Precalculus Enhanced with Graphing Utilities 8th edition Sullivan Solutions Manual

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Chapter 1
Graphs

Section 1.1

1. 0

2. 

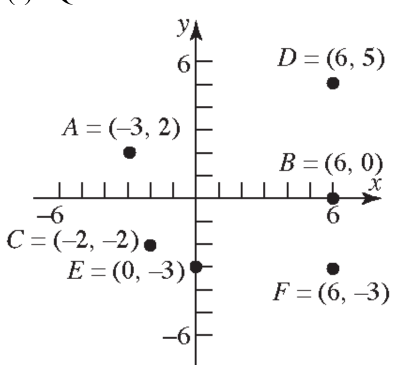
3. x-coordinate; y-coordinate; quadrants

4. False; points that lie in Quadrant IV will have a positive x-coordinate and a negative y-coordinate. The point $(-1,4)$ lies in Quadrant II.

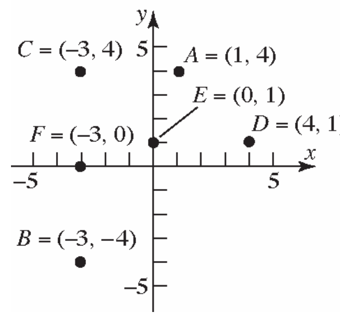
5. d

6. c

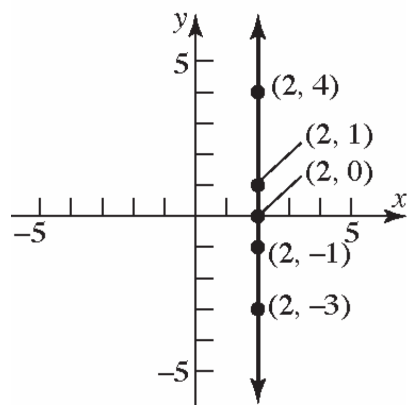
7. (a) Quadrant II
(b) x-axis
(c) Quadrant III
(d) Quadrant I
(e) y-axis
(f) Quadrant IV



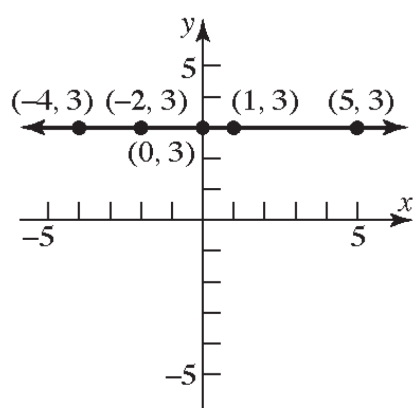
8. (a) Quadrant I
(b) Quadrant III
(c) Quadrant II
(d) Quadrant I
(e) y-axis
(f) x-axis



9. The points will be on a vertical line that is two units to the right of the y-axis.



10. The points will be on a horizontal line that is three units above the x-axis.



11. $(-1,4)$; Quadrant II

12. (3, 4); Quadrant I

13. (3, 1); Quadrant I

14. $(-6,-4)$; Quadrant III

15. $X\min=-11$
$X\max=5$
$X\text{ scl}=1$
$Y\min=-3$
$Y\max=6$
$Y\text{ scl}=1$

16. $X\min=-3$
$X\max=7$
$X\text{ scl}=1$
$Y\min=-4$
$Y\max=9$
$Y\text{ scl}=1$

17. $X\min=-30$
$X\max=50$
$X\text{ scl}=10$
$Y\min=-90$
$Y\max=50$
$Y\text{ scl}=10$

18. $X\min=-90$
$X\max=30$
$X\text{ scl}=10$
$Y\min=-50$
$Y\max=70$
$Y\text{ scl}=10$

19. $X\min=-10$
$X\max=110$
$X\text{ scl}=10$
$Y\min=-10$
$Y\max=160$
$Y\text{ scl}=10$

20. $X\min=-20$
$X\max=110$
$X\text{ scl}=10$
$Y\min=-10$
$Y\max=60$
$Y\text{ scl}=10$

21. $X\min=-6$
$X\max=6$
$X\text{ scl}=2$
$Y\min=-4$
$Y\max=4$
$Y\text{ scl}=2$

22. $X\min=-3$
$X\max=3$
$X\text{ scl}=1$
$Y\min=-2$
$Y\max=2$
$Y\text{ scl}=1$

23. $X\min=-6$
$X\max=6$
$X\text{ scl}=2$
$Y\min=-1$
$Y\max=3$
$Y\text{ scl}=1$

24. $X\min=-9$
$X\max=9$
$X\text{ scl}=3$
$Y\min=-12$
$Y\max=4$
$Y\text{ scl}=4$

25. $X\min=3$
$X\max=9$
$X\text{ scl}=1$
$Y\min=2$
$Y\max=10$
$Y\text{ scl}=2$

26. $X\min=-22$
$X\max=-10$
$X\text{ scl}=2$
$Y\min=4$
$Y\max=8$
$Y\text{ scl}=1$

27. $y = x^4 - \sqrt{x}$

$0 = 0^4 - \sqrt{0}$ $\quad 1 = 1^4 - \sqrt{1}$ $\quad 0 = (-1)^4 - \sqrt{-1}$

$0 = 0$ $\quad 1 \neq 0$ $\quad 0 \neq 1 - \sqrt{-1}$

(0, 0) is on the graph of the equation.

28. $y = x^3 - 2\sqrt{x}$

$0 = 0^3 - 2\sqrt{0}$ $\quad 1 = 1^3 - 2\sqrt{1}$ $\quad -1 = 1^3 - 2\sqrt{1}$

$0 = 0$ $\quad 1 \neq -1$ $\quad -1 = -1$

(0, 0) and (1, –1) are on the graph of the equation.

29. $y^2 = x^2 + 9$

$3^2 = 0^2 + 9$ $\quad 0^2 = 3^2 + 9$ $\quad 0^2 = (-3)^2 + 9$

$9 = 9$ $\quad 0 \neq 18$ $\quad 0 \neq 18$

(0, 3) is on the graph of the equation.

30. $y^3 = x + 1$

$2^3 = 1 + 1$ $\quad 1^3 = 0 + 1$ $\quad 0^3 = -1 + 1$

$8 \neq 2$ $\quad 1 = 1$ $\quad 0 = 0$

(0, 1) and (–1, 0) are on the graph of the equation.

31. $x^2 + y^2 = 4$

$0^2 + 2^2 = 4$ $\quad (-2)^2 + 2^2 = 4$ $\quad \sqrt{2}^2 + \sqrt{2}^2 = 4$

$4 = 4$ $\quad 8 \neq 4$ $\quad 4 = 4$

(0, 2) and $\left(\sqrt{2}, \sqrt{2}\right)$ are on the graph of the equation.

32. $x^2 + 4y^2 = 4$

$0^2 + 4 \cdot 1^2 = 4$ $\quad 2^2 + 4 \cdot 0^2 = 4$ $\quad 2^2 + 4\left(\frac{1}{2}\right)^2 = 4$

$4 = 4$ $\quad 4 = 4$ $\quad 5 \neq 4$

(0, 1) and (2, 0) are on the graph of the equation.

33. (–1, 0), (1, 0)

34. (0, 1)

35. $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right), (0, 1)$

36. (–2, 0), (2, 0), (0, –3)

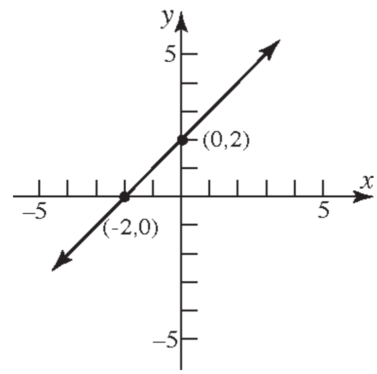
37. $(1,0)$, $(0,2)$, $(0,-2)$

38. (2, 0), (0, 2), (–2, 0), (0, –2)

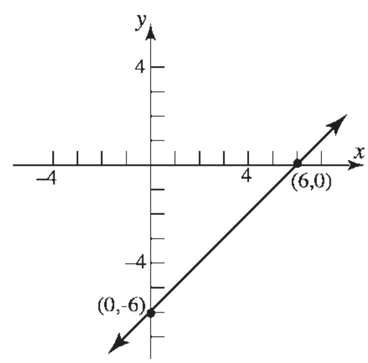
39. (–4,0), (–1,0), (4, 0), (0, –3)

40. (–2, 0), (2, 0), (0, 3)

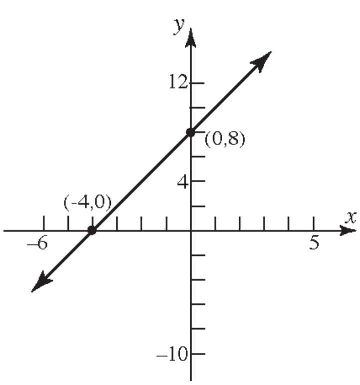
41. $y = x + 2$



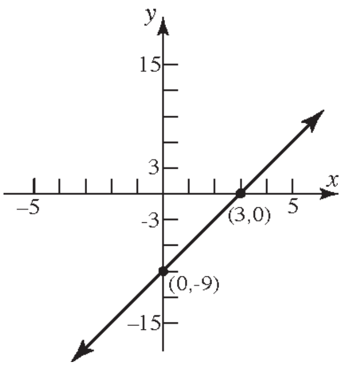
42. $y = x - 6$



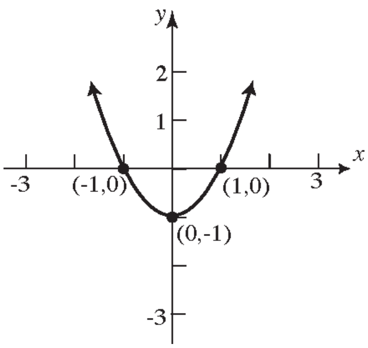
43. $y = 2x + 8$



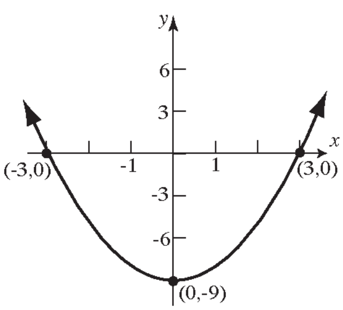
44. $y = 3x - 9$



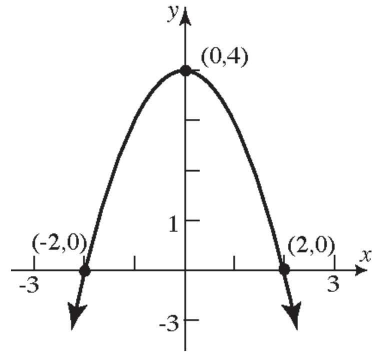
45. $y = x^2 - 1$



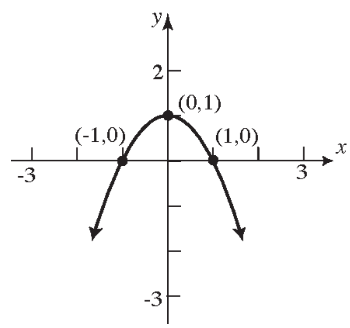
46. $y = x^2 - 9$



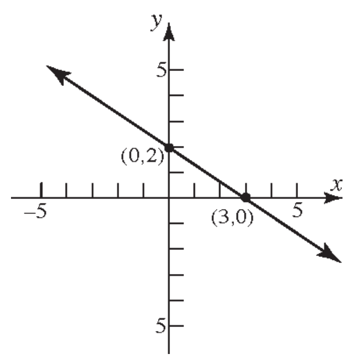
47. $y = -x^2 + 4$



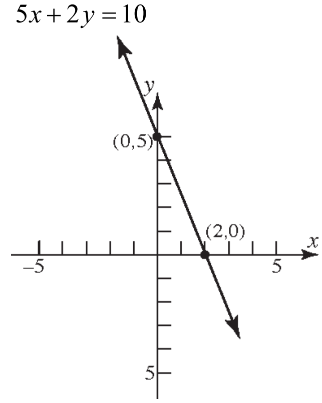
48. $y = -x^2 + 1$



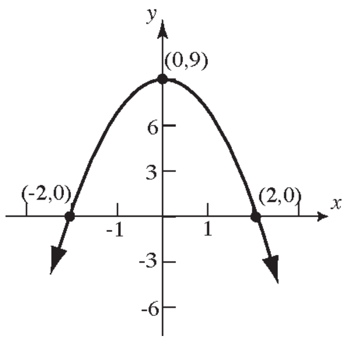
49. $2x + 3y = 6$



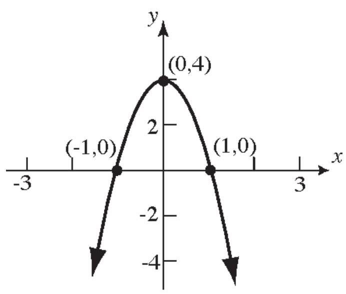
50. $5x + 2y = 10$



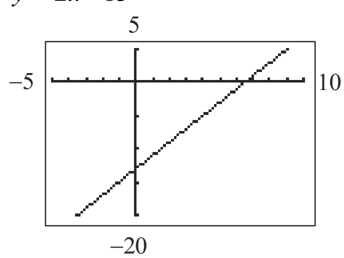
51. $9x^2 + 4y = 36$

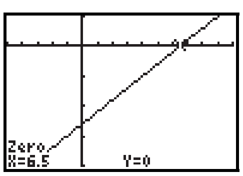


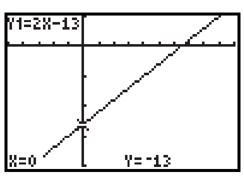
52. $4x^2 + y = 4$



53. $y = 2x - 13$

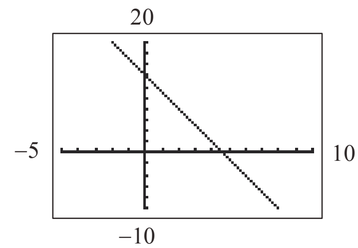


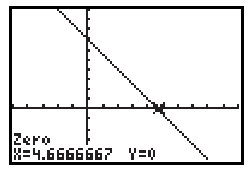


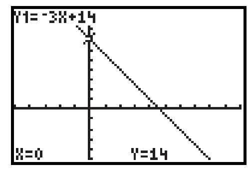


The x-intercept is $x = 6.5$ and the y-intercept is $y = -13$.

54. $y = -3x + 14$

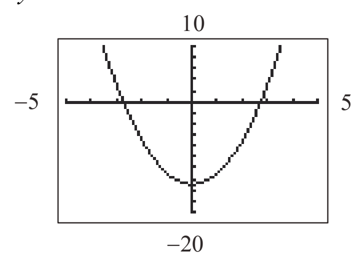


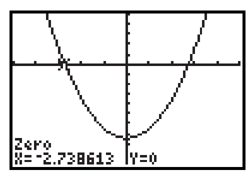


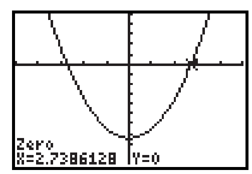


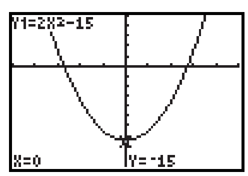
The x-intercept is $x = 4.67$ and the y-intercept is $y = 14$.

55. $y = 2x^2 - 15$



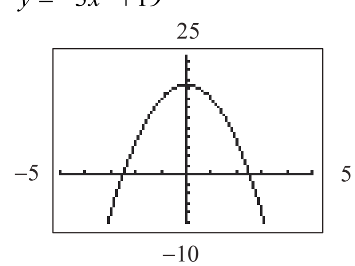


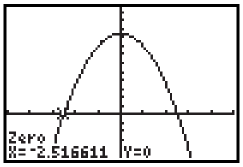


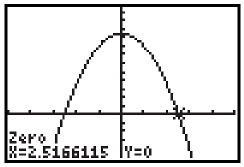


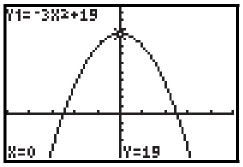
The x-intercepts are $x = -2.74$ and $x = 2.74$. The y-intercept is $y = -15$.

56. $y = -3x^2 + 19$



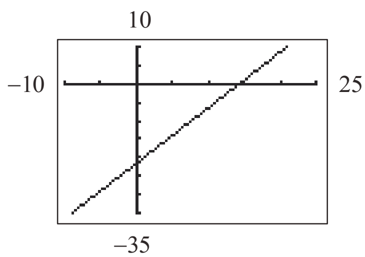


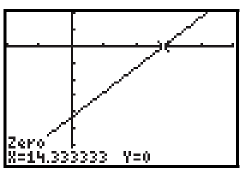


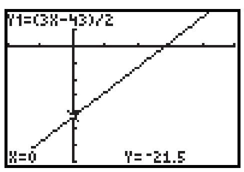


The x-intercepts are $x = -2.52$ and $x = 2.52$. The y-intercept is $y = 19$.

57. $3x - 2y = 43$ or $y = \dfrac{3x-43}{2}$

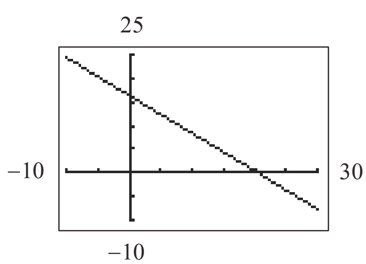


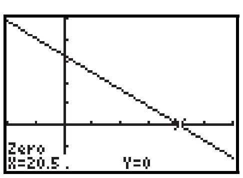


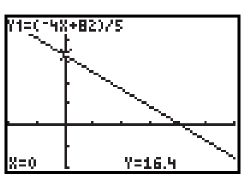


The x-intercept is $x = 14.33$ and the y-intercept is $y = -21.5$.

58. $4x + 5y = 82$ or $y = \dfrac{-4x+82}{5}$

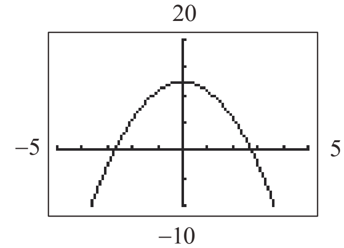


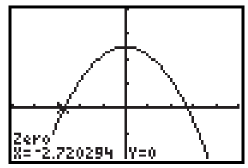


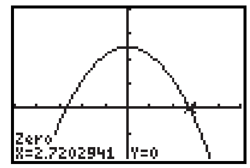


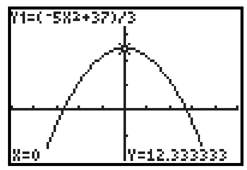
The x-intercept is $x = 20.5$ and the y-intercept is $y = 16.4$.

59. $5x^2 + 3y = 37$ or $y = \dfrac{-5x^2+37}{3}$



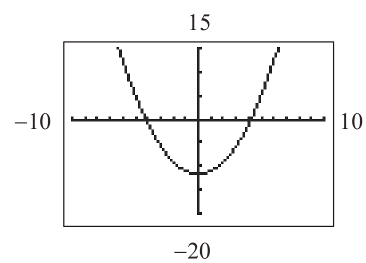


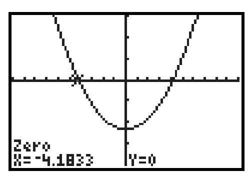


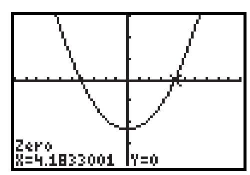


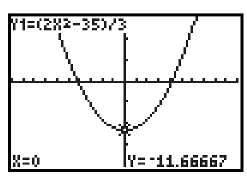
The x-intercepts are $x = -2.72$ and $x = 2.72$. The y-intercept is $y = 12.33$.

60. $2x^2 - 3y = 35$ or $y = \dfrac{2x^2-35}{3}$









The x-intercepts are $x = -4.18$ and $x = 4.18$. The y-intercept is $y = -11.67$.

61. If $(2, 5)$ is shifted 3 units right then the x coordinate would be $2+3$. If it is shifted 2 units down then the y-coordinate would be $5+(-2)$.

Thus the new point would be
$(2+3, 5+(-2)) = (5,3)$.

62. If $(-1, 6)$ is shifted 2 units left then the x coordinate would be $-1+(-2)$. If it is shifted 4 units up then the y-coordinate would be $6+4$. Thus the new point would be $(-1+(-2), 6+4) = (-3,10)$.

63. **a.** 25 feet

b. 23.2 ft; 34.1 ft

c. (0,6), The shot is released at a height of 6 feet; (48.7, 0), the shot hits the ground after traveling a horizontal distance of 48.7 feet.

64. **a.** 20 meters

b. 12 seconds; 36 meters

c. (0, 2), The discus is released at a height of 2 meters; (18, 0), the discus hits the ground after 18 seconds.

65. **a.** \$19.95; \$19.95

b. \$182.45

c. (0, 19.95), The membership plan costs \$19.95 per month.

66. **a.** 1.5 miles

b. 1 mile

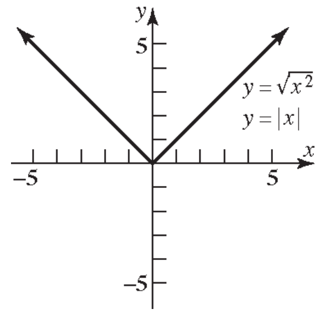
c. (0, 2), Caleb's friend lives 2 miles from his house; (28, 0), it takes Caleb 28 minutes to ride home.

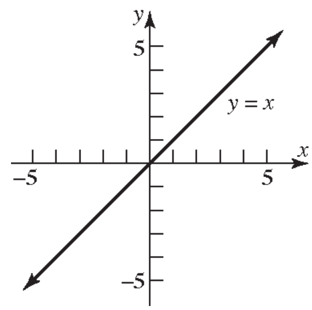
67. $(-1,0), (1,0), (0,-1), (0,1)$

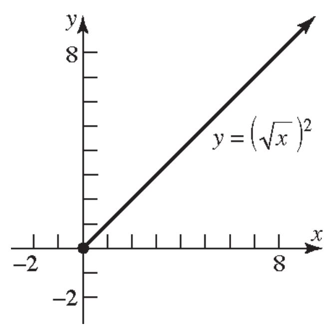
68. $(-2,0), (3,0), (0,-2), (0,0), (0,2)$

69. Answers will vary

70. **a.**







b. Since $\sqrt{x^2} = |x|$ for all x, the graphs of $y = \sqrt{x^2}$ and $y = |x|$ are the same.

c. For $y = \left(\sqrt{x}\right)^2$, the domain of the variable x is $x \geq 0$; for $y = x$, the domain of the variable x is all real numbers. Thus, $\left(\sqrt{x}\right)^2 = x$ only for $x \geq 0$.

d. For $y = \sqrt{x^2}$, the range of the variable y is $y \geq 0$; for $y = x$, the range of the variable y is all real numbers. Also, $\sqrt{x^2} = x$ only if $x \geq 0$.

71. Answers will vary

72. Answers will vary. A complete graph presents enough of the graph to the viewer so they can "see" the rest of the graph as an obvious continuation of what is shown.

73. Answers will vary.

Section 1.2

1. $|5-(-3)| = |8| = 8$

2. $\sqrt{3^2+4^2} = \sqrt{25} = 5$

3. $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$
Since the sum of the squares of two of the sides of the triangle equals the square of the third side, the triangle is a right triangle.

4. $(0,0)$

5. $\frac{1}{2}bh$

6. true

7. midpoint

8. False; the distance between two points is never negative.

9. True; $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

10. a

11. $d(P_1,P_2) = \sqrt{(2-0)^2+(1-0)^2}$
$= \sqrt{2^2+1^2} = \sqrt{4+1} = \sqrt{5}$

12. $d(P_1,P_2) = \sqrt{(-2-0)^2+(1-0)^2}$
$= \sqrt{(-2)^2+1^2} = \sqrt{4+1} = \sqrt{5}$

13. $d(P_1,P_2) = \sqrt{(-2-1)^2+(2-1)^2}$
$= \sqrt{(-3)^2+1^2} = \sqrt{9+1} = \sqrt{10}$

14. $d(P_1,P_2) = \sqrt{(2-(-1))^2+(2-1)^2}$
$= \sqrt{3^2+1^2} = \sqrt{9+1} = \sqrt{10}$

15. $d(P_1,P_2) = \sqrt{(5-3)^2+(4-(-4))^2}$
$= \sqrt{2^2+(8)^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$

16. $d(P_1,P_2) = \sqrt{(2-(-1))^2+(4-0)^2}$
$= \sqrt{(3)^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

17. $d(P_1,P_2) = \sqrt{(4-(-7))^2+(0-3)^2}$
$= \sqrt{11^2+(-3)^2} = \sqrt{121+9} = \sqrt{130}$

18. $d(P_1,P_2) = \sqrt{(4-2)^2+(2-(-3))^2}$
$= \sqrt{2^2+5^2} = \sqrt{4+25} = \sqrt{29}$

19. $d(P_1,P_2) = \sqrt{(6-5)^2+(1-(-2))^2}$
$= \sqrt{1^2+3^2} = \sqrt{1+9} = \sqrt{10}$

20. $d(P_1,P_2) = \sqrt{(6-(-4))^2+(2-(-3))^2}$
$= \sqrt{10^2+5^2} = \sqrt{100+25}$
$= \sqrt{125} = 5\sqrt{5}$

21. $d(P_1,P_2) = \sqrt{(2.3-(-0.2))^2+(1.1-(0.3))^2}$
$= \sqrt{2.5^2+0.8^2} = \sqrt{6.25+0.64}$
$= \sqrt{6.89} \approx 2.62$

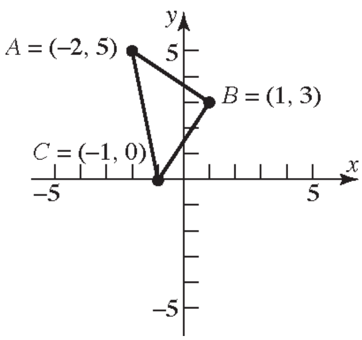
22. $d(P_1,P_2) = \sqrt{(-0.3-1.2)^2+(1.1-2.3)^2}$
$= \sqrt{(-1.5)^2+(-1.2)^2} = \sqrt{2.25+1.44}$
$= \sqrt{3.69} \approx 1.92$

23. $d(P_1,P_2) = \sqrt{(0-a)^2+(0-b)^2}$
$= \sqrt{(-a)^2+(-b)^2} = \sqrt{a^2+b^2}$

24. $d(P_1,P_2) = \sqrt{(0-a)^2+(0-a)^2}$
$= \sqrt{(-a)^2+(-a)^2}$
$= \sqrt{a^2+a^2} = \sqrt{2a^2} = |a|\sqrt{2}$

25. $A=(-2,5),\ B=(1,3),\ C=(-1,0)$

$$d(A,B)=\sqrt{(1-(-2))^2+(3-5)^2}$$
$$=\sqrt{3^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$$
$$d(B,C)=\sqrt{(-1-1)^2+(0-3)^2}$$
$$=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$$
$$d(A,C)=\sqrt{(-1-(-2))^2+(0-5)^2}$$
$$=\sqrt{1^2+(-5)^2}=\sqrt{1+25}=\sqrt{26}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

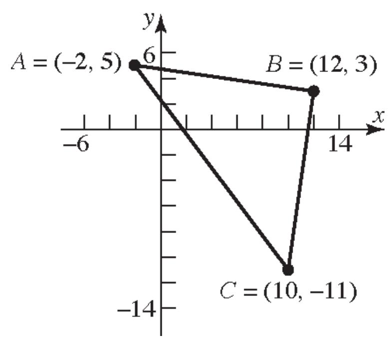
$$[d(A,B)]^2+[d(B,C)]^2=[d(A,C)]^2$$
$$\left(\sqrt{13}\right)^2+\left(\sqrt{13}\right)^2=\left(\sqrt{26}\right)^2$$
$$13+13=26$$
$$26=26$$

The area of a triangle is $A=\frac{1}{2}\cdot bh$. In this problem,

$$A=\frac{1}{2}\cdot[d(A,B)]\cdot[d(B,C)]$$
$$=\frac{1}{2}\cdot\sqrt{13}\cdot\sqrt{13}=\frac{1}{2}\cdot 13$$
$$=\frac{13}{2}\text{ square units}$$

26. $A=(-2,5),\ B=(12,3),\ C=(10,-11)$

$$d(A,B)=\sqrt{(12-(-2))^2+(3-5)^2}$$
$$=\sqrt{14^2+(-2)^2}$$
$$=\sqrt{196+4}=\sqrt{200}$$
$$=10\sqrt{2}$$
$$d(B,C)=\sqrt{(10-12)^2+(-11-3)^2}$$
$$=\sqrt{(-2)^2+(-14)^2}$$
$$=\sqrt{4+196}=\sqrt{200}$$
$$=10\sqrt{2}$$
$$d(A,C)=\sqrt{(10-(-2))^2+(-11-5)^2}$$
$$=\sqrt{12^2+(-16)^2}$$
$$=\sqrt{144+256}=\sqrt{400}$$
$$=20$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$[d(A,B)]^2+[d(B,C)]^2=[d(A,C)]^2$$
$$\left(10\sqrt{2}\right)^2+\left(10\sqrt{2}\right)^2=(20)^2$$
$$200+200=400$$
$$400=400$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

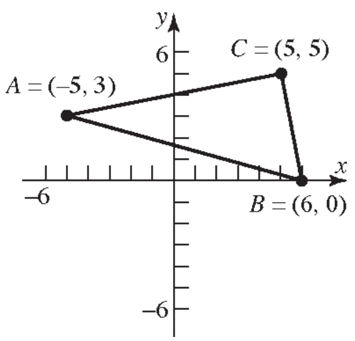
$$A=\frac{1}{2}\cdot[d(A,B)]\cdot[d(B,C)]$$
$$=\frac{1}{2}\cdot 10\sqrt{2}\cdot 10\sqrt{2}$$
$$=\frac{1}{2}\cdot 100\cdot 2=100\text{ square units}$$

27. $A=(-5,3),\ B=(6,0),\ C=(5,5)$

$$d(A,B)=\sqrt{(6-(-5))^2+(0-3)^2}=\sqrt{11^2+(-3)^2}=\sqrt{121+9}=\sqrt{130}$$

$$d(B,C)=\sqrt{(5-6)^2+(5-0)^2}=\sqrt{(-1)^2+5^2}=\sqrt{1+25}=\sqrt{26}$$

$$d(A,C)=\sqrt{(5-(-5))^2+(5-3)^2}=\sqrt{10^2+2^2}=\sqrt{100+4}=\sqrt{104}=2\sqrt{26}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$[d(A,C)]^2+[d(B,C)]^2=[d(A,B)]^2$$
$$\left(\sqrt{104}\right)^2+\left(\sqrt{26}\right)^2=\left(\sqrt{130}\right)^2$$
$$104+26=130$$
$$130=130$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

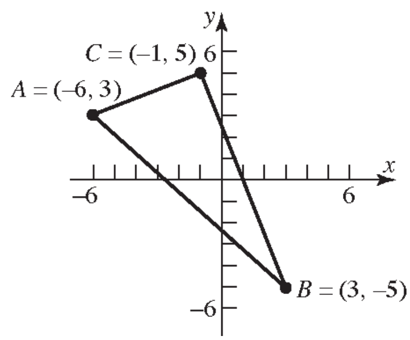
$$A=\frac{1}{2}\cdot[d(A,C)]\cdot[d(B,C)]=\frac{1}{2}\cdot\sqrt{104}\cdot\sqrt{26}=\frac{1}{2}\cdot 2\sqrt{26}\cdot\sqrt{26}=\frac{1}{2}\cdot 2\cdot 26$$
$= 26$ square units

28. $A=(-6,3),\ B=(3,-5),\ C=(-1,5)$

$$d(A,B)=\sqrt{(3-(-6))^2+(-5-3)^2}=\sqrt{9^2+(-8)^2}=\sqrt{81+64}=\sqrt{145}$$

$$d(B,C)=\sqrt{(-1-3)^2+(5-(-5))^2}=\sqrt{(-4)^2+10^2}=\sqrt{16+100}=\sqrt{116}=2\sqrt{29}$$

$$d(A,C)=\sqrt{(-1-(-6))^2+(5-3)^2}=\sqrt{5^2+2^2}=\sqrt{25+4}=\sqrt{29}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$[d(A,C)]^2+[d(B,C)]^2=[d(A,B)]^2$$
$$\left(\sqrt{29}\right)^2+\left(2\sqrt{29}\right)^2=\left(\sqrt{145}\right)^2$$
$$29+4\cdot 29=145$$
$$29+116=145$$
$$145=145$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

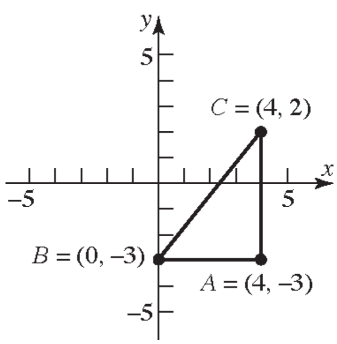
$$A=\frac{1}{2}\cdot[d(A,C)]\cdot[d(B,C)]=\frac{1}{2}\cdot\sqrt{29}\cdot 2\sqrt{29}=\frac{1}{2}\cdot 2\cdot 29$$
$= 29$ square units

29. $A=(4,-3),\ B=(0,-3),\ C=(4,2)$

$$\begin{aligned} d(A,B) &= \sqrt{(0-4)^2+\left(-3-(-3)\right)^2} \\ &= \sqrt{(-4)^2+0^2} = \sqrt{16+0} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

$$\begin{aligned} d(B,C) &= \sqrt{\left(4-0\right)^2+\left(2-(-3)\right)^2} \\ &= \sqrt{4^2+5^2} = \sqrt{16+25} \\ &= \sqrt{41} \end{aligned}$$

$$\begin{aligned} d(A,C) &= \sqrt{(4-4)^2+\left(2-(-3)\right)^2} \\ &= \sqrt{0^2+5^2} = \sqrt{0+25} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$\begin{aligned} \left[d(A,B)\right]^2+\left[d(A,C)\right]^2 &= \left[d(B,C)\right]^2 \\ 4^2+5^2 &= \left(\sqrt{41}\right)^2 \\ 16+25 &= 41 \\ 41 &= 41 \end{aligned}$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

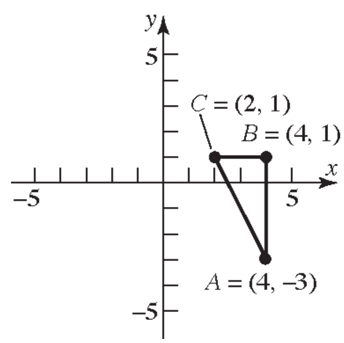
$$\begin{aligned} A &= \frac{1}{2}\cdot\left[d(A,B)\right]\cdot\left[d(A,C)\right] \\ &= \frac{1}{2}\cdot 4\cdot 5 \\ &= 10 \text{ square units} \end{aligned}$$

30. $A=(4,-3),\ B=(4,1),\ C=(2,1)$

$$\begin{aligned} d(A,B) &= \sqrt{(4-4)^2+\left(1-(-3)\right)^2} \\ &= \sqrt{0^2+4^2} \\ &= \sqrt{0+16} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

$$\begin{aligned} d(B,C) &= \sqrt{\left(2-4\right)^2+\left(1-1\right)^2} \\ &= \sqrt{(-2)^2+0^2} = \sqrt{4+0} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} d(A,C) &= \sqrt{(2-4)^2+\left(1-(-3)\right)^2} \\ &= \sqrt{(-2)^2+4^2} = \sqrt{4+16} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$\begin{aligned} \left[d(A,B)\right]^2+\left[d(B,C)\right]^2 &= \left[d(A,C)\right]^2 \\ 4^2+2^2 &= \left(2\sqrt{5}\right)^2 \\ 16+4 &= 20 \\ 20 &= 20 \end{aligned}$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

$$\begin{aligned} A &= \frac{1}{2}\cdot\left[d(A,B)\right]\cdot\left[d(B,C)\right] \\ &= \frac{1}{2}\cdot 4\cdot 2 \\ &= 4 \text{ square units} \end{aligned}$$

31. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{3+5}{2},\frac{-4+4}{2}\right)=\left(\frac{8}{2},\frac{0}{2}\right)=(4,0)$$

32. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-2+2}{2},\frac{0+4}{2}\right)=\left(\frac{0}{2},\frac{4}{2}\right)=(0,2)$$

33. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-1+8}{2},\frac{4+0}{2}\right)=\left(\frac{7}{2},\frac{4}{2}\right)=\left(\frac{7}{2},2\right)$$

34. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{2+4}{2},\frac{-3+2}{2}\right)=\left(\frac{6}{2},\frac{-1}{2}\right)=\left(3,-\frac{1}{2}\right)$$

35. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{7+9}{2},\frac{-5+1}{2}\right)=\left(\frac{16}{2},\frac{-4}{2}\right)=(8,-2)$$

36. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-4+2}{2},\frac{-3+2}{2}\right)=\left(\frac{-2}{2},\frac{-1}{2}\right)=\left(-1,-\frac{1}{2}\right)$$

37. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{a+0}{2},\frac{b+0}{2}\right)=\left(\frac{a}{2},\frac{b}{2}\right)$$

38. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{a+0}{2},\frac{a+0}{2}\right)=\left(\frac{a}{2},\frac{a}{2}\right)$$

39. $P_1=(1,3);P_2=(5,15)$

$$d(P_1,P_2)=\sqrt{(5-1)^2+(15-3)^2}=\sqrt{(4)^2+(12)^2}=\sqrt{16+144}=\sqrt{160}=4\sqrt{10}$$

The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{1+5}{2},\frac{3+15}{2}\right)=\left(\frac{6}{2},\frac{18}{2}\right)=(3,9)$$

40. $P_1 = (-8,-4); P_2 = (2,3)$

$$\begin{aligned} d(P_1, P_2) &= \sqrt{(2-(-8))^2 + (3-(-4))^2} \\ &= \sqrt{(10)^2 + (7)^2} \\ &= \sqrt{100+49} \\ &= \sqrt{149} \end{aligned}$$

The coordinates of the midpoint are:

$$\begin{aligned} (x,y) &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{-8+2}{2}, \frac{-4+3}{2}\right) \\ &= \left(\frac{-6}{2}, \frac{-1}{2}\right) \\ &= \left(-3, -\frac{1}{2}\right) \end{aligned}$$

41. $P_1 = (-4,6); P_2 = (4,-8)$

$$\begin{aligned} d(P_1, P_2) &= \sqrt{(4-(-4))^2 + (-8-6)^2} \\ &= \sqrt{(8)^2 + (-14)^2} \\ &= \sqrt{64+196} \\ &= \sqrt{260} = 2\sqrt{65} \end{aligned}$$

The coordinates of the midpoint are:

$$\begin{aligned} (x,y) &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{-4+4}{2}, \frac{6+(-8)}{2}\right) \\ &= \left(\frac{0}{2}, \frac{-2}{2}\right) \\ &= (0,-1) \end{aligned}$$

42. $P_1 = (0,6); P_2 = (3,-8)$

$$\begin{aligned} d(P_1, P_2) &= \sqrt{(3-0)^2 + (-8-6)^2} \\ &= \sqrt{(3)^2 + (-14)^2} \\ &= \sqrt{9+196} \\ &= \sqrt{205} \end{aligned}$$

The coordinates of the midpoint are:

$$\begin{aligned} (x,y) &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{0+3}{2}, \frac{6+(-8)}{2}\right) \\ &= \left(\frac{3}{2}, \frac{-2}{2}\right) \\ &= \left(\frac{3}{2}, -1\right) \end{aligned}$$

43. **a.** If we use a right triangle to solve the problem, we know the hypotenuse is 13 units in length. One of the legs of the triangle will be 2+3=5. Thus the other leg will be:

$$\begin{aligned} 5^2 + b^2 &= 13^2 \\ 25 + b^2 &= 169 \\ b^2 &= 144 \\ b &= 12 \end{aligned}$$

Thus the coordinates will have an y value of $-1-12=-13$ and $-1+12=11$. So the points are $(3,11)$ and $(3,-13)$.

b. Consider points of the form $(3,y)$ that are a distance of 13 units from the point $(-2,-1)$.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ &= \sqrt{(3-(-2))^2 + (-1-y)^2} \\ &= \sqrt{(5)^2 + (-1-y)^2} \\ &= \sqrt{25+1+2y+y^2} \\ &= \sqrt{y^2+2y+26} \end{aligned}$$

$$\begin{aligned} 13 &= \sqrt{y^2+2y+26} \\ 13^2 &= \left(\sqrt{y^2+2y+26}\right)^2 \\ 169 &= y^2+2y+26 \\ 0 &= y^2+2y-143 \\ 0 &= (y-11)(y+13) \end{aligned}$$

$$y-11=0 \quad \text{or} \quad y+13=0$$
$$y=11 \qquad\qquad y=-13$$

Thus, the points $(3,11)$ and $(3,-13)$ are a distance of 13 units from the point $(-2,-1)$.

44. **a.** If we use a right triangle to solve the problem, we know the hypotenuse is 17 units in length. One of the legs of the triangle will be 2+6=8. Thus the other leg will be:

$$8^2 + b^2 = 17^2$$
$$64 + b^2 = 289$$
$$b^2 = 225$$
$$b = 15$$

Thus the coordinates will have an x value of $1-15=-14$ and $1+15=16$. So the points are $(-14,-6)$ and $(16,-6)$.

b. Consider points of the form $(x,-6)$ that are a distance of 17 units from the point $(1,2)$.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(1-x)^2+(2-(-6))^2} \\ &= \sqrt{x^2-2x+1+(8)^2} \\ &= \sqrt{x^2-2x+1+64} \\ &= \sqrt{x^2-2x+65} \end{aligned}$$

$$17 = \sqrt{x^2-2x+65}$$
$$17^2 = \left(\sqrt{x^2-2x+65}\right)^2$$
$$289 = x^2-2x+65$$
$$0 = x^2-2x-224$$
$$0 = (x+14)(x-16)$$
$$x+14=0 \quad \text{or} \quad x-16=0$$
$$x=-14 \qquad\qquad x=16$$

Thus, the points $(-14,-6)$ and $(16,-6)$ are a distance of 13 units from the point $(1,2)$.

45. Points on the x-axis have a y-coordinate of 0. Thus, we consider points of the form $(x,0)$ that are a distance of 6 units from the point $(4,-3)$.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(4-x)^2+(-3-0)^2} \\ &= \sqrt{16-8x+x^2+(-3)^2} \\ &= \sqrt{16-8x+x^2+9} \\ &= \sqrt{x^2-8x+25} \end{aligned}$$

$$6 = \sqrt{x^2-8x+25}$$
$$6^2 = \left(\sqrt{x^2-8x+25}\right)^2$$
$$36 = x^2-8x+25$$
$$0 = x^2-8x-11$$
$$\begin{aligned} x &= \frac{-(-8)\pm\sqrt{(-8)^2-4(1)(-11)}}{2(1)} \\ &= \frac{8\pm\sqrt{64+44}}{2} = \frac{8\pm\sqrt{108}}{2} \\ &= \frac{8\pm 6\sqrt{3}}{2} = 4\pm 3\sqrt{3} \end{aligned}$$
$$x = 4+3\sqrt{3} \quad \text{or} \quad x = 4-3\sqrt{3}$$

Thus, the points $\left(4+3\sqrt{3},0\right)$ and $\left(4-3\sqrt{3},0\right)$ are on the x-axis and a distance of 6 units from the point $(4,-3)$.

46. Points on the y-axis have an x-coordinate of 0. Thus, we consider points of the form $(0,y)$ that are a distance of 6 units from the point $(4,-3)$.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(4-0)^2+(-3-y)^2} \\ &= \sqrt{4^2+9+6y+y^2} \\ &= \sqrt{16+9+6y+y^2} \\ &= \sqrt{y^2+6y+25} \end{aligned}$$

$$6 = \sqrt{y^2+6y+25}$$
$$6^2 = \left(\sqrt{y^2+6y+25}\right)^2$$
$$36 = y^2+6y+25$$
$$0 = y^2+6y-11$$
$$\begin{aligned} y &= \frac{(-6)\pm\sqrt{(6)^2-4(1)(-11)}}{2(1)} \\ &= \frac{-6\pm\sqrt{36+44}}{2} = \frac{-6\pm\sqrt{80}}{2} \\ &= \frac{-6\pm 4\sqrt{5}}{2} = -3\pm 2\sqrt{5} \end{aligned}$$
$$y = -3+2\sqrt{5} \quad \text{or} \quad y = -3-2\sqrt{5}$$

Thus, the points $\left(0,-3+2\sqrt{5}\right)$ and $\left(0,-3-2\sqrt{5}\right)$

are on the y-axis and a distance of 6 units from the point $(4,-3)$.

47. For the x coordinate of B: $\frac{2+x_2}{2}=5$

$$2+x_2=10$$
$$x_2=8$$

For the y coordinate of B: $\frac{1+y_2}{2}=4$

$$1+y_2=8$$
$$y_2=7$$

Thus, the point B is $(8,7)$

48. Let the coordinates of point B be (x,y). Using the midpoint formula, we can write

$(2,3)=\left(\frac{-1+x}{2},\frac{8+y}{2}\right)$.

This leads to two equations we can solve.

$$\frac{-1+x}{2}=2 \qquad \frac{8+y}{2}=3$$
$$-1+x=4 \qquad 8+y=6$$
$$x=5 \qquad y=-2$$

Point B has coordinates $(5,-2)$.

49. $M=(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$.

$P_1=(x_1,y_1)=(-3,6)$ and $(x,y)=(-1,4)$, so

$$x=\frac{x_1+x_2}{2} \quad \text{and} \quad y=\frac{y_1+y_2}{2}$$
$$-1=\frac{-3+x_2}{2} \qquad 4=\frac{6+y_2}{2}$$
$$-2=-3+x_2 \qquad 8=6+y_2$$
$$1=x_2 \qquad 2=y_2$$

Thus, $P_2=(1,2)$.

50. $M=(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$.

$P_2=(x_2,y_2)=(7,-2)$ and $(x,y)=(5,-4)$, so

$$x=\frac{x_1+x_2}{2} \quad \text{and} \quad y=\frac{y_1+y_2}{2}$$
$$5=\frac{x_1+7}{2} \qquad -4=\frac{y_1+(-2)}{2}$$
$$10=x_1+7 \qquad -8=y_1+(-2)$$
$$3=x_1 \qquad -6=y_1$$

Thus, $P_1=(3,-6)$.

51. The midpoint of AB is: $D=\left(\frac{0+6}{2},\frac{0+0}{2}\right)$

$$=(3,0)$$

The midpoint of AC is: $E=\left(\frac{0+4}{2},\frac{0+4}{2}\right)$

$$=(2,2)$$

The midpoint of BC is: $F=\left(\frac{6+4}{2},\frac{0+4}{2}\right)$

$$=(5,2)$$

$$d(C,D)=\sqrt{(0-4)^2+(3-4)^2}$$
$$=\sqrt{(-4)^2+(-1)^2}=\sqrt{16+1}=\sqrt{17}$$
$$d(B,E)=\sqrt{(2-6)^2+(2-0)^2}$$
$$=\sqrt{(-4)^2+2^2}=\sqrt{16+4}$$
$$=\sqrt{20}=2\sqrt{5}$$
$$d(A,F)=\sqrt{(2-0)^2+(5-0)^2}$$
$$=\sqrt{2^2+5^2}=\sqrt{4+25}$$
$$=\sqrt{29}$$

52. Let $P_1=(0,0),\ P_2=(0,4),\ P=(x,y)$

$$d(P_1,P_2)=\sqrt{(0-0)^2+(4-0)^2}$$
$$=\sqrt{16}=4$$
$$d(P_1,P)=\sqrt{(x-0)^2+(y-0)^2}$$
$$=\sqrt{x^2+y^2}=4$$
$$\rightarrow\ x^2+y^2=16$$
$$d(P_2,P)=\sqrt{(x-0)^2+(y-4)^2}$$
$$=\sqrt{x^2+(y-4)^2}=4$$
$$\rightarrow\ x^2+(y-4)^2=16$$

Therefore,

$$y^2 = (y-4)^2$$
$$y^2 = y^2 - 8y + 16$$
$$8y = 16$$
$$y = 2$$

which gives

$$x^2 + 2^2 = 16$$
$$x^2 = 12$$
$$x = \pm 2\sqrt{3}$$

Two triangles are possible. The third vertex is $\left(-2\sqrt{3}, 2\right)$ or $\left(2\sqrt{3}, 2\right)$.

53. $d(P_1,P_2) = \sqrt{(-4-2)^2 + (1-1)^2}$
$= \sqrt{(-6)^2 + 0^2}$
$= \sqrt{36}$
$= 6$

$d(P_2,P_3) = \sqrt{(-4-(-4))^2 + (-3-1)^2}$
$= \sqrt{0^2 + (-4)^2}$
$= \sqrt{16}$
$= 4$

$d(P_1,P_3) = \sqrt{(-4-2)^2 + (-3-1)^2}$
$= \sqrt{(-6)^2 + (-4)^2}$
$= \sqrt{36+16}$
$= \sqrt{52}$
$= 2\sqrt{13}$

Since $[d(P_1,P_2)]^2 + [d(P_2,P_3)]^2 = [d(P_1,P_3)]^2$, the triangle is a right triangle.

54. $d(P_1,P_2) = \sqrt{(6-(-1))^2 + (2-4)^2}$
$= \sqrt{7^2 + (-2)^2}$
$= \sqrt{49+4}$
$= \sqrt{53}$

$d(P_2,P_3) = \sqrt{(4-6)^2 + (-5-2)^2}$
$= \sqrt{(-2)^2 + (-7)^2}$
$= \sqrt{4+49}$
$= \sqrt{53}$

$d(P_1,P_3) = \sqrt{(4-(-1))^2 + (-5-4)^2}$
$= \sqrt{5^2 + (-9)^2}$
$= \sqrt{25+81}$
$= \sqrt{106}$

Since $[d(P_1,P_2)]^2 + [d(P_2,P_3)]^2 = [d(P_1,P_3)]^2$, the triangle is a right triangle.
Since $d(P_1,P_2) = d(P_2,P_3)$, the triangle is isosceles.
Therefore, the triangle is an isosceles right triangle.

55. $d(P_1,P_2) = \sqrt{(0-(-2))^2 + (7-(-1))^2}$
$= \sqrt{2^2 + 8^2} = \sqrt{4+64} = \sqrt{68}$
$= 2\sqrt{17}$

$d(P_2,P_3) = \sqrt{(3-0)^2 + (2-7)^2}$
$= \sqrt{3^2 + (-5)^2} = \sqrt{9+25}$
$= \sqrt{34}$

$d(P_1,P_3) = \sqrt{(3-(-2))^2 + (2-(-1))^2}$
$= \sqrt{5^2 + 3^2} = \sqrt{25+9}$
$= \sqrt{34}$

Since $d(P_2,P_3) = d(P_1,P_3)$, the triangle is isosceles.

Since $[d(P_1,P_3)]^2 + [d(P_2,P_3)]^2 = [d(P_1,P_2)]^2$, the triangle is also a right triangle.
Therefore, the triangle is an isosceles right triangle.

56. $d(P_1,P_2) = \sqrt{(-4-7)^2 + (0-2)^2}$
$= \sqrt{(-11)^2 + (-2)^2}$
$= \sqrt{121+4} = \sqrt{125}$
$= 5\sqrt{5}$

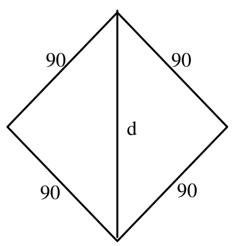
$d(P_2,P_3) = \sqrt{(4-(-4))^2 + (6-0)^2}$
$= \sqrt{8^2 + 6^2} = \sqrt{64+36}$
$= \sqrt{100}$
$= 10$

$$d(P_1,P_3)=\sqrt{(4-7)^2+(6-2)^2}$$
$$=\sqrt{(-3)^2+4^2}=\sqrt{9+16}$$
$$=\sqrt{25}$$
$$=5$$

Since $[d(P_1,P_3)]^2+[d(P_2,P_3)]^2=[d(P_1,P_2)]^2$, the triangle is a right triangle.

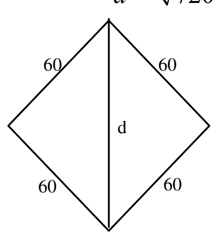
57. Using the Pythagorean Theorem:

$$90^2+90^2=d^2$$
$$8100+8100=d^2$$
$$16200=d^2$$
$$d=\sqrt{16200}=90\sqrt{2}\approx 127.28 \text{ feet}$$

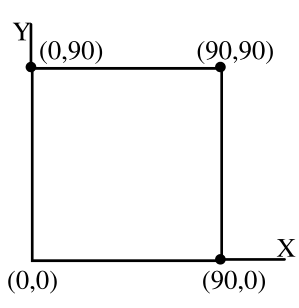


58. Using the Pythagorean Theorem:

$$60^2+60^2=d^2$$
$$3600+3600=d^2 \rightarrow 7200=d^2$$
$$d=\sqrt{7200}=60\sqrt{2}\approx 84.85 \text{ feet}$$



59. a. First: (90, 0), Second: (90, 90), Third: (0, 90)



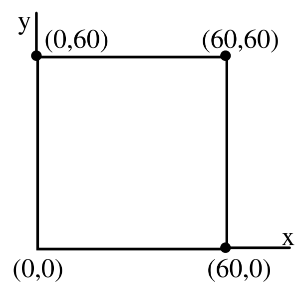
b. Using the distance formula:

$$d=\sqrt{(310-90)^2+(15-90)^2}$$
$$=\sqrt{220^2+(-75)^2}=\sqrt{54025}$$
$$=5\sqrt{2161}\approx 232.43 \text{ feet}$$

c. Using the distance formula:

$$d=\sqrt{(300-0)^2+(300-90)^2}$$
$$=\sqrt{300^2+210^2}=\sqrt{134100}$$
$$=30\sqrt{149}\approx 366.20 \text{ feet}$$

60. a. First: (60, 0), Second: (60, 60) Third: (0, 60)



b. Using the distance formula:

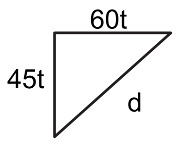
$$d=\sqrt{(180-60)^2+(20-60)^2}$$
$$=\sqrt{120^2+(-40)^2}=\sqrt{16000}$$
$$=40\sqrt{10}\approx 126.49 \text{ feet}$$

c. Using the distance formula:

$$d=\sqrt{(220-0)^2+(220-60)^2}$$
$$=\sqrt{220^2+160^2}=\sqrt{74000}$$
$$=20\sqrt{185}\approx 272.03 \text{ feet}$$

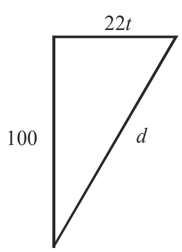
61. The Focus heading east moves a distance $60t$ after t hours. The truck heading south moves a distance $40t$ after t hours. Their distance apart after t hours is:

$$\begin{aligned} d &= \sqrt{(60t)^2 + (45t)^2} \\ &= \sqrt{3600t^2 + 2025t^2} \\ &= \sqrt{5625t^2} \\ &= 75t \text{ miles} \end{aligned}$$



62. $\dfrac{15 \text{ miles}}{1 \text{ hr}} \cdot \dfrac{5280 \text{ ft}}{1 \text{ mile}} \cdot \dfrac{1 \text{ hr}}{3600 \text{ sec}} = 22 \text{ ft/sec}$

$$\begin{aligned} d &= \sqrt{100^2 + (22t)^2} \\ &= \sqrt{10000 + 484t^2} \text{ feet} \end{aligned}$$



63. **a.** The shortest side is between $P_1 = (2.6, 1.5)$ and $P_2 = (2.7, 1.7)$. The estimate for the desired intersection point is:

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{2.6 + 2.7}{2}, \frac{1.5 + 1.7}{2}\right) \\ &= \left(\frac{5.3}{2}, \frac{3.2}{2}\right) \\ &= (2.65, 1.6) \end{aligned}$$

b. Using the distance formula:

$$\begin{aligned} d &= \sqrt{(2.65 - 1.4)^2 + (1.6 - 1.3)^2} \\ &= \sqrt{(1.25)^2 + (0.3)^2} \\ &= \sqrt{1.5625 + 0.09} \\ &= \sqrt{1.6525} \\ &\approx 1.285 \text{ units} \end{aligned}$$

64. Let $P_1 = (2014, 110.21)$ and $P_2 = (2018, 138.43)$. The midpoint is:

$$\begin{aligned} (x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{2014 + 2018}{2}, \frac{110.21 + 138.43}{2}\right) \\ &= \left(\frac{4032}{2}, \frac{248.64}{2}\right) \\ &= (2016, 124.32) \end{aligned}$$

The estimate for 2016 is \$124.32 billion. The estimate net sales of Costco Wholesale Corporation in 2016 is higher than the reported value of \$116.07 billion.

65. For 2010 we have the ordered pair $(2010, 22113)$ and for 2018 we have the ordered pair $(2018, 25465)$. The midpoint is

$$\begin{aligned} (\text{year}, \$) &= \left(\frac{2010 + 2018}{2}, \frac{22113 + 25465}{2}\right) \\ &= \left(\frac{4028}{2}, \frac{47578}{2}\right) \\ &= (2014, 23789) \end{aligned}$$

Using the midpoint, we estimate the poverty level in 2014 to be \$23,789. This is lower than the actual value.

66. Let $P_1 = (0, 0)$, $P_2 = (a, 0)$, and $P_3 = \left(\dfrac{a}{2}, \dfrac{\sqrt{3}\, a}{2}\right)$. Then

$$\begin{aligned} d(P_1, P_2) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(a - 0)^2 + (0 - 0)^2} = \sqrt{a^2} = |a| \\ d(P_2, P_3) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(\frac{a}{2} - a\right)^2 + \left(\frac{\sqrt{3}\, a}{2} - 0\right)^2} \\ &= \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{4a^2}{4}} = \sqrt{a^2} = |a| \\ d(P_1, P_3) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{\sqrt{3}\, a}{2} - 0\right)^2} \\ &= \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{4a^2}{4}} = \sqrt{a^2} = |a| \end{aligned}$$

Since the lengths of the three sides are all equal,

the triangle is an equilateral triangle.
The midpoints of the saids are

$$M_{P_1P_2} = \left(\frac{0+a}{2}, \frac{0+0}{2}\right) = \left(\frac{a}{2}, 0\right)$$

$$M_{P_2P_3} = \left(\frac{a+\frac{a}{2}}{2}, \frac{0+\frac{\sqrt{3}\,a}{2}}{2}\right) = \left(\frac{3a}{4}, \frac{\sqrt{3}\,a}{4}\right)$$

$$M_{P_1P_3} = \left(\frac{0+\frac{a}{2}}{2}, \frac{0+\frac{\sqrt{3}a}{2}}{2}\right) = \left(\frac{a}{4}, \frac{\sqrt{3}\,a}{4}\right)$$

Then,

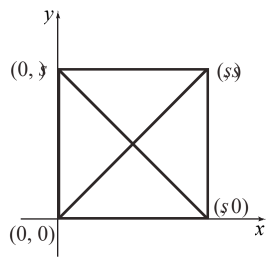
$$d\left(M_{P_1P_2}, M_{P_2P_3}\right) = \sqrt{\left(\frac{3a}{4}-\frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}\,a}{4}-0\right)^2}$$
$$= \sqrt{\left(\frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$= \sqrt{\frac{a^2}{16}+\frac{3a^2}{16}} = \frac{|a|}{2}$$

$$d\left(M_{P_2P_3}, M_{P_1P_3}\right) = \sqrt{\left(\frac{3a}{4}-\frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}\,a}{4}-\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$= \sqrt{\left(\frac{a}{2}\right)^2 + 0^2}$$
$$= \sqrt{\frac{a^2}{4}} = \frac{|a|}{2}$$

$$d\left(M_{P_1P_2}, M_{P_1P_3}\right) = \sqrt{\left(\frac{a}{4}-\frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}\,a}{4}-0\right)^2}$$
$$= \sqrt{\left(-\frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$= \sqrt{\frac{a^2}{16}+\frac{3a^2}{16}} = \frac{|a|}{2}$$

Since the lengths of the sides of the triangle formed by the midpoints are all equal, the triangle is equilateral.

67. Let $P_1 = (0,0)$, $P_2 = (0,s)$, $P_3 = (s,0)$, and $P_4 = (s,s)$ be the vertices of the square.



The points P_1 and P_4 are endpoints of one diagonal and the points P_2 and P_3 are the endpoints of the other diagonal.

$$M_{P_1P_4} = \left(\frac{0+s}{2}, \frac{0+s}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$$

$$M_{P_2P_3} = \left(\frac{0+s}{2}, \frac{s+0}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$$

The midpoints of the diagonals are the same. Therefore, the diagonals of a square intersect at their midpoints.

68. Let $P = (a, 2a)$. Then

$$\sqrt{(a+5)^2 + (2a-1)^2} = \sqrt{(a-4)^2 + (2a+4)^2}$$
$$(a+5)^2 + (2a-1)^2 = (a-4)^2 + (2a+4)^2$$
$$5a^2 + 6a + 26 = 5a^2 + 8a + 32$$
$$6a + 26 = 8a + 32$$
$$-2a = 6$$
$$a = -3$$

Then $P = (-3, -6)$.

69. Arrange the parallelogram on the coordinate plane so that the vertices are
$P_1 = (0,0), P_2 = (a,0), P_3 = (a+b, c)$ and $P_4 = (b,c)$
Then the lengths of the sides are:

$$d(P_1, P_2) = \sqrt{(a-0)^2 + (0-0)^2}$$
$$= \sqrt{a^2} = |a|$$

$$d(P_2, P_3) = \sqrt{[(a+b)-a]^2 + (c-0)^2}$$
$$= \sqrt{b^2 + c^2}$$

$$d(P_3, P_4) = \sqrt{[b-(a+b)]^2 + (c-c)^2}$$
$$= \sqrt{a^2} = |a|$$

and

$$d(P_1,P_4)=\sqrt{(b-0)^2+(c-0)^2}$$
$$=\sqrt{b^2+c^2}$$

P_1 and P_3 are the endpoints of one diagonal, and P_2 and P_4 are the endpoints of the other diagonal. The lengths of the diagonals are

$$d(P_1,P_3)=\sqrt{[(a+b)-0]^2+(c-0)^2}$$
$$=\sqrt{a^2+2ab+b^2+c^2}$$

and

$$d(P_2,P_4)=\sqrt{(b-a)^2+(c-0)^2}\ .$$
$$=\sqrt{a^2-2ab+b^2+c^2}$$

Sum of the squares of the sides:

$$a^2+(\sqrt{b^2+c^2})^2+a^2+(\sqrt{b^2+c^2})^2$$
$$=2a^2+2b^2+2c^2$$

Sum of the squares of the diagonals:

$$\left(\sqrt{a^2+2ab+b^2+c^2}\right)^2+\left(\sqrt{a^2-2ab+b^2+c^2}\right)^2$$
$$=2a^2+2b^2+2c^2$$

Section 1.3

1. $2(x+3)-1=-7$
   $2(x+3)=-6$
   $x+3=-3$
   $x=-6$
   The solution set is $\{-6\}$.

2. $x^2-9=0$
   $x^2=9$
   $x=\pm\sqrt{9}=\pm3$
   The solution set is $\{-3,3\}$.

3. intercepts

4. $y=0$

5. y-axis

6. 4

7. $(-3,4)$

8. True

9. False; the y-coordinate of a point at which the graph crosses or touches the x-axis is always 0. The x-coordinate of such a point is an x-intercept.

10. False; a graph can be symmetric with respect to both coordinate axes (in such cases it will also be symmetric with respect to the origin).
    For example: $x^2+y^2=1$

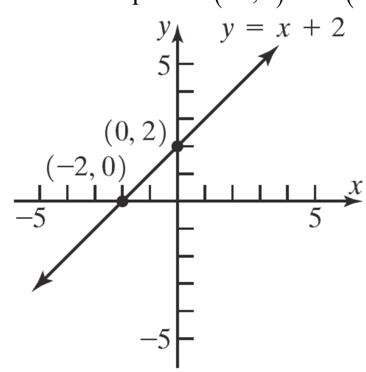
11. d

12. c

13. $y=x+2$

x-intercept:	y-intercept:
$0=x+2$	$y=0+2$
$-2=x$	$y=2$

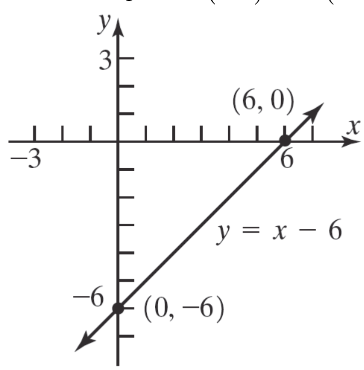
The intercepts are $(-2,0)$ and $(0,2)$.



14. $y=x-6$

x-intercept:	y-intercept:
$0=x-6$	$y=0-6$
$6=x$	$y=-6$

The intercepts are $(6,0)$ and $(0,-6)$.

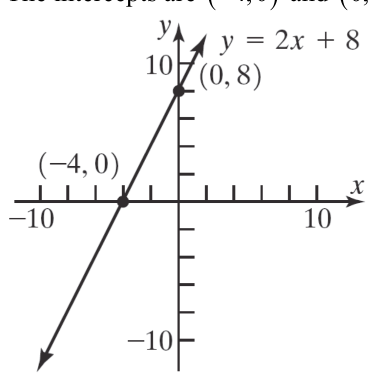


15. $y = 2x + 8$

x-intercept:
$0 = 2x + 8$
$2x = -8$
$x = -4$

y-intercept:
$y = 2(0) + 8$
$y = 8$

The intercepts are $(-4,0)$ and $(0,8)$.

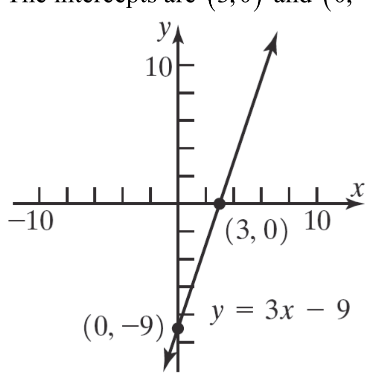


16. $y = 3x - 9$

x-intercept:
$0 = 3x - 9$
$3x = 9$
$x = 3$

y-intercept:
$y = 3(0) - 9$
$y = -9$

The intercepts are $(3,0)$ and $(0,-9)$.

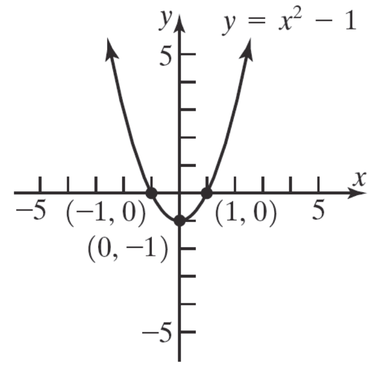


17. $y = x^2 - 1$

x-intercepts:
$0 = x^2 - 1$
$x^2 = 1$
$x = \pm 1$

y-intercept:
$y = 0^2 - 1$
$y = -1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,-1)$.

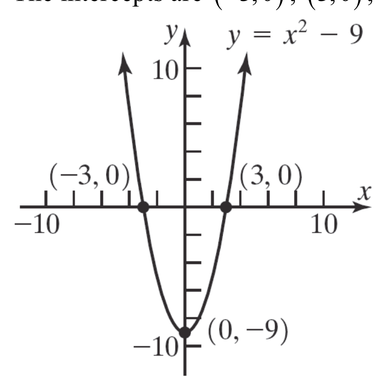


18. $y = x^2 - 9$

x-intercepts:
$0 = x^2 - 9$
$x^2 = 9$
$x = \pm 3$

y-intercept:
$y = 0^2 - 9$
$y = -9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,-9)$.

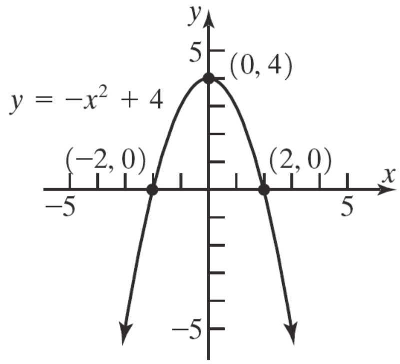


19. $y = -x^2 + 4$

x-intercepts:
$0 = -x^2 + 4$
$x^2 = 4$
$x = \pm 2$

y-intercepts:
$y = -(0)^2 + 4$
$y = 4$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,4)$.

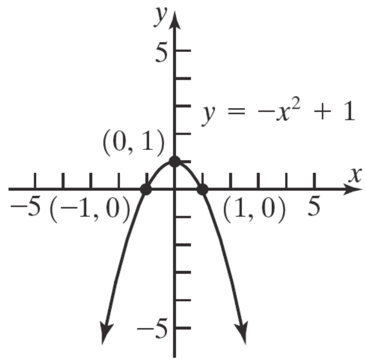


20. $y=-x^2+1$

x-intercepts:
$$0=-x^2+1$$
$$x^2=1$$
$$x=\pm 1$$

y-intercept:
$$y=-(0)^2+1$$
$$y=1$$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,1)$.

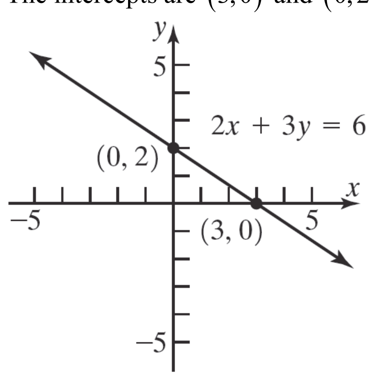


21. $2x+3y=6$

x-intercepts:
$$2x+3(0)=6$$
$$2x=6$$
$$x=3$$

y-intercept:
$$2(0)+3y=6$$
$$3y=6$$
$$y=2$$

The intercepts are $(3,0)$ and $(0,2)$.

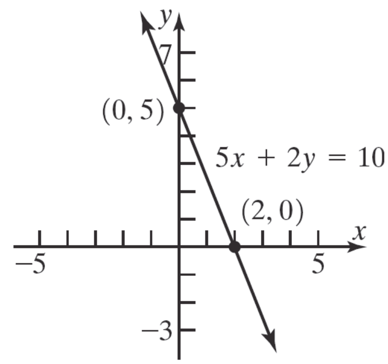


22. $5x+2y=10$

x-intercepts:
$$5x+2(0)=10$$
$$5x=10$$
$$x=2$$

y-intercept:
$$5(0)+2y=10$$
$$2y=10$$
$$y=5$$

The intercepts are $(2,0)$ and $(0,5)$.

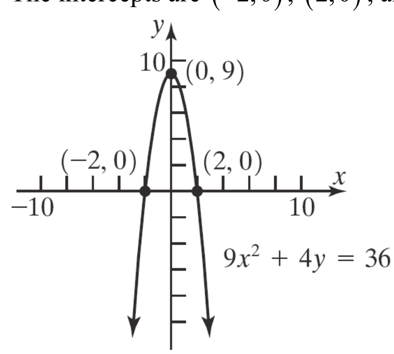


23. $9x^2+4y=36$

x-intercepts:
$$9x^2+4(0)=36$$
$$9x^2=36$$
$$x^2=4$$
$$x=\pm 2$$

y-intercept:
$$9(0)^2+4y=36$$
$$4y=36$$
$$y=9$$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,9)$.

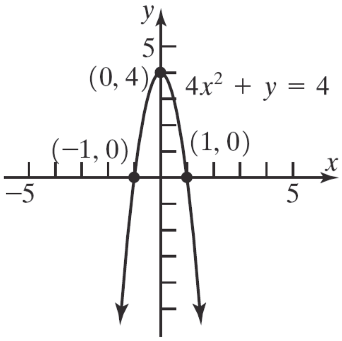


24. $4x^2+y=4$

x-intercepts:
$$4x^2+0=4$$
$$4x^2=4$$
$$x^2=1$$
$$x=\pm 1$$

y-intercept:
$$4(0)^2+y=4$$
$$y=4$$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,4)$.



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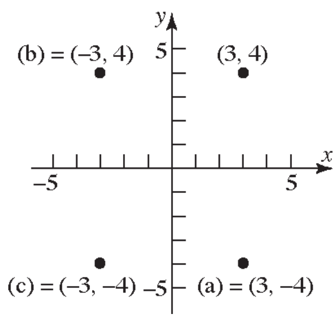

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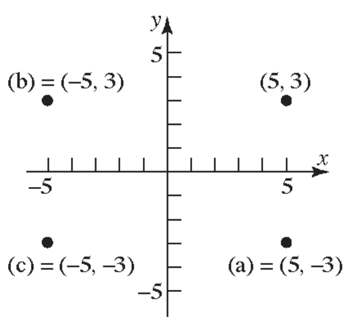
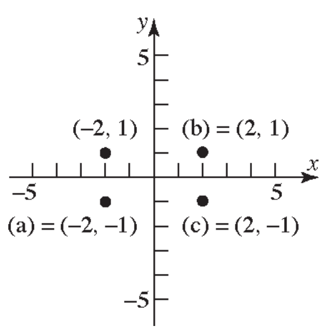
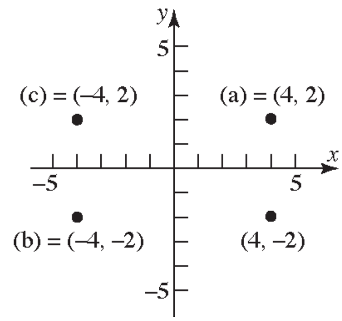


27.



28.



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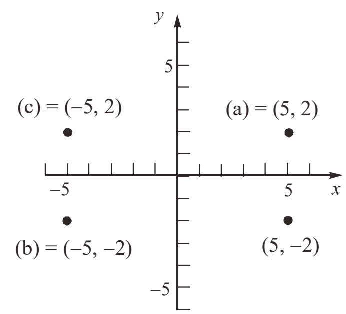

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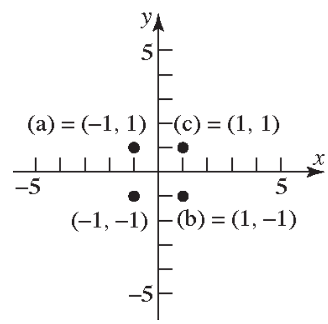

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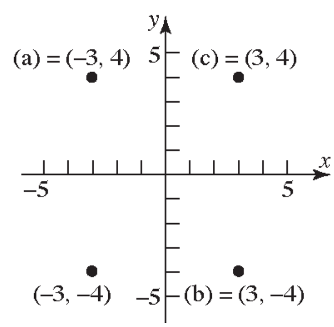

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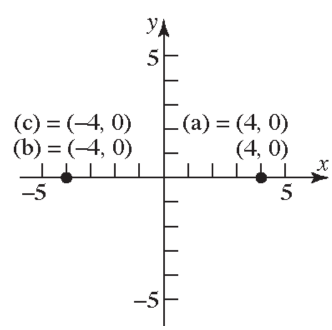

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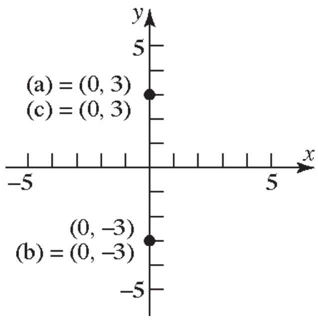

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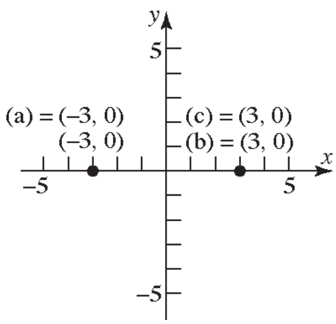


35. **a.** Intercepts: $(-1,0)$ and $(1,0)$

b. Symmetric with respect to the x-axis, y-axis, and the origin.

36. **a.** Intercepts: $(0,1)$

b. Not symmetric to the x-axis, the y-axis, nor the origin

37. **a.** Intercepts: $\left(-\frac{\pi}{2},0\right)$, $(0,1)$, and $\left(\frac{\pi}{2},0\right)$

b. Symmetric with respect to the y-axis.

38. **a.** Intercepts: $(-2,0)$, $(0,-3)$, and $(2,0)$

b. Symmetric with respect to the y-axis.

39. **a.** Intercepts: $(0,0)$

b. Symmetric with respect to the x-axis.

40. **a.** Intercepts: $(-2,0)$, $(0,2)$, $(0,-2)$, and $(2,0)$

b. Symmetric with respect to the x-axis, y-axis, and the origin.

41. **a.** Intercepts: $(-2,0)$, $(0,0)$, and $(2,0)$

b. Symmetric with respect to the origin.

42. **a.** Intercepts: $(-4,0)$, $(0,0)$, and $(4,0)$

b. Symmetric with respect to the origin.

43. **a.** x-intercepts: $[-2,1]$, y-intercept 0

b. Not symmetric to x-axis, y-axis, or origin.

44. **a.** x-intercepts: $[-1,2]$, y-intercept 0

b. Not symmetric to x-axis, y-axis, or origin.

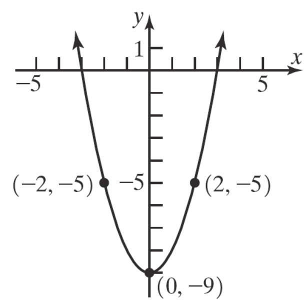
45. **a.** Intercepts: none

b. Symmetric with respect to the origin.

46. **a.** Intercepts: none

b. Symmetric with respect to the x-axis.

47.



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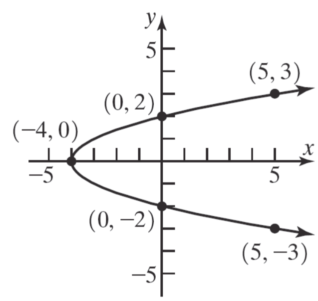

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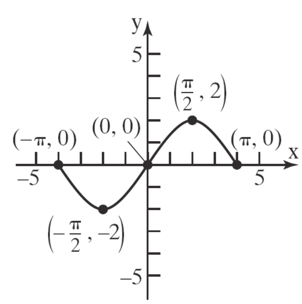

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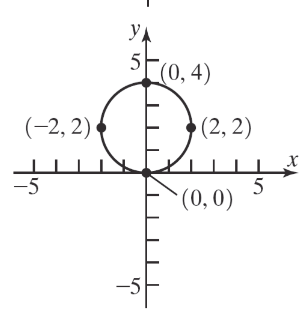


51. $y^2 = x + 16$

x-intercepts:

$0^2 = x + 16$

$-16 = x$

y-intercepts:

$y^2 = 0 + 16$

$y^2 = 16$

$y = \pm 4$

The intercepts are $(-16, 0)$, $(0, -4)$ and $(0, 4)$.

Test x-axis symmetry: Let $y = -y$

$(-y)^2 = x + 16$

$y^2 = x + 16$ same

Test y-axis symmetry: Let $x = -x$

$y^2 = -x + 16$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-y)^2 = -x + 16$

$y^2 = -x + 16$ different

Therefore, the graph will have x-axis symmetry.

52. $y^2 = x + 9$

x-intercepts:

$(0)^2 = -x + 9$

$0 = -x + 9$

$x = 9$

y-intercepts:

$y^2 = 0 + 9$

$y^2 = 9$

$y = \pm 3$

The intercepts are $(-9, 0)$, $(0, -3)$ and $(0, 3)$.

Test x-axis symmetry: Let $y = -y$

$(-y)^2 = x + 9$

$y^2 = x + 9$ same

Test y-axis symmetry: Let $x = -x$

$y^2 = -x + 9$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-y)^2 = -x + 9$

$y^2 = -x + 9$ different

Therefore, the graph will have x-axis symmetry.

53. $y = \sqrt[3]{x}$

x-intercepts:

$0 = \sqrt[3]{x}$

$0 = x$

y-intercepts:

$y = \sqrt[3]{0} = 0$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \sqrt[3]{x}$ different

Test y-axis symmetry: Let $x = -x$

$y = \sqrt[3]{-x} = -\sqrt[3]{x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \sqrt[3]{-x} = -\sqrt[3]{x}$

$y = \sqrt[3]{x}$ same

Therefore, the graph will have origin symmetry.

54. $y = \sqrt[5]{x}$

x-intercepts:

$0 = \sqrt[3]{x}$

$0 = x$

y-intercepts:

$y = \sqrt[5]{0} = 0$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \sqrt[5]{x}$ different

Test y-axis symmetry: Let $x = -x$

$y = \sqrt[5]{-x} = -\sqrt[5]{x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \sqrt[5]{-x} = -\sqrt[5]{x}$

$y = \sqrt[5]{x}$ same

Therefore, the is symmetric with respect to the origin.

55. $x^2 + y - 9 = 0$

x-intercepts:

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

y-intercepts:

$0^2 + y - 9 = 0$

$y = 9$

The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, 9)$.

Test x-axis symmetry: Let $y = -y$

$x^2 - y - 9 = 0$ different

Test y-axis symmetry: Let $x = -x$

$(-x)^2 + y - 9 = 0$

$x^2 + y - 9 = 0$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$(-x)^2 - y - 9 = 0$

$x^2 - y - 9 = 0$ different

Therefore, the graph has y-axis symmetry.

56. $x^2 - y - 4 = 0$

x-intercepts:
$$x^2 - 0 - 4 = 0$$
$$x^2 = 4$$
$$x = \pm 2$$

y-intercept:
$$0^2 - y - 4 = 0$$
$$-y = 4$$
$$y = -4$$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,-4)$.

Test x-axis symmetry: Let $y = -y$
$$x^2 - (-y) - 4 = 0$$
$$x^2 + y - 4 = 0 \text{ different}$$

Test y-axis symmetry: Let $x = -x$
$$(-x)^2 - y - 4 = 0$$
$$x^2 - y - 4 = 0 \text{ same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$
$$(-x)^2 - (-y) - 4 = 0$$
$$x^2 + y - 4 = 0 \text{ different}$$

Therefore, the graph has y-axis symmetry.

57. $25x^2 + 4y^2 = 100$

x-intercepts:
$$25x^2 + 4(0)^2 = 100$$
$$25x^2 = 100$$
$$x^2 = 4$$
$$x = \pm 2$$

y-intercepts:
$$25(0)^2 + 4y^2 = 100$$
$$4y^2 = 25$$
$$y^2 = 5$$
$$y = \pm 5$$

The intercepts are $(-2,0)$, $(2,0)$, $(0,-5)$, and $(0,5)$.

Test x-axis symmetry: Let $y = -y$
$$25x^2 + 4(-y)^2 = 100$$
$$25x^2 + 4y^2 = 100 \text{ same}$$

Test y-axis symmetry: Let $x = -x$
$$25(-x)^2 + 4y^2 = 100$$
$$25x^2 + 4y^2 = 100 \text{ same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$
$$25(-x)^2 + 4(-y)^2 = 100$$
$$25x^2 + 4y^2 = 100 \text{ same}$$

Therefore, the graph has x-axis, y-axis, and origin symmetry.

58. $4x^2 + y^2 = 4$

x-intercepts:
$$4x^2 + 0^2 = 4$$
$$4x^2 = 4$$
$$x^2 = 1$$
$$x = \pm 1$$

y-intercepts:
$$4(0)^2 + y^2 = 4$$
$$y^2 = 4$$
$$y = \pm 2$$

The intercepts are $(-1,0)$, $(1,0)$, $(0,-2)$, and $(0,2)$.

Test x-axis symmetry: Let $y = -y$
$$4x^2 + (-y)^2 = 4$$
$$4x^2 + y^2 = 4 \text{ same}$$

Test y-axis symmetry: Let $x = -x$
$$4(-x)^2 + y^2 = 4$$
$$4x^2 + y^2 = 4 \text{ same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$
$$4(-x)^2 + (-y)^2 = 4$$
$$4x^2 + y^2 = 4 \text{ same}$$

Therefore, the graph has x-axis, y-axis, and origin symmetry.

59. $y = x^3 - 64$

x-intercepts:
$$0 = x^3 - 64$$
$$x^3 = 64$$
$$x = 4$$

y-intercepts:
$$y = 0^3 - 64$$
$$y = -64$$

The intercepts are $(4,0)$ and $(0,-64)$.

Test x-axis symmetry: Let $y = -y$
$$-y = x^3 - 64 \text{ different}$$

Test y-axis symmetry: Let $x = -x$
$$y = (-x)^3 - 64$$
$$y = -x^3 - 64 \text{ different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$
$$-y = (-x)^3 - 64$$
$$y = x^3 + 64 \text{ different}$$

Therefore, the graph has no symmetry.

60. $y = x^4 - 1$

x-intercepts:
$0 = x^4 - 1$
$x^4 = 1$
$x = \pm 1$

y-intercepts:
$y = 0^4 - 1$
$y = -1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,-1)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^4 - 1$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^4 - 1$

$y = x^4 - 1$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^4 - 1$

$-y = x^4 - 1$ different

Therefore, the graph has y-axis symmetry.

61. $y = x^2 - 2x - 8$

x-intercepts:
$0 = x^2 - 2x - 8$
$0 = (x-4)(x+2)$
$x = 4$ or $x = -2$

y-intercepts:
$y = 0^2 - 2(0) - 8$
$y = -8$

The intercepts are $(4,0)$, $(-2,0)$, and $(0,-8)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^2 - 2x - 8$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^2 - 2(-x) - 8$

$y = x^2 + 2x - 8$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^2 - 2(-x) - 8$

$-y = x^2 + 2x - 8$ different

Therefore, the graph has no symmetry.

62. $y = x^2 + 4$

x-intercepts:
$0 = x^2 + 4$
$x^2 = -4$
no real solution

y-intercepts:
$y = 0^2 + 4$
$y = 4$

The only intercept is $(0,4)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^2 + 4$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^2 + 4$

$y = x^2 + 4$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^2 + 4$

$-y = x^2 + 4$ different

Therefore, the graph has y-axis symmetry.

63. $y = \dfrac{4x}{x^2 + 16}$

x-intercepts:
$0 = \dfrac{4x}{x^2 + 16}$
$4x = 0$
$x = 0$

y-intercepts:
$y = \dfrac{4(0)}{0^2 + 16} = \dfrac{0}{16} = 0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \dfrac{4x}{x^2 + 16}$ different

Test y-axis symmetry: Let $x = -x$

$y = \dfrac{4(-x)}{(-x)^2 + 16}$

$y = -\dfrac{4x}{x^2 + 16}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \dfrac{4(-x)}{(-x)^2 + 16}$

$-y = -\dfrac{4x}{x^2 + 16}$

$y = \dfrac{4x}{x^2 + 16}$ same

Therefore, the graph has origin symmetry.

64. $y=\dfrac{x^2-4}{2x}$

x-intercepts:

$0=\dfrac{x^2-4}{2x}$

$x^2-4=0$

$x^2=4$

$x=\pm 2$

y-intercepts:

$y=\dfrac{0^2-4}{2(0)}=\dfrac{-4}{0}$

undefined

The intercepts are $(-2,0)$ and $(2,0)$.

Test x-axis symmetry: Let $y=-y$

$-y=\dfrac{x^2-4}{2x}$ different

Test y-axis symmetry: Let $x=-x$

$y=\dfrac{(-x)^2-4}{2(-x)}$

$y=-\dfrac{x^2-4}{2x}$ different

Test origin symmetry: Let $x=-x$ and $y=-y$

$-y=\dfrac{(-x)^2-4}{2(-x)}$

$-y=\dfrac{x^2-4}{-2x}$

$y=\dfrac{x^2-4}{2x}$ same

Therefore, the graph has origin symmetry.

65. $y=\dfrac{-x^3}{x^2-9}$

x-intercepts:

$0=\dfrac{-x^3}{x^2-9}$

$-x^3=0$

$x=0$

y-intercepts:

$y=\dfrac{-0^3}{0^2-9}=\dfrac{0}{-9}=0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y=-y$

$-y=\dfrac{-x^3}{x^2-9}$

$y=\dfrac{x^3}{x^2-9}$ different

Test y-axis symmetry: Let $x=-x$

$y=\dfrac{-(-x)^3}{(-x)^2-9}$

$y=\dfrac{x^3}{x^2-9}$ different

Test origin symmetry: Let $x=-x$ and $y=-y$

$-y=\dfrac{-(-x)^3}{(-x)^2-9}$

$-y=\dfrac{x^3}{x^2-9}$

$y=\dfrac{-x^3}{x^2-9}$ same

Therefore, the graph has origin symmetry.

66. $y=\dfrac{x^4+1}{2x^5}$

x-intercepts:

$0=\dfrac{x^4+1}{2x^5}$

$x^4=-1$

no real solution

y-intercepts:

$y=\dfrac{0^4+1}{2(0)^5}=\dfrac{1}{0}$

undefined

There are no intercepts for the graph of this equation.

Test x-axis symmetry: Let $y=-y$

$-y=\dfrac{x^4+1}{2x^5}$ different

Test y-axis symmetry: Let $x=-x$

$y=\dfrac{(-x)^4+1}{2(-x)^5}$

$y=\dfrac{x^4+1}{-2x^5}$ different

Test origin symmetry: Let $x=-x$ and $y=-y$

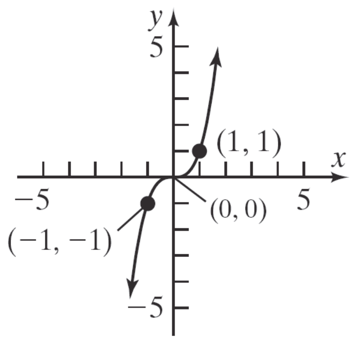
$-y=\dfrac{(-x)^4+1}{2(-x)^5}$

$-y=\dfrac{x^4+1}{-2x^5}$

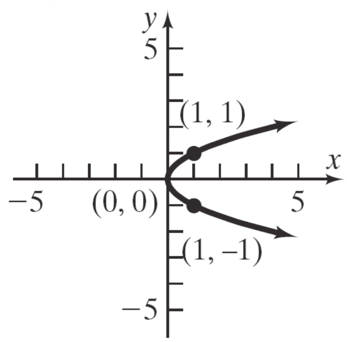
$y=\dfrac{x^4+1}{2x^5}$ same

Therefore, the graph has origin symmetry.

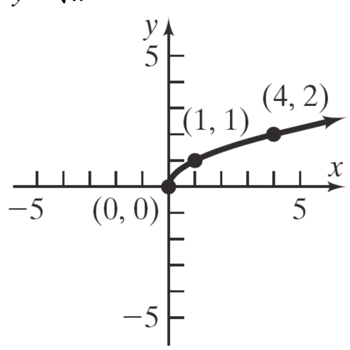
67. $y = x^3$



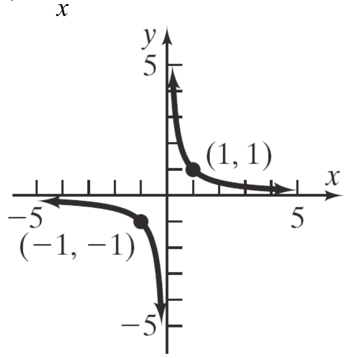
68. $x = y^2$



69. $y = \sqrt{x}$



70. $y = \dfrac{1}{x}$



71. If the point $(a, 4)$ is on the graph of $y = x^2 + 3x$, then we have

$$4 = a^2 + 3a$$
$$0 = a^2 + 3a - 4$$
$$0 = (a+4)(a-1)$$
$$a + 4 = 0 \quad \text{or} \quad a - 1 = 0$$
$$a = -4 \qquad\qquad a = 1$$

Thus, $a = -4$ or $a = 1$.

72. If the point $(a, -5)$ is on the graph of $y = x^2 + 6x$, then we have

$$-5 = a^2 + 6a$$
$$0 = a^2 + 6a + 5$$
$$0 = (a+5)(a+1)$$
$$a + 5 = 0 \quad \text{or} \quad a + 1 = 0$$
$$a = -5 \qquad\qquad a = -1$$

Thus, $a = -5$ or $a = -1$.

73. **a.**
$$0 = x^2 - 5$$
$$x^2 = 5$$
$$x = \pm\sqrt{5}$$

The x-intercepts are $x = -\sqrt{5}$ and $x = \sqrt{5}$.

$$y = (0)^2 - 5 = -5$$

The y-intercept is $y = -5$.

The intercepts are $\left(-\sqrt{5}, 0\right)$, $\left(\sqrt{5}, 0\right)$, and $(0, -5)$.

b. x-axis (replace y by $-y$):
$$-y = x^2 - 5$$
$$y = 5 - x^2 \quad \text{different}$$

y-axis (replace x by $-x$):
$$y = (-x)^2 - 5 = x^2 - 5 \quad \text{same}$$

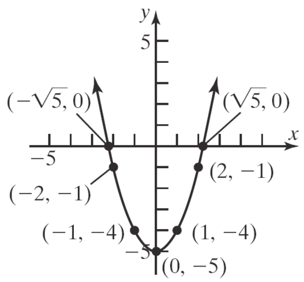
origin (replace x by $-x$ and y by $-y$):
$$-y = (-x)^2 - 5$$
$$y = 5 - x^2 \quad \text{different}$$

The equation has y-axis symmetry.

c. $y = x^2 - 5$

Additional points:

x	$y = x^2 - 5$	(x, y)
1	$y = 1^2 - 5 = -4$	$(1, -4)$
-1	from symmetry	$(-1, -4)$
2	$y = 2^2 - 5 = -1$	$(2, -1)$
-2	from symmetry	$(-2, -1)$



74. a. $0 = x^2 - 8$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

The x-intercepts are $x = -2\sqrt{2}$ and $x = 2\sqrt{2}$.

$y = (0)^2 - 8 = -8$

The y-intercept is $y = -8$.

The intercepts are $(-2\sqrt{2}, 0)$, $(2\sqrt{2}, 0)$, and $(0, -8)$.

b. x-axis (replace y by $-y$):

$-y = x^2 - 8$

$y = 8 - x^2$ different

y-axis (replace x by $-x$):

$y = (-x)^2 - 8 = x^2 - 8$ same

origin (replace x by $-x$ and y by $-y$):

$-y = (-x)^2 - 8$

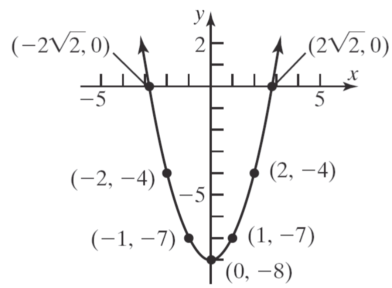
$y = 8 - x^2$ different

The equation has y-axis symmetry.

c. $y = x^2 - 8$

Additional points:

x	$y = x^2 - 5$	(x, y)
1	$y = 1^2 - 8 = -7$	$(1, -7)$
-1	from symmetry	$(-1, -7)$
2	$y = 2^2 - 8 = -4$	$(2, -4)$
-2	from symmetry	$(-2, -4)$



75. a. $x - (0)^2 = -9$

$x = -9$

The x-intercept is $x = -9$.

$(0) - y^2 = -9$

$-y^2 = -9$

$y^2 = 9 \rightarrow y = \pm 3$

The y-intercepts are $y = -3$ and $y = 3$.

The intercepts are $(-9, 0)$, $(0, -3)$, and $(0, 3)$.

b. x-axis (replace y by $-y$):

$x - (-y)^2 = -9$

$x - y^2 = -9$ same

y-axis (replace x by $-x$):

$-x - y^2 = -9$

$x + y^2 = 9$ different

origin (replace x by $-x$ and y by $-y$):

$-x - (-y)^2 = -9$

$-x - y^2 = -9$

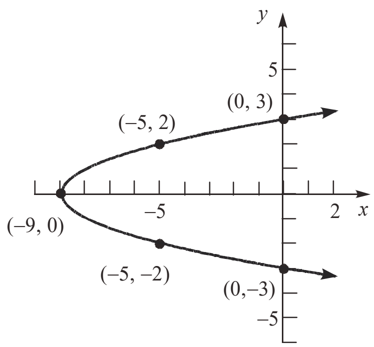
$x + y^2 = 9$ different

The equation has x-axis symmetry.

c. $x - y^2 = -9$ or $x = y^2 - 9$

Additional points:

y	$x = y^2 - 9$	(x, y)
2	$x = 2^2 - 9 = -5$	$(-5, 2)$
−2	from symmetry	$(-5, -2)$



76. a. $x + (0)^2 = 4$

$x = 4$

The x-intercept is $x = 4$.

$(0) + y^2 = 4$

$y^2 = 4 \rightarrow y = \pm 2$

The y-intercepts are $y = -2$ and $y = 2$.

The intercepts are $(4, 0)$, $(0, -2)$, and $(0, 2)$.

b. x-axis (replace y by $-y$):

$x + (-y)^2 = 4$

$x + y^2 = 4$ same

y-axis (replace x by $-x$):

$-x + y^2 = 4$

$x - y^2 = -4$ different

origin (replace x by $-x$ and y by $-y$):

$-x + (-y)^2 = 4$

$-x + y^2 = 4$

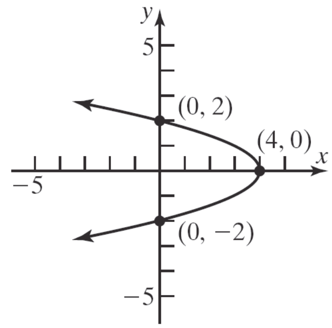
$x - y^2 = -4$ different

The equation has x-axis symmetry.

c. $x + y^2 = 4$ or $x = 4 - y^2$

Additional points:

y	$x = 4 - y^2$	(x, y)
1	$x = 4 - 1^2 = 3$	$(3, 1)$
−1	from symmetry	$(3, -1)$



77. a. $x^2 + (0)^2 = 9$

$x^2 = 9$

$x = \pm 3$

The x-intercepts are $x = -3$ and $x = 3$.

$(0)^2 + y^2 = 9$

$y^2 = 9$

$y = \pm 3$

The y-intercepts are $y = -3$ and $y = 3$.

The intercepts are $(-3, 0)$, $(3, 0)$, $(0, -3)$, and $(0, 3)$.

b. x-axis (replace y by $-y$):

$x^2 + (-y)^2 = 9$

$x^2 + y^2 = 9$ same

y-axis (replace x by $-x$):

$(-x)^2 + y^2 = 9$

$x^2 + y^2 = 9$ same

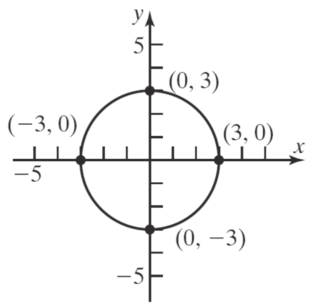
origin (replace x by $-x$ and y by $-y$):

$(-x)^2 + (-y)^2 = 9$

$x^2 + y^2 = 9$ same

The equation has x-axis symmetry, y-axis symmetry, and origin symmetry.

c. $x^2 + y^2 = 9$



78. a. $x^2 + (0)^2 = 16$

$x^2 = 16$

$x = \pm 4$

The x-intercepts are $x = -4$ and $x = 4$.

$(0)^2 + y^2 = 16$

$y^2 = 16$

$y = \pm 4$

The y-intercepts are $y = -4$ and $y = 4$.

The intercepts are $(-4,0)$, $(4,0)$, $(0,-4)$, and $(0,4)$.

b. x-axis (replace y by $-y$):

$x^2 + (-y)^2 = 16$

$x^2 + y^2 = 16$ same

y-axis (replace x by $-x$):

$(-x)^2 + y^2 = 16$

$x^2 + y^2 = 16$ same

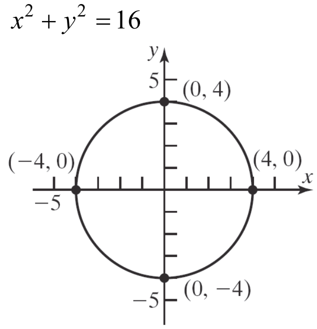
origin (replace x by $-x$ and y by $-y$):

$(-x)^2 + (-y)^2 = 16$

$x^2 + y^2 = 16$ same

The equation has x-axis symmetry, y-axis symmetry, and origin symmetry.

c. $x^2 + y^2 = 16$



79. a. $0 = x^3 - 4x$

$0 = x\left(x^2 - 4\right)$

$x = 0$ or $x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

The x-intercepts are $x = 0$, $x = -2$, and $x = 2$.

$y = 0^3 - 4(0) = 0$

The y-intercept is $y = 0$.

The intercepts are $(0,0)$, $(-2,0)$, and $(2,0)$.

b. x-axis (replace y by $-y$):

$-y = x^3 - 4x$

$y = 4x - x^3$ different

y-axis (replace x by $-x$):

$y = (-x)^3 - 4(-x)$

$y = -x^3 + 4x$ different

origin (replace x by $-x$ and y by $-y$):

$-y = (-x)^3 - 4(-x)$

$-y = -x^3 + 4x$

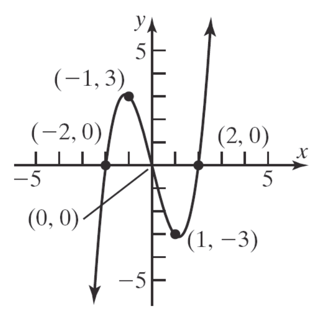
$y = x^3 - 4x$ same

The equation has origin symmetry.

c. $y = x^3 - 4x$

Additional points:

x	$y = x^3 - 4x$	(x, y)
1	$y = 1^3 - 4(1) = -3$	$(1,-3)$
-1	from symmetry	$(-1,3)$



80. a. $0 = x^3 - x$

$0 = x\left(x^2 - 1\right)$

$x = 0 \quad \text{or} \quad x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

The x-intercepts are $x = 0$, $x = -1$, and $x = 1$.

$y = 0^3 - (0) = 0$

The y-intercept is $y = 0$.

The intercepts are $(0,0)$, $(-1,0)$, and $(1,0)$.

b. x-axis (replace y by $-y$):

$-y = x^3 - x$

$y = x - x^3$ different

y-axis (replace x by $-x$):

$y = (-x)^3 - (-x)$

$y = -x^3 + x$ different

origin (replace x by $-x$ and y by $-y$):

$-y = (-x)^3 - (-x)$

$-y = -x^3 + x$

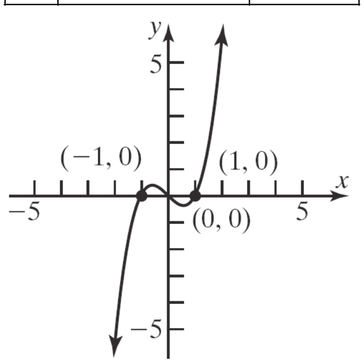
$y = x^3 - x$ same

The equation has origin symmetry.

c. $y = x^3 - x$

Additional points:

x	$y = x^3 - 4x$	(x, y)
2	$y = 2^3 - (2) = 6$	$(2,6)$
-2	from symmetry	$(-2,-6)$



81. For a graph with origin symmetry, if the point (a,b) is on the graph, then so is the point $(-a,-b)$. Since the point $(1,2)$ is on the graph of an equation with origin symmetry, the point $(-1,-2)$ must also be on the graph.

82. For a graph with y-axis symmetry, if the point (a,b) is on the graph, then so is the point $(-a,b)$. Since 6 is an x-intercept in this case, the point $(6,0)$ is on the graph of the equation. Due to the y-axis symmetry, the point $(-6,0)$ must also be on the graph. Therefore, -6 is another x-intercept.

83. For a graph with origin symmetry, if the point (a,b) is on the graph, then so is the point $(-a,-b)$. Since -4 is an x-intercept in this case, the point $(-4,0)$ is on the graph of the equation. Due to the origin symmetry, the point $(4,0)$ must also be on the graph. Therefore, 4 is another x-intercept.

84. For a graph with x-axis symmetry, if the point (a,b) is on the graph, then so is the point $(a,-b)$. Since 2 is a y-intercept in this case, the point $(0,2)$ is on the graph of the equation. Due to the x-axis symmetry, the point $(0,-2)$ must also be on the graph. Therefore, -2 is another y-intercept.

85. a. $\left(x^2 + y^2 - x\right)^2 = x^2 + y^2$

x-intercepts:

$\left(x^2 + (0)^2 - x\right)^2 = x^2 + (0)^2$

$\left(x^2 - x\right)^2 = x^2$

$x^4 - 2x^3 + x^2 = x^2$

$x^4 - 2x^3 = 0$

$x^3(x - 2) = 0$

$x^3 = 0 \quad \text{or} \quad x - 2 = 0$

$x = 0 \qquad\qquad x = 2$

y-intercepts:

$$\left((0)^2 + y^2 - 0\right)^2 = (0)^2 + y^2$$
$$\left(y^2\right)^2 = y^2$$
$$y^4 = y^2$$
$$y^4 - y^2 = 0$$
$$y^2\left(y^2 - 1\right) = 0$$
$$y^2 = 0 \quad \text{or} \quad y^2 - 1 = 0$$
$$y = 0 \qquad\qquad y^2 = 1$$
$$y = \pm 1$$

The intercepts are $(0,0)$, $(2,0)$, $(0,-1)$, and $(0,1)$.

b. Test x-axis symmetry: Let $y = -y$

$$\left(x^2 + (-y)^2 - x\right)^2 = x^2 + (-y)^2$$
$$\left(x^2 + y^2 - x\right)^2 = x^2 + y^2 \quad \text{same}$$

Test y-axis symmetry: Let $x = -x$

$$\left((-x)^2 + y^2 - (-x)\right)^2 = (-x)^2 + y^2$$
$$\left(x^2 + y^2 + x\right)^2 = x^2 + y^2 \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$\left((-x)^2 + (-y)^2 - (-x)\right)^2 = (-x)^2 + (-y)^2$$
$$\left(x^2 + y^2 + x\right)^2 = x^2 + y^2 \quad \text{different}$$

Thus, the graph will have x-axis symmetry.

86. a. $16y^2 = 120x - 225$

y-intercepts:

$$16y^2 = 120(0) - 225$$
$$16y^2 = -225$$
$$y^2 = -\frac{225}{16}$$

no real solution

x-intercepts:

$$16(0)^2 = 120x - 225$$
$$0 = 120x - 225$$
$$-120x = -225$$
$$x = \frac{-225}{-120} = \frac{15}{8}$$

The only intercept is $\left(\frac{15}{8}, 0\right)$.

b. Test x-axis symmetry: Let $y = -y$

$$16(-y)^2 = 120x - 225$$
$$16y^2 = 120x - 225 \quad \text{same}$$

Test y-axis symmetry: Let $x = -x$

$$16y^2 = 120(-x) - 225$$
$$16y^2 = -120x - 225 \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$16(-y)^2 = 120(-x) - 225$$
$$16y^2 = -120x - 225 \quad \text{different}$$

Thus, the graph has x-axis symmetry.

87. Let $y = 0$.

$$(x^2 + 0^2)^2 = a^2(x^2 - 0^2)$$
$$x^4 = a^2(x^2)$$
$$x^4 - a^2x^2 = 0$$
$$x^2(x^2 - a^2) = 0$$
$$x^2 = 0 \quad \text{or} \quad (x^2 - a^2) = 0$$
$$x = 0 \quad \text{or} \quad x^2 = a^2$$
$$x = -a, a$$

Let $x = 0$.

$$(0^2 + y^2)^2 = a^2(0^2 - y^2)$$
$$y^4 = a^2(-y^2)$$
$$y^4 + a^2y^2 = 0$$
$$y^2(y^2 + a^2) = 0$$
$$y = 0$$

(Note that the solutions to $y^2 + a^2 = 0$ are not real)

So the intercepts are are (0,0), (a,0) and (-a,0).

Test x-axis symmetry: Replace y by -y

$$(x^2 + (-y)^2)^2 = a^2(x^2 - (-y)^2)$$
$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \text{ equivalent}$$

Test y-axis symmetry: replace x by -x

$$((-x)^2+y^2)^2=a^2((-x)^2-y^2)$$

$$(x^2+y^2)^2=a^2(x^2-y^2) \text{ equivalent}$$

Test origin symmetry: replace x by -x and y by -y

$$((-x)^2+(-y)^2)^2=a^2((-x)^2-(-y)^2)$$

$$(x^2+y^2)^2=a^2(x^2-y^2) \text{ equivalent}$$

The graph is symmetric by respect to the x-axis, the y-axis, and the origin.

88. Let $y = 0$.

$$(x^2+0^2-ax)^2=b^2(x^2+0^2)$$

$$x^4-2ax^3+a^2x^2-b^2x^2=0$$

$$x^2\left[(x-(a+b)\right]\left[(x-(a-b)\right]=0$$

$$x=0 \text{ or } x=a+b$$

$$\text{or } x=a-b$$

Let $x = 0$.

$$(0^2+y^2-a\cdot 0)^2=b^2(0^2+y^2)$$

$$y^4-b^2y^2=0$$

$$y^2(y+b)(y-b)=0$$

$$y=0, y=-b, y=b$$

So the intercepts are (0,0), (a-b,0), (a+b,0), (0,-b), (0, b).

Test x-axis symmetry: replace y by -y

$$\left[x^2+(-y)^2-ax\right]^2=b^2\left[x^2+(-y)^2\right]$$

$$(x^2+y^2-ax)^2=b^2(x^2+y^2) \text{ Equivalent}$$

Test y-axis symmetry: replace x by -x

$$\left[(-x)^2+y^2-a(-x)\right]^2=b^2\left[(-x)^2+y^2\right]$$

$$(x^2+y^2+ax)^2=b^2(x^2+y^2) \text{ Not equivalent}$$

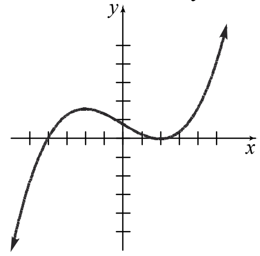
Test origin symmetry: replace x by -x and y by -y

$$\left[(-x)^2+(-y)^2-a(-x)\right]^2=b^2\left[(-x)^2+(-y)^2\right]$$

$$(x^2+y^2+ax)^2=b^2(x^2+y^2) \text{ No equivalent}$$

The graph is symmetric with respect to the x-axis only.

89. Answers will vary. One example:



90. Answers will vary.
Case 1: Graph has x-axis and y-axis symmetry, show origin symmetry.

(x,y) on graph $\to (x,-y)$ on graph

(from x-axis symmetry)

$(x,-y)$ on graph $\to (-x,-y)$ on graph

(from y-axis symmetry)

Since the point $(-x,-y)$ is also on the graph, the graph has origin symmetry.

Case 2: Graph has x-axis and origin symmetry, show y-axis symmetry.

(x,y) on graph $\to (x,-y)$ on graph

(from x-axis symmetry)

$(x,-y)$ on graph $\to (-x,y)$ on graph

(from origin symmetry)

Since the point $(-x,y)$ is also on the graph, the graph has y-axis symmetry.

Case 3: Graph has y-axis and origin symmetry, show x-axis symmetry.

(x,y) on graph $\to (-x,y)$ on graph

(from y-axis symmetry)

$(-x,y)$ on graph $\to (x,-y)$ on graph

(from origin symmetry)

Since the point $(x,-y)$ is also on the graph, the graph has x-axis symmetry.

91. Answers may vary. The graph must contain the points $(-2,5)$, $(-1,3)$, and $(0,2)$. For the graph to be symmetric about the y-axis, the graph must also contain the points $(2,5)$ and $(1,3)$ (note that (0, 2) is on the y-axis).

For the graph to also be symmetric with respect to the x-axis, the graph must also contain the points $(-2,-5)$, $(-1,-3)$, $(0,-2)$, $(2,-5)$, and $(1,-3)$. Recall that a graph with two of the symmetries (x-axis, y-axis, origin) will necessarily have the third. Therefore, if the original graph with y-axis symmetry also has x-axis symmetry, then it will also have origin symmetry.

Section 1.4

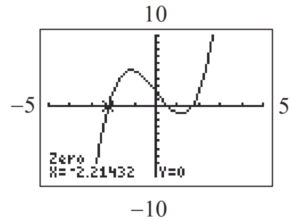
1.
$$2x^2+5x+2=0$$
$$(2x+1)(x+2)=0$$
$$2x+1=0 \quad \text{or} \quad x+2=0$$
$$2x=-1 \qquad x=-2$$
$$x=-\frac{1}{2}$$
The solution set is $\left\{-2,-\frac{1}{2}\right\}$.

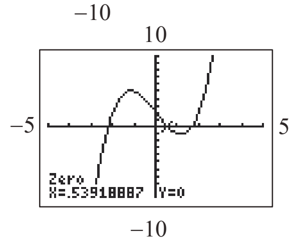
2.
$$2x+3=4(x-1)+1$$
$$2x+3=4x-4+1$$
$$2x+3=4x-3$$
$$-2x+3=-3$$
$$-2x=-6$$
$$x=3$$
The solution set is $\{3\}$.

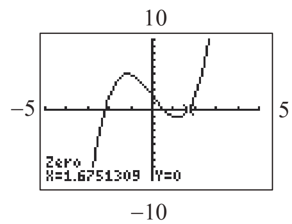
3. ZERO (ROOT)

4. False; exact solutions are obtained, in general, if the solution is a rational number. For irrational solutions, approximate answers are typically given, though some utilities can express certain irrational expressions in exact form.

5. $x^3-4x+2=0$; Use ZERO (or ROOT) on the graph of $y_1=x^3-4x+2$.

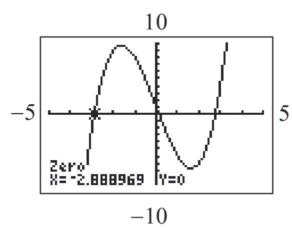


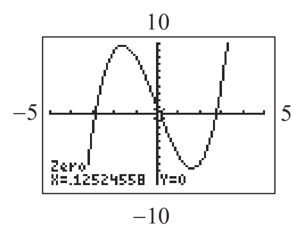


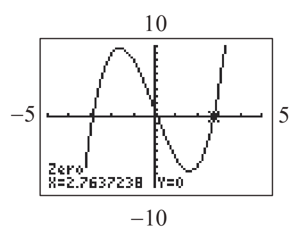


The solution set is $\{-2.21, 0.54, 1.68\}$.

6. $x^3-8x+1=0$; Use ZERO (or ROOT) on the graph of $y_1=x^3-8x+1$.

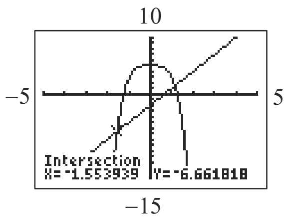


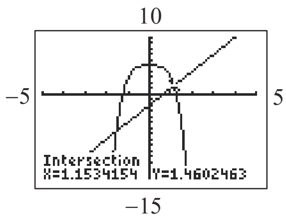




The solution set is $\{-2.89, 0.13, 2.76\}$.

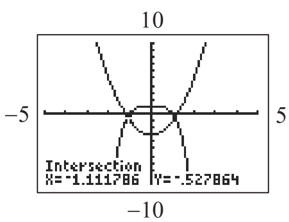
7. $-2x^4+5=3x-2$; Use INTERSECT on the graphs of $y_1=-2x^4+5$ and $y_2=3x-2$.

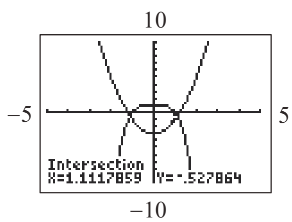




The solution set is $\{-1.55, 1.15\}$.

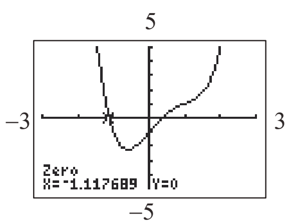
8. $-x^4+1=2x^2-3$; Use INTERSECT on the graphs of $y_1=-x^4+1$ and $y_2=2x^2-3$.

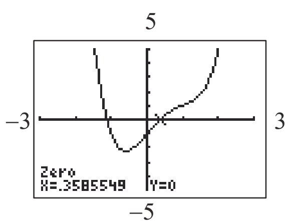




The solution set is $\{-1.11, 1.11\}$.

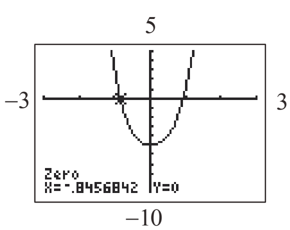
9. $x^4-2x^3+3x-1=0$; Use ZERO (or ROOT) on the graph of $y_1=x^4-2x^3+3x-1$.

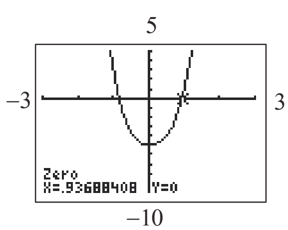




The solution set is $\{-1.12, 0.36\}$.

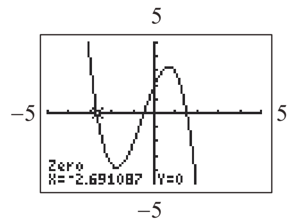
10. $3x^4-x^3+4x^2-5=0$; Use ZERO (or ROOT) on the graph of $y_1=3x^4-x^3+4x^2-5$.

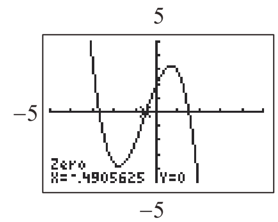


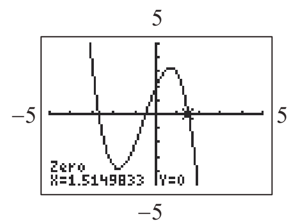


The solution set is $\{-0.85, 0.94\}$.

11. $-x^3-\frac{5}{3}x^2+\frac{7}{2}x+2=0$;
Use ZERO (or ROOT) on the graph of $y_1=-x^3-(5/3)x^2+(7/2)x+2$.

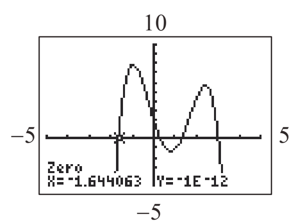


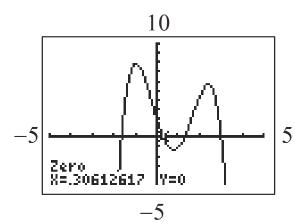


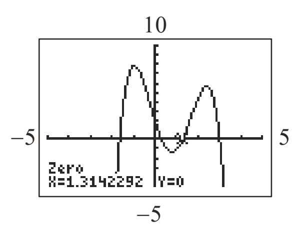


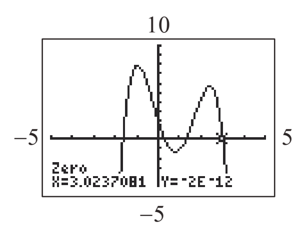
The solution set is $\{-2.69, -0.49, 1.51\}$.

12. $-x^4+3x^3+\frac{7}{3}x^2-\frac{15}{2}x+2=0$; Use ZERO (or ROOT) on the graph of $y_1=-x^4+3x^3+(7/3)x^2-(15/2)x+2$.



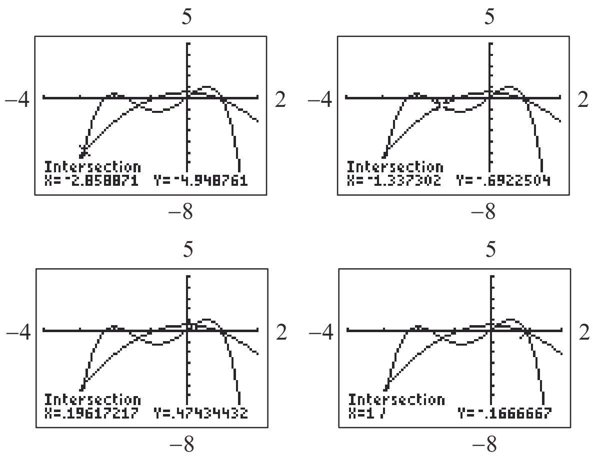






The solution set is $\{-1.64, 0.31, 1.31, 3.02\}$.

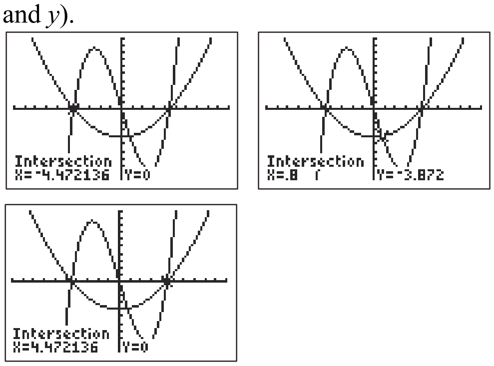
13. $-\frac{2}{3}x^4-2x^3+\frac{5}{2}x=-\frac{2}{3}x^2+\frac{1}{2}$
Use INTERSECT on the graphs of $y_1=-(2/3)x^4-2x^3+(5/2)x$ and $y_2=-(2/3)x^2+1/2$.



The solution set is $\{-2.86, -1.34, 0.20, 1.00\}$.

14. $\frac{1}{4}x^3 - 5x = \frac{1}{5}x^2 - 4$

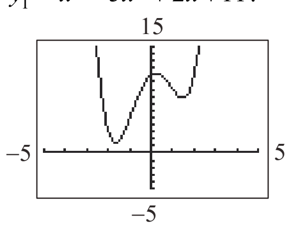
Use INTERSECT on the graphs of $y_1 = x^3/4 - 5x$ and $y_2 = x^2/5 - 4$ and a standard viewing window (-10 to 10 for both x and y).



The solution set is $\{-4.47, 0.80, 4.47\}$.

15. $x^4 - 5x^2 + 2x + 11 = 0$

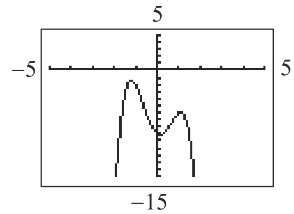
Use ZERO (or ROOT) on the graph of $y_1 = x^4 - 5x^2 + 2x + 11$.



There are no real solutions.

16. $-3x^4 + 8x^2 - 2x - 9 = 0$

Use ZERO (or ROOT) on the graph of $y_1 = -3x^4 + 8x^2 - 2x - 9$.



There are no real solutions.

17.
$$\begin{aligned}
2(3+2x) &= 3(x-4)\\
6+4x &= 3x-12\\
6+4x-6 &= 3x-12-6\\
4x &= 3x-18\\
4x-3x &= 3x-18-3x\\
x &= -18
\end{aligned}$$
The solution set is $\{-18\}$.

18.
$$\begin{aligned}
3(2-x) &= 2x-1\\
6-3x &= 2x-1\\
6-3x-6 &= 2x-1-6\\
-3x &= 2x-7\\
-3x-2x &= 2x-7-2x\\
-5x &= -7\\
\frac{-5x}{-5} &= \frac{-7}{-5}\\
x &= \frac{7}{5}
\end{aligned}$$
The solution set is $\left\{\frac{7}{5}\right\}$.

19.
$$\begin{aligned}
8x-(2x+1) &= 3x-13\\
8x-2x-1 &= 3x-13\\
6x-1 &= 3x-13\\
6x-1+1 &= 3x-13+1\\
6x &= 3x-12\\
6x-3x &= 3x-12-3x\\
3x &= -12\\
\frac{3x}{3} &= \frac{-12}{3}\\
x &= -4
\end{aligned}$$
The solution set is $\{-4\}$.

20. $$\begin{aligned} 5-(2x-1)&=10-x\\ 5-2x+1&=10-x\\ -2x+6&=10-x\\ -2x+6+2x&=10-x+2x\\ 6&=10+x\\ 6-10&=10+x-10\\ -4&=x \end{aligned}$$
The solution set is $\{-4\}$.

21. $$\begin{aligned} \frac{x+1}{3}+\frac{x+2}{7}&=5\\ 21\left(\frac{x+1}{3}\right)+21\left(\frac{x+2}{7}\right)&=21(5)\\ 7x+7+3x+6&=105\\ 10x+13&=105\\ 10x+13-13&=105-13\\ 10x&=92\\ \frac{10x}{10}&=\frac{92}{10}\\ x&=\frac{46}{5} \end{aligned}$$
The solution set is $\left\{\frac{46}{5}\right\}$.

22. $$\begin{aligned} \frac{2x+1}{3}+16&=3x\\ 3\left(\frac{2x+1}{3}+16\right)&=3\cdot 3x\\ 2x+1+48&=9x\\ 2x+49&=9x\\ 2x+49-2x&=9x-2x\\ 49&=7x\\ \frac{49}{7}&=\frac{7x}{7}\\ x&=7 \end{aligned}$$
The solution set is $\{7\}$.

23. $$\begin{aligned} \frac{5}{y}+\frac{4}{y}&=3\\ \frac{9}{y}&=3\\ \left(\frac{9}{y}\right)^{-1}&=3^{-1}\\ \frac{y}{9}&=\frac{1}{3}\\ 9\cdot\frac{y}{9}&=9\cdot\frac{1}{3}\\ y&=3 \end{aligned}$$
The solution set is $\{3\}$.

24. $$\begin{aligned} \frac{4}{y}-5&=\frac{18}{2y}\\ 2y\left(\frac{4}{y}-5\right)&=2y\left(\frac{18}{2y}\right)\\ 8-10y&=18\\ 8-10y-8&=18-8\\ -10y&=10\\ \frac{-10y}{-10}&=\frac{10}{-10}\\ y&=-1 \end{aligned}$$
The solution set is $\{-1\}$.

25. $$\begin{aligned} (x+7)(x-1)&=(x+1)^2\\ x^2-x+7x-7&=x^2+2x+1\\ x^2+6x-7&=x^2+2x+1\\ x^2+6x-7-x^2&=x^2+2x+1-x^2\\ 6x-7&=2x+1\\ 6x-7-2x&=2x+1-2x\\ 4x-7&=1\\ 4x-7+7&=1+7\\ 4x&=8\\ \frac{4x}{4}&=\frac{8}{4}\Rightarrow x=2 \end{aligned}$$
The solution set is $\{2\}$.

26. $(x+2)(x-3)=(x-3)^2$

$$x^2+2x-3x-6=x^2-6x+9$$
$$x^2-x-6=x^2-6x+9$$
$$x^2-x-6-x^2=x^2-6x+9-x^2$$
$$-x-6=-6x+9$$
$$-x-6+6x=-6x+9+6x$$
$$5x-6=9$$
$$5x-6+6=9+6$$
$$5x=15$$
$$\frac{5x}{5}=\frac{15}{5}$$
$$x=3$$

The solution set is $\{3\}$.

27. $x^2-3x-28=0$

$$(x-7)(x+4)=28$$
$$x-7=0 \quad \text{or} \quad x+4=0$$
$$x=7 \qquad\qquad x=-4$$

The solution set is $\{-4,7\}$.

28. $x^2-7x-18=0$

$$(x-9)(x+2)=0$$
$$x-9=0 \quad \text{or} \quad x+2=0$$
$$x=9 \qquad\qquad x=-2$$

The solution set is $\{-2,9\}$.

29. $3x^2=4x+4$

$$3x^2-4x-4=0$$
$$(3x+2)(x-2)=0$$
$$3x+2=0 \quad \text{or} \quad x-2=0$$
$$3x=-2 \qquad\qquad x=2$$
$$x=-\frac{2}{3}$$

The solution set is $\left\{-\frac{2}{3},2\right\}$.

30. $5x^2=13x+6$

$$5x^2-13x-6=0$$
$$(5x+2)(x-3)=0$$
$$5x+2=0 \quad \text{or} \quad x-3=0$$
$$5x=-2 \qquad\qquad x=3$$
$$x=-\frac{2}{5}$$

The solution set is $\left\{-\frac{2}{5},3\right\}$.

31. $x^3+x^2-4x-4=0$

$$\left(x^3+x^2\right)+(-4x-4)=0$$
$$x^2(x+1)-4(x+1)=0$$
$$(x+1)\left(x^2-4\right)=0$$
$$(x+1)(x-2)(x+2)=0$$
$$x+1=0 \quad \text{or} \quad x-2=0 \quad \text{or} \quad x+2=0$$
$$x=-1 \qquad\qquad x=2 \qquad\qquad x=-2$$

The solution set is $\{-2,-1,2\}$.

32. $x^3+2x^2-9x-18=0$

$$\left(x^3+2x^2\right)+(-9x-18)=0$$
$$x^2(x+2)-9(x+2)=0$$
$$(x+2)\left(x^2-9\right)=0$$
$$(x+2)(x-3)(x+3)=0$$
$$x+2=0 \quad \text{or} \quad x-3=0 \quad \text{or} \quad x+3=0$$
$$x=-2 \qquad\qquad x=3 \qquad\qquad x=-3$$

The solution set is $\{-3,-2,3\}$.

33. $\sqrt{x+1}=4$

$$\left(\sqrt{x+1}\right)^2=4^2$$
$$x+1=16$$
$$x+1-1=16-1$$
$$x=15$$

Check:

$$\sqrt{15+1}=4$$
$$\sqrt{16}=4$$
$$4=4 \text{ T}$$

The solution set is $\{15\}$.

34. $\sqrt{x-2}=3$

$$\left(\sqrt{x-2}\right)^2=3^2$$
$$x-2=9$$
$$x-2+2=9+2$$
$$x=11$$

Check:

$$\sqrt{11-2}=3$$
$$\sqrt{9}=3$$
$$3=3 \quad \text{T}$$

The solution set is $\{11\}$.

35. $\dfrac{2}{x+2}+\dfrac{3}{x-1}=\dfrac{-8}{5}$

LCD = $5(x-1)(x+2)$

Multiply both sides of the equation by the LCD. This yields the equation

$$2\cdot 5(x-1)+3\cdot 5(x+2)=-8(x-1)(x+2)$$
$$10(x-1)+15(x+2)=-8\left(x^2+x-2\right)$$
$$10x-10+15x+30=-8x^2-8x+16$$
$$25x+20=-8x^2-8x+16$$
$$8x^2+33x+4=0$$
$$(8x+1)(x+4)=0$$
$$8x+1=0 \quad \text{or} \quad x+4=0$$
$$8x=-1 \qquad x=-4$$
$$x=-\frac{1}{8}$$

The solution set is $\left\{-4,-\dfrac{1}{8}\right\}$.

36. $\dfrac{1}{x+1}-\dfrac{5}{x-4}=\dfrac{21}{4}$

LCD = $4(x-4)(x+1)$

Multiply both sides of the equation by the LCD. This gives the equation

$$4(x-4)-5\cdot 4(x+1)=21(x-4)(x+1)$$
$$4(x-4)-20(x+1)=21\left(x^2-3x-4\right)$$
$$4x-16-20x-20=21x^2-63x-84$$
$$-16x-36=21x^2-63x-84$$
$$0=21x^2-47x-48$$
$$0=(x-3)(21x+16)$$
$$x-3=0 \quad \text{or} \quad 21x+16=0$$
$$x=3 \qquad 21x=-16$$
$$x=-\frac{16}{21}$$

The solution set is $\left\{-\dfrac{16}{21},3\right\}$.

Section 1.5

1. **a.** 2; 2; 1
b. -2
c. 0
d. ii, iv
e. ii

2. undefined; 0

3. 3; 2

x-intercept: $2x+3(0)=6$
$$2x=6$$
$$x=3$$

y-intercept: $2(0)+3y=6$
$$3y=6$$
$$y=2$$

4. True

5. False; the slope is $\frac{3}{2}$.

$$2y=3x+5$$
$$y=\frac{3}{2}x+\frac{5}{2}$$

6. True; $2(1)+(2)\overset{?}{=}4$
$$2+2\overset{?}{=}4$$
$$4=4 \quad \text{True}$$

7. $m_1=m_2$; y-intercepts; $m_1\cdot m_2=-1$

8. 2

9. $-\dfrac{1}{2}$

10. d

11. b

12. d

13. a. $\text{Slope} = \dfrac{1-0}{2-0} = \dfrac{1}{2}$

b. If x increases by 2 units, y will increase by 1 unit.

14. a. $\text{Slope} = \dfrac{1-0}{-2-0} = -\dfrac{1}{2}$

b. If x increases by 2 units, y will decrease by 1 unit.

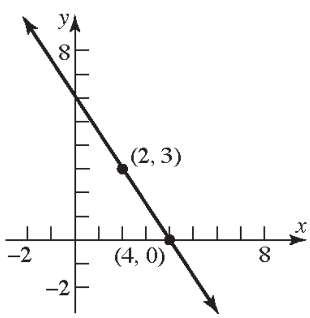
15. a. $\text{Slope} = \dfrac{1-2}{1-(-2)} = -\dfrac{1}{3}$

b. If x increases by 3 units, y will decrease by 1 unit.

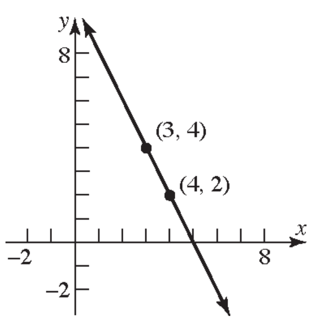
16. a. $\text{Slope} = \dfrac{2-1}{2-(-1)} = \dfrac{1}{3}$

b. If x increases by 3 units, y will increase by 1 unit.

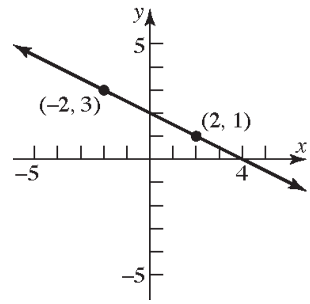
17. $\text{Slope} = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{0-3}{4-2} = -\dfrac{3}{2}$



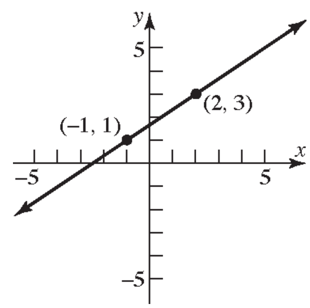
18. $\text{Slope} = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{4-2}{3-4} = \dfrac{2}{-1} = -2$



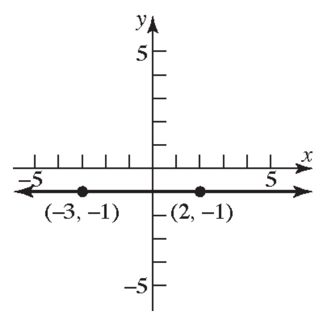
19. $\text{Slope} = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{1-3}{2-(-2)} = \dfrac{-2}{4} = -\dfrac{1}{2}$



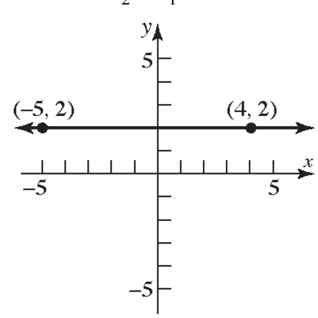
20. $\text{Slope} = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{3-1}{2-(-1)} = \dfrac{2}{3}$



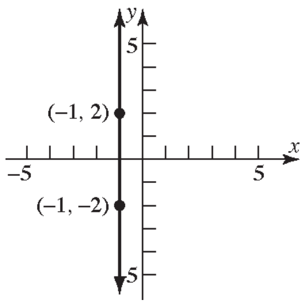
21. $\text{Slope} = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{-1-(-1)}{2-(-3)} = \dfrac{0}{5} = 0$



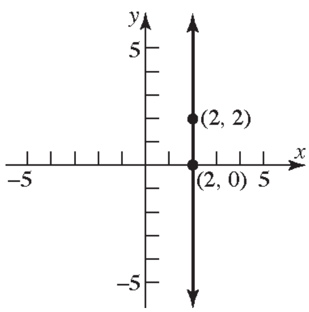
22. $\text{Slope} = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{2-2}{-5-4} = \dfrac{0}{-9} = 0$



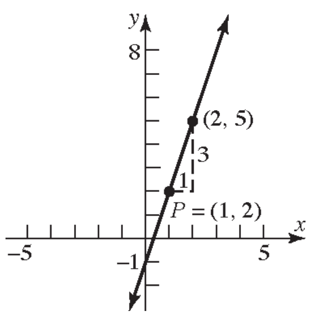
23. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-2}{-1-(-1)} = \dfrac{-4}{0}$ undefined.



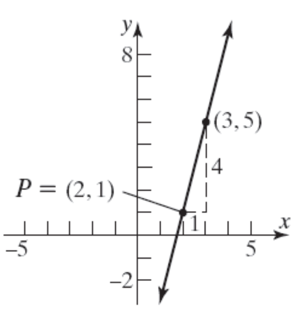
24. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-0}{2-2} = \dfrac{2}{0}$ undefined.



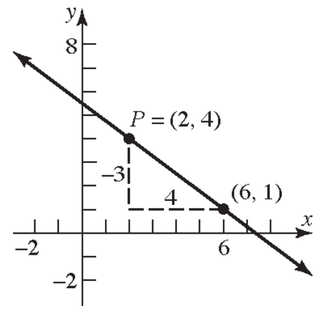
25. $P = (1,2); m = 3$



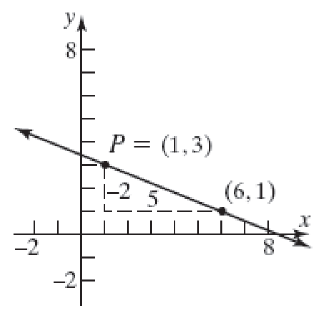
26. $P = (2,1); m = 4$



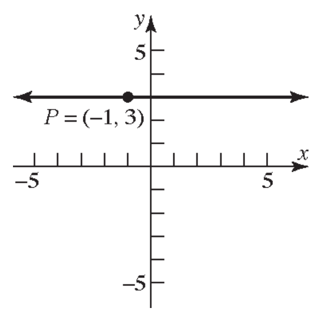
27. $P = (2,4); m = -\dfrac{3}{4}$



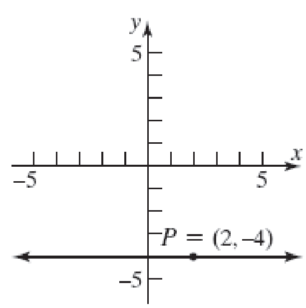
28. $P = (1,3); m = -\dfrac{2}{5}$



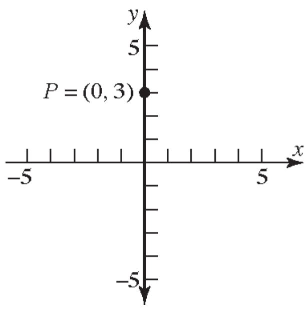
29. $P = (-1,3); m = 0$



30. $P = (2,-4); m = 0$

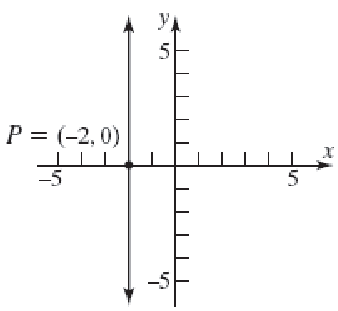


31. $P = (0,3)$; slope undefined



(note: the line is the y-axis)

32. $P = (-2,0)$; slope undefined



33. $P = (1,2); m = 3$; $y - 2 = 3(x - 1)$

34. $P = (2,1); m = 4$; $y - 1 = 4(x - 2)$

35. $P = (2,4); m = -\frac{3}{4}$; $y - 4 = -\frac{3}{4}(x - 2)$

36. $P = (1,3); m = -\frac{2}{5}$; $y - 3 = -\frac{2}{5}(x - 1)$

37. $P = (-1,3); m = 0$; $y - 3 = 0$

38. $P = (2,-4); m = 0$; $y + 4 = 0$

39. Slope $= 4 = \frac{4}{1}$; point: $(1,2)$

If x increases by 1 unit, then y increases by 4 units.

Answers will vary. Three possible points are:

$x = 1 + 1 = 2$ and $y = 2 + 4 = 6$

$(2,6)$

$x = 2 + 1 = 3$ and $y = 6 + 4 = 10$

$(3,10)$

$x = 3 + 1 = 4$ and $y = 10 + 4 = 14$

$(4,14)$

40. Slope $= 2 = \frac{2}{1}$; point: $(-2,3)$

If x increases by 1 unit, then y increases by 2 units.

Answers will vary. Three possible points are:

$x = -2 + 1 = -1$ and $y = 3 + 2 = 5$

$(-1,5)$

$x = -1 + 1 = 0$ and $y = 5 + 2 = 7$

$(0,7)$

$x = 0 + 1 = 1$ and $y = 7 + 2 = 9$

$(1,9)$

41. Slope $= -\frac{3}{2} = \frac{-3}{2}$; point: $(2,-4)$

If x increases by 2 units, then y decreases by 3 units.

Answers will vary. Three possible points are:

$x = 2 + 2 = 4$ and $y = -4 - 3 = -7$

$(4,-7)$

$x = 4 + 2 = 6$ and $y = -7 - 3 = -10$

$(6,-10)$

$x = 6 + 2 = 8$ and $y = -10 - 3 = -13$

$(8,-13)$

42. Slope $= \frac{4}{3}$; point: $(-3,2)$

If x increases by 3 units, then y increases by 4 units.

Answers will vary. Three possible points are:

$x = -3 + 3 = 0$ and $y = 2 + 4 = 6$

$(0,6)$

$x = 0 + 3 = 3$ and $y = 6 + 4 = 10$

$(3,10)$

$x = 3 + 3 = 6$ and $y = 10 + 4 = 14$

$(6,14)$

43. Slope $= -2 = \frac{-2}{1}$; point: $(-2,-3)$

If x increases by 1 unit, then y decreases by 2 units.
Answers will vary. Three possible points are:

$x = -2+1 = -1$ and $y = -3-2 = -5$

$(-1,-5)$

$x = -1+1 = 0$ and $y = -5-2 = -7$

$(0,-7)$

$x = 0+1 = 1$ and $y = -7-2 = -9$

$(1,-9)$

44. Slope $= -1 = \frac{-1}{1}$; point: $(4,1)$

If x increases by 1 unit, then y decreases by 1 unit.
Answers will vary. Three possible points are:

$x = 4+1 = 5$ and $y = 1-1 = 0$

$(5,0)$

$x = 5+1 = 6$ and $y = 0-1 = -1$

$(6,-1)$

$x = 6+1 = 7$ and $y = -1-1 = -2$

$(7,-2)$

45. (0, 0) and (2, 1) are points on the line.

Slope $= \frac{1-0}{2-0} = \frac{1}{2}$

y-intercept is 0; using $y = mx+b$:

$$y = \frac{1}{2}x + 0$$
$$2y = x$$
$$0 = x - 2y$$
$$x - 2y = 0 \text{ or } y = \frac{1}{2}x$$

46. (0, 0) and (–2, 1) are points on the line.

Slope $= \frac{1-0}{-2-0} = \frac{1}{-2} = -\frac{1}{2}$

y-intercept is 0; using $y = mx+b$:

$$y = -\frac{1}{2}x + 0$$
$$2y = -x$$
$$x + 2y = 0$$
$$x + 2y = 0 \text{ or } y = -\frac{1}{2}x$$

47. (–1, 3) and (1, 1) are points on the line.

Slope $= \frac{1-3}{1-(-1)} = \frac{-2}{2} = -1$

Using $y - y_1 = m(x - x_1)$

$$y - 1 = -1(x-1)$$
$$y - 1 = -x + 1$$
$$y = -x + 2$$
$$x + y = 2 \text{ or } y = -x + 2$$

48. (–1, 1) and (2, 2) are points on the line.

Slope $= \frac{2-1}{2-(-1)} = \frac{1}{3}$

Using $y - y_1 = m(x - x_1)$

$$y - 1 = \frac{1}{3}\left(x - (-1)\right)$$
$$y - 1 = \frac{1}{3}(x+1)$$
$$y - 1 = \frac{1}{3}x + \frac{1}{3}$$
$$y = \frac{1}{3}x + \frac{4}{3}$$
$$x - 3y = -4 \text{ or } y = \frac{1}{3}x + \frac{4}{3}$$

49. $y - y_1 = m(x - x_1),\ m = 2$

$$y - 3 = 2(x-3)$$
$$y - 3 = 2x - 6$$
$$y = 2x - 3$$
$$2x - y = 3 \text{ or } y = 2x - 3$$

50. $y - y_1 = m(x - x_1),\ m = -1$

$$y - 2 = -1(x-1)$$
$$y - 2 = -x + 1$$
$$y = -x + 3$$
$$x + y = 3 \text{ or } y = -x + 3$$

51. $y - y_1 = m(x - x_1),\ m = -\frac{1}{2}$

$$y - 2 = -\frac{1}{2}(x-1)$$
$$y - 2 = -\frac{1}{2}x + \frac{1}{2}$$
$$y = -\frac{1}{2}x + \frac{5}{2}$$
$$x + 2y = 5 \text{ or } y = -\frac{1}{2}x + \frac{5}{2}$$

52. $y-y_1=m(x-x_1),\ m=1$
$$y-1=1(x-(-1))$$
$$y-1=x+1$$
$$y=x+2$$
$$x-y=-2 \text{ or } y=x+2$$

53. Slope = 3; containing (–2, 3)
$$y-y_1=m(x-x_1)$$
$$y-3=3(x-(-2))$$
$$y-3=3x+6$$
$$y=3x+9$$
$$3x-y=-9 \text{ or } y=3x+9$$

54. Slope = 2; containing the point (4, –3)
$$y-y_1=m(x-x_1)$$
$$y-(-3)=2(x-4)$$
$$y+3=2x-8$$
$$y=2x-11$$
$$2x-y=11 \text{ or } y=2x-11$$

55. Slope $=\frac{1}{2}$; containing the point (3, 1)
$$y-y_1=m(x-x_1)$$
$$y-1=\frac{1}{2}(x-3)$$
$$y-1=\frac{1}{2}x-\frac{3}{2}$$
$$y=\frac{1}{2}x-\frac{1}{2}$$
$$x-2y=1 \text{ or } y=\frac{1}{2}x-\frac{1}{2}$$

56. Slope $=-\frac{2}{3}$; containing (1, –1)
$$y-y_1=m(x-x_1)$$
$$y-(-1)=-\frac{2}{3}(x-1)$$
$$y+1=-\frac{2}{3}x+\frac{2}{3}$$
$$y=-\frac{2}{3}x-\frac{1}{3}$$
$$2x+3y=-1 \text{ or } y=-\frac{2}{3}x-\frac{1}{3}$$

57. Containing (1, 3) and (–1, 2)
$$m=\frac{2-3}{-1-1}=\frac{-1}{-2}=\frac{1}{2}$$
$$y-y_1=m(x-x_1)$$
$$y-3=\frac{1}{2}(x-1)$$
$$y-3=\frac{1}{2}x-\frac{1}{2}$$
$$y=\frac{1}{2}x+\frac{5}{2}$$
$$x-2y=-5 \text{ or } y=\frac{1}{2}x+\frac{5}{2}$$

58. Containing the points (–3, 4) and (2, 5)
$$m=\frac{5-4}{2-(-3)}=\frac{1}{5}$$
$$y-y_1=m(x-x_1)$$
$$y-5=\frac{1}{5}(x-2)$$
$$y-5=\frac{1}{5}x-\frac{2}{5}$$
$$y=\frac{1}{5}x+\frac{23}{5}$$
$$x-5y=-23 \text{ or } y=\frac{1}{5}x+\frac{23}{5}$$

59. Slope = –3; y-intercept =3
$$y=mx+b$$
$$y=-3x+3$$
$$3x+y=3 \text{ or } y=-3x+3$$

60. Slope = –2; y-intercept = –2
$$y=mx+b$$
$$y=-2x+(-2)$$
$$2x+y=-2 \text{ or } y=-2x-2$$

61. x-intercept = –4; y-intercept = 4
Points are (–4, 0) and (0, 4)
$$m=\frac{4-0}{0-(-4)}=\frac{4}{4}=1$$
$$y=mx+b$$
$$y=1x+4$$
$$y=x+4$$
$$x-y=-4 \text{ or } y=x+4$$

62. x-intercept = 2; y-intercept = –1
Points are (2,0) and (0,–1)
$$m = \frac{-1-0}{0-2} = \frac{-1}{-2} = \frac{1}{2}$$
$$y = mx + b$$
$$y = \frac{1}{2}x - 1$$
$$x - 2y = 2 \text{ or } y = \frac{1}{2}x - 1$$

63. Slope undefined; containing the point (2, 4)
This is a vertical line.
$x = 2$ No slope-intercept form.

64. Slope undefined; containing the point (3, 8)
This is a vertical line.
$x = 3$ No slope-intercept form.

65. Horizontal lines have slope $m = 0$ and take the form $y = b$. Therefore, the horizontal line passing through the point $(-3,2)$ is $y = 2$.

66. Vertical lines have an undefined slope and take the form $x = a$. Therefore, the vertical line passing through the point $(4,-5)$ is $x = 4$.

67. Parallel to $y = 2x$; Slope = 2
Containing (–1, 2)
$$y - y_1 = m(x - x_1)$$
$$y - 2 = 2(x - (-1))$$
$$y - 2 = 2x + 2 \rightarrow y = 2x + 4$$
$$2x - y = -4 \text{ or } y = 2x + 4$$

68. Parallel to $y = -3x$; Slope = –3; Containing the point (–1, 2)
$$y - y_1 = m(x - x_1)$$
$$y - 2 = -3(x - (-1))$$
$$y - 2 = -3x - 3 \rightarrow y = -3x - 1$$
$$3x + y = -1 \text{ or } y = -3x - 1$$

69. Parallel to $x - 2y = -5$;
Slope $= \frac{1}{2}$; Containing the point $(0,0)$
$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{1}{2}(x - 0) \rightarrow y = \frac{1}{2}x$$
$$x - 2y = 0 \text{ or } y = \frac{1}{2}x$$

70. Parallel to $2x - y = -2$; Slope = 2
Containing the point (0, 0)
$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2(x - 0)$$
$$y = 2x$$
$$2x - y = 0 \text{ or } y = 2x$$

71. Parallel to $x = 5$; Containing (4,2)
This is a vertical line.
$x = 4$ No slope-intercept form.

72. Parallel to $y = 5$; Containing the point (4, 2)
This is a horizontal line. Slope = 0
$y = 2$

73. Perpendicular to $y = \frac{1}{2}x + 4$; Containing (1, –2)
Slope of perpendicular = –2
$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -2(x - 1)$$
$$y + 2 = -2x + 2 \rightarrow y = -2x$$
$$2x + y = 0 \text{ or } y = -2x$$

74. Perpendicular to $y = 2x - 3$; Containing the point (1, –2)
Slope of perpendicular $= -\frac{1}{2}$
$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{1}{2}(x - 1)$$
$$y + 2 = -\frac{1}{2}x + \frac{1}{2} \rightarrow y = -\frac{1}{2}x - \frac{3}{2}$$
$$x + 2y = -3 \text{ or } y = -\frac{1}{2}x - \frac{3}{2}$$

75. Perpendicular to $x - 2y = -5$; Containing the point (0, 4)
Slope of perpendicular = –2
$$y = mx + b$$
$$y = -2x + 4$$
$$2x + y = 4 \text{ or } y = -2x + 4$$

76. Perpendicular to $2x + y = 2$; Containing the point (–3, 0)

Slope of perpendicular $= \frac{1}{2}$

$y - y_1 = m(x - x_1)$

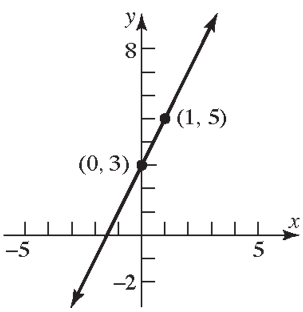
$y - 0 = \frac{1}{2}(x - (-3)) \rightarrow y = \frac{1}{2}x + \frac{3}{2}$

$x - 2y = -3$ or $y = \frac{1}{2}x + \frac{3}{2}$

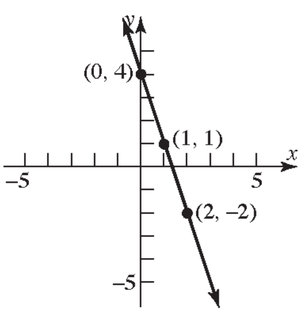
77. Perpendicular to $x = 8$; Containing (3, 4)
Slope of perpendicular = 0 (horizontal line)
$y = 4$

78. Perpendicular to $y = 8$;
Containing the point (3, 4)
Slope of perpendicular is undefined (vertical line). $x = 3$ No slope-intercept form.

79. $y = 2x + 3$; Slope = 2; y-intercept = 3

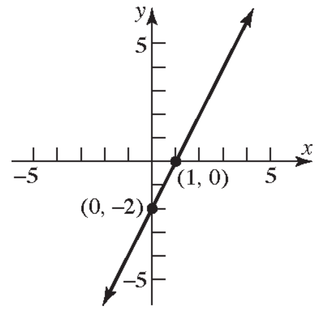


80. $y = -3x + 4$; Slope = –3; y-intercept = 4



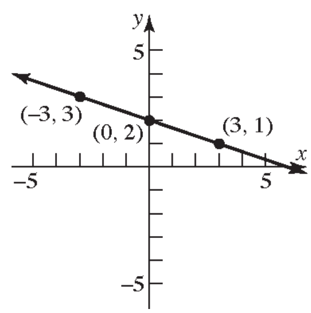
81. $\frac{1}{2}y = x - 1$; $y = 2x - 2$

Slope = 2; y-intercept = –2

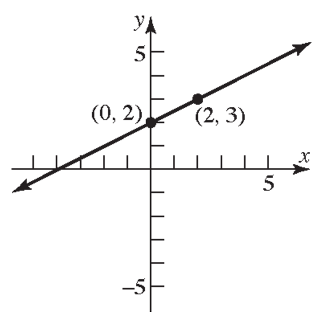


82. $\frac{1}{3}x + y = 2$; $y = -\frac{1}{3}x + 2$

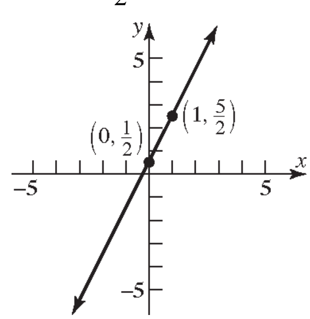
Slope $= -\frac{1}{3}$; y-intercept = 2



83. $y = \frac{1}{2}x + 2$; Slope $= \frac{1}{2}$; y-intercept = 2

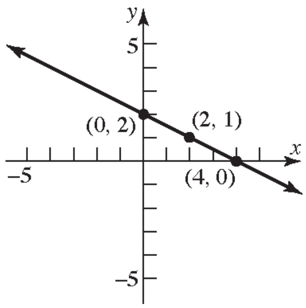


84. $y = 2x + \frac{1}{2}$; Slope = 2; y-intercept $= \frac{1}{2}$



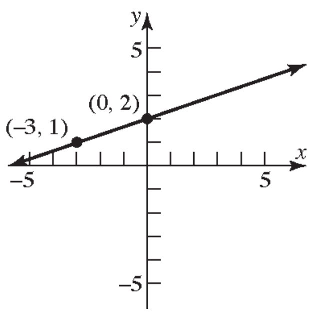
85. $x+2y=4$; $2y=-x+4 \rightarrow y=-\dfrac{1}{2}x+2$

Slope $=-\dfrac{1}{2}$; y-intercept $=2$



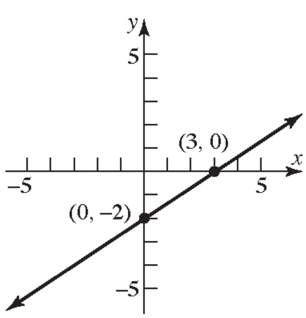
86. $-x+3y=6$; $3y=x+6 \rightarrow y=\dfrac{1}{3}x+2$

Slope $=\dfrac{1}{3}$; y-intercept $=2$



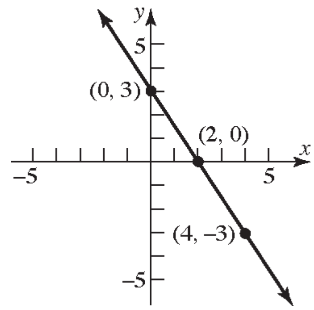
87. $2x-3y=6$; $-3y=-2x+6 \rightarrow y=\dfrac{2}{3}x-2$

Slope $=\dfrac{2}{3}$; y-intercept $=-2$



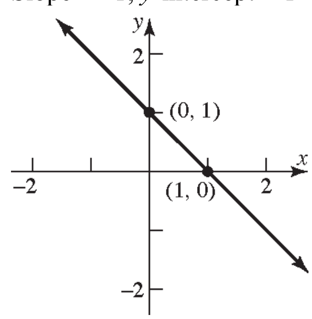
88. $3x+2y=6$; $2y=-3x+6 \rightarrow y=-\dfrac{3}{2}x+3$

Slope $=-\dfrac{3}{2}$; y-intercept $=3$



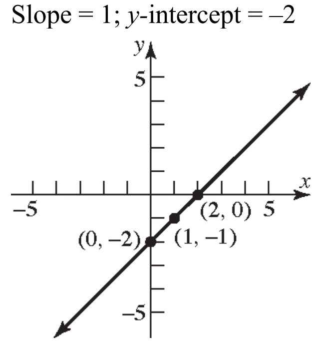
89. $x+y=1$; $y=-x+1$

Slope = –1; y-intercept = 1



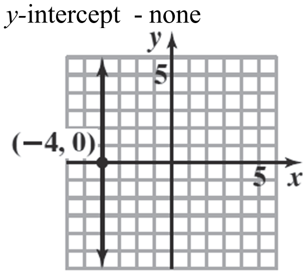
90. $x-y=2$; $y=x-2$

Slope = 1; y-intercept = –2

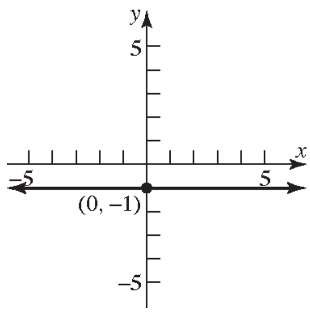


91. $x=-4$; Slope is undefined

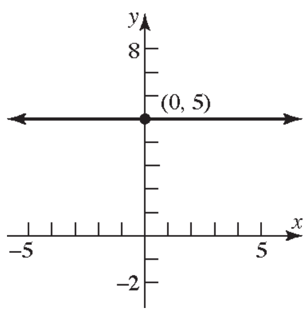
y-intercept - none



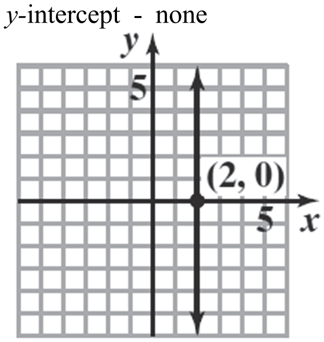
92. $y=-1$; Slope = 0; y-intercept $=-1$



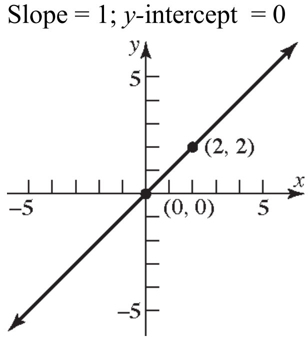
93. $y=5$; Slope = 0; y-intercept = 5



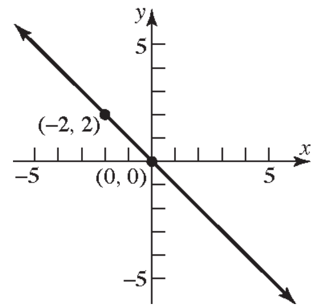
94. $x=2$; Slope is undefined
y-intercept - none



95. $y-x=0$; $y=x$
Slope = 1; y-intercept $=0$

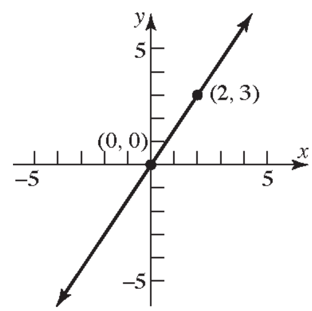


96. $x+y=0$; $y=-x$
Slope $=-1$; y-intercept $=0$



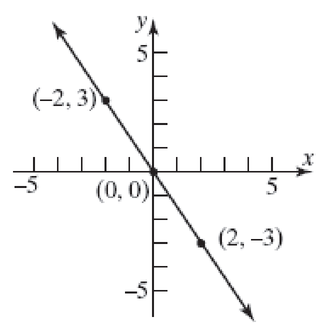
97. $2y-3x=0$; $2y=3x \rightarrow y=\frac{3}{2}x$

Slope $=\frac{3}{2}$; y-intercept = 0



98. $3x+2y=0$; $2y=-3x \rightarrow y=-\frac{3}{2}x$

Slope $=-\frac{3}{2}$; y-intercept = 0



99. a. x-intercept: $2x+3(0)=6$

$$2x=6$$
$$x=3$$

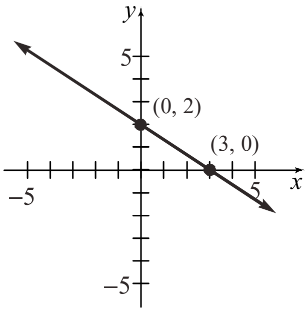
The point $(3,0)$ is on the graph.

y-intercept: $2(0)+3y=6$

$$3y=6$$
$$y=2$$

The point $(0,2)$ is on the graph.

b.



100. a. x-intercept: $3x-2(0)=6$

$$3x=6$$
$$x=2$$

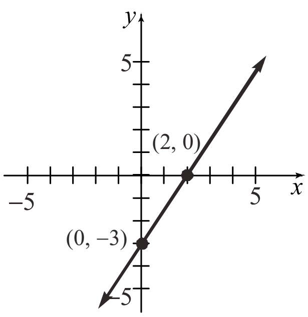
The point $(2,0)$ is on the graph.

y-intercept: $3(0)-2y=6$

$$-2y=6$$
$$y=-3$$

The point $(0,-3)$ is on the graph.

b.



101. a. x-intercept: $-4x+5(0)=40$

$$-4x=40$$
$$x=-10$$

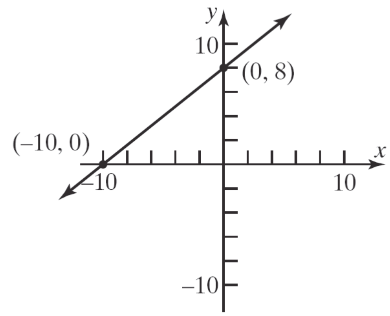
The point $(-10,0)$ is on the graph.

y-intercept: $-4(0)+5y=40$

$$5y=40$$
$$y=8$$

The point $(0,8)$ is on the graph.

b.



102. a. x-intercept: $6x-4(0)=24$

$$6x=24$$
$$x=4$$

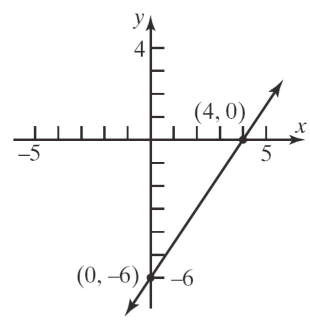
The point $(4,0)$ is on the graph.

y-intercept: $6(0)-4y=24$

$$-4y=24$$
$$y=-6$$

The point $(0,-6)$ is on the graph.

b.



103. a. x-intercept: $7x+2(0)=21$

$$7x=21$$
$$x=3$$

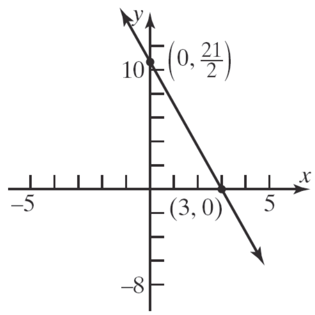
The point $(3,0)$ is on the graph.

y-intercept: $7(0)+2y=21$

$$2y=21$$
$$y=\frac{21}{2}$$

The point $\left(0,\frac{21}{2}\right)$ is on the graph.

b.



104. a. x-intercept: $5x+3(0)=18$

$$5x=18$$
$$x=\frac{18}{5}$$

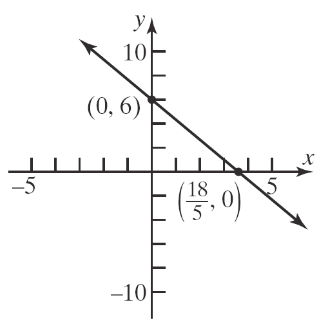
The point $\left(\frac{18}{5},0\right)$ is on the graph.

y-intercept: $5(0)+3y=18$

$$3y=18$$
$$y=6$$

The point $(0,6)$ is on the graph.

b.



105. a. x-intercept: $\frac{1}{2}x+\frac{1}{3}(0)=1$

$$\frac{1}{2}x=1$$
$$x=2$$

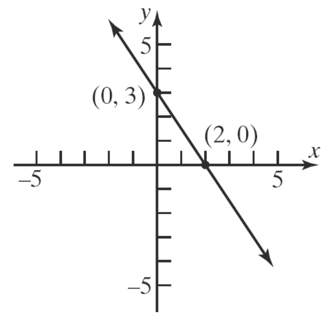
The point $(2,0)$ is on the graph.

y-intercept: $\frac{1}{2}(0)+\frac{1}{3}y=1$

$$\frac{1}{3}y=1$$
$$y=3$$

The point $(0,3)$ is on the graph.

b.



106. a. x-intercept: $x-\frac{2}{3}(0)=4$

$$x=4$$

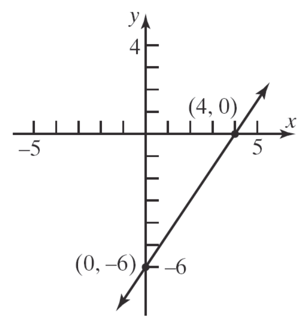
The point $(4,0)$ is on the graph.

y-intercept: $(0)-\frac{2}{3}y=4$

$$-\frac{2}{3}y=4$$
$$y=-6$$

The point $(0,-6)$ is on the graph.

b.



107. a. x-intercept: $0.2x-0.5(0)=1$

$$0.2x=1$$
$$x=5$$

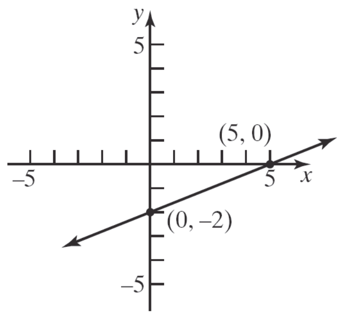
The point $(5,0)$ is on the graph.

y-intercept: $0.2(0)-0.5y=1$

$$-0.5y=1$$
$$y=-2$$

The point $(0,-2)$ is on the graph.

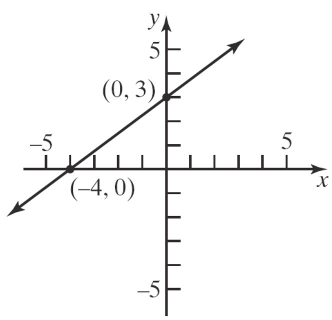
b.



108. a. x-intercept: $-0.3x+0.4(0)=1.2$
$$-0.3x=1.2$$
$$x=-4$$
The point $(-4,0)$ is on the graph.

y-intercept: $-0.3(0)+0.4y=1.2$
$$0.4y=1.2$$
$$y=3$$
The point $(0,3)$ is on the graph.

b.



109. The equation of the x-axis is $y=0$. (The slope is 0 and the y-intercept is 0.)

110. The equation of the y-axis is $x=0$. (The slope is undefined.)

111. The slopes are the same but the y-intercepts are different. Therefore, the two lines are parallel.

112. The slopes are opposite-reciprocals. That is, their product is -1. Therefore, the lines are perpendicular.

113. The slopes are different and their product does not equal -1. Therefore, the lines are neither parallel nor perpendicular.

114. The slopes are different and their product does not equal -1 (in fact, the signs are the same so the product is positive). Therefore, the lines are neither parallel nor perpendicular.

115. Intercepts: $(0,2)$ and $(-2,0)$. Thus, slope = 1.
$y=x+2$ or $x-y=-2$

116. Intercepts: $(0,1)$ and $(1,0)$. Thus, slope = –1.
$y=-x+1$ or $x+y=1$

117. Intercepts: $(3,0)$ and $(0,1)$. Thus, slope = $-\frac{1}{3}$.

$y=-\frac{1}{3}x+1$ or $x+3y=3$

118. Intercepts: $(0,-1)$ and $(-2,0)$. Thus, slope = $-\frac{1}{2}$.

$y=-\frac{1}{2}x-1$ or $x+2y=-2$

119. $P_1=(-2,5)$, $P_2=(1,3)$: $m_1=\frac{5-3}{-2-1}=\frac{2}{-3}=-\frac{2}{3}$

$P_2=(1,3)$, $P_3=(-1,0)$: $m_2=\frac{3-0}{1-(-1)}=\frac{3}{2}$

Since $m_1\cdot m_2=-1$, the line segments $\overline{P_1P_2}$ and $\overline{P_2P_3}$ are perpendicular. Thus, the points P_1, P_2, and P_3 are vertices of a right triangle.

120. $P_1=(1,-1)$, $P_2=(4,1)$, $P_3=(2,2)$, $P_4=(5,4)$

$m_{12}=\frac{1-(-1)}{4-1}=\frac{2}{3}$; $m_{24}=\frac{4-1}{5-4}=3$;

$m_{34}=\frac{4-2}{5-2}=\frac{2}{3}$; $m_{13}=\frac{2-(-1)}{2-1}=3$

Each pair of opposite sides are parallel (same slope) and adjacent sides are not perpendicular. Therefore, the vertices are for a parallelogram.

121. $P_1=(-1,0)$, $P_2=(2,3)$, $P_3=(1,-2)$, $P_4=(4,1)$

$m_{12}=\frac{3-0}{2-(-1)}=\frac{3}{3}=1$; $m_{24}=\frac{1-3}{4-2}=-1$;

$m_{34}=\frac{1-(-2)}{4-1}=\frac{3}{3}=1$; $m_{13}=\frac{-2-0}{1-(-1)}=-1$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). Therefore, the vertices are for a rectangle.

122. $P_1=(0,0)$, $P_2=(1,3)$, $P_3=(4,2)$, $P_4=(3,-1)$

$m_{12}=\dfrac{3-0}{1-0}=3$; $m_{23}=\dfrac{2-3}{4-1}=-\dfrac{1}{3}$;

$m_{34}=\dfrac{-1-2}{3-4}=3$; $m_{14}=\dfrac{-1-0}{3-0}=-\dfrac{1}{3}$

$d_{12}=\sqrt{(1-0)^2+(3-0)^2}=\sqrt{1+9}=\sqrt{10}$

$d_{23}=\sqrt{(4-1)^2+(2-3)^2}=\sqrt{9+1}=\sqrt{10}$

$d_{34}=\sqrt{(3-4)^2+(-1-2)^2}=\sqrt{1+9}=\sqrt{10}$

$d_{14}=\sqrt{(3-0)^2+(-1-0)^2}=\sqrt{9+1}=\sqrt{10}$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). In addition, the length of all four sides is the same. Therefore, the vertices are for a square.

123. Let x = number of miles driven, and let C = cost in dollars.
Total cost = (cost per mile)(number of miles) + fixed cost
$C=0.60x+39$
When $x=110$, $C=(0.60)(110)+39=\$105.00$.
When $x=230$, $C=(0.60)(230)+39=\$177.00$.

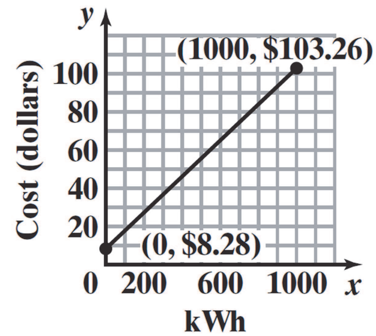
124. Let x = number of pairs of jeans manufactured, and let C = cost in dollars.
Total cost = (cost per pair)(number of pairs) + fixed cost
$C=20x+1200$
When $x=400$, $C=(20)(400)+1200=\$9200$.
When $x=740$, $C=(20)(740)+1200=\$16,000$.

125. Let x = number of miles driven annually, and let C = cost in dollars.
Total cost = (approx cost per mile)(number of miles) + fixed cost
$C=0.14x+4252$

126. Let x = profit in dollars, and let S = salary in dollars.
Weekly salary = (% share of profit)(profit) + weekly pay
$S=0.05x+525$

127. **a.** $C=0.0905x+8.28$; $0\le x\le 1000$

b.



c. For 200 kWh,
$C=0.0905(200)+8.28=\$26.38$

d. For 500 kWh,
$C=0.0905(500)+8.28=\$53.53$

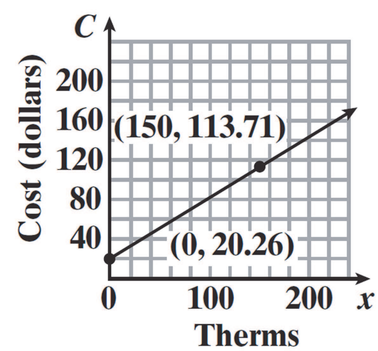
e. For each usage increase of 1 kWh, the monthly charge increases by \$0.0905 (that is, 9.05 cents).

128. **a.** $C=0.623x+20.26$

b. For 90 therms,
$C=0.623(90)+20.26=\$76.33$

c. For 150 therms,
$C=0.623(150)+20.26=\$113.71$

d.



e. For each usage increase of 1 therm the monthly charge increases by \$0.623 (that is, 62.3 cents).

129. $(°C, °F) = (0, 32)$; $(°C, °F) = (100, 212)$

$$\text{slope} = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

$$°F - 32 = \frac{9}{5}(°C - 0)$$

$$°F - 32 = \frac{9}{5}(°C)$$

$$°C = \frac{5}{9}(°F - 32)$$

If $°F = 70$, then

$$°C = \frac{5}{9}(70-32) = \frac{5}{9}(38)$$

$$°C \approx 21.1°$$

130. a. $K = °C + 273$

b. $°C = \frac{5}{9}(°F - 32)$

$$K = \frac{5}{9}(°F - 32) + 273$$

$$K = \frac{5}{9}°F - \frac{160}{9} + 273$$

$$K = \frac{5}{9}°F + \frac{2297}{9}$$

131. a. The y-intercept is (0, 30), so $b = 30$. Since the ramp drops 2 inches for every 25 inches of run, the slope is $m = \frac{-2}{25} = -\frac{2}{25}$. Thus, the equation is $y = -\frac{2}{25}x + 30$.

b. Let $y = 0$.

$$0 = -\frac{2}{25}x + 30$$

$$\frac{2}{25}x = 30$$

$$\frac{25}{2}\left(\frac{2}{25}x\right) = \frac{25}{2}(30)$$

$$x = 375$$

The x-intercept is (375, 0). This means that the ramp meets the floor 375 inches (or 31.25 feet) from the base of the platform.

c. No. From part (b), the run is 31.25 feet which exceeds the required maximum of 30 feet.

d. First, design requirements state that the maximum slope is a drop of 1 inch for each 12 inches of run. This means $|m| \le \frac{1}{12}$.

Second, the run is restricted to be no more than 30 feet = 360 inches. For a rise of 30 inches, this means the minimum slope is $\frac{30}{360} = \frac{1}{12}$. That is, $|m| \ge \frac{1}{12}$. Thus, the only possible slope is $|m| = \frac{1}{12}$. The diagram indicates that the slope is negative. Therefore, the only slope that can be used to obtain the 30-inch rise and still meet design requirements is $m = -\frac{1}{12}$. In words, for every 12 inches of run, the ramp must drop *exactly* 1 inch.

132. a. Let x represent the percent of internet ad spending. Let y represent the percent of print ad spending. Then the points (0.19, 0.26) and (0.35, 0.16) are on the line. Thus, $m = \frac{16-26}{35-19} = -\frac{10}{16} = -0.625$. Using the point-slope formula we have

$$y - 26 = -0.625(x - 19)$$

$$y - 26 = -0.625x + 11.875$$

$$y = -0.625x + 37.875$$

b. x-intercept: $0 = -0.625x + 37.875$

$$-37.875 = -0.625x$$

$$60.6 = x$$

y-intercept: $y = -0.625(0) + 37.875$

$$= 37.875$$

The intercepts are (60.6, 0) and (0, 37.875).

c. y-intercept: When Internet ads account for 0% of U.S. advertisement spending, print ads account for 37.875% of the spending.
x-intercept: When Internet ads account for 60.6% of U.S. advertisement spending, print ads account for 0% of the spending.

d. Let $x = 39.2$.

$$y = -0.625(39.2) + 37.875 = 13.4\%$$

133. a. Let x = number of boxes to be sold, and A = money, in dollars, spent on advertising. We have the points

$$(x_1, A_1) = (100{,}000,\ 40{,}000);$$

$(x_2, A_2) = (200{,}000,\ 60{,}000)$

$$\text{slope} = \frac{60{,}000 - 40{,}000}{200{,}000 - 100{,}000} = \frac{20{,}000}{100{,}000} = \frac{1}{5}$$

$$A - 40{,}000 = \frac{1}{5}(x - 100{,}000)$$

$$A - 40{,}000 = \frac{1}{5}x - 20{,}000$$

$$A = \frac{1}{5}x + 20{,}000$$

b. If $x = 300{,}000$, then

$$A = \frac{1}{5}(300{,}000) + 20{,}000 = \$80{,}000$$

c. Each additional box sold requires an additional \$0.20 in advertising.

134. $2x - y = C$

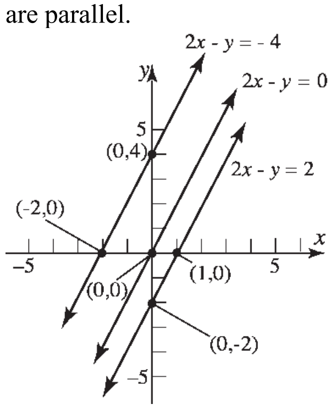
Graph the lines:

$2x - y = -4$

$2x - y = 0$

$2x - y = 2$

All the lines have the same slope, 2. The lines are parallel.



135. Put each linear equation in slope/intercept form.

$$x + 2y = 5 \qquad 2x - 3y + 4 = 0 \qquad ax + y = 0$$
$$2y = -x + 5 \qquad -3y = -2x - 4 \qquad y = -ax$$
$$y = -\frac{1}{2}x + \frac{5}{2} \qquad y = \frac{2}{3}x + \frac{4}{3}$$

If the slope of $y = -ax$ equals the slope of either of the other two lines, then no triangle is formed.

So, $-a = -\frac{1}{2} \Rightarrow a = \frac{1}{2}$ and $-a = \frac{2}{3} \Rightarrow a = -\frac{2}{3}$.

Also if all three lines intersect at a single point, then no triangle is formed. So, we find where $y = -\frac{1}{2}x + \frac{5}{2}$ and $y = \frac{2}{3}x + \frac{4}{3}$ intersect.

$$-\frac{1}{2}x + \frac{5}{2} = \frac{2}{3}x + \frac{4}{3}$$
$$-\frac{7}{6}x = -\frac{7}{6}$$
$$x = 1$$
$$-\frac{1}{2}(1) + \frac{5}{2} = 2$$

The two lines intersect at (1, 2). If $y = -ax$ also contains the point (1, 2), then $2 = -a \cdot 1 \Rightarrow a = -2$.

The three numbers are $\frac{1}{2}, -\frac{2}{3}$, and -2.

136. The slope of the line containing (a,b) and (b,a) is

$$\frac{a-b}{b-a} = -1$$

The slope of the line $y = x$ is 1.

The two lines are perpendicular.

The midpoint of (a,b) and (b,a) is

$$M = \left(\frac{a+b}{2}, \frac{b+a}{2}\right).$$

Since the x and y coordinates of M are equal, M lies on the line $y = x$.

Note: $\frac{a+b}{2} = \frac{b+a}{2}$

137. The three midpoints are

$$\left(\frac{0+a}{2}, \frac{0+0}{2}\right) = \left(\frac{a}{2}, 0\right), \left(\frac{a+b}{2}, \frac{0+c}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$$

and $\left(\frac{0+b}{2}, \frac{0+c}{2}\right) = \left(\frac{b}{2}, \frac{c}{2}\right)$.

Line 1 from (0,0) to $\left(\frac{a+b}{2},\frac{c}{2}\right)$

$$m=\frac{\frac{c}{2}-0}{\frac{a+b}{2}-0}=\frac{c}{a+b};$$

$$y-0=\frac{c}{a+b}(x-0)$$

$$y=\frac{c}{a+b}x_1$$

Line 2 from (a, 0) to $\left(\frac{b}{2},\frac{c}{2}\right)$

$$m=\frac{\frac{c}{2}-0}{\frac{b}{2}-a}=\frac{\frac{c}{2}}{\frac{b-2a}{2}}=\frac{c}{b-2a}$$

$$y-0=\frac{c}{b-2a}(x-a)$$

$$y=\frac{c}{b-2a}(x-a)$$

Line 3 from $\left(\frac{a}{2},0\right)$ to (b,c)

$$m=\frac{c-0}{b-\frac{a}{2}}=\frac{2c}{2b-a}$$

$$y-0=\frac{2c}{2b-a}\left(x-\frac{a}{2}\right)$$

$$y=\frac{2c}{2b-a}\left(x-\frac{a}{2}\right)$$

Find where line 1 and line 2 intersect:

$$\frac{c}{a+b}x=\frac{c}{b-2a}(x-a)$$

$$\frac{b-2a}{a+b}x=x-a$$

$$\frac{b-2a-a-b}{a+b}x=-a$$

$$\frac{-3a}{a+b}x=-a$$

$$x=\frac{a+b}{3};$$

Substitute into line 1:

$$y=\frac{c}{a+b}\cdot\frac{a+b}{3}=\frac{c}{3}.$$

So, line 1 and line 2 intersect at $\left(\frac{a+b}{3},\frac{c}{3}\right)$.

Show that line 3 contains the point $\left(\frac{a+b}{3},\frac{c}{3}\right)$:

$$y=\frac{2c}{2b-a}\left(\frac{a+b}{3}-\frac{a}{2}\right)=\frac{2c}{2b-a}\cdot\frac{2b-a}{6}=\frac{c}{3}$$ So

the three lines intersect at $\left(\frac{a+b}{3},\frac{c}{3}\right)$.

138. Refer to Figure 43. Assume $m_1m_2=-1$. Then

$$\begin{aligned}\left[d(A,B)\right]^2&=(1-1)^2+(m_1-m_2)^2\\&=(m_1-m_2)^2\\&=m_1^{\,2}-2m_1m_2+m_2^{\,2}\\&=m_1^{\,2}-2(-1)+m_2^{\,2}\\&=m_1^{\,2}+m_2^{\,2}+2\end{aligned}$$

Now,

$$\left[d(O,B)\right]^2=(1-0)^2+(m_1-0)^2=1+m_1^{\,2}$$

$$\left[d(O,A)\right]^2=(1-0)^2+(m_2-0)^2=1+m_2^{\,2},$$

So

$$\begin{aligned}\left[d(O,B)\right]^2+\left[d(O,A)\right]^2&=1+m_1^{\,2}+1+m_2^{\,2}\\&=m_1^{\,2}+m_2^{\,2}+2=\left[d(A,B)\right]^2\end{aligned}$$

By the converse of the Pythagorean Theorem, ΔAOB is a right triangle with right angle at vertex *O*. Thus lines OA and OB are perpendicular.

139. (b), (c), (e) and (g)
The line has positive slope and positive *y*-intercept.

140. (a), (c), and (g)
The line has negative slope and positive *y*-intercept.

141. (c)
The equation $x-y=-2$ has slope 1 and *y*-intercept $(0, 2)$. The equation $x-y=1$ has slope 1 and *y*-intercept $(0,-1)$. Thus, the lines are parallel with positive slopes. One line has a positive *y*-intercept and the other with a negative *y*-intercept.

142. (d)
The equation $y-2x=2$ has slope 2 and y-intercept $(0, 2)$. The equation $x+2y=-1$ has slope $-\frac{1}{2}$ and y-intercept $\left(0,-\frac{1}{2}\right)$. The lines are perpendicular since $2\left(-\frac{1}{2}\right)=-1$. One line has a positive y-intercept and the other with a negative y-intercept.

143 – 145. Answers will vary.

146. No, the equation of a vertical line cannot be written in slope-intercept form because the slope is undefined.

147. No, a line does not need to have both an x-intercept and a y-intercept. Vertical and horizontal lines have only one intercept (unless they are a coordinate axis). Every line must have at least one intercept.

148. Two lines with equal slopes and equal y-intercepts are coinciding lines (i.e. the same).

149. Two lines that have the same x-intercept and y-intercept (assuming the x-intercept is not 0) are the same line since a line is uniquely defined by two distinct points.

150. No. Two lines with the same slope and different x-intercepts are distinct parallel lines and have no points in common.
Assume Line 1 has equation $y=mx+b_1$ and Line 2 has equation $y=mx+b_2$,
Line 1 has x-intercept $-\frac{b_1}{m}$ and y-intercept b_1.
Line 2 has x-intercept $-\frac{b_2}{m}$ and y-intercept b_2.
Assume also that Line 1 and Line 2 have unequal x-intercepts.
If the lines have the same y-intercept, then $b_1=b_2$.

$$b_1=b_2 \Rightarrow \frac{b_1}{m}=\frac{b_2}{m} \Rightarrow -\frac{b_1}{m}=-\frac{b_2}{m}$$

But $-\frac{b_1}{m}=-\frac{b_2}{m} \Rightarrow$ Line 1 and Line 2 have the same x-intercept, which contradicts the original assumption that the lines have unequal x-intercepts. Therefore, Line 1 and Line 2 cannot have the same y-intercept.

151. Yes. Two distinct lines with the same y-intercept, but different slopes, can have the same x-intercept if the x-intercept is $x=0$.
Assume Line 1 has equation $y=m_1x+b$ and Line 2 has equation $y=m_2x+b$,
Line 1 has x-intercept $-\frac{b}{m_1}$ and y-intercept b.
Line 2 has x-intercept $-\frac{b}{m_2}$ and y-intercept b.
Assume also that Line 1 and Line 2 have unequal slopes, that is $m_1 \neq m_2$.
If the lines have the same x-intercept, then $-\frac{b}{m_1}=-\frac{b}{m_2}$.

$$\begin{aligned} -\frac{b}{m_1}&=-\frac{b}{m_2} \\ -m_2b&=-m_1b \\ -m_2b+m_1b&=0 \end{aligned}$$

But $-m_2b+m_1b=0 \Rightarrow b(m_1-m_2)=0$
$\Rightarrow b=0$
or $m_1-m_2=0 \Rightarrow m_1=m_2$

Since we are assuming that $m_1 \neq m_2$, the only way that the two lines can have the same x-intercept is if $b=0$.

152. Answers will vary.

153. $m=\frac{y_2-y_1}{x_2-x_1}=\frac{-4-2}{1-(-3)}=\frac{-6}{4}=-\frac{3}{2}$
It appears that the student incorrectly found the slope by switching the direction of one of the subtractions.

Section 1.6

1. add; $\left(\frac{1}{2}\cdot 10\right)^2=25$

2. $(x-2)^2=9$

$$\begin{aligned} x-2&=\pm\sqrt{9} \\ x-2&=\pm 3 \\ x&=2\pm 3 \\ x=5 \quad &\text{or} \quad x=-1 \end{aligned}$$

The solution set is $\{-1, 5\}$.

3. **a.** Center: $(0,0)$; radius 1; $x^2+y^2=1$

b. $x^2+y^2=9$

c. $x^2+y^2=25$

d. Center: $(2,1)$; $(x-2)^2+(y-1)^2=9$

e. Center: $(-2,1)$; $(x+2)^2+(y-1)^2=9$

f. Center: $(-2,-2)$; $(x+2)^2+(y+2)^2=9$

g. Center: $(5,-7)$; radius 6

4. **a.** Center: $(4,0)$; $(x-4)^2+y^2=9$; No y-intercepts; x-intercepts: 1,7

b. Center: $(4,-3)$; $(x-4)^2+(y+3)^2=9$; No y-intercepts; x-intercepts: 4

c. $(x-4)^2+(y+4)^2=9$; No y-intercepts; No x-intercepts

d. One x-intercept

e. No y-intercepts

5. False. For example, $x^2+y^2+2x+2y+8=0$ is not a circle. It has no real solutions.

6. radius

7. True; $r^2=9\to r=3$

8. False; the center of the circle $(x+3)^2+(y-2)^2=13$ is $(-3,2)$.

9. d

10. a

11. Center = (2, 1)
Radius = distance from (0,1) to (2,1)

$$=\sqrt{(2-0)^2+(1-1)^2}=\sqrt{4}=2$$

Equation: $(x-2)^2+(y-1)^2=4$

12. Center = (1, 2)
Radius = distance from (1,0) to (1,2)

$$=\sqrt{(1-1)^2+(2-0)^2}=\sqrt{4}=2$$

Equation: $(x-1)^2+(y-2)^2=4$

13. Center = midpoint of (1, 2) and (4, 2)

$$=\left(\frac{1+4}{2},\frac{2+2}{2}\right)=\left(\frac{5}{2},2\right)$$

Radius = distance from $\left(\frac{5}{2},2\right)$ to (4,2)

$$=\sqrt{\left(4-\frac{5}{2}\right)^2+(2-2)^2}=\sqrt{\frac{9}{4}}=\frac{3}{2}$$

Equation: $\left(x-\frac{5}{2}\right)^2+(y-2)^2=\frac{9}{4}$

14. Center = midpoint of (0, 1) and (2, 3)

$$=\left(\frac{0+2}{2},\frac{1+3}{2}\right)=(1,2)$$

Radius = distance from $(1,2)$ to (2,3)

$$=\sqrt{(2-1)^2+(3-2)^2}=\sqrt{2}$$

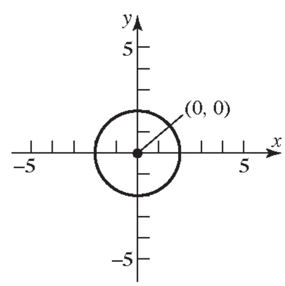
Equation: $(x-1)^2+(y-2)^2=2$

15. $(x-h)^2+(y-k)^2=r^2$

$(x-0)^2+(y-0)^2=2^2$

$x^2+y^2=4$

General form: $x^2+y^2-4=0$

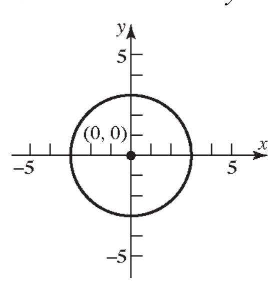


16. $(x-h)^2+(y-k)^2=r^2$

$(x-0)^2+(y-0)^2=3^2$

$x^2+y^2=9$

General form: $x^2+y^2-9=0$



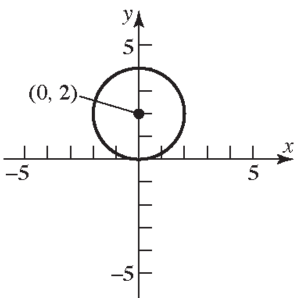
17. $(x-h)^2+(y-k)^2=r^2$

$$(x-0)^2+(y-2)^2=2^2$$

$$x^2+(y-2)^2=4$$

General form: $x^2+y^2-4y+4=4$

$$x^2+y^2-4y=0$$



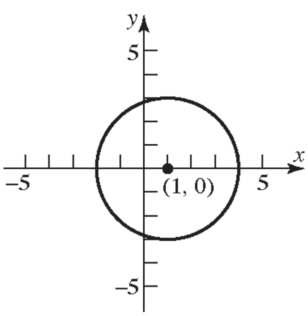
18. $(x-h)^2+(y-k)^2=r^2$

$$(x-1)^2+(y-0)^2=3^2$$

$$(x-1)^2+y^2=9$$

General form: $x^2-2x+1+y^2=9$

$$x^2+y^2-2x-8=0$$



19. $(x-h)^2+(y-k)^2=r^2$

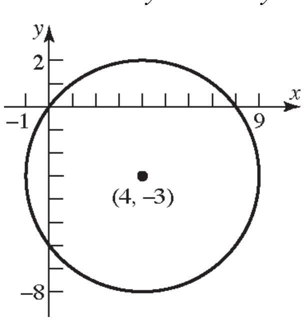
$$(x-4)^2+(y-(-3))^2=5^2$$

$$(x-4)^2+(y+3)^2=25$$

General form:

$$x^2-8x+16+y^2+6y+9=25$$

$$x^2+y^2-8x+6y=0$$



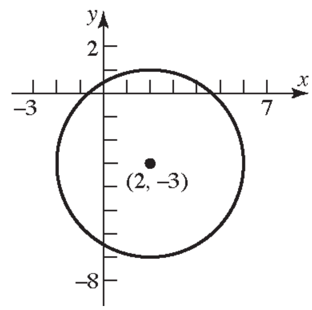
20. $(x-h)^2+(y-k)^2=r^2$

$$(x-2)^2+(y-(-3))^2=4^2$$

$$(x-2)^2+(y+3)^2=16$$

General form: $x^2-4x+4+y^2+6y+9=16$

$$x^2+y^2-4x+6y-3=0$$



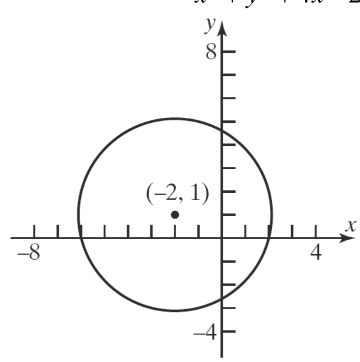
21. $(x-h)^2+(y-k)^2=r^2$

$$(x-(-2))^2+(y-1)^2=4^2$$

$$(x+2)^2+(y-1)^2=16$$

General form: $x^2+4x+4+y^2-2y+1=16$

$$x^2+y^2+4x-2y-11=0$$



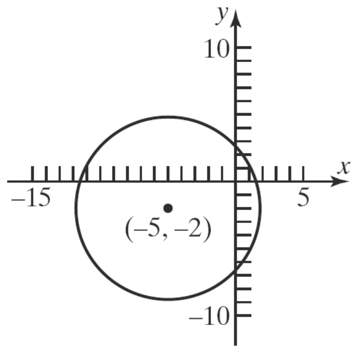
22. $(x-h)^2+(y-k)^2=r^2$

$$(x-(-5))^2+(y-(-2))^2=7^2$$

$$(x+5)^2+(y+2)^2=49$$

General form: $x^2 + 10x + 25 + y^2 + 4y + 4 = 49$

$$x^2 + y^2 + 10x + 4y - 20 = 0$$



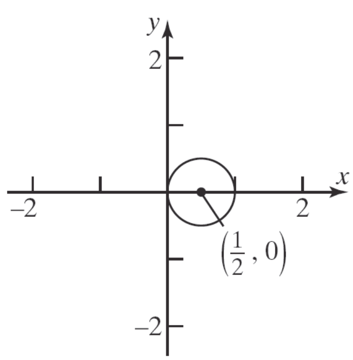
23. $(x-h)^2 + (y-k)^2 = r^2$

$$\left(x - \frac{1}{2}\right)^2 + (y-0)^2 = \left(\frac{1}{2}\right)^2$$

$$\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$$

General form: $x^2 - x + \frac{1}{4} + y^2 = \frac{1}{4}$

$$x^2 + y^2 - x = 0$$



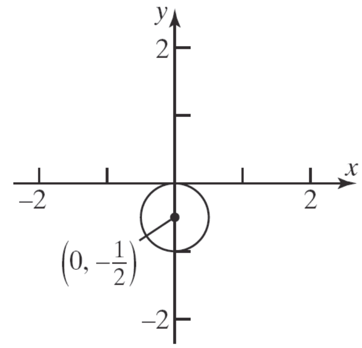
24. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + \left(y - \left(-\frac{1}{2}\right)\right)^2 = \left(\frac{1}{2}\right)^2$$

$$x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{4}$$

General form: $x^2 + y^2 + y + \frac{1}{4} = \frac{1}{4}$

$$x^2 + y^2 + y = 0$$



25. $(x-h)^2 + (y-k)^2 = r^2$

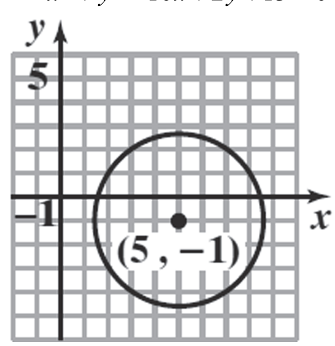
$$(x-5)^2 + (y-(-1))^2 = \left(\sqrt{13}\right)^2$$

$$(x-5)^2 + (y+1)^2 = 13$$

General form:

$$x^2 - 10x + 25 + y^2 + 2y + 1 = 13$$

$$x^2 + y^2 - 10x + 2y + 13 = 0$$



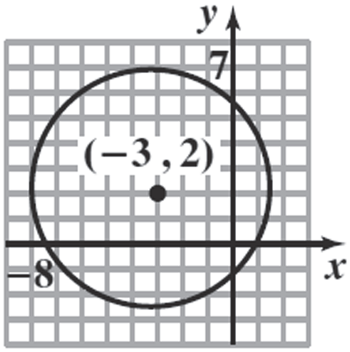
26. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-(-3))^2 + (y-2)^2 = \left(2\sqrt{5}\right)^2$$

$$(x+3)^2 + (y-2)^2 = 20$$

General form: $x^2+6x+9+y^2-4y+4=20$

$$x^2+y^2+6x-4y-7=0$$

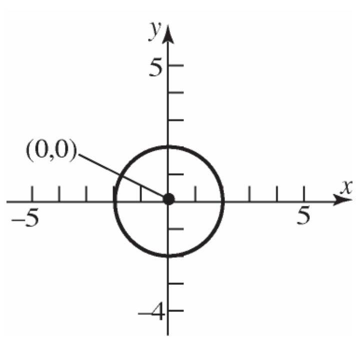


27. $x^2+y^2=4$

$x^2+y^2=2^2$

a. Center: $(0,0)$; Radius $=2$

b.



c. x-intercepts: $x^2+(0)^2=4$

$$x^2=4$$
$$x=\pm\sqrt{4}=\pm 2$$

y-intercepts: $(0)^2+y^2=4$

$$y^2=4$$
$$y=\pm\sqrt{4}=\pm 2$$

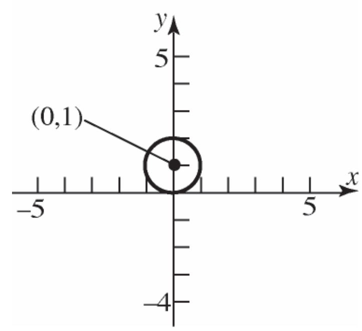
The intercepts are $(-2,0)$, $(2,0)$, $(0,-2)$, and $(0,2)$.

28. $x^2+(y-1)^2=1$

$x^2+(y-1)^2=1^2$

a. Center:$(0,1)$; Radius $=1$

b.



c. x-intercepts: $x^2+(0-1)^2=1$

$$x^2+1=1$$
$$x^2=0$$
$$x=\pm\sqrt{0}=0$$

y-intercepts: $(0)^2+(y-1)^2=1$

$$(y-1)^2=1$$
$$y-1=\pm\sqrt{1}$$
$$y-1=\pm 1$$
$$y=1\pm 1$$
$$y=2 \quad \text{or} \quad y=0$$

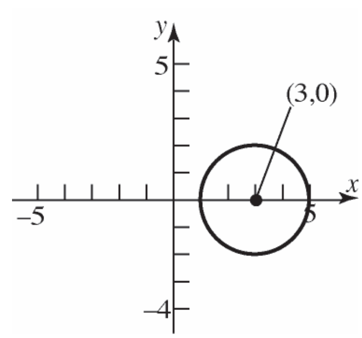
The intercepts are $(0,0)$ and $(0,2)$.

29. $2(x-3)^2+2y^2=8$

$(x-3)^2+y^2=4$

a. Center: $(3, 0)$; Radius $=2$

b.



c. x-intercepts: $(x-3)^2+(0)^2=4$

$$(x-3)^2=4$$
$$x-3=\pm\sqrt{4}$$
$$x-3=\pm 2$$
$$x=3\pm 2$$
$$x=5 \quad \text{or} \quad x=1$$

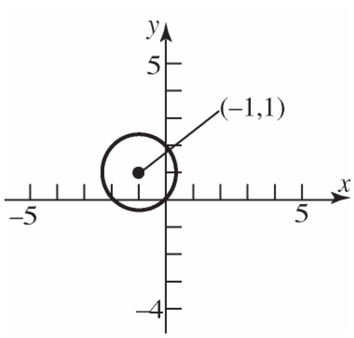
y-intercepts: $(0-3)^2+y^2=4$
$$(-3)^2+y^2=4$$
$$9+y^2=4$$
$$y^2=-5$$
No real solution.

The intercepts are $(1,0)$ and $(5,0)$.

30. $3(x+1)^2+3(y-1)^2=6$
$$(x+1)^2+(y-1)^2=2$$

a. Center: $(-1,1)$; Radius $=\sqrt{2}$

b.



c. x-intercepts: $(x+1)^2+(0-1)^2=2$
$$(x+1)^2+(-1)^2=2$$
$$(x+1)^2+1=2$$
$$(x+1)^2=1$$
$$x+1=\pm\sqrt{1}$$
$$x+1=\pm 1$$
$$x=-1\pm 1$$
$$x=0 \quad\text{or}\quad x=-2$$

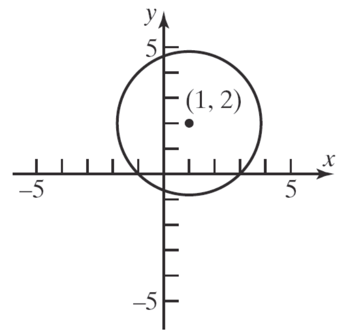
y-intercepts: $(0+1)^2+(y-1)^2=2$
$$(1)^2+(y-1)^2=2$$
$$1+(y-1)^2=2$$
$$(y-1)^2=1$$
$$y-1=\pm\sqrt{1}$$
$$y-1=\pm 1$$
$$y=1\pm 1$$
$$y=2 \quad\text{or}\quad y=0$$

The intercepts are $(-2,0)$, $(0,0)$, and $(0,2)$.

31. $x^2+y^2-2x-4y-4=0$
$$x^2-2x+y^2-4y=4$$
$$(x^2-2x+1)+(y^2-4y+4)=4+1+4$$
$$(x-1)^2+(y-2)^2=3^2$$

a. Center: $(1, 2)$; Radius $=3$

b.



c. x-intercepts: $(x-1)^2+(0-2)^2=3^2$
$$(x-1)^2+(-2)^2=3^2$$
$$(x-1)^2+4=9$$
$$(x-1)^2=5$$
$$x-1=\pm\sqrt{5}$$
$$x=1\pm\sqrt{5}$$

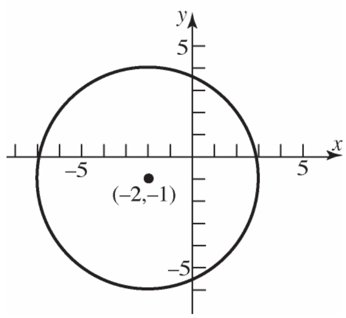
y-intercepts: $(0-1)^2+(y-2)^2=3^2$
$$(-1)^2+(y-2)^2=3^2$$
$$1+(y-2)^2=9$$
$$(y-2)^2=8$$
$$y-2=\pm\sqrt{8}$$
$$y-2=\pm 2\sqrt{2}$$
$$y=2\pm 2\sqrt{2}$$

The intercepts are $(1-\sqrt{5},0)$, $(1+\sqrt{5},0)$, $(0, 2-2\sqrt{2})$, and $(0, 2+2\sqrt{2})$.

32. $x^2+y^2+4x+2y-20=0$
$$x^2+4x+y^2+2y=20$$
$$(x^2+4x+4)+(y^2+2y+1)=20+4+1$$
$$(x+2)^2+(y+1)^2=5^2$$

a. Center: $(-2,-1)$; Radius $=5$

b.



c. x-intercepts: $(x+2)^2+(0+1)^2=5^2$

$$(x+2)^2+1=25$$
$$(x+2)^2=24$$
$$x+2=\pm\sqrt{24}$$
$$x+2=\pm 2\sqrt{6}$$
$$x=-2\pm 2\sqrt{6}$$

y-intercepts: $(0+2)^2+(y+1)^2=5^2$

$$4+(y+1)^2=25$$
$$(y+1)^2=21$$
$$y+1=\pm\sqrt{21}$$
$$y=-1\pm\sqrt{21}$$

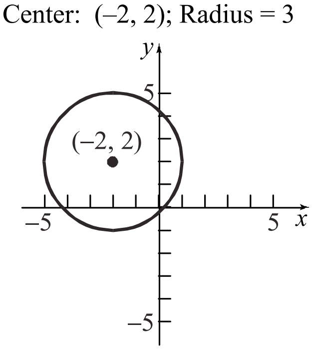
The intercepts are $\left(-2-2\sqrt{6},0\right)$, $\left(-2+2\sqrt{6},0\right)$, $\left(0,-1-\sqrt{21}\right)$, and $\left(0,-1+\sqrt{21}\right)$.

33.

$$x^2+y^2+4x-4y-1=0$$
$$x^2+4x+y^2-4y=1$$
$$(x^2+4x+4)+(y^2-4y+4)=1+4+4$$
$$(x+2)^2+(y-2)^2=3^2$$

a. Center: (–2, 2); Radius = 3

b.



c. x-intercepts: $(x+2)^2+(0-2)^2=3^2$

$$(x+2)^2+4=9$$
$$(x+2)^2=5$$
$$x+2=\pm\sqrt{5}$$
$$x=-2\pm\sqrt{5}$$

y-intercepts: $(0+2)^2+(y-2)^2=3^2$

$$4+(y-2)^2=9$$
$$(y-2)^2=5$$
$$y-2=\pm\sqrt{5}$$
$$y=2\pm\sqrt{5}$$

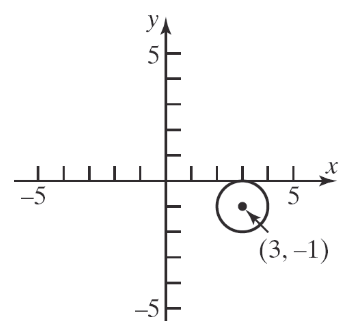
The intercepts are $\left(-2-\sqrt{5},0\right)$, $\left(-2+\sqrt{5},0\right)$, $\left(0,2-\sqrt{5}\right)$, and $\left(0,2+\sqrt{5}\right)$.

34.

$$x^2+y^2-6x+2y+9=0$$
$$x^2-6x+y^2+2y=-9$$
$$(x^2-6x+9)+(y^2+2y+1)=-9+9+1$$
$$(x-3)^2+(y+1)^2=1^2$$

a. Center: (3, –1); Radius = 1

b.



c. x-intercepts: $(x-3)^2+(0+1)^2=1^2$

$$(x-3)^2+1=1$$
$$(x-3)^2=0$$
$$x-3=0$$
$$x=3$$

y-intercepts: $(0-3)^2+(y+1)^2=1^2$

$$9+(y+1)^2=1$$
$$(y+1)^2=-8$$

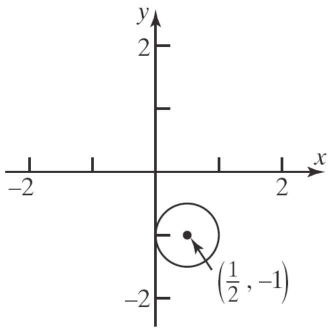
No real solution.

The intercept only intercept is $(3,0)$.

35.
$$x^2+y^2-x+2y+1=0$$
$$x^2-x+y^2+2y=-1$$
$$\left(x^2-x+\frac{1}{4}\right)+(y^2+2y+1)=-1+\frac{1}{4}+1$$
$$\left(x-\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{1}{2}\right)^2$$

a. Center: $\left(\frac{1}{2},-1\right)$; Radius $=\frac{1}{2}$

b.



c. x-intercepts: $\left(x-\frac{1}{2}\right)^2+(0+1)^2=\left(\frac{1}{2}\right)^2$
$$\left(x-\frac{1}{2}\right)^2+1=\frac{1}{4}$$
$$\left(x-\frac{1}{2}\right)^2=-\frac{3}{4}$$

No real solutions

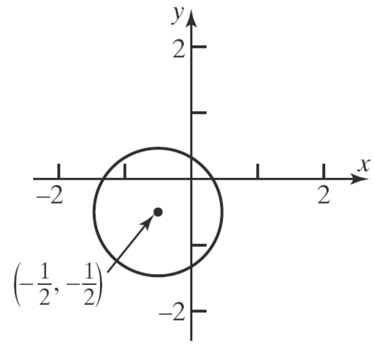
y-intercepts: $\left(0-\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{1}{2}\right)^2$
$$\frac{1}{4}+(y+1)^2=\frac{1}{4}$$
$$(y+1)^2=0$$
$$y+1=0$$
$$y=-1$$

The only intercept is $(0,-1)$.

36.
$$x^2+y^2+x+y-\frac{1}{2}=0$$
$$x^2+x+y^2+y=\frac{1}{2}$$
$$\left(x^2+x+\frac{1}{4}\right)+\left(y^2+y+\frac{1}{4}\right)=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}$$
$$\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1^2$$

a. Center: $\left(-\frac{1}{2},-\frac{1}{2}\right)$; Radius $=1$

b.



c. x-intercepts: $\left(x+\frac{1}{2}\right)^2+\left(0+\frac{1}{2}\right)^2=1^2$
$$\left(x+\frac{1}{2}\right)^2+\frac{1}{4}=1$$
$$\left(x+\frac{1}{2}\right)^2=\frac{3}{4}$$
$$x+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}$$
$$x=\frac{-1\pm\sqrt{3}}{2}$$

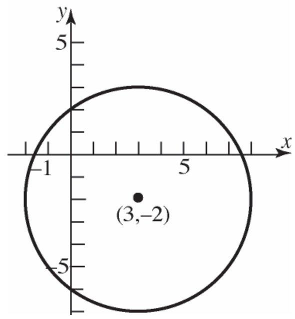
y-intercepts: $\left(0+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1^2$
$$\frac{1}{4}+\left(y+\frac{1}{2}\right)^2=1$$
$$\left(y+\frac{1}{2}\right)^2=\frac{3}{4}$$
$$y+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}$$
$$y=\frac{-1\pm\sqrt{3}}{2}$$

The intercepts are $\left(\frac{-1-\sqrt{3}}{2},0\right)$, $\left(\frac{-1+\sqrt{3}}{2},0\right)$, $\left(0,\frac{-1-\sqrt{3}}{2}\right)$, and $\left(0,\frac{-1+\sqrt{3}}{2}\right)$.

37.
$$2x^2+2y^2-12x+8y-24=0$$
$$x^2+y^2-6x+4y=12$$
$$x^2-6x+y^2+4y=12$$
$$(x^2-6x+9)+(y^2+4y+4)=12+9+4$$
$$(x-3)^2+(y+2)^2=5^2$$

a. Center: (3,–2); Radius = 5

b.



c. x-intercepts: $(x-3)^2+(0+2)^2=5^2$

$$(x-3)^2+4=25$$
$$(x-3)^2=21$$
$$x-3=\pm\sqrt{21}$$
$$x=3\pm\sqrt{21}$$

y-intercepts: $(0-3)^2+(y+2)^2=5^2$

$$9+(y+2)^2=25$$
$$(y+2)^2=16$$
$$y+2=\pm 4$$
$$y=-2\pm 4$$
$$y=2 \quad \text{or} \quad y=-6$$

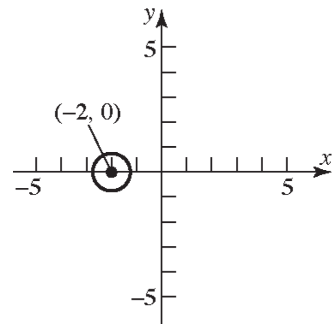
The intercepts are $\left(3-\sqrt{21},0\right)$, $\left(3+\sqrt{21},0\right)$, $\left(0,-6\right)$, and $\left(0,2\right)$.

38. a. $2x^2+2y^2+8x+7=0$

$$2x^2+8x+2y^2=-7$$
$$x^2+4x+y^2=-\frac{7}{2}$$
$$(x^2+4x+4)+y^2=-\frac{7}{2}+4$$
$$(x+2)^2+y^2=\frac{1}{2}$$
$$(x+2)^2+y^2=\left(\frac{\sqrt{2}}{2}\right)^2$$

Center: (–2, 0); Radius = $\dfrac{\sqrt{2}}{2}$

b.



c. x-intercepts: $(x+2)^2+\left(0\right)^2=\frac{1}{2}$

$$(x+2)^2=\frac{1}{2}$$
$$x+2=\pm\sqrt{\frac{1}{2}}$$
$$x+2=\pm\frac{\sqrt{2}}{2}$$
$$x=-2\pm\frac{\sqrt{2}}{2}$$

y-intercepts: $(0+2)^2+y^2=\frac{1}{2}$

$$4+y^2=\frac{1}{2}$$
$$y^2=-\frac{7}{2}$$

No real solutions.

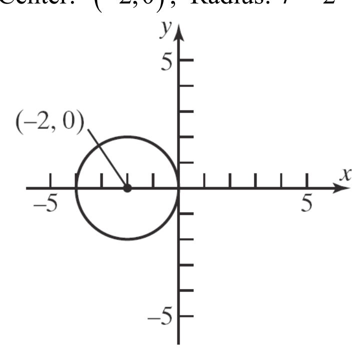
The intercepts are $\left(-2-\frac{\sqrt{2}}{2},0\right)$ and $\left(-2+\frac{\sqrt{2}}{2},0\right)$.

39. $2x^2+8x+2y^2=0$

$$x^2+4x+y^2=0$$
$$x^2+4x+4+y^2=0+4$$
$$(x+2)^2+y^2=2^2$$

a. Center: $\left(-2,0\right)$; Radius: $r=2$

b.



c. x-intercepts:
$$\begin{aligned}(x+2)^2+(0)^2&=2^2\\(x+2)^2&=4\\(x+2)^2&=\pm\sqrt{4}\\x+2&=\pm 2\\x&=-2\pm 2\\x=0 \quad &\text{or} \quad x=-4\end{aligned}$$

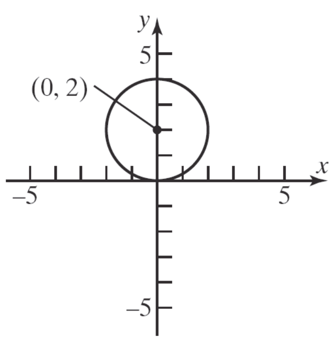
y-intercepts:
$$\begin{aligned}(0+2)^2+y^2&=2^2\\4+y^2&=4\\y^2&=0\\y&=0\end{aligned}$$

The intercepts are $(-4,0)$ and $(0,0)$.

40.
$$\begin{aligned}3x^2+3y^2-12y&=0\\x^2+y^2-4y&=0\\x^2+y^2-4y+4&=0+4\\x^2+(y-2)^2&=4\end{aligned}$$

a. Center: $(0,2)$; Radius: $r=2$

b.



c. x-intercepts:
$$\begin{aligned}x^2+(0-2)^2&=4\\x^2+4&=4\\x^2&=0\\x&=0\end{aligned}$$

y-intercepts:
$$\begin{aligned}0^2+(y-2)^2&=4\\(y-2)^2&=4\\y-2&=\pm\sqrt{4}\\y-2&=\pm 2\\y&=2\pm 2\\y=4 \quad &\text{or} \quad y=0\end{aligned}$$

The intercepts are $(0,0)$ and $(0,4)$.

41. Center at (0, 0); containing point (–2, 3).
$$r=\sqrt{(-2-0)^2+(3-0)^2}=\sqrt{4+9}=\sqrt{13}$$
Equation:
$$\begin{aligned}(x-0)^2+(y-0)^2&=\left(\sqrt{13}\right)^2\\x^2+y^2&=13\end{aligned}$$

42. Center at (1, 0); containing point (–3, 2).
$$r=\sqrt{(-3-1)^2+(2-0)^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$$
Equation:
$$\begin{aligned}(x-1)^2+(y-0)^2&=\left(\sqrt{20}\right)^2\\(x-1)^2+y^2&=20\end{aligned}$$

43. Endpoints of a diameter are (1, 4) and (–3, 2).
The center is at the midpoint of that diameter:

Center: $\left(\dfrac{1+(-3)}{2},\dfrac{4+2}{2}\right)=(-1,3)$

Radius: $r=\sqrt{(1-(-1))^2+(4-3)^2}=\sqrt{4+1}=\sqrt{5}$

Equation:
$$\begin{aligned}(x-(-1))^2+(y-3)^2&=\left(\sqrt{5}\right)^2\\(x+1)^2+(y-3)^2&=5\end{aligned}$$

44. Endpoints of a diameter are (4, 3) and (0, 1).
The center is at the midpoint of that diameter:

Center: $\left(\dfrac{4+0}{2},\dfrac{3+1}{2}\right)=(2,2)$

Radius: $r=\sqrt{(4-2)^2+(3-2)^2}=\sqrt{4+1}=\sqrt{5}$

Equation:
$$\begin{aligned}(x-2)^2+(y-2)^2&=\left(\sqrt{5}\right)^2\\(x-2)^2+(y-2)^2&=5\end{aligned}$$

45.
$$\begin{aligned}C&=2\pi r\\16\pi&=2\pi r\\r&=8\end{aligned}$$

$$\begin{aligned}(x-2)^2+(y-(-4))^2&=(8)^2\\(x-2)^2+(y+4)^2&=64\end{aligned}$$

46.
$$\begin{aligned}A&=\pi r^2\\49\pi&=\pi r^2\\r&=7\end{aligned}$$

$$\begin{aligned}(x-(-5))^2+(y-6)^2&=(7)^2\\(x+5)^2+(y-6)^2&=49\end{aligned}$$

47. (c); Center: $(1,-2)$; Radius = 2

48. (d) ; Center: $(-3,3)$; Radius = 3

49. (b) ; Center: $(-1,2)$; Radius = 2

50. (a) ; Center: $(3,-3)$; Radius = 3

51. The centers of the circles are: (4,-2) and (-1,5).

The slope is $m=\dfrac{5-(-2)}{-1-4}=\dfrac{7}{-5}=-\dfrac{7}{5}$. Use the slope and one point to find the equation of the line.

$$y-(-2)=-\frac{7}{5}(x-4)$$
$$y+2=-\frac{7}{5}x+\frac{28}{5}$$
$$5y+10=-7x+28$$
$$7x+5y=18$$

52. Find the centers of the two circles:

$$x^2+y^2-4x+6y+4=0$$
$$(x^2-4x+4)+(y^2+6y+9)=-4+4+9$$
$$(x-2)^2+(y+3)^2=9$$

Center: $(2,-3)$

$$x^2+y^2+6x+4y+9=0$$
$$(x^2+6x+9)+(y^2+4y+4)=-9+9+4$$
$$(x+3)^2+(y+2)^2=4$$

Center: $(-3,-2)$

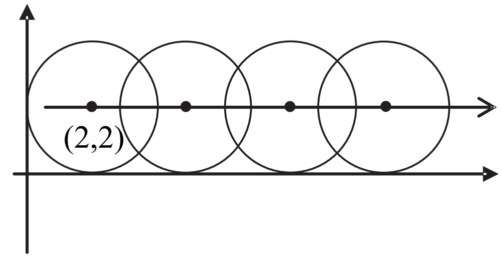
Find the slope of the line containing the centers:

$$m=\frac{-2-(-3)}{-3-2}=-\frac{1}{5}$$

Find the equation of the line containing the centers:

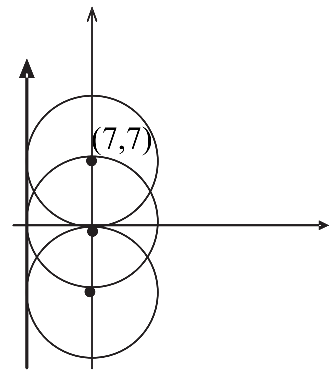
$$y+3=-\frac{1}{5}(x-2)$$
$$5y+15=-x+2$$
$$x+5y=-13$$
$$x+5y+13=0$$

53. Consider the following diagram:



Therefore, the path of the center of the circle has the equation $y=2$.

54. Consider the following diagram:



Therefore the path of the center of the circle has the equation $x=7$.

55. Let the upper-right corner of the square be the point (x,y). The circle and the square are both centered about the origin. Because of symmetry, we have that $x=y$ at the upper-right corner of the square. Therefore, we get

$$x^2+y^2=9$$
$$x^2+x^2=9$$
$$2x^2=9$$
$$x^2=\frac{9}{2}$$
$$x=\sqrt{\frac{9}{2}}=\frac{3\sqrt{2}}{2}$$

The length of one side of the square is $2x$. Thus, the area is

$$A=s^2=\left(2\cdot\frac{3\sqrt{2}}{2}\right)^2=\left(3\sqrt{2}\right)^2=18 \text{ square units.}$$

56. The area of the shaded region is the area of the circle, less the area of the square. Let the upper-right corner of the square be the point (x, y).

The circle and the square are both centered about the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square.

Therefore, we get

$$x^2 + y^2 = 36$$
$$x^2 + x^2 = 36$$
$$2x^2 = 36$$
$$x^2 = 18$$
$$x = 3\sqrt{2}$$

The length of one side of the square is $2x$. Thus, the area of the square is $\left(2\cdot 3\sqrt{2}\right)^2 = 72$ square units. From the equation of the circle, we have $r = 6$. The area of the circle is $\pi r^2 = \pi\left(6\right)^2 = 36\pi$ square units.

Therefore, the area of the shaded region is $A = 36\pi - 72$ square units.

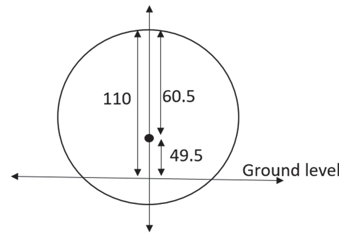
57. The diameter of the Ferris wheel was 250 feet, so the radius was 125 feet. The maximum height was 264 feet, so the center was at a height of $264 - 125 = 139$ feet above the ground. Since the center of the wheel is on the y-axis, it is the point (0, 139). Thus, an equation for the wheel is:

$$\left(x-0\right)^2 + \left(y-139\right)^2 = 125^2$$
$$x^2 + \left(y-139\right)^2 = 15{,}625$$

58. The diameter of the wheel is 520 feet, so the radius is 260 feet. The maximum height is 550 feet, so the center of the wheel is at a height of $550 - 260 = 290$ feet above the ground. Since the center of the wheel is on the y-axis, it is the point (0, 290). Thus, an equation for the wheel is:

$$\left(x-0\right)^2 + \left(y-290\right)^2 = 260^2$$
$$x^2 + \left(y-290\right)^2 = 67{,}600$$

59.

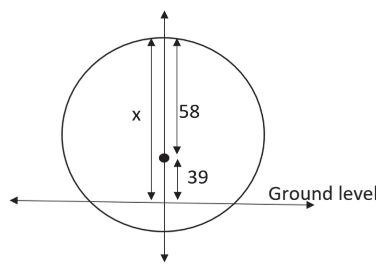


Refer to figure. Since the radius of the building is 60.5 m and the height of the building is 110 m, then the center of the building is 49.5 m above the ground, so the y-coordinate of the center is 49.5. The equation of the circle is given by

$$x^2 + (y-49.5)^2 = 60.5^2 = 3660.25$$

60. Complete the square to find the equation of the circle representing the formula for the building.

$$x^2 + y^2 - 78y + 1521 = 1843 + 1521 = 3364$$
$$x^2 + (y-39)^2 = 58^2$$



Refer to figure. The y coordinate of the center is 39. The radius is 58. Thus the height of the building is 58 + 39 = 97 m.

61. Center at (2, 3); tangent to the x-axis.
$r = 3$

Equation: $(x-2)^2 + (y-3)^2 = 3^2$
$(x-2)^2 + (y-3)^2 = 9$

62. Center at (–3, 1); tangent to the y-axis.
$r = 3$

Equation: $(x+3)^2 + (y-1)^2 = 3^2$
$(x+3)^2 + (y-1)^2 = 9$

63. Center at (–1, 3); tangent to the line $y = 2$. This means that the circle contains the point (–1, 2), so the radius is $r = 1$.

Equation: $(x+1)^2 + (y-3)^2 = (1)^2$
$(x+1)^2 + (y-3)^2 = 1$

64. Center at (4, –2); tangent to the line $x = 1$. This means that the circle contains the point (1, –2), so the radius is $r = 3$.

Equation: $(x-4)^2+(y+2)^2=(3)^2$

$$(x-4)^2+(y+2)^2=9$$

65. a. Substitute $y = mx + b$ into $x^2 + y^2 = r^2$:

$$x^2+(mx+b)^2=r^2$$
$$x^2+m^2x^2+2bmx+b^2=r^2$$
$$(1+m^2)x^2+2bmx+b^2-r^2=0$$

This equation has one solution if and only if the discriminant is zero.

$$(2bm)^2-4(1+m^2)(b^2-r^2)=0$$
$$4b^2m^2-4b^2+4r^2-4b^2m^2+4m^2r^2=0$$
$$-4b^2+4r^2+4m^2r^2=0$$
$$-b^2+r^2+m^2r^2=0$$
$$r^2(1+m^2)=b^2$$

b. From part (a) we know $(1+m^2)x^2+2bmx+b^2-r^2=0$. Using the quadratic formula, since the discriminant is zero, we get:

$$x=\frac{-2bm}{2(1+m^2)}=\frac{-bm}{\left(\frac{b^2}{r^2}\right)}=\frac{-bmr^2}{b^2}=\frac{-mr^2}{b}$$

$$y=m\left(\frac{-mr^2}{b}\right)+b$$
$$=\frac{-m^2r^2}{b}+b=\frac{-m^2r^2+b^2}{b}=\frac{r^2}{b}$$

The point of tangency is $\left(\frac{-mr^2}{b},\frac{r^2}{b}\right)$.

c. The slope of the tangent line is m.
The slope of the line joining the point of tangency and the center (0,0) is:

$$\frac{\left(\frac{r^2}{b}-0\right)}{\left(\frac{-mr^2}{b}-0\right)}=\frac{r^2}{b}\cdot\frac{b}{-mr^2}=-\frac{1}{m}$$

The two lines are perpendicular.

66. Let (h, k) be the center of the circle.

$$x-2y+4=0$$
$$2y=x+4$$
$$y=\frac{1}{2}x+2$$

The slope of the tangent line is $\frac{1}{2}$. The slope from (h, k) to (0, 2) is –2.

$$\frac{2-k}{0-h}=-2$$
$$2-k=2h$$

The other tangent line is $y = 2x - 7$, and it has slope 2.

The slope from (h, k) to (3, –1) is $-\frac{1}{2}$.

$$\frac{-1-k}{3-h}=-\frac{1}{2}$$
$$2+2k=3-h$$
$$2k=1-h$$
$$h=1-2k$$

Solve the two equations in h and k:

$$2-k=2(1-2k)$$
$$2-k=2-4k$$
$$3k=0$$
$$k=0$$
$$h=1-2(0)=1$$

The center of the circle is (1, 0).

67. The slope of the line containing the center (0,0) and $\left(1, 2\sqrt{2}\right)$ is

$\frac{2\sqrt{2}-0}{1-0}=2\sqrt{2}$. Then the slope of the tangent line is $\frac{-1}{2\sqrt{2}}=-\frac{\sqrt{2}}{4}$.

So the equation of the tangent line is:

$$y-2\sqrt{2}=-\frac{\sqrt{2}}{4}(x-1)$$
$$y-2\sqrt{2}=-\frac{\sqrt{2}}{4}x+\frac{\sqrt{2}}{4}$$
$$4y-8\sqrt{2}=-\sqrt{2}x+\sqrt{2}$$
$$\sqrt{2}x+4y=9\sqrt{2}$$

68. $x^2 + y^2 - 4x + 6y + 4 = 0$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -4 + 4 + 9$$

$$(x-2)^2 + (y+3)^2 = 9$$

Center: $(2, -3)$

The slope of the line containing the center and $\left(3, 2\sqrt{2}-3\right)$ is $\dfrac{2\sqrt{2}-3-(-3)}{3-2} = \dfrac{2\sqrt{2}}{1} = 2\sqrt{2}$

Then the slope of the tangent line is: $\dfrac{-1}{2\sqrt{2}} = -\dfrac{\sqrt{2}}{4}$

So, the equation of the tangent line is

$$y - \left(2\sqrt{2}-3\right) = -\frac{\sqrt{2}}{4}(x-3)$$

$$y - 2\sqrt{2} + 3 = -\frac{\sqrt{2}}{4}x + \frac{3\sqrt{2}}{4}$$

$$4y - 8\sqrt{2} + 12 = -\sqrt{2}x + 3\sqrt{2}$$

$$\sqrt{2}x + 4y = 11\sqrt{2} - 12$$

69. The center of the circle is $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$ and the radius is $\dfrac{1}{2}\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$. Then the equation of the circle is $\left(x - \dfrac{x_1+x_2}{2}\right)^2 + \left(y - \dfrac{y_1+y_2}{2}\right)^2 = \dfrac{1}{4}\left[(x_1-x_2)^2 + (y_1-y_2)^2\right]$. Expanding, gives

$$x^2 - x(x_1+x_2) + \frac{(x_1+x_2)^2}{4} + y^2 - y(y_1+y_2) + \frac{(y_1+y_2)^2}{4} = \frac{1}{4}\left[x_1^2 - 2x_1x_2 + x_2^2 + y_1^2 - 2y_1y_2 + y_2^2\right]$$

$$4x^2 - 4x_1x - 4x_2x + x_1^2 + 2x_1x_2 + x_2^2 + 4y^2 - 4y_1y - 4y_2y + y_1^2 + 2y_1y_2 + y_2^2 = x_1^2 - 2x_1x_2 + x_2^2 + y_1^2 - 2y_1y_2 + y_2^2$$

$$4x^2 - 4x_1x - 4x_2x + 4x_1x_2 + 4y^2 - 4y_1y - 4y_2y + 4y_1y_2 = 0$$

$$x^2 - x_1x - x_2x + x_1x_2 + y^2 - y_1y - y_2y + y_1y_2 = 0$$

$$x(x-x_1) - x_2(x+x_1) + y(y-y_1) - y_2(y+y_1) = 0$$

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$

70. Complete the square to get $\left(x + \dfrac{d}{2}\right)^2 + \left(y + \dfrac{e}{2}\right)^2 = \dfrac{d^2 + e^2 - 4f}{4}$. The slope of the line between the center $\left(-\dfrac{d}{2}, -\dfrac{e}{2}\right)$ and the point of tangency (x_0, y_0) is $m = \dfrac{y_0 + \frac{e}{2}}{x_0 + \frac{d}{2}}$. So the slope of the tangent line is $m_{\tan} = -\dfrac{x_0 + \frac{d}{2}}{y_0 + \frac{e}{2}}$.

Therefore, the equation of the tangent line is $y - y_0 = -\dfrac{x_0 + \frac{d}{2}}{y_0 + \frac{e}{2}}(x - x_0)$ which is equivalent to

$$(x-x_0)\left(x_0 + \tfrac{d}{2}\right) + (y-y_0)\left(y_0 + \tfrac{e}{2}\right) = 0$$

$$x_0x + \frac{d}{2}x - x_0^2 - \frac{d}{2}x_0 + y_0y + \frac{e}{2}y - y_0^2 - \frac{e}{2}y_0 = 0$$

$$x_0x + y_0y + \frac{d}{2}x + \frac{e}{2}y - \left(x_0^2 + y_0^2 + \frac{d}{2}x_0 + \frac{e}{2}y_0\right) = 0$$

Because (x_0, y_0) is on the circle, $x_0^2 + y_0^2 + dx_0 + ey_0 + f = 0$, and

$x_0^2 + y_0^2 + \dfrac{d}{2}x_0 + \dfrac{e}{2}y_0 = -\dfrac{d}{2}x_0 - \dfrac{e}{2}y_0 - f$ Substituting this result gives

$$x_0^{\,2} + y_0^{\,2} + \frac{d}{2}x_0 + \frac{e}{2}y_0 - \left(-\frac{d}{2}x_0 - \frac{e}{2}y_0 - f\right) = 0$$

$$x_0^{\,2} + y_0^{\,2} + \frac{d}{2}x_0 + \frac{e}{2}y_0 + \frac{d}{2}x_0 + \frac{e}{2}y_0 + f = 0$$

$$x_0^{\,2} + y_0^{\,2} + d\left(\frac{x + x_0}{2}\right) + e\left(\frac{y + y_0}{2}\right) + f = 0$$

71. (b), (c), (e) and (g)
We need $h,k > 0$ and $(0,0)$ on the graph.

72. (b), (e) and (g)
We need $h < 0$, $k = 0$, and $|h| > r$.

73. Answers will vary.

74. The student has the correct radius, but the signs of the coordinates of the center are incorrect. The student needs to write the equation in the standard form $(x-h)^2 + (y-k)^2 = r^2$.

$$(x+3)^2 + (y-2)^2 = 16$$

$$(x-(-3))^2 + (y-2)^2 = 4^2$$

Thus, $(h,k) = (-3,2)$ and $r = 4$.

Chapter 1 Review

1. $(0,0),(4,2)$

a. $\text{distance} = \sqrt{(4-0)^2 + (2-0)^2}$
$= \sqrt{16+4} = \sqrt{20}$
$= 2\sqrt{5}$

b. $\text{midpoint} = \left(\frac{0+4}{2}, \frac{0+2}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = (2,1)$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$

d. When x increases by 2 units, y increases by 1 unit.

2. $(1,-1),(-2,3)$

a. $\text{distance} = \sqrt{(-2-1)^2 + (3-(-1))^2}$
$= \sqrt{9+16} = \sqrt{25} = 5$

b. $\text{midpoint} = \left(\frac{1+(-2)}{2}, \frac{-1+3}{2}\right)$
$= \left(\frac{-1}{2}, \frac{2}{2}\right) = \left(-\frac{1}{2}, 1\right)$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{3-(-1)}{-2-1} = \frac{4}{-3} = -\frac{4}{3}$

d. When x increases by 3 units, y decreases by 4 units.

3. $(4,-4),(4,8)$

a. $\text{distance} = \sqrt{(4-4)^2 + (8-(-4))^2}$
$= \sqrt{0+144} = \sqrt{144} = 12$

b. $\text{midpoint} = \left(\frac{4+4}{2}, \frac{-4+8}{2}\right)$
$= \left(\frac{8}{2}, \frac{4}{2}\right) = (4,2)$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{8-(-4)}{4-4} = \frac{12}{0}$, undefined

d. Undefined slope means the points lie on a vertical line, in this case $x = 4$.

4. $(-2,-1),(3,-1)$

a. $\text{distance} = \sqrt{(3-(-2))^2 + (-1-(-1))^2}$
$= \sqrt{25+0} = \sqrt{25} = 5$

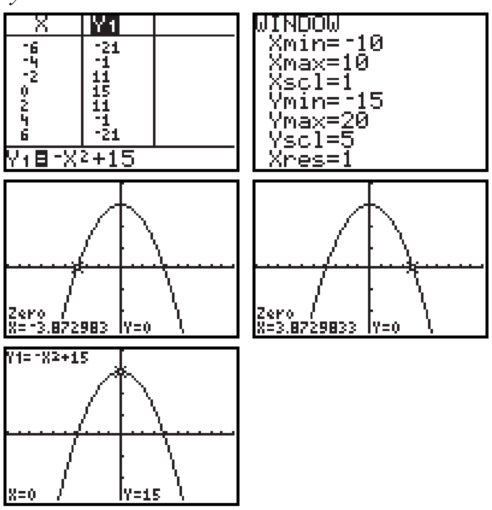
b. $\text{midpoint} = \left(\dfrac{-2+3}{2}, \dfrac{-1+(-1)}{2}\right)$
$= \left(\dfrac{1}{2}, -\dfrac{2}{2}\right) = \left(\dfrac{1}{2}, -1\right)$

c. $\text{slope} = \dfrac{\Delta y}{\Delta x} = \dfrac{-1-(-1)}{3-(-2)} = \dfrac{0}{5} = 0$

d. Slope = 0 means the points lie on a horizontal line, in this case $y = -1$.

5. $(-4,0),(0,0),(2,0)$, $(0,-2),(0,0),(0,2)$

6. $y = -x^2 + 15$



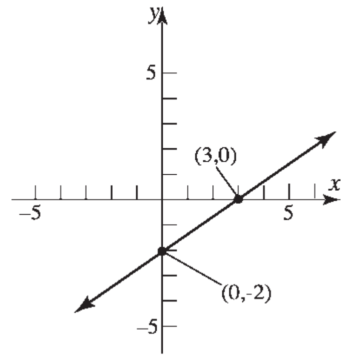
The intercepts are $(0,15)$, $(-3.87,0)$, and $(3.87,0)$.

7. $2x - 3y = 6$
$-3y = -2x + 6$
$y = \dfrac{2}{3}x - 2$

x-intercepts:
$2x - 3(0) = 6$
$2x = 6$
$x = 3$

y-intercept:
$2(0) - 3y = 6$
$-3y = 6$
$y = -2$

The intercepts are $(3,0)$ and $(0,-2)$.

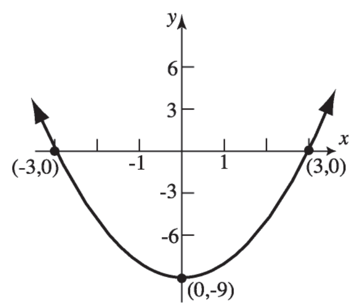


8. $y = x^2 - 9$

x-intercepts:
$0 = x^2 - 9$
$0 = (x-3)(x+3)$
$x = 3$ or $x = -3$

y-intercept:
$y = 0^2 - 9$
$y = -9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,-9)$.

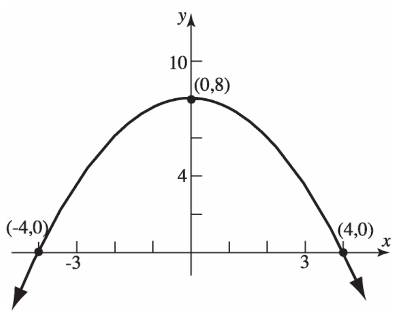


9. $x^2 + 2y = 16$
$2y = -x^2 + 16$
$y = -\dfrac{1}{2}x^2 + 8$

x-intercepts:
$x^2 + 2(0) = 16$
$x^2 = 16$
$x = \pm 4$

y-intercept:
$0^2 + 2y = 16$
$2y = 16$
$y = 8$

The intercepts are $(-4,0)$, $(4,0)$, and $(0,8)$.



10. $2x = 3y^2$

x-axis: $2x = 3(-y)^2$

$2x = 3y^2$ same

y-axis: $2(-x) = 3y^2$

$-2x = 3y^2$ different

origin: $2(-x) = 3(-y)^2$

$-2x = 3y^2$ different

Therefore, the graph of the equation has x-axis symmetry.

11. $x^2 + 4y^2 = 16$

x-axis: $x^2 + 4(-y)^2 = 16$

$x^2 + 4y^2 = 16$ same

y-axis: $(-x)^2 + 4y^2 = 16$

$x^2 + 4y^2 = 16$ same

origin: $(-x)^2 + 4(-y)^2 = 16$

$x^2 + 4y^2 = 16$ same

Therefore, the graph of the equation has x-axis, y-axis, and origin symmetry.

12. $y = x^4 - 3x^2 - 4$

x-intercepts:

$0 = x^4 - 3x^2 - 4$

$0 = (x^2 - 4)(x^2 + 1)$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

y-intercepts:

$y = (0)^4 - 3(0)^2 - 4$

$= -4$

The intercepts are $(-2, 0)$, $(2, 0)$, $(0, -4)$, and $(0, 2)$.

Test *x*-axis symmetry: Let $y = -y$

$-y = x^4 - 3x^2 - 4$

$y = -x^4 + 3x^2 + 4$ different

Test *y*-axis symmetry: Let $x = -x$

$y = (-x)^4 - 3(-x)^2 - 4$

$y = x^4 - 3x^2 - 4$ same

Test origin symmetry: Let $x = -x$ and $y = -y$.

$-y = (-x)^4 - 3(-x)^2 - 4$

$-y = x^4 - 3x^2 - 4$

$y = -x^4 + 3x^2 + 4$ different

Therefore, the graph will have *y*-axis symmetry.

13. $y = x^3 - x$

x-axis: $(-y) = x^3 - x$

$-y = x^3 - x$ different

y-axis: $y = (-x)^3 - (-x)$

$y = -x^3 + x$ different

origin: $(-y) = (-x)^3 - (-x)$

$-y = -x^3 + x$

$y = x^3 - x$ same

Therefore, the equation of the graph has origin symmetry.

14. $x^2 + x + y^2 + 2y = 0$

x-axis: $x^2 + x + (-y)^2 + 2(-y) = 0$

$x^2 + x + y^2 - 2y = 0$ different

y-axis: $(-x)^2 + (-x) + y^2 + 2y = 0$

$x^2 - x + y^2 + 2y = 0$ different

origin:

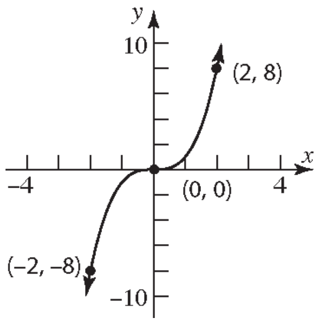
$(-x)^2 + (-x) + (-y)^2 + 2(-y) = 0$

$x^2 - x + y^2 - 2y = 0$ different

Therefore, the graph of the equation has none of the three symmetries.

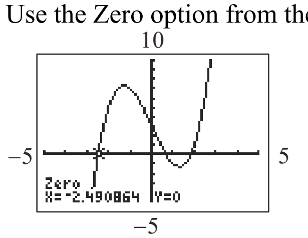
15.

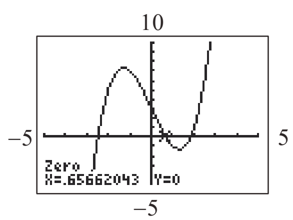
x	$y = x^3$	(x, y)
-2	$y = (-2)^3 = -8$	$(-2, -8)$
0	$y = (0)^3 = 0$	$(0, 0)$
2	$y = (2)^3 = 8$	$(2, 8)$

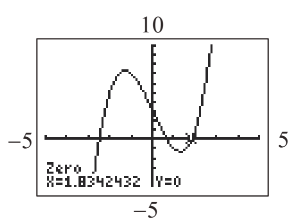


16. $x^3 - 5x + 3 = 0$

Use the Zero option from the CALC menu.



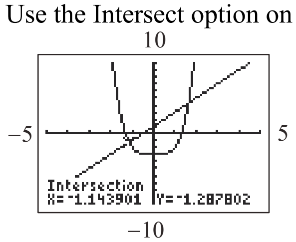


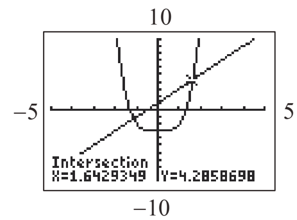


The solution set is $\{-2.49, 0.66, 1.83\}$.

17. $x^4 - 3 = 2x + 1$

Use the Intersect option on the CALC menu.

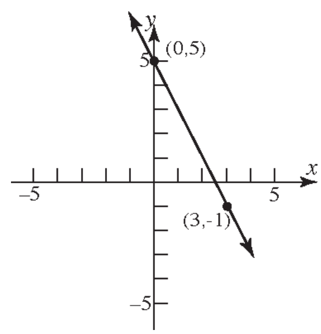




The solution set is $\{-1.14, 1.64\}$.

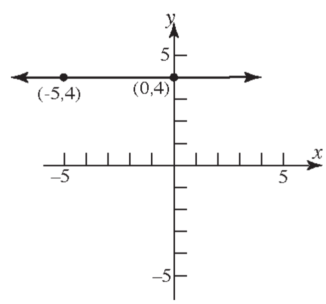
18. Slope = –2; containing (3,–1)

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -2(x - 3)$$
$$y + 1 = -2x + 6$$
$$y = -2x + 5$$
$$2x + y = 5 \text{ or } y = -2x + 5$$

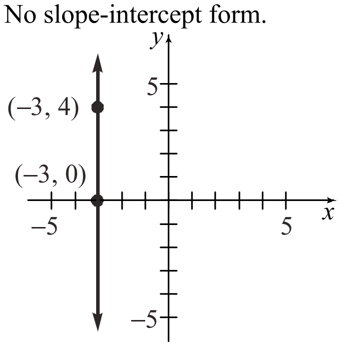


19. Slope = 0; containing the point (–5, 4)

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 0(x - (-5))$$
$$y - 4 = 0$$
$$y = 4$$

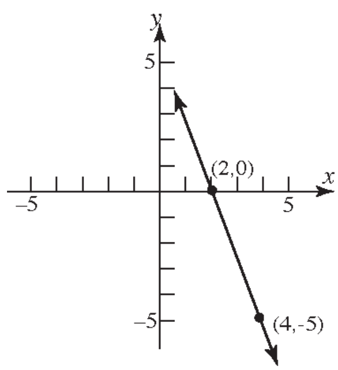


20. Slope undefined; containing (–3, 4).
This is a vertical line.
$x = -3$
No slope-intercept form.



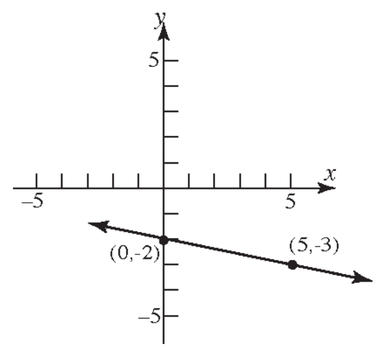
21. x-intercept = 2 or the point (2, 0);
Containing the point (4, –5)

$$m = \frac{-5-0}{4-2} = \frac{-5}{2} = -\frac{5}{2}$$
$$y - y_1 = m(x - x_1)$$
$$y - 0 = -\frac{5}{2}(x-2)$$
$$y = -\frac{5}{2}x + 5$$
$$5x + 2y = 10 \text{ or } y = -\frac{5}{2}x + 5$$



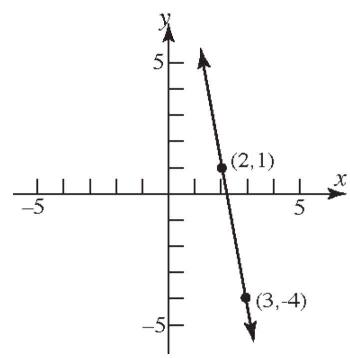
22. y-intercept = –2; containing (5,–3)
Points are (5,–3) and (0,–2)

$$m = \frac{-2-(-3)}{0-5} = \frac{1}{-5} = -\frac{1}{5}$$
$$y = mx + b$$
$$y = -\frac{1}{5}x - 2$$
$$x + 5y = -10 \text{ or } y = -\frac{1}{5}x - 2$$



23. Containing the points (3,–4) and (2, 1)

$$m = \frac{1-(-4)}{2-3} = \frac{5}{-1} = -5$$
$$y - y_1 = m(x - x_1)$$
$$y - (-4) = -5(x-3)$$
$$y + 4 = -5x + 15$$
$$y = -5x + 11$$
$$5x + y = 11 \text{ or } y = -5x + 11$$



24. Parallel to $2x-3y=-4$;

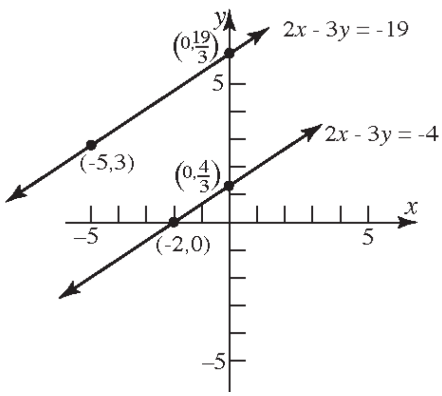
Slope $=\frac{2}{3}$; containing (–5,3)

$$y-y_1=m(x-x_1)$$

$$y-3=\frac{2}{3}(x-(-5)) \rightarrow y-3=\frac{2}{3}x+\frac{10}{3}$$

$$y=\frac{2}{3}x+\frac{19}{3}$$

$2x-3y=-19$ or $y=\frac{2}{3}x+\frac{19}{3}$



25. Perpendicular to $3x-y=-4$;

Slope of perpendicular $=-\frac{1}{3}$

Containing the point (–2, 4)

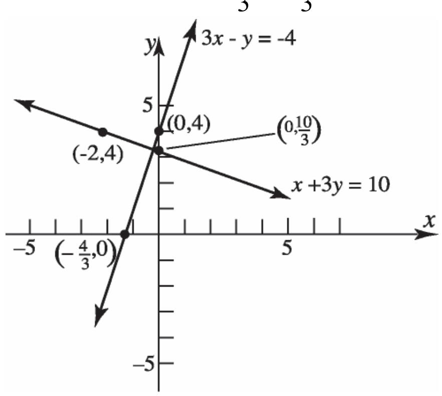
$$y-y_1=m(x-x_1)$$

$$y-4=-\frac{1}{3}(x-(-2))$$

$$y-4=-\frac{1}{3}x-\frac{2}{3}$$

$$y=-\frac{1}{3}x+\frac{10}{3}$$

$x+3y=10$ or $y=-\frac{1}{3}x+\frac{10}{3}$



26. $(x-h)^2+(y-k)^2=r^2$

$$\left(x-(-2)\right)^2+\left(y-3\right)^2=4^2$$

$$\left(x+2\right)^2+\left(y-3\right)^2=16$$

27. $(x-h)^2+(y-k)^2=r^2$

$$\left(x-(-1)\right)^2+\left(y-(-2)\right)^2=1^2$$

$$\left(x+1\right)^2+\left(y+2\right)^2=1$$

28. $x^2+y^2-2x+4y-4=0$

$$x^2-2x+y^2+4y=4$$

$$(x^2-2x+1)+(y^2+4y+4)=4+1+4$$

$$(x-1)^2+(y+2)^2=9$$

$$(x-1)^2+(y+2)^2=3^2$$

Center: (1,–2)   Radius = 3

$$x^2+0^2-2x+4(0)-4=0$$

$$x^2-2x-4=0$$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)}=\frac{2\pm\sqrt{20}}{2}$$

$$=\frac{2\pm2\sqrt{5}}{2}=1\pm\sqrt{5}$$

$$0^2+y^2-2(0)+4y-4=0$$

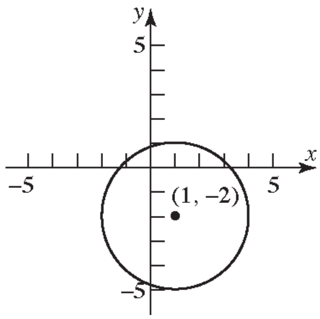
$$y^2+4y-4=0$$

$$y=\frac{-4\pm\sqrt{4^2-4(1)(-4)}}{2(1)}=\frac{-4\pm\sqrt{32}}{2}$$

$$=\frac{-4\pm4\sqrt{2}}{2}=-2\pm2\sqrt{2}$$

Intercepts:

$\left(1-\sqrt{5},0\right)$, $\left(1+\sqrt{5},0\right)$, $\left(0,-2-2\sqrt{2}\right)$, and $\left(0,-2+2\sqrt{2}\right)$



29. $3x^2+3y^2-6x+12y=0$

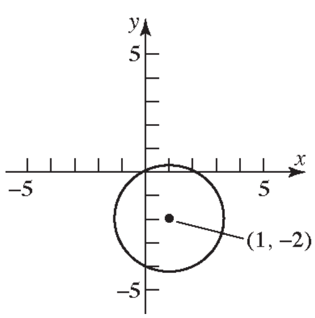
$$x^2+y^2-2x+4y=0$$
$$x^2-2x+y^2+4y=0$$
$$(x^2-2x+1)+(y^2+4y+4)=1+4$$
$$(x-1)^2+(y+2)^2=5$$
$$(x-1)^2+(y+2)^2=\left(\sqrt{5}\right)^2$$

Center: $(1,–2)$ Radius = $\sqrt{5}$

$$3x^2+3(0)^2-6x+12(0)=0$$
$$3x^2-6x=0$$
$$3x(x-2)=0$$
$$x=0 \quad \text{or} \quad x=2$$

$$3(0)^2+3y^2-6(0)+12y=0$$
$$3y^2+12y=0$$
$$3y(y+4)=0$$
$$y=0 \quad \text{or} \quad y=-4$$

<u>Intercepts:</u>
$(0,0)$, $(2,0)$, and $(0,-4)$



30. Given the points
$A=(-2,0),\ B=(-4,4),\ C=(8,5)$.

a. Find the distance between each pair of points.

$$d(A,B)=\sqrt{(-4-(-2))^2+(4-0)^2}$$
$$=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$$
$$d(B,C)=\sqrt{(8-(-4))^2+(5-4)^2}$$
$$=\sqrt{144+1}=\sqrt{145}$$
$$d(A,C)=\sqrt{(8-(-2))^2+(5-0)^2}$$
$$=\sqrt{100+25}=\sqrt{125}=5\sqrt{5}$$
$$\left(\sqrt{20}\right)^2+\left(\sqrt{125}\right)^2=\left(\sqrt{145}\right)^2$$
$$20+125=145$$
$$145=145$$

The Pythagorean Theorem is satisfied, so this is a right triangle.

b. Find the slopes:

$$m_{AB}=\frac{4-0}{-4-(-2)}=\frac{4}{-2}=-2$$
$$m_{BC}=\frac{5-4}{8-(-4)}=\frac{1}{12}$$
$$m_{AC}=\frac{5-0}{8-(-2)}=\frac{5}{10}=\frac{1}{2}$$
$$m_{AB}\cdot m_{AC}=-2\cdot\frac{1}{2}=-1$$

Since the product of the slopes is –1, the sides of the triangle are perpendicular and the triangle is a right triangle.

31. slope of $\overline{AB}=\frac{1-5}{6-2}=-1$;

slope of $\overline{BC}=\frac{-1-1}{8-6}=-1$

Since lines AB and BC are parallel, and have a point in common (B), they are the same line. Therefore, A, B, and C are collinear.

32. A circle with center $(-1,2)$ has equation

$(x+1)^2+(y-2)^2=r^2$

point $A(1,5)\Rightarrow(1+1)^2+(5-2)^2=r^2$

$4+9=r^2\Rightarrow r=\sqrt{13}$

point $B(2,4)\Rightarrow(2+1)^2+(4-2)^2=r^2$

$9+4=r^2\Rightarrow r=\sqrt{13}$

point $C(-3,5)\Rightarrow(-3+1)^2+(5-2)^2=r^2$

$4+9=r^2\Rightarrow r=\sqrt{13}$

Therefore the points *A*, *B* and *C* lie on a circle with center point $(-1,2)$ and radius = $\sqrt{13}$.

33. Endpoints of the diameter are (–3, 2) and (5,–6). The center is at the midpoint of the diameter:

Center: $\left(\dfrac{-3+5}{2},\dfrac{2+(-6)}{2}\right)=(1,-2)$

Radius: $r=\sqrt{(1-(-3))^2+(-2-2)^2}=\sqrt{16+16}$

$=\sqrt{32}=4\sqrt{2}$

Equation: $(x-1)^2+(y+2)^2=\left(4\sqrt{2}\right)^2$

$x^2-2x+1+y^2+4y+4=32$

$x^2+y^2-2x+4y-27=0$

34. Using the distance formula on the points (–3, 2) and (5, *y*) yields

$d=\sqrt{(5-(-3))^2+(y-2)^2}=\sqrt{64+(y-2)^2}$

Now set $d=10$ and solve for *y*.

$10=\sqrt{64+(y-2)^2}$

$10^2=\left(\sqrt{64+(y-2)^2}\right)^2$

$100=64+(y-2)^2$

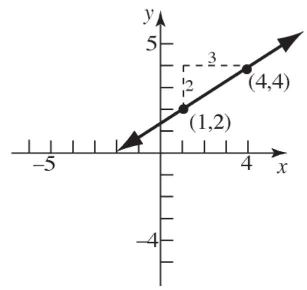
$36=(y-2)^2$

$\pm6=y-2$

so $y-2=6\rightarrow y=8$

and $y-2=-6\rightarrow y=-4$

35. slope = $\dfrac{2}{3}$, containing the point (1,2)



36. Answers will vary.

37. **a.** $x=0$ is a vertical line passing through the origin, that is, $x=0$ is the equation of the *y*-axis.

b. $y=0$ is a horizontal line passing through the origin, that is, $y=0$ is the equation of the *x*-axis.

c. $x+y=0$ implies that $y=-x$. Thus, the graph of $x+y=0$ is the line passing through the origin with slope = –1.

d. $xy=0$ implies that either $x=0$, $y=0$, or both are 0. $x=0$ is the y-axis and $y=0$ is the x-axis. Thus, the graph of $xy=0$ is a graph consisting of the coordinate axes.

e. $x^2+y^2=0$ implies that both $x=0$ and $y=0$. Thus, the graph of $x^2+y^2=0$ is a graph consisting of a single point - the origin.

Chapter 1 Test

1. **a.** $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(4-(-2))^2+(5-(-3))^2}$

$=\sqrt{6^2+8^2}$

$=\sqrt{36+64}$

$=\sqrt{100}$

$=10$

b. $M = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$= \left(\dfrac{4+(-2)}{2}, \dfrac{5+(-3)}{2}\right)$

$= \left(\dfrac{2}{2}, \dfrac{2}{2}\right)$

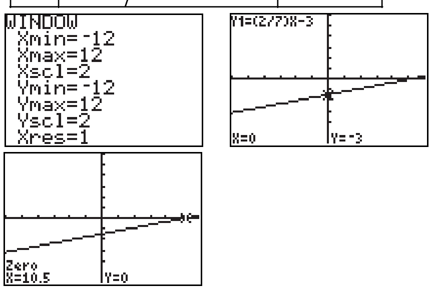
$= (1,1)$

2. $2x - 7y = 21$

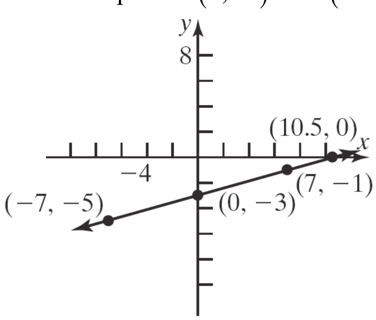
$-7y = -2x + 21$

$y = \dfrac{2}{7}x - 3$

x	$y = \frac{2}{7}x - 3$	(x, y)
-7	$y = \frac{2}{7}(-7) - 3 = -5$	$(-7, -5)$
0	$y = \frac{2}{7}(0) - 3 = -3$	$(0, -3)$
7	$y = \frac{2}{7}(7) - 3 = -1$	$(7, -1)$

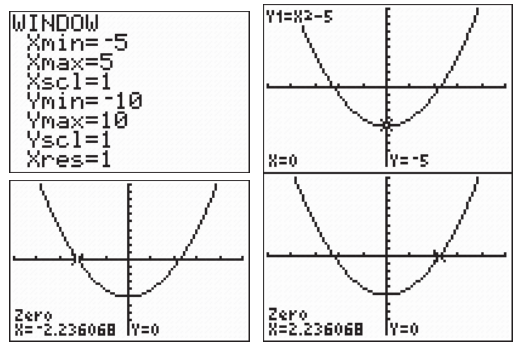


The intercepts are $(0, -3)$ and $(10.5, 0)$.

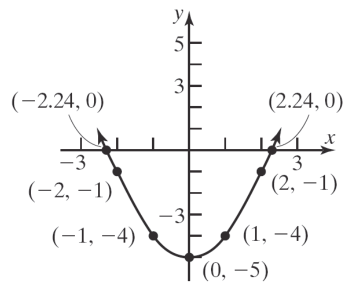


3. $y = x^2 - 5$

x	$y = x^2 - 5$	(x, y)
-3	$y = (-3)^2 - 5 = 4$	$(-3, 4)$
-1	$y = (-1)^2 - 5 = -4$	$(-1, -4)$
0	$y = (0)^2 - 5 = -5$	$(0, -5)$
1	$y = (1)^2 - 5 = -4$	$(1, -4)$
3	$y = (3)^2 - 5 = 4$	$(3, 4)$

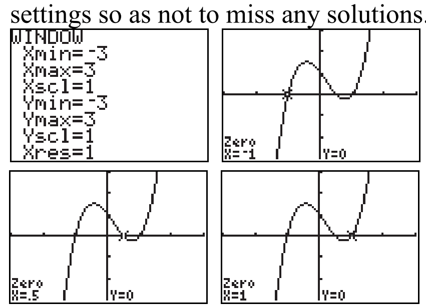


The intercepts are $(0, -5)$, $\approx(-2.24, 0)$, and $\approx(2.24, 0)$.



4. $2x^3 - x^2 - 2x + 1 = 0$

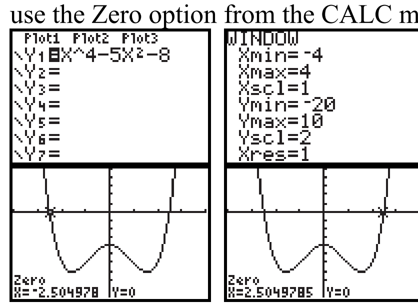
Since this equation has 0 on one side, we will use the Zero option from the CALC menu. It will be important to carefully select the window settings so as not to miss any solutions.



The solutions to the equation are -1, 0.5, and 1.

5. $x^4 - 5x^2 - 8 = 0$

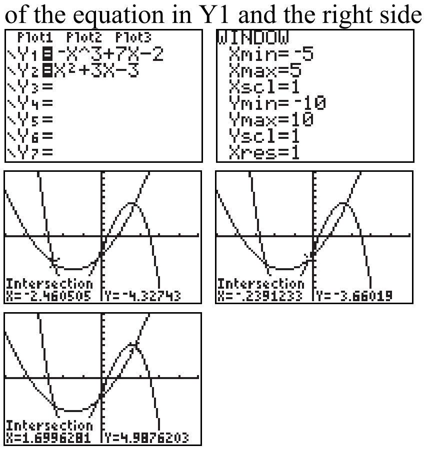
Since this equation has 0 on one side, we will use the Zero option from the CALC menu.



The solutions, rounded to two decimal places, are -2.50 and 2.50.

6. $-x^3+7x-2=x^2+3x-3$

Since there are nonzero expressions on both sides of the equation, we will use the Intersect option from the CALC menu. Enter the left side of the equation in Y1 and the right side in Y2.

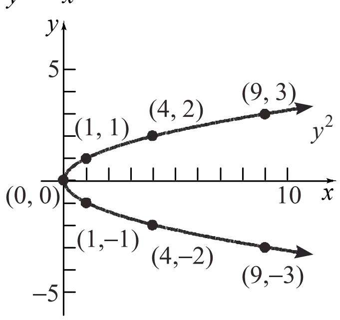


The solutions, rounded to two decimal places, are -2.46, -0.24, and 1.70.

7. **a.** $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-1-3}{5-(-1)}=\dfrac{-4}{6}=-\dfrac{2}{3}$

b. If x increases by 3 units, y will decrease by 2 units.

8. $y^2=x$



9. $x^2+y=9$

x-intercepts:

$x^2+0=9$

$x^2=9$

$x=\pm 3$

y-intercept:

$(0)^2+y=9$

$y=9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,9)$.

Test x-axis symmetry: Let $y=-y$

$x^2+(-y)=9$

$x^2-y=9$ different

Test y-axis symmetry: Let $x=-x$

$(-x)^2+y=9$

$x^2+y=9$ same

Test origin symmetry: Let $x=-x$ and $y=-y$

$(-x)^2+(-y)=9$

$x^2-y=9$ different

Therefore, the graph will have y-axis symmetry.

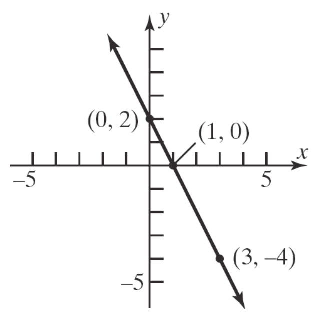
10. Slope $=-2$; containing $(3,-4)$

$y-y_1=m(x-x_1)$

$y-(-4)=-2(x-3)$

$y+4=-2x+6$

$y=-2x+2$



11. $(x-h)^2+(y-k)^2=r^2$

$(x-4)^2+(y-(-3))^2=5^2$

$(x-4)^2+(y+3)^2=25$

General form:

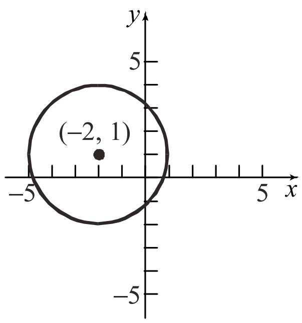
$(x-4)^2+(y+3)^2=25$

$x^2-8x+16+y^2+6y+9=25$

$x^2+y^2-8x+6y=0$

12.
$$x^2 + y^2 + 4x - 2y - 4 = 0$$
$$x^2 + 4x + y^2 - 2y = 4$$
$$(x^2 + 4x + 4) + (y^2 - 2y + 1) = 4 + 4 + 1$$
$$(x+2)^2 + (y-1)^2 = 3^2$$

Center: (–2, 1); Radius = 3



13. $2x + 3y = 6$
$$3y = -2x + 6$$
$$y = -\frac{2}{3}x + 2$$

Parallel line

Any line parallel to $2x + 3y = 6$ has slope $m = -\frac{2}{3}$. The line contains $(1, -1)$:
$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -\frac{2}{3}(x - 1)$$
$$y + 1 = -\frac{2}{3}x + \frac{2}{3}$$
$$y = -\frac{2}{3}x - \frac{1}{3}$$

Perpendicular line

Any line perpendicular to $2x + 3y = 6$ has slope $m = \frac{3}{2}$. The line contains $(0, 3)$:
$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{3}{2}(x - 0)$$
$$y - 3 = \frac{3}{2}x$$
$$y = \frac{3}{2}x + 3$$

Chapter 1 Project

Internet-based Project