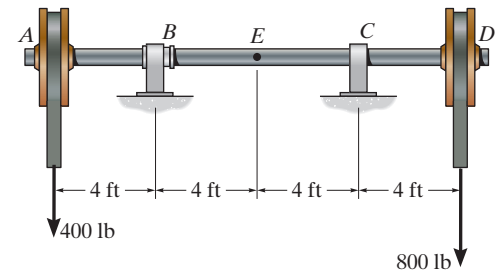


***Mechanics of Materials 11th edition Hibbeler Solutions
Manual***

SKU: 9780137605521-SOLUTIONS

1-1.

The shaft is supported by a smooth thrust bearing at *B* and a journal bearing at *C*. Determine the resultant internal loadings acting on the cross section at *E*.



SOLUTION

Support Reactions: We will only need to compute C_y by writing the moment equation of equilibrium about *B* with reference to the free-body diagram of the entire shaft, Fig. *a*.

$$\zeta + \sum M_B = 0; \quad C_y(8) + 400(4) - 800(12) = 0 \quad C_y = 1000 \text{ lb}$$

Internal Loadings: Using the result for C_y , section *DE* of the shaft will be considered. Referring to the free-body diagram, Fig. *b*,

$$\pm \sum F_x = 0; \quad N_E = 0$$

$$+\uparrow \sum F_y = 0; \quad V_E + 1000 - 800 = 0 \quad V_E = -200 \text{ lb}$$

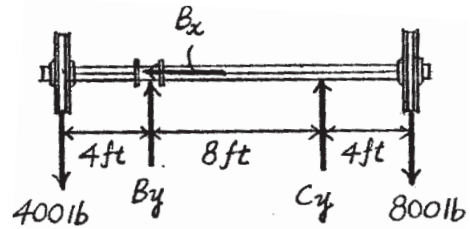
$$\zeta + \sum M_E = 0; \quad 1000(4) - 800(8) - M_E = 0$$

$$M_E = -2400 \text{ lb} \cdot \text{ft} = -2.40 \text{ kip} \cdot \text{ft} \quad \text{Ans.}$$

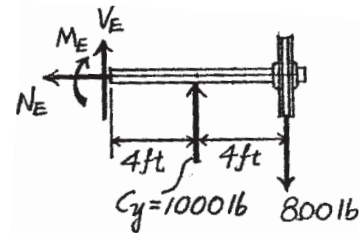
The negative signs indicates that V_E and M_E act in the opposite sense to that shown on the free-body diagram.

Ans.

Ans.



(a)



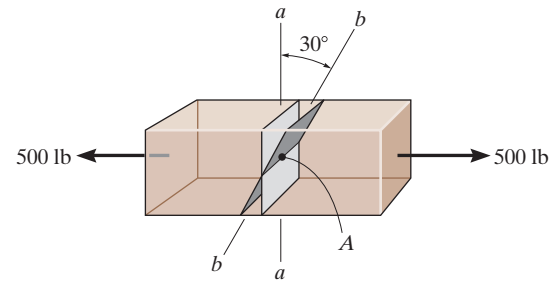
(b)

Ans:

$$C_y = 1000 \text{ lb}, N_E = 0, V_E = 200 \text{ lb}, M_E = 2.40 \text{ kip} \cdot \text{ft}$$

1-2.

Determine the resultant internal normal and shear force in the member on (a) section $a-a$ and (b) section $b-b$, each of which passes through the centroid A . The 500-lb load is applied along the centroidal axis of the member.



SOLUTION

(a)

$$\begin{aligned} \pm \Sigma F_x = 0; \quad N_a - 500 &= 0 \\ N_a &= 500 \text{ lb} \end{aligned}$$

$$+\downarrow \Sigma F_y = 0; \quad V_a = 0$$

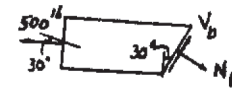
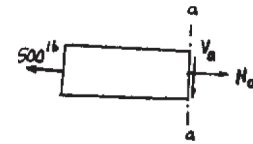
Ans.

(b)

$$\begin{aligned} \nearrow \Sigma F_x = 0; \quad N_b - 500 \cos 30^\circ &= 0 \\ N_b &= 433 \text{ lb} \end{aligned}$$

$$\begin{aligned} +\nearrow \Sigma F_y = 0; \quad V_b - 500 \sin 30^\circ &= 0 \\ V_b &= 250 \text{ lb} \end{aligned}$$

Ans.



Ans.

Ans.

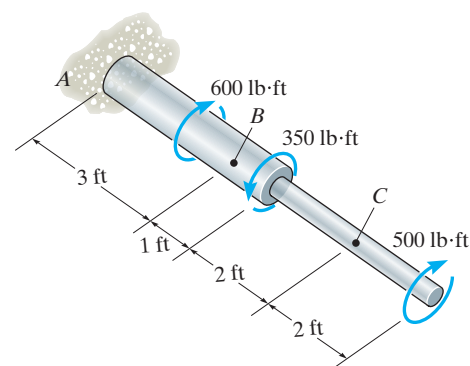
Ans:

(a) $N_a = 500 \text{ lb}$, $V_a = 0$,

(b) $N_b = 433 \text{ lb}$, $V_b = 250 \text{ lb}$

1-3.

Determine the resultant internal torque acting on the cross sections through points *B* and *C*.



SOLUTION

$$\Sigma M_x = 0; \quad T_B + 350 - 500 = 0$$

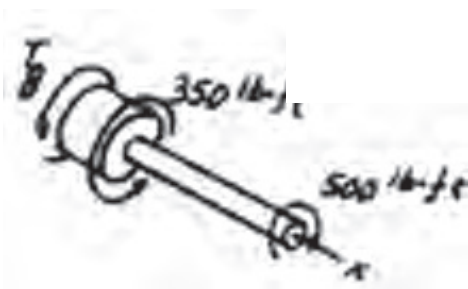
$$T_B = 150 \text{ lb} \cdot \text{ft}$$

Ans.

$$\Sigma M_x = 0; \quad T_C - 500 = 0$$

$$T_C = 500 \text{ lb} \cdot \text{ft}$$

Ans.



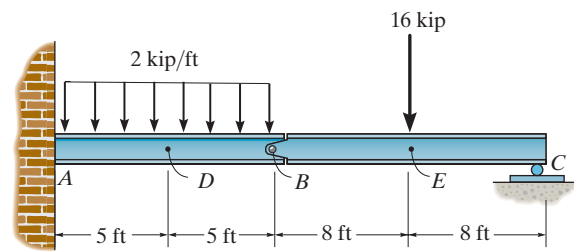
Ans:

$$T_B = 150 \text{ lb} \cdot \text{ft}$$

$$T_C = 500 \text{ lb} \cdot \text{ft}$$

***1-4.**

Determine the resultant internal loadings in the beam at cross sections through points *D* and *E*. Point *E* is just to the left of the 16-kip load.



SOLUTION

Support Reactions: We will only need to calculate B_x , B_y and C_y by consider the equilibrium of member *BC* with reference to its FBD shown in Fig. *b*.

$$\zeta + \sum M_B = 0; \quad C_y(16) - 16(8) = 0 \quad C_y = 8 \text{ kip}$$

$$\zeta + \sum M_C = 0; \quad 16(8) - B_y(16) = 0 \quad B_y = 8 \text{ kip}$$

$$\pm \sum F_x = 0 \quad B_x = 0$$

Internal Loadings: For point *D*, consider segment *BD* of member *AB*. Using the results of B_x and B_y , the FBD of this segment shown in Fig. *c* is referred.

$$\pm \sum F_x = 0; \quad N_D = 0 \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad V_D - 10 - 8 = 0 \quad V_D = 18.0 \text{ kip} \quad \text{Ans.}$$

$$\zeta + \sum M_D = 0; \quad -M_D - 10(2.5) - 8(5) = 0 \quad M_D = -65.0 \text{ kip} \cdot \text{ft} \quad \text{Ans.}$$

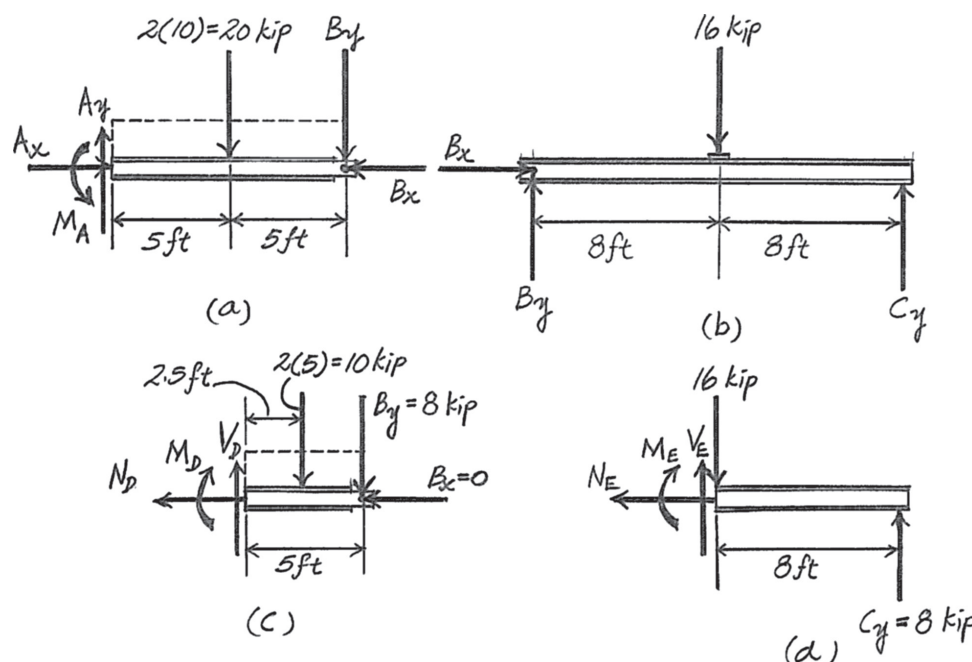
The negative sign indicates that M_D acts in the sense opposite to that shown in FBD.

For point *E*, segment *CE* of member *BC* using the result of C_y will be considered. Referring to its FBD, Fig. *d*

$$\pm \sum F_x = 0; \quad N_E = 0 \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad V_E + 8 - 16 = 0 \quad V_E = 8.00 \text{ kip} \quad \text{Ans.}$$

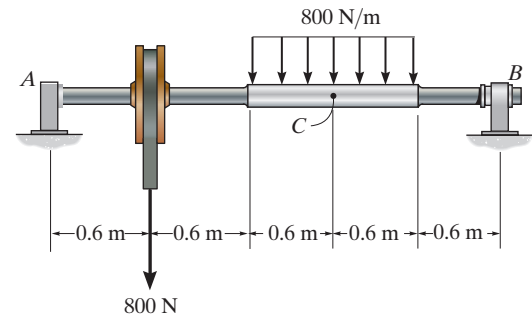
$$\zeta + \sum M_E = 0; \quad 8(8) - M_E = 0 \quad M_E = 64.0 \text{ kip} \cdot \text{ft} \quad \text{Ans.}$$



Ans:
 $N_D = 0$
 $V_D = 18.0 \text{ kip}$
 $M_D = 65.0 \text{ kip} \cdot \text{ft}$
 $N_E = 0$
 $V_E = 8.00 \text{ kip}$
 $M_E = 64.0 \text{ kip} \cdot \text{ft}$

1-5.

The shaft is supported by a smooth thrust bearing at B and a smooth journal bearing at A . Determine the resultant internal loadings acting on the cross section at C .



SOLUTION

Support Reactions: We will only need to compute B_y by writing the moment equation of equilibrium about A with reference to the FBD of the entire shaft, Fig. a .

$$\zeta + \Sigma M_A = 0; \quad B_y(3) - 800(0.6) - 960(1.8) = 0 \quad B_y = 736 \text{ N}$$

Internal Loadings: Using the result of B_y , segment BC of the shaft will be considered. Referring to its FBD, Fig. b ,

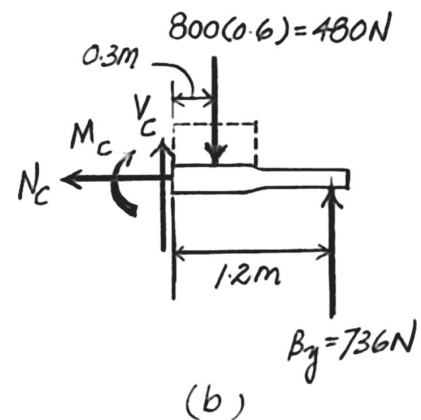
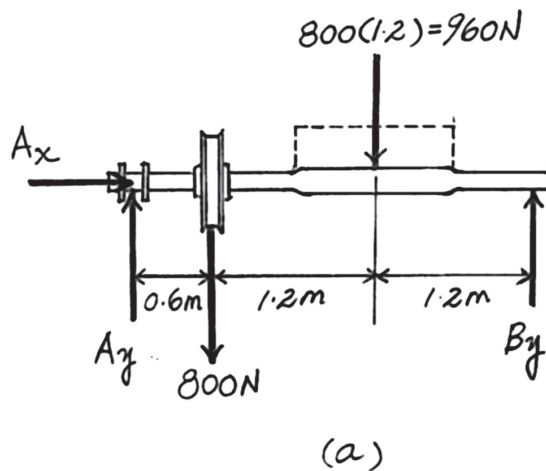
$$\rightarrow \Sigma F_x = 0; \quad N_C = 0 \quad \text{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad V_C + 736 - 480 = 0 \quad V_C = -256 \text{ N} \quad \text{Ans.}$$

$$\zeta + \Sigma M_C = 0; \quad 736(1.2) - 480(0.3) - M_C = 0$$

$$M_C = 739.2 \text{ N} \cdot \text{m} = 739 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

The negative sign indicates that V_C acts in the opposite sense to that shown in FBD.



Ans:

$$B_y = 736 \text{ N}$$

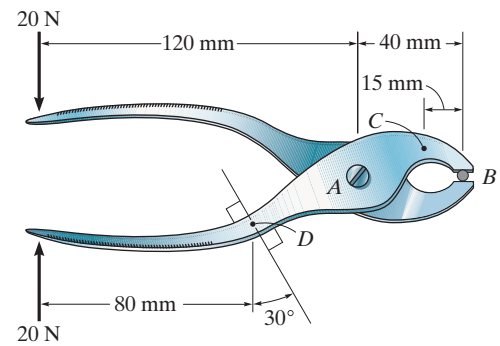
$$N_C = 0$$

$$V_C = 256 \text{ N}$$

$$M_C = 739 \text{ N} \cdot \text{m}$$

1-6.

Determine the resultant internal loading on the cross section through point C of the pliers. There is a pin at A , and the jaws at B are smooth.



SOLUTION

$$+\uparrow \Sigma F_y = 0; \quad -V_C + 60 = 0; \quad V_C = 60 \text{ N}$$

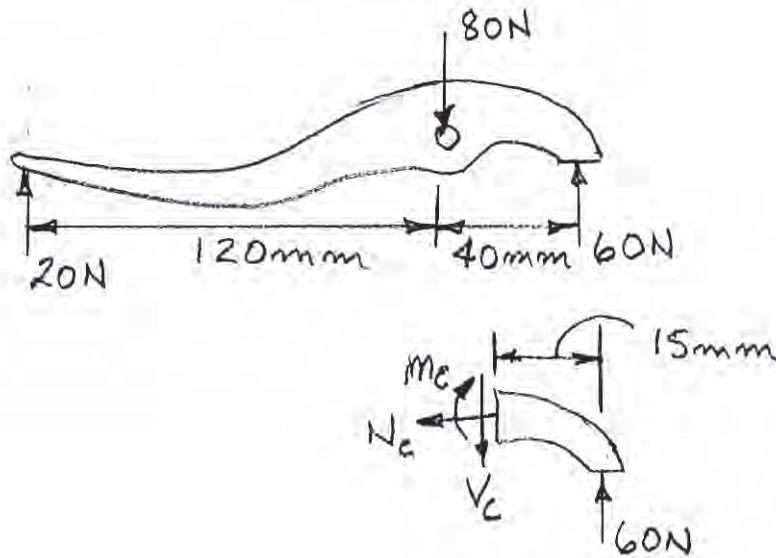
Ans.

$$\rightarrow \Sigma F_x = 0; \quad N_C = 0$$

Ans.

$$+\curvearrowright \Sigma M_C = 0; \quad -M_C + 60(0.015) = 0; \quad M_C = 0.9 \text{ N}\cdot\text{m}$$

Ans.



Ans:

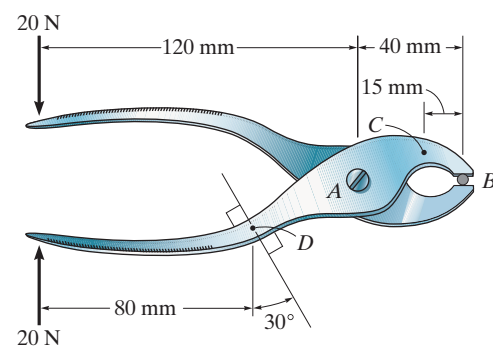
$$V_C = 60 \text{ N}$$

$$N_C = 0$$

$$M_C = 0.9 \text{ N}\cdot\text{m}$$

1-7.

Determine the resultant internal loading on the cross section through point D of the pliers. There is a pin at A , and the jaws at B are smooth.



SOLUTION

$$\downarrow + \Sigma F_y = 0; \quad V_D - 20 \cos 30^\circ = 0; \quad V_D = 17.3 \text{ N}$$

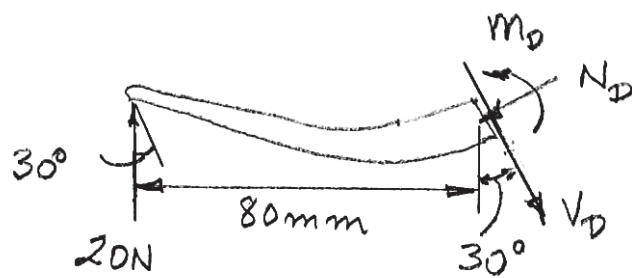
Ans.

$$+\swarrow \Sigma F_x = 0; \quad N_D - 20 \sin 30^\circ = 0; \quad N_D = 10 \text{ N}$$

Ans.

$$+\curvearrowright \Sigma M_D = 0; \quad M_D - 20(0.08) = 0; \quad M_D = 1.60 \text{ N} \cdot \text{m}$$

Ans.



Ans:

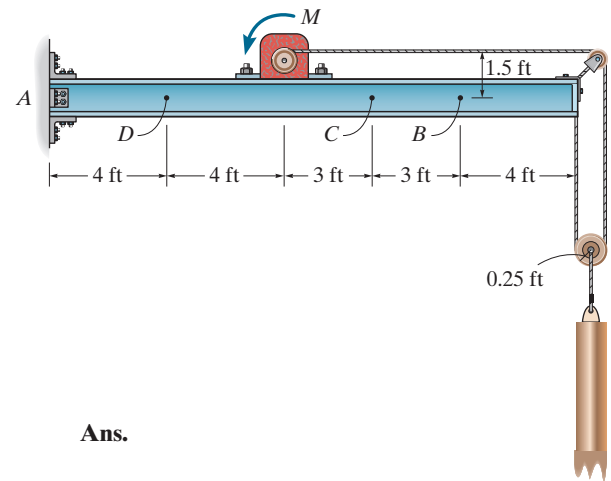
$$V_D = 17.3 \text{ N}$$

$$N_D = 10 \text{ N}$$

$$M_D = 1.60 \text{ N} \cdot \text{m}$$

***1-8.**

The 800-lb load is being hoisted at a constant speed using the motor M , which has a weight of 90 lb. Determine the resultant internal loadings acting on the cross section through point B in the beam. The beam has a weight of 40 lb/ft and is fixed to the wall at A .



SOLUTION

$$\rightarrow \Sigma F_x = 0; \quad -N_B - 0.4 = 0$$

$$N_B = -0.4 \text{ kip}$$

Ans.

$$+\uparrow \Sigma F_y = 0; \quad V_B - 0.8 - 0.16 = 0$$

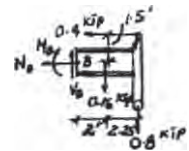
$$V_B = 0.960 \text{ kip}$$

Ans.

$$\zeta + \Sigma M_B = 0; \quad -M_B - 0.16(2) - 0.8(4.25) + 0.4(1.5) = 0$$

$$M_B = -3.12 \text{ kip} \cdot \text{ft}$$

Ans.



Ans:

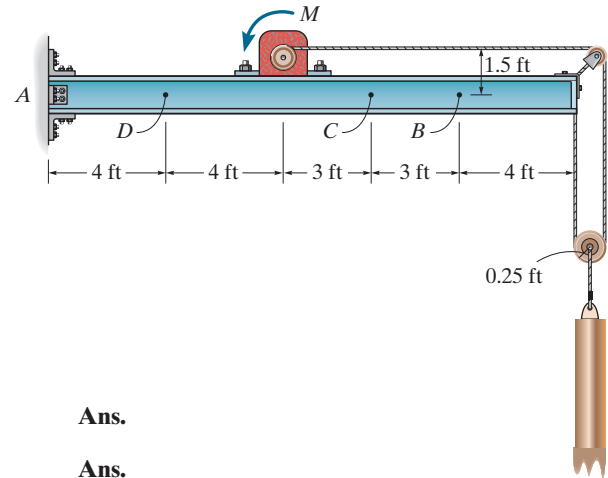
$$N_B = 0.4 \text{ kip}$$

$$V_B = 0.960 \text{ kip}$$

$$M_B = 3.12 \text{ kip} \cdot \text{ft}$$

1-9.

Determine the resultant internal loadings acting on the cross section through points *C* and *D* of the beam.



SOLUTION

For point *C*:

$$\leftarrow \Sigma F_x = 0; \quad N_C + 0.4 = 0; \quad N_C = -0.4 \text{ kip}$$

Ans.

$$+\uparrow \Sigma F_y = 0; \quad V_C - 0.8 - 0.04(7) = 0; \quad V_C = 1.08 \text{ kip}$$

Ans.

$$\zeta + \Sigma M_C = 0; \quad -M_C - 0.8(7.25) - 0.04(7)(3.5) + 0.4(1.5) = 0$$

$$M_C = -6.18 \text{ kip} \cdot \text{ft}$$

Ans.

For point *D*:

$$\leftarrow \Sigma F_x = 0; \quad N_D = 0$$

Ans.

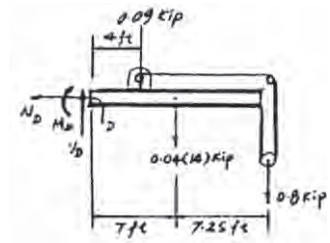
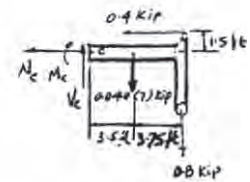
$$+\uparrow \Sigma F_y = 0; \quad V_D - 0.09 - 0.04(14) - 0.8 = 0; \quad V_D = 1.45 \text{ kip}$$

Ans.

$$\zeta + \Sigma M_D = 0; \quad -M_D - 0.09(4) - 0.04(14)(7) - 0.8(14.25) = 0$$

$$M_D = -15.7 \text{ kip} \cdot \text{ft}$$

Ans.



Ans:

$$N_C = -0.4 \text{ kip}$$

$$V_C = 1.08 \text{ kip}$$

$$M_C = -6.18 \text{ kip} \cdot \text{ft}$$

$$N_D = 0$$

$$V_D = 1.45 \text{ kip}$$

$$M_D = -15.7 \text{ kip} \cdot \text{ft}$$

1–10.

Determine the resultant internal normal force acting on the cross section through point *A* in each column. In (a), segment *BC* weighs 180 lb/ft and segment *CD* weighs 250 lb/ft. In (b), the column has a mass of 200 kg/m.

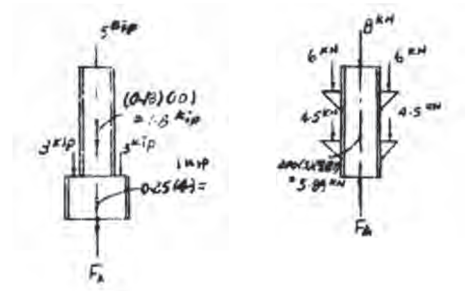
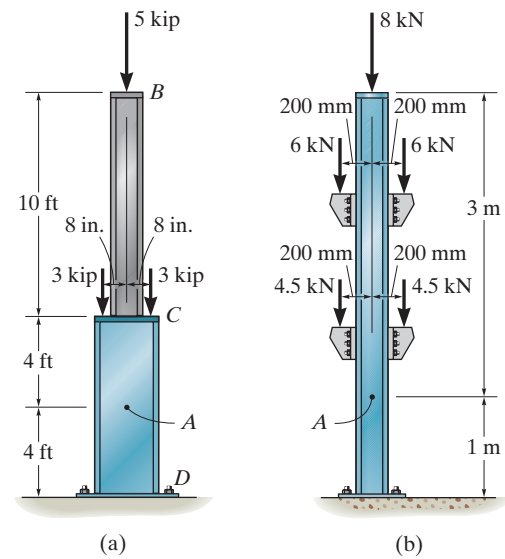
SOLUTION

$$(a) \quad +\uparrow \Sigma F_y = 0; \quad F_A - 1.0 - 3 - 3 - 1.8 - 5 = 0$$

$$F_A = 13.8 \text{ kip}$$

$$(b) \quad +\uparrow \Sigma F_y = 0; \quad F_A - 4.5 - 4.5 - 5.89 - 6 - 6 - 8 = 0$$

$$F_A = 34.9 \text{ kN}$$



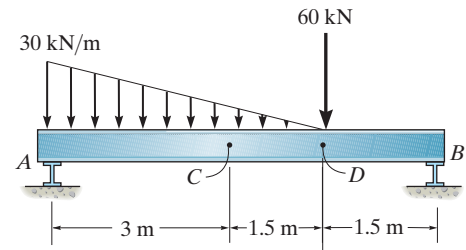
Ans:

$$(a) \quad F_A = 13.8 \text{ kip}$$

$$(b) \quad F_A = 34.9 \text{ kN}$$

1-11.

Determine the resultant internal loadings on the cross section at point C. Assume the support reactions at A and B are vertical.



SOLUTION

Support Reactions: Only B_y needs to be computed. Referring to the FBD of the entire beam and writing the moment equation of equilibrium about A, Fig. a,

$$\zeta + \sum M_A = 0; \quad B_y(6) - 67.5(1.5) - 60(4.5) = 0 \quad B_y = 61.875 \text{ kN}$$

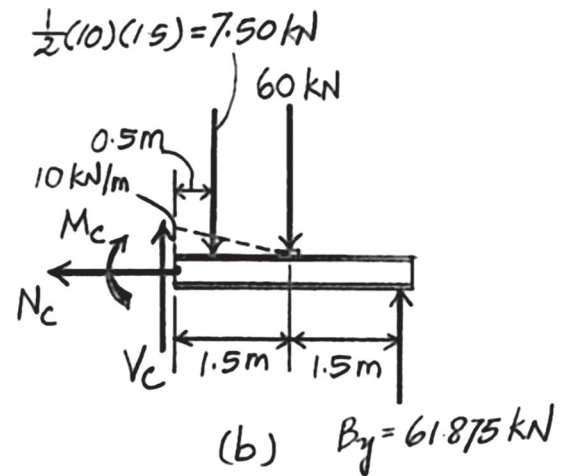
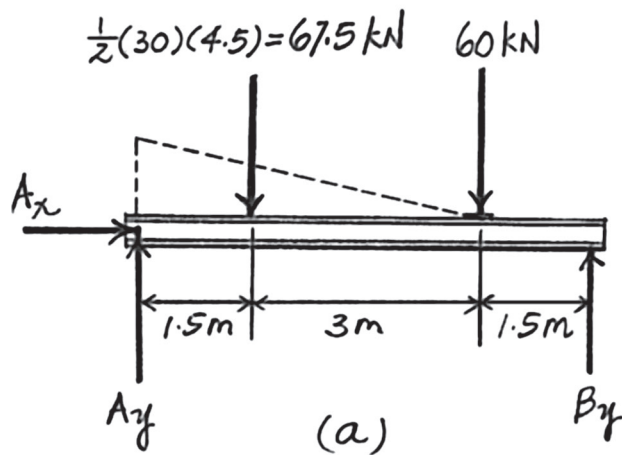
Internal Loadings: Using the result of B_y and referring to the FBD of segment BC of the beam, Fig. b.

$$\rightarrow \sum F_x = 0; \quad N_C = 0 \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad V_C + 61.875 - 7.50 - 60 = 0 \quad V_C = 5.625 \text{ kN} \quad \text{Ans.}$$

$$\zeta + \sum M_C = 0; \quad 61.875(3) - 7.50(0.5) - 60(1.5) - M_C = 0$$

$$M_C = 91.875 \text{ kN} \cdot \text{m} = 91.9 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$



Ans:

$$B_y = 61.875 \text{ kN}$$

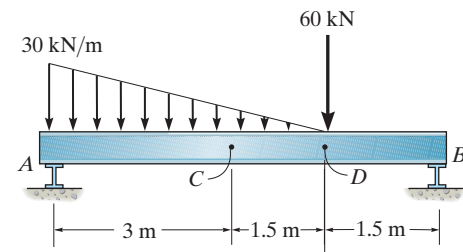
$$N_C = 0$$

$$V_C = 5.625 \text{ kN}$$

$$M_C = 91.9 \text{ kN} \cdot \text{m}$$

***1-12.**

Determine the resultant internal loadings on the cross section at point D . Assume point D is just to the left of the 60-kN force. Assume the support reactions at A and B vertical.



SOLUTION

Support Reactions: Only B_y needs to be computed. Referring to the FBD of the entire beam and writing the moment equation of equilibrium about A , Fig. a ,

$$\zeta + \sum M_A = 0; \quad B_y(6) - 67.5(1.5) - 60(4.5) = 0 \quad B_y = 61.875 \text{ kN}$$

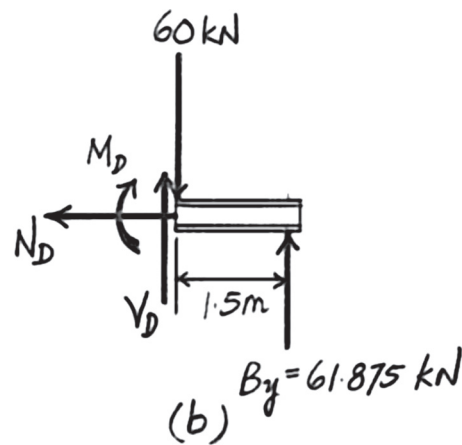
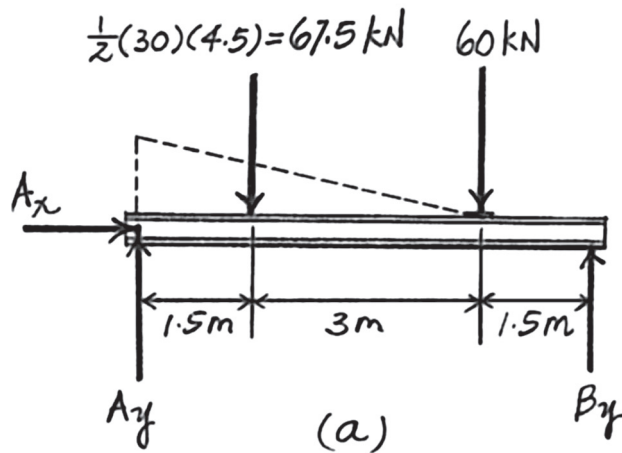
Internal Loadings: using the result of B_y and referring to the FBD of segment BD of the beam, Fig. b ,

$$\rightarrow \sum F_x = 0; \quad N_D = 0 \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad V_D + 61.875 - 60 = 0 \quad V_D = -1.875 \text{ kN} \quad \text{Ans.}$$

$$\zeta + \sum M_D = 0; \quad 61.875(1.5) - M_D = 0 \quad M_D = 92.8125 \text{ kN} \cdot \text{m} = 92.8 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$

The negative sign indicates that V_D acts in the sense that opposite to that shown in the FBD.



Ans:

$$B_y = 61.875 \text{ kN}$$

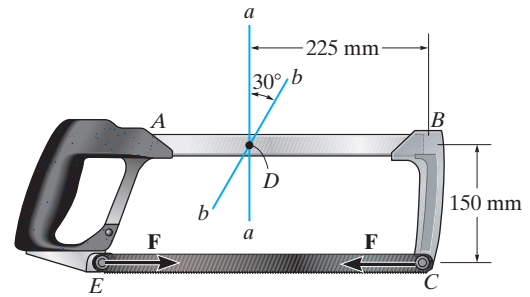
$$N_D = 0$$

$$V_D = -1.875 \text{ kN}$$

$$M_D = 92.8 \text{ kN} \cdot \text{m}$$

1–13.

The blade of the hacksaw is subjected to a pretension force of $F = 100$ N. Determine the resultant internal loadings acting on section $a-a$ that passes through point D .



SOLUTION

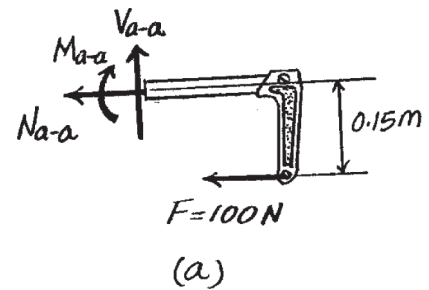
Internal Loadings: Referring to the free-body diagram of the section of the hacksaw shown in Fig. a ,

$$\leftarrow + \sum F_x = 0; \quad N_{a-a} + 100 = 0 \quad N_{a-a} = -100 \text{ N} \quad \text{Ans.}$$

$$+ \uparrow \sum F_y = 0; \quad V_{a-a} = 0 \quad \text{Ans.}$$

$$\curvearrowleft + \sum M_D = 0; \quad -M_{a-a} - 100(0.15) = 0 \quad M_{a-a} = -15 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

The negative sign indicates that N_{a-a} and M_{a-a} act in the opposite sense to that shown on the free-body diagram.

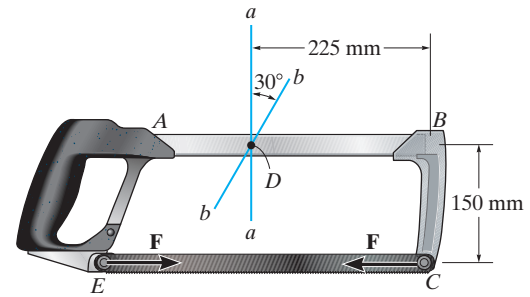


Ans:

$$N_{a-a} = -100 \text{ N}, \quad V_{a-a} = 0, \quad M_{a-a} = -15 \text{ N} \cdot \text{m}$$

1-14.

The blade of the hacksaw is subjected to a pretension force of $F = 100$ N. Determine the resultant internal loadings acting on section $b-b$ that passes through point D .



SOLUTION

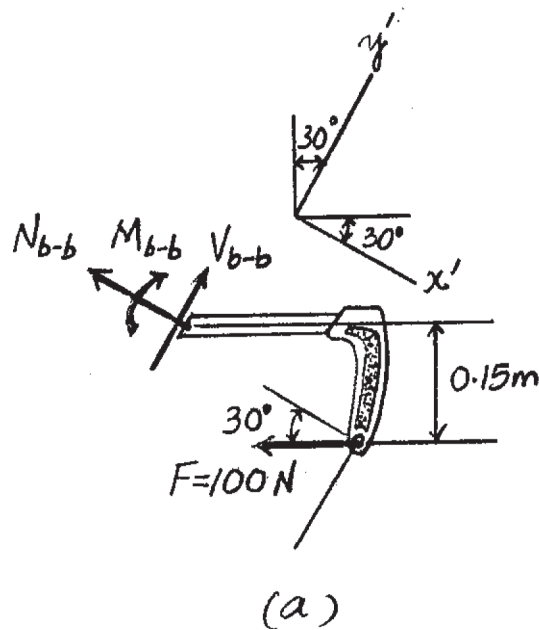
Internal Loadings: Referring to the free-body diagram of the section of the hacksaw shown in Fig. a ,

$$\Sigma F_x = 0; \quad N_{b-b} + 100 \cos 30^\circ = 0 \quad N_{b-b} = -86.6 \text{ N} \quad \text{Ans.}$$

$$\Sigma F_y = 0; \quad V_{b-b} - 100 \sin 30^\circ = 0 \quad V_{b-b} = 50 \text{ N} \quad \text{Ans.}$$

$$\zeta + \Sigma M_D = 0; \quad -M_{b-b} - 100(0.15) = 0 \quad M_{b-b} = -15 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

The negative sign indicates that N_{b-b} and M_{b-b} act in the opposite sense to that shown on the free-body diagram.

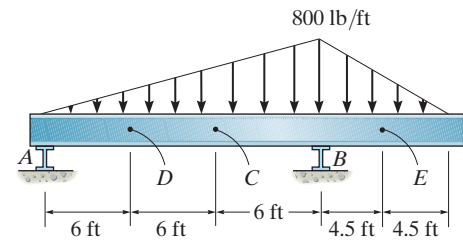


Ans:

$$N_{b-b} = -86.6 \text{ N}, \quad V_{b-b} = 50 \text{ N} \\ M_{b-b} = -15 \text{ N} \cdot \text{m}$$

1-15.

Determine the resultant internal loadings on the cross section at point C. Assume the reactions at the supports A and B are vertical.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. a,

$$\zeta + \Sigma M_B = 0; \quad \frac{1}{2}(0.8)(18)(6) - \frac{1}{2}(0.8)(9)(3) - A_y(18) = 0 \quad A_y = 1.80 \text{ kip}$$

Internal Loadings: Referring to the FBD of the left beam segment sectioned through point C, Fig. b,

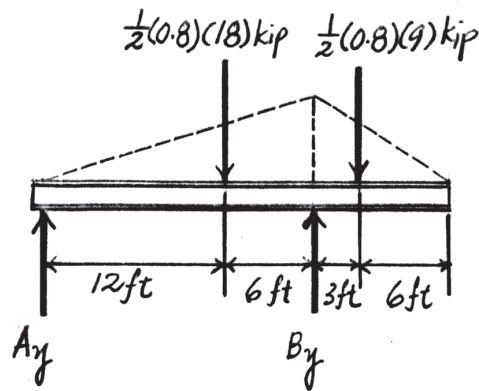
$$\pm \Sigma F_x = 0; \quad N_C = 0 \quad \text{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 1.80 - \frac{1}{2}(0.5333)(12) - V_C = 0 \quad V_C = -1.40 \text{ kip} \quad \text{Ans.}$$

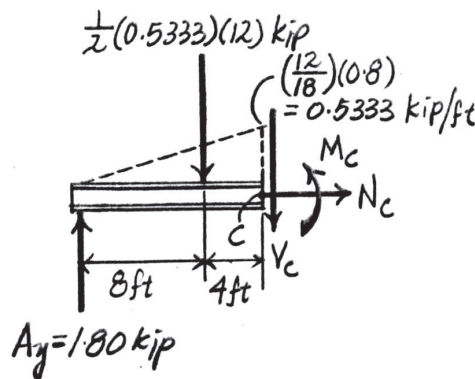
$$\zeta + \Sigma M_C = 0; \quad M_C + \frac{1}{2}(0.5333)(12)(4) - 1.80(12) = 0$$

$$M_C = 8.80 \text{ kip} \cdot \text{ft} \quad \text{Ans.}$$

The negative sign indicates that V_C acts in the sense opposite to that shown on the FBD.



(a)



(b)

Ans:

$$A_y = 1.80 \text{ kip}$$

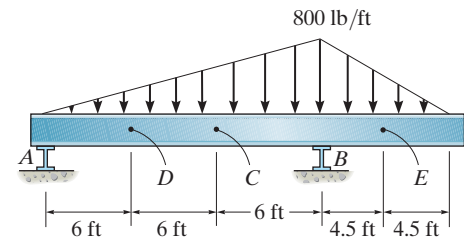
$$N_C = 0$$

$$V_C = 1.40 \text{ kip}$$

$$M_C = 8.80 \text{ kip} \cdot \text{ft}$$

***1-16.**

Determine the resultant internal loadings on the cross section at points D and E . Assume the reactions at the supports A and B are vertical.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. a ,

$$\zeta + \Sigma M_B = 0; \quad \frac{1}{2}(0.8)(18)(6) - \frac{1}{2}(0.8)(9)(3) - A_y(18) = 0 \quad A_y = 1.80 \text{ kip}$$

Internal Loadings: Referring to the FBD of the left segment of the beam section through D , Fig. b ,

$$\pm \Sigma F_x = 0; \quad N_D = 0$$

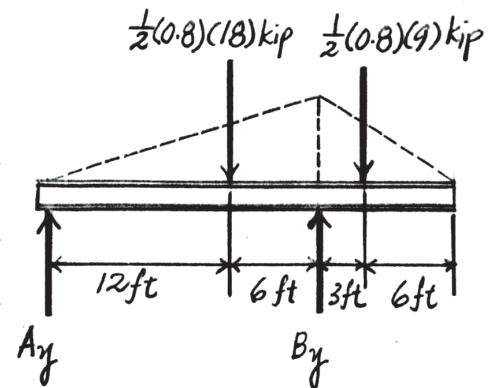
$$+\uparrow \Sigma F_y = 0; \quad 1.80 - \frac{1}{2}(0.2667)(6) - V_D = 0 \quad V_D = 1.00 \text{ kip}$$

$$\zeta + \Sigma M_D = 0; \quad M_D + \frac{1}{2}(0.2667)(6)(2) - 1.80(6) = 0$$

$$M_D = 9.20 \text{ kip} \cdot \text{ft}$$

Ans.

Ans.



(a)

Referring to the FBD of the right segment of the beam sectioned through E , Fig. c ,

$$\pm \Sigma F_x = 0; \quad N_E = 0$$

$$+\uparrow \Sigma F_y = 0; \quad V_E - \frac{1}{2}(0.4)(4.5) = 0 \quad V_E = 0.900 \text{ kip}$$

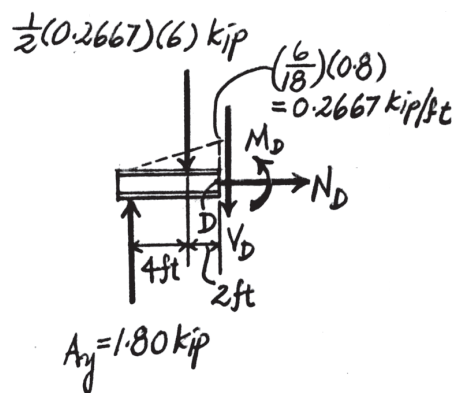
$$\zeta + \Sigma M_E = 0; \quad -M_E - \frac{1}{2}(0.4)(4.5)(1.5) = 0 \quad M_E = -1.35 \text{ kip} \cdot \text{ft}$$

Ans.

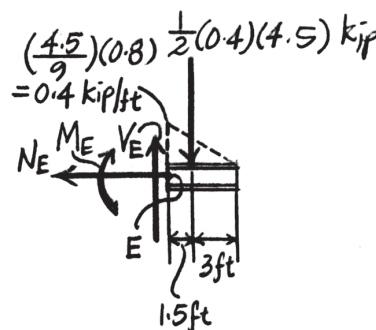
Ans.

Ans.

The negative sign indicates that M_E act in the sense opposite to that shown in Fig. c .



(b)



(c)

Ans:

$$A_y = 1.80 \text{ kip},$$

$$N_D = 0,$$

$$V_D = 1.00 \text{ kip},$$

$$M_D = 9.20 \text{ kip} \cdot \text{ft}$$

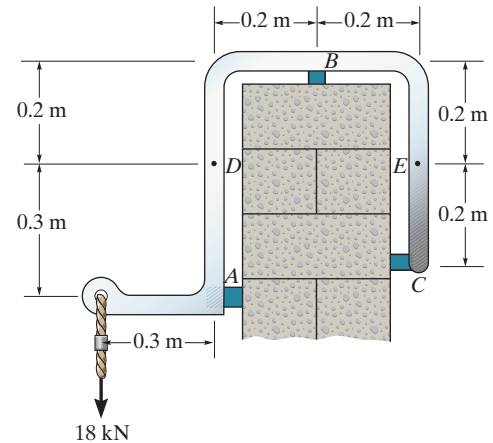
$$N_E = 0,$$

$$V_E = 0.900 \text{ kip},$$

$$M_E = -1.35 \text{ kip} \cdot \text{ft}$$

1-17.

The sky hook is used to support the cable of a scaffold over the side of a building. If it consists of a smooth rod that contacts the parapet of a wall at points A , B , and C , determine the normal force, shear force, and moment on the cross section at points D and E .



SOLUTION

Support Reactions:

$$+\uparrow \Sigma F_y = 0; \quad N_B - 18 = 0 \quad N_B = 18.0 \text{ kN}$$

$$\zeta + \Sigma M_C = 0; \quad 18(0.7) - 18.0(0.2) - N_A(0.1) = 0$$

$$N_A = 90.0 \text{ kN}$$

$$\rightarrow \Sigma F_x = 0; \quad N_C - 90.0 = 0 \quad N_C = 90.0 \text{ kN}$$

Equations of Equilibrium: For point D

$$\rightarrow \Sigma F_x = 0; \quad V_D - 90.0 = 0$$

$$V_D = 90.0 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad N_D - 18 = 0$$

$$N_D = 18.0 \text{ kN}$$

$$\zeta + \Sigma M_D = 0; \quad M_D + 18(0.3) - 90.0(0.3) = 0$$

$$M_D = 21.6 \text{ kN} \cdot \text{m}$$

Equations of Equilibrium: For point E

$$\rightarrow \Sigma F_x = 0; \quad 90.0 - V_E = 0$$

$$V_E = 90.0 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad N_E = 0$$

$$\zeta + \Sigma M_E = 0; \quad 90.0(0.2) - M_E = 0$$

$$M_E = 18.0 \text{ kN} \cdot \text{m}$$

Ans.

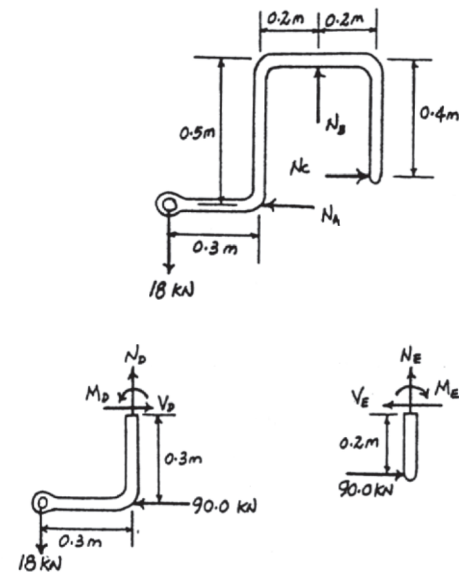
Ans.

Ans.

Ans.

Ans.

Ans.



Ans:

$$V_D = 90.0 \text{ kN}$$

$$N_D = 18.0 \text{ kN}$$

$$M_D = 21.6 \text{ kN} \cdot \text{m}$$

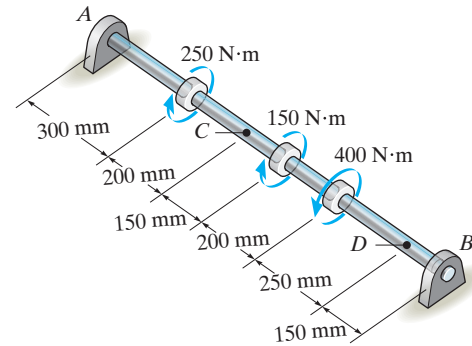
$$V_E = 90.0 \text{ kN}$$

$$N_E = 0$$

$$M_E = 18.0 \text{ kN} \cdot \text{m}$$

1–18.

Determine the resultant internal torque acting on the cross section through points *C* and *D*. The support bearings at *A* and *B* allow free turning of the shaft.



SOLUTION

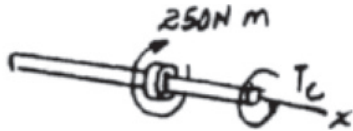
$$\Sigma M_x = 0; \quad T_C - 250 = 0$$

$$T_C = 250 \text{ N} \cdot \text{m}$$

Ans.

$$\Sigma M_x = 0; \quad T_D = 0$$

Ans.



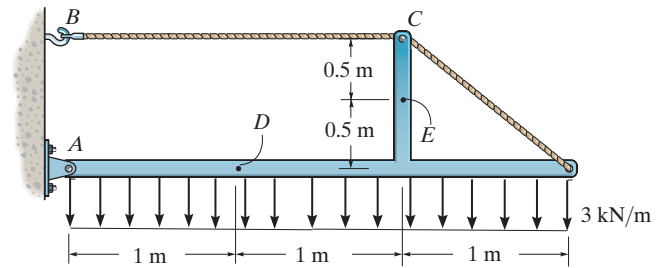
Ans:

$$T_C = 250 \text{ N} \cdot \text{m}$$

$$T_D = 0$$

***1–20.**

Determine the resultant internal loadings on the cross section at point D . The cable passes over a smooth peg at C .



SOLUTION

Support Reactions: Referring to the FBD of the entire assembly shown in Fig. a ,

$$\zeta + \Sigma M_A = 0; \quad T(1) - 9(1.5) = 0 \quad T = 13.5 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 9 = 0 \quad A_y = 9 \text{ kN}$$

$$+\rightarrow \Sigma F_x = 0; \quad A_x - 13.5 \text{ kN} = 0 \quad A_x = 13.5 \text{ kN}$$

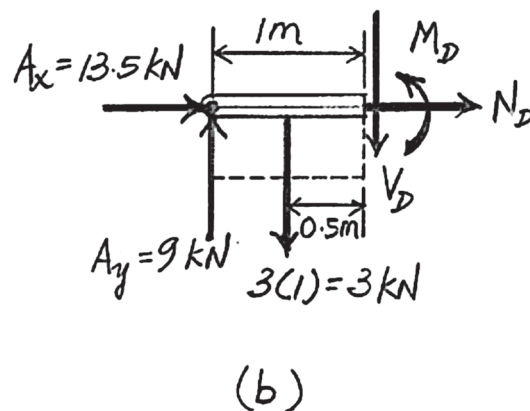
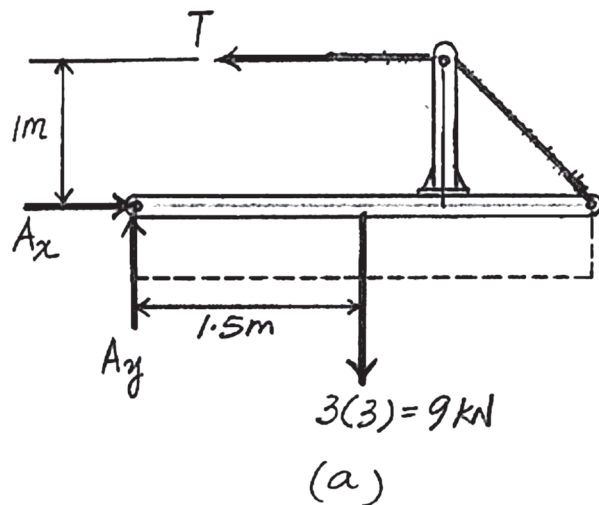
Internal Loadings: Using the results of A_x and A_y , segment AD will be considered. Referring to its FBD, Fig. b ,

$$+\rightarrow \Sigma F_x = 0; \quad N_D + 13.5 = 0 \quad N_D = -13.5 \text{ kN} \quad \text{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 9 - 3 - V_D = 0 \quad V_D = 6.00 \text{ kN} \quad \text{Ans.}$$

$$\zeta + \Sigma M_D = 0; \quad M_D + 3(0.5) - 9(1) = 0 \quad M_D = 7.50 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$

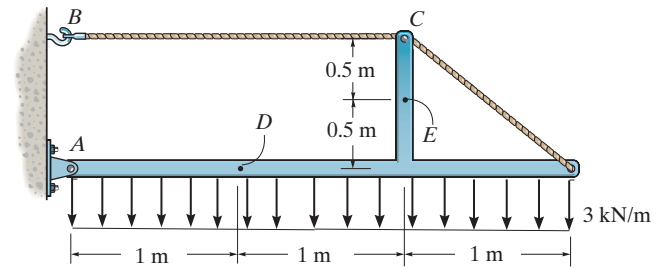
The negative sign indicates N_D acts in the sense opposite to that shown in FBD.



Ans:
 $N_D = -13.5 \text{ kN}$
 $V_D = 6.00 \text{ kN}$
 $M_D = 7.50 \text{ kN} \cdot \text{m}$

1-21.

Determine the resultant internal loadings on the cross section at point E . The cable passes over a smooth peg at C .



SOLUTION

Support Reactions: Only Tension T in the cable needs to be calculated. Referring to the FBD of the entire assembly shown in Fig. a and writing the moment equation of equilibrium about A ,

$$\zeta + \sum M_A = 0; \quad T(1) - 9(1.5) = 0 \quad T = 13.5 \text{ kN}$$

Internal Loadings: Using the result of T and referring to the FBD of segment CE , Fig. b ,

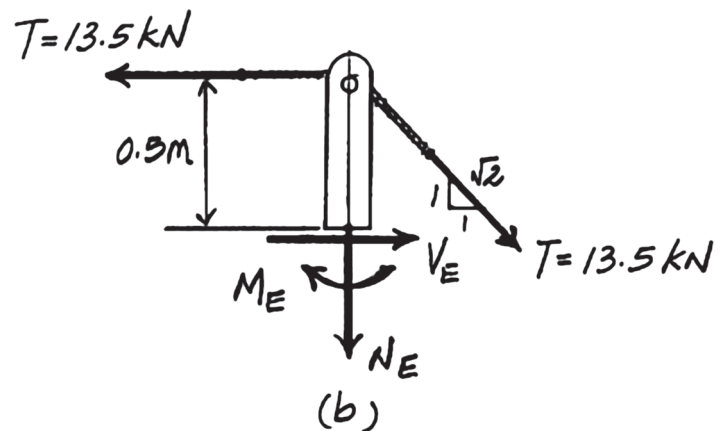
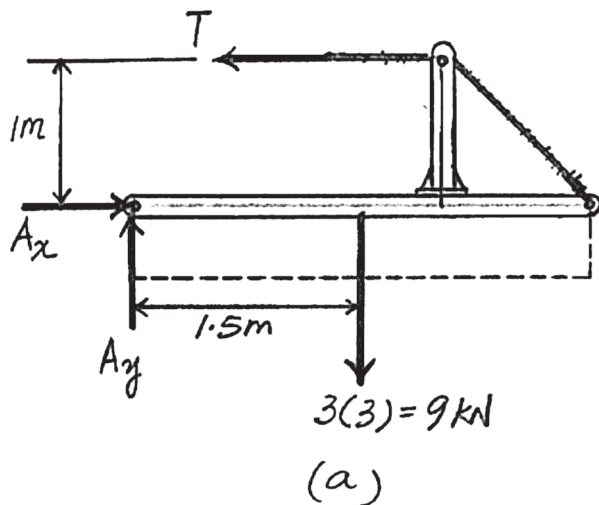
$$+\rightarrow \sum F_x = 0; \quad V_E + 13.5\left(\frac{1}{\sqrt{2}}\right) - 13.5 = 0 \quad V_E = 3.954 \text{ kN} = 3.95 \text{ kN} \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad -N_E - 13.5\left(\frac{1}{\sqrt{2}}\right) = 0 \quad N_E = -9.546 \text{ kN} = -9.55 \text{ kN} \quad \text{Ans.}$$

$$\zeta + \sum M_E = 0; \quad 13.5(0.5) - 13.5\left(\frac{1}{\sqrt{2}}\right)(0.5) - M_E = 0$$

$$M_E = 1.977 \text{ kN} \cdot \text{m} = 1.98 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$

The negative sign indicates that N_E acts in the sense that opposite to that shown in the FBD.



Ans:

$$V_E = 3.95 \text{ kN}$$

$$N_E = -9.55 \text{ kN}$$

$$M_E = 1.98 \text{ kN} \cdot \text{m}$$

1-22.

The metal stud punch is subjected to a force of 120 N on the handle. Determine the magnitude of the reactive force at the pin A and in the short link BC . Also, determine the resultant internal loadings acting on the cross section at point D .

SOLUTION

Member:

$$\zeta + \Sigma M_A = 0; \quad F_{BC} \cos 30^\circ (50) - 120(500) = 0$$

$$F_{BC} = 1385.6 \text{ N} = 1.39 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 1385.6 - 120 \cos 30^\circ = 0$$

$$A_y = 1489.56 \text{ N}$$

$$\leftarrow \Sigma F_x = 0; \quad A_x - 120 \sin 30^\circ = 0; \quad A_x = 60 \text{ N}$$

$$F_A = \sqrt{1489.56^2 + 60^2}$$

$$= 1491 \text{ N} = 1.49 \text{ kN}$$

Segment:

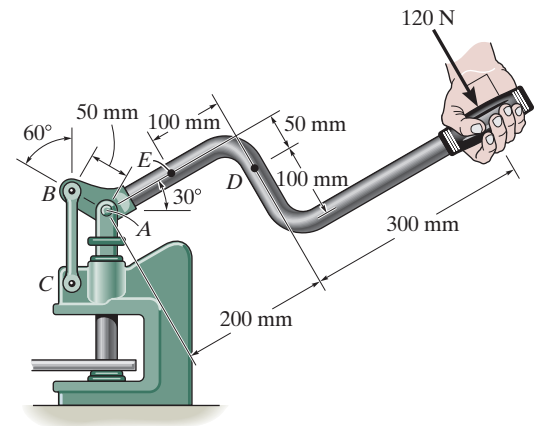
$$\nearrow \Sigma F_{x'} = 0; \quad N_D - 120 = 0$$

$$N_D = 120 \text{ N}$$

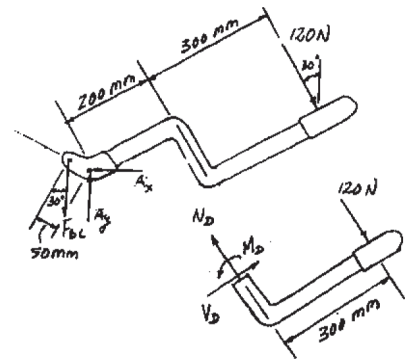
$$+\nearrow \Sigma F_{y'} = 0; \quad V_D = 0$$

$$\zeta + \Sigma M_D = 0; \quad M_D - 120(0.3) = 0$$

$$M_D = 36.0 \text{ N} \cdot \text{m}$$



Ans.



Ans.

Ans.

Ans.

Ans.

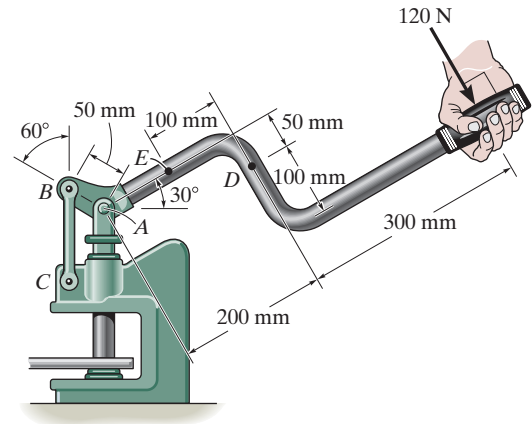
Ans:

$$F_{BC} = 1.39 \text{ kN}, F_A = 1.49 \text{ kN}, N_D = 120 \text{ N},$$

$$V_D = 0, M_D = 36.0 \text{ N} \cdot \text{m}$$

1-23.

The metal stud punch is subjected to a force of 120 N on the handle. Determine the resultant internal loadings acting on the cross section of the handle arm at point *E*, and on the cross section of the short link *BC*.



SOLUTION

Member:

$$\zeta + \Sigma M_A = 0; \quad F_{BC} \cos 30^\circ (50) - 120(500) = 0$$

$$F_{BC} = 1385.6 \text{ N} = 1.3856 \text{ kN}$$

Segment:

$$\nabla \Sigma F_x = 0; \quad N_E = 0$$

$$\curvearrowright + \Sigma F_y = 0; \quad V_E - 120 = 0; \quad V_E = 120 \text{ N}$$

$$\zeta + \Sigma M_E = 0; \quad M_E - 120(0.4) = 0; \quad M_E = 48.0 \text{ N} \cdot \text{m}$$

Short link:

$$\leftarrow \Sigma F_x = 0; \quad V = 0$$

$$+\uparrow \Sigma F_y = 0; \quad 1.3856 - N = 0; \quad N = 1.39 \text{ kN}$$

$$\zeta + \Sigma M_H = 0; \quad M = 0$$

Ans.

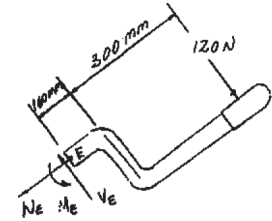
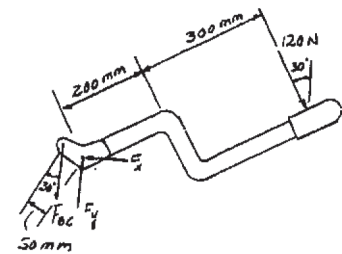
Ans.

Ans.

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Ans.

Ans.

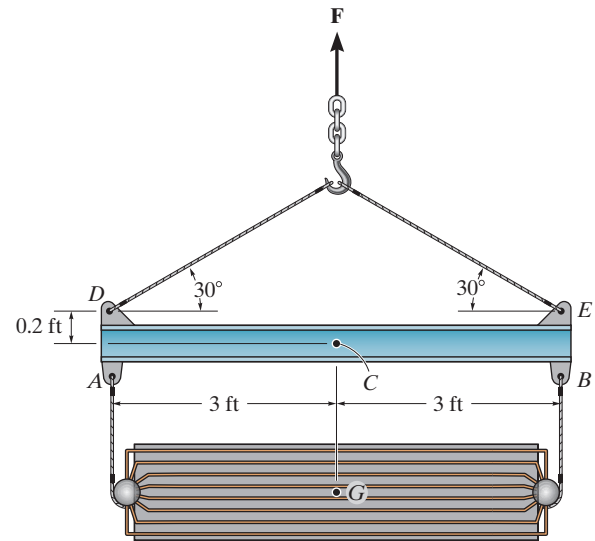


Ans:

$N_E = 0, V_E = 120 \text{ N}, M_E = 48.0 \text{ N} \cdot \text{m},$
Short link: $V = 0, N = 1.39 \text{ kN}, M = 0$

***1-24.**

Determine the resultant internal loadings acting on the cross section at point C. The cooling unit has a total weight of 52 kip and a center of gravity at G.



SOLUTION

From FBD (a)

$$\zeta + \Sigma M_A = 0; \quad T_B(6) - 52(3) = 0; \quad T_B = 26 \text{ kip}$$

From FBD (b)

$$\zeta + \Sigma M_D = 0; \quad T_E \sin 30^\circ(6) - 26(6) = 0; \quad T_E = 52 \text{ kip}$$

From FBD (c)

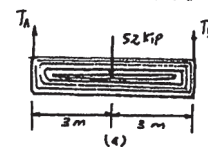
$$\pm \Sigma F_x = 0; \quad -N_C - 52 \cos 30^\circ = 0; \quad N_C = -45.0 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0; \quad V_C + 52 \sin 30^\circ - 26 = 0; \quad V_C = 0$$

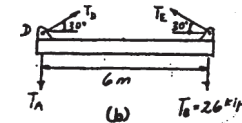
$$\zeta + \Sigma M_C = 0; \quad 52 \cos 30^\circ(0.2) + 52 \sin 30^\circ(3) - 26(3) - M_C = 0$$

$$M_C = 9.00 \text{ kip} \cdot \text{ft}$$

Ans.

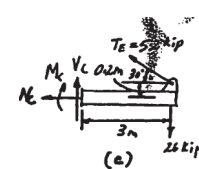


Ans.



Ans.

Ans.



Ans.

Ans:

$$T_B = 26 \text{ kip}$$

$$T_E = 52 \text{ kip}$$

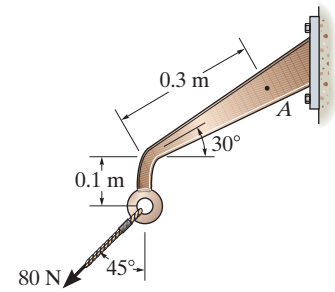
$$N_C = 45.0 \text{ kip,}$$

$$V_C = 0$$

$$M_C = 9.00 \text{ kip} \cdot \text{ft}$$

1–25.

A force of 80 N is supported by the bracket. Determine the resultant internal loadings acting on the cross section at point A.



SOLUTION

Equations of Equilibrium:

$$+\nearrow \Sigma F_{x'} = 0; \quad N_A - 80 \cos 15^\circ = 0$$

$$N_A = 77.3 \text{ N}$$

Ans.

$$\curvearrowleft \Sigma F_{y'} = 0; \quad V_A - 80 \sin 15^\circ = 0$$

$$V_A = 20.7 \text{ N}$$

Ans.

$$\zeta + \Sigma M_A = 0; \quad M_A + 80 \cos 45^\circ (0.3 \cos 30^\circ) - 80 \sin 45^\circ (0.1 + 0.3 \sin 30^\circ) = 0$$

$$M_A = -0.555 \text{ N} \cdot \text{m}$$

Ans.

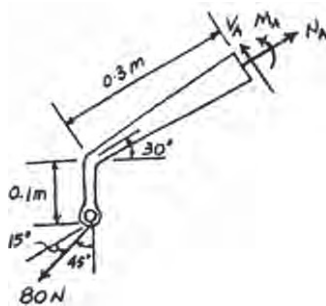
or

$$\zeta + \Sigma M_A = 0; \quad M_A + 80 \sin 15^\circ (0.3 + 0.1 \sin 30^\circ) - 80 \cos 15^\circ (0.1 \cos 30^\circ) = 0$$

$$M_A = -0.555 \text{ N} \cdot \text{m}$$

Ans.

Negative sign indicates that M_A acts in the opposite direction to that shown on FBD.



Ans:

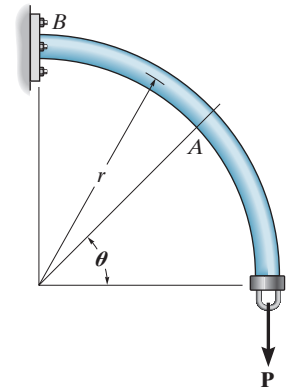
$$N_A = 77.3 \text{ N}$$

$$V_A = 20.7 \text{ N}$$

$$M_A = -0.555 \text{ N} \cdot \text{m}$$

1-26.

The curved rod has a radius r and is fixed to the wall at B . Determine the resultant internal loadings acting on the cross section at point A which is located at an angle θ from the horizontal.



SOLUTION

Equations of Equilibrium: For point A

$$\rightarrow + \sum F_x = 0; \quad P \cos \theta - N_A = 0$$

$$N_A = P \cos \theta$$

Ans.

$$\uparrow + \sum F_y = 0; \quad V_A - P \sin \theta = 0$$

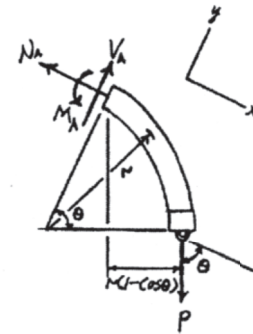
$$V_A = P \sin \theta$$

Ans.

$$\curvearrowleft + \sum M_A = 0; \quad M_A - P[r(1 - \cos \theta)] = 0$$

$$M_A = Pr(1 - \cos \theta)$$

Ans.



Ans:

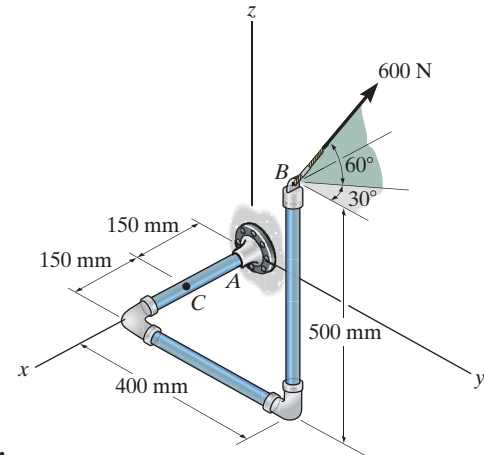
$$N_A = P \cos \theta$$

$$V_A = P \sin \theta$$

$$M_A = Pr(1 - \cos \theta)$$

1-27.

The pipe assembly is subjected to a force of 600 N at B. Determine the resultant internal loading acting on the cross section at point C.



SOLUTION

Internal Loading: Referring to the free-body diagram of the section of the pipe shown in Fig. a,

$$\Sigma F_x = 0; (N_C)_x - 600 \cos 60^\circ \sin 30^\circ = 0 \quad (N_C)_x = 150 \text{ N} \quad \text{Ans.}$$

$$\Sigma F_y = 0; (V_C)_y + 600 \cos 60^\circ \cos 30^\circ = 0 \quad (V_C)_y = -260 \text{ N} \quad \text{Ans.}$$

$$\Sigma F_z = 0; (V_C)_z + 600 \sin 60^\circ = 0 \quad (V_C)_z = -520 \text{ N} \quad \text{Ans.}$$

$$\Sigma M_x = 0; (T_C)_x + 600 \sin 60^\circ (0.4) - 600 \cos 60^\circ \cos 30^\circ (0.5) = 0$$

$$(T_C)_x = -77.9 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

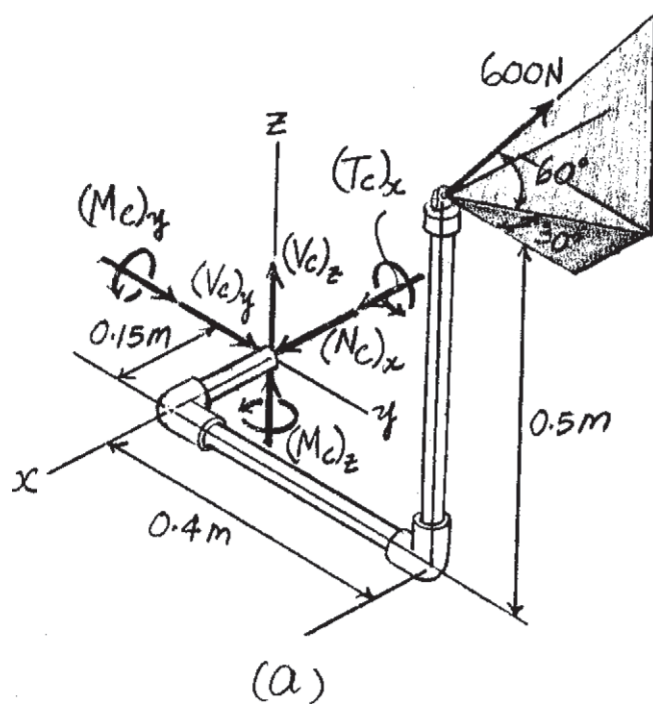
$$\Sigma M_y = 0; (M_C)_y - 600 \sin 60^\circ (0.15) - 600 \cos 60^\circ \sin 30^\circ (0.5) = 0$$

$$(M_C)_y = 153 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

$$\Sigma M_z = 0; (M_C)_z + 600 \cos 60^\circ \cos 30^\circ (0.15) + 600 \cos 60^\circ \sin 30^\circ (0.4) = 0$$

$$(M_C)_z = -99.0 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

The negative signs indicate that $(V_C)_y$, $(V_C)_z$, $(T_C)_x$, and $(M_C)_z$ act in the opposite sense to that shown on the free-body diagram.



Ans:

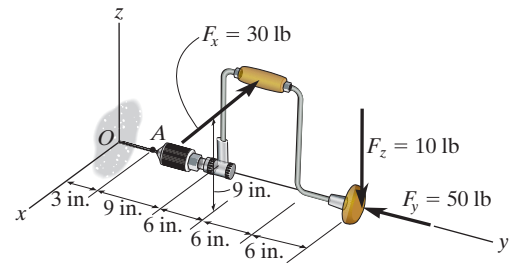
$$(N_C)_x = 150 \text{ N}, (V_C)_y = -260 \text{ N},$$

$$(V_C)_z = -520 \text{ N}, (T_C)_x = -77.9 \text{ N} \cdot \text{m},$$

$$(M_C)_y = 153 \text{ N} \cdot \text{m}, (M_C)_z = -99.0 \text{ N} \cdot \text{m}$$

***1-28.**

If the drill bit jams when the handle of the hand drill is subjected to the forces shown, determine the resultant internal loadings acting on the cross section of the drill bit at A.



SOLUTION

Internal Loading: Referring to the free-body diagram of the section of the drill and brace shown in Fig. a,

$$\Sigma F_x = 0; \quad (V_A)_x - 30 = 0 \quad (V_A)_x = 30 \text{ lb} \quad \text{Ans.}$$

$$\Sigma F_y = 0; \quad (N_A)_y - 50 = 0 \quad (N_A)_y = 50 \text{ lb} \quad \text{Ans.}$$

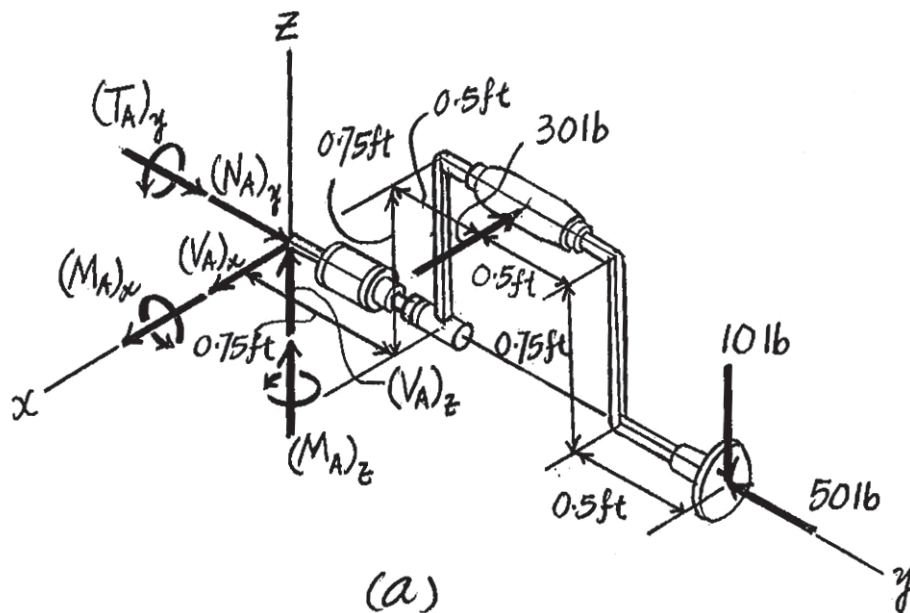
$$\Sigma F_z = 0; \quad (V_A)_z - 10 = 0 \quad (V_A)_z = 10 \text{ lb} \quad \text{Ans.}$$

$$\Sigma M_x = 0; \quad (M_A)_x - 10(2.25) = 0 \quad (M_A)_x = 22.5 \text{ lb} \cdot \text{ft} \quad \text{Ans.}$$

$$\Sigma M_y = 0; \quad (T_A)_y - 30(0.75) = 0 \quad (T_A)_y = 22.5 \text{ lb} \cdot \text{ft} \quad \text{Ans.}$$

$$\Sigma M_z = 0; \quad (M_A)_z + 30(1.25) = 0 \quad (M_A)_z = -37.5 \text{ lb} \cdot \text{ft} \quad \text{Ans.}$$

The negative sign indicates that $(M_A)_z$ acts in the opposite sense to that shown on the free-body diagram.

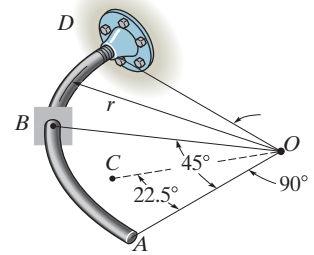


Ans:

$$\begin{aligned} (V_A)_x &= 30 \text{ lb}, \\ (N_A)_y &= 50 \text{ lb}, \\ (V_A)_z &= 10 \text{ lb}, \\ (M_A)_x &= 22.5 \text{ lb} \cdot \text{ft}, \\ (T_A)_y &= 22.5 \text{ lb} \cdot \text{ft}, \\ (M_A)_z &= -37.5 \text{ lb} \cdot \text{ft} \end{aligned}$$

1-29.

The curved rod AD of radius r has a weight per length of w . If it lies in the horizontal plane, determine the resultant internal loadings acting on the cross section at point B . *Hint:* The distance from the centroid C of segment AB to point O is $CO = 0.9745r$.



SOLUTION

$$\Sigma F_z = 0; \quad V_B - \frac{\pi}{4}rw = 0; \quad V_B = 0.785wr$$

Ans.

$$\Sigma F_x = 0; \quad N_B = 0$$

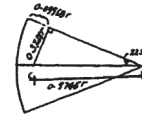
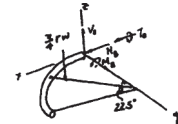
Ans.

$$\Sigma M_x = 0; \quad T_B - \frac{\pi}{4}rw(0.09968r) = 0; \quad T_B = 0.0783wr^2$$

Ans.

$$\Sigma M_y = 0; \quad M_B + \frac{\pi}{4}rw(0.3729r) = 0; \quad M_B = -0.293wr^2$$

Ans.



Ans:

$$V_B = 0.785wr,$$

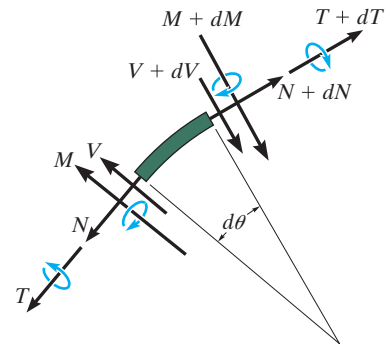
$$N_B = 0,$$

$$T_B = 0.0783wr^2,$$

$$M_B = -0.293wr^2$$

1–30.

A differential element taken from a curved bar is shown in the figure. Show that $dN/d\theta = V$, $dV/d\theta = -N$, $dM/d\theta = -T$, and $dT/d\theta = M$.



SOLUTION

$$\Sigma F_x = 0;$$

$$N \cos \frac{d\theta}{2} + V \sin \frac{d\theta}{2} - (N + dN) \cos \frac{d\theta}{2} + (V + dV) \sin \frac{d\theta}{2} = 0 \quad (1)$$

$$\Sigma F_y = 0;$$

$$N \sin \frac{d\theta}{2} - V \cos \frac{d\theta}{2} + (N + dN) \sin \frac{d\theta}{2} + (V + dV) \cos \frac{d\theta}{2} = 0 \quad (2)$$

$$\Sigma M_x = 0;$$

$$T \cos \frac{d\theta}{2} + M \sin \frac{d\theta}{2} - (T + dT) \cos \frac{d\theta}{2} + (M + dM) \sin \frac{d\theta}{2} = 0 \quad (3)$$

$$\Sigma M_y = 0;$$

$$T \sin \frac{d\theta}{2} - M \cos \frac{d\theta}{2} + (T + dT) \sin \frac{d\theta}{2} + (M + dM) \cos \frac{d\theta}{2} = 0 \quad (4)$$

Since $\frac{d\theta}{2}$ is small, then $\sin \frac{d\theta}{2} = \frac{d\theta}{2}$, $\cos \frac{d\theta}{2} = 1$

Eq. (1) becomes $Vd\theta - dN + \frac{dVd\theta}{2} = 0$

Neglecting the second order term, $Vd\theta - dN = 0$

$$\frac{dN}{d\theta} = V \quad \text{QED}$$

Eq. (2) becomes $Nd\theta + dV + \frac{Nd\theta}{2} = 0$

Neglecting the second order term, $Nd\theta + dV = 0$

$$\frac{dV}{d\theta} = -N \quad \text{QED}$$

Eq. (3) becomes $Md\theta - dT + \frac{Md\theta}{2} = 0$

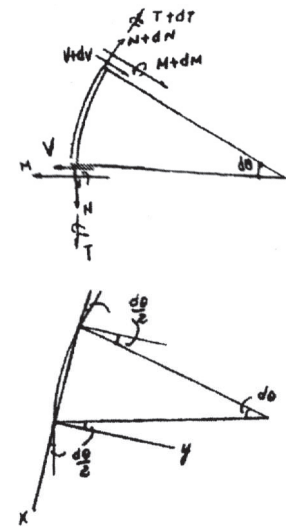
Neglecting the second order term, $Md\theta - dT = 0$

$$\frac{dT}{d\theta} = M \quad \text{QED}$$

Eq. (4) becomes $Td\theta + dM + \frac{Td\theta}{2} = 0$

Neglecting the second order term, $Td\theta + dM = 0$

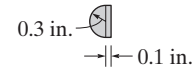
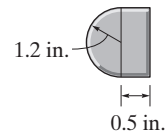
$$\frac{dM}{d\theta} = -T \quad \text{QED}$$



Ans:
N/A

1–31.

A 175-lb woman stands on a vinyl floor wearing stiletto high-heel shoes. If the heel has the dimensions shown, determine the average normal stress she exerts on the floor and compare it with the average normal stress developed when a man having the same weight is wearing flat-heeled shoes. Assume the entire weight is supported only by the heel of one shoe.



SOLUTION

Stiletto shoes:

$$A = \frac{1}{2}(\pi)(0.3)^2 + (0.6)(0.1) = 0.2014 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{175 \text{ lb}}{0.2014 \text{ in}^2} = 869 \text{ psi}$$

Ans.

Flat-heeled shoes:

$$A = \frac{1}{2}(\pi)(1.2)^2 + 2.4(0.5) = 3.462 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{175 \text{ lb}}{3.462 \text{ in}^2} = 50.5 \text{ psi}$$

Ans.

Ans:

Stiletto shoes:

$$\sigma = 869 \text{ psi}$$

Flat-heeled shoes:

$$\sigma = 50.5 \text{ psi}$$

***1-32.**

Determine the largest intensity w of the uniform loading that can be applied to the frame without causing either the average normal stress or the average shear stress at section $b-b$ to exceed $\sigma = 15 \text{ MPa}$ and $\tau = 16 \text{ MPa}$, respectively. Member CB has a square cross section of 30 mm on each side.

SOLUTION

Support Reactions: FBD(a)

$$\zeta + \Sigma M_A = 0; \quad \frac{4}{5}F_{BC}(3) - 3w(1.5) = 0 \quad F_{BC} = 1.875w$$

Equations of Equilibrium: For section $b-b$, FBD(b)

$$\pm \Sigma F_x = 0; \quad \frac{4}{5}(1.875w) - V_{b-b} = 0 \quad V_{b-b} = 1.50w$$

$$+ \uparrow \Sigma F_y = 0; \quad \frac{3}{5}(1.875w) - N_{b-b} = 0 \quad N_{b-b} = 1.125w$$

Average Normal Stress and Shear Stress: The cross-sectional area of section $b-b$, $A' = \frac{5A}{3}$; where $A = (0.03)(0.03) = 0.90(10^{-3}) \text{ m}^2$.

$$\text{Then } A' = \frac{5}{3}(0.90)(10^{-3}) = 1.50(10^{-3}) \text{ m}^2.$$

Assume failure due to normal stress.

$$(\sigma_{b-b})_{\text{Allow}} = \frac{N_{b-b}}{A'}; \quad 15(10^6) = \frac{1.125w}{1.50(10^{-3})}$$

$$w = 20000 \text{ N/m} = 20.0 \text{ kN/m}$$

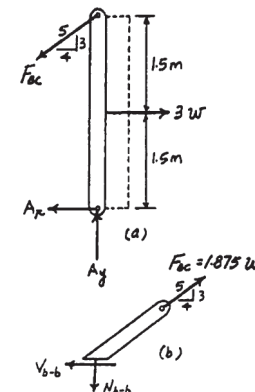
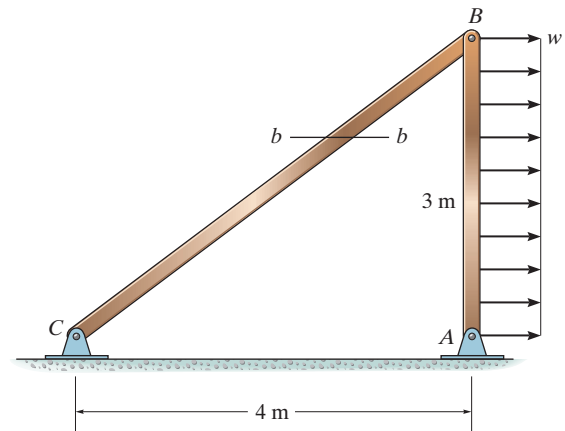
Ans.

Assume failure due to shear stress.

$$(\tau_{b-b})_{\text{Allow}} = \frac{V_{b-b}}{A'}; \quad 16(10^6) = \frac{1.50w}{1.50(10^{-3})}$$

$$w = 16000 \text{ N/m} = 16.0 \text{ kN/m (Controls)}$$

Ans.



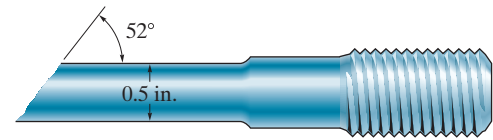
Ans:

$$w = 20.0 \text{ kN/m}$$

$$w = 16.0 \text{ kN/m (Controls)}$$

1-33.

The specimen failed in a tension test at an angle of 52° when the axial load was 19.80 kip. If the diameter of the specimen is 0.5 in., determine the average normal and average shear stress acting on the area of the inclined failure plane. Also, what is the average normal stress acting on the *cross section* when failure occurs?



SOLUTION

$$+\swarrow \Sigma F_x = 0; \quad V - 19.80 \cos 52^\circ = 0$$

$$V = 12.19 \text{ kip}$$

$$+\nwarrow \Sigma F_y = 0; \quad N - 19.80 \sin 52^\circ = 0$$

$$N = 15.603 \text{ kip}$$

Inclined plane:

$$\sigma' = \frac{P}{A}; \quad \sigma' = \frac{15.603}{\frac{\pi(0.25)^2}{\sin 52^\circ}} = 62.6 \text{ ksi}$$

Ans.

$$\tau'_{\text{avg}} = \frac{V}{A}; \quad \tau'_{\text{avg}} = \frac{12.19}{\frac{\pi(0.25)^2}{\sin 52^\circ}} = 48.9 \text{ ksi}$$

Ans.

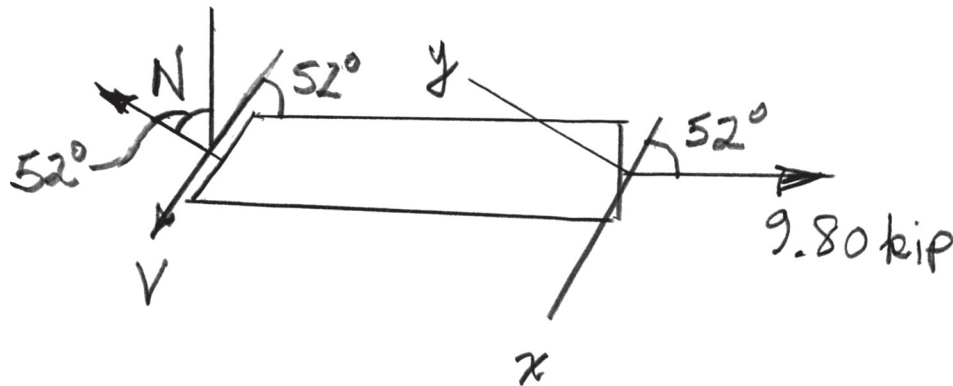
Cross section:

$$\sigma = \frac{P}{A}; \quad \sigma = \frac{19.80}{\pi(0.25)^2} = 101 \text{ ksi}$$

Ans.

$$\tau_{\text{avg}} = \frac{V}{A}; \quad \tau_{\text{avg}} = 0$$

Ans.



Ans:

Inclined plane:

$$\sigma' = 62.6 \text{ ksi}$$

$$\tau'_{\text{avg}} = 48.9 \text{ ksi}$$

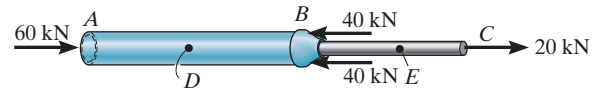
Cross section:

$$\sigma = 101 \text{ ksi}$$

$$\tau_{\text{avg}} = 0$$

1–34.

The built-up shaft consists of a pipe AB and solid rod BC . The pipe has an inner diameter of 25 mm and outer diameter of 30 mm. The rod has a diameter of 15 mm. Determine the average normal stress at points D and E and represent the stress on a volume element located at each of these points.



SOLUTION

Internal Loadings: Referring to the FBD of the rod and the pipe shown in Fig. a and b respectively,

$$\Sigma F_x = 0; \quad 20 - F_E = 0 \quad F_E = 20 \text{ kN}$$

$$\Sigma F_x = 0; \quad 60 - F_D = 0 \quad F_D = 60 \text{ kN}$$

Normal stress: The cross-sectional area of the rod and the pipe are

$$A_r = \frac{\pi}{4}(0.015^2) = 56.25(10^{-6})\pi \text{ m}^2$$

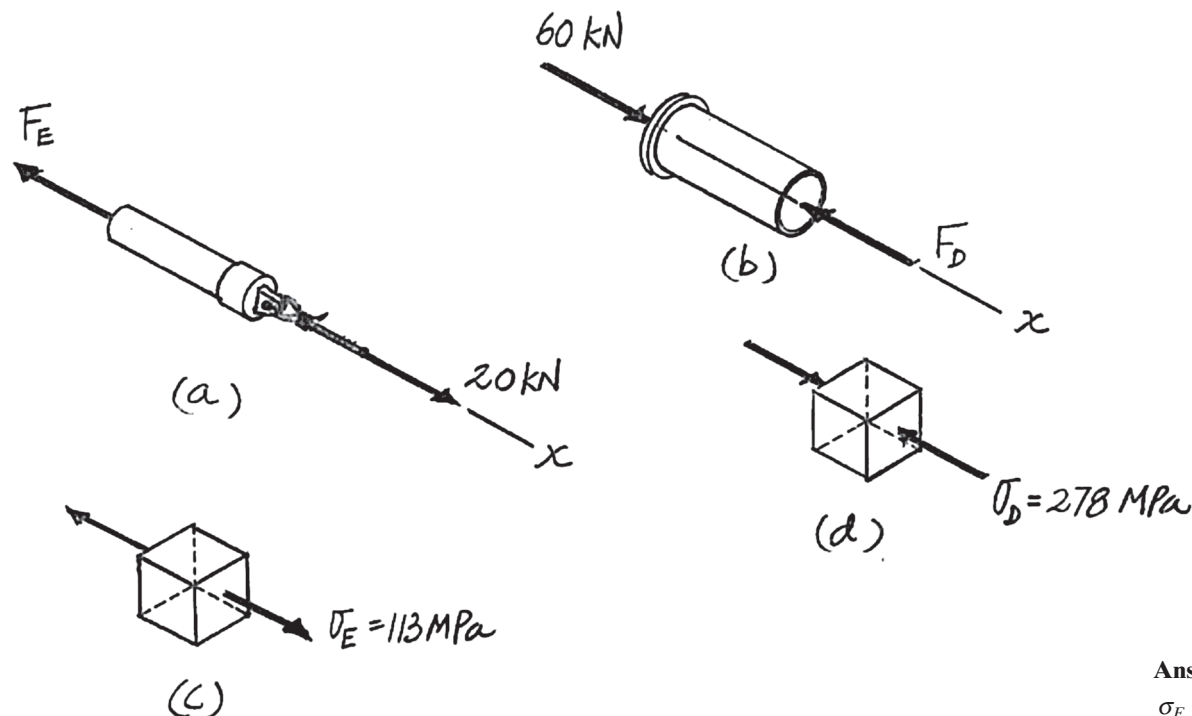
$$A_p = \frac{\pi}{4}(0.03^2 - 0.025^2) = 68.75(10^{-6})\pi \text{ m}^2$$

Then

$$\sigma_E = \frac{F_E}{A_r} = \frac{20(10^3)}{56.25(10^{-6})\pi} = 113.18(10^6) \text{ Pa (T)} = 113 \text{ MPa (T)} \quad \text{Ans.}$$

$$\sigma_D = \frac{F_D}{A_p} = \frac{60(10^3)}{68.75(10^{-6})\pi} = 277.80(10^6) \text{ Pa (C)} = 278 \text{ MPa (C)} \quad \text{Ans.}$$

The state of stress at points D and E can be represented by the volume element shown in Fig. d and c respectively.



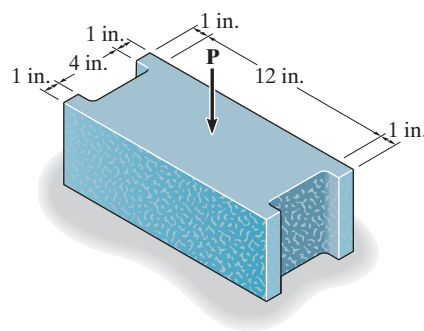
Ans:

$$\sigma_E = 113 \text{ MPa (T)}$$

$$\sigma_D = 278 \text{ MPa (C)}$$

1–35.

If the material fails when the average normal stress reaches 120 psi, determine the largest centrally applied vertical load **P** the block can support.



SOLUTION

Average Normal Stress: The cross-sectional area of the block is

$$A = 12(6) - 2[4(1)] = 76 \text{ in}^2$$

Thus,

$$\sigma_{\text{allow}} = \frac{N_{\text{allow}}}{A}, \quad 120 = \frac{P_{\text{allow}}}{76}$$

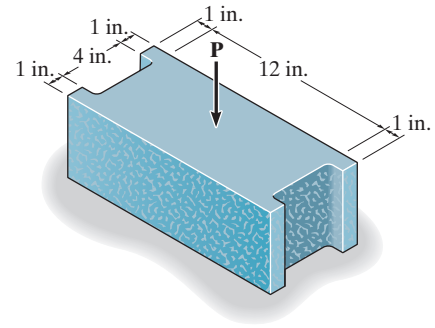
$$P_{\text{allow}} = 9120 \text{ lb} = 9.12 \text{ kip}$$

Ans.

Ans:
 $P_{\text{allow}} = 9.12 \text{ kip}$

***1-36.**

If the block is subjected to a centrally applied force of $P = 6$ kip, determine the average normal stress in the material. Show the stress acting on a differential volume element of the material.



SOLUTION

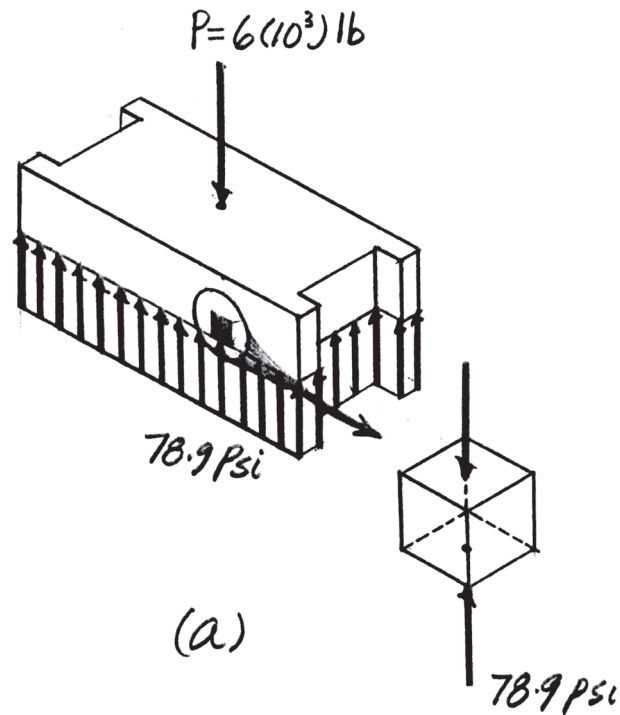
Average Normal Stress: The cross-sectional area of the block is

$$A = 14(6) - 2[4(1)] = 76 \text{ in}^2$$

Thus,

$$\sigma = \frac{N}{A} = \frac{6(10^3)}{76} = 78.947 \text{ psi} = 78.9 \text{ psi} \quad \text{Ans.}$$

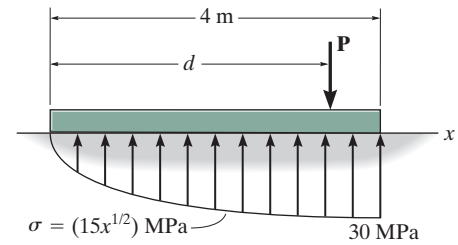
The average normal stress acting on the differential volume element is shown in Fig. *a*.



Ans:
 $\sigma = 78.9 \text{ psi}$

1-37.

The plate has a width of 0.5 m. If the stress distribution at the support varies as shown, determine the force P applied to the plate and the distance d to where it is applied.



SOLUTION

The resultant force dF of the bearing pressure acting on the plate of area $dA = b dx = 0.5 dx$, Fig. a ,

$$dF = \sigma_b dA = (15x^{1/2})(10^6)(0.5dx) = 7.5(10^6)x^{1/2} dx$$

$$+\uparrow \Sigma F_y = 0; \quad \int dF - P = 0$$

$$\int_0^{4\text{ m}} 7.5(10^6)x^{1/2} dx - P = 0$$

$$P = 40(10^6) \text{ N} = 40 \text{ MN}$$

Ans.

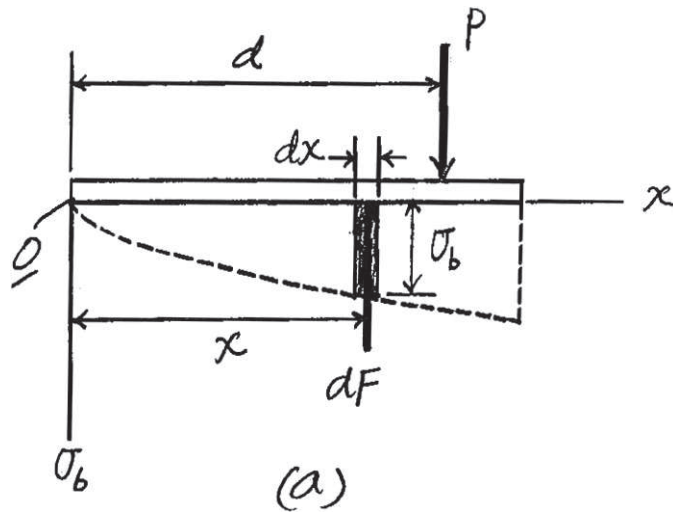
Equilibrium requires

$$\zeta + \Sigma M_O = 0; \quad \int x dF - Pd = 0$$

$$\int_0^{4\text{ m}} x[7.5(10^6)x^{1/2} dx] - 40(10^6) d = 0$$

$$d = 2.40 \text{ m}$$

Ans.

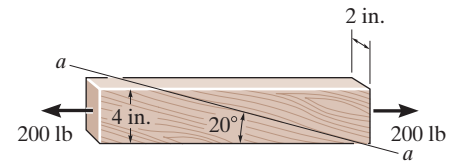


Ans:

$$P = 40 \text{ MN}, d = 2.40 \text{ m}$$

1-38.

The member is subjected to a tensile force of 200 lb. Determine the average normal and average shear stress in the wood fibers, which are oriented along plane $a-a$ at 20° with the axis of the member.



SOLUTION

Internal Loadings: Referring to the FBD of the lower segment of the board sectioned through plane $a-a$, Fig. a ,

$$\Sigma F_x = 0; \quad N - 200 \sin 20^\circ = 0 \quad N = 68.40 \text{ lb}$$

$$\Sigma F_y = 0; \quad 200 \cos 20^\circ - V = 0 \quad V = 187.94 \text{ lb}$$

Average Normal and Shear Stress: The area of plane $a-a$ is

$$A = 2 \left(\frac{4}{\sin 20^\circ} \right) = 23.39 \text{ in}^2$$

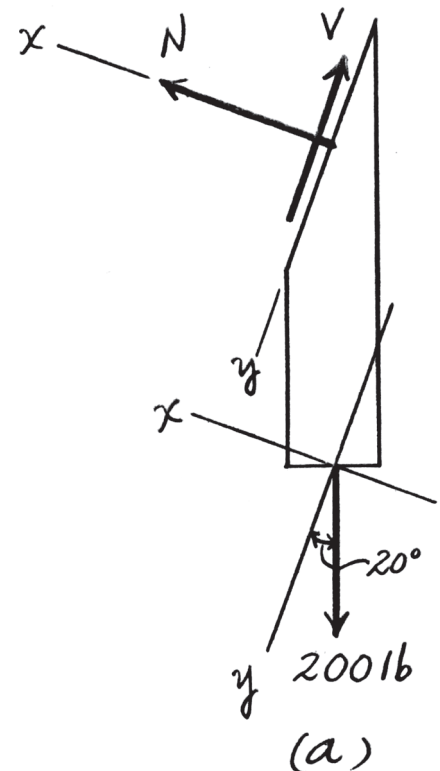
Then,

$$\sigma = \frac{N}{A} = \frac{68.40}{23.39} = 2.92 \text{ psi}$$

$$\tau = \frac{V}{A} = \frac{187.94}{23.39} = 8.03 \text{ psi}$$

Ans.

Ans.



Ans:

$$\sigma = 2.92 \text{ psi},$$

$$\tau = 8.03 \text{ psi}$$

1-39.

The boom has a uniform weight of 600 lb and is hoisted into position using the cable BC . If the cable has a diameter of 0.5 in., plot the average normal stress in the cable as a function of the boom position θ for $0^\circ \leq \theta \leq 90^\circ$.

SOLUTION

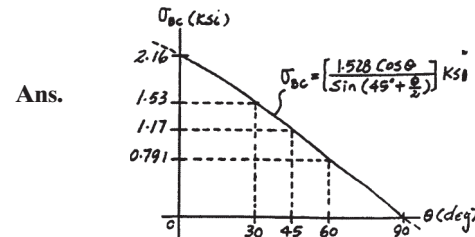
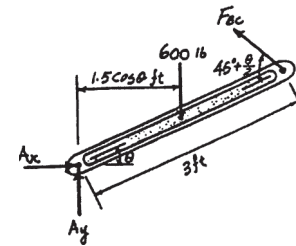
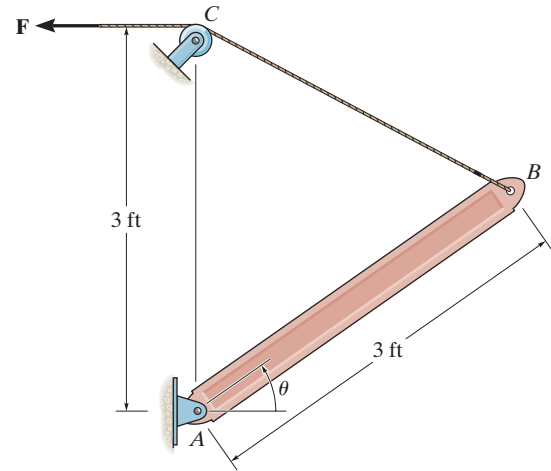
Support Reactions:

$$\zeta + \Sigma M_A = 0; \quad F_{BC} \sin \left(45^\circ + \frac{\theta}{2} \right) (3) - 600(1.5 \cos \theta) = 0$$

$$F_{BC} = \frac{300 \cos \theta}{\sin \left(45^\circ + \frac{\theta}{2} \right)}$$

Average Normal Stress:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{300 \cos \theta}{\frac{\pi}{4} (0.5^2)} = \left\{ \frac{1.528 \cos \theta}{\sin \left(45^\circ + \frac{\theta}{2} \right)} \right\} \text{ ksi}$$



Ans:

$$\sigma_{BC} = \left\{ \frac{1.528 \cos \theta}{\sin \left(45^\circ + \frac{\theta}{2} \right)} \right\} \text{ ksi}$$

*1–40.

Determine the average normal stress in each of the 20-mm-diameter bars of the truss. Set $P = 40$ kN.

SOLUTION

Internal Loadings: The force developed in each member of the truss can be determined by using the method of joints. First, consider the equilibrium of joint C , Fig. a ,

$$\pm \rightarrow \Sigma F_x = 0; \quad 40 - F_{BC} \left(\frac{4}{5} \right) = 0 \quad F_{BC} = 50 \text{ kN (C)}$$

$$+\uparrow \Sigma F_y = 0; \quad 50 \left(\frac{3}{5} \right) - F_{AC} = 0 \quad F_{AC} = 30 \text{ kN (T)}$$

Subsequently, the equilibrium of joint B , Fig. b , is considered

$$\pm \rightarrow \Sigma F_x = 0; \quad 50 \left(\frac{4}{5} \right) - F_{AB} = 0 \quad F_{AB} = 40 \text{ kN (T)}$$

Average Normal Stress: The cross-sectional area of each of the bars is

$$A = \frac{\pi}{4} (0.02^2) = 0.3142(10^{-3}) \text{ m}^2. \text{ We obtain,}$$

$$(\sigma_{\text{avg}})_{BC} = \frac{F_{BC}}{A} = \frac{50(10^3)}{0.3142(10^{-3})} = 159 \text{ MPa}$$

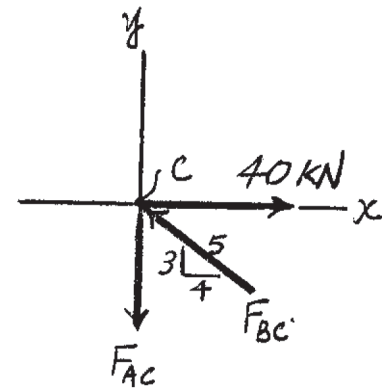
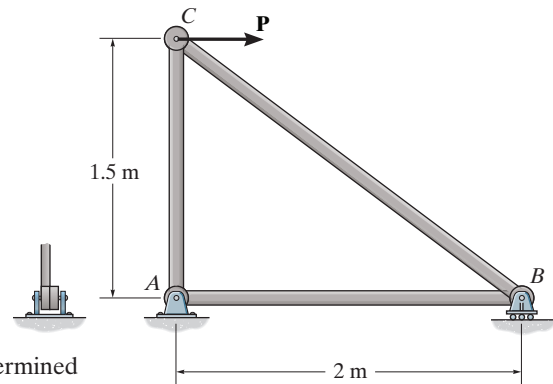
Ans.

$$(\sigma_{\text{avg}})_{AC} = \frac{F_{AC}}{A} = \frac{30(10^3)}{0.3142(10^{-3})} = 95.5 \text{ MPa}$$

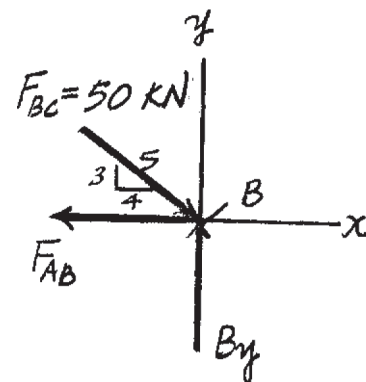
Ans.

$$(\sigma_{\text{avg}})_{AB} = \frac{F_{AB}}{A} = \frac{40(10^3)}{0.3142(10^{-3})} = 127 \text{ MPa}$$

Ans.



(a)



(b)

Ans:

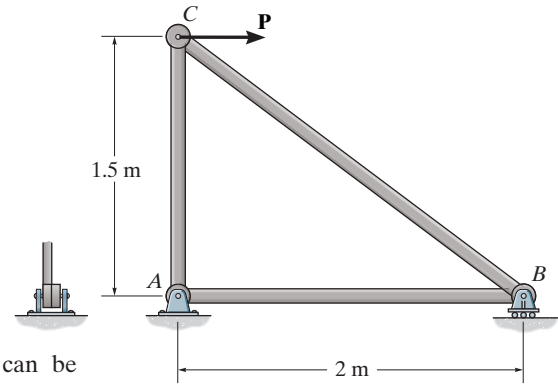
$$(\sigma_{\text{avg}})_{BC} = 159 \text{ MPa,}$$

$$(\sigma_{\text{avg}})_{AC} = 95.5 \text{ MPa,}$$

$$(\sigma_{\text{avg}})_{AB} = 127 \text{ MPa}$$

1-41.

If the average normal stress in each of the 20-mm-diameter bars is not allowed to exceed 150 MPa, determine the maximum force P that can be applied to joint C .



SOLUTION

Internal Loadings: The force developed in each member of the truss can be determined by using the method of joints. First, consider the equilibrium of joint C , Fig. a .

$$\pm \rightarrow \Sigma F_x = 0; \quad P - F_{BC} \left(\frac{4}{5} \right) = 0 \quad F_{BC} = 1.25P(C)$$

$$+\uparrow \Sigma F_y = 0; \quad 1.25P \left(\frac{3}{5} \right) - F_{AC} = 0 \quad F_{AC} = 0.75P(T)$$

Subsequently, the equilibrium of joint B , Fig. b , is considered.

$$\pm \rightarrow \Sigma F_x = 0; \quad 1.25P \left(\frac{4}{5} \right) - F_{AB} = 0 \quad F_{AB} = P(T)$$

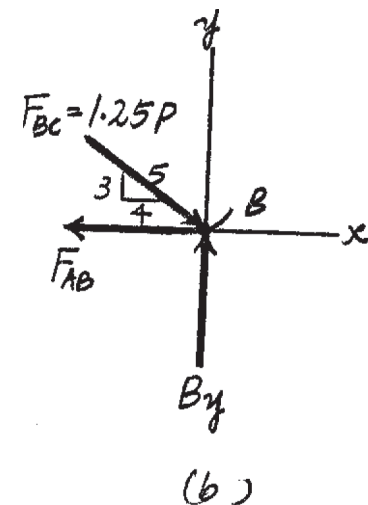
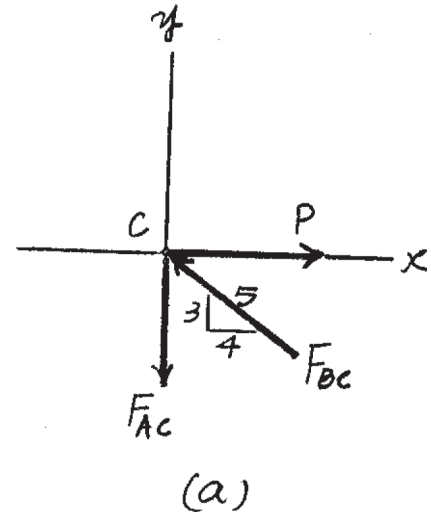
Average Normal Stress: Since the cross-sectional area and the allowable normal stress of each bar are the same, member BC , which is subjected to the maximum normal force, is the critical member. The cross-sectional area of each of the bars is

$$A = \frac{\pi}{4} (0.02^2) = 0.3142(10^{-3}) \text{ m}^2. \text{ We have,}$$

$$(\sigma_{\text{avg}})_{\text{allow}} = \frac{F_{BC}}{A}; \quad 150(10^6) = \frac{1.25P}{0.3142(10^{-3})}$$

$$P = 37\,699 \text{ N} = 37.7 \text{ kN}$$

Ans.



Ans:
 $P = 37.7 \text{ kN}$

1-42.

Determine the maximum average shear stress in pin A of the truss. A horizontal force of $P = 40$ kN is applied to joint C . Each pin has a diameter of 25 mm and is subjected to double shear.

SOLUTION

Internal Loadings: The forces acting on pins A and B are equal to the support reactions at A and B . Referring to the free-body diagram of the entire truss, Fig. a ,

$$\begin{aligned} \Sigma M_A = 0; & \quad B_y(2) - 40(1.5) = 0 & \quad B_y = 30 \text{ kN} \\ \pm \Sigma F_x = 0; & \quad 40 - A_x = 0 & \quad A_x = 40 \text{ kN} \\ +\uparrow \Sigma F_y = 0; & \quad 30 - A_y = 0 & \quad A_y = 30 \text{ kN} \end{aligned}$$

Thus,

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{40^2 + 30^2} = 50 \text{ kN}$$

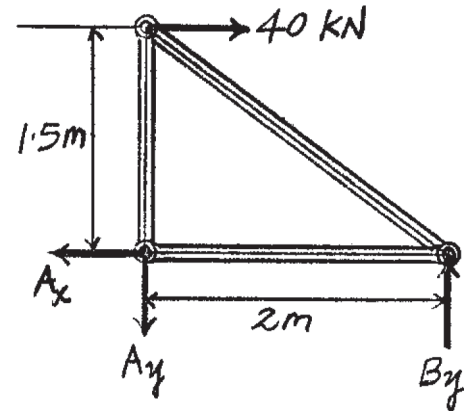
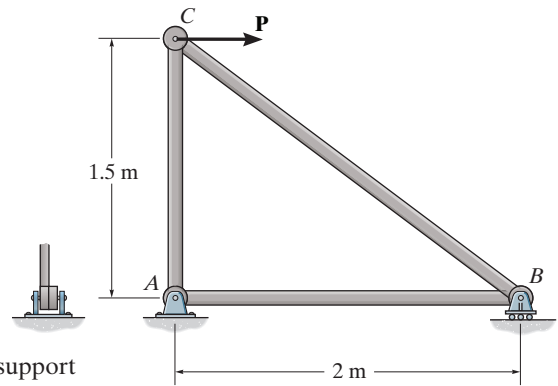
Since pin A is in double shear, Fig. b , the shear forces developed on the shear planes of pin A are

$$V_A = \frac{F_A}{2} = \frac{50}{2} = 25 \text{ kN}$$

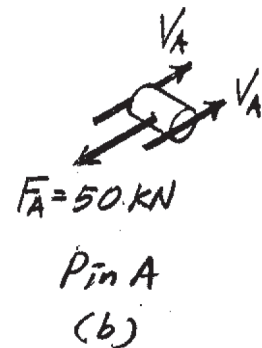
Average Shear Stress: The area of the shear plane for pin A is $A_A = \frac{\pi}{4}(0.025^2) = 0.4909(10^{-3}) \text{ m}^2$. We have

$$(\tau_{\text{avg}})_A = \frac{V_A}{A_A} = \frac{25(10^3)}{0.4909(10^{-3})} = 50.9 \text{ MPa}$$

Ans.



(a)



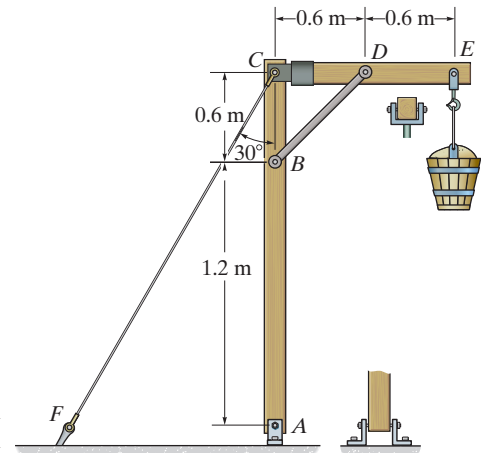
Ans:

$$F_A = 50 \text{ kN}$$

$$(\tau_{\text{avg}})_A = 50.9 \text{ MPa}$$

1-43.

The 150-kg bucket is suspended from end E of the frame. Determine the average normal stress in the 6-mm diameter wire CF and the 15-mm diameter short strut BD .



SOLUTION

Internal Loadings: The normal force developed in rod BD and cable CF can be determined by writing the moment equations of equilibrium about C and A with reference to the free-body diagram of member CE and the entire frame shown in Figs. a and b , respectively.

$$\zeta + \Sigma M_C = 0; \quad F_{BD} \sin 45^\circ (0.6) - 150(9.81)(1.2) = 0 \quad F_{BD} = 4162.03 \text{ N}$$

$$\zeta + \Sigma M_A = 0; \quad F_{CF} \sin 30^\circ (1.8) - 150(9.81)(1.2) = 0 \quad F_{CF} = 1962 \text{ N}$$

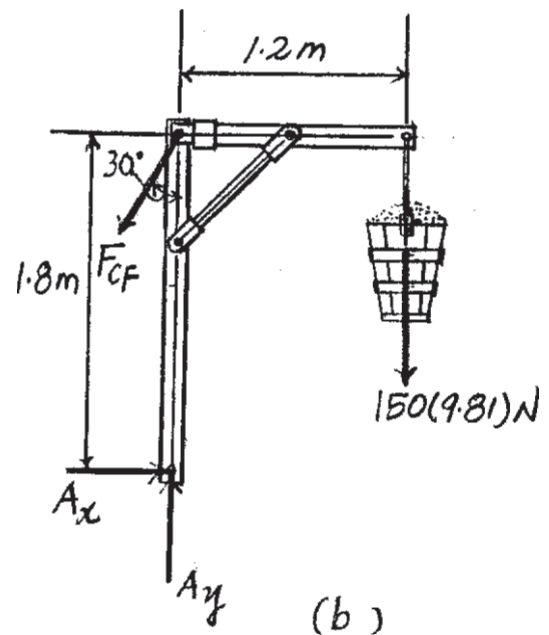
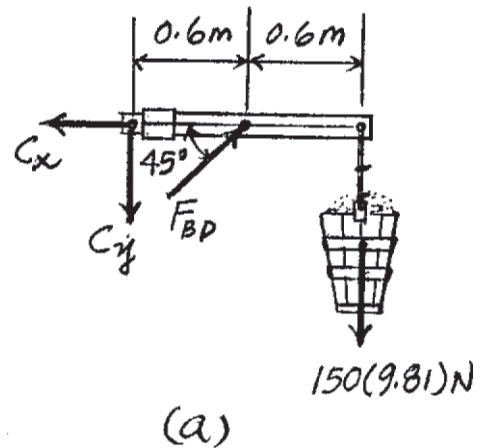
Average Normal Stress: The cross-sectional areas of rod BD and cable CF are $A_{BD} = \frac{\pi}{4} (0.015^2) = 0.1767(10^{-3}) \text{ m}^2$ and $A_{CF} = \frac{\pi}{4} (0.006^2) = 28.274(10^{-6}) \text{ m}^2$. We have

$$(\sigma_{\text{avg}})_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{4162.03}{0.1767(10^{-3})} = 23.6 \text{ MPa}$$

Ans.

$$(\sigma_{\text{avg}})_{CF} = \frac{F_{CF}}{A_{CF}} = \frac{1962}{28.274(10^{-6})} = 69.4 \text{ MPa}$$

Ans.



Ans:

$$(\sigma_{\text{avg}})_{BD} = 23.6 \text{ MPa}, (\sigma_{\text{avg}})_{CF} = 69.4 \text{ MPa}$$

***1-44.**

The 150-kg bucket is suspended from end E of the frame. If the diameters of the pins at A and D are 6 mm and 10 mm, respectively, determine the average shear stress developed in these pins. Each pin is subjected to double shear.

SOLUTION

Internal Loading: The forces exerted on pins D and A are equal to the support reaction at D and A. First, consider the free-body diagram of member CE shown in Fig. a.

$$\zeta + \Sigma M_C = 0; \quad F_{BD} \sin 45^\circ (0.6) - 150(9.81)(1.2) = 0 \quad F_{BD} = 4162.03 \text{ N}$$

Subsequently, the free-body diagram of the entire frame shown in Fig. b will be considered.

$$\zeta + \Sigma M_A = 0; \quad F_{CF} \sin 30^\circ (1.8) - 150(9.81)(1.2) = 0 \quad F_{CF} = 1962 \text{ N}$$

$$\rightarrow \Sigma F_x = 0; \quad A_x - 1962 \sin 30^\circ = 0 \quad A_x = 981 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 1962 \cos 30^\circ - 150(9.81) = 0 \quad A_y = 3170.64 \text{ N}$$

Thus, the forces acting on pins D and A are

$$F_D = F_{BD} = 4162.03 \text{ N} \quad F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{981^2 + 3170.64^2} = 3318.93 \text{ N}$$

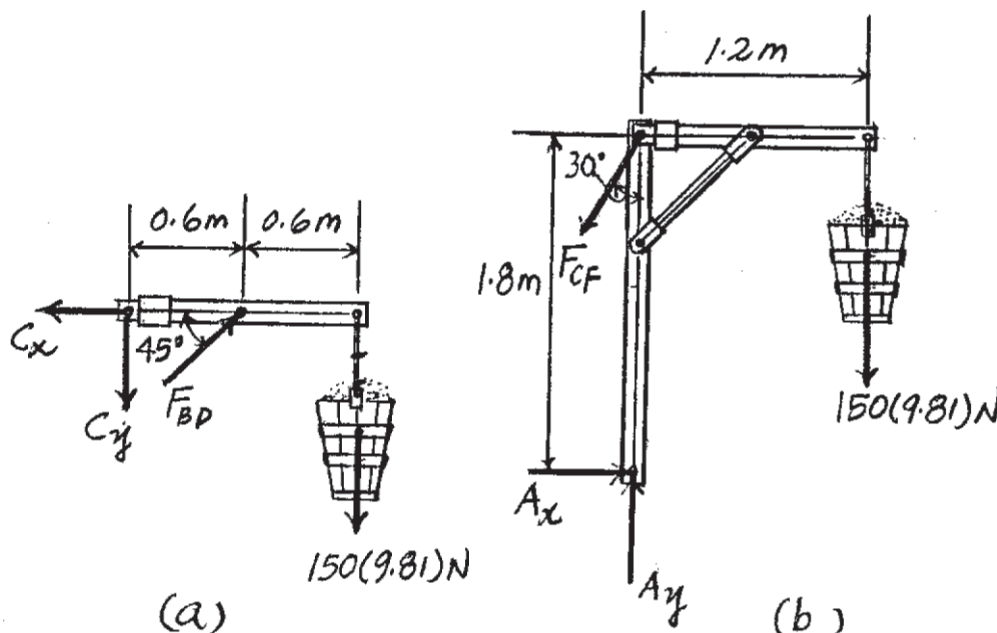
Since both pins are in double shear

$$V_D = \frac{F_D}{2} = 2081.02 \text{ N} \quad V_A = \frac{F_A}{2} = 1659.47 \text{ N}$$

Average Shear Stress: The cross-sectional areas of the shear plane of pins D and A are $A_D = \frac{\pi}{4} (0.01^2) = 78.540(10^{-6}) \text{ m}^2$ and $A_A = \frac{\pi}{4} (0.006^2) = 28.274(10^{-6}) \text{ m}^2$. We obtain

$$(\tau_{\text{avg}})_A = \frac{V_A}{A_A} = \frac{1659.47}{28.274(10^{-6})} = 58.7 \text{ MPa} \quad \text{Ans.}$$

$$(\tau_{\text{avg}})_D = \frac{V_D}{A_D} = \frac{2081.02}{78.540(10^{-6})} = 26.5 \text{ MPa} \quad \text{Ans.}$$



Ans:

$$(\tau_{\text{avg}})_A = 58.7 \text{ MPa}$$

$$(\tau_{\text{avg}})_D = 26.5 \text{ MPa}$$

1-45.

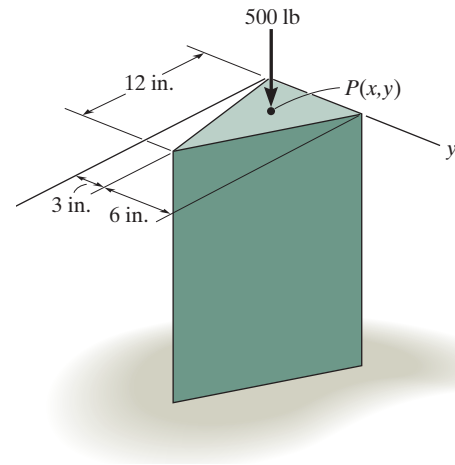
If the pedestal is subjected to a compressive force of 500 lb, specify the x and y coordinates for the location of point $P(x, y)$, where the load must be applied on the cross section, so that the average normal stress is uniform. Calculate the stress and sketch its distribution acting on the cross section at a location removed from the point of load application.

SOLUTION

$$x = \frac{\frac{1}{2}(3)(12)\left(\frac{12}{3}\right) + \frac{1}{2}(6)(12)\left(\frac{12}{3}\right)}{\frac{1}{2}(9)(12)} = 4 \text{ in.}$$

$$y = \frac{\frac{1}{2}(3)(12)(3)\left(\frac{2}{3}\right) + \frac{1}{2}(6)(12)\left(3 + \frac{6}{3}\right)}{\frac{1}{2}(9)(12)} = 4 \text{ in.}$$

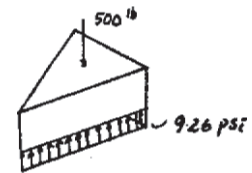
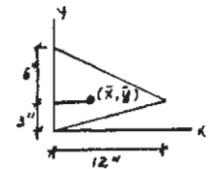
$$\sigma = \frac{P}{A} = \frac{500}{\frac{1}{2}(9)(12)} = 9.26 \text{ psi}$$



Ans.

Ans.

Ans.



Ans:

$x = 4 \text{ in.}$

$y = 4 \text{ in.}$

$\sigma = 9.26 \text{ psi}$

1-46.

The beam is supported by two rods AB and CD that have cross-sectional areas of 12 mm^2 and 8 mm^2 , respectively. If $d = 1 \text{ m}$, determine the average normal stress in each rod.

SOLUTION

$$\zeta + \Sigma M_A = 0; \quad F_{CD}(3) - 6(1) = 0$$

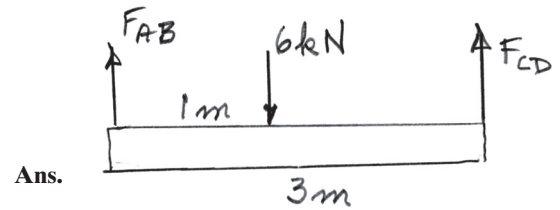
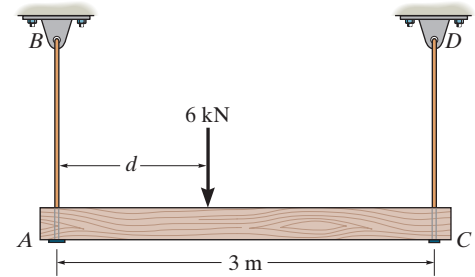
$$F_{CD} = 2 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} - 6 + 2 = 0$$

$$F_{AB} = 4 \text{ kN}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{4(10^3)}{12(10^{-6})} = 333 \text{ MPa}$$

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{2(10^3)}{8(10^{-6})} = 250 \text{ MPa}$$



Ans.

Ans:
 $\sigma_{AB} = 333 \text{ MPa}$,
 $\sigma_{CD} = 250 \text{ MPa}$

1-47.

The beam is supported by two rods AB and CD that have cross-sectional areas of 12 mm^2 and 8 mm^2 , respectively. Determine the position d of the 6-kN load so that the average normal stress in each rod is the same.

SOLUTION

$$\zeta + \Sigma M_O = 0; \quad F_{CD}(3 - d) - F_{AB}(d) = 0$$

$$\sigma = \frac{F_{AB}}{12} = \frac{F_{CD}}{8}$$

$$F_{AB} = 1.5 F_{CD}$$

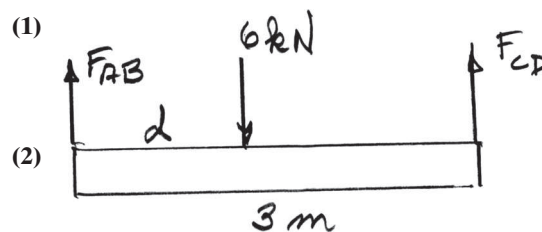
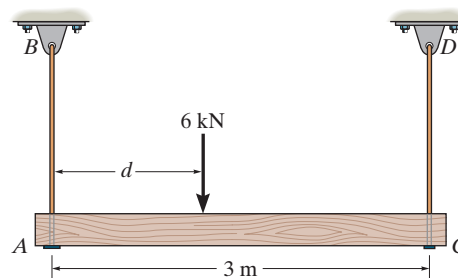
From Eqs. (1) and (2),

$$F_{CD}(3 - d) - 1.5 F_{CD}(d) = 0$$

$$F_{CD}(3 - d - 1.5d) = 0$$

$$3 - 2.5d = 0$$

$$d = 1.20 \text{ m}$$

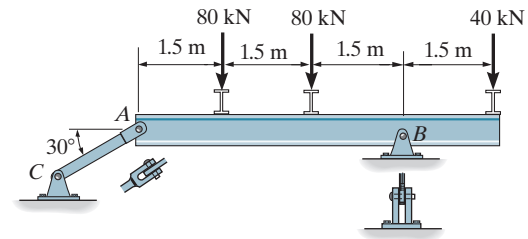


Ans.

Ans:
 $d = 1.20 \text{ m}$

***1-48.**

The beam is supported by a pin at B and a short link AC . Determine the average shear stress developed in the pins at A , B , and C . All pins are in double shear as shown, and each has a diameter of 30 mm.



SOLUTION

Internal Loadings: The force acting on pin A (or C) and B can be determined by considering the equilibrium of the entire beam, Fig a .

$$\zeta + \sum M_B = 0; \quad 80(1.5) + 80(3) - 40(1.5) - F_{AC} \sin 30^\circ(4.5) = 0$$

$$F_{AC} = 133.33 \text{ kN}$$

$$\zeta + \sum M_A = 0; \quad B_y(4.5) - 80(1.5) - 80(3) - 40(6) = 0 \quad B_y = 133.33 \text{ kN}$$

$$\rightarrow \sum F_x = 0; \quad 133.33 \cos 30^\circ - B_x = 0 \quad B_x = 115.47 \text{ kN}$$

Thus,

$$F_B = \sqrt{B_x^2 + B_y^2} = \sqrt{115.47^2 + 133.33^2} = 176.38 \text{ kN}$$

Since pins A (or C) and B are subjected to double shear as shown in their FBDs, Fig. b and c respectively, the shear planes are subjected to

$$V_A = \frac{F_{AC}}{2} = \frac{133.33}{2} = 66.67 \text{ kN} \quad V_B = \frac{F_B}{2} = \frac{176.38}{2} = 88.19 \text{ kN}$$

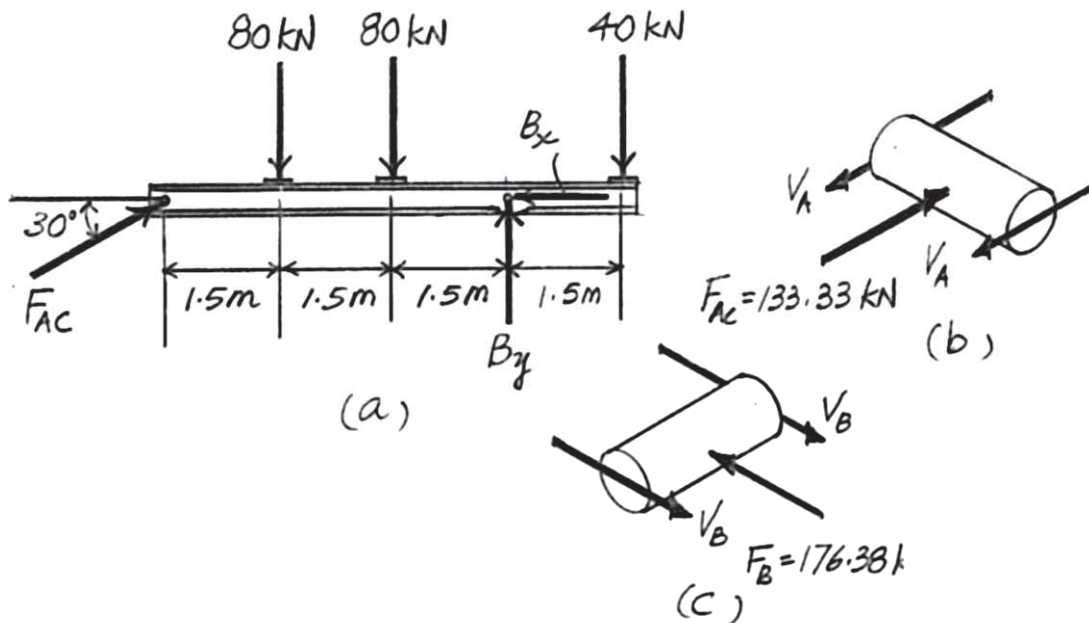
Average Shear Stress: The area of shear plane for pins A and B are

$$A_A = A_B = \frac{\pi}{4} (0.03^2) = 0.225(10^{-3})\pi \text{ m}^2$$

Then

$$(\tau_{\text{avg}})_C = (\tau_{\text{avg}})_A = \frac{V_A}{A_A} = \frac{66.67(10^3)}{0.225(10^{-3})\pi} = 94.31(10^6) \text{ Pa} = 94.3 \text{ MPa} \quad \text{Ans.}$$

$$(\tau_{\text{avg}})_B = \frac{V_B}{A_B} = \frac{88.19(10^3)}{0.225(10^{-3})\pi} = 124.77(10^6) \text{ Pa} = 125 \text{ MPa} \quad \text{Ans.}$$



Ans:

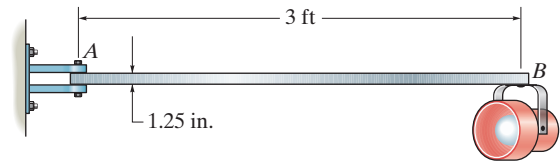
$$(\tau_{\text{avg}})_A = 94.3 \text{ MPa}$$

$$(\tau_{\text{avg}})_C = 94.3 \text{ MPa}$$

$$(\tau_{\text{avg}})_B = 125 \text{ MPa}$$

1–49.

The railcar lamp is supported by the $\frac{1}{8}$ -in.-diameter pin at A . If the lamp weighs 4 lb, and the extension arm AB has a weight of 0.5 lb/ft, determine the average shear stress in the pin needed to support the lamp. *Hint:* The shear force in the pin is caused by the couple moment required for equilibrium at A .



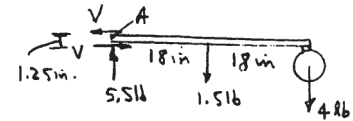
SOLUTION

$$\zeta + \Sigma M_A = 0; \quad V(1.25) - 1.5(18) - 4(36) = 0$$

$$V = 136.8 \text{ lb}$$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{136.8}{\frac{\pi}{4} \left(\frac{1}{8}\right)^2} = 11.1 \text{ ksi}$$

Ans.



Ans:
 $\tau_{\text{avg}} = 11.1 \text{ ksi}$

1–50.

The plastic block is subjected to an axial compressive force of 600 N. Assuming that the caps at the top and bottom distribute the load uniformly throughout the block, determine the average normal and average shear stress acting along section $a-a$.

SOLUTION

Along $a-a$:

$$+\nearrow \Sigma F_x = 0; \quad V - 600 \sin 30^\circ = 0$$

$$V = 300 \text{ N}$$

$$+\searrow \Sigma F_y = 0; \quad -N + 600 \cos 30^\circ = 0$$

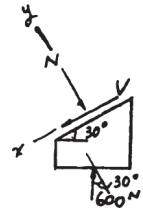
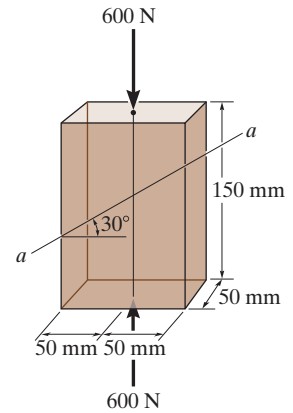
$$N = 519.6 \text{ N}$$

$$\sigma_{a-a} = \frac{519.6}{(0.05)\left(\frac{0.1}{\cos 30^\circ}\right)} = 90.0 \text{ kPa}$$

$$\tau_{a-a} = \frac{300}{(0.05)\left(\frac{0.1}{\cos 30^\circ}\right)} = 52.0 \text{ kPa}$$

Ans.

Ans.



Ans:

$$\sigma_{a-a} = 90.0 \text{ kPa,}$$

$$\tau_{a-a} = 52.0 \text{ kPa}$$

1-51.

During a tension test, the wooden specimen is subjected to an average normal stress of 2 ksi. Determine the axial force P applied to the specimen. Also, find the average shear stress developed along section $a-a$ of the specimen.

SOLUTION

Internal Loading: The normal force developed on the cross section of the middle portion of the specimen can be obtained by considering the free-body diagram shown in Fig. a .

$$+\uparrow \Sigma F_y = 0; \quad \frac{P}{2} + \frac{P}{2} - N = 0 \quad N = P$$

Referring to the free-body diagram shown in fig. b , the shear force developed in the shear plane $a-a$ is

$$+\uparrow \Sigma F_y = 0; \quad \frac{P}{2} - V_{a-a} = 0 \quad V_{a-a} = \frac{P}{2}$$

Average Normal Stress and Shear Stress: The cross-sectional area of the specimen is $A = 1(2) = 2 \text{ in}^2$. We have

$$\sigma_{\text{avg}} = \frac{N}{A}; \quad 2(10^3) = \frac{P}{2}$$

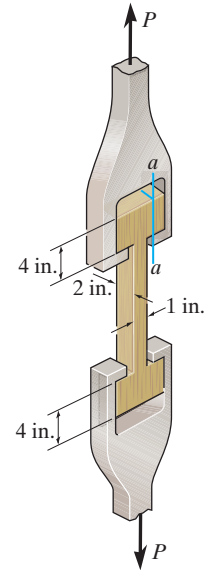
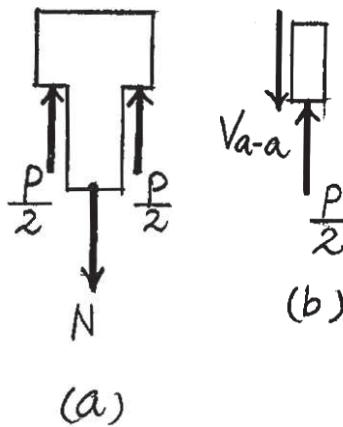
$$P = 4(10^3) \text{ lb} = 4 \text{ kip}$$

Ans.

Using the result of P , $V_{a-a} = \frac{P}{2} = \frac{4(10^3)}{2} = 2(10^3) \text{ lb}$. The area of the shear plane is $A_{a-a} = 2(4) = 8 \text{ in}^2$. We obtain

$$(\tau_{a-a})_{\text{avg}} = \frac{V_{a-a}}{A_{a-a}} = \frac{2(10^3)}{8} = 250 \text{ psi}$$

Ans.

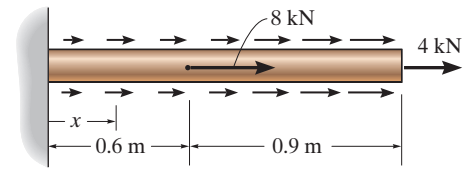


Ans:

$$P = 4 \text{ kip}, (\tau_{a-a})_{\text{avg}} = 250 \text{ psi}$$

***1-52.**

The bar has a cross-sectional area of $400(10^{-6})\text{m}^2$. If it is subjected to a uniform axial distributed loading along its length of 9 kN/m , and to two concentrated loads as shown, determine the average normal stress in the bar as a function of x for $0.6\text{ m} < x \leq 1.5\text{ m}$.



SOLUTION

Internal Loading: Referring to a FBD of the right segment of the bar sectioned at x ,

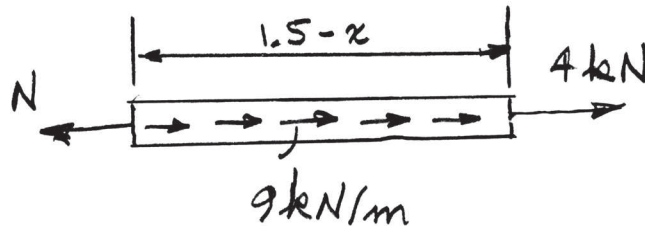
$$\pm \rightarrow \Sigma F_x = 0; \quad 4 + 9(1.5 - x) - N = 0$$

$$N = \{17.5 - 9x\} \text{ kN}$$

Average Normal Stress:

$$\begin{aligned} \sigma &= \frac{N}{A} = \frac{(17.5 - 9x)(10^3)}{400(10^{-6})} \\ &= \{43.75 - 22.5x\} \text{ MPa} \end{aligned}$$

Ans.

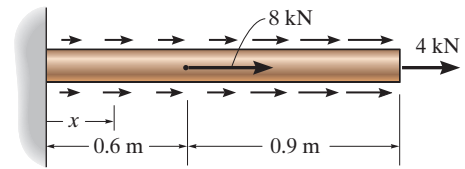


Ans:

$$\sigma = \{43.75 - 22.5x\} \text{ MPa}$$

1-53.

The bar has a cross-sectional area of $400(10^{-6})\text{ m}^2$. If it is subjected to a triangular axial distributed loading along its length which is 0 at $x = 0$ and 9 kN/m at $x = 1.5\text{ m}$, and to two concentrated loads as shown, determine the average normal stress in the bar as a function of x for $0 \leq x < 0.6\text{ m}$.



SOLUTION

Internal Loading: Referring to the FBD of the right segment of the bar sectioned at x , Fig. *a*,

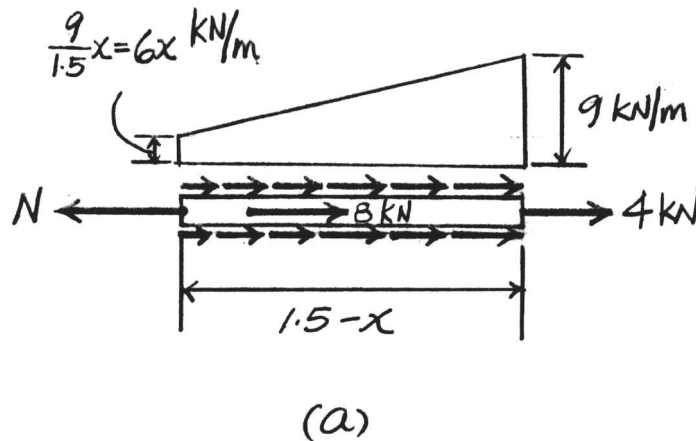
$$\rightarrow \Sigma F_x = 0; \quad 8 + 4 + \frac{1}{2}(6x + 9)(1.5 - x) = 0$$

$$N = \{18.75 - 3x^2\} \text{ kN}$$

Average Normal Stress:

$$\begin{aligned} \sigma &= \frac{N}{A} = \frac{(18.75 - 3x^2)(10^3)}{400(10^{-6})} \\ &= \{43.75 - 22.5x\} \text{ MPa} \end{aligned}$$

Ans.

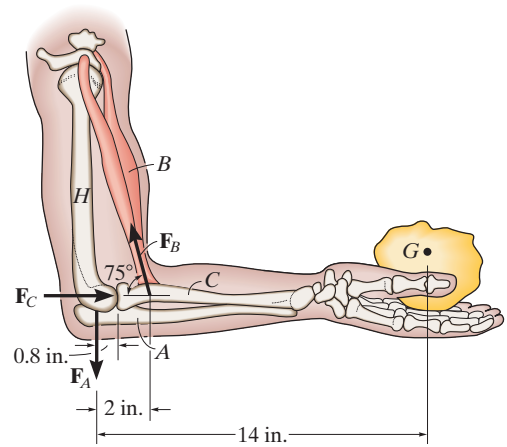


Ans:

$$\sigma = \{43.75 - 22.5x\} \text{ MPa}$$

1-54.

When the hand is holding the 5-lb stone, the humerus H , assumed to be smooth, exerts normal forces F_C and F_A on the radius C and ulna A , respectively, as shown. If the smallest cross-sectional area of the ligament at B is 0.30 in^2 , determine the greatest average tensile stress to which it is subjected.



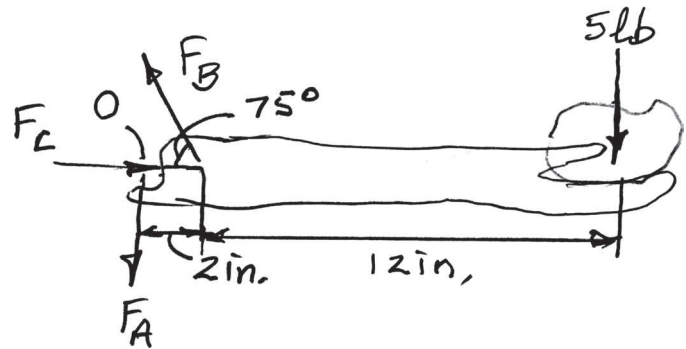
SOLUTION

$$\zeta + \Sigma M_O = 0; \quad F_B \sin 75^\circ (2) - 5(14) = 0$$

$$F_B = 36.235 \text{ lb}$$

$$\sigma = \frac{P}{A} = \frac{36.235}{0.30} = 121 \text{ psi}$$

Ans.



Ans:

$$\sigma = 121 \text{ psi}$$

1-55.

The 2-Mg concrete pipe has a center of mass at point G . If it is suspended from cables AB and AC , determine the average normal stress in the cables. The diameters of AB and AC are 12 mm and 10 mm, respectively.

SOLUTION

Internal Loadings: The normal force developed in cables AB and AC can be determined by considering the equilibrium of the hook for which the free-body diagram is shown in Fig. a .

$$\Sigma F_{x'} = 0; \quad 2000(9.81) \cos 45^\circ - F_{AB} \cos 15^\circ = 0 \quad F_{AB} = 14\,362.83 \text{ N (T)}$$

$$\Sigma F_{y'} = 0; \quad 2000(9.81) \sin 45^\circ - 14\,362.83 \sin 15^\circ - F_{AC} = 0 \quad F_{AC} = 10\,156.06 \text{ N (T)}$$

Average Normal Stress: The cross-sectional areas of cables AB and AC are

$$A_{AB} = \frac{\pi}{4} (0.012^2) = 0.1131(10^{-3}) \text{ m}^2 \quad \text{and} \quad A_{AC} = \frac{\pi}{4} (0.01^2) = 78.540(10^{-6}) \text{ m}^2.$$

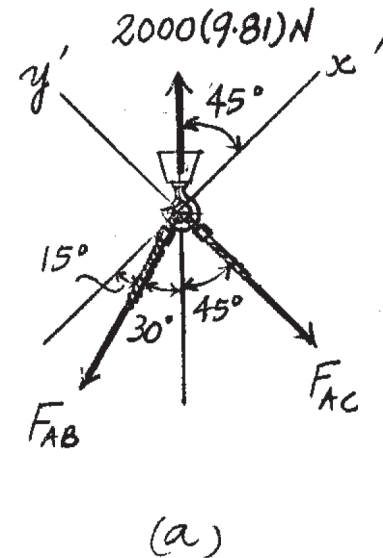
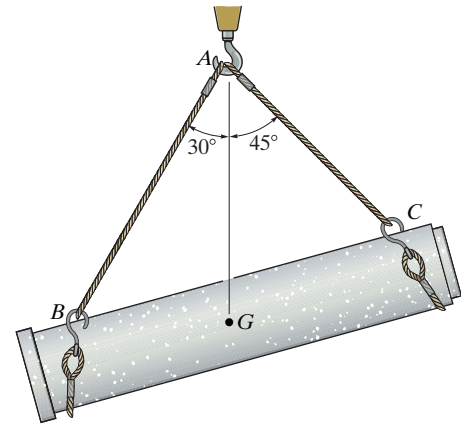
We have

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{14\,362.83}{0.1131(10^{-3})} = 127 \text{ MPa}$$

$$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{10\,156.06}{78.540(10^{-6})} = 129 \text{ MPa}$$

Ans.

Ans.



Ans:

$$\sigma_{AB} = 127 \text{ MPa}, \sigma_{AC} = 129 \text{ MPa}$$

***1-56.**

The 2-Mg concrete pipe has a center of mass at point G . If it is suspended from cables AB and AC , determine the diameter of cable AB so that the average normal stress in this cable is the same as in the 10-mm-diameter cable AC .

SOLUTION

Internal Loadings: The normal force in cables AB and AC can be determined by considering the equilibrium of the hook for which the free-body diagram is shown in Fig. a .

$$\Sigma F_{x'} = 0; 2000(9.81) \cos 45^\circ - F_{AB} \cos 15^\circ = 0 \quad F_{AB} = 14.36 \text{ kN} \quad \text{Ans.}$$

$$\Sigma F_{y'} = 0; 2000(9.81) \sin 45^\circ - 14\,362.83 \sin 15^\circ - F_{AC} = 0 \quad F_{AC} = 10.16 \text{ kN} \quad \text{Ans.}$$

Average Normal Stress: The cross-sectional areas of cables AB and AC are $A_{AB} = \frac{\pi}{4} d_{AB}^2$ and $A_{AC} = \frac{\pi}{4} (0.01)^2 = 78.540(10^{-6}) \text{ m}^2$.

Here, we require

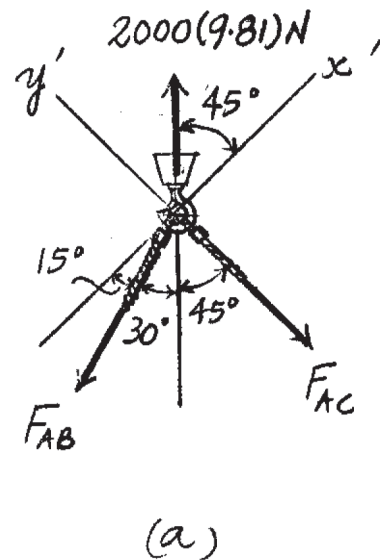
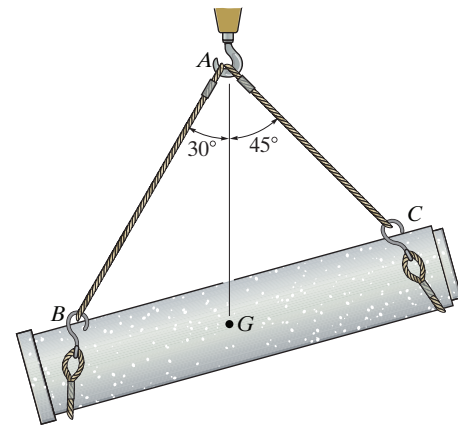
$$\sigma_{AB} = \sigma_{AC}$$

$$\frac{F_{AB}}{A_{AB}} = \frac{F_{AC}}{A_{AC}}$$

$$\frac{14\,362.83}{\frac{\pi}{4} d_{AB}^2} = \frac{10\,156.06}{78.540(10^{-6})}$$

$$d_{AB} = 0.01189 \text{ m} = 11.9 \text{ mm}$$

Ans.



Ans:

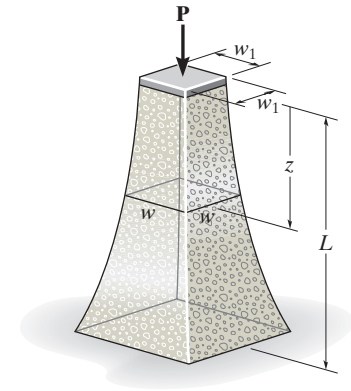
$$F_{AB} = 14.36 \text{ kN}$$

$$F_{AC} = 10.16 \text{ kN}$$

$$d_{AB} = 11.9 \text{ mm}$$

1-57.

The pier is made of material having a specific weight γ . If it has a square cross section, determine its width w as a function of z so that the average normal stress in the pier remains constant. The pier supports a constant load P at its top where its width is w_1 .



SOLUTION

Assume constant stress σ_1 , then at the top,

$$\sigma_1 = \frac{P}{w_1^2} \quad (1)$$

For an increase in z the area must increase,

$$dA = \frac{dW}{\sigma_1} = \frac{\gamma A dz}{\sigma_1} \quad \text{or} \quad \frac{dA}{A} = \frac{\gamma}{\sigma_1} dz$$

For the top section:

$$\int_{A_1}^A \frac{dA}{A} = \frac{\gamma}{\sigma_1} \int_0^z dz$$

$$\ln \frac{A}{A_1} = \frac{\gamma}{\sigma_1} z$$

$$A = A_1 e^{\left(\frac{\gamma}{\sigma_1}\right)z}$$

$$A = w^2$$

$$A_1 = w_1^2$$

$$w = w_1 e^{\left(\frac{\gamma}{2\sigma_1}\right)z}$$

From Eq. (1),

$$w = w_1 e^{\left[\frac{w_1^2 \gamma}{2P}\right]z} \quad \text{Ans.}$$

Ans:

$$w = w_1 e^{\left[\frac{w_1^2 \gamma}{2P}\right]z}$$

1-58.

The anchor bolt was pulled out of the concrete wall and the failure surface formed part of a frustum and cylinder. This indicates a shear failure occurred along the cylinder BC and tension failure along the frustum AB . If the shear and normal stresses along these surfaces have the magnitudes shown, determine the force $\mathbf{P}$ that is applied to the bolt.

SOLUTION

Average Normal Stress:

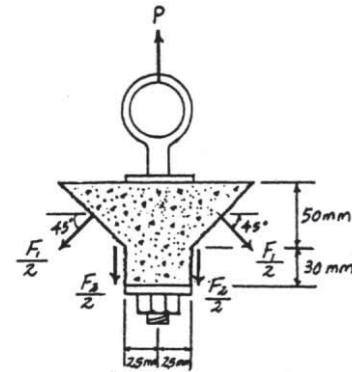
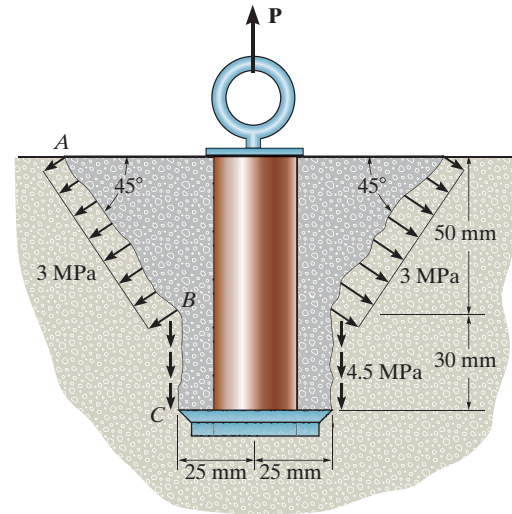
$$\begin{aligned} \text{For the frustum, } A &= 2\pi\bar{x}L = 2\pi(0.025 + 0.025)(\sqrt{0.05^2 + 0.05^2}) \\ &= 0.02221 \text{ m}^2 \\ \sigma &= \frac{P}{A}; \quad 3(10^6) = \frac{F_1}{0.02221} \\ F_1 &= 66.64 \text{ kN} \end{aligned}$$

Average Shear Stress:

$$\begin{aligned} \text{For the cylinder, } A &= \pi(0.05)(0.03) = 0.004712 \text{ m}^2 \\ \tau_{\text{avg}} &= \frac{V}{A}; \quad 4.5(10^6) = \frac{F_2}{0.004712} \\ F_2 &= 21.21 \text{ kN} \end{aligned}$$

Equation of Equilibrium:

$$\begin{aligned} +\uparrow \Sigma F_y &= 0; \quad P - 21.21 - 66.64 \sin 45^\circ = 0 \\ P &= 68.3 \text{ kN} \end{aligned}$$

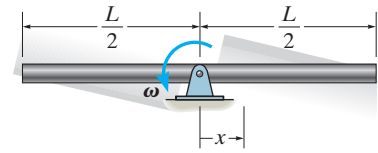


Ans.

Ans:
 $P = 68.3 \text{ kN}$

1-59.

The uniform bar, having a cross-sectional area of A and mass per unit length of m , is pinned at its center. If it is rotating in the horizontal plane at a constant angular rate of ω , determine the average normal stress in the bar as a function of x .



SOLUTION

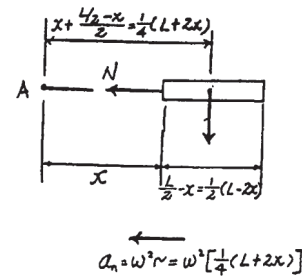
Equation of Motion:

$$\begin{aligned} \leftarrow \Sigma F_x = ma_N; \quad N &= m \left[\frac{1}{2} (L - 2x) \right] \omega^2 \left[\frac{1}{4} (L + 2x) \right] \\ &= \frac{m\omega^2}{8} (L^2 - 4x^2) \end{aligned}$$

Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{m\omega^2}{8A} (L^2 - 4x^2)$$

Ans.

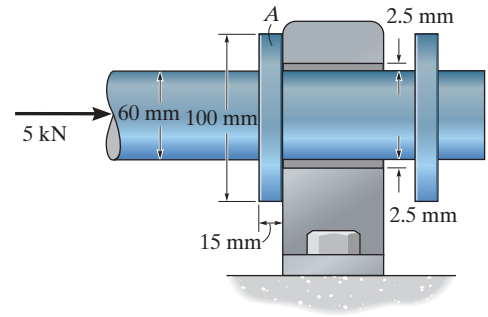


Ans:

$$\sigma = \frac{m\omega^2}{8A} (L^2 - 4x^2)$$

*1-60.

If the shaft is subjected to an axial force of 5 kN, determine the bearing stress acting on the collar A.



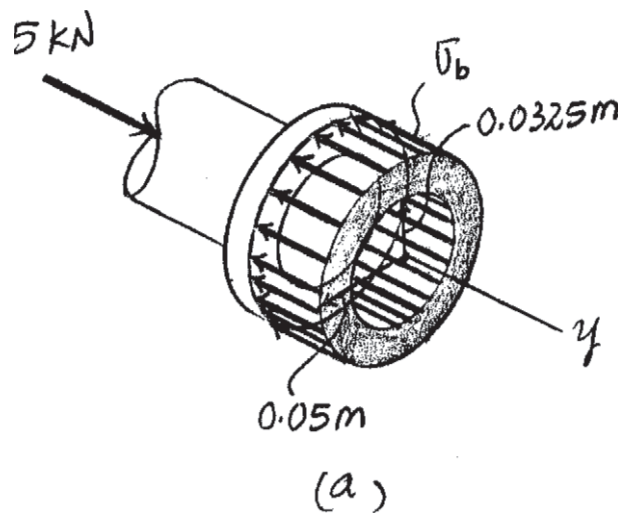
SOLUTION

Bearing Stress: The bearing area on the collar, shown shaded in Fig. *a*, is $A_b = \pi(0.05^2 - 0.0325^2) = 4.536(10^{-3}) \text{ m}^2$. Referring to the free-body diagram of the collar, Fig. *a*, and writing the force equation of equilibrium along the axis of the shaft,

$$\Sigma F_y = 0; \quad 5(10^3) - \sigma_b[4.536(10^{-3})] = 0$$

$$\sigma_b = 1.10 \text{ MPa}$$

Ans.

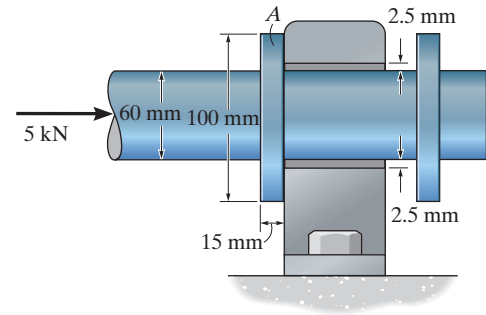


Ans:

$$\sigma_b = 1.10 \text{ MPa}$$

1-61.

If the 60-mm diameter shaft is subjected to an axial force of 5 kN, determine the average shear stress developed in the shear plane where the collar *A* and shaft are connected.



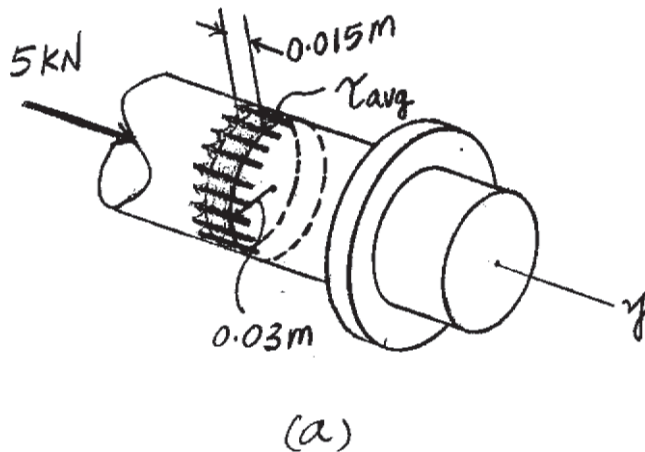
SOLUTION

Average Shear Stress: The area of the shear plane, shown shaded in Fig. *a*, is $A = 2\pi(0.03)(0.015) = 2.827(10^{-3})\text{m}^2$. Referring to the free-body diagram of the shaft, Fig. *a*, and writing the force equation of equilibrium along the axis of the shaft,

$$\Sigma F_y = 0; 5(10^3) - \tau_{\text{avg}}[2.827(10^{-3})] = 0$$

$$\tau_{\text{avg}} = 1.77 \text{ MPa}$$

Ans.

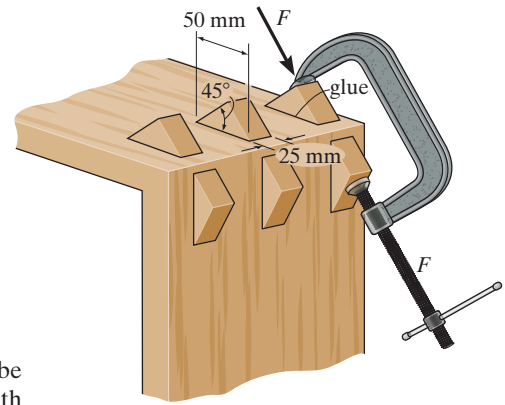


Ans:

$$\tau_{\text{avg}} = 1.77 \text{ MPa}$$

1-62.

The triangular blocks are glued along each side of the joint. A C-clamp placed between two of the blocks is used to draw the joint tight. If the glue can withstand a maximum average shear stress of 800 kPa, determine the maximum allowable clamping force F .



SOLUTION

Internal Loadings: The shear force developed on the glued shear plane can be obtained by writing the force equation of equilibrium along the x axis with reference to the free-body diagram of the triangular block, Fig. a .

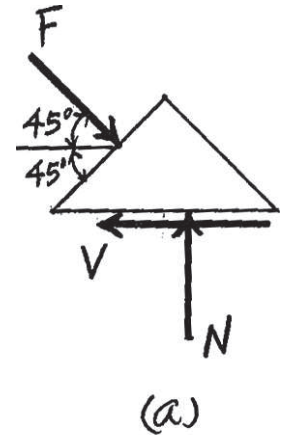
$$\rightarrow \Sigma F_x = 0; \quad F \cos 45^\circ - V = 0 \quad V = \frac{\sqrt{2}}{2} F$$

Average Normal and Shear Stress: The area of the glued shear plane is $A = 0.05(0.025) = 1.25(10^{-3})\text{m}^2$. We obtain

$$\tau_{\text{avg}} = \frac{V}{A}; \quad 800(10^3) = \frac{\frac{\sqrt{2}}{2} F}{1.25(10^{-3})}$$

$$F = 1414 \text{ N} = 1.41 \text{ kN}$$

Ans.

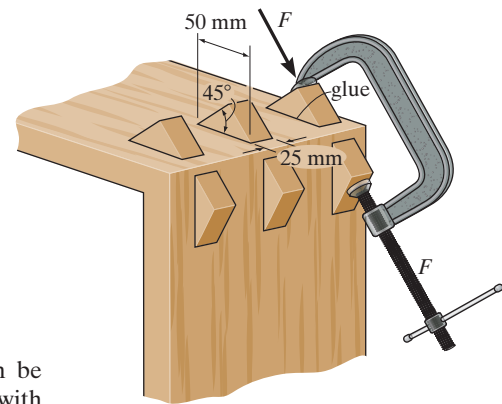


Ans:

$$F = 1.41 \text{ kN}$$

1-63.

The triangular blocks are glued along each side of the joint. A C-clamp placed between two of the blocks is used to draw the joint tight. If the clamping force is $F = 900 \text{ N}$, determine the average shear stress developed within the glued shear plane.



SOLUTION

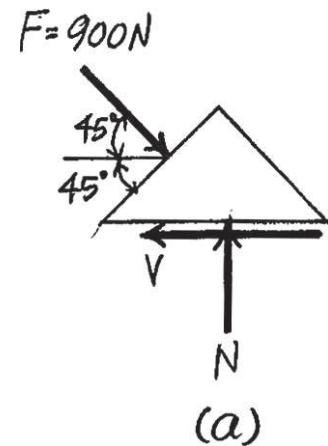
Internal Loadings: The shear force developed on the glued shear plane can be obtained by writing the force equation of equilibrium along the x axis with reference to the free-body diagram of the triangular block, Fig. a .

$$\rightarrow \Sigma F_x = 0; \quad 900 \cos 45^\circ - V = 0 \quad V = 636.40 \text{ N}$$

Average Normal and Shear Stress: The area of the glued shear plane is $A = 0.05(0.025) = 1.25(10^{-3}) \text{ m}^2$. We obtain

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{636.40}{1.25(10^{-3})} = 509 \text{ kPa}$$

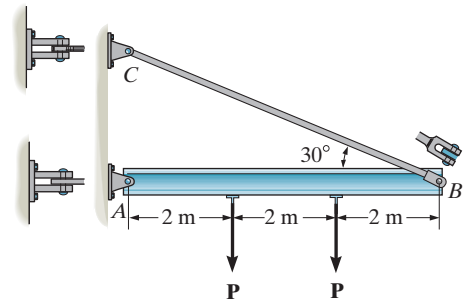
Ans.



Ans:
 $\tau_{\text{avg}} = 509 \text{ kPa}$

***1-64.**

If $P = 20$ kN, determine the average shear stress developed in the pins at A and C . All pins are subjected to double shear as shown, and each has a diameter of 18 mm.



SOLUTION

Referring to the FBD of member AB , Fig. a

$$\zeta + \Sigma M_A = 0; \quad F_{BC} \sin 30^\circ (6) - 20(2) - 20(4) = 0 \quad F_{BC} = 40 \text{ kN}$$

$$\rightarrow \Sigma F_x = 0; \quad A_x - 40 \cos 30^\circ = 0 \quad A_x = 34.64 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 20 - 20 + 40 \sin 30^\circ \quad A_y = 20 \text{ kN}$$

Thus, the force acting on pin A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{34.64^2 + 20^2} = 40 \text{ kN}$$

Pins A and C are subjected to double shear. Referring to their FBDs in Figs. b and c ,

$$V_A = \frac{F_A}{2} = \frac{40}{2} = 20 \text{ kN} \quad V_C = \frac{F_{BC}}{2} = \frac{40}{2} = 20 \text{ kN}$$

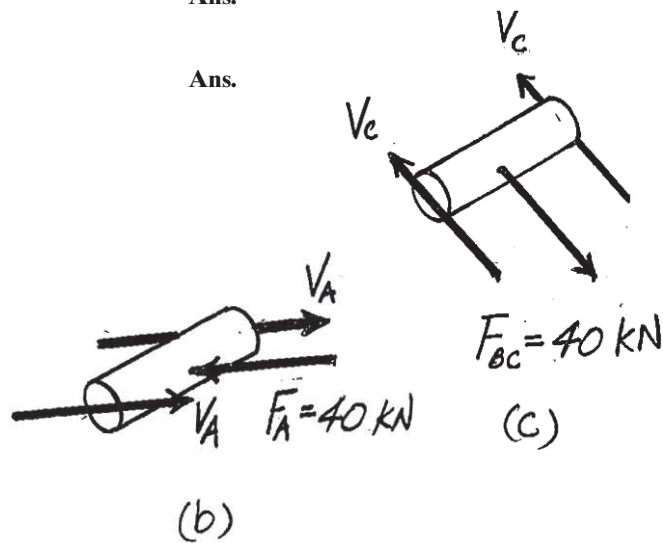
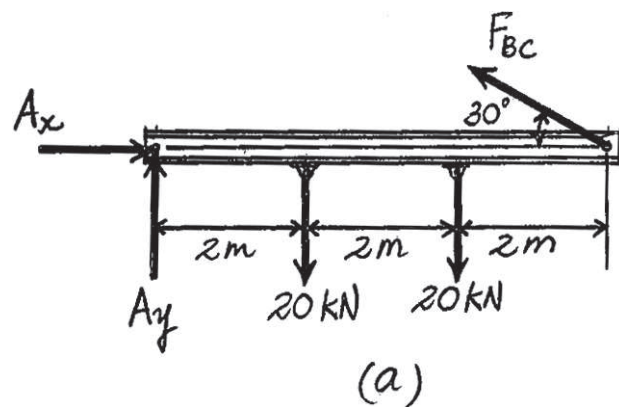
The cross-sectional area of Pins A and C are $A_A = A_C = \frac{\pi}{4} (0.018^2) = 81(10^{-6})\pi \text{ m}^2$. Thus

$$\tau_A = \frac{V_A}{A_A} = \frac{20(10^3)}{81(10^{-6})\pi} = 78.59(10^6) \text{ Pa} = 78.6 \text{ MPa}$$

Ans.

$$\tau_C = \frac{V_C}{A_C} = \frac{20(10^3)}{81(10^{-6})\pi} = 78.59(10^6) \text{ Pa} = 78.6 \text{ MPa}$$

Ans.



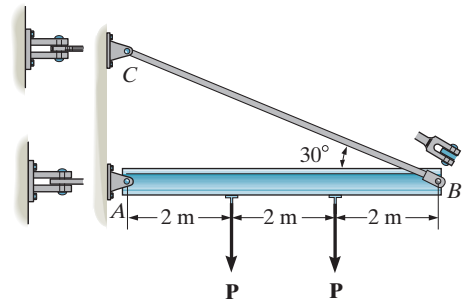
Ans:

$$\tau_A = 78.6 \text{ MPa}$$

$$\tau_C = 78.6 \text{ MPa}$$

1-65.

Determine the maximum magnitude P of the load the beam will support if the average shear stress in each pin is not allowed to exceed 60 MPa. All pins are subjected to double shear as shown, and each has a diameter of 18 mm.



SOLUTION

Referring to the FBD of member AB , Fig. a ,

$$\zeta + \Sigma M_A = 0; \quad F_{BC} \sin 30^\circ (6) - P(2) - P(4) = 0 \quad F_{BC} = 2P$$

$$\rightarrow \Sigma F_x = 0; \quad A_x - 2P \cos 30^\circ = 0 \quad A_x = 1.732P$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - P - P + 2P \sin 30^\circ = 0 \quad A_y = P$$

Thus, the force acting on pin A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(1.732P)^2 + P^2} = 2P$$

All pins are subjected to same force and double shear. Referring to the FBD of the pin, Fig. b ,

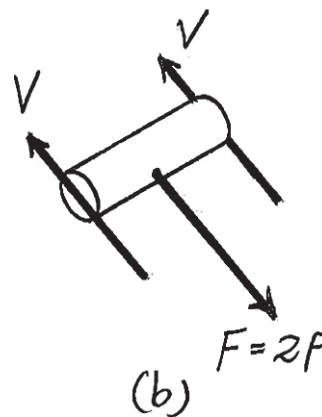
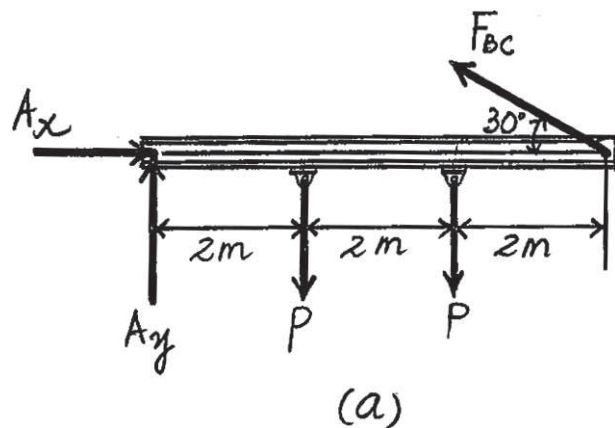
$$V = \frac{F}{2} = \frac{2P}{2} = P$$

The cross-sectional area of the pin is $A = \frac{\pi}{4} (0.018^2) = 81.0(10^{-6})\pi \text{ m}^2$. Thus,

$$\tau_{\text{allow}} = \frac{V}{A}; \quad 60(10^6) = \frac{P}{81.0(10^{-6})\pi}$$

$$P = 15\,268 \text{ N} = 15.3 \text{ kN}$$

Ans.

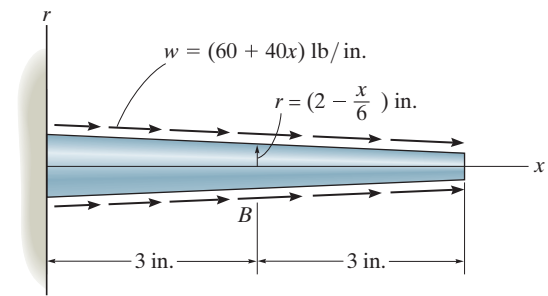


Ans:

$$P = 15.3 \text{ kN}$$

1-66.

The tapered rod has a radius of $r = (2 - x/6)$ in. and is subjected to the distributed loading of $w = (60 + 40x)$ lb/in. Determine the average normal stress at the center of the rod, B .



SOLUTION

$$A = \pi \left(2 - \frac{3}{6} \right)^2 = 7.069 \text{ in}^2$$

$$\Sigma F_x = 0; \quad N - \int_3^6 (60 + 40x) dx = 0; \quad N = 720 \text{ lb}$$

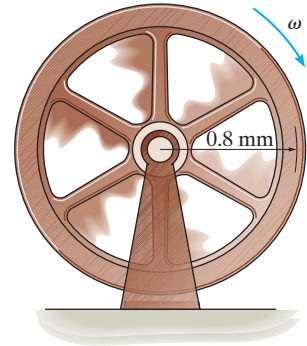
$$\sigma = \frac{720}{7.069} = 102 \text{ psi}$$

Ans.

Ans:
 $\sigma = 102 \text{ psi}$

1–67.

Determine the greatest constant angular velocity ω of the flywheel so that the average normal stress in its rim does not exceed $\sigma = 15 \text{ MPa}$. Assume the rim is a thin ring having a thickness of 3 mm, width of 20 mm, and a mass of 30 kg/m. Rotation occurs in the horizontal plane. Neglect the effect of the spokes in the analysis. *Hint:* Consider a free-body diagram of a semicircular segment of the ring. The center of mass for this segment is located at $\bar{r} = 2r/\pi$ from the center.



SOLUTION

$$+\downarrow \Sigma F_n = m(a_G)_n;$$

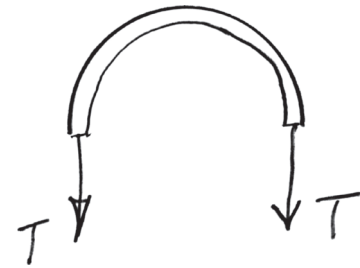
$$2T = m(\bar{r})\omega^2$$

$$2\sigma A = m\left(\frac{2r}{\pi}\right)\omega^2$$

$$2(15(10^6))(0.003)(0.020) = \pi(0.8)(30)\left(\frac{2(0.8)}{\pi}\right)\omega^2$$

$$\omega = 6.85 \text{ rad/s}$$

Ans.

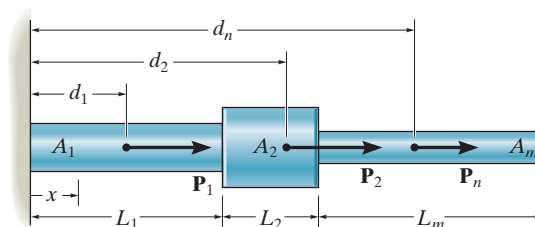


Ans:

$$\omega = 6.85 \text{ rad/s}$$

***1-68.**

Consider the general problem of a bar made from m segments, each having a constant cross-sectional area A_m and length L_m . If there are n loads on the bar as shown, write a computer program that can be used to determine the average normal stress at any specified location x . Show an application of the program using the values $L_1 = 4$ ft, $d_1 = 2$ ft, $P_1 = 400$ lb, $A_1 = 3$ in², $L_2 = 2$ ft, $d_2 = 6$ ft, $P_2 = -300$ lb, $A_2 = 1$ in².



SOLUTION

N/A

1-69.

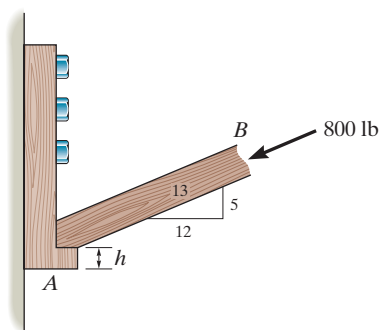
If A and B are both made of wood and are $\frac{3}{8}$ in. thick, determine to the nearest $\frac{1}{4}$ in. the smallest dimension h of the vertical segment so that it does not fail in shear. The allowable shear stress for the segment is $\tau_{\text{allow}} = 300$ psi.

SOLUTION

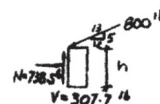
$$\tau_{\text{allow}} = 300 = \frac{307.7}{\left(\frac{3}{8}\right)h}$$

$$h = 2.74 \text{ in.}$$

$$\text{Use } h = 2\frac{3}{4} \text{ in.}$$



Ans.

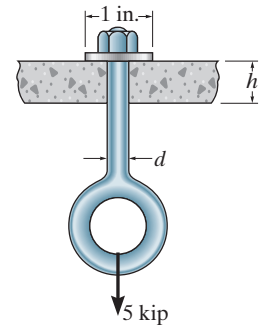


Ans:

$$\text{Use } h = 2\frac{3}{4} \text{ in.}$$

1–70.

The eye bolt is used to support the load of 5 kip. Determine its diameter d to the nearest $\frac{1}{8}$ in. and the required thickness h to the nearest $\frac{1}{8}$ in. of the support so that the washer will not penetrate or shear through it. The allowable normal stress for the bolt is $\sigma_{\text{allow}} = 21$ ksi and the allowable shear stress for the supporting material is $\tau_{\text{allow}} = 5$ ksi.



SOLUTION

Allowable Normal Stress: Design of bolt size

$$\sigma_{\text{allow}} = \frac{P}{A_b}; \quad 21.0(10^3) = \frac{5(10^3)}{\frac{\pi}{4}d^2}$$

$$d = 0.5506 \text{ in.}$$

$$\text{Use } d = \frac{5}{8} \text{ in.}$$

Ans.

Allowable Shear Stress: Design of support thickness

$$\tau_{\text{allow}} = \frac{V}{A}; \quad 5(10^3) = \frac{5(10^3)}{\pi(1)(h)}$$

$$\text{Use } h = \frac{3}{8} \text{ in.}$$

Ans.

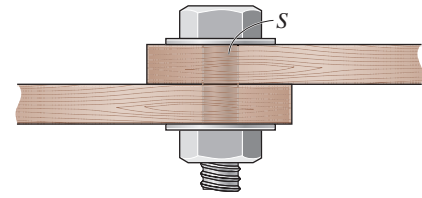
Ans:

$$d = \frac{5}{8} \text{ in.}$$

$$h = \frac{3}{8} \text{ in.}$$

1-71.

The connection is made using a bolt and nut and two washers. If the allowable bearing stress of the washers on the boards is $(\sigma_b)_{\text{allow}} = 2 \text{ ksi}$, and the allowable tensile stress within the bolt shank S is $(\sigma_t)_{\text{allow}} = 18 \text{ ksi}$, determine the maximum allowable tension in the bolt shank. The bolt shank has a diameter of 0.31 in., and the washers have an outer diameter (hole) of 0.75 in. and inner diameter (hole) of 0.50 in.



SOLUTION

Allowable Normal Stress: Assume tension failure

$$\sigma_{\text{allow}} = \frac{P}{A}; \quad 18 = \frac{P}{\frac{\pi}{4}(0.31^2)}$$

$$P = 1.36 \text{ kip}$$

Allowable Bearing Stress: Assume bearing failure

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \quad 2 = \frac{P}{\frac{\pi}{4}(0.75^2 - 0.50^2)}$$

$$P = 0.491 \text{ kip (Controls)}$$

Ans.

Ans:

$$P = 1.36 \text{ kip}$$

$$P = 0.491 \text{ kip (Controls)}$$

*1-72.

Determine the required cross-sectional area of member BC if the allowable normal stress is $\sigma_{\text{allow}} = 24 \text{ ksi}$.

SOLUTION

$$\zeta + \Sigma M_A = 0; \quad -400(6) - 800(8.485) + 2(8.485)(D_y) = 0$$

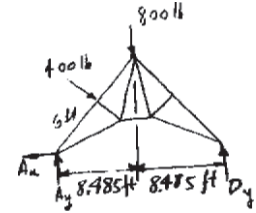
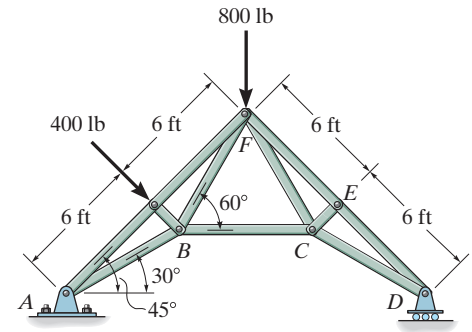
$$D_y = 541.42 \text{ lb}$$

$$\zeta + \Sigma M_F = 0; \quad 541.42(8.485) - F_{BC}(5.379) = 0$$

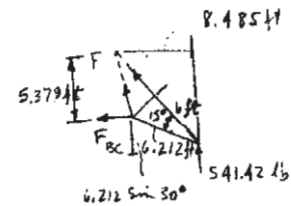
$$F_{BC} = 854.01 \text{ lb}$$

$$\sigma = \frac{P}{A}; \quad 24000 = \frac{854.01}{A}$$

$$A = 0.0356 \text{ in}^2$$



Ans.

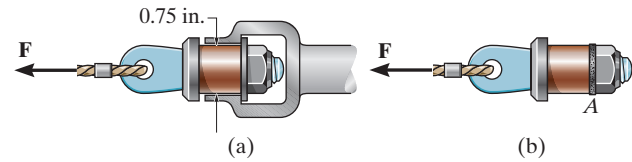


Ans:

$$A = 0.0356 \text{ in}^2$$

1-73.

The steel swivel bushing is held in place using a nut and washer as shown in Fig. (a). Failure of the washer *A* can cause the push rod to separate as shown in Fig. (b). If the maximum average shear stress is $\tau_{\max} = 21$ ksi, determine the force *F* that must be applied to the bushing. The washer is $\frac{1}{16}$ in. thick.



SOLUTION

$$\tau_{\text{avg}} = \frac{V}{A}; \quad 21(10^3) = \frac{F}{2\pi(0.375)(\frac{1}{16})}$$

$$F = 3092.5 \text{ lb} = 3.09 \text{ kip}$$

Ans.

Ans:
 $F = 3.09 \text{ kip}$

1-74.

The spring mechanism is used as a shock absorber for a load applied to the drawbar AB . Determine the force in each spring when the 50-kN force is applied. Each spring is originally unstretched and the drawbar slides along the smooth guide posts CG and EF . The ends of all springs are attached to their respective members. Also, what is the required diameter of the shank of bolts CG and EF if the allowable stress for the bolts is $\sigma_{\text{allow}} = 150 \text{ MPa}$?

SOLUTION

Equations of Equilibrium:

$$\zeta + \Sigma M_H = 0; \quad -F_{BF}(200) + F_{AG}(200) = 0$$

$$F_{BF} = F_{AG} = F$$

$$+\uparrow \Sigma F_y = 0; \quad 2F + F_H - 50 = 0 \quad (1)$$

Required,

$$\Delta_H = \Delta_B; \quad \frac{F_H}{80} = \frac{F}{60}$$

$$F = 0.75 F_H \quad (2)$$

Solving Eqs. (1) and (2) yields,

$$F_H = 20.0 \text{ kN}$$

$$F_{BF} = F_{AG} = F = 15.0 \text{ kN}$$

Allowable Normal Stress: Design of bolt shank size.

$$\sigma_{\text{allow}} = \frac{P}{A}; \quad 150(10^6) = \frac{15.0(10^3)}{\frac{\pi}{4}d^2}$$

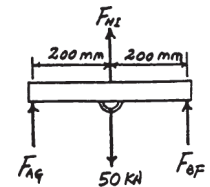
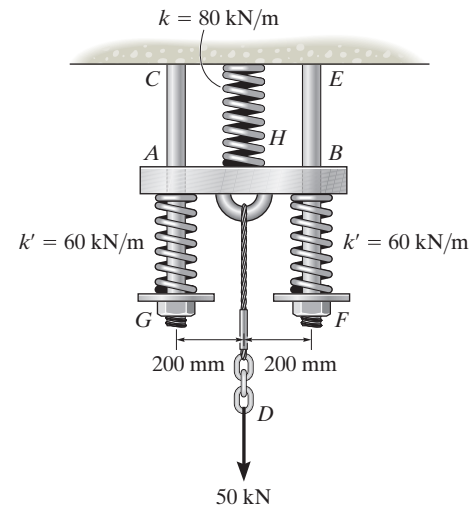
$$d = 0.01128 \text{ m} = 11.3 \text{ mm}$$

$$d_{EF} = d_{CG} = 11.3 \text{ mm}$$

Ans.

Ans.

Ans.



Ans:

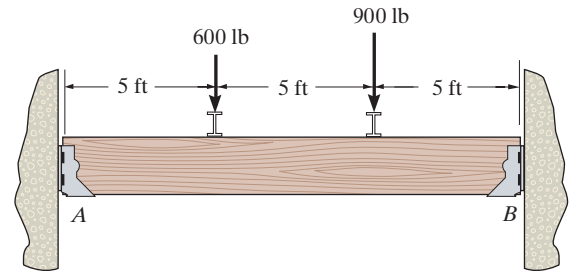
$$F_H = 20.0 \text{ kN},$$

$$F_{BF} = F_{AG} = 15.0 \text{ kN},$$

$$d_{EF} = d_{CG} = 11.3 \text{ mm}$$

1-75.

The hangers support the joist in such a way that the four nails on each hanger can be assumed to support an equal portion of the load. If the joist is subjected to the loading shown, determine the average shear stress in each nail of the hanger at ends *A* and *B*. Each nail has a diameter of $\frac{3}{16}$ in. The hangers only support vertical loads.



SOLUTION

Support Reactions: Referring to the FBD of the beam, Fig *a*,

$$\zeta + \Sigma M_A = 0; \quad B_y(15) - 600(5) - 900(10) = 0 \quad B_y = 800 \text{ lb}$$

$$\zeta + \Sigma M_B = 0; \quad 900(5) + 600(10) - A_y(15) = 0 \quad A_y = 700 \text{ lb}$$

Average Shear Stress: Since each of the vertical reactions A_y and B_y are supported equally by four nails, the shear force supported by each nail at hangers *A* and *B* are

$$V_A = \frac{A_y}{4} = \frac{700}{4} = 175 \text{ lb} \quad V_B = \frac{B_y}{4} = \frac{800}{4} = 200 \text{ lb}$$

The area of shear plane for nails at *A* and *B* are

$$A_A = A_B = \frac{\pi}{4} \left(\frac{3}{16} \right)^2 = 0.02761 \text{ in}^2$$

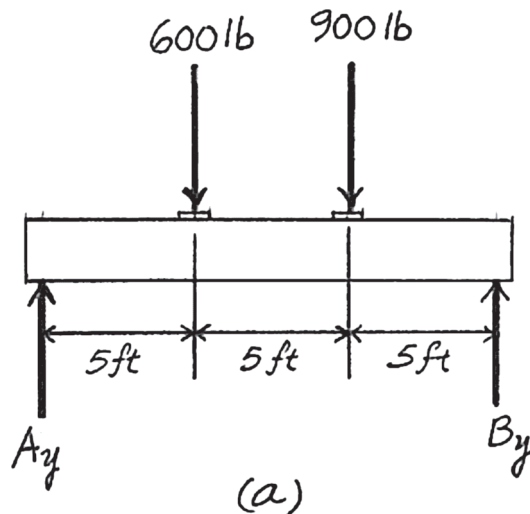
Then

$$(\tau_{\text{avg}})_A = \frac{V_A}{A_A} = \frac{175}{0.02761} = 6.338(10^3) \text{ psi} = 6.34 \text{ ksi}$$

Ans.

$$(\tau_{\text{avg}})_B = \frac{V_B}{A_B} = \frac{200}{0.02761} = 7.243(10^3) \text{ psi} = 7.24 \text{ ksi}$$

Ans.



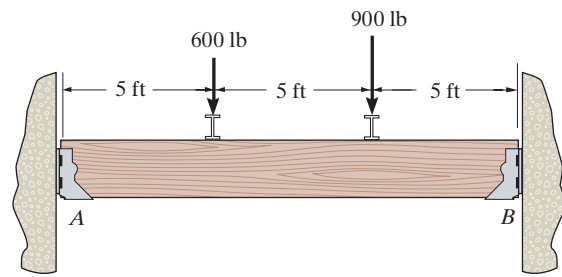
Ans:

$$(\tau_{\text{avg}})_A = 6.34 \text{ ksi}$$

$$(\tau_{\text{avg}})_B = 7.24 \text{ ksi}$$

***1-76.**

The hangers support the joist in such a way that the four nails on each hanger can be assumed to support an equal portion of the load. Determine the smallest diameter of the nails at A and B to the nearest $\frac{1}{16}$ in. if the allowable shear stress for the nails is $\tau_{\text{allow}} = 6.5$ ksi. The hangers only support vertical loads.



SOLUTION

Support Reactions: Referring to the FBD of the beam, Fig a ,

$$\zeta + \Sigma M_A = 0; \quad B_y(15) - 600(5) - 900(10) = 0 \quad B_y = 800 \text{ lb}$$

$$\zeta + \Sigma M_B = 0; \quad 900(5) + 600(10) - A_y(15) = 0 \quad A_y = 700 \text{ lb}$$

Allowable Shear Stress: Since each of the vertical reactions A_y and B_y are supported equally by four nails, the shear force supported by each nail at hangers A and B are

$$V_A = \frac{A_y}{4} = \frac{700}{4} = 175 \text{ lb} \quad V_B = \frac{B_y}{4} = \frac{800}{4} = 200 \text{ lb}$$

Then

$$(\tau_{\text{avg}})_{\text{allow}} = \frac{V_A}{A_A}; \quad 6.5(10^3) = \frac{175}{\frac{\pi}{4}d_A^2} \quad d_A = 0.1851 \text{ in.}$$

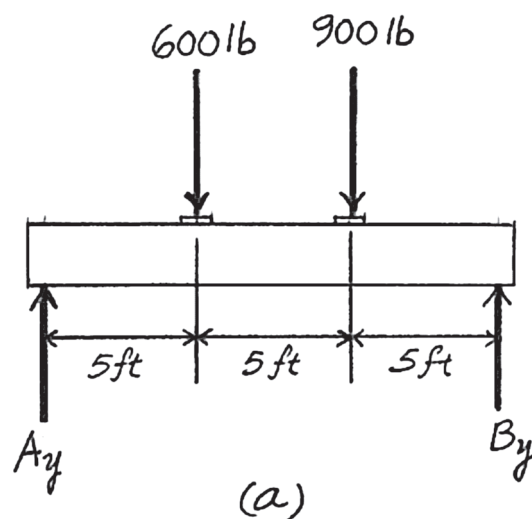
$$\text{use } d_A = \frac{3}{16} \text{ in.}$$

Ans.

$$(\tau_{\text{avg}})_{\text{allow}} = \frac{V_B}{A_B}; \quad 6.5(10^3) = \frac{200}{\frac{\pi}{4}d_B^2} \quad d_B = 0.1979 \text{ in.}$$

$$\text{use } d_B = \frac{1}{4} \text{ in.}$$

Ans.



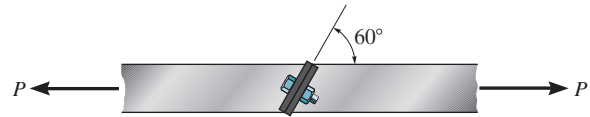
Ans:

$$d_A = \frac{3}{16} \text{ in.}$$

$$d_B = \frac{1}{4} \text{ in.}$$

1-77.

The tension member is fastened together using *two* bolts, one on each side of the member. Each bolt has a diameter of 0.3 in. Determine the maximum load P that can be applied to the member if the allowable shear stress for a bolt is $\tau_{\text{allow}} = 12$ ksi and the allowable average normal stress is $\sigma_{\text{allow}} = 20$ ksi.



SOLUTION

$$\uparrow + \Sigma F_y = 0; \quad N - P \sin 60^\circ = 0$$

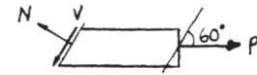
$$P = 1.1547 N$$

(1)

$$\rightarrow + \Sigma F_x = 0; \quad V - P \cos 60^\circ = 0$$

$$P = 2 V$$

(2)



Assume failure due to shear:

$$\tau_{\text{allow}} = 12 = \frac{V}{(2) \frac{\pi}{4} (0.3)^2}$$

$$V = 1.696 \text{ kip}$$

From Eq. (2),

$$P = 3.39 \text{ kip}$$

Assume failure due to normal force:

$$\sigma_{\text{allow}} = 20 = \frac{N}{(2) \frac{\pi}{4} (0.3)^2}$$

$$N = 2.827 \text{ kip}$$

From Eq. (1),

$$P = 3.26 \text{ kip} \quad (\text{Controls})$$

Ans.

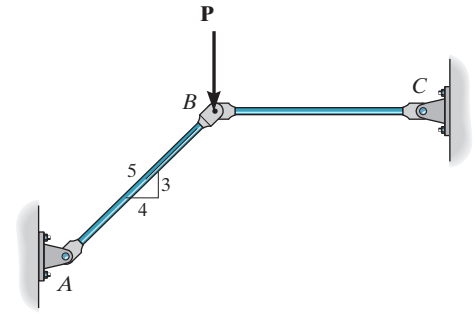
Ans:

$$P = 3.39 \text{ kip}$$

$$P = 3.26 \text{ kip} \quad (\text{Controls})$$

1-78.

The two aluminum rods support the vertical force of $P = 60 \text{ kN}$. Determine their required diameters if the failure stress for the aluminum is $\sigma_{\text{fail}} = 400 \text{ MPa}$. Use F.S. = 2.



SOLUTION

Internal Loadings: The normal force developed in members AB and BC can be determined by considering the equilibrium of joint B , Fig a .

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} \left(\frac{3}{5} \right) - 60 = 0 \quad F_{AB} = 100 \text{ kN (C)}$$

$$+\rightarrow \Sigma F_x = 0; \quad 100 \left(\frac{4}{5} \right) - F_{BC} = 0 \quad F_{BC} = 80 \text{ kN (C)}$$

Allowable Normal Stress: Using the F.S. = 2,

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{\text{F.S.}} = \frac{400 \text{ MPa}}{2} = 200 \text{ MPa}$$

Then,

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \quad 200(10^6) = \frac{100(10^3)}{\frac{\pi}{4} d_{AB}^2}$$

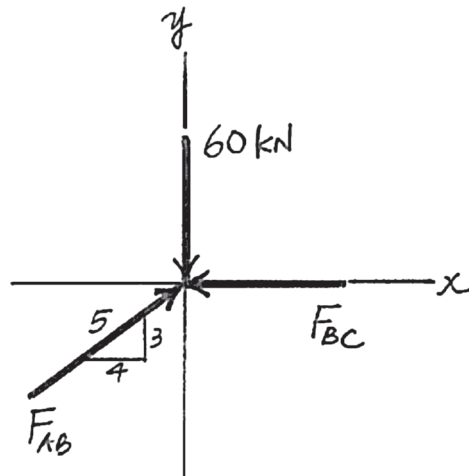
$$d_{AB} = 0.02523 \text{ m} = 25.2 \text{ mm}$$

Ans.

$$\sigma_{\text{allow}} = \frac{F_{BC}}{A_{BC}}; \quad 200(10^6) = \frac{80(10^3)}{\frac{\pi}{4} d_{BC}^2}$$

$$d_{BC} = 0.02257 \text{ m} = 22.6 \text{ mm}$$

Ans.



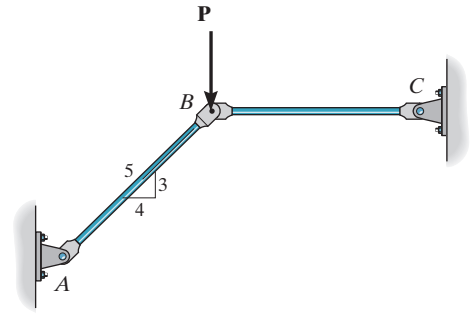
Ans:

$$d_{AB} = 25.2 \text{ mm}$$

$$d_{BC} = 22.6 \text{ mm}$$

1-79.

The two aluminum rods AB and BC have diameters of 25 and 20 mm, respectively. Determine the largest vertical force P that can be supported. The failure stress for the aluminum is $\sigma_{\text{fail}} = 400 \text{ MPa}$. Use F.S. = 2.



SOLUTION

Internal Loadings: The normal force developed in members AB and BC can be determined by considering the equilibrium of joint B , Fig a .

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} \left(\frac{3}{5} \right) - P = 0 \quad F_{AB} = \frac{5}{3} P \quad (C)$$

$$+\rightarrow \Sigma F_x = 0; \quad \left(\frac{5}{3} P \right) \left(\frac{4}{5} \right) - F_{BC} = 0 \quad F_{BC} = \frac{4}{3} P \quad (C)$$

Allowable Normal Stress: Using the F.S. = 2,

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{\text{F.S.}} = \frac{400 \text{ MPa}}{2} = 200 \text{ MPa}$$

The cross-sectional area of members AB and BC are

$$A_{AB} = \frac{\pi}{4} (0.025^2) = 0.15625(10^{-3}) \pi \text{ m}^2 \quad A_{BC} = \frac{\pi}{4} (0.02^2) = 0.1(10^{-3}) \pi \text{ m}^2$$

Then,

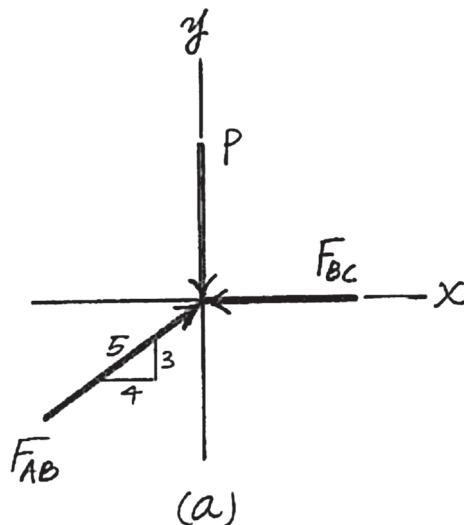
$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \quad 200(10^6) = \frac{\frac{5}{3} P}{0.15625(10^{-3}) \pi}$$

$$P = 58.90(10^3) \text{ N} = 58.90 \text{ kN}$$

$$\sigma_{\text{allow}} = \frac{F_{BC}}{A_{BC}}; \quad 200(10^6) = \frac{\frac{4}{3} P}{0.1(10^{-3}) \pi}$$

$$P = 47.12(10^3) \text{ N} = 47.1 \text{ kN (Controls)}$$

Ans.



Ans:

$$P = 58.90 \text{ kN}$$

$$P = 47.1 \text{ kN (Controls)}$$

*1-80.

The cotter pin is used to hold the two rods together. Determine the smallest thickness t of the pin and the smallest diameter d of the rods. All parts are made of steel for which the failure normal stress is $\sigma_{\text{fail}} = 500 \text{ MPa}$ and the failure shear stress is $\tau_{\text{fail}} = 375 \text{ MPa}$. Use a factor of safety of $(\text{F.S.})_t = 2.50$ in tension and $(\text{F.S.})_s = 1.75$ in shear.

SOLUTION

Allowable Normal Stress: Design of rod size

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{\text{F.S.}} = \frac{P}{A}; \quad \frac{500(10^6)}{2.5} = \frac{30(10^3)}{\frac{\pi}{4}d^2}$$

$$d = 0.01382 \text{ m} = 13.8 \text{ mm}$$

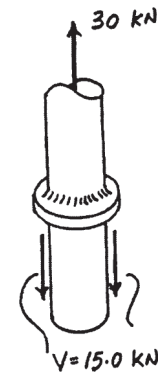
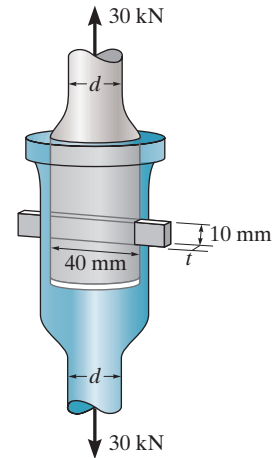
Ans.

Allowable Shear Stress: Design of cotter size.

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S.}} = \frac{V}{A}; \quad \frac{375(10^6)}{1.75} = \frac{15.0(10^3)}{(0.01)t}$$

$$t = 0.0070 \text{ m} = 7.00 \text{ mm}$$

Ans.



Ans:

$$d = 13.8 \text{ mm},$$

$$t = 7.00 \text{ mm}$$

1-81.

The 200-mm-diameter aluminum cylinder supports a compressive load of 300 kN. Determine the average normal and shear stress acting on section $a-a$. Show the results on a differential element located on the section.

SOLUTION

Referring to the FBD of the upper segment of the cylinder sectional through $a-a$ shown in Fig. a ,

$$+\nearrow \Sigma F_x = 0; \quad N_{a-a} - 300 \cos 30^\circ = 0 \quad N_{a-a} = 259.81 \text{ kN}$$

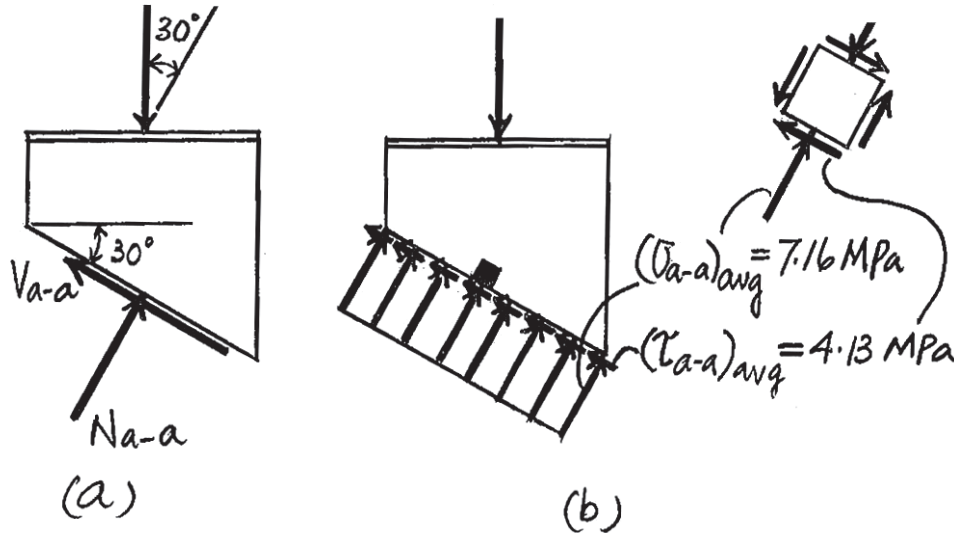
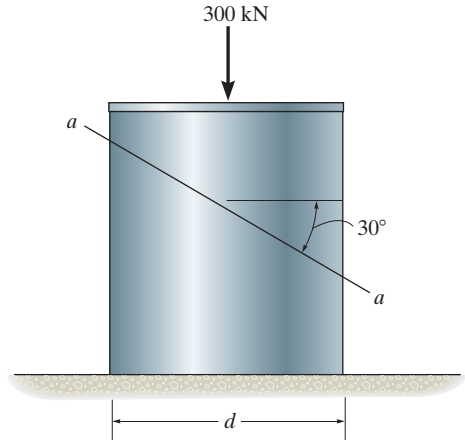
$$+\searrow \Sigma F_y = 0; \quad V_{a-a} - 300 \sin 30^\circ = 0 \quad V_{a-a} = 150 \text{ kN}$$

Section $a-a$ of the cylinder is an ellipse with $a = 0.1 \text{ m}$ and $b = \frac{0.1}{\cos 30^\circ} \text{ m}$. Thus,
 $A_{a-a} = \pi ab = \pi(0.1)\left(\frac{0.1}{\cos 30^\circ}\right) = 0.03628 \text{ m}^2$.

$$(\sigma_{a-a})_{\text{avg}} = \frac{N_{a-a}}{A_{a-a}} = \frac{259.81(10^3)}{0.03628} = 7.162(10^6) \text{ Pa} = 7.16 \text{ MPa} \quad \text{Ans.}$$

$$(\tau_{a-a})_{\text{avg}} = \frac{V_{a-a}}{A_{a-a}} = \frac{150(10^3)}{0.03628} = 4.135(10^6) \text{ Pa} = 4.13 \text{ MPa} \quad \text{Ans.}$$

The differential element representing the state of stress of a point on section $a-a$ is shown in Fig. b



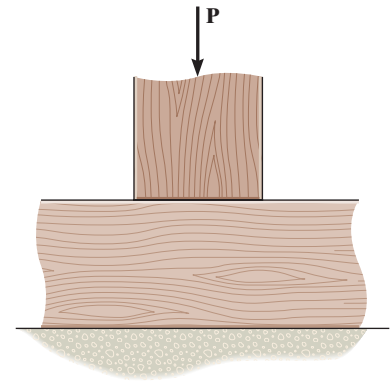
Ans:

$$(\sigma_{a-a})_{\text{avg}} = 7.16 \text{ MPa}$$

$$(\tau_{a-a})_{\text{avg}} = 4.13 \text{ MPa}$$

1–82.

The 60 mm × 60 mm oak post is supported on the pine block. If the allowable bearing stresses for these materials are $\sigma_{\text{oak}} = 43 \text{ MPa}$ and $\sigma_{\text{pine}} = 25 \text{ MPa}$, determine the greatest load P that can be supported. If a rigid bearing plate is used between these materials, determine its required area so that the maximum load P can be supported. What is this load?



SOLUTION

For failure of pine block:

$$\sigma = \frac{P}{A}; \quad 25(10^6) = \frac{P}{(0.06)(0.06)}$$

$$P = 90 \text{ kN}$$

Ans.

For failure of oak post:

$$\sigma = \frac{P}{A}; \quad 43(10^6) = \frac{P}{(0.06)(0.06)}$$

$$P = 154.8 \text{ kN}$$

Area of plate based on strength of pine block:

$$\sigma = \frac{P}{A}; \quad 25(10^6) = \frac{154.8(10^3)}{A}$$

$$A = 6.19(10^{-3}) \text{ m}^2$$

Ans.

$$P_{\text{max}} = 155 \text{ kN}$$

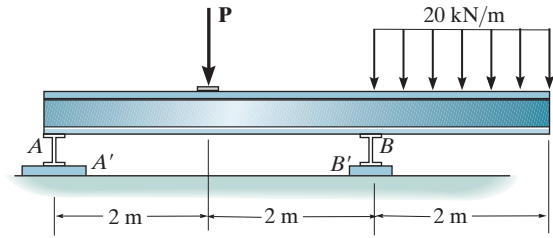
Ans.

Ans:

$$P = 90 \text{ kN}, A = 6.19(10^{-3}) \text{ m}^2, P_{\text{max}} = 155 \text{ kN}$$

1-83.

If the allowable bearing stress for the plates under the supports at A and B is $(\sigma_b)_{\text{allow}} = 1.8 \text{ MPa}$, determine the size of *square* bearing plates A' and B' required to support the load. Dimension the plates to the nearest mm. The reactions at the supports are vertical. Take $P = 150 \text{ kN}$.



SOLUTION

Support Reactions: The vertical force acting on bearing plates A' and B' can be determined by considering the equilibrium of the beam, Fig a .

$$\zeta + \Sigma M_A = 0; \quad F_B(4) - 150(2) - 40(5) = 0 \quad F_B = 125 \text{ kN}$$

$$\zeta + \Sigma M_B = 0; \quad 150(2) - 40(1) - F_A(4) = 0 \quad F_A = 65 \text{ kN}$$

Allowable Bearing Stress: For plate A' ,

$$(\sigma_b)_{\text{allow}} = \frac{F_A}{A_{A'}}; \quad 1.8(10^6) = \frac{65(10^3)}{a_{A'}^2}$$

$$a_{A'} = 0.1900 \text{ m} = 190 \text{ mm}$$

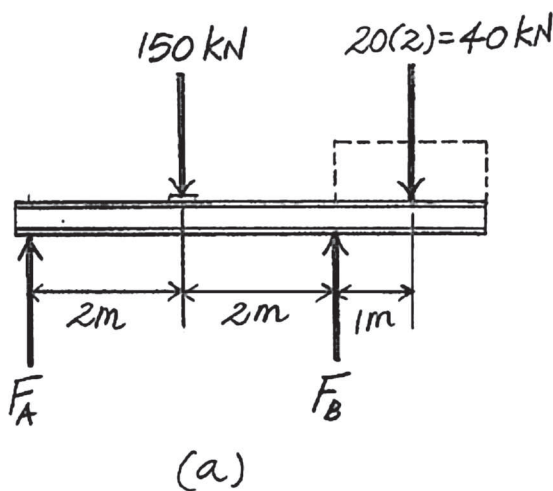
Ans.

For plate B' ,

$$(\sigma_b)_{\text{allow}} = \frac{F_B}{A_{B'}}; \quad 1.8(10^6) = \frac{125(10^3)}{a_{B'}^2}$$

$$a_{B'} = 0.2635 \text{ m} = 264 \text{ mm}$$

Ans.



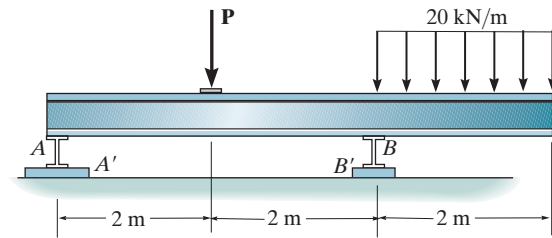
Ans:

$$a_{A'} = 190 \text{ mm}$$

$$a_{B'} = 264 \text{ mm}$$

***1-84.**

If the allowable bearing stress for the plates under the supports at A and B is $(\sigma_b)_{\text{allow}} = 1.8 \text{ MPa}$, determine the maximum load P that can be applied to the beam. The plates A' and B' have square cross sections of $200 \text{ mm} \times 200 \text{ mm}$ and $300 \text{ mm} \times 300 \text{ mm}$, respectively.



SOLUTION

Support Reactions: The vertical force acting on bearing plates A' and B' can be determined by considering the equilibrium of the beam, Fig. a .

$$\zeta + \sum M_A = 0; \quad F_B(4) - P(2) - 40(5) = 0 \quad F_B = 0.5P + 50$$

$$\zeta + \sum M_B = 0; \quad P(2) - 40(1) - F_A(4) = 0 \quad F_A = 0.5P - 10$$

Allowable Bearing Stress: For plate A' ,

$$\sigma_{\text{allow}} = \frac{F_A}{A_{A'}}; \quad 1.8(10^6) = \frac{(0.5P - 10)(10^3)}{0.2(0.2)}$$

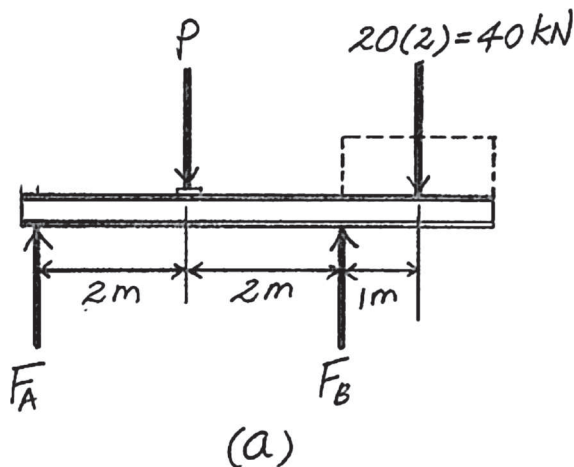
$$P = 164 \text{ kN (Controls)}$$

Ans.

For plate B' ,

$$\sigma_{\text{allow}} = \frac{F_B}{A_{B'}}; \quad 1.8(10^6) = \frac{(0.5P + 50)(10^3)}{0.3(0.3)}$$

$$P = 224 \text{ kN}$$



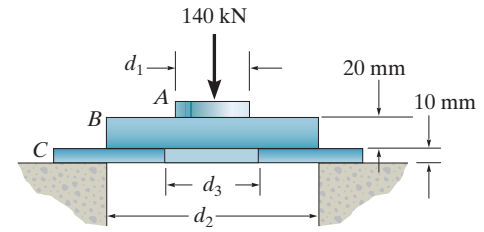
Ans:

$$P = 164 \text{ kN (Controls)}$$

$$P = 224 \text{ kN}$$

1-85.

The assembly consists of three disks *A*, *B*, and *C* that are used to support the load of 140 kN. Determine the smallest diameter d_1 of the top disk, the largest diameter d_2 of the opening, and the largest diameter d_3 of the hole in the bottom disk. The allowable bearing stress for the material is $(\sigma_b)_{\text{allow}} = 350 \text{ MPa}$ and allowable shear stress is $\tau_{\text{allow}} = 125 \text{ MPa}$.



SOLUTION

Allowable Shear Stress: Assume shear failure for disk *C*.

$$\tau_{\text{allow}} = \frac{V}{A}; \quad 125(10^6) = \frac{140(10^3)}{\pi d_2(0.01)}$$

$$d_2 = 0.03565 \text{ m} = 35.7 \text{ mm}$$

Ans.

Allowable Bearing Stress: Assume bearing failure for disk *C*.

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \quad 350(10^6) = \frac{140(10^3)}{\frac{\pi}{4}(0.03565^2 - d_3^2)}$$

$$d_3 = 0.02760 \text{ m} = 27.6 \text{ mm}$$

Ans.

Allowable Bearing Stress: Assume bearing failure for disk *B*.

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \quad 350(10^6) = \frac{140(10^3)}{\frac{\pi}{4}d_1^2}$$

$$d_1 = 0.02257 \text{ m} = 22.6 \text{ mm}$$

Since $d_3 = 27.6 \text{ mm} > d_1 = 22.6 \text{ mm}$, disk *B* might fail due to shear.

$$\tau = \frac{V}{A} = \frac{140(10^3)}{\pi(0.02257)(0.02)} = 98.7 \text{ MPa} < \tau_{\text{allow}} = 125 \text{ MPa} \text{ (OK)}$$

Therefore,

$$d_1 = 22.6 \text{ mm}$$

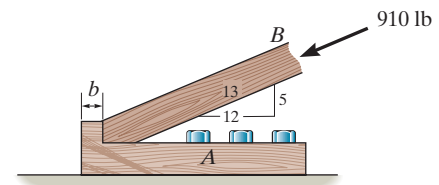
Ans.

Ans:

$$\begin{aligned} d_2 &= 35.7 \text{ mm}, \\ d_3 &= 27.6 \text{ mm}, \\ d_1 &= 22.6 \text{ mm} \end{aligned}$$

1-86.

Member B is subjected to a compressive force of 910 lb. If A and B are both made of wood and are $\frac{1}{2}$ in. thick, determine to the nearest $\frac{1}{8}$ in. the smallest dimension b of the horizontal segment so that it does not fail in shear. The allowable average shear stress for the segment is $\tau_{\text{allow}} = 275$ psi.



SOLUTION

Support Reactions: Referring to the FBD of member B , Fig a ,

$$\rightarrow \Sigma F_x = 0; \quad F - 910 \left(\frac{12}{13} \right) = 0 \quad F = 840 \text{ lb}$$

Allowable Average Shear Stress: Referring to the FBD of the horizontal segment, Fig. b ,

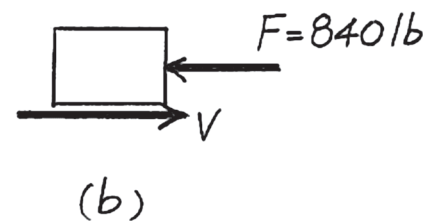
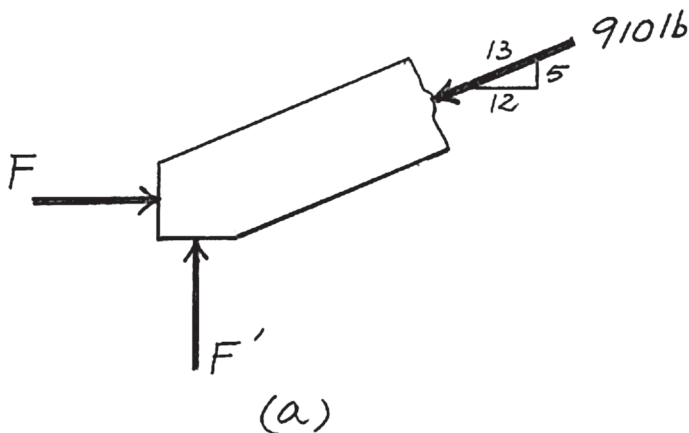
$$\rightarrow \Sigma F_x = 0; \quad V - 840 = 0 \quad V = 840 \text{ lb}$$

Then,

$$(\tau_{\text{avg}})_{\text{allow}} = \frac{V}{A}; \quad 275 = \frac{840}{b \left(\frac{1}{2} \right)} \quad b = 6.109 \text{ in.}$$

$$\text{use } b = 6 \frac{1}{8} \text{ in.}$$

Ans.

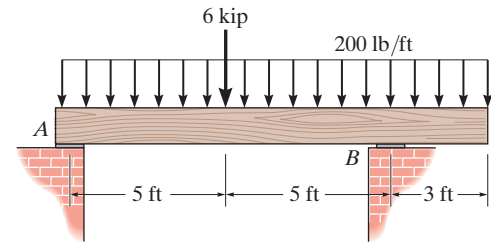


Ans:

$$b = 6 \frac{1}{8} \text{ in.}$$

1-87.

The beam is made from southern pine and is supported by base plates resting on brickwork. If the allowable bearing stresses for the materials are $(\sigma_{\text{pine}})_{\text{allow}} = 2.81 \text{ ksi}$ and $(\sigma_{\text{brick}})_{\text{allow}} = 6.70 \text{ ksi}$, determine the required length of the base plates at A and B to the nearest $\frac{1}{4}$ inch in order to support the load shown. The plates are 3 in. wide.



SOLUTION

The design must be based on strength of the pine.

At A :

$$\sigma = \frac{P}{A}; \quad 2810 = \frac{3910}{l_A(3)}$$

$$\text{Use } l_A = \frac{1}{2} \text{ in.} \quad l_A = 0.464 \text{ in.}$$

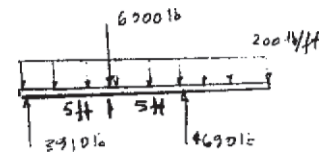
Ans.

At B :

$$\sigma = \frac{P}{A}; \quad 2810 = \frac{4690}{l_B(3)}$$

$$\text{Use } l_B = \frac{3}{4} \text{ in.} \quad l_B = 0.556 \text{ in.}$$

Ans.

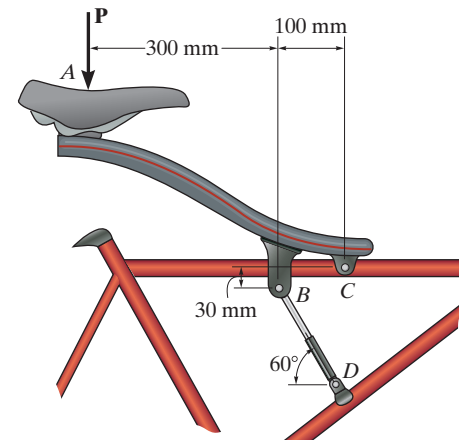


Ans:

$$l_A = \frac{1}{2} \text{ in.}, l_B = \frac{3}{4} \text{ in.}$$

***1-88.**

The soft-ride suspension system of the mountain bike is pinned at C and supported by the shock absorber BD . If it is designed to support a load $P = 1500$ N, determine the required minimum diameter of pins B and C . Use a factor of safety of 2 against failure. The pins are made of material having a failure shear stress of $\tau_{\text{fail}} = 150$ MPa, and each pin is subjected to double shear.



SOLUTION

Internal Loadings: The forces acting on pins B and C can be determined by considering the equilibrium of the free-body diagram of the soft-ride suspension system shown in Fig. a .

$$\zeta + \Sigma M_C = 0; \quad 1500(0.4) - F_{BD} \sin 60^\circ(0.1) - F_{BD} \cos 60^\circ(0.03) = 0$$

$$F_{BD} = 5905.36 \text{ N}$$

$$\rightarrow \Sigma F_x = 0; \quad C_x - 5905.36 \cos 60^\circ = 0 \quad C_x = 2952.68 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 5905.36 \sin 60^\circ - 1500 - C_y = 0 \quad C_y = 3614.20 \text{ N}$$

Thus,

$$F_B = F_{BD} = 5905.36 \text{ N} \quad F_C = \sqrt{C_x^2 + C_y^2} = \sqrt{2952.68^2 + 3614.20^2} = 4666.98 \text{ N}$$

Since **both** pins are in double shear,

$$V_B = \frac{F_B}{2} = \frac{5905.36}{2} = 2952.68 \text{ N} \quad V_C = \frac{F_C}{2} = \frac{4666.98}{2} = 2333.49 \text{ N}$$

Allowable Shear Stress:

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S.}} = \frac{150}{2} = 75 \text{ MPa}$$

Using this result,

$$\tau_{\text{allow}} = \frac{V_B}{A_B}; \quad 75(10^6) = \frac{2952.68}{\frac{\pi}{4} d_B^2}$$

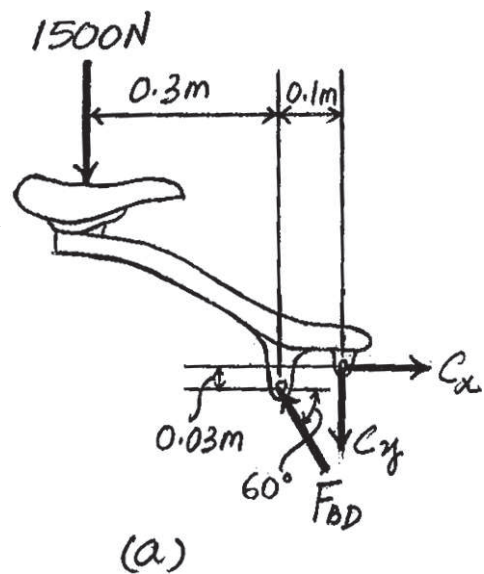
$$d_B = 0.007080 \text{ m} = 7.08 \text{ mm}$$

Ans.

$$\tau_{\text{allow}} = \frac{V_C}{A_C}; \quad 75(10^6) = \frac{2333.49}{\frac{\pi}{4} d_C^2}$$

$$d_C = 0.006294 \text{ m} = 6.29 \text{ mm}$$

Ans.



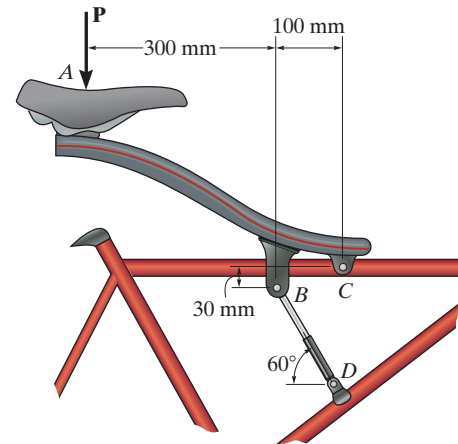
Ans:

$$d_B = 7.08 \text{ mm}$$

$$d_C = 6.29 \text{ mm}$$

1-89.

The soft-ride suspension system of the mountain bike is pinned at C and supported by the shock absorber BD . If it is designed to support a load of $P = 1500$ N, determine the factor of safety of pins B and C against failure if they are made of a material having a shear failure stress of $\tau_{\text{fail}} = 150$ MPa. Pin B has a diameter of 7.5 mm, and pin C has a diameter of 6.5 mm. Both pins are subjected to double shear.



SOLUTION

Internal Loadings: The forces acting on pins B and C can be determined by considering the equilibrium of the free-body diagram of the soft-ride suspension system shown in Fig. a .

$$+\sum M_C = 0; \quad 1500(0.4) - F_{BD} \sin 60^\circ(0.1) - F_{BD} \cos 60^\circ(0.03) = 0$$

$$F_{BD} = 5905.36 \text{ N}$$

$$+\rightarrow \sum F_x = 0; \quad C_x - 5905.36 \cos 60^\circ = 0 \quad C_x = 2952.68 \text{ N}$$

$$+\uparrow \sum F_y = 0; \quad 5905.36 \sin 60^\circ - 1500 - C_y = 0 \quad C_y = 3614.20 \text{ N}$$

Thus,

$$F_B = F_{BD} = 5905.36 \text{ N} \quad F_C = \sqrt{C_x^2 + C_y^2} = \sqrt{2952.68^2 + 3614.20^2} = 4666.98 \text{ N}$$

Since both pins are in double shear,

$$V_B = \frac{F_B}{2} = \frac{5905.36}{2} = 2952.68 \text{ N} \quad V_C = \frac{F_C}{2} = \frac{4666.98}{2} = 2333.49 \text{ N}$$

Allowable Shear Stress: The areas of the shear plane for pins B and C are

$$A_B = \frac{\pi}{4}(0.0075^2) = 44.179(10^{-6})\text{m}^2 \quad \text{and} \quad A_C = \frac{\pi}{4}(0.0065^2) = 33.183(10^{-6})\text{m}^2.$$

We obtain

$$(\tau_{\text{avg}})_B = \frac{V_B}{A_B} = \frac{2952.68}{44.179(10^{-6})} = 66.84 \text{ MPa}$$

$$(\tau_{\text{avg}})_C = \frac{V_C}{A_C} = \frac{2333.49}{33.183(10^{-6})} = 70.32 \text{ MPa}$$

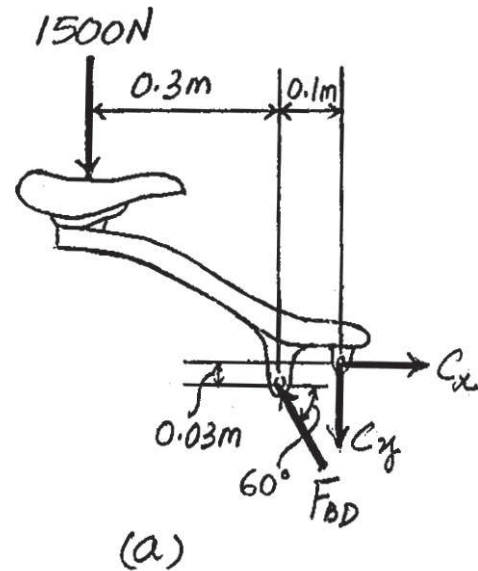
Using these results,

$$(\text{F.S.})_B = \frac{\tau_{\text{fail}}}{(\tau_{\text{avg}})_B} = \frac{150}{66.84} = 2.24$$

Ans.

$$(\text{F.S.})_C = \frac{\tau_{\text{fail}}}{(\tau_{\text{avg}})_C} = \frac{150}{70.32} = 2.13$$

Ans.



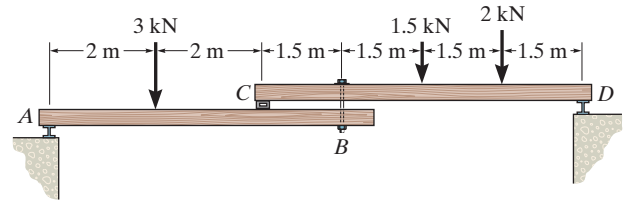
Ans:

$$(\text{F.S.})_B = 2.24$$

$$(\text{F.S.})_C = 2.13$$

1-90.

The compound wooden beam is connected together by a bolt at B . Assuming that the connections at A , B , C , and D exert only vertical forces on the members, determine the required diameter of the bolt at B and the required outer diameter of its washers if the allowable tensile stress for the bolt is $(\sigma_t)_{\text{allow}} = 150 \text{ MPa}$ and the allowable bearing stress for the wood is $(\sigma_b)_{\text{allow}} = 28 \text{ MPa}$. Assume that the hole in the washers has the same diameter as the bolt.



SOLUTION

From FBD (a):

$$\zeta + \Sigma M_D = 0; \quad F_B(4.5) + 1.5(3) + 2(1.5) - F_C(6) = 0$$

$$4.5 F_B - 6 F_C = -7.5 \quad (1)$$

From FBD (b):

$$\zeta + \Sigma M_A = 0; \quad F_B(5.5) - F_C(4) - 3(2) = 0$$

$$5.5 F_B - 4 F_C = 6 \quad (2)$$

Solving Eqs. (1) and (2) yields

$$F_B = 4.40 \text{ kN}; \quad F_C = 4.55 \text{ kN}$$

For bolt:

$$\sigma_{\text{allow}} = 150(10^6) = \frac{4.40(10^3)}{\frac{\pi}{4}(d_B)^2}$$

$$d_B = 0.00611 \text{ m}$$

$$= 6.11 \text{ mm}$$

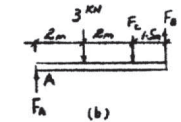
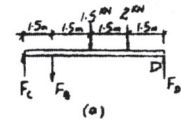
Ans.

For washer:

$$\sigma_{\text{allow}} = 28(10^4) = \frac{4.40(10^3)}{\frac{\pi}{4}(d_w^2 - 0.00611^2)}$$

$$d_w = 0.0154 \text{ m} = 15.4 \text{ mm}$$

Ans.



Ans:

$$d_B = 6.11 \text{ mm}$$

$$d_w = 15.4 \text{ mm}$$

1-91.

The 60-kg flowerpot is suspended from wires AB and BC . If the wires have a failure normal stress of $\sigma_{\text{fail}} = 380 \text{ MPa}$, determine the minimum diameter of each wire. Use a factor of safety of 2.

SOLUTION

Internal Loading: The normal force developed in cables AB and BC can be determined by considering the equilibrium of joint B , Fig a .

$$+\rightarrow \Sigma F_x = 0; \quad F_{BC} \cos 45^\circ - F_{AB} \cos 60^\circ = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC} \sin 45^\circ + F_{AB} \sin 60^\circ - 60(9.81) = 0 \quad (2)$$

Solving Eqs. (1) and (2),

$$F_{AB} = 430.89 \text{ N} \quad F_{BC} = 304.68 \text{ N}$$

Allowable Normal Stress: Using the F.S. = 2,

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{\text{F.S.}} = \frac{380 \text{ MPa}}{2} = 190 \text{ MPa}$$

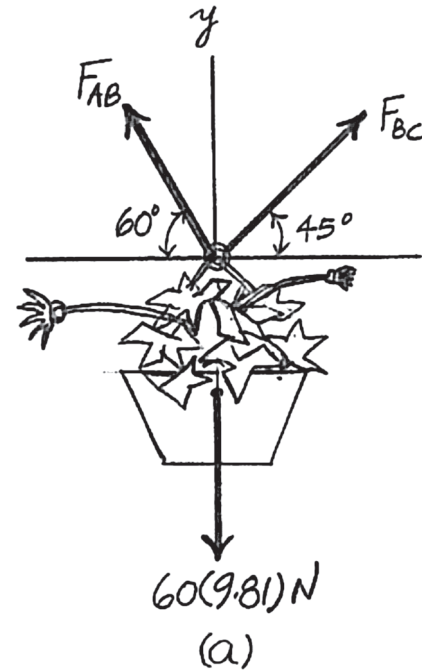
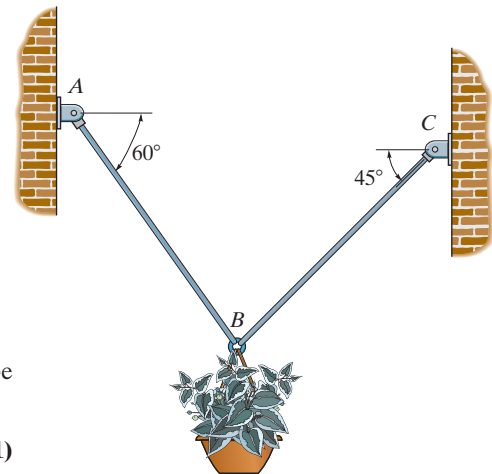
Then,

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \quad 190(10^6) = \frac{430.89}{\frac{\pi}{4} d_{AB}^2}$$

$$d_{AB} = 1.6993(10^{-3}) \text{ m} = 1.70 \text{ mm}$$

$$\sigma_{\text{allow}} = \frac{F_{BC}}{A_{BC}}; \quad 190(10^6) = \frac{304.68}{\frac{\pi}{4} d_{BC}^2}$$

$$d_{BC} = 1.4289(10^{-3}) \text{ m} = 1.43 \text{ mm}$$



Ans.

Ans.

Ans:

$$F_{AB} = 430.89 \text{ N}$$

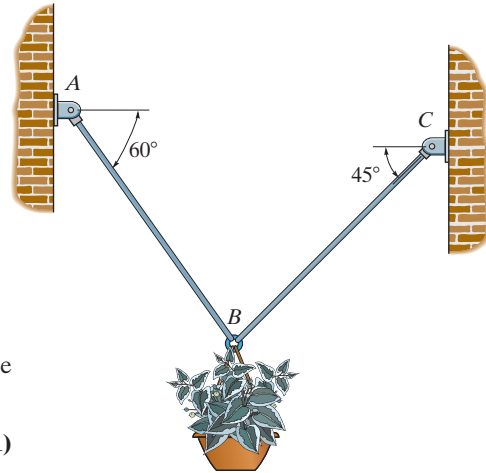
$$F_{BC} = 304.68 \text{ N}$$

$$d_{AB} = 1.70 \text{ mm}$$

$$d_{BC} = 1.43 \text{ mm}$$

***1-92.**

The 60-kg flowerpot is suspended from wires AB and BC which have diameters of 1.75 mm and 1.5 mm, respectively. If the wires have a failure normal stress of $\sigma_{\text{fail}} = 380$ MPa, determine the factor of safety of each wire.



SOLUTION

Internal Loading: The normal force developed in cables AB and BC can be determined by considering the equilibrium of joint B , Fig a .

$$\rightarrow \Sigma F_x = 0; \quad F_{BC} \cos 45^\circ - F_{AB} \cos 60^\circ = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC} \sin 45^\circ + F_{AB} \sin 60^\circ - 60(9.81) = 0 \quad (2)$$

Solving Eqs. (1) and (2),

$$F_{AB} = 430.89 \text{ N} \quad F_{BC} = 304.68 \text{ N}$$

Average Normal Stress: The cross-sectional area of wires AB and BC are

$$A_{AB} = \frac{\pi}{4} (0.00175^2) = 0.765625(10^{-6}) \pi \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} (0.0015^2) = 0.5625(10^{-6}) \pi \text{ m}^2$$

Then,

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{430.89}{0.765625(10^{-6}) \pi} = 179.14(10^6) \text{ Pa} = 179.14 \text{ MPa}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{304.68}{0.5625(10^{-6}) \pi} = 172.41(10^6) \text{ Pa} = 172.41 \text{ MPa}$$

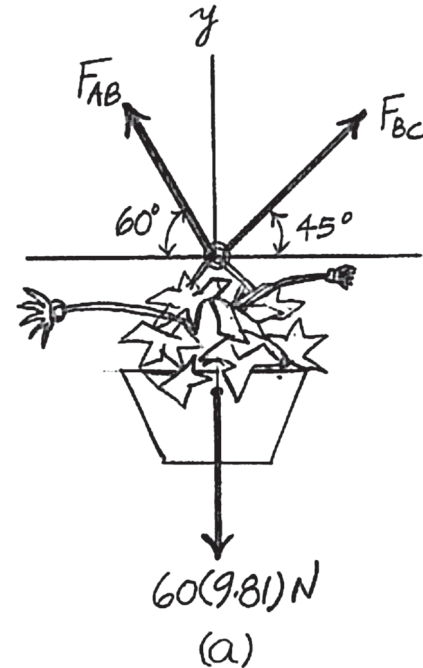
Using these result

$$(F.S.)_{AB} = \frac{\sigma_{\text{fail}}}{\sigma_{AB}} = \frac{380}{179.14} = 2.1212 = 2.12$$

Ans.

$$(F.S.)_{BC} = \frac{\sigma_{\text{fail}}}{\sigma_{BC}} = \frac{380}{172.41} = 2.2040 = 2.20$$

Ans.



Ans:

$$F_{AB} = 430.89 \text{ N}$$

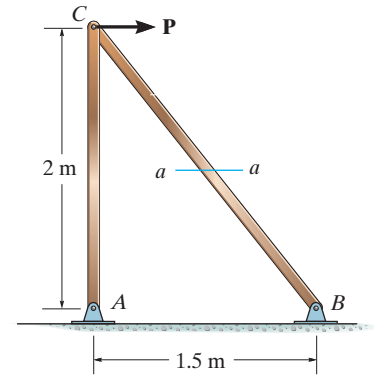
$$F_{BC} = 304.68 \text{ N}$$

$$(F.S.)_{AB} = 2.12$$

$$(F.S.)_{BC} = 2.20$$

1-93.

Determine the largest load P that can be applied to the frame without causing either the average normal stress or the average shear stress at section $a-a$ to exceed $\sigma = 180 \text{ MPa}$ and $\tau = 75 \text{ MPa}$, respectively. Member CB has a square cross section of 40 mm on each side.



SOLUTION

Method of Joint: Consider the equilibrium of joint C, Fig a ,

$$\rightarrow \Sigma F_x = 0; \quad P - F_{BC} \left(\frac{3}{5} \right) = 0 \quad F_{BC} = \frac{5}{3} P$$

Internal Loading: The normal and shear forces on section $a-a$ can be determined by considering the equilibrium of upper segment of member BC, Fig b .

$$\rightarrow \Sigma F_x = 0; \quad \left(\frac{5}{3} P \right) \left(\frac{3}{5} \right) - V_{a-a} = 0 \quad V_{a-a} = P$$

$$+\uparrow \Sigma F_y = 0; \quad N_{a-a} - \left(\frac{5}{3} P \right) \left(\frac{4}{5} \right) = 0 \quad N_{a-a} = \frac{4}{3} P$$

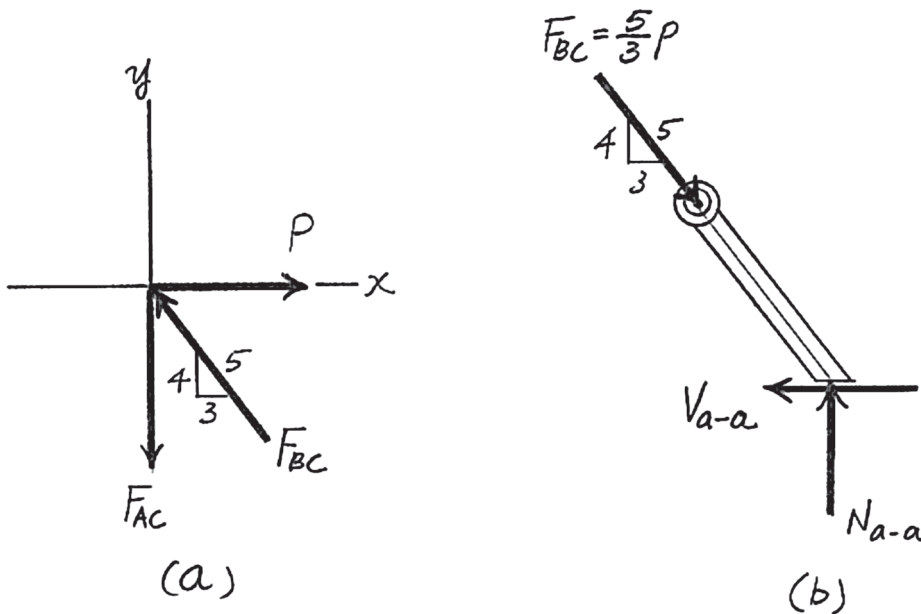
Allowable Normal and Shear Stresses: The area of section $a-a$ is

$$A_{a-a} = 0.04 \left(\frac{0.04}{\frac{4}{5}} \right) = 0.002 \text{ m}^2$$

Then,

$$(\sigma_{\text{avg}})_{\text{allow}} = \frac{N_{a-a}}{A_{a-a}}; \quad 180(10^6) = \frac{\frac{4}{3} P}{0.002} \quad P = 270(10^3) \text{ N} = 270 \text{ kN}$$

$$(\tau_{\text{avg}})_{\text{allow}} = \frac{V_{a-a}}{A_{a-a}}; \quad 75(10^6) = \frac{P}{0.002} \quad P = 150(10^3) \text{ N} = 150 \text{ kN (Controls) Ans.}$$



Ans:

$$P = 270 \text{ kN}$$

$$P = 150 \text{ kN (Controls)}$$

1-94.

The aluminum bracket A is used to support the centrally applied load of 8 kip. If it has a thickness of 0.5 in., determine the smallest height h in order to prevent a shear failure. The failure shear stress is $\tau_{\text{fail}} = 23$ ksi. Use a factor of safety for shear of F.S. = 2.5.

SOLUTION

Equation of Equilibrium:

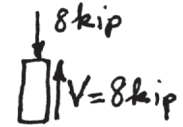
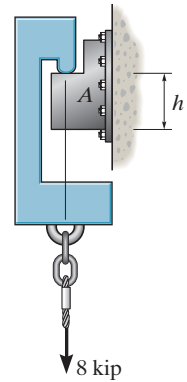
$$+\uparrow \Sigma F_y = 0; \quad V - 8 = 0 \quad V = 8.00 \text{ kip}$$

Allowable Shear Stress: Design of the support size

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S.}} = \frac{V}{A}; \quad \frac{23(10^3)}{2.5} = \frac{8.00(10^3)}{h(0.5)}$$

$$h = 1.74 \text{ in.}$$

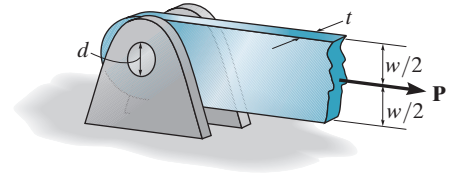
Ans.



Ans:
 $h = 1.74 \text{ in.}$

1-95.

If the allowable tensile stress for the bar is $(\sigma_t)_{\text{allow}} = 21 \text{ ksi}$, and the allowable shear stress for the pin is $\tau_{\text{allow}} = 12 \text{ ksi}$, determine the diameter of the pin so that the load P will be a maximum. What is this load? Assume the hole in the bar has the same diameter d as the pin. Take $t = \frac{1}{4} \text{ in.}$ and $w = 2 \text{ in.}$



SOLUTION

Allowable Normal Stress: The effective cross-sectional area A' for the bar must be considered here by taking into account the reduction in cross sectional area introduced by the hole. Here $A' = (2 - d)(\frac{1}{4})$.

$$(\sigma_t)_{\text{allow}} = \frac{P}{A'}; \quad 21(10^3) = \frac{P_{\text{max}}}{(2 - d)(\frac{1}{4})} \quad (1)$$

Allowable Shear Stress: The pin is subjected to double shear and therefore, $V = \frac{P_{\text{max}}}{2}$

$$\tau_{\text{allow}} = \frac{V}{A}; \quad 12(10^3) = \frac{P_{\text{max}}/2}{\frac{\pi}{4}d^2} \quad (2)$$

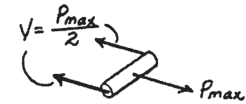
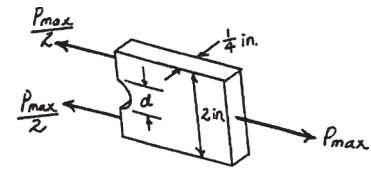
Solving Eq. (1) and (2) yields:

$$d = 0.620 \text{ in.}$$

Ans.

$$P_{\text{max}} = 7.25 \text{ kip}$$

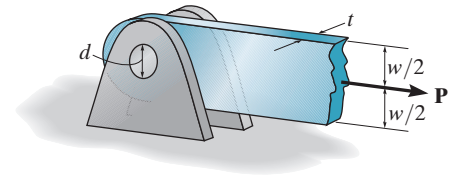
Ans.



Ans:
 $d = 0.620 \text{ in.},$
 $P_{\text{max}} = 7.25 \text{ kip}$

***1-96.**

The bar is connected to the support using a pin having a diameter of $d = 1$ in. If the allowable tensile stress for the bar is $(\sigma_t)_{\text{allow}} = 20$ ksi, and the allowable bearing stress between the pin and the bar is $(\sigma_b)_{\text{allow}} = 30$ ksi, determine the dimensions w and t so that the gross area of the cross section is $wt = 2 \text{ in}^2$ and the load P is a maximum. What is this maximum load? Assume the hole in the bar has the same diameter as the pin.



SOLUTION

Allowable Normal Stress: The effective cross-sectional area A' for the bar must be considered here by taking into account the reduction in cross-sectional area introduced by the hole. Here $A' = (w - 1)t = wt - t = (2 - t) \text{ in}^2$ where $wt = 2 \text{ in}^2$.

$$(\sigma_t)_{\text{allow}} = \frac{P}{A'}; \quad 20(10^3) = \frac{P_{\text{max}}}{2 - t} \quad (1)$$

Allowable Bearing Stress: The projected area

$$A_p = (1)t = t \text{ in}^2.$$

$$(\sigma_b)_{\text{allow}} = \frac{P}{A_p}; \quad 30(10^3) = \frac{P_{\text{max}}}{t} \quad (2)$$

Solving Eq. (1) and (2) yields:

$$t = 0.800 \text{ in.}$$

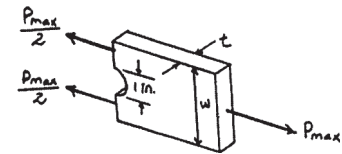
Ans.

$$P_{\text{max}} = 24.0 \text{ kip}$$

Ans.

And $w = 2.50 \text{ in.}$

Ans.



Ans:

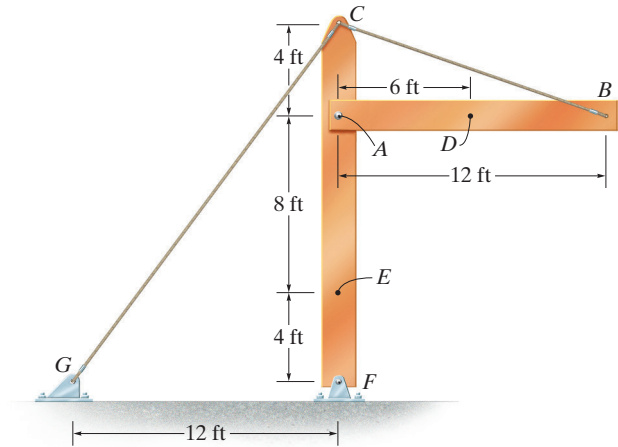
$$t = 0.800 \text{ in.,}$$

$$P_{\text{max}} = 24.0 \text{ kip,}$$

$$w = 2.50 \text{ in.}$$

R1-1.

The beam AB is pin supported at A and supported by a cable BC . A separate cable CG is used to hold up the frame. If the beam AB and the column FC have a weight per unit length of 120 lb/ft and 180 lb/ft, respectively, determine the resultant internal loadings acting on cross sections located at points D and E .



SOLUTION

Segment AD :

$$\begin{aligned} \pm \sum F_x &= 0; & N_D + 2.16 &= 0; & N_D &= -2.16 \text{ kip} \\ +\downarrow \sum F_y &= 0; & V_D + 0.72 - 0.72 &= 0; & V_D &= 0 \\ \zeta + \sum M_D &= 0; & M_D - 0.72(3) &= 0; & M_D &= 2.16 \text{ kip} \cdot \text{ft} \end{aligned}$$

Segment FE :

$$\begin{aligned} \pm \sum F_x &= 0; & V_E - 0.54 &= 0; & V_E &= 0.540 \text{ kip} \\ +\downarrow \sum F_y &= 0; & N_E + 0.72 - 5.04 &= 0; & N_E &= 4.32 \text{ kip} \\ \zeta + \sum M_E &= 0; & -M_E + 0.54(4) &= 0; & M_E &= 2.16 \text{ kip} \cdot \text{ft} \end{aligned}$$

Ans.

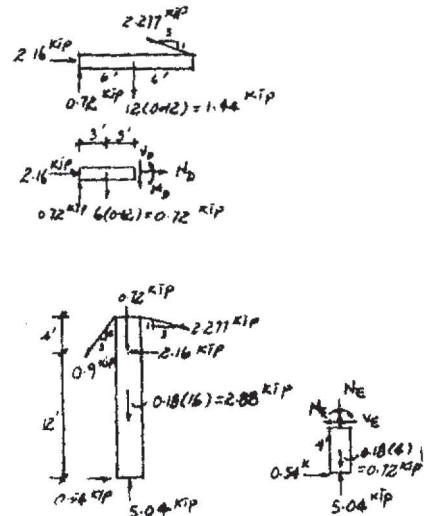
Ans.

Ans.

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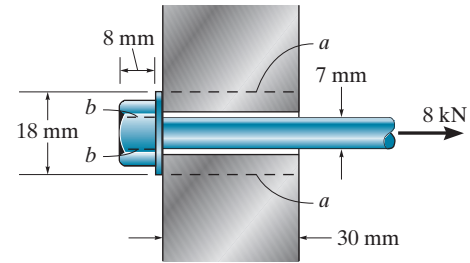


Ans:

$$\begin{aligned} N_D &= -2.16 \text{ kip}, & V_D &= 0, & M_D &= 2.16 \text{ kip} \cdot \text{ft}, \\ V_E &= 0.540 \text{ kip}, & N_E &= 4.32 \text{ kip}, & M_E &= 2.16 \text{ kip} \cdot \text{ft} \end{aligned}$$

R1-2.

The long bolt passes through the 30-mm-thick plate. If the force in the bolt shank is 8 kN, determine the average normal stress in the shank, the average shear stress along the cylindrical area of the plate defined by the section lines $a-a$, and the average shear stress in the bolt head along the cylindrical area defined by the section lines $b-b$.



SOLUTION

$$\sigma_s = \frac{P}{A} = \frac{8 (10^3)}{\frac{\pi}{4} (0.007)^2} = 208 \text{ MPa}$$

Ans.

$$(\tau_{\text{avg}})_a = \frac{V}{A} = \frac{8 (10^3)}{\pi (0.018)(0.030)} = 4.72 \text{ MPa}$$

Ans.

$$(\tau_{\text{avg}})_b = \frac{V}{A} = \frac{8 (10^3)}{\pi (0.007)(0.008)} = 45.5 \text{ MPa}$$

Ans.

Ans:

$$\sigma_s = 208 \text{ MPa}, (\tau_{\text{avg}})_a = 4.72 \text{ MPa},$$

$$(\tau_{\text{avg}})_b = 45.5 \text{ MPa}$$

R1-3.

Determine the required thickness of member BC to the nearest $\frac{1}{16}$ in., and the diameter of the pins at A and B if the allowable normal stress for member BC is $\sigma_{\text{allow}} = 29 \text{ ksi}$ and the allowable shear stress for the pins is $\tau_{\text{allow}} = 10 \text{ ksi}$.

SOLUTION

Referring to the FBD of member AB , Fig. a ,

$$\zeta + \Sigma M_A = 0; \quad 2(8)(4) - F_{BC} \sin 60^\circ (8) = 0 \quad F_{BC} = 9.238 \text{ kip}$$

$$\pm \Sigma F_x = 0; \quad 9.238 \cos 60^\circ - A_x = 0 \quad A_x = 4.619 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0; \quad 9.238 \sin 60^\circ - 2(8) + A_y = 0 \quad A_y = 8.00 \text{ kip}$$

Thus, the force acting on pin A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{4.619^2 + 8.00^2} = 9.238 \text{ kip}$$

Pin A is subjected to single shear, Fig. c , while pin B is subjected to double shear, Fig. b .

$$V_A = F_A = 9.238 \text{ kip} \quad V_B = \frac{F_{BC}}{2} = \frac{9.238}{2} = 4.619 \text{ kip}$$

For member BC

$$\sigma_{\text{allow}} = \frac{F_{BC}}{A_{BC}}; \quad 29 = \frac{9.238}{1.5(t)} \quad t = 0.2124 \text{ in.}$$

$$\text{Use } t = \frac{1}{4} \text{ in.}$$

For pin A ,

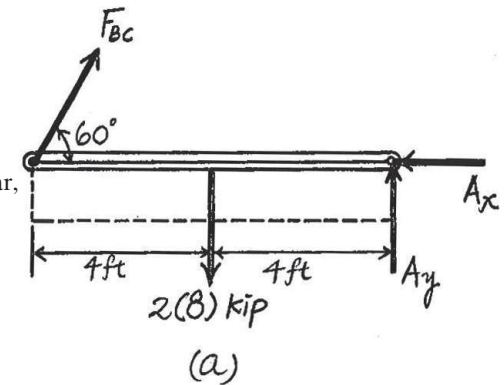
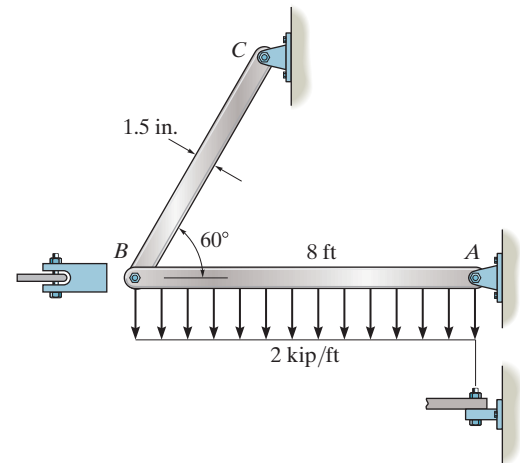
$$\tau_{\text{allow}} = \frac{V_A}{A_A}; \quad 10 = \frac{9.238}{\frac{\pi}{4} d_A^2} \quad d_A = 1.085 \text{ in.}$$

$$\text{Use } d_A = 1\frac{1}{8} \text{ in.}$$

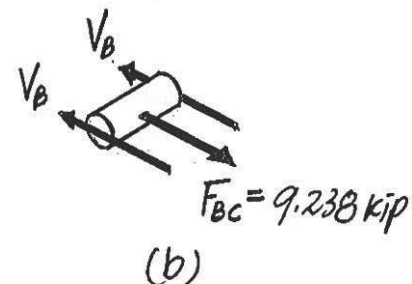
For pin B ,

$$\tau_{\text{allow}} = \frac{V_B}{A_B}; \quad 10 = \frac{4.619}{\frac{\pi}{4} d_B^2} \quad d_B = 0.7669 \text{ in.}$$

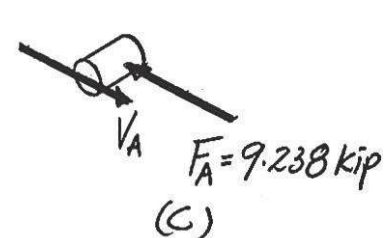
$$\text{Use } d_B = \frac{13}{16} \text{ in.}$$



Ans.



Ans.



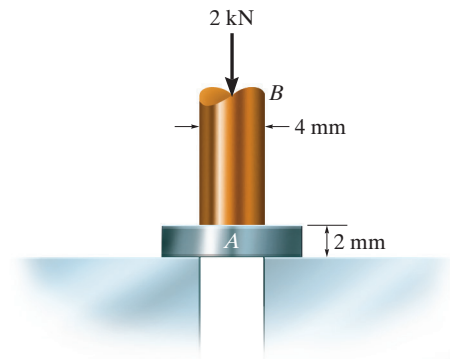
Ans.

Ans:

$$\text{Use } t = \frac{1}{4} \text{ in., } d_A = 1\frac{1}{8} \text{ in., } d_B = \frac{13}{16} \text{ in.}$$

***R1-4.**

The circular punch B exerts a force of 2 kN on the top of the plate A . Determine the average shear stress in the plate.



SOLUTION

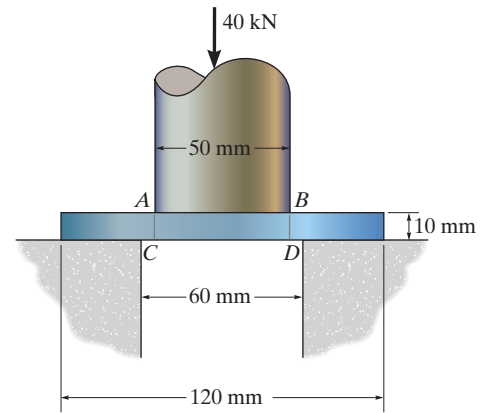
Average Shear Stress: The shear area $A = \pi(0.004)(0.002) = 8.00(10^{-6})\pi \text{ m}^2$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{2(10^3)}{8.00(10^{-6})\pi} = 79.6 \text{ MPa} \quad \text{Ans.}$$

Ans:
 $\tau_{\text{avg}} = 79.6 \text{ MPa}$

R1-5.

Determine the average punching shear stress the circular shaft creates in the circular metal plate through section AC and BD . Also, what is the average bearing stress developed on the surface of the plate under the shaft?



SOLUTION

Average Shear and Bearing Stress: The area of the shear plane and the bearing area on the punch are $A_V = \pi(0.05)(0.01) = 0.5(10^{-3})\pi \text{ m}^2$ and $A_b = \frac{\pi}{4}(0.12^2 - 0.06^2) = 2.7(10^{-3})\pi \text{ m}^2$. We obtain

$$\tau_{\text{avg}} = \frac{P}{A_V} = \frac{40(10^3)}{0.5(10^{-3})\pi} = 25.5 \text{ MPa}$$

Ans.

$$\sigma_b = \frac{P}{A_b} = \frac{40(10^3)}{2.7(10^{-3})\pi} = 4.72 \text{ MPa}$$

Ans.

Ans:

$$\tau_{\text{avg}} = 25.5 \text{ MPa}, \sigma_b = 4.72 \text{ MPa}$$

R1-6.

The 150 mm by 150 mm block of aluminum supports a compressive load of 6 kN. Determine the average normal stress and shear stress acting on the plane through section $a-a$. Show the results on a volume element located on the plane.

SOLUTION

Equation of Equilibrium:

$$+\nearrow \Sigma F_x = 0; \quad V_{a-a} - 6 \cos 60^\circ = 0 \quad V_{a-a} = 3.00 \text{ kN}$$

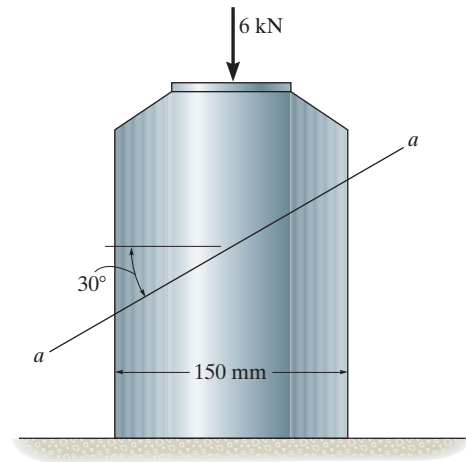
$$\curvearrowleft + \Sigma F_y = 0; \quad N_{a-a} - 6 \sin 60^\circ = 0 \quad N_{a-a} = 5.196 \text{ kN}$$

Average Normal Stress and Shear Stress: The cross sectional Area at section $a-a$ is

$$A = \left(\frac{0.15}{\sin 60^\circ} \right) (0.15) = 0.02598 \text{ m}^2.$$

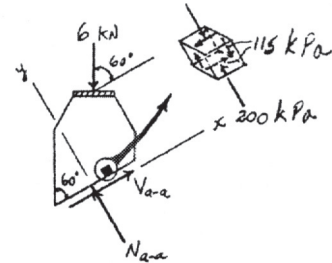
$$\sigma_{a-a} = \frac{N_{a-a}}{A} = \frac{5.196(10^3)}{0.02598} = 200 \text{ kPa}$$

$$\tau_{a-a} = \frac{V_{a-a}}{A} = \frac{3.00(10^3)}{0.02598} = 115 \text{ kPa}$$



Ans.

Ans.

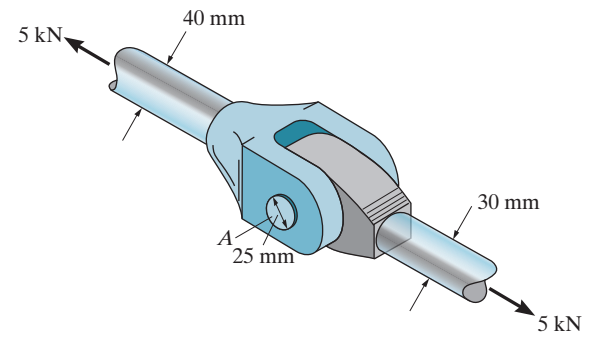


Ans:

$$\sigma_{a-a} = 200 \text{ kPa}, \tau_{a-a} = 115 \text{ kPa}$$

R1-7.

The yoke-and-rod connection is subjected to a tensile force of 5 kN. Determine the average normal stress in each rod and the average shear stress in the pin *A* between the members.



SOLUTION

For the 40 - mm - dia. rod:

$$\sigma_{40} = \frac{P}{A} = \frac{5 (10^3)}{\frac{\pi}{4} (0.04)^2} = 3.98 \text{ MPa}$$

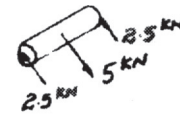
For the 30 - mm - dia. rod:

$$\sigma_{30} = \frac{V}{A} = \frac{5 (10^3)}{\frac{\pi}{4} (0.03)^2} = 7.07 \text{ MPa}$$

Average shear stress for pin *A*:

$$\tau_{\text{avg}} = \frac{P}{A} = \frac{2.5 (10^3)}{\frac{\pi}{4} (0.025)^2} = 5.09 \text{ MPa}$$

Ans.



Ans.

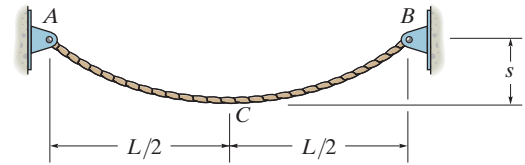
Ans.

Ans:

$$\sigma_{40} = 3.98 \text{ MPa}, \sigma_{30} = 7.07 \text{ MPa}, \tau_{\text{avg}} = 5.09 \text{ MPa}$$

***R1-8.**

The cable has a specific weight γ (weight/volume) and cross-sectional area A . Assuming the sag s is small, so that the cable's length is approximately L and its weight can be distributed uniformly along the horizontal axis, determine the average normal stress in the cable at its lowest point C .



SOLUTION

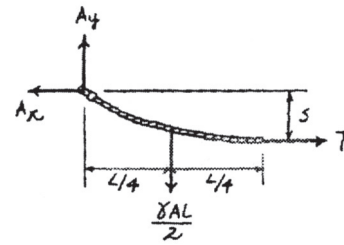
Equation of Equilibrium:

$$\zeta + \sum M_A = 0; \quad Ts - \frac{\gamma AL}{2} \left(\frac{L}{4} \right) = 0$$

$$T = \frac{\gamma AL^2}{8s}$$

Average Normal Stress:

$$\sigma = \frac{T}{A} = \frac{\frac{\gamma AL^2}{8s}}{A} = \frac{\gamma L^2}{8s}$$



Ans.

Ans:

$$\sigma = \frac{\gamma L^2}{8s}$$