

***Algebra and Trigonometry - Enhanced with Graphing
Utilities 9th edition Sullivan Solutions Manual***

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INSTRUCTOR'S SOLUTIONS MANUAL

GEX INC.

ALGEBRA & TRIGONOMETRY ENHANCED WITH GRAPHING UTILITIES NINTH EDITION

Michael Sullivan

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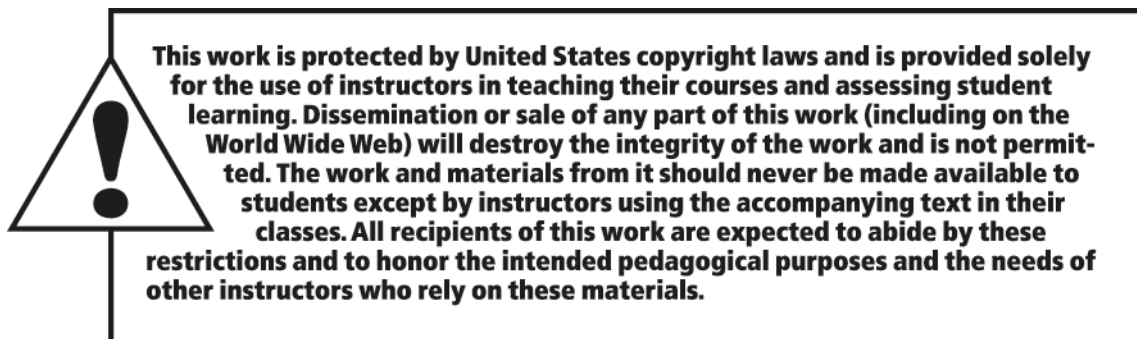
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Chapter R

Review

Section R.1

1. rational
2. $4 + 5 \cdot 6 - 3 = 4 + 30 - 3 = 31$
3. Distributive
4. c
5. a
6. b
7. True
8. False; The Zero-Product Property states that if a product equals 0, then at least one of the factors must equal 0.
9. False; 6 is the Greatest Common Factor of 12 and 18. The Least Common Multiple is the smallest value that both numbers will divide evenly. The LCM for 12 and 18 is 36.
10. True
11. $A \cup B = \{1, 3, 4, 5, 9\} \cup \{2, 4, 6, 7, 8\}$
 $= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
12. $A \cup C = \{1, 3, 4, 5, 9\} \cup \{1, 3, 4, 6\}$
 $= \{1, 3, 4, 5, 6, 9\}$
13. $A \cap B = \{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\} = \{4\}$
14. $A \cap C = \{1, 3, 4, 5, 9\} \cap \{1, 3, 4, 6\} = \{1, 3, 4\}$
15. $(A \cup B) \cap C$
 $= (\{1, 3, 4, 5, 9\} \cup \{2, 4, 6, 7, 8\}) \cap \{1, 3, 4, 6\}$
 $= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \cap \{1, 3, 4, 6\}$
 $= \{1, 3, 4, 6\}$
16. $(A \cap B) \cup C$
 $= (\{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\}) \cup \{1, 3, 4, 6\}$
 $= \{4\} \cup \{1, 3, 4, 6\}$
 $= \{1, 3, 4, 6\}$
17. $\overline{A} = \{0, 2, 6, 7, 8\}$
18. $\overline{C} = \{0, 2, 5, 7, 8, 9\}$
19. $\overline{A \cap B} = \overline{\{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\}}$
 $= \overline{\{4\}} = \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$
20. $\overline{B \cup C} = \overline{\{2, 4, 6, 7, 8\} \cup \{1, 3, 4, 6\}}$
 $= \overline{\{1, 2, 3, 4, 6, 7, 8\}} = \{0, 5, 9\}$
21. $\overline{A} \cup \overline{B} = \{0, 2, 6, 7, 8\} \cup \{0, 1, 3, 5, 9\}$
 $= \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$
22. $\overline{B} \cap \overline{C} = \{0, 1, 3, 5, 9\} \cap \{0, 2, 5, 7, 8, 9\}$
 $= \{0, 5, 9\}$
23. a. $\{2, 5\}$
b. $\{-6, 2, 5\}$
c. $\left\{-6, \frac{1}{2}, -1.333\dots = -1.\overline{3}, 2, 5\right\}$
d. $\{\pi\}$
e. $\left\{-6, \frac{1}{2}, -1.333\dots = -1.\overline{3}, \pi, 2, 5\right\}$
24. a. $\{1\}$
b. $\{0, 1\}$
c. $\left\{-\frac{5}{3}, 2.060606\dots = 2.\overline{06}, 1.25, 0, 1\right\}$
d. $\{\sqrt{5}\}$

Chapter R: Review

- e. $\left\{-\frac{5}{3}, 2.060606\dots = 2.\overline{06}, 1.25, 0, 1, \sqrt{5}\right\}$
25. a. $\{1\}$
 b. $\{0, 1\}$
 c. $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$
 d. None
 e. $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$
26. a. None
 b. $\{-1\}$
 c. $\{-1.3, -1.2, -1.1, -1\}$
 d. None
 e. $\{-1.3, -1.2, -1.1, -1\}$
27. a. None
 b. None
 c. None
 d. $\left\{\sqrt{2}, \pi, \sqrt{2}+1, \pi+\frac{1}{2}\right\}$
 e. $\left\{\sqrt{2}, \pi, \sqrt{2}+1, \pi+\frac{1}{2}\right\}$
28. a. None
 b. None
 c. $\left\{\frac{1}{2}+10.3\right\}$
 d. $\left\{-\sqrt{2}, \pi+\sqrt{2}\right\}$
 e. $\left\{-\sqrt{2}, \pi+\sqrt{2}, \frac{1}{2}+10.3\right\}$
29. a. 18.953 b. 18.952
30. a. 25.861 b. 25.861
31. a. 28.653 b. 28.653
32. a. 99.052 b. 99.052
33. a. 0.063 b. 0.062
34. a. 0.054 b. 0.053
35. a. 9.999 b. 9.998
36. a. 1.001 b. 1.000
37. a. 0.429 b. 0.428
38. a. 0.556 b. 0.555
39. a. 34.733 b. 34.733
40. a. 16.200 b. 16.200
41. $3+2=5$
42. $5\cdot 2=10$
43. $x+2=3\cdot 4$
44. $3+y=2+2$
45. $3y=1+2$
46. $2x=4\cdot 6$
47. $x-2=6$
48. $2-y=6$
49. $\frac{x}{2}=6$
50. $\frac{2}{x}=6$
51. $9-4+2=5+2=7$
52. $6-4+3=2+3=5$
53. $-6+4\cdot 3=-6+12=6$
54. $8-4\cdot 2=8-8=0$
55. $18-5\cdot 2=18-10=8$
56. $100-10\cdot 2=100-20=80$
57. $4+\frac{1}{3}=\frac{12+1}{3}=\frac{13}{3}$
58. $2-\frac{1}{2}=\frac{4-1}{2}=\frac{3}{2}$

$$\begin{aligned} 59. \quad 6 - [3 \cdot 5 + 2 \cdot (3 - 2)] &= 6 - [15 + 2 \cdot (1)] \\ &= 6 - 17 \\ &= -11 \end{aligned}$$

$$\begin{aligned} 60. \quad 2 \cdot [8 - 3 \cdot (4 + 2)] - 3 &= 2 \cdot [8 - 3 \cdot (6)] - 3 \\ &= 2 \cdot [8 - 18] - 3 \\ &= 2 \cdot [-10] - 3 \\ &= -20 - 3 \\ &= -23 \end{aligned}$$

$$\begin{aligned} 61. \quad 4 \cdot (9 + 5) - 6 \cdot 7 + 3 &= 4 \cdot (14) - 42 + 3 \\ &= 56 - 42 + 3 \\ &= 14 + 3 \\ &= 17 \end{aligned}$$

$$\begin{aligned} 62. \quad 1 - (4 \cdot 3 - 2 + 2) &= 1 - (12 - 2 + 2) \\ &= 1 - 12 \\ &= -11 \end{aligned}$$

$$\begin{aligned} 63. \quad 10 - [6 - 2 \cdot 2 + (8 - 3)] \cdot 2 &= 10 - [6 - 4 + 5] \cdot 2 \\ &= 10 - [2 + 5] \cdot 2 \\ &= 10 - [7] \cdot 2 \\ &= 10 - 14 \\ &= -4 \end{aligned}$$

$$\begin{aligned} 64. \quad 2 - 5 \cdot 4 - [6 \cdot (3 - 4)] &= 2 - 20 - [6 \cdot (-1)] \\ &= -18 - [-6] \\ &= -18 + 6 \\ &= -12 \end{aligned}$$

$$65. \quad (5 - 3) \frac{1}{2} = (2) \frac{1}{2} = 1$$

$$66. \quad (5 + 4) \frac{1}{3} = (9) \frac{1}{3} = 3$$

$$67. \quad \frac{4 + 8}{5 - 3} = \frac{12}{2} = 6$$

$$68. \quad \frac{2 - 4}{5 - 3} = \frac{-2}{2} = -1$$

$$69. \quad \frac{3}{5} \cdot \frac{10}{21} = \frac{3 \cdot 2 \cdot 5}{5 \cdot 3 \cdot 7} = \frac{\cancel{3} \cdot 2 \cdot \cancel{5}}{\cancel{5} \cdot \cancel{3} \cdot 7} = \frac{2}{7}$$

$$70. \quad \frac{5}{9} \cdot \frac{3}{10} = \frac{5 \cdot 3}{3 \cdot 3 \cdot 5 \cdot 2} = \frac{\cancel{5} \cdot \cancel{3}}{\cancel{3} \cdot 3 \cdot \cancel{5} \cdot 2} = \frac{1}{6}$$

$$71. \quad \frac{6}{25} \cdot \frac{10}{27} = \frac{2 \cdot 3 \cdot 5 \cdot 2}{5 \cdot 5 \cdot 3 \cdot 9} = \frac{2 \cdot \cancel{3} \cdot \cancel{5} \cdot 2}{\cancel{5} \cdot 5 \cdot \cancel{3} \cdot 9} = \frac{4}{45}$$

$$72. \quad \frac{21}{25} \cdot \frac{100}{3} = \frac{3 \cdot 7 \cdot 4 \cdot 25}{25 \cdot 3} = \frac{\cancel{3} \cdot 7 \cdot 4 \cdot \cancel{25}}{\cancel{25} \cdot \cancel{3}} = 28$$

$$73. \quad \frac{3}{4} + \frac{2}{5} = \frac{15 + 8}{20} = \frac{23}{20}$$

$$74. \quad \frac{4}{3} + \frac{1}{2} = \frac{8 + 3}{6} = \frac{11}{6}$$

$$75. \quad \frac{7}{8} + \frac{4}{7} = \frac{49 + 32}{56} = \frac{81}{56}$$

$$76. \quad \frac{8}{9} + \frac{15}{2} = \frac{16 + 135}{18} = \frac{151}{18}$$

$$77. \quad \frac{5}{18} + \frac{1}{12} = \frac{10 + 3}{36} = \frac{13}{36}$$

$$78. \quad \frac{2}{15} + \frac{8}{9} = \frac{6 + 40}{45} = \frac{46}{45}$$

$$79. \quad \frac{5}{24} - \frac{8}{15} = \frac{25 - 64}{120} = -\frac{39}{120} = -\frac{13}{40}$$

$$80. \quad \frac{3}{14} - \frac{2}{21} = \frac{9 - 4}{42} = \frac{5}{42}$$

$$81. \quad \frac{3}{20} - \frac{2}{15} = \frac{9 - 8}{60} = \frac{1}{60}$$

$$82. \quad \frac{6}{35} - \frac{3}{14} = \frac{12 - 15}{70} = -\frac{3}{70}$$

$$83. \quad \left(\frac{5}{18} \right) \left(\frac{11}{27} \right) = \frac{5}{18} \cdot \frac{27}{11} = \frac{5 \cdot 9 \cdot 3}{9 \cdot 2 \cdot 11} = \frac{5 \cdot \cancel{9} \cdot 3}{\cancel{9} \cdot 2 \cdot 11} = \frac{15}{22}$$

$$84. \quad \left(\frac{5}{21} \right) \left(\frac{2}{35} \right) = \frac{5}{21} \cdot \frac{35}{2} = \frac{5 \cdot 7 \cdot 5}{7 \cdot 3 \cdot 2} = \frac{5 \cdot \cancel{7} \cdot 5}{\cancel{7} \cdot 3 \cdot 2} = \frac{25}{6}$$

Chapter R: Review

$$85. \frac{1}{3} \cdot \frac{4}{7} + \frac{17}{21} = \frac{4}{21} + \frac{17}{21} = \frac{21}{21} = 1$$

$$86. \frac{2}{3} + \frac{4}{5} \cdot \frac{1}{6} = \frac{2}{3} + \frac{2 \cdot 2}{5 \cdot 3 \cdot 2} = \frac{2}{3} + \frac{2 \cdot \cancel{2}}{5 \cdot 3 \cdot \cancel{2}} = \frac{2}{3} + \frac{2}{15}$$

$$= \frac{2}{3} \cdot \frac{5}{5} + \frac{2}{15} = \frac{10}{15} + \frac{2}{15} = \frac{10+2}{15} = \frac{12}{15}$$

$$= \frac{4 \cdot 3}{5 \cdot 3} = \frac{4 \cdot \cancel{3}}{5 \cdot \cancel{3}} = \frac{4}{5}$$

$$87. 2 \cdot \frac{3}{4} + \frac{3}{8} = \frac{2}{1} \cdot \frac{3}{4} + \frac{3}{8} = \frac{6}{4} + \frac{3}{8} = \frac{6}{4} \cdot \frac{2}{2} + \frac{3}{8}$$

$$= \frac{12}{8} + \frac{3}{8} = \frac{12+3}{8} = \frac{15}{8}$$

$$88. 3 \cdot \frac{5}{6} - \frac{1}{2} = \frac{3}{1} \cdot \frac{5}{6} - \frac{1}{2} = \frac{3 \cdot 5}{3 \cdot 2} - \frac{1}{2} = \frac{\cancel{3} \cdot 5}{\cancel{3} \cdot 2} - \frac{1}{2}$$

$$= \frac{5}{2} - \frac{1}{2} = \frac{5-1}{2} = \frac{4}{2} = 2$$

$$89. 6(x+4) = 6x+24$$

$$90. 4(2x-1) = 8x-4$$

$$91. x(x-4) = x^2-4x$$

$$92. 4x(x+3) = 4x^2+12x$$

$$93. 2\left(\frac{3}{4}x - \frac{1}{2}\right) = 2 \cdot \frac{3}{4}x - 2 \cdot \frac{1}{2} = \frac{2 \cdot 3x}{2 \cdot 2} - \frac{2}{2}$$

$$= \frac{\cancel{2} \cdot 3x}{\cancel{2} \cdot 2} - \frac{2}{2} = \frac{3}{2}x - 1$$

$$94. 3\left(\frac{2}{3}x + \frac{1}{6}\right) = 3 \cdot \frac{2}{3}x + 3 \cdot \frac{1}{6} = \frac{3 \cdot 2x}{3} + \frac{3}{3 \cdot 2}$$

$$= \frac{\cancel{3} \cdot 2x}{\cancel{3}} + \frac{\cancel{3}}{\cancel{3} \cdot 2} = 2x + \frac{1}{2}$$

$$95. (x+2)(x+4) = x^2+4x+2x+8$$

$$= x^2+6x+8$$

$$96. (x+5)(x+1) = x^2+x+5x+5$$

$$= x^2+6x+5$$

$$97. (x+9)(2x-7) = 2x^2-7x+18x-63$$

$$= 2x^2+11x-63$$

$$98. (3x-1)(x+5) = 3x^2+15x-x-5$$

$$= 3x^2+14x-5$$

$$99. (x-8)(x-2) = x^2-2x-8x+16$$

$$= x^2-10x+16$$

$$100. (x-4)(x-2) = x^2-2x-4x+8$$

$$= x^2-6x+8$$

$$101. 3x(x-5k) = 3x^2-60x$$

$$3x^2-15xk = 3x^2-60x$$

$$-15xk = -60x$$

$$k = 4$$

$$102. (x-k)(x+3k) = x^2+4x-12$$

$$x^2+3kx-kx-3k^2 = x^2+4x-12$$

$$x^2+x(3k-k)-3k^2 = x^2+4x-12$$

$$x^2+x(3k-k)-3k^2 = x^2+4x-12$$

$$x^2+x(2k)-3k^2 = x^2+4x-12$$

$$2k = 4$$

$$k = 2$$

$$103. 2x+3x = 2 \cdot x+3 \cdot x$$

$$= (2+3) \cdot x$$

$$= (5) \cdot x$$

$$= 5x$$

$$104. 2+3 \cdot 4 = 2+12 = 14$$

since multiplication comes before addition in the order of operations for real numbers.

$$(2+3) \cdot 4 = 5 \cdot 4 = 20$$

since operations inside parentheses come before multiplication in the order of operations for real numbers.

$$105. 2(3 \cdot 4) = 2(12) = 24$$

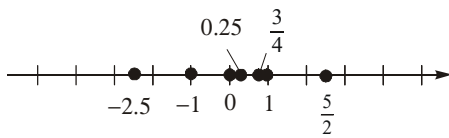
$$(2 \cdot 3) \cdot (2 \cdot 4) = (6)(8) = 48$$

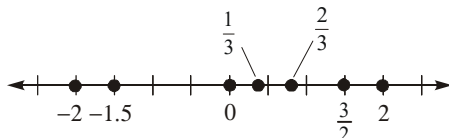
$$106. \frac{4+3}{2+5} = \frac{7}{7} = 1, \text{ but}$$

$$\frac{4}{2} + \frac{3}{5} = \frac{4 \cdot 5 + 3 \cdot 2}{10} = \frac{20+6}{10} = \frac{26}{10} = \frac{13}{5} = 2.6$$

107. Subtraction is not commutative; for example: $2 - 3 = -1 \neq 1 = 3 - 2$.
108. Subtraction is not associative; for example: $(5 - 2) - 1 = 2 \neq 4 = 5 - (2 - 1)$.
109. Division is not commutative; for example:
 $\frac{2}{3} \neq \frac{3}{2}$.
110. Division is not associative; for example: $(12 \div 2) \div 2 = 6 \div 2 = 3$, but
 $12 \div (2 \div 2) = 12 \div 1 = 12$.
111. The Symmetric Property implies that if $2 = x$, then $x = 2$.
112. From the *principle of substitution*, if $x = 5$, then
 $(x)(x) = (5)(5)$
 $\Rightarrow x^2 = 25$
 $\Rightarrow x^2 + x = 25 + 5$
 $\Rightarrow x^2 + x = 30$
113. There are no real numbers that are both rational and irrational, since an irrational number, by definition, is a number that cannot be expressed as the ratio of two integers; that is, not a rational number.
 Every real number is either a rational number or an irrational number, since the decimal form of a real number either involves an infinitely repeating pattern of digits or an infinite, non-repeating string of digits.
114. The sum of an irrational number and a rational number must be irrational. Otherwise, the irrational number would then be the difference of two rational numbers, and therefore would have to be rational.
115. Answers will vary.
116. Since 1 day = 24 hours, we compute
 $\frac{12997}{24} = 541.54\overline{16}$.
 Now we only need to consider the decimal part of the answer in terms of a 24 hour day. That is,
 $(0.54\overline{16})(24) \approx 13$ hours. So it must be 13 hours later than 12 noon, which makes the time 1 AM CST.
117. Answers will vary.

Section R.2

- variable
- origin
- strict
- base; exponent (or power)
- 1.2345678×10^3
- d
- a
- b
- True
- False; the absolute value of a real number is nonnegative. $|0| = 0$ which is not a positive number.
- False; a number in scientific notation is expressed as the product of a number, x , $1 \leq x < 10$ or $-10 < x \leq -1$, and a power of 10.
- True
- 

A number line with arrows at both ends. Ticks are marked at intervals of 0.5. Labels below the line are -2.5, -1, 0, 1, and 5/2. Above the line, points are marked with dots and labeled: 0.25 (at 1/4), 3/4 (at 3/4), and 5/2 (at 2.5).
- 

A number line with arrows at both ends. Ticks are marked at intervals of 0.5. Labels below the line are -2, -1.5, 0, 3/2, and 2. Above the line, points are marked with dots and labeled: 1/3 (at 1/3), 2/3 (at 2/3), and 3/2 (at 1.5).
- $\frac{1}{2} > 0$
- $5 < 6$
- $-1 > -2$
- $-3 < -\frac{5}{2}$
- $\pi > 3.14$
- $\sqrt{2} > 1.41$

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21. $\frac{1}{2} = 0.5$

22. $\frac{1}{3} > 0.33$

23. $\frac{2}{3} < 0.67$

24. $\frac{1}{4} = 0.25$

25. $x > 0$

26. $z < 0$

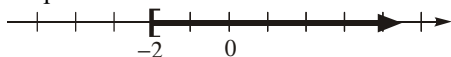
27. $x < 2$

28. $y > -5$

29. $x \leq 1$

30. $x \geq 2$

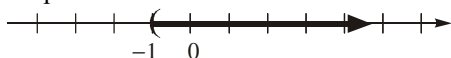
31. Graph on the number line: $x \geq -2$



32. Graph on the number line: $x < 4$



33. Graph on the number line: $x > -1$



34. Graph on the number line: $x \leq 7$



35. $d(C, D) = d(0, 1) = |1 - 0| = |1| = 1$

36. $d(C, A) = d(0, -3) = |-3 - 0| = |-3| = 3$

37. $d(D, E) = d(1, 3) = |3 - 1| = |2| = 2$

38. $d(C, E) = d(0, 3) = |3 - 0| = |3| = 3$

39. $d(A, E) = d(-3, 3) = |3 - (-3)| = |6| = 6$

40. $d(D, B) = d(1, -1) = |-1 - 1| = |-2| = 2$

41. $x + 2y = -2 + 2 \cdot 3 = -2 + 6 = 4$

42. $3x + y = 3(-2) + 3 = -6 + 3 = -3$

43. $5xy + 2 = 5(-2)(3) + 2 = -30 + 2 = -28$

44. $-2x + xy = -2(-2) + (-2)(3) = 4 - 6 = -2$

45. $\frac{2x}{x-y} = \frac{2(-2)}{-2-3} = \frac{-4}{-5} = \frac{4}{5}$

46. $\frac{x+y}{x-y} = \frac{-2+3}{-2-3} = \frac{1}{-5} = -\frac{1}{5}$

47. $\frac{3x+2y}{2+y} = \frac{3(-2)+2(3)}{2+3} = \frac{-6+6}{5} = \frac{0}{5} = 0$

48. $\frac{2x-3}{y} = \frac{2(-2)-3}{3} = \frac{-4-3}{3} = -\frac{7}{3}$

49. $|x+y| = |3+(-2)| = |1| = 1$

50. $|x-y| = |3-(-2)| = |5| = 5$

51. $|x|+|y| = |3|+|-2| = 3+2 = 5$

52. $|x|-|y| = |3|-|-2| = 3-2 = 1$

53. $\frac{|x|}{x} = \frac{|3|}{3} = \frac{3}{3} = 1$

54. $\frac{|y|}{y} = \frac{|-2|}{-2} = \frac{2}{-2} = -1$

55. $|4x-5y| = |4(3)-5(-2)|$
 $= |12+10|$
 $= |22|$
 $= 22$

56. $|3x+2y| = |3(3)+2(-2)| = |9-4| = |5| = 5$

57. $||4x|-|5y|| = ||4(3)|-|5(-2)||$
 $= ||12|-|10||$
 $= |12-10|$
 $= |2|$
 $= 2$

$$\begin{aligned}
 58. \quad 3|x| + 2|y| &= 3|3| + 2|-2| \\
 &= 3 \cdot 3 + 2 \cdot 2 \\
 &= 9 + 4 \\
 &= 13
 \end{aligned}$$

$$59. \quad \frac{x^2 - 1}{x}$$

Part (c) must be excluded. The value $x = 0$ must be excluded from the domain because it causes division by 0.

$$60. \quad \frac{x^2 + 1}{x}$$

Part (c) must be excluded. The value $x = 0$ must be excluded from the domain because it causes division by 0.

$$61. \quad \frac{x}{x^2 - 9} = \frac{x}{(x-3)(x+3)}$$

Part (a), $x = 3$, must be excluded because it causes the denominator to be 0.

$$62. \quad \frac{x}{x^2 + 9}$$

None of the given values are excluded. The domain is all real numbers.

$$63. \quad \frac{x^2}{x^2 + 1}$$

None of the given values are excluded. The domain is all real numbers.

$$64. \quad \frac{x^3}{x^2 - 1} = \frac{x^3}{(x-1)(x+1)}$$

Parts (b) and (d) must be excluded. The values $x = 1$, and $x = -1$ must be excluded from the domain because they cause division by 0.

$$65. \quad \frac{x^2 + 5x - 10}{x^3 - x} = \frac{x^2 + 5x - 10}{x(x-1)(x+1)}$$

Parts (b), (c), and (d) must be excluded. The values $x = 0$, $x = 1$, and $x = -1$ must be excluded from the domain because they cause division by 0.

$$66. \quad \frac{-9x^2 - x + 1}{x^3 + x} = \frac{-9x^2 - x + 1}{x(x^2 + 1)}$$

Part (c) must be excluded. The value $x = 0$ must

be excluded from the domain because it causes division by 0.

$$67. \quad \frac{4}{x-5}$$

$x = 5$ must be excluded because it makes the denominator equal 0.

$$\text{Domain} = \{x | x \neq 5\}$$

$$68. \quad \frac{-6}{x+4}$$

$x = -4$ must be excluded since it makes the denominator equal 0.

$$\text{Domain} = \{x | x \neq -4\}$$

$$69. \quad \frac{x}{x+4}$$

$x = -4$ must be excluded since it makes the denominator equal 0.

$$\text{Domain} = \{x | x \neq -4\}$$

$$70. \quad \frac{x-2}{x-6}$$

$x = 6$ must be excluded since it makes the denominator equal 0.

$$\text{Domain} = \{x | x \neq 6\}$$

$$71. \quad C = \frac{5}{9}(F - 32) = \frac{5}{9}(32 - 32) = \frac{5}{9}(0) = 0^\circ\text{C}$$

$$72. \quad C = \frac{5}{9}(F - 32) = \frac{5}{9}(212 - 32) = \frac{5}{9}(180) = 100^\circ\text{C}$$

$$73. \quad C = \frac{5}{9}(F - 32) = \frac{5}{9}(77 - 32) = \frac{5}{9}(45) = 25^\circ\text{C}$$

$$\begin{aligned}
 74. \quad C &= \frac{5}{9}(F - 32) = \frac{5}{9}(-4 - 32) \\
 &= \frac{5}{9}(-36) \\
 &= -20^\circ\text{C}
 \end{aligned}$$

$$75. \quad (-9)^2 = (-9)(-9) = 81$$

$$76. \quad -4^2 = -(4)^2 = -16$$

$$77. \quad 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

Chapter R: Review

$$78. -4^{-2} = -\frac{1}{4^2} = -\frac{1}{16}$$

$$79. 3^{-6} \cdot 3^4 = 3^{-6+4} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$80. 4^{-2} \cdot 4^3 = 4^{-2+3} = 4^1 = 4$$

$$81. (4^{-3})^{-1} = 4^{(-3)(-1)} = 4^3 = 64$$

$$82. (2^{-1})^{-3} = 2^{(-1)(-3)} = 2^3 = 8$$

$$83. \sqrt{100} = \sqrt{10^2} = 10$$

$$84. \sqrt{36} = \sqrt{6^2} = 6$$

$$85. \sqrt{(-4)^2} = |-4| = 4$$

$$86. \sqrt{(-3)^2} = |-3| = 3$$

$$87. (9x^4)^2 = 9^2 (x^4)^2 = 81x^8$$

$$88. (-4x^2)^{-1} = \frac{1}{-4x^2} = -\frac{1}{4x^2}$$

$$89. (x^2 y^{-1})^2 = (x^2)^2 \cdot (y^{-1})^2 = x^4 y^{-2} = \frac{x^4}{y^2}$$

$$90. (x^{-1} y)^3 = (x^{-1})^3 \cdot y^3 = x^{-3} y^3 = \frac{y^3}{x^3}$$

$$91. \frac{x^2 y^5}{x^3 y^4} = x^{2-3} y^{5-4} = x^{-1} y^1 = \frac{y}{x}$$

$$92. \frac{x^{-2} y}{x y^2} = x^{-2-1} y^{1-2} = x^{-3} y^{-1} = \frac{1}{x^3 y}$$

$$\begin{aligned} 93. \frac{(-4)^2 y^5 (xz)^3}{(-3)^3 x y^7 z^2} &= \frac{16 y^5 x^3 z^3}{-27 x y^7 z^2} \\ &= -\frac{16}{27} x^{3-1} y^{5-7} z^{3-2} \\ &= -\frac{16}{27} x^2 y^{-2} z^1 \\ &= -\frac{16x^2 z}{27y^2} \end{aligned}$$

$$\begin{aligned} 94. \frac{4x^{-2} (yz)^{-1}}{2^3 x^4 y} &= \frac{4x^{-2} y^{-1} z^{-1}}{8x^4 y} \\ &= \frac{4}{8} x^{-2-4} y^{-1-1} z^{-1} \\ &= \frac{1}{2} x^{-6} y^{-2} z^{-1} \\ &= \frac{1}{2x^6 y^2 z} \end{aligned}$$

$$95. \left(\frac{2x^{-3}}{3y^{-1}} \right)^{-2} = \left(\frac{3y}{2x^3} \right)^{-2} = \left(\frac{3x^3}{2y} \right)^2 = \frac{3^2 x^6}{2^2 y^2} = \frac{9x^6}{4y^2}$$

$$\begin{aligned} 96. \left(\frac{5x^{-2}}{6y^{-2}} \right)^{-3} &= \left(\frac{5y^2}{6x^2} \right)^{-3} = \left(\frac{6x^2}{5y^2} \right)^3 \\ &= \frac{6^3 (x^2)^3}{5^3 (y^2)^3} = \frac{216x^6}{125y^6} \end{aligned}$$

$$97. 2xy^{-1} = \frac{2x}{y} = \frac{2(2)}{(-1)} = -4$$

$$98. -3x^{-1}y = \frac{-3y}{x} = \frac{-3(-1)}{(2)} = \frac{3}{2}$$

$$99. x^2 + y^2 = (2)^2 + (-1)^2 = 4 + 1 = 5$$

$$100. x^2 y^2 = (2)^2 (-1)^2 = 4 \cdot 1 = 4$$

$$101. (xy)^2 = (2 \cdot (-1))^2 = (-2)^2 = 4$$

$$102. (x+y)^2 = (2+(-1))^2 = (1)^2 = 1$$

$$103. \sqrt{x^2} = |x| = |2| = 2$$

$$104. (\sqrt{x})^2 = x = 2$$

$$105. \sqrt{x^2 + y^2} = \sqrt{(2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$$

$$106. \sqrt{x^2} + \sqrt{y^2} = |x| + |y| = |2| + |-1| = 2 + 1 = 3$$

$$107. x^y = 2^{-1} = \frac{1}{2}$$

108. $y^x = (-1)^2 = 1$

109. If $x = 2$,

$$2x^3 - 3x^2 + 5x - 4 = 2 \cdot 2^3 - 3 \cdot 2^2 + 5 \cdot 2 - 4$$

$$= 16 - 12 + 10 - 4$$

$$= 10$$

If $x = 1$,

$$2x^3 - 3x^2 + 5x - 4 = 2 \cdot 1^3 - 3 \cdot 1^2 + 5 \cdot 1 - 4$$

$$= 2 - 3 + 5 - 4$$

$$= 0$$

110. If $x = 1$,

$$4x^3 + 3x^2 - x + 2 = 4 \cdot 1^3 + 3 \cdot 1^2 - 1 + 2$$

$$= 4 + 3 - 1 + 2$$

$$= 8$$

If $x = 2$,

$$4x^3 + 3x^2 - x + 2 = 4 \cdot 2^3 + 3 \cdot 2^2 - 2 + 2$$

$$= 32 + 12 - 2 + 2$$

$$= 44$$

111. $\frac{(666)^4}{(222)^4} = \left(\frac{666}{222}\right)^4 = 3^4 = 81$

112. $(0.1)^3(20)^3 = \left(\frac{1}{10}\right)^3 \cdot (2 \cdot 10)^3$

$$= \frac{1}{10^3} \cdot 2^3 \cdot 10^3$$

$$= 2^3 = 8$$

113. $(8.2)^6 \approx 304,006.671$

114. $(3.7)^5 \approx 693.440$

115. $(6.1)^{-3} \approx 0.004$

116. $(2.2)^{-5} \approx 0.019$

117. $(-2.8)^6 \approx 481.890$

118. $-(2.8)^6 \approx -481.890$

119. $(-8.11)^{-4} \approx 0.000$

120. $-(8.11)^{-4} \approx -0.000$

121. $454.2 = 4.542 \times 10^2$

122. $32.14 = 3.214 \times 10^1$

123. $0.013 = 1.3 \times 10^{-2}$

124. $0.00421 = 4.21 \times 10^{-3}$

125. $32,155 = 3.2155 \times 10^4$

126. $21,210 = 2.121 \times 10^4$

127. $0.000423 = 4.23 \times 10^{-4}$

128. $0.0514 = 5.14 \times 10^{-2}$

129. $6.15 \times 10^4 = 61,500$

130. $9.7 \times 10^3 = 9700$

131. $1.214 \times 10^{-3} = 0.001214$

132. $9.88 \times 10^{-4} = 0.000988$

133. $1.1 \times 10^8 = 110,000,000$

134. $4.112 \times 10^2 = 411.2$

135. $8.1 \times 10^{-2} = 0.081$

136. $6.453 \times 10^{-1} = 0.6453$

137. $A = lw$

138. $P = 2(l + w)$

139. $C = \pi d$

140. $A = \frac{1}{2}bh$

141. $A = \frac{\sqrt{3}}{4}x^2$

142. $P = 3x$

143. $V = \frac{4}{3}\pi r^3$

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144. $S = 4\pi r^2$

145. $V = x^3$

146. $S = 6x^2$

147. a. If $x = 1000$,
 $C = 4000 + 2x$
 $= 4000 + 2(1000)$
 $= 4000 + 2000$
 $= \$6000$
 The cost of producing 1000 watches is \$6000.

b. If $x = 2000$,
 $C = 4000 + 2x$
 $= 4000 + 2(2000)$
 $= 4000 + 4000$
 $= \$8000$
 The cost of producing 2000 watches is \$8000.

148. $210 + 80 - 120 + 25 - 60 - 32 - 5 = \98
 His balance at the end of the month was \$98.

149. We want the difference between x and 4 to be at least 6 units. Since we don't care whether the value for x is larger or smaller than 4, we take the absolute value of the difference. We want the inequality to be non-strict since we are dealing with an 'at least' situation. Thus, we have
 $|x - 4| \geq 6$

150. We want the difference between x and 2 to be more than 5 units. Since we don't care whether the value for x is larger or smaller than 2, we take the absolute value of the difference. We want the inequality to be strict since we are dealing with a 'more than' situation. Thus, we have
 $|x - 2| > 5$

151. a. $|x - 110| = |108 - 110| = |-2| = 2 \leq 5$
 108 volts is acceptable.

b. $|x - 110| = |104 - 110| = |-6| = 6 > 5$
 104 volts is not acceptable.

152. a. $|x - 220| = |214 - 220| = |-6| = 6 \leq 8$
 214 volts is acceptable.

b. $|x - 220| = |209 - 220| = |-11| = 11 > 8$
 209 volts is not acceptable.

153. a. $|x - 3| = |2.999 - 3|$
 $= |-0.001|$
 $= 0.001 \leq 0.01$
 A radius of 2.999 centimeters is acceptable.

b. $|x - 3| = |2.89 - 3|$
 $= |-0.11|$
 $= 0.11 \not\leq 0.01$
 A radius of 2.89 centimeters is not acceptable.

154. a. $|x - 98.6| = |97 - 98.6|$
 $= |-1.6|$
 $= 1.6 \geq 1.5$
 97°F is unhealthy.

b. $|x - 98.6| = |100 - 98.6|$
 $= |1.4|$
 $= 1.4 < 1.5$
 100°F is not unhealthy.

155. The distance from Earth to the Moon is about
 $4 \times 10^8 = 400,000,000$ meters.

156. The height of Mt. Everest is about
 $8848 = 8.848 \times 10^3$ meters.

157. The wavelength of visible light is about
 $5 \times 10^{-7} = 0.0000005$ meters.

158. The diameter of an atom is about
 $1 \times 10^{-10} = 0.0000000001$ meters.

159. The diameter is about $0.0403 = 4.03 \times 10^{-2}$ inches.

160. The tiniest motor is less than $0.00004 = 4 \times 10^{-5}$ millimeters tall.

161. $186,000 \cdot 60 \cdot 60 \cdot 24 \cdot 365$

$$= (1.86 \times 10^5) (6 \times 10^1)^2 (2.4 \times 10^1) (3.65 \times 10^2)$$

$$= 586.5696 \times 10^{10} = 5.865696 \times 10^{12}$$

There are about 5.9×10^{12} miles in one light-year.

162. $\frac{93,000,000}{186,000} = \frac{9.3 \times 10^7}{1.86 \times 10^5} = 5 \times 10^2$

$$= 500 \text{ seconds} \approx 8 \text{ min. } 20 \text{ sec.}$$

It takes about 8 minutes 20 seconds for a beam of light to reach Earth from the Sun.

163. $\frac{1}{3} = 0.333333 \dots > 0.333$

$\frac{1}{3}$ is larger by approximately 0.0003333 ...

164. $\frac{2}{3} = 0.666666 \dots > 0.666$

$\frac{2}{3}$ is larger by approximately 0.000666 ...

165. $(5.24 \times 10^6)(6.5 \times 10^{13}) = 34.06 \times 10^{19}$
 $= 3.406 \times 10^{20}$

166. $\frac{1.62 \times 10^{-4}}{4.5 \times 10^{-10}} = \frac{1.62}{4.5} \times \frac{10^{-4}}{10^{-10}} = 0.36 \times 10^6$
 $= 3.6 \times 10^5$

167. No. For any positive number a , the value $\frac{a}{2}$ is smaller and therefore closer to 0.

168. We are given that $1 < x^2 < 10$. This implies that $1 < x < \sqrt{10}$. Since $x < \sqrt{10} \approx 3.162$ and $x > \pi \approx 3.142$, the number could be 3.15 or 3.16 (which are between 1 and 10 as required). The number could also be 3.14 since numbers such as 3.146 which lie between π and $\sqrt{10}$ would equal 3.14 when truncated to two decimal places.

169. Answers will vary.

170. Answers will vary.
 $5 < 8$ is a true statement because 5 is further to the left than 8 on a real number line.

Section R.3

1. right; hypotenuse

2. $A = \frac{1}{2}bh$

3. $C = 2\pi r$

4. similar

5. c

6. b

7. True.

8. True. $6^2 + 8^2 = 36 + 64 = 100 = 10^2$

9. False; the surface area of a sphere of radius r is given by $V = 4\pi r^2$.

10. True. The lengths of the corresponding sides are equal.

11. True. Two corresponding angles are equal.

12. False. The sides are not proportional.

13. $a = 5, b = 12,$
 $c^2 = a^2 + b^2$
 $= 5^2 + 12^2$
 $= 25 + 144$
 $= 169 \Rightarrow c = 13$

14. $a = 6, b = 8,$
 $c^2 = a^2 + b^2$
 $= 6^2 + 8^2$
 $= 36 + 64$
 $= 100 \Rightarrow c = 10$

15. $a = 10, b = 24,$
 $c^2 = a^2 + b^2$
 $= 10^2 + 24^2$
 $= 100 + 576$
 $= 676 \Rightarrow c = 26$

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16. $a = 4$, $b = 3$,

$$c^2 = a^2 + b^2$$

$$= 4^2 + 3^2$$

$$= 16 + 9$$

$$= 25 \Rightarrow c = 5$$

17. $a = 7$, $b = 24$,

$$c^2 = a^2 + b^2$$

$$= 7^2 + 24^2$$

$$= 49 + 576$$

$$= 625 \Rightarrow c = 25$$

18. $a = 14$, $b = 48$,

$$c^2 = a^2 + b^2$$

$$= 14^2 + 48^2$$

$$= 196 + 2304$$

$$= 2500 \Rightarrow c = 50$$

19. $5^2 = 3^2 + 4^2$

$$25 = 9 + 16$$

$$25 = 25$$

The given triangle is a right triangle. The hypotenuse is 5.

20. $10^2 = 6^2 + 8^2$

$$100 = 36 + 64$$

$$100 = 100$$

The given triangle is a right triangle. The hypotenuse is 10.

21. $6^2 = 4^2 + 5^2$

$$36 = 16 + 25$$

$$36 = 41 \text{ false}$$

The given triangle is not a right triangle.

22. $3^2 = 2^2 + 2^2$

$$9 = 4 + 4$$

$$9 = 8 \text{ false}$$

The given triangle is not a right triangle.

23. $25^2 = 7^2 + 24^2$

$$625 = 49 + 576$$

$$625 = 625$$

The given triangle is a right triangle. The hypotenuse is 25.

24. $26^2 = 10^2 + 24^2$

$$676 = 100 + 576$$

$$676 = 676$$

The given triangle is a right triangle. The hypotenuse is 26.

25. $6^2 = 3^2 + 4^2$

$$36 = 9 + 16$$

$$36 = 25 \text{ false}$$

The given triangle is not a right triangle.

26. $7^2 = 5^2 + 4^2$

$$49 = 25 + 16$$

$$49 = 41 \text{ false}$$

The given triangle is not a right triangle.

27. $A = l \cdot w = 6 \cdot 7 = 42 \text{ in}^2$

28. $A = l \cdot w = 9 \cdot 4 = 36 \text{ cm}^2$

29. $A = \frac{1}{2}b \cdot h = \frac{1}{2}(14)(4) = 28 \text{ in}^2$

30. $A = \frac{1}{2}b \cdot h = \frac{1}{2}(4)(9) = 18 \text{ cm}^2$

31. $A = \pi r^2 = \pi(5)^2 = 25\pi \text{ m}^2$

$$C = 2\pi r = 2\pi(5) = 10\pi \text{ m}$$

32. $A = \pi r^2 = \pi(2)^2 = 4\pi \text{ ft}^2$

$$C = 2\pi r = 2\pi(2) = 4\pi \text{ ft}$$

33. $V = lwh = 6 \cdot 8 \cdot 5 = 240 \text{ ft}^3$

$$S = 2lw + 2lh + 2wh$$

$$= 2(6)(8) + 2(6)(5) + 2(8)(5)$$

$$= 96 + 60 + 80$$

$$= 236 \text{ ft}^2$$

34. $V = lwh = 9 \cdot 4 \cdot 8 = 288 \text{ in}^3$

$$S = 2lw + 2lh + 2wh$$

$$= 2(9)(4) + 2(9)(8) + 2(4)(8)$$

$$= 72 + 144 + 64$$

$$= 280 \text{ in}^2$$

$$35. \quad V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 5^3 = \frac{500}{3}\pi \text{ cm}^3$$

$$S = 4\pi r^2 = 4\pi \cdot 5^2 = 100\pi \text{ cm}^2$$

$$36. \quad V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 3^3 = 36\pi \text{ ft}^3$$

$$S = 4\pi r^2 = 4\pi \cdot 3^2 = 36\pi \text{ ft}^2$$

$$37. \quad V = \pi r^2 h = \pi(9)^2(8) = 648\pi \text{ in}^3$$

$$S = 2\pi r^2 + 2\pi rh$$

$$= 2\pi(9)^2 + 2\pi(9)(8)$$

$$= 162\pi + 144\pi$$

$$= 306\pi \text{ in}^2$$

$$38. \quad V = \pi r^2 h = \pi(8)^2(9) = 576\pi \text{ in}^3$$

$$S = 2\pi r^2 + 2\pi rh$$

$$= 2\pi(8)^2 + 2\pi(8)(9)$$

$$= 128\pi + 144\pi$$

$$= 272\pi \text{ in}^2$$

39. The diameter of the circle is 2, so its radius is 1.

$$A = \pi r^2 = \pi(1)^2 = \pi \text{ square units}$$

40. The diameter of the circle is 2, so its radius is 1.

$$A = 2^2 - \pi(1)^2 = 4 - \pi \text{ square units}$$

41. The diameter of the circle is the length of the diagonal of the square.

$$d^2 = 2^2 + 2^2$$

$$= 4 + 4$$

$$= 8$$

$$d = \sqrt{8} = 2\sqrt{2}$$

$$r = \frac{d}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

The area of the circle is:

$$A = \pi r^2 = \pi(\sqrt{2})^2 = 2\pi \text{ square units}$$

42. The diameter of the circle is the length of the diagonal of the square.

$$d^2 = 2^2 + 2^2$$

$$= 4 + 4$$

$$= 8$$

$$d = \sqrt{8} = 2\sqrt{2}$$

$$r = \frac{d}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

The area is:

$$A = \pi(\sqrt{2})^2 - 2^2 = 2\pi - 4 \text{ square units } 43.$$

Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{8}{4} = \frac{x}{2}$$

$$\frac{8 \cdot 2}{4} = x$$

$$4 = x$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A = 90^\circ$, $B = 60^\circ$, and $C = 30^\circ$.

44. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{6}{12} = \frac{x}{16}$$

$$\frac{6 \cdot 16}{12} = x$$

$$8 = x$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A = 30^\circ$, $B = 75^\circ$, and $C = 75^\circ$.

45. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{30}{20} = \frac{x}{45}$$

$$\frac{30 \cdot 45}{20} = x$$

$$\frac{135}{2} = x \text{ or } x = 67.5$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A = 60^\circ$, $B = 95^\circ$, and $C = 25^\circ$.

Chapter R: Review

46. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{8}{10} = \frac{x}{50}$$

$$\frac{8 \cdot 50}{10} = x$$

$$40 = x$$

In addition, corresponding angles must have the same angle measure. Therefore, we have

$$A = 50^\circ, B = 125^\circ, \text{ and } C = 5^\circ.$$

47. The total distance traveled is 4 times the circumference of the wheel.
Total Distance = $4C = 4(\pi d) = 4\pi \cdot 16$
 $= 64\pi \approx 201.1 \text{ inches} \approx 16.8 \text{ feet}$

48. The distance traveled in one revolution is the circumference of the disk 4π .

The number of revolutions =

$$\frac{\text{dist. traveled}}{\text{circumference}} = \frac{20}{4\pi} = \frac{5}{\pi} \approx 1.6 \text{ revolutions}$$

49. Area of the border = area of EFGH – area of ABCD
 $= 10^2 - 6^2 = 100 - 36 = 64 \text{ ft}^2$
50. FG = 4 feet; BG = 4 feet and BC = 10 feet, so CG = 6 feet. The area of the triangle CGF is:
- $$A = \frac{1}{2} \cdot (4)(6) = 12 \text{ ft}^2$$

51. Area of the window = area of the rectangle + area of the semicircle.

$$A = (6)(4) + \frac{1}{2} \cdot \pi \cdot 2^2 = 24 + 2\pi \approx 30.28 \text{ ft}^2$$

Perimeter of the window = 2 heights + width + one-half the circumference.

$$P = 2(6) + 4 + \frac{1}{2} \cdot \pi(4) = 12 + 4 + 2\pi$$

$$= 16 + 2\pi \approx 22.28 \text{ feet}$$

52. Area of the deck = area of the pool and deck – area of the pool.

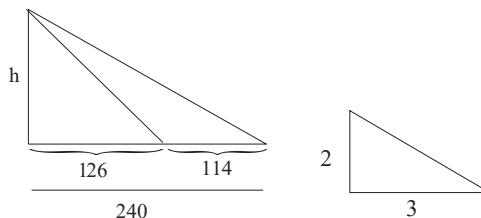
$$A = \pi(13)^2 - \pi(10)^2 = 169\pi - 100\pi$$

$$= 69\pi \text{ ft}^2 \approx 216.77 \text{ ft}^2$$

The amount of fence is the circumference of the circle with radius 13 feet.

$$C = 2\pi(13) = 26\pi \text{ ft} \approx 81.68 \text{ ft}$$

53. We can form similar triangles using the Great Pyramid's height/shadow and Thales' height/shadow:



This allows us to write

$$\frac{h}{240} = \frac{2}{3}$$

$$h = \frac{2 \cdot 240}{3} = 160$$

The height of the Great Pyramid is 160 paces.

54. Let x = the approximate distance from San Juan to Hamilton and y = the approximate distance from Hamilton to Fort Lauderdale. Using similar triangles, we get

$$\frac{1046}{58} = \frac{x}{53.5} \quad \frac{1046}{58} = \frac{y}{57}$$

$$\frac{1046 \cdot 53.5}{58} = x \quad \frac{1046 \cdot 57}{58} = y$$

$$964.8 \approx x \quad 1028.0 \approx y$$

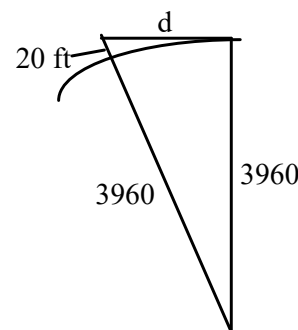
The approximate distance between San Juan and Hamilton is 965 miles and the approximate distance between Hamilton and Fort Lauderdale is 1028 miles.

55. Convert 20 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$20 \text{ feet} = 20 \text{ feet} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.003788 \text{ miles}$$

$$d^2 = (3960 + 0.003788)^2 - 3960^2 = 30 \text{ sq. miles}$$

$$d \approx 5.477 \text{ miles}$$

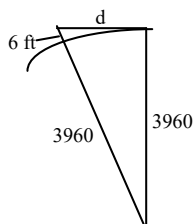


56. Convert 6 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$6 \text{ feet} = 6 \text{ feet} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.001136 \text{ miles}$$

$$d^2 = (3960 + 0.001136)^2 - 3960^2 = 9 \text{ sq. miles}$$

$$d \approx 3 \text{ miles}$$



57. Convert 100 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$100 \text{ feet} = 100 \text{ feet} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.018939 \text{ miles}$$

$$d^2 = (3960 + 0.018939)^2 - 3960^2 \approx 150 \text{ sq. miles}$$

$$d \approx 12.2 \text{ miles}$$

Convert 150 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$150 \text{ feet} = 150 \text{ feet} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.028409 \text{ miles}$$

$$d^2 = (3960 + 0.028409)^2 - 3960^2 \approx 225 \text{ sq. miles}$$

$$d \approx 15.0 \text{ miles}$$

58. Given $m > 0$, $n > 0$ and $m > n$,

if $a = m^2 - n^2$, $b = 2mn$ and $c = m^2 + n^2$, then

$$a^2 + b^2 = (m^2 - n^2)^2 + (2mn)^2$$

$$= m^4 - 2m^2n^2 + n^4 + 4m^2n^2$$

$$= m^4 + 2m^2n^2 + n^4$$

$$\text{and } c^2 = (m^2 + n^2)^2 = m^4 + 2m^2n^2 + n^4$$

$\therefore a^2 + b^2 = c^2 \rightarrow a, b$ and c represent the sides of a right triangle.

59. $V = \pi r^2 h$

$$= \pi(10)^2(4.5)$$

$$= 450\pi \text{ ft}^3$$

So, $1 \text{ ft}^3 \approx 7.48052 \text{ gal}$ so

$$(450\pi \text{ ft}^3)(7.48052 \text{ gal/ft}^3) \approx 10,575 \text{ gal}$$

60. $10000(5.61458) = 56145.8 \text{ ft}^3$

$$V = \pi r^2 h$$

$$56145.8 = \pi(25)^2 h$$

$$h = \frac{56145.8}{625\pi} = 28.6 \text{ ft}$$

61. $A = \pi r^2$

$$A_2 = \pi(2r)^2$$

$$= \pi 4r^2$$

$$= 4\pi r^2 = 4A$$

If you double the radius, the area is four times the original area.

62. $V = \frac{4}{3}\pi r^3$

$$V_2 = \frac{4}{3}\pi(2r)^3$$

$$= \frac{4}{3}\pi \cdot 8r^3$$

$$= 8 \cdot \frac{4}{3}\pi r^3 = 8V$$

If you double the radius the volume is 8 times the original volume.

63. Let l = length of the rectangle and w = width of the rectangle.

Notice that

$$(l+w)^2 - (l-w)^2$$

$$= [(l+w) + (l-w)][(l+w) - (l-w)]$$

$$= (2l)(2w) = 4lw = 4A$$

$$\text{So } A = \frac{1}{4}[(l+w)^2 - (l-w)^2]$$

Since $(l-w)^2 \geq 0$, the largest area will occur when $l-w=0$ or $l=w$; that is, when the rectangle is a square. But

$$1000 = 2l + 2w = 2(l+w)$$

$$500 = l + w = 2l$$

$$250 = l = w$$

The largest possible area is $250^2 = 62500 \text{ sq ft}$.

A circular pool with circumference = 1000 feet

yields the equation: $2\pi r = 1000 \Rightarrow r = \frac{500}{\pi}$

The area enclosed by the circular pool is:

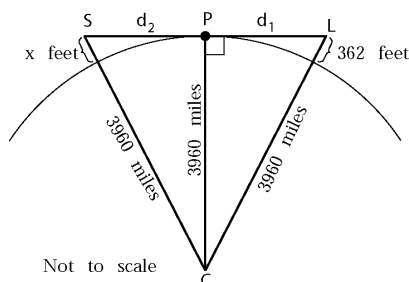
$$A = \pi r^2 = \pi \left(\frac{500}{\pi} \right)^2 = \frac{500^2}{\pi} \approx 79577.47 \text{ ft}^2$$

Thus, a circular pool will enclose the most area.

Chapter R: Review

64. Consider the diagram showing the lighthouse at point L, relative to the center of Earth, using the radius of Earth as 3960 miles. Let P refer to the furthest point on the horizon from which the light is visible. Note also that

$$362 \text{ feet} = \frac{362}{5280} \text{ miles.}$$



Apply the Pythagorean Theorem to $\triangle CPL$:

$$(3960)^2 + (d_1)^2 = \left(3960 + \frac{362}{5280}\right)^2$$

$$(d_1)^2 = \left(3960 + \frac{362}{5280}\right)^2 - (3960)^2$$

$$d_1 = \sqrt{\left(3960 + \frac{362}{5280}\right)^2 - (3960)^2} \approx 23.30 \text{ mi.}$$

Therefore, the light from the lighthouse can be seen at point P on the horizon, where point P is approximately 23.30 miles away from the lighthouse. Brochure information is slightly overstated.

Verify the ship information:

Let S refer to the ship's location, and let x equal the height, in feet, of the ship.

We need $d_1 + d_2 \geq 40$.

Since $d_1 \approx 23.30$ miles we need

$$d_2 \geq 40 - 23.30 = 16.70 \text{ miles.}$$

Apply the Pythagorean Theorem to $\triangle CPS$:

$$(3960)^2 + (16.7)^2 = (3960 + x)^2$$

$$\sqrt{(3960)^2 + (16.7)^2} = 3960 + x$$

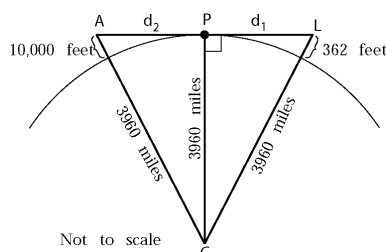
$$\sqrt{(3960)^2 + (16.7)^2} - 3960 = x$$

$$x \approx 0.035 \text{ miles}$$

$$x \approx 185.93 \text{ feet.}$$

The ship would have to be at least 186 feet tall to see the lighthouse from 40 miles away.

Verify the airplane information:



Let A refer to the airplane's location. The distance from the plane to point P is d_2 .

We want to show that $d_1 + d_2 \geq 120$.

Assume the altitude of the airplane is

$$10,000 \text{ feet} = \frac{10000}{5280} \text{ miles.}$$

Apply the Pythagorean Theorem to $\triangle CPA$:

$$(3960)^2 + (d_2)^2 = \left(3960 + \frac{10000}{5280}\right)^2$$

$$(d_2)^2 = \left(3960 + \frac{10000}{5280}\right)^2 - (3960)^2$$

$$d_2 = \sqrt{\left(3960 + \frac{10000}{5280}\right)^2 - (3960)^2} \approx 122.49 \text{ miles.}$$

$$\text{re, } d_1 + d_2 \approx 23.30 + 122.49 = 145.79 \geq 120.$$

The brochure information is slightly understated.

Note that a plane at an altitude of 6233 feet could see the lighthouse from 120 miles away.

Section R.4

- 4; 3
- $x^4 - 16$
- $x^3 - 8$
- a
- c
- False; monomials cannot have negative degrees.
- True
- False; the dividend = (quotient)(divisor) + remainder

9. $2x^3$ Monomial; Variable: x ; Coefficient: 2; Degree: 3
10. $-4x^2$ Monomial; Variable: x ; Coefficient: -4 ; Degree: 2
11. $\frac{8}{x} = 8x^{-1}$ Not a monomial; when written in the form ax^k , the variable has a negative exponent.
12. $-2x^{-3}$ Not a monomial; when written in the form ax^k , the variable has a negative exponent.
13. $-2xy^2$ Monomial; Variables: x, y ; Coefficient: -2 ; Degree: 3
14. $5x^2y^3$ Monomial; Variables: x, y ; Coefficient: 5; Degree: 5
15. $\frac{8x}{y} = 8xy^{-1}$ Not a monomial; when written in the form $ax^n y^m$, the exponent on the variable y is negative.
16. $-\frac{2x^2}{y^3} = -2x^2y^{-3}$ Not a monomial; when written in the form $ax^n y^m$, the exponent on the variable y is negative.
17. $x^2 + y^2$ Not a monomial; the expression contains more than one term. This expression is a binomial.
18. $3x^2 + 4$ Not a monomial; the expression contains more than one term. This expression is a binomial.
19. $3x^2 - 5$ Polynomial; Degree: 2
20. $1 - 4x$ Polynomial; Degree: 1
21. 5 Polynomial; Degree: 0
22. $-\pi$ Polynomial; Degree: 0
23. $3x^2 - \frac{5}{x}$ Not a polynomial; the variable in the denominator results in an exponent that is not a nonnegative integer.
24. $\frac{3}{x} + 2$ Not a polynomial; the variable in the denominator results in an exponent that is not a nonnegative integer.
25. $2y^3 - \sqrt{2}$ Polynomial; Degree: 3
26. $10z^2 + z$ Polynomial; Degree: 2
27. $\frac{x^2 + 5}{x^3 - 1}$ Not a polynomial; the polynomial in the denominator has a degree greater than 0.
28. $\frac{3x^3 + 2x - 1}{x^2 + x + 1}$ Not a polynomial; the polynomial in the denominator has a degree greater than 0.
29. $(x^2 + 6x + 8) + (3x^2 - 4x + 7)$
 $= (x^2 + 3x^2) + (6x - 4x) + (8 + 7)$
 $= 4x^2 + 2x + 15$
30. $(x^3 + 3x^2 + 2) + (x^2 - 4x + 4)$
 $= x^3 + (3x^2 + x^2) + (-4x) + (2 + 4)$
 $= x^3 + 4x^2 - 4x + 6$
31. $(x^3 - 2x^2 + 5x + 10) - (2x^2 - 4x + 3)$
 $= x^3 - 2x^2 + 5x + 10 - 2x^2 + 4x - 3$
 $= x^3 + (-2x^2 - 2x^2) + (5x + 4x) + (10 - 3)$
 $= x^3 - 4x^2 + 9x + 7$
32. $(x^2 - 3x - 4) - (x^3 - 3x^2 + x + 5)$
 $= x^2 - 3x - 4 - x^3 + 3x^2 - x - 5$
 $= -x^3 + (x^2 + 3x^2) + (-3x - x) + (-4 - 5)$
 $= -x^3 + 4x^2 - 4x - 9$
33. $(6x^5 + x^3 + x) + (5x^4 - x^3 + 3x^2)$
 $= 6x^5 + 5x^4 + 3x^2 + x$
34. $(10x^5 - 8x^2) + (3x^3 - 2x^2 + 6)$
 $= 10x^5 + 3x^3 - 10x^2 + 6$
35. $(x^2 - 6x + 4) + 3(2x^2 + x - 5)$
 $= x^2 - 6x + 4 + 6x^2 + 3x - 15$
 $= 7x^2 - 3x - 11$

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$$\begin{aligned} 36. \quad & -2(x^2 + x + 1) + (-5x^2 - x + 2) \\ & = -2x^2 - 2x - 2 - 5x^2 - x + 2 \\ & = -7x^2 - 3x \end{aligned}$$

$$\begin{aligned} 37. \quad & 6(x^3 + x^2 - 3) - 4(2x^3 - 3x^2) \\ & = 6x^3 + 6x^2 - 18 - 8x^3 + 12x^2 \\ & = -2x^3 + 18x^2 - 18 \end{aligned}$$

$$\begin{aligned} 38. \quad & 8(4x^3 - 3x^2 - 1) - 6(4x^3 + 8x - 2) \\ & = 32x^3 - 24x^2 - 8 - 24x^3 - 48x + 12 \\ & = 8x^3 - 24x^2 - 48x + 4 \end{aligned}$$

$$\begin{aligned} 39. \quad & (x^2 - x + 2) + (2x^2 - 3x + 5) - (x^2 + 1) \\ & = x^2 - x + 2 + 2x^2 - 3x + 5 - x^2 - 1 \\ & = 2x^2 - 4x + 6 \end{aligned}$$

$$\begin{aligned} 40. \quad & (x^2 + 1) - (4x^2 + 5) + (x^2 + x - 2) \\ & = x^2 + 1 - 4x^2 - 5 + x^2 + x - 2 \\ & = -2x^2 + x - 6 \end{aligned}$$

$$\begin{aligned} 41. \quad & 7(y^2 - 5y + 3) - 4(3 - y^2) \\ & = 7y^2 - 35y + 21 - 12 + 4y^2 \\ & = 11y^2 - 35y + 9 \end{aligned}$$

$$\begin{aligned} 42. \quad & 8(1 - y^3) + 4(1 + y + y^2 + y^3) \\ & = 8 - 8y^3 + 4 + 4y + 4y^2 + 4y^3 \\ & = -4y^3 + 4y^2 + 4y + 12 \end{aligned}$$

$$43. \quad x^2(x^2 + 2x - 5) = x^4 + 2x^3 - 5x^2$$

$$44. \quad 4x^2(x^3 - x + 2) = 4x^5 - 4x^3 + 8x^2$$

$$45. \quad -2x^2(4x^3 + 5) = -8x^5 - 10x^2$$

$$46. \quad 5x^3(3x - 4) = 15x^4 - 20x^3$$

$$\begin{aligned} 47. \quad & (x+1)(x^2 + 2x - 4) \\ & = x(x^2 + 2x - 4) + 1(x^2 + 2x - 4) \\ & = x^3 + 2x^2 - 4x + x^2 + 2x - 4 \\ & = x^3 + 3x^2 - 2x - 4 \end{aligned}$$

$$\begin{aligned} 48. \quad & (2x-3)(x^2 + x + 1) \\ & = 2x(x^2 + x + 1) - 3(x^2 + x + 1) \\ & = 2x^3 + 2x^2 + 2x - 3x^2 - 3x - 3 \\ & = 2x^3 - x^2 - x - 3 \end{aligned}$$

$$\begin{aligned} 49. \quad & (x+2)(x+4) = x^2 + 4x + 2x + 8 \\ & = x^2 + 6x + 8 \end{aligned}$$

$$\begin{aligned} 50. \quad & (x+3)(x+5) = x^2 + 5x + 3x + 15 \\ & = x^2 + 8x + 15 \end{aligned}$$

$$\begin{aligned} 51. \quad & (2x+7)(x+5) = 2x^2 + 7x + 10x + 35 \\ & = 2x^2 + 17x + 35 \end{aligned}$$

$$\begin{aligned} 52. \quad & (3x+1)(2x+1) = 6x^2 + 3x + 2x + 1 \\ & = 6x^2 + 5x + 1 \end{aligned}$$

$$\begin{aligned} 53. \quad & (x-4)(x+2) = x^2 + 2x - 4x - 8 \\ & = x^2 - 2x - 8 \end{aligned}$$

$$\begin{aligned} 54. \quad & (x+4)(x-2) = x^2 - 2x + 4x - 8 \\ & = x^2 + 2x - 8 \end{aligned}$$

$$\begin{aligned} 55. \quad & (x-6)(x-3) = x^2 - 6x - 3x + 18 \\ & = x^2 - 9x + 18 \end{aligned}$$

$$\begin{aligned} 56. \quad & (x-5)(x-1) = x^2 - x - 5x + 5 \\ & = x^2 - 6x + 5 \end{aligned}$$

$$\begin{aligned} 57. \quad & (2x+3)(x-2) = 2x^2 - 4x + 3x - 6 \\ & = 2x^2 - x - 6 \end{aligned}$$

$$\begin{aligned} 58. \quad & (2x-4)(3x+1) = 6x^2 + 2x - 12x - 4 \\ & = 6x^2 - 10x - 4 \end{aligned}$$

$$\begin{aligned} 59. \quad & (-3x+4)(x-2) = -3x^2 + 4x + 6x - 8 \\ & = -3x^2 + 10x - 8 \end{aligned}$$

$$\begin{aligned} 60. \quad & (-3x-1)(x+1) = -3x^2 - 3x - x - 1 \\ & = -3x^2 - 4x - 1 \end{aligned}$$

$$\begin{aligned} 61. \quad & (-x-5)(-2x-7) = 2x^2 + 10x + 7x + 35 \\ & = 2x^2 + 17x + 35 \end{aligned}$$

$$\begin{aligned} 62. \quad (-2x-3)(3-x) &= -6x + 2x^2 - 9 + 3x \\ &= 2x^2 - 3x - 9 \end{aligned}$$

$$\begin{aligned} 63. \quad (x-2y)(x+y) &= x^2 + xy - 2xy - 2y^2 \\ &= x^2 - xy - 2y^2 \end{aligned}$$

$$\begin{aligned} 64. \quad (2x+3y)(x-y) &= 2x^2 - 2xy + 3xy - 3y^2 \\ &= 2x^2 + xy - 3y^2 \end{aligned}$$

$$\begin{aligned} 65. \quad (-2x-3y)(3x+2y) &= -6x^2 - 4xy - 9xy - 6y^2 \\ &= -6x^2 - 13xy - 6y^2 \end{aligned}$$

$$\begin{aligned} 66. \quad (x-3y)(-2x+y) &= -2x^2 + xy + 6xy - 3y^2 \\ &= -2x^2 + 7xy - 3y^2 \end{aligned}$$

$$67. \quad (x-7)(x+7) = x^2 - 7^2 = x^2 - 49$$

$$68. \quad (x-1)(x+1) = x^2 - 1^2 = x^2 - 1$$

$$69. \quad (2x+3)(2x-3) = (2x)^2 - 3^2 = 4x^2 - 9$$

$$70. \quad (3x+2)(3x-2) = (3x)^2 - 2^2 = 9x^2 - 4$$

$$71. \quad (x+4)^2 = x^2 + 2 \cdot x \cdot 4 + 4^2 = x^2 + 8x + 16$$

$$72. \quad (x+5)^2 = x^2 + 2 \cdot x \cdot 5 + 5^2 = x^2 + 10x + 25$$

$$73. \quad (x-4)^2 = x^2 - 2 \cdot x \cdot 4 + 4^2 = x^2 - 8x + 16$$

$$74. \quad (x-5)^2 = x^2 - 2 \cdot x \cdot 5 + 5^2 = x^2 - 10x + 25$$

$$75. \quad (3x+4)(3x-4) = (3x)^2 - 4^2 = 9x^2 - 16$$

$$76. \quad (5x-3)(5x+3) = (5x)^2 - 3^2 = 25x^2 - 9$$

$$\begin{aligned} 77. \quad (2x-3)^2 &= (2x)^2 - 2(2x)(3) + 3^2 \\ &= 4x^2 - 12x + 9 \end{aligned}$$

$$\begin{aligned} 78. \quad (3x-4)^2 &= (3x)^2 - 2(3x)(4) + 4^2 \\ &= 9x^2 - 24x + 16 \end{aligned}$$

$$79. \quad (x+y)(x-y) = (x)^2 - (y)^2 = x^2 - y^2$$

$$80. \quad (x+3y)(x-3y) = (x)^2 - (3y)^2 = x^2 - 9y^2$$

$$81. \quad (3x+y)(3x-y) = (3x)^2 - (y)^2 = 9x^2 - y^2$$

$$82. \quad (3x+4y)(3x-4y) = (3x)^2 - (4y)^2 = 9x^2 - 16y^2$$

$$83. \quad (x+y)^2 = x^2 + 2xy + y^2$$

$$84. \quad (x-y)^2 = x^2 - 2xy + y^2$$

$$\begin{aligned} 85. \quad (x-2y)^2 &= x^2 + 2(x \cdot (-2y)) + (2y)^2 \\ &= x^2 - 4xy + 4y^2 \end{aligned}$$

$$\begin{aligned} 86. \quad (2x+3y)^2 &= (2x)^2 + 2(2x \cdot 3y) + (3y)^2 \\ &= 4x^2 + 12xy + 9y^2 \end{aligned}$$

$$\begin{aligned} 87. \quad (x-2)^3 &= x^3 - 3 \cdot x^2 \cdot 2 + 3 \cdot x \cdot 2^2 - 2^3 \\ &= x^3 - 6x^2 + 12x - 8 \end{aligned}$$

$$\begin{aligned} 88. \quad (x+1)^3 &= x^3 + 3 \cdot x^2 \cdot 1 + 3 \cdot x \cdot 1^2 + 1^3 \\ &= x^3 + 3x^2 + 3x + 1 \end{aligned}$$

$$\begin{aligned} 89. \quad (2x+1)^3 &= (2x)^3 + 3(2x)^2(1) + 3(2x) \cdot 1^2 + 1^3 \\ &= 8x^3 + 12x^2 + 6x + 1 \end{aligned}$$

$$\begin{aligned} 90. \quad (3x-2)^3 &= (3x)^3 - 3(3x)^2(2) + 3(3x) \cdot 2^2 - 2^3 \\ &= 27x^3 - 54x^2 + 36x - 8 \end{aligned}$$

$$\begin{array}{r} 4x^2 - 11x + 23 \\ 91. \quad x+2 \overline{) 4x^3 - 3x^2 + x + 1} \\ \underline{4x^3 + 8x^2} \\ -11x^2 + x \\ \underline{-11x^2 - 22x} \\ 23x + 1 \\ \underline{23x + 46} \\ -45 \end{array}$$

Check:

$$\begin{aligned} (x+2)(4x^2 - 11x + 23) &+ (-45) \\ &= 4x^3 - 11x^2 + 23x + 8x^2 - 22x + 46 - 45 \\ &= 4x^3 - 3x^2 + x + 1 \end{aligned}$$

The quotient is $4x^2 - 11x + 23$; the remainder is -45 .

Chapter R: Review

$$\begin{array}{r}
 3x^2 - 7x + 15 \\
 92. \quad x + 2 \overline{) 3x^3 - x^2 + x - 2} \\
 \underline{3x^3 + 6x^2} \\
 -7x^2 + x \\
 \underline{-7x^2 - 14x} \\
 15x - 2 \\
 \underline{15x + 30} \\
 -32
 \end{array}$$

Check:

$$\begin{aligned}
 (x+2)(3x^2-7x+15) + (-32) \\
 = 3x^3 - 7x^2 + 15x + 6x^2 - 14x + 30 - 32 \\
 = 3x^3 - x^2 + x - 2
 \end{aligned}$$

The quotient is $3x^2 - 7x + 15$; the remainder is -32 .

$$\begin{array}{r}
 4x - 3 \\
 93. \quad x^2 \overline{) 4x^3 - 3x^2 + x + 1} \\
 \underline{4x^3} \\
 -3x^2 + x + 1 \\
 \underline{-3x^2} \\
 x + 1
 \end{array}$$

Check:

$$(x^2)(4x-3) + (x+1) = 4x^3 - 3x^2 + x + 1$$

The quotient is $4x - 3$; the remainder is $x + 1$.

$$\begin{array}{r}
 3x - 1 \\
 94. \quad x^2 \overline{) 3x^3 - x^2 + x - 2} \\
 \underline{3x^3} \\
 -x^2 + x - 2 \\
 \underline{-x^2} \\
 x - 2
 \end{array}$$

Check:

$$(x^2)(3x-1) + (x-2) = 3x^3 - x^2 + x - 2$$

The quotient is $3x - 1$; the remainder is $x - 2$.

$$\begin{array}{r}
 5x^2 - 13 \\
 95. \quad x^2 + 2 \overline{) 5x^4 + 0x^3 - 3x^2 + x + 1} \\
 \underline{5x^4} + 10x^2 \\
 -13x^2 + x + 1 \\
 \underline{-13x^2} - 26 \\
 x + 27
 \end{array}$$

Check:

$$\begin{aligned}
 (x^2+2)(5x^2-13) + (x+27) \\
 = 5x^4 + 10x^2 - 13x^2 - 26 + x + 27 \\
 = 5x^4 - 3x^2 + x + 1
 \end{aligned}$$

The quotient is $5x^2 - 13$; the remainder is $x + 27$.

$$\begin{array}{r}
 5x^2 - 11 \\
 96. \quad x^2 + 2 \overline{) 5x^4 + 0x^3 - x^2 + x - 2} \\
 \underline{5x^4} + 10x^2 \\
 -11x^2 + x - 2 \\
 \underline{-11x^2} - 22 \\
 x + 20
 \end{array}$$

Check:

$$\begin{aligned}
 (x^2+2)(5x^2-11) + (x+20) \\
 = 5x^4 + 10x^2 - 11x^2 - 22 + x + 20 \\
 = 5x^4 - x^2 + x - 2
 \end{aligned}$$

The quotient is $5x^2 - 11$; the remainder is $x + 20$.

$$\begin{array}{r}
 2x^2 \\
 97. \quad 2x^3 - 1 \overline{) 4x^5 + 0x^4 + 0x^3 - 3x^2 + x + 1} \\
 \underline{4x^5} - 2x^2 \\
 -x^2 + x + 1
 \end{array}$$

Check:

$$\begin{aligned}
 (2x^3-1)(2x^2) + (-x^2+x+1) \\
 = 4x^5 - 2x^2 - x^2 + x + 1 = 4x^5 - 3x^2 + x + 1
 \end{aligned}$$

The quotient is $2x^2$; the remainder is $-x^2 + x + 1$.

$$98. \quad 3x^3 - 1 \overline{) 3x^5 + 0x^4 + 0x^3 - x^2 + x - 2}$$

$$\begin{array}{r} 3x^5 \\ \hline -x^2 \\ \hline x - 2 \end{array}$$

Check:

$$(3x^3 - 1)(x^2) + (x - 2) = 3x^5 - x^2 + x - 2$$

The quotient is x^2 ; the remainder is $x - 2$.

$$99. \quad 2x^2 + x + 1 \overline{) 2x^4 - 3x^3 + 0x^2 + x + 1}$$

$$\begin{array}{r} 2x^4 + x^3 + x^2 \\ \hline -4x^3 - x^2 + x \\ \hline -4x^3 - 2x^2 - 2x \\ \hline x^2 + 3x + 1 \\ \hline x^2 + \frac{1}{2}x + \frac{1}{2} \\ \hline \frac{5}{2}x + \frac{1}{2} \end{array}$$

Check:

$$(2x^2 + x + 1)(x^2 - 2x + \frac{1}{2}) + \frac{5}{2}x + \frac{1}{2}$$

$$= 2x^4 - 4x^3 + x^2 + x^3 - 2x^2 + \frac{1}{2}x$$

$$+ x^2 - 2x + \frac{1}{2} + \frac{5}{2}x + \frac{1}{2}$$

$$= 2x^4 - 3x^3 + x + 1$$

The quotient is $x^2 - 2x + \frac{1}{2}$; the remainder is

$$\frac{5}{2}x + \frac{1}{2}.$$

$$100. \quad 3x^2 + x + 1 \overline{) 3x^4 - x^3 + 0x^2 + x - 2}$$

$$\begin{array}{r} 3x^4 + x^3 + x^2 \\ \hline -2x^3 - x^2 + x \\ \hline -2x^3 - \frac{2}{3}x^2 - \frac{2}{3}x \\ \hline -\frac{1}{3}x^2 + \frac{5}{3}x - 2 \\ \hline -\frac{1}{3}x^2 - \frac{1}{9}x - \frac{1}{9} \\ \hline \frac{16}{9}x - \frac{17}{9} \end{array}$$

Check:

$$(3x^2 + x + 1)(x^2 - \frac{2}{3}x - \frac{1}{9}) + (\frac{16}{9}x - \frac{17}{9})$$

$$= 3x^4 + x^3 + x^2 - 2x^3 - \frac{2}{3}x^2 - \frac{2}{3}x$$

$$- \frac{1}{3}x^2 - \frac{1}{9}x - \frac{1}{9} + \frac{16}{9}x - \frac{17}{9}$$

$$= 3x^4 - x^3 + x - 2$$

The quotient is $x^2 - \frac{2}{3}x - \frac{1}{9}$; the remainder is

$$\frac{16}{9}x - \frac{17}{9}.$$

$$101. \quad x - 1 \overline{) -4x^2 - 3x - 3}$$

$$\begin{array}{r} -4x^2 + x^2 + 0x - 4 \\ \hline -4x^3 + 4x^2 \\ \hline -3x^2 \\ \hline -3x^2 + 3x \\ \hline -3x - 4 \\ \hline -3x + 3 \\ \hline -7 \end{array}$$

Check:

$$(x - 1)(-4x^2 - 3x - 3) + (-7)$$

$$= -4x^3 - 3x^2 - 3x + 4x^2 + 3x + 3 - 7$$

$$= -4x^3 + x^2 - 4$$

The quotient is $-4x^2 - 3x - 3$; the remainder is -7 .

$$102. \quad x - 1 \overline{) -3x^3 - 3x^2 - 3x - 5}$$

$$\begin{array}{r} -3x^4 + 3x^3 \\ \hline -3x^3 \\ \hline -3x^3 + 3x^2 \\ \hline -3x^2 - 2x \\ \hline -3x^2 + 3x \\ \hline -5x - 1 \\ \hline -5x + 5 \\ \hline -6 \end{array}$$

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Check:

$$\begin{aligned} & (x-1)(-3x^3-3x^2-3x-5)+(-6) \\ &= -3x^4-3x^3-3x^2-5x+3x^3+3x^2 \\ & \quad +3x+5-6 \\ &= -3x^4-2x-1 \end{aligned}$$

The quotient is $-3x^3-3x^2-3x-5$; the remainder is -6 .

$$\begin{array}{r} 103. \quad x^2+x+1 \overline{) x^4+0x^3-x^2+0x+1} \\ \underline{x^4+0x^3+x^2+0x+1} \\ -x^3-2x^2 \\ \underline{-x^3-x^2-x} \\ -x^2+x+1 \\ \underline{-x^2-x-1} \\ 2x+2 \end{array}$$

Check:

$$\begin{aligned} & (x^2+x+1)(x^2-x-1)+2x+2 \\ &= x^4+x^3+x^2-x^3-x^2-x-x^2-x \\ & \quad -1+2x+2 \\ &= x^4-x^2+1 \end{aligned}$$

The quotient is x^2-x-1 ; the remainder is $2x+2$.

$$\begin{array}{r} 104. \quad x^2-x+1 \overline{) x^4+0x^3-x^2+0x+1} \\ \underline{x^4-0x^3+x^2+0x+1} \\ x^3-2x^2 \\ \underline{x^3-x^2+x} \\ -x^2-x+1 \\ \underline{-x^2+x-1} \\ -2x+2 \end{array}$$

Check:

$$\begin{aligned} & (x^2-x+1)(x^2+x-1)+(-2x+2) \\ &= x^4+x^3-x^2-x^3-x^2+x+x^2+x \\ & \quad -1-2x+2 \\ &= x^4-x^2+1 \end{aligned}$$

The quotient is x^2+x-1 ; the remainder is $-2x+2$.

$$\begin{array}{r} 105. \quad x-a \overline{) x^3+0x^2+0x-a^3} \\ \underline{x^3-ax^2} \\ ax^2 \\ \underline{ax^2-a^2x} \\ a^2x-a^3 \\ \underline{a^2x-a^3} \\ 0 \end{array}$$

Check:

$$\begin{aligned} & (x-a)(x^2+ax+a^2)+0 \\ &= x^3+ax^2+a^2x-ax^2-a^2x-a^3 \\ &= x^3-a^3 \end{aligned}$$

The quotient is x^2+ax+a^2 ; the remainder is 0.

$$\begin{array}{r} 106. \quad x-a \overline{) x^5+0x^4+0x^3+0x^2+0x-a^5} \\ \underline{x^5-ax^4} \\ ax^4 \\ \underline{ax^4-a^2x^3} \\ a^2x^3 \\ \underline{a^2x^3-a^3x^2} \\ a^3x^2 \\ \underline{a^3x^2-a^4x} \\ a^4x-a^5 \\ \underline{a^4x-a^5} \\ 0 \end{array}$$

Section R.5: Factoring Polynomials

Check:

$$\begin{aligned}(x-a)(x^4+ax^3+a^2x^2+a^3x+a^4)+0 \\&= x^5+ax^4+a^2x^3+a^3x^2+a^4x-ax^4 \\&\quad -a^2x^3-a^3x^2-a^4x-a^5 \\&= x^5-a^5\end{aligned}$$

The quotient is $x^4+ax^3+a^2x^2+a^3x+a^4$; the remainder is 0.

107. $(3x-2k)(4x+3k)=12x^2+kx-96$
 $12x^2-8kx+9kx-6k^2=12x^2+kx-96$
 $kx-6k^2=kx-96$
 $-6k^2=-96$
 $k^2=16$
 $k=\pm 4$
108. The products $(x+y)(x-y)$ and $(z+w)(z-w)$ will each result in a binomial that is the difference of squares. The product of those resulting binomials will have 4 terms.
109. When we multiply polynomials $p_1(x)$ and $p_2(x)$, each term of $p_1(x)$ will be multiplied by each term of $p_2(x)$. So when the highest-powered term of $p_1(x)$ multiplies by the highest powered term of $p_2(x)$, the exponents on the variables in those terms will add according to the basic rules of exponents. Therefore, the highest powered term of the product polynomial will have degree equal to the sum of the degrees of $p_1(x)$ and $p_2(x)$.
110. When we add two polynomials $p_1(x)$ and $p_2(x)$, where the degree of $p_1(x) \neq$ the degree of $p_2(x)$, each term of $p_1(x)$ will be added to each term of $p_2(x)$. Since only the terms with equal degrees will combine via addition, the degree of the sum polynomial will be the degree of the highest powered term overall, that is, the degree of the polynomial that had the higher degree.
111. When we add two polynomials $p_1(x)$ and $p_2(x)$, where the degree of $p_1(x) =$ the degree

of $p_2(x)$, the new polynomial will have degree $\leq$ the degree of $p_1(x)$ and $p_2(x)$.

112. Answers will vary.

113. Answers will vary.

Section R.5

- $3x(x-2)(x+2)$
- prime
- c
- b
- d
- c
- True; x^2+4 is prime over the set of real numbers.
- False; $3x^3-2x^2-6x+4=(3x-2)(x^2-2)$
- $3x+6=3(x+2)$
- $7x-14=7(x-2)$
- $ax^2+a=a(x^2+1)$
- $ax-a=a(x-1)$
- $x^3+x^2+x=x(x^2+x+1)$
- $x^3-x^2+x=x(x^2-x+1)$
- $2x^2-2x=2x(x-1)$
- $3x^2-3x=3x(x-1)$
- $3x^2y-6xy^2+12xy=3xy(x-2y+4)$
- $60x^2y-48xy^2+72x^3y=12xy(5x-4y+6x^2)$
- $x^2-1=x^2-1^2=(x-1)(x+1)$
- $x^2-4=x^2-2^2=(x-2)(x+2)$

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21. $4x^2 - 1 = (2x)^2 - 1^2 = (2x-1)(2x+1)$

22. $9x^2 - 1 = (3x)^2 - 1^2 = (3x-1)(3x+1)$

23. $x^2 - 16 = x^2 - 4^2 = (x-4)(x+4)$

24. $x^2 - 25 = x^2 - 5^2 = (x-5)(x+5)$

25. $25x^2 - 4 = (5x-2)(5x+2)$

26. $36x^2 - 9 = 9(4x^2 - 1) = 9(2x-1)(2x+1)$

27. $x^2 + 2x + 1 = (x+1)^2$

28. $x^2 - 4x + 4 = (x-2)^2$

29. $x^2 + 4x + 4 = (x+2)^2$

30. $x^2 - 2x + 1 = (x-1)^2$

31. $x^2 - 10x + 25 = (x-5)^2$

32. $x^2 + 10x + 25 = (x+5)^2$

33. $4x^2 + 4x + 1 = (2x+1)^2$

34. $9x^2 + 6x + 1 = (3x+1)^2$

35. $16x^2 + 8x + 1 = (4x+1)^2$

36. $25x^2 + 10x + 1 = (5x+1)^2$

37. $x^3 - 27 = x^3 - 3^3 = (x-3)(x^2 + 3x + 9)$

38. $x^3 + 125 = x^3 + 5^3 = (x+5)(x^2 - 5x + 25)$

39. $x^3 + 27 = x^3 + 3^3 = (x+3)(x^2 - 3x + 9)$

40. $27 - 8x^3 = 3^3 - (2x)^3$
 $= (3-2x)(9+6x+4x^2)$
 $= -(2x-3)(4x^2+6x+9)$

41. $8x^3 + 27 = (2x)^3 + 3^3$
 $= (2x+3)(4x^2 - 6x + 9)$

42. $64 - 27x^3 = 4^3 - (3x)^3$
 $= (4-3x)(16+12x+9x^2)$
 $= -(3x-4)(9x^2+12x+16)$

43. $x^2 + 5x + 6 = (x+2)(x+3)$

44. $x^2 + 6x + 8 = (x+2)(x+4)$

45. $x^2 + 7x + 6 = (x+6)(x+1)$

46. $x^2 + 9x + 8 = (x+8)(x+1)$

47. $x^2 + 7x + 10 = (x+2)(x+5)$

48. $x^2 + 11x + 10 = (x+10)(x+1)$

49. $x^2 - 10x + 16 = (x-2)(x-8)$

50. $x^2 - 17x + 16 = (x-16)(x-1)$

51. $x^2 - 7x - 8 = (x+1)(x-8)$

52. $x^2 - 2x - 8 = (x+2)(x-4)$

53. $x^2 + 7x - 8 = (x+8)(x-1)$

54. $x^2 + 2x - 8 = (x+4)(x-2)$

55. $2x^2 + 4x + 3x + 6 = 2x(x+2) + 3(x+2)$
 $= (x+2)(2x+3)$

56. $3x^2 - 3x + 2x - 2 = 3x(x-1) + 2(x-1)$
 $= (x-1)(3x+2)$

57. $5x^2 - 15x + x - 3 = 5x(x-3) + 1(x-3)$
 $= (x-3)(5x+1)$

58. $3x^2 + 6x - x - 2 = 3x(x+2) - 1(x+2)$
 $= (x+2)(3x-1)$

59. $6x^2 + 21x + 8x + 28 = 3x(2x+7) + 4(2x+7)$
 $= (2x+7)(3x+4)$

Section R.5: Factoring Polynomials

$$60. \quad 9x^2 - 6x + 3x - 2 = 3x(3x - 2) + 1(3x - 2) \\ = (3x - 2)(3x + 1)$$

$$61. \quad 3x^2 + 4x + 1 = (3x + 1)(x + 1)$$

$$62. \quad 2x^2 + 3x + 1 = (2x + 1)(x + 1)$$

$$63. \quad 2z^2 + 9z + 7 = (2z + 7)(z + 1)$$

$$64. \quad 6z^2 + 5z + 1 = (3z + 1)(2z + 1)$$

$$65. \quad 5x^2 + 6x - 8 = (5x - 4)(x + 2)$$

$$66. \quad 3x^2 + 10x + 8 = (3x + 4)(x + 2)$$

$$67. \quad 5x^2 - 6x - 8 = (5x + 4)(x - 2)$$

$$68. \quad 3x^2 - 10x + 8 = (3x - 4)(x - 2)$$

$$69. \quad 5x^2 + 22x + 8 = (5x + 2)(x + 4)$$

$$70. \quad 3x^2 - 14x + 8 = (3x - 2)(x - 4)$$

$$71. \quad 5x^2 + 18x - 8 = (5x - 2)(x + 4)$$

$$72. \quad 3x^2 - 10x - 8 = (3x + 2)(x - 4)$$

73. Since b is 10 then we need half of 10 squared to be the last term in our trinomial. Thus

$$\frac{1}{2}(10) = 5; (5)^2 = 25$$

$$x^2 + 10x + 25 = (x + 5)^2$$

74. Since b is 14 then we need half of 14 squared to be the last term in our trinomial. Thus

$$\frac{1}{2}(14) = 7; (7)^2 = 49$$

$$p^2 + 14p + 49 = (p + 7)^2$$

75. Since b is -6 then we need half of -6 squared to be the last term in our trinomial. Thus

$$\frac{1}{2}(-6) = -3; (-3)^2 = 9$$

$$y^2 - 6y + 9 = (y - 3)^2$$

76. Since b is -4 then we need half of -4 squared to be the last term in our trinomial. Thus

$$\frac{1}{2}(-4) = -2; (-2)^2 = 4$$

$$x^2 - 4x + 4 = (x - 2)^2$$

77. Since b is $-\frac{1}{2}$ then we need half of $-\frac{1}{2}$ squared to be the last term in our trinomial. Thus

$$\frac{1}{2}\left(-\frac{1}{2}\right) = -\frac{1}{4}; \left(-\frac{1}{4}\right)^2 = \frac{1}{16}$$

$$x^2 - \frac{1}{2}x + \frac{1}{16} = \left(x - \frac{1}{4}\right)^2$$

78. Since b is $\frac{1}{3}$ then we need half of $\frac{1}{3}$ squared to be the last term in our trinomial. Thus

$$\frac{1}{2}\left(\frac{1}{3}\right) = \frac{1}{6}; \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

$$x^2 + \frac{1}{3}x + \frac{1}{36} = \left(x + \frac{1}{6}\right)^2$$

$$79. \quad x^2 - 36 = (x - 6)(x + 6)$$

$$80. \quad x^2 - 9 = (x - 3)(x + 3)$$

$$81. \quad 2 - 8x^2 = 2(1 - 4x^2) = 2(1 - 2x)(1 + 2x)$$

$$82. \quad 3 - 27x^2 = 3(1 - 9x^2) = 3(1 - 3x)(1 + 3x)$$

$$83. \quad x^2 + 11x + 10 = (x + 1)(x + 10)$$

$$84. \quad x^2 + 5x + 4 = (x + 4)(x + 1)$$

$$85. \quad x^2 - 10x + 21 = (x - 7)(x - 3)$$

$$86. \quad x^2 - 6x + 8 = (x - 2)(x - 4)$$

$$87. \quad 4x^2 - 8x + 32 = 4(x^2 - 2x + 8)$$

$$88. \quad 3x^2 - 12x + 15 = 3(x^2 - 4x + 5)$$

89. $x^2 + 4x + 16$ is prime over the reals because there are no factors of 16 whose sum is 4.

$$90. \quad x^2 + 12x + 36 = (x + 6)^2$$

$$91. \quad 15 + 2x - x^2 = -(x^2 - 2x - 15) = -(x - 5)(x + 3)$$

Chapter R: Review

92. $14 + 6x - x^2 = -(x^2 - 6x - 14)$ is prime over the integers because there are no factors of -14 whose sum is -6 .
93. $3x^2 - 12x - 36 = 3(x^2 - 4x - 12)$
 $= 3(x - 6)(x + 2)$
94. $x^3 + 8x^2 - 20x = x(x^2 + 8x - 20)$
 $= x(x + 10)(x - 2)$
95. $y^4 + 11y^3 + 30y^2 = y^2(y^2 + 11y + 30)$
 $= y^2(y + 5)(y + 6)$
96. $3y^3 - 18y^2 - 48y = 3y(y^2 - 6y - 16)$
 $= 3y(y + 2)(y - 8)$
97. $4x^2 + 12x + 9 = (2x + 3)^2$
98. $9x^2 - 12x + 4 = (3x - 2)^2$
99. $6x^2 + 8x + 2 = 2(3x^2 + 4x + 1)$
 $= 2(3x + 1)(x + 1)$
100. $8x^2 + 6x - 2 = 2(4x^2 + 3x - 1)$
 $= 2(4x - 1)(x + 1)$
101. $x^4 - 81 = (x^2)^2 - 9^2 = (x^2 - 9)(x^2 + 9)$
 $= (x - 3)(x + 3)(x^2 + 9)$
102. $x^4 - 1 = (x^2)^2 - 1^2 = (x^2 - 1)(x^2 + 1)$
 $= (x - 1)(x + 1)(x^2 + 1)$
103. $x^6 - 2x^3 + 1 = (x^3 - 1)^2$
 $= [(x - 1)(x^2 + x + 1)]^2$
 $= (x - 1)^2(x^2 + x + 1)^2$
104. $x^6 + 2x^3 + 1 = (x^3 + 1)^2$
 $= [(x + 1)(x^2 - x + 1)]^2$
 $= (x + 1)^2(x^2 - x + 1)^2$
105. $x^7 - x^5 = x^5(x^2 - 1) = x^5(x - 1)(x + 1)$
106. $x^8 - x^5 = x^5(x^3 - 1) = x^5(x - 1)(x^2 + x + 1)$
107. $16x^2 + 24x + 9 = (4x + 3)^2$
108. $9x^2 - 24x + 16 = (3x - 4)^2$
109. $5 + 16x - 16x^2 = -(16x^2 - 16x - 5)$
 $= -(4x - 5)(4x + 1)$
110. $5 + 11x - 16x^2 = -(16x^2 - 11x - 5)$
 $= -(16x + 5)(x - 1)$
111. $4y^2 - 16y + 15 = (2y - 5)(2y - 3)$
112. $9y^2 + 9y - 4 = (3y + 4)(3y - 1)$
113. $1 - 8x^2 - 9x^4 = -(9x^4 + 8x^2 - 1)$
 $= -(9x^2 - 1)(x^2 + 1)$
 $= -(3x - 1)(3x + 1)(x^2 + 1)$
114. $4 - 14x^2 - 8x^4 = -2(4x^4 + 7x^2 - 2)$
 $= -2(4x^2 - 1)(x^2 + 2)$
 $= -2(2x - 1)(2x + 1)(x^2 + 2)$
115. $x(x + 3) - 6(x + 3) = (x + 3)(x - 6)$
116. $5(3x - 7) + x(3x - 7) = (3x - 7)(x + 5)$
117. $(x + 2)^2 - 5(x + 2) = (x + 2)[(x + 2) - 5]$
 $= (x + 2)(x - 3)$
118. $(x - 1)^2 - 2(x - 1) = (x - 1)[(x - 1) - 2]$
 $= (x - 1)(x - 3)$
119. $(3x - 2)^3 - 27$
 $= (3x - 2)^3 - 3^3$
 $= [(3x - 2) - 3][(3x - 2)^2 + 3(3x - 2) + 9]$
 $= (3x - 5)(9x^2 - 12x + 4 + 9x - 6 + 9)$
 $= (3x - 5)(9x^2 - 3x + 7)$

$$\begin{aligned}
 120. \quad & (5x+1)^3 - 1 \\
 &= (5x+1)^3 - 1^3 \\
 &= [(5x+1)-1][(5x+1)^2 + (1)(5x+1)+1] \\
 &= 5x(25x^2 + 10x + 1 + 5x + 1 + 1) \\
 &= 5x(25x^2 + 15x + 3)
 \end{aligned}$$

$$\begin{aligned}
 121. \quad & 3(x^2 + 10x + 25) - 4(x+5) \\
 &= 3(x+5)^2 - 4(x+5) \\
 &= (x+5)[3(x+5) - 4] \\
 &= (x+5)(3x+15-4) \\
 &= (x+5)(3x+11)
 \end{aligned}$$

$$\begin{aligned}
 122. \quad & 7(x^2 - 6x + 9) + 5(x-3) \\
 &= 7(x-3)^2 + 5(x-3) \\
 &= (x-3)[7(x-3) + 5] \\
 &= (x-3)(7x-21+5) \\
 &= (x-3)(7x-16)
 \end{aligned}$$

$$\begin{aligned}
 123. \quad & x^3 + 2x^2 - x - 2 = x^2(x+2) - 1(x+2) \\
 &= (x+2)(x^2 - 1) \\
 &= (x+2)(x-1)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 124. \quad & x^3 - 3x^2 - x + 3 = x^2(x-3) - 1(x-3) \\
 &= (x-3)(x^2 - 1) \\
 &= (x-3)(x-1)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 125. \quad & x^4 - x^3 + x - 1 = x^3(x-1) + 1(x-1) \\
 &= (x-1)(x^3 + 1) \\
 &= (x-1)(x+1)(x^2 - x + 1)
 \end{aligned}$$

$$\begin{aligned}
 126. \quad & x^4 + x^3 + x + 1 = x^3(x+1) + 1(x+1) \\
 &= (x+1)(x^3 + 1) \\
 &= (x+1)(x+1)(x^2 - x + 1) \\
 &= (x+1)^2(x^2 - x + 1)
 \end{aligned}$$

$$\begin{aligned}
 127. \quad & 2(3x+4)^2 + (2x+3) \cdot 2(3x+4) \cdot 3 \\
 &= 2(3x+4)((3x+4) + (2x+3) \cdot 3) \\
 &= 2(3x+4)(3x+4+6x+9) \\
 &= 2(3x+4)(9x+13)
 \end{aligned}$$

$$\begin{aligned}
 128. \quad & 5(2x+1)^2 + (5x-6) \cdot 2(2x+1) \cdot 2 \\
 &= (2x+1)(5(2x+1) + (5x-6) \cdot 4) \\
 &= (2x+1)(10x+5+20x-24) \\
 &= (2x+1)(30x-19)
 \end{aligned}$$

$$\begin{aligned}
 129. \quad & 2x(2x+5) + x^2 \cdot 2 = 2x((2x+5) + x) \\
 &= 2x(2x+5+x) \\
 &= 2x(3x+5)
 \end{aligned}$$

$$\begin{aligned}
 130. \quad & 3x^2(8x-3) + x^3 \cdot 8 = x^2(3(8x-3) + 8x) \\
 &= x^2(24x-9+8x) \\
 &= x^2(32x-9)
 \end{aligned}$$

$$\begin{aligned}
 131. \quad & 2(x+3)(x-2)^3 + (x+3)^2 \cdot 3(x-2)^2 \\
 &= (x+3)(x-2)^2(2(x-2) + (x+3) \cdot 3) \\
 &= (x+3)(x-2)^2(2x-4+3x+9) \\
 &= (x+3)(x-2)^2(5x+5) \\
 &= 5(x+3)(x-2)^2(x+1)
 \end{aligned}$$

$$\begin{aligned}
 132. \quad & 4(x+5)^3(x-1)^2 + (x+5)^4 \cdot 2(x-1) \\
 &= 2(x+5)^3(x-1)(2(x-1) + (x+5)) \\
 &= 2(x+5)^3(x-1)(2x-2+x+5) \\
 &= 2(x+5)^3(x-1)(3x+3) \\
 &= 2 \cdot 3(x+5)^3(x-1)(x+1) \\
 &= 6(x+5)^3(x-1)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 133. \quad & (4x-3)^2 + x \cdot 2(4x-3) \cdot 4 \\
 &= (4x-3)((4x-3) + 8x) \\
 &= (4x-3)(4x-3+8x) \\
 &= (4x-3)(12x-3) \\
 &= 3(4x-3)(4x-1)
 \end{aligned}$$

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$$\begin{aligned}
 134. \quad & 3x^2(3x+4)^2 + x^3 \cdot 2(3x+4) \cdot 3 \\
 &= 3x^2(3x+4)((3x+4)+2x) \\
 &= 3x^2(3x+4)(3x+4+2x) \\
 &= 3x^2(3x+4)(5x+4)
 \end{aligned}$$

$$\begin{aligned}
 135. \quad & 2(3x-5) \cdot 3(2x+1)^3 + (3x-5)^2 \cdot 3(2x+1)^2 \cdot 2 \\
 &= 6(3x-5)(2x+1)^2((2x+1)+(3x-5)) \\
 &= 6(3x-5)(2x+1)^2(2x+1+3x-5) \\
 &= 6(3x-5)(2x+1)^2(5x-4)
 \end{aligned}$$

$$\begin{aligned}
 136. \quad & 3(4x+5)^2 \cdot 4(5x+1)^2 + (4x+5)^3 \cdot 2(5x+1) \cdot 5 \\
 &= 2(4x+5)^2(5x+1)(6(5x+1)+5(4x+5)) \\
 &= 2(4x+5)^2(5x+1)(30x+6+20x+25) \\
 &= 2(4x+5)^2(5x+1)(50x+31)
 \end{aligned}$$

137.

$$\begin{aligned}
 x^{-4} + x^{-2} &= x^{-4} + x^{-2}x^{-4}x^4 \\
 &= x^{-4}(1 + x^{-2}x^4) \\
 &= x^{-4}(1 + x^{-2+4}) \\
 &= x^{-4}(1 + x^2)
 \end{aligned}$$

138.

$$\begin{aligned}
 & 2x^{-5} - 6x^{-4} + 8x^{-3} \\
 &= 2x^{-5} - 6x^{-4}x^{-1}x + 8x^{-3}x^{-2}x^2 \\
 &= 2(x^{-5} - 3x^{-5}x + 4x^{-5}x^2) \\
 &= 2x^{-5}(1 - 3x + 4x^2)
 \end{aligned}$$

139.

$$\begin{aligned}
 & x^{-2}(x+1) + x^{-1}(x-1) \\
 &= x^{-2}(x+1) + x^{-2}x(x-1) \\
 &= x^{-2}[(x+1) + x(x-1)] \\
 &= x^{-2}(x+1+x^2-x) \\
 &= x^{-2}(x^2+1)
 \end{aligned}$$

$$\begin{aligned}
 & x(x+3)^{-1} + 4x^2(x+3)^{-2} \\
 &= x(x+3)^{-2}(x+3) + 4x^2(x+3)^{-2} \\
 140. \quad &= (x+3)^{-2}(x(x+3) + 4x^2) \\
 &= x(x+3)^{-2}(x+3+4x) \\
 &= x(x+3)^{-2}(5x+3)
 \end{aligned}$$

141. The possible factorizations are
 $(x \pm 1)(x \pm 4) = x^2 \pm 5x + 4$ or
 $(x \pm 2)(x \pm 2) = x^2 \pm 4x + 4$, none of which equals $x^2 + 4$.

142. The possible factorizations are
 $(x \pm 1)^2 = x^2 \pm 2x + 1$, neither of which equals $x^2 + x + 1$.

143. Answers will vary.

144. Answers will vary.

Section R.6

1. quotient; divisor; remainder

$$2 \overline{) -3 }$$

3. d

4. a

5. True

6. True

$$\begin{array}{r}
 2 \overline{) 1 } \\
 \underline{2 } \\
 1
 \end{array}$$

Quotient: $x^2 - 5x - 5$
Remainder: 0

$$\begin{array}{r}
 -1 \overline{) 1 } \\
 \underline{-1 } \\
 1
 \end{array}$$

Quotient: $x^2 + x - 4$
Remainder: 5

Section R.6: Synthetic Division

$$\begin{array}{r|rrrr} 9. & 3 & 2 & -1 & 3 \\ & & 9 & 33 & 96 \\ \hline & 3 & 11 & 32 & 99 \end{array}$$

Quotient: $3x^2 + 11x + 32$
Remainder: 99

$$\begin{array}{r|rrrr} 10. & -2 & -4 & 2 & -1 & 1 \\ & & 8 & -20 & 42 \\ \hline & -4 & 10 & -21 & 43 \end{array}$$

Quotient: $-4x^2 + 10x - 21$
Remainder: 43

$$\begin{array}{r|rrrrrr} 11. & -3 & 1 & 0 & -4 & 0 & 1 & 0 \\ & & -3 & 9 & -15 & 45 & -138 \\ \hline & 1 & -3 & 5 & -15 & 46 & -138 \end{array}$$

Quotient: $x^4 - 3x^3 + 5x^2 - 15x + 46$
Remainder: -138

$$\begin{array}{r|rrrrr} 12. & 2 & 1 & 0 & 1 & 0 & 2 \\ & & 2 & 4 & 10 & 20 \\ \hline & 1 & 2 & 5 & 10 & 22 \end{array}$$

Quotient: $x^3 + 2x^2 + 5x + 10$
Remainder: 22

$$\begin{array}{r|rrrrrrr} 13. & 1 & 4 & 0 & -3 & 0 & 1 & 0 & 5 \\ & & 4 & 4 & 1 & 1 & 2 & 2 \\ \hline & 4 & 4 & 1 & 1 & 2 & 2 & 7 \end{array}$$

Quotient: $4x^5 + 4x^4 + x^3 + x^2 + 2x + 2$
Remainder: 7

$$\begin{array}{r|rrrrrr} 14. & -1 & 1 & 0 & 5 & 0 & 0 & -10 \\ & & -1 & 1 & -6 & 6 & -6 \\ \hline & 1 & -1 & 6 & -6 & 6 & -16 \end{array}$$

Quotient: $x^4 - x^3 + 6x^2 - 6x + 6$
Remainder: -16

$$\begin{array}{r|rrrr} 15. & -1.1 & 0.1 & 0 & 0.2 & 0 \\ & & -0.11 & 0.121 & -0.3531 \\ \hline & 0.1 & -0.11 & 0.321 & -0.3531 \end{array}$$

Quotient: $0.1x^2 - 0.11x + 0.321$
Remainder: -0.3531

$$\begin{array}{r|rrrr} 16. & -2.1 & 0.1 & 0 & -0.2 \\ & & -0.21 & 0.441 \\ \hline & 0.1 & -0.21 & 0.241 \end{array}$$

Quotient: $0.1x - 0.21$
Remainder: 0.241

$$\begin{array}{r|rrrrrr} 17. & 2 & 1 & 0 & 0 & 0 & 0 & -32 \\ & & 2 & 4 & 8 & 16 & 32 \\ \hline & 1 & 2 & 4 & 8 & 16 & 0 \end{array}$$

Quotient: $x^4 + 2x^3 + 4x^2 + 8x + 16$
Remainder: 0

$$\begin{array}{r|rrrrrr} 18. & -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ & & -1 & 1 & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & -1 & 1 & 0 \end{array}$$

Quotient: $x^4 - x^3 + x^2 - x + 1$
Remainder: 0

$$\begin{array}{r|rrrr} 19. & 2 & 4 & -3 & -8 & 4 \\ & & 8 & 10 & 4 \\ \hline & 4 & 5 & 2 & 8 \end{array}$$

Remainder = $8 \neq 0$. Therefore, $x - 2$ is not a factor of $4x^3 - 3x^2 - 8x + 4$.

$$\begin{array}{r|rrrr} 20. & -3 & -4 & 5 & 0 & 8 \\ & & 12 & -51 & 153 \\ \hline & -4 & 17 & -51 & 161 \end{array}$$

Remainder = $161 \neq 0$. Therefore, $x + 3$ is not a factor of $-4x^3 + 5x^2 + 8$.

$$\begin{array}{r|rrrrr} 21. & 3 & 2 & -6 & 0 & -7 & 21 \\ & & 6 & 0 & 0 & -21 \\ \hline & 2 & 0 & 0 & -7 & 0 \end{array}$$

Remainder = 0. Therefore, $x - 3$ is a factor of $2x^4 - 6x^3 - 7x + 21$.

$$\begin{array}{r|rrrrr} 22. & 2 & 4 & 0 & -15 & 0 & -4 \\ & & 8 & 16 & 2 & 4 \\ \hline & 4 & 8 & 1 & 2 & 0 \end{array}$$

Remainder = 0. Therefore, $x - 2$ is a factor of $4x^4 - 15x^2 - 4$.

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$$\begin{array}{r} 23. \quad -2 \overline{) 5 \quad 0 \quad 0 \quad 43 \quad 0 \quad 0 \quad 24} \\ \underline{-10 \quad 20 \quad -40 \quad -6 \quad 12 \quad -24} \\ 3 \quad -10 \quad 20 \quad 3 \quad -6 \quad 12 \quad 0 \end{array}$$

Remainder = 0. Therefore, $x+3$ is a factor of $5x^6 + 43x^3 + 24$.

$$\begin{array}{r} 24. \quad -3 \overline{) 2 \quad 0 \quad -18 \quad 0 \quad 1 \quad 0 \quad -9} \\ \underline{-6 \quad 18 \quad 0 \quad 0 \quad -3 \quad 9} \\ 2 \quad -6 \quad 0 \quad 0 \quad 1 \quad -3 \quad 0 \end{array}$$

Remainder = 0. Therefore, $x+2$ is a factor of $2x^6 - 18x^4 + x^2 - 9$. **25.**

$$\begin{array}{r} -4 \overline{) 1 \quad 0 \quad -16 \quad -1 \quad 0 \quad 19} \\ \underline{-4 \quad 16 \quad 0 \quad 4 \quad -16} \\ 1 \quad -4 \quad 0 \quad -1 \quad 4 \quad 3 \end{array}$$

Remainder = $1 \neq 0$. Therefore, $x+4$ is not a factor of $x^5 - 16x^3 - x^2 + 19$.

$$\begin{array}{r} 26. \quad -4 \overline{) 1 \quad 0 \quad -16 \quad 0 \quad 1 \quad 0 \quad -16} \\ \underline{-4 \quad 16 \quad 0 \quad 0 \quad -4 \quad 16} \\ 1 \quad -4 \quad 0 \quad 0 \quad 1 \quad -4 \quad 0 \end{array}$$

Remainder = 0. Therefore, $x+4$ is a factor of $x^6 - 16x^4 + x^2 - 16$.

$$\begin{array}{r} 27. \quad \frac{1}{3} \overline{) 3 \quad -1 \quad 0 \quad 6 \quad -2} \\ \underline{1 \quad 0 \quad 0 \quad 2} \\ 3 \quad 0 \quad 0 \quad 6 \quad 0 \end{array}$$

Remainder = 0; therefore $x - \frac{1}{3}$ is a factor of $3x^4 - x^3 + 6x - 2$.

$$\begin{array}{r} 28. \quad -\frac{1}{3} \overline{) 3 \quad 1 \quad 0 \quad -3 \quad 1} \\ \underline{-1 \quad 0 \quad 0 \quad 1} \\ 3 \quad 0 \quad 0 \quad -3 \quad 2 \end{array}$$

Remainder = $2 \neq 0$; therefore $x + \frac{1}{3}$ is not a factor of $3x^4 + x^3 - 3x + 1$.

$$\begin{array}{r} 29. \quad -2 \overline{) 1 \quad -2 \quad 3 \quad 5} \\ \underline{-2 \quad 8 \quad -22} \\ 1 \quad -4 \quad 11 \quad -17 \end{array}$$

$$\frac{x^3 - 2x^2 + 3x + 5}{x + 2} = x^2 - 4x + 11 + \frac{-17}{x + 2}$$

$$a + b + c + d = 1 - 4 + 11 - 17 = -9$$

$$\begin{array}{r} 30. \quad h \overline{) 1 \quad 3 - h \quad -2h \quad -2h^2 \quad h^3} \\ \underline{h \quad 3h \quad h^2 \quad -h^3} \\ 1 \quad 3 \quad h \quad -h^2 \quad 0 \end{array}$$

$x^3 + 3x^2 + hx - h^2$ is the quotient and 0 is the remainder.

$$\begin{array}{r} 31. \quad -y \overline{) 1 \quad 3y \quad -3y^2 \quad -y^3 \quad 4y^4} \\ \underline{-y \quad -2y^2 \quad 5y^3 \quad 4y^4} \\ 1 \quad 2y \quad -5y^2 \quad 4y^3 \quad 0 \end{array}$$

Yes, $x+y$ is a factor of

$$x^4 + 3x^3y - 3x^2y^2 - xy^3 + 4y^4.$$

32. Answers will vary.

Section R.7

1. lowest terms

2. Least Common Multiple

3. d

4. a

$$\begin{aligned} 5. \text{ True; } \frac{\frac{1}{3} + \frac{1}{x}}{\frac{1}{5} - \frac{1}{x}} &= \frac{\frac{x+3}{3x}}{\frac{x-5}{5x}} \\ &= \frac{x+3}{3x} \cdot \frac{5x}{x-5} = \frac{5(x+3)}{3(x-5)} \end{aligned}$$

6. False;

$$2x^3 + 6x^2 = 2x^2(x+3)$$

$$6x^4 + 4x^3 = 2x^3(3x+2)$$

$$LCM = 2x^3(x+3)(3x+2)$$

$$7. \quad \frac{3x+9}{x^2-9} = \frac{3(x+3)}{(x-3)(x+3)} = \frac{3}{x-3}$$

$$8. \quad \frac{4x^2+8x}{12x+24} = \frac{4x(x+2)}{12(x+2)} = \frac{x}{3}$$

$$9. \quad \frac{x^2-2x}{3x-6} = \frac{x(x-2)}{3(x-2)} = \frac{x}{3}$$

$$10. \frac{15x^2 + 24x}{3x^2} = \frac{3x(5x+8)}{3x^2} = \frac{5x+8}{x}$$

$$11. \frac{24x^2}{12x^2 - 6x} = \frac{24x^2}{6x(2x-1)} = \frac{4x}{2x-1}$$

$$12. \frac{x^2 + 4x + 4}{x^2 - 4} = \frac{(x+2)(x+2)}{(x-2)(x+2)} = \frac{x+2}{x-2}$$

$$13. \frac{y^2 - 49}{3y^2 - 18y - 21} = \frac{(y+7)(y-7)}{3(y^2 - 6y - 7)}$$

$$= \frac{(y+7)(y-7)}{3(y-7)(y+1)}$$

$$= \frac{y+7}{3(y+1)}$$

$$14. \frac{3y^2 - y - 2}{3y^2 + 5y + 2} = \frac{(3y+2)(y-1)}{(3y+2)(y+1)} = \frac{y-1}{y+1}$$

$$15. \frac{x^2 + 4x - 12}{x^2 - 4x + 4} = \frac{(x+6)(x-2)}{(x-2)(x-2)} = \frac{x+6}{x-2}$$

$$16. \frac{x - x^2}{x^2 + x - 2} = \frac{-x(x-1)}{(x+2)(x-1)} = \frac{-x}{x+2} = -\frac{x}{x+2}$$

$$17. \frac{x^2 + x - 20}{4 - x} = \frac{(x+5)(x-4)}{2 - x}$$

$$= \frac{(x+5)(x-4)}{-1(x-4)}$$

$$= -(x+5)$$

$$18. \frac{2x^2 + 5x - 3}{1 - 2x} = \frac{(2x-1)(x+3)}{-1(2x-1)} = -(x+3) = -x-3$$

$$19. \frac{3x+6}{5x^2} \cdot \frac{x}{x^2-4} = \frac{3(x+2)}{5x^2} \cdot \frac{x}{(x-2)(x+2)}$$

$$= \frac{3}{5x(x-2)}$$

$$20. \frac{3}{2x} \cdot \frac{x^2}{6x+10} = \frac{3}{2} \cdot \frac{x}{2(3x+5)} = \frac{3x}{4(3x+5)}$$

$$21. \frac{4x^2}{x^2-16} \cdot \frac{x^3-64}{2x}$$

$$= \frac{4x^2}{(x-4)(x+4)} \cdot \frac{(x-4)(x^2+4x+16)}{2x}$$

$$= \frac{2x \cdot 2x(x-4)(x^2+4x+16)}{2x(x-4)(x+4)}$$

$$= \frac{2x(x^2+4x+16)}{x+4}$$

$$22. \frac{12}{x^2+x} \cdot \frac{x^3+1}{4x-2} = \frac{12}{x(x+1)} \cdot \frac{(x+1)(x^2-x+1)}{2(2x-1)}$$

$$= \frac{2 \cdot 6(x+1)(x^2-x+1)}{2x(x+1)(2x-1)}$$

$$= \frac{6(x^2-x+1)}{x(2x-1)}$$

$$23. \frac{4x-8}{-3x} \cdot \frac{12}{12-6x} = \frac{4(x-2)}{-3x} \cdot \frac{12}{6(2-x)}$$

$$= \frac{4(x-2)}{-3x} \cdot \frac{2}{(-1)(x-2)}$$

$$= \frac{8}{3x}$$

$$24. \frac{6x-27}{5x} \cdot \frac{2}{4x-18} = \frac{3(2x-9)}{5x} \cdot \frac{2}{2(2x-9)} = \frac{3}{5x}$$

$$25. \frac{x^2-4x-12}{x^2+2x-48} \cdot \frac{x^2+4x-32}{x^2+10x+16}$$

$$= \frac{(x-6)(x+2)}{(x+8)(x-6)} \cdot \frac{(x+8)(x-4)}{(x+8)(x+2)}$$

$$= \frac{x-4}{x+8}$$

$$26. \frac{x^2+x-6}{x^2+4x-5} \cdot \frac{x^2-25}{x^2+2x-15}$$

$$= \frac{(x-2)(x+3)}{(x+5)(x-1)} \cdot \frac{(x+5)(x-5)}{(x+5)(x-3)}$$

$$= \frac{(x-2)(x+3)(x-5)}{(x+5)(x-1)(x-3)}$$

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$$\begin{aligned} 27. \quad \frac{\frac{6x}{x^2-4}}{\frac{3x-9}{2x+4}} &= \frac{6x}{x^2-4} \cdot \frac{2x+4}{3x-9} \\ &= \frac{6x}{(x-2)(x+2)} \cdot \frac{2(x+2)}{3(x-3)} \\ &= \frac{4x}{(x-2)(x-3)} \end{aligned}$$

$$\begin{aligned} 28. \quad \frac{\frac{12x}{5x+20}}{\frac{4x^2}{x^2-16}} &= \frac{12x}{5x+20} \cdot \frac{x^2-16}{4x^2} \\ &= \frac{12x}{5(x+4)} \cdot \frac{(x+4)(x-4)}{4x^2} \\ &= \frac{3(x-4)}{5x} \end{aligned}$$

$$\begin{aligned} 29. \quad \frac{\frac{8x}{x^2-1}}{\frac{10x}{x+1}} &= \frac{8x}{x^2-1} \cdot \frac{x+1}{10x} \\ &= \frac{8x}{(x-1)(x+1)} \cdot \frac{x+1}{10x} \\ &= \frac{4}{5(x-1)} \end{aligned}$$

$$\begin{aligned} 30. \quad \frac{\frac{x-2}{4x}}{\frac{x^2-4x+4}{12x}} &= \frac{x-2}{4x} \cdot \frac{12x}{x^2-4x+4} \\ &= \frac{x-2}{4x} \cdot \frac{12x}{(x-2)(x-2)} \\ &= \frac{3}{x-2} \end{aligned}$$

$$\begin{aligned} 31. \quad \frac{\frac{4-x}{4+x}}{\frac{4x}{x^2-16}} &= \frac{4-x}{4+x} \cdot \frac{x^2-16}{4x} \\ &= \frac{4-x}{4+x} \cdot \frac{(x+4)(x-4)}{4x} \\ &= \frac{(4-x)(x-4)}{4x} \\ &= -\frac{(x-4)^2}{4x} \end{aligned}$$

$$\begin{aligned} 32. \quad \frac{\frac{3+x}{x^2-9}}{9x^3} &= \frac{3+x}{x^2-9} \cdot \frac{9x^3}{x^2-9} \\ &= \frac{3+x}{3-x} \cdot \frac{9x^3}{(x+3)(x-3)} \\ &= \frac{9x^3}{(3-x)(x-3)} \\ &= \frac{9x^3}{-(x-3)^2} \\ &= -\frac{9x^3}{(x-3)^2} \end{aligned}$$

$$\begin{aligned} 33. \quad \frac{\frac{x^2+7x+12}{x^2-7x+12}}{\frac{x^2+x-12}{x^2-x-12}} &= \frac{x^2+7x+12}{x^2-7x+12} \cdot \frac{x^2-x-12}{x^2+x-12} \\ &= \frac{(x+3)(x+4)}{(x-3)(x-4)} \cdot \frac{(x-4)(x+3)}{(x+4)(x-3)} \\ &= \frac{(x+3)^2}{(x-3)^2} \end{aligned}$$

$$\begin{aligned} 34. \quad \frac{\frac{x^2+7x+6}{x^2+x-6}}{\frac{x^2+5x-6}{x^2+5x+6}} &= \frac{x^2+7x+6}{x^2+x-6} \cdot \frac{x^2+5x+6}{x^2+5x-6} \\ &= \frac{(x+6)(x+1)}{(x+3)(x-2)} \cdot \frac{(x+2)(x+3)}{(x+6)(x-1)} \\ &= \frac{(x+1)(x+2)}{(x-2)(x-1)} \end{aligned}$$

$$\begin{aligned} 35. \quad \frac{\frac{5x^2-7x-6}{2x^2+3x-5}}{\frac{15x^2+14x+3}{2x^2+13x+20}} &= \frac{5x^2-7x-6}{2x^2+3x-5} \cdot \frac{2x^2+13x+20}{15x^2+14x+3} \\ &= \frac{(5x+3)(x-2)}{(x-1)(2x+5)} \cdot \frac{(2x+5)(x+4)}{(5x+3)(3x+1)} \\ &= \frac{(x-2)(x+4)}{(x-1)(3x+1)} \end{aligned}$$

$$\begin{aligned}
 36. \quad \frac{\frac{9x^2+3x-2}{12x^2+5x-2}}{\frac{9x^2-6x+1}{8x^2-10x-3}} &= \frac{9x^2+3x-2}{12x^2+5x-2} \cdot \frac{8x^2-10x-3}{9x^2-6x+1} \\
 &= \frac{(3x+2)(3x-1)}{(3x+2)(4x-1)} \cdot \frac{(4x+1)(2x-3)}{(3x-1)(3x-1)} \\
 &= \frac{(4x+1)(2x-3)}{(4x-1)(3x-1)}
 \end{aligned}$$

$$37. \quad \frac{x}{2} + \frac{5}{2} = \frac{x+5}{2}$$

$$38. \quad \frac{3}{x} - \frac{6}{x} = \frac{3-6}{x} = \frac{-3}{x} = -\frac{3}{x}$$

$$39. \quad \frac{x^2}{2x-3} - \frac{4}{2x-3} = \frac{x^2-4}{2x-3} = \frac{(x+2)(x-2)}{2x-3}$$

$$40. \quad \frac{3x^2}{2x-1} - \frac{9}{2x-1} = \frac{3x^2-9}{2x-1} = \frac{3(x^2-3)}{2x-1}$$

$$41. \quad \frac{x+5}{x-4} + \frac{3x-2}{x-4} = \frac{x+5+3x-2}{x-4} = \frac{4x+3}{x-4}$$

$$42. \quad \frac{2x-5}{3x+2} + \frac{x+4}{3x+2} = \frac{2x-5+x+4}{3x+2} = \frac{3x-1}{3x+2}$$

$$\begin{aligned}
 43. \quad \frac{3x+5}{2x-1} - \frac{2x-4}{2x-1} &= \frac{(3x+5)-(2x-4)}{2x-1} \\
 &= \frac{3x+5-2x+4}{2x-1} \\
 &= \frac{x+9}{2x-1}
 \end{aligned}$$

$$\begin{aligned}
 44. \quad \frac{5x-4}{3x+4} - \frac{x+1}{3x+4} &= \frac{(5x-4)-(x+1)}{3x+4} \\
 &= \frac{5x-4-x-1}{3x+4} \\
 &= \frac{4x-5}{3x+4}
 \end{aligned}$$

$$45. \quad \frac{4}{x-2} + \frac{x}{2-x} = \frac{4}{x-2} - \frac{x}{x-2} = \frac{4-x}{x-2}$$

$$46. \quad \frac{6}{x-1} - \frac{x}{1-x} = \frac{6}{x-1} + \frac{x}{x-1} = \frac{x+6}{x-1}$$

$$\begin{aligned}
 47. \quad \frac{7}{x-3} - \frac{3}{x+1} &= \frac{7(x+1)}{(x-3)(x+1)} - \frac{3(x-3)}{(x+1)(x-3)} \\
 &= \frac{7x+7-3x+9}{(x+1)(x-3)} \\
 &= \frac{4x+16}{(x+1)(x-3)} \\
 &= \frac{4(x+4)}{(x+1)(x-3)}
 \end{aligned}$$

$$\begin{aligned}
 48. \quad \frac{2}{x+5} - \frac{5}{x-5} &= \frac{2(x-5)}{(x+5)(x-5)} - \frac{5(x+5)}{(x+5)(x-5)} \\
 &= \frac{2x-10-5x-25}{(x+5)(x-5)} \\
 &= \frac{-3x-35}{(x+5)(x-5)} \\
 &= -\frac{3x+35}{(x+5)(x-5)}
 \end{aligned}$$

$$\begin{aligned}
 49. \quad \frac{x}{x+1} + \frac{2x-3}{x-1} &= \frac{x(x-1)}{(x+1)(x-1)} + \frac{(2x-3)(x+1)}{(x-1)(x+1)} \\
 &= \frac{x^2-x+2x^2-x-3}{(x-1)(x+1)} \\
 &= \frac{3x^2-2x-3}{(x-1)(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 50. \quad \frac{3x}{x-4} + \frac{2x}{x+3} &= \frac{3x(x+3)}{(x-4)(x+3)} + \frac{2x(x-4)}{(x-4)(x+3)} \\
 &= \frac{3x^2+9x+2x^2-8x}{(x-4)(x+3)} \\
 &= \frac{5x^2+x}{(x-4)(x+3)} \\
 &= \frac{x(5x+1)}{(x-4)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 51. \quad \frac{x-3}{x+2} - \frac{x+4}{x-2} &= \frac{(x-3)(x-2)}{(x+2)(x-2)} - \frac{(x+4)(x+2)}{(x-2)(x+2)} \\
 &= \frac{x^2-5x+6-(x^2+6x+8)}{(x+2)(x-2)} \\
 &= \frac{x^2-5x+6-x^2-6x-8}{(x+2)(x-2)} \\
 &= \frac{-11x-2}{(x+2)(x-2)} \quad \text{or} \quad \frac{-(11x+2)}{(x+2)(x-2)}
 \end{aligned}$$

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$$\begin{aligned}
 52. \quad \frac{2x-3}{x-1} - \frac{2x+1}{x+1} &= \frac{(2x-3)(x+1)}{(x-1)(x+1)} - \frac{(2x+1)(x-1)}{(x+1)(x-1)} \\
 &= \frac{2x^2 - x - 3 - (2x^2 - x - 1)}{(x+1)(x-1)} \\
 &= \frac{2x^2 - x - 3 - 2x^2 + x + 1}{(x+1)(x-1)} \\
 &= \frac{-2}{(x+1)(x-1)} \\
 &= -\frac{2}{(x+1)(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 53. \quad \frac{x}{x^2-4} + \frac{1}{x} &= \frac{x^2 + x^2 - 4}{x(x^2 - 4)} \\
 &= \frac{2x^2 - 4}{x(x^2 - 4)} \\
 &= \frac{2(x^2 - 2)}{x(x-2)(x+2)}
 \end{aligned}$$

$$\begin{aligned}
 54. \quad \frac{x-1}{x^3} + \frac{x}{x^2+1} &= \frac{(x-1)(x^2+1) + x^4}{x^3(x^2+1)} \\
 &= \frac{x^3 - x^2 + x - 1 + x^4}{x^3(x^2+1)} \\
 &= \frac{x^4 + x^3 - x^2 + x - 1}{x^3(x^2+1)}
 \end{aligned}$$

$$\begin{aligned}
 55. \quad x^2 - 4 &= (x+2)(x-2) \\
 x^2 - x - 2 &= (x+1)(x-2) \\
 \text{Therefore, LCM} &= (x+2)(x-2)(x+1).
 \end{aligned}$$

$$\begin{aligned}
 56. \quad x^2 - x - 12 &= (x+3)(x-4) \\
 x^2 - 8x + 16 &= (x-4)(x-4) \\
 \text{Therefore, LCM} &= (x+3)(x-4)^2.
 \end{aligned}$$

$$\begin{aligned}
 57. \quad x^3 - x &= x(x^2 - 1) = x(x+1)(x-1) \\
 x^2 - x &= x(x-1) \\
 \text{Therefore, LCM} &= x(x+1)(x-1).
 \end{aligned}$$

$$\begin{aligned}
 58. \quad 3x^2 - 27 &= 3(x^2 - 9) = 3(x+3)(x-3) \\
 2x^2 - x - 15 &= (2x+5)(x-3) \\
 \text{Therefore, LCM} &= 3(2x+5)(x-3)(x+3).
 \end{aligned}$$

$$\begin{aligned}
 59. \quad 4x^3 - 4x^2 + x &= x(4x^2 - 4x + 1) \\
 &= x(2x-1)(2x-1) \\
 2x^3 - x^2 &= x^2(2x-1) \\
 x^3 & \\
 \text{Therefore, LCM} &= x^3(2x-1)^2.
 \end{aligned}$$

$$\begin{aligned}
 60. \quad x-3 & \\
 x^2 + 3x &= x(x+3) \\
 x^3 - 9x &= x(x^2 - 9) = x(x+3)(x-3) \\
 \text{Therefore, LCM} &= x(x+3)(x-3).
 \end{aligned}$$

$$\begin{aligned}
 61. \quad x^3 - x &= x(x^2 - 1) = x(x+1)(x-1) \\
 x^3 - 2x^2 + x &= x(x^2 - 2x + 1) = x(x-1)^2 \\
 x^3 - 1 &= (x-1)(x^2 + x + 1) \\
 \text{Therefore, LCM} &= x(x+1)(x-1)^2(x^2 + x + 1).
 \end{aligned}$$

$$\begin{aligned}
 62. \quad x^2 + 4x + 4 &= (x+2)^2 \\
 x^3 + 2x^2 &= x^2(x+2) \\
 (x+2)^3 & \\
 \text{Therefore, LCM} &= x^2(x+2)^3.
 \end{aligned}$$

$$\begin{aligned}
 63. \quad \frac{x}{x^2-7x+6} - \frac{x}{x^2-2x-24} & \\
 &= \frac{x}{(x-6)(x-1)} - \frac{x}{(x-6)(x+4)} \\
 &= \frac{x(x+4)}{(x-6)(x-1)(x+4)} - \frac{x(x-1)}{(x-6)(x+4)(x-1)} \\
 &= \frac{x^2 + 4x - x^2 + x}{(x-6)(x+4)(x-1)} = \frac{5x}{(x-6)(x+4)(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 64. \quad \frac{x}{x-3} - \frac{x+1}{x^2+5x-24} & \\
 &= \frac{x}{(x-3)} - \frac{x+1}{(x-3)(x+8)} \\
 &= \frac{x(x+8)}{(x-3)(x+8)} - \frac{x+1}{(x-3)(x+8)} \\
 &= \frac{x^2 + 8x - x - 1}{(x-3)(x+8)} = \frac{x^2 + 7x - 1}{(x-3)(x+8)}
 \end{aligned}$$

$$\begin{aligned}
65. \quad & \frac{4x}{x^2-4} - \frac{2}{x^2+x-6} \\
&= \frac{4x}{(x-2)(x+2)} - \frac{2}{(x+3)(x-2)} \\
&= \frac{4x(x+3)}{(x-2)(x+2)(x+3)} - \frac{2(x+2)}{(x+3)(x-2)(x+2)} \\
&= \frac{4x^2+12x-2x-4}{(x-2)(x+2)(x+3)} \\
&= \frac{4x^2+10x-4}{(x-2)(x+2)(x+3)} \\
&= \frac{2(2x^2+5x-2)}{(x-2)(x+2)(x+3)}
\end{aligned}$$

$$\begin{aligned}
66. \quad & \frac{3x}{x-1} - \frac{x-4}{x^2-2x+1} = \frac{3x}{(x-1)} - \frac{x-4}{(x-1)^2} \\
&= \frac{3x(x-1)}{(x-1)(x-1)} - \frac{x-4}{(x-1)^2} \\
&= \frac{3x^2-3x-x+4}{(x-1)^2} \\
&= \frac{3x^2-4x+4}{(x-1)^2}
\end{aligned}$$

$$\begin{aligned}
67. \quad & \frac{3}{(x-1)^2(x+1)} + \frac{2}{(x-1)(x+1)^2} \\
&= \frac{3(x+1)+2(x-1)}{(x-1)^2(x+1)^2} \\
&= \frac{3x+3+2x-2}{(x-1)^2(x+1)^2} \\
&= \frac{5x+1}{(x-1)^2(x+1)^2}
\end{aligned}$$

$$\begin{aligned}
68. \quad & \frac{2}{(x+2)^2(x-1)} - \frac{6}{(x+2)(x-1)^2} \\
&= \frac{2(x-1)-6(x+2)}{(x+2)^2(x-1)^2} \\
&= \frac{2x-2-6x-12}{(x+2)^2(x-1)^2} \\
&= \frac{-4x-14}{(x+2)^2(x-1)^2} \\
&= \frac{-2(2x+7)}{(x+2)^2(x-1)^2}
\end{aligned}$$

$$\begin{aligned}
69. \quad & \frac{x+4}{x^2-x-2} - \frac{2x+3}{x^2+2x-8} \\
&= \frac{x+4}{(x-2)(x+1)} - \frac{2x+3}{(x+4)(x-2)} \\
&= \frac{(x+4)(x+4)}{(x-2)(x+1)(x+4)} - \frac{(2x+3)(x+1)}{(x+4)(x-2)(x+1)} \\
&= \frac{x^2+8x+16-(2x^2+5x+3)}{(x-2)(x+1)(x+4)} \\
&= \frac{-x^2+3x+13}{(x-2)(x+1)(x+4)}
\end{aligned}$$

$$\begin{aligned}
70. \quad & \frac{2x-3}{x^2+8x+7} - \frac{x-2}{(x+1)^2} \\
&= \frac{2x-3}{(x+1)(x+7)} - \frac{x-2}{(x+1)^2} \\
&= \frac{(2x-3)(x+1)}{(x+1)(x+7)(x+1)} - \frac{(x-2)(x+7)}{(x+1)^2(x+7)} \\
&= \frac{2x^2-x-3-(x^2+5x-14)}{(x+1)^2(x+7)} \\
&= \frac{x^2-6x+11}{(x+1)^2(x+7)}
\end{aligned}$$

$$\begin{aligned}
71. \quad & \frac{1}{x} - \frac{2}{x^2+x} + \frac{3}{x^3-x^2} \\
&= \frac{1}{x} - \frac{2}{x(x+1)} + \frac{3}{x^2(x-1)} \\
&= \frac{x(x+1)(x-1)-2x(x-1)+3(x+1)}{x^2(x+1)(x-1)} \\
&= \frac{x(x^2-1)-2x^2+2x+3x+3}{x^2(x+1)(x-1)} \\
&= \frac{x^3-x-2x^2+5x+3}{x^2(x+1)(x-1)} \\
&= \frac{x^3-2x^2+4x+3}{x^2(x+1)(x-1)}
\end{aligned}$$

$$\begin{aligned}
 72. \quad & \frac{x}{(x-1)^2} + \frac{2}{x} - \frac{x+1}{x^3-x^2} \\
 &= \frac{x}{(x-1)^2} + \frac{2}{x} - \frac{x+1}{x^2(x-1)} \\
 &= \frac{x^3+2x(x-1)^2-(x+1)(x-1)}{x^2(x-1)^2} \\
 &= \frac{x^3+2x(x^2-2x+1)-(x^2-1)}{x^2(x-1)^2} \\
 &= \frac{x^3+2x^3-4x^2+2x-x^2+1}{x^2(x-1)^2} \\
 &= \frac{3x^3-5x^2+2x+1}{x^2(x-1)^2}
 \end{aligned}$$

$$\begin{aligned}
 73. \quad & \frac{1}{h} \left(\frac{1}{x+h} - \frac{1}{x} \right) = \frac{1}{h} \left(\frac{1 \cdot x}{(x+h)x} - \frac{1(x+h)}{x(x+h)} \right) \\
 &= \frac{1}{h} \left(\frac{x-x-h}{x(x+h)} \right) \\
 &= \frac{-h}{hx(x+h)} \\
 &= \frac{-1}{x(x+h)}
 \end{aligned}$$

$$\begin{aligned}
 74. \quad & \frac{1}{h} \left(\frac{1}{(x+h)^2} - \frac{1}{x^2} \right) \\
 &= \frac{1}{h} \left(\frac{1 \cdot x^2}{(x+h)^2 x^2} - \frac{1(x+h)^2}{x^2(x+h)^2} \right) \\
 &= \frac{1}{h} \left(\frac{x^2 - (x^2 + 2xh + h^2)}{x^2(x+h)^2} \right) \\
 &= \frac{-2xh - h^2}{hx^2(x+h)^2} \\
 &= \frac{h(-2x-h)}{hx^2(x+h)^2} \\
 &= \frac{-2x-h}{x^2(x+h)^2} \\
 &= -\frac{2x+h}{x^2(x+h)^2}
 \end{aligned}$$

$$\begin{aligned}
 75. \quad & \frac{1+\frac{1}{x}}{1-\frac{1}{x}} = \frac{\left(\frac{x}{x} + \frac{1}{x}\right)}{\left(\frac{x}{x} - \frac{1}{x}\right)} = \frac{\left(\frac{x+1}{x}\right)}{\left(\frac{x-1}{x}\right)} = \frac{x+1}{x} \cdot \frac{x}{x-1} = \frac{x+1}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 76. \quad & \frac{4+\frac{1}{x^2}}{3-\frac{1}{x^2}} = \frac{\left(\frac{4x^2}{x^2} + \frac{1}{x^2}\right)}{\left(\frac{3x^2}{x^2} - \frac{1}{x^2}\right)} = \frac{\left(\frac{4x^2+1}{x^2}\right)}{\left(\frac{3x^2-1}{x^2}\right)} \\
 &= \frac{4x^2+1}{x^2} \cdot \frac{x^2}{3x^2-1} \\
 &= \frac{4x^2+1}{3x^2-1}
 \end{aligned}$$

$$\begin{aligned}
 77. \quad & \frac{2-\frac{x+1}{x}}{3+\frac{x-1}{x+1}} = \frac{\frac{2x}{x} - \frac{x+1}{x}}{\frac{3(x+1)}{x+1} + \frac{x-1}{x+1}} = \frac{\frac{2x-x-1}{x}}{\frac{3x+3+x-1}{x+1}} \\
 &= \frac{\frac{x-1}{x}}{\frac{4x+2}{x+1}} = \frac{x-1}{x} \cdot \frac{x+1}{2(2x+1)} \\
 &= \frac{(x-1)(x+1)}{2x(2x+1)}
 \end{aligned}$$

$$\begin{aligned}
 78. \quad & \frac{1-\frac{x}{x+1}}{2-\frac{x-1}{x}} = \frac{\left(\frac{x+1}{x+1} - \frac{x}{x+1}\right)}{\left(\frac{2x}{x} - \frac{x-1}{x}\right)} = \frac{\left(\frac{1}{x+1}\right)}{\left(\frac{x+1}{x}\right)} \\
 &= \frac{1}{x+1} \cdot \frac{x}{x+1} \\
 &= \frac{x}{(x+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 79. \quad & \frac{\frac{x+4}{x-2} - \frac{x-3}{x+1}}{x+1} \\
 &= \frac{\left(\frac{(x+4)(x+1)}{(x-2)(x+1)} - \frac{(x-3)(x-2)}{(x+1)(x-2)}\right)}{x+1} \\
 &= \frac{\left(\frac{x^2+5x+4-(x^2-5x+6)}{(x-2)(x+1)}\right)}{x+1} \\
 &= \frac{10x-2}{(x-2)(x+1)} \cdot \frac{1}{x+1} \\
 &= \frac{2(5x-1)}{(x-2)(x+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 80. \quad & \frac{\frac{x-2}{x+1} - \frac{x}{x-2}}{x+3} \\
 &= \frac{\left(\frac{(x-2)(x-2)}{(x+1)(x-2)} - \frac{x(x+1)}{(x-2)(x+1)} \right)}{x+3} \\
 &= \frac{\left(\frac{x^2 - 4x + 4 - (x^2 + x)}{(x-2)(x+1)} \right)}{x+3} \\
 &= \frac{-5x + 4}{(x-2)(x+1)} \cdot \frac{1}{x+3} \\
 &= \frac{-5x + 4}{(x-2)(x+1)(x+3)} \\
 &= \frac{-(5x-4)}{(x-2)(x+1)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 81. \quad & \frac{\frac{x-2}{x+2} + \frac{x-1}{x+1}}{\frac{x}{x+1} - \frac{2x-3}{x}} \\
 &= \frac{\left(\frac{(x-2)(x+1)}{(x+2)(x+1)} + \frac{(x-1)(x+2)}{(x+1)(x+2)} \right)}{\left(\frac{x^2}{(x+1)(x)} - \frac{(2x-3)(x+1)}{x(x+1)} \right)} \\
 &= \frac{\left(\frac{x^2 - x - 2 + x^2 + x - 2}{(x+2)(x+1)} \right)}{\left(\frac{x^2 - (2x^2 - x - 3)}{x(x+1)} \right)} \\
 &= \frac{\left(\frac{2x^2 - 4}{(x+2)(x+1)} \right)}{\left(\frac{-x^2 + x + 3}{x(x+1)} \right)} \\
 &= \frac{2(x^2 - 2)}{(x+2)(x+1)} \cdot \frac{x(x+1)}{-(x^2 - x - 3)} \\
 &= \frac{2x(x^2 - 2)}{-(x+2)(x^2 - x - 3)} \\
 &= \frac{-2x(x^2 - 2)}{(x+2)(x^2 - x - 3)}
 \end{aligned}$$

$$\begin{aligned}
 82. \quad & \frac{\frac{2x+5}{x} - \frac{x}{x-3}}{\frac{x^2}{x-3} - \frac{(x+1)^2}{x+3}} \\
 &= \frac{\left(\frac{(2x+5)(x-3)}{x(x-3)} - \frac{x(x)}{x(x-3)} \right)}{\left(\frac{x^2(x+3)}{(x-3)(x+3)} - \frac{(x-3)(x+1)^2}{(x-3)(x+3)} \right)} \\
 &= \frac{\left(\frac{2x^2 - x - 15 - x^2}{x(x-3)} \right)}{\left(\frac{x^3 + 3x^2 - (x^3 - x^2 - 5x - 3)}{(x-3)(x+3)} \right)} \\
 &= \frac{\left(\frac{x^2 - x - 15}{x(x-3)} \right)}{\left(\frac{4x^2 + 5x + 3}{(x-3)(x+3)} \right)} \\
 &= \frac{x^2 - x - 15}{x(x-3)} \cdot \frac{(x-3)(x+3)}{4x^2 + 5x + 3} \\
 &= \frac{(x^2 - x - 15)(x+3)}{x(4x^2 + 5x + 3)}
 \end{aligned}$$

$$\begin{aligned}
 83. \quad & 1 - \frac{1}{1 - \frac{1}{x}} = 1 - \frac{1}{\frac{x-1}{x}} \\
 &= 1 - \frac{x}{x-1} \\
 &= \frac{x-1-x}{x-1} \\
 &= \frac{-1}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 84. \quad & 1 - \frac{1}{1 - \frac{1}{1-x}} = 1 - \frac{1}{\frac{1-x-1}{1-x}} = 1 - \frac{1}{\frac{-x}{1-x}} \\
 &= 1 - \frac{1-x}{-x} = 1 + \frac{1-x}{x} \\
 &= \frac{x+1-x}{x} \\
 &= \frac{1}{x}
 \end{aligned}$$

$$\begin{aligned}
 85. \quad \frac{2(x-1)^{-1} + 3}{3(x-1)^{-1} + 2} &= \frac{\frac{2}{x-1} + 3}{\frac{3}{x-1} + 2} = \frac{\frac{2}{x-1} + \frac{3(x-1)}{x-1}}{\frac{3}{x-1} + \frac{2(x-1)}{x-1}} \\
 &= \frac{\frac{2+3(x-1)}{x-1}}{\frac{3+2(x-1)}{x-1}} \\
 &= \frac{2+3(x-1)}{3+2(x-1)} \cdot \frac{x-1}{x-1} \\
 &= \frac{2+3(x-1)}{3+2(x-1)} = \frac{2+3x-3}{3+2x-2} \\
 &= \frac{3x-1}{2x+1}
 \end{aligned}$$

$$\begin{aligned}
 86. \quad \frac{4(x+2)^{-1} - 3}{3(x+2)^{-1} - 1} &= \frac{\frac{4}{x+2} - 3}{\frac{3}{x+2} - 1} = \frac{\frac{4}{x+2} - \frac{3(x+2)}{x+2}}{\frac{3}{x+2} - \frac{1(x+2)}{x+2}} \\
 &= \frac{\frac{4-3(x+2)}{x+2}}{\frac{3-(x+2)}{x+2}} \\
 &= \frac{4-3(x+2)}{3-(x+2)} \cdot \frac{x+2}{x+2} \\
 &= \frac{4-3(x+2)}{3-(x+2)} = \frac{4-3x-6}{3-x-2} \\
 &= \frac{-3x-2}{-x+1} = \frac{3x+2}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 87. \quad \frac{(2x+3) \cdot 3 - (3x-5) \cdot 2}{(3x-5)^2} &= \frac{6x+9-6x+10}{(3x-5)^2} \\
 &= \frac{19}{(3x-5)^2}
 \end{aligned}$$

$$\begin{aligned}
 88. \quad \frac{(4x+1) \cdot 5 - (5x-2) \cdot 4}{(5x-2)^2} &= \frac{20x+5-20x+8}{(5x-2)^2} \\
 &= \frac{13}{(5x-2)^2}
 \end{aligned}$$

$$\begin{aligned}
 89. \quad \frac{x \cdot 2x - (x^2+1) \cdot 1}{(x^2+1)^2} &= \frac{2x^2 - x^2 - 1}{(x^2+1)^2} \\
 &= \frac{x^2 - 1}{(x^2+1)^2} \\
 &= \frac{(x-1)(x+1)}{(x^2+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 90. \quad \frac{x \cdot 2x - (x^2-4) \cdot 1}{(x^2-4)^2} &= \frac{2x^2 - x^2 + 4}{(x^2-4)^2} = \frac{x^2 + 4}{(x^2-4)^2} \\
 &= \frac{x^2 + 4}{(x+2)^2 (x-2)^2}
 \end{aligned}$$

$$\begin{aligned}
 91. \quad \frac{(3x+1) \cdot 2x - x^2 \cdot 3}{(3x+1)^2} &= \frac{6x^2 + 2x - 3x^2}{(3x+1)^2} \\
 &= \frac{3x^2 + 2x}{(3x+1)^2} \\
 &= \frac{x(3x+2)}{(3x+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 92. \quad \frac{(2x-5) \cdot 3x^2 - x^3 \cdot 2}{(2x-5)^2} &= \frac{6x^3 - 15x^2 - 2x^3}{(2x-5)^2} \\
 &= \frac{4x^3 - 15x^2}{(2x-5)^2} \\
 &= \frac{x^2(4x-15)}{(2x-5)^2}
 \end{aligned}$$

$$\begin{aligned}
 93. \quad \frac{(x^2+1) \cdot 3 - (3x+4) \cdot 2x}{(x^2+1)^2} &= \frac{3x^2 + 3 - 6x^2 - 8x}{(x^2+1)^2} \\
 &= \frac{-3x^2 - 8x + 3}{(x^2+1)^2} \\
 &= \frac{-(3x^2 + 8x - 3)}{(x^2+1)^2} \\
 &= -\frac{(3x-1)(x+3)}{(x^2+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 94. \quad \frac{(x^2+9) \cdot 2 - (2x-5) \cdot 2x}{(x^2+9)^2} &= \frac{2x^2+18-4x^2+10x}{(x^2+9)^2} \\
 &= \frac{-2x^2+10x+18}{(x^2+9)^2} \\
 &= \frac{-2(x^2-5x-9)}{(x^2+9)^2}
 \end{aligned}$$

$$\begin{aligned}
 95. \quad \frac{1}{f} &= (n-1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \\
 \frac{1}{f} &= (n-1) \left(\frac{R_2 + R_1}{R_1 \cdot R_2} \right) \\
 \frac{R_1 \cdot R_2}{f} &= (n-1)(R_2 + R_1) \\
 \frac{f}{R_1 \cdot R_2} &= \frac{1}{(n-1)(R_2 + R_1)} \\
 f &= \frac{R_1 \cdot R_2}{(n-1)(R_2 + R_1)} \\
 f &= \frac{0.1(0.2)}{(1.5-1)(0.2+0.1)} \\
 &= \frac{0.02}{0.5(0.3)} = \frac{0.02}{0.15} = \frac{2}{15} \text{ meters}
 \end{aligned}$$

$$\begin{aligned}
 96. \quad \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_1 R_2 R_3} \\
 R &= \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2} \\
 &= \frac{5 \cdot 4 \cdot 10}{4 \cdot 10 + 5 \cdot 10 + 5 \cdot 4} \\
 &= \frac{200}{110} = \frac{20}{11} \text{ ohms}
 \end{aligned}$$

$$\begin{aligned}
 97. \quad \frac{x^2+3xk-3x-9k}{x^2+2x-15} &= \frac{x(x+3k)-3(x+3k)}{(x-3)(x+5)} \\
 \frac{(x-3)(x+3k)}{(x-3)(x+5)} &= \frac{(x+3k)}{(x+5)} \\
 &= \frac{x+12}{x+5} \\
 x+3k &= x+12 \\
 3k &= 12 \\
 k &= 4
 \end{aligned}$$

$$\begin{aligned}
 98. \quad \frac{A}{x+2} + \frac{7}{x-3} &= \frac{12x-1}{x^2-x+6} \\
 A(x-3) + 7(x+2) &= 12x-1 \\
 Ax-3A+7x+14 &= 12x-1 \\
 x(A+7)-3A+14 &= 12x-1 \\
 &\text{so} \\
 A+7 &= 12 \\
 A &= 5
 \end{aligned}$$

$$\begin{aligned}
 99. \quad 1 + \frac{1}{x} &= \frac{x+1}{x} \Rightarrow a=1, b=1, c=0 \\
 1 + \frac{1}{1+\frac{1}{x}} &= 1 + \frac{1}{\left(\frac{x+1}{x}\right)} = 1 + \frac{x}{x+1} \\
 &= \frac{x+1+x}{x+1} = \frac{2x+1}{x+1} \\
 \Rightarrow a &= 2, b=1, c=1
 \end{aligned}$$

$$\begin{aligned}
 1 + \frac{1}{1+\frac{1}{1+\frac{1}{x}}} &= 1 + \frac{1}{\left(\frac{2x+1}{x+1}\right)} = 1 + \frac{x+1}{2x+1} \\
 &= \frac{2x+1+x+1}{2x+1} = \frac{3x+2}{2x+1} \\
 \Rightarrow a &= 3, b=2, c=1
 \end{aligned}$$

$$\begin{aligned}
 1 + \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}} &= 1 + \frac{1}{\left(\frac{3x+2}{2x+1}\right)} = 1 + \frac{2x+1}{3x+2} \\
 &= \frac{3x+2+2x+1}{3x+2} = \frac{5x+3}{3x+2} \\
 \Rightarrow a &= 5, b=3, c=2
 \end{aligned}$$

If we continue this process, the values of a , b and c produce the following sequences:

$a: 1, 2, 3, 5, 8, 13, 21, \dots$

$b: 1, 1, 2, 3, 5, 8, 13, 21, \dots$

$c: 0, 1, 1, 2, 3, 5, 8, 13, 21, \dots$

In each case we have a *Fibonacci Sequence*, where the next value in the list is obtained from

the sum of the previous 2 values in the list.

Chapter R: Review

100. Answers will vary.

101. Answers will vary.

Section R.8

1. 9; -9

2. 4; $|-4| = 4$

3. index

4. cube root

5. b

6. d

7. c

8. c

9. true

10. False; $\sqrt[4]{(-3)^4} = |-3| = 3$

11. $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$

12. $\sqrt[4]{16} = \sqrt[4]{2^4} = 2$

13. $\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

14. $\sqrt[3]{-1} = \sqrt[3]{(-1)^3} = -1$

15. $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

16. $\sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}$

17. $\sqrt{700} = \sqrt{100 \cdot 7} = 10\sqrt{7}$

18. $\sqrt{45x^3} = \sqrt{9 \cdot 5 \cdot x^2 \cdot x} = 3x\sqrt{5x}$

19. $\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = 2\sqrt[3]{4}$

20. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = 3\sqrt[3]{2}$

21. $\sqrt[3]{-8x^4} = \sqrt[3]{-8x^3 \cdot x} = -2x\sqrt[3]{x}$

22. $\sqrt[3]{192x^5} = \sqrt[3]{64 \cdot 3x^3 \cdot x^2} = 4x\sqrt[3]{3x^2}$

23. $\sqrt[4]{243} = \sqrt[4]{81 \cdot 3} = 3\sqrt[4]{3}$

24. $\sqrt[4]{48x^5} = \sqrt[4]{16x^4 \cdot 3x} = 2x\sqrt[4]{3x}$

25. $\sqrt[4]{x^{12}y^8} = \sqrt[4]{(x^3)^4(y^2)^4} = x^3y^2$

26. $\sqrt[5]{x^{10}y^5} = \sqrt[5]{(x^2)^5y^5} = x^2y$

27. $\sqrt[4]{\frac{x^9y^7}{xy^3}} = \sqrt[4]{x^8y^4} = x^2y$

28. $\sqrt[3]{\frac{3xy^2}{81x^4y^2}} = \sqrt[3]{\frac{1}{27x^3}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27x^3}} = \frac{1}{3x}$

29. $\sqrt{64x} = 8\sqrt{x}$

30. $\sqrt{9x^5} = 3\sqrt{x^4 \cdot x} = 3x^2\sqrt{x}$

31. $\sqrt[4]{162x^9y^{12}} = \sqrt[4]{2(3)^4x(x^2)^4(y^3)^4}$
 $= 3x^2y^3\sqrt[4]{2x}$

32. $\sqrt[3]{-40x^{14}y^{10}} = \sqrt[3]{5(-2)^3x^2(x^4)^3y(y^3)^3}$
 $= -2x^4y^3\sqrt[3]{5x^2y}$

33. $\sqrt{15x^2}\sqrt{5x} = \sqrt{75x^2 \cdot x} = \sqrt{25 \cdot 3 \cdot x^2 \cdot x} = 5x\sqrt{3x}$

34. $\sqrt{5x}\sqrt{20x^3} = \sqrt{100x^4} = 10x^2$

35. $(\sqrt{5}\sqrt[3]{9})^2 = (\sqrt{5})^2(\sqrt[3]{9})^2$
 $= 5 \cdot \sqrt[3]{9^2} = 5\sqrt[3]{81} = 5 \cdot 3\sqrt[3]{3} = 15\sqrt[3]{3}$

36. $(\sqrt[3]{3}\sqrt{10})^4 = (\sqrt[3]{3})^4(\sqrt{10})^4$
 $= \sqrt[3]{3^4} \cdot 10^2 = 3\sqrt[3]{3} \cdot 100 = 300\sqrt[3]{3}$

37. $(3\sqrt{6})(2\sqrt{2}) = 6\sqrt{12} = 6\sqrt{4 \cdot 3} = 12\sqrt{3}$

38. $(5\sqrt{8})(-3\sqrt{3}) = -15\sqrt{24} = -30\sqrt{6}$

39. $3\sqrt{2} + 4\sqrt{2} = (3+4)\sqrt{2} = 7\sqrt{2}$

$$40. \quad 6\sqrt{5} - 4\sqrt{5} = (6-4)\sqrt{5} = 2\sqrt{5}$$

$$\begin{aligned} 41. \quad -\sqrt{48} + 5\sqrt{12} &= -\sqrt{16 \cdot 3} + 5\sqrt{4 \cdot 3} \\ &= -4\sqrt{3} + 5 \cdot 2\sqrt{3} \\ &= (-4+10)\sqrt{3} \\ &= 6\sqrt{3} \end{aligned}$$

$$\begin{aligned} 42. \quad 2\sqrt{12} - 3\sqrt{27} &= 2\sqrt{4 \cdot 3} - 3\sqrt{9 \cdot 3} \\ &= 4\sqrt{3} - 9\sqrt{3} \\ &= (4-9)\sqrt{3} \\ &= -5\sqrt{3} \end{aligned}$$

$$\begin{aligned} 43. \quad (\sqrt{3}+3)(\sqrt{3}-1) &= (\sqrt{3})^2 + 3\sqrt{3} - \sqrt{3} - 3 \\ &= 3 + 2\sqrt{3} - 3 \\ &= 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} 44. \quad (\sqrt{5}-2)(\sqrt{5}+3) &= (\sqrt{5})^2 - 2\sqrt{5} + 3\sqrt{5} - 6 \\ &= 5 + \sqrt{5} - 6 \\ &= \sqrt{5} - 1 \end{aligned}$$

$$\begin{aligned} 45. \quad 5\sqrt[3]{2} - 2\sqrt[3]{54} &= 5\sqrt[3]{2} - 2 \cdot 3\sqrt[3]{2} \\ &= 5\sqrt[3]{2} - 6\sqrt[3]{2} \\ &= (5-6)\sqrt[3]{2} \\ &= -\sqrt[3]{2} \end{aligned}$$

$$\begin{aligned} 46. \quad 9\sqrt[3]{24} - \sqrt[3]{81} &= 9 \cdot 2\sqrt[3]{3} - 3\sqrt[3]{3} \\ &= 18\sqrt[3]{3} - 3\sqrt[3]{3} \\ &= (18-3)\sqrt[3]{3} \\ &= 15\sqrt[3]{3} \end{aligned}$$

$$\begin{aligned} 47. \quad (\sqrt{x}-1)^2 &= (\sqrt{x})^2 - 2\sqrt{x} + 1 \\ &= x - 2\sqrt{x} + 1 \end{aligned}$$

$$\begin{aligned} 48. \quad (\sqrt{x} + \sqrt{5})^2 &= (\sqrt{x})^2 + 2(\sqrt{x})(\sqrt{5}) + (\sqrt{5})^2 \\ &= x + 2\sqrt{5x} + 5 \end{aligned}$$

$$\begin{aligned} 49. \quad \sqrt[3]{16x^4} - \sqrt[3]{2x} &= \sqrt[3]{8x^3 \cdot 2x} - \sqrt[3]{2x} \\ &= 2x\sqrt[3]{2x} - \sqrt[3]{2x} \\ &= (2x-1)\sqrt[3]{2x} \end{aligned}$$

$$\begin{aligned} 50. \quad \sqrt[4]{32x} + \sqrt[4]{2x^5} &= \sqrt[4]{16 \cdot 2x} + \sqrt[4]{x^4 \cdot 2x} \\ &= 2\sqrt[4]{2x} + x\sqrt[4]{2x} \\ &= (2+x)\sqrt[4]{2x} \quad \text{or} \quad (x+2)\sqrt[4]{2x} \end{aligned}$$

$$\begin{aligned} 51. \quad \sqrt{8x^3} - 3\sqrt{50x} &= \sqrt{4x^2 \cdot 2x} - 3\sqrt{25 \cdot 2x} \\ &= 2x\sqrt{2x} - 15\sqrt{2x} \\ &= (2x-15)\sqrt{2x} \end{aligned}$$

$$\begin{aligned} 52. \quad 3x\sqrt{9y} + 4\sqrt{25y} &= 9x\sqrt{y} + 20\sqrt{y} \\ &= (9x+20)\sqrt{y} \end{aligned}$$

$$\begin{aligned} 53. \quad \sqrt[3]{16x^4y} - 3x\sqrt[3]{2xy} + 5\sqrt[3]{-2xy^4} \\ &= \sqrt[3]{8x^3 \cdot 2xy} - 3x\sqrt[3]{2xy} + 5\sqrt[3]{-y^3 \cdot 2xy} \\ &= 2x\sqrt[3]{2xy} - 3x\sqrt[3]{2xy} - 5y\sqrt[3]{2xy} \\ &= (2x-3x-5y)\sqrt[3]{2xy} \\ &= (-x-5y)\sqrt[3]{2xy} \quad \text{or} \quad -(x+5y)\sqrt[3]{2xy} \end{aligned}$$

$$\begin{aligned} 54. \quad 8xy - \sqrt{25x^2y^2} + \sqrt[3]{8x^3y^3} &= 8xy - 5xy + 2xy \\ &= (8-5+2)xy \\ &= 5xy \end{aligned}$$

$$55. \quad \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$56. \quad \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$57. \quad \frac{-\sqrt{3}}{\sqrt{5}} = \frac{-\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-\sqrt{15}}{5}$$

$$58. \quad \frac{-\sqrt{3}}{\sqrt{8}} = \frac{-\sqrt{3}}{2\sqrt{2}} = \frac{-\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{6}}{2 \cdot 2} = \frac{-\sqrt{6}}{4}$$

$$\begin{aligned} 59. \quad \frac{\sqrt{3}}{5-\sqrt{2}} &= \frac{\sqrt{3}}{5-\sqrt{2}} \cdot \frac{5+\sqrt{2}}{5+\sqrt{2}} \\ &= \frac{\sqrt{3}(5+\sqrt{2})}{25-2} \\ &= \frac{\sqrt{3}(5+\sqrt{2})}{23} \quad \text{or} \quad \frac{5\sqrt{3}+\sqrt{6}}{23} \end{aligned}$$

Chapter R: Review

$$\begin{aligned}
 60. \quad \frac{\sqrt{2}}{\sqrt{7}+2} &= \frac{\sqrt{2}}{\sqrt{7}+2} \cdot \frac{\sqrt{7}-2}{\sqrt{7}-2} \\
 &= \frac{\sqrt{2}(\sqrt{7}-2)}{7-4} \\
 &= \frac{\sqrt{2}(\sqrt{7}-2)}{3} \quad \text{or} \quad \frac{\sqrt{14}-2\sqrt{2}}{3}
 \end{aligned}$$

$$\begin{aligned}
 61. \quad \frac{2-\sqrt{5}}{2+3\sqrt{5}} &= \frac{2-\sqrt{5}}{2+3\sqrt{5}} \cdot \frac{2-3\sqrt{5}}{2-3\sqrt{5}} \\
 &= \frac{4-2\sqrt{5}-6\sqrt{5}+15}{4-45} \\
 &= \frac{19-8\sqrt{5}}{-41} = \frac{8\sqrt{5}-19}{41}
 \end{aligned}$$

$$\begin{aligned}
 62. \quad \frac{\sqrt{3}-1}{2\sqrt{3}+3} &= \frac{\sqrt{3}-1}{2\sqrt{3}+3} \cdot \frac{2\sqrt{3}-3}{2\sqrt{3}-3} \\
 &= \frac{6-2\sqrt{3}-3\sqrt{3}+3}{12-9} = \frac{9-5\sqrt{3}}{3}
 \end{aligned}$$

$$\begin{aligned}
 63. \quad \frac{5}{\sqrt{2}-1} &= \frac{5}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} \\
 &= \frac{5\sqrt{2}+5}{2-1} = 5\sqrt{2}+5
 \end{aligned}$$

$$\begin{aligned}
 64. \quad \frac{-3}{\sqrt{5}+4} &= \frac{-3}{\sqrt{5}+4} \cdot \frac{\sqrt{5}-4}{\sqrt{5}-4} \\
 &= \frac{-3\sqrt{5}+12}{5-16} = \frac{-3\sqrt{5}+12}{-11} \\
 &= \frac{3\sqrt{5}-12}{11}
 \end{aligned}$$

$$65. \quad \frac{5}{\sqrt[3]{2}} = \frac{5}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{5\sqrt[3]{4}}{2}$$

$$66. \quad \frac{-2}{\sqrt[3]{9}} = \frac{-2}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{-2\sqrt[3]{3}}{3}$$

$$\begin{aligned}
 67. \quad \frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} &= \frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} \cdot \frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}-\sqrt{x}} \\
 &= \frac{(x+h)-2\sqrt{x(x+h)}+x}{(x+h)-x} \\
 &= \frac{x+h-2\sqrt{x^2+xh}+x}{x+h-x} \\
 &= \frac{2x+h-2\sqrt{x^2+xh}}{h}
 \end{aligned}$$

$$\begin{aligned}
 68. \quad \frac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}-\sqrt{x-h}} &= \frac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}-\sqrt{x-h}} \cdot \frac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}+\sqrt{x-h}} \\
 &= \frac{(x+h)+2\sqrt{(x-h)(x+h)}+(x-h)}{(x+h)-(x-h)} \\
 &= \frac{x+h+2\sqrt{x^2-h^2}+x-h}{x+h-x+h} \\
 &= \frac{2x+2\sqrt{x^2-h^2}}{2h} \\
 &= \frac{x+\sqrt{x^2-h^2}}{h}
 \end{aligned}$$

$$\begin{aligned}
 69. \quad \frac{\sqrt{11}+1}{2} &= \frac{\sqrt{11}+1}{2} \cdot \frac{\sqrt{11}-1}{\sqrt{11}-1} \\
 &= \frac{11-1}{2(\sqrt{11}-1)} = \frac{10}{2(\sqrt{11}-1)} \\
 &= \frac{5}{\sqrt{11}-1}
 \end{aligned}$$

$$\begin{aligned}
 70. \quad \frac{5-\sqrt{43}}{3} &= \frac{5-\sqrt{43}}{3} \cdot \frac{5+\sqrt{43}}{5+\sqrt{43}} = \frac{25-43}{3(5+\sqrt{43})} \\
 &= \frac{-18}{3(5+\sqrt{43})} = \frac{-6}{5+\sqrt{43}} = -\frac{6}{5+\sqrt{43}}
 \end{aligned}$$

$$\begin{aligned}
 71. \quad \frac{\sqrt{6}-\sqrt{15}}{\sqrt{15}} &= \frac{\sqrt{6}-\sqrt{15}}{\sqrt{15}} \cdot \frac{\sqrt{6}+\sqrt{15}}{\sqrt{6}+\sqrt{15}} \\
 &= \frac{6-15}{\sqrt{90}-15} = \frac{-9}{3\sqrt{10}-15} \\
 &= \frac{-9}{3(\sqrt{10}-5)} = -\frac{3}{\sqrt{10}-5}
 \end{aligned}$$

$$72. \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{5-3}{5-\sqrt{15}} = \frac{2}{5-\sqrt{15}}$$

$$73. \frac{\sqrt{x}-\sqrt{c}}{x-c} = \frac{\sqrt{x}-\sqrt{c}}{x-c} \cdot \frac{\sqrt{x}+\sqrt{c}}{\sqrt{x}+\sqrt{c}}$$

$$= \frac{x-c}{(x-c)(\sqrt{x}+\sqrt{c})} = \frac{1}{\sqrt{x}+\sqrt{c}}$$

$$74. \frac{\sqrt{x}-2}{x-4} = \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$= \frac{x-4}{(x-4)(\sqrt{x}+2)} = \frac{1}{\sqrt{x}+2}$$

$$75. \frac{\sqrt{x-7}-1}{x-8} = \frac{\sqrt{x-7}-1}{x-8} \cdot \frac{\sqrt{x-7}+1}{\sqrt{x-7}+1}$$

$$= \frac{x-7-1}{(x-8)(\sqrt{x-7}+1)}$$

$$= \frac{x-8}{(x-8)(\sqrt{x-7}+1)}$$

$$= \frac{1}{\sqrt{x-7}+1}$$

$$76. \frac{4-\sqrt{x-9}}{x-25} = \frac{4-\sqrt{x-9}}{x-25} \cdot \frac{4+\sqrt{x-9}}{4+\sqrt{x-9}}$$

$$= \frac{16-(x-9)}{(x-25)(4+\sqrt{x-9})}$$

$$= \frac{25-x}{(x-25)(4+\sqrt{x-9})}$$

$$= -\frac{x-25}{(x-25)(4+\sqrt{x-9})}$$

$$= -\frac{1}{4+\sqrt{x-9}}$$

$$77. 8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$$

$$78. 4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$$

$$79. (-64)^{1/3} = \sqrt[3]{-64} = -4$$

$$80. 16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$$

$$81. 100^{3/2} = (\sqrt{100})^3 = 10^3 = 1000$$

$$82. 25^{3/2} = (\sqrt{25})^3 = 5^3 = 125$$

$$83. 4^{-3/2} = \frac{1}{4^{3/2}} = \frac{1}{(\sqrt{4})^3} = \frac{1}{2^3} = \frac{1}{8}$$

$$84. 16^{-3/2} = \frac{1}{16^{3/2}} = \frac{1}{(\sqrt{16})^3} = \frac{1}{4^3} = \frac{1}{64}$$

$$85. \left(\frac{9}{8}\right)^{3/2} = \left(\sqrt{\frac{9}{8}}\right)^3 = \left(\frac{3}{2\sqrt{2}}\right)^3 = \frac{3^3}{2^3(\sqrt{2})^3}$$

$$= \frac{27}{8 \cdot 2\sqrt{2}} = \frac{27}{16\sqrt{2}} = \frac{27}{16\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{27\sqrt{2}}{32}$$

$$86. \left(\frac{27}{8}\right)^{2/3} = \left(\sqrt[3]{\frac{27}{8}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$87. \left(\frac{8}{9}\right)^{-3/2} = \left(\frac{9}{8}\right)^{3/2} = \left(\sqrt{\frac{9}{8}}\right)^3 = \left(\frac{3}{2\sqrt{2}}\right)^3$$

$$= \frac{3^3}{2^3(\sqrt{2})^3} = \frac{27}{8 \cdot 2\sqrt{2}} = \frac{27}{16\sqrt{2}}$$

$$= \frac{27}{16\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{27\sqrt{2}}{32}$$

$$88. \left(\frac{8}{27}\right)^{-2/3} = \left(\frac{27}{8}\right)^{2/3} = \left(\sqrt[3]{\frac{27}{8}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$89. (-1000)^{-1/3} = \left(-\frac{1}{1000}\right)^{1/3} = \sqrt[3]{-\frac{1}{1000}} = -\frac{1}{10}$$

$$90. -25^{-1/2} = -\left(\frac{1}{25}\right)^{1/2} = -\sqrt{\frac{1}{25}} = -\frac{1}{5}$$

Chapter R: Review

$$91. \left(-\frac{64}{125}\right)^{-2/3} = \left(-\frac{125}{64}\right)^{2/3} = \left(\sqrt[3]{-\frac{125}{64}}\right)^2 \\ = \left(-\frac{5}{4}\right)^2 = \frac{25}{16}$$

$$92. -81^{-3/4} = -\left(\frac{1}{81}\right)^{3/4} = -\left(\sqrt[4]{\frac{1}{81}}\right)^3 \\ = -\left(\frac{1}{3}\right)^3 = -\frac{1}{27}$$

$$93. x^{3/4} x^{1/3} x^{-1/2} = x^{3/4+1/3-1/2} = x^{7/12}$$

$$94. x^{2/3} x^{1/2} x^{-1/4} = x^{2/3+1/2-1/4} = x^{11/12}$$

$$95. (x^3 y^6)^{1/3} = (x^3)^{1/3} (y^6)^{1/3} = xy^2$$

$$96. (x^4 y^8)^{3/4} = (x^4)^{3/4} (y^8)^{3/4} = x^3 y^6$$

$$97. \frac{(x^2 y)^{1/3} (xy^2)^{2/3}}{x^{2/3} y^{2/3}} = \frac{(x^2)^{1/3} (y)^{1/3} (x)^{2/3} (y^2)^{2/3}}{x^{2/3} y^{2/3}} \\ = \frac{x^{2/3} y^{1/3} x^{2/3} y^{4/3}}{x^{2/3} y^{2/3}} \\ = x^{2/3+2/3-2/3} y^{1/3+4/3-2/3} \\ = x^{2/3} y^1 = x^{2/3} y$$

$$98. \frac{(xy)^{1/4} (x^2 y^2)^{1/2}}{(x^2 y)^{3/4}} = \frac{x^{1/4} y^{1/4} (x^2)^{1/2} (y^2)^{1/2}}{(x^2)^{3/4} y^{3/4}} \\ = \frac{x^{1/4} y^{1/4} xy}{x^{3/2} y^{3/4}} \\ = x^{1/4+1-3/2} y^{1/4+1-3/4} \\ = x^{-1/4} y^{1/2} = \frac{y^{1/2}}{x^{1/4}}$$

$$99. \frac{(16x^2 y^{-1/3})^{3/4}}{(xy^2)^{1/4}} = \frac{16^{3/4} (x^2)^{3/4} (y^{-1/3})^{3/4}}{x^{1/4} (y^2)^{1/4}} \\ = \frac{(\sqrt[4]{16})^3 x^{3/2} y^{-1/4}}{x^{1/4} y^{1/2}} \\ = 2^3 x^{3/2-1/4} y^{-1/4-1/2} \\ = 8x^{5/4} y^{-3/4} \\ = \frac{8x^{5/4}}{y^{3/4}}$$

$$100. \frac{(4x^{-1} y^{1/3})^{3/2}}{(xy)^{3/2}} = \frac{4^{3/2} (x^{-1})^{3/2} (y^{1/3})^{3/2}}{x^{3/2} y^{3/2}} \\ = \frac{(\sqrt{4})^3 x^{-3/2} y^{1/2}}{x^{3/2} y^{3/2}} \\ = 2^3 x^{-3/2-3/2} y^{1/2-3/2} \\ = 8x^{-3} y^{-1} \\ = \frac{8}{x^3 y}$$

$$101. \frac{x}{(1+x)^{1/2}} + 2(1+x)^{1/2} = \frac{x+2(1+x)^{1/2} (1+x)^{1/2}}{(1+x)^{1/2}} \\ = \frac{x+2(1+x)}{(1+x)^{1/2}} \\ = \frac{x+2+2x}{(1+x)^{1/2}} \\ = \frac{3x+2}{(1+x)^{1/2}}$$

$$102. \frac{1+x}{2x^{1/2}} + x^{1/2} = \frac{1+x+x^{1/2} \cdot 2x^{1/2}}{2x^{1/2}} \\ = \frac{1+x+2x}{2x^{1/2}} = \frac{3x+1}{2x^{1/2}}$$

$$\begin{aligned}
 103. \quad & 2x(x^2+1)^{1/2} + x^2 \cdot \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x \\
 &= 2x(x^2+1)^{1/2} + \frac{x^3}{(x^2+1)^{1/2}} \\
 &= \frac{2x(x^2+1)^{1/2} \cdot (x^2+1)^{1/2} + x^3}{(x^2+1)^{1/2}} \\
 &= \frac{2x(x^2+1)^{1/2+1/2} + x^3}{(x^2+1)^{1/2}} = \frac{2x(x^2+1)^1 + x^3}{(x^2+1)^{1/2}} \\
 &= \frac{2x^3 + 2x + x^3}{(x^2+1)^{1/2}} = \frac{3x^3 + 2x}{(x^2+1)^{1/2}} \\
 &= \frac{x(3x^2 + 2)}{(x^2+1)^{1/2}}
 \end{aligned}$$

$$\begin{aligned}
 104. \quad & (x+1)^{1/3} + x \cdot \frac{1}{3}(x+1)^{-2/3}, x \neq -1 \\
 &= (x+1)^{1/3} + \frac{x}{3(x+1)^{2/3}} \\
 &= \frac{3(x+1)^{2/3}(x+1)^{1/3} + x}{3(x+1)^{2/3}} \\
 &= \frac{3(x+1)^{2/3+1/3} + x}{3(x+1)^{2/3}} = \frac{3(x+1)^1 + x}{3(x+1)^{2/3}} \\
 &= \frac{3x+3+x}{3(x+1)^{2/3}} = \frac{4x+3}{3(x+1)^{2/3}}
 \end{aligned}$$

$$\begin{aligned}
 105. \quad & \sqrt{4x+3} \cdot \frac{1}{2\sqrt{x-5}} + \sqrt{x-5} \cdot \frac{1}{5\sqrt{4x+3}}, x > 5 \\
 &= \frac{\sqrt{4x+3}}{2\sqrt{x-5}} + \frac{\sqrt{x-5}}{5\sqrt{4x+3}} \\
 &= \frac{\sqrt{4x+3} \cdot 5 \cdot \sqrt{4x+3} + \sqrt{x-5} \cdot 2 \cdot \sqrt{x-5}}{10\sqrt{x-5}\sqrt{4x+3}} \\
 &= \frac{5(4x+3) + 2(x-5)}{10\sqrt{(x-5)(4x+3)}} \\
 &= \frac{20x+15+2x-10}{10\sqrt{(x-5)(4x+3)}} \\
 &= \frac{22x+5}{10\sqrt{(x-5)(4x+3)}}
 \end{aligned}$$

$$\begin{aligned}
 106. \quad & \frac{\sqrt[3]{8x+1}}{3\sqrt[3]{(x-2)^2}} + \frac{\sqrt[3]{x-2}}{24\sqrt[3]{(8x+1)^2}}, x \neq 2, x \neq -\frac{1}{8} \\
 &= \frac{8\sqrt[3]{8x+1} \cdot \sqrt[3]{(8x+1)^2} + \sqrt[3]{x-2} \cdot \sqrt[3]{(x-2)^2}}{24\sqrt[3]{(x-2)^2} \cdot \sqrt[3]{(8x+1)^2}} \\
 &= \frac{8\sqrt[3]{(8x+1)^3} + \sqrt[3]{(x-2)^3}}{24\sqrt[3]{(x-2)^2} \cdot \sqrt[3]{(8x+1)^2}} \\
 &= \frac{8(8x+1) + x-2}{24\sqrt[3]{(x-2)^2(8x+1)^2}} \\
 &= \frac{64x+8+x-2}{24\sqrt[3]{(x-2)^2(8x+1)^2}} \\
 &= \frac{65x+6}{24\sqrt[3]{(x-2)^2(8x+1)^2}}
 \end{aligned}$$

$$\begin{aligned}
 107. \quad & \frac{\left(\sqrt{1+x} - x \cdot \frac{1}{2\sqrt{1+x}}\right)}{1+x} = \frac{\left(\sqrt{1+x} - \frac{x}{2\sqrt{1+x}}\right)}{1+x} \\
 &= \frac{\left(\frac{2\sqrt{1+x}\sqrt{1+x} - x}{2\sqrt{1+x}}\right)}{1+x} \\
 &= \frac{2(1+x) - x}{2(1+x)^{1/2}} \cdot \frac{1}{1+x} \\
 &= \frac{2+x}{2(1+x)^{3/2}}
 \end{aligned}$$

$$\begin{aligned}
 108. \quad & \frac{\left(\sqrt{x^2+1} - x \cdot \frac{2x}{2\sqrt{x^2+1}}\right)}{x^2+1} \\
 &= \frac{\left(\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}\right)}{x^2+1} \\
 &= \frac{\left(\sqrt{x^2+1} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} - \frac{x^2}{\sqrt{x^2+1}}\right)}{x^2+1} \\
 &= \frac{\left(\frac{x^2+1-x^2}{\sqrt{x^2+1}}\right)}{x^2+1} = \frac{1}{\sqrt{x^2+1}} \cdot \frac{1}{x^2+1} \\
 &= \frac{1}{(x^2+1)^{3/2}}
 \end{aligned}$$

Chapter R: Review

$$\begin{aligned}
 109. \quad & \frac{(x+4)^{1/2} - 2x(x+4)^{-1/2}}{x+4} \\
 &= \frac{\left((x+4)^{1/2} - \frac{2x}{(x+4)^{1/2}} \right)}{x+4} \\
 &= \frac{\left((x+4)^{1/2} \cdot \frac{(x+4)^{1/2}}{(x+4)^{1/2}} - \frac{2x}{(x+4)^{1/2}} \right)}{x+4} \\
 &= \frac{\left(\frac{x+4-2x}{(x+4)^{1/2}} \right)}{x+4} \\
 &= \frac{-x+4}{(x+4)^{1/2}} \cdot \frac{1}{x+4} \\
 &= \frac{-x+4}{(x+4)^{3/2}} \\
 &= \frac{4-x}{(x+4)^{3/2}}
 \end{aligned}$$

$$\begin{aligned}
 110. \quad & \frac{(9-x^2)^{1/2} + x^2(9-x^2)^{-1/2}}{9-x^2}, -3 < x < 3 \\
 &= \frac{\left((9-x^2)^{1/2} + \frac{x^2}{(9-x^2)^{1/2}} \right)}{9-x^2} \\
 &= \frac{\left(\frac{(9-x^2)^{1/2} \cdot (9-x^2)^{1/2} + x^2}{(9-x^2)^{1/2}} \right)}{9-x^2} \\
 &= \frac{(9-x^2)^{1/2} \cdot (9-x^2)^{1/2} + x^2}{(9-x^2)^{1/2}} \cdot \frac{1}{9-x^2} \\
 &= \frac{9-x^2+x^2}{(9-x^2)^{1/2}} \cdot \frac{1}{9-x^2} \\
 &= \frac{9}{(9-x^2)^{3/2}}
 \end{aligned}$$

$$\begin{aligned}
 111. \quad & \frac{\frac{x^2}{(x^2-1)^{1/2}} - (x^2-1)^{1/2}}{x^2}, x < -1 \text{ or } x > 1 \\
 &= \frac{\left(\frac{x^2 - (x^2-1)^{1/2} \cdot (x^2-1)^{1/2}}{(x^2-1)^{1/2}} \right)}{x^2} \\
 &= \frac{x^2 - (x^2-1)^{1/2} \cdot (x^2-1)^{1/2}}{(x^2-1)^{1/2}} \cdot \frac{1}{x^2} \\
 &= \frac{x^2 - (x^2-1)}{(x^2-1)^{1/2}} \cdot \frac{1}{x^2} \\
 &= \frac{x^2 - x^2 + 1}{(x^2-1)^{1/2}} \cdot \frac{1}{x^2} \\
 &= \frac{1}{x^2(x^2-1)^{1/2}}
 \end{aligned}$$

$$\begin{aligned}
 112. \quad & \frac{(x^2+4)^{1/2} - x^2(x^2+4)^{-1/2}}{x^2+4} \\
 &= \frac{\left((x^2+4)^{1/2} - \frac{x^2}{(x^2+4)^{1/2}} \right)}{x^2+4} \\
 &= \frac{\left(\frac{(x^2+4)^{1/2} \cdot (x^2+4)^{1/2} - x^2}{(x^2+4)^{1/2}} \right)}{x^2+4} \\
 &= \frac{(x^2+4)^{1/2} \cdot (x^2+4)^{1/2} - x^2}{(x^2+4)^{1/2}} \cdot \frac{1}{x^2+4} \\
 &= \frac{x^2+4-x^2}{(x^2+4)^{1/2}} \cdot \frac{1}{x^2+4} = \frac{4}{(x^2+4)^{3/2}}
 \end{aligned}$$

$$\begin{aligned}
 113. \quad & \frac{\frac{1+x^2}{2\sqrt{x}} - 2x\sqrt{x}}{(1+x^2)^2}, x > 0 \\
 & = \frac{\left(\frac{1+x^2 - (2\sqrt{x})(2x\sqrt{x})}{2\sqrt{x}} \right)}{(1+x^2)^2} \\
 & = \frac{1+x^2 - (2\sqrt{x})(2x\sqrt{x})}{2\sqrt{x}} \cdot \frac{1}{(1+x^2)^2} \\
 & = \frac{1+x^2-4x^2}{2\sqrt{x}} \cdot \frac{1}{(1+x^2)^2} = \frac{1-3x^2}{2\sqrt{x}(1+x^2)^2}
 \end{aligned}$$

$$\begin{aligned}
 114. \quad & \frac{2x(1-x^2)^{1/3} + \frac{2}{3}x^3(1-x^2)^{-2/3}}{(1-x^2)^{2/3}}, x \neq -1, x \neq 1 \\
 & = \frac{\left(2x(1-x^2)^{1/3} + \frac{2x^3}{3(1-x^2)^{2/3}} \right)}{(1-x^2)^{2/3}} \\
 & = \frac{\left(\frac{2x(1-x^2)^{1/3} \cdot 3(1-x^2)^{2/3} + 2x^3}{3(1-x^2)^{2/3}} \right)}{(1-x^2)^{2/3}} \\
 & = \frac{6x(1-x^2)^{1/3+2/3} + 2x^3}{3(1-x^2)^{2/3}} \cdot \frac{1}{(1-x^2)^{2/3}} \\
 & = \frac{6x(1-x^2) + 2x^3}{3(1-x^2)^{2/3+2/3}} = \frac{6x-6x^3+2x^3}{3(1-x^2)^{4/3}} \\
 & = \frac{6x-4x^3}{3(1-x^2)^{4/3}} = \frac{2x(3-2x^2)}{3(1-x^2)^{4/3}}
 \end{aligned}$$

$$\begin{aligned}
 115. \quad & (x+1)^{3/2} + x \cdot \frac{3}{2}(x+1)^{1/2} \\
 & = (x+1)^{1/2} \left(x+1 + \frac{3}{2}x \right) \\
 & = (x+1)^{1/2} \left(\frac{5}{2}x+1 \right) \\
 & = \frac{1}{2}(x+1)^{1/2} (5x+2)
 \end{aligned}$$

$$\begin{aligned}
 116. \quad & (x^2+4)^{4/3} + x \cdot \frac{4}{3}(x^2+4)^{1/3} \cdot 2x \\
 & = (x^2+4)^{1/3} \left(x^2+4 + \frac{8}{3}x^2 \right) \\
 & = (x^2+4)^{1/3} \left(\frac{11}{3}x^2+4 \right) \\
 & = \frac{1}{3}(x^2+4)^{1/3} (11x^2+12)
 \end{aligned}$$

$$\begin{aligned}
 117. \quad & 6x^{1/2}(x^2+x) - 8x^{3/2} - 8x^{1/2} \\
 & = 2x^{1/2}(3(x^2+x) - 4x - 4) \\
 & = 2x^{1/2}(3x^2 - x - 4) \\
 & = 2x^{1/2}(3x-4)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 118. \quad & 6x^{1/2}(2x+3) + x^{3/2} \cdot 8 \\
 & = 2x^{1/2}(3(2x+3) + 4x) \\
 & = 2x^{1/2}(10x+9)
 \end{aligned}$$

$$\begin{aligned}
 119. \quad & 3(x^2+4)^{4/3} + x \cdot 4(x^2+4)^{1/3} \cdot 2x \\
 & = (x^2+4)^{1/3} [3(x^2+4) + 8x^2] \\
 & = (x^2+4)^{1/3} [3x^2+12+8x^2] \\
 & = (x^2+4)^{1/3} (11x^2+12)
 \end{aligned}$$

$$\begin{aligned}
 120. \quad & 2x(3x+4)^{4/3} + x^2 \cdot 4(3x+4)^{1/3} \\
 & = 2x(3x+4)^{1/3} [(3x+4) + 2x] \\
 & = 2x(3x+4)^{1/3} (5x+4)
 \end{aligned}$$

$$\begin{aligned}
 121. \quad & 4(3x+5)^{1/3}(2x+3)^{3/2} + 3(3x+5)^{4/3}(2x+3)^{1/2} \\
 & = (3x+5)^{1/3}(2x+3)^{1/2} [4(2x+3) + 3(3x+5)] \\
 & = (3x+5)^{1/3}(2x+3)^{1/2} (8x+12+9x+15) \\
 & = (3x+5)^{1/3}(2x+3)^{1/2} (17x+27)
 \end{aligned}$$

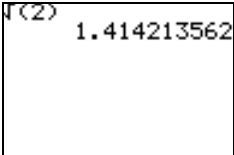
where $x \geq -\frac{3}{2}$.

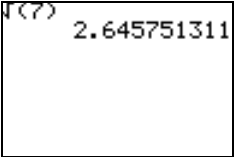
Chapter R: Review

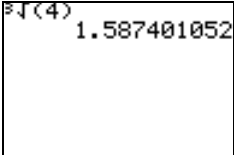
$$\begin{aligned}
 122. \quad & 6(6x+1)^{1/3}(4x-3)^{3/2} + 6(6x+1)^{4/3}(4x-3)^{1/2} \\
 &= 6(6x+1)^{1/3}(4x-3)^{1/2}[(4x-3) + (6x+1)] \\
 &= 6(6x+1)^{1/3}(4x-3)^{1/2}(10x-2) \\
 &= 6(6x+1)^{1/3}(4x-3)^{1/2}(2)(5x-1) \\
 &= 12(6x+1)^{1/3}(4x-3)^{1/2}(5x-1) \\
 &\text{where } x \geq \frac{3}{4}.
 \end{aligned}$$

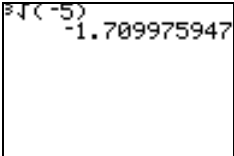
$$\begin{aligned}
 123. \quad & 3x^{-1/2} + \frac{3}{2}x^{1/2}, x > 0 \\
 &= \frac{3}{x^{1/2}} + \frac{3}{2}x^{1/2} \\
 &= \frac{3 \cdot 2 + 3x^{1/2} \cdot x^{1/2}}{2x^{1/2}} = \frac{6 + 3x}{2x^{1/2}} = \frac{3(x+2)}{2x^{1/2}}
 \end{aligned}$$

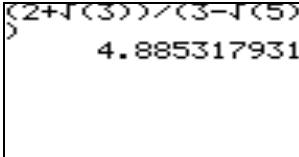
$$\begin{aligned}
 124. \quad & 8x^{1/3} - 4x^{-2/3}, x \neq 0 \\
 &= 8x^{1/3} - \frac{4}{x^{2/3}} \\
 &= \frac{8x^{1/3} \cdot x^{2/3} - 4}{x^{2/3}} = \frac{8x - 4}{x^{2/3}} = \frac{4(2x-1)}{x^{2/3}}
 \end{aligned}$$

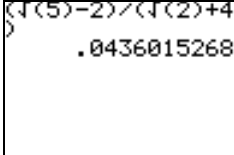
$$125. \quad \sqrt{2} \approx 1.41$$


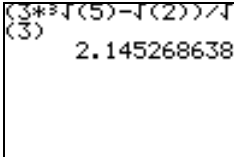
$$126. \quad \sqrt{7} \approx 2.65$$


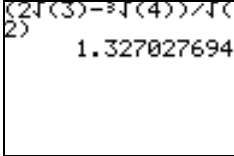
$$127. \quad \sqrt[3]{4} \approx 1.59$$


$$128. \quad \sqrt[3]{-5} \approx -1.71$$


$$129. \quad \frac{2+\sqrt{3}}{3-\sqrt{5}} \approx 4.89$$


$$130. \quad \frac{\sqrt{5}-2}{\sqrt{2}+4} \approx 0.04$$


$$131. \quad \frac{3\sqrt[3]{5}-\sqrt{2}}{\sqrt{3}} \approx 2.15$$


$$132. \quad \frac{2\sqrt{3}-\sqrt[3]{4}}{\sqrt{2}} \approx 1.33$$


$$133. \quad \text{a. } V = 40(12)^2 \sqrt{\frac{96}{12}} - 0.608 \approx 15,660.4 \text{ gallons}$$

$$\text{b. } V = 40(1)^2 \sqrt{\frac{96}{1}} - 0.608 \approx 390.7 \text{ gallons}$$

$$134. \quad \text{a. } v = \sqrt{64 \cdot 4 + 0^2} = \sqrt{256} = 16 \text{ feet per second}$$

$$\text{b. } v = \sqrt{64 \cdot 16 + 0^2} = \sqrt{1024} = 32 \text{ feet per second}$$

$$\text{c. } v = \sqrt{64 \cdot 2 + 4^2} = \sqrt{144} = 12 \text{ feet per second}$$

$$135. \quad T = 2\pi\sqrt{\frac{64}{32}} = 2\pi\sqrt{2} \approx 8.89 \text{ seconds}$$

$$\begin{aligned}
 136. \quad T &= 2\pi\sqrt{\frac{16}{32}} = 2\pi\sqrt{\frac{1}{2}} = \frac{2\pi}{\sqrt{2}} \\
 &= \pi\sqrt{2} \approx 4.44 \text{ seconds}
 \end{aligned}$$

$$\begin{array}{r}
 137. \quad \sqrt{3} \overline{) 1 \quad -4 \quad -3 \quad 13} \\
 \underline{ \sqrt{3} \quad 3-4\sqrt{3} \quad -12} \\
 1 \quad \sqrt{3}-4 \quad -4\sqrt{3} \quad 1
 \end{array}$$

The quotient is $x^2 + (\sqrt{3} - 4)x - 4\sqrt{3}$

The remainder is 1

$$\begin{array}{r}
 138. \quad 1+\sqrt{2} \overline{) 1 \quad -9 \quad 13 \quad 7} \\
 \underline{ 1+\sqrt{2} \quad -6-7\sqrt{2} \quad -7} \\
 1 \quad \sqrt{3}-8 \quad -7-7\sqrt{2} \quad 0
 \end{array}$$

Yes, $1+\sqrt{2}$ is a factor of $x^3 - 9x^2 + 13x + 7$.

139. Answers may vary. One possibility follows: If

$$a = -5, \text{ then } \sqrt{a^2} = \sqrt{(-5)^2} = \sqrt{25} = 5 \neq a.$$

Since we use the principal square root, which is always non-negative,

$$\sqrt{a^2} = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

which is the definition of $|a|$, so $\sqrt{a^2} = |a|$.

Chapter 1

Graphs, Equations, and Inequalities

Section 1.1

1. 0



2.

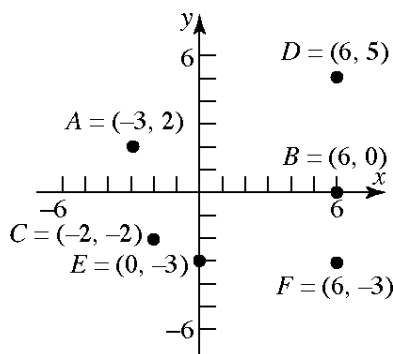
3. x -coordinate; y -coordinate; quadrants

4. False; points that lie in Quadrant IV will have a positive x -coordinate and a negative y -coordinate.
The point $(-1, 4)$ lies in Quadrant II.

5. d

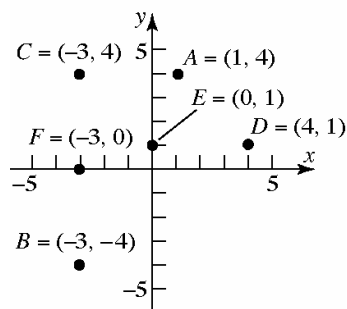
6. c

7. (a) Quadrant II
(b) x -axis
(c) Quadrant III
(d) Quadrant I
(e) y -axis
(f) Quadrant IV

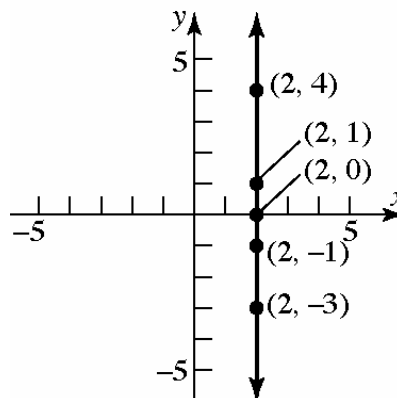


8. (a) Quadrant I
(b) Quadrant III
(c) Quadrant II
(d) Quadrant I
(e) y -axis

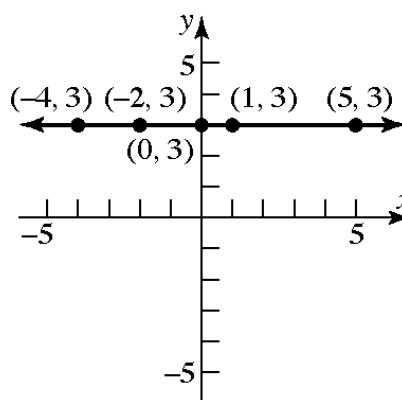
(f) x -axis



9. The points will be on a vertical line that is two units to the right of the y -axis.



10. The points will be on a horizontal line that is three units above the x -axis.



11. $(-1, 4)$; Quadrant II

12. $(3, 4)$; Quadrant I

13. $(3, 1)$; Quadrant I

Section 1.1: Graphing Utilities; Introduction to Graphing Equations

14. $(-6, -4)$; Quadrant III

15. $X \min = -11$

$X \max = 5$

$X \text{ scl} = 1$

$Y \min = -3$

$Y \max = 6$

$Y \text{ scl} = 1$

16. $X \min = -3$

$X \max = 7$

$X \text{ scl} = 1$

$Y \min = -4$

$Y \max = 9$

$Y \text{ scl} = 1$

17. $X \min = -30$

$X \max = 50$

$X \text{ scl} = 10$

$Y \min = -90$

$Y \max = 50$

$Y \text{ scl} = 10$

18. $X \min = -90$

$X \max = 30$

$X \text{ scl} = 10$

$Y \min = -50$

$Y \max = 70$

$Y \text{ scl} = 10$

19. $X \min = -10$

$X \max = 110$

$X \text{ scl} = 10$

$Y \min = -10$

$Y \max = 160$

$Y \text{ scl} = 10$

20. $X \min = -20$

$X \max = 110$

$X \text{ scl} = 10$

$Y \min = -10$

$Y \max = 60$

$Y \text{ scl} = 10$

21. $X \min = -6$

$X \max = 6$

$X \text{ scl} = 2$

$Y \min = -4$

$Y \max = 4$

$Y \text{ scl} = 2$

22. $X \min = -3$

$X \max = 3$

$X \text{ scl} = 1$

$Y \min = -2$

$Y \max = 2$

$Y \text{ scl} = 1$

23. $X \min = -6$

$X \max = 6$

$X \text{ scl} = 2$

$Y \min = -1$

$Y \max = 3$

$Y \text{ scl} = 1$

24. $X \min = -9$

$X \max = 9$

$X \text{ scl} = 3$

$Y \min = -12$

$Y \max = 4$

$Y \text{ scl} = 4$

25. $X \min = 3$

$X \max = 9$

$X \text{ scl} = 1$

$Y \min = 2$

$Y \max = 10$

$Y \text{ scl} = 2$

26. $X \min = -22$

$X \max = -10$

$X \text{ scl} = 2$

$Y \min = 4$

$Y \max = 8$

$Y \text{ scl} = 1$

Chapter 1: Graphs, Equations, and Inequalities

27. $y = x^4 - \sqrt{x}$

$$0 = 0^4 - \sqrt{0} \quad 1 = 1^4 - \sqrt{1} \quad 0 = (-1)^4 - \sqrt{-1}$$

$$0 = 0 \quad 1 \neq 0 \quad 0 \neq 1 - \sqrt{-1}$$

(0, 0) is on the graph of the equation.

28. $y = x^3 - 2\sqrt{x}$

$$0 = 0^3 - 2\sqrt{0} \quad 1 = 1^3 - 2\sqrt{1} \quad -1 = 1^3 - 2\sqrt{1}$$

$$0 = 0 \quad 1 \neq -1 \quad -1 = -1$$

(0, 0) and (1, -1) are on the graph of the equation.

29. $y^2 = x^2 + 9$

$$3^2 = 0^2 + 9 \quad 0^2 = 3^2 + 9 \quad 0^2 = (-3)^2 + 9$$

$$9 = 9 \quad 0 \neq 18 \quad 0 \neq 18$$

(0, 3) is on the graph of the equation.

30. $y^3 = x + 1$

$$2^3 = 1 + 1 \quad 1^3 = 0 + 1 \quad 0^3 = -1 + 1$$

$$8 \neq 2 \quad 1 = 1 \quad 0 = 0$$

(0, 1) and (-1, 0) are on the graph of the equation.

31. $x^2 + y^2 = 4$

$$0^2 + 2^2 = 4 \quad (-2)^2 + 2^2 = 4 \quad \sqrt{2}^2 + \sqrt{2}^2 = 4$$

$$4 = 4 \quad 8 \neq 4 \quad 4 = 4$$

(0, 2) and $(\sqrt{2}, \sqrt{2})$ are on the graph of the equation.

32. $x^2 + 4y^2 = 4$

$$0^2 + 4 \cdot 1^2 = 4 \quad 2^2 + 4 \cdot 0^2 = 4 \quad 2^2 + 4 \left(\frac{1}{2}\right)^2 = 4$$

$$4 = 4 \quad 4 = 4 \quad 5 \neq 4$$

(0, 1) and (2, 0) are on the graph of the equation.

33. (-1, 0), (1, 0)

34. (0, 1)

35. $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right), (0, 1)$

36. (-2, 0), (2, 0), (0, -3)

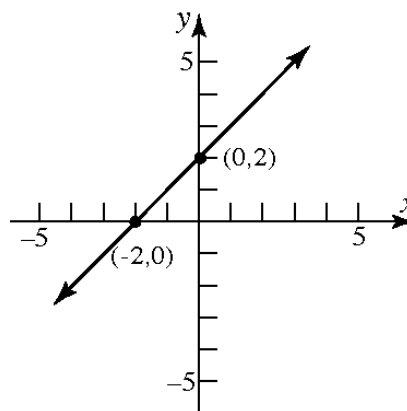
37. (1, 0), (0, 2), (0, -2)

38. (2, 0), (0, 2), (-2, 0), (0, -2)

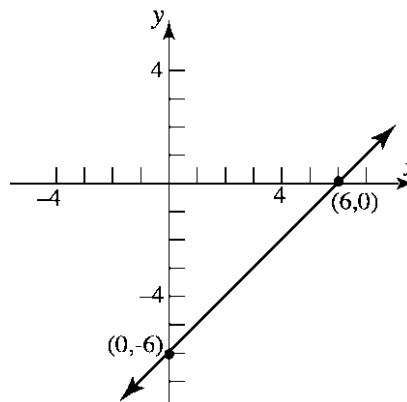
39. (-4, 0), (-1, 0), (4, 0), (0, -3)

40. (-2, 0), (2, 0), (0, 3)

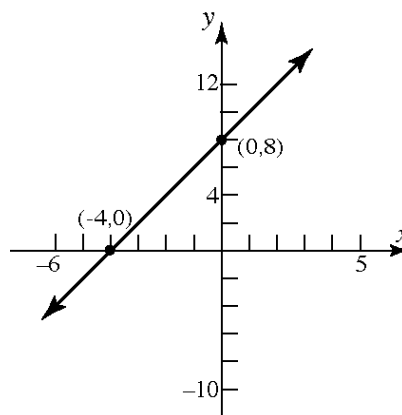
41. $y = x + 2$



42. $y = x - 6$

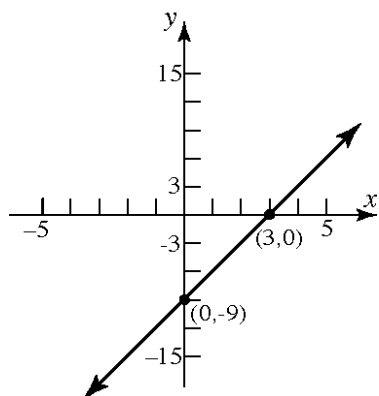


43. $y = 2x + 8$

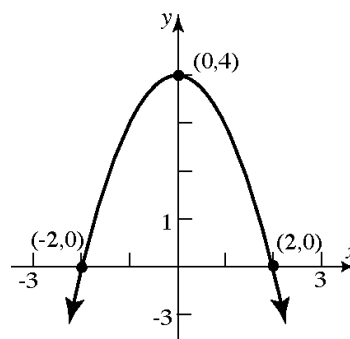


Section 1.1: Graphing Utilities; Introduction to Graphing Equations

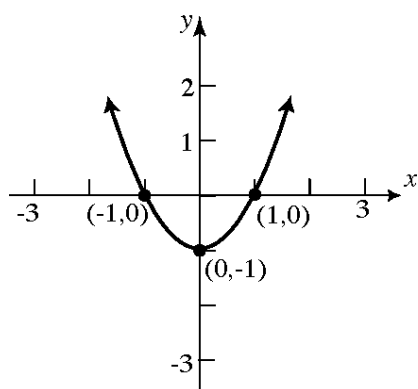
44. $y = 3x - 9$



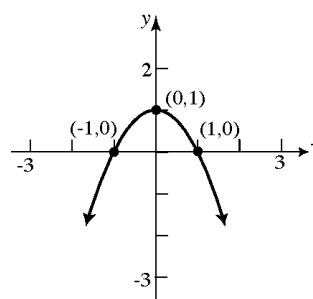
47. $y = -x^2 + 4$



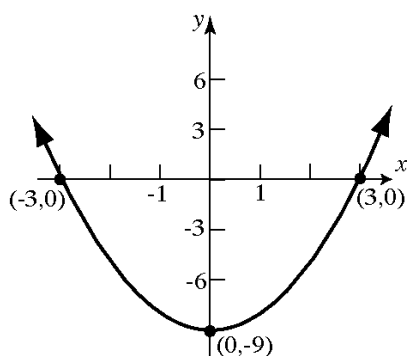
45. $y = x^2 - 1$



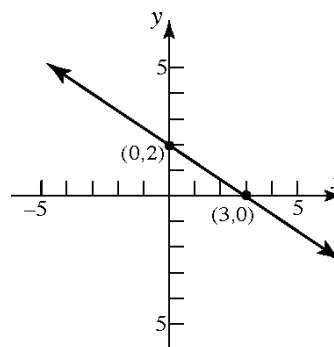
48. $y = -x^2 + 1$



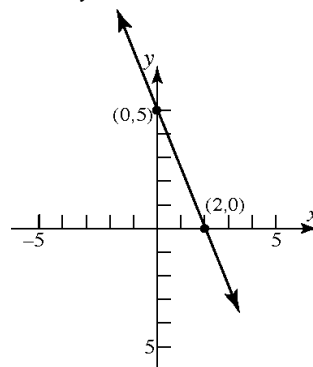
46. $y = x^2 - 9$



49. $2x + 3y = 6$

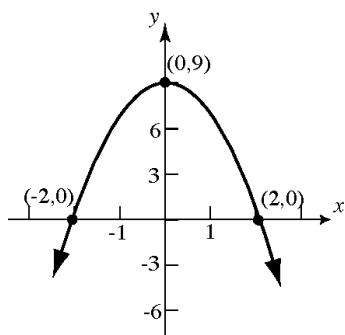


50. $5x + 2y = 10$

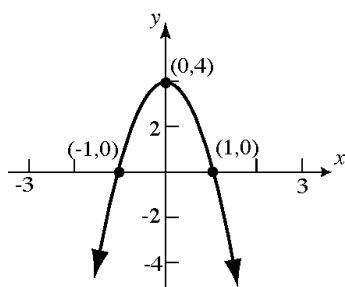


Chapter 1: Graphs, Equations, and Inequalities

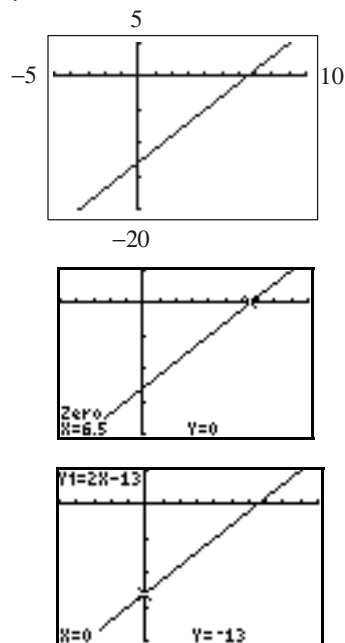
51. $9x^2 + 4y = 36$



52. $4x^2 + y = 4$

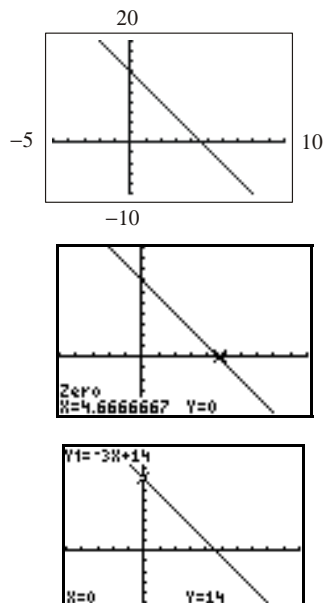


53. $y = 2x - 13$



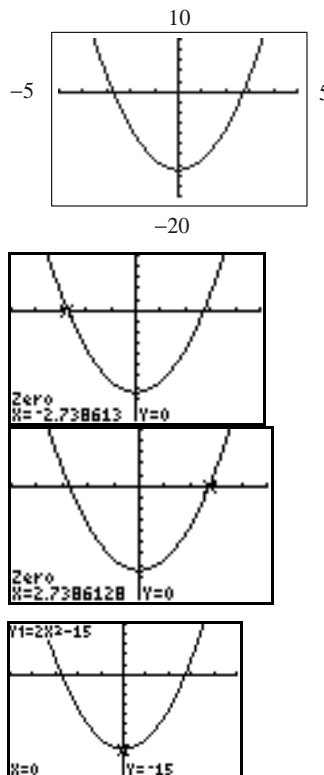
The x-intercept is $x = 6.5$ and the y-intercept is $y = -13$.

54. $y = -3x + 14$



The x-intercept is $x = 4.67$ and the y-intercept is $y = 14$.

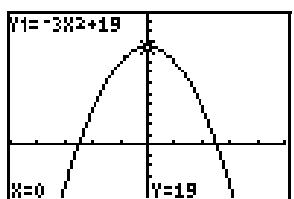
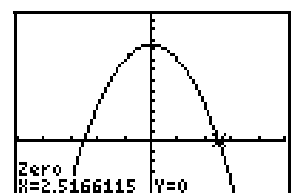
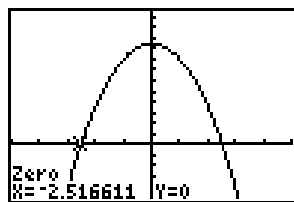
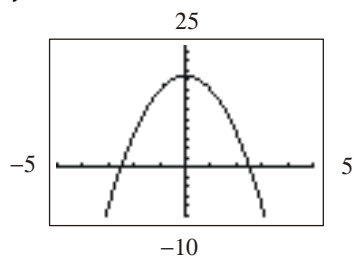
55. $y = 2x^2 - 15$



The x-intercepts are $x = -2.74$ and $x = 2.74$.
The y-intercept is $y = -15$.

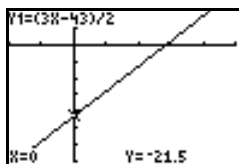
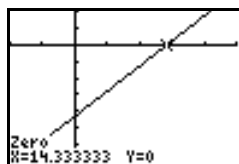
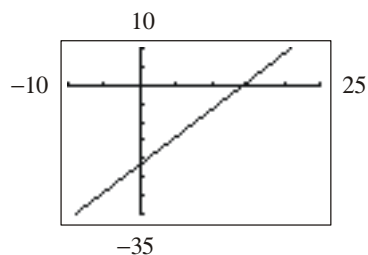
Section 1.1: Graphing Utilities; Introduction to Graphing Equations

56. $y = -3x^2 + 19$



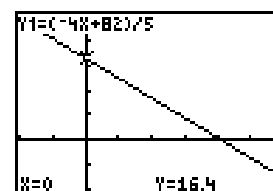
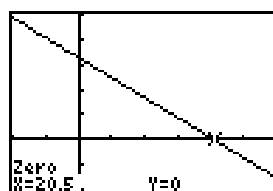
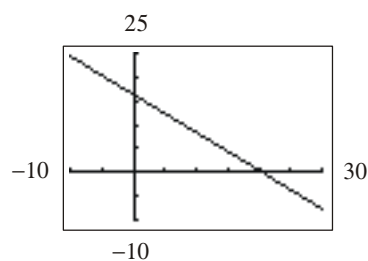
The x-intercepts are $x = -2.52$ and $x = 2.52$.
The y-intercept is $y = 19$.

57. $3x - 2y = 43$ or $y = \frac{3x - 43}{2}$



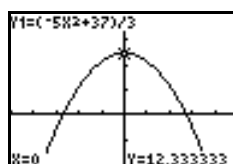
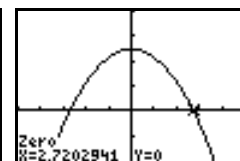
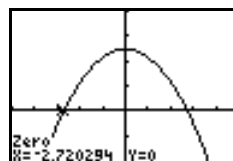
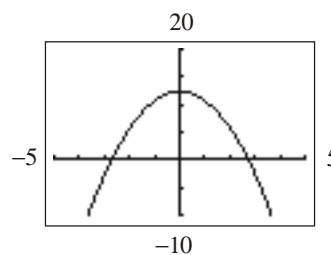
The x-intercept is $x = 14.33$ and the y-intercept is $y = -21.5$.

58. $4x + 5y = 82$ or $y = \frac{-4x + 82}{5}$



The x-intercept is $x = 20.5$ and the y-intercept is $y = 16.4$.

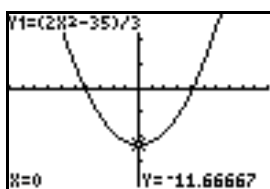
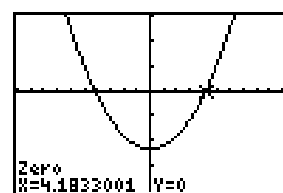
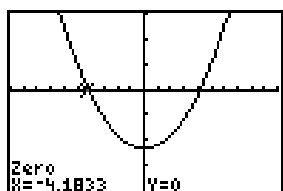
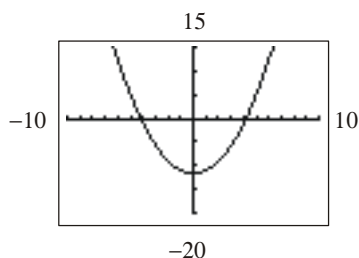
59. $5x^2 + 3y = 37$ or $y = \frac{-5x^2 + 37}{3}$



The x-intercepts are $x = -2.72$ and $x = 2.72$.
The y-intercept is $y = 12.33$.

Chapter 1: Graphs, Equations, and Inequalities

60. $2x^2 - 3y = 35$ or $y = \frac{2x^2 - 35}{3}$



The x-intercepts are $x = -4.18$ and $x = 4.18$.
The y-intercept is $y = -11.67$.

61. If $(2, 5)$ is shifted 3 units right then the x coordinate would be $2 + 3$. If it is shifted 2 units down then the y-coordinate would be $5 + (-2)$. Thus the new point would be $(2 + 3, 5 + (-2)) = (5, 3)$.

62. If $(-1, 6)$ is shifted 2 units left then the x coordinate would be $-1 + (-2)$. If it is shifted 4 units up then the y-coordinate would be $6 + 4$. Thus the new point would be $(-1 + (-2), 6 + 4) = (-3, 10)$.

63. a. 25 feet
b. 23.2 ft; 34.1 ft

- c. $(0, 6)$, The shot is released at a height of 6 feet; $(48.7, 0)$, the shot hits the ground after traveling a horizontal distance of 48.7 feet.

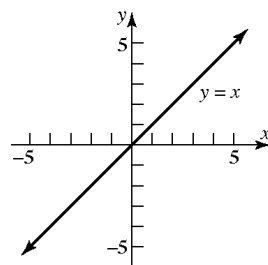
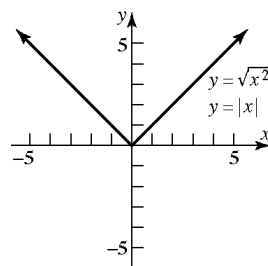
64. a. 20 meters
b. 12 seconds; 36 meters
c. $(0, 2)$, The discus is released at a height of 2 meters; $(18, 0)$, the discus hits the ground after 18 seconds.
65. a. \$19.95; \$19.95
b. \$182.45
c. $(0, 19.95)$, The membership plan costs \$19.95 per month.
66. a. 1.5 miles
b. 1 mile
c. $(0, 2)$, Caleb's friend lives 2 miles from his house; $(28, 0)$, it takes Caleb 28 minutes to ride home.

67. $(-1, 0), (1, 0), (0, -1), (0, 1)$

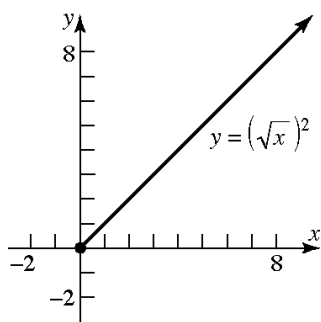
68. $(-2, 0), (3, 0), (0, -2), (0, 0), (0, 2)$

69. Answers will vary

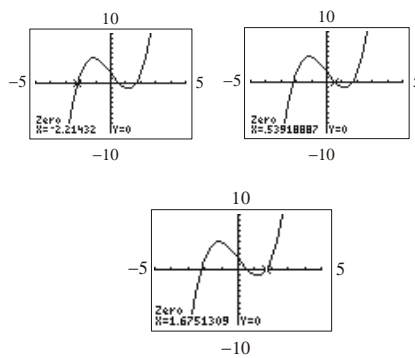
70. a.



Section 1.2: Solving Equations Using a Graphing Utility; Linear and Rational Equations



- b. Since $\sqrt{x^2} = |x|$ for all x , the graphs of $y = \sqrt{x^2}$ and $y = |x|$ are the same.
- c. For $y = (\sqrt{x})^2$, the domain of the variable x is $x \geq 0$; for $y = x$, the domain of the variable x is all real numbers. Thus, $(\sqrt{x})^2 = x$ only for $x \geq 0$.
- d. For $y = \sqrt{x^2}$, the range of the variable y is $y \geq 0$; for $y = x$, the range of the variable y is all real numbers. Also, $\sqrt{x^2} = x$ only if $x \geq 0$.
71. Answers will vary
72. Answers will vary. A complete graph presents enough of the graph to the viewer so they can "see" the rest of the graph as an obvious continuation of what is shown.
73. Answers will vary.
5. b
6. identity
7. linear; first-degree
8. d
9. True; equations that are contradictions have no solution.
10. False; when using a graphing utility, we often obtain decimal approximations to solutions.
11. Divide both sides by 7. The solution set is $\{3\}$.
12. Divide both sides by 6. The solution set is $\{-4\}$.
13. Subtract 35 from both sides, then divide both sides by 5. The solution set is $\{-7\}$.
14. Subtract 18 from both sides, then divide both sides by 6. The solution set is $\{-3\}$.
15. Add 3 to both sides, then divide both sides by 2. The solution set is $\left\{\frac{3}{2}\right\}$.
16. Subtract 4 from both sides, then divide both sides by 3. The solution set is $\left\{-\frac{4}{3}\right\}$.
17. Multiply both sides by 4. The solution set is $\left\{\frac{7}{5}\right\}$.
18. Multiply both sides by $\frac{3}{2}$. The solution set is $\left\{\frac{27}{4}\right\}$.
19. $x^3 - 4x + 2 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = x^3 - 4x + 2$.



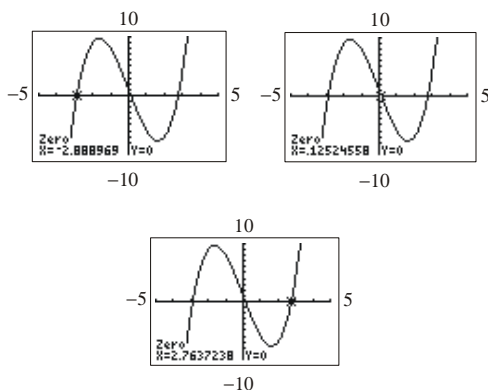
The solution set is $\{-2.21, 0.54, 1.68\}$.

Section 1.2

- Using the distributive property:
 $-3(x-5) = -3x + 15$
- $2x - 3 - 5(x+1) = 2(3) - 3 - 5[(3)+1]$
 $= 6 - 3 - 5(4) = 6 - 3 - 20$
 $= 3 - 10 = -17$
- No, $x = 4$ is not in the domain of $\frac{3}{x-4}$ because it makes the denominator equal 0.
- $\text{LCM} = (x+1)(x-1)(x+3) = x^3 + 3x^2 - x - 3$

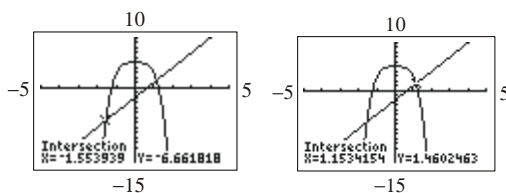
Chapter 1: Graphs, Equations, and Inequalities

20. $x^3 - 8x + 1 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = x^3 - 8x + 1$.



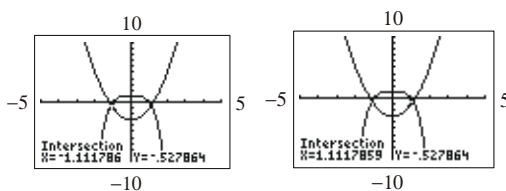
The solution set is $\{-2.89, 0.13, 2.76\}$.

21. $-2x^4 + 5 = 3x - 2$; Use INTERSECT on the graphs of $y_1 = -2x^4 + 5$ and $y_2 = 3x - 2$.



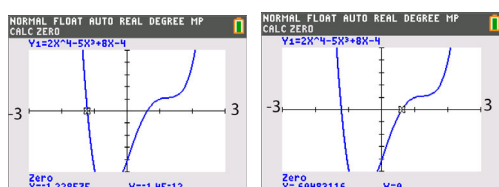
The solution set is $\{-1.55, 1.15\}$.

22. $-x^4 + 1 = 2x^2 - 3$; Use INTERSECT on the graphs of $y_1 = -x^4 + 1$ and $y_2 = 2x^2 - 3$.



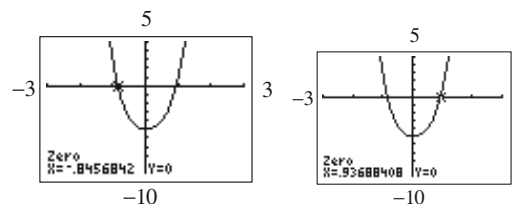
The solution set is $\{-1.11, 1.11\}$.

23. $2x^4 - 5x^3 + 8x - 4 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = 2x^4 - 5x^3 + 8x - 4$.



The solution set is $\{-1.23, 0.60\}$.

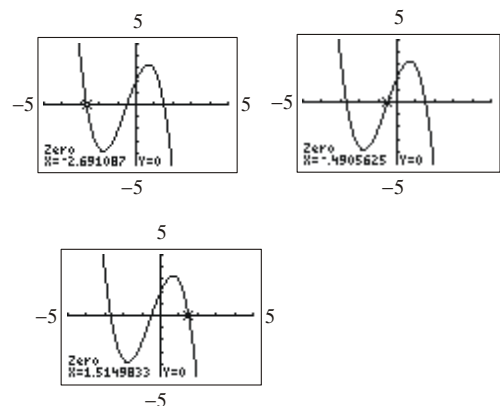
24. $3x^4 - x^3 + 4x^2 - 5 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = 3x^4 - x^3 + 4x^2 - 5$.



The solution set is $\{-0.85, 0.94\}$.

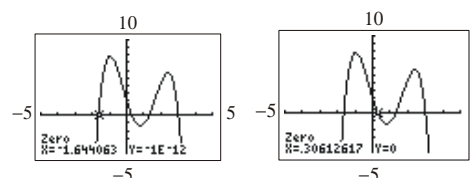
25. $-x^3 - \frac{5}{3}x^2 + \frac{7}{2}x + 2 = 0$;

Use ZERO (or ROOT) on the graph of $y_1 = -x^3 - (5/3)x^2 + (7/2)x + 2$.

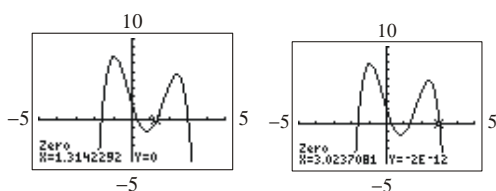


The solution set is $\{-2.69, -0.49, 1.51\}$.

26. $-x^4 + 3x^3 + \frac{7}{3}x^2 - \frac{15}{2}x + 2 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = -x^4 + 3x^3 + (7/3)x^2 - (15/2)x + 2$.



Section 1.2: Solving Equations Using a Graphing Utility; Linear and Rational Equations



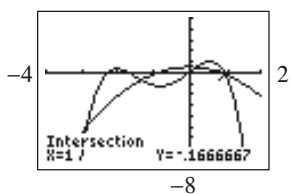
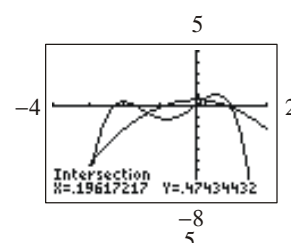
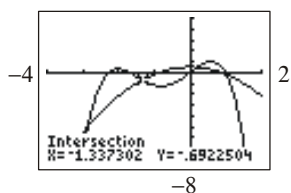
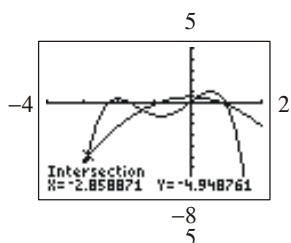
The solution set is $\{-1.64, 0.31, 1.31, 3.02\}$.

27. $-\frac{2}{3}x^4 - 2x^3 + \frac{5}{2}x = -\frac{2}{3}x^2 + \frac{1}{2}$

Use INTERSECT on the graphs of

$$y_1 = -(2/3)x^4 - 2x^3 + (5/2)x \text{ and}$$

$$y_2 = -(2/3)x^2 + 1/2.$$



The solution set is $\{-2.86, -1.34, 0.20, 1.00\}$.

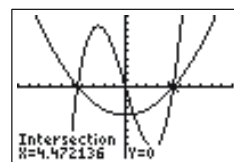
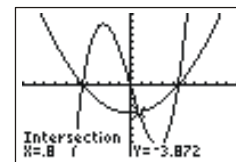
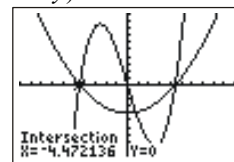
28. $\frac{1}{4}x^3 - 5x = \frac{1}{5}x^2 - 4$

Use INTERSECT on the graphs of

$$y_1 = x^3/4 - 5x \text{ and } y_2 = x^2/5 - 4 \text{ and a}$$

standard viewing window (-10 to 10 for both x

and y).

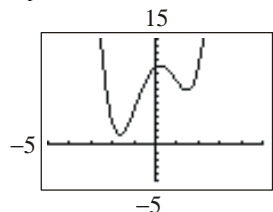


The solution set is $\{-4.47, 0.80, 4.47\}$.

29. $x^4 - 5x^2 + 2x + 11 = 0$

Use ZERO (or ROOT) on the graph of

$$y_1 = x^4 - 5x^2 + 2x + 11.$$

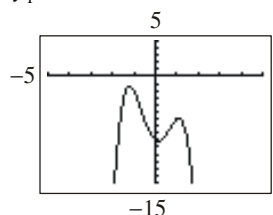


There are no real solutions.

30. $-3x^4 + 8x^2 - 2x - 9 = 0$

Use ZERO (or ROOT) on the graph of

$$y_1 = -3x^4 + 8x^2 - 2x - 9.$$



There are no real solutions.

31. $3x + 4 = x$

$$3x + 4 - 4 = x - 4$$

$$3x = x - 4$$

$$3x - x = x - 4 - x$$

$$2x = -4$$

$$\frac{2x}{2} = \frac{-4}{2}$$

$$x = -2$$

The solution set is $\{-2\}$.

Chapter 1: Graphs, Equations, and Inequalities

$$\begin{aligned}32. \quad 2x + 9 &= 5x \\2x + 9 - 9 &= 5x - 9 \\2x &= 5x - 9 \\2x - 5x &= 5x - 9 - 5x \\-3x &= -9 \\\frac{-3x}{-3} &= \frac{-9}{-3} \\x &= 3\end{aligned}$$

The solution set is $\{3\}$.

$$\begin{aligned}33. \quad 2t - 6 &= 3 - t \\2t - 6 + 6 &= 3 - t + 6 \\2t &= 9 - t \\2t + t &= 9 - t + t \\3t &= 9 \\\frac{3t}{3} &= \frac{9}{3} \\t &= 3\end{aligned}$$

The solution set is $\{3\}$.

$$\begin{aligned}34. \quad 5y + 6 &= -18 - y \\5y + 6 - 6 &= -18 - y - 6 \\5y &= -y - 24 \\5y + y &= -y - 24 + y \\6y &= -24 \\\frac{6y}{6} &= \frac{-24}{6} \\y &= -4\end{aligned}$$

The solution set is $\{-4\}$.

$$\begin{aligned}35. \quad 6 - x &= 2x + 9 \\6 - x - 6 &= 2x + 9 - 6 \\-x &= 2x + 3 \\-x - 2x &= 2x + 3 - 2x \\-3x &= 3 \\\frac{-3x}{-3} &= \frac{3}{-3} \\x &= -1\end{aligned}$$

The solution set is $\{-1\}$.

$$\begin{aligned}36. \quad 3 - 2x &= 2 - x \\3 - 2x - 3 &= 2 - x - 3 \\-2x &= -x - 1 \\-2x + x &= -x - 1 + x \\-x &= -1 \\-1(-x) &= -1(-1) \\x &= 1\end{aligned}$$

The solution set is $\{1\}$.

$$\begin{aligned}37. \quad 3(5 + 3x) &= 8(x - 1) \\15 + 9x &= 8x - 8 \\15 + 9x - 15 &= 8x - 8 - 15 \\9x &= 8x - 23 \\9x - 8x &= 8x - 23 - 8x \\x &= -23\end{aligned}$$

The solution set is $\{-23\}$.

$$\begin{aligned}38. \quad 3(2 - x) &= 2x - 1 \\6 - 3x &= 2x - 1 \\6 - 3x - 6 &= 2x - 1 - 6 \\-3x &= 2x - 7 \\-3x - 2x &= 2x - 7 - 2x \\-5x &= -7 \\\frac{-5x}{-5} &= \frac{-7}{-5} \\x &= \frac{7}{5}\end{aligned}$$

The solution set is $\left\{\frac{7}{5}\right\}$.

$$\begin{aligned}39. \quad 8x - (3x + 2) &= 3x - 10 \\8x - 3x - 2 &= 3x - 10 \\5x - 2 &= 3x - 10 \\5x - 2 + 2 &= 3x - 10 + 2 \\5x &= 3x - 8 \\5x - 3x &= 3x - 8 - 3x \\2x &= -8 \\\frac{2x}{2} &= \frac{-8}{2} \\x &= -4\end{aligned}$$

The solution set is $\{-4\}$.

Section 1.2: Solving Equations Using a Graphing Utility; Linear and Rational Equations

40. $7 - (2x - 1) = 10$

$$7 - 2x + 1 = 10$$

$$-2x + 8 = 10$$

$$-2x + 8 - 8 = 10 - 8$$

$$-2x = 2$$

$$\frac{-2x}{-2} = \frac{2}{-2}$$

$$x = -1$$

The solution set is $\{-1\}$.

41. $2(3x - 5) + 6(x - 3) = -3(4 - 5x) + 5x - 6$

$$6x - 10 + 6x - 18 = -12 + 15x + 5x - 6$$

$$12x - 28 = 20x - 18$$

$$-8x - 28 = -18$$

$$-8x = 10$$

$$x = -\frac{5}{4}$$

The solution set is $\left\{-\frac{5}{4}\right\}$.

42. $5(x - 2) - 2(3x + 1) = 4(1 - 2x) + x$

$$5x - 10 - 6x - 2 = 4 - 8x + x$$

$$-x - 12 = 4 - 7x$$

$$6x - 12 = 4$$

$$6x = 16$$

$$x = \frac{8}{3}$$

The solution set is $\left\{\frac{8}{3}\right\}$.

43. $\frac{3}{2}x + 2 = \frac{1}{2} - \frac{1}{2}x$

$$2\left(\frac{3}{2}x + 2\right) = 2\left(\frac{1}{2} - \frac{1}{2}x\right)$$

$$3x + 4 = 1 - x$$

$$3x + 4 - 4 = 1 - x - 4$$

$$3x = -x - 3$$

$$3x + x = -x - 3 + x$$

$$4x = -3$$

$$\frac{4x}{4} = \frac{-3}{4}$$

$$x = -\frac{3}{4}$$

The solution set is $\left\{-\frac{3}{4}\right\}$.

44. $\frac{1}{3}x = 2 - \frac{2}{3}x$

$$3\left(\frac{1}{3}x\right) = 3\left(2 - \frac{2}{3}x\right)$$

$$x = 6 - 2x$$

$$x + 2x = 6 - 2x + 2x$$

$$3x = 6$$

$$\frac{3x}{3} = \frac{6}{3}$$

$$x = 2$$

The solution set is $\{2\}$.

45. $\frac{2}{3}p = \frac{1}{2}p - \frac{1}{3}$

$$6\left(\frac{2}{3}p\right) = 6\left(\frac{1}{2}p - \frac{1}{3}\right)$$

$$4p = 3p - 2$$

$$4p - 3p = 3p - 2 - 3p$$

$$p = -2$$

The solution set is $\{-2\}$.

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$$\begin{aligned}
 46. \quad & \frac{1}{2} - \frac{1}{3}p = \frac{4}{3} \\
 & 6\left(\frac{1}{2} - \frac{1}{3}p\right) = 6 \cdot \frac{4}{3} \\
 & 3 - 2p = 8 \\
 & 3 - 2p - 3 = 8 - 3 \\
 & -2p = 5 \\
 & \frac{-2p}{-2} = \frac{5}{-2} \\
 & p = -\frac{5}{2}
 \end{aligned}$$

The solution set is $\left\{-\frac{5}{2}\right\}$.

$$\begin{aligned}
 47. \quad & 0.2m = 0.9 + 0.5m \\
 & 0.2m - 0.5m = 0.9 + 0.5m - 0.5m \\
 & -0.3m = 0.9 \\
 & \frac{-0.3m}{-0.3} = \frac{0.9}{-0.3} \\
 & m = -3
 \end{aligned}$$

The solution set is $\{-3\}$.

$$\begin{aligned}
 48. \quad & 0.9t = 1 + t \\
 & 0.9t - t = 1 + t - t \\
 & -0.1t = 1 \\
 & \frac{-0.1t}{-0.1} = \frac{1}{-0.1} \\
 & t = -10
 \end{aligned}$$

The solution set is $\{-10\}$.

$$\begin{aligned}
 49. \quad & \frac{x+1}{3} + \frac{x+2}{7} = 2 \\
 & 21\left(\frac{x+1}{3}\right) + 21\left(\frac{x+2}{7}\right) = 21(2) \\
 & 7x + 7 + 3x + 6 = 42 \\
 & 10x + 13 = 42 \\
 & 10x + 13 - 13 = 42 - 13 \\
 & 10x = 29 \\
 & \frac{10x}{10} = \frac{29}{10} \\
 & x = \frac{29}{10}
 \end{aligned}$$

The solution set is $\left\{\frac{29}{10}\right\}$.

$$\begin{aligned}
 50. \quad & \frac{2x+1}{3} + 16 = 3x \\
 & 3\left(\frac{2x+1}{3} + 16\right) = 3 \cdot 3x \\
 & 2x + 1 + 48 = 9x \\
 & 2x + 49 = 9x \\
 & 2x + 49 - 2x = 9x - 2x \\
 & 49 = 7x \\
 & \frac{49}{7} = \frac{7x}{7} \\
 & x = 7
 \end{aligned}$$

The solution set is $\{7\}$.

$$\begin{aligned}
 51. \quad & \frac{2}{y} + \frac{4}{y} = 3 \\
 & \frac{6}{y} = 3 \\
 & \left(\frac{6}{y}\right)^{-1} = 3^{-1} \\
 & \frac{y}{6} = \frac{1}{3} \\
 & 6 \cdot \frac{y}{6} = 6 \cdot \frac{1}{3} \\
 & y = 2
 \end{aligned}$$

The solution set is $\{2\}$.

$$\begin{aligned}
 52. \quad & \frac{4}{y} - 5 = \frac{5}{2y} \\
 & 2y\left(\frac{4}{y} - 5\right) = 2y\left(\frac{5}{2y}\right) \\
 & 8 - 10y = 5 \\
 & 8 - 10y - 8 = 5 - 8 \\
 & -10y = -3 \\
 & \frac{-10y}{-10} = \frac{-3}{-10} \\
 & y = \frac{3}{10}
 \end{aligned}$$

The solution set is $\left\{\frac{3}{10}\right\}$.

Section 1.2: Solving Equations Using a Graphing Utility; Linear and Rational Equations

$$53. \quad \frac{1}{2} + \frac{2}{x} = \frac{3}{4}$$

$$4x \left(\frac{1}{2} + \frac{2}{x} \right) = 4x \left(\frac{3}{4} \right)$$

$$2x + 8 = 3x$$

$$2x + 8 - 2x = 3x - 2x$$

$$8 = x \quad \text{or} \quad x = 8$$

The solution set is $\{8\}$.

$$54. \quad \frac{3}{x} - \frac{1}{3} = \frac{1}{6}$$

$$6x \left(\frac{3}{x} - \frac{1}{3} \right) = 6x \left(\frac{1}{6} \right)$$

$$18 - 2x = x$$

$$18 - 2x + 2x = x + 2x$$

$$18 = 3x$$

$$\frac{18}{3} = \frac{3x}{3}$$

$$6 = x \quad \text{or} \quad x = 6$$

The solution set is $\{6\}$.

$$55. \quad (x+7)(x-1) = (x+1)^2$$

$$x^2 - x + 7x - 7 = x^2 + 2x + 1$$

$$x^2 + 6x - 7 = x^2 + 2x + 1$$

$$x^2 + 6x - 7 - x^2 = x^2 + 2x + 1 - x^2$$

$$6x - 7 = 2x + 1$$

$$6x - 7 - 2x = 2x + 1 - 2x$$

$$4x - 7 = 1$$

$$4x - 7 + 7 = 1 + 7$$

$$4x = 8$$

$$\frac{4x}{4} = \frac{8}{4} \Rightarrow x = 2$$

The solution set is $\{2\}$.

$$56. \quad (x+2)(x-3) = (x+3)^2$$

$$x^2 + 2x - 3x - 6 = x^2 + 6x + 9$$

$$x^2 - x - 6 = x^2 + 6x + 9$$

$$x^2 - x - 6 - x^2 = x^2 + 6x + 9 - x^2$$

$$-x - 6 = 6x + 9$$

$$-x - 6 - 6x = 6x + 9 - 6x$$

$$-7x - 6 = 9$$

$$-7x - 6 + 6 = 9 + 6$$

$$-7x = 15$$

$$\frac{-7x}{-7} = \frac{15}{-7}$$

$$x = -\frac{15}{7}$$

The solution set is $\left\{-\frac{15}{7}\right\}$.

$$57. \quad x(2x-3) = (2x+1)(x-4)$$

$$2x^2 - 3x = 2x^2 - 7x - 4$$

$$2x^2 - 3x - 2x^2 = 2x^2 - 7x - 4 - 2x^2$$

$$-3x = -7x - 4$$

$$-3x + 7x = -7x - 4 + 7x$$

$$4x = -4$$

$$\frac{4x}{4} = \frac{-4}{4} \Rightarrow x = -1$$

The solution set is $\{-1\}$.

$$58. \quad x(1+2x) = (2x-1)(x-2)$$

$$x + 2x^2 = 2x^2 - 5x + 2$$

$$2x^2 + x - 2x^2 = 2x^2 - 5x + 2 - 2x^2$$

$$x = -5x + 2$$

$$x + 5x = -5x + 2 + 5x$$

$$6x = 2$$

$$\frac{6x}{6} = \frac{2}{6} \Rightarrow x = \frac{1}{3}$$

The solution set is $\left\{\frac{1}{3}\right\}$.

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59. $p(p^2 + 3) = 12 + p^3$

$$p^3 + 3p = 12 + p^3$$

$$p^3 + 3p - p^3 = 12 + p^3 - p^3$$

$$3p = 12$$

$$\frac{3p}{3} = \frac{12}{3}$$

$$p = 4$$

The solution set is $\{4\}$.

60. $w(4 - w^2) = 8 - w^3$

$$4w - w^3 = 8 - w^3$$

$$4w = 8$$

$$w = 2$$

The solution set is $\{2\}$.

61. $\frac{x}{x-2} + 3 = \frac{2}{x-2}$

$$(x-2)\left(\frac{x}{x-2} + 3\right) = (x-2)\left(\frac{2}{x-2}\right)$$

$$x + 3(x-2) = 2$$

$$x + 3x - 6 = 2$$

$$4x - 6 = 2$$

$$4x = 8$$

$$x = 2$$

This solution is not in the domain of the variable so it must be discarded. The equation has no solution. The solution set is $\{\}$ or $\emptyset$.

62. $\frac{2x}{x+3} = \frac{-6}{x+3} - 2$

$$(x+3)\left(\frac{2x}{x+3}\right) = (x+3)\left(\frac{-6}{x+3} - 2\right)$$

$$2x = -6 - 2(x+3)$$

$$2x = -6 - 2x - 6$$

$$2x = -2x - 12$$

$$4x = -12$$

$$x = -3$$

This solution is not in the domain of the variable so it must be discarded. The equation has no solution. The solution set is $\{\}$ or $\emptyset$.

63. $\frac{2x}{x^2-4} = \frac{4}{x^2-4} - \frac{3}{x+2}$

$$(x^2-4)\left(\frac{2x}{x^2-4}\right) = (x^2-4)\left(\frac{4}{x^2-4} - \frac{3}{x+2}\right)$$

$$2x = 4 - 3(x-2)$$

$$2x = 4 - 3x + 6$$

$$2x = 10 - 3x$$

$$5x = 10$$

$$x = 2$$

This solution is not in the domain of the variable so it must be discarded. The equation has no solution. The solution set is $\{\}$ or $\emptyset$.

64. $\frac{x}{x^2-9} + \frac{4}{x+3} = \frac{3}{x^2-9}$

$$(x^2-9)\left(\frac{x}{x^2-9} + \frac{4}{x+3}\right) = (x^2-9)\left(\frac{3}{x^2-9}\right)$$

$$x + 4(x-3) = 3$$

$$x + 4x - 12 = 3$$

$$5x - 12 = 3$$

$$5x = 15$$

$$x = 3$$

This solution is not in the domain of the variable so it must be discarded. The equation has no solution. The solution set is $\{\}$ or $\emptyset$.

65. $\frac{x}{x+2} = \frac{3}{2}$

$$2x = 3(x+2)$$

$$2x = 3x + 6$$

$$-x = 6$$

$$x = -6$$

The solution set is $\{-6\}$.

66. $\frac{3x}{x-1} = 2$

$$3x = 2(x-1)$$

$$3x = 2x - 2$$

$$x = -2$$

The solution set is $\{-2\}$.

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$$67. \quad \frac{7}{3x+10} = \frac{2}{x-3}$$

$$7(x-3) = 2(3x+10)$$

$$7x - 21 = 6x + 20$$

$$7x = 6x + 41$$

$$x = 41$$

The solution set is $\{41\}$.

$$68. \quad \frac{-4}{x+4} = \frac{-3}{x+6}$$

$$-4(x+6) = -3(x+4)$$

$$-4x - 24 = -3x - 12$$

$$-4x = -3x + 12$$

$$-x = 12$$

$$x = -12$$

The solution set is $\{-12\}$.

$$69. \quad \frac{6t+7}{4t-1} = \frac{3t+8}{2t-4}$$

$$(6t+7)(2t-4) = (3t+8)(4t-1)$$

$$12t^2 + 14t - 24t - 28 = 12t^2 + 32t - 3t - 8$$

$$12t^2 - 10t - 28 = 12t^2 + 29t - 8$$

$$-10t - 28 = 29t - 8$$

$$-10t = 29t + 20$$

$$-39t = 20$$

$$t = \frac{20}{-39}$$

$$t = -\frac{20}{39}$$

The solution set is $\left\{-\frac{20}{39}\right\}$.

$$70. \quad \frac{8w+5}{10w-7} = \frac{4w-3}{5w+7}$$

$$(8w+5)(5w+7) = (4w-3)(10w-7)$$

$$40w^2 + 25w + 56w + 35 = 40w^2 - 30w - 28w + 21$$

$$40w^2 + 81w + 35 = 40w^2 - 58w + 21$$

$$81w + 35 = -58w + 21$$

$$81w = -58w - 14$$

$$139w = -14$$

$$w = -\frac{14}{139}$$

The solution set is $\left\{-\frac{14}{139}\right\}$.

$$71. \quad \frac{4}{x-2} = \frac{-3}{x+5} + \frac{7}{(x+5)(x-2)}$$

$$\text{LCD} = (x-2)(x+5)$$

$$\frac{4(x+5)}{(x-2)(x+5)} = \frac{-3(x-2)}{(x-2)(x+5)} + \frac{7}{(x+5)(x-2)}$$

$$4(x+5) = -3(x-2) + 7$$

$$4x + 20 = -3x + 6 + 7$$

$$4x + 20 = -3x + 13$$

$$4x = -3x - 7$$

$$7x = -7$$

$$x = -1$$

The solution set is $\{-1\}$.

$$72. \quad \frac{-4}{2x+3} + \frac{1}{x-1} = \frac{1}{(2x+3)(x-1)}$$

$$\text{LCD} = (2x+3)(x-1)$$

$$\frac{-4(x-1)}{(2x+3)(x-1)} + \frac{1(2x+3)}{(2x+3)(x-1)} = \frac{1}{(2x+3)(x-1)}$$

$$-4(x-1) + (2x+3) = 1$$

$$-4x + 4 + 2x + 3 = 1$$

$$-2x + 7 = 1$$

$$-2x = -6$$

$$x = 3$$

The solution set is $\{3\}$.

$$73. \quad \frac{2}{y+3} + \frac{3}{y-4} = \frac{5}{y+6}$$

$$\text{LCD} = (y+3)(y-4)(y+6)$$

$$(y+3)(y-4)(y+6) \left(\frac{2}{y+3} + \frac{3}{y-4} = \frac{5}{y+6} \right)$$

$$2(y-4)(y+6) + 3(y+3)(y+6) = 5(y+3)(y-4)$$

$$2(y^2 + 2y - 24) + 3(y^2 + 9y + 18) = 5(y^2 - y - 12)$$

$$2y^2 + 4y - 48 + 3y^2 + 27y + 54 = 5y^2 - 5y - 60$$

$$5y^2 + 31y + 6 = 5y^2 - 5y - 60$$

$$31y + 6 = -5y - 60$$

$$31y = -5y - 66$$

$$36y = -66$$

$$y = -\frac{66}{36}$$

$$y = -\frac{11}{6}$$

The solution set is $\left\{-\frac{11}{6}\right\}$.

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$$\begin{aligned}
 74. \quad & \frac{5}{5z-11} + \frac{4}{2z-3} = \frac{-3}{5-z} \\
 & \text{LCD} = (5z-11)(2z-3)(5-z) \\
 & (5z-11)(2z-3)(5-z) \left(\frac{5}{5z-11} + \frac{4}{2z-3} = \frac{-3}{5-z} \right) \\
 & 5(2z-3)(5-z) + 4(5z-11)(5-z) = -3(5z-11)(2z-3) \\
 & -10z^2 + 65z - 75 - 20z^2 + 144z - 220 = -30z^2 + 111z - 99 \\
 & -30z^2 + 209z - 295 = -30z^2 + 111z - 99 \\
 & 209z - 295 = 111z - 99 \\
 & 209z = 111z + 196 \\
 & 98z = 196 \\
 & z = 2
 \end{aligned}$$

The solution set is $\{2\}$.

$$\begin{aligned}
 75. \quad & \frac{x}{x^2-9} - \frac{x-4}{x^2+3x} = \frac{10}{x^2-3x} \\
 & \text{LCD} = x(x+3)(x-3) \\
 & x(x+3)(x-3) \left(\frac{x}{x^2-9} - \frac{x-4}{x^2+3x} = \frac{10}{x^2-3x} \right) \\
 & x^2 - (x-4)(x-3) = 10(x+3) \\
 & x^2 - (x^2 - 7x + 12) = 10x + 30 \\
 & x^2 - x^2 + 7x - 12 = 10x + 30 \\
 & 7x - 12 = 10x + 30 \\
 & -3x = 42 \\
 & -x = 14 \\
 & x = -14
 \end{aligned}$$

The solution set is $\{-14\}$.

$$\begin{aligned}
 76. \quad & \frac{x+1}{x^2+2x} - \frac{x+4}{x^2+x} = \frac{-3}{x^2+3x+2} \\
 & \text{LCD} = x(x+2)(x+1) \\
 & x(x+2)(x+1) \left(\frac{x+1}{x^2+2x} - \frac{x+4}{x^2+x} = \frac{-3}{x^2+3x+2} \right) \\
 & (x+1)(x+1) - (x+4)(x+2) = -3(x) \\
 & x^2 + 2x + 1 - (x^2 + 6x + 8) = -3x \\
 & x^2 + 2x + 1 - x^2 - 6x - 8 = -3x \\
 & -4x - 7 = -3x \\
 & -4x = -3x + 7 \\
 & -x = 7 \\
 & x = -7
 \end{aligned}$$

The solution set is $\{-7\}$.

$$\begin{aligned}
 77. \quad & y = 5x + 4 \\
 & 2 = 5a + 4 \\
 & -2 = 5a \\
 & a = -\frac{2}{5}
 \end{aligned}$$

$$\begin{aligned}
 78. \quad & y = x^2 + 3x \\
 & b = 2^2 + 3(2) = 4 + 6 = 10
 \end{aligned}$$

$$\begin{aligned}
 79. \quad & 2x + 3y = 6 \\
 & 2a + 3b = 6 \\
 & 3b = -2a + 6 \\
 & b = -\frac{2}{3}a + 2
 \end{aligned}$$

$$\begin{aligned}
 80. \quad & y = mx + b \\
 & 0 = m(2) + b \\
 & 5 = m(0) + b \rightarrow 5 = b \\
 & \text{Therefore,} \\
 & 0 = 2m + 5 \\
 & -2m = 5 \\
 & m = -\frac{5}{2}
 \end{aligned}$$

$$\begin{aligned}
 81. \quad & ax - b = c, \quad a \neq 0 \\
 & ax - b + b = c + b \\
 & ax = c + b \\
 & \frac{ax}{a} = \frac{c+b}{a} \\
 & x = \frac{b+c}{a}
 \end{aligned}$$

$$\begin{aligned}
 82. \quad & 1 - ax = b, \quad a \neq 0 \\
 & 1 - ax - 1 = b - 1 \\
 & -ax = b - 1 \\
 & \frac{-ax}{-a} = \frac{b-1}{-a} \\
 & x = \frac{b-1}{-a} = \frac{1-b}{a}
 \end{aligned}$$

Section 1.2: Solving Equations Using a Graphing Utility; Linear and Rational Equations

$$83. \frac{x}{a} + \frac{x}{b} = c, \quad a \neq 0, b \neq 0, a \neq -b$$

$$ab \left(\frac{x}{a} + \frac{x}{b} \right) = ab \cdot c$$

$$bx + ax = abc$$

$$x(a+b) = abc$$

$$\frac{x(a+b)}{a+b} = \frac{abc}{a+b}$$

$$x = \frac{abc}{a+b}$$

$$84. \frac{a}{x} + \frac{b}{x} = c, \quad c \neq 0$$

$$x \left(\frac{a}{x} + \frac{b}{x} \right) = x \cdot c$$

$$a + b = cx$$

$$\frac{a+b}{c} = \frac{cx}{c}$$

$$x = \frac{a+b}{c}$$

such that $a \neq -b$

$$85. \frac{1}{x-a} + \frac{1}{x+a} = \frac{2}{x-1}$$

Multiply both sides by the LCD,

$(x-a)(x+a)(x-1)$, to get:

$$(x+a)(x-1) + (x-a)(x-1) = 2(x-a)(x+a)$$

$$x^2 + ax - x - a + x^2 - ax - x + a = 2x^2 - 2a^2$$

$$2x^2 - 2x = 2x^2 - 2a^2$$

$$2x^2 - 2x - 2x^2 = 2x^2 - 2a^2 - 2x^2$$

$$-2x = -2a^2$$

$$\frac{-2x}{-2} = \frac{-2a^2}{-2}$$

$$x = a^2$$

where $a \neq -1, 0, 1$.

$$86. \frac{b+c}{x+a} = \frac{b-c}{x-a}$$

$$(b+c)(x-a) = (b-c)(x+a)$$

$$bx + cx - ab - ac = bx - cx + ab - ac$$

$$bx + cx - ab - ac - bx = bx - cx + ab - ac - bx$$

$$cx - ab - ac = -cx + ab - ac$$

$$cx - ab - ac + cx = -cx + ab - ac + cx$$

$$2cx - ab - ac = ab - ac$$

$$2cx - ab - ac + ab + ac = ab - ac + ab + ac$$

$$2cx = 2ab$$

$$\frac{2cx}{2c} = \frac{2ab}{2c}$$

$$x = \frac{ab}{c}$$

where $a \neq 0, c \neq 0$, and $b \neq \pm c$.

$$87. x + 2a = 16 + ax - 6a$$

when $x = 4$:

$$4 + 2a = 16 + a(4) - 6a$$

$$4 + 2a = 16 + 4a - 6a$$

$$4 + 2a = 16 - 2a$$

$$4a = 12$$

$$a = 3$$

$$88. x + 2b = x - 4 + 2bx$$

when $x = 2$:

$$2 + 2b = 2 - 4 + 2b(2)$$

$$2 + 2b = 2 - 4 + 4b$$

$$2 + 2b = -2 + 4b$$

$$4 = 2b$$

$$\frac{4}{2} = b \Rightarrow b = 2$$

89. Solving for R:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$RR_1R_2 \left(\frac{1}{R} \right) = RR_1R_2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$R_1R_2 = RR_2 + RR_1$$

$$R_1R_2 = R(R_2 + R_1)$$

$$\frac{R_1R_2}{R_2 + R_1} = \frac{R(R_2 + R_1)}{R_2 + R_1}$$

$$\frac{R_1R_2}{R_1 + R_2} = R$$

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90. Solving for r :

$$A = P(1 + rt)$$

$$A = P + Prt$$

$$A - P = Prt$$

$$\frac{A - P}{Pt} = \frac{Prt}{Pt}$$

$$r = \frac{A - P}{Pt}$$

91. Solving for R :

$$F = \frac{mv^2}{R}$$

$$RF = R \left(\frac{mv^2}{R} \right)$$

$$RF = mv^2$$

$$\frac{RF}{F} = \frac{mv^2}{F} \Rightarrow R = \frac{mv^2}{F}$$

92. Solving for T :

$$PV = nRT$$

$$\frac{PV}{nR} = \frac{nRT}{nR}$$

$$T = \frac{PV}{nR}$$

93. Solving for r :

$$S = \frac{a}{1 - r}$$

$$(1 - r) \cdot S = (1 - r) \cdot \frac{a}{1 - r}$$

$$(1 - r)S = a$$

$$\frac{(1 - r)S}{S} = \frac{a}{S}$$

$$1 - r = \frac{a}{S}$$

$$1 - r - 1 = \frac{a}{S} - 1$$

$$-r = \frac{a}{S} - 1$$

$$r = 1 - \frac{a}{S} \quad \text{or} \quad r = \frac{S - a}{S}$$

94. Solving for t :

$$v = -gt + v_0$$

$$v - v_0 = -gt + v_0 - v_0$$

$$v - v_0 = -gt$$

$$\frac{v - v_0}{-g} = \frac{-gt}{-g}$$

$$\frac{v_0 - v}{g} = t \quad \text{or} \quad t = \frac{v_0 - v}{g}$$

95. Let x = amount invested in bonds.

Invested in CDs: $x - 3000$

$$x + (x - 3,000) = 20,000$$

$$2x - 3,000 = 20,000$$

$$2x = 23,000$$

$$x = 11,500$$

\$11,500 will be invested in bonds and \$8,500 will be invested in CD's.

96. Let x = amount that Sean receives.

Amount for Jorge: $x - 3000$

$$x + (x - 3,000) = 10,000$$

$$2x - 3,000 = 10,000$$

$$2x = 13,000$$

$$x = 6,500$$

Sean will receive \$6,500 and Jorge will receive \$3,500.

97. Let x = Sandra's regular hourly wage.

$$40x + (1.5x)(8) = 910$$

$$40x + 12x = 910$$

$$52x = 910$$

$$x = \frac{910}{52} = 17.50$$

Sandra's regular hourly wage is \$17.50.

98. Let x = Leigh's regular hourly wage.

$$40x + 6(1.5x) + 4(2x) = 1083$$

$$40x + 9x + 8x = 1083$$

$$57x = 1083$$

$$x = \frac{1083}{57} = 19$$

Leigh's regular hourly wage is \$19.00.

99. Let x = final exam score.

Compute the final average and set equal to 80.

$$\left(\frac{1}{7} \right) (80 + 83 + 71 + 61 + 95 + x + x) = 80$$

Now solve for x :

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$$\left(\frac{1}{7}\right)(390 + 2x) = 80$$

$$390 + 2x = 560$$

$$2x = 170$$

$$x = 85$$

Camilla needs to score an 85 on the final exam to get an average of 80 in the course.

- 100.** Let x = final exam score.

$$\left(\frac{1}{3}\right)\left(\frac{86 + 80 + 84 + 90}{4}\right) + \left(\frac{2}{3}\right)x = 80$$

$$\frac{340}{12} + \frac{2}{3}x = 80$$

$$12\left(\frac{340}{12} + \frac{2}{3}x\right) = 12(80)$$

$$340 + 8x = 960$$

$$8x = 620$$

$$x = \frac{620}{8} = 77.5$$

Ali must score 78 on the final to earn a B.

Following the same approach as above, we get

$$340 + 8x = 12(90)$$

$$340 + 8x = 1080$$

$$8x = 740$$

$$x = \frac{740}{8} = 92.5$$

Ali must score 93 on the final to earn an A.

- 101.** Let x represent the original price of the phone.
Then $0.15x$ represents the reduction in the price of the phone.

original price – reduction = new price

$$x - 0.15x = 799$$

$$0.85x = 799$$

$$x = 940$$

The original price of the phone was \$940.

The amount of the savings is

$$0.15(\$940) = \$141.$$

- 102.** Let x represent the original price of the car.
Then $0.12x$ represents the reduction in the price of the car.

original price – reduction = new price

$$x - 0.12x = 28,160$$

$$0.88x = 28,160$$

$$x = 32,000$$

The original price of the car was \$32,000.

The amount of the savings is

$$\$32,000 - \$28,160 = \$3,840.$$

- 103.** Let x represent the price the theater pays for the candy.

Then $2.75x$ represents the markup on the candy.

The selling price of the candy is \$3.00.

supplier price + markup = selling price

$$x + 2.75x = 4.50$$

$$3.75x = 4.50$$

$$x = 1.20$$

The theater paid \$1.20 for the candy.

- 104.** Let x represent the dealer's cost of the new car.

$$x = 0.85(\$34,000) = \$28,900$$

If the dealer accepts \$100 over cost, then you will pay

$$\$28,900 + \$100 = \$29,000.$$

- 105.** Let a = number of adult patrons.

Child patrons: $3030 - a$

$$12.00a + 8.50(3030 - a) = 27,155$$

$$12a + 25,755 - 8.50a = 27,155$$

$$3.50a + 25,755 = 27,155$$

$$3.50a = 1,400$$

$$\frac{3.50a}{3.50} = \frac{1,400}{3.50}$$

$$a = 400$$

The theater had 400 adult patrons.

- 106.** Let x = original price of the boots.

Discount: $0.3x$

original price – discount = sale price

$$x - 0.3x = 399$$

$$0.7x = 399$$

$$\frac{0.7x}{0.7} = \frac{399}{0.7}$$

$$x = 570$$

The original price of the boots was \$570.

- 107.** Let w = width.

Length: $l = w + 8$

Perimeter: $P = 2l + 2w$

$$2l + 2w = 60$$

$$2(w + 8) + 2w = 60$$

$$2w + 16 + 2w = 60$$

$$4w + 16 = 60$$

$$4w = 44$$

$$w = 11$$

The rectangle has a width of 11 feet and a length of 19 feet.

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108. Let w = width.

Length: $l = 2w$

Perimeter: $P = 2l + 2w$

$$2l + 2w = 42$$

$$2(2w) + 2w = 42$$

$$4w + 2w = 42$$

$$6w = 42$$

$$w = 7$$

The rectangle has a width of 7 meters and a length of 14 meters.

109. Let x = the total number of people who owned a smart phone based on the recent survey. Then $0.534x$ represents the number of people who owned a smart phone that ran the Google Android OS.

$$0.534x = 124.4$$

$$x = \frac{124.4}{0.534} \approx 233$$

There were about 233 million people who owned a smart phone at the time of the survey.

110. Let W be the percentage that used WhatsApp.

Then:

$$62 = 2W - 2$$

$$64 = 2W$$

$$W = 32$$

Therefore, 32% of U.S. adults aged 18-29 used WhatsApp.

111. Let x = amount that Scott gets.

Amount for Alice: $\frac{3}{4}x$

Amount for Tricia: $\frac{1}{2}x$

$$x + \left(\frac{3}{4}x\right) + \left(\frac{1}{2}x\right) = 900,000$$

$$\frac{9}{4}x = 900,000$$

$$x = \frac{4}{9}(900,000)$$

$$x = 400,000$$

Scott receives \$400,000; Alice receives \$300,000; Tricia receives \$200,000.

112. Let x = amount that Judy pays.

Amount Tom pays: $\frac{2}{3}x$

$$x + \left(\frac{2}{3}x\right) = 18$$

$$\frac{5}{3}x = 18$$

$$\frac{3}{5} \cdot \frac{5}{3}x = \frac{3}{5}(18)$$

$$x = 10.80$$

Judy pays \$10.80 and Tom pays \$7.20.

113. An isosceles triangle has three equal sides. Therefore: $4x + 10 = 2x + 40 = 3x + 18$. Solve each set separately:

$$4x + 10 = 2x + 40$$

$$2x = 30$$

$$x = 15$$

$$4x + 10 = 3x + 18$$

$$x = 8$$

$$2x + 40 = 3x + 18$$

$$-x = -22$$

$$x = 22$$

Since 22 is the largest of the numbers then the largest perimeter is:

$$4(22) + 10 + 2(22) + 40 + 3(22) + 18 = 266$$

$$114. \quad \frac{3}{4}x - \frac{1}{5}\left(\frac{1}{2} - 3x\right) + 1 = \frac{1}{4}\left(\frac{1}{20}x + 6\right) - \frac{4}{5}$$

$$\frac{3}{4}x - \frac{1}{10} + \frac{3}{5}x + 1 = \frac{1}{80}x + \frac{3}{2} - \frac{4}{5}$$

Multiply both sides by the LCD 80 to clear fractions.

$$60x - 8 + 48x + 80 = x + 120 - 64$$

$$108x + 72 = x + 58$$

$$107x = -16$$

$$x = -\frac{16}{107}$$

115. If a hexagon is inscribed in a circle then the sides of the hexagon are equal to the radius of the circle. Let the $P = 6r$ be the perimeter of the hexagon. Let r be the radius of the circle.

$$6r = r + 10$$

$$5r = 10$$

$$r = 2$$

Thus $r = 2$ inches is the radius of the circle where the perimeter of the hexagon is 10 inches more than the radius.

116. In order to solve $\frac{5}{x+3} + 3 = \frac{8+x}{x+3}$, we multiply each term by the expression " $x+3$ " to get
- $$(x+3)\left(\frac{5}{x+3} + 3\right) = \left(\frac{8+x}{x+3}\right)(x+3).$$

Now, provided $x \neq -3$, we can cancel the denominators to get

$$5 + (x+3)(3) = 8+x$$

$$5 + 3x + 9 = 8 + x \Rightarrow 2x = -6 \Rightarrow x = -3$$

However, we already stated that $x \neq -3$. So we have a contradiction.

117. Answers will vary. One example is
 $3x+1 = 3x+6$.

118. Answers will vary. In general, 'solve' means to find values for a variable that make the equation true, 'evaluate' means to plug in a specific value for a variable in an expression and simplify, and 'simplify' means to collect like terms and write an expression in a simpler form.

Ex: Solve $3(x+2) - x = 8$.

Solution: $3(x+2) - x = 8$

$$3x + 6 - x = 8$$

$$2x + 6 = 8$$

$$2x = 2$$

$$x = 1$$

Ex: Evaluate $3(x+2) - x$ for $x = -2$.

$$\begin{aligned} \text{Solution: } 3(x+2) - x &= 3(-2+2) - (-2) \\ &= 3(0) + 2 \\ &= 2 \end{aligned}$$

Ex: Simplify $3(x+2) - x$.

Solution: $3(x+2) - x$

$$3x + 6 - x$$

$$2x + 6$$

119. In obtaining step (7) we divided by $x-2$. Since $x=2$ from step (1), we actually divided by 0.

Section 1.3

1. $x^2 - 5x - 6 = (x-6)(x+1)$

2. $2x^2 - x - 3 = (2x-3)(x+1)$

3. $\left\{-\frac{5}{3}, 3\right\}$

4. True

5. $\frac{1}{2} \cdot 5 = \frac{5}{2}; \left(\frac{5}{2}\right)^2 = \frac{25}{4}; x^2 + 5x + \frac{25}{4}$
 $x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)^2$

6. discriminant; negative

7. False; a quadratic equation may have no real solutions.

8. False; If $x^2 = p$ then x could also be negative.

9. b

10. d

11. $x^2 - 9x = 0$

$$x(x-9) = 0$$

$$x = 0 \text{ or } x - 9 = 0$$

$$x = 0 \text{ or } x = 9$$

The solution set is $\{0, 9\}$.

12. $x^2 + 4x = 0$

$$x(x+4) = 0$$

$$x = 0 \text{ or } x + 4 = 0$$

$$x = 0 \text{ or } x = -4$$

The solution set is $\{-4, 0\}$.

13. $x^2 - 25 = 0$

$$(x+5)(x-5) = 0$$

$$x+5 = 0 \text{ or } x-5 = 0$$

$$x = -5 \text{ or } x = 5$$

The solution set is $\{-5, 5\}$.

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14. $x^2 - 9 = 0$

$$(x+3)(x-3) = 0$$

$$x+3=0 \quad \text{or} \quad x-3=0$$

$$x=-3 \quad \text{or} \quad x=3$$

The solution set is $\{-3, 3\}$.

15. $z^2 + z - 6 = 0$

$$(z+3)(z-2) = 0$$

$$z+3=0 \quad \text{or} \quad z-2=0$$

$$z=-3 \quad \text{or} \quad z=2$$

The solution set is $\{-3, 2\}$.

16. $v^2 + 7v + 6 = 0$

$$(v+6)(v+1) = 0$$

$$v+6=0 \quad \text{or} \quad v+1=0$$

$$v=-6 \quad \text{or} \quad v=-1$$

The solution set is $\{-6, -1\}$

17. $2x^2 - 5x - 3 = 0$

$$(2x+1)(x-3) = 0$$

$$2x+1=0 \quad \text{or} \quad x-3=0$$

$$x = -\frac{1}{2} \quad \text{or} \quad x = 3$$

The solution set is $\left\{-\frac{1}{2}, 3\right\}$

18. $3x^2 + 5x + 2 = 0$

$$(3x+2)(x+1) = 0$$

$$3x+2=0 \quad \text{or} \quad x+1=0$$

$$x = -\frac{2}{3} \quad \text{or} \quad x = -1$$

The solution set is $\left\{-1, -\frac{2}{3}\right\}$.

19. $-5w^2 + 180 = 0$

$$-5(w^2 - 36) = 0$$

$$-5(w+6)(w-6) = 0$$

$$w+6=0 \quad \text{or} \quad w-6=0$$

$$w=-6 \quad \text{or} \quad w=6$$

The solution set is $\{-6, 6\}$.

20. $2y^2 - 50 = 0$

$$2(y^2 - 25) = 0$$

$$2(y+5)(y-5) = 0$$

$$y+5=0 \quad \text{or} \quad y-5=0$$

$$y=-5 \quad \text{or} \quad y=5$$

The solution set is $\{-5, 5\}$.

21. $x(x+3) - 10 = 0$

$$x^2 + 3x - 10 = 0$$

$$(x-2)(x+5) = 0$$

$$x-2=0 \quad \text{or} \quad x+5=0$$

$$x=2 \quad \text{or} \quad x=-5$$

The solution set is $\{-5, 2\}$.

22. $x(x+4) = 12$

$$x^2 + 4x - 12 = 0$$

$$(x+6)(x-2) = 0$$

$$x+6=0 \quad \text{or} \quad x-2=0$$

$$x=-6 \quad \text{or} \quad x=2$$

The solution set is $\{-6, 2\}$.

23. $4x^2 + 9 = 12x$

$$4x^2 - 12x + 9 = 0$$

$$(2x-3)^2 = 0$$

$$2x-3=0$$

$$x = \frac{3}{2}$$

The solution set is $\left\{\frac{3}{2}\right\}$.

24. $25x^2 + 16 = 40x$

$$25x^2 - 40x + 16 = 0$$

$$(5x-4)^2 = 0$$

$$5x-4=0$$

$$x = \frac{4}{5}$$

The solution set is $\left\{\frac{4}{5}\right\}$.

Section 1.3: Quadratic Equations

25. $6(p^2 - 1) = 5p$

$$6p^2 - 6 = 5p$$

$$6p^2 - 5p - 6 = 0$$

$$(3p + 2)(2p - 3) = 0$$

$$3p + 2 = 0 \quad \text{or} \quad 2p - 3 = 0$$

$$p = -\frac{2}{3} \quad \text{or} \quad p = \frac{3}{2}$$

The solution set is $\left\{-\frac{2}{3}, \frac{3}{2}\right\}$.

26. $2(2u^2 - 4u) + 3 = 0$

$$4u^2 - 8u + 3 = 0$$

$$(2u - 1)(2u - 3) = 0$$

$$2u - 1 = 0 \quad \text{or} \quad 2u - 3 = 0$$

$$u = \frac{1}{2} \quad \text{or} \quad u = \frac{3}{2}$$

The solution set is $\left\{\frac{1}{2}, \frac{3}{2}\right\}$.

27. $6x - 5 = \frac{6}{x}$

$$(6x - 5)x = \left(\frac{6}{x}\right)x$$

$$6x^2 - 5x = 6$$

$$6x^2 - 5x - 6 = 0$$

$$(3x + 2)(2x - 3) = 0$$

$$3x + 2 = 0 \quad \text{or} \quad 2x - 3 = 0$$

$$x = -\frac{2}{3} \quad \text{or} \quad x = \frac{3}{2}$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{2}{3}, \frac{3}{2}\right\}$.

28. $x + \frac{12}{x} = 7$

$$\left(x + \frac{12}{x}\right)x = 7x$$

$$x^2 + 12 = 7x$$

$$x^2 - 7x + 12 = 0$$

$$(x - 3)(x - 4) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = 3 \quad \text{or} \quad x = 4$$

Neither of these values causes a denominator to equal zero, so the solution set is $\{3, 4\}$.

29. $\frac{4(x-2)}{x-3} + \frac{3}{x} = \frac{-3}{x(x-3)}$

$$\left(\frac{4(x-2)}{x-3} + \frac{3}{x}\right)x(x-3) = \left(\frac{-3}{x(x-3)}\right)x(x-3)$$

$$4x(x-2) + 3(x-3) = -3$$

$$4x^2 - 8x + 3x - 9 = -3$$

$$4x^2 - 5x - 6 = 0$$

$$(4x + 3)(x - 2) = 0$$

$$4x + 3 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -\frac{3}{4} \quad \text{or} \quad x = 2$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{3}{4}, 2\right\}$.

30. $\frac{5}{x+4} = 4 + \frac{3}{x-2}$

$$\left(\frac{5}{x+4}\right)(x+4)(x-2) = \left(4 + \frac{3}{x-2}\right)(x+4)(x-2)$$

$$5(x-2) = 4(x+4)(x-2) + 3(x+4)$$

$$5x - 10 = 4(x^2 + 2x - 8) + 3x + 12$$

$$5x - 10 = 4x^2 + 8x - 32 + 3x + 12$$

$$0 = 4x^2 + 6x - 10$$

$$0 = 2(2x^2 + 3x - 5)$$

$$0 = 2(2x + 5)(x - 1)$$

$$2x + 5 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = -\frac{5}{2} \quad \text{or} \quad x = 1$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{5}{2}, 1\right\}$.

31. $x^2 = 25$

$$x = \pm\sqrt{25}$$

$$x = \pm 5$$

The solution set is $\{-5, 5\}$.

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32. $x^2 = 36$

$$x = \pm\sqrt{36}$$

$$x = \pm 6$$

The solution set is $\{-6, 6\}$.

33. $(x-1)^2 = 4$

$$x-1 = \pm\sqrt{4}$$

$$x-1 = \pm 2$$

$$x-1 = 2 \text{ or } x-1 = -2$$

$$x = 3 \text{ or } x = -1$$

The solution set is $\{-1, 3\}$.

34. $(x+2)^2 = 1$

$$x+2 = \pm\sqrt{1}$$

$$x+2 = \pm 1$$

$$x+2 = 1 \text{ or } x+2 = -1$$

$$x = -1 \text{ or } x = -3$$

The solution set is $\{-3, -1\}$.

35. $\left(\frac{1}{3}h+4\right)^2 = 16$

$$\frac{1}{3}h+4 = \pm\sqrt{16}$$

$$\frac{1}{3}h+4 = \pm 4$$

$$\frac{1}{3}h+4 = 4 \text{ or } \frac{1}{3}h+4 = -4$$

$$\frac{1}{3}h = 0 \text{ or } \frac{1}{3}h = -8$$

$$h = 0 \text{ or } h = -24$$

The solution set is $\{-24, 0\}$.

36. $(3z-2)^2 = 4$

$$3z-2 = \pm\sqrt{4}$$

$$3z-2 = \pm 2$$

$$3z-2 = 2 \text{ or } 3z-2 = -2$$

$$3z = 4 \text{ or } 3z = 0$$

$$z = \frac{4}{3} \text{ or } z = 0$$

The solution set is $\left\{0, \frac{4}{3}\right\}$.

37. $x^2 + 4x = 21$

$$x^2 + 4x + 4 = 21 + 4$$

$$(x+2)^2 = 25$$

$$x+2 = \pm\sqrt{25}$$

$$x+2 = \pm 5$$

$$x = -2 \pm 5$$

$$x = 3 \text{ or } x = -7$$

The solution set is $\{-7, 3\}$.

38. $x^2 - 6x = 13$

$$x^2 - 6x + 9 = 13 + 9$$

$$(x-3)^2 = 22$$

$$x-3 = \pm\sqrt{22}$$

$$x = 3 \pm \sqrt{22}$$

The solution set is $\{3 - \sqrt{22}, 3 + \sqrt{22}\}$.

39. $x^2 - \frac{1}{2}x - \frac{3}{16} = 0$

$$x^2 - \frac{1}{2}x = \frac{3}{16}$$

$$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{3}{16} + \frac{1}{16}$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{1}{4}$$

$$x - \frac{1}{4} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

$$x = \frac{1}{4} \pm \frac{1}{2}$$

$$x = \frac{3}{4} \text{ or } x = -\frac{1}{4}$$

The solution set is $\left\{-\frac{1}{4}, \frac{3}{4}\right\}$.

$$40. \quad x^2 + \frac{2}{3}x - \frac{1}{3} = 0$$

$$x^2 + \frac{2}{3}x = \frac{1}{3}$$

$$x^2 + \frac{2}{3}x + \frac{1}{9} = \frac{1}{3} + \frac{1}{9}$$

$$\left(x + \frac{1}{3}\right)^2 = \frac{4}{9}$$

$$x + \frac{1}{3} = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$$

$$x = -\frac{1}{3} \pm \frac{2}{3}$$

$$x = \frac{1}{3} \text{ or } x = -1$$

The solution set is $\left\{-1, \frac{1}{3}\right\}$.

$$41. \quad 3x^2 + x - \frac{1}{2} = 0$$

$$x^2 + \frac{1}{3}x - \frac{1}{6} = 0$$

$$x^2 + \frac{1}{3}x = \frac{1}{6}$$

$$x^2 + \frac{1}{3}x + \frac{1}{36} = \frac{1}{6} + \frac{1}{36}$$

$$\left(x + \frac{1}{6}\right)^2 = \frac{7}{36}$$

$$x + \frac{1}{6} = \pm\sqrt{\frac{7}{36}}$$

$$x + \frac{1}{6} = \pm\frac{\sqrt{7}}{6}$$

$$x = \frac{-1 \pm \sqrt{7}}{6}$$

The solution set is $\left\{\frac{-1 - \sqrt{7}}{6}, \frac{-1 + \sqrt{7}}{6}\right\}$.

$$42. \quad 2x^2 - 3x - 1 = 0$$

$$x^2 - \frac{3}{2}x - \frac{1}{2} = 0$$

$$x^2 - \frac{3}{2}x = \frac{1}{2}$$

$$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{1}{2} + \frac{9}{16}$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$$x - \frac{3}{4} = \pm\sqrt{\frac{17}{16}}$$

$$x - \frac{3}{4} = \pm\frac{\sqrt{17}}{4}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

The solution set is $\left\{\frac{3 - \sqrt{17}}{4}, \frac{3 + \sqrt{17}}{4}\right\}$.

$$43. \quad x^2 + 2x - 13 = 0$$

$$a = 1, \quad b = 2, \quad c = -13$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-13)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 52}}{2}$$

$$= \frac{-2 \pm \sqrt{56}}{2} = \frac{-2 \pm 2\sqrt{14}}{2} = -1 \pm \sqrt{14}$$

The solution set is $\{-1 - \sqrt{14}, -1 + \sqrt{14}\}$.

$$44. \quad x^2 + 4x + 2 = 0$$

$$a = 1, \quad b = 4, \quad c = 2$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 8}}{2}$$

$$= \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

The solution set is $\{-2 - \sqrt{2}, -2 + \sqrt{2}\}$.

$$45. \quad x^2 - 4x - 1 = 0$$

$$a = 1, \quad b = -4, \quad c = -1$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)} = \frac{4 \pm \sqrt{16 + 4}}{2}$$

$$= \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

The solution set is $\{2 - \sqrt{5}, 2 + \sqrt{5}\}$.

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46. $x^2 + 6x + 1 = 0$

$a = 1, \quad b = 6, \quad c = 1$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(1)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 4}}{2}$$

$$= \frac{-6 \pm \sqrt{32}}{2} = \frac{-6 \pm 4\sqrt{2}}{2} = -3 \pm 2\sqrt{2}$$

The solution set is $\{-3 - 2\sqrt{2}, -3 + 2\sqrt{2}\}$.

47. $2x^2 - 5x + 3 = 0$

$a = 2, \quad b = -5, \quad c = 3$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25 - 24}}{4} = \frac{5 \pm \sqrt{1}}{4} = \frac{5 \pm 1}{4}$$

$x = \frac{5+1}{4}$ or $x = \frac{5-1}{4}$

$x = \frac{6}{4}$ or $x = \frac{4}{4}$

$x = \frac{3}{2}$ or $x = 1$

The solution set is $\left\{1, \frac{3}{2}\right\}$.

48. $2x^2 + 5x + 3 = 0$

$a = 2, \quad b = 5, \quad c = 3$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25 - 24}}{4} = \frac{-5 \pm \sqrt{1}}{4} = \frac{-5 \pm 1}{4}$$

$x = \frac{-5+1}{4}$ or $x = \frac{-5-1}{4}$

$x = \frac{-4}{4}$ or $x = \frac{-6}{4}$

$x = -1$ or $x = -\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2}, -1\right\}$.

49. $4y^2 - y + 2 = 0$

$a = 4, \quad b = -1, \quad c = 2$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(2)}}{2(4)}$$

$$= \frac{1 \pm \sqrt{1 - 32}}{8} = \frac{1 \pm \sqrt{-31}}{8}$$

No real solution.

50. $4t^2 + t + 1 = 0$

$a = 4, \quad b = 1, \quad c = 1$

$$t = \frac{-1 \pm \sqrt{1^2 - 4(4)(1)}}{2(4)}$$

$$= \frac{-1 \pm \sqrt{1 - 16}}{8} = \frac{-1 \pm \sqrt{-15}}{8}$$

No real solution.

51. $9x^2 + 8x = 5$

$9x^2 + 8x - 5 = 0$

$a = 9, \quad b = 8, \quad c = -5$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(9)(-5)}}{2(9)}$$

$$= \frac{-8 \pm \sqrt{64 + 180}}{18} = \frac{-8 \pm \sqrt{244}}{18}$$

$$= \frac{-8 \pm 2\sqrt{61}}{18} = \frac{-4 \pm \sqrt{61}}{9}$$

The solution set is $\left\{\frac{-4 - \sqrt{61}}{9}, \frac{-4 + \sqrt{61}}{9}\right\}$.

52. $2x^2 = 1 - 2x$

$2x^2 + 2x - 1 = 0$

$a = 2, \quad b = 2, \quad c = -1$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)} = \frac{-2 \pm \sqrt{4 + 8}}{4}$$

$$= \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

The solution set is $\left\{\frac{-1 - \sqrt{3}}{2}, \frac{-1 + \sqrt{3}}{2}\right\}$.

53. $4x^2 = 9x$

$$4x^2 - 9x = 0$$

$$x(4x - 9) = 0$$

$$x = 0 \quad \text{or} \quad 4x - 9 = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{9}{4}$$

The solution set is $\left\{0, \frac{9}{4}\right\}$.

54. $5x = 4x^2$

$$0 = 4x^2 - 5x$$

$$0 = x(4x - 5)$$

$$x = 0 \quad \text{or} \quad 4x - 5 = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{5}{4}$$

The solution set is $\left\{0, \frac{5}{4}\right\}$.

55. $9t^2 - 6t + 1 = 0$

$$a = 9, \quad b = -6, \quad c = 1$$

$$t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)}$$

$$= \frac{6 \pm \sqrt{36 - 36}}{18} = \frac{6 \pm 0}{18} = \frac{1}{3}$$

The solution set is $\left\{\frac{1}{3}\right\}$.

56. $4u^2 - 6u + 9 = 0$

$$a = 4, \quad b = -6, \quad c = 9$$

$$u = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$$

$$= \frac{6 \pm \sqrt{36 - 144}}{8} = \frac{6 \pm \sqrt{-108}}{8}$$

No real solution.

57. $\frac{3}{4}x^2 - \frac{1}{4}x - \frac{1}{2} = 0$

$$4\left(\frac{3}{4}x^2 - \frac{1}{4}x - \frac{1}{2}\right) = 4(0)$$

$$3x^2 - x - 2 = 0$$

$$a = 3, \quad b = -1, \quad c = -2$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{1 \pm \sqrt{1 + 24}}{6} = \frac{1 \pm \sqrt{25}}{6} = \frac{1 \pm 5}{6}$$

$$x = \frac{1+5}{6} \quad \text{or} \quad x = \frac{1-5}{6}$$

$$x = \frac{6}{6} \quad \text{or} \quad x = \frac{-4}{6}$$

$$x = 1 \quad \text{or} \quad x = -\frac{2}{3}$$

The solution set is $\left\{-\frac{2}{3}, 1\right\}$.

58. $\frac{2}{3}x^2 - x - 3 = 0$

$$3\left(\frac{2}{3}x^2 - x - 3\right) = 3(0)$$

$$2x^2 - 3x - 9 = 0$$

$$a = 2, \quad b = -3, \quad c = -9$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-9)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9 + 72}}{4} = \frac{3 \pm \sqrt{81}}{4} = \frac{3 \pm 9}{4}$$

$$x = \frac{3+9}{4} \quad \text{or} \quad x = \frac{3-9}{4}$$

$$x = \frac{12}{4} \quad \text{or} \quad x = \frac{-6}{4}$$

$$x = 3 \quad \text{or} \quad x = -\frac{3}{2}$$

The solution set is $\left\{-\frac{3}{2}, 3\right\}$.

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$$59. \quad \frac{5}{3}x^2 - x = \frac{1}{3}$$

$$3\left(\frac{5}{3}x^2 - x\right) = 3\left(\frac{1}{3}\right)$$

$$5x^2 - 3x = 1$$

$$5x^2 - 3x - 1 = 0$$

$$a = 5, \quad b = -3, \quad c = -1$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-1)}}{2(5)}$$

$$= \frac{3 \pm \sqrt{9+20}}{10} = \frac{3 \pm \sqrt{29}}{10}$$

$$\text{The solution set is } \left\{ \frac{3 - \sqrt{29}}{10}, \frac{3 + \sqrt{29}}{10} \right\}.$$

$$60. \quad \frac{3}{5}x^2 - x = \frac{1}{5}$$

$$5\left(\frac{3}{5}x^2 - x\right) = 5\left(\frac{1}{5}\right)$$

$$3x^2 - 5x = 1$$

$$3x^2 - 5x - 1 = 0$$

$$a = 3, \quad b = -5, \quad c = -1$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{25+12}}{6} = \frac{5 \pm \sqrt{37}}{6}$$

$$\text{The solution set is } \left\{ \frac{5 - \sqrt{37}}{6}, \frac{5 + \sqrt{37}}{6} \right\}.$$

$$61. \quad 2x(x+2) = 3$$

$$2x^2 + 4x - 3 = 0$$

$$a = 2, \quad b = 4, \quad c = -3$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)} = \frac{-4 \pm \sqrt{16+24}}{4}$$

$$= \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}$$

$$\text{The solution set is } \left\{ \frac{-2 - \sqrt{10}}{2}, \frac{-2 + \sqrt{10}}{2} \right\}.$$

$$62. \quad 3x(x+2) = 1$$

$$3x^2 + 6x - 1 = 0$$

$$a = 3, \quad b = 6, \quad c = -1$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(-1)}}{2(3)} = \frac{-6 \pm \sqrt{36+12}}{6}$$

$$= \frac{-6 \pm \sqrt{48}}{6} = \frac{-6 \pm 4\sqrt{3}}{6} = \frac{-3 \pm 2\sqrt{3}}{3}$$

$$\text{The solution set is } \left\{ \frac{-3 - 2\sqrt{3}}{3}, \frac{-3 + 2\sqrt{3}}{3} \right\}.$$

$$63. \quad 4 + \frac{1}{x} - \frac{1}{x^2} = 0$$

$$x^2 \left(4 + \frac{1}{x} - \frac{1}{x^2} \right) = x^2 (0)$$

$$4x^2 + x - 1 = 0$$

$$a = 4, \quad b = 1, \quad c = -1$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-1 \pm \sqrt{1+16}}{8} = \frac{-1 \pm \sqrt{17}}{8}$$

Neither of these values causes a denominator to equal zero, so the solution set is

$$\left\{ \frac{-1 - \sqrt{17}}{8}, \frac{-1 + \sqrt{17}}{8} \right\}.$$

$$64. \quad 2 + \frac{8}{x} + \frac{3}{x^2} = 0$$

$$x^2 \left(2 + \frac{8}{x} + \frac{3}{x^2} \right) = x^2 (0)$$

$$2x^2 + 8x + 3 = 0$$

$$a = 2, \quad b = 8, \quad c = 3$$

$$x = \frac{-(8) \pm \sqrt{8^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-8 \pm \sqrt{64-24}}{4} = \frac{-8 \pm \sqrt{40}}{4}$$

$$= \frac{-8 \pm 2\sqrt{10}}{4} = \frac{-4 \pm \sqrt{10}}{2}$$

Neither of these values causes a denominator to equal zero, so the solution set is

$$\left\{ \frac{-4 - \sqrt{10}}{2}, \frac{-4 + \sqrt{10}}{2} \right\}.$$

65. $\frac{3x}{x-2} + \frac{1}{x} = 4$

$$\left(\frac{3x}{x-2} + \frac{1}{x}\right)x(x-2) = 4x(x-2)$$

$$3x(x) + (x-2) = 4x^2 - 8x$$

$$3x^2 + x - 2 = 4x^2 - 8x$$

$$0 = x^2 - 9x + 2$$

$$a = 1, \quad b = -9, \quad c = 2$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{9 \pm \sqrt{81-8}}{2} = \frac{9 \pm \sqrt{73}}{2}$$

Neither of these values causes a denominator to equal zero, so the solution set is

$$\left\{ \frac{9 - \sqrt{73}}{2}, \frac{9 + \sqrt{73}}{2} \right\}.$$

66. $\frac{2x}{x-3} + \frac{1}{x} = 4$

$$\left(\frac{2x}{x-3} + \frac{1}{x}\right)x(x-3) = 4x(x-3)$$

$$2x(x) + (x-3) = 4x^2 - 12x$$

$$2x^2 + x - 3 = 4x^2 - 12x$$

$$0 = 2x^2 - 13x + 3$$

$$a = 2, \quad b = -13, \quad c = 3$$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{13 \pm \sqrt{169-24}}{4} = \frac{13 \pm \sqrt{145}}{4}$$

Neither of these values causes a denominator to equal zero, so the solution set is

$$\left\{ \frac{13 - \sqrt{145}}{4}, \frac{13 + \sqrt{145}}{4} \right\}.$$

67. $x^2 - 4.1x + 2.2 = 0$

$$a = 1, \quad b = -4.1, \quad c = 2.2$$

$$x = \frac{-(-4.1) \pm \sqrt{(-4.1)^2 - 4(1)(2.2)}}{2(1)}$$

$$= \frac{4.1 \pm \sqrt{16.81-8.8}}{2} = \frac{4.1 \pm \sqrt{8.01}}{2}$$

$$x \approx 3.47 \quad \text{or} \quad x \approx 0.63$$

The solution set is $\{0.63, 3.47\}$.

68. $x^2 + 3.9x + 1.8 = 0$

$$a = 1, \quad b = 3.9, \quad c = 1.8$$

$$x = \frac{-3.9 \pm \sqrt{(3.9)^2 - 4(1)(1.8)}}{2(1)}$$

$$= \frac{-3.9 \pm \sqrt{15.21-7.2}}{2} = \frac{-3.9 \pm \sqrt{8.01}}{2}$$

$$x \approx -0.53 \quad \text{or} \quad x \approx -3.37$$

The solution set is $\{-3.37, -0.53\}$.

69. $x^2 + \sqrt{3}x - 3 = 0$

$$a = 1, \quad b = \sqrt{3}, \quad c = -3$$

$$x = \frac{-\sqrt{3} \pm \sqrt{(\sqrt{3})^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-\sqrt{3} \pm \sqrt{3+12}}{2} = \frac{-\sqrt{3} \pm \sqrt{15}}{2}$$

$$x \approx 1.07 \quad \text{or} \quad x \approx -2.80$$

The solution set is $\{-2.80, 1.07\}$.

70. $x^2 + \sqrt{2}x - 2 = 0$

$$a = 1, \quad b = \sqrt{2}, \quad c = -2$$

$$x = \frac{-\sqrt{2} \pm \sqrt{(\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2+8}}{2} = \frac{-\sqrt{2} \pm \sqrt{10}}{2}$$

$$x \approx 0.87 \quad \text{or} \quad x \approx -2.29$$

The solution set is $\{-2.29, 0.87\}$.

71. $\pi x^2 - x - \pi = 0$

$$a = \pi, \quad b = -1, \quad c = -\pi$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(\pi)(-\pi)}}{2(\pi)}$$

$$= \frac{1 \pm \sqrt{1+4\pi^2}}{2\pi}$$

$$x \approx 1.17 \quad \text{or} \quad x \approx -0.85$$

The solution set is $\{-0.85, 1.17\}$.

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72. $\pi x^2 + \pi x - 2 = 0$

$a = \pi, \quad b = \pi, \quad c = -2$

$$x = \frac{-\pi \pm \sqrt{(\pi)^2 - 4(\pi)(-2)}}{2(\pi)}$$

$$= \frac{-\pi \pm \sqrt{\pi^2 + 8\pi}}{2\pi}$$

$x \approx 0.44$ or $x \approx -1.44$

The solution set is $\{-1.44, 0.44\}$.

73. $2x^2 - 6x + 7 = 0$

$a = 2, \quad b = -6, \quad c = 7$

$$b^2 - 4ac = (-6)^2 - 4(2)(7) = 36 - 56 = -20$$

Since the $b^2 - 4ac < 0$, the equation has no real solution.

74. $x^2 + 4x + 7 = 0$

$a = 1, \quad b = 4, \quad c = 7$

$$b^2 - 4ac = (4)^2 - 4(1)(7) = 16 - 28 = -12$$

Since the $b^2 - 4ac < 0$, the equation has no real solution.

75. $9x^2 - 30x + 25 = 0$

$a = 9, \quad b = -30, \quad c = 25$

$$b^2 - 4ac = (-30)^2 - 4(9)(25) = 900 - 900 = 0$$

Since $b^2 - 4ac = 0$, the equation has one repeated real solution.

76. $25x^2 - 20x + 4 = 0$

$a = 25, \quad b = -20, \quad c = 4$

$$b^2 - 4ac = (-20)^2 - 4(25)(4) = 400 - 400 = 0$$

Since $b^2 - 4ac = 0$, the equation has one repeated real solution.

77. $3x^2 + 5x - 8 = 0$

$a = 3, \quad b = 5, \quad c = -8$

$$b^2 - 4ac = (5)^2 - 4(3)(-8) = 25 + 96 = 121$$

Since $b^2 - 4ac > 0$, the equation has two unequal real solutions.

78. $2x^2 - 3x - 7 = 0$

$a = 2, \quad b = -3, \quad c = -7$

$$b^2 - 4ac = (-3)^2 - 4(2)(-7) = 9 + 56 = 65$$

Since $b^2 - 4ac > 0$, the equation has two unequal real solutions.

79. $x^2 - 5 = 0$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

The solution set is $\{-\sqrt{5}, \sqrt{5}\}$.

80. $x^2 - 6 = 0$

$$x^2 = 6$$

$$x = \pm\sqrt{6}$$

The solution set is $\{-\sqrt{6}, \sqrt{6}\}$.

81. $16x^2 - 8x + 1 = 0$

$$(4x - 1)(4x - 1) = 0$$

$$4x - 1 = 0$$

$$x = \frac{1}{4}$$

The solution set is $\left\{\frac{1}{4}\right\}$.

82. $9x^2 - 12x + 4 = 0$

$$(3x - 2)(3x - 2) = 0$$

$$3x - 2 = 0$$

$$x = \frac{2}{3}$$

The solution set is $\left\{\frac{2}{3}\right\}$.

83. $10x^2 - 19x - 15 = 0$

$$(5x + 3)(2x - 5) = 0$$

$$5x + 3 = 0 \quad \text{or} \quad 2x - 5 = 0$$

$$x = -\frac{3}{5} \quad \text{or} \quad x = \frac{5}{2}$$

The solution set is $\left\{-\frac{3}{5}, \frac{5}{2}\right\}$.

84. $6x^2 + 7x - 20 = 0$

$$(3x - 4)(2x + 5) = 0$$

$$3x - 4 = 0 \quad \text{or} \quad 2x + 5 = 0$$

$$x = \frac{4}{3} \quad \text{or} \quad x = -\frac{5}{2}$$

The solution set is $\left\{-\frac{5}{2}, \frac{4}{3}\right\}$.

85. $2 + z = 6z^2$

$$0 = 6z^2 - z - 2$$

$$0 = (3z - 2)(2z + 1)$$

$$3z - 2 = 0 \text{ or } 2z + 1 = 0$$

$$z = \frac{2}{3} \text{ or } z = -\frac{1}{2}$$

The solution set is $\left\{-\frac{1}{2}, \frac{2}{3}\right\}$.

86. $2 = y + 6y^2$

$$0 = 6y^2 + y - 2$$

$$0 = (3y + 2)(2y - 1)$$

$$3y + 2 = 0 \text{ or } 2y - 1 = 0$$

$$y = -\frac{2}{3} \text{ or } y = \frac{1}{2}$$

The solution set is $\left\{-\frac{2}{3}, \frac{1}{2}\right\}$.

87. $x^2 + \sqrt{2}x = \frac{1}{2}$

$$x^2 + \sqrt{2}x - \frac{1}{2} = 0$$

$$2\left(x^2 + \sqrt{2}x - \frac{1}{2}\right) = 2(0)$$

$$2x^2 + 2\sqrt{2}x - 1 = 0$$

$$a = 2, \quad b = 2\sqrt{2}, \quad c = -1$$

$$x = \frac{-(2\sqrt{2}) \pm \sqrt{(2\sqrt{2})^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-2\sqrt{2} \pm \sqrt{8+8}}{4} = \frac{-2\sqrt{2} \pm \sqrt{16}}{4}$$

$$= \frac{-2\sqrt{2} \pm 4}{4} = \frac{-\sqrt{2} \pm 2}{2}$$

The solution set is $\left\{\frac{-\sqrt{2}-2}{2}, \frac{-\sqrt{2}+2}{2}\right\}$

88. $\frac{1}{2}x^2 = \sqrt{2}x + 1$

$$\frac{1}{2}x^2 - \sqrt{2}x - 1 = 0$$

$$2\left(\frac{1}{2}x^2 - \sqrt{2}x - 1\right) = 2(0)$$

$$x^2 - 2\sqrt{2}x - 2 = 0$$

$$a = 1, \quad b = -2\sqrt{2}, \quad c = -2$$

$$x = \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{2\sqrt{2} \pm \sqrt{8+8}}{2} = \frac{2\sqrt{2} \pm \sqrt{16}}{2}$$

$$= \frac{2\sqrt{2} \pm 4}{2} = \frac{\sqrt{2} \pm 2}{1}$$

The solution set is $\{\sqrt{2} - 2, \sqrt{2} + 2\}$.

89. $x^2 + x = 4$

$$x^2 + x - 4 = 0$$

$$a = 1, \quad b = 1, \quad c = -4$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$

The solution set is $\left\{\frac{-1-\sqrt{17}}{2}, \frac{-1+\sqrt{17}}{2}\right\}$.

90. $x^2 + x = 1$

$$x^2 + x - 1 = 0$$

$$a = 1, \quad b = 1, \quad c = -1$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

The solution set is $\left\{\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right\}$.

91.
$$\frac{x}{x-2} + \frac{2}{x+1} = \frac{7x+1}{x^2-x-2}$$

$$\frac{x}{x-2} + \frac{2}{x+1} = \frac{7x+1}{(x-2)(x+1)}$$

$$\left(\frac{x}{x-2} + \frac{2}{x+1}\right)(x-2)(x+1) = \left(\frac{7x+1}{(x-2)(x+1)}\right)(x-2)(x+1)$$

$$x(x+1) + 2(x-2) = 7x+1$$

$$x^2 + x + 2x - 4 = 7x + 1$$

$$x^2 + 3x - 4 = 7x + 1$$

$$x^2 - 4x - 5 = 0$$

$$(x+1)(x-5) = 0$$

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$$x+1=0 \quad \text{or} \quad x-5=0$$

$$x=-1 \quad \text{or} \quad x=5$$

The value $x = -1$ causes a denominator to equal zero, so we disregard it. Thus, the solution set is $\{5\}$.

92.

$$\frac{3x}{x+2} + \frac{1}{x-1} = \frac{4-7x}{x^2+x-2}$$

$$\frac{3x}{x+2} + \frac{1}{x-1} = \frac{4-7x}{(x+2)(x-1)}$$

$$\left(\frac{3x}{x+2} + \frac{1}{x-1}\right)(x+2)(x-1) = \left(\frac{4-7x}{(x+2)(x-1)}\right)(x+2)(x-1)$$

$$3x(x-1) + (x+2) = 4-7x$$

$$3x^2 - 3x + x + 2 = 4 - 7x$$

$$3x^2 - 2x + 2 = 4 - 7x$$

$$3x^2 + 5x - 2 = 0$$

$$(3x-1)(x+2) = 0$$

$$3x-1=0 \quad \text{or} \quad x+2=0$$

$$x = \frac{1}{3} \quad \text{or} \quad x = -2$$

The value $x = -2$ causes a denominator to equal zero, so we disregard it. Thus, the solution set is $\left\{\frac{1}{3}\right\}$.

93. Since this is a right triangle then we can use the Pythagorean Theorem. So

$$(2x+3)^2 = (2x-5)^2 + (x+7)^2$$

$$4x^2 + 12x + 9 = 4x^2 - 20x + 25 + x^2 + 14x + 49$$

$$12x + 9 = x^2 - 6x + 74$$

$$0 = x^2 - 18x + 65$$

$$0 = (x-5)(x-13)$$

$$x-5=0 \quad \text{or} \quad x-13=0$$

$$x=5 \quad \text{or} \quad x=13$$

This means there are 2 possible that meet these requirements. Substituting x into the given sides gives:

When $x = 5$: 5m, 12m, 13m

When $x = 13$: 20m, 21m, 29m

Thus there are 2 solutions.

94. Since this is a right triangle then we can use the Pythagorean Theorem. So

$$(4x+5)^2 = (3x+13)^2 + x^2$$

$$16x^2 + 40x + 25 = 9x^2 + 78x + 169 + x^2$$

$$6x^2 - 38x - 144 = 0$$

$$2(3x^2 - 19x - 72) = 0$$

$$2(3x+8)(x-9) = 0$$

$$3x+8=0 \quad \text{or} \quad x-9=0$$

$$x = -\frac{8}{3} \quad \text{or} \quad x = 9$$

This means there are 2 possible solutions that meet these requirements. Substituting x into the given sides gives:

When $x = 9$: 41m, 40m, 9m

When $x = -\frac{8}{3}$ at least one side of the triangle

has a negative measurement which is impossible. Thus there is only 1 triangle possible

95. Let w represent the width of window.

Then $l = w + 2$ represents the length of the window.

Since the area is 143 square feet, we have:

$$w(w+2) = 143$$

$$w^2 + 2w - 143 = 0$$

$$(w+13)(w-11) = 0$$

$$\cancel{w = -13} \quad \text{or} \quad w = 11$$

Discard the negative solution since width cannot be negative. The width of the rectangular window is 11 feet and the length is 13 feet.

96. Let w represent the width of window.

Then $l = w + 1$ represents the length of the window.

Since the area is 306 square centimeters, we have: $w(w+1) = 306$

$$w^2 + w - 306 = 0$$

$$(w+18)(w-17) = 0$$

$$\cancel{w = -18} \quad \text{or} \quad w = 17$$

Discard the negative solution since width cannot be negative. The width of the rectangular window is 17 centimeters and the length is 18 centimeters.

97. Let l represent the length of the rectangle.

Let w represent the width of the rectangle.

The perimeter is 26 meters and the area is 40

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square meters.

$$2l + 2w = 26$$

$$l + w = 13 \quad \text{so} \quad w = 13 - l$$

$$lw = 40$$

$$l(13 - l) = 40$$

$$13l - l^2 = 40$$

$$l^2 - 13l + 40 = 0$$

$$(l - 8)(l - 5) = 0$$

$$l = 8 \quad \text{or} \quad l = 5$$

$$w = 5 \quad w = 8$$

The dimensions are 5 meters by 8 meters.

98. Let r represent the radius of the circle. Since the field is a square with area 1250 square feet, the length of a side of the square is $\sqrt{1250} = 25\sqrt{2}$ feet. The length of the diagonal is $2r$.

Use the Pythagorean Theorem to solve for r :

$$(2r)^2 = (25\sqrt{2})^2 + (25\sqrt{2})^2$$

$$4r^2 = 1250 + 1250$$

$$4r^2 = 2500$$

$$r^2 = 625$$

$$r = 25$$

The shortest radius setting for the sprinkler is 25 feet.

99. Let x = length of side of original sheet in feet.

Length of box: $x - 2$ feet

Width of box: $x - 2$ feet

Height of box: 1 foot

$$V = l \cdot w \cdot h$$

$$4 = (x - 2)(x - 2)(1)$$

$$4 = x^2 - 4x + 4$$

$$0 = x^2 - 4x$$

$$0 = x(x - 4)$$

$$x = 0 \quad \text{or} \quad x = 4$$

Discard $x = 0$ since that is not a feasible length for the original sheet. Therefore, the original sheet should measure 4 feet on each side.

100. Let x = width of original sheet in feet.

Length of sheet: $2x$

Length of box: $2x - 2$ feet

Width of box: $x - 2$ feet

Height of box: 1 foot

$$V = l \cdot w \cdot h$$

$$4 = (2x - 2)(x - 2)(1)$$

$$4 = 2x^2 - 6x + 4$$

$$0 = 2x^2 - 6x$$

$$0 = x^2 - 3x$$

$$0 = x(x - 3)$$

$$x = 0 \quad \text{or} \quad x = 3$$

Discard $x = 0$ since that is not a feasible length for the original sheet. Therefore, the original sheet is 3 feet wide and 6 feet long.

101. a. When the ball strikes the ground, the distance from the ground will be 0.

Therefore, we solve

$$96 + 80t - 16t^2 = 0$$

$$-16t^2 + 80t + 96 = 0$$

$$t^2 - 5t - 6 = 0$$

$$(t - 6)(t + 1) = 0$$

$$t = 6 \quad \text{or} \quad t = -1$$

Discard the negative solution since the time of flight must be positive. The ball will strike the ground after 6 seconds.

- b. When the ball passes the top of the building, it will be 96 feet from the ground. Therefore, we solve

$$96 + 80t - 16t^2 = 96$$

$$-16t^2 + 80t = 0$$

$$t^2 - 5t = 0$$

$$t(t - 5) = 0$$

$$t = 0 \quad \text{or} \quad t = 5$$

The ball is at the top of the building at time $t = 0$ when it is thrown. It will pass the top of the building on the way down after 5 seconds.

102. a. To find when the object will be 15 meters above the ground, we solve

$$-4.9t^2 + 20t = 15$$

$$-4.9t^2 + 20t - 15 = 0$$

$$a = -4.9, \quad b = 20, \quad c = -15$$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-15)}}{2(-4.9)}$$

$$= \frac{-20 \pm \sqrt{106}}{-9.8} = \frac{20 \pm \sqrt{106}}{9.8}$$

$$t \approx 0.99 \quad \text{or} \quad t \approx 3.09$$

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The object will be 15 meters above the ground after about 0.99 seconds (on the way up) and about 3.09 seconds (on the way down).

- b. The object will strike the ground when the distance from the ground is 0. Therefore, we solve

$$-4.9t^2 + 20t = 0$$

$$t(-4.9t + 20) = 0$$

$$t = 0 \quad \text{or} \quad -4.9t + 20 = 0$$

$$-4.9t = -20$$

$$t \approx 4.08$$

The object will strike the ground after about 4.08 seconds.

c. $-4.9t^2 + 20t = 100$

$$-4.9t^2 + 20t - 100 = 0$$

$$a = -4.9, \quad b = 20, \quad c = -100$$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-100)}}{2(-4.9)}$$

$$= \frac{-20 \pm \sqrt{-1560}}{-9.8}$$

There is no real solution. The object never reaches a height of 100 meters.

103. Let x represent the number of centimeters the length and width should be reduced.

$12 - x$ = the new length, $7 - x$ = the new width.

The new volume is 90% of the old volume.

$$(12 - x)(7 - x)(3) = 0.9(12)(7)(3)$$

$$3x^2 - 57x + 252 = 226.8$$

$$3x^2 - 57x + 25.2 = 0$$

$$x^2 - 19x + 8.4 = 0$$

$$x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(8.4)}}{2(1)} = \frac{19 \pm \sqrt{327.4}}{2}$$

$$x \approx 0.45 \quad \text{or} \quad x \approx 18.55$$

Since 18.55 exceeds the dimensions, it is discarded. The dimensions of the new chocolate bar are: 11.55 cm by 6.55 cm by 3 cm.

104. Let x represent the number of centimeters the length and width should be reduced.
 $12 - x$ = the new length, $7 - x$ = the new width.
 The new volume is 80% of the old volume.

$$(12 - x)(7 - x)(3) = 0.8(12)(7)(3)$$

$$3x^2 - 57x + 252 = 201.6$$

$$3x^2 - 57x + 50.4 = 0$$

$$x^2 - 19x + 16.8 = 0$$

$$x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(16.8)}}{2(1)} = \frac{19 \pm \sqrt{293.8}}{2}$$

$$x \approx 0.93 \quad \text{or} \quad x \approx 18.07$$

Since 18.07 exceeds the dimensions, it is discarded. The dimensions of the new chocolate bar are: 11.07 cm by 6.07 cm by 3 cm.

105. Let x represent the width of the border measured in feet. The radius of the pool is 5 feet. Then $x + 5$ represents the radius of the circle, including both the pool and the border. The total area of the pool and border is

$$A_T = \pi(x + 5)^2.$$

The area of the pool is $A_P = \pi(5)^2 = 25\pi$.

The area of the border is

$$A_B = A_T - A_P = \pi(x + 5)^2 - 25\pi.$$

Since the concrete is 3 inches or 0.25 feet thick, the volume of the concrete in the border is

$$0.25A_B = 0.25(\pi(x + 5)^2 - 25\pi)$$

Solving the volume equation:

$$0.25(\pi(x + 5)^2 - 25\pi) = 27$$

$$\pi(x^2 + 10x + 25 - 25) = 108$$

$$\pi x^2 + 10\pi x - 108 = 0$$

$$x = \frac{-10\pi \pm \sqrt{(10\pi)^2 - 4(\pi)(-108)}}{2(\pi)}$$

$$= \frac{-31.42 \pm \sqrt{100\pi^2 + 432\pi}}{6.28}$$

$$x \approx 2.71 \quad \text{or} \quad x \approx -12.71$$

Discard the negative solution. The width of the border is roughly 2.71 feet.

106. Let x represent the width of the border measured in feet. The radius of the pool is 5 feet. Then $x + 5$ represents the radius of the circle, including both the pool and the border. The total area of the pool and border is

$$A_T = \pi(x + 5)^2.$$

The area of the pool is $A_P = \pi(5)^2 = 25\pi$.

The area of the border is

$$A_B = A_T - A_P = \pi(x + 5)^2 - 25\pi.$$

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Since the concrete is 4 inches $= \frac{1}{3}$ foot thick, the volume of the concrete in the border is

$$\frac{1}{3}A_B = \frac{1}{3}(\pi(x+5)^2 - 25\pi)$$

Solving the volume equation:

$$\frac{1}{3}(\pi(x+5)^2 - 25\pi) = 27$$

$$\pi(x^2 + 10x + 25 - 25) = 81$$

$$\begin{aligned} \pi x^2 + 10\pi x - 81 &= 0 \\ x &= \frac{-10\pi \pm \sqrt{(10\pi)^2 - 4(\pi)(-81)}}{2(\pi)} \\ &= \frac{-31.42 \pm \sqrt{100\pi^2 + 324\pi}}{6.28} \end{aligned}$$

$$x \approx 2.13 \text{ or } x \approx -12.13$$

Discard the negative solution. The width of the border is approximately 2.13 feet.

- 107.** Let x represent the width of the border measured in feet.

The total area is $A_T = (6+2x)(10+2x)$.

The area of the garden is $A_G = 6 \cdot 10 = 60$.

The area of the border is

$$A_B = A_T - A_G = (6+2x)(10+2x) - 60.$$

Since the concrete is 3 inches or 0.25 feet thick, the volume of the concrete in the border is

$$0.25A_B = 0.25((6+2x)(10+2x) - 60)$$

Solving the volume equation:

$$0.25((6+2x)(10+2x) - 60) = 27$$

$$60 + 32x + 4x^2 - 60 = 108$$

$$4x^2 + 32x - 108 = 0$$

$$x^2 + 8x - 27 = 0$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-27)}}{2(1)} = \frac{-8 \pm \sqrt{172}}{2}$$

$$x \approx 2.56 \text{ or } x \approx -10.56$$

Discard the negative solution. The width of the border is approximately 2.56 feet.

- 108.** Let x = the width and $2x$ = the length of the patio. The height is $\frac{1}{3}$ foot and the concrete available is $8(27) = 216$ cubic feet..

$$V = lwh = x(2x) \cdot \frac{1}{3} = 216$$

$$\frac{2}{3}x^2 = 216$$

$$x^2 = 324 \Rightarrow x = \pm 18$$

The dimensions of the patio are 18 feet by 36 feet.

- 109.** Let x = the length of a 12.9-inch iPad Pro in a 16:9 format.

Then $\frac{9}{16}x$ = the width of the iPad. The diagonal of the 12.9-inch iPad is 9.7 inches, so by the Pythagorean theorem we have:

$$x^2 + \left(\frac{9}{16}x\right)^2 = 12.9^2$$

$$x^2 + \frac{81}{256}x^2 = 166.41$$

$$256\left(x^2 + \frac{81}{256}x^2\right) = 256(166.41)$$

$$256x^2 + 81x^2 = 42600.96$$

$$337x^2 = 42600.96 \Rightarrow x^2 = \frac{42600.96}{337}$$

$$x = \pm \sqrt{\frac{42600.96}{337}} \approx \pm 11.24$$

Since the length cannot be negative, the length of the iPad is $\sqrt{\frac{42600.96}{337}}$ inches and the width is

$$\frac{9}{16}\sqrt{\frac{42600.96}{337}} \approx 6.32 \text{ inches. Thus, the area of the}$$

iPad is $\sqrt{\frac{42600.96}{337}} \cdot \frac{9}{16}\sqrt{\frac{42600.96}{337}} = 71.11$ square inches.

Let y = the length of a 14.4-inch 3:2 format

Microsoft Surface Pro. Then $\frac{2}{3}y$ = the width of

the Surface Pro. The diagonal of a 14.4-inch Surface Pro is 14.4 inches, so by the Pythagorean theorem we have:

$$y^2 + \left(\frac{2}{3}y\right)^2 = 14.4^2$$

$$y^2 + \frac{4}{9}y^2 = 207.36$$

$$9\left(y^2 + \frac{4}{9}y^2\right) = 9(207.36)$$

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$$9y^2 + 4y^2 = 1866.24$$

$$13y^2 = 1866.24$$

$$y^2 = \frac{1866.24}{13}$$

$$y = \pm \sqrt{\frac{1866.24}{13}} \approx \pm 11.98$$

Since the length cannot be negative, the length of the Surface Pro is 11.98 inches and the width is

$$\frac{2}{3} \sqrt{\frac{1866.24}{13}} \approx 7.99 \text{ inches. Thus, the area of the}$$

14.4-inch 3:2 format Surface Pro is

$$\sqrt{\frac{1866.24}{13}} \cdot \frac{2}{3} \sqrt{\frac{1866.24}{13}}$$

≈ 95.7 square inches.

The Surface Pro format has the larger screen since its area is larger.

- 110.** Let x = the length of a 8.3-inch iPad Mini in a 4:3 format.

Then $\frac{3}{4}x$ = the width of the iPad. The diagonal

of the 8.3-inch iPad is 8.3 inches, so by the Pythagorean theorem we have:

$$x^2 + \left(\frac{3}{4}x\right)^2 = 8.3^2$$

$$x^2 + \frac{9}{16}x^2 = 68.89$$

$$16\left(x^2 + \frac{9}{16}x^2\right) = 16(68.89)$$

$$16x^2 + 9x^2 = 1102.24$$

$$25x^2 = 1102.24$$

$$x^2 = 44.0896$$

$$x = \pm \sqrt{44.0896} = \pm 6.64$$

Since the length cannot be negative, the length of the iPad is 6.64 inches and the width is

$$\frac{3}{4}(6.64) = 4.98 \text{ inches. Thus, the area of the}$$

iPad is $(6.64)(4.98) = 33.1$ square inches.

Let y = the length of a 8-inch 16:10 format

Amazon Fire HD 8™. Then $\frac{10}{16}y$ = the width of

the Fire. The diagonal of a 8-inch Fire is 8 inches, so by the Pythagorean theorem we have:

$$y^2 + \left(\frac{10}{16}y\right)^2 = 8^2$$

$$y^2 + \frac{100}{256}y^2 = 64$$

$$256\left(y^2 + \frac{100}{256}y^2\right) = 256(64)$$

$$256y^2 + 100y^2 = 16384$$

$$356y^2 = 16384$$

$$y^2 = \frac{16384}{356}$$

$$y = \pm \sqrt{\frac{16384}{356}} \approx \pm 6.78399$$

Since the length cannot be negative, the length of

the Fire is $\sqrt{\frac{16384}{356}} \approx 6.78399$ inches and the

width is $\frac{10}{16} \sqrt{\frac{16384}{356}} \approx 4.240$ inches. Thus, the area

of the Amazon Fire is

$$(6.78399)(4.240) \approx 28.8 \text{ square inches.}$$

The iPad Mini™ 4:3 format has the larger screen since its area is larger.

- 111.** Let h be 1.1. Then

$$1.1 = -0.00025x^2 + 0.04x$$

$$0 = -0.00025x^2 + 0.04x - 1.1$$

$$x = \frac{-0.04 \pm \sqrt{(0.04)^2 - 4(-0.00025)(-1.1)}}{2(-0.00025)}$$

$$= 35.3 \text{ ft or } 124.7 \text{ ft}$$

124.7 ft does not make sense in the context of the problem, so the answer is 35.3 ft.

- 112.** Since d is expressed in billions, we will set $d = 12$ and solve for x using the Quadratic Formula.

$$d = 0.33x^2 + 0.26x + 2.29$$

$$12 = 0.33x^2 + 0.26x + 2.29$$

$$0 = 0.33x^2 + 0.26x - 9.71$$

$$x = \frac{-0.26 \pm \sqrt{(0.26)^2 - 4(0.33)(-9.71)}}{2(0.33)}$$

$$= \frac{-0.26 \pm \sqrt{13.0772}}{0.66}$$

$$x \approx 5.085 \text{ or } x \approx -5.873$$

So the nearest year when the earnings were 12 occurred about 5 years after 2018 or 2023. The negative value -5.873 has no meaning.

113. We will set $g = 2.97$ and solve for h using the Quadratic Formula.

$$g = -0.0006x^2 + 0.015x + 3.04$$

$$2.97 = -0.0006x^2 + 0.015x + 3.04$$

$$0 = -0.0006x^2 + 0.015x + 0.07$$

$$x = \frac{-0.015 \pm \sqrt{(0.015)^2 - 4(-0.0006)(0.07)}}{2(-0.0006)}$$

$$= \frac{-0.015 \pm \sqrt{0.000393}}{-0.0012}$$

$$x \approx 29 \text{ or } x \approx -4.02$$

So the estimated numbers of hours worked by a student with a GPA of 2.97 is 29 hours. The value -4.02 has no meaning since it is negative.

114. Let x be the numbers of members in the fraternity and s be the share paid by each member. Then $s = \frac{1470}{x}$. If there are 7 members who cannot contribute then the share goes up by \$5. So we have the following equation:

$$s + 5 = \frac{1470}{x-7} \text{ or } (s+5)(x-7) = 1470$$

Solving these two equations together:

$$(s+5)(x-7) = 1470 \text{ and } s = \frac{1470}{x}$$

$$\left(\frac{1470}{x} + 5\right)(x-7) = 1470$$

$$1470 - \frac{10290}{x} + 5x - 35 = 1470$$

$$5x - \frac{10290}{x} - 35 = 0$$

$$5x^2 - 35x - 10290 = 0$$

$$5x^2 - 35x - 10290 = 0$$

$$x^2 - 7x - 2058 = 0$$

$$(x+42)(x-49) = 0$$

$$x = -42 \text{ or } x = 49$$

Since x is the number of members, it must be positive so the number of members is 49.

115. Let a be the age the individual is able to start saving money. Then we need to find where the models are equal. Solving these two equations together:

$$-25a^2 + 2400a - 30700 = 160a + 7840$$

$$25a^2 - 2240a + 38540 = 0$$

$$a = \frac{2240 \pm \sqrt{(2240)^2 - 4(25)(38540)}}{2(25)}$$

$$a = \frac{2240 \pm \sqrt{1163600}}{50}$$

$$a = \frac{2240 \pm 1078.7}{50}$$

$$a = \frac{2240 + 1078.7}{50} \text{ or } a = \frac{2240 - 1078.7}{50}$$

$$a = 66.4 \text{ or } a = 23.2$$

Since x is the age to start saving, it makes sense that the answer is approximate at age 23.

116. We will set the equation equal to 10 and solve:

$$0.003x^2 - 0.034x + 8.086 = 10$$

$$0.003x^2 - 0.034x - 1.914 = 0$$

$$x = \frac{0.034 \pm \sqrt{(-0.034)^2 - 4(0.003)(-1.914)}}{2(0.003)}$$

$$x = \frac{0.034 \pm \sqrt{0.024124}}{0.006}$$

$$x = \frac{0.034 \pm 0.15532}{0.006}$$

$$x = \frac{0.034 - 0.15532}{0.006} \text{ or } x = \frac{0.034 + 0.15532}{0.006}$$

$$x = -20.22 \text{ or } x = 31.55$$

The percentage will reach 10% approximately 32 years after 1960 which is 1992.

117. $\frac{1}{2}n(n+1) = 703$

$$n(n+1) = 1406$$

$$n^2 + n - 1406 = 0$$

$$(n-37)(n+38) = 0$$

$$n = 37 \text{ or } n = -38$$

Since the number of consecutive integers cannot be negative, we discard the negative value. We must add 37 consecutive integers, beginning at 1, in order to get a sum of 703.

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118. $\frac{1}{2}n(n-3) = 65$

$$n(n-3) = 130$$

$$n^2 - 3n - 130 = 0$$

$$(n-13)(n+10) = 0$$

$$n = 13 \text{ or } n = -10$$

Since the number of sides cannot be negative, we discard the negative value. A polygon with 65 diagonals will have 13 sides.

$$\frac{1}{2}n(n-3) = 80$$

$$n(n-3) = 160$$

$$n^2 - 3n - 160 = 0$$

$$a = 1, b = -3, c = -160$$

$$n = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-160)}}{2(1)} = \frac{3 \pm \sqrt{649}}{2}$$

Neither solution is an integer, so there is no polygon that has 80 diagonals.

119. The roots of a quadratic equation are

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_1 + x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} + \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b - \sqrt{b^2 - 4ac} - b + \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2b}{2a}$$

$$= -\frac{b}{a}$$

120. The roots of a quadratic equation are

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_1 \cdot x_2 = \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} \right) \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} \right)$$

$$= \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{(2a)^2} = \frac{b^2 - b^2 + 4ac}{4a^2}$$

$$= \frac{4ac}{4a^2}$$

$$= \frac{c}{a}$$

121. In order to have one repeated solution, we need the discriminant to be 0.

$$b^2 - 4ac = 0$$

$$1^2 - 4(k)(k) = 0$$

$$1 - 4k^2 = 0$$

$$4k^2 = 1$$

$$k^2 = \frac{1}{4}$$

$$k = \pm \sqrt{\frac{1}{4}}$$

$$k = \frac{1}{2} \text{ or } k = -\frac{1}{2}$$

122. In order to have one repeated solution, we need the discriminant to be 0.

$$b^2 - 4ac = 0$$

$$(-k)^2 - 4(1)(4) = 0$$

$$k^2 - 16 = 0$$

$$(k-4)(k+4) = 0$$

$$k = 4 \text{ or } k = -4$$

123. For $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^2 - bx + c = 0$:

$$x = \frac{b \pm \sqrt{(-b)^2 - 4ac}}{2a} = -\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

124. For $ax^2 + bx + c = 0$:

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

For $cx^2 + bx + a = 0$:

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$$\begin{aligned} x_1^* &= \frac{-b - \sqrt{b^2 - 4(c)(a)}}{2c} = \frac{-b - \sqrt{b^2 - 4ac}}{2c} \\ &= \frac{-b - \sqrt{b^2 - 4ac}}{2c} \cdot \frac{-b + \sqrt{b^2 - 4ac}}{-b + \sqrt{b^2 - 4ac}} \\ &= \frac{b^2 - (b^2 - 4ac)}{2c(-b + \sqrt{b^2 - 4ac})} = \frac{4ac}{2c(-b + \sqrt{b^2 - 4ac})} \\ &= \frac{2a}{-b + \sqrt{b^2 - 4ac}} \\ &= \frac{1}{x_2} \end{aligned}$$

and

$$\begin{aligned} x_2^* &= \frac{-b + \sqrt{b^2 - 4(c)(a)}}{2c} = \frac{-b + \sqrt{b^2 - 4ac}}{2c} \\ &= \frac{-b + \sqrt{b^2 - 4ac}}{2c} \cdot \frac{-b - \sqrt{b^2 - 4ac}}{-b - \sqrt{b^2 - 4ac}} \\ &= \frac{b^2 - (b^2 - 4ac)}{2c(-b - \sqrt{b^2 - 4ac})} = \frac{4ac}{2c(-b - \sqrt{b^2 - 4ac})} \\ &= \frac{2a}{-b - \sqrt{b^2 - 4ac}} \\ &= \frac{1}{x_1} \end{aligned}$$

125. If x = original width and y = original length, then

$xy = 1$ or $x = \frac{1}{y}$. The ratio of side lengths is

$\frac{x}{y} = \frac{1}{y^2}$. Folding along the longest side results

in sides of length $x = \frac{1}{y}$ and $\frac{y}{2}$ whose ratio is

$$\frac{\frac{y}{2}}{\frac{1}{y}} = \frac{y^2}{2}$$

Equating the ratios gives

$$\frac{1}{y^2} = \frac{y^2}{2}$$

$$y^4 = 2$$

$$y = \sqrt[4]{2} \text{ m}$$

So

$$x = \frac{1}{\sqrt[4]{2}} = \frac{\sqrt[4]{8}}{2} \text{ m.}$$

126. a. $x^2 = 9$ and $x = 3$ are not equivalent because they do not have the same solution set. In the first equation we can also have $x = -3$.
- b. $x = \sqrt{9}$ and $x = 3$ are equivalent because $\sqrt{9} = 3$.
- c. $(x-1)(x-2) = (x-1)^2$ and $x-2 = x-1$ are not equivalent because they do not have the same solution set.
The first equation has the solution set $\{1\}$ while the second equation has no solutions.
127. Answers will vary. Methods may include the quadratic formula, completing the square, graphing, etc.
128. Answers will vary. Knowing the discriminant allows us to know how many real solutions the equation will have.
129. Answers will vary. One possibility:
Two distinct: $x^2 - 3x - 18 = 0$
One repeated: $x^2 - 14x + 49 = 0$
No real: $x^2 + x + 4 = 0$
130. Answers will vary.

Section 1.4

- True; for example, $i^2 = -1$.
- $(2+i)(2-i) = 4 - i^2 = 4 - (-1) = 5$
- False; a quadratic equation in the complex number system either has two real solutions, one double root solution, or two complex solutions.
- real; imaginary; imaginary unit
- False; the conjugate of $2+5i$ is $2-5i$.
- True; the set of real numbers is a subset of the set of complex numbers.
- False; if $2-3i$ is a solution of a quadratic equation with real coefficients, then its conjugate, $2+3i$, is also a solution.
- b

Chapter 1: Graphs, Equations, and Inequalities

9. a

10. c

$$11. (2-3i)+(6+8i)=(2+6)+(-3+8)i=8+5i$$

$$12. (4+5i)+(-8+2i)=(4+(-8))+(5+2)i \\ =-4+7i$$

$$13. (-3+2i)-(4-4i)=(-3-4)+(2-(-4))i \\ =-7+6i$$

$$14. (3-4i)-(-3-4i)=(3-(-3))+(-4-(-4))i \\ =6+0i=6$$

$$15. (2-5i)-(8+6i)=(2-8)+(-5-6)i \\ =-6-11i$$

$$16. (-8+4i)-(2-2i)=(-8-2)+(4-(-2))i \\ =-10+6i$$

$$17. 3(2-6i)=6-18i$$

$$18. -4(2+8i)=-8-32i$$

$$19. -3i(7+6i)=-21i-18i^2=-21i-18(-1) \\ =18-21i$$

$$20. 3i(-3+4i)=-9i+12i^2=-9i+12(-1)=-12-9i$$

$$21. (3-4i)(2+i)=6+3i-8i-4i^2 \\ =6-5i-4(-1) \\ =10-5i$$

$$22. (5+3i)(2-i)=10-5i+6i-3i^2 \\ =10+i-3(-1) \\ =13+i$$

$$23. (-5-i)(-5+i)=25-5i+5i-i^2 \\ =25-(-1) \\ =26$$

$$24. (-3+i)(3+i)=-9-3i+3i+i^2 \\ =-9+(-1) \\ =-10$$

$$25. \frac{10}{3-4i} = \frac{10}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{30+40i}{9+12i-12i-16i^2} \\ = \frac{30+40i}{9-16(-1)} = \frac{30+40i}{25} \\ = \frac{30}{25} + \frac{40}{25}i \\ = \frac{6}{5} + \frac{8}{5}i$$

$$26. \frac{13}{5-12i} = \frac{13}{5-12i} \cdot \frac{5+12i}{5+12i} \\ = \frac{65+156i}{25+60i-60i-144i^2} \\ = \frac{65+156i}{25-144(-1)} = \frac{65+156i}{169} \\ = \frac{65}{169} + \frac{156}{169}i \\ = \frac{5}{13} + \frac{12}{13}i$$

$$27. \frac{2+i}{i} = \frac{2+i}{i} \cdot \frac{-i}{-i} = \frac{-2i-i^2}{-i^2} \\ = \frac{-2i-(-1)}{-(-1)} = \frac{1-2i}{1} = 1-2i$$

$$28. \frac{2-i}{-2i} = \frac{2-i}{-2i} \cdot \frac{i}{i} = \frac{2i-i^2}{-2i^2} \\ = \frac{2i-(-1)}{-2(-1)} = \frac{1+2i}{2} = \frac{1}{2} + i$$

$$29. \frac{6-i}{1+i} = \frac{6-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{6-6i-i+i^2}{1-i+i-i^2} \\ = \frac{6-7i+(-1)}{1-(-1)} = \frac{5-7i}{2} = \frac{5}{2} - \frac{7}{2}i$$

$$30. \frac{2+3i}{1-i} = \frac{2+3i}{1-i} \cdot \frac{1+i}{1+i} = \frac{2+2i+3i+3i^2}{1+i-i-i^2} \\ = \frac{2+5i+3(-1)}{1-(-1)} = \frac{-1+5i}{2} = -\frac{1}{2} + \frac{5}{2}i$$

$$31. \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 = \frac{1}{4} + 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right) + \frac{3}{4}i^2 \\ = \frac{1}{4} + \frac{\sqrt{3}}{2}i + \frac{3}{4}(-1) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

Section 1.4: Complex Numbers; Quadratic Equations in the Complex Number System

$$\begin{aligned} 32. \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^2 &= \frac{3}{4} - 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}i\right) + \frac{1}{4}i^2 \\ &= \frac{3}{4} - \frac{\sqrt{3}}{2}i + \frac{1}{4}(-1) = \frac{1}{2} - \frac{\sqrt{3}}{2}i \end{aligned}$$

$$33. (1+i)^2 = 1+2i+i^2 = 1+2i+(-1) = 2i$$

$$34. (1-i)^2 = 1-2i+i^2 = 1-2i+(-1) = -2i$$

$$35. i^{23} = i^{22+1} = i^{22} \cdot i = (i^2)^{11} \cdot i = (-1)^{11}i = -i$$

$$36. i^{14} = (i^2)^7 = (-1)^7 = -1$$

$$\begin{aligned} 37. i^{-20} &= \frac{1}{i^{20}} = \frac{1}{i^{20}} = \frac{1}{(i^2)^{10}} \\ &= \frac{1}{(-1)^{10}} = \frac{1}{1} = 1 \end{aligned}$$

$$\begin{aligned} 38. i^{-23} &= \frac{1}{i^{23}} = \frac{1}{i^{22+1}} = \frac{1}{i^{22} \cdot i} = \frac{1}{(i^2)^{11} \cdot i} \\ &= \frac{1}{(-1)^{11}i} = \frac{1}{-i} = \frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = i \end{aligned}$$

$$39. i^6 - 5 = (i^2)^3 - 5 = (-1)^3 - 5 = -1 - 5 = -6$$

$$40. 4+i^3 = 4+i^2 \cdot i = 4+(-1)i = 4-i$$

$$\begin{aligned} 41. 6i^3 - 4i^5 &= i^3(6-4i^2) \\ &= i^2 \cdot i(6-4(-1)) = -1 \cdot i(10) = -10i \end{aligned}$$

$$\begin{aligned} 42. 4i^3 - 2i^2 + 1 &= 4i^2 \cdot i - 2i^2 + 1 \\ &= 4(-1)i - 2(-1) + 1 \\ &= -4i + 2 + 1 \\ &= 3 - 4i \end{aligned}$$

$$\begin{aligned} 43. (1+i)^3 &= (1+i)(1+i)(1+i) = (1+2i+i^2)(1+i) \\ &= (1+2i-1)(1+i) = 2i(1+i) \\ &= 2i+2i^2 = 2i+2(-1) \\ &= -2+2i \end{aligned}$$

$$44. (3i)^4 + 1 = 81i^4 + 1 = 81(1) + 1 = 82$$

$$45. i^7(1+i^2) = i^7(1+(-1)) = i^7(0) = 0$$

$$46. 2i^4(1+i^2) = 2(1)(1+(-1)) = 2(0) = 0$$

$$\begin{aligned} 47. i^8 + i^6 - i^4 - i^2 &= (i^2)^4 + (i^2)^3 + (i^2)^2 + i^2 \\ &= (-1)^4 + (-1)^3 + (-1)^2 - 1 \\ &= 1 - 1 + 1 - 1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} 48. i^7 + i^5 + i^3 + i &= (i^2)^3 \cdot i + (i^2)^2 \cdot i + i^2 \cdot i + i \\ &= (-1)^3 \cdot i + (-1)^2 \cdot i + (-1) \cdot i + i \\ &= -i + i - i + i \\ &= 0 \end{aligned}$$

$$49. \sqrt{-4} = 2i$$

$$50. \sqrt{-9} = 3i$$

$$51. \sqrt{-25} = 5i$$

$$52. \sqrt{-64} = 8i$$

$$53. \sqrt{-12} = i\sqrt{4 \cdot 3} = 2\sqrt{3}i$$

$$54. \sqrt{-18} = i\sqrt{9 \cdot 2} = 3\sqrt{2}i$$

$$55. \sqrt{-200} = i\sqrt{100 \cdot 2} = 10\sqrt{2}i$$

$$56. \sqrt{-45} = i\sqrt{9 \cdot 5} = 3\sqrt{5}i$$

$$\begin{aligned} 57. \sqrt{(3+4i)(4i-3)} &= \sqrt{12i-9+16i^2-12i} \\ &= \sqrt{-9+16(-1)} \\ &= \sqrt{-25} \\ &= 5i \end{aligned}$$

$$\begin{aligned} 58. \sqrt{(4+3i)(3i-4)} &= \sqrt{12i-16+9i^2-12i} \\ &= \sqrt{-16+9(-1)} \\ &= \sqrt{-25} \\ &= 5i \end{aligned}$$

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59. $x^2 + 4 = 0$

$$x^2 = -4$$

$$x = \pm\sqrt{-4}$$

$$x = \pm 2i$$

The solution set is $\{-2i, 2i\}$.

60. $x^2 - 4 = 0$

$$(x+2)(x-2) = 0$$

$$x = -2 \text{ or } x = 2$$

The solution set is $\{-2, 2\}$.

61. $x^2 - 16 = 0$

$$(x+4)(x-4) = 0$$

$$x = -4 \text{ or } x = 4$$

The solution set is $\{-4, 4\}$.

62. $x^2 + 25 = 0$

$$x^2 = -25$$

$$x = \pm\sqrt{-25} = \pm 5i$$

The solution set is $\{-5i, 5i\}$.

63. $x^2 - 6x + 13 = 0$

$$a = 1, b = -6, c = 13,$$

$$b^2 - 4ac = (-6)^2 - 4(1)(13) = 36 - 52 = -16$$

$$x = \frac{-(-6) \pm \sqrt{-16}}{2(1)} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

The solution set is $\{3 - 2i, 3 + 2i\}$.

64. $x^2 + 4x + 8 = 0$

$$a = 1, b = 4, c = 8$$

$$b^2 - 4ac = 4^2 - 4(1)(8) = 16 - 32 = -16$$

$$x = \frac{-4 \pm \sqrt{-16}}{2(1)} = \frac{-4 \pm 4i}{2} = -2 \pm 2i$$

The solution set is $\{-2 - 2i, -2 + 2i\}$.

65. $x^2 - 6x + 10 = 0$

$$a = 1, b = -6, c = 10$$

$$b^2 - 4ac = (-6)^2 - 4(1)(10) = 36 - 40 = -4$$

$$x = \frac{-(-6) \pm \sqrt{-4}}{2(1)} = \frac{6 \pm 2i}{2} = 3 \pm i$$

The solution set is $\{3 - i, 3 + i\}$.

66. $x^2 - 2x + 5 = 0$

$$a = 1, b = -2, c = 5$$

$$b^2 - 4ac = (-2)^2 - 4(1)(5) = 4 - 20 = -16$$

$$x = \frac{-(-2) \pm \sqrt{-16}}{2(1)} = \frac{2 \pm 4i}{2} = 1 \pm 2i$$

The solution set is $\{1 - 2i, 1 + 2i\}$.

67. $25x^2 - 10x + 2 = 0$

$$a = 25, b = -10, c = 2$$

$$b^2 - 4ac = (-10)^2 - 4(25)(2) = 100 - 200 = -100$$

$$x = \frac{-(-10) \pm \sqrt{-100}}{2(25)} = \frac{10 \pm 10i}{50} = \frac{1}{5} \pm \frac{1}{5}i$$

The solution set is $\left\{ \frac{1}{5} - \frac{1}{5}i, \frac{1}{5} + \frac{1}{5}i \right\}$.

68. $10x^2 + 6x + 1 = 0$

$$a = 10, b = 6, c = 1$$

$$b^2 - 4ac = 6^2 - 4(10)(1) = 36 - 40 = -4$$

$$x = \frac{-6 \pm \sqrt{-4}}{2(10)} = \frac{-6 \pm 2i}{20} = -\frac{3}{10} \pm \frac{1}{10}i$$

The solution set is $\left\{ -\frac{3}{10} - \frac{1}{10}i, -\frac{3}{10} + \frac{1}{10}i \right\}$.

69. $5x^2 + 1 = 2x$

$$5x^2 - 2x + 1 = 0$$

$$a = 5, b = -2, c = 1$$

$$b^2 - 4ac = (-2)^2 - 4(5)(1) = 4 - 20 = -16$$

$$x = \frac{-(-2) \pm \sqrt{-16}}{2(5)} = \frac{2 \pm 4i}{10} = \frac{1}{5} \pm \frac{2}{5}i$$

The solution set is $\left\{ \frac{1}{5} - \frac{2}{5}i, \frac{1}{5} + \frac{2}{5}i \right\}$.

70. $13x^2 + 1 = 6x$

$$13x^2 - 6x + 1 = 0$$

$$a = 13, b = -6, c = 1$$

$$b^2 - 4ac = (-6)^2 - 4(13)(1) = 36 - 52 = -16$$

$$x = \frac{-(-6) \pm \sqrt{-16}}{2(13)} = \frac{6 \pm 4i}{26} = \frac{3}{13} \pm \frac{2}{13}i$$

The solution set is $\left\{ \frac{3}{13} - \frac{2}{13}i, \frac{3}{13} + \frac{2}{13}i \right\}$.

Section 1.4: Complex Numbers; Quadratic Equations in the Complex Number System

71. $x^2 + x + 1 = 0$

$a = 1, b = 1, c = 1,$

$b^2 - 4ac = 1^2 - 4(1)(1) = 1 - 4 = -3$

$x = \frac{-1 \pm \sqrt{-3}}{2(1)} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

The solution set is $\left\{-\frac{1}{2} - \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i\right\}.$

72. $x^2 - x + 1 = 0$

$a = 1, b = -1, c = 1$

$b^2 - 4ac = (-1)^2 - 4(1)(1) = 1 - 4 = -3$

$x = \frac{-(-1) \pm \sqrt{-3}}{2(1)} = \frac{1 \pm \sqrt{3}i}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

The solution set is $\left\{\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} + \frac{\sqrt{3}}{2}i\right\}.$

73. $x^3 - 64 = 0$

$(x - 4)(x^2 + 4x + 16) = 0$

$x - 4 = 0 \Rightarrow x = 4$

or $x^2 + 4x + 16 = 0$

$a = 1, b = 4, c = 16$

$b^2 - 4ac = 4^2 - 4(1)(16) = 16 - 64 = -48$

$x = \frac{-4 \pm \sqrt{-48}}{2(1)} = \frac{-4 \pm 4\sqrt{3}i}{2} = -2 \pm 2\sqrt{3}i$

The solution set is $\{4, -2 - 2\sqrt{3}i, -2 + 2\sqrt{3}i\}.$

74. $x^3 + 27 = 0$

$(x + 3)(x^2 - 3x + 9) = 0$

$x + 3 = 0 \Rightarrow x = -3$

or $x^2 - 3x + 9 = 0$

$a = 1, b = -3, c = 9$

$b^2 - 4ac = (-3)^2 - 4(1)(9) = 9 - 36 = -27$

$x = \frac{-(-3) \pm \sqrt{-27}}{2(1)} = \frac{3 \pm 3\sqrt{3}i}{2} = \frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$

The solution set is $\left\{-3, \frac{3}{2} - \frac{3\sqrt{3}}{2}i, \frac{3}{2} + \frac{3\sqrt{3}}{2}i\right\}.$

75. $x^4 = 16$

$x^4 - 16 = 0$

$(x^2 - 4)(x^2 + 4) = 0$

$(x - 2)(x + 2)(x^2 + 4) = 0$

$x - 2 = 0$ or $x + 2 = 0$ or $x^2 + 4 = 0$

$x = 2$ or $x = -2$ or $x^2 = -4$

$x = 2$ or $x = -2$ or $x = \pm\sqrt{-4} = \pm 2i$

The solution set is $\{-2, 2, -2i, 2i\}.$

76. $x^4 = 1$

$x^4 - 1 = 0$

$(x^2 - 1)(x^2 + 1) = 0$

$(x - 1)(x + 1)(x^2 + 1) = 0$

$x - 1 = 0$ or $x + 1 = 0$ or $x^2 + 1 = 0$

$x = 1$ or $x = -1$ or $x^2 = -1$

$x = 1$ or $x = -1$ or $x = \pm\sqrt{-1} = \pm i$

The solution set is $\{-1, 1, -i, i\}.$

77. $x^4 + 13x^2 + 36 = 0$

$(x^2 + 9)(x^2 + 4) = 0$

$x^2 + 9 = 0$ or $x^2 + 4 = 0$

$x^2 = -9$ or $x^2 = -4$

$x = \pm\sqrt{-9}$ or $x = \pm\sqrt{-4}$

$x = \pm 3i$ or $x = \pm 2i$

The solution set is $\{-3i, 3i, -2i, 2i\}.$

78. $x^4 + 3x^2 - 4 = 0$

$(x^2 - 1)(x^2 + 4) = 0$

$(x - 1)(x + 1)(x^2 + 4) = 0$

$x - 1 = 0$ or $x + 1 = 0$ or $x^2 + 4 = 0$

$x = 1$ or $x = -1$ or $x^2 = -4$

$x = 1$ or $x = -1$ or $x = \pm\sqrt{-4} = \pm 2i$

The solution set is $\{-1, 1, -2i, 2i\}.$

79. $3x^2 - 3x + 4 = 0$

$a = 3, b = -3, c = 4$

$b^2 - 4ac = (-3)^2 - 4(3)(4) = 9 - 48 = -39$

The equation has two complex solutions that are conjugates of each other.

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80. $2x^2 - 4x + 1 = 0$

$a = 2, b = -4, c = 1$

$b^2 - 4ac = (-4)^2 - 4(2)(1) = 16 - 8 = 8$

The equation has two unequal real number solutions.

81. $2x^2 + 3x = 4$

$2x^2 + 3x - 4 = 0$

$a = 2, b = 3, c = -4$

$b^2 - 4ac = 3^2 - 4(2)(-4) = 9 + 32 = 41$

The equation has two unequal real solutions.

82. $x^2 + 6 = 2x$

$x^2 - 2x + 6 = 0$

$a = 1, b = -2, c = 6$

$b^2 - 4ac = (-2)^2 - 4(1)(6) = 4 - 24 = -20$

The equation has two complex solutions that are conjugates of each other.

83. $9x^2 - 12x + 4 = 0$

$a = 9, b = -12, c = 4$

$b^2 - 4ac = (-12)^2 - 4(9)(4) = 144 - 144 = 0$

The equation has a repeated real solution.

84. $4x^2 + 12x + 9 = 0$

$a = 4, b = 12, c = 9$

$b^2 - 4ac = 12^2 - 4(4)(9) = 144 - 144 = 0$

The equation has a repeated real solution.

85. The other solution is $\overline{2+3i} = 2-3i$.

86. The other solution is $\overline{4-i} = 4+i$.

87. $z + \bar{z} = 3-4i + \overline{3-4i} = 3-4i + 3+4i = 6$

88. $w - \bar{w} = 8+3i - \overline{(8+3i)}$
 $= 8+3i - (8-3i)$
 $= 8+3i - 8+3i$
 $= 0+6i = 6i$

89. $z \cdot \bar{z} = (3-4i)\overline{(3-4i)}$
 $= (3-4i)(3+4i)$
 $= 9+12i-12i-16i^2$
 $= 9-16(-1) = 25$

90. $\overline{z-w} = \overline{3-4i-(8+3i)}$
 $= \overline{3-4i-8-3i}$
 $= \overline{-5-7i}$
 $= -5+7i$

91. $Z = \frac{V}{I} = \frac{18+i}{3-4i} = \frac{18+i}{3-4i} \cdot \frac{3+4i}{3+4i}$
 $= \frac{54+72i+3i+4i^2}{9+12i-12i-16i^2} = \frac{54+75i-4}{9+16}$
 $= \frac{50+75i}{25} = 2+3i$

The impedance is $2+3i$ ohms.

92. $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{2+i} + \frac{1}{4-3i} = \frac{(4-3i)+(2+i)}{(2+i)(4-3i)}$
 $= \frac{6-2i}{8-6i+4i-3i^2} = \frac{6-2i}{8-2i+3} = \frac{6-2i}{11-2i}$

So, $Z = \frac{11-2i}{6-2i} = \frac{11-2i}{6-2i} \cdot \frac{6+2i}{6+2i}$
 $= \frac{66+22i-12i-4i^2}{36+12i-12i-4i^2} = \frac{66+10i+4}{36+4}$
 $= \frac{70+10i}{40} = \frac{7}{4} + \frac{1}{4}i$

The total impedance is $\frac{7}{4} + \frac{1}{4}i$ ohms.

93. $z + \bar{z} = (a+bi) + \overline{(a+bi)}$
 $= a+bi + a-bi$
 $= 2a$

$z - \bar{z} = a+bi - \overline{(a+bi)}$
 $= a+bi - (a-bi)$
 $= a+bi - a+bi$
 $= 2bi$

94. $\overline{\overline{z}} = \overline{a+bi} = \overline{a-bi} = a+bi = z$

95. $\overline{z+w} = \overline{(a+bi)+(c+di)}$
 $= \overline{(a+c)+(b+d)i}$
 $= (a+c) - (b+d)i$
 $= (a-bi) + (c-di)$
 $= \overline{a+bi} + \overline{c+di}$
 $= \bar{z} + \bar{w}$

Section 1.5: Radical Equations; Equations Quadratic in Form; Absolute Value Equations; Factorable Equations

$$\begin{aligned}
 96. \quad \overline{z \cdot w} &= \overline{(a+bi) \cdot (c+di)} \\
 &= \overline{ac + adi + bci + bdi^2} \\
 &= \overline{(ac - bd) + (ad + bc)i} \\
 &= (ac - bd) - (ad + bc)i \\
 \overline{z} \cdot \overline{w} &= \overline{a+bi} \cdot \overline{c+di} \\
 &= (a-bi)(c-di) \\
 &= ac - adi - bci + bdi^2 \\
 &= (ac - bd) - (ad + bc)i
 \end{aligned}$$

Section 1.5

1. True

$$2. \quad (\sqrt[3]{x})^3 = x$$

$$3. \quad 2x^2 - 7x - 4 = (x-4)(2x+1)$$

$$\begin{aligned}
 4. \quad x^3 + 4x^2 - 9x - 36 &= x^2(x+4) - 9(x+4) \\
 &= (x+4)(x^2 - 9) \\
 &= (x+4)(x-3)(x+3)
 \end{aligned}$$

5. The distance from the origin to -4 on a real number line is 4 units.

6. False; you can also use the Quadratic Formula or completing the square.

7. quadratic in form

8. True

9. a

10. c

$$11. \quad \sqrt{y+6} = 4$$

$$(\sqrt{y+6})^2 = 4^2$$

$$y+6 = 16 \rightarrow y = 10$$

$$\text{Check: } \sqrt{22+6} = \sqrt{28} = 5$$

The solution set is $\{22\}$.

$$12. \quad \sqrt{t-3} = 7$$

$$(\sqrt{t-3})^2 = 7^2$$

$$t-3 = 49 \rightarrow t = 52$$

$$\text{Check: } \sqrt{52-3} = \sqrt{49} = 7$$

The solution set is $\{52\}$.

$$13. \quad \sqrt{2t-1} = 1$$

$$(\sqrt{2t-1})^2 = 1^2$$

$$2t-1 = 1 \rightarrow 2t = 2 \rightarrow t = 1$$

$$\text{Check: } \sqrt{2(1)-1} = \sqrt{1} = 1$$

The solution set is $\{1\}$.

$$14. \quad \sqrt{3t+4} = 2$$

$$(\sqrt{3t+4})^2 = 2^2$$

$$3t+4 = 4 \rightarrow 3t = 0 \rightarrow t = 0$$

$$\text{Check: } \sqrt{3(0)+4} = \sqrt{4} = 2$$

The solution set is $\{0\}$.

$$15. \quad \sqrt{3t+4} = -6$$

Since the principal square root is never negative, the equation has no real solution.

$$16. \quad \sqrt{5t+3} = -2$$

Since the principal square root is never negative, the equation has no real solution.

$$17. \quad \sqrt[3]{1-2x} - 3 = 0$$

$$\sqrt[3]{1-2x} = 3$$

$$(\sqrt[3]{1-2x})^3 = 3^3$$

$$1-2x = 27 \rightarrow -2x = 26 \rightarrow x = -13$$

$$\text{Check: } \sqrt[3]{1-2(-13)} - 3 = \sqrt[3]{27} - 3 = 0$$

The solution set is $\{-13\}$.

$$18. \quad \sqrt[3]{1-2x} - 1 = 0$$

$$\sqrt[3]{1-2x} = 1$$

$$(\sqrt[3]{1-2x})^3 = 1^3$$

$$1-2x = 1 \rightarrow -2x = 0 \rightarrow x = 0$$

$$\text{Check: } \sqrt[3]{1-2(0)} - 1 = \sqrt[3]{1} - 1 = 0$$

The solution set is $\{0\}$.

Chapter 1: Graphs, Equations, and Inequalities

19. $\sqrt[4]{5x-4} = 2$

$$\left(\sqrt[4]{5x-4}\right)^4 = 2^4$$

$$5x-4 = 16$$

$$5x = 20$$

$$x = 4$$

Check:

$$\sqrt[4]{5(4)-4} = \sqrt[4]{16} = 2 \text{ T}$$

The solution set is $\{4\}$.

20. $\sqrt[5]{2x-3} = -1$

$$\left(\sqrt[5]{2x-3}\right)^5 = (-1)^5$$

$$2x-3 = -1$$

$$2x = 2$$

$$x = 1$$

Check:

$$\sqrt[5]{2(1)-3} = \sqrt[5]{-1} = -1 \text{ T}$$

The solution set is $\{1\}$.

21. $\sqrt[5]{x^2+2x} = -1$

$$\left(\sqrt[5]{x^2+2x}\right)^5 = (-1)^5$$

$$x^2+2x = -1$$

$$x^2+2x+1 = 0$$

$$(x+1)^2 = 0$$

$$x+1 = 0$$

$$x = -1$$

Check:

$$\sqrt[5]{(-1)^2+2(-1)} = \sqrt[5]{1-2} = \sqrt[5]{-1} = -1 \text{ T}$$

The solution set is $\{-1\}$.

22. $\sqrt[4]{x^2+16} = \sqrt{5}$

$$\left(\sqrt[4]{x^2+16}\right)^4 = (\sqrt{5})^4$$

$$x^2+16 = 25$$

$$x^2 = 9$$

$$x = \pm 3$$

Check:

$$\sqrt[4]{(-3)^2+16} = \sqrt[4]{9+16} = \sqrt[4]{25} = \sqrt{5} \text{ T}$$

$$\sqrt[4]{(3)^2+16} = \sqrt[4]{9+16} = \sqrt[4]{25} = \sqrt{5} \text{ T}$$

The solution set is $\{-3, 3\}$.

23. $x = 8\sqrt{x}$

$$(x)^2 = (8\sqrt{x})^2$$

$$x^2 = 64x$$

$$x^2 - 64x = 0$$

$$x(x-64) = 0$$

$$x = 0 \text{ or } x = 64$$

Check:

$$0 = 8\sqrt{0} \quad 64 = 8\sqrt{64}$$

$$0 = 0 \text{ T} \quad 64 = 8 \cdot 8$$

$$64 = 64 \text{ T}$$

The solution set is $\{0, 64\}$.

24. $x = 3\sqrt{x}$

$$(x)^2 = (3\sqrt{x})^2$$

$$x^2 = 9x$$

$$x^2 - 9x = 0$$

$$x(x-9) = 0$$

$$x = 0 \text{ or } x = 9$$

Check:

$$0 = 3\sqrt{0} \quad 9 = 3\sqrt{9}$$

$$0 = 0 \text{ T} \quad 9 = 3 \cdot 3$$

$$9 = 9 \text{ T}$$

The solution set is $\{0, 9\}$.

25. $\sqrt{15-2x} = x$

$$(\sqrt{15-2x})^2 = x^2$$

$$15-2x = x^2 \rightarrow x^2+2x-15 = 0$$

$$(x+5)(x-3) = 0 \rightarrow x = -5 \text{ or } x = 3$$

$$\text{Check } -5: \sqrt{15-2(-5)} = \sqrt{25}$$

$$= 5 \neq -5$$

$$\text{Check } 3: \sqrt{15-2(3)} = \sqrt{9} = 3 = 3$$

The solution set is $\{3\}$.

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26. $\sqrt{12-x} = x$
 $(\sqrt{12-x})^2 = x^2$
 $12-x = x^2 \rightarrow x^2 + x - 12 = 0$
 $(x+4)(x-3) = 0 \rightarrow x = -4 \text{ or } x = 3$
 Check -4: $\sqrt{12-(-4)} = \sqrt{16}$
 $= 4 \neq -4$
 Check 3: $\sqrt{12-3} = \sqrt{9} = 3 = 3$
 The solution set is $\{3\}$.

27. $\sqrt{3(x+10)} - 4 = x$
 $\sqrt{3(x+10)} = x+4$
 $(\sqrt{3(x+10)})^2 = (x+4)^2$
 $3x+30 = x^2 + 8x + 16$
 $0 = x^2 + 5x - 14$
 $0 = (x+7)(x-2)$
 $x = -7 \text{ or } x = 2$
 Check -7: $\sqrt{3(-7+10)} - 4 = \sqrt{9} - 4 = -1 \neq -7$
 Check 2: $\sqrt{3(2+10)} - 4 = \sqrt{36} - 4 = 2 = 2$
 Discard $x = -7$ as extraneous.
 The solution set is $\{2\}$.

28. $\sqrt{1-x} - 3 = x+2$
 $\sqrt{1-x} = x+5$
 $(\sqrt{1-x})^2 = (x+5)^2$
 $1-x = x^2 + 10x + 25$
 $0 = x^2 + 11x + 24$
 $0 = (x+3)(x+8)$
 $x = -3 \text{ or } x = -8$
 Check -3: $\sqrt{1-(-3)} - 3 = -3+2 \rightarrow -1 = -1$
 Check -8: $\sqrt{1-(-8)} - 3 = -8+2 \rightarrow 0 = -6$
 Discard $x = -8$ as extraneous.
 The solution set is $\{-3\}$.

29. $\sqrt{x^2-x-4} = x+2$
 $(\sqrt{x^2-x-4})^2 = (x+2)^2$
 $x^2-x-4 = x^2+4x+4$
 $-8 = 5x \rightarrow -\frac{8}{5} = x$
 Check:

$$\sqrt{\left(-\frac{8}{5}\right)^2 - \left(-\frac{8}{5}\right) - 4} = \left(-\frac{8}{5}\right) + 2$$

$$\sqrt{\frac{64}{25} + \frac{8}{5} - 4} = \frac{2}{5}$$

$$\sqrt{\frac{4}{25}} = \frac{2}{5}$$

$$\frac{2}{5} = \frac{2}{5} \quad \text{T}$$

The solution set is $\left\{-\frac{8}{5}\right\}$.

30. $\sqrt{x^2-x-8} = x+5$
 $(\sqrt{x^2-x-8})^2 = (x+5)^2$
 $x^2-x-8 = x^2+10x+25$
 $-11x = 33$
 $x = -3$

Check:
 $\sqrt{(-3)^2 - (-3) - 8} = (-3) + 5$
 $\sqrt{9+3-8} = -3+5$
 $\sqrt{4} = 2$
 $2 = 2 \quad \text{T}$

The solution set is $\{-3\}$.

31. $3 + \sqrt{3x+1} = x$
 $\sqrt{3x+1} = x-3$
 $(\sqrt{3x+1})^2 = (x-3)^2$
 $3x+1 = x^2-6x+9$
 $0 = x^2-9x+8$
 $0 = (x-1)(x-8)$
 $x = 1 \text{ or } x = 8$

Check 1: $3 + \sqrt{3(1)+1} = 3 + \sqrt{4} = 5 \neq 1$
 Check 8: $3 + \sqrt{3(8)+1} = 3 + \sqrt{25} = 8 = 8$
 Discard $x = 1$ as extraneous.
 The solution set is $\{8\}$.

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32. $2 + \sqrt{12 - 2x} = x$

$$\sqrt{12 - 2x} = x - 2$$

$$(\sqrt{12 - 2x})^2 = (x - 2)^2$$

$$12 - 2x = x^2 - 4x + 4$$

$$0 = x^2 - 2x - 8$$

$$(x + 2)(x - 4) = 0$$

$$x = -2 \text{ or } x = 4$$

Check -2: $2 + \sqrt{12 - 2(-2)} = 2 + \sqrt{16} = 6 \neq -2$

Check 4: $2 + \sqrt{12 - 2(4)} = 2 + \sqrt{4} = 4 = 4$

Discard $x = -2$ as extraneous.

The solution set is $\{4\}$.

33. $\sqrt{3x - 5} - \sqrt{x + 7} = 2$

$$\sqrt{3x - 5} = 2 + \sqrt{x + 7}$$

$$(\sqrt{3x - 5})^2 = (2 + \sqrt{x + 7})^2$$

$$3x - 5 = 4 + 4\sqrt{x + 7} + x + 7$$

$$2x - 16 = 4\sqrt{x + 7}$$

$$(2x - 16)^2 = (4\sqrt{x + 7})^2$$

$$4x^2 - 64x + 256 = 16(x + 7)$$

$$4x^2 - 64x + 256 = 16x + 112$$

$$4x^2 - 80x + 144 = 0$$

$$x^2 - 20x + 36 = 0$$

$$(x - 2)(x - 18) = 0 \rightarrow x = 2 \text{ or } x = 18$$

Check 2: $\sqrt{3(2) - 5} - \sqrt{2 + 7}$

$$= \sqrt{1} - \sqrt{9} = 1 - 3 = -2 \neq 2$$

Check 18: $\sqrt{3(18) - 5} - \sqrt{18 + 7}$

$$= \sqrt{49} - \sqrt{25} = 7 - 5 = 2 = 2$$

Discard $x = 2$ as extraneous.

The solution set is $\{18\}$.

34. $\sqrt{3x + 7} + \sqrt{x + 2} = 1$

$$\sqrt{3x + 7} = 1 - \sqrt{x + 2}$$

$$(\sqrt{3x + 7})^2 = (1 - \sqrt{x + 2})^2$$

$$3x + 7 = 1 - 2\sqrt{x + 2} + x + 2$$

$$2x + 4 = -2\sqrt{x + 2}$$

$$-x - 2 = \sqrt{x + 2}$$

$$(-x - 2)^2 = (\sqrt{x + 2})^2$$

$$x^2 + 4x + 4 = x + 2$$

$$x^2 + 3x + 2 = 0$$

$$(x + 1)(x + 2) = 0$$

$$x = -1 \text{ or } x = -2$$

Check -1: $\sqrt{3(-1) + 7} + \sqrt{-1 + 2}$

$$= \sqrt{4} + \sqrt{1} = 2 + 1 = 3 \neq 1$$

Check -2: $\sqrt{3(-2) + 7} + \sqrt{-2 + 2}$

$$= \sqrt{1} + \sqrt{0} = 1 + 0 = 1 = 1$$

Discard $x = -1$ as extraneous.

The solution set is $\{-2\}$.

35. $\sqrt{3x + 1} - \sqrt{x - 1} = 2$

$$\sqrt{3x + 1} = 2 + \sqrt{x - 1}$$

$$(\sqrt{3x + 1})^2 = (2 + \sqrt{x - 1})^2$$

$$3x + 1 = 4 + 4\sqrt{x - 1} + x - 1$$

$$2x - 2 = 4\sqrt{x - 1}$$

$$(2x - 2)^2 = (4\sqrt{x - 1})^2$$

$$4x^2 - 8x + 4 = 16(x - 1)$$

$$x^2 - 2x + 1 = 4x - 4$$

$$x^2 - 6x + 5 = 0$$

$$(x - 1)(x - 5) = 0 \rightarrow x = 1 \text{ or } x = 5$$

Check 1: $\sqrt{3(1) + 1} - \sqrt{1 - 1}$

$$= \sqrt{4} - \sqrt{0} = 2 - 0 = 2 = 2$$

Check 5: $\sqrt{3(5) + 1} - \sqrt{5 - 1}$

$$= \sqrt{16} - \sqrt{4} = 4 - 2 = 2 = 2$$

The solution set is $\{1, 5\}$.

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$$\begin{aligned}
 36. \quad & \sqrt{2x+3} - \sqrt{x+1} = 1 \\
 & \sqrt{2x+3} = 1 + \sqrt{x+1} \\
 & (\sqrt{2x+3})^2 = (1 + \sqrt{x+1})^2 \\
 & 2x+3 = 1 + 2\sqrt{x+1} + x+1 \\
 & x+1 = 2\sqrt{x+1} \\
 & (x+1)^2 = (2\sqrt{x+1})^2 \\
 & x^2 + 2x + 1 = 4(x+1) \\
 & x^2 + 2x + 1 = 4x + 4 \\
 & x^2 - 2x - 3 = 0 \\
 & (x+1)(x-3) = 0 \rightarrow x = -1 \text{ or } x = 3 \\
 & \text{Check } -1: \sqrt{2(-1)+3} - \sqrt{-1+1} = 1 - 0 = 1 \\
 & \text{Check } 3: \sqrt{2(3)+3} - \sqrt{3+1} = 3 - 2 = 1 \\
 & \text{The solution set is } \{-1, 3\}.
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & \sqrt{3-2\sqrt{x}} = \sqrt{x} \\
 & (\sqrt{3-2\sqrt{x}})^2 = (\sqrt{x})^2 \\
 & 3 - 2\sqrt{x} = x \\
 & -2\sqrt{x} = x - 3 \\
 & (-2\sqrt{x})^2 = (x-3)^2 \\
 & 4x = x^2 - 6x + 9 \\
 & 0 = x^2 - 10x + 9 \\
 & 0 = (x-9)(x-1) \\
 & x = 1 \text{ or } x = 9 \\
 & \text{Check:} \\
 & \sqrt{3-2\sqrt{1}} = \sqrt{1} \quad \sqrt{3-2\sqrt{9}} = \sqrt{9} \\
 & \sqrt{3-2} = 1 \quad \sqrt{3-2 \cdot 3} = 3 \\
 & \sqrt{1} = 1 \quad \sqrt{-3} \neq 3 \\
 & 1 = 1 \text{ T} \\
 & \text{Discard } x = 9 \text{ as extraneous. The solution set is } \{1\}.
 \end{aligned}$$

$$\begin{aligned}
 38. \quad & \sqrt{10+3\sqrt{x}} = \sqrt{x} \\
 & (\sqrt{10+3\sqrt{x}})^2 = (\sqrt{x})^2 \\
 & 10 + 3\sqrt{x} = x \\
 & 3\sqrt{x} = x - 10 \\
 & (3\sqrt{x})^2 = (x-10)^2 \\
 & 9x = x^2 - 20x + 100 \\
 & 0 = x^2 - 29x + 100 \\
 & 0 = (x-4)(x-25) \\
 & x = 4 \text{ or } x = 25 \\
 & \text{Check:} \\
 & \sqrt{10+3\sqrt{4}} = \sqrt{4} \quad \sqrt{10+3\sqrt{25}} = \sqrt{25} \\
 & \sqrt{10+3 \cdot 2} = 2 \quad \sqrt{10+3 \cdot 5} = 5 \\
 & \sqrt{16} = 2 \quad \sqrt{25} = 5 \\
 & 4 \neq 2 \quad 5 = 5 \text{ T} \\
 & \text{Discard } x = 4 \text{ as extraneous. The solution set is } \{25\}.
 \end{aligned}$$

$$\begin{aligned}
 39. \quad & (3x+1)^{1/2} = 4 \\
 & ((3x+1)^{1/2})^2 = (4)^2 \\
 & 3x+1 = 16 \rightarrow 3x = 15 \rightarrow x = 5 \\
 & \text{Check:} \\
 & (3(5)+1)^{1/2} = 4 \\
 & 16^{1/2} = 4 \\
 & 4 = 4 \text{ T} \\
 & \text{The solution set is } \{5\}.
 \end{aligned}$$

$$\begin{aligned}
 40. \quad & (3x-5)^{1/2} = 2 \\
 & ((3x-5)^{1/2})^2 = (2)^2 \\
 & 3x-5 = 4 \rightarrow 3x = 9 \rightarrow x = 3 \\
 & \text{Check:} \\
 & (3(3)-5)^{1/2} = 2 \\
 & 4^{1/2} = 2 \\
 & 2 = 2 \text{ T} \\
 & \text{The solution set is } \{3\}.
 \end{aligned}$$

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41. $(5x-2)^{1/3} = 2$

$$\left((5x-2)^{1/3}\right)^3 = (2)^3$$

$$5x-2=8 \rightarrow 5x=10 \rightarrow x=2$$

Check:

$$(5(2)-2)^{1/3} = 2$$

$$8^{1/3} = 2$$

$$2 = 2$$

The solution set is $\{2\}$.

42. $(2x+1)^{1/3} = -1$

$$\left((2x+1)^{1/3}\right)^3 = (-1)^3$$

$$2x+1=-1 \rightarrow 2x=-2 \rightarrow x=-1$$

Check:

$$(2(-1)+1)^{1/3} = -1$$

$$(-1)^{1/3} = -1$$

$$-1 = -1$$

The solution set is $\{-1\}$.

43. $(x^2+9)^{1/2} = 5$

$$\left((x^2+9)^{1/2}\right)^2 = (5)^2$$

$$x^2+9=25 \rightarrow x^2=16$$

$$x=-4 \text{ or } x=4$$

Check:

$$\left((-4)^2+9\right)^{1/2} = 5 \quad \left((4)^2+9\right)^{1/2} = 5$$

$$25^{1/2} = 5 \quad 25^{1/2} = 5$$

$$5 = 5 \quad 5 = 5$$

The solution set is $\{-4, 4\}$.

44. $(x^2-16)^{1/2} = 9$

$$\left((x^2-16)^{1/2}\right)^2 = (9)^2$$

$$x^2-16=81 \rightarrow x^2=97$$

$$x=-\sqrt{97} \text{ or } x=\sqrt{97}$$

$$\left((- \sqrt{97})^2-16\right)^{1/2} = 9 \quad \left((\sqrt{97})^2-16\right)^{1/2} = 9$$

$$(97-16)^{1/2} = 9 \quad (97-16)^{1/2} = 9$$

$$81^{1/2} = 9$$

$$81^{1/2} = 9$$

$$9 = 9$$

$$9 = 9$$

The solution set is $\{-\sqrt{97}, \sqrt{97}\}$.

45. $t^4-16=0$

$$(t^2+4)(t^2-4)=0$$

$$(t^2+4)(t+2)(t-2)=0$$

$t^2+4=0$ has no real solution, so we only need to consider

$$t+2=0 \quad \text{or} \quad t-2=0$$

$$t=-2 \quad t=2$$

The solution set is $\{-2, 2\}$.

46. $y^4-4=0$

$$(y^2+2)(y^2-2)=0$$

$$(y^2+2)(y+\sqrt{2})(y-\sqrt{2})=0$$

$$y^2+2=0 \Rightarrow y^2=-2, \text{ which is impossible}$$

$$\text{or } (y+\sqrt{2})(y-\sqrt{2})=0 \Rightarrow y=\pm\sqrt{2}$$

The solution set is $\{-\sqrt{2}, \sqrt{2}\}$.

47. $x^4-5x^2+4=0$

$$(x^2-4)(x^2-1)=0$$

$$x^2-4=0 \text{ or } x^2-1=0$$

$$x=\pm 2 \text{ or } x=\pm 1$$

The solution set is $\{-2, -1, 1, 2\}$.

48. $x^4-10x^2+24=0$

$$(x^2-6)(x^2-4)=0$$

$$x^2-6=0 \quad \text{or} \quad x^2-4=0$$

$$x^2=6 \quad x^2=4$$

$$x=\pm\sqrt{6} \quad x=\pm 2$$

The solution set is $\{-\sqrt{6}, -2, 2, \sqrt{6}\}$.

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49. $6x^4 - 5x^2 - 1 = 0$
 $(6x^2 + 1)(x^2 - 1) = 0$
 $6x^2 + 1 = 0$ or $x^2 - 1 = 0$
 $6x^2 = -1$, which is impossible
or $x = \pm 1$
The solution set is $\{-1, 1\}$.

50. $2x^4 - 5x^2 - 12 = 0$
 $(2x^2 + 3)(x^2 - 4) = 0$
 $2x^2 + 3 = 0$ or $x^2 - 4 = 0$
 $2x^2 = -3$, which is impossible
or $x = \pm 2$
The solution set is $\{-2, 2\}$.

51. $x^6 + 7x^3 - 8 = 0$
 $(x^3 + 8)(x^3 - 1) = 0$
 $x^3 + 8 = 0$ or $x^3 - 1 = 0$
 $x^3 = -8 \rightarrow x = -2$
or $x^3 = 1 \rightarrow x = 1$
The solution set is $\{-2, 1\}$.

52. $x^6 - 7x^3 - 8 = 0$
 $(x^3 - 8)(x^3 + 1) = 0$
 $x^3 - 8 = 0$ or $x^3 + 1 = 0$
 $x^3 = 8 \rightarrow x = 2$
or $x^3 = -1 \rightarrow x = -1$
The solution set is $\{-1, 2\}$.

53. $(x+2)^2 + 7(x+2) + 12 = 0$
let $p = x+2 \rightarrow p^2 = (x+2)^2$
 $p^2 + 7p + 12 = 0$
 $(p+3)(p+4) = 0$
 $p+3 = 0$ or $p+4 = 0$
 $p = -3 \rightarrow x+2 = -3 \rightarrow x = -5$
or $p = -4 \rightarrow x+2 = -4 \rightarrow x = -6$
The solution set is $\{-6, -5\}$.

54. $(2x+5)^2 - (2x+5) - 6 = 0$
let $p = 2x+5 \rightarrow p^2 = (2x+5)^2$
 $p^2 - p - 6 = 0$
 $(p-3)(p+2) = 0$
 $p-3 = 0$ or $p+2 = 0$
 $p = 3 \rightarrow 2x+5 = 3 \rightarrow x = -1$
or $p = -2 \rightarrow 2x+5 = -2 \rightarrow x = -\frac{7}{2}$
The solution set is $\{-\frac{7}{2}, -1\}$.

55. $2(s+1)^2 - 5(s+1) = 3$
let $p = s+1 \rightarrow p^2 = (s+1)^2$
 $2p^2 - 5p = 3$
 $2p^2 - 5p - 3 = 0$
 $(2p+1)(p-3) = 0$
 $2p+1 = 0$ or $p-3 = 0$
 $p = -\frac{1}{2} \rightarrow s+1 = -\frac{1}{2} \rightarrow s = -\frac{3}{2}$
or $p = 3 \rightarrow s+1 = 3 \rightarrow s = 2$
The solution set is $\{-\frac{3}{2}, 2\}$.

56. $3(1-y)^2 + 5(1-y) + 2 = 0$
let $p = 1-y \rightarrow p^2 = (1-y)^2$
 $3p^2 + 5p + 2 = 0$
 $(3p+2)(p+1) = 0$
 $3p+2 = 0$ or $p+1 = 0$
 $p = -\frac{2}{3} \rightarrow 1-y = -\frac{2}{3} \rightarrow y = \frac{5}{3}$
or $p = -1 \rightarrow 1-y = -1 \rightarrow y = 2$
The solution set is $\{\frac{5}{3}, 2\}$.

57. $x - 4\sqrt{x} = 0$
 $x = 4\sqrt{x}$
 $(x)^2 = (4\sqrt{x})^2$
 $x^2 = 16x \rightarrow 0 = 16x - x^2$
 $0 = x(16-x) \rightarrow x = 0$
or $16-x = 0 \rightarrow x = 16$

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Check:

$$x = 0: 0 - 4\sqrt{0} = 0$$

$$0 = 0$$

$$x = 16: (16) - 4\sqrt{16} = 0$$

$$16 - 16 = 0$$

$$0 = 0$$

The solution set is $\{0, 16\}$.

58. $x - 8\sqrt{x} = 0$

$$8\sqrt{x} = x$$

$$(8\sqrt{x})^2 = (x)^2$$

$$64x = x^2 \rightarrow 0 = x^2 - 64x$$

$$0 = x(x - 64) \rightarrow x = 0 \text{ or } x = 64$$

Check:

$$x = 0: 0 - 8\sqrt{0} = 0$$

$$0 = 0$$

$$x = 64: 64 - 8\sqrt{64} = 0$$

$$64 - 64 = 0$$

The solution set is $\{0, 64\}$.

59. $x + \sqrt{x} = 20$

$$\text{let } p = \sqrt{x} \rightarrow p^2 = x$$

$$p^2 + p = 20$$

$$p^2 + p - 20 = 0$$

$$(p + 5)(p - 4) = 0$$

$$p + 5 = 0 \text{ or } p - 4 = 0$$

$$p = -5 \rightarrow \sqrt{x} = -5 \text{ non-real}$$

$$\text{or } p = 4 \rightarrow \sqrt{x} = 4 \rightarrow x = 16$$

Check:

$$x = 16: 16 + \sqrt{16} = 20$$

$$16 + 4 = 20$$

The solution set is $\{16\}$.

60. $x + \sqrt{x} = 6$

$$\text{let } p = \sqrt{x} \rightarrow p^2 = x$$

$$p^2 + p = 6$$

$$p^2 + p - 6 = 0$$

$$(p + 3)(p - 2) = 0$$

$$p + 3 = 0 \text{ or } p - 2 = 0$$

$$p = -3 \rightarrow \sqrt{x} = -3 \text{ non-real}$$

$$\text{or } p = 2 \rightarrow \sqrt{x} = 2 \rightarrow x = 4$$

Check:

$$x = 4: 4 + \sqrt{4} = 6$$

$$4 + 2 = 6$$

The solution set is $\{4\}$.

61. $z^{1/2} - 4t^{1/4} + 4 = 0$

$$\text{let } p = z^{1/4} \rightarrow p^2 = z^{1/2}$$

$$p^2 - 4p + 4 = 0$$

$$(p - 2)(p - 2) = 0$$

$$p - 2 = 0$$

$$p = 2 \rightarrow z^{1/4} = 2 \rightarrow z = 16$$

Check:

$$z = 16: 16^{1/2} - 4(16)^{1/4} + 4 = 0$$

$$4 - 8 + 4 = 0 \rightarrow 0 = 0$$

The solution set is $\{16\}$.

62. $t^{1/2} - 2t^{1/4} + 1 = 0$

$$\text{let } p = t^{1/4} \rightarrow p^2 = t^{1/2}$$

$$p^2 - 2p + 1 = 0$$

$$(p - 1)(p - 1) = 0$$

$$p - 1 = 0$$

$$p = 1 \rightarrow t^{1/4} = 1 \rightarrow t = 1$$

Check:

$$t = 1: 1^{1/2} - 2(1)^{1/4} + 1 = 0$$

$$1 - 2 + 1 = 0 \rightarrow 0 = 0$$

The solution set is $\{1\}$.

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63. $4x^{1/2} - 9x^{1/4} + 4 = 0$

let $p = x^{1/4} \rightarrow p^2 = x^{1/2}$

$4p^2 - 9p + 4 = 0$

$p = \frac{9 \pm \sqrt{81 - 64}}{8} = \frac{9 \pm \sqrt{17}}{8}$

$x^{1/4} = \frac{9 \pm \sqrt{17}}{8} \rightarrow x = \left(\frac{9 \pm \sqrt{17}}{8} \right)^4$

Check:

$4 \left(\left(\frac{9 + \sqrt{17}}{8} \right)^4 \right)^{1/2} - 9 \left(\left(\frac{9 + \sqrt{17}}{8} \right)^4 \right)^{1/4} + 4 = 0$

$4 \left(\frac{9 + \sqrt{17}}{8} \right)^2 - 9 \left(\frac{9 + \sqrt{17}}{8} \right) + 4 = 0$

$4 \frac{(9 + \sqrt{17})^2}{64} - 9 \left(\frac{9 + \sqrt{17}}{8} \right) + 4 = 0$

$64 \left(4 \frac{(9 + \sqrt{17})^2}{64} - 9 \left(\frac{9 + \sqrt{17}}{8} \right) + 4 \right) = (0)(64)$

$4(9 + \sqrt{17})^2 - 72(9 + \sqrt{17}) + 256 = 0$

$4(81 + 18\sqrt{17} + 17) - 72(9 + \sqrt{17}) + 256 = 0$

$324 + 72\sqrt{17} + 68 - 648 - 72\sqrt{17} + 256 = 0$

$0 = 0$

$4 \left(\left(\frac{9 - \sqrt{17}}{8} \right)^4 \right)^{1/2} - 9 \left(\left(\frac{9 - \sqrt{17}}{8} \right)^4 \right)^{1/4} + 4 = 0$

$4 \left(\frac{9 - \sqrt{17}}{8} \right)^2 - 9 \left(\frac{9 - \sqrt{17}}{8} \right) + 4 = 0$

$4(81 - 18\sqrt{17} + 17) - 72(9 - \sqrt{17}) + 256 = 0$

$324 - 72\sqrt{17} + 68 - 648 + 72\sqrt{17} + 256 = 0$

$0 = 0$

The solution set is $\left\{ \left(\frac{9 - \sqrt{17}}{8} \right)^4, \left(\frac{9 + \sqrt{17}}{8} \right)^4 \right\}$.

64. $x^{1/2} - 3x^{1/4} + 2 = 0$

let $p = x^{1/4} \rightarrow p^2 = x^{1/2}$

$p^2 - 3p + 2 = 0$

$(p - 2)(p - 1) = 0$

$p = 2 \rightarrow x^{1/4} = 2 \rightarrow x = 16$

or $p = 1 \rightarrow x^{1/4} = 1 \rightarrow x = 1$

Check:

$x = 16: 16^{1/2} - 3(16)^{1/4} + 2 = 0$

$4 - 6 + 2 = 0 \rightarrow 0 = 0$

$x = 1: 1^{1/2} - 3(1)^{1/4} + 2 = 0$

$1 - 3 + 2 = 0 \rightarrow 0 = 0$

The solution set is $\{1, 16\}$.

65. $\frac{1}{(x+1)^2} = \frac{1}{x+1} + 2$

let $p = \frac{1}{x+1} \rightarrow p^2 = \left(\frac{1}{x+1} \right)^2$

$p^2 = p + 2 \rightarrow p^2 - p - 2 = 0$

$(p + 1)(p - 2) = 0 \rightarrow p = -1 \text{ or } p = 2$

$p = -1 \rightarrow \frac{1}{x+1} = -1 \rightarrow 1 = -x - 1 \rightarrow x = -2$

or

$p = 2 \rightarrow \frac{1}{x+1} = 2 \rightarrow 1 = 2x + 2 \rightarrow x = -\frac{1}{2}$

Check:

$x = -2: \frac{1}{(-2+1)^2} = \frac{1}{-2+1} + 2$

$1 = -1 + 2$

$1 = 1$

$x = -\frac{1}{2}: \frac{1}{\left(-\frac{1}{2}+1\right)^2} = \frac{1}{\left(-\frac{1}{2}+1\right)} + 2$

$4 = 2 + 2$

$4 = 4$

The solution set is $\left\{ -2, -\frac{1}{2} \right\}$.

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$$66. \frac{1}{(x-1)^2} + \frac{1}{x-1} = 12$$

$$\text{let } p = \frac{1}{x-1} \rightarrow p^2 = \left(\frac{1}{x-1}\right)^2$$

$$p^2 + p = 12$$

$$p^2 + p - 12 = 0$$

$$(p+4)(p-3) = 0$$

$$p = -4 \quad \text{or} \quad p = 3$$

$$\frac{1}{x-1} = -4 \quad \frac{1}{x-1} = 3$$

$$1 = -4x + 4$$

$$1 = 3x - 3$$

$$4x = 3$$

$$4 = 3x$$

$$x = \frac{3}{4}$$

$$x = \frac{4}{3}$$

Check:

$$x = \frac{3}{4}: \frac{1}{\left(\frac{3}{4}-1\right)^2} + \frac{1}{\left(\frac{3}{4}-1\right)} = 12$$

$$\frac{1}{\left(\frac{1}{4}\right)^2} + \frac{1}{\left(-\frac{1}{4}\right)} = 12$$

$$16 - 4 = 12$$

$$12 = 12$$

$$x = \frac{4}{3}: \frac{1}{\left(\frac{4}{3}-1\right)^2} + \frac{1}{\left(\frac{4}{3}-1\right)} = 12$$

$$\frac{1}{\left(\frac{1}{3}\right)^2} + \frac{1}{\left(\frac{1}{3}\right)} = 12$$

$$9 + 3 = 12$$

$$12 = 12$$

The solution set is $\left\{\frac{3}{4}, \frac{4}{3}\right\}$.

$$67. 3x^{-2} - 7x^{-1} - 6 = 0$$

$$\text{let } p = x^{-1} \rightarrow p^2 = x^{-2}$$

$$3p^2 - 7p - 6 = 0$$

$$(3p+2)(p-3) = 0$$

$$p = -\frac{2}{3} \quad \text{or} \quad p = 3$$

$$p = -\frac{2}{3} \rightarrow x^{-1} = -\frac{2}{3} \rightarrow (x^{-1})^{-1} = \left(-\frac{2}{3}\right)^{-1}$$

$$\rightarrow x = -\frac{3}{2}$$

$$p = 3 \rightarrow x^{-1} = 3 \rightarrow (x^{-1})^{-1} = (3)^{-1} \rightarrow x = \frac{1}{3}$$

Check:

$$x = -\frac{3}{2}: 3\left(-\frac{3}{2}\right)^{-2} - 7\left(-\frac{3}{2}\right)^{-1} - 6 = 0$$

$$3\left(\frac{4}{9}\right) - 7\left(-\frac{2}{3}\right) - 6 = 0$$

$$\frac{4}{3} + \frac{14}{3} - 6 = 0$$

$$0 = 0$$

$$x = \frac{1}{3}: 3\left(\frac{1}{3}\right)^{-2} - 7\left(\frac{1}{3}\right)^{-1} - 6 = 0$$

$$3(9) - 7(3) - 6 = 0$$

$$27 - 21 - 6 = 0$$

$$0 = 0$$

The solution set is $\left\{-\frac{3}{2}, \frac{1}{3}\right\}$.

$$68. 2x^{-2} - 3x^{-1} - 4 = 0$$

$$\text{let } p = x^{-1} \rightarrow p^2 = x^{-2}$$

$$2p^2 - 3p - 4 = 0$$

$$p = \frac{3 \pm \sqrt{9+32}}{4} = \frac{3 \pm \sqrt{41}}{4}$$

$$p = \frac{3 + \sqrt{41}}{4} \rightarrow x^{-1} = \frac{3 + \sqrt{41}}{4}$$

$$(x^{-1})^{-1} = \left(\frac{3 + \sqrt{41}}{4}\right)^{-1}$$

$$x = \frac{4}{3 + \sqrt{41}} \left(\frac{3 - \sqrt{41}}{3 - \sqrt{41}}\right) = \frac{12 - 4\sqrt{41}}{-32}$$

$$= \frac{-3 + \sqrt{41}}{8}$$

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$$p = \frac{3 - \sqrt{41}}{4} \rightarrow x^{-1} = \frac{3 - \sqrt{41}}{4}$$

$$(x^{-1})^{-1} = \left(\frac{3 - \sqrt{41}}{4} \right)^{-1}$$

$$x = \frac{4}{3 - \sqrt{41}} \left(\frac{3 + \sqrt{41}}{3 + \sqrt{41}} \right) = \frac{12 + 4\sqrt{41}}{-32}$$

$$= \frac{-3 - \sqrt{41}}{8}$$

Check:

$$x = \frac{-3 + \sqrt{41}}{8}:$$

$$2 \left(\frac{-3 + \sqrt{41}}{8} \right)^{-2} - 3 \left(\frac{-3 + \sqrt{41}}{8} \right)^{-1} - 4 = 0$$

$$2 \left(\frac{64}{(-3 + \sqrt{41})^2} \right) - 3 \left(\frac{8}{-3 + \sqrt{41}} \right) - 4 = 0$$

$$2(64) - 3(8)(-3 + \sqrt{41}) - 4(-3 + \sqrt{41})^2 = 0$$

$$128 + 72 - 24\sqrt{41} - 4(9 - 6\sqrt{41} + 41) = 0$$

$$128 + 72 - 24\sqrt{41} - 36 + 24\sqrt{41} - 164 = 0$$

$$0 = 0$$

$$x = \frac{-3 - \sqrt{41}}{8}:$$

$$2 \left(\frac{-3 - \sqrt{41}}{8} \right)^{-2} - 3 \left(\frac{-3 - \sqrt{41}}{8} \right)^{-1} - 4 = 0$$

$$2 \left(\frac{64}{(-3 - \sqrt{41})^2} \right) - 3 \left(\frac{8}{-3 - \sqrt{41}} \right) - 4 = 0$$

$$2(64) - 3(8)(-3 - \sqrt{41}) - 4(-3 - \sqrt{41})^2 = 0$$

$$128 + 72 + 24\sqrt{41} - 4(9 + 6\sqrt{41} + 41) = 0$$

$$128 + 72 + 24\sqrt{41} - 36 - 24\sqrt{41} - 164 = 0$$

$$0 = 0$$

The solution set is $\left\{ \frac{-3 - \sqrt{41}}{8}, \frac{-3 + \sqrt{41}}{8} \right\}$.

69. $2x^{2/3} - 5x^{1/3} - 3 = 0$

let $p = x^{1/3} \rightarrow p^2 = x^{2/3}$

$$2p^2 - 5p - 3 = 0 \rightarrow (2p + 1)(p - 3) = 0$$

$$p = -\frac{1}{2} \text{ or } p = 3$$

$$p = -\frac{1}{2} \rightarrow x^{1/3} = -\frac{1}{2}$$

$$\rightarrow (x^{1/3})^3 = \left(-\frac{1}{2}\right)^3 \rightarrow x = -\frac{1}{8}$$

or

$$p = 3 \rightarrow x^{1/3} = 3 \rightarrow (x^{1/3})^3 = (3)^3$$

$$\rightarrow x = 27$$

Check:

$$x = -\frac{1}{8}: 2\left(-\frac{1}{8}\right)^{2/3} - 5\left(-\frac{1}{8}\right)^{1/3} - 3 = 0$$

$$2\left(\frac{1}{4}\right) - 5\left(-\frac{1}{2}\right) - 3 = 0$$

$$\frac{1}{2} + \frac{5}{2} - 3 = 0$$

$$3 - 3 = 0 \rightarrow 0 = 0$$

$$x = 27: 2(27)^{2/3} - 5(27)^{1/3} - 3 = 0$$

$$2(9) - 5(3) - 3 = 0$$

$$18 - 15 - 3 = 0$$

$$3 - 3 = 0 \rightarrow 0 = 0$$

The solution set is $\left\{ -\frac{1}{8}, 27 \right\}$.

70. $3x^{4/3} + 5x^{2/3} - 2 = 0$

let $p = x^{2/3} \rightarrow p^2 = x^{4/3}$

$$3p^2 + 5p - 2 = 0$$

$$(3p - 1)(p + 2) = 0$$

$$p = \frac{1}{3} \text{ or } p = -2$$

$$p = \frac{1}{3} \rightarrow x^{2/3} = \frac{1}{3}$$

$$(x^{2/3})^3 = \left(\frac{1}{3}\right)^3$$

$$x^2 = \frac{1}{27} \rightarrow x = \pm \sqrt{\frac{1}{27}} = \pm \frac{\sqrt{3}}{9}$$

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or

$$p = -2 \rightarrow x^{2/3} = -2$$

$$(x^{2/3})^{3/2} = (-2)^{3/2}$$

$$x = (-2)^{3/2} \text{ which is not a real number}$$

Check:

$$3\left(\frac{\sqrt{3}}{9}\right)^{4/3} + 5\left(\frac{\sqrt{3}}{9}\right)^{2/3} - 2 = 0$$

$$3\left(\frac{1}{9}\right) + 5\left(\frac{1}{3}\right) - 2 = 0$$

$$\frac{1}{3} + \frac{5}{3} - 2 = 0$$

$$0 = 0$$

$$3\left(-\frac{\sqrt{3}}{9}\right)^{4/3} + 5\left(-\frac{\sqrt{3}}{9}\right)^{2/3} - 2 = 0$$

$$3\left(\frac{1}{9}\right) + 5\left(\frac{1}{3}\right) - 2 = 0$$

$$\frac{1}{3} + \frac{5}{3} - 2 = 0$$

$$0 = 0$$

The solution set is $\left\{-\frac{\sqrt{3}}{9}, \frac{\sqrt{3}}{9}\right\}$.

71. $|2x+3|=5$
 $2x+3=5$ or $2x+3=-5$
 $2x=2$ or $2x=-8$
 $x=1$ or $x=-4$
 The solution set is $\{-4, 1\}$.

72. $|3x-1|=2$
 $3x-1=2$ or $3x-1=-2$
 $3x=3$ or $3x=-1$
 $x=1$ or $x=-\frac{1}{3}$
 The solution set is $\left\{-\frac{1}{3}, 1\right\}$.

73. $|1-4t|+8=13 \Rightarrow |1-4t|=5$
 $1-4t=5$ or $1-4t=-5$
 $-4t=4$ or $-4t=-6$
 $t=-1$ or $t=\frac{3}{2}$

The solution set is $\left\{-1, \frac{3}{2}\right\}$.

74. $|1-2z|+6=9 \Rightarrow |1-2z|=3$
 $1-2z=3$ or $1-2z=-3$
 $-2z=2$ or $-2z=-4$
 $z=-1$ or $z=2$

The solution set is $\{-1, 2\}$.

75. $|-7x|=21$
 $-7x=21$ or $-7x=-21$
 $x=-3$ or $x=3$
 The solution set is $\{-3, 3\}$.

76. $|-x|=1$
 $-x=1$ or $-x=-1$
 The solution set is $\{-1, 1\}$.

77. $4-|2x|=3 \Rightarrow |2x|=1$
 $2x=1$ or $2x=-1$
 $x=\frac{1}{2}$ or $x=-\frac{1}{2}$
 The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$.

78. $5-\left|\frac{1}{2}x\right|=3 \Rightarrow 2=\left|\frac{1}{2}x\right|$
 $\frac{1}{2}x=2$ or $\frac{1}{2}x=-2$
 $x=4$ or $x=-4$
 The solution set is $\{-4, 4\}$.

79. $\frac{8}{7}|x|=3$
 $|x|=\frac{21}{8} \rightarrow x=\frac{21}{8}$ or $x=-\frac{21}{8}$
 The solution set is $\left\{-\frac{21}{8}, \frac{21}{8}\right\}$.

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80. $\frac{3}{4}|x| = 9$

$$|x| = 12 \rightarrow x = 12 \text{ or } x = -12$$

The solution set is $\{-12, 12\}$.

81. $\left|\frac{x}{3} + \frac{2}{5}\right| = 2$

$$\frac{x}{3} + \frac{2}{5} = 2 \quad \text{or} \quad \frac{x}{3} + \frac{2}{5} = -2$$

$$5x + 6 = 30 \quad \text{or} \quad 5x + 6 = -30$$

$$5x = 24 \quad \text{or} \quad 5x = -36$$

$$x = \frac{24}{5} \quad \text{or} \quad x = -\frac{36}{5}$$

The solution set is $\left\{-\frac{36}{5}, \frac{24}{5}\right\}$.

82. $\left|\frac{x}{2} - \frac{1}{3}\right| = 1$

$$\frac{x}{2} - \frac{1}{3} = 1 \quad \text{or} \quad \frac{x}{2} - \frac{1}{3} = -1$$

$$3x - 2 = 6 \quad \text{or} \quad 3x - 2 = -6$$

$$3x = 8 \quad \text{or} \quad 3x = -4$$

$$x = \frac{8}{3} \quad \text{or} \quad x = -\frac{4}{3}$$

The solution set is $\left\{-\frac{4}{3}, \frac{8}{3}\right\}$.

83. $|u - 2| = -\frac{1}{2}$

impossible, since absolute value always yields a non-negative number.

84. $|2 - v| = -1$

impossible, since absolute value always yields a non-negative number.

85. $|x^2 - 9| = 0$

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

The solution set is $\{-3, 3\}$.

86. $|x^2 - 16| = 0$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$x = \pm 4$$

The solution set is $\{-4, 4\}$.

87. $|x^2 - 2x| = 3$

$$x^2 - 2x = 3 \quad \text{or} \quad x^2 - 2x = -3$$

$$x^2 - 2x - 3 = 0 \quad \text{or} \quad x^2 - 2x + 3 = 0$$

$$(x-3)(x+1) = 0 \quad \text{or} \quad x = \frac{2 \pm \sqrt{4-12}}{2}$$

$$= \frac{2 \pm \sqrt{-8}}{2} \text{ no real sol.}$$

$$x = 3 \quad \text{or} \quad x = -1$$

The solution set is $\{-1, 3\}$.

88. $|x^2 + x| = 12$

$$x^2 + x = 12 \quad \text{or} \quad x^2 + x = -12$$

$$x^2 + x - 12 = 0 \quad \text{or} \quad x^2 + x + 12 = 0$$

$$(x-3)(x+4) = 0 \quad \text{or} \quad x = \frac{-1 \pm \sqrt{1-48}}{2}$$

$$= \frac{1 \pm \sqrt{-47}}{2} \text{ no real sol.}$$

$$x = 3 \quad \text{or} \quad x = -4$$

The solution set is $\{-4, 3\}$.

89. $x^3 - 9x = 0$

$$x(x^2 - 9) = 0$$

$$x(x-3)(x+3) = 0$$

$$x = 0 \quad \text{or} \quad x - 3 = 0 \quad x + 3 = 0$$

$$x = 3 \quad x = -3$$

The solution set is $\{-3, 0, 3\}$.

90. $x^4 - 81x^2 = 0$

$$x^2(x^2 - 81) = 0$$

$$x^2(x-9)(x+9) = 0$$

$$x^2 = 0 \quad \text{or} \quad x - 9 = 0 \quad \text{or} \quad x + 9 = 0$$

$$x = 0 \quad x = 9 \quad x = -9$$

The solution set is $\{-9, 0, 9\}$.

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91. $x^3 + x^2 - 20x = 0$

$$x(x^2 + x - 20) = 0$$

$$x(x+5)(x-4) = 0$$

$$x = 0 \text{ or } x + 5 = 0 \text{ or } x - 4 = 0$$

$$x = -5 \quad x = 4$$

The solution set is $\{-5, 0, 4\}$.

92. $x^3 + 6x^2 - 7x = 0$

$$x(x^2 + 6x - 7) = 0$$

$$x(x+7)(x-1) = 0$$

$$x = 0 \text{ or } x + 7 = 0 \text{ or } x - 1 = 0$$

$$x = -7 \quad x = 1$$

The solution set is $\{-7, 0, 1\}$.

93. $x^3 + x^2 - x - 1 = 0$

$$x^2(x+1) - 1(x+1) = 0$$

$$(x+1)(x^2 - 1) = 0$$

$$(x+1)(x-1)(x+1) = 0$$

$$x + 1 = 0 \text{ or } x - 1 = 0$$

$$x = -1 \quad x = 1$$

The solution set is $\{-1, 1\}$.

94. $x^3 + 4x^2 - x - 4 = 0$

$$x^2(x+4) - 1(x+4) = 0$$

$$(x+4)(x^2 - 1) = 0$$

$$(x+4)(x-1)(x+1) = 0$$

$$x + 4 = 0 \text{ or } x - 1 = 0 \text{ or } x + 1 = 0$$

$$x = -4 \quad x = 1 \quad x = -1$$

The solution set is $\{-4, -1, 1\}$.

95. $x^3 - 3x^2 - 16x + 48 = 0$

$$x^2(x-3) - 16(x-3) = 0$$

$$(x-3)(x^2 - 16) = 0$$

$$(x-3)(x-4)(x+4) = 0$$

$$x - 3 = 0 \text{ or } x - 4 = 0 \text{ or } x + 4 = 0$$

$$x = 3 \quad x = 4 \quad x = -4$$

The solution set is $\{-4, 4, 3\}$.

96. $x^3 - 3x^2 - x + 3 = 0$

$$x^2(x-3) - 1(x-3) = 0$$

$$(x-3)(x^2 - 1) = 0$$

$$(x-3)(x-1)(x+1) = 0$$

$$x - 3 = 0 \text{ or } x - 1 = 0 \text{ or } x + 1 = 0$$

$$x = 3 \quad x = 1 \quad x = -1$$

The solution set is $\{-1, 1, 3\}$.

97. $2x^3 + 4 = x^2 + 8x$

$$2x^3 - x^2 - 8x + 4 = 0$$

$$x^2(2x-1) - 4(2x-1) = 0$$

$$(2x-1)(x^2 - 4) = 0$$

$$(2x-1)(x-2)(x+2) = 0$$

$$2x - 1 = 0 \text{ or } x - 2 = 0 \text{ or } x + 2 = 0$$

$$2x = 1 \quad x = 2 \quad x = -2$$

$$x = \frac{1}{2}$$

The solution set is $\{-2, \frac{1}{2}, 2\}$.

98. $3x^3 + 4x^2 = 27x + 36$

$$3x^3 + 4x^2 - 27x - 36 = 0$$

$$x^2(3x+4) - 9(3x+4) = 0$$

$$(3x+4)(x^2 - 9) = 0$$

$$(3x+4)(x-3)(x+3) = 0$$

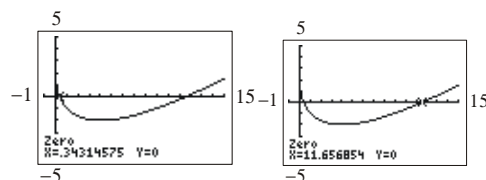
$$3x + 4 = 0 \text{ or } x - 3 = 0 \text{ or } x + 3 = 0$$

$$3x = -4 \quad x = 3 \quad x = -3$$

$$x = -\frac{4}{3}$$

The solution set is $\{-3, -\frac{4}{3}, 3\}$.

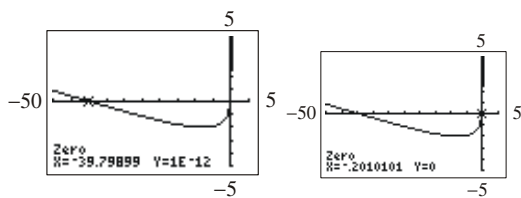
99. $x - 4x^{1/2} + 2 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = x - 4\sqrt{x} + 2$.



The solution set is $\{0.34, 11.66\}$.

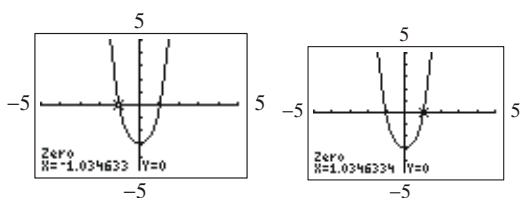
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100. $x^{2/3} + 4x^{1/3} + 2 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = x^{2/3} + 4x^{1/3} + 2$.



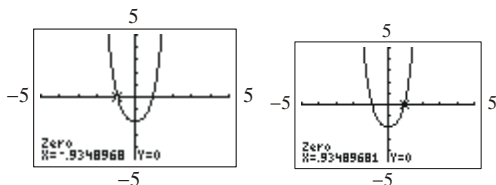
The solution set is $\{-39.80, -0.20\}$.

101. $x^4 + \sqrt{3}x^2 - 3 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = x^4 + \sqrt{3}x^2 - 3$.



The solution set is $\{-1.03, 1.03\}$.

102. $x^4 + \sqrt{2}x^2 - 2 = 0$; Use ZERO (or ROOT) on the graph of $y_1 = x^4 + \sqrt{2}x^2 - 2$.



The solution set is $\{-0.93, 0.93\}$.

103. $3x^2 + 7x - 20 = 0$
 $(3x - 5)(x + 4) = 0$
 $3x - 5 = 0$ or $x + 4 = 0$
 $3x = 5$ $x = -4$
 $x = \frac{5}{3}$

The solution set is $\{-4, \frac{5}{3}\}$.

104. $2x^2 - 13x + 21 = 0$
 $(2x - 7)(x - 3) = 0$
 $2x - 7 = 0$ or $x - 3 = 0$
 $2x = 7$ $x = 3$
 $x = \frac{7}{2}$

The solution set is $\{\frac{7}{2}, 3\}$.

105. $5a^3 - 45a = -2a^2 + 18$
 $5a^3 + 2a^2 - 45a - 18 = 0$
 $a^2(5a + 2) - 9(5a + 2) = 0$
 $(a^2 - 9)(5a + 2) = 0$
 $(a - 3)(a + 3)(5a + 2) = 0$
 $a - 3 = 0$ or $a + 3 = 0$ or $5a + 2 = 0$
 $a = 3$ $a = -3$ $5a = -2$
 $a = -\frac{2}{5}$

The solution set is $\{-3, -\frac{2}{5}, 3\}$.

106. $3z^3 - 12z = -5z^2 + 20$
 $3z^3 + 5z^2 - 12z - 20 = 0$
 $z^2(3z + 5) - 4(3z + 5) = 0$
 $(z^2 - 4)(3z + 5) = 0$
 $(z - 2)(z + 2)(3z + 5) = 0$
 $z - 2 = 0$ or $z + 2 = 0$ $3z + 5 = 0$
 $z = 2$ $z = -2$ $3z = -5$
 $z = -\frac{5}{3}$

The solution set is $\{-2, -\frac{5}{3}, 2\}$.

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107. $-3|5x-2|+9=0$

$$-3|5x-2|=-9$$

$$|5x-2|=3$$

$$5x-2=3 \quad \text{or} \quad 5x-2=-3$$

$$5x=5 \qquad 5x=-1$$

$$x=1 \qquad x=-\frac{1}{5}$$

The solution set is $\left\{-\frac{1}{5}, 1\right\}$.

108. $\frac{1}{4}|2x-3|=\frac{3}{2}$

$$|2x-3|=6$$

$$2x-3=6 \quad \text{or} \quad 2x-3=-6$$

$$2x=9 \qquad 2x=-3$$

$$x=\frac{9}{2} \qquad x=-\frac{3}{2}$$

The solution set is $\left\{-\frac{3}{2}, \frac{9}{2}\right\}$.

109. $4(w-3)=w+3$

$$4w-12=w+3$$

$$3w=15$$

$$w=5$$

The solution set is $\{5\}$.

110. $6(k+3)-2k=12$

$$6k+18-2k=12$$

$$4k=-6$$

$$k=-\frac{3}{2}$$

The solution set is $\left\{-\frac{3}{2}\right\}$.

111. $\left(\frac{v}{v+1}\right)^2 + \frac{2v}{v+1} = 8$

Let $u = \frac{v}{v+1}$. Rewrite the equation:

$$u^2 + 2u = 8$$

$$u^2 + 2u - 8 = 0$$

$$(u-2)(u+4) = 0$$

$$u = 2 \quad \text{or} \quad u = -4$$

Go back in terms of v and solve:

$$\frac{v}{v+1} = 2 \quad \text{or} \quad \frac{v}{v+1} = -4$$

$$v = 2v+2 \qquad v = -4v-4$$

$$-v = 2 \qquad 5v = -4$$

$$v = -2 \qquad v = -\frac{4}{5}$$

The solution set is $\left\{-2, -\frac{4}{5}\right\}$.

112. $\left(\frac{y}{y-1}\right)^2 - \frac{6y}{y-1} = 7$

Let $u = \frac{y}{y-1}$. Rewrite the equation:

$$u^2 - 6u = 7$$

$$u^2 - 6u - 7 = 0$$

$$(u-7)(u+1) = 0$$

$$u = 7 \quad \text{or} \quad u = -1$$

Go back in terms of y and solve:

$$\frac{y}{y-1} = 7 \quad \text{or} \quad \frac{y}{y-1} = -1$$

$$y = 7y-7 \qquad y = -y+1$$

$$-6y = -7 \qquad 2y = 1$$

$$y = \frac{7}{6} \qquad y = \frac{1}{2}$$

The solution set is $\left\{\frac{1}{2}, \frac{7}{6}\right\}$.

113. $|-3x+2|=x+10$

$$-3x+2=-(x+10) \quad \text{or} \quad -3x+2=x+10$$

$$-3x+2=-x-10 \qquad -4x=8$$

$$-2x=-12 \qquad x=-2$$

$$x=6$$

The solution set is $\{-2, 6\}$.

Section 1.5: Radical Equations; Equations Quadratic in Form; Absolute Value Equations; Factorable Equations

114. $|4x-3|=x+2$

$$4x-3=-(x+2) \quad \text{or} \quad 4x-3=x+2$$

$$4x-3=-x-2 \quad 3x=5$$

$$5x=1 \quad x=\frac{5}{3}$$

$$x=\frac{1}{5}$$

The solution set is $\left\{\frac{1}{5}, \frac{5}{3}\right\}$.

115. $\sqrt{2x+5}-x=1$

$$\sqrt{2x+5}=x+1$$

$$(\sqrt{2x+5})^2=(x+1)^2$$

$$2x+5=x^2+2x+1$$

$$x^2-4=0$$

$$(x-2)(x+2)=0$$

$$x=2 \quad \text{or} \quad x=-2$$

Check:

$$\sqrt{2(-2)+5}-(-2)=1 \quad \sqrt{2(2)+5}-(2)=1$$

$$\sqrt{1}+2=1 \quad \sqrt{9}-2=1$$

$$3 \neq 1 \quad 1=1 \quad \text{T}$$

The solution set is $\{2\}$.

116. $\sqrt{3x+1}-2x=-6$

$$\sqrt{3x+1}=2x-6$$

$$(\sqrt{3x+1})^2=(2x-6)^2$$

$$3x+1=4x^2-24x+36$$

$$4x^2-27x+35=0$$

$$(4x-7)(x-5)=0$$

$$4x-7=0 \quad \text{or} \quad x-5=0$$

$$4x=7 \quad x=5$$

$$x=\frac{7}{4}$$

Check:

$$\sqrt{3\left(\frac{7}{4}\right)+1}-2\left(\frac{7}{4}\right)=-6 \quad \sqrt{3(5)+1}-2(5)=-6$$

$$4-10=-6$$

$$\frac{5}{2}-\frac{7}{2}=-6$$

$$-6=-6 \quad \text{T}$$

$$-1 \neq -6$$

The solution set is $\{5\}$.

117. $3m^2+6m=-1$

$$3m^2+6m+1=0$$

$$a=3, \quad b=6, \quad c=1$$

$$m=\frac{-6 \pm \sqrt{6^2-4(3)(1)}}{2(3)}=\frac{-6 \pm \sqrt{24}}{6}$$

$$=\frac{-6 \pm 2\sqrt{6}}{6}=\frac{-3 \pm \sqrt{6}}{3}$$

The solution set is $\left\{\frac{-3-\sqrt{6}}{3}, \frac{-3+\sqrt{6}}{3}\right\}$.

118. $4y^2-8y=3$

$$4y^2-8y-3=0$$

$$a=4, \quad b=-8, \quad c=-3$$

$$y=\frac{-(-8) \pm \sqrt{(-8)^2-4(4)(-3)}}{2(4)}$$

$$=\frac{8 \pm \sqrt{112}}{8}=\frac{8 \pm 4\sqrt{7}}{8}=\frac{2 \pm \sqrt{7}}{2}$$

The solution set is $\left\{\frac{2-\sqrt{7}}{2}, \frac{2+\sqrt{7}}{2}\right\}$.

119. $|x^2+x-1|=1$

$$x^2+x-1=1 \quad \text{or} \quad x^2+x-1=-1$$

$$x^2+x-2=0 \quad x^2+x=0$$

$$(x+2)(x-1)=0 \quad x(x+1)=0$$

$$x=-2 \quad \text{or} \quad x=1 \quad x=0 \quad \text{or} \quad x=-1$$

The solution set is $\{-2, -1, 0, 1\}$.

120. $|x^2+3x-2|=2$

$$x^2+3x-2=2 \quad \text{or} \quad x^2+3x-2=-2$$

$$x^2+3x-4=0 \quad x^2+3x=0$$

$$(x+4)(x-1)=0 \quad x(x+3)=0$$

$$x=-4 \quad \text{or} \quad x=1 \quad x=0 \quad \text{or} \quad x=-3$$

The solution set is $\{-4, -3, 0, 1\}$.

Chapter 1: Graphs, Equations, and Inequalities

121. $\sqrt[4]{5x^2 - 6} = x$

$$\left(\sqrt[4]{5x^2 - 6}\right)^4 = x^4$$

$$5x^2 - 6 = x^4$$

$$x^4 - 5x^2 + 6 = 0$$

$$(x^2 - 3)(x^2 - 2) = 0$$

$$x^2 - 3 = 0 \quad \text{or} \quad x^2 - 2 = 0$$

$$x^2 = 3 \quad x^2 = 2$$

$$x = \pm\sqrt{3} \quad x = \pm\sqrt{2}$$

Since an even root cannot be negative, we can exclude $-\sqrt{3}$ and $-\sqrt{2}$ as solutions.

Check:

$$\sqrt[4]{5(\sqrt{2})^2} - 6 = \sqrt{2} \quad \sqrt[4]{5(\sqrt{3})^2} - 6 = \sqrt{3}$$

$$\sqrt[4]{4} = \sqrt{2} \quad \sqrt[4]{9} = \sqrt{3}$$

$$\sqrt[4]{2^2} = \sqrt{2} \quad \sqrt[4]{3^2} = \sqrt{3}$$

$$\sqrt{2} = \sqrt{2} \quad \sqrt{3} = \sqrt{3} \quad \text{T}$$

The solution set is $\{\sqrt{2}, \sqrt{3}\}$.

122. $\sqrt[4]{4 - 3x^2} = x$

$$\left(\sqrt[4]{4 - 3x^2}\right)^4 = x^4$$

$$4 - 3x^2 = x^4$$

$$x^4 + 3x^2 - 4 = 0$$

$$(x^2 + 4)(x^2 - 1) = 0$$

$$x^2 + 4 = 0 \quad \text{or} \quad x^2 - 1 = 0$$

$$x^2 = -4 \quad x^2 = 1$$

$$\text{no real solution} \quad x = \pm 1$$

Since an even root cannot be negative, we can exclude -1 as a solution.

Check:

$$\sqrt[4]{4 - 3(1)^2} = 1$$

$$\sqrt[4]{1} = 1$$

$$1 = 1 \quad \text{T}$$

The solution set is $\{1\}$.

123. $t^4 - 16 = 0$

$$(t^2 - 4)(t^2 + 4) = 0$$

$$t^2 - 4 = 0 \quad \text{or} \quad t^2 + 4 = 0$$

$$t^2 = 4 \quad t^2 = -4$$

$$t = \pm 2 \quad t = \pm 2i$$

The solution set is $\{-2i, 2i, -2, 2\}$.

124. $y^4 - 81 = 0$

$$(y^2 - 9)(y^2 + 9) = 0$$

$$y^2 - 9 = 0 \quad \text{or} \quad y^2 + 9 = 0$$

$$y^2 = 9 \quad y^2 = -9$$

$$y = \pm 3 \quad y = \pm 3i$$

The solution set is $\{-3i, 3i, -3, 3\}$.

125. $x^6 - 9x^3 + 8 = 0$

$$(x^3 - 8)(x^3 - 1) = 0$$

$$(x - 2)(x^2 + 2x + 4)(x - 1)(x^2 + x + 1) = 0$$

$$x - 2 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = 2 \quad x = 1$$

or $x^2 + 2x + 4 = 0$

$$a = 1, b = 2, c = 4$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)} = \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2i\sqrt{3}}{2} = -1 \pm i\sqrt{3}$$

or

$$x^2 + x + 1 = 0$$

$$a = 1, b = 1, c = 1$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

The solution set is $\left\{-\frac{1}{2} - \frac{\sqrt{3}}{2}i, -1 - i\sqrt{3},\right.$

$\left.-\frac{1}{2} + \frac{\sqrt{3}}{2}i, -1 + i\sqrt{3}, 1, 2\right\}$.

Section 1.5: Radical Equations; Equations Quadratic in Form; Absolute Value Equations; Factorable Equations

$$126. \quad z^6 + 28z^3 + 27 = 0$$

$$(z^3 + 27)(z^3 + 1) = 0$$

$$(z + 3)(z^2 - 3z + 9)(z + 1)(z^2 - z + 1) = 0$$

$$z + 3 = 0 \quad \text{or} \quad z + 1 = 0$$

$$z = -3 \quad \quad \quad z = -1$$

$$\text{or} \quad z^2 - 3z + 9 = 0$$

$$a = 1, b = -3, c = 9$$

$$z = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{-27}}{2} = \frac{3 \pm 3i\sqrt{3}}{2}$$

$$\text{or} \quad z^2 - z + 1 = 0$$

$$a = 1, b = -1, c = 1$$

$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

$$\text{The solution set is } \left\{ \frac{3}{2} - \frac{3\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i, \right.$$

$$\left. \frac{3}{2} + \frac{3\sqrt{3}}{2}i, \frac{1}{2} + \frac{\sqrt{3}}{2}i, -3, -1 \right\}.$$

$$127. \quad k^2 - k = 12 \Rightarrow k^2 - k - 12 = 0$$

$$(k - 4)(k + 3) = 0$$

$$k = 4 \quad \text{or} \quad k = -3$$

$$\frac{x+3}{x-3} = 4 \quad \quad \quad \frac{x+3}{x-3} = -3$$

$$x + 3 = 4x - 12 \quad \quad \quad x + 3 = -3x + 9$$

$$3x = 15$$

$$x = 5$$

$$x = \frac{3}{2}$$

And since neither of these x values causes a denominator to equal zero, the solution set is

$$\left\{ \frac{3}{2}, 5 \right\}.$$

$$128. \quad k^2 - 3k = 28 \Rightarrow k^2 - 3k - 28 = 0$$

$$(k + 4)(k - 7) = 0$$

$$k = -4 \quad \text{or} \quad k = 7$$

$$\frac{x+3}{x-4} = -4 \quad \quad \quad \frac{x+3}{x-4} = 7$$

$$x + 3 = -4x + 16 \quad \quad \quad x + 3 = 7x - 28$$

$$x = \frac{13}{5} \quad \quad \quad x = \frac{31}{6}$$

And since neither of these x values causes a denominator to equal zero, the solution set is

$$\left\{ \frac{13}{5}, \frac{31}{6} \right\}.$$

$$129. \quad |8 - 3x| = |2x - 7|$$

$$8 - 3x = 2x - 7 \quad \text{or} \quad 8 - 3x = -(2x - 7)$$

$$-3x - 2x = -7 - 8 \quad \quad \quad 8 - 3x = -2x + 7$$

$$-5x = -15 \quad \quad \quad -3x + 2x = 7 - 8$$

$$x = -3 \quad \quad \quad -x = -1$$

$$x = 1$$

The solution set is $\{-3, 1\}$.

$$130. \quad |5x + 3| = |12 - 4x|$$

$$5x + 3 = 12 - 4x \quad \text{or} \quad 5x + 3 = -(12 - 4x)$$

$$5x + 4x = 12 - 3 \quad \quad \quad 5x + 3 = -12 + 4x$$

$$9x = 9 \quad \quad \quad 5x - 4x = -12 - 3$$

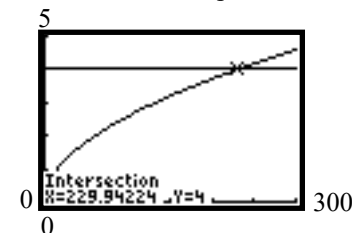
$$x = 1 \quad \quad \quad x = -15$$

The solution set is $\{1, -15\}$.

$$131. \quad \text{Graph the equations } y_1 = \sqrt{x}/4 + x/1100 \text{ and}$$

$$y_2 = 4; \text{ then use INTERSECT to find the}$$

x -coordinate of the points of intersection:



The distance to the water's surface is approximately 229.94 feet.

Chapter 1: Graphs, Equations, and Inequalities

132. $T = \sqrt[4]{\frac{LH^2}{25}}$

Let $T = 4$ and $H = 10$, and solve for L .

$$4 = \sqrt[4]{\frac{L(10)^2}{25}}$$

$$4 = \sqrt[4]{4L}$$

$$(4)^4 = (\sqrt[4]{4L})^4$$

$$256 = 4L$$

$$64 = L$$

The crushing load is 64 tons.

133. $T = 2\pi\sqrt{\frac{l}{32}}$

Let $T = 16.5$ and solve for l .

$$16.5 = 2\pi\sqrt{\frac{l}{32}}$$

$$\frac{16.5}{2\pi} = \sqrt{\frac{l}{32}}$$

$$\left(\frac{16.5}{2\pi}\right)^2 = \left(\sqrt{\frac{l}{32}}\right)^2$$

$$\left(\frac{16.5}{2\pi}\right)^2 = \frac{l}{32}$$

$$l = 32\left(\frac{16.5}{2\pi}\right)^2 \approx 220.7$$

The length was approximately 221 feet.

134. $\sqrt{3x+5} - \sqrt{x-2} = \sqrt{x+3}$
 $(\sqrt{3x+5} - \sqrt{x-2})^2 = (\sqrt{x+3})^2$

$$3x+5-2\sqrt{(3x+5)(x-2)}+(x-2)=x+3$$

$$4x+3-2\sqrt{(3x+5)(x-2)}=x+3$$

$$2\sqrt{3x^2-x-10}=3x$$

$$4(3x^2-x-10)=9x^2$$

$$3x^2-4x-40=0$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(3)(-40)}}{6}$$

$$= \frac{4 \pm \sqrt{496}}{6} = \frac{4 \pm 4\sqrt{31}}{6}$$

$$= \frac{4 \pm 4\sqrt{31}}{6}$$

$$= \frac{2 \pm 2\sqrt{31}}{3}$$

Since $x > 2$, the negative solution is extraneous.

The solution set is $\left\{\frac{2+2\sqrt{31}}{3}\right\}$.

135.

$$\sqrt[4]{\sqrt[3]{\sqrt{x-7}-10}+18}=2$$

$$\left(\sqrt[4]{\sqrt[3]{\sqrt{x-7}-10}+18}\right)^4=(2)^4$$

$$\sqrt[3]{\sqrt{x-7}-10}+18=16$$

$$\left(\sqrt[3]{\sqrt{x-7}-10}\right)^3=(-2)^3$$

$$\sqrt{x-7}-10=-8$$

$$(\sqrt{x-7})^2=(2)^2$$

$$x-7=4$$

$$x=11$$

The solution set is $\{11\}$

136. $12x^{7/5} + 3x^{3/5} = 13x^{9/10}$

$$12x^{14/10} + 3x^{6/10} - 13x^{9/10} = 0$$

$$x^{9/10}(12x-13x^{1/2}+3)=0$$

$$x^{9/10}=0 \Rightarrow x=0$$

To solve $12x-13x^{1/2}+3=0$, let $u=x^{1/2}$

$$\text{Then } 12u^2-13u+3=0$$

$$(4u-3)(3u-1)=0$$

$$u=x^{1/2}=\frac{3}{4} \text{ or } x^{1/2}=\frac{1}{3}$$

$$x=\frac{9}{16} \quad x=\frac{1}{9}$$

The solution set is $\left\{0, \frac{9}{16}, \frac{1}{9}\right\}$

137. $z^6 + 28z^3 + 27 = 0$

$$(z^3+27)(z^3+1)=0$$

$$(z+3)(z^2-3z+9)(z+1)(z^2-z+1)=0$$

$$z+3=0 \quad \text{or } z+1=0$$

$$z=-3 \quad \text{or } z=-1$$

Section 1.6: Problem Solving; Interest, Mixture, Uniform Motion, Constant Rate Job Applications

$$a = 1, b = -3, c = 9$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2}$$

$$= \frac{3 \pm \sqrt{-27}}{2} = \frac{3 \pm 3\sqrt{3}i}{2}$$

Also,

$$a = 1, b = -1, c = 1$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2}$$

$$= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}i}{2}$$

$$\text{The solution set is } \left\{ -3, -1, \frac{1 \pm \sqrt{3}i}{2}, \frac{3 \pm 3\sqrt{3}i}{2} \right\}.$$

138. Answers will vary, one example is $\sqrt{x+1} = -1$

139. Answers will vary, one example is

$$x - \sqrt{x} - 2 = 0$$

140. Answers will vary.

141. $|x|$ represents the distance from x to the origin.

Since distance is never negative, $|x| = -2$ has no real solution.

142. Mya did not check her solutions and included the extraneous solution, $x = -1$.

$$\sqrt{2x+3} - x = 0$$

$$\sqrt{2x+3} = x$$

$$(\sqrt{2x+3})^2 = x^2$$

$$2x+3 = x^2$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3 \text{ or } x = -1$$

Check:

$$\sqrt{2(3)+3} - 3 = 0 \qquad \sqrt{2(-1)+3} - (-1) = 0$$

$$\sqrt{9} - 3 = 0 \qquad \sqrt{1} + 1 = 0$$

$$3 - 3 = 0 \qquad 1 + 1 = 0$$

$$0 = 0 \quad \text{T} \qquad 2 \neq 0$$

The solution set is $\{3\}$.

Section 1.6

1. mathematical modeling

2. interest

3. uniform motion

4. False; the amount charged for the use of principal is the interest.

5. True; this is the uniform motion formula.

6. a

7. b

8. c

9. Let A represent the area of the circle and r the radius. The area of a circle is the product of π times the square of the radius: $A = \pi r^2$

10. Let C represent the circumference of a circle and r the radius. The circumference of a circle is the product of π times twice the radius: $C = 2\pi r$

11. Let A represent the area of the square and s the length of a side. The area of the square is the square of the length of a side: $A = s^2$

12. Let P represent the perimeter of a square and s the length of a side. The perimeter of a square is four times the length of a side: $P = 4s$

13. Let F represent the force, m the mass, and a the acceleration. Force equals the product of the mass times the acceleration: $F = ma$

14. Let P represent the pressure, F the force, and A the area. Pressure is the force per unit area: $P = \frac{F}{A}$

15. Let W represent the work, F the force, and d the distance. Work equals force times distance: $W = Fd$

16. Let K represent the kinetic energy, m the mass, and v the velocity. Kinetic energy is one-half the product of the mass and the square of the velocity: $K = \frac{1}{2}mv^2$

Chapter 1: Graphs, Equations, and Inequalities

17. C = total variable cost in dollars, x = number of dishwashers manufactured: $C = 150x$
18. R = total revenue in dollars, x = number of dishwashers sold: $R = 250x$
19. Let x represent the amount of money invested in bonds. Then $50,000 - x$ represents the amount of money invested in CD's. Since the total interest is to be \$6,000, we have:

$$\begin{aligned}0.15x + 0.07(50,000 - x) &= 6,000 \\(100)(0.15x + 0.07(50,000 - x)) &= (6,000)(100) \\15x + 7(50,000 - x) &= 600,000 \\15x + 350,000 - 7x &= 600,000 \\8x + 350,000 &= 600,000 \\8x &= 250,000 \\x &= 31,250\end{aligned}$$

\$31,250 should be invested in bonds at 15% and \$18,750 should be invested in CD's at 7%.

20. Let x represent the amount of money invested in bonds. Then $50,000 - x$ represents the amount of money invested in CD's. Since the total interest is to be \$7,000, we have:
- $$\begin{aligned}0.15x + 0.07(50,000 - x) &= 7,000 \\(100)(0.15x + 0.07(50,000 - x)) &= (7,000)(100) \\15x + 7(50,000 - x) &= 700,000 \\15x + 350,000 - 7x &= 700,000 \\8x + 350,000 &= 700,000 \\8x &= 350,000 \\x &= 43,750\end{aligned}$$

\$43,750 should be invested in bonds at 15% and \$6,250 should be invested in CD's at 7%.

21. Let x represent the amount of money loaned at 4%. Then $12,000 - x$ represents the amount of money loaned at 5.5%. Since the total interest is to be \$600, we have:
- $$\begin{aligned}0.04x + 0.055(12,000 - x) &= 600 \\0.04x + 660 - 0.055x &= 600 \\-0.015x + 660 &= 600 \\-0.015x &= -60 \\x &= 4,000\end{aligned}$$
- \$4,000 is loaned at 4%.

22. Let x represent the interest rate.
- $$\begin{aligned}16,350 &= 15,000 + 15,000(x)(2) \\16,350 &= 15,000 + 30,000x \\1,350 &= 30,000x \\x &= \frac{1,350}{30,000} \\x &= 0.045 \\x &= 4.5\%\end{aligned}$$

The interest rate was 4.5% per year..

23. Let x represent the number of pounds of Earl Grey tea. Then $100 - x$ represents the number of pounds of orange pekoe tea.
- $$\begin{aligned}6x + 4(100 - x) &= 5.50(100) \\6x + 400 - 4x &= 550 \\2x + 400 &= 550 \\2x &= 150 \\x &= 75\end{aligned}$$

75 pounds of Earl Grey tea must be blended with 25 pounds of orange pekoe.

24. Let x represent the number of pounds of the first kind of coffee. Then $100 - x$ represents the number of pounds of the second kind of coffee.
- $$\begin{aligned}2.75x + 5(100 - x) &= 4.10(100) \\2.75x + 500 - 5x &= 410 \\-2.25x + 500 &= 410 \\-2.25x &= -90 \\x &= 40\end{aligned}$$

40 pounds of the first kind of coffee must be blended with 60 pounds of the second kind of coffee.

25. Let x represent the number of pounds of cashews. Then $x + 60$ represents the number of pounds in the mixture.
- $$\begin{aligned}9x + 4.50(60) &= 7.75(x + 60) \\9x + 270 &= 7.75x + 465 \\1.25x &= 195 \\x &= 156\end{aligned}$$

156 pounds of cashews must be added to the 60 pounds of almonds.

Section 1.6: Problem Solving; Interest, Mixture, Uniform Motion, Constant Rate Job Applications

26. Let x represent the number of caramels in the box. Then $30 - x$ represents the number of cremes in the box.

$$\text{Revenue} - \text{Cost} = \text{Profit}$$

$$12.50 - (0.25x + 0.45(30 - x)) = 3.00$$

$$12.50 - (0.25x + 13.5 - 0.45x) = 3.00$$

$$12.50 - (13.5 - 0.20x) = 3.00$$

$$12.50 - 13.50 + 0.20x = 3.00$$

$$-1.00 + 0.20x = 3.00$$

$$0.20x = 4.00$$

$$x = 20$$

The box should contain 20 caramels and 10 cremes.

27. Let r represent the speed of the current.

	Rate	Time	Distance
Upstream	$16 - r$	$\frac{20}{60} = \frac{1}{3}$	$\frac{16 - r}{3}$
Downstream	$16 + r$	$\frac{15}{60} = \frac{1}{4}$	$\frac{16 + r}{4}$

Since the distance is the same in each direction:

$$\frac{16 - r}{3} = \frac{16 + r}{4}$$

$$4(16 - r) = 3(16 + r)$$

$$64 - 4r = 48 + 3r$$

$$16 = 7r$$

$$r = \frac{16}{7} \approx 2.286$$

The speed of the current is approximately 2.286 miles per hour.

28. Let r represent the speed of the motorboat.

	Rate	Time	Distance
Upstream	$r - 3$	5	$5(r - 3)$
Downstream	$r + 3$	2.5	$2.5(r + 3)$

The distance is the same in each direction:

$$5(r - 3) = 2.5(r + 3)$$

$$5r - 15 = 2.5r + 7.5$$

$$2.5r = 22.5$$

$$r = 9$$

The speed of the motorboat is 9 miles per hour.

29. Let r represent the speed of the current.

	Rate	Time	Distance
Upstream	$15 - r$	$\frac{10}{15 - r}$	10
Downstream	$15 + r$	$\frac{10}{15 + r}$	10

Since the total time is 1.5 hours, we have:

$$\frac{10}{15 - r} + \frac{10}{15 + r} = 1.5$$

$$10(15 + r) + 10(15 - r) = 1.5(15 - r)(15 + r)$$

$$150 + 10r + 150 - 10r = 1.5(225 - r^2)$$

$$300 = 1.5(225 - r^2)$$

$$200 = 225 - r^2$$

$$r^2 - 25 = 0$$

$$(r - 5)(r + 5) = 0$$

$$r = 5 \text{ or } r = -5$$

Speed must be positive, so disregard $r = -5$.

The speed of the current is 5 miles per hour.

30. Let r represent the rate of the slower car. Then $r + 10$ represents the rate of the faster car.

	Rate	Time	Distance
Slower car	r	3.5	$3.5r$
Faster car	$r + 10$	3	$3(r + 10)$

$$3.5r = 3(r + 10)$$

$$3.5r = 3r + 30$$

$$0.5r = 30$$

$$r = 60$$

The slower car travels at a rate of 60 miles per hour. The faster car travels at a rate of 70 miles per hour. The distance is $(70)(3) = 210$ miles.

31. Let r represent Karen's normal walking speed.

	Rate	Time	Distance
With walkway	$r + 2.5$	$\frac{50}{r + 2.5}$	50
Against walkway	$r - 2.5$	$\frac{50}{r - 2.5}$	50

Since the total time is 48 seconds:

$$\frac{50}{r + 2.5} + \frac{50}{r - 2.5} = 48$$

$$50(r - 2.5) + 50(r + 2.5) = 48(r - 2.5)(r + 2.5)$$

$$50r - 125 + 50r + 125 = 48(r^2 - 6.25)$$

$$100r = 48r^2 - 300$$

$$0 = 48r^2 - 100r - 300$$

$$0 = 12r^2 - 25r - 75$$

$$r = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(12)(-75)}}{2(12)}$$

$$= \frac{25 \pm \sqrt{4225}}{24}$$

$$r \approx 3.75 \text{ or } r \approx -1.67$$

Speed must be positive, so disregard $r \approx -1.67$.

Karen's normal walking speed is approximately 3.75 feet per second.

Chapter 1: Graphs, Equations, and Inequalities

32. Let r represent the speed of the airport walkway.

	Rate	Time	Distance
Walking with	$1.5 + r$	$\frac{280}{1.5 + r}$	280
Standing still	r	$\frac{280}{r}$	280

Walking with the walkway takes 60 seconds less time than standing still on the walkway:

$$\begin{aligned}\frac{280}{1.5 + r} &= \frac{280}{r} - 60 \\ 280r &= 280(1.5 + r) - 60r(r + 1.5) \\ 280r &= 420 + 280r - 60r^2 - 90r\end{aligned}$$

$$60r^2 + 90r - 420 = 0$$

$$2r^2 + 3r - 14 = 0$$

$$(2r + 7)(r - 2) = 0$$

$$2r + 7 = 0 \quad \text{or} \quad r - 2 = 0$$

$$r = -\frac{7}{2} \quad \text{or} \quad r = 2$$

Speed must be positive, so disregard $r = -\frac{7}{2}$.

The speed of the airport walkway is 2 meters per second.

33. Let w represent the width of a regulation doubles tennis court. Then $2w + 6$ represents the length. The area is 2808 square feet:

$$w(2w + 6) = 2808$$

$$2w^2 + 6w = 2808$$

$$2w^2 + 6w - 2808 = 0$$

$$w^2 + 3w - 1404 = 0$$

$$(w + 39)(w - 36) = 0$$

$$w + 39 = 0 \quad \text{or} \quad w - 36 = 0$$

$$w = -39 \quad \text{or} \quad w = 36$$

The width must be positive, so disregard $w = -39$.

The width of a regulation doubles tennis court is 36 feet and the length is $2(36) + 6 = 78$ feet.

34. Let L represent the length of the pickleball court. The width is 2 feet less than half the length,

$$W = \frac{L}{2} - 2.$$

The area of the court is $LW = 880$.

$$L\left(\frac{L}{2} - 2\right) = 880$$

$$L^2 - 4L = 1,760$$

$$L^2 - 4L - 1,760 = 0$$

Using quadratic formula to solve L ,

$$L = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1760)}}{2(1)}$$

$$L = \frac{4 \pm \sqrt{16 + 7040}}{2}$$

$$L = \frac{4 \pm 84}{2}$$

$$L = 44 \quad \text{or} \quad L = -40$$

The length must be positive, so disregard

$$L = -40.$$

The width of the pickleball court is

$$W = \frac{44}{2} - 2 = 22 - 2 = 20.$$

The dimensions of the court are 44 feet by 20 feet.

35. Let t represent the time it takes to do the job together.

	Time to do job	Part of job done in one minute
Trent	30	$\frac{1}{30}$
Lois	20	$\frac{1}{20}$
Together	t	$\frac{1}{t}$

$$\frac{1}{30} + \frac{1}{20} = \frac{1}{t}$$

$$2t + 3t = 60$$

$$5t = 60$$

$$t = 12$$

Working together, the job can be done in 12 minutes.

36. Let t represent the time it takes April to do the job working alone.

	Time to do job	Part of job done in one hour
Patrice	10	$\frac{1}{10}$
April	t	$\frac{1}{t}$
Together	6	$\frac{1}{6}$

$$\frac{1}{10} + \frac{1}{t} = \frac{1}{6}$$

$$3t + 30 = 5t$$

$$2t = 30$$

$$t = 15$$

April would take 15 hours to paint the rooms.

Section 1.6: Problem Solving; Interest, Mixture, Uniform Motion, Constant Rate Job Applications

37. l = length of the garden
 w = width of the garden

- a. The length of the garden is to be twice its width. Thus, $l = 2w$.
 The dimensions of the fence are $l + 4$ and $w + 4$.

The perimeter is 46 feet, so:

$$2(l + 4) + 2(w + 4) = 46$$

$$2(2w + 4) + 2(w + 4) = 46$$

$$4w + 8 + 2w + 8 = 46$$

$$6w + 16 = 46$$

$$6w = 30$$

$$w = 5$$

The dimensions of the garden are 5 feet by 10 feet.

- b. Area = $l \cdot w = 5 \cdot 10 = 50$ square feet
- c. If the dimensions of the garden are the same, then the length and width of the fence are also the same ($l + 4$). The perimeter is 46 feet, so:
- $$2(l + 4) + 2(l + 4) = 46$$
- $$2l + 8 + 2l + 8 = 46$$
- $$4l + 16 = 46$$
- $$4l = 30$$
- $$l = 7.5$$

The dimensions of the garden are 7.5 feet by 7.5 feet.

- d. Area = $l \cdot w = 7.5(7.5) = 56.25$ square feet.

38. l = length of the pond
 w = width of the pond

- a. The pond is to be a square. Thus, $l = w$.
 The dimensions of the fenced area are $w + 6$ on each side. The perimeter is 100 feet, so:

$$4(w + 6) = 100$$

$$4w + 24 = 100$$

$$4w = 76$$

$$w = 19$$

The dimensions of the pond are 19 feet by 19 feet.

- b. The length of the pond is to be three times the width. Thus, $l = 3w$. The dimensions of the fenced area are $w + 6$ and $l + 6$. The perimeter is 100 feet, so:

$$2(w + 6) + 2(l + 6) = 100$$

$$2(w + 6) + 2(3w + 6) = 100$$

$$2w + 12 + 6w + 12 = 100$$

$$8w + 24 = 100$$

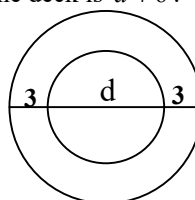
$$8w = 76$$

$$w = 9.5$$

$$l = 3(9.5) = 28.5$$

The dimensions of the pond are 9.5 feet by 28.5 feet.

- c. If the pond is circular, the diameter is d and the diameter of the circle with the pond and the deck is $d + 6$.



The perimeter is 100 feet, so:

$$\pi(d + 6) = 100$$

$$\pi d + 6\pi = 100$$

$$\pi d = 100 - 6\pi$$

$$d = \frac{100}{\pi} - 6 \approx 25.83$$

The diameter of the pond is 25.83 feet.

- d. Area_{square} = $l \cdot w = 19(19) = 361$ ft².
 Area_{rectangle} = $l \cdot w = 28.5(9.5) = 270.75$ ft².
 Area_{circle} = $\pi r^2 = \pi \left(\frac{25.83}{2} \right)^2 \approx 524$ ft².

The circular pond has the largest area.

39. Let t represent the time it takes for the defensive back to catch the tight end.

	Time to run 100 yards	Time	Rate	Distance
Tight End	12 sec	t	$\frac{100}{12} = \frac{25}{3}$	$\frac{25}{3}t$
Def. Back	10 sec	t	$\frac{100}{10} = 10$	$10t$

Since the defensive back has to run 5 yards farther, we have:

$$\frac{25}{3}t + 5 = 10t$$

$$25t + 15 = 30t$$

$$15 = 5t$$

$$t = 3 \rightarrow 10t = 30$$

The defensive back will catch the tight end at the 45 yard line ($15 + 30 = 45$).

Chapter 1: Graphs, Equations, and Inequalities

40. Let x represent the number of highway miles traveled. Then $30,000 - x$ represents the number of city miles traveled.

$$\begin{aligned}\frac{x}{40} + \frac{30,000 - x}{25} &= 900 \\ 200\left(\frac{x}{40} + \frac{30,000 - x}{25}\right) &= 200(900) \\ 5x + 240,000 - 8x &= 180,000 \\ -3x + 240,000 &= 180,000 \\ -3x &= -60,000 \\ x &= 20,000\end{aligned}$$

Therese is allowed to claim 20,000 miles as a business expense.

41. Let x represent the number of gallons of pure water. Then $x + 1$ represents the number of gallons in the 60% solution.

$$\begin{aligned}(\%)(\text{gallons}) + (\%)(\text{gallons}) &= (\%)(\text{gallons}) \\ 0(x) + 1(1) &= 0.60(x + 1) \\ 1 &= 0.6x + 0.6 \\ 0.4 &= 0.6x \\ x &= \frac{4}{6} = \frac{2}{3}\end{aligned}$$

$\frac{2}{3}$ gallon of pure water should be added.

42. Let x represent the number of liters to be drained and replaced with pure antifreeze.

$$\begin{aligned}(\%)(\text{liters}) + (\%)(\text{liters}) &= (\%)(\text{liters}) \\ 1(x) + 0.40(15 - x) &= 0.60(15) \\ x + 6 - 0.40x &= 9 \\ 0.60x &= 3 \\ x &= 5\end{aligned}$$

5 liters should be drained and replaced with pure antifreeze.

43. Let x represent the number of ounces of water to be evaporated; the amount of salt remains the same. Therefore, we get

$$\begin{aligned}0.04(32) &= 0.06(32 - x) \\ 1.28 &= 1.92 - 0.06x \\ 0.06x &= 0.64 \\ x &= \frac{0.64}{0.06} = \frac{64}{6} = \frac{32}{3} = 10\frac{2}{3}\end{aligned}$$

$10\frac{2}{3} \approx 10.67$ ounces of water need to be evaporated.

44. Let x represent the number of gallons of water to be evaporated; the amount of salt remains the same.

$$\begin{aligned}0.03(240) &= 0.05(240 - x) \\ 7.2 &= 12 - 0.05x \\ 0.05x &= 4.8 \\ x &= \frac{4.8}{0.05} = 96\end{aligned}$$

96 gallons of water need to be evaporated.

45. Let x represent the number of grams of pure gold. Then $60 - x$ represents the number of grams of 12 karat gold to be used.

$$\begin{aligned}x + \frac{1}{2}(60 - x) &= \frac{2}{3}(60) \\ x + 30 - 0.5x &= 40 \\ 0.5x &= 10 \\ x &= 20\end{aligned}$$

20 grams of pure gold should be mixed with 40 grams of 12 karat gold.

46. Let x represent the number of atoms of oxygen. $2x$ represents the number of atoms of hydrogen. $x + 1$ represents the number of atoms of carbon.

$$\begin{aligned}x + 2x + x + 1 &= 45 \\ 4x &= 44 \\ x &= 11\end{aligned}$$

There are 11 atoms of oxygen and 22 atoms of hydrogen in the sugar molecule.

47. Let t represent the time it takes for Mike to catch up with Dan. Since the distances are the same, we have:

$$\begin{aligned}\frac{1}{6}t &= \frac{1}{9}(t + 1) \\ 3t &= 2t + 2 \\ t &= 2\end{aligned}$$

Mike will pass Dan after 2 minutes, which is a distance of $\frac{1}{3}$ mile.

48. Let t represent the time of flight with the wind. The distance is the same in each direction:

$$\begin{aligned}330t &= 270(5 - t) \\ 330t &= 1350 - 270t \\ 600t &= 1350 \\ t &= 2.25\end{aligned}$$

The distance the plane can fly and still return safely is $330(2.25) = 742.5$ miles.

Section 1.6: Problem Solving; Interest, Mixture, Uniform Motion, Constant Rate Job Applications

49. Let t represent the time the auxiliary pump needs to run. Since the two pumps are emptying one tanker, we have:

$$\frac{3}{4} + \frac{t}{9} = 1$$

$$27 + 4t = 36$$

$$4t = 9$$

$$t = \frac{9}{4} = 2.25$$

The auxiliary pump must run for 2.25 hours. It must be started at 9:45 a.m.

50. Let x represent the number of pounds of pure cement. Then $x + 20$ represents the number of pounds in the 40% mixture.

$$x + 0.25(20) = 0.40(x + 20)$$

$$x + 5 = 0.4x + 8$$

$$0.6x = 3$$

$$x = \frac{30}{6} = 5$$

5 pounds of pure cement should be added.

51. Let t represent the time for the tub to fill with the faucets on and the stopper removed. Since one tub is being filled, we have:

$$\frac{t}{15} + \left(-\frac{t}{20}\right) = 1$$

$$4t - 3t = 60$$

$$t = 60$$

60 minutes is required to fill the tub.

52. Let t be the time the 5 horsepower pump needs to run to finish emptying the pool. Since the two pumps are emptying one pool, we have:

$$\frac{t+2}{5} + \frac{2}{8} = 1$$

$$4(2+t) + 5 = 20$$

$$8 + 4t + 5 = 20$$

$$4t = 7$$

$$t = 1.75$$

The 5 horsepower pump must run for an additional 1.75 hours or 1 hour and 45 minutes to empty the pool.

53. Let t represent the time spent running. Then $5 - t$ represents the time spent biking.

	Rate	Time	Distance
Run	6	t	$6t$
Bike	25	$5 - t$	$25(5 - t)$

The total distance is 87 miles:

$$6t + 25(5 - t) = 87$$

$$6t + 125 - 25t = 87$$

$$-19t + 125 = 87$$

$$-19t = -38$$

$$t = 2$$

The time spent running is 2 hours, so the distance of the run is $6(2) = 12$ miles. The distance of the bicycle race is $25(5 - 2) = 75$ miles.

54. Let r represent the speed of the eastbound cyclist. Then $r + 5$ represents the speed of the westbound cyclist.

	Rate	Time	Distance
Eastbound	r	6	$6r$
Westbound	$r + 5$	6	$6(r + 5)$

The total distance is 246 miles:

$$6r + 6(r + 5) = 246$$

$$6r + 6r + 30 = 246$$

$$12r + 30 = 246$$

$$12r = 216$$

$$r = 18$$

The speed of the eastbound cyclist is 18 miles per hour, and the speed of the westbound cyclist is $18 + 5 = 23$ miles per hour.

55. Burke's rate is $\frac{100}{12}$ meters/sec. In 9.81 seconds,

Burke will run $\frac{100}{12}(9.81) = 81.75$ meters. Bolt would win by $100 - 81.75 = 18.25$ meters.

56. $A = 2\pi r^2 + 2\pi r h$. Since $A = 58.9\pi$ square inches and $h = 6.4$ inches,

$$2\pi r^2 + 2\pi r(6.4) = 58.9\pi$$

$$2\pi r^2 + 12.8\pi r - 58.9\pi = 0$$

$$2r^2 + 12.8r - 58.9 = 0$$

$$r = \frac{-12.8 \pm \sqrt{(12.8)^2 - 4(2)(-58.9)}}{2(2)}$$

$$= \frac{-12.8 \pm \sqrt{635.04}}{4}$$

$$r = 3.1 \text{ or } r = -9.5$$

The radius of the coffee can is 3.1 inches.

Chapter 1: Graphs, Equations, and Inequalities

57. Let x be the side length of the square piece of sheet metal.
 Since a 1-foot square is cut from each corner and the edges are turned up, then
 $L = x - 2$, $W = x - 2$, and $H = 1$.
 The volume of the box is $LWH = 4$.
 $(x - 2)(x - 2)(1) = 4$
 $(x - 2)^2 = 4$
 $x - 2 = \pm 2$
 $x = 4$, since length cannot be negative
 The dimensions should be 4 feet by 4 feet.
58. Let x be the width of the rectangular sheet. Then the length is $2x$ feet.
 Since a 1-foot square is cut from each corner and the edges are turned up, then
 $L = 2x - 2$, $W = x - 2$, and $H = 1$.
 The volume of the box is $LWH = 4$.
 $(2x - 2)(x - 2)(1) = 4$
 $2x^2 - 4x - 2x + 4 = 4$
 $2x^2 - 6x = 0$
 $2x(x - 3) = 0$
 $x - 3 = 0$
 $x = 3$
 Since width cannot be zero. So $W = 3$
 The length of the box, $L = 2(3) = 6$.
 The dimensions should be 6 feet by 3 feet.
59. Answers will vary.
60. Let x be the original selling price of the shirt.
 Profit = Revenue - Cost
 $4 = x - 0.40x - 20 \rightarrow 24 = 0.60x \rightarrow x = 40$
 The original price should be \$40 to ensure a profit of \$4 after the sale.
 If the sale is 50% off, the profit is:
 $40 - 0.50(40) - 20 = 40 - 20 - 20 = 0$
 At 50% off there will be no profit.
61. Let t_1 and t_2 represent the times for the two segments of the trip. Since Atlanta is halfway between Chicago and Miami, the distances are equal.

$$45t_1 = 55t_2$$

$$t_1 = \frac{55}{45}t_2$$

$$t_1 = \frac{11}{9}t_2$$

Computing the average speed:

$$\begin{aligned} \text{Avg Speed} &= \frac{\text{Distance}}{\text{Time}} = \frac{45t_1 + 55t_2}{t_1 + t_2} \\ &= \frac{45\left(\frac{11}{9}t_2\right) + 55t_2}{\frac{11}{9}t_2 + t_2} = \frac{55t_2 + 55t_2}{\left(\frac{11t_2 + 9t_2}{9}\right)} \\ &= \frac{110t_2}{\left(\frac{20t_2}{9}\right)} = \frac{990t_2}{20t_2} \\ &= \frac{99}{2} = 49.5 \text{ miles per hour} \end{aligned}$$

The average speed for the trip from Chicago to Miami is 49.5 miles per hour.

62. The time traveled with the tail wind was:

$$t = \frac{919}{550} \approx 1.67091 \text{ hours.}$$

Since they were 20 minutes $\left(\frac{1}{3} \text{ hour}\right)$ early, the time in still air would have been:

$$\begin{aligned} 1.67091 \text{ hrs} + 20 \text{ min} &= (1.67091 + 0.33333) \text{ hrs} \\ &\approx 2.00424 \text{ hrs} \end{aligned}$$

Thus, with no wind, the ground speed is

$$\begin{aligned} \frac{919}{2.00424} &\approx 458.53. \text{ Therefore, the tail wind is} \\ 550 - 458.53 &= 91.47 \text{ knots.} \end{aligned}$$

63. It is impossible to mix two solutions with a lower concentration and end up with a new solution with a higher concentration.

Algebraic Solution:

Let x = the number of liters of 25% solution.

$$(\%)(\text{liters}) + (\%)(\text{liters}) = (\%)(\text{liters})$$

$$0.25x + 0.48(20) = 0.58(20 + x)$$

$$0.25x + 9.6 = 10.6 + 0.58x$$

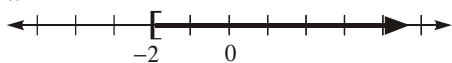
$$-0.33x = 1$$

$$x \approx -3.03 \text{ liters}$$

(not possible)

Section 1.7

1. $x \geq -2$



2. True; the absolute value of a number is the distance the number lies from 0 on a real number line. Negative numbers lie to the left of 0 on a real number line, but still have a positive distance from 0.

3. $A \cup B = \{a, b, c, d, e, i, o, u\}$

4. negative

5. closed interval

6. $-a \leq u \leq a$

7. $(-\infty, a]$

8. True; this follows from the addition property for inequalities.

9. True; this follows from the multiplication property for inequalities.

10. False; since both sides of the inequality are being divided by a negative number, the sense, or direction, of the inequality must be reversed.

That is, $\frac{a}{c} > \frac{b}{c}$.

11. a

12. c

13. Interval: $[0, 2]$

Inequality: $0 \leq x \leq 2$

14. Interval: $(-1, 2)$

Inequality: $-1 < x < 2$

15. Interval: $[2, \infty)$

Inequality: $x \geq 2$

16. Interval: $(-\infty, 0]$

Inequality: $x \leq 0$

17. Interval: $[0, 3)$

Inequality: $0 \leq x < 3$

18. Interval: $(-1, 1]$

Inequality: $-1 < x \leq 1$

19. a. $3 < 5$

$3 + 3 < 5 + 3$

$6 < 8$

b. $3 < 5$

$3 - 5 < 5 - 5$

$-2 < 0$

c. $3 < 5$

$3(3) < 3(5)$

$9 < 15$

d. $3 < 5$

$-2(3) > -2(5)$

$-6 > -10$

20. a. $2 > 1$

$2 + 3 > 1 + 3$

$5 > 4$

b. $2 > 1$

$2 - 5 > 1 - 5$

$-3 > -4$

c. $2 > 1$

$3(2) > 3(1)$

$6 > 3$

d. $2 > 1$

$-2(2) < -2(1)$

$-4 < -2$

21. a. $2x + 1 < 2$

$2x + 1 + 3 < 2 + 3$

$2x + 4 < 5$

b. $2x + 1 < 2$

$2x + 1 - 5 < 2 - 5$

$2x - 4 < -3$

Chapter 1: Graphs, Equations, and Inequalities

c. $2x+1 < 2$
 $3(2x+1) < 3(2)$
 $6x+3 < 6$

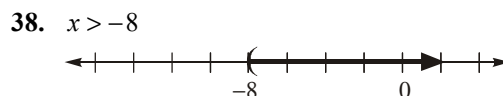
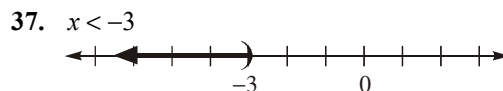
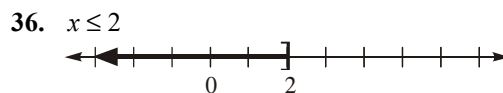
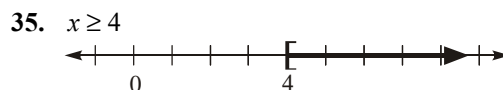
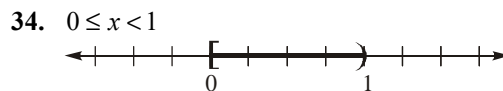
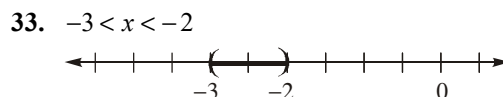
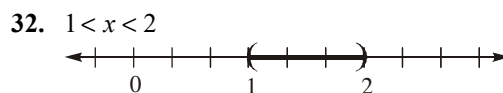
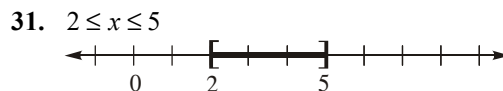
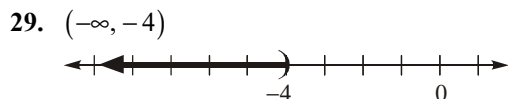
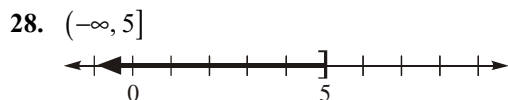
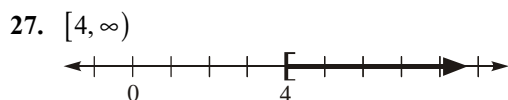
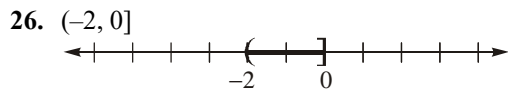
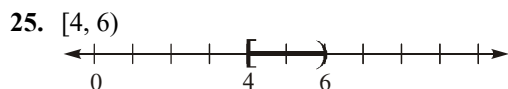
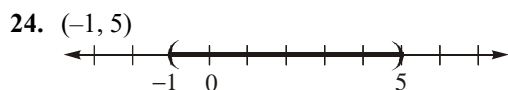
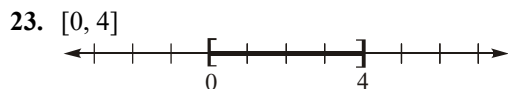
d. $2x+1 < 2$
 $-2(2x+1) > -2(2)$
 $-4x-2 > -4$

22. a. $1-2x > 5$
 $1-2x+3 > 5+3$
 $4-2x > 8$

b. $1-2x > 5$
 $1-2x-5 > 5-5$
 $-2x-4 > 0$

c. $1-2x > 5$
 $3(1-2x) > 3(5)$
 $3-6x > 15$

d. $1-2x > 5$
 $-2(1-2x) < -2(5)$
 $-2+4x < -10$



39. If $x < 5$, then $x-5 < 0$.

40. If $x < -4$, then $x+4 < 0$.

41. If $x > -4$, then $x+4 > 0$.

42. If $x > 6$, then $x-6 > 0$.

43. If $x \geq -4$, then $3x \geq -12$.

44. If $x \leq 3$, then $2x \leq 6$.

45. If $x > 6$, then $-2x < -12$.

46. If $x > -2$, then $-4x < 8$.

47. If $8x < 40$, then $x < 5$.

48. If $3x \leq 12$, then $x \leq 4$.

49. If $-\frac{1}{2}x \leq 3$, then $x \geq -6$.

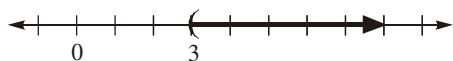
50. If $-\frac{1}{4}x > 1$, then $x < -4$.

51. $3x - 7 > 2$

$$3x > 9$$

$$x > 3$$

$$\{x | x > 3\} \text{ or } (3, \infty)$$



52. $2x + 5 > 1$

$$2x > -4$$

$$x > -2$$

$$\{x | x > -2\} \text{ or } (-2, \infty)$$

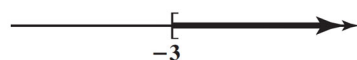


53. $3 - 5x \leq 18$

$$-5x \leq 15$$

$$x \geq -3$$

$$\{x | x \geq -3\} \text{ or } [-3, \infty)$$

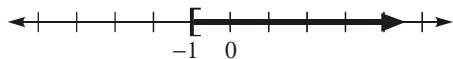


54. $2 - 3x \leq 5$

$$-3x \leq 3$$

$$x \geq -1$$

$$\{x | x \geq -1\} \text{ or } [-1, \infty)$$

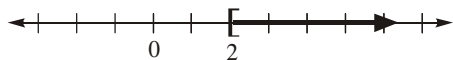


55. $3x - 1 \geq 3 + x$

$$2x \geq 4$$

$$x \geq 2$$

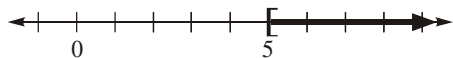
$$\{x | x \geq 2\} \text{ or } [2, \infty)$$



56. $2x - 2 \geq 3 + x$

$$x \geq 5$$

$$\{x | x \geq 5\} \text{ or } [5, \infty)$$



57. $-2(x + 3) < 8$

$$-2x - 6 < 8$$

$$-2x < 14$$

$$x > -7$$

$$\{x | x > -7\} \text{ or } (-7, \infty)$$



58. $-3(1 - x) < 12$

$$-3 + 3x < 12$$

$$3x < 15$$

$$x < 5$$

$$\{x | x < 5\} \text{ or } (-\infty, 5)$$



59. $4 - 3(1 - x) \leq 3$

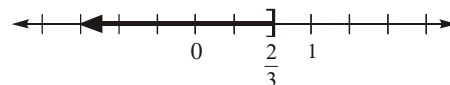
$$4 - 3 + 3x \leq 3$$

$$3x + 1 \leq 3$$

$$3x \leq 2$$

$$x \leq \frac{2}{3}$$

$$\{x | x \leq \frac{2}{3}\} \text{ or } \left(-\infty, \frac{2}{3}\right]$$



60. $8 - 4(2 - x) \leq -2x$

$$8 - 8 + 4x \leq -2x$$

$$4x \leq -2x$$

$$6x \leq 0$$

$$x \leq 0$$

$$\{x | x \leq 0\} \text{ or } (-\infty, 0]$$



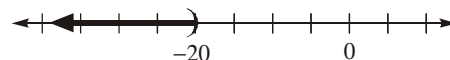
61. $\frac{1}{2}(x - 4) > x + 8$

$$\frac{1}{2}x - 2 > x + 8$$

$$-\frac{1}{2}x > 10$$

$$x < -20$$

$$\{x | x < -20\} \text{ or } (-\infty, -20)$$



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62. $3x + 4 > \frac{1}{3}(x - 2)$

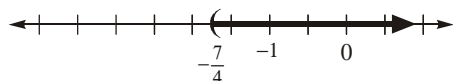
$$3x + 4 > \frac{1}{3}x - \frac{2}{3}$$

$$9x + 12 > x - 2$$

$$8x > -14$$

$$x > -\frac{7}{4}$$

$$\left\{x \mid x > -\frac{7}{4}\right\} \text{ or } \left(-\frac{7}{4}, \infty\right)$$



63. $-1 \leq 3x - 7 \leq 5$

$$6 \leq 3x \leq 12$$

$$2 \leq x \leq 4$$

$$\{x \mid 2 \leq x \leq 4\} \text{ or } [2, 4]$$

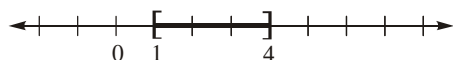


64. $4 \leq 2x + 2 \leq 10$

$$2 \leq 2x \leq 8$$

$$1 \leq x \leq 4$$

$$\{x \mid 1 \leq x \leq 4\} \text{ or } [1, 4]$$

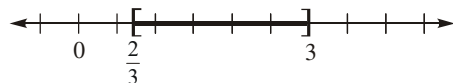


65. $-5 \leq 4 - 3x \leq 2$

$$-9 \leq -3x \leq -2$$

$$3 \geq x \geq \frac{2}{3}$$

$$\left\{x \mid \frac{2}{3} \leq x \leq 3\right\} \text{ or } \left[\frac{2}{3}, 3\right]$$

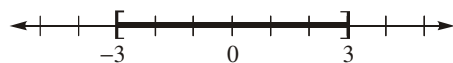


66. $-3 \leq 3 - 2x \leq 9$

$$-6 \leq -2x \leq 6$$

$$3 \geq x \geq -3$$

$$\{x \mid -3 \leq x \leq 3\} \text{ or } [-3, 3]$$



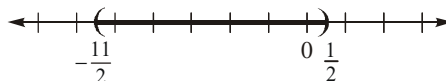
67. $-3 < \frac{2x-1}{4} < 0$

$$-12 < 2x - 1 < 0$$

$$-11 < 2x < 1$$

$$-\frac{11}{2} < x < \frac{1}{2}$$

$$\left\{x \mid -\frac{11}{2} < x < \frac{1}{2}\right\} \text{ or } \left(-\frac{11}{2}, \frac{1}{2}\right)$$



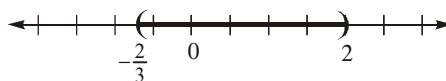
68. $0 < \frac{3x+2}{2} < 4$

$$0 < 3x + 2 < 8$$

$$-2 < 3x < 6$$

$$-\frac{2}{3} < x < 2$$

$$\left\{x \mid -\frac{2}{3} < x < 2\right\} \text{ or } \left(-\frac{2}{3}, 2\right)$$

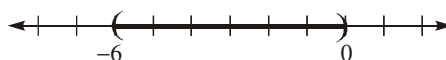


69. $1 < 1 - \frac{1}{2}x < 4$

$$0 < -\frac{1}{2}x < 3$$

$$0 > x > -6 \text{ or } -6 < x < 0$$

$$\{x \mid -6 < x < 0\} \text{ or } (-6, 0)$$



70. $0 < 1 - \frac{1}{3}x < 1$

$$-1 < -\frac{1}{3}x < 0$$

$$3 > x > 0 \text{ or } 0 < x < 3$$

$$\{x \mid 0 < x < 3\} \text{ or } (0, 3)$$



71. $(x+2)(x-3) > (x-1)(x+1)$

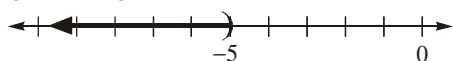
$$x^2 - x - 6 > x^2 - 1$$

$$-x - 6 > -1$$

$$-x > 5$$

$$x < -5$$

$$\{x \mid x < -5\} \text{ or } (-\infty, -5)$$



72. $(x-1)(x+1) > (x-3)(x+4)$

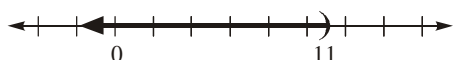
$$x^2 - 1 > x^2 + x - 12$$

$$-1 > x - 12$$

$$-x > -11$$

$$x < 11$$

$$\{x \mid x < 11\} \text{ or } (-\infty, 11)$$

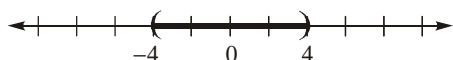


73. $|2x| < 8$

$$-8 < 2x < 8$$

$$-4 < x < 4$$

$$\{x \mid -4 < x < 4\} \text{ or } (-4, 4)$$

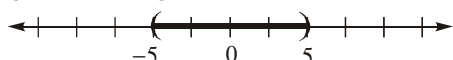


74. $|3x| < 15$

$$-15 < 3x < 15$$

$$-5 < x < 5$$

$$\{x \mid -5 < x < 5\} \text{ or } (-5, 5)$$

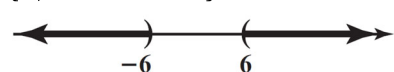


75. $|7x| > 42$

$$7x < -42 \text{ or } 7x > 42$$

$$x < -6 \text{ or } x > 6$$

$$\{x \mid x < -6 \text{ or } x > 6\} \text{ or } (-\infty, -6) \cup (6, \infty)$$

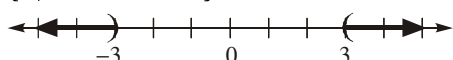


76. $|2x| > 6$

$$2x < -6 \text{ or } 2x > 6$$

$$x < -3 \text{ or } x > 3$$

$$\{x \mid x < -3 \text{ or } x > 3\} \text{ or } (-\infty, -3) \cup (3, \infty)$$



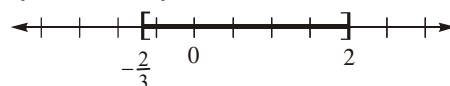
77. $|3t - 2| \leq 4$

$$-4 \leq 3t - 2 \leq 4$$

$$-2 \leq 3t \leq 6$$

$$-\frac{2}{3} \leq t \leq 2$$

$$\left\{t \mid -\frac{2}{3} \leq t \leq 2\right\} \text{ or } \left[-\frac{2}{3}, 2\right]$$



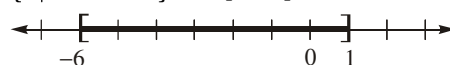
78. $|2u + 5| \leq 7$

$$-7 \leq 2u + 5 \leq 7$$

$$-12 \leq 2u \leq 2$$

$$-6 \leq u \leq 1$$

$$\{u \mid -6 \leq u \leq 1\} \text{ or } [-6, 1]$$



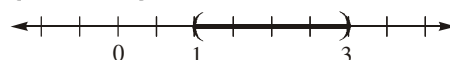
79. $|x - 2| + 2 < 3$

$$|x - 2| < 1$$

$$-1 < x - 2 < 1$$

$$1 < x < 3$$

$$\{x \mid 1 < x < 3\} \text{ or } (1, 3)$$



80. $|x + 4| + 3 < 5$

$$|x + 4| < 2$$

$$-2 < x + 4 < 2$$

$$-6 < x < -2$$

$$\{x \mid -6 < x < -2\} \text{ or } (-6, -2)$$

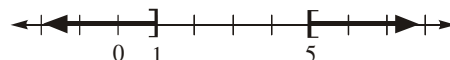


81. $|x - 3| \geq 2$

$$x - 3 \leq -2 \text{ or } x - 3 \geq 2$$

$$x \leq 1 \text{ or } x \geq 5$$

$$\{x \mid x \leq 1 \text{ or } x \geq 5\} \text{ or } (-\infty, 1] \cup [5, \infty)$$



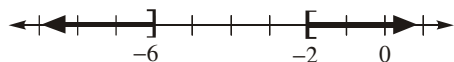
82. $|x + 4| \geq 2$

$$x + 4 \leq -2 \text{ or } x + 4 \geq 2$$

$$x \leq -6 \text{ or } x \geq -2$$

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$$\{x \mid x \leq -6 \text{ or } x \geq -2\} \text{ or } (-\infty, -6] \cup [-2, \infty)$$



$$83. |5 - 2x| > |-7|$$

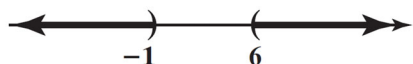
$$|5 - 2x| > 7$$

$$5 - 2x < -7 \text{ or } 5 - 2x > 7$$

$$-2x < -12 \text{ or } -2x > 2$$

$$x > 6 \text{ or } x < -1$$

$$\{x \mid x < -1 \text{ or } x > 6\} \text{ or } (-\infty, -1) \cup (6, \infty)$$



$$84. |2 - 3x| > |-1|$$

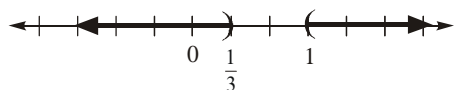
$$|2 - 3x| > 1$$

$$2 - 3x < -1 \text{ or } 2 - 3x > 1$$

$$-3x < -3 \text{ or } -3x > -1$$

$$x > 1 \text{ or } x < \frac{1}{3}$$

$$\{x \mid x < \frac{1}{3} \text{ or } x > 1\} \text{ or } (-\infty, \frac{1}{3}) \cup (1, \infty)$$



$$85. |1 - 4x| - 7 < -2$$

$$|1 - 4x| < 5$$

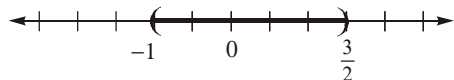
$$-5 < 1 - 4x < 5$$

$$-6 < -4x < 4$$

$$\frac{-6}{-4} > x > \frac{4}{-4}$$

$$\frac{3}{2} > x > -1 \text{ or } -1 < x < \frac{3}{2}$$

$$\{x \mid -1 < x < \frac{3}{2}\} \text{ or } (-1, \frac{3}{2})$$



$$86. |1 - 2x| - 4 < -1$$

$$|1 - 2x| < 3$$

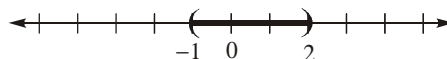
$$-3 < 1 - 2x < 3$$

$$-4 < -2x < 2$$

$$\frac{-4}{-2} > x > \frac{2}{-2}$$

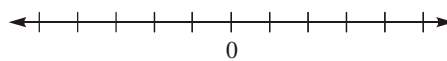
$$2 > x > -1 \text{ or } -1 < x < 2$$

$$\{x \mid -1 < x < 2\} \text{ or } (-1, 2)$$



$$87. |9x + 5| < -2$$

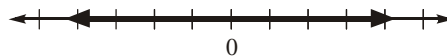
No solution since absolute value is always non-negative.



$$88. |3x - 4| \geq 0$$

All real numbers since absolute value is always non-negative.

$$\{x \mid -\infty < x < \infty\} \text{ or } (-\infty, \infty)$$

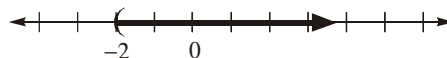


$$89. 3 - 4x < 11$$

$$-4x < 8$$

$$x > -2$$

$$\{x \mid x > -2\} \text{ or } (-2, \infty)$$

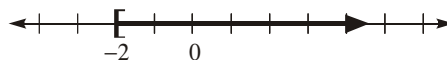


$$90. 1 - 3x \leq 7$$

$$-3x \leq 6$$

$$x \geq -2$$

$$\{x \mid x \geq -2\} \text{ or } [-2, \infty)$$



91. $|2x+1|-5 \geq -1$

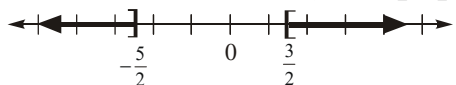
$|2x+1| \geq 4$

$2x+1 \leq -4 \quad \text{or} \quad 2x+1 \geq 4$

$2x \leq -5 \quad 2x \geq 3$

$x \leq -\frac{5}{2} \quad x \geq \frac{3}{2}$

$\{x \mid x \leq -\frac{5}{2} \text{ or } x \geq \frac{3}{2}\} \text{ or } \left(-\infty, -\frac{5}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$



92. $|5x+2|-3 > 9$

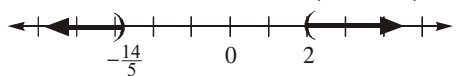
$|5x+2| > 12$

$5x+2 < -12 \quad \text{or} \quad 5x+2 > 12$

$5x < -14 \quad 5x > 10$

$x < -\frac{14}{5} \quad x > 2$

$\{x \mid x < -\frac{14}{5} \text{ or } x > 2\} \text{ or } \left(-\infty, -\frac{14}{5}\right) \cup (2, \infty)$



93. $\frac{x}{2} \geq 1 - \frac{x}{4}$

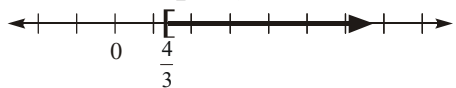
$4 \cdot \frac{x}{2} \geq 4 \cdot \left(1 - \frac{x}{4}\right)$

$2x \geq 4 - x$

$3x \geq 4$

$x \geq \frac{4}{3}$

$\{x \mid x \geq \frac{4}{3}\} \text{ or } \left[\frac{4}{3}, \infty\right)$



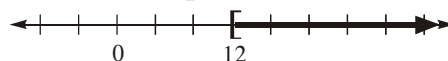
94. $\frac{x}{3} \geq 2 + \frac{x}{6}$

$6 \cdot \frac{x}{3} \geq 6 \left(2 + \frac{x}{6}\right)$

$2x \geq 12 + x$

$x \geq 12$

$\{x \mid x \geq 12\} \text{ or } [12, \infty)$



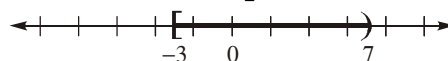
95. $-\frac{1}{3} \leq \frac{x+1}{6} < \frac{4}{3}$

$6 \left(-\frac{1}{3}\right) \leq 6 \cdot \frac{x+1}{6} < 6 \cdot \frac{4}{3}$

$-2 \leq x+1 < 8$

$-3 \leq x < 7$

$\{x \mid -3 \leq x < 7\} \text{ or } [-3, 7)$



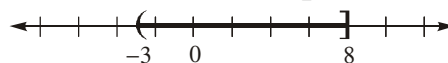
96. $-\frac{3}{2} < \frac{x-3}{4} \leq \frac{5}{4}$

$4 \left(-\frac{3}{2}\right) < 4 \cdot \frac{x-3}{4} \leq 4 \cdot \frac{5}{4}$

$-6 < x-3 \leq 5$

$-3 < x \leq 8$

$\{x \mid -3 < x \leq 8\} \text{ or } (-3, 8]$



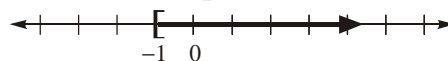
97. $x(4x+3) \leq (2x+1)^2$

$4x^2 + 3x \leq 4x^2 + 4x + 1$

$-x \leq 1$

$x \geq -1$

$\{x \mid x \geq -1\} \text{ or } [-1, \infty)$



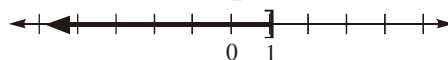
98. $x(9x-5) \leq (3x-1)^2$

$9x^2 - 5x \leq 9x^2 - 6x + 1$

$-5x \leq -6x + 1$

$x \leq 1$

$\{x \mid x \leq 1\} \text{ or } (-\infty, 1]$



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$$99. \quad |(3x-2)-7| < \frac{1}{2}$$

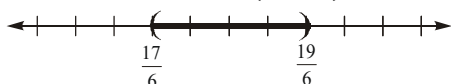
$$|3x-9| < \frac{1}{2}$$

$$-\frac{1}{2} < 3x-9 < \frac{1}{2}$$

$$\frac{17}{2} < 3x < \frac{19}{2}$$

$$\frac{17}{6} < x < \frac{19}{6}$$

$$\left\{x \mid \frac{17}{6} < x < \frac{19}{6}\right\} \quad \text{or} \quad \left(\frac{17}{6}, \frac{19}{6}\right)$$



$$100. \quad |(4x-1)-11| < \frac{1}{4}$$

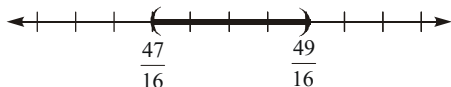
$$|4x-12| < \frac{1}{4}$$

$$-\frac{1}{4} < 4x-12 < \frac{1}{4}$$

$$\frac{47}{4} < 4x < \frac{49}{4}$$

$$\frac{47}{16} < x < \frac{49}{16}$$

$$\left\{x \mid \frac{47}{16} < x < \frac{49}{16}\right\} \quad \text{or} \quad \left(\frac{47}{16}, \frac{49}{16}\right)$$



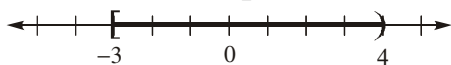
$$101. \quad -3 < 5-2x \leq 11$$

$$-8 < -2x \leq 6$$

$$4 > x \geq -3$$

$$-3 \leq x < 4$$

$$\{x \mid -3 \leq x < 4\} \quad \text{or} \quad [-3, 4)$$



$$102. \quad 2 \leq 3-2(x+1) < 8$$

$$2 \leq 3-2x-2 < 8$$

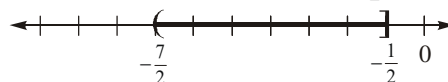
$$2 \leq 1-2x < 8$$

$$1 \leq -2x < 7$$

$$-\frac{1}{2} \geq x > -\frac{7}{2}$$

$$-\frac{7}{2} < x \leq -\frac{1}{2}$$

$$\left\{x \mid -\frac{7}{2} < x \leq -\frac{1}{2}\right\} \quad \text{or} \quad \left(-\frac{7}{2}, -\frac{1}{2}\right]$$



$$103. \quad 7-|x-1| > 4$$

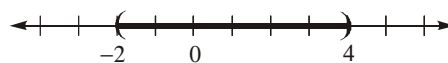
$$-|x-1| > -3$$

$$|x-1| < 3$$

$$-3 < x-1 < 3$$

$$-2 < x < 4$$

$$\{x \mid -2 < x < 4\} \quad \text{or} \quad (-2, 4)$$



$$104. \quad 9-|x+3| \geq 5$$

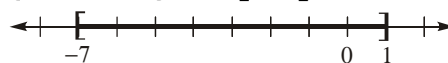
$$-|x+3| \geq -4$$

$$|x+3| \leq 4$$

$$-4 \leq x+3 \leq 4$$

$$-7 \leq x \leq 1$$

$$\{x \mid -7 \leq x \leq 1\} \quad \text{or} \quad [-7, 1]$$



$$105. \quad -3 < x+5 < 2x$$

$$-3 < x+5 \quad \text{and} \quad x+5 < 2x$$

$$-8 < x$$

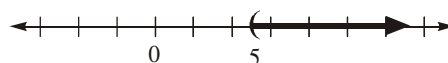
$$5 < x$$

$$x > -8$$

$$x > 5$$

We need both $x > -8$ and $x > 5$. Therefore, $x > 5$ is sufficient to satisfy both inequalities.

$$\{x \mid x > 5\} \quad \text{or} \quad (5, \infty)$$



106. $2 < x - 3 < 2x$

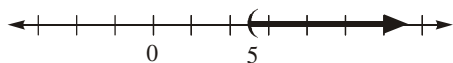
$2 < x - 3 \text{ and } x - 3 < 2x$

$5 < x \quad -3 < x$

$x > 5 \quad x > -3$

We need both $x > 5$ and $x > -3$. Therefore,
 $x > 5$ is sufficient to satisfy both inequalities.

$\{x \mid x > 5\} \text{ or } (5, \infty)$



107. $x + 2 < 2x - 1 < 5x$

$x + 2 < 2x - 1 \text{ and } 2x - 1 < 5x$

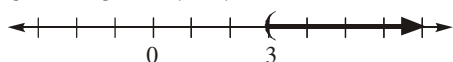
$2 < x - 1 \quad -3x - 1 < 0$

$3 < x \quad -3x < 1$

$x > 3 \quad x > -\frac{1}{3}$

We need both $x > -\frac{1}{3}$ and $x > 3$. Therefore,
 $x > 3$ is sufficient to satisfy both inequalities.

$\{x \mid x > 3\} \text{ or } (3, \infty)$



108. $2x - 1 < 3x + 5 < 5x - 7$

$2x - 1 < 3x + 5 \text{ and } 3x + 5 < 5x - 7$

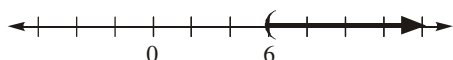
$-x - 1 < 5 \quad -2x + 5 < -7$

$-x < 6 \quad -2x < -12$

$x > -6 \quad x > 6$

We need both $x > -6$ and $x > 6$. Therefore,
 $x > 6$ is sufficient to satisfy both inequalities.

$\{x \mid x > 6\} \text{ or } (6, \infty)$



109. $|x - 2| < 0.5$

$-0.5 < x - 2 < 0.5$

$-0.5 + 2 < x < 0.5 + 2$

$1.5 < x < 2.5$

Solution set: $\{x \mid 1.5 < x < 2.5\}$

110. $|x - (-1)| < 1$

$-1 < x - (-1) < 1$

$-1 < x + 1 < 1$

$-1 - 1 < x < 1 - 1$

$-2 < x < 0$

Solution set: $\{x \mid -2 < x < 0\}$

111. $|x - (-3)| > 2$

$x - (-3) < -2 \text{ or } x - (-3) > 2$

$x + 3 < -2 \text{ or } x + 3 > 2$

$x < -5 \text{ or } x > -1$

Solution set: $\{x \mid x < -5 \text{ or } x > -1\}$

112. $|x - 2| > 3$

$x - 2 < -3 \text{ or } x - 2 > 3$

$x < -1 \text{ or } x > 5$

Solution set: $\{x \mid x < -1 \text{ or } x > 5\}$

113. $21 < \text{young adult's age} < 30$

114. $40 \leq \text{middle-aged} < 60$

115. A temperature x that differs from 98.6°F by at least 1.5°F .

$|x - 98.6^\circ| \geq 1.5^\circ$

$x - 98.6^\circ \leq -1.5^\circ \text{ or } x - 98.6^\circ \geq 1.5^\circ$

$x \leq 97.1^\circ \text{ or } x \geq 100.1^\circ$

The temperatures that are considered unhealthy are those that are less than 97.1°F or greater than 100.1°F , inclusive.

116. The length L must be within 0.0025 of 5.375 inches..

$|L - 5.375| \leq 0.0025$

$-0.0025 \leq L - 5.375 \leq 0.0025$

$5.3725 \leq L \leq 5.3775$

The lengths must be between 5.3725 and 5.3775 inches, inclusive.

117. The percentage must be within 3.2 percentage points of 77 percent. The inequality that represents this would be:

$|x - 77| \leq 3.2$

$-3.2 \leq x - 77 \leq 3.2$

$73.8 \leq x \leq 80.2$

The actual percentage is likely to fall between 73.8% and 80.2% inclusive.

118. The speed x varies from 707 mph by up to 55 mph.

a. $|x - 707| \leq 55$

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- b. $-55 \leq x - 707 \leq 55$
 $-55 \leq x - 707 \leq 55$
 $652 \leq x \leq 762$
 The speed of sound is between 652 and 762 miles per hour, depending on conditions.
119. a. Let x = age at death.
 $x - 30 \geq 52.2$
 $x \geq 82.2$
 Therefore, the average life expectancy for a 30-year-old man in 2023 will be greater than or equal to 82.2 years.
- b. Let x = age at death.
 $x - 30 \geq 55.8$
 $x \geq 85.8$
 Therefore, the average life expectancy for a 30-year-old woman in 2023 will be greater than or equal to 85.8 years.
- c. By the given information, a woman can expect to live $85.8 - 82.2 = 3.6$ years longer.
120. $V = 20T$
 $80^\circ \leq T \leq 120^\circ$
 $80^\circ \leq \frac{V}{20} \leq 120^\circ$
 $1600 \leq V \leq 2400$
 The volume ranges from 1600 to 2400 cubic centimeters, inclusive.
121. Let P represent the selling price and C represent the commission.
 Calculating the commission:
 $C = 45,000 + 0.25(P - 900,000)$
 $= 45,000 + 0.25P - 225,000$
 $= 0.25P - 180,000$
 Calculate the commission range, given the price range:
 $900,000 \leq P \leq 1,100,000$
 $0.25(900,000) \leq 0.25P \leq 0.25(1,100,000)$
 $225,000 \leq 0.25P \leq 275,000$
 $225,000 - 180,000 \leq 0.25P - 180,000 \leq 275,000 - 180,000$
 $45,000 \leq C \leq 95,000$
 The agent's commission ranges from \$45,000 to \$95,000, inclusive.
 $\frac{45,000}{900,000} = 0.05 = 5\%$ to $\frac{95,000}{1,100,000} = 0.086 = 8.6\%$, inclusive.
 As a percent of selling price, the commission ranges from 5% to 8.6%, inclusive.
122. Let C represent the commission.
 Calculate the commission range:
 $25 + 0.4(200) \leq C \leq 25 + 0.4(3000)$
 $105 \leq C \leq 1225$
 The commissions are at least \$105 and at most \$1225.
123. Let W = weekly wages and T = tax withheld.
 Calculating the withholding tax range, given the range of weekly wages:
 $500 \leq W \leq 700$
 $500 - 312.5 \leq W - 312.5 \leq 700 - 312.5$
 $187.50 \leq W - 312.5 \leq 387.5$
 $0.12(187.5) \leq 0.12(W - 312.5) \leq 0.12(387.5)$
 $22.50 \leq 0.12(W - 312.5) \leq 46.5$
 $22.5 + 1100 \leq 0.12(W - 312.5) + 1100 \leq 46.5 + 1100$
 $1122.50 \leq T \leq 1146.50$
 The amount withheld varies from \$1122.50 to \$1146.50, inclusive.
124. Let x represent the length of time you should exercise on the last two days.
 $25 + 35 + 0 + 40 + 15 + x = 150$
 $115 + x = 150$
 $x = 35$
 You will stay within the guidelines by exercising from 35 minutes total on the last two days.
125. Let x represent the amount paid for international minutes and y represent the number of international minutes. The range of the bills is \$69.50 to \$140.75. The rate plan is \$60. Thus the range of costs of the international minutes is: $9.50 \leq x \leq 80.75$. The cost per min is \$0.25.
 $\frac{9.50}{0.25} \leq y \leq \frac{80.75}{0.25}$
 $38 \leq y \leq 323$
 The minutes varies from 38 to 323 minutes, inclusive.
126. Let C represent the amount paid for the fares. The range of the fare is \$20.93 to \$40.44. of miles. The number of miles is 23.
 $20.93 \leq 23x \leq 40.44$
 $0.91 \leq x \leq 1.76$
 The cost per mile varies from \$0.91 per mile to \$1.76 per mile, inclusive.

127. You have already consumed 40 grams of fat.
Let C represent the number of cookies. Then we have the following equation:

$$40 + 8C \leq 64$$

$$8C \leq 24$$

$$C \leq 3$$

You may eat up to 3 cookies and keep the total fat content of your meal not more than 64g.

128. You have already consumed 730 calories. Let x represent the number of apple sauce orders you can eat. Then we have the following equation:

$$730 + 50x \leq 830$$

$$50x \leq 100$$

$$x \leq 2$$

You may eat up to 2 orders of apple sauce and keep your calories below or equal to 830.

129. a. Let T represent the score on the last test and G represent the course grade.
Calculating the course grade and solving for the last test:

$$G = \frac{68 + 82 + 87 + 89 + T}{5}$$

$$G = \frac{326 + T}{5}$$

$$5G = 326 + T$$

$$T = 5G - 326$$

Calculating the range of scores on the last test, given the grade range:

$$80 \leq G < 90$$

$$400 \leq 5G < 450$$

$$74 \leq 5G - 326 < 124$$

$$74 \leq T < 124$$

To get a grade of B, you need at least a 74 on the fifth test.

- b. Let T represent the score on the last test and G represent the course grade.
Calculating the course grade and solving for the last test:

$$G = \frac{68 + 82 + 87 + 89 + 2T}{6}$$

$$G = \frac{326 + 2T}{6}$$

$$G = \frac{163 + T}{3}$$

$$T = 3G - 163$$

Calculating the range of scores on the last test, given the grade range:

$$80 \leq G < 90$$

$$240 \leq 3G < 270$$

$$77 \leq 3G - 163 < 107$$

$$77 \leq T < 107$$

To get a grade of B, you need at least a 77 on the fifth test.

130. Let T represent the test scores of the people in the top 2.5%.

$$T > 1.96(12) + 100 = 123.52$$

People in the top 2.5% will have test scores greater than 123.52. That is, $T > 123.52$ or $(123.52, \infty)$.

131. Since $a < b$,

$$\frac{a}{2} < \frac{b}{2} \quad \text{and} \quad \frac{a}{2} < \frac{b}{2}$$

$$\frac{a}{2} + \frac{a}{2} < \frac{a}{2} + \frac{b}{2} \quad \text{and} \quad \frac{a}{2} + \frac{b}{2} < \frac{b}{2} + \frac{b}{2}$$

$$a < \frac{a+b}{2} \quad \text{and} \quad \frac{a+b}{2} < b$$

$$\text{So, } a < \frac{a+b}{2} < b.$$

132. From problem 131, $a < \frac{a+b}{2} < b$, so

$$d\left(a, \frac{a+b}{2}\right) = \frac{a+b}{2} - a = \frac{a+b-2a}{2} = \frac{b-a}{2} \quad \text{and}$$

$$d\left(b, \frac{a+b}{2}\right) = b - \frac{a+b}{2} = \frac{2b-a-b}{2} = \frac{b-a}{2}.$$

Therefore, $\frac{a+b}{2}$ is equidistant from a and b .

133. If $0 < a < b$, then

$$ab > a^2 > 0 \quad \text{and} \quad b^2 > ab > 0$$

$$(\sqrt{ab})^2 > a^2 \quad \text{and} \quad b^2 > (\sqrt{ab})^2$$

$$\sqrt{ab} > a \quad \text{and} \quad b > \sqrt{ab}$$

Therefore, $a < \sqrt{ab} < b$.

134. Show that $\sqrt{ab} < \frac{a+b}{2}$.

$$\begin{aligned} \frac{a+b}{2} - \sqrt{ab} &= \frac{1}{2}(a - 2\sqrt{ab} + b) \\ &= \frac{1}{2}(\sqrt{a} - \sqrt{b})^2 > 0, \text{ since } a \neq b. \end{aligned}$$

Therefore, $\sqrt{ab} < \frac{a+b}{2}$.

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135. For $0 < a < b$, $\frac{1}{h} = \frac{1}{2} \left(\frac{1}{a} + \frac{1}{b} \right)$

$$h \cdot \frac{1}{h} = \frac{1}{2} \left(\frac{b+a}{ab} \right) \cdot h$$

$$1 = \frac{1}{2} \left(\frac{b+a}{ab} \right) \cdot h$$

$$h = \frac{2ab}{a+b}$$

$$h - a = \frac{2ab}{a+b} - a = \frac{2ab - a(a+b)}{a+b}$$

$$= \frac{2ab - a^2 - ab}{a+b} = \frac{ab - a^2}{a+b}$$

$$= \frac{a(b-a)}{a+b} > 0$$

Therefore, $h > a$.

$$b - h = b - \frac{2ab}{a+b} = \frac{b(a+b) - 2ab}{a+b}$$

$$= \frac{ab + b^2 - 2ab}{a+b} = \frac{b^2 - ab}{a+b}$$

$$= \frac{b(b-a)}{a+b} > 0$$

Therefore, $h < b$, and we have $a < h < b$.

136. Show that $h = \frac{(\text{geometric mean})^2}{\text{arithmetic mean}} = \frac{(\sqrt{ab})^2}{\left(\frac{1}{2}(a+b)\right)}$

$$\frac{1}{h} = \frac{1}{2} \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$\frac{2}{h} = \frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab}$$

$$\frac{h}{2} = \frac{ab}{a+b}$$

$$h = 2 \cdot \frac{ab}{a+b} = \frac{(\sqrt{ab})^2}{\left(\frac{1}{2}(a+b)\right)}$$

137. $x - 4 < 2x - 3 \leq \frac{x+5}{3}$

$$3x - 12 < 6x - 9 \leq x + 5$$

$$3x - 12 < 6x - 9 \quad \text{and} \quad 6x - 9 \leq x + 5$$

$$-3 < 3x$$

$$5x \leq 14$$

$$-1 < x$$

$$x \leq \frac{14}{5}$$

This is equivalent to $-1 < x \leq \frac{14}{5}$. The solution

set, in interval notation, is $\left(-1, \frac{14}{5}\right]$.

138. The largest value of $2x^2 - 3$ occurs at the largest value for $|x|$.

$$2 \leq 5 - x \leq 9$$

$$-3 \leq -x \leq 4$$

$$-4 \leq x \leq 3$$

$$3 \geq x \geq -4 \quad \text{or} \quad -4 \leq x \leq 3$$

The largest value for $2x^2 - 3$ is

$$2(-4)^2 - 3 = 32 - 3 = 29.$$

139. Answers will vary. One possibility:

$$\text{No solution: } 4x + 6 \leq 2(x - 5) + 2x$$

$$\text{One solution: } 3x + 5 \leq 2(x + 3) + 1 \leq 3(x + 2) - 1$$

140. Answers will vary.

141. Since $x^2 \geq 0$, we have

$$x^2 + 1 \geq 0 + 1$$

$$x^2 + 1 \geq 1$$

Therefore, the expression $x^2 + 1$ can never be less than -5 .

142. Answers will vary.

Chapter 1 Review Exercises

1. $2 - \frac{x}{3} = 8$

$$6 - x = 24$$

$$x = -18$$

The solution set is $\{-18\}$.

2. $-2(5 - 3x) + 8 = 4 + 5x$

$$-10 + 6x + 8 = 4 + 5x$$

$$6x - 2 = 4 + 5x$$

$$x = 6$$

The solution set is $\{6\}$.

$$\begin{aligned}
 3. \quad \frac{3x}{4} - \frac{x}{3} &= \frac{1}{12} \\
 9x - 4x &= 1 \\
 5x &= 1 \\
 x &= \frac{1}{5}
 \end{aligned}$$

The solution set is $\left\{\frac{1}{5}\right\}$

$$\begin{aligned}
 4. \quad \frac{x}{x-1} &= \frac{6}{5} \\
 5x &= 6x - 6 \\
 6 &= x
 \end{aligned}$$

Since $x = 6$ does not cause a denominator to equal zero, the solution set is $\{6\}$.

$$\begin{aligned}
 5. \quad x(1-x) &= 6 \\
 x - x^2 &= 6 \\
 0 &= x^2 - x + 6 \\
 b^2 - 4ac &= (-1)^2 - 4(1)(6) \\
 &= 1 - 24 = -23
 \end{aligned}$$

Therefore, there are no real solutions.

$$\begin{aligned}
 6. \quad \frac{1-3x}{4} &= \frac{x+6}{3} + \frac{1}{2} \\
 (12)\left(\frac{1-3x}{4}\right) &= \left(\frac{x+6}{3} + \frac{1}{2}\right)(12) \\
 3(1-3x) &= 4(x+6) + 6 \\
 3-9x &= 4x+24+6 \\
 -13x &= 27 \\
 x &= -\frac{27}{13}
 \end{aligned}$$

The solution set is $\left\{-\frac{27}{13}\right\}$.

$$\begin{aligned}
 7. \quad (x-1)(2x+3) &= 3 \\
 2x^2 + x - 3 &= 3 \\
 2x^2 + x - 6 &= 0 \\
 (2x-3)(x+2) &= 0 \Rightarrow x = \frac{3}{2} \text{ or } x = -2 \\
 \text{The solution set is } &\left\{-2, \frac{3}{2}\right\}.
 \end{aligned}$$

$$\begin{aligned}
 8. \quad 2x+3 &= 4x^2 \\
 0 &= 4x^2 - 2x - 3 \\
 x &= \frac{2 \pm \sqrt{4+48}}{8} = \frac{2 \pm \sqrt{52}}{8} \\
 &= \frac{2 \pm 2\sqrt{13}}{8} = \frac{1 \pm \sqrt{13}}{4}
 \end{aligned}$$

The solution set is $\left\{\frac{1-\sqrt{13}}{4}, \frac{1+\sqrt{13}}{4}\right\}$.

$$\begin{aligned}
 9. \quad \sqrt[3]{x^2-1} &= 2 \\
 \left(\sqrt[3]{x^2-1}\right)^3 &= (2)^3 \\
 x^2 - 1 &= 8 \\
 x^2 &= 9 \Rightarrow x = \pm 3
 \end{aligned}$$

Check: $x = -3$

$$\sqrt[3]{(-3)^2-1} = 2$$

$$\sqrt[3]{9-1} = 2$$

$$\sqrt[3]{8} = 2$$

$$2 = 2$$

Check: $x = 3$

$$\sqrt[3]{(3)^2-1} = 2$$

$$\sqrt[3]{9-1} = 2$$

$$\sqrt[3]{8} = 2$$

$$2 = 2$$

The solution set is $\{-3, 3\}$.

$$\begin{aligned}
 10. \quad x(x+1) + 2 &= 0 \\
 x^2 + x + 2 &= 0 \\
 x &= \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm \sqrt{-7}}{2} \\
 \text{No real solutions.}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad 3x^2 - x + 1 &= 0 \\
 x &= \frac{1 \pm \sqrt{1-12}}{6} = \frac{1 \pm \sqrt{-11}}{6} \\
 \text{No real solutions.}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad x^4 - 5x^2 + 4 &= 0 \\
 (x^2 - 4)(x^2 - 1) &= 0 \\
 x^2 - 4 = 0 \text{ or } x^2 - 1 &= 0 \\
 x = \pm 2 \text{ or } x = \pm 1 \\
 \text{The solution set is } &\{-2, -1, 1, 2\}.
 \end{aligned}$$

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$$\begin{aligned}
 13. \quad & \sqrt{2x-3} + x = 3 \\
 & \sqrt{2x-3} = 3 - x \\
 & 2x - 3 = 9 - 6x + x^2 \\
 & x^2 - 8x + 12 = 0 \Rightarrow (x-2)(x-6) = 0 \\
 & x = 2 \text{ or } x = 6
 \end{aligned}$$

Check $x = 2$:

$$\sqrt{2(2)-3} + 2 = \sqrt{1} + 2 = 3$$

Check $x = 6$:

$$\sqrt{2(6)-3} + 6 = \sqrt{9} + 6 = 9 \neq 3$$

The solution set is $\{2\}$.

$$\begin{aligned}
 14. \quad & \sqrt[4]{2x+3} = 2 \\
 & (\sqrt[4]{2x+3})^4 = 2^4 \\
 & 2x + 3 = 16 \\
 & 2x = 13 \\
 & x = \frac{13}{2}
 \end{aligned}$$

Check $x = \frac{13}{2}$:

$$\sqrt[4]{2\left(\frac{13}{2}\right) + 3} = \sqrt[4]{13+3} = \sqrt[4]{16} = 2$$

The solution set is $\left\{\frac{13}{2}\right\}$.

$$\begin{aligned}
 15. \quad & \sqrt{x+1} + \sqrt{x-1} = \sqrt{2x+1} \\
 & (\sqrt{x+1} + \sqrt{x-1})^2 = (\sqrt{2x+1})^2 \\
 & x+1 + 2\sqrt{x+1}\sqrt{x-1} + x-1 = 2x+1 \\
 & 2x + 2\sqrt{x+1}\sqrt{x-1} = 2x+1 \\
 & 2\sqrt{x+1}\sqrt{x-1} = 1 \\
 & (2\sqrt{x+1}\sqrt{x-1})^2 = (1)^2 \\
 & 4(x+1)(x-1) = 1 \\
 & 4x^2 - 4 = 1 \\
 & 4x^2 = 5 \\
 & x^2 = \frac{5}{4} \\
 & x = \pm \frac{\sqrt{5}}{2}
 \end{aligned}$$

Check $x = \frac{\sqrt{5}}{2}$:

$$\sqrt{\frac{\sqrt{5}}{2} + 1} + \sqrt{\frac{\sqrt{5}}{2} - 1} = \sqrt{2\left(\frac{\sqrt{5}}{2}\right) + 1}$$

$$1.79890743995 = 1.79890743995$$

Check $x = -\frac{\sqrt{5}}{2}$:

$$\sqrt{-\frac{\sqrt{5}}{2} + 1} + \sqrt{-\frac{\sqrt{5}}{2} - 1} = \sqrt{2\left(-\frac{\sqrt{5}}{2}\right) + 1},$$

The second solution is not possible because it makes the radicand negative.

The solution set is $\left\{\frac{\sqrt{5}}{2}\right\}$.

$$16. \quad 2x^{1/2} - 3 = 0$$

$$2x^{1/2} = 3$$

$$(2x^{1/2})^2 = 3^2$$

$$4x = 9$$

$$x = \frac{9}{4}$$

Check $x = \frac{9}{4}$:

$$2\left(\frac{9}{4}\right)^{1/2} - 3 = 2\left(\frac{3}{2}\right) - 3 = 3 - 3 = 0$$

The solution set is $\left\{\frac{9}{4}\right\}$.

$$17. \quad x^{-6} - 7x^{-3} - 8 = 0$$

$$\text{let } p = x^{-3} \Rightarrow p^2 = x^{-6}$$

$$p^2 - 7p - 8 = 0$$

$$(p-8)(p+1) = 0$$

$$p = 8 \text{ or } p = -1$$

$$p = 8 \Rightarrow x^{-3} = 8$$

$$(x^{-3})^{-1/3} = (8)^{-1/3} \Rightarrow x = \frac{1}{2}$$

$$p = -1 \Rightarrow x^{-3} = -1$$

$$(x^{-3})^{-1/3} = (-1)^{-1/3} \Rightarrow x = -1$$

Check $x = \frac{1}{2}$:

$$\left(\frac{1}{2}\right)^{-6} - 7\left(\frac{1}{2}\right)^{-3} - 8 = 64 - 56 - 8 = 0$$

Check $x = -1$:

$$(-1)^{-6} - 7(-1)^{-3} - 8 = 1 + 7 - 8 = 0$$

The solution set is $\left\{-1, \frac{1}{2}\right\}$.

$$18. \quad x^2 + m^2 = 2mx + (nx)^2$$

$$x^2 + m^2 = 2mx + n^2x^2$$

$$x^2 - n^2x^2 - 2mx + m^2 = 0$$

$$(1 - n^2)x^2 - 2mx + m^2 = 0$$

$$x = \frac{2m \pm \sqrt{4m^2 - 4m^2(1 - n^2)}}{2(1 - n^2)}$$

$$= \frac{2m \pm \sqrt{4m^2(1 - (1 - n^2))}}{2(1 - n^2)}$$

$$= \frac{2m \pm 2m\sqrt{1 - (1 - n^2)}}{2(1 - n^2)}$$

$$= \frac{m \pm m\sqrt{n^2}}{1 - n^2} = \frac{m \pm mn}{1 - n^2} = \frac{m(1 \pm n)}{1 - n^2}$$

$$x = \frac{m(1 + n)}{1 - n^2} = \frac{m(1 + n)}{(1 + n)(1 - n)} = \frac{m}{1 - n}$$

or

$$x = \frac{m(1 - n)}{1 - n^2} = \frac{m(1 - n)}{(1 + n)(1 - n)} = \frac{m}{1 + n}$$

The solution set is $\left\{\frac{m}{1 - n}, \frac{m}{1 + n}\right\}$, $n \neq 1$, $n \neq -1$.

$$19. \quad 10a^2x^2 - 2abx - 36b^2 = 0$$

$$5a^2x^2 - abx - 18b^2 = 0$$

$$(5ax + 9b)(ax - 2b) = 0$$

$$5ax + 9b = 0 \quad \text{or} \quad ax - 2b = 0$$

$$5ax = -9b \quad ax = 2b$$

$$x = -\frac{9b}{5a} \quad x = \frac{2b}{a}$$

The solution set is $\left\{-\frac{9b}{5a}, \frac{2b}{a}\right\}$, $a \neq 0$.

$$20. \quad \sqrt{x^2 + 3x + 7} - \sqrt{x^2 - 3x + 9} + 2 = 0$$

$$\sqrt{x^2 + 3x + 7} = \sqrt{x^2 - 3x + 9} - 2$$

$$\left(\sqrt{x^2 + 3x + 7}\right)^2 = \left(\sqrt{x^2 - 3x + 9} - 2\right)^2$$

$$x^2 + 3x + 7 = x^2 - 3x + 9 - 4\sqrt{x^2 - 3x + 9} + 4$$

$$6x - 6 = -4\sqrt{x^2 - 3x + 9}$$

$$(6(x - 1))^2 = (-4\sqrt{x^2 - 3x + 9})^2$$

$$36(x^2 - 2x + 1) = 16(x^2 - 3x + 9)$$

$$36x^2 - 72x + 36 = 16x^2 - 48x + 144$$

$$20x^2 - 24x - 108 = 0$$

$$5x^2 - 6x - 27 = 0$$

$$(5x + 9)(x - 3) = 0 \Rightarrow x = -\frac{9}{5} \quad \text{or} \quad x = 3$$

Check $x = -\frac{9}{5}$:

$$\sqrt{\left(-\frac{9}{5}\right)^2 + 3\left(-\frac{9}{5}\right) + 7} - \sqrt{\left(-\frac{9}{5}\right)^2 - 3\left(-\frac{9}{5}\right) + 9} + 2$$

$$= \sqrt{\frac{81}{25} - \frac{27}{5} + 7} - \sqrt{\frac{81}{25} + \frac{27}{5} + 9} + 2$$

$$= \sqrt{\frac{81 - 135 + 175}{25}} - \sqrt{\frac{81 + 135 + 225}{25}} + 2$$

$$= \sqrt{\frac{121}{25}} - \sqrt{\frac{441}{25}} + 2 = \frac{11}{5} - \frac{21}{5} + 2 = 0$$

Check $x = 3$:

$$\sqrt{(3)^2 + 3(3) + 7} - \sqrt{(3)^2 - 3(3) + 9} + 2$$

$$= \sqrt{9 + 9 + 7} - \sqrt{9 - 9 + 9} + 2$$

$$= \sqrt{25} - \sqrt{9} + 2 = 2 + 2$$

$$= 4 \neq 0$$

The solution set is $\left\{-\frac{9}{5}\right\}$.

$$21. \quad |2x + 3| = 7$$

$$2x + 3 = 7 \quad \text{or} \quad 2x + 3 = -7$$

$$2x = 4 \quad \text{or} \quad 2x = -10$$

$$x = 2 \quad \text{or} \quad x = -5$$

The solution set is $\{-5, 2\}$.

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22. $|2-3x|+2=9$

$$|2-3x|=7$$

$$2-3x=7 \quad \text{or} \quad 2-3x=-7$$

$$-3x=5 \quad -3x=-9$$

$$x=-\frac{5}{3} \quad x=3$$

The solution set is $\left\{-\frac{5}{3}, 3\right\}$

23. $2x^3=3x^2$

$$2x^3-3x^2=0$$

$$x^2(2x-3)=0$$

$$x^2=0 \Rightarrow x=0$$

$$2x-3=0 \Rightarrow x=\frac{3}{2}$$

The solution set is $\left\{0, \frac{3}{2}\right\}$.

24. $2x^3+5x^2-8x-20=0$

$$x^2(2x+5)-4(2x+5)=0$$

$$(2x+5)(x^2-4)=0$$

$$2x+5=0 \quad \text{or} \quad x^2-4=0$$

$$x=-\frac{5}{2} \quad \text{or} \quad x=\pm 2$$

The solution set is $\left\{-\frac{5}{2}, -2, 2\right\}$.

25. $\frac{1}{x-1} + \frac{3}{x+2} = \frac{11}{x^2+x-2}$

$$\frac{(x+2)+3(x-1)}{(x-1)(x+2)} = \frac{11}{(x-1)(x+2)}$$

$$(x+2)+3(x-1)=11$$

$$x+2+3x-3=11$$

$$4x-1=11$$

$$4x=12$$

$$x=3$$

Since 3 does not make any denominator equal to 0, the solution set is $\{3\}$.

26. $(x-2)^2=9$

$$x-2=\pm\sqrt{9}$$

$$x-2=\pm 3$$

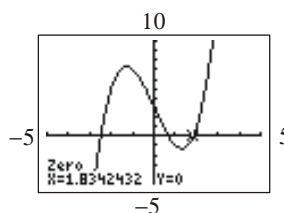
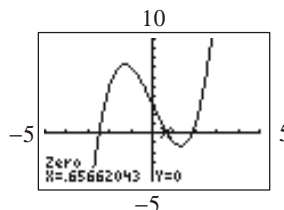
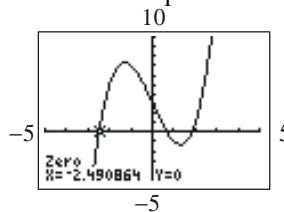
$$x=2\pm 3$$

$$x=5 \quad \text{or} \quad x=-1$$

The solution set is $\{-1, 5\}$.

27. $x^3-5x+3=0$

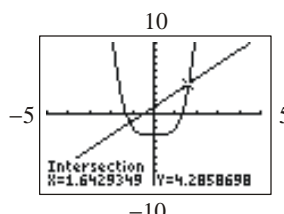
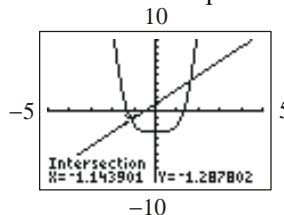
Use the Zero option from the CALC menu.



The solution set is $\{-2.49, 0.66, 1.83\}$.

28. $x^4-3=2x+1$

Use the Intersect option on the CALC menu.



The solution set is $\{-1.14, 1.64\}$.

$$29. \quad \frac{2x-3}{5} + 2 \leq \frac{x}{2}$$

$$2(2x-3) + 10(2) \leq 5x$$

$$4x - 6 + 20 \leq 5x$$

$$14 \leq x$$

$$x \geq 14$$

$$\{x | x \geq 14\} \text{ or } [14, \infty)$$



$$30. \quad -9 \leq \frac{2x+3}{-4} \leq 7$$

$$36 \geq 2x+3 \geq -28$$

$$33 \geq 2x \geq -31$$

$$\frac{33}{2} \geq x \geq -\frac{31}{2}$$

$$-\frac{31}{2} \leq x \leq \frac{33}{2}$$

$$\left\{x \mid -\frac{31}{2} \leq x \leq \frac{33}{2}\right\} \text{ or } \left[-\frac{31}{2}, \frac{33}{2}\right]$$



$$31. \quad 2 < \frac{3-3x}{12} < 6$$

$$24 < 3-3x < 72$$

$$21 < -3x < 69$$

$$-7 > x > -23$$

$$\{x | -23 < x < -7\} \text{ or } (-23, -7)$$



$$32. \quad |3x+4| < \frac{1}{2}$$

$$-\frac{1}{2} < 3x+4 < \frac{1}{2}$$

$$-\frac{9}{2} < 3x < -\frac{7}{2}$$

$$-\frac{3}{2} < x < -\frac{7}{6}$$

$$\left\{x \mid -\frac{3}{2} < x < -\frac{7}{6}\right\} \text{ or } \left(-\frac{3}{2}, -\frac{7}{6}\right)$$



$$33. \quad |2x-5| \geq 9$$

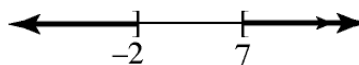
$$2x-5 \leq -9 \text{ or } 2x-5 \geq 9$$

$$2x \leq -4 \text{ or } 2x \geq 14$$

$$x \leq -2 \text{ or } x \geq 7$$

$$\{x | x \leq -2 \text{ or } x \geq 7\} \text{ or}$$

$$(-\infty, -2] \text{ or } [7, \infty)$$



$$34. \quad 2 + |2-3x| \leq 4$$

$$|2-3x| \leq 2$$

$$-2 \leq 2-3x \leq 2$$

$$-4 \leq -3x \leq 0$$

$$\frac{4}{3} \geq x \geq 0$$

$$0 \leq x \leq \frac{4}{3}$$

$$\left\{x \mid 0 \leq x \leq \frac{4}{3}\right\} \text{ or } \left[0, \frac{4}{3}\right]$$



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35. $1 - |2 - 3x| < -4$

$$-|2 - 3x| < -5$$

$$|2 - 3x| > 5$$

$$2 - 3x < -5 \quad \text{or} \quad 2 - 3x > 5$$

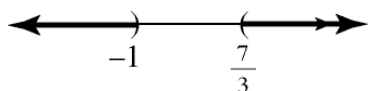
$$7 < 3x \quad \text{or} \quad -3 > 3x$$

$$\frac{7}{3} < x \quad \text{or} \quad -1 > x$$

$$x < -1 \quad \text{or} \quad x > \frac{7}{3}$$

$$\left\{ x \mid x < -1 \text{ or } x > \frac{7}{3} \right\} \text{ or}$$

$$(-\infty, -1) \text{ or } \left(\frac{7}{3}, \infty \right)$$



36. $(6 + 3i) - (2 - 4i) = (6 - 2) + (3 - (-4))i = 4 + 7i$

37. $4(3 - i) + 3(-5 + 2i) = 12 - 4i - 15 + 6i = -3 + 2i$

38. $\frac{3}{3+i} = \frac{3}{3+i} \cdot \frac{3-i}{3-i} = \frac{9-3i}{9-3i+3i-i^2} = \frac{9-3i}{10}$
 $= \frac{9}{10} - \frac{3}{10}i$

39. $i^{50} = i^{48} \cdot i^2 = (i^4)^{12} \cdot i^2 = 1^{12}(-1) = -1$

40. $(2 + 3i)^3 = (2 + 3i)^2(2 + 3i)$
 $= (4 + 12i + 9i^2)(2 + 3i)$
 $= (-5 + 12i)(2 + 3i)$
 $= -10 - 15i + 24i + 36i^2$
 $= -46 + 9i$

41. $x^2 + x + 1 = 0$

$$a = 1, b = 1, c = 1,$$

$$b^2 - 4ac = 1^2 - 4(1)(1) = 1 - 4 = -3$$

$$x = \frac{-1 \pm \sqrt{-3}}{2(1)} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

The solution set is $\left\{ -\frac{1}{2} - \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i \right\}$.

42. $2x^2 + x - 2 = 0$

$$a = 2, b = 1, c = -2,$$

$$b^2 - 4ac = 1^2 - 4(2)(-2) = 1 + 16 = 17$$

$$x = \frac{-1 \pm \sqrt{17}}{2(2)} = \frac{-1 \pm \sqrt{17}}{4}$$

The solution set is $\left\{ \frac{-1 - \sqrt{17}}{4}, \frac{-1 + \sqrt{17}}{4} \right\}$.

43. $x^2 + 3 = x$

$$x^2 - x + 3 = 0$$

$$a = 1, b = -1, c = 3,$$

$$b^2 - 4ac = (-1)^2 - 4(1)(3) = 1 - 12 = -11$$

$$x = \frac{-(-1) \pm \sqrt{-11}}{2(1)} = \frac{1 \pm \sqrt{11}i}{2} = \frac{1}{2} \pm \frac{\sqrt{11}}{2}i$$

The solution set is $\left\{ \frac{1}{2} - \frac{\sqrt{11}}{2}i, \frac{1}{2} + \frac{\sqrt{11}}{2}i \right\}$.

44. $x(1 + x) = 2$

$$x^2 + x - 2 = 0$$

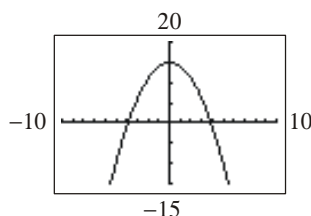
$$(x + 2)(x - 1) = 0 \Rightarrow x = -2 \text{ or } x = 1$$

The solution set is $\{-2, 1\}$.

45. $y = -x^2 + 15$

X	Y1
-5	-21
-4	-11
-3	11
-2	15
-1	14
0	-21
1	-11
2	11
3	15
4	-21
5	-11

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-15
Ymax=20
Yscl=5
Zres=1



46. $(-4, 0), (0, 0), (2, 0), (0, -2), (0, 0), (0, 2)$

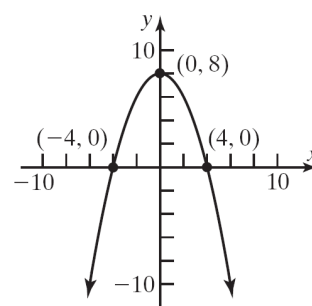
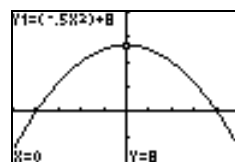
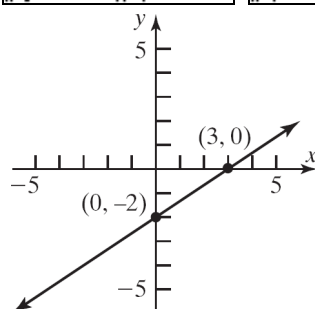
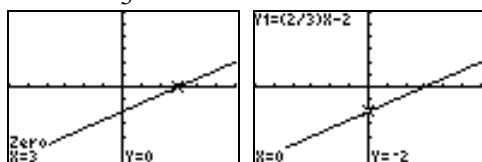
x-intercepts: $-4, 0, 2$

y-intercepts: $-2, 0, 2$

47. $2x - 3y = 6$

$$-3y = -2x + 6$$

$$y = \frac{2}{3}x - 2$$



50. $p = 2l + 2w$

51. $I = P \cdot r \cdot t \Rightarrow I = (9000)(0.07)(1) = \630

52. Let x represent the amount of money invested in bonds. Then $70,000 - x$ represents the amount of money invested in CD's.

Since the total interest is to be \$5000, we have:

$$0.08x + 0.05(70,000 - x) = 5000$$

$$(100)(0.08x + 0.05(70,000 - x)) = (5000)(100)$$

$$8x + 350,000 - 5x = 500,000$$

$$3x + 350,000 = 500,000$$

$$3x = 150,000$$

$$x = 50,000$$

\$50,000 should be invested in bonds at 8% and \$20,000 should be invested in CD's at 5%.

53. Using $s = vt$, we have $t = 3$ and $v = 1100$.

Finding the distance s in feet:

$$s = 1100(3) = 3300$$

The storm is 3300 feet away.

54. $1600 \leq I \leq 3600$

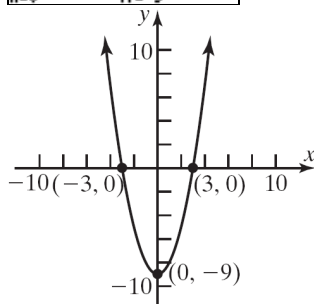
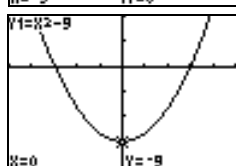
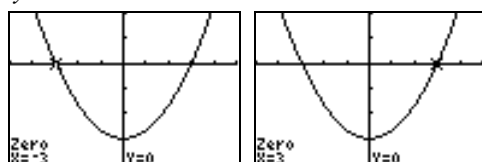
$$1600 \leq \frac{900}{x^2} \leq 3600$$

$$\frac{1}{1600} \geq \frac{x^2}{900} \geq \frac{1}{3600}$$

$$\frac{9}{16} \geq x^2 \geq \frac{1}{4}$$

$$\frac{3}{4} \geq x \geq \frac{1}{2}$$

48. $y = x^2 - 9$



49. $x^2 + 2y = 16$

$$2y = -x^2 + 16$$

$$y = -\frac{1}{2}x^2 + 8$$



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The range of distances is from 0.5 meters to 0.75 meters, inclusive.

55. Let s represent the distance the plane can travel.

	With wind	Against wind
Rate	$250 + 30 = 280$	$250 - 30 = 220$
Time	$\frac{(s/2)}{280}$	$\frac{(s/2)}{220}$
Dist.	$\frac{s}{2}$	$\frac{s}{2}$

Since the total time is at most 5 hours, we have:

$$\frac{(s/2)}{280} + \frac{(s/2)}{220} \leq 5$$

$$\frac{s}{560} + \frac{s}{440} \leq 5$$

$$11s + 14s \leq 5(6160)$$

$$25s \leq 30,800$$

$$s \leq 1232$$

The plane can travel at most 1232 miles or 616 miles one way and return 616 miles.

56. Let t represent the time it takes the helicopter to reach the raft.

	Raft	Helicopter
Rate	5	90
Time	t	t
Dist.	$5t$	$90t$

Since the total distance is 150 miles, we have:

$$5t + 90t = 150$$

$$95t = 150$$

$$t \approx 1.58 \text{ hours} \approx 1 \text{ hour and 35 minutes}$$

The helicopter will reach the raft in about 1 hour and 35 minutes.

57. Let d represent the distance flown by the bee traveling at 3 meters per second.

$$\frac{d}{3} = \frac{150 - d}{5} \quad (\text{Times needed to meet are equal.})$$

$$5d = 450 - 3d$$

$$8d = 450$$

$$d = 56.25 \text{ meters} \Rightarrow t = \frac{56.25}{3} = 18.75 \text{ seconds}$$

The bees meet for the first time after 18.75 seconds.

The bees will meet a second time on the second lap. The first bee will have traveled $150 + x$ meters and the second bee will have traveled $150 + (150 - x)$ meters.

Solving for time, we have:

$$\frac{150 + x}{3} = \frac{150 + (150 - x)}{5}$$

$$\frac{150 + x}{3} = \frac{300 - x}{5}$$

$$750 + 5x = 900 - 3x$$

$$8x = 150$$

$$x = 18.75 \text{ meters into the second lap}$$

$$t = \frac{168.75}{3} = 56.25 \text{ seconds}$$

The bees meet the second time after 56.25 seconds (37.5 seconds later).

58. Given that $s = 1280 - 32t - 16t^2$,

- a. The object hits the ground when $s = 0$.

$$0 = 1280 - 32t - 16t^2$$

$$t^2 + 2t - 80 = 0$$

$$(t + 10)(t - 8) = 0 \Rightarrow t = -10, t = 8$$

The object hits the ground after 8 seconds.

- b. After 4 seconds, the object's height is

$$s = 1280 - 32(4) - 16(4)^2 = 896 \text{ feet.}$$

59. Let t represent the time it takes Clarissa to complete the job by herself.

	Clarissa	Shawna
Time to do job alone	t	$t + 5$
Part of job done in 1 day	$\frac{1}{t}$	$\frac{1}{t + 5}$
Time on job (days)	6	6
Part of job done by each person	$\frac{6}{t}$	$\frac{6}{t + 5}$

Since the two people paint one house, we have:

$$\frac{6}{t} + \frac{6}{t + 5} = 1$$

$$6(t + 5) + 6t = t(t + 5)$$

$$6t + 30 + 6t = t^2 + 5t$$

$$t^2 - 7t - 30 = 0$$

$$(t - 10)(t + 3) = 0 \Rightarrow t = 10 \text{ or } t = -3$$

It takes Clarissa 10 days to paint the house when working by herself.

60. Let x represent the amount of the \$8-per-pound coffee.

Amt. of coffee (pounds)	Price (\$)	Total \$
20	4	$(20)(4)$
x	8	$(8)(x)$
$20 + x$	5	$(5)(20 + x)$

$$80 + 8x = (5)(20 + x)$$

$$80 + 8x = 100 + 5x$$

$$3x = 20$$

$$x = \frac{20}{3} = 6\frac{2}{3}$$

Add $6\frac{2}{3}$ pounds of \$8/lb coffee to get $26\frac{2}{3}$ pounds of \$5/lb coffee.

61. Let x represent the amount of water evaporated.

% salt	Tot. amt.	amt. of salt
2%	64	$(0.02)(64)$
0%	x	$(0.00)(x)$
10%	$64 - x$	$(0.10)(64 - x)$

$$(0.02)(64) - (0.00)(x) = (0.10)(64 - x)$$

$$1.28 = 6.4 - 0.10x$$

$$0.10x = 5.12$$

$$x = 51.2$$

51.2 ounces of water must be evaporated.

62. Let the length of leg 1 = x .
Then the length of leg 2 = $17 - x$.
By the Pythagorean Theorem we have

$$x^2 + (17 - x)^2 = (13)^2$$

$$x^2 + x^2 - 34x + 289 = 169$$

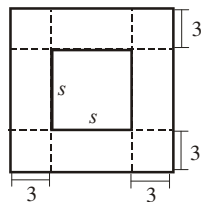
$$2x^2 - 34x + 120 = 0$$

$$x^2 - 17x + 60 = 0$$

$$(x - 12)(x - 5) = 0 \Rightarrow x = 12 \text{ or } x = 5$$

the legs are 5 cm and 12 cm long.

63. a. Consider the following diagram:



$$4(s + 6) = 50$$

$$4s + 24 = 50$$

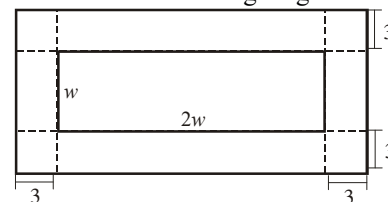
$$4s = 26$$

$$s = 6.5$$

The painting is 6.5 inches by 6.5 inches.

$s + 6 = 12.5$, so the frame is 12.5 inches by 12.5 inches.

- b. Consider the following diagram:



$$2(2w + 6) + 2(w + 6) = 50$$

$$4w + 12 + 2w + 12 = 50$$

$$6w = 26$$

$$w = \frac{26}{6} = 4\frac{1}{3} \Rightarrow l = 2w = 8\frac{2}{3}$$

The painting is $8\frac{2}{3}$ inches by $4\frac{1}{3}$ inches.

The frame is $14\frac{2}{3}$ inches by $10\frac{1}{3}$ inches.

64. Let t represent the time it takes the smaller pump to finish filling the tank.

	3hp Pump	8hp Pump
Time to do job alone	12	8
Part of job done in 1 hr	$\frac{1}{12}$	$\frac{1}{8}$
Time on job (hrs)	$t + 4$	4
Part of job done by each pump	$\frac{t + 4}{12}$	$\frac{4}{8}$

Since the two pumps fill one tank, we have:

$$\frac{t + 4}{12} + \frac{4}{8} = 1$$

$$\frac{t + 4}{12} = \frac{1}{2}$$

$$t + 4 = 6$$

$$t = 2$$

It takes the small pump a total of 2 more hours to fill the tank.

Chapter 1: Graphs, Equations, and Inequalities

65. Let $w = 4$. Solve for the length:

$$l^2 = 4(l + 4)$$

$$l^2 = 4l + 16$$

$$l^2 - 4l - 16 = 0$$

$$l = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-16)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{80}}{2} = 2 \pm 2\sqrt{5} \approx 6.47$$

The length of the plasterboard should be cut to a length of approximately 6.47 feet.

66. Let x represent the number of passengers over 20. Then $20 + x$ represents the total number of passengers, and $15 - 0.1x$ represents the fare for each passenger. Solving the equation for total cost, \$482.40, we have:

$$(20 + x)(15 - 0.1x) = 482.40$$

$$300 + 13x - 0.1x^2 = 482.40$$

$$-0.1x^2 + 13x - 182.40 = 0$$

$$x^2 - 130x + 1824 = 0$$

$$(x - 114)(x - 16) = 0 \Rightarrow x = 114 \text{ or } x = 16$$

Since the capacity of the bus is 44, we discard the 114. Therefore, $20 + 16 = 36$ people went on the trip; each person paid $15 - 0.1(16) = \$13.40$.

67. Let r_S represent Scott's rate and let r_T represent Todd's rate. The time for Scott to run 95 meters is the same as for Todd to run 100 meters.

$$\frac{95}{r_S} = \frac{100}{r_T}$$

$$r_S = 0.95r_T$$

$$d_S = t \cdot r_S = t(0.95r_T) = 0.95d_T$$

If Todd starts from 5 meters behind the start:

$$d_T = 105$$

$$d_S = 0.95d_T = 0.95(105) = 99.75$$

- The race does not end in a tie.
- Todd wins the race.
- Todd wins by 0.25 meters.
- To end in a tie:
 $100 = 0.95(100 + x)$
 $100 = 95 + 0.95x$
 $5 = 0.95x$
 $x = 5.263$ meters

- $95 = 0.95(100)$ Therefore, the race ends in a tie.

f, g, h. Race is a tie.

68. In each problem, we need to use the Least Common Multiple of the expressions $x - 2$ and $x^2 - 4$, namely $x^2 - 4$.

- We use $x^2 - 4$ as the LCD in order to combine the given expressions.
- We multiply each side of the equation by $x^2 - 4$ in order to clear out the denominators before solving the equation.
- We use $x^2 - 4$ as the LCD in order to combine the terms on the left hand side before solving the inequality.

Chapter 1 Test

1. $2x^2 + 6x = x - 3$

$$2x^2 + 5x + 3 = 0$$

$$(2x + 3)(x + 1) = 0$$

$$2x + 3 = 0 \quad \text{or} \quad x + 1 = 0$$

$$2x = -3 \quad x = -1$$

$$x = -\frac{3}{2}$$

The solution set is $\{-\frac{3}{2}, -1\}$.

2. $x + 1 = \sqrt{x + 7}$

$$(x + 1)^2 = (\sqrt{x + 7})^2$$

$$x^2 + 2x + 1 = x + 7$$

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -3 \quad x = 2$$

Check:

$$-3 + 1 \stackrel{?}{=} \sqrt{-3 + 7} \quad 2 + 1 \stackrel{?}{=} \sqrt{2 + 7}$$

$$-2 \stackrel{?}{=} \sqrt{4} \quad 3 \stackrel{?}{=} \sqrt{9}$$

$$-2 \neq 2 \quad 3 = 3 \text{ T}$$

The solution set is $\{2\}$.

$$3. \quad 2 - \frac{3}{m} = \frac{2}{m+2}$$

$$\text{LCD: } m(m+2)$$

$$\text{Restricted values: } m = -2, m = 0$$

$$\frac{2m(m+2)}{m(m+2)} - \frac{3(m+2)}{m(m+2)} = \frac{2m}{m(m+2)}$$

$$2m(m+2) - 3(m+2) = 2m$$

$$2m^2 + 4m - 3m - 6 = 2m$$

$$2m^2 - m - 6 = 0$$

$$(2m+3)(m-2) = 0$$

$$2m+3=0 \quad \text{or} \quad m-2=0$$

$$m = -\frac{3}{2} \quad m = 2$$

Since neither solution is a restricted value, the solution set is $\{-\frac{3}{2}, 2\}$.

$$4. \quad 5x - 8 = -4(x-1) + 6$$

$$5x - 8 = -4x + 4 + 6$$

$$5x - 8 = -4x + 10$$

$$9x = 18$$

$$x = 2$$

The solution set is $\{2\}$.

$$5. \quad 5|3-2b| - 7 = 8$$

$$5|3-2b| = 15$$

$$|3-2b| = 3$$

$$3-2b = 3 \quad \text{or} \quad 3-2b = -3$$

$$-2b = 0 \quad -2b = -6$$

$$b = 0 \quad b = 3$$

The solution set is $\{0, 3\}$.

$$6. \quad x^4 + x^2 = 3x^2 + 8$$

$$x^4 - 2x^2 - 8 = 0$$

Let $u = x^2$. Then $u^2 = (x^2)^2 = x^4$, and we have

$$u^2 - 2u - 8 = 0$$

$$(u-4)(u+2) = 0$$

$$u = 4 \quad \text{or} \quad u = -2$$

Since we are solving for x , we get

$$x^2 = 4 \quad \text{or} \quad x^2 = -2$$

$$x = \pm 2 \quad x = \pm\sqrt{-2}$$

no real solution

The solution set is $\{-2, 2\}$.

$$7. \quad x^2 - 4x + 2 = 0$$

$$a = 1, b = -4, c = 2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16-8}}{2}$$

$$= \frac{4 \pm \sqrt{8}}{2}$$

$$= \frac{4 \pm 2\sqrt{2}}{2}$$

$$= 2 \pm \sqrt{2}$$

The solution set is $\{2 - \sqrt{2}, 2 + \sqrt{2}\}$.

$$8. \quad 2x^2 + x - 1 = x(x+7) + 2$$

$$2x^2 + x - 1 = x^2 + 7x + 2$$

$$x^2 - 6x - 3 = 0$$

$$a = 1, b = -6, c = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36+12}}{2}$$

$$= \frac{6 \pm \sqrt{48}}{2}$$

$$= \frac{6 \pm 4\sqrt{3}}{2}$$

$$= 3 \pm 2\sqrt{3}$$

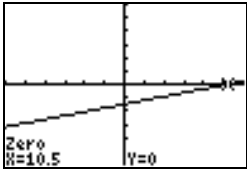
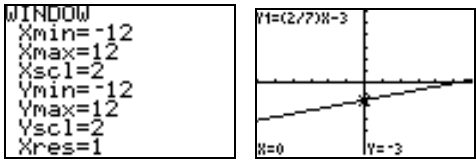
The solution set is $\{3 - 2\sqrt{3}, 3 + 2\sqrt{3}\}$.

$$9. \quad 2x - 7y = 21$$

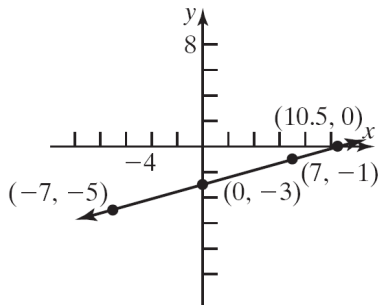
$$-7y = -2x + 21$$

$$y = \frac{2}{7}x - 3$$

x	$y = \frac{2}{7}x - 3$	(x, y)
-7	$y = \frac{2}{7}(-7) - 3 = -5$	$(-7, -5)$
0	$y = \frac{2}{7}(0) - 3 = -3$	$(0, -3)$
7	$y = \frac{2}{7}(7) - 3 = -1$	$(7, -1)$

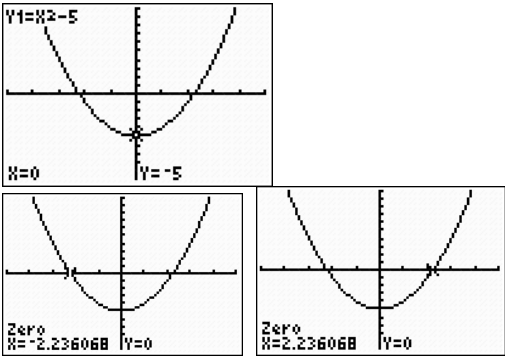
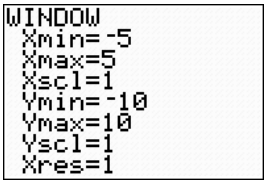


The intercepts are $(0, -3)$ and $(10.5, 0)$.

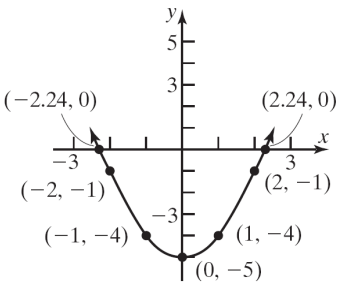


10. $y = x^2 - 5$

x	$y = x^2 - 5$	(x, y)
-3	$y = (-3)^2 - 5 = 4$	$(-3, 4)$
-1	$y = (-1)^2 - 5 = -4$	$(-1, -4)$
0	$y = (0)^2 - 5 = -5$	$(0, -5)$
1	$y = (1)^2 - 5 = -4$	$(1, -4)$
3	$y = (3)^2 - 5 = 4$	$(3, 4)$

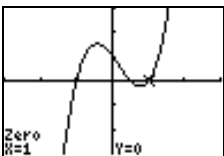
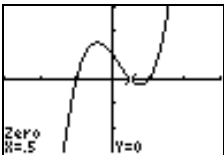
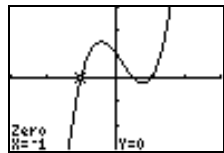
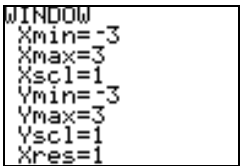


The intercepts are $(0, -5)$, $\approx (-2.24, 0)$, and $\approx (2.24, 0)$.



11. $2x^3 - x^2 - 2x + 1 = 0$

Since this equation has 0 on one side, we will use the Zero option from the CALC menu. It will be important to carefully select the window settings so as not to miss any solutions.



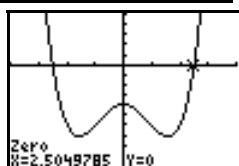
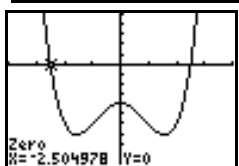
The solutions to the equation are -1 , 0.5 , and 1 .

12. $x^4 - 5x^2 - 8 = 0$

Since this equation has 0 on one side, we will use the Zero option from the CALC menu.

```
Plot1 Plot2 Plot3
Y1 X^4-5X^2-8
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

```
WINDOW
Xmin=-4
Xmax=4
Xscl=1
Ymin=-20
Ymax=10
Yscl=2
Xres=1
```



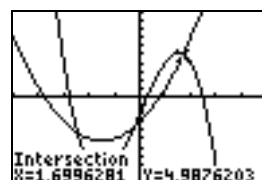
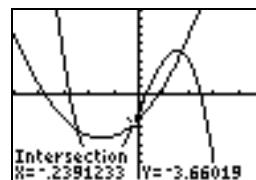
The solutions, rounded to two decimal places, are -2.50 and 2.50 .

13. $-x^3 + 7x - 2 = x^2 + 3x - 3$

Since there are nonzero expressions on both sides of the equation, we will use the Intersect option from the CALC menu. Enter the left side of the equation in Y1 and the right side in Y2.

```
Plot1 Plot2 Plot3
Y1 -X^3+7X-2
Y2 X^2+3X-3
Y3=
Y4=
Y5=
Y6=
Y7=
```

```
WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1
```



The solutions, rounded to two decimal places, are -2.46 , -0.24 , and 1.70 .

14. $\frac{2x+3}{4} < -2$

$$4 \cdot \frac{2x+3}{4} < 4 \cdot (-2)$$

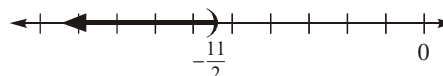
$$2x+3 < -8$$

$$2x = -11$$

$$x < -\frac{11}{2}$$

Solution set: $\{x \mid x < -\frac{11}{2}\}$

Interval: $(-\infty, -\frac{11}{2})$



15. $|2x+3| - 4 \geq 3$

$$|2x+3| - 4 + 4 \geq 3 + 4$$

$$|2x+3| \geq 7$$

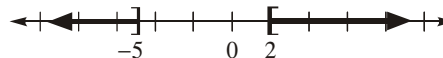
$$2x+3 \leq -7 \quad \text{or} \quad 2x+3 \geq 7$$

$$2x \leq -10 \quad 2x \geq 4$$

$$x \leq -5 \quad x \geq 2$$

Solution set: $\{x \mid x \leq -5 \text{ or } x \geq 2\}$

Interval: $(-\infty, -5] \cup [2, \infty)$



Chapter 1: Graphs, Equations, and Inequalities

16. $-7 < 3 - 5x \leq 8$

$$-7 - 3 < 3 - 5x - 3 \leq 8 - 3$$

$$-10 < -5x \leq 5$$

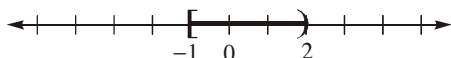
$$\frac{-10}{-5} > \frac{-5x}{-5} \geq \frac{5}{-5}$$

$$2 > x \geq -1$$

$$-1 \leq x < 2$$

Solution set: $\{x \mid -1 \leq x < 2\}$

Interval: $[-1, 2)$



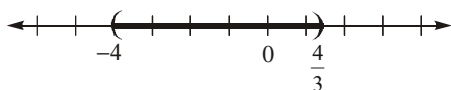
17. $|3x + 4| < 8$

$$-8 < 3x + 4 < 8$$

$$-12 < 3x < 4$$

$$-4 < x < \frac{4}{3}$$

$$\left\{x \mid -4 < x < \frac{4}{3}\right\} \text{ or } \left(-4, \frac{4}{3}\right)$$



18. $2(3 - 7i) - (4 + 11i) = 6 - 14i - 4 - 11i$

$$= 6 - 4 - 14i - 11i$$

$$= 2 - 25i$$

19. $(3 + 10i)(8 + i) = 3(8 + i) + 10i(8 + i)$

$$= 24 + 3i + 80i + 10i^2$$

$$= 24 + 83i + 10(-1)$$

$$= 14 + 83i$$

20. $\frac{2+i}{5-3i} = \frac{2+i}{5-3i} \cdot \frac{5+3i}{5+3i}$

$$= \frac{10 + 6i + 5i + 3i^2}{25 - 9i^2}$$

$$= \frac{10 + 11i + 3(-1)}{25 - 9(-1)}$$

$$= \frac{7 + 11i}{34}$$

$$= \frac{7}{34} + \frac{11}{34}i$$

21. $4x^2 - 4x + 5 = 0$

$$a = 4, b = -4, c = 5$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(5)}}{2(4)} = \frac{4 \pm \sqrt{16 - 80}}{8}$$

$$= \frac{4 \pm \sqrt{-64}}{8} = \frac{4 \pm 8i}{8} = \frac{1}{2} \pm i$$

The solution set is $\left\{\frac{1}{2} - i, \frac{1}{2} + i\right\}$.

22. We can use a table to summarize the given information.

	minutes per customer	# of cust. in 1 minute
Jamie	5	$\frac{1}{5}$
Scott	8.5	$\frac{1}{8.5}$
Together	t	$\frac{1}{t}$

This leads to the following:

(Jamie's rate) + (Scott's rate) = (rate together)

$$\frac{1}{5} + \frac{1}{8.5} = \frac{1}{t}$$

$$\frac{8.5 + 5}{42.5} = \frac{1}{t}$$

$$\frac{13.5}{42.5} = \frac{1}{t}$$

$$13.5t = 42.5$$

$$t = \frac{42.5}{13.5} = \frac{85}{27}$$

Since $65 \cdot \frac{85}{27} \approx 204.63$, it will take Jamie and

Scott about 204.63 minutes (3.41 hours) to check out 65 customers working together.

23. Let x = pounds of banana chips. Then the total pounds of the mix will be $x + 40$.

Since there is to be no loss in revenue, we get

$$\text{revenue}_{\text{new mix}} = \text{revenue}_{\text{banana}} + \text{revenue}_{\text{cherry mix}}$$

$$(\text{price})(\text{lbs})_{\text{new}} = (\text{price})(\text{lbs})_{\text{ban.}} + (\text{price})(\text{lbs})_{\text{cher.}}$$

$$(6.75)(x + 40) = (2.75)(x) + (9.25)(40)$$

$$6.75x + 270 = 2.75x + 370$$

$$4x = 100$$

$$x = 25$$

The retailer needs to mix 25 pounds of banana chips with the 40 pounds of the original mix.

24. Let x = sale price. We know that the discount is $0.42(275.00) = 115.50$. Therefore,

sale price = original price – discount

$$x = 275 - 115.50$$

$$x = 159.50$$

The sale price is \$159.50.

25. Here we need the simple interest formula, $I = P \cdot r \cdot t$. In this case we have $r = 0.04$, $t = \frac{3 \text{ months}}{12 \text{ months/yr}} = \frac{1}{4} \text{ yr}$, and $P = 10,000$.

$$I = P \cdot r \cdot t$$

$$= (10,000)(0.04)\left(\frac{1}{4}\right)$$

$$= 100$$

Mekhi will earn \$100.00 in interest after 3 months.

Chapter 1 Project

Internet-based Project – Answers will vary.

Chapter 2

Graphs

Section 2.1

1. $|5 - (-3)| = |8| = 8$

2. $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$

3. $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$
Since the sum of the squares of two of the sides of the triangle equals the square of the third side, the triangle is a right triangle.

4. $(0, 0)$

5. $\frac{1}{2}bh$

6. true

7. midpoint

8. False; the distance between two points is never negative.

9. True; $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

10. a

11. $d(P_1, P_2) = \sqrt{(2-0)^2 + (1-0)^2}$
 $= \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$

12. $d(P_1, P_2) = \sqrt{(-2-0)^2 + (1-0)^2}$
 $= \sqrt{(-2)^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$

13. $d(P_1, P_2) = \sqrt{(-2-1)^2 + (2-1)^2}$
 $= \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$

14. $d(P_1, P_2) = \sqrt{(2-(-1))^2 + (2-1)^2}$
 $= \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$

15. $d(P_1, P_2) = \sqrt{(5-3)^2 + (4-(-4))^2}$
 $= \sqrt{2^2 + (8)^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$

16. $d(P_1, P_2) = \sqrt{(2-(-1))^2 + (4-0)^2}$
 $= \sqrt{(3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

17. $d(P_1, P_2) = \sqrt{(4-(-7))^2 + (0-3)^2}$
 $= \sqrt{11^2 + (-3)^2} = \sqrt{121+9} = \sqrt{130}$

18. $d(P_1, P_2) = \sqrt{(4-2)^2 + (2-(-3))^2}$
 $= \sqrt{2^2 + 5^2} = \sqrt{4+25} = \sqrt{29}$

19. $d(P_1, P_2) = \sqrt{(6-5)^2 + (1-(-2))^2}$
 $= \sqrt{1^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$

20. $d(P_1, P_2) = \sqrt{(6-(-4))^2 + (2-(-3))^2}$
 $= \sqrt{10^2 + 5^2} = \sqrt{100+25}$
 $= \sqrt{125} = 5\sqrt{5}$

21. $d(P_1, P_2) = \sqrt{(2.3-(-0.2))^2 + (1.1-(0.3))^2}$
 $= \sqrt{2.5^2 + 0.8^2} = \sqrt{6.25+0.64}$
 $= \sqrt{6.89} \approx 2.62$

22. $d(P_1, P_2) = \sqrt{(-0.3-1.2)^2 + (1.1-2.3)^2}$
 $= \sqrt{(-1.5)^2 + (-1.2)^2} = \sqrt{2.25+1.44}$
 $= \sqrt{3.69} \approx 1.92$

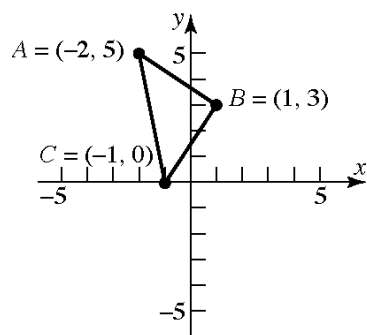
23. $d(P_1, P_2) = \sqrt{(0-a)^2 + (0-b)^2}$
 $= \sqrt{(-a)^2 + (-b)^2} = \sqrt{a^2 + b^2}$

24. $d(P_1, P_2) = \sqrt{(0-a)^2 + (0-a)^2}$
 $= \sqrt{(-a)^2 + (-a)^2}$
 $= \sqrt{a^2 + a^2} = \sqrt{2a^2} = |a|\sqrt{2}$

Section 2.1: The Distance and Midpoint Formulas

25. $A = (-2, 5)$, $B = (1, 3)$, $C = (-1, 0)$

$$\begin{aligned} d(A, B) &= \sqrt{(1 - (-2))^2 + (3 - 5)^2} \\ &= \sqrt{3^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13} \\ d(B, C) &= \sqrt{(-1 - 1)^2 + (0 - 3)^2} \\ &= \sqrt{(-2)^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13} \\ d(A, C) &= \sqrt{(-1 - (-2))^2 + (0 - 5)^2} \\ &= \sqrt{1^2 + (-5)^2} = \sqrt{1 + 25} = \sqrt{26} \end{aligned}$$



Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

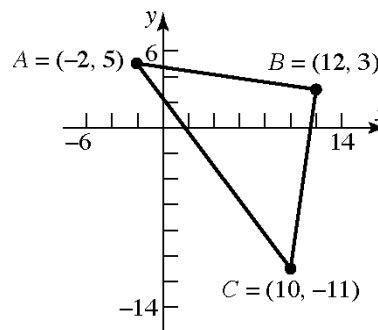
$$\begin{aligned} [d(A, B)]^2 + [d(B, C)]^2 &= [d(A, C)]^2 \\ (\sqrt{13})^2 + (\sqrt{13})^2 &= (\sqrt{26})^2 \\ 13 + 13 &= 26 \\ 26 &= 26 \end{aligned}$$

The area of a triangle is $A = \frac{1}{2} \cdot bh$. In this problem,

$$\begin{aligned} A &= \frac{1}{2} \cdot [d(A, B)] \cdot [d(B, C)] \\ &= \frac{1}{2} \cdot \sqrt{13} \cdot \sqrt{13} = \frac{1}{2} \cdot 13 \\ &= \frac{13}{2} \text{ square units} \end{aligned}$$

26. $A = (-2, 5)$, $B = (12, 3)$, $C = (10, -11)$

$$\begin{aligned} d(A, B) &= \sqrt{(12 - (-2))^2 + (3 - 5)^2} \\ &= \sqrt{14^2 + (-2)^2} \\ &= \sqrt{196 + 4} = \sqrt{200} \\ &= 10\sqrt{2} \\ d(B, C) &= \sqrt{(10 - 12)^2 + (-11 - 3)^2} \\ &= \sqrt{(-2)^2 + (-14)^2} \\ &= \sqrt{4 + 196} = \sqrt{200} \\ &= 10\sqrt{2} \\ d(A, C) &= \sqrt{(10 - (-2))^2 + (-11 - 5)^2} \\ &= \sqrt{12^2 + (-16)^2} \\ &= \sqrt{144 + 256} = \sqrt{400} \\ &= 20 \end{aligned}$$



Verifying that $\triangle ABC$ is a right triangle by the Pythagorean Theorem:

$$\begin{aligned} [d(A, B)]^2 + [d(B, C)]^2 &= [d(A, C)]^2 \\ (10\sqrt{2})^2 + (10\sqrt{2})^2 &= (20)^2 \\ 200 + 200 &= 400 \\ 400 &= 400 \end{aligned}$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$\begin{aligned} A &= \frac{1}{2} \cdot [d(A, B)] \cdot [d(B, C)] \\ &= \frac{1}{2} \cdot 10\sqrt{2} \cdot 10\sqrt{2} \\ &= \frac{1}{2} \cdot 100 \cdot 2 = 100 \text{ square units} \end{aligned}$$