

***Thomas' Calculus 15th edition Hass Solutions
Manual***

SKU: 9780137615582-SOLUTIONS

INSTRUCTOR'S SOLUTIONS MANUAL

JENNIFER A. BLUE

SUNY Empire State College

THOMAS' CALCULUS LATE TRANSCENDENTALS FIFTEENTH EDITION

Based on the original work by

George B. Thomas, Jr

Massachusetts Institute of Technology

as revised by

Joel Hass

University of California, Davis

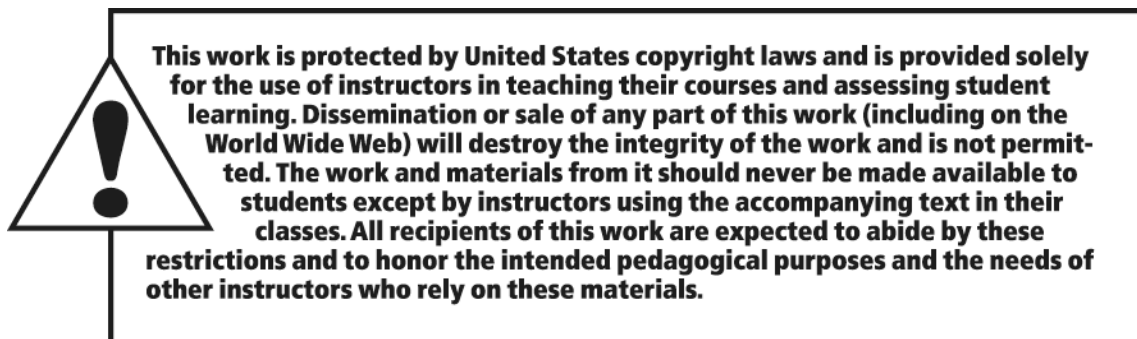
Christopher Heil

Georgia Institute of Technology

Maurice D. Weir

Naval Postgraduate School





The author and publisher of this book have used their best efforts in preparing this book. These efforts include the development, research, and testing of the theories and programs to determine their effectiveness. The author and publisher make no warranty of any kind, expressed or implied, with regard to these programs or the documentation contained in this book. The author and publisher shall not be liable in any event for incidental or consequential damages in connection with, or arising out of, the furnishing, performance, or use of these programs.

Reproduced by Pearson from electronic files supplied by the author.

Copyright © 2023, 2018, 2014 by Pearson Education, Inc. or its affiliates, 221 River Street, Hoboken, NJ 07030. All Rights Reserved. Manufactured in the United States of America. This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise. For information regarding permissions, request forms, and the appropriate contacts within the Pearson Education Global Rights and Permissions department, please visit www.pearsoned.com/permissions/.

PEARSON and REVEL are exclusive trademarks owned by Pearson Education, Inc. or its affiliates in the U.S. and/or other countries.

Unless otherwise indicated herein, any third-party trademarks, logos, or icons that may appear in this work are the property of their respective owners, and any references to third-party trademarks, logos, icons, or other trade dress are for demonstrative or descriptive purposes only. Such references are not intended to imply any sponsorship, endorsement, authorization, or promotion of Pearson's products by the owners of such marks, or any relationship between the owner and Pearson Education, Inc., or its affiliates, authors, licensees, or distributors.



ISBN-13: 978-0-13-761600-8
ISBN-10: 0-13-761600-7

TABLE OF CONTENTS

1 Functions 1

- 1.1 Functions and Their Graphs 1
- 1.2 Combining Functions; Shifting and Scaling Graphs 9
- 1.3 Trigonometric Functions 19
- 1.4 Graphing with Software 27
 - Practice Exercises 32
 - Additional and Advanced Exercises 40

2 Limits and Continuity 43

- 2.1 Rates of Change and Tangents to Curves 43
- 2.2 Limit of a Function and Limit Laws 47
- 2.3 The Precise Definition of a Limit 57
- 2.4 One-Sided Limits 65
- 2.5 Continuity 70
- 2.6 Limits Involving Infinity; Asymptotes of Graphs 75
 - Practice Exercises 86
 - Additional and Advanced Exercises 92

3 Derivatives 99

- 3.1 Tangents and the Derivative at a Point 99
- 3.2 The Derivative as a Function 105
- 3.3 Differentiation Rules 116
- 3.4 The Derivative as a Rate of Change 121
- 3.5 Derivatives of Trigonometric Functions 127
- 3.6 The Chain Rule 134
- 3.7 Implicit Differentiation 146
- 3.8 Related Rates 155
- 3.9 Linearization and Differentials 160
 - Practice Exercises 167
 - Additional and Advanced Exercises 180

4 Applications of Derivatives 187

- 4.1 Extreme Values of Functions on Closed Intervals 187
- 4.2 The Mean Value Theorem 195
- 4.3 Monotonic Functions and the First Derivative Test 201
- 4.4 Concavity and Curve Sketching 213
- 4.5 Applied Optimization 236
- 4.6 Newton's Method 251
- 4.7 Antiderivatives 255
 - Practice Exercises 263
 - Additional and Advanced Exercises 277

5 Integrals 285

- 5.1 Area and Estimating with Finite Sums 285
- 5.2 Sigma Notation and Limits of Finite Sums 290
- 5.3 The Definite Integral 296
- 5.4 The Fundamental Theorem of Calculus 312
- 5.5 Indefinite Integrals and the Substitution Method 322
- 5.6 Definite Integral Substitutions and the Area Between Curves 328
 - Practice Exercises 345
 - Additional and Advanced Exercises 355

6 Applications of Definite Integrals 361

- 6.1 Volumes Using Cross-Sections 361
- 6.2 Volumes Using Cylindrical Shells 373
- 6.3 Arc Length 384
- 6.4 Areas of Surfaces of Revolution 391
- 6.5 Work and Fluid Forces 397
- 6.6 Moments and Centers of Mass 408
 - Practice Exercises 422
 - Additional and Advanced Exercises 432

7 Transcendental Functions 439

- 7.1 Inverse Functions and Their Derivatives 439
- 7.2 Natural Logarithms 449
- 7.3 Exponential Functions 457
- 7.4 Exponential Change and Separable Differential Equations 472
- 7.5 Indeterminate Forms and L'Hôpital's Rule 478
- 7.6 Inverse Trigonometric Functions 487
- 7.7 Hyperbolic Functions 501
- 7.8 Relative Rates of Growth 510
 - Practice Exercises 515
 - Additional and Advanced Exercises 529

8 Techniques of Integration 533

- 8.1 Using Basic Integration Formulas 533
- 8.2 Integration by Parts 544
- 8.3 Trigonometric Integrals 558
- 8.4 Trigonometric Substitutions 567
- 8.5 Integration of Rational Functions by Partial Fractions 577
- 8.6 Integral Tables and Computer Algebra Systems 588
- 8.7 Numerical Integration 599
- 8.8 Improper Integrals 612
- 8.9 Probability 626
 - Practice Exercises 634
 - Additional and Advanced Exercises 648

9 First-Order Differential Equations 657

- 9.1 Solutions, Slope Fields, and Euler's Method 657
- 9.2 First-Order Linear Equations 667
- 9.3 Applications 671
- 9.4 Graphical Solutions of Autonomous Equations 675
- 9.5 Systems of Equations and Phase Planes 683
 - Practice Exercises 688
 - Additional and Advanced Exercises 696

10 Infinite Sequences and Series 699

- 10.1 Sequences 699
- 10.2 Infinite Series 711
- 10.3 The Integral Test 720
- 10.4 Comparison Tests 728
- 10.5 Absolute Convergence; The Ratio and Root Tests 738
- 10.6 Alternating Series and Conditional Convergence 745
- 10.7 Power Series 755
- 10.8 Taylor and Maclaurin Series 767
- 10.9 Convergence of Taylor Series 773
- 10.10 The Binomial Series and Applications of Taylor Series 781
 - Practice Exercises 791
 - Additional and Advanced Exercises 801

11 Parametric Equations and Polar Coordinates 807

- 11.1 Parametrizations of Plane Curves 807
- 11.2 Calculus with Parametric Curves 816
- 11.3 Polar Coordinates 825
- 11.4 Graphing Polar Coordinate Equations 831
- 11.5 Areas and Lengths in Polar Coordinates 839
- 11.6 Conic Sections 845
- 11.7 Conics in Polar Coordinates 856
 - Practice Exercises 866
 - Additional and Advanced Exercises 877

12 Vectors and the Geometry of Space 883

- 12.1 Three-Dimensional Coordinate Systems 883
- 12.2 Vectors 888
- 12.3 The Dot Product 894
- 12.4 The Cross Product 900
- 12.5 Lines and Planes in Space 908
- 12.6 Cylinders and Quadric Surfaces 916
 - Practice Exercises 922
 - Additional and Advanced Exercises 930

13 Vector-Valued Functions and Motion in Space 937

- 13.1 Curves in Space and Their Tangents 937
- 13.2 Integrals of Vector Functions; Projectile Motion 944
- 13.3 Arc Length in Space 953
- 13.4 Curvature and Normal Vectors of a Curve 957
- 13.5 Tangential and Normal Components of Acceleration 965
- 13.6 Velocity and Acceleration in Polar Coordinates 972
 - Practice Exercises 974
 - Additional and Advanced Exercises 981

14 Partial Derivatives 985

- 14.1 Functions of Several Variables 985
- 14.2 Limits and Continuity in Higher Dimensions 995
- 14.3 Partial Derivatives 1003
- 14.4 The Chain Rule 1012
- 14.5 Directional Derivatives and Gradient Vectors 1025
- 14.6 Tangent Planes and Differentials 1030
- 14.7 Extreme Values and Saddle Points 1038
- 14.8 Lagrange Multipliers 1055
- 14.9 Taylor's Formula for Two Variables 1066
- 14.10 Partial Derivatives with Constrained Variables 1070
 - Practice Exercises 1073
 - Additional and Advanced Exercises 1090

15 Multiple Integrals 1097

- 15.1 Double and Iterated Integrals over Rectangles 1097
- 15.2 Double Integrals over General Regions 1100
- 15.3 Area by Double Integration 1115
- 15.4 Double Integrals in Polar Form 1120
- 15.5 Triple Integrals in Rectangular Coordinates 1126
- 15.6 Moments and Centers of Mass 1132
- 15.7 Triple Integrals in Cylindrical and Spherical Coordinates 1139
- 15.8 Substitutions in Multiple Integrals 1152
 - Practice Exercises 1160
 - Additional and Advanced Exercises 1168

16 Integrals and Vector Fields 1175

- 16.1 Line Integrals 1175
- 16.2 Vector Fields and Line Integrals: Work, Circulation, and Flux 1181
- 16.3 Path Independence, Conservative Fields, and Potential Functions 1193
- 16.4 Green's Theorem in the Plane 1199
- 16.5 Surfaces and Area 1207
- 16.6 Surface Integrals 1217
- 16.7 Stokes' Theorem 1228
- 16.8 The Divergence Theorem and a Unified Theory 1235
 - Practice Exercises 1242
 - Additional and Advanced Exercises 1252

17 Second-Order Differential Equations 1257

- 17.1 Second-Order Linear Equations 1257
- 17.2 Nonhomogeneous Linear Equations 1263
- 17.3 Applications 1275
- 17.4 Euler Equations 1281
- 17.5 Power-Series Solutions 1284

18 Complex Functions 1295

- 18.1 Complex Numbers 1295
- 18.2 Functions of a Complex Variable 1297
- 18.3 Derivatives 1298
- 18.4 The Cauchy-Riemann Equations 1299
- 18.5 Complex Power Series 1299
- 18.6 Some Complex Functions 1300
- 18.7 Conformal Maps 1302
 - Additional and Advanced Exercises 1303

19 Fourier Series and Wavelets 1305

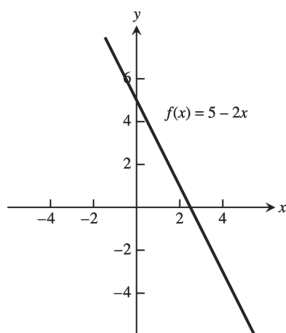
- 19.1 Periodic Functions 1305
- 19.2 Summing Sines and Cosines 1305
- 19.3 Vectors and Approximation in Three and More Dimensions 1307
- 19.4 Approximation of Functions 1310
- 19.5 Advanced Topic: The Haar System and Wavelets 1317
 - Additional and Advanced Exercises 1321

CHAPTER 1 FUNCTIONS

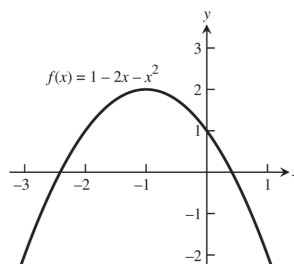
1.1 FUNCTIONS AND THEIR GRAPHS

1. domain = $(-\infty, \infty)$; range = $[1, \infty)$
2. domain = $[0, \infty)$; range = $(-\infty, 1]$
3. domain = $[-2, \infty)$; y in range and $y = \sqrt{5x+10} \geq 0 \Rightarrow y$ can be any positive real number $\Rightarrow$ range = $[0, \infty)$.
4. domain = $(-\infty, 0] \cup [3, \infty)$; y in range and $y = \sqrt{x^2 - 3x} \geq 0 \Rightarrow y$ can be any positive real number $\Rightarrow$ range = $[0, \infty)$.
5. domain = $(-\infty, 3) \cup (3, \infty)$; y in range and $y = \frac{4}{3-t}$, now if $t < 3 \Rightarrow 3-t > 0 \Rightarrow \frac{4}{3-t} > 0$, or if $t > 3 \Rightarrow 3-t < 0 \Rightarrow \frac{4}{3-t} < 0 \Rightarrow y$ can be any nonzero real number $\Rightarrow$ range = $(-\infty, 0) \cup (0, \infty)$.
6. domain = $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$; y in range and $y = \frac{2}{t^2-16}$, now if $t < -4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0$, or if $-4 < t < 4 \Rightarrow -16 \leq t^2 - 16 < 0 \Rightarrow -\frac{2}{16} \geq \frac{2}{t^2-16}$, or if $t > 4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0 \Rightarrow y$ can be any nonzero real number $\Rightarrow$ range = $(-\infty, -\frac{1}{8}] \cup (0, \infty)$.
7. (a) Not the graph of a function of x since it fails the vertical line test.
(b) Is the graph of a function of x since any vertical line intersects the graph at most once.
8. (a) Not the graph of a function of x since it fails the vertical line test.
(b) Not the graph of a function of x since it fails the vertical line test.
9. base = x ; $(\text{height})^2 + \left(\frac{x}{2}\right)^2 = x^2 \Rightarrow \text{height} = \frac{\sqrt{3}}{2}x$; area is $a(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$;
perimeter is $p(x) = x + x + x = 3x$.
10. s = side length $\Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$; and area is $a = s^2 \Rightarrow a = \frac{1}{2}d^2$.
11. Let D = diagonal length of a face of the cube and ℓ = the length of an edge. Then $\ell^2 + D^2 = d^2$ and $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$. The surface area is $6\ell^2 = \frac{6d^2}{3} = 2d^2$ and the volume is $\ell^3 = \left(\frac{d}{\sqrt{3}}\right)^3 = \frac{d^3}{3\sqrt{3}}$.
12. The coordinates of P are $(x, \sqrt{x})$ so the slope of the line joining P to the origin is $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} (x > 0)$.
Thus, $(x, \sqrt{x}) = \left(\frac{1}{m^2}, \frac{1}{m}\right)$.
13. $2x + 4y = 5 \Rightarrow y = -\frac{1}{2}x + \frac{5}{4}$; $L = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + \left(-\frac{1}{2}x + \frac{5}{4}\right)^2} = \sqrt{x^2 + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{25}{16}}$
 $= \sqrt{\frac{5}{4}x^2 - \frac{5}{4}x + \frac{25}{16}} = \sqrt{\frac{20x^2 - 20x + 25}{16}} = \frac{\sqrt{20x^2 - 20x + 25}}{4}$
14. $y = \sqrt{x-3} \Rightarrow y^2 + 3 = x$; $L = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(y^2 + 3 - 4)^2 + y^2} = \sqrt{(y^2 - 1)^2 + y^2}$
 $= \sqrt{y^4 - 2y^2 + 1 + y^2} = \sqrt{y^4 - y^2 + 1}$

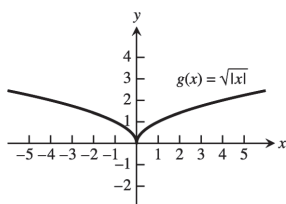
15. The domain is $(-\infty, \infty)$.



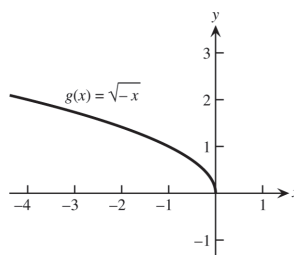
16. The domain is $(-\infty, \infty)$.



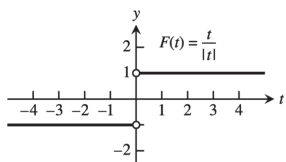
17. The domain is $(-\infty, \infty)$.



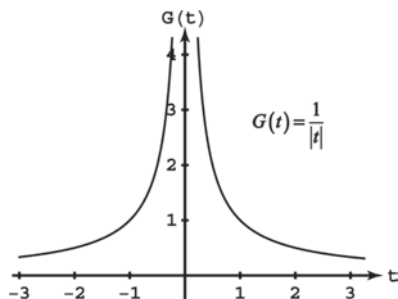
18. The domain is $(-\infty, 0]$.



19. The domain is $(-\infty, 0) \cup (0, \infty)$.



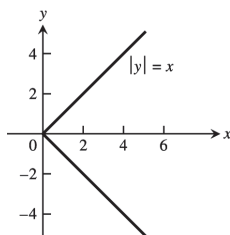
20. The domain is $(-\infty, 0) \cup (0, \infty)$.



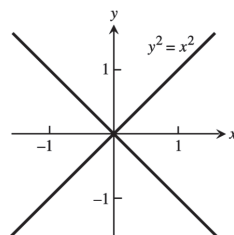
21. The domain is $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$.

22. The range is $[5, \infty)$.

23. Neither graph passes the vertical line test.
(a)

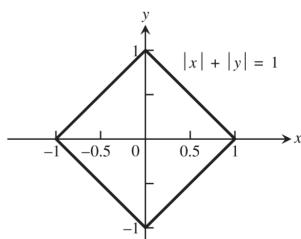


- (b)

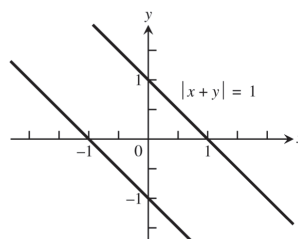


24. Neither graph passes the vertical line test.

(a)



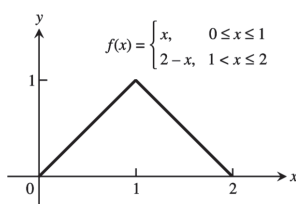
(b)



$$|x+y|=1 \Leftrightarrow \begin{cases} x+y=1 \\ \text{or} \\ x+y=-1 \end{cases} \Leftrightarrow \begin{cases} y=1-x \\ \text{or} \\ y=-1-x \end{cases}$$

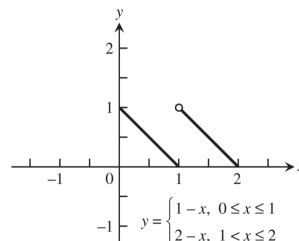
- 25.

x	0	1	2
y	0	1	0

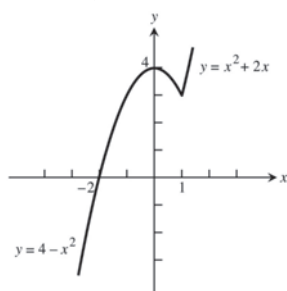


- 26.

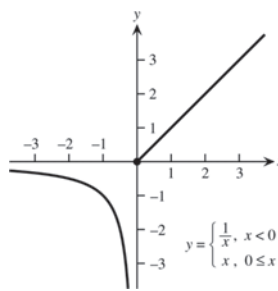
x	0	1	2
y	1	0	0



27.
$$F(x) = \begin{cases} 4-x^2, & x \leq 1 \\ x^2+2x, & x > 1 \end{cases}$$



28.
$$G(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & 0 \leq x \end{cases}$$



29. (a) Line through (0, 0) and (1, 1):
- $y = x$
- ; Line through (1, 1) and (2, 0):
- $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ -x+2, & 1 < x \leq 2 \end{cases}$$

(b)
$$f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \\ 2, & 2 \leq x < 3 \\ 0, & 3 \leq x \leq 4 \end{cases}$$

30. (a) Line through (0, 2) and (2, 0):
- $y = -x + 2$

Line through (2, 1) and (5, 0): $m = \frac{0-1}{5-2} = -\frac{1}{3}$, so $y = -\frac{1}{3}(x-2) + 1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x+2, & 0 < x \leq 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \leq 5 \end{cases}$$

(b) Line through $(-1, 0)$ and $(0, -3)$: $m = \frac{-3-0}{0-(-1)} = -3$, so $y = -3x - 3$

Line through $(0, 3)$ and $(2, -1)$: $m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x-3, & -1 < x \leq 0 \\ -2x+3, & 0 < x \leq 2 \end{cases}$$

31. (a) Line through $(-1, 1)$ and $(0, 0)$: $y = -x$

Line through $(0, 1)$ and $(1, 1)$: $y = 1$

Line through $(1, 1)$ and $(3, 0)$: $m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$, so $y = -\frac{1}{2}(x-1) + 1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x & -1 \leq x < 0 \\ 1 & 0 < x \leq 1 \\ -\frac{1}{2}x + \frac{3}{2} & 1 < x < 3 \end{cases}$$

(b) Line through $(-2, -1)$ and $(0, 0)$: $y = \frac{1}{2}x$

Line through $(0, 2)$ and $(1, 0)$: $y = -2x + 2$

Line through $(1, -1)$ and $(3, -1)$: $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x & -2 \leq x \leq 0 \\ -2x+2 & 0 < x \leq 1 \\ -1 & 1 < x \leq 3 \end{cases}$$

32. (a) Line through $(\frac{T}{2}, 0)$ and $(T, 1)$: $m = \frac{1-0}{T-(T/2)} = \frac{2}{T}$, so $y = \frac{2}{T}(x - \frac{T}{2}) + 0 = \frac{2}{T}x - 1$

$$f(x) = \begin{cases} 0, & 0 \leq x \leq \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \leq T \end{cases}$$

$$(b) f(x) = \begin{cases} A, & 0 \leq x < \frac{T}{2} \\ -A, & \frac{T}{2} \leq x < T \\ A, & T \leq x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \leq x \leq 2T \end{cases}$$

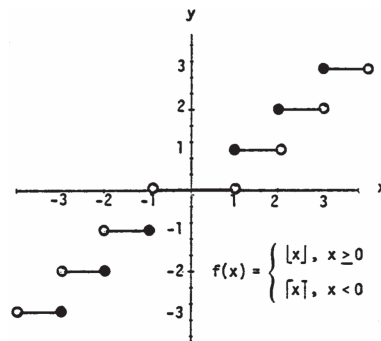
33. (a) $\lfloor x \rfloor = 0$ for $x \in [0, 1)$

(b) $\lceil x \rceil = 0$ for $x \in (-1, 0]$

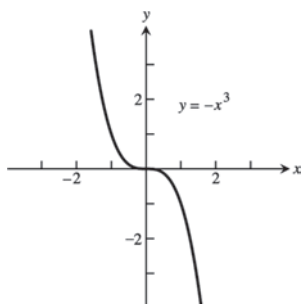
34. $\lfloor x \rfloor = \lceil x \rceil$ only when x is an integer.

35. For any real number x , $n \leq x \leq n+1$, where n is an integer. Now: $n \leq x \leq n+1 \Rightarrow -(n+1) \leq -x \leq -n$.
By definition: $\lceil -x \rceil = -n$ and $\lfloor x \rfloor = n \Rightarrow -\lfloor x \rfloor = -n$. So $\lceil -x \rceil = -\lfloor x \rfloor$ for all real x .

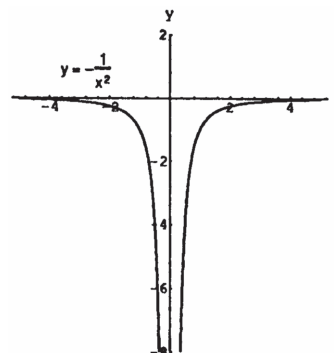
36. To find $f(x)$ you delete the decimal or fractional portion of x , leaving only the integer part.



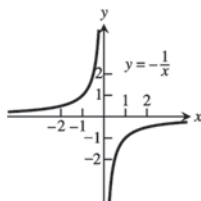
37. Symmetric about the origin
 Dec: $-\infty < x < \infty$
 Inc: nowhere



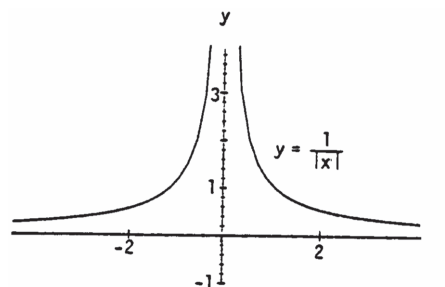
38. Symmetric about the y-axis
 Dec: $-\infty < x < 0$
 Inc: $0 < x < \infty$



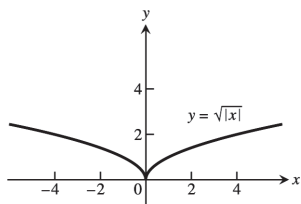
39. Symmetric about the origin
 Dec: nowhere
 Inc: $-\infty < x < 0$
 $0 < x < \infty$



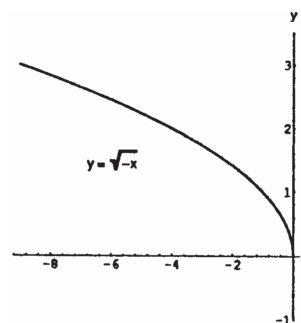
40. Symmetric about the y-axis
 Dec: $0 < x < \infty$
 Inc: $-\infty < x < 0$



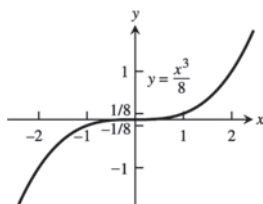
41. Symmetric about the y-axis
 Dec: $-\infty < x \leq 0$
 Inc: $0 \leq x < \infty$



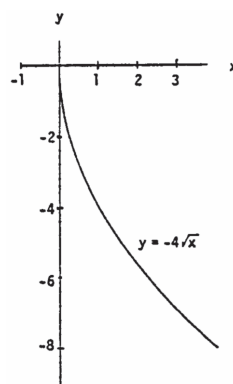
42. No symmetry
 Dec: $-\infty < x \leq 0$
 Inc: nowhere



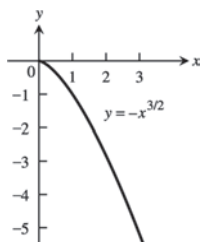
43. Symmetric about the origin
Dec: nowhere
Inc: $-\infty < x < \infty$



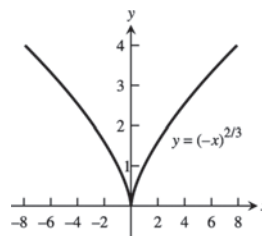
44. No symmetry
Dec: $0 \leq x < \infty$
Inc: nowhere



45. No symmetry
Dec: $0 \leq x < \infty$
Inc: nowhere



46. Symmetric about the y-axis
Dec: $-\infty < x \leq 0$
Inc: $0 \leq x < \infty$



47. Since a horizontal line not through the origin is symmetric with respect to the y-axis, but not with respect to the origin, the function is even.

48. $f(x) = x^{-5} = \frac{1}{x^5}$ and $f(-x) = (-x)^{-5} = \frac{1}{(-x)^5} = -\left(\frac{1}{x^5}\right) = -f(x)$. Thus the function is odd.

49. Since $f(x) = x^2 + 1 = (-x)^2 + 1 = f(-x)$. The function is even.

50. Since $[f(x) = x^2 + x] \neq [f(-x) = (-x)^2 - x]$ and $[f(x) = x^2 + x] \neq [-f(x) = -(x)^2 - x]$ the function is neither even nor odd.

51. Since $g(x) = x^3 + x$, $g(-x) = -x^3 - x = -(x^3 + x) = -g(x)$. So the function is odd.

52. $g(x) = x^4 + 3x^2 - 1 = (-x)^4 + 3(-x)^2 - 1 = g(-x)$, thus the function is even.

53. $g(x) = \frac{1}{x^2 - 1} = \frac{1}{(-x)^2 - 1} = g(-x)$. Thus the function is even.

54. $g(x) = \frac{x}{x^2 - 1}$; $g(-x) = -\frac{x}{x^2 - 1} = -g(x)$. So the function is odd.

55. $h(t) = \frac{1}{t-1}$; $h(-t) = \frac{1}{-t-1}$; $-h(t) = \frac{1}{1-t}$. Since $h(t) \neq -h(t)$ and $h(t) \neq h(-t)$, the function is neither even nor odd.

56. Since $|t^3| = |(-t)^3|$, $h(t) = h(-t)$ and the function is even.
57. $h(t) = 2t + 1$, $h(-t) = -2t + 1$. So $h(t) \neq h(-t)$. $-h(t) = -2t - 1$, so $h(t) \neq -h(t)$. The function is neither even nor odd.
58. $h(t) = 2|t| + 1$ and $h(-t) = 2|-t| + 1 = 2|t| + 1$. So $h(t) = h(-t)$ and the function is even.
59. $g(x) = \sin 2x$; $g(-x) = -\sin 2x = -g(x)$. So the function is odd.
60. $g(x) = \sin x^2$; $g(-x) = \sin x^2 = g(x)$. So the function is even.
61. $g(x) = \cos 3x$; $g(-x) = \cos 3x = g(x)$. So the function is even.
62. $g(x) = 1 + \cos x$; $g(-x) = 1 + \cos x = g(x)$. So the function is even.
63. $s = kt \Rightarrow 25 = k(75) \Rightarrow k = \frac{1}{3} \Rightarrow s = \frac{1}{3}t$; $60 = \frac{1}{3}t \Rightarrow t = 180$
64. $K = c v^2 \Rightarrow 12960 = c(18)^2 \Rightarrow c = 40 \Rightarrow K = 40v^2$; $K = 40(10)^2 = 4000$ joules
65. $r = \frac{k}{s} \Rightarrow 6 = \frac{k}{4} \Rightarrow k = 24 \Rightarrow r = \frac{24}{s}$; $10 = \frac{24}{s} \Rightarrow s = \frac{12}{5}$
66. $P = \frac{k}{V} \Rightarrow 14.7 = \frac{k}{1000} \Rightarrow k = 14700 \Rightarrow P = \frac{14700}{V}$; $23.4 = \frac{14700}{V} \Rightarrow V = \frac{24500}{39} \approx 628.2$ in.³
67. $V = f(x) = x(14 - 2x)(22 - 2x) = 4x^3 - 72x^2 + 308x$; $0 < x < 7$
68. (a) Let h = height of the triangle. Since the triangle is isosceles, $(\overline{AB})^2 + (\overline{AB})^2 = 2^2 \Rightarrow \overline{AB} = \sqrt{2}$. So,
 $h^2 + 1^2 = (\sqrt{2})^2 \Rightarrow h = 1 \Rightarrow B$ is at $(0, 1) \Rightarrow$ slope of $AB = -1 \Rightarrow$ The equation of AB is
 $y = f(x) = -x + 1$; $x \in [0, 1]$.
- (b) $A(x) = 2xy = 2x(-x + 1) = -2x^2 + 2x$; $x \in [0, 1]$
69. (a) Graph h because it is an even function and rises less rapidly than does Graph g .
 (b) Graph f because it is an odd function.
 (c) Graph g because it is an even function and rises more rapidly than does Graph h .
70. (a) Graph f because it is linear.
 (b) Graph g because it contains $(0, 1)$.
 (c) Graph h because it is a nonlinear odd function.

71. (a) From the graph,
- $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow x \in (-2, 0) \cup (4, \infty)$

(b) $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow \frac{x}{2} - 1 - \frac{4}{x} > 0$

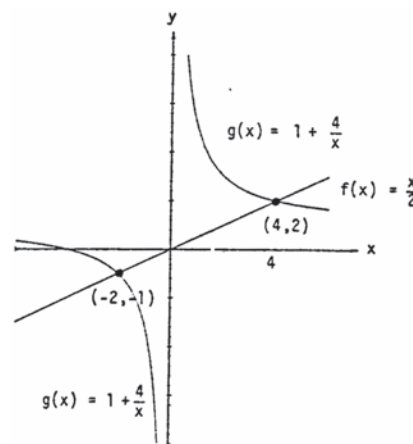
$$x > 0: \frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} > 0 \Rightarrow \frac{(x-4)(x+2)}{2x} > 0$$

$$\Rightarrow x > 4 \text{ since } x \text{ is positive;}$$

$$x < 0: \frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} < 0 \Rightarrow \frac{(x-4)(x+2)}{2x} < 0$$

$$\Rightarrow x < -2 \text{ since } x \text{ is negative;}$$

$$\text{sign of } (x-4)(x+2)$$

Solution interval: $(-2, 0) \cup (4, \infty)$ 

72. (a) From the graph,
- $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow x \in (-\infty, -5) \cup (-1, 1)$

(b) Case $x < -1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} > 2$

$$\Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5.$$

Thus, $x \in (-\infty, -5)$ solves the inequality.

Case $-1 < x < 1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} < 2$

$$\Rightarrow 3x + 3 > 2x - 2 \Rightarrow x > -5 \text{ which}$$

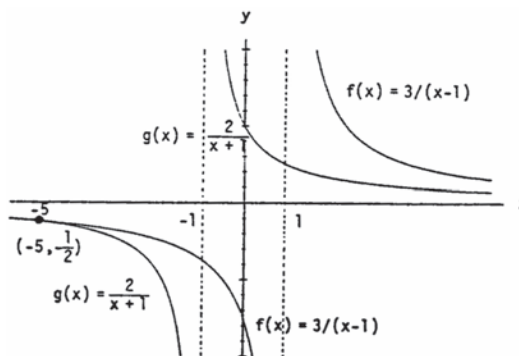
$$\text{is true if } x > -1. \text{ Thus, } x \in (-1, 1)$$

$$\text{solves the inequality.}$$

Case $1 < x$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5$

which is never true if $1 < x$,

so no solution here.

In conclusion, $x \in (-\infty, -5) \cup (-1, 1)$.

73. A curve symmetric about the
- x
- axis will not pass the vertical line test because the points
- (x, y)
- and
- $(x, -y)$
- lie on the same vertical line. The graph of the function
- $y = f(x) = 0$
- is the
- x
- axis, a horizontal line for which there is a single
- y
- value, 0, for any
- x
- .

74. price = $40 + 5x$, quantity = $300 - 25x \Rightarrow R(x) = (40 + 5x)(300 - 25x)$

75. $x^2 + x^2 = h^2 \Rightarrow x = \frac{h}{\sqrt{2}} = \frac{\sqrt{2}h}{2}$; cost = $5(2x) + 10h \Rightarrow C(h) = 10\left(\frac{\sqrt{2}h}{2}\right) + 10h = 5h(\sqrt{2} + 2)$

76. (a) Note that 2 mi = 10,560 ft, so there are
- $\sqrt{800^2 + x^2}$
- feet of river cable at \$180 per foot and
- $(10,560 - x)$
- feet of land cable at \$100 per foot. The cost is
- $C(x) = 180\sqrt{800^2 + x^2} + 100(10,560 - x)$
- .

(b) $C(0) = \$1,200,000$

$C(500) \approx \$1,175,812$

$C(1000) \approx \$1,186,512$

$C(1500) \approx \$1,212,000$

$C(2000) \approx \$1,243,732$

$C(2500) \approx \$1,278,479$

$C(3000) \approx \$1,314,870$

Values beyond this are all larger. It would appear that the least expensive location is less than 2000 feet from the point P .

1.2 COMBINING FUNCTIONS; SHIFTING AND SCALING GRAPHS

1. $D_f: -\infty < x < \infty, D_g: x \geq 1 \Rightarrow D_{f+g} = D_{fg}: x \geq 1$
2. $D_f: x+1 \geq 0 \Rightarrow x \geq -1, D_g: x-1 \geq 0 \Rightarrow x \geq 1$. Therefore $D_{f+g} = D_{fg}: x \geq 1$
3. $D_f: -\infty < x < \infty, D_g: -\infty < x < \infty, D_{f/g}: -\infty < x < \infty, D_{g/f}: -\infty < x < \infty$
4. $D_f: -\infty < x < \infty, D_g: x \geq 0, D_{f/g}: x \geq 0, D_{g/f}: x \geq 0$
5. (a) 2 (b) 22 (c) $x^2 + 2$
 (d) $(x+5)^2 - 3 = x^2 + 10x + 22$ (e) 5 (f) -2
 (g) $x+10$ (h) $(x^2 - 3)^2 - 3 = x^4 - 6x^2 + 6$
6. (a) $-\frac{1}{3}$ (b) 2 (c) $\frac{1}{x+1} - 1 = \frac{-x}{x+1}$
 (d) $\frac{1}{x}$ (e) 0 (f) $\frac{3}{4}$
 (g) $x-2$ (h) $\frac{1}{\frac{1}{x+1} + 1} = \frac{1}{\frac{x+2}{x+1}} = \frac{x+1}{x+2}$
7. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(4-x)) = f(3(4-x)) = f(12-3x) = (12-3x)+1 = 13-3x$
8. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^2)) = f(2(x^2)-1) = f(2x^2-1) = 3(2x^2-1)+4 = 6x^2+1$
9. $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\frac{1}{x}\right)\right) = f\left(\frac{1}{\frac{1}{x}+4}\right) = f\left(\frac{x}{1+4x}\right) = \sqrt{\frac{x}{1+4x}+1} = \sqrt{\frac{5x+1}{1+4x}}$
10. $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\sqrt{2-x}\right)\right) = f\left(\frac{(\sqrt{2-x})^2}{(\sqrt{2-x})^2+1}\right) = f\left(\frac{2-x}{3-x}\right) = \frac{\frac{2-x}{3-x}+2}{3-\frac{2-x}{3-x}} = \frac{8-3x}{7-2x}$
11. (a) $(f \circ g)(x)$ (b) $(j \circ g)(x)$ (c) $(g \circ g)(x)$
 (d) $(j \circ j)(x)$ (e) $(g \circ h \circ f)(x)$ (f) $(h \circ j \circ f)(x)$
12. (a) $(f \circ j)(x)$ (b) $(g \circ h)(x)$ (c) $(h \circ h)(x)$
 (d) $(f \circ f)(x)$ (e) $(j \circ g \circ f)(x)$ (f) $(g \circ f \circ h)(x)$
13.

$g(x)$	$f(x)$	$(f \circ g)(x)$
(a) $x-7$	$\sqrt{x}$	$\sqrt{x-7}$
(b) $x+2$	$3x$	$3(x+2) = 3x+6$
(c) x^2	$\sqrt{x-5}$	$\sqrt{x^2-5}$
(d) $\frac{x}{x-1}$	$\frac{x}{x-1}$	$\frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{x}{x-(x-1)} = x$
(e) $\frac{1}{x-1}$	$1+\frac{1}{x}$	x
(f) $\frac{1}{x}$	$\frac{1}{x}$	x

14. (a) $(f \circ g)(x) = |g(x)| = \frac{1}{|x-1|}$
 (b) $(f \circ g)(x) = \frac{g(x)-1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{x}{x+1} = \frac{1}{g(x)} \Rightarrow \frac{1}{x+1} = \frac{1}{g(x)}$, so $g(x) = x+1$.
 (c) Since $(f \circ g)(x) = \sqrt{g(x)} = |x|$, $g(x) = x^2$.
 (d) Since $(f \circ g)(x) = f(\sqrt{x}) = |x|$, $f(x) = x^2$. (Note that the domain of the composition is $[0, \infty)$.)

The completed table is shown. Note that the absolute value sign in part (d) is optional.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
(a)	$\frac{1}{x-1}$	$ x $	$\frac{1}{ x-1 }$
(b)	$x+1$	$\frac{x-1}{x}$	$\frac{x}{x+1}$
(c)	x^2	$\sqrt{x}$	$ x $
(d)	$\sqrt{x}$	x^2	$ x $

15. (a) $f(g(-1)) = f(1) = 1$ (b) $g(f(0)) = g(-2) = 2$ (c) $f(f(-1)) = f(0) = -2$
 (d) $g(g(2)) = g(0) = 0$ (e) $g(f(-2)) = g(1) = -1$ (f) $f(g(1)) = f(-1) = 0$
16. (a) $f(g(0)) = f(-1) = 2 - (-1) = 3$, where $g(0) = 0 - 1 = -1$
 (b) $g(f(3)) = g(-1) = -(-1) = 1$, where $f(3) = 2 - 3 = -1$
 (c) $g(g(-1)) = g(1) = 1 - 1 = 0$, where $g(-1) = -(-1) = 1$
 (d) $f(f(2)) = f(0) = 2 - 0 = 2$, where $f(2) = 2 - 2 = 0$
 (e) $g(f(0)) = g(2) = 2 - 1 = 1$, where $f(0) = 2 - 0 = 2$
 (f) $f\left(g\left(\frac{1}{2}\right)\right) = f\left(-\frac{1}{2}\right) = 2 - \left(-\frac{1}{2}\right) = \frac{5}{2}$, where $g\left(\frac{1}{2}\right) = \frac{1}{2} - 1 = -\frac{1}{2}$

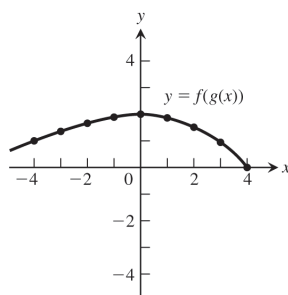
17. (a) $(f \circ g)(x) = f(g(x)) = \sqrt{\frac{1}{x} + 1} = \sqrt{\frac{1+x}{x}}$
 $(g \circ f)(x) = g(f(x)) = \frac{1}{\sqrt{x+1}}$
 (b) Domain $(f \circ g)$: $(-\infty, -1] \cup (0, \infty)$, domain $(g \circ f)$: $(-1, \infty)$
18. (a) $(f \circ g)(x) = f(g(x)) = 1 - 2\sqrt{x} + x$
 $(g \circ f)(x) = g(f(x)) = 1 - |x|$
 (b) Domain $(f \circ g)$: $[0, \infty)$, domain $(g \circ f)$: $(-\infty, \infty)$
19. $(f \circ g)(x) = x \Rightarrow f(g(x)) = x \Rightarrow \frac{g(x)}{g(x)-2} = x \Rightarrow g(x) = (g(x)-2)x = x \cdot g(x) - 2x$
 $\Rightarrow g(x) - x \cdot g(x) = -2x \Rightarrow g(x) = -\frac{2x}{1-x} = \frac{2x}{x-1}$

20. $(f \circ g)(x) = x+2 \Rightarrow f(g(x)) = x+2 \Rightarrow 2(g(x))^3 - 4 = x+2 \Rightarrow (g(x))^3 = \frac{x+6}{2} \Rightarrow g(x) = \sqrt[3]{\frac{x+6}{2}}$

21. $V = V(s) = V(s(t)) = V(2t-3)$
 $= (2t-3)^2 + 2(2t-3) + 3$
 $= 4t^2 - 8t + 6$

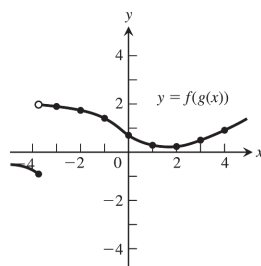
22. (a)

x	-4	-3	-2	-1	0	1	2	3	4
$g(x)$	-2	-1	-0.5	-0.2	0	0.2	0.5	1	2
$f(g(x))$	1	1.3	1.6	1.8	2	1.8	1.5	1	0



(b)

x	-4	-3	-2	-1	0	1	2	3	4
$g(x)$	1.5	0.3	-0.7	-1.5	-2.4	-2.8	-3	-2.7	-2
$f(g(x))$	-0.8	1.9	1.7	1.5	0.7	0.3	0.2	0.5	0.9



23. (a) $y = -(x+7)^2$

(b) $y = -(x-4)^2$

24. (a) $y = x^2 + 3$

(b) $y = x^2 - 5$

25. (a) Position 4

(b) Position 1

(c) Position 2

(d) Position 3

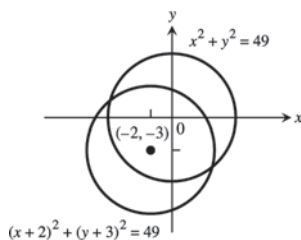
26. (a) $y = -(x-1)^2 + 4$

(b) $y = -(x+2)^2 + 3$

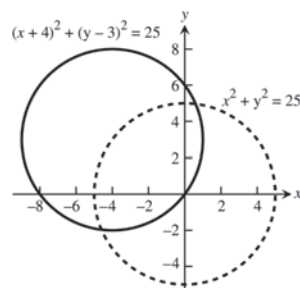
(c) $y = -(x+4)^2 - 1$

(d) $y = -(x-2)^2$

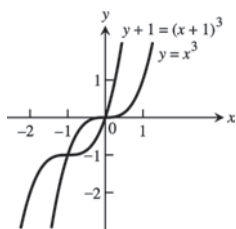
27.



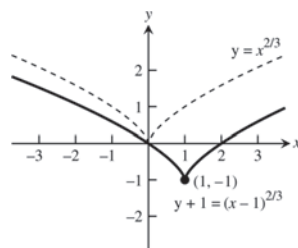
28.



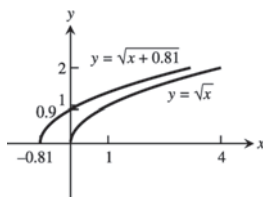
29.



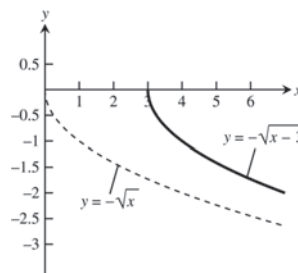
30.



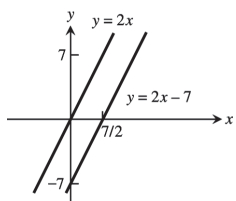
31.



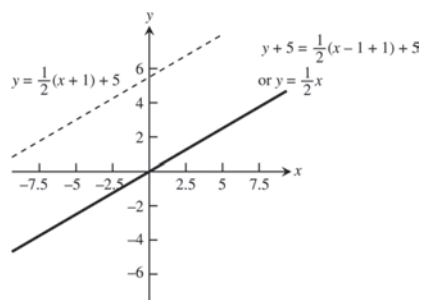
32.



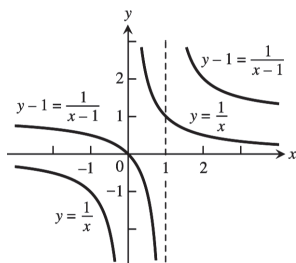
33.



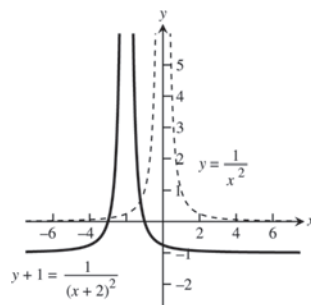
34.



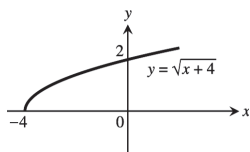
35.



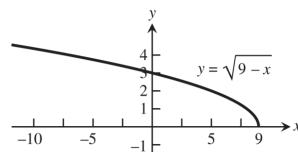
36.



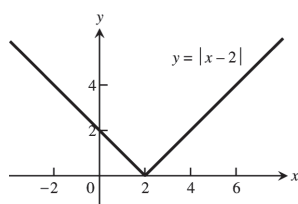
37.



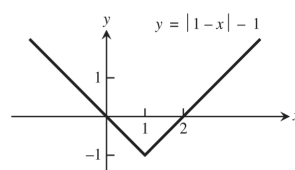
38.



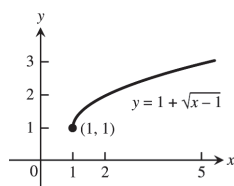
39.



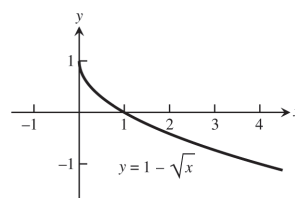
40.



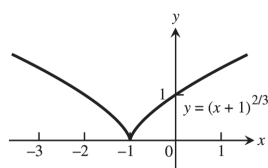
41.



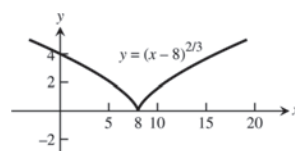
42.



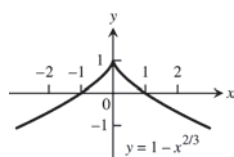
43.



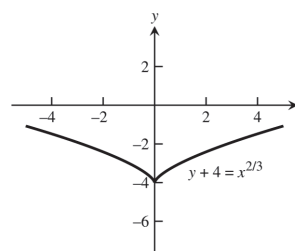
44.



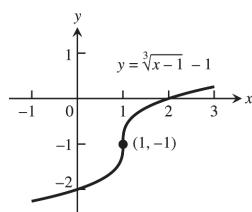
45.



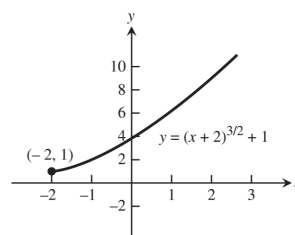
46.



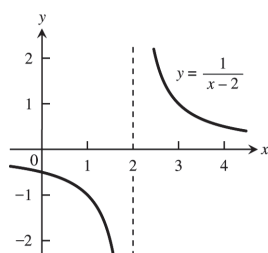
47.



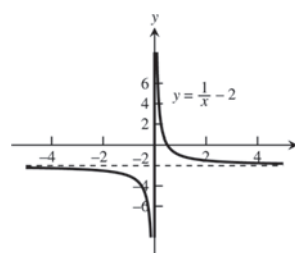
48.



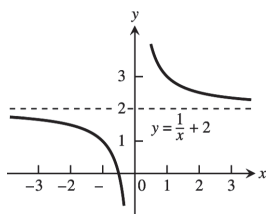
49.



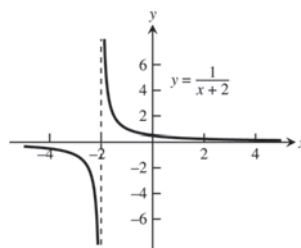
50.



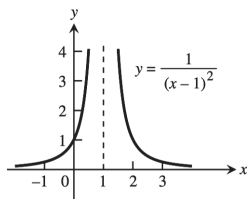
51.



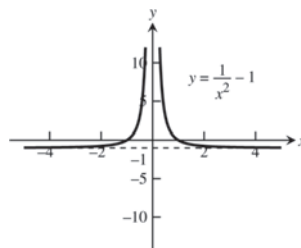
52.



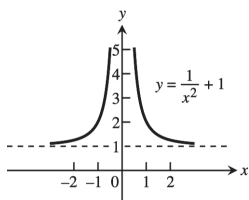
53.



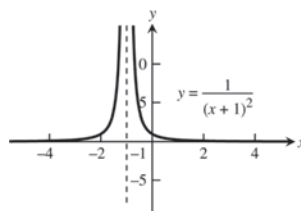
54.



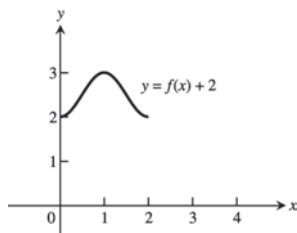
55.



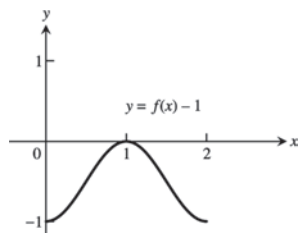
56.



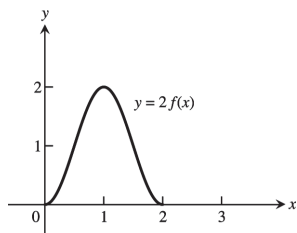
57. (a) domain: $[0, 2]$; range: $[2, 3]$



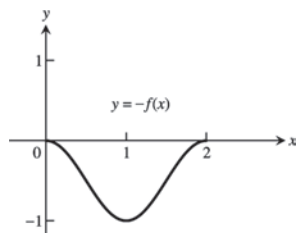
(b) domain: $[0, 2]$; range: $[-1, 0]$



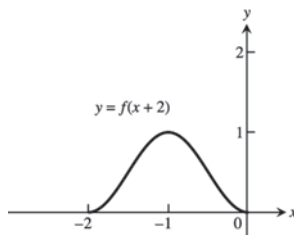
(c) domain: $[0, 2]$; range: $[0, 2]$



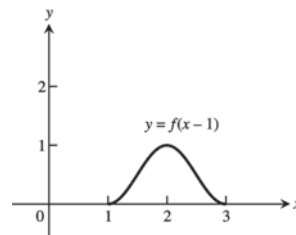
(d) domain: $[0, 2]$; range: $[-1, 0]$



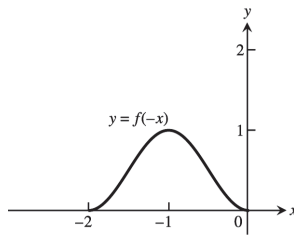
- (e) domain:
- $[-2, 0]$
- ; range:
- $[0, 1]$



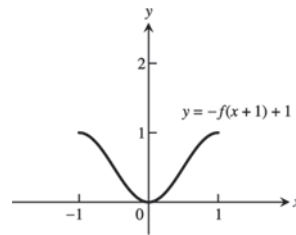
- (f) domain:
- $[1, 3]$
- ; range:
- $[0, 1]$



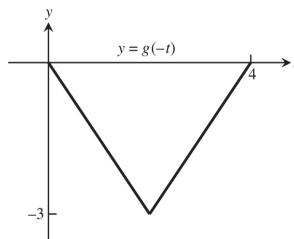
- (g) domain:
- $[-2, 0]$
- ; range:
- $[0, 1]$



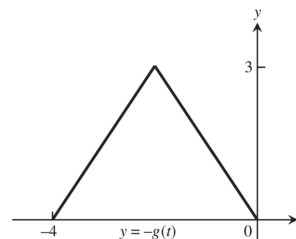
- (h) domain:
- $[-1, 1]$
- ; range:
- $[0, 1]$



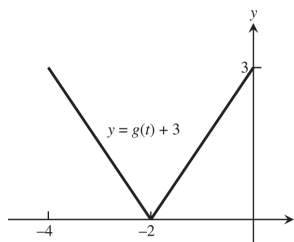
58. (a) domain:
- $[0, 4]$
- ; range:
- $[-3, 0]$



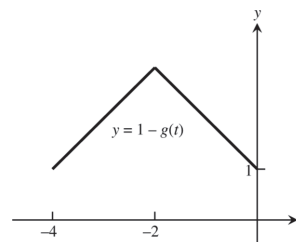
- (b) domain:
- $[-4, 0]$
- ; range:
- $[0, 3]$



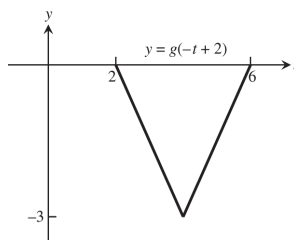
- (c) domain:
- $[-4, 0]$
- ; range:
- $[0, 3]$



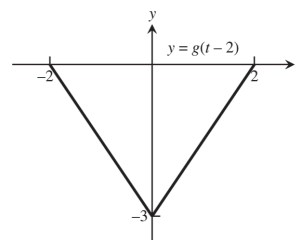
- (d) domain:
- $[-4, 0]$
- ; range:
- $[1, 4]$

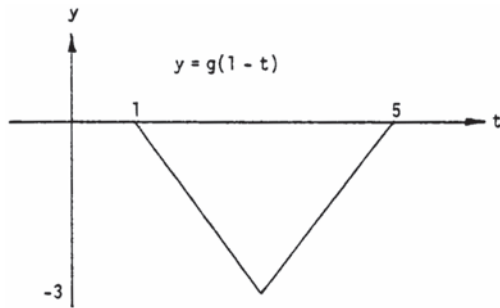
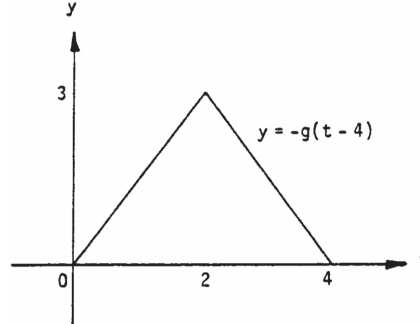


- (e) domain:
- $[2, 4]$
- ; range:
- $[-3, 0]$



- (f) domain:
- $[-2, 2]$
- ; range:
- $[-3, 0]$



(g) domain: $[1, 5]$; range: $[-3, 0]$ (h) domain: $[0, 4]$; range: $[0, 3]$ 

59. $y = 3x^2 - 3$

60. $y = (2x)^2 - 1 = 4x^2 - 1$

61. $y = \frac{1}{2} \left(1 + \frac{1}{x^2} \right) = \frac{1}{2} + \frac{1}{2x^2}$

62. $y = 1 + \frac{1}{(x/3)^2} = 1 + \frac{9}{x^2}$

63. $y = \sqrt{4x+1}$

64. $y = 3\sqrt{x+1}$

65. $y = \sqrt{4 - \left(\frac{x}{2}\right)^2} = \frac{1}{2} \sqrt{16 - x^2}$

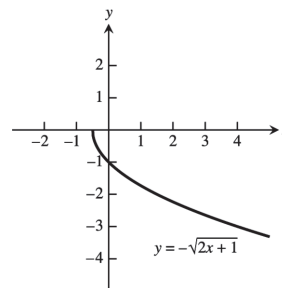
66. $y = \frac{1}{3} \sqrt{4 - x^2}$

67. $y = 1 - (3x)^3 = 1 - 27x^3$

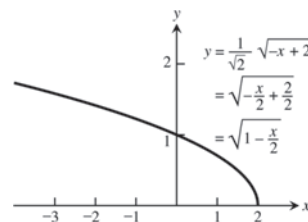
68. $y = 1 - \left(\frac{x}{2}\right)^3 = 1 - \frac{x^3}{8}$

69. Let $y = -\sqrt{2x+1} = f(x)$ and let $g(x) = x^{1/2}$,
 $h(x) = \left(x + \frac{1}{2}\right)^{1/2}$, $i(x) = \sqrt{2} \left(x + \frac{1}{2}\right)^{1/2}$, and
 $j(x) = -\left[\sqrt{2} \left(x + \frac{1}{2}\right)^{1/2}\right] = f(x)$. The graph of $h(x)$

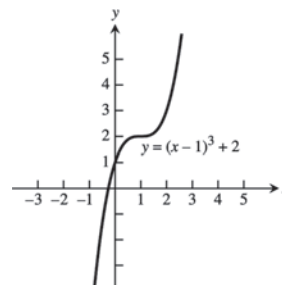
is the graph of $g(x)$ shifted left $\frac{1}{2}$ unit; the graph of $i(x)$ is the graph of $h(x)$ stretched vertically by a factor of $\sqrt{2}$; and the graph of $j(x) = f(x)$ is the graph of $i(x)$ reflected across the x -axis.



70. Let $y = \sqrt{1 - \frac{x}{2}} = f(x)$. Let $g(x) = (-x)^{1/2}$,
 $h(x) = (-x+2)^{1/2}$, and $i(x) = \frac{1}{\sqrt{2}}(-x+2)^{1/2} =$
 $\sqrt{1 - \frac{x}{2}} = f(x)$. The graph of $g(x)$ is the graph of $y = \sqrt{x}$ reflected across the x -axis. The graph of $h(x)$ is the graph of $g(x)$ shifted right two units. And the graph of $i(x)$ is the graph of $h(x)$ compressed vertically by a factor of $\sqrt{2}$.

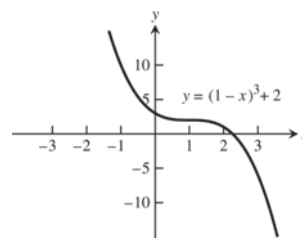


71. $y = f(x) = x^3$. Shift $f(x)$ one unit right followed by a shift two units up to get $g(x) = (x-1)^3 + 2$.

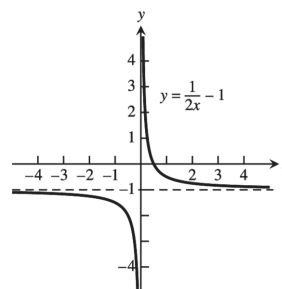


72. $y = (1-x)^3 + 2 = -[(x-1)^3 + (-2)] = f(x)$.

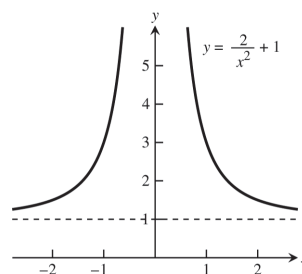
Let $g(x) = x^3$, $h(x) = (x-1)^3$, $i(x) = (x-1)^3 + (-2)$, and $j(x) = -[(x-1)^3 + (-2)]$. The graph of $h(x)$ is the graph of $g(x)$ shifted right one unit; the graph of $i(x)$ is the graph of $h(x)$ shifted down two units; and the graph of $f(x)$ is the graph of $i(x)$ reflected across the x -axis.



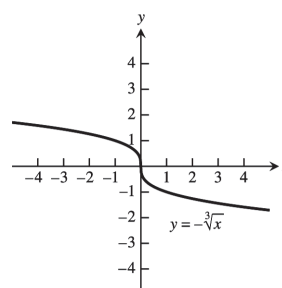
73. Compress the graph of $f(x) = \frac{1}{x}$ horizontally by a factor of 2 to get $g(x) = \frac{1}{2x}$. Then shift $g(x)$ vertically down 1 unit to get $h(x) = \frac{1}{2x} - 1$.



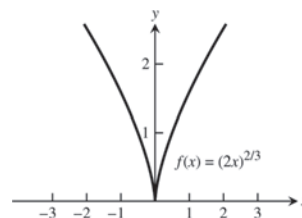
74. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{2}{x^2} + 1 = \frac{1}{\left(\frac{x^2}{2}\right)} + 1$
 $= \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2} + 1 = \frac{1}{\left[\left(\frac{1}{\sqrt{2}}\right)x\right]^2} + 1$. Since $\sqrt{2} \approx 1.4$, we see
 that the graph of $f(x)$ stretched horizontally by a factor of 1.4 and shifted up 1 unit is the graph of $g(x)$.



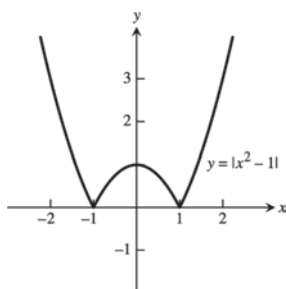
75. Reflect the graph of $y = f(x) = \sqrt[3]{x}$ across the x -axis to get $g(x) = -\sqrt[3]{x}$.



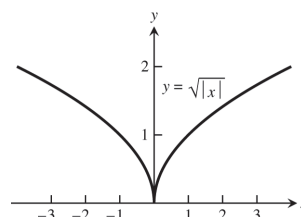
76. $y = f(x) = (-2x)^{2/3} = [(-1)(2)x]^{2/3} = (-1)^{2/3} (2x)^{2/3} = (2x)^{2/3}$. So the graph of $f(x)$ is the graph of $g(x) = x^{2/3}$ compressed horizontally by a factor of 2.



77.



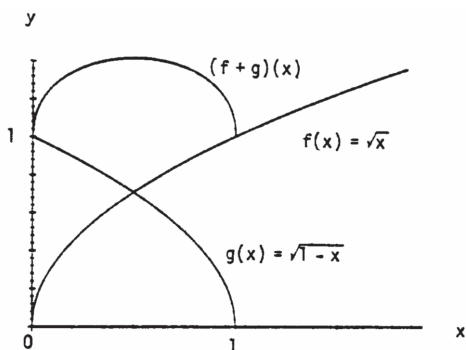
78.



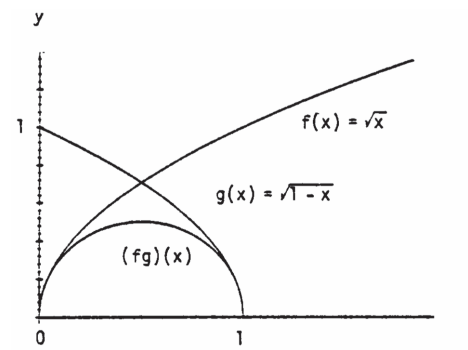
79. (a) $(fg)(-x) = f(-x)g(-x) = f(x)(-g(x)) = -(fg)(x)$, odd
 (b) $\left(\frac{f}{g}\right)(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f}{g}\right)(x)$, odd
 (c) $\left(\frac{g}{f}\right)(-x) = \frac{g(-x)}{f(-x)} = \frac{-g(x)}{f(x)} = -\left(\frac{g}{f}\right)(x)$, odd
 (d) $f^2(-x) = f(-x)f(-x) = f(x)f(x) = f^2(x)$, even
 (e) $g^2(-x) = (g(-x))^2 = (-g(x))^2 = g^2(x)$, even
 (f) $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x)$, even
 (g) $(g \circ f)(-x) = g(f(-x)) = g(f(x)) = (g \circ f)(x)$, even
 (h) $(f \circ f)(-x) = f(f(-x)) = f(f(x)) = (f \circ f)(x)$, even
 (i) $(g \circ g)(-x) = g(g(-x)) = g(-g(x)) = -g(g(x)) = -(g \circ g)(x)$, odd

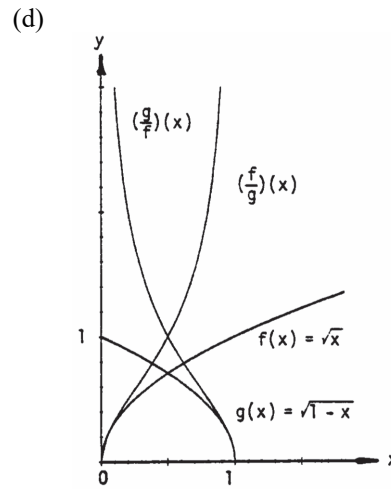
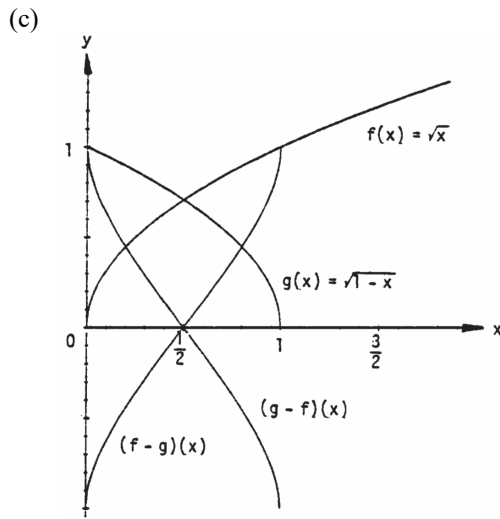
80. Yes, $f(x) = 0$ is both even and odd since $f(-x) = 0 = f(x)$ and $f(-x) = 0 = -f(x)$.

81. (a)

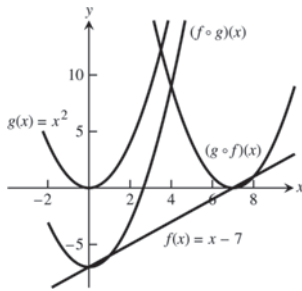


(b)





82.



1.3 TRIGONOMETRIC FUNCTIONS

- (a) $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$ m (b) $s = r\theta = (10)(110^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}$ m
- $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$ radians and $\frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right) = 225^\circ$
- $\theta = 80^\circ \Rightarrow \theta = 80^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{4\pi}{9} \Rightarrow s = (6)\left(\frac{4\pi}{9}\right) = 8.4$ in. (since the diameter = 12 in. $\Rightarrow$ radius = 6 in.)
- $d = 1$ meter $\Rightarrow r = 50$ cm $\Rightarrow \theta = \frac{s}{r} = \frac{30}{50} = 0.6$ rad or $0.6\left(\frac{180^\circ}{\pi}\right) \approx 34^\circ$

5.

θ	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin \theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos \theta$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan \theta$	0	$\sqrt{3}$	0	und.	-1
$\cot \theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	-1
$\sec \theta$	-1	-2	1	und.	$-\sqrt{2}$
$\csc \theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

6.

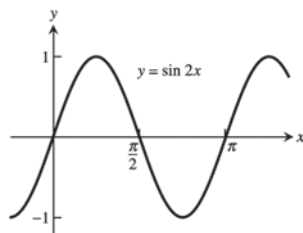
θ	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin \theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan \theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot \theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec \theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc \theta$	1	$-\frac{2}{\sqrt{3}}$	-2	$\sqrt{2}$	2

7. $\cos x = -\frac{4}{5}, \tan x = -\frac{3}{4}$

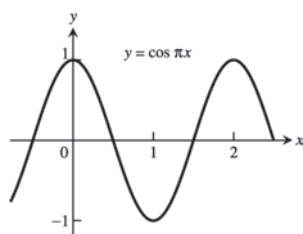
9. $\sin x = -\frac{\sqrt{8}}{3}, \tan x = -\sqrt{8}$

11. $\sin x = -\frac{1}{\sqrt{5}}, \cos x = -\frac{2}{\sqrt{5}}$

13.

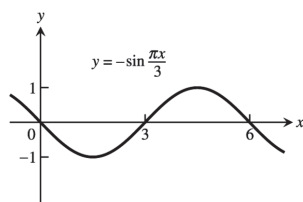

period = π

15.



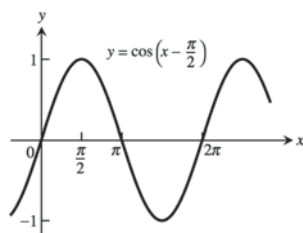
period = 2

17.



period = 6

19.

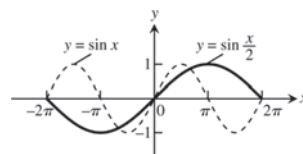

period = 2π

8. $\sin x = \frac{2}{\sqrt{5}}, \cos x = \frac{1}{\sqrt{5}}$

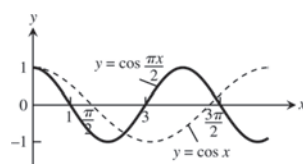
10. $\sin x = \frac{12}{13}, \tan x = -\frac{12}{5}$

12. $\cos x = -\frac{\sqrt{3}}{2}, \tan x = \frac{1}{\sqrt{3}}$

14.

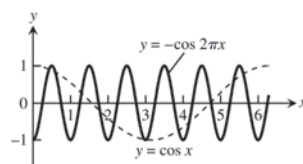

period = 4π

16.



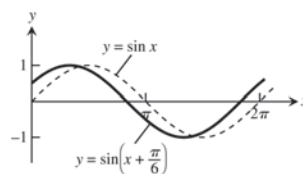
period = 4

18.

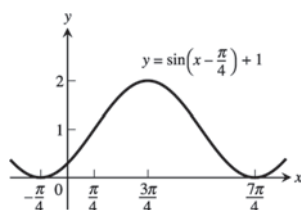


period = 1

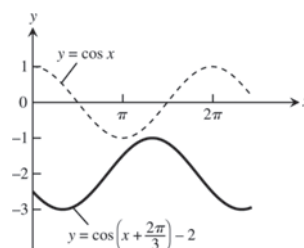
20.

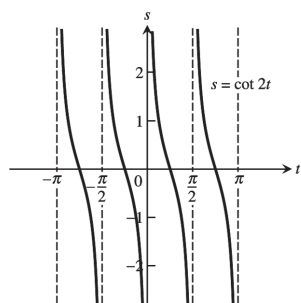

period = 2π

21.

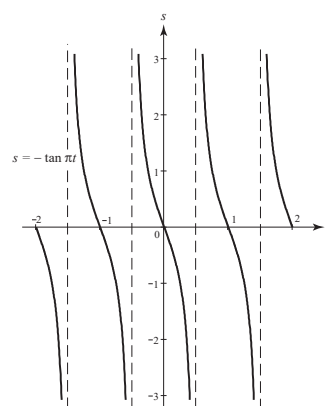

 period = 2π

22.

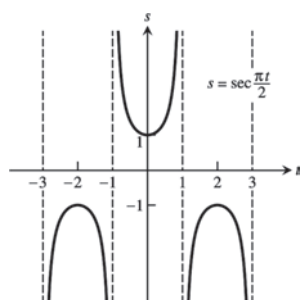
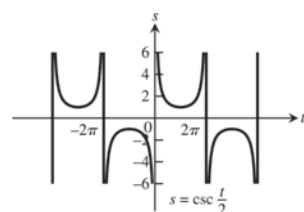

 period = 2π

 23. period = $\frac{\pi}{2}$, symmetric about the origin


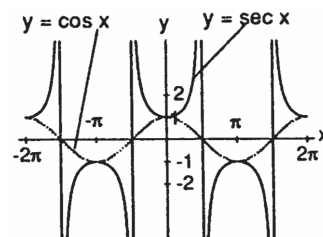
24. period = 1, symmetric about the origin



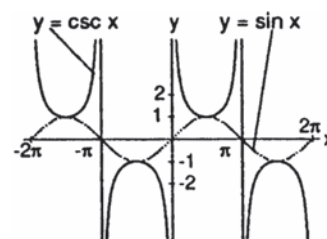
25. period = 4, symmetric about the s-axis


 26. period = 4π , symmetric about the origin


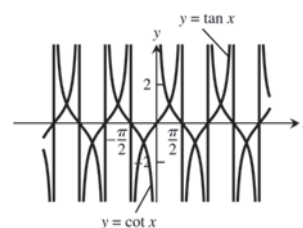
27. (a) $\cos x$ and $\sec x$ are positive for x in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$; and $\cos x$ and $\sec x$ are negative for x in the intervals $(-\frac{3\pi}{2}, -\frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$. $\sec x$ is undefined when $\cos x$ is 0. The range of $\sec x$ is $(-\infty, -1] \cup [1, \infty)$; the range of $\cos x$ is $[-1, 1]$.



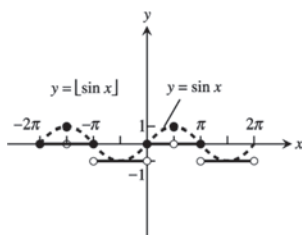
- (b) $\sin x$ and $\csc x$ are positive for x in the intervals $(-\frac{3\pi}{2}, -\pi)$ and $(0, \pi)$; and $\sin x$ and $\csc x$ are negative for x in the intervals $(-\pi, 0)$ and $(\pi, \frac{3\pi}{2})$. $\csc x$ is undefined when $\sin x$ is 0. The range of $\csc x$ is $(-\infty, -1] \cup [1, \infty)$; the range of $\sin x$ is $[-1, 1]$.



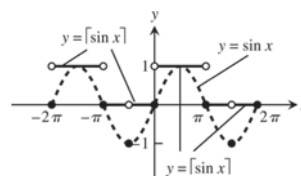
28. Since $\cot x = \frac{1}{\tan x}$, $\cot x$ is undefined when $\tan x = 0$ and is zero when $\tan x$ is undefined. As $\tan x$ approaches zero through positive values, $\cot x$ approaches infinity. Also, $\cot x$ approaches negative infinity as $\tan x$ approaches zero through negative values.



29. $D: -\infty < x < \infty$; $R: y = -1, 0, 1$



30. $D: -\infty < x < \infty$; $R: y = -1, 0, 1$



31. $\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\left(-\frac{\pi}{2}\right) - \sin x \sin\left(-\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(-1) = \sin x$
32. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(1) = -\sin x$
33. $\sin\left(x + \frac{\pi}{2}\right) = \sin x \cos\left(\frac{\pi}{2}\right) + \cos x \sin\left(\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(1) = \cos x$
34. $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(-1) = -\cos x$
35. $\cos(A - B) = \cos(A + (-B)) = \cos A \cos(-B) - \sin A \sin(-B) = \cos A \cos B - \sin A(-\sin B)$
 $= \cos A \cos B + \sin A \sin B$
36. $\sin(A - B) = \sin(A + (-B)) = \sin A \cos(-B) + \cos A \sin(-B) = \sin A \cos B + \cos A(-\sin B)$
 $= \sin A \cos B - \cos A \sin B$
37. If $B = A$, $A - B = 0 \Rightarrow \cos(A - B) = \cos 0 = 1$. Also $\cos(A - B) = \cos(A - A) = \cos A \cos A + \sin A \sin A$
 $= \cos^2 A + \sin^2 A$. Therefore, $\cos^2 A + \sin^2 A = 1$.
38. If $B = 2\pi$, then $\cos(A + 2\pi) = \cos A \cos 2\pi - \sin A \sin 2\pi = (\cos A)(1) - (\sin A)(0) = \cos A$ and
 $\sin(A + 2\pi) = \sin A \cos 2\pi + \cos A \sin 2\pi = (\sin A)(1) + (\cos A)(0) = \sin A$. The result agrees with the fact that the cosine and sine functions have period 2π .
39. $\cos(\pi + x) = \cos \pi \cos x - \sin \pi \sin x = (-1)(\cos x) - (0)(\sin x) = -\cos x$

40. $\sin(2\pi - x) = \sin 2\pi \cos(-x) + \cos(2\pi) \sin(-x) = (0)(\cos(-x)) + (1)(\sin(-x)) = -\sin x$
41. $\sin\left(\frac{3\pi}{2} - x\right) = \sin\left(\frac{3\pi}{2}\right)\cos(-x) + \cos\left(\frac{3\pi}{2}\right)\sin(-x) = (-1)(\cos x) + (0)(\sin(-x)) = -\cos x$
42. $\cos\left(\frac{3\pi}{2} + x\right) = \cos\left(\frac{3\pi}{2}\right)\cos x - \sin\left(\frac{3\pi}{2}\right)\sin x = (0)(\cos x) - (-1)(\sin x) = \sin x$
43. $\sin \frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin \frac{\pi}{4} \cos \frac{\pi}{3} + \cos \frac{\pi}{4} \sin \frac{\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$
44. $\cos \frac{11\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \cos \frac{\pi}{4} \cos \frac{2\pi}{3} - \sin \frac{\pi}{4} \sin \frac{2\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2} + \sqrt{6}}{4}$
45. $\cos \frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos \frac{\pi}{3} \cos\left(-\frac{\pi}{4}\right) - \sin \frac{\pi}{3} \sin\left(-\frac{\pi}{4}\right) = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1 + \sqrt{3}}{2\sqrt{2}}$
46. $\sin \frac{5\pi}{12} = \sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1 + \sqrt{3}}{2\sqrt{2}}$
47. $\cos^2 \frac{\pi}{8} = \frac{1 + \cos\left(\frac{2\pi}{8}\right)}{2} = \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4}$
48. $\cos^2 \frac{5\pi}{12} = \frac{1 + \cos\left(\frac{10\pi}{12}\right)}{2} = \frac{1 + \left(-\frac{\sqrt{3}}{2}\right)}{2} = \frac{2 - \sqrt{3}}{4}$
49. $\sin^2 \frac{\pi}{12} = \frac{1 - \cos\left(\frac{2\pi}{12}\right)}{2} = \frac{1 - \frac{\sqrt{3}}{2}}{2} = \frac{2 - \sqrt{3}}{4}$
50. $\sin^2 \frac{3\pi}{8} = \frac{1 - \cos\left(\frac{6\pi}{8}\right)}{2} = \frac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{2} = \frac{2 + \sqrt{2}}{4}$
51. $\sin^2 \theta = \frac{3}{4} \Rightarrow \sin \theta = \pm \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
52. $\sin^2 \theta = \cos^2 \theta \Rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{\cos^2 \theta}{\cos^2 \theta} \Rightarrow \tan^2 \theta = 1 \Rightarrow \tan \theta = \pm 1 \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
53. $\sin 2\theta - \cos \theta = 0 \Rightarrow 2 \sin \theta \cos \theta - \cos \theta = 0 \Rightarrow \cos \theta (2 \sin \theta - 1) = 0 \Rightarrow \cos \theta = 0$ or $2 \sin \theta - 1 = 0$
 $\Rightarrow \cos \theta = 0$ or $\sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$, or $\theta = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$
54. $\cos 2\theta + \cos \theta = 0 \Rightarrow 2 \cos^2 \theta - 1 + \cos \theta = 0 \Rightarrow 2 \cos^2 \theta + \cos \theta - 1 = 0 \Rightarrow (\cos \theta + 1)(2 \cos \theta - 1) = 0$
 $\Rightarrow \cos \theta + 1 = 0$ or $2 \cos \theta - 1 = 0 \Rightarrow \cos \theta = -1$ or $\cos \theta = \frac{1}{2} \Rightarrow \theta = \pi$ or $\theta = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$
55. $\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} = \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
56. $\tan(A - B) = \frac{\sin(A - B)}{\cos(A - B)} = \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B + \sin A \sin B} = \frac{\frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} + \frac{\sin A \sin B}{\cos A \cos B}} = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
57. According to the figure in the text, we have the following: By the law of cosines, $c^2 = a^2 + b^2 - 2ab \cos \theta$
 $= 1^2 + 1^2 - 2 \cos(A - B) = 2 - 2 \cos(A - B)$. By distance formula, $c^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$
 $= \cos^2 A - 2 \cos A \cos B + \cos^2 B + \sin^2 A - 2 \sin A \sin B + \sin^2 B = 2 - 2(\cos A \cos B + \sin A \sin B)$. Thus
 $c^2 = 2 - 2 \cos(A - B) = 2 - 2(\cos A \cos B + \sin A \sin B) \Rightarrow \cos(A - B) = \cos A \cos B + \sin A \sin B$.

58. (a) $\cos(A - B) = \cos A \cos B + \sin A \sin B$
 $\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$ and $\cos \theta = \sin\left(\frac{\pi}{2} - \theta\right)$
Let $\theta = A + B$
 $\sin(A + B) = \cos\left[\frac{\pi}{2} - (A + B)\right] = \cos\left[\left(\frac{\pi}{2} - A\right) - B\right] = \cos\left(\frac{\pi}{2} - A\right) \cos B + \sin\left(\frac{\pi}{2} - A\right) \sin B$
 $= \sin A \cos B + \cos A \sin B$
- (b) $\cos(A - B) = \cos A \cos B + \sin A \sin B$
 $\cos(A - (-B)) = \cos A \cos(-B) + \sin A \sin(-B)$
 $\Rightarrow \cos(A + B) = \cos A \cos(-B) + \sin A \sin(-B) = \cos A \cos B + \sin A(-\sin B) = \cos A \cos B - \sin A \sin B$
Because the cosine function is even and the sine function is odd.

59. $c^2 = a^2 + b^2 - 2ab \cos C = 2^2 + 3^2 - 2(2)(3) \cos(60^\circ) = 4 + 9 - 12 \cos(60^\circ) = 13 - 12\left(\frac{1}{2}\right) = 7$.
Thus, $c = \sqrt{7} \approx 2.65$.

60. $c^2 = a^2 + b^2 - 2ab \cos C = 2^2 + 3^2 - 2(2)(3) \cos(40^\circ) = 13 - 12 \cos(40^\circ)$. Thus, $c = \sqrt{13 - 12 \cos 40^\circ} \approx 1.951$.

61. From the figures in the text, we see that $\sin B = \frac{h}{c}$. If C is an acute angle, then $\sin C = \frac{h}{b}$. On the other hand, if C is obtuse (as in the figure on the right in the text), then $\sin C = \sin(\pi - C) = \frac{h}{b}$. Thus, in either case,
 $h = b \sin C = c \sin B \Rightarrow ah = ab \sin C = ac \sin B$.

By the law of cosines, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ and $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$. Moreover, since the sum of the interior angles of triangle is π , we have $\sin A = \sin(\pi - (B + C)) = \sin(B + C) = \sin B \cos C + \cos B \sin C$
 $= \left(\frac{h}{c}\right) \left[\frac{a^2 + b^2 - c^2}{2ab}\right] + \left[\frac{a^2 + c^2 - b^2}{2ac}\right] \left(\frac{h}{b}\right) = \left(\frac{h}{2abc}\right)(2a^2 + b^2 - c^2 + c^2 - b^2) = \frac{ah}{bc} \Rightarrow ah = bc \sin A$.

Combining our results we have $ah = ab \sin C$, $ah = ac \sin B$, and $ah = bc \sin A$. Dividing by abc gives
 $\frac{h}{bc} = \frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}$.
law of sines

62. By the law of sines, $\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sqrt{3}/2}{c}$. By Exercise 59 we know that $c = \sqrt{7}$. Thus $\sin B = \frac{3\sqrt{3}}{2\sqrt{7}} \approx 0.982$.

63. From the figure at the right and the law of cosines,

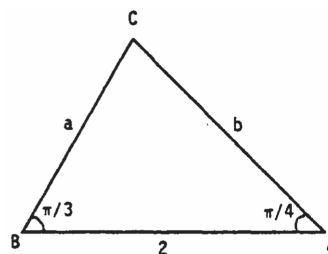
$$b^2 = a^2 + 2^2 - 2(2a) \cos B$$

$$= a^2 + 4 - 4a\left(\frac{1}{2}\right) = a^2 - 2a + 4.$$

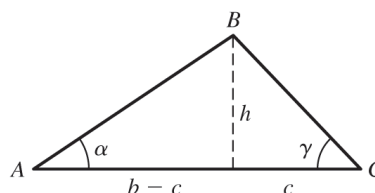
Applying the law of sines to the figure, $\frac{\sin A}{a} = \frac{\sin B}{b}$
 $\Rightarrow \frac{\sqrt{2}/2}{a} = \frac{\sqrt{3}/2}{b} \Rightarrow b = \frac{\sqrt{3}}{\sqrt{2}} a$. Thus, combining results,

$$a^2 - 2a + 4 = b^2 = \frac{3}{2}a^2 \Rightarrow 0 = \frac{1}{2}a^2 + 2a - 4 \Rightarrow 0 = a^2 + 4a - 8.$$

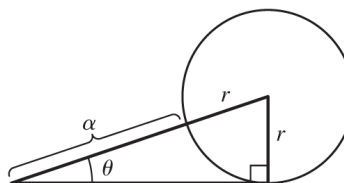
From the quadratic formula and the fact that $a > 0$, we have $a = \frac{-4 + \sqrt{4^2 - 4(1)(-8)}}{2} = \frac{4\sqrt{3} - 4}{2} \approx 1.464$.



$$\begin{aligned}
 64. \quad \tan \gamma &= \frac{h}{c} \Rightarrow c = \frac{h}{\tan \gamma} \\
 \tan \alpha &= \frac{h}{b-c} = \frac{h}{b - \frac{h}{\tan \gamma}} = \frac{h \tan \gamma}{b \tan \gamma - h} \Rightarrow \\
 b \tan \alpha \tan \gamma - h \tan \alpha &= h \tan \gamma \Rightarrow \\
 b \tan \alpha \tan \gamma &= h \tan \alpha + h \tan \gamma \Rightarrow \\
 b \tan \alpha \tan \gamma &= h(\tan \alpha + \tan \gamma) \Rightarrow \\
 h &= \frac{b \tan \alpha \tan \gamma}{\tan \alpha + \tan \gamma}
 \end{aligned}$$

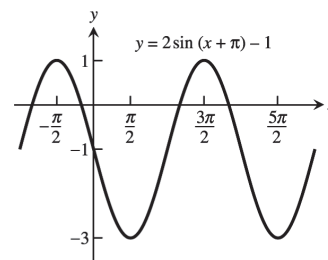


$$\begin{aligned}
 65. \quad \sin \theta &= \frac{r}{\alpha + r} \\
 \Rightarrow \alpha \sin \theta + r \sin \theta &= r \\
 \Rightarrow \alpha \sin \theta &= r - r \sin \theta = r(1 - \sin \theta) \\
 \Rightarrow r &= \frac{\alpha \sin \theta}{1 - \sin \theta}
 \end{aligned}$$

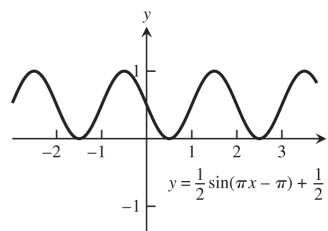


66. (a) The graphs of $y = \sin x$ and $y = x$ nearly coincide when x is near the origin (when the calculator is in radians mode).
 (b) In degree mode, when x is near zero degrees the sine of x is much closer to zero than x itself. The curves look like intersecting straight lines near the origin when the calculator is in degree mode.

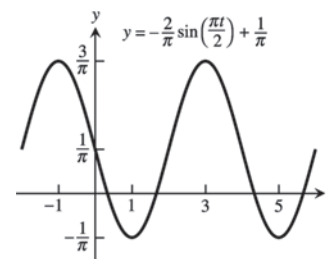
$$67. \quad A = 2, B = 2\pi, C = -\pi, D = -1$$



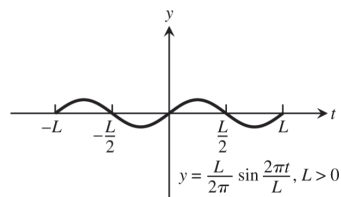
$$68. \quad A = \frac{1}{2}, B = 2, C = 1, D = \frac{1}{2}$$



$$69. \quad A = -\frac{2}{\pi}, B = 4, C = 0, D = \frac{1}{\pi}$$



70. $A = \frac{L}{2\pi}$, $B = L$, $C = 0$, $D = 0$



71–74. Example CAS commands:

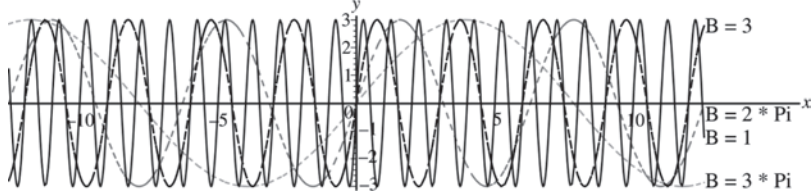
Maple:

```
f := x -> A*sin((2*Pi/B)*(x-C))+D1;
A:=3; C:=0; D1:=0;
f_list := [seq(f(x), B=[1,3,2*Pi,5*Pi])];
plot(f_list, x=-4*Pi..4*Pi, scaling=constrained,
     color=[red,blue,green,cyan], linestyle=[1,3,4,7],
     legend=["B=1", "B=3", "B=2*Pi", "B=3*Pi"],
     title="#71 (Section 1.3)");
```

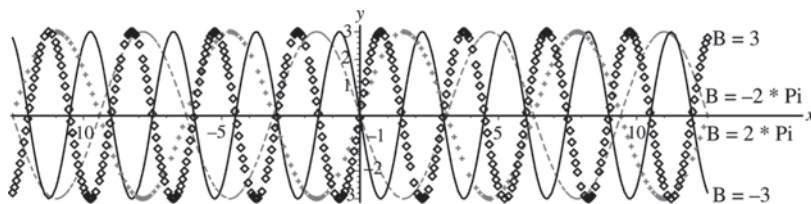
Mathematica:

```
Clear[a, b, c, d, f, x]
f[x_] := a Sin[2π/b (x - c)] + d
Plot[f[x]/., {a -> 3, b -> 1, c -> 0, d -> 0}, {x, -4π, 4π}]
```

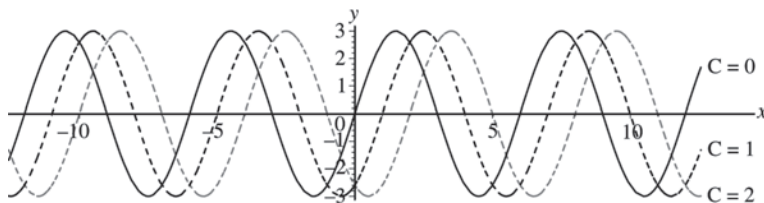
71. (a) The graph stretches horizontally.



(b) The period remains the same: period = $|B|$. The graph has a horizontal shift of $\frac{1}{2}$ period.



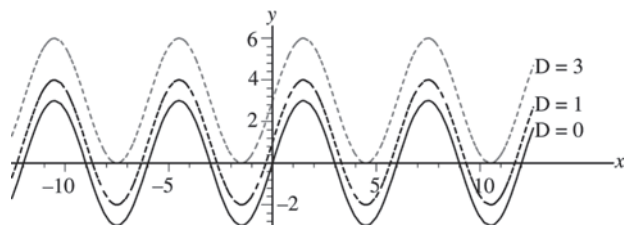
72. (a) The graph is shifted right C units.



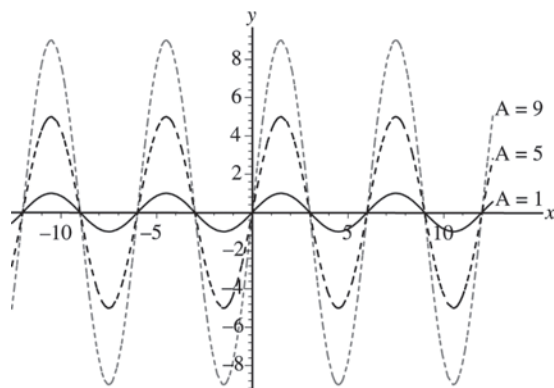
(b) The graph is shifted left C units.

(c) A shift of $\pm$ one period will produce no apparent shift. $|C| = 6$

73. (a) The graph shifts upwards $|D|$ units for $D > 0$.
 (b) The graph shifts down $|D|$ units for $D < 0$.



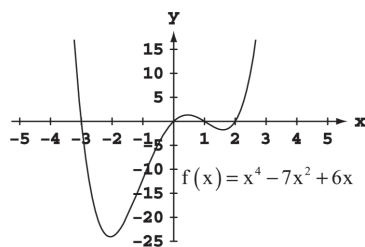
74. (a) The graph stretches $|A|$ units. (b) For $A < 0$, the graph is inverted.



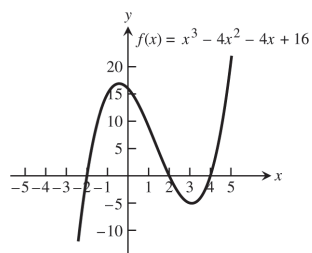
1.4 GRAPHING WITH SOFTWARE

- 1–4. The most appropriate viewing window displays the maxima, minima, intercepts, and end behavior of the graphs and has little unused space.

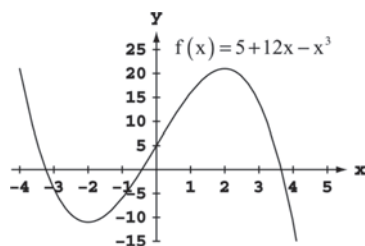
1. d.



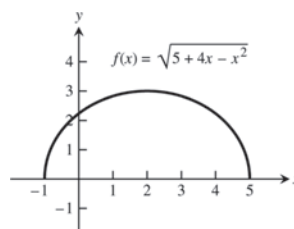
2. c.



3. d.

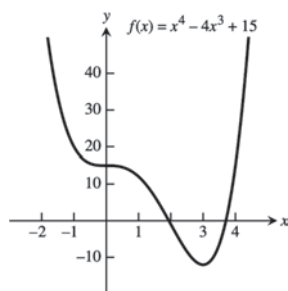


4. b.

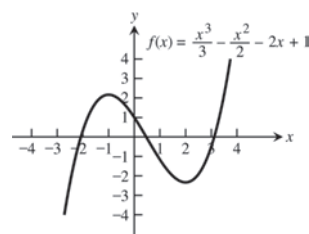


5–30. For any display there are many appropriate display widows. The graphs given as answers in Exercises 5–30 are not unique in appearance.

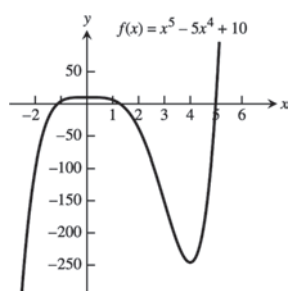
5. $[-2, 5]$ by $[-15, 40]$



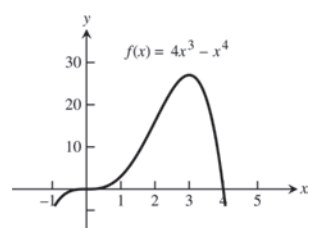
6. $[-4, 4]$ by $[-4, 4]$



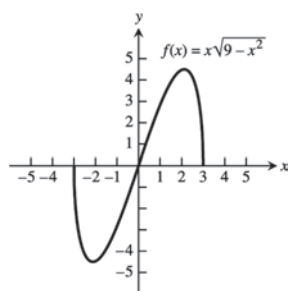
7. $[-2, 6]$ by $[-250, 50]$



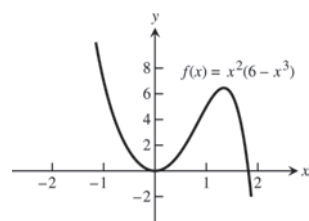
8. $[-1, 5]$ by $[-5, 30]$



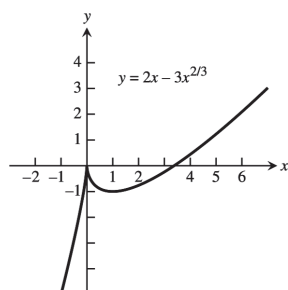
9. $[-4, 4]$ by $[-5, 5]$



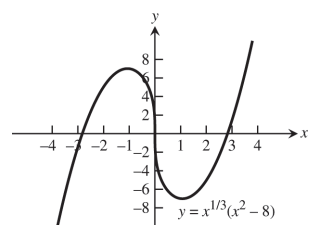
10. $[-2, 2]$ by $[-2, 8]$



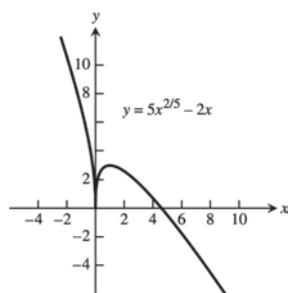
11. $[-2, 6]$ by $[-5, 4]$



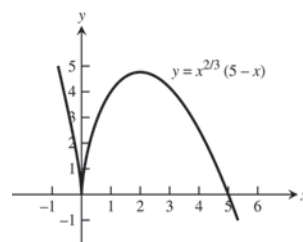
12. $[-4, 4]$ by $[-8, 8]$



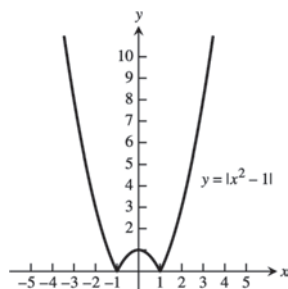
13. $[-1, 6]$ by $[-1, 4]$



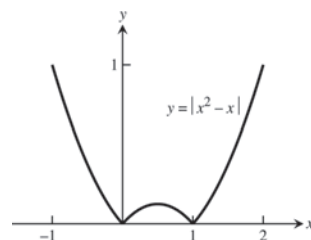
14. $[-1, 6]$ by $[-1, 5]$



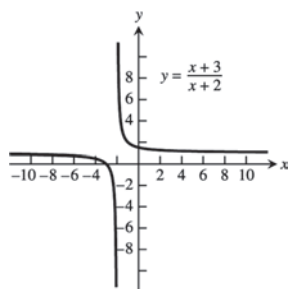
15. $[-3, 3]$ by $[0, 10]$



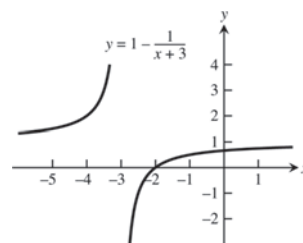
16. $[-1, 2]$ by $[0, 1]$



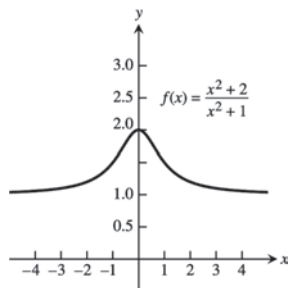
17. $[-10, 10]$ by $[-10, 10]$



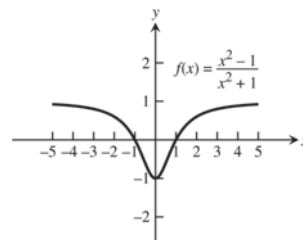
18. $[-5, 1]$ by $[-2, 4]$

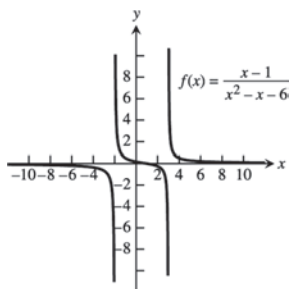
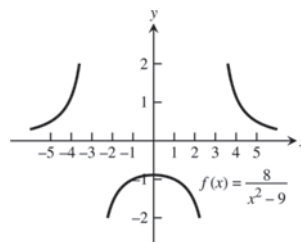
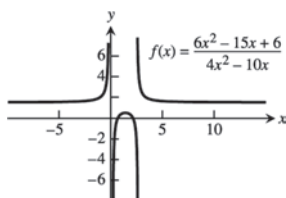
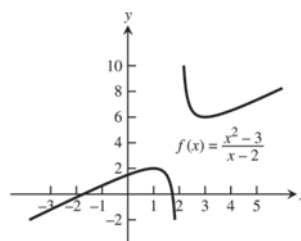
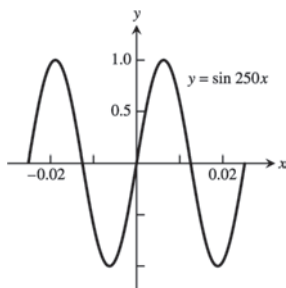
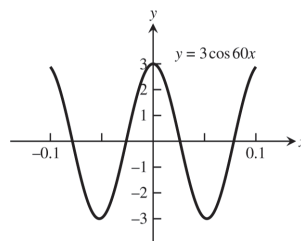
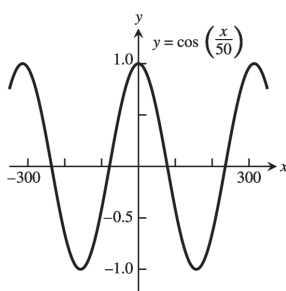
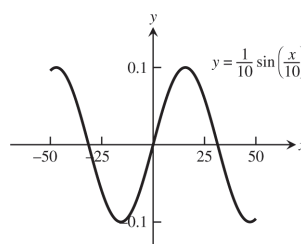


19. $[-4, 4]$ by $[0, 3]$

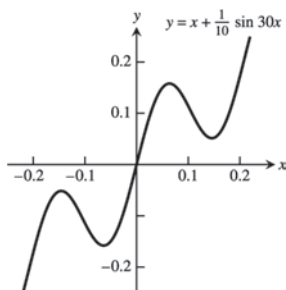


20. $[-5, 5]$ by $[-2, 2]$

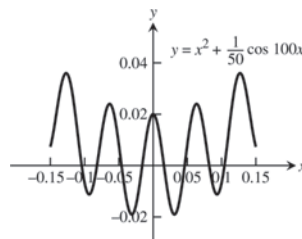


21. $[-10, 10]$ by $[-6, 6]$

22. $[-5, 5]$ by $[-2, 2]$

23. $[-6, 10]$ by $[-6, 6]$

24. $[-3, 5]$ by $[-2, 10]$

25. $[-0.03, 0.03]$ by $[-1.25, 1.25]$

26. $[-0.1, 0.1]$ by $[-3, 3]$

27. $[-300, 300]$ by $[-1.25, 1.25]$

28. $[-50, 50]$ by $[-0.1, 0.1]$


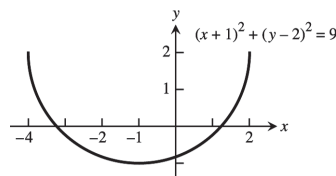
29. $[-0.25, 0.25]$ by $[-0.3, 0.3]$



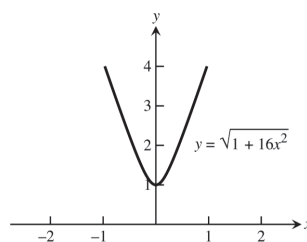
30. $[-0.15, 0.15]$ by $[-0.02, 0.05]$



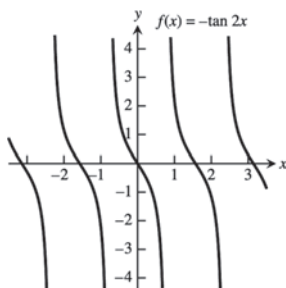
31. $x^2 + 2x = 4 + 4y - y^2 \Rightarrow y = 2 \pm \sqrt{-x^2 - 2x + 8}$.
 The lower half is produced by graphing
 $y = 2 - \sqrt{-x^2 - 2x + 8}$.



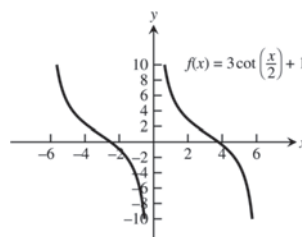
32. $y^2 - 16x^2 = 1 \Rightarrow y = \pm \sqrt{1 + 16x^2}$. The upper branch
 is produced by graphing $y = \sqrt{1 + 16x^2}$.



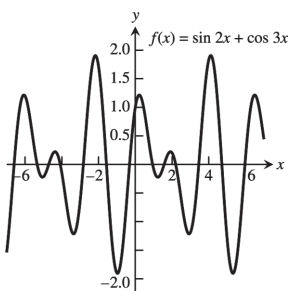
33.



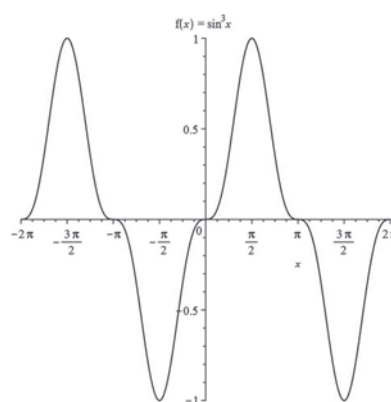
34.



35.



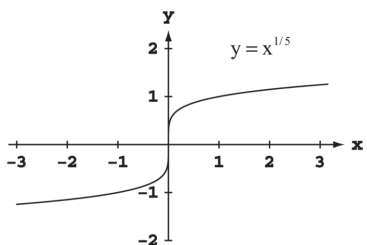
36.



CHAPTER 1 PRACTICE EXERCISES

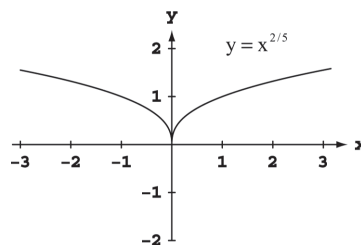
- The area is $A = \pi r^2$ and the circumference is $C = 2\pi r$. Thus, $r = \frac{C}{2\pi} \Rightarrow A = \pi \left(\frac{C}{2\pi} \right)^2 = \frac{C^2}{4\pi}$.
- The surface area is $S = 4\pi r^2 \Rightarrow r = \left(\frac{S}{4\pi} \right)^{1/2}$. The volume is $V = \frac{4}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$. Substitution into the formula for surface area gives $S = 4\pi r^2 = 4\pi \left(\frac{3V}{4\pi} \right)^{2/3}$.
- The coordinates of a point on the parabola are (x, x^2) . The angle of inclination θ joining this point to the origin satisfies the equation $\tan \theta = \frac{x^2}{x} = x$. Thus the point has coordinates $(x, x^2) = (\tan \theta, \tan^2 \theta)$.
- $\tan \theta = \frac{\text{rise}}{\text{run}} = \frac{h}{500} \Rightarrow h = 500 \tan \theta$ ft

5.



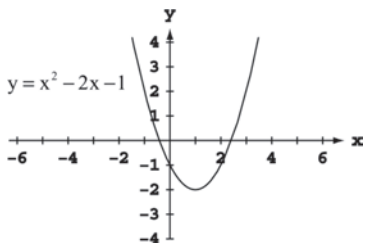
Symmetric about the origin

6.



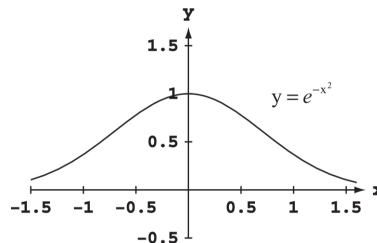
Symmetric about the y-axis

7.



Neither

8.



Symmetric about the y-axis

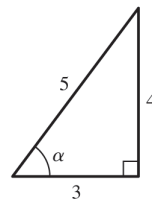
- $y(-x) = (-x)^2 + 1 = x^2 + 1 = y(x)$; even
- $y(-x) = (-x)^5 - (-x)^3 - (-x) = -x^5 + x^3 + x = -y(x)$; odd
- $y(-x) = 1 - \cos(-x) = 1 - \cos x = y(x)$; even
- $y(-x) = \sec(-x) \tan(-x) = \frac{\sin(-x)}{\cos^2(-x)} = \frac{-\sin x}{\cos^2 x} = -\sec x \tan x = -y(x)$; odd
- $y(-x) = \frac{(-x)^4 + 1}{(-x)^3 - 2(-x)} = \frac{x^4 + 1}{-x^3 + 2x} = -\frac{x^4 + 1}{x^3 - 2x} = -y(x)$; odd
- $y(-x) = (-x) - \sin(-x) = (-x) + \sin x = -(x - \sin x) = -y(x)$; odd
- $y(-x) = -x + \cos(-x) = -x + \cos x$; neither even nor odd

16. $y(-x) = (-x)\cos(-x) = -x\cos x = -y(x)$; odd
17. Since f and g are odd $\Rightarrow f(-x) = -f(x)$ and $g(-x) = -g(x)$.
- (a) $(f \cdot g)(-x) = f(-x)g(-x) = [-f(x)] [-g(x)] = f(x)g(x) = (f \cdot g)(x) \Rightarrow f \cdot g$ is even.
- (b) $f^3(-x) = f(-x)f(-x)f(-x) = [-f(x)] [-f(x)] [-f(x)] = -f(x) \cdot f(x) \cdot f(x) = -f^3(x) \Rightarrow f^3$ is odd.
- (c) $f(\sin(-x)) = f(-\sin(x)) = -f(\sin(x)) \Rightarrow f(\sin(x))$ is odd.
- (d) $g(\sec(-x)) = g(\sec(x)) \Rightarrow g(\sec(x))$ is even.
- (e) $|g(-x)| = |-g(x)| = |g(x)| \Rightarrow |g|$ is even.
18. Let $f(a-x) = f(a+x)$ and define $g(x) = f(x+a)$. Then $g(-x) = f((-x)+a) = f(a-x) = f(a+x) = f(x+a) = g(x) \Rightarrow g(x) = f(x+a)$ is even.
19. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
- (b) Since $|x|$ attains all nonnegative values, the range is $[-2, \infty)$.
20. (a) Since the square root requires $1-x \geq 0$, the domain is $(-\infty, 1]$.
- (b) Since $\sqrt{1-x}$ attains all nonnegative values, the range is $[-2, \infty)$.
21. (a) Since the square root requires $16-x^2 \geq 0$, the domain is $[-4, 4]$.
- (b) For values of x in the domain, $0 \leq 16-x^2 \leq 16$, so $0 \leq \sqrt{16-x^2} \leq 4$. The range is $[0, 4]$.
22. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
- (b) Since 3^{2-x} attains all positive values, the range is $(1, \infty)$.
23. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
- (b) Since $2e^{-x}$ attains all positive values, the range is $(-3, \infty)$.
24. (a) The function is equivalent to $y = \tan 2x$, so we require $2x \neq \frac{k\pi}{2}$ for odd integers k . The domain is given by $x \neq \frac{k\pi}{4}$ for odd integers k .
- (b) Since the tangent function attains all values, the range is $(-\infty, \infty)$.
25. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
- (b) The sine function attains values from -1 to 1 , so $-2 \leq 2\sin(3x+\pi) \leq 2$ and hence $-3 \leq 2\sin(3x+\pi)-1 \leq 1$. The range is $[-3, 1]$.
26. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
- (b) The function is equivalent to $y = \sqrt[5]{x^2}$, which attains all nonnegative values. The range is $[0, \infty)$.
27. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
- (b) The cosine function attains values from -1 to 1 , so $-1 \leq \cos(x-3) \leq 1$ and hence $0 \leq \cos(x-3)+1 \leq 2$. The range is $[0, 2]$.
28. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
- (b) The cube root attains all real values, so the range is $(-\infty, \infty)$.
29. $y = 5 - \sqrt{(x-3)(x+1)}$ so the domain $= (-\infty, -1] \cup [3, \infty)$; $\sqrt{(x-3)(x+1)} \geq 0$ and can be any positive number, so the range $= (-\infty, 5]$.

30. $y = 2 + \frac{3x^2}{x^2+4}$ so the domain $= (-\infty, \infty)$; $0 \leq \frac{3x^2}{x^2+4} < 3$ so the range $= [2, 5)$.

31. $y = 4 \sin\left(\frac{1}{x}\right)$ so the domain $= (-\infty, 0) \cup (0, \infty)$; if $\frac{2}{3\pi} \leq x \leq \frac{2}{\pi}$, then $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$, so the range $= [-4, 4]$.

32. $y = 3 \cos x + 4 \sin x$ so the domain $= (-\infty, \infty)$;
 and $\sqrt{3^2 + 4^2} = 5$ so $3 \cos x + 4 \sin x = 5\left(\frac{3}{5} \cos x + \frac{4}{5} \sin x\right)$
 $= 5(\cos \alpha \cos x + \sin \alpha \sin x) = 5 \cos(\alpha - x)$, where $-1 \leq \cos(\alpha - x) \leq 1$
 so the range $= [-5, 5]$.



33. (a) Increasing because volume increases as radius increases.
 (b) Neither, since the greatest integer function is composed of horizontal (constant) line segments.
 (c) Decreasing because as the height increases, the atmospheric pressure decreases.
 (d) Increasing because the kinetic (motion) energy increases as the particles velocity increases.
34. (a) Increasing on $[2, \infty)$ (b) Increasing on $[-1, \infty)$
 (c) Increasing on $(-\infty, \infty)$ (d) Increasing on $\left[\frac{1}{2}, \infty\right)$
35. (a) The function is defined for $-4 \leq x \leq 4$, so the domain is $[-4, 4]$.
 (b) The function is equivalent to $y = \sqrt{|x|}$, $-4 \leq x \leq 4$, which attains values from 0 to 2 for x in the domain.
 The range is $[0, 2]$.
36. (a) The function is defined for $-2 \leq x \leq 2$, so the domain is $[-2, 2]$.
 (b) The range is $[-1, 1]$.

37. First piece: Line through $(0, 1)$ and $(1, 0)$; $m = \frac{0-1}{1-0} = \frac{-1}{1} = -1 \Rightarrow y = -x + 1 = 1 - x$

Second piece: Line through $(1, 1)$ and $(2, 0)$; $m = \frac{0-1}{2-1} = \frac{-1}{1} = -1 \Rightarrow y = -(x-1) + 1 = -x + 2 = 2 - x$

$$f(x) = \begin{cases} 1-x, & 0 \leq x < 1 \\ 2-x, & 1 \leq x \leq 2 \end{cases}$$

38. First piece: Line through $(0, 0)$ and $(2, 5)$; $m = \frac{5-0}{2-0} = \frac{5}{2} \Rightarrow y = \frac{5}{2}x$

Second piece: Line through $(2, 5)$ and $(4, 0)$; $m = \frac{0-5}{4-2} = \frac{-5}{2} = -\frac{5}{2} \Rightarrow y = -\frac{5}{2}(x-2) + 5 = -\frac{5}{2}x + 10 = 10 - \frac{5x}{2}$

$$f(x) = \begin{cases} \frac{5}{2}x, & 0 \leq x < 2 \\ 10 - \frac{5x}{2}, & 2 \leq x \leq 4 \end{cases} \quad (\text{Note: } x = 2 \text{ can be included on either piece.})$$

39. (a) $(f \circ g)(-1) = f(g(-1)) = f\left(\frac{1}{\sqrt{-1}+2}\right) = f(1) = \frac{1}{1} = 1$

(b) $(g \circ f)(2) = g(f(2)) = g\left(\frac{1}{2}\right) = \frac{1}{\sqrt{\frac{1}{2}}+2} = \frac{1}{\sqrt{2.5}} \text{ or } \sqrt{\frac{2}{5}}$

(c) $(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = x, x \neq 0$

(d) $(g \circ g)(x) = g(g(x)) = g\left(\frac{1}{\sqrt{x}+2}\right) = \frac{1}{\sqrt{\frac{1}{\sqrt{x}+2}}+2} = \frac{\sqrt[4]{x+2}}{\sqrt{1+2\sqrt{x}+2}}$

40. (a) $(f \circ g)(-1) = f(g(-1)) = f(\sqrt[3]{-1+1}) = f(0) = 2 - 0 = 2$

(b) $(g \circ f)(2) = f(g(2)) = g(2-2) = g(0) = \sqrt[3]{0+1} = 1$

(c) $(f \circ f)(x) = f(f(x)) = f(2-x) = 2 - (2-x) = x$

(d) $(g \circ g)(x) = g(g(x)) = g(\sqrt[3]{x+1}) = \sqrt[3]{\sqrt[3]{x+1}+1}$

41. (a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x+2}) = 2 - (\sqrt{x+2})^2 = -x, x \geq -2$

$(g \circ f)(x) = g(f(x)) = g(2-x^2) = \sqrt{(2-x^2)+2} = \sqrt{4-x^2}$

(b) Domain of $f \circ g$: $[-2, \infty)$.

(c) Range of $f \circ g$: $(-\infty, 2]$.

Domain of $g \circ f$: $[-2, 2]$.

Range of $g \circ f$: $[0, 2]$.

42. (a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{1-x}) = \sqrt{\sqrt{1-x}} = \sqrt[4]{1-x}$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{1-\sqrt{x}}$

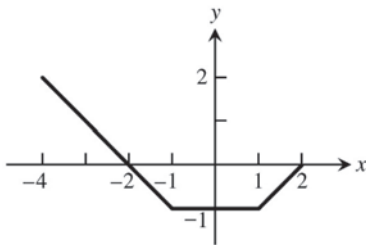
(b) Domain of $f \circ g$: $(-\infty, 1]$

(c) Range of $f \circ g$: $[0, \infty)$

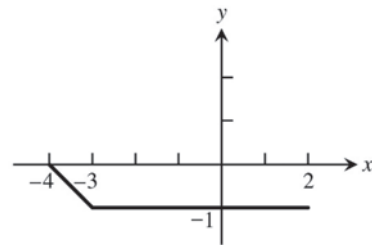
Domain of $g \circ f$: $[0, 1]$

Range of $g \circ f$: $[0, 1]$

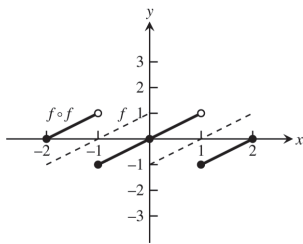
43. $y = f(x)$



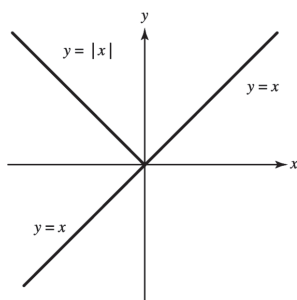
$y = (f \circ f)(x)$



44.

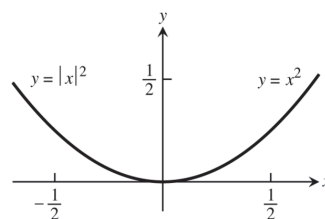


45.



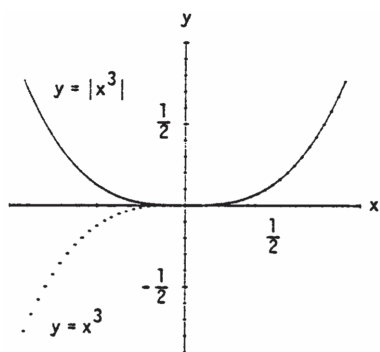
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y -axis. The graph of $f_2(x)$ to the left of the y -axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y -axis.

46.



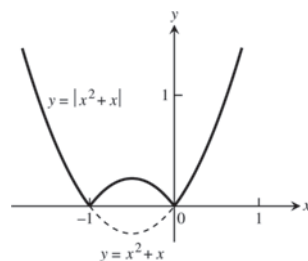
It does not change the graph.

47.



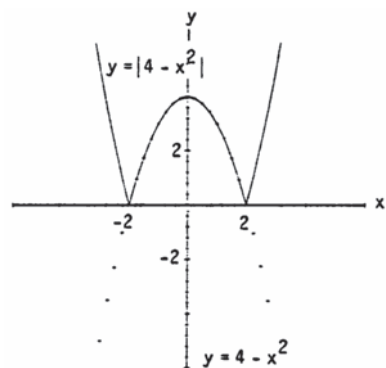
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x -axis.

48.



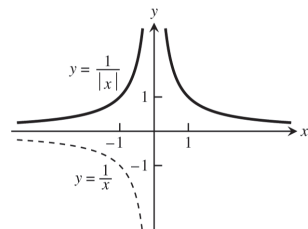
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x -axis.

49.



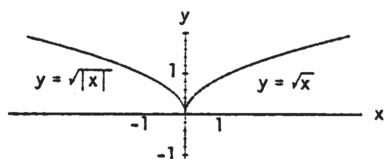
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x -axis.

50.



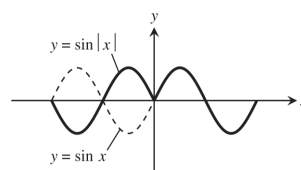
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y -axis. The graph of $f_2(x)$ to the left of the y -axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y -axis.

51.



The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y -axis. The graph of $f_2(x)$ to the left of the y -axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y -axis.

52.



The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y -axis. The graph of $f_2(x)$ to the left of the y -axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y -axis.

53. (a) $y = g(x-3) + \frac{1}{2}$

(c) $y = g(-x)$

(e) $y = 5 \cdot g(x)$

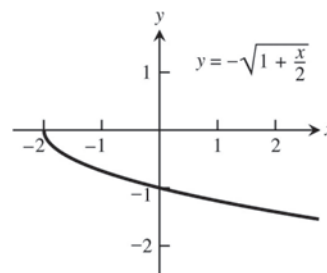
(b) $y = g\left(x + \frac{2}{3}\right) - 2$

(d) $y = -g(x)$

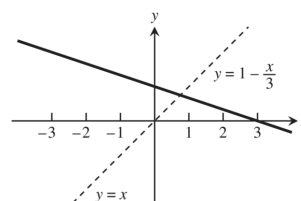
(f) $y = g(5x)$

54. (a) Shift the graph of f right 5 units.(b) Horizontally compress the graph of f by a factor of 4.(c) Horizontally compress the graph of f by a factor of 3 and then reflect the graph about the y -axis.(d) Horizontally compress the graph of f by a factor of 2 and then shift the graph left $\frac{1}{2}$ unit.(e) Horizontally stretch the graph of f by a factor of 3 and then shift the graph down 4 units.(f) Vertically stretch the graph of f by a factor of 3, then reflect the graph about the x -axis, and finally shift the graph up $\frac{1}{4}$ unit.

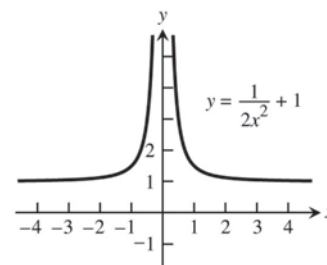
55. Reflect the graph of $y = \sqrt{x}$ about the x -axis followed by a horizontal compression by a factor of $\frac{1}{2}$, then shift left 2 units.



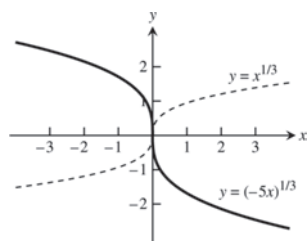
56. Reflect the graph of $y = x$ about the x -axis, followed by a vertical compression of the graph by a factor of 3, then shift the graph up 1 unit.



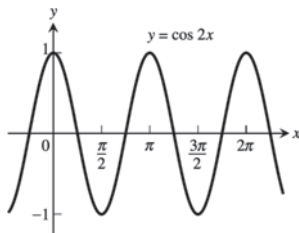
57. Vertical compression of the graph of $y = \frac{1}{x^2}$ by a factor of 2, then shift the graph up 1 unit.



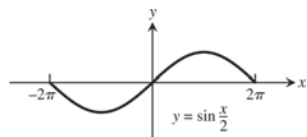
58. Reflect the graph of $y = x^{1/3}$ about the y -axis, then compress the graph horizontally by a factor of 5.



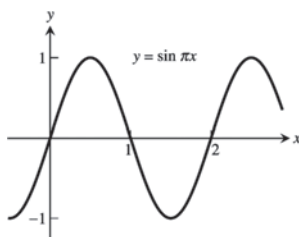
59.

period = π

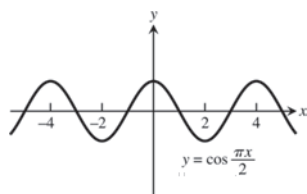
60.

period = 4π

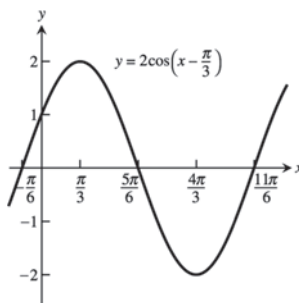
61.

period = 2

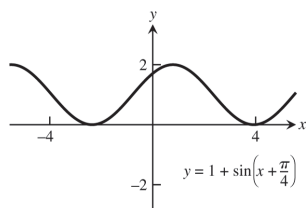
62.

period = 4

63.

period = 2π

64.

period = 2π

65. (a) $\sin B = \sin \frac{\pi}{3} = \frac{b}{c} = \frac{b}{2} \Rightarrow b = 2 \sin \frac{\pi}{3} = 2 \left(\frac{\sqrt{3}}{2} \right) = \sqrt{3}$. By the Pythagorean theorem,

$$a^2 + b^2 = c^2 \Rightarrow a = \sqrt{c^2 - b^2} = \sqrt{4 - 3} = 1.$$

- (b) $\sin B = \sin \frac{\pi}{3} = \frac{b}{c} = \frac{2}{c} \Rightarrow c = \frac{2}{\sin \frac{\pi}{3}} = \frac{2}{(\frac{\sqrt{3}}{2})} = \frac{4}{\sqrt{3}}$. Thus, $a = \sqrt{c^2 - b^2} = \sqrt{\left(\frac{4}{\sqrt{3}}\right)^2 - (2)^2} = \sqrt{\frac{16}{3} - 4} = \sqrt{\frac{4}{3}} = \frac{2\sqrt{3}}{3}$.

66. (a) $\sin A = \frac{a}{c} \Rightarrow a = c \sin A$

- (b) $\tan A = \frac{a}{b} \Rightarrow a = b \tan A$

67. (a) $\tan B = \frac{b}{a} \Rightarrow a = \frac{b}{\tan B}$

(b) $\sin A = \frac{a}{c} \Rightarrow c = \frac{a}{\sin A}$

68. (a) $\sin A = \frac{a}{c}$

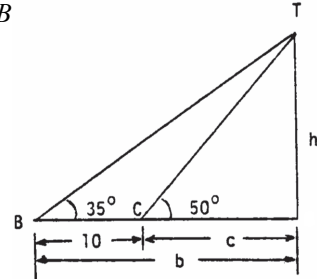
(b) $\sin A = \frac{a}{c} = \frac{\sqrt{c^2 - b^2}}{c}$

69. Let h = height of vertical pole, and let b and c denote the distances of points B and C from the base of the pole, measured along the flat ground, respectively. Then, $\tan 50^\circ = \frac{h}{c}$, $\tan 35^\circ = \frac{h}{b}$, and $b - c = 10$.

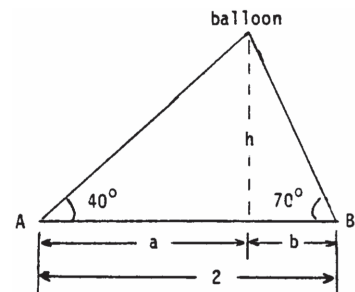
Thus, $h = c \tan 50^\circ$ and $h = b \tan 35^\circ = (c + 10) \tan 35^\circ$

$\tan 35^\circ \Rightarrow c \tan 50^\circ = (c + 10) \tan 35^\circ \Rightarrow c(\tan 50^\circ - \tan 35^\circ) = 10 \tan 35^\circ$

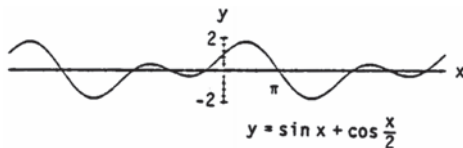
$\Rightarrow c = \frac{10 \tan 35^\circ}{\tan 50^\circ - \tan 35^\circ} \Rightarrow h = c \tan 50^\circ = \frac{10 \tan 35^\circ \tan 50^\circ}{\tan 50^\circ - \tan 35^\circ} \approx 16.98 \text{ m.}$



70. Let h = height of balloon above ground. From the figure at the right, $\tan 40^\circ = \frac{h}{a}$, $\tan 70^\circ = \frac{h}{b}$, and $a + b = 2$. Thus, $h = b \tan 70^\circ \Rightarrow h = (2 - a) \tan 70^\circ$ and $h = a \tan 40^\circ \Rightarrow (2 - a) \tan 70^\circ = a \tan 40^\circ$
 $\Rightarrow a(\tan 40^\circ + \tan 70^\circ) = 2 \tan 70^\circ \Rightarrow a = \frac{2 \tan 70^\circ}{\tan 40^\circ + \tan 70^\circ} \Rightarrow h = a \tan 40^\circ$
 $= \frac{2 \tan 70^\circ \tan 40^\circ}{\tan 40^\circ + \tan 70^\circ} \approx 1.3 \text{ km.}$



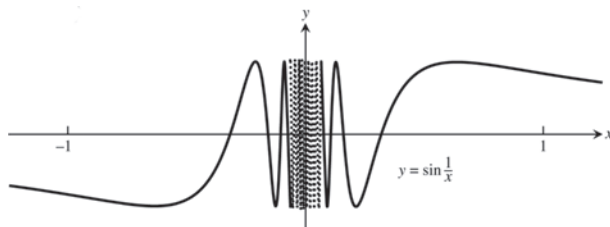
71. (a)



- (b) The period appears to be 4π .

- (c) $f(x + 4\pi) = \sin(x + 4\pi) + \cos\left(\frac{x + 4\pi}{2}\right) = \sin(x + 2\pi) + \cos\left(\frac{x}{2} + 2\pi\right) = \sin x + \cos \frac{x}{2}$
 since the period of sine and cosine is 2π . Thus, $f(x)$ has period 4π .

72. (a)



- (b) Domain: $(-\infty, 0) \cup (0, \infty)$; Range: $[-1, 1]$

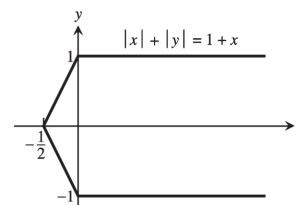
- (c) f is not periodic. For suppose f has period p . Then $f\left(\frac{1}{2\pi} + kp\right) = f\left(\frac{1}{2\pi}\right) = \sin 2\pi = 0$ for all integers k .

Choose k so large that $\frac{1}{2\pi} + kp > \frac{1}{\pi} \Rightarrow 0 < \frac{1}{(1/(2\pi)) + kp} < \pi$. But then $f\left(\frac{1}{2\pi} + kp\right) = \sin\left(\frac{1}{(1/(2\pi)) + kp}\right) > 0$ which is a contradiction. Thus f has no period, as claimed.

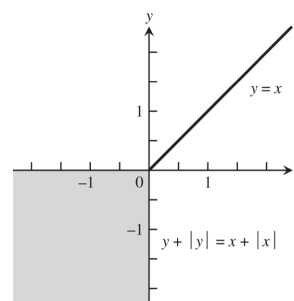
CHAPTER 1 ADDITIONAL AND ADVANCED EXERCISES

- There are (infinitely) many such function pairs. For example, $f(x) = 3x$ and $g(x) = 4x$ satisfy $f(g(x)) = f(4x) = 3(4x) = 12x = 4(3x) = g(3x) = g(f(x))$.
- Yes, there are many such function pairs. For example, if $g(x) = (2x+3)^3$ and $f(x) = x^{1/3}$, then $(f \circ g)(x) = f(g(x)) = f((2x+3)^3) = ((2x+3)^3)^{1/3} = 2x+3$.
- If f is odd and defined at x , then $f(-x) = -f(x)$. Thus $g(-x) = f(-x) - 2 = -f(x) - 2$ whereas $-g(x) = -(f(x) - 2) = -f(x) + 2$. Then g cannot be odd because $g(-x) = -g(x) \Rightarrow -f(x) - 2 = -f(x) + 2 \Rightarrow 4 = 0$, which is a contradiction. Also, $g(x)$ is not even unless $f(x) = 0$ for all x . On the other hand, if f is even, then $g(x) = f(x) - 2$ is also even: $g(-x) = f(-x) - 2 = f(x) - 2 = g(x)$.
- If g is odd and $g(0)$ is defined, then $g(0) = g(-0) = -g(0)$. Therefore, $2g(0) = 0 \Rightarrow g(0) = 0$.

- For (x, y) in the 1st quadrant, $|x| + |y| = 1 + x$
 $\Leftrightarrow x + y = 1 + x \Leftrightarrow y = 1$. For (x, y) in the 2nd quadrant, $|x| + |y| = x +$
 $1 \Leftrightarrow -x + y = x + 1 \Leftrightarrow y = 2x + 1$. In the 3rd quadrant, $|x| + |y| = x + 1$
 $\Leftrightarrow -x - y = x + 1 \Leftrightarrow y = -2x - 1$. In the 4th quadrant,
 $|x| + |y| = x + 1 \Leftrightarrow x + (-y) = x + 1 \Leftrightarrow y = -1$. The graph is given at the right.



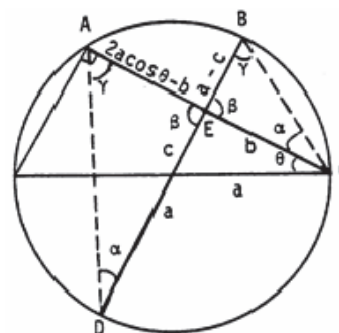
- We use reasoning similar to Exercise 5.
 - 1st quadrant: $y + |y| = x + |x| \Leftrightarrow 2y = 2x \Leftrightarrow y = x$.
 - 2nd quadrant: $y + |y| = x + |x| \Leftrightarrow 2y = x + (-x) = 0 \Leftrightarrow y = 0$.
 - 3rd quadrant: $y + |y| = x + |x| \Leftrightarrow y + (-y) = x + (-x) \Leftrightarrow 0 = 0 \Rightarrow$ all points in the 3rd quadrant satisfy the equation.
 - 4th quadrant: $y + |y| = x + |x| \Leftrightarrow y + (-y) = 2x \Leftrightarrow 0 = x$.
 Combining these results we have the graph given at the right.



- $\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x = (1 - \cos x)(1 + \cos x) \Rightarrow (1 - \cos x) = \frac{\sin^2 x}{1 + \cos x} \Rightarrow \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$
 - Using the definition of the tangent function and the double angle formulas, we have

$$\tan^2\left(\frac{x}{2}\right) = \frac{\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)} = \frac{\frac{1 - \cos\left(2\left(\frac{x}{2}\right)\right)}{2}}{\frac{1 + \cos\left(2\left(\frac{x}{2}\right)\right)}{2}} = \frac{1 - \cos x}{1 + \cos x}.$$

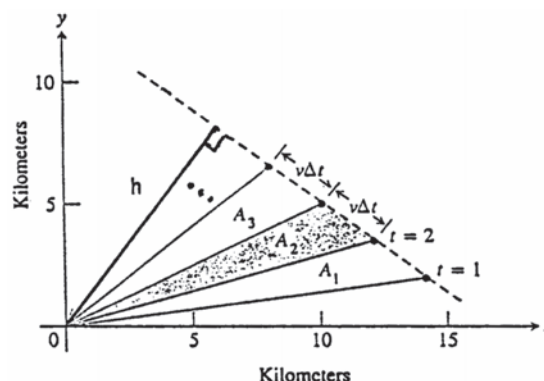
- The angles labeled γ in the accompanying figure are equal since both angles subtend arc CD . Similarly, the two angles labeled α are equal since they both subtend arc AB . Thus, triangles AED and BEC are similar which implies $\frac{a-c}{b} = \frac{2a \cos \theta - b}{a+c} \Rightarrow (a-c)(a+c) = b(2a \cos \theta - b)$
 $\Rightarrow a^2 - c^2 = 2ab \cos \theta - b^2$
 $\Rightarrow c^2 - a^2 + b^2 = 2ab \cos \theta.$



9. As in the proof of the law of sines of Section 1.3, Exercise 61, $ah = bc \sin A = ab \sin C = ac \sin B$
 $\Rightarrow$ the area of $ABC = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}ah = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B$.
10. As in Section 1.3, Exercise 61, $(\text{Area of } ABC)^2 = \frac{1}{4}(\text{base})^2(\text{height})^2 = \frac{1}{4}a^2h^2 = \frac{1}{4}a^2b^2 \sin^2 C$
 $= \frac{1}{4}a^2b^2(1 - \cos^2 C)$. By the law of cosines, $c^2 = a^2 + b^2 - 2ab \cos C \Rightarrow \cos C = \frac{a^2 + b^2 - c^2}{2ab}$. Thus,
 $(\text{area of } ABC)^2 = \frac{1}{4}a^2b^2(1 - \cos^2 C) = \frac{1}{4}a^2b^2 \left(1 - \left(\frac{a^2 + b^2 - c^2}{2ab} \right)^2 \right) = \frac{a^2b^2}{4} \left(1 - \frac{(a^2 + b^2 - c^2)^2}{4a^2b^2} \right)$
 $= \frac{1}{16} (4a^2b^2 - (a^2 + b^2 - c^2)^2) = \frac{1}{16} [(2ab + (a^2 + b^2 - c^2))(2ab - (a^2 + b^2 - c^2))]$
 $= \frac{1}{16} [(a+b)^2 - c^2)(c^2 - (a-b)^2)] = \frac{1}{16} [(a+b+c)((a+b)-c)(c+(a-b))(c-(a-b))]$
 $= \left[\left(\frac{a+b+c}{2} \right) \left(\frac{-a+b+c}{2} \right) \left(\frac{a-b+c}{2} \right) \left(\frac{a+b-c}{2} \right) \right] = s(s-a)(s-b)(s-c)$, where $s = \frac{a+b+c}{2}$.
Therefore, the area of ABC equals $\sqrt{s(s-a)(s-b)(s-c)}$.
11. If f is even and odd, then $f(-x) = -f(x)$ and $f(-x) = f(x) \Rightarrow f(x) = -f(x)$ for all x in the domain of f .
Thus $2f(x) = 0 \Rightarrow f(x) = 0$.
12. (a) As suggested, let $E(x) = \frac{f(x) + f(-x)}{2} \Rightarrow E(-x) = \frac{f(-x) + f(-(-x))}{2} = \frac{f(x) + f(-x)}{2} = E(x) \Rightarrow E$ is an even
function. Define $O(x) = f(x) - E(x) = f(x) - \frac{f(x) + f(-x)}{2} = \frac{f(x) - f(-x)}{2}$. Then $O(-x) = \frac{f(-x) - f(-(-x))}{2}$
 $= \frac{f(-x) - f(x)}{2} = -\left(\frac{f(x) - f(-x)}{2} \right) = -O(x) \Rightarrow O$ is an odd function $\Rightarrow f(x) = E(x) + O(x)$ is the sum of an
even and an odd function.
- (b) Part (a) shows that $f(x) = E(x) + O(x)$ is the sum of an even and an odd function. If also
 $f(x) = E_1(x) + O_1(x)$, where E_1 is even and O_1 is odd, then $f(x) - f(x) = 0$
 $= (E_1(x) + O_1(x)) - (E(x) + O(x))$. Thus, $E(x) - E_1(x) = O_1(x) - O(x)$ for all x in the domain of f (which
is the same as the domain of $E - E_1$ and $O - O_1$). Now $(E - E_1)(-x) = E(-x) - E_1(-x) = E(x) - E_1(x)$
(since E and E_1 are even) $= (E - E_1)(x) \Rightarrow E - E_1$ is even. Likewise, $(O_1 - O)(-x) = O_1(-x) - O(-x)$
 $= -O_1(x) - (-O(x))$ (since O and O_1 are odd) $= -(O_1(x) - O(x)) = -(O_1 - O)(x) \Rightarrow O_1 - O$ is odd.
Therefore, $E - E_1$ and $O_1 - O$ are both even and odd so they must be zero at each x in the domain of f by
Exercise 11. That is, $E_1 = E$ and $O_1 = O$, so the decomposition of f found in part (a) is unique.
13. $y = ax^2 + bx + c = a \left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} \right) - \frac{b^2}{4a} + c = a \left(x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a} + c$
- (a) If $a > 0$ the graph is a parabola that opens upward. Increasing a causes a vertical stretching and a shift of
the vertex toward the y -axis and upward. If $a < 0$ the graph is a parabola that opens downward.
Decreasing a causes a vertical stretching and a shift of the vertex toward the y -axis and downward.
- (b) If $a > 0$ the graph is a parabola that opens upward. If also $b > 0$, then increasing b causes a shift of the
graph downward to the left; if $b < 0$, then decreasing b causes a shift of the graph downward and to the
right.
If $a < 0$ the graph is a parabola that opens downward. If $b > 0$, increasing b shifts the graph upward to the
right. If $b < 0$, decreasing b shifts the graph upward to the left.
- (c) Changing c (for fixed a and b) by Δc shifts the graph upward Δc units if $\Delta c > 0$, and downward $-\Delta c$
units if $\Delta c < 0$.

14. (a) If $a > 0$, the graph rises to the right of the vertical line $x = -b$ and falls to the left. If $a < 0$, the graph falls to the right of the line $x = -b$ and rises to the left. If $a = 0$, the graph reduces to the horizontal line $y = c$. As $|a|$ increases, the slope at any given point $x = x_0$ increases in magnitude and the graph becomes steeper. As $|a|$ decreases, the slope at x_0 decreases in magnitude and the graph rises or falls more gradually.
- (b) Increasing b shifts the graph to the left; decreasing b shifts it to the right.
- (c) Increasing c shifts the graph upward; decreasing c shifts it downward.

15. Each of the triangles pictured has the same base $b = v\Delta t = v(1 \text{ sec})$. Moreover, the height of each triangle is the same value h . Thus $\frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}bh = A_1 = A_2 = A_3 = \dots$. In conclusion, the object sweeps out equal areas in each one second interval.



16. (a) Using the midpoint formula, the coordinates of P are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$. Thus the slope of $\overline{OP} = \frac{\Delta y}{\Delta x} = \frac{b/2}{a/2} = \frac{b}{a}$.
- (b) The slope of $\overline{AB} = \frac{b-0}{0-a} = -\frac{b}{a}$. The line segments $\overline{AB}$ and $\overline{OP}$ are perpendicular when the product of their slopes is $-1 = \left(\frac{b}{a}\right)\left(-\frac{b}{a}\right) = -\frac{b^2}{a^2}$. Thus, $b^2 = a^2 \Rightarrow a = b$ (since both are positive). Therefore, $\overline{AB}$ is perpendicular to $\overline{OP}$ when $a = b$.
17. From the figure we see that $0 \leq \theta \leq \frac{\pi}{2}$ and $AB = AD = 1$. From trigonometry we have the following:
 $\sin \theta = \frac{EB}{AB} = EB$, $\cos \theta = \frac{AE}{AD} = AE$, $\tan \theta = \frac{CD}{AD} = CD$, and $\tan \theta = \frac{EB}{AE} = \frac{\sin \theta}{\cos \theta}$. We can see that:
 $\text{area } \triangle AEB < \text{area sector } \widehat{DB} < \text{area } \triangle ADC \Rightarrow \frac{1}{2}(AE)(EB) < \frac{1}{2}(AD)^2 \theta < \frac{1}{2}(AD)(CD)$
 $\Rightarrow \frac{1}{2} \sin \theta \cos \theta < \frac{1}{2}(1)^2 \theta < \frac{1}{2}(1)(\tan \theta) \Rightarrow \frac{1}{2} \sin \theta \cos \theta < \frac{1}{2} \theta < \frac{1}{2} \frac{\sin \theta}{\cos \theta}$.
18. $(f \circ g)(x) = f(g(x)) = a(cx + d) + b = acx + ad + b$ and $(g \circ f)(x) = g(f(x)) = c(ax + b) + d = acx + cb + d$. Thus $(f \circ g)(x) = (g \circ f)(x) \Rightarrow acx + ad + b = acx + cb + d \Rightarrow ad + b = cb + d$. Note that $f(d) = ad + b$ and $g(b) = cb + d$, thus $(f \circ g)(x) = (g \circ f)(x)$ if $f(d) = g(b)$.