

***Using and Understanding Mathematics 8th edition  
Bennett Solutions Manual***

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# INSTRUCTOR'S SOLUTIONS MANUAL

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## USING AND UNDERSTANDING MATHEMATICS A QUANTITATIVE REASONING APPROACH EIGHTH EDITION

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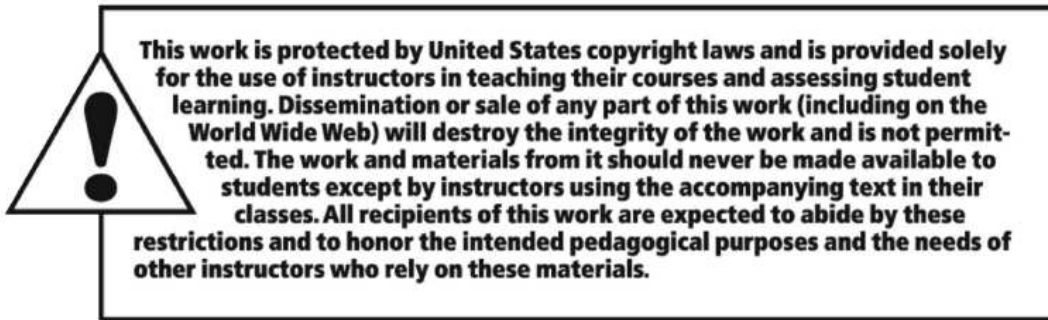
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## UNIT 1A: LIVING IN THE MEDIA AGE

### THINK ABOUT IT

- Pg. 21. Not guilty does not mean innocent; it means not enough evidence to prove guilt. If defendants were required to prove innocence, there would be many cases where they would be unable to provide such proof even though they were, in fact, innocent. This relates to the fallacy of appeal to ignorance in the sense that lack of proof of guilt does not mean innocence, and lack of proof of innocence does not mean guilt.
- Pg. 23. Opinions will vary. One argument is that character questions should be allowed in court if answers to those questions may show bias or ulterior motives for testimony given by a witness. This would be a good topic for a discussion either during or outside of class.

### QUICK QUIZ

- QQ1. **a.** By the definition used in this book, an argument always contains at least one premise and a conclusion.
- QQ2. **c.** By definition, a fallacy is a deceptive argument.
- QQ3. **b.** An argument must contain a conclusion.
- QQ4. **a.** Circular reasoning is an argument where the premise and the conclusion say essentially the same thing.
- QQ5. **b.** Using the fact that a statement is unproved to imply that it is false is appeal to ignorance.
- QQ6. **b.** “I don’t support the President’s tax plan” is the conclusion because the premise “I don’t trust his motives” supports that conclusion.
- QQ7. **b.** This is a personal attack because the premise (I don’t trust his motives) attacks the character of the President, and says nothing about the substance of his tax plan.
- QQ8. **c.** This is limited choice because the argument does not allow for the possibility that you are a fan of, say, boxing.
- QQ9. **b.** Just because  $A$  preceded  $B$  does not necessarily imply that  $A$  caused  $B$ .
- QQ10. **a.** By definition, a straw man is an argument that distorts (or misrepresents) the real issue.

### READING QUESTIONS

- Logic is the study of the methods and principles of reasoning. Arguing logically may or may not change a person’s position, but it can give either person insight into the other’s thinking.
- An argument refers to a reasoned or thoughtful process. An argument uses a set of facts or assumptions, called premises, to support a conclusion.
- A fallacy is an argument in which the conclusion is not well supported by the premises. The examples of fallacies given in the text are appeal to popularity, false cause, appeal to ignorance, hasty generalization, limited choice, appeal to emotion, personal attack, circular reasoning, diversion (red herring), and straw man. Examples will vary.
- The five steps (See the information box on page 24 in the text for a summary of each step.) to evaluating media information are:
  - Consider the source.
  - Check the date.
  - Validate accuracy.
  - Watch for hidden agendas.
  - Don’t miss the big picture.

### DOES IT MAKE SENSE?

- Does not make sense. Raising one’s voice has nothing to do with logical arguments.
- Does not make sense. Logical arguments always contain at least one premise and a conclusion.
- Makes sense. A logical person would not put much faith in an argument that uses premises he believes to be false to support a conclusion.

8. Makes sense. There's nothing wrong with stating the conclusion of an argument before laying out the premises.
9. Does not make sense. One can disagree with the conclusion of a well-stated argument regardless of whether it is fallacious.
10. Makes sense. Despite the fact that an argument may be poorly constructed and fallacious, it still may have a believable conclusion.

### BASIC SKILLS AND CONCEPTS

11. a. *Premise:* Apple's iPhone outsells all other smartphones. *Conclusion:* They must be the best smartphone on the market.  
b. The fact that many people buy the iPhone does not necessarily mean it is the best smartphone.
12. a. *Premise:* I became sick soon after eating at Burger Hut. *Conclusion:* Burger Hut food made me sick.  
b. The argument doesn't prove that Burger Hut food was the cause of the sickness.
13. a. *Premise:* Decades of searching have not revealed life on other planets. *Conclusion:* Life in the universe must be confined to Earth.  
b. Failure to find life on other planets does not imply that life does not exist elsewhere in the universe.
14. a. *Premise:* I saw three people use food stamps to buy expensive steaks. *Conclusion:* Abuse of food stamps is widespread.  
b. The conclusion is based on relatively little evidence.
15. a. *Premise:* He refused to testify by invoking his Fifth Amendment rights. *Conclusion:* He must be guilty.  
b. There are many reasons that someone might have for refusing to testify (being guilty is only one of them), and thus this is the fallacy of limited choice.
16. a. *Premise:* Thousands of unarmed people, many of them children, are killed by firearms every year.  
*Conclusion:* The sale of all guns should be banned.  
b. The conclusion is reached on the basis of an emotional statement.
17. a. *Premise:* Senator Smith has accepted contributions from companies that sell genetically modified crop seeds. *Conclusion:* Senator Smith's bill is a sham.  
b. A claim about Senator Smith's personal behavior is used to criticize his bill.
18. a. *Premise:* It's illegal to drive faster than the speed limit and breaking the law makes you a criminal.  
*Conclusion:* Drivers who exceed the speed limit are criminals.  
b. The conclusion is a restatement of the premise.
19. a. *Premise:* Good grades are needed to get into college, and a college diploma is necessary for a good career.  
*Conclusion:* Attendance should count in high school grades.  
b. The premise (which is often true) directs attention away from the conclusion.
20. a. *Premise:* The mayor wants to raise taxes to fund social programs. *Conclusion:* She must not believe in the value of hard work.  
b. The mayor is characterized (perhaps wrongly) by one quality, on which the conclusion is based.
21. false; Explanations will vary.
22. false; Explanations will vary.
23. false; Explanations will vary.
24. true; Explanations will vary.

### FURTHER APPLICATIONS

25. *Premise:* Eating oysters for dinner, followed by a nightmare. *Conclusion:* Oysters cause nightmares. This argument suffers from the **false cause** fallacy. We cannot conclude that the former caused the latter simply because they happened together.

26. *Premise:* Chinese restaurants in America outnumber McDonald's by nearly three to one. *Conclusion:* Chinese food is preferable to hamburgers. This is a blatant **appeal to popularity**. No argument concerning actual food preference is given.
27. *Premise:* All the nurses in Belvedere Hospital are women. *Conclusion:* Women are better qualified for medical jobs. The conclusion has been reached with a **hasty generalization**, because a small number of female nurses were used as evidence to support a claim about all men and women.
28. *Premise:* The governor wants to sell public lands to an energy exploration company. *Premise:* He is an untrustworthy opportunist. *Conclusion:* I oppose the land sale. This is a **personal attack** on the governor's past transgressions, which should play little part in the logical decision about whether to oppose the land sale.
29. *Premise:* My uncle never drank alcohol and lived to be 93. *Conclusion:* Avoiding alcohol leads to greater longevity. **False cause** is at play here, as the abstinence from alcohol may have nothing to do with the uncle's longevity, even though both are occurring at the same time.
30. *Premise:* The state has no right to take a life. *Conclusion:* The death penalty should be abolished. Both the premise and conclusion say essentially the same thing; this is **circular reasoning**.
31. *Premise:* Five hundred million copies of *Don Quixote* have been sold. *Conclusion:* *Don Quixote* is popular. This is an **appeal to popularity**.
32. *Premise:* I live near an oil well and have never felt an earthquake. *Conclusion:* Claims that fracking causes earthquakes are ridiculous. This is an **appeal to ignorance**: the lack of knowledge of cases where oil wells do not cause earthquakes does not mean they do not cause earthquakes.
33. *Premise:* After I last gave to a charity, an audit showed that most of the money was used to pay its administrators in the front office. *Conclusion:* Charities cannot be trusted. The conclusion has been reached with a **hasty generalization**, because a small number charities are not passing donations on to the intended recipients does not mean that all charities do not pass on donations to the intended recipients.
34. *Premise:* Prison overcrowding is a crisis. *Conclusion:* Capital punishment must be used to reduce the overcrowding. This is **limited choice**: the premise does not allow for the possibility that there are other ways to alleviate prison overcrowding.
35. *Premise:* The senator is a member of the National Rifle Association. *Conclusion:* I'm sure she opposes a ban on large capacity magazines. This is a **personal attack** on members of the National Rifle Association. The argument also distorts the position of the National Rifle Association (not all members would oppose a ban on large capacity magazines); this is also a **straw man**.
36. *Premise:* Wider highways can relieve traffic congestion. *Conclusion:* We should build wider highways to benefit the tourist industry. This is a **diversion**, because the argument begins with traffic congestions and ends discussing tourism.
37. *Premise:* Some Democrats support doubling the federal minimum wage. *Conclusion:* Democrats think that everyone should have the same income. The argument distorts the position of the Democrats; this is a **straw man**.
38. *Premise:* The giant sea squid has never been observed in its habitat. *Conclusion:* It must be extinct in the wild. **Appeal to ignorance** is used here – the lack of proof of the existence of the giant sea squid does not imply it is extinct.
39. *Premise:* My little boy loves dolls, and my little girl loves trucks. *Conclusion:* There's no truth to the claim that boys prefer mechanical toys while girls prefer maternal toys. Using one child of each gender to come up with a conclusion about all children is **hasty generalization**. It can also be seen as an **appeal to ignorance**: the lack of examples of boys enjoying mechanical toys (and girls enjoying maternal toys) does not mean that they don't enjoy these toys.

40. *Premise:* Some Republicans want to reduce government regulations. *Conclusion:* Republicans don't think that government can improve society. The argument distorts the position of the Republicans; this is a **straw man**.
41. The example shows the fallacy of division because the fact that Jake is an American does not mean that he acts the same as all other Americans.
42. The example shows the gambler's fallacy because it uses the fact that one type of weather occurred on previous consecutive days to draw a conclusion about the next day's weather.
43. The example shows the fallacy of slippery slope because it assumes that the fact that troops have been sent to three countries means it's inevitable that they'll be sent to more.
44. The example shows the middle ground fallacy because it assumes that the best policy must be in between the two positions advocated by the senators.

## UNIT 1B: PROPOSITIONS AND TRUTH VALUES

### THINK ABOUT IT

- Pg. 29. Answers will vary. This would be a good topic for a discussion either during or outside of class.
- Pg. 32. We needed 8 rows for 3 propositions; adding a fourth proposition means two possible truth values for each of those 8 rows, or 16 rows total. The conjunction is true only if all four propositions are true.
- Pg. 34. The precise definitions of logic sometimes differ from our "everyday" intuition. There is no possible way that Jones could personally eliminate all poverty on Earth, regardless of whether she is elected. Thus, at the time you heard her make this promise, you would certainly conclude that she was being less than truthful. Nevertheless, according to the rules of logic, the only way her statement can be false is if she is elected, in which case she would be unable to follow through on the promise. If she is not elected, her claim is true (at least according to the laws of logic).

### QUICK QUIZ

- QQ1. **c.** This is a proposition because it is a complete sentence making a claim, which could be true or false.
- QQ2. **a.** The truth value of a proposition's negation (*not p*) can always be determined by the truth value of the proposition.
- QQ3. **c.** Conditional statements are, by definition, in the form of *if p, then q*.
- QQ4. **c.** The table will require eight rows because there are two possible truth values for each of the propositions *x*, *y*, and *z*.
- QQ5. **c.** Because it is not stated otherwise, we are dealing with the inclusive or (and thus either *p* is true, or *q* is true, or both are true).
- QQ6. **a.** The conjunction *p and q* is true only when both are true, and since *p* is false, *p and q* must also be false.
- QQ7. **b.** This is the correct rephrasing of the original conjunction.
- QQ8. **c.** This is the *contrapositive* of the original conjunction.
- QQ9. **b.** Statements are logically equivalent only when they have the same truth values.
- QQ10. **a.** Rewriting the statement in *if p, then q* form gives, "if you want to win, then you've got to play."

### READING QUESTIONS

1. A proposition makes a claim (either an assertion or a denial) that may be either true or false. It must have the structure of a complete sentence. Examples will vary.
2. The negation of a proposition *p* is another proposition that makes the opposite claim of *p*. Examples will vary.

3. Given two propositions  $p$  and  $q$ , the statement  $p$  and  $q$  is called their conjunction, the statement  $p$  or  $q$  is called their disjunction, and a statement of the form *if  $p$ , then  $q$*  is called a conditional proposition (or implication). Examples will vary.
4. The inclusive *or* means “either or both,” while the exclusive *or* means “one or the other, but not both.”
5. Truth tables for  $p$  and  $q$ ,  $p$  or  $q$ , and *if  $p$ , then  $q$* .

| $p$ | $q$ | $p$ and $q$ |
|-----|-----|-------------|
| T   | T   | T           |
| T   | F   | F           |
| F   | T   | F           |
| F   | F   | F           |

| $p$ | $q$ | $p$ or $q$ |
|-----|-----|------------|
| T   | T   | T          |
| T   | F   | T          |
| F   | T   | T          |
| F   | F   | F          |

| $p$ | $q$ | <i>if <math>p</math>, then <math>q</math></i> |
|-----|-----|-----------------------------------------------|
| T   | T   | T                                             |
| T   | F   | F                                             |
| F   | T   | T                                             |
| F   | F   | T                                             |

6. See the information box and truth table on page 35 in the text.

### DOES IT MAKE SENSE?

7. Does not make sense. Propositions are never questions.
8. Makes sense. The mayor’s stance on banning guns indicates he supports gun control.
9. Makes sense. If restated in *if  $p$ , then  $q$*  form, this statement would read, “If we catch him, then he will be dead or alive.” Clearly this is true, as it covers all the possibilities. (One could argue semantics, and say that a dead person is not caught, but rather discovered. Splitting hairs like this might lead one to claim the statement does not make sense).
10. Does not make sense. The first statement is in the *if  $p$ , then  $q$*  form, and the second is the converse (i.e. *if  $q$ , then  $p$* ). Since the converse of an *if ... then* statement is not logically equivalent to the original statement, this doesn’t make sense.
11. Does not make sense. Not all statements fall under the purview of logical analysis.
12. Does not make sense. The converse of a statement is not always false if the original statement is true.

### BASIC SKILLS AND CONCEPTS

13. Since it’s a complete sentence that makes a claim (whether true or false is immaterial), it’s a proposition.
14. No claim is made with this statement, so it’s not a proposition.
15. No claim is made with this statement, so it’s not a proposition.
16. This is a complete sentence that makes a claim, so it’s a proposition.
17. Questions are never propositions.
18. This is a proposition as we can assign a truth value to it, and it’s a complete sentence.
19. The negation is *Asia is not in the northern hemisphere*. The original proposition is true; the negation is false.
20. The negation is *Peru is not in the northern hemisphere*. The original proposition is false; the negation is true.
21. The negation is *The Beatles were a German band*. The original proposition is true; the negation is false.
22. The negation is *Earth is not the center of the universe*. The original proposition is false; the negation is true.
23. Sarah did go to dinner.
24. The mayor appears to approve of the rallies. Whether the mayor approves of them is debatable, given the limited information.
25. Taxes will not be lowered.
27. Sue wants new trees planted in the park.
26. Logging will continue.
28. The senator does not support the bill.

29. This is the truth table for the conjunction  $q$  and  $r$ .

| $q$ | $r$ | $q$ and $r$ |
|-----|-----|-------------|
| T   | T   | T           |
| T   | F   | F           |
| F   | T   | F           |
| F   | F   | F           |

30. This is the truth table for the conjunction  $p$  and  $s$ .

| $p$ | $s$ | $p$ and $s$ |
|-----|-----|-------------|
| T   | T   | T           |
| T   | F   | F           |
| F   | T   | F           |
| F   | F   | F           |

31. “Dogs are animals” is true and “oak trees are plants” is also true. Since both propositions are true, the conjunction is true.
32. “ $12 + 6 = 18$ ” is true, but “ $3 \times 5 = 8$ ” is false. The conjunction is false because both propositions in a conjunction must be true for the entire statement to be true.
33. “Venus is a planet” is true and “The Sun is a star” is also true. Since both propositions are true, the conjunction is true.
34. “Emily Dickinson was a poet” is true, but “Kanye West is a Major League pitcher” is false. The conjunction is false because both propositions in a conjunction must be true for the entire statement to be true.
35. “All birds can fly” is false and “some fish live in trees” is also false. The conjunction is false because both propositions in a conjunction must be true for the entire statement to be true.
36. “Not all men are tall” is true and “Not all women are short” is also true, since both propositions are true, the conjunction is true.

37. This is the truth table for  $q$  and  $r$  and  $s$ .

| $q$ | $r$ | $s$ | $q$ and $r$ and $s$ |
|-----|-----|-----|---------------------|
| T   | T   | T   | T                   |
| T   | T   | F   | F                   |
| T   | F   | T   | F                   |
| T   | F   | F   | F                   |
| F   | T   | T   | F                   |
| F   | T   | F   | F                   |
| F   | F   | T   | F                   |
| F   | F   | F   | F                   |

38. This is the truth table for  $p$  and  $q$  and  $r$  and  $s$ .

| $p$ | $q$ | $r$ | $s$ | $p$ and $q$ and $r$ and $s$ |
|-----|-----|-----|-----|-----------------------------|
| T   | T   | T   | T   | T                           |
| T   | T   | T   | F   | F                           |
| T   | T   | F   | T   | F                           |
| T   | T   | F   | F   | F                           |
| T   | F   | T   | T   | F                           |
| T   | F   | T   | F   | F                           |
| T   | F   | F   | T   | F                           |
| T   | F   | F   | F   | F                           |
| F   | T   | T   | T   | F                           |
| F   | T   | T   | F   | F                           |
| F   | T   | F   | T   | F                           |
| F   | T   | F   | F   | F                           |
| F   | F   | T   | T   | F                           |
| F   | F   | T   | F   | F                           |
| F   | F   | F   | T   | F                           |
| F   | F   | F   | F   | F                           |

39. The exclusive *or* is used here as it is unlikely that the proposition means you might both walk or ride a bike to the park.
40. The exclusive *or* is used here because you probably can’t have both the salad and soup.
41. The exclusive *or* is used here in the sense of which book will be read first.
42. Oil changes are good for either 3 months or 5,000 miles, whichever comes first, so this is the exclusive use of *or*.

43. The inclusive *or* is used here as you probably would be thrilled to both scuba dive or surf on your next vacation.
44. Most insurance policies that cover “fire or theft” allow for the coverage of both at the same time, so this is the inclusive *or*.

45. This is the truth table for the disjunction  $r$  or  $s$ .

| $r$ | $s$ | $r$ or $s$ |
|-----|-----|------------|
| T   | T   | T          |
| T   | F   | T          |
| F   | T   | T          |
| F   | F   | F          |

46. This is the truth table for the disjunction  $p$  or  $r$ .

| $p$ | $r$ | $p$ or $r$ |
|-----|-----|------------|
| T   | T   | T          |
| T   | F   | T          |
| F   | T   | T          |
| F   | F   | F          |

47. This is the truth table for  $p$  and (*not*  $p$ ).

| $p$ | <i>not</i> $p$ | $p$ and ( <i>not</i> $p$ ) |
|-----|----------------|----------------------------|
| T   | F              | F                          |
| F   | T              | F                          |

48. This is the truth table for  $q$  or (*not*  $q$ ).

| $q$ | <i>not</i> $q$ | $q$ or ( <i>not</i> $q$ ) |
|-----|----------------|---------------------------|
| T   | F              | T                         |
| F   | T              | T                         |

49. This is the truth table for  $p$  or  $q$  or  $r$ .

| $p$ | $q$ | $r$ | $p$ or $q$ or $r$ |
|-----|-----|-----|-------------------|
| T   | T   | T   | T                 |
| T   | T   | F   | T                 |
| T   | F   | T   | T                 |
| T   | F   | F   | T                 |
| F   | T   | T   | T                 |
| F   | T   | F   | T                 |
| F   | F   | T   | T                 |
| F   | F   | F   | F                 |

50. This is the truth table for  $p$  or (*not*  $p$ ) or  $q$ .

| $p$ | ( <i>not</i> $p$ ) | $q$ | $p$ or ( <i>not</i> $p$ ) or $q$ |
|-----|--------------------|-----|----------------------------------|
| T   | F                  | T   | T                                |
| T   | F                  | F   | T                                |
| F   | T                  | T   | T                                |
| F   | T                  | F   | T                                |

51. “Elephants are animals” is true and “elephants are plants” is false. The disjunction is true because a disjunction is true when at least one of its propositions is true.
52. “The Nile River is in Europe” is false and “the Ganges River is in Asia” is true. The disjunction is true because a disjunction is true when at least one of its propositions is true.
53. Both “ $3 \times 5 = 15$ ” and “ $3 + 5 = 8$ ” are true. The disjunction is true because a disjunction is true when at least one of its propositions is true.
54. Both “ $2 + 2 = 5$ ” and “ $3 + 3 = 7$ ” are false. The disjunction is false because a disjunction is false when all of its propositions are false.
55. Both “Cars swim” and “dolphins fly” are false. The disjunction is false because a disjunction is false when all of its propositions are false.
56. “Oranges are round” is true and “bananas are round” is false. The disjunction is true because at least one of the propositions is true.

57. This is the truth table for *if p, then r*.

| <i>p</i> | <i>r</i> | <i>if p, then r</i> |
|----------|----------|---------------------|
| T        | T        | T                   |
| T        | F        | F                   |
| F        | T        | T                   |
| F        | F        | T                   |

58. This is the truth table for *if q, then s*.

| <i>q</i> | <i>s</i> | <i>if q, then s</i> |
|----------|----------|---------------------|
| T        | T        | T                   |
| T        | F        | F                   |
| F        | T        | T                   |
| F        | F        | T                   |

59. *Hypothesis*: Trout can swim. *Conclusion*: Trout are fish. Both propositions are true, and the conditional proposition (implication) is true.
60. *Hypothesis*: Paris is in France. *Conclusion*: New York is in America. Both propositions are true, and the conditional proposition (implication) is true.
61. *Hypothesis*: Paris is in France. *Conclusion*: New York is in China. The hypothesis is true, the conclusion is false, and the conditional proposition is false.
62. *Hypothesis*: Paris is in Mongolia. *Conclusion*: New York is in America. The hypothesis is false, the conclusion is true, and the conditional proposition is true.
63. *Hypothesis*: Trees can walk. *Conclusion*: Birds wear wigs. The hypothesis is false, the conclusion is false, and the conditional proposition is true.
64. *Hypothesis*:  $2 \times 3 = 6$ ; *Conclusion*:  $2 + 3 = 6$ . Since the hypothesis is true, and the conclusion is false, the implication is false.
65. *Hypothesis*: Dogs can swim. *Conclusion*: Dogs are fish. The hypothesis is true, the conclusion is false, and the conditional proposition is false.
66. *Hypothesis*: Dogs are fish. *Conclusion*: Dogs can swim. The hypothesis is false, the conclusion is true, and the conditional proposition is true.
67. If it snows (*p*), then I get cold (*q*).
68. If a person lives in Boston (*p*), then that person lives in Massachusetts (*q*).
69. If you are breathing (*p*), then you are alive (*q*).
70. If you are alive (*p*), then you are breathing (*q*).
71. If a creature is a spider (*p*), then it has eight legs (*q*).
72. If a person is a medical doctor (*p*), then that person knows anatomy (*q*).
73. *Converse*: If Tara's family owns a car, then they own a Cadillac. *Inverse*: If Tara's family does not own a Cadillac, then they do not own a car. *Contrapositive*: If Tara's family does not own a car, then they do not own a Cadillac. The original proposition and the contrapositive are equivalent. The converse and inverse are equivalent.
74. *Converse*: If the patient is breathing, then the patient is alive. *Inverse*: If the patient is not alive, then the patient is not breathing. *Contrapositive*: If the patient is not breathing, then the patient is not alive. The converse and inverse are always logically equivalent, and the contrapositive is always logically equivalent to the original statement.
75. *Converse*: If Helen is a U.S. citizen, then she is the U.S. President. *Inverse*: If Helen is not the U.S. President, then she is not a U.S. citizen. *Contrapositive*: If Helen is not a U.S. citizen, then she is not the U.S. President. The original proposition and the contrapositive are equivalent. The converse and inverse are equivalent.

76. *Converse*: If the lights are on, then I am using electricity. *Inverse*: If I am not using electricity, then the lights are not on. *Contrapositive*: If the lights are not on, then I am not using electricity. The converse and inverse are always logically equivalent, and the contrapositive is always logically equivalent to the original statement.
77. *Converse*: If there is gas in the tank, then the engine is running. *Inverse*: If the engine is not running, then there is no gas in the tank. *Contrapositive*: If there is no gas in the tank, then the engine is not running. The original proposition and the contrapositive are equivalent. The converse and inverse are equivalent.
78. *Converse*: If the oceans rise, then the polar ice caps will have melted. *Inverse*: If the polar ice caps do not melt, then the oceans will not rise. *Contrapositive*: If the oceans do not rise, then the polar ice caps will not have melted. The converse and inverse are always logically equivalent, and the contrapositive is always logically equivalent to the original statement.

### FURTHER APPLICATIONS

79. If I let go of what I am, then I become what I might be.
80. If we wait for some other person or some other time, then change will not come.
81. If one is making policy, then it must be rooted in unbiased science.
82. If I have six hours to chop down a tree, then I will spend the first four sharpening the axe.
83. "If Sue lives in Cleveland, then she lives in Ohio," where it is assumed that Sue lives in Cincinnati. (Answers will vary.) Because Sue lives in Cincinnati, the hypothesis is false, while the conclusion is true, and this means the implication is true. The converse, "If Sue lives in Ohio, then she lives in Cleveland," is false, because the hypothesis is true, but the conclusion is false.
84. "If  $2 + 2 = 4$ , then  $3 + 3 = 6$ ." (Answers will vary.) The implication is true, because the hypothesis is true and the conclusion is true. The converse, "If  $3 + 3 = 6$ , then  $2 + 2 = 4$ ," is also true for the same reason.
85. "If Ramon lives in Albuquerque, then he lives in New Mexico" where it is assumed that Ramon lives in Albuquerque. (Answers will vary.) The implication is true, because the hypothesis is true and the conclusion is true. The contrapositive, "If Ramon does not live in New Mexico, then he does not live in Albuquerque", is logically equivalent to the original conditional, so it is also true.
86. "If Delaware is in America, then Maryland is in Canada." (Answers will vary.) The hypothesis is true, while the conclusion is false, and this means the implication is false. In the inverse, "If Delaware is not in America, then Maryland is not in Canada," the hypothesis is false, while the conclusion is true, and this means the implication is true.
87. "If it is a fruit, then it is an apple." (Answers will vary.) The implication is false because, when the hypothesis is true, the conclusion may be false (it could be an orange). In the converse, "If it is an apple, then it is a fruit.", when the hypothesis is true, the conclusion is true, and this means the implication is true.
88. (1) If the payer does not know that you remarried, then alimony you receive is taxable.  
(2) If the payer knows that you remarried, then alimony you receive is not taxable.  
(3) If you pay alimony to another party, then it is not deductible on your return.
89. (a) Doing what you do best and being happy is sufficient for being further along in life than most people.  
(b) Being further along in life than most people is necessary for doing what you do best and being happy.
90. (a) Concentrating on what you don't have is sufficient for never having enough.  
(b) Never having enough is necessary for concentrating on what you don't have.
91. (a) Not liking what everyone pretends to love is sufficient for not feeling stupid.  
(b) Not feeling stupid is necessary for not liking what everyone else pretends to love.
92. (a) Not imagining is sufficient for not doing. (b) Not doing is necessary for not imagining.
93. emphasis 95. emphasis 97. literal
94. literal 96. emphasis 98. literal

99. Following is a truth table for both
- $\text{not } (p \text{ and } q)$
- and
- $(\text{not } p) \text{ or } (\text{not } q)$
- .

| $p$ | $q$ | $p \text{ and } q$ | $\text{not } (p \text{ and } q)$ | $(\text{not } p) \text{ or } (\text{not } q)$ |
|-----|-----|--------------------|----------------------------------|-----------------------------------------------|
| T   | T   | T                  | F                                | F                                             |
| T   | F   | F                  | T                                | T                                             |
| F   | T   | F                  | T                                | T                                             |
| F   | F   | F                  | T                                | T                                             |

Since both statements have the same truth values (compare the last two columns of the table), they are logically equivalent.

100. Following is a truth table for both
- $\text{not } (p \text{ or } q)$
- and
- $(\text{not } p) \text{ and } (\text{not } q)$
- .

| $p$ | $q$ | $p \text{ or } q$ | $\text{not } (p \text{ or } q)$ | $(\text{not } p) \text{ and } (\text{not } q)$ |
|-----|-----|-------------------|---------------------------------|------------------------------------------------|
| T   | T   | T                 | F                               | F                                              |
| T   | F   | T                 | F                               | F                                              |
| F   | T   | T                 | F                               | F                                              |
| F   | F   | F                 | T                               | T                                              |

Since both statements have the same truth values (compare the last two columns in the table), they are logically equivalent.

101. Following is a truth table for both
- $\text{not } (p \text{ and } q)$
- and
- $(\text{not } p) \text{ and } (\text{not } q)$
- .

| $p$ | $q$ | $p \text{ and } q$ | $\text{not } (p \text{ and } q)$ | $(\text{not } p) \text{ and } (\text{not } q)$ |
|-----|-----|--------------------|----------------------------------|------------------------------------------------|
| T   | T   | T                  | F                                | F                                              |
| T   | F   | F                  | T                                | F                                              |
| F   | T   | F                  | T                                | F                                              |
| F   | F   | F                  | T                                | T                                              |

Note that the last two columns in the truth table don't agree, and thus the statements are not logically equivalent.

102. Following is a truth table for
- $\text{not } (p \text{ or } q)$
- and
- $(\text{not } p) \text{ or } (\text{not } q)$
- .

| $p$ | $q$ | $p \text{ or } q$ | $\text{not } (p \text{ or } q)$ | $(\text{not } p) \text{ or } (\text{not } q)$ |
|-----|-----|-------------------|---------------------------------|-----------------------------------------------|
| T   | T   | T                 | F                               | F                                             |
| T   | F   | T                 | F                               | T                                             |
| F   | T   | T                 | F                               | T                                             |
| F   | F   | F                 | T                               | T                                             |

Note that the last two columns in the truth table don't agree, so the statements are not logically equivalent.

103. Following is a truth table for
- $(p \text{ and } q) \text{ or } r$
- and
- $(p \text{ or } r) \text{ and } (p \text{ or } q)$
- .

| $p$ | $q$ | $r$ | $p \text{ and } q$ | $(p \text{ and } q) \text{ or } r$ | $p \text{ or } r$ | $p \text{ or } q$ | $(p \text{ or } r) \text{ and } (p \text{ or } q)$ |
|-----|-----|-----|--------------------|------------------------------------|-------------------|-------------------|----------------------------------------------------|
| T   | T   | T   | T                  | T                                  | T                 | T                 | T                                                  |
| T   | T   | F   | T                  | T                                  | T                 | T                 | T                                                  |
| T   | F   | T   | F                  | T                                  | T                 | T                 | T                                                  |
| T   | F   | F   | F                  | F                                  | T                 | T                 | T                                                  |
| F   | T   | T   | F                  | T                                  | T                 | T                 | T                                                  |
| F   | T   | F   | F                  | F                                  | F                 | T                 | F                                                  |
| F   | F   | T   | F                  | T                                  | T                 | F                 | F                                                  |
| F   | F   | F   | F                  | F                                  | F                 | F                 | F                                                  |

Since the fifth and eighth column of the table don't agree, these two statements are not logically equivalent.

104. Following is a truth table for  $(p \text{ or } q) \text{ and } r$  and  $(p \text{ and } r) \text{ or } (q \text{ and } r)$ .

| $p$ | $q$ | $r$ | $p \text{ or } q$ | $(p \text{ or } q) \text{ and } r$ | $p \text{ and } r$ | $q \text{ and } r$ | $(p \text{ and } r) \text{ or } (q \text{ and } r)$ |
|-----|-----|-----|-------------------|------------------------------------|--------------------|--------------------|-----------------------------------------------------|
| T   | T   | T   | T                 | T                                  | T                  | T                  | T                                                   |
| T   | T   | F   | T                 | F                                  | F                  | F                  | F                                                   |
| T   | F   | T   | T                 | T                                  | T                  | F                  | T                                                   |
| T   | F   | F   | T                 | F                                  | F                  | F                  | F                                                   |
| F   | T   | T   | T                 | T                                  | F                  | T                  | T                                                   |
| F   | T   | F   | T                 | F                                  | F                  | F                  | F                                                   |
| F   | F   | T   | F                 | F                                  | F                  | F                  | F                                                   |
| F   | F   | F   | F                 | F                                  | F                  | F                  | F                                                   |

Since the fifth and eighth columns agree, the statements are logically equivalent.

105. Given the implication *if  $p$ , then  $q$*  the contrapositive is *if  $(\text{not } q)$  then  $(\text{not } p)$* . The converse is *if  $q$ , then  $p$*  and the inverse of the converse is *if  $(\text{not } q)$  then  $(\text{not } p)$* , which is the contrapositive. Similarly, the contrapositive is also the converse of the inverse.

## UNIT 1C: SETS AND VENN DIAGRAMS

### THINK ABOUT IT

Pg. 40. The set of students in the mathematics class could be described by writing each student's name within the braces, separated by commas. The set of countries you have visited would be written with the names of the countries within the braces. Additional examples will vary.

Pg. 46. The student should see that the statement some teachers do not have tattoos leaves both questions posed unanswered. Thus, from the statement given, it is not possible to know whether some teachers have tattoos. From this, it also follows that we cannot be sure that none of the teachers have tattoos.

Pg. 47. Changing the circle for employed to unemployed is fine, since a teenager is either one or the other.

Pg. 49. The two sets in this case are the opposites of the two sets chosen for Figure 1C.14 in the text, so they work equally well.

### QUICK QUIZ

QQ1. **b.** The ellipsis is a convenient way to represent all the other states in the U.S. without having to write them all down.

QQ2. **c.**  $3\frac{1}{2}$  is a rational number (a ratio of two integers), but it is not an integer.

QQ3. **a.** When the circle labeled  $C$  is contained within the circle labeled  $D$ , it indicates that  $C$  is a subset of  $D$ .

QQ4. **b.** Since the set of cats is disjoint from the set of dogs, the two circles should be drawn as non-overlapping circles.

QQ5. **a.** Because all apples are fruit, the set  $A$  should be drawn within the set  $B$  (the set of apples is a subset of the set of fruits).

QQ6. **c.** Some cross-country runners may also be swimmers, so their sets should be overlapping.

QQ7. **a.** The X is placed in the region where *business executives* and *working mothers* overlap to indicate that there is at least one member in that region.

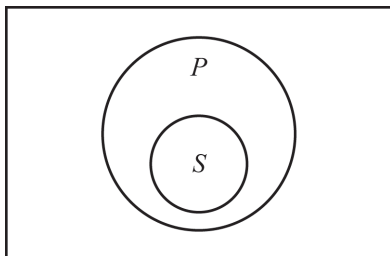
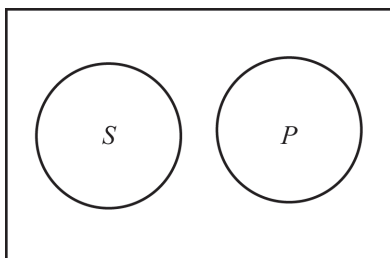
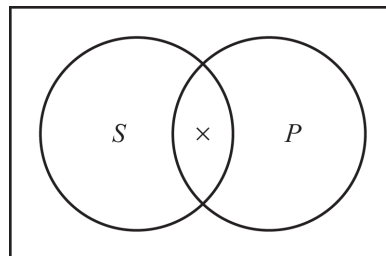
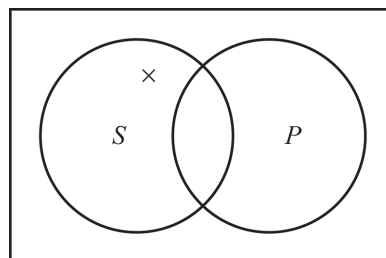
QQ8. **c.** The region X is within both *males* and *athletes*, but not within *Republicans*.

QQ9. **a.** The central region is common to all three sets, and so represents those who are male, Republican, and an athlete.

QQ10. **c.** The sum of the entries in the column labeled Low Birth Weight is 32.

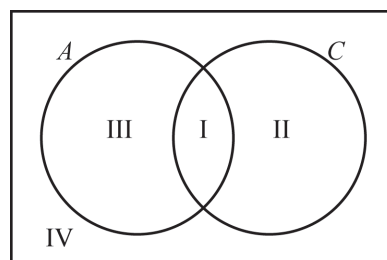
## READING QUESTIONS

1. A set is a collection of objects. Sets are often described by listing their members within a pair of braces,  $\{ \}$ .
2. Venn diagrams use circles to represent sets. The Venn diagram for  $A$  being a subset of  $B$  shows the circle for  $A$  inside the circle for  $B$ . The Venn diagram for disjoint sets consists of separated circles that do not touch. The Venn diagram for overlapping sets consists of two overlapping circles.
3. All  $S$  are  $P$ .

No  $S$  are  $P$ .Some  $S$  are  $P$ .Some  $S$  are not  $P$ .

4. You need to determine where the given proposition is implying “all are”, “none are”, “some are”, or “some are not”.
5. Draw three overlapping circles as shown in Figure 1C.11 in the text. Each of the eight regions represent a distinct combination of the three properties indicated by the three circles.
6. In the following table, region I represents elements that have both property  $A$  and property  $C$ . Region II represents elements that have both property  $C$  and property  $B$ . Region III represents elements that have both property  $A$  and property  $D$ . Region IV represents elements that have both property  $D$  and property  $B$ . To create a Venn diagram, use a circle to represent one row property and another circle to represent one column property. The values in the regions on the cell are then entered into the areas of the Venn diagram that represent both properties. One possible Venn diagram is shown, where the circles represent properties  $A$  and  $C$ , respectively.

|     | $A$ | $B$ |
|-----|-----|-----|
| $C$ | I   | II  |
| $D$ | III | IV  |



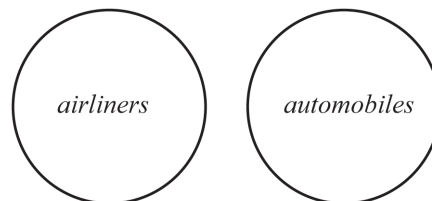
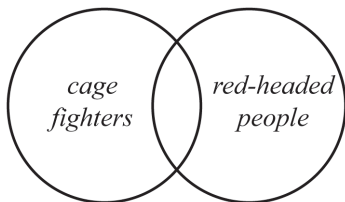
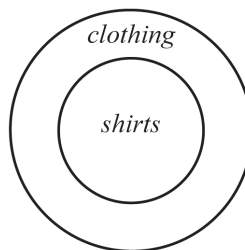
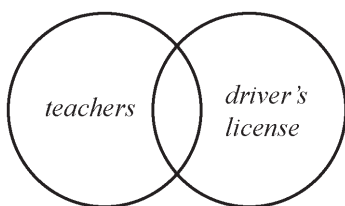
## DOES IT MAKE SENSE?

7. Does not make sense. There are people that live in houses in Chicago, and would not be an element of the set of those who rent apartments.
8. Does not make sense. The set of jabbers is a subset of the set of wocks, but this does not mean there could be no wocks outside the set of jabbers.
9. Does not make sense. The number of students in a class is a whole number, and whole numbers are not in the set of irrational numbers.

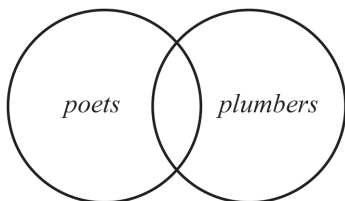
10. Makes sense. The students that ride a bike could be represented by the inside of the circle and those that did not ride a bike would be represented by the area outside of the circle, but inside the rectangle, or vice versa.
11. Does not make sense. A Venn diagram shows only the relationship between members of sets, but does not have much to say about the truth value of a categorical proposition.
12. Makes sense. Vegetarians could be represented by the inside of one circle and non-vegetarians represented by the area outside of the same circle, or vice versa. Republicans and women could be represented in the same way using the other two circles.

### BASIC SKILLS AND CONCEPTS

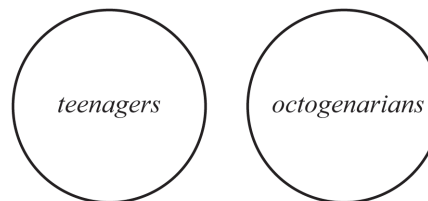
13. 888 is a natural number.
14.  $-23$  is an integer.
15.  $3/4$  is a rational number.
16.  $-6/5$  is a rational number.
17. 3.414 is a rational number.
18. 0 is a whole number.
19.  $\pi$  is a real number.
20.  $\sqrt{8}$  is a real number.
21.  $-45.12$  is a rational number.
22.  $\sqrt{98}$  is a real number.
23.  $\pi/4$  is a real number.
24.  $-34/19.2$  is a rational number.
25.  $-123/79$  is a rational number.
26.  $-923.66$  is a rational number.
27.  $\pi/129$  is a real number.
28. 93,145,095 is a natural number.
29.  $\{1, 2, 3, \dots, 30, 31\}$
30.  $\{23, 25, 27, 29, 31, 33, 35\}$
31.  $\{\text{Alabama, Arkansas, Louisiana, Tennessee}\}$
32.  $\{6, 9, 12, 15, 18, 21, 24\}$
33.  $\{16, 25, 36\}$
34.  $\{\text{Kansas, Kentucky}\}$
35.  $\{6, 12, 18, 24, 30\}$
36.  $\{a, e, i, o, u\}$
37. Because some teachers have driver's licenses, the circles should overlap.
38. Because some cage fighters are red-headed, the circles should overlap.
39. Shirts are clothing, and thus the set of shirts is a subset of the set of clothing. This means one circle should be contained within the other.
40. No airliner is an automobile, so these sets are disjoint, and the circles should not overlap.



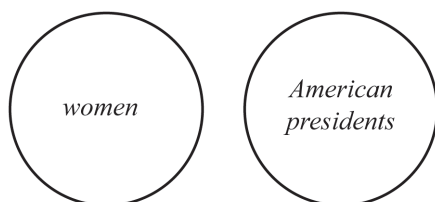
41. Some poets are also plumbers, so the circles should overlap.



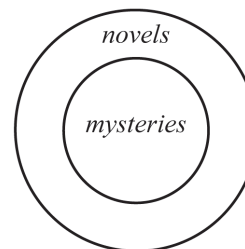
43. No teenager is an octogenarian, so these sets are disjoint, and the circles should not overlap.



42. At the time of writing, no woman has been an American president so these sets are disjoint, and the circles should not overlap. This could change in the future.

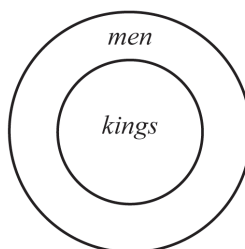


44. Mystery novels are a subset of all novels, so one circle should be placed within the other.



45. b. The subject is *kings*, and the predicate is *men*.

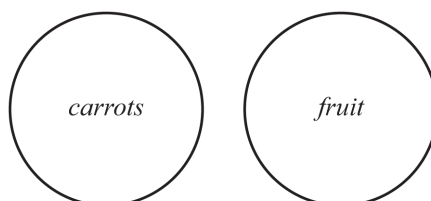
c.



- d. No, the diagram does not show evidence that there is a king that is not a man.

46. b. The subject is *carrots*, and the predicate is *fruit*.

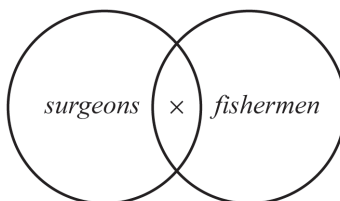
c.



- d. No, since the sets are disjoint, they would have no common members.

47. b. The subject is *surgeons*, and the predicate is *fisherman*.

c.

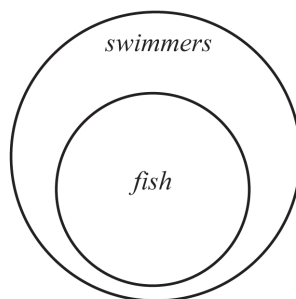


- d. No, the diagram gives no evidence that there are fishermen that are not surgeons.

48. a. All fish are swimmers.

b. The subject is *fish*, and the predicate is *swimmers*.

c.

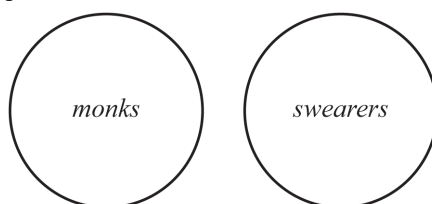


d. No, the diagram does not show evidence that there is a fish that is not a swimmer.

49. a. No monks swear.

b. The subject is *monks*, and the predicate is *swearers*.

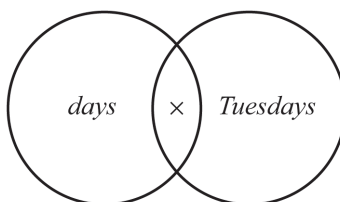
c.



d. No, since the sets are disjoint, they would have no common members.

50. b. The subject is *days*, and the predicate is *Tuesdays*.

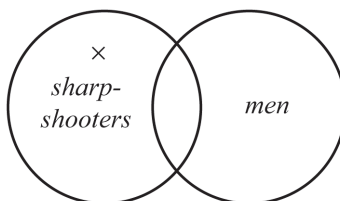
c.



d. No, the diagram gives no evidence that there are days that are not Tuesdays.

51. b. The subject is *sharpshooters*, and the predicate is *men*.

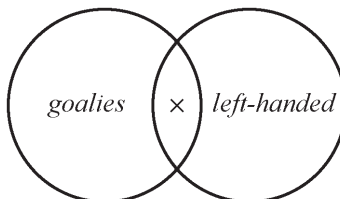
c.



d. No, the diagram gives no evidence that there is a sharpshooter that is a man.

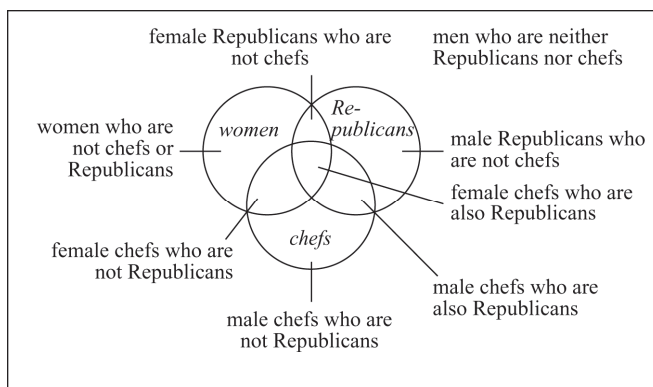
52. b. The subject is *goalies*, and the predicate is *left-handed*.

c.

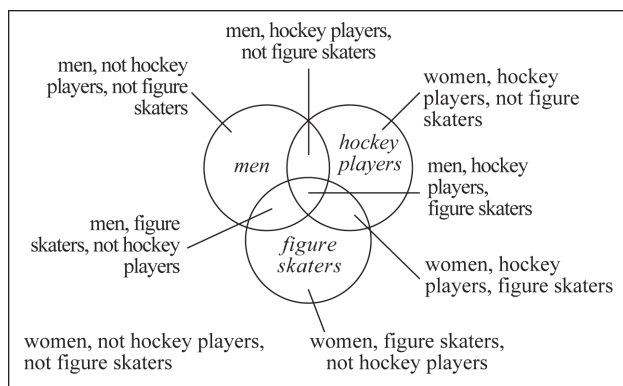


d. No, the diagram gives no evidence that there are right-handed goalies.

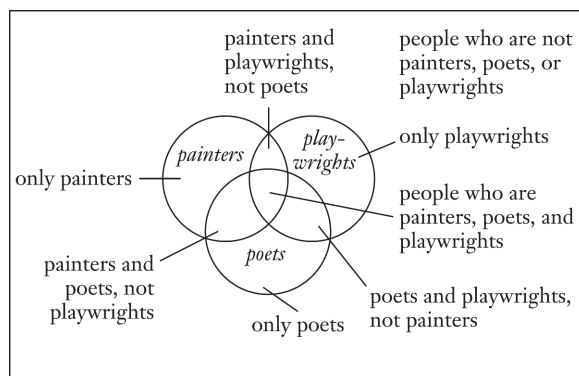
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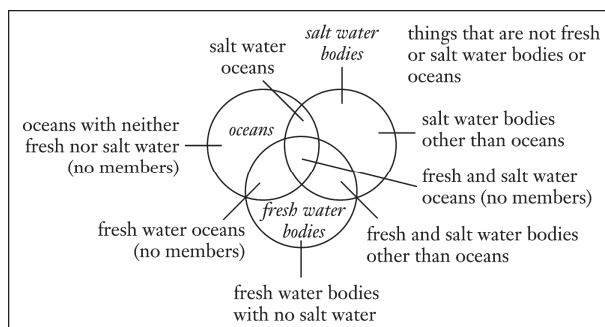
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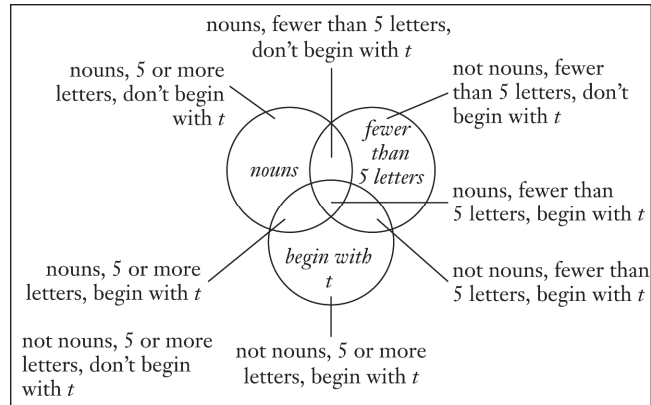
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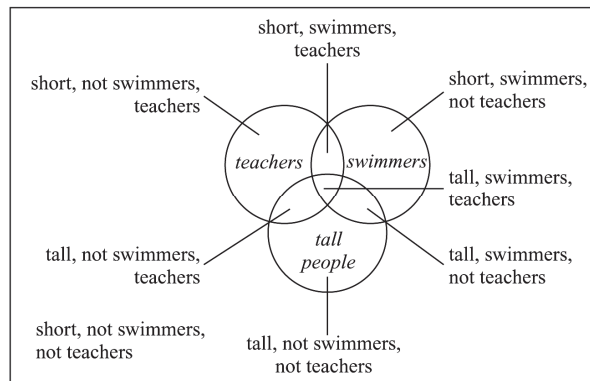
56.



57.



58.



59.

|              | Women                  | Men                    |
|--------------|------------------------|------------------------|
| Right-handed | $200 - 18 = 182$       | $150 - 18 = 132$       |
| Left-Handed  | $0.09 \times 200 = 18$ | $0.12 \times 150 = 18$ |

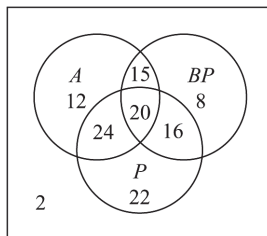
60.

|       | Women | Men |
|-------|-------|-----|
| White | 44%   | 38% |
| Black | 90%   | 79% |

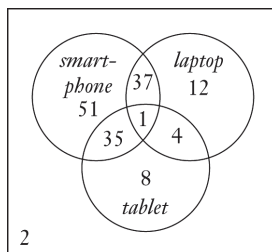
61. a. There are  $15 + 16 = 31$  people at the party that are under 30.  
 b. There are 28 single people at the party that are not under 30.  
 c. There are  $22 + 15 = 37$  married people at the party.  
 d. There are  $22 + 15 + 16 + 28 = 81$  people at the party.
62. a. There are 22 married people at the party that are not under 30.  
 b. There are  $16 + 28 = 44$  single people at the party.  
 c. There are 16 single people at the party who are under 30.  
 d. There are  $22 + 28 = 50$  people at the party that are not under 30.
63. a. There are 16 people at the conference that are employed, have a college degree, and are not local.  
 b. There are  $4 + 20 = 24$  people at the conference that are unemployed and local.  
 c. There are 8 people at the conference that are employed, local, and don't have a college degree.  
 d. There are  $4 + 20 + 8 + 11 = 43$  people that are local at the conference.

64. a. There are 11 people at the conference that are employed, have a college degree, and are local.  
 b. There are  $6 + 16 = 22$  people at the conference that are employed and not local.  
 c. There are 3 people at the conference that are unemployed, not local, and don't have a college degree.  
 d. There are  $4 + 20 + 9 + 8 + 11 + 16 + 6 + 3 = 77$  people at the conference.

65. a.



- b. Add the numbers in the regions that are contained in the  $P$  and  $BP$  circles, to find that 105 people took pain medication or blood pressure medication.  
 c. Add the number of people that are in the  $BP$  circle, but outside the  $A$  circle, to arrive at 24 people.  
 d. Add the number of people that are in the  $BP$  circle. There are 59 patients that took (at least) blood pressure medication.  
 e. Use the region that is contained in the  $P$  circle, but not in the  $A$  and  $BP$  circles, to find that 22 patients took only pain medication.  
 f. Add the numbers in the regions where two circles overlap, excluding the region where all three circles overlap, to find that 55 patients took exactly two medications.
66. a.



- b. Add the number of people that are in any of the regions contained within the circle *smart phone*. There are 124 such people.  
 c. Add the number of people that are in any of the regions contained within the two circles *tablet* and *laptop*. There are 97 such people.  
 d. Use the region that is contained by both the *smart phone* and *laptop* circles, but not contained in the *tablet* circle. There are 37 such people.  
 e. Add the number of people that are in the region of any two intersecting circles, excluding the region common to all three circles. There are 76 such people.  
 f. Use the regions that is contained by either the *smart phone* and *laptop* circles, but not contained in the *tablet* circle. There are 100 such people.

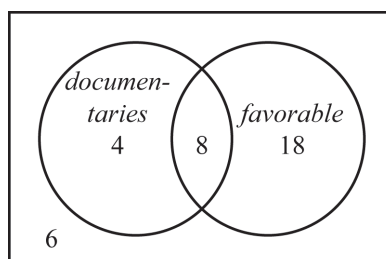
### FURTHER APPLICATIONS

67. a.

|               | Favorable Review | Non-favorable Review | Total |
|---------------|------------------|----------------------|-------|
| Documentaries | 8                | $12 - 8 = 4$         | 12    |
| Feature Films | $24 - 6 = 18$    | 6                    | 24    |
| Total         | $8 + 18 = 26$    | $4 + 6 = 10$         | 36    |

67. (continued)

b.



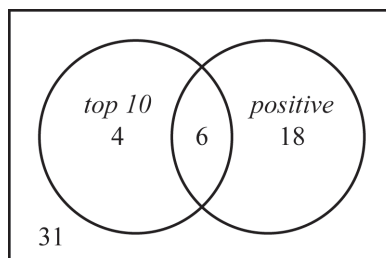
c. 4 documentaries received unfavorable reviews.

d. 18 feature films received favorable reviews.

68. a.

|            | Tested Positive | Tested Negative | Total          |
|------------|-----------------|-----------------|----------------|
| Top 10     | 6               | $10 - 6 = 4$    | 10             |
| Not top 10 | $24 - 6 = 18$   | $35 - 4 = 31$   | $18 + 31 = 49$ |
| Total      | 24              | 35              | $10 + 49 = 59$ |

b.



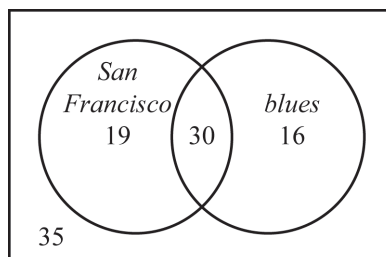
c. 31 runners that tested negative did not finish in the top 10.

d. 59 runners were tested.

69. a.

|               | Blues          | Country        | Total           |
|---------------|----------------|----------------|-----------------|
| Nashville     | $51 - 35 = 16$ | 35             | $100 - 49 = 51$ |
| San Francisco | 30             | 19             | $30 + 19 = 49$  |
| Total         | $16 + 30 = 46$ | $35 + 19 = 54$ | 100             |

b.



c. 16 Nashville respondents preferred blues.

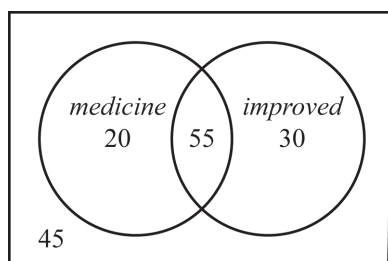
d. 46 respondents preferred blues.

70. a.

|          | Improve        | Did not improve | Total           |
|----------|----------------|-----------------|-----------------|
| Medicine | 55             | $75 - 55 = 20$  | 75              |
| Placebo  | $75 - 45 = 30$ | 45              | 75              |
| Total    | $55 + 30 = 85$ | $20 + 45 = 65$  | $75 + 75 = 150$ |

70. (continued)

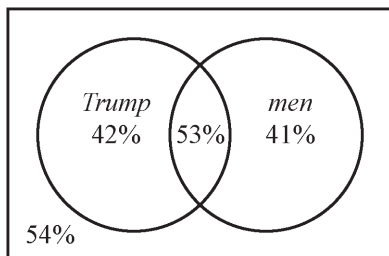
b.



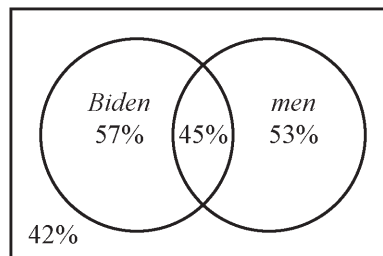
c. 20 people who received medicine did not improve.

d. 30 who received the placebo improved.

71. Answers will vary.



72. Answers will vary



73. a. The two-way table should look like this:

|                   | Subject did not lie. | Subject lied. | Total          |
|-------------------|----------------------|---------------|----------------|
| Polygraph: lie    | $57 - 42 = 15$       | 42            | 57             |
| Polygraph: no lie | 32                   | $51 - 42 = 9$ | $32 + 9 = 41$  |
| Total             | $15 + 32 = 47$       | 51            | $57 + 41 = 98$ |

b.  $\frac{42 + 32}{98} = 0.755 = 75.5\%$

c.  $\frac{15 + 9}{98} = 0.245 = 24.5\%$

d. Answers will vary.

74. a. The two-way table should look like this:

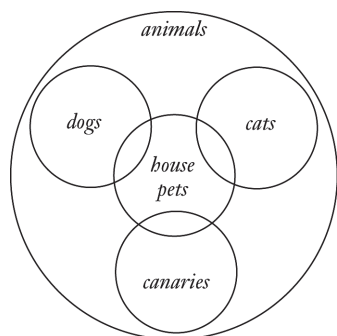
|                            | Irregular seat belt use | Regular seat belt use | Total                |
|----------------------------|-------------------------|-----------------------|----------------------|
| Texted while driving       | 1737                    | $3785 - 1737 = 2048$  | 3785                 |
| Did not text while driving | $3682 - 1737 = 1945$    | 2775                  | $1945 + 2775 = 4720$ |
| Total                      | 3682                    | $2048 + 2775 = 4823$  | $3785 + 4720 = 8505$ |

b.  $\frac{1737}{8505} = 0.204 = 20.4\%$

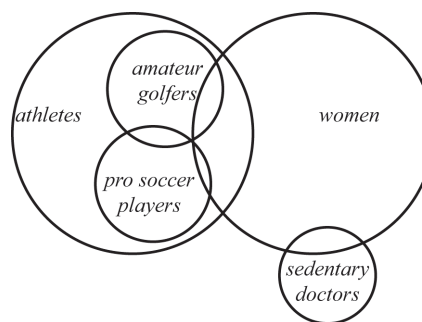
c.  $\frac{2048}{8505} = 0.241 = 24.1\%$

d. Answers will vary.

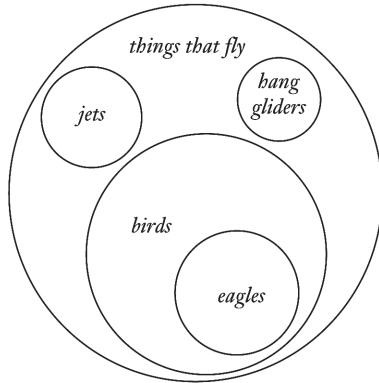
75.



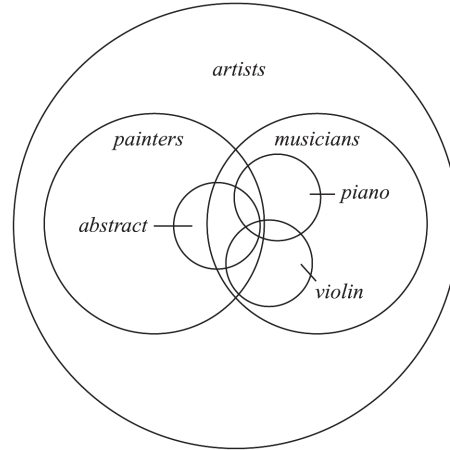
76.



77.



78.



## UNIT 1D: ANALYZING ARGUMENTS

### THINK ABOUT IT

Pg. 56. Clear market research would be the best evidence on which to build the case. For example, use focus groups to react to the story, or show sample movie clips to groups of typical viewers.

Pg. 57. Answers will vary. Recall that inductive arguments make a case for a general conclusion from more specific premises, while deductive arguments make a case for a specific conclusion from more general premises. This would be a good topic for a discussion either during or outside of class.

Pg. 59. Changing John Kennedy to a person that was not president does not affect the argument's structure or the truth of the premises. Since the conclusion is now false, it is invalid.

Pg. 61. Changing from heroin to aspirin does not affect the argument's structure, so it is still valid. It is now also sound, because this change makes both premises true.

### QUICK QUIZ

QQ1. **b.** The only way to prove a statement true beyond all doubt is with a valid and sound deductive argument.

QQ2. **c.** A deductive argument that is valid has a logical structure that implies its conclusion from its premises.

QQ3. **c.** If a deductive argument is not valid, it cannot be sound.

QQ4. **a.** Premise 1 claims the set of *knights* is a subset of the set of *heroes*, and Premise 2 claims Paul is a hero, which means the X must reside within the *hero* circle. However, we cannot be sure whether the X should fall within or outside the *knights* circle, so it belongs on the border.

QQ5. **c.** Diagram *a* in question 4 is the correct diagram for its argument, and since X lies on the border of the *knights* circle, Paul may or may not be a knight.

QQ6. **b.** The argument is of the form *denying the conclusion*, and one can always conclude *p* is not true in such arguments. (Whether the argument is sound is another question).

QQ7. **c.** This argument is of the form *affirming the conclusion*, and it is always invalid, which means we can conclude nothing about *p*.

QQ8. **c.** A chain of conditionals from *a* to *d* is necessary before we can claim the argument is valid.

QQ9. **b.** The side opposite the right angle in a right triangle is always the longest, and it's called the hypotenuse.

QQ10. **b.** The Pythagorean theorem states that  $c^2 = 4^2 + 5^2 = 16 + 25 = 41$ .

### READING QUESTIONS

1. An inductive argument makes a case for a general conclusion from more specific premises, while a deductive argument makes a case for a specific conclusion from more general premises.

2. An inductive argument is strong if it makes a compelling case for its conclusion. It is weak if its conclusion is not well supported by its premises. An inductive argument cannot prove its conclusion true, so we evaluate it only in terms of its strength, not validity.
3. An argument is valid if its conclusion follows necessarily from its premises, regardless of the truth of those premises or conclusions. An argument is sound if it is valid and its premises are all true, otherwise it is unsound. By definition, sound deductive arguments must be valid.
4. First, draw a Venn diagram that represents all the information contained in the premises. If the diagram must be drawn in a way that confirms the conclusion, then the argument is valid. Otherwise, the argument is not valid.
5. Examples will vary. The four basic conditional arguments are: Affirming the Hypothesis, Affirming the Conclusion, Denying the Hypothesis, and Denying the Conclusion.
6. Examples will vary. A chain of conditionals follows the pattern shown below.  
*Premise: if  $p$ , then  $q$ .*  
*Premise: if  $q$ , then  $r$ .*  
*Conclusion: if  $p$ , then  $r$ .*
7. Inductive logic cannot be used to prove a mathematical theorem. No matter how many true cases are provided, a false case may be just around the corner.
8. The process of seeking inductive evidence can be very useful when you are having difficulty remembering whether a particular theorem or mathematical rule applies. It often helps to try a few test cases and see if the rule works.

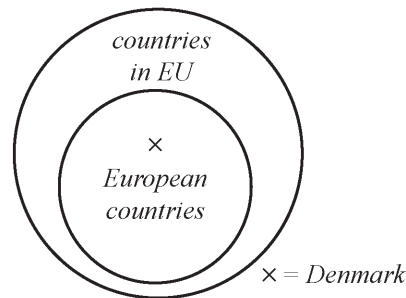
**DOES IT MAKE SENSE?**

9. Does not make sense. One cannot prove a conclusion beyond all doubt with an inductive argument.
10. Makes sense. An inductive argument is judged on its strength.
11. Makes sense. As long as the logic of a deductive argument is valid, if one accepts the truth (or soundness) of the premises, the conclusion necessarily follows.
12. Does not make sense. A deductive argument which is valid is not necessarily sound, and therefore the conclusion may not be true.
13. Does not make sense. This argument is of the form *affirming the conclusion*, and it is always invalid.
14. Make sense. Mathematicians generally do not claim a theorem to be true until it is proved with a valid and sound deductive argument.

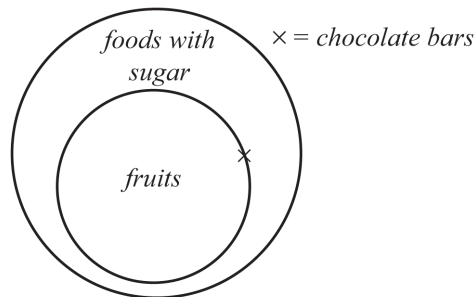
**BASIC SKILLS AND CONCEPTS**

15. This is an inductive argument because it makes the case for a general conclusion based on many specific observations.
16. This is a deductive argument because a specific conclusion is deduced from more general premises.
17. This is an inductive argument because it makes the case for a general conclusion based on many specific observations.
18. This is an inductive argument because it makes the case for a general conclusion based on many specific observations.
19. This is a deductive argument because a specific conclusion is deduced from more general premises.
20. This is a deductive argument because a specific conclusion is deduced from more general premises.
21. This is a deductive argument because a specific conclusion is deduced from more general premises.
22. This is an inductive argument because it makes the case for a general conclusion based on many specific observations.

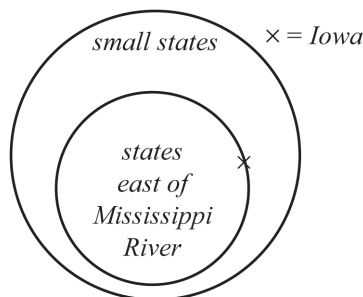
23. The premises are true, though the argument is weak (it speaks to only six of many painters). The conclusion is false.
24. The premises are generally true, and the argument is weak. The conclusion may not be true (it is possible to eat dessert and not gain weight).
25. The premises are true, the argument *seems* moderately strong, and the conclusion is false.
26. The premises are true, the argument seems moderately strong, and the conclusion is true.
27. The premises are true and the argument is moderately strong. The conclusion is correct.
28. The premises are true, and the argument is moderately strong. The conclusion is true.
29. a. The argument is valid.



- b. The argument is not sound since the first premise is false.
30. a. The argument is not valid.

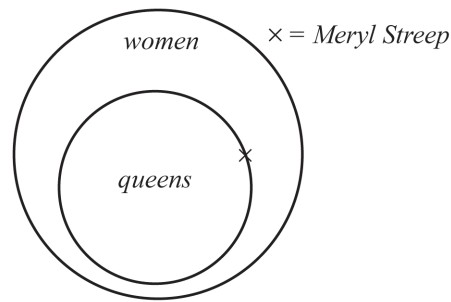


- b. The diagram shows the argument is invalid, even though the premises are true. Because it is invalid, the argument cannot be sound.
31. a. The argument is not valid.



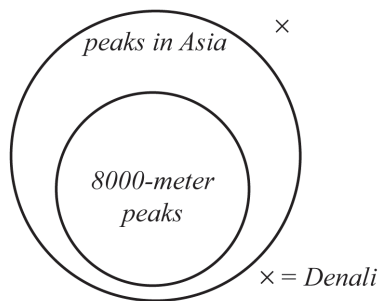
- b. The diagram shows the argument is invalid, even though the premises are questionable. Because it is invalid, the argument cannot be sound.

32. a. The argument is not valid.



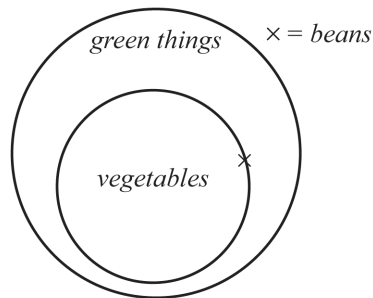
b. As shown in the diagram, the argument is not valid because we cannot place the X within the *queens* circle based on the second premise alone. Though the premises are true, the argument is not sound.

33. a. The argument is valid.



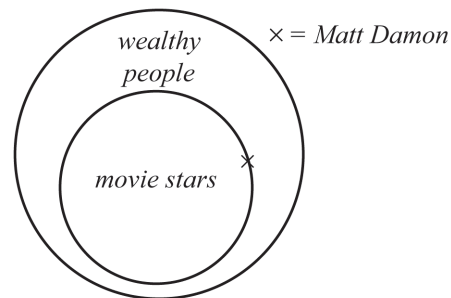
b. The premises are true, and the argument is sound.

34. a. The argument is not valid.



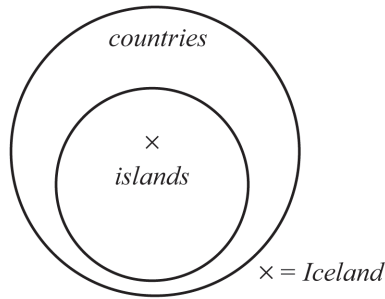
b. The argument is not sound since the first premise is false.

35. a. The argument is not valid.



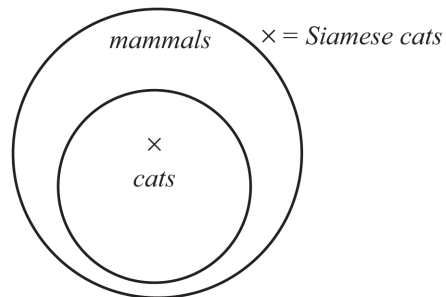
b. The diagram shows the argument is invalid, so the argument cannot be sound.

36. a. The argument is valid.



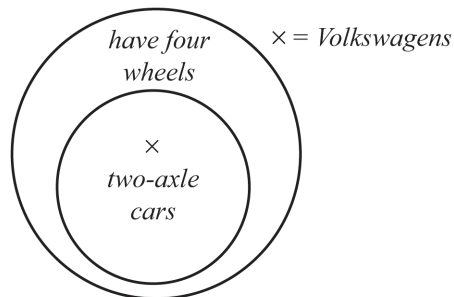
- b. The first premise is not true, so the argument is not sound.

37. a. Affirming the hypothesis – this form is always valid, as confirmed by the diagram.



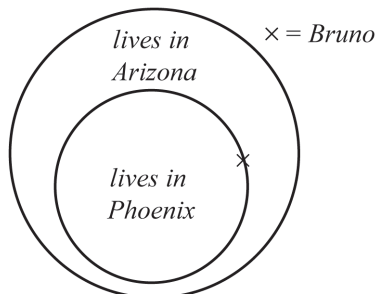
- b. The premises are true, and thus the argument is sound.

38. a. Affirming the hypothesis – this form is always valid, as confirmed by the diagram.



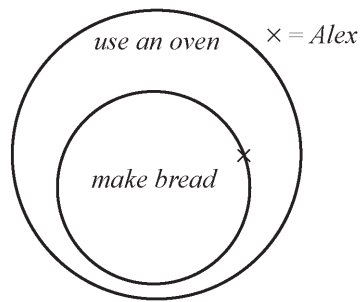
- b. The premises are true, and thus the argument is sound.

39. a. Affirming the conclusion – this form is always invalid, as confirmed by the diagram.



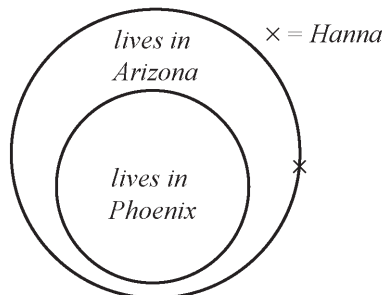
- b. Since it is invalid, the argument cannot be sound.

40. a. Affirming the conclusion – this form is always invalid, as confirmed by the diagram.



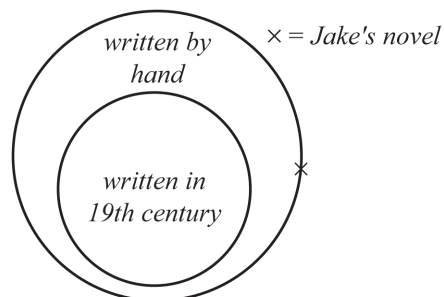
- b. Since it is invalid, the argument cannot be sound.

41. a. Denying the hypothesis – this form is always invalid, as confirmed by the diagram.



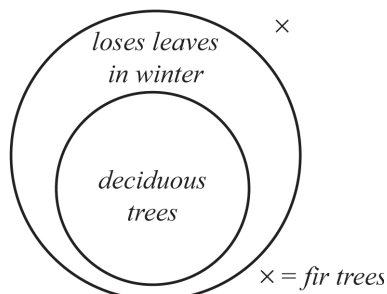
- b. Since it is invalid, the argument cannot be sound.

42. a. Denying the hypothesis – this form is always invalid, as confirmed by the diagram.



- b. Since it is invalid, the argument cannot be sound.

43. a. Denying the conclusion – this form is always valid, as confirmed by the diagram.



- b. The premises are true, and thus the argument is sound.