Precalculus A Right Triangle Approach 5th edition Ratti Solutions Manual

SKU: 9780137519415-SOLUTIONS

INSTRUCTOR'S SOLUTIONS MANUAL

BEVERLY FUSFIELD

PRECALCULUS
A RIGHT TRIANGLE APPROACH
FIFTH EDITION

J. S. Ratti

University of South Florida

Marcus McWaters

University of South Florida

Leslaw A. Skrzypek

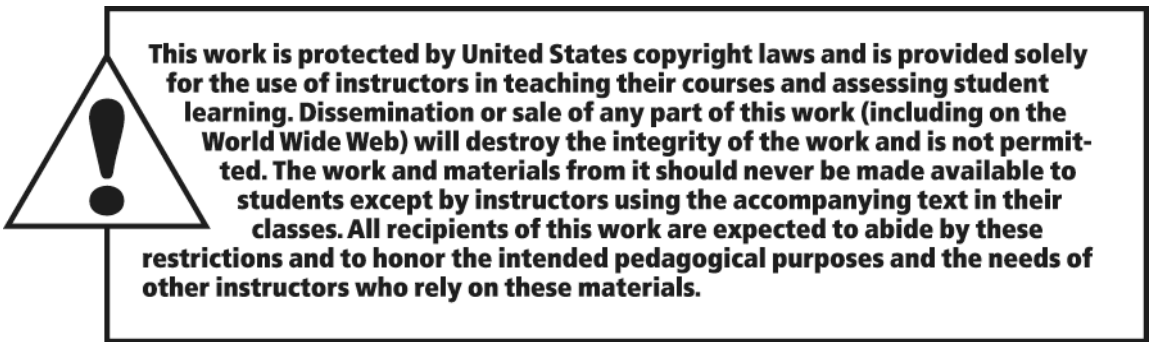
University of South Florida

Jessica Bernards

Portland Community College

Wendy Fresh

Portland Community College



ISBN-13: 978-0-13-751980-4
ISBN-10: 0-13-751980-X

CONTENTS

Chapter P Basic Concepts of Algebra

P.1 The Real Numbers and Their Properties

Practice Problems

1. Let $x = 2.132132132\ldots$
Then, $1000x = 2132.132132\ldots$
$1000x = 2132.132132\ldots$
$x = \quad 2.132132\ldots$
$999x = 2130$
$$x = \frac{2130}{999} = \frac{710}{333}$$

2. a. Natural numbers: 2, 7

b. Whole numbers: 0, 2, 7

c. Integers: $-6, -\frac{21}{7} = -3, 0, 2, 7$

d. Rational numbers:
$$-6, -\frac{21}{7}(=-3), -\frac{1}{2}, 0, \frac{4}{3}, 2, 7$$

e. Irrational numbers: $\sqrt{3}, \sqrt{17}$

f. Real numbers: the set B

3. a. $2^3 = 2 \cdot 2 \cdot 2 = 8$

b. $(3a)^2 = (3a)(3a) = 9a^2$

c. $\left(\frac{1}{2}\right)^4 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$

4. a. $-2 < 0$ true because -2 is to the left of 0 on the number line.

b. $5 \le 7$ true because 5 is to the left of 7 on the number line.

c. $-4 > -1$ false because -4 is to the left of -1 on the number line.

5. $A = \{-3, -1, 0, 1, 3\}$, $B = \{-4, -2, 0, 2, 4\}$
$A \cap B = \{0\}$
$A \cup B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

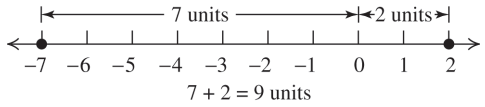
6. a. $I_1 \cup I_2 = (-\infty, \infty)$

b. $I_1 \cap I_2 = [-2, 5)$

7. a. $|-10| = 10$

b. $|3 - 4| = |-1| = 1$

c. $|2(-3) + 7| = |-6 + 7| = |1| = 1$

8.


$d(-7, 2) = |-7 - 2| = |-9| = 9$

9. a. $(-3) \cdot 5 + 20 = -15 + 20 = 5$

b. $5 - 12 \div 6 \cdot 2 = 5 - 2 \cdot 2 = 5 - 4 = 1$

c. $\frac{9-1}{4} - 5 \cdot 7 = \frac{8}{4} - 5 \cdot 7 = 2 - 35 = -33$

d. $-3 + (x-4)^2$ for $x = 6$
$-3 + (6-4)^2 = -3 + 2^2 = -3 + 4 = 1$

10. a. $\frac{7}{4} + \frac{3}{8} = \frac{7 \cdot 2}{4 \cdot 2} + \frac{3}{8} = \frac{7 \cdot 2 + 3}{8} = \frac{17}{8}$

b. $\frac{8}{3} - \frac{2}{5} = \frac{8 \cdot 5}{3 \cdot 5} - \frac{2 \cdot 3}{5 \cdot 3} = \frac{8 \cdot 5 - 2 \cdot 3}{3 \cdot 5} = \frac{34}{15}$

c. $\frac{9}{14} \cdot \frac{7}{3} = \frac{9 \cdot 7}{14 \cdot 3} = \frac{3 \cdot \cancel{3} \cdot \cancel{7}}{2 \cdot \cancel{7} \cdot \cancel{3}} = \frac{3}{2}$

d. $\frac{\frac{5}{8}}{\frac{15}{16}} = \frac{5}{8} \cdot \frac{16}{15} = \frac{5 \cdot 16}{8 \cdot 15} = \frac{\cancel{5} \cdot 2 \cdot \cancel{8}}{\cancel{8} \cdot 3 \cdot \cancel{5}} = \frac{2}{3}$

11. a. $(x-2) \div 3 + x = (3-2) \div 3 + 3$
$= 1 \div 3 + 3 = \frac{1}{3} + 3 = \frac{10}{3}$

b. $7 - \frac{x}{|y|} = 7 - \frac{-1}{|3|} = 7 + \frac{1}{3} = \frac{22}{3}$

12. $°\text{C} = \frac{39}{3} + 4 = 13 + 4 = 17°\text{C}$

Concepts and Vocabulary

1. Whole numbers are formed by adding the number zero to the set of natural numbers.

2. The number -3 is an integer, but it is also a rational number and a real number.

3. If $a < b$, then a is to the left of b on the number line.

4. If a real number is not a rational number, it is an irrational number.

5. True

6. False. $\frac{-5}{2} = -2\frac{1}{2}$

7. True

8. False. An example is $\left(2+\sqrt{2}\right)+\left(2-\sqrt{2}\right)=4$, which is rational.

Building Skills

9. $0.\overline{3}$, repeating

10. $0.\overline{6}$, repeating

11. -0.8, terminating

12. -0.25, terminating

13. $0.\overline{27}$, repeating

14. $0.\overline{3}$, repeating

15. $3.1\overline{6}$, repeating

16. $2.7\overline{3}$, repeating

17. $3.75 = \frac{375}{100} = \frac{15}{4}$

18. $-2.35 = -\frac{235}{100} = -\frac{47}{20}$

19. $10x = 53.\overline{3}$
$x = 5.\overline{3}$
$9x = 48$
$x = \frac{48}{9} = \frac{16}{3}$
$-5.\overline{3} = -\frac{16}{3}$

20. $10x = 96.\overline{6}$
$x = 9.\overline{6}$
$9x = 87$
$x = \frac{87}{9} = \frac{29}{3}$

21. $100x = 213.\overline{13}$
$x = 2.\overline{13}$
$99x = 211$
$x = \frac{211}{99}$

22. $100x = 323.\overline{23}$
$x = 3.\overline{23}$
$99x = 320$
$x = \frac{320}{99}$

23. $100x = 452.3\overline{23}$
$x = 4.5\overline{23}$
$99x = 447.8$
$x = \frac{447.8}{99} = \frac{4478}{990} = \frac{2239}{495}$

24. $100x = 142.35\overline{35}$
$x = 1.42\overline{35}$
$99x = 140.93$
$x = \frac{140.93}{99} = \frac{14,093}{9900}$

25. Rational **26.** Rational

27. Rational **28.** Rational

29. Rational **30.** Rational

31. Irrational **32.** Irrational

Exercises 33-38 refer to the set

$$A = \left\{-19, -\frac{12}{3}, -\sqrt{3}, 0, 2, \sqrt{10}, \frac{17}{4}, 11\right\}$$

33. Natural numbers: 2, 11

34. Whole numbers: 0, 2, 11

35. Integers: $-19, -\frac{12}{3} = -4, 0, 2, 11$

36. Rational numbers: $-19, -\frac{12}{3}, 0, 2, \frac{17}{4}, 11$

37. Irrational numbers: $\sqrt{3}, \sqrt{10}$

38. Real numbers: All numbers in set A are real numbers.

39. 10^3
base: 10; exponent: 3
$10^3 = 10 \cdot 10 \cdot 10 = 1000$

40. 5^4
base: 5; exponent: 4
$5^4 = 5 \cdot 5 \cdot 5 \cdot 5 = 625$

41. $\left(\frac{2}{3}\right)^3$
base: $\frac{2}{3}$; exponent: 3
$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$

42. $\left(\frac{5}{2}\right)^4$
base: $\frac{5}{2}$; exponent: 4
$\left(\frac{5}{2}\right)^4 = \frac{5}{2} \cdot \frac{5}{2} \cdot \frac{5}{2} \cdot \frac{5}{2} = \frac{625}{16}$

43. $(-2)^3$

base: –2; exponent: 3

$(-2)^3 = (-2)\cdot(-2)\cdot(-2) = -8$

44. $\left(-\frac{1}{2}\right)^4$

base: $-\frac{1}{2}$; exponent: 4

$\left(-\frac{1}{2}\right)^4 = \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) = \frac{1}{16}$

45. $2\cdot 3^4$

base: 3; exponent: 4

$2\cdot 3^4 = 2\cdot 3\cdot 3\cdot 3\cdot 3 = 2\cdot 81 = 162$

46. $5\cdot\left(\frac{1}{3}\right)^3$

base: $\frac{1}{3}$; exponent: 3

$5\cdot\left(\frac{1}{3}\right)^3 = 5\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} = 5\cdot\frac{1}{27} = \frac{5}{27}$

47. $-2\cdot 3^4$

base: 3; exponent: 4

$-2\cdot 3^4 = -2\cdot 3\cdot 3\cdot 3\cdot 3 = -2\cdot 81 = -162$

48. $-3\cdot(-2)^4$

base: –2; exponent: 4

$-3\cdot(-2)^4 = -3(-2)(-2)(-2)(-2)$
$= -3\cdot 16 = -48$

49. $-3\cdot(-2)^5$

base: –2; exponent: 5

$-3\cdot(-2)^5 = -3(-2)(-2)(-2)(-2)(-2)$
$= -3\cdot(-32) = 96$

50. $-5\cdot(-3)^2$

base: –3; exponent: 2

$-5\cdot(-3)^2 = -5(-3)(-3) = -5\cdot 9 = -45$

51. $3 > -2$

52. $-3 < -2$

53. $\frac{1}{2} \ge \frac{1}{2}$

54. $x < x+1$

55. $5 \le 2x$

56. $x - 1 > 2$

57. $-x > 0$

58. $x < 0$

59. $2x + 7 \le 14$

60. $2x + 3 \le 5$

61. $4 = \frac{24}{6}$

62. $-5 < -2$

63. $-4 < 0$

64. $-\frac{9}{2} = -4\frac{1}{2}$

65. $A \cup B = \{-4, -3, -2, 0, 1, 2, 3, 4\}$

66. $A \cap B = \{0, 2, 4\}$

67. $A \cap C = \{-4, -2, 0, 2\}$

68. $B \cup C = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

69. $(B \cap C) \cup A = \{-3, 0, 2\} \cup A$
$= \{-4, -3, -2, 0, 2, 4\}$

70. $(A \cup C) \cap B = \{-4, -3, -2, -1, 0, 2, 4\} \cap B$
$= \{-3, 0, 2, 4\}$

71. $(A \cup B) \cap C = \{-4, -3, -2, 0, 1, 2, 3, 4\} \cap C$
$= \{-4, -3, -2, 0, 2\}$

72. $(A \cup B) \cup C = \{-4, -3, -2, 0, 1, 2, 3, 4\} \cup C$
$= \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

73. $I_1 \cup I_2 = (-2, 5)$; $I_1 \cap I_2 = [1, 3]$

74. $I_1 \cup I_2 = [1, 7]$; $I_1 \cap I_2 = (3, 5)$

75. $I_1 \cup I_2 = (-6, 10)$; $I_1 \cap I_2 = \varnothing$

76. $I_1 \cup I_2 = (-\infty, \infty)$; $I_1 \cap I_2 = \varnothing$

77. $I_1 \cup I_2 = (-\infty, \infty)$; $I_1 \cap I_2 = [2, 5)$

78. $I_1 \cup I_2 = (-2, \infty)$; $I_1 \cap I_2 = (0, \infty)$

79. $I_1 \cup I_2 = (-\infty, \infty)$; $I_1 \cap I_2 = [-1, 3) \cup [5, 7]$

80. $I_1 \cup I_2 = (-\infty, 2] \cup (6, \infty)$; $I_1 \cap I_2 = [-3, 0]$

81. $|-4| = 4$

82. $-|-17| = -17$

83. $\left|\frac{5}{-7}\right| = \frac{5}{7}$

84. $\left|\frac{-3}{5}\right| = \frac{3}{5}$

85. $\left|5 - \sqrt{2}\right| = 5 - \sqrt{2}$

86. $\left|\sqrt{2} - 5\right| = 5 - \sqrt{2}$

87. $\left|\sqrt{3} - 2\right| = 2 - \sqrt{3}$

88. $|3 - \pi| = \pi - 3$

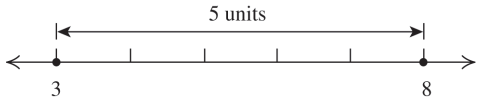
89. $\frac{8}{|-8|} = \frac{8}{8} = 1$

90. $\frac{-8}{|8|} = \frac{-8}{8} = -1$

91. $\left|5-\left|-7\right|\right| = \left|5-7\right| = \left|-2\right| = 2$

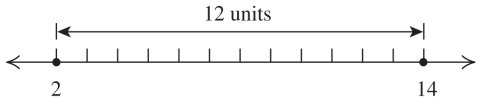
92. $\left|\left|4\right|-\left|7\right|\right| = \left|4-7\right| = \left|-3\right| = 3$

93.



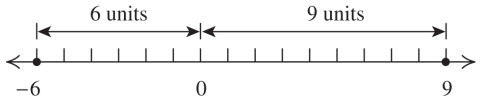
$d(3,8) = |3-8| = |-5| = 5$

94.



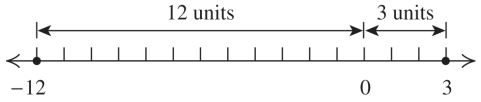
$d(2,14) = |2-14| = |-12| = 12$

95.



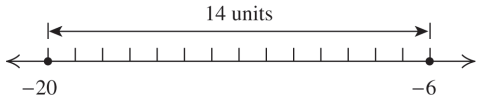
$d(-6,9) = |-6-9| = |-15| = 15$

96.



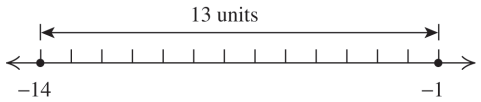
$d(-12,3) = |-12-3| = |-15| = 15$

97.



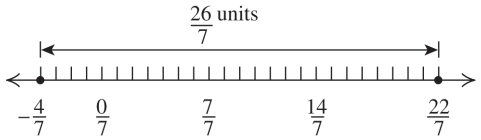
$d(-20,-6) = |-20-(-6)| = |-14| = 14$

98.



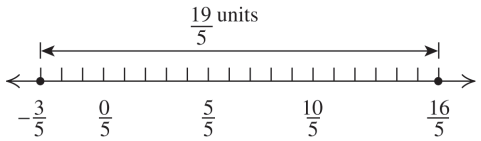
$d(-14,-1) = |-14-(-1)| = |-13| = 13$

99.



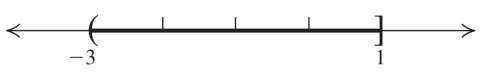
$$d\left(\frac{22}{7},-\frac{4}{7}\right) = \left|\frac{22}{7}-\left(-\frac{4}{7}\right)\right| = \left|\frac{26}{7}\right| = \frac{26}{7}$$

100.



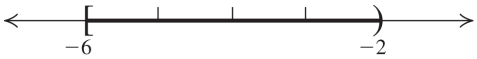
$$d\left(\frac{16}{5},-\frac{3}{5}\right) = \left|\frac{16}{5}-\left(-\frac{3}{5}\right)\right| = \left|\frac{19}{5}\right| = \frac{19}{5}$$

101.



$-3 < x \le 1$

102.



$-6 \le x < -2$

103.

$x \ge -3$

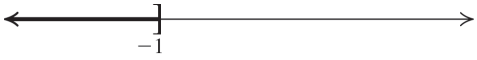
104.

$x \ge 0$

105.

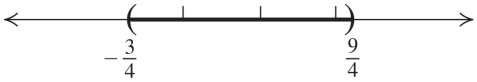
$x \le 5$

106.



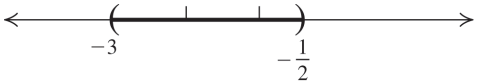
$x \le -1$

107.



$$-\frac{3}{4} < x < \frac{9}{4}$$

108.



$$-3 < x < -\frac{1}{2}$$

109. $4(x+1) = 4x+4$

110. $(-3)(2-x) = -6+3x$

111. $5(x-y+1) = 5x-5y+5$

112. $2(3x+5-y) = 6x+10-2y$

113. Additive inverse: -5; reciprocal: $\frac{1}{5}$

114. Additive inverse: $\frac{2}{3}$; reciprocal: $-\frac{3}{2}$

115. Additive inverse: 0; no reciprocal

116. Additive inverse: -1.7; reciprocal: $\frac{10}{17}$

117. Additive inverse

118. Additive inverse

119. Multiplicative identity

120. Multiplicative identity

121. Associative property of multiplication

122. Associative property of multiplication

123. Multiplicative inverse

124. Multiplicative inverse

125. Additive identity

126. Additive identity

127. Associative property of addition

128. Commutative and associative properties of addition

129. $\frac{3}{5}+\frac{4}{3}=\frac{3\cdot 3}{5\cdot 3}+\frac{4\cdot 5}{3\cdot 5}=\frac{9+20}{15}=\frac{29}{15}$

130. $\frac{7}{10}+\frac{3}{4}=\frac{7\cdot 2}{10\cdot 2}+\frac{3\cdot 5}{4\cdot 5}=\frac{14+15}{20}=\frac{29}{20}$

131. $\frac{6\cdot 7}{5\cdot 7}+\frac{5\cdot 5}{7\cdot 5}=\frac{42+25}{35}=\frac{67}{35}$

132. $\frac{9}{2}+\frac{5}{12}=\frac{9\cdot 6}{2\cdot 6}+\frac{5}{12}=\frac{54+5}{12}=\frac{59}{12}$

133. $\frac{5}{6}+\frac{3}{10}=\frac{5\cdot 5}{6\cdot 5}+\frac{3\cdot 3}{10\cdot 3}=\frac{25+9}{30}=\frac{34}{30}=\frac{17}{15}$

134. $\frac{8}{15}+\frac{2}{9}=\frac{8\cdot 3}{15\cdot 3}+\frac{2\cdot 5}{9\cdot 5}=\frac{24+10}{45}=\frac{34}{45}$

135. $\frac{5}{8}-\frac{9}{10}=\frac{5\cdot 5}{8\cdot 5}-\frac{9\cdot 4}{10\cdot 4}=\frac{25-36}{40}=-\frac{11}{40}$

136. $\frac{7}{8}-\frac{1}{5}=\frac{7\cdot 5}{8\cdot 5}-\frac{1\cdot 8}{5\cdot 8}=\frac{35-8}{40}=\frac{27}{40}$

137. $\frac{5}{9}-\frac{7}{11}=\frac{5\cdot 11}{9\cdot 11}-\frac{7\cdot 9}{11\cdot 9}=\frac{55-63}{99}=-\frac{8}{99}$

138. $\frac{5}{8}-\frac{7}{12}=\frac{5\cdot 3}{8\cdot 3}-\frac{7\cdot 2}{12\cdot 2}=\frac{15-14}{24}=\frac{1}{24}$

139. $\frac{2}{5}-\frac{1}{2}=\frac{2\cdot 2}{5\cdot 2}-\frac{1\cdot 5}{2\cdot 5}=\frac{4-5}{10}=-\frac{1}{10}$

140. $\frac{1}{4}-\frac{1}{6}=\frac{1\cdot 3}{4\cdot 3}-\frac{1\cdot 2}{6\cdot 2}=\frac{3-2}{12}=\frac{1}{12}$

141. $\frac{3}{4}\cdot\frac{8}{27}=\frac{3\cdot 8}{4\cdot 27}=\frac{\cancel{3}\cdot 2\cdot\cancel{4}}{\cancel{4}\cdot\cancel{3}\cdot 9}=\frac{2}{9}$

142. $\frac{9}{7}\cdot\frac{14}{27}=\frac{9\cdot 14}{7\cdot 27}=\frac{\cancel{9}\cdot 2\cdot\cancel{7}}{\cancel{7}\cdot 3\cdot\cancel{9}}=\frac{2}{3}$

143. $\frac{\frac{8}{5}}{\frac{16}{15}}=\frac{8}{5}\cdot\frac{15}{16}=\frac{8\cdot 15}{5\cdot 16}=\frac{\cancel{8}\cdot 3\cdot\cancel{5}}{\cancel{5}\cdot 2\cdot\cancel{8}}=\frac{3}{2}$

144. $\frac{\frac{5}{6}}{\frac{15}{6}}=\frac{5}{6}\cdot\frac{6}{15}=\frac{5\cdot 6}{6\cdot 15}=\frac{\cancel{5}\cdot\cancel{6}}{\cancel{6}\cdot 3\cdot\cancel{5}}=\frac{1}{3}$

145. $\frac{\frac{7}{8}}{\frac{21}{16}}=\frac{7}{8}\cdot\frac{16}{21}=\frac{7\cdot 16}{8\cdot 21}=\frac{\cancel{7}\cdot 2\cdot\cancel{8}}{\cancel{8}\cdot 3\cdot\cancel{7}}=\frac{2}{3}$

146. $\frac{\frac{3}{10}}{\frac{7}{15}}=\frac{3}{10}\cdot\frac{15}{7}=\frac{3\cdot 15}{10\cdot 7}=\frac{3\cdot 3\cdot\cancel{5}}{2\cdot\cancel{5}\cdot 7}=\frac{9}{14}$

147. $5\cdot\frac{3}{10}-\frac{1}{2}=\frac{\cancel{5}}{1}\cdot\frac{3}{2\cdot\cancel{5}}-\frac{1}{2}=\frac{3}{2}-\frac{1}{2}=\frac{2}{2}=1$

148. $2\cdot\frac{7}{2}-\frac{3}{2}=\frac{14}{2}-\frac{3}{2}=\frac{11}{2}$

149. $3\cdot\frac{2}{15}-\frac{1}{3}=\frac{3\cdot 2}{1\cdot 15}-\frac{1}{3}=\frac{\cancel{3}\cdot 2}{1\cdot\cancel{3}\cdot 5}-\frac{1}{3}=\frac{2}{5}-\frac{1}{3}$
$=\frac{2\cdot 3}{5\cdot 3}-\frac{1\cdot 5}{3\cdot 5}=\frac{6}{15}-\frac{5}{15}=\frac{1}{15}$

150. $2\cdot\frac{5}{3}-\frac{3}{2}=\frac{10}{3}-\frac{3}{2}=\frac{20}{6}-\frac{9}{6}=\frac{11}{6}$

151. $2(x+y)-3y=2(3+(-5))-3(-5)$
$=2(-2)-(-15)=-4+15=11$

152. $-2(x+y)+5y=-2(3+(-5))+5(-5)$
$=-2(-2)+(-25)$
$=4+(-25)=-21$

153. $3|x|-2|y|=3|3|-2|-5|=3(3)-2(5)$
$=9-10=-1$

154. $7|x-y|=7|3-(-5)|=7|8|=7(8)=56$

155. $\frac{x-3y}{2}+xy=\frac{3-3(-5)}{2}+3(-5)$
$=\frac{3-(-15)}{2}+(-15)$
$=\frac{18}{2}+(-15)=9+(-15)=-6$

156. $\dfrac{y+3}{x} - xy = \dfrac{-5+3}{3} - 3(-5)$
$= \dfrac{-2}{3} - (-15) = \dfrac{43}{3}$

157. $\dfrac{2(1-2x)}{y} - (-x)y = \dfrac{2(1-2(3))}{-5} - (-3)(-5)$
$= \dfrac{2(-5)}{-5} - 15 = 2 - 15 = -13$

158. $\dfrac{3(2-x)}{y} - (1-xy) = \dfrac{3(2-3)}{-5} - (1-3(-5))$
$= \dfrac{3(-1)}{-5} - (1-(-15))$
$= \dfrac{3}{5} - 16 = -\dfrac{77}{5}$

159. $\dfrac{\frac{14}{x}+\frac{1}{2}}{\frac{-y}{4}} = \dfrac{\frac{14}{3}+\frac{1}{2}}{\frac{-(-5)}{4}} = \dfrac{31}{6}\cdot\dfrac{4}{5} = \dfrac{62}{15}$

160. $\dfrac{\frac{4}{-y}+\frac{8}{x}}{\frac{y}{2}} = \dfrac{\frac{4}{-(-5)}+\frac{8}{3}}{\frac{-5}{2}} = \left(\dfrac{4}{5}+\dfrac{8}{3}\right)\cdot\left(-\dfrac{2}{5}\right)$
$= \dfrac{52}{15}\cdot\left(-\dfrac{2}{5}\right) = -\dfrac{104}{75}$

161. $\dfrac{x}{y} + \dfrac{x}{3} = \dfrac{3x+xy}{3y}$

162. $(x+2)(x+3) = x^2+3x+2x+6 = x^2+5x+6$

163. $5(x+3) = 5x+5(3) = 5x+15$.

164. $(25x)(4x) = 100x^2$

165. $x-(3y+2) = x-3y-2$

166. $2x-(4y-5) = 2x-4y+5$

167. $\dfrac{x+y}{x} = \dfrac{x}{x}+\dfrac{y}{x} = 1+\dfrac{y}{x}$

168. $\dfrac{x+y}{x+z} = \dfrac{x}{x+z}+\dfrac{y}{x+z}$

169. $(x+1)(y+1) = xy+x+y+1$

170. $\dfrac{\frac{1}{x}}{\frac{1}{y}} = \dfrac{1}{x}\cdot\dfrac{y}{1} = \dfrac{y}{x}$

171. $d(P, M) = \left|\dfrac{a+b}{2}-a\right| = \left|\dfrac{a+b-2a}{2}\right| = \left|\dfrac{-a+b}{2}\right|$
$d(Q, M) = \left|b-\dfrac{a+b}{2}\right| = \left|\dfrac{2b-a+b}{2}\right| = \left|\dfrac{b-a}{2}\right|$
Since
$d(P, M) = \left|\dfrac{-a+b}{2}\right| = \left|\dfrac{b-a}{2}\right| = d(Q, M)$, M is the midpoint of the line segment PQ.

172. Answers may vary. Using the hint, we have $-\dfrac{4}{13} = -\dfrac{40}{130}$ and $\dfrac{7}{13} = \dfrac{70}{130}$. Therefore,
$-\dfrac{40}{130} < -\dfrac{31}{130} < -\dfrac{30}{130} < \cdots < 0 < \cdots$
$< \dfrac{1}{130} < \dfrac{10}{130} < \cdots < \dfrac{69}{130} < \dfrac{70}{130}$.

Applying the Concepts

173. a. people who own either MP3 players or people who own DVD players.

b. people who own both MP3 players and DVD players.

174. a. A = {Ford Bronco, Honda Accord}

b. B = {Ford Bronco}

c. C = {Ford Bronco, Honda Accord, Audi A4}

d. $A \cap B$ = {Ford Bronco}

e. $B \cap C$ = {Ford Bronco}

f. $A \cup B$ = {Ford Bronco, Honda Accord}

g. $A \cup C$ = {Ford Bronco, Honda Accord, Audi A4}

175. $119.5 \le x \le 134.5$

176. $49 \le x \le 95$

177. a. $|124-120| = 4$

b. $|137-120| = 17$

c. $|114-120| = |-6| = 6$

178. a. $|72-56| = 16$ MB per hour

b. $|700-380| = 320$ MB per hour

179. $°\text{C} = \frac{42}{3} + 4 = 14 + 4 = 18°\text{C}$

180. $22 = \frac{c}{3} + 4 \Rightarrow 18 = \frac{c}{3} \Rightarrow c = 54$

If the Celsius temperature is 22°, then 54 chirps will be counted.

181. Let x = the number of calories from broccoli. Then we have
$522.5 - 55x = 0 \Rightarrow 522.5 = 55x \Rightarrow 9.5 = x$
The number of grams of broccoli is
$9.5 \times 100 = 950$ grams.

182. Let x = the number of orders of french fries. The number of calories lost from broccoli is $6 \times 55 = 330$. Then we have
$165x - 330 = 0 \Rightarrow 165x = 330 \Rightarrow x = 2$
So, Carmen will have to eat 2 orders of french fries.

Beyond the Basics

183. True **184.** True

185. a. False. For example, $0 \cdot \sqrt{2} = 0$.

b. The products of a nonzero rational number and an irrational number is an irrational number.

186. False. For example, $\sqrt{2} \cdot \sqrt{2} = 2$.

187. False. For example, $\left(2 + \sqrt{3}\right) - \left(6 + \sqrt{3}\right) = -4$.

188. False. For example, $\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{4} = 2$.

189. True **190.** True

For exercises 191–192, use the following definition: An integer P is *even* if $p = 2n$ for some integer n. An integer q is odd if $q = 2k + 1$ for some integer k.

191. a. If a is odd, then $a = 2m + 1$ for some integer m.

$$a^2 = (2m+1)^2 = (2m+1)(2m+1)$$
$$= 4m^2 + 4m + 1 = 4m(m+1) + 1$$
$$= 2(2m(m+1)) + 1,$$

which is of the format $2k + 1$. Therefore, a^2 is an odd integer.

b. Note that this statement is the contrapositive of the statement in part (a). That is, if the statement is "if p, then q", the contrapositive is "if not q, then not p." The contrapositive of "If an integer a is odd, then a^2 is also an odd integer" is "If a^2 is not an odd integer (i.e., an even integer), then a is also not an odd integer (i.e., an even integer." Logically, these two statements are true. That means that the original statement and its contrapositive are either both true or both false. We already know from part (a) that the original statement is true. Therefore, the contrapositive is also true. Thus, if b^2 is even, then b is also an even integer.

192. $2q^2$ is even because it is of the form $2n$, so it follows that p^2 is even. Then, from exercise 191(b), p is also even. Therefore, $p = 2n$, for some integer n. Substituting, we have
$2q^2 = p^2 = (2n)^2 \Rightarrow 2q^2 = 2 \cdot 2n^2 \Rightarrow$
$q^2 = 2n^2$, and, thus, q is even using exercise 191 (b).

Getting Ready for the Next Section

GR1. a. $a^2 \cdot a^3 = a^{\underline{5}}$

b. $a^4 \cdot a^7 = a^{\underline{11}}$

c. $a^m \cdot a^n = a^{\underline{m+n}}$

GR2. a. $\frac{b^3}{b} = b^{\underline{2}}$

b. $\frac{b^7}{b^3} = b^{\underline{4}}$

c. $\frac{b^m}{b^n} = b^{\underline{m-n}}$

GR3. a. $\left(a^2\right)^3 = a^{\underline{6}}$

b. $\left(a^4\right)^2 = a^{\underline{8}}$

c. $\left(a^m\right)^n = a^{\underline{m \cdot n}} = a^{\underline{mn}}$

GR4. a. $(ab)^2 = a^{\underline{2}}b^{\underline{2}}$

b. $(ab)^4 = a^{\underline{4}}b^{\underline{4}}$

c. $(ab)^n = a^{\underline{n}}b^{\underline{n}}$

GR5. $\frac{7}{3}+\frac{3}{2}=\frac{7\cdot 2}{3\cdot 2}+\frac{3\cdot 3}{2\cdot 3}=\frac{14+9}{6}=\frac{23}{6}$

GR6. $\frac{3}{6}-\frac{2}{3}=\frac{3}{6}-\frac{2\cdot 2}{3\cdot 2}=\frac{3-4}{6}=-\frac{1}{6}$

GR7. $\frac{3}{2}\cdot\frac{12}{7}=\frac{3\cdot 12}{2\cdot 7}=\frac{3\cdot \cancel{2}\cdot 6}{\cancel{2}\cdot 7}=\frac{18}{7}$

GR8. $\frac{3}{5}\div\frac{12}{15}=\frac{3}{5}\cdot\frac{15}{12}=\frac{3\cdot 15}{5\cdot 12}=\frac{\cancel{3}\cdot 3\cdot \cancel{5}}{\cancel{5}\cdot\cancel{3}\cdot 4}=\frac{3}{4}$

P.2 Integer Exponents and Scientific Notation

Practice Exercises

1. a. $2^{-1}=\frac{1}{2}$

b. $\left(\frac{4}{5}\right)^0=1$

c. $\left(\frac{3}{2}\right)^{-2}=\frac{1}{(3/2)^2}=\frac{1}{9/4}=\frac{4}{9}$

2. a. $x^2\cdot 3x^7=3x^9$

b. $\left(2^2x^3\right)\left(4x^{-3}\right)=4x^3\cdot 4x^{-3}=16x^{3+(-3)}$
$=16x^0=16\cdot 1=16$

3. a. $\frac{3^4}{3^0}=3^{4-0}=3^4=81$

b. $\frac{5}{5^{-2}}=5^{1-(-2)}=5^3=125$

c. $\frac{2x^3}{3x^{-4}}=\frac{2x^{3-(-4)}}{3}=\frac{2x^7}{3}$

4. a. $\left(7^{-5}\right)^0=7^{(-5)(0)}=7^0=1$

b. $\left(7^0\right)^{-5}=7^{(0)(-5)}=7^0=1$

c. $\left(x^{-1}\right)^8=x^{(-1)(8)}=x^{-8}=\frac{1}{x^8}$

d. $\left(x^{-2}\right)^{-5}=x^{(-2)(-5)}=x^{10}$

5. a. $\left(\frac{1}{2}x\right)^{-1}=\left(\frac{1}{2}\right)^{-1}(x)^{-1}=2\left(\frac{1}{x}\right)=\frac{2}{x}$

b. $\left(5x^{-1}\right)^2=5^2\left(x^{-1}\right)^2=5^2x^{(-1)(2)}$
$=25x^{-2}=\frac{25}{x^2}$

c. $\left(xy^2\right)^3=x^3y^{2(3)}=x^3y^6$

d. $\left(x^{-2}y\right)^{-3}=x^{(-2)(-3)}y^{-3}=\frac{x^6}{y^3}$

6. a. $\left(\frac{1}{3}\right)^2=\frac{1^2}{3^2}=\frac{1}{9}$

b. $\left(\frac{10}{7}\right)^{-2}=\left(\frac{7}{10}\right)^2=\frac{7^2}{10^2}=\frac{49}{100}$

7. a. $\left(2x^4\right)^{-2}=\left(\frac{1}{2x^4}\right)^2=\frac{1}{4x^8}$

b. $\frac{x^2(-y)^3}{\left(xy^2\right)^3}=\frac{-x^2y^3}{x^3y^6}=-\frac{1}{xy^3}$

8. $732,000=7.32\times 10^5$

9. $\frac{2.1\times 10^{10}}{3.5\times 10^8}=\frac{2.1}{3.5}\times\frac{10^{10}}{10^8}=\frac{2.10}{3.5}\times 10^2$
$\approx 0.60\times 10^2=\$60$ per person

Concepts and Vocabulary

1. In the expression 7^2, the number 2 is called the exponent.

2. In the expression -3^7, the base is 3.

3. The number $\frac{1}{4^{-2}}$ simplifies to be the positive integer 16.

4. The power-of-a-product rule allows us to rewrite $(5a)^3$ as 5^3a^3.

5. False. $(-11)^{10}=11^{10}$

6. False. When $\left(x^2\right)^3$ is simplified, the expression becomes x^6.

7. True

8. False

Building Skills

9. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

10. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

11. $\left(\frac{1}{2}\right)^{-4} = 2^4 = 16$

12. $\left(-\frac{1}{2}\right)^{-2} = (-2)^2 = 4$

13. $7^0 = 1$

14. $(-8)^0 = 1$

15. $-(-7)^0 = -1$

16. $\left(\sqrt{2}\right)^0 = 1$

17. $\left(2^3\right)^2 = 2^{3\cdot 2} = 2^6 = 64$

18. $\left(3^2\right)^3 = 3^{2\cdot 3} = 3^6 = 729$

19. $\left(3^2\right)^{-2} = 3^{2(-2)} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

20. $\left(7^2\right)^{-1} = 7^{2(-1)} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

21. $\left(5^{-2}\right)^3 = \left(\frac{1}{5^2}\right)^3 = \left(\frac{1}{25}\right)^3 = \frac{1}{15,625}$

22. $\left(5^{-1}\right)^3 = \left(\frac{1}{5}\right)^3 = \left(\frac{1}{5}\right)^3 = \frac{1}{125}$

23. $\left(4^{-3}\right)\cdot\left(4^5\right) = 4^{(-3+5)} = 4^2 = 16$

24. $\left(7^{-2}\right)\cdot\left(7^3\right) = 7^{(-2+3)} = 7^1 = 7$

25. $\left(\sqrt{3}\right)^0 + 10^0 = 1 + 1 = 2$

26. $\left(\sqrt{5}\right)^0 - 9^0 = 1 - 1 = 0$

27. $3^{-2} + \left(\frac{1}{3}\right)^2 = \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$

28. $5^{-2} + \left(\frac{1}{5}\right)^2 = \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^2 = \frac{1}{25} + \frac{1}{25} = \frac{2}{25}$

29. $-2^{-3} = -\frac{1}{2^3} = -\frac{1}{8}$

30. $-3^{-2} = -\frac{1}{3^2} = -\frac{1}{9}$

31. $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$

32. $(-2)^{-3} = \frac{1}{(-2)^3} = -\frac{1}{8}$

33. $\frac{2^{11}}{2^{10}} = 2^{(11-10)} = 2^1 = 2$

34. $\frac{3^6}{3^8} = 3^{(6-8)} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

35. $\frac{\left(5^3\right)^4}{5^{12}} = \frac{5^{12}}{5^{12}} = 1$

36. $\frac{\left(9^5\right)^2}{9^8} = \frac{9^{10}}{9^8} = 9^{(10-8)} = 9^2 = 81$

37. $\frac{2^5\cdot 3^{-2}}{2^4\cdot 3^{-3}} = 2^{(5-4)}\cdot 3^{(-2-(-3))} = 2^1\cdot 3^1 = 6$

38. $\frac{4^{-2}\cdot 5^3}{4^{-3}\cdot 5} = 4^{(-2-(-3))}\cdot 5^{(3-1)}$

$= 4^1\cdot 5^2 = 4\cdot 25 = 100$

39. $\frac{-5^{-2}}{2^{-1}} = -\frac{1}{5^2}\cdot 2^1 = -\frac{1}{25}\cdot 2 = -\frac{2}{25}$

40. $\frac{-7^{-2}}{3^{-1}} = -\frac{1}{7^2}\cdot 3^1 = -\frac{1}{49}\cdot 3 = -\frac{3}{49}$

41. $\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$

42. $\left(\frac{1}{5}\right)^{-1} = 5$

43. $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$

44. $\left(\frac{3}{2}\right)^{-2} = \left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$

45. $\left(\frac{11}{7}\right)^{-2} = \frac{1}{\left(\frac{11}{7}\right)^2} = \frac{1}{\frac{121}{49}} = \frac{49}{121}$

46. $\left(\frac{13}{5}\right)^{-2} = \frac{1}{\left(\frac{13}{5}\right)^2} = \frac{1}{\frac{169}{25}} = \frac{25}{169}$

47. $x^4y^0 = x^4 \cdot 1 = x^4$

48. $x^{-1}y^0 = \frac{1}{x} \cdot 1 = \frac{1}{x}$

49. $x^{-1}y = \frac{1}{x} \cdot y = \frac{y}{x}$

50. $x^2y^{-2} = x^2 \cdot \frac{1}{y^2} = \frac{x^2}{y^2}$

51. $x^{-1}y^{-2} = \frac{1}{xy^2}$

52. $x^{-3}y^{-2} = \frac{1}{x^3y^2}$

53. $\left(x^{-3}\right)^4 = x^{-12} = \frac{1}{x^{12}}$

54. $\left(x^{-5}\right)^2 = x^{-10} = \frac{1}{x^{10}}$

55. $\left(x^{-11}\right)^{-3} = x^{(-11)\cdot(-3)} = x^{33}$

56. $\left(x^{-4}\right)^{-12} = x^{(-4)\cdot(-12)} = x^{48}$

57. $-3(xy)^5 = -3x^5y^5$

58. $-8(xy)^6 = -8x^6y^6$

59. $4\left(xy^{-1}\right)^2 = 4x^2y^{-2} = \frac{4x^2}{y^2}$

60. $6\left(x^{-1}y\right)^3 = 6x^{-3}y^3 = \frac{6y^3}{x^3}$

61. $3\left(x^{-1}y\right)^{-5} = 3x^{(-1)\cdot(-5)}y^{-5} = \frac{3x^5}{y^5}$

62. $-5\left(xy^{-1}\right)^{-6} = -5x^{-6}y^{(-1)\cdot(-6)} = -\frac{5y^6}{x^6}$

63. $\frac{\left(x^3\right)^2}{\left(x^2\right)^5} = \frac{x^{3\cdot2}}{x^{2\cdot5}} = \frac{x^6}{x^{10}} = x^{6-10} = x^{-4} = \frac{1}{x^4}$

64. $\frac{x^2}{\left(x^3\right)^4} = \frac{x^2}{x^{3\cdot4}} = \frac{x^2}{x^{12}} = x^{2-12} = x^{-10} = \frac{1}{x^{10}}$

65. $\left(\frac{2xy}{x^2}\right)^3 = \frac{2^3x^3y^3}{x^{2\cdot3}} = \frac{8x^3y^3}{x^6} = 8x^{3-6}y^3$
$= 8x^{-3}y^3 = \frac{8y^3}{x^3}$

66. $\left(\frac{5xy}{x^3}\right)^4 = \frac{5^4x^4y^4}{x^{3\cdot4}} = \frac{625x^4y^4}{x^{12}} = 625x^{4-12}y^4$
$= 625x^{-8}y^4 = \frac{625y^4}{x^8}$

67. $\left(\frac{-3x^2y}{x}\right)^5 = \frac{(-3)^5x^{2\cdot5}y^5}{x^5} = \frac{-243x^{10}y^5}{x^5}$
$= -243x^{10-5}y^5 = -243x^5y^5$

68. $\left(\frac{-2xy^2}{y}\right)^3 = \frac{(-2)^3x^3y^{2\cdot3}}{y^3} = \frac{-8x^3y^6}{y^3}$
$= -8x^3y^{6-3} = -8x^3y^3$

69. $\left(\frac{-3x}{5}\right)^{-2} = \frac{(-3)^{-2}x^{-2}}{5^{-2}} = \frac{\frac{1}{9}\cdot\frac{1}{x^2}}{\frac{1}{25}} = \frac{\frac{1}{9x^2}}{\frac{1}{25}} = \frac{25}{9x^2}$

70. $\left(\frac{-5y}{3}\right)^{-4} = \frac{(-5)^{-4}y^{-4}}{3^{-4}} = \frac{\frac{1}{5^4}\cdot\frac{1}{y^4}}{\frac{1}{3^4}} = \frac{\frac{1}{625y^4}}{\frac{1}{81}}$
$= \frac{81}{625y^4}$

71. $\left(\frac{4x^{-2}}{xy^5}\right)^3 = \frac{4^3x^{-2\cdot3}}{x^3y^{5\cdot3}} = \frac{64x^{-6}}{x^3y^{15}} = \frac{64}{x^9y^{15}}$

72. $\left(\frac{3x^2y}{y^3}\right)^5 = \frac{3^5x^{2\cdot5}y^5}{y^{3\cdot5}} = \frac{243x^{10}y^5}{y^{15}} = \frac{243x^{10}}{y^{10}}$

73. $\frac{x^3y^{-3}}{x^{-2}y} = x^{3-(-2)}y^{-3-1} = x^5y^{-4} = \frac{x^5}{y^4}$

74. $\frac{x^2y^{-2}}{x^{-1}y^2} = x^{2-(-1)}y^{-2-2} = x^3y^{-4} = \frac{x^3}{y^4}$

75. $\frac{27x^{-3}y^5}{9x^{-4}y^7} = 3x^{-3-(-4)}y^{5-7} = 3x^1y^{-2} = \frac{3x}{y^2}$

76. $\dfrac{15x^5y^{-2}}{3x^7y^{-3}} = 5x^{5-7}y^{-2-(-3)} = 5x^{-2}y^1 = \dfrac{5y}{x^2}$

77. $\dfrac{1}{x^3}\left(x^2\right)^3 x^{-4} = x^{-3}\left(x^{2\cdot 3}\right)x^{-4} = x^{-3}\left(x^6\right)x^{-4}$

$= x^{-3+6-4} = x^{-1} = \dfrac{1}{x}$

78. $\left(8a^3b\right)^{-4}\left(\dfrac{2a}{b}\right)^{12} = 8^{-4}a^{3(-4)}b^{-4}\cdot\dfrac{2^{12}a^{12}}{b^{12}}$

$= \left(2^3\right)^{-4}a^{-12}b^{-4}\cdot 2^{12}a^{12}b^{-12}$

$= 2^{3(-4)}a^{-12}b^{-4}\cdot 2^{12}a^{12}b^{-12}$

$= 2^{-12}\cdot 2^{12}\cdot a^{-12}\cdot a^{12}\cdot b^{-4}\cdot b^{-12}$

$= 2^{-12+12}\cdot a^{-12+12}\cdot b^{-4-12}$

$= 2^0a^0b^{-16} = b^{-16} = \dfrac{1}{b^{16}}$

79. $\left(-xy^2\right)^3\left(-2x^2y^2\right)^{-4} = \dfrac{\left(-xy^2\right)^3}{\left(-2x^2y^2\right)^4}$

$= \dfrac{(-x)^3\left(y^2\right)^3}{(-2)^4\left(x^2\right)^4\left(y^2\right)^4}$

$= \dfrac{-x^3\cdot y^{2\cdot 3}}{16x^{2\cdot 4}y^{2\cdot 4}} = \dfrac{-x^3y^6}{16x^8y^8}$

$= -\dfrac{1}{16}\left(x^{3-8}\right)\left(y^{6-8}\right)$

$= -\dfrac{1}{16}x^{-5}y^{-2} = -\dfrac{1}{16x^5y^2}$

80. $\left[\dfrac{\left(-x^2y\right)^3y^{-4}}{(xy)^5}\right]^{-2} = \left[\dfrac{\left(-x^{2\cdot 3}\right)y^3y^{-4}}{x^5y^5}\right]^{-2}$

$= \left(\dfrac{-x^6y^{-1}}{x^5y^5}\right)^{-2}$

$= \left(-x^{6-5}y^{-1-5}\right)^{-2}$

$= \left(-xy^{-6}\right)^{-2} = (-x)^{-2}\left(y^{-6}\right)^{-2}$

$= x^{-2}y^{(-6)(-2)} = x^{-2}y^{12} = \dfrac{y^{12}}{x^2}$

81. $\left(4x^3y^2z\right)^2\left(x^3y^2z\right)^{-7} = \dfrac{\left(4x^3y^2z\right)^2}{\left(x^3y^2z\right)^7}$

$= \dfrac{4^2\left(x^3\right)^2\left(y^2\right)^2z^2}{\left(x^3\right)^7\left(y^2\right)^7z^7}$

$= \dfrac{16x^{3\cdot 2}y^{2\cdot 2}z^2}{x^{3\cdot 7}y^{2\cdot 7}z^7}$

$= \dfrac{16x^6y^4z^2}{x^{21}y^{14}z^7}$

$= 16x^{6-21}y^{4-14}z^{2-7}$

$= 16x^{-15}y^{-10}z^{-5}$

$= \dfrac{16}{x^{15}y^{10}z^5}$

82. $\dfrac{(2xyz)^2}{\left(x^3y\right)^2(xz)^{-1}} = \dfrac{2^2x^2y^2z^2}{x^{3\cdot 2}y^2x^{-1}z^{-1}} = \dfrac{4x^2y^2z^2}{x^6y^2x^{-1}z^{-1}}$

$= \dfrac{4x^2y^2z^2}{x^5y^2z^{-1}} = 4x^{2-5}y^{2-2}z^{2-(-1)}$

$= 4x^{-3}y^0z^3 = \dfrac{4z^3}{x^3}$

83. $\dfrac{5a^{-2}bc^2}{a^4b^{-3}c^2} = 5a^{-2-4}b^{1-(-3)}c^{2-2}$

$= 5a^{-6}b^4\cdot 1 = \dfrac{5b^4}{a^6}$

84. $\dfrac{(-3)^2a^5(bc)^2}{(-2)^3a^2b^3c^4} = \dfrac{9a^5b^2c^2}{-8a^2b^3c^4} = -\dfrac{9}{8}a^{5-2}b^{2-3}c^{2-4}$

$= -\dfrac{9}{8}a^3b^{-1}c^{-2} = -\dfrac{9a^3}{8bc^2}$

85. $\left(\dfrac{xy^{-3}z^{-2}}{x^2y^{-4}z^3}\right)^{-3} = \dfrac{x^{-3}y^{(-3)(-3)}z^{(-2)(-3)}}{x^{2(-3)}y^{(-4)(-3)}z^{3(-3)}}$

$= \dfrac{x^{-3}y^9z^6}{x^{-6}y^{12}z^{-9}}$

$= x^{-3-(-6)}y^{9-12}z^{6-(-9)}$

$= x^3y^{-3}z^{15} = \dfrac{x^3z^{15}}{y^3}$

86. $\left(\dfrac{xy^{-2}z^{-1}}{x^{-5}yz^{-8}}\right)^{-1} = \dfrac{x^{-1}y^{(-2)(-1)}z^{(-1)(-1)}}{x^{(-5)(-1)}y^{-1}z^{(-8)(-1)}}$

$= \dfrac{x^{-1}y^{2}z^{1}}{x^{5}y^{-1}z^{8}} = x^{-1-5}y^{2-(-1)}z^{1-8}$

$= x^{-6}y^{3}z^{-7} = \dfrac{y^{3}}{x^{6}z^{7}}$

87. $125 = 1.25\times10^{2}$

88. $247 = 2.47\times10^{2}$

89. $850,000 = 8.5\times10^{5}$

90. $205,000 = 2.05\times10^{5}$

91. $0.007 = 7.0\times10^{-3}$

92. $0.0019 = 1.9\times10^{-3}$

93. $0.00000275 = 2.75\times10^{-6}$

94. $0.0000038 = 3.8\times10^{-6}$

Applying the Concepts

95. $135 \text{ ft}^3 \times 2^3 = 1080 \text{ ft}^3$

96. $675 \text{ in.}^3 \times \left(\dfrac{1}{3}\right)^3 = 25 \text{ in.}^3$

97. a. $(2x)^2 = 4x^2 = 4A = 2^2A$

b. $(3x)^2 = 9x^2 = 9A = 3^2A$

98. a. $\pi\left(\dfrac{2d}{2}\right)^2 = \pi d^2 = 4\pi\left(\dfrac{d}{2}\right)^2 = 2^2A$

b. $\pi\left(\dfrac{3d}{2}\right)^2 = 9\pi\left(\dfrac{d}{2}\right)^2 = 9\pi\left(\dfrac{d}{2}\right)^2 = 3^2A$

99. $F = Sw^2 = 25,000(0.25)^2 = 1562.5 \text{ lb}$

100. $F = S\pi\left(\dfrac{d}{2}\right)^2 = 10,000(3.14)\left(\dfrac{1.5}{2}\right)^2$

$\approx 17,662.5 \text{ lbs.}$

101. $99 \text{ billion} = 99,000,000,000 = 9.9\times10^{10}$

$330 \text{ million} = 330,000,000 = 3.3\times10^{8}$

$\dfrac{99 \text{ billion}}{330 \text{ million}} = \dfrac{9.9\times10^{10}}{3.3\times10^{8}} = \dfrac{9.9}{3.3}\times\dfrac{10^{10}}{10^{8}}$

$= 3\times10^{2} = 300$

The cost is about \$300 per person.

102. $56 \text{ billion} = 56,000,000,000 = 5.6\times10^{10}$

$330 \text{ million} = 330,000,000 = 3.3\times10^{8}$

$\dfrac{56 \text{ billion}}{330 \text{ million}} = \dfrac{95.6\times10^{10}}{3.3\times10^{8}} = \dfrac{5.6}{3.3}\times\dfrac{10^{10}}{10^{8}}$

$\approx 1.7\times10^{2} = 170$

The cost is about \$170 per person.

103. $365.25 \text{ days} \times \dfrac{24 \text{ hr}}{\text{day}} \times \dfrac{60 \text{ min}}{\text{hr}} \times \dfrac{60 \text{ sec}}{\text{min}}$

$= 365.25\times24\times60\times60 \text{ sec}$

$= 31,557,600 \text{ sec}$

$= 3.15576\times10^{7} \text{ sec}$

104. $366 \text{ days} \times \dfrac{24 \text{ hr}}{\text{day}} \times \dfrac{60 \text{ min}}{\text{hr}} \times \dfrac{60 \text{ sec}}{\text{min}}$

$= 366\times24\times60\times60 \text{ sec}$

$= 31,622,400 \text{ sec}$

$= 3.16224\times10^{7} \text{ sec}$

105.

Celestial Body	Equatorial Diameter (km)	Scientific Notation
Earth	12,700	1.27×10^4 km
Moon	3480	3.48×10^3 km
Sun	1,390,000	1.39×10^6 km
Jupiter	134,000	1.34×10^5 km
Mercury	4800	4.8×10^3 km

106. $37,600,000,000,000,000,000,000 =$

3.76×10^{22} atoms

107. $602,000,000,000,000,000,000,000 =$

6.02×10^{20} atoms

108. $0.00000000000000000266 =$

2.66×10^{-20} kg

109. $0.00000000000000000000167 =$

1.67×10^{-21} kg

110. $380,000,000 = 3.8\times10^{8}$ m

111. $5,980,000,000,000,000,000,000,000 =$

5.98×10^{24} kg

112. $2,000,000,000,000,000,000,000,000,000,000 = 2.0\times10^{30}$ kg

Beyond the Basics

113. $2^x = 32$

a. $2^{x+2} = 2^x \cdot 2^2 = 32 \cdot 4 = 128$

b. $2^{x-1} = 2^x \cdot 2^{-1} = 32 \cdot \frac{1}{2} = 16$

114. $3^x = 81$

a. $3^{x+1} = 3^x \cdot 3^1 = 81 \cdot 3 = 243$

b. $3^{x-2} = 3^x \cdot 3^{-2} = 81 \cdot \left(\frac{1}{3}\right)^2 = 81 \cdot \frac{1}{9} = 9$

115. $5^x = 11$

a. $5^{x+1} = 5^x \cdot 5^1 = 11 \cdot 5 = 55$

b. $5^{x-2} = 5^x \cdot 5^{-2} = 11 \cdot \left(\frac{1}{5}\right)^2 = 11 \cdot \frac{1}{25} = \frac{11}{25}$

116. $a^x = b$

a. $a^{x+2} = a^x \cdot a^2 = b \cdot a^2 = a^2 b$

b. $a^{x-1} = a^x \cdot a^{-1} = b \cdot \frac{1}{a} = \frac{b}{a}$

117.
$$\frac{3^{2-n} \cdot 9^{2n-2}}{3^{3n}} = \frac{3^{2-n} \cdot \left(3^2\right)^{2n-2}}{3^{3n}} = \frac{3^{2-n} \cdot 3^{2(2n-2)}}{3^{3n}} = \frac{3^{3n-2}}{3^{3n}} = 3^{(3n-2)-3n} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

118.
$$\frac{2^{m+1} \cdot 3^{2m-n} \cdot 5^{m+n+2} \cdot 6^n}{6^m \cdot 10^{n+1} \cdot 15^m} = \frac{2^m \cdot 2^1 \cdot 3^{2m} \cdot 3^{-n} \cdot 5^m \cdot 5^n \cdot 5^2 \cdot (2 \cdot 3)^n}{(2 \cdot 3)^m \cdot (2 \cdot 5)^{n+1} \cdot (3 \cdot 5)^m} = \frac{2^m \cdot 2^1 \cdot 3^{2m} \cdot 3^{-n} \cdot 5^m \cdot 5^n \cdot 5^2 \cdot 2^n \cdot 3^n}{2^m \cdot 3^m \cdot 2^{n+1} \cdot 5^{n+1} \cdot 3^m \cdot 5^m}$$
$$= \frac{\cancel{2^m} \cdot \cancel{2^1} \cdot 3^{2m} \cdot 3^{-n} \cdot \cancel{5^m} \cdot \cancel{5^n} \cdot 5^2 \cdot \cancel{2^n} \cdot 3^n}{\cancel{2^m} \cdot 3^m \cdot \cancel{2^n} \cdot \cancel{2^1} \cdot \cancel{5^n} \cdot 5^1 \cdot 3^m \cdot \cancel{5^m}} = \frac{3^{2m} \cdot 5^2}{3^m \cdot 3^m \cdot 5^1} = \frac{3^{2m} \cdot 5^2}{3^{2m} \cdot 5^1} = 5$$

119.
$$\frac{2^{x(y-z)}}{2^{y(x-z)}} \div \left(\frac{2^y}{2^x}\right)^z = \frac{2^{xy-xz}}{2^{xy-yz}} \div \frac{2^{yz}}{2^{xz}} = \frac{\frac{2^{xy}}{2^{xz}}}{\frac{2^{xy}}{2^{yz}}} \cdot \frac{2^{xz}}{2^{yz}} = \frac{2^{yz}}{2^{xz}} \cdot \frac{2^{xz}}{2^{yz}} = 1$$

120.
$$\left(\frac{a^x}{a^y}\right)^{\frac{1}{xy}} \cdot \left(\frac{a^y}{a^z}\right)^{\frac{1}{yz}} \cdot \left(\frac{a^z}{a^x}\right)^{\frac{1}{xz}} = \frac{a^{1/y}}{a^{1/x}} \cdot \frac{a^{1/z}}{a^{1/y}} \cdot \frac{a^{1/x}}{a^{1/z}} = 1$$

Getting Ready for the Next Section

GR1. a. $x^2 \cdot x^5 = x^{2+5} = x^7$

b. $(2x)\left(-5x^2\right) = -10x^3$

c. $\left(2y^2\right)\left(3y^3\right)\left(4y^5\right) = 24y^{2+3+5} = 24y^{10}$

GR2. a. $2x^2 + 5x^2 = 7x^2$

b. $3x^2 - 4x^2 = -x^2$

c. $3x^3 - 5x^3 + 11x^3 = 9x^3$

GR3. $x(2x+5) = x(2x) + x(5) = 2x^2 + 5x$

GR4. $2x(7-3x) = 2x(7) + 2x(-3x) = 14x - 6x^2$

GR5. $-5x\left(x^2+1\right) = (-5x)\left(x^2\right) + (-5x)(1) = -5x^3 - 5x$

GR6. True. This is an example of the commutative property of multiplication.

GR7. False. $5x^2$ and $3x^3$ are not like terms, so they cannot be combined.

GR8. $2x^2\left(5x^3 - 3x + 4\right) = 10x^{2+3} - 6x^{2+1} + 8x^2 = 10x^5 - 6x^3 + 8x^2$

P.3 Polynomials

Practice Problems

1. $16(7)^2 + 15(7) = 889$ ft

2. Using the horizontal method:

$$\begin{aligned}&\left(7x^3+2x^2-5\right)+\left(-2x^3+3x^2+2x+1\right)\\&=\left(7x^3-2x^3\right)+\left(2x^2+3x^2\right)+2x+(-5+1)\\&=5x^3+5x^2+2x-4\end{aligned}$$

3. To use the column method, first change the sign in each term in the second polynomial and then add.

$$\left(3x^4-5x^3+2x^2+7\right)-\left(-2x^4+3x^2+x-5\right)$$

$$\begin{array}{rrrrrr} 3x^4 & -5x^3 & +2x^2 & & +x & +7 \\ +2x^4 & & -3x^2 & -x & & +5 \\ \hline 5x^4 & -5x^3 & -x^2 & -x & & +12 \end{array}$$

$$\left(3x^4-5x^3+2x^2+7\right)-\left(-2x^4+3x^2+x-5\right)$$
$$=5x^4-5x^3-x^2-x+12$$

4. $$\begin{aligned}&-2x^3\left(4x^2+2x-5\right)\\&=-2x^3\left(4x^2\right)-2x^3(2x)-2x^3(-5)\\&=-8x^5-4x^4+10x^3\end{aligned}$$

5. $$\begin{aligned}&\left(5x^2+2x\right)\left(-2x^2+x-7\right)\\&=5x^2\left(-2x^2+x-7\right)+2x\left(-2x^2+x-7\right)\\&=-10x^4+5x^3-35x^2-4x^3+2x^2-14x\\&=-10x^4+x^3-33x^2-14x\end{aligned}$$

6. a. $$\begin{aligned}(4x-1)(x+7)&=4x^2+28x-x-7\\&=4x^2+27x-7\end{aligned}$$

b. $$\begin{aligned}(3x-2)(2x-5)&=6x^2-15x-4x+10\\&=6x^2-19x+10\end{aligned}$$

7. $$\begin{aligned}(3x+2)^2&=(3x)^2+2(3x)(2)+2^2\\&=9x^2+12x+4\end{aligned}$$

8. $$(1-2x)(1+2x)=1^2-(2x)^2=1-4x^2$$

9. a. $$(x+2y)(x-2y)=x^2-(2y)^2=x^2-4y^2$$

b. $$\begin{aligned}&(2x-y)^3\\&=(2x)^3-3(2x)^2y+3(2x)y^2-y^3\\&=8x^3-3\left(4x^2\right)y+6xy^2-y^3\\&=8x^3-12x^2y+6xy^2-y^3\end{aligned}$$

Concepts and Vocabulary

1. The polynomial $-3x^7+2x^2-9x+4$ has leading coefficient -3 and degree 7.

2. When a polynomial is written so that the exponents in each term decrease from left to right, it is said to be in standard form.

3. When a polynomial in x of degree 3 is added to a polynomial in x of degree 4, the resulting polynomial has degree 4.

4. When a polynomial in x of degree 3 is multiplied by a polynomial in x of degree 4, the resulting polynomial has degree 7.

5. True

6. True. This is true if A or B or both are zero.

7. False. This is not a polynomial because the term $-3\sqrt{x}$ does not have an exponent that is either a positive integer or zero.

8. True

Building Skills

9. Polynomial; x^2+2x+1

10. Not a polynomial

11. Not a polynomial

12. A polynomial; $x^7+3x^5+3x^4-2x+1$

13. Not a polynomial

14. Not a polynomial

15. A polynomial; in standard form

16. A polynomial; in standard form

17. Degree: 1; terms: $7x$, 3

18. Degree: 2; terms: $-3x^2, 7$

19. Degree: 4; terms: $-x^4, x^2, 2x, -9$

20. Degree: 7; terms: $9x^7, 2x^3, x, -21$

21. $$\begin{aligned}&\left(x^3+2x^2-5x+3\right)+\left(-x^3+2x-4\right)\\&=\left(x^3+\left(-x^3\right)\right)+2x^2+(-5x+2x)+(3-4)\\&=2x^2-3x-1\end{aligned}$$

22. $$\begin{aligned}&\left(x^3-3x+1\right)+\left(x^3-x^2+x-3\right)\\&=\left(x^3+x^3\right)-x^2+(-3x+x)+(1-3)\\&=2x^3-x^2-2x-2\end{aligned}$$

23. $$\begin{aligned}&\left(2x^3-x^2+x-5\right)-\left(x^3-4x+3\right)\\&=\left(2x^3-x^3\right)-x^2+(x-(-4x))+(-5-3)\\&=x^3-x^2+5x-8\end{aligned}$$

24. $\left(-x^3+2x-4\right)-\left(x^3+3x^2-7x+2\right)$
$=\left(-x^3-x^3\right)-3x^2+\left(2x-(-7x)\right)+(-4-2)$
$=-2x^3-3x^2+9x-6$

25. $\left(-2x^4+3x^2-7x\right)-\left(8x^4+6x^3-9x^2-17\right)$
$=\left(-2x^4-8x^4\right)-7x-6x^3$
$+\left(3x^2-\left(-9x^2\right)\right)-(-17)$
$=-10x^4-6x^3+12x^2-7x+17$

26. $3\left(x^2-2x+2\right)+2\left(5x^2-x+4\right)$
$=3x^2-6x+6+10x^2-2x+8$
$=13x^2-8x+14$

27. $-2\left(3x^2+x+1\right)+6\left(-3x^2-2x-2\right)$
$=-6x^2-2x-2-18x^2-12x-12$
$=-24x^2-14x-14$

28. $2\left(5x^2-x+3\right)-4\left(3x^2+7x+1\right)$
$=10x^2-2x+6-12x^2-28x-4$
$=-2x^2-30x+2$

29. $\left(3y^3-4y+2\right)+(2y+1)-\left(y^3-y^2+4\right)$
$=3y^3-4y+2+2y+1-y^3+y^2-4$
$=2y^3+y^2-2y-1$

30. $\left(5y^2+3y-1\right)-\left(y^2-2y+3\right)+\left(2y^2+y+5\right)$
$=5y^2+3y-1-y^2+2y-3+2y^2+y+5$
$=6y^2+6y+1$

31. $6x(2x+3)=12x^2+18x$

32. $7x(3x-4)=21x^2-28x$

33. $(x+1)\left(x^2+2x+2\right)$
$=x\left(x^2+2x+2\right)+1\left(x^2+2x+2\right)$
$=x^3+2x^2+2x+x^2+2x+2$
$=x^3+3x^2+4x+2$

34. $(x-5)\left(2x^2-3x+1\right)$
$=x\left(2x^2-3x+1\right)-5\left(2x^2-3x+1\right)$
$=2x^3-3x^2+x-10x^2+15x-5$
$=2x^3-13x^2+16x-5$

35. $(3x-2)\left(x^2-x+1\right)$
$=3x\left(x^2-x+1\right)-2\left(x^2-x+1\right)$
$=3x^3-3x^2+3x-2x^2+2x-2$
$=3x^3-5x^2+5x-2$

36. $(2x+1)\left(x^2-3x+4\right)$
$=2x\left(x^2-3x+4\right)+1\left(x^2-3x+4\right)$
$=2x^3-6x^2+8x+x^2-3x+4$
$=2x^3-5x^2+5x+4$

37. $(x+1)(x+2)=x(x+2)+1(x+2)$
$=x^2+2x+x+2=x^2+3x+2$

38. $(x+2)(x+3)=x(x+3)+2(x+3)$
$=x^2+3x+2x+6$
$=x^2+5x+6$

39. $(3x+2)(3x+1)=9x^2+3x+6x+2$
$=9x^2+9x+2$

40. $(x+3)(2x+5)=2x^2+5x+6x+15$
$=2x^2+11x+15$

41. $(-4x+5)(x+3)=-4x^2-12x+5x+15$
$=-4x^2-7x+15$

42. $(-2x+1)(x-5)=-2x^2+10x+x-5$
$=-2x^2+11x-5$

43. $(3x-2)(2x-1)=6x^2-3x-4x+2$
$=6x^2-7x+2$

44. $(x-1)(5x-3)=5x^2-3x-5x+3$
$=5x^2-8x+3$

45. $(2x-3a)(2x+5a)=4x^2+10ax-6ax-15a^2$
$=4x^2+4ax-15a^2$

46. $(5x-2a)(x+5a)=5x^2+25ax-2ax-10a^2$
$=5x^2+23ax-10a^2$

47. $(x+2)^2-x^2=x^2+4x+4-x^2=4x+4$

48. $(x-3)^2-x^2=x^2-6x+9-x^2=-6x+9$

49. $(4x+1)^2=16x^2+8x+1$

50. $(3x+2)^2=9x^2+12x+4$

51. $\left(x+\frac{3}{4}\right)^2 = x^2 + 2\left(\frac{3}{4}\right)x + \left(\frac{3}{4}\right)^2$
$= x^2 + \frac{3}{2}x + \frac{9}{16}$

52. $\left(x+\frac{2}{5}\right)^2 = x^2 + 2\left(\frac{2}{5}\right)x + \left(\frac{2}{5}\right)^2$
$= x^2 + \frac{4}{5}x + \frac{4}{25}$

53. $(3x-4y)^2 = (3x)^2 - 2(3x)(4y) + (4y)^2$
$= 9x^2 - 24xy + 16y^2$

54. $(2x+5y)^2 = (2x)^2 + 2(2x)(5y) + (5y)^2$
$= 4x^2 + 20xy + 25y^2$

55. $\left(x^2+2\right)^2 = \left(x^2\right)^2 + 2\left(x^2\right)(2) + 2^2$
$= x^4 + 4x^2 + 4$

56. $\left(3-x^2\right)^2 = 3^2 - 2(3)\left(x^2\right) + \left(x^2\right)^2$
$= 9 - 6x^2 + x^4 = x^4 - 6x^2 + 9$

57. $\left(\frac{x}{2}+\frac{2}{x}\right)^2 = \left(\frac{x}{2}\right)^2 + 2\left(\frac{x}{2}\right)\left(\frac{2}{x}\right) + \left(\frac{2}{x}\right)^2$
$= \frac{x^2}{4} + 2 + \frac{4}{x^2} = \frac{x^2}{4} + \frac{4}{x^2} + 2$

58. $\left(\frac{3}{y}-\frac{y}{3}\right)^2 = \left(\frac{3}{y}\right)^2 - 2\left(\frac{3}{y}\right)\left(\frac{y}{3}\right) + \left(\frac{y}{3}\right)^2$
$= \frac{9}{y^2} - 2 + \frac{y^2}{9} = \frac{9}{y^2} + \frac{y^2}{9} - 2$

59. $(x+2)^3 = x^3 + 3x^2(2) + 3x(2)^2 + 2^3$
$= x^3 + 6x^2 + 12x + 8$

60. $(2x-1)^3 = (2x)^3 - 3(2x)^2(1) + 3(2x)(1) - 1^3$
$= 8x^3 - 12x^2 + 6x - 1$

61. $(x+3)^3 - x^3 = x^3 + 3x^2(3) + 3x(3) + 3^3 - x^3$
$= 9x^2 + 27x + 27$

62. $(x-2)^3 - x^3 = x^3 - 3x^2(2) + 3x(2) - 2^3 - x^3$
$= -6x^2 + 12x - 8$

63. $(x+2y)^3 = x^3 + 3x^2(2y) + 3x(2y)^2 + 2^3$
$= x^3 + 6x^2y + 12xy^2 + 8$

64. $(2x+3y)^3$
$= (2x)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2 + (3y)^3$
$= 8x^3 + 36x^2y + 54xy^2 + 27y^3$

65. $(5+2x)(5-2x) = 5^2 - (2x)^2 = 25 - 4x^2$

66. $(3-4x)(3+4x) = 3^2 - (4x)^2 = 9 - 16x^2$

67. $(2x+3y)(2x-3y) = (2x)^2 - (3y)^2$
$= 4x^2 - 9y^2$

68. $(5x-2y)(5x+2y) = (5x)^2 - (2y)^2$
$= 25x^2 - 4y^2$

69. $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right) = x^2 - \left(\frac{1}{x}\right)^2 = x^2 - \frac{1}{x^2}$

70. $\left(\frac{y}{2}-\frac{2}{y}\right)\left(\frac{y}{2}+\frac{2}{y}\right) = \left(\frac{y}{2}\right)^2 - \left(\frac{2}{y}\right)^2$
$= \frac{y^2}{4} - \frac{4}{y^2}$

71. $\left(x^2+3\right)\left(x^2-3\right) = \left(x^2\right)^2 - 3^2 = x^4 - 9$

72. $\left(x^3-2\right)\left(x^3+2\right) = \left(x^3\right)^2 - 2^2 = x^6 - 4$

73. $(2x-3)\left(x^2-3x+5\right)$
$= 2x\left(x^2-3x+5\right) - 3\left(x^2-3x+5\right)$
$= 2x^3 - 6x^2 + 10x - 3x^2 + 9x - 15$
$= 2x^3 - 9x^2 + 19x - 15$

74. $(x-2)\left(x^2-4x-3\right)$
$= x\left(x^2-4x-3\right) - 2\left(x^2-4x-3\right)$
$= x^3 - 4x^2 - 3x - 2x^2 + 8x + 6$
$= x^3 - 6x^2 + 5x + 6$

75. $(1+y)\left(1-y+y^2\right)$
$= 1\left(1-y+y^2\right) + y\left(1-y+y^2\right)$
$= 1 - y + y^2 + y - y^2 + y^3 = y^3 + 1$

76. $(y+4)\left(y^2-4y+16\right)$
$= y\left(y^2-4y+16\right) + 4\left(y^2-4y+16\right)$
$= y^3 - 4y^2 + 16y + 4y^2 - 16y + 64$
$= y^3 + 64$

77. $(x-6)\left(x^2+6x+36\right)$
$= x\left(x^2+6x+36\right)-6\left(x^2+6x+36\right)$
$= x^3+6x^2+36x-6x^2-36x-216$
$= x^3-216$

78. $(x-1)\left(x^2+x+1\right)$
$= x\left(x^2+x+1\right)-1\left(x^2+x+1\right)$
$= x^3+x^2+x-x^2-x-1 = x^3-1$

79. $(x+2y)(3x+5y) = 3x^2+5xy+6xy+10y^2$
$= 3x^2+11xy+10y^2$

80. $(2x+y)(7x+2y) = 14x^2+4xy+7xy+2y^2$
$= 14x^2+11xy+2y^2$

81. $(2x-y)(3x+7y) = 6x^2+14xy-3xy-7y^2$
$= 6x^2+11xy-7y^2$

82. $(x-3y)(2x+5y) = 2x^2+5xy-6xy-15y^2$
$= 2x^2-xy-15y^2$

83. $(x-y)^2(x+y)^2 = (x-y)(x-y)(x+y)(x+y)$
$= \left[(x-y)(x+y)\right]\left[(x-y)(x+y)\right]$
$= \left(x^2-y^2\right)\left(x^2-y^2\right)$
$= x^4-2x^2y^2+y^4$

84. $(2x+y)^2(2x-y)^2$
$= (2x+y)(2x+y)(2x-y)(2x-y)$
$= \left[(2x-y)(2x+y)\right]\left[(2x-y)(2x+y)\right]$
$= \left(4x^2-y^2\right)\left(4x^2-y^2\right)$
$= 16x^4-8x^2y^2+y^4$

85. $(x+y)(x-2y)^2 = (x+y)\left(x^2-4xy+4y^2\right)$
$= x\left(x^2-4xy+4y^2\right)+y\left(x^2-4xy+4y^2\right)$
$= x^3-4x^2y+4xy^2+x^2y-4xy^2+4y^3$
$= x^3-3x^2y+4y^3$

86. $(x-y)(x+2y)^2 = (x-y)\left(x^2+4xy+4y^2\right)$
$= x\left(x^2+4xy+4y^2\right)-y\left(x^2+4xy+4y^2\right)$
$= x^3+4x^2y+4xy^2-x^2y-4xy^2-4y^3$
$= x^3+3x^2y-4y^3$

87. $(x-2y)^3(x+2y)$
$= \left(x^3+3x^2(-2y)+3x(-2y)^2+(-2y)^3\right)$
$\cdot(x+2y)$
$= \left(x^3-6x^2y+12xy^2-8y^3\right)(x+2y)$
$= x\left(x^3-6x^2y+12xy^2-8y^3\right)$
$+\ 2y\left(x^3-6x^2y+12xy^2-8y^3\right)$
$= x^4-6x^3y+12x^2y^2-8xy^3$
$+\ 2x^3y-12x^2y^2+24xy^3-16y^4$
$= x^4-4x^3y+16xy^3-16y^4$

88. $(2x+y)^3(2x-y)$
$= \left[(2x)^3+3(2x)^2y+3(2x)y^2+y^3\right]$
$\cdot(2x-y)$
$= \left(8x^3+12x^2y+6xy^2+y^3\right)(2x-y)$
$= 2x\left(8x^3+12x^2y+6xy^2+y^3\right)$
$-\ y\left(8x^3+12x^2y+6xy^2+y^3\right)$
$= 16x^4+24x^3y+12x^2y^2+2xy^3$
$-\ 8x^3y-12x^2y^2-6xy^3-y^4$
$= 16x^4+16x^3y-4xy^3-y^4$

For exercises 89–94, use
$a^2+b^2 = (a+b)^2-2ab = (a-b)^2+2ab.$

89. $x+y=4,\ xy=3$
$x^2+y^2 = (x+y)^2-2xy$
$= 4^2-2\cdot 3 = 16-6 = 10$

90. $x-y=3,\ xy=2$
$x^2+y^2 = (x-y)^2+2xy$
$= 3^2+2\cdot 2 = 9+4 = 13$

91. a. $x+\dfrac{1}{x}=3$

Let $a=x$ and $b=\dfrac{1}{x}$. Then $ab = x\left(\dfrac{1}{x}\right)=1.$

$x^2+\dfrac{1}{x^2} = \left(x+\dfrac{1}{x}\right)^2-2x\left(\dfrac{1}{x}\right)$
$= 3^2-2 = 9-2 = 7$

b. Let $a = x^2$ and $b = \frac{1}{x^2}$. Using the result from part a, we have

$$x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2x^2\left(\frac{1}{x^2}\right)$$
$$= 7^2 - 2 = 49 - 2 = 47$$

92. a. $x - \frac{1}{x} = 2$

Let $a = x$ and $b = \frac{1}{x}$. Then $ab = x\left(\frac{1}{x}\right) = 1$.

$$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2x\left(\frac{1}{x}\right)$$
$$= 2^2 + 2 = 4 + 2 = 6$$

b. Let $a = x^2$ and $b = \frac{1}{x^2}$. Using the result from part a, we have

$$x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2x^2\left(\frac{1}{x^2}\right)$$
$$= 6^2 - 2 = 36 - 2 = 34$$

93. Let $a = 3x$ and $b = 2y$. Then $ab = 6xy$.

$$9x^2 + 4y^2 = (3x + 2y)^2 - 2(6xy)$$
$$= 12^2 - 2 \cdot 6 \cdot 6 = 144 - 72 = 72$$

94. Let $a = 3x$ and $b = 7y$. Then $ab = 21xy$.

$$9x^2 + 49y^2 = (3x - 7y)^2 + 2(21xy)$$
$$= 10^2 + 2(21)(-1)$$
$$= 100 - 42 = 58$$

Applying the Concepts

95. $0.11\left(5^2\right) + 0.02(5) + 5.57 = \$8.425$ in 2015 (five years after 2010)

96. $0.035\left(4^2\right) + 0.15(4) + 5.17 = 6.33$

In 2012 (four years after 2008), theater grosses were 6.33 billion dollars.

97. $0.1\left(40^2\right) + 40 + 50 = \$250.00$

98. $(15 - 10)^2 + 5(15) = 5^2 + 75$
$= 25 + 75 = \$100.00$

99. $d = 16\left(5^2\right) + 20(5) = 16(25) + 100 = 500$ feet

100. $d = 16\left(2^2\right) + 10(2) = 16(4) + 20 = 84$ feet

101. a. $-x + 22.50$

b. $(30 + 10x)(22.50 - x)$
$= 30(22.50) - 30x + 225x - 10x^2$
$= 675 + 195x - 10x^2$
$= -10x^2 + 195x + 675$

102. a. $10n + 250$

b. $(50 - 2n)(10n + 250)$
$= 50(10n) + 50(250) - 2n(10n) - 2n(250)$
$= 500n + 12,500 - 20n^2 - 500n$
$= -20n^2 + 12,500$

Beyond the Basics

103. $(a + b + c)^2 = (a + b + c)(a + b + c)$
$= a(a + b + c) + b(a + b + c) + c(a + b + c)$
$= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2$
$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

104. $\left(x^2 + x + 1\right)^2 = \left(x^2\right)^2 + x^2 + 1^2 + 2x^2x + 2x(1) + 2x^2(1)$
$= x^4 + x^2 + 1 + 2x^3 + 2x + 2x^2$
$= x^4 + 2x^3 + 3x^2 + 2x + 1$

105. $(2x + y - z)^2 = (2x)^2 + y^2 + (-z)^2 + 2(2x)y + 2y(-z) + 2(2x)(-z)$
$= 4x^2 + y^2 + z^2 + 4xy - 2yz - 4xz$

$(2x - y + z)^2 = (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y) + 2(-y)z + 2(2x)z$
$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

(*continued on next page*)

(*continued*)

$$(2x+y-z)^2-(2x-y+z)^2=\left(4x^2+y^2+z^2+4xy-2yz-4xz\right)-\left(4x^2+y^2+z^2-4xy-2yz+4xz\right)$$
$$=8xy-8xz$$

Alternatively, recognize the difference of two squares.

$$(2x+y-z)^2-(2x-y+z)^2=\left[(2x+y-z)+(2x-y+z)\right]\left[(2x+y-z)-(2x-y+z)\right]$$
$$=(4x)(2y-2z)=8xy-8xz$$

106. $a+b+c=8;\ ab+bc+ac=12$

$$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ac=a^2+b^2+c^2+2(ab+bc+ac)$$
$$8^2=a^2+b^2+c^2+2(12)$$
$$64=a^2+b^2+c^2+24$$
$$40=a^2+b^2+c^2$$

107. $x+y+z=12;\ x^2+y^2+z^2=44$

$$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2xz=x^2+y^2+z^2+2(xy+yz+xz)$$
$$12^2=44+2(xy+yz+xz)$$
$$144=44+2(xy+yz+xz)$$
$$100=2(xy+yz+xz)$$
$$50=xy+yz+xz$$

108. $x^2+y^2+z^2=64;\ \ xy+yz+xz=18$

$$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2xz=x^2+y^2+z^2+2(xy+yz+xz)$$
$$(x+y+z)^2=64+2(18)$$
$$(x+y+z)^2=100$$
$$x+y+z=\sqrt{100}=10$$

109. $(a+b+c)\left(a^2+b^2+c^2-ab-bc-ac\right)$

$$=a\left(a^2+b^2+c^2-ab-bc-ac\right)+b\left(a^2+b^2+c^2-ab-bc-ac\right)+c\left(a^2+b^2+c^2-ab-bc-ac\right)$$
$$=a^3+ab^2+ac^2-a^2b-abc-a^2c+a^2b+b^3+bc^2-ab^2-b^2c-abc+a^2c+b^2c+c^3-abc-bc^2-ac^2$$
$$=a^3+b^3+c^3-3abc$$

110. From exercise 109,

$$(0)\left(a^2+b^2+c^2-ab-bc-ac\right)=a^3+b^3+c^3-3abc\Rightarrow 0=a^3+b^3+c^3-3abc\Rightarrow a^3+b^3+c^3=3abc$$

111. Let $a=x-y$, $b=y-z$, and $c=z-x$. Then $a+b+c=(x-y)+(y-z)+(z-x)=0$.

From exercise 110, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$. So,

$$(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x).$$

112. Let $a=2x-3y$, $b=3y-5z$, and $c=5z-3x$. Then $a+b+c=(2x-3y)+(3y-5z)+(5z-3x)=0$.

From exercise 110, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$. So,

$$(2x-3y)^3+(3y-5z)^3+(5z-2x)^3=3(2x-3y)(3y-5z)(5z-2x).$$

113. From exercise 109, $(a+b+c)\left(a^2+b^2+c^2-ab-bc-ac\right)=a^3+b^3+c^3-3abc.$

If $a+b+c=8$ and $ab+bc+ac=19,$ we have

$(a+b+c)\left(a^2+b^2+c^2-ab-bc-ac\right)=(a+b+c)\left(a^2+b^2+c^2-(ab+bc+ac)\right)=a^3+b^3+c^3-3abc \Rightarrow$

$8\left(a^2+b^2+c^2-19\right)=a^3+b^3+c^3-3abc \quad (1)$

From exercise 103,

$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ac=a^2+b^2+c^2+2(ab+bc+ac) \Rightarrow$

$8^2=a^2+b^2+c^2+2(19) \Rightarrow 64=a^2+b^2+c^2+38 \Rightarrow a^2+b^2+c^2=26$

Substituting into equation (1) gives us $8(26-19)=a^3+b^3+c^3-3abc \Rightarrow 56=a^3+b^3+c^3-3abc.$

114. From exercise 109, $(a+b+c)\left(a^2+b^2+c^2-ab-bc-ac\right)=a^3+b^3+c^3-3abc.$

If $a+b+c=9$ and $ab+bc+ca=11,$ we have

$(a+b+c)\left(a^2+b^2+c^2-ab-bc-ac\right)=(a+b+c)\left(a^2+b^2+c^2-(ab+bc+ac)\right)=a^3+b^3+c^3-3abc \Rightarrow$

$9\left(a^2+b^2+c^2-11\right)=a^3+b^3+c^3-3abc \quad (1)$

From exercise 103,

$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ac=a^2+b^2+c^2+2(ab+bc+ac) \Rightarrow$

$9^2=a^2+b^2+c^2+2(11) \Rightarrow 81=a^2+b^2+c^2+22 \Rightarrow a^2+b^2+c^2=59$

Substituting into equation (1) gives us $9(59-11)=a^3+b^3+c^3-3abc \Rightarrow 432=a^3+b^3+c^3-3abc.$

Getting Ready for the Next Section

GR1. $x(2x+5)=x(2x)+x(5)=2x^2+5x$

GR2. $2x(7-3x)=2x(7)+2x(-3x)=14x-6x^2$

GR3. $-5x\left(x^2+1\right)=-5x\left(x^2\right)-5x(1)=-5x^3-5x$

GR4. True. This is an example of the commutative property for multiplication.

	a	*b*	*a* + *b*	*ab*
GR5.	3	4	7	12
GR6.	–3	5	2	–15
GR7.	4	2	6	8
GR8.	3	–5	–2	–15
GR9.	2	5	7	10
GR10.	3	5	8	15
GR11.	–5	7	2	–35
GR12.	5	–7	–2	–35
GR13.	–2	–3	–5	6
GR14.	2	3	5	6

P.4 Factoring Polynomials

Practice Problems

1. a. $6x^5+14x^3=2x^3\left(3x^2+7\right)$

b. $7x^5+21x^4+35x^2=7x^2\left(x^3+3x^2+5\right)$

c. $5x^2(x-y)+2(x-y)=(x-y)\left(5x^2+2\right)$

2. a $x^2+6x+8=(x+4)(x+2)$

b. $x^2-3x-10=(x-5)(x+2)$

3. a. $x^2+4x+4=(x+2)^2$

b. $9x^2-6x+1=(3x-1)^2$

4. a. $x^2-16=(x-4)(x+4)$

b. $4x^2-25=(2x-5)(2x+5)$

5. $x^4-81=\left(x^2-9\right)\left(x^2+9\right)$

$=(x-3)(x+3)\left(x^2+9\right)$

6. a. $x^3 - 125 = (x-5)\left(x^2+5x+25\right)$

b. $27x^3 + 8 = (3x+2)\left(9x^2-6x+4\right)$

7. Following the reasoning in Example 7, in four years, the company will have invested the initial 12 million dollars, plus an additional 4 million dollars. Thus, the total investment is 16 million dollars. To find the profit or loss with 16 million dollars invested, let $x = 16$ in the profit-loss polynomial:

$$\begin{aligned}&(0.012)(x-14)\left(x^2+14x+196\right)\\ &= (0.012)(16-14)\left(16^2+14\cdot 16+196\right)\\ &= 16.224\end{aligned}$$

The company will have made a profit of 16.224 million dollars in four years.

8. a. $\begin{aligned}x^3+3x^2+x+3 &= x^2(x+3)+1(x+3)\\ &= (x+3)\left(x^2+1\right)\end{aligned}$

b. $\begin{aligned}28x^3-20x^2-7x+5 &= 4x^2(7x-5)-(7x-5)\\ &= (7x-5)\left(4x^2-1\right)\\ &= (7x-5)(2x+1)(2x-1)\end{aligned}$

c. $\begin{aligned}x^2-y^2-2y-1 &= x^2-\left(y^2+2y+1\right)\\ &= x^2-(y+1)^2\\ &= \left(x-(y+1)\right)\left(x+(y+1)\right)\\ &= (x-y-1)(x+y+1)\end{aligned}$

9. $\begin{aligned}x^4+5x^2+9 &= x^4+6x^2-x^2+9\\ &= x^4+6x^2+9-x^2\\ &= \left(x^2+3\right)^2-x^2\\ &= \left[\left(x^2+3\right)+x\right]\left[\left(x^2+3\right)-x\right]\\ &= \left(x^2+x+3\right)\left(x^2-x+3\right)\end{aligned}$

10. a. $\begin{aligned}5x^2+11x+2 &= 5x^2+10x+x+2\\ &= \left(5x^2+10x\right)+(x+2)\\ &= 5x(x+2)+1(x+2)\\ &= (5x+1)(x+2)\end{aligned}$

b. $\begin{aligned}4x^2+x-3 &= 4x^2+4x-3x-3\\ &= \left(4x^2+4x\right)-(3x+3)\\ &= 4x(x+1)-3(x+1)\\ &= (4x-3)(x+1)\end{aligned}$

Concepts and Vocabulary

1. The polynomials $x+2$ and $x-2$ are called factors of the polynomial x^2-4.

2. The polynomial $3y$ is the greatest common monomial factor of the polynomial $3y^2+6y$.

3. The GCF of the polynomial $10x^3+30x^2$ is $10x^2$.

4. A polynomial that cannot be factored as a product of two polynomials (excluding constant polynomials ± 1) is said to be irreducible.

5. True

6. True

7. False. The polynomial x^2-4 can be factored as $(x+2)(x-2)$. Therefore, it is not irreducible.

8. True

Building Skills

9. $8x-24 = 8(x-3)$

10. $5x+25 = 5(x+5)$

11. $-6x^2+12x = -6x(x-2)$

12. $-3x^2+21 = -3(x^2-7)$

13. $7x^2+14x^3 = 7x^2(1+2x)$

14. $9x^3-18x^4 = 9x^3(1-2x)$

15. $x^4+2x^3+x^2 = x^2\left(x^2+2x+1\right)$

16. $x^4-5x^3+7x^2 = x^2\left(x^2-5x+7\right)$

17. $3x^3-x^2 = x^2(3x-1)$

18. $2x^3+2x^2 = 2x^2(x+1)$

19. $8ax^3+4ax^2 = 4ax^2(2x+1)$

20. $ax^4-2ax^2+ax = ax\left(x^3-2x+1\right)$

21. $x(x-y)+3(x-y) = (x-y)(x+3)$

22. $2a(a+1)-3(a+1) = (a+1)(2a-3)$

23. $3x(2x+y)+5y(2x+y)=(2x+y)(3x+5y)$

24. $2x(5x-3y)+7y(5x-3y)$
$=(5x-3y)(2x+7y)$

25. $(y+2)^2-3(y+2)=(y+2)(y+2-3)$
$=(y+2)(y-1)$

26. $(y+1)^2+(y+1)=(y+1)((y+1)+1)$
$=(y+1)(y+2)$

27. $x^3+3x^2+x+3=x^2(x+3)+1(x+3)$
$=(x+3)(x^2+1)$

28. $x^3+5x^2+x+5=x^2(x+5)+1(x+5)$
$=(x+5)(x^2+1)$

29. $x^3-5x^2+x-5=x^2(x-5)+1(x-5)$
$=(x-5)(x^2+1)$

30. $x^3-7x^2+x-7=x^2(x-7)+1(x-7)$
$=(x-7)(x^2+1)$

31. $6x^3+4x^2+3x+2=2x^2(3x+2)+1(3x+2)$
$=(3x+2)(2x^2+1)$

32. $3x^3+6x^2+x+2=3x^2(x+2)+1(x+2)$
$=(x+2)(3x^2+1)$

33. $12x^7+4x^5+3x^4+x^2$
$=4x^5(3x^2+1)+x^2(3x^2+1)$
$=(3x^2+1)(4x^5+x^2)$
$=x^2(3x^2+1)(4x^3+1)$

34. $3x^7+3x^5+x^4+x^2=3x^5(x^2+1)+x^2(x^2+1)$
$=(x^2+1)(3x^5+x^2)$
$=x^2(x^2+1)(3x^3+1)$

35. $xy+ab+bx+ay=(xy+bx)+(ab+ay)$
$=x(y+b)+a(b+y)$
$=x(y+b)+a(y+b)$
$=(b+y)(x+a)$

36. $2a^2x-b-bx+2a^2=2a^2x+2a^2-(b+bx)$
$=2a^2(x+1)-b(1+x)$
$=2a^2(x+1)-b(x+1)$
$=(x+1)(2a^2-b)$

37. $x^2+7x+12=(x+3)(x+4)$

38. $x^2+8x+15=(x+3)(x+5)$

39. $x^2-6x+8=(x-2)(x-4)$

40. $x^2-9x+14=(x-2)(x-7)$

41. $x^2-3x-4=(x+1)(x-4)$

42. $x^2-5x-6=(x+1)(x-6)$

43. Irreducible **44.** Irreducible

45. $2x^2+x-36=(2x+9)(x-4)$

46. $2x^2+3x-27=(2x+9)(x-3)$

47. $6x^2+17x+12=(2x+3)(3x+4)$

48. $8x^2-10x-3=(2x-3)(4x+1)$

49. $3x^2-11x-4=(3x+1)(x-4)$

50. $5x^2+7x+2=(5x+2)(x+1)$

51. Irreducible **52.** Irreducible

53. $x^2-xy-20y^2=(x+4y)(x-5y)$

54. $8p^2-2pq-15q^2=(4p+5q)(2p-3q)$

55. $15x^2-28+x=15x^2+x-28$
$=(3x-4)(5x+7)$

56. $24y+35+4y^2=4y^2+24y+35$
$=(2y+7)(2y+5)$

57. $x^2+6x+9=(x+3)^2$

58. $x^2+8x+16=(x+4)^2$

59. $9x^2+6x+1=(3x+1)^2$

60. $36x^2+12x+1=(6x+1)^2$

61. $25x^2-20x+4=(5x-2)^2$

62. $64x^2+32x+4=4(16x^2+8x+1)$
$=4(4x+1)^2$

63. $49x^2+42x+9=(7x+3)^2$

64. $9x^2+24x+16=(3x+4)^2$

65. $x^2-64=(x-8)(x+8)$

66. $x^2-121=(x-11)(x+11)$

67. $4x^2-1=(2x-1)(2x+1)$

68. $9x^2-1=(3x-1)(3x+1)$

69. $16x^2-9=(4x-3)(4x+3)$

70. $25x^2-49=(5x-7)(5x+7)$

71. $x^4-1=\left(x^2-1\right)\left(x^2+1\right)=(x-1)(x+1)\left(x^2+1\right)$

72. $x^4-81=\left(x^2-9\right)\left(x^2+9\right)$
$=(x-3)(x+3)\left(x^2+9\right)$

73. $20x^4-5=5\left(4x^4-1\right)=5\left(2x^2-1\right)\left(2x^2+1\right)$

74. $12x^4-75=3\left(4x^4-25\right)=3\left(2x^2-5\right)\left(2x^2+5\right)$

75. $x^3+64=x^3+4^3=(x+4)\left(x^2-4x+16\right)$

76. $x^3+125=x^3+5^3=(x+5)\left(x^2-5x+25\right)$

77. $x^3-27=x^3-3^3=(x-3)\left(x^2+3x+9\right)$

78. $x^3-216=x^3-6^3=(x-6)\left(x^2+6x+36\right)$

79. $8-x^3=2^3-x^3=(2-x)\left(4+2x+x^2\right)$

80. $27-x^3=3^3-x^3=(3-x)\left(9+3x+x^2\right)$

81. $8x^3-27=(2x)^3-3^3=(2x-3)\left(4x^2+6x+9\right)$

82. $8x^3-125=(2x)^3-5^3$
$=(2x-5)\left(4x^2+10x+25\right)$

83. $40x^3+5=5\left(8x^3+1\right)=5\left((2x)^3+1^3\right)$
$=5(2x+1)\left(4x^2-2x+1\right)$

84. $7x^3+56=7\left(x^3+8\right)=7\left(x^3+2^3\right)$
$=7(x+2)\left(x^2-2x+4\right)$

85. $x^4+x^2+25=x^4+10x^2-9x^2+25$
$=\left(x^4+10x^2+25\right)-9x^2$
$=\left(x^2+5\right)^2-9x^2$
$=\left[\left(x^2+5\right)+3x\right]\left[\left(x^2+5\right)-3x\right]$
$=\left(x^2+3x+5\right)\left(x^2-3x+5\right)$

86. $x^4+x^2+1=x^4+2x^2-x^2+1$
$=\left(x^4+2x^2+1\right)-x^2$
$=\left(x^2+1\right)^2-x^2$
$=\left[\left(x^2+1\right)+x\right]\left[\left(x^2+1\right)-x\right]$
$=\left(x^2+x+1\right)\left(x^2-x+1\right)$

87. $x^4+15x^2+64=x^4+16x^2-x^2+64$
$=\left(x^4+16x^2+64\right)-x^2$
$=\left(x^2+8\right)^2-x^2$
$=\left[\left(x^2+8\right)+x\right]\left[\left(x^2+8\right)-x\right]$
$=\left(x^2+x+8\right)\left(x^2-x+8\right)$

88. $x^4-7x^2+9=x^4-6x^2-x^2+9$
$=\left(x^4-6x^2+9\right)-x^2$
$=\left(x^2-3\right)^2-x^2$
$=\left[\left(x^2-3\right)+x\right]\left[\left(x^2-3\right)-x\right]$
$=\left(x^2+x-3\right)\left(x^2-x-3\right)$

89. $x^4-x^2+16=x^4+8x^2-9x^2+16$
$=\left(x^4+8x^2+16\right)-9x^2$
$=\left(x^2+4\right)^2-9x^2$
$=\left[\left(x^2+4\right)+3x\right]\left[\left(x^2+4\right)-3x\right]$
$=\left(x^2+3x+4\right)\left(x^2-3x+4\right)$

90. $x^4 - 16x^2 + 36 = x^4 - 12x^2 - 4x^2 + 36$
$= \left(x^4 - 12x^2 + 36\right) - 4x^2$
$= \left(x^2 - 6\right)^2 - 4x^2$
$= \left[\left(x^2 - 6\right) + 2x\right]\left[\left(x^2 - 6\right) - 2x\right]$
$= \left(x^2 + 2x - 6\right)\left(x^2 - 2x - 6\right)$

91. $x^4 + 4 = x^4 + 4x^2 - 4x^2 + 4$
$= \left(x^4 + 4x^2 + 4\right) - 4x^2$
$= \left(x^2 + 2\right)^2 - 4x^2$
$= \left[\left(x^2 + 2\right) + 2x\right]\left[\left(x^2 + 2\right) - 2x\right]$
$= \left(x^2 + 2x + 2\right)\left(x^2 - 2x + 2\right)$

92. $x^4 + 64 = x^4 + 16x^2 - 16x^2 + 64$
$= \left(x^4 + 16x^2 + 64\right) - 16x^2$
$= \left(x^2 + 8\right)^2 - 16x^2$
$= \left[\left(x^2 + 8\right) + 4x\right]\left[\left(x^2 + 8\right) - 4x\right]$
$= \left(x^2 + 4x + 8\right)\left(x^2 - 4x + 8\right)$

93. $1 - 16x^2 = (1 + 4x)(1 - 4x)$

94. $4 - 25x^2 = (2 - 5x)(2 + 5x)$

95. $x^2 - 6x + 9 = (x - 3)^2$

96. $x^2 - 8x + 16 = (x - 4)^2$

97. $4x^2 + 4x + 1 = (2x + 1)^2$

98. $16x^2 + 8x + 1 = (4x + 1)^2$

99. $2x^2 - 8x - 10 = 2\left(x^2 - 4x - 5\right)$
$= 2(x + 1)(x - 5)$

100. $5x^2 - 10x - 40 = 5\left(x^2 - 2x - 8\right)$
$= 5\left(x + 2\right)\left(x - 4\right)$

101. $2x^2 + 3x - 20 = (2x - 5)(x + 4)$

102. $2x^2 - 7x - 30 = (2x + 5)(x - 6)$

103. $x^2 - 24x + 36$ is irreducible.

104. $x^2 - 20x + 25$ is irreducible.

105. $3x^5 + 12x^4 + 12x^3 = 3x^3(x^2 + 4x + 4)$
$= 3x^3(x + 2)^2$

106. $2x^5 + 16x^4 + 32x^3 = 2x^3(x^2 + 8x + 16)$
$= 2x^3(x + 4)^2$

107. $9x^2 - 1 = (3x + 1)(3x - 1)$

108. $16x^2 - 25 = (4x + 5)(4x - 5)$

109. $16x^2 + 24x + 9 = (4x + 3)^2$

110. $4x^2 + 20x + 25 = (2x + 5)^2$

111. $x^2 + 15$ is irreducible.

112. $x^2 + 24$ is irreducible.

113. $45x^3 + 8x^2 - 4x = x\left(45x^2 + 8x - 4\right)$
$= x\left(5x + 2\right)\left(9x - 2\right)$

114. $25x^3 + 40x^2 - 9x = x\left(25x^2 + 40x - 9\right)$
$= x\left(5x + 9\right)\left(5x - 1\right)$

115. $ax^2 - 7a^2x - 8a^3 = a\left(x^2 - 7ax - 8a^2\right)$
$= a\left(x + a\right)\left(x - 8a\right)$

116. $ax^2 - 10a^2x - 24a^3 = a\left(x^2 - 10ax - 24a^2\right)$
$= a\left(x + 2a\right)\left(x - 12a\right)$

117. $x^2 - 16a^2 + 8x + 16 = \left(x^2 + 8x + 16\right) - 16a^2$
$= \left(x + 4\right)^2 - 16a^2$
$= \left(x + 4 + 4a\right)\left(x + 4 - 4a\right)$

118. $x^2 - 36a^2 + 6x + 9 = \left(x^2 + 6x + 9\right) - 36a^2$
$= \left(x + 3\right)^2 - 36a^2$
$= \left(x + 3 + 6a\right)\left(x + 3 - 6a\right)$

119. $3x^5 + 12x^4y + 12x^3y^2 = 3x^3\left(x^2 + 4xy + 4y^2\right)$
$= 3x^3\left(x + 2y\right)^2$

120. $4x^5 + 24x^4y + 36x^3y^2 = 4x^3\left(x^2 + 6xy + 9y^2\right)$
$= 4x^3\left(x + 3y\right)^2$

121. $x^2 - 25a^2 + 4x + 4 = \left(x^2 + 4x + 4\right) - 25a^2$
$= \left(x + 2\right)^2 - 25a^2$
$= \left(x + 2 + 5a\right)\left(x + 2 - 5a\right)$

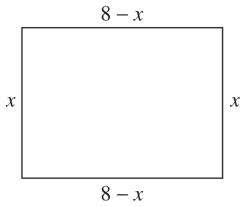
122. $x^2 - 16a^2 + 4x + 4 = (x^2 + 4x + 4) - 16a^2$
$= (x+2)^2 - 16a^2$
$= (x + 2 + 4a)(x + 2 - 4a)$

123. $18x^6 + 12x^5y + 2x^4y^2 = 2x^4(9x^2 + 6xy + y^2)$
$= 2x^4(3x + y)^2$

124. $12x^6 + 12x^5y + 3x^4y^2 = 3x^4(4x^2 + 4xy + y^2)$
$= 3x^4(2x + y)^2$

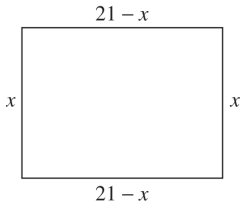
Applying the Concepts

125.



If one side of the garden is x feet, and the perimeter is 16 feet, then $\frac{16-2x}{2} = 8 - x$ gives the other dimension of the rectangle. So, the area of the garden is $x(8-x)$.

126.



If one side of the tray is x inches, and the perimeter is 42 inches, then $\frac{42-2x}{2} = 21 - x$ gives the other dimension of the tray. So, the area of the tray is $x(21-x)$.

127. a. Mateo will break even when the profit or loss is 0, so we must determine in how many months that will occur. Evaluate the polynomial that represents Mateo's profit or loss for different values until the value of the polynomial is 0. The number of months is represented by x.

x	$10x^3 - 30x^2 - 90x - 50$	Polynomial value
0	$10(0)^3 - 30(0)^2 - 90(0) - 50$	−50
1	$10(1)^3 - 30(1)^2 - 90(1) - 50$	−160
2	$10(2)^3 - 30(2)^2 - 90(2) - 50$	−270
3	$10(3)^3 - 30(3)^2 - 90(3) - 50$	−320
4	$10(4)^3 - 30(4)^2 - 90(4) - 50$	−250
5	$10(5)^3 - 30(5)^2 - 90(5) - 50$	0

Mateo will break even in 5 months.

b. Evaluate the profit or loss after 10 months by letting $x = 10$.
$10(10)^3 - 30(10)^2 - 90(10) - 50 = 6050$
After 10 months, there is a profit of \$6050.

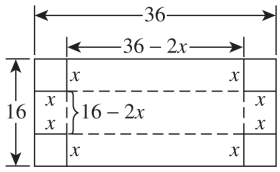
128. a. The company will break even when the profit or loss is 0, so we must determine in how many months that will occur. Evaluate the polynomial that represents the company's profit or loss for different values until the value of the polynomial is 0. The number of months is represented by x.

x	$90x^3 - 360x^2 + 630x - 2520$	Polynomial value
0	$90(0)^3 - 360(0)^2 + 630(0) - 2520$	−2520
1	$90(1)^3 - 360(1)^2 + 630(1) - 2520$	−2160
2	$90(2)^3 - 360(2)^2 + 630(2) - 2520$	−1980
3	$90(3)^3 - 360(3)^2 + 630(3) - 2520$	−1440
4	$90(4)^3 - 360(4)^2 + 630(4) - 2520$	0

The company will break even in 4 months.

b. Evaluate the profit or loss after 10 months by letting $x = 10$.
$90(10)^3 - 360(10)^2 + 630(10) - 2520$
$= 57,780$
After 10 months, there is a profit of \$57,780.

Use the figure below for exercises 129 and 130.

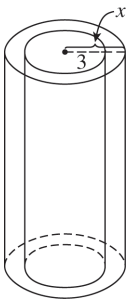


129. If x = the length of the cut corner, then $36 - 2x$ = the length of the box, and $16 - 2x$ = the width of the box. The height of the box is x. So, $v = x(36-2x)(16-2x) = x(2(18-x))(2(8-x)) = 4x(18-x)(8-x)$.

130. $S.A. = (36-2x)(16-2x) + 2x(36-2x) + 2x(16-2x) = 576 - 104x + 4x^2 + 72x - 4x^2 + 32x - 4x^2$
$= 576 - 4x^2 = 4(144 - x^2) = 4(12-x)(12+x)$

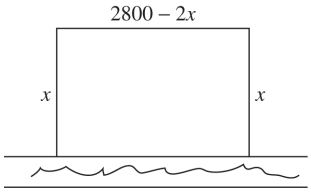
131. The area of the outside circle = $4\pi\,\text{cm}^2$. The area of the inside circle = $\pi x^2\,\text{cm}^2$. So, the area of the disk = area of outside circle – area of inside circle = $4\pi - \pi x^2 = \pi(4-x^2) = \pi(2-x)(2+x)\,\text{cm}^2$.

132.



The volume of the inside cylinder is $3^2\pi \cdot 8 = 72\pi\,\text{ft}^3$, and the volume of the outside cylinder is $x^2\pi \cdot 8 = 8\pi x^2\,\text{ft}^3$.
So the volume between the cylinders is
$8\pi x^2 - 72\pi = 8\pi(x^2 - 9) = 8\pi(x-3)(x+3)\,\text{ft}^3$.

133.



If one side of the fence is x feet, and the rancher needs a total of 2800 feet of fencing, then the width of the fence is $2800 - 2x$. So, the area of the pen is $x(2800-2x) = 2800x - 2x^2 = 2x(1400-x)\,\text{ft}^2$.

134. The area of the figure = the area of the rectangle plus the area of the circle. Find the length of the rectangle as follows: The circumference of the circle = $2\pi\left(\frac{x}{2}\right) = \pi x$. The perimeter of the figure is 48, so the length of the rectangle is $\frac{48-\pi x}{2}$. The area of the circle is $\frac{\pi x^2}{4}$ and the area of the rectangle is $x\left(24 - \frac{\pi x}{2}\right) = 24x - \frac{\pi x^2}{2}$. So. the area of the figure is

$$\frac{\pi x^2}{4} + \left(24x - \frac{\pi x^2}{2}\right) = \frac{1}{4}(96x - \pi x^2) = \frac{1}{4}x(96-\pi x)\,\text{in.}^2$$

Beyond the Basics

135. $x^3 + y^3 + x + y = (x+y)\left(x^2 - xy + y^2\right) + x + y$
$= (x+y)\left(x^2 - xy + y^2 + 1\right)$

136. $x^4 - 11x^2 + 1 = x^4 - 2x^2 + 1 - 9x^2 = \left(x^2 - 1\right)^2 - 9x^2$
$= \left(x^2 - 1 - 3x\right)\left(x^2 - 1 + 3x\right) = \left(x^2 - 3x - 1\right)\left(x^2 + 3x - 1\right)$

137. $x^4+y^4+x^2y^2 = x^4+2x^2y^2+y^4-x^2y^2 = \left(x^2+y^2\right)^2-x^2y^2$
$= \left(x^2+y^2-xy\right)\left(x^2+y^2+xy\right) = \left(x^2-xy+y^2\right)\left(x^2+xy+y^2\right)$

138. $x^4+4y^4 = \left(x^2\right)^2+\left(2y^2\right)^2+4x^2y^2-4x^2y^2$
$= \left(x^2\right)^2+4x^2y^2+\left(2y^2\right)^2-4x^2y^2$
$= \left(x^2+2y^2\right)^2-4x^2y^2$
$= \left(x^2+2y^2-2xy\right)\left(x^2+2y^2+2xy\right)$
$= \left(x^2-2xy+2y^2\right)\left(x^2+2xy+2y^2\right)$

139. $x^6-64 = \left(x^3\right)^2-8^2 = \left(x^3-8\right)\left(x^3+8\right)$
$= (x-2)(x+2x+4)(x+2)(x-2x+4)$
$= (x-2)(x+2)(x+2x+4)(x-2x+4)$

140. $x^6-26x^3-27 = \left(x^3-27\right)\left(x^3+1\right) = (x-3)\left(x^2+3x+9\right)(x+1)\left(x^2-x+1\right)$

141. $\left(x^2-3x\right)^2-38\left(x^2-3x\right)-80 = \left[\left(x^2-3x\right)-40\right]\left[\left(x^2-3x\right)+2\right]$
$= \left(x^2-3x-40\right)\left(x^2-3x+2\right)$
$= (x-8)(x+5)(x-2)(x-1)$

142. $4x^2-y^2-z^2+2yz = 4x^2-\left(y^2+z^2-2yz\right) = 4x^2-\left(y^2-2yz+z^2\right)$
$= 4x^2-(y-z)^2 = (2x-(y-z))(2x+(y-z))$
$= (2x-y+z)(2x+y-z)$

143. From exercise 109 in Section P.3, we have $(a+b+c)\left(a^2+b^2+c^2-ab-bc-ac\right) = a^3+b^3+c^3-3abc.$

Then, $a^3+b^3+c^3-3abc = (a+b+c)\left(a^2+b^2+c^2-ab-bc-ac\right)$
$= \frac{1}{2}(a+b+c)\cdot 2\left(a^2+b^2+c^2-ab-bc-ac\right)$
$= \frac{1}{2}(a+b+c)\left(2a^2+2b^2+2c^2-2ab-2bc-2ac\right)$
$= \frac{1}{2}(a+b+c)\left(\left(a^2-2ab+b^2\right)+\left(b^2-2bc+c^2\right)+\left(c^2-2ca+a^2\right)\right)$
$= \frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

If $a^3+b^3+c^3-3abc = 0,$ then either $a+b+c=0$ or $(a-b)^2+(b-c)^2+(c-a)^2 = 0 \Rightarrow a=b=c.$

144. See exercise 111 in section P.3.
$(x-y)^3+(y-z)^3+(z-x)^3 = 3(x-y)(y-z)(z-x)$

145. Let $a = z(x-y) = xz-yz,\ b = x(y-z) = xy-xz,$ and $c = y(z-x) = yz-xy.$ Then
$a+b+c = xz-yz+xy-xz+yz-xy = 0,$ and we can apply Section P.3 exercise 110.
$z^3(x-y)^3+x^3(y-z)^3+y^3(z-x)^3 = 3xyz(x-y)(y-z)(z-x)$

146. $\dfrac{\left(x^2-y^2\right)^3+\left(y^2-z^2\right)^3+\left(z^2-x^2\right)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$

In the numerator, let $a=x^2-y^2$, $b=y^2-z^2$, and $c=z^2-x^2$. Then $a+b+c=0$, and we can apply Section P.3 exercise 111. Use the result from exercise 144 above for the denominator.

$$\begin{aligned}\frac{\left(x^2-y^2\right)^3+\left(y^2-z^2\right)^3+\left(z^2-x^2\right)^3}{(x-y)^3+(y-z)^3+(z-x)^3}&=\frac{3\left(x^2-y^2\right)\left(y^2-z^2\right)\left(z^2-x^2\right)}{3(x-y)(y-z)(z-x)}\\&=\frac{(x+y)(x-y)(y+z)(y-z)(z+x)(z-x)}{(x-y)(y-z)(z-x)}\\&=(x+y)(y+z)(z+x)\end{aligned}$$

Getting Ready for the Next Section

GR1. $3x^2+9=3\left(x^2+3\right)$

GR2. $21x-7x^2=7x(3-x)$

GR3. $x^2+2x-15=(x+5)(x-3)$

GR4. $x^2-5x+4=(x-4)(x-1)$

GR5. $\dfrac{2x}{4x+6}=\dfrac{2x}{2(2x+3)}=\dfrac{x}{2x+3}$

GR6. $\dfrac{2x^3+3x}{x^2+x}=\dfrac{x\left(2x^2+3\right)}{x(x+1)}=\dfrac{2x^2+3}{x+1}$

GR7. $\dfrac{x^2+3x+2}{x^2+4x+3}=\dfrac{(x+1)(x+2)}{(x+1)(x+3)}=\dfrac{x+2}{x+3}$

GR8. $\dfrac{x^2-x-6}{x^2-4}=\dfrac{(x-3)(x+2)}{(x-2)(x+2)}=\dfrac{x-3}{x-2}$

P.5 Rational Expressions

Practice Problems

1. $\dfrac{0.05(1-0.1)}{0.95(0.1)+(0.05)(1-0.1)}\approx 0.32$

The likelihood that a student who tests positive is a nonuser when the test that is used is 95% accurate is about 32%.

2. a. Note that $x\neq 0,\ x\neq -4$.

$$\frac{2x^3+8x^2}{3x^2+12x}=\frac{2x^{\cancel{2}}\cancel{(x+4)}}{3\cancel{x}\,\cancel{(x+4)}}=\frac{2x}{3}$$

b. Note that $x\neq -2$.

$$\begin{aligned}\frac{x^2-4}{x^2+4x+4}&=\frac{(x-2)(x+2)}{(x+2)^2}\\&=\frac{(x-2)\,\cancel{(x+2)}}{(x+2)\,\cancel{(x+2)}}=\frac{x-2}{x+2}\end{aligned}$$

3. Note that $x\neq 0,\ x\neq -4,\ x\neq -1$

$$\begin{aligned}\frac{\dfrac{x^2-2x-3}{7x^3+28x^2}}{\dfrac{4x+4}{2x^4+8x^3}}&=\frac{x^2-2x-3}{7x^3+28x^2}\div\frac{4x+4}{2x^4+8x^3}\\&=\frac{x^2-2x-3}{7x^3+28x^2}\cdot\frac{2x^4+8x^3}{4x+4}\\&=\frac{(x-3)\,\cancel{(x+1)}}{7x^2\,\cancel{(x+4)}}\cdot\frac{2\cancel{x^3}\,\cancel{(x+4)}}{\cancel{4}\,\cancel{(x+1)}}\\&=\frac{x(x-3)}{14}\end{aligned}$$

4. a. Note that $x\neq 6,\ x\neq -6$.

$$\begin{aligned}\frac{5x+22}{x^2-36}+\frac{2(x+10)}{x^2-36}&=\frac{5x+22+2x+20}{x^2-36}\\&=\frac{7x+42}{x^2-36}\\&=\frac{7\cancel{(x+6)}}{(x-6)\,\cancel{(x+6)}}=\frac{7}{x-6}\end{aligned}$$

b. Note that $x\neq -4,\ x\neq 3$.

$$\begin{aligned}\frac{4x+1}{x^2+x-12}-\frac{3x+4}{x^2+x-12}&=\frac{(4x+1)-(3x+4)}{x^2+x-12}\\&=\frac{x-3}{x^2+x-12}\\&=\frac{\cancel{x-3}}{(x+4)\,\cancel{(x-3)}}\\&=\frac{1}{x+4}\end{aligned}$$

5. a. Note that $x \neq -2,\ x \neq 5$.

$$\frac{2x}{x+2}+\frac{3x}{x-5}$$
$$=\frac{2x(x-5)}{(x+2)(x-5)}+\frac{3x(x+2)}{(x+2)(x-5)}$$
$$=\frac{2x^2-10x+3x^2+6x}{(x+2)(x-5)}=\frac{5x^2-4x}{(x+2)(x-5)}$$
$$=\frac{x(5x-4)}{(x+2)(x-5)}$$

b. Note that $x \neq 4,\ x \neq -3$.

$$\frac{5x}{x-4}-\frac{2x}{x+3}$$
$$=\frac{5x(x+3)}{(x-4)(x+3)}-\frac{2x(x-4)}{(x-4)(x+3)}$$
$$=\frac{5x^2+15x-2x^2+8x}{(x-4)(x+3)}=\frac{3x^2+23x}{(x-4)(x+3)}$$
$$=\frac{x(3x+23)}{(x-4)(x+3)}$$

6. a. $\dfrac{x^2+3x}{x^2(x+2)^2(x-2)},\ \dfrac{4x^2+1}{3x(x-2)^2}$

$\text{LCD}=3x^2(x+2)^2(x-2)^2$

b. $\dfrac{2x-1}{x^2-25},\ \dfrac{3-7x^2}{x^2+4x-5}$

$x^2-25=(x-5)(x+5)$

$x^2+4x-5=(x-1)(x+5)$

$\text{LCD}=(x-1)(x-5)(x+5)$

7. a. Note that $x \neq 2,\ x \neq -2$.

$$\frac{4}{x^2-4x+4}+\frac{x}{x^2-4}$$
$$=\frac{4}{(x-2)^2}+\frac{x}{(x-2)(x+2)}$$ The LCD is $(x-2)^2(x+2)$.
$$=\frac{4(x+2)}{(x-2)^2(x+2)}+\frac{x(x-2)}{(x-2)^2(x+2)}$$
$$=\frac{(4x+8)+(x^2-2x)}{(x-2)^2(x+2)}=\frac{x^2+2x+8}{(x-2)^2(x+2)}$$

b. Note that $x \neq 0,\ x \neq 5$.

$$\frac{2x}{3(x-5)^2}-\frac{6x}{2(x^2-5x)}$$
$$=\frac{2x}{3(x-5)^2}-\frac{6x}{2x(x-5)}$$ The LCD is $6x(x-5)^2$.
$$=\frac{2x(2x)}{3\cdot 2x(x-5)^2}-\frac{6x(3)(x-5)}{3\cdot 2x(x-5)^2}$$
$$=\frac{4x^2-18x^2+90x}{6x(x-5)^2}=\frac{-14x^2+90x}{6x(x-5)^2}$$
$$=\frac{2x(-7x+45)}{6x(x-5)^2}=\frac{-7x+45}{3(x-5)^2}$$

8. Note that $x \neq 0,\ x \neq 5,\ x \neq -5$.

$$\frac{\frac{5}{3x}+\frac{1}{3}}{\frac{x^2-25}{3x}}=\frac{\left(\frac{5}{3x}+\frac{1}{3}\right)\cdot 3x}{\left(\frac{x^2-25}{3x}\right)\cdot 3x}=\frac{5+x}{x^2-25}$$
$$=\frac{\cancel{x+5}}{(x-5)\cancel{(x+5)}}=\frac{1}{x-5}$$

9. Note that $x \neq \dfrac{4}{5}$.

$$\frac{5x}{3x-\frac{4}{2-\frac{1}{3}}}=\frac{5x}{3x-\frac{4}{\frac{5}{3}}}=\frac{5x}{3x-4\left(\frac{3}{5}\right)}=\frac{5x}{3x-\frac{12}{5}}$$
$$=\frac{5x\cdot 5}{\left(3x-\frac{12}{5}\right)\cdot 5}=\frac{25x}{15x-12}$$

Concepts and Vocabulary

1. The least common denominator of two rational expressions is the polynomial of least degree that contains each denominator as a factor.

2. The first step in finding the LCD of two rational expressions is to factor the denominators completely.

3. If the denominators of two rational expressions are x^2-2x and x^2-x-2, then the LCD is $x(x-2)(x+1)$.

4. A rational expression that contains another rational expression in its numerator or denominator is called a complex rational expression.

5. False **6.** True

7. False **8.** True

Building Skills

9. $\dfrac{2x+2}{x^2+2x+1}=\dfrac{2(x+1)}{(x+1)^2}=\dfrac{2}{x+1}, x\neq -1$

10. $\dfrac{3x-6}{x^2-4x+4}=\dfrac{3(x-2)}{(x-2)^2}=\dfrac{3}{x-2}, x\neq 2$

11. $\dfrac{3x+3}{x^2-1}=\dfrac{3(x+1)}{(x-1)(x+1)}=\dfrac{3}{x-1}, x\neq -1, x\neq 1$

12. $\dfrac{10-5x}{4-x^2}=\dfrac{5(2-x)}{(2-x)(2+x)}=\dfrac{5}{2+x},\ x\neq -2, x\neq 2$

13. $\dfrac{2x-6}{9-x^2}=\dfrac{2(x-3)}{(3-x)(3+x)}=\dfrac{-2(3-x)}{(3-x)(3+x)}$
$=-\dfrac{2}{3+x}, x\neq -3, x\neq 3$

14. $\dfrac{15+3x}{x^2-25}=\dfrac{3(5+x)}{(x-5)(x+5)}=\dfrac{3}{x-5},\ x\neq -5, x\neq 5$

15. $\dfrac{2x-1}{1-2x}=\dfrac{-1(1-2x)}{1-2x}=-1, x\neq \dfrac{1}{2}$

16. $\dfrac{2-5x}{5x-2}=\dfrac{-1(5x-2)}{5x-2}=-1, x\neq \dfrac{2}{5}$

17. $\dfrac{x^2-6x+9}{4x-12}=\dfrac{(x-3)^2}{4(x-3)}=\dfrac{x-3}{4}, x\neq 3$

18. $\dfrac{x^2-10x+25}{3x-15}=\dfrac{(x-5)^2}{3(x-5)}=\dfrac{x-5}{3}, x\neq 5$

19. $\dfrac{7x^2+7x}{x^2+2x+1}=\dfrac{7x(x+1)}{(x+1)^2}=\dfrac{7x}{x+1}, x\neq -1$

20. $\dfrac{4x^2+12x}{x^2+6x+9}=\dfrac{4x(x+3)}{(x+3)^2}=\dfrac{4x}{x+3}, x\neq -3$

21. $\dfrac{x^2-11x+10}{x^2+6x-7}=\dfrac{(x-10)(x-1)}{(x+7)(x-1)}$
$=\dfrac{x-10}{x+7},\ x\neq -7, x\neq 1$

22. $\dfrac{x^2+2x-15}{x^2-7x+12}=\dfrac{(x+5)(x-3)}{(x-4)(x-3)}$
$=\dfrac{x+5}{x-4},\ x\neq 3, x\neq 4$

23. $\dfrac{6x^4+14x^3+4x^2}{6x^4-10x^3-4x^2}=\dfrac{2x^2\left(3x^2+7x+2\right)}{2x^2\left(3x^2-5x-2\right)}$
$=\dfrac{2x^2(3x+1)(x+2)}{2x^2(3x+1)(x-2)}$
$=\dfrac{x+2}{x-2}, x\neq -\dfrac{1}{3}, x\neq 0, x\neq 2$

24. $\dfrac{3x^3+x^2}{3x^4-11x^3-4x^2}=\dfrac{x^2(3x+1)}{x^2\left(3x^2-11x-4\right)}$
$=\dfrac{x^2(3x+1)}{x^2(3x+1)(x-4)}$
$=\dfrac{1}{x-4},\ x\neq -\dfrac{1}{3}, x\neq 0, x\neq 4$

25. $\dfrac{x-3}{2x+4}\cdot\dfrac{10x+20}{5x-15}=\dfrac{x-3}{2(x+2)}\cdot\dfrac{10(x+2)}{5(x-3)}=1$

26. $\dfrac{6x+4}{2x-8}\cdot\dfrac{x-4}{9x+6}=\dfrac{2(3x+2)}{2(x-4)}\cdot\dfrac{x-4}{3(3x+2)}=\dfrac{1}{3}$

27. $\dfrac{2x+6}{4x-8}\cdot\dfrac{x^2+x-6}{x^2-9}=\dfrac{2(x+3)}{4(x-2)}\cdot\dfrac{(x+3)(x-2)}{(x+3)(x-3)}$
$=\dfrac{x+3}{2(x-3)}$

28. $\dfrac{25x^2-9}{4-2x}\cdot\dfrac{4-x^2}{10x-6}$
$=\dfrac{(5x+3)(5x-3)}{2(2-x)}\cdot\dfrac{(2-x)(2+x)}{2(5x-3)}$
$=\dfrac{(5x+3)(x+2)}{4}$

29. $\dfrac{x^2-7x}{x^2-6x-7}\cdot\dfrac{x^2-1}{x^2}$
$=\dfrac{x(x-7)}{(x+1)(x-7)}\cdot\dfrac{(x+1)(x-1)}{x^2}=\dfrac{x-1}{x}$

30. $\dfrac{x^2-9}{x^2-6x+9}\cdot\dfrac{5x-15}{x+3}=\dfrac{(x-3)(x+3)}{(x-3)(x-3)}\cdot\dfrac{5(x-3)}{x+3}$
$=5$

31. $\dfrac{x^2-x-6}{x^2+3x+2}\cdot\dfrac{x^2-1}{x^2-9}$
$=\dfrac{(x-3)(x+2)}{(x+1)(x+2)}\cdot\dfrac{(x-1)(x+1)}{(x-3)(x+3)}=\dfrac{x-1}{x+3}$

32. $\dfrac{x^2+2x-8}{x^2+x-20}\cdot\dfrac{x^2-16}{x^2+5x+4}=\dfrac{(x-2)(x+4)}{(x-4)(x+5)}\cdot\dfrac{(x-4)(x+4)}{(x+1)(x+4)}=\dfrac{(x-2)(x+4)}{(x+5)(x+1)}$

33. $\dfrac{2-x}{x+1}\cdot\dfrac{x^2+3x+2}{x^2-4}=\dfrac{-1(x-2)}{x+1}\cdot\dfrac{(x+1)(x+2)}{(x-2)(x+2)}=-1$

34. $\dfrac{3-x}{x+5}\cdot\dfrac{x^2+8x+15}{x^2-9}=\dfrac{-1(x-3)}{x+5}\cdot\dfrac{(x+3)(x+5)}{(x-3)(x+3)}=-1$

35. $\dfrac{x+2}{6}\div\dfrac{4x+8}{9}=\dfrac{x+2}{6}\cdot\dfrac{9}{4(x+2)}=\dfrac{3}{8}$

36. $\dfrac{x+3}{20}\div\dfrac{4x+12}{9}=\dfrac{x+3}{20}\cdot\dfrac{9}{4(x+3)}=\dfrac{9}{80}$

37. $\dfrac{x^2-9}{x}\div\dfrac{2x+6}{5x^2}=\dfrac{(x-3)(x+3)}{x}\cdot\dfrac{5x^2}{2(x+3)}=\dfrac{5x(x-3)}{2}$

38. $\dfrac{x^2-1}{3x}\div\dfrac{7x-7}{x^2+x}=\dfrac{(x-1)(x+1)}{3x}\cdot\dfrac{x(x+1)}{7(x-1)}=\dfrac{(x+1)^2}{21}$

39. $\dfrac{x^2+2x-3}{x^2+8x+16}\div\dfrac{x-1}{3x+12}=\dfrac{(x-1)(x+3)}{(x+4)^2}\cdot\dfrac{3(x+4)}{x-1}=\dfrac{3(x+3)}{x+4}$

40. $\dfrac{x^2+5x+6}{x^2+6x+9}\div\dfrac{x^2+3x+2}{x^2+7x+12}=\dfrac{(x+2)(x+3)}{(x+3)(x+3)}\cdot\dfrac{(x+3)(x+4)}{(x+1)(x+2)}=\dfrac{x+4}{x+1}$

41. $\left(\dfrac{x^2-9}{x^3+8}\div\dfrac{x+3}{x^3+2x^2-x-2}\right)\left(\dfrac{1}{x^2-1}\right)=\dfrac{(x-3)(x+3)}{(x+2)(x^2-2x+4)}\cdot\dfrac{(x^2-1)(x+2)}{x+3}\cdot\dfrac{1}{x^2-1}=\dfrac{x-3}{x^2-2x+4}$

42. $\left(\dfrac{x^2-25}{x^2-3x-4}\div\dfrac{x^2+3x-10}{x^2-1}\right)\left(\dfrac{x-2}{x-5}\right)=\dfrac{(x-5)(x+5)}{(x-4)(x+1)}\cdot\dfrac{(x-1)(x+1)}{(x+5)(x-2)}\cdot\dfrac{x-2}{x-5}=\dfrac{x-1}{x-4}$

43. $\dfrac{x}{5}+\dfrac{3}{5}=\dfrac{x+3}{5}$

44. $\dfrac{7}{4}-\dfrac{x}{4}=\dfrac{7-x}{4}$

45. $\dfrac{x}{2x+1}+\dfrac{4}{2x+1}=\dfrac{x+4}{2x+1}$

46. $\dfrac{2x}{7x-3}+\dfrac{x}{7x-3}=\dfrac{3x}{7x-3}$

47. $\dfrac{x^2}{x+1}-\dfrac{x^2-1}{x+1}=\dfrac{x^2-(x^2-1)}{x+1}=\dfrac{1}{x+1}$

48. $\dfrac{2x+7}{3x+2}-\dfrac{x-2}{3x+2}=\dfrac{(2x+7)-(x-2)}{3x+2}=\dfrac{x+9}{3x+2}$

49. $\dfrac{4}{3-x}+\dfrac{2x}{x-3}=\dfrac{4}{-1(x-3)}+\dfrac{2x}{x-3}$
$=\dfrac{-4+2x}{x-3}=\dfrac{2(x-2)}{x-3}$

50. $\dfrac{-2}{1-x}+\dfrac{2-x}{x-1}=\dfrac{-2}{1-x}+\dfrac{2-x}{-1(1-x)}$
$=\dfrac{-2}{1-x}-\dfrac{2-x}{1-x}=\dfrac{-4+x}{1-x}$
$=\dfrac{x-4}{1-x}$ or $-\dfrac{x-4}{x-1}$

51. $\dfrac{5x}{x^2+1}+\dfrac{2x}{x^2+1}=\dfrac{7x}{x^2+1}$

52. $\dfrac{x}{2(x-1)^2}+\dfrac{3x}{2(x-1)^2}=\dfrac{4x}{2(x-1)^2}=\dfrac{2x}{(x-1)^2}$

53. $\dfrac{7x}{2(x-3)}+\dfrac{x}{2(x-3)}=\dfrac{8x}{2(x-3)}=\dfrac{4x}{x-3}$

54. $\dfrac{4x}{4(x+5)^2}+\dfrac{8x}{4(x+5)^2}=\dfrac{12x}{4(x+5)^2}=\dfrac{3x}{(x+5)^2}$

55. $\dfrac{x}{x^2-4}-\dfrac{2}{x^2-4}=\dfrac{x-2}{(x-2)(x+2)}=\dfrac{1}{x+2}$

56. $\dfrac{5x}{x^2-1}-\dfrac{5}{x^2-1}=\dfrac{5x-5}{(x-1)(x+1)}=\dfrac{5(x-1)}{(x-1)(x+1)}=\dfrac{5}{x+1}$

57. $\dfrac{x-2}{2x+1}-\dfrac{x}{2x-1}=\dfrac{(x-2)(2x-1)}{(2x+1)(2x-1)}-\dfrac{x(2x+1)}{(2x+1)(2x-1)}=\dfrac{(2x^2-5x+2)-(2x^2+x)}{(2x+1)(2x-1)}$

$=\dfrac{-6x+2}{(2x+1)(2x-1)}=\dfrac{2(1-3x)}{(2x+1)(2x-1)}$

58. $\dfrac{2x-1}{4x+1}-\dfrac{2x}{4x-1}=\dfrac{(2x-1)(4x-1)}{(4x+1)(4x-1)}-\dfrac{2x(4x+1)}{(4x+1)(4x-1)}=\dfrac{(8x^2-6x+1)-(8x^2+2x)}{(4x+1)(4x-1)}=\dfrac{1-8x}{(4x+1)(4x-1)}$

59. $\dfrac{-x}{x+2}+\dfrac{x-2}{x}-\dfrac{x}{x-2}=\dfrac{-x^2(x-2)}{x(x+2)(x-2)}+\dfrac{(x-2)^2(x+2)}{x(x+2)(x-2)}-\dfrac{x^2(x+2)}{x(x+2)(x-2)}$

$=\dfrac{(-x^3+2x^2)}{x(x+2)(x-2)}+\dfrac{(x^3-2x^2-4x+8)}{x(x+2)(x-2)}+\dfrac{-(x^3+2x^2)}{x(x+2)(x-2)}$

$=\dfrac{-x^3-2x^2-4x+8}{x(x+2)(x-2)}=-\dfrac{x^3+2x^2+4x-8}{x(x+2)(x-2)}$

60. $\dfrac{3x}{x-1}+\dfrac{x+1}{x}-\dfrac{2x}{x+1}=\dfrac{3x^2(x+1)}{x(x-1)(x+1)}+\dfrac{(x+1)^2(x-1)}{x(x-1)(x+1)}-\dfrac{2x^2(x-1)}{x(x-1)(x+1)}$

$=\dfrac{3x^3+3x^2}{x(x-1)(x+1)}+\dfrac{x^3+x^2-x-1}{x(x-1)(x+1)}-\dfrac{2x^3-2x^2}{x(x-1)(x+1)}=\dfrac{2x^3+6x^2-x-1}{x(x-1)(x+1)}$

In exercises 61–68, to find the LCD, first factor each denominator completely, form a product of the distinct irreducible factors of each polynomial, and then attach to each factor in this product the greatest exponent that appears on this factor in any of the factored denominators.

61. The denominators are $3x-6=3(x-2)$ and $4x-8=4(x-2)\Rightarrow$ LCD $=3\cdot4(x-2)=12(x-2)$

62. The denominators are $7+21x=7(1+3x)$ and $3+9x=3(1+3x)\Rightarrow$ LCD $=7\cdot3(1+3x)=21(1+3x)$

63. The denominators are $4x^2-1=(2x+1)(2x-1)$ and $(2x+1)^2=(2x+1)(2x+1)\Rightarrow$ LCD $=(2x-1)(2x+1)^2$

64. The denominators are $(3x-1)^2=(3x-1)(3x-1)$ and $9x^2-1=(3x-1)(3x+1)\Rightarrow$ LCD $=(3x-1)^2(3x+1)$

65. The denominators are $x^2+3x+2=(x+1)(x+2)$ and $x^2-1=(x-1)(x+1)\Rightarrow$ LCD $=(x-1)(x+1)(x+2)$

66. The denominators are $x^2-x-6=(x-3)(x+2)$ and $x^2-9=(x-3)(x+3)\Rightarrow$ LCD $=(x-3)(x+3)(x+2)$

67. The denominators are $x^2-5x+4=(x-1)(x-4)$ and
$x^2+x-2=(x-1)(x+2)\Rightarrow$ LCD $=(x-1)(x-4)(x+2)$

68. The denominators are $x^2 - 2x - 3 = (x-3)(x+1)$ and

$x^2 + 3x + 2 = (x+1)(x+2) \Rightarrow \text{LCD} = (x-3)(x+1)(x+2)$

For exercises 69–84, remember to find the LCD first, and then rewrite each rational expression as a rational expression with the LCD as the denominator. Next add or subtract numerators, following the rules for order of operations. Finally simplify the final rational expression. Leave your answer in factored form.

69. $\dfrac{5}{x-3} + \dfrac{2x}{x^2-9} = \dfrac{5}{x-3} + \dfrac{2x}{(x-3)(x+3)} = \dfrac{5(x+3)}{(x-3)(x+3)} + \dfrac{2x}{(x-3)(x+3)} = \dfrac{5x+15+2x}{(x-3)(x+3)} = \dfrac{7x+15}{(x-3)(x+3)}$

70. $\dfrac{3x}{x-1} + \dfrac{x}{x^2-1} = \dfrac{3x}{x-1} + \dfrac{x}{(x-1)(x+1)} = \dfrac{3x(x+1)}{(x-1)(x+1)} + \dfrac{x}{(x-1)(x+1)} = \dfrac{3x^2+3x+x}{(x-1)(x+1)} = \dfrac{3x^2+4x}{(x-1)(x+1)}$
$= \dfrac{x(3x+4)}{(x-1)(x+1)}$

71. $\dfrac{2x}{x^2-4} - \dfrac{x}{x+2} = \dfrac{2x}{(x-2)(x+2)} - \dfrac{x}{x+2} = \dfrac{2x}{(x-2)(x+2)} - \dfrac{x(x-2)}{(x+2)(x-2)}$
$= \dfrac{2x-(x^2-2x)}{(x-2)(x+2)} = \dfrac{-x^2+4x}{(x-2)(x+2)} = \dfrac{x(4-x)}{(x-2)(x+2)}$

72. $\dfrac{3x-1}{x^2-16} - \dfrac{2x+1}{x-4} = \dfrac{3x-1}{(x-4)(x+4)} - \dfrac{2x+1}{x-4} = \dfrac{3x-1}{(x-4)(x+4)} - \dfrac{(2x+1)(x+4)}{(x-4)(x+4)}$
$= \dfrac{3x-1-(2x^2+9x+4)}{(x-4)(x+4)} = \dfrac{-2x^2-6x-5}{(x-4)(x+4)} = -\dfrac{2x^2+6x+5}{(x-4)(x+4)}$

73. $\dfrac{x-2}{x^2+3x-10} + \dfrac{x+3}{x^2+x-6} = \dfrac{x-2}{(x-2)(x+5)} + \dfrac{x+3}{(x-2)(x+3)} = \dfrac{1}{x+5} + \dfrac{1}{x-2}$
$= \dfrac{x-2}{(x+5)(x-2)} + \dfrac{x+5}{(x+5)(x-2)} = \dfrac{2x+3}{(x-2)(x+5)}$

74. $\dfrac{x+3}{x^2-x-2} + \dfrac{x-1}{x^2+2x+1} = \dfrac{x+3}{(x-2)(x+1)} + \dfrac{x-1}{(x+1)(x+1)} = \dfrac{(x+3)(x+1)}{(x-2)(x+1)(x+1)} + \dfrac{(x-1)(x-2)}{(x+1)(x+1)(x-2)}$
$= \dfrac{(x^2+4x+3)+(x^2-3x+2)}{(x-2)(x+1)(x+1)} = \dfrac{2x^2+x+5}{(x-2)(x+1)^2}$

75. $\dfrac{2x-3}{9x^2-1} + \dfrac{4x-1}{(3x-1)^2} = \dfrac{2x-3}{(3x-1)(3x+1)} + \dfrac{4x-1}{(3x-1)^2} = \dfrac{(2x-3)(3x-1)}{(3x+1)(3x-1)^2} + \dfrac{(4x-1)(3x+1)}{(3x+1)(3x-1)^2}$
$= \dfrac{(6x^2-11x+3)+(12x^2+x-1)}{(3x+1)(3x-1)^2} = \dfrac{18x^2-10x+2}{(3x+1)(3x-1)^2} = \dfrac{2(9x^2-5x+1)}{(3x+1)(3x-1)^2}$

76. $\dfrac{3x+1}{(2x+1)^2} + \dfrac{x+3}{4x^2-1} = \dfrac{3x+1}{(2x+1)^2} + \dfrac{x+3}{(2x-1)(2x+1)} = \dfrac{(3x+1)(2x-1)}{(2x+1)^2(2x-1)} + \dfrac{(x+3)(2x+1)}{(2x-1)(2x+1)^2}$
$= \dfrac{(6x^2-x-1)+(2x^2+7x+3)}{(2x-1)(2x+1)^2} = \dfrac{8x^2+6x+2}{(2x-1)(2x+1)^2} = \dfrac{2(4x^2+3x+1)}{(2x-1)(2x+1)^2}$

77. $\dfrac{x-3}{x^2-25} - \dfrac{x-3}{x^2+9x+20} = \dfrac{x-3}{(x-5)(x+5)} - \dfrac{x-3}{(x+4)(x+5)} = \dfrac{(x-3)(x+4)}{(x-5)(x+5)(x+4)} - \dfrac{(x-3)(x-5)}{(x+4)(x+5)(x-5)}$
$= \dfrac{(x^2+x-12)-(x^2-8x+15)}{(x-5)(x+5)(x+4)} = \dfrac{9x-27}{(x-5)(x+5)(x+4)} = \dfrac{9(x-3)}{(x-5)(x+5)(x+4)}$

78. $\dfrac{2x}{x^2-16}-\dfrac{2x-7}{x^2-7x+12}=\dfrac{2x}{(x-4)(x+4)}-\dfrac{2x-7}{(x-4)(x-3)}=\dfrac{2x(x-3)}{(x-4)(x+4)(x-3)}-\dfrac{(2x-7)(x+4)}{(x-4)(x+4)(x-3)}$

$=\dfrac{(2x^2-6x)-(2x^2+x-28)}{(x-4)(x+4)(x-3)}=\dfrac{-7x+28}{(x-4)(x+4)(x-3)}$

$=\dfrac{-7(x-4)}{(x-4)(x+4)(x-3)}=-\dfrac{7}{(x+4)(x-3)}$

79. $\dfrac{3}{x^2-4}+\dfrac{1}{2-x}-\dfrac{1}{2+x}=-\dfrac{3}{4-x^2}+\dfrac{1}{2-x}-\dfrac{1}{2+x}=-\dfrac{3}{(2-x)(2+x)}+\dfrac{1}{2-x}-\dfrac{1}{2+x}$

$=\dfrac{-3}{(2-x)(2+x)}+\dfrac{1(2+x)}{(2-x)(2+x)}-\dfrac{1(2-x)}{(2+x)(2-x)}=\dfrac{-3+(2+x)-(2-x)}{(2-x)(2+x)}=\dfrac{2x-3}{(2-x)(2+x)}$

80. $\dfrac{2}{5+x}+\dfrac{5}{x^2-25}+\dfrac{7}{5-x}=\dfrac{2}{5+x}-\dfrac{5}{25-x^2}+\dfrac{7}{5-x}=\dfrac{2}{5+x}-\dfrac{5}{(5-x)(5+x)}+\dfrac{7}{5-x}$

$=\dfrac{2(5-x)}{(5+x)(5-x)}-\dfrac{5}{(5-x)(5+x)}+\dfrac{7(5+x)}{(5-x)(5+x)}$

$=\dfrac{(10-2x)-5+(35+7x)}{(5+x)(5-x)}=\dfrac{5x+40}{(5+x)(5-x)}=\dfrac{5(x+8)}{(5+x)(5-x)}$

81. $\dfrac{x+3a}{x-5a}-\dfrac{x+5a}{x-3a}=\dfrac{(x+3a)(x-3a)}{(x-5a)(x-3a)}-\dfrac{(x+5a)(x-5a)}{(x-3a)(x-5a)}=\dfrac{(x^2-9a^2)-(x^2-25a^2)}{(x-5a)(x-3a)}=\dfrac{16a^2}{(x-5a)(x-3a)}$

82. $\dfrac{3x-a}{2x-a}-\dfrac{2x+a}{3x+a}=\dfrac{(3x-a)(3x+a)}{(2x-a)(3x+a)}-\dfrac{(2x+a)(2x-a)}{(3x+a)(2x-a)}=\dfrac{(9x^2-a^2)-(4x^2-a^2)}{(2x-a)(3x+a)}=\dfrac{5x^2}{(2x-a)(3x+a)}$

83. $\dfrac{1}{x+h}-\dfrac{1}{x}=\dfrac{x}{x(x+h)}-\dfrac{1(x+h)}{x(x+h)}$

$=\dfrac{x-(x+h)}{x(x+h)}=-\dfrac{h}{x(x+h)}$

84. $\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}=\dfrac{x^2}{x^2(x+h)^2}-\dfrac{1(x+h)^2}{x^2(x+h)^2}$

$=\dfrac{x^2-(x+h)^2}{x^2(x+h)^2}$

$=\dfrac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2}$

$=\dfrac{-2xh-h^2}{x^2(x+h)^2}=-\dfrac{h(2x+h)}{x^2(x+h)^2}$

85. $\dfrac{\dfrac{2}{x}}{\dfrac{3}{x^2}}=\dfrac{2}{x}\cdot\dfrac{x^2}{3}=\dfrac{2x}{3}$

86. $\dfrac{\dfrac{-6}{x}}{\dfrac{2}{x^3}}=\dfrac{-6}{x}\cdot\dfrac{x^3}{2}=-3x^2$

87. Using Method 1:

$\dfrac{\dfrac{1}{x}}{1-\dfrac{1}{x}}=\dfrac{\dfrac{1}{x}}{\dfrac{x}{x}-\dfrac{1}{x}}=\dfrac{\dfrac{1}{x}}{\dfrac{x-1}{x}}=\dfrac{1}{x}\cdot\dfrac{x}{x-1}$

$=\dfrac{1}{x-1},\ x\neq 0,\ 1$

88. Using Method 1:

$\dfrac{\dfrac{1}{x^2}}{\dfrac{1}{x^2}-1}=\dfrac{\dfrac{1}{x^2}}{\dfrac{1}{x^2}-\dfrac{x^2}{x^2}}=\dfrac{\dfrac{1}{x^2}}{\dfrac{1-x^2}{x^2}}=\dfrac{1}{x^2}\cdot\dfrac{x^2}{1-x^2}$

$=\dfrac{1}{1-x^2}=\dfrac{1}{(1-x)(1+x)},\ x\neq -1,\ 0,\ 1$

89. Using Method 2:

$\dfrac{\dfrac{1}{x}-1}{\dfrac{1}{x}+1}=\dfrac{\left(\dfrac{1}{x}-1\right)x}{\left(\dfrac{1}{x}+1\right)x}=\dfrac{1-x}{1+x},\ x\neq 0,\ x\neq -1$