

***Operations Research 11th edition Taha Solutions
Manual***

SKU: 9780137625789-SOLUTIONS

CHAPTER 2

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Modeling with linear programming

2-1

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Chapter 2

1

- (a) $x_2 - x_1 \geq 1$ **or** $-x_1 + x_2 \geq 1$
 (b) $x_1 + 2x_2 \geq 3$ **and** $x_1 + 2x_2 \leq 6$
 $6 \times 2 + 4 \times 2 = 20 < 24$
 $1 \times 1 + 2 \times 2 = 6 = 6$
 $-1 \times 1 + 1 \times 2 = 0 < 1$
 $1 \times 2 = 2 < 1$ } *feasible*
 (c) $x_2 \geq x_1$ **or** $x_1 - x_2 \leq 0$
 (d) $x_1 + x_2 \geq 3$
 (e) $\frac{x_2}{x_1 + x_2} \leq .5$ **or** $.5 x_1 - x_2 \geq 0$

2

- (a) $(x_1, x_2) = (1, 4)$
 $(x_1, x_2) \geq 0$
 $6 \times 1 + 4 \times 4 = 22 < 24$
 $1 \times 1 + 2 \times 4 = 9 > 6 \Rightarrow$ infeasible
 (b) $(x_1, x_2) = (2, 2)$
 $(x_1, x_2) \geq 0$
 $6 \times 2 + 4 \times 2 = 20 < 24$
 $1 \times 2 + 2 \times 2 = 6 = 6$
 $-1 \times 2 + 1 \times 2 = 0 < 1$
 $1 \times 2 = 2 = 2$
Feasible, $z = 5 \times 2 + 4 \times 2 = \18
 (c) $(x_1, x_2) = (3, 1.5)$
 $(x_1, x_2) \geq 0$
 $6 \times 3 + 4 \times 1.5 = 24 < 24$
 $1 \times 3 + 2 \times 1.5 = 6 = 6$
 $-1 \times 3 + 1 \times 1.5 = -1.5 < 1$
 $1 \times 1.5 = 1.5 < 2$
Feasible, $z = 5 \times 3 + 4 \times 1.5 = \21
 (c) $(x_1, x_2) = (2, 1)$
 $(x_1, x_2) \geq 0$
 $6 \times 2 + 4 \times 1 = 16 < 24$
 $1 \times 2 + 2 \times 1 = 4 < 6$
 $-1 \times 2 + 1 \times 1 = -1 < 1$
 $1 \times 1 = 1 < 2$
Feasible, $z = 5 \times 2 + 4 \times 1 = \14
 (c) $(x_1, x_2) = (2, -1)$
 $x_2 < 0 \Rightarrow$ infeasible

3

$(x_1, x_2) = (2, 2)$: Let s_1 and s_2 be the unused daily amounts of M1 and M2.

For M1: $s_1 = 24 - (6x_1 + 4x_2) = 24 - 2 \times 10 = 4$ tons/day

For M2: $s_2 = 6 - (x_1 + 2x_2) = 6 - 2 \times 3 = 0$ ton/day

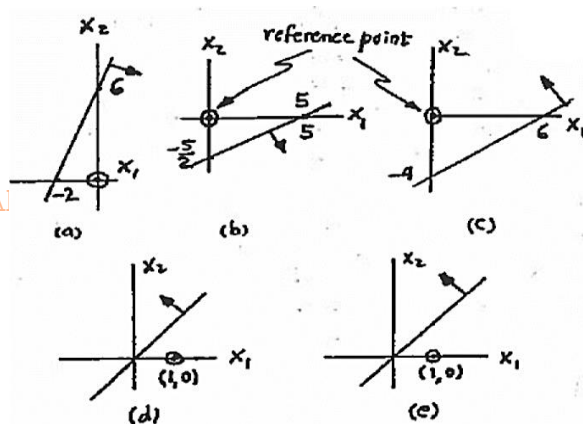
4

Quantity discount results in a nonlinear function:

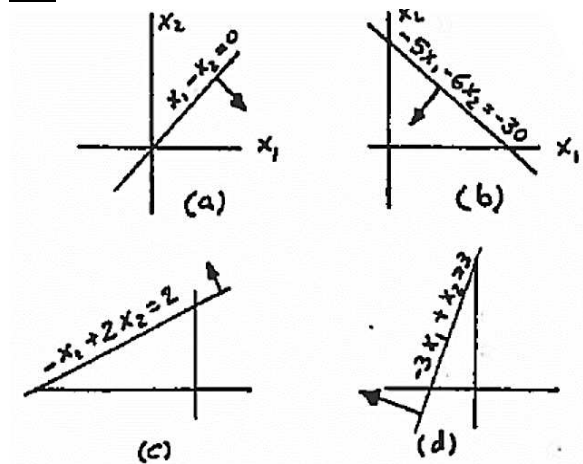
$$z = \begin{cases} 5x_1 + 4x_2, & 0 \leq x_1 \leq 2 \\ 4.5x_1 + 4x_2, & x_1 \geq 2 \end{cases}$$

The situation cannot be an LP. Mixed integer programming (Chapter 9) can handle this nonlinearity.

5



6



continue...

Chapter 2

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(a)

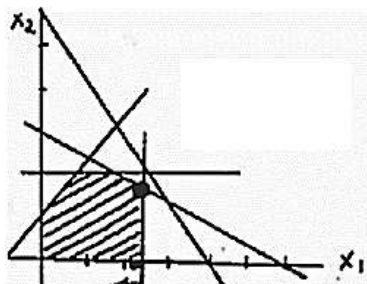
$$x_1 \leq 2.5$$

Optimum:

$$x_1 = 2.5$$

$$x_2 = 1.75$$

$$z = \$13$$



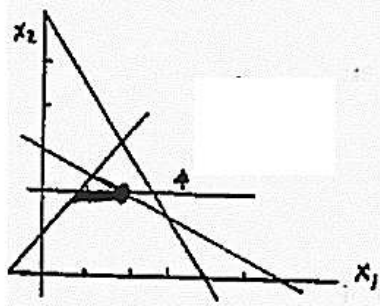
(b)

$$x_2 \geq 2$$

Optimum:

$$x_1 = 2, x_2 = 2$$

$$z = \$18$$



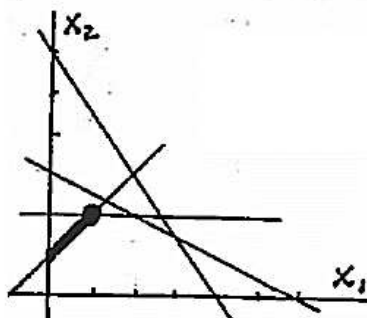
(c)

$$-x_1 + x_2 = 1$$

Optimum:

$$x_1 = 1, x_2 = 2$$

$$z = \$13$$



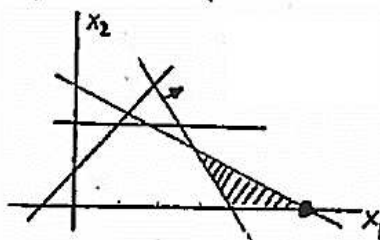
(d)

$$6x_1 + 4x_2 \geq 24$$

Optimum:

$$x_1 = 6, x_2 = 0$$

$$z = \$30$$



(e)

infeasible space

8

 x_1 = daily units of product 1

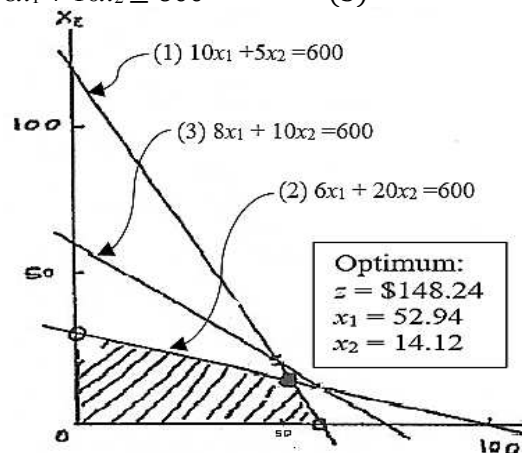
 x_2 = daily units of product 2

$$\text{Max } z = 2x_1 + 3x_2 \text{ s.t.}$$

$$10x_1 + 5x_2 \leq 600 \quad (1)$$

$$6x_1 + 20x_2 \leq 600 \quad (2)$$

$$8x_1 + 10x_2 \leq 600 \quad (3)$$



9

 x_1 = nbr. of units of A

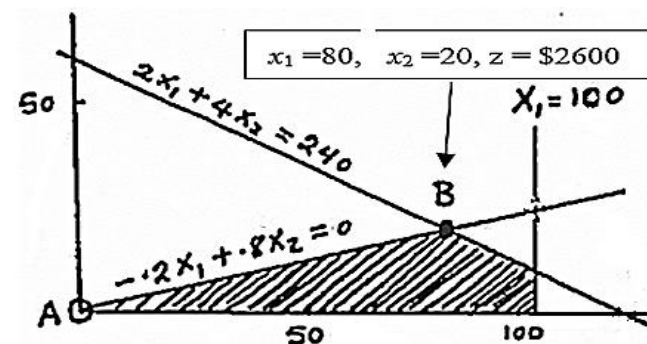
 x_2 = nbr. of units of B

$$\text{Max } z = 20x_1 + 50x_2 \text{ s.t. } x_1, x_2 \geq 0$$

$$\frac{x_1}{x_1 + x_2} \geq .8 \text{ or } -.2x_1 + .8x_2 \leq 0$$

$$x_1 \leq 100$$

$$2x_1 + 4x_2 \leq 240$$



Chapter 2

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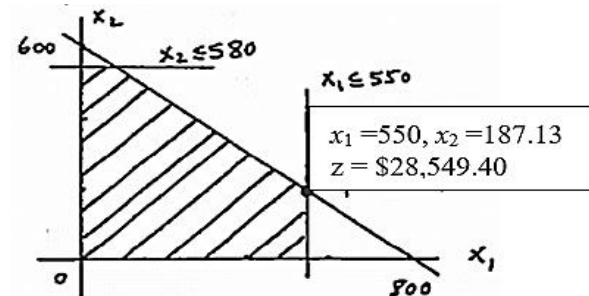
x_1 = nbr. of sheets/day

x_2 = nbr. of bars/day

Max $z = 40x_1 + 35x_2$ s.t.

$$\frac{x_1}{800} + \frac{x_2}{600} \leq 1$$

$$0 \leq x_1 \leq 550, 0 \leq x_2 \leq 580$$



11

x_1 = \$ invested in A

x_2 = \$ invested in B

Max $z = .05x_1 + .08x_2$ s.t.

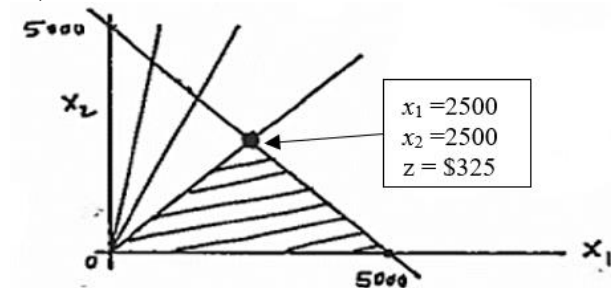
$$x_1 \geq .25(x_1 + x_2)$$

$$x_2 \leq .5(x_1 + x_2)$$

$$x_1 \geq .5x_2$$

$$x_1 + x_2 \leq 5000$$

$$x_1, x_2 \geq 0$$



12

x_1 = nbr of practical courses

x_2 = nbr of humanistic courses

Max $z = 1500x_1 + .1000x_2$ s.t.

$$x_1 \geq .25(x_1 + x_2)$$

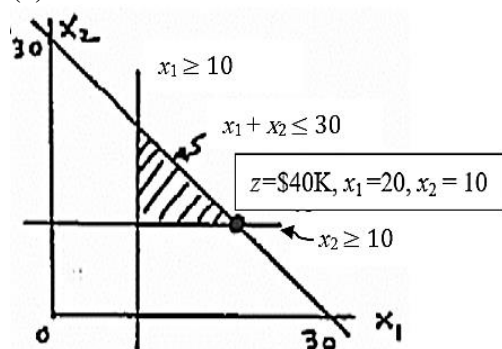
$$x_1 + x_2 \leq 30$$

$$x_1 \geq 10,$$

continued...

12 cont'd

(a)



(b) Change $x_1 + x_2 \leq 30$ to $x_1 + x_2 \leq 31$

Optimum: $z = \$41,500$, $\Delta z = 41500 - 40,000 = \1500

Conclusion: any additional course will be 'practical'

13

x_1 = units of solution A

x_2 = units of solution B

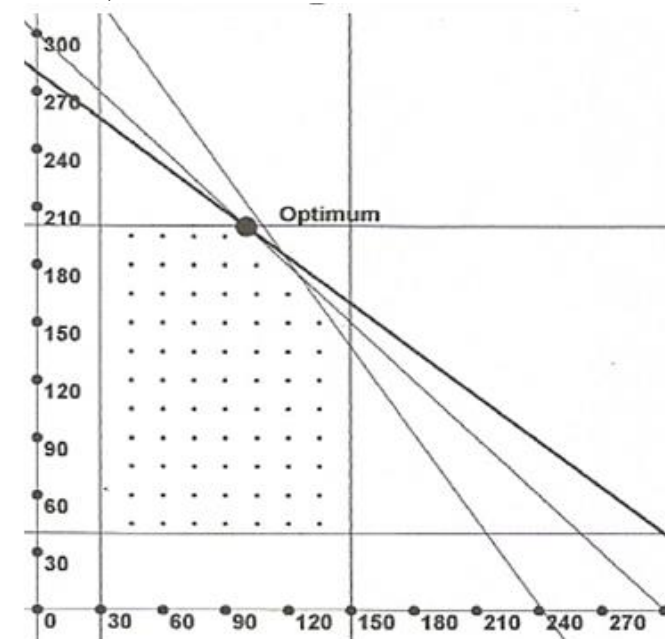
Max $z = 8x_1 + 10x_2$ s.t.

$$.5x_1 + .5x_2 \leq 159$$

$$.6x_1 + .4x_2 \leq 145$$

$$x_1 \geq 30, x_1 \leq 150$$

$$x_2 \geq 40, x_2 \leq 200$$



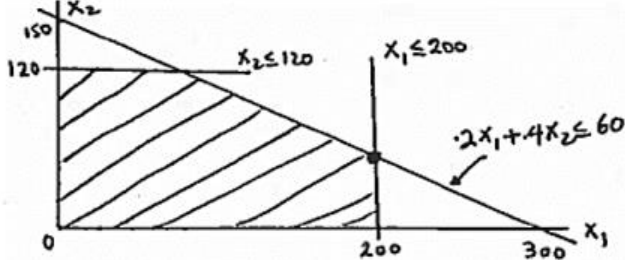
Optimum: $z = 2800$, $x_1 = 100$, $x_2 = 200$

continued...

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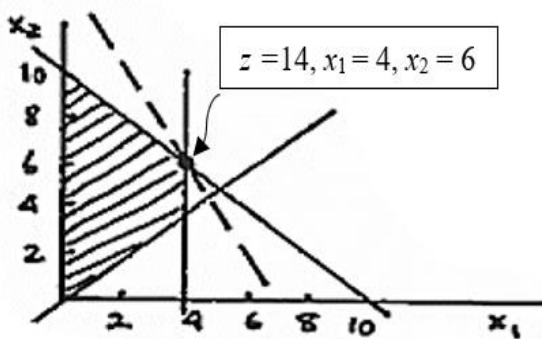
x_1 = nbr of grano boxes
 x_2 = nbr of wheatie boxes
 Max $z = x_1 + 1.35x_2$ s.t.
 $.2x_1 + .4x_2 \leq 60$
 $.6x_1 + .4x_2 \leq 145$
 $0 \leq x_1 \leq 200$
 $0 \leq x_2 \leq 120$



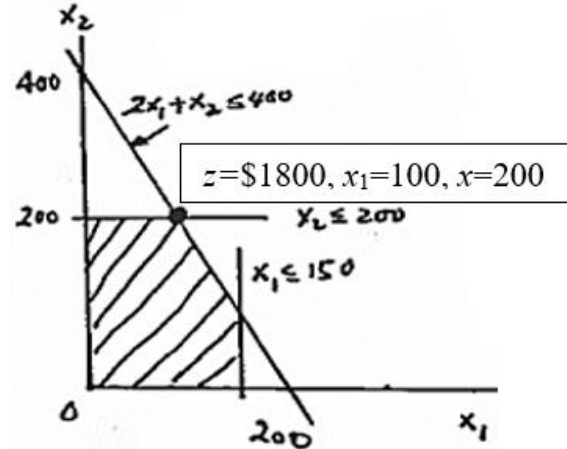
Optimum: $x_1 = 200, x_2 = 50, z = \267.50
 Area allocation: 67% grano, 33% wheatie

15

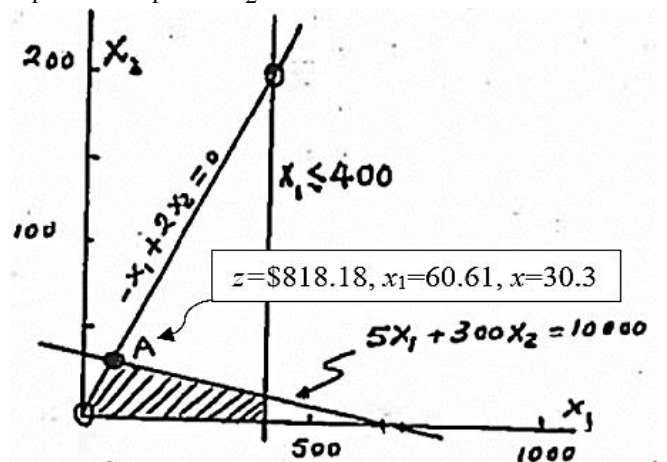
x_1 = play hours/day
 x_2 = work hours/day
 Max $z = 2x_1 + x_2$ s.t.
 $x_1 + x_2 \leq 10$
 $x_1 - x_2 \leq 0$
 $x_1 \leq 4$

**16**

x_1 = daily nbr. of type 1 hats
 x_2 = daily nbr. of type 2 hats
 Max $z = 8x_1 + 5x_2$ s.t.
 $2x_1 + x_2 \leq 400$
 $0 \leq x_1 \leq 150$
 $0 \leq x_2 \leq 200$

**17**

x_1 = radio minutes
 x_2 = TV minutes
 Max $z = x_1 + 25x_2$ s.t.
 $15x_1 + 300x_2 \leq 10,000$
 $\frac{x_1}{x_2} \geq 2$ or $-x_1 + 2x_2 \leq 0$
 $x_1 \leq 400, x_1 \geq 0, x_2 \geq 0$



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x_1 =consumed tons of C1/hr

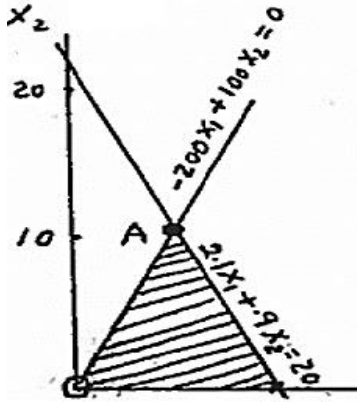
x_2 = consumed tons of C2/hr

Max $z = 12000x_1 + 9000x_2$ s.t.

$1800x_1 + 2100x_2 \leq 2000(x_1 + x_2) \Rightarrow -200x_1 + 100x_2 \leq 20$

$2.1x_1 + 9x_2 \leq 20$

$x_1, x_2 \geq 0$



(a) optimum:

$z = 153,846$ lb, $x_1 = 5.128$ ton/hr, $x_2 = 10.256$ ton/hr

optimal ration = $5.128/10.256 = .5$

(b) $2.1x_1 + 9x_2 \leq (20+1)$

$z = 161,538$ lb, $\Delta z = 161,538 - 153,846 = 7,692$ lb

19

x_1 =nbr of radio commercials beyond the first

x_2 = nbr of TV commercials beyond the first

Max $z = 2000x_1 + 3000x_2 + 5000 + 2000$ s.t.

$300(x_1+1) + 2000(x_2+1) \leq 20,000$

$300(x_1+1) \leq .8 \times 20,000$

$2000(x_2+1) \leq .8 \times 20,000$

or,

Max $z = 2000x_1 + 3000x_2 + 7000$ s.t.

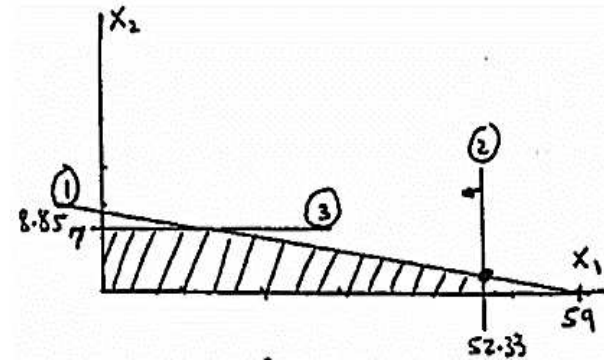
$300x_1 + 200x_2 \leq 17,700$

$300x_1 \leq 15,700$

$2000x_2 \leq 14,000$

$x_1, x_2 \geq 0$

19 cont'd



Optimum: Ratio = $52.33 + 1 = 53.33$

TV = $1 + 1 = 2$, $z = 114,666.67$

20

x_1 =nbr of T-shirts/hr

x_2 = nbr of jackets/hr

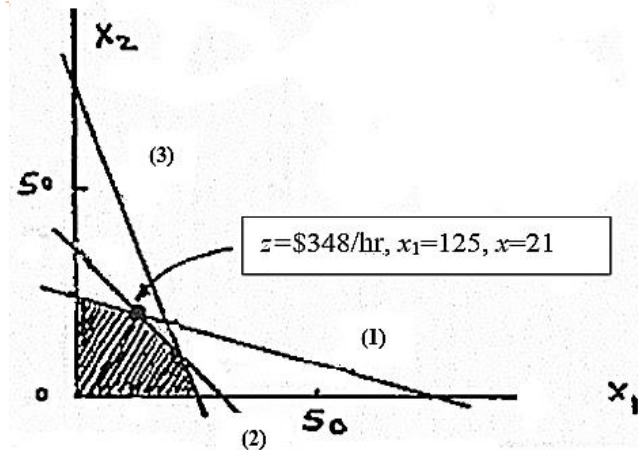
Max $z = 8x_1 + 12x_2$ s.t.

$20x_1 + 60x_2 \leq 25 \times 60 = 1500$ (1)

$70x_1 + 60x_2 \leq 35 \times 60 = 2100$ (2)

$12x_1 + 4x_2 \leq 5 \times 60 = 300$ (3)

$x_1, x_2 \geq 0$



$z = \$348/\text{hr}$, $x_1 = 125$, $x_2 = 21$

21

x_1 =nbr of desks/day

x_2 = nbr of chair/day

Max $z = 50x_1 + 100x_2$ s.t.

$x_1/200 + x_2/80 \leq 1$ (1)

$x_1/150 + x_2/110 \leq 1$ (2)

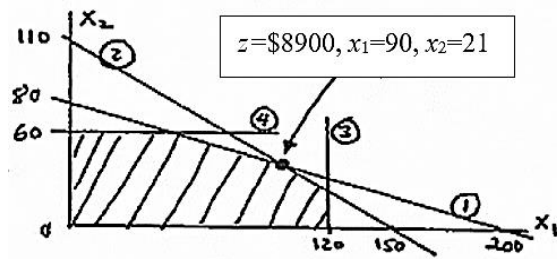
$x_1 \leq 120$, $x_2 \leq 60$ (3&4)

$x_1, x_2 \geq 0$

continued...

Chapter 2

21 cont'd



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 x_1 = nbr of HiFi1 units x_2 = nbr of HiFi2 units**constraints:**Let s_i be a slack variable, $i=1,2,3$

$$6x_1 + 4x_2 + s_1 = 480 \times 9 = 432 \quad (1)$$

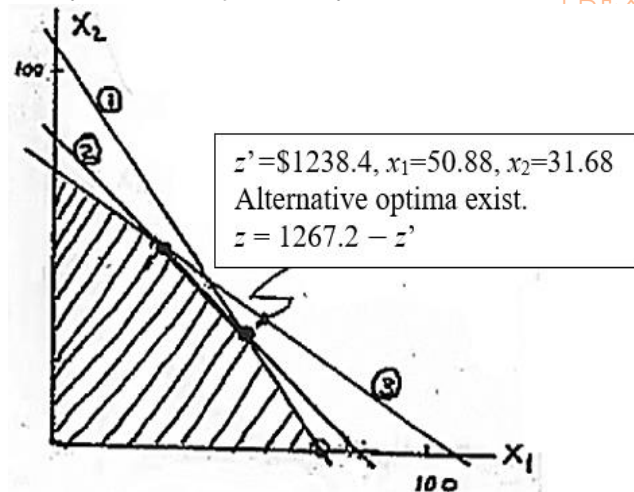
$$5x_1 + 5x_2 + s_2 = 35 \times 60 = 2100 \quad (2)$$

$$4x_1 + 6x_2 + s_3 = 5 \times 60 = 300 \quad (3)$$

$$x_1, x_2 \geq 0$$

Objective function:Let $z = s_1 + s_2 + s_3$ ($= 1267.2 - 15x_1 - 15x_2$, per constraints 1&2&3), hence

$$\text{Min } z = s_1 + s_2 + s_3 \equiv \text{Max } z' = 15x_1 + 15x_2$$



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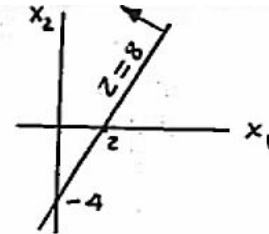
Corner point	(x_1, x_2)	z
A	(0, 0)	0
B	(4, 0)	20
C	(3, 1.5)	21 (optimum)
D	(2, 2)	18
E	(1, 2)	13
F	(0, 1)	4

24 – 27

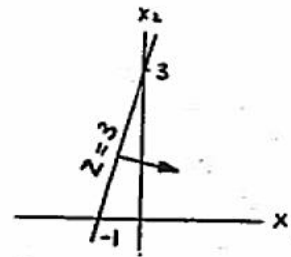
TORA experiments

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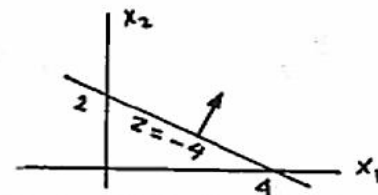
(a)



(b)



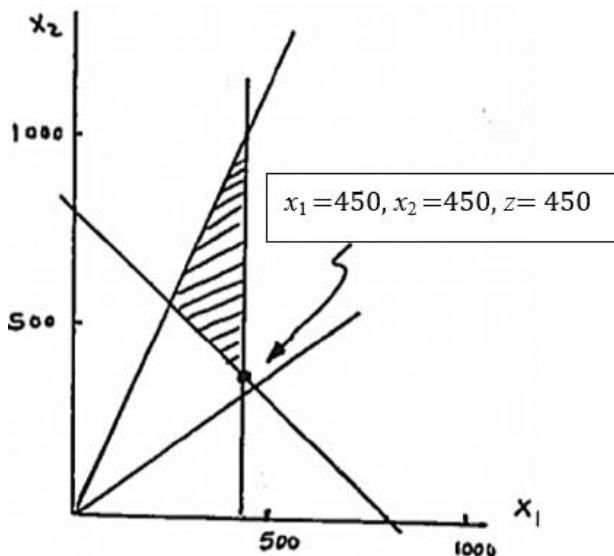
(c)



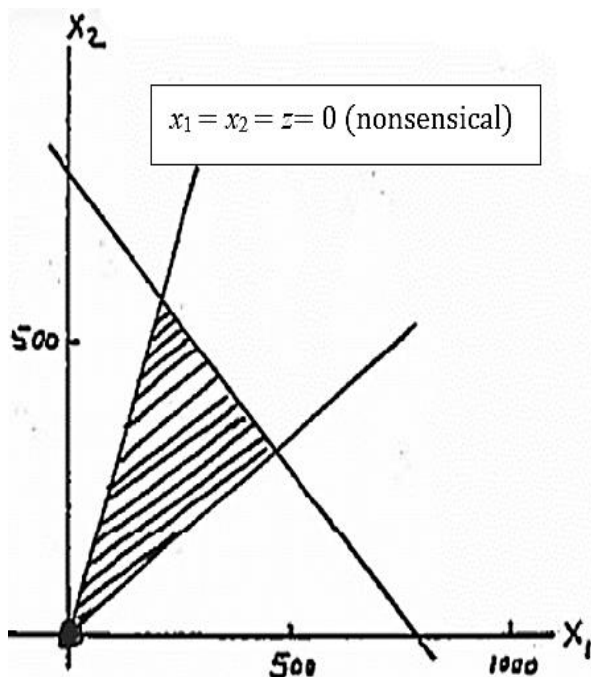
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Additional constraint: $x_1 \leq 450$



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31

x_1 = nbr of hours in store 1

x_2 = nbr of hours in store 2

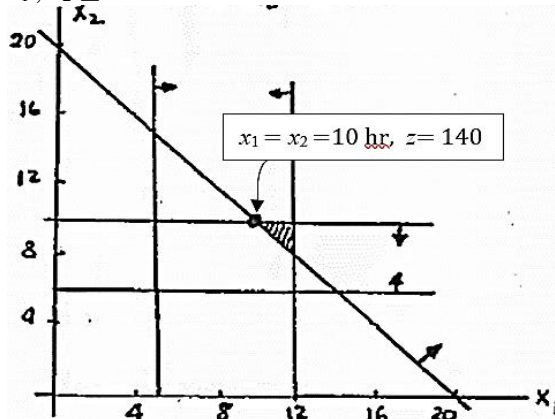
Min $z = 8x_1 + 6x_2$ s.t.

$x_1 + x_2 \geq 20$

$5 \leq x_1 \leq 12$

$6 \leq x_2 \leq 10$

$x_1, x_2 \geq 0$



32

x_1 = amount in 1K bbl/day from Iran

x_2 = amount in 1K bbl/day from Dubai

Refinery capacity = $(x_1 + x_2)$

Min $z = x_1 + x_2$ s.t.

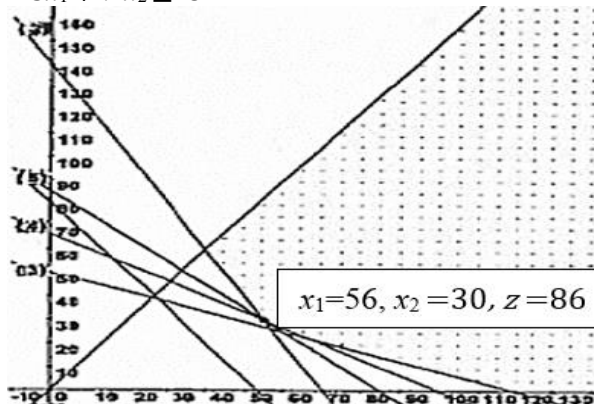
$x_1 \geq .4(x_1 + x_2)$ or $.6x_1 - .4x_2 \geq 0$

$.2x_1 + .1x_2 \geq 14$

$.25x_1 + .6x_2 \geq 30$

$.1x_1 + .15x_2 \geq 10$

$.15x_1 + .1x_2 \geq 8$



Chapter 2

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x_1 = amount in \$1K invested in blue chips

x_2 = amount in \$1K invested in high-tech

Refinery capacity = $(x_1 + x_2)$

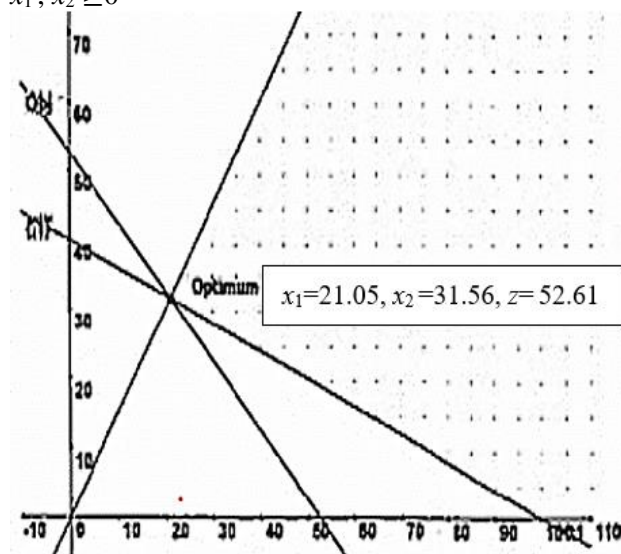
Min $z = x_1 + x_2$ s.t.

$$.1x_1 + .25x_2 \geq 10$$

$$.6x_1 - .4x_2 \geq 0$$

$$x_1, x_2 \geq 0$$

$$x_1 = .33, x_2 = .67, z = 86.67$$



34

x_1 = ratio of scrap A in alloy

x_2 = ratio of scrap B in alloy

Min $z = 100x_1 + 80x_2$ s.t.

$$.06x_1 + .03x_2 \geq .03$$

$$.06x_1 + .03x_2 \leq .06$$

$$.03x_1 + .06x_2 \geq .03$$

$$.03x_1 + .06x_2 \leq .05$$

$$.04x_1 + .03x_2 \geq .03$$

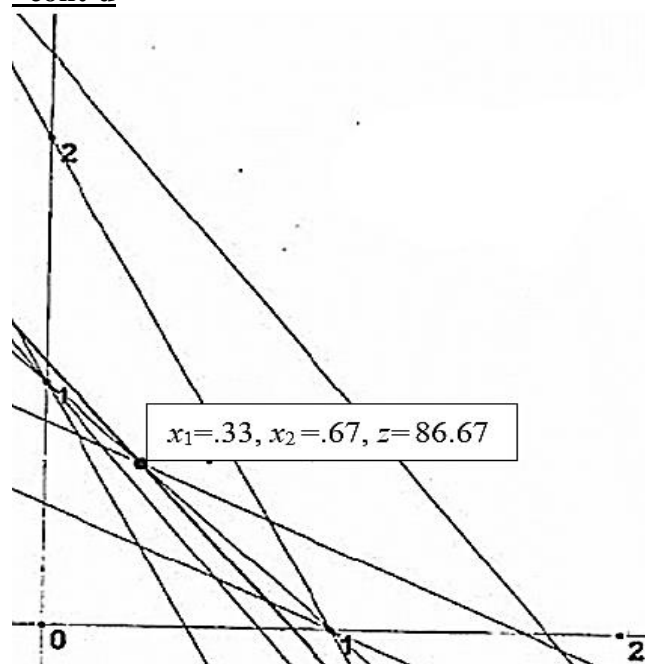
$$.04x_1 + .03x_2 \leq .07$$

$$x_1 + x_2 = 1$$

$$x_1, x_2 \geq 0$$

continued...

cont'd



35-39

AMPL-Solver-TORA exercises

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(a) x_j = executed portion of project j

$$\text{Max } z = 32.4x_1 + 35.8x_2 + 17.75x_3 + 14.8x_4 + 18.2x_5 + 12.35x_6$$

s.t.

$$10.5x_1 + 8.3x_2 + 10.2x_3 + 7.2x_4 + 12.3x_5 + 9.2x_6 \leq 60$$

$$14.4x_1 + 12.6x_2 + 14.2x_3 + 10.5x_4 + 10.1x_5 + 7.8x_6 \leq 70$$

$$2.2x_1 + 9.5x_2 + 5.6x_3 + 7.5x_4 + 8.3x_5 + 6.9x_6 \leq 35$$

$$2.4x_1 + 3.1x_2 + 4.2x_3 + 5x_4 + 6.3x_5 + 5.1x_6 \leq 20$$

$$0 \leq x_i \leq 1, j = 1, 2, \dots, 6$$

$$\text{Solution: } x_1 = x_2 = x_3 = x_4 = 1, x_5 = .84, x_6 = 0, z = 116.06$$

(b) Add the constraint $x_2 \leq x_6$

$$\text{Solution: } x_1 = x_2 = x_3 = x_4 = x_6 = 1, x_5 = .70 + .03, z = 113.68$$

(c) Let s_i = unused funds at the end of year i and change the RHS of constraints 2, 3, and 4 to

$$70 + s_2, 35 + s_3, \text{ and } 20 + s_4.$$

$$\text{Solution: } x_1 = x_2 = x_3 = x_4 = x_5 = 1, x_6 = .71, z = 127.72$$

continued...

Chapter 2

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Solution interpretation:

i	s_i	$s_i - s_{i-1}$	Decision
1	4.96	--	--
2	7.62	+2.66	Don't borrow from year 1
3	4.62	-3.00	Borrow 3 from year 2
4	0	-24.6	Borrow 4.62 from year 2

Availing excess money in current year for later years resulted in completing the first five projects + 71% of project 6. Total revenue increased from \$116,060 to \$127,720.

(d) Declare slack s_i unrestricted and re-solve.

Solution: $s_1 = 2.3$, $s_2 = .4$, $s_3 = -5$, $s_4 = -6.1$, $z = 131.3 \Rightarrow$ additional funds are needed in years 3 and 4.

Increase in return = $131.3 - 116.06 = 15.24$. Ignoring time value of money, the amount borrowed [= $5 + 6.1 - (2.3 + .4) = 8.4$] yields a rate of return = $(15.28 - 8.4) / 8.8 \approx 81\%$

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x_i = \$-amt invested in project i ($=1, 2, 3, 4$)

y_j = \$-amt invested in in bank in year j ($=1, 2, 3, 4, 5$)

Max $z = y_5$

s.t.

$$x_1 + x_2 + x_4 + y_1 \leq 10,000$$

$$.5x_1 + .6x_2 - x_3 + .4x_4 + 1.065 y_1 - y_2 = 0$$

$$.3x_1 + .2x_2 + .8x_3 + .6x_4 + 1.065 y_2 - y_3 = 0$$

$$1.8x_1 + 1.5x_2 + 1.9x_3 + 1.86x_4 + 1.065 y_3 - y_4 = 0$$

$$1.2x_1 + 1.3x_2 + .8x_3 + .95x_4 + 1.065 y_4 - y_5 = 0$$

all vars ≥ 0

Solution: $x_1 = 0$, $x_2 = \$10,000$, $x_3 = \$600$, $x_4 = 0$

$y_1 = y_2 = 0$, $y_3 = \$6800$, $y_4 = \$33,642$

$y_5 = z = \$53,628.73$ at the start of year 5

42

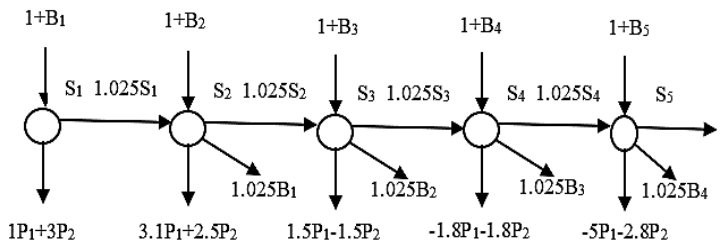
P_i = undertaken fraction of project i ($=1, 2$)

B_j = million \$ borrowed in quarter j ($=1, 2, 3, 4$)

S_j = surplus million \$ at the start of quarter j ($=1, 2, 3, 4$)

continued...

42 cont'd



(a)

Max $z = S_5$

s.t.

$$P_1 + 3P_2 - S_5 - B_1 = 1$$

$$3.1P_1 + 2.5P_2 - 1.02S_1 + S_2 + 1.025B_1 - B_2 = 1$$

$$1.5P_1 - 1.5P_2 - 1.02S_2 + S_3 + 1.025B_2 - B_3 = 1$$

$$-1.8P_1 - 1.8P_2 - 1.02S_3 + S_4 + 1.025B_3 - B_4 = 1$$

$$-5P_1 - 2.8P_2 - 1.02S_4 + S_5 + 1.025B_4 = 1$$

$$0 \leq P_i \leq 1, i = 1, 2$$

$$0 \leq B_j \leq 1, j = 1, 2, 3, 4$$

Solution: $z = 5.8366$, $P_1 = .7113$, $P_2 = 0$

$$B_1 = 0, B_2 = .9104, B_3 = 1, B_4 = 0$$

(b) $B_1 = 0$, $B_2 = .9104$, $B_3 = 1$, $B_4 = 0$

$$S_1 = .2887, S_2 = 0, S_3 = 0, S_4 = 1.2553$$

The solution shows that $B_i S_i = 0$, which means that it is not optimal to borrow and end up with surplus in any quarter. The result also makes sense because the cost of borrowing ($=2.5\%$) is higher than the return on surplus funds ($=2\%$).

43

Assume that the investment program ends at the start of year 11, so that the 6-year bond can be exercised in years 1 through 5 only. Similarly, the 9-year bond can be used in years 1 and 2 only. From years 6 on, the only investments available are insured saving at 7.5%.

I_i = insured savings in year i ($=1, 2, \dots, 10$)

G_i = 6-year bond i ($=1, 2, \dots, 5$)

M_i = 9-year bond i ($=1, 2$)

Objective: Maximize accumulation at end of year 10

$$\text{Max } z = 1.075I_{10} + 1.079G_5 + 1.085M_2$$

continued...

Chapter 2

43 cont'd

Cash flow constraints:

$$I_1 + .98G_1 + 1.02M_1 = 2$$

$$I_2 + .98G_2 + 1.02M_2 = 2 + 1.075 I_1 + .079G_1 + .085M_1$$

$$I_3 + .98G_3 = 2.5 + 1.075 I_2 + .079 (G_1 + G_2) + .085(M_1 + M_2)$$

$$I_4 + .98G_4 = 2.5 + 1.075 I_3 + .079 (G_1 + G_2 + G_3) + .085(M_1 + M_2)$$

$$I_5 + .98G_5 = 3 + 1.075 I_4 + .079 (G_1 + G_2 + G_3 + G_4) + .085(M_1 + M_2)$$

$$I_6 = 3.5 + 1.075 I_5 + .079 (G_1 + G_2 + G_3 + G_4 + G_5) + .085(M_1 + M_2)$$

$$I_7 = 3.5 + 1.075 I_6 + 1.079G_1 + .079 (G_2 + G_3 + G_4 + G_5) + .085(M_1 + M_2)$$

$$I_8 = 4 + 1.075 I_7 + 1.079G_2 + .079 (G_3 + G_4 + G_5) + .085(M_1 + M_2)$$

$$I_9 = 4 + 1.075 I_8 + 1.079G_3 + .079(G_4 + G_5) + .085(M_1 + M_2)$$

$$I_{10} = 5 + 1.075 I_9 + 1.079G_4 + .079G_5 + 1.085M_1 + .085M_2$$

all vars nonnegative

Solution: $z = 46.85$

$$I_1 \text{ to } I_5 = 0, I_6 = 4.63, I_7 = 9.61, I_8 = 15.47,$$

$$I_9 = 24.67, I_{10} = 37.52$$

$$G_1 = G_2 = 0, G_3 = 2.91, G_4 = 3.14, G_5 = 3.90,$$

$$M_1 = 1.96, M_2 = 2.12$$

Year	Recommendation
1	invest all in 9-yr bond
2	ditto
3	invest all in 6-yr bond
4	ditto
5	ditto
6	invest all in savings
7	ditto
8	ditto
9	ditto
10	ditto

44

x_{iA} = \$1K-amt invested in plan A, year i ($=1,2,3$)

x_{iB} = \$1K-amt invested in plan B, year i ($=1,2$)

Max $z = 1.7x_{3A} + 3x_{2B}$ s.t.

$$x_{1A} + 3x_{1B} \leq 100$$

$$-1.7x_{1A} + 3x_{2A} + x_{2B} = 0$$

$$-3x_{1B} - 1.7x_{2A} + x_{3A} = 0$$

all vars nonnegative

Solution: all values in \$1K

$$z = 46.85, x_{1A} = 100, x_{2B} = 170,$$

all other vars = 0

Alternative solution: $x_{1B} = 100, x_{3A} = 300,$

all other vars = 0

45

x_i = \$-amt allocated to choice i ($=1,2,3,4$)

y = minimum return

$$\text{Max } z = \min \begin{cases} -3x_1 + 4x_2 - 7x_3 + 15x_4 \\ 5x_1 - 3x_2 + 9x_3 + 4x_4 \\ 3x_1 - 9x_2 + 10x_3 - 8x_4 \end{cases} \text{ s.t.}$$

$$x_1 + x_2 + x_3 + x_4 \leq 500$$

$$x_1, x_2, x_3, x_4 \geq 0$$

The problem can be converted to an LP as

Max $z = y$

$$y = \min \begin{cases} -3x_1 + 4x_2 - 7x_3 + 15x_4 \\ 5x_1 - 3x_2 + 9x_3 + 4x_4 \\ 3x_1 - 9x_2 + 10x_3 - 8x_4 \end{cases} \Rightarrow$$

$$-3x_1 + 4x_2 - 7x_3 + 15x_4 \geq y$$

$$5x_1 - 3x_2 + 9x_3 + 4x_4 \geq y$$

$$3x_1 - 9x_2 + 10x_3 - 8x_4 \geq y$$

$$x_1 + x_2 + x_3 + x_4 \leq 500$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Solution: $z = \$1175, x_1 = x_2 = 0, x_3 = \$287.5, x_4 = \$212.5$

Chapter 2

46

$$i = \begin{cases} 1, & \text{regular savings} \\ 2, & \text{3-month CD} \\ 3, & \text{6-month CD} \end{cases}$$

x_{it} = \$-deposit in plan i at the start of month t

$$i = \begin{cases} 1, 2, \dots, 12 & \text{if } i = 1 \\ 1, 2, \dots, 10 & \text{if } i = 2 \\ 1, 2, \dots, 7 & \text{if } i = 3 \end{cases}$$

d_t = \$demand for period t

y_1 = initial amt on hand to ensure feasibility

r_i = interest rate for plan i ($i = 1, 2, 3$)

$$J_i = \begin{cases} 12, & \text{if } i = 1 \\ 10, & \text{if } i = 2 \\ 7, & \text{if } i = 3 \end{cases} \quad P_i = \begin{cases} 1, & \text{if } i = 1 \\ 3, & \text{if } i = 2 \\ 6, & \text{if } i = 3 \end{cases}$$

$$\text{Max } z = \sum_{t=1}^{12} \sum_{i=1}^3 r_i x_{i,t-P_i} - y_1 \quad \text{s.t.}$$

$$y_1 - x_{11} - x_{21} - x_{31} \geq d_1$$

$$1000 + \sum_{i=1|t>P_i}^3 (1+r_i)x_{i,t-P_i} - \sum_{i=1|t \leq J_i}^3 x_{it} \geq d_t, t=2, \dots, 12$$

all variables nonnegative

Solution: (see file *amplProb2-46.txt*)

y_1 = \$1200, z = \$1136.29

interest amt = $1200 - 1136.29 = \$63.71$

Deposits:

t	x1t	x2t	x3t
1	0	0	0
2	0	200	0
3	286.48	313.53	0
4	0	587.43	0
5	314.37	289.30	0
6	0	734.69	0
7	0	98.20	0
8	0	294.60	
9	0	848.60	
10	0	0	
11	0		
12	0		

47

x_{w1} = wrenches/wk (regular time)

x_{w2} = wrenches/wk (overtime)

x_{w3} = wrenches/wk (subcontracting)

x_{c1} = chisels/wk (regular time)

x_{c2} = chisels /wk (overtime)

x_{c3} = chisels /wk (subcontracting)

Min $z = 2x_{w1} + 2.8x_{w2} + 3x_{w3} + 2.1x_{c1} + 3.2x_{c2} + 4.2x_{c3}$

s.t.

$$x_{w1} \leq 550, x_{w2} \leq 250, x_{c1} \leq 620, x_{c2} \leq 280$$

$$x_{w1} + x_{w2} + x_{w3} \geq 1500$$

$$x_{c1} + x_{c2} + x_{c3} \geq 1200$$

$$\frac{x_{c1} + x_{c2} + x_{c3}}{x_{w1} + x_{w2} + x_{w3}} \geq 2$$

$$\Rightarrow 2(x_{w1} + x_{w2} + x_{w3}) - (x_{c1} + x_{c2} + x_{c3}) \leq 0$$

all vars nonnegative

(a) Solution: $z = \$14,918$

$$x_{w1} = 550, x_{w2} = 250, x_{w3} = 700$$

$$x_{c1} = 620, x_{c2} = 280, x_{c3} = 2100$$

(b) Increasing marginal unit costs ensures using the less expensive capacity before the more expensive ones. Else, if the cost functions are not monotonically increasing, additional constraints are needed to ensure the capacity restriction is satisfied.

48

x_j = units produced of product j ($=1, 2, 3, 4$)

Unit profit:

Product 1: $75 - 2 \times 10 - 3 \times 5 - 7 \times 4 = \12

Product 2: $70 - 3 \times 10 - 2 \times 5 - 3 \times 4 = \18

Product 3: $55 - 4 \times 10 - 1 \times 5 - 2 \times 4 = \2

Product 4: $45 - 2 \times 10 - 2 \times 5 - 1 \times 4 = \11

Max $z = 12x_1 + 18x_2 + 2x_3 + 11x_4$ s.t.

$$2x_1 + 3x_2 + 4x_3 + 2x_4 \leq 500$$

$$3x_1 + 2x_2 + 1x_3 + 2x_4 \leq 380$$

$$7x_1 + 3x_2 + 2x_3 + 1x_4 \leq 4500$$

Solution: $z = \$2950$

$$x_1 = 0, x_2 = 133.33, x_3 = 0, x_4 = 50$$

Chapter 2

49

x_j =units produced of model j ($=1,2,3,4$)

Max $z = 30x_1 + 20x_2 + 50x_3$ s.t.

$$(1) 2x_1 + 3x_2 + 5x_3 \leq 4000$$

$$(2) 4x_1 + 2x_2 + 7x_3 \leq 6000$$

$$(3) x_1 + .5x_2 + (1/3)x_3 \leq 1500$$

$$(4) x_1/3 = x_2/2 \rightarrow 2x_1 - 3x_2 = 0$$

$$(5) x_2/3 = x_3/5 \rightarrow 5x_2 - 2x_3 = 0$$

$$x_1 \geq 200, x_2 \geq 200, x_3 \geq 150$$

all vars nonnegative

Solution: $z = \$4108.08$

$$x_1 = 324.32, x_2 = 216.22, x_3 = 540.54$$

50

For $i = (1,2,3)$ and $j = (1,2)$

x_{ij} = cartons at start of month i from supplier j

I_i = end inventory in period i

c_{ij} = price per unit of x_{ij}

h = holding cost per unit per month

C = supplier capacity per month

d_i = demand for month i

$$\text{Min } z = \sum_{i=1}^3 \sum_{j=1}^2 c_{ij} x_{ij} + h \left[\sum_{i=1}^3 \left(\sum_{j=1}^2 \frac{(I_{i-1} + x_{ij}) + I_i}{2} \right) \right] \text{ s.t.}$$

$$\sum_{j=1}^2 I_{i-1} + x_{ij} + I_i = d_i, i = 1, 2, 3$$

$$x_{ij} \leq C, \text{ for all } i \text{ and } j$$

Solution:

i	x_{i1}	x_{i2}	I_i
1	400	100	0
2	400	400	200
3	200	0	0

51

x_i = production in quarter i

I_i = end inventory in quarter i

Min $z = 20x_1 + 22x_2 + 24x_3 + 26x_4 + 3.5(I_1 + I_2 + I_3)$ s.t.

$$x_1 = 300 + I_1, I_1 + x_2 = 400 + I_2, I_2 + x_3 = 450 + I_3,$$

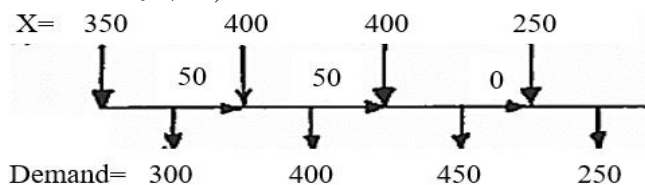
$$I_3 + x_4 = 250$$

$$x_i \leq 400, i = 1, 2, 3, I_i \leq 100, i = 1, 2, I_0 = I_4 = 0$$

continued...

51 cont'd

Solution: $z = \$32,250$



52

For $i = (1,2)$ and $j = (1,2,3)$

x_{ij} = quantity of product i in month j

I_{ij} = end inventory of product i in month j

$$\text{Min } z = 30 \sum_{j=1}^3 x_{1j} + 28 \sum_{j=1}^3 x_{2j} + .9 \sum_{j=1}^3 I_{1j} + .75 \sum_{j=1}^3 I_{2j} \text{ s.t.}$$

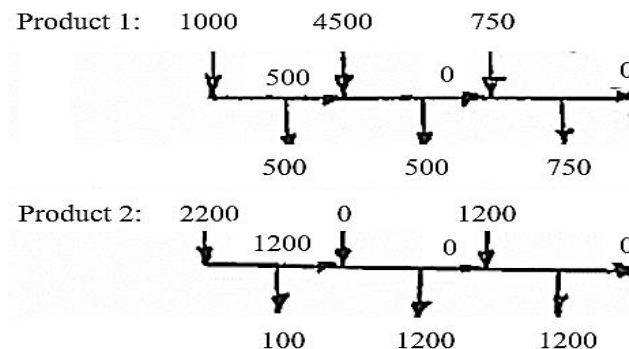
$$\frac{x_{1j}}{1.25} + x_{2j} \leq \begin{cases} 3000, j = 1 \\ 3500, j = 2 \\ 3000, j = 3 \end{cases}$$

$$I_{1,j-1} + x_{1j} - I_{1j} \leq \begin{cases} 500, j = 1 \\ 5000, j = 2 \\ 750, j = 3 \end{cases} \quad I_{i0} = 0, i = 1, 2$$

$$I_{2,j-1} + x_{2j} - I_{2j} \leq \begin{cases} 1000, j = 1 \\ 1200, j = 2 \\ 1200, j = 3 \end{cases}$$

all vars nonnegative

Solution: Cost $z = \$39720$



Chapter 2

53

For $i = (1, 2)$ and $j = (1, 2, 3)$

x_{ij} = quantity by operation i in month j

I_{ij} = entering inventory of operation i in month j

$$\text{Min } z = .2 \sum_{j=1}^2 I_{1j} + .4 \sum_{j=1}^2 I_{2j} \\ + 10x_{11} + 12x_{12} + 11x_{13} + 15x_{21} + 18x_{22} + 16x_{23}$$

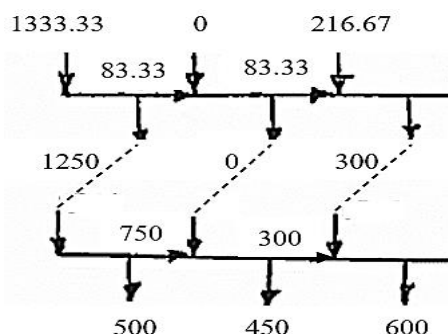
s.t.

$$.6x_{11} \leq 800, .6x_{12} \leq 700, .6x_{13} \leq 550$$

$$.8x_{21} \leq 1000, .8x_{22} \leq 850, .8x_{23} \leq 700$$

$$\left. \begin{aligned} x_{1j} + I_{1,j-1} &= x_{2j} + I_{1j} \\ x_{2j} + I_{2,j-1} &= x_{2j} + I_{2j} + d_j \end{aligned} \right\}, I_{i0} = 0 \text{ for } i=1, 2$$

Solution: Cost $z = \$39,720$



54

x_j = units of product j , $j = 1, 2$

y_i^- = unused hours of machine i

y_i^+ = overtime hours of machine i $i=1, 2$

y_i^+ = overtime hours of machine i

$$\text{Max } z = 110x_1 + 118x_2 - 100(y_1^+ + y_2^+) \text{ s.t.}$$

$$\frac{x_1}{5} + \frac{x_2}{5} + y_1^- - y_1^+ = 8$$

$$\frac{x_1}{8} + \frac{x_2}{4} + y_2^- - y_2^+ = 8$$

all vars. nonnegative

Solution: Revenue $z = \$6,232$

$$x_1 = 56, x_2 = 4, y_1^+ = 4 \text{ hrs}, y_2^+ = 0, y_1^- = 0, y_2^- = 0$$

55

x_i = nbr of 8-hr shift buses in period i

y_i = nbr of 12-hr-shift buses in period i

p = regular pay rate/hr

8-hour pay = $\$8p$

12-hour pay = $8p + 4 \times (1.5p) = \$14p$

$$\text{Min } z = p \left(8 \sum_{i=1}^6 x_i + 14 \sum_{i=1}^6 y_i \right) \text{ s.t.}$$

x						y							
1	2	3	4	5	6	1	2	3	4	5	6		
1						1				1	1	$\geq$	4
1	1					1	1				1	$\geq$	8
	1	1				1	1	1				$\geq$	10
		1	1				1	1	1			$\geq$	7
			1	1				1	1	1		$\geq$	12
				1	1				1	1	1	$\geq$	4

Solution: $z = \$196p$

$x_1 = 4, x_2 = 4, x_3 = 0, x_4 = 2, x_5 = 4, x_6 = 0$, sum = 14 8-hr

$y_1 = 0, y_2 = 0, y_3 = 6, y_4 = 0, y_5 = 0, y_6 = 0$, sum = 6 12-hr

Total nbr of buses = 14 8-hr and 6 12-hr = 20

8-hr buses only solution (see Example 2.4-5):

nbr of buses = 26

cost: $8p \times 26 = \$208p$

Conclusion: A mix of 8-hr and 12-hr buses is cheaper.

Chapter 2

56

x_i = nbr of volunteers starting in hour i

$$\text{Min } z = \sum_{i=1}^{14} x_i \text{ s.t.}$$

	x														
	1	2	3	4	5	6	7	8	9	10	11	12	13		
8:00	1													$\geq$	4
9:00	1	1												$\geq$	4
10:00	1	1												$\geq$	6
11:00		1	1											$\geq$	6
12:00			1	1										$\geq$	8
1:00			1	1	1									$\geq$	8
2:00				1	1	1								$\geq$	6
3:00					1	1	1							$\geq$	6
4:00						1	1	1						$\geq$	4
5:00							1	1	1	1				$\geq$	4
6:00								1	1	1	1			$\geq$	6
7:00									1	1	1	1		$\geq$	6
8:00											1	1	1	$\geq$	8
9:00												1	1	$\geq$	8

Solution: 32 volunteers

$$x_1=4, x_3=2, x_4=6, x_6=2,$$

$$x_7=4, x_{10}=6, x_{12}=8$$

All other variables are zero

57

Add the constraints $x_5=0$ and $x_{11}=0$ in Problem 2-56.

The solution in 2-56 happened to satisfy these constraints.

58

x_i = nbr of casualties starting on day i ($i=1$:Monday...)

$$\text{Min } z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \text{ s.t.}$$

	x								
	1	2	3	4	5	6	7		
M	1			1	1	1	1	$\geq$	20
T	1	1			1	1	1	$\geq$	14
W	1	1	1			1	1	$\geq$	10
Th	1	1	1	1			1	$\geq$	15
F	1	1	1	1	1			$\geq$	18
Sat		1	1	1	1	1		$\geq$	10
Sun			1	1	1	1	1	$\geq$	12

Solution: $z = 20$ worker

$$x_1 = 8, x_4 = 6, x_5 = 4, x_6 = 1, x_7 = 1, \text{ all others} = 0$$

59

x_i = nbr of casualties starting on day i

($i=1$:Monday...)

$$\text{Min } z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

s.t.

	x										
	1	2	3	4	5	6	7	8	9		
8:01	1									$\geq$	2
9:01	1	1								$\geq$	2
10:01	1	1	1							$\geq$	3
11:01		1	1	1						$\geq$	4
12:01			1	1						$\geq$	4
1:01				1		1				$\geq$	3
2:01						1	1			$\geq$	3
3:01						1	1	1		$\geq$	3
4:01							1	1	1	$\geq$	3

Solution: $z = 9$ worker

$$x_1 = 2, x_3 = 1, x_4 = 3, x_7 = 3, \text{ all others} = 0$$

Chapter 2

60

Let x_i = Nbr. of workers starting shift on day i , lasting for 7 days

y_{ij} = Nbr. of worker starting shift on day i and starting their 2 days off on day j , $i \neq j$

Thus, of the x_i workers who start on Monday, y_{12} will take T and W off, y_{13} will take W and Th off, as the following table shows:

	x1	x2	x3	4	x5	x6	x7
1	start x1 on M	y_{12}	$y_{12}+y_{13}$	$y_{13}+y_{14}$	$y_{14}+y_{15}$	$y_{15}+y_{16}$	$y_{16}+y_{17}$
2	y_{27}	start x2 on T	y_{23}	$y_{23}+y_{24}$	$y_{24}+y_{25}$	$y_{25}+y_{26}$	$y_{26}+y_{27}$
3	$y_{31}+y_{37}$	y_{31}	start x3 on W	y_{34}	$y_{34}+y_{35}$	$y_{35}+y_{36}$	$y_{36}+y_{37}$
4	$y_{41}+y_{47}$	$y_{41}+y_{42}$	y_{42}	start x4 on Th	y_{45}	$y_{45}+y_{46}$	$y_{46}+y_{47}$
5	$y_{51}+y_{57}$	$y_{51}+y_{52}$	$y_{53}+y_{54}$	$y_{54}+y_{55}$	start x5 on F	y_{56}	$y_{56}+y_{57}$
6	$y_{61}+y_{67}$	$y_{61}+y_{62}$	$y_{62}+y_{63}$	$y_{63}+y_{64}$	y_{64}	x6 start on S	y_{67}
7	y_{71}	$y_{71}+y_{72}$	$y_{72}+y_{73}$	$y_{73}+y_{74}$	$y_{74}+y_{75}$	y_{75}	x7 start on Su
	12	18	20	28	32	40	40

Let

b_i = min number of workers needed for day i

$$s = \sum_{i=1}^7 x_i$$

LP model: Minimize $z = s$, subject to

$$s - \sum_{i=1}^7 x_i = 0$$

$$s - \sum_{i \neq j} \{ \text{all } y_{ij} \text{ in column (day) } i \} \geq b_i, i = 1, 2, \dots, 7$$

$$x_i - \sum_{\substack{j=1 \\ j \neq i}}^7 y_{ij} = 0, i = 1, 2, \dots, 7$$

continued...

continued...

Chapter 2

60 cont'd

Solution: 42 workers

Starting on	Nbr	Nbr off						
Day		Non	Tue	Wed	Th	Fri	Sat	Sun
Mon	16		16	16				
Tu	8				8	8		
Wed	8	8	8					
Thu	0							
Fri	6			6	6			
Sat	2	2						2
Sun	2					2	2	
Nbr off		10	24	22	14	10	2	2
Nbr at work		32	18	20	28	32	40	40
Surplus		22	0	0	0	0	0	0

Chapter 2

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x_e = nbr of efficiency apartments
 x_d = nbr of duplexes
 x_s = nbr of single-family homes
 x_r = retail space in ft²

$$\text{Max } z = 600x_e + 750x_d + 1200x_s + 100x_r$$

s.t.

$$x_e \leq 500, x_d \leq 300, x_s \leq 250$$

$$x_r \geq 10x_e + 15x_d + 18x_s$$

$$x_r \leq 10,000$$

$$x_d \geq \frac{x_e + x_s}{2}$$

all var nonnegative

$$\text{Solution: } z = \$1,595,714.29$$

$$x_e = 207.14 \approx 207, x_d = 228.57 \approx 229, x_s = 250,$$

$$x_r = 10,000$$

LP does not generate integer solution, hence is the rounding. See Chapter 9 for ILP.

62

x_i = acquired portion of property i

Each site is represented by a separate LP. The site that yields the smallest objective value is selected.

Site 1 LP:

$$\text{Min } z = 25 + x_1 + 2.1x_2 + 2.35x_3 + 1.85x_4 + 2.95x_5$$

s.t.

$$x_4 \geq .75, x_i \leq 1, i=1, 2, \dots, 5$$

$$20x_1 + 50x_2 + 50x_3 + 30x_4 + 60x_5 \geq 200$$

$$\text{Solution: } z = \$34.6625M$$

$$x_1 = .875, x_2 = 1, x_3 = 1, x_4 = .75, x_5 = 1$$

Site 2 LP:

$$\text{Min } z = 27 + x_1 + 2.8x_2 + 1.9x_3 + 2.85x_4 + 2.5x_5$$

s.t.

$$x_3 \geq .5, x_i \leq 1, i=1, 2, 3, 4$$

$$80x_1 + 60x_2 + 50x_3 + 70x_4 \geq 200$$

$$\text{Solution: } z = \$34.35M$$

$$x_1 = 1, x_2 = 1, x_3 = .5, x_4 = .5$$

Select site 2

63

x_{ij} = portion of project i completed in year j

$$\text{Years of income for project } i = \sum_{j=1}^5 (5-j)x_{ij}$$

$$\begin{aligned} \text{Max } z = & .05(4x_{11} + 3x_{12} + 2x_{13}) + \\ & .07(3x_{22} + 2x_{23} + x_{24}) + \\ & .15(4x_{31} + 3x_{32} + 2x_{33} + x_{34}) + \\ & .02(2x_{43} + x_{44}) \end{aligned}$$

s.t.

Completion:

$$x_{11} + x_{12} + x_{13} = 1 \quad (\text{project 1})$$

$$x_{43} + x_{44} = 1 \quad (\text{project 4})$$

$$.25 \leq x_{22} + x_{23} + x_{24} + x_{25} \leq 1 \quad (\text{project 2})$$

$$.25 \leq x_{31} + x_{32} + x_{33} + x_{34} + x_{35} \leq 1 \quad (\text{project 3})$$

Budget:

$$5x_{11} + 15x_{31} \leq 3 \quad (\text{year 1})$$

$$5x_{12} + 8x_{22} + 15x_{32} \leq 6 \quad (\text{year 2})$$

$$5x_{13} + 8x_{23} + 15x_{33} + 1.2x_{43} \leq 7 \quad (\text{year 3})$$

$$8x_{24} + 15x_{34} + 1.2x_{44} \leq 7 \quad (\text{year 4})$$

$$8x_{25} + 15x_{35} \leq 7 \quad (\text{year 5})$$

all vars nonnegative

$$\text{Solution: } z = \$523,750$$

$$x_{11} = .6, x_{12} = .4 \quad (\text{Project 1 completed in yr 2})$$

$$x_{24} = .225, x_{25} = .025 \quad (\text{Project 2 25\% done in yr 5})$$

$$x_{32} = .267, x_{33} = .387, x_{34} = .346 \quad (\text{Project 3 completed in yr 5})$$

$$x_{43} = 1 \quad (\text{Project 4 completed in yr 3})$$

64

x_L = nbr of low income units

x_m = nbr of middle income units

x_u = nbr of upper income units

x_p = nbr of public housing units

x_s = nbr of school rooms

x_r = nbr of retail units

x_c = nbr of condemned homes

$$\text{Max } z = 7x_L + 12x_m + 20x_u + 5x_p - 10x_s + 15x_r - 7x_c \quad \text{s.t.}$$

$$100 \leq x_L \leq 200, 125 \leq x_m \leq 190,$$

$$75 \leq x_u \leq 260, 0 \leq x_p \leq 600, 0 \leq x_c \leq 2/.045$$

$$.05x_L + .07x_m + .03x_u + .025x_p + .045x_s + .1x_r \leq$$

$$.85(50 + .25x_c)$$

$$x_r \geq .023x_L + .034x_m + .046x_u + .023x_p + .034x_s$$

$$25x_s \geq 1.3x_L + 1.2x_m + .5x_u + 1.4x_p$$

continued...

Chapter 2

64 cont'd

Solution: $z = \$8290.30$

$x_L = 100$, $x_m = 125$, $x_u = 227.0$, $x_p = 300$, $x_s = 32.54$,
 $x_r = 25$, $x_c = 0$.

65

x_1 = nbr of single-family homes

x_2 = nbr of double-family homes

x_3 = nbr of triple-family homes

x_4 = nbr of recreations areas

Max $z = 10,000x_1 + 12,000x_2 + 15,000x_3$ s.t.

$2x_1 + 3x_2 + 4x_3 + x_4 \leq .85 \times 800$

$\frac{x_1}{x_1 + x_2 + x_3} \geq .5$ or $.5x_1 - .5x_2 - .5x_3 \geq 0$

$x_4 \geq \frac{x_1 + 2x_2 + 3x_3}{200}$ or $200x_4 - x_1 - 2x_2 - 3x_3 \geq 0$

$1000x_1 + 1200x_2 + 1400x_3 + 450x_4 \geq 100,000$

$400x_1 + 600x_2 + 840x_3 + 450x_4 \leq 200,000$

all vars nonnegative

Solution: $z = \$3,391,521.20$

$x_1 = 339.15$, $x_2 = 0$, $x_3 = 0$, $x_4 = 1.69$ areas

66

New land use constraint: $x_1 + x_2 + x_3 + x_4$

$2x_1 + 3x_2 + 4x_3 + x_4 \leq .85 \times (800 + 100)$

Solution: $z = \$3,815,461.35$

$x_1 = 381.54$, $x_2 = 0$, $x_3 = 0$, $x_4 = 1.91$ areas

$\Delta z = 3,815,461.35 - 3,391,521.20 = \$423,940$

$\Delta z < \$450,000$. The purchasing cost of 100 new acres. Purchase of additional acres is not recommended.

67

x_s = tons of strawberry/day

x_g = tons of grapes/day

x_a = tons of apples/day

x_A = cans of drink A/day

x_B = cans of drink B/day

x_C = cans of drink C/day

x_{sA} = lb strawberry in drink A/day

x_{sB} = lb strawberry in drink B/day

x_{gA} = lb grapes in drink A/day

x_{gB} = lb grapes in drink B/day

x_{gC} = lb grapes in drink C/day

x_{aB} = lb apples in drink B/day

x_{aC} = lb apples in drink C/day

Max $z = 1.15x_A + 1.25x_B + 1.2x_C - 200x_s - 100x_g - 90x_a$
 s.t.

$x_a \leq 200$, $x_g \leq 100$, $x_s \leq 150$

$x_{sA} + x_{sB} = 1500x_s$

$x_{gA} + x_{gB} + x_{gC} = 1200x_g$

$x_{aB} + x_{aC} = 1000x_a$

$x_A = x_{sA} + x_{gA}$

$x_B = x_{sB} + x_{gB} + x_{aB}$

$x_C = x_{gC} + x_{aC}$

$x_{sA} = x_{gA}$

$x_{sB} = x_{gB}$

$x_{sB} = .5x_{aB}$

$3x_{gC} = 2x_{aC}$

Solution: $x_A = x_{sA} + x_{gA}$

$x_B = x_{sB} + x_{gB} + x_{aB}$

$x_A = 90,000$ cans, $x_B = 300,000$ cans, $x_C = 0$

		j		
	i	A	B	C
x_{ij}	s	45,000	75,000	0
	g	45,000	75,000	0
	a	0	150,000	0

Chapter 2

68

x_s = lb of screws/package

x_b = lb of bolts/package

x_n = lb of nuts/package

x_w = lb of washers/package

Min $z = 1.1x_s + 1.5x_b + (70/80)x_n + (20/30)x_w$ s.t.

$Y = x_s + x_b + x_n + x_w$

$x_s \geq .1Y$, $x_b \geq .25Y$, $x_b \leq 50x_w$, $x_b \leq 10x_n$

$x_n \leq .15Y$, $Y \geq 1$

all vars nonnegative

Solution: $z = \$1.12$

$Y = 1$, $x_s = .5$, $x_b = .25$, $x_n = .15$, $x_w = .1$

69

x_{oA} , x_{oB} , x_{oC} = lb of oats in cereal A, B, C

x_{rA} , x_{rC} = lb of rasins in cereal A, C

x_{cB} , x_{cC} = lb of coconuts in cereal B, C

x_{aA} , x_{aB} , x_{aC} = lb of almond in cereal A, B, C

Max $z = \frac{1}{5}(2W_A + 2.5W_B + 3W_C) - \frac{1}{2000}(100Y_o + 120Y_r + 110Y_c + 200Y_a)$ s.t.

$Y_o = x_{oA} + x_{oB} + x_{oC}$

$Y_r = x_{rA} + x_{rC}$

$Y_c = x_{cB} + x_{cC}$

$Y_a = x_{aA} + x_{aB} + x_{aC}$

$W_A = x_{oA} + x_{rA} + x_{aA}$

$W_B = x_{oB} + x_{cB} + x_{aB}$

$W_C = x_{oC} + x_{rC} + x_{cC} + x_{aC}$

$W_A \geq 500 \times 5 (= 2500)$

$W_B \geq 600 \times 5 (= 3000)$

$W_C \geq 800 \times 5 (= 4000)$

$Y_o \leq 5 \times 2000 (= 10,000)$

$Y_r \leq 2 \times 2000 (= 4,000)$

$Y_c \leq 1 \times 2000 (= 2,000)$

$Y_a \leq 1 \times 2000 (= 2,000)$

continued...

69 cont'd

$x_{oA} = \frac{50}{5}x_{rA}$, $x_{oA} = \frac{50}{2}x_{aA}$

$x_{oB} = \frac{60}{5}x_{cB}$, $x_{oB} = \frac{60}{3}x_{aB}$

$x_{oC} = \frac{60}{3}x_{rC}$, $x_{oC} = \frac{60}{4}x_{cC}$, $x_{oC} = \frac{60}{2}x_{aC}$

all vars nonnegative

Solution: $z = \$5384.84/\text{day}$

$W_A = 2500$ lb or 500 boxes/day

$W_B = 3000$ lb or 600 boxes/day

$W_C = 5793.45$ lb or ≈ 1158 boxes/day

$Y_o = 10,000$ lb or 5 tons/day

$Y_r = 471.19$ lb or .214 ton/day

$Y_c = 428.16$ lb or .236 ton/day

$Y_a = 394.11$ lb or .197 ton/day

70

For $i = 1, 2$,

x_{Ai} = bbl of gasoline A in fuel i

x_{Bi} = bbl of gasoline B in fuel i

x_{Ci} = bbl of gasoline C in fuel i

x_{Di} = bbl of gasoline D in fuel i

Max $z = 200F_1 + 250F_2 - (120Y_A + 90Y_B + 100Y_C + 150Y_D)$ s.t.

$Y_A = x_{A1} + x_{A2}$, $Y_B = x_{B1} + x_{B2}$

$Y_C = x_{C1} + x_{C2}$, $Y_D = x_{D1} + x_{D2}$

$F_1 = x_{A1} + x_{B1} + x_{C1} + x_{D1}$

$F_2 = x_{A2} + x_{B2} + x_{C2} + x_{D2}$

$x_{A1} = x_{B1}$, $x_{A1} = .5x_{C1}$, $x_{A1} = .25x_{D1}$

$x_{A2} = x_{B2}$, $x_{A2} = 2x_{C2}$, $x_{A2} = (2/3)x_{D2}$

$Y_A \leq 1000$, $Y_B \leq 1200$, $Y_C \leq 900$, $Y_D \leq 1500$

$F_1 \geq 200$, $F_2 \geq 400$

Solution: $z = 495,416.67$

$Y_A = 958.33$ bbl/day, $Y_B = 958.33$ bbl/day

$Y_C = 516.67$ bbl/day, $Y_D = 1500$ bbl/day

$F_1 = 200$ bbl/day, $F_2 = 3733.33$ bbl/day

Chapter 2

71

A = bbl crude A/day

B = bbl crude B/day

R = bbl regular/day

P = bbl premium/day

J = bbl jet/day

+/- superscript $\equiv (R, P, J)$ surplus/shortage

$$\begin{aligned} \text{Max } z = & 50(R - R^+) + 70(P - P^+) + 120(J - J^+) \\ & - (10R^- + 15P^- + 20J^-) - (2R^+ + 3P^+ + 4J^+) \\ & - (30A + 40B) \quad \text{s.t.} \end{aligned}$$

$$A \leq 2500, B \leq 3000$$

$$R = .2A + .25B, R + R^- - R^+ = 500$$

$$P = .1A + .3B, P + P^- - P^+ = 700$$

$$J = .25A + .1B, J + J^- - J^+ = 400$$

$$\text{Solution: } z = \$21,852.94$$

$$A = 1176.47 \text{ bbl/day}, B = 1058.82 \text{ bbl/day}$$

$$R = 500 \text{ bbl/day}, P = 435.29 \text{ bbl/day}$$

$$J = 400 \text{ bbl/day}$$

72

NR = nafta bbl/day in regular

NP = nafta bbl/day in premium

NJ = nafta bbl/day in jet

LR = light oil bbl/day in regular

LP = light oil bbl/day in premium

LJ = light oil bbl/day in jet

+/- superscript $\equiv (R, P, J)$ surplus/shortage

72 cont'd

$$\begin{aligned} \text{Max } z = & 50(R - R^+) + 70(P - P^+) + 120(J - J^+) \\ & - (10R^- + 15P^- + 20J^-) - (2R^+ + 3P^+ + 4J^+) \\ & - (30A + 40B) \quad \text{s.t.} \end{aligned}$$

$$A \leq 2500, B \leq 3000$$

$$R + R^- - R^+ = 500$$

$$P + P^- - P^+ = 700$$

$$J + J^- - J^+ = 400$$

$$.35A + .45B = NR + NP + NJ$$

$$.6A + .5B = LR + LP + LJ$$

$$R = NR + LR, P = NP + LP, J = NJ + LJ$$

$$\text{Solution: } z = \$71,473.68$$

$$A = 1684.21 \text{ bbl/day}, B = 0$$

$$R = 500 \text{ bbl/day}, P = 700 \text{ bbl/day}$$

$$J = 400 \text{ bbl/day}$$

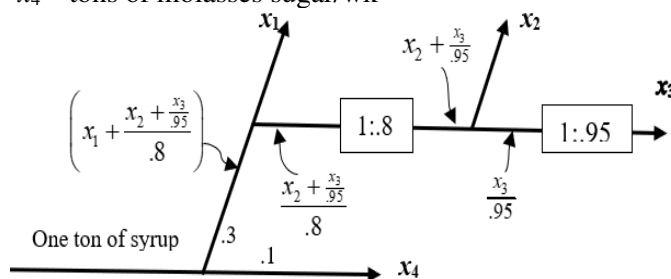
73

x_1 = tons of brown sugar/wk

x_2 = tons of white sugar/wk

x_3 = tons of powdered sugar/wk

x_4 = tons of molasses sugar/wk



$$\text{Max } z = 150x_1 + 200x_2 + 230x_3 + 35x_4 \quad \text{s.t.}$$

$$x_4 \leq 4000 \times .1 (= 400)$$

$$x_1 + \left(\frac{x_2 + \frac{x_3}{.95}}{.8} \right) \leq .3 \times 4000 \text{ or } .76x_1 + .95x_2 + x_3 \leq 912$$

$$x_1 \geq 25, x_2 \geq 25, x_3 \geq 25, x_4 \geq 25$$

$$\text{Solution: } z = \$222,667.50$$

$$x_1 = 25, x_2 = 25, x_3 = 869.25, x_4 = 400 / \text{wk}$$

2-21

Chapter 2

74

A = bbl/hr of stock A

B = bbl/hr of sock B

for $i = (1, 2)$:

Y_{Ai} = bbl/hr of A in gasoline i

Y_{Bi} = bbl/hr of B in gasoline i

Max $z = 7(Y_{A1} + Y_{B1}) + 10(Y_{A2} + Y_{B2})$ s.t.

$A = Y_{A1} + Y_{A2}, A \leq 450$

$B = Y_{B1} + Y_{B2}, B \leq 700$

$98Y_{A1} + 89Y_{B1} \geq 91(Y_{A1} + Y_{B1})$

$98Y_{A2} + 89Y_{B2} \geq 93(Y_{A2} + Y_{B2})$

$10 \geq 12(Y_{A1} + Y_{B1})$

$10Y_{A2} + 8Y_{B2} \geq 12(Y_{A2} + Y_{B2})$

all vars no $Y_{A1} + 8Y_{B1}$ negative

Solution: $z = \$10,675$

$A = 450$ bl/hr, $B = 700$ bl/hr

gas 1 produced = $Y_{A1} + Y_{B1}$
 $= 61.11 + 213.89 = 275$ bbl/hr

gas 2 produced = $Y_{A2} + Y_{B2}$
 $= 385.89 + 486.11 = 875$ bbl/hr

75

S = steel scrap tons/day

A = aluminum scrap tons/day

C = cast iron scrap tons/day

Ab = aluminum briquette tons/day

Sb = silicon briquette tons/day

a = tons of aluminum/day

g = tons of graphite/day

s = tons of silicon/day

continued...

75 cont'd

aI = tons of alum. ingots-I/day

aII = tons of alum. ingots-II/day

gI = tons of graph. ingots-I/day

gII = tons of graph. ingots-II/day

sI = tons of silicon ingots-I/day

sII = tons of silicon ingots-II/day

I_1 = tons of ingot-I/day

I_2 = tons of ingot-II/day

Min $z = 100S + 150A + 75C + 900Ab + 380Sb$ s.t.

$a = .1S + .95A + Ab$

$g = .05S + .01A + .15C$

$s = .04S + .02A + .08C + Sb$

$I_1 = aI + gI + sI$

$I_2 = aII + gII + sII$

$aI + aII \leq a, sI + sII \leq s, gI + gII \leq g$

$.081I_1 \leq aI \leq .108I_1$

$.015I_1 \leq gI \leq .03I_1$

$.025I_1 \leq sI \leq \infty$

$.062I_2 \leq aII \leq .089I_2$

$.015I_2 \leq gII \leq \infty$

$.025I_2 \leq sII \leq .041I_2$

$I_1 \geq 130, I_2 \geq 250$

Solution: $z = \$117,435.65$

$S = 0, A = 38.2, C = 1489.41$

$Ab = Sb = 0$

$I_1 = 130, I_2 = 250$

$a = 36.29, g = 223.79, s = 119.92$

76

For $i = 1, 2, 3; j = A, B; k = I, II, III, IV$:

x_{ij} = tons of ore i allocated to alloy j

W_j = tons of alloy j produced

M_k = tons of metal k extracted from all three ores

continued...

Chapter 2

76 cont'd

$$\text{Min } z = 200W_A + 300W_B - 30(x_{1A} + x_{1B}) - 40(x_{2A} + x_{2B}) - 50(x_{3A} + x_{3B}) \text{ s.t.}$$

Alloy A specs:

$$\left. \begin{aligned} M_I &= .2x_{1A} + .1x_{2A} + .05x_{3A} \\ M_I &\leq .8W_A \end{aligned} \right\} \equiv .2x_{1A} + .1x_{2A} + .05x_{3A} \leq .8W_A$$

$$M_{II}: .1x_{1A} + .2x_{2A} + .05x_{3A} \leq .3W_A$$

$$M_{IV}: .3x_{1A} + .3x_{2A} + .2x_{3A} \leq .5W_A$$

Alloy B specs:

$$M_{II}: .1x_{1B} + .2x_{2B} + .05x_{3B} \geq .4W_B$$

$$M_{II}: .1x_{1B} + .2x_{2B} + .05x_{3B} \leq .6W_B$$

$$M_{III}: .3x_{1B} + .3x_{2B} + .7x_{3B} \geq .3W_B$$

$$M_{IV}: .3x_{1B} + .3x_{2B} + .2x_{3B} \leq .7W_B$$

Ore constraints:

$$x_{1A} + x_{1B} \leq 1000$$

$$x_{2A} + x_{2B} \leq 2000$$

$$x_{3A} + x_{3B} \leq 3000$$

$$\text{Solution: } z = 400,000$$

$$W_A = 1800, W_B = 1000$$

$$x_{1A} = 1000, x_{1B} = 0, x_{2A} = 0$$

$$x_{2B} = 2000, x_{3A} = 3000, x_{3B} = 0$$

77

x_i = Shelf space (in²) allocated to cereal i

$$\text{Max } z = 1.1x_1 + 1.3x_2 + 1.08x_3 + 1.25x_4 + 1.25x_5 \text{ s.t.}$$

$$16x_1 + 24x_2 + 14x_3 + 22x_4 + 20x_5 \leq 5000$$

$$x_1 \leq 100, x_2 \leq 85, x_3 \leq 140, x_4 \leq 80, x_5 \leq 90$$

all vars nonnegative

$$\text{Solution: } z = \$314/\text{day}$$

$$x_1 = 100, x_2 = 0, x_3 = 140, x_4 = 0, x_5 = 44$$

78

x_i = nbr of ads in issue i ($=1,2,3,4$)

$$\text{Min } z = S_1^- + S_2^- + S_3^- + S_4^- \text{ s.t.}$$

$$(-30 + 60 + 30)x_1 + S_1^- - S_1^+ = .51(400)$$

$$(80 + 30 - 45)x_2 + S_2^- - S_2^+ = .51(400)$$

$$(40 + 10)x_3 + S_3^- - S_3^+ = .51(400)$$

$$(90 - 25)x_4 + S_4^- - S_4^+ = .51(400)$$

$$1500(x_1 + x_2 + x_3 + x_4) \leq 100,000$$

all vars nonnegative

$$\text{Solution: } x_1 = 3.4, x_2 = 3.14, x_3 = 4.08, x_4 = 3.14$$

79

For $i=1,2$ and $j=1,2,3$

x_{ij} = units of part j produced by department i

$$\text{Max } z = \min\{x_{11} + x_{21}, x_{12} + x_{22}, x_{13} + x_{23}\}$$

or

$$\text{Max } z = y \text{ s.t.}$$

$$y \leq x_{11} + x_{21}$$

$$y \leq x_{12} + x_{22}$$

$$y \leq x_{13} + x_{23}$$

$$\frac{x_{11}}{8} + \frac{x_{12}}{5} + \frac{x_{13}}{10} \leq 100$$

$$\frac{x_{21}}{6} + \frac{x_{22}}{12} + \frac{x_{23}}{4} \leq 80$$

all vars nonnegative

Solution:

$$\text{nbr of assembly units} = y = 556.2 = 556$$

$$x_{11} = 354.78, x_{12} = 0, x_{13} = 556.52$$

$$x_{21} = 201.79, x_{22} = 556.52, x_{23} = 556.52 = 0$$

80

x_i = tons of coal i ($=1,2,3$)

$$\text{Min } z = 30x_1 + 35x_2 + 33x_3 \text{ s.t.}$$

$$2500x_1 + 1500x_2 + 1600x_3 \leq 2000(x_1 + x_2 + x_3)$$

$$x_1 \leq 30, x_2 \leq 30, x_3 \leq 30$$

$$x_1 + x_2 + x_3 \geq 50$$

continued...

Chapter 2

80 cont'd

Solution: $z = 1361.11$

$x_1 = 22.22, x_2 = 0, x_3 = 27.78$

81

t_i = green time in sec for highway i ($=1, 2, 3$)

$$\max z = 3\left(\frac{500}{3600}\right)t_1 + 4\left(\frac{600}{3600}\right)t_2 + 5\left(\frac{400}{3600}\right)t_3$$

$$\left(\frac{500}{3600}\right)t_1 + \left(\frac{600}{3600}\right)t_2 + \left(\frac{400}{3600}\right)t_3 \leq \left(\frac{510}{3600}\right)(2.2 \times 60 - 3 \times 10)$$

$$t_1 + t_2 + t_3 + 3 \times 10 \leq 2.2 \times 20$$

Solution: $z = \$58.04/\text{hr}$

$t_1 = 25, t_2 = 43.6, t_3 = 33.4$

82

y_i = observation i ($=1, 2, \dots, 10$)

fitted straight line: $\hat{y}_i = ai + b$

Let $d_i = |y_i - (ai + b)|, i = 1, 2, \dots, 10$

$$\text{Min } z = d_1 + d_2 + \dots + d_{10}$$

$$\sum_{i=1}^{10} |y_i - (ai + b)| \text{ s.t.}$$

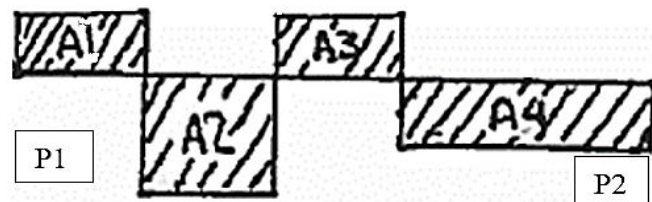
$$y_i - (ai + b) \leq d_i$$

$$y_i - (ai + b) \geq -d_i$$

all $d_i \geq 0, a$ and b unrestricted

Solution: $\hat{y}_i = 2.85714i + 6.42857$

83



$A1 = (2 \text{ mile} \times 1760 \text{ yard/mile}) \times 10 \text{ yard} \times 50 \text{ yard}$
 $= +1760 \text{K cubic yard}$

$A2 = -3520 \text{K}, A3 = +1760 \text{K}, A4 = -3520 \text{K}$

continued...

83 cont'd

center-to-center distances in miles =

(5)A2 (6)A4

(1)A1	2	7
(2)A2	2	3
(3)P1	3	8
(4)P2	7	2

\$ transportation/cubic yard =

(5)A2 (6)A4

(1)A1	.2+2×.15=.50	.2+7×.15=1.25
(2)A2	.2+2×.15=.50	.2+3×.15=.65
(3)P1	(1.5+.2)+3×.15=2.15	(1.5+.2)+8×.15=2.90
(4)P2	(1.9+.2)+7×.15=3.15	(1.9+.2)+2×.15=2.40

Solution below in **bf**: see file *solverProb2-83.xlsx*

x15	x16	x25	x26	x35	x36	x45	x46	z
1760	0	0	1760	1760	0	0	1760	10032
0.5	1.25	0.5	0.65	2.15	2.9	3.15	2.4	10032
1	1							<= 1760
		1	1					<= 1760
				1	1			<= 20000
						1	1	<= 15000
		1		1		1		>= 3520
	1		1		1		1	>= 3520

$x_{15} = A1 \rightarrow A2, x_{26} = A3 \rightarrow A4$

$x_{55} = P1 \rightarrow A2, x_{46} = P2 \rightarrow A4$

84

x_{ij} = blue regulars on front i defending line j

y_{ij} = blue reserves on front i defending line j

t_{ij} = delay days on front i defending line j

Max $z = \min\{t_{11} + t_{12} + t_{13}, t_{21} + t_{22} + t_{23}\}$ s.t.

$$x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} \leq 200$$

$$y_{12} + y_{13} + y_{22} + y_{23} \leq 200$$

$$t_{11} = .5 + 8.8 \frac{x_{11}}{30}, t_{12} = .75 + 7.9 \frac{x_{12} + y_{12}}{60}$$

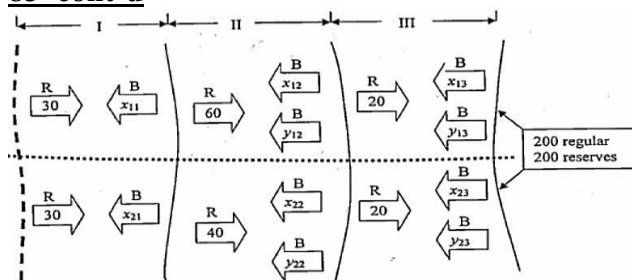
$$t_{13} = .55 + 10.2 \frac{x_{13} + y_{13}}{20}, t_{21} = 1.1 + 10.5 \frac{x_{21}}{30}$$

$$t_{22} = 1.3 + 8.1 \frac{x_{22} + y_{22}}{40}, t_{23} = 1.5 + 9.2 \frac{x_{23} + y_{23}}{20}$$

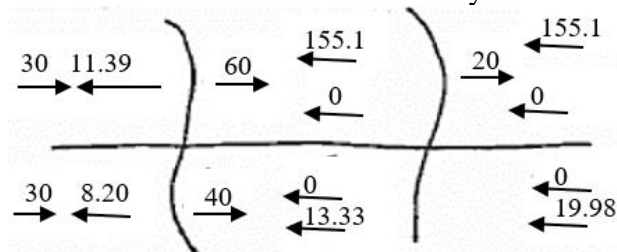
continued...

Chapter 2

83 cont'd



Solution: Battle duration = 87.65 days



85

x_i = efficiency of plant i ($i=1,2,3,4$)

Min $z = .2 \times 500x_1 + .25 \times 3000x_2$

$+ .15 \times 6000x_3 + .18 \times 1000x_4$ s.t.

$.94 \times 500(1-x_1) + 3000(1-x_2) \leq .0009 \times 220000$

$.94^2 \times 500(1-x_1) + .94 \times 3000(1-x_2) + 6000(1-x_3) \leq .0008 \times 200000$

$.94^3 \times 500(1-x_1) + .94^2 \times 3000(1-x_2) + .94 \times 6000(1-x_3) + 1000(1-x_4) \leq .0008 \times 210000$

$0 \leq x_1 \leq .99, 0 \leq x_2 \leq .99, 0 \leq x_3 \leq .99, 0 \leq x_4 \leq .99$

Solution: cost/hr = \$1891.41

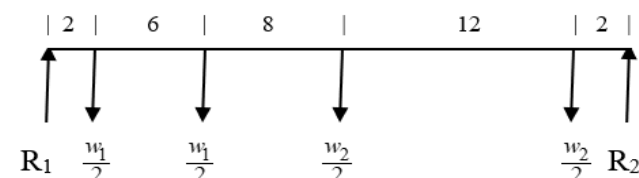
efficiency: $x_1 = .99, x_2 = .9661, x_3 = .99, x_4 = .9824$

86

w_i = capacity of yoke i in kips

R_1 = reaction in kips at left end

R_2 = reaction in kips at right end



continued...

86 cont'd

Max $z = w_1 + w_2$ s.t.

$R_1 + R_2 = w_1 + w_2$

$2\left(\frac{w_1}{2}\right) + 8\left(\frac{w_1}{2}\right) + 16\left(\frac{w_2}{2}\right) + 28\left(\frac{w_2}{2}\right) = 30R_2$

$R_1 \leq 25, R_2 \leq 25, \frac{w_1}{2} \leq 20, \frac{w_2}{2} \leq 20$

Solution: $w_1 = 20.59$ kips, $w_2 = 29.41$ kips

87

For $i=1,2,3, j=1,2,3,4$:

x_{ij} = nbr of aircraft type i allocated to route j

S_j = nbr of passengers denied service on route j

Min $z = 1000(3x_{11}) + 1100(2x_{12}) + 1200(2x_{13}) + 1500(x_{14}) + 800(4x_{21}) + 900(3x_{22}) + 1000(3x_{23}) + 1000(2x_{24}) + 600(5x_{31}) + 800(5x_{32}) + 800(4x_{33}) + 900(2x_{34}) + 40S_1 + 50S_2 + 45S_3 + 70S_4$ s.t.

$\sum_{j=1}^4 x_{1j} \leq 5, \sum_{j=1}^4 x_{2j} \leq 5, \sum_{j=1}^4 x_{3j} \leq 5,$

$50((3x_{11}) + 30(4x_{21}) + 20(5x_{31}) + S_1) \leq 1000$

$50(2x_{12}) + 30(3x_{22}) + 20(5x_{32}) + S_2 \leq 2000$

$50(2x_{13}) + 30(3x_{23}) + 20(4x_{33}) + S_3 \leq 900$

$50(x_{14}) + 30(3x_{24}) + 20(2x_{34}) + S_4 \leq 1200$

all vars nonnegative

Solution: cost = \$221,900

Type	Route	Nbr planes
1	1	5
2	4	8
3	1	2.5 ≈ 3
3	2	7.5 ≈ 8

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