

Fluid Mechanics 3rd edition Hibbeler Solutions Manual

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Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-1.

Show that Pascal's law applies within a fluid that is accelerating, provided there are no shearing stresses acting within the fluid.

SOLUTION

Consider the free-body diagram of a triangular element of fluid as shown in Fig. 2-2b. If this element has acceleration components of a_x, a_y, a_z , then since $dm = \rho dV$, the equations of motion in the y and z directions give

$$\Sigma F_y = dma_y; \quad p_y(\Delta x)(\Delta s \sin \theta) - [p(\Delta x \Delta s)] \sin \theta = \rho \left(\frac{1}{2} \Delta x (\Delta s \cos \theta) (\Delta s \sin \theta) \right) a_y$$

$$\Sigma F_z = dma_z; \quad p_z(\Delta x)(\Delta s \cos \theta) - [p(\Delta x \Delta s)] \cos \theta - \gamma \left[\frac{1}{2} \Delta x (\Delta s \cos \theta) (\Delta s \sin \theta) \right] = \rho \left(\frac{1}{2} \Delta x (\Delta s \cos \theta) (\Delta s \sin \theta) \right) a_z$$

Dividing by $\Delta x \Delta s$ and letting $\Delta s \rightarrow 0$, so the element reduces in size, we obtain

$$p_y = p$$

$$p_z = p$$

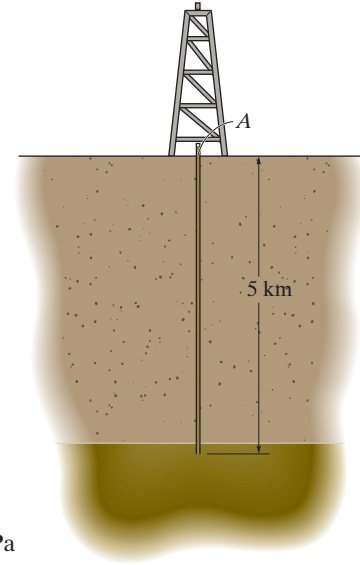
By a similar argument, the element can be rotated 90° about the z axis and $\Sigma F_x = dma_x$ can be applied to show $p_x = p$. Since the angle θ of the inclined face is *arbitrary*, this indeed shows that the pressure at a point is the *same in all directions* for any fluid that has no shearing stress acting within it.

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2-2.

The oil derrick has drilled 5 km into the ground before it strikes a crude oil reservoir. When this happens, the pressure at the well head A becomes 25 MPa. Drilling “mud” is to be placed into the entire length of pipe to displace the oil and balance this pressure. What should be the density of the mud so that the pressure at A becomes zero? Take $\rho_o = 880 \text{ kg/m}^3$.



SOLUTION

Consider the case when the crude oil is pushing out at A where $p_A = 25(10^6) \text{ Pa}$, Fig. a . Here, $\rho_o = 880 \text{ kg/m}^3$, hence $p_o = \rho_o gh = (880 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5000 \text{ m}) = 43.164(10^6) \text{ Pa}$

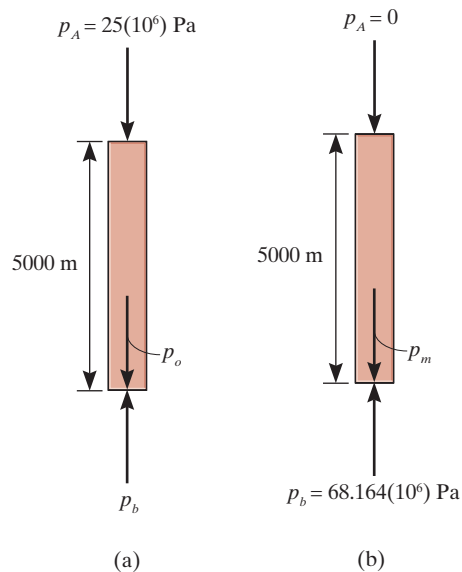
$$p_b = p_A + p_o = 25(10^6) \text{ Pa} + 43.164(10^6) \text{ Pa} = 68.164(10^6) \text{ Pa}$$

It is required that $p_A = 0$, Fig. b . Thus,

$$p_b = p_m = \rho_m gh \quad \text{TBEXAM.COM}$$

$$68.164(10^6) \frac{\text{N}}{\text{m}^2} = \rho_m (9.81 \text{ m/s}^2)(5000 \text{ m})$$

$$\rho_m = 1390 \text{ kg/m}^3 = 1.39 \text{ Mg/m}^3 \quad \text{Ans.}$$



Ans:
 $\rho_m = 1.39 \text{ Mg/m}^3$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–3.

In 1896, S. Riva-Rocci developed the prototype of the current sphygmomanometer, a device used to measure blood pressure. When it was worn as a cuff around the upper arm and inflated, the air pressure within the cuff was connected to a mercury manometer. If the reading for the high (or systolic) pressure is 120 mm and for the low (or diastolic) pressure is 80 mm, determine these pressures in psi and pascals.

SOLUTION

Mercury is considered to be incompressible. From Appendix A, the density of mercury is $\rho_{\text{Hg}} = 13\,550 \text{ kg/m}^3$. Thus, the systolic pressure is

$$p_s = \rho_{\text{Hg}} g h_s = (13\,550 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.12 \text{ m}) = 15.95(10^3) \text{ Pa} \\ = 16.0 \text{ kPa} \quad \textbf{Ans.}$$

$$p_s = \left[15.95(10^3) \frac{\text{N}}{\text{m}^2} \right] \left(\frac{1 \text{ lb}}{4.4482 \text{ N}} \right) \left(\frac{0.3048 \text{ m}}{1 \text{ ft}} \right)^2 \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 = 2.31 \text{ psi} \quad \textbf{Ans.}$$

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The diastolic pressure is

$$p_d = \rho_{\text{Hg}} g h_d = (13\,550 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.08 \text{ m}) = 10.63(10^3) \text{ Pa} \\ = 10.6 \text{ kPa} \quad \textbf{Ans.}$$

$$p_d = \left[10.63(10^3) \frac{\text{N}}{\text{m}^2} \right] \left(\frac{1 \text{ lb}}{4.4482 \text{ N}} \right) \left(\frac{0.3048 \text{ m}}{1 \text{ ft}} \right)^2 \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 = 1.54 \text{ psi} \quad \textbf{Ans.}$$

Ans:

$$p_s = 16.0 \text{ kPa} = 2.31 \text{ psi}$$

$$p_d = 10.6 \text{ kPa} = 1.54 \text{ psi}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-4.**

Show why water would not be a good fluid to use for a barometer, by calculating the height to which standard atmospheric pressure will elevate it in a glass tube. Compare this result with that of mercury. Take $\gamma_w = 62.4 \text{ lb/ft}^3$, $\gamma_{\text{Hg}} = 847 \text{ lb/ft}^3$.

SOLUTION

For water barometer, Fig. *a*,

$$p_w = \gamma_w h_w = p_{\text{atm}}$$

$$\left(62.4 \frac{\text{lb}}{\text{ft}^3}\right) h_w = \left(14.7 \frac{\text{lb}}{\text{in}^2}\right) \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)^2$$

$$h_w = 33.92 \text{ ft} = 33.9 \text{ ft}$$

Ans.

For mercury barometer, Fig. *b*,

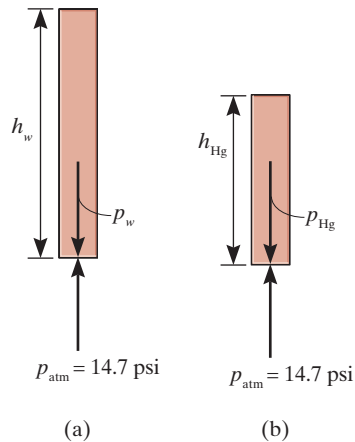
$$p_{\text{Hg}} = \gamma_{\text{Hg}} h_{\text{Hg}} = p_{\text{atm}}$$

$$\left(847 \frac{\text{lb}}{\text{ft}^3}\right) h_{\text{Hg}} = \left(14.7 \frac{\text{lb}}{\text{in}^2}\right) \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)^2$$

$$h_{\text{Hg}} = (2.4992 \text{ ft}) \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) = 30.0 \text{ in.}$$

Ans.

A water barometer is not suitable since it requires a very long tube.



Ans:

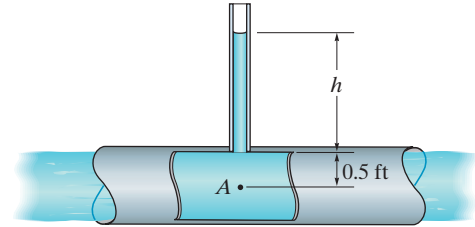
$$h_w = 33.9 \text{ ft}$$

$$h_{\text{Hg}} = 30.0 \text{ in.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-5.

If the piezometer measures a gage pressure of 10 psi at point A , determine the height h of the water in the tube. Compare this height with that using mercury. Take $\rho_w = 1.94 \text{ slug/ft}^3$ and $\rho_{\text{Hg}} = 26.3 \text{ slug/ft}^3$.



SOLUTION

Here, the absolute pressure to be measured is

$$p = p_g + p_{\text{atm}} = \left(10 \frac{\text{lb}}{\text{in}^2}\right) \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)^2 + p_{\text{atm}} = (1440 + p_{\text{atm}}) \frac{\text{lb}}{\text{ft}^2}$$

For the water piezometer, Fig. a ,

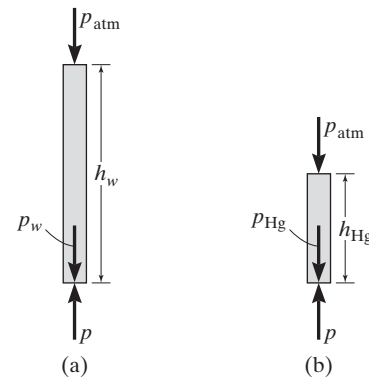
$$p = p_{\text{atm}} + p_w; \quad (1440 + p_{\text{atm}}) \frac{\text{lb}}{\text{ft}^2} = p_{\text{atm}} + (1.94 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(h + 0.5 \text{ ft})$$

$$h_w = 22.55 \text{ ft} = 22.6 \text{ ft} \quad \text{Ans.}$$

For the mercury piezometer, Fig. b ,

$$p = p_{\text{atm}} + p_{\text{Hg}}; \quad (1440 + p_{\text{atm}}) \frac{\text{lb}}{\text{ft}^2} = p_{\text{atm}} + (26.3 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(h + 0.5 \text{ ft})$$

$$h_{\text{Hg}} = 1.20 \text{ ft} \quad \text{Ans.}$$



Ans:

$$h_w = 22.6 \text{ ft}$$

$$h_{\text{Hg}} = 1.20 \text{ ft}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–6.

If the absolute pressure in a tank is 140 kPa, determine the pressure head in mm of mercury. The atmospheric pressure is 100 kPa.

SOLUTION

$$p_{\text{abs}} = p_{\text{atm}} + p_g$$

$$140 \text{ kPa} = 100 \text{ kPa} + p_g$$

$$p_g = 40 \text{ kPa}$$

From Appendix A, $\rho_{\text{Hg}} = 13\,550 \text{ kg/m}^3$.

$$p_g = \gamma_{\text{Hg}} h_{\text{Hg}}$$

$$40(10^3) \text{ N/m}^2 = (13\,550 \text{ kg/m}^3)(9.81 \text{ m/s}^2)h_{\text{Hg}}$$

$$h_{\text{Hg}} = 0.3009 \text{ m} = 301 \text{ mm}$$

Ans.

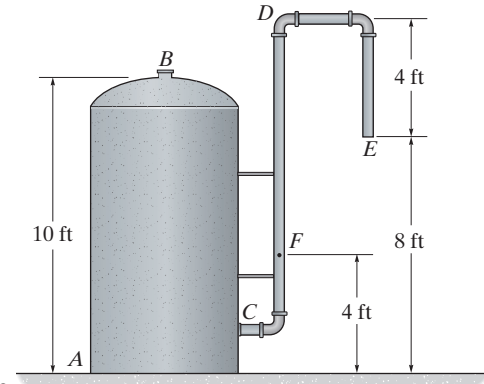
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Ans:
 $h_{\text{Hg}} = 301 \text{ mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-7.

The field storage tank is filled with oil. The standpipe is connected to the tank at C , and the system is open to the atmosphere at B and E . Determine the maximum pressure in the tank in psi if the oil reaches a level of F in the pipe. Also, at what level should the oil be in the tank, so that the maximum pressure occurs in the tank? What is this value? Take $\rho_o = 1.78 \text{ slug/ft}^3$.



SOLUTION

Since the top of the tank is open to the atmosphere, the free surface of the oil in the tank will be the same height as that of point F . Thus, the maximum pressure which occurs at the base of the tank (level A) is

$$\begin{aligned}(p_A)_g &= \gamma h \\ &= (1.78 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(4 \text{ ft}) \\ &= 229.26 \frac{\text{lb}}{\text{ft}^2} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 = 1.59 \text{ psi} \quad \text{Ans.}\end{aligned}$$

Maximum pressure occurs at the base of the tank (level A) when the oil reaches level B . TBEXAM.COM **Ans.**

$$\begin{aligned}(p_A)_{\max} &= \gamma h \\ &= (1.78 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(10 \text{ ft}) \\ &= 573.16 \text{ lb/ft}^2 \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 = 3.98 \text{ psi} \quad \text{Ans.}\end{aligned}$$

Ans:

$$(p_A)_g = 1.59 \text{ psi}$$

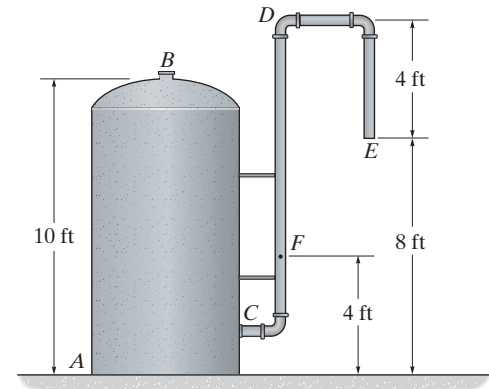
Maximum pressure occurs at base A when the oil reaches level B .

$$(p_A)_{\max} = 3.98 \text{ psi}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-8.**

The field storage tank is filled with oil. The standpipe is connected to the tank at C and open to the atmosphere at E . Determine the maximum pressure that can be developed in the tank if the oil has a density of 1.78 slug/ft^3 . Where does this maximum pressure occur? Assume that there is no air trapped in the tank and that the top of the tank at B is closed.



SOLUTION

Level D is the highest the oil is allowed to rise in the tube, and the maximum gage pressure occurs at the base of the tank (level A).

$$\begin{aligned}(p_A)_{\max} &= \gamma h \\ &= (1.78 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(8 \text{ ft} + 4 \text{ ft}) \\ &= \left(687.79 \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 = 4.78 \text{ psi}\end{aligned}$$

Ans.

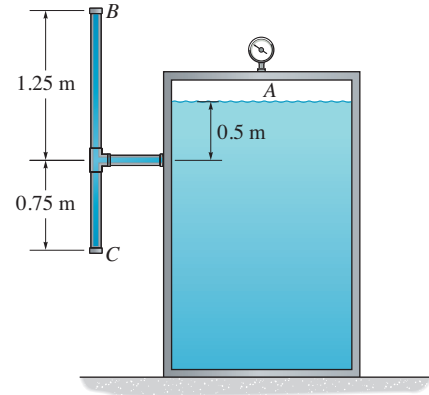
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Ans:
 $(p_A)_{\max} = 4.78 \text{ psi}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–9.

The pipes connected to the closed tank are completely filled with water. If the pressure at A is 300 kPa, determine the force acting on the inside of the end caps at B and C if the pipe has an inner diameter of 60 mm.



SOLUTION

$$p_B = 300(10^3) \text{ N/m}^2 + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.5 \text{ m}) - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.25 \text{ m}) = 292.64(10^3) \text{ N/m}^2$$

$$p_C = 300(10^3) \text{ N/m}^2 + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.5 \text{ m}) + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.75 \text{ m}) = 312.26(10^3) \text{ N/m}^2$$

Thus, the forces due to pressure acting on the cap at B and C are

$$F_B = p_B A = [292.64(10^3) \text{ N/m}^2] [\pi (0.03 \text{ m})^2] = 827.43 \text{ N} = 827 \text{ N}$$

Ans.

$$F_C = p_C A = [312.26(10^3) \text{ N/m}^2] [\pi (0.03 \text{ m})^2] = 882.90 \text{ N} = 883 \text{ N}$$

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Ans.

Ans:

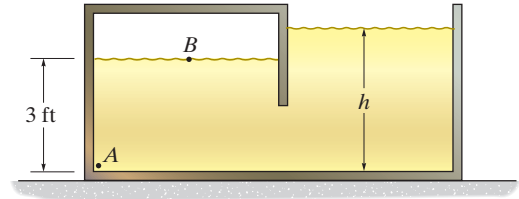
$$F_B = 827 \text{ N}$$

$$F_C = 883 \text{ N}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–10.

Determine the gage pressure developed at point A and at the air surface B within the enclosure if $h = 4.5 \text{ ft}$. The liquid is alcohol, where $\gamma_{al} = 47.2 \text{ lb/ft}^3$.



SOLUTION

The atmospheric gage pressure is $(p_{\text{atm}})_g = 0$. Thus the gage pressures at points A and B are

$$\begin{aligned}(p_A)_g &= (p_{\text{atm}})_g + \gamma_{al} h_A \\ &= 0 + (47.2 \text{ lb/ft}^3)(4.5 \text{ ft}) \\ &= (212.4) \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\ &= 1.475 \text{ psi} = 1.48 \text{ psi}\end{aligned}$$

Ans.

$$\begin{aligned}(p_B)_g &= (p_A)_g - \gamma_{al} h_B \\ &= 212.4 \text{ lb/ft}^2 - (47.2 \text{ lb/ft}^3)(3 \text{ ft}) \\ &= (70.8 \text{ lb/ft}^2) \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\ &= 0.4917 \text{ psi} = 0.492 \text{ psi}\end{aligned}$$

Ans.

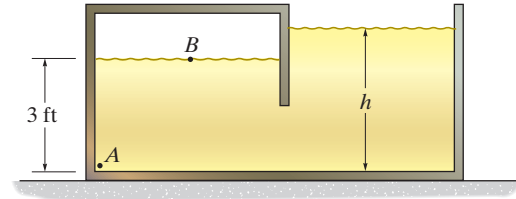
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Ans:
 $(p_A)_g = 1.48 \text{ psi}$
 $(p_B)_g = 0.492 \text{ psi}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–11.

If the gage pressure in the enclosure is $p_B = 0.7 \text{ psi}$, determine the gage pressure developed at point A and the height h of the alcohol level outside the enclosure. Take $\gamma_{al} = 47.2 \text{ lb/ft}^3$.



SOLUTION

The gage pressure at point A is

$$(p_A)_g = (p_B)_g + \gamma_{al} h_{AB}$$

$$= 0.7 \text{ psi} + (47.2 \text{ lb/ft}^3)(3 \text{ ft}) \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2$$

$$= 1.683 \text{ psi} = 1.68 \text{ psi}$$

Ans.

The atmospheric gage pressure is $(p_{\text{atm}})_g = 0$. Thus,

$$(p_B)_g = (p_{\text{atm}})_g + \gamma_{al} h_B$$

$$(0.7 \text{ lb/in}^2) \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right)^2 = 0 + (47.2 \text{ lb/ft}^3)(h - 3 \text{ ft})$$

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$$h = 5.136 \text{ ft} = 5.14 \text{ ft}$$

Ans.

Ans:

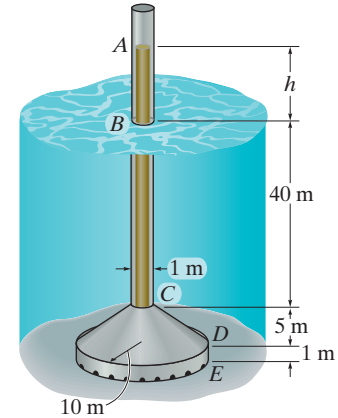
$$(p_A)_g = 1.68 \text{ psi}$$

$$h = 5.14 \text{ ft}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-12.**

The structure shown is used for the temporary storage of crude oil at sea for later loading into ships. When it is not filled with oil, the water level in the stem is at B (sea level). Why? As the oil is loaded into the stem, the water is displaced through exit ports at E . If the stem is filled with oil, that is, to the depth of C , determine the height h of the oil level above sea level. Take $\rho_o = 900 \text{ kg/m}^3$, $\rho_w = 1020 \text{ kg/m}^3$.



SOLUTION

The water level remains at B when empty because the gage pressure at B must be zero. It is required that the pressure at C caused by the water and oil be the same. Then

$$(p_C)_w = (p_C)_o$$

$$\rho_w g h_w = \rho_o g h_o$$

$$(1020 \text{ kg/m}^3)(g)(40 \text{ m}) = (900 \text{ kg/m}^3)(g)(40 \text{ m} + h)$$

$$h = 5.33 \text{ m}$$

Ans.

Ans.

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Ans:

Because the gage pressure at B must be zero.

$$h = 5.33 \text{ m}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–13.

If the water in the structure in Prob. 2–12 is displaced by crude oil to the level D at the bottom of the cone, then how high h will the oil extend above sea level? Take $\rho_o = 900 \text{ kg/m}^3$, $\rho_w = 1020 \text{ kg/m}^3$.

SOLUTION

It is required that the pressure at D caused by the water and oil be the same.

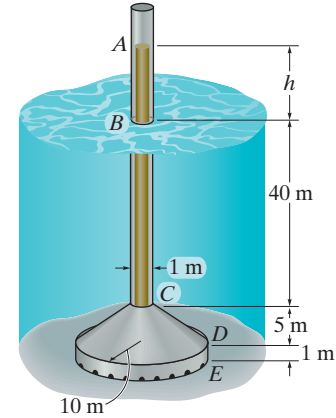
$$(p_D)_w = (p_D)_o$$

$$\rho_w g h_w = \rho_o g h_o$$

$$(1020 \text{ kg/m}^3)(g)(45 \text{ m}) = (900 \text{ kg/m}^3)(g)(45 \text{ m} + h)$$

$$h = 6.00 \text{ m}$$

Ans.



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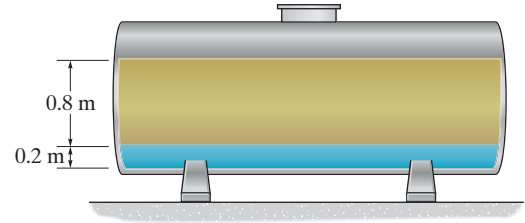
Ans:

$$h = 6.00 \text{ m}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–14.

The tank is filled with water and gasoline at a temperature of 20°C to the depths shown. If the absolute air pressure at the top of the tank is 200 kPa , determine the gage pressure at the bottom of the tank. Would the result be different if the tank had a flat bottom rather than a curved one? The atmospheric pressure is 101 kPa .



SOLUTION

The absolute pressure at the bottom of tank is

$$(p_b)_{\text{abs}} = p_{\text{air}} + p_g + p_w$$

Here, p_g and p_w can be determined with $\rho_g = 726 \text{ kg/m}^3$ and $\rho_w = 998.3 \text{ kg/m}^3$ at $T = 20^\circ\text{C}$ (table in Appendix A). Then

$$\begin{aligned} (p_b)_{\text{abs}} &= 200(10^3) \text{ N/m}^2 + (726 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.8 \text{ m}) + (998.3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.2 \text{ m}) \\ &= 207.66 \text{ kPa} \end{aligned}$$

Thus, the gage pressure at the bottom of the tank can be determined using

$$\begin{aligned} (p_b)_{\text{abs}} &= (p_b)_{\text{gage}} + p_{\text{atm}} \\ 207.66 \text{ kPa} &= (p_b)_{\text{gage}} + 101 \text{ kPa} \\ (p_b)_{\text{gage}} &= 106.66 \text{ kPa} = 107 \text{ kPa} \end{aligned}$$

Ans.

No, the pressure at the bottom of the tank does not depend on its shape.

Ans.

Ans:

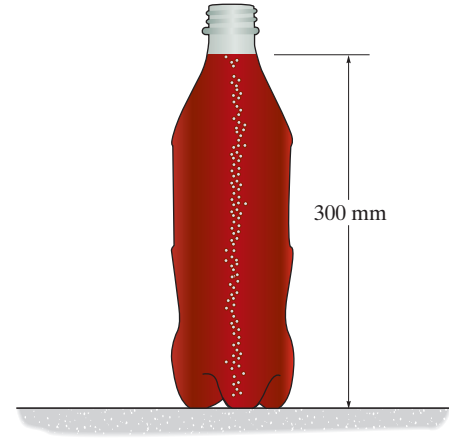
$$(p_b)_{\text{gage}} = 107 \text{ kPa}$$

The pressure at the bottom of the tank does not depend on its shape.

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–15.

At the bottom of an open bottle a bubble within carbonated water has a diameter of 0.2 mm. Determine the bubble's diameter when it reaches the surface. The temperature of the water and bubbles is 10°C , and the atmospheric pressure is 101 kPa. Assume that the density of the water is the same as that of pure water.



SOLUTION

Applying the ideal gas law, $p = \rho RT$, of which T is constant in this case. Thus,

$$\frac{p}{\rho} = \text{constant}$$

Since $\rho = \frac{m}{V}$, where m , the mass of the CO_2 in the bubble, is also constant, then

$$\frac{p}{m/V} = \text{constant}$$

$$pV = \text{constant} \quad (1)$$

At $T = 10^\circ\text{C}$, $\rho_w = 999.7 \text{ kg/m}^3$. The pressure due to the static water can be determined using $p = \gamma h$. Then the absolute pressure at the bottom of the bottle can be determined from

$$\begin{aligned} p_b &= p_{\text{atm}} + \gamma_w h_w \\ &= 101(10^3) \text{ N/m}^2 + (999.7 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \text{ m}) \\ &= 103.94(10^3) \text{ Pa} = 103.94 \text{ kPa} \end{aligned}$$

At the surface of the bottle, the absolute pressure is

$$p_s = p_{\text{atm}} = 101 \text{ kPa}$$

Using Eq. (1), we can write

$$\begin{aligned} p_b V_b &= p_s V_s \\ (103.94 \text{ kPa}) \left[\frac{4}{3} \pi (0.1 \text{ mm})^3 \right] &= (101 \text{ kPa}) \left[\frac{4}{3} \pi \left(\frac{d_b}{2} \right)^3 \right] \\ d_b &= 0.201923 \text{ mm} \\ &= 0.202 \text{ mm} \end{aligned}$$

Ans.

Ans:
 $d_b = 0.202 \text{ mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-16.**

If air at 25°C is at an atmospheric pressure of 104 kPa at the ground floor of a skyscraper, determine the absolute pressure at the top of the building, which has an elevation of 300 m . Assume that the temperature is constant and that air is compressible. Work the problem again assuming that air is incompressible. Round off the answers to five significant figures.

SOLUTION

From the table in Appendix A, $R = 286.9 \text{ J/[kg} \cdot \text{K}]$. Here, $z_0 = 0$, $z = 300 \text{ m}$ and $T_0 = (25 + 273) \text{ K} = 298 \text{ K}$. Considering air to be compressible, apply Eq. 2-8.

$$p = p_0 e^{-(g/RT_0)(z-z_0)}$$

$$p = (104 \text{ kPa}) e^{-9.81/[(286.9)(298)](300-0)}$$

$$= 100.48 \text{ kPa (abs.)}$$

Ans.

If air is considered incompressible, the ideal gas law gives

$$p_0 = \rho_0 R T_0; \quad 104(10^3) \text{ N/m}^2 = \rho_0 (286.9 \text{ J/[kg} \cdot \text{K}])(298 \text{ K})$$

$$\rho_0 = 1.2164 \text{ kg/m}^3$$

Then

$$p = p_0 - \rho_0 g h$$

$$= 104(10^3) \text{ N/m}^2 - (1.2164 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(300 \text{ m})$$

$$= 100.42 \text{ kPa (abs.)}$$

Ans.

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Ans:

For compressible air, $p = 100.48 \text{ kPa (abs.)}$

For incompressible air, $p = 100.42 \text{ kPa (abs.)}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–17.

If air at 75°F is at an atmospheric pressure of 15.1 psi at the ground floor of a skyscraper, determine the absolute pressure at the top of the building, which has an elevation of 1200 ft . Assume that the temperature is constant and that air is compressible. Work the problem again assuming that air is incompressible. Round off the answers to four significant figures.

SOLUTION

From the table in Appendix A, $R = 1716 \text{ ft} \cdot \text{lb}/[\text{slug} \cdot ^\circ\text{R}]$. Here, $z_0 = 0$, $z = 1200 \text{ ft}$ and $T_0 = (75 + 460)^\circ\text{R} = 535^\circ\text{R}$. Considering air to be compressible, apply Eq. 2-8.

$$p = p_0 e^{-(g/RT_0)(z - z_0)}$$

$$p = (15.1 \text{ psia}) e^{-[32.2/1716/535](1200 - 0)}$$

$$= 14.48 \text{ psia}$$

Ans.

If air is considered incompressible, the ideal gas law gives

$$p_0 = \rho_0 R T_0; \quad (15.1 \text{ lb/in}^2) \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right)^2 = \rho_0 (1716 \text{ ft} \cdot \text{lb}/[\text{slug} \cdot ^\circ\text{R}]) (535^\circ\text{R})$$

$$\rho_0 = 0.002368 \text{ slug/ft}^3$$

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Then

$$p = p_0 - \gamma_0 h$$

$$= (15.1 \text{ lb/in}^2) \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right)^2 - (0.002368 \text{ slug/ft}^3) (32.2 \text{ ft/s}^2) (1200 \text{ ft})$$

$$= (2082.88 \text{ lb/ft}^2) \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 = 14.46 \text{ psia}$$

Ans.

Ans:

For compressible air, $p = 14.48 \text{ psia}$

For incompressible air, $p = 14.46 \text{ psia}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–18.

As the weather balloon ascends, measurements indicate that the absolute temperature begins to decrease linearly from T_0 at $z = 0$ to T_f at $z = h$. If the absolute pressure of the air at $z = 0$ is p_0 , determine the pressure as a function of z .

SOLUTION

We will first determine the absolute temperature as a function of z :

$$T = \left[T_0 - \left(\frac{T_0 - T_f}{h} \right) z \right] = \frac{T_0 h - (T_0 - T_f) z}{h}$$

Using this result to apply the ideal gas law,

$$\begin{aligned} p &= \rho RT; \quad \rho = \frac{p}{RT} = \frac{p}{R \left[\frac{T_0 h - (T_0 - T_f) z}{h} \right]} \\ &= \frac{ph}{R [T_0 h - (T_0 - T_f) z]} \end{aligned}$$

However,

$$dp = -\gamma dz = -\rho g dz$$

Substitute the result of ρ into this equation:

$$\begin{aligned} dp &= -\frac{phg dz}{R [T_0 h - (T_0 - T_f) z]} \\ \frac{dp}{p} &= -\frac{gh}{R} \left[\frac{dz}{T_0 h - (T_0 - T_f) z} \right] \end{aligned}$$

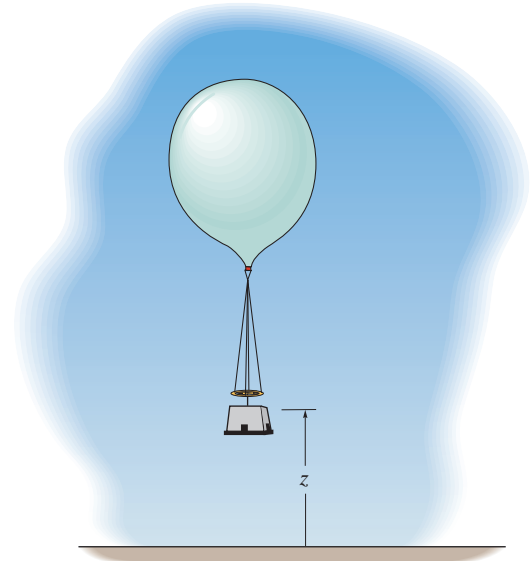
When $z = 0, p = p_0$. Then

$$\begin{aligned} \int_{p_0}^p \frac{dp}{p} &= -\frac{gh}{R} \int_0^z \frac{dz}{T_0 h - (T_0 - T_f) z} \\ \ln p \Big|_{p_0}^p &= -\frac{gh}{R} \left\{ -\frac{1}{T_0 - T_f} \ln [T_0 h - (T_0 - T_f) z] \right\} \Big|_0^z \\ \ln \frac{p}{p_0} &= \frac{gh}{R(T_0 - T_f)} \ln \left[\frac{T_0 h - (T_0 - T_f) z}{T_0 h} \right] \\ \ln \frac{p}{p_0} &= \ln \left\{ \left[\frac{T_0 h - (T_0 - T_f) z}{T_0 h} \right]^{\frac{gh}{R(T_0 - T_f)}} \right\} \\ \frac{p}{p_0} &= \left[\frac{T_0 h - (T_0 - T_f) z}{T_0 h} \right]^{\frac{gh}{R(T_0 - T_f)}} \\ p &= p_0 \left[1 - \left(\frac{T_0 - T_f}{T_0 h} \right) z \right]^{\frac{gh}{R(T_0 - T_f)}} \end{aligned}$$

Ans.

Ans:

$$p = p_0 \left[1 - \left(\frac{T_0 - T_f}{T_0 h} \right) z \right]^{\frac{gh}{R(T_0 - T_f)}}$$



Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–19.

As the balloon ascends, measurements indicate that the temperature begins to decrease at a constant rate, from $T = 60^\circ\text{F}$ at $z = 0$ to $T = 50^\circ\text{F}$ at $z = 2500 \text{ ft}$. If the absolute pressure of the air at $z = 0$ is $p = 14.7 \text{ psi}$, plot the variation of pressure (vertical axis) versus altitude for $0 \leq z \leq 2500 \text{ ft}$. Give values for increments of $\Delta z = 500 \text{ ft}$.

SOLUTION

We will first determine the absolute temperature as a function of z .

$$T = \left[520 - \left(\frac{520 - 510}{2500} \right) z \right] ^\circ\text{R} = (520 - 0.004z) ^\circ\text{R}$$

Using this result to apply the ideal gas law with $R = 1716 \text{ ft} \cdot \text{lb}/[\text{slug} \cdot ^\circ\text{R}]$

$$p = \rho RT; \quad \rho = \frac{p}{RT} = \frac{p}{1716(520 - 0.004z)} = \frac{p}{892.32(10^3) - 6.864z}$$

However,

$$dp = -\gamma dz = -\rho g dz$$

Substitute the result of ρ into this equation:

$$dp = -\frac{p(32.2)}{892.32(10^3) - 6.864z} dz$$

$$\frac{dp}{p} = -\frac{32.2 dz}{892.32(10^3) - 6.864z}$$

When $z = 0$, $p = \left(14.7 \frac{\text{lb}}{\text{in}^2} \right) \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right)^2 = 2116.8 \text{ lb/ft}^2$. Then

$$\int_{2116.8}^p \frac{dp}{p} = -32.2 \int_0^z \frac{dz}{892.32(10^3) - 6.864z}$$

$$\ln p \Big|_{2116.8}^p = -32.2 \left\{ -\frac{1}{6.864} \ln [892.32(10^3) - 6.864z] \right\} \Big|_0^z$$

$$\ln \left(\frac{p}{2116.8} \right) = 4.6911 \ln \left[\frac{892.32(10^3) - 6.864z}{892.32(10^3)} \right]$$

$$\ln \left(\frac{p}{2116.8} \right) = \ln \left\{ \left[\frac{892.32(10^3) - 6.864z}{892.32(10^3)} \right]^{4.6911} \right\}$$

$$\frac{p}{2116.8} = \left[\frac{892.32(10^3) - 6.864z}{892.32(10^3)} \right]^{4.6911}$$

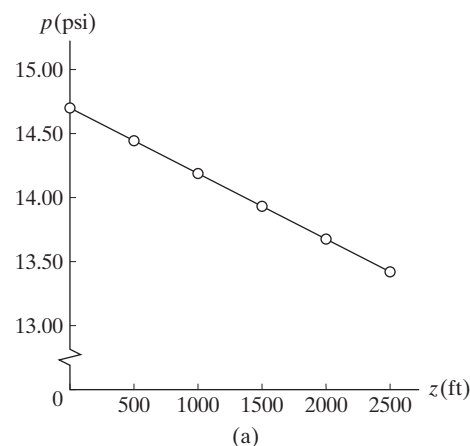
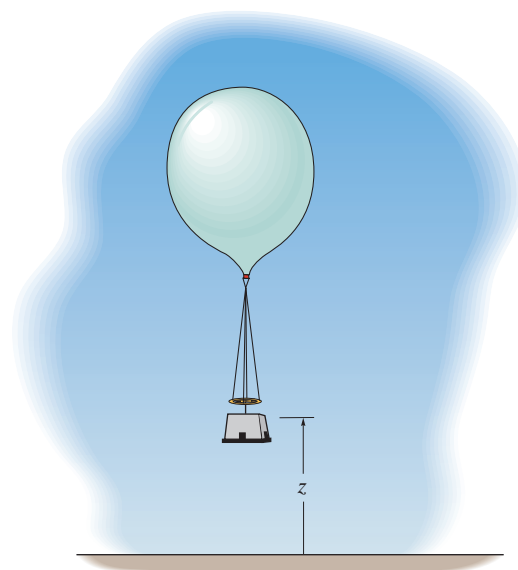
$$p = \left\{ 2116.8 [1 - 7.6923(10^{-6})z]^{4.6911} \frac{\text{lb}}{\text{ft}^2} \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2$$

$$p = \{ 14.7 [1 - 7.6923(10^{-6})z]^{4.6911} \} \text{ psi where } z \text{ is in ft.}$$

For $0 \leq z \leq 2500 \text{ ft}$,

$z(\text{ft})$	0	500	1000	1500	2000	2500
$p(\text{psi})$	14.70	14.44	14.18	13.92	13.67	13.42

The plot of p vs z is shown in Fig. *a*.



Ans:

$$p = \{ 14.7 [1 - 7.69(10^{-6})z]^{4.69} \} \text{ psi}$$

where z is in ft.

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–20.**

The Burj Khalifa in Dubai is currently the world's tallest building. If air at 40°C is at an atmospheric pressure of 105 kPa at the ground floor (sea level), determine the absolute pressure at the top of the building, which has an elevation of 828 m . Assume that the temperature is constant and that air is compressible. Work the problem again assuming that air is incompressible. Round off the answers to four significant figures.

SOLUTION

For compressible air, with $R = 286.9 \text{ J/(kg} \cdot \text{K)}$ (Appendix A), $T_0 = 40^\circ\text{C} + 273 = 313 \text{ K}$, $z_0 = 0$, and $z = 828 \text{ m}$,

$$p = p_0 e^{-(g/RT_0)(z-z_0)}$$

$$p = (105 \text{ kPa}) e^{-9.81/[(286.9)(313)](828-0)}$$

$$= 95.92 \text{ kPa}$$

Ans.

For incompressible air, with $\rho = 1.127 \text{ kg/m}^3$ at $T = 40^\circ\text{C}$ (Appendix A),

$$p = p_0 - \rho gh$$

$$p = 105(10^3) \text{ N/m}^2 - (1.127 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(828 \text{ m})$$

$$= 95.85 \text{ kPa}$$

Ans.

Ans:

For compressible air, $p = 95.92 \text{ kPa}$

For incompressible air, $p = 95.85 \text{ kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–21.

The Burj Khalifa in Dubai is currently the world's tallest building. If air at 100°F is at an atmospheric pressure of 14.7 psi at the ground floor (sea level), determine the absolute pressure at the top of the building, which has an elevation of 2717 ft . Assume that the temperature is constant and that air is compressible. Work the problem again assuming that air is incompressible. Round off the answers to four significant figures.

SOLUTION

For compressible air, with $R = 1716 \text{ ft} \cdot \text{lb}/[\text{slug} \cdot ^\circ\text{R}]$ (Appendix A),
 $T_0 = (100 + 460)^\circ\text{R} = 560^\circ\text{R}$

$$p = p_0 e^{-(g/RT_0)(z - z_0)}$$

$$p = (14.7 \text{ psia}) e^{32.2/1716/560(2717 - 0)}$$

$$= 13.42 \text{ psia}$$

Ans.

For incompressible air, with $\rho = 0.00220 \text{ slug/ft}^3$ at $T = 100^\circ\text{F}$ (Appendix A),

$$p = p_0 - \gamma h$$

$$= 14.7 \text{ lb/in}^2 - (0.00220 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(2717 \text{ ft}) \left(\frac{1 \text{ ft}}{12 \text{ in}} \right)^2$$

$$= 13.36 \text{ psia}$$

Ans.

Ans:

For compressible air, $p = 13.42 \text{ psia}$

For incompressible air, $p = 13.36 \text{ psia}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–22.

A liquid has a density that varies with depth h , such that $\rho = (0.001h + 1.60) \text{ slug/ft}^3$, where h is in feet. Plot the variation of the pressure due to its weight (vertical axis) versus depth for $0 \leq h \leq 100 \text{ ft}$. Give values for increments of 20 ft.

SOLUTION

For the compressible liquid,

$$dp = -\gamma dz = -\rho g dz$$

However, $z = h_0 - h$ where h_0 is a constant, Fig. *a*. Then $dz = -dh$. Substituting this result into the above equation, it becomes

$$dp = \rho g dh$$

Integrating this equation using the gage pressure $p = 0$ at $h = 0$ and p at h , then

$$\int_0^p dp = \int_0^h (0.001h + 1.60)(32.2) dh$$

$$p = \left[(0.0161h^2 + 51.52h) \frac{\text{lb}}{\text{ft}^2} \right] \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2$$

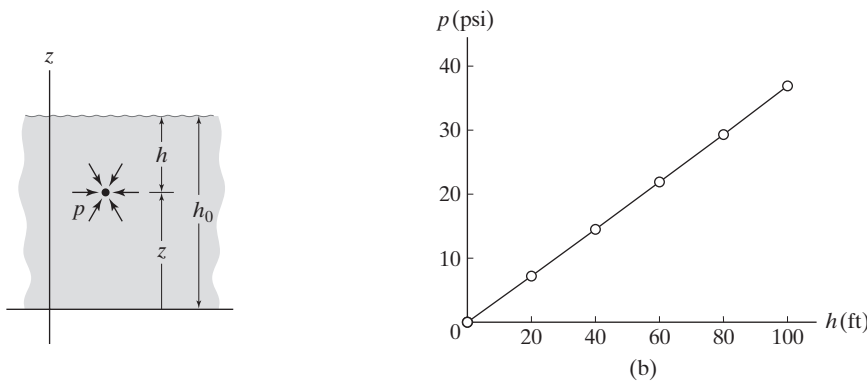
$$= [0.1118(10^{-3})h^2 + 0.3578h] \text{ psi} \quad \text{where } h \text{ is in feet.}$$

Ans.

For $0 \leq h \leq 100 \text{ ft}$,

$h(\text{ft})$	0	20	40	60	80	100
$p(\text{psi})$	0	7.20	14.5	21.9	29.3	36.9

The plot of p vs h is shown in Fig. *b*.



Ans:

$$p = [0.112(10^{-3})h^2 + 0.358h] \text{ psi}$$

where h is in feet.

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–23.

In the troposphere the absolute temperature of the air varies with elevation such that $T = T_0 - Cz$, where C is a constant. Using Fig. 2–11, determine the constants, T_0 and C . If $p_0 = 101 \text{ kPa}$ at $z_0 = 0$, determine the absolute pressure in the air at an elevation of 5 km.

SOLUTION

From Fig. 2–11, $T_C = 15^\circ\text{C}$ at $z = 0$. Then

$$15^\circ\text{C} = T_0 - C(0)$$

$$T_0 = 15^\circ\text{C}$$

Ans.

Also, $T_C = -56.5^\circ\text{C}$ at $z = 11.0(10^3) \text{ m}$. Then

$$-56.5^\circ\text{C} = 15^\circ\text{C} - C[11.0(10^3)\text{m}]$$

$$C = 6.50(10^{-3})^\circ\text{C/m}$$

Ans.

Thus,

$$T_C = [15 - 6.50(10^{-3})z]^\circ\text{C}$$

The absolute temperature is therefore

$$T = (15 - 6.50(10^{-3})z) + 273 = [288 - 6.50(10^{-3})z] \text{ K} \quad (1)$$

Substitute the ideal gas law $p = \rho RT$ or $\rho = \frac{p}{RT}$ into $dp = -\gamma dz = -\rho g dz$,

$$dp = -\frac{p}{RT} g dz$$

$$\frac{dp}{p} = -\frac{g}{RT} dz$$

From the table in Appendix A, gas constant for air is $R = 286.9 \text{ J/(kg} \cdot \text{K)}$.

Also, $p = 101 \text{ kPa}$ at $z = 0$. Then

$$\begin{aligned} \int_{101(10^3) \text{ Pa}}^p \frac{dp}{p} &= -\left(\frac{9.81 \text{ m/s}^2}{286.9 \text{ J/(kg} \cdot \text{K)}}\right) \int_0^z \frac{dz}{288 - 6.50(10^{-3})z} \\ \ln p \Big|_{101(10^3) \text{ Pa}}^p &= 5.2605 \ln [288 - 6.50(10^{-3})z] \Big|_0^z \\ \ln \left[\frac{p}{101(10^3)} \right] &= 5.2605 \ln \left[\frac{288 - 6.50(10^{-3})z}{288} \right] \\ p &= 101(10^3) \left[\frac{288 - 6.50(10^{-3})z}{288} \right]^{5.2605} \end{aligned}$$

At $z = 5(10^3) \text{ m}$,

$$\begin{aligned} p &= 101(10^3) \left\{ \frac{288 - [6.50(10^{-3})][5(10^3)]}{288} \right\}^{5.2605} \\ &= 53.8(10^3) \text{ Pa} = 53.8 \text{ kPa} \end{aligned}$$

Ans.

Ans:

$$T_0 = 15^\circ\text{C}$$

$$C = 6.50(10^{-3})^\circ\text{C/m}$$

$$p = 53.8 \text{ kPa (abs.)}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-24.**

Determine the temperature at an elevation of $z = 15 \text{ km}$. Also, what is the pressure at this elevation? Assume the stratosphere begins at $z = 11 \text{ km}$ (see Fig. 2-11), and take $p_0 = 101.3 \text{ kPa}$ at $z_0 = 0$.

SOLUTION

Within the troposphere, $0 \leq z < 11.0(10^3) \text{ m}$, the temperature T_C as a function of z is $T_C = T_0 - Cz$ (the equation of the straight line shown in Fig. 2-11). From this figure, we obtain $T_C = 15^\circ\text{C}$ at $z = 0$. Then

$$15^\circ\text{C} = T_0 - C(0)$$

$$T_0 = 15^\circ\text{C}$$

Also, $T_C = -56.5^\circ\text{C}$ at $z = 11.0(10^3) \text{ m}$. Then

$$-56.5^\circ\text{C} = 15^\circ\text{C} - C[11.0(10^3) \text{ m}]$$

$$C = 6.50(10^{-3})^\circ\text{C/m}$$

Thus,

$$T_C = [15 - 6.50(10^{-3})z]^\circ\text{C}$$

The absolute temperature is therefore

$$T = 15 - 6.50(10^{-3})z + 273 = [288 - 6.50(10^{-3})z] \text{ K} \quad (1)$$

Substitute the ideal gas law $p = \rho RT$ or $\rho = \frac{p}{RT}$ into $dp = -\gamma dz = -\rho g dz$,

$$dp = -\frac{p}{RT} g dz$$

$$\frac{dp}{p} = -\frac{g}{RT} dz \quad (2)$$

From the table in Appendix A, the gas constant for air is $R = 286.9 \text{ J/kg} \cdot \text{K}$. Also, $p = 101.3 \text{ kPa}$ at $z = 0$. Then substitute Eq. (1) into (2).

$$\begin{aligned} \int_{101.3(10^3) \text{ Pa}}^p \frac{dp}{p} &= -\left(\frac{9.81 \text{ m/s}^2}{286.9 \text{ J/kg} \cdot \text{K}}\right) \int_0^z \frac{dz}{288 - 6.50(10^{-3})z} \\ \ln p \Big|_{101.3(10^3) \text{ Pa}}^p &= 5.2605 \ln [288 - 6.50(10^{-3})z] \Big|_0^z \\ \ln \left[\frac{p}{101.3(10^3)} \right] &= 5.2605 \ln \left[\frac{288 - 6.50(10^{-3})z}{288} \right] \\ p &= 101.3(10^3) \left[\frac{288 - 6.50(10^{-3})z}{288} \right]^{5.2605} \end{aligned}$$

***2-24. Continued**

At $z = 11.0(10^3)$ m,

$$p = 101.3(10^3) \left\{ \frac{288 - [6.50(10^{-3})][11.0(10^3)]}{288} \right\}^{5.2605} = 22.58(10^3) \text{ Pa}$$

At $z = 15(10^3)$ m, it is into the stratosphere in the region $11(10^3) \text{ m} < z < 20.1(10^3) \text{ m}$, of which the temperature is constant, Fig. 2-11. Thus,

$$T_C = -56.5^\circ\text{C}$$

Ans.

Integrate Eq. (2) using this result and $T = -56.5^\circ\text{C} + 273 = 216.5 \text{ K}$.

$$\int_{22.58(10^3) \text{ Pa}}^p \frac{dp}{p} = - \left[\frac{9.81 \text{ m/s}^2}{(286.9 \text{ J/kg} \cdot \text{K})(216.5 \text{ K})} \right] \int_{11(10^3) \text{ m}}^z dz$$

$$\ln p \Big|_{22.58(10^3) \text{ Pa}}^p = -0.1579(10^{-3}) z \Big|_{11(10^3) \text{ m}}^z$$

$$\ln \frac{p}{22.58(10^3)} = 0.1579(10^{-3}) [11(10^3) - z]$$

$$p = [22.58(10^3) e^{0.1579(10^{-3})[11(10^3) - z]}] \text{ Pa}$$

At $z = 15(10^3)$ m,

$$p = [22.58(10^3) e^{0.1579(10^{-3})[11(10^3) - 15(10^3)]}] \text{ Pa}$$

$$= 12.00(10^3) \text{ Pa} = 12.0 \text{ kPa}$$

Ans.

Ans:

$$T_C = -56.5^\circ\text{C}$$

$$p = 12.0 \text{ kPa (abs.)}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–25.

A heavy cylindrical glass is inverted and then placed down at the bottom of a 13-ft-deep swimming pool. Determine the height Δh of water within the glass when it is at the bottom. Assume the air in the glass remains at the same temperature as the atmosphere. *Hint:* Account for the change in volume of air in the glass due to the pressure change. The atmospheric pressure is $p_{\text{atm}} = 14.7 \text{ psi}$.

SOLUTION

When submerged, the density and hence the volume of the air change due to pressure changes. Applying the ideal gas law,

$$p = \rho RT$$

However, $\rho = \frac{m}{V}$. Then

$$p = \frac{m}{V}RT$$

$$pV = mRT$$

Since the mass and temperature of the air are constant, mRT is also constant. Thus,

$$p_1 V_1 = p_2 V_2 \quad (1)$$

When $p_1 = p_{\text{atm}} = \left(14.7 \frac{\text{lb}}{\text{in}^2}\right) \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)^2 = 2116.8 \frac{\text{lb}}{\text{ft}^2}$, $V_1 = \pi(0.25 \text{ ft})^2(1 \text{ ft}) = 0.0625\pi \text{ ft}^3$. When submerged, the water rises to the height of Δh . Thus,

$$p_2 = p_{\text{atm}} + \gamma_w h = 2116.8 \frac{\text{lb}}{\text{ft}^2} + \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)(13 \text{ ft} - \Delta h)$$

$$= (2928 - 62.4\Delta h) \frac{\text{lb}}{\text{ft}^2}$$

$$V_2 = \pi(0.25 \text{ ft})^2(1 \text{ ft} - \Delta h) = [0.0625\pi(1 - \Delta h)] \text{ ft}^3$$

Substitute these values into Eq. (1).

$$\left(2116.8 \frac{\text{lb}}{\text{ft}^2}\right)(0.0625\pi \text{ ft}^3) = \left[(2928 - 62.4\Delta h) \frac{\text{lb}}{\text{ft}^2}\right][0.0625\pi(1 - \Delta h)] \text{ ft}^3$$

$$2116.8 = (2928 - 62.4\Delta h)(1 - \Delta h)$$

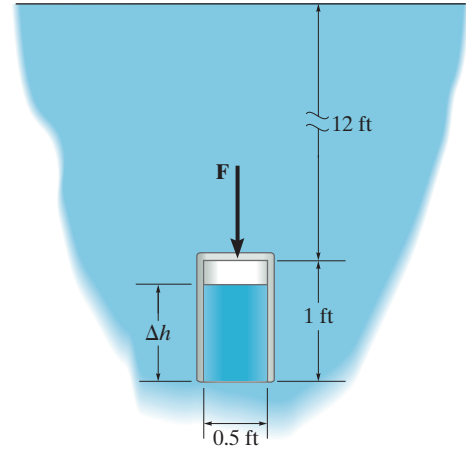
$$62.4\Delta h^2 - 2990.4\Delta h + 811.2 = 0$$

Choose $\Delta h < 1 \text{ ft}$,

$$\Delta h = 0.2728 \text{ ft}$$

$$= 3.27 \text{ in.}$$

Ans.



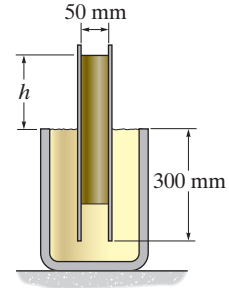
Ans:

$\Delta h = 3.27 \text{ in.}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–26.

The 150-mm-diameter container is filled to the top with glycerin, and a 50-mm-diameter pipe is inserted within it to a depth of 300 mm. If 0.00075 m^3 of kerosene is then poured into the pipe, causing the displaced glycerin to overflow, determine the height h to which the kerosene rises from the top of the glycerin.



SOLUTION

The height of the kerosene column in the pipe, Fig. *a*, is

$$h_{ke} = \frac{V_{ke}}{\pi r^2} = \frac{0.00075 \text{ m}^3}{\pi (0.025 \text{ m})^2} = \left(\frac{1.2}{\pi} \right) \text{ m}$$

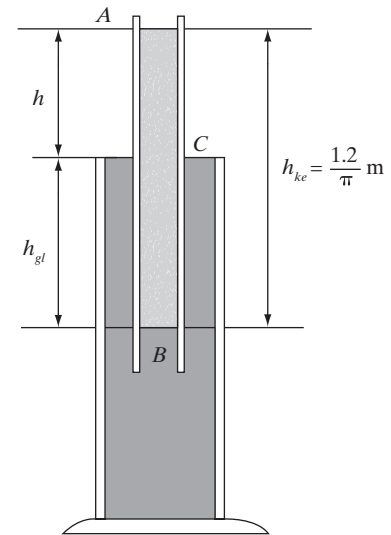
From Appendix A, $\rho_{ke} = 814 \text{ kg/m}^3$ and $\rho_{gl} = 1260 \text{ kg/m}^3$. Writing the manometer equation from *A* to *B* to *C* by referring to Fig. *a*,

$$p_{\text{atm}} + \rho_{ke} g h_{ke} - \rho_{gl} g h_{gl} = p_{\text{atm}}$$

$$h_{gl} = \left(\frac{\rho_{ke}}{\rho_{gl}} \right) h_{ke} = \left(\frac{814 \text{ kg/m}^3}{1260 \text{ kg/m}^3} \right) \left(\frac{1.2}{\pi} \text{ m} \right) = 0.2468 \text{ m}$$

Thus,

$$h = h_{ke} - h_{gl} = \frac{1.2}{\pi} \text{ m} - 0.2468 \text{ m} = 0.1352 \text{ m} = 135 \text{ mm} \quad \textbf{Ans.}$$



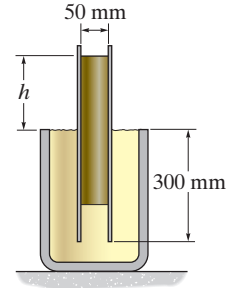
(a)

Ans:
 $h = 135 \text{ mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–27.

The 150-mm-diameter container is filled to the top with glycerin, and a 50-mm-diameter pipe is inserted within it to a depth of 300 mm. Determine the maximum volume of kerosene that can be poured into the pipe while causing the displaced glycerin to overflow, so the kerosene does not come out from the bottom end. How high h does the kerosene rise above the glycerin?



SOLUTION

From Appendix A, $\rho_{ke} = 814 \text{ kg/m}^3$ and $\rho_{gl} = 1260 \text{ kg/m}^3$. The kerosene is required to hit the bottom of the pipe as shown in Fig. *a*. Write the manometer equation from *A* to *B* to *C*.

$$p_{\text{atm}} + \rho_{ke}gh_{ke} - \rho_{gl}gh_{gl} = p_{\text{atm}}$$

$$h_{ke} = \frac{\rho_{gl}}{\rho_{ke}}h_{gl}$$

Here, $h_{ke} = (h + 0.3) \text{ m}$ and $h_{gl} = 0.3 \text{ m}$. Then

$$(h + 0.3) \text{ m} = \left(\frac{1260 \text{ kg/m}^3}{814 \text{ kg/m}^3} \right) (0.3 \text{ m})$$

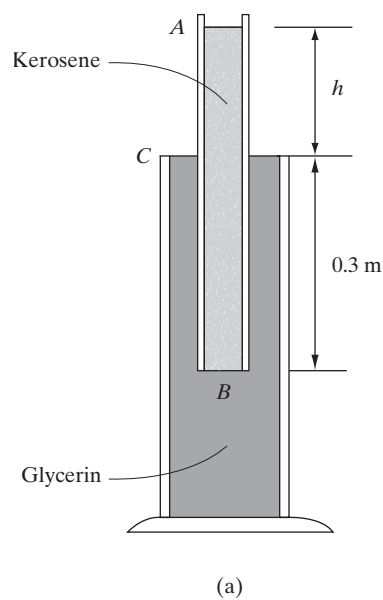
$$h = 0.1644 \text{ m} = 164 \text{ mm}$$

Ans.

Thus, the volume of the kerosene in the pipe is

$$\begin{aligned} V_{ke} &= \pi r^2 h_{ke} = \pi (0.025 \text{ m})^2 (0.1644 \text{ m} + 0.3 \text{ m}) = 0.9118 (10^{-3}) \text{ m}^3 \\ &= 0.912 (10^{-3}) \text{ m}^3 \end{aligned}$$

Ans.



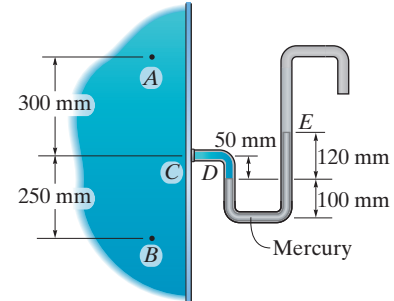
Ans:

$$h = 164 \text{ mm}, V_{ke} = 0.912 (10^{-3}) \text{ m}^3$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–28.**

Butyl carbitol, used in the production of plastics, is stored in a tank having the U-tube manometer. If the U-tube is filled with mercury to level E , determine the pressure in the tank at point B . Take $S_{\text{Hg}} = 13.55$, and $S_{bc} = 0.957$.



SOLUTION

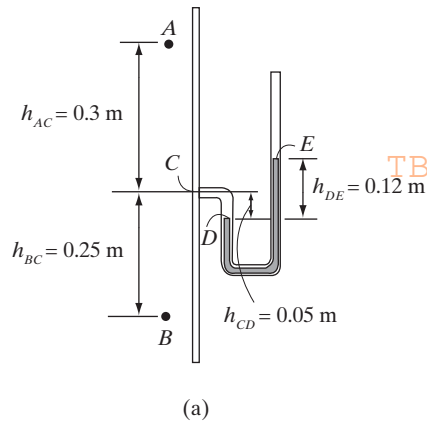
Referring to Fig. a , the manometer rule gives

$$p_E + \rho_{\text{Hg}}gh_{DE} + \rho_{bc}g(-h_{CD} + h_{BC}) = p_B$$

$$0 + 13.55(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.120 \text{ m}) + 0.957(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(-0.05 \text{ m} + 0.25 \text{ m}) = p_B$$

$$p_B = 17.83(10^3) \text{ Pa} = 17.8 \text{ kPa}$$

Ans.



Ans:
 $p_B = 17.8 \text{ kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–29.

Determine the level h' of water in the tube if the depths of oil and water in the tank are 0.6 m and 0.8 m, respectively, and the height of mercury in the tube is $h = 0.08 \text{ m}$. Take $\rho_o = 900 \text{ kg/m}^3$, $\rho_w = 1000 \text{ kg/m}^3$, and $\rho_{\text{Hg}} = 13\,550 \text{ kg/m}^3$.

SOLUTION

Referring to Fig. *a*, $h_{AB} = 0.6 \text{ m}$, $h_{BC} = 0.8 \text{ m} - h'$ and $h_{CD} = h = 0.08 \text{ m}$. Then the manometer rule gives

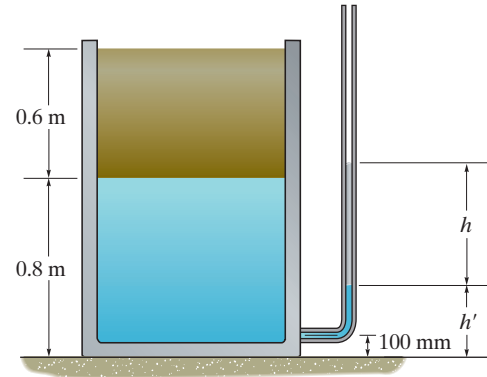
$$p_A + \rho_o g h_{AB} + \rho_w g h_{BC} - \rho_{\text{Hg}} g h_{CD} = p_D$$

Here, $p_A = p_D = 0$, since points *A* and *D* are exposed to the atmosphere.

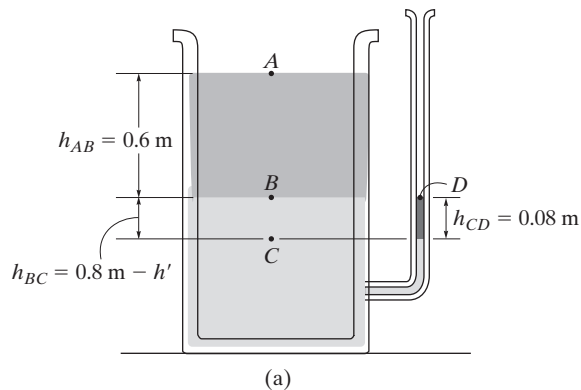
$$\begin{aligned} 0 + (900 \text{ kg/m}^3)(g)(0.6 \text{ m}) + (1000 \text{ kg/m}^3)(g)(0.8 \text{ m} - h') \\ - (13\,550 \text{ kg/m}^3)(g)(0.08 \text{ m}) = 0 \\ h' = 0.256 \text{ m} = 256 \text{ mm} \end{aligned}$$

Ans.

Note: Since $0.1 \text{ m} < h' < 0.8 \text{ m}$, the solution is **OK!**



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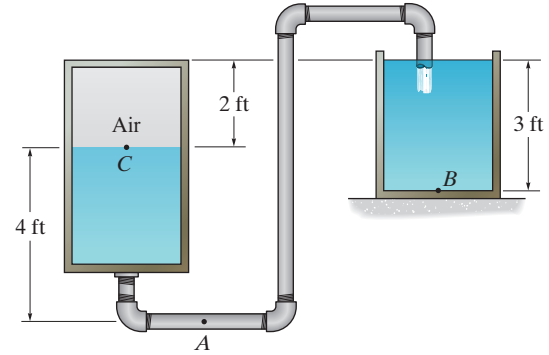
Ans:

$$h' = 256 \text{ mm}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–30.

Determine the pressures at points *A* and *B*. The containers are filled with water.



SOLUTION

$$p_A = \gamma_w h_A = (62.4 \text{ lb/ft}^3)(2\text{ft} + 4\text{ft}) = \left(374.4 \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 = 2.60 \text{ psi} \quad \text{Ans.}$$

$$p_B = \gamma_w h_B = (62.4 \text{ lb/ft}^3)(3 \text{ ft}) = \left(187.2 \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 = 1.30 \text{ psi} \quad \text{Ans.}$$

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Ans:

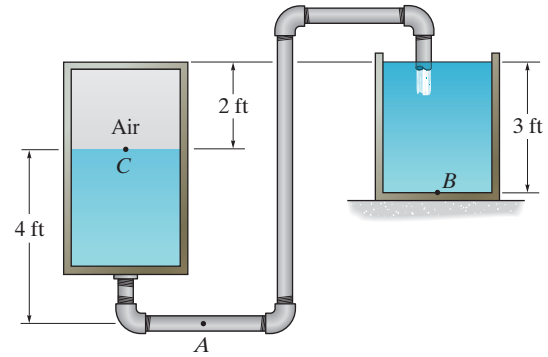
$$p_A = 2.60 \text{ psi}$$

$$p_B = 1.30 \text{ psi}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–31.

Determine the pressure at point *C*. The containers are filled with water.



SOLUTION

$$p_C = \gamma h_C = (62.4 \text{ lb/ft}^3)(2 \text{ ft}) = \left(124.8 \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 0.870 \text{ psi} \quad \text{Ans.}$$

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Ans:
 $p_C = 0.870 \text{ psi}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-32.**

Determine the pressure at the center A of the pipe, which is filled with water. The mercury in the inclined-tube manometer has the level shown, and the other end of the tube is opened to the atmosphere. $S_{\text{Hg}} = 13.55$.

SOLUTION

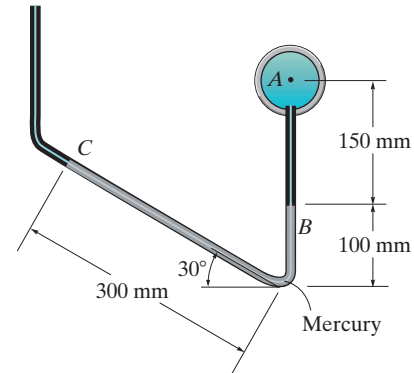
Referring to Fig. a , the manometer rule gives

$$p_A + \rho_w g h_{AB} - \rho_{\text{Hg}} g h_{BC} = p_C$$

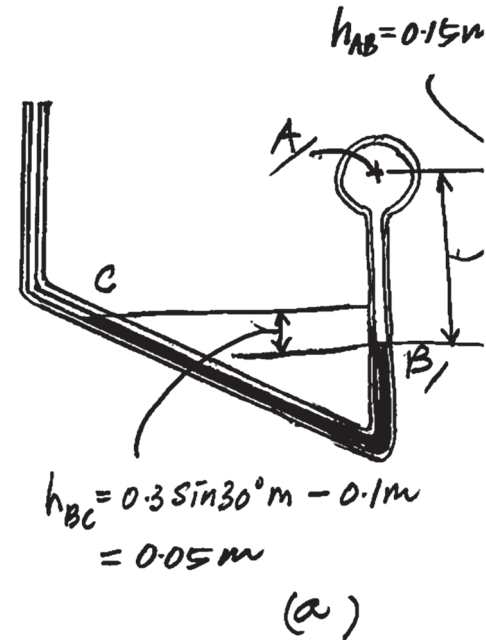
Here, the tube is opened to the atmosphere. Then, $p_C = p_{\text{atm}} = 0$. Thus

$$p_A + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.15 \text{ m}) - 13.55(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.05 \text{ m}) = 0$$

$$p_A = 5.1748(10^3) \text{ Pa} = 5.17 \text{ kPa}$$



Ans.



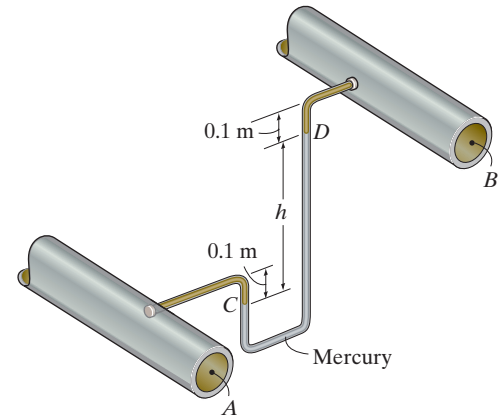
Ans:

$$p_A = 5.17 \text{ kPa}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–33.

The two pipes contain hexylene glycol, which causes the level of mercury in the manometer to be at $h = 0.3 \text{ m}$. Determine the differential pressure in the pipes, $p_A - p_B$. Take $\rho_{hgl} = 923 \text{ kg/m}^3$, $\rho_{Hg} = 13\,550 \text{ kg/m}^3$. Neglect the diameter of the pipes.

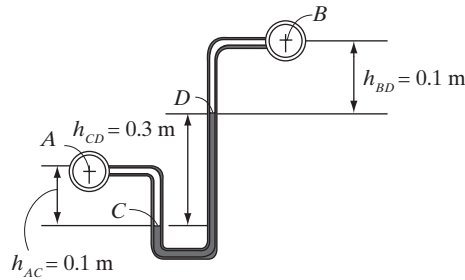


SOLUTION

Referring to Fig. *a*, the manometer rule gives

$$\begin{aligned}
 p_A + \rho_{hgl}gh_{AC} - \rho_{Hg}gh_{CD} - \rho_{hgl}gh_{BD} &= p_B \\
 p_A + (923 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.1 \text{ m}) - (13\,550 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \text{ m}) \\
 - (923 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.1 \text{ m}) &= p_B \\
 p_A - p_B &= 39.88(10^3) \text{ Pa} = 39.9 \text{ kPa}
 \end{aligned}$$

Ans.



(a)

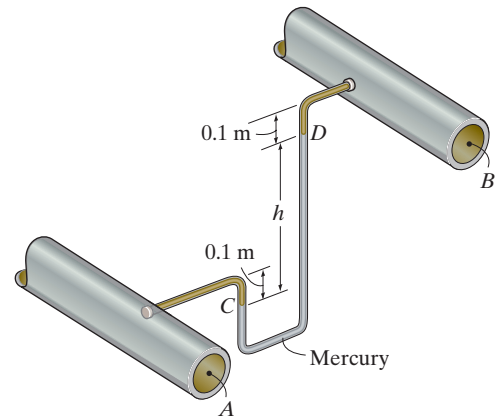
Ans:

$$p_A - p_B = 39.9 \text{ kPa}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–34.

The two pipes contain hexylene glycol, which causes the differential pressure reading of the mercury in the manometer to be at $h = 0.3 \text{ m}$. If the pressure in pipe A increases by 6 kPa , and the pressure in pipe B decreases by 2 kPa , determine the new differential reading h' of the manometer. Take $\rho_{hgl} = 923 \text{ kg/m}^3$, $\rho_{Hg} = 13\,550 \text{ kg/m}^3$. Neglect the diameter of the pipes.



SOLUTION

As shown in Fig. a , the mercury level is at C and D . Applying the manometer rule,

$$\begin{aligned} p_A + \rho_{hgl}gh_{AC} - \rho_{Hg}gh_{CD} - \rho_{hgl}gh_{DB} &= p_B \\ p_A + (923 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.1 \text{ m}) - (13\,550 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \text{ m}) \\ &\quad - (923 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.1 \text{ m}) = p_B \\ p_A - p_B &= 39\,877.65 \text{ Pa} \end{aligned} \quad (1)$$

When the pressure at A and B changes, the mercury level will be at C' and D' , Fig. a . Then, the manometer rule gives

$$\begin{aligned} (p_A + \Delta p_A) + \rho_{hgl}gh_{AC'} - \rho_{Hg}gh_{C'D'} - \rho_{hgl}gh_{D'B} &= (p_B - \Delta p_B) \\ [p_A + 6(10^3) \text{ N/m}^2] + (923 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.1 \text{ m} + \Delta h) \\ &\quad - (13\,550 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \text{ m} + 2\Delta h) \\ &\quad - (923 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.1 \text{ m} - \Delta h) = [p_B - 2(10^3) \text{ N/m}^2] \\ p_A - p_B &= 31\,877.65 + 247741.74 \Delta h \end{aligned} \quad (2)$$

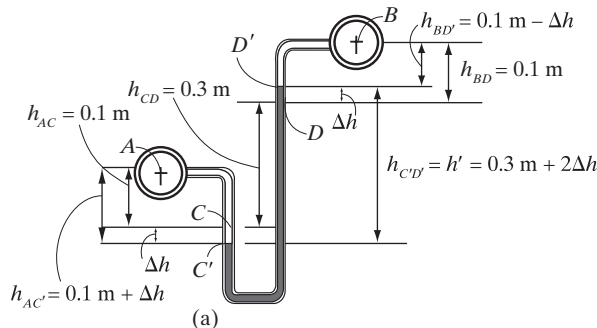
Equating Eqs. (1) and (2), we obtain

$$39\,877.65 = 31\,877.65 + 247741.74 \Delta h$$

$$\Delta h = 0.03229$$

Thus,

$$\begin{aligned} h' &= 0.3 \text{ m} + 2\Delta h \\ &= 0.3 \text{ m} + 2(0.03229 \text{ m}) = 0.36458 \text{ m} = 365 \text{ mm} \end{aligned} \quad \text{Ans.}$$

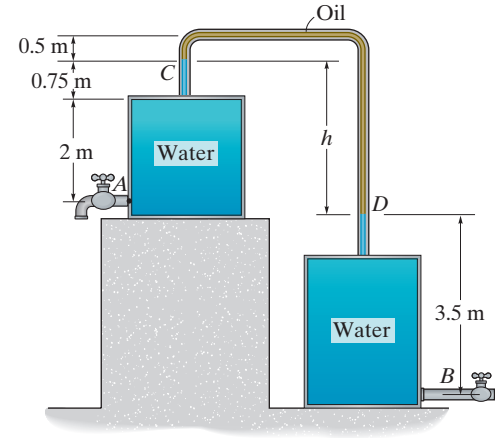


Ans:
 $h' = 365 \text{ mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–35.

The pressure in the tank at the closed valve A is 300 kPa. If the differential elevation in the oil level is $h = 2.5 \text{ m}$, determine the pressure in the pipe at B . Take $\rho_o = 900 \text{ kg/m}^3$.



SOLUTION

Referring to Fig. a , the manometer rule gives

$$p_A - \rho_w g h_{AC} + \rho_o g h_{CD} + \rho_w g h_{BD} = p_B$$

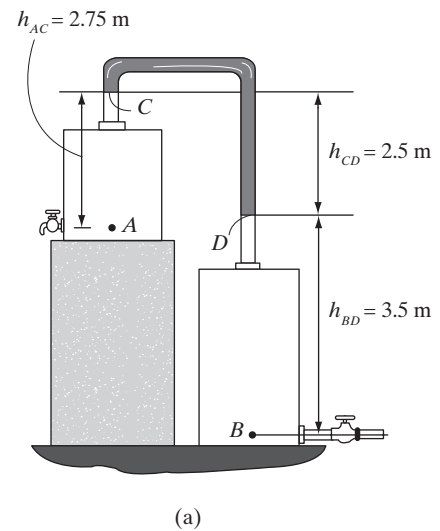
$$300(10^3) \text{ N/m}^2 - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.75 \text{ m}) + (900 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.5 \text{ m})$$

$$+ (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3.5 \text{ m}) = p_B$$

$$p_B = 329.43(10^3) \text{ Pa} = 329 \text{ kPa}$$

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Ans.



Ans:
 $p_B = 329 \text{ kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-36.**

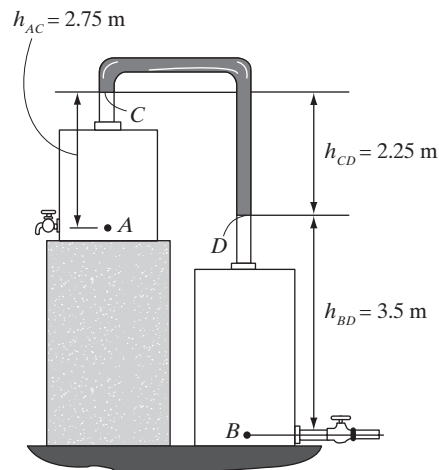
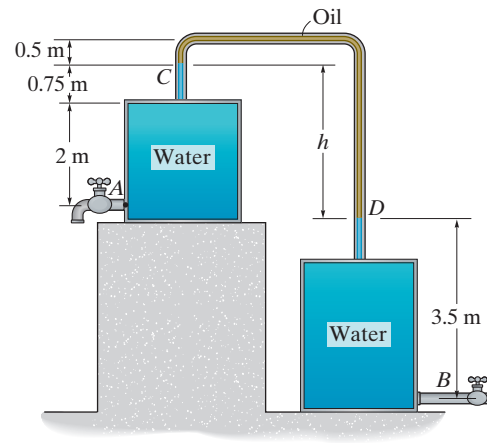
The pressure in the tank at B is 600 kPa. If the differential elevation of the oil is $h = 2.25 \text{ m}$, determine the pressure at the closed valve A . Take $\rho_o = 900 \text{ kg/m}^3$.

SOLUTION

Referring to Fig. a , the manometer rule gives

$$\begin{aligned}
 p_A - \rho_w g h_{AC} + \rho_o g h_{CD} + \rho_w g h_{BD} &= p_B \\
 p_A - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.75 \text{ m}) + (900 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.25 \text{ m}) \\
 + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3.5 \text{ m}) &= 600(10^3) \text{ N/m}^2 \\
 p_A &= 572.78(10^3) \text{ Pa} = 573 \text{ kPa}
 \end{aligned}$$

Ans.



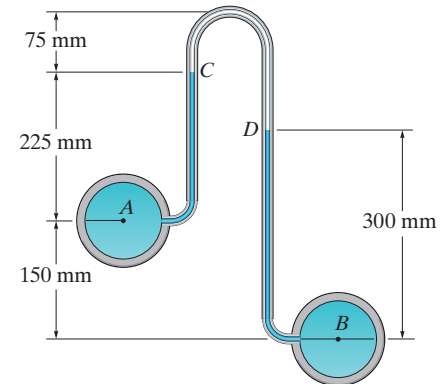
(a)

Ans:
 $p_A = 573 \text{ kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–37.

The inverted U-tube manometer is used to measure the difference in pressure between water flowing in the pipes at A and B . If the top segment is filled with air, and the water levels in each segment are as indicated, determine the pressure difference between A and B . $\rho_w = 1000 \text{ kg/m}^3$.



SOLUTION

Notice that the pressure throughout the air in the tube is constant. Referring to Fig. a ,

$$p_A = (p_w)_1 + p_a = \rho_w g(h_w)_1 + p_a$$

And

$$p_B = (p_w)_2 + p_a = \rho_w g(h_w)_2 + p_a$$

Therefore,

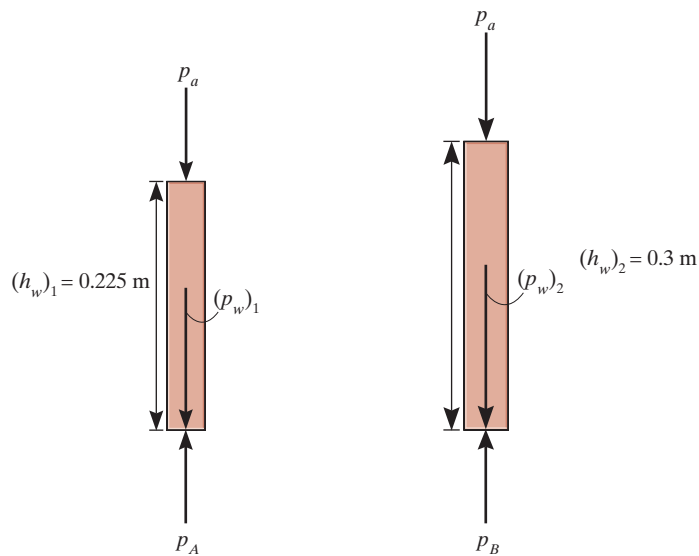
$$\begin{aligned} p_B - p_A &= [\rho_w g(h_w)_2 + p_a] - [\rho_w g(h_w)_1 + p_a] \\ &= \rho_w g[(h_w)_2 - (h_w)_1] \\ &= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \text{ m} - 0.225 \text{ m}) \\ &= 735.75 \text{ Pa} = 736 \text{ Pa} \end{aligned}$$

Ans.

Also, using the manometer equation,

$$p_A - \rho_w g h_{AC} + \rho_w g h_{DB} = p_B$$

$$p_B - p_A = \rho_w g[h_{DB} - h_{AC}]$$



(a)

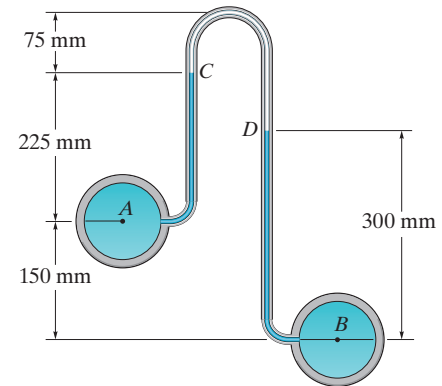
Ans:

$$p_B - p_A = 736 \text{ Pa}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–38.

Solve Prob. 2–37 if the top segment is filled with an oil for which $\rho_o = 800 \text{ kg/m}^3$.



SOLUTION

Referring to Fig. *a*, write the manometer equation starting at *A* and ending at *B*.

$$p_A - \rho_w g(h_w)_1 + \rho_o g h_o + \rho_w g(h_w)_2 = p_B$$

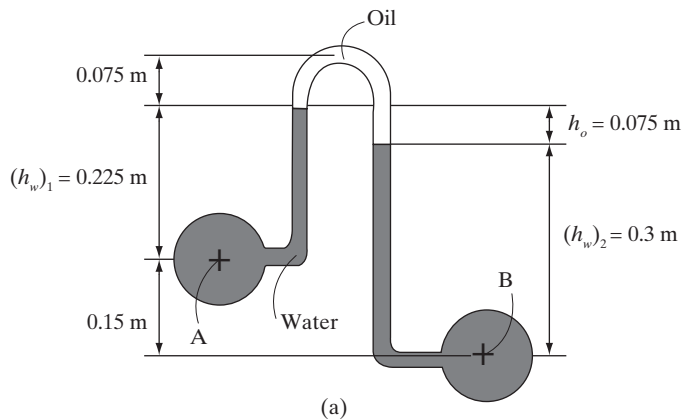
$$p_B - p_A = \rho_w g[(h_w)_2 - (h_w)_1] + \rho_o g h_o$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \text{ m} - 0.225 \text{ m}) + (800 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.075 \text{ m})$$

$$= 1.324(10^3) \text{ Pa} = 1.32 \text{ kPa}$$

Ans.

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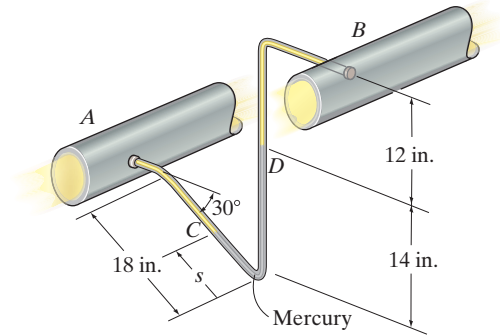
Ans:

$$p_B - p_A = 1.32 \text{ kPa}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–39.

Trichlorethylene, flowing through both pipes, is to be added to jet fuel produced in a refinery. A careful monitoring of pressure is required through the use of the inclined-tube manometer. If the pressure at A is 30 psi and the pressure at B is 25 psi, determine the position s that defines the level of mercury in the inclined-tube manometer. Take $S_{\text{Hg}} = 13.55$ and $S_t = 1.466$. Neglect the diameter of the pipes.



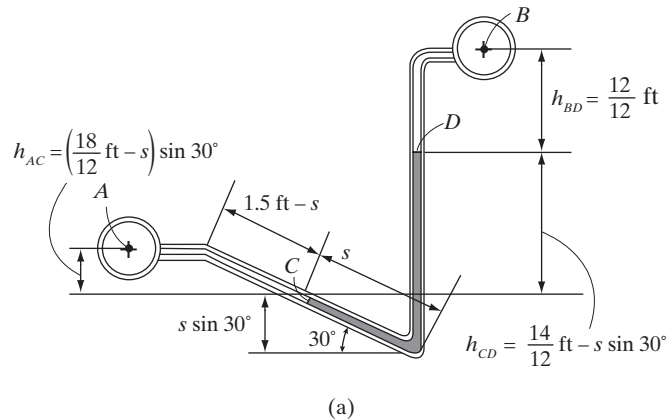
SOLUTION

Referring to Fig. *a*, the manometer rule gives

$$\begin{aligned}
 p_A + \gamma_t h_{AC} - \gamma_{\text{Hg}} h_{CD} - \gamma_t h_{BD} &= p_B \\
 \left(\frac{30 \text{ lb}}{\text{in}^2} \right) \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right)^2 + 1.466(62.4 \text{ lb/ft}^3) \left(\frac{18}{12} \text{ ft} - s \right) \sin 30^\circ - 13.55(62.4 \text{ lb/ft}^3) \left(\frac{14}{12} \text{ ft} - s \sin 30^\circ \right) \\
 - 1.466(62.4 \text{ lb/ft}^3) \left(\frac{12}{12} \text{ ft} \right) &= 25 \frac{\text{lb}}{\text{in}^2} \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right)^2 \\
 s &= 0.7674 \text{ ft} \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right) = 9.208 \text{ in.} = 9.21 \text{ in.}
 \end{aligned}$$

Ans.

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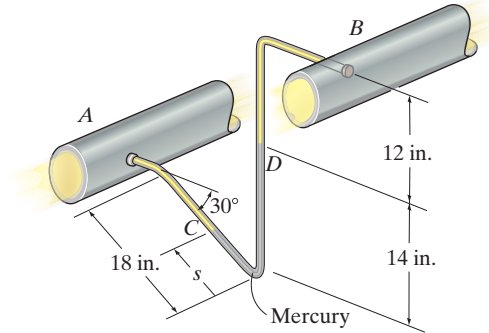
(a)

Ans:
 $s = 9.21 \text{ in.}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–40.**

Trichlorethylene, flowing through both pipes, is to be added to jet fuel produced in a refinery. A careful monitoring of pressure is required through the use of the inclined-tube manometer. If the pressure at A is 30 psi and $s = 7 \text{ in.}$, determine the pressure at B . Take $S_{\text{Hg}} = 13.55$ and $S_t = 1.466$. Neglect the diameter of the pipes.



SOLUTION

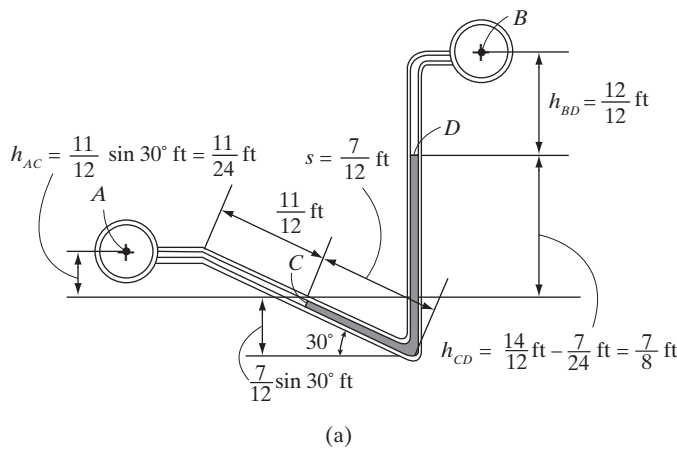
Referring to Fig. a , the manometer rule gives

$$p_A + \gamma_t h_{AC} - \gamma_{\text{Hg}} h_{CD} - \gamma_t h_{BD} = p_B$$

$$\frac{30 \text{ lb}}{\text{in}^2} \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right)^2 + 1.466(62.4 \text{ lb/ft}^3) \left(\frac{11}{24} \text{ ft} \right) - 13.55(62.4 \text{ lb/ft}^3) \left(\frac{7}{8} \text{ ft} \right) - 1.466(62.4 \text{ lb/ft}^3) \left(\frac{12}{12} \text{ ft} \right) = p_B$$

$$p_B = 3530.62 \text{ lb/ft}^2 \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 = 24.52 \text{ psi} = 24.5 \text{ psi} \quad \text{Ans.}$$

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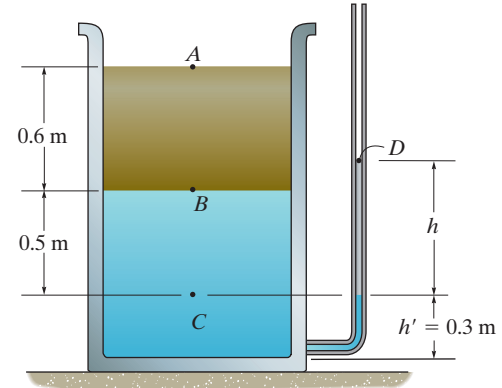


Ans:
 $p_B = 24.5 \text{ psi}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–41.

Determine the height h of the mercury in the tube if the level of water in the tube is $h' = 0.3 \text{ m}$ and the depths of the oil and water in the tank are 0.6 m and 0.5 m , respectively. Take $\rho_o = 900 \text{ kg/m}^3$, $\rho_w = 1000 \text{ kg/m}^3$, and $\rho_{\text{Hg}} = 13\,550 \text{ kg/m}^3$.



SOLUTION

Referring to Fig. *a*, $h_{AB} = 0.6 \text{ m}$, $h_{BC} = 0.5 \text{ m}$ and $h_{CD} = h$. Then the manometer rule gives

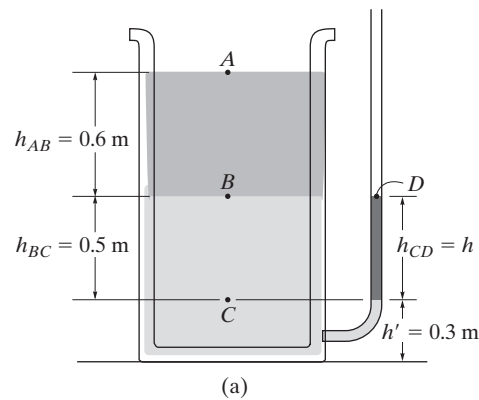
$$p_A + \rho_o g h_{AB} + \rho_w g h_{BC} - \rho_{\text{Hg}} g h_{CD} = p_D$$

Here, $p_A = p_D = 0$, since points *A* and *D* are exposed to the atmosphere.

$$0 + (900 \text{ kg/m}^3)(g)(0.6 \text{ m}) + (1000 \text{ kg/m}^3)(g)(0.5 \text{ m}) - (13\,550 \text{ kg/m}^3)(g)(h) = 0$$

$$h = 0.07675 \text{ m} = 76.8 \text{ mm}$$

Ans.

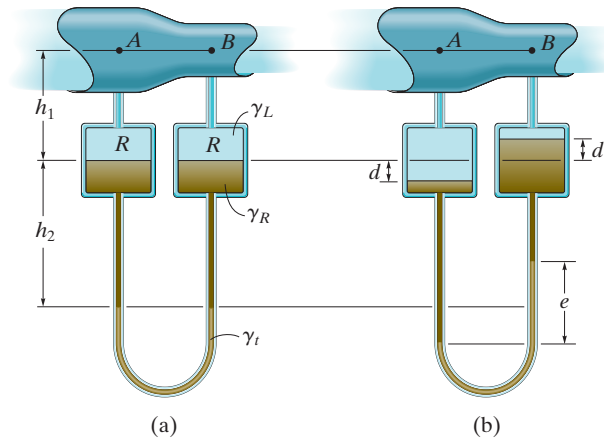


Ans:
 $h = 76.8 \text{ mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-42.

The *micro-manometer* is used to measure small differences in pressure. The reservoirs R and upper portion of the lower tubes are filled with a liquid having a specific weight of γ_R , whereas the lower portion is filled with a liquid having a specific weight of γ_L , Fig. *a*. When the liquid flows through the venturi meter, the levels of the liquids with respect to the original levels are shown in Fig. *b*. If the cross-sectional area of each reservoir is A_R and the cross-sectional area of the U-tube is A_t , determine the pressure difference $p_A - p_B$ as a function of e . The liquid in the venturi meter has a specific weight of γ_L .



SOLUTION

Write the manometer equation starting at A and ending at B , Fig. *c*.

$$p_A + \gamma_L(h_1 + d) + \gamma_R\left(h_2 - d + \frac{e}{2}\right) - \gamma_L e$$

$$- \gamma_R\left(h_2 - \frac{e}{2} + d\right) - \gamma_L(h_1 - d) = p_B$$

$$p_A - p_B = 2\gamma_R d - 2\gamma_L d + \gamma_L e - \gamma_R e$$

Since the same amount of liquid leaving the left reservoir will enter into the left tube,

$$A_R d = A_t \left(\frac{e}{2}\right)$$

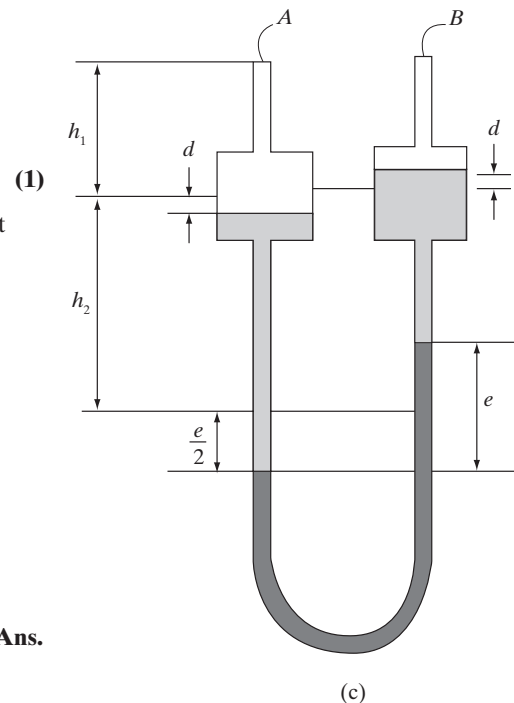
$$d = \left(\frac{A_t}{2A_R}\right)e$$

Substitute this result into Eq. (1).

$$p_A - p_B = 2\gamma_R \left(\frac{A_t}{2A_R}\right)e - 2\gamma_L \left(\frac{A_t}{2A_R}\right)e + \gamma_L e - \gamma_R e$$

$$= e \left[\left(\frac{A_t}{A_R}\right)\gamma_R - \left(\frac{A_t}{A_R}\right)\gamma_L + \gamma_L - \gamma_R \right]$$

$$= e \left[\gamma_L - \left(1 - \frac{A_t}{A_R}\right)\gamma_R - \left(\frac{A_t}{A_R}\right)\gamma_L \right]$$



Ans.

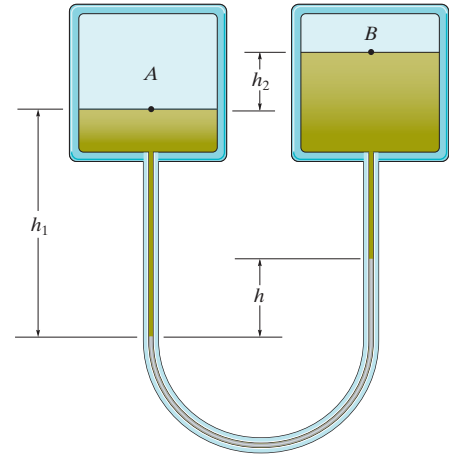
Ans:

$$p_A - p_B = e \left[\gamma_L - \left(1 - \frac{A_t}{A_R}\right)\gamma_R - \left(\frac{A_t}{A_R}\right)\gamma_L \right]$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–43.

A *micro-manometer* works on the principles shown. Here there are two reservoirs filled with kerosene, each having a cross-sectional area of 300 mm^2 . The connecting tube has a cross-sectional area of 15 mm^2 and contains mercury. Determine h if the pressure difference $p_A - p_B = 40 \text{ Pa}$. Take $\rho_{\text{Hg}} = 13\,550 \text{ kg/m}^3$, $\rho_{ke} = 814 \text{ kg/m}^3$. *Hint:* Both h_1 and h_2 can be eliminated from the analysis.



SOLUTION

Referring to Fig. *a*, write the manometer equation starting at *A* and ending at *B*.

$$p_A + \rho_1 g h_1 - \rho_2 g h - \rho_1 g (h_1 - h + h_2) = p_B$$

$$p_A - p_B = \rho_2 g h - \rho_1 g h + \rho_1 g h_2 \quad (1)$$

Since the same amount of liquid leaving the left reservoir will enter the left tube,

$$A_R \left(\frac{h_2}{2} \right) = A_t \left(\frac{h}{2} \right)$$

$$h_2 = \left(\frac{A_t}{A_R} \right) h \quad \text{TBEXAM.COM}$$

Substitute this result into Eq. (1).

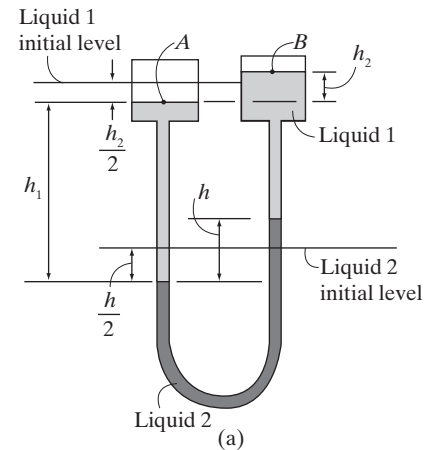
$$p_A - p_B = \rho_2 g h - \rho_1 g h + \rho_1 g \left(\frac{A_t}{A_R} \right) h$$

$$p_A - p_B = h \left[\rho_2 g - \left(1 - \frac{A_t}{A_R} \right) \rho_1 g \right] \quad (2)$$

When $\rho_1 = \rho_{ke} = 814 \text{ kg/m}^3$, $\rho_2 = \rho_{\text{Hg}} = 13\,550 \text{ kg/m}^3$ and $p_A - p_B = 40 \text{ Pa}$,

$$40 \text{ N/m}^2 = h \left[(13\,550 \text{ kg/m}^3)(9.81 \text{ m/s}^2) - \left(1 - \frac{15}{300} \right) (814 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \right]$$

$$h = 0.3191(10^{-3}) \text{ m} = 0.319 \text{ mm} \quad \text{Ans.}$$

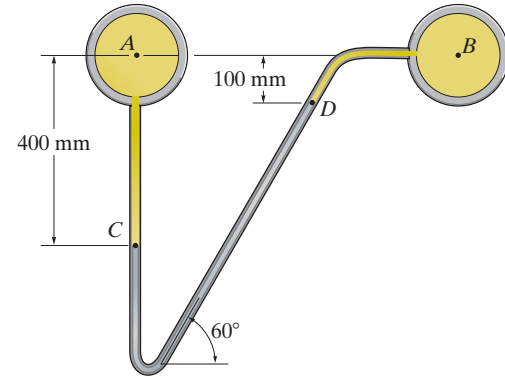


Ans:
 $h = 0.319 \text{ mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-44.**

Determine the difference in pressure $p_A - p_B$ between the centers A and B of the closed pipes, which are filled with kerosene. The mercury in the inclined-tube manometer has the level shown. Take $S_{\text{Hg}} = 13.55$ and $S_k = 0.82$.



SOLUTION

Referring to Fig. *a*, $h_{AC} = 0.4 \text{ m}$, $h_{CD} = 0.4 \text{ m} - 0.1 \text{ m} = 0.3 \text{ m}$ and $h_{BD} = 0.1 \text{ m}$. Then the manometer rule gives

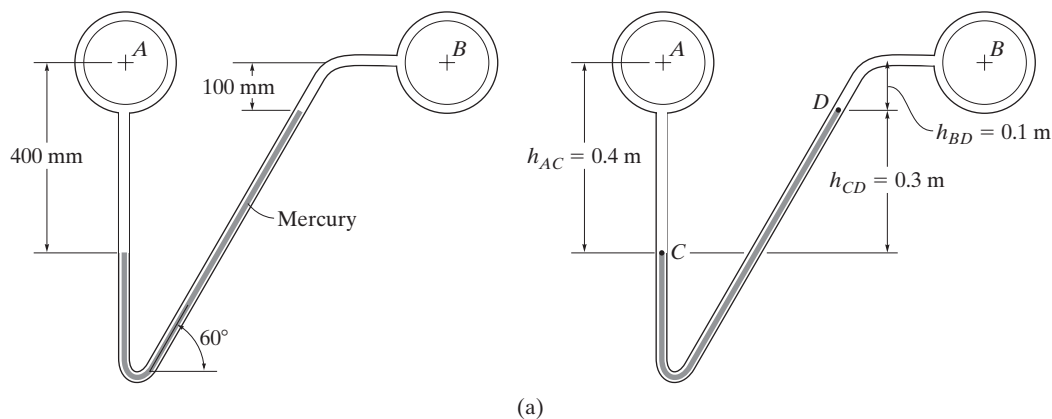
$$p_A + \rho_k g h_{AC} - \rho_{\text{Hg}} g h_{CD} - \rho_k g h_{BD} = p_B$$

$$p_A + 0.82(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.4 \text{ m}) - 13.55(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \text{ m})$$

$$- 0.82(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.1 \text{ m}) = p_B$$

$$p_A - p_B = 37.46(10^3) \text{ Pa} = 37.5 \text{ kPa}$$

Ans.



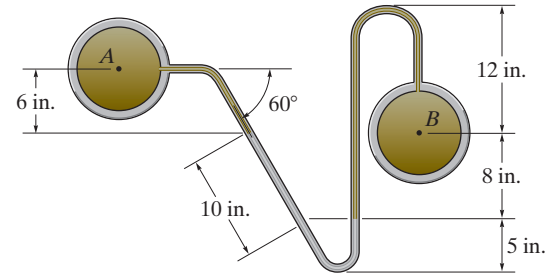
Ans:

$$p_A - p_B = 37.5 \text{ kPa}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–45.

The pipes at A and B contain oil and the inclined-tube manometer is filled with oil and mercury. Determine the pressure difference between A and B . Take $\rho_o = 1.70 \text{ slug/ft}^3$ and $\rho_{\text{Hg}} = 26.3 \text{ slug/ft}^3$.



SOLUTION

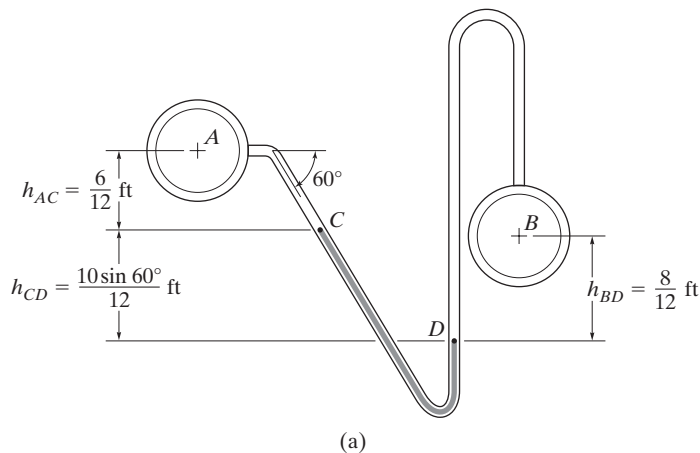
Referring to Fig. a , $h_{AC} = \frac{6}{12} \text{ ft}$, $h_{CD} = \frac{10 \sin 60^\circ}{12} \text{ ft}$ and $h_{BD} = \frac{8}{12} \text{ ft}$. Then the manometer rule gives

$$p_A + \rho_o g h_{AC} + \rho_{\text{Hg}} g h_{CD} - \rho_o g h_{BD} = p_B$$

$$p_A + \rho_o g (h_{AC} - h_{BD}) + \rho_{\text{Hg}} g h_{CD} = p_B$$

$$p_A + (1.70 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2) \left(\frac{6}{12} \text{ ft} - \frac{8}{12} \text{ ft} \right) + (26.3 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2) \left(\frac{10 \sin 60^\circ}{12} \text{ ft} \right) = p_B$$

$$p_B - p_A = \left(602.05 \frac{\text{lb}}{\text{ft}^2} \right) \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 = 4.18 \text{ psi} \quad \textbf{Ans.}$$

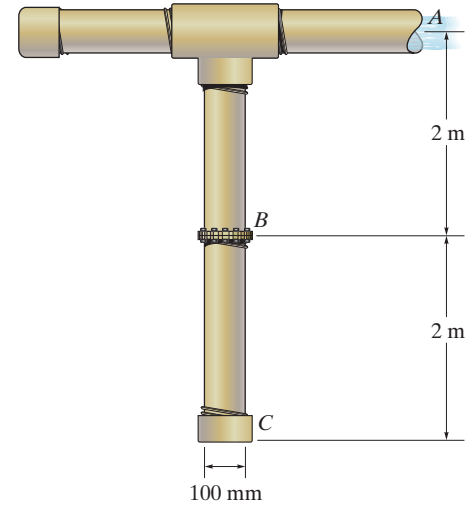


Ans:
 $p_B - p_A = 4.18 \text{ psi}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-46.

The vertical pipe segment has an inner diameter of 100 mm and is capped at its end and suspended from the horizontal pipe as shown. If it is filled with water and the pressure at A is 80 kPa, determine the resultant force that must be resisted by the bolts at B in order to hold the flanges together. Neglect the weight of the pipe but not the water within it.



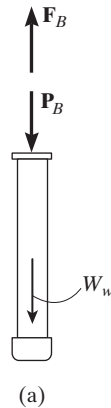
SOLUTION

The forces acting on segment BC of the pipe are indicated on its free-body diagram, Fig. a . Here, $\mathbf{F}_B$ is the force that must be resisted by the bolt, W_w is the weight of the water in segment BC of the pipe, and $\mathbf{P}_B$ is the resultant force of pressure acting on the cross section at B .

$$+\uparrow \Sigma F_y = 0; \quad F_B - W_w - p_B A_B = 0$$

$$\begin{aligned} F_B &= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m})(\pi)(0.05 \text{ m})^2 \\ &\quad + [80(10^3) \text{ N/m}^2 + 1000 \text{ kg/m}^3(9.81 \text{ m/s}^2)(2 \text{ m})]\pi(0.05 \text{ m})^2 \\ &= 937 \text{ N} \end{aligned}$$

Ans.

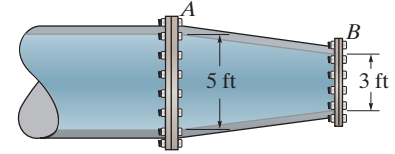


Ans:
 $F_B = 937 \text{ N}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–47.

Nitrogen in the chamber is at a pressure of 60 psi. Determine the total force the bolts at joints *A* and *B* must resist to maintain the pressure. There is a cover plate at *B* having a diameter of 3 ft.



SOLUTION

The force that must be resisted by the bolts at *A* and *B* can be obtained by considering the free-body diagrams in Figs. *a* and *b*, respectively. For the bolts at *B*, Fig. *b*,

$$\pm \Sigma F_x = 0; \quad p_B A_B - F_B = 0$$

$$\begin{aligned} F_B &= p_B A_B = (60 \text{ lb/in}^2) \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right)^2 [(\pi)(1.5 \text{ ft})^2] \\ &= 61\,073 \text{ lb} = 61.1 \text{ kip} \end{aligned}$$

Ans.

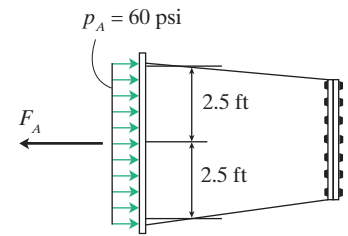
For the bolts at *A*, Fig. *a*,

$$\pm \Sigma F_x = 0; \quad p_A A_A - F_A = 0$$

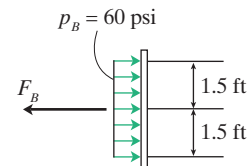
$$\begin{aligned} F_A &= p_A A_A = (60 \text{ lb/in}^2) \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right)^2 [(\pi)(2.5 \text{ ft})^2] \\ &= 169\,646 \text{ lb} = 170 \text{ kip} \end{aligned}$$

Ans.

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(a)



(b)

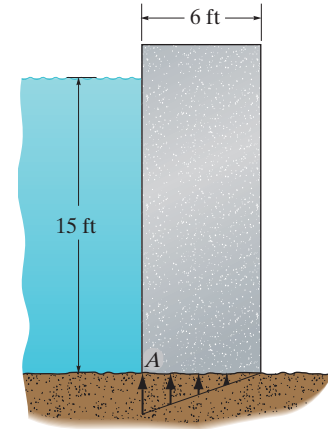
Ans:

$$F_B = 61.1 \text{ kip}, F_A = 170 \text{ kip}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–48.**

Seepage is assumed to occur beneath the concrete wall, producing a linear distribution of hydrostatic pressure as shown. Determine the resultant force on a 1-ft-wide portion of the wall and its location, measured to the right and upward from point A.



SOLUTION

Since the width of the dam considered is $b = 1 \text{ ft}$, the intensity of the distributed load at the base A of the dam is

$$w_A = \gamma_w h_A b = (62.4 \text{ lb/ft}^3)(15 \text{ ft})(1 \text{ ft}) = 936 \text{ lb/ft}$$

Then, the resultant forces of the triangular distributed loads shown in Fig. a are

$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2} (936 \text{ lb/ft})(15 \text{ ft}) = 7020 \text{ lb}$$

$$F_v = \frac{1}{2} w_A L = \frac{1}{2} (936 \text{ lb/ft})(6 \text{ ft}) = 2808 \text{ lb}$$

Then, the magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(7020 \text{ lb})^2 + (2808 \text{ lb})^2} = 7560.77 \text{ lb} = 7.56 \text{ kip} (10^3 \text{ lb}) \quad \text{Ans.}$$

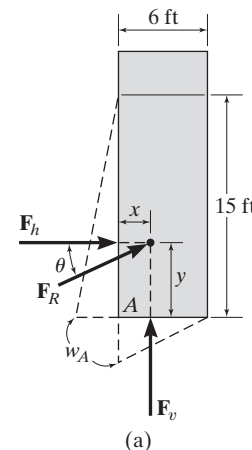
and its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{2808 \text{ lb}}{7020 \text{ lb}}\right) = 21.8^\circ \nearrow \quad \text{Ans.}$$

The location of the resultant force from A is

$$x = \frac{1}{3}(6 \text{ ft}) = 2 \text{ ft} \quad \text{Ans.}$$

$$y = \frac{1}{3}(15 \text{ ft}) = 5 \text{ ft} \quad \text{Ans.}$$



Ans:
 $F_R = 7.56 \text{ kip} (10^3 \text{ lb})$
 $\theta = 21.8^\circ \nearrow$
 $x = 2 \text{ ft}$
 $y = 5 \text{ ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-49.

The storage tank contains oil and water acting at the depths shown. Determine the resultant force that both of these liquids exert on the side ABC of the tank if the side has a width of 1.25 m. Also, determine the location of this resultant, measured from the top surface of the oil. Take $\rho_o = 900 \text{ kg/m}^3$.

SOLUTION

Loading. Since the side of the tank has a constant width, then the intensities of the distributed loading at B and C , Fig. a , are

$$w_B = \rho_o g h_{AB} b = (900 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.75 \text{ m})(1.25 \text{ m}) = 8.277 \text{ kN/m}$$

$$w_C = w_B + \rho_w g h_{BC} b = 8.277 \text{ kN/m} + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.5 \text{ m})(1.25 \text{ m}) = 26.77 \text{ kN/m}$$

Resultant Force. The resultant force can be determined by adding the shaded triangular and rectangular areas in Fig. b . The resultant force is therefore

$$\begin{aligned} F_R &= F_1 + F_2 + F_3 \\ &= \frac{1}{2}(0.75 \text{ m})(8.277 \text{ kN/m}) + (1.5 \text{ m})(8.277 \text{ kN/m}) + \frac{1}{2}(1.5 \text{ m})(18.39 \text{ kN/m}) \\ &= 3.104 \text{ kN} + 12.42 \text{ kN} + 13.80 \text{ kN} = 29.32 \text{ kN} = 29.3 \text{ kN} \end{aligned} \quad \text{Ans.}$$

As shown, each of these three parallel resultants acts through the centroid of its respective area.

$$y_1 = \frac{2}{3}(0.75 \text{ m}) = 0.5 \text{ m}$$

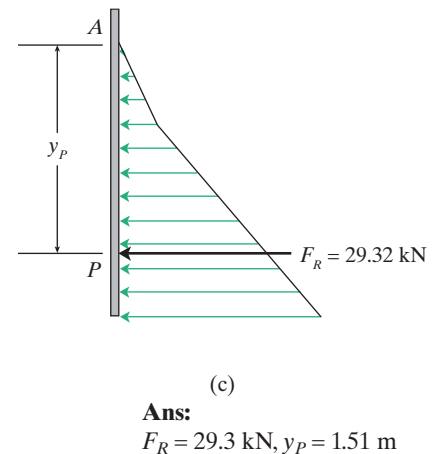
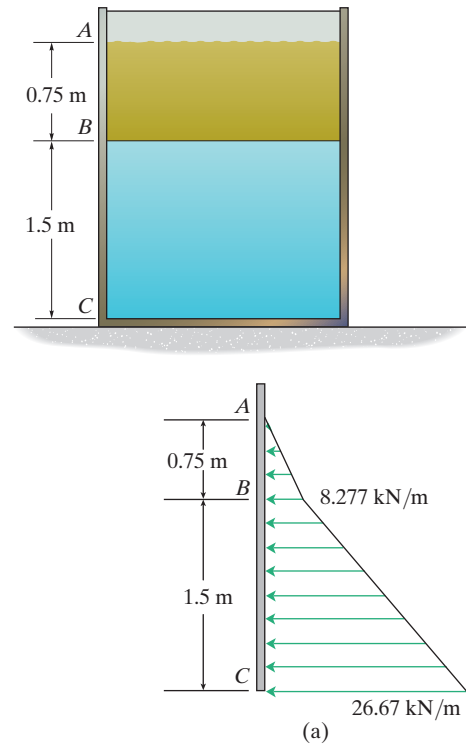
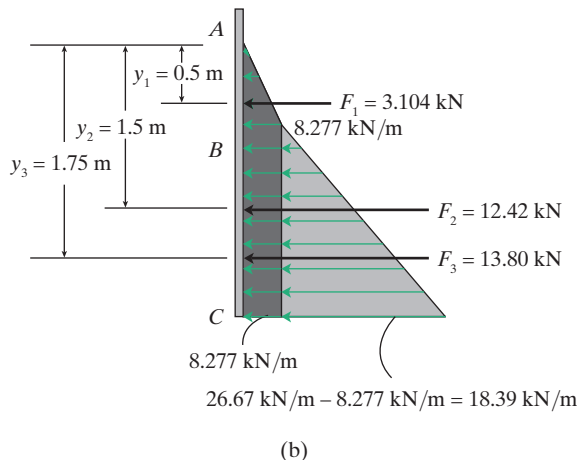
$$y_2 = 0.75 \text{ m} + \frac{1}{2}(1.5 \text{ m}) = 1.5 \text{ m}$$

$$y_3 = 0.75 \text{ m} + \frac{2}{3}(1.5 \text{ m}) = 1.75 \text{ m}$$

The location of the resultant force is determined by equating the moment of the resultant about A , Fig. c , to the moments of the component forces about A , Fig. b . We have

$$y_P F_R = \sum y F; \quad y_P (29.32 \text{ kN}) = (0.5 \text{ m})(3.104 \text{ kN}) + (1.5 \text{ m})(12.42 \text{ kN}) + (1.75 \text{ m})(13.80 \text{ kN})$$

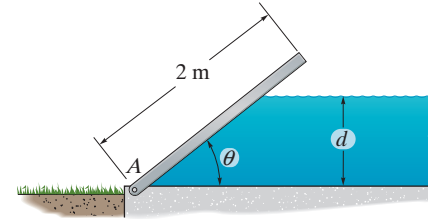
$$y_P = 1.51 \text{ m} \quad \text{Ans.}$$



Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–50.

The uniform swamp gate has a mass of 4 Mg and a width of 1.5 m. Determine the angle θ for equilibrium if the water rises to a depth of $d = 1.5 \text{ m}$.



SOLUTION

Since the gate has a constant width of $b = 1.5 \text{ m}$, the intensity of the distributed load at A can be computed from

$$\begin{aligned} w_A &= \rho_w g h_A b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.5 \text{ m})(1.5 \text{ m}) \\ &= 22.07(10^3) \text{ N/m} \end{aligned}$$

The resulting triangular distributed load is shown on the free-body diagram of the gate, Fig. *a*.

$$F = \frac{1}{2} w_A L = \frac{1}{2} \left[22.07(10^3) \text{ N/m} \right] \left(\frac{1.5 \text{ m}}{\sin \theta} \right) = \frac{16.554(10^3)}{\sin \theta}$$

Referring to the free-body diagram of the gate, Fig. *a*,

$$\begin{aligned} \zeta + \Sigma M_A &= 0; \quad \left[\frac{16.554(10^3)}{\sin \theta} \right] \left[\frac{1}{3} \left(\frac{1.5 \text{ m}}{\sin \theta} \right) \right] - [4000(9.81) \text{ N}] \cos \theta (1 \text{ m}) = 0 \\ \sin^2 \theta \cos \theta &= 0.2109 \end{aligned}$$

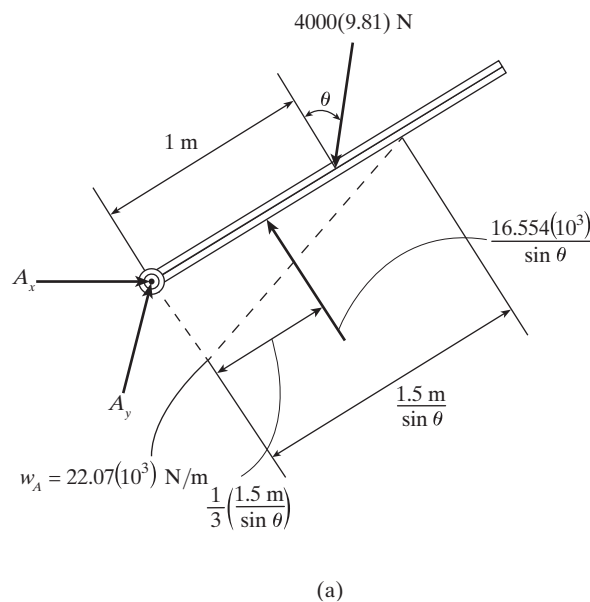
Solving numerically,

$$\theta = 29.49^\circ \text{ or } 77.18^\circ$$

Since $\frac{1.5 \text{ m}}{\sin 77.18^\circ} = 1.54 \text{ m} < 2 \text{ m}$ and $\frac{1.5 \text{ m}}{\sin 29.49^\circ} = 3.05 \text{ m} > 2 \text{ m}$ only one solution is valid.

$$\theta = 77.2^\circ$$

Ans.

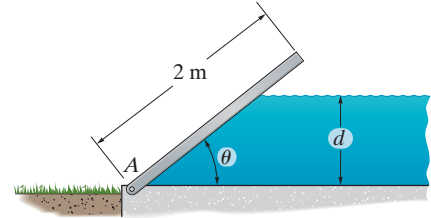


Ans:
 $\theta = 77.2^\circ$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-51.

The uniform swamp gate has a mass of 3 Mg and a width of 1.5 m. Determine the depth of the water d if the gate is held in equilibrium at an angle of $\theta = 60^\circ$.



SOLUTION

Since the gate has a constant width of $b = 1.5 \text{ m}$, the intensity of the distributed load at A can be computed from

$$\begin{aligned} w_A &= \rho_w g h_A b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(d)(1.5 \text{ m}) \\ &= 14\,715d \text{ N/m} \end{aligned}$$

The resulting triangular distributed load is shown on the free-body diagram of the gate, Fig. a .

$$F = \frac{1}{2} w_A L = \frac{1}{2} (14\,715d) \left(\frac{d}{\sin 60^\circ} \right) = \frac{7357.5}{\sin 60^\circ} d^2$$

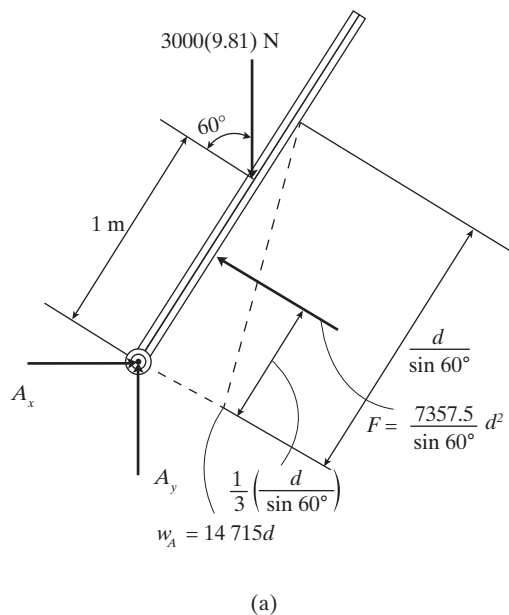
Referring to the free-body diagram of the gate, Fig. a ,

$$\zeta + \Sigma M_A = 0; \quad \left(\frac{7357.5}{\sin 60^\circ} d^2 \right) \left[\frac{1}{3} \left(\frac{d}{\sin 60^\circ} \right) \right] - [3000(9.81) \text{ N}] \cos 60^\circ (1 \text{ m}) = 0$$

$$d = 1.6510 \text{ m} = 1.65 \text{ m}$$

Ans.

Since $\frac{1.6510 \text{ m}}{\sin 60^\circ} = 1.906 \text{ m} < 2 \text{ m}$, this result is valid.

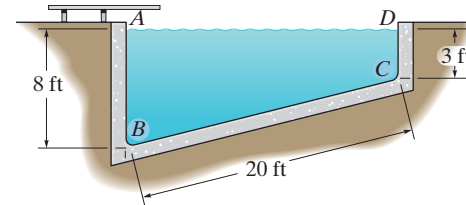


Ans:
 $d = 1.65 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-52.**

A swimming pool has a width of 12 ft and a side profile as shown. Determine the resultant force the water exerts on walls AB and DC , and on the bottom BC .



SOLUTION I

Since the swimming pool has a constant width of $b = 12 \text{ ft}$, the intensities of the distributed load at B and C can be computed from

$$w_B = \gamma h_{AB} b = (62.4 \text{ lb/ft}^3)(8 \text{ ft})(12 \text{ ft}) = 5990.4 \text{ lb/ft}$$

$$w_C = \gamma h_{DC} b = (62.4 \text{ lb/ft}^3)(3 \text{ ft})(12 \text{ ft}) = 2246.4 \text{ lb/ft}$$

Using these results, the distributed loads acting on walls AB and DC and bottom BC are shown in Figs. a , b , and c .

$$F_{AB} = \frac{1}{2} w_B h_{AB} = \frac{1}{2} (5990.4 \text{ lb/ft})(8 \text{ ft}) = 23\,962 \text{ lb} = 24.0 \text{ kip} \quad \text{Ans.}$$

$$F_{DC} = \frac{1}{2} w_C h_{DC} = \frac{1}{2} (2246.4 \text{ lb/ft})(3 \text{ ft}) = 3369.6 \text{ lb} = 3.37 \text{ kip} \quad \text{Ans.}$$

$$\begin{aligned} F_{BC} &= \frac{1}{2} (w_B + w_C) L_{BC} = \frac{1}{2} [5990.4 \text{ lb/ft} + 2246.4 \text{ lb/ft}] (20 \text{ ft}) \\ &= 82\,368 \text{ lb} = 82.4 \text{ kip} \quad \text{Ans.} \end{aligned}$$

SOLUTION II

The same result can also be obtained as follows. For wall AB ,

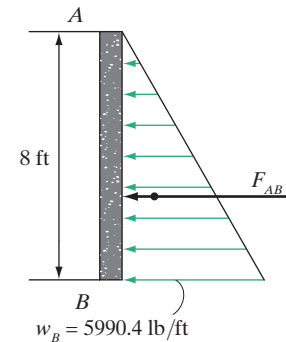
$$F_{AB} = \gamma \bar{h}_{AB} A_{AB} = (62.4 \text{ lb/ft}^3)(4 \text{ ft}) [8 \text{ ft}(12 \text{ ft})] = 23\,962 \text{ lb} = 24.0 \text{ kip} \quad \text{Ans.}$$

For wall DC ,

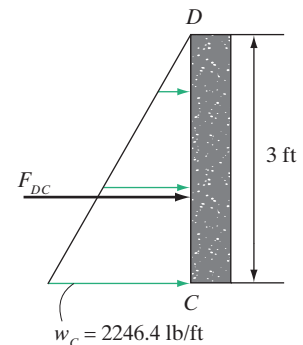
$$F_{DC} = \gamma \bar{h}_{DC} A_{DC} = (62.4 \text{ lb/ft}^3)(1.5 \text{ ft}) [3 \text{ ft}(12 \text{ ft})] = 3369.6 \text{ lb} = 3.37 \text{ kip} \quad \text{Ans.}$$

For floor BC ,

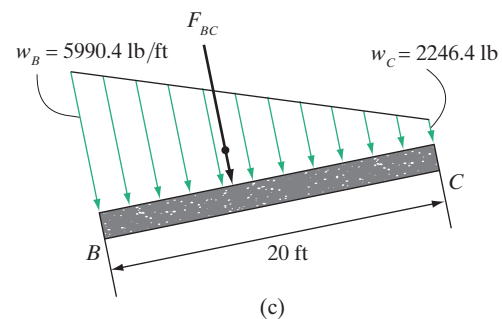
$$F_{BC} = \gamma \bar{h}_{BC} A_{BC} = (62.4 \text{ lb/ft}^3)(5.5 \text{ ft}) [20 \text{ ft}(12 \text{ ft})] = 82\,368 \text{ lb} = 82.4 \text{ kip} \quad \text{Ans.}$$



(a)



(b)



(c)

Ans:

$$F_{AB} = 24.0 \text{ kip}$$

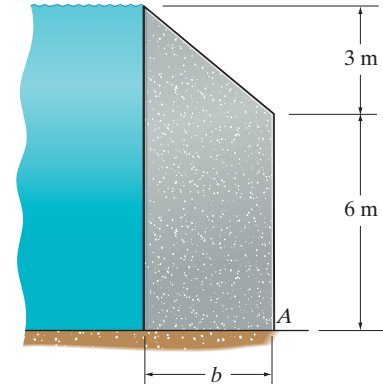
$$F_{DC} = 3.37 \text{ kip}$$

$$F_{BC} = 82.4 \text{ kip}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–53.

Determine the smallest thickness b of the concrete gravity dam that will prevent the dam from overturning due to water pressure acting on the face of the dam. The density of concrete is $\rho_c = 2.40 \text{ Mg/m}^3$. *Hint:* Work the problem using a 1-m width of the dam.



SOLUTION

If we consider the dam as having a width of $a = 1 \text{ m}$, the intensity of the distributed load at the base of the dam is

$$w_b = \rho_w g h a = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(9 \text{ m})(1 \text{ m}) = 88.29(10^3) \text{ N/m}$$

The resultant force of the triangular distributed load shown on the FBD of the dam, Fig. *a*, is

$$F = \frac{1}{2} w_b h = \frac{1}{2} [88.29(10^3) \text{ N/m}](9 \text{ m}) = 397.30(10^3) \text{ N}$$

The dam can be subdivided into a triangular and a rectangular part, and the weight of each of these parts is

$$W_t = \rho_c g V_t = (2400 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \left[\frac{1}{2} (b)(3 \text{ m})(1 \text{ m}) \right] = [35.32(10^3)b] \text{ N}$$

$$W_r = \rho_c g V_r = (2400 \text{ kg/m}^3)(9.81 \text{ m/s}^2) [b(6 \text{ m})(1 \text{ m})] = [141.26(10^3)b] \text{ N}$$

The dam will tip about point *A*. Writing the moment equation of equilibrium about point *A*, Fig. *a*,

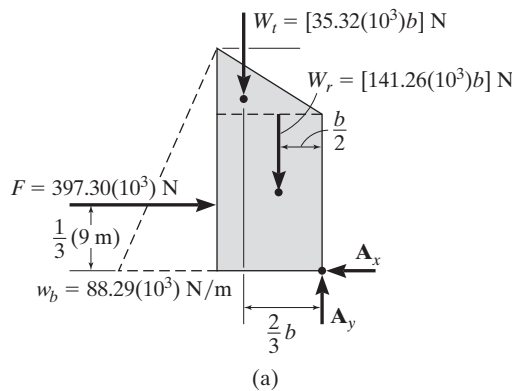
$$(+\Sigma M_A = 0; [141.26(10^3)b] \left(\frac{b}{2} \right) + [35.32(10^3)b] \left(\frac{2}{3}b \right)$$

$$- [397.30(10^3) \text{ N}] \left[\frac{1}{3}(9 \text{ m}) \right] = 0$$

$$94\,176b^2 - 1\,191\,915 = 0$$

$$b = 3.5576 \text{ m} = 3.56 \text{ m}$$

Ans.

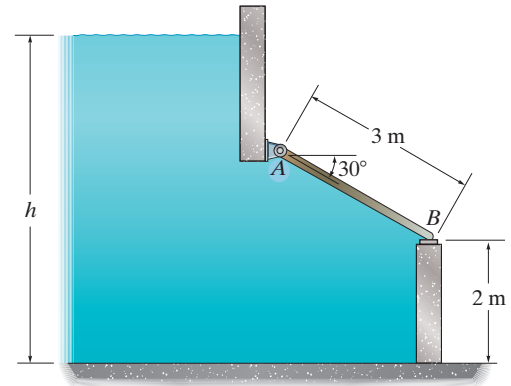


Ans:
 $b = 3.56 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-54.

The uniform control gate AB is pinned at A and rests on the smooth surface at B . If the gate has a mass of 8.50 Mg , determine the minimum depth of water h in the reservoir that will cause the gate to be on the verge of opening. The gate has a width of 1 m .



SOLUTION

The depths of points A and B are $h_A = h - 2 \text{ m} - 3 \text{ m} \sin 30^\circ = (h - 3.5 \text{ m})$ and $h_B = (h - 2 \text{ m})$. Thus, the intensities of the distributed load at points A and B are

$$w_A = \rho_w g h_A b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(h - 3.5 \text{ m})(1 \text{ m}) = [9810(h - 3.5)] \text{ N/m}$$

$$w_B = \rho_w g h_B b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(h - 2 \text{ m})(1 \text{ m}) = [9810(h - 2)] \text{ N/m}$$

Thus,

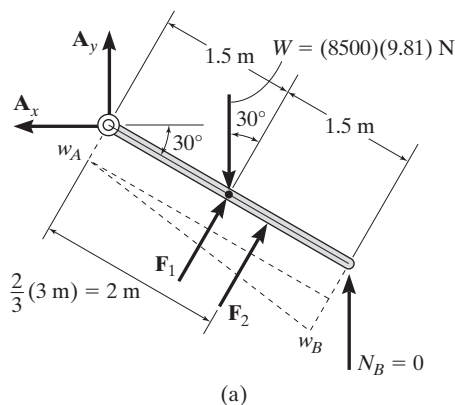
$$F_1 = w_A l_{AB} = [9810(h - 3.5)](3 \text{ m}) = [29.43(10^3)(h - 3.5)] \text{ N}$$

$$F_2 = \frac{1}{2}(w_B - w_A)l_{AB} = \frac{1}{2}[9810(h - 2) - 9810(h - 3.5)](3 \text{ m}) = 22.0725(10^3) \text{ N}$$

Since the gate is required to be on the verge to open, $N_B = 0$. Write the moment equation of equilibrium about point A by referring to the FBD of the gate, Fig. a .

$$\begin{aligned} \zeta + \Sigma M_A = 0; & \quad [29.43(10^3)(h - 3.5)](1.5 \text{ m}) + [22.0725(10^3) \text{ N}](2 \text{ m}) \\ & \quad - [8500(9.81) \text{ N}](\cos 30^\circ)(1.5 \text{ m}) = 0 \\ & \quad h = 4.9537 \text{ m} = 4.95 \text{ m} \end{aligned}$$

Ans.

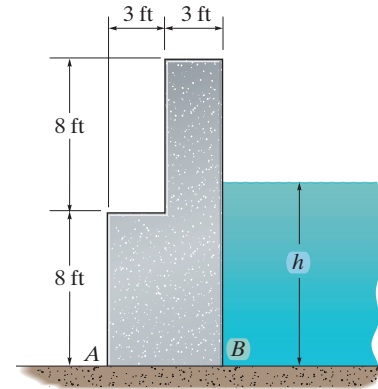


Ans:
 $h = 4.95 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-55.

Determine the critical height h of the water level before the concrete gravity dam starts to tip over due to water pressure acting on its face. The specific weight of concrete is $\gamma_c = 150 \text{ lb/ft}^3$. *Hint:* Work the problem using a 1-ft width of the dam.



SOLUTION

We will consider the dam to have a width of $b = 1 \text{ ft}$. Then the intensity of the distributed load at the base of the dam is

$$w_B = \gamma_w h b = (62.4 \text{ lb/ft}^3)(h)(1 \text{ ft}) = (62.4h) \text{ lb/ft}.$$

The resulting triangular distributed load is shown on the free-body diagram of the dam, Fig. *a*.

$$F = \frac{1}{2} w_B h = \frac{1}{2} (62.4h) h = (31.2h^2) \text{ lb}$$

It is convenient to subdivide the dam into two parts. The weight of each part is

$$W_1 = \gamma_c V_1 = (150 \text{ lb/ft}^3)[(3 \text{ ft})(16 \text{ ft})(1 \text{ ft})] = 7200 \text{ lb}$$

$$W_2 = \gamma_c V_2 = (150 \text{ lb/ft}^3)[(3 \text{ ft})(8 \text{ ft})(1 \text{ ft})] = 3600 \text{ lb}$$

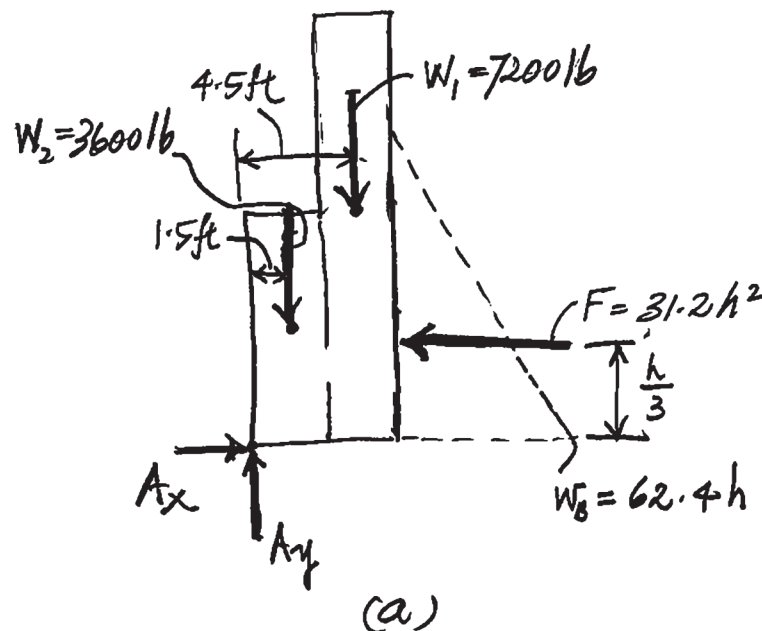
The dam will overturn about point A. Referring to the free-body diagram of the dam, Fig. *a*,

$$\zeta + \sum M_A = 0; \quad (31.2h^2)\left(\frac{h}{3}\right) - (3600 \text{ lb})(1.5 \text{ ft}) - (7200 \text{ lb})(4.5 \text{ ft}) = 0$$

$$h = 15.38 \text{ ft} = 15.4 \text{ ft}$$

Ans.

Since $h < 16 \text{ ft}$, the solution is O.K!



Ans:

$$h = 15.4 \text{ ft}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-56.**

Determine the critical height h of the water level before the concrete gravity dam starts to tip over due to water pressure acting on its face. Assume water also seeps under the base of the dam, producing a uniform distribution of hydrostatic pressure. The specific weight of concrete is $\gamma_c = 150 \text{ lb/ft}^3$. *Hint: Work the problem using a 1-ft width of the dam.*

SOLUTION

We will consider the dam to have a width of $b = 1 \text{ ft}$. Then the intensity of the distributed load at the base of the dam is

$$w_B = \gamma_w h b = (62.4 \text{ lb/ft}^3)(h)(1 \text{ ft}) = (62.4 h) \text{ lb/ft}$$

The resultant forces of the triangular distributed load and the uniform distributed load due to the pressure of the seepage water shown on the FBD of the dam, Fig. a , are

$$F_1 = \frac{1}{2} w_B h = \frac{1}{2} (62.4 h) h = (31.2 h^2) \text{ lb}$$

$$F_2 = w_B L = (62.4 h)(6 \text{ ft}) = (374.4 h) \text{ lb}$$

It is convenient to subdivide the dam into two parts. The weight of each part is

$$W_1 = \gamma_c V_1 = (150 \text{ lb/ft}^3) [(3 \text{ ft})(16 \text{ ft})(1 \text{ ft})] = 7200 \text{ lb}$$

$$W_2 = \gamma_c V_2 = (150 \text{ lb/ft}^3) [(3 \text{ ft})(8 \text{ ft})(1 \text{ ft})] = 3600 \text{ lb}$$

The dam will overturn about point A . Referring to the FBD of the dam, Fig. a ,

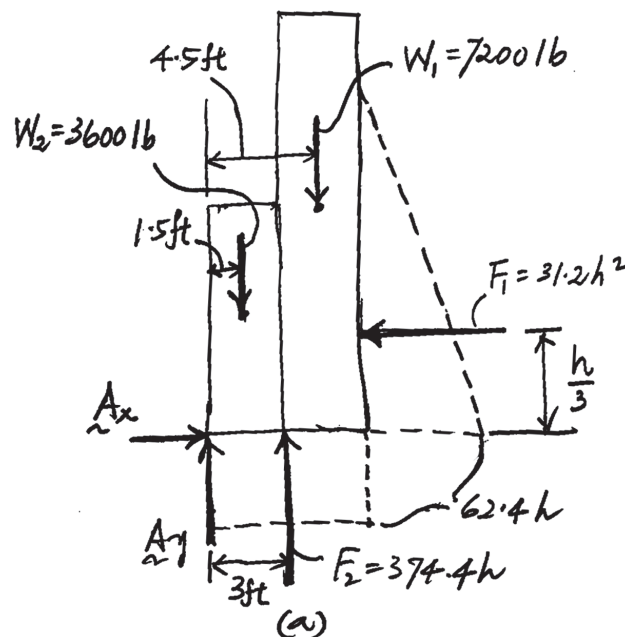
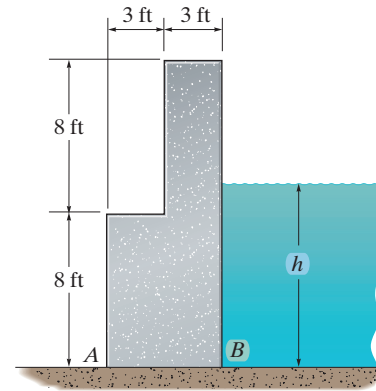
$$\zeta + \sum M_A = 0; \quad 31.2 h^2 \left(\frac{h}{3} \right) + (374.4 h)(3 \text{ ft}) - (3600 \text{ lb})(1.5 \text{ ft}) - (7200 \text{ lb})(4.5 \text{ ft}) = 0$$

$$10.4 h^3 + 1123.2 h - 37800 = 0$$

Solving numerically and choosing a real root in the range of $0 < h < 16 \text{ ft}$,

$$h = 13.054 \text{ ft} = 13.1 \text{ ft}$$

Ans.



Ans:
 $h = 13.1 \text{ ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-57.

The gate is 2 ft wide and is pinned at A and held in place by a smooth latch bolt at B that exerts a force normal to the gate. Determine this force caused by the water and the resultant force on the pin for equilibrium.

SOLUTION

Since the gate has a width of $b = 2 \text{ ft}$, the intensities of the distributed loads at A and B can be computed from

$$w_A = \gamma_w h_A b = (62.4 \text{ lb/ft}^3)(3 \text{ ft})(2 \text{ ft}) = 374.4 \text{ lb/ft}$$

$$w_B = \gamma_w h_B b = (62.4 \text{ lb/ft}^3)(6 \text{ ft})(2 \text{ ft}) = 748.8 \text{ lb/ft}$$

The resulting trapezoidal distributed load is shown on the free-body diagram of the gate, Fig. a . This load can be subdivided into two parts. The resultant force of each part is

$$F_1 = w_A L_{AB} = (374.4 \text{ lb/ft})(3\sqrt{2} \text{ ft}) = 1123.2\sqrt{2} \text{ lb}$$

$$F_2 = \frac{1}{2}(w_B - w_A)L_{AB} = \frac{1}{2}(748.8 \text{ lb/ft} - 374.4 \text{ lb/ft})(3\sqrt{2} \text{ ft}) = 561.6\sqrt{2} \text{ lb}$$

Considering the free-body diagram of the gate, Fig. a ,

$$\zeta + \Sigma M_A = 0; \quad 1123.2\sqrt{2} \text{ lb} \left(\frac{1}{2} 3\sqrt{2} \text{ ft} \right) + 561.6\sqrt{2} \text{ lb} \left(\frac{2}{3} 3\sqrt{2} \text{ ft} \right) - N_B (3\sqrt{2} \text{ ft}) = 0$$

$$N_B = 1323.7 \text{ lb} = 1.32 \text{ kip} \quad \text{Ans.}$$

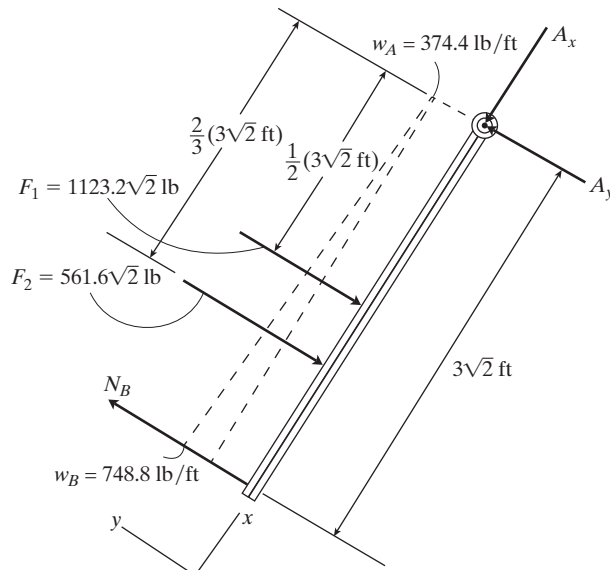
$$\Sigma F_x = 0; \quad A_x = 0$$

$$\Uparrow + \Sigma F_y = 0; \quad 1323.7 \text{ lb} - 1123.2\sqrt{2} \text{ lb} - 561.6\sqrt{2} \text{ lb} + A_y = 0$$

$$A_y = 1058.96 \text{ lb} = 1.059 \text{ kip}$$

Thus,

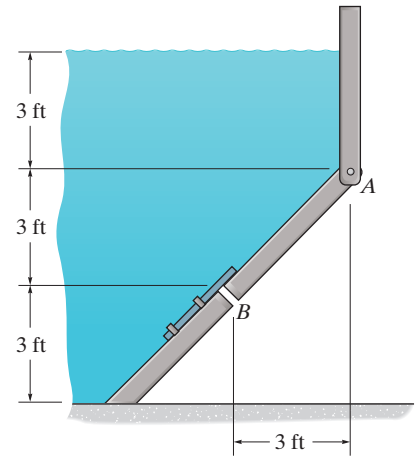
$$F_A = \sqrt{(0)^2 + (1.059 \text{ kip})^2} = 1.06 \text{ kip} \quad \text{Ans.}$$



$$\text{Ans:}$$

$$N_B = 1.32 \text{ kip}$$

$$F_A = 1.06 \text{ kip}$$



Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–58.

The uniform rectangular relief gate AB has a weight of 800 lb and a width of 2 ft. Determine the components of reaction at the pin B and the normal reaction at the smooth support A .

SOLUTION

Here, $h_B = 9 \text{ ft}$ and $h_A = 9 \text{ ft} + 6 \text{ ft} \sin 60^\circ = 14.20 \text{ ft}$. Thus, the intensities of the distributed load at B and A are

$$w_B = \gamma_w h_B b = (62.4 \text{ lb/ft}^3)(9 \text{ ft})(2 \text{ ft}) = 1123.2 \text{ lb/ft}$$

$$w_A = \gamma_w h_A b = (62.4 \text{ lb/ft}^3)(14.20 \text{ ft})(2 \text{ ft}) = 1771.68 \text{ lb/ft}$$

Thus,

$$(F_p)_1 = (1123.2 \text{ lb/ft})(6 \text{ ft}) = 6739.2 \text{ lb}$$

$$(F_p)_2 = \frac{1}{2}(1771.68 \text{ lb/ft} - 1123.2 \text{ lb/ft})(6 \text{ ft}) = 1945.44 \text{ lb}$$

Write the moment equation of equilibrium about B by referring to the FBD of the gate, Fig. a .

$$\zeta + \Sigma M_B = 0; \quad (800 \text{ lb}) \cos 60^\circ (3 \text{ ft}) + (6739.2 \text{ lb})(3 \text{ ft}) + (1945.44 \text{ lb})(4 \text{ ft})$$

$$- N_A \cos 60^\circ (6 \text{ ft}) = 0$$

$$N_A = 9733.12 \text{ lb} = 9.73 \text{ kip}$$

Ans.

Using this result to write the force equations of equilibrium along x and y axes,

$$\leftarrow \Sigma F_x = 0; \quad B_x - (6739.2 \text{ lb}) \sin 60^\circ - (1945.44 \text{ lb}) \sin 60^\circ = 0$$

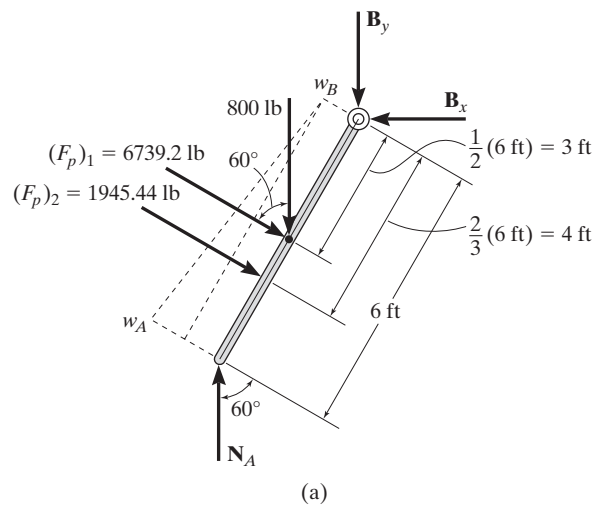
$$B_x = 7521.12 \text{ lb} = 7.52 \text{ kip}$$

Ans.

$$+\uparrow \Sigma F_y = 0; \quad 9733.12 \text{ lb} - 800 \text{ lb} - (6739.2 \text{ lb}) \cos 60^\circ - (1945.44 \text{ lb}) \cos 60^\circ - B_y = 0$$

$$B_y = 4590.80 \text{ lb} = 4.59 \text{ kip}$$

Ans.



Ans:

$$N_A = 9.73 \text{ kip}$$

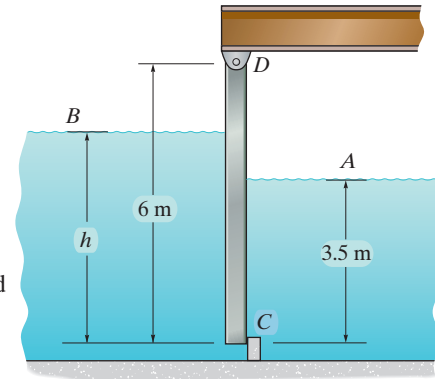
$$B_x = 7.52 \text{ kip}$$

$$B_y = 4.59 \text{ kip}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-59.

The tide gate opens automatically when the tide water at B subsides, allowing the marsh at A to drain. For the water level of $h = 4 \text{ m}$, determine the horizontal reaction at the smooth stop C . The gate has a width of 2 m . At what height h will the gate be on the verge of opening?



SOLUTION

Since the gate has a constant width of $b = 2 \text{ m}$, the intensities of the distributed load on the left and right sides of the gate at C are

$$(w_C)_L = \rho_w g h_{BC}(b) = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4 \text{ m})(2 \text{ m}) \\ = 78.48(10^3) \text{ N/m}$$

$$(w_C)_R = \rho_w g h_{AC}(b) = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3.5 \text{ m})(2 \text{ m}) \\ = 68.67(10^3) \text{ N/m}$$

The resultant triangular distributed load on the left and right sides of the gate is shown on its free-body diagram, Fig. a ,

$$F_L = \frac{1}{2}(w_C)_L L_{BC} = \frac{1}{2}(78.48(10^3) \text{ N/m})(4 \text{ m}) = 156.96(10^3) \text{ N}$$

$$F_R = \frac{1}{2}(w_C)_R L_{AC} = \frac{1}{2}(68.67(10^3) \text{ N/m})(3.5 \text{ m}) = 120.17(10^3) \text{ N}$$

These results can also be obtained as follows:

$$F_L = \gamma \bar{h}_L A_L = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m})[(4 \text{ m})(2 \text{ m})] = 156.96(10^3) \text{ N}$$

$$F_R = \gamma \bar{h}_R A_R = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.75 \text{ m})[3.5 \text{ m}(2 \text{ m})] = 120.17(10^3) \text{ N}$$

Referring to the free-body diagram of the gate in Fig. a ,

$$\zeta + \Sigma M_D = 0; \quad [156.96(10^3) \text{ N}]\left[2 \text{ m} + \frac{2}{3}(4 \text{ m})\right] - [120.17(10^3) \text{ N}]\left[2.5 \text{ m} + \frac{2}{3}(3.5 \text{ m})\right] - F_C(6 \text{ m}) = 0$$

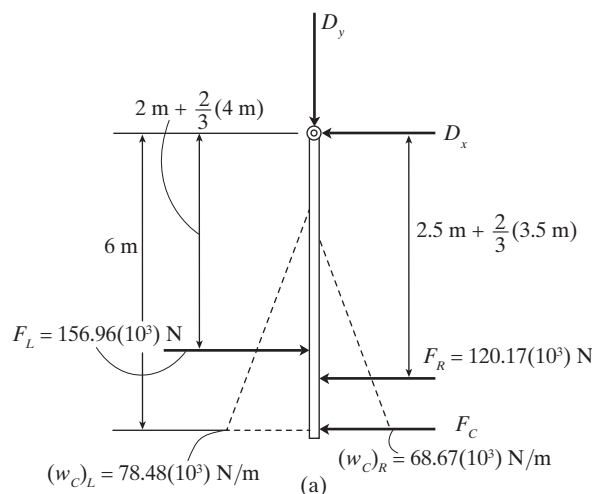
$$F_C = 25.27(10^3) \text{ N} = 25.3 \text{ kN}$$

Ans.

When $h = 3.5 \text{ m}$, the water levels are equal. Since $F_C = 0$, the gate will be on the verge of opening.

$$h = 3.5 \text{ m}$$

Ans.



Ans:
 $F_C = 25.3 \text{ kN}$
 $h = 3.5 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–60.**

The tide gate opens automatically when the tide water at B subsides, allowing the marsh at A to drain. Determine the horizontal reaction at the smooth stop C as a function of the depth h of the water level. Starting at $h = 6 \text{ m}$, plot values for each decrement of 0.5 m until the gate begins to open. The gate has a width of 2 m .

SOLUTION

Since the gate has a constant width of $b = 2 \text{ m}$, the intensities of the distributed loads on the left and right sides of the gate at C are

$$(w_C)_L = \rho_w g h_{BC} b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(h)(2 \text{ m}) = 19.62(10^3)h$$

$$(w_C)_R = \rho_w g h_{AC} b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3.5 \text{ m})(2 \text{ m}) = 68.67(10^3) \text{ N/m}$$

The resultant forces of the triangular distributed loads on the left and right sides of the gate shown on its FBD, Fig. a , are

$$F_L = \frac{1}{2}(w_C)_L h_{BC} = \frac{1}{2}[19.62(10^3)h]h = 9.81(10^3)h^2$$

$$F_R = \frac{1}{2}(w_C)_R h_{AC} = \frac{1}{2}[68.67(10^3) \text{ N/m}](3.5 \text{ m}) = 120.17(10^3) \text{ N}$$

Consider the moment equilibrium about D by referring to the FBD of the gate, Fig. a .

$$\zeta + \Sigma M_D = 0; \quad [9.81(10^3)h^2]\left(6 \text{ m} - h + \frac{2}{3}h\right) - 120.17(10^3)\left[2.5 \text{ m} + \frac{2}{3}(3.5 \text{ m})\right] - F_C(6 \text{ m}) = 0$$

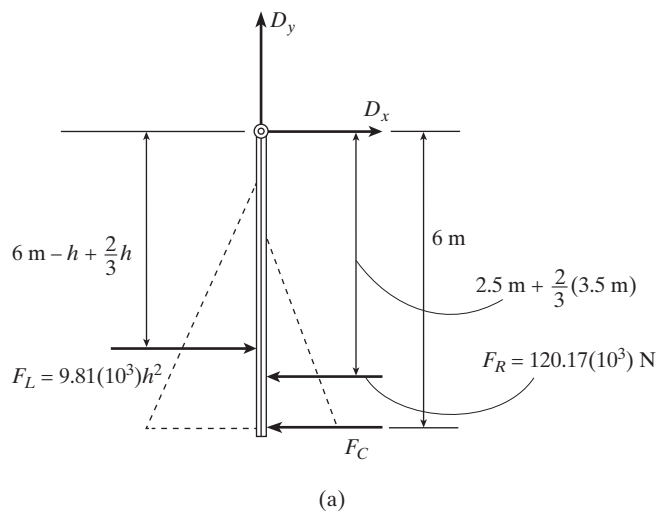
$$58.86(10^3)h^2 - 3.27(10^3)h^3 - 580.83(10^3) - 6F_C = 0$$

$$F_C = (9.81h^2 - 0.545h^3 - 96.806)(10^3) \text{ N}$$

$$F_C = (9.81h^2 - 0.545h^3 - 96.8) \text{ kN where } h \text{ is in meters.}$$

Ans.

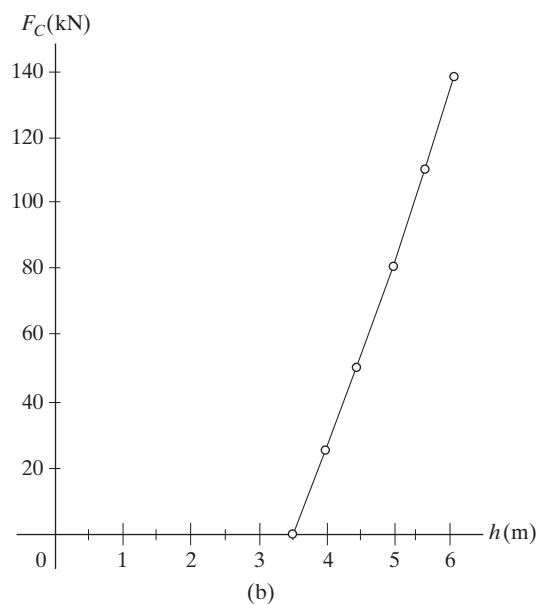
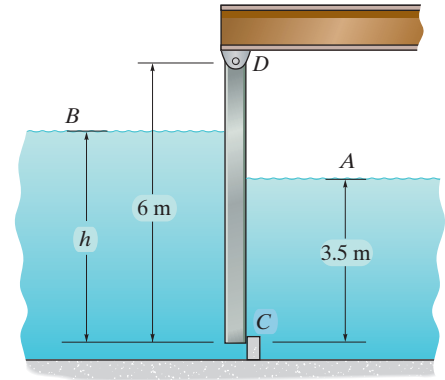
The gate will be on the verge of opening when the water level on both sides of the gate are equal, that is, when $h = 3.5 \text{ m}$. The plot of F_C vs h is shown in Fig. b .



Ans:

$$F_C = (9.81h^2 - 0.545h^3 - 96.8) \text{ kN}$$

where h is in meters.

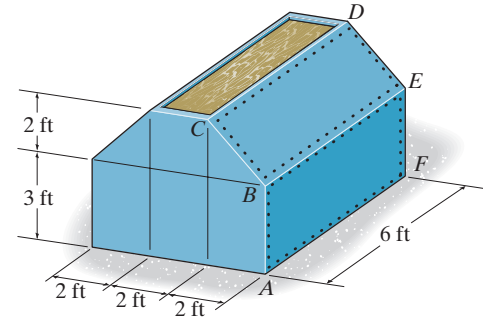


$h(\text{m})$	3.5	4	4.5	5.0	5.5	6.0
$F_C(\text{kN})$	0	25.3	52.2	80.3	109	139

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-61.

The bin is used to store carbon tetrachloride, a cleaning agent for metal parts. If it is filled to the top, determine the magnitude of the resultant force this liquid exerts on each of the two side plates, $AFE B$ and $BEDC$, and the location of the center of pressure on each plate, measured from BE . Take $\rho_{ct} = 3.09 \text{ slug/ft}^3$.



SOLUTION

Since the side plate has a width of $b = 6 \text{ ft}$, the intensities of the distributed load can be computed from

$$w_B = \rho g h_B b = (3.09 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(2 \text{ ft})(6 \text{ ft}) = 1193.976 \text{ lb/ft}$$

$$w_A = \rho g h_A b = (3.09 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(5 \text{ ft})(6 \text{ ft}) = 2984.94 \text{ lb/ft}$$

The resulting distributed load on plates $BEDC$ and $AFE B$ are shown in Figs. *a* and *b*, respectively. For plate $BEDC$,

$$F_{BEDC} = \frac{1}{2}(w_B)L_{BC} = \frac{1}{2}(1193.976 \text{ lb/ft})(2\sqrt{2} \text{ ft}) = 1688.54 \text{ lb} = 1.69 \text{ kip} \quad \text{Ans.}$$

And the center of pressure of this plate from BE is

$$y_P = \frac{1}{3}(2\sqrt{2} \text{ ft}) = 0.943 \text{ ft} \quad \text{Ans.}$$

For $AFE B$,

$$F_1 = w_B L_{AB} = (1193.976 \text{ lb/ft})(3 \text{ ft}) = 3581.93 \text{ lb}$$

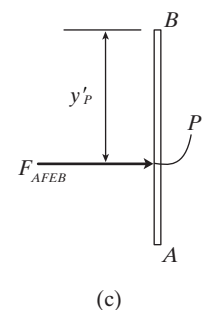
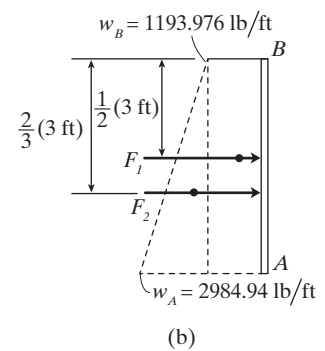
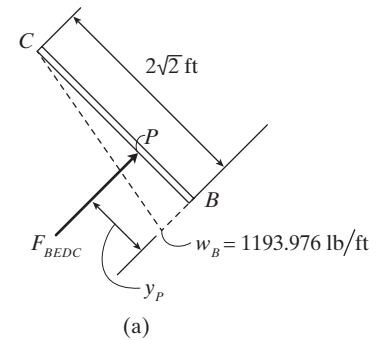
$$F_2 = \frac{1}{2}(w_A - w_B)L_{AB} = \frac{1}{2}(2984.94 \text{ lb/ft} - 1193.976 \text{ lb/ft})(3 \text{ ft}) = 2686.45 \text{ lb}$$

$$F_{AFE B} = F_1 + F_2 = 3581.93 \text{ lb} + 2686.45 \text{ lb} = 6268.37 \text{ lb} = 6.27 \text{ kip} \quad \text{Ans.}$$

The location of the center of pressure measured from BE can be obtained by equating the sum of the moments of the forces in Figs. *b* and *c*.

$$\zeta + M_{R_B} = \Sigma M_B; \quad (6268.37 \text{ lb})y'_P = (3581.93 \text{ lb})\left[\frac{1}{2}(3 \text{ ft})\right] + (2686.45 \text{ lb})\left[\frac{2}{3}(3 \text{ ft})\right]$$

$$y'_P = 1.714 \text{ ft} = 1.71 \text{ ft} \quad \text{Ans.}$$



Ans:

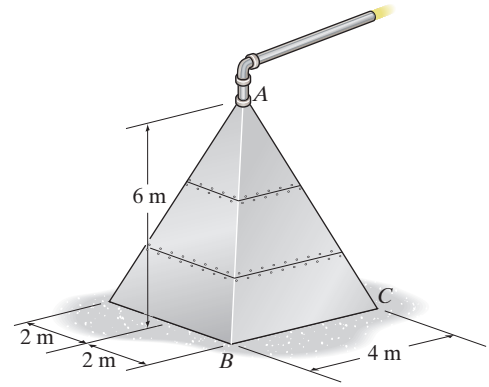
$$F_{BEDC} = 1.69 \text{ kip}, y_P = 0.943 \text{ ft}$$

$$F_{AFE B} = 6.27 \text{ kip}, y'_P = 1.71 \text{ ft}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-62.

Ethyl alcohol is pumped into the tank, which has the shape of a four-sided pyramid. When the tank is completely filled, determine the resultant force acting on each side, and its location measured from the top A along the side. Use the formula method. Take $\rho_{ea} = 789 \text{ kg/m}^3$.



SOLUTION

The geometry of the side wall of the tank is shown in Fig. *a*. In this case, it is convenient to calculate the resultant force as follows.

$$F_R = \gamma_{ea} \bar{h} A = (789 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \left[\frac{2}{3}(6 \text{ m}) \right] \left(\frac{1}{2} \right) (4 \text{ m})(\sqrt{40})$$

$$= 390.1(10^3) \text{ N} = 390 \text{ kN}$$

Ans.

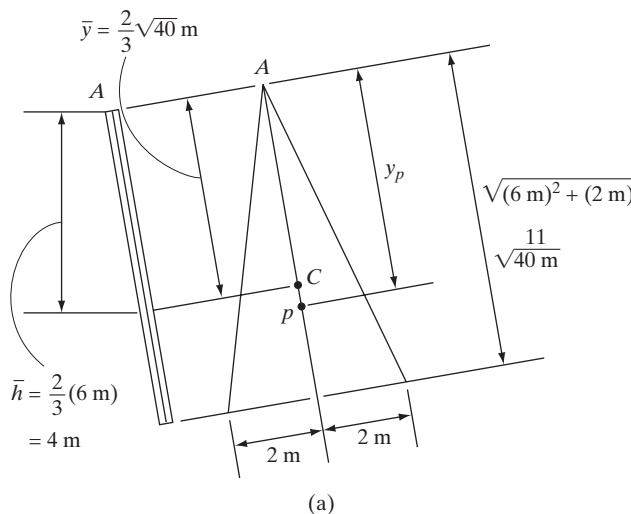
The location of the center of pressure can be determined from

$$y_P = \frac{\bar{I}_x}{\bar{y}_A} + \bar{y}$$

$$= \frac{\frac{1}{36}(4 \text{ m})(\sqrt{40} \text{ m})^3}{\frac{2}{3}(\sqrt{40} \text{ m})\left(\frac{1}{2}(4 \text{ m})(\sqrt{40} \text{ m})\right)} + \left(\frac{2}{3}\right)(\sqrt{40} \text{ m})$$

$$= 4.74 \text{ m}$$

Ans.



Ans:

$$F_R = 390 \text{ kN}$$

$$y_P = 4.74 \text{ m}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-63.

Determine the critical height h of the water level that causes the concrete gravity dam to be on the verge of tipping over. Assume water also seeps under the base of the dam and produces a uniform pressure under the dam. The density of concrete is $\rho_c = 2400 \text{ kg/m}^3$. *Hint:* Work the problem using a 1-m width of the dam.

SOLUTION

We will consider the dam as having a width of $b = 1 \text{ m}$. Then the intensity of the distributed load at the base of the dam is

$$w_B = \rho_w g h b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(h)(1 \text{ m}) = (9810h) \text{ N/m}$$

The resultant forces of the triangular distributed load and the uniform distributed load due to the pressure of seepage water shown on the FBD of the dam, Fig. *a*, are

$$F_1 = \frac{1}{2} w_B h = \frac{1}{2} (9810h)(h) = (4905h^2) \text{ N}$$

$$F_2 = w_B L_B = (9810h)(6 \text{ m}) = (58\,860h) \text{ N}$$

The weight of the concrete dam is

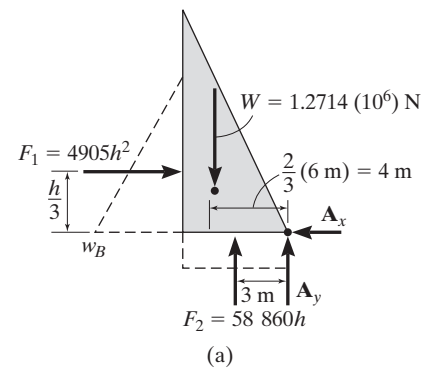
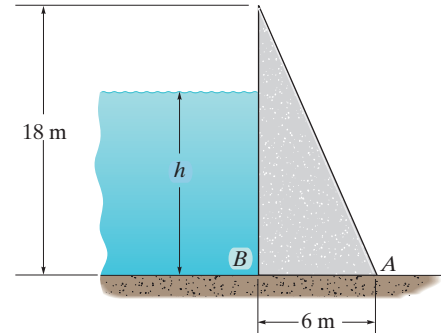
$$\begin{aligned} W &= \rho_c g V = (2400 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \left[\frac{1}{2} (6 \text{ m})(18 \text{ m})(1 \text{ m}) \right] \\ &= 1.2714(10^6) \text{ N} \end{aligned}$$

The dam will overturn about point *A*. Write the moment equation of equilibrium about point *A* by referring to Fig. *a*.

$$\begin{aligned} \zeta + \Sigma M_A &= 0; \quad [1.2714(10^6) \text{ N}](4 \text{ m}) - 4905h^2 \left(\frac{h}{3} \right) - 58\,860h(3 \text{ m}) = 0 \\ 1635h^3 + 176.58(10^3)h - 5.0855(10^6) &= 0 \end{aligned}$$

Solving numerically,

$$h = 12.1583 \text{ m} = 12.2 \text{ m} < 18 \text{ m} \quad \text{Ans.}$$



Ans:
 $h = 12.2 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-64.**

The pressure of the air at A within the closed tank is 200 kPa. Determine the resultant force acting on the plates BC and CD caused by the water. The tank has a width of 1.75 m.

SOLUTION

$$\begin{aligned} p_C &= p_B = p_A + \rho g h_{AB} \\ &= 200(10^3) \text{ N/m}^2 + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m}) \\ &= 219.62(10^3) \text{ Pa} \end{aligned}$$

$$\begin{aligned} p_D &= p_A + \rho g h_{AD} \\ &= 200(10^3) \text{ N/m}^2 + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3.5 \text{ m}) \\ &= 234.335(10^3) \text{ Pa} \end{aligned}$$

Since plates BC and CD have a constant width of $b = 1.75 \text{ m}$, the intensities of the distributed load at points B (or C) and D are

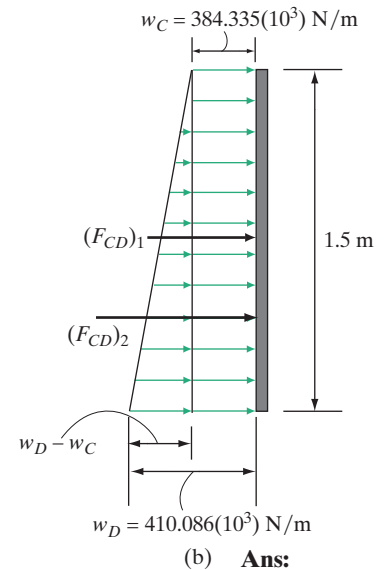
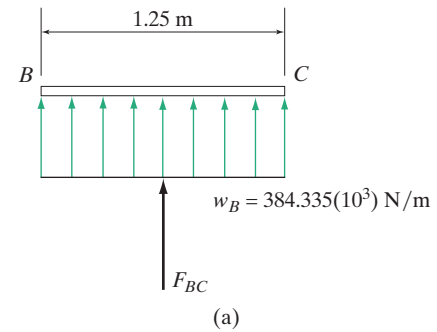
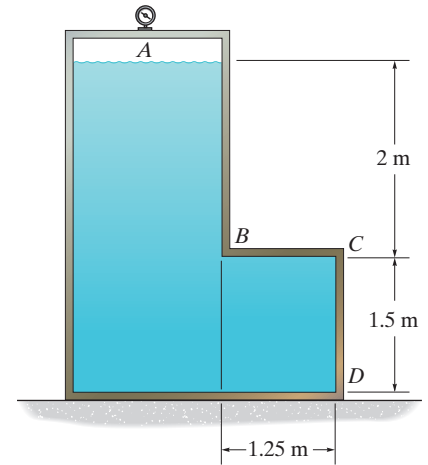
$$w_C = w_B = p_B b = (219.62(10^3) \text{ N/m}^2)(1.75 \text{ m}) = 384.335(10^3) \text{ N/m}$$

$$w_D = p_D b = (234.335(10^3) \text{ N/m}^2)(1.75 \text{ m}) = 410.086(10^3) \text{ N/m}$$

Using these results, the distributed loads acting on plates BC and CD are shown in Figs. a and b , respectively.

$$F_{BC} = w_B L_{BC} = \left[384.335(10^3) \frac{\text{N}}{\text{m}} \right] (1.25 \text{ m}) = 480.42(10^3) \text{ N} = 480 \text{ kN} \quad \text{Ans.}$$

$$\begin{aligned} F_{CD} &= (F_{CD})_1 + (F_{CD})_2 = w_C L_{CD} + \frac{1}{2}(w_D - w_C)L_{CD} \\ &= \left[384.335(10^3) \frac{\text{N}}{\text{m}} \right] (1.5 \text{ m}) + \frac{1}{2} \left[410.086(10^3) \frac{\text{N}}{\text{m}} - 384.335(10^3) \frac{\text{N}}{\text{m}} \right] (1.5 \text{ m}) \\ &= 595.82(10^3) \text{ N} = 596 \text{ kN} \quad \text{Ans.} \end{aligned}$$



Ans:
 $F_{BC} = 480 \text{ kN}$,
 $F_{CD} = 596 \text{ kN}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-65.

The uniform plate, which is hinged at C , is used to control the level of the water at A to maintain its constant depth of 6 m. If the plate has a width of 1.5 m and a mass of 30 Mg, determine the required minimum height h of the water at B so that seepage will not occur at D .

SOLUTION

Referring to the geometry shown in Fig. a ,

$$\frac{x}{5} = \frac{h}{4}; \quad x = \frac{5}{4}h$$

The intensities of the distributed load shown in the FBD of the gate, Fig. b , are

$$w_1 = \rho_w g h_1 b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m})(1.5 \text{ m}) = 29.43(10^3) \text{ N/m}$$

$$w_2 = \rho_w g h_2 b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(6 \text{ m})(1.5 \text{ m}) = 88.29(10^3) \text{ N/m}$$

$$w_3 = \rho_w g h_3 b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(h)(1.5 \text{ m}) = [14.715(10^3)h] \text{ N/m}$$

Then, the resultant forces of these distributed loads are

$$F_1 = w_1 l_{CD} = [29.43(10^3) \text{ N/m}](5 \text{ m}) = 147.15(10^3) \text{ N}$$

$$F_2 = \frac{1}{2}(w_2 - w_1)l_{CD} = \frac{1}{2}[88.29(10^3) \text{ N/m} - 29.43(10^3) \text{ N/m}](5 \text{ m}) = 147.15(10^3) \text{ N}$$

$$F_3 = \frac{1}{2}w_3 l_{BD} = \frac{1}{2}[14.715(10^3)h]\left(\frac{5}{4}h\right) = [9.196875(10^3)h^2] \text{ N}$$

and act at

$$d_1 = \frac{1}{2}(5 \text{ m}) = 2.5 \text{ m} \quad d_2 = \frac{2}{3}(5 \text{ m}) = 3.3333 \text{ m}$$

$$d_3 = 5 \text{ m} - \frac{1}{3}\left(\frac{5}{4}h\right) = (5 - 0.4167h) \text{ m}$$

The seepage is on the verge of occurring when the gate is about to open. Thus, it is required that $N_D = 0$. Write the moment equation of equilibrium about point C by referring to Fig. a .

$$(+\Sigma M_C = 0; [147.15(10^3) \text{ N}](2.5 \text{ m}) + [147.15(10^3) \text{ N}](3.3333 \text{ m})$$

$$- [30(10^3)(9.81) \text{ N}]\left(\frac{3}{5}\right)(2.5 \text{ m})$$

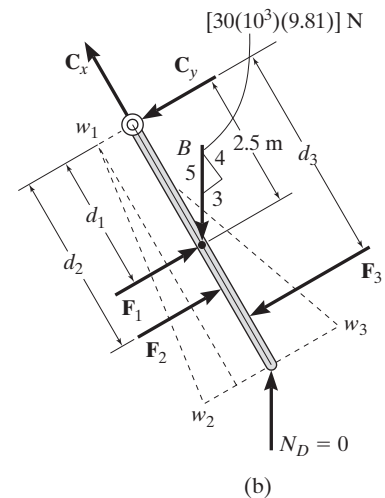
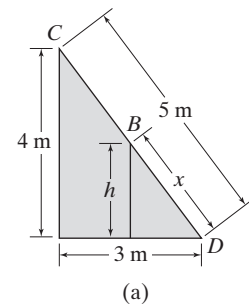
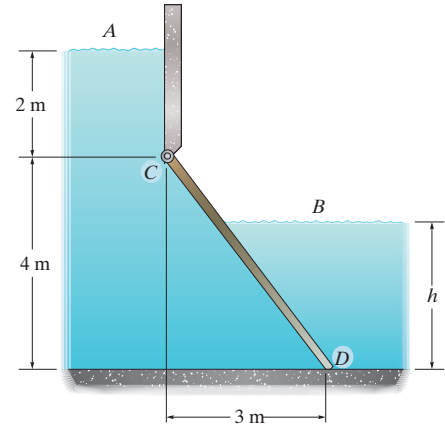
$$- [9.196875(10^3)h^2](5 - 0.4167h) = 0$$

$$3.8320h^3 - 45.9844h^2 + 416.925 = 0$$

Solving numerically,

$$h = 3.5987 \text{ m} = 3.60 \text{ m}$$

Ans.

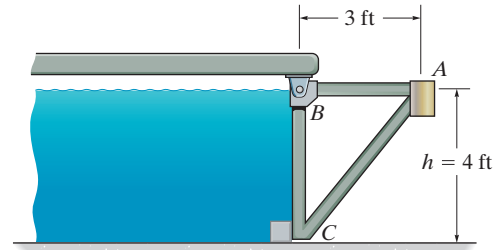


Ans:
 $h = 3.60 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–66.

Determine the weight of block A if the rectangular gate BC begins to open when the water level reaches the top of the channel, $h = 4 \text{ ft}$. The gate has a width of 2 ft . There is a smooth stop block at C .



SOLUTION

Since the gate has a constant width of $b = 2 \text{ ft}$, the intensity of the distributed load at C can be computed from

$$w_C = \gamma_w h_C b = (62.4 \text{ lb/ft}^3)(4 \text{ ft})(2 \text{ ft}) = 499.2 \text{ lb/ft}$$

The resultant triangular distributed load is shown on the free-body diagram of the gate, Fig. a , and the resultant force of this load is

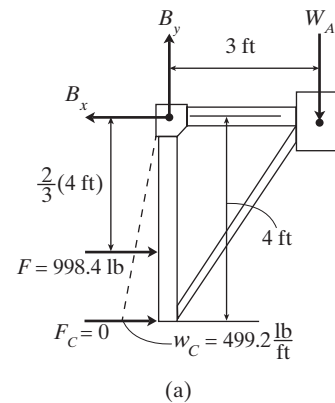
$$F = \frac{1}{2} w_C h_C = \frac{1}{2} (499.2 \text{ lb/ft})(4 \text{ ft}) = 998.4 \text{ lb}$$

Referring to the free-body diagram of the gate, Fig. a ,

$$\zeta + \Sigma M_B = 0; \quad 998.4 \text{ lb} \left[\frac{2}{3} (4 \text{ ft}) \right] - W_A (3 \text{ ft}) = 0$$

$$W_A = 887.47 \text{ lb} = 887 \text{ lb}$$

Ans.

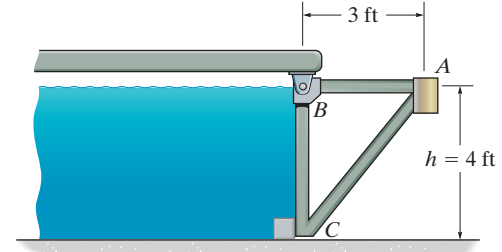


Ans:
 $W_A = 887 \text{ lb}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-67.

Determine the weight of block A so that the 2-ft-radius circular gate BC begins to open when the water level reaches the top of the channel, $h = 4 \text{ ft}$. There is a smooth stop block at C .



SOLUTION

Since the gate is circular in shape, it is convenient to compute the resultant force as follows.

$$F_R = \gamma_w \bar{h} A$$

$$= (62.4 \text{ lb/ft}^3)(2 \text{ ft})(\pi)(2 \text{ ft})^2 = 499.2\pi \text{ lb}$$

The location of the center of pressure can be determined from

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$

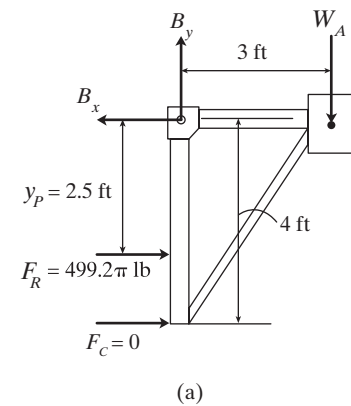
$$= \frac{\left(\frac{\pi(2 \text{ ft})^4}{4}\right)}{(2 \text{ ft})(\pi)(2 \text{ ft})^2} + 2 \text{ ft} = 2.50 \text{ ft}$$

Referring to the free-body diagram of the gate, Fig. a ,

$$\zeta + \Sigma M_B = 0; \quad 499.2\pi \text{ lb}(2.5 \text{ ft}) - W_A(3 \text{ ft}) = 0$$

$$W_A = 1306.90 \text{ lb} = 1.31 \text{ kip}$$

Ans.



Ans:
 $W_A = 1.31 \text{ kip}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-68.**

The tapered settling tank is filled to the top with alcohol. Determine the resultant force the alcohol exerts on the trapezoidal clean-out plate. How far from the top of the tank does this force act on the plate? Use the formula method. Take $\gamma_{al} = 49.3 \text{ lb/ft}^3$.

SOLUTION

Referring to the geometry of the plate shown in Fig. *a*,

$$A = \frac{1}{2}(2 \text{ ft})(3 \text{ ft}) + (4 \text{ ft})(3 \text{ ft}) = 15 \text{ ft}^2$$

$$\bar{y} = \frac{(4.5 \text{ ft})[(4 \text{ ft})(3 \text{ ft})] + (5 \text{ ft})\left[\frac{1}{2}(2 \text{ ft})(3 \text{ ft})\right]}{15 \text{ ft}^2} = 4.60 \text{ ft}$$

$$\begin{aligned} \bar{I}_x &= \frac{1}{12}(4 \text{ ft})(3 \text{ ft})^3 + (4 \text{ ft})(3 \text{ ft})(4.60 \text{ ft} - 4.5 \text{ ft})^2 \\ &\quad + \frac{1}{36}(2 \text{ ft})(3 \text{ ft})^3 + \frac{1}{2}(2 \text{ ft})(3 \text{ ft})(5 \text{ ft} - 4.60 \text{ ft})^2 \\ &= 11.1 \text{ ft}^4 \end{aligned}$$

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Here, $\bar{h} = \bar{y} = 4.60 \text{ ft}$. Then, the resultant force is given by

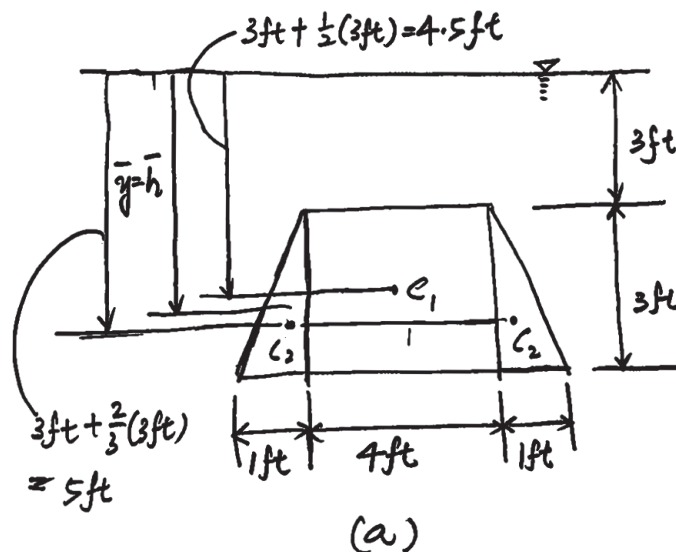
$$\begin{aligned} F_R &= \gamma_{al} \bar{h} A = (49.3 \text{ lb/ft}^3)(4.60 \text{ ft})(15 \text{ ft}^2) \\ &= 3.4017 (10^3) \text{ lb} = 3.40 \text{ kip } (10^3) \text{ lb} \end{aligned}$$

Ans.

And it acts at

$$\begin{aligned} y_P &= \bar{y} + \frac{\bar{I}_x}{\bar{y} A} = 4.60 \text{ ft} + \frac{11.1 \text{ ft}^4}{(4.60 \text{ ft})(15 \text{ ft}^2)} \\ &= 4.7609 \text{ ft} = 4.76 \text{ ft} \end{aligned}$$

Ans.

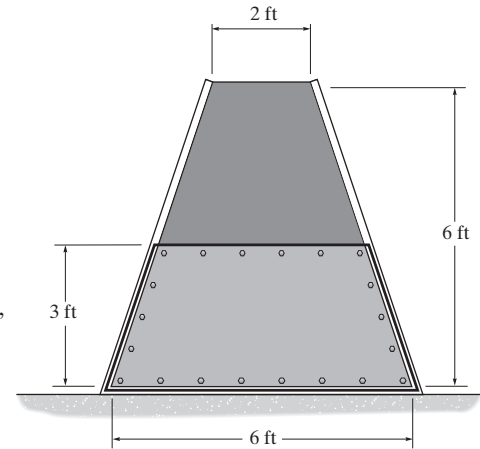


Ans:
 $F_R = 3.40 \text{ kip}$
 $y_P = 4.76 \text{ ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-69.

The tapered settling tank is filled to the top with alcohol. Determine the resultant force the alcohol exerts on the trapezoidal clean-out plate. How far from the top of the tank does this force act on the plate? Use the integration method. Take $\gamma_{al} = 49.3 \text{ lb/ft}^3$.



SOLUTION

With respect to x and y axes established, the equation of side AB of the plate, Fig. a , is

$$\frac{y-3}{x-2} = \frac{6-3}{3-2}; \quad 2x = \frac{2}{3}(y+3)$$

Thus, the area of the differential element shown shaded in Fig. a is $dA = 2xdy = \frac{2}{3}(y+3)dy$. The pressure acting on this differential element is $p = \gamma_{al}h = (49.3 \text{ lb/ft}^3)y = (49.3y) \text{ lb/ft}^2$. Thus, the resultant force acting on the portion of the plate contacting alcohol is

$$\begin{aligned} F_R &= \int_A p dA = \int_{3 \text{ ft}}^{6 \text{ ft}} (49.3y) \left[\frac{2}{3}(y+3)dy \right] \\ &= 32.8667 \left(\frac{y^3}{3} + \frac{3y^2}{2} \right) \bigg|_{3 \text{ ft}}^{6 \text{ ft}} \\ &= 3.4017(10^3) \text{ lb} = 3.40 \text{ kip} (10^3) \text{ lb} \end{aligned}$$

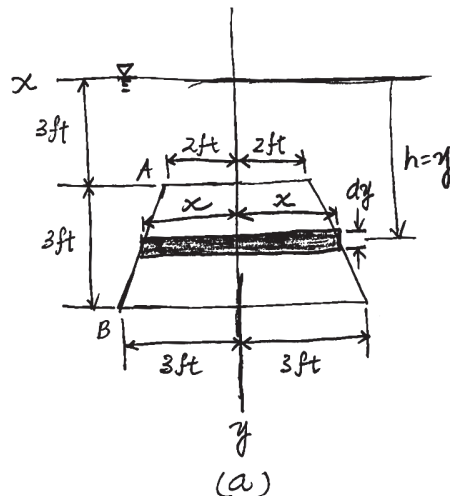
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Ans.

And it acts at

$$\begin{aligned} y_P &= \frac{\int_A y p dA}{F_R} = \frac{1}{3.4017(10^3)} \int_{3 \text{ ft}}^{6 \text{ ft}} y (49.3y) \left[\frac{2}{3}(y+3)dy \right] \\ &= \frac{1}{3.4017(10^3)} \left[32.8667 \left(\frac{y^4}{4} + y^3 \right) \right] \bigg|_{3 \text{ ft}}^{6 \text{ ft}} \\ &= 4.7609 \text{ ft} = 4.76 \text{ ft} \end{aligned}$$

Ans.



Ans:

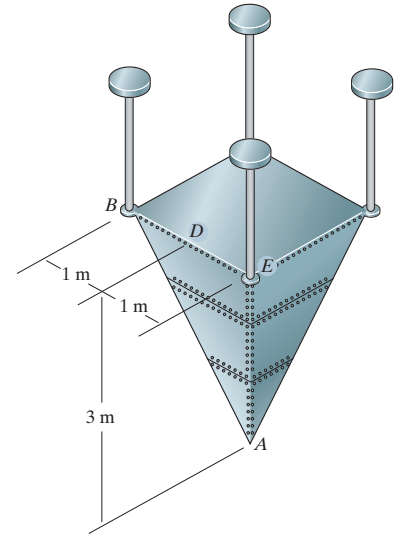
$$F_R = 3.40 \text{ kip}$$

$$y_P = 4.76 \text{ ft}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-70.

Water is stored in the silo, which has the shape of a four-sided pyramid. When the tank is completely full, determine the resultant force acting on each side, and its location measured from the top D along the side. Use the formula method.



SOLUTION

The geometry of the side wall of the silo is shown in Fig. a . Thus

$$A = \frac{1}{2}(2 \text{ m})(\sqrt{10} \text{ m}) = \sqrt{10} \text{ m}^2$$

$$\bar{y} = \frac{1}{3}\sqrt{10} \text{ m} \quad \bar{h} = \frac{1}{3}(3 \text{ m}) = 1 \text{ m}$$

$$\bar{I}_x = \frac{1}{36}(2 \text{ m})(\sqrt{10} \text{ m})^3 = \frac{5\sqrt{10}}{9} \text{ m}^4$$

The resultant force is

$$\begin{aligned} F_R &= \rho_w g \bar{h} A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1 \text{ m})(\sqrt{10} \text{ m}^2) \\ &= 31.022(10^3) \text{ N} = 31.0 \text{ kN} \end{aligned}$$

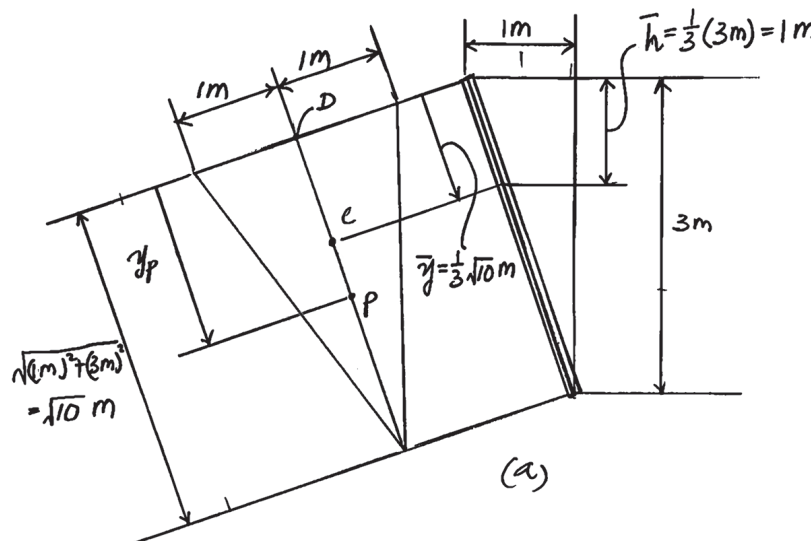
Ans.

And it acts at

$$y_P = \bar{y} + \frac{\bar{I}_x}{\bar{y}A} = \frac{1}{3}\sqrt{10} \text{ m} + \frac{\frac{5\sqrt{10}}{9} \text{ m}^4}{\left(\frac{1}{3}\sqrt{10} \text{ m}\right)(\sqrt{10} \text{ m}^2)}$$

$$= 1.5811 \text{ m} = 1.58 \text{ m}$$

Ans.



Ans:

$$F_R = 31.0 \text{ kN}$$

$$y_P = 1.58 \text{ m}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-71.

Water is stored in the silo, which has the shape of a four-sided pyramid. When the tank is completely full, determine the resultant force acting on each side and its location measured from top D along the side. Use the integration method.

SOLUTION

With respect to x and y axes established, the equation of side AB of the plate, Fig. a , is

$$\frac{y-0}{x-1} = \frac{\sqrt{10}-0}{0-1}; \quad x = 1 - \frac{y}{\sqrt{10}}$$

Also,

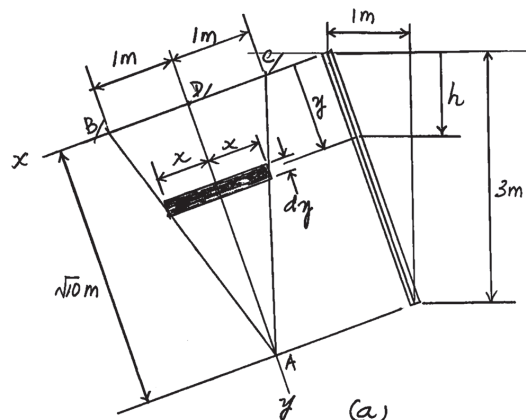
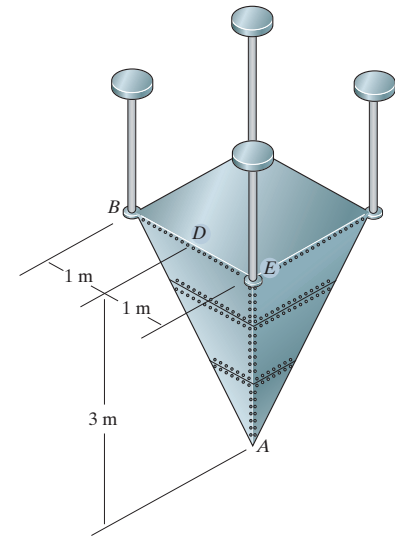
$$\frac{3-h}{3} = \frac{\sqrt{10}-y}{\sqrt{10}}; \quad h = \frac{3}{\sqrt{10}}y$$

Thus, the area of the differential element shown shaded in Fig. a is $dA = 2xdy = 2\left(1 - \frac{y}{\sqrt{10}}\right)dy$. The pressure acting on this differential element is $p = \rho_w gh = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left(\frac{3}{\sqrt{10}}y\right) = \left(\frac{29.43(10^3)}{\sqrt{10}}y\right) \text{ N/m}^2$. Thus, the resultant force acting on the entire plate is

$$\begin{aligned} F_R &= \int_A p dA = \int_0^{\sqrt{10} \text{ m}} \left[\frac{29.43(10^3)}{\sqrt{10}} y \right] \left[2\left(1 - \frac{y}{\sqrt{10}}\right) dy \right] \\ &= \frac{58.86(10^3)}{\sqrt{10}} \left(\frac{y^2}{2} - \frac{y^3}{3\sqrt{10}} \right) \bigg|_0^{\sqrt{10} \text{ m}} \\ &= 31.022(10^3) \text{ N} = 31.0 \text{ kN} \end{aligned}$$

And it acts at

$$\begin{aligned} y_P &= \frac{\int_A y p dA}{F_R} = \frac{1}{31.022(10^3)} \int_0^{\sqrt{10} \text{ m}} y \left[\frac{29.43(10^3)}{\sqrt{10}} y \right] \left[2\left(1 - \frac{y}{\sqrt{10}}\right) dy \right] \\ &= 0.6 \left(\frac{y^3}{3} - \frac{y^4}{4\sqrt{10}} \right) \bigg|_0^{\sqrt{10} \text{ m}} \\ &= 1.5811 \text{ m} = 1.58 \text{ m} \end{aligned}$$



Ans.

Ans.

Ans:

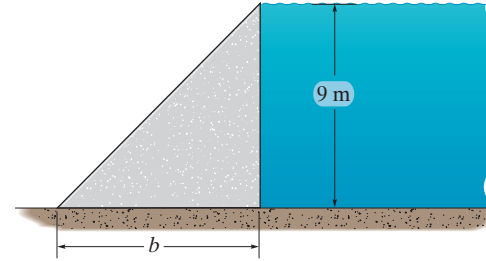
$$F_R = 31.0 \text{ kN}$$

$$y_P = 1.58 \text{ m}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-72.**

Determine the smallest base length b of the concrete gravity dam that will prevent the dam from overturning due to water pressure acting on the face of the dam. The density of concrete is $\rho_c = 2.4 \text{ Mg/m}^3$. *Hint:* Work the problem using a 1-m width of dam.



SOLUTION

If we consider the dam as having a width of $a = 1 \text{ m}$, the intensity of the distributed load at the base of the dam is

$$\begin{aligned} w_b &= \rho_w g h a = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(9 \text{ m})(1 \text{ m}) \\ &= 88.29(10^3) \text{ N/m} \end{aligned}$$

The resultant force of the triangular distributed load shown on the free-body diagram of the dam, Fig. *a*, is

$$F = \frac{1}{2} w_b h = \frac{1}{2} [88.29(10^3) \text{ N/m}] (9 \text{ m}) = 397.305(10^3) \text{ N}$$

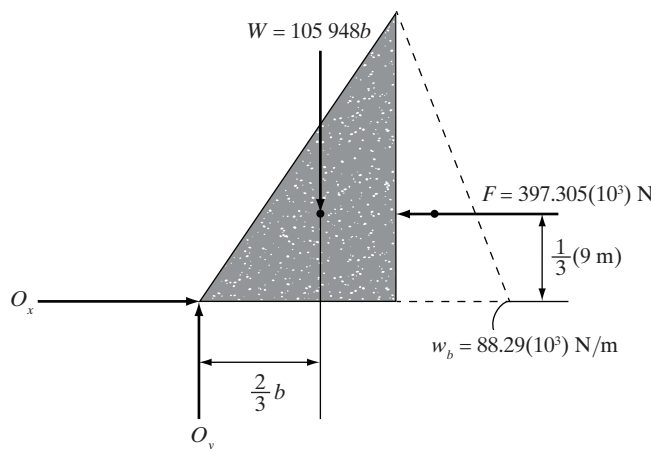
The weight of the dam is given by

$$\begin{aligned} W &= \rho_c g V = [2.4(10^3) \text{ kg/m}^3] (9.81 \text{ m/s}^2) \left[\frac{1}{2} (9 \text{ m})(1 \text{ m})b \right] \\ &= 105\,948b \end{aligned}$$

The dam will overturn about point O . Referring to the free-body diagram of the dam, Fig. *a*,

$$\begin{aligned} \zeta + \Sigma M_O &= 0; \quad [397.305(10^3) \text{ N}] \left[\frac{1}{3} (9 \text{ m}) \right] - 105\,948b \left(\frac{2}{3} b \right) = 0 \\ b &= 4.108 \text{ m} = 4.11 \text{ m} \end{aligned}$$

Ans.



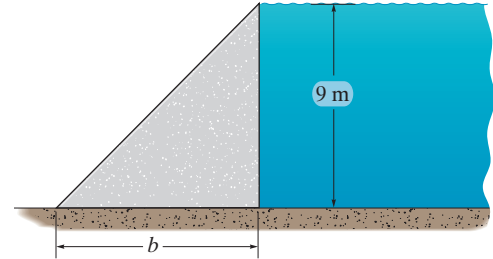
(a)

Ans:
 $b = 4.11 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-73.

Determine the smallest base length b of the concrete gravity dam that will prevent the dam from overturning due to water pressure acting on the face of the dam. Assume water also seeps under the base of the dam and produces a uniform pressure under the dam. The density of concrete is $\rho_c = 2.4 \text{ Mg/m}^3$. *Hint:* Work the problem using a 1-m width of dam.



SOLUTION

If we consider the dam as having a width of $a = 1 \text{ m}$, the intensity of the distributed load at the base of the dam is

$$w_b = \rho g h a = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(9 \text{ m})(1 \text{ m}) = 88.29(10^3) \text{ N/m}$$

The resultant forces of the triangular distributed load and the uniform distributed load due to the pressure of the seepage water shown on the FBD of the dam, Fig. *a*, is

$$F_1 = \frac{1}{2} w_b h = \frac{1}{2} [88.29(10^3) \text{ N/m}](9 \text{ m}) = 397.305(10^3) \text{ N}$$

$$F_2 = w_b b = [88.29(10^3) \text{ N/m}] b = 88.29(10^3) b$$

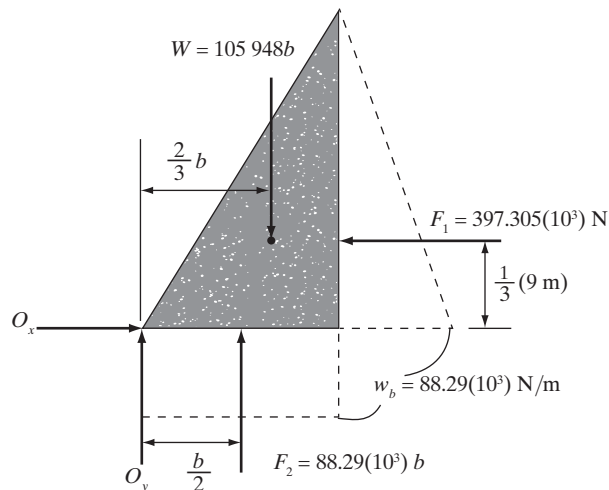
The weight of the dam is given by

$$\begin{aligned} W &= \rho_c g V = [2.4(10^3) \text{ kg/m}^3](9.81 \text{ m/s}^2) \left[\frac{1}{2} (b)(9 \text{ m})(1 \text{ m}) \right] \\ &= 105\,948b \end{aligned}$$

The dam will overturn about point O . Referring to the FBD of the dam, Fig. *a*,

$$\begin{aligned} \zeta + \Sigma M_O = 0; \quad & [397.305(10^3) \text{ N}] \left[\frac{1}{3}(9 \text{ m}) \right] + [88.29(10^3) b] \left(\frac{b}{2} \right) - 105\,948b \left(\frac{2}{3}b \right) = 0 \\ & b = 6.708 \text{ m} = 6.71 \text{ m} \end{aligned}$$

Ans.



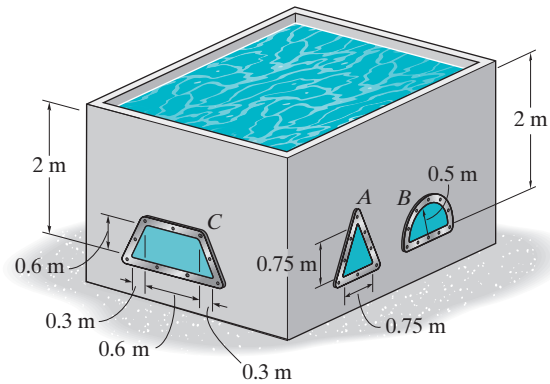
(a)

Ans:
 $b = 6.71 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–74.

The tank is filled with water. Determine the resultant force acting on the triangular plate *A* and the location of the center of pressure, measured from the water surface. Solve the problem using the formula method.



SOLUTION

The resultant force is

$$F_R = \rho_w g \bar{h} A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.75 \text{ m}) \left[\frac{1}{2} (0.75 \text{ m})(0.75 \text{ m}) \right]$$

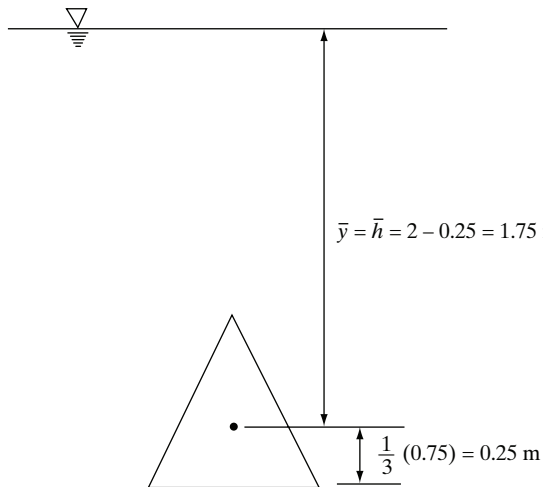
$$= 4828.36 \text{ N} = 4.83 \text{ kN} \quad \text{Ans.}$$

$$\bar{I}_x = \frac{1}{36} (bh^3) = \frac{1}{36} (0.75 \text{ m})(0.75 \text{ m})^3 = 8.7891(10^{-3}) \text{ m}^4$$

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{8.7891(10^{-3}) \text{ m}^4}{(1.75 \text{ m}) \left[\frac{1}{2} (0.75 \text{ m})(0.75 \text{ m}) \right]} + 1.75 \text{ m}$$

$$= 1.768 \text{ m} = 1.77 \text{ m} \quad \text{Ans.}$$

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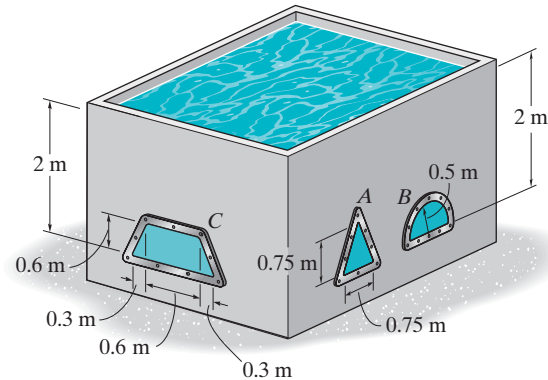


Ans:
 $F_R = 4.83 \text{ kN}$
 $y_P = 1.77 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-75.

Solve Prob. 2-74 using the integration method.



SOLUTION

From the geometry shown in Fig. *a*,

$$\frac{y}{0.75 \text{ m}} = \frac{0.375 \text{ m} - x}{0.375 \text{ m}} \quad x = (0.375 - 0.5y) \text{ m}$$

Referring to Fig. *b*, the pressure as a function of y can be written as

$$p = \rho_w g h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m} - y) = [9810(2 - y)] \text{ N/m}^2$$

This pressure acts on the element of area $dA = 2xdy = 2(0.375 - 0.5y)dy$.

Thus,

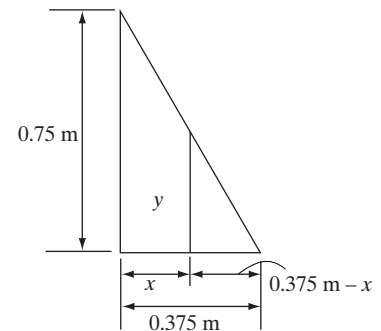
$$\begin{aligned} dF &= p dA = [9810(2 - y) \text{ N/m}^2] [2(0.375 - 0.5y)dy] \\ &= 19\,620(0.5y^2 - 1.375y + 0.75)dy \end{aligned}$$

Then

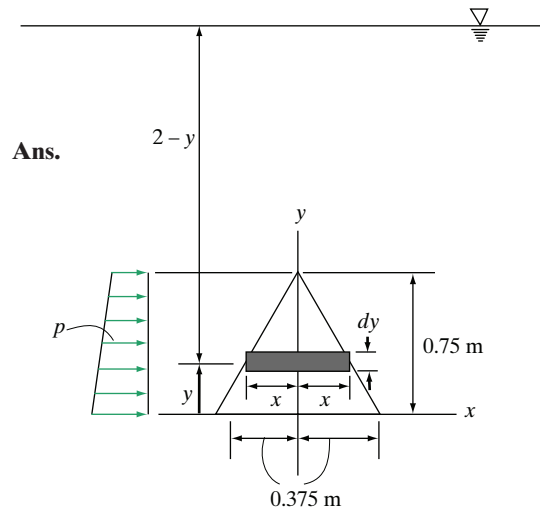
$$\begin{aligned} F_R &= \int dF = 19\,620 \int_0^{0.75 \text{ m}} (0.5y^2 - 1.375y + 0.75) dy \\ &= 19\,620 \left[\frac{0.5y^3}{3} - \frac{1.375y^2}{2} + 0.75y \right] \bigg|_0^{0.75 \text{ m}} \\ &= 4828.36 \text{ N} = 4.83 \text{ kN} \end{aligned}$$

And

$$\begin{aligned} y_P &= \frac{\int (2 - y) dF}{F_R} \\ &= \frac{\int_0^{0.75 \text{ m}} (2 - y) [19\,620(0.5y^2 - 1.375y + 0.75) dy]}{4828.36} \\ &= \frac{19\,620 \int_0^{0.75 \text{ m}} (-0.5y^3 + 2.375y^2 - 3.5y + 1.5) dy}{4828.36} \\ &= \frac{19\,620 \left(-0.125y^4 + 0.79167y^3 - 1.75y^2 + 1.5y \right) \bigg|_0^{0.75 \text{ m}}}{4828.36} \\ &= 1.768 = 1.77 \text{ m} \end{aligned}$$



(a)



(b)

Ans.

Ans.

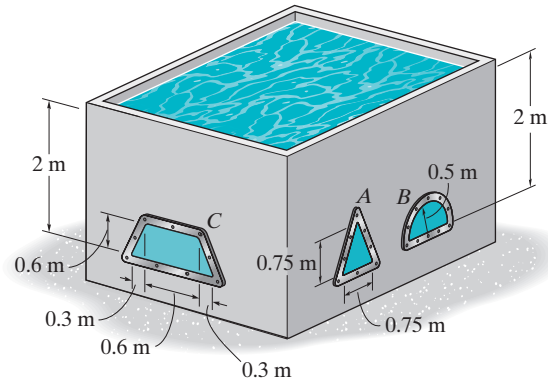
Ans:

$$\begin{aligned} F_R &= 4.83 \text{ kN} \\ y_P &= 1.77 \text{ m} \end{aligned}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-76.**

The tank is filled with water. Determine the resultant force acting on the semicircular plate B and the location of the center of pressure, measured from the water surface. Solve the problem using the formula method.



SOLUTION

The resultant force is

$$F_R = \rho_w g \bar{h} A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \left[\left(2 - \frac{2}{3\pi} \right) \text{ m} \right] \left[\frac{1}{2} \pi (0.5 \text{ m})^2 \right]$$

$$= 6887.26 \text{ N} = 6.89 \text{ kN}$$

Ans.

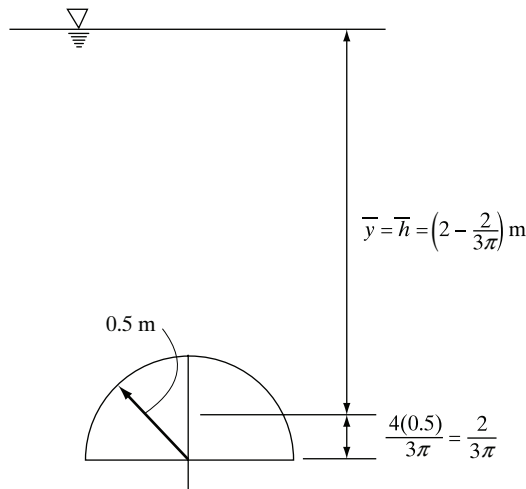
$$\bar{I}_x = 0.1098 r^4 = 0.1098 (0.5 \text{ m})^4 = 6.8625 (10^{-3}) \text{ m}^4$$

Then

$$y_P = \frac{\bar{I}_x}{\bar{y} A} + \bar{y} = \frac{6.8625 (10^{-3}) \text{ m}^4}{\left[\left(2 - \frac{2}{3\pi} \right) \text{ m} \right] \left[\frac{1}{2} \pi (0.5 \text{ m})^2 \right]} + \left(2 - \frac{2}{3\pi} \right) \text{ m} = 1.798 \text{ m}$$

$$= 1.80 \text{ m}$$

Ans.



Ans:

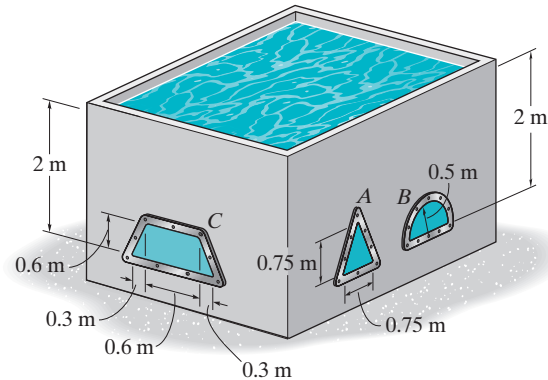
$$F_R = 6.89 \text{ kN}$$

$$y_P = 1.80 \text{ m}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-77.

Solve Prob. 2-76 using the integration method.



SOLUTION

Referring to Fig. *a*, the pressure as a function of y can be written as

$$p = \rho_w g h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m} - y) = [9810(2 - y)] \text{ N/m}^2$$

This pressure acts on the strip element of area $dA = 2xdy$. Here, $x = (0.25 - y^2)^{1/2}$. Thus, $dA = 2(0.25 - y^2)^{1/2} dy$. Then

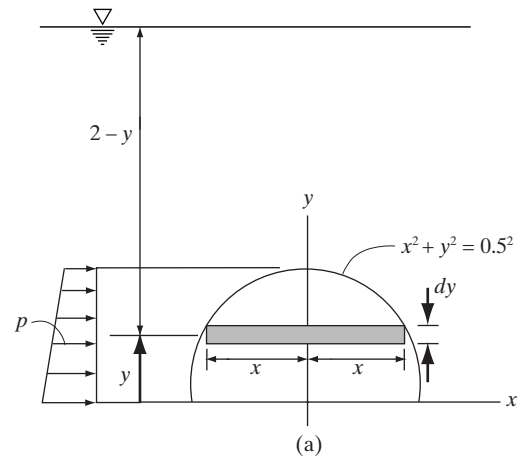
$$\begin{aligned} dF &= p dA = 9810(2 - y)[2(0.25 - y^2)^{1/2} dy] \\ &= 19620[2(0.25 - y^2)^{1/2} - y(0.25 - y^2)^{1/2}] dy \end{aligned}$$

Then

$$\begin{aligned} F_R &= \int dF = 19620 \int_0^{0.5 \text{ m}} [2(0.25 - y^2)^{1/2} - y(0.25 - y^2)^{1/2}] dy \\ &= 19620 \left[y(0.25 - y^2)^{1/2} + 0.25 \sin^{-1} \frac{y}{0.5} + \frac{1}{3}(0.25 - y^2)^{3/2} \right]_0^{0.5 \text{ m}} \\ &= 6887.26 \text{ N} \\ &= 6.89 \text{ kN} \end{aligned}$$

And

$$\begin{aligned} y_P &= \frac{\int (2 - y) dF}{F_R} \\ &= \frac{\int_0^{0.5 \text{ m}} (2 - y) \{ 19620[2(0.25 - y^2)^{1/2} - y(0.25 - y^2)^{1/2}] dy \}}{6887.26} \\ &= \frac{19620 \int_0^{0.5 \text{ m}} [4(0.25 - y^2)^{1/2} - 4y(0.25 - y^2)^{1/2} + y^2(0.25 - y^2)^{1/2}] dy}{6887.26} \\ &= \frac{19620 \left[2y(0.25 - y^2)^{1/2} + 0.5 \sin^{-1} \frac{y}{0.5} + \frac{4}{3}(0.25 - y^2)^{3/2} \right. \\ &\quad \left. - \frac{y}{4}(0.25 - y^2)^{3/2} + \frac{y}{32}(0.25 - y^2)^{1/2} + \frac{1}{128} \sin^{-1} \frac{y}{0.5} \right]_0^{0.5 \text{ m}}}{6887.26} \\ &= \frac{12380.29}{6887.26} = 1.798 \text{ m} = 1.80 \text{ m} \end{aligned}$$



Ans.

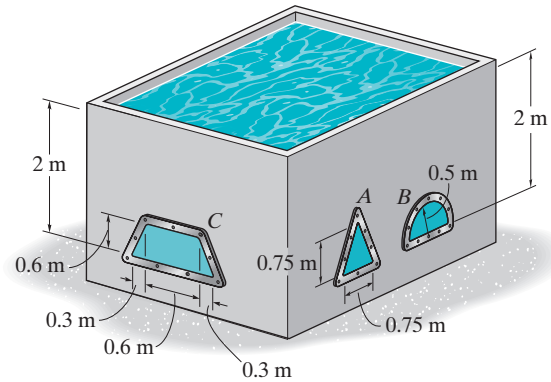
Ans.

Ans:
 $F_R = 6.89 \text{ kN}$
 $y_P = 1.80 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-78.

The tank is filled with water. Determine the resultant force acting on the trapezoidal plate *C* and the location of the center of pressure, measured from the water surface. Solve the problem using the formula method.



SOLUTION

Referring to the geometry shown in Fig. *a*,

$$A = (0.6 \text{ m})(0.6 \text{ m}) + \frac{1}{2}(0.6 \text{ m})(0.6 \text{ m}) = 0.54 \text{ m}^2$$

$$\bar{y} = \frac{(1.7 \text{ m})(0.6 \text{ m})(0.6 \text{ m}) + (1.8 \text{ m})\left[\frac{1}{2}(0.6 \text{ m})(0.6 \text{ m})\right]}{0.54 \text{ m}^2} = 1.7333 \text{ m}$$

$$\begin{aligned} \bar{I}_x &= \frac{1}{12}(0.6 \text{ m})(0.6 \text{ m})^3 + (0.6 \text{ m})(0.6 \text{ m})(1.7333 \text{ m} - 1.7 \text{ m})^2 \\ &\quad + \frac{1}{36}(0.6 \text{ m})(0.6 \text{ m})^3 + \frac{1}{2}(0.6 \text{ m})(0.6 \text{ m})(1.8 \text{ m} - 1.7333 \text{ m})^2 \\ &= 0.0156 \text{ m}^4 \end{aligned}$$

The resultant force is

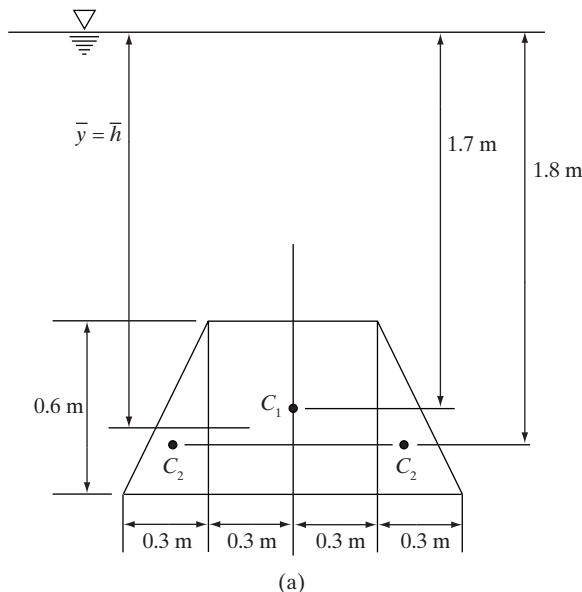
$$F_R = \rho_w g \bar{h} A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.7333 \text{ m})(0.54 \text{ m}^2) = 9182.16 \text{ N} \\ = 9.18 \text{ kN}$$

Ans.

And it acts at

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{0.0156 \text{ m}^4}{(1.7333 \text{ m})(0.54 \text{ m}^2)} + 1.7333 \text{ m} = 1.75 \text{ m}$$

Ans.



Ans:
 $F_R = 9.18 \text{ kN}$
 $y_P = 1.75 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-79.

Solve Prob. 2-78 using the integration method.

SOLUTION

Referring to the geometry shown in Fig. *a*,

$$\frac{0.6 \text{ m} - y}{0.6 \text{ m}} = \frac{x - 0.3 \text{ m}}{0.3 \text{ m}}; \quad x = (0.6 - 0.5y) \text{ m}$$

Referring to Fig. *b*, the pressure as a function of *y* can be written as

$$p = \rho_w g h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 - y) \text{ m} = [9810(2 - y)] \text{ N/m}^2$$

This pressure acts on the element of area $dA = 2xdy = 2(0.6 - 0.5y)dy = (1.2 - y)dy$. Thus

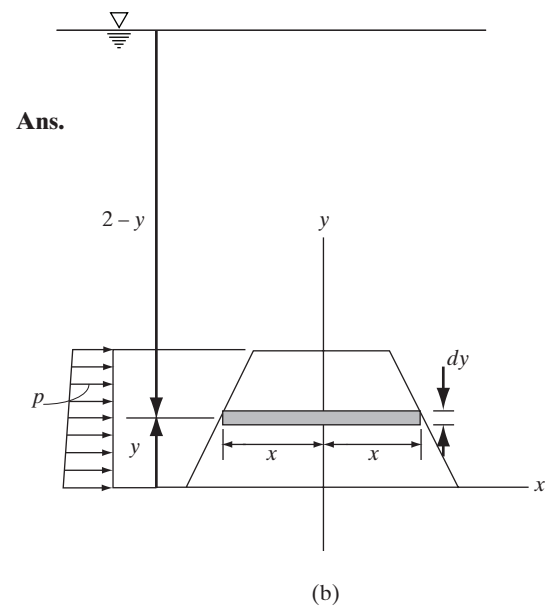
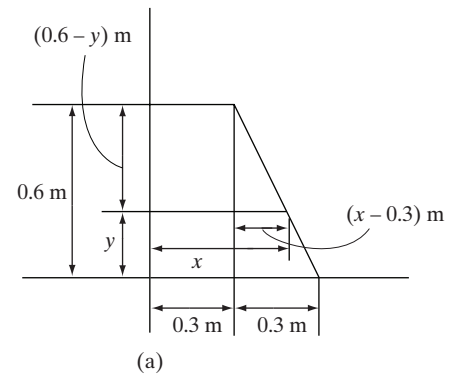
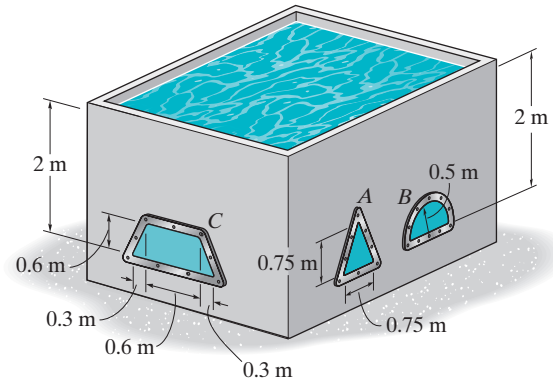
$$\begin{aligned} dF &= p dA = 9810(2 - y)(1.2 - y)dy \\ &= 9810(y^2 - 3.2y + 2.4)dy \end{aligned}$$

Then

$$\begin{aligned} F_R &= \int dF = 9810 \int_0^{0.6 \text{ m}} (y^2 - 3.2y + 2.4) dy \\ &= 9810 \left(\frac{y^3}{3} - 1.6y^2 + 2.4y \right) \bigg|_0^{0.6 \text{ m}} \\ &= 9182.16 \text{ N} = 9.18 \text{ kN} \end{aligned}$$

And it acts at

$$\begin{aligned} y_P &= \frac{\int (2 - y) dF}{F_R} \\ &= \frac{\int_0^{0.6 \text{ m}} (2 - y)[9810(y^2 - 3.2y + 2.4)dy]}{9182.16} \\ &= \frac{9810 \int_0^{0.6 \text{ m}} (-y^3 + 5.2y^2 - 8.8y + 4.8)dy}{9182.16} \\ &= \frac{9810 \left(-\frac{y^4}{4} + 1.7333y^3 - 4.4y^2 + 4.8y \right) \bigg|_0^{0.6 \text{ m}}}{9182.16} \\ &= 1.75 \text{ m} \end{aligned}$$



Ans.

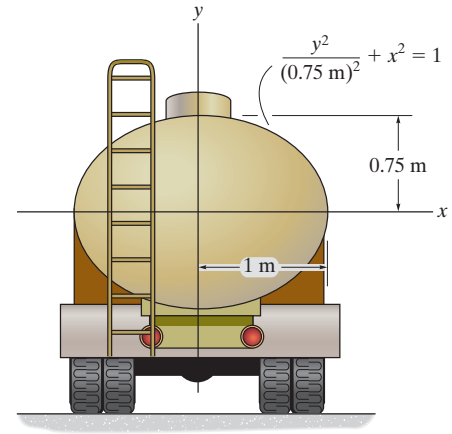
Ans.

Ans:
 $F_R = 9.18 \text{ kN}$
 $y_P = 1.75 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-80.**

The tank truck is filled to its top, $y = 0.75 \text{ m}$, with water. Determine the magnitude of the resultant force on the elliptical back plate of the tank, and the location of the center of pressure measured from the top of the tank. Solve the problem using the formula method.



SOLUTION

Using the table on the inside back cover for the area and moment of inertia about the centroidal x axis of the elliptical plate, we get

$$F_R = \rho_w g \bar{h} A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.75 \text{ m})(\pi)(0.75 \text{ m})(1 \text{ m})$$

$$= 17.3 \text{ kN}$$

Ans.

The center of pressure is at

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$

$$= \frac{\left[\frac{1}{4} \pi (1 \text{ m})(0.75 \text{ m})^3 \right]}{(0.75 \text{ m})\pi(1 \text{ m})(0.75 \text{ m})} + 0.75 \text{ m}$$

$$= 0.9375 \text{ m} = 0.938 \text{ m}$$

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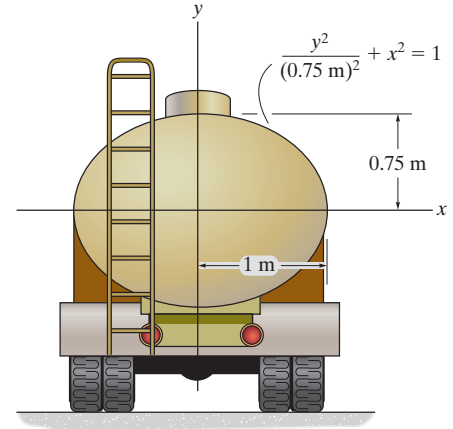
Ans.

Ans:
 $F_R = 17.3 \text{ kN}$
 $y_P = 0.938 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–81.

Solve Prob. 2–80 using the integration method.



SOLUTION

By integration of a horizontal strip of area,

$$dF = p dA = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.75 \text{ m} - y)(2x dy)$$

$$F_R = 19\,620 \int_{-0.75 \text{ m}}^{0.75 \text{ m}} (0.75 - y) \left(1 - \frac{y^2}{(0.75)^2}\right)^{\frac{1}{2}} dy$$

$$= 19\,620 \left[\int_{-0.75 \text{ m}}^{0.75 \text{ m}} \sqrt{(0.75)^2 - y^2} dy - \frac{1}{0.75} \int_{-0.75 \text{ m}}^{0.75 \text{ m}} y \sqrt{(0.75)^2 - y^2} dy \right]$$

$$= \frac{19\,620}{2} \left[y \sqrt{(0.75)^2 - y^2} + (0.75)^2 \sin^{-1} \frac{y}{0.75} \right]_{-0.75}^{0.75} - \frac{19\,620}{0.75} \left[-\frac{1}{3} \sqrt{((0.75)^2 - y^2)^3} \right]_{-0.75}^{0.75}$$

$$= \frac{19\,620\pi(0.75)^2}{2} - 0 = 17\,336 \text{ N} = 17.3 \text{ kN}$$

Ans.

$$y_P = \frac{19\,620 \int_{-0.75 \text{ m}}^{0.75 \text{ m}} y(0.75 - y) \left(1 - \frac{y^2}{(0.75)^2}\right)^{\frac{1}{2}} dy}{17\,336 \text{ N}} = -0.1875 \text{ m from the } x \text{ axis}$$

$$y_P = 0.75 \text{ m} + 0.1875 \text{ m} = 0.9375 \text{ m} = 0.938 \text{ m from the top of the tank}$$

Ans.

Ans:

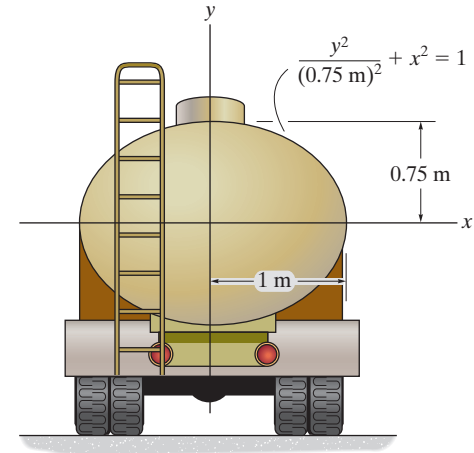
$$F_R = 17.3 \text{ kN}$$

$$y_P = 0.938 \text{ m}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-82.

The tank truck is half filled with water. Determine the magnitude of the resultant force on the elliptical back plate of the tank, and the location of the center of pressure measured from the x axis. Solve the problem using the formula method. *Hint:* The centroid of a semi-ellipse measured from the x axis is $\bar{y} = 4b/3\pi$ and the area and moment of inertia about the x axis are $\pi ab/2$ and $\pi ab^3/8$, respectively, where a and b are the major and minor axes.



SOLUTION

The area and moment of inertia about the x axis of the half-ellipse plate are

$$A = \frac{\pi}{2}ab = \frac{\pi}{2}(1 \text{ m})(0.75 \text{ m}) = 0.375\pi \text{ m}^2$$

$$I_x = \frac{1}{2} \left(\frac{\pi}{4}ab^3 \right) = \frac{1}{2} \left[\frac{\pi}{4}(1 \text{ m})(0.75 \text{ m})^3 \right] = 0.05273\pi \text{ m}^4$$

Thus, the moment of inertia of the half of ellipse about its centroidal $\bar{x}$ axis can be determined by using the parallel-axis theorem.

$$\begin{aligned} I_x &= \bar{I}_x + A\bar{y}^2 \\ 0.05273\pi \text{ m}^4 &= \bar{I}_x + (0.375 \text{ m}^2) \left[\frac{4(0.75 \text{ m})}{3\pi} \right]^2 \\ \bar{I}_x &= 0.046304 \text{ m}^4 \end{aligned}$$

Since $\bar{h} = \frac{4(0.75 \text{ m})}{3\pi} = 0.3183 \text{ m}$, then

$$\begin{aligned} F_R &= \gamma \bar{h} A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3183 \text{ m})(0.375\pi \text{ m}^2) \\ &= 3.679(10^3) \text{ N} = 3.68 \text{ kN} \end{aligned}$$

Ans.

Since $\bar{y} = \bar{h} = 0.3183 \text{ m}$,

$$\begin{aligned} y_P &= \frac{\bar{I}_x}{\bar{y}A} + \bar{y} \\ &= \frac{0.046304 \text{ m}^4}{(0.3183 \text{ m})(0.375\pi \text{ m}^2)} + 0.3183 \text{ m} \\ &= 0.4418 \text{ m} = 442 \text{ mm} \end{aligned}$$

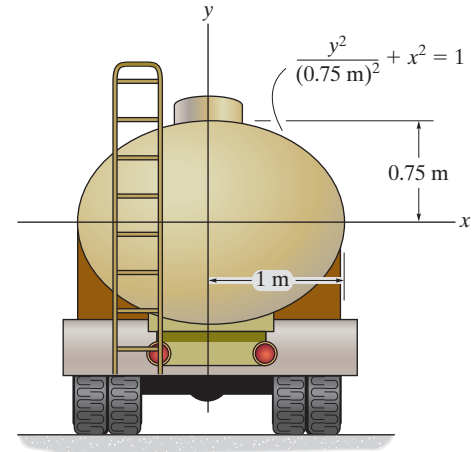
Ans.

Ans:
 $F_R = 3.68 \text{ kN}$
 $y_P = 442 \text{ mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–83.

Solve Prob. 2–82 using the integration method.



SOLUTION

Using a horizontal strip of area dA ,

$$dF = p dA$$

$$dF = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(-y)(2x dy)$$

$$F_R = -19\,620 \int_{-0.75 \text{ m}}^0 (y) \left(1 - \frac{y^2}{0.75^2}\right)^{\frac{1}{2}} dy$$

$$= -\frac{19\,620}{0.75} \int_{-0.75 \text{ m}}^0 y \sqrt{0.75^2 - y^2} dy$$

$$= \frac{26\,160}{3} \left[\sqrt{(0.75^2 - y^2)^3} \right]_{-0.75 \text{ m}}^0$$

$$= 3678.75 \text{ N} = 3.68 \text{ kN}$$

Ans.

$$y_P = -\frac{-19\,620 \int_{-0.75 \text{ m}}^0 y(y) \left(1 - \frac{y^2}{0.75^2}\right)^{\frac{1}{2}} dy}{3678.75 \text{ N}} = 0.4418 \text{ m} = 442 \text{ mm}$$

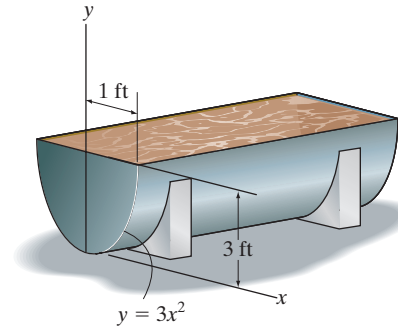
Ans.

Ans:
 $F_R = 3.68 \text{ kN}$
 $y_P = 442 \text{ mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-84.**

The cleaning trough is filled to its top with a solvent having a density of 2.78 slug/ft^3 . Determine the magnitude of the resultant force acting on the parabolic end plate, and the location of the center of pressure measured from the top. Solve the problem using the formula method.



SOLUTION

From the table in the inside back cover, the area and moment of inertia of the parabolic plate about the centroidal $\bar{x}$ axis are

$$A = \frac{2}{3}bh = \frac{2}{3}(2 \text{ ft})(3 \text{ ft}) = 4 \text{ ft}^2$$

$$\bar{I}_x = \frac{8}{175}bh^3 = \frac{8}{175}(2 \text{ ft})(3 \text{ ft})^3 = 2.4686 \text{ ft}^4$$

Also, $\bar{h} = \frac{2}{5}h = \frac{2}{5}(3 \text{ ft}) = 1.2 \text{ ft}$. Then the resultant force is given by

$$\begin{aligned} F_R &= \gamma \bar{h} A = (2.78 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(1.2 \text{ ft})(4 \text{ ft}^2) \\ &= 429.68 \text{ lb} = 430 \text{ lb} \end{aligned}$$

Ans.

And it acts at

$$\begin{aligned} y_P &= \bar{y} + \frac{\bar{I}_x}{\bar{y}A} \\ &= 1.2 \text{ ft} + \frac{2.4686 \text{ ft}^4}{(1.2 \text{ ft})(4 \text{ ft}^2)} \\ &= 1.7143 \text{ ft} = 1.71 \text{ ft} \end{aligned}$$

Ans.

Ans:
 $F_R = 430 \text{ lb}$
 $y_P = 1.71 \text{ ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-85.

Solve Prob. 2-84 using the integration method.

SOLUTION

The area of the differential element shown shaded in Fig. *a* is $dA = 2xdy$. Here $x = \frac{1}{\sqrt{3}} y^{\frac{1}{2}}$. Then $dA = 2 \left(\frac{1}{\sqrt{3}} y^{\frac{1}{2}} \right) dy = \frac{2}{\sqrt{3}} y^{\frac{1}{2}} dy$. The pressure acting on this element is $p = \gamma h$ where $h = (3 - y) \text{ ft}$. Thus, $p = (2.78 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(3 - y) \text{ ft} = 89.516(3 - y)$. Thus, the resultant force acting on the entire plate is

$$\begin{aligned} F_R &= \int_A p dA = \int_0^{3 \text{ ft}} 89.516(3 - y) \left(\frac{2}{\sqrt{3}} y^{\frac{1}{2}} dy \right) \\ &= \frac{179.032}{\sqrt{3}} \int_0^{3 \text{ ft}} \left(3y^{\frac{1}{2}} - y^{\frac{3}{2}} \right) dy \\ &= \frac{179.032}{\sqrt{3}} \left(2y^{\frac{3}{2}} - \frac{2}{5} y^{\frac{5}{2}} \right) \bigg|_0^{3 \text{ ft}} \\ &= 429.68 \text{ lb} = 430 \text{ lb} \end{aligned}$$

Ans.

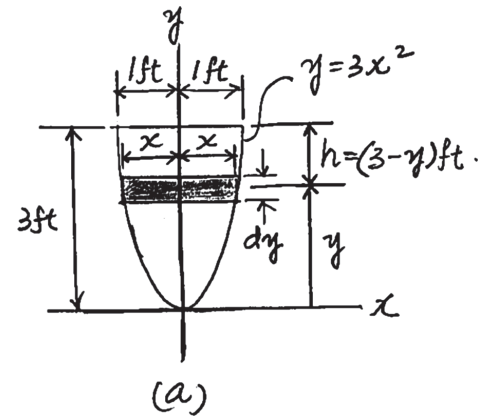
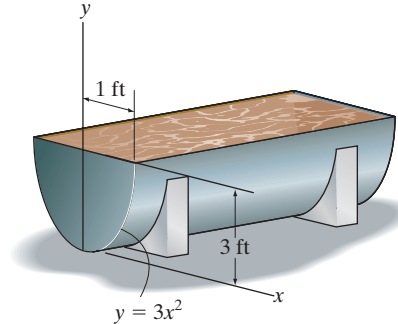
And it acts at

$$\begin{aligned} y'_p &= \frac{\int_A y p dA}{F_R} = \frac{1}{429.68} \int_0^{3 \text{ ft}} y [89.516(3 - y)] \left(\frac{2}{\sqrt{3}} y^{\frac{1}{2}} dy \right) \\ &= 0.2406 \int_0^{3 \text{ ft}} \left(3y^{\frac{3}{2}} - y^{\frac{5}{2}} \right) dy \\ &= 0.2406 \left(\frac{6}{5} y^{\frac{5}{2}} - \frac{2}{7} y^{\frac{7}{2}} \right) \bigg|_0^{3 \text{ ft}} \\ &= 1.2857 \text{ ft} \end{aligned}$$

Here, y'_p is measured from the bottom of the trough. Thus, from the top of the trough,

$$y_p = 3 \text{ ft} - y'_p = 3 \text{ ft} - 1.2857 \text{ ft} = 1.7143 \text{ ft} = 1.71 \text{ ft}$$

Ans.



Ans:

$$\begin{aligned} F_R &= 430 \text{ lb} \\ y_p &= 1.71 \text{ ft} \end{aligned}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-86.

The tank is filled to its top with lubricating oil. Determine the resultant force acting on the semicircular plate ABC , and its location on the plate measured from the base AC of the tank. Use the formula method. Take $\gamma_o = 54.9 \text{ lb/ft}^3$.

SOLUTION

Referring to the dimensions shown in Fig. a ,

$$\bar{r} = \frac{4r}{3\pi} = \frac{4(6 \text{ ft})}{3\pi} = \frac{8}{\pi} \text{ ft}$$

$$\bar{y} = 18 \text{ ft} - \frac{8}{\pi} \text{ ft} = 15.4535 \text{ ft}$$

$$\bar{h} = (15.4535 \text{ ft}) \sin 60^\circ = 13.3831 \text{ ft}$$

Also,

$$A = \frac{1}{2}(\pi r^2) = \frac{1}{2}[\pi(6 \text{ ft})^2] = 18\pi \text{ ft}^2$$

$$\bar{I}_x = 0.1098r^4 = 0.1098(6 \text{ ft})^4 = 142.30 \text{ ft}^4$$

The resultant force can now be determined:

$$\begin{aligned} F_R &= \gamma_o \bar{h} A = (54.9 \text{ lb/ft}^3)(13.3831 \text{ ft})(18\pi \text{ ft}^2) \\ &= 41.548(10^3) \text{ lb} = 41.5 \text{ kip} \end{aligned}$$

Ans.

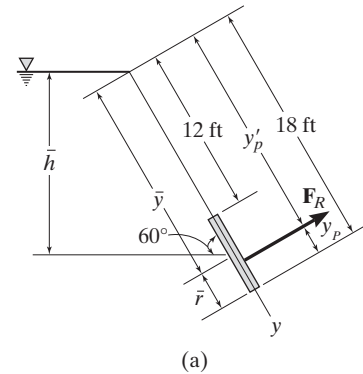
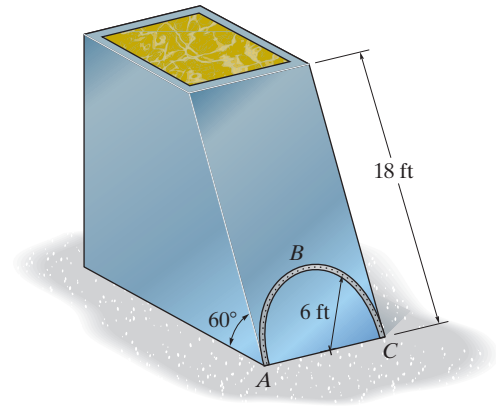
And it acts at

$$y'_p = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{142.30 \text{ ft}^4}{(15.4535 \text{ ft})(18\pi \text{ ft}^2)} + 15.4535 \text{ ft} = 15.6163 \text{ ft}$$

Thus,

$$y_p = 18 \text{ ft} - 15.6163 \text{ ft} = 2.3837 \text{ ft} = 2.38 \text{ ft}$$

Ans.



(a)

Ans:
 $F_R = 41.5 \text{ kip}$
 $y_p = 2.38 \text{ ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-87.

Solve Prob. 2-86 using the integration method.

SOLUTION

With respect to the x and y axes established, the equation of the semicircular plate, Fig. *a*, is

$$\begin{aligned}(x - 0)^2 + (y - 18)^2 &= 6^2 \\ x^2 + (y - 18)^2 &= 36 \\ x &= \sqrt{36 - (y - 18)^2}\end{aligned}$$

The area of the differential element shown shaded in Fig. *a* is $dA = 2x dy = 2\sqrt{36 - (y - 18)^2} dy$. The pressure acting on this differential element is $p = \gamma h = (54.9 \text{ lb/ft}^3)(y \sin 60^\circ) = 47.5448y$. Thus, the resultant force acting on the entire semicircular plate is

$$\begin{aligned}F_R &= \int_A p dA = \int_{12 \text{ ft}}^{18 \text{ ft}} (47.5448y) [2\sqrt{36 - (y - 18)^2} dy] \\ &= 95.0896 \int_{12 \text{ ft}}^{18 \text{ ft}} y \sqrt{36 - (y - 18)^2} dy \\ &= 95.0896 \left\{ -\frac{1}{3} \sqrt{[36 - (y - 18)^2]^3} \right\}_{12 \text{ ft}}^{18 \text{ ft}} + 9 \left[(y - 18) \sqrt{36 - (y - 18)^2} + 36 \sin^{-1} \left(\frac{y - 18}{6} \right) \right]_{12 \text{ ft}}^{18 \text{ ft}} \\ &= 41.548(10^3) \text{ lb} = 41.5 \text{ kip}\end{aligned}$$

Ans.

And it acts at

$$y'_p = \frac{\int_A y p dA}{F_R}$$

where

$$\begin{aligned}\int_A y p dA &= \int_{12 \text{ ft}}^{18 \text{ ft}} y (47.5448y) [2\sqrt{36 - (y - 18)^2} dy] \\ &= 95.0896 \int_{12 \text{ ft}}^{18 \text{ ft}} y^2 \sqrt{36 - (y - 18)^2} dy \\ &= 95.0896 \left\{ -\left(\frac{y - 18}{4} \right) \sqrt{[36 - (y - 18)^2]^3} + \frac{9}{2} (y - 18) \sqrt{36 - (y - 18)^2} + 162 \sin^{-1} \left(\frac{y - 18}{6} \right) \right. \\ &\quad \left. + \frac{324}{2} \left[(y - 18) \sqrt{36 - (y - 18)^2} + 36 \sin^{-1} \left(\frac{y - 18}{6} \right) \right] \right\}_{12 \text{ ft}}^{18 \text{ ft}} \\ &= 648\,829.85 \text{ lb} \cdot \text{ft}\end{aligned}$$

Then

$$y'_p = \frac{648\,829.85 \text{ lb} \cdot \text{ft}}{41.548(10^3) \text{ lb}} = 15.6163 \text{ ft}$$

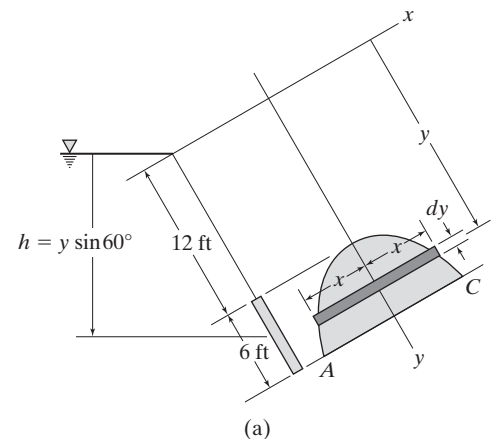
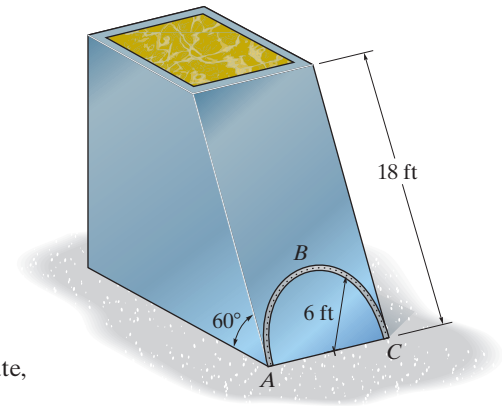
Thus,

$$y_p = 18 - y'_p = 2.3837 \text{ ft} = 2.38 \text{ ft}$$

Ans.

Ans:

$$\begin{aligned}F_R &= 41.5 \text{ kip} \\ y_p &= 2.38 \text{ ft}\end{aligned}$$



Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-88.**

The tank is filled with water. Determine the resultant force acting on the trapezoidal plate and the location of the center of pressure, measured from the water surface. Solve the problem using the formula method.

SOLUTION

Referring to the geometry shown in Fig. *a*,

$$A = \frac{1}{2}(2 \text{ m})(1.5 \text{ m}) + (1 \text{ m})(1.5 \text{ m}) = 3.00 \text{ m}^2$$

$$\bar{h} = \bar{y} = \frac{(4.25 \text{ m})[(1 \text{ m})(1.5 \text{ m})] + (4.00 \text{ m})\left[\frac{1}{2}(2 \text{ m})(1.5 \text{ m})\right]}{(1 \text{ m})(1.5 \text{ m}) + \frac{1}{2}(2 \text{ m})(1.5 \text{ m})} = 4.125 \text{ m}$$

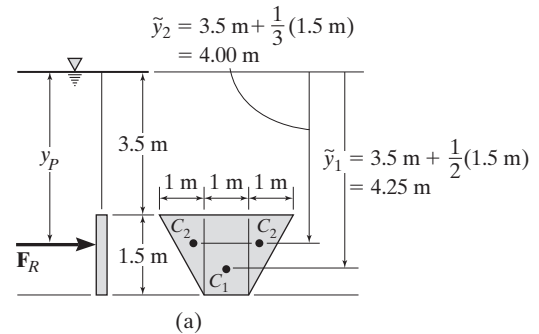
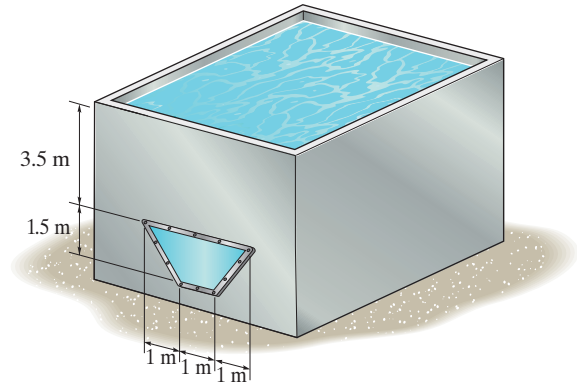
$$\begin{aligned} \bar{I}_x &= \frac{1}{12}(1 \text{ m})(1.5 \text{ m})^3 + (1 \text{ m})(1.5 \text{ m})(4.25 \text{ m} - 4.125 \text{ m})^2 \\ &\quad + \frac{1}{36}(2 \text{ m})(1.5 \text{ m})^3 + \left[\frac{1}{2}(2 \text{ m})(1.5 \text{ m})\right](4.125 \text{ m} - 4.00 \text{ m})^2 \\ &= 0.515625 \text{ m}^4 \end{aligned}$$

The resultant force can now be determined:

$$\begin{aligned} F_R &= \gamma_w \bar{h} A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4.125 \text{ m})(3.00 \text{ m}^2) \\ &= 121.40(10^3) \text{ N} = 121 \text{ kN} \end{aligned}$$

And it acts at

$$\begin{aligned} y_P &= \frac{\bar{I}_x}{\bar{y} A} + \bar{y} = \frac{0.515625 \text{ m}^4}{(4.125 \text{ m})(3.00 \text{ m}^2)} + 4.125 \text{ m} \\ &= 4.1667 \text{ m} = 4.17 \text{ m} \end{aligned}$$



Ans.

Ans.

Ans:
 $F_R = 121 \text{ kN}$
 $y_P = 4.17 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–89.

Solve Prob. 2–88 using the integration method.

SOLUTION

With respect to the x and y axes established, the equation of side EF of the plate, Fig. a , is

$$\frac{y - 3.5}{x - 1.5} = \frac{5 - 3.5}{0.5 - 1.5}; \quad x = 3.8333 - 0.6667y$$

The area of the differential element shown shaded in Fig. a is $dA = 2x dy = 2(3.8333 - 0.6667y)dy = (7.6667 - 1.3333y)dy$. The pressure acting on this differential element is $p = \gamma h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)y = 9810y$. Thus, the resultant force acting on the entire plate is

$$\begin{aligned} F_R &= \int_A p dA = \int_{3.5 \text{ m}}^{5 \text{ m}} (9810y)(7.6667 - 1.3333y) dy \\ &= 9810 \int_{3.5 \text{ m}}^{5 \text{ m}} (7.6667y - 1.3333y^2) dy \\ &= 9810 \left(3.8333y^2 - 0.4444y^3 \right) \bigg|_{3.5 \text{ m}}^{5 \text{ m}} \\ &= 121.40(10^3) \text{ N} = 121 \text{ kN} \end{aligned}$$

Ans.

And it acts at

$$y_P = \frac{\int_A y p dA}{F_R}$$

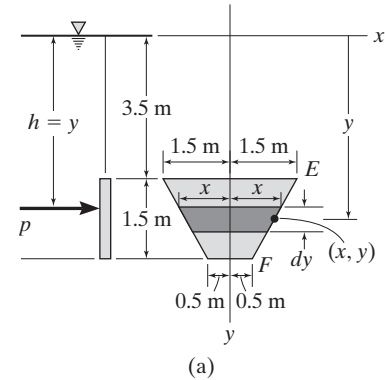
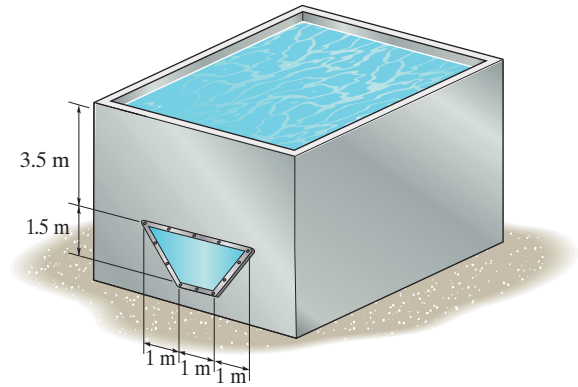
where

$$\begin{aligned} \int_A y p dA &= \int_{3.5 \text{ m}}^{5 \text{ m}} y(9810y)(7.6667 - 1.3333y) dy \\ &= 9810 \int_{3.5 \text{ m}}^{5 \text{ m}} (7.6667y^2 - 1.3333y^3) dy \\ &= 9810 \left(2.5556y^3 - 0.3333y^4 \right) \bigg|_{3.5 \text{ m}}^{5 \text{ m}} \\ &= 505.83(10^3) \text{ N} \cdot \text{m} \end{aligned}$$

Thus,

$$y_P = \frac{505.83(10^3) \text{ N} \cdot \text{m}}{121.40(10^3) \text{ N}} = 4.1667 \text{ m} = 4.17 \text{ m}$$

Ans.

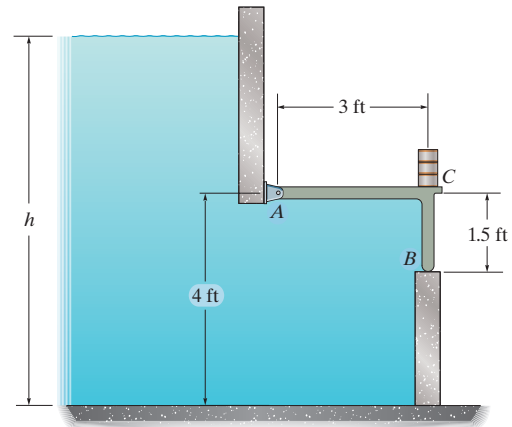


Ans:
 $F_R = 121 \text{ kN}$
 $y_P = 4.17 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-90.

The control gate ACB is pinned at A and rests on the smooth surface at B . Determine the amount of weight that should be placed at C in order to maintain a reservoir depth of $h = 10 \text{ ft}$. The gate has a width of 3 ft . Neglect its weight.



SOLUTION

The intensities of the distributed load at C and B shown in Fig. a are

$$w_C = \gamma_w h_C b = (62.4 \text{ lb/ft}^3)(6 \text{ ft})(3 \text{ ft}) = 1123.2 \text{ lb/ft}$$

$$w_B = \gamma_w h_B b = (62.4 \text{ lb/ft}^3)(7.5 \text{ ft})(3 \text{ ft}) = 1404 \text{ lb/ft}$$

Thus,

$$F_1 = (1123.2 \text{ lb/ft})(3 \text{ ft}) = 3369.6 \text{ lb}$$

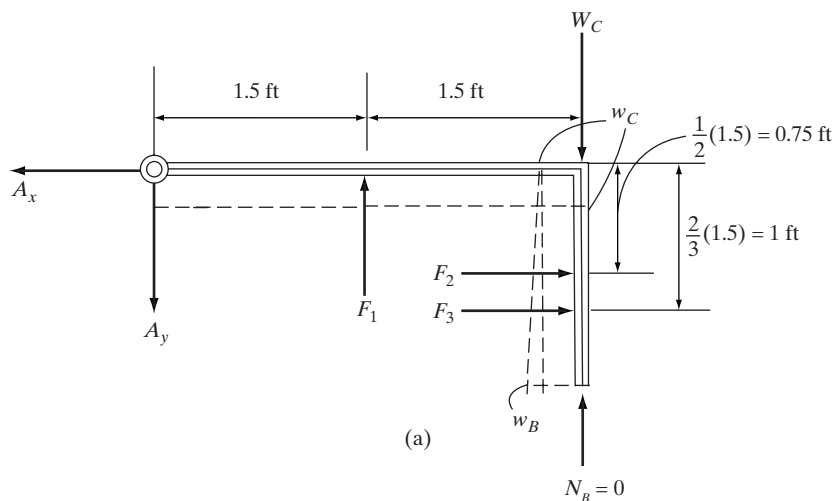
$$F_2 = (1123.2 \text{ lb/ft})(1.5 \text{ ft}) = 1684.8 \text{ lb}$$

$$F_3 = \frac{1}{2}[(1404 \text{ lb/ft} - 1123.2 \text{ lb/ft})](1.5 \text{ ft}) = 210.6 \text{ lb}$$

Since the gate is about to be opened, $N_B = 0$. Write the moment equation of equilibrium about point A by referring to Fig. a .

$$\zeta + \Sigma M_A = 0; \quad (3369.6 \text{ lb})(1.5 \text{ ft}) + (1684.8 \text{ lb})(0.75 \text{ ft}) + (210.6 \text{ lb})(1 \text{ ft}) - W_C(3 \text{ ft}) = 0$$

$$W_C = 2176.2 \text{ lb} = 2.18 \text{ kip} \quad \text{Ans.}$$

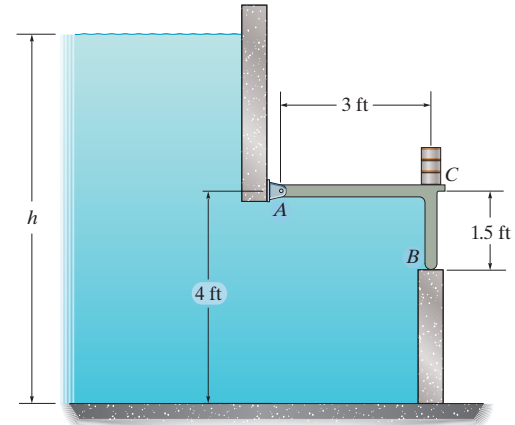


Ans:
 $W_C = 2.18 \text{ kip}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-91.

The control gate ACB is pinned at A and rests on the smooth surface at B . If the counterweight C is 2000 lb, determine the maximum depth of water h in the reservoir before the gate begins to open. The gate has a width of 3 ft. Neglect its weight.



SOLUTION

The intensities of the distributed loads at C and B are shown in Fig. a .

$$w_C = \gamma_w h_C b = (62.4 \text{ lb/ft}^3)(h - 4 \text{ ft})(3 \text{ ft}) = [187.2(h - 4)] \text{ lb/ft}$$

$$w_B = \gamma_w h_B b = (62.4 \text{ lb/ft}^3)(h - 2.5 \text{ ft})(3 \text{ ft}) = [187.2(h - 2.5)] \text{ lb/ft}$$

Thus,

$$F_1 = (187.2(h - 4) \text{ lb/ft})(3 \text{ ft}) = 561.6(h - 4) \text{ lb}$$

$$F_2 = (187.2(h - 4) \text{ lb/ft})(1.5 \text{ ft}) = 280.8(h - 4) \text{ lb}$$

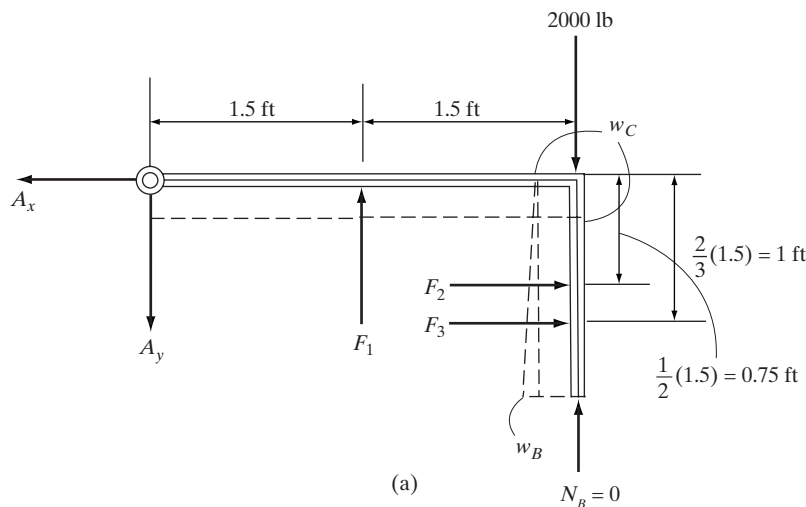
$$F_3 = \frac{1}{2}[187.2(h - 2.5) \text{ lb/ft} - (187.2(h - 4) \text{ lb/ft})](1.5 \text{ ft}) = 210.6 \text{ lb}$$

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Since the gate is required to be opened, $N_B = 0$. Write the moment equation of equilibrium about point A by referring to Fig. a .

$$\begin{aligned} \zeta + \Sigma M_A = 0; \quad & [561.6(h - 4) \text{ lb}](1.5 \text{ ft}) + [280.8(h - 4) \text{ lb}](0.75 \text{ ft}) \\ & + (210.6 \text{ lb})(1 \text{ ft}) - (2000 \text{ lb})(3 \text{ ft}) = 0 \\ & 1053(h - 4) = 5789.4 \\ & h = 9.498 \text{ ft} = 9.50 \text{ ft} \end{aligned}$$

Ans.

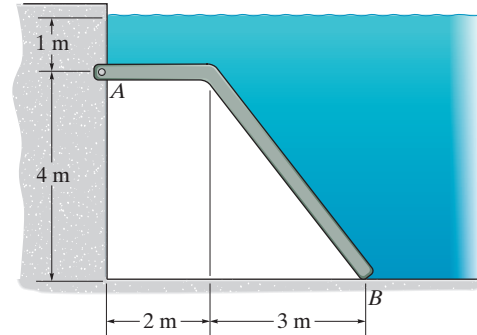


Ans:
 $h = 9.50 \text{ ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-92.**

The bent plate is 1.5 m wide and is pinned at A and rests on a smooth support at B . Determine the horizontal and vertical components of reaction at A and the vertical reaction at the smooth support B . The fluid is water.



SOLUTION

Since the gate has a width of $b = 1.5 \text{ m}$, the intensities of the distributed loads at A and B can be computed from

$$w_A = \rho_w g h_A b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1 \text{ m})(1.5 \text{ m}) = 14.715(10^3) \text{ N/m}$$

$$w_B = \rho_w g h_B b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m})(1.5 \text{ m}) = 73.575(10^3) \text{ N/m}$$

Using these results, the distributed load acting on the plate is shown on the free-body diagram of the gate, Fig. a .

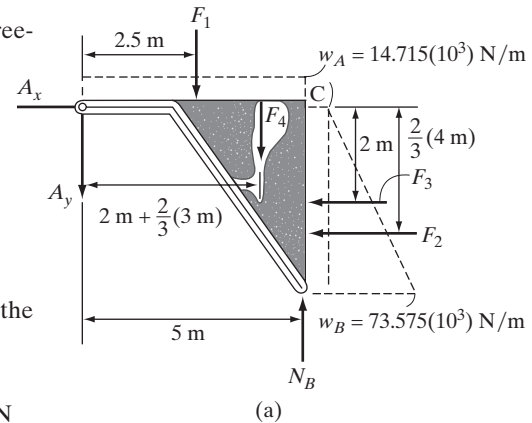
$$F_1 = w_A L_{AB} = (14.715(10^3) \text{ N/m})(5 \text{ m}) = 73.575(10^3) \text{ N}$$

$$F_2 = \frac{1}{2}(w_B - w_A)L_{BC} = \frac{1}{2}(73.575(10^3) \text{ N/m} - 14.715(10^3) \text{ N/m})(4 \text{ m}) = 117.72(10^3) \text{ N}$$

$$F_3 = w_A L_{BC} = (14.715(10^3) \text{ N/m})(4 \text{ m}) = 58.86(10^3) \text{ N}$$

F_4 on the free-body diagram is equal to the weight of the water contained in the shaded triangular block, Fig. a .

$$F_4 = \rho_w g V = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left[\frac{1}{2}(3 \text{ m})(4 \text{ m})(1.5 \text{ m})\right] = 88.29(10^3) \text{ N}$$



Considering the free-body diagram of the gate, Fig. a ,

$$\begin{aligned} \zeta + \Sigma M_A = 0; \quad N_B(5 \text{ m}) - 73.575(10^3) \text{ N}(2.5 \text{ m}) - 58.86(10^3) \text{ N}(2 \text{ m}) - 117.72(10^3) \text{ N}\left(\frac{2}{3}(4 \text{ m})\right) \\ - 88.29(10^3) \text{ N}\left(2 \text{ m} + \frac{2}{3}(3 \text{ m})\right) = 0 \end{aligned}$$

$$N_B = 193.748(10^3) \text{ N} = 194 \text{ kN}$$

Ans.

$$\rightarrow \Sigma F_x = 0; \quad A_x - 58.86(10^3) \text{ N} - 117.72(10^3) \text{ N} = 0$$

$$A_x = 176.58(10^3) \text{ N} = 177 \text{ kN}$$

Ans.

$$+\uparrow \Sigma F_y = 0; \quad -A_y - 73.575(10^3) \text{ N} - 88.29(10^3) \text{ N} + 193.748(10^3) \text{ N} = 0$$

$$A_y = 31.88(10^3) \text{ N} = 31.9 \text{ kN}$$

Ans.

Ans:

$$N_B = 194 \text{ kN}$$

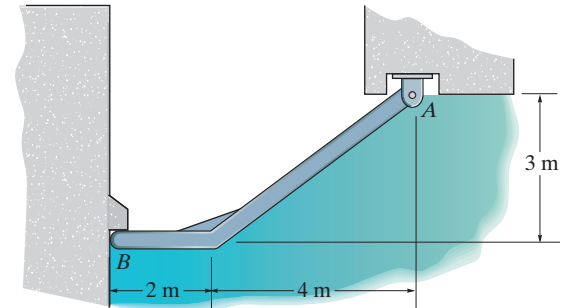
$$A_x = 177 \text{ kN}$$

$$A_y = 31.9 \text{ kN}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-93.

The bent plate is 2 m wide. It is supported by the pin at A and smooth support at B . Determine the horizontal and vertical components of reaction at A and normal reaction at B if the tank is filled with water.



SOLUTION

The horizontal component of the resultant force acting on the gate is equal to the pressure force on the vertically projected area of the gate. Referring to Fig. a ,

$$w_B = \rho_w g h_B b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3 \text{ m})(2 \text{ m}) = 58.86(10^3) \text{ N/m}$$

Thus

$$F_h = \frac{1}{2}[58.86(10^3) \text{ N/m}](3 \text{ m}) = 88.29(10^3) \text{ N} = 88.29 \text{ kN}$$

It acts at

$$\tilde{y} = \frac{2}{3}(3 \text{ m}) = 2 \text{ m}$$

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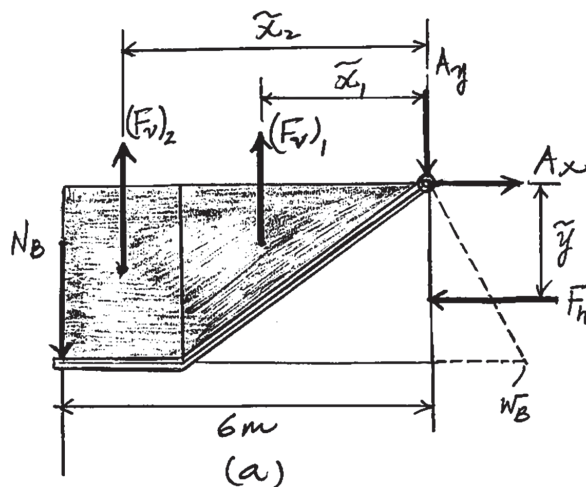
The vertical component of the resultant force acting on the gate is equal to the weight of the imaginary column of water above the gate (shown shaded in Fig. a) but acts upward.

$$(F_v)_1 = \rho_w g V_1 = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left[\frac{1}{2}(4 \text{ m})(3 \text{ m})(2 \text{ m})\right] = 117.72(10^3) \text{ N} = 117.72 \text{ kN}$$

$$(F_v)_2 = \rho_w g V_2 = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(2 \text{ m})(3 \text{ m})(2 \text{ m})] = 117.72(10^3) \text{ N} = 117.72 \text{ kN}$$

They act at

$$\tilde{x}_1 = \frac{2}{3}(4 \text{ m}) = 2.6667 \text{ m} \quad \tilde{x}_2 = 4 \text{ m} + \frac{1}{2}(2 \text{ m}) = 5 \text{ m}$$



2-93. Continued

Considering the equilibrium of the FBD of the gate in Fig. *a*,

$$\zeta + M_A = 0; \quad N_B(6 \text{ m}) - (88.29 \text{ kN})(2 \text{ m}) - (117.72 \text{ kN})(2.6667 \text{ m})$$

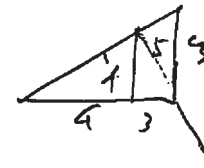
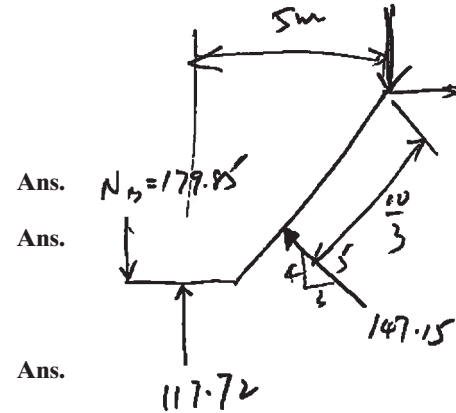
$$-(117.72 \text{ kN})(5 \text{ m}) = 0$$

$$N_B = 179.85 \text{ kN} = 180 \text{ kN}$$

$$\rightarrow \Sigma F_x = 0; \quad A_x - 88.29 \text{ kN} = 0 \quad A_x = 88.29 \text{ kN} = 88.3 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad 117.72 \text{ kN} + 117.72 \text{ kN} - 179.85 \text{ kN} - A_y = 0$$

$$A_y = 55.59 \text{ kN} = 55.6 \text{ kN}$$



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Ans:

$$N_B = 180 \text{ kN}$$

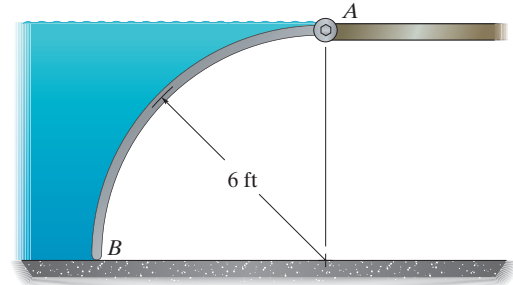
$$A_x = 88.3 \text{ kN}$$

$$A_y = 55.6 \text{ kN}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-94.

The thin quarter-circular arched gate is 3 ft wide, is pinned at A , and rests on the smooth support at B . Determine the reactions at these supports due to the water pressure. The centroid of a quarter-circular area measured from its diametral axis is $\bar{y} = 4r/3\pi$, where r is its radius.



SOLUTION

Referring to the geometry shown in Fig. a ,

$$A_{ADB} = (6 \text{ ft})(6 \text{ ft}) - \frac{\pi}{4}(6 \text{ ft})^2 = (36 - 9\pi) \text{ ft}^2$$

$$\bar{x} = \frac{(3 \text{ ft})[(6 \text{ ft})(6 \text{ ft})] - \left(\frac{8}{\pi} \text{ ft}\right)\left[\frac{\pi}{4}(6 \text{ ft})^2\right]}{(36 - 9\pi) \text{ ft}^2} = 4.6598 \text{ ft}$$

The horizontal component of the resultant force acting on the gate is equal to the pressure force on the vertically projected area of the gate. Referring to Fig. b ,

$$w_B = \gamma_w h_B b = (62.4 \text{ lb/ft}^3)(6 \text{ ft})(3 \text{ ft}) = 1123.2 \text{ lb/ft}$$

Thus,

$$F_h = \frac{1}{2}(1123.2 \text{ lb/ft})(6 \text{ ft}) = 3369.6 \text{ lb}$$

The vertical component of the resultant force acting on the gate is equal to the weight of the column of water above the gate (shown shaded in Fig. b).

$$F_v = \gamma_w V = \gamma_w A_{ADB} b = (62.4 \text{ lb/ft}^3)[(36 - 9\pi) \text{ ft}^2](3 \text{ ft}) = 1446.24 \text{ lb}$$

Considering the equilibrium of the FBD of the gate in Fig. b ,

$$\zeta + \Sigma M_A = 0; (1446.24 \text{ lb})(4.6598 \text{ ft}) + (3369.6 \text{ lb})(4 \text{ ft}) - N_B(6 \text{ ft}) = 0$$

$$N_B = 3369.6 \text{ lb} = 3.37$$

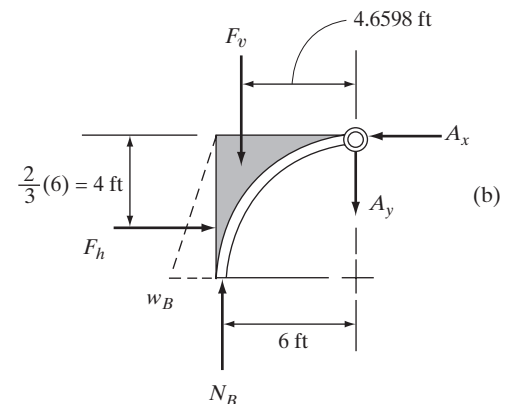
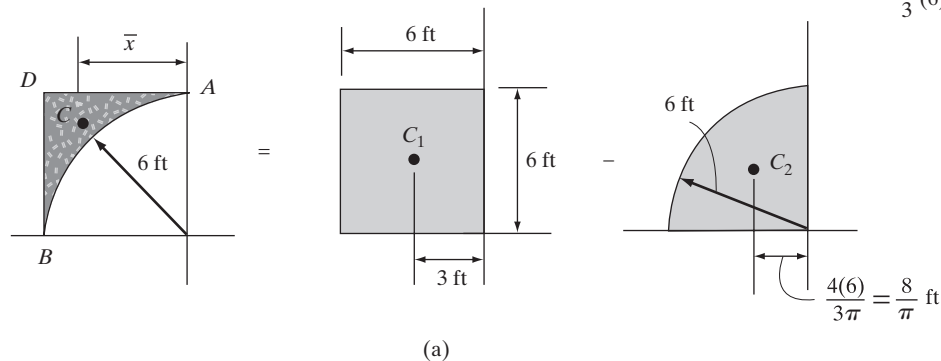
Ans.

$$\rightarrow \Sigma F_x = 0; 3369.6 \text{ lb} - A_x = 0 \quad A_x = 3369.6 \text{ lb} = 3.37$$

Ans.

$$+\uparrow \Sigma F_y = 0; 3369.6 \text{ lb} - 1446.24 \text{ lb} - A_y = 0 \quad A_y = 1923.36 \text{ lb} = 1.92$$

Ans.



Ans:

$$N_B = 3.37 \text{ kip}$$

$$A_x = 3.37 \text{ kip}$$

$$A_y = 1.92 \text{ kip}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-95.

Water is confined in the vertical chamber, which is 3 m wide. Determine the resultant force it exerts on the arched base AB .

SOLUTION

Due to symmetry, the resultant force that the water exerts on arch AB will be vertically downward, and its magnitude is equal to the weight of the water column above the arch which is shown shaded in Fig. a . This water column can be subdivided into three parts as indicated in Fig. a . From the geometry of Fig. a ,

$$\phi = \sin^{-1}\left(\frac{3 \text{ m}}{8 \text{ m}}\right) = 0.3844 \text{ rad}$$

$$h = \sqrt{(8 \text{ m})^2 - (3 \text{ m})^2} = \sqrt{55} \text{ m}$$

Then the area of each part is

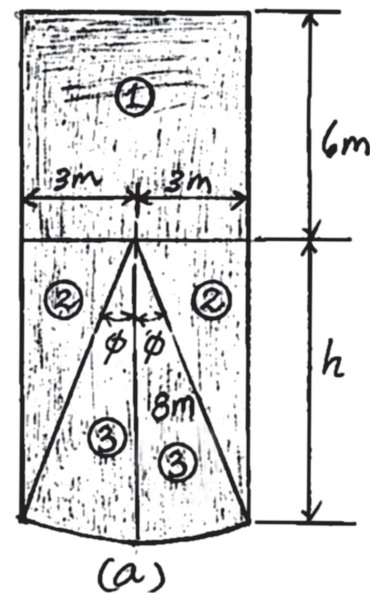
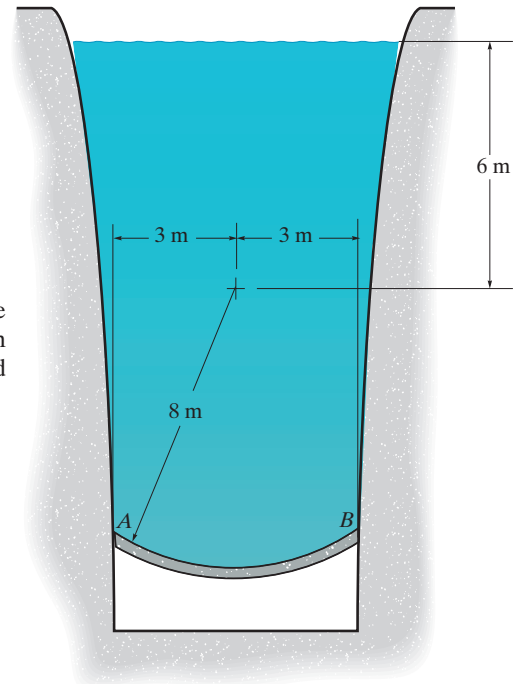
$$A_1 = (6 \text{ m})(6 \text{ m}) = 36 \text{ m}^2 \quad A_2 = \frac{1}{2}(6 \text{ m})(\sqrt{55} \text{ m}) = 3\sqrt{55} \text{ m}^2$$

$$A_3 = \frac{1}{2}r^2\theta = \frac{1}{2}(8 \text{ m})^2 [2(0.3844)] = 24.60 \text{ m}^2$$

Therefore, the resultant force is given by

$$\begin{aligned} F_R = W = \rho_w g V &= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) [(36 \text{ m}^2 + 3\sqrt{55} \text{ m}^2 + 24.60 \text{ m}^2)(3 \text{ m})] \\ &= 2.4383(10^6) \text{ N} = 2.44 \text{ MN} \end{aligned}$$

Ans.



Ans:
 $F_R = 2.44 \text{ MN}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-96.**

The wall is in the form of a parabola. Determine the magnitude and direction of the resultant force the water exerts on the wall if it is 8 ft wide.

SOLUTION

The horizontal loading on the wall is due to the pressure on the vertical projected area of the wall, Fig. *a*. Since the wall has a constant width of $b = 8 \text{ ft}$, the intensity of the horizontal distributed load at the base of the wall is

$$w = \gamma_w h b = (62.4 \text{ lb/ft}^3)(12 \text{ ft})(8 \text{ ft}) = 5.9904(10^3) \text{ lb/ft}$$

Thus,

$$F_h = \frac{1}{2} w h = \frac{1}{2} [5.9904(10^3) \text{ lb/ft}](12 \text{ ft}) = 35.9424(10^3) \text{ lb}$$

The vertical force acting on the wall is equal to the weight of the water contained in the block above the wall (shown shaded in Fig. *a*). From the inside back cover of the book, the volume of this block (parabolic cross-section) is

$$V = \frac{2}{3} a h b = \frac{2}{3} (12 \text{ ft})(12 \text{ ft})(8 \text{ ft}) = 768 \text{ ft}^3$$

Thus,

$$F_v = \gamma_w V = (62.4 \text{ lb/ft}^3)(768 \text{ ft}^3) = 47.9232(10^3) \text{ lb}$$

Then the magnitude of the resultant force is

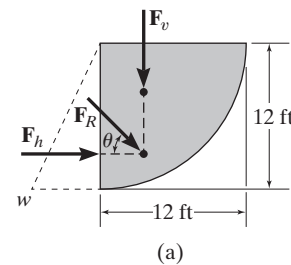
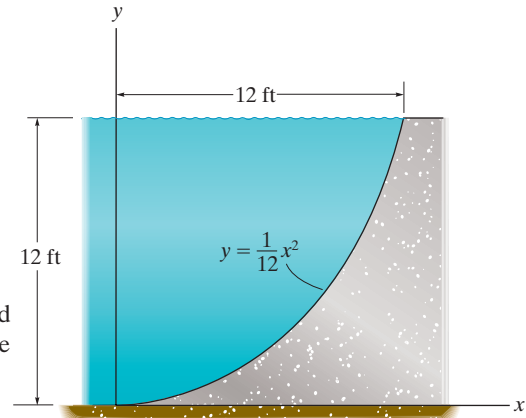
$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{[35.9424(10^3) \text{ lb}]^2 + [47.9232(10^3) \text{ lb}]^2} = 59.904(10^3) \text{ lb} = 59.9 \text{ kip}$$

Ans.

And its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left[\frac{47.9232(10^3) \text{ lb}}{35.9424(10^3) \text{ lb}}\right] = 53.13^\circ = 53.1^\circ \swarrow$$

Ans.



Ans:

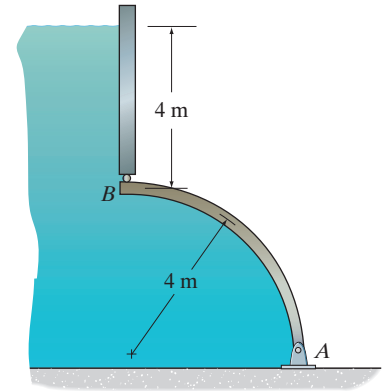
$$F_R = 59.9 \text{ kip}$$

$$\theta = 53.1^\circ \swarrow$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-97.

Determine the horizontal and vertical components of reaction at the hinge A and the normal reaction at B caused by the water pressure. The plate AB has a width of 2 m.



SOLUTION

The vertical component of the resultant force the water exerts on the plate is equal to the weight of the water column shown shaded in Fig. a , but it acts upward. This column can be subdivided into parts 1 and 2 as shown in Figs. b and c respectively. Part 2 is a hole and so should be considered as a negative part. Thus the area of the column, Fig. a , is $\Sigma A = (4 \text{ m})(8 \text{ m}) - \frac{\pi(4 \text{ m})^2}{4} = 19.43 \text{ m}^2$ and the horizontal distance measured from its centroid to point A is

$$\bar{x} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{(2 \text{ m})[(4 \text{ m})(8 \text{ m})] - \left[4 \text{ m} - \frac{4(4 \text{ m})}{3\pi}\right] \left[\frac{\pi(4 \text{ m})^2}{4}\right]}{19.43 \text{ m}^2} = 1.8045 \text{ m}$$

The magnitude of the vertical component of the resultant force is

$$(F_R)_v = \rho_w g V = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(19.43 \text{ m}^2)(2 \text{ m})] = 381.29(10^3) \text{ N} = 381.29 \text{ kN}$$

The intensities of the distributed load at points A and B are

$$w_A = \rho_w g h_A b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(8 \text{ m})(2 \text{ m}) = 156.96(10^3) \text{ N/m}$$

$$w_B = \rho_w g h_B b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4 \text{ m})(2 \text{ m}) = 78.48(10^3) \text{ N/m}$$

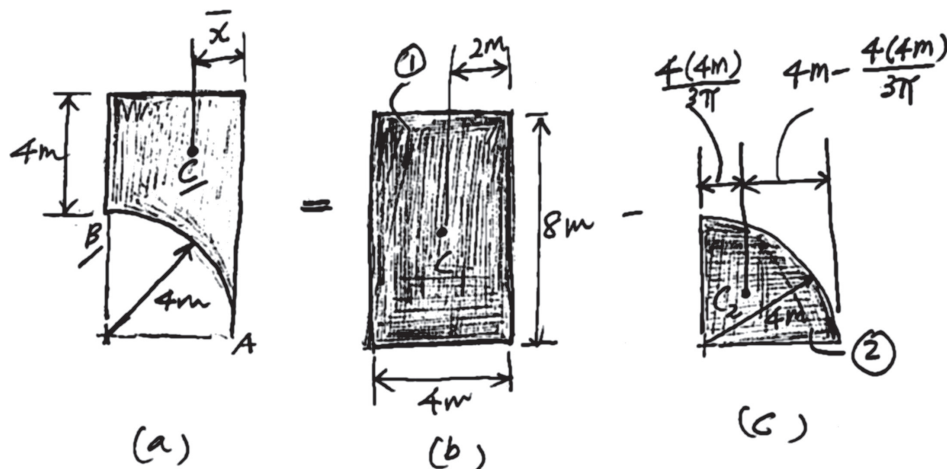
Then the horizontal components of the resultant force are

$$[(F_R)_h]_1 = [78.48(10^3) \text{ N/m}](4 \text{ m}) = 313.92(10^3) \text{ N} = 313.92 \text{ kN}$$

$$[(F_R)_h]_2 = \frac{1}{2}[156.96(10^3) \text{ N/m} - 78.48(10^3) \text{ N/m}](4 \text{ m}) = 156.96(10^3) \text{ N} = 156.96 \text{ kN}$$

And they act at

$$\bar{y}_1 = \frac{1}{2}(4 \text{ m}) = 2 \text{ m} \quad \bar{y}_2 = \frac{1}{3}(4 \text{ m}) = 1.3333 \text{ m}$$



2-97. Continued

Referring to the FBD of the plate, Fig. *d*,

$$\zeta + \sum M_A = 0; \quad N_B(4 \text{ m}) - (381.29 \text{ kN})(1.8045 \text{ m}) - (313.92 \text{ kN})(2 \text{ m}) \\ - (156.96 \text{ kN})(1.3333 \text{ m}) = 0$$

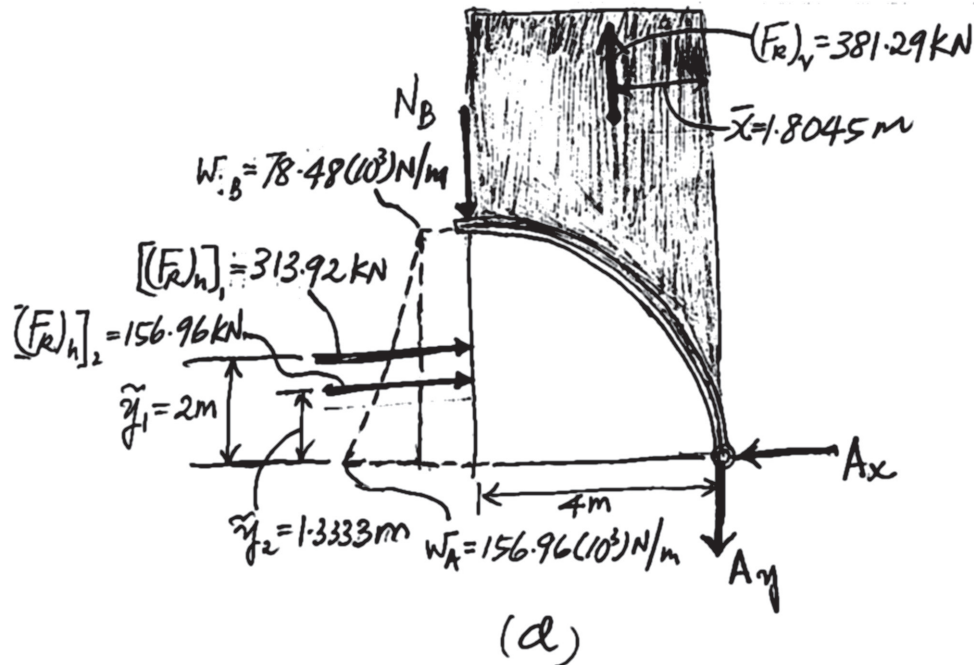
$$N_B = 381.29 \text{ kN} = 381 \text{ kN} \quad \text{Ans.}$$

$$\pm \sum F_x = 0; \quad 313.92 \text{ kN} + 156.96 \text{ kN} - A_x = 0$$

$$A_x = 470.88 \text{ kN} = 471 \text{ kN} \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad 381.29 \text{ kN} - 381.29 \text{ kN} - A_y = 0$$

$$A_y = 0 \quad \text{Ans.}$$

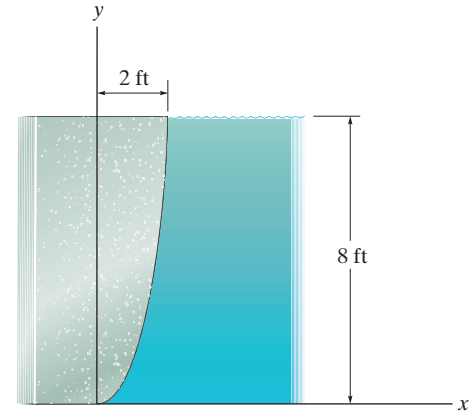


Ans:
 $N_B = 381 \text{ kN}$
 $A_x = 471 \text{ kN}$
 $A_y = 0$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-98.

The 6-ft-wide wall has the parabolic shape shown. Determine the magnitude and direction of the resultant force the water exerts on the wall.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. *a*, the intensity of the distributed load at the base of the wall is

$$w_A = \gamma_w h_A b = (62.4 \text{ lb/ft}^3)(8 \text{ ft})(6 \text{ ft}) = 2.9952(10^3) \text{ lb/ft}$$

Thus, the horizontal component of the resultant force is

$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2} [2.9952(10^3) \text{ lb/ft}](8 \text{ ft}) = 11.9808(10^3) \text{ lb}$$

The vertical component of the resultant force is equal to the weight of the imaginary column of water above surface *AB* of the wall (shown shaded in Fig. *a*) but acts upwards. The volume of this column of water is

$$V = \frac{2}{3} a h b = \frac{2}{3} (2 \text{ ft})(8 \text{ ft})(6 \text{ ft}) = 64 \text{ ft}^3$$

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Thus,

$$F_v = \gamma_w V = (62.4 \text{ lb/ft}^3)(64 \text{ ft}^3) = 3.9936(10^3) \text{ lb}$$

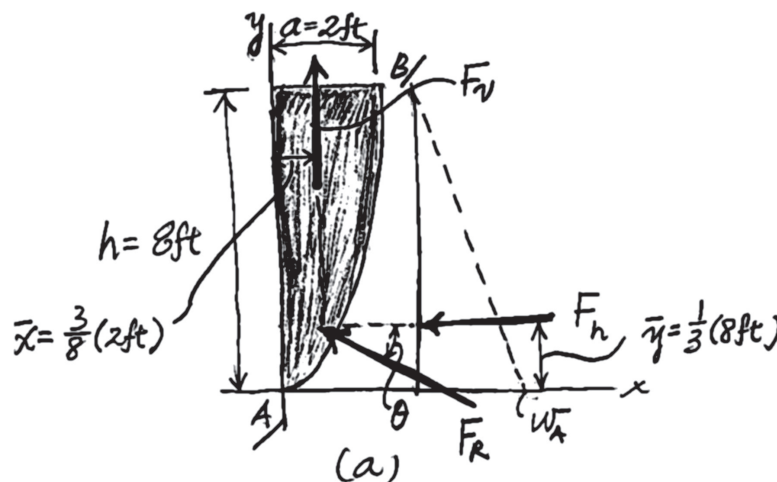
The magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{[11.9808(10^3) \text{ lb}]^2 + [3.9936(10^3) \text{ lb}]^2}$$

$$= 12.6289(10^3) \text{ lb} = 12.6(10^3) \text{ lb} \quad \text{Ans.}$$

and its direction is

$$\theta = \tan^{-1} \left(\frac{F_v}{F_h} \right) = \tan^{-1} \left[\frac{3.9936(10^3) \text{ lb}}{11.9808(10^3) \text{ lb}} \right] = 18.43^\circ = 18.4^\circ \quad \text{Ans.}$$



Ans:

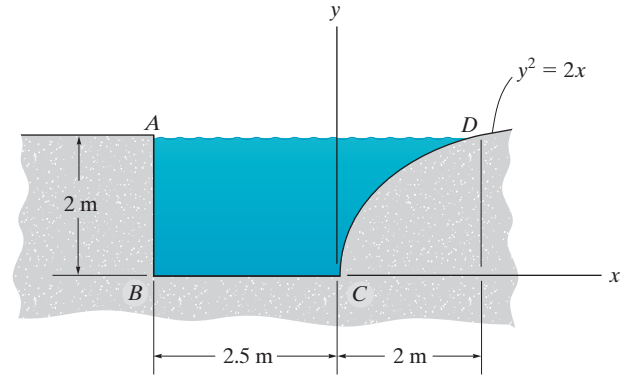
$$F_R = 12.6(10^3) \text{ lb}$$

$$\theta = 18.4^\circ$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-99.

Determine the magnitude of the resultant force the water exerts on AB , BC , and CD of the enclosure, which is 3 m wide.



SOLUTION

Horizontal Component. The horizontal component of the force on CD is the same as the force on AB . Its magnitude can be determined from

$$F_{AB} = (F_{CD})_h = \gamma_w \bar{h} A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1 \text{ m})(2 \text{ m}(3 \text{ m}))$$

$$= 58.86(10^3) \text{ N} = 58.9 \text{ kN} \quad \text{Ans.}$$

Vertical Component. The force on BC and the vertical component of the force on CD is equal to the weight of the water contained in blocks $ABCEA$ and $CDEC$ (shown shaded in Fig. *a*). Here, $A_{ABCEA} = 2 \text{ m}(2.5 \text{ m}) = 5 \text{ m}^2$ and $A_{CDEC} = \frac{1}{3}bh = \frac{1}{3}(2 \text{ m})(2 \text{ m}) = 1.3333 \text{ m}^2$. Then, TBEXAM.COM

$$F_{BC} = \gamma_w V_{ABCEA} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[5 \text{ m}^2(3 \text{ m})]$$

$$= 147.15(10^3) \text{ N} = 147 \text{ kN} \quad \text{Ans.}$$

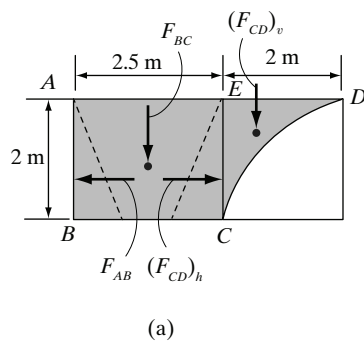
$$(F_{CD})_v = \gamma_w V_{CDEC} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[1.3333 \text{ m}^2(3 \text{ m})]$$

$$= 39.24(10^3) \text{ N}$$

Therefore,

$$F_{CD} = \sqrt{(F_{CD})_h^2 + (F_{CD})_v^2} = \sqrt{[58.86(10^3) \text{ N}]^2 + [39.24(10^3) \text{ N}]^2}$$

$$= 70.74(10^3) \text{ N} = 70.7 \text{ kN} \quad \text{Ans.}$$



Ans:

$$F_{AB} = 58.9 \text{ kN}$$

$$F_{BC} = 147 \text{ kN}$$

$$F_{CD} = 70.7 \text{ kN}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-100.**

Determine the magnitude and direction of the resultant hydrostatic force the water exerts on the parabolic face AB of the wall if it is 3 m wide.

SOLUTION

The horizontal loading on the wall is due to the pressure on the vertical projected area of the wall, Fig. a . Since the wall has a constant width of $b = 3 \text{ m}$, the intensity of the horizontal distributed load at the base of the wall is

$$w = \rho_w g h b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(8 \text{ m})(3 \text{ m}) = 235.44(10^3) \text{ N/m}$$

Thus,

$$F_h = \frac{1}{2} w h = \frac{1}{2} [235.44(10^3) \text{ N/m}](8 \text{ m}) = 941.76(10^3) \text{ N} = 941.76 \text{ kN}$$

The vertical force acting on the wall is equal to the weight of water contained in the imaginary block above the wall (shown shaded in Fig. a), but acts vertically upward. From the inside back cover of the book, the volume of this block (exparabolic cross-section) is

$$V = \frac{1}{3} a h b = \frac{1}{3} (4 \text{ m})(8 \text{ m})(3 \text{ m}) = 32.0 \text{ m}^3$$

Thus,

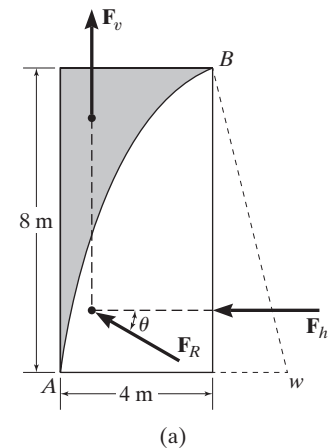
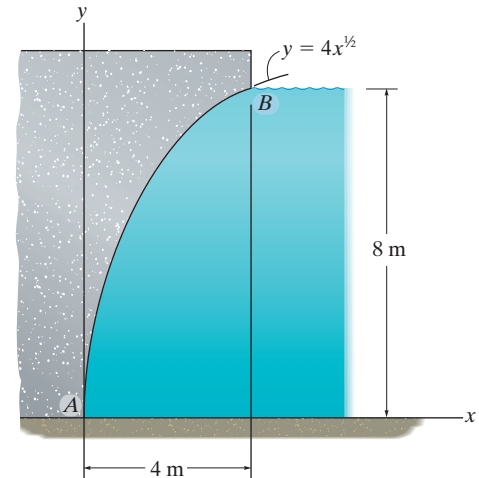
$$\begin{aligned} F_v &= \rho_w g V = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(32.0 \text{ m}^3) \\ &= 313.92(10^3) \text{ N} = 313.92 \text{ kN} \end{aligned}$$

Then the magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(941.76 \text{ kN})^2 + (313.92 \text{ kN})^2} = 992.70 \text{ kN} = 993 \text{ kN} \text{ Ans.}$$

And its direction is

$$\theta = \tan^{-1} \left(\frac{F_v}{F_h} \right) = \tan^{-1} \left(\frac{313.92 \text{ kN}}{941.76 \text{ kN}} \right) = 18.43^\circ = 18.4^\circ \quad \text{Ans.}$$

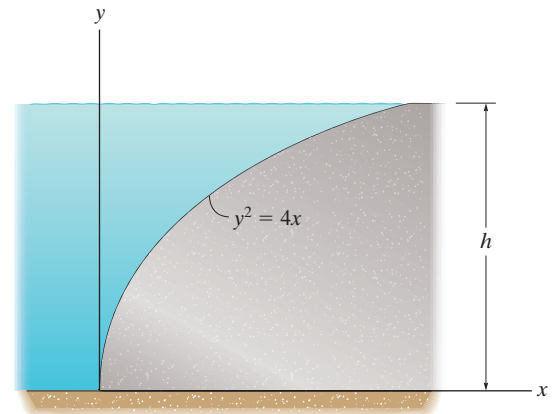


Ans:
 $F_R = 993 \text{ kN}$
 $\theta = 18.4^\circ \quad \swarrow$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-101.

The 5-m-wide wall is in the form of a parabola. If the depth of the water is $h = 4 \text{ m}$, determine the magnitude and direction of the resultant force on the wall.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4 \text{ m})(5 \text{ m}) = 196.2(10^3) \text{ N/m}$$

Thus,

$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2} [196.2(10^3) \text{ N/m}](4 \text{ m}) = 392.4(10^3) \text{ N} = 392.4 \text{ kN}$$

It acts at

$$\bar{y} = \frac{1}{3} h_A = \frac{1}{3} (4 \text{ m}) = \frac{4}{3} \text{ m}$$

The vertical component of the resultant force is equal to the weight of the column of water above surface *AB* of the wall (shown shaded in Fig. *a*). The volume of this column of water is

$$V = \frac{1}{3} a h b = \frac{1}{3} (4 \text{ m})(4 \text{ m})(5 \text{ m}) = 26.67 \text{ m}^3$$

Thus,

$$F_v = \rho_w g V = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(26.67 \text{ m}^3) = 261.6(10^3) \text{ N} = 261.6 \text{ kN}$$

It acts at

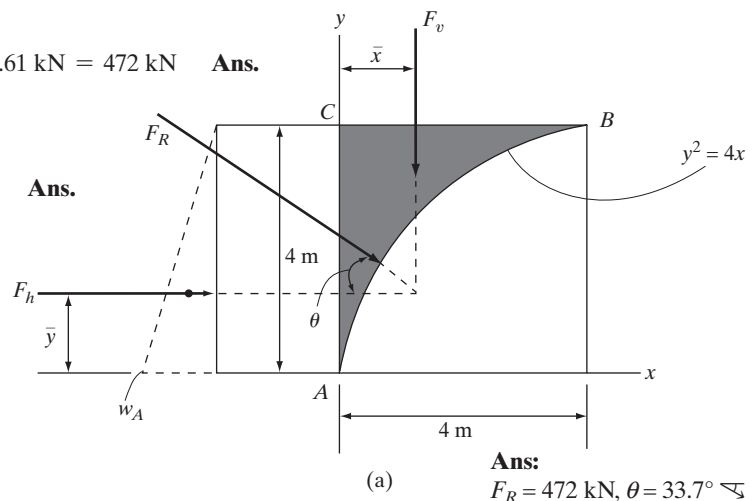
$$\bar{x} = \frac{3}{10} a = \frac{3}{10} (4 \text{ m}) = \frac{6}{5} \text{ m}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(392.4 \text{ kN})^2 + (261.6 \text{ kN})^2} = 471.61 \text{ kN} = 472 \text{ kN} \quad \text{Ans.}$$

And its direction is

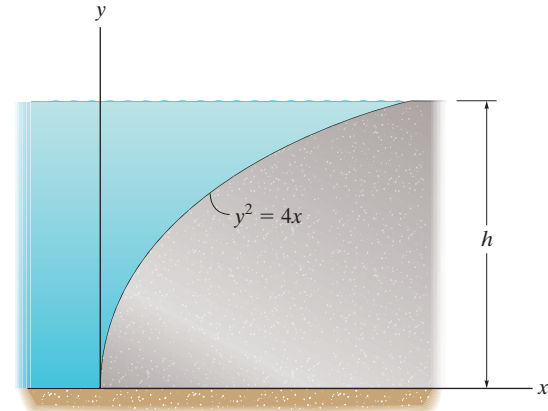
$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{261.6 \text{ kN}}{392.4 \text{ kN}}\right) = 33.69^\circ = 33.7^\circ \quad \text{Ans.}$$



Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-102.

The 5-m-wide wall is in the form of a parabola. Determine the magnitude of the resultant force on the wall as a function of depth h of the water. Plot the results of force (vertical axis) versus depth h for $0 \leq h \leq 4 \text{ m}$. Give values for increments of $\Delta h = 0.5 \text{ m}$.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(h)(5 \text{ m}) = 49.05(10^3)h$$

Thus,

$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2} [49.05(10^3)h]h = 24.525(10^3)h^2$$

The vertical component of the resultant force is equal to the weight of the column of water above surface *AB* of the wall (shown shaded in Fig. *a*). The volume of this column of water is

$$V = \frac{1}{3} a h b = \frac{1}{3} \left(\frac{h^2}{4} \right) (h)(5 \text{ m}) = \frac{5}{12} h^3$$

Thus,

$$F_v = \rho_w g V = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \left(\frac{5}{12} h^3 \right) = 4087.5 h^3$$

Then the magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2}$$

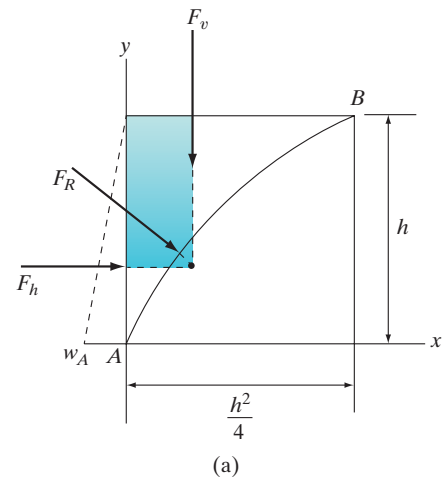
$$F_R = \sqrt{[24.525(10^3)h^2]^2 + [4087.5h^3]^2}$$

$$F_R = \sqrt{601.48(10^6)h^4 + 16.71(10^6)h^6}$$

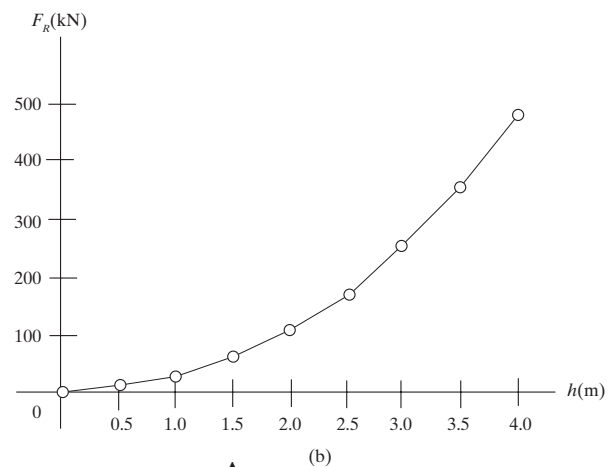
The plot of F_R vs h is shown in Fig. *b*.

$$F_R = \left[\sqrt{601(10^6)h^4 + 16.7(10^6)h^6} \right] \text{ N}$$

where h is in m.



$h(\text{m})$	0	0.5	1.0	1.5	2.0	2.5	3.0
$F_R(\text{kN})$	0	6.15	24.9	56.9	103	166	247
	3.5	4.0					
	348	472					



Ans:

$$F_R = \left[\sqrt{601(10^6)h^4 + 16.7(10^6)h^6} \right] \text{ N}$$

where h is in m.

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-103.

Gate AB has a width of 0.5 m and a radius of 1 m . Determine the horizontal and vertical components of reaction at the pin A and the horizontal reaction at the smooth stop B due to the water pressure.

SOLUTION

Vertical Component. This component is equal to the weight of the water contained in the block shown shaded in Fig. a , but it acts upward. This block can be subdivided into parts (1) and (2) as shown in Figs. b and c . Part (2) is a hole and should be considered as a negative part. Thus, the area of the block, Fig. a , is $\Sigma A = (1 \text{ m})(1 \text{ m}) - \frac{\pi}{4}(1 \text{ m})^2 = 0.2146 \text{ m}^2$ and the horizontal distance measured from its centroid to point A is

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{0.5 \text{ m}(1 \text{ m})(1 \text{ m}) - \frac{4(1 \text{ m})}{3\pi} \left[\frac{\pi}{4}(1 \text{ m})^2 \right]}{0.2146 \text{ m}^2} = 0.7766 \text{ m}$$

The magnitude of the vertical component is TBEXAM.COM

$$\begin{aligned} (F_R)_v &= \rho_w g V = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[0.2146 \text{ m}^2(0.5 \text{ m})] \\ &= 1052.62 \text{ N} \end{aligned}$$

Horizontal Component. The intensity of the distributed load at B is

$$w_B = \rho_w g h_B b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1 \text{ m})(0.5 \text{ m}) = 4.905(10^3) \text{ N/m}$$

Then,

$$(F_R)_h = \frac{1}{2} w_B h_B = \frac{1}{2} [4.905(10^3) \text{ N/m}](1 \text{ m}) = 2452.5 \text{ N}$$

Considering the free-body diagram of the gate in Fig. d ,

$$\zeta + \Sigma M_A = 0; \quad (2452.5 \text{ N}) \left[\frac{2}{3}(1 \text{ m}) \right] + (1052.62 \text{ N})(0.7766 \text{ m}) - F_B(1 \text{ m}) = 0$$

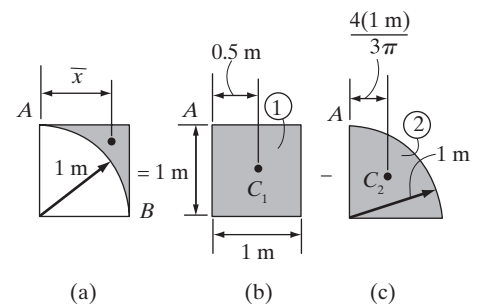
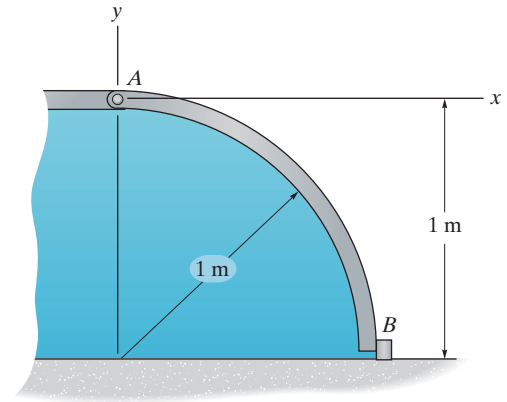
$$F_B = 2452.5 \text{ N} = 2.45 \text{ kN}$$

$$\rightarrow \Sigma F_x = 0; \quad 2452.5 \text{ N} - 2452.5 \text{ N} - A_x = 0$$

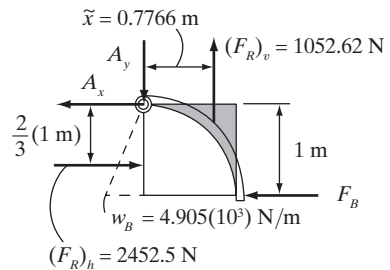
$$A_x = 0$$

$$+\uparrow \Sigma F_y = 0; \quad 1052.62 \text{ N} - A_y = 0$$

$$A_y = 1052.62 \text{ N} = 1.05 \text{ kN}$$



Ans.



Ans.

Ans.

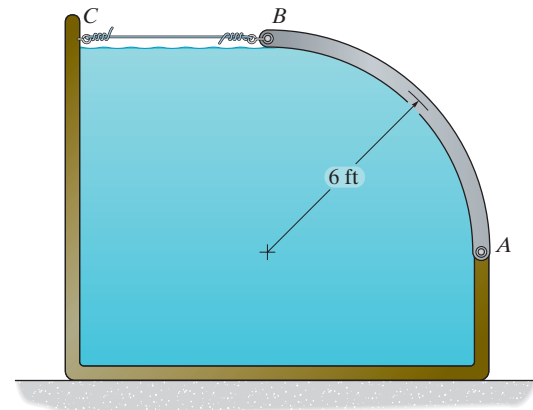
(d)

Ans:
 $F_B = 2.45 \text{ kN}$
 $A_x = 0$
 $A_y = 1.05 \text{ kN}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-104.**

A quarter-circular plate is pinned at A and tied to the tank's wall using the cable BC . If the tank and plate are 4 ft wide, determine the horizontal and vertical components of reaction at A , and the tension in the cable due to the water pressure.



SOLUTION

Referring to the geometry shown in Fig. a ,

$$A_{ADB} = (6 \text{ ft})(6 \text{ ft}) - \frac{\pi}{4}(6 \text{ ft})^2 = (36 - 9\pi) \text{ ft}^2$$

$$\bar{x} = \frac{(3 \text{ ft})[(6 \text{ ft})(6 \text{ ft})] - \left[\left(6 - \frac{8}{\pi} \right) \text{ ft} \right] \left[\frac{\pi}{4} (6 \text{ ft})^2 \right]}{(36 - 9\pi) \text{ ft}^2} = 1.3402 \text{ ft}$$

The horizontal component of the resultant force acting on the shell is equal to the pressure force on the vertically projected area of the shell. Referring to Fig. b ,

$$w_A = \gamma_w h_A b = (62.4 \text{ lb/ft}^3)(6 \text{ ft})(4 \text{ ft}) = 1497.6 \text{ lb/ft}$$

Thus,

$$F_h = \frac{1}{2} (1497.6 \text{ lb/ft})(6 \text{ ft}) = 4492.8 \text{ lb}$$

The vertical component of the resultant force acting on the shell is equal to the weight of the imaginary column of water above the shell (shown shaded in Fig. b), but acts upwards.

$$F_v = \gamma_w V = \gamma_w A_{ADB} b = (62.4 \text{ lb/ft}^3) [(36 - 9\pi) \text{ ft}^2] (4 \text{ ft}) = 1928.33 \text{ lb}$$

Write the moment equation of equilibrium about A by referring to Fig. b .

$$\zeta + \Sigma M_A = 0; T_{BC}(6 \text{ ft}) - (1928.33 \text{ lb})(1.3402 \text{ ft}) - (4492.8 \text{ lb})(2 \text{ ft}) = 0$$

$$T_{BC} = 1928.33 \text{ lb} = 1.93 \text{ kip}$$

Ans.

$$\rightarrow \Sigma F_x = 0; -A_x + 4492.8 \text{ lb} - 1928.33 \text{ lb} = 0$$

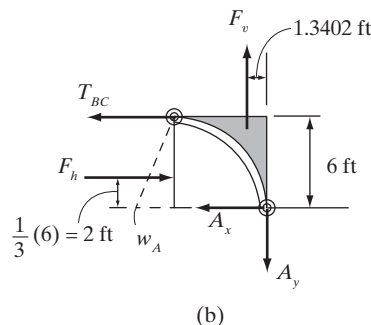
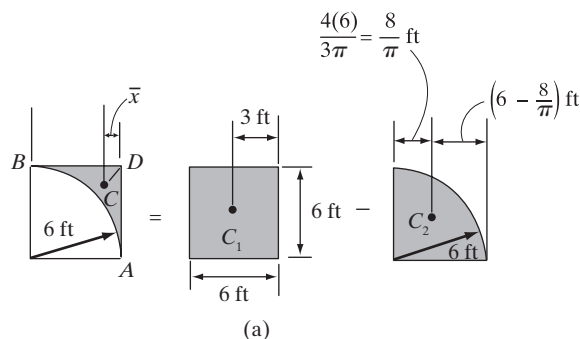
$$A_x = 2564.5 \text{ lb} = 2.56 \text{ kip}$$

Ans.

$$+\uparrow \Sigma F_y = 0; 1928.33 \text{ lb} - A_y = 0$$

$$A_y = 1928.33 \text{ lb} = 1.93 \text{ kip}$$

Ans.



Ans:
 $T_{BC} = 1.93 \text{ kip}$
 $A_x = 2.56 \text{ kip}$
 $A_y = 1.93 \text{ kip}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-105.

The canal transports water and has the cross section shown. Determine the magnitude and direction of the resultant force per unit length acting on wall AB , and the location of the center of pressure on the wall, measured with respect to the x and y axes. *Hint:* The line of action of the resultant force intersects the wall at this point.

SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of curve AB of the canal. Referring to Fig. a ,

$$w_A = \gamma_w h_A b = (62.4 \text{ lb/ft}^3)(9 \text{ ft})(1 \text{ ft}) = 561.6 \text{ lb/ft}$$

Thus,

$$F_h = \frac{1}{2}(561.6 \text{ lb/ft})(9 \text{ ft}) = 2527.2 \text{ lb}$$

And it acts at

$$\bar{y} = \frac{1}{3}(9 \text{ ft}) = 3 \text{ ft}$$

The vertical component of the resultant force is equal to the weight of the column of water above curve AB of the canal.

$$F_v = \int dF = \int \gamma_w dV = \gamma_w \int b dA = \gamma_w \int (1)(x dy)$$

However, $y = \frac{1}{3}x^3$ or $x = 3^{\frac{1}{3}}y^{\frac{1}{3}}$. Then

$$F_v = 62.4 \int_0^{9 \text{ ft}} 3^{\frac{1}{3}} y^{\frac{1}{3}} dy = 62.4 \left(3^{\frac{1}{3}} \right) \left[\frac{3}{4} y^{\frac{4}{3}} \right]_0^{9 \text{ ft}} = 1263.6 \text{ lb}$$

The location of its point of application can be determined from

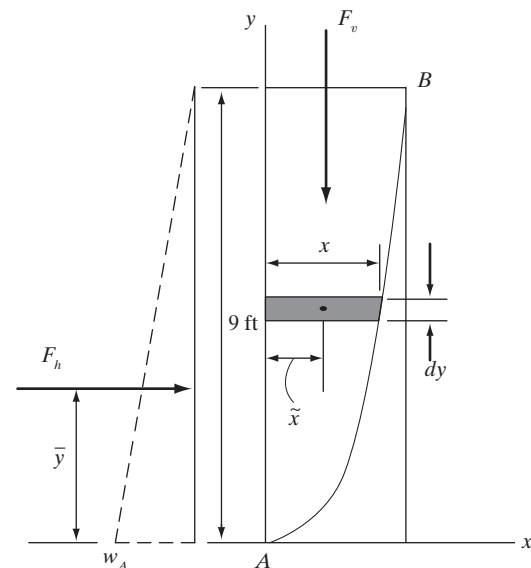
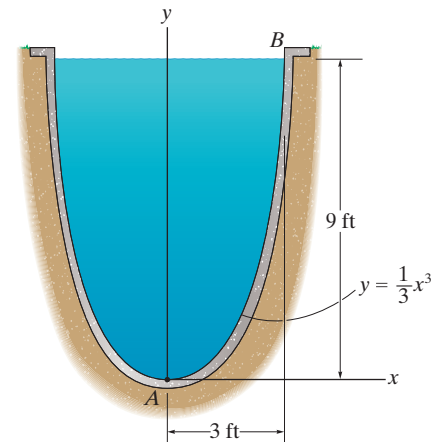
$$\bar{x} = \frac{\int \tilde{x} dF}{F_v} \quad \text{where } \tilde{x} = \frac{x}{2} \text{ and } dF = \gamma_w x dy = 62.4 x dy$$

Thus,

$$\begin{aligned} \bar{x} &= \frac{\int_0^{9 \text{ ft}} \left(\frac{x}{2} \right) (62.4 x dy)}{1263.6} \\ &= \frac{31.2 \int_0^{9 \text{ ft}} x^2 dy}{1263.6} \\ &= \frac{31.2 \int_0^{9 \text{ ft}} 3^{\frac{2}{3}} y^{\frac{2}{3}} dy}{1263.6} \\ &= \frac{31.2 \left(3^{\frac{2}{3}} \right) \left(\frac{3}{5} y^{\frac{5}{3}} \right) \Big|_0^{9 \text{ ft}}}{1263.6} \\ &= 1.20 \text{ ft} \end{aligned}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(2527.2 \text{ lb})^2 + (1263.6 \text{ lb})^2} = 2825.50 \text{ lb} = 2.83 \text{ kip} \quad \text{Ans.}$$



(a)

2–105. Continued

And its direction is defined by

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{1263.6 \text{ lb}}{2527.2 \text{ lb}}\right) = 26.57^\circ = 26.6^\circ \quad \swarrow \quad \textbf{Ans.}$$

The equation of the line of action of F_R is

$$y - \bar{y} = m(x - \bar{x}); y - 3 = -\tan 26.57^\circ(x - 1.20)$$

$$y = -0.5x + 3.6$$

The intersection point of the line of action of F_R and surface AB can be obtained by solving simultaneously this equation and that of AB .

$$\frac{1}{3}x^3 = -0.5x + 3.6$$

$$\frac{1}{3}x^3 + 0.5x - 3.6 = 0$$

Solving numerically,

$$x_p = 1.9851 \text{ ft} = 1.99 \text{ ft} \quad \textbf{Ans.}$$

When $x_p = 1.9851 \text{ ft}$,

$$y_p = \frac{1}{3}(1.9851^3) = 2.61 \text{ ft} \quad \textbf{Ans.}$$

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Ans:

$$F_R = 2.83 \text{ kip}$$

$$\theta = 26.6^\circ \quad \swarrow$$

$$x_p = 1.99 \text{ ft}$$

$$y_p = 2.61 \text{ ft}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-106.

The semicircular gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the torque T that must be applied at the pin A in order to open the gate. The gate has a mass of 8 Mg with center of mass at G . It is 4 m wide.

SOLUTION

The horizontal loading on the gate is due to the pressure on the vertical projected area of the gate, Fig. a . Since the gate has a constant width of $b = 4 \text{ m}$, the intensity of the distributed load at point B is

$$w_B = \rho_w g h_B b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(6 \text{ m})(4 \text{ m}) = 235.44(10^3) \text{ N/m}$$

Thus,

$$F_h = \frac{1}{2} w_B h_B = \frac{1}{2} [235.44(10^3) \text{ N/m}](6 \text{ m}) = 706.32(10^3) \text{ N}$$

and it acts at

$$\bar{y} = \frac{2}{3}(6 \text{ m}) = 4 \text{ m}$$

The upward force on BE and downward force on CE is equal to the weight of water contained in blocks $BACDEB$ (imaginary) and $CDEC$, respectively. Thus, the net upward force on BEC is equal to the weight of water contained in block $BACEB$ shown shaded in Fig. a . Thus,

$$\begin{aligned} F_v &= \rho_w g V_{BACEB} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \left[\frac{\pi}{2} (3 \text{ m})^2 (4 \text{ m}) \right] \\ &= 176.58(10^3) \pi \text{ N} \end{aligned}$$

And it acts at

$$\bar{x} = \frac{4(3 \text{ m})}{3\pi} = \frac{4}{\pi} \text{ m}$$

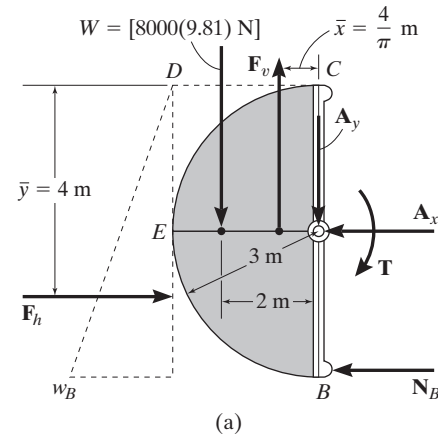
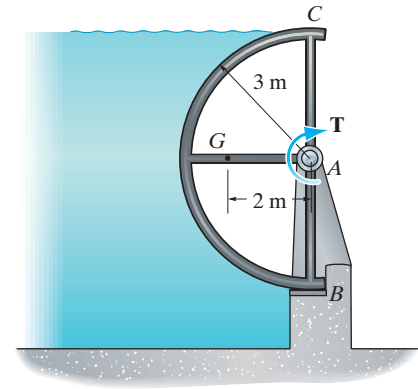
When the gate is on the verge of opening, $N_B = 0$. Write the moment equation of equilibrium about point A by referring to the FBD of the gate, Fig. a .

$$\begin{aligned} \zeta + \Sigma M_A &= 0; \quad [8000(9.81) \text{ N}](2 \text{ m}) + [706.32(10^3) \text{ N}](4 \text{ m} - 3 \text{ m}) \\ &\quad - [176.58(10^3) \pi \text{ N}]\left(\frac{4}{\pi} \text{ m}\right) - T = 0 \end{aligned}$$

$$T = 156.96(10^3) \text{ N} \cdot \text{m} = 157 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$

This solution can be simplified if one realizes that the resultant force due to the water pressure on the gate will act perpendicular to the circular surface, thus acting through center A of the semicircular gate and so producing no moment about this point.

$$\begin{aligned} \zeta + \Sigma M_A &= 0; \quad [8000(9.81) \text{ N}](2 \text{ m}) - T = 0 \\ T &= 156.96(10^3) \text{ N} \cdot \text{m} = 157 \text{ kN} \cdot \text{m} \quad \text{Ans.} \end{aligned}$$

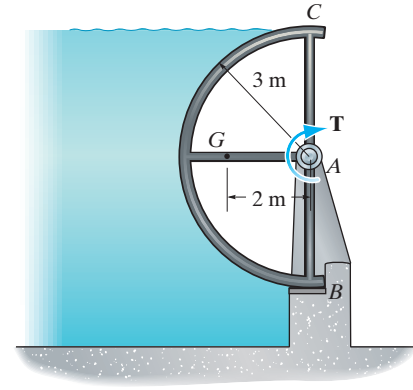


$$\begin{aligned} \text{Ans:} \\ T &= 157 \text{ kN} \cdot \text{m} \end{aligned}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-107.

The semicircular gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the horizontal and vertical components of reaction at pin A and the normal reaction at B . The gate has a mass of 8 Mg with center of mass at G . It is 4 m wide. Take $T = 0$.



SOLUTION

The horizontal loading on the gate is due to the pressure on the vertical projected area of the gate, Fig. a . Since the gate has a constant width of $b = 4 \text{ m}$, the intensity of the distributed load at point B is

$$w_B = \rho_w g h_B b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(6 \text{ m})(4 \text{ m}) = 235.44(10^3) \text{ N/m}$$

Thus,

$$F_h = \frac{1}{2} w_B h_B = \frac{1}{2} [235.44(10^3) \text{ N/m}] (6 \text{ m}) = 706.32(10^3) \text{ N}$$

and it acts at

$$\bar{y} = \frac{2}{3} (6 \text{ m}) = 4 \text{ m} \quad \text{TBEXAM.COM}$$

The upward force on BE and downward force on CE is equal to the weight of water contained in blocks $BACDEB$ (imaginary) and $CDEC$, respectively. Thus, the net upward force on BEC is equal to the weight of water contained in block $BACEB$ shown shaded in Fig. a . Thus,

$$\begin{aligned} F_v &= \rho_w g V_{BACEB} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \left[\frac{\pi}{2} (3 \text{ m})^2 (4 \text{ m}) \right] \\ &= 176.58(10^3) \pi \text{ N} \end{aligned}$$

And it acts at

$$\bar{x} = \frac{4(3 \text{ m})}{3\pi} = \frac{4}{\pi} \text{ m}$$

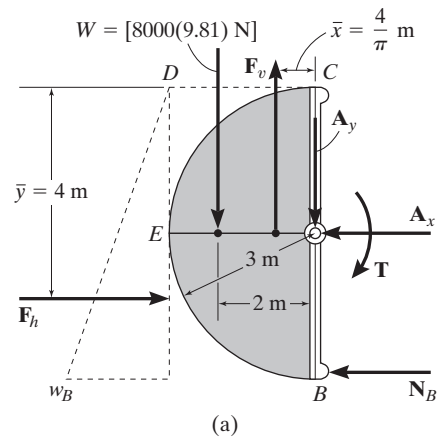
Write the moment equation of equilibrium about point A and B with $T = 0$ by referring to the FBD of the gate, Fig. a .

$$\begin{aligned} \zeta + \Sigma M_A &= 0; \quad [8000(9.81) \text{ N}](2 \text{ m}) + [706.32(10^3) \text{ N}](4 \text{ m} - 3 \text{ m}) \\ &\quad - [176.58(10^3) \pi \text{ N}]\left(\frac{4}{\pi} \text{ m}\right) - N_B(3 \text{ m}) = 0 \\ N_B &= 52.32(10^3) \text{ N} = 52.3 \text{ kN} \end{aligned} \quad \text{Ans.}$$

$$\begin{aligned} \zeta + \Sigma M_B &= 0; \quad A_x(3 \text{ m}) + [8000(9.81) \text{ N}](2 \text{ m}) - [176.58(10^3) \pi \text{ N}]\left(\frac{4}{\pi} \text{ m}\right) \\ &\quad - [706.32(10^3) \text{ N}](6 \text{ m} - 4 \text{ m}) = 0 \\ A_x &= 654(10^3) \text{ N} = 654 \text{ kN} \end{aligned} \quad \text{Ans.}$$

Write the force equation of equilibrium along y axis.

$$\begin{aligned} + \uparrow \Sigma F_y &= 0; \quad 176.58(10^3) \pi \text{ N} - 8000(9.81) \text{ N} - A_y = 0 \\ A_y &= 476.26(10^3) \text{ N} = 476 \text{ kN} \end{aligned} \quad \text{Ans.}$$

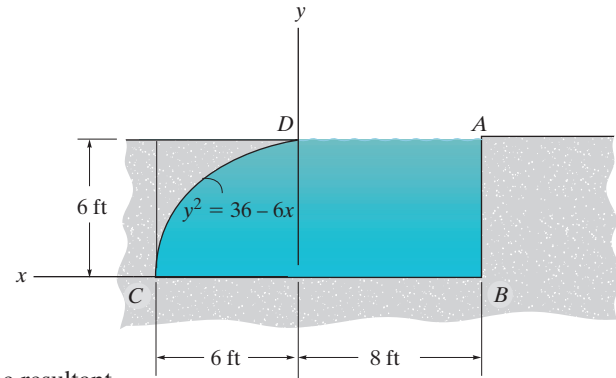


$$\begin{aligned} \text{Ans:} \\ N_B &= 52.3 \text{ kN} \\ A_x &= 654 \text{ kN} \\ A_y &= 476 \text{ kN} \end{aligned}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-108.**

Determine the magnitude of the resultant force the water exerts on AB , BC , and CD of the enclosure, which is 10 ft wide.



SOLUTION

The horizontal component of the resultant force on CD is the same as the resultant force on AB . Its magnitude can be determined from

$$\begin{aligned} F_{AB} = (F_{CD})_h &= \gamma_w \bar{h} A = (62.4 \text{ lb/ft}^3)(3 \text{ ft})[(6 \text{ ft})(10 \text{ ft})] \\ &= 11.232(10^3) \text{ lb} = 11.2 \text{ kip } (10^3) \text{ lb} \end{aligned} \quad \text{Ans.}$$

The vertical component of the resultant force on CD is equal to the weight of water column above CD (shown shaded in Fig. *a*) but acts upward.

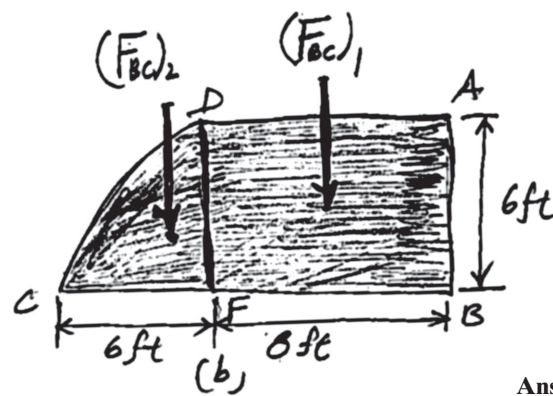
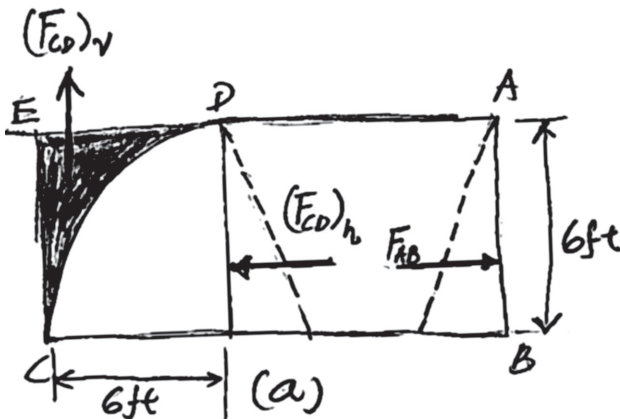
$$(F_{CD})_v = \gamma_w \nabla_{CDEC} = (62.4 \text{ lb/ft}^3) \left[\frac{1}{3} (6 \text{ ft})(6 \text{ ft})(10 \text{ ft}) \right] = 7.488(10^3) \text{ lb.}$$

Thus,

$$\begin{aligned} F_{CD} &= \sqrt{(F_{CD})_h^2 + (F_{CD})_v^2} = \sqrt{[11.232(10^3) \text{ lb}]^2 + [7.488(10^3) \text{ lb}]^2} \\ &= 13.499(10^3) \text{ lb} = 13.5 \text{ kip } (10^3) \text{ lb} \end{aligned} \quad \text{Ans.}$$

The force on BC is equal to the weight of water column above BC shown shaded in Fig. *b*.

$$\begin{aligned} F_{BC} &= (F_{BC})_1 + (F_{BC})_2 \\ &= \gamma_w \nabla_{ABFDA} + \gamma_w \nabla_{CDFC} \\ &= (62.4 \text{ lb/ft}^3) [(6 \text{ ft})(8 \text{ ft})(10 \text{ ft})] + (62.4 \text{ lb/ft}^3) \left[\frac{2}{3} (6 \text{ ft})(6 \text{ ft})(10 \text{ ft}) \right] \\ &= 44.928(10^3) \text{ lb} = 44.9 \text{ kip } (10^3) \text{ lb} \end{aligned} \quad \text{Ans.}$$

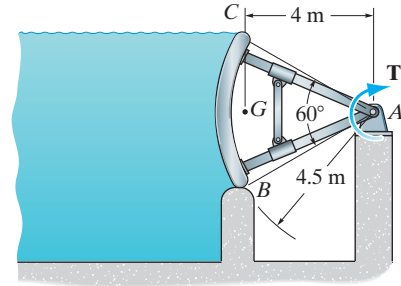


Ans:
 $F_{AB} = 11.2 \text{ kip}$
 $F_{CD} = 13.5 \text{ kip}$
 $F_{BC} = 44.9 \text{ kip}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-109.

The Tainter gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the torque T that must be applied at the pin A in order to open the gate. The gate has a mass of 5 Mg and a center of mass at G . It is 3 m wide.



SOLUTION

Horizontal Component. This component can be determined by applying

$$(F_{BC})_h = \gamma_w \bar{h} A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4.5 \sin 30^\circ \text{ m}) \left[2(4.5 \sin 30^\circ \text{ m})(3 \text{ m}) \right] \\ = 297.98(10^3) \text{ N}$$

Vertical Component. The upward force on BE and downward force on CE is equal to the weight of the water contained in blocks $BCDEB$ and $CEDC$, respectively. Thus, the net upward force on BEC is equal to the weight of the water contained in block $BCEB$, shown shaded in Fig. a . This block can be subdivided into parts (1) and (2), Figs. b and c , respectively. However, part (2) is a hole and should be considered as a negative part. The area of block $BCEB$ is $\Sigma A = \left[\frac{\pi}{6} (4.5 \text{ m})^2 \right] - \frac{1}{2} (4.5 \text{ m})(4.5 \cos 30^\circ \text{ m}) = 1.8344 \text{ m}^2$ and the horizontal distance measured from its centroid to point A is

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{\left(\frac{9}{\pi} \text{ m} \right) \left[\frac{\pi}{6} (4.5 \text{ m})^2 \right] - \frac{2}{3} (4.5 \cos 30^\circ \text{ m}) \left[\frac{1}{2} (4.5 \text{ m})(4.5 \cos 30^\circ \text{ m}) \right]}{1.8344 \text{ m}^2} \\ = 4.1397 \text{ m}$$

The magnitude of the vertical component is

$$(F_{BC})_v = \gamma_w V_{BCEB} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) [1.8344 \text{ m}^2 (3 \text{ m})] \\ = 53.985(10^3) \text{ N}$$

When the gate is on the verge of opening, $N_B = 0$. Referring to the free-body diagram of the gate in Fig. d ,

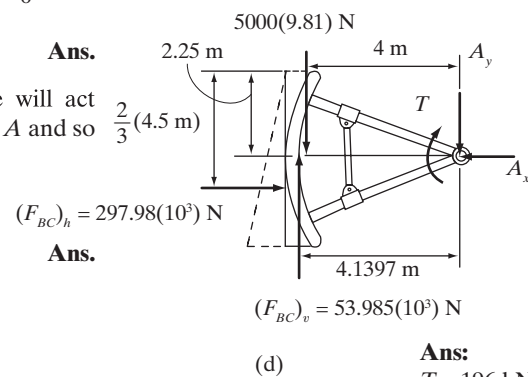
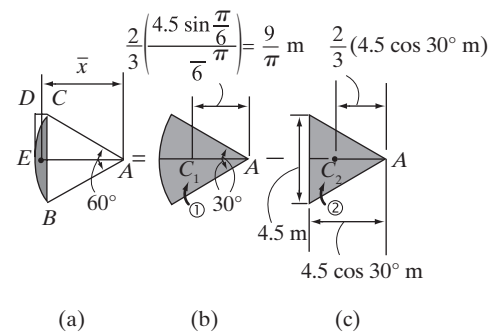
$$\zeta + \Sigma M_A = 0; \quad [5000(9.81) \text{ N}](4 \text{ m}) + [297.98(10^3) \text{ N}] \left[\frac{2}{3} (4.5 \text{ m}) - 2.25 \text{ m} \right] \\ - [53.985(10^3) \text{ N}](4.1397 \text{ m}) - T = 0$$

$$T = 196.2(10^3) \text{ N} \cdot \text{m} = 196 \text{ kN} \cdot \text{m}$$

This solution can be simplified if one realizes that the resultant force will act perpendicular to the circular surface. Therefore, F_{BC} will act through point A and so produces no moment about this point. Hence,

$$\zeta + \Sigma M_A = 0; \quad [5000(9.81) \text{ N}](4 \text{ m}) - T = 0$$

$$T = 196.2(10^3) \text{ N} \cdot \text{m} = 196 \text{ kN} \cdot \text{m}$$

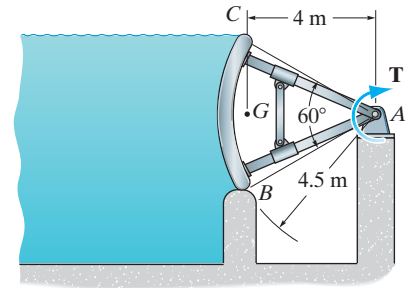


Ans:
 $T = 196 \text{ kN} \cdot \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-110.

The Tainter gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the horizontal and vertical components of reaction at pin A and the vertical reaction at the smooth spillway crest B . The gate has a mass of 5 Mg and a center of mass at G . It is 3 m wide. Take $T = 0$.



SOLUTION

Horizontal Component. This component can be determined from

$$(F_{BC})_h = \gamma_w \bar{h} A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4.5 \sin 30^\circ \text{ m})[2(4.5 \sin 30^\circ \text{ m})(3 \text{ m})] \\ = 297.98(10^3) \text{ N}$$

Vertical Component. The upward force on BE and downward force on CE is equal to the weight of the water contained in blocks $BCDEB$ and $CEDC$, respectively. Thus, the net upward force on BEC is equal to the weight of the water contained in block $BCEB$, shown shaded in Fig. a . This block can be subdivided into parts (1) and (2), Figs. b and c , respectively. However, part (2) is a hole and should be considered as a negative part. The area of block $BCEB$ is $\Sigma A = \left[\frac{\pi}{6} (4.5 \text{ m})^2 \right] - \frac{1}{2} (4.5 \text{ m})(4.5 \cos 30^\circ \text{ m}) = 1.8344 \text{ m}^2$ and the horizontal distance measured from its centroid to point A is

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{\left(\frac{9}{\pi} \text{ m} \right) \left[\frac{\pi}{6} (4.5 \text{ m})^2 \right] - \frac{2}{3} (4.5 \cos 30^\circ \text{ m}) \left[\frac{1}{2} (4.5 \text{ m})(4.5 \cos 30^\circ \text{ m}) \right]}{1.8344 \text{ m}^2} \\ = 4.1397 \text{ m}$$

Thus, the magnitude of the vertical component is

$$(F_{BC})_v = \gamma_w V_{BCEB} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[1.8344 \text{ m}^2(3 \text{ m})] \\ = 53.985(10^3) \text{ N}$$

Considering the free-body diagram of the gate in Fig. d ,

$$\zeta + \Sigma M_A = 0; \quad [5000(9.81) \text{ N}](4 \text{ m}) + [297.98(10^3) \text{ N}] \left[\frac{2}{3} (4.5 \text{ m}) - 2.25 \text{ m} \right] \\ - [53.985(10^3) \text{ N}](4.1397 \text{ m}) - N_B(4.5 \cos 30^\circ \text{ m}) = 0$$

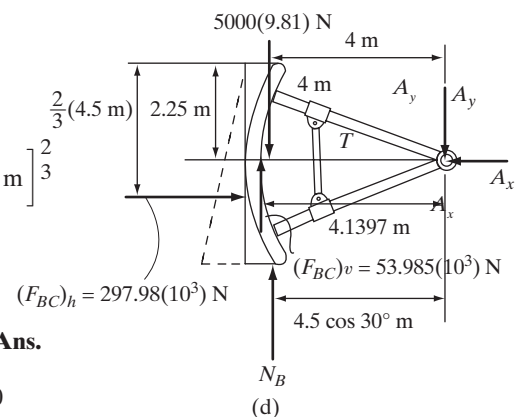
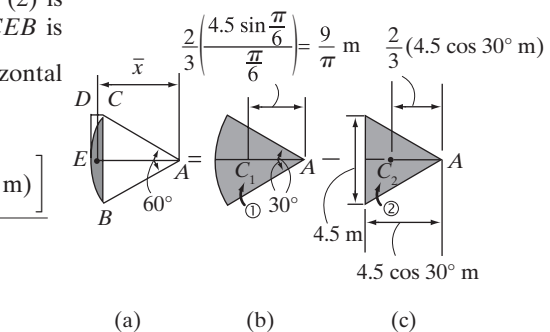
$$N_B = 50.345(10^3) \text{ N} = 50.3 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad 50.345(10^3) \text{ N} + 53.985(10^3) \text{ N} - 5000(9.81) \text{ N} - A_y = 0$$

$$A_y = 55.28(10^3) \text{ N} = 55.3 \text{ kN}$$

$$\rightarrow \Sigma F_x = 0; \quad 297.98(10^3) \text{ N} - A_x$$

$$A_x = 297.98(10^3) \text{ N} = 298 \text{ kN}$$



Ans.

Ans.

Ans.

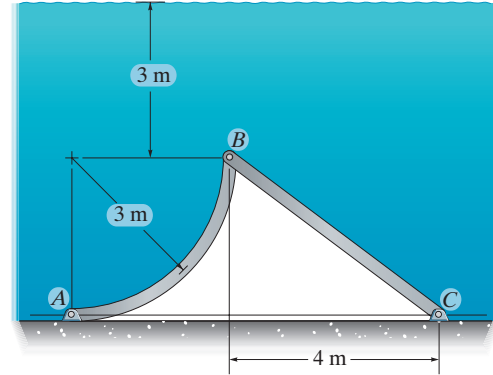
Ans:

$$N_B = 50.3 \text{ kN} \\ A_y = 55.3 \text{ kN} \\ A_x = 298 \text{ kN}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-111.

The curved and flat plates are pin connected at A , B , and C . They are submerged in water at the depth shown. Determine the horizontal and vertical components of reaction at pin B . The plates have a width of 4 m.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force on the vertically projected area of the plate. Referring to Fig. a ,

$$w_B = \rho_w g h_B b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3 \text{ m})(4 \text{ m}) = 117.72(10^3) \text{ N/m}$$

$$w_A = w_C = \rho_w g h_C b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(6 \text{ m})(4 \text{ m}) = 235.44(10^3) \text{ N/m}$$

Thus,

$$(F_h)_{AB_1} = (F_h)_{BC_1} = [117.72(10^3) \text{ N/m}](3 \text{ m}) = 353.16(10^3) \text{ N} = 353.16 \text{ kN}$$

$$(F_h)_{AB_2} = (F_h)_{BC_2} = \frac{1}{2}[235.44(10^3) \text{ N/m} - 117.72(10^3) \text{ N/m}](3 \text{ m}) = 176.58(10^3) \text{ N} = 176.58 \text{ kN}$$

They act at

$$\tilde{y}_1 = \tilde{y}_3 = \frac{1}{2}(3) = 1.5 \text{ m} \quad \tilde{y}_2 = \tilde{y}_4 = \frac{1}{3}(3 \text{ m}) = 1 \text{ m}$$

The vertical component of the resultant force is equal to the weight of the column of water above the plates shown shaded in Fig. a .

$$(F_v)_{AB_1} = \rho_w g V_{AB_1} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(3 \text{ m})(3 \text{ m})(4 \text{ m})] = 353.16(10^3) \text{ N} = 353.16 \text{ kN}$$

$$(F_v)_{AB_2} = \rho_w g V_{AB_2} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left[\frac{1}{4}\pi(3 \text{ m})^2(4 \text{ m})\right] = 88.29\pi(10^3) \text{ N} = 88.29\pi \text{ kN}$$

$$(F_v)_{BC_1} = \rho_w g V_{BC_1} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(3 \text{ m})(4 \text{ m})(4 \text{ m})] = 470.88(10^3) \text{ N} = 470.88 \text{ kN}$$

$$(F_v)_{BC_2} = \rho_w g V_{BC_2} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left[\frac{1}{2}(3 \text{ m})(4 \text{ m})(4 \text{ m})\right] = 235.44(10^3) \text{ N} = 235.44 \text{ kN}$$

They act at

$$\tilde{x}_1 = \frac{1}{2}(3 \text{ m}) = 1.5 \text{ m} \quad \tilde{x}_2 = \frac{4(3 \text{ m})}{3\pi} = \frac{4}{\pi} \text{ m} \quad \tilde{x}_3 = \frac{1}{2}(4 \text{ m}) = 2 \text{ m} \quad \tilde{x}_4 = \frac{1}{3}(4 \text{ m}) = \frac{4}{3} \text{ m}$$

Referring to Fig. a and writing the moment equations of equilibrium about A and C ,

$$\begin{aligned} \zeta + \Sigma M_A = 0; \quad & B_x(3 \text{ m}) - B_y(3 \text{ m}) - (353.16 \text{ kN})(1.5 \text{ m}) - (88.29\pi \text{ kN})\left(\frac{4}{\pi} \text{ m}\right) \\ & - (353.16 \text{ kN})(1.5 \text{ m}) - (176.58 \text{ kN})(1 \text{ m}) = 0 \\ & B_x - B_y = 529.74 \end{aligned} \quad (1)$$

2-111. Continued

$$\begin{aligned} \zeta + \Sigma M_C = 0; & \quad (470.88 \text{ kN})(2 \text{ m}) + (235.44 \text{ kN})\left(\frac{4}{3} \text{ m}\right) + (353.16 \text{ kN})(1.5 \text{ m}) \\ & + (176.58 \text{ kN})(1 \text{ m}) - B_x(3 \text{ m}) - B_y(4 \text{ m}) = 0 \\ & \quad 3B_x - 4B_y = 1962 \end{aligned} \quad (2)$$

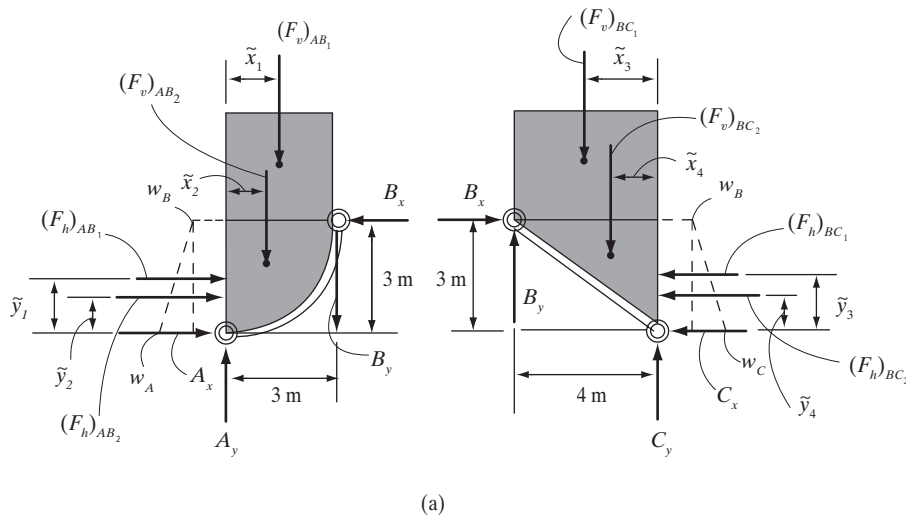
Solving Eqs. (1) and (2),

$$B_x = 582.99 \text{ kN} = 583 \text{ kN}$$

Ans.

$$B_y = 53.3 \text{ kN}$$

Ans.



Ans:

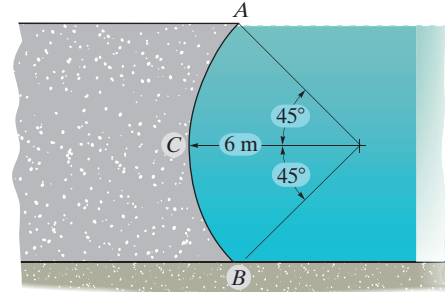
$$B_x = 583 \text{ kN}$$

$$B_y = 53.3 \text{ kN}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

*2-112.

Determine the magnitude of the resultant force the water exerts on the curved vertical wall. The wall is 3 m wide.



SOLUTION

The vertical downward force on AC and vertical upward force on BC are equal to the weight of water column $ACDA$ and $ABCD$ respectively. Thus, the net upward force on ACB is equal to the weight of water column $ABCA$ shown shaded in Fig. a .

Here, $A_{ABCA} = \frac{1}{2}(6 \text{ m})^2 \left(\frac{\pi}{2} \right) - \frac{1}{2}(6 \text{ m})(6 \text{ m}) = (9\pi - 18) \text{ m}^2$. Then,

$$(F_{ACB})_v = \rho_w g V_{ABCA} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\{(9\pi - 18) \text{ m}^2\}(3 \text{ m})$$

$$= 302.37(10^3) \text{ N} = 302.37 \text{ kN}$$

From Fig. b , $\bar{h} = \frac{1}{2}(6\sqrt{2} \text{ m}) = 3\sqrt{2} \text{ m}$. Then, the horizontal component of the resultant force on ACB can be determined by applying TBEXAM.COM

$$(F_{ACB})_h = \rho_w g \bar{h} A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3\sqrt{2} \text{ m})[(6\sqrt{2} \text{ m})(3 \text{ m})]$$

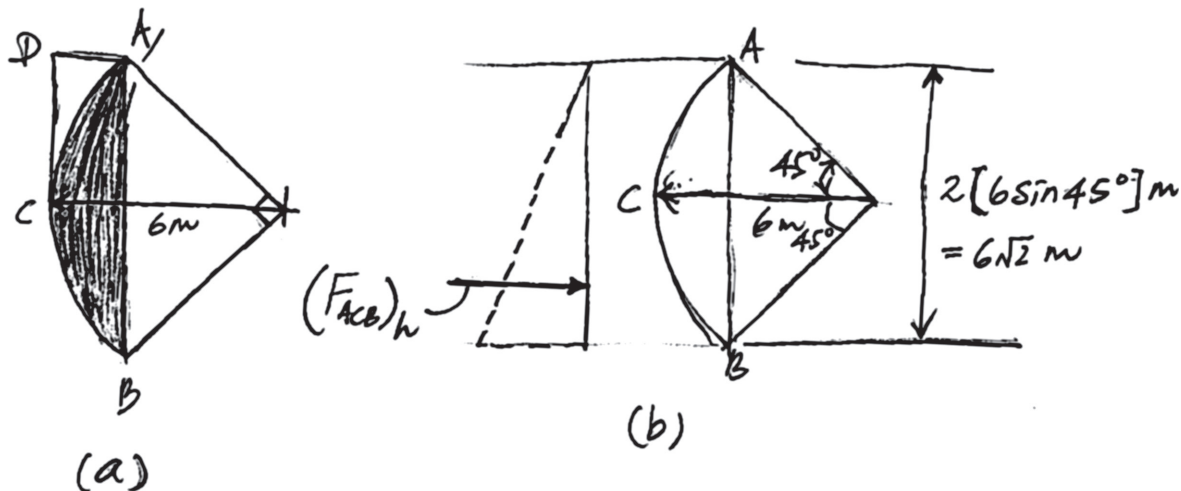
$$= 1059.48(10^3) \text{ N} = 1059.48 \text{ kN}$$

Thus, the magnitude of the resultant force on ACB is

$$F_{ACB} = \sqrt{(F_{ACB})_h^2 + (F_{ACB})_v^2} = \sqrt{(1059.48 \text{ kN})^2 + (302.37 \text{ kN})^2}$$

$$= 1101.78 \text{ kN} = 1.10 \text{ MN}$$

Ans.

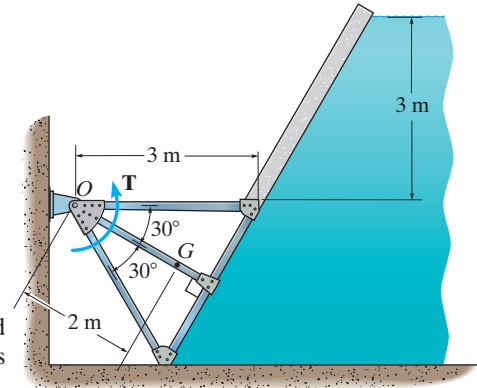


Ans:
 $F_{ACB} = 1.10 \text{ MN}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-113.

The sluice gate for a water channel is 2 m wide and in the closed position, as shown. Determine the magnitude of the resultant force of the water acting on the gate. Also, what is the smallest torque T that must be applied to open the gate if its mass is 6 Mg with its center of mass at G ?



SOLUTION

The horizontal loading on the gate is due to the pressure on the vertical projected area of the gate, Fig. *a*. Since the gate has a constant width of $b = 2 \text{ m}$, the intensities of the horizontal distributed load at points *A* and *B* are

$$w_A = \rho_w g h_{AB} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3 \text{ m})(2 \text{ m}) = 58.86(10^3) \text{ N/m}$$

$$w_B = \rho_w g h_{BB} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3 \text{ m} + 3 \sin 60^\circ \text{ m})(2 \text{ m}) = 109.83(10^3) \text{ N/m}$$

Thus,

$$(F_h)_1 = w_A h_{AB} = [58.86(10^3) \text{ N/m}](3 \sin 60^\circ \text{ m}) = 152.92(10^3) \text{ N}$$

$$(F_h)_2 = \frac{1}{2} (w_B - w_A) h_{AB} = \frac{1}{2} [109.83(10^3) \text{ N/m} - 58.86(10^3) \text{ N/m}](3 \sin 60^\circ \text{ m}) = 66.22(10^3) \text{ N}$$

Then

$$F_h = (F_h)_1 + (F_h)_2 = 152.92(10^3) \text{ N} + 66.22(10^3) \text{ N} = 219.14(10^3) \text{ N} = 219.14 \text{ kN}$$

Here, $(F_h)_1$ and $(F_h)_2$ act at

$$\tilde{y}_1 = \frac{1}{2} (3 \sin 60^\circ \text{ m}) = 1.2990 \text{ m} \quad \tilde{y}_2 = \frac{2}{3} (3 \sin 60^\circ \text{ m}) = 1.7320 \text{ m}$$

The vertical force is equal to the weight of the water contained in the imaginary block above the gate (shown shaded in Fig. *a*), but acts upward.

$$(F_v)_1 = \rho_w g V_1 = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) [(3 \text{ m})(1.5 \text{ m})(2 \text{ m})] = 88.29(10^3) \text{ N}$$

$$(F_v)_2 = \rho_w g V_2 = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \left[\frac{1}{2} (3 \sin 60^\circ \text{ m})(1.5 \text{ m})(2 \text{ m}) \right] = 38.23(10^3) \text{ N}$$

Then

$$F_v = (F_v)_1 + (F_v)_2 = 88.29(10^3) + 38.23(10^3) = 126.52(10^3) \text{ N} = 126.52 \text{ kN}$$

Here, $(F_v)_1$ and $(F_v)_2$ act at

$$\tilde{x}_1 = 1.5 \text{ m} + \frac{1}{2} (1.5 \text{ m}) = 2.25 \text{ m} \quad \tilde{x}_2 = 1.5 \text{ m} + \frac{1}{3} (1.5 \text{ m}) = 2 \text{ m}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(219.14 \text{ kN})^2 + (126.52 \text{ kN})^2} = 253.04 \text{ kN} = 253 \text{ kN} \quad \text{Ans.}$$

When the gate is on the verge of opening, $N_B = 0$. Write the moment equation of equilibrium about point *O* by referring to the FBD of the gate, Fig. *a*.

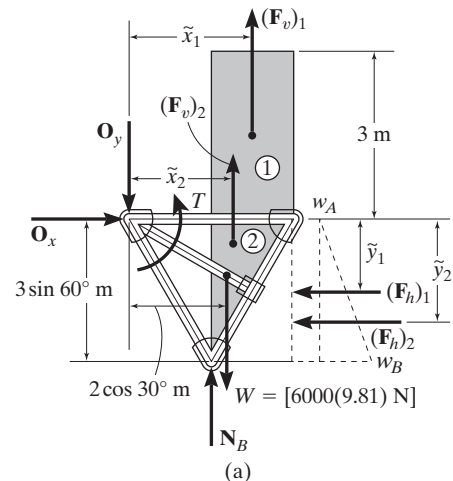
$$\begin{aligned} \zeta + \Sigma M_O = 0; & \quad T + [88.29(10^3) \text{ N}](2.25 \text{ m}) + [38.23(10^3) \text{ N}](2 \text{ m}) \\ & \quad - [152.92(10^3) \text{ N}](1.2990 \text{ m}) - [66.22(10^3) \text{ N}](1.7320 \text{ m}) \\ & \quad - [6000(9.81) \text{ N}](2 \cos 30^\circ \text{ m}) = 0 \\ & \quad T = 140.18(10^3) \text{ N} \cdot \text{m} = 140 \text{ kN} \cdot \text{m} \end{aligned}$$

Ans.

Ans:

$$F_R = 253 \text{ kN}$$

$$T = 140 \text{ kN} \cdot \text{m}$$



Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-114.

The stopper in the shape of a frustum is used to plug the 100-mm-diameter hole in the tank that contains amyl acetate. If the greatest vertical force the stopper can resist is 100 N, determine the depth d before it becomes unplugged. Take ρ_{aa}

$= 863 \text{ kg/m}^3$. *Hint:* The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

SOLUTION

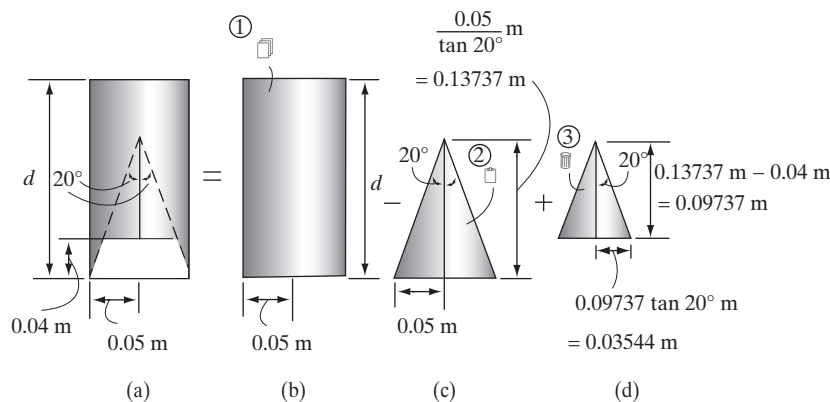
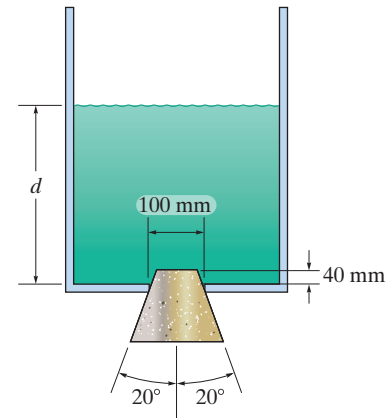
The vertical downward force on the frustum stopper is due to the weight of the liquid contained in the block shown shaded in Fig. *a*. This block can be subdivided into parts (1), (2), and (3), shown in Figs. *b*, *c*, and *d*, respectively. Part (2) is a hole and should be considered as a negative part. Thus, the volume of the shaded block in Fig. *a* is

$$\begin{aligned} V &= V_1 - V_2 + V_3 \\ &= \pi(0.05 \text{ m})^2 d - \frac{1}{3}\pi(0.05 \text{ m})^2(0.13737 \text{ m}) + \frac{1}{3}\pi(0.03544 \text{ m})^2(0.09737 \text{ m}) \\ &= [2.5(10^{-3})\pi d - 0.2316(10^{-3})] \text{ m}^3 \end{aligned}$$

The vertical force on the stopper is required to be equal to 100 N. Then,

$$\begin{aligned} F &= \rho_w g V \\ 100 \text{ N} &= (863 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[2.5(10^{-3})\pi d - 0.2316(10^{-3})] \text{ m}^3 \\ d &= 1.5334 \text{ m} = 1.53 \text{ m} \end{aligned}$$

Ans.



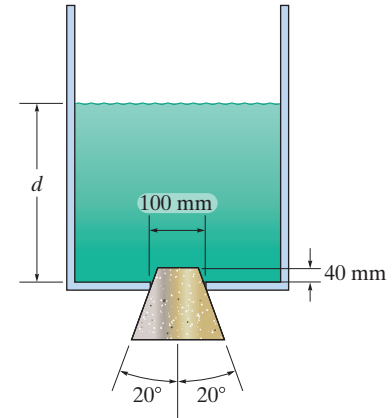
Ans:
 $d = 1.53 \text{ m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-115.

The stopper in the shape of a frustum is used to plug the 100-mm-diameter hole in the tank that contains amyl acetate. Determine the vertical force this liquid exerts on the stopper. Take $d = 0.6 \text{ m}$ and $\rho_{aa} = 863 \text{ kg/m}^3$. *Hint:* The

volume of a cone is $V = \frac{1}{3} \pi r^2 h$.



SOLUTION

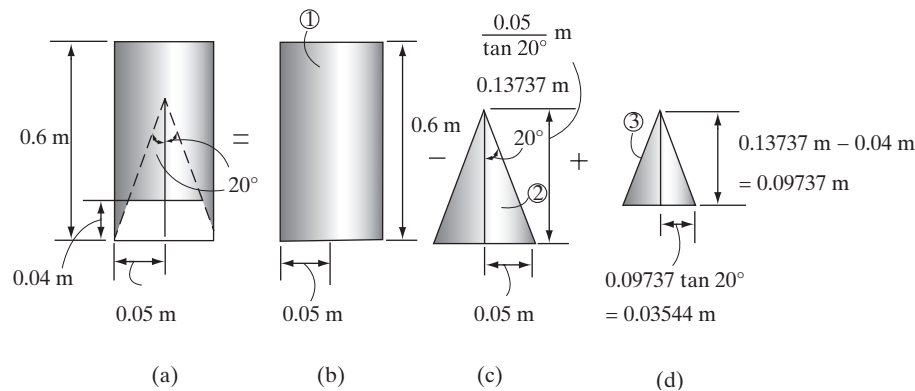
The vertical downward force on the frustum stopper is due to the weight of the liquid contained in the block shown shaded in Fig. *a*. This block can be subdivided into parts (1), (2), and (3), shown in Figs. *b*, *c*, and *d*, respectively. Part (2) is a hole and should be considered as a negative part. Thus, the volume of the shaded block in Fig. *a* is

$$\begin{aligned} V &= V_1 - V_2 + V_3 \\ &= \pi(0.05 \text{ m})^2(0.6 \text{ m}) - \frac{1}{3}\pi(0.05 \text{ m})^2(0.13737 \text{ m}) + \frac{1}{3}\pi(0.03544 \text{ m})^2(0.09737 \text{ m}) \\ &= 4.4808(10^{-3}) \text{ m}^3 \end{aligned}$$

Then,

$$\begin{aligned} F &= \rho_w g V \\ &= (863 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[4.4808(10^{-3}) \text{ m}^3] \\ &= 37.93 \text{ N} = 37.9 \text{ N} \end{aligned}$$

Ans.

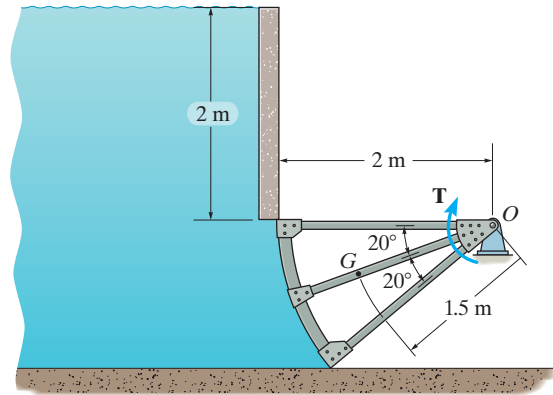


Ans:
 $F = 37.9 \text{ N}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-116.**

The Tainter gate for a water channel is 1.5 m wide and in the closed position, as shown. Determine the magnitude of the resultant force of the water acting on the gate. Also, what is the smallest torque T that must be applied to open the gate if its weight is 30 kN and its center of gravity is at G ?



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the gate. Referring to Fig. *a*,

$$w_1 = \rho_w g h_1 b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m})(1.5 \text{ m}) = 29.43(10^3) \text{ N/m}$$

$$w_2 = \rho_w g h_2 b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m} + 2 \text{ m} \sin 40^\circ)(1.5 \text{ m}) = 48.347(10^3) \text{ N/m}$$

Then

$$(F_h)_1 = [29.43(10^3) \text{ N/m}](2 \sin 40^\circ \text{ m}) = 37.834(10^3) \text{ N} = 37.834 \text{ kN}$$

$$(F_h)_2 = \frac{1}{2}[(48.347 - 29.43)(10^3) \text{ N/m}](2 \sin 40^\circ \text{ m}) = 12.160(10^3) \text{ N} = 12.160 \text{ kN}$$

$$F_h = (F_h)_1 + (F_h)_2 = 37.834(10^3) \text{ N} + 12.160(10^3) \text{ N} = 49.994(10^3) \text{ N} = 49.994 \text{ kN}$$

Also,

$$\tilde{y}_1 = \frac{1}{2}(2 \text{ m} \sin 40^\circ) = 0.6428 \text{ m} \quad \text{and} \quad \tilde{y}_2 = \frac{2}{3}(2 \text{ m} \sin 40^\circ) = 0.8571 \text{ m}$$

The vertical component of the resultant force is equal to the weight of the imaginary column of water above the gate (shown shaded in Fig. *a*), but acts upward. The volume of this column of water is

$$\begin{aligned} V &= \left[(2 \text{ m} - 2 \text{ m} \cos 40^\circ)(2 \text{ m}) + \frac{1}{2}(2 \text{ m})^2 \left(\frac{40^\circ}{180^\circ} \pi \text{ rad} \right) - \frac{1}{2}(2 \text{ m} \cos 40^\circ)(2 \text{ m} \sin 40^\circ) \right] (1.5 \text{ m}) \\ &= 2.0209 \text{ m}^3 \end{aligned}$$

$$F_v = \rho_w g V = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.0209 \text{ m}^3) = 19.825(10^3) \text{ N} = 19.825 \text{ kN}$$

Referring to Figs. *b* and *c*,

$$\tilde{r} = \frac{2}{3} \left(\frac{2 \text{ m} \sin 20^\circ}{\frac{20^\circ}{180^\circ} \pi} \right) = 1.3064 \text{ m} \quad \tilde{x}_2 = 1.3064 \text{ m} \cos 20^\circ = 1.2276 \text{ m}$$

$$\tilde{x}_1 = 2 \text{ m} \cos 40^\circ + \left(\frac{2 \text{ m} - 2 \text{ m} \cos 40^\circ}{2} \right) = 1.7660 \text{ m}$$

$$\tilde{x}_3 = \frac{2}{3}(2 \text{ m} \cos 40^\circ) = 1.0214 \text{ m}$$

Thus, F_v acts at

$$\begin{aligned} \bar{x} &= \frac{(1.7660 \text{ m})(2 \text{ m} - 2 \text{ m} \cos 40^\circ)(2 \text{ m}) + (1.2276 \text{ m}) \left[\frac{1}{2}(2 \text{ m})^2 \left(\frac{40^\circ}{180^\circ} \pi \text{ rad} \right) \right] - (1.0214 \text{ m}) \left[\frac{1}{2}(2 \text{ m} \cos 40^\circ)(2 \text{ m} \sin 40^\circ) \right]}{(2 \text{ m} - 2 \text{ m} \cos 40^\circ)(2 \text{ m}) + \frac{1}{2}(2 \text{ m})^2 \left(\frac{40^\circ}{180^\circ} \pi \text{ rad} \right) - \frac{1}{2}(2 \text{ m} \cos 40^\circ)(2 \text{ m} \sin 40^\circ)} \\ &= 1.7524 \text{ m} \end{aligned}$$

2-116. Continued

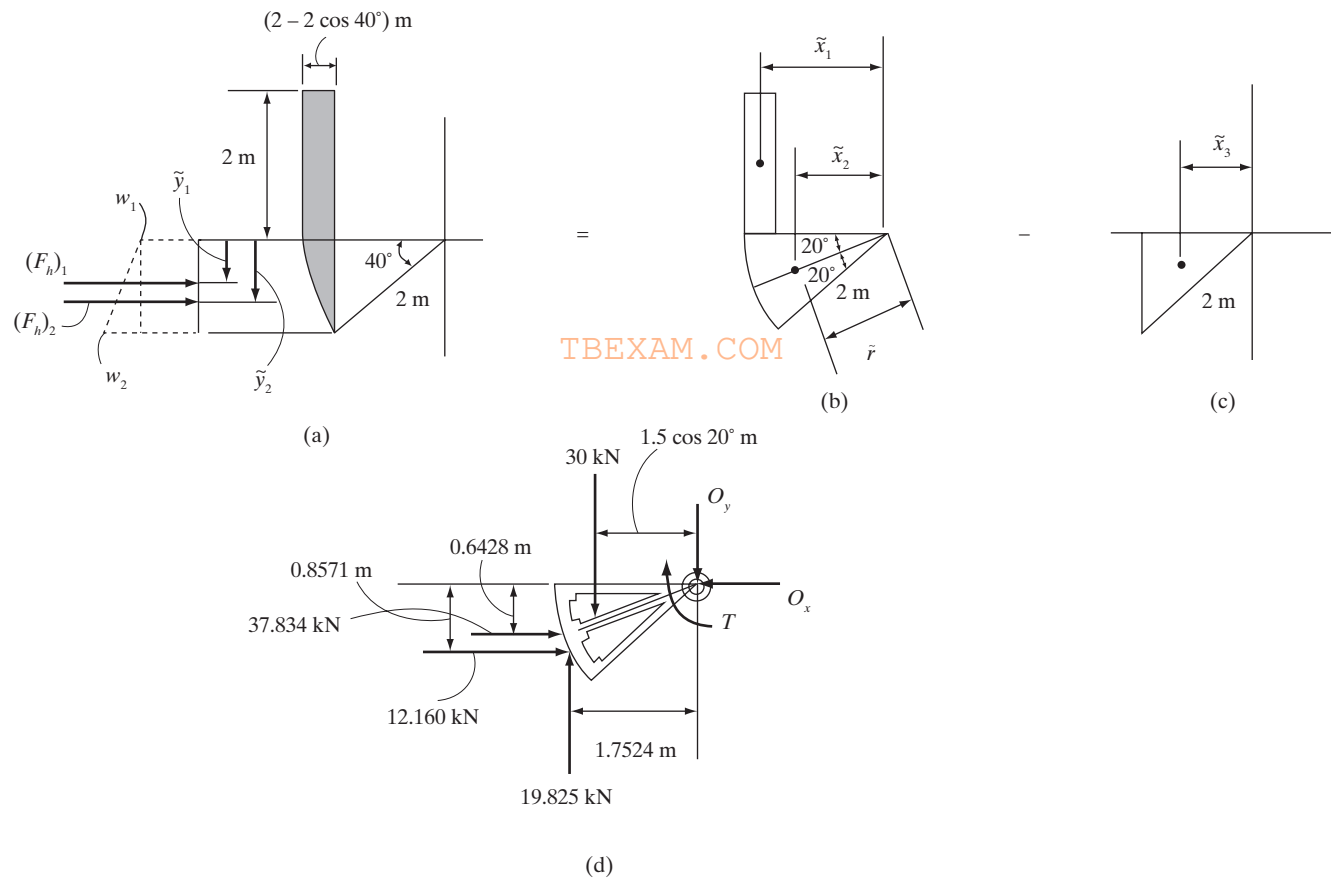
The magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(49.994 \text{ kN})^2 + (19.825 \text{ kN})^2} = 53.78 \text{ kN} = 53.8 \text{ kN} \quad \text{Ans.}$$

Referring to the FBD of the gate shown in Fig *d*,

$$\begin{aligned} \zeta + \Sigma M_O = 0; & \quad (30 \text{ kN})(1.5 \cos 20^\circ \text{ m}) + (37.834 \text{ kN})(0.6428 \text{ m}) + (12.160 \text{ kN})(0.8571 \text{ m}) \\ & \quad - (19.825 \text{ kN})(1.7524 \text{ m}) - T = 0 \\ & \quad T = 42.29 \text{ kN} \cdot \text{m} = 42.3 \text{ kN} \cdot \text{m} \quad \text{Ans.} \end{aligned}$$

Note that the resultant force of the water acting on the gate must act normal to its circular surface, and therefore it will pass through the pin at *O*. Therefore, it produces no moment about the pin.

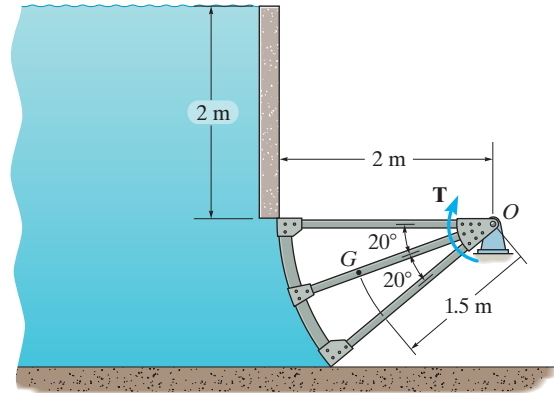


$$\begin{aligned} \text{Ans:} \\ F_R &= 53.8 \text{ kN} \\ T &= 42.3 \text{ kN} \cdot \text{m} \end{aligned}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-117.

Solve the first part of Prob. 2-116 by the integration method using polar coordinates.



SOLUTION

Referring to Fig. a , $h = (2 + 2 \sin \theta) \text{ m}$. Thus, the pressure acting on the gate as a function of θ is

$$p = \rho_w g h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 + 2 \sin \theta) \text{ m} = [19\,620(1 + \sin \theta)] \text{ N/m}^2$$

This pressure is acting on the element of area $dA = b ds = 1.5 ds = 1.5(2 d\theta) = 3 d\theta$.

Thus,

$$dF = p dA = 19\,620(1 + \sin \theta)(3 d\theta) = 58.86(10^3)(1 + \sin \theta) d\theta$$

The horizontal and vertical components of $d\mathbf{F}$ are

$$\begin{aligned} (dF)_h &= 58.86(10^3)(1 + \sin \theta) \cos \theta d\theta \\ &= 58.86(10^3)(\cos \theta + \sin \theta \cos \theta) d\theta \end{aligned}$$

$$\begin{aligned} (dF)_v &= 58.86(10^3)(1 + \sin \theta) \sin \theta d\theta \\ &= 58.86(10^3)(\sin \theta + \sin^2 \theta) d\theta \end{aligned}$$

Since $\sin 2\theta = 2 \sin \theta \cos \theta$ and $\cos 2\theta = 1 - 2 \sin^2 \theta$, then

$$(dF)_h = 58.86(10^3) \left(\cos \theta + \frac{1}{2} \sin 2\theta \right) d\theta$$

$$(dF)_v = 58.86(10^3) \left(\sin \theta + \frac{1}{2} - \frac{1}{2} \cos 2\theta \right) d\theta$$

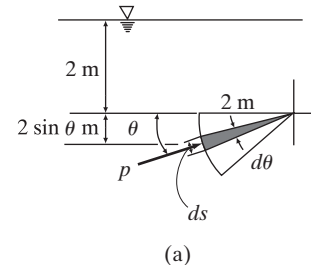
The horizontal and vertical components of the resultant force are

$$\begin{aligned} F_h &= \int (dF)_h = 58.86(10^3) \int_0^{\frac{2\pi}{9}} \left(\cos \theta + \frac{1}{2} \sin 2\theta \right) d\theta \\ &= 58.86(10^3) \left[\sin \theta - \frac{1}{4} \cos 2\theta \right] \bigg|_0^{\frac{2\pi}{9}} \\ &= 49.994(10^3) \text{ N} = 49.994 \text{ kN} \end{aligned}$$

$$\begin{aligned} F_v &= \int (dF)_v = 58.86(10^3) \int_0^{\frac{2\pi}{9}} \left(\sin \theta + \frac{1}{2} - \frac{1}{2} \cos 2\theta \right) d\theta \\ &= 58.86(10^3) \left(-\cos \theta + \frac{1}{2} \theta - \frac{1}{4} \sin 2\theta \right) \bigg|_0^{\frac{2\pi}{9}} \\ &= 19.825(10^3) \text{ N} = 19.825 \text{ kN} \end{aligned}$$

Thus, the magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(49.994 \text{ kN})^2 + (19.825 \text{ kN})^2} = 53.78 \text{ kN} = 53.8 \text{ kN} \quad \text{Ans.}$$

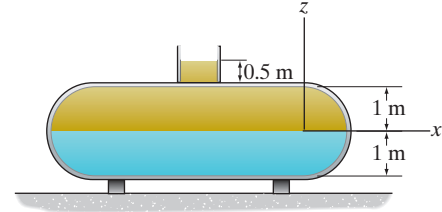


Ans:
 $F_R = 53.8 \text{ kN}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-118.

The cylindrical tank with hemispherical ends is filled with gasoline and water to the levels shown. Determine the horizontal and vertical components of the resultant force on its hemispherical end. Take $\rho_g = 726 \text{ kg/m}^3$.



SOLUTION

The vertical component of the resultant force is equal to the total weight of the gasoline and water contained in the quarter sphere. Here,

$$(F_v)_g = \rho_g g V_g = [(726 \text{ kg/m}^3)(9.81 \text{ m/s}^2)] \left[\frac{1}{4} \left(\frac{4}{3} \right) (\pi) (1 \text{ m})^3 \right] = 2.374(10^3) \pi \text{ N}$$

$$(F_v)_w = \rho_w g V_w = [(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)] \left[\frac{1}{4} \left(\frac{4}{3} \right) (\pi) (1 \text{ m})^3 \right] = 3.270(10^3) \pi \text{ N}$$

Then

$$\begin{aligned} F_v &= (F_v)_g + (F_v)_w = 2.374(10^3) \pi \text{ N} + 3.270(10^3) \pi \text{ N} \\ &= 5.644(10^3) \pi \text{ N} = 17.7 \text{ kN} \end{aligned}$$

Ans.

The equation of the pressure as a function of z for gasoline, Fig. *a*, is

$$p_g = \rho_g g h_g = (726 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.5 \text{ m} - z) = 7.122(10^3)(1.5 - z)$$

When $h_g = 1.5 \text{ m}$ ($z = 0$), $p_g|_{z=0} = 7.122(10^3)(1.5 - 0) = 10.683(10^3) \text{ N/m}^2$.

Then for water realizing that $h_w = -z$,

$$\begin{aligned} p_w &= p_g|_{z=0} + \rho_w g h_w \\ &= 10.683(10^3) \text{ N/m}^2 + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(-z) \\ &= [10.683(10^3) - 9.81(10^3)z] \text{ N/m}^2 \end{aligned}$$

Here, the differential force dF acting on the differential element of area $dA = 2ydz = 2\sqrt{1 - z^2}dz$ is $dF = p dA = p[2\sqrt{1 - z^2}dz]$, Fig. *b*. For gasoline,

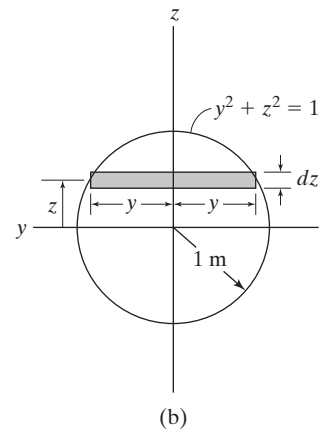
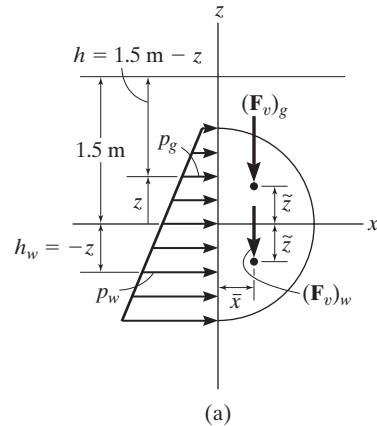
$$\begin{aligned} dF_g &= 7.122(10^3)(1.5 - z)[2\sqrt{1 - z^2}dz] \\ &= 14.244(10^3)[1.5\sqrt{1 - z^2} - z\sqrt{1 - z^2}]dz \end{aligned}$$

Integrating,

$$\begin{aligned} (F_g)_h &= 14.244(10^3) \int_0^1 [1.5\sqrt{1 - z^2} - z\sqrt{1 - z^2}] dz \\ &= 14.244(10^3) \left\{ \frac{1.5}{2} [z\sqrt{1 - z^2} + \sin^{-1}z] - \left[-\frac{1}{3}\sqrt{(1 - z^2)^3} \right] \right\} \bigg|_0^1 \\ &= 12.033(10^3) \text{ N} \end{aligned}$$

For water,

$$\begin{aligned} dF_w &= (10.683(10^3) - 9.81(10^3)z)[2\sqrt{1 - z^2}dz] \\ &= 19.62(10^3)[1.089\sqrt{1 - z^2} - z\sqrt{1 - z^2}]dz \end{aligned}$$



2-118. Continued

Integrating,

$$\begin{aligned}(F_w)_h &= 19.62(10^3) \int_{-1\text{ m}}^0 [1.089\sqrt{1-z^2} - z\sqrt{1-z^2}] dz \\ &= 19.62(10^3) \left\{ \frac{1.089}{2} [z\sqrt{1-z^2} + \sin^{-1}z] - \left[-\frac{1}{3}\sqrt{(1-z^2)^3} \right] \right\} \bigg|_{-1\text{ m}}^0 \\ &= 23.321(10^3) \text{ N}\end{aligned}$$

Then

$$\begin{aligned}F_h &= (F_g)_h + (F_w)_h = 12.033(10^3) \text{ N} + 23.321(10^3) \text{ N} \\ &= 35.354(10^3) \text{ N} = 35.4 \text{ kN}\end{aligned}$$

Ans.

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Ans:

$$F_v = 17.7 \text{ kN}$$

$$F_h = 35.4 \text{ kN}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-119.

The hollow spherical float controls the level of water within the tank. If the water is at the level shown, determine the horizontal and vertical components of the force acting on the supporting arm at the pin A , and the normal force on the smooth support B . Neglect the weight of the float.

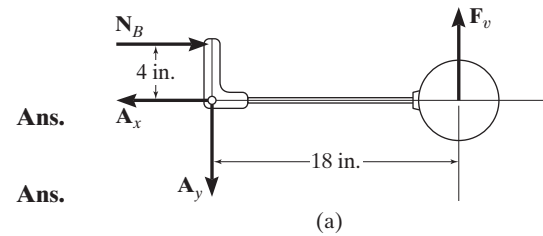
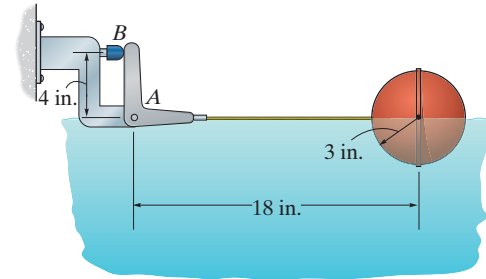
SOLUTION

The vertical buoyant force acting on the spherical float is

$$F_v = \gamma_w V_{\text{sub}} = (62.4 \text{ lb/ft}^3) \left\{ \frac{1}{2} \left[\frac{4}{3} \pi \left(\frac{3}{12} \text{ ft} \right)^3 \right] \right\} = 0.65\pi \text{ lb}$$

Consider the equilibrium of the FBD of the floating system shown in Fig. a .

$$\begin{aligned} \curvearrowright + \Sigma M_A &= 0; & (0.65\pi \text{ lb})(18 \text{ in.}) - N_B(4 \text{ in.}) &= 0 \\ & & N_B &= 2.925\pi \text{ lb} = 9.19 \text{ lb} \\ \rightarrow \Sigma F_x &= 0; & 2.925\pi \text{ lb} - A_x &= 0 \\ & & A_x &= 2.925\pi \text{ lb} = 9.19 \text{ lb} \\ + \uparrow \Sigma F_y &= 0; & 0.65\pi \text{ lb} - A_y &= 0 \\ & & A_y &= 0.65\pi \text{ lb} = 2.04 \text{ lb} \end{aligned}$$



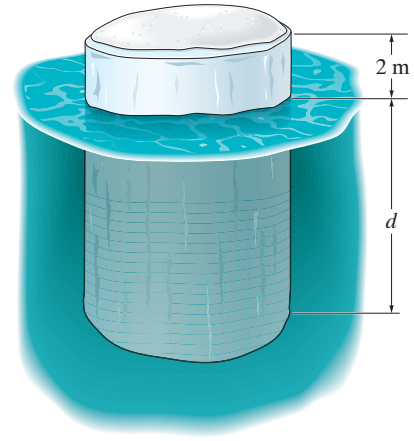
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Ans:
 $N_B = 9.19 \text{ lb}$
 $A_x = 9.19 \text{ lb}$
 $A_y = 2.04 \text{ lb}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-120.**

Consider an iceberg to be in the form of a cylinder of arbitrary diameter and floating in the ocean as shown. If the cylinder extends 2 m above the ocean's surface, determine the depth of the cylinder below the surface. The density of ocean water is $\rho_w = 1024 \text{ kg/m}^3$, and the density of the ice is $\rho_i = 935 \text{ kg/m}^3$.



SOLUTION

The weight of the iceberg is

$$W = \rho_i g V_i = (935 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[\pi r^2(2 + d)]$$

The buoyant force is

$$F_b = \rho_w g V_{\text{sub}} = (1024 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(\pi r^2 d)$$

Referring to the FBD of the iceberg, Fig. *a*, equilibrium requires,

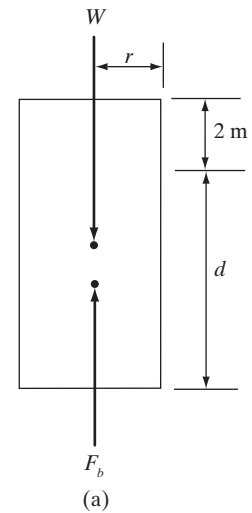
$$+\uparrow \Sigma F_y = 0; \quad F_b - W = 0$$

$$(1024 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(\pi r^2 d) - (935 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[\pi r^2(2 + d)] = 0$$

$$1024 d = 935(2 + d)$$

$$d = 21.01 \text{ m} = 21.0 \text{ m}$$

Ans.



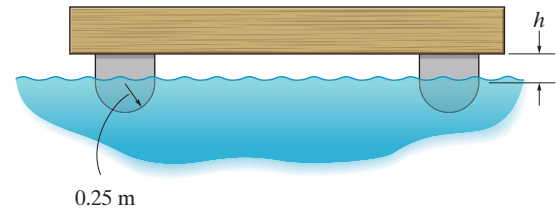
Ans:

$$d = 21.0 \text{ m}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–121.

The raft consists of a uniform platform having a mass of 2 Mg and four floats, each having a mass of 120 kg and a length of 4 m. Determine the height h at which the platform floats from the water surface. Take $\rho_w = 1 \text{ Mg/m}^3$.



SOLUTION

Each float must support a weight of

$$W = \left[\frac{1}{4}(2000 \text{ kg}) + 120 \text{ kg} \right] 9.81 \text{ m/s}^2 = 6082.2 \text{ N}$$

For equilibrium, the buoyant force on each float is required to be

$$+\uparrow \Sigma F_y = 0; \quad F_b - 6082.2 \text{ N} = 0 \quad F_b = 6082.2 \text{ N}$$

Therefore, the volume of water that must be displaced to generate this force is

$$F_b = \gamma V; \quad 6082.2 \text{ N} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)V$$

$$V = 0.620 \text{ m}^3$$

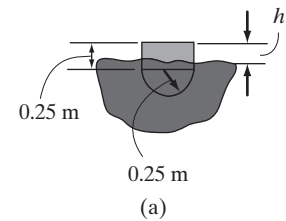
Since the semicircular segment of a float has a volume of $\frac{1}{2}(\pi)(0.25 \text{ m})^2(4 \text{ m}) = 0.3927 \text{ m}^3 < 0.620 \text{ m}^3$, then it must be fully submerged to develop F_b . As shown in Fig. *a*, we require

$$0.620 \text{ m}^3 = \frac{1}{2}(\pi)(0.25 \text{ m})^2(4 \text{ m}) + (0.25 \text{ m} - h)(0.5 \text{ m})(4 \text{ m})$$

Thus,

$$h = 0.136 \text{ m} = 136 \text{ mm} < 250 \text{ mm}$$

Ans.

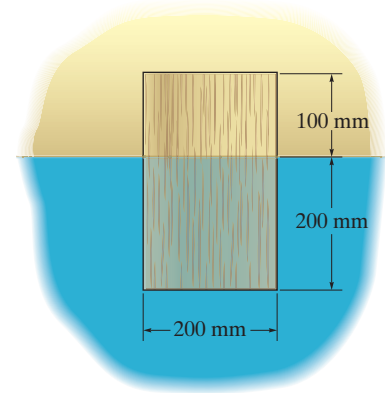


Ans:
 $h = 136 \text{ mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-122.

The cylinder floats in the water and oil to the level shown. Determine the weight of the cylinder. $\rho_o = 910 \text{ kg/m}^3$.



SOLUTION

The buoyant forces on the cylinder due to the oil and water are

$$(F_b)_o = \rho_o g (V_{\text{sub}})_o = (910 \text{ kg/m}^3)(9.81 \text{ m/s}^2) [\pi (0.1 \text{ m})^2 (0.1 \text{ m})] = 8.9271 \pi \text{ N}$$

$$(F_b)_w = \rho_w g (V_{\text{sub}})_w = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) [\pi (0.1 \text{ m})^2 (0.2 \text{ m})] = 19.62 \pi \text{ N}$$

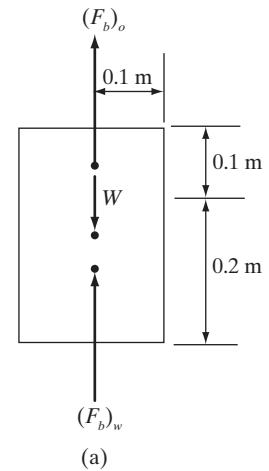
Referring to the FBD of the cylinder, Fig. *a*, equilibrium requires

$$+\uparrow \Sigma F_y = 0; \quad 8.9271 \pi \text{ N} + 19.62 \pi \text{ N} - W = 0$$

$$W = 89.68 \text{ N} = 89.7 \text{ N}$$

Ans.

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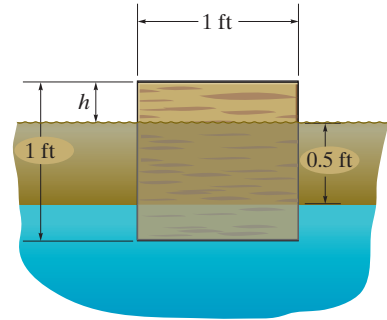


Ans:
 $W = 89.7 \text{ N}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–123.

The wood block has a specific weight of 45 lb/ft^3 . Determine the depth h at which it floats in the oil–water system. The block is 1 ft wide. Take $\rho_o = 1.75 \text{ slug/ft}^3$.



SOLUTION

The weight of the block is

$$W = \gamma_b V_b = (45 \text{ lb/ft}^3)(1 \text{ ft})^3 = 45 \text{ lb}$$

Assume that the block floats in both oil and water, Fig. *a*. Then, the volume of the water and oil being displaced is

$$(V_o)_{\text{Disp}} = 0.5 \text{ ft}(1 \text{ ft})(1 \text{ ft}) = 0.5 \text{ ft}^3$$

$$(V_w)_{\text{Disp}} = (0.5 \text{ ft} - h)(1 \text{ ft})(1 \text{ ft}) = (0.5 - h) \text{ ft}^3$$

Thus, the buoyant forces on the block due to the oil and water are

$$(F_b)_o = \gamma_o (V_o)_{\text{Disp}} = (1.75 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(0.5 \text{ ft}^3) = 28.175 \text{ lb}$$

$$(F_b)_w = \gamma_w (V_w)_{\text{Disp}} = (62.4 \text{ lb/ft}^3)(0.5 - h) \text{ ft}^3 = (31.2 - 62.4h) \text{ lb}$$

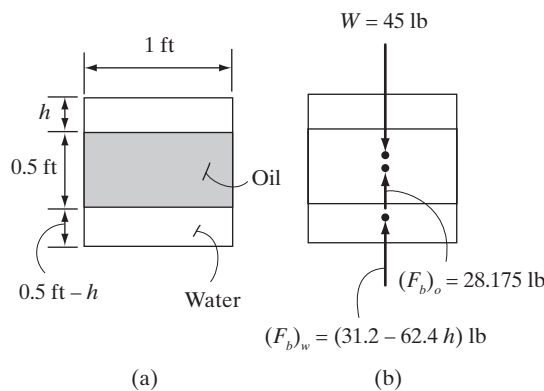
Considering the free-body diagram in Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \quad (31.2 - 62.4h) \text{ lb} + 28.175 \text{ lb} - 45 \text{ lb} = 0$$

$$h = 0.2304 \text{ ft} = 0.230 \text{ ft}$$

Ans.

Since $h < 0.5 \text{ ft}$, the assumption was correct and the result is valid.

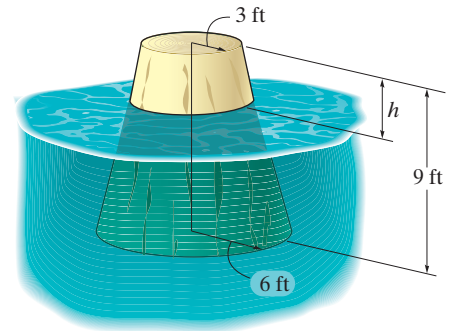


Ans:
 $h = 0.230 \text{ ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-124.**

Determine the height at which the oak block will float above the water surface. The specific weight of the wood is $\gamma_o = 48 \text{ lb/ft}^3$.



SOLUTION

Referring to the geometry shown in Fig. *a*,

$$\frac{y}{9+y} = \frac{3}{6}; \quad y = 9 \text{ ft}$$

And

$$\frac{r}{3} = \frac{h+9}{9}; \quad r = \frac{1}{3}(h+9) \text{ ft}$$

Thus, the volume of the oak block is

$$V_O = \frac{1}{3}\pi(6 \text{ ft})^2(18 \text{ ft}) - \frac{1}{3}\pi(3 \text{ ft})^2(9 \text{ ft}) = 189\pi \text{ ft}^3$$

And the submerged volume of the wooden block is

$$V_{\text{sub}} = \frac{1}{3}\pi(6 \text{ ft})^2(18 \text{ ft}) - \frac{1}{3}\pi\left[\frac{1}{3}(h+9)\right]^2(h+9) = \frac{\pi}{27}[5832 - (h+9)^3] \text{ ft}^3$$

Then the weight of the wooden block and the buoyant force are

$$W_O = \gamma_O V_O = (48 \text{ lb/ft}^3)(189\pi \text{ ft}^3) = 9072\pi \text{ lb}$$

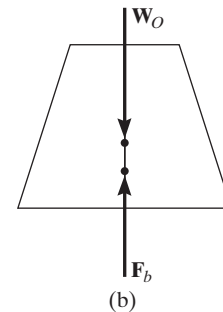
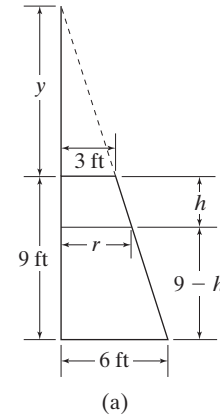
$$F_b = \gamma_w V_{\text{sub}} = (62.4 \text{ lb/ft}^3) \left\{ \frac{\pi}{27}[5832 - (h+9)^3] \right\} = 2.3111\pi[5832 - (h+9)^3] \text{ lb}$$

Referring to the FBD of the oak block, Fig. *b*, equilibrium requires

$$+\uparrow \Sigma F_y = 0; \quad 2.3111\pi[5832 - (h+9)^3] \text{ lb} - 9072\pi \text{ lb} = 0$$

$$h = 3.4000 \text{ ft} = 3.40 \text{ ft}$$

Ans.

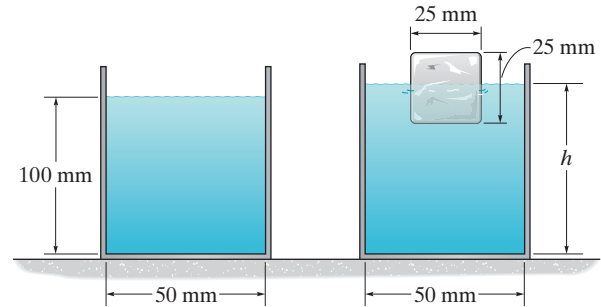


Ans:
 $h = 3.40 \text{ ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-125.

A glass having a diameter of 50 mm is filled with water to the level shown. If an ice cube with 25-mm sides is placed into the glass, determine the new height h of the water surface. Take $\rho_w = 1000 \text{ kg/m}^3$ and $\rho_{ice} = 920 \text{ kg/m}^3$. What will the water level h be when the ice cube completely melts?



SOLUTION

Since the ice floats, the buoyant force is equal to the weight of the ice cube, which is

$$F_b = W_i = \rho_i V_i g = (920 \text{ kg/m}^3)(0.025 \text{ m})^3(9.81 \text{ m/s}^2) = 0.1410 \text{ N}$$

This buoyant force is also equal to the weight of the water displaced by the submerged ice cube at a depth h_s .

$$F_b = \rho_w g V_s; \quad 0.1410 \text{ N} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(0.025 \text{ m})^2 h_s]$$

$$h_s = 0.023 \text{ m}$$

Referring to Fig. *a*,

$$V_1 = V_2 - V_3$$

$$[\pi(0.025 \text{ m})^2](0.1 \text{ m}) = [\pi(0.025 \text{ m})^2]h - (0.02 \text{ m})^2(0.023 \text{ m})$$

$$h = 0.1073 \text{ m} = 107 \text{ mm} \quad \text{Ans.}$$

The mass of the ice cube is

$$M_i = \rho_i V_i = (920 \text{ kg/m}^3)(0.025 \text{ m})^3 = 0.014375 \text{ kg}$$

Thus, the rise in water level due to the additional water of the melting ice cubes can be determined from

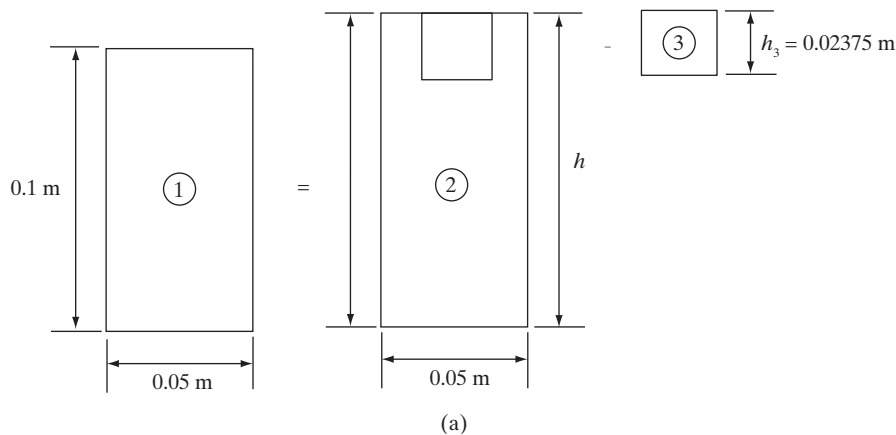
$$M_i = \rho_w V_w; \quad 0.014375 \text{ kg} = (1000 \text{ kg/m}^3)[\pi(0.025 \text{ m})^2 \Delta h]$$

$$\Delta h = 0.007321$$

Thus,

$$h' = 0.1 \text{ m} + 0.007321 \text{ m} = 107 \text{ mm} \quad \text{Ans.}$$

Note: The water level h remains unchanged as the cube melts.



Ans:
 $h = 107 \text{ mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-126.

Water in the cylindrical container is originally at a height of $h = 3 \text{ ft}$. If a 2-ft-diameter cylindrical disk made of material having a specific weight of 40 lb/ft^3 is placed in the water, determine the new level h of the water.

SOLUTION

The weight of the disk is

$$W = \gamma_d V_d = (40 \text{ lb/ft}^3)[\pi(1 \text{ ft})^2(1 \text{ ft})] = 40\pi \text{ lb}$$

The buoyant force on the disk is equal to the weight of the displaced water.

$$F_b = \gamma_w V_{\text{disp}} = (62.4 \text{ lb/ft}^3) V_{\text{disp}} = (62.4 V_{\text{disp}}) \text{ lb}$$

Consider the force equilibrium along vertical by referring to the FBD of the disk, Fig. *a*,

$$+\uparrow \sum F_y = 0; \quad (62.4 V_{\text{disp}}) \text{ lb} - 40\pi \text{ lb} = 0 \quad V_{\text{disp}} = 2.0138 \text{ ft}^3$$

The volume of water is

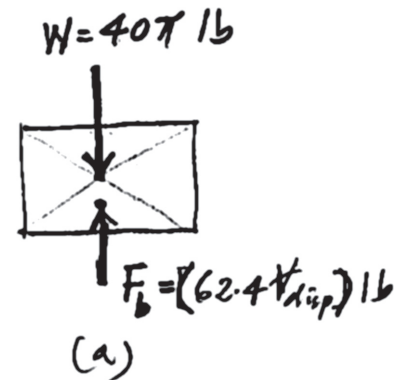
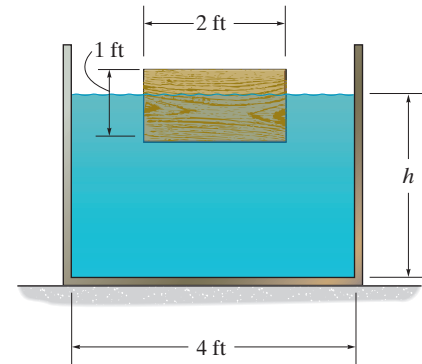
$$V_w = \pi(2 \text{ ft})^2(3 \text{ ft}) = 12\pi \text{ ft}^3$$

The level of the water in the container is at height h when the disk is floating on the water. Thus,

$$V_w = V' - V_{\text{disp}}; \quad 12\pi \text{ ft}^3 = \pi(2 \text{ ft})^2(h) - 2.0138 \text{ ft}^3$$

$$h = 3.1603 \text{ ft} = 3.16 \text{ ft}$$

Ans.



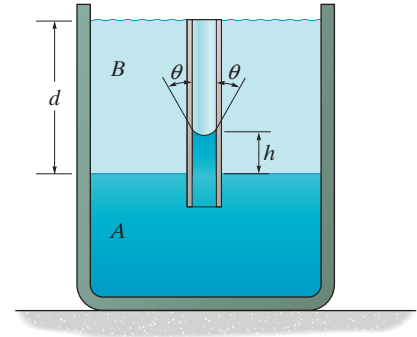
Ans:

$$h = 3.16 \text{ ft}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–127.

An open-ended tube having an inner radius r is placed in a wetting liquid A having a density ρ_A . The top of the tube is just below the surface of a surrounding liquid B , which has a density ρ_B , where $\rho_A > \rho_B$. If the surface tension σ causes liquid A to make a wetting angle θ with the tube wall as shown, determine the rise h of liquid A within the tube. Show that the result is independent of the depth d of liquid B .



SOLUTION

The volume of the column of liquid A in the rise is $V = \pi r^2 h$. Thus, its weight is

$$W = \gamma_A V = \rho_A g (\pi r^2 h) = \pi g \rho_A r^2 h$$

The force on the top and bottom of the column is $p_t A = \rho_B g (d - h) (\pi r^2)$ and $p_b A = \rho_B g (d) (\pi r^2)$. The difference in these forces is the buoyant force.

$$F_B = \gamma_B V = \rho_B g (\pi r^2 h) = \pi g \rho_B r^2 h$$

The force equilibrium along the vertical, Fig. *a*, requires

$$+\uparrow \Sigma F_y = 0; \quad \sigma(2\pi r) \cos \theta + \pi g \rho_B r^2 h - \pi g \rho_A r^2 h = 0$$

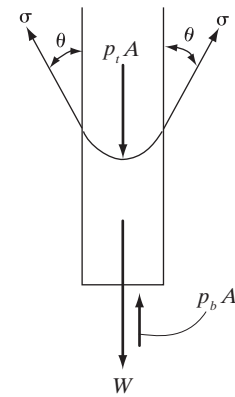
$$2\pi r \sigma \cos \theta + \pi g r^2 h (\rho_B - \rho_A) = 0$$

$$2\pi r \sigma \cos \theta = (\rho_A - \rho_B) \pi g r^2 h$$

$$h = \frac{2\sigma \cos \theta}{gr(\rho_A - \rho_B)}$$

Ans.

Note that the result is independent of d .



(a)

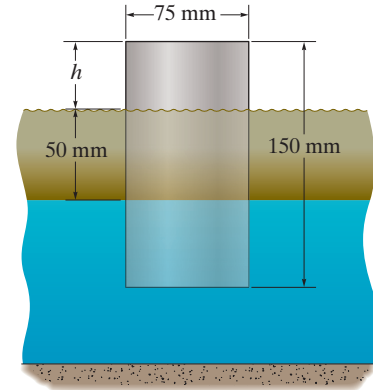
Ans:

$$h = \frac{2\sigma \cos \theta}{gr(\rho_A - \rho_B)}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-128.**

The cylinder has a diameter of 75 mm and a mass of 600 g. If it is placed in the tank, which contains oil and water, determine the height h above the surface of the oil at which it will float if maintained in the vertical position. Take $\rho_o = 980 \text{ kg/m}^3$.



SOLUTION

Since the cylinder floats, the buoyant force is equal to the weight of the cylinder.

$$F_b = (0.6 \text{ kg})(9.81 \text{ m/s}^2) = 5.886 \text{ N}$$

Assuming that the cylinder is submerged below the oil layer, then, the buoyant force produced by the oil layer is

$$\begin{aligned} (F_b)_o &= \rho_o g (V_s)_o = (980 \text{ kg/m}^3)(9.81 \text{ m/s}^2) [\pi (0.0375 \text{ m})^2 (0.05 \text{ m})] \\ &= 2.124 \text{ N} < F_b \quad \textbf{(O.K!)} \end{aligned}$$

The buoyant force produced by the water layer is

$$\begin{aligned} (F_b)_w &= \rho_w g (V_s)_w = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) [\pi (0.0375 \text{ m})^2 (0.15 \text{ m} - 0.05 \text{ m} - h)] \\ &= 43.339 (0.1 - h) \text{ N} \end{aligned}$$

We require

$$\begin{aligned} F_b &= (F_b)_{oil} + (F_b)_w \\ 5.886 \text{ N} &= 2.124 \text{ N} + 43.339 (0.1 - h) \text{ N} \\ h &= 13.2 \text{ mm} \end{aligned}$$

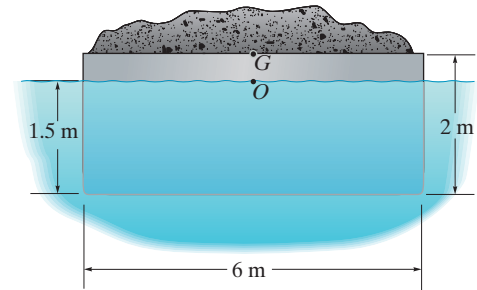
Ans.

Ans:
 $h = 13.2 \text{ mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-129.

When loaded with gravel, the barge floats in water at the depth shown. If its center of gravity is at G , determine whether the barge will restore itself when a wave causes it to roll slightly at 9° .



SOLUTION

When the barge tips 9° , the submerged portion is trapezoidal in shape, as shown in Fig. *a*. The new center of buoyancy, C'_b , is located at the centroid of this area. Then

$$\bar{x} = \frac{(0)(6)(1.0248) + (1)\left[\frac{1}{2}(6)(0.9503)\right]}{(6)(1.0248) + \frac{1}{2}(6)(0.9503)} = 0.3168 \text{ m}$$

$$\bar{y} = \frac{\frac{1}{2}(1.0248)(6)(1.0248) + \left[1.0248 + \frac{1}{3}(0.9503)\right]\left[\frac{1}{2}(6)(0.9503)\right]}{(6)(1.0248) + \frac{1}{2}(6)(0.9503)} = 0.7751 \text{ m}$$

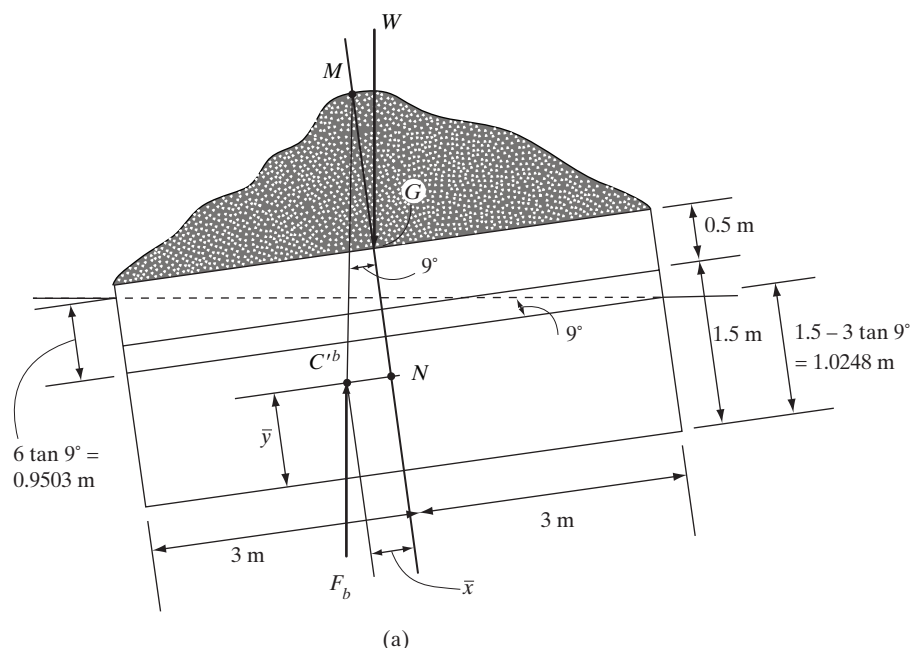
The intersection point M of the line of action of $\mathbf{F}_b$ and the centerline of the barge is the metacenter, Fig. *a*. From the geometry of triangle MNC'_b , we have

$$MN = \frac{\bar{x}}{\tan 9^\circ} = \frac{0.3168}{\tan 9^\circ} = 2 \text{ m}$$

Also,

$$GN = 2 - \bar{y} = 2 - 0.7751 = 1.2249 \text{ m}$$

Since $MN > GN$, point M is above G . Therefore, **the barge will restore itself.**

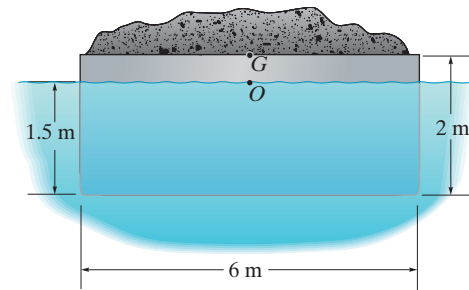


Ans:
The barge will restore itself.

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-130.

When loaded with gravel, the barge floats in water at the depth shown. If its center of gravity is at G , determine whether the barge will restore itself when a wave causes it to tip slightly.



SOLUTION

The barge is tilted counterclockwise slightly and the new center of buoyancy C'_b is located to the left of the old one. The metacenter M is at the intersection point of the center line of the barge and the line of action of F_b , Fig. *a*. The location of C'_b can be obtained by referring to Fig. *b*.

$$\bar{x} = \frac{(1 \text{ m}) \left[\frac{1}{2} (6 \text{ m}) (6 \tan \phi \text{ m}) \right]}{(1.5 \text{ m})(6 \text{ m})} = 2 \tan \phi \text{ m}$$

Then

$$\delta = \bar{x} \cos \phi = 2 \text{ m} \tan \phi \cos \phi = (2 \text{ m}) \left(\frac{\sin \phi}{\cos \phi} \right) (\cos \phi) = (2 \sin \phi) \text{ m}$$

Since ϕ is very small, $\sin \phi = \phi$, hence

$$\delta = 2\phi \text{ m} \quad (1)$$

From the geometry shown in Fig. *a*,

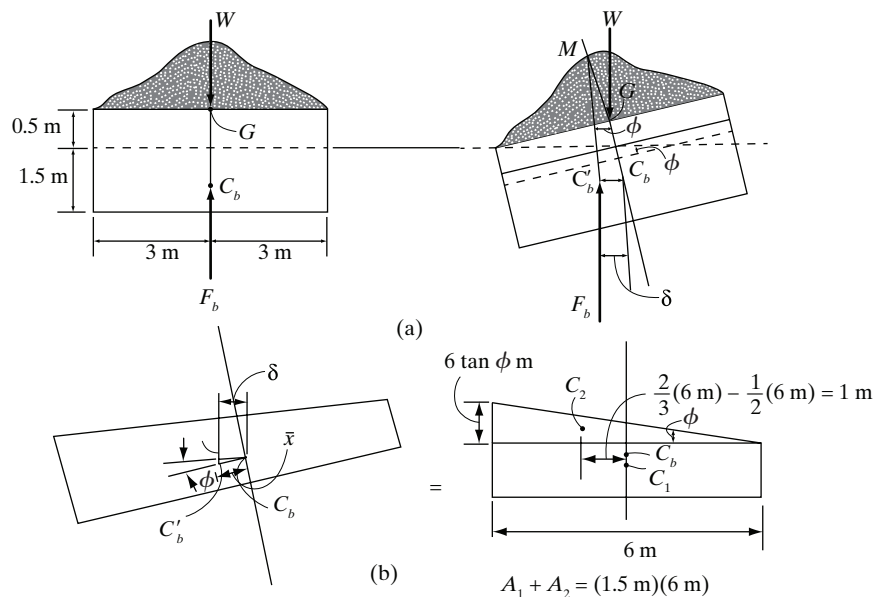
$$\delta = MC_b \sin \phi = MC_b \phi \quad (2)$$

Equating Eqs. (1) and (2),

$$2\phi = MC_b \phi$$

$$MC_b = 2 \text{ m}$$

Here, $GC_b = 2 \text{ m} - 0.75 \text{ m} = 1.25 \text{ m}$. Since $MC_b > GC_b$, the barge is in stable equilibrium. Thus, it will restore itself if tilted slightly. **Ans.**



Ans:
The barge will restore itself.

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-131.

The bucket of paint rests on the floor of the lift. Determine the maximum pressure developed in the paint if the lift is moving upward with (a) a constant velocity of 8 ft/s , and (b) an acceleration of 8 ft/s^2 . Take $\gamma_p = 78.6 \text{ lb/ft}^3$. The top of the bucket full of paint is open to the atmosphere.

SOLUTION

- a) When the lift is travelling upward with a constant velocity of 8 ft/s , $a_c = 0$. Thus,

$$p = \gamma_p h \left(1 + \frac{a_c}{g} \right) = (78.6 \text{ lb/ft}^3)(1.5 \text{ ft})(1 + 0)$$

$$= (117.9 \text{ lb/ft}^2) \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 = 0.819 \text{ psi}$$

Ans.

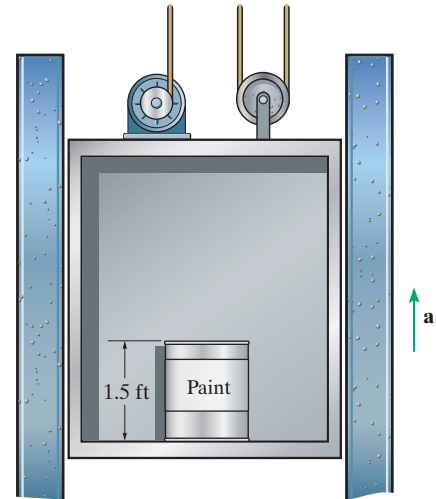
- b) When the lift is travelling upward with $a_c = 8 \text{ ft/s}^2$,

$$p = \gamma_p h \left(1 + \frac{a_c}{g} \right) = (78.6 \text{ lb/ft}^3)(1.5 \text{ ft}) \left[1 + \frac{8 \text{ ft/s}^2}{32.2 \text{ ft/s}^2} \right]$$

$$= (147.19 \text{ lb/ft}^2) \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2$$

$$= 1.02 \text{ psi}$$

Ans.



Ans:

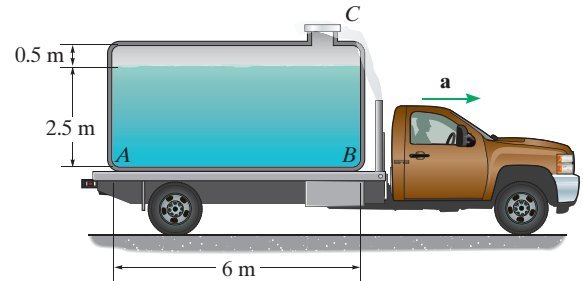
a) $p = 0.819 \text{ psi}$

b) $p = 1.02 \text{ psi}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-132.**

The truck carries a closed container filled with sewage to the height 2.5 m. If it has a constant acceleration of 3 m/s^2 , determine the pressure at the bottom corner *A*. Take $\rho_{\text{sew}} = 1080 \text{ kg/m}^3$. The cover *C* is open to the atmosphere.



SOLUTION

The angle of inclination of the sewerage surface in the accelerated tank can be determined from

$$\tan \theta = \frac{a_c}{g} = \frac{3 \text{ m/s}^2}{9.81 \text{ m/s}^2}; \quad \theta = 17.00^\circ > \tan^{-1} \left(\frac{0.5 \text{ m}}{3 \text{ m}} \right) = 9.46^\circ$$

Therefore, the sewerage surface in the accelerated tank is as shown in Fig. *a*. If the tank has a width of w , the volume of the sewerage can be determined using its level when the truck is at rest indicated by the dashed line in Fig. *a*.

$$V_{\text{sew}} = (6 \text{ m})(2.5 \text{ m})w = 15w \text{ m}^3$$

Since the volume of the sewerage remains the same, from the geometry of Fig. *a*,

$$V_{\text{sew}} = \left[(6 \text{ m})(b) + \frac{1}{2}(6 \text{ m} + a)(3 \text{ m} - b) \right] w = 15w$$

$$6b + 3a - ab = 12 \quad (1)$$

Also,

$$\tan 17.00^\circ = \frac{3 \text{ m} - b}{6 \text{ m} - a}$$

$$b = 0.3058a + 1.1651 \quad (2)$$

Substitute Eq. (2) into (1),

$$0.3058a^2 - 3.6697a + 5.0092 = 0$$

Solving for $a < 6 \text{ m}$,

$$a = 1.5706 \text{ m} \quad b = 1.6454 \text{ m}$$

***2-132. Continued**

To determine the pressure at A , the sewerage surface must be extended using an imaginary surface as shown in Fig. a . Here, $h_A = 3 \text{ m} + (1.5706 \text{ m}) \tan 1700^\circ = 3.4803 \text{ m}$. Thus,

$$\begin{aligned} p_A &= \rho_{\text{sew}} g h_A = (1080 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3.4803 \text{ m}) \\ &= 36.87(10^3) \text{ Pa} = 36.9 \text{ kPa} \end{aligned} \quad \textbf{Ans.}$$

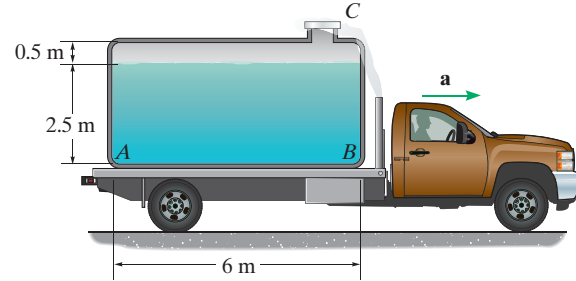
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Ans:
 $p_A = 36.9 \text{ kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-133.

The truck carries a closed container filled with sewage to the height 2.5 m. If it has a constant acceleration of 3 m/s^2 , determine the pressure at the bottom corner B . Take $\rho_{\text{sew}} = 1080 \text{ kg/m}^3$. The cover C is open to the atmosphere.



SOLUTION

The angle of inclination of the sewerage surface in the accelerated tank can be determined from

$$\tan \theta = \frac{a_c}{g} = \frac{3 \text{ m/s}^2}{9.81 \text{ m/s}^2}; \quad \theta = 17.00^\circ > \tan^{-1} \left(\frac{0.5 \text{ m}}{3 \text{ m}} \right) = 9.46^\circ$$

Therefore, the sewerage surface in the accelerated tank is as shown in Fig. a . If the tank has a width of w , the volume of the sewerage can be determined using its level when the truck is at rest indicated by the dashed line in Fig. a .

$$V_{\text{sew}} = (6 \text{ m})(2.5 \text{ m})w = 15w \text{ m}^3$$

Since the volume of the sewerage remains the same, from the geometry of Fig. a ,

$$V_{\text{sew}} = \left[(6 \text{ m})(b) + \frac{1}{2}(6 \text{ m} + a)(3 \text{ m} - b) \right] w = 15w$$

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$$6b + 3a - ab = 12$$

(1)

Also,

$$\tan 17.00^\circ = \frac{3 \text{ m} - b}{6 \text{ m} - a}$$

$$b = 0.3058a + 1.1651$$

(2)

Substitute Eq. (2) into (1),

$$0.3058a^2 - 3.6697a + 5.0092 = 0$$

Solving for $a < 6 \text{ m}$,

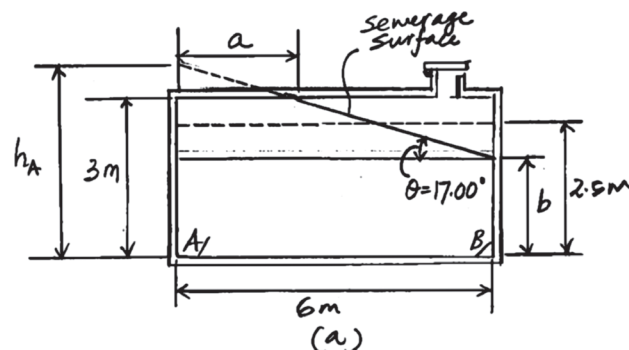
$$a = 1.5706 \text{ m} \quad b = 1.6454 \text{ m}$$

Here, $h_B = b = 1.6454 \text{ m}$. Then the pressure at B is given by

$$p_B = \rho_{\text{sew}} g h_B = (1080 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.6454 \text{ m})$$

$$= 17.43(10^3) \text{ Pa} = 17.4 \text{ kPa}$$

Ans.

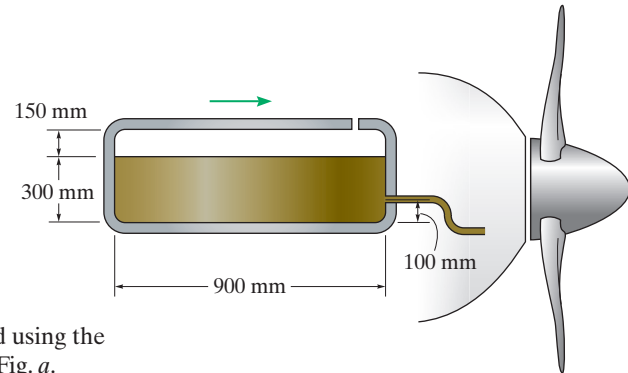


Ans:
 $p_B = 17.4 \text{ kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-134.

The fuel tank, supply line, and engine for an airplane are shown. If the gas tank is filled to the level shown, determine the largest constant acceleration a_c that the plane can have without causing the engine to be starved of fuel. The plane is accelerating to the right for this to happen. Suggest a safer location for attaching the fuel line.



SOLUTION

If the width of the tank is w , the volume of the fuel can be determined using the fuel level when the airplane is at rest indicated by the dashed line in Fig. *a*.

$$V_f = (0.9 \text{ m})(0.3 \text{ m})w = 0.27w$$

It is required that the fuel level is about to drop lower than the supply line. In this case, the fuel level is indicated by the solid line in Fig. *a*.

$$V_f = (0.9 \text{ m})(0.45 \text{ m})w - \frac{1}{2}(0.45 \text{ m} - 0.1 \text{ m})(0.9 \text{ m} - b)w = 0.27w$$

$$b = 0.1286 \text{ m}$$

Thus,

$$\tan \theta = \frac{0.45 \text{ m} - 0.1 \text{ m}}{0.9 \text{ m} - 0.1286 \text{ m}} = 0.4537$$

And so

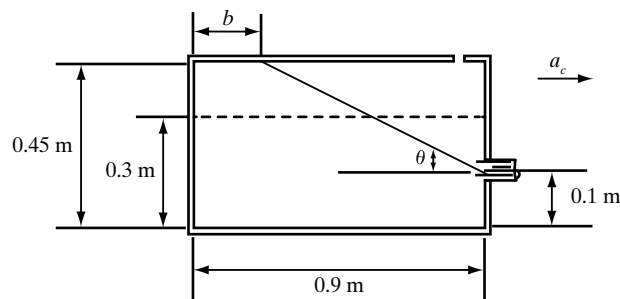
$$\tan \theta = \frac{a_c}{g}; \quad 0.4537 = \frac{a_c}{9.81 \text{ m/s}^2}$$

$$a_c = 4.45 \text{ m/s}^2$$

Ans.

The safer location for attaching the fuel line is at the bottom of the tank.

Ans.



Ans:

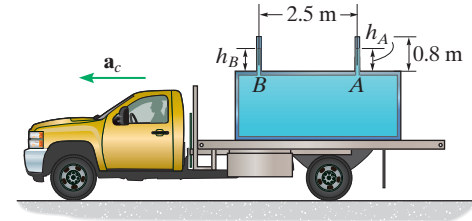
$$a_c = 4.45 \text{ m/s}^2$$

The safer location is at the bottom of the tank.

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-135.

Determine the level of water in vent tubes A and B if the container accelerates at $a_c = 1.2 \text{ m/s}^2$. When it is at rest, $h_A = h_B = 0.5 \text{ m}$. The cross section of tube B is twice that of tube A .



SOLUTION

The imaginary water surface in the accelerated tank is shown in Fig. a . The inclination angle of this surface is given by

$$\tan \theta = \frac{a_c}{g} = \frac{1.2 \text{ m/s}^2}{9.81 \text{ m/s}^2}; \quad \theta = 6.974^\circ$$

Since the volume of the water drops in tube B must be the same to that rises in tube A , $\Delta h_A = 2\Delta h_B$. Subsequently, from the similar triangle, Fig. a ,

$$\frac{x}{2.5 \text{ m} - x} = \frac{\Delta h_B}{\Delta h_A} = \frac{\Delta h_B}{2\Delta h_B}$$

$$2x = 2.5 - x$$

$$x = 0.8333 \text{ m}$$

From the geometry of the triangles,

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$$\Delta h_A = (2.5 \text{ m} - 0.8333 \text{ m}) \tan 6.974^\circ = 0.2039 \text{ m}$$

$$\Delta h_B = (0.8333 \text{ m}) \tan 6.974^\circ = 0.1019 \text{ m}$$

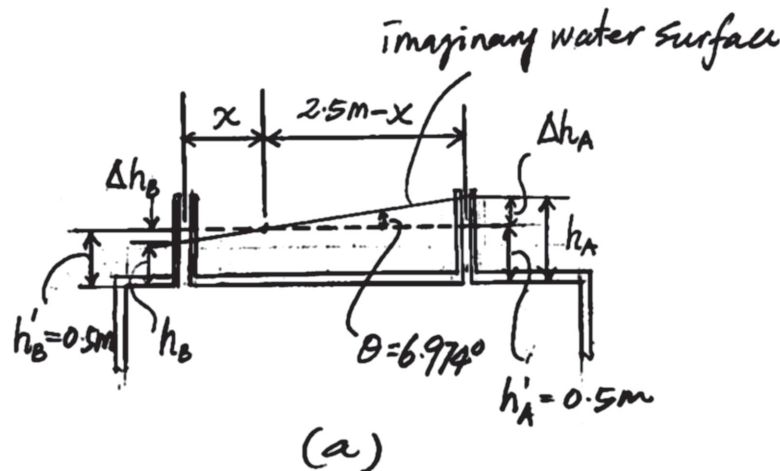
Since $\Delta h_A < 0.3 \text{ m}$, the water will not spill.

$$h_A = h'_A + \Delta h_A = 0.5 \text{ m} + 0.2039 \text{ m} = 0.7039 \text{ m}$$

Ans.

$$h_B = h'_B - \Delta h_B = 0.5 \text{ m} - 0.1019 \text{ m} = 0.3981 \text{ m}$$

Ans.



Ans:

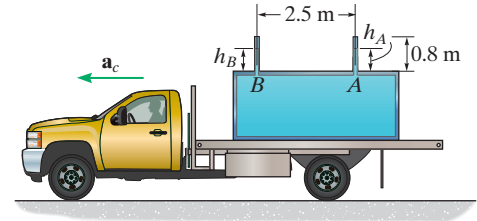
$$h_A = 0.7039 \text{ m}$$

$$h_B = 0.3981 \text{ m}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-136.**

Determine the constant acceleration of the container so that no water will spill from vent tube A. When it is at rest, $h_A = h_B = 0.5 \text{ m}$. The cross section of tube B is twice that of tube A.



SOLUTION

The imaginary water surface in the accelerated tank is shown in Fig. *a*. Since the volume of the water drops in tube B must be the same to that rises in tube A, $\Delta h_A = 2\Delta h_B$. Subsequently, from the similar triangle, Fig. *a*,

$$\frac{x}{2.5 \text{ m} - x} = \frac{\Delta h_B}{\Delta h_A} = \frac{\Delta h_B}{2\Delta h_B}$$

$$2x = 2.5 - x$$

$$x = 0.8333 \text{ m}$$

Since the water is required to be on the verge to spill out from tube A, $\Delta h_A = h_A - h'_A = 0.8 \text{ m} - 0.5 \text{ m} = 0.3 \text{ m}$. Thus, from the geometry of the triangle, the inclination angle of the water surface is given by

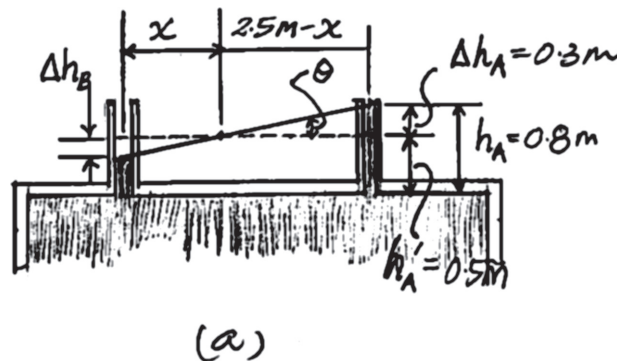
$$\theta = \tan^{-1} \left(\frac{\Delta h_A}{2.5 \text{ m} - x} \right) = \tan^{-1} \left(\frac{0.3 \text{ m}}{2.5 \text{ m} - 0.8333 \text{ m}} \right) = 10.20^\circ$$

The maximum allowable acceleration of the tank can be determined from

$$\tan \theta = \frac{a_c}{g}; \quad \tan 10.20^\circ = \frac{a_c}{9.81 \text{ m/s}^2}$$

$$a_c = 1.7658 \text{ m/s}^2 = 1.77 \text{ m/s}^2$$

Ans.



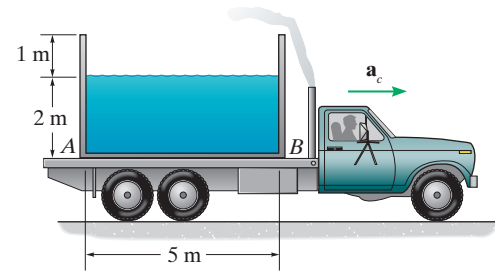
Ans:

$$a_c = 1.77 \text{ m/s}^2$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-137.

The truck carries an open container of water as shown. If it has a constant acceleration of 2 m/s^2 , determine the angle of inclination of the surface of the water and the pressure at the bottom corners A and B .



Ans.

SOLUTION

The free surface of the water in the accelerated tank is shown in Fig. *a*.

$$\tan \theta = \frac{a_c}{g} = \frac{2 \text{ m/s}^2}{9.81 \text{ m/s}^2}$$

$$\theta = 11.52^\circ = 11.5^\circ$$

From the geometry in Fig. *a*,

$$\Delta h_A = \Delta h_B = (2.5 \text{ m}) \tan 11.52^\circ$$

$$= 0.5097 \text{ m} < 1 \text{ m}$$

Thus,

$$p_A = \rho_w g h_A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m} + 0.5097 \text{ m})$$

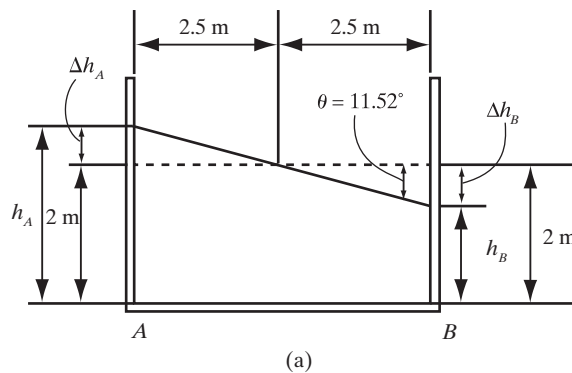
$$= 24.6 \text{ kPa}$$

Ans.

$$p_B = \rho_w g h_B = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m} - 0.5097 \text{ m})$$

$$= 14.6 \text{ kPa}$$

Ans.



Ans:

$$\theta = 11.5^\circ$$

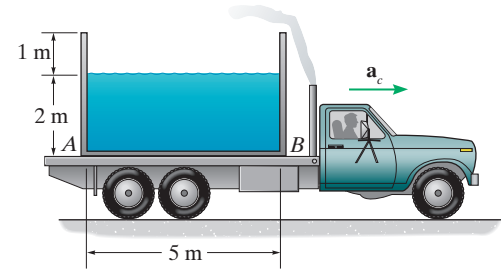
$$p_A = 24.6 \text{ kPa}$$

$$p_B = 14.6 \text{ kPa}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-138.

The truck carries an open container of water as shown. Determine the maximum constant acceleration it can have without causing the water to spill out of the container.



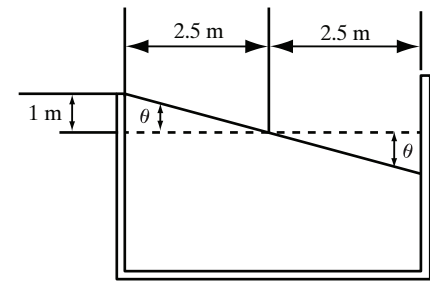
SOLUTION

The water is required to be on the verge to spill from the left side wall. The surface of the water under this condition is shown in Fig. *a*.

$$\tan \theta = \frac{1 \text{ m}}{2.5 \text{ m}} = \frac{a_c}{9.81 \text{ m/s}^2}$$

$$a_c = 3.92 \text{ m/s}^2$$

Ans.



(a)

Ans:

$$a_c = 3.92 \text{ m/s}^2$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-139.

The cart is allowed to roll freely down the inclined plane due to its weight. Show that the slope of the surface of the liquid, θ , during the motion is $\theta = \phi$.

SOLUTION

Referring to the free-body diagram of the container in Fig. *a*,

$$+\sum F_{x'} = ma_{x'}$$

$$W \sin \phi = \frac{W}{g} a_c$$

$$a_c = g \sin \phi$$

Referring to Fig. *b*,

$$a_x = -(g \sin \phi) \cos \phi$$

$$a_y = -(g \sin \phi) \sin \phi$$

We will now apply Newton's equations of motion, Fig. *c*.

$$+\sum F_x = ma_x; \quad -\left(p + \frac{\partial p}{\partial x} dx\right) dy dz + p dy dz = \frac{\gamma(dx dy dz)}{g} a_x$$

$$\frac{\partial p}{\partial x} = -\frac{\gamma}{g} a_x$$

In *y* direction,

$$+\uparrow \sum F_y = ma_y; \quad p dx dz - \left(p + \frac{\partial p}{\partial y} dy\right) dx dz - \gamma dx dy dz = \frac{\gamma dx dy dz}{g} a_y$$

$$\frac{\partial p}{\partial y} = -\gamma \left(1 + \frac{a_y}{g}\right)$$

At the surface, p is constant, so that $dp = 0 = \left(\frac{\partial p}{\partial x}\right) dx + \left(\frac{\partial p}{\partial y}\right) dy$

$$\frac{\gamma dx}{g} a_x = -\gamma \left(1 + \frac{a_y}{g}\right)$$

$$\frac{dy}{dx} = -\frac{a_x}{g + a_y} = \frac{g \sin \phi \cos \phi}{g - g \sin \phi \sin \phi} = \frac{\sin \phi \cos \phi}{\cos^2 \phi} = \frac{\sin \phi}{\cos \phi} = \tan \phi$$

Since at the surface,

$$\frac{dy}{dx} = \tan \theta$$

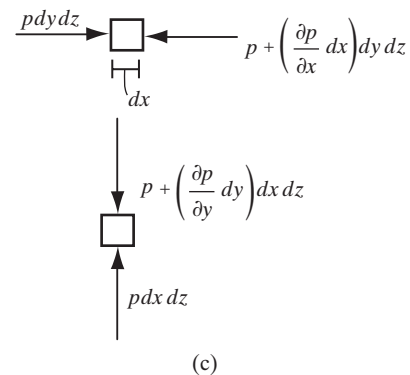
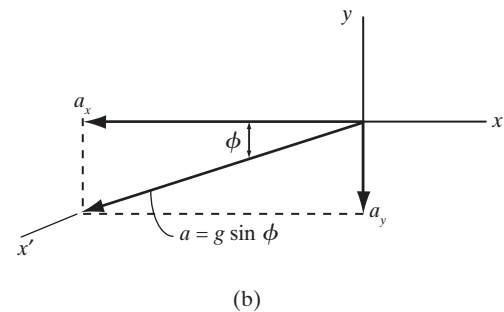
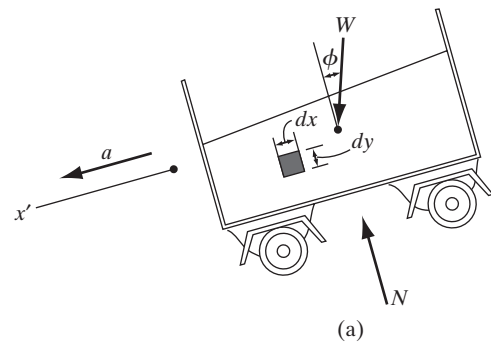
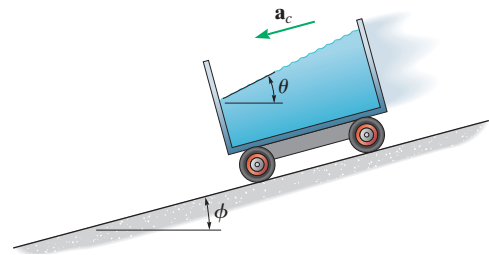
then

$$\tan \theta = \tan \phi$$

or

$$\theta = \phi$$

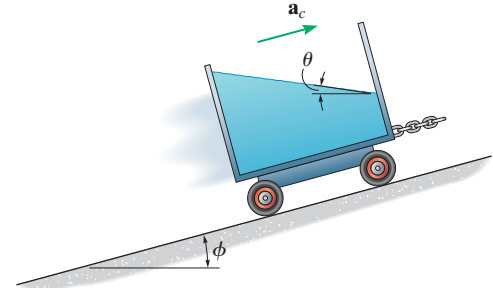
Q.E.D.



Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-140.**

The cart is given a constant acceleration a_c up the inclined plane. Show that the lines of constant pressure *within* the liquid have a slope of $\tan \theta = (a_c \cos \phi)/(a_c \sin \phi + g)$.



SOLUTION

As in the preceding solution, we determine that

$$\frac{dy}{dx} = -\frac{a_x}{g + a_y} \quad (1)$$

Here, the slope of the surface of the liquid is

$$\frac{dy}{dx} = -\tan \theta \quad (2)$$

Equating Eqs. (1) and (2), we obtain

$$\tan \theta = \frac{a_x}{g + a_y} \quad (3)$$

By establishing the x and y axes shown in Fig. a ,

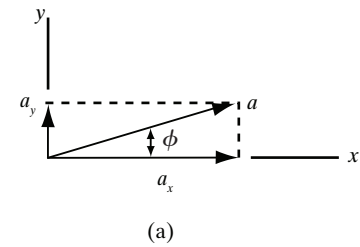
$$a_x = a_c \cos \phi$$

$$a_y = a_c \sin \phi$$

Substituting these values into Eq. (3),

$$\tan \theta = \frac{a_c \cos \phi}{a_c \sin \phi + g}$$

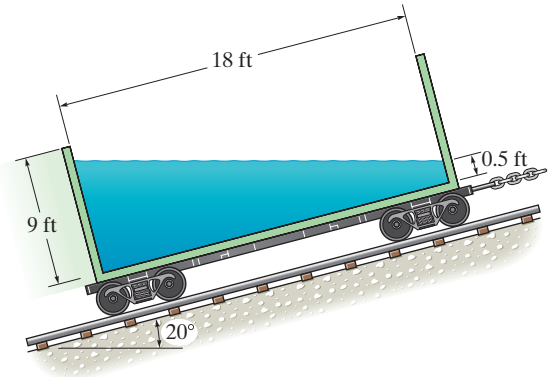
Q.E.D.



Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-141.

The open railcar is used to transport water up the 20° incline. When the car is at rest, the water level is as shown. Determine the maximum constant acceleration the car can have when it is pulled up the incline so that no water will spill out.



SOLUTION

The volume of the water can be determined by using the water level when the car is at rest, indicated by dashed line in Fig. *a*. Here

$$a = 0.5 \text{ ft} + 18 \tan 20^\circ \text{ ft} = 7.0515 \text{ ft}$$

If the width of the car is w ,

$$V_w = \frac{1}{2}(0.5 \text{ ft} + 7.0515 \text{ ft})(18 \text{ ft})w = 67.9632 w$$

It is required that the water is about to spill out. In this case, the water is indicated by the solid line in Fig. *a*,

$$V_w = \frac{1}{2}(9 \text{ ft})(b)w = 67.9632 w$$

$$b = 15.1029 \text{ ft} < 18 \text{ ft} \quad \text{TBEXAM.COM (O.K.)}$$

Then,

$$\theta = \tan^{-1}\left(\frac{9 \text{ ft}}{15.1029 \text{ ft}}\right) - 20^\circ = 10.7912^\circ$$

Consider the vertical block of water of weight $dW = \gamma_w h dA$ shown shaded in Fig. *a*.

$$+\uparrow \Sigma F_y = ma_y; \quad p dA - \gamma_w h dA = \left(\frac{\gamma_w h dA}{g}\right) a_c \sin 20^\circ$$

$$p = \frac{\gamma_w h}{g} a_c \sin 20^\circ + \gamma_w h$$

$$p = \frac{\gamma_w h}{g} (a_c \sin 20^\circ + g) \quad (1)$$

Consider the horizontal block of water of weight $dW = \gamma_w x dA$ shown shaded in Fig. *a*.

$$+\rightarrow \Sigma F_x = ma_x; \quad p_2 dA - p_1 dA = \left(\frac{\gamma_w x dA}{g}\right) a_c \cos 20^\circ$$

$$p_2 - p_1 = \frac{\gamma_w x}{g} a_c \cos 20^\circ \quad (2)$$

However, from Eq. (1), p_2 is at h_2 and p_1 is at h_1 , so that

$$p_2 - p_1 = \frac{\gamma_w}{g} (h_2 - h_1) (a_c \sin 20^\circ + g)$$

Substituting this result into Eq. (2), we have

$$\frac{\gamma_w}{g} (h_2 - h_1) (a_c \sin 20^\circ + g) = \frac{\gamma_w x}{g} a_c \cos 20^\circ$$

$$\frac{h_2 - h_1}{x} = \frac{a_c \cos 20^\circ}{a_c \sin 20^\circ + g}$$

2-141. Continued

However, $\tan \theta = \frac{h_2 - h_1}{x}$. Thus

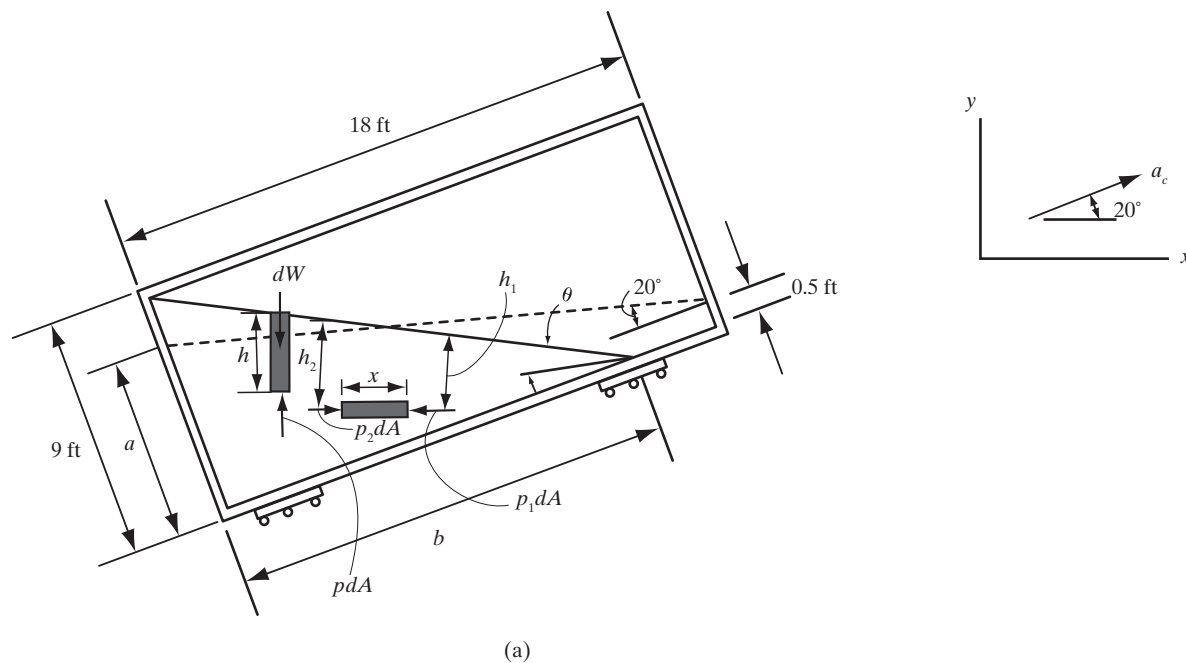
$$\tan \theta = \frac{a_c \cos 20^\circ}{a_c \sin 20^\circ + g}$$

Here, $\theta = 10.7912^\circ$. Then

$$\tan 10.7912^\circ = \frac{a_c \cos 20^\circ}{a_c \sin 20^\circ + 32.2 \text{ ft/s}^2}$$

$$a_c = 7.02 \text{ ft/s}^2$$

Ans.



Ans:
 $a_c = 7.02 \text{ ft/s}^2$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-142.

The open railcar is used to transport water up the 20° incline. When the car is at rest, the water level is as shown. Determine the maximum constant deceleration the car can have when it is pulled up the incline so that no water will spill out.

SOLUTION

The volume of water can be determined using the water level when the car is at rest indicated by the dashed line in Fig. *a*. Here,

$$a = 0.5 \text{ ft} + 18 \text{ ft} \tan 20^\circ = 7.0515 \text{ ft}$$

If the width of the car is w ,

$$V_w = \frac{1}{2}(0.5 \text{ ft} + 7.0515 \text{ ft})(18 \text{ ft})w = 67.9632 w$$

It is required that the water is about to spill out. In this case, the water level is indicated by the solid line in Fig. *a*,

$$V_w = \frac{1}{2}(9 \text{ ft})(b)(w) = 67.9632 w$$

$$b = 15.1029 \text{ ft} < 18 \text{ ft}$$

Then

$$\theta = \tan^{-1}\left(\frac{9 \text{ ft}}{15.1029 \text{ ft}}\right) + 20^\circ = 50.7912^\circ$$

Consider the vertical block of water of weight $dW = \gamma_w h dA$ shown shaded in Fig. *a*.

$$+\uparrow \Sigma F_y = ma_y; \quad p dA - \gamma_w h dA = \left(\frac{\gamma_w h dA}{g}\right)(-a_c \sin 20^\circ)$$

$$p = \frac{\gamma_w h}{g}(g - a_c \sin 20^\circ) \quad (1)$$

Consider the horizontal block of water of weight $dW = \gamma_w x dA$ shown shaded in Fig. *a*.

$$+\rightarrow \Sigma F_x = ma_x; \quad p_1 dA - p_2 dA = \left(\frac{\gamma_w x dA}{g}\right)(-a_c \cos 20^\circ)$$

$$p_2 - p_1 = \frac{\gamma_w x}{g} a_c \cos 20^\circ \quad (2)$$

However, from Eq. (1), since p_1 is at h_1 and p_2 is at h_2 ,

$$p_2 - p_1 = \frac{\gamma_w}{g}(h_2 - h_1)(g - a_c \sin 20^\circ)$$

Substituting this result into Eq. (2),

$$\frac{\gamma_w}{g}(h_2 - h_1)(g - a_c \sin 20^\circ) = \frac{\gamma_w x}{g} a_c \cos 20^\circ$$

$$\frac{h_2 - h_1}{x} = \frac{a_c \cos 20^\circ}{g - a_c \sin 20^\circ}$$

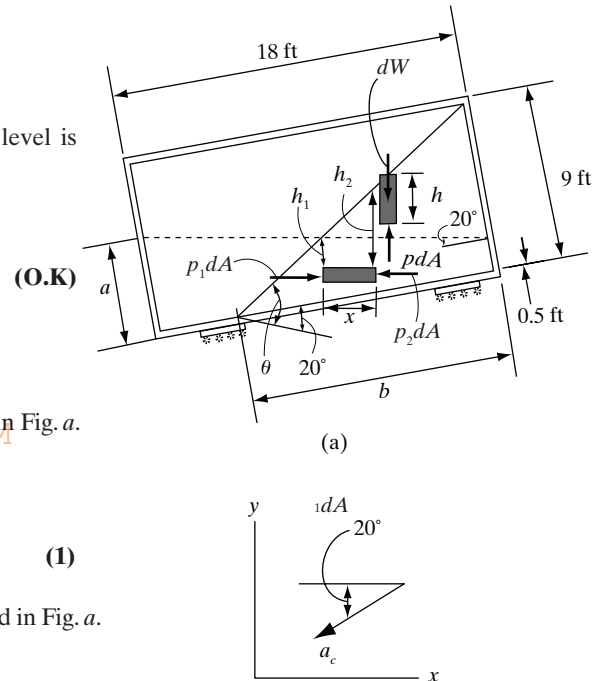
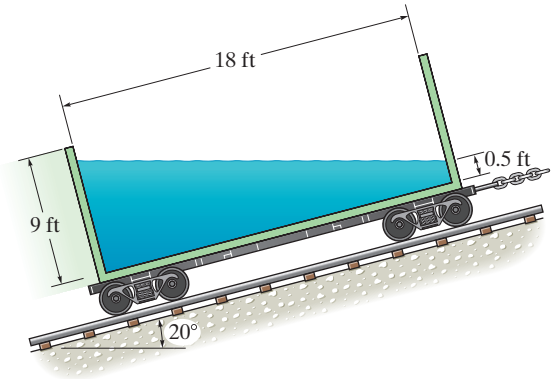
However, $\tan \theta = \frac{h_2 - h_1}{x}$. Thus

$$\tan \theta = \frac{a_c \cos 20^\circ}{g - a_c \sin 20^\circ}$$

Here, $\theta = 50.7912^\circ$. Then

$$\tan 50.7912^\circ = \frac{a_c \cos 20^\circ}{32.2 \text{ ft/s}^2 - a_c \sin 20^\circ}$$

$$a_c = 29.04 \text{ ft/s}^2 = 29.0 \text{ ft/s}^2$$



Ans.

Ans:
 $a_c = 29.0 \text{ ft/s}^2$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-143.

The beaker is filled to a height of $h = 0.1 \text{ m}$ with kerosene and placed on the platform. What is the maximum angular velocity ω it can have so that no kerosene spills out of the beaker?

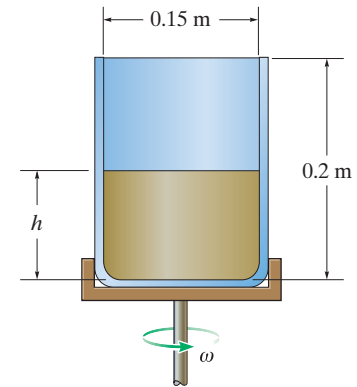
SOLUTION

When the kerosene is about to spill out of the beaker, Fig. *a*, $h/2 = 0.1 \text{ m}$ or $h = 0.2 \text{ m}$.

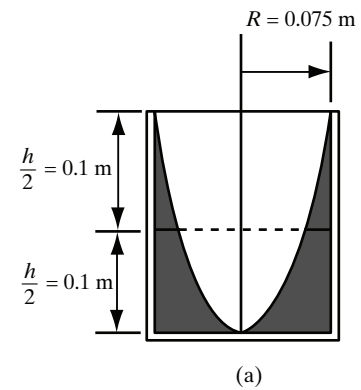
$$h = \frac{\omega^2}{2g} r^2$$

$$0.2 \text{ m} = \frac{\omega^2}{2(9.81 \text{ m/s}^2)} \left(\frac{0.15 \text{ m}}{2} \right)^2$$

$$\omega = 26.4 \text{ rad/s}$$



Ans.



Ans:
 $\omega = 26.4 \text{ rad/s}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2-144.**

The beaker is filled to a height of $h = 0.1 \text{ m}$ with kerosene and placed on the platform. To what height h' does the kerosene rise against the wall of the beaker when the platform has an angular velocity of $\omega = 15 \text{ rad/s}$?

SOLUTION

$$H = \frac{\omega^2}{2g} r^2$$

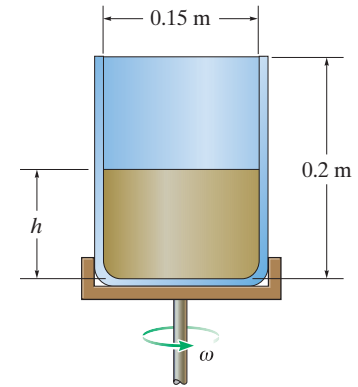
$$H = \frac{(15 \text{ rad/s})^2}{2(9.81 \text{ m/s}^2)} \left(\frac{0.15 \text{ m}}{2} \right)^2$$

$$= 0.0645 \text{ m}$$

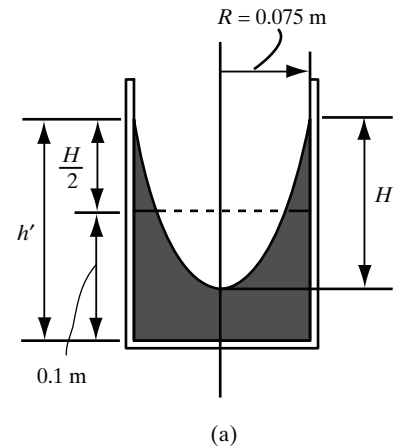
From Fig. *a*, we observe that

$$h' = 0.1 \text{ m} + \frac{0.0645}{2} \text{ m}$$

$$h' = 0.132 \text{ m} = 132 \text{ mm}$$



Ans.



Ans:
 $h' = 132 \text{ mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-145.

The glass is filled with water to a height of $d = 0.1 \text{ m}$. To what height d' does the water rise against the wall of the glass when the platform has an angular velocity of $\omega = 15 \text{ rad/s}$?

SOLUTION

From the geometry shown in Fig. *a*,

$$\frac{d''}{d'' + 0.2 \text{ m}} = \frac{0.05 \text{ m}}{0.1 \text{ m}}; \quad d'' = 0.2 \text{ m}$$

Then,

$$\frac{r}{0.05 \text{ m}} = \frac{0.3 \text{ m}}{0.2 \text{ m}}; \quad r = 0.075 \text{ m}$$

Thus, the volume of the empty space in the container shown shaded in Fig. *a* is

$$V_{\text{es}} = \frac{1}{3}\pi(0.1 \text{ m})^2(0.4 \text{ m}) - \frac{1}{3}\pi(0.075 \text{ m})^2(0.3 \text{ m}) = 0.77083\pi(10^{-3}) \text{ m}^3$$

The parabolic profile of the free water surface is shown in Fig. *b*. Then

$$\begin{aligned} h &= \left(\frac{\omega^2}{2g}\right)r^2 \\ h &= \left[\frac{(15 \text{ rad/s})^2}{2(9.81 \text{ m/s}^2)}\right]R^2 \\ &= 11.4679R^2 \end{aligned}$$

Since the empty space in the glass must remain the same, the shaded volume shown in Fig. *b* [paraboloid segment (1) and frustum segment (2)] must be equal to this volume. Here, the volume of the paraboloid is equal to one half the volume of the cylinder of the same radius and height.

$$\begin{aligned} V_1 + V_2 &= V_{\text{es}} \\ \frac{1}{2}[\pi R^2(11.4679R^2)] + \left[\frac{1}{3}\pi(0.1 \text{ m})^2(0.4 \text{ m}) - \frac{1}{3}\pi R^2(d' + 0.2)\right] &= 0.77083\pi(10^{-3}) \text{ m}^3 \\ 5.7339R^4 - \frac{1}{3}R^2(d' + 0.2) + 0.5625(10^{-3}) &= 0 \end{aligned} \quad (1)$$

From the geometry shown in Fig. *b*,

$$\frac{d' + 0.2 \text{ m}}{0.2} = \frac{R}{0.05}; \quad d' + 0.2 = 4R$$

Substitute this result into Eq. (1).

$$5.7339R^4 - \frac{4}{3}R^3 + 0.5625(10^{-3}) = 0$$

Solving numerically,

$$R = 0.087852 \text{ m} < 0.1 \text{ m}$$

Thus,

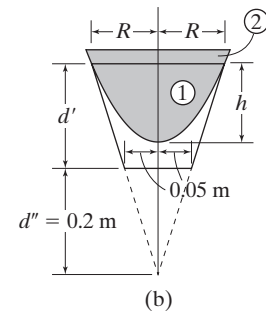
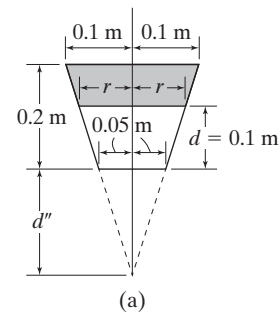
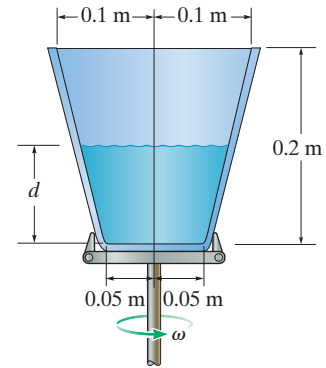
$$\begin{aligned} d' + 0.2 &= 4(0.087852) \\ d' &= 0.1514 \text{ m} = 0.151 \text{ m} \end{aligned}$$

Ans.

Also,

$$h = 11.4679(0.087852^2) = 0.0885 \text{ m} < d' \quad \textbf{(O.K!)}$$

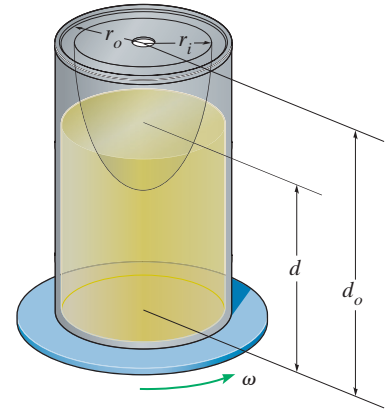
Ans:
 $d' = 0.151 \text{ m}$



Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-146.

The drum has a hole in the center of its lid and is filled to a height d with a liquid having a density ρ . If the drum is then placed on the rotating platform and it attains an angular velocity of ω , determine the inner radius r_i of the liquid where it contacts the lid.



SOLUTION

The volume of the paraboloid empty space must be the same as the volume of the empty space when the liquid is not spinning. Since the volume of the paraboloid is equal to one half the volume of the cylinder of the same radius and height, then

$$\begin{aligned} V_{\text{parab}} &= V_{\text{es}} \\ \frac{1}{2}(\pi r_i^2 h) &= \pi r_o^2 (d_o - d) \\ r_i^2 h &= 2r_o^2 (d_o - d) \end{aligned} \quad (1)$$

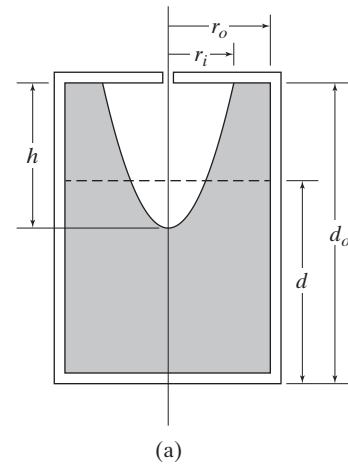
So then,

$$h = \frac{\omega^2}{2g} r_i^2 \quad (2)$$

Substitute Eq. (2) into (1).

$$\begin{aligned} r_i^2 \left(\frac{\omega^2}{2g} \right) r_i^2 &= 2r_o^2 (d_o - d) \\ r_i^4 &= \frac{4gr_o^2 (d_o - d)}{\omega^2} \\ r_i &= \left[\frac{4gr_o^2 (d_o - d)}{\omega^2} \right]^{1/4} \end{aligned}$$

Ans.



Ans:

$$r_i = \left[\frac{4gr_o^2 (d_o - d)}{\omega^2} \right]^{1/4}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2-147.

The tube is filled with water to the level $h = 1 \text{ ft}$. Determine the pressure at point O when the tube has an angular velocity of $\omega = 8 \text{ rad/s}$.

SOLUTION

The level of the water in the tube will not change. Therefore, the imaginary surface will be as shown in Fig. *a*.

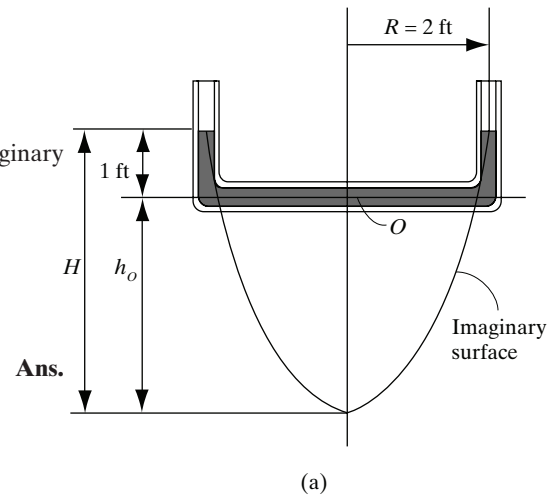
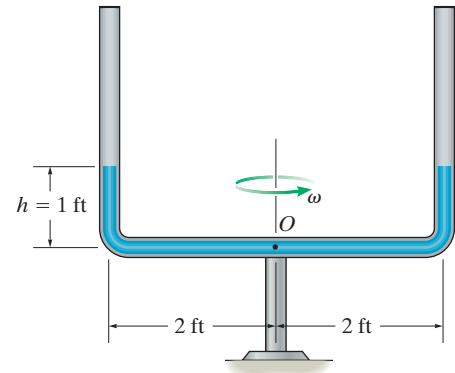
$$H = \frac{\omega^2 R^2}{2g} = \frac{(8 \text{ rad/s})^2 (2 \text{ ft})^2}{2(32.2 \text{ ft/s}^2)} = 3.9752 \text{ ft}$$

We observe from Fig. *a* that

$$h_O = H - 1 \text{ ft} = 3.9752 \text{ ft} - 1 \text{ ft} = 2.9752 \text{ ft}$$

Finally, the pressure at O must be negative since it is 2.9752 ft above the imaginary surface of the liquid.

$$\begin{aligned} p_O &= \gamma h_O = (62.4 \text{ lb/ft}^3)(-2.9752 \text{ ft}) \\ &= \left(-185.65 \frac{\text{lb}}{\text{ft}^2}\right) \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 \\ &= -1.29 \text{ psi} \end{aligned}$$



Ans:
 $p_O = -1.29 \text{ psi}$