

***Algebra and Trigonometry Enhanced with Graphing
Utilities 8th edition Sullivan Solutions Manual***

SKU: 9780135813485-SOLUTIONS

Chapter 1

Graphs, Equations, and Inequalities

Section 1.1

1. 0



2.

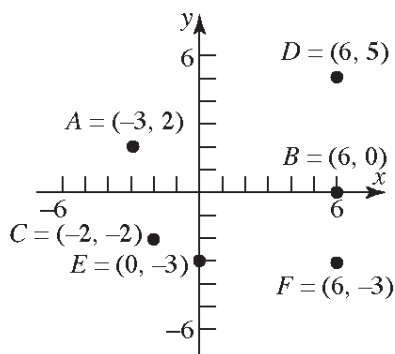
3. x -coordinate; y -coordinate; quadrants

4. False; points that lie in Quadrant IV will have a positive x -coordinate and a negative y -coordinate.
The point $(-1, 4)$ lies in Quadrant II.

5. d

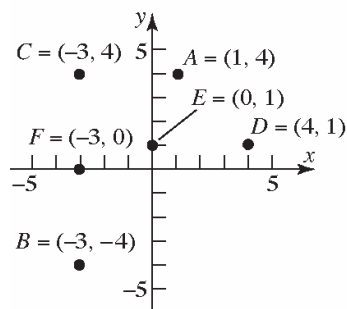
6. c

7. (a) Quadrant II
(b) x -axis
(c) Quadrant III
(d) Quadrant I
(e) y -axis
(f) Quadrant IV

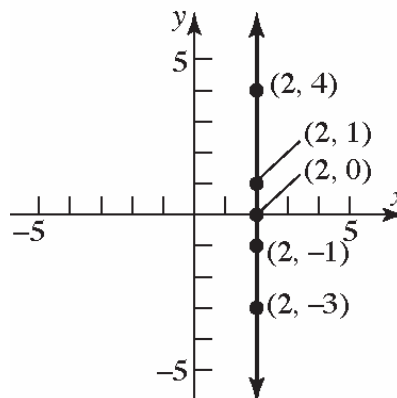


8. (a) Quadrant I
(b) Quadrant III
(c) Quadrant II
(d) Quadrant I
(e) y -axis

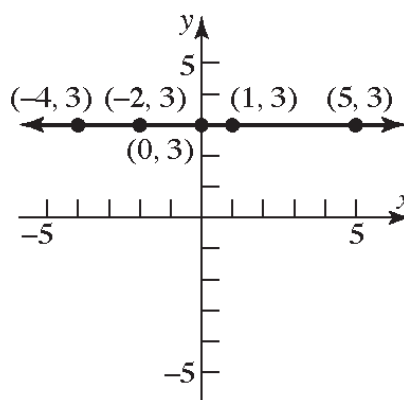
(f) x -axis



9. The points will be on a vertical line that is two units to the right of the y -axis.



10. The points will be on a horizontal line that is three units above the x -axis.



11. $(-1, 4)$; Quadrant II

12. $(3, 4)$; Quadrant I

13. $(3, 1)$; Quadrant I

Section 1.1: Graphing Utilities; Introduction to Graphing Equations

14. $(-6, -4)$; Quadrant III

15. $X \min = -11$

$X \max = 5$

$X \text{ scl} = 1$

$Y \min = -3$

$Y \max = 6$

$Y \text{ scl} = 1$

16. $X \min = -3$

$X \max = 7$

$X \text{ scl} = 1$

$Y \min = -4$

$Y \max = 9$

$Y \text{ scl} = 1$

17. $X \min = -30$

$X \max = 50$

$X \text{ scl} = 10$

$Y \min = -90$

$Y \max = 50$

$Y \text{ scl} = 10$

18. $X \min = -90$

$X \max = 30$

$X \text{ scl} = 10$

$Y \min = -50$

$Y \max = 70$

$Y \text{ scl} = 10$

19. $X \min = -10$

$X \max = 110$

$X \text{ scl} = 10$

$Y \min = -10$

$Y \max = 160$

$Y \text{ scl} = 10$

20. $X \min = -20$

$X \max = 110$

$X \text{ scl} = 10$

$Y \min = -10$

$Y \max = 60$

$Y \text{ scl} = 10$

21. $X \min = -6$

$X \max = 6$

$X \text{ scl} = 2$

$Y \min = -4$

$Y \max = 4$

$Y \text{ scl} = 2$

22. $X \min = -3$

$X \max = 3$

$X \text{ scl} = 1$

$Y \min = -2$

$Y \max = 2$

$Y \text{ scl} = 1$

23. $X \min = -6$

$X \max = 6$

$X \text{ scl} = 2$

$Y \min = -1$

$Y \max = 3$

$Y \text{ scl} = 1$

24. $X \min = -9$

$X \max = 9$

$X \text{ scl} = 3$

$Y \min = -12$

$Y \max = 4$

$Y \text{ scl} = 4$

25. $X \min = 3$

$X \max = 9$

$X \text{ scl} = 1$

$Y \min = 2$

$Y \max = 10$

$Y \text{ scl} = 2$

26. $X \min = -22$

$X \max = -10$

$X \text{ scl} = 2$

$Y \min = 4$

$Y \max = 8$

$Y \text{ scl} = 1$

Chapter 1: Graphs, Equations, and Inequalities

27. $y = x^4 - \sqrt{x}$

$$0 = 0^4 - \sqrt{0} \quad 1 = 1^4 - \sqrt{1} \quad 0 = (-1)^4 - \sqrt{-1}$$

$$0 = 0 \quad 1 \neq 0 \quad 0 \neq 1 - \sqrt{-1}$$

(0, 0) is on the graph of the equation.

28. $y = x^3 - 2\sqrt{x}$

$$0 = 0^3 - 2\sqrt{0} \quad 1 = 1^3 - 2\sqrt{1} \quad -1 = 1^3 - 2\sqrt{1}$$

$$0 = 0 \quad 1 \neq -1 \quad -1 = -1$$

(0, 0) and (1, -1) are on the graph of the equation.

29. $y^2 = x^2 + 9$

$$3^2 = 0^2 + 9 \quad 0^2 = 3^2 + 9 \quad 0^2 = (-3)^2 + 9$$

$$9 = 9 \quad 0 \neq 18 \quad 0 \neq 18$$

(0, 3) is on the graph of the equation.

30. $y^3 = x + 1$

$$2^3 = 1 + 1 \quad 1^3 = 0 + 1 \quad 0^3 = -1 + 1$$

$$8 \neq 2 \quad 1 = 1 \quad 0 = 0$$

(0, 1) and (-1, 0) are on the graph of the equation.

31. $x^2 + y^2 = 4$

$$0^2 + 2^2 = 4 \quad (-2)^2 + 2^2 = 4 \quad \sqrt{2}^2 + \sqrt{2}^2 = 4$$

$$4 = 4 \quad 8 \neq 4 \quad 4 = 4$$

(0, 2) and $(\sqrt{2}, \sqrt{2})$ are on the graph of the equation.

32. $x^2 + 4y^2 = 4$

$$0^2 + 4 \cdot 1^2 = 4 \quad 2^2 + 4 \cdot 0^2 = 4 \quad 2^2 + 4 \left(\frac{1}{2}\right)^2 = 4$$

$$4 = 4 \quad 4 = 4 \quad 5 \neq 4$$

(0, 1) and (2, 0) are on the graph of the equation.

33. (-1, 0), (1, 0)

34. (0, 1)

35. $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right), (0, 1)$

36. (-2, 0), (2, 0), (0, -3)

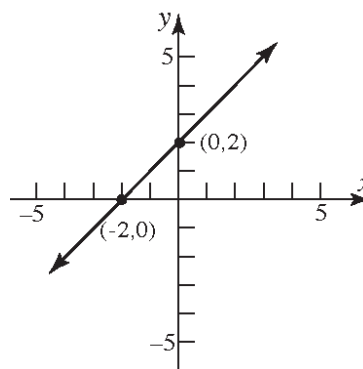
37. (1, 0), (0, 2), (0, -2)

38. (2, 0), (0, 2), (-2, 0), (0, -2)

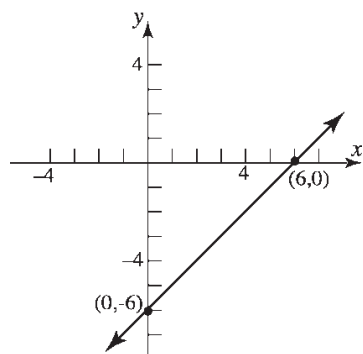
39. (-4, 0), (-1, 0), (4, 0), (0, -3)

40. (-2, 0), (2, 0), (0, 3)

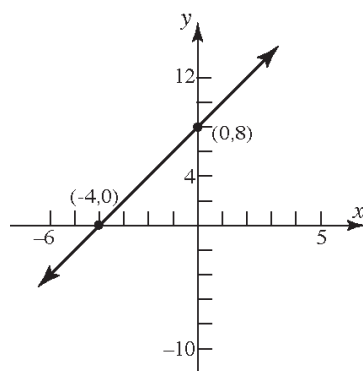
41. $y = x + 2$



42. $y = x - 6$

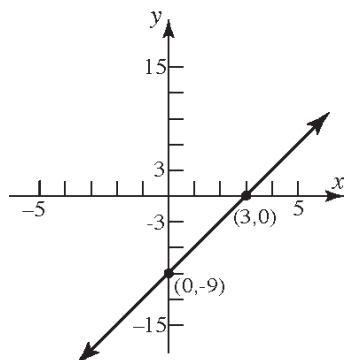


43. $y = 2x + 8$

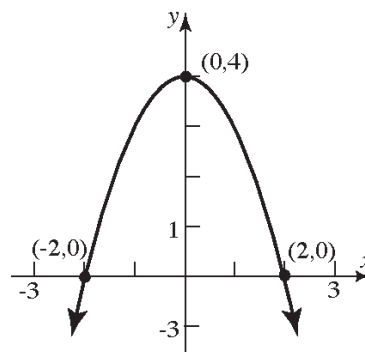


Section 1.1: Graphing Utilities; Introduction to Graphing Equations

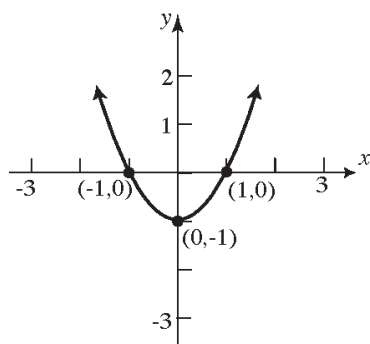
44. $y = 3x - 9$



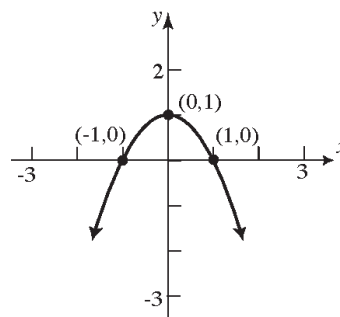
47. $y = -x^2 + 4$



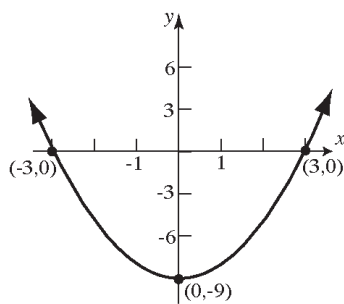
45. $y = x^2 - 1$



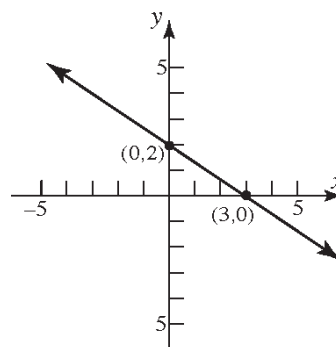
48. $y = -x^2 + 1$



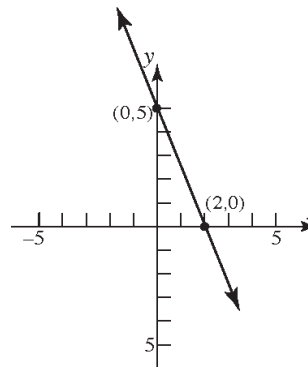
46. $y = x^2 - 9$



49. $2x + 3y = 6$

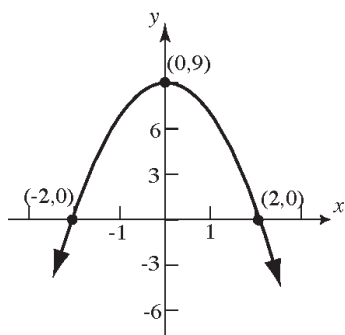


50. $5x + 2y = 10$

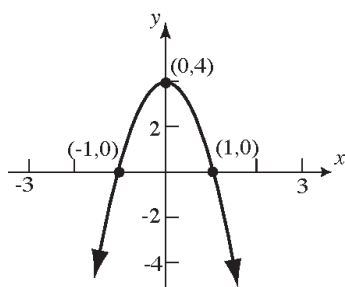


Chapter 1: Graphs, Equations, and Inequalities

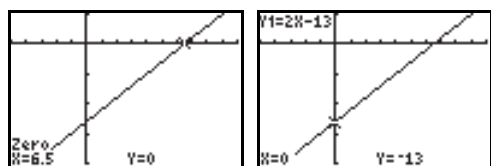
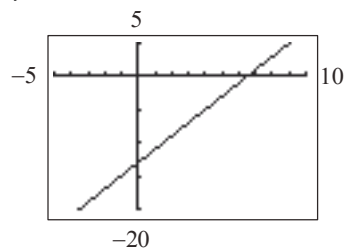
51. $9x^2 + 4y = 36$



52. $4x^2 + y = 4$

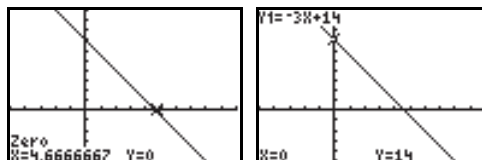
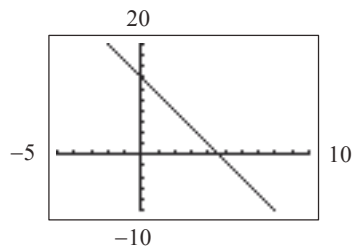


53. $y = 2x - 13$



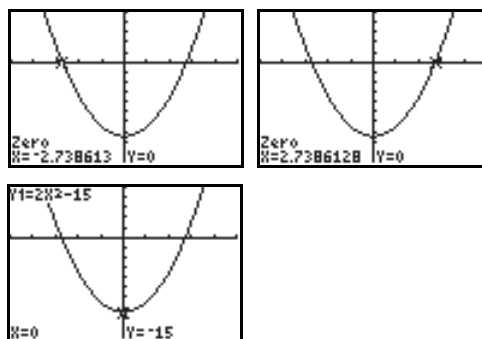
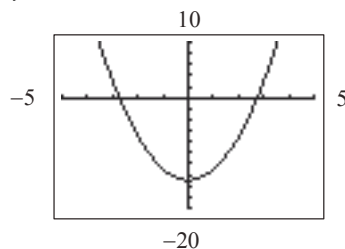
The x-intercept is $x = 6.5$ and the y-intercept is $y = -13$.

54. $y = -3x + 14$



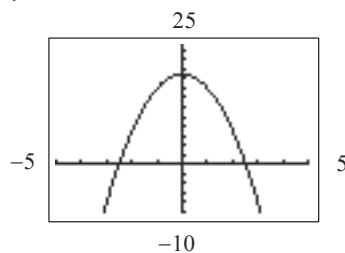
The x-intercept is $x = 4.67$ and the y-intercept is $y = 14$.

55. $y = 2x^2 - 15$

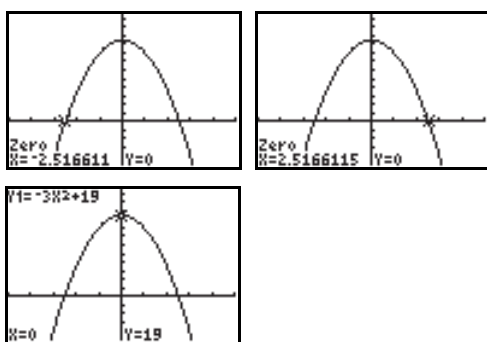


The x-intercepts are $x = -2.74$ and $x = 2.74$. The y-intercept is $y = -15$.

56. $y = -3x^2 + 19$

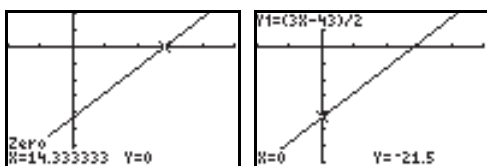
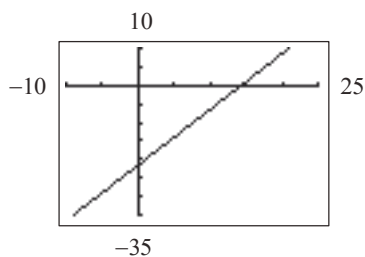


Section 1.1: Graphing Utilities; Introduction to Graphing Equations



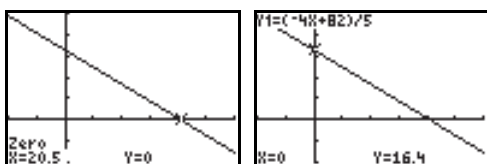
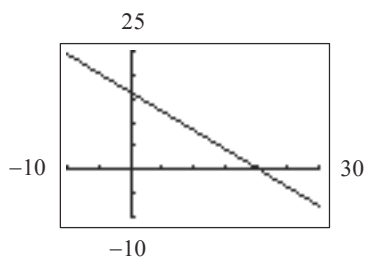
The x-intercepts are $x = -2.52$ and $x = 2.52$.
The y-intercept is $y = 19$.

57. $3x - 2y = 43$ or $y = \frac{3x - 43}{2}$



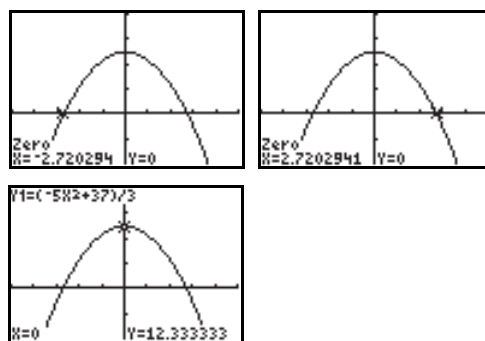
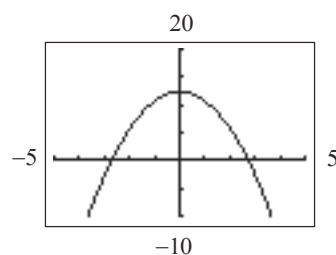
The x-intercept is $x = 14.33$ and the y-intercept is $y = -21.5$.

58. $4x + 5y = 82$ or $y = \frac{-4x + 82}{5}$



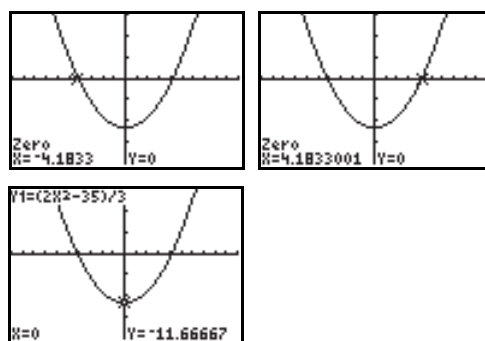
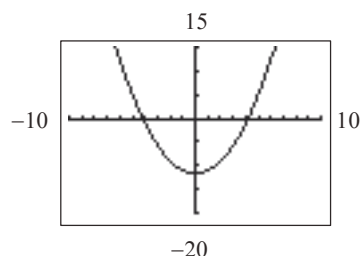
The x-intercept is $x = 20.5$ and the y-intercept is $y = 16.4$.

59. $5x^2 + 3y = 37$ or $y = \frac{-5x^2 + 37}{3}$



The x-intercepts are $x = -2.72$ and $x = 2.72$.
The y-intercept is $y = 12.33$.

60. $2x^2 - 3y = 35$ or $y = \frac{2x^2 - 35}{3}$



The x-intercepts are $x = -4.18$ and $x = 4.18$.
The y-intercept is $y = -11.67$.

61. If $(2, 5)$ is shifted 3 units right then the x coordinate would be $2 + 3$. If it is shifted 2 units down then the y-coordinate would be $5 + (-2)$.

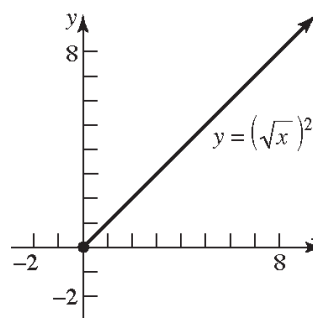
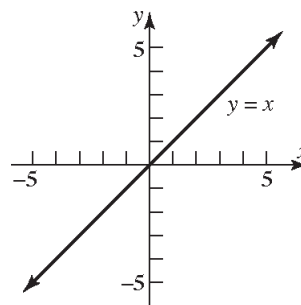
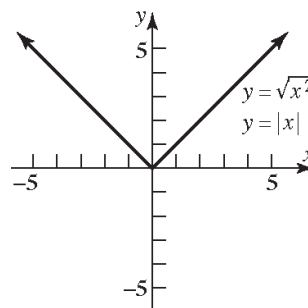
Chapter 1: Graphs, Equations, and Inequalities

Thus the new point would be
 $(2 + 3, 5 + (-2)) = (5, 3)$.

62. If $(-1, 6)$ is shifted 2 units left then the x coordinate would be $-1 + (-2)$. If it is shifted 4 units up then the y -coordinate would be $6 + 4$. Thus the new point would be $(-1 + (-2), 6 + 4) = (-3, 10)$.

63. a. 25 feet
 b. 23.2 ft; 34.1 ft
 c. $(0, 6)$, The shot is released at a height of 6 feet; $(48.7, 0)$, the shot hits the ground after traveling a horizontal distance of 48.7 feet.
64. a. 20 meters
 b. 12 seconds; 36 meters
 c. $(0, 2)$, The discus is released at a height of 2 meters; $(18, 0)$, the discus hits the ground after 18 seconds.
65. a. \$19.95; \$19.95
 b. \$182.45
 c. $(0, 19.95)$, The membership plan costs \$19.95 per month.
66. a. 1.5 miles
 b. 1 mile
 c. $(0, 2)$, Caleb's friend lives 2 miles from his house; $(28, 0)$, it takes Caleb 28 minutes to ride home.
67. $(-1, 0), (1, 0), (0, -1), (0, 1)$
68. $(-2, 0), (3, 0), (0, -2), (0, 0), (0, 2)$
69. Answers will vary

70. a.



- b. Since $\sqrt{x^2} = |x|$ for all x , the graphs of $y = \sqrt{x^2}$ and $y = |x|$ are the same.
- c. For $y = (\sqrt{x})^2$, the domain of the variable x is $x \geq 0$; for $y = x$, the domain of the variable x is all real numbers. Thus, $(\sqrt{x})^2 = x$ only for $x \geq 0$.
- d. For $y = \sqrt{x^2}$, the range of the variable y is $y \geq 0$; for $y = x$, the range of the variable y is all real numbers. Also, $\sqrt{x^2} = x$ only if $x \geq 0$.

71. Answers will vary

Section 1.2: Solving Equations Using a Graphing Utility; Linear and Rational Equations

72. Answers will vary. A complete graph presents enough of the graph to the viewer so they can “see” the rest of the graph as an obvious continuation of what is shown.
73. Answers will vary.

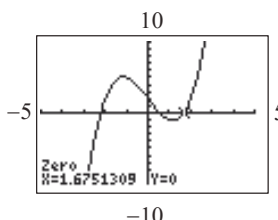
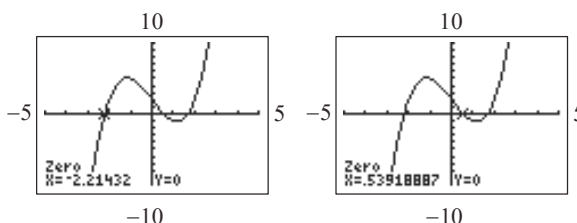
Section 1.2

1. Using the distributive property:
 $-3(x-5) = -3x + 15$
2. $2x - 3 - 5(x+1) = 2(3) - 3 - 5[(3)+1]$
 $= 6 - 3 - 5(4) = 6 - 3 - 20$
 $= 3 - 10 = -17$
3. No, $x = 4$ is not in the domain of $\frac{3}{x-4}$ because it makes the denominator equal 0.
4. $\text{LCM} = (x-1)(x+3) = x^2 + 2x - 3$
5. b
6. identity
7. linear; first-degree
8. d
9. True; equations that are contradictions have no solution.
10. False; when using a graphing utility, we often obtain decimal approximations to solutions.
11. Divide both sides by 7. The solution set is $\{3\}$.
12. Divide both sides by 6. The solution set is $\{-4\}$.
13. Subtract 35 from both sides, then divide both sides by 5. The solution set is $\{-7\}$.
14. Subtract 18 from both sides, then divide both sides by 6. The solution set is $\{-3\}$.
15. Add 3 to both sides, then divide both sides by 2.
The solution set is $\left\{\frac{3}{2}\right\}$.
16. Subtract 4 from both sides, then divide both sides by 3. The solution set is $\left\{-\frac{4}{3}\right\}$.

17. Multiply both sides by 4. The solution set is $\left\{\frac{7}{5}\right\}$.

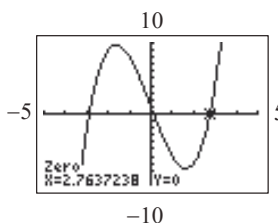
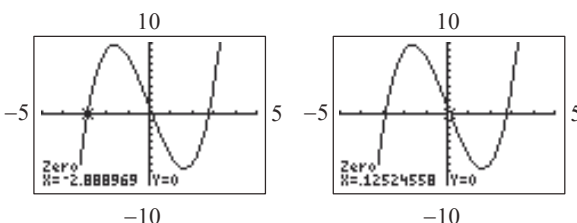
18. Multiply both sides by $\frac{3}{2}$. The solution set is $\left\{\frac{27}{4}\right\}$.

19. $x^3 - 4x + 2 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = x^3 - 4x + 2$.



The solution set is $\{-2.21, 0.54, 1.68\}$.

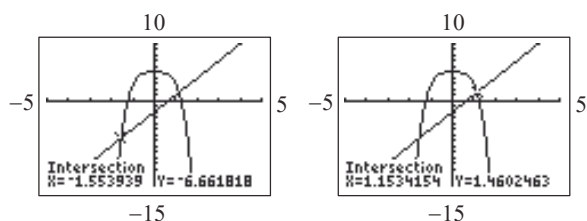
20. $x^3 - 8x + 1 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = x^3 - 8x + 1$.



The solution set is $\{-2.89, 0.13, 2.76\}$.

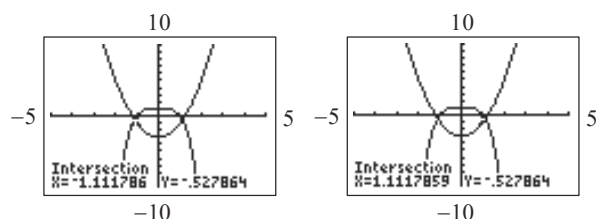
21. $-2x^4 + 5 = 3x - 2$; Use INTERSECT on the graphs of $y_1 = -2x^4 + 5$ and $y_2 = 3x - 2$.

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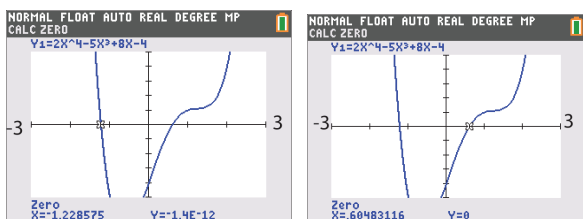
The solution set is $\{-1.55, 1.15\}$.

22. $-x^4 + 1 = 2x^2 - 3$; Use INTERSECT on the graphs of $y_1 = -x^4 + 1$ and $y_2 = 2x^2 - 3$.



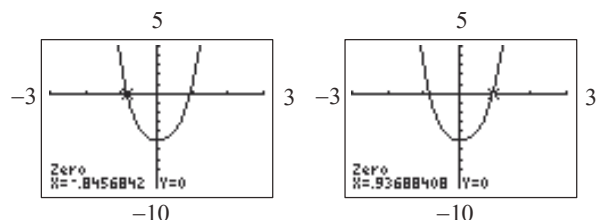
The solution set is $\{-1.11, 1.11\}$.

23. $2x^4 - 5x^3 + 8x - 4 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = 2x^4 - 5x^3 + 8x - 4$.



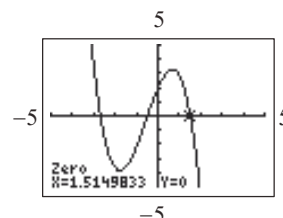
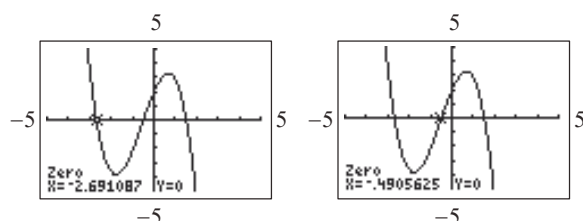
The solution set is $\{-1.23, 0.60\}$.

24. $3x^4 - x^3 + 4x^2 - 5 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = 3x^4 - x^3 + 4x^2 - 5$.



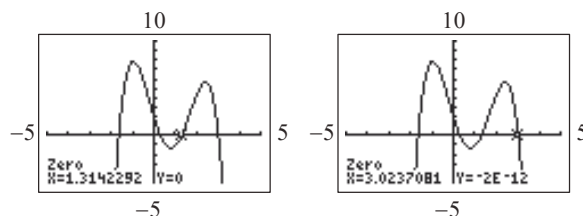
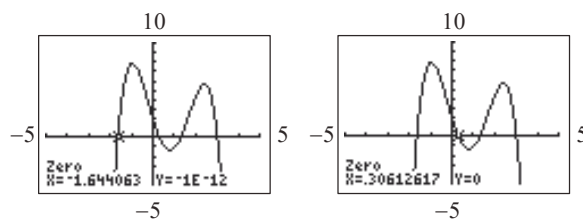
The solution set is $\{-0.85, 0.94\}$.

25. $-x^3 - \frac{5}{3}x^2 + \frac{7}{2}x + 2 = 0$;
Use ZERO (or ROOT) on the graph of $y_1 = -x^3 - (5/3)x^2 + (7/2)x + 2$.



The solution set is $\{-2.69, -0.49, 1.51\}$.

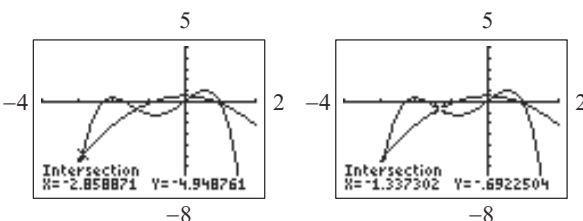
26. $-x^4 + 3x^3 + \frac{7}{3}x^2 - \frac{15}{2}x + 2 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = -x^4 + 3x^3 + (7/3)x^2 - (15/2)x + 2$.



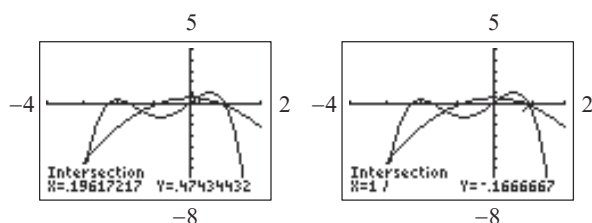
The solution set is $\{-1.64, 0.31, 1.31, 3.02\}$.

27. $-\frac{2}{3}x^4 - 2x^3 + \frac{5}{2}x = -\frac{2}{3}x^2 + \frac{1}{2}$

Use INTERSECT on the graphs of $y_1 = -(2/3)x^4 - 2x^3 + (5/2)x$ and $y_2 = -(2/3)x^2 + 1/2$.



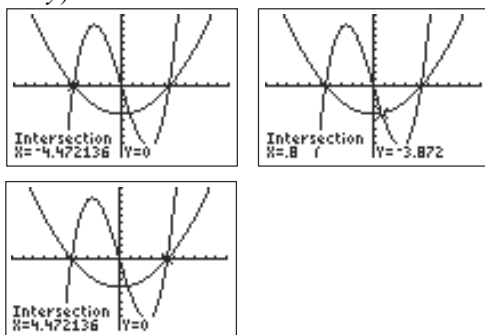
Section 1.2: Solving Equations Using a Graphing Utility; Linear and Rational Equations



The solution set is $\{-2.86, -1.34, 0.20, 1.00\}$.

28. $\frac{1}{4}x^3 - 5x = \frac{1}{5}x^2 - 4$

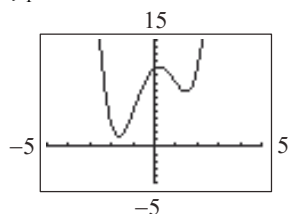
Use INTERSECT on the graphs of $y_1 = x^3/4 - 5x$ and $y_2 = x^2/5 - 4$ and a standard viewing window (-10 to 10 for both x and y).



The solution set is $\{-4.47, 0.80, 4.47\}$.

29. $x^4 - 5x^2 + 2x + 11 = 0$

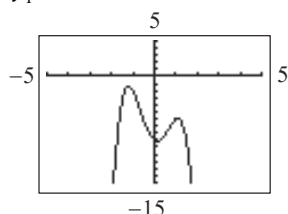
Use ZERO (or ROOT) on the graph of $y_1 = x^4 - 5x^2 + 2x + 11$.



There are no real solutions.

30. $-3x^4 + 8x^2 - 2x - 9 = 0$

Use ZERO (or ROOT) on the graph of $y_1 = -3x^4 + 8x^2 - 2x - 9$.



There are no real solutions.

31. $3x + 4 = x$

$$3x + 4 - 4 = x - 4$$

$$3x = x - 4$$

$$3x - x = x - 4 - x$$

$$2x = -4$$

$$\frac{2x}{2} = \frac{-4}{2}$$

$$x = -2$$

The solution set is $\{-2\}$.

32. $2x + 9 = 5x$

$$2x + 9 - 9 = 5x - 9$$

$$2x = 5x - 9$$

$$2x - 5x = 5x - 9 - 5x$$

$$-3x = -9$$

$$\frac{-3x}{-3} = \frac{-9}{-3}$$

$$x = 3$$

The solution set is $\{3\}$.

33. $2t - 6 = 3 - t$

$$2t - 6 + 6 = 3 - t + 6$$

$$2t = 9 - t$$

$$2t + t = 9 - t + t$$

$$3t = 9$$

$$\frac{3t}{3} = \frac{9}{3}$$

$$t = 3$$

The solution set is $\{3\}$.

34. $5y + 6 = -18 - y$

$$5y + 6 - 6 = -18 - y - 6$$

$$5y = -y - 24$$

$$5y + y = -y - 24 + y$$

$$6y = -24$$

$$\frac{6y}{6} = \frac{-24}{6}$$

$$y = -4$$

The solution set is $\{-4\}$.

Chapter 1: Graphs, Equations, and Inequalities

35. $6 - x = 2x + 9$

$6 - x - 6 = 2x + 9 - 6$

$-x = 2x + 3$

$-x - 2x = 2x + 3 - 2x$

$-3x = 3$

$\frac{-3x}{-3} = \frac{3}{-3}$

$x = -1$

The solution set is $\{-1\}$.

36. $3 - 2x = 2 - x$

$3 - 2x - 3 = 2 - x - 3$

$-2x = -x - 1$

$-2x + x = -x - 1 + x$

$-x = -1$

$-1(-x) = -1(-1)$

$x = 1$

The solution set is $\{1\}$.

37. $3(5 + 3x) = 8(x - 1)$

$15 + 9x = 8x - 8$

$15 + 9x - 15 = 8x - 8 - 15$

$9x = 8x - 23$

$9x - 8x = 8x - 23 - 8x$

$x = -23$

The solution set is $\{-23\}$.

38. $3(2 - x) = 2x - 1$

$6 - 3x = 2x - 1$

$6 - 3x - 6 = 2x - 1 - 6$

$-3x = 2x - 7$

$-3x - 2x = 2x - 7 - 2x$

$-5x = -7$

$\frac{-5x}{-5} = \frac{-7}{-5}$

$x = \frac{7}{5}$

The solution set is $\left\{\frac{7}{5}\right\}$.

39. $8x - (3x + 2) = 3x - 10$

$8x - 3x - 2 = 3x - 10$

$5x - 2 = 3x - 10$

$5x - 2 + 2 = 3x - 10 + 2$

$5x = 3x - 8$

$5x - 3x = 3x - 8 - 3x$

$2x = -8$

$\frac{2x}{2} = \frac{-8}{2}$

$x = -4$

The solution set is $\{-4\}$.

40. $7 - (2x - 1) = 10$

$7 - 2x + 1 = 10$

$-2x + 8 = 10$

$-2x + 8 - 8 = 10 - 8$

$-2x = 2$

$\frac{-2x}{-2} = \frac{2}{-2}$

$x = -1$

The solution set is $\{-1\}$.

41. $2(3x - 5) + 6(x - 3) = -3(4 - 5x) + 5x - 6$

$6x - 10 + 6x - 18 = -12 + 15x + 5x - 6$

$12x - 28 = 20x - 18$

$-8x - 28 = -18$

$-8x = 10$

$x = -\frac{5}{4}$

The solution set is $\left\{-\frac{5}{4}\right\}$.

42. $5(x - 2) - 2(3x + 1) = 4(1 - 2x) + x$

$5x - 10 - 6x - 2 = 4 - 8x + x$

$-x - 12 = 4 - 7x$

$6x - 12 = 4$

$6x = 16$

$x = \frac{8}{3}$

The solution set is $\left\{\frac{8}{3}\right\}$.

Section 1.2: Solving Equations Using a Graphing Utility; Linear and Rational Equations

$$\begin{aligned}
 43. \quad \frac{3}{2}x + 2 &= \frac{1}{2} - \frac{1}{2}x \\
 2\left(\frac{3}{2}x + 2\right) &= 2\left(\frac{1}{2} - \frac{1}{2}x\right) \\
 3x + 4 &= 1 - x \\
 3x + 4 - 4 &= 1 - x - 4 \\
 3x &= -x - 3 \\
 3x + x &= -x - 3 + x \\
 4x &= -3 \\
 \frac{4x}{4} &= \frac{-3}{4} \\
 x &= -\frac{3}{4}
 \end{aligned}$$

The solution set is $\left\{-\frac{3}{4}\right\}$.

$$\begin{aligned}
 44. \quad \frac{1}{3}x &= 2 - \frac{2}{3}x \\
 3\left(\frac{1}{3}x\right) &= 3\left(2 - \frac{2}{3}x\right) \\
 x &= 6 - 2x \\
 x + 2x &= 6 - 2x + 2x \\
 3x &= 6 \\
 \frac{3x}{3} &= \frac{6}{3} \\
 x &= 2
 \end{aligned}$$

The solution set is $\{2\}$.

$$\begin{aligned}
 45. \quad \frac{2}{3}p &= \frac{1}{2}p - \frac{1}{3} \\
 6\left(\frac{2}{3}p\right) &= 6\left(\frac{1}{2}p - \frac{1}{3}\right) \\
 4p &= 3p - 2 \\
 4p - 3p &= 3p - 2 - 3p \\
 p &= -2 \\
 \text{The solution set is } &\{-2\}.
 \end{aligned}$$

$$\begin{aligned}
 46. \quad \frac{1}{2} - \frac{1}{3}p &= \frac{4}{3} \\
 6\left(\frac{1}{2} - \frac{1}{3}p\right) &= 6 \cdot \frac{4}{3} \\
 3 - 2p &= 8 \\
 3 - 2p - 3 &= 8 - 3 \\
 -2p &= 5 \\
 \frac{-2p}{-2} &= \frac{5}{-2} \\
 p &= -\frac{5}{2}
 \end{aligned}$$

The solution set is $\left\{-\frac{5}{2}\right\}$.

$$\begin{aligned}
 47. \quad 0.2m &= 0.9 + 0.5m \\
 0.2m - 0.5m &= 0.9 + 0.5m - 0.5m \\
 -0.3m &= 0.9 \\
 \frac{-0.3m}{-0.3} &= \frac{0.9}{-0.3} \\
 m &= -3
 \end{aligned}$$

The solution set is $\{-3\}$.

$$\begin{aligned}
 48. \quad 0.9t &= 1 + t \\
 0.9t - t &= 1 + t - t \\
 -0.1t &= 1 \\
 \frac{-0.1t}{-0.1} &= \frac{1}{-0.1} \\
 t &= -10
 \end{aligned}$$

The solution set is $\{-10\}$.

$$\begin{aligned}
 49. \quad \frac{x+1}{3} + \frac{x+2}{7} &= 2 \\
 21\left(\frac{x+1}{3}\right) + 21\left(\frac{x+2}{7}\right) &= 21(2) \\
 7x + 7 + 3x + 6 &= 42 \\
 10x + 13 &= 42 \\
 10x + 13 - 13 &= 42 - 13 \\
 10x &= 29 \\
 \frac{10x}{10} &= \frac{29}{10} \\
 x &= \frac{29}{10} \\
 \text{The solution set is } &\left\{\frac{29}{10}\right\}.
 \end{aligned}$$

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$$\begin{aligned}
 50. \quad & \frac{2x+1}{3} + 16 = 3x \\
 & 3\left(\frac{2x+1}{3} + 16\right) = 3 \cdot 3x \\
 & 2x+1+48 = 9x \\
 & 2x+49 = 9x \\
 & 2x+49-2x = 9x-2x \\
 & 49 = 7x \\
 & \frac{49}{7} = \frac{7x}{7} \\
 & x = 7 \\
 & \text{The solution set is } \{7\}.
 \end{aligned}$$

$$\begin{aligned}
 51. \quad & \frac{2}{y} + \frac{4}{y} = 3 \\
 & \frac{6}{y} = 3 \\
 & \left(\frac{6}{y}\right)^{-1} = 3^{-1} \\
 & \frac{y}{6} = \frac{1}{3} \\
 & 6 \cdot \frac{y}{6} = 6 \cdot \frac{1}{3} \\
 & y = 2 \\
 & \text{The solution set is } \{2\}.
 \end{aligned}$$

$$\begin{aligned}
 52. \quad & \frac{4}{y} - 5 = \frac{5}{2y} \\
 & 2y\left(\frac{4}{y} - 5\right) = 2y\left(\frac{5}{2y}\right) \\
 & 8-10y = 5 \\
 & 8-10y-8 = 5-8 \\
 & -10y = -3 \\
 & \frac{-10y}{-10} = \frac{-3}{-10} \\
 & y = \frac{3}{10} \\
 & \text{The solution set is } \left\{\frac{3}{10}\right\}.
 \end{aligned}$$

$$\begin{aligned}
 53. \quad & \frac{1}{2} + \frac{2}{x} = \frac{3}{4} \\
 & 4x\left(\frac{1}{2} + \frac{2}{x}\right) = 4x\left(\frac{3}{4}\right) \\
 & 2x+8 = 3x \\
 & 2x+8-2x = 3x-2x \\
 & 8 = x \quad \text{or} \quad x = 8 \\
 & \text{The solution set is } \{8\}.
 \end{aligned}$$

$$\begin{aligned}
 54. \quad & \frac{3}{x} - \frac{1}{3} = \frac{1}{6} \\
 & 6x\left(\frac{3}{x} - \frac{1}{3}\right) = 6x\left(\frac{1}{6}\right) \\
 & 18-2x = x \\
 & 18-2x+2x = x+2x \\
 & 18 = 3x \\
 & \frac{18}{3} = \frac{3x}{3} \\
 & 6 = x \quad \text{or} \quad x = 6 \\
 & \text{The solution set is } \{6\}.
 \end{aligned}$$

$$\begin{aligned}
 55. \quad & (x+7)(x-1) = (x+1)^2 \\
 & x^2 - x + 7x - 7 = x^2 + 2x + 1 \\
 & x^2 + 6x - 7 = x^2 + 2x + 1 \\
 & x^2 + 6x - 7 - x^2 = x^2 + 2x + 1 - x^2 \\
 & 6x - 7 = 2x + 1 \\
 & 6x - 7 - 2x = 2x + 1 - 2x \\
 & 4x - 7 = 1 \\
 & 4x - 7 + 7 = 1 + 7 \\
 & 4x = 8 \\
 & \frac{4x}{4} = \frac{8}{4} \Rightarrow x = 2 \\
 & \text{The solution set is } \{2\}.
 \end{aligned}$$

Section 1.2: Solving Equations Using a Graphing Utility; Linear and Rational Equations

56. $(x+2)(x-3) = (x+3)^2$

$$x^2 + 2x - 3x - 6 = x^2 + 6x + 9$$

$$x^2 - x - 6 = x^2 + 6x + 9$$

$$x^2 - x - 6 - x^2 = x^2 + 6x + 9 - x^2$$

$$-x - 6 = 6x + 9$$

$$-x - 6 - 6x = 6x + 9 - 6x$$

$$-7x - 6 = 9$$

$$-7x - 6 + 6 = 9 + 6$$

$$-7x = 15$$

$$\frac{-7x}{-7} = \frac{15}{-7}$$

$$x = -\frac{15}{7}$$

The solution set is $\left\{-\frac{15}{7}\right\}$.

57. $x(2x-3) = (2x+1)(x-4)$

$$2x^2 - 3x = 2x^2 - 7x - 4$$

$$2x^2 - 3x - 2x^2 = 2x^2 - 7x - 4 - 2x^2$$

$$-3x = -7x - 4$$

$$-3x + 7x = -7x - 4 + 7x$$

$$4x = -4$$

$$\frac{4x}{4} = \frac{-4}{4} \Rightarrow x = -1$$

The solution set is $\{-1\}$.

58. $x(1+2x) = (2x-1)(x-2)$

$$x + 2x^2 = 2x^2 - 5x + 2$$

$$2x^2 + x - 2x^2 = 2x^2 - 5x + 2 - 2x^2$$

$$x = -5x + 2$$

$$x + 5x = -5x + 2 + 5x$$

$$6x = 2$$

$$\frac{6x}{6} = \frac{2}{6} \Rightarrow x = \frac{1}{3}$$

The solution set is $\left\{\frac{1}{3}\right\}$.

59. $p(p^2+3) = 12 + p^3$

$$p^3 + 3p = 12 + p^3$$

$$p^3 + 3p - p^3 = 12 + p^3 - p^3$$

$$3p = 12$$

$$\frac{3p}{3} = \frac{12}{3}$$

$$p = 4$$

The solution set is $\{4\}$.

60. $w(4-w^2) = 8 - w^3$

$$4w - w^3 = 8 - w^3$$

$$4w = 8$$

$$w = 2$$

The solution set is $\{2\}$.

61. $\frac{x}{x-2} + 3 = \frac{2}{x-2}$

$$(x-2)\left(\frac{x}{x-2} + 3\right) = (x-2)\left(\frac{2}{x-2}\right)$$

$$x + 3(x-2) = 2$$

$$x + 3x - 6 = 2$$

$$4x - 6 = 2$$

$$4x = 8$$

$$x = 2$$

This solution is not in the domain of the variable so it must be discarded. The equation has no solution. The solution set is $\{\}$ or $\emptyset$.

62. $\frac{2x}{x+3} = \frac{-6}{x+3} - 2$

$$(x+3)\left(\frac{2x}{x+3}\right) = (x+3)\left(\frac{-6}{x+3} - 2\right)$$

$$2x = -6 - 2(x+3)$$

$$2x = -6 - 2x - 6$$

$$2x = -2x - 12$$

$$4x = -12$$

$$x = -3$$

This solution is not in the domain of the variable so it must be discarded. The equation has no solution. The solution set is $\{\}$ or $\emptyset$.

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$$\begin{aligned}
 63. \quad \frac{2x}{x^2-4} &= \frac{4}{x^2-4} - \frac{3}{x+2} \\
 (x^2-4)\left(\frac{2x}{x^2-4}\right) &= (x^2-4)\left(\frac{4}{x^2-4} - \frac{3}{x+2}\right) \\
 2x &= 4 - 3(x-2) \\
 2x &= 4 - 3x + 6 \\
 2x &= 10 - 3x \\
 5x &= 10 \\
 x &= 2
 \end{aligned}$$

This solution is not in the domain of the variable so it must be discarded. The equation has no solution. The solution set is $\{ \}$ or $\emptyset$.

$$\begin{aligned}
 64. \quad \frac{x}{x^2-9} + \frac{4}{x+3} &= \frac{3}{x^2-9} \\
 (x^2-9)\left(\frac{x}{x^2-9} + \frac{4}{x+3}\right) &= (x^2-9)\left(\frac{3}{x^2-9}\right) \\
 x+4(x-3) &= 3 \\
 x+4x-12 &= 3 \\
 5x-12 &= 3 \\
 5x &= 15 \\
 x &= 3
 \end{aligned}$$

This solution is not in the domain of the variable so it must be discarded. The equation has no solution. The solution set is $\{ \}$ or $\emptyset$.

$$\begin{aligned}
 65. \quad \frac{x}{x+2} &= \frac{3}{2} \\
 2x &= 3(x+2) \\
 2x &= 3x+6 \\
 -x &= 6 \\
 x &= -6
 \end{aligned}$$

The solution set is $\{-6\}$.

$$\begin{aligned}
 66. \quad \frac{3x}{x-1} &= 2 \\
 3x &= 2(x-1) \\
 3x &= 2x-2 \\
 x &= -2 \\
 \text{The solution set is } &\{-2\}.
 \end{aligned}$$

$$\begin{aligned}
 67. \quad \frac{7}{3x+10} &= \frac{2}{x-3} \\
 7(x-3) &= 2(3x+10) \\
 7x-21 &= 6x+20 \\
 7x &= 6x+41 \\
 x &= 41 \\
 \text{The solution set is } &\{41\}.
 \end{aligned}$$

$$\begin{aligned}
 68. \quad \frac{-4}{x+4} &= \frac{-3}{x+6} \\
 -4(x+6) &= -3(x+6) \\
 -4x-24 &= -3x-12 \\
 -4x &= -3x+12 \\
 -x &= 12 \\
 x &= -12 \\
 \text{The solution set is } &\{-12\}.
 \end{aligned}$$

$$\begin{aligned}
 69. \quad \frac{6t+7}{4t-1} &= \frac{3t+8}{2t-4} \\
 (6t+7)(2t-4) &= (3t+8)(4t-1) \\
 12t^2+14t-24t-28 &= 12t^2+32t-3t-8 \\
 12t^2-10t-28 &= 12t^2+29t-8 \\
 -10t-28 &= 29t-8 \\
 -10t &= 29t+20 \\
 -39t &= 20 \\
 t &= \frac{20}{-39} \\
 t &= -\frac{20}{39} \\
 \text{The solution set is } &\left\{-\frac{20}{39}\right\}.
 \end{aligned}$$

$$\begin{aligned}
 70. \quad \frac{8w+5}{10w-7} &= \frac{4w-3}{5w+7} \\
 (8w+5)(5w+7) &= (4w-3)(10w-7) \\
 40w^2+25w+56w+35 &= 40w^2-30w-28w+21 \\
 40w^2+81w+35 &= 40w^2-58w+21 \\
 81w+35 &= -58w+21 \\
 81w &= -58w-14 \\
 139w &= -14 \\
 w &= -\frac{14}{139} \\
 \text{The solution set is } &\left\{-\frac{14}{139}\right\}.
 \end{aligned}$$

Section 1.2: Solving Equations Using a Graphing Utility; Linear and Rational Equations

$$71. \frac{4}{x-2} = \frac{-3}{x+5} + \frac{7}{(x+5)(x-2)}$$

$$\text{LCD} = (x-2)(x+5)$$

$$\frac{4(x+5)}{(x-2)(x+5)} = \frac{-3(x-2)}{(x-2)(x+5)} + \frac{7}{(x+5)(x-2)}$$

$$4(x+5) = -3(x-2) + 7$$

$$4x + 20 = -3x + 6 + 7$$

$$4x + 20 = -3x + 13$$

$$4x = -3x - 7$$

$$7x = -7$$

$$x = -1$$

The solution set is $\{-1\}$.

$$72. \frac{-4}{2x+3} + \frac{1}{x-1} = \frac{1}{(2x+3)(x-1)}$$

$$\text{LCD} = (2x+3)(x-1)$$

$$\frac{-4(x-1)}{(2x+3)(x-1)} + \frac{1(2x+3)}{(2x+3)(x-1)} = \frac{1}{(2x+3)(x-1)}$$

$$-4(x-1) + (2x+3) = 1$$

$$-4x + 4 + 2x + 3 = 1$$

$$-2x + 7 = 1$$

$$-2x = -6$$

$$x = 3$$

The solution set is $\{3\}$.

$$73. \frac{2}{y+3} + \frac{3}{y-4} = \frac{5}{y+6}$$

$$\text{LCD} = (y+3)(y-4)(y+6)$$

$$(y+3)(y-4)(y+6) \left(\frac{2}{y+3} + \frac{3}{y-4} = \frac{5}{y+6} \right)$$

$$2(y-4)(y+6) + 3(y+3)(y+6) = 5(y+3)(y-4)$$

$$2(y^2 + 2y - 24) + 3(y^2 + 9y + 18) = 5(y^2 - y - 12)$$

$$2y^2 + 4y - 48 + 3y^2 + 27y + 54 = 5y^2 - 5y - 60$$

$$5y^2 + 31y + 6 = 5y^2 - 5y - 60$$

$$31y + 6 = -5y - 60$$

$$31y = -5y - 66$$

$$36y = -66$$

$$y = -\frac{66}{36}$$

$$y = -\frac{11}{6}$$

The solution set is $\left\{-\frac{11}{6}\right\}$.

Chapter 1: Graphs, Equations, and Inequalities

$$74. \frac{5}{5z-11} + \frac{4}{2z-3} = \frac{-3}{5-z}$$

$$\text{LCD} = (5z-11)(2z-3)(5-z)$$

$$(5z-11)(2z-3)(5-z) \left(\frac{5}{5z-11} + \frac{4}{2z-3} = \frac{-3}{5-z} \right)$$

$$5(2z-3)(5-z) + 4(5z-11)(5-z) = -3(5z-11)(2z-3)$$

$$-10z^2 + 65z - 75 - 20z^2 + 144z - 220 = -30z^2 + 111z - 99$$

$$-30z^2 + 209z - 295 = -30z^2 + 111z - 99$$

$$209z - 295 = 111z - 99$$

$$209z = 111z + 196$$

$$98z = 196$$

$$z = 2$$

The solution set is $\{2\}$.

$$75. \frac{x}{x^2-9} - \frac{x-4}{x^2+3x} = \frac{10}{x^2-3x}$$

$$\text{LCD} = x(x+3)(x-3)$$

$$x(x+3)(x-3) \left(\frac{x}{x^2-9} - \frac{x-4}{x^2+3x} = \frac{10}{x^2-3x} \right)$$

$$x^2 - (x-4)(x-3) = 10(x+3)$$

$$x^2 - (x^2 - 7x + 12) = 10x + 30$$

$$x^2 - x^2 + 7x - 12 = 10x + 30$$

$$7x - 12 = 10x + 30$$

$$-3x = 42$$

$$-x = 14$$

$$x = -14$$

The solution set is $\{-14\}$.

$$76. \frac{x+1}{x^2+2x} - \frac{x+4}{x^2+x} = \frac{-3}{x^2+3x+2}$$

$$\text{LCD} = x(x+2)(x+1)$$

$$x(x+2)(x+1) \left(\frac{x+1}{x^2+2x} - \frac{x+4}{x^2+x} = \frac{-3}{x^2+3x+2} \right)$$

$$(x+1)(x+1) - (x+4)(x+2) = -3(x)$$

$$x^2 + 2x + 1 - (x^2 + 6x + 8) = -3x$$

$$x^2 + 2x + 1 - x^2 - 6x - 8 = -3x$$

$$-4x - 7 = -3x$$

$$-4x = -3x + 7$$

$$-x = 7$$

$$x = -7$$

The solution set is $\{-7\}$.

$$77. y = 5x + 4$$

$$2 = 5a + 4$$

$$-2 = 5a$$

$$a = -\frac{2}{5}$$

$$78. y = x^2 + 3x$$

$$b = 2^2 + 3(2) = 4 + 6 = 10$$

$$79. 2x + 3y = 6$$

$$2a + 3b = 6$$

$$3b = -2a + 6$$

$$b = -\frac{2}{3}a + 2$$

$$80. y = mx + b$$

$$0 = m(2) + b$$

$$5 = m(0) + b \rightarrow 5 = b$$

Therefore,

$$0 = 2m + 5$$

$$-2m = 5$$

$$m = -\frac{5}{2}$$

$$81. ax - b = c, \quad a \neq 0$$

$$ax - b + b = c + b$$

$$ax = c + b$$

$$\frac{ax}{a} = \frac{c+b}{a}$$

$$x = \frac{b+c}{a}$$

$$82. 1 - ax = b, \quad a \neq 0$$

$$1 - ax - 1 = b - 1$$

$$-ax = b - 1$$

$$\frac{-ax}{-a} = \frac{b-1}{-a}$$

$$x = \frac{b-1}{-a} = \frac{1-b}{a}$$

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$$83. \frac{x}{a} + \frac{x}{b} = c, \quad a \neq 0, b \neq 0, a \neq -b$$

$$ab \left(\frac{x}{a} + \frac{x}{b} \right) = ab \cdot c$$

$$bx + ax = abc$$

$$x(a+b) = abc$$

$$\frac{x(a+b)}{a+b} = \frac{abc}{a+b}$$

$$x = \frac{abc}{a+b}$$

$$84. \frac{a}{x} + \frac{b}{x} = c, \quad c \neq 0$$

$$x \left(\frac{a}{x} + \frac{b}{x} \right) = x \cdot c$$

$$a + b = cx$$

$$\frac{a+b}{c} = \frac{cx}{c}$$

$$x = \frac{a+b}{c}$$

such that $a \neq -b$

$$85. \frac{1}{x-a} + \frac{1}{x+a} = \frac{2}{x-1}$$

Multiply both sides by the LCD,

$(x-a)(x+a)(x-1)$, to get:

$$(x+a)(x-1) + (x-a)(x-1) = 2(x-a)(x+a)$$

$$x^2 + ax - x - a + x^2 - ax - x + a = 2x^2 - 2a^2$$

$$2x^2 - 2x = 2x^2 - 2a^2$$

$$2x^2 - 2x - 2x^2 = 2x^2 - 2a^2 - 2x^2$$

$$-2x = -2a^2$$

$$\frac{-2x}{-2} = \frac{-2a^2}{-2}$$

$$x = a^2$$

where $a \neq -1, 0, 1$.

$$86. \frac{b+c}{x+a} = \frac{b-c}{x-a}$$

$$(b+c)(x-a) = (b-c)(x+a)$$

$$bx + cx - ab - ac = bx - cx + ab - ac$$

$$bx + cx - ab - ac - bx = bx - cx + ab - ac - bx$$

$$cx - ab - ac = -cx + ab - ac$$

$$cx - ab - ac + cx = -cx + ab - ac + cx$$

$$2cx - ab - ac = ab - ac$$

$$2cx - ab - ac + ab + ac = ab - ac + ab + ac$$

$$2cx = 2ab$$

$$\frac{2cx}{2c} = \frac{2ab}{2c}$$

$$x = \frac{ab}{c}$$

where $a \neq 0, c \neq 0$, and $b \neq \pm c$.

$$87. x + 2a = 16 + ax - 6a$$

when $x = 4$:

$$4 + 2a = 16 + a(4) - 6a$$

$$4 + 2a = 16 + 4a - 6a$$

$$4 + 2a = 16 - 2a$$

$$4a = 12$$

$$a = 3$$

$$88. x + 2b = x - 4 + 2bx$$

when $x = 2$:

$$2 + 2b = 2 - 4 + 2b(2)$$

$$2 + 2b = 2 - 4 + 4b$$

$$2 + 2b = -2 + 4b$$

$$4 = 2b$$

$$\frac{4}{2} = b \Rightarrow b = 2$$

89. Solving for R:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$RR_1R_2 \left(\frac{1}{R} \right) = RR_1R_2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$R_1R_2 = RR_2 + RR_1$$

$$R_1R_2 = R(R_2 + R_1)$$

$$\frac{R_1R_2}{R_2 + R_1} = \frac{R(R_2 + R_1)}{R_2 + R_1}$$

$$\frac{R_1R_2}{R_1 + R_2} = R$$

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90. Solving for r :

$$A = P(1 + rt)$$

$$A = P + Prt$$

$$A - P = Prt$$

$$\frac{A - P}{Pt} = \frac{Prt}{Pt}$$

$$r = \frac{A - P}{Pt}$$

91. Solving for R :

$$F = \frac{mv^2}{R}$$

$$RF = R \left(\frac{mv^2}{R} \right)$$

$$RF = mv^2$$

$$\frac{RF}{F} = \frac{mv^2}{F} \Rightarrow R = \frac{mv^2}{F}$$

92. Solving for T :

$$PV = nRT$$

$$\frac{PV}{nR} = \frac{nRT}{nR}$$

$$T = \frac{PV}{nR}$$

93. Solving for r :

$$S = \frac{a}{1 - r}$$

$$(1 - r) \cdot S = (1 - r) \cdot \frac{a}{1 - r}$$

$$(1 - r)S = a$$

$$\frac{(1 - r)S}{S} = \frac{a}{S}$$

$$1 - r = \frac{a}{S}$$

$$1 - r - 1 = \frac{a}{S} - 1$$

$$-r = \frac{a}{S} - 1$$

$$r = 1 - \frac{a}{S} \quad \text{or} \quad r = \frac{S - a}{S}$$

94. Solving for t :

$$v = -gt + v_0$$

$$v - v_0 = -gt + v_0 - v_0$$

$$v - v_0 = -gt$$

$$\frac{v - v_0}{-g} = \frac{-gt}{-g}$$

$$\frac{v_0 - v}{g} = t \quad \text{or} \quad t = \frac{v_0 - v}{g}$$

95. Let x = amount invested in bonds.

Invested in CDs: $x - 3000$

$$x + (x - 3,000) = 20,000$$

$$2x - 3,000 = 20,000$$

$$2x = 23,000$$

$$x = 11,500$$

\$11,500 will be invested in bonds and \$8,500 will be invested in CD's.

96. Let x = amount that Sean receives.

Amount for George: $x - 3000$

$$x + (x - 3,000) = 10,000$$

$$2x - 3,000 = 10,000$$

$$2x = 13,000$$

$$x = 6,500$$

Sean will receive \$6,500 and George will receive \$3,500.

97. Let x = Sandra's regular hourly wage.

$$40x + (1.5x)(8) = 910$$

$$40x + 12x = 910$$

$$52x = 910$$

$$x = \frac{910}{52} = 17.50$$

Sandra's regular hourly wage is \$17.50.

98. Let x = Leigh's regular hourly wage.

$$40x + 6(1.5x) + 4(2x) = 1083$$

$$40x + 9x + 8x = 1083$$

$$57x = 1083$$

$$x = \frac{1083}{57} = 19$$

Leigh's regular hourly wage is \$19.00.

99. Let x = final exam score.

Compute the final average and set equal to 80.

$$\left(\frac{1}{7} \right) (80 + 83 + 71 + 61 + 95 + x + x) = 80$$

Now solve for x :

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$$\left(\frac{1}{7}\right)(390 + 2x) = 80$$

$$390 + 2x = 560$$

$$2x = 170$$

$$x = 85$$

Brooke needs to score an 85 on the final exam to get an average of 80 in the course.

- 100.** Let x = final exam score.

$$\left(\frac{1}{3}\right)\left(\frac{86 + 80 + 84 + 90}{4}\right) + \left(\frac{2}{3}\right)x = 80$$

$$\frac{340}{12} + \frac{2}{3}x = 80$$

$$12\left(\frac{340}{12} + \frac{2}{3}x\right) = 12(80)$$

$$340 + 8x = 960$$

$$8x = 620$$

$$x = \frac{620}{8} = 77.5$$

Mike must score 78 on the final to earn a B.

Following the same approach as above, we get

$$340 + 8x = 12(90)$$

$$340 + 8x = 1080$$

$$8x = 740$$

$$x = \frac{740}{8} = 92.5$$

Mike must score 93 on the final to earn an A.

- 101.** Let x represent the original price of the phone.
Then $0.15x$ represents the reduction in the price of the phone.

original price – reduction = new price

$$x - 0.15x = 799$$

$$0.85x = 799$$

$$x = 940$$

The original price of the phone was \$940.

The amount of the savings is

$$0.15(\$940) = \$141.$$

- 102.** Let x represent the original price of the car.
Then $0.12x$ represents the reduction in the price of the car.

original price – reduction = new price

$$x - 0.12x = 28,160$$

$$0.88x = 28,160$$

$$x = 32,000$$

The original price of the car was \$32,000.

The amount of the savings is

$$\$32,000 - \$28,160 = \$3,840.$$

- 103.** Let x represent the price the theater pays for the candy.

Then $2.75x$ represents the markup on the candy.

The selling price of the candy is \$3.00.

supplier price + markup = selling price

$$x + 2.75x = 4.50$$

$$3.75x = 4.50$$

$$x = 1.20$$

The theater paid \$1.20 for the candy.

- 104.** Let x represent the dealer's cost of the new car.

$$x = 0.85(\$34,000) = \$28,900$$

If the dealer accepts \$100 over cost, then you will pay

$$\$28,900 + \$100 = \$29,000.$$

- 105.** Let a = number of adult patrons.

Child patrons: $5200 - a$

$$8.50a + 6.00(5200 - a) = 32,200$$

$$8.50a + 31,200 - 6.00a = 32,200$$

$$2.50a + 31,200 = 32,200$$

$$2.50a = 1,000$$

$$\frac{2.50a}{2.50} = \frac{1,000}{2.50}$$

$$a = 400$$

The theater had 400 adult patrons.

- 106.** Let x = original price of the boots.

Discount: $0.3x$

original price – discount = sale price

$$x - 0.3x = 399$$

$$0.7x = 399$$

$$\frac{0.7x}{0.7} = \frac{399}{0.7}$$

$$x = 570$$

The original price of the boots was \$570.

- 107.** Let w = width.

Length: $l = w + 8$

Perimeter: $P = 2l + 2w$

$$2l + 2w = 60$$

$$2(w + 8) + 2w = 60$$

$$2w + 16 + 2w = 60$$

$$4w + 16 = 60$$

$$4w = 44$$

$$w = 11$$

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The rectangle has a width of 11 feet and a length of 19 feet. **108.** Let w = width.

Length: $l = 2w$

Perimeter: $P = 2l + 2w$

$$2l + 2w = 42$$

$$2(2w) + 2w = 42$$

$$4w + 2w = 42$$

$$6w = 42$$

$$w = 7$$

The rectangle has a width of 7 meters and a length of 14 meters.

- 109.** Let x = the total number of people who owned a smart phone in 2019. Then $0.534x$ represents the number of people who owned a smart phone that ran the Google Android OS.
 $0.534x = 124.4$

$$x = \frac{124.4}{0.534} \approx 233$$

There were about 233 million people who owned a smart phone in 2019.

- 110.** Let T be the percentage that used Twitter. Then:
 $67 = 2T - 9$
 $76 = 2T$
 $T = 38$
Therefore, 38% of U.S. adults aged 18-29 used Twitter.

- 111.** Let x = amount that Scott gets.

Amount for Alice: $\frac{3}{4}x$

Amount for Tricia: $\frac{1}{2}x$

$$x + \left(\frac{3}{4}x\right) + \left(\frac{1}{2}x\right) = 900,000$$

$$\frac{9}{4}x = 900,000$$

$$x = \frac{4}{9}(900,000)$$

$$x = 400,000$$

Scott receives \$400,000; Alice receives \$300,000; Tricia receives \$200,000.

- 112.** Let x = amount that Judy pays.

Amount Tom pays: $\frac{2}{3}x$

$$x + \left(\frac{2}{3}x\right) = 18$$

$$\frac{5}{3}x = 18$$

$$\frac{3}{5} \cdot \frac{5}{3}x = \frac{3}{5}(18)$$

$$x = 10.80$$

Judy pays \$10.80 and Tom pays \$7.20.

- 113.** An isosceles triangle has three equal sides. Therefore: $4x + 10 = 2x + 40 = 3x + 18$. Solve each set separately:

$$4x + 10 = 2x + 40$$

$$2x = 30$$

$$x = 15$$

$$4x + 10 = 3x + 18$$

$$x = 8$$

$$2x + 40 = 3x + 18$$

$$-x = -22$$

$$x = 22$$

Since 22 is the largest of the numbers then the largest perimeter is:

$$4(22) + 10 + 2(22) + 40 + 3(22) + 18 = 266$$

$$\mathbf{114.} \quad \frac{3}{4}x - \frac{1}{5}\left(\frac{1}{2} - 3x\right) + 1 = \frac{1}{4}\left(\frac{1}{20}x + 6\right) - \frac{4}{5}$$

$$\frac{3}{4}x - \frac{1}{10} + \frac{3}{5}x + 1 = \frac{1}{80}x + \frac{3}{2} - \frac{4}{5}$$

Multiply both sides by the LCD 80 to clear fractions.

$$60x - 8 + 48x + 80 = x + 120 - 64$$

$$108x + 72 = x + 58$$

$$107x = -16$$

$$x = -\frac{16}{107}$$

- 115.** If a hexagon is inscribed in a circle then the sides of the hexagon are equal to the radius of the circle. Let the $P = 6r$ be the perimeter of the hexagon. Let r be the radius of the circle.

$$6r = r + 10$$

$$5r = 10$$

$$r = 2$$

Thus $r = 2$ inches is the radius of the circle where the perimeter of the hexagon is 10 inches more than the radius.

116. In order to solve $\frac{5}{x+3} + 3 = \frac{8+x}{x+3}$, we multiply each term by the expression " $x+3$ " to get
- $$(x+3)\left(\frac{5}{x+3} + 3\right) = \left(\frac{8+x}{x+3}\right)(x+3).$$

Now, provided $x \neq -3$, we can cancel the denominators to get

$$5 + (x+3)(3) = 8+x$$

$$5 + 3x + 9 = 8 + x \Rightarrow 2x = -6 \Rightarrow x = -3$$

However, we already stated that $x \neq -3$. So we have a contradiction.

117. Answers will vary. One example is $3x+1 = 3x+6$.

118. Answers will vary. In general, 'solve' means to find values for a variable that make the equation true, 'evaluate' means to plug in a specific value for a variable in an expression and simplify, and 'simplify' means to collect like terms and write an expression in a simpler form.

Ex: Solve $3(x+2) - x = 8$.

Solution: $3(x+2) - x = 8$

$$3x + 6 - x = 8$$

$$2x + 6 = 8$$

$$2x = 2$$

$$x = 1$$

Ex: Evaluate $3(x+2) - x$ for $x = -2$.

$$\begin{aligned}\text{Solution: } 3(x+2) - x &= 3(-2+2) - (-2) \\ &= 3(0) + 2 \\ &= 2\end{aligned}$$

Ex: Simplify $3(x+2) - x$.

$$\begin{aligned}\text{Solution: } 3(x+2) - x \\ 3x + 6 - x \\ 2x + 6\end{aligned}$$

119. In obtaining step (7) we divided by $x-2$. Since $x=2$ from step (1), we actually divided by 0.

Section 1.3

1. $x^2 - 5x - 6 = (x-6)(x+1)$

2. $2x^2 - x - 3 = (2x-3)(x+1)$

3. $\left\{-\frac{5}{3}, 3\right\}$

4. True

5. $\frac{1}{2} \cdot 5 = \frac{5}{2}; \left(\frac{5}{2}\right)^2 = \frac{25}{4}; x^2 + 5x + \frac{25}{4}$
 $x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)^2$

6. discriminant; negative

7. False; a quadratic equation may have no real solutions.

8. False; If $x^2 = p$ then x could also be negative.

9. b

10. d

11. $x^2 - 9x = 0$

$$x(x-9) = 0$$

$$x = 0 \text{ or } x - 9 = 0$$

$$x = 0 \text{ or } x = 9$$

The solution set is $\{0, 9\}$.

12. $x^2 + 4x = 0$

$$x(x+4) = 0$$

$$x = 0 \text{ or } x + 4 = 0$$

$$x = 0 \text{ or } x = -4$$

The solution set is $\{-4, 0\}$.

13. $x^2 - 25 = 0$

$$(x+5)(x-5) = 0$$

$$x + 5 = 0 \text{ or } x - 5 = 0$$

$$x = -5 \text{ or } x = 5$$

The solution set is $\{-5, 5\}$.

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14. $x^2 - 9 = 0$

$$(x+3)(x-3) = 0$$

$$x+3=0 \quad \text{or} \quad x-3=0$$

$$x=-3 \quad \text{or} \quad x=3$$

The solution set is $\{-3, 3\}$.

15. $z^2 + z - 6 = 0$

$$(z+3)(z-2) = 0$$

$$z+3=0 \quad \text{or} \quad z-2=0$$

$$z=-3 \quad \text{or} \quad z=2$$

The solution set is $\{-3, 2\}$.

16. $v^2 + 7v + 6 = 0$

$$(v+6)(v+1) = 0$$

$$v+6=0 \quad \text{or} \quad v+1=0$$

$$v=-6 \quad \text{or} \quad v=-1$$

The solution set is $\{-6, -1\}$

17. $2x^2 - 5x - 3 = 0$

$$(2x+1)(x-3) = 0$$

$$2x+1=0 \quad \text{or} \quad x-3=0$$

$$x = -\frac{1}{2} \quad \text{or} \quad x = 3$$

The solution set is $\left\{-\frac{1}{2}, 3\right\}$

18. $3x^2 + 5x + 2 = 0$

$$(3x+2)(x+1) = 0$$

$$3x+2=0 \quad \text{or} \quad x+1=0$$

$$x = -\frac{2}{3} \quad \text{or} \quad x = -1$$

The solution set is $\left\{-1, -\frac{2}{3}\right\}$.

19. $-5w^2 + 180 = 0$

$$-5(w^2 - 36) = 0$$

$$-5(w+6)(w-6) = 0$$

$$w+6=0 \quad \text{or} \quad w-6=0$$

$$w=-6 \quad \text{or} \quad w=6$$

The solution set is $\{-6, 6\}$.

20. $2y^2 - 50 = 0$

$$2(y^2 - 25) = 0$$

$$2(y+5)(y-5) = 0$$

$$y+5=0 \quad \text{or} \quad y-5=0$$

$$y=-5 \quad \text{or} \quad y=5$$

The solution set is $\{-5, 5\}$.

21. $x(x+3) - 10 = 0$

$$x^2 + 3x - 10 = 0$$

$$(x-2)(x+5) = 0$$

$$x-2=0 \quad \text{or} \quad x+5=0$$

$$x=2 \quad \text{or} \quad x=-5$$

The solution set is $\{-5, 2\}$.

22. $x(x+4) = 12$

$$x^2 + 4x - 12 = 0$$

$$(x+6)(x-2) = 0$$

$$x+6=0 \quad \text{or} \quad x-2=0$$

$$x=-6 \quad \text{or} \quad x=2$$

The solution set is $\{-6, 2\}$.

23. $4x^2 + 9 = 12x$

$$4x^2 - 12x + 9 = 0$$

$$(2x-3)^2 = 0$$

$$2x-3=0$$

$$x = \frac{3}{2}$$

The solution set is $\left\{\frac{3}{2}\right\}$.

24. $25x^2 + 16 = 40x$

$$25x^2 - 40x + 16 = 0$$

$$(5x-4)^2 = 0$$

$$5x-4=0$$

$$x = \frac{4}{5}$$

The solution set is $\left\{\frac{4}{5}\right\}$.

Section 1.3: Quadratic Equations

25. $6(p^2 - 1) = 5p$
 $6p^2 - 6 = 5p$
 $6p^2 - 5p - 6 = 0$
 $(3p + 2)(2p - 3) = 0$
 $3p + 2 = 0$ or $2p - 3 = 0$
 $p = -\frac{2}{3}$ or $p = \frac{3}{2}$
 The solution set is $\left\{-\frac{2}{3}, \frac{3}{2}\right\}$.

26. $2(2u^2 - 4u) + 3 = 0$
 $4u^2 - 8u + 3 = 0$
 $(2u - 1)(2u - 3) = 0$
 $2u - 1 = 0$ or $2u - 3 = 0$
 $u = \frac{1}{2}$ or $u = \frac{3}{2}$
 The solution set is $\left\{\frac{1}{2}, \frac{3}{2}\right\}$.

27. $6x - 5 = \frac{6}{x}$
 $(6x - 5)x = \left(\frac{6}{x}\right)x$
 $6x^2 - 5x = 6$
 $6x^2 - 5x - 6 = 0$
 $(3x + 2)(2x - 3) = 0$
 $3x + 2 = 0$ or $2x - 3 = 0$
 $x = -\frac{2}{3}$ or $x = \frac{3}{2}$
 Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{2}{3}, \frac{3}{2}\right\}$.

28. $x + \frac{12}{x} = 7$
 $\left(x + \frac{12}{x}\right)x = 7x$
 $x^2 + 12 = 7x$
 $x^2 - 7x + 12 = 0$
 $(x - 3)(x - 4) = 0$
 $x - 3 = 0$ or $x - 4 = 0$
 $x = 3$ or $x = 4$

Neither of these values causes a denominator to equal zero, so the solution set is $\{3, 4\}$.

29. $\frac{4(x-2)}{x-3} + \frac{3}{x} = \frac{-3}{x(x-3)}$
 $\left(\frac{4(x-2)}{x-3} + \frac{3}{x}\right)x(x-3) = \left(\frac{-3}{x(x-3)}\right)x(x-3)$
 $4x(x-2) + 3(x-3) = -3$
 $4x^2 - 8x + 3x - 9 = -3$
 $4x^2 - 5x - 6 = 0$
 $(4x + 3)(x - 2) = 0$
 $4x + 3 = 0$ or $x - 2 = 0$
 $x = -\frac{3}{4}$ or $x = 2$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{3}{4}, 2\right\}$.

30. $\frac{5}{x+4} = 4 + \frac{3}{x-2}$
 $\left(\frac{5}{x+4}\right)(x+4)(x-2) = \left(4 + \frac{3}{x-2}\right)(x+4)(x-2)$
 $5(x-2) = 4(x+4)(x-2) + 3(x+4)$
 $5x - 10 = 4(x^2 + 2x - 8) + 3x + 12$
 $5x - 10 = 4x^2 + 8x - 32 + 3x + 12$
 $0 = 4x^2 + 6x - 10$
 $0 = 2(2x^2 + 3x - 5)$
 $0 = 2(2x + 5)(x - 1)$
 $2x + 5 = 0$ or $x - 1 = 0$
 $x = -\frac{5}{2}$ or $x = 1$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{5}{2}, 1\right\}$.

31. $x^2 = 25$
 $x = \pm\sqrt{25}$
 $x = \pm 5$
 The solution set is $\{-5, 5\}$.

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32. $x^2 = 36$

$$x = \pm\sqrt{36}$$

$$x = \pm 6$$

The solution set is $\{-6, 6\}$.

33. $(x-1)^2 = 4$

$$x-1 = \pm\sqrt{4}$$

$$x-1 = \pm 2$$

$$x-1 = 2 \text{ or } x-1 = -2$$

$$x = 3 \text{ or } x = -1$$

The solution set is $\{-1, 3\}$.

34. $(x+2)^2 = 1$

$$x+2 = \pm\sqrt{1}$$

$$x+2 = \pm 1$$

$$x+2 = 1 \text{ or } x+2 = -1$$

$$x = -1 \text{ or } x = -3$$

The solution set is $\{-3, -1\}$.

35. $\left(\frac{1}{3}h+4\right)^2 = 16$

$$\frac{1}{3}h+4 = \pm\sqrt{16}$$

$$\frac{1}{3}h+4 = \pm 4$$

$$\frac{1}{3}h+4 = 4 \text{ or } \frac{1}{3}h+4 = -4$$

$$\frac{1}{3}h = 0 \text{ or } \frac{1}{3}h = -8$$

$$h = 0 \text{ or } h = -24$$

The solution set is $\{-24, 0\}$.

36. $(3z-2)^2 = 4$

$$3z-2 = \pm\sqrt{4}$$

$$3z-2 = \pm 2$$

$$3z-2 = 2 \text{ or } 3z-2 = -2$$

$$3z = 4 \text{ or } 3z = 0$$

$$z = \frac{4}{3} \text{ or } z = 0$$

The solution set is $\left\{0, \frac{4}{3}\right\}$.

37. $x^2 + 4x = 21$

$$x^2 + 4x + 4 = 21 + 4$$

$$(x+2)^2 = 25$$

$$x+2 = \pm\sqrt{25}$$

$$x+2 = \pm 5$$

$$x = -2 \pm 5$$

$$x = 3 \text{ or } x = -7$$

The solution set is $\{-7, 3\}$.

38. $x^2 - 6x = 13$

$$x^2 - 6x + 9 = 13 + 9$$

$$(x-3)^2 = 22$$

$$x-3 = \pm\sqrt{22}$$

$$x = 3 \pm \sqrt{22}$$

The solution set is $\{3 - \sqrt{22}, 3 + \sqrt{22}\}$.

39. $x^2 - \frac{1}{2}x - \frac{3}{16} = 0$

$$x^2 - \frac{1}{2}x = \frac{3}{16}$$

$$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{3}{16} + \frac{1}{16}$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{1}{4}$$

$$x - \frac{1}{4} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

$$x = \frac{1}{4} \pm \frac{1}{2}$$

$$x = \frac{3}{4} \text{ or } x = -\frac{1}{4}$$

The solution set is $\left\{-\frac{1}{4}, \frac{3}{4}\right\}$.

$$40. \quad x^2 + \frac{2}{3}x - \frac{1}{3} = 0$$

$$x^2 + \frac{2}{3}x = \frac{1}{3}$$

$$x^2 + \frac{2}{3}x + \frac{1}{9} = \frac{1}{3} + \frac{1}{9}$$

$$\left(x + \frac{1}{3}\right)^2 = \frac{4}{9}$$

$$x + \frac{1}{3} = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$$

$$x = -\frac{1}{3} \pm \frac{2}{3}$$

$$x = \frac{1}{3} \text{ or } x = -1$$

The solution set is $\left\{-1, \frac{1}{3}\right\}$.

$$41. \quad 3x^2 + x - \frac{1}{2} = 0$$

$$x^2 + \frac{1}{3}x - \frac{1}{6} = 0$$

$$x^2 + \frac{1}{3}x = \frac{1}{6}$$

$$x^2 + \frac{1}{3}x + \frac{1}{36} = \frac{1}{6} + \frac{1}{36}$$

$$\left(x + \frac{1}{6}\right)^2 = \frac{7}{36}$$

$$x + \frac{1}{6} = \pm\sqrt{\frac{7}{36}}$$

$$x + \frac{1}{6} = \pm\frac{\sqrt{7}}{6}$$

$$x = \frac{-1 \pm \sqrt{7}}{6}$$

The solution set is $\left\{\frac{-1 - \sqrt{7}}{6}, \frac{-1 + \sqrt{7}}{6}\right\}$.

$$42. \quad 2x^2 - 3x - 1 = 0$$

$$x^2 - \frac{3}{2}x - \frac{1}{2} = 0$$

$$x^2 - \frac{3}{2}x = \frac{1}{2}$$

$$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{1}{2} + \frac{9}{16}$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$$x - \frac{3}{4} = \pm\sqrt{\frac{17}{16}}$$

$$x - \frac{3}{4} = \pm\frac{\sqrt{17}}{4}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

The solution set is $\left\{\frac{3 - \sqrt{17}}{4}, \frac{3 + \sqrt{17}}{4}\right\}$.

$$43. \quad x^2 - 4x + 2 = 0$$

$$a = 1, \quad b = 2, \quad c = -13$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-13)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 52}}{2}$$

$$= \frac{-2 \pm \sqrt{56}}{2} = \frac{-2 \pm 2\sqrt{14}}{2} = -1 \pm \sqrt{14}$$

The solution set is $\{-1 - \sqrt{14}, -1 + \sqrt{14}\}$.

$$44. \quad x^2 + 4x + 2 = 0$$

$$a = 1, \quad b = 4, \quad c = 2$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 8}}{2}$$

$$= \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

The solution set is $\{-2 - \sqrt{2}, -2 + \sqrt{2}\}$.

$$45. \quad x^2 - 4x - 1 = 0$$

$$a = 1, \quad b = -4, \quad c = -1$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)} = \frac{4 \pm \sqrt{16 + 4}}{2}$$

$$= \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

The solution set is $\{2 - \sqrt{5}, 2 + \sqrt{5}\}$.

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46. $x^2 + 6x + 1 = 0$

$a = 1, \quad b = 6, \quad c = 1$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(1)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 4}}{2}$$

$$= \frac{-6 \pm \sqrt{32}}{2} = \frac{-6 \pm 4\sqrt{2}}{2} = -3 \pm 2\sqrt{2}$$

The solution set is $\{-3 - 2\sqrt{2}, -3 + 2\sqrt{2}\}$.

47. $2x^2 - 5x + 3 = 0$

$a = 2, \quad b = -5, \quad c = 3$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25 - 24}}{4} = \frac{5 \pm \sqrt{1}}{4} = \frac{5 \pm 1}{4}$$

$x = \frac{5+1}{4}$ or $x = \frac{5-1}{4}$

$x = \frac{6}{4}$ or $x = \frac{4}{4}$

$x = \frac{3}{2}$ or $x = 1$

The solution set is $\left\{1, \frac{3}{2}\right\}$.

48. $2x^2 + 5x + 3 = 0$

$a = 2, \quad b = 5, \quad c = 3$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25 - 24}}{4} = \frac{-5 \pm \sqrt{1}}{4} = \frac{-5 \pm 1}{4}$$

$x = \frac{-5+1}{4}$ or $x = \frac{-5-1}{4}$

$x = \frac{-4}{4}$ or $x = \frac{-6}{4}$

$x = -1$ or $x = -\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2}, -1\right\}$.

49. $4y^2 - y + 2 = 0$

$a = 4, \quad b = -1, \quad c = 2$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(2)}}{2(4)}$$

$$= \frac{1 \pm \sqrt{1 - 32}}{8} = \frac{1 \pm \sqrt{-31}}{8}$$

No real solution.

50. $4t^2 + t + 1 = 0$

$a = 4, \quad b = 1, \quad c = 1$

$$t = \frac{-1 \pm \sqrt{1^2 - 4(4)(1)}}{2(4)}$$

$$= \frac{-1 \pm \sqrt{1 - 16}}{8} = \frac{-1 \pm \sqrt{-15}}{8}$$

No real solution.

51. $9x^2 + 8x = 5$

$9x^2 + 8x - 5 = 0$

$a = 9, \quad b = 8, \quad c = -5$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(9)(-5)}}{2(9)}$$

$$= \frac{-8 \pm \sqrt{64 + 180}}{18} = \frac{-8 \pm \sqrt{244}}{18}$$

$$= \frac{-8 \pm 2\sqrt{61}}{18} = \frac{-4 \pm \sqrt{61}}{9}$$

The solution set is $\left\{\frac{-4 - \sqrt{61}}{9}, \frac{-4 + \sqrt{61}}{9}\right\}$.

52. $2x^2 = 1 - 2x$

$2x^2 + 2x - 1 = 0$

$a = 2, \quad b = 2, \quad c = -1$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)} = \frac{-2 \pm \sqrt{4 + 8}}{4}$$

$$= \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

The solution set is $\left\{\frac{-1 - \sqrt{3}}{2}, \frac{-1 + \sqrt{3}}{2}\right\}$.

53. $4x^2 = 9x$

$4x^2 - 9x = 0$

$x(4x - 9) = 0$

$x = 0 \text{ or } 4x - 9 = 0$

$x = 0 \text{ or } x = \frac{9}{4}$

The solution set is $\left\{0, \frac{9}{4}\right\}$.

54. $5x = 4x^2$

$0 = 4x^2 - 5x$

$0 = x(4x - 5)$

$x = 0 \text{ or } 4x - 5 = 0$

$x = 0 \text{ or } x = \frac{5}{4}$

The solution set is $\left\{0, \frac{5}{4}\right\}$.

55. $9t^2 - 6t + 1 = 0$

$a = 9, \quad b = -6, \quad c = 1$

$$t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)}$$

$$= \frac{6 \pm \sqrt{36 - 36}}{18} = \frac{6 \pm 0}{18} = \frac{1}{3}$$

The solution set is $\left\{\frac{1}{3}\right\}$.

56. $4u^2 - 6u + 9 = 0$

$a = 4, \quad b = -6, \quad c = 9$

$$u = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$$

$$= \frac{6 \pm \sqrt{36 - 144}}{8} = \frac{6 \pm \sqrt{-108}}{8}$$

No real solution.

57. $\frac{3}{4}x^2 - \frac{1}{4}x - \frac{1}{2} = 0$

$$4\left(\frac{3}{4}x^2 - \frac{1}{4}x - \frac{1}{2}\right) = 4(0)$$

$3x^2 - x - 2 = 0$

$a = 3, \quad b = -1, \quad c = -2$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{1 \pm \sqrt{1 + 24}}{6} = \frac{1 \pm \sqrt{25}}{6} = \frac{1 \pm 5}{6}$$

$x = \frac{1+5}{6} \text{ or } x = \frac{1-5}{6}$

$x = \frac{6}{6} \text{ or } x = \frac{-4}{6}$

$x = 1 \text{ or } x = -\frac{2}{3}$

The solution set is $\left\{-\frac{2}{3}, 1\right\}$.

58. $\frac{2}{3}x^2 - x - 3 = 0$

$$3\left(\frac{2}{3}x^2 - x - 3\right) = 3(0)$$

$2x^2 - 3x - 9 = 0$

$a = 2, \quad b = -3, \quad c = -9$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-9)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9 + 72}}{4} = \frac{3 \pm \sqrt{81}}{4} = \frac{3 \pm 9}{4}$$

$x = \frac{3+9}{4} \text{ or } x = \frac{3-9}{4}$

$x = \frac{12}{4} \text{ or } x = \frac{-6}{4}$

$x = 3 \text{ or } x = -\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2}, 3\right\}$.

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$$59. \quad \frac{5}{3}x^2 - x = \frac{1}{3}$$

$$3\left(\frac{5}{3}x^2 - x\right) = 3\left(\frac{1}{3}\right)$$

$$5x^2 - 3x = 1$$

$$5x^2 - 3x - 1 = 0$$

$$a = 5, \quad b = -3, \quad c = -1$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-1)}}{2(5)}$$

$$= \frac{3 \pm \sqrt{9+20}}{10} = \frac{3 \pm \sqrt{29}}{10}$$

$$\text{The solution set is } \left\{ \frac{3 - \sqrt{29}}{10}, \frac{3 + \sqrt{29}}{10} \right\}.$$

$$60. \quad \frac{3}{5}x^2 - x = \frac{1}{5}$$

$$5\left(\frac{3}{5}x^2 - x\right) = 5\left(\frac{1}{5}\right)$$

$$3x^2 - 5x = 1$$

$$3x^2 - 5x - 1 = 0$$

$$a = 3, \quad b = -5, \quad c = -1$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{25+12}}{6} = \frac{5 \pm \sqrt{37}}{6}$$

$$\text{The solution set is } \left\{ \frac{5 - \sqrt{37}}{6}, \frac{5 + \sqrt{37}}{6} \right\}.$$

$$61. \quad 2x(x+2) = 3$$

$$2x^2 + 4x - 3 = 0$$

$$a = 2, \quad b = 4, \quad c = -3$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)} = \frac{-4 \pm \sqrt{16+24}}{4}$$

$$= \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}$$

$$\text{The solution set is } \left\{ \frac{-2 - \sqrt{10}}{2}, \frac{-2 + \sqrt{10}}{2} \right\}.$$

$$62. \quad 3x(x+2) = 1$$

$$3x^2 + 6x - 1 = 0$$

$$a = 3, \quad b = 6, \quad c = -1$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(-1)}}{2(3)} = \frac{-6 \pm \sqrt{36+12}}{6}$$

$$= \frac{-6 \pm \sqrt{48}}{6} = \frac{-6 \pm 4\sqrt{3}}{6} = \frac{-3 \pm 2\sqrt{3}}{3}$$

$$\text{The solution set is } \left\{ \frac{-3 - 2\sqrt{3}}{3}, \frac{-3 + 2\sqrt{3}}{3} \right\}.$$

$$63. \quad 4 + \frac{1}{x} - \frac{1}{x^2} = 0$$

$$x^2 \left(4 + \frac{1}{x} - \frac{1}{x^2} \right) = x^2 (0)$$

$$4x^2 + x - 1 = 0$$

$$a = 4, \quad b = 1, \quad c = -1$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-1 \pm \sqrt{1+16}}{8} = \frac{-1 \pm \sqrt{17}}{8}$$

Neither of these values causes a denominator to equal zero, so the solution set is

$$\left\{ \frac{-1 - \sqrt{17}}{8}, \frac{-1 + \sqrt{17}}{8} \right\}.$$

$$64. \quad 2 + \frac{8}{x} + \frac{3}{x^2} = 0$$

$$x^2 \left(2 + \frac{8}{x} + \frac{3}{x^2} \right) = x^2 (0)$$

$$2x^2 + 8x + 3 = 0$$

$$a = 2, \quad b = 8, \quad c = 3$$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-8 \pm \sqrt{64-24}}{4} = \frac{-8 \pm \sqrt{40}}{4}$$

$$= \frac{-8 \pm 2\sqrt{10}}{4} = \frac{-4 \pm \sqrt{10}}{2}$$

Neither of these values causes a denominator to equal zero, so the solution set is

$$\left\{ \frac{-4 - \sqrt{10}}{2}, \frac{-4 + \sqrt{10}}{2} \right\}.$$

$$65. \quad \frac{3x}{x-2} + \frac{1}{x} = 4$$

$$\left(\frac{3x}{x-2} + \frac{1}{x} \right) x(x-2) = 4x(x-2)$$

$$3x(x) + (x-2) = 4x^2 - 8x$$

$$3x^2 + x - 2 = 4x^2 - 8x$$

$$0 = x^2 - 9x + 2$$

$$a = 1, \quad b = -9, \quad c = 2$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{9 \pm \sqrt{81-8}}{2} = \frac{9 \pm \sqrt{73}}{2}$$

Neither of these values causes a denominator to equal zero, so the solution set is

$$\left\{ \frac{9 - \sqrt{73}}{2}, \frac{9 + \sqrt{73}}{2} \right\}.$$

$$66. \quad \frac{2x}{x-3} + \frac{1}{x} = 4$$

$$\left(\frac{2x}{x-3} + \frac{1}{x} \right) x(x-3) = 4x(x-3)$$

$$2x(x) + (x-3) = 4x^2 - 12x$$

$$2x^2 + x - 3 = 4x^2 - 12x$$

$$0 = 2x^2 - 13x + 3$$

$$a = 2, \quad b = -13, \quad c = 3$$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{13 \pm \sqrt{169-24}}{4} = \frac{13 \pm \sqrt{145}}{4}$$

Neither of these values causes a denominator to equal zero, so the solution set is

$$\left\{ \frac{13 - \sqrt{145}}{4}, \frac{13 + \sqrt{145}}{4} \right\}.$$

$$67. \quad x^2 - 4.1x + 2.2 = 0$$

$$a = 1, \quad b = -4.1, \quad c = 2.2$$

$$x = \frac{-(-4.1) \pm \sqrt{(-4.1)^2 - 4(1)(2.2)}}{2(1)}$$

$$= \frac{4.1 \pm \sqrt{16.81-8.8}}{2} = \frac{4.1 \pm \sqrt{8.01}}{2}$$

$$x \approx 3.47 \quad \text{or} \quad x \approx 0.63$$

The solution set is $\{0.63, 3.47\}$.

$$68. \quad x^2 + 3.9x + 1.8 = 0$$

$$a = 1, \quad b = 3.9, \quad c = 1.8$$

$$x = \frac{-3.9 \pm \sqrt{(3.9)^2 - 4(1)(1.8)}}{2(1)}$$

$$= \frac{-3.9 \pm \sqrt{15.21-7.2}}{2} = \frac{-3.9 \pm \sqrt{8.01}}{2}$$

$$x \approx -0.53 \quad \text{or} \quad x \approx -3.37$$

The solution set is $\{-3.37, -0.53\}$.

$$69. \quad x^2 + \sqrt{3}x - 3 = 0$$

$$a = 1, \quad b = \sqrt{3}, \quad c = -3$$

$$x = \frac{-\sqrt{3} \pm \sqrt{(\sqrt{3})^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-\sqrt{3} \pm \sqrt{3+12}}{2} = \frac{-\sqrt{3} \pm \sqrt{15}}{2}$$

$$x \approx 1.07 \quad \text{or} \quad x \approx -2.80$$

The solution set is $\{-2.80, 1.07\}$.

$$70. \quad x^2 + \sqrt{2}x - 2 = 0$$

$$a = 1, \quad b = \sqrt{2}, \quad c = -2$$

$$x = \frac{-\sqrt{2} \pm \sqrt{(\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2+8}}{2} = \frac{-\sqrt{2} \pm \sqrt{10}}{2}$$

$$x \approx 0.87 \quad \text{or} \quad x \approx -2.29$$

The solution set is $\{-2.29, 0.87\}$.

$$71. \quad \pi x^2 - x - \pi = 0$$

$$a = \pi, \quad b = -1, \quad c = -\pi$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(\pi)(-\pi)}}{2(\pi)}$$

$$= \frac{1 \pm \sqrt{1+4\pi^2}}{2\pi}$$

$$x \approx 1.17 \quad \text{or} \quad x \approx -0.85$$

The solution set is $\{-0.85, 1.17\}$.

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72. $\pi x^2 + \pi x - 2 = 0$

$a = \pi, \quad b = \pi, \quad c = -2$

$$x = \frac{-\pi \pm \sqrt{(\pi)^2 - 4(\pi)(-2)}}{2(\pi)}$$

$$= \frac{-\pi \pm \sqrt{\pi^2 + 8\pi}}{2\pi}$$

$x \approx 0.44$ or $x \approx -1.44$

The solution set is $\{-1.44, 0.44\}$.

73. $2x^2 - 6x + 7 = 0$

$a = 2, \quad b = -6, \quad c = 7$

$b^2 - 4ac = (-6)^2 - 4(2)(7) = 36 - 56 = -20$

Since the $b^2 - 4ac < 0$, the equation has no real solution.

74. $x^2 + 4x + 7 = 0$

$a = 1, \quad b = 4, \quad c = 7$

$b^2 - 4ac = (4)^2 - 4(1)(7) = 16 - 28 = -12$

Since the $b^2 - 4ac < 0$, the equation has no real solution.

75. $9x^2 - 30x + 25 = 0$

$a = 9, \quad b = -30, \quad c = 25$

$b^2 - 4ac = (-30)^2 - 4(9)(25) = 900 - 900 = 0$

Since $b^2 - 4ac = 0$, the equation has one repeated real solution.

76. $25x^2 - 20x + 4 = 0$

$a = 25, \quad b = -20, \quad c = 4$

$b^2 - 4ac = (-20)^2 - 4(25)(4) = 400 - 400 = 0$

Since $b^2 - 4ac = 0$, the equation has one repeated real solution.

77. $3x^2 + 5x - 8 = 0$

$a = 3, \quad b = 5, \quad c = -8$

$b^2 - 4ac = (5)^2 - 4(3)(-8) = 25 + 96 = 121$

Since $b^2 - 4ac > 0$, the equation has two unequal real solutions.

78. $2x^2 - 3x - 7 = 0$

$a = 2, \quad b = -3, \quad c = -7$

$b^2 - 4ac = (-3)^2 - 4(2)(-7) = 9 + 56 = 65$

Since $b^2 - 4ac > 0$, the equation has two unequal real solutions.

79. $x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

The solution set is $\{-\sqrt{5}, \sqrt{5}\}$.

80. $x^2 - 6 = 0$

$x^2 = 6$

$x = \pm\sqrt{6}$

The solution set is $\{-\sqrt{6}, \sqrt{6}\}$.

81. $16x^2 - 8x + 1 = 0$

$(4x - 1)(4x - 1) = 0$

$4x - 1 = 0$

$x = \frac{1}{4}$

The solution set is $\left\{\frac{1}{4}\right\}$.

82. $9x^2 - 12x + 4 = 0$

$(3x - 2)(3x - 2) = 0$

$3x - 2 = 0$

$x = \frac{2}{3}$

The solution set is $\left\{\frac{2}{3}\right\}$.

83. $10x^2 - 19x - 15 = 0$

$(5x + 3)(2x - 5) = 0$

$5x + 3 = 0$ or $2x - 5 = 0$

$x = -\frac{3}{5}$ or $x = \frac{5}{2}$

The solution set is $\left\{-\frac{3}{5}, \frac{5}{2}\right\}$.

84. $6x^2 + 7x - 20 = 0$

$(3x - 4)(2x + 5) = 0$

$3x - 4 = 0$ or $2x + 5 = 0$

$x = \frac{4}{3}$ or $x = -\frac{5}{2}$

The solution set is $\left\{-\frac{5}{2}, \frac{4}{3}\right\}$.

85. $2 + z = 6z^2$

$$0 = 6z^2 - z - 2$$

$$0 = (3z - 2)(2z + 1)$$

$$3z - 2 = 0 \text{ or } 2z + 1 = 0$$

$$z = \frac{2}{3} \text{ or } z = -\frac{1}{2}$$

The solution set is $\left\{-\frac{1}{2}, \frac{2}{3}\right\}$.

86. $2 = y + 6y^2$

$$0 = 6y^2 + y - 2$$

$$0 = (3y + 2)(2y - 1)$$

$$3y + 2 = 0 \text{ or } 2y - 1 = 0$$

$$y = -\frac{2}{3} \text{ or } y = \frac{1}{2}$$

The solution set is $\left\{-\frac{2}{3}, \frac{1}{2}\right\}$.

87. $x^2 + \sqrt{2}x = \frac{1}{2}$

$$x^2 + \sqrt{2}x - \frac{1}{2} = 0$$

$$2\left(x^2 + \sqrt{2}x - \frac{1}{2}\right) = 2(0)$$

$$2x^2 + 2\sqrt{2}x - 1 = 0$$

$$a = 2, \quad b = 2\sqrt{2}, \quad c = -1$$

$$x = \frac{-2\sqrt{2} \pm \sqrt{(2\sqrt{2})^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-2\sqrt{2} \pm \sqrt{8+8}}{4} = \frac{-2\sqrt{2} \pm \sqrt{16}}{4}$$

$$= \frac{-2\sqrt{2} \pm 4}{4} = \frac{-\sqrt{2} \pm 2}{2}$$

The solution set is $\left\{\frac{-\sqrt{2}-2}{2}, \frac{-\sqrt{2}+2}{2}\right\}$

88. $\frac{1}{2}x^2 = \sqrt{2}x + 1$

$$\frac{1}{2}x^2 - \sqrt{2}x - 1 = 0$$

$$2\left(\frac{1}{2}x^2 - \sqrt{2}x - 1\right) = 2(0)$$

$$x^2 - 2\sqrt{2}x - 2 = 0$$

$$a = 1, \quad b = -2\sqrt{2}, \quad c = -2$$

$$x = \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{2\sqrt{2} \pm \sqrt{8+8}}{2} = \frac{2\sqrt{2} \pm \sqrt{16}}{2}$$

$$= \frac{2\sqrt{2} \pm 4}{2} = \frac{\sqrt{2} \pm 2}{1}$$

The solution set is $\{\sqrt{2}-2, \sqrt{2}+2\}$.

89. $x^2 + x = 4$

$$x^2 + x - 4 = 0$$

$$a = 1, \quad b = 1, \quad c = -4$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$

The solution set is $\left\{\frac{-1-\sqrt{17}}{2}, \frac{-1+\sqrt{17}}{2}\right\}$.

90. $x^2 + x = 1$

$$x^2 + x - 1 = 0$$

$$a = 1, \quad b = 1, \quad c = -1$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

The solution set is $\left\{\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right\}$.

$$\begin{aligned}
 91. \quad & \frac{x}{x-2} + \frac{2}{x+1} = \frac{7x+1}{x^2-x-2} \\
 & \frac{x}{x-2} + \frac{2}{x+1} = \frac{7x+1}{(x-2)(x+1)} \\
 & \left(\frac{x}{x-2} + \frac{2}{x+1} \right) (x-2)(x+1) = \left(\frac{7x+1}{(x-2)(x+1)} \right) (x-2)(x+1) \\
 & x(x+1) + 2(x-2) = 7x+1 \\
 & x^2 + x + 2x - 4 = 7x+1 \\
 & x^2 + 3x - 4 = 7x+1 \\
 & x^2 - 4x - 5 = 0 \\
 & (x+1)(x-5) = 0 \\
 & x+1 = 0 \quad \text{or} \quad x-5 = 0 \\
 & x = -1 \quad \text{or} \quad x = 5
 \end{aligned}$$

The value $x = -1$ causes a denominator to equal zero, so we disregard it. Thus, the solution set is $\{5\}$.

$$\begin{aligned}
 92. \quad & \frac{3x}{x+2} + \frac{1}{x-1} = \frac{4-7x}{x^2+x-2} \\
 & \frac{3x}{x+2} + \frac{1}{x-1} = \frac{4-7x}{(x+2)(x-1)} \\
 & \left(\frac{3x}{x+2} + \frac{1}{x-1} \right) (x+2)(x-1) = \left(\frac{4-7x}{(x+2)(x-1)} \right) (x+2)(x-1) \\
 & 3x(x-1) + (x+2) = 4-7x \\
 & 3x^2 - 3x + x + 2 = 4-7x \\
 & 3x^2 - 2x + 2 = 4-7x \\
 & 3x^2 + 5x - 2 = 0 \\
 & (3x-1)(x+2) = 0 \\
 & 3x-1 = 0 \quad \text{or} \quad x+2 = 0 \\
 & x = \frac{1}{3} \quad \text{or} \quad x = -2
 \end{aligned}$$

The value $x = -2$ causes a denominator to equal zero, so we disregard it. Thus, the solution set is $\left\{ \frac{1}{3} \right\}$.

93. Since this is a right triangle then we can use the Pythagorean Theorem. So

$$\begin{aligned}
 & (2x+3)^2 = (2x-5)^2 + (x+7)^2 \\
 & 4x^2 + 12x + 9 = 4x^2 - 20x + 25 + x^2 + 14x + 49 \\
 & 12x + 9 = x^2 - 6x + 74 \\
 & 0 = x^2 - 18x + 65 \\
 & 0 = (x-5)(x-13) \\
 & x-5 = 0 \quad \text{or} \quad x-13 = 0 \\
 & x = 5 \quad \text{or} \quad x = 13
 \end{aligned}$$

This means there are 2 possible that meet these requirements. Substituting x into the given sides gives:

When $x = 5$: 5m, 12m, 13m
 When $x = 13$: 20m, 21m, 29m
 Thus there are 2 solutions.

Section 1.3: Quadratic Equations

94. Since this is a right triangle then we can use the Pythagorean Theorem. So

$$\begin{aligned}(4x+5)^2 &= (3x+13)^2 + x^2 \\ 16x^2 + 40x + 25 &= 9x^2 + 78x + 169 + x^2 \\ 6x^2 - 38x - 144 &= 0 \\ 2(3x^2 - 19x - 72) &= 0 \\ 2(3x+8)(x-9) &= 0 \\ 3x+8 &= 0 \quad \text{or} \quad x-9 = 0 \\ x &= -\frac{8}{3} \quad \text{or} \quad x = 9\end{aligned}$$

This means there are 2 possible solutions that meet these requirements. Substituting x into the given sides gives:

When $x = 9$: 41m, 40m, 9m

When $x = -\frac{8}{3}$ at least one side of the triangle

has a negative measurement which is impossible. Thus there is only 1 triangle possible

95. Let w represent the width of window. Then $l = w + 2$ represents the length of the window.

Since the area is 143 square feet, we have:

$$\begin{aligned}w(w+2) &= 143 \\ w^2 + 2w - 143 &= 0 \\ (w+13)(w-11) &= 0\end{aligned}$$

$$\cancel{w = -13} \quad \text{or} \quad w = 11$$

Discard the negative solution since width cannot be negative. The width of the rectangular window is 11 feet and the length is 13 feet.

96. Let w represent the width of window. Then $l = w + 1$ represents the length of the window. Since the area is 306 square centimeters, we have: $w(w+1) = 306$

$$\begin{aligned}w^2 + w - 306 &= 0 \\ (w+18)(w-17) &= 0\end{aligned}$$

$$\cancel{w = -18} \quad \text{or} \quad w = 17$$

Discard the negative solution since width cannot be negative. The width of the rectangular window is 17 centimeters and the length is 18 centimeters.

97. Let l represent the length of the rectangle. Let w represent the width of the rectangle. The perimeter is 26 meters and the area is 40

square meters.

$$2l + 2w = 26$$

$$l + w = 13 \quad \text{so} \quad w = 13 - l$$

$$lw = 40$$

$$l(13-l) = 40$$

$$13l - l^2 = 40$$

$$l^2 - 13l + 40 = 0$$

$$(l-8)(l-5) = 0$$

$$l = 8 \quad \text{or} \quad l = 5$$

$$w = 5 \quad w = 8$$

The dimensions are 5 meters by 8 meters.

98. Let r represent the radius of the circle. Since the field is a square with area 1250 square feet, the length of a side of the square is $\sqrt{1250} = 25\sqrt{2}$ feet. The length of the diagonal is $2r$.

Use the Pythagorean Theorem to solve for r :

$$(2r)^2 = (25\sqrt{2})^2 + (25\sqrt{2})^2$$

$$4r^2 = 1250 + 1250$$

$$4r^2 = 2500$$

$$r^2 = 625$$

$$r = 25$$

The shortest radius setting for the sprinkler is 25 feet.

99. Let x = length of side of original sheet in feet. Length of box: $x - 2$ feet
Width of box: $x - 2$ feet
Height of box: 1 foot
 $V = l \cdot w \cdot h$

$$4 = (x-2)(x-2)(1)$$

$$4 = x^2 - 4x + 4$$

$$0 = x^2 - 4x$$

$$0 = x(x-4)$$

$$x = 0 \quad \text{or} \quad x = 4$$

Discard $x = 0$ since that is not a feasible length for the original sheet. Therefore, the original sheet should measure 4 feet on each side.

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- 100.** Let x = width of original sheet in feet.

Length of sheet: $2x$

Length of box: $2x - 2$ feet

Width of box: $x - 2$ feet

Height of box: 1 foot

$$V = l \cdot w \cdot h$$

$$4 = (2x - 2)(x - 2)(1)$$

$$4 = 2x^2 - 6x + 4$$

$$0 = 2x^2 - 6x$$

$$0 = x^2 - 3x$$

$$0 = x(x - 3)$$

$$x = 0 \text{ or } x = 3$$

Discard $x = 0$ since that is not a feasible length for the original sheet. Therefore, the original sheet is 3 feet wide and 6 feet long.

- 101. a.** When the ball strikes the ground, the distance from the ground will be 0.

Therefore, we solve

$$96 + 80t - 16t^2 = 0$$

$$-16t^2 + 80t + 96 = 0$$

$$t^2 - 5t - 6 = 0$$

$$(t - 6)(t + 1) = 0$$

$$t = 6 \text{ or } t = -1$$

Discard the negative solution since the time of flight must be positive. The ball will strike the ground after 6 seconds.

- b.** When the ball passes the top of the building, it will be 96 feet from the ground. Therefore, we solve

$$96 + 80t - 16t^2 = 96$$

$$-16t^2 + 80t = 0$$

$$t^2 - 5t = 0$$

$$t(t - 5) = 0$$

$$t = 0 \text{ or } t = 5$$

The ball is at the top of the building at time $t = 0$ when it is thrown. It will pass the top of the building on the way down after 5 seconds.

- 102. a.** To find when the object will be 15 meters above the ground, we solve

$$-4.9t^2 + 20t = 15$$

$$-4.9t^2 + 20t - 15 = 0$$

$$a = -4.9, b = 20, c = -15$$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-15)}}{2(-4.9)}$$

$$= \frac{-20 \pm \sqrt{106}}{-9.8} = \frac{20 \pm \sqrt{106}}{9.8}$$

$$t \approx 0.99 \text{ or } t \approx 3.09$$

The object will be 15 meters above the ground after about 0.99 seconds (on the way up) and about 3.09 seconds (on the way down).

- b.** The object will strike the ground when the distance from the ground is 0. Therefore, we solve

$$-4.9t^2 + 20t = 0$$

$$t(-4.9t + 20) = 0$$

$$t = 0 \text{ or } -4.9t + 20 = 0$$

$$-4.9t = -20$$

$$t \approx 4.08$$

The object will strike the ground after about 4.08 seconds.

- c.** $-4.9t^2 + 20t = 100$

$$-4.9t^2 + 20t - 100 = 0$$

$$a = -4.9, b = 20, c = -100$$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-100)}}{2(-4.9)}$$

$$= \frac{-20 \pm \sqrt{-1560}}{-9.8}$$

There is no real solution. The object never reaches a height of 100 meters.

- 103.** Let x represent the number of centimeters the length and width should be reduced.

$12 - x$ = the new length, $7 - x$ = the new width.

The new volume is 90% of the old volume.

$$(12 - x)(7 - x)(3) = 0.9(12)(7)(3)$$

$$3x^2 - 57x + 252 = 226.8$$

$$3x^2 - 57x + 25.2 = 0$$

$$x^2 - 19x + 8.4 = 0$$

$$x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(8.4)}}{2(1)} = \frac{19 \pm \sqrt{327.4}}{2}$$

$$x \approx 0.45 \text{ or } x \approx 18.55$$

Since 18.55 exceeds the dimensions, it is discarded. The dimensions of the new chocolate bar are: 11.55 cm by 6.55 cm by 3 cm.

- 104.** Let x represent the number of centimeters the length and width should be reduced.
 $12 - x$ = the new length, $7 - x$ = the new width.
 The new volume is 80% of the old volume.
 $(12 - x)(7 - x)(3) = 0.8(12)(7)(3)$

$$3x^2 - 57x + 252 = 201.6$$

$$3x^2 - 57x + 50.4 = 0$$

$$x^2 - 19x + 16.8 = 0$$

$$x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(16.8)}}{2(1)} = \frac{19 \pm \sqrt{293.8}}{2}$$

$$x \approx 0.93 \text{ or } x \approx 18.07$$

Since 18.07 exceeds the dimensions, it is discarded. The dimensions of the new chocolate bar are: 11.07 cm by 6.07 cm by 3 cm.

- 105.** Let x represent the width of the border measured in feet. The radius of the pool is 5 feet. Then $x + 5$ represents the radius of the circle, including both the pool and the border. The total area of the pool and border is

$$A_T = \pi(x + 5)^2.$$

The area of the pool is $A_P = \pi(5)^2 = 25\pi$.

The area of the border is

$$A_B = A_T - A_P = \pi(x + 5)^2 - 25\pi.$$

Since the concrete is 3 inches or 0.25 feet thick, the volume of the concrete in the border is

$$0.25A_B = 0.25(\pi(x + 5)^2 - 25\pi)$$

Solving the volume equation:

$$0.25(\pi(x + 5)^2 - 25\pi) = 27$$

$$\pi(x^2 + 10x + 25 - 25) = 108$$

$$\pi x^2 + 10\pi x - 108 = 0$$

$$x = \frac{-10\pi \pm \sqrt{(10\pi)^2 - 4(\pi)(-108)}}{2(\pi)}$$

$$= \frac{-31.42 \pm \sqrt{100\pi^2 + 432\pi}}{6.28}$$

$$x \approx 2.71 \text{ or } x \approx -12.71$$

Discard the negative solution. The width of the border is roughly 2.71 feet.

- 106.** Let x represent the width of the border measured in feet. The radius of the pool is 5 feet. Then $x + 5$ represents the radius of the circle, including both the pool and the border. The total area of the pool and border is

$$A_T = \pi(x + 5)^2.$$

The area of the pool is $A_P = \pi(5)^2 = 25\pi$.

The area of the border is

$$A_B = A_T - A_P = \pi(x + 5)^2 - 25\pi.$$

Since the concrete is 4 inches = $\frac{1}{3}$ foot thick, the volume of the concrete in the border is

$$\frac{1}{3}A_B = \frac{1}{3}(\pi(x + 5)^2 - 25\pi)$$

Solving the volume equation:

$$\frac{1}{3}(\pi(x + 5)^2 - 25\pi) = 27$$

$$\pi(x^2 + 10x + 25 - 25) = 81$$

$$\pi x^2 + 10\pi x - 81 = 0$$

$$x = \frac{-10\pi \pm \sqrt{(10\pi)^2 - 4(\pi)(-81)}}{2(\pi)}$$

$$= \frac{-31.42 \pm \sqrt{100\pi^2 + 324\pi}}{6.28}$$

$$x \approx 2.13 \text{ or } x \approx -12.13$$

Discard the negative solution. The width of the border is approximately 2.13 feet.

- 107.** Let x represent the width of the border measured in feet.

The total area is $A_T = (6 + 2x)(10 + 2x)$.

The area of the garden is $A_G = 6 \cdot 10 = 60$.

The area of the border is

$$A_B = A_T - A_G = (6 + 2x)(10 + 2x) - 60.$$

Since the concrete is 3 inches or 0.25 feet thick, the volume of the concrete in the border is

$$0.25A_B = 0.25((6 + 2x)(10 + 2x) - 60)$$

Solving the volume equation:

$$0.25((6 + 2x)(10 + 2x) - 60) = 27$$

$$60 + 32x + 4x^2 - 60 = 108$$

$$4x^2 + 32x - 108 = 0$$

$$x^2 + 8x - 27 = 0$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-27)}}{2(1)} = \frac{-8 \pm \sqrt{172}}{2}$$

$$x \approx 2.56 \text{ or } x \approx -10.56$$

Discard the negative solution. The width of the border is approximately 2.56 feet.

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- 108.** Let x = the width and $2x$ = the length of the patio. The height is $\frac{1}{3}$ foot and the concrete available is $8(27) = 216$ cubic feet..

$$V = lwh = x(2x) \cdot \frac{1}{3} = 216$$

$$\frac{2}{3}x^2 = 216$$

$$x^2 = 324 \Rightarrow x = \pm 18$$

The dimensions of the patio are 18 feet by 36 feet.

- 109.** Let x = the length of a 12.9-inch iPad Pro in a 16:94:3 format.

Then $\frac{9}{16}x$ = the width of the iPad. The diagonal of the 12.9-inch iPad is 9.7 inches, so by the Pythagorean theorem we have:

$$x^2 + \left(\frac{9}{16}x\right)^2 = 12.9^2$$

$$x^2 + \frac{81}{256}x^2 = 166.41$$

$$256\left(x^2 + \frac{81}{256}x^2\right) = 256(166.41)$$

$$256x^2 + 81x^2 = 42600.96$$

$$337x^2 = 42600.96 \Rightarrow x^2 = \frac{42600.96}{337}$$

$$x = \pm\sqrt{\frac{42600.96}{337}} \approx \pm 11.24$$

Since the length cannot be negative, the length of the iPad is $\sqrt{\frac{42600.96}{337}}$ inches and the width is

$$\frac{9}{16}\sqrt{\frac{42600.96}{337}} \approx 6.32 \text{ inches. Thus, the area of the}$$

iPad is $\sqrt{\frac{42600.96}{337}} \cdot \frac{9}{16}\sqrt{\frac{42600.96}{337}} = 71.11$ square inches.

Let y = the length of a 12-inch 3:2 format

Microsoft Surface Pro. Then $\frac{2}{3}y$ = the width of the Surface Pro. The diagonal of a 12-inch Surface Pro is 12 inches, so by the Pythagorean theorem we have:

$$y^2 + \left(\frac{2}{3}y\right)^2 = 12^2$$

$$y^2 + \frac{4}{9}y^2 = 151.29$$

$$9\left(y^2 + \frac{4}{9}y^2\right) = 9(151.29)$$

$$9y^2 + 4y^2 = 1361.61$$

$$13y^2 = 1361.61$$

$$y^2 = \frac{1361.61}{13}$$

$$y = \pm\sqrt{\frac{1361.61}{13}} \approx \pm 10.23$$

Since the length cannot be negative, the length of the Surface Pro is 10.23 inches and the width is

$$\frac{2}{3}\sqrt{\frac{1361.61}{13}} \approx 6.82 \text{ inches. Thus, the area of the}$$

12.3-inch 3:2 format Surface Pro is

$$\sqrt{\frac{1361.61}{13}} \cdot \frac{2}{3}\sqrt{\frac{1361.61}{13}}$$

$$\approx 69.83 \text{ square inches.}$$

The iPad Pro format has the larger screen since its area is larger.

- 110.** Let x = the length of a 7.9-inch iPad Mini in a 4:3 format.

Then $\frac{3}{4}x$ = the width of the iPad. The diagonal of the 7.9-inch iPad is 7.9 inches, so by the Pythagorean theorem we have:

$$x^2 + \left(\frac{3}{4}x\right)^2 = 7.9^2$$

$$x^2 + \frac{9}{16}x^2 = 62.41$$

$$16\left(x^2 + \frac{9}{16}x^2\right) = 16(62.41)$$

$$16x^2 + 9x^2 = 998.56$$

$$25x^2 = 998.56$$

$$x^2 = 39.9424$$

$$x = \pm\sqrt{39.9424} = \pm 6.32$$

Since the length cannot be negative, the length of the iPad is 6.32 inches and the width is

$$\frac{3}{4}(6.32) = 4.74 \text{ inches. Thus, the area of the}$$

iPad is $(6.32)(4.74) = 29.9568$ square inches.

Let y = the length of a 8-inch 16:10 format

Amazon Fire HD 8™. Then $\frac{10}{16}y$ = the width of

the Fire. The diagonal of a 8-inch Fire is 8 inches, so by the Pythagorean theorem we have:

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$$\begin{aligned}
 y^2 + \left(\frac{10}{16}y\right)^2 &= 8^2 \\
 y^2 + \frac{100}{256}y^2 &= 64 \\
 256\left(y^2 + \frac{100}{256}y^2\right) &= 256(64) \\
 256y^2 + 100y^2 &= 16384 \\
 356y^2 &= 16384 \\
 y^2 &= \frac{16384}{356} \\
 y &= \pm\sqrt{\frac{16384}{356}} \approx \pm 6.78399
 \end{aligned}$$

Since the length cannot be negative, the length of

the Fire is $\sqrt{\frac{16384}{356}} \approx 6.78399$ inches and the

width is $\frac{10}{16}\sqrt{\frac{16384}{356}} \approx 4.240$ inches. Thus, the area

of the Amazon Fire is

$$(6.78399)(4.240) \approx 28.76 \text{ square inches.}$$

The iPad Mini™ 4:3 format has the larger screen since its area is larger.

- 111.** Let h be 1.1. Then

$$1.1 = -0.00025x^2 + 0.04x$$

$$0 = -0.00025x^2 + 0.04x - 1.1$$

$$x = \frac{-0.04 \pm \sqrt{(0.04)^2 - 4(-0.00025)(-1.1)}}{2(-0.00025)}$$

$$= 35.3 \text{ ft or } 124.7 \text{ ft}$$

124.7 ft does not make sense in the context of the problem, so the answer is 35.3 ft.

- 112.** Since d is expressed in 1000's we will set $d = 15$ and solve for x using the Quadratic Formula.

$$d = -0.012x^2 + 0.828x + 15.750$$

$$25 = -0.012x^2 + 0.828x + 15.750$$

$$0 = -0.012x^2 + 0.828x - 9.25$$

$$x = \frac{-0.828 \pm \sqrt{(0.828)^2 - 4(-0.012)(-9.25)}}{2(-0.012)}$$

$$= \frac{-0.828 \pm \sqrt{.241584}}{-0.024}$$

$$x \approx 54.98 \text{ or } x \approx 14.02$$

So the nearest year when the difference was \$25,000 occurred about 14 years after 1980 or 1994. The value 55 has no meaning since it is in the future.

- 113.** We will set $g = 2.97$ and solve for h using the Quadratic Formula.

$$g = -0.0006x^2 + 0.015x + 3.04$$

$$2.97 = -0.0006x^2 + 0.015x + 3.04$$

$$0 = -0.0006x^2 + 0.015x + 0.07$$

$$x = \frac{-0.015 \pm \sqrt{(0.015)^2 - 4(-0.0006)(0.07)}}{2(-0.0006)}$$

$$= \frac{-0.015 \pm \sqrt{0.000393}}{-0.0012}$$

$$x \approx 29 \text{ or } x \approx -4.02$$

So the estimated numbers of hours worked by a student with a GPA of 2.97 is 29 hours. The value -4.02 has no meaning since it is negative.

- 114.** Let x be the numbers of members in the fraternity and s be the share paid by each

member. Then $s = \frac{1470}{x}$. If there are 7

members who cannot contribute then the share goes up by \$5. So we have the following equation:

$$s + 5 = \frac{1470}{x-7} \text{ or } (s+5)(x-7) = 1470$$

Solving these two equations together:

$$(s+5)(x-7) = 1470 \text{ and } s = \frac{1470}{x}$$

$$\left(\frac{1470}{x} + 5\right)(x-7) = 1470$$

$$1470 - \frac{10290}{x} + 5x - 35 = 1470$$

$$5x - \frac{10290}{x} - 35 = 0$$

$$5x^2 - 35x - 10290 = 0$$

$$5x^2 - 35x - 10290 = 0$$

$$x^2 - 7x - 2058 = 0$$

$$(x+42)(x-49) = 0$$

$$x = -42 \text{ or } x = 49$$

Since x is the number of members, it must be positive so the number of members is 49.

Chapter 1: Graphs, Equations, and Inequalities

- 115.** Let a be the age the individual is able to start saving money. Then we need to find where the models are equal. Solving these two equations together:

$$-25a^2 + 2400a - 30700 = 160a + 7840$$

$$25a^2 - 2240a + 38540 = 0$$

$$a = \frac{2240 \pm \sqrt{(2240)^2 - 4(25)(38540)}}{2(25)}$$

$$a = \frac{2240 \pm \sqrt{1163600}}{50}$$

$$a = \frac{2240 \pm 1078.7}{50}$$

$$a = \frac{2240 + 1078.7}{50} \quad \text{or} \quad a = \frac{2240 - 1078.7}{50}$$

$$a = 66.4 \quad \text{or} \quad a = 23.2$$

Since x is the age to start saving, it makes sense that the answer is approximate at age 23.

- 116.** We will set the equation equal to 10 and solve:

$$0.003x^2 - 0.034x + 8.086 = 10$$

$$0.003x^2 - 0.034x - 1.914 = 0$$

$$x = \frac{0.034 \pm \sqrt{(-0.034)^2 - 4(0.003)(-1.914)}}{2(0.003)}$$

$$x = \frac{0.034 \pm \sqrt{.024124}}{0.006}$$

$$x = \frac{0.034 \pm .15532}{0.006}$$

$$x = \frac{0.034 - 0.15532}{0.006} \quad \text{or} \quad x = \frac{0.034 + 0.15532}{0.006}$$

$$x = -20.22 \quad \text{or} \quad x = 31.55$$

The percentage will reach 10% approximately 32 years after 1960 which is 1992.

117.
$$\frac{1}{2}n(n+1) = 703$$

$$n(n+1) = 1406$$

$$n^2 + n - 1406 = 0$$

$$(n-37)(n+38) = 0$$

$$n = 37 \quad \text{or} \quad n = -38$$

Since the number of consecutive integers cannot be negative, we discard the negative value. We must add 37 consecutive integers, beginning at 1, in order to get a sum of 703.

118.
$$\frac{1}{2}n(n-3) = 65$$

$$n(n-3) = 130$$

$$n^2 - 3n - 130 = 0$$

$$(n-13)(n+10) = 0$$

$$n = 13 \quad \text{or} \quad n = -10$$

Since the number of sides cannot be negative, we discard the negative value. A polygon with 65 diagonals will have 13 sides.

$$\frac{1}{2}n(n-3) = 80$$

$$n(n-3) = 160$$

$$n^2 - 3n - 160 = 0$$

$$a = 1, b = -3, c = -160$$

$$n = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-160)}}{2(1)} = \frac{3 \pm \sqrt{646}}{2}$$

Neither solution is an integer, so there is no polygon that has 80 diagonals.

- 119.** The roots of a quadratic equation are

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_1 + x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} + \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b - \sqrt{b^2 - 4ac} - b + \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2b}{2a}$$

$$= -\frac{b}{a}$$

- 120.** The roots of a quadratic equation are

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_1 \cdot x_2 = \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} \right) \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} \right)$$

$$= \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{(2a)^2} = \frac{b^2 - b^2 + 4ac}{4a^2}$$

$$= \frac{4ac}{4a^2}$$

$$= \frac{c}{a}$$

121. In order to have one repeated solution, we need the discriminant to be 0.

$$b^2 - 4ac = 0$$

$$1^2 - 4(k)(k) = 0$$

$$1 - 4k^2 = 0$$

$$4k^2 = 1$$

$$k^2 = \frac{1}{4}$$

$$k = \pm \sqrt{\frac{1}{4}}$$

$$k = \frac{1}{2} \quad \text{or} \quad k = -\frac{1}{2}$$

122. In order to have one repeated solution, we need the discriminant to be 0.

$$b^2 - 4ac = 0$$

$$(-k)^2 - 4(1)(4) = 0$$

$$k^2 - 16 = 0$$

$$(k - 4)(k + 4) = 0$$

$$k = 4 \quad \text{or} \quad k = -4$$

123. For $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^2 - bx + c = 0$:

$$\begin{aligned} x &= \frac{b \pm \sqrt{(-b)^2 - 4ac}}{2a} \\ &= -\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right) \end{aligned}$$

124. For $ax^2 + bx + c = 0$:

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

For $cx^2 + bx + a = 0$:

$$\begin{aligned} x_1^* &= \frac{-b - \sqrt{b^2 - 4(c)(a)}}{2c} = \frac{-b - \sqrt{b^2 - 4ac}}{2c} \\ &= \frac{-b - \sqrt{b^2 - 4ac}}{2c} \cdot \frac{-b + \sqrt{b^2 - 4ac}}{-b + \sqrt{b^2 - 4ac}} \\ &= \frac{b^2 - (b^2 - 4ac)}{2c(-b + \sqrt{b^2 - 4ac})} = \frac{4ac}{2c(-b + \sqrt{b^2 - 4ac})} \\ &= \frac{2a}{-b + \sqrt{b^2 - 4ac}} \\ &= \frac{1}{x_2} \end{aligned}$$

and

$$\begin{aligned} x_2^* &= \frac{-b + \sqrt{b^2 - 4(c)(a)}}{2c} = \frac{-b + \sqrt{b^2 - 4ac}}{2c} \\ &= \frac{-b + \sqrt{b^2 - 4ac}}{2c} \cdot \frac{-b - \sqrt{b^2 - 4ac}}{-b - \sqrt{b^2 - 4ac}} \\ &= \frac{b^2 - (b^2 - 4ac)}{2c(-b - \sqrt{b^2 - 4ac})} = \frac{4ac}{2c(-b - \sqrt{b^2 - 4ac})} \\ &= \frac{2a}{-b - \sqrt{b^2 - 4ac}} \\ &= \frac{1}{x_1} \end{aligned}$$

125. If x = original width and y = original length, then

$$xy = 1 \quad \text{or} \quad x = \frac{1}{y}. \quad \text{The ratio of side lengths is}$$

$$\frac{x}{y} = \frac{1}{y^2}. \quad \text{Folding along the longest side results}$$

in sides of length $x = \frac{1}{y}$ and $\frac{y}{2}$ whose ratio is

$$\frac{\frac{y}{2}}{\frac{1}{y}} = \frac{y^2}{2}$$

Equating the ratios gives

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$$\frac{1}{y^2} = \frac{y^2}{2}$$

$$y^4 = 2$$

$$y = \sqrt[4]{2} \text{ m}$$

So

$$x = \frac{1}{\sqrt[4]{2}} = \frac{\sqrt[4]{8}}{2} \text{ m.}$$

126. a. $x^2 = 9$ and $x = 3$ are not equivalent because they do not have the same solution set. In the first equation we can also have $x = -3$.
- b. $x = \sqrt{9}$ and $x = 3$ are equivalent because $\sqrt{9} = 3$.
- c. $(x-1)(x-2) = (x-1)^2$ and $x-2 = x-1$ are not equivalent because they do not have the same solution set.
The first equation has the solution set $\{1\}$ while the second equation has no solutions.
127. Answers will vary. Methods may include the quadratic formula, completing the square, graphing, etc.
128. Answers will vary. Knowing the discriminant allows us to know how many real solutions the equation will have.
129. Answers will vary. One possibility:
Two distinct: $x^2 - 3x - 18 = 0$
One repeated: $x^2 - 14x + 49 = 0$
No real: $x^2 + x + 4 = 0$
130. Answers will vary.

Section 1.4

1. Integers: $\{-3, 0\}$
Rationals: $\left\{-3, 0, \frac{6}{5}\right\}$
2. True; the set of real numbers consists of all rational and irrational numbers.

$$\begin{aligned} 3. \quad \frac{3}{2+\sqrt{3}} &= \frac{3}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} \\ &= \frac{3(2-\sqrt{3})}{2^2 - (\sqrt{3})^2} \\ &= \frac{3(2-\sqrt{3})}{4-3} \\ &= 3(2-\sqrt{3}) \end{aligned}$$

4. real; imaginary; imaginary unit
5. False; the conjugate of $2+5i$ is $2-5i$.
6. True; the set of real numbers is a subset of the set of complex numbers.
7. False; if $2-3i$ is a solution of a quadratic equation with real coefficients, then its conjugate, $2+3i$, is also a solution.
8. b
9. a
10. c
11. $(2-3i) + (6+8i) = (2+6) + (-3+8)i = 8+5i$
12. $(4+5i) + (-8+2i) = (4+(-8)) + (5+2)i = -4+7i$
13. $(-3+2i) - (4-4i) = (-3-4) + (2-(-4))i = -7+6i$
14. $(3-4i) - (-3-4i) = (3-(-3)) + (-4-(-4))i = 6+0i = 6$
15. $(2-5i) - (8+6i) = (2-8) + (-5-6)i = -6-11i$
16. $(-8+4i) - (2-2i) = (-8-2) + (4-(-2))i = -10+6i$
17. $3(2-6i) = 6-18i$
18. $-4(2+8i) = -8-32i$
19. $-3i(7+6i) = -21i-18i^2 = -21i-18(-1) = 18-21i$

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$$20. \quad 3i(-3+4i) = -9i + 12i^2 = -9i + 12(-1) = -12 - 9i$$

$$\begin{aligned} 21. \quad (3-4i)(2+i) &= 6 + 3i - 8i - 4i^2 \\ &= 6 - 5i - 4(-1) \\ &= 10 - 5i \end{aligned}$$

$$\begin{aligned} 22. \quad (5+3i)(2-i) &= 10 - 5i + 6i - 3i^2 \\ &= 10 + i - 3(-1) \\ &= 13 + i \end{aligned}$$

$$\begin{aligned} 23. \quad (-5-i)(-5+i) &= 25 - 5i + 5i - i^2 \\ &= 25 - (-1) \\ &= 26 \end{aligned}$$

$$\begin{aligned} 24. \quad (-3+i)(3+i) &= -9 - 3i + 3i + i^2 \\ &= -9 + (-1) \\ &= -10 \end{aligned}$$

$$\begin{aligned} 25. \quad \frac{10}{3-4i} &= \frac{10}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{30+40i}{9+12i-12i-16i^2} \\ &= \frac{30+40i}{9-16(-1)} = \frac{30+40i}{25} \\ &= \frac{30}{25} + \frac{40}{25}i \\ &= \frac{6}{5} + \frac{8}{5}i \end{aligned}$$

$$\begin{aligned} 26. \quad \frac{13}{5-12i} &= \frac{13}{5-12i} \cdot \frac{5+12i}{5+12i} \\ &= \frac{65+156i}{25+60i-60i-144i^2} \\ &= \frac{65+156i}{25-144(-1)} = \frac{65+156i}{169} \\ &= \frac{65}{169} + \frac{156}{169}i \\ &= \frac{5}{13} + \frac{12}{13}i \end{aligned}$$

$$\begin{aligned} 27. \quad \frac{2+i}{i} &= \frac{2+i}{i} \cdot \frac{-i}{-i} = \frac{-2i-i^2}{-i^2} \\ &= \frac{-2i-(-1)}{-(-1)} = \frac{1-2i}{1} = 1-2i \end{aligned}$$

$$\begin{aligned} 28. \quad \frac{2-i}{-2i} &= \frac{2-i}{-2i} \cdot \frac{i}{i} = \frac{2i-i^2}{-2i^2} \\ &= \frac{2i-(-1)}{-2(-1)} = \frac{1+2i}{2} = \frac{1}{2} + i \end{aligned}$$

$$\begin{aligned} 29. \quad \frac{6-i}{1+i} &= \frac{6-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{6-6i-i+i^2}{1-i+i-i^2} \\ &= \frac{6-7i+(-1)}{1-(-1)} = \frac{5-7i}{2} = \frac{5}{2} - \frac{7}{2}i \end{aligned}$$

$$\begin{aligned} 30. \quad \frac{2+3i}{1-i} &= \frac{2+3i}{1-i} \cdot \frac{1+i}{1+i} = \frac{2+2i+3i+3i^2}{1+i-i-i^2} \\ &= \frac{2+5i+3(-1)}{1-(-1)} = \frac{-1+5i}{2} = -\frac{1}{2} + \frac{5}{2}i \end{aligned}$$

$$\begin{aligned} 31. \quad \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 &= \frac{1}{4} + 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right) + \frac{3}{4}i^2 \\ &= \frac{1}{4} + \frac{\sqrt{3}}{2}i + \frac{3}{4}(-1) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \end{aligned}$$

$$\begin{aligned} 32. \quad \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^2 &= \frac{3}{4} - 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}i\right) + \frac{1}{4}i^2 \\ &= \frac{3}{4} - \frac{\sqrt{3}}{2}i + \frac{1}{4}(-1) = \frac{1}{2} - \frac{\sqrt{3}}{2}i \end{aligned}$$

$$33. \quad (1+i)^2 = 1 + 2i + i^2 = 1 + 2i + (-1) = 2i$$

$$34. \quad (1-i)^2 = 1 - 2i + i^2 = 1 - 2i + (-1) = -2i$$

$$35. \quad i^{23} = i^{22+1} = i^{22} \cdot i = (i^2)^{11} \cdot i = (-1)^{11}i = -i$$

$$36. \quad i^{14} = (i^2)^7 = (-1)^7 = -1$$

$$\begin{aligned} 37. \quad i^{-20} &= \frac{1}{i^{20}} = \frac{1}{i^{20}} = \frac{1}{(i^2)^{10}} \\ &= \frac{1}{(-1)^{10}} = \frac{1}{1} = 1 \end{aligned}$$

$$\begin{aligned} 38. \quad i^{-23} &= \frac{1}{i^{23}} = \frac{1}{i^{22+1}} = \frac{1}{i^{22} \cdot i} = \frac{1}{(i^2)^{11} \cdot i} \\ &= \frac{1}{(-1)^{11}i} = \frac{1}{-i} = \frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = i \end{aligned}$$

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$$39. i^6 - 5 = (i^2)^3 - 5 = (-1)^3 - 5 = -1 - 5 = -6$$

$$40. 4 + i^3 = 4 + i^2 \cdot i = 4 + (-1)i = 4 - i$$

$$41. 6i^3 - 4i^5 = i^3(6 - 4i^2) \\ = i^2 \cdot i(6 - 4(-1)) = -1 \cdot i(10) = -10i$$

$$42. 4i^3 - 2i^2 + 1 = 4i^2 \cdot i - 2i^2 + 1 \\ = 4(-1)i - 2(-1) + 1 \\ = -4i + 2 + 1 \\ = 3 - 4i$$

$$43. (1+i)^3 = (1+i)(1+i)(1+i) = (1+2i+i^2)(1+i) \\ = (1+2i-1)(1+i) = 2i(1+i) \\ = 2i+2i^2 = 2i+2(-1) \\ = -2+2i$$

$$44. (3i)^4 + 1 = 81i^4 + 1 = 81(1) + 1 = 82$$

$$45. i^7(1+i^2) = i^7(1+(-1)) = i^7(0) = 0$$

$$46. 2i^4(1+i^2) = 2(1)(1+(-1)) = 2(0) = 0$$

$$47. i^8 + i^6 - i^4 - i^2 = (i^2)^4 + (i^2)^3 + (i^2)^2 + i^2 \\ = (-1)^4 + (-1)^3 + (-1)^2 - 1 \\ = 1 - 1 + 1 - 1 \\ = 0$$

$$48. i^7 + i^5 + i^3 + i = (i^2)^3 \cdot i + (i^2)^2 \cdot i + i^2 \cdot i + i \\ = (-1)^3 \cdot i + (-1)^2 \cdot i + (-1) \cdot i + i \\ = -i + i - i + i \\ = 0$$

$$49. \sqrt{-4} = 2i$$

$$50. \sqrt{-9} = 3i$$

$$51. \sqrt{-25} = 5i$$

$$52. \sqrt{-64} = 8i$$

$$53. \sqrt{-12} = i\sqrt{4 \cdot 3} = 2\sqrt{3}i$$

$$54. \sqrt{-18} = i\sqrt{9 \cdot 2} = 3\sqrt{2}i$$

$$55. \sqrt{-200} = i\sqrt{100 \cdot 2} = 10\sqrt{2}i$$

$$56. \sqrt{-45} = i\sqrt{9 \cdot 5} = 3\sqrt{5}i$$

$$57. \sqrt{(3+4i)(4i-3)} = \sqrt{12i-9+16i^2-12i} \\ = \sqrt{-9+16(-1)} \\ = \sqrt{-25} \\ = 5i$$

$$58. \sqrt{(4+3i)(3i-4)} = \sqrt{12i-16+9i^2-12i} \\ = \sqrt{-16+9(-1)} \\ = \sqrt{-25} \\ = 5i$$

$$59. x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \pm\sqrt{-4}$$

$$x = \pm 2i$$

The solution set is $\{-2i, 2i\}$.

$$60. x^2 - 4 = 0$$

$$(x+2)(x-2) = 0$$

$$x = -2 \text{ or } x = 2$$

The solution set is $\{-2, 2\}$.

$$61. x^2 - 16 = 0$$

$$(x+4)(x-4) = 0$$

$$x = -4 \text{ or } x = 4$$

The solution set is $\{-4, 4\}$.

$$62. x^2 + 25 = 0$$

$$x^2 = -25$$

$$x = \pm\sqrt{-25} = \pm 5i$$

The solution set is $\{-5i, 5i\}$.

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63. $x^2 - 6x + 13 = 0$
 $a = 1, b = -6, c = 13,$
 $b^2 - 4ac = (-6)^2 - 4(1)(13) = 36 - 52 = -16$
 $x = \frac{-(-6) \pm \sqrt{-16}}{2(1)} = \frac{6 \pm 4i}{2} = 3 \pm 2i$
 The solution set is $\{3 - 2i, 3 + 2i\}.$

64. $x^2 + 4x + 8 = 0$
 $a = 1, b = 4, c = 8$
 $b^2 - 4ac = 4^2 - 4(1)(8) = 16 - 32 = -16$
 $x = \frac{-4 \pm \sqrt{-16}}{2(1)} = \frac{-4 \pm 4i}{2} = -2 \pm 2i$
 The solution set is $\{-2 - 2i, -2 + 2i\}.$

65. $x^2 - 6x + 10 = 0$
 $a = 1, b = -6, c = 10$
 $b^2 - 4ac = (-6)^2 - 4(1)(10) = 36 - 40 = -4$
 $x = \frac{-(-6) \pm \sqrt{-4}}{2(1)} = \frac{6 \pm 2i}{2} = 3 \pm i$
 The solution set is $\{3 - i, 3 + i\}.$

66. $x^2 - 2x + 5 = 0$
 $a = 1, b = -2, c = 5$
 $b^2 - 4ac = (-2)^2 - 4(1)(5) = 4 - 20 = -16$
 $x = \frac{-(-2) \pm \sqrt{-16}}{2(1)} = \frac{2 \pm 4i}{2} = 1 \pm 2i$
 The solution set is $\{1 - 2i, 1 + 2i\}.$

67. $25x^2 - 10x + 2 = 0$
 $a = 25, b = -10, c = 2$
 $b^2 - 4ac = (-10)^2 - 4(25)(2) = 100 - 200 = -100$
 $x = \frac{-(-10) \pm \sqrt{-100}}{50} = \frac{10 \pm 10i}{50} = \frac{1}{5} \pm \frac{1}{5}i$
 The solution set is $\left\{ \frac{1}{5} - \frac{1}{5}i, \frac{1}{5} + \frac{1}{5}i \right\}.$

68. $10x^2 + 6x + 1 = 0$
 $a = 10, b = 6, c = 1$
 $b^2 - 4ac = 6^2 - 4(10)(1) = 36 - 40 = -4$
 $x = \frac{-6 \pm \sqrt{-4}}{2(10)} = \frac{-6 \pm 2i}{20} = -\frac{3}{10} \pm \frac{1}{10}i$
 The solution set is $\left\{ -\frac{3}{10} - \frac{1}{10}i, -\frac{3}{10} + \frac{1}{10}i \right\}.$

69. $5x^2 + 1 = 2x$
 $5x^2 - 2x + 1 = 0$
 $a = 5, b = -2, c = 1$
 $b^2 - 4ac = (-2)^2 - 4(5)(1) = 4 - 20 = -16$
 $x = \frac{-(-2) \pm \sqrt{-16}}{2(5)} = \frac{2 \pm 4i}{10} = \frac{1}{5} \pm \frac{2}{5}i$
 The solution set is $\left\{ \frac{1}{5} - \frac{2}{5}i, \frac{1}{5} + \frac{2}{5}i \right\}.$

70. $13x^2 + 1 = 6x$
 $13x^2 - 6x + 1 = 0$
 $a = 13, b = -6, c = 1$
 $b^2 - 4ac = (-6)^2 - 4(13)(1) = 36 - 52 = -16$
 $x = \frac{-(-6) \pm \sqrt{-16}}{2(13)} = \frac{6 \pm 4i}{26} = \frac{3}{13} \pm \frac{2}{13}i$
 The solution set is $\left\{ \frac{3}{13} - \frac{2}{13}i, \frac{3}{13} + \frac{2}{13}i \right\}.$

71. $x^2 + x + 1 = 0$
 $a = 1, b = 1, c = 1,$
 $b^2 - 4ac = 1^2 - 4(1)(1) = 1 - 4 = -3$
 $x = \frac{-1 \pm \sqrt{-3}}{2(1)} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$
 The solution set is $\left\{ -\frac{1}{2} - \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i \right\}.$

72. $x^2 - x + 1 = 0$
 $a = 1, b = -1, c = 1$
 $b^2 - 4ac = (-1)^2 - 4(1)(1) = 1 - 4 = -3$
 $x = \frac{-(-1) \pm \sqrt{-3}}{2(1)} = \frac{1 \pm \sqrt{3}i}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$
 The solution set is $\left\{ \frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} + \frac{\sqrt{3}}{2}i \right\}.$

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73. $x^3 - 64 = 0$

$$(x-4)(x^2+4x+16)=0$$

$$x-4=0 \Rightarrow x=4$$

$$\text{or } x^2+4x+16=0$$

$$a=1, b=4, c=16$$

$$b^2-4ac=4^2-4(1)(16)=16-64=-48$$

$$x = \frac{-4 \pm \sqrt{-48}}{2(1)} = \frac{-4 \pm 4\sqrt{3}i}{2} = -2 \pm 2\sqrt{3}i$$

The solution set is $\{4, -2-2\sqrt{3}i, -2+2\sqrt{3}i\}$.

74. $x^3 + 27 = 0$

$$(x+3)(x^2-3x+9)=0$$

$$x+3=0 \Rightarrow x=-3$$

$$\text{or } x^2-3x+9=0$$

$$a=1, b=-3, c=9$$

$$b^2-4ac=(-3)^2-4(1)(9)=9-36=-27$$

$$x = \frac{-(-3) \pm \sqrt{-27}}{2(1)} = \frac{3 \pm 3\sqrt{3}i}{2} = \frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$$

The solution set is $\left\{-3, \frac{3}{2} - \frac{3\sqrt{3}}{2}i, \frac{3}{2} + \frac{3\sqrt{3}}{2}i\right\}$.

75. $x^4 = 16$

$$x^4 - 16 = 0$$

$$(x^2-4)(x^2+4)=0$$

$$(x-2)(x+2)(x^2+4)=0$$

$$x-2=0 \text{ or } x+2=0 \text{ or } x^2+4=0$$

$$x=2 \text{ or } x=-2 \text{ or } x^2=-4$$

$$x=2 \text{ or } x=-2 \text{ or } x=\pm\sqrt{-4}=\pm 2i$$

The solution set is $\{-2, 2, -2i, 2i\}$.

76. $x^4 = 1$

$$x^4 - 1 = 0$$

$$(x^2-1)(x^2+1)=0$$

$$(x-1)(x+1)(x^2+1)=0$$

$$x-1=0 \text{ or } x+1=0 \text{ or } x^2+1=0$$

$$x=1 \text{ or } x=-1 \text{ or } x^2=-1$$

$$x=1 \text{ or } x=-1 \text{ or } x=\pm\sqrt{-1}=\pm i$$

The solution set is $\{-1, 1, -i, i\}$.

77. $x^4 + 13x^2 + 36 = 0$

$$(x^2+9)(x^2+4)=0$$

$$x^2+9=0 \text{ or } x^2+4=0$$

$$x^2=-9 \text{ or } x^2=-4$$

$$x=\pm\sqrt{-9} \text{ or } x=\pm\sqrt{-4}$$

$$x=\pm 3i \text{ or } x=\pm 2i$$

The solution set is $\{-3i, 3i, -2i, 2i\}$.

78. $x^4 + 3x^2 - 4 = 0$

$$(x^2-1)(x^2+4)=0$$

$$(x-1)(x+1)(x^2+4)=0$$

$$x-1=0 \text{ or } x+1=0 \text{ or } x^2+4=0$$

$$x=1 \text{ or } x=-1 \text{ or } x^2=-4$$

$$x=1 \text{ or } x=-1 \text{ or } x=\pm\sqrt{-4}=\pm 2i$$

The solution set is $\{-1, 1, -2i, 2i\}$.

79. $3x^2 - 3x + 4 = 0$

$$a=3, b=-3, c=4$$

$$b^2-4ac=(-3)^2-4(3)(4)=9-48=-39$$

The equation has two complex solutions that are conjugates of each other.

80. $2x^2 - 4x + 1 = 0$

$$a=2, b=-4, c=1$$

$$b^2-4ac=(-4)^2-4(2)(1)=16-8=8$$

The equation has two unequal real number solutions.

81. $2x^2 + 3x = 4$

$$2x^2 + 3x - 4 = 0$$

$$a=2, b=3, c=-4$$

$$b^2-4ac=3^2-4(2)(-4)=9+32=41$$

The equation has two unequal real solutions.

82. $x^2 + 6 = 2x$

$$x^2 - 2x + 6 = 0$$

$$a=1, b=-2, c=6$$

$$b^2-4ac=(-2)^2-4(1)(6)=4-24=-20$$

The equation has two complex solutions that are conjugates of each other.

Section 1.4: Complex Numbers; Quadratic Equations in the Complex Number System

83. $9x^2 - 12x + 4 = 0$
 $a = 9, b = -12, c = 4$
 $b^2 - 4ac = (-12)^2 - 4(9)(4) = 144 - 144 = 0$
 The equation has a repeated real solution.

84. $4x^2 + 12x + 9 = 0$
 $a = 4, b = 12, c = 9$
 $b^2 - 4ac = 12^2 - 4(4)(9) = 144 - 144 = 0$
 The equation has a repeated real solution.

85. The other solution is $\overline{2+3i} = 2-3i$.

86. The other solution is $\overline{4-i} = 4+i$.

87. $z + \bar{z} = 3 - 4i + \overline{3 - 4i} = 3 - 4i + 3 + 4i = 6$

88. $w - \bar{w} = 8 + 3i - \overline{(8 + 3i)}$
 $= 8 + 3i - (8 - 3i)$
 $= 8 + 3i - 8 + 3i$
 $= 0 + 6i = 6i$

89. $z \cdot \bar{z} = (3 - 4i)(\overline{3 - 4i})$
 $= (3 - 4i)(3 + 4i)$
 $= 9 + 12i - 12i - 16i^2$
 $= 9 - 16(-1) = 25$

90. $\overline{z - w} = \overline{3 - 4i - (8 + 3i)}$
 $= \overline{3 - 4i - 8 - 3i}$
 $= \overline{-5 - 7i}$
 $= -5 + 7i$

91. $Z = \frac{V}{I} = \frac{18+i}{3-4i} = \frac{18+i}{3-4i} \cdot \frac{3+4i}{3+4i}$
 $= \frac{54+72i+3i+4i^2}{9+12i-12i-16i^2} = \frac{54+75i-4}{9+16}$
 $= \frac{50+75i}{25} = 2+3i$

The impedance is $2+3i$ ohms.

92. $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{2+i} + \frac{1}{4-3i} = \frac{(4-3i)+(2+i)}{(2+i)(4-3i)}$
 $= \frac{6-2i}{8-6i+4i-3i^2} = \frac{6-2i}{8-2i+3} = \frac{6-2i}{11-2i}$

So, $Z = \frac{11-2i}{6-2i} = \frac{11-2i}{6-2i} \cdot \frac{6+2i}{6+2i}$
 $= \frac{66+22i-12i-4i^2}{36+12i-12i-4i^2} = \frac{66+10i+4}{36+4}$
 $= \frac{70+10i}{40} = \frac{7}{4} + \frac{1}{4}i$

The total impedance is $\frac{7}{4} + \frac{1}{4}i$ ohms.

93. $z + \bar{z} = (a+bi) + \overline{(a+bi)}$
 $= a+bi + a-bi$
 $= 2a$

$z - \bar{z} = a+bi - \overline{(a+bi)}$
 $= a+bi - (a-bi)$
 $= a+bi - a+bi$
 $= 2bi$

94. $\overline{\overline{z}} = \overline{a+bi} = \overline{a-bi} = a+bi = z$

95. $\overline{z+w} = \overline{(a+bi)+(c+di)}$
 $= \overline{(a+c)+(b+d)i}$
 $= \overline{(a+c) - (b+d)i}$
 $= \overline{(a-bi)+(c-di)}$
 $= \overline{a+bi+c+di}$
 $= \overline{z+w}$

96. $\overline{z \cdot w} = \overline{(a+bi) \cdot (c+di)}$
 $= \overline{ac+adi+bc+bd i^2}$
 $= \overline{(ac-bd)+(ad+bc)i}$
 $= \overline{(ac-bd) - (ad+bc)i}$

$\overline{z} \cdot \overline{w} = \overline{a+bi} \cdot \overline{c+di}$
 $= (a-bi)(c-di)$
 $= ac - adi - bci + bd i^2$
 $= (ac-bd) - (ad+bc)i$

97. $(a+bi)^2 = -(a-bi)^2$
 $a^2 + 2abi + (bi)^2 = -(a^2 - 2abi + (bi)^2)$
 $a^2 + 2abi + b^2 i^2 = -(a^2 - 2abi + b^2 i^2)$
 $a^2 + 2abi - b^2 = -a^2 + 2abi + b^2$

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$$a^2 - b^2 = -(a^2 - b^2)$$

$$2(a^2 - b^2) = 0$$

$$a^2 = b^2 \rightarrow b = \pm a$$

Any complex number of the form $a + ai$ or $a - ai$ will work.

98. Let $u = \sqrt[3]{2}$ in $x^3 + 2 = 0$ so that $x^3 + u^3 = 0$.

Then, $(x+u)(x^2 - ux + u^2) = 0$. From the first factor we find $x = -u = -\sqrt[3]{2}$. From the second factor, use the quadratic formula to get

$$x = \frac{-(-u) \pm \sqrt{(-u)^2 - 4 \cdot 1 \cdot u^2}}{2 \cdot 1}$$

$$= \frac{u \pm \sqrt{-3u^2}}{2} = \frac{u}{2} \pm \frac{u\sqrt{3}}{2}i = \frac{\sqrt[3]{2}}{2} \pm \frac{\sqrt[3]{2}\sqrt{3}}{2}i$$

The solution set is:

$$\left\{ -\sqrt[3]{2}, \frac{\sqrt[3]{2}}{2} \pm \frac{\sqrt[3]{2}\sqrt{3}}{2}i \right\}$$

99. $(x+5)(y-5) = (x+y)^2$; let $u = x+5$ (so $x = u-5$ and $v = y-5$ so $y = v+5$).

Substituting gives $uv = (u+v)^2$ or

$u^2 + uv + v^2 = 0$ which is quadratic in u . Using the quadratic formula gives

$$x = \frac{-v \pm \sqrt{v^2 - 4 \cdot 1 \cdot v^2}}{2 \cdot 1} = \frac{-v \pm |v|\sqrt{-3}}{2}. \text{ Since } x$$

is a real number, u must also be a real number. This is only possible if $v = 0$ which then makes $u = 0$. Therefore, $x = 0 - 5 = -5$ and $y = 0 + 5 = 5$, so $x - y = -5 - 5 = -10$

- 100 – 102. Answers will vary.

103. Answers will vary. A complex number is the sum or difference of two numbers (real and imaginary parts of the complex number) just as a binomial is the sum or difference of two monomial terms. We multiply two binomials by using the FOIL method, an approach we can also use to multiply two complex numbers.

104. Although the set of real numbers is a subset of the set of complex numbers, not all rules that work in the real number system can be used in the larger complex number system. The rule that allows us to write the product of two square roots as the square root of the product only works in the real number system. That is, $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ only when $\sqrt{a}$ and $\sqrt{b}$ are real

numbers. In the complex number system we must first convert the radicals to complex form.

In this case this means we need to write $\sqrt{-9}$ as $\sqrt{-1 \cdot 9} = \sqrt{9} \cdot \sqrt{-1} = 3i$. Then we can multiply to get $\sqrt{-9} \cdot \sqrt{-9} = 3i \cdot 3i = 9i^2 = 9(-1) = -9$.

Section 1.5

1. True

2. $(\sqrt[3]{x})^3 = x$

3. $2x^2 - 7x - 4 = (x-4)(2x+1)$

4. $x^3 + 4x^2 - 9x - 36 = x^2(x+4) - 9(x+4)$
 $= (x+4)(x^2 - 9)$
 $= (x+4)(x-3)(x+3)$

5. The distance from the origin to -4 on a real number line is 4 units.

6. False; you can also use the Quadratic Formula or completing the square.

7. quadratic in form

8. True

9. a

10. c

11. $\sqrt{y+6} = 4$

$$(\sqrt{y+6})^2 = 4^2$$

$$y+6 = 16 \rightarrow y = 10$$

$$\text{Check: } \sqrt{22+6} = \sqrt{28} = 5$$

The solution set is $\{22\}$.

Section 1.5: Radical Equations; Equations Quadratic in Form; Absolute Value Equations; Factorable Equations

$$12. \sqrt{t-3} = 7$$

$$(\sqrt{t-3})^2 = 7^2$$

$$t-3 = 49 \rightarrow t = 52$$

$$\text{Check: } \sqrt{52-3} = \sqrt{49} = 7$$

The solution set is $\{52\}$.

$$13. \sqrt{2t-1} = 1$$

$$(\sqrt{2t-1})^2 = 1^2$$

$$2t-1 = 1 \rightarrow 2t = 2 \rightarrow t = 1$$

$$\text{Check: } \sqrt{2(1)-1} = \sqrt{1} = 1$$

The solution set is $\{1\}$.

$$14. \sqrt{3t+4} = 2$$

$$(\sqrt{3t+4})^2 = 2^2$$

$$3t+4 = 4 \rightarrow 3t = 0 \rightarrow t = 0$$

$$\text{Check: } \sqrt{3(0)+4} = \sqrt{4} = 2$$

The solution set is $\{0\}$.

$$15. \sqrt{3t+4} = -6$$

Since the principal square root is never negative, the equation has no real solution.

$$16. \sqrt{5t+3} = -2$$

Since the principal square root is never negative, the equation has no real solution.

$$17. \sqrt[3]{1-2x} - 3 = 0$$

$$\sqrt[3]{1-2x} = 3$$

$$(\sqrt[3]{1-2x})^3 = 3^3$$

$$1-2x = 27 \rightarrow -2x = 26 \rightarrow x = -13$$

$$\text{Check: } \sqrt[3]{1-2(-13)} - 3 = \sqrt[3]{27} - 3 = 0$$

The solution set is $\{-13\}$.

$$18. \sqrt[3]{1-2x} - 1 = 0$$

$$\sqrt[3]{1-2x} = 1$$

$$(\sqrt[3]{1-2x})^3 = 1^3$$

$$1-2x = 1 \rightarrow -2x = 0 \rightarrow x = 0$$

$$\text{Check: } \sqrt[3]{1-2(0)} - 1 = \sqrt[3]{1} - 1 = 0$$

The solution set is $\{0\}$.

$$19. \sqrt[4]{5x-4} = 2$$

$$(\sqrt[4]{5x-4})^4 = 2^4$$

$$5x-4 = 16$$

$$5x = 20$$

$$x = 4$$

Check:

$$\sqrt[4]{5(4)-4} = \sqrt[4]{16} = 2 \text{ T}$$

The solution set is $\{4\}$.

$$20. \sqrt[5]{2x-3} = -1$$

$$(\sqrt[5]{2x-3})^5 = (-1)^5$$

$$2x-3 = -1$$

$$2x = 2$$

$$x = 1$$

Check:

$$\sqrt[5]{2(1)-3} = \sqrt[5]{-1} = -1 \text{ T}$$

The solution set is $\{1\}$.

$$21. \sqrt[5]{x^2+2x} = -1$$

$$(\sqrt[5]{x^2+2x})^5 = (-1)^5$$

$$x^2+2x = -1$$

$$x^2+2x+1 = 0$$

$$(x+1)^2 = 0$$

$$x+1 = 0$$

$$x = -1$$

Check:

$$\sqrt[5]{(-1)^2+2(-1)} = \sqrt[5]{1-2} = \sqrt[5]{-1} = -1 \text{ T}$$

The solution set is $\{-1\}$.

$$22. \sqrt[4]{x^2+16} = \sqrt{5}$$

$$(\sqrt[4]{x^2+16})^4 = (\sqrt{5})^4$$

$$x^2+16 = 25$$

$$x^2 = 9$$

$$x = \pm 3$$

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Check:

$$\sqrt[4]{(-3)^2 + 16} = \sqrt[4]{9 + 16} = \sqrt[4]{25} = \sqrt{5} \text{ T}$$

$$\sqrt[4]{(3)^2 + 16} = \sqrt[4]{9 + 16} = \sqrt[4]{25} = \sqrt{5} \text{ T}$$

The solution set is $\{-3, 3\}$.

23. $x = 8\sqrt{x}$

$$(x)^2 = (8\sqrt{x})^2$$

$$x^2 = 64x$$

$$x^2 - 64x = 0$$

$$x(x - 64) = 0$$

$$x = 0 \text{ or } x = 64$$

Check:

$$0 = 8\sqrt{0} \quad 64 = 8\sqrt{64}$$

$$0 = 0 \text{ T} \quad 64 = 8 \cdot 8$$

$$64 = 64 \text{ T}$$

The solution set is $\{0, 64\}$.

24. $x = 3\sqrt{x}$

$$(x)^2 = (3\sqrt{x})^2$$

$$x^2 = 9x$$

$$x^2 - 9x = 0$$

$$x(x - 9) = 0$$

$$x = 0 \text{ or } x = 9$$

Check:

$$0 = 3\sqrt{0} \quad 9 = 3\sqrt{9}$$

$$0 = 0 \text{ T} \quad 9 = 3 \cdot 3$$

$$9 = 9 \text{ T}$$

The solution set is $\{0, 9\}$.

25. $\sqrt{15 - 2x} = x$

$$(\sqrt{15 - 2x})^2 = x^2$$

$$15 - 2x = x^2 \rightarrow x^2 + 2x - 15 = 0$$

$$(x + 5)(x - 3) = 0 \rightarrow x = -5 \text{ or } x = 3$$

$$\text{Check } -5: \sqrt{15 - 2(-5)} = \sqrt{25}$$

$$= 5 \neq -5$$

$$\text{Check } 3: \sqrt{15 - 2(3)} = \sqrt{9} = 3 = 3$$

The solution set is $\{3\}$.

26. $\sqrt{12 - x} = x$

$$(\sqrt{12 - x})^2 = x^2$$

$$12 - x = x^2 \rightarrow x^2 + x - 12 = 0$$

$$(x + 4)(x - 3) = 0 \rightarrow x = -4 \text{ or } x = 3$$

$$\text{Check } -4: \sqrt{12 - (-4)} = \sqrt{16}$$

$$= 4 \neq -4$$

$$\text{Check } 3: \sqrt{12 - 3} = \sqrt{9} = 3 = 3$$

The solution set is $\{3\}$.

27. $\sqrt{3(x + 10)} - 4 = x$

$$\sqrt{3(x + 10)} = x + 4$$

$$(\sqrt{3(x + 10)})^2 = (x + 4)^2$$

$$3x + 30 = x^2 + 8x + 16$$

$$0 = x^2 + 5x - 14$$

$$0 = (x + 7)(x - 2)$$

$$x = -7 \text{ or } x = 2$$

$$\text{Check } -7: \sqrt{3(-7 + 10)} - 4 = \sqrt{9} - 4 = -1 \neq -7$$

$$\text{Check } 2: \sqrt{3(2 + 10)} - 4 = \sqrt{36} - 4 = 2 = 2$$

Discard $x = -7$ as extraneous.

The solution set is $\{2\}$.

28. $\sqrt{1 - x} - 3 = x + 2$

$$\sqrt{1 - x} = x + 5$$

$$(\sqrt{1 - x})^2 = (x + 5)^2$$

$$1 - x = x^2 + 10x + 25$$

$$0 = x^2 + 11x + 24$$

$$0 = (x + 3)(x + 8)$$

$$x = -3 \text{ or } x = -8$$

$$\text{Check } -3: \sqrt{1 - (-3)} - 3 = -3 + 2 \rightarrow -1 = -1$$

$$\text{Check } -8: \sqrt{1 - (-8)} - 3 = -8 + 2 \rightarrow 0 = -6$$

Discard $x = -8$ as extraneous.

The solution set is $\{-3\}$.

29. $\sqrt{x^2 - x - 4} = x + 2$

$$(\sqrt{x^2 - x - 4})^2 = (x + 2)^2$$

$$x^2 - x - 4 = x^2 + 4x + 4$$

$$-8 = 5x \rightarrow -\frac{8}{5} = x$$

Check:

Section 1.5: Radical Equations; Equations Quadratic in Form; Absolute Value Equations; Factorable Equations

$$\begin{aligned}\sqrt{\left(-\frac{8}{5}\right)^2 - \left(-\frac{8}{5}\right) - 4} &= \left(-\frac{8}{5}\right) + 2 \\ \sqrt{\frac{64}{25} + \frac{8}{5} - 4} &= \frac{2}{5} \\ \sqrt{\frac{4}{25}} &= \frac{2}{5} \\ \frac{2}{5} &= \frac{2}{5} \quad \text{T}\end{aligned}$$

The solution set is $\left\{-\frac{8}{5}\right\}$.

$$\begin{aligned}30. \quad \sqrt{x^2 - x - 8} &= x + 5 \\ \left(\sqrt{x^2 - x - 8}\right)^2 &= (x + 5)^2 \\ x^2 - x - 8 &= x^2 + 10x + 25 \\ -11x &= 33 \\ x &= -3\end{aligned}$$

Check:

$$\begin{aligned}\sqrt{(-3)^2 - (-3) - 8} &= (-3) + 5 \\ \sqrt{9 + 3 - 8} &= -3 + 5 \\ \sqrt{4} &= 2 \\ 2 &= 2 \quad \text{T}\end{aligned}$$

The solution set is $\{-3\}$.

$$\begin{aligned}31. \quad 3 + \sqrt{3x + 1} &= x \\ \sqrt{3x + 1} &= x - 3 \\ \left(\sqrt{3x + 1}\right)^2 &= (x - 3)^2 \\ 3x + 1 &= x^2 - 6x + 9 \\ 0 &= x^2 - 9x + 8 \\ 0 &= (x - 1)(x - 8) \\ x &= 1 \quad \text{or} \quad x = 8\end{aligned}$$

Check 1: $3 + \sqrt{3(1) + 1} = 3 + \sqrt{4} = 5 \neq 1$

Check 8: $3 + \sqrt{3(8) + 1} = 3 + \sqrt{25} = 8 = 8$

Discard $x = 1$ as extraneous.

The solution set is $\{8\}$.

$$\begin{aligned}32. \quad 2 + \sqrt{12 - 2x} &= x \\ \sqrt{12 - 2x} &= x - 2 \\ \left(\sqrt{12 - 2x}\right)^2 &= (x - 2)^2 \\ 12 - 2x &= x^2 - 4x + 4 \\ 0 &= x^2 - 2x - 8 \\ (x + 2)(x - 4) &= 0 \\ x &= -2 \quad \text{or} \quad x = 4\end{aligned}$$

Check -2 : $2 + \sqrt{12 - 2(-2)} = 2 + \sqrt{16} = 6 \neq -2$

Check 4 : $2 + \sqrt{12 - 2(4)} = 2 + \sqrt{4} = 4 = 4$

Discard $x = -2$ as extraneous.

The solution set is $\{4\}$.

$$\begin{aligned}33. \quad \sqrt{3x - 5} - \sqrt{x + 7} &= 2 \\ \sqrt{3x - 5} &= 2 + \sqrt{x + 7} \\ \left(\sqrt{3x - 5}\right)^2 &= \left(2 + \sqrt{x + 7}\right)^2 \\ 3x - 5 &= 4 + 4\sqrt{x + 7} + x + 7 \\ 2x - 16 &= 4\sqrt{x + 7} \\ (2x - 16)^2 &= \left(4\sqrt{x + 7}\right)^2\end{aligned}$$

$$4x^2 - 64x + 256 = 16(x + 7)$$

$$4x^2 - 64x + 256 = 16x + 112$$

$$4x^2 - 80x + 144 = 0$$

$$x^2 - 20x + 36 = 0$$

$$(x - 2)(x - 18) = 0 \rightarrow x = 2 \quad \text{or} \quad x = 18$$

Check 2: $\sqrt{3(2) - 5} - \sqrt{2 + 7}$
 $= \sqrt{1} - \sqrt{9} = 1 - 3 = -2 \neq 2$

Check 18: $\sqrt{3(18) - 5} - \sqrt{18 + 7}$
 $= \sqrt{49} - \sqrt{25} = 7 - 5 = 2 = 2$

Discard $x = 2$ as extraneous.

The solution set is $\{18\}$.

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$$\begin{aligned}
 34. \quad & \sqrt{3x+7} + \sqrt{x+2} = 1 \\
 & \sqrt{3x+7} = 1 - \sqrt{x+2} \\
 & (\sqrt{3x+7})^2 = (1 - \sqrt{x+2})^2 \\
 & 3x+7 = 1 - 2\sqrt{x+2} + x+2 \\
 & 2x+4 = -2\sqrt{x+2} \\
 & -x-2 = \sqrt{x+2} \\
 & (-x-2)^2 = (\sqrt{x+2})^2 \\
 & x^2 + 4x + 4 = x+2 \\
 & x^2 + 3x + 2 = 0 \\
 & (x+1)(x+2) = 0 \\
 & x = -1 \text{ or } x = -2 \\
 \text{Check -1: } & \sqrt{3(-1)+7} + \sqrt{-1+2} \\
 & = \sqrt{4} + \sqrt{1} = 2 + 1 = 3 \neq 1 \\
 \text{Check -2: } & \sqrt{3(-2)+7} + \sqrt{-2+2} \\
 & = \sqrt{1} + \sqrt{0} = 1 + 0 = 1 = 1 \\
 \text{Discard } x = -1 & \text{ as extraneous.} \\
 \text{The solution set is } & \{-2\}.
 \end{aligned}$$

$$\begin{aligned}
 35. \quad & \sqrt{3x+1} - \sqrt{x-1} = 2 \\
 & \sqrt{3x+1} = 2 + \sqrt{x-1} \\
 & (\sqrt{3x+1})^2 = (2 + \sqrt{x-1})^2 \\
 & 3x+1 = 4 + 4\sqrt{x-1} + x-1 \\
 & 2x-2 = 4\sqrt{x-1} \\
 & (2x-2)^2 = (4\sqrt{x-1})^2 \\
 & 4x^2 - 8x + 4 = 16(x-1) \\
 & x^2 - 2x + 1 = 4x - 4 \\
 & x^2 - 6x + 5 = 0 \\
 & (x-1)(x-5) = 0 \rightarrow x = 1 \text{ or } x = 5 \\
 \text{Check 1: } & \sqrt{3(1)+1} - \sqrt{1-1} \\
 & = \sqrt{4} - \sqrt{0} = 2 - 0 = 2 = 2 \\
 \text{Check 5: } & \sqrt{3(5)+1} - \sqrt{5-1} \\
 & = \sqrt{16} - \sqrt{4} = 4 - 2 = 2 = 2 \\
 \text{The solution set is } & \{1, 5\}.
 \end{aligned}$$

$$\begin{aligned}
 36. \quad & \sqrt{2x+3} - \sqrt{x+1} = 1 \\
 & \sqrt{2x+3} = 1 + \sqrt{x+1} \\
 & (\sqrt{2x+3})^2 = (1 + \sqrt{x+1})^2 \\
 & 2x+3 = 1 + 2\sqrt{x+1} + x+1 \\
 & x+1 = 2\sqrt{x+1} \\
 & (x+1)^2 = (2\sqrt{x+1})^2 \\
 & x^2 + 2x + 1 = 4(x+1) \\
 & x^2 + 2x + 1 = 4x + 4 \\
 & x^2 - 2x - 3 = 0 \\
 & (x+1)(x-3) = 0 \rightarrow x = -1 \text{ or } x = 3 \\
 \text{Check -1: } & \sqrt{2(-1)+3} - \sqrt{-1+1} = 1 - 0 = 1 \\
 \text{Check 3: } & \sqrt{2(3)+3} - \sqrt{3+1} = 3 - 2 = 1 \\
 \text{The solution set is } & \{-1, 3\}.
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & \sqrt{3-2\sqrt{x}} = \sqrt{x} \\
 & (\sqrt{3-2\sqrt{x}})^2 = (\sqrt{x})^2 \\
 & 3 - 2\sqrt{x} = x \\
 & -2\sqrt{x} = x - 3 \\
 & (-2\sqrt{x})^2 = (x-3)^2 \\
 & 4x = x^2 - 6x + 9 \\
 & 0 = x^2 - 10x + 9 \\
 & 0 = (x-9)(x-1) \\
 & x = 1 \text{ or } x = 9 \\
 \text{Check:} & \\
 & \sqrt{3-2\sqrt{1}} = \sqrt{1} \quad \sqrt{3-2\sqrt{9}} = \sqrt{9} \\
 & \sqrt{3-2} = 1 \quad \sqrt{3-2 \cdot 3} = 3 \\
 & \sqrt{1} = 1 \quad \sqrt{-3} \neq 3 \\
 & 1 = 1 \quad \text{ } \\
 \text{Discard } x = 9 & \text{ as extraneous. The solution set is } \{1\}.
 \end{aligned}$$

Section 1.5: Radical Equations; Equations Quadratic in Form; Absolute Value Equations; Factorable Equations

$$\begin{aligned}
 38. \quad & \sqrt{10+3\sqrt{x}} = \sqrt{x} \\
 & \left(\sqrt{10+3\sqrt{x}}\right)^2 = (\sqrt{x})^2 \\
 & 10+3\sqrt{x} = x \\
 & 3\sqrt{x} = x-10 \\
 & (3\sqrt{x})^2 = (x-10)^2 \\
 & 9x = x^2 - 20x + 100 \\
 & 0 = x^2 - 29x + 100 \\
 & 0 = (x-4)(x-25) \\
 & x = 4 \quad \text{or} \quad x = 25
 \end{aligned}$$

Check:

$$\begin{array}{ll}
 \sqrt{10+3\sqrt{4}} = \sqrt{4} & \sqrt{10+3\sqrt{25}} = \sqrt{25} \\
 \sqrt{10+3\cdot 2} = 2 & \sqrt{10+3\cdot 5} = 5 \\
 \sqrt{16} = 2 & \sqrt{25} = 5 \\
 4 \neq 2 & 5 = 5 \quad \text{T}
 \end{array}$$

Discard $x = 4$ as extraneous. The solution set is $\{25\}$.

$$\begin{aligned}
 39. \quad & (3x+1)^{1/2} = 4 \\
 & \left((3x+1)^{1/2}\right)^2 = (4)^2 \\
 & 3x+1 = 16 \rightarrow 3x = 15 \rightarrow x = 5
 \end{aligned}$$

Check:

$$\begin{aligned}
 (3(5)+1)^{1/2} &= 4 \\
 16^{1/2} &= 4 \\
 4 &= 4 \quad \text{T}
 \end{aligned}$$

The solution set is $\{5\}$.

$$\begin{aligned}
 40. \quad & (3x-5)^{1/2} = 2 \\
 & \left((3x-5)^{1/2}\right)^2 = (2)^2 \\
 & 3x-5 = 4 \rightarrow 3x = 9 \rightarrow x = 3
 \end{aligned}$$

Check:

$$\begin{aligned}
 (3(3)-5)^{1/2} &= 2 \\
 4^{1/2} &= 2 \\
 2 &= 2 \quad \text{T}
 \end{aligned}$$

The solution set is $\{3\}$.

$$\begin{aligned}
 41. \quad & (5x-2)^{1/3} = 2 \\
 & \left((5x-2)^{1/3}\right)^3 = (2)^3 \\
 & 5x-2 = 8 \rightarrow 5x = 10 \rightarrow x = 2
 \end{aligned}$$

Check:

$$\begin{aligned}
 (5(2)-2)^{1/3} &= 2 \\
 8^{1/3} &= 2 \\
 2 &= 2
 \end{aligned}$$

The solution set is $\{2\}$.

$$\begin{aligned}
 42. \quad & (2x+1)^{1/3} = -1 \\
 & \left((2x+1)^{1/3}\right)^3 = (-1)^3 \\
 & 2x+1 = -1 \rightarrow 2x = -2 \rightarrow x = -1
 \end{aligned}$$

Check:

$$\begin{aligned}
 (2(-1)+1)^{1/3} &= -1 \\
 (-1)^{1/3} &= -1 \\
 -1 &= -1
 \end{aligned}$$

The solution set is $\{-1\}$.

$$\begin{aligned}
 43. \quad & (x^2+9)^{1/2} = 5 \\
 & \left((x^2+9)^{1/2}\right)^2 = (5)^2 \\
 & x^2+9 = 25 \rightarrow x^2 = 16 \\
 & x = -4 \quad \text{or} \quad x = 4
 \end{aligned}$$

Check:

$$\begin{array}{ll}
 \left((-4)^2+9\right)^{1/2} = 5 & \left((4)^2+9\right)^{1/2} = 5 \\
 25^{1/2} = 5 & 25^{1/2} = 5 \\
 5 = 5 & 5 = 5
 \end{array}$$

The solution set is $\{-4, 4\}$.

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44. $(x^2 - 16)^{1/2} = 9$

$$\left((x^2 - 16)^{1/2}\right)^2 = (9)^2$$

$$x^2 - 16 = 81 \rightarrow x^2 = 97$$

$$x = -\sqrt{97} \text{ or } x = \sqrt{97}$$

$$\left((- \sqrt{97})^2 - 16\right)^{1/2} = 9 \quad \left((\sqrt{97})^2 - 16\right)^{1/2} = 9$$

$$(97 - 16)^{1/2} = 9 \quad (97 - 16)^{1/2} = 9$$

$$81^{1/2} = 9 \quad 81^{1/2} = 9$$

$$9 = 9 \quad 9 = 9$$

The solution set is $\{-\sqrt{97}, \sqrt{97}\}$.

45. $t^4 - 16 = 0$

$$(t^2 + 4)(t^2 - 4) = 0$$

$$(t^2 + 4)(t + 2)(t - 2) = 0$$

$t^2 + 4 = 0$ has no real solution, so we only need to consider

$$t + 2 = 0 \quad \text{or} \quad t - 2 = 0$$

$$t = -2 \quad t = 2$$

The solution set is $\{-2, 2\}$

46. $y^4 - 4 = 0$

$$(y^2 + 2)(y^2 - 2) = 0$$

$$(y^2 + 2)(y + \sqrt{2})(y - \sqrt{2}) = 0$$

$$y^2 + 2 = 0 \Rightarrow y^2 = -2, \text{ which is impossible}$$

$$\text{or } (y + \sqrt{2})(y - \sqrt{2}) = 0 \Rightarrow y = \pm\sqrt{2}$$

The solution set is $\{-\sqrt{2}, \sqrt{2}\}$.

47. $x^4 - 5x^2 + 4 = 0$

$$(x^2 - 4)(x^2 - 1) = 0$$

$$x^2 - 4 = 0 \text{ or } x^2 - 1 = 0$$

$$x = \pm 2 \text{ or } x = \pm 1$$

The solution set is $\{-2, -1, 1, 2\}$.

48. $x^4 - 10x^2 + 24 = 0$

$$(x^2 - 6)(x^2 - 4) = 0$$

$$x^2 - 6 = 0 \quad \text{or} \quad x^2 - 4 = 0$$

$$x^2 = 6 \quad x^2 = 4$$

$$x = \pm\sqrt{6} \quad x = \pm 2$$

The solution set is $\{-\sqrt{6}, -2, 2, \sqrt{6}\}$.

49. $6x^4 - 5x^2 - 1 = 0$

$$(6x^2 + 1)(x^2 - 1) = 0$$

$$6x^2 + 1 = 0 \text{ or } x^2 - 1 = 0$$

$$6x^2 = -1, \text{ which is impossible}$$

$$\text{or } x = \pm 1$$

The solution set is $\{-1, 1\}$.

50. $2x^4 - 5x^2 - 12 = 0$

$$(2x^2 + 3)(x^2 - 4) = 0$$

$$2x^2 + 3 = 0 \text{ or } x^2 - 4 = 0$$

$$2x^2 = -3, \text{ which is impossible}$$

$$\text{or } x = \pm 2$$

The solution set is $\{-2, 2\}$.

51. $x^6 + 7x^3 - 8 = 0$

$$(x^3 + 8)(x^3 - 1) = 0$$

$$x^3 + 8 = 0 \text{ or } x^3 - 1 = 0$$

$$x^3 = -8 \rightarrow x = -2$$

$$\text{or } x^3 = 1 \rightarrow x = 1$$

The solution set is $\{-2, 1\}$.

52. $x^6 - 7x^3 - 8 = 0$

$$(x^3 - 8)(x^3 + 1) = 0$$

$$x^3 - 8 = 0 \text{ or } x^3 + 1 = 0$$

$$x^3 = 8 \rightarrow x = 2$$

$$\text{or } x^3 = -1 \rightarrow x = -1$$

The solution set is $\{-1, 2\}$.

Section 1.5: Radical Equations; Equations Quadratic in Form; Absolute Value Equations; Factorable Equations

53. $(x+2)^2 + 7(x+2) + 12 = 0$

let $p = x+2 \rightarrow p^2 = (x+2)^2$

$p^2 + 7p + 12 = 0$

$(p+3)(p+4) = 0$

$p+3 = 0$ or $p+4 = 0$

$p = -3 \rightarrow x+2 = -3 \rightarrow x = -5$

or $p = -4 \rightarrow x+2 = -4 \rightarrow x = -6$

The solution set is $\{-6, -5\}$.

54. $(2x+5)^2 - (2x+5) - 6 = 0$

let $p = 2x+5 \rightarrow p^2 = (2x+5)^2$

$p^2 - p - 6 = 0$

$(p-3)(p+2) = 0$

$p-3 = 0$ or $p+2 = 0$

$p = 3 \rightarrow 2x+5 = 3 \rightarrow x = -1$

or $p = -2 \rightarrow 2x+5 = -2 \rightarrow x = -\frac{7}{2}$

The solution set is $\{-\frac{7}{2}, -1\}$.

55. $2(s+1)^2 - 5(s+1) = 3$

let $p = s+1 \rightarrow p^2 = (s+1)^2$

$2p^2 - 5p = 3$

$2p^2 - 5p - 3 = 0$

$(2p+1)(p-3) = 0$

$2p+1 = 0$ or $p-3 = 0$

$p = -\frac{1}{2} \rightarrow s+1 = -\frac{1}{2} \rightarrow s = -\frac{3}{2}$

or $p = 3 \rightarrow s+1 = 3 \rightarrow s = 2$

The solution set is $\{-\frac{3}{2}, 2\}$.

56. $3(1-y)^2 + 5(1-y) + 2 = 0$

let $p = 1-y \rightarrow p^2 = (1-y)^2$

$3p^2 + 5p + 2 = 0$

$(3p+2)(p+1) = 0$

$3p+2 = 0$ or $p+1 = 0$

$p = -\frac{2}{3} \rightarrow 1-y = -\frac{2}{3} \rightarrow y = \frac{5}{3}$

or $p = -1 \rightarrow 1-y = -1 \rightarrow y = 2$

The solution set is $\{\frac{5}{3}, 2\}$.

57. $x - 4\sqrt{x} = 0$

$x = 4\sqrt{x}$

$(x)^2 = (4\sqrt{x})^2$

$x^2 = 16x \rightarrow 0 = 16x - x^2$

$0 = x(16-x) \rightarrow x = 0$

or $16-x = 0 \rightarrow x = 16$

Check:

$x = 0: 0 - 4\sqrt{0} = 0$

$0 = 0$

$x = 16: (16) - 4\sqrt{16} = 0$

$16 - 16 = 0$

$0 = 0$

The solution set is $\{0, 16\}$.

58. $x - 8\sqrt{x} = 0$

$8\sqrt{x} = x$

$(8\sqrt{x})^2 = (x)^2$

$64x = x^2 \rightarrow 0 = x^2 - 64x$

$0 = x(x-64) \rightarrow x = 0$ or $x = 64$

Check:

$x = 0: 0 - 8\sqrt{0} = 0$

$0 = 0$

$x = 64: 64 - 8\sqrt{64} = 0$

$64 - 64 = 0$

The solution set is $\{0, 64\}$.

Chapter 1: Graphs, Equations, and Inequalities

59. $x + \sqrt{x} = 20$

let $p = \sqrt{x} \rightarrow p^2 = x$

$$p^2 + p = 20$$

$$p^2 + p - 20 = 0$$

$$(p+5)(p-4) = 0$$

$$p+5 = 0 \text{ or } p-4 = 0$$

$$p = -5 \rightarrow \sqrt{x} = -5 \text{ non-real}$$

$$\text{or } p = 4 \rightarrow \sqrt{x} = 4 \rightarrow x = 16$$

Check:

$$x = 16: 16 + \sqrt{16} = 20$$

$$16 + 4 = 20$$

The solution set is $\{16\}$.

60. $x + \sqrt{x} = 6$

let $p = \sqrt{x} \rightarrow p^2 = x$

$$p^2 + p = 6$$

$$p^2 + p - 6 = 0$$

$$(p+3)(p-2) = 0$$

$$p+3 = 0 \text{ or } p-2 = 0$$

$$p = -3 \rightarrow \sqrt{x} = -3 \text{ non-real}$$

$$\text{or } p = 2 \rightarrow \sqrt{x} = 2 \rightarrow x = 4$$

Check:

$$x = 4: 4 + \sqrt{4} = 6$$

$$4 + 2 = 6$$

The solution set is $\{4\}$.

61. $z^{1/2} - 4z^{1/4} + 4 = 0$

let $p = z^{1/4} \rightarrow p^2 = z^{1/2}$

$$p^2 - 4p + 4 = 0$$

$$(p-2)(p-2) = 0$$

$$p-2 = 0$$

$$p = 2 \rightarrow z^{1/4} = 2 \rightarrow z = 16$$

Check:

$$z = 16: 16^{1/2} - 4(16)^{1/4} + 4 = 0$$

$$4 - 8 + 4 = 0 \rightarrow 0 = 0$$

The solution set is $\{16\}$.

62. $t^{1/2} - 2t^{1/4} + 1 = 0$

let $p = t^{1/4} \rightarrow p^2 = t^{1/2}$

$$p^2 - 2p + 1 = 0$$

$$(p-1)(p-1) = 0$$

$$p-1 = 0$$

$$p = 1 \rightarrow t^{1/4} = 1 \rightarrow t = 1$$

Check:

$$t = 1: 1^{1/2} - 2(1)^{1/4} + 1 = 0$$

$$1 - 2 + 1 = 0 \rightarrow 0 = 0$$

The solution set is $\{1\}$.

63. $4x^{1/2} - 9x^{1/4} + 4 = 0$

let $p = x^{1/4} \rightarrow p^2 = x^{1/2}$

$$4p^2 - 9p + 4 = 0$$

$$p = \frac{9 \pm \sqrt{81 - 64}}{8} = \frac{9 \pm \sqrt{17}}{8}$$

$$x^{1/4} = \frac{9 \pm \sqrt{17}}{8} \rightarrow x = \left(\frac{9 \pm \sqrt{17}}{8} \right)^4$$

Check:

$$4 \left(\left(\frac{9 + \sqrt{17}}{8} \right)^4 \right)^{1/2} - 9 \left(\left(\frac{9 + \sqrt{17}}{8} \right)^4 \right)^{1/4} + 4 = 0$$

$$4 \left(\frac{9 + \sqrt{17}}{8} \right)^2 - 9 \left(\frac{9 + \sqrt{17}}{8} \right) + 4 = 0$$

$$4 \frac{(9 + \sqrt{17})^2}{64} - 9 \left(\frac{9 + \sqrt{17}}{8} \right) + 4 = 0$$

$$64 \left(4 \frac{(9 + \sqrt{17})^2}{64} - 9 \left(\frac{9 + \sqrt{17}}{8} \right) + 4 \right) = (0)(64)$$

$$4(9 + \sqrt{17})^2 - 72(9 + \sqrt{17}) + 256 = 0$$

$$4(81 + 18\sqrt{17} + 17) - 72(9 + \sqrt{17}) + 256 = 0$$

$$324 + 72\sqrt{17} + 68 - 648 - 72\sqrt{17} + 256 = 0$$

$$0 = 0$$

Section 1.5: Radical Equations; Equations Quadratic in Form; Absolute Value Equations; Factorable Equations

$$4\left(\left(\frac{9-\sqrt{17}}{8}\right)^4\right)^{1/2} - 9\left(\left(\frac{9-\sqrt{17}}{8}\right)^4\right)^{1/4} + 4 = 0$$

$$4\left(\frac{9-\sqrt{17}}{8}\right)^2 - 9\left(\frac{9-\sqrt{17}}{8}\right) + 4 = 0$$

$$4(81 - 18\sqrt{17} + 17) - 72(9 - \sqrt{17}) + 256 = 0$$

$$324 - 72\sqrt{17} + 68 - 648 + 72\sqrt{17} + 256 = 0$$

$$0 = 0$$

The solution set is $\left\{\left(\frac{9-\sqrt{17}}{8}\right)^4, \left(\frac{9+\sqrt{17}}{8}\right)^4\right\}$.

64. $x^{1/2} - 3x^{1/4} + 2 = 0$
 let $p = x^{1/4} \rightarrow p^2 = x^{1/2}$
 $p^2 - 3p + 2 = 0$
 $(p-2)(p-1) = 0$
 $p = 2 \rightarrow x^{1/4} = 2 \rightarrow x = 16$
 or $p = 1 \rightarrow x^{1/4} = 1 \rightarrow x = 1$

Check:

$$x = 16: 16^{1/2} - 3(16)^{1/4} + 2 = 0$$

$$4 - 6 + 2 = 0 \rightarrow 0 = 0$$

$$x = 1: 1^{1/2} - 3(1)^{1/4} + 2 = 0$$

$$1 - 3 + 2 = 0 \rightarrow 0 = 0$$

The solution set is $\{1, 16\}$.

65. $\frac{1}{(x+1)^2} = \frac{1}{x+1} + 2$
 let $p = \frac{1}{x+1} \rightarrow p^2 = \left(\frac{1}{x+1}\right)^2$
 $p^2 = p + 2 \rightarrow p^2 - p - 2 = 0$
 $(p+1)(p-2) = 0 \rightarrow p = -1 \text{ or } p = 2$
 $p = -1 \rightarrow \frac{1}{x+1} = -1 \rightarrow 1 = -x - 1 \rightarrow x = -2$
 or
 $p = 2 \rightarrow \frac{1}{x+1} = 2 \rightarrow 1 = 2x + 2 \rightarrow x = -\frac{1}{2}$

Check:

$$x = -2: \frac{1}{(-2+1)^2} = \frac{1}{-2+1} + 2$$

$$1 = -1 + 2$$

$$1 = 1$$

$$x = -\frac{1}{2}: \frac{1}{\left(-\frac{1}{2}+1\right)^2} = \frac{1}{\left(-\frac{1}{2}+1\right)} + 2$$

$$4 = 2 + 2$$

$$4 = 4$$

The solution set is $\{-2, -\frac{1}{2}\}$.

66. $\frac{1}{(x-1)^2} + \frac{1}{x-1} = 12$

$$\text{let } p = \frac{1}{x-1} \rightarrow p^2 = \left(\frac{1}{x-1}\right)^2$$

$$p^2 + p = 12$$

$$p^2 + p - 12 = 0$$

$$(p+4)(p-3) = 0$$

$$p = -4 \quad \text{or} \quad p = 3$$

$$\frac{1}{x-1} = -4$$

$$\frac{1}{x-1} = 3$$

$$1 = -4x + 4$$

$$1 = 3x - 3$$

$$4x = 3$$

$$4 = 3x$$

$$x = \frac{3}{4}$$

$$x = \frac{4}{3}$$

Check:

$$x = \frac{3}{4}: \frac{1}{\left(\frac{3}{4}-1\right)^2} + \frac{1}{\left(\frac{3}{4}-1\right)} = 12$$

$$\frac{1}{\left(\frac{1}{4}\right)^2} + \frac{1}{\left(-\frac{1}{4}\right)} = 12$$

$$16 - 4 = 12$$

$$12 = 12$$

$$x = \frac{4}{3}: \frac{1}{\left(\frac{4}{3}-1\right)^2} + \frac{1}{\left(\frac{4}{3}-1\right)} = 12$$

$$\frac{1}{\left(\frac{1}{3}\right)^2} + \frac{1}{\left(\frac{1}{3}\right)} = 12$$

$$9 + 3 = 12$$

$$12 = 12$$

The solution set is $\left\{\frac{3}{4}, \frac{4}{3}\right\}$.

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67. $3x^{-2} - 7x^{-1} - 6 = 0$

let $p = x^{-1} \rightarrow p^2 = x^{-2}$

$3p^2 - 7p - 6 = 0$

$(3p+2)(p-3) = 0$

$p = -\frac{2}{3}$ or $p = 3$

$p = -\frac{2}{3} \rightarrow x^{-1} = -\frac{2}{3} \rightarrow (x^{-1})^{-1} = \left(-\frac{2}{3}\right)^{-1}$

$\rightarrow x = -\frac{3}{2}$

$p = 3 \rightarrow x^{-1} = 3 \rightarrow (x^{-1})^{-1} = (3)^{-1} \rightarrow x = \frac{1}{3}$

Check:

$x = -\frac{3}{2}: 3\left(-\frac{3}{2}\right)^{-2} - 7\left(-\frac{3}{2}\right)^{-1} - 6 = 0$

$3\left(\frac{4}{9}\right) - 7\left(-\frac{2}{3}\right) - 6 = 0$

$\frac{4}{3} + \frac{14}{3} - 6 = 0$

$0 = 0$

$x = \frac{1}{3}: 3\left(\frac{1}{3}\right)^{-2} - 7\left(\frac{1}{3}\right)^{-1} - 6 = 0$

$3(9) - 7(3) - 6 = 0$

$27 - 21 - 6 = 0$

$0 = 0$

The solution set is $\left\{-\frac{3}{2}, \frac{1}{3}\right\}$.

68. $2x^{-2} - 3x^{-1} - 4 = 0$

let $p = x^{-1} \rightarrow p^2 = x^{-2}$

$2p^2 - 3p - 4 = 0$

$p = \frac{3 \pm \sqrt{9+32}}{4} = \frac{3 \pm \sqrt{41}}{4}$

$p = \frac{3 + \sqrt{41}}{4} \rightarrow x^{-1} = \frac{3 + \sqrt{41}}{4}$

$(x^{-1})^{-1} = \left(\frac{3 + \sqrt{41}}{4}\right)^{-1}$

$x = \frac{4}{3 + \sqrt{41}} \left(\frac{3 - \sqrt{41}}{3 - \sqrt{41}}\right) = \frac{12 - 4\sqrt{41}}{-32}$

$= \frac{-3 + \sqrt{41}}{8}$

$p = \frac{3 - \sqrt{41}}{4} \rightarrow x^{-1} = \frac{3 - \sqrt{41}}{4}$

$(x^{-1})^{-1} = \left(\frac{3 - \sqrt{41}}{4}\right)^{-1}$

$x = \frac{4}{3 - \sqrt{41}} \left(\frac{3 + \sqrt{41}}{3 + \sqrt{41}}\right) = \frac{12 + 4\sqrt{41}}{-32}$

$= \frac{-3 - \sqrt{41}}{8}$

Check:

$x = \frac{-3 + \sqrt{41}}{8}$:

$2\left(\frac{-3 + \sqrt{41}}{8}\right)^{-2} - 3\left(\frac{-3 + \sqrt{41}}{8}\right)^{-1} - 4 = 0$

$2\left(\frac{64}{(-3 + \sqrt{41})^2}\right) - 3\left(\frac{8}{-3 + \sqrt{41}}\right) - 4 = 0$

$2(64) - 3(8)(-3 + \sqrt{41}) - 4(-3 + \sqrt{41})^2 = 0$

$128 + 72 - 24\sqrt{41} - 4(9 - 6\sqrt{41} + 41) = 0$

$128 + 72 - 24\sqrt{41} - 36 + 24\sqrt{41} - 164 = 0$
 $0 = 0$

$x = \frac{-3 - \sqrt{41}}{8}$:

$2\left(\frac{-3 - \sqrt{41}}{8}\right)^{-2} - 3\left(\frac{-3 - \sqrt{41}}{8}\right)^{-1} - 4 = 0$

$2\left(\frac{64}{(-3 - \sqrt{41})^2}\right) - 3\left(\frac{8}{-3 - \sqrt{41}}\right) - 4 = 0$

$2(64) - 3(8)(-3 - \sqrt{41}) - 4(-3 - \sqrt{41})^2 = 0$

$128 + 72 + 24\sqrt{41} - 4(9 + 6\sqrt{41} + 41) = 0$

$128 + 72 + 24\sqrt{41} - 36 - 24\sqrt{41} - 164 = 0$
 $0 = 0$

The solution set is $\left\{\frac{-3 - \sqrt{41}}{8}, \frac{-3 + \sqrt{41}}{8}\right\}$.

Section 1.5: Radical Equations; Equations Quadratic in Form; Absolute Value Equations; Factorable Equations

69. $2x^{2/3} - 5x^{1/3} - 3 = 0$

let $p = x^{1/3} \rightarrow p^2 = x^{2/3}$

$2p^2 - 5p - 3 = 0 \rightarrow (2p+1)(p-3) = 0$

$p = -\frac{1}{2}$ or $p = 3$

$p = -\frac{1}{2} \rightarrow x^{1/3} = -\frac{1}{2}$

$\rightarrow (x^{1/3})^3 = \left(-\frac{1}{2}\right)^3 \rightarrow x = -\frac{1}{8}$

or

$p = 3 \rightarrow x^{1/3} = 3 \rightarrow (x^{1/3})^3 = (3)^3$

$\rightarrow x = 27$

Check:

$x = -\frac{1}{8}: 2\left(-\frac{1}{8}\right)^{2/3} - 5\left(-\frac{1}{8}\right)^{1/3} - 3 = 0$

$2\left(\frac{1}{4}\right) - 5\left(-\frac{1}{2}\right) - 3 = 0$

$\frac{1}{2} + \frac{5}{2} - 3 = 0$

$3 - 3 = 0 \rightarrow 0 = 0$

$x = 27: 2(27)^{2/3} - 5(27)^{1/3} - 3 = 0$

$2(9) - 5(3) - 3 = 0$

$18 - 15 - 3 = 0$

$3 - 3 = 0 \rightarrow 0 = 0$

The solution set is $\left\{-\frac{1}{8}, 27\right\}$.

70. $3x^{4/3} + 5x^{2/3} - 2 = 0$

let $p = x^{2/3} \rightarrow p^2 = x^{4/3}$

$3p^2 + 5p - 2 = 0$

$(3p-1)(p+2) = 0$

$p = \frac{1}{3}$ or $p = -2$

$p = \frac{1}{3} \rightarrow x^{2/3} = \frac{1}{3}$

$(x^{2/3})^3 = \left(\frac{1}{3}\right)^3$

$x^2 = \frac{1}{27} \rightarrow x = \pm \sqrt{\frac{1}{27}} = \pm \frac{\sqrt{3}}{9}$

or

$p = -2 \rightarrow x^{2/3} = -2$

$(x^{2/3})^{3/2} = (-2)^{3/2}$

$x = (-2)^{3/2}$ which is not a real number

Check:

$3\left(\frac{\sqrt{3}}{9}\right)^{4/3} + 5\left(\frac{\sqrt{3}}{9}\right)^{2/3} - 2 = 0$

$3\left(\frac{1}{9}\right) + 5\left(\frac{1}{3}\right) - 2 = 0$

$\frac{1}{3} + \frac{5}{3} - 2 = 0$

$0 = 0$

$3\left(-\frac{\sqrt{3}}{9}\right)^{4/3} + 5\left(-\frac{\sqrt{3}}{9}\right)^{2/3} - 2 = 0$

$3\left(\frac{1}{9}\right) + 5\left(\frac{1}{3}\right) - 2 = 0$

$\frac{1}{3} + \frac{5}{3} - 2 = 0$

$0 = 0$

The solution set is $\left\{-\frac{\sqrt{3}}{9}, \frac{\sqrt{3}}{9}\right\}$.

71. $|2x+3| = 5$

$2x+3 = 5$ or $2x+3 = -5$

$2x = 2$ or $2x = -8$

$x = 1$ or $x = -4$

The solution set is $\{-4, 1\}$.

72. $|3x-1| = 2$

$3x-1 = 2$ or $3x-1 = -2$

$3x = 3$ or $3x = -1$

$x = 1$ or $x = -\frac{1}{3}$

The solution set is $\left\{-\frac{1}{3}, 1\right\}$.

Chapter 1: Graphs, Equations, and Inequalities

73. $|1 - 4t| + 8 = 13 \Rightarrow |1 - 4t| = 5$

$$1 - 4t = 5 \quad \text{or} \quad 1 - 4t = -5$$

$$-4t = 4 \quad \text{or} \quad -4t = -6$$

$$t = -1 \quad \text{or} \quad t = \frac{3}{2}$$

The solution set is $\left\{-1, \frac{3}{2}\right\}$.

74. $|1 - 2z| + 6 = 9 \Rightarrow |1 - 2z| = 3$

$$1 - 2z = 3 \quad \text{or} \quad 1 - 2z = -3$$

$$-2z = 2 \quad \text{or} \quad -2z = -4$$

$$z = -1 \quad \text{or} \quad z = 2$$

The solution set is $\{-1, 2\}$.

75. $|-7x| = 21$

$$-7x = 21 \quad \text{or} \quad -7x = -21$$

$$x = -3 \quad \text{or} \quad x = 3$$

The solution set is $\{-3, 3\}$.

76. $|-x| = 1$

$$-x = 1 \quad \text{or} \quad -x = -1$$

The solution set is $\{-1, 1\}$.

77. $4 - |2x| = 3 \Rightarrow |2x| = 1$

$$2x = 1 \quad \text{or} \quad 2x = -1$$

$$x = \frac{1}{2} \quad \text{or} \quad x = -\frac{1}{2}$$

The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$.

78. $5 - \left|\frac{1}{2}x\right| = 3 \Rightarrow 2 = \left|\frac{1}{2}x\right|$

$$\frac{1}{2}x = 2 \quad \text{or} \quad \frac{1}{2}x = -2$$

$$x = 4 \quad \text{or} \quad x = -4$$

The solution set is $\{-4, 4\}$.

79. $\frac{8}{7}|x| = 3$

$$|x| = \frac{21}{8} \rightarrow x = \frac{21}{8} \quad \text{or} \quad x = -\frac{21}{8}$$

The solution set is $\left\{-\frac{21}{8}, \frac{21}{8}\right\}$.

80. $\frac{3}{4}|x| = 9$

$$|x| = 12 \rightarrow x = 12 \quad \text{or} \quad x = -12$$

The solution set is $\{-12, 12\}$.

81. $\left|\frac{x}{3} + \frac{2}{5}\right| = 2$

$$\frac{x}{3} + \frac{2}{5} = 2 \quad \text{or} \quad \frac{x}{3} + \frac{2}{5} = -2$$

$$5x + 6 = 30 \quad \text{or} \quad 5x + 6 = -30$$

$$5x = 24 \quad \text{or} \quad 5x = -36$$

$$x = \frac{24}{5} \quad \text{or} \quad x = -\frac{36}{5}$$

The solution set is $\left\{-\frac{36}{5}, \frac{24}{5}\right\}$.

82. $\left|\frac{x}{2} - \frac{1}{3}\right| = 1$

$$\frac{x}{2} - \frac{1}{3} = 1 \quad \text{or} \quad \frac{x}{2} - \frac{1}{3} = -1$$

$$3x - 2 = 6 \quad \text{or} \quad 3x - 2 = -6$$

$$3x = 8 \quad \text{or} \quad 3x = -4$$

$$x = \frac{8}{3} \quad \text{or} \quad x = -\frac{4}{3}$$

The solution set is $\left\{-\frac{4}{3}, \frac{8}{3}\right\}$.

83. $|u - 2| = -\frac{1}{2}$

impossible, since absolute value always yields a non-negative number.

84. $|2 - v| = -1$

impossible, since absolute value always yields a non-negative number.

85. $|x^2 - 9| = 0$

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

The solution set is $\{-3, 3\}$.

Section 1.5: Radical Equations; Equations Quadratic in Form; Absolute Value Equations; Factorable Equations

86. $|x^2 - 16| = 0$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$x = \pm 4$$

The solution set is $\{-4, 4\}$.

87. $|x^2 - 2x| = 3$

$$x^2 - 2x = 3 \quad \text{or} \quad x^2 - 2x = -3$$

$$x^2 - 2x - 3 = 0 \quad \text{or} \quad x^2 - 2x + 3 = 0$$

$$(x-3)(x+1) = 0 \quad \text{or} \quad x = \frac{2 \pm \sqrt{4-12}}{2}$$

$$= \frac{2 \pm \sqrt{-8}}{2} \quad \text{no real sol.}$$

$$x = 3 \quad \text{or} \quad x = -1$$

The solution set is $\{-1, 3\}$.

88. $|x^2 + x| = 12$

$$x^2 + x = 12 \quad \text{or} \quad x^2 + x = -12$$

$$x^2 + x - 12 = 0 \quad \text{or} \quad x^2 + x + 12 = 0$$

$$(x-3)(x+4) = 0 \quad \text{or} \quad x = \frac{-1 \pm \sqrt{1-48}}{2}$$

$$= \frac{1 \pm \sqrt{-47}}{2} \quad \text{no real sol.}$$

$$x = 3 \quad \text{or} \quad x = -4$$

The solution set is $\{-4, 3\}$.

89. $x^3 - 9x = 0$

$$x(x^2 - 9) = 0$$

$$x(x-3)(x+3) = 0$$

$$x = 0 \quad \text{or} \quad x - 3 = 0 \quad x + 3 = 0$$

$$x = 3 \quad x = -3$$

The solution set is $\{-3, 0, 3\}$.

90. $x^4 - 81x^2 = 0$

$$x^2(x^2 - 81) = 0$$

$$x^2(x-9)(x+9) = 0$$

$$x^2 = 0 \quad \text{or} \quad x - 9 = 0 \quad \text{or} \quad x + 9 = 0$$

$$x = 0 \quad x = 9 \quad x = -9$$

The solution set is $\{-9, 0, 9\}$.

91. $x^3 + x^2 - 20x = 0$

$$x(x^2 + x - 20) = 0$$

$$x(x+5)(x-4) = 0$$

$$x = 0 \quad \text{or} \quad x + 5 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = -5 \quad x = 4$$

The solution set is $\{-5, 0, 4\}$.

92. $x^3 + 6x^2 - 7x = 0$

$$x(x^2 + 6x - 7) = 0$$

$$x(x+7)(x-1) = 0$$

$$x = 0 \quad \text{or} \quad x + 7 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = -7 \quad x = 1$$

The solution set is $\{-7, 0, 1\}$.

93. $x^3 + x^2 - x - 1 = 0$

$$x^2(x+1) - 1(x+1) = 0$$

$$(x+1)(x^2 - 1) = 0$$

$$(x+1)(x-1)(x+1) = 0$$

$$x+1 = 0 \quad \text{or} \quad x-1 = 0$$

$$x = -1 \quad x = 1$$

The solution set is $\{-1, 1\}$.

94. $x^3 + 4x^2 - x - 4 = 0$

$$x^2(x+4) - 1(x+4) = 0$$

$$(x+4)(x^2 - 1) = 0$$

$$(x+4)(x-1)(x+1) = 0$$

$$x+4 = 0 \quad \text{or} \quad x-1 = 0 \quad \text{or} \quad x+1 = 0$$

$$x = -4 \quad x = 1 \quad x = -1$$

The solution set is $\{-4, -1, 1\}$.

95. $x^3 - 3x^2 - 16x + 48 = 0$

$$x^2(x-3) - 16(x-3) = 0$$

$$(x-3)(x^2 - 16) = 0$$

$$(x-3)(x-4)(x+4) = 0$$

$$x-3 = 0 \quad \text{or} \quad x-4 = 0 \quad \text{or} \quad x+4 = 0$$

$$x = 3 \quad x = 4 \quad x = -4$$

The solution set is $\{-4, 3, 4\}$.

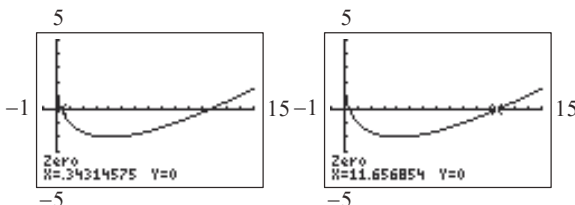
Chapter 1: Graphs, Equations, and Inequalities

96. $x^3 - 3x^2 - x + 3 = 0$
 $x^2(x-3) - 1(x-3) = 0$
 $(x-3)(x^2-1) = 0$
 $(x-3)(x-1)(x+1) = 0$
 $x-3=0$ or $x-1=0$ or $x+1=0$
 $x=3$ $x=1$ $x=-1$
 The solution set is $\{-1, 1, 3\}$.

97. $2x^3 + 4 = x^2 + 8x$
 $2x^3 - x^2 - 8x + 4 = 0$
 $x^2(2x-1) - 4(2x-1) = 0$
 $(2x-1)(x^2-4) = 0$
 $(2x-1)(x-2)(x+2) = 0$
 $2x-1=0$ or $x-2=0$ or $x+2=0$
 $2x=1$ $x=2$ $x=-2$
 $x = \frac{1}{2}$
 The solution set is $\left\{-2, \frac{1}{2}, 2\right\}$.

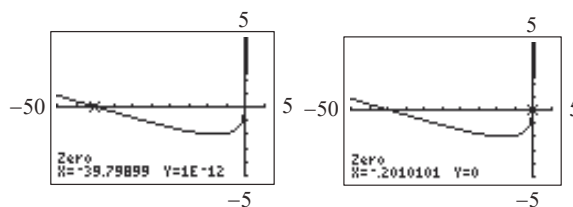
98. $3x^3 + 4x^2 = 27x + 36$
 $3x^3 + 4x^2 - 27x - 36 = 0$
 $x^2(3x+4) - 9(3x+4) = 0$
 $(3x+4)(x^2-9) = 0$
 $(3x+4)(x-3)(x+3) = 0$
 $3x+4=0$ or $x-3=0$ or $x+3=0$
 $3x=-4$ $x=3$ $x=-3$
 $x = -\frac{4}{3}$
 The solution set is $\left\{-3, -\frac{4}{3}, 3\right\}$.

99. $x - 4x^{1/2} + 2 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = x - 4\sqrt{x} + 2$.



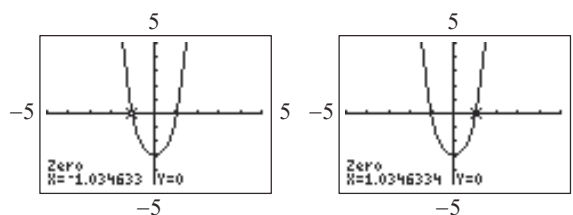
The solution set is $\{0.34, 11.66\}$.

100. $x^{2/3} + 4x^{1/3} + 2 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = x^{2/3} + 4x^{1/3} + 2$.



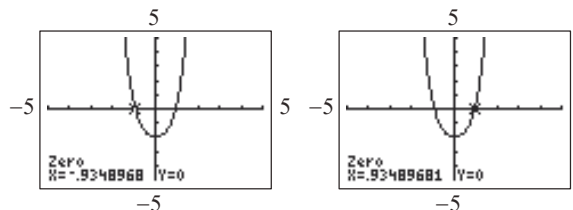
The solution set is $\{-39.80, -0.20\}$.

101. $x^4 + \sqrt{3}x^2 - 3 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = x^4 + \sqrt{3}x^2 - 3$.



The solution set is $\{-1.03, 1.03\}$.

102. $x^4 + \sqrt{2}x^2 - 2 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = x^4 + \sqrt{2}x^2 - 2$.



The solution set is $\{-0.93, 0.93\}$.

103. $3x^2 + 7x - 20 = 0$
 $(3x-5)(x+4) = 0$
 $3x-5=0$ or $x+4=0$
 $3x=5$ $x=-4$
 $x = \frac{5}{3}$

The solution set is $\left\{-4, \frac{5}{3}\right\}$.

Section 1.5: Radical Equations; Equations Quadratic in Form; Absolute Value Equations; Factorable Equations

104. $2x^2 - 13x + 21 = 0$

$$(2x - 7)(x - 3) = 0$$

$$2x - 7 = 0 \quad \text{or} \quad x - 3 = 0$$

$$2x = 7 \quad x = 3$$

$$x = \frac{7}{2}$$

The solution set is $\left\{\frac{7}{2}, 3\right\}$.

105. $5a^3 - 45a = -2a^2 + 18$

$$5a^3 + 2a^2 - 45a - 18 = 0$$

$$a^2(5a + 2) - 9(5a + 2) = 0$$

$$(a^2 - 9)(5a + 2) = 0$$

$$(a - 3)(a + 3)(5a + 2) = 0$$

$$a - 3 = 0 \quad \text{or} \quad a + 3 = 0 \quad \text{or} \quad 5a + 2 = 0$$

$$a = 3 \quad a = -3 \quad 5a = -2$$

$$a = -\frac{2}{5}$$

The solution set is $\left\{-3, -\frac{2}{5}, 3\right\}$.

106. $3z^3 - 12z = -5z^2 + 20$

$$3z^3 + 5z^2 - 12z - 20 = 0$$

$$z^2(3z + 5) - 4(3z + 5) = 0$$

$$(z^2 - 4)(3z + 5) = 0$$

$$(z - 2)(z + 2)(3z + 5) = 0$$

$$z - 2 = 0 \quad \text{or} \quad z + 2 = 0 \quad 3z + 5 = 0$$

$$z = 2 \quad z = -2 \quad 3z = -5$$

$$z = -\frac{5}{3}$$

The solution set is $\left\{-2, -\frac{5}{3}, 2\right\}$.

107. $-3|5x - 2| + 9 = 0$

$$-3|5x - 2| = -9$$

$$|5x - 2| = 3$$

$$5x - 2 = 3 \quad \text{or} \quad 5x - 2 = -3$$

$$5x = 5 \quad 5x = -1$$

$$x = 1 \quad x = -\frac{1}{5}$$

The solution set is $\left\{-\frac{1}{5}, 1\right\}$.

108. $\frac{1}{4}|2x - 3| = \frac{3}{2}$

$$|2x - 3| = 6$$

$$2x - 3 = 6 \quad \text{or} \quad 2x - 3 = -6$$

$$2x = 9 \quad 2x = -3$$

$$x = \frac{9}{2} \quad x = -\frac{3}{2}$$

The solution set is $\left\{-\frac{3}{2}, \frac{9}{2}\right\}$.

109. $4(w - 3) = w + 3$

$$4w - 12 = w + 3$$

$$3w = 15$$

$$w = 5$$

The solution set is $\{5\}$.

110. $6(k + 3) - 2k = 12$

$$6k + 18 - 2k = 12$$

$$4k = -6$$

$$k = -\frac{3}{2}$$

The solution set is $\left\{-\frac{3}{2}\right\}$.

$$111. \left(\frac{v}{v+1} \right)^2 + \frac{2v}{v+1} = 8$$

Let $u = \frac{v}{v+1}$. Rewrite the equation:

$$u^2 + 2u = 8$$

$$u^2 + 2u - 8 = 0$$

$$(u-2)(u+4) = 0$$

$$u = 2 \text{ or } u = -4$$

Go back in terms of v and solve:

$$\frac{v}{v+1} = 2 \quad \text{or} \quad \frac{v}{v+1} = -4$$

$$v = 2v + 2 \quad v = -4v - 4$$

$$-v = 2 \quad 5v = -4$$

$$v = -2 \quad v = -\frac{4}{5}$$

The solution set is $\left\{ -2, -\frac{4}{5} \right\}$.

$$112. \left(\frac{y}{y-1} \right)^2 - \frac{6y}{y-1} = 7$$

Let $u = \frac{y}{y-1}$. Rewrite the equation:

$$u^2 - 6u = 7$$

$$u^2 - 6u - 7 = 0$$

$$(u-7)(u+1) = 0$$

$$u = 7 \text{ or } u = -1$$

Go back in terms of y and solve:

$$\frac{y}{y-1} = 7 \quad \text{or} \quad \frac{y}{y-1} = -1$$

$$y = 7y - 7 \quad y = -y + 1$$

$$-6y = -7 \quad 2y = 1$$

$$y = \frac{7}{6} \quad y = \frac{1}{2}$$

The solution set is $\left\{ \frac{1}{2}, \frac{7}{6} \right\}$.

$$113. |-3x+2| = x+10$$

$$-3x+2 = -(x+10) \quad \text{or} \quad -3x+2 = x+10$$

$$-3x+2 = -x-10 \quad -4x = 8$$

$$-2x = -12 \quad x = -2$$

$$x = 6$$

The solution set is $\{-2, 6\}$.

$$114. |4x-3| = x+2$$

$$4x-3 = -(x+2) \quad \text{or} \quad 4x-3 = x+2$$

$$4x-3 = -x-2 \quad 3x = 5$$

$$5x = 1 \quad x = \frac{5}{3}$$

$$x = \frac{1}{5}$$

The solution set is $\left\{ \frac{1}{5}, \frac{5}{3} \right\}$.

$$115. \sqrt{2x+5} - x = 1$$

$$\sqrt{2x+5} = x+1$$

$$(\sqrt{2x+5})^2 = (x+1)^2$$

$$2x+5 = x^2 + 2x + 1$$

$$x^2 - 4 = 0$$

$$(x-2)(x+2) = 0$$

$$x = 2 \quad \text{or} \quad x = -2$$

Check:

$$\sqrt{2(-2)+5} - (-2) = 1 \quad \sqrt{2(2)+5} - (2) = 1$$

$$\sqrt{1} + 2 = 1 \quad \sqrt{9} - 2 = 1$$

$$3 \neq 1 \quad 1 = 1 \quad \text{T}$$

The solution set is $\{2\}$.

$$116. \sqrt{3x+1} - 2x = -6$$

$$\sqrt{3x+1} = 2x-6$$

$$(\sqrt{3x+1})^2 = (2x-6)^2$$

$$3x+1 = 4x^2 - 24x + 36$$

$$4x^2 - 27x + 35 = 0$$

$$(4x-7)(x-5) = 0$$

$$4x-7 = 0 \quad \text{or} \quad x-5 = 0$$

$$4x = 7 \quad x = 5$$

$$x = \frac{7}{4}$$

Check:

$$\sqrt{3\left(\frac{7}{4}\right)+1} - 2\left(\frac{7}{4}\right) = -6 \quad \sqrt{3(5)+1} - 2(5) = -6$$

$$4 - 10 = -6$$

$$\frac{5}{2} - \frac{7}{2} = -6$$

$$-6 = -6 \quad \text{T}$$

$$-1 \neq -6$$

The solution set is $\{5\}$.

Section 1.5: Radical Equations; Equations Quadratic in Form; Absolute Value Equations; Factorable Equations

117. $3m^2 + 6m = -1$

$$3m^2 + 6m + 1 = 0$$

$$a = 3, b = 6, c = 1$$

$$m = \frac{-6 \pm \sqrt{6^2 - 4(3)(1)}}{2(3)} = \frac{-6 \pm \sqrt{24}}{6}$$

$$= \frac{-6 \pm 2\sqrt{6}}{6} = \frac{-3 \pm \sqrt{6}}{3}$$

The solution set is $\left\{ \frac{-3 - \sqrt{6}}{3}, \frac{-3 + \sqrt{6}}{3} \right\}$.

118. $4y^2 - 8y = 3$

$$4y^2 - 8y - 3 = 0$$

$$a = 4, b = -8, c = -3$$

$$y = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(-3)}}{2(4)}$$

$$= \frac{8 \pm \sqrt{112}}{8} = \frac{8 \pm 4\sqrt{7}}{8} = \frac{2 \pm \sqrt{7}}{2}$$

The solution set is $\left\{ \frac{2 - \sqrt{7}}{2}, \frac{2 + \sqrt{7}}{2} \right\}$.

119. $|x^2 + x - 1| = 1$

$$x^2 + x - 1 = 1 \quad \text{or} \quad x^2 + x - 1 = -1$$

$$x^2 + x - 2 = 0 \quad x^2 + x = 0$$

$$(x+2)(x-1) = 0 \quad x(x+1) = 0$$

$$x = -2 \text{ or } x = 1 \quad x = 0 \text{ or } x = -1$$

The solution set is $\{-2, -1, 0, 1\}$.

120. $|x^2 + 3x - 2| = 2$

$$x^2 + 3x - 2 = 2 \quad \text{or} \quad x^2 + 3x - 2 = -2$$

$$x^2 + 3x - 4 = 0 \quad x^2 + 3x = 0$$

$$(x+4)(x-1) = 0 \quad x(x+3) = 0$$

$$x = -4 \text{ or } x = 1 \quad x = 0 \text{ or } x = -3$$

The solution set is $\{-4, -3, 0, 1\}$.

121. $\sqrt[4]{5x^2 - 6} = x$

$$\left(\sqrt[4]{5x^2 - 6}\right)^4 = x^4$$

$$5x^2 - 6 = x^4$$

$$x^4 - 5x^2 + 6 = 0$$

$$(x^2 - 3)(x^2 - 2) = 0$$

$$x^2 - 3 = 0 \quad \text{or} \quad x^2 - 2 = 0$$

$$x^2 = 3 \quad x^2 = 2$$

$$x = \pm\sqrt{3} \quad x = \pm\sqrt{2}$$

Since an even root cannot be negative, we can exclude $-\sqrt{3}$ and $-\sqrt{2}$ as solutions.

Check:

$$\sqrt[4]{5(\sqrt{2})^2 - 6} = \sqrt{2} \quad \sqrt[4]{5(\sqrt{3})^2 - 6} = \sqrt{3}$$

$$\sqrt[4]{4} = \sqrt{2} \quad \sqrt[4]{9} = \sqrt{3}$$

$$\sqrt[4]{2^2} = \sqrt{2} \quad \sqrt[4]{3^2} = \sqrt{3}$$

$$\sqrt{2} = \sqrt{2} \text{ T} \quad \sqrt{3} = \sqrt{3} \text{ T}$$

The solution set is $\{\sqrt{2}, \sqrt{3}\}$.

122. $\sqrt[4]{4 - 3x^2} = x$

$$\left(\sqrt[4]{4 - 3x^2}\right)^4 = x^4$$

$$4 - 3x^2 = x^4$$

$$x^4 + 3x^2 - 4 = 0$$

$$(x^2 + 4)(x^2 - 1) = 0$$

$$x^2 + 4 = 0 \quad \text{or} \quad x^2 - 1 = 0$$

$$x^2 = -4 \quad x^2 = 1$$

$$\text{no real solution} \quad x = \pm 1$$

Since an even root cannot be negative, we can exclude -1 as a solution.

Check:

$$\sqrt[4]{4 - 3(1)^2} = 1$$

$$\sqrt[4]{1} = 1$$

$$1 = 1 \text{ T}$$

The solution set is $\{1\}$.

Chapter 1: Graphs, Equations, and Inequalities

123. $t^4 - 16 = 0$

$$(t^2 - 4)(t^2 + 4) = 0$$

$$t^2 - 4 = 0 \quad \text{or} \quad t^2 + 4 = 0$$

$$t^2 = 4 \quad t^2 = -4$$

$$t = \pm 2 \quad t = \pm 2i$$

The solution set is $\{-2i, 2i, -2, 2\}$.

124. $y^4 - 81 = 0$

$$(y^2 - 9)(y^2 + 9) = 0$$

$$y^2 - 9 = 0 \quad \text{or} \quad y^2 + 9 = 0$$

$$y^2 = 9 \quad y^2 = -9$$

$$y = \pm 3 \quad y = \pm 3i$$

The solution set is $\{-3i, 3i, -3, 3\}$.

125. $x^6 - 9x^3 + 8 = 0$

$$(x^3 - 8)(x^3 - 1) = 0$$

$$(x - 2)(x^2 + 2x + 4)(x - 1)(x^2 + x + 1) = 0$$

$$x - 2 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = 2 \quad x = 1$$

or $x^2 + 2x + 4 = 0$

$$a = 1, b = 2, c = 4$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)} = \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2i\sqrt{3}}{2} = -1 \pm i\sqrt{3}$$

or

$$x^2 + x + 1 = 0$$

$$a = 1, b = 1, c = 1$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

The solution set is $\left\{-\frac{1}{2} - \frac{\sqrt{3}}{2}i, -1 - i\sqrt{3},\right.$

$-\frac{1}{2} + \frac{\sqrt{3}}{2}i, -1 + i\sqrt{3}, 1, 2\}$.

126. $z^6 + 28z^3 + 27 = 0$

$$(z^3 + 27)(z^3 + 1) = 0$$

$$(z + 3)(z^2 - 3z + 9)(z + 1)(z^2 - z + 1) = 0$$

$$z + 3 = 0 \quad \text{or} \quad z + 1 = 0$$

$$z = -3 \quad z = -1$$

or $z^2 - 3z + 9 = 0$

$$a = 1, b = -3, c = 9$$

$$z = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{-27}}{2} = \frac{3 \pm 3i\sqrt{3}}{2}$$

or $z^2 - z + 1 = 0$

$$a = 1, b = -1, c = 1$$

$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

The solution set is $\left\{\frac{3}{2} - \frac{3\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i,\right.$

$\frac{3}{2} + \frac{3\sqrt{3}}{2}i, \frac{1}{2} + \frac{\sqrt{3}}{2}i, -3, -1\}$.

127. $k^2 - k = 12 \Rightarrow k^2 - k - 12 = 0$

$$(k - 4)(k + 3) = 0$$

$$k = 4 \quad \text{or} \quad k = -3$$

$$\frac{x+3}{x-3} = 4 \quad \frac{x+3}{x-3} = -3$$

$$x + 3 = 4x - 12 \quad x + 3 = -3x + 9$$

$$3x = 15$$

$$x = 5$$

$$x = \frac{3}{2}$$

And since neither of these x values causes a denominator to equal zero, the solution set is

$\left\{\frac{3}{2}, 5\right\}$.

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128. $k^2 - 3k = 28 \Rightarrow k^2 - 3k - 28 = 0$

$(k+4)(k-7) = 0$

$k = -4$ or $k = 7$

$\frac{x+3}{x-4} = -4$ or $\frac{x+3}{x-4} = 7$

$x+3 = -4x+16$ or $x+3 = 7x-28$

$x = \frac{13}{5}$ or $x = \frac{31}{6}$

And since neither of these x values causes a denominator to equal zero, the solution set is

$\left\{\frac{13}{5}, \frac{31}{6}\right\}$.

129. $|8-3x| = |2x-7|$

$8-3x = 2x-7$ or $8-3x = -(2x-7)$

$-3x-2x = -7-8$ or $8-3x = -2x+7$

$-5x = -15$ or $-3x+2x = 7-8$

$x = -3$ or $-x = -1$

$x = 1$

The solution set is $\{-3, 1\}$.

130. $|5x+3| = |12-4x|$

$5x+3 = 12-4x$ or $5x+3 = -(12-4x)$

$5x+4x = 12-3$ or $5x+3 = -12+4x$

$9x = 9$ or $5x-4x = -12-3$

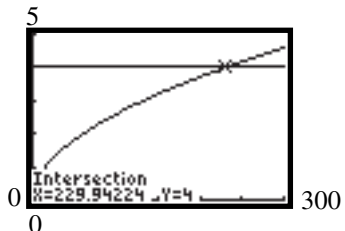
$x = 1$ or $x = -15$

The solution set is $\{1, -15\}$.

131. Graph the equations $y_1 = \sqrt{x}/4 + x/1100$ and

$y_2 = 4$; then use INTERSECT to find the

x -coordinate of the points of intersection:



The distance to the water's surface is approximately 229.94 feet.

132. $T = \sqrt[4]{\frac{LH^2}{25}}$

Let $T = 4$ and $H = 10$, and solve for L .

$4 = \sqrt[4]{\frac{L(10)^2}{25}}$

$4 = \sqrt[4]{4L}$

$(4)^4 = (\sqrt[4]{4L})^4$

$256 = 4L$

$64 = L$

The crushing load is 64 tons.

133. $T = 2\pi\sqrt{\frac{l}{32}}$

Let $T = 16.5$ and solve for l .

$16.5 = 2\pi\sqrt{\frac{l}{32}}$

$\frac{16.5}{2\pi} = \sqrt{\frac{l}{32}}$

$\left(\frac{16.5}{2\pi}\right)^2 = \left(\sqrt{\frac{l}{32}}\right)^2$

$\left(\frac{16.5}{2\pi}\right)^2 = \frac{l}{32}$

$l = 32\left(\frac{16.5}{2\pi}\right)^2 \approx 220.7$

The length was approximately 221 feet.

134. $\sqrt{3x+5} - \sqrt{x-2} = \sqrt{x+3}$

$(\sqrt{3x+5} - \sqrt{x-2})^2 = (\sqrt{x+3})^2$

$3x+5-2\sqrt{(3x+5)(x-2)}+(x-2)=x+3$

$4x+3-2\sqrt{(3x+5)(x-2)}=x+3$

$2\sqrt{3x^2-x-10}=3x$

$4(3x^2-x-10)=9x^2$

$3x^2-4x-40=0$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(3)(-40)}}{6}$

$= \frac{4 \pm \sqrt{496}}{6} = \frac{4 \pm 4\sqrt{31}}{6}$

$= \frac{4 \pm 4\sqrt{31}}{6}$

$= \frac{2 \pm 2\sqrt{31}}{3}$

Chapter 1: Graphs, Equations, and Inequalities

Since $x > 2$, the negative solution is extraneous.

The solution set is $\left\{\frac{2+2\sqrt{31}}{3}\right\}$.

135.

$$\begin{aligned}\sqrt[4]{\sqrt[3]{\sqrt{x-7}-10}+18} &= 2 \\ \left(\sqrt[4]{\sqrt[3]{\sqrt{x-7}-10}+18}\right)^4 &= (2)^4 \\ \sqrt[3]{\sqrt{x-7}-10}+18 &= 16 \\ \sqrt[3]{\sqrt{x-7}-10} &= (-2)^3 \\ \sqrt{x-7}-10 &= -8 \\ (\sqrt{x-7})^2 &= (2)^2 \\ x-7 &= 4 \\ x &= 11\end{aligned}$$

The solution set is $\{11\}$

136.

$$\begin{aligned}12x^{7/5} + 3x^{2/5} &= 13x^{9/10} \\ 12x^{14/10} + 3x^{4/10} - 13x^{9/10} &= 0 \\ x^{9/10}(12x - 13x^{1/2} + 3) &= 0 \\ x^{9/10} = 0 &\Rightarrow x = 0 \\ \text{To solve } 12x - 13x^{1/2} + 3 = 0, &\text{ let } u = x^{1/2} \\ \text{Then } 12u^2 - 13u + 3 &= 0 \\ (4u - 3)(3u - 1) &= 0 \\ u = x^{1/2} = \frac{3}{4} &\text{ or } x^{1/2} = \frac{1}{3} \\ x = \frac{9}{16} &\quad x = \frac{1}{9}\end{aligned}$$

The solution set is $\left\{0, \frac{9}{16}, \frac{1}{9}\right\}$

137.

$$\begin{aligned}z^6 + 28z^3 + 27 &= 0 \\ (z^3 + 27)(z^3 + 1) &= 0 \\ (z+3)(z^2 - 3z + 9)(z+1)(z^2 - z + 1) &= 0 \\ z+3 = 0 &\quad \text{or } z+1 = 0 \\ z = -3 &\quad \text{or } z = -1\end{aligned}$$

$$a = 1, b = -3, c = 9$$

$$\begin{aligned}x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2} \\ &= \frac{3 \pm \sqrt{-27}}{2} = \frac{3 \pm 3\sqrt{3}i}{2}\end{aligned}$$

Also,

$$a = 1, b = -1, c = 1$$

$$\begin{aligned}x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2} \\ &= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}i}{2}\end{aligned}$$

The solution set is $\left\{-3, -1, \frac{1 \pm \sqrt{3}i}{2}, \frac{3 \pm 3\sqrt{3}i}{2}\right\}$.

138. Answers will vary, one example is $\sqrt{x+1} = -1$

139. Answers will vary, one example is $x - \sqrt{x} - 2 = 0$

140. Answers will vary.

141. $|x|$ represents the distance from x to the origin. Since distance is never negative, $|x| = -2$ has no real solution.

142. Jane did not check her solutions and included the extraneous solution, $x = -1$.

$$\begin{aligned}\sqrt{2x+3} - x &= 0 \\ \sqrt{2x+3} &= x \\ (\sqrt{2x+3})^2 &= x^2 \\ 2x+3 &= x^2 \\ x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0 \\ x = 3 &\quad \text{or } x = -1\end{aligned}$$

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Check:

$$\sqrt{2(3)+3}-3=0 \quad \sqrt{2(-1)+3}-(-1)=0$$

$$\sqrt{9}-3=0 \quad \sqrt{1}+1=0$$

$$3-3=0 \quad 1+1=0$$

$$0=0 \quad \text{T} \quad 2 \neq 0$$

The solution set is $\{3\}$.

Section 1.6

1. mathematical modeling
2. interest
3. uniform motion
4. False; the amount charged for the use of principal is the interest.
5. True; this is the uniform motion formula.
6. a
7. b
8. c
9. Let A represent the area of the circle and r the radius. The area of a circle is the product of π times the square of the radius: $A = \pi r^2$
10. Let C represent the circumference of a circle and r the radius. The circumference of a circle is the product of π times twice the radius: $C = 2\pi r$
11. Let A represent the area of the square and s the length of a side. The area of the square is the square of the length of a side: $A = s^2$
12. Let P represent the perimeter of a square and s the length of a side. The perimeter of a square is four times the length of a side: $P = 4s$
13. Let F represent the force, m the mass, and a the acceleration. Force equals the product of the mass times the acceleration: $F = ma$
14. Let P represent the pressure, F the force, and A the area. Pressure is the force per unit area: $P = \frac{F}{A}$
15. Let W represent the work, F the force, and d the distance. Work equals force times distance: $W = Fd$
16. Let K represent the kinetic energy, m the mass, and v the velocity. Kinetic energy is one-half the product of the mass and the square of the velocity: $K = \frac{1}{2}mv^2$
17. C = total variable cost in dollars, x = number of dishwashers manufactured: $C = 150x$
18. R = total revenue in dollars, x = number of dishwashers sold: $R = 250x$
19. Let x represent the amount of money invested in bonds. Then $50,000 - x$ represents the amount of money invested in CD's. Since the total interest is to be \$6,000, we have:

$$0.15x + 0.07(50,000 - x) = 6,000$$

$$(100)(0.15x + 0.07(50,000 - x)) = (6,000)(100)$$

$$15x + 7(50,000 - x) = 600,000$$

$$15x + 350,000 - 7x = 600,000$$

$$8x + 350,000 = 600,000$$

$$8x = 250,000$$

$$x = 31,250$$

\$31,250 should be invested in bonds at 15% and \$18,750 should be invested in CD's at 7%.
20. Let x represent the amount of money invested in bonds. Then $50,000 - x$ represents the amount of money invested in CD's. Since the total interest is to be \$7,000, we have:

$$0.15x + 0.07(50,000 - x) = 7,000$$

$$(100)(0.15x + 0.07(50,000 - x)) = (7,000)(100)$$

$$15x + 7(50,000 - x) = 700,000$$

$$15x + 350,000 - 7x = 700,000$$

$$8x + 350,000 = 700,000$$

$$8x = 350,000$$

$$x = 43,750$$

\$43,750 should be invested in bonds at 15% and \$6,250 should be invested in CD's at 7%.

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21. Let x represent the amount of money loaned at 8%. Then $12,000 - x$ represents the amount of money loaned at 18%. Since the total interest is to be \$1,000, we have:

$$\begin{aligned} 0.08x + 0.18(12,000 - x) &= 1,000 \\ (100)(0.08x + 0.18(12,000 - x)) &= (1,000)(100) \\ 8x + 18(12,000 - x) &= 100,000 \\ 8x + 216,000 - 18x &= 100,000 \\ -10x + 216,000 &= 100,000 \\ -10x &= -116,000 \\ x &= 11,600 \end{aligned}$$

\$11,600 is loaned at 8% and \$400 is at 18%.

22. Let x represent the amount of money loaned at 16%. Then $1,000,000 - x$ represents the amount of money loaned at 19%. Since the total interest is to be \$1,000,000(0.18), we have:

$$\begin{aligned} 0.16x + 0.19(1,000,000 - x) &= 1,000,000(0.18) \\ 0.16x + 190,000 - 0.19x &= 180,000 \\ -0.03x + 190,000 &= 180,000 \\ -0.03x &= -10,000 \\ x &= \frac{-10,000}{-0.03} \\ x &= \$333,333.33 \end{aligned}$$

Wendy can lend \$333,333.33 at 16%.

23. Let x represent the number of pounds of Earl Grey tea. Then $100 - x$ represents the number of pounds of orange pekoe tea.

$$6x + 4(100 - x) = 5.50(100)$$

$$6x + 400 - 4x = 550$$

$$2x + 400 = 550$$

$$2x = 150$$

$$x = 75$$

75 pounds of Earl Grey tea must be blended with 25 pounds of orange pekoe.

24. Let x represent the number of pounds of the first kind of coffee. Then $100 - x$ represents the number of pounds of the second kind of coffee.

$$2.75x + 5(100 - x) = 4.10(100)$$

$$2.75x + 500 - 5x = 410$$

$$-2.25x + 500 = 410$$

$$-2.25x = -90$$

$$x = 40$$

40 pounds of the first kind of coffee must be blended with 60 pounds of the second kind of coffee.

25. Let x represent the number of pounds of cashews. Then $x + 60$ represents the number of pounds in the mixture.

$$9x + 4.50(60) = 7.75(x + 60)$$

$$9x + 270 = 7.75x + 465$$

$$1.25x = 195$$

$$x = 156$$

156 pounds of cashews must be added to the 60 pounds of almonds.

26. Let x represent the number of caramels in the box. Then $30 - x$ represents the number of cremes in the box.

$$\text{Revenue} - \text{Cost} = \text{Profit}$$

$$12.50 - (0.25x + 0.45(30 - x)) = 3.00$$

$$12.50 - (0.25x + 13.5 - 0.45x) = 3.00$$

$$12.50 - (13.5 - 0.20x) = 3.00$$

$$12.50 - 13.50 + 0.20x = 3.00$$

$$-1.00 + 0.20x = 3.00$$

$$0.20x = 4.00$$

$$x = 20$$

The box should contain 20 caramels and 10 cremes.

27. Let r represent the speed of the current.

	Rate	Time	Distance
Upstream	$16 - r$	$\frac{20}{60} = \frac{1}{3}$	$\frac{16 - r}{3}$
Downstream	$16 + r$	$\frac{15}{60} = \frac{1}{4}$	$\frac{16 + r}{4}$

Since the distance is the same in each direction:

$$\frac{16 - r}{3} = \frac{16 + r}{4}$$

$$4(16 - r) = 3(16 + r)$$

$$64 - 4r = 48 + 3r$$

$$16 = 7r$$

$$r = \frac{16}{7} \approx 2.286$$

The speed of the current is approximately 2.286 miles per hour.

28. Let r represent the speed of the motorboat.

	Rate	Time	Distance
Upstream	$r - 3$	5	$5(r - 3)$
Downstream	$r + 3$	2.5	$2.5(r + 3)$

The distance is the same in each direction:

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$$5(r-3) = 2.5(r+3)$$

$$5r - 15 = 2.5r + 7.5$$

$$2.5r = 22.5$$

$$r = 9$$

The speed of the motorboat is 9 miles per hour.

29. Let r represent the speed of the current.

	Rate	Time	Distance
Upstream	$15 - r$	$\frac{10}{15 - r}$	10
Downstream	$15 + r$	$\frac{10}{15 + r}$	10

Since the total time is 1.5 hours, we have:

$$\frac{10}{15 - r} + \frac{10}{15 + r} = 1.5$$

$$10(15 + r) + 10(15 - r) = 1.5(15 - r)(15 + r)$$

$$150 + 10r + 150 - 10r = 1.5(225 - r^2)$$

$$300 = 1.5(225 - r^2)$$

$$200 = 225 - r^2$$

$$r^2 - 25 = 0$$

$$(r - 5)(r + 5) = 0$$

$$r = 5 \text{ or } r = -5$$

Speed must be positive, so disregard $r = -5$.

The speed of the current is 5 miles per hour.

30. Let r represent the rate of the slower car. Then $r + 10$ represents the rate of the faster car.

	Rate	Time	Distance
Slower car	r	3.5	$3.5r$
Faster car	$r + 10$	3	$3(r + 10)$

$$3.5r = 3(r + 10)$$

$$3.5r = 3r + 30$$

$$0.5r = 30$$

$$r = 60$$

The slower car travels at a rate of 60 miles per hour. The faster car travels at a rate of 70 miles per hour. The distance is $(70)(3) = 210$ miles.

31. Let r represent Karen's normal walking speed.

	Rate	Time	Distance
With walkway	$r + 2.5$	$\frac{50}{r + 2.5}$	50
Against walkway	$r - 2.5$	$\frac{50}{r - 2.5}$	50

Since the total time is 48 seconds:

$$\frac{50}{r + 2.5} + \frac{50}{r - 2.5} = 48$$

$$50(r - 2.5) + 50(r + 2.5) = 48(r - 2.5)(r + 2.5)$$

$$50r - 125 + 50r + 125 = 48(r^2 - 6.25)$$

$$100r = 48r^2 - 300$$

$$0 = 48r^2 - 100r - 300$$

$$0 = 12r^2 - 25r - 75$$

$$r = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(12)(-75)}}{2(12)}$$

$$= \frac{25 \pm \sqrt{4225}}{24}$$

$$r \approx 3.75 \text{ or } r \approx -1.67$$

Speed must be positive, so disregard $r \approx -1.67$.

Karen's normal walking speed is approximately 3.75 feet per second.

32. Let r represent the speed of the airport walkway.

	Rate	Time	Distance
Walking with	$1.5 + r$	$\frac{280}{1.5 + r}$	280
Standing still	r	$\frac{280}{r}$	280

Walking with the walkway takes 60 seconds less time than standing still on the walkway:

$$\frac{280}{1.5 + r} = \frac{280}{r} - 60$$

$$280r = 280(1.5 + r) - 60r(r + 1.5)$$

$$280r = 420 + 280r - 60r^2 - 90r$$

$$60r^2 + 90r - 420 = 0$$

$$2r^2 + 3r - 14 = 0$$

$$(2r + 7)(r - 2) = 0$$

$$2r + 7 = 0 \quad \text{or} \quad r - 2 = 0$$

$$r = -\frac{7}{2} \quad \text{or} \quad r = 2$$

Speed must be positive, so disregard $r = -\frac{7}{2}$.

The speed of the airport walkway is 2 meters per second.

33. Let w represent the width of a regulation doubles tennis court. Then $2w + 6$ represents the length. The area is 2808 square feet:

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$$w(2w + 6) = 2808$$

$$2w^2 + 6w = 2808$$

$$2w^2 + 6w - 2808 = 0$$

$$w^2 + 3w - 1404 = 0$$

$$(w + 39)(w - 36) = 0$$

$$w + 39 = 0 \quad \text{or} \quad w - 36 = 0$$

$$w = -39 \quad \text{or} \quad w = 36$$

The width must be positive, so disregard $w = -39$.

The width of a regulation doubles tennis court is 36 feet and the length is $2(36) + 6 = 78$ feet.

34. Let t represent the time it takes the Brother HL-L8350CDW to complete the print job alone. Then $t + 9$ represents the time it takes the Xerox VersaLink C500 to complete the print job alone.

	Time to do job	Part of job done in one minute
Brother	t	$\frac{1}{t}$
Xerox	$t + 9$	$\frac{1}{t + 9}$
Together	20	$\frac{1}{20}$

$$\frac{1}{t} + \frac{1}{t + 9} = \frac{1}{20}$$

$$20(t + 9) + 20t = t(t + 9)$$

$$20t + 180 + 20t = t^2 + 9t$$

$$0 = t^2 - 31t - 180$$

$$0 = (t - 36)(t + 5)$$

$$t - 36 = 0 \quad \text{or} \quad t + 5 = 0$$

$$t = 36 \quad \text{or} \quad t = -5$$

Time must be positive, so disregard $t = -5$.

The Brother HL-L8350CDW takes 36 minutes to

complete the job alone, printing $\frac{1440}{36} = 40$

pages per minute. Xerox VersaLink C500 takes

$36 + 9 = 45$ minutes to complete the job alone,

printing $\frac{1440}{45} = 32$ pages per minute.

35. Let t represent the time it takes to do the job together.

	Time to do job	Part of job done in one minute
Trent	30	$\frac{1}{30}$
Lois	20	$\frac{1}{20}$
Together	t	$\frac{1}{t}$

$$\frac{1}{30} + \frac{1}{20} = \frac{1}{t}$$

$$2t + 3t = 60$$

$$5t = 60$$

$$t = 12$$

Working together, the job can be done in 12 minutes.

36. Let t represent the time it takes April to do the job working alone.

	Time to do job	Part of job done in one hour
Patrice	10	$\frac{1}{10}$
April	t	$\frac{1}{t}$
Together	6	$\frac{1}{6}$

$$\frac{1}{10} + \frac{1}{t} = \frac{1}{6}$$

$$3t + 30 = 5t$$

$$2t = 30$$

$$t = 15$$

April would take 15 hours to paint the rooms.

37. l = length of the garden
 w = width of the garden

- a. The length of the garden is to be twice its width. Thus, $l = 2w$.

The dimensions of the fence are $l + 4$ and $w + 4$.

The perimeter is 46 feet, so:

$$2(l + 4) + 2(w + 4) = 46$$

$$2(2w + 4) + 2(w + 4) = 46$$

$$4w + 8 + 2w + 8 = 46$$

$$6w + 16 = 46$$

$$6w = 30$$

$$w = 5$$

The dimensions of the garden are 5 feet by 10 feet.

- b. Area = $l \cdot w = 5 \cdot 10 = 50$ square feet
c. If the dimensions of the garden are the same, then the length and width of the fence are also the same $(l + 4)$. The perimeter is 46 feet, so:

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$$2(l+4)+2(l+4)=46$$

$$2l+8+2l+8=46$$

$$4l+16=46$$

$$4l=30$$

$$l=7.5$$

The dimensions of the garden are 7.5 feet by 7.5 feet.

d. $\text{Area} = l \cdot w = 7.5(7.5) = 56.25$ square feet.

38. l = length of the pond

w = width of the pond

a. The pond is to be a square. Thus, $l = w$.

The dimensions of the fenced area are $w+6$ on each side. The perimeter is 100 feet, so:

$$4(w+6)=100$$

$$4w+24=100$$

$$4w=76$$

$$w=19$$

The dimensions of the pond are 19 feet by 19 feet.

b. The length of the pond is to be three times the width. Thus, $l = 3w$. The dimensions of the fenced area are $w+6$ and $l+6$. The perimeter is 100 feet, so:

$$2(w+6)+2(l+6)=100$$

$$2(w+6)+2(3w+6)=100$$

$$2w+12+6w+12=100$$

$$8w+24=100$$

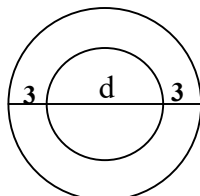
$$8w=76$$

$$w=9.5$$

$$l=3(9.5)=28.5$$

The dimensions of the pond are 9.5 feet by 28.5 feet.

c. If the pond is circular, the diameter is d and the diameter of the circle with the pond and the deck is $d+6$.



The perimeter is 100 feet, so:

$$\pi(d+6)=100$$

$$\pi d+6\pi=100$$

$$\pi d=100-6\pi$$

$$d=\frac{100}{\pi}-6 \approx 25.83$$

The diameter of the pond is 25.83 feet.

d. $\text{Area}_{\text{square}} = l \cdot w = 19(19) = 361 \text{ ft}^2$.

$$\text{Area}_{\text{rectangle}} = l \cdot w = 28.5(9.5) = 270.75 \text{ ft}^2$$

$$\text{Area}_{\text{circle}} = \pi r^2 = \pi \left(\frac{25.83}{2} \right)^2 \approx 524 \text{ ft}^2$$

The circular pond has the largest area.

39. Let t represent the time it takes for the defensive back to catch the tight end.

	Time to run 100 yards	Time	Rate	Distance
Tight End	12 sec	t	$\frac{100}{12} = \frac{25}{3}$	$\frac{25}{3}t$
Def. Back	10 sec	t	$\frac{100}{10} = 10$	$10t$

Since the defensive back has to run 5 yards farther, we have:

$$\frac{25}{3}t + 5 = 10t$$

$$25t + 15 = 30t$$

$$15 = 5t$$

$$t = 3 \rightarrow 10t = 30$$

The defensive back will catch the tight end at the 45 yard line ($15 + 30 = 45$).

40. Let x represent the number of highway miles traveled. Then $30,000 - x$ represents the number of city miles traveled.

$$\frac{x}{40} + \frac{30,000-x}{25} = 900$$

$$200 \left(\frac{x}{40} + \frac{30,000-x}{25} \right) = 200(900)$$

$$5x + 240,000 - 8x = 180,000$$

$$-3x + 240,000 = 180,000$$

$$-3x = -60,000$$

$$x = 20,000$$

Therese is allowed to claim 20,000 miles as a business expense.

41. Let x represent the number of gallons of pure water. Then $x+1$ represents the number of gallons in the 60% solution.

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$$(\%)(\text{gallons}) + (\%)(\text{gallons}) = (\%)(\text{gallons})$$

$$0(x) + 1(1) = 0.60(x + 1)$$

$$1 = 0.6x + 0.6$$

$$0.4 = 0.6x$$

$$x = \frac{4}{6} = \frac{2}{3}$$

$\frac{2}{3}$ gallon of pure water should be added.

42. Let x represent the number of liters to be drained and replaced with pure antifreeze.

$$(\%)(\text{liters}) + (\%)(\text{liters}) = (\%)(\text{liters})$$

$$1(x) + 0.40(15 - x) = 0.60(15)$$

$$x + 6 - 0.40x = 9$$

$$0.60x = 3$$

$$x = 5$$

5 liters should be drained and replaced with pure antifreeze.

43. Let x represent the number of ounces of water to be evaporated; the amount of salt remains the same. Therefore, we get

$$0.04(32) = 0.06(32 - x)$$

$$1.28 = 1.92 - 0.06x$$

$$0.06x = 0.64$$

$$x = \frac{0.64}{0.06} = \frac{64}{6} = \frac{32}{3} = 10\frac{2}{3}$$

$10\frac{2}{3} \approx 10.67$ ounces of water need to be evaporated.

44. Let x represent the number of gallons of water to be evaporated; the amount of salt remains the same.

$$0.03(240) = 0.05(240 - x)$$

$$7.2 = 12 - 0.05x$$

$$0.05x = 4.8$$

$$x = \frac{4.8}{0.05} = 96$$

96 gallons of water need to be evaporated.

45. Let x represent the number of grams of pure gold. Then $60 - x$ represents the number of grams of 12 karat gold to be used.

$$x + \frac{1}{2}(60 - x) = \frac{2}{3}(60)$$

$$x + 30 - 0.5x = 40$$

$$0.5x = 10$$

$$x = 20$$

20 grams of pure gold should be mixed with 40 grams of 12 karat gold.

46. Let x represent the number of atoms of oxygen. $2x$ represents the number of atoms of hydrogen. $x + 1$ represents the number of atoms of carbon.

$$x + 2x + x + 1 = 45$$

$$4x = 44$$

$$x = 11$$

There are 11 atoms of oxygen and 22 atoms of hydrogen in the sugar molecule.

47. Let t represent the time it takes for Mike to catch up with Dan. Since the distances are the same, we have:

$$\frac{1}{6}t = \frac{1}{9}(t + 1)$$

$$3t = 2t + 2$$

$$t = 2$$

Mike will pass Dan after 2 minutes, which is a distance of $\frac{1}{3}$ mile.

48. Let t represent the time of flight with the wind. The distance is the same in each direction:

$$330t = 270(5 - t)$$

$$330t = 1350 - 270t$$

$$600t = 1350$$

$$t = 2.25$$

The distance the plane can fly and still return safely is $330(2.25) = 742.5$ miles.

49. Let t represent the time the auxiliary pump needs to run. Since the two pumps are emptying one tanker, we have:

$$\frac{3}{4} + \frac{t}{9} = 1$$

$$27 + 4t = 36$$

$$4t = 9$$

$$t = \frac{9}{4} = 2.25$$

The auxiliary pump must run for 2.25 hours. It must be started at 9:45 a.m.

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50. Let x represent the number of pounds of pure cement. Then $x + 20$ represents the number of pounds in the 40% mixture.

$$x + 0.25(20) = 0.40(x + 20)$$

$$x + 5 = 0.4x + 8$$

$$0.6x = 3$$

$$x = \frac{30}{6} = 5$$

5 pounds of pure cement should be added.

51. Let t represent the time for the tub to fill with the faucets on and the stopper removed. Since one tub is being filled, we have:

$$\frac{t}{15} + \left(-\frac{t}{20}\right) = 1$$

$$4t - 3t = 60$$

$$t = 60$$

60 minutes is required to fill the tub.

52. Let t be the time the 5 horsepower pump needs to run to finish emptying the pool. Since the two pumps are emptying one pool, we have:

$$\frac{t+2}{5} + \frac{2}{8} = 1$$

$$4(2+t) + 5 = 20$$

$$8 + 4t + 5 = 20$$

$$4t = 7$$

$$t = 1.75$$

The 5 horsepower pump must run for an additional 1.75 hours or 1 hour and 45 minutes to empty the pool.

53. Let t represent the time spent running. Then $5 - t$ represents the time spent biking.

	Rate	Time	Distance
Run	6	t	$6t$
Bike	25	$5 - t$	$25(5 - t)$

The total distance is 87 miles:

$$6t + 25(5 - t) = 87$$

$$6t + 125 - 25t = 87$$

$$-19t + 125 = 87$$

$$-19t = -38$$

$$t = 2$$

The time spent running is 2 hours, so the distance of the run is $6(2) = 12$ miles. The distance of the bicycle race is $25(5 - 2) = 75$ miles.

54. Let r represent the speed of the eastbound cyclist. Then $r + 5$ represents the speed of the westbound cyclist.

	Rate	Time	Distance
Eastbound	r	6	$6r$
Westbound	$r + 5$	6	$6(r + 5)$

The total distance is 246 miles:

$$6r + 6(r + 5) = 246$$

$$6r + 6r + 30 = 246$$

$$12r + 30 = 246$$

$$12r = 216$$

$$r = 18$$

The speed of the eastbound cyclist is 18 miles per hour, and the speed of the westbound cyclist is $18 + 5 = 23$ miles per hour.

55. Burke's rate is $\frac{100}{12}$ meters/sec. In 9.81 seconds,

Burke will run $\frac{100}{12}(9.81) = 81.75$ meters. Bolt would win by $100 - 81.75 = 18.25$ meters.

56. $A = 2\pi r^2 + 2\pi r h$. Since $A = 58.9\pi$ square inches and $h = 6.4$ inches,

$$2\pi r^2 + 2\pi r(6.4) = 58.9\pi$$

$$2\pi r^2 + 12.8\pi r - 58.9\pi = 0$$

$$2r^2 + 12.8r - 58.9 = 0$$

$$r = \frac{-12.8 \pm \sqrt{(12.8)^2 - 4(2)(-58.9)}}{2(2)}$$

$$= \frac{-12.8 \pm \sqrt{635.04}}{4}$$

$$r = 3.1 \text{ or } \cancel{r = -9.5}$$

The radius of the coffee can is 3.1 inches.

57. Let the individual times to complete the project be E for Elaine, B for Brian, and D for either daughter. Using the respective rates gives

$$\frac{1}{E} + \frac{1}{B} = \frac{1}{2}, \frac{1}{E} + \frac{1}{D} + \frac{1}{D} = \frac{1}{2} \text{ (or } \frac{1}{E} + \frac{2}{D} = \frac{1}{2}),$$

and $\frac{1}{B} + \frac{1}{D} = \frac{1}{4}$. From the first two equations,

$$\frac{1}{B} = \frac{2}{D}. \text{ Substituting into the third equation}$$

$$\text{gives } \frac{2}{D} + \frac{1}{D} = \frac{1}{4} \text{ or } \frac{3}{D} = \frac{1}{4} \rightarrow D = 12 \text{ hours.}$$

$$\text{Then } \frac{1}{E} + \frac{2}{12} = \frac{1}{2} \rightarrow E = 3 \text{ hours and}$$

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$\frac{1}{B} + \frac{1}{12} = \frac{1}{4} \rightarrow B = 6$ hours. The combined rate of Elaine, Brian, and one of their daughters is $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} = \frac{7}{12}$ project per hour, so it will take them $\frac{12}{7}$ hours to complete the project.

58. If x = liters of original solution, then there were originally $0.2x$ liters of salt and $0.8x$ liters of pure water. Over time, the solution loses $0.25(0.8x) = 0.2x$ liters of pure water. She adds 20 liters of salt so the total amount of salt is $0.2x + 20$ liters. She also adds 10 liters of pure water, so the total amount of pure water is $0.8x - 0.2x + 10 = 0.6x + 10$ liters. The resulting concentration is $33 \frac{1}{3}\%$ which means $\frac{0.2x + 20}{0.2x + 20 + 0.6x + 10} = \frac{1}{3}$ or $\frac{0.2x + 20}{0.8x + 30} = \frac{1}{3}$ or $0.6x + 60 = 0.8x + 30 \rightarrow x = 150$. There were initially 150 liters of solution in the vat.

59. The speed of the train relative to the man is $30 - 4 = 26$ miles per hour. The time is $5 \text{ sec} = \frac{5}{60} \text{ min} = \frac{5}{3600} \text{ h} = \frac{1}{720} \text{ h}$.

$$d = r t$$

$$= 26 \left(\frac{1}{720} \right)$$

$$= \frac{26}{720} \text{ miles}$$

$$= \frac{26}{720} \cdot 5280 \approx 190.67 \text{ feet}$$

The freight train is about 190.67 feet long.

60. Answers will vary.

61. Let x be the original selling price of the shirt.
Profit = Revenue - Cost
 $4 = x - 0.40x - 20 \rightarrow 24 = 0.60x \rightarrow x = 40$
The original price should be \$40 to ensure a profit of \$4 after the sale.

If the sale is 50% off, the profit is:

$$40 - 0.50(40) - 20 = 40 - 20 - 20 = 0$$

At 50% off there will be no profit.

62. Let t_1 and t_2 represent the times for the two segments of the trip. Since Atlanta is halfway between Chicago and Miami, the distances are equal.

$$45t_1 = 55t_2$$

$$t_1 = \frac{55}{45}t_2$$

$$t_1 = \frac{11}{9}t_2$$

Computing the average speed:

$$\text{Avg Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{45t_1 + 55t_2}{t_1 + t_2}$$

$$= \frac{45 \left(\frac{11}{9}t_2 \right) + 55t_2}{\frac{11}{9}t_2 + t_2} = \frac{55t_2 + 55t_2}{\left(\frac{11t_2 + 9t_2}{9} \right)}$$

$$= \frac{110t_2}{\left(\frac{20t_2}{9} \right)} = \frac{990t_2}{20t_2}$$

$$= \frac{99}{2} = 49.5 \text{ miles per hour}$$

The average speed for the trip from Chicago to Miami is 49.5 miles per hour.

63. The time traveled with the tail wind was:

$$t = \frac{919}{550} \approx 1.67091 \text{ hours}$$

Since they were 20 minutes $\left(\frac{1}{3} \text{ hour} \right)$ early, the time in still air would have been:

$$1.67091 \text{ hrs} + 20 \text{ min} = (1.67091 + 0.33333) \text{ hrs} \approx 2.00424 \text{ hrs}$$

Thus, with no wind, the ground speed is

$$\frac{919}{2.00424} \approx 458.53. \text{ Therefore, the tail wind is}$$

$$550 - 458.53 = 91.47 \text{ knots}$$

64. It is impossible to mix two solutions with a lower concentration and end up with a new solution with a higher concentration.

Algebraic Solution:

Let x = the number of liters of 25% solution.

$$(\%)(\text{liters}) + (\%)(\text{liters}) = (\%)(\text{liters})$$

$$0.25x + 0.48(20) = 0.58(20 + x)$$

$$0.25x + 9.6 = 10.6 + 0.58x$$

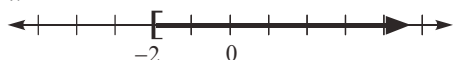
$$-0.33x = 1$$

$$x \approx -3.03 \text{ liters}$$

(not possible)

Section 1.7

1. $x \geq -2$



2. True; the absolute value of a number is the distance the number lies from 0 on a real number line. Negative numbers lie to the left of 0 on a real number line, but still have a positive distance from 0.

3. $A \cup B = \{a, b, c, d, e, i, o, u\}$

4. negative

5. closed interval

6. $-a \leq u \leq a$

7. $(-\infty, a]$

8. True; this follows from the addition property for inequalities.

9. True; this follows from the multiplication property for inequalities.

10. False; since both sides of the inequality are being divided by a negative number, the sense, or direction, of the inequality must be reversed.

That is, $\frac{a}{c} > \frac{b}{c}$.

11. a

12. c

13. Interval: $[0, 2]$

Inequality: $0 \leq x \leq 2$

14. Interval: $(-1, 2)$

Inequality: $-1 < x < 2$

15. Interval: $[2, \infty)$

Inequality: $x \geq 2$

16. Interval: $(-\infty, 0]$

Inequality: $x \leq 0$

17. Interval: $[0, 3)$

Inequality: $0 \leq x < 3$

18. Interval: $(-1, 1]$

Inequality: $-1 < x \leq 1$

19. a. $3 < 5$

$$3 + 3 < 5 + 3$$

$$6 < 8$$

b. $3 < 5$

$$3 - 5 < 5 - 5$$

$$-2 < 0$$

c. $3 < 5$

$$3(3) < 3(5)$$

$$9 < 15$$

d. $3 < 5$

$$-2(3) > -2(5)$$

$$-6 > -10$$

20. a. $2 > 1$

$$2 + 3 > 1 + 3$$

$$5 > 4$$

b. $2 > 1$

$$2 - 5 > 1 - 5$$

$$-3 > -4$$

c. $2 > 1$

$$3(2) > 3(1)$$

$$6 > 3$$

Chapter 1: Graphs, Equations, and Inequalities

d. $2 > 1$

$$-2(2) < -2(1)$$

$$-4 < -2$$

21. a. $2x + 1 < 2$

$$2x + 1 + 3 < 2 + 3$$

$$2x + 4 < 5$$

b. $2x + 1 < 2$

$$2x + 1 - 5 < 2 - 5$$

$$2x - 4 < -3$$

c. $2x + 1 < 2$

$$3(2x + 1) < 3(2)$$

$$6x + 3 < 6$$

d. $2x + 1 < 2$

$$-2(2x + 1) > -2(2)$$

$$-4x - 2 > -4$$

22. a. $1 - 2x > 5$

$$1 - 2x + 3 > 5 + 3$$

$$4 - 2x > 8$$

b. $1 - 2x > 5$

$$1 - 2x - 5 > 5 - 5$$

$$-2x - 4 > 0$$

c. $1 - 2x > 5$

$$3(1 - 2x) > 3(5)$$

$$3 - 6x > 15$$

d. $1 - 2x > 5$

$$-2(1 - 2x) < -2(5)$$

$$-2 + 4x < -10$$

23. $[0, 4]$



24. $(-1, 5)$



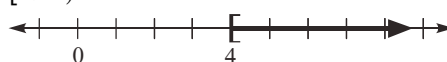
25. $[4, 6)$



26. $(-2, 0]$



27. $[4, \infty)$



28. $(-\infty, 5]$



29. $(-\infty, -4)$



30. $(1, \infty)$



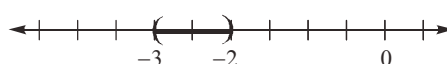
31. $2 \leq x \leq 5$



32. $1 < x < 2$



33. $-3 < x < -2$



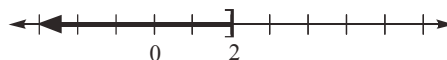
34. $0 \leq x < 1$



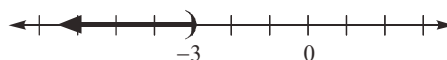
35. $x \geq 4$



36. $x \leq 2$



37. $x < -3$



38. $x > -8$



39. If $x < 5$, then $x - 5 < 0$.

40. If $x < -4$, then $x + 4 < 0$.

41. If $x > -4$, then $x + 4 > 0$.

42. If $x > 6$, then $x - 6 > 0$.

43. If $x \geq -4$, then $3x \geq -12$.

44. If $x \leq 3$, then $2x \leq 6$.

45. If $x > 6$, then $-2x < -12$.

46. If $x > -2$, then $-4x < 8$.

47. If $8x < 40$, then $x < 5$.

48. If $3x \leq 12$, then $x \leq 4$.

49. If $-\frac{1}{2}x \leq 3$, then $x \geq -6$.

50. If $-\frac{1}{4}x > 1$, then $x < -4$.

51. $3x - 7 > 2$

$3x > 9$

$x > 3$

$\{x \mid x > 3\}$ or $(3, \infty)$



52. $2x + 5 > 1$

$2x > -4$

$x > -2$

$\{x \mid x > -2\}$ or $(-2, \infty)$

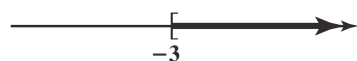


53. $3 - 5x \leq 18$

$-5x \leq 15$

$x \geq -3$

$\{x \mid x \geq -3\}$ or $[-3, \infty)$

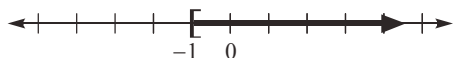


54. $2 - 3x \leq 5$

$-3x \leq 3$

$x \geq -1$

$\{x \mid x \geq -1\}$ or $[-1, \infty)$

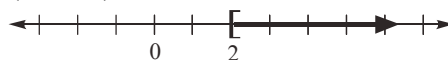


55. $3x - 1 \geq 3 + x$

$2x \geq 4$

$x \geq 2$

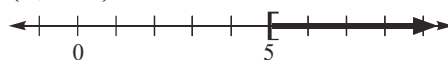
$\{x \mid x \geq 2\}$ or $[2, \infty)$



56. $2x - 2 \geq 3 + x$

$x \geq 5$

$\{x \mid x \geq 5\}$ or $[5, \infty)$



57. $-2(x + 3) < 8$

$-2x - 6 < 8$

$-2x < 14$

$x > -7$

$\{x \mid x > -7\}$ or $(-7, \infty)$



58. $-3(1 - x) < 12$

$-3 + 3x < 12$

$3x < 15$

$x < 5$

$\{x \mid x < 5\}$ or $(-\infty, 5)$



59. $4 - 3(1 - x) \leq 3$

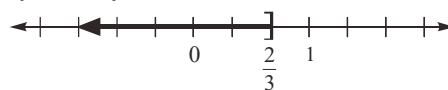
$4 - 3 + 3x \leq 3$

$3x + 1 \leq 3$

$3x \leq 2$

$x \leq \frac{2}{3}$

$\{x \mid x \leq \frac{2}{3}\}$ or $(-\infty, \frac{2}{3}]$



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60. $8 - 4(2 - x) \leq -2x$

$$8 - 8 + 4x \leq -2x$$

$$4x \leq -2x$$

$$6x \leq 0$$

$$x \leq 0$$

$$\{x \mid x \leq 0\} \text{ or } (-\infty, 0]$$



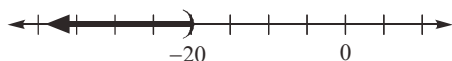
61. $\frac{1}{2}(x - 4) > x + 8$

$$\frac{1}{2}x - 2 > x + 8$$

$$-\frac{1}{2}x > 10$$

$$x < -20$$

$$\{x \mid x < -20\} \text{ or } (-\infty, -20)$$



62. $3x + 4 > \frac{1}{3}(x - 2)$

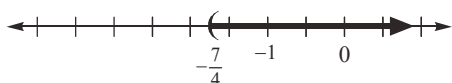
$$3x + 4 > \frac{1}{3}x - \frac{2}{3}$$

$$9x + 12 > x - 2$$

$$8x > -14$$

$$x > -\frac{7}{4}$$

$$\left\{x \mid x > -\frac{7}{4}\right\} \text{ or } \left(-\frac{7}{4}, \infty\right)$$



63. $-1 \leq 3x - 7 \leq 5$

$$6 \leq 3x \leq 12$$

$$2 \leq x \leq 4$$

$$\{x \mid 2 \leq x \leq 4\} \text{ or } [2, 4]$$



64. $4 \leq 2x + 2 \leq 10$

$$2 \leq 2x \leq 8$$

$$1 \leq x \leq 4$$

$$\{x \mid 1 \leq x \leq 4\} \text{ or } [1, 4]$$

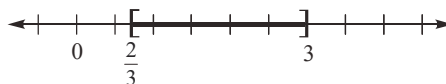


65. $-5 \leq 4 - 3x \leq 2$

$$-9 \leq -3x \leq -2$$

$$3 \geq x \geq \frac{2}{3}$$

$$\left\{x \mid \frac{2}{3} \leq x \leq 3\right\} \text{ or } \left[\frac{2}{3}, 3\right]$$



66. $-3 \leq 3 - 2x \leq 9$

$$-6 \leq -2x \leq 6$$

$$3 \geq x \geq -3$$

$$\{x \mid -3 \leq x \leq 3\} \text{ or } [-3, 3]$$



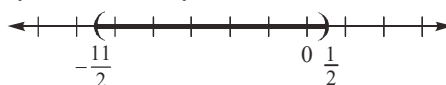
67. $-3 < \frac{2x - 1}{4} < 0$

$$-12 < 2x - 1 < 0$$

$$-11 < 2x < 1$$

$$-\frac{11}{2} < x < \frac{1}{2}$$

$$\left\{x \mid -\frac{11}{2} < x < \frac{1}{2}\right\} \text{ or } \left(-\frac{11}{2}, \frac{1}{2}\right)$$



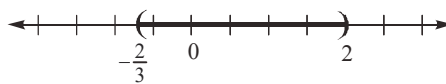
68. $0 < \frac{3x + 2}{2} < 4$

$$0 < 3x + 2 < 8$$

$$-2 < 3x < 6$$

$$-\frac{2}{3} < x < 2$$

$$\left\{x \mid -\frac{2}{3} < x < 2\right\} \text{ or } \left(-\frac{2}{3}, 2\right)$$



69. $1 < 1 - \frac{1}{2}x < 4$

$$0 < -\frac{1}{2}x < 3$$

$$0 > x > -6 \text{ or } -6 < x < 0$$

$$\{x \mid -6 < x < 0\} \text{ or } (-6, 0)$$



70. $0 < 1 - \frac{1}{3}x < 1$

$$-1 < -\frac{1}{3}x < 0$$

$$3 > x > 0 \quad \text{or} \quad 0 < x < 3$$

$$\{x \mid 0 < x < 3\} \text{ or } (0, 3)$$



71. $(x+2)(x-3) > (x-1)(x+1)$

$$x^2 - x - 6 > x^2 - 1$$

$$-x - 6 > -1$$

$$-x > 5$$

$$x < -5$$

$$\{x \mid x < -5\} \text{ or } (-\infty, -5)$$



72. $(x-1)(x+1) > (x-3)(x+4)$

$$x^2 - 1 > x^2 + x - 12$$

$$-1 > x - 12$$

$$-x > -11$$

$$x < 11$$

$$\{x \mid x < 11\} \text{ or } (-\infty, 11)$$

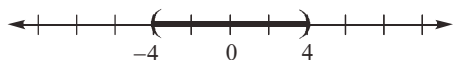


73. $|2x| < 8$

$$-8 < 2x < 8$$

$$-4 < x < 4$$

$$\{x \mid -4 < x < 4\} \text{ or } (-4, 4)$$

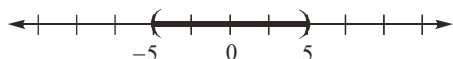


74. $|3x| < 15$

$$-15 < 3x < 15$$

$$-5 < x < 5$$

$$\{x \mid -5 < x < 5\} \text{ or } (-5, 5)$$



75. $|7x| > 42$

$$7x < -42 \quad \text{or} \quad 7x > 42$$

$$x < -6 \quad \text{or} \quad x > 6$$

$$\{x \mid x < -6 \text{ or } x > 6\} \text{ or } (-\infty, -6) \cup (6, \infty)$$

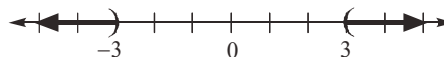


76. $|2x| > 6$

$$2x < -6 \quad \text{or} \quad 2x > 6$$

$$x < -3 \quad \text{or} \quad x > 3$$

$$\{x \mid x < -3 \text{ or } x > 3\} \text{ or } (-\infty, -3) \cup (3, \infty)$$



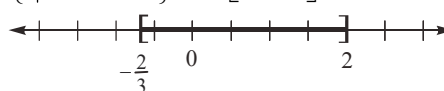
77. $|3t - 2| \leq 4$

$$-4 \leq 3t - 2 \leq 4$$

$$-2 \leq 3t \leq 6$$

$$-\frac{2}{3} \leq t \leq 2$$

$$\left\{t \mid -\frac{2}{3} \leq t \leq 2\right\} \text{ or } \left[-\frac{2}{3}, 2\right]$$



78. $|2u + 5| \leq 7$

$$-7 \leq 2u + 5 \leq 7$$

$$-12 \leq 2u \leq 2$$

$$-6 \leq u \leq 1$$

$$\{u \mid -6 \leq u \leq 1\} \text{ or } [-6, 1]$$



79. $|x - 2| + 2 < 3$

$$|x - 2| < 1$$

$$-1 < x - 2 < 1$$

$$1 < x < 3$$

$$\{x \mid 1 < x < 3\} \text{ or } (1, 3)$$



80. $|x + 4| + 3 < 5$

$$|x + 4| < 2$$

$$-2 < x + 4 < 2$$

$$-6 < x < -2$$

$$\{x \mid -6 < x < -2\} \text{ or } (-6, -2)$$



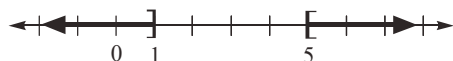
Chapter 1: Graphs, Equations, and Inequalities

81. $|x-3| \geq 2$

$$x-3 \leq -2 \text{ or } x-3 \geq 2$$

$$x \leq 1 \text{ or } x \geq 5$$

$$\{x | x \leq 1 \text{ or } x \geq 5\} \text{ or } (-\infty, 1] \cup [5, \infty)$$

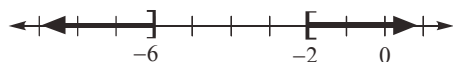


82. $|x+4| \geq 2$

$$x+4 \leq -2 \text{ or } x+4 \geq 2$$

$$x \leq -6 \text{ or } x \geq -2$$

$$\{x | x \leq -6 \text{ or } x \geq -2\} \text{ or } (-\infty, -6] \cup [-2, \infty)$$



83. $|5-2x| > |-7|$

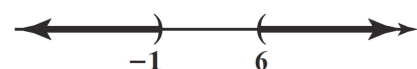
$$|5-2x| > 7$$

$$5-2x < -7 \text{ or } 5-2x > 7$$

$$-2x < -12 \text{ or } -2x > 2$$

$$x > 6 \text{ or } x < -1$$

$$\{x | x < -1 \text{ or } x > 6\} \text{ or } (-\infty, -1) \cup (6, \infty)$$



84. $|2-3x| > |-1|$

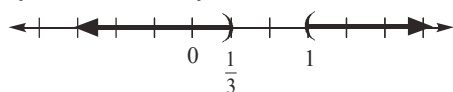
$$|2-3x| > 1$$

$$2-3x < -1 \text{ or } 2-3x > 1$$

$$-3x < -3 \text{ or } -3x > -1$$

$$x > 1 \text{ or } x < \frac{1}{3}$$

$$\{x | x < \frac{1}{3} \text{ or } x > 1\} \text{ or } (-\infty, \frac{1}{3}) \cup (1, \infty)$$



85. $|1-4x| - 7 < -2$

$$|1-4x| < 5$$

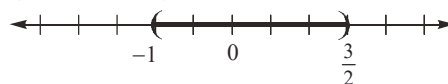
$$-5 < 1-4x < 5$$

$$-6 < -4x < 4$$

$$-\frac{6}{-4} > x > \frac{4}{-4}$$

$$\frac{3}{2} > x > -1 \text{ or } -1 < x < \frac{3}{2}$$

$$\{x | -1 < x < \frac{3}{2}\} \text{ or } (-1, \frac{3}{2})$$



86. $|1-2x| - 4 < -1$

$$|1-2x| < 3$$

$$-3 < 1-2x < 3$$

$$-4 < -2x < 2$$

$$-\frac{4}{-2} > x > \frac{2}{-2}$$

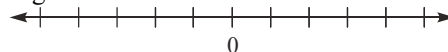
$$2 > x > -1 \text{ or } -1 < x < 2$$

$$\{x | -1 < x < 2\} \text{ or } (-1, 2)$$



87. $|9x+5| < -2$

No solution since absolute value is always non-negative.



88. $|3x-4| \geq 0$

All real numbers since absolute value is always non-negative.

$$\{x | -\infty < x < \infty\} \text{ or } (-\infty, \infty)$$



89. $3-4x < 11$

$$-4x < 8$$

$$x > -2$$

$$\{x | x > -2\} \text{ or } (-2, \infty)$$

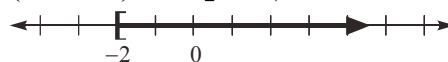


90. $1-3x \leq 7$

$$-3x \leq 6$$

$$x \geq -2$$

$$\{x | x \geq -2\} \text{ or } [-2, \infty)$$



91. $|2x+1|-5 \geq -1$

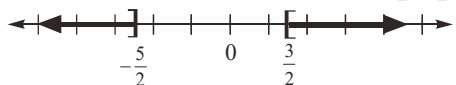
$|2x+1| \geq 4$

$2x+1 \leq -4 \quad \text{or} \quad 2x+1 \geq 4$

$2x \leq -5 \quad 2x \geq 3$

$x \leq -\frac{5}{2} \quad x \geq \frac{3}{2}$

$\left\{x \mid x \leq -\frac{5}{2} \text{ or } x \geq \frac{3}{2}\right\} \text{ or } \left(-\infty, -\frac{5}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$



92. $|5x+2|-3 > 9$

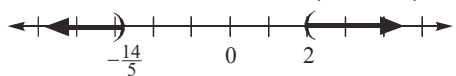
$|5x+2| > 12$

$5x+2 < -12 \quad \text{or} \quad 5x+2 > 12$

$5x < -14 \quad 5x > 10$

$x < -\frac{14}{5} \quad x > 2$

$\left\{x \mid x < -\frac{14}{5} \text{ or } x > 2\right\} \text{ or } \left(-\infty, -\frac{14}{5}\right) \cup (2, \infty)$



93. $\frac{x}{2} \geq 1 - \frac{x}{4}$

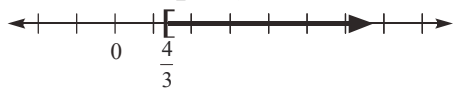
$4 \cdot \frac{x}{2} \geq 4 \cdot \left(1 - \frac{x}{4}\right)$

$2x \geq 4 - x$

$3x \geq 4$

$x \geq \frac{4}{3}$

$\left\{x \mid x \geq \frac{4}{3}\right\} \text{ or } \left[\frac{4}{3}, \infty\right)$



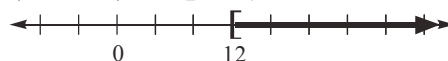
94. $\frac{x}{3} \geq 2 + \frac{x}{6}$

$6 \cdot \frac{x}{3} \geq 6 \left(2 + \frac{x}{6}\right)$

$2x \geq 12 + x$

$x \geq 12$

$\{x \mid x \geq 12\} \text{ or } [12, \infty)$



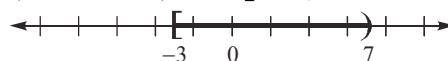
95. $-\frac{1}{3} \leq \frac{x+1}{6} < \frac{4}{3}$

$6 \left(-\frac{1}{3}\right) \leq 6 \cdot \frac{x+1}{6} < 6 \cdot \frac{4}{3}$

$-2 \leq x+1 < 8$

$-3 \leq x < 7$

$\{x \mid -3 \leq x < 7\} \text{ or } [-3, 7)$



96. $-\frac{3}{2} < \frac{x-3}{4} \leq \frac{5}{4}$

$4 \left(-\frac{3}{2}\right) < 4 \cdot \frac{x-3}{4} \leq 4 \cdot \frac{5}{4}$

$-6 < x-3 \leq 5$

$-3 < x \leq 8$

$\{x \mid -3 < x \leq 8\} \text{ or } (-3, 8]$



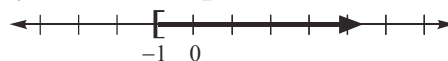
97. $x(4x+3) \leq (2x+1)^2$

$4x^2 + 3x \leq 4x^2 + 4x + 1$

$-x \leq 1$

$x \geq -1$

$\{x \mid x \geq -1\} \text{ or } [-1, \infty)$



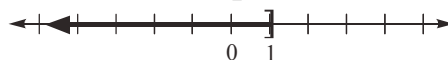
98. $x(9x-5) \leq (3x-1)^2$

$9x^2 - 5x \leq 9x^2 - 6x + 1$

$-5x \leq -6x + 1$

$x \leq 1$

$\{x \mid x \leq 1\} \text{ or } (-\infty, 1]$



Chapter 1: Graphs, Equations, and Inequalities

$$99. \quad |(3x-2)-7| < \frac{1}{2}$$

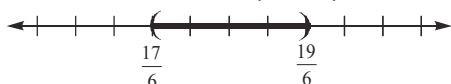
$$|3x-9| < \frac{1}{2}$$

$$-\frac{1}{2} < 3x-9 < \frac{1}{2}$$

$$\frac{17}{2} < 3x < \frac{19}{2}$$

$$\frac{17}{6} < x < \frac{19}{6}$$

$$\left\{x \mid \frac{17}{6} < x < \frac{19}{6}\right\} \quad \text{or} \quad \left(\frac{17}{6}, \frac{19}{6}\right)$$



$$100. \quad |(4x-1)-11| < \frac{1}{4}$$

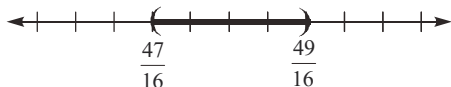
$$|4x-12| < \frac{1}{4}$$

$$-\frac{1}{4} < 4x-12 < \frac{1}{4}$$

$$\frac{47}{4} < 4x < \frac{49}{4}$$

$$\frac{47}{16} < x < \frac{49}{16}$$

$$\left\{x \mid \frac{47}{16} < x < \frac{49}{16}\right\} \quad \text{or} \quad \left(\frac{47}{16}, \frac{49}{16}\right)$$



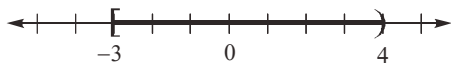
$$101. \quad -3 < 5-2x \leq 11$$

$$-8 < -2x \leq 6$$

$$4 > x \geq -3$$

$$-3 \leq x < 4$$

$$\{x \mid -3 \leq x < 4\} \quad \text{or} \quad [-3, 4)$$



$$102. \quad 2 \leq 3-2(x+1) < 8$$

$$2 \leq 3-2x-2 < 8$$

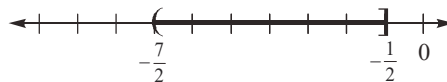
$$2 \leq 1-2x < 8$$

$$1 \leq -2x < 7$$

$$-\frac{1}{2} \geq x > -\frac{7}{2}$$

$$-\frac{7}{2} < x \leq -\frac{1}{2}$$

$$\left\{x \mid -\frac{7}{2} < x \leq -\frac{1}{2}\right\} \quad \text{or} \quad \left(-\frac{7}{2}, -\frac{1}{2}\right]$$



$$103. \quad 7-|x-1| > 4$$

$$-|x-1| > -3$$

$$|x-1| < 3$$

$$-3 < x-1 < 3$$

$$-2 < x < 4$$

$$\{x \mid -2 < x < 4\} \quad \text{or} \quad (-2, 4)$$



$$104. \quad 9-|x+3| \geq 5$$

$$-|x+3| \geq -4$$

$$|x+3| \leq 4$$

$$-4 \leq x+3 \leq 4$$

$$-7 \leq x \leq 1$$

$$\{x \mid -7 \leq x \leq 1\} \quad \text{or} \quad [-7, 1]$$



$$105. \quad -3 < x+5 < 2x$$

$$-3 < x+5 \quad \text{and} \quad x+5 < 2x$$

$$-8 < x$$

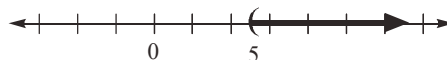
$$5 < x$$

$$x > -8$$

$$x > 5$$

We need both $x > -8$ and $x > 5$. Therefore, $x > 5$ is sufficient to satisfy both inequalities.

$$\{x \mid x > 5\} \quad \text{or} \quad (5, \infty)$$



106. $2 < x - 3 < 2x$

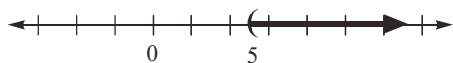
$2 < x - 3 \text{ and } x - 3 < 2x$

$5 < x \quad -3 < x$

$x > 5 \quad x > -3$

We need both $x > 5$ and $x > -3$. Therefore,
 $x > 5$ is sufficient to satisfy both inequalities.

$\{x \mid x > 5\} \text{ or } (5, \infty)$



107. $x + 2 < 2x - 1 < 5x$

$x + 2 < 2x - 1 \text{ and } 2x - 1 < 5x$

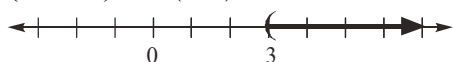
$2 < x - 1 \quad -3x - 1 < 0$

$3 < x \quad -3x < 1$

$x > 3 \quad x > -\frac{1}{3}$

We need both $x > -\frac{1}{3}$ and $x > 3$. Therefore,
 $x > 3$ is sufficient to satisfy both inequalities.

$\{x \mid x > 3\} \text{ or } (3, \infty)$



108. $2x - 1 < 3x + 5 < 5x - 7$

$2x - 1 < 3x + 5 \text{ and } 3x + 5 < 5x - 7$

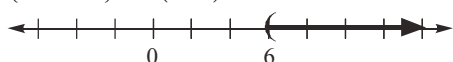
$-x - 1 < 5 \quad -2x + 5 < -7$

$-x < 6 \quad -2x < -12$

$x > -6 \quad x > 6$

We need both $x > -6$ and $x > 6$. Therefore,
 $x > 6$ is sufficient to satisfy both inequalities.

$\{x \mid x > 6\} \text{ or } (6, \infty)$



109. $|x - 2| < 0.5$

$-0.5 < x - 2 < 0.5$

$-0.5 + 2 < x < 0.5 + 2$

$1.5 < x < 2.5$

Solution set: $\{x \mid 1.5 < x < 2.5\}$

110. $|x - (-1)| < 1$

$-1 < x - (-1) < 1$

$-1 < x + 1 < 1$

$-1 - 1 < x < 1 - 1$

$-2 < x < 0$

Solution set: $\{x \mid -2 < x < 0\}$

111. $|x - (-3)| > 2$

$x - (-3) < -2 \text{ or } x - (-3) > 2$

$x + 3 < -2 \text{ or } x + 3 > 2$

$x < -5 \text{ or } x > -1$

Solution set: $\{x \mid x < -5 \text{ or } x > -1\}$

112. $|x - 2| > 3$

$x - 2 < -3 \text{ or } x - 2 > 3$

$x < -1 \text{ or } x > 5$

Solution set: $\{x \mid x < -1 \text{ or } x > 5\}$

113. $21 < \text{young adult's age} < 30$

114. $40 \leq \text{middle-aged} < 60$

115. A temperature x that differs from 98.6°F by at least 1.5° .

$|x - 98.6^\circ| \geq 1.5^\circ$

$x - 98.6^\circ \leq -1.5^\circ \text{ or } x - 98.6^\circ \geq 1.5^\circ$

$x \leq 97.1^\circ \text{ or } x \geq 100.1^\circ$

The temperatures that are considered unhealthy are those that are less than 97.1°F or greater than 100.1°F , inclusive.

116. The length L must be within 0.0025 of 5.375 inches..

$|L - 5.375| \leq 0.0025$

$-0.0025 \leq L - 5.375 \leq 0.0025$

$5.3725 \leq L \leq 5.3775$

The lengths must be between 5.3725 and 5.3775 inches, inclusive.

117. The percentage must be within 3.9 percentage points of 64 percent. The inequality that represents this would be:

$|x - 64| \leq 3.9$

$-3.9 \leq x - 64 \leq 3.9$

$60.1 \leq x \leq 67.9$

The actual percentage is likely to fall between 60.1% and 67.9% inclusive.

118. The speed x varies from 707 mph by up to 55 mph.

a. $|x - 707| \leq 55$

Chapter 1: Graphs, Equations, and Inequalities

- b. $-55 \leq x - 707 \leq 55$
 $-55 \leq x - 707 \leq 55$
 $652 \leq x \leq 762$
 The speed of sound is between 652 and 762 miles per hour, depending on conditions.
119. a. Let x = age at death.
 $x - 30 \geq 51.7$
 $x \geq 81.7$
 Therefore, the average life expectancy for a 30-year-old male in 2019 will be greater than or equal to 81.7 years.
- b. Let x = age at death.
 $x - 30 \geq 55.5$
 $x \geq 85.5$
 Therefore, the average life expectancy for a 30-year-old female in 2019 will be greater than or equal to 85.5 years.
- c. By the given information, a female can expect to live $85.5 - 81.7 = 3.8$ years longer.
120. $V = 20T$
 $80^\circ \leq T \leq 120^\circ$
 $80^\circ \leq \frac{V}{20} \leq 120^\circ$
 $1600 \leq V \leq 2400$
 The volume ranges from 1600 to 2400 cubic centimeters, inclusive.
121. Let P represent the selling price and C represent the commission.
 Calculating the commission:
 $C = 45,000 + 0.25(P - 900,000)$
 $= 45,000 + 0.25P - 225,000$
 $= 0.25P - 180,000$
 Calculate the commission range, given the price range:
 $900,000 \leq P \leq 1,100,000$
 $0.25(900,000) \leq 0.25P \leq 0.25(1,100,000)$
 $225,000 \leq 0.25P \leq 275,000$
 $225,000 - 180,000 \leq 0.25P - 180,000 \leq 275,000 - 180,000$
 $45,000 \leq C \leq 95,000$
 The agent's commission ranges from \$45,000 to \$95,000, inclusive.
 $\frac{45,000}{900,000} = 0.05 = 5\%$ to $\frac{95,000}{1,100,000} = 0.086 = 8.6\%$, inclusive.
 As a percent of selling price, the commission ranges from 5% to 8.6%, inclusive.
122. Let C represent the commission.
 Calculate the commission range:
 $25 + 0.4(200) \leq C \leq 25 + 0.4(3000)$
 $105 \leq C \leq 1225$
 The commissions are at least \$105 and at most \$1225.
123. Let W = weekly wages and T = tax withheld.
 Calculating the withholding tax range, given the range of weekly wages:
 $900 \leq W \leq 1100$
 $900 - 832 \leq W - 832 \leq 1100 - 832$
 $68 \leq W - 832 \leq 268$
 $0.22(68) \leq 0.22(W - 832) \leq 0.22(268)$
 $14.96 \leq 0.22(W - 832) \leq 58.96$
 $14.96 + 87.34 \leq 0.22(W - 832) + 87.34 \leq 58.96 + 87.34$
 $102.3 \leq T \leq 146.3$
 The amount withheld varies from \$102.30 to \$146.30, inclusive.
124. Let x represent the length of time Sue should exercise on the seventh day.
 $200 \leq 40 + 45 + 0 + 50 + 25 + 35 + x \leq 300$
 $200 \leq 195 + x \leq 300$
 $5 \leq x \leq 105$
 Sue will stay within the ACSM guidelines by exercising from 5 to 105 minutes.
125. Let K represent the monthly usage in kilowatt-hours and let C represent the monthly customer bill.
 Calculating the bill: $C = 0.0936K + 30$
 Calculating the range of kilowatt-hours, given the range of bills:
 $137.64 \leq C \leq 221.88$
 $137.64 \leq 30 + 0.0936W \leq 221.88$
 $107.64 \leq 0.0936K \leq 191.88$
 $1150 \leq K \leq 2050$
 The usage varies from 1150.00 kilowatt-hours to 2050.00 kilowatt-hours, inclusive.
126. Let W represent the amount of sewer/water used (in thousands of gallons). Let C represent the customer charge (in dollars).
 Calculating the charge:
 $C = 23.55 + 0.40W$
 Calculating the range of water usage, given the range of charges:

$$30.35 \leq C \leq 36.75$$

$$30.35 \leq 23.55 + 0.40W \leq 36.75$$

$$6.8 \leq 0.40K \leq 13.2$$

$$17 \leq K \leq 33$$

The range of water usage ranged from 17,000 to 33,000 gallons.

- 127.** You have already consumed 21 grams of fat. Let C represent the number of cookies. Then we have the following equation:

$$21 + 5C \leq 46$$

$$5C \leq 25$$

$$C \leq 5$$

You may eat up to 5 cookies and keep the total fat content of your meal not more than 46g.

- 128.** You have already consumed 150 grams of sodium. Let H represent the number of hamburgers. Then we have the following equation:

$$150 + 380H \leq 1290$$

$$380C \leq 1140$$

$$C \leq 3$$

You may eat up to 3 hamburgers and keep the total sodium content of your meal not more than 1290g.

- 129. a.** Let T represent the score on the last test and G represent the course grade. Calculating the course grade and solving for the last test:

$$G = \frac{68 + 82 + 87 + 89 + T}{5}$$

$$G = \frac{326 + T}{5}$$

$$5G = 326 + T$$

$$T = 5G - 326$$

Calculating the range of scores on the last test, given the grade range:

$$80 \leq G < 90$$

$$400 \leq 5G < 450$$

$$74 \leq 5G - 326 < 124$$

$$74 \leq T < 124$$

To get a grade of B, you need at least a 74 on the fifth test.

- b.** Let T represent the score on the last test and G represent the course grade. Calculating the course grade and solving for the last test:

$$G = \frac{68 + 82 + 87 + 89 + 2T}{6}$$

$$G = \frac{326 + 2T}{6}$$

$$G = \frac{163 + T}{3}$$

$$T = 3G - 163$$

Calculating the range of scores on the last test, given the grade range:

$$80 \leq G < 90$$

$$240 \leq 3G < 270$$

$$77 \leq 3G - 163 < 107$$

$$77 \leq T < 107$$

To get a grade of B, you need at least a 77 on the fifth test.

- 130.** Let T represent the test scores of the people in the top 2.5%.

$$T > 1.96(12) + 100 = 123.52$$

People in the top 2.5% will have test scores greater than 123.52. That is, $T > 123.52$ or $(123.52, \infty)$.

- 131.** Since $a < b$,

$$\frac{a}{2} < \frac{b}{2} \quad \text{and} \quad \frac{a}{2} < \frac{b}{2}$$

$$\frac{a}{2} + \frac{a}{2} < \frac{a}{2} + \frac{b}{2} \quad \text{and} \quad \frac{a}{2} + \frac{b}{2} < \frac{b}{2} + \frac{b}{2}$$

$$a < \frac{a+b}{2} \quad \text{and} \quad \frac{a+b}{2} < b$$

$$\text{So, } a < \frac{a+b}{2} < b.$$

- 132.** From problem 131, $a < \frac{a+b}{2} < b$, so

$$d\left(a, \frac{a+b}{2}\right) = \frac{a+b}{2} - a = \frac{a+b-2a}{2} = \frac{b-a}{2} \quad \text{and}$$

$$d\left(b, \frac{a+b}{2}\right) = b - \frac{a+b}{2} = \frac{2b-a-b}{2} = \frac{b-a}{2}.$$

Therefore, $\frac{a+b}{2}$ is equidistant from a and b .

- 133.** If $0 < a < b$, then

$$ab > a^2 > 0 \quad \text{and} \quad b^2 > ab > 0$$

$$(\sqrt{ab})^2 > a^2 \quad \text{and} \quad b^2 > (\sqrt{ab})^2$$

$$\sqrt{ab} > a \quad \text{and} \quad b > \sqrt{ab}$$

Therefore, $a < \sqrt{ab} < b$.

Chapter 1: Graphs, Equations, and Inequalities

134. Show that $\sqrt{ab} < \frac{a+b}{2}$.

$$\begin{aligned}\frac{a+b}{2} - \sqrt{ab} &= \frac{1}{2}(a - 2\sqrt{ab} + b) \\ &= \frac{1}{2}(\sqrt{a} - \sqrt{b})^2 > 0, \text{ since } a \neq b.\end{aligned}$$

135. For $0 < a < b$, $\frac{1}{h} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$

$$h \cdot \frac{1}{h} = \frac{1}{2}\left(\frac{b+a}{ab}\right) \cdot h$$

$$1 = \frac{1}{2}\left(\frac{b+a}{ab}\right) \cdot h$$

$$h = \frac{2ab}{a+b}$$

$$\begin{aligned}h - a &= \frac{2ab}{a+b} - a = \frac{2ab - a(a+b)}{a+b} \\ &= \frac{2ab - a^2 - ab}{a+b} = \frac{ab - a^2}{a+b} \\ &= \frac{a(b-a)}{a+b} > 0\end{aligned}$$

Therefore, $h > a$.

$$\begin{aligned}b - h &= b - \frac{2ab}{a+b} = \frac{b(a+b) - 2ab}{a+b} \\ &= \frac{ab + b^2 - 2ab}{a+b} = \frac{b^2 - ab}{a+b} \\ &= \frac{b(b-a)}{a+b} > 0\end{aligned}$$

Therefore, $h < b$, and we have $a < h < b$.

136. Show that $h = \frac{(\text{geometric mean})^2}{\text{arithmetic mean}} = \frac{(\sqrt{ab})^2}{\left(\frac{1}{2}(a+b)\right)}$

$$\frac{1}{h} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$\frac{2}{h} = \frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab}$$

$$\frac{h}{2} = \frac{ab}{a+b}$$

$$h = 2 \cdot \frac{ab}{a+b} = \frac{(\sqrt{ab})^2}{\left(\frac{1}{2}(a+b)\right)}$$

Therefore, $\sqrt{ab} < \frac{a+b}{2}$.

137. $x - 4 < 2x - 3 \leq \frac{x+5}{3}$

$$3x - 12 < 6x - 9 \leq x + 5$$

$$3x - 12 < 6x - 9 \quad \text{and} \quad 6x - 9 \leq x + 5$$

$$-3 < 3x \qquad 5x \leq 14$$

$$-1 < x \qquad x \leq \frac{14}{5}$$

This is equivalent to $-1 < x \leq \frac{14}{5}$. The solution

set, in interval notation, is $\left(-1, \frac{14}{5}\right]$.

138. The largest value of $2x^2 - 3$ occurs at the largest value for $|x|$.

$$2 \leq 5 - x \leq 9$$

$$-3 \leq -x \leq 4$$

$$-4 \leq x \leq 3$$

$$3 \geq x \geq -4 \text{ or } -4 \leq x \leq 3$$

The largest value for $2x^2 - 3$ is

$$2(-4)^2 - 3 = 32 - 3 = 29.$$

139. Answers will vary. One possibility:

$$\text{No solution: } 4x + 6 \leq 2(x - 5) + 2x$$

$$\text{One solution: } 3x + 5 \leq 2(x + 3) + 1 \leq 3(x + 2) - 1$$

140. Answers will vary.

141. Since $x^2 \geq 0$, we have

$$x^2 + 1 \geq 0 + 1$$

$$x^2 + 1 \geq 1$$

Therefore, the expression $x^2 + 1$ can never be less than -5 .

142. Answers will vary.

Chapter 1 Review Exercises

1. $2 - \frac{x}{3} = 8$

$6 - x = 24$

$x = -18$

The solution set is $\{-18\}$.

2. $-2(5 - 3x) + 8 = 4 + 5x$

$-10 + 6x + 8 = 4 + 5x$

$6x - 2 = 4 + 5x$

$x = 6$

The solution set is $\{6\}$.

3. $\frac{3x}{4} - \frac{x}{3} = \frac{1}{12}$

$9x - 4x = 1$

$5x = 1$

$x = \frac{1}{5}$

The solution set is $\{\frac{1}{5}\}$

4. $\frac{x}{x-1} = \frac{6}{5}$

$5x = 6x - 6$

$6 = x$

Since $x = 6$ does not cause a denominator to equal zero, the solution set is $\{6\}$.

5. $x(1 - x) = 6$

$x - x^2 = 6$

$0 = x^2 - x + 6$

$b^2 - 4ac = (-1)^2 - 4(1)(6)$

$= 1 - 24 = -23$

Therefore, there are no real solutions.

6. $\frac{1-3x}{4} = \frac{x+6}{3} + \frac{1}{2}$

$(12)\left(\frac{1-3x}{4}\right) = \left(\frac{x+6}{3} + \frac{1}{2}\right)(12)$

$3(1-3x) = 4(x+6) + 6$

$3 - 9x = 4x + 24 + 6$

$-13x = 27$

$x = -\frac{27}{13}$

The solution set is $\{-\frac{27}{13}\}$.

7. $(x-1)(2x+3) = 3$

$2x^2 + x - 3 = 3$

$2x^2 + x - 6 = 0$

$(2x-3)(x+2) = 0 \Rightarrow x = \frac{3}{2} \text{ or } x = -2$

The solution set is $\{-2, \frac{3}{2}\}$.

8. $2x + 3 = 4x^2$

$0 = 4x^2 - 2x - 3$

$x = \frac{2 \pm \sqrt{4 + 48}}{8} = \frac{2 \pm \sqrt{52}}{8}$

$= \frac{2 \pm 2\sqrt{13}}{8} = \frac{1 \pm \sqrt{13}}{4}$

The solution set is $\left\{\frac{1-\sqrt{13}}{4}, \frac{1+\sqrt{13}}{4}\right\}$.

9. $\sqrt[3]{x^2 - 1} = 2$

$\left(\sqrt[3]{x^2 - 1}\right)^3 = (2)^3$

$x^2 - 1 = 8$

$x^2 = 9 \Rightarrow x = \pm 3$

Check: $x = -3$

$\sqrt[3]{(-3)^2 - 1} = 2$

$\sqrt[3]{9 - 1} = 2$

$\sqrt[3]{8} = 2$

$2 = 2$

The solution set is $\{-3, 3\}$.

Check: $x = 3$

$\sqrt[3]{(3)^2 - 1} = 2$

$\sqrt[3]{9 - 1} = 2$

$\sqrt[3]{8} = 2$

$2 = 2$

10. $x(x+1) + 2 = 0$

$x^2 + x + 2 = 0$

$x = \frac{-1 \pm \sqrt{1 - 8}}{2} = \frac{-1 \pm \sqrt{-7}}{2}$

No real solutions.

11. $3x^2 - x + 1 = 0$

$x = \frac{1 \pm \sqrt{1 - 12}}{6} = \frac{1 \pm \sqrt{-11}}{6}$

No real solutions.

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12. $x^4 - 5x^2 + 4 = 0$

$$(x^2 - 4)(x^2 - 1) = 0$$

$$x^2 - 4 = 0 \text{ or } x^2 - 1 = 0$$

$$x = \pm 2 \text{ or } x = \pm 1$$

The solution set is $\{-2, -1, 1, 2\}$.

13. $\sqrt{2x-3} + x = 3$

$$\sqrt{2x-3} = 3 - x$$

$$2x - 3 = 9 - 6x + x^2$$

$$x^2 - 8x + 12 = 0 \Rightarrow (x-2)(x-6) = 0$$

$$x = 2 \text{ or } x = 6$$

Check $x = 2$:

$$\sqrt{2(2)-3} + 2 = \sqrt{1} + 2 = 3$$

Check $x = 6$:

$$\sqrt{2(6)-3} + 6 = \sqrt{9} + 6 = 9 \neq 3$$

The solution set is $\{2\}$.

14. $\sqrt[4]{2x+3} = 2$

$$(\sqrt[4]{2x+3})^4 = 2^4$$

$$2x + 3 = 16$$

$$2x = 13$$

$$x = \frac{13}{2}$$

Check $x = \frac{13}{2}$:

$$\sqrt[4]{2\left(\frac{13}{2}\right)+3} = \sqrt[4]{13+3} = \sqrt[4]{16} = 2$$

The solution set is $\left\{\frac{13}{2}\right\}$.

15. $\sqrt{x+1} + \sqrt{x-1} = \sqrt{2x+1}$

$$(\sqrt{x+1} + \sqrt{x-1})^2 = (\sqrt{2x+1})^2$$

$$x+1 + 2\sqrt{x+1}\sqrt{x-1} + x-1 = 2x+1$$

$$2x + 2\sqrt{x+1}\sqrt{x-1} = 2x+1$$

$$2\sqrt{x+1}\sqrt{x-1} = 1$$

$$(2\sqrt{x+1}\sqrt{x-1})^2 = (1)^2$$

$$4(x+1)(x-1) = 1$$

$$4x^2 - 4 = 1$$

$$4x^2 = 5$$

$$x^2 = \frac{5}{4}$$

$$x = \pm \frac{\sqrt{5}}{2}$$

Check $x = \frac{\sqrt{5}}{2}$:

$$\sqrt{\frac{\sqrt{5}}{2}+1} + \sqrt{\frac{\sqrt{5}}{2}-1} = \sqrt{2\left(\frac{\sqrt{5}}{2}\right)+1}$$

$$1.79890743995 = 1.79890743995$$

Check $x = -\frac{\sqrt{5}}{2}$:

$$\sqrt{-\frac{\sqrt{5}}{2}+1} + \sqrt{-\frac{\sqrt{5}}{2}-1} = \sqrt{2\left(-\frac{\sqrt{5}}{2}\right)+1},$$

The second solution is not possible because it makes the radicand negative.

The solution set is $\left\{\frac{\sqrt{5}}{2}\right\}$.

16. $2x^{1/2} - 3 = 0$

$$2x^{1/2} = 3$$

$$(2x^{1/2})^2 = 3^2$$

$$4x = 9$$

$$x = \frac{9}{4}$$

Check $x = \frac{9}{4}$:

$$2\left(\frac{9}{4}\right)^{1/2} - 3 = 2\left(\frac{3}{2}\right) - 3 = 3 - 3 = 0$$

The solution set is $\left\{\frac{9}{4}\right\}$.

17. $x^{-6} - 7x^{-3} - 8 = 0$

let $p = x^{-3} \Rightarrow p^2 = x^{-6}$

$$p^2 - 7p - 8 = 0$$

$$(p-8)(p+1) = 0$$

$$p = 8 \text{ or } p = -1$$

$$p = 8 \Rightarrow x^{-3} = 8$$

$$(x^{-3})^{-1/3} = (8)^{-1/3} \Rightarrow x = \frac{1}{2}$$

$$p = -1 \Rightarrow x^{-3} = -1$$

$$(x^{-3})^{-1/3} = (-1)^{-1/3} \Rightarrow x = -1$$

Check $x = \frac{1}{2}$:

$$\left(\frac{1}{2}\right)^{-6} - 7\left(\frac{1}{2}\right)^{-3} - 8 = 64 - 56 - 8 = 0$$

Check $x = -1$:

$$(-1)^{-6} - 7(-1)^{-3} - 8 = 1 + 7 - 8 = 0$$

The solution set is $\left\{-1, \frac{1}{2}\right\}$.

18. $x^2 + m^2 = 2mx + (nx)^2$

$$x^2 + m^2 = 2mx + n^2x^2$$

$$x^2 - n^2x^2 - 2mx + m^2 = 0$$

$$(1-n^2)x^2 - 2mx + m^2 = 0$$

$$x = \frac{2m \pm \sqrt{4m^2 - 4m^2(1-n^2)}}{2(1-n^2)}$$

$$= \frac{2m \pm \sqrt{4m^2(1-(1-n^2))}}{2(1-n^2)}$$

$$= \frac{2m \pm 2m\sqrt{1-(1-n^2)}}{2(1-n^2)}$$

$$= \frac{m \pm m\sqrt{n^2}}{1-n^2} = \frac{m \pm mn}{1-n^2} = \frac{m(1 \pm n)}{1-n^2}$$

$$x = \frac{m(1+n)}{1-n^2} = \frac{m(1+n)}{(1+n)(1-n)} = \frac{m}{1-n}$$

or

$$x = \frac{m(1-n)}{1-n^2} = \frac{m(1-n)}{(1+n)(1-n)} = \frac{m}{1+n}$$

The solution set is $\left\{\frac{m}{1-n}, \frac{m}{1+n}\right\}$, $n \neq 1$, $n \neq -1$.

19. $10a^2x^2 - 2abx - 36b^2 = 0$

$$5a^2x^2 - abx - 18b^2 = 0$$

$$(5ax+9b)(ax-2b) = 0$$

$$5ax+9b=0 \quad \text{or} \quad ax-2b=0$$

$$5ax = -9b \quad ax = 2b$$

$$x = -\frac{9b}{5a} \quad x = \frac{2b}{a}$$

The solution set is $\left\{-\frac{9b}{5a}, \frac{2b}{a}\right\}$, $a \neq 0$.

20. $\sqrt{x^2+3x+7} - \sqrt{x^2-3x+9} + 2 = 0$

$$\sqrt{x^2+3x+7} = \sqrt{x^2-3x+9} - 2$$

$$\left(\sqrt{x^2+3x+7}\right)^2 = \left(\sqrt{x^2-3x+9} - 2\right)^2$$

$$x^2+3x+7 = x^2-3x+9 - 4\sqrt{x^2-3x+9} + 4$$

$$6x-6 = -4\sqrt{x^2-3x+9}$$

$$(6(x-1))^2 = (-4\sqrt{x^2-3x+9})^2$$

$$36(x^2-2x+1) = 16(x^2-3x+9)$$

$$36x^2 - 72x + 36 = 16x^2 - 48x + 144$$

$$20x^2 - 24x - 108 = 0$$

$$5x^2 - 6x - 27 = 0$$

$$(5x+9)(x-3) = 0 \Rightarrow x = -\frac{9}{5} \text{ or } x = 3$$

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Check $x = -\frac{9}{5}$:

$$\begin{aligned} & \sqrt{\left(-\frac{9}{5}\right)^2 + 3\left(-\frac{9}{5}\right) + 7} - \sqrt{\left(-\frac{9}{5}\right)^2 - 3\left(-\frac{9}{5}\right) + 9 + 2} \\ &= \sqrt{\frac{81}{25} - \frac{27}{5} + 7} - \sqrt{\frac{81}{25} + \frac{27}{5} + 9 + 2} \\ &= \sqrt{\frac{81 - 135 + 175}{25}} - \sqrt{\frac{81 + 135 + 225}{25}} + 2 \\ &= \sqrt{\frac{121}{25}} - \sqrt{\frac{441}{25}} + 2 = \frac{11}{5} - \frac{21}{5} + 2 = 0 \end{aligned}$$

Check $x = 3$:

$$\begin{aligned} & \sqrt{(3)^2 + 3(3) + 7} - \sqrt{(3)^2 - 3(3) + 9 + 2} \\ &= \sqrt{9 + 9 + 7} - \sqrt{9 - 9 + 9 + 2} \\ &= \sqrt{25} - \sqrt{9} + 2 = 2 + 2 \\ &= 4 \neq 0 \end{aligned}$$

The solution set is $\left\{-\frac{9}{5}\right\}$.

21. $|2x + 3| = 7$

$$2x + 3 = 7 \quad \text{or} \quad 2x + 3 = -7$$

$$2x = 4 \quad \text{or} \quad 2x = -10$$

$$x = 2 \quad \text{or} \quad x = -5$$

The solution set is $\{-5, 2\}$.

22. $|2 - 3x| + 2 = 9$

$$|2 - 3x| = 7$$

$$2 - 3x = 7 \quad \text{or} \quad 2 - 3x = -7$$

$$-3x = 5 \quad \quad \quad -3x = -9$$

$$x = -\frac{5}{3} \quad \quad \quad x = 3$$

The solution set is $\left\{-\frac{5}{3}, 3\right\}$

23. $2x^3 = 3x^2$

$$2x^3 - 3x^2 = 0$$

$$x^2(2x - 3) = 0$$

$$x^2 = 0 \Rightarrow x = 0$$

$$2x - 3 = 0 \Rightarrow x = \frac{3}{2}$$

The solution set is $\left\{0, \frac{3}{2}\right\}$.

24. $2x^3 + 5x^2 - 8x - 20 = 0$

$$x^2(2x + 5) - 4(2x + 5) = 0$$

$$(2x + 5)(x^2 - 4) = 0$$

$$2x + 5 = 0 \quad \text{or} \quad x^2 - 4 = 0$$

$$x = -\frac{5}{2} \quad \text{or} \quad x = \pm 2$$

The solution set is $\left\{-\frac{5}{2}, -2, 2\right\}$.

25. $\frac{1}{x-1} + \frac{3}{x+2} = \frac{11}{x^2 + x - 2}$

$$\frac{(x+2) + 3(x-1)}{(x-1)(x+2)} = \frac{11}{(x-1)(x+2)}$$

$$(x+2) + 3(x-1) = 11$$

$$x + 2 + 3x - 3 = 11$$

$$4x - 1 = 11$$

$$4x = 12$$

$$x = 3$$

Since 3 does not make any denominator equal to 0, the solution set is $\{3\}$.

26. $(x-2)^2 = 9$

$$x - 2 = \pm\sqrt{9}$$

$$x - 2 = \pm 3$$

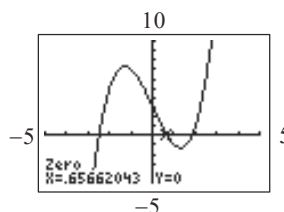
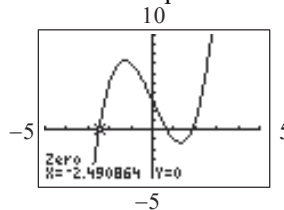
$$x = 2 \pm 3$$

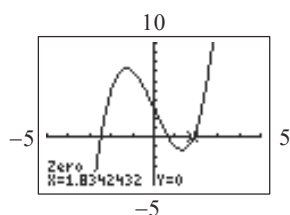
$$x = 5 \quad \text{or} \quad x = -1$$

The solution set is $\{-1, 5\}$.

27. $x^3 - 5x + 3 = 0$

Use the Zero option from the CALC menu.

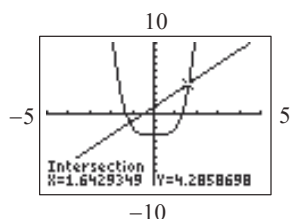
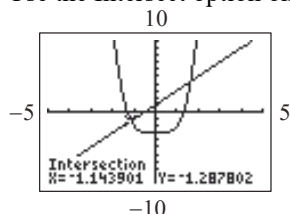




The solution set is $\{-2.49, 0.66, 1.83\}$.

28. $x^4 - 3 = 2x + 1$

Use the Intersect option on the CALC menu.



The solution set is $\{-1.14, 1.64\}$.

29. $\frac{2x-3}{5} + 2 \leq \frac{x}{2}$

$$2(2x-3) + 10(2) \leq 5x$$

$$4x - 6 + 20 \leq 5x$$

$$14 \leq x$$

$$x \geq 14$$

$$\{x \mid x \geq 14\} \text{ or } [14, \infty)$$



30. $-9 \leq \frac{2x+3}{-4} \leq 7$

$$36 \geq 2x+3 \geq -28$$

$$33 \geq 2x \geq -31$$

$$\frac{33}{2} \geq x \geq -\frac{31}{2}$$

$$-\frac{31}{2} \leq x \leq \frac{33}{2}$$

$$\left\{x \mid -\frac{31}{2} \leq x \leq \frac{33}{2}\right\} \text{ or } \left[-\frac{31}{2}, \frac{33}{2}\right]$$



31. $2 < \frac{3-3x}{12} < 6$

$$24 < 3-3x < 72$$

$$21 < -3x < 69$$

$$-7 > x > -23$$

$$\{x \mid -23 < x < -7\} \text{ or } (-23, -7)$$



32. $|3x+4| < \frac{1}{2}$

$$-\frac{1}{2} < 3x+4 < \frac{1}{2}$$

$$-\frac{9}{2} < 3x < -\frac{7}{2}$$

$$-\frac{3}{2} < x < -\frac{7}{6}$$

$$\left\{x \mid -\frac{3}{2} < x < -\frac{7}{6}\right\} \text{ or } \left(-\frac{3}{2}, -\frac{7}{6}\right)$$



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33. $|2x-5| \geq 9$

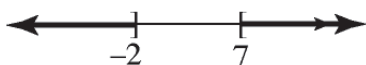
$$2x-5 \leq -9 \text{ or } 2x-5 \geq 9$$

$$2x \leq -4 \text{ or } 2x \geq 14$$

$$x \leq -2 \text{ or } x \geq 7$$

$$\{x \mid x \leq -2 \text{ or } x \geq 7\} \text{ or}$$

$$(-\infty, -2] \text{ or } [7, \infty)$$



34. $2 + |2-3x| \leq 4$

$$|2-3x| \leq 2$$

$$-2 \leq 2-3x \leq 2$$

$$-4 \leq -3x \leq 0$$

$$\frac{4}{3} \geq x \geq 0$$

$$0 \leq x \leq \frac{4}{3}$$

$$\left\{x \mid 0 \leq x \leq \frac{4}{3}\right\} \text{ or } \left[0, \frac{4}{3}\right]$$



35. $1 - |2-3x| < -4$

$$-|2-3x| < -5$$

$$|2-3x| > 5$$

$$2-3x < -5 \text{ or } 2-3x > 5$$

$$7 < 3x \text{ or } -3 > 3x$$

$$\frac{7}{3} < x \text{ or } -1 > x$$

$$x < -1 \text{ or } x > \frac{7}{3}$$

$$\left\{x \mid x < -1 \text{ or } x > \frac{7}{3}\right\} \text{ or}$$

$$(-\infty, -1) \text{ or } \left(\frac{7}{3}, \infty\right)$$



36. $(6+3i) - (2-4i) = (6-2) + (3-(-4))i = 4+7i$

37. $4(3-i) + 3(-5+2i) = 12-4i-15+6i = -3+2i$

38. $\frac{3}{3+i} = \frac{3}{3+i} \cdot \frac{3-i}{3-i} = \frac{9-3i}{9-3i+3i-i^2} = \frac{9-3i}{10}$
 $= \frac{9}{10} - \frac{3}{10}i$

39. $i^{50} = i^{48} \cdot i^2 = (i^4)^{12} \cdot i^2 = 1^{12}(-1) = -1$

40. $(2+3i)^3 = (2+3i)^2(2+3i)$
 $= (4+12i+9i^2)(2+3i)$
 $= (-5+12i)(2+3i)$
 $= -10-15i+24i+36i^2$
 $= -46+9i$

41. $x^2 + x + 1 = 0$

$$a = 1, b = 1, c = 1,$$

$$b^2 - 4ac = 1^2 - 4(1)(1) = 1 - 4 = -3$$

$$x = \frac{-1 \pm \sqrt{-3}}{2(1)} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

The solution set is $\left\{-\frac{1}{2} - \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i\right\}$.

42. $2x^2 + x - 2 = 0$

$$a = 2, b = 1, c = -2,$$

$$b^2 - 4ac = 1^2 - 4(2)(-2) = 1 + 16 = 17$$

$$x = \frac{-1 \pm \sqrt{17}}{2(2)} = \frac{-1 \pm \sqrt{17}}{4}$$

The solution set is $\left\{\frac{-1 - \sqrt{17}}{4}, \frac{-1 + \sqrt{17}}{4}\right\}$.

43. $x^2 + 3 = x$

$$x^2 - x + 3 = 0$$

$$a = 1, b = -1, c = 3,$$

$$b^2 - 4ac = (-1)^2 - 4(1)(3) = 1 - 12 = -11$$

$$x = \frac{-(-1) \pm \sqrt{-11}}{2(1)} = \frac{1 \pm \sqrt{11}i}{2} = \frac{1}{2} \pm \frac{\sqrt{11}}{2}i$$

The solution set is $\left\{\frac{1}{2} - \frac{\sqrt{11}}{2}i, \frac{1}{2} + \frac{\sqrt{11}}{2}i\right\}$.

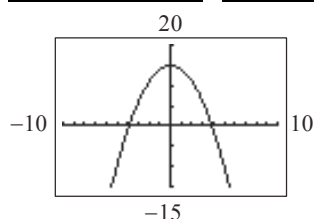
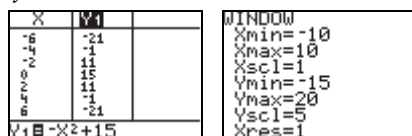
44. $x(1+x) = 2$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0 \Rightarrow x = -2 \text{ or } x = 1$$

The solution set is $\{-2, 1\}$.

45. $y = -x^2 + 15$



46. $(-4, 0), (0, 0), (2, 0), (0, -2), (0, 0), (0, 2)$

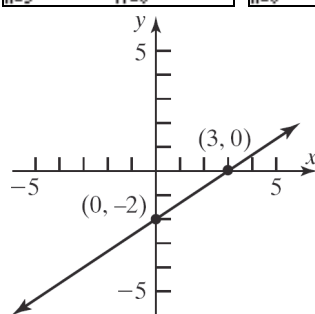
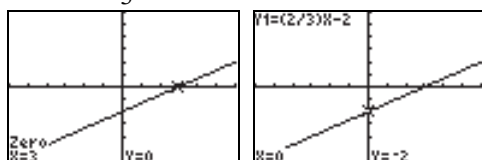
x-intercepts: $-4, 0, 2$

y-intercepts: $-2, 0, 2$

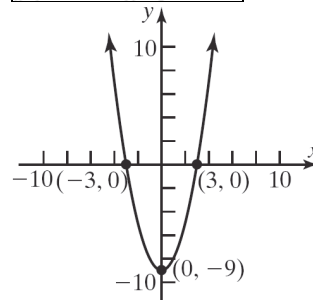
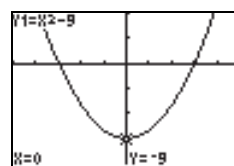
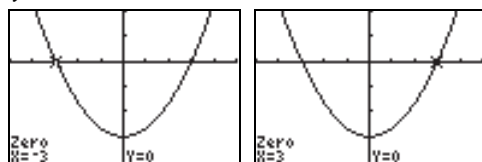
47. $2x - 3y = 6$

$$-3y = -2x + 6$$

$$y = \frac{2}{3}x - 2$$



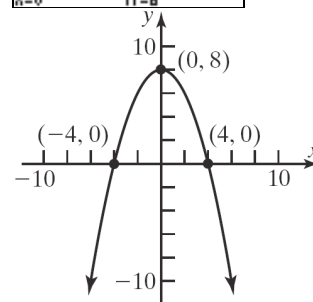
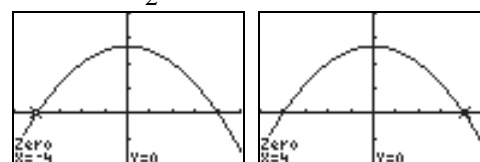
48. $y = x^2 - 9$



49. $x^2 + 2y = 16$

$$2y = -x^2 + 16$$

$$y = -\frac{1}{2}x^2 + 8$$



50. $p = 2l + 2w$

51. $I = P \cdot r \cdot t \Rightarrow I = (9000)(0.07)(1) = \630

52. Let x represent the amount of money invested in bonds. Then $70,000 - x$ represents the amount of money invested in CD's. Since the total interest is to be \$5000, we have:

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$$\begin{aligned}
 0.08x + 0.05(70,000 - x) &= 5000 \\
 (100)(0.08x + 0.05(70,000 - x)) &= (5000)(100) \\
 8x + 350,000 - 5x &= 500,000 \\
 3x + 350,000 &= 500,000 \\
 3x &= 150,000 \\
 x &= 50,000
 \end{aligned}$$

\$50,000 should be invested in bonds at 8% and \$20,000 should be invested in CD's at 5%.

53. Using $s = vt$, we have $t = 3$ and $v = 1100$.

Finding the distance s in feet:

$$s = 1100(3) = 3300$$

The storm is 3300 feet away.

54. $1600 \leq I \leq 3600$

$$1600 \leq \frac{900}{x^2} \leq 3600$$

$$\frac{1}{1600} \geq \frac{x^2}{900} \geq \frac{1}{3600}$$

$$\frac{9}{16} \geq x^2 \geq \frac{1}{4}$$

$$\frac{3}{4} \geq x \geq \frac{1}{2}$$

The range of distances is from 0.5 meters to 0.75 meters, inclusive.

55. Let s represent the distance the plane can travel.

	With wind	Against wind
Rate	$250 + 30 = 280$	$250 - 30 = 220$
Time	$\frac{(s/2)}{280}$	$\frac{(s/2)}{220}$
Dist.	$\frac{s}{2}$	$\frac{s}{2}$

Since the total time is at most 5 hours, we have:

$$\frac{(s/2)}{280} + \frac{(s/2)}{220} \leq 5$$

$$\frac{s}{560} + \frac{s}{440} \leq 5$$

$$11s + 14s \leq 5(6160)$$

$$25s \leq 30,800$$

$$s \leq 1232$$

The plane can travel at most 1232 miles or 616 miles one way and return 616 miles.

56. Let t represent the time it takes the helicopter to reach the raft.

	Raft	Helicopter
Rate	5	90
Time	t	t
Dist.	$5t$	$90t$

Since the total distance is 150 miles, we have:

$$5t + 90t = 150$$

$$95t = 150$$

$$t \approx 1.58 \text{ hours} \approx 1 \text{ hour and 35 minutes}$$

The helicopter will reach the raft in about 1 hour and 35 minutes.

57. Let d represent the distance flown by the bee traveling at 3 meters per second.

$$\frac{d}{3} = \frac{150 - d}{5} \quad (\text{Times needed to meet are equal.})$$

$$5d = 450 - 3d$$

$$8d = 450$$

$$d = 56.25 \text{ meters} \Rightarrow t = \frac{56.25}{3} = 18.75 \text{ seconds}$$

The bees meet for the first time after 18.75 seconds.

The bees will meet a second time on the second lap. The first bee will have traveled $150 + x$ meters and the second bee will have traveled $150 + (150 - x)$ meters.

Solving for time, we have:

$$\frac{150 + x}{3} = \frac{150 + (150 - x)}{5}$$

$$\frac{150 + x}{3} = \frac{300 - x}{5}$$

$$750 + 5x = 900 - 3x$$

$$8x = 150$$

$$x = 18.75 \text{ meters into the second lap}$$

$$t = \frac{168.75}{3} = 56.25 \text{ seconds}$$

The bees meet the second time after 56.25 seconds (37.5 seconds later).

58. Given that $s = 1280 - 32t - 16t^2$,

- a. The object hits the ground when $s = 0$.

$$0 = 1280 - 32t - 16t^2$$

$$t^2 + 2t - 80 = 0$$

$$(t + 10)(t - 8) = 0 \Rightarrow t = -10, t = 8$$

The object hits the ground after 8 seconds.

- b. After 4 seconds, the object's height is

$$s = 1280 - 32(4) - 16(4)^2 = 896 \text{ feet.}$$

59. Let t represent the time it takes Clarissa to complete the job by herself.

	Clarissa	Shawna
Time to do job alone	t	$t + 5$
Part of job done in 1 day	$\frac{1}{t}$	$\frac{1}{t + 5}$
Time on job (days)	6	6
Part of job done by each person	$\frac{6}{t}$	$\frac{6}{t + 5}$

Since the two people paint one house, we have:

$$\frac{6}{t} + \frac{6}{t + 5} = 1$$

$$6(t + 5) + 6t = t(t + 5)$$

$$6t + 30 + 6t = t^2 + 5t$$

$$t^2 - 7t - 30 = 0$$

$$(t - 10)(t + 3) = 0 \Rightarrow t = 10 \text{ or } t = -3$$

It takes Clarissa 10 days to paint the house when working by herself.

60. Let x represent the amount of the \$8-per-pound coffee.

Amt. of coffee (pounds)	Price (\$)	Total \$
20	4	$(20)(4)$
x	8	$(8)(x)$
$20 + x$	5	$(5)(20 + x)$

$$80 + 8x = (5)(20 + x)$$

$$80 + 8x = 100 + 5x$$

$$3x = 20$$

$$x = \frac{20}{3} = 6\frac{2}{3}$$

Add $6\frac{2}{3}$ pounds of \$8/lb coffee to get $26\frac{2}{3}$ pounds of \$5/lb coffee.

61. Let x represent the amount of water evaporated.

% salt	Tot. amt.	amt. of salt
2%	64	$(0.02)(64)$
0%	x	$(0.00)(x)$
10%	$64 - x$	$(0.10)(64 - x)$

$$(0.02)(64) - (0.00)(x) = (0.10)(64 - x)$$

$$1.28 = 6.4 - 0.10x$$

$$0.10x = 5.12$$

$$x = 51.2$$

51.2 ounces of water must be evaporated.

62. Let the length of leg 1 = x .
Then the length of leg 2 = $17 - x$.
By the Pythagorean Theorem we have

$$x^2 + (17 - x)^2 = (13)^2$$

$$x^2 + x^2 - 34x + 289 = 169$$

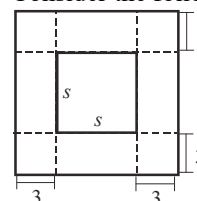
$$2x^2 - 34x + 120 = 0$$

$$x^2 - 17x + 60 = 0$$

$$(x - 12)(x - 5) = 0 \Rightarrow x = 12 \text{ or } x = 5$$

the legs are 5 cm and 12 cm long.

63. a. Consider the following diagram:



$$4(s + 6) = 50$$

$$4s + 24 = 50$$

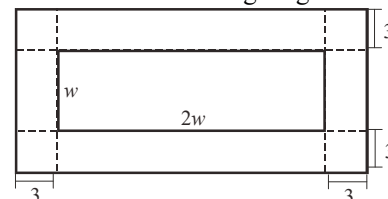
$$4s = 26$$

$$s = 6.5$$

The painting is 6.5 inches by 6.5 inches.

$s + 6 = 12.5$, so the frame is 12.5 inches by 12.5 inches.

- b. Consider the following diagram:



$$2(2w + 6) + 2(w + 6) = 50$$

$$4w + 12 + 2w + 12 = 50$$

$$6w = 26$$

$$w = \frac{26}{6} = 4\frac{1}{3} \Rightarrow l = 2w = 8\frac{2}{3}$$

The painting is $8\frac{2}{3}$ inches by $4\frac{1}{3}$ inches.

The frame is $14\frac{2}{3}$ inches by $10\frac{1}{3}$ inches.

Chapter 1: Graphs, Equations, and Inequalities

64. Let t represent the time it takes the smaller pump to finish filling the tank.

	3hp Pump	8hp Pump
Time to do job alone	12	8
Part of job done in 1 hr	$\frac{1}{12}$	$\frac{1}{8}$
Time on job (hrs)	$t + 4$	4
Part of job done by each pump	$\frac{t+4}{12}$	$\frac{4}{8}$

Since the two pumps fill one tank, we have:

$$\frac{t+4}{12} + \frac{4}{8} = 1$$

$$\frac{t+4}{12} = \frac{1}{2}$$

$$t+4 = 6$$

$$t = 2$$

It takes the small pump a total of 2 more hours to fill the tank.

65. Let $w = 4$. Solve for the length:

$$l^2 = 4(l+4)$$

$$l^2 = 4l + 16$$

$$l^2 - 4l - 16 = 0$$

$$l = \frac{-(-4) + \sqrt{(-4)^2 - 4(1)(-16)}}{2(1)}$$

$$= \frac{4 + \sqrt{80}}{2} = 2 + 2\sqrt{5} \approx 6.47$$

The length of the plasterboard should be cut to a length of approximately 6.47 feet.

66. Let x represent the number of passengers over 20. Then $20 + x$ represents the total number of passengers, and $15 - 0.1x$ represents the fare for each passenger. Solving the equation for total cost, \$482.40, we have:

$$(20 + x)(15 - 0.1x) = 482.40$$

$$300 + 13x - 0.1x^2 = 482.40$$

$$-0.1x^2 + 13x - 182.40 = 0$$

$$x^2 - 130x + 1824 = 0$$

$$(x - 114)(x - 16) = 0 \Rightarrow x = 114 \text{ or } x = 16$$

Since the capacity of the bus is 44, we discard

the 114. Therefore, $20 + 16 = 36$ people went on the trip; each person paid $15 - 0.1(16) = \$13.40$.

67. Let r_S represent Scott's rate and let r_T represent Todd's rate. The time for Scott to run 95 meters is the same as for Todd to run 100 meters.

$$\frac{95}{r_S} = \frac{100}{r_T}$$

$$r_S = 0.95r_T$$

$$d_S = t \cdot r_S = t(0.95r_T) = 0.95d_T$$

If Todd starts from 5 meters behind the start:

$$d_T = 105$$

$$d_S = 0.95d_T = 0.95(105) = 99.75$$

- The race does not end in a tie.
- Todd wins the race.
- Todd wins by 0.25 meters.
- To end in a tie:
 $100 = 0.95(100 + x)$
 $100 = 95 + 0.95x$
 $5 = 0.95x$
 $x = 5.263$ meters
- $95 = 0.95(100)$ Therefore, the race ends in a tie.

f,g,h. Race is a tie.

68. In each problem, we need to use the Least Common Multiple of the expressions $x - 2$ and $x^2 - 4$, namely $x^2 - 4$.

- We use $x^2 - 4$ as the LCD in order to combine the given expressions.
- We multiply each side of the equation by $x^2 - 4$ in order to clear out the denominators before solving the equation.
- We use $x^2 - 4$ as the LCD in order to combine the terms on the left hand side before solving the inequality.

Chapter 1 Test

1. $2x^2 + 6x = x - 3$

$2x^2 + 5x + 3 = 0$

$(2x+3)(x+1) = 0$

$2x+3=0 \quad \text{or} \quad x+1=0$

$2x = -3 \quad x = -1$

$x = -\frac{3}{2}$

The solution set is $\{-\frac{3}{2}, -1\}$.

2. $x+1 = \sqrt{x+7}$

$(x+1)^2 = (\sqrt{x+7})^2$

$x^2 + 2x + 1 = x + 7$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x+3=0 \quad \text{or} \quad x-2=0$

$x = -3 \quad x = 2$

Check:

$-3+1 \stackrel{?}{=} \sqrt{-3+7} \quad 2+1 \stackrel{?}{=} \sqrt{2+7}$

$-2 \stackrel{?}{=} \sqrt{4} \quad 3 \stackrel{?}{=} \sqrt{9}$

$-2 \neq 2 \quad 3 = 3 \quad \text{T}$

The solution set is $\{2\}$.

3. $2 - \frac{3}{m} = \frac{2}{m+2}$

LCD: $m(m+2)$

Restricted values: $m = -2, m = 0$

$\frac{2m(m+2)}{m(m+2)} - \frac{3(m+2)}{m(m+2)} = \frac{2m}{m(m+2)}$

$2m(m+2) - 3(m+2) = 2m$

$2m^2 + 4m - 3m - 6 = 2m$

$2m^2 - m - 6 = 0$

$(2m+3)(m-2) = 0$

$2m+3=0 \quad \text{or} \quad m-2=0$

$m = -\frac{3}{2} \quad m = 2$

Since neither solution is a restricted value, the solution set is $\{-\frac{3}{2}, 2\}$.

4. $5x - 8 = -4(x-1) + 6$

$5x - 8 = -4x + 4 + 6$

$5x - 8 = -4x + 10$

$9x = 18$

$x = 2$

The solution set is $\{2\}$.

5. $5|3-2b| - 7 = 8$

$5|3-2b| = 15$

$|3-2b| = 3$

$3-2b = 3 \quad \text{or} \quad 3-2b = -3$

$-2b = 0 \quad -2b = -6$

$b = 0 \quad b = 3$

The solution set is $\{0, 3\}$.

6. $x^4 + x^2 = 3x^2 + 8$

$x^4 - 2x^2 - 8 = 0$

Let $u = x^2$. Then $u^2 = (x^2)^2 = x^4$, and we have

$u^2 - 2u - 8 = 0$

$(u-4)(u+2) = 0$

$u = 4 \quad \text{or} \quad u = -2$

Since we are solving for x , we get

$x^2 = 4 \quad \text{or} \quad x^2 = -2$

$x = \pm 2 \quad x = \pm\sqrt{-2}$

no real solution

The solution set is $\{-2, 2\}$.

7. $x^2 - 4x + 2 = 0$

$a = 1, b = -4, c = 2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$

$= \frac{4 \pm \sqrt{16-8}}{2}$

$= \frac{4 \pm \sqrt{8}}{2}$

$= \frac{4 \pm 2\sqrt{2}}{2}$

$= 2 \pm \sqrt{2}$

The solution set is $\{2 - \sqrt{2}, 2 + \sqrt{2}\}$.

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8. $2x^2 + x - 1 = x(x + 7) + 2$

$$2x^2 + x - 1 = x^2 + 7x + 2$$

$$x^2 - 6x - 3 = 0$$

$$a = 1, b = -6, c = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 + 12}}{2}$$

$$= \frac{6 \pm \sqrt{48}}{2}$$

$$= \frac{6 \pm 4\sqrt{3}}{2}$$

$$= 3 \pm 2\sqrt{3}$$

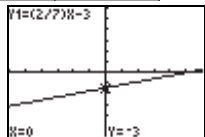
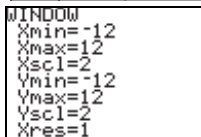
The solution set is $\{3 - 2\sqrt{3}, 3 + 2\sqrt{3}\}$.

9. $2x - 7y = 21$

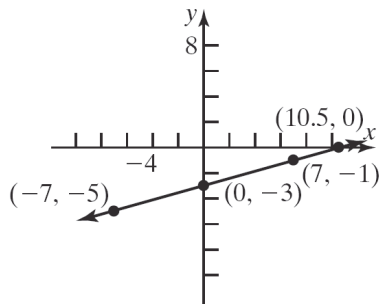
$$-7y = -2x + 21$$

$$y = \frac{2}{7}x - 3$$

x	$y = \frac{2}{7}x - 3$	(x, y)
-7	$y = \frac{2}{7}(-7) - 3 = -5$	$(-7, -5)$
0	$y = \frac{2}{7}(0) - 3 = -3$	$(0, -3)$
7	$y = \frac{2}{7}(7) - 3 = -1$	$(7, -1)$

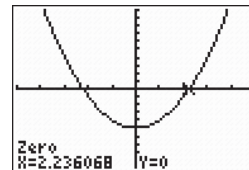
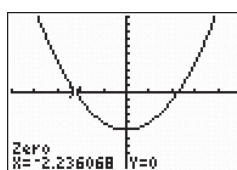
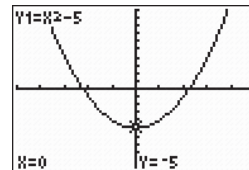
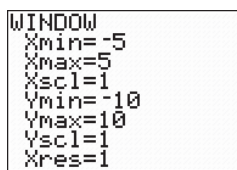


The intercepts are $(0, -3)$ and $(10.5, 0)$.

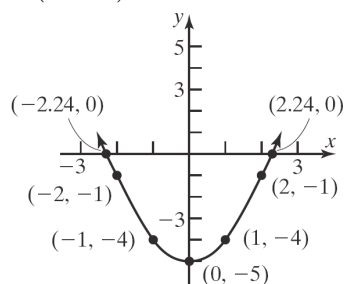


10. $y = x^2 - 5$

x	$y = x^2 - 5$	(x, y)
-3	$y = (-3)^2 - 5 = 4$	$(-3, 4)$
-1	$y = (-1)^2 - 5 = -4$	$(-1, -4)$
0	$y = (0)^2 - 5 = -5$	$(0, -5)$
1	$y = (1)^2 - 5 = -4$	$(1, -4)$
3	$y = (3)^2 - 5 = 4$	$(3, 4)$

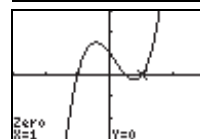
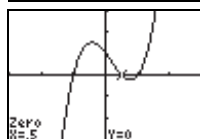
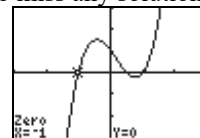
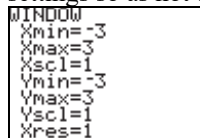


The intercepts are $(0, -5)$, $\approx (-2.24, 0)$, and $\approx (2.24, 0)$.



11. $2x^3 - x^2 - 2x + 1 = 0$

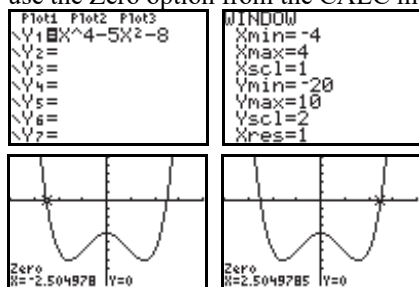
Since this equation has 0 on one side, we will use the Zero option from the CALC menu. It will be important to carefully select the window settings so as not to miss any solutions.



The solutions to the equation are -1 , 0.5 , and 1 .

12. $x^4 - 5x^2 - 8 = 0$

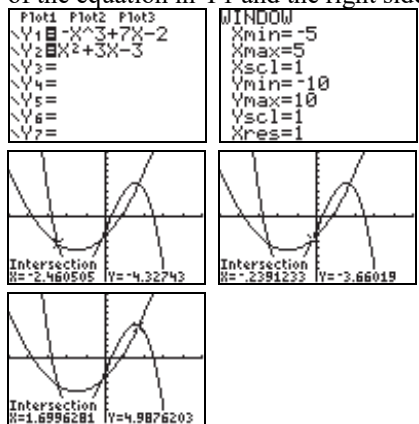
Since this equation has 0 on one side, we will use the Zero option from the CALC menu.



The solutions, rounded to two decimal places, are -2.50 and 2.50.

13. $-x^3 + 7x - 2 = x^2 + 3x - 3$

Since there are nonzero expressions on both sides of the equation, we will use the Intersect option from the CALC menu. Enter the left side of the equation in Y1 and the right side in Y2.



The solutions, rounded to two decimal places, are -2.46, -0.24, and 1.70.

14. $\frac{2x+3}{4} < -2$

$$4 \cdot \frac{2x+3}{4} < 4 \cdot (-2)$$

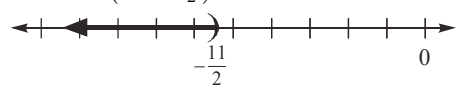
$$2x+3 < -8$$

$$2x = -11$$

$$x < -\frac{11}{2}$$

Solution set: $\{x \mid x < -\frac{11}{2}\}$

Interval: $(-\infty, -\frac{11}{2})$



15. $|2x+3| - 4 \geq 3$

$$|2x+3| - 4 + 4 \geq 3 + 4$$

$$|2x+3| \geq 7$$

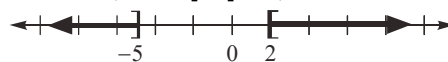
$$2x+3 \leq -7 \quad \text{or} \quad 2x+3 \geq 7$$

$$2x \leq -10 \quad 2x \geq 4$$

$$x \leq -5 \quad x \geq 2$$

Solution set: $\{x \mid x \leq -5 \text{ or } x \geq 2\}$

Interval: $(-\infty, -5] \cup [2, \infty)$



16. $-7 < 3 - 5x \leq 8$

$$-7 - 3 < 3 - 5x - 3 \leq 8 - 3$$

$$-10 < -5x \leq 5$$

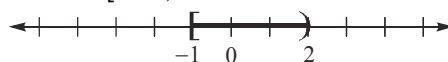
$$\frac{-10}{-5} > \frac{-5x}{-5} \geq \frac{5}{-5}$$

$$2 > x \geq -1$$

$$-1 \leq x < 2$$

Solution set: $\{x \mid -1 \leq x < 2\}$

Interval: $[-1, 2)$



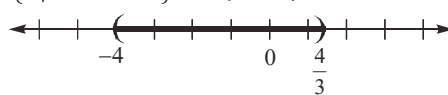
17. $|3x+4| < 8$

$$-8 < 3x+4 < 8$$

$$-12 < 3x < 4$$

$$-4 < x < \frac{4}{3}$$

Solution set: $\{x \mid -4 < x < \frac{4}{3}\}$ or $(-4, \frac{4}{3})$



18. $2(3-7i) - (4+11i) = 6-14i-4-11i$

$$= 6-4-14i-11i$$

$$= 2-25i$$

19. $(3+10i)(8+i) = 3(8+i) + 10i(8+i)$

$$= 24+3i+80i+10i^2$$

$$= 24+83i+10(-1)$$

$$= 14+83i$$

Chapter 1: Graphs, Equations, and Inequalities

$$\begin{aligned}
 20. \quad \frac{2+i}{5-3i} &= \frac{2+i}{5-3i} \cdot \frac{5+3i}{5+3i} \\
 &= \frac{10+6i+5i+3i^2}{25-9i^2} \\
 &= \frac{10+11i+3(-1)}{25-9(-1)} \\
 &= \frac{7+11i}{34} \\
 &= \frac{7}{34} + \frac{11}{34}i
 \end{aligned}$$

$$21. \quad 4x^2 - 4x + 5 = 0$$

$$a = 4, b = -4, c = 5$$

$$\begin{aligned}
 x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(5)}}{2(4)} = \frac{4 \pm \sqrt{16-80}}{8} \\
 &= \frac{4 \pm \sqrt{-64}}{8} = \frac{4 \pm 8i}{8} = \frac{1}{2} \pm i
 \end{aligned}$$

$$\text{The solution set is } \left\{ \frac{1}{2} - i, \frac{1}{2} + i \right\}.$$

22. We can use a table to summarize the given information.

	minutes per customer	# of cust. in 1 minute
Jamie	5	$\frac{1}{5}$
Scott	8.5	$\frac{1}{8.5}$
Together	t	$\frac{1}{t}$

This leads to the following:

$$(\text{Jamie's rate}) + (\text{Scott's rate}) = (\text{rate together})$$

$$\frac{1}{5} + \frac{1}{8.5} = \frac{1}{t}$$

$$\frac{8.5+5}{42.5} = \frac{1}{t}$$

$$\frac{13.5}{42.5} = \frac{1}{t}$$

$$13.5t = 42.5$$

$$t = \frac{42.5}{13.5} = \frac{85}{27}$$

Since $65 \cdot \frac{85}{27} \approx 204.63$, it will take Jamie and

Scott about 204.63 minutes (3.41 hours) to check out 65 customers working together.

23. Let x = pounds of banana chips. Then the total pounds of the mix will be $x + 40$.
 Since there is to be no loss in revenue, we get
 $\text{revenue}_{\text{new mix}} = \text{revenue}_{\text{banana}} + \text{revenue}_{\text{cherry mix}}$
 $(\text{price})(\text{lbs})_{\text{new}} = (\text{price})(\text{lbs})_{\text{ban.}} + (\text{price})(\text{lbs})_{\text{cher.}}$
 $(6.75)(x + 40) = (2.75)(x) + (9.25)(40)$
 $6.75x + 270 = 2.75x + 370$
 $4x = 100$
 $x = 25$

The retailer needs to mix 25 pounds of banana chips with the 40 pounds of the original mix.

24. Let x = sale price. We know that the discount is $0.42(275.00) = 115.50$. Therefore,
 $\text{sale price} = \text{original price} - \text{discount}$
 $x = 275 - 115.50$
 $x = 159.50$
 The sale price is \$159.50.

25. Here we need the simple interest formula,
 $I = P \cdot r \cdot t$. In this case we have $r = 0.04$,
 $t = \frac{3 \text{ months}}{12 \text{ months/yr}} = \frac{1}{4} \text{ yr}$, and $P = 10,000$.
 $I = P \cdot r \cdot t$
 $= (10,000)(0.04)\left(\frac{1}{4}\right)$
 $= 100$
 Glenn will earn \$100.00 in interest after 3 months.

Chapter 1 Projects

Project IInternet-based ProjectProject II

$$1. \quad T = \frac{n}{Cnp + L + M}, \quad n = 3, L = 5, M = 1, C = 0.2$$

$$T = \frac{3}{0.2(3)p + 5 + 1} = \frac{3}{0.6p + 6} = \frac{1}{0.2p + 2}$$

2. All of the times given in problem 1 were in seconds, so $T = 0.1$ board per second needs to be used as the value for T in the equation found in problem 1.

$$0.1 = \frac{1}{0.2p + 2}$$

$$(0.2p + 2)(0.1) = 1$$

$$0.02p + 0.2 = 1$$

$$0.02p = 0.8$$

$$p = 40 \text{ parts per board}$$

3. $T = 0.15$ board per second

$$0.15 = \frac{1}{0.2p + 2}$$

$$(0.2p + 2)(0.15) = 1$$

$$0.03p + 0.3 = 1$$

$$0.03p = 0.7$$

$$p \approx 23.3 \text{ parts per board}$$

Thus, only 23 parts per board will work.

For problems 4 – 6, C is requested, so solve for C first:

$$T = \frac{n}{Cnp + L + M}$$

$$(Cnp + L + M)T = n$$

$$CnpT + LT + MT = n$$

$$CnpT = n - LT - MT$$

$$C = \frac{n - LT - MT}{npT}$$

4. $T = 0.06, n = 3, p = 100, M = 1, L = 5$

$$C = \frac{3 - 5(0.06) - 1(0.06)}{3(100)(0.06)} \approx 0.147 \text{ sec}$$

5. $T = 0.06, n = 3, p = 150, M = 1, L = 5$

$$C = \frac{3 - 5(0.06) - 1(0.06)}{3(150)(0.06)} \approx 0.098 \text{ sec}$$

6. $T = 0.06, n = 3, p = 200, M = 1, L = 5$

$$C = \frac{3 - 5(0.06) - 1(0.06)}{3(200)(0.06)} \approx 0.073 \text{ sec}$$

7. As the number of parts per board increases, the tact time decreases, if all the other factors remain constant.