Precalculus 7th edition Blitzer Solutions Manual

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INSTRUCTOR'S SOLUTIONS MANUAL

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PRECALCULUS

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PRECALCULUS 7E

Chapter P
Fundamental Concepts of Algebra

Section P.1

Check Point Exercises

1. $8+6(x-3)^2 = 8+6(13-3)^2$
$= 8+6(10)^2$
$= 8+6(100)$
$= 8+600$
$= 608$

2. **a.** Since 2016 is 16 years after 2000, substitute 16 for x.
$T = -x^2+361x+3193$
$= -(16)^2+361(16)+3193$
$= 8713$
The average cost of tuition and fees at public U.S. colleges for the school year ending in 2016 was \$8713.

b. The formula underestimates the actual answer by \$65.

3. The elements common to {3, 4, 5, 6, 7} and {3, 7, 8, 9} are 3 and 7.
$\{3,4,5,6,7\} \cap \{3,7,8,9\} = \{3,7\}$

4. The union is the set containing all the elements of either set.
$\{3,4,5,6,7\} \cup \{3,7,8,9\} = \{3,4,5,6,7,8,9\}$

5. $\left\{-9,\ -1.3,\ 0,\ 0.\overline{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}\right\}$

a. Natural numbers: $\sqrt{9}$ because $\sqrt{9}=3$

b. Whole numbers: $0,\ \sqrt{9}$

c. Integers: $-9,\ 0,\ \sqrt{9}$

d. Rational numbers: $-9,\ -1.3,\ 0,\ 0.\overline{3},\ \sqrt{9}$

e. Irrational numbers: $\frac{\pi}{2},\ \sqrt{10}$

f. Real numbers: $-9,\ -1.3,\ 0,\ 0.\overline{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}$

6. **a.** $\left|1-\sqrt{2}\right|$
Because $\sqrt{2} \approx 1.4$, the number inside the absolute value bars is negative. The absolute value of x when $x < 0$ is $-x$. Thus,
$\left|1-\sqrt{2}\right| = -\left(1-\sqrt{2}\right) = \sqrt{2}-1$

b. $|\pi - 3|$
Because $\pi \approx 3.14$, the number inside the absolute value bars is positive. The absolute value of a positive number is the number itself. Thus,
$|\pi - 3| = \pi - 3$.

c. $\frac{|x|}{x}$
Because $x > 0$, $|x| = x$.
Thus, $\frac{|x|}{x} = \frac{x}{x} = 1$

7. $|-4-(5)| = |-9| = 9$
The distance between –4 and 5 is 9.

8. $7(4x^2+3x)+2(5x^2+x)$
$= 7(4x^2+3x)+2(5x^2+x)$
$= 28x^2+21x+10x^2+2x$
$= 38x^2+23x$

9. $6+4[7-(x-2)]$
$= 6+4[7-x+2)]$
$= 6+4[9-x]$
$= 6+36-4x$
$= 42-4x$

Concept and Vocabulary Check P.1

C1. expression

C2. b to the nth power; base; exponent

C3. formula; modeling; models

C4. intersection; $A \cap B$

C5. union; $A \cup B$

C6. natural

C7. whole

C8. integers

C9. rational

C10. Irrational

C11. rational; irrational

C12. absolute value; x, $-x$

C13. $b+a$; ba

C14. $a+(b+c)$; $(ab)c$

C15. $ab+ac$

C16. 0; inverse; 0; identity

C17. inverse; 1; identity

C18. simplified

C19. a

Exercise Set P.1

1. $7+5(10)=7+50=57$

2. $8+6(5)=8+30=38$

3. $6(3)-8=18-8=10$

4. $8(3)-4=24-4=20$

5. $8^2+3(8)=64+24=88$

6. $6^2+5(6)=36+30=66$

7. $7^2-6(7)+3=49-42+3=7+3=10$

8. $8^2-7(8)+4=64-56+4=8+4=12$

9. $4+5(9-7)^3=4+5(2)^3$
$=4+5(8)=4+40=44$

10. $6+5(8-6)^3=6+5(2)^3$
$=6+5(8)$
$=6+40=46$

11. $8^2-3(8-2)=64-3(6)$
$=64-18=46$

12. $8^2-4(8-3)=64-4(5)=64-20=44$

13. $\dfrac{5(x+2)}{2x-14}=\dfrac{5(10+2)}{2(10)-14}$
$=\dfrac{5(12)}{6}$
$=5\cdot 2$
$=10$

14. $\dfrac{7(x-3)}{2x-16}=\dfrac{7(9-3)}{2(9)-16}=\dfrac{7(6)}{2}=7\cdot 3=21$

15. $\dfrac{2x+3y}{x+1}; x=-2, y=4$
$=\dfrac{2(-2)+3(4)}{-2+1}=\dfrac{-4+12}{-1}=\dfrac{8}{-1}=-8$

16. $\dfrac{2x+y}{xy-2x}$; $x=-2$ and $y=4$
$\dfrac{2(-2)+4}{(-2)(4)-2(-2)}=\dfrac{-4+4}{-8+4}=\dfrac{0}{4}=0$

17. $C=\dfrac{5}{9}(50-32)=\dfrac{5}{9}(18)=10$
50°F is equivalent to 10°C.

18. $C=\dfrac{5}{9}(F-32)=\dfrac{5}{9}(86-32)=\dfrac{5}{9}(54)=30$
86°F is equivalent to 30°C.

19. $h=4+60t-16t^2=4+60(2)-16(2)^2$
$=4+120-16(4)=4+120-64$
$=124-64=60$
Two seconds after it is kicked, the ball's height is 60 feet.

20. $h=4+60t-16t^2$
$=4+60(3)-16(3)^2$
$=4+180-16(9)$
$=4+180-144$
$=184-144=40$
Three seconds after it is kicked, the ball's height is 40 feet.

21. $\{1,2,3,4\} \cap \{2,4,5\} = \{2,4\}$

22. $\{1,3,7\} \cap \{2,3,8\} = \{3\}$

23. $\{s,e,t\} \cap \{t,e,s\} = \{s,e,t\}$

24. $\{r,e,a,l\} \cap \{l,e,a,r\} = \{r,e,a,l\}$

25. $\{1,3,5,7\} \cap \{2,4,6,8,10\} = \{\ \}$
The empty set is also denoted by $\varnothing$.

26. $\{1,3,5,7\} \cap \{-5,-3,-1\} = \{\ \}$ or $\varnothing$

27. $\{a,b,c,d\} \cap \varnothing = \varnothing$

28. $\{w,y,z\} \cap \varnothing = \varnothing$

29. $\{1,2,3,4\} \cup \{2,4,5\} = \{1,2,3,4,5\}$

30. $\{1,3,7,8\} \cup \{2,3,8\} = \{1,2,3,7,8\}$

31. $\{1,3,5,7\} \cup \{2,4,6,8,10\}$
$= \{1,2,3,4,5,6,7,8,10\}$

32. $\{0,1,3,5\} \cup \{2,4,6\} = \{0,1,2,3,4,5,6\}$

33. $\{a,e,i,o,u\} \cup \varnothing = \{a,e,i,o,u\}$

34. $\{e,m,p,t,y\} \cup \varnothing = \{e,m,p,t,y\}$

35. **a.** $\sqrt{100}$

b. $0, \sqrt{100}$

c. $-9, 0, \sqrt{100}$

d. $-9, -\frac{4}{5}, 0, 0.25, 9.2, \sqrt{100}$

e. $\sqrt{3}$

f. $-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}$

36. **a.** $\sqrt{49}$

b. $0, \sqrt{49}$

c. $-7, 0, \sqrt{49}$

d. $-7, -0.\overline{6}, 0, \sqrt{49}$

e. $\sqrt{50}$

f. $-7, -0.\overline{6}, 0, \sqrt{49}, \sqrt{50}$

37. **a.** $\sqrt{64}$

b. $0, \sqrt{64}$

c. $-11, 0, \sqrt{64}$

d. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{64}$

e. $\sqrt{5}, \pi$

f. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}$

38. **a.** $\sqrt{4}$

b. $0, \sqrt{4}$

c. $-5, 0, \sqrt{4}$

d. $-5, -0.\overline{3}, 0, \sqrt{4}$

e. $\sqrt{2}$

f. $-5, -0.\overline{3}, 0, \sqrt{2}, \sqrt{4}$

39. 0

40. Answers will vary. An example is $\frac{1}{2}$.

41. Answers will vary. An example is 2.

42. Answers will vary. An example is -2.

43. true; –13 is to the left of –2 on the number line.

44. false; –6 is to the left of 2 on the number line.

45. true; 4 is to the right of –7 on the number line.

46. true; –13 is to the left of –5 on the number line.

47. true; $-\pi = -\pi$

48. true; –3 is to the right of –13 on the number line.

49. true; 0 is to the right of –6 on the number line.

50. true; 0 is to the right of –13 on the number line.

51. $|300| = 300$

52. $|-203| = 203$

53. $|12-\pi| = 12-\pi$

54. $|7-\pi| = 7-\pi$

55. $\left|\sqrt{2}-5\right| = 5-\sqrt{2}$

56. $\left|\sqrt{5}-13\right| = 13-\sqrt{5}$

57. $\dfrac{-3}{|-3|} = \dfrac{-3}{3} = -1$

58. $\dfrac{-7}{|-7|} = \dfrac{-7}{7} = -1$

59. $\big||-3|-|-7|\big| = |3-7| = |-4| = 4$

60. $\big||-5|-|-13|\big| = |5-13| = |-8| = 8$

61. $|x+y| = |2+(-5)| = |-3| = 3$

62. $|x-y| = |2-(-5)| = |7| = 7$

63. $|x|+|y| = |2|+|-5| = 2+5 = 7$

64. $|x|-|y| = |2|-|-5| = 2-5 = -3$

65. $\dfrac{y}{|y|} = \dfrac{-5}{|-5|} = \dfrac{-5}{5} = -1$

66. $\dfrac{|x|}{x}+\dfrac{|y|}{y} = \dfrac{|2|}{2}+\dfrac{|-5|}{-5} = \dfrac{2}{2}+\dfrac{5}{-5} = 1+(-1) = 0$

67. The distance is $|2-17| = |-15| = 15$.

68. The distance is $|4-15| = |-11| = 11$.

69. The distance is $|-2-5| = |-7| = 7$.

70. The distance is $|-6-8| = |-14| = 14$.

71. The distance is $|-19-(-4)| = |-19+4| = |-15| = 15$.

72. The distance is $|-26-(-3)| = |-26+3| = |-23| = 23$.

73. The distance is
$|-3.6-(-1.4)| = |-3.6+1.4| = |-2.2| = 2.2$.

74. The distance is
$|-5.4-(-1.2)| = |-5.4+1.2| = |-4.2| = 4.2$.

75. $6+(-4) = (-4)+6$;
commutative property of addition

76. $11\cdot(7+4) = 11\cdot 7+11\cdot 4$;
distributive property of multiplication over addition

77. $6+(2+7) = (6+2)+7$;
associative property of addition

78. $6\cdot(2\cdot 3) = 6\cdot(3\cdot 2)$;
commutative property of multiplication

79. $(2+3)+(4+5) = (4+5)+(2+3)$;
commutative property of addition

80. $7\cdot(11\cdot 8) = (11\cdot 8)\cdot 7$;
commutative property of multiplication

81. $2(-8+6) = -16+12$;
distributive property of multiplication over addition

82. $-8(3+11) = -24+(-88)$;
distributive property of multiplication over addition

83. $\dfrac{1}{x+3}(x+3) = 1; x \neq -3$,
inverse property of multiplication

84. $(x+4)+\big[-(x+4)\big] = 0$;
inverse property of addition

85.
$$\begin{aligned} 5(3x+4)-4 &= 5\cdot 3x+5\cdot 4-4 \\ &= 15x+20-4 \\ &= 15x+16 \end{aligned}$$

86.
$$\begin{aligned} 2(5x+4)-3 &= 2\cdot 5x+2\cdot 4-3 \\ &= 10x+8-3 \\ &= 10x+5 \end{aligned}$$

87.
$$\begin{aligned} 5(3x-2)+12x &= 5\cdot 3x-5\cdot 2+12x \\ &= 15x-10+12x \\ &= 27x-10 \end{aligned}$$

88.
$$\begin{aligned} 2(5x-1)+14x &= 2\cdot 5x-2\cdot 1+14x \\ &= 10x-2+14x \\ &= 24x-2 \end{aligned}$$

89. $7(3y-5)+2(4y+3)$
$=7\cdot 3y-7\cdot 5+2\cdot 4y+2\cdot 3$
$=21y-35+8y+6$
$=29y-29$

90. $4(2y-6)+3(5y+10)$
$=4\cdot 2y-4\cdot 6+3\cdot 5y+3\cdot 10$
$=8y-24+15y+30$
$=23y+6$

91. $5(3y-2)-(7y+2)=15y-10-7y-2$
$=8y-12$

92. $4(5y-3)-(6y+3)=20y-12-6y-3$
$=14y-15$

93. $7-4\left[3-(4y-5)\right]=7-4[3-4y+5]$
$=7-4[8-4y]$
$=7-32+16y$
$=16y-25$

94. $6-5[8-(2y-4)]=6-5[8-2y+4]$
$=6-5[12-2y]$
$=6-60+10y$
$=10y-54$

95. $18x^2+4-\left[6\left(x^2-2\right)+5\right]$
$=18x^2+4-\left[6x^2-12+5\right]$
$=18x^2+4-\left[6x^2-7\right]$
$=18x^2+4-6x^2+7$
$=18x^2-6x^2+4+7$
$=(18-6)x^2+11=12x^2+11$

96. $14x^2+5-\left[7\left(x^2-2\right)+4\right]$
$=14x^2+5-\left[7x^2-14+4\right]$
$=14x^2+5-\left[7x^2-10\right]$
$=14x^2+5-7x^2+10$
$=14x^2-7x^2+5+10$
$=(14-7)x^2+15$
$=7x^2+15$

97. $-(-14x)=14x$

98. $-(-17y)=17y$

99. $-(2x-3y-6)=-2x+3y+6$

100. $-(5x-13y-1)=-5x+13y+1$

101. $\frac{1}{3}(3x)+\left[(4y)+(-4y)\right]=x+0=x$

102. $\frac{1}{2}(2y)+\left[(-7x)+7x\right]=y+0=y$

103. $|-6| \square |-3|$
$6 \square 3$
$6>3$
Since $6>3$, $|-6|>|-3|$.

104. $|-20| \square |-50|$
$20 \square 50$
$20<50$
Since $20<50$, $|-20|<|-50|$.

105. $\left|\frac{3}{5}\right| \square |-0.6|$
$|0.6| \square |-0.6|$
$0.6 \square 0.6$
$0.6 = 0.6$
Since $0.6=0.6$, $\left|\frac{3}{5}\right|=|-0.6|$.

106. $\left|\frac{5}{2}\right| \square |-2.5|$
$|2.5| \square |-2.5|$
$2.5 \square 2.5$
$2.5 = 2.5$
Since $2.5=2.5$, $\left|\frac{5}{2}\right|=|-2.5|$.

107. $\frac{30}{40}-\frac{3}{4} \square \frac{14}{15}\cdot\frac{15}{14}$

$\frac{30}{40}-\frac{30}{40} \square \frac{\cancel{14}}{\cancel{15}}\cdot\frac{\cancel{15}}{\cancel{14}}$

$0 \square 1$

$0<1$

Since $0<1$, $\frac{30}{40}-\frac{3}{4} < \frac{14}{15}\cdot\frac{15}{14}$.

108. $\frac{17}{18}\cdot\frac{18}{17} \square \frac{50}{60}-\frac{5}{6}$

$\frac{\cancel{17}}{\cancel{18}}\cdot\frac{\cancel{18}}{\cancel{17}} \square \frac{50}{60}-\frac{50}{60}$

$1 \square 0$

$1>0$

Since $1>0$, $\frac{17}{18}\cdot\frac{18}{17} > \frac{50}{60}-\frac{5}{6}$.

109. $\frac{8}{13}\div\frac{8}{13} \square |-1|$

$\frac{8}{13}\cdot\frac{13}{8} \square 1$

$1 \square 1$

$1=1$

Since $1=1$, $\frac{8}{13}\div\frac{8}{13}=|-1|$.

110. $|-2| \square \frac{4}{17}\div\frac{4}{17}$

$2 \square \frac{4}{17}\cdot\frac{17}{4}$

$2 \square 1$

$2>1$

Since $2>1$, $|-2| > \frac{4}{17}\div\frac{4}{17}$.

111. $8^2-16\div 2^2\cdot 4-3 = 64-16\div 4\cdot 4-3$

$= 64-4\cdot 4-3$

$= 64-16-3$

$= 48-3$

$= 45$

112. $10^2-100\div 5^2\cdot 2-3 = 100-100\div 25\cdot 2-3$

$= 100-4\cdot 2-3$

$= 100-8-3$

$= 92-3$

$= 89$

113. $\frac{5\cdot 2-3^2}{[3^2-(-2)]^2} = \frac{5\cdot 2-9}{[9-(-2)]^2}$

$= \frac{10-9}{[9+2]^2}$

$= \frac{10-9}{11^2}$

$= \frac{1}{121}$

114. $\frac{10\div 2+3\cdot 4}{(12-3\cdot 2)^2} = \frac{5+12}{(12-6)^2}$

$= \frac{17}{6^2}$

$= \frac{17}{36}$

115. $8-3[-2(2-5)-4(8-6)] = 8-3[-2(-3)-4(2)]$

$= 8-3[6-8]$

$= 8-3[-2]$

$= 8+6$

$= 14$

116. $8-3[-2(5-7)-5(4-2)] = 8-3[-2(-2)-5(2)]$

$= 8-3[4-10]$

$= 8-3[-6]$

$= 8+18$

$= 26$

117. $\frac{2(-2)-4(-3)}{5-8} = \frac{-4+12}{-3}$

$= \frac{8}{-3}$

$= -\frac{8}{3}$

118. $\frac{6(-4)-5(-3)}{9-10} = \frac{-24+15}{-1}$

$= \frac{-9}{-1}$

$= 9$

119. $$\frac{(5-6)^2-2|3-7|}{89-3\cdot 5^2}=\frac{(-1)^2-2|-4|}{89-3\cdot 25}$$
$$=\frac{1-2(4)}{89-75}$$
$$=\frac{1-8}{14}$$
$$=\frac{-7}{14}$$
$$=-\frac{1}{2}$$

120. $$\frac{12\div 3\cdot 5|2^2+3^2|}{7+3-6^2}=\frac{12\div 3\cdot 5|4+9|}{7+3-36}$$
$$=\frac{4\cdot 5|13|}{10-36}$$
$$=\frac{20(13)}{-26}$$
$$=\frac{260}{-26}$$
$$=-10$$

121. $x-(x+4)=x-x-4=-4$

122. $x-(8-x)=x-8+x=2x-8$

123. $6(-5x)=-30x$

124. $10(-4x)=-40x$

125. $5x-2x=3x$

126. $6x-(-2x)=6x+2x=8x$

127. $8x-(3x+6)=8x-3x-6=5x-6$

128. $8-3(x+6)=8-3x-18=-3x-10$

129. a. $$H=\frac{7}{10}(220-a)$$
$$H=\frac{7}{10}(220-20)$$
$$=\frac{7}{10}(200)$$
$$=140$$
The lower limit of the heart rate for a 20-year-old with this exercise goal is 140 beats per minute.

b. $$H=\frac{4}{5}(220-a)$$
$$H=\frac{4}{5}(220-20)$$
$$=\frac{4}{5}(200)$$
$$=160$$
The upper limit of the heart rate for a 20-year-old with this exercise goal is 160 beats per minute.

130. a. $$H=\frac{1}{2}(220-a)$$
$$H=\frac{1}{2}(220-30)$$
$$=\frac{1}{2}(190)$$
$$=95$$
The lower limit of the heart rate for a 30-year-old with this exercise goal is 95 beats per minute.

b. $$H=\frac{3}{5}(220-a)$$
$$H=\frac{3}{5}(220-30)$$
$$=\frac{3}{5}(190)$$
$$=114$$
The upper limit of the heart rate for a 30-year-old with this exercise goal is 114 beats per minute.

131. a. Model 1
$$T=975x+13,547$$
$$=975(18)+13,547$$
$$=31,097$$
Model 1 estimates the cost to have been \$31,097 in 2018, which overestimates the actual cost by \$366.

Model 2
$$T=32x^2+331x+15,479$$
$$=32(18)^2+331(18)+15,479$$
$$=31,805$$
Model 2 estimates the cost to have been \$31,805 in 2018, which overestimates the actual cost by \$1074.

b. Model 2

$$T = 32x^2 + 331x + 15,479$$
$$= 32(30)^2 + 331(30) + 15,479$$
$$= 54,209$$

Model 2 estimates the cost will be \$54,209 in 2030.

132. a. Model 1

$$T = 975x + 13,547$$
$$= 975(10) + 13,547$$
$$= 23,297$$

Model 1 estimates the cost to have been \$23,297 in 2010, which overestimates the actual cost by \$1028.

Model 2

$$T = 32x^2 + 331x + 15,479$$
$$= 32(10)^2 + 331(10) + 15,479$$
$$= 21,989$$

Model 2 estimates the cost to have been \$21,989 in 2010, which underestimates the actual cost by \$280.

b. Model 2

$$T = 32x^2 + 331x + 15,479$$
$$= 32(25)^2 + 331(25) + 15,479$$
$$= 43,754$$

Model 2 estimates the cost will be \$43,754 in 2025.

133. a.
$$0.015x + 0.0175(6000 - x)$$
$$= 0.015x + 105 - 0.0175x$$
$$= 105 - 0.0025x$$

b.
$$0.015x + 0.0175(6000 - x)$$
$$= 0.015(4400) + 0.0175(6000 - 4400)$$
$$= 0.015(4400) + 0.0175(1600)$$
$$= 94$$

$$105 - 0.0025x$$
$$= 105 - 0.0025(4400)$$
$$= 94$$

Both expressions determine the total interest is \$94

c.
$$105 - 0.0025x$$
$$= 105 - 0.0025(3000)$$
$$= 97.5$$

The total interest is \$97.50

134. a.
$$0.06t + 0.5(50 - t)$$
$$= 0.06t + 25 - 0.5t$$
$$= 25 - 0.44t$$

b.
$$0.06t + 0.5(50 - t)$$
$$= 0.06t + 0.5(50 - 20)$$
$$= 0.06(20) + 0.5(30)$$
$$= 16.2$$

$$25 - 0.44t$$
$$= 25 - 0.44(20)$$
$$= 16.2$$

Both expressions determine the total distance is 16.2 miles.

c.
$$25 - 0.44t$$
$$= 25 - 0.44(25)$$
$$= 14$$

The total distance is 14 miles

135. – 143. Answers will vary.

144. does not make sense; Explanations will vary. Sample explanation: Models do not always accurately predict future values.

145. does not make sense; Explanations will vary. Sample explanation: To use the model, substitute 0 for x.

146. makes sense

147. does not make sense; Explanations will vary. Sample explanation: The commutative property changes order and the associative property changes groupings.

148. false; Changes to make the statement true will vary. A sample change is: Some rational numbers are not integers.

149. false; Changes to make the statement true will vary. A sample change is: All whole numbers are integers.

150. true

151. false; Changes to make the statement true will vary. A sample change is: Some irrational numbers are negative.

152. false; Changes to make the statement true will vary. A sample change is: The term x has a coefficient of 1.

153. false; Changes to make the statement true will vary. A sample change is:
$5+3(x-4)=5+3x-12=3x-7$.

154. false; Changes to make the statement true will vary. A sample change is: $-x-x=-2x$.

155. true

156. $\sqrt{2}\approx 1.4$
$1.4<1.5$
$\sqrt{2}<1.5$

157. $-\pi > -3.5$

158. $-\dfrac{3.14}{2}=-1.57$
$-\dfrac{\pi}{2}\approx -1.571$
$-1.57 > -1.571$
$-\dfrac{3.14}{2} > -\dfrac{\pi}{2}$

159. **a.** $b^4\cdot b^3=(b\cdot b\cdot b\cdot b)(b\cdot b\cdot b)=b^7$

b. $b^5\cdot b^5=(b\cdot b\cdot b\cdot b\cdot b)(b\cdot b\cdot b\cdot b\cdot b)=b^{10}$

c. add the exponents

160. **a.** $\dfrac{b^7}{b^3}=\dfrac{b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b}{b\cdot b\cdot b}=b^4$

b. $\dfrac{b^8}{b^2}=\dfrac{b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b}{b\cdot b}=b^6$

c. subtract the exponents

161. $6.2\times 10^3=6.2\times 10\times 10\times 10=6200$
It moves the decimal point 3 places to the right.

Section P.2

Check Point Exercises

1. **a.** $\left(2x^3y^6\right)^4=(2)^4\left(x^3\right)^4\left(y^6\right)^4=16x^{12}y^{24}$

b. $\left(-6x^2y^5\right)\left(3xy^3\right)=(-6)\cdot 3\cdot x^2\cdot x\cdot y^5\cdot y^3$
$=-18x^3y^8$

c. $\dfrac{100x^{12}y^2}{20x^{16}y^{-4}}=\left(\dfrac{100}{20}\right)\left(\dfrac{x^{12}}{x^{16}}\right)\left(\dfrac{y^2}{y^{-4}}\right)$
$=5x^{12-16}y^{2-(-4)}$
$=5x^{-4}y^6$
$=\dfrac{5y^6}{x^4}$

d. $\left(\dfrac{5x}{y^4}\right)^{-2}=\dfrac{(5)^{-2}(x)^{-2}}{\left(y^4\right)^{-2}}$
$=\dfrac{(5)^{-2}(x)^{-2}}{\left(y^4\right)^{-2}}$
$=\dfrac{5^{-2}x^{-2}}{y^{-8}}$
$=\dfrac{y^8}{5^2x^2}$
$=\dfrac{y^8}{25x^2}$

2. **a.** $-2.6\times 10^9=-2,600,000,000$

b. $3.017\times 10^{-6}=0.000003017$

3. **a.** $5,210,000,000=5.21\times 10^9$

b. $-0.00000006893=-6.893\times 10^{-8}$

4. $410\times 10^7=\left(4.1\times 10^2\right)\times 10^7$
$=4.1\times\left(10^2\times 10^7\right)$
$=4.1\times 10^9$

5 **a.** $\left(7.1\times10^{5}\right)\left(5\times10^{-7}\right)$

$= 7.1\cdot5\times10^{5}\cdot10^{-7}$

$= 35.5\times10^{-2}$

$= \left(3.55\times10^{1}\right)\times10^{-2}$

$= 3.55\times\left(10^{1}\times10^{-2}\right)$

$= 3.55\times10^{-1}$

b. $\dfrac{1.2\times10^{6}}{3\times10^{-3}} = \dfrac{1.2}{3}\cdot\dfrac{10^{6}}{10^{-3}}$

$= 0.4\times10^{6-(-3)}$

$= 0.4\times10^{9}$

$= 4\times10^{8}$

6. $\dfrac{13,500,000,000,000}{309,000,000} = \dfrac{1.35\times10^{13}}{3.09\times10^{8}} = \dfrac{1.35}{3.09}\cdot\dfrac{10^{13}}{10^{8}}$

$= 0.44\times10^{5}$

$= 44,000$

Each citizen would have to pay \$44,000 in 2010. This would be an increase of \$33,000, per citizen, between 2010 and 2020.

Concept and Vocabulary Check P.2

C1. b^{m+n}; add

C2. b^{m-n}; subtract

C3. 1

C4. $\dfrac{1}{b^{n}}$

C5. false

C6. b^{n}

C7. true

C8. a number greater than or equal to 1 and less than 10; integer

C9. true

C10. False

Exercise Set P.2

1. $5^{2}\cdot2 = (5\cdot5)\cdot2 = 25\cdot2 = 50$

2. $6^{2}\cdot2 = (6\cdot6)\cdot2 = 36\cdot2 = 72$

3. $(-2)^{6} = (-2)(-2)(-2)(-2)(-2)(-2) = 64$

4. $(-2)^{4} = (-2)(-2)(-2)(-2) = 16$

5. $-2^{6} = -2\cdot2\cdot2\cdot2\cdot2\cdot2 = -64$

6. $-2^{4} = -2\cdot2\cdot2\cdot2 = -16$

7. $(-3)^{0} = 1$

8. $(-9)^{0} = 1$

9. $-3^{0} = -1$

10. $-9^{0} = -1$

11. $4^{-3} = \dfrac{1}{4^{3}} = \dfrac{1}{4\cdot4\cdot4} = \dfrac{1}{64}$

12. $2^{-6} = \dfrac{1}{2^{6}} = \dfrac{1}{2\cdot2\cdot2\cdot2\cdot2\cdot2} = \dfrac{1}{64}$

13. $2^{2}\cdot2^{3} = 2^{2+3} = 2^{5} = 2\cdot2\cdot2\cdot2\cdot2 = 32$

14. $3^{3}\cdot3^{2} = 3^{3+2} = 3^{5} = 3\cdot3\cdot3\cdot3\cdot3 = 243$

15. $(2^{2})^{3} = 2^{2\cdot3} = 2^{6} = 2\cdot2\cdot2\cdot2\cdot2\cdot2 = 64$

16. $(3^{3})^{2} = 3^{3\cdot2} = 3^{6} = 3\cdot3\cdot3\cdot3\cdot3\cdot3 = 729$

17. $\dfrac{2^{8}}{2^{4}} = 2^{8-4} = 2^{4} = 2\cdot2\cdot2\cdot2 = 16$

18. $\dfrac{3^{8}}{3^{4}} = 3^{8-4} = 3^{4} = 3\cdot3\cdot3\cdot3 = 81$

19. $3^{-3}\cdot3 = 3^{-3+1} = 3^{-2} = \dfrac{1}{3^{2}} = \dfrac{1}{3\cdot3} = \dfrac{1}{9}$

20. $2^{-3}\cdot2 = 2^{-3+1} = 2^{-2} = \dfrac{1}{2^{2}} = \dfrac{1}{2\cdot2} = \dfrac{1}{4}$

21. $\dfrac{2^{3}}{2^{7}} = 2^{3-7} = 2^{-4} = \dfrac{1}{2^{4}} = \dfrac{1}{2\cdot2\cdot2\cdot2} = \dfrac{1}{16}$

22. $\dfrac{3^{4}}{3^{7}} = 3^{4-7} = 3^{-3} = \dfrac{1}{3^{3}} = \dfrac{1}{3\cdot3\cdot3} = \dfrac{1}{27}$

23. $x^{-2}y = \frac{1}{x^2} \cdot y = \frac{y}{x^2}$

24. $xy^{-3} = x \cdot \frac{1}{y^3} = \frac{x}{y^3}$

25. $x^0 y^5 = 1 \cdot y^5 = y^5$

26. $x^7 \cdot y^0 = x^7 \cdot 1 = x^7$

27. $x^3 \cdot x^7 = x^{3+7} = x^{10}$

28. $x^{11} \cdot x^5 = x^{11+5} = x^{16}$

29. $x^{-5} \cdot x^{10} = x^{-5+10} = x^5$

30. $x^{-6} \cdot x^{12} = x^{-6+12} = x^6$

31. $(x^3)^7 = x^{3 \cdot 7} = x^{21}$

32. $(x^{11})^5 = x^{11 \cdot 5} = x^{55}$

33. $(x^{-5})^3 = x^{-5 \cdot 3} = x^{-15} = \frac{1}{x^{15}}$

34. $(x^{-6})^4 = x^{-6 \cdot 4} = x^{-24} = \frac{1}{x^{24}}$

35. $\frac{x^{14}}{x^7} = x^{14-7} = x^7$

36. $\frac{x^{30}}{x^{10}} = x^{30-10} = x^{20}$

37. $\frac{x^{14}}{x^{-7}} = x^{14-(-7)} = x^{14+7} = x^{21}$

38. $\frac{x^{30}}{x^{-10}} = x^{30-(-10)} = x^{30+10} = x^{40}$

39. $(8x^3)^2 = 8^2 (x^3)^2 = 8^2 x^{3 \cdot 2} = 64x^6$

40. $(6x^4)^2 = (6)^2 (x^4)^2 = 6^2 x^{4 \cdot 2} = 36x^8$

41. $\left(-\frac{4}{x}\right)^3 = \frac{(-4)^3}{x^3} = -\frac{64}{x^3}$

42. $\left(-\frac{6}{y}\right)^3 = \frac{(-6)^3}{y^3} = -\frac{216}{y^3}$

43. $(-3x^2y^5)^2 = (-3)^2 (x^2)^2 \cdot (y^5)^2$
$= 9x^{2 \cdot 2} y^{5 \cdot 2}$
$= 9x^4 y^{10}$

44. $(-3x^4y^6)^3 = (-3)^3 (x^4)^3 (y^6)^3$
$= -27x^{4 \cdot 3} y^{6 \cdot 3}$
$= -27x^{12} y^{18}$

45. $(3x^4)(2x^7) = 3 \cdot 2x^4 \cdot x^7 = 6x^{4+7} = 6x^{11}$

46. $(11x^5)(9x^{12}) = 11 \cdot 9x^5 x^{12} = 99x^{5+12} = 99x^{17}$

47. $(-9x^3y)(-2x^6y^4) = (-9)(-2)x^3x^6yy^4$
$= 18x^{3+6}y^{1+4}$
$= 18x^9y^5$

48. $(-5x^4y)(-6x^7y^{11}) = (-5)(-6)x^4x^7yy^{11}$
$= 30x^{4+7}y^{1+11}$
$= 30x^{11}y^{12}$

49. $\frac{8x^{20}}{2x^4} = \left(\frac{8}{2}\right)\left(\frac{x^{20}}{x^4}\right) = 4x^{20-4} = 4x^{16}$

50. $\frac{20x^{24}}{10x^6} = \left(\frac{20}{10}\right)\left(\frac{x^{24}}{x^6}\right) = 2x^{24-6} = 2x^{18}$

51. $\frac{25a^{13} \cdot b^4}{-5a^2 \cdot b^3} = \left(\frac{25}{-5}\right)\left(\frac{a^{13}}{a^2}\right)\left(\frac{b^4}{b^3}\right)$
$= -5a^{13-2}b^{4-3}$
$= -5a^{11}b$

52. $\frac{35a^{14}b^6}{-7a^7b^3} = \left(\frac{35}{-7}\right)\left(\frac{a^{14}}{a^7}\right)\left(\frac{b^6}{b^3}\right)$
$= -5a^{14-7}b^{6-3}$
$= -5a^7b^3$

53. $\frac{14b^7}{7b^{14}} = \left(\frac{14}{7}\right)\left(\frac{b^7}{b^{14}}\right) = 2 \cdot b^{7-14} = 2b^{-7} = \frac{2}{b^7}$

54. $\dfrac{20b^{10}}{10b^{20}} = \left(\dfrac{20}{10}\right)\left(\dfrac{b^{10}}{b^{20}}\right)$
$= 2b^{10-20}$
$= 2b^{-10}$
$= \dfrac{2}{b^{10}}$

55. $(4x^3)^{-2} = (4^{-2})(x^3)^{-2}$
$= 4^{-2}x^{-6}$
$= \dfrac{1}{4^2 x^6}$
$= \dfrac{1}{16x^6}$

56. $(10x^2)^{-3} = 10^{-3}x^{2\cdot(-3)}$
$= 10^{-3}x^{-6}$
$= \dfrac{1}{10^3 x^6}$
$= \dfrac{1}{1000x^6}$

57. $\dfrac{24x^3 \cdot y^5}{32x^7 y^{-9}} = \dfrac{3}{4}x^{3-7}y^{5-(-9)}$
$= \dfrac{3}{4}x^{-4}y^{14}$
$= \dfrac{3y^{14}}{4x^4}$

58. $\dfrac{10x^4y^9}{30x^{12}y^{-3}} = \dfrac{1}{3}x^{4-12}y^{9-(-3)}$
$= \dfrac{1}{3}x^{-8}y^{12}$
$= \dfrac{y^{12}}{3x^8}$

59. $\left(\dfrac{5x^3}{y}\right)^{-2} = \dfrac{5^{-2}x^{-6}}{y^{-2}} = \dfrac{y^2}{25x^6}$

60. $\left(\dfrac{3x^4}{y}\right)^{-3} = \left(\dfrac{y}{3x^4}\right)^3$
$= \dfrac{y^3}{3^3 x^{4\cdot 3}}$
$= \dfrac{y^3}{27x^{12}}$

61. $\left(\dfrac{-15a^4b^2}{5a^{10}b^{-3}}\right)^3 = \left(\dfrac{-3b^{2-(-3)}}{a^{10-4}}\right)^3$
$= \left(\dfrac{-3b^5}{a^6}\right)^3$
$= \dfrac{-27b^{15}}{a^{18}}$

62. $\left(\dfrac{-30a^{14}b^8}{10a^{17}b^{-2}}\right)^3 = \left(\dfrac{-3b^{8-(-2)}}{a^{17-14}}\right)^3$
$= \left(\dfrac{-3b^{10}}{a^3}\right)^3$
$= \dfrac{-27b^{30}}{a^9}$

63. $\left(\dfrac{3a^{-5}b^2}{12a^3b^{-4}}\right)^0 = 1$

64. $\left(\dfrac{4a^{-5}b^3}{12a^3b^{-5}}\right)^0 = 1$

65. $3.8\times10^2 = 380$

66. $9.2\times10^2 = 920$

67. $6\times10^{-4} = 0.0006$

68. $7\times10^{-5} = 0.00007$

69. $-7.16\times10^6 = -7{,}160{,}000$

70. $-8.17\times10^6 = -8{,}170{,}000$

71. $7.9\times10^{-1} = 0.79$

72. $6.8\times10^{-1} = 0.68$

73. $-4.15\times10^{-3} = -0.00415$

74. $-3.14\times10^{-3} = -0.00314$

75. $-6.00001\times10^{10} = -60,000,100,000$

76. $-7.00001\times10^{10} = -70,000,100,000$

77. $32,000 = 3.2\times10^{4}$

78. $64,000 = 6.4\times10^{4}$

79. $638,000,000,000,000,000$
$= 6.38\times10^{17}$

80. $579,000,000,000,000,000 = 5.79\times10^{17}$

81. $-5716 = -5.716\times10^{3}$

82. $-3829 = -3.829\times10^{3}$

83. $0.0027 = 2.7\times10^{-3}$

84. $0.0083 = 8.3\times10^{-3}$

85. $-0.00000000504 = -5.04\times10^{-9}$

86. $-0.00000000405 = -4.05\times10^{-9}$

87. $\left(3\times10^{4}\right)\left(2.1\times10^{3}\right) = (3\times2.1)\left(10^{4}\times10^{3}\right)$
$= 6.3\times10^{4+3} = 6.3\times10^{7}$

88. $\left(2\times10^{4}\right)\left(4.1\times10^{3}\right) = 8.2\times10^{7}$

89. $\left(1.6\times10^{15}\right)\left(4\times10^{-11}\right) = (1.6\times4)\left(10^{15}\times10^{-11}\right)$
$= 6.4\times10^{15+(-11)}$
$= 6.4\times10^{4}$

90. $\left(1.4\times10^{15}\right)\left(3\times10^{-11}\right) = (1.4\times3)\left(10^{15}\times10^{-11}\right)$
$= 4.2\times10^{15+(-11)}$
$= 4.2\times10^{4}$

91. $\left(6.1\times10^{-8}\right)\left(2\times10^{-4}\right) = (6.1\times2)\left(10^{-8}\times10^{-4}\right)$
$= 12.2\times10^{-8+(-4)}$
$= 12.2\times10^{-12}$
$= 1.22\times10^{-11}$

92. $\left(5.1\times10^{-8}\right)\left(3\times10^{-4}\right) = 15.3\times10^{-12}$
$= 1.53\times10^{-11}$

93. $\left(4.3\times10^{8}\right)\left(6.2\times10^{4}\right)$
$= (4.3\times6.2)\left(10^{8}\times10^{4}\right)$
$= 26.66\times10^{8+4}$
$= 26.66\times10^{12}$
$= 2.666\times10^{13} \approx 2.67\times10^{13}$

94. $\left(8.2\times10^{8}\right)\left(4.6\times10^{4}\right)$
$= 37.72\times10^{8+4} = 37.72\times10^{12}$
$= 3.772\times10^{13} \approx 3.77\times10^{13}$

95. $\dfrac{8.4\times10^{8}}{4\times10^{5}} = \dfrac{8.4}{4}\times\dfrac{10^{8}}{10^{5}}$
$= 2.1\times10^{8-5} = 2.1\times10^{3}$

96. $\dfrac{6.9\times10^{8}}{3\times10^{5}} = 2.3\times10^{8-5} = 2.3\times10^{3}$

97. $\dfrac{3.6\times10^{4}}{9\times10^{-2}} = \dfrac{3.6}{9}\times\dfrac{10^{4}}{10^{-2}}$
$= 0.4\times10^{4-(-2)}$
$= 0.4\times10^{6} = 4\times10^{5}$

98. $\dfrac{1.2\times10^{4}}{2\times10^{-2}} = 0.6\times10^{4-(-2)} = 0.6\times10^{6}$
$= \left(6\times10^{-1}\right)\times10^{6} = 6\times10^{5}$

99. $\dfrac{4.8\times10^{-2}}{2.4\times10^{6}} = \dfrac{4.8}{2.4}\times\dfrac{10^{-2}}{10^{6}}$
$= 2\times10^{-2-6} = 2\times10^{-8}$

100. $\dfrac{7.5\times10^{-2}}{2.5\times10^{6}} = 3\times10^{-2-6} = 3\times10^{-8}$

101. $\dfrac{2.4\times10^{-2}}{4.8\times10^{-6}} = \dfrac{2.4}{4.8}\times\dfrac{10^{-2}}{10^{-6}}$
$= 0.5\times10^{-2-(-6)}$
$= 0.5\times10^{4} = 5\times10^{3}$

102. $\dfrac{1.5\times10^{-2}}{5\times10^{-6}} = 0.5\times10^{-2-(-6)}$

$= 0.5\times10^{4} = 5\times10^{3}$

103. $\dfrac{480,000,000,000}{0.00012} = \dfrac{4.8\times10^{11}}{1.2\times10^{-4}}$

$= \dfrac{4.8}{1.2}\times\dfrac{10^{11}}{10^{-4}}$

$= 4\times10^{11-(-4)}$

$= 4\times10^{15}$

104. $\dfrac{282,000,000,000}{0.00141} = \dfrac{2.82\times10^{11}}{1.41\times10^{-3}}$

$= 2\times10^{11-(-3)}$

$= 2\times10^{14}$

105. $\dfrac{0.00072\times0.003}{0.00024}$

$= \dfrac{\left(7.2\times10^{-4}\right)\left(3\times10^{-3}\right)}{2.4\times10^{-4}}$

$= \dfrac{7.2\times3}{2.4}\times\dfrac{10^{-4}\cdot10^{-3}}{10^{-4}} = 9\times10^{-3}$

106. $\dfrac{66000\times0.001}{0.003\times0.002} = \dfrac{\left(6.6\times10^{4}\right)\left(1\times10^{-3}\right)}{\left(3\times10^{-3}\right)\left(2\times10^{-3}\right)}$

$= \dfrac{6.6\times10^{1}}{6\times10^{-6}} = 1.1\times10^{1-(-6)}$

$= 1.1\times10^{7}$

107. $\dfrac{\left(x^{-2}y\right)^{-3}}{\left(x^{2}y^{-1}\right)^{3}} = \dfrac{x^{6}y^{-3}}{x^{6}y^{-3}}$

$= x^{6-6}y^{-3-(-3)} = x^{0}y^{0} = 1$

108. $\dfrac{\left(xy^{-2}\right)^{-2}}{\left(x^{-2}y\right)^{-3}} = \dfrac{x^{-2}y^{4}}{x^{6}y^{-3}}$

$= x^{-2-6}y^{4-(-3)} = x^{-8}y^{7} = \dfrac{y^{7}}{x^{8}}$

109. $\left(2x^{-3}yz^{-6}\right)(2x)^{-5} = 2x^{-3}yz^{-6}\cdot2^{-5}x\cdot^{-5}$

$= 2^{-4}x^{-8}yz^{-6} = \dfrac{y}{2^{4}x^{8}z^{6}} = \dfrac{y}{16x^{8}z^{6}}$

110. $\left(3x^{-4}yz^{-7}\right)(3x)^{-3} = 3x^{-4}yz^{-7}\cdot3^{-3}x\cdot^{-3}$

$= 3^{-2}x^{-7}yz^{-7} = \dfrac{y}{3^{2}x^{7}z^{7}} = \dfrac{y}{9x^{7}z^{7}}$

111. $\left(\dfrac{x^{3}y^{4}z^{5}}{x^{-3}y^{-4}z^{-5}}\right)^{-2} = \left(x^{6}y^{8}z^{10}\right)^{-2}$

$= x^{-12}y^{-16}z^{-20} = \dfrac{1}{x^{12}y^{16}z^{20}}$

112. $\left(\dfrac{x^{4}y^{5}z^{6}}{x^{-4}y^{-5}z^{-6}}\right)^{-4} = \left(x^{8}y^{10}z^{12}\right)^{-4}$

$= x^{-32}y^{-40}z^{-48} = \dfrac{1}{x^{32}y^{40}z^{48}}$

113. $\dfrac{\left(2^{-1}x^{-2}y^{-1}\right)^{-2}\left(2x^{-4}y^{3}\right)^{-2}\left(16x^{-3}y^{3}\right)^{0}}{\left(2x^{-3}y^{-5}\right)^{2}}$

$= \dfrac{\left(2^{2}x^{2}y^{2}\right)\left(2^{-2}x^{8}y^{-6}\right)(1)}{\left(2^{2}x^{-6}y^{-10}\right)}$

$= \dfrac{x^{18}y^{6}}{4}$

114. $\dfrac{\left(2^{-1}x^{-3}y^{-1}\right)^{-2}\left(2x^{-6}y^{4}\right)^{-2}\left(9x^{3}y^{-3}\right)^{0}}{\left(2x^{-4}y^{-6}\right)^{2}}$

$= \dfrac{\left(2^{2}x^{6}y^{2}\right)\left(2^{-2}x^{12}y^{-8}\right)(1)}{\left(2^{2}x^{-8}y^{-12}\right)}$

$= \dfrac{x^{26}y^{6}}{4}$

115. **a.** 5.19×10^{11}

b. 4.8×10^{7}

c. $\dfrac{5.19\times10^{11}}{4.8\times10^{7}} = \dfrac{5.19}{4.8}\times\dfrac{10^{11}}{10^{7}}$

$\approx 1.0813\times10^{4}$

$\approx 10,813$

$10,813 yearly per person

d. $\dfrac{10,813}{12} \approx 901$

$901 monthly per person

116. a. 3.3×10^{10}

b. 2.57×10^{7}

c. $$\frac{3.3\times10^{10}}{2.57\times10^{7}}=\frac{3.3}{2.57}\times\frac{10^{10}}{10^{7}}$$
$$\approx 1.284\times10^{3}$$
$$\approx 1284$$
\$1284 yearly per person

d. $\frac{1284}{12}=107$
\$107 monthly per person

117. Medicaid
$$\frac{1.98\times10^{11}}{5.34\times10^{7}}=\frac{1.98}{5.34}\times\frac{10^{11}}{10^{7}}$$
$$\approx 0.3708\times10^{4}$$
$$\approx 3708$$

Medicare
$$\frac{2.94\times10^{11}}{4.23\times10^{7}}=\frac{2.94}{4.23}\times\frac{10^{11}}{10^{7}}$$
$$\approx 0.6950\times10^{4}$$
$$\approx 6950$$

Medicare provides the greater yearly per person benefit by, \$6950 \$3708, or \$3242.

118. a. 2.55×10^{13}

b. 2.54×10^{11}

c. $$\frac{2.55\times10^{13}}{2.54\times10^{11}}=\frac{2.55}{2.54}\times\frac{10^{13}}{10^{11}}$$
$$\approx 1.00\times10^{2}$$
$$\approx 100$$
approximately 100 years

119. a. 2.55×10^{13}

b. 6×10^{4}

c. $$\frac{2.55\times10^{13}}{6\times10^{4}}=\frac{2.55}{6}\times\frac{10^{13}}{10^{4}}$$
$$=0.425\times10^{9}$$
$$=425,000,000$$
425,000,000 Americans

120. a. 9.84×10^{11}

b. 3.2×10^{7}

c. $$\frac{9.84\times10^{11}}{3.2\times10^{7}}=\frac{9.84}{3.2}\times\frac{10^{11}}{10^{7}}$$
$$=3.075\times10^{4}$$
$$=30,750$$
30,750 years

121. – 128. Answers will vary.

129. does not make sense; Explanations will vary.
Sample explanation: $36(x^3)^9 = 36x^{27}$ not $36x^{12}$.

130. makes sense

131. does not make sense; Explanations will vary.
Sample explanation: 4.6×10^{12} represents over 4 trillion. The entire world population is measured in billions (10^9).

132. makes sense

133. false; Changes to make the statement true will vary.
A sample change is: $4^{-2} > 4^{-3}$.

134. true

135. false; Changes to make the statement true will vary.
A sample change is: $(-2)^4 \neq 2^{-4}$ because $16 \neq \frac{1}{16}$.

136. false; Changes to make the statement true will vary.
A sample change is: $5^2 \cdot 5^{-2} = 2^5 \cdot 2^{-5}$.

137. false; Changes to make the statement true will vary.
A sample change is: $534.7 \neq 5347$.

138. false; Changes to make the statement true will vary.
A sample change is:
$$\frac{8\times10^{30}}{2\times10^{-5}} = 4\times10^{30-(-5)} = 4\times10^{35}.$$

139. false; Changes to make the statement true will vary.
A sample change is:
$(7\times10^5)+(2\times10^{-3}) = 700,000.002$.

140. true

141. The doctor has gathered:

$$2^{-1}+2^{-2}=\frac{1}{2}+\frac{1}{2^2}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$$

So, $1-\frac{3}{4}=\frac{1}{4}$ is remaining.

142. $b^A = MN, b^C = M, b^D = N$

$b^A = b^C b^D$

$A = C + D$

143. $\frac{70 \text{ bts}}{\cancel{\text{min}}} \cdot \frac{60 \cancel{\text{min}}}{\cancel{\text{hr}}} \cdot \frac{24 \cancel{\text{hrs}}}{\cancel{\text{day}}} \cdot \frac{365 \cancel{\text{days}}}{\cancel{\text{yr}}} \cdot 80 \cancel{\text{yrs}}$

$= 70 \cdot 60 \cdot 24 \cdot 365 \cdot 80$ beats

$= 2943360000$ beats

$= 2.94336 \times 10^9$ beats

$\approx 2.94 \times 10^9$ beats

The heartbeats approximately 2.94×10^9 times over a lifetime of 80 years.

144. Answers will vary.

145. **a.** $\sqrt{16} \cdot \sqrt{4} = 4 \cdot 2 = 8$

b. $\sqrt{16 \cdot 4} = \sqrt{64} = 8$

c. $\sqrt{16} \cdot \sqrt{4} = \sqrt{16 \cdot 4}$

146. **a.** $\sqrt{300} \approx 17.32$

b. $10\sqrt{3} \approx 17.32$

c. $\sqrt{300} = 10\sqrt{3}$

147. **a.** $21x + 10x = 31x$

b. $21\sqrt{2} + 10\sqrt{2} = 31\sqrt{2}$

Section P.3

Check Point Exercises

1. **a.** $\sqrt{81} = 9$

b. $-\sqrt{9} = -3$

c. $\sqrt{\frac{1}{25}} = \frac{1}{5}$

d. $\sqrt{36+64} = \sqrt{100} = 10$

e. $\sqrt{36} + \sqrt{64} = 6 + 8 = 14$

2. **a.** $\sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

b.
$$\begin{aligned}\sqrt{5x} \cdot \sqrt{10x} &= \sqrt{5x \cdot 10x} \\ &= \sqrt{50x^2} \\ &= \sqrt{25 \cdot 2x^2} \\ &= \sqrt{25x^2} \cdot \sqrt{2} \\ &= 5x\sqrt{2}\end{aligned}$$

3. **a.** $\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$

b.
$$\begin{aligned}\frac{\sqrt{150x^3}}{\sqrt{2x}} &= \sqrt{\frac{150x^3}{2x}} \\ &= \sqrt{75x^2} \\ &= \sqrt{25x^2} \cdot \sqrt{3} \\ &= 5x\sqrt{3}\end{aligned}$$

4. **a.**
$$\begin{aligned}8\sqrt{13} + 9\sqrt{13} &= (8+9)\sqrt{3} \\ &= 17\sqrt{13}\end{aligned}$$

b.
$$\begin{aligned}&\sqrt{17x} - 20\sqrt{17x} \\ &= 1\sqrt{17x} - 20\sqrt{17x} \\ &= (1-20)\sqrt{17x} \\ &= -19\sqrt{17x}\end{aligned}$$

5. **a.**
$$\begin{aligned}&5\sqrt{27} + \sqrt{12} \\ &= 5\sqrt{9 \cdot 3} + \sqrt{4 \cdot 3} \\ &= 5 \cdot 3\sqrt{3} + 2\sqrt{3} \\ &= 15\sqrt{3} + 2\sqrt{3} \\ &= (15+2)\sqrt{3} \\ &= 17\sqrt{3}\end{aligned}$$

b.
$$\begin{aligned}&6\sqrt{18x} - 4\sqrt{8x} \\ &= 6\sqrt{9 \cdot 2x} - 4\sqrt{4 \cdot 2x} \\ &= 6 \cdot 3\sqrt{2x} - 4 \cdot 2\sqrt{2x} \\ &= 18\sqrt{2x} - 8\sqrt{2x} \\ &= (18-8)\sqrt{2x} \\ &= 10\sqrt{2x}\end{aligned}$$

6. a. If we multiply numerator and denominator by $\sqrt{3}$, the denominator becomes $\sqrt{3}\cdot\sqrt{3}=\sqrt{9}=3$. Therefore, multiply by 1, choosing $\frac{\sqrt{3}}{\sqrt{3}}$ for 1.

$$\frac{5}{\sqrt{3}}=\frac{5}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{5\sqrt{3}}{\sqrt{9}}=\frac{5\sqrt{3}}{3}$$

b. The *smallest* number that will produce a perfect square in the denominator of $\frac{6}{\sqrt{12}}$ is $\sqrt{3}$ because $\sqrt{12}\cdot\sqrt{3}=\sqrt{36}=6$. So multiply by 1, choosing $\frac{\sqrt{3}}{\sqrt{3}}$ for 1.

$$\frac{6}{\sqrt{12}}=\frac{6}{\sqrt{12}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{6\sqrt{3}}{\sqrt{36}}=\frac{6\sqrt{3}}{6}=\sqrt{3}$$

7. Multiply by $\frac{4-\sqrt{5}}{4-\sqrt{5}}$.

$$\begin{aligned}\frac{8}{4+\sqrt{5}}&=\frac{8}{4+\sqrt{5}}\cdot\frac{4-\sqrt{5}}{4-\sqrt{5}}\\&=\frac{8(4-\sqrt{5})}{4^2-(\sqrt{5})^2}\\&=\frac{8(4-\sqrt{5})}{16-5}\\&=\frac{8(4-\sqrt{5})}{11}\text{ or }\frac{32-8\sqrt{5}}{11}\end{aligned}$$

8. a. $\sqrt[3]{40}=\sqrt[3]{8\cdot5}=\sqrt[3]{8}\cdot\sqrt[3]{5}=2\sqrt[3]{5}$

b. $\sqrt[5]{8}\cdot\sqrt[5]{8}=\sqrt[5]{64}=\sqrt[5]{32}\cdot\sqrt[5]{2}=2\sqrt[5]{2}$

c. $\sqrt[3]{\frac{125}{27}}=\frac{\sqrt[3]{125}}{\sqrt[3]{27}}=\frac{5}{3}$

9.
$$\begin{aligned}&3\sqrt[3]{81}-4\sqrt[3]{3}\\&=3\sqrt[3]{27\cdot3}-4\sqrt[3]{3}\\&=3\cdot3\sqrt[3]{3}-4\sqrt[3]{3}\\&=9\sqrt[3]{3}-4\sqrt[3]{3}\\&=(9-4)\sqrt[3]{3}\\&=5\sqrt[3]{3}\end{aligned}$$

10. a. $25^{\frac{1}{2}}=\sqrt{25}=5$

b. $8^{\frac{1}{3}}=\sqrt[3]{8}=2$

c. $-81^{\frac{1}{4}}=-\sqrt[4]{81}=-3$

d. $(-8)^{\frac{1}{3}}=\sqrt[3]{-8}=-2$

e. $27^{-\frac{1}{3}}=\frac{1}{27^{\frac{1}{3}}}=\frac{1}{\sqrt[3]{27}}=\frac{1}{3}$

11. a. $27^{\frac{4}{3}}=\left(\sqrt[3]{27}\right)^4=(3)^4=81$

b. $4^{\frac{3}{2}}=\left(\sqrt{4}\right)^3=(2)^3=8$

c. $32^{-\frac{2}{5}}=\frac{1}{32^{\frac{2}{5}}}=\frac{1}{\left(\sqrt[5]{32}\right)^2}=\frac{1}{2^2}=\frac{1}{4}$

12. a.
$$\begin{aligned}&\left(2x^{4/3}\right)\left(5x^{8/3}\right)\\&=2\cdot5x^{4/3}\cdot x^{8/3}\\&=10x^{(4/3)+(8/3)}\\&=10x^{12/3}\\&=10x^4\end{aligned}$$

b.
$$\begin{aligned}\frac{20x^4}{5x^{3/2}}&=\left(\frac{20}{5}\right)\left(\frac{x^4}{x^{3/2}}\right)\\&=4x^{4-(3/2)}\\&=4x^{(8/2)-(3/2)}\\&=4x^{5/2}\end{aligned}$$

13. $\sqrt[6]{x^3}=x^{3/6}=x^{1/2}=\sqrt{x}$

Concept and Vocabulary Check P.3

C1. principal

C2. 8^2

C3. $|a|$

C4. $\sqrt{a}\cdot\sqrt{b}$

C5. $\frac{\sqrt{a}}{\sqrt{b}}$

C6. $18\sqrt{3}$

C7. 5; $6\sqrt{3}$

C8. $7-\sqrt{3}$

C9. $\sqrt{10}+\sqrt{2}$

C10. index; radicand

C11. $(-2)^5$

C12. a; $|a|$

C13. $\sqrt[n]{a}$

C14. 2; 8

Exercise Set P.3

1. $\sqrt{36}=\sqrt{6^2}=6$

2. $\sqrt{25}=\sqrt{5^2}=5$

3. $-\sqrt{36}=-\sqrt{6^2}=-6$

4. $-\sqrt{25}=-\sqrt{5^2}=-5$

5. $\sqrt{-36}$, The square root of a negative number is not real.

6. $\sqrt{-25}$, The square root of a negative number is not real.

7. $\sqrt{25-16}=\sqrt{9}=3$

8. $\sqrt{144+25}=\sqrt{169}=13$

9. $\sqrt{25}-\sqrt{16}=5-4=1$

10. $\sqrt{144}+\sqrt{25}=12+5=17$

11. $\sqrt{(-13)^2}=\sqrt{169}=13$

12. $\sqrt{(-17)^2}=\sqrt{289}=17$

13. $\sqrt{50}=\sqrt{25\cdot 2}=\sqrt{25}\sqrt{2}=5\sqrt{2}$

14. $\sqrt{27}=\sqrt{9\cdot 3}=\sqrt{9}\sqrt{3}=3\sqrt{3}$

15.
$$\begin{aligned}\sqrt{45x^2}&=\sqrt{9x^2\cdot 5}\\&=\sqrt{9x^2}\sqrt{5}\\&=\sqrt{9}\sqrt{x^2}\sqrt{5}\\&=3|x|\sqrt{5}\end{aligned}$$

16.
$$\begin{aligned}\sqrt{125x^2}&=\sqrt{25x^2\cdot 5}\\&=\sqrt{25x^2}\sqrt{5}\\&=\sqrt{25}\sqrt{x^2}\sqrt{5}\\&=5|x|\sqrt{5}\end{aligned}$$

17.
$$\begin{aligned}\sqrt{2x}\cdot\sqrt{6x}&=\sqrt{2x\cdot 6x}\\&=\sqrt{12x^2}\\&=\sqrt{4x^2}\cdot\sqrt{3}\\&=2x\sqrt{3}\end{aligned}$$

18.
$$\begin{aligned}\sqrt{10x}\cdot\sqrt{8x}&=\sqrt{10x\cdot 8x}\\&=\sqrt{80x^2}\\&=\sqrt{16x^2}\cdot\sqrt{5}\\&=4x\sqrt{5}\end{aligned}$$

19. $\sqrt{x^3}=\sqrt{x^2}\cdot\sqrt{x}=x\sqrt{x}$

20. $\sqrt{y^3}=\sqrt{y^2}\cdot\sqrt{y}=y\sqrt{y}$

21.
$$\begin{aligned}\sqrt{2x^2}\cdot\sqrt{6x}&=\sqrt{2x^2\cdot 6x}\\&=\sqrt{12x^3}\\&=\sqrt{4x^2}\cdot\sqrt{3x}\\&=2x\sqrt{3x}\end{aligned}$$

22.
$$\begin{aligned}\sqrt{6x}\cdot\sqrt{3x^2}&=\sqrt{6x\cdot 3x^2}\\&=\sqrt{18x^3}\\&=\sqrt{9x^2}\cdot\sqrt{2x}\\&=3x\sqrt{2x}\end{aligned}$$

23. $\sqrt{\frac{1}{81}}=\frac{\sqrt{1}}{\sqrt{81}}=\frac{1}{9}$

24. $\sqrt{\frac{1}{49}} = \frac{\sqrt{1}}{\sqrt{49}} = \frac{1}{7}$

25. $\sqrt{\frac{49}{16}} = \frac{\sqrt{49}}{\sqrt{16}} = \frac{7}{4}$

26. $\sqrt{\frac{121}{9}} = \frac{\sqrt{121}}{\sqrt{9}} = \frac{11}{3}$

27. $\frac{\sqrt{48x^3}}{\sqrt{3x}} = \sqrt{\frac{48x^3}{3x}} = \sqrt{16x^2} = 4x$

28. $\frac{\sqrt{72x^3}}{\sqrt{8x}} = \sqrt{\frac{72x^3}{8x}} = \sqrt{9x^2} = 3x$

29. $\frac{\sqrt{150x^4}}{\sqrt{3x}} = \sqrt{\frac{150x^4}{3x}}$
$= \sqrt{50x^3}$
$= \sqrt{25x^2} \cdot \sqrt{2x}$
$= 5x\sqrt{2x}$

30. $\frac{\sqrt{24x^4}}{\sqrt{3x}} = \sqrt{\frac{24x^4}{3x}}$
$= \sqrt{8x^3}$
$= \sqrt{4x^2} \cdot \sqrt{2x}$
$= 2x\sqrt{2x}$

31. $\frac{\sqrt{200x^3}}{\sqrt{10x^{-1}}} = \sqrt{\frac{200x^3}{10x^{-1}}}$
$= \sqrt{20x^{3-(-1)}}$
$= \sqrt{20x^4}$
$= \sqrt{4 \cdot 5x^4}$
$= 2x^2\sqrt{5}$

32. $\frac{\sqrt{500x^3}}{\sqrt{10x^{-1}}} = \sqrt{\frac{500x^3}{10x^{-1}}} = \sqrt{50x^{3-(-1)}}$
$= \sqrt{50x^4} = \sqrt{25 \cdot 2x^4} = 5x^2\sqrt{2}$

33. $7\sqrt{3} + 6\sqrt{3} = (7+6)\sqrt{3} = 13\sqrt{3}$

34. $8\sqrt{5} + 11\sqrt{5} = (8+11)\sqrt{5} = 19\sqrt{5}$

35. $6\sqrt{17x} - 8\sqrt{17x} = (6-8)\sqrt{17x} = -2\sqrt{17x}$

36. $4\sqrt{13x} - 6\sqrt{13x} = (4-6)\sqrt{13x} = -2\sqrt{13x}$

37. $\sqrt{8} + 3\sqrt{2} = \sqrt{4 \cdot 2} + 3\sqrt{2}$
$= 2\sqrt{2} + 3\sqrt{2}$
$= (2+3)\sqrt{2}$
$= 5\sqrt{2}$

38. $\sqrt{20} + 6\sqrt{5} = \sqrt{4 \cdot 5} + 6\sqrt{5}$
$= 2\sqrt{5} + 6\sqrt{5}$
$= (2+6)\sqrt{5}$
$= 8\sqrt{5}$

39. $\sqrt{50x} - \sqrt{8x} = \sqrt{25 \cdot 2x} - \sqrt{4 \cdot 2x}$
$= 5\sqrt{2x} - 2\sqrt{2x}$
$= (5-2)\sqrt{2x}$
$= 3\sqrt{2x}$

40. $\sqrt{63x} - \sqrt{28x} = \sqrt{9 \cdot 7x} - \sqrt{4 \cdot 7x}$
$= 3\sqrt{7x} - 2\sqrt{7x}$
$= (3-2)\sqrt{7x}$
$= \sqrt{7x}$

41. $3\sqrt{18} + 5\sqrt{50} = 3\sqrt{9 \cdot 2} + 5\sqrt{25 \cdot 2}$
$= 3 \cdot 3\sqrt{2} + 5 \cdot 5\sqrt{2}$
$= 9\sqrt{2} + 25\sqrt{2}$
$= (9+25)\sqrt{2}$
$= 34\sqrt{2}$

42. $4\sqrt{12} - 2\sqrt{75} = 4\sqrt{4 \cdot 3} - 2\sqrt{25 \cdot 3}$
$= 4 \cdot 2\sqrt{3} - 2 \cdot 5\sqrt{3}$
$= 8\sqrt{3} - 10\sqrt{3}$
$= (8-10)\sqrt{3}$
$= -2\sqrt{3}$

43. $3\sqrt{8} - \sqrt{32} + 3\sqrt{72} - \sqrt{75}$
$= 3\sqrt{4 \cdot 2} - \sqrt{16 \cdot 2} + 3\sqrt{36 \cdot 2} - \sqrt{25 \cdot 3}$
$= 3 \cdot 2\sqrt{2} - 4\sqrt{2} + 3 \cdot 6\sqrt{2} - 5\sqrt{3}$
$= 6\sqrt{2} - 4\sqrt{2} + 18\sqrt{2} - 5\sqrt{3}$
$= 20\sqrt{2} - 5\sqrt{3}$

44. $3\sqrt{54}-2\sqrt{24}-\sqrt{96}+4\sqrt{63}$
$=3\sqrt{9\cdot 6}-2\sqrt{4\cdot 6}-\sqrt{16\cdot 6}+4\sqrt{9\cdot 7}$
$=3\cdot 3\sqrt{6}-2\cdot 2\sqrt{6}-4\sqrt{6}+4\cdot 3\sqrt{7}$
$=9\sqrt{6}-4\sqrt{6}-4\sqrt{6}+12\sqrt{7}$
$=\sqrt{6}+12\sqrt{7}$

45. $\dfrac{1}{\sqrt{7}}=\dfrac{1}{\sqrt{7}}\cdot\dfrac{\sqrt{7}}{\sqrt{7}}=\dfrac{\sqrt{7}}{7}$

46. $\dfrac{2}{\sqrt{10}}=\dfrac{2}{\sqrt{10}}\cdot\dfrac{\sqrt{10}}{\sqrt{10}}=\dfrac{2\sqrt{10}}{10}=\dfrac{\sqrt{10}}{5}$

47. $\dfrac{\sqrt{2}}{\sqrt{5}}=\dfrac{\sqrt{2}}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{\sqrt{10}}{5}$

48. $\dfrac{\sqrt{7}}{\sqrt{3}}=\dfrac{\sqrt{7}}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{\sqrt{21}}{3}$

49. $\dfrac{13}{3+\sqrt{11}}=\dfrac{13}{3+\sqrt{11}}\cdot\dfrac{3-\sqrt{11}}{3-\sqrt{11}}$
$=\dfrac{13(3-\sqrt{11})}{3^2-(\sqrt{11})^2}$
$=\dfrac{13(3-\sqrt{11})}{9-11}$
$=\dfrac{13(3-\sqrt{11})}{-2}$

50. $\dfrac{3}{3+\sqrt{7}}=\dfrac{3}{3+\sqrt{7}}\cdot\dfrac{3-\sqrt{7}}{3-\sqrt{7}}$
$=\dfrac{3(3-\sqrt{7})}{3^2-(\sqrt{7})^2}$
$=\dfrac{3(3-\sqrt{7})}{9-7}$
$=\dfrac{3(3-\sqrt{7})}{2}$

51. $\dfrac{7}{\sqrt{5}-2}=\dfrac{7}{\sqrt{5}-2}\cdot\dfrac{\sqrt{5}+2}{\sqrt{5}+2}$
$=\dfrac{7(\sqrt{5}+2)}{(\sqrt{5})^2-2^2}$
$=\dfrac{7(\sqrt{5}+2)}{5-4}$
$=7(\sqrt{5}+2)$

52. $\dfrac{5}{\sqrt{3}-1}=\dfrac{5}{\sqrt{3}-1}\cdot\dfrac{\sqrt{3}+1}{\sqrt{3}+1}$
$=\dfrac{5(\sqrt{3}+1)}{(\sqrt{3})^2-1^2}$
$=\dfrac{5(\sqrt{3}+1)}{3-1}$
$=\dfrac{5(\sqrt{3}+1)}{2}$

53. $\dfrac{6}{\sqrt{5}+\sqrt{3}}=\dfrac{6}{\sqrt{5}+\sqrt{3}}\cdot\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$
$=\dfrac{6(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$
$=\dfrac{6(\sqrt{5}-\sqrt{3})}{5-3}$
$=\dfrac{6(\sqrt{5}-\sqrt{3})}{2}$
$=3(\sqrt{5}-\sqrt{3})$

54. $\dfrac{11}{\sqrt{7}-\sqrt{3}}=\dfrac{11}{\sqrt{7}-\sqrt{3}}\cdot\dfrac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$
$=\dfrac{11(\sqrt{7}+\sqrt{3})}{(\sqrt{7})^2-(\sqrt{3})^2}$
$=\dfrac{11(\sqrt{7}+\sqrt{3})}{7-3}$
$=\dfrac{11(\sqrt{7}+\sqrt{3})}{4}$

55. $\sqrt[3]{125}=\sqrt[3]{5^3}=5$

56. $\sqrt[3]{8}=\sqrt[3]{2^3}=2$

57. $\sqrt[3]{-8}=\sqrt[3]{(-2)^3}=-2$

58. $\sqrt[3]{-125}=\sqrt[3]{(-5)^3}=-5$

59. $\sqrt[4]{-16}$ is not a real number.

60. $\sqrt[4]{-81}$ is not a real number.

61. $\sqrt[4]{(-3)^4}=|-3|=3$

62. $\sqrt[4]{(-2)^4}=|-2|=2$

63. $\sqrt[5]{(-3)^5} = -3$

64. $\sqrt[5]{(-2)^5} = -2$

65. $\sqrt[5]{-\frac{1}{32}} = \sqrt[5]{-\frac{1}{2^5}} = -\frac{1}{2}$

66. $\sqrt[6]{\frac{1}{64}} = \frac{\sqrt[6]{1}}{\sqrt[6]{2^6}} = \frac{1}{2}$

67. $\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = \sqrt[3]{8}\sqrt[3]{4} = 2 \cdot \sqrt[3]{4}$

68. $\sqrt[3]{150}$ cannot be simplified further.

69. $\sqrt[3]{x^4} = \sqrt[3]{x^3 \cdot x} = x \cdot \sqrt[3]{x}$

70. $\sqrt[3]{x^5} = \sqrt[3]{x^3 x^2} = x\sqrt[3]{x^2}$

71. $\sqrt[3]{9} \cdot \sqrt[3]{6} = \sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27}\sqrt[3]{2} = 3\sqrt[3]{2}$

72. $\sqrt[3]{12} \cdot \sqrt[3]{4} = \sqrt[3]{48} = \sqrt[3]{8 \cdot 6} = 2\sqrt[3]{6}$

73. $\frac{\sqrt[5]{64x^6}}{\sqrt[5]{2x}} = \sqrt[5]{\frac{64x^6}{2x}} = \sqrt[5]{32x^5} = 2x$

74. $\frac{\sqrt[4]{162x^5}}{\sqrt[4]{2x}} = \sqrt[4]{\frac{162x^5}{2x}} = \sqrt[4]{81x^4} = 3x$

75. $4\sqrt[5]{2} + 3\sqrt[5]{2} = 7\sqrt[5]{2}$

76. $6\sqrt[5]{3} + 2\sqrt[5]{3} = 8\sqrt[5]{3}$

77. $5\sqrt[3]{16} + \sqrt[3]{54} = 5\sqrt[3]{8 \cdot 2} + \sqrt[3]{27 \cdot 2}$
$= 5 \cdot 2\sqrt[3]{2} + 3\sqrt[3]{2}$
$= 10\sqrt[3]{2} + 3\sqrt[3]{2}$
$= 13\sqrt[3]{2}$

78. $3\sqrt[3]{24} + \sqrt[3]{81} = \sqrt[3]{8 \cdot 3} + \sqrt[3]{27 \cdot 3}$
$= 3 \cdot 2\sqrt[3]{3} + 3\sqrt[3]{3}$
$= 6\sqrt[3]{3} + 3\sqrt[3]{3}$
$= 9\sqrt[3]{3}$

79. $\sqrt[3]{54xy^3} - y\sqrt[3]{128x}$
$= \sqrt[3]{27 \cdot 2xy^3} - y\sqrt[3]{64 \cdot 2x}$
$= 3y\sqrt[3]{2x} - 4y\sqrt[3]{2x}$
$= -y\sqrt[3]{2x}$

80. $\sqrt[3]{24xy^3} - y\sqrt[3]{81x}$
$= \sqrt[3]{8 \cdot 3xy^3} - y\sqrt[3]{27 \cdot 3x}$
$= 2y\sqrt[3]{3x} - 3y\sqrt[3]{3x}$
$= -y\sqrt[3]{3x}$

81. $\sqrt{2} + \sqrt[3]{8} = \sqrt{2} + 2$

82. $\sqrt{3} + \sqrt[3]{15}$ will not simplify.

83. $36^{1/2} = \sqrt{36} = 6$

84. $121^{1/2} = \sqrt{121} = 11$

85. $8^{1/3} = \sqrt[3]{8} = 2$

86. $27^{1/3} = \sqrt[3]{27} = 3$

87. $125^{2/3} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

88. $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 4$

89. $32^{-4/5} = \frac{1}{32^{4/5}} = \frac{1}{2^4} = \frac{1}{16}$

90. $16^{-5/2} = \frac{1}{16^{5/2}} = \frac{1}{(\sqrt{16})^5} = \frac{1}{4^5} = \frac{1}{1024}$

91. $\left(7x^{1/3}\right)\left(2x^{1/4}\right) = 7 \cdot 2x^{1/3} \cdot x^{1/4}$
$= 14 \cdot x^{1/3+1/4}$
$= 14x^{7/12}$

92. $(3x^{2/3})(4x^{3/4}) = 3 \cdot 4x^{2/3} \cdot x^{3/4}$
$= 12 \cdot x^{2/3+3/4}$
$= 12x^{17/12}$

93. $\dfrac{20x^{1/2}}{5x^{1/4}} = \left(\dfrac{20}{5}\right)\left(\dfrac{x^{1/2}}{x^{1/4}}\right)$
$= 4 \cdot x^{1/2-1/4}$
$= 4x^{1/4}$

94. $\dfrac{72x^{3/4}}{9x^{1/3}} = \left(\dfrac{72}{9}\right)\left(\dfrac{x^{3/4}}{x^{1/3}}\right) = 8 \cdot x^{3/4-1/3} = 8x^{5/12}$

95. $\left(x^{2/3}\right)^3 = x^{2/3 \cdot 3} = x^2$

96. $(x^{4/5})^5 = x^{4/5 \cdot 5} = x^4$

97. $(25x^4y^6)^{1/2} = 25^{1/2}x^{4 \cdot 1/2}y^{6 \cdot 1/2} = 5x^2|y|^3$

98. $(125x^9y^6)^{1/3} = 125^{1/3}x^{9/3}y^{6/3} = 5x^3y^2$

99. $\dfrac{\left(3y^{\frac{1}{4}}\right)^3}{y^{\frac{1}{12}}} = \dfrac{27y^{\frac{3}{4}}}{y^{\frac{1}{12}}} = 27y^{\frac{3}{4}-\frac{1}{12}}$
$= 27y^{\frac{8}{12}} = 27y^{\frac{2}{3}}$

100. $\dfrac{\left(2y^{1/5}\right)^4}{y^{3/10}} = \dfrac{2^4\left(y^{1/5}\right)^4}{y^{3/10}}$
$= \dfrac{16y^{4/5}}{y^{3/10}} = 16y^{4/5-3/10} = 16y^{1/2}$

101. $\sqrt[4]{5^2} = 5^{2/4} = 5^{1/2} = \sqrt{5}$

102. $\sqrt[4]{7^2} = 7^{2/4} = 7^{1/2} = \sqrt{7}$

103. $\sqrt[3]{x^6} = x^{6/3} = x^2$

104. $\sqrt[4]{x^{12}} = x^{12/4} = |x|^3$

105. $\sqrt[6]{x^4} = \sqrt[6/2]{x^{4/2}} = \sqrt[3]{x^2}$

106. $\sqrt[9]{x^6} = \sqrt[9/3]{x^{6/3}} = \sqrt[3]{x^2}$

107. $\sqrt[9]{x^6y^3} = x^{\frac{6}{9}}y^{\frac{3}{9}} = x^{\frac{2}{3}}y^{\frac{1}{3}} = \sqrt[3]{x^2y}$

108. $\sqrt[12]{x^4y^8} = |x|^{\frac{4}{12}}|y|^{\frac{8}{12}} = |x|^{\frac{1}{3}}|y|^{\frac{2}{3}} = \sqrt[3]{|x|y^2}$

109. $\sqrt[3]{\sqrt[4]{16} + \sqrt{625}} = \sqrt[3]{2 + 25} = \sqrt[3]{27} = 3$

110. $\sqrt[3]{\sqrt{\sqrt{169} + \sqrt{9}} + \sqrt{\sqrt[3]{1000} + \sqrt[3]{216}}}$
$= \sqrt[3]{\sqrt{13+3} + \sqrt{10+6}}$
$= \sqrt[3]{\sqrt{16} + \sqrt{16}}$
$= \sqrt[3]{4+4} = \sqrt[3]{8}$
$= 2$

111. $\left(49x^{-2}y^4\right)^{-1/2}\left(xy^{1/2}\right)$
$= (49)^{-1/2}\left(x^{-2}\right)^{-1/2}\left(y^4\right)^{-1/2}\left(xy^{1/2}\right)$
$= \dfrac{1}{49^{1/2}}x^{(-2)(-1/2)}y^{(4)(-1/2)}\left(xy^{1/2}\right)$
$= \dfrac{1}{7}x^1y^{-2} \cdot xy^{1/2} = \dfrac{1}{7}x^{1+1}y^{-2+(1/2)}$
$= \dfrac{1}{7}x^2y^{-3/2} = \dfrac{x^2}{7y^{3/2}}$

112. $\left(8x^{-6}y^3\right)^{1/3}\left(x^{5/6}y^{-1/3}\right)^6$
$= 8^{1/3}x^{(-6)(1/3)}y^{(3)(1/3)}x^{(5/6)(6)}y^{(-1/3)(6)}$
$= 2x^{-2}y^1x^5y^{-2} = 2x^{-2+5}y^{1+(-2)}$
$= 2x^3y^{-1} = \dfrac{2x^3}{y}$

113. $\left(\dfrac{x^{-5/4}y^{1/3}}{x^{-3/4}}\right)^{-6} = \left(x^{(-5/4)-(-3/4)}y^{1/3}\right)^{-6}$
$= \left(x^{-2/4}y^{1/3}\right)^{-6} = x^{(-2/4)(-6)}y^{(1/3)(-6)}$
$= x^3y^{-2} = \dfrac{x^3}{y^2}$

114. $\left(\dfrac{x^{1/2}y^{-7/4}}{y^{-5/4}}\right)^{-4} = \left(x^{1/2}y^{(-7/4)-(-5/4)}\right)^{-4}$
$= \left(x^{1/2}y^{-2/4}\right)^{-4} = x^{(1/2)(-4)}y^{(-2/4)(-4)}$
$= x^{-2}y^2 = \dfrac{y^2}{x^2}$

115. a.
$$P = \frac{x\left(13+\sqrt{x}\right)}{5\sqrt{x}}$$
$$= \frac{25\left(13+\sqrt{25}\right)}{5\sqrt{25}}$$
$$= \frac{25(13+5)}{5(5)}$$
$$= 18$$

18% of 25-year-olds must pay more taxes.

b.
$$P = \frac{x\left(13+\sqrt{x}\right)}{5\sqrt{x}}$$
$$= \frac{13x+x\sqrt{x}}{5\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$$
$$= \frac{13x\sqrt{x}+x^2}{5x}$$
$$= \frac{13\sqrt{x}+x}{5}$$

c.
$$P = \frac{13\sqrt{x}+x}{5}$$
$$= \frac{13\sqrt{25}+25}{5}$$
$$= \frac{13(5)+25}{5}$$
$$= 18$$

Yes we get the same amount, 18%.
No, this does not prove we have correctly rationalized the denominator. Explanations will vary.

116. a. For 2020:
$$E = 5\sqrt{x}+34.1$$
$$= 5\sqrt{10}+34.1$$

For 2050:
$$E = 5\sqrt{x}+34.1$$
$$= 5\sqrt{40}+34.1$$
$$= 5\cdot 2\sqrt{10}+34.1$$
$$= 10\sqrt{10}+34.1$$

Difference:
$$(10\sqrt{10}+34.1)-(5\sqrt{10}+34.1)$$
$$= 10\sqrt{10}+34.1-5\sqrt{10}-34.1$$
$$= 10\sqrt{10}-5\sqrt{10}+34.1-34.1$$
$$= 5\sqrt{10}$$

The difference is $5\sqrt{10}$.

b. $5\sqrt{10} \approx 15.8$
This underestimates the difference projected by the graph of $65.8-47.3=18.5$ by 2.7. This represents a difference of 2.7 million people.

117.
$$\frac{2}{\sqrt{5}-1} \cdot \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{2(\sqrt{5}+1)}{5-1}$$
$$= \frac{2(\sqrt{5}+1)}{4}$$
$$= \frac{\sqrt{5}+1}{2}$$
$$\approx 1.62$$

About 1.62 to 1.

118.
$$R_a = R_f\sqrt{1-\left(\frac{v}{c}\right)^2}$$
$$= R_f\sqrt{1-\left(\frac{0.9c}{c}\right)^2}$$
$$= R_f\sqrt{1-(0.9)^2}$$
$$= R_f\sqrt{0.19}$$
$$\approx 0.44R_f$$

$$R_a = 0.44R_f$$
$$44 = 0.44R_f$$
$$\frac{44}{0.44} = \frac{0.44R_f}{0.44}$$
$$100 = R_f$$

If you are gone for 44 weeks, then 100 weeks will have passed for your friend.

119. Perimeter:

$$\begin{aligned} P &= 2l + 2w \\ &= 2 \cdot \sqrt{125} + 2 \cdot 2\sqrt{20} \\ &= 2 \cdot \sqrt{25 \cdot 5} + 4\sqrt{4 \cdot 5} \\ &= 2 \cdot 5\sqrt{5} + 4 \cdot 2\sqrt{5} \\ &= 10\sqrt{5} + 8\sqrt{5} \\ &= 18\sqrt{5} \text{ feet} \end{aligned}$$

Area:

$$\begin{aligned} A &= lw \\ &= \sqrt{125} \cdot 2\sqrt{20} \\ &= 2\sqrt{125 \cdot 20} \\ &= 2\sqrt{2500} \\ &= 2 \cdot 50 \\ &= 100 \text{ square feet} \end{aligned}$$

120. Perimeter:

$$\begin{aligned} P &= 2l + 2w \\ &= 2 \cdot 4\sqrt{20} + 2 \cdot \sqrt{80} \\ &= 8\sqrt{4 \cdot 5} + 2\sqrt{16 \cdot 5} \\ &= 8 \cdot 2\sqrt{5} + 2 \cdot 4\sqrt{5} \\ &= 16\sqrt{5} + 8\sqrt{5} \\ &= 24\sqrt{5} \text{ feet} \end{aligned}$$

Area:

$$\begin{aligned} A &= lw \\ &= 4\sqrt{20} \cdot \sqrt{80} \\ &= 4\sqrt{20 \cdot 80} \\ &= 4\sqrt{1600} \\ &= 4 \cdot 40 \\ &= 160 \text{ square feet} \end{aligned}$$

121. – 128. Answers will vary.

129. makes sense

130. makes sense

131. does not make sense; Explanations will vary. Sample explanation: $2\sqrt{20} + 4\sqrt{75}$ simplifies to $4\sqrt{5} + 20\sqrt{3}$ and thus the radical terms are not common.

132. does not make sense; Explanations will vary. Sample explanation: Finding the *n*th root first often gives smaller numbers on the middle step.

133. false; Changes to make the statement true will vary. A sample change is: $7^{1/2} \cdot 7^{1/2} = 7^1 = 7$.

134. false; Changes to make the statement true will vary. A sample change is: $(8)^{-\frac{1}{3}} = \frac{1}{(8)^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$.

135. false; Changes to make the statement true will vary. The cube root of –8 is the real number –2.

136. false; Changes to make the statement true will vary. A sample change is: $\frac{\sqrt{20}}{8} = \frac{\sqrt{5}}{4}$.

137. $\left(5+\sqrt{\boxed{3}}\right)\left(5-\sqrt{\boxed{3}}\right) = 22$

$$25-\boxed{3} = 22$$

$$\boxed{3} = 3$$

138. $\sqrt{\boxed{25}x^{\boxed{14}}} = 5x^7$

139. $\sqrt{13+\sqrt{2}+\frac{7}{3+\sqrt{2}}}$

$$= \sqrt{13+\sqrt{2}+\frac{7}{3+\sqrt{2}}\cdot\frac{3-\sqrt{2}}{3-\sqrt{2}}}$$

$$= \sqrt{13+\sqrt{2}+\frac{21-7\sqrt{2}}{9-2}}$$

$$= \sqrt{13+\sqrt{2}+\frac{21-7\sqrt{2}}{7}}$$

$$= \sqrt{13+\sqrt{2}+3-\sqrt{2}}$$

$$= \sqrt{16}$$

$$= 4$$

140. a. $3^{\frac{1}{2}} \boxed{>} 3^{\frac{1}{3}}$

Calculator Check: $1.7321 > 1.4422$

b. $\sqrt{7}+\sqrt{18} \boxed{>} \sqrt{7+18}$

Calculator Check: $6.8884 > 5$

141. a. $\frac{ab}{a^2+ab+b^2}+\left(\frac{ac-ad-bc+bd}{ac-ad+bc-bd}\div\frac{a^3-b^3}{a^3+b^3}\right) = \frac{ab}{a^2+ab+b^2}+\left(\frac{a(c-d)-b(c-d)}{a(c-d)+b(c-d)}\cdot\frac{a^3+b^3}{a^3-b^3}\right)$

$$= \frac{ab}{a^2+ab+b^2}+\left(\frac{\cancel{(c-d)}\cancel{(a-b)}}{\cancel{(c-d)}\cancel{(a+b)}}\cdot\frac{\cancel{(a+b)}\left(a^2-ab+b^2\right)}{\cancel{(a-b)}\left(a^2+ab+b^2\right)}\right) = \frac{ab}{a^2+ab+b^2}+\frac{a^2-ab+b^2}{a^2+ab+b^2}$$

$$= \frac{ab+a^2-ab+b^2}{a^2+ab+b^2} = \frac{a^2+b^2}{a^2+ab+b^2}$$

Her son is 8 years old.

b. Son's portion:

$$\frac{8^{-\frac{4}{3}}+2^{-2}}{16^{-\frac{3}{4}}+2^{-1}}=\frac{\frac{1}{\left(\sqrt[3]{8}\right)^4}+\frac{1}{2^2}}{\frac{1}{\left(\sqrt[4]{16}\right)^3}+\frac{1}{2}}$$

$$=\frac{\frac{1}{2^4}+\frac{1}{4}}{\frac{1}{2^3}+\frac{1}{2}}$$

$$=\frac{\frac{1}{16}+\frac{1}{4}}{\frac{1}{8}+\frac{1}{2}}$$

$$=\frac{\frac{5}{16}}{\frac{5}{8}}$$

$$=\frac{8}{16}$$

$$=\frac{1}{2}$$

Mom's portion:

$$\frac{1}{2}\left(1-\frac{1}{2}\right)=\frac{1}{2}\left(\frac{1}{2}\right)=\frac{1}{4}$$

142. $(2x^3y^2)(5x^4y^7)=10x^7y^9$

143. $2x^4(8x^4+3x)=2x^4(8x^4)+2x^4(3x)$

$$=16x^8+6x^5$$

144. $2x(x^2+4x+5)+3(x^2+4x+5)$

$=2x^3+8x^2+10x+3x^2+12x+15$

$=2x^3+8x^2+3x^2+10x+12x+15$

$=2x^3+11x^2+22x+15$

Section P.4

Check Point Exercises

1. **a.** $(-17x^3 + 4x^2 - 11x - 5) + (16x^3 - 3x^2 + 3x - 15)$

$= (-17x^3 + 16x^3) + (4x^2 - 3x^2) + (-11x + 3x) + (-5 - 15)$

$= -x^3 + x^2 - 8x - 20$

b. $(13x^2 - 9x^2 - 7x + 1) - (-7x^3 + 2x^2 - 5x + 9)$

$= (13x^3 - 9x^2 - 7x + 1) + (7x^3 - 2x^2 + 5x - 9)$

$= (13x^3 + 7x^3) + (-9x^2 - 2x^2) + (-7x + 5x) + (1 - 9)$

$= 20x^3 - 11x^2 - 2x - 8$

2. $(5x - 2)(3x^2 - 5x + 4)$

$= 5x(3x^2 - 5x + 4) - 2(3x^2 - 5x + 4)$

$= 5x \cdot 3x^2 - 5x \cdot 5x + 5x \cdot 4 - 2 \cdot 3x^2 + 2 \cdot 5x - 2 \cdot 4$

$= 15x^3 - 25x^2 + 20x - 6x^2 + 10x - 8$

$= 15x^3 - 31x^2 + 30x - 8$

3. $(7x - 5)(4x - 3) = 7x \cdot 4x + 7x(-3) + (-5)4x + (-5)(-3)$

$= 28x^2 - 21x - 20x + 15$

$= 28x^2 - 41x + 15$

4. **a.** $(7x - 6y)(3x - y) = (7x)(3x) + (7x)(-y) + (-6y)(3x) + (-6y)(-y)$

$= 21x^2 - 7xy - 18xy + 6y^2$

$= 21x^2 - 25xy + 6y^2$

b. $(2x + 4y)^2 = (2x)^2 + 2(2x)(4y) + (4y)^2$

$= 4x^2 + 16xy + 16y^2$

5. **a.** $(3x + 2 + 5y)(3x + 2 - 5y) = (3x + 2)^2 - (5y)^2$

$= 9x^2 + 12x + 4 - 25y^2$

$= 9x^2 + 12x - 25y^2 + 4$

b. $(2x + y + 3)^2 = (2x + y)^2 + 2(2x + y)(3) + 3^2$

$= 4x^2 + 4xy + y^2 + 12x + 6y + 9$

$= 4x^2 + 4xy + 12x + y^2 + 6y + 9$

Concept and Vocabulary Check P.4

C1. whole

C2. standard

C3. monomial

C4. binomial

C5. trinomial

C6. n

C7. like;

C8. distributive; $4x^3 - 8x^2 + 6$; $7x^3$

C9. $5x$; 3; like

C10. $3x^2$; $5x$; $21x$; 35

C11. $A^2 - B^2$; minus

C12. $A^2 + 2AB + B^2$; squared; product of the terms; squared

C13. $A^2 - 2AB + B^2$; minus; product of the terms; plus

C14. $n+m$

Exercise Set P.4

1. yes; $2x + 3x^2 - 5 = 3x^2 + 2x - 5$

2. no; The term $3x^{-1}$ does not have a whole number exponent.

3. no; The form of a polynomial involves addition and subtraction, not division.

4. yes; $x^2 - x^3 + x^4 - 5 = x^4 - x^3 + x^2 - 5$

5. $3x^2$ has degree 2
$-5x$ has degree 1
4 has degree 0
$3x^2 - 5x + 4$ has degree 2.

6. $-4x^3$ has degree 3
$7x^2$ has degree 2
-11 has degree 0
$-4x^3 + 7x^2 - 11$ has degree 3.

7. x^2 has degree 2
$-4x^3$ has degree 3
$9x$ has degree 1
$-12x^4$ has degree 4
63 has degree 0
$x^2-4x^3+9x-12x^4+63$ has degree 4.

8. x^2 has degree 2
$-8x^3$ has degree 3
$15x^4$ has degree 4
91 has degree 0
$x^2-8x^3+15x^4+91$ has degree 4.

9. $(-6x^3+5x^2-8x+9)+(17x^3+2x^2-4x-13)=(-6x^3+17x^3)+(5x^2+2x^2)+(-8x-4x)+(9-13)$
$=11x^3+7x^2-12x-4$
The degree is 3.

10. $(-7x^3+6x^2-11x+13)+(19x^3-11x^2+7x-17)=(-7x^3+19x^3)+(6x^2-11x^2)+(-11x+7x)+(13-17)$
$=12x^3-5x^2-4x-4$
The degree is 3.

11. $(17x^3-5x^2+4x-3)-(5x^3-9x^2-8x+11)=(17x^3-5x^2+4x-3)+(-5x^3+9x^2+8x-11)$
$=(17x^3-5x^3)+(-5x^2+9x^2)+(4x+8x)+(-3-11)$
$=12x^3+4x^2+12x-14$
The degree is 3.

12. $(18x^4-2x^3-7x+8)-(9x^4-6x^3-5x+7)=(18x^4-2x^3-7x+8)+(-9x^4+6x^3+5x-7)$
$=(18x^4-9x^4)+(-2x^3+6x^3)+(-7x+5x)+(8-7)$
$=9x^4+4x^3-2x+1$
The degree is 4.

13. $(5x^2-7x-8)+(2x^2-3x+7)-(x^2-4x-3)=(5x^2-7x-8)+(2x^2-3x+7)+(-x^2+4x+3)$
$=(5x^2+2x^2-x^2)+(-7x-3x+4x)+(-8+7+3)$
$=6x^2-6x+2$
The degree is 2.

14. $(8x^2+7x-5)-(3x^2-4x)-(-6x^3-5x^2+3)=(8x^2+7x-5)+(-3x^2+4x)+(6x^3+5x^2-3)$
$=6x^3+(8x^2-3x^2+5x^2)+(7x+4x)+(-5-3)$
$=6x^3+10x^2+11x-8$
The degree is 3.

15. $(x+1)(x^2-x+1)=x(x^2)-x\cdot x+x\cdot 1+1(x^2)-1\cdot x+1\cdot 1$
$=x^3-x^2+x+x^2-x+1$
$=x^3+1$

16. $(x+5)(x^2-5x+25) = x(x^2)-x(5x)+x(25)+5(x^2)-5(5x)+5(25)$
$= x^3-5x^2+25x+5x^2-25x+125)$
$= x^3+125$

17. $(2x-3)(x^2-3x+5) = (2x)(x^2)+(2x)(-3x)+(2x)(5)+(-3)(x^2)+(-3)(-3x)+(-3)(5)$
$= 2x^3-6x^2+10x-3x^2+9x-15$
$= 2x^3-9x^2+19x-15$

18. $(2x-1)(x^2-4x+3) = (2x)(x^2)+(2x)(-4x)+(2x)(3)+(-1)(x^2)+(-1)(-4x)+(-1)(3)$
$= 2x^3-8x^2+6x-x^2+4x-3$
$= 2x^3-9x^2+10x-3$

19. $(x+7)(x+3) = x^2+3x+7x+21 = x^2+10x+21$

20. $(x+8)(x+5) = x^2+5x+8x+40 = x^2+13x+40$

21. $(x-5)(x+3) = x^2+3x-5x-15 = x^2-2x-15$

22. $(x-1)(x+2) = x^2+2x-x-2 = x^2+x-2$

23. $(3x+5)(2x+1) = (3x)(2x)+3x(1)+5(2x)+5 = 6x^2+3x+10x+5 = 6x^2+13x+5$

24. $(7x+4)(3x+1) = (7x)(3x)+7x(1)+4(3x)+4(1) = 21x^2+7x+12x+4 = 21x^2+19x+4$

25. $(2x-3)(5x+3) = (2x)(5x)+(2x)(3)+(-3)(5x)+(-3)(3) = 10x^2+6x-15x-9 = 10x^2-9x-9$

26. $(2x-5)(7x+2) = (2x)(7x)+(2x)(2)+(-5)(7x)+(-5)(2) = 14x^2+4x-35x-10 = 14x^2-31x-10$

27. $(5x^2-4)(3x^2-7) = (5x^2)(3x^2)+(5x^2)(-7)+(-4)(3x^2)+(-4)(-7) = 15x^4-35x^2-12x^2+28 = 15x^4-47x^2+28$

28. $(7x^2-2)(3x^2-5) = (7x^2)(3x^2)+(7x^2)(-5)+(-2)(3x^2)+(-2)(-5) = 21x^4-35x^2-6x^2+10 = 21x^4-41x^2+10$

29. $(8x^3+3)(x^2-5) = (8x^3)(x^2)+(8x^3)(-5)+(3)(x^2)+(3)(-5) = 8x^5-40x^3+3x^2-15$

30. $(7x^3+5)(x^2-2) = (7x^3)(x^2)+(7x^3)(-2)+(5)(x^2)+(5)(-2) = 7x^5-14x^3+5x^2-10$

31. $(x+3)(x-3) = x^2-3^2 = x^2-9$

32. $(x+5)(x-5) = x^2-5^2 = x^2-25$

33. $(3x+2)(3x-2) = (3x)^2-2^2 = 9x^2-4$

34. $(2x+5)(2x-5) = (2x)^2-5^2 = 4x^2-25$

35. $(5-7x)(5+7x) = 5^2-(7x)^2 = 25-49x^2$

36. $(4-3x)(4+3x) = 4^2-(3x)^2 = 16-9x^2$

37. $(4x^2+5x)(4x^2-5x)=(4x^2)^2-(5x)^2=16x^4-25x^2$

38. $(3x^2+4x)(3x^2-4x)=(3x^2)^2-(4x)^2=9x^4-16x^2$

39. $\left(1-y^5\right)\left(1+y^5\right)=(1)^2-\left(y^5\right)^2=1-y^{10}$

40. $\left(2-y^5\right)\left(2+y^5\right)=(2)^2-\left(y^5\right)^2=4-y^{10}$

41. $(x+2)^2=x^2+2\cdot x\cdot 2+2^2=x^2+4x+4$

42. $(x+5)^2=x^2+2\cdot x\cdot 5+5^2=x^2+10x+25$

43. $(2x+3)^2=(2x)^2+2(2x)(3)+3^2=4x^2+12x+9$

44. $(3x+2)^2=(3x)^2+2(3x)(2)+2^2=9x^2+12x+4$

45. $(x-3)^2=x^2-2\cdot x\cdot 3+3^2=x^2-6x+9$

46. $(x-4)^2=x^2-2\cdot x\cdot 4+4^2=x^2-8x+16$

47. $(4x^2-1)^2=(4x^2)^2-2(4x^2)(1)+1^2=16x^4-8x^2+1$

48. $(5x^2-3)^2=(5x^2)^2-2(5x^2)(3)+3^2=25x^4-30x^2+9$

49. $(7-2x)^2=7^2-2(7)(2x)+(2x)^2=49-28x+4x^2=4x^2-28x+49$

50. $(9-5x)^2=9^2-2(9)(5x)+(5x)^2=81-90x+25x^2$ or $25x^2-90x+81$

51. $(x+1)^3=x^3+3\cdot x^2\cdot 1+3x\cdot 1^2+1^3=x^3+3x^2+3x+1$

52. $(x+2)^3=x^3+3\cdot x^2\cdot 2+3\cdot x\cdot 2^2+2^3=x^3+6x^2+12x+8$

53. $(2x+3)^3=(2x)^3+3\cdot(2x)^2\cdot 3+3(2x)\cdot 3^2+3^3=8x^3+36x^2+54x+27$

54. $(3x+4)^3=(3x)^3+3(3x)^2\cdot 4+3(3x)\cdot 4^2+4^3=27x^3+108x^2+144x+64$

55. $(x-3)^3=x^3-3\cdot x^3\cdot 3+3\cdot x\cdot 3^2-3^3=x^3-9x^2+27x-27$

56. $(x-1)^3=x^3-3x^2\cdot 1+3x\cdot 1^2-1^3=x^3-3x^2+3x-1$

57. $(3x-4)^3=(3x)^3-3(3x)^2\cdot 4+3(3x)\cdot 4^2-4^3=27x^3-108x^2+144x-64$

58. $(2x-3)^3=(2x)^3-3(2x)^2\cdot 3+3(2x)\cdot 3^2-3^3=8x^3-36x^2+54x-27$

59.
$$\begin{aligned}(x+5y)(7x+3y)&=x(7x)+x(3y)+(5y)(7x)+(5y)(3y)\\&=7x^2+3xy+35xy+15y^2\\&=7x^2+38xy+15y^2\end{aligned}$$

60.
$$\begin{aligned}(x+9y)(6x+7y)&=x(6x)+x(7y)+(9y)(6x)+(9y)(7y)\\&=6x^2+7xy+54xy+63y^2\\&=6x^2+61xy+63y^2\end{aligned}$$

61. $(x-3y)(2x+7y) = x(2x)+x(7y)+(-3y)(2x)+(-3y)(7y)$
$= 2x^2+7xy-6xy-21y^2$
$= 2x^2+xy-21y^2$

62. $(3x-y)(2x+5y) = (3x)(2x)+(3x)(5y)+(-y)(2x)+(-y)(5y)$
$= 6x^2+15xy-2xy-5y^2$
$= 6x^2+13xy-5y^2$

63. $(3xy-1)(5xy+2) = (3xy)(5xy)+(3xy)(2)+(-1)(5xy)+(-1)(2)$
$= 15x^2y^2+6xy-5xy-2$
$= 15x^2y^2+xy-2$

64. $(7x^2y+1)(2x^2y-3) = (7x^2y)(2x^2y)+(7x^2y)(-3)+(1)2x^2y+(1)(-3)$
$= 14x^4y^2-21x^2y+2x^2y-3$
$= 14x^4y^2-19x^2y-3$

65. $(7x+5y)^2 = (7x)^2+2(7x)(5y)+(5y)^2 = 49x^2+70xy+25y^2$

66. $(9x+7y)^2 = (9x)^2+2(9x)(7y)+(7y)^2 = 81x^2+126xy+49y^2$

67. $(x^2y^2-3)^2 = (x^2y^2)^2-2(x^2y^2)(3)+3^2 = x^4y^4-6x^2y^2+9$

68. $(x^2y^2-5)^2 = (x^2y^2)^2-2(x^2y^2)(5)+5^2 = x^4y^4-10x^2y^2+25$

69. $(x-y)(x^2+xy+y^2) = x(x^2)+x(xy)+x(y^2)+(-y)(x^2)+(-y)(xy)+(-y)(y^2)$
$= x^3+x^2y+xy^2-x^2y-xy^2-y^3$
$= x^3-y^3$

70. $(x+y)(x^2-xy+y^2) = x(x^2)+x(-xy)+x(y^2)+y(x^2)+y(-xy)+y(y^2)$
$= x^3-x^2y+xy^2+x^2y-xy^2+y^3$
$= x^3+y^3$

71. $(3x+5y)(3x-5y) = (3x)^2-(5y)^2 = 9x^2-25y^2$

72. $(7x+3y)(7x-3y) = (7x)^2-(3y)^2 = 49x^2-9y^2$

73. $(x+y+3)(x+y-3) = (x+y)^2-3^2 = x^2+2xy+y^2-9$

74. $(x+y+5)(x+y-5) = (x+y)^2-5^2 = x^2+2xy+y^2-25$

75. $(3x+7-5y)(3x+7+5y) = (3x+7)^2-(5y)^2 = 9x^2+42x+49-25y^2$

76. $(5x+7y-2)(5x+7y+2) = (5x+7y)^2-2^2 = 25x^2+70xy+49y^2-4$

77. $[5y-(2x+3)][5y+(2x+3)]=(5y)^2-(2x+3)^2=25y^2-(4x^2+12x+9)=25y^2-4x^2-12x-9$

78. $[8y+(7-3x)][8y-(7-3x)]=(8y)^2-(7-3x)^2=64y^2-(49-42x+9x^2)=64y^2-49+42x-9x^2$

79. $(x+y+1)^2=(x+y)^2+2(x+y)+1=x^2+2xy+y^2+2x+2y+1$

80. $(x+y+2)^2=(x+y)^2+2(x+y)(2)+2^2=x^2+2xy+y^2+4x+4y+4$

81. $(2x+y+1)^2=(2x+y)^2+2(2x+y)+1=4x^2+4xy+y^2+4x+2y+1$

82. $(5x+1+6y)^2=(5x+1)^2+2(5x+1)(6y)+(6y)^2=25x^2+10x+60xy+1+12y+36y^2$

83.
$$\begin{aligned}(3x+4y)^2-(3x-4y)^2&=\left[(3x)^2+2(3x)(4y)+(4y)^2\right]-\left[(3x)^2-2(3x)(4y)+(4y)^2\right]\\&=\left(9x^2+24xy+16y^2\right)-\left(9x^2-24xy+16y^2\right)\\&=9x^2+24xy+16y^2-9x^2+24xy-16y^2\\&=48xy\end{aligned}$$

84.
$$\begin{aligned}(5x+2y)^2-(5x-2y)^2&=\left[(5x)^2+2(5x)(2y)+(2y)^2\right]-\left[(5x)^2-2(5x)(2y)+(2y)^2\right]\\&=\left(25x^2+20xy+4y^2\right)-\left(25x^2-20xy+4y^2\right)\\&=25x^2+20xy+4y^2-25x^2+20xy-4y^2\\&=40xy\end{aligned}$$

85.
$$\begin{aligned}&(5x-7)(3x-2)-(4x-5)(6x-1)\\&=\left[15x^2-10x-21x+14\right]-\left[24x^2-4x-30x+5\right]\\&=\left(15x^2-31x+14\right)-\left(24x^2-34x+5\right)\\&=15x^2-31x+14-24x^2+34x-5\\&=-9x^2+3x+9\end{aligned}$$

86.
$$\begin{aligned}&(3x+5)(2x-9)-(7x-2)(x-1)\\&=\left(6x^2-27x+10x-45\right)-\left(7x^2-7x-2x+2\right)\\&=\left(6x^2-17x-45\right)-\left(7x^2-9x+2\right)\\&=6x^2-17x-45-7x^2+9x-2\\&=-x^2-8x-47\end{aligned}$$

87. $(2x+5)(2x-5)\left(4x^2+25\right)$

$=\left[(2x)^2-5^2\right]\left(4x^2+25\right)$

$=\left(4x^2-25\right)\left(4x^2+25\right)$

$=\left(4x^2\right)^2-(25)^2$

$=16x^4-625$

88. $(3x+4)(3x-4)\left(9x^2+16\right)$

$=\left[(3x)^2-4^2\right]\left(9x^2+16\right)$

$=\left(9x^2-16\right)\left(9x^2+16\right)$

$=\left(9x^2\right)^2-(16)^2$

$=81x^4-256$

89. $\dfrac{(2x-7)^5}{(2x-7)^3}=(2x-7)^{5-3}$

$=(2x-7)^2$

$=(2x)^2-2(2x)(7)+(7)^2$

$=4x^2-28x+49$

90. $\dfrac{(5x-3)^6}{(5x-3)^4}=(5x-3)^{6-4}$

$=(5x-3)^2$

$=(5x)^2-2(5x)(3)+(3)^2$

$=25x^2-30x+9$

91. **a.** $S=0.2x^3-1.5x^2+3.4x+25+\left(0.1x^3-1.3x^2+3.3x+5\right)$

$S=0.2x^3-1.5x^2+3.4x+25+0.1x^3-1.3x^2+3.3x+5$

$S=0.3x^3-2.8x^2+6.7x+30$

b. $S=0.3x^3-2.8x^2+6.7x+30$

$S=0.3(5)^3-2.8(5)^2+6.7(5)+30$

$S=31$

The model gives a score of 31 for the group in the 45-54 age range which is the same as the score displayed by the bar graph.

92. **a.** $S = 0.1x^3 - 1.2x^2 + 2.4x + 17 + \left(0.2x^3 - 1.6x^2 + 4.3x + 13\right)$

$S = 0.1x^3 - 1.2x^2 + 2.4x + 17 + 0.2x^3 - 1.6x^2 + 4.3x + 13$

$S = 0.3x^3 - 2.8x^2 + 6.7x + 30$

b. $S = 0.3x^3 - 2.8x^2 + 6.7x + 30$

$S = 0.3(6)^3 - 2.8(6)^2 + 6.7(6) + 30$

$S = 34.2$

The model gives a score of 34.2 for the group in the 55-64 age range. This overestimates the score shown on the bar graph by 0.2.

93. $x(8-2x)(10-2x) = x\left(80 - 36x + 4x^2\right)$

$= 80x - 36x^2 + 4x^3$

$= 4x^3 - 36x^2 + 80x$

94. $x(8-2x)(5-2x) = x\left(40 - 26x + 4x^2\right)$

$= 40x - 26x^2 + 4x^3$

$= 4x^3 - 26x^2 + 40x$

95. $(x+9)(x+3) - (x+5)(x+1)$

$= x^2 + 12x + 27 - \left(x^2 + 6x + 5\right)$

$= x^2 + 12x + 27 - x^2 - 6x - 5$

$= 6x + 22$

96. $(x+4)(x+3) - (x+2)(x+1)$

$= x^2 + 7x + 12 - \left(x^2 + 3x + 2\right)$

$= x^2 + 7x + 12 - x^2 - 3x - 2$

$= 4x + 10$

97. – 102. Answers will vary.

103. makes sense

104. does not make sense; Explanations will vary. Sample explanation: FOIL is used to multiply two binomials.

105. makes sense

106. makes sense, although answers may vary

107. $(x+3)(x-1) + ((x+3) - x)(x - (x-1))$

$= (x+3)(x-1) + 3(x - x + 1)$

$= x^2 - x + 3x - 3 + 3$

$= x^2 + 2x$

108. $(2x-1)x(x+3) - x(x-2)x$

$= (2x^2 + 5x - 3)(x+2) - x^2(x-2)$

$= 2x^3 + 5x^2 - 3x - x^3 + 2x^2$

$= x^3 + 7x^2 - 3x$

109. $(x+5)(2x+1)(x+2)-3\cdot x(x+5)$

$=(2x^2+11x+5)(x+2)-3x^2-15x$

$=2x^3+15x^2+27x+10-3x^2-15x$

$=2x^3+12x^2+12x+10$

110. $(y^n+2)(y^n-2)-(y^n-3)^2$

$=y^{2n}-4-(y^{2n}-6y^n+9)$

$=y^{2n}-4-y^{2n}+6y^n-9$

$=6y^n-13$

111. $(x+3)(x+\boxed{4})=x^2+7x+12$

112. $(x-\boxed{2})(x-12)=x^2-14x+24$

113. $(4x+1)(2x-\boxed{3})=8x^2-10x-3$

Section P.5

Check Point Exercises

1. **a.** $10x^3-4x^2$

$=2x^2(5x)-2x^2(2)$

$=2x^2(5x-2)$

b. $2x(x-7)+3(x-7)$

$=(x-7)(2x+3)$

2. $x^3+5x^2-2x-10$

$=(x^3+5x^2)-(2x+10)$

$=x^2(x+5)-2(x+5)$

$=(x+5)(x^2-2)$

3. Find two numbers whose product is 40 and whose sum is 13. The required integers are 8 and 5. Thus, $x^2+13x+40=(x+8)(x+5)$ or $(x+5)(x+8)$.

4. Find two numbers whose product is –14 and whose sum is –5. The required integers are –7 and 2. Thus, $x^2-5x-14=(x-7)(x+2)$ or $(x+2)(x-7)$.

5. Find two First terms whose product is $6x^2$.

$6x^2+19x-7=(6x\quad)(x\quad)$

$6x^2+19x-7=(3x\quad)(2x\quad)$

Find two Last terms whose product is –7.
The possible factors are 1(–7) and –1(7).

Try various combinations of these factors to find the factorization in which the sum of the Outside and Inside products is $19x$.