = \pi r^2 h$$

$\frac{V}{\pi r^2} = h$ Divide by πr^2.

15. **(a)** Solve $A = \frac{1}{2}h(b + B)$ for b.

$2A = h(b + B)$ Multiply by 2.

$\frac{2A}{h} = b + B$ Divide by h.

$\frac{2A}{h} - B = b,$ Subtract B.

or $b = \frac{2A - hB}{h}$

(b) Solve $A = \frac{1}{2}h(b + B)$ for B.

$2A = h(b + B)$ Multiply by 2.

$\frac{2A}{h} = b + B$ Divide by h.

$\frac{2A}{h} - b = B$ Subtract b.

Another method to solve for B is the following.

$2A = h(b + B)$ Multiply by 2.

$2A = hb + hB$ Distributive prop.

$2A - hb = hB$ Subtract hb.

$\frac{2A - hb}{h} = B$ Divide by h.

16. Solve $V = \frac{1}{3}\pi r^2 h$ for h.

$$V = \frac{1}{3}\pi r^2 h$$

$\frac{3V}{\pi r^2} = h$ Divide by $\frac{1}{3}\pi r^2$.

17. Solve $F=\frac{9}{5}C+32$ for C.

$$F-32=\frac{9}{5}C$$
$$\frac{5}{9}(F-32)=\frac{5}{9}\left(\frac{9}{5}C\right)$$
$$\frac{5}{9}(F-32)=C$$

18. Solve $C=\frac{5}{9}(F-32)$ for F.

$$\frac{9}{5}C=\frac{9}{5}\cdot\frac{5}{9}(F-32) \quad \text{Multiply by } \frac{9}{5}.$$
$$\frac{9}{5}C=F-32$$
$$\frac{9}{5}C+32=F \quad \text{Add 32.}$$

19. **(a)** Solve $ax+b=0$ for x.

$$ax+b=0$$
$$ax=-b$$
$$x=\frac{-b}{a}$$

(b) Solve $ax+b=0$ for a.

$$ax+b=0$$
$$ax=-b$$
$$a=\frac{-b}{x}$$

20. **(a)** Solve $y=mx+b$ for x.

$$y=mx+b$$
$$y-b=mx$$
$$\frac{y-b}{m}=x$$

(b) Solve $y=mx+b$ for m.

$$y=mx+b$$
$$y-b=mx$$
$$\frac{y-b}{x}=m$$

21. Solve $A=P(1+rt)$ for t.

$$A=P(1+rt)$$
$$\frac{A}{P}=1+rt$$
$$\frac{A}{P}-1=rt$$
$$\frac{\frac{A}{P}-1}{r}=t$$

This can also be written as $t=\frac{A-P}{Pr}$.

22. Solve $M=C(1+r)$ for r.

$$M=C(1+r)$$
$$\frac{M}{C}=1+r$$
$$\frac{M}{C}-1=r$$

This can also be written as $r=\frac{M-C}{C}$.

23. $A=\frac{1}{2}bh$

$$2A=2\left(\frac{1}{2}bh\right) \quad \text{Multiply by 2.}$$
$$2A=bh$$
$$\frac{2A}{b}=\frac{bh}{b} \quad \text{Divide by } b.$$
$$\frac{2A}{b}=h$$
$$\frac{2A}{b}=\frac{2}{1}\cdot\frac{A}{b}$$
$$=2\left(\frac{A}{b}\right) \quad \text{This choice is A.}$$
$$=2A\left(\frac{1}{b}\right) \quad \text{This is choice B.}$$

To get choice C, divide $A=\frac{1}{2}bh$ by $\frac{1}{2}b$.

$\frac{A}{\frac{1}{2}b}=\frac{\frac{1}{2}bh}{\frac{1}{2}b}$ gives us $h=\frac{A}{\frac{1}{2}b}$.

Choice D, $h=\frac{\frac{1}{2}A}{b}$, can be multiplied by $\frac{2}{2}$ on the right side to get $h=\frac{A}{2b}$, so it is *not* equivalent to $h=\dfrac{2A}{b}$. Therefore, the correct answer is D.

24. $C=\dfrac{5}{9}(F-32)$

$C=\dfrac{5}{9}F-\dfrac{160}{9}$ This is choice A.

$C=\dfrac{5}{9}\cdot\dfrac{F}{1}-\dfrac{160}{9}$

$C=\dfrac{5F}{9}-\dfrac{160}{9}$ This is choice B.

$C=\dfrac{5F-160}{9}$ This is choice C.

Therefore, the correct answer is D.

25.
$$4x+y=1$$
$$4x+y-4x=1-4x$$
$$y=-4x+1$$

26.
$$3x+y=9$$
$$y=-3x+9$$

27.
$$x-2y=-6$$
$$x-2y-x=-6-x$$
$$-2y=-x-6$$
$$y=\frac{-x-6}{-2}$$

Simplified, this is $y=\dfrac{1}{2}x+3$.

28.
$$x-5y=-20$$
$$-5y=-20-x$$
$$y=\frac{-20-x}{-5}$$

Simplified, this is $y=\dfrac{1}{5}x+4$.

29.
$$4x+9y=11$$
$4x+9y-4x=11-4x$ Subtract $4x$.
$$9y=-4x+11$$
$\dfrac{9y}{9}=\dfrac{-4x+11}{9}$ Divide by 9.
$$y=\frac{-4x+11}{9}$$
$y=\dfrac{-4x}{9}+\dfrac{11}{9}$ $\quad \dfrac{a+b}{c}=\dfrac{a}{c}+\dfrac{b}{c}$

Simplified, this is $y=-\dfrac{4}{9}x+\dfrac{11}{9}$.

30.
$$7x+8y=11$$
$7x+8y-7x=11-7x$ Subtract $7x$.
$$8y=-7x+11$$
$\dfrac{8y}{8}=\dfrac{-7x+11}{8}$ Divide by 8.
$$y=\frac{-7x+11}{8}$$
$y=\dfrac{-7x}{8}+\dfrac{11}{8}$ $\quad \dfrac{a+b}{c}=\dfrac{a}{c}+\dfrac{b}{c}$

Simplified, this is $y=-\dfrac{7}{8}x+\dfrac{11}{8}$.

31.
$$-3x+2y=5$$
$-3x+2y+3x=5+3x$ Add $3x$.
$$2y=3x+5$$
$y=\dfrac{3x+5}{2}$ Divide by 2.

Simplified, this is $y=\dfrac{3}{2}x+\dfrac{5}{2}$.

32.
$$-5x+3y=12$$
$3y=5x+12$ Add $5x$.
$y=\dfrac{5x+12}{3}$ Divide by 3.

Simplified, this is $y=\dfrac{5}{3}x+4$.

33.
$$6x-5y=7$$
$6x-5y-6x=7-6x$ Subtract $6x$.
$$-5y=-6x+7$$
$y=\dfrac{-6x+7}{-5}$ Divide by -5.

Simplified, this is $y=\dfrac{6}{5}x-\dfrac{7}{5}$.

34.
$$8x-3y=4$$
$-3y=-8x+4$ Subtract $8x$.
$y=\dfrac{-8x+4}{-3}$ Divide by -3.

Simplified, this is $y=\dfrac{8}{3}x-\dfrac{4}{3}$.

35. $$\frac{1}{2}x-\frac{1}{3}y=1$$
$$6\cdot\left(\frac{1}{2}x-\frac{1}{3}y\right)=6\cdot 1 \quad \text{Multiply by 6.}$$
$$3x-2y=6$$
$$3x-2y-3x=6-3x$$
$$-2y=-3x+6$$
$$y=\frac{-3x+6}{-2}$$
Simplified, this is $y=\frac{3}{2}x-3$.

36. $$\frac{2}{3}x-\frac{2}{5}y=2$$
$$\frac{1}{3}x-\frac{1}{5}y=1 \quad \text{Divide by 2.}$$
$$5x-3y=15 \quad \text{Multiply by 15.}$$
$$-3y=15-5x$$
$$y=\frac{15-5x}{-3}$$
Simplified, this is $y=\frac{5}{3}x-5$.

37. Use the formula $d=rt$ to find the time.
$$d=rt$$
$$\frac{d}{r}=t$$
$$\frac{300}{81}=t$$
$$3.7\approx t$$
Therefore, a cheetah can run 300 ft (100 yd) in approximately 3.7 seconds.

38. Solve $d=rt$ for t.
$$t=\frac{d}{r}$$
$$t=\frac{10{,}500}{500}=21 \quad \text{Let } d=10{,}500,\ r=500.$$
The travel time is 21 hr.

39. Solve $d=rt$ for r.
$$r=\frac{d}{t}$$
$$r=\frac{54}{0.28}\approx 193 \quad \text{Let } d=54,\ t=0.28.$$
The kick's speed was approximately 193 feet per second.

40. Solve $d=rt$ for r.
$$r=\frac{d}{t}$$
$$r=\frac{520}{10}=52 \quad \text{Let } d=520,\ t=10.$$
The driver's rate was 52 mph.

41. Use the formula $F=\frac{9}{5}C+32$.
$$F=\frac{9}{5}(49)+32 \quad \text{Let } C=49.$$
$$=88.2+32$$
$$=120.2$$
The corresponding temperature is 120°F.

42. Use the formula $C=\frac{5}{9}(F-32)$.
$$C=\frac{5}{9}(-41-32) \quad \text{Let } F=-41.$$
$$=\frac{5}{9}(-73)$$
$$\approx -41$$
The corresponding temperature is about -41°C.

43. Solve $P=4s$ for s.
$$s=\frac{P}{4}$$
To find s, substitute 920 for P.
$$s=\frac{920}{4}=230$$
The length of each side is 230 m.

44. Use $V=\pi r^2 h$.

Replace r by $\frac{35}{2}=17.5$ and h by 588.
$$V=\pi(17.5)^2(588)$$
$$\approx 565{,}722.3$$
To the nearest whole number, the volume is $565{,}722\ \text{ft}^3$.

45. Use the formula $C=2\pi r$.

(a) $480\pi=2\pi r$ Let $C=480\pi$.
$$\frac{480\pi}{2\pi}=r \quad \text{Divide by } 2\pi.$$
The radius of the circle is 240 inches.

(b) The diameter is twice the radius.

$d = 2r$

$d = 2(240)$

$d = 480$

The diameter of the circle is 480 inches.

46. **(a)** $d = 2r = 2(2.5) = 5$

The diameter is 5 inches.

(b) Use the formula $C = 2\pi r$.

$C = 2\pi r$

$C = 2\pi(2.5) = 5\pi$

The circumference is 5π in.

47. Use $V = LWH$.

Let $V = 187$, $L = 11$, and $W = 8.5$.

$187 = 11(8.5)H$

$187 = 93.5H$

$2 = H$ Divide by 93.5.

The ream is 2 inches thick.

48. Use $V = LWH$.

Let $V = 238$, $W = 8.5$, and $H = 2$.

$238 = L(8.5)(2)$

$238 = L(17)$

$14 = L$ Divide by 17.

The length of a legal sheet of paper is 14 inches.

49. 18% of 780 is what number?

$$\text{partial amount } a = 0.18 \cdot 780 = 140.4$$

50. 23% of 480 is what number?

$$\text{partial amount } a = 0.23 \cdot 480 = 110.4$$

51. 42% of what number is 294?

$$\frac{\text{partial amount } a}{\text{percent } p} = \frac{294}{0.42} = 700$$

52. 18% of what number is 108?

$$\frac{\text{partial amount } a}{\text{percent } p} = \frac{108}{0.18} = 600$$

53. 120% of what number is 510?

$$\frac{\text{partial amount } a}{\text{percent } p} = \frac{510}{1.20} = 425$$

54. 140% of what number is 315?

$$\frac{\text{partial amount } a}{\text{percent } p} = \frac{315}{1.40} = 225$$

55. 12 is what percent of 48?

$$\frac{\text{partial amount } a}{\text{whole amount } b} = \frac{12}{48} = 0.25 = 25\%$$

56. 15 is what percent of 150?

$$\frac{\text{partial amount } a}{\text{whole amount } b} = \frac{15}{150} = 0.10 = 10\%$$

57. 36 is what percent of 30?

$$\frac{\text{partial amount } a}{\text{whole amount } b} = \frac{36}{30} = 1.20 = 120\%$$

58. 96 is what percent of 48?

$$\frac{\text{partial amount } a}{\text{whole amount } b} = \frac{96}{48} = 2.00 = 200\%$$

59. 4 is what percent of 20?

$$\frac{\text{partial amount } a}{\text{whole amount } b} = \frac{4}{20} = 0.20 = 20\%$$

60. 12 is what percent of 50?

$$\frac{\text{partial amount } a}{\text{whole amount } b} = \frac{12}{50} = 0.24 = 24\%$$

61. $12 - 3 = 9$ is what percent of 12?

$$\frac{\text{partial amount } a}{\text{whole amount } b} = \frac{9}{12} = 0.75 = 75\%$$

62. $30 - 15 = 15$ is what percent of 30?

$$\frac{\text{partial amount } a}{\text{whole amount } b} = \frac{15}{30} = 0.5 = 50\%$$

63. The mixture is 36 oz and that part which is alcohol is 9 oz. Thus, the percent of alcohol is

$$\frac{9}{36}=\frac{1}{4}=\frac{25}{100}=25\%.$$

The percent of water is $100\%-25\%=75\%$.

64. Let x = the amount of pure acid in the mixture. Then x can be found by multiplying the total amount of the mixture by the percent of acid given as a decimal (0.35).

$$x=40(0.35)=14$$

There are 14 L of pure acid. Since there are 40 L altogether, there are $40-14$, or 26 L of pure water in the mixture.

65. Find what percent \$6300 is of \$210,000.

$$\frac{6300}{210{,}000}=0.03=3\%$$

The agent received a 3% rate of commission.

66. Solve $I=prt$ for r.

$$r=\frac{I}{pt}$$

$$r=\frac{25.50}{3400(1)}$$

$$=0.0075=0.75\%$$

The interest rate on this deposit is 0.75%.

67. The partial amount \$1950 is 65% of the total amount needed.

$p\cdot b=a$	percent · whole = partial
$0.65\cdot b=1950$	Let $p=0.65$, $a=1950$.
$\frac{0.65b}{0.65}=\frac{1950}{0.65}$	Divide by 0.65.
$b=3000$	

The total amount needed for the used car is \$3000.

68. The partial amount \$525 is 70% of the total amount needed.

$p\cdot b=a$	percent · whole = partial
$0.70\cdot b=525$	Let $p=0.70$, $a=525$.
$\frac{0.70b}{0.70}=\frac{525}{0.70}$	Divide by 0.70.
$b=750$	

The total amount needed for the deposit is \$750.

69. **(a)** Seattle:

$$\text{Pct.}=\frac{W}{W+L}=\frac{90}{90+72}=\frac{90}{162}\approx .556$$

(b) L.A. Angels:

$$\text{Pct.}=\frac{W}{W+L}=\frac{73}{73+89}=\frac{73}{162}\approx .451$$

(c) Texas:

$$\text{Pct.}=\frac{W}{W+L}=\frac{68}{68+94}=\frac{68}{162}\approx .420$$

(d) Oakland:

$$\text{Pct.}=\frac{W}{W+L}=\frac{60}{60+102}=\frac{60}{162}\approx .370$$

70. **(a)** San Diego:

$$\text{Pct.}=\frac{W}{W+L}=\frac{89}{89+73}=\frac{89}{162}\approx .549$$

(b) San Francisco:

$$\text{Pct.}=\frac{W}{W+L}=\frac{81}{81+81}=\frac{81}{162}=.500$$

(c) Arizona:

$$\text{Pct.}=\frac{W}{W+L}=\frac{74}{74+88}=\frac{74}{162}\approx .457$$

(d) Colorado:

$$\text{Pct.}=\frac{W}{W+L}=\frac{68}{68+94}=\frac{68}{162}\approx .420$$

71. $\frac{35.1\text{ million}}{121\text{ million}}\approx 0.29$

In 2021, about 29% of the U.S. households that owned at least one Internet-enabled device owned only a smart TV.

72. $\frac{50.8\text{ million}}{121\text{ million}}\approx 0.42$

In 2021, about 42% of the U.S. households that owned at least one Internet-enabled device owned only a video game console.

73. $0.163(121)=19.723$

In 2021, about 19.7 million U.S. households that owned at least one Internet-enabled device owned all three types of Internet-enabled devices.

74. $0.325(121)=39.325$

In 2021, about 39.3 million U.S. households that owned at least one Internet-enabled device owned both a video game console and a smart TV.

75. $0.29(292{,}017) = 84{,}684.93$

To the nearest dollar, \$84,685 will be spent to provide housing.

76. $0.16(292{,}017) = 46{,}722.72$

To the nearest dollar, \$46,723 will be spent to provide child care and education.

77. $0.06(292{,}017) = 17{,}521.02$

To the nearest dollar, \$17,521 will be spent on clothing.

78. $\dfrac{\$52{,}600}{\$292{,}017} \approx 0.1801$

So the food cost is about 18%, which agrees with the percent shown in the graph.

79. Let x = the percent increase (as a decimal).

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$
$$x = \frac{15.12 - 13.50}{13.50}$$
$$x = \frac{1.62}{13.50}$$
$$x = 0.12$$

The percent increase was 12%.

80. Let x = the percent decrease (as a decimal).

$$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$
$$x = \frac{70.00 - 56.00}{70.00}$$
$$x = \frac{14.00}{70.00}$$
$$x = 0.20$$

The percent discount was 20%.

81. Let x = the percent decrease (as a decimal).

$$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$
$$x = \frac{1{,}337{,}726 - 1{,}261{,}726}{1{,}337{,}726}$$
$$x = \frac{76{,}000}{1{,}337{,}726}$$
$$x \approx 0.05681$$

The percent decrease was 5.7%.

82. Let x = the percent increase (as a decimal).

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$
$$x = \frac{9{,}509{,}934 - 9{,}470{,}661}{9{,}470{,}661}$$
$$x = \frac{39{,}273}{9{,}470{,}661}$$
$$x \approx 0.0041$$

The percent increase was 0.4%.

83. $$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$
$$= \frac{17.99 - 16.17}{17.99}$$
$$= \frac{1.82}{17.99} \approx 0.101$$

The percent discount was 10.1%.

84. $$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$
$$= \frac{44.98 - 29.96}{44.98}$$
$$= \frac{15.02}{44.98} \approx 0.334$$

The percent discount was 33.4%.

85. (a) $$S = \frac{4k+7}{k+90}$$
$$S = \frac{4(20)+7}{(20)+90}$$
$$S \approx 0.79$$

A 20 kg child has a body surface area of approximately 0.79 m^2.

(b) $$S = \frac{4k+7}{k+90}$$
$$S = \frac{4(26)+7}{(26)+90}$$
$$S \approx 0.96$$

A 26 kg child has a body surface area of approximately 0.96 m^2.

86. (a) $$S = \frac{4k+7}{k+90}$$
$$S = \frac{4(13.608)+7}{(13.608)+90}$$
$$S \approx 0.59$$

A 30 lb child (13.608 kg) has a body surface area of approximately 0.59 m^2.

(b) $S = \dfrac{4k+7}{k+90}$

$S = \dfrac{4(11.793)+7}{(11.793)+90}$

$S \approx 0.53$

A 26 lb child (11.793 kg) has a body surface area of approximately 0.53 m^2.

87. **(a)** Recall from exercise 85, part a, that a 20 kg child has a body surface area of approximately 0.79 m^2.

$C = \dfrac{\text{body surface area in m}^2}{1.7} \times D$

$C = \dfrac{0.79}{1.7} \times 250$

$C \approx 116$

A 20 kg child requires a dose of approximately 116 mg.

(b) Recall from exercise 85, part b, that a 26 kg child has a body surface area of approximately 0.96 m^2.

$C = \dfrac{\text{body surface area in m}^2}{1.7} \times D$

$C = \dfrac{0.96}{1.7} \times 250$

$C \approx 141$

A 26 kg child requires a dose of approximately 141 mg.

88. **(a)** Recall from exercise 86, part a, that a 30 lb child has a body surface area of approximately 0.59 m^2.

$C = \dfrac{\text{body surface area in m}^2}{1.7} \times D$

$C = \dfrac{0.59}{1.7} \times 500$

$C \approx 174$

A 30 lb child requires a dose of approximately 174 mg.

(b) Recall from exercise 86, part b, that a 26 lb child has a body surface area of approximately 0.53 m^2.

$C = \dfrac{\text{body surface area in m}^2}{1.7} \times D$

$C = \dfrac{0.53}{1.7} \times 500$

$C \approx 156$

A 26 lb child requires a dose of approximately 156 mg.

89. **(a)** $\dfrac{7x+8}{3} = 12$

$3 \cdot \left(\dfrac{7x+8}{3}\right) = 12 \cdot 3$

$7x + 8 = 36$

(b) $\dfrac{ax+k}{c} = t$

$c \cdot \left(\dfrac{ax+k}{c}\right) = t \cdot c$

$ax + k = tc$

90. **(a)** $7x + 8 = 36$

$7x + 8 - 8 = 36 - 8$

(b) $ax + k = tc$

$ax + k - k = tc - k$

91. **(a)** $7x + 8 - 8 = 36 - 8$

$7x = 28$

(b) $ax + k - k = tc - k$

$ax = tc - k$

92. **(a)** $7x = 28$

$\dfrac{7x}{7} = \dfrac{28}{7}$

$x = 4$

The solution set is $\{4\}$.

(b) $ax = tc - k$

$\dfrac{ax}{a} = \dfrac{tc-k}{a}$

$x = \dfrac{tc-k}{a}$

93. In 92(b), the restriction that $a \neq 0$ must be placed on the variables. This is necessary because if $a = 0$, then the denominator is 0, which is not allowed.

94.
$$28 = \frac{7}{2}(a+13)$$
$$28 = \frac{7}{2}(a+13)$$
$$28 = \frac{7(a+13)}{2}$$
$$2\cdot 28 = \left(\frac{7(a+13)}{2}\right)\cdot 2$$
$$56 = 7(a+13)$$
$$\frac{56}{7} = \frac{7(a+13)}{7}$$
$$8 = a+13$$
$$8-13 = a+13-13$$
$$-5 = a$$
The solution set is $\{-5\}$.

95.
$$S = \frac{n}{2}(a+\ell)$$
$$S = \frac{n}{2}(a+\ell)$$
$$S = \frac{n(a+\ell)}{2}$$
$$2\cdot S = \left(\frac{n(a+\ell)}{2}\right)\cdot 2$$
$$2S = n(a+\ell)$$
$$\frac{2S}{n} = \frac{n(a+\ell)}{n}$$
$$\frac{2S}{n} = a+\ell$$
$$\frac{2S}{n}-\ell = a+\ell-\ell$$
$$\frac{2S}{n}-\ell = a, \quad \text{or} \quad a = \frac{2S-n\ell}{n}$$
The restriction $n \neq 0$ must be placed on the variables.

1.3 Applications of Linear Equations

Classroom Examples, Now Try Exercises

1. **(a)** The sum of a number and 6 is 28.
$$\begin{array}{ccc} \text{The sum of a number and 6} & \text{is} & 28. \\ \downarrow & \downarrow & \downarrow \\ x+6 & = & 28 \end{array}$$
An equation is $x+6=28$.

(b)
$$\begin{array}{ccccc} \text{The product of a number and 7} & \text{is} & \text{twice the number} & \text{plus} & 12. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7x & = & 2x & + & 12 \end{array}$$
An equation is $7x = 2x+12$.

(c)
$$\begin{array}{ccccc} \text{The quotient of a number and 6,} & \text{added to} & \text{twice the number,} & \text{is} & 7. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \frac{x}{6} & + & 2x & = & 7 \end{array}$$
An equation is $\frac{x}{6}+2x=7$, or equivalently, $2x+\frac{x}{6}=7$.

N1. **(a)**
$$\begin{array}{ccc} \text{The quotient of a number and 10} & \text{is} & \text{twice the number.} \\ \downarrow & \downarrow & \downarrow \\ \frac{x}{10} & = & 2x \end{array}$$
An equation is $\frac{x}{10} = 2x$.

(b)
$$\begin{array}{ccccc} \text{The product of a number and 5,} & \text{decreased by} & 7, & \text{is} & \text{zero.} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 5x & - & 7 & = & 0 \end{array}$$
An equation is $5x-7=0$.

2. **(a)** $5x-3(x+2)=7$ is an *equation* because it has an equality symbol.
$$5x-3(x+2)=7$$
$$5x-3x-6=7 \quad \text{Distributive property}$$
$$2x-6=7 \quad \text{Combine like terms.}$$
$$2x=13 \quad \text{Add 6.}$$
$$x=\frac{13}{2} \quad \text{Divide by 2.}$$
The solution set is $\left\{\frac{13}{2}\right\}$.

(b) $5x-3(x+2)$ is an *expression* because there is no equality symbol.

$5x-3(x+2)$
$= 5x-3x-6$ Distributive property
$= 2x-6$ Combine like terms.

N2. (a) $3(x-5)+2x-1$ is an *expression* because there is no equality symbol.

$3(x-5)+2x-1$
$= 3x-15+2x-1$ Distributive property
$= 5x-16$ Combine like terms.

(b) $3(x-5)+2x=1$ is an *equation* because it has an equality symbol.

$3(x-5)+2x=1$
$3x-15+2x=1$ Distributive property
$5x-15=1$ Combine like terms.
$5x=16$ Add 15.
$x=\frac{16}{5}$ Divide by 5.

The solution set is $\left\{\frac{16}{5}\right\}$.

3. *Step 2*
The length and perimeter are given in terms of the width W. The length L is 5 cm more than the width, so
$L=W+5.$
The perimeter P is 5 times the width, so
$P=5W.$
Step 3
Use the formula for perimeter of a rectangle.
$P=2L+2W$
$5W=2(W+5)+2W$ $P=5W;\ L=W+5$
Step 4
Solve the equation.
$5W=2W+10+2W$ Distributive property
$5W=4W+10$ Combine terms.
$W=10$ Subtract $4W$.
Step 5
The width is 10 and the length is
$L=W+5=10+5=15.$
The rectangle is 10 cm by 15 cm.
Step 6
15 is 5 more than 10 and
$P=2(10)+2(15)=50$ is five times 10.

N3. *Step 2*
The length and perimeter are given in terms of the width W. The length L is 2 ft more than twice the width, so $L=2W+2.$
The perimeter P is 34, so $P=34.$
Step 3
Use the formula for perimeter of a rectangle.
$P=2L+2W$
$34=2(2W+2)+2W$ *$P=34;\ L=2W+2$*
Step 4
Solve the equation.
$34=4W+4+2W$ Distributive property
$34=6W+4$ Combine terms.
$30=6W$ Subtract 4.
$5=W$ Divide by 6.
Step 5
The width is 5 and the length is
$L=2W+2=10+2=12.$
The rectangle is 5 ft by 12 ft.
Step 6
12 is 2 more than twice 5 and
$P=2(12)+2(5)=34.$

4. *Step 2*
Let $x=$ the number of home runs for Judge.
Then $x-16=$ the number of home runs for Schwarber.
Step 3
The sum of their home runs is 108, so an equation is
$x+(x-16)=108.$
Step 4
Solve the equation.
$2x-16=108$
$2x=124$ Add 16.
$x=62$ Divide by 2.
Step 5
Judge had 62 home runs and Schwarber had $62-16=46$ home runs.
Step 6
46 is 16 less than 62, and the sum of 46 and 62 is 108.

N4. *Step 2*
Let $x=$ the number of TDs for Stafford.
Then $x+2=$ the number of TDs for Brady.
Step 3
The sum of their TDs is 84, so an equation is
$x+(x+2)=84.$
Step 4
Solve the equation.
$2x+2=84$
$2x=82$ Subtract 2.
$x=41$ Divide by 2.

Step 5
Stafford had 41 TDs and Brady had $41+2=43$ TDs.
Step 6
43 is 2 more than 41, and the sum of 41 and 43 is 84.

5. *Step 2*
Let $x =$ the Medicare spending in 2010.
Because $53.2\% = 53.2(0.01) = 0.532$, $0.532x =$ the additional Medicare spending in 2021.
Step 3

The number in 2010	plus	the increase	is	692.
↓	↓	↓	↓	↓
x	$+$	$0.532x$	$=$	692

Step 4
Solve the equation.

$$\begin{aligned} 1x + 0.532x &= 692 && \text{Identity property} \\ 1.532x &= 692 && \text{Combine like terms.} \\ x &\approx 451.7 && \text{Divide by } 1.532. \end{aligned}$$

Step 5
The approximate Medicare spending in 2010 was \$451.7 billion.
Step 6
Check that the increase, $692 - 451.7 = 240.3$, is about 53.2% of 451.7, $53.2\% \cdot 451.7 \approx 240.3$, as required.

N5. *Step 2*
Let $x =$ the number of Introductory Statistics students in the fall of 2015. Since $700\% = 700(0.01) = 7$, $7x =$ the number of additional students in 2023.
Step 3

The number in 2015	plus	the increase	is	96.
↓	↓	↓	↓	↓
x	$+$	$7x$	$=$	96

Step 4
Solve the equation.

$$\begin{aligned} 1x + 7x &= 96 && \text{Identity property} \\ 8x &= 96 && \text{Combine like terms.} \\ x &= 12 && \text{Divide by 8.} \end{aligned}$$

Step 5
There were 12 students in the fall of 2015.
Step 6
Check that the increase, $96 - 12 = 84$, is 700% of 12, $700\% \cdot 12 = 700(0.01)(12) = 84$, as required.

6. Let $x =$ the amount invested at 2.5%.
Then $34{,}000 - x =$ the amount invested at 2%.
Use $I = prt$ with $t = 1$.

Principal	Rate (as a decimal)	Interest
x	0.025	$0.025x$
$34{,}000 - x$	0.02	$0.02(34{,}000 - x)$
34,000	← Totals →	772.50

The last column gives the equation.

$$0.025x + 0.02(34{,}000 - x) = 772.50$$

Solve the equation.

$$\begin{aligned} 0.025x + 680 - 0.02x &= 772.50 && \text{Dist. prop.} \\ 0.005x + 680 &= 772.50 && \text{Combine terms.} \\ 0.005x &= 92.5 && \text{Subtract 680.} \\ x &= 18{,}500 && \text{Divide by 0.005.} \end{aligned}$$

The person invested \$18,500 at 2.5% and $\$34{,}000 - \$18{,}500 = \$15{,}500$ at 2%.

N6. Let $x =$ the amount invested at 3%.
Then $20{,}000 - x =$ the amount invested at 2.5%.
Use $I = prt$ with $t = 1$.
Make a table to organize the information.

Principal	Rate (as a decimal)	Interest
x	0.03	$0.03x$
$20{,}000 - x$	0.025	$0.025(20{,}000 - x)$
20,000	← Totals →	575

The last column gives the equation.

$$0.03x + 0.025(20{,}000 - x) = 575$$

Solve the equation.

$$\begin{aligned} 0.03x + 500 - 0.025x &= 575 && \text{Dist. prop.} \\ 0.005x + 500 &= 575 && \text{Combine terms.} \\ 0.005x &= 75 && \text{Subtract 500.} \\ x &= 15{,}000 && \text{Divide by 0.005.} \end{aligned}$$

The person invested \$15,000 at 3% and $\$20{,}000 - \$15{,}000 = \$5{,}000$ at 2.5%.

7. Let $x =$ the amount of \$8 per lb candy. Then $x + 100 =$ the amount of \$7 per lb candy.
Make a table to organize the information.

Number of Pounds	Price per Pound	Value
x	\$8	$8x$
100	\$4	400
$x + 100$	\$7	$7(x + 100)$

The last column gives the equation.

$8x+400=7(x+100)$ Dist. prop.

$8x+400=7x+700$

$x+400=700$ Subtract 7x.

$x=300$ Divide by 400.

300 lb of candy worth \$8 per lb should be used.

N7. Let $x=$ the number of liters of 20% acid solution needed. Make a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
5	0.30	$0.30(5)=1.5$
x	0.20	$0.20x$
$x+5$	0.24	$0.24(x+5)$

Acid in 30% solution + acid in 20% solution = acid in 24% solution.

$1.5 + 0.20x = 0.24(x+5)$

Solve the equation.

$1.5+0.20x=0.24x+1.2$

$0.3=0.04x$

$x=7.5$

$7\frac{1}{2}$ liters of the 20% solution are needed.

Check

30% of 5 is 1.5 and 20% of 7.5 is 1.5;
$1.5+1.5=3.0,$ which is the same as 24% of $(5+7.5)$.

8. Let $x=$ the number of liters of water.

Number of Liters	Percent (as a decimal)	Liters of Pure Antifreeze
x	$0\%=0$	0
20	$50\%=0.50$	0.50(20)
$x+20$	$40\%=0.40$	$0.40(x+20)$

The last column gives the equation.

$0+0.50(20)=0.40(x+20)$

$10=0.4x+8$

$2=0.4x$ Subtract 8.

$5=x$ Divide by 0.4.

5 L of water are needed.

N8. Let $x=$ the number of gallons of pure antifreeze.

Number of Liters	Percent (as a decimal)	Liters of Pure Antifreeze
x	$100\%=1$	$1x$
3	$30\%=0.30$	0.30(3)
$x+3$	$40\%=0.40$	$0.40(x+3)$

The last column gives the equation.

$1x+0.30(3)=0.40(x+3)$

$1.0x+0.9=0.4x+1.2$

$0.6x+0.9=1.2$ Subtract $0.4x$.

$0.6x=0.3$ Subtract 0.9.

$x=0.5$ Divide by 0.6.

$\frac{1}{2}$ gallon of pure antifreeze is needed.

Exercises

1. **(a)** 15 more than a number $x+15$

(b) 15 is more than a number. $15>x$

2. **(a)** 5 greater than a number $x+5$

(b) 5 is greater than a number. $5>x$

3. **(a)** 8 less than a number $x-8$

(b) 8 is less than a number. $8<x$

4. **(a)** 6 less than a number $x-6$

(b) 6 is less than a number. $6<x$

5. Because the unknown number comes first in the verbal phrase, it must come first in the mathematical expression. The correct expression is $x-7$.

6. Because the unknown number comes first in the verbal phrase, it must be placed in the numerator of the mathematical expression. The correct expression is $\frac{n}{12}$.

7. Twice a number, decreased by 13
$2x-13$

8. Triple a number, decreased by 14
$3x-14$

9. 12 increased by four times a number
$12+4x$

10. 15 more than one-half of a number
$\frac{1}{2}x+15$

11. The product of 8 and 16 less than a number
$8(x-16)$

12. The product of 8 more than a number and 5 less than the number
$(x+8)(x-5)$

13. The quotient of three times a number and 10
$\frac{3x}{10}$

14. The quotient of 9 and five times a nonzero number: $\frac{9}{5x}, (x \neq 0)$

15. The sentence "the sum of a number and 6 is -31" can be translated as $x+6=-31$.
Solve for x.
$x+6=-31$
$x=-37$ Subtract 6.
The number is -37.

16. The sentence "the sum of a number and -4 is 18" can be translated as $x+(-4)=18$.
Solve for x.
$x+(-4)=18$
$x=22$ Add 4.
The number is 22.

17. The sentence "if the product of a number and -4 is subtracted from the number, the result is 9 more than the number" can be translated as $x-(-4x)=x+9$.
Solve for x.
$x-(-4x)=x+9$
$x+4x=x+9$
$4x=9$
$x=\frac{9}{4}$
The number is $\frac{9}{4}$.

18. The sentence "if the quotient of a number and 6 is added to twice the number, the result is 8 less than the number" can be translated as
$2x+\frac{x}{6}=x-8$.
Solve for x.
$2x+\frac{x}{6}=x-8$
$12x+x=6x-48$ Multiply by 6.
$13x=6x-48$
$7x=-48$ Subtract $6x$.
$x=-\frac{48}{7}$ Divide by 7.
The number is $-\frac{48}{7}$.

19. The sentence "when $\frac{2}{3}$ of a number is subtracted from 12, the result is 10" can be translated as $12-\frac{2}{3}x=10$.
Solve for x.
$12-\frac{2}{3}x=10$
$36-2x=30$ Multiply by 3.
$-2x=-6$ Subtract 36.
$x=3$ Divide by -2.
The number is 3.

20. The sentence "when 75% of a number is added to 6, the result is 3 more than the number" can be translated as $6+0.75x=x+3$.
Solve for x.
$6+0.75x=x+3$
$600+75x=100x+300$ Multiply by 100.
$600-25x=300$ Subtract $100x$.
$-25x=-300$ Subtract 600.
$x=12$ Divide by -25.
The number is 12.

21. $5(x+3)-8(2x-6)$ is an *expression* because there is no equality symbol.
$5(x+3)-8(2x-6)$
$=5x+15-16x+48$ Distributive property
$=-11x+63$ Combine like terms.

22. $-7(x+4)+13(x-6)$ has no equality symbol, so it is an *expression*.
$-7(x+4)+13(x-6)$
$=-7x-28+13x-78$ Distributive prop.
$=6x-106$ Combine like terms.

23. $5(x+3)-8(2x-6)=12$ has an equality symbol, so this represents an *equation.*

$$\begin{aligned} 5(x+3)-8(2x-6)&=12 \\ 5x+15-16x+48&=12 && \text{Distributive prop.} \\ -11x+63&=12 && \text{Combine terms.} \\ -11x&=-51 && \text{Subtract 63.} \\ x&=\frac{51}{11} && \text{Divide by } -11. \end{aligned}$$

The solution set is $\left\{\frac{51}{11}\right\}$.

24. $-7(x+4)+13(x-6)=18$ has an equality symbol, so it is an *equation.*

$$\begin{aligned} -7(x+4)+13(x-6)&=18 \\ -7x-28+13x-78&=18 \\ 6x-106&=18 \\ 6x&=124 \\ x&=\frac{124}{6}=\frac{62}{3} \end{aligned}$$

The solution set is $\left\{\frac{62}{3}\right\}$.

25. $\frac{1}{2}x-\frac{1}{6}x+\frac{3}{2}-8$ is an *expression* because there is no equality symbol.

$$\begin{aligned} &\frac{1}{2}x-\frac{1}{6}x+\frac{3}{2}-8 \\ &=\frac{3}{6}x-\frac{1}{6}x+\frac{3}{2}-\frac{16}{2} && \text{Common denom.} \\ &=\frac{2}{6}x-\frac{13}{2} && \text{Combine like terms.} \\ &=\frac{1}{3}x-\frac{13}{2} && \text{Reduce.} \end{aligned}$$

26. $\frac{1}{3}x+\frac{1}{5}x-\frac{1}{2}+7$ is an *expression* because there is no equality symbol.

$$\begin{aligned} &\frac{1}{3}x+\frac{1}{5}x-\frac{1}{2}+7 \\ &=\frac{5}{15}x+\frac{3}{15}x-\frac{1}{2}+\frac{14}{2} && \text{Common denom.} \\ &=\frac{8}{15}x+\frac{13}{2} && \text{Combine like terms.} \end{aligned}$$

27. *Step 1*
We are asked to find the number of patents each corporation secured.

Step 2
Let $x=$ the number of patents IBM secured.
Then $x-2316$ = the number of patents that Samsung secured.

Step 3
A total of 15,048 patents were secured, so
$\underline{x}+\underline{x-2316}=15{,}048$

Step 4
$$\begin{aligned} 2x-2316&=15{,}048 \\ 2x&=17{,}364 \\ x&=\underline{8682} \end{aligned}$$

Step 5
IBM secured 8682 patents and Samsung secured $8682-2316=\underline{6366}$ patents.

Step 6
The number of Samsung patents was 2316 fewer than the number of IBM patents. The total number of patents was
$8682+\underline{6366}=\underline{15{,}048}$.

28. *Step 1*
We are asked to find the number of travelers to each country.

Step 2
Let $x=$ the number of travelers to Canada (in millions). Then $x+7.9=$ the number of travelers to Mexico.

Step 3
A total of 12.9 million U.S. residents traveled to Mexico and Canada, so
$$\underline{x}+\underline{(x+7.9)}=12.9.$$

Step 4
$$\begin{aligned} 2x+7.9&=12.9 \\ 2x&=5 \\ x&=\underline{2.5} \end{aligned}$$

Step 5
There were 2.5 million travelers to Canada and $2.5+7.9=\underline{10.4\text{ million}}$ travelers to Mexico.

Step 6
The number of travelers to Mexico was 7.9 million more than the number of travelers to Canada. The total number of travelers was
$2.5+\underline{10.4}=\underline{12.9\text{ million}}$.

29. *Step 2*
Let $W=$ the width of the base. Then $2W-65$ is the length of the base.

Step 3
The perimeter of the base is 860 feet. Using $P=2L+2W$ gives us
$$2(2W-65)+2W=860.$$

Step 4

$$4W - 130 + 2W = 860$$
$$6W - 130 = 860$$
$$6W = 990$$
$$W = \frac{990}{6} = 165$$

Step 5
The width of the base is 165 feet and the length of the base is $2(165) - 65 = 265$ feet.

Step 6
$2L + 2W = 2(265) + 2(165) = 530 + 330 = 860,$
which is the perimeter of the base.

30. *Step 2*
Let $L =$ the length of the top floor. Then
$\frac{1}{2}L + 20$ is the width of the top floor.
Step 3
The perimeter of the top floor is 520 feet. Using $P = 2L + 2W$ gives us
$2L + 2\left(\frac{1}{2}L + 20\right) = 520.$

Step 4

$$2L + L + 40 = 520$$
$$3L + 40 = 520$$
$$3L = 480$$
$$L = 160$$

Step 5
The length of the top floor is 160 feet and the width of the top floor is
$\frac{1}{2}(160) + 20 = 100$ feet.
Step 6
$2L + 2W = 2(160) + 2(100) = 320 + 200 = 520,$
which is the perimeter of the top floor.

31. *Step 2*
Let x = the width of the painting. Then
$x + 5.54$ = the height of the painting.
Step 3
The perimeter of the painting is 108.44 inches. Using $P = 2L + 2W$ gives us
$2(x + 5.54) + 2x = 108.44.$

Step 4

$$2x + 11.08 + 2x = 108.44$$
$$4x + 11.08 = 108.44$$
$$4x = 97.36$$
$$x = 24.34$$

Step 5
The width of the painting is 24.34 inches and the height is $24.34 + 5.54 = 29.88$ inches.
Step 6
29.88 is 5.54 more than 24.34 and
$2(29.88) + 2(24.34) = 108.44,$ as required.

32. *Step 2*
Let W = the width of the rectangle. Then
$W + 12$ = the length of the rectangle.
Step 3
The perimeter, $2L + 2W$, is 16 times the width, so $2(W + 12) + 2W = 16(W)$.

Step 4

$$2W + 24 + 2W = 16W$$
$$4W + 24 = 16W$$
$$24 = 12W$$
$$2 = W$$

Step 5
The width of the rectangle is 2 centimeters. The length is $2 + 12 = 14$ centimeters.
Step 6
The length is 12 more than the width. The perimeter is $2(14) + 2(2) = 32,$ which is 16 times the width, as required.

33. *Step 2*
Let x = the length of the middle side. Then the shortest side is $x - 75$ and the longest side is $x + 375$.
Step 3
The perimeter of the Bermuda Triangle is 3075 miles. Using $P = a + b + c$ gives us
$x + (x - 75) + (x + 375) = 3075.$

Step 4

$$3x + 300 = 3075$$
$$3x = 2775 \quad \text{Subtract 300.}$$
$$x = 925 \quad \text{Divide by 3.}$$

Step 5
The length of the middle side is 925 miles. The length of the shortest side is
$x - 75 = 925 - 75 = 850$ miles. The length of the longest side is
$x + 375 = 925 + 375 = 1300$ miles.
Step 6
The answer checks because
$925 + 850 + 1300 = 3075$ miles, which is the correct perimeter.

34. *Step 2*
Let x = the length of one of the sides of equal length.

Step 3
The perimeter of the triangle is 931.5 feet.
Using $P = a + b + c$ gives us
$x + x + 438 = 931.5.$
Step 4
$$\begin{aligned} 2x + 438 &= 931.5 \\ 2x &= 493.5 \quad \text{Subtract 438.} \\ x &= 246.75 \quad \text{Divide by 2.} \end{aligned}$$
Step 5
The two walls are each 246.75 feet long.
Step 6
The answer checks because
$246.75 + 246.75 + 438 = 931.5,$ which is the correct perimeter.

35. *Step 2*
Let x = the amount of revenue for Walmart. Then $x - 103$ is the amount of revenue for Amazon (in billions).
Step 3
The total revenue was \$1043 billion, so
$x + (x - 103) = 1043.$
Step 4
$$\begin{aligned} 2x - 103 &= 1043 \\ 2x &= 1146 \quad \text{Add 103.} \\ x &= 573 \quad \text{Divide by 2.} \end{aligned}$$
Step 5
The amount of revenue for Walmart was \$573 billion. The amount of revenue for Amazon was $x - 103 = 573 - 103 = \$470$ billion.
Step 6
The answer checks because $573 + 470 = \$1043$ billion, which is the correct total revenue.

36. *Step 2*
Let x = the number of performances of *Phantom.* Then $x - 3670 =$ the number of performances of *Chicago.*
Step 3
There were 23,070 total performances, so
$x + (x - 3670) = 23,070.$
Step 4
$$\begin{aligned} 2x - 3670 &= 23,070 \\ 2x &= 26,740 \\ x &= 13,370 \end{aligned}$$
Step 5
There were 13,370 performances of *Phantom* and $13,370 - 3670 = 9700$ performances of *Chicago.*
Step 6
The total number of performances is 23,070, as required.

37. *Step 2*
Let $x =$ the height of the Eiffel Tower.
Then $x - 804 =$ the height of the Leaning Tower of Pisa.
Step 3
Together these heights are 1164 ft, so
$x + (x - 804) = 1164.$
Step 4
$$\begin{aligned} 2x - 804 &= 1164 \\ 2x &= 1968 \\ x &= 984 \end{aligned}$$
Step 5
The height of the Eiffel Tower is 984 feet and the height of the Leaning Tower of Pisa is $984 - 804 = 180$ feet.
Step 6
180 feet is 804 feet shorter than 984 feet and the sum of 180 feet and 984 feet is 1164 feet.

38. *Step 2*
Let $x =$ the Mets' payroll (in millions).
Then $x + 24 =$ the Dodgers' payroll (in millions).
Step 3
The two payrolls totaled \$530.2 million, so
$x + (x + 24) = 530.2$
Step 4
$$\begin{aligned} 2x + 24 &= 530.2 \\ 2x &= 506.2 \\ x &= 253.1 \end{aligned}$$
Step 5
In 2017, the Mets' payroll was \$253.1 million and the Dodgers' payroll was $253.1 + 24 = \$277.1$ million.
Step 6
\$277.1 million is \$24 million more than \$253.1 million and the sum of \$277.1 million and \$253.1 million is \$530.2 million.

39. *Step 2*
Let $x =$ number of followers for Nike (in millions). Then $x + 6 =$ number of followers for National Geographic (in millions).
Step 3
There were 506 million total followers, so
$x + (x + 6) = 506.$
Step 4
$$\begin{aligned} 2x + 6 &= 506 \\ 2x &= 500 \\ x &= 250 \end{aligned}$$
Step 5
Nike had 250 million followers, so National Geographic had $250 + 6 = 256$ million followers.

Step 6
256 million is 6 million more than 250 million, and the total is $256+250=506$ million followers.

40. *Step 2*
Let $x=$ the number of hits Williams got. Then $x+276=$ the number of hits Hornsby got.
Step 3
Their base hits totaled 5584, so $x+(x+276)=5584$.
Step 4
$$\begin{aligned} 2x+276&=5584\\ 2x&=5308\\ x&=2654 \end{aligned}$$
Step 5
Williams got 2654 base hits, and Hornsby got $2654+276=2930$ base hits.
Step 6
2930 is 276 more than 2654 and the total is $2654+2930=5584$.

41. Let $x=$ the percent increase.
$$\begin{aligned} x&=\frac{\text{amount of increase}}{\text{original amount}}\\ &=\frac{9.8-8}{8}\\ &=\frac{1.8}{8}\\ &=0.225 \end{aligned}$$
The percent increase is 22.5%.

42. Let $x=$ the percent increase.
$$\begin{aligned} x&=\frac{\text{amount of increase}}{\text{original amount}}\\ &=\frac{405-332}{332}\\ &=\frac{73}{332}\\ &\approx 0.219 \end{aligned}$$
The percent increase is approximately 22.0%.

43. Let $x=$ the approximate number of users in 2022. Since x is 185.7% more than the 2015 number, solve the equation below to find the number of users in 2022.
$$\begin{aligned} x&=700+1.857(700)\\ &=700+1299.9\\ &=1999.9 \end{aligned}$$
To the nearest million, WhatsApp had 2000 million users in 2022.

44. Let $x=$ the payroll in 2022. Since x is 12.4% more than the 2020 payroll, solve the equation below to find the payroll in 2022.
$$\begin{aligned} x&=37.6+0.124(37.6)\\ &=37.6+4.6624\\ &\approx 42.3 \end{aligned}$$
To the nearest tenth of a million, the payroll was $42.3 million in 2022.

45. Let $x=$ the percent decrease.
$$\begin{aligned} x&=\frac{\text{amount of decrease}}{\text{original amount}}\\ &=\frac{16{,}000{,}000-350{,}000}{16{,}000{,}000}\\ &=\frac{15{,}650{,}000}{16{,}000{,}000}\\ &\approx 0.978 \end{aligned}$$
The percent decrease is approximately 97.8%.

46. Let $x=$ the percent decrease.
$$\begin{aligned} x&=\frac{\text{amount of decrease}}{\text{original amount}}\\ &=\frac{84.5-68}{84.5}\\ &=\frac{16.5}{84.5}\\ &\approx 0.195 \end{aligned}$$
The percent decrease is approximately 19.5%.

47. Let $x=$ the amount of the receipts excluding tax. Since the sales tax is 9% of x, solve the equation below to find the amount of the tax.
$$\begin{aligned} x+0.09x&=2725\\ 1x+0.09x&=2725\\ 1.09x&=2725\\ x+\frac{2725}{1.09}&=2500 \end{aligned}$$
Thus, the tax was $0.09(2500)=\$225$.

48. Let $x=$ the amount of commission. Since x is 6% of the selling price, $x=0.06(159{,}000)=9540$.
So after the real estate agent was paid, they had $159{,}000-9540=\$149{,}460$.

49. We cannot expect the final mixture to be worth more than the more expensive of the two ingredients.

50. Let $x =$ the number of liters of 30% acid solution. Make a chart.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
x	0.30	$0.30x$
15	0.50	$0.50(15) = 7.5$
$x+15$	0.60	$0.60(x+15)$

$$\begin{array}{ccccc} \text{Acid in 30\%} & + & \text{acid in 50\%} & = & \text{acid in 60\%} \\ \text{solution} & & \text{solution} & & \text{solution.} \\ 0.30x & + & 7.5 & = & 0.60(x+15) \end{array}$$

Solve the equation.

$$\begin{aligned} 3x+75 &= 6x+90 && \text{Multiply by 10.} \\ -3x &= 15 && \text{Subtract } 6x \text{ and 75.} \\ x &= -5 && \text{Divide by } -3. \end{aligned}$$

The solution, -5, is impossible since the number of liters of 30% acid solution cannot be negative. Therefore, this problem has no solution.

51. Let $x =$ the amount invested at 3%. Then $12{,}000 - x =$ the amount invested at 4%. Complete the table. Use $I = prt$ with $t = 1$.

Principal	Rate (as a decimal)	Interest
x	0.03	$0.03x$
$12{,}000 - x$	0.04	$0.04(12{,}000 - x)$
12,000	← Totals →	440

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest} & + & \text{interest} & = & \text{total} \\ \text{at 3\%} & & \text{at 4\%} & & \text{interest.} \\ 0.03x & + & 0.04(12{,}000-x) & = & 440 \end{array}$$

Multiply by 100.

$$\begin{aligned} 3x + 4(12{,}000 - x) &= 44{,}000 \\ 3x + 48{,}000 - 4x &= 44{,}000 \\ -x &= -4000 \\ x &= 4000 \end{aligned}$$

He should invest \$4000 at 3% and $12{,}000 - 4000 = \$8000$ at 4%.

Check

\$4000 at 3% = \$120 and

\$8000 at 4% = \$320; \$120 + \$320 = \$440.

52. Let $x =$ the amount invested at 2%. Then $60{,}000 - x =$ the amount invested at 3%. Complete the table. Use $I = prt$ with $t = 1$.

Principal	Rate (as a decimal)	Interest
x	0.02	$0.02x$
$60{,}000 - x$	0.03	$0.03(60{,}000 - x)$
60,000	← Totals →	1600

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest} & + & \text{interest} & = & \text{total} \\ \text{at 2\%} & & \text{at 3\%} & & \text{interest.} \\ 0.02x & + & 0.03(60{,}000-x) & = & 1600 \end{array}$$

Multiply by 100.

$$\begin{aligned} 2x + 3(60{,}000 - x) &= 160{,}000 \\ 2x + 180{,}000 - 3x &= 160{,}000 \\ -x &= -20{,}000 \\ x &= 20{,}000 \end{aligned}$$

She invested \$20,000 at 2% and $60{,}000 - x = 60{,}000 - 20{,}000 = \$40{,}000$ at 3%.

Check

\$20,000 at 2% = \$400 and

\$40,000 at 3% = \$1200; \$400 + \$1200 = \$1600.

53. Let $x =$ the amount invested at 4.5%. Then $2x - 1000 =$ the amount invested at 3%. Use $I = prt$ with $t = 1$. Make a table.

Principal	Rate (as a decimal)	Interest
x	0.045	$0.045x$
$2x - 1000$	0.03	$0.03(2x - 1000)$
	Total →	1020

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest} & + & \text{interest} & = & \text{total} \\ \text{at 4.5\%} & & \text{at 3\%} & & \text{interest.} \\ 0.045x & + & 0.03(2x-1000) & = & 1020 \end{array}$$

Multiply by 1000.

$$\begin{aligned} 45x + 30(2x - 1000) &= 1{,}020{,}000 \\ 45x + 60x - 30{,}000 &= 1{,}020{,}000 \\ 105x &= 1{,}050{,}000 \\ x &= \frac{1{,}050{,}000}{105} = 10{,}000 \end{aligned}$$

She invested \$10,000 at 4.5% and $2x - 1000 = 2(10{,}000) - 1000 = \$19{,}000$ at 3%.

Check

\$19,000 is \$1000 less than two times \$10,000.

\$10,000 at 4.5% = \$450

\$19,000 at 3% = \$570; \$450 + \$570 = \$1020.

54. Let $x =$ the amount invested at 3.5%. Then $3x + 5000 =$ the amount invested at 4%. Use $I = prt$ with $t = 1$. Make a table.

Principal	Rate (as a decimal)	Interest
x	0.035	$0.035x$
$3x + 5000$	0.04	$0.04(3x + 5000)$
	Total $\rightarrow$	1440

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest} & & \text{interest} & & \\ \text{at } 3.5\% & + & \text{at } 4\% & = & 1440. \\ 0.035x & + & 0.04(3x+5000) & = & 1440 \end{array}$$

Multiply by 1000.

$$35x + 40(3x + 5000) = 1{,}440{,}000$$
$$35x + 120x + 200{,}000 = 1{,}440{,}000$$
$$155x = 1{,}240{,}000$$
$$x = \frac{1{,}240{,}000}{155} = 8000$$

He invested \$8000 at 3.5% and $3x + 5000 = 3(8000) + 5000 = \$29{,}000$ at 4%.

Check

\$29,000 is \$5000 more than three times \$8000.

$\$8000$ at $3.5\% = \$280$ and

$\$29{,}000$ at $4\% = \$1160$; $\$280 + \$1160 = \$1440$.

55. Let $x =$ the amount of additional money to be invested at 3%.

Use $I = prt$ with $t = 1$. Make a table.

Use the fact that the yearly return on the two investments is 4%.

Principal	Rate (as a decimal)	Interest
12,000	0.06	0.06(12,000)
x	0.03	$0.03x$
$12{,}000 + x$	0.04	$0.04(12{,}000 + x)$

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest} & & \text{interest} & & \text{interest} \\ \text{at } 6\% & + & \text{at } 3\% & = & \text{at } 4\%. \\ 0.06(12{,}000) & + & 0.03x & = & 0.04(12{,}000+x) \end{array}$$

Multiply by 100.

$$6(12{,}000) + 3x = 4(12{,}000 + x)$$
$$72{,}000 + 3x = 48{,}000 + 4x$$
$$24{,}000 = x$$

\$24,000 should be invested at 3%.

Check

$\$12{,}000$ at $6\% = \$720$ and

$\$24{,}000$ at $3\% = \$720$; $\$720 + \$720 = \$1440$, which is the same as $(\$12{,}000 + \$24{,}000)$ at 4%.

56. Let $x =$ the amount of additional money to be invested at 5%.

Use $I = prt$ with $t = 1$. Make a table. Use the fact that the yearly return on the two investments is 6%.

Principal	Rate (as a decimal)	Interest
17,000	0.065	0.065(17,000)
x	0.05	$0.05x$
$17{,}000 + x$	0.06	$0.06(17{,}000 + x)$

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest} & & \text{interest} & & \text{interest} \\ \text{at } 6.5\% & + & \text{at } 5\% & = & \text{at } 6\% \\ 0.065(17{,}000) & + & 0.05x & = & 0.06(17{,}000+x) \end{array}$$

Multiply by 1000.

$$65(17{,}000) + 50x = 60(17{,}000 + x)$$
$$1{,}105{,}000 + 50x = 1{,}020{,}000 + 60x$$
$$85{,}000 = 10x$$
$$8500 = x$$

They should deposit \$8500 at 6%.

Check

$\$17{,}000$ at $6.5\% = \$1105$ and

$\$8500$ at $5\% = \$425$;

$\$1105 + \$425 = \$1530$, which is the same as $(\$17{,}000 + \$8500)$ at 6%.

57. Let $x =$ the number of liters of 10% acid solution needed. Make a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
10	0.04	$0.04(10) = 0.4$
x	0.10	$0.10x$
$x + 10$	0.06	$0.06(x + 10)$

$$\begin{array}{ccccc} \text{Acid in } 4\% & & \text{acid in } 10\% & & \text{acid in } 6\% \\ \text{solution} & + & \text{solution} & = & \text{solution.} \\ 0.4 & + & 0.10x & = & 0.06(x+10) \end{array}$$

Solve the equation.

$0.4 + 0.10x = 0.06x + 0.6$	Distributive prop.
$0.4 + 0.04x = 0.6$	Subtract $0.06x$.
$0.04x = 0.2$	Subtract 0.4.
$x = 5$	Divide by 0.04.

Five liters of the 10% solution are needed.

Check
4% of 10 is 0.4 and 10% of 5 is 0.5;
$0.4+0.5=0.9,$ which is the same as 6% of $(10+5)$.

58. Let $x=$ the number of liters of 14% alcohol solution needed. Make a chart.

Liters of Solution	Percent (as a decimal)	Liters of Pure Alcohol
x	0.14	$0.14x$
20	0.50	$0.50(20)=10$
$x+20$	0.30	$0.30(x+20)$

$$\begin{array}{ccccc} \text{Alcohol in} & & \text{alcohol in} & & \text{alcohol in} \\ \text{14\% solution} & + & \text{50\% solution} & = & \text{30\% solution.} \\ 0.14x & + & 10 & = & 0.30(x+20) \end{array}$$

Solve the equation.

$$\begin{array}{rll} 14x+1000=30(x+20) & \text{Multiply by } 100. \\ 14x+1000=30x+600 & \text{Distributive property} \\ -16x=-400 & \text{Subtract 30x and } 1000. \\ x=25 & \text{Divide by } -16. \end{array}$$

25 L of 14% solution must be added.
Check
14% of 25 is 3.5 and 50% of 20 is 10;
$3.5+10=13.5,$ which is the same as 30% of $(25+20)$.

59. Let $x=$ the number of liters of the 20% alcohol solution. Make a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Alcohol
12	0.12	$0.12(12)=1.44$
x	0.20	$0.20x$
$x+12$	0.14	$0.14(x+12)$

$$\begin{array}{ccccc} \text{Alcohol in} & & \text{alcohol in} & & \text{alcohol in} \\ \text{12\% solution} & + & \text{20\% solution} & = & \text{14\% solution.} \\ 1.44 & + & 0.20x & = & 0.14(x+12) \end{array}$$

Solve the equation.

$$\begin{array}{rll} 144+20x=14(x+12) & \text{Multiply by } 100. \\ 144+20x=14x+168 & \text{Distributive property} \\ 6x=24 & \text{Subtract } 14x \text{ and } 144. \\ x=4 & \text{Divide by } 6. \end{array}$$

4L of 20% alcohol solution are needed.
Check
12% of 12 is 1.44 and 20% of 4 is 0.8;
$1.44+0.8=2.24,$ which is the same as 14% of $(12+4)$.

60. Let $x=$ the number of liters of 10% alcohol solution. Make a chart.

Liters of Solution	Percent (as a decimal)	Liters of Pure Alcohol
x	0.10	$0.10x$
40	0.50	$0.50(40)=20$
$x+40$	0.40	$0.40(x+40)$

$$\begin{array}{ccccc} \text{Alcohol in} & & \text{alcohol in} & & \text{alcohol in} \\ \text{10\% soln} & + & \text{50\% soln} & = & \text{40\% soln.} \\ 0.10x & + & 20 & = & 0.40(x+40) \end{array}$$

Solve the equation.

$$\begin{array}{rll} 1x+200=4(x+40) & \text{Multiply by } 10. \\ x+200=4x+160 & \text{Distributive property} \\ -3x=-40 & \text{Subtract } 4x \text{ and } 200. \\ x=\frac{40}{3} \text{ or } 13\frac{1}{3} & \text{Divide by } -3. \end{array}$$

$13\frac{1}{3}$ L of 10% solution should be added.
Check
50% of 40 is 20 and 10% of $\frac{40}{3}$ is $\frac{4}{3}$;
$20+\frac{4}{3}=21\frac{1}{3},$ which is the same as 40% of $\left(\frac{40}{3}+40\right)$.

61. Let $x=$ the amount of pure dye used (pure dye is 100% dye). Make a table.

Gallons of Solution	Percent (as a decimal)	Gallons of Pure Dye
x	1	$1x=x$
4	0.25	$0.25(4)=1$
$x+4$	0.40	$0.40(x+4)$

Write the equation from the last column in the table.
$x+1=0.4(x+4)$
Solve the equation.

$$\begin{array}{rll} x+1=0.4(x+4) & \\ x+1=0.4x+1.6 & \text{Distributive property} \\ 0.6x=0.6 & \text{Subtract 0.4x and } 1. \\ x=1 & \text{Divide by } 0.6. \end{array}$$

One gallon of pure (100%) dye is needed.
Check
100% of 1 is 1 and 25% of 4 is 1; $1+1=2,$ which is the same as 40% of $(1+4)$.

62. Let $x =$ the number of gallons of water. Make a chart.

Gallons of Solution	Percent (as a decimal)	Gallons of Pure Insecticide
x	0	$0(x) = 0$
6	0.04	$0.04(6) = 0.24$
$x + 6$	0.03	$0.03(x + 6)$

$$\begin{array}{ccccc} \text{Insecticide} & & \text{insecticide} & & \text{insecticide} \\ \text{in water} & + & \text{in 4\% solution} & = & \text{in 3\% solution.} \\ 0 & + & 0.24 & = & 0.03(x+6) \end{array}$$

Solve the equation.

$0 + 24 = 3(x + 6)$ Multiply by 100.

$24 = 3x + 18$ Distributive property

$6 = 3x$ Subtract 18.

$2 = x$ Divide by 3.

2 gallons of water should be added.
Check
4% of 6 is 0.24, which is the same as 3% of $(2 + 6)$.

63. Let $x =$ the amount of \$6 per lb nuts. Make a table.

Pounds of Nuts	Cost per lb	Total Cost
50	\$2	$2(50) = 100$
x	\$6	$6x$
$x + 50$	\$5	$5(x + 50)$

The total value of the \$2 per lb nuts and the \$6 per lb nuts must equal the value of the \$5 per lb nuts.

$100 + 6x = 5(x + 50)$

Solve the equation.

$100 + 6x = 5x + 250$

$x = 150$

The retailer should use 150 lb of \$6 nuts.
Check
50 pounds of the \$2 per lb nuts are worth \$100 and 150 pounds of the \$6 per lb nuts are worth \$900; $\$100 + \$900 = \$1000$, which is the same as $(50 + 150)$ pounds worth \$5 per lb.

64. Let $x =$ the number of ounces of 2¢ per oz tea. Make a table.

Ounces of Tea	Cost per oz	Total Cost
x	2¢ or 0.02	$0.02x$
100	5¢ or 0.05	$0.05(100) = 5$
$x + 100$	3¢ or 0.03	$0.03(x + 100)$

$$\begin{array}{ccccc} \text{Cost of} & & \text{cost of} & & \text{cost of} \\ \text{2¢ tea} & + & \text{5¢ tea} & = & \text{3¢ tea.} \\ 0.02x & + & 5 & = & 0.03(x+100) \end{array}$$

Solve the equation.

$2x + 500 = 3(x + 100)$ Multiply by 100.

$2x + 500 = 3x + 300$ Distributive property

$200 = x$ Subtract $2x$ and 300.

200 oz of 2¢ per oz tea should be used.
Check
200 oz of 2¢ per oz tea is worth \$4 and 100 oz of 50 per oz tea is worth \$5; $\$4 + \$5 = \$9$, which is the same value as $(200 + 100)$ oz of 3¢ per oz tea.

65. (a) Let $x =$ the amount invested at 3%.
$800 - x =$ the amount invested at 6%.

(b) Let $y =$ the amount of 3% acid used.
$800 - y =$ the amount of 6% acid used.

66. (a) Organize the information in a table.

Principal	Percent (as a decimal)	Interest
x	0.03	$0.03x$
$800 - x$	0.06	$0.06(800 - x)$
800	0.0525	$0.0525(800)$

The amount of interest earned at 3% and 6% is found in the last column of the table, $0.03x$ and $0.06(800 - x)$.

(b) Organize the information in a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
y	0.03	$0.03y$
$800 - y$	0.06	$0.06(800 - y)$
800	0.0525	$0.0525(800)$

The amount of pure acid in the 3% and 6% mixtures is found in the last column of the table, $0.03y$ and $0.06(800 - y)$.

67. Refer to the tables for Exercise 66. In each case, the last column gives the equation.

(a) $0.03x + 0.06(800 - x) = 0.0525(800)$

(b) $0.03y + 0.06(800 - y) = 0.0525(800)$

68. In both cases, multiply by 10,000 to clear the decimals.

(a)
$$\begin{aligned} 0.03x + 0.06(800 - x) &= 0.0525(800) \\ 300x + 600(800 - x) &= 525(800) \\ 300x + 480{,}000 - 600x &= 420{,}000 \\ -300x &= -60{,}000 \\ x &= 200 \end{aligned}$$

Jack invested \$200 at 3% and $800 - x = 800 - 200 = \$600$ at 6%.

(b)
$$\begin{aligned} 0.03y + 0.06(800 - y) &= 0.0525(800) \\ 300y + 600(800 - y) &= 525(800) \\ 300y + 480{,}000 - 600y &= 420{,}000 \\ -300y &= -60{,}000 \\ y &= 200 \end{aligned}$$

Jill used 200 L of 3% acid solution and $800 - y = 800 - 200 = 600$ L of 6% acid solution.

(c) The processes used to solve Problems A and B were virtually the same. Aside from the variables chosen, the problem information was organized in similar tables and the equations solved were the same.

1.4 Further Applications of Linear Equations

Classroom Examples, Now Try Exercises

1. Let $x =$ the number of dimes.
Then $26 - x =$ the number of half-dollars.

Number of Coins	Denomination	Value
x	0.10	$0.10x$
$26 - x$	0.50	$0.50(26 - x)$
	Total →	8.60

Multiply the number of coins by the denominations, and add the results to get 8.60.

$$\begin{aligned} 0.10x + 0.50(26 - x) &= 8.60 \\ 1x + 5(26 - x) &= 86 \quad \text{Multiply by 10.} \\ 1x + 130 - 5x &= 86 \\ -4x &= -44 \\ x &= 11 \end{aligned}$$

The child has 11 dimes and $26 - 11 = 15$ half-dollars.

Check
The number of coins is $11 + 15 = 26$ and the value of the coins is $\$0.10(11) + \$0.50(15) = \$8.60$, as required.

N1. Let $x =$ the number of dimes.
Then $52 - x =$ the number of nickels.

Number of Coins	Denomination	Value
x	0.10	$0.10x$
$52 - x$	0.05	$0.05(52 - x)$
	Total →	3.70

Multiply the number of coins by the denominations, and add the results to get 3.70.

$$\begin{aligned} 0.10x + 0.05(52 - x) &= 3.70 \\ 10x + 5(52 - x) &= 370 \quad \text{Multiply by 100.} \\ 10x + 260 - 5x &= 370 \\ 5x &= 110 \\ x &= 22 \end{aligned}$$

The cashier has 22 dimes and $52 - 22 = 30$ nickels.
Check
The number of coins is $22 + 30 = 52$ and the value of the coins is $\$0.10(22) + \$0.05(30) = \$3.70$, as required.

2. Let $x =$ the amount of time needed for the cars to be 420 mi apart.
Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
Northbound Car	60	x	$60x$
Southbound Car	45	x	$45x$
Total			420

The total distance traveled is the sum of the distances traveled by each car, since they are traveling in opposite directions. This total is 420 mi.

$$\begin{aligned} 60x + 45x &= 420 \\ 105x &= 420 \\ x &= \frac{420}{105} = 4 \end{aligned}$$

The cars will be 420 mi apart in 4 hr.
Check
The northbound car traveled $60(4) = 240$ miles. The southbound car traveled $45(4) = 180$ miles, for a total of $240 + 180 = 420$, as required.

N2. Let $x =$ the amount of time needed for the trains to be 387.5 km apart.
Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
First Train	80	x	$80x$
Second Train	75	x	$75x$
Total			387.5

The total distance traveled is the sum of the distances traveled by each train, since they are traveling in opposite directions. This total is 387.5 km.

$$80x + 75x = 387.5$$
$$155x = 387.5$$
$$x = \frac{387.5}{155} = 2.5$$

The trains will be 387.5 km apart in 2.5, or $2\frac{1}{2}$ hr.

Check
The first train traveled $80(2.5) = 200$ km. The second train traveled $75(2.5) = 187.5$ km, for a total of $200 + 187.5 = 387.5$, as required.

3. Let $x =$ the driving rate.
Then $x - 12 =$ the bus rate.
Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
Car	x	$\frac{1}{2}$	$\frac{1}{2}x$
Bus	$x-12$	$\frac{3}{4}$	$\frac{3}{4}(x-12)$

The distances are equal.

$$\frac{1}{2}x = \frac{3}{4}(x-12)$$
$$2x = 3(x-12) \quad \text{Multiply by 4.}$$
$$2x = 3x - 36$$
$$36 = x$$

The distance traveled to work is

$$\frac{1}{2}x = \frac{1}{2}(36) = 18 \text{ miles.}$$

Check
The distance traveled to work by bus is

$$\frac{3}{4}(x-12) = \frac{3}{4}(36-12) = \frac{3}{4}(24) = 18 \text{ miles,}$$

which is the same as we found above (by car).

N3. Let $x =$ the driving rate.
Then $x - 30 =$ the bicycling rate.
Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
Car	x	$\frac{1}{2}$	$\frac{1}{2}x$
Bike	$x-30$	$1\frac{1}{2} = \frac{3}{2}$	$\frac{3}{2}(x-30)$

The distances are equal.

$$\frac{1}{2}x = \frac{3}{2}(x-30)$$
$$1x = 3(x-30) \quad \text{Multiply by 2.}$$
$$x = 3x - 90$$
$$90 = 2x$$
$$45 = x$$

The distance the accountant travels to work is

$$\frac{1}{2}x = \frac{1}{2}(45) = 22.5 \text{ miles.}$$

Check
The distance the accountant travels to work by bike is

$$\frac{3}{2}(x-30) = \frac{3}{2}(45-30) = \frac{3}{2}(15) = 22.5 \text{ miles,}$$

which is the same as we found above (by car).

4. The sum of the three measures must equal 180°.

$$x + (x+61) + (2x+7) = 180$$
$$4x + 68 = 180$$
$$4x = 112$$
$$x = 28$$

The angles measure $28°$, $28 + 61 = 89°$, and $2(28) + 7 = 63°$.

Check
Since $28° + 89° + 63° = 180°$, the answers are correct.

N4. The sum of the three measures must equal 180°.

$$x + (x+11) + (3x-36) = 180$$
$$5x - 25 = 180$$
$$5x = 205$$
$$x = 41$$

The angles measure $41°$, $41 + 11 = 52°$, and $3(41) - 36 = 87°$.

Check
Since $41° + 52° + 87° = 180°$, the answers are correct.

5. $x =$ the first of the three consecutive integers
$x+1 =$ the second integer
$x+2 =$ the third integer
Write the sum of the first and third numbers, $x+(x+2)$, and set it equal to 49 less than 3 times the second.

$$\begin{aligned} x+(x+2) &= 3(x+1)-49 \\ 2x+2 &= 3x+3-49 \\ 2x+2 &= 3x-46 \\ 2 &= x-46 \\ 48 &= x \end{aligned}$$

The three consecutive numbers are 48, 49, and 50.
Check
The sum of the first and third consecutive integers is $48+50=98$.
49 less than 3 times the second, yields $3(49)-49=98,$ as desired.

N5. $x =$ the first of the three consecutive integers
$x+1 =$ the second integer
$x+2 =$ the third integer
Write the sum of the first and second numbers, $x+(x+1)$, and set it equal to 43 less than 3 times the third.

$$\begin{aligned} x+(x+1) &= 3(x+2)-43 \\ 2x+1 &= 3x+6-43 \\ 2x+1 &= 3x-37 \\ 1 &= x-37 \\ 38 &= x \end{aligned}$$

The three consecutive numbers are 38, 39, and 40.
Check
The sum of the first and second consecutive integers is $38+39=77$.
43 less than 3 times the third, yields $3(40)-43=77,$ as desired.

Exercises

1. The total amount is

$$\begin{aligned} 12(0.10)+18(0.25) &= 1.20+4.50 \\ &= \$5.70. \end{aligned}$$

2. Use $d=rt$, or $t=\frac{d}{r}$.
Substitute 7700 for *d* and 550 for *r*.

$$t=\frac{7700}{550}=14$$

Its travel time is 14 hours.

3. Use $d=rt$, or $r=\frac{d}{t}$. Substitute 300 for *d* and 10 for *t*.

$$r=\frac{300}{10}=30$$

Their rate was 30 mph.

4. Use $P=4s$ or $s=\frac{P}{4}$.
Substitute 160 for *P*.

$$s=\frac{160}{4}=40$$

The length of each side of the square is 40 in. This is also the length of each side of the equilateral triangle. To find the perimeter of the equilateral triangle, use $P=3s$.
Substitute 40 for *s*.

$$P=3(40)=120$$

The perimeter would be 120 inches.

5. Use $P=3s$ or $s=\frac{P}{3}$.
Substitute 27 for *P*.

$$s=\frac{27}{3}=9$$

The length of each side of the triangle is 9 in. This is also the length of each side of the square. To find the area of the square, use $A=s^2$.
Substitute 9 for *s*.

$$A=(9)^2=81$$

The area would be 81 square inches.

6. Use $A=\pi r^2$.
Substitute 25π for *A*.

$$\begin{aligned} 25\pi &= \pi r^2 \\ 25 &= r^2 \\ 5 &= r \end{aligned}$$

The length of the radius of the circle is 5 ft. This is also the length of each side of the square. To find the perimeter of the square, use $P=4s$.
Substitute 5 for *s*.

$$P=4(5)=20$$

The perimeter would be 20 feet.

7. The right side of the equation should be 180, not 90. The correct equation is

$$x+(x+1)+(x+2)=180$$

$$3x + 3 = 180$$
$$3x = 177$$
$$x = 59$$

The angles measure 59°, 60°, and 61°.

8. The number of dimes should be $21 - x$, not $x - 21$. The correct equation is

$$0.05x + 0.10(21 - x) = 14.5$$
$$0.05x + 2.1 - 0.10x = 1.45$$
$$-0.05x = -0.65$$
$$x = 13$$

There are 13 nickels and $21 - 13 = 8$ dimes.

9. Let $x =$ the number of pennies. Then x is also the number of dimes, and $44 - 2x$ is the number of quarters.

Number of Coins	Denomination	Value
x	0.01	$0.01x$
x	0.10	$0.10x$
$44 - 2x$	0.25	$0.25(44 - 2x)$
44	← Totals →	4.37

The sum of the values must equal the total value.

$$0.01x + 0.10x + 0.25(44 - 2x) = 4.37$$
$$x + 10x + 25(44 - 2x) = 437$$

Multiply both sides by 100.

$$x + 10x + 1100 - 50x = 437$$
$$-39x + 1100 = 437$$
$$-39x = -663$$
$$x = 17$$

There are 17 pennies, 17 dimes, and $44 - 2(17) = 10$ quarters.

Check

The number of coins is $17 + 17 + 10 = 44$ and the value of the coins is $\$0.01(17) + \$0.10(17) + \$0.25(10) = \$4.37$, as required.

10. Let $x =$ the number of nickels and the number of quarters. Then $2x$ is the number of half-dollars.

Number of Coins	Denomination	Value
x	0.05	$0.05x$
x	0.25	$0.25x$
$2x$	0.50	$0.50(2x)$
	Total →	2.60

The sum of the values must equal the total value.

$$0.05x + 0.25x + 0.50(2x) = 2.60$$
$$5x + 25x + 50(2x) = 260$$

Multiply both sides by 100.

$$5x + 25x + 100x = 260$$
$$130x = 260$$
$$x = 2$$

There were 2 nickels, 2 quarters, and $2(2) = 4$ half-dollars.

Check

The number of coins is $2 + 2 + 4 = 8$ and the value of the coins is $\$0.05(2) + \$0.25(2) + \$0.50(4) = \$2.60$, as required.

11. Let $x =$ the number of loonies. Then $37 - x$ is the number of toonies.

Number of Coins	Denomination	Value
x	1	$1x$
$37 - x$	2	$2(37 - x)$
37	← Totals →	51

The sum of the values must equal the total value.

$$1x + 2(37 - x) = 51$$
$$x + 74 - 2x = 51$$
$$-x + 74 = 51$$
$$23 = x$$

They have 23 loonies and $37 - 23 = 14$ toonies.

Check

The number of coins is $23 + 14 = 37$ and the value of the coins is $\$1(23) + \$2(14) = \$51$, as required.

12. Let $x =$ the number of \$1 bills. Then $119 - x$ is the number of \$5 bills.

Number of Bills	Denomination	Value
x	1	$1x$
$119 - x$	5	$5(119 - x)$
119	← Totals →	347

The sum of the values equals the total value.

$$1x + 5(119 - x) = 347$$
$$x + 595 - 5x = 347$$
$$-4x = -248$$
$$x = 62$$

The cash drawer has 62 \$1 bills and $119 - 62 = 57$ \$5 bills.

Check
Number of bills is $62+57=119$ and the value is $\$1(62)+\$5(57)=\$62+\$285=\$347,$ as required.

13. Let $x=$ the number of \$10 coins. Then $41-x$ is the number of \$20 coins.

Number of Coins	Denomination	Value
x	10	$10x$
$41-x$	20	$20(41-x)$
41	← Totals →	540

The sum of the values must equal the total value.

$$\begin{aligned} 10x+20(41-x)&=540\\ 10x+820-20x&=540\\ -10x&=-280\\ x&=28 \end{aligned}$$

The collector has 28 \$10 coins and $41-28=13$ \$20 coins.

Check
The number of coins is $28+13=41$ and the value of the coins is $\$10(28)+\$20(13)=\$540,$ as required.

14. Let $x=$ the number of two-cent pieces. Then $3x$ is the number of three-cent pieces.

Number of Coins	Denomination	Value
x	0.02	$0.02x$
$3x$	0.03	$0.03(3x)$
	Total →	2.42

The sum of the values must equal the total value.

$$\begin{aligned} 0.02x+0.03(3x)&=2.42\\ 2x+3(3x)&=242 \end{aligned}$$

Multiply both sides by 100.

$$\begin{aligned} 2x+9x&=242\\ 11x&=242\\ x&=22 \end{aligned}$$

The collector has 22 two-cent pieces and $3(22)=66$ three-cent pieces.

Check
66 is three times 22 and the value of the coins is $\$0.02(22)+\$0.03(66)=\$2.42,$ as required.

15. Let $x=$ the number of adult tickets sold. Then $1460-x=$ the number of student/senior tickets.

Cost of Ticket	Number Sold	Amount Collected
\$25	x	$25x$
\$19	$1460-x$	$19(1460-x)$
Totals	1460	\$32,972

Write the equation from the last column of the table.

$$\begin{aligned} 25x+19(1460-x)&=32{,}972\\ 25x+27{,}740-19x&=32{,}972\\ 6x&=5232\\ x&=872 \end{aligned}$$

872 adult tickets were sold; $1460-872=588$ student and senior tickets were sold.

Check
The number of tickets sold was $872+588=1460$ and the amount collected was

$$\begin{aligned} \$25(872)+\$19(588)&=\$21{,}800+\$11{,}172\\ &=\$32{,}972, \end{aligned}$$

as required.

16. Let $x=$ the number of student tickets sold. Then $480-x=$ the number of nonstudent tickets sold.

Cost of Ticket	Number Sold	Amount Collected
\$5	x	$5x$
\$8	$480-x$	$8(480-x)$
Totals	480	\$2895

Write the equation from the last column of the table.

$$\begin{aligned} 5x+8(480-x)&=2895\\ 5x+3840-8x&=2895\\ -3x&=-945\\ x&=315 \end{aligned}$$

315 student tickets were sold; $480-315=165$ nonstudent tickets were sold.

Check
The number of tickets sold was $315+165=480$ and the amount collected was $\$5(315)+\$8(165)=\$1575+\$1320=\$2895,$ as required.

17. $d=rt,$ so

$$r=\frac{d}{t}=\frac{100}{12.37}\approx 8.08$$

Her rate was about 8.08 m/sec.

18. $d = rt$, so

$$r = \frac{d}{t} = \frac{400}{51.46} \approx 7.77$$

Her rate was about 7.77 m/sec.

19. $d = rt$, so

$$r = \frac{d}{t} = \frac{200}{19.62} \approx 10.19$$

His rate was about 10.19 m/sec.

20. $d = rt$, so

$$r = \frac{d}{t} = \frac{400}{43.85} \approx 9.12$$

His rate was about 9.12 m/sec.

21. Let $t =$ the time until they are 110 mi apart. Use the formula $d = rt$. Complete the table.

	Rate	Time	Distance
First Steamer	22	t	$22t$
Second Steamer	22	t	$22t$
			110

The total distance traveled is the sum of the distances traveled by each steamer, since they are traveling in opposite directions. This total is 110 mi.

$$22t + 22t = 110$$
$$44t = 110$$
$$t = \frac{110}{44} = \frac{5}{2} \quad \text{or} \quad 2\frac{1}{2}$$

It will take them $2\frac{1}{2}$ hr.

Check

Each steamer traveled $22(2.5) = 55$ miles for a total of $2(55) = 110$ miles, as required.

22. Let $t =$ the time it takes for the trains to be 315 km apart. Use the formula $d = rt$. Complete the table.

	Rate	Time	Distance
First Train	85	t	$85t$
Second Train	95	t	$95t$
			315

The total distance traveled is the sum of the distances traveled by each train, since they are traveling in opposite directions. This total is 315 km.

$$85t + 95t = 315$$
$$180t = 315$$
$$t = \frac{315}{180} = \frac{7}{4} \quad \text{or} \quad 1\frac{3}{4}$$

It will take the trains $1\frac{3}{4}$ hr before they are 315 km apart.

Check

The first train traveled $85(1.75) = 148.75$ km and the second train traveled $95(1.75) = 166.25$. The sum is 315 km, as required.

23. Let $t =$ Mulder's time. Then $t - \frac{1}{2} =$ Scully's time.

	Rate	Time	Distance
Mulder	65	t	$65t$
Scully	68	$t - \frac{1}{2}$	$68\left(t - \frac{1}{2}\right)$

The distances are equal.

$$65t = 68\left(t - \frac{1}{2}\right)$$
$$65t = 68t - 34$$
$$-3t = -34$$
$$t = \frac{34}{3} \quad \text{or} \quad 11\frac{1}{3}$$

Mulder's time will be $11\frac{1}{3}$ hr. Since he left at 8:30 A.M., $11\frac{1}{3}$ hr or 11 hr 20 min later is 7:50 P.M.

Check

Mulder's distance was

$65\left(\frac{34}{3}\right) = 736\frac{2}{3}$ miles. Scully's distance was

$68\left(\frac{34}{3} - \frac{1}{2}\right) = 68\left(\frac{65}{6}\right) = 736\frac{2}{3}$, as required.

24. Let $x =$ Lois' travel time.

Since Clark leaves 15 minutes after Lois, and $\frac{15}{60} = \frac{1}{4}$ hr, $x - \frac{1}{4} =$ time for Clark.

Complete the table using the formula $rt = d$.

	Rate	Time	Distance
Lois	35	x	$35x$
Clark	40	$x - \frac{1}{4}$	$40\left(x - \frac{1}{4}\right)$

Since Lois and Clark are going in opposite directions, we add their distances to get 140 mi.

$$35x+40\left(x-\frac{1}{4}\right)=140$$
$$35x+40x-10=140$$
$$75x=150$$
$$x=2$$

They will be 140 mi apart at
8 A.M. + 2 hr = 10 A.M.

Check
Lois' distance was $35(2)=70$. Clark's distance was $40\left(2-\frac{1}{4}\right)=40\left(\frac{7}{4}\right)=70$. The sum is 140 miles, as required.

25. Let $x=$ her average rate on Sunday. Then $x+5=$ her average rate on Saturday.

	Rate	Time	Distance
Saturday	$x+5$	3.6	$3.6(x+5)$
Sunday	x	4	$4x$

The distances are equal.

$$3.6(x+5)=4x$$
$$3.6x+18=4x$$
$$18=0.4x \qquad \text{Subtract } 3.6x.$$
$$x=\frac{18}{0.4}=45$$

Her average rate on Sunday was 45 mph.
Check
On Sunday, 4 hours at 45 mph = 180 miles. On Saturday, 3.6 hours at 50 mph = 180 miles. The distances are equal.

26. Let $x=$ her biking rate.
Then $x-7=$ her walking rate.

	Rate	Time	Distance
Walking	$x-7$	$\frac{40}{60}=\frac{2}{3}$ hr	$\frac{2}{3}(x-7)$
Biking	x	$\frac{12}{60}=\frac{1}{5}$ hr	$\frac{1}{5}x$

The distances are equal.

$$\frac{2}{3}(x-7)=\frac{1}{5}x$$
$$10(x-7)=3x \qquad \text{Multiply by 15.}$$
$$10x-70=3x$$
$$7x=70$$
$$x=10$$

The distance from her house to the train station is $\frac{1}{5}x=\frac{1}{5}(10)=2$ miles.

Check
The distance walking is $(3\text{ mph})\left(\frac{2}{3}\text{hr}\right)=2$ mi.
The distance biking is $(10\text{ mph})\left(\frac{1}{5}\text{hr}\right)=2$ mi.
The distances are equal.

27. Let $x=$ Anne's time.
Then $x+\frac{1}{2}=$ Johnny's time.

	Rate	Time	Distance
Anne	60	x	$60x$
Johnny	50	$x+\frac{1}{2}$	$50\left(x+\frac{1}{2}\right)$

The total distance is 80.

$$60x+50\left(x+\frac{1}{2}\right)=80$$
$$60x+50x+25=80$$
$$110x=55$$
$$x=\frac{55}{110}=\frac{1}{2}$$

They will meet $\frac{1}{2}$ hr after Anne leaves.

Check
Anne travels $60\left(\frac{1}{2}\right)=30$ miles. Johnny travels $50\left(\frac{1}{2}+\frac{1}{2}\right)=50$ miles. The sum of the distances is 80 miles, as required.

28. Let $x=$ her rate (speed) during the first part of the trip. Then $x-25=$ her rate during rush-hour traffic. Make a table using the formula $rt=d$.

	Rate	Time	Distance
First Part	x	2	$2x$
Rush-Hour	$x-25$	$\frac{1}{2}$	$\frac{1}{2}(x-25)$

The total distance was 125 miles.

$$2x+\frac{1}{2}(x-25)=125$$
$$4x+x-25=250 \qquad \text{Multiply by 2.}$$
$$5x=275$$
$$x=55$$

The rate during the first part of the trip was 55 mph.

Check
The distance traveled during the first part of the trip was $55(2) = 110$ miles. The distance traveled during the second part of the trip was $(55-25)(0.5) = 15$ miles. The sum of the distances is 125 miles, as required.

29. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned}(x-30)+(2x-120)+\left(\frac{1}{2}x+15\right)&=180\\ \frac{7}{2}x-135&=180\\ 7x-270&=360\\ 7x&=630\\ x&=90\end{aligned}$$

With $x = 90$, the three angle measures become $(90-30)° = 60°$, $[2(90)-120]° = 60°$, and $\left[\frac{1}{2}(90)+15\right]° = 60°$.

30. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned}(x+15)+(10x-20)+(x+5)&=180\\ 12x&=180\\ x&=15\end{aligned}$$

With $x = 15$, the three angle measures become $(15+15)° = 30°$, $(10\cdot 15-20)° = 130°$, and $(15+5)° = 20°$.

31. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned}(3x+7)+(9x-4)+(4x+1)&=180\\ 16x+4&=180\\ 16x&=176\\ x&=11\end{aligned}$$

With $x = 11$, the three angle measures become $(3\cdot 11+7)° = 40°$, $(9\cdot 11-4)° = 95°$, and $(4\cdot 11+1)° = 45°$.

32. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned}(2x+7)+(x+61)+x&=180\\ 4x+68&=180\\ 4x&=112\\ x&=28\end{aligned}$$

With $x = 28$, the three angle measures become $(2\cdot 28+7)° = 63°$, $(28+61)° = 89°$, and 28°.

33. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned}(3x-36)+(x+11)+x&=180\\ 5x-25&=180\\ 5x&=205\\ x&=41\end{aligned}$$

With $x = 41$, the other angle measures become $(3\cdot 41-36)° = 87°$, and $(41+11)° = 52°$.

34. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned}(x+10)+(12x-30)+(x+4)&=180\\ 14x-16&=180\\ 14x&=196\\ x&=14\end{aligned}$$

With $x = 14$, the three angle measures become $(14+10)° = 24°$, $(12\cdot 14-30)° = 138°$, and $(14+4)° = 18°$.

35. Vertical angles have equal measure.

$$\begin{aligned}8x+2&=7x+17\\ x&=15\end{aligned}$$

$8\cdot 15+2 = 122$ and $7\cdot 15+17 = 122$.
The angles are both 122°.

36. Vertical angles have equal measure.

$$\begin{aligned}9-5x&=25-3x\\ 9&=25+2x\\ -16&=2x\\ -8&=x\end{aligned}$$

$9-5(-8) = 49$ and $25-3(-8) = 49$.
The angles are both 49°.

37. The sum of the two angles is 90°.

$$\begin{aligned}(5x-1)+2x&=90\\ 7x-1&=90\\ 7x&=91\\ x&=13\end{aligned}$$

The measures of the two angles are $[5(13)-1]° = 64°$ and $[2(13)]° = 26°$.

38. The sum of the two angles is 90°.

$$\begin{aligned}(3x-9)+6x&=90\\9x-9&=90\\9x&=99\\x&=11\end{aligned}$$

The measures of the two angles are $[3(11)-9]° = 24°$ and $[6(11)]° = 66°$.

39. Supplementary angles have an angle measure sum of 180°.

$$\begin{aligned}(3x+5)+(5x+15)&=180\\8x+20&=180\\8x&=160\\x&=20\end{aligned}$$

With $x = 20$, the two angle measures become $(3\cdot 20+5)° = 65°$ and $(5\cdot 20+15)° = 115°$.

40. Supplementary angles have an angle measure sum of 180°.

$$\begin{aligned}(3x+1)+(7x+49)&=180\\10x+50&=180\\10x&=130\\x&=13\end{aligned}$$

With $x = 13$, the two angle measures become $(3\cdot 13+1)° = 40°$ and $(7\cdot 13+49)° = 140°$.

41. Let $x =$ the first consecutive integer. Then $x+1$ will be the second consecutive integer, and $x+2$ will be the third consecutive integer.
The sum of the first and twice the second is 17 more than twice the third.

$$\begin{aligned}x+2(x+1)&=2(x+2)+17\\x+2x+2&=2x+4+17\\3x+2&=2x+21\\x&=19\end{aligned}$$

Since $x = 19$, $x+1 = 20$, and $x+2 = 21$.
The three consecutive integers are 19, 20, and 21.

42. Let $x =$ the first consecutive integer. Then $x+1$ will be the second consecutive integer, and $x+2$ will be the third consecutive integer.
The sum of the first and twice the third is 39 more than twice the second.

$$\begin{aligned}x+2(x+2)&=2(x+1)+39\\x+2x+4&=2x+2+39\\3x+4&=2x+41\\x&=37\end{aligned}$$

Since $x = 37$, $x+1 = 38$, and $x+2 = 39$.
The three consecutive integers are 37, 38, and 39.

43. Let $x =$ the first integer. Then $x+1$, $x+2$, and $x+3$ are the next three consecutive integers.
The sum of the first three integers is 54 more than the fourth.

$$\begin{aligned}x+(x+1)+(x+2)&=(x+3)+54\\3x+3&=x+57\\2x&=54\\x&=27\end{aligned}$$

The four consecutive integers are 27, 28, 29, and 30.

44. Let $x =$ the first integer. Then $x+1$, $x+2$, and $x+3$ are the next three consecutive integers.
The sum of the last three integers is 86 more than the first.

$$\begin{aligned}(x+1)+(x+2)+(x+3)&=x+86\\3x+6&=x+86\\2x&=80\\x&=40\end{aligned}$$

The four consecutive integers are 40, 41, 42, and 43.

45. Let $x =$ the guest's room number. Then $x+2$ will be the room number of the room next to the guest.
The sum of these room numbers will be 2430.

$$\begin{aligned}x+(x+2)&=2430\\2x+2&=2430\\2x&=2428\\x&=1214\end{aligned}$$

The guest's room number is 1214 and the room next door is numbered $1214+2=1216$.

46. Let $x =$ the first room number. Then $x+2$ will be the second room number.
The sum of these room numbers will be 1618.

$$\begin{aligned}x+(x+2)&=1618\\2x+2&=1618\\2x&=1616\\x&=808\end{aligned}$$

The two room numbers are 808 and $808+2=810$.

47. Let $x =$ the first even integer. Then $x+2$ and $x+4$ are the next two consecutive even integers.
The sum of the least integer and the middle integer is 26 more than the greatest integer.

$$x+(x+2)=(x+4)+26$$
$$2x+2=x+30$$
$$x+2=30$$
$$x=28$$

The three even integers are 28, 30 and 32.

48. Let $x =$ the first even integer. Then $x+2$ and $x+4$ are the next two consecutive even integers.
The sum of the least integer and the greatest integer is 12 more than the middle integer.

$$x+(x+4)=(x+2)+12$$
$$2x+4=x+14$$
$$x+4=14$$
$$x=10$$

The three even integers are 10, 12, and 14.

49. Let $x =$ the first odd integer. Then $x+2$ and $x+4$ are the next two consecutive odd integers.
The sum of the least integer and the middle integer is 19 more than the greatest integer.

$$x+(x+2)=(x+4)+19$$
$$2x+2=x+23$$
$$x+2=23$$
$$x=21$$

The three even integers are 21, 23, and 25.

50. Let $x =$ the first odd integer. Then $x+2$ and $x+4$ are the next two consecutive odd integers.
The sum of the least integer and the greatest integer is 13 more than the middle integer.

$$x+(x+4)=(x+2)+13$$
$$2x+4=x+15$$
$$x+4=15$$
$$x=11$$

The three even integers are 11, 13, and 15.

51. The sum of the measures of the angles of a triangle is 180°.

$$x+2x+60=180$$
$$3x+60=180$$
$$3x=120$$
$$x=40$$

The measures of the unknown angles are 40° and $2x=80°$.

52. Two angles which form a straight line add to 180°, so $180°-60°=120°$. The measure of the unknown angle is 120°.

53. The sum of the measures of the unknown angles in Exercise 51 is $40°+80°=120°$. This sum is equal to the measure of the angle in Exercise 52.

54. The sum of the measures of angles ① and ② is equal to the measure of angle ③.

Summary Exercises Applying Problem-Solving Techniques

1. Let $x =$ the width of the rectangle. Then $x+3$ is the length of the rectangle.
If the length were decreased by 2 inches and the width were increased by 1 inch, the perimeter would be 24 inches. Use the formula $P=2L+2W$, and substitute 24 for P, $(x+3)-2$ or $x+1$ for L, and $x+1$ for W.

$$P=2L+2W$$
$$24=2(x+1)+2(x+1)$$
$$24=2x+2+2x+2$$
$$24=4x+4$$
$$20=4x$$
$$5=x$$

The width of the rectangle is 5 inches, and the length is $5+3=8$ inches.

2. Let $x =$ the width. Then $2x$ is the length (twice the width).
Use $P=2L+2W$ and add one more width to cut the area into two parts. Thus, an equation is shown below. Solve the equation.

$$2L+2W+W=210$$
$$2(2x)+2x+x=210$$
$$7x=210$$
$$x=30$$

The width is 30 meters, and the length is $2(30)=60$ meters.

3. Let $x =$ the regular price of the item.
The sale price after a 28.57% (or 0.2857) discount was \$199.99, so an equation is $x-0.2857x=199.99$.
Solve the equation

$$x-0.2857x=199.99.$$
$$0.7143x=199.99$$
$$x=\frac{199.99}{0.7143}$$
$$x\approx 279.98$$

To the nearest cent, the regular price was \$279.98.

4. Let $x =$ the regular price of the smart TV. The sale price after a discount of 40% (or 0.40) was \$255, so an equation is $x - 0.40x = 255$. Solve the equation.

$$x - 0.40x = 255$$
$$0.60x = 255$$
$$x = 425$$

The regular price of the smart TV was \$425.

5. Let $x =$ the amount invested at 4%. Then $2x$ is the amount invested at 5%. Use $I = prt$ with $t = 1$ yr. Make a table.

Principal	Rate (as a decimal)	Interest
x	0.04	$0.04x$
$2x$	0.05	$0.05(2x) = 0.10x$
	Total $\rightarrow$	112

The last column gives the equation.
Interest at 4% + interest at 5% = total interest.

$$0.04x \quad + \quad 0.10x \quad = \quad 112$$

Multiply by 100.

$$4x + 10x = 11{,}200$$
$$14x = 11{,}200$$
$$x = 800$$

\$800 is invested at 4% and $2(\$800) = \$1600$ at 5%.

Check
$\$800$ at $4\% = \$32$ and
$\$1600$ at $5\% = \$80$; $\$32 + \$80 = \$112$

6. Let $x =$ the amount invested at 3%. Then $x + 2000$ is the amount invested at 4%. Use $I = prt$ with $t = 1$ yr. Make a table.

Principal	Rate (as a decimal)	Interest
x	0.03	$0.03x$
$x + 2000$	0.04	$0.04(x + 2000)$
	Total $\rightarrow$	920

The last column gives the equation.
Interest at 3% + interest at 4% = total interest.

$$0.03x \quad + 0.04(x + 2000) = \quad 920$$

Multiply by 100.

$$3x + 4(x + 2000) = 92{,}000$$
$$3x + 4x + 8000 = 92{,}000$$
$$7x = 84{,}000$$
$$x = 12{,}000$$

\$12,000 is invested at 3% and \$14,000 is invested at 4%.

Check
$\$12{,}000$ at $3\% = \$360$ and
$\$14{,}000$ at $4\% = \$560$; $\$360 + \$560 = \$920$

7. Let $x =$ the number of points scored by Young in the 2021–2022 season. Then $x - 37 =$ the number of points scored by DeRozan in the 2021–2022 season. The total number of points scored by both was 4273.

$$x + (x - 37) = 4273$$
$$2x - 37 = 4273$$
$$2x = 4310$$
$$x = 2155$$

Young scored 2155 points and DeRozan scored $2155 - 37 = 2118$ points.

Check
2118 is 37 fewer points than 2155 points, and $2118 + 2155 = 4273$.

8. Let $x =$ the number of films released in 2011. Then $x - 198 =$ the number released in 2021. A total of 1004 were released in both years.

$$x + (x - 198) = 1004$$
$$2x - 198 = 1004$$
$$2x = 1202$$
$$x = 601$$

In 2011, 601 films were released. In 2021, $601 - 198 = 403$ films were released.

Check
403 is 198 less than 601, and $601 + 403 = 1004$.

9. Let $t =$ the time it will take until John and Pat meet. Use $d = rt$ and make a table.

	Rate	Time	Distance
John	60	t	$60t$
Pat	28	t	$28t$

The total distance is 440 miles.

$$60t + 28t = 440$$
$$88t = 440$$
$$t = 5$$

It will take 5 hours for John and Pat to meet.

Check
John traveled $60(5) = 300$ miles and Pat traveled $28(5) = 140$ miles; $300 + 140 = 440$, as required.

10. Let $x =$ the length of the side of the square that is being cut from the 12 by 16 sheet of tin. The width of the tin decreases by x at the left and right ends of the tin because small squares have been cut out at each corner. Thus, $W = 12 - 2x$

is the width of the box made by cutting out the small squares. Similarly, $L = 16 - 2x$ is the length of the box. The length L of the box is twice its width W minus 5.

$$L = 2W - 5$$
$$16 - 2x = 2(12 - 2x) - 5$$

Solve for x.

$$16 - 2x = 2(12 - 2x) - 5$$
$$16 - 2x = 24 - 4x - 5$$
$$16 - 2x = 19 - 4x$$
$$2x = 3$$
$$x = 1\frac{1}{2}$$

The length of a side of the square is $x = 1\frac{1}{2}$ cm.

Check

The length of the box will be

$$16 - 2x = 16 - 2\left(1\frac{1}{2}\right) = 16 - 3 = 13.$$

Its width will be

$$12 - 2x = 12 - 2\left(1\frac{1}{2}\right) = 12 - 3 = 9.$$

Twice the width minus 5, $2W - 5 = 2 \cdot 9 - 5 = 13,$ is equal to the length of the box as desired.

11. Let $x =$ the number of liters of the 5% drug solution.

Liters of Solution	Percent (as a decimal)	Liters of Pure Drug
20	0.10	$20(0.10) = 2$
x	0.05	$0.05x$
$20 + x$	0.08	$0.08(20 + x)$

Drug in 10% + drug in 5% = drug in 8%.

$$2 + 0.05x = 0.08(20 + x)$$

Multiply by 100.

$$200 + 5x = 8(20 + x)$$
$$200 + 5x = 160 + 8x$$
$$40 = 3x$$
$$x = \frac{40}{3} \text{ or } 13\frac{1}{3}$$

The pharmacist should add $13\frac{1}{3}$ L.

Check

10% of 20 is 2 and 5% of $\frac{40}{3}$ is $\frac{2}{3}$; $2 + \frac{2}{3} = \frac{8}{3}$, which is the same as 8% of $\left(20 + \frac{40}{3}\right)$.

12. Let $x =$ the number of kilograms of the metal that is 20% tin.

Kilograms of Metal	Percent Tin (as a decimal)	Kilograms of Pure Tin
80	0.70	$80(0.70) = 56$
x	0.20	$0.20x$
$80 + x$	0.50	$0.50(80 + x)$

Tin in 70% + tin in 20% = tin in 50%.

$$56 + 0.20x = 0.50(80 + x)$$

Multiply by 100.

$$560 + 2x = 5(80 + x)$$
$$560 + 2x = 400 + 5x$$
$$160 = 3x$$
$$x = \frac{160}{3} \text{ or } 53\frac{1}{3}$$

$53\frac{1}{3}$ kilograms should be added.

Check

70% of 80 is 56 and 20% of $\frac{160}{3}$ is $\frac{32}{3}$;

$56 + \frac{32}{3} = 66\frac{2}{3}$, which is the same as 50% of $\left(80 + \frac{160}{3}\right)$.

13. Let $x =$ the number of \$5 bills. Then $126 - x$ is the number of \$10 bills.

Number of Bills	Denomination	Value
x	5	$5x$
$126 - x$	10	$10(126 - x)$
126	← Totals →	840

The sum of the values must equal the total value.

$$5x + 10(126 - x) = 840$$
$$5x + 1260 - 10x = 840$$
$$-5x = -420$$
$$x = 84$$

There are 84 \$5 bills and $126 - 84 = 42$ \$10 bills.

Check
The number of bills is $84+42=126$ and the value of the bills is $\$5(84)+\$10(42)=\$840,$ as required.

14. Let $x=$ the number of \$10 tickets sold. Then $2460-x$ = the number of \$13 tickets sold.

Number Sold	Cost of Ticket	Amount Collected
x	\$10	$10x$
$2460-x$	\$13	$13(2460-x)$
2460	← Totals →	\$27,030

Write the equation from the last column of the table.

$$10x+13(2460-x)=27,030$$
$$10x+31,980-13x=27,030$$
$$-3x=-4950$$
$$x=1650$$

1650 \$10 tickets were sold and $2460-1650=810$ \$13 tickets were sold.
Check
The number of tickets sold was $1650+810=2460.$
The amount collected was

$$\$10(1650)+\$13(810)=\$16,500+\$10,530$$
$$=\$27,030$$

as required.

15. The sum of the measures of the three angles of a triangle is 180°.

$$x+(6x-50)+(x-10)=180$$
$$8x-60=180$$
$$8x=240$$
$$x=30$$

With $x=30,$ the three angle measures become $(6\cdot 30-50)°=130°,$ $(30-10)°=20°,$ and 30°.

16. In the figure, the two angles are supplementary, so their sum is 180°.

$$(10x+7)+(7x+3)=180$$
$$17x+10=180$$
$$17x=170$$
$$x=10$$

The two angle measures are $10(10)+7=107°$ and $7(10)+3=73°.$

17. Let $x=$ the least integer. Then $x+1$ is the middle integer and $x+2$ is the greatest integer. "The sum of the least and greatest of three consecutive integers is 32 more than the middle integer" translates to

$$x+(x+2)=32+(x+1).$$
$$2x+2=x+33$$
$$x=31$$

The three consecutive integers are 31, 32, and 33.
Check
The sum of the least and greatest integers is $31+33=64,$ which is the same as 32 more than the middle integer.

18. Let $x=$ the first odd integer. Then $x+2$ is the next odd integer.
"If the lesser of two consecutive odd integers is doubled, the result is 7 more than the greater of the two integers" translates to

$$2(x)=7+(x+2).$$
$$2x=x+9$$
$$x=9$$

The two consecutive odd integers are 9 and 11.
Check
Doubling the lesser gives us $2(9)=18,$ which is equal to 7 more than 11.

19. Let $x=$ the length of the shortest side. Then $2x$ is the length of the middle side and $3x-2$ is the length of the longest side.
The perimeter is 34 inches. Using $P=a+b+c$ gives us

$$x+2x+(3x-2)=34.$$
$$6x-2=34$$
$$6x=36$$
$$x=6$$

The lengths of the three sides are 6 inches, $2(6)=12$ inches, and $3(6)-2=16$ inches.
Check
The sum of the lengths of the three sides is $6+12+16=34$ inches, as required.

20. Let $x=$ the length of the rectangle. Then the perimeter is $43+x.$

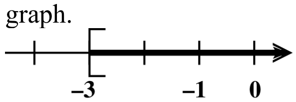
$$P=2L+2W$$
$$43+x=2x+2(10) \quad \textit{Let } W=10.$$
$$43+x=2x+20$$
$$23=x$$

The length of the rectangle is 23 inches.
Check
$P=2L+2W=2(23)+2(10)=66$ inches, which is 43 inches more than the length.

1.5 Linear Inequalities in One Variable

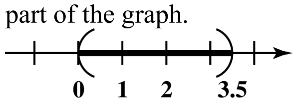
Classroom Examples, Now Try Exercises

1. (a) The statement $x \ge -3$ says that x can represent any number greater than or equal to -3. Graph it by placing a bracket at -3 and drawing an arrow to the right. The bracket shows that -3 is a part of the graph.



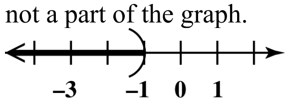
This interval is written with a square bracket at -3 as $[-3, \infty)$.

(b) The statement $0 < x < 3.5$ says that x can represent any number between 0 and 3.5 with both 0 and 3.5 excluded. Graph it by placing parentheses at both 0 and 3.5. Fill in the points in between them. The parentheses show that 0 and -3 are not part of the graph.



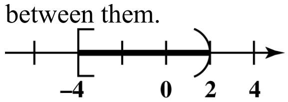
This interval is written with parentheses as $(0, 3.5)$.

N1. (a) The statement $x < -1$ says that x can represent any number less than -1, but x cannot equal -1. Graph it by placing a right parenthesis at -1 and drawing an arrow to the left. The parenthesis shows that -1 is not a part of the graph.



This interval is written with a parenthesis at -1 as $(-\infty, -1)$.

(b) The statement $-4 \le x < 2$ says that x can represent any number between -4 and 2 with -4 included and 2 excluded. Graph it by placing a left bracket at -4 and a right parenthesis at 2. Fill in the points in between them.



This interval is written as $[-4, 2)$.

2. $x - 5 > 1$

$x > 6$ Add 5.

Check

Substitute 6 for x in $x - 5 = 1$.

$6 - 5 \stackrel{?}{=} 1$

$1 = 1$ True

This shows that 6 is the boundary point. Choose 0 and 7 as test points.

$x - 5 > 1$

Let $x = 0$.	Let $x = 7$.
$0 - 5 \stackrel{?}{>} 1$	$7 - 5 \stackrel{?}{>} 1$
$-5 > 1$ False	$2 > 1$ True
0 is not in the solution set.	7 is in the solution set.

The check confirms that $(6, \infty)$ is the solution set.

N2. $x - 10 > -7$

$x > 3$ Add 10.

Check

Substitute 3 for x in $x - 10 = -7$.

$3 - 10 \stackrel{?}{=} -7$

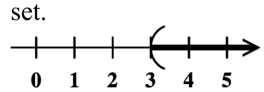
$-7 = -7$ True

This shows that 3 is the boundary point. Choose 0 and 7 as test points.

$x - 10 > -7$

Let $x = 0$.	Let $x = 7$.
$0 - 10 \stackrel{?}{>} -7$	$7 - 10 \stackrel{?}{>} -7$
$-10 > -7$ False	$-3 > -7$ True
0 is not in the solution set.	7 is in the solution set.

The check confirms that $(3, \infty)$ is the solution set.



3. $5x + 3 \ge 4x - 1$

$x + 3 \ge -1$ Subtract $4x$.

$x \ge -4$ Subtract -3.

Check

Substitute -4 for x in $5x + 3 = 4x - 1$.

$5(-4) + 3 \stackrel{?}{>} 4(-4) - 1$

$-17 = -17$ True

So -4 satisfies the equality part of $\ge$. Choose -5 and 0 as test points.

$5x+3 \geq 4x-1$

$5(-5)+3 \overset{?}{\geq} 4(-5)-1$ Let $x=-5$.

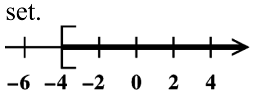
$-22 \geq -21$ False

-5 is not in the solution set.

$5(0)+3 \overset{?}{\geq} 4(0)-1$ Let $x=0$.

$3 \geq -1$ True

0 is in the solution set.
The check confirms that $[-4, \infty)$ is the solution set.



N3. $4x+1 \geq 5x$

$1 \geq x$ Subtract $4x$.

$x \leq 1$ Equivalent inequality.

Check
Substitute 1 for x in $4x+1=5x$.

$4(1)+1 \overset{?}{=} 5(1)$

$5=5$ True

So 1 satisfies the equality part of $\geq$. Choose 0 and 2 as test points.

$4x+1 \geq 5x$

$4(0)+1 \overset{?}{>} 5(0)$ Let $x=0$.

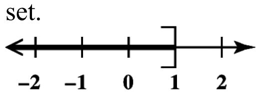
$1 \geq 0$ True

0 is in the solution set.

$4(2)+1 \overset{?}{>} 5(2)$ Let $x=2$.

$9 \geq 10$ False

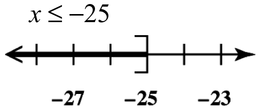
2 is not in the solution set.
The check confirms that $(-\infty, 1]$ is the solution set.



4. (a) $4x \leq -100$

$\frac{4x}{4} \leq \frac{-100}{4}$ Divide by $4>0$; do not reverse the symbol.

$x \leq -25$

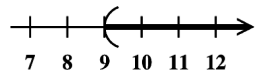


The solution set is the interval $(-\infty, -25]$.

(b) $-9x < -81$

$\frac{-81}{-9} > \frac{-81}{-9}$ Divide by $-9<0$; reverse the symbol.

$x > 9$

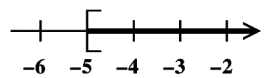


The solution set is the interval $(9, \infty)$.

N4. (a) $8x \geq -40$

$\frac{8x}{8} \geq \frac{-40}{8}$ Divide by $8>0$; do not reverse the symbol.

$x \geq -5$

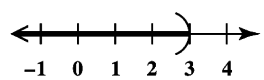


The solution set is the interval $[-5, \infty)$.

(b) $-20x > -60$

$\frac{-20x}{-20} < \frac{-60}{-20}$ Divide by $-20<0$; reverse the symbol.

$x < 3$



The solution set is the interval $(-\infty, 3)$.

5. $6(x-1)+3x \geq -x-3(x+2)$

$6x-6+3x \geq -x-3x-6$

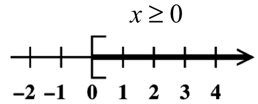
$9x-6 \geq -4x-6$

$13x-6 \geq -6$

$13x \geq 0$

$\frac{13x}{13} \geq \frac{0}{13}$

$x \geq 0$



The solution set is the interval $[0, \infty)$.

N5. $5-2(x-4) \leq 11-4x$

$5-2x+8 \leq 11-4x$

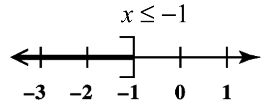
$-2x+13 \leq 11-4x$

$2x+13 \leq 11$

$2x \leq -2$

$\frac{2x}{2} \leq \frac{-2}{2}$

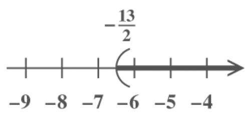
$x \leq -1$



The solution set is the interval $(-\infty, -1]$.

6. Solve the following inequality.

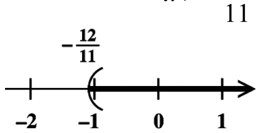
$$\begin{array}{rll} \frac{1}{4}(x+3)+2 < \frac{3}{4}(x+8) & & \\ 4\left[\frac{1}{4}(x+3)+2\right] < 4\left[\frac{3}{4}(x+8)\right] & & \text{Multiply by 4.} \\ (x+3)+8 < 3(x+8) & & \\ x+11 < 3x+24 & & \\ -2x+11 < 24 & & \text{Subtract } 3x. \\ -2x < 13 & & \text{Subtract } 11. \\ x > -\frac{13}{2} & & \text{Divide by } -2; \text{ reverse symbol.} \end{array}$$



The solution set is the interval $\left(-\frac{13}{2}, \infty\right)$.

N6.

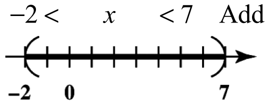
$$\begin{array}{rll} \frac{3}{4}(x-2)+\frac{1}{2} > \frac{1}{5}(x-8) & & \\ 20\left[\frac{3}{4}(x-2)+\frac{1}{2}\right] > 20\left[\frac{1}{5}(x-8)\right] & & \text{Mult. by 20.} \\ 15(x-2)+10 > 4(x-8) & & \\ 15x-30+10 > 4x-32 & & \\ 15x-20 > 4x-32 & & \\ 11x-20 > -32 & & \text{Subtract } 4x. \\ 11x > -12 & & \text{Add 20.} \\ x > -\frac{12}{11} & & \text{Divide by } 11. \end{array}$$



The solution set is the interval $\left(-\frac{12}{11}, \infty\right)$.

7.

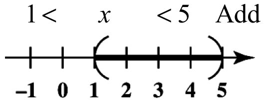
$$\begin{array}{rcll} -4 < & x-2 & < 5 & \\ -2 < & x & < 7 & \text{Add 2 to each part.} \end{array}$$



The solution set is the interval $(-2, 7)$.

N7.

$$\begin{array}{rcll} -1 < & x-2 & < 3 & \\ 1 < & x & < 5 & \text{Add 2 to each part.} \end{array}$$



The solution set is the interval (1, 5).

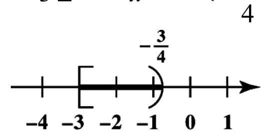
8.

$$\begin{array}{rcll} 5 \le & -3x-4 & \le 9 & \\ 9 \le & -3x & \le 13 & \text{Add 4 to each part.} \\ \frac{9}{-3} \ge & \frac{-3x}{-3} & \ge \frac{13}{-3} & \text{Divide by } -3; \text{ reverse symbol.} \\ -\frac{13}{3} \le & x & \le -3 & \end{array}$$

The solution set is the interval $\left[-\frac{13}{3}, -3\right]$.

N8.

$$\begin{array}{rcll} -2 < & -4x-5 & \le 7 & \\ 3 < & -4x & \le 12 & \text{Add 5 to each part.} \\ \frac{3}{-4} > & \frac{-4x}{-4} & \ge \frac{12}{-4} & \text{Divide by } -4. \text{ Reverse inequalities.} \\ -\frac{3}{4} > & x & \ge -3 & \text{Reduce.} \\ -3 \le & x & < -\frac{3}{4} & \text{Equivalent inequality.} \end{array}$$



The solution set is the interval $\left[-3, -\frac{3}{4}\right)$.

9. *Step 2*
Let h = the number of hours she can rent the leaf blower.
Step 3
She must pay \$15, plus \$3.50h, to rent the leaf blower for h hours, and this amount must be *no more than* \$36.

$$\underbrace{\text{Cost of renting}}_{\downarrow\atop 15+3.50h} \quad \underbrace{\text{is no more than}}_{\downarrow\atop \le} \quad \underbrace{36 \text{ dollars.}}_{\downarrow\atop 36}$$

Step 4

$$\begin{array}{rl} 3.50h \le 21 & \text{Subtract 15.} \\ h \le 6 & \text{Divide by } 3.50. \end{array}$$

Step 5
She can use the leaf blower for a maximum of 6 hr. (She may use it for less time, as indicated by the inequality $h \le 6$.)
Step 6
If Marge uses the leaf blower for 6 hr, she will spend $15+3.50(6)=36$, the maximum amount.

N9. *Step 2*
Let x = the number of months she belongs to the health club.
Step 3
She must pay \$40, plus \$35x, to belong to the health club for x months, and this amount must be *no more than* \$355.

$$\underbrace{\text{Cost of belonging}}_{\downarrow} \quad \underbrace{\text{is no more than}}_{\downarrow} \quad \underbrace{\text{355 dollars.}}_{\downarrow}$$
$$40+35x \quad \le \quad 355$$

Step 4
$35x \le 315$ Subtract 40.
$x \le 9$ Divide by 35.
Step 5
She can belong to the health club for a maximum of 9 months. (She may belong for less time, as indicated by the inequality $x \le 9$.)
Step 6
If Sara belongs for 9 months, she will spend $40+35(9)=355$, the maximum amount.

10. Let x = the grade the student must make on the fourth test.
To find the average of the four tests, add them and divide by 4. This average must be at least 90, that is, greater than or equal to 90.

$$\frac{92+90+84+x}{4} \ge 90$$
$$\frac{266+x}{4} \ge 90$$
$$266+x \ge 360 \quad \text{Multiply by 4.}$$
$$x \ge 94 \quad \text{Subtract 266.}$$

The student must score at least 94 on the fourth test.

Check $\frac{92+90+84+94}{4} = \frac{360}{4} = 90$

A score of 94 or more will give an average of at least 90, as required.

N10. Let x = the grade the student must make on the fourth test.
To find the average of the four tests, add them and divide by 4. This average must be at least 90, that is, greater than or equal to 90.

$$\frac{82+97+93+x}{4} \ge 90$$
$$\frac{272+x}{4} \ge 90$$
$$272+x \ge 360 \quad \text{Multiply by 4.}$$
$$x \ge 88 \quad \text{Subtract 272.}$$

The fourth test score must be at least 88.

Check $\frac{82+97+93+88}{4} = \frac{360}{4} = 90$

A score of 88 or more will give an average of at least 90, as required.

Exercises

1. $x \le 3$
In interval notation, this inequality is written $(-\infty, 3]$. The bracket indicates that 3 is included. The answer is choice D.

2. $x > 3$
In interval notation, this inequality is written $(3, \infty)$. The parenthesis indicates that 3 is not included. The answer is choice C.

3. $x < 3$
In interval notation, this inequality is written $(-\infty, 3)$. The parenthesis indicates that 3 is not included. The graph of this inequality is shown in choice B.

4. $x \ge 3$
In interval notation, this inequality is written $[3, \infty)$. The bracket indicates that 3 is included. The graph of this inequality is shown in choice A.

5. $-3 \le x \le 3$
In interval notation, this inequality is written $[-3, 3]$. The brackets indicates that -3 and 3 are included. The answer is choice F.

6. $-3 < x < 3$
In interval notation, this inequality is written $(-3, 3)$. The parentheses indicate that neither -3 nor 3 is included. The answer is choice E.

7. **(a)** The optimal category corresponds to an LDL number of "less than 100."

$$\underbrace{\text{LDL number}}_{\downarrow} \quad \underbrace{\text{is less than}}_{\downarrow} \quad \underbrace{100.}_{\downarrow}$$
$$x \quad < \quad 100$$

(b) The near optimal/above optimal category corresponds to an LDL number between "100–129." This is represented by the inequality below.

$$\underbrace{\text{LDL number}}_{\downarrow}$$
$$100 \le x \le 129$$

(c) The borderline high category corresponds to an LDL number between "130–159." This is represented by the inequality below.

$$\underbrace{\text{LDL number}}_{\downarrow}$$
$$130 \le x \le 159$$

(d) The high category corresponds to an LDL number between "160–189." This is represented by the inequality below.

$$\underbrace{\text{LDL number}}_{\downarrow}$$
$$160 \le x \le 189$$

(e) The very high category corresponds to an LDL number of "190 and above." This can also be written as "greater than or equal to 190."

$$\underbrace{\text{LDL number}}_{\substack{\downarrow \\ x}} \quad \underbrace{\text{is greater than or equal to}}_{\substack{\downarrow \\ \ge}} \quad \underbrace{190.}_{\substack{\downarrow \\ 190}}$$

8. **(a)** The normal category corresponds to a triglycerides number of "less than 150."

$$\underbrace{\text{Triglycerides number}}_{\substack{\downarrow \\ x}} \quad \underbrace{\text{is less than}}_{\substack{\downarrow \\ <}} \quad \underbrace{150.}_{\substack{\downarrow \\ 150}}$$

(b) The middle high category corresponds to a triglycerides number between "150–199." This is represented by the inequality below.

$$\underbrace{\text{triglycerides number}}_{\downarrow}$$
$$150 \le x \le 199$$

(c) The high category corresponds to a triglycerides number of "200–499." This is represented by the inequality below.

$$\underbrace{\text{triglycerides number}}_{\downarrow}$$
$$200 \le x \le 499$$

(d) The very high category corresponds to a triglycerides number of "500 or higher." This can also be written as "greater than or equal to 500."

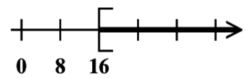
$$\underbrace{\text{Triglycerides number}}_{\substack{\downarrow \\ x}} \quad \underbrace{\text{is greater than or equal to}}_{\substack{\downarrow \\ \ge}} \quad \underbrace{500.}_{\substack{\downarrow \\ 500}}$$

9. Since $4 > 0$, the student should not have reversed the direction of the inequality symbol when dividing by 4. We reverse the inequality symbol only when multiplying or dividing by a *negative* number. The solution set is $[-16, \infty)$.

10. The student divided by -2, a *negative* number, which requires reversing the direction of the inequality symbol. The solution set is $(9, \infty)$.

11. $x - 4 \ge 12$

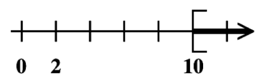
$x \ge 16$ Add 4.



The solution set is the interval $[16, \infty)$.

12. $x - 3 \ge 7$

$x \ge 10$ Add 3.

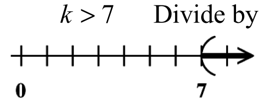


The solution set is the interval $[10, \infty)$.

13. $3k + 1 > 22$

$3k > 21$ Subtract 1.

$k > 7$ Divide by 3.

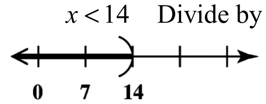


The solution set is the interval $(7, \infty)$.

14. $5x + 6 < 76$

$5x < 70$ Subtract 6.

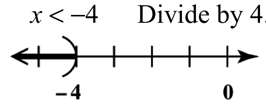
$x < 14$ Divide by 5.



The solution set is the interval $(-\infty, 14)$.

15. $4x < -16$

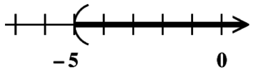
$x < -4$ Divide by 4.



The solution set is the interval $(-\infty, -4)$.

16. $2x > -10$

$x > -5$ Divide by 2.



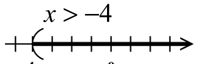
The solution set is the interval $(-5, \infty)$.

17. $-4x < 16$

Divide both sides by -4, and reverse the inequality symbol.

$$\frac{-4x}{-4} > \frac{16}{-4}$$

$$x > -4$$



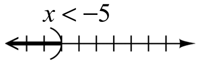
The solution set is the interval $(-4, \infty)$.

18. $-5x > 25$

Divide both sides by -5, and reverse the inequality symbol.

$$\frac{-5x}{-5} < \frac{25}{-5}$$

$$x < -5$$



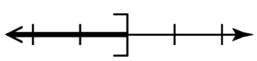
The solution set is the interval $(-\infty, -5)$.

19. $-\frac{3}{4}x \geq 30$

Multiply both sides by $-\frac{4}{3}$ and reverse the inequality symbol.

$$-\frac{4}{3}\left(-\frac{3}{4}x\right) \leq -\frac{4}{3}(30)$$

$$x \leq -40$$



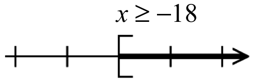
The solution set is the interval $(-\infty, -40]$.

20. $-\frac{2}{3}x \leq 12$

Multiply both sides by $-\frac{3}{2}$ and reverse the inequality symbol.

$$-\frac{3}{2}\left(-\frac{2}{3}x\right) \geq -\frac{3}{2}(12)$$

$$x \geq -18$$



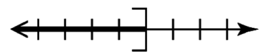
The solution set is the interval $[-18, \infty)$.

21. $-1.3x \geq -5.2$

Divide both sides by -1.3, and reverse the inequality symbol.

$$\frac{-1.3x}{-1.3} \leq \frac{-5.2}{-1.3}$$

$$x \leq 4$$



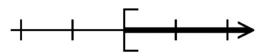
The solution set is the interval $(-\infty, 4]$.

22. $-2.5x \leq -1.25$

Divide both sides by -2.5, and reverse the inequality symbol.

$$\frac{-2.5x}{-2.5} \geq \frac{-1.25}{-2.5}$$

$$x \geq 0.5$$



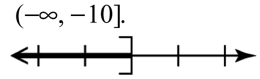
The solution set is the interval $[0.5, \infty)$.

23. $5x + 2 \leq -48$

$5x \leq -50$ Subtract 2.

$x \leq -10$ Divide by 5.

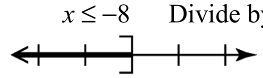
Check that the solution set is the interval $(-\infty, -10]$.



24. $4x + 1 \leq -31$

$4x \leq -32$ Subtract 1.

$x \leq -8$ Divide by 4.



The solution set is the interval $(-\infty, -8]$.

25. $\frac{3k-1}{4} > 5$

$4\left(\frac{3k-1}{4}\right) > 4(5)$ Multiply by 4.

$3k - 1 > 20$

$3k > 21$ Add 1.

$k > 7$ Divide by 3.

Check Let $k = 7$ in the *equation* $\frac{3k-1}{4} = 5$.

$$\frac{3(7)-1}{4} \stackrel{?}{=} 5$$

$$\frac{20}{4} \stackrel{?}{=} 5$$

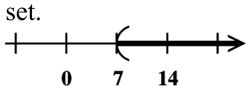
$5 = 5$ True

This shows that 7 is the boundary point. Now test a number on each side of 7. We choose 0 and 10.

$$\frac{3k-1}{4} > 5$$

Let $k=0$.	Let $k=10$.
$\frac{3(0)-1}{4} \overset{?}{>} 5$	$\frac{3(10)-1}{4} \overset{?}{>} 5$
$\frac{-1}{4} > 5$ False	$\frac{29}{4} > 5$ True
0 is not in the solution set.	10 is in the solution set.

The check confirms that $(7, \infty)$ is the solution set.



26. $\dfrac{5x-6}{8} < 8$

$$8\left(\frac{5x-6}{8}\right) < 8\cdot 8 \quad \text{Multiply by 8.}$$
$$5x-6 < 64$$
$$5x < 70 \quad \text{Add 6.}$$
$$x < 14 \quad \text{Divide by 5.}$$

Check Let $x = 14$ in the *equation* $\dfrac{5x-6}{8} = 8$.

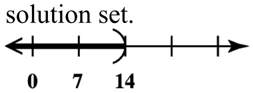
$$\frac{5(14)-6}{8} \overset{?}{=} 8$$
$$\frac{64}{8} \overset{?}{=} 8$$
$$8 = 8 \quad \text{True}$$

This shows that 14 is the boundary point. Now test a number on each side of 14. We choose 0 and 20.

$$\frac{5x-6}{8} < 8$$

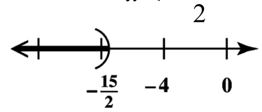
Let $x=0$.	Let $x=20$.
$\frac{5(0)-6}{8} \overset{?}{<} 8$	$\frac{5(20)-6}{8} \overset{?}{<} 8$
$-\frac{6}{8} < 8$ True	$\frac{94}{8}\left(\text{or } 11\frac{6}{8}\right) < 8$ False
0 is in the solution set.	20 is not in the solution set.

The check confirms that $(-\infty, 14)$ is the solution set.



27. Multiply both sides by -4, and reverse the inequality symbol.

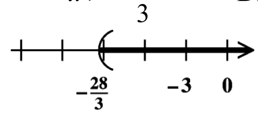
$$\frac{2x-5}{-4} > 5$$
$$-4\left(\frac{2x-5}{-4}\right) < -4(5)$$
$$2x-5 < -20$$
$$2x < -15 \quad \text{Add 5.}$$
$$x < -\frac{15}{2} \quad \text{Divide by 2.}$$



The solution set is the interval $\left(-\infty, -\dfrac{15}{2}\right)$.

28. Multiply both sides by -5, and reverse the inequality symbol.

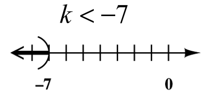
$$\frac{3x-2}{-5} < 6$$
$$3x-2 > -30$$
$$3x > -28 \quad \text{Add 2.}$$
$$x > -\frac{28}{3} \quad \text{Divide by 3.}$$



The solution set is the interval $\left(-\dfrac{28}{3}, \infty\right)$.

29. $3k+1 < -20$

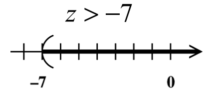
$$3k < -21 \quad \text{Subtract 1.}$$
$$k < -7 \quad \text{Divide by 3.}$$



The solution set is the interval $(-\infty, -7)$.

30. $5z+6 > -29$

$$5z > -35 \quad \text{Subtract 6.}$$
$$z > -7 \quad \text{Divide by 5.}$$



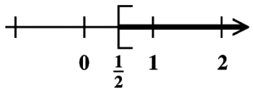
The solution set is the interval $(-7, \infty)$.

31. $6x - 4 \ge -2x$

$8x - 4 \ge 0$ Add $2x$.

$8x \ge 4$ Add 4.

$x \ge \frac{4}{8} = \frac{1}{2}$ Divide by 8.



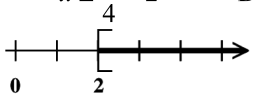
The solution set is the interval $\left[\frac{1}{2}, \infty\right)$.

32. $2x - 8 \ge -2x$

$4x - 8 \ge 0$ Add $2x$.

$4x \ge 8$ Add 8.

$x \ge \frac{8}{4} = 2$ Divide by 4.



The solution set is the interval $[2, \infty)$.

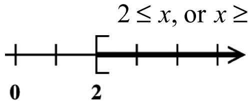
33. $x - 2(x-4) \le 3x$

$x - 2x + 8 \le 3x$

$-x + 8 \le 3x$

$8 \le 4x$ Add x.

$2 \le x$, or $x \ge 2$



The solution set is the interval $[2, \infty)$.

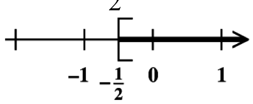
34. $x - 3(x+1) \le 4x$

$x - 3x - 3 \le 4x$

$-2x - 3 \le 4x$

$-3 \le 6x$ Add $2x$.

$-\frac{1}{2} \le x$, or $x \ge -\frac{1}{2}$



The solution set is the interval $\left[-\frac{1}{2}, \infty\right)$.

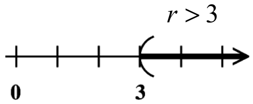
35. $-(4+r) + 2 - 3r < -14$

$-4 - r + 2 - 3r < -14$

$-4r - 2 < -14$

$-4r < -12$

$r > 3$ Reverse symbol.



The solution set is the interval $(3, \infty)$.

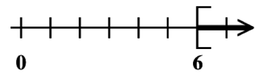
36. $-(9+x) - 5 + 4x \ge 4$

$-9 - x - 5 + 4x \ge 4$

$-14 + 3x \ge 4$

$3x \ge 18$

$x \ge 6$



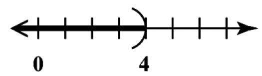
The solution set is the interval $[6, \infty)$.

37. $-3(x-6) > 2x - 2$

$-3x + 18 > 2x - 2$ Distributive property

$-5x > -20$ Subtract $2x$ and 18.

$x < 4$ Reverse symbol.



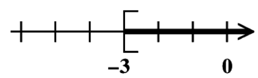
The solution set is the interval $(-\infty, 4)$.

38. $-2(x+4) \le 6x + 16$

$-2x - 8 \le 6x + 16$

$-8x \le 24$

$x \ge -3$ Reverse symbol.



The solution set is the interval $[-3, \infty)$.

39. Multiply both sides by 6 to clear the fractions.

$\frac{2}{3}(3x-1) \ge \frac{3}{2}(2x-3)$

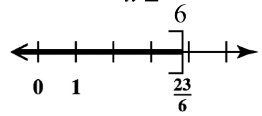
$6 \cdot \frac{2}{3}(3x-1) \ge 6 \cdot \frac{3}{2}(2x-3)$

$4(3x-1) \ge 9(2x-3)$

$12x - 4 \ge 18x - 27$

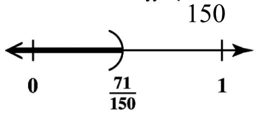
$-6x \ge -23$

$x \le \frac{23}{6}$ Reverse symbol.



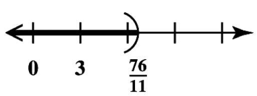
The solution set is the interval $\left(-\infty, \frac{23}{6}\right]$.

40. $$\frac{7}{5}(10x-1)<\frac{2}{3}(6x+5)$$
$$15\cdot\frac{7}{5}(10x-1)<15\cdot\frac{2}{3}(6x+5)$$
$$21(10x-1)<10(6x+5)$$
$$210x-21<60x+50$$
$$150x<71$$
$$x<\frac{71}{150}$$



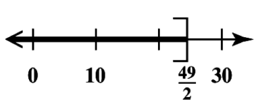
The solution set is the interval $\left(-\infty,\frac{71}{150}\right)$.

41. Multiply each term by 4 to clear the fractions.
$$-\frac{1}{4}(p+6)+\frac{3}{2}(2p-5)<10$$
$$-1(p+6)+6(2p-5)<40$$
$$-p-6+12p-30<40$$
$$11p-36<40$$
$$11p<76$$
$$p<\frac{76}{11}$$



The solution set is the interval $\left(-\infty,\frac{76}{11}\right)$.

42. $$\frac{3}{5}(t-2)-\frac{1}{4}(2t-7)\le 3$$
$$20\cdot\frac{3}{5}(t-2)-20\cdot\frac{1}{4}(2t-7)\le 20\cdot 3$$
$$12(t-2)-5(2t-7)\le 60$$
$$12t-24-10t+35\le 60$$
$$2t\le 49$$
$$t\le\frac{49}{2}$$

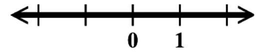


The solution set is the interval $\left(-\infty,\frac{49}{2}\right]$.

43. $$3(2x-4)-4x<2x+3$$
$$6x-12-4x<2x+3$$
$$2x-12<2x+3$$
$$-12<3 \qquad \text{True}$$

The statement is true for all values of x.

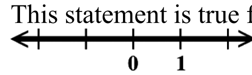
Therefore, the original inequality is true for any real number.



The solution set is the interval $(-\infty,\infty)$.

44. $$7(4-x)+5x<2(16-x)$$
$$28-7x+5x<32-2x$$
$$28-2x<32-2x$$
$$28<32 \qquad \text{True}$$

This statement is true for all values of x.



The solution set is the interval $(-\infty,\infty)$.

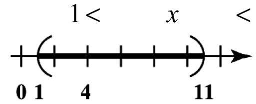
45. $$8\left(\frac{1}{2}x+3\right)<8\left(\frac{1}{2}x-1\right)$$
$$4x+24<4x-8$$
$$24<-8 \qquad \text{False}$$

This is a false statement, so the inequality is a contradiction. The solution set is $\varnothing$.

46. $$10\left(\frac{1}{5}x+2\right)<10\left(\frac{1}{5}x+1\right)$$
$$2x+20<2x+10$$
$$20<10 \qquad \text{False}$$

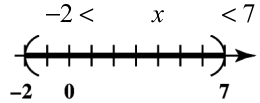
This is a false statement, so the inequality is a contradiction. The solution set is $\varnothing$.

47. The goal is to isolate the variable x.
$$-4< \quad x-5 \quad <6$$
$$-4+5< \quad x-5+5 \quad <6+5 \qquad \text{Add 5.}$$
$$1< \quad x \quad <11$$



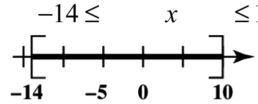
The solution set is the interval (1, 11).

48. The goal is to isolate the variable x.
$$-1< \quad x+1 \quad <8$$
$$-1-1< \quad x+1-1 \quad <8-1 \qquad \text{Subtract 1.}$$
$$-2< \quad x \quad <7$$



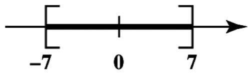
The solution set is the interval $(-2, 7)$.

49. $$-9\le \quad x+5 \quad \le 15$$
$$-9-5\le \quad x+5-5 \quad \le 15-5 \qquad \text{Subtract 5.}$$
$$-14\le \quad x \quad \le 10$$



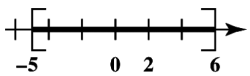
The solution set is the interval $[-14, 10]$.

50.
$$\begin{aligned} -4 &\le x+3 \le 10 \\ -4-3 &\le x+3-3 \le 10-3 \quad \text{Subtract 3.} \\ -7 &\le x \le 7 \end{aligned}$$



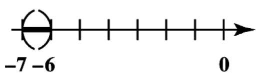
The solution set is the interval $[-7, 7]$.

51.
$$\begin{aligned} -6 &\le 2x+4 \le 16 \\ -10 &\le 2x \le 12 \quad \text{Subtract 4.} \\ -5 &\le x \le 6 \quad \text{Divide by 2.} \end{aligned}$$



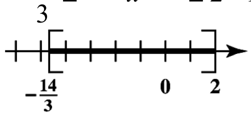
The solution set is the interval $[-5, 6]$.

52.
$$\begin{aligned} -15 &< 3x+6 < -12 \\ -21 &< 3x < -18 \quad \text{Subtract 6.} \\ -7 &< x < -6 \quad \text{Divide by 3.} \end{aligned}$$



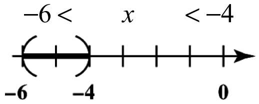
The solution set is the interval $(-7, -6)$.

53.
$$\begin{aligned} -19 &\le 3x-5 \le 1 \\ -14 &\le 3x < 6 \quad \text{Add 5.} \\ -\frac{14}{3} &\le x \le 2 \quad \text{Divide by 3.} \end{aligned}$$



The solution set is the interval $\left[-\frac{14}{3}, 2\right]$.

54.
$$\begin{aligned} -16 &< 3x+2 < -10 \\ -18 &< 3x < -12 \quad \text{Subtract 2.} \\ -6 &< x < -4 \quad \text{Divide by 3.} \end{aligned}$$

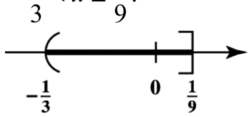


The solution set is the interval $(-6, -4)$.

55.
$$\begin{aligned} 4 &\le -9x+5 < 8 \\ -1 &\le -9x < 3 \quad \text{Subtract 5.} \\ \frac{1}{9} &\ge x > -\frac{1}{3} \quad \text{Divide by } -9. \text{ Reverse inequalities.} \end{aligned}$$

The last inequality may be written as

$$-\frac{1}{3} < x \le \frac{1}{9}.$$

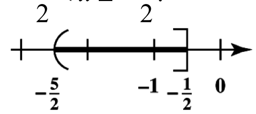


The solution set is the interval $\left(-\frac{1}{3}, \frac{1}{9}\right]$.

56.
$$\begin{aligned} 4 &\le -2x+3 < 8 \\ 1 &\le -2x < 5 \quad \text{Subtract 3.} \\ -\frac{1}{2} &\ge x > -\frac{5}{2} \quad \text{Divide by } -2. \text{ Reverse inequalities.} \end{aligned}$$

The last inequality may be written as

$$-\frac{5}{2} < x \le -\frac{1}{2}.$$

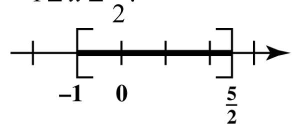


The solution set is the interval $\left(-\frac{5}{2}, -\frac{1}{2}\right]$.

57.
$$\begin{aligned} -8 &\le -4x+2 \le 6 \\ -10 &\le -4x \le 4 \quad \text{Subtract 2.} \\ \frac{5}{2} &\ge x \ge -1 \quad \text{Divide by } -4. \text{ Reverse inequalities.} \end{aligned}$$

The last inequality may be written as

$$-1 \le x \le \frac{5}{2}.$$

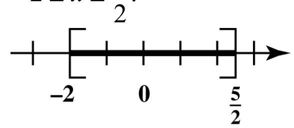


The solution set is the interval $\left[-1, \frac{5}{2}\right]$.

58.
$$\begin{aligned} -12 &\le -6x+3 \le 15 \\ -15 &\le -6x \le 12 \quad \text{Subtract 3.} \\ \frac{5}{2} &\ge x \ge -2 \quad \text{Divide by } -6. \text{ Reverse inequalities.} \end{aligned}$$

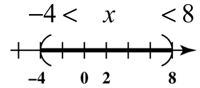
The last inequality may be written as

$$-2 \le x \le \frac{5}{2}.$$



The solution set is the interval $\left[-2, \frac{5}{2}\right]$.

59.
$$\begin{aligned} -3 &< \frac{3}{4}x < 6 \\ -12 &< 3x < 24 \quad \text{Multiply by 4.} \\ -4 &< x < 8 \quad \text{Divide by 3.} \end{aligned}$$



The solution set is the interval $(-4, 8)$.