Water Resources Engineering 4th edition Chin Solutions Manual

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SOLUTIONS MANUAL

Water-Resources Engineering

Fourth Edition

David A. Chin

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Chapter 1

Introduction

1.1. The mean annual rainfall in Boston is approximately $\boxed{1050 \text{ mm}}$, and the mean annual evapotranspiration is in the range of $\boxed{380\text{–}630 \text{ mm}}$ (USGS). On the basis of rainfall, this indicates a $\boxed{\text{subhumid}}$ climate. The mean annual rainfall in Santa Fe is approximately $\boxed{360 \text{ mm}}$ and the mean annual evapotranspiration is $\boxed{< 380 \text{ mm}}$. On the basis of rainfall, this indicates an $\boxed{\text{arid}}$ climate.

Chapter 2

Fundamentals of Flow in Closed Conduits

2.1. From the given data: $D_1 = 0.1$ m, $D_2 = 0.15\,\text{m}$, $V_1 = 2\,\text{m/s}$. Using these data, the following preliminary calculations are useful:

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.1)^2 = 0.007854\,\text{m}^2, \qquad A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.15)^2 = 0.01767\,\text{m}^2$$

Volumetric flow rate, Q, is given by

$$Q = A_1V_1 = (0.007854)(2) = \boxed{0.0157\,\text{m}^3/\text{s}}$$

According to the continuity equation,

$$A_1V_1 = A_2V_2 = Q \quad \rightarrow \quad V_2 = \frac{Q}{A_2} = \frac{0.0157}{0.01767} = \boxed{0.889\,\text{m/s}}$$

At 20°C, the density of water, ρ, is 998 kg/m^3, and the mass flow rate, $\dot{m}$, is given by

$$\dot{m} = \rho Q = (998)(0.0157) = \boxed{15.7\,\text{kg/s}}$$

2.2. From the given data: $D_1 = 0.2\,\text{m}$, $D_2 = 0.3\,\text{m}$, and $V_1 = 0.75\,\text{m/s}$. Using these data, the following preliminary calculations are useful:

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.03142\,\text{m}^2, \qquad A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.3)^2 = 0.07069\,\text{m}^2$$

(a) According to the continuity equation,

$$A_1V_1 = A_2V_2 \quad \rightarrow \quad V_2 = \frac{A_1V_1}{A_2} = \frac{(0.03142)(0.75)}{0.07069} = \boxed{0.333\,\text{m/s}}$$

(b) The volume flow rate, Q, is given by

$$Q = A_1V_1 = (0.03142)(0.75) = 0.02357\,\text{m}^3/\text{s} = \boxed{23.6\,\text{L/s}}$$

2.3. From the given data: $D_1 = 200$ mm, $D_2 = 100$ mm, $V_1 = 1$ m/s, and

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.0314\,\mathrm{m}^2$$
$$A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.1)^2 = 0.00785\,\mathrm{m}^2$$

The flow rate, Q_1, in the 200-mm pipe is given by

$$Q_1 = A_1V_1 = (0.0314)(1) = 0.0314\,\mathrm{m}^3/\mathrm{s}$$

and hence the flow rate, Q_2, in the 100-mm pipe is

$$Q_2 = \frac{Q_1}{2} = \frac{0.0314}{2} = \boxed{0.0157\,\mathrm{m}^3/\mathrm{s}}$$

The average velocity, V_2, in the 100-mm pipe is

$$V_2 = \frac{Q_2}{A_2} = \frac{0.0157}{0.00785} = \boxed{2\,\mathrm{m/s}}$$

2.4. The velocity distribution in the pipe is

$$v(r) = v_0\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{1}$$

and the average velocity, $\overline{V}$, is defined as

$$\overline{V} = \frac{1}{A}\int_A V\,\mathrm{d}A, \qquad A = \pi R^2, \qquad \mathrm{d}A = 2\pi r\,\mathrm{d}r \tag{2}$$

Combining Equations 1 and 2 yields

$$\overline{V} = \frac{1}{\pi R^2}\int_0^R v_0\left[1 - \left(\frac{r}{R}\right)^2\right]2\pi r\,\mathrm{d}r = \frac{2v_0}{R^2}\left[\int_0^R r\,\mathrm{d}r - \int_0^R \frac{r^3}{R^2}\,\mathrm{d}r\right] = \frac{2v_0}{R^2}\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right]$$
$$= \frac{2v_0}{R^2}\frac{R^2}{4} = \boxed{\frac{v_0}{2}}$$

The flow rate, Q, is therefore given by

$$Q = A\overline{V} = \boxed{\frac{\pi R^2 V_0}{2}}$$

2.5.

$$\beta = \frac{1}{A\bar{V}^2}\int_A v^2\,dA = \frac{4}{\pi R^2 V_0^2}\int_0^R V_0^2\left[1 - \frac{2r^2}{R^2} + \frac{r^4}{R^4}\right]2\pi r dr$$
$$= \frac{8}{R^2}\left[\int_0^R r dr - \int_0^R \frac{2r^3}{R^2}dr + \int_0^R \frac{r^5}{R^4}dr\right] = \frac{8}{R^2}\left[\frac{R^2}{2} - \frac{R^4}{2R^2} + \frac{R^6}{6R^4}\right]$$
$$= \boxed{\frac{4}{3}}$$

2.6. $D = 0.2\,\text{m}$, $Q = 60\,\text{L/s} = 0.06\,\text{m}^3/\text{s}$, $L = 100\,\text{m}$, $p_1 = 500\,\text{kPa}$, $p_2 = 400\,\text{kPa}$, $\gamma = 9.79\,\text{kN/m}^3$.

$$R = \frac{D}{4} = \frac{0.2}{4} = 0.05\,\text{m}$$
$$\Delta h = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{500-400}{9.79} = 10.2\,\text{m}$$
$$\tau_0 = \frac{\gamma R \Delta h}{L} = \frac{(9.79\times 10^3)(0.05)(10.2)}{100} = \boxed{49.9\ \text{N/m}^2}$$
$$A = \frac{\pi D^2}{4} = \frac{\pi(0.2)^2}{4} = 0.0314\,\text{m}^2$$
$$V = \frac{Q}{A} = \frac{0.06}{0.0314} = 1.91\,\text{m/s}$$
$$f = \frac{8\tau_0}{\rho V^2} = \frac{8(49.9)}{(998)(1.91)^2} = \boxed{0.11}$$

2.7. $T = 20°\text{C}$, $V = 2$ m/s, $D = 0.25$ m, horizontal pipe, ductile iron. For ductile iron pipe, $k_\text{s} = 0.26$ mm, and

$$\frac{k_\text{s}}{D} = \frac{0.26}{250} = 0.00104$$
$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998.2)(2)(0.25)}{(1.002\times 10^{-3})} = 4.981\times 10^5$$

From the Moody diagram:

$$\boxed{f = 0.0202\ \text{(flow is not fully turbulent)}}$$

Using the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_\text{s}/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

Substituting for k_s/D and Re gives

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{0.00104}{3.7} + \frac{2.51}{4.981\times 10^5\sqrt{f}}\right)$$

By trial and error leads to

$$\boxed{f = 0.0204}$$

Using the Swamee-Jain equation,

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_\text{s}/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}}\right] = -2\log\left[\frac{0.00104}{3.7} + \frac{5.74}{(4.981\times 10^5)^{0.9}}\right]$$

which leads to

$$\boxed{f = 0.0205}$$

The head loss, h_f, over 100 m of pipeline is given by

$$h_\text{f} = f\frac{L}{D}\frac{V^2}{2g} = 0.0204\frac{100}{0.25}\frac{(2)^2}{2(9.81)} = 1.66\,\text{m}$$

Therefore the pressure drop, Δp, is given by

$$\Delta p = \gamma h_\mathrm{f} = (9.79)(1.66) = \boxed{16.3\,\mathrm{kPa}}$$

If the pipe is 1 m lower at the downstream end, f would not change, but the pressure drop, Δp, would then be given by

$$\Delta p = \gamma(h_\mathrm{f} - 1.0) = 9.79(1.66 - 1) = \boxed{6.46\,\mathrm{kPa}}$$

2.8. From the given data: $D = 25$ mm, $k_\mathrm{s} = 0.1$ mm, $\theta = 10°$, $p_1 = 550$ kPa, and $L = 100$ m. At 20°C, $\nu = 1.00 \times 10^{-6}$ m^2/s, $\gamma = 9.79$ kN/m^3, and

$$\frac{k_\mathrm{s}}{D} = \frac{0.1}{25} = 0.004$$
$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.025)^2 = 4.909 \times 10^{-4}\,\mathrm{m}^2$$
$$h_\mathrm{f} = f\frac{L}{D}\frac{Q^2}{2gA^2} = f\frac{100}{0.025}\frac{Q^2}{2(9.81)(4.909 \times 10^{-4})^2} = 8.46 \times 10^8 fQ^2$$

The energy equation applied over 100 m of pipe is

$$\frac{p_1}{\gamma} + \frac{V^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V^2}{2g} + z_2 + h_\mathrm{f}$$

which simplifies to

$$p_2 = p_1 - \gamma(z_2 - z_1) - \gamma h_\mathrm{f}$$
$$p_2 = 550 - 9.79(100 \sin 10°) - 9.79(8.46 \times 10^8 fQ^2)$$
$$p_2 = 380.0 - 8.28 \times 10^9 fQ^2$$

(a) For $Q = 2$ L/min $= 3.333 \times 10^{-5}$ m^3/s,

$$V = \frac{Q}{A} = \frac{3.333 \times 10^{-5}}{4.909 \times 10^{-4}} = 0.06790\,\mathrm{m/s}$$
$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(0.06790)(0.025)}{1 \times 10^{-6}} = 1698$$

Since Re < 2000, the flow is laminar when $Q = 2$ L/min. Hence,

$$f = \frac{64}{\mathrm{Re}} = \frac{64}{1698} = 0.03770$$
$$p_2 = 380.0 - 8.28 \times 10^9(0.03770)(3.333 \times 10^{-5})^2 = 380\,\mathrm{kPa}$$

Therefore, when the flow is 2 L/min, the pressure at the downstream section is $\boxed{380\text{ kPa}}$.
For $Q = 20$ L/min $= 3.333 \times 10^{-4}$ m^3/s,

$$V = \frac{Q}{A} = \frac{3.333 \times 10^{-4}}{4.909 \times 10^{-4}} = 0.6790\,\mathrm{m/s}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(0.6790)(0.025)}{1\times10^{-6}} = 16980$$

Since Re > 5000, the flow is turbulent when Q = 20 L/min. Hence,

$$f = \frac{0.25}{\left[\log\left(\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{0.004}{3.7} + \frac{5.74}{16980^{0.9}}\right)\right]^2} = 0.0342$$

$$p_2 = 380.0 - 8.28\times10^9(0.0342)(3.333\times10^{-4})^2 = 349\,\text{kPa}$$

Therefore, when the flow is 2 L/min, the pressure at the downstream section is $\boxed{349\text{ kPa}}$.

(b) Using the Colebrook equation with Q = 20 L/min,

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right] = -2\log\left[\frac{0.004}{3.7} + \frac{2.51}{16980\sqrt{f}}\right]$$

which yields $\boxed{f = 0.0337}$. Comparing this with the Swamee-Jain result of $\boxed{f = 0.0342}$ indicates a difference of $\boxed{1.5\%}$, which is more than the 1% claimed by Swamee-Jain.

2.9. The Colebrook equation is given by

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

Inverting and squaring this equation gives

$$\boxed{f = \frac{0.25}{\left\{\log\left[\left(\frac{k_s/D}{3.7}\right) + \frac{2.51}{\text{Re}\sqrt{f}}\right]\right\}^2}}$$

This equation is "slightly more convenient" than the Colebrook formula since it is quasi-explicit in f, whereas the Colebrook formula gives $1/\sqrt{f}$.

2.10. The Colebrook equation is preferable since it provides greater accuracy than interpolating from the Moody diagram.

2.11. From the given data: SG = 0.87, $\nu = 1.20\times10^{-4}\,\text{m}^2/\text{s}$, $D = 125\,\text{mm}$, $L = 80\,\text{m}$, $\theta = 8°$, $p_2 = 150\,\text{kPa}$, and $\tau_0 = 180\,\text{Pa}$. Quantities derived directly from the given data are: $\Delta z = L\sin\theta = 11.13\,\text{m}$ and $\gamma = \text{SG}\cdot g = 8.532\,\text{kN/m}^3$. The shear stress, τ_0 is related to the friction factor, f, as follows

$$f = \frac{\tau_0}{\frac{1}{8}\rho V^2} \quad\rightarrow\quad fV^2 = \frac{8\tau_0}{\rho} \tag{1}$$

The energy equation applied between the upstream and downstream ends of the pipe is as follows:

$$\frac{p_1}{\gamma} + \frac{fL}{D}\frac{V^2}{2g} = \frac{p_2}{\gamma} + \Delta z \tag{2}$$

Combining Equations 1 and 2, rearranging to make p_1 the subject of the formula, and substituting given parameter values yields

$$p_1 = p_2 + \gamma\Delta z + \frac{4\tau_0 L}{D} = 150 + (8.532)(11.13) + \frac{4(180)(80)}{0.125}[\ \times10^{-3}\ \text{kPa/Pa}] = \boxed{706\,\text{kPa}}$$

2.12. From the given data: $Q = 500\,\text{L/s} = 0.5\,\text{m}^3/\text{s}$, $D = 750\,\text{mm}$, $k_\text{s} = 1.5\,\text{mm}$, $L = 3\,\text{km}$, and $\Delta z = 1.5\,\text{m}$. For crude oil at 20°C, $\rho = 856\,\text{kg/m}^3$, $\mu = 7.2\,\text{mPa·s}$, $\nu = \mu/\rho = 8.411 \times 10^{-6}\,\text{m}^2/\text{s}$, and $\gamma = \rho g = 8.395\,\text{kN/m}^3$. The following preliminary calculations are useful:

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.5)^2}{4} = 0.4418\,\text{m}^2, \qquad V = \frac{Q}{A} = \frac{0.5}{0.4418} = 1.132\,\text{m/s}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.132)(0.750)}{8.411 \times 10^{-6}} = 1.009 \times 10^5, \qquad \frac{k_\text{s}}{D} = \frac{1.5}{750} = 2.00 \times 10^{-3}$$

Using the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_\text{s}/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right] \rightarrow \frac{1}{\sqrt{f}} = -2\log\left[\frac{2.00 \times 10^{-3}}{3.7} + \frac{2.51}{1.009 \times 10^5\sqrt{f}}\right]$$

which yields $f = 0.0251$. Substituting into the Darcy–Weisbach equation (Equation **??**) gives the head loss due to friction, h_f, over the length of the pipe as

$$h_\text{f} = \frac{fL}{D}\frac{V^2}{2g} = \frac{(0.0251)(3000)}{0.750}\frac{1.132^2}{2(9.807)} = 6.555\,\text{m}$$

(a) The pressure change, Δp, can be determined using the energy equation, which gives

$$-\left(\frac{\Delta p}{\gamma} + \Delta z\right) = h_\text{f} \quad \rightarrow \quad \Delta p = -\gamma(h_\text{f} + \Delta z)$$

Substituting the given and derived data gives

$$\Delta p = -(8.395)(6.555 + 1.5) = \boxed{-67.6\,\text{kPa}}$$

(b) The rate, P, at which energy is being consumed is given by

$$P = \gamma Q h_\text{f} = (8.395)(0.50)(6.555) = \boxed{27.5\,\text{kW}}$$

(c) If the roughness height is reduced by 70%, then $k_\text{s}/D = 0.3(2.00 \times 10^{-3}) = 6 \times 10^{-4}$. Repeating the calculations in Parts (a) and (b) for the new value of k_s/D gives

$$\Delta p = -58.0\,\text{kPa}, \qquad P = 22.7\,\text{kW}$$

Therefore, the pressure change is reduced by $\boxed{14\%}$, and the rate of energy loss is reduced by $\boxed{17\%}$.

2.13. From the given data: $\text{SG} = 0.87$, $\nu = 1.20 \times 10^{-4}\,\text{m}^2/\text{s}$, $D = 50\,\text{mm}$, $A = \pi D^2/4 = 1.963 \times 10^{-3}\,\text{m}^2$, $k_\text{s} \approx 0$, and $\tau_0 = 150\,\text{Pa}$. For the given fluid, $\rho = \text{SG} \cdot 1000 = 870\,\text{kg/m}^3$. The shear stress, τ_0 is related to the friction factor, f, as follows

$$f = \frac{\tau_0}{\frac{1}{8}\rho V^2} \quad \rightarrow \quad fV^2 = \frac{8\tau_0}{\rho}$$

If the flow is turbulent, the f can be evaluated using the Colebrook equation such that

$$f_{\text{CE}}\left(\text{Re}, \frac{k_s}{D}\right)\left(\frac{VD}{\nu}, 0\right)V^2 = \frac{8\tau_0}{\rho} \quad \rightarrow \quad f_{\text{CE}}\left(\text{Re}, \frac{k_s}{D}\right)\left(\frac{V(0.050)}{1.20\times 10^{-4}}, 0\right)V^2 = \frac{8(150)}{870} \quad \rightarrow \quad V = 5.38\,\text{m/s}$$

This gives a Reynolds number (VD/ν) equal to 2241 and a flow rate (AV) of 10.6 L/s. Since $2000 < \text{Re} < 4000$ the flow is in the transition stage and the assumption of turbulent flow is not validated. Try laminar flow where $f = 64/\text{Re}$, which gives

$$\frac{64}{\frac{VD}{\nu}}V^2 = \frac{8\tau_0}{\rho} \quad \rightarrow \quad \frac{64}{\frac{V(0.050)}{1.20\times 10^{-4}}}V^2 = \frac{8(150)}{870} \quad \rightarrow \quad V = 8.98\,\text{m/s}$$

This gives a Reynolds number (VD/ν) equal to 3742 and a flow rate (AV) of 17.6 L/s. Since $2000 < \text{Re} < 4000$ the flow is in the transition stage and the assumption of laminar flow is not validated. These results indicate that the flow is likely in the transitional range. The flow rate given by the turbulent-flow analysis is 10.6 L/s, and the flow rate given by the laminar-flow analysis is 17.6 L/s. Therefore the flow rate is expected to vary in the range of $\boxed{\text{10.6–17.6 L/s}}$.

2.14. From the given data: $D = 0.5$ m, $p_1 = 600$ kPa, $Q = 0.50\,\text{m}^3/\text{s}$, $z_1 = 120$ m, $z_2 = 100$ m, $\gamma = 9.79\,\text{kN/m}^3$, $L = 1000$ m, $k_s = 0.26$ mm (ductile iron). The following preliminary calculation is useful:

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.5)^2 = 0.1963\,\text{m}^2, \qquad V = \frac{Q}{A} = \frac{0.50}{0.1963} = 2.55\,\text{m/s}$$

Using the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

where $k_s/D = 0.26/500 = 0.00052$, and at 20°C,

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(998)(2.55)(0.5)}{1.00\times 10^{-3}} = 1.27\times 10^6$$

Substituting k_s/D and Re into the Colebrook equation gives

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{0.00052}{3.7} + \frac{2.51}{1.27\times 10^6\sqrt{f}}\right) \quad \rightarrow \quad f = 0.0172$$

Applying the energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f$$

Since $V_1 = V_2$, and h_f is given by the Darcy-Weisbach equation, then the energy equation can be written as

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + f\frac{L}{D}\frac{V^2}{2g}$$

Substituting known values leads to

$$\frac{600}{9.79} + 120 = \frac{p_2}{9.79} + 100 + 0.0172\frac{1000}{0.5}\frac{(2.55)^2}{2(9.81)} \quad \rightarrow \quad \boxed{p_2 = 684\,\text{kPa}}$$

If p is the (static) pressure at the top of a 30 m high building, then

$$p = p_2 - 30\gamma = 684 - 30(9.79) = 390\,\text{kPa}$$

$\boxed{\text{This (static) water pressure is adequate for service.}}$

2.15. From the given data: $Q = 50\,\text{L/s} = 0.05\,\text{m}^3/\text{s}$, $D = 200\,\text{mm}$, $k_\text{s} = 0.12\,\text{mm}$, $L_1 = 1\,\text{km}$, $L_2 = 3\,\text{km}$, $p_\text{A} = 500\,\text{kPa}$, $p_\text{B} = 350\,\text{kPa}$. For water at 20°C, $\nu = 1.00 \times 10^{-6}\,\text{m}^2/\text{s}$ and $\gamma = 9.789\,\text{kN/m}^3$. From the given data:

$$A = \frac{\pi D^2}{4} = 0.03141\,\text{m}^2, \qquad V = \frac{Q}{A} = 1.592\,\text{m/s}, \qquad \text{Re} = \frac{VD}{\nu} = 3.183 \times 10^5, \qquad \frac{k_\text{s}}{D} = 6 \times 10^{-4}$$

Using the Colebrook equation with the above values of Re and k_s/D gives $f = 0.0187$.

(a) Apply the energy equation between the reservoir and Location A:

$$h_\text{p} = \left[f\frac{L_1}{D} + 1\right]\frac{V^2}{2g} + \frac{p_\text{A}}{\gamma} + z_\text{A} = \left[(0.0187)\frac{1000}{0.2} + 1\right]\frac{1.592^2}{2(9.907)} + \frac{500}{9.789} + 20 = 83.27\,\text{m}$$

Apply the energy equation between the reservoir and Location B:

$$h_\text{p} = \left[f\frac{L_2}{D} + 1\right]\frac{V^2}{2g} + \frac{p_\text{B}}{\gamma} + z_\text{B} = \left[(0.0187)\frac{3000}{0.2} + 1\right]\frac{1.592^2}{2(9.907)} + \frac{350}{9.789} - 20 = 52.08\,\text{m}$$

Since both of the above conditions must be met, the required pump head is $h_\text{p} = 74.29\,\text{m}$, and the required power, P, is given by

$$P = \gamma Q h_\text{p} = (9.789)(0.05)(83.27) = \boxed{40.8\,\text{kW}}$$

(b) If the flow is fully turbulent, $\text{Re} \rightarrow \infty$ and the Colebrook equation gives $f = 0.0174$. This friction factor differs by around 7% from the actual friction factor ($= 0.0187$), therefore the flow is $\boxed{\text{not fully turbulent}}$.

2.16. From the given data: $D = 0.2\,\text{m}$, $Q = 0.03\,\text{m}^3/\text{s}$, $k_\text{s} = 0.01\,\text{mm}$, $p_1 = 500\,\text{kPa}$, $z_1 = 10.00\,\text{m}$, $z_2 = 10.50\,\text{m}$, and $L = 100\,\text{m}$. For water at 20°C, $\gamma = 9.789\,\text{kN/m}^3$ and $\nu = 10^{-6}\,\text{m}^2/\text{s}$. The following preliminary calculations are useful:

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.2)^2 = 0.03142\,\text{m}^2, \qquad V = \frac{Q}{A} = \frac{0.03}{0.03142} = 0.9548\,\text{m/s}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(0.9548)(0.2)}{10^{-6}} = 190960$$

(a) Using the Jain equation to calculate the friction factor gives:

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_\mathrm{s}}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right] = -2\log\left[\frac{0.01}{3.7(200)} + \frac{5.74}{190960^{0.9}}\right] \quad \rightarrow \quad f = 0.0161$$

Calculate the head loss using the Darcy-Weisbach equation:

$$h_\mathrm{f} = f\frac{L}{D}\frac{V^2}{2g} = 0.0161\frac{100}{0.2}\frac{0.9548^2}{2(9.807)} = 0.3742\,\mathrm{m}$$

Using the definition of the head loss:

$$\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right) = h_\mathrm{f}$$

$$\rightarrow \quad \left(\frac{500}{9.807} + 10.00\right) - \left(\frac{p_2}{9.789} + 10.50\right) = 0.3742 \quad \rightarrow \quad \boxed{p_2 = 491\,\mathrm{kPa}}$$

(b) This pressure $\boxed{\text{is adequate}}$ for water supply.

2.17. From the given data: $Q = 100\,\mathrm{L/s}$, $L = 100\,\mathrm{m}$, $D = 200\,\mathrm{mm}$, $\Delta z = 0.8\,\mathrm{m}$, and $\Delta p = -90\,\mathrm{kPa}$. For water at 20°C, $\nu = 1.00 \times 10^{-6}\,\mathrm{m^2/s}$ and $\gamma = 9.79\,\mathrm{kN/m^3}$. The following preliminary calculations are useful,

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.2)^2}{4} = 3.142 \times 10^{-2}\,\mathrm{m}^2, \qquad V = \frac{Q}{A} = \frac{0.1}{3.142 \times 10^{-2}} = 3.183\,\mathrm{m/s}$$

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(3.183)(0.200)}{1.00 \times 10^{-6}} = 6.366 \times 10^5$$

(a) According to the energy equation,

$$h_\mathrm{f} = -\left(\frac{\Delta p}{\gamma} + \Delta z\right) = -\left(\frac{-90}{9.79} + 0.8\right) = 8.393\,\mathrm{m}$$

The corresponding friction factor can be calculated using the Darcy-Weisbach equation, which gives

$$h_\mathrm{f} = \frac{fL}{D}\frac{V^2}{2g} \quad \rightarrow \quad f = \frac{h_\mathrm{f}\cdot D \cdot 2g}{L \cdot V^2} = \frac{(8.393)(0.200)(2 \times 9.807)}{(100)(3.183)^2} = \boxed{0.0325}$$

(b) Using the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_\mathrm{s}/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right] \rightarrow \frac{1}{\sqrt{0.0325}} = -2\log\left[\frac{k_\mathrm{s}/200}{3.7} + \frac{2.51}{6.366 \times 10^5\sqrt{0.0325}}\right]$$

which yields $\boxed{k_\mathrm{s} = 1.2\,\mathrm{mm}}$. According to Table 2.1, k_s for ductile iron is typically around 0.1 mm or less, so the interior surface of the pipe is likely to be in $\boxed{\text{poor condition}}$.

2.18. The head loss, h_f, in the pipe is estimated by

$$h_\mathrm{f} = \left(\frac{p_\mathrm{main}}{\gamma} + z_\mathrm{main}\right) - \left(\frac{p_\mathrm{outlet}}{\gamma} + z_\mathrm{outlet}\right)$$

where p_main = 400 kPa, z_main = 0 m, p_outlet = 0 kPa, and z_outlet = 2.0 m. Therefore,

$$h_\mathrm{f} = \left(\frac{400}{9.79} + 0\right) - (0 + 2.0) = 38.9\,\mathrm{m}$$

Also, since D = 25 mm, L = 20 m, k_s = 0.15 mm (from Table 2.1), $\nu = 1.00 \times 10^{-6}$ m^2/s (at 20°C), the combined Darcy-Weisbach and Colebrook equation (Equation 2.36) yields,

$$\begin{aligned}
Q &= -2.221D^2\sqrt{\frac{gDh_\mathrm{f}}{L}}\log\left(\frac{k_\mathrm{s}/D}{3.7} + \frac{1.775\nu}{D\sqrt{gDh_\mathrm{f}/L}}\right) \\
&= -2.221(0.025)^2\sqrt{\frac{(9.81)(0.025)(38.9)}{20}}\log\left[\frac{0.15/25}{3.7} + \frac{1.775(1.00\times10^{-6})}{(0.025)\sqrt{(9.81)(0.025)(38.9)/20}}\right] \\
&= 0.00265\,\mathrm{m^3/s} = 2.65\,\mathrm{L/s}
\end{aligned}$$

The faucet can therefore be expected to deliver $\boxed{2.65\text{ L/s}}$ when fully open.

2.19. From the given data: $Q = 300\,\mathrm{L/s} = 0.300\,\mathrm{m^3/s}$, $L = 40\,\mathrm{m}$, and $h_\mathrm{f} = 45\,\mathrm{m}$. Assume that $\nu = 10^{-6}\,\mathrm{m^2/s}$ (at 20°C) and take $k_\mathrm{s} = 0.15\,\mathrm{mm}$ (from Table 2.1). Substituting these data into Equation 2.36 gives

$$\begin{aligned}
Q &= -2.221D^2\sqrt{\frac{gDh_\mathrm{f}}{L}}\log\left(\frac{k_\mathrm{s}/D}{3.7} + \frac{1.775\nu}{D\sqrt{gDh_\mathrm{f}/L}}\right) \\
0.3 &= -2.221D^2\sqrt{\frac{(9.81)D(45)}{(40)}}\log\left(\frac{0.00015}{3.7D} + \frac{1.775(10^{-6})}{D\sqrt{(9.81)D(45)/(40)}}\right)
\end{aligned}$$

This is an implicit equation in D that can be solved numerically to yield $\boxed{D = 166\,\mathrm{mm}}$.

2.20. Since k_s = 0.15 mm, L = 40 m, Q = 0.3 m^3/s, h_f = 45 m, $\nu = 1.00 \times 10^{-6}$ m^2/s, the Swamee-Jain approximation (Equation 2.37 gives

$$\begin{aligned}
D &= 0.66\left[k_\mathrm{s}^{1.25}\left(\frac{LQ^2}{gh_\mathrm{f}}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_\mathrm{f}}\right)^{5.2}\right]^{0.04} \\
&= 0.66\left\{(0.00015)^{1.25}\left[\frac{(40)(0.3)^2}{(9.81)(45)}\right]^{4.75} + (1.00\times10^{-6})(0.3)^{9.4}\left[\frac{40}{(9.81)(45)}\right]^{5.2}\right\}^{0.04} \\
&= 0.171\,\mathrm{m} = \boxed{171\,\mathrm{mm}}
\end{aligned}$$

The calculated pipe diameter (171 mm) is about 3% higher than calculated by the Colebrook equation (166 mm).

2.21. From the given data: $k_s/D = 1 \times 10^{-4}$ and $\mathrm{Re} = 10^4 - 10^8$. This problem can be done using MATLAB or a similar program.

(a) Using the Haaland equation, the maximum percentage difference with the Colebrook equation occurs at $\mathrm{Re} = 1.526 \times 10^5$ where $f_\mathrm{C} = 0.0172$, $f_\mathrm{H} = 0.0169$, and the percentage error is $\boxed{-1.37\%}$.

(b) Using the Swamee–Jain equation, the maximum percentage difference with the Colebrook equation occurs at $\mathrm{Re} = 2.20 \times 10^6$ where $f_\mathrm{C} = 0.0127$, $f_\mathrm{SJ} = 0.0128$, and the percentage error is $\boxed{0.59\%}$.

(c) Based on these results, the $\boxed{\text{Swamee-Jain equation}}$ would be preferable.

2.22. The kinetic energy correction factor, α, is defined by

$$\int_A \rho \frac{v^3}{2} dA = \alpha \rho \frac{V^3}{2} A$$

or

$$\alpha = \frac{\int_A v^3 dA}{V^3 A} \tag{1}$$

Using the velocity distribution in Problem 2.4 gives

$$\begin{aligned}
\int_A v^3 dA &= \int_0^R V_0^3 \left[1 - \left(\frac{r}{R}\right)^2\right]^2 2\pi r \; dr \\
&= 2\pi V_0^3 \int_0^R \left[1 - 3\left(\frac{r}{R}\right)^2 + 3\left(\frac{r}{R}\right)^4 - \left(\frac{r}{R}\right)^6\right] r \; dr \\
&= 2\pi V_0^3 \int_0^R \left[r - \frac{3r^3}{R^2} + \frac{3r^5}{R^4} - \frac{r^7}{R^6}\right] dr \\
&= 2\pi V_0^3 \left[\frac{r^2}{2} - \frac{3r^4}{4R^2} + \frac{r^6}{2R^4} - \frac{r^8}{8R^6}\right]_0^R \\
&= 2\pi R^2 V_0^3 \left[\frac{1}{2} - \frac{3}{4} + \frac{1}{2} - \frac{1}{8}\right] \\
&= \frac{\pi R^2 V_0^3}{4}
\end{aligned} \tag{2}$$

The average velocity, V, was calculated in Problem 2.4 as

$$V = \frac{V_0}{2}$$

hence

$$V^3 A = \left(\frac{V_0}{2}\right)^3 \pi R^2 = \frac{\pi R^2 V_0^3}{8} \tag{3}$$

Combining Equations 1 to 3 gives

$$\alpha = \frac{\pi R^2 V_0^3/4}{\pi R^2 V_0^3/8} = \boxed{2}$$

2.23. The kinetic energy correction factor, α, is defined by

$$\alpha = \frac{\int_A v^3 dA}{V^3 A} \tag{1}$$

Using the given velocity distribution gives

$$\int_A v^3 dA = \int_0^R v_0^3 \left(1 - \frac{r}{R}\right)^{\frac{3}{7}} 2\pi r \, dr = 2\pi v_0^3 \int_0^R \left(1 - \frac{r}{R}\right)^{\frac{3}{7}} r \, dr \tag{2}$$

To facilitate integration, let

$$x = 1 - \frac{r}{R} \tag{3}$$

which gives

$$r = R(1 - x) \tag{4}$$

$$dr = -R \, dx \tag{5}$$

Combining Equations 2 to 5 gives

$$\begin{aligned}
\int_A v^3 dA &= 2\pi v_0^3 \int_0^1 x^{\frac{3}{7}} R(1-x)(-R)dx \\
&= 2\pi R^2 v_0^3 \int_0^1 x^{\frac{3}{7}}(1-x)dx = 2\pi R^2 v_0^3 \int_0^1 (x^{\frac{3}{7}} - x^{\frac{10}{7}})dx \\
&= 2\pi R^2 v_0^3 \left[\frac{7}{10}x^{\frac{10}{7}} - \frac{7}{17}x^{\frac{17}{7}}\right]_0^1 \\
&= 0.576\pi R^2 v_0^3
\end{aligned} \tag{6}$$

The average velocity, V, is given by (using the same substitution as above)

$$\begin{aligned}
V &= \frac{1}{A}\int_A v \, dA \\
&= \frac{1}{\pi R^2}\int_0^R v_0 \left(1 - \frac{r}{R}\right)^{\frac{1}{7}} 2\pi r \, dr = \frac{2v_0}{R^2}\int_1^0 x^{\frac{1}{7}} R(1-x)(-R)dx \\
&= 2v_0 \int_0^1 (x^{\frac{1}{7}} - x^{\frac{8}{7}})dx = 2v_0 \left[\frac{7}{8}x^{\frac{8}{7}} - \frac{7}{15}x^{\frac{15}{7}}\right]_0^1 \\
&= 0.817 v_0
\end{aligned} \tag{7}$$

Using this result,

$$V^3 A = (0.817 v_0)^3 \pi R^2 = 0.545\pi R^2 v_0^3 \tag{8}$$

Combining Equations 1, 6, and 8 gives

$$\alpha = \frac{0.576\pi R^2 v_0^3}{0.545\pi R^2 v_0^3} = \boxed{1.06}$$

The momentum correction factor, β, is defined by

$$\beta = \frac{\int_A v^2 dA}{AV^2} \tag{9}$$

In this case,

$$AV^2 = \pi R^2(0.817v_0)^2 = 0.667\pi R^2 v_0^2 \tag{10}$$

and

$$\begin{aligned}\int_A v^2 dA &= \int_0^R v_0^2 \left(1 - \frac{r}{R}\right)^{\frac{2}{7}} 2\pi r\ dr \\ &= 2\pi v_0^2 \int_1^0 x^{\frac{2}{7}} R(1-x)(-R)dx = 2\pi R^2 v_0^2 \int_0^1 (x^{\frac{2}{7}} - x^{\frac{9}{7}})dx \\ &= 2\pi R^2 v_0^2 \left[\frac{7}{9}x^{\frac{9}{7}} - \frac{7}{16}x^{\frac{16}{7}}\right]_0^1 = 0.681\pi R^2 v_0^2 \end{aligned} \tag{11}$$

Combining Equations 9 to 11 gives

$$\beta = \frac{0.681\pi R^2 v_0^2}{0.667\pi R^2 v_0^2} = \boxed{1.02}$$

2.24. The kinetic energy correction factor, α, is defined by

$$\alpha = \frac{\int_A v^3 dA}{V^3 A} \tag{1}$$

Using the velocity distribution given by Equation **??** gives

$$\int_A v^3\,\mathrm{d}A = \int_0^R V_0^3 \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} 2\pi r\,\mathrm{d}r = 2\pi V_0^3 \int_0^R \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} r\,\mathrm{d}r \tag{2}$$

Let

$$x = 1 - \frac{r}{R}, \qquad r = R(1-x), \qquad \mathrm{d}r = -R\,\mathrm{d}x \tag{3}$$

Combining Equations 2 to 3 gives

$$\begin{aligned}\int_A v^3\,\mathrm{d}A &= 2\pi V_0^3 \int_0^1 x^{\frac{3}{n}} R(1-x)(-R)\,\mathrm{d}x = 2\pi R^2 V_0^3 \int_0^1 x^{\frac{3}{n}}(1-x)\,\mathrm{d}x \\ &= 2\pi R^2 V_0^3 \int_0^1 (x^{\frac{3}{n}} - x^{\frac{3+n}{n}})\,\mathrm{d}x = 2\pi R^2 V_0^3 \left[\frac{n}{3+n}x^{\frac{3+n}{n}} - \frac{n}{3+2n}x^{\frac{3+2n}{n}}\right]_0^1 \\ &= \frac{2n^2}{(3+n)(3+2n)}\pi R^2 V_0^3 \end{aligned} \tag{4}$$

The average velocity, V, is given by

$$\begin{aligned} V &= \frac{1}{A}\int_A v\,\mathrm{d}A = \frac{1}{\pi R^2}\int_0^R V_0 \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} 2\pi r\,\mathrm{d}r = \frac{2V_0}{R^2}\int_1^0 x^{\frac{1}{n}} R(1-x)(-R)\,\mathrm{d}x \\ &= 2V_0 \int_0^1 (x^{\frac{1}{n}} - x^{\frac{1+n}{n}})\,\mathrm{d}x = 2V_0 \left[\frac{n}{1+n}x^{\frac{1+n}{n}} - \frac{n}{1+2n}x^{\frac{1+2n}{n}}\right]_0^1 \\ &= \left[\frac{2n^2}{(1+n)(1+2n)}\right] V_0 \end{aligned} \tag{5}$$

Using this result,

$$V^3 A = \left[\frac{2n^2}{(1+n)(1+2n)}\right]^3 V_0^3 \pi R^2 = \frac{8n^6}{(1+n)^3(1+2n)^3}\pi R^2 V_0^3 \tag{6}$$

Combining Equations 1, 4, and 6 gives

$$\alpha = \frac{\dfrac{2n^2}{(3+n)(3+2n)}\pi R^2 V_0^3}{\dfrac{8n^6}{(1+n)^3(1+2n)^3}\pi R^2 V_0^3} = \boxed{\frac{(1+n)^3(1+2n)^3}{4n^4(3+n)(3+2n)}}$$

Putting $n = 7$ gives $\boxed{\alpha = 1.06}$, the same result obtained in Problem 2.23.

2.25. (a) For the logarithmic velocity distribution,

$$V_0 = (1 + 1.326\sqrt{f})V \tag{1}$$

For the power-law distribution, the average velocity can be calculated by integration,

$$V = \frac{1}{\pi R^2}\int_0^R V_0 \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} 2\pi r \, \mathrm{d}r$$

To facilitate integration let $x = 1 - r/R$ and hence $\mathrm{d}r = -R\,\mathrm{d}x$, which gives

$$V = -2V_0 \int_1^0 x^{\frac{1}{n}}(1-x)\,\mathrm{d}x = 2V_0\left[\frac{n}{n+1} - \frac{n}{2n+1}\right] \tag{2}$$

Combining Equations 1 and 2 gives the following requirement for both the average velocity (V) and the maximum velocity (V_0) to be equal

$$\boxed{\left[\frac{n}{n+1} - \frac{n}{2n+1}\right] = \frac{1}{2(1+1.326\sqrt{f})}} \tag{3}$$

(b) For $f = 0.02$, Equation 3 gives

$$\left[\frac{n}{n+1} - \frac{n}{2n+1}\right] = \frac{1}{2(1+1.326\sqrt{0.02})} \quad \rightarrow \quad n = \boxed{8.32}$$

(c) Normalizing the logarithmic velocity distribution by V_0 and using the calculated value for f gives

$$\frac{u}{V_0} = 1 - \frac{2.04\sqrt{f}}{1+1.326\sqrt{f}}\log\left[\frac{1}{1-(r/R)}\right] \quad \rightarrow \quad \boxed{\frac{u}{V_0} = 1 - 0.243\log\left[\frac{1}{1-(r/R)}\right]} \tag{4}$$

The normalized power-law velocity distribution given by Equation 2.56 can be expressed as

$$\frac{u}{V_0} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \quad \rightarrow \quad \boxed{\frac{u}{V_0} = \left(1 - \frac{r}{R}\right)^{0.120}} \tag{5}$$

The logarithmic and power-law velocity distributions are plotted and compared in Figure 2.1.

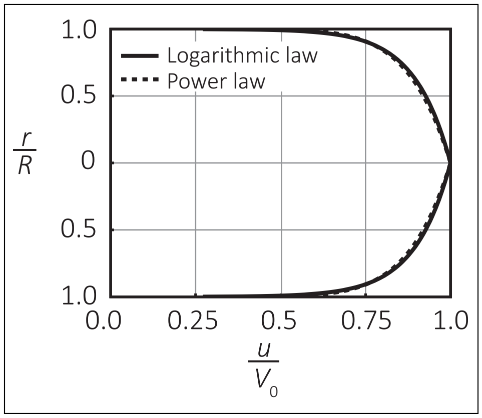

Figure 2.1: Comparison of turbulent velocity distributions

It is apparent from Figure 2.1 that the logarithmic and power-law velocity distributions are in very close agreement

2.26. From the given data: $p = 1\,\text{MPa}$, $D_\text{p} = 65\,\text{mm}$, $L = 300\,\text{m}$, $k_\text{s} = 0.5\,\text{mm}$, $k_\text{s}/D_\text{p} = 7.692 \times 10^{-3}$, $\Delta z = 2.8\,\text{m}$, $h = 30\,\text{m}$ and $k_\text{n} = 0.04$. For water at 20°C, $\gamma = 9.789\,\text{kN}{\cdot}\text{m}^3$ and $\nu = 1.004 \times 10^{-6}\,\text{m}^2/\text{s}$. The velocity of the jet, V_j, can be calculated from the energy equation, as follows

$$\frac{V_\text{j}^2}{2g} = h \quad \rightarrow \quad V_\text{j} = \sqrt{2gh} = \sqrt{2(9.807)(30)} = 24.26\,\text{m/s}$$

Equations that must be satisfied are the energy equation between the beginning and end of the hose, and also the continuity equation across the nozzle. These equations are given by,

$$\frac{p}{\gamma} + \frac{V_\text{p}^2}{2g} - \frac{fL}{D_\text{p}}\frac{V_\text{p}^2}{2g} - k_\text{n}\frac{V_\text{j}^2}{2g} = \frac{V_\text{j}^2}{2g} + \Delta z \quad \rightarrow \quad \frac{p}{\gamma} + \left[1 - \frac{fL}{D_\text{p}}\right]\frac{V_\text{p}^2}{2g} = (1 + k_\text{n})\frac{V_\text{j}^2}{2g} + \Delta z \qquad (1)$$

$$V_\text{p}A_\text{p} = V_\text{j}A_\text{j} \quad \rightarrow \quad V_\text{p} = \frac{D_\text{j}^2}{D_\text{p}^2}V_\text{j} = \frac{D_\text{j}^2}{0.065^2}(24.26) \quad \rightarrow \quad V_\text{p} = 5742D_\text{j}^2 \qquad (2)$$

The friction factor, f, calculated using the Colebrook equation is given by

$$f = f_\text{CE}\left(\text{Re}, \frac{k_\text{s}}{D}\right)\left(\frac{k_\text{s}}{D_\text{p}}, \text{Re}_\text{p}\right) \quad \rightarrow \quad f = f_\text{CE}\left(\text{Re}, \frac{k_\text{s}}{D}\right)\left(7.692 \times 10^{-3}, \frac{V_\text{p}(0.065)}{1.004 \times 10^{-6}}\right) \qquad (3)$$

Substitute Equation 3 and the given values into Equation 1, solve for V_p, and then use this value of V_p in Equation 2 to obtain D_j. The result is $V_\text{p} = 2.880\,\text{m/s}$ and $D_\text{j} = 0.0224\,\text{m} \approx \boxed{22\text{ mm}}$.

2.27. From the given data: $D = 200\,\text{mm}$, $L = 1\,\text{km}$, $\Delta z = 7\,\text{m}$, $p = 140\,\text{kPa}$, and $Q = 60\,\text{L/s} = 0.06\,\text{m}^3/\text{s}$. For water at 20°C, $\nu = 1.004 \times 10^{-6}\,\text{m}^2/\text{s}$ and $\gamma = 9.789\,\text{kN/m}^3$. The following preliminary calculations are useful,

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.2)^2}{4} = 3.142 \times 10^{-2}\,\mathrm{m}^2, \qquad V = \frac{0.06}{3.142 \times 10^{-2}} = 1.910\,\mathrm{m/s}$$

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(1.910)(0.2)}{1.004 \times 10^{-6}} = 3.805 \times 10^5, \qquad h_\mathrm{v} = \frac{V^2}{2g} = \frac{1.910^2}{2(9.807)} = 0.1860\,\mathrm{m}$$

In order to find the pipe roughness, apply the energy equation between the pressure source and the outlet, which gives

$$\frac{p}{\gamma} + \Delta z - \frac{fL}{D}\frac{V^2}{2g} = \frac{V^2}{2g} \quad \rightarrow \quad f = \frac{\dfrac{p}{\gamma} + \Delta z - \dfrac{V^2}{2g}}{\dfrac{L}{D}\dfrac{V^2}{2g}} = \frac{\dfrac{140}{9.789} + 7 - 0.1860}{\dfrac{1000}{0.2}0.1860} = 0.0227$$

Using the Colebrook equation,

$$f_\mathrm{CE}\left(\mathrm{Re}, \frac{k_\mathrm{s}}{D}\right)\left(\mathrm{Re}, \frac{k_\mathrm{s}}{D}\right) = 0.0227 \quad \rightarrow \quad f_\mathrm{CE}\left(\mathrm{Re}, \frac{k_\mathrm{s}}{D}\right)\left(3.805 \times 10^5, \frac{k_\mathrm{s}}{200}\right) = 0.0227 \quad \rightarrow \quad k_\mathrm{s} = 0.324\,\mathrm{mm}$$

With the new connection, $Q_{12} = 80\,\mathrm{L/s}$, $Q_{23} = 60\,\mathrm{L/s}$, and the following preliminary calculations are useful,

$$V_{12} = \frac{Q_{12}}{A} = \frac{0.08}{3.142 \times 10^{-2}} = 2.547\,\mathrm{m/s}, \qquad V_{23} = \frac{Q_{23}}{A} = \frac{0.06}{3.142 \times 10^{-2}} = 1.910\,\mathrm{m/s}$$

$$h_{\mathrm{v}12} = \frac{V_{12}^2}{2g} = \frac{2.547^2}{2(9.807)} = 0.3306\,\mathrm{m}, \qquad h_{\mathrm{v}23} = \frac{V_{23}^2}{2g} = \frac{1.910^2}{2(9.807)} = 0.1860\,\mathrm{m}$$

$$\mathrm{Re}_{12} = \frac{V_{12}D}{\nu} = \frac{(2.547)(0.2)}{1.004 \times 10^{-6}} = 5.073 \times 10^5, \quad \mathrm{Re}_{23} = \frac{V_{23}D}{\nu} = \frac{(1.910)(0.2)}{1.004 \times 10^{-6}} = 3.805 \times 10^5$$

$$f_{12} = f_\mathrm{CE}\left(\mathrm{Re}, \frac{k_\mathrm{s}}{D}\right)\left(\mathrm{Re}_{12}, \frac{k_\mathrm{s}}{D}\right) = 0.0226, \qquad f_{23} = f_\mathrm{CE}\left(\mathrm{Re}, \frac{k_\mathrm{s}}{D}\right)\left(\mathrm{Re}_{23}, \frac{k_\mathrm{s}}{D}\right) = 0.0227$$

$$L_{12} = \frac{L}{2} = \frac{1000}{2} = 500\,\mathrm{m}, \qquad L_{23} = \frac{L}{2} = \frac{1000}{2} = 500\,\mathrm{m}$$

Applying the energy equation from the beginning to the end of the pipeline gives

$$\frac{p_1}{\gamma} + \Delta z - \frac{f_{12}L_{12}}{D}\frac{V_{12}^2}{2g} - \frac{f_{23}L_{23}}{D}\frac{V_{23}^2}{2g} = \frac{V_{23}^2}{2g}$$

which yields

$$\begin{aligned} p_1 &= \gamma\left[-\Delta z + \frac{f_{12}L_{12}}{D}\frac{V_{12}^2}{2g} + \frac{f_{23}L_{23}}{D}\frac{V_{23}^2}{2g} + \frac{V_{23}^2}{2g}\right] \\ &= (9.789)\left[-7 + \frac{(0.0226)(500)}{0.2}(0.3306) + \frac{(0.0227)(500)}{0.2}(0.1860) + 0.1860\right] = \boxed{219\,\mathrm{kPa}} \end{aligned}$$

2.28. $p_1 = 30$ kPa, $p_2 = 500$ kPa, therefore head, h_p, added by pump is given by

$$h_\mathrm{p} = \frac{p_2 - p_1}{\gamma} = \frac{500 - 30}{9.79} = \boxed{48.0\,\mathrm{m}}$$

Power, P, added by pump is given by

$$P = \gamma Q h_\mathrm{p} = (9.79)(Q)(48.0) = \boxed{470 \text{ kW per m}^3/\text{s}}$$

2.29. $Q = 60\,\mathrm{L/s} = 0.06\,\mathrm{m^3/s}$, $D = 0.2$ m, $k_\mathrm{s} = 0.9$ mm (riveted steel), $k_\mathrm{s}/D = 0.9/200 = 0.00450$, for 90° bend $K = 0.3$, for the entrance $K = 1.0$, at 20°C $\rho = 998$ kg/m^3, and $\mu = 1.00 \times 10^{-3}$ Pa·s, therefore

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.2)^2 = 0.0314\,\mathrm{m}^2$$

$$V = \frac{Q}{A} = \frac{0.06}{0.0314} = 1.91\,\mathrm{m/s}$$

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{(998)(1.91)(0.2)}{1.00 \times 10^{-3}} = 3.81 \times 10^5$$

Substituting k_s/D and Re into the Colebrook equation gives

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{0.00450}{3.7} + \frac{2.51}{3.81 \times 10^5\sqrt{f}}\right)$$

which leads to

$$f = 0.0297$$

Minor head loss, h_m, is given by

$$h_m = \sum K\frac{V^2}{2g} = (1.0 + 0.3)\frac{(1.91)^2}{2(9.81)} = 0.242\,\mathrm{m}$$

If friction losses, h_f, account for 90% of the total losses, then

$$h_\mathrm{f} = f\frac{L}{D}\frac{V^2}{2g} = 9h_m$$

which means that

$$0.0297\frac{L}{0.2}\frac{(1.91)^2}{2(9.81)} = 9(0.242)$$

Solving for L gives

$$\boxed{L = 78.9\,\mathrm{m}}$$

For pipe lengths shorter than the length calculated in this problem, the word "minor" should not be used.

2.30. From the given data: $p_0 = 480$ kPa, $v_0 = 5$ m/s, $z_0 = 2.44$ m, $D = 19$ mm $= 0.019$ m, $L = 40$ m, $z_1 = 7.62$ m, and $\sum K_m = 3.5$. For copper tubing it can be assumed that $k_s = 0.0023$ mm. Applying the energy and Darcy-Weisbach equations between the water main and the faucet gives

$$\frac{p_0}{\gamma} + z_0 - h_f - h_m = \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1$$

$$\frac{480}{9.79} + 2.44 - \frac{f(40)}{0.019}\frac{v^2}{2(9.81)} - 3.5\frac{v^2}{2(9.81)} = \frac{0}{\gamma} + \frac{v^2}{2(9.81)} + 7.62$$

which simplifies to

$$v = \frac{6.622}{\sqrt{107.3f - 0.2141}} \qquad (1)$$

The Colebrook equation, with $\nu = 1 \times 10^{-6}$ m²/s gives

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_s}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f}}\right]$$

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{0.0025}{3.7(19)} + \frac{2.51}{\frac{v(0.019)}{1\times10^{-6}}\sqrt{f}}\right]$$

$$\frac{1}{\sqrt{f}} = -2\log\left[3.556 \times 10^{-5} + \frac{1.321 \times 10^{-4}}{v\sqrt{f}}\right] \qquad (2)$$

Combining Equations 1 and 2 gives

$$\frac{1}{\sqrt{f}} = -2\log\left[3.556 \times 10^{-5} + \frac{1.995 \times 10^{-5}\sqrt{107.3f - 0.2141}}{\sqrt{f}}\right]$$

which yields

$$f = 0.0189$$

Substituting into Equation 1 yields

$$v = \frac{6.622}{\sqrt{107.3(0.0189) - 0.2141}} = 4.92\,\text{m/s}$$

$$Q = Av = \left(\frac{\pi}{4}0.019^2\right)(4.92) = 0.00139\,\text{m}^3/\text{s} = \boxed{1.39\,\text{L/s}\ (= 22\ \text{gpm})}$$

This flow is $\boxed{\text{very high}}$ for a faucet. The flow would be $\boxed{\text{reduced}}$ if other faucets are open, this is due to increased pipe flow and frictional resistance between the water main and the faucet.

2.31. From the given data: $D = 50$ mm $= 0.050$ m, $L = 30.0$ m, and $k_s = 0$. Assume water at 20°C, $\gamma = 9.79$ kN/m³ and $\nu = 1.00 \times 10^{-6}$ m²/s. The local loss coefficients are given in the following table:

Location	Type of Fixture	k_{local}
1	sharp-edged entrance	0.5
2, 3, 5	90° elbow	1.2
4	tee	1.9
6	globe valve	4.0

Hence, between the water main and the faucet the total local loss coefficient is $k = 0.5 + 3(1.2) + 1.9 + 4 = 10$. The energy equation between points 1 and 6 is given by

$$\frac{p_1}{\gamma} + z_1 - \left[f\frac{L}{D} + \sum k\right]\frac{V^2}{2g} = \frac{V^2}{2g} + z_6$$

which can be conveniently expressed as

$$\frac{p_1}{\gamma} - \left[f\frac{L}{D} + \left(\sum k\right) + 1\right]\frac{V^2}{2g} = (z_6 - z_1) \tag{1}$$

The friction factor, f, can be determined from the following sequence of calculations,

$$\begin{aligned}
A &= \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.050)^2 = 0.001963\,\text{m}^2 \\
V &= \frac{Q}{A} = \frac{Q}{0.001963} = 509.3Q \\
\text{Re} &= \frac{VD}{\nu} = \frac{509.3Q(0.050)}{1.00\times10^{-6}} = 2.547\times10^7 Q \\
f &= \frac{0.25}{\left[\log\left(\frac{k_\text{s}}{3.7D} + \frac{5.75}{\text{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(0 + \frac{5.75}{(2.547\times10^7Q)^{0.9}}\right)\right]^2} = \frac{0.3086}{(6.563+\log Q)^2} \qquad (2)
\end{aligned}$$

Combining Equations 1 and 2 with the given data yields

$$\frac{400}{9.79} + 0 - \left[\frac{0.3086}{(6.563+\log Q)^2}\frac{30.0}{0.050} + 10 + 1\right]\frac{(509.3Q)^2}{2(9.81)} = 15$$

which yields $Q = 0.00991$ m^3/s = $\boxed{9.91 \text{ L/s}}$ = 595 L/min.

2.32. From the given data: $z_1 = -1.5$ m, $z_2 = 40$ m, $p_1 = 450$ kPa, $\sum k = 10.0$, $Q = 20$ L/s = 0.02 m^3/s, $D = 150$ mm (PVC), $L = 60$ m, $T = 20°$C, and $p_2 = 150$ kPa. The combined energy and Darcy-Weisbach equations give

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_\text{p} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left[\frac{fL}{D} + \sum k\right]\frac{V^2}{2g} \tag{1}$$

where

$$V_1 = V_2 = V = \frac{Q}{A} = \frac{0.02}{\frac{\pi(0.15)^2}{4}} = 1.13\,\text{m/s} \tag{2}$$

At 20°C, $\nu = 1.00\times10^{-6}$ m^2/s, and

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.13)(0.15)}{1.00\times10^{-6}} = 169500$$

Since PVC pipe is smooth ($k_\text{s} = 0$), the friction factor, f, is given by

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right) = -2\log\left(\frac{2.51}{169500\sqrt{f}}\right)$$

which yields

$$f = 0.0162 \qquad (3)$$

Taking $\gamma = 9.79$ kN/m^3 and combining Equations 1 to 3 yields

$$\frac{450}{9.79} + \frac{1.13^2}{2(9.81)} + (-1.5) + h_\mathrm{p} = \frac{150}{9.79} + \frac{1.13^2}{2(9.81)} + 40 + \left[\frac{(0.0162)(60)}{0.15} + 10\right]\frac{1.13^2}{2(9.81)}$$

which gives

$$h_\mathrm{p} = 11.9\,\mathrm{m}$$

Since $h_\mathrm{p} > 0$, a $\boxed{\text{booster pump is required}}$. The power, P, to be supplied by the pump is given by

$$P = \gamma Q h_\mathrm{p} = (9.79)(0.02)(11.9) = \boxed{2.3\,\mathrm{kW}}$$

2.33. From the given data: $D_\mathrm{p} = 20$ mm, $D_\mathrm{e} = 12$ mm, $k_\mathrm{s} = 0.003$ mm, and $Q = 50$ L/min $= 8.333 \times 10^{-4}\,\mathrm{m^3/s}$. For water at 20°C, $\gamma = 9.79\,\mathrm{kN/m^3}$ and $\nu = 1.00 \times 10^{-6}\,\mathrm{m^2/s}$. The following preliminary calculations are useful,

$$A_\mathrm{p} = \frac{\pi D_\mathrm{p}^2}{4} = \frac{\pi(0.020)^2}{4} = 3.142 \times 10^{-4}\,\mathrm{m^2}, \qquad V_\mathrm{p} = \frac{Q}{A_\mathrm{p}} = \frac{8.333 \times 10^{-4}}{3.142 \times 10^{-4}} = 2.653\,\mathrm{m/s}$$

$$\mathrm{Re} = \frac{VD_\mathrm{p}}{\nu} = \frac{(2.653)(0.020)}{1.00 \times 10^{-6}} = 5.305 \times 10^4, \qquad \frac{k_\mathrm{s}}{D_\mathrm{p}} = \frac{0.003}{20} = 1.50 \times 10^{-4}$$

$$\Delta z = 2\,\mathrm{m} + 3\,\mathrm{m} = 5\,\mathrm{m}, \qquad L = 3.5\,\mathrm{m} + 2\,\mathrm{m} + 5\,\mathrm{m} + 3\,\mathrm{m} + 4\,\mathrm{m} = 17.5\,\mathrm{m}$$

$$\sum K = 9 + 3(1.2) + 10 = 22.6, \qquad A_\mathrm{e} = \frac{\pi D_\mathrm{e}^2}{4} = \frac{\pi(0.012)^2}{4} = 1.131 \times 10^{-4}\,\mathrm{m^2}$$

$$V_\mathrm{e} = \frac{Q}{A_\mathrm{e}} = \frac{8.333 \times 10^{-4}}{1.131 \times 10^{-4}} = 7.368\,\mathrm{m/s}$$

Using $\mathrm{Re} = 5.305 \times 10^4$ and $k_\mathrm{s}/D = 1.50 \times 10^{-4}$ in the Colebrook equation yields $f = 0.0212$. Applying the energy equation between the water main and the faucet exit to obtain the required pressure, p_m, in the main gives

$$\frac{p_\mathrm{m}}{\gamma} - \left[\frac{fL}{D} + \sum K\right]\frac{V_\mathrm{p}^2}{2g} = \frac{V_\mathrm{e}^2}{2g} + \Delta z$$

$$\frac{p_\mathrm{m}}{9.79} - \left[\frac{(0.0212)(17.5)}{0.020} + 22.6\right]\frac{2.653^2}{2(9.807)} = \frac{7.368^2}{2(9.807)} + 5 \quad \rightarrow \quad p_\mathrm{m} = \boxed{220\,\mathrm{kPa}}$$

2.34. From the given data: $Q = 6\,\mathrm{L/s} = 0.006\,\mathrm{m^3/s}$, $D_1 = 75\,\mathrm{mm}$, and $D_2 = 50\,\mathrm{mm}$. For water at 20°C, $\rho = 998\,\mathrm{kg/m^3}$, and $\gamma = 9790\,\mathrm{N/m^3}$. The following preliminary calculations are useful:

$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi(0.050)^2}{4} = 1.963 \times 10^{-3}\,\mathrm{m^2}, \qquad A_1 = \frac{\pi D_1^2}{4} = \frac{\pi(0.075)^2}{4} = 4.418 \times 10^{-3}\,\mathrm{m^2}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.006}{1.963 \times 10^{-3}} = 3.056\,\mathrm{m/s}, \qquad V_1 = \frac{Q}{A_1} = \frac{0.006}{4.418 \times 10^{-3}} = 1.358\,\mathrm{m/s}$$

$$\frac{D_2}{D_1} = \frac{50}{75} = 0.667$$

(a) Using the head-loss coefficient given by Equation **??**, since $D_2/D_1 < 0.76$, the head loss, h_ℓ, at the sudden contraction is calculated as follows,

$$K = 0.42\left(1 - \frac{D_2^2}{D_1^2}\right) = 0.42\left(1 - \frac{50^2}{75^2}\right) = 0.2333$$

$$h_\ell = K\frac{V_2^2}{2g} = (0.2333)\frac{3.056^2}{2(9.807)} = 0.1111\,\text{m}$$

Therefore, the head loss at the sudden expansion is approximately equal to $\boxed{0.11\text{ m}}$.

(b) The pressure change, Δp, at the expansion is calculated by applying the energy equation across the expansion, such that

$$-\left(\frac{\Delta p}{\gamma} + \frac{\Delta V^2}{2g}\right) = h_\ell \quad \rightarrow \quad \Delta p = -\left(\gamma h_\ell + \tfrac{1}{2}\rho\Delta V^2\right) \tag{1}$$

Substituting the given and derived data into Equation 1 gives

$$\Delta p = -\left[(9790)(0.1111) + \tfrac{1}{2}(998)(3.056^2 - 1.358^2)\right]\ [\times 10^{-3}\text{ kPa/Pa}] = -1.278\,\text{kPa}$$

Therefore, it is expected that the pressure will decrease by 1.28 kPa across the sudden expansion.

(c) If the local head loss is not taken into account, then $h_\ell = 0$ and Equation 1 gives

$$\Delta p = -\left[0 + \tfrac{1}{2}(998)(3.056^2 - 1.358^2)\right]\ [\times 10^{-3}\text{ kPa/Pa}] = -0.1906\,\text{kPa}$$

Therefore, if the local head loss is neglected, the pressure would be predicted to decrease by approximately $\boxed{0.19\text{ kPa}}$.

2.35. From the given data: $D = 41\,\text{mm}$, $p = 350\,\text{kPa}$, $D_\text{e} = 30\,\text{mm}$, $z_\text{e} = 1\,\text{m}$, $h = 5\,\text{m}$, $K_\text{b} = 1.2$, $K_\text{n} = 0.05$, $k_\text{s} = 0.1\,\text{mm}$, and $L = 15\,\text{m} + 3\,\text{m} = 18\,\text{m}$. For water at 20°C, $\gamma = 9.79\,\text{kN/m}^3$ and $\nu = 1.00 \times 10^{-6}\,\text{m}^2/\text{s}$. Applying the Bernoulli equation to the fountain gives

$$\frac{V_\text{e}^2}{2g} = h \quad \rightarrow \quad V_\text{e} = \sqrt{2gh} = \sqrt{2(9.807)(5)} = 9.903\,\text{m/s}$$

Applying the continuity equation gives the velocity, V, is the pipe as

$$V = \frac{A_\text{e}}{A}\,V_\text{e} = \frac{D_\text{e}^2}{D^2}\,V_\text{e} = \frac{30^2}{41^2}(9.903) = 5.302\,\text{m/s}$$

Using the values of the given and derived parameters, the following preliminary calculations are useful,

$$\text{Re} = \frac{VD}{\nu} = \frac{(5.302)(0.041)}{1.00 \times 10^{-6}} = 2.174 \times 10^5, \qquad \frac{k_\text{s}}{D} = \frac{0.1}{41} = 2.439 \times 10^{-3}$$

$$\frac{V^2}{2g} = \frac{5.302^2}{2(9.807)} = 1.433\,\text{m}, \qquad \frac{V_\text{e}^2}{2g} = \frac{9.903^2}{2(9.807)} = 5.000\,\text{m}$$

Substituting the given and derived parameters into the Colebrook equation (Equation 2.28) gives

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_\text{s}/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right] \quad \rightarrow \quad \frac{1}{\sqrt{f}} = -2\log\left[\frac{2.439\times10^{-3}}{3.7} + \frac{2.51}{2.174\times10^5\sqrt{f}}\right]$$

which yields $f = 0.0254$. Applying the energy equation between the water main and the nozzle exit gives

$$\frac{p}{\gamma} - \left(\frac{fL}{D} + K_\text{v}\right)\frac{V^2}{2g} - K_\text{n}\frac{V_\text{e}^2}{2g} = \frac{V_\text{e}^2}{2g} + z_\text{e}$$

which gives

$$K_\text{v} = \frac{\dfrac{p}{\gamma} - K_\text{n}\dfrac{V_\text{e}^2}{2g} - \dfrac{V_\text{e}^2}{2g} - z_\text{e}}{\dfrac{V^2}{2g}} - \frac{fL}{D} = \frac{\dfrac{350}{9.79} - (1.2)(5) - 5 - 1}{1.433} - \frac{(0.0254)(18)}{0.041} = \boxed{9.42}$$

2.36. From the given data: $p_0 = 400\,\text{kPa}$, $L = 50\,\text{m}$, $k_\text{s} = 0.1\,\text{mm}$, $K = 30$, and $Q = 100\,\text{L/min} = 1.667\times10^{-3}\,\text{m}^3/\text{s}$. For water at 15°C, $\nu = 1.14\times10^{-6}\,\text{m}^2/\text{s}$, and $\gamma = 9.798\,\text{kN/m}^3$. The following functions can be defined in terms of the unknown diameter, D,

$$(1)\ A(D) = \frac{\pi D^2}{4}, \qquad (2)\ V(D) = \frac{Q}{A(D)} = \frac{1.667\times10^{-3}}{A(D)}$$

$$(3)\ \text{Re}(D) = \frac{V(D)D}{\nu} = \frac{V(D)D}{1.14\times10^{-6}}, \qquad (4)\ f(D) = f_\text{CE}\left(\frac{k_\text{s}}{D}, \text{Re}(D)\right)$$

$$(5)\ h_\text{v}(D) = \frac{V(D)^2}{2g}$$

where f_CE is the friction factor derived from the Colebrook equation. Applying the energy equation with the defined functions gives

$$\frac{p_0}{\gamma} - \left[\frac{f(D)L}{D} + K\right]h_\text{v}(D) = h_\text{v}(D)$$

$$\frac{400}{9.798} - \left[\frac{f(D)(50)}{D} + 30\right]h_\text{v}(D) = h_\text{v}(D) \quad \rightarrow \quad D = 0.0264\,\text{m} = \boxed{26.4\,\text{mm}}$$

2.37. From the given data: $\Delta z = 42\,\text{m} + 1.5\,\text{m} = 43.5\,\text{m}$, $L = 1\,\text{km}$, $D = 600\,\text{mm}$, $k_\text{s} = 1\,\text{mm}$, and $K_\text{b} = 0.8$. For water at 20°C, $\nu = 1.00\times10^{-6}\,\text{m}^2/\text{s}$, and $\gamma = 9.79\,\text{kN/m}^3$. The value of h can be calculated for any given combination of p and Q. The calculations will be illustrated for $Q = 0.3\,\text{m}^3/\text{s}$ and $p_\text{B} = 450\,\text{kPa}$. The following preliminary calculations are useful,

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.6)^2}{4} = 0.2827\,\text{m}^2, \qquad V = \frac{Q}{A} = \frac{1}{0.2827} = 1.061\,\text{m/s}$$

$$\frac{k_s}{D} = \frac{1}{600} = 1.667 \times 10^{-3}, \qquad \text{Re} = \frac{VD}{\nu} = \frac{(1.061)(0.6)}{1.00 \times 10^{-6}} = 6.366 \times 10^5$$

Substituting the given and derived parameters into the Colebrook equation (Equation 2.28) gives

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right] \quad \rightarrow \quad \frac{1}{\sqrt{f}} = -2\log\left[\frac{1.667 \times 10^{-3}}{3.7} + \frac{2.51}{6.366 \times 10^5\sqrt{f}}\right]$$

which yields $f = 0.0227$. Applying the energy equation between Points A and B gives

$$h + \Delta z - \left[\frac{fL}{D} + 5K_b\right]\frac{V^2}{2g} = \frac{p_B}{\gamma}$$

$$h + 43.5 - \left[\frac{(0.0227)(1000)}{0.6} + 5(0.8)\right]\frac{1.061^2}{2(9.807)} = \frac{450}{9.79} \quad \rightarrow \quad h = 4.86\,\text{m}$$

These calculations can be repeated for the given extreme values of p_B and Q leading to the following results:

Q (m^3/s)	p_B (kPa)	h (m)
0.3	330	−7.39
0.3	450	4.86
1.0	330	16.59
1.0	450	28.85

Since water will always be maintained in the tank, the range of h is $\boxed{\text{0 to 28.9 m}}$. At the minimum flow rate, the minimum pressure (when $h = 0$ m) is approximately $\boxed{\text{403 kPa}}$.

2.38. from the given data: $L = 25$ m, $D = 19$ mm, $p_0 = 200$ kPa, $\Delta z = 0.9$ m, $k_s = 0.1$ mm, $D_e = 10$ mm, and $K_e = 0.05$. For water at 15°C, $\nu = 1.140 \times 10^{-6}\ \text{m}^2/\text{s}$, and $\gamma = 9.798\ \text{kN/m}^3$. The following preliminary calculations are useful,

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.019)^2}{4} = 2.835 \times 10^{-4}\ \text{m}^2, \qquad r = \frac{D^4}{D_e^4} = 13.03$$

$$\frac{k_s}{D} = \frac{0.1}{19} = 5.263 \times 10^{-3}, \qquad A_e = \frac{\pi D_e^2}{4} = \frac{\pi(0.010)^2}{4} = 7.853 \times 10^{-5}\ \text{m}^2$$

For any given flow rate, Q, the following functions can be defined,

$$(1)\ V(Q) = \frac{Q}{A} = \frac{Q}{2.835 \times 10^{-4}}, \qquad (2)\ h_v(Q) = \frac{V(Q)^2}{2g}$$

$$(3)\ \text{Re}(Q) = \frac{V(Q)D}{\nu} = \frac{V(Q)(0.019)}{1.140 \times 10^{-6}}, \qquad (4)\ f(Q) = f_{CE}\left[\frac{k_s}{D}, \text{Re}(Q)\right]$$

where f_{CE} represents the value of f derived using the Colebrook equation.

(a) Applying the energy equation between the spigot and the hose exit (without a nozzle) gives

$$\frac{p_0}{\gamma}+h_\mathrm{v}-\frac{f(Q)L}{D}h_\mathrm{v}=h_\mathrm{v} \quad\rightarrow\quad \frac{200}{9.798}-\frac{f(Q)(25)}{0.019}h_\mathrm{v}(Q)=0 \quad\rightarrow\quad Q=8.667\times10^{-4}\,\mathrm{m^3/s}$$

Therefore the flow rate is $Q = 8.667 \times 10^{-4}\,\mathrm{m^3/s} = \boxed{52\ \mathrm{L/min}}$. The corresponding exit velocity, V_e, is given by

$$V_\mathrm{e}=\frac{Q}{A}=\frac{8.667\times10^{-4}}{2.835\times10^{-4}}=\boxed{3.06\,\mathrm{m/s}}$$

(b) Applying the energy equation between the spigot and the hose exit (with a nozzle) gives

$$\frac{p_0}{\gamma}+h_\mathrm{v}-\frac{f(Q)L}{D}h_\mathrm{v}-K_\mathrm{e}rh_\mathrm{v}=rh_\mathrm{v} \quad\rightarrow\quad \frac{200}{9.798}+\left[1-\frac{f(Q)(25)}{0.019}-K_\mathrm{e}r-r\right]h_\mathrm{v}(Q)=0$$

which gives $Q = 7.592 \times 10^{-4}\,\mathrm{m^3/s} = \boxed{46\ \mathrm{L/min}}$. The corresponding exit velocity, V_e, is given by

$$V_\mathrm{e}=\frac{Q}{A_\mathrm{e}}=\frac{7.592\times10^{-4}}{7.853\times10^{-5}}=\boxed{9.67\,\mathrm{m/s}}$$

(c) The main benefit of a nozzle is that it produces a jet that can deliver the liquid to a $\boxed{\text{further location}}$ (from the exit) than is possible without the nozzle.

2.39. From the given data: $L = 1\,\mathrm{km}$, $D = 500\,\mathrm{mm}$, $\Delta z = 20\,\mathrm{m}$, $K = 45$, $P = 250\,\mathrm{kW}$, and $Q = 500\,\mathrm{L/s} = 0.5\,\mathrm{m^3/s}$. For water at 20°C, $\gamma = 9.79\,\mathrm{kN/m^3}$, and $\nu = 1.00 \times 10^{-6}\,\mathrm{m^2/s}$. The following preliminary calculations are useful,

$$A=\frac{\pi D^2}{4}=\frac{\pi(0.0.5)^2}{4}=0.1963\,\mathrm{m}^2, \qquad V=\frac{Q}{A}=\frac{0.5}{0.1963}=2.547\,\mathrm{m/s}$$

$$h_\mathrm{v}=\frac{V^2}{2g}=\frac{2.547^2}{2(9.807)}=0.3306\,\mathrm{m}, \qquad \mathrm{Re}=\frac{VD}{\nu}=\frac{(2.547)(0.5)}{1.00\times10^{-6}}=1.273\times10^6$$

$$h_\mathrm{p}=\frac{P}{\gamma Q}=\frac{250}{(9.79)(0.5)}=51.07\,\mathrm{m}$$

(a) The friction factor as a function of k_s can be expressed as

$$f(k_\mathrm{s})=f_\mathrm{CE}\left(\frac{k_\mathrm{s}}{D},\mathrm{Re}\right)=f_\mathrm{CE}\left(\frac{k_\mathrm{s}}{500},1.273\times10^6\right)$$

where f_CE represents the value of f derived using the Colebrook equation. Applying the energy equation between the lower and upper reservoirs gives

$$0-\left[\frac{fL}{D}+K\right]h_\mathrm{v}+h_\mathrm{p}=\Delta z$$

$$0-\left[\frac{f(k_\mathrm{s})(1000)}{0.5}+45\right](0.3306)+51.07=20 \quad\rightarrow\quad k_\mathrm{s}=\boxed{1.2\,\mathrm{mm}}$$

The value of the friction factor corresponding to $k_\mathrm{s} = 1.2\,\mathrm{mm}$ is $f = 0.0245$. This friction factor will remain the same as long as the flow rate remains the same.

(b) If K is reduced by 70%, then $K = 0.3(45) = 13.5$. Applying the energy equation between the lower and upper reservoirs gives

$$0 - \left[\frac{fL}{D} + K\right] h_\mathrm{v} + h_\mathrm{p} = \Delta z$$

$$0 - \left[\frac{(0.0245)(1000)}{0.5} + 13.5\right](0.3306) + h_\mathrm{p} = 20 \quad \rightarrow \quad h_\mathrm{p} = 40.66\,\mathrm{m}$$

The corresponding pump power, P, is given by

$$P = \gamma Q h_\mathrm{p} = (9.79)(0.5)(40.66) = \boxed{199\,\mathrm{kW}}$$

Therefore the required pumping power is reduced by approximately 20%.

2.40. (a) Diameter of pipe, $D = 0.75$ m, area, A given by

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.75)^2 = 0.442\,\mathrm{m}^2$$

and velocity, V, in pipe

$$V = \frac{Q}{A} = \frac{1}{0.442} = 2.26\,\mathrm{m/s}$$

Energy equation between reservoir and A:

$$7 + h_\mathrm{p} - h_\mathrm{f} = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A \tag{1}$$

where $p_A = 350$ kPa, $\gamma = 9.79$ kN/m^3, $V_A = 2.26$ m/s, $z_A = 10$ m, and

$$h_\mathrm{f} = \frac{fL}{D}\frac{V^2}{2g}$$

where f depends on Re and k_s/D. At 20°C, $\nu = 1.00 \times 10^{-6}$ m^2/s and

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(2.26)(0.75)}{1.00 \times 10^{-6}} = 1.70 \times 10^6$$

$$\frac{k_s}{D} = \frac{0.26}{750} = 3.47 \times 10^{-4}$$

Using the Swamee-Jain equation,

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_\mathrm{s}/D}{3.7} + \frac{5.74}{\mathrm{Re}^{0.9}}\right] = -2\log\left[\frac{3.47 \times 10^{-4}}{3.7} + \frac{5.74}{(1.70 \times 10^6)^{0.9}}\right] = 7.93$$

which leads to

$$f = 0.0159$$

The head loss, h_f, between the reservoir and A is therefore given by

$$h_\mathrm{f} = \frac{fL}{D}\frac{V^2}{2g} = \frac{(0.0159)(1000)}{0.75}\frac{(2.26)^2}{2(9.81)} = 5.52\,\mathrm{m}$$

Substituting into Equation 1 yields

$$7 + h_\mathrm{p} - 5.52 = \frac{350}{9.81} + \frac{2.26^2}{2(9.81)} + 10$$

which leads to

$$\boxed{h_\mathrm{p} = 44.5\,\mathrm{m}}$$

(b) Power, P, supplied by the pump is given by

$$P = \gamma Q h_\mathrm{p} = (9.79)(1)(44.5) = \boxed{436\,\mathrm{kW}}$$

(c) Energy equation between A and B is given by

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A - h_\mathrm{f} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

and since $V_A = V_B$,

$$\begin{aligned} p_B &= p_A + \gamma(z_A - z_B - h_\mathrm{f}) = 350 + 9.79(10 - 4 - 5.52) \\ &= \boxed{355\,\mathrm{kPa}} \end{aligned}$$

2.41. From the given data: $Q = 0.7\,\mathrm{m^3/s}$, $D = 600\,\mathrm{mm}$, $k_\mathrm{s} = 1\,\mathrm{mm}$, $L_\mathrm{0A} = 1\,\mathrm{km}$, $L_\mathrm{AB} = 1.5\,\mathrm{km}$, $L_\mathrm{BC} = 1.8\,\mathrm{km}$, , $L_\mathrm{0C} = 4.3\,\mathrm{km}$, $z_\mathrm{A} = -10\,\mathrm{m}$, $z_\mathrm{B} = 12\,\mathrm{m}$, $z_\mathrm{C} = 5\,\mathrm{m}$, and $p_\mathrm{C} = 400\,\mathrm{kPa}$. For water at 20°C, $\gamma = 9.79\,\mathrm{kN/m^3}$ and $\nu = 1.00 \times 10^{-6}\,\mathrm{m^2/s}$. The following preliminary calculations are useful,

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.6)^2}{4} = 0.2827\,\mathrm{m^2}, \qquad V = \frac{Q}{A} = \frac{0.7}{0.2827} = 2.476\,\mathrm{m/s}$$

$$h_\mathrm{v} = \frac{V^2}{2g} = \frac{2.476^2}{2(9.807)} = 0.3125\,\mathrm{m}, \qquad \frac{k_\mathrm{s}}{D} = \frac{1}{600} = 1.667 \times 10^{-3}$$

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(2.476)(0.6)}{1.00 \times 10^{-6}} = 1.485 \times 10^6$$

Substituting the given and derived parameters into the Colebrook equation (Equation 2.28) gives

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_\mathrm{s}/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right] \quad \rightarrow \quad \frac{1}{\sqrt{f}} = -2\log\left[\frac{1.667 \times 10^{-3}}{3.7} + \frac{2.51}{1.485 \times 10^6\sqrt{f}}\right]$$

which yields $f = 0.0225$. Using the energy equation, the head added by the pump, h_p, is given by

$$0 + h_\mathrm{p} - \frac{fL_\mathrm{0C}}{D}h_\mathrm{v} = \frac{p_\mathrm{C}}{\gamma} + z_\mathrm{C} \rightarrow h_\mathrm{p} - \frac{(0.0225)(4300)}{0.6}(0.3125) = \frac{400}{9.79} + 5 \quad \rightarrow \quad h_\mathrm{p} = 96.18\,\mathrm{m}$$

The maximum pressure is expected at A. Applying the energy equation between the reservoir as A gives,

$$0 + h_\mathrm{p} - \frac{fL_\mathrm{0A}}{D}h_\mathrm{v} = \frac{p_\mathrm{A}}{\gamma} + z_\mathrm{A} \rightarrow 96.18 - \frac{(0.0225)(1000)}{0.6}(0.3125) = \frac{p_\mathrm{A}}{9.79} - 10 \quad \rightarrow \quad p_\mathrm{A} = 925\,\mathrm{kPa}$$

The minimum pressure could occur at B. Applying the energy equation between the reservoir as B gives,

$$0+h_\mathrm{p}-\frac{fL_{0\mathrm{B}}}{D}h_\mathrm{v}=\frac{p_\mathrm{B}}{\gamma}+z_\mathrm{B}\rightarrow 96.18-\frac{(0.0225)(2500)}{0.6}(0.3125)=\frac{p_\mathrm{B}}{9.79}+12 \quad\rightarrow\quad p_\mathrm{B}=538\,\mathrm{kPa}$$

(a) Based on evaluations of the pressures at key points in the pipeline, the estimated range of pressures is $\boxed{\text{400–925 kPa}}$.

(b) Expressing the maximum pressure in the pipeline as a function of the pipe diameter yields that the maximum pressure in the pipeline (at Point A) is equal to 600 kPa when D = 0.873 m = $\boxed{\text{873 mm}}$. This was done using MATLAB. Any similar software will yield the same result.

2.42. From the given data: $L=230\,\mathrm{m}$, $k_\mathrm{s}=0.2\,\mathrm{mm}$, $\Delta z=18\,\mathrm{m}$, $p=400\,\mathrm{kPa}$, and $Q=50\,\mathrm{L/s}=0.05\,\mathrm{m^3/s}$. For water at 20°C, $\gamma=9.789\,\mathrm{kN/m^3}$, and $\nu=1.004\times10^{-6}\,\mathrm{m^2/s}$. Applying the energy equation between the discharge side of the pump and the reservoir, and using the Colebrook equation to calculate f, yields

$$\frac{p}{\gamma}-\frac{fL}{D}\frac{V^2}{2g}-\frac{V^2}{2g}=\Delta z \quad\rightarrow\quad \frac{p}{\gamma}-\left[\frac{f_\mathrm{CE}\left(\mathrm{Re},\frac{k_\mathrm{s}}{D}\right)\left(\frac{4Q}{\pi D\nu},\frac{k_\mathrm{s}}{D}\right)L}{D}+1\right]\frac{8Q^2}{\pi^2D^4g}=\Delta z$$

Substituting the given data:

$$\frac{400}{9.789}-\left[\frac{f_\mathrm{CE}\left(\mathrm{Re},\frac{k_\mathrm{s}}{D}\right)\left(\frac{4(0.05)}{\pi D(1.004\times10^{-6})},\frac{0.0002}{D}\right)(230)}{D}+1\right]\frac{8(0.05)^2}{\pi^2D^4(9.807)}=18 \quad\rightarrow\quad D=0.136\,\mathrm{m}$$

Therefore, the required pipe diameter is approximately $\boxed{\text{136 mm.}}$

2.43. From the given data: $\Delta z=4\,\mathrm{m}$, $A_\mathrm{t}=10\,\mathrm{m^2}$, $D=75\,\mathrm{mm}$, $L=65\,\mathrm{m}$, $k_\mathrm{s}=0.5\,\mathrm{mm}$, $p=550\,\mathrm{kPa}$, $h_1=0.5\,\mathrm{m}$, and $h_2=3\,\mathrm{m}$. For water at 20°C, $\gamma=9.789\,\mathrm{kN/m^3}$. The following preliminary calculations are useful,

$$A=\frac{\pi D^2}{4}=\frac{\pi(0.075)^2}{4}=4.418\times10^{-3}\,\mathrm{m}^2, \qquad \frac{k_\mathrm{s}}{D}=\frac{0.5}{75}=6.667\times10^{-3}$$

$$f=f_\mathrm{CE}\left(\mathrm{Re},\frac{k_\mathrm{s}}{D}\right)\left(\frac{k_\mathrm{s}}{D},\infty\right)=0.0332, \qquad a=\frac{p}{\gamma}-4=\frac{550}{9.789}-4=52.19\,\mathrm{m}$$

and

$$b=A_\mathrm{t}^2\frac{fL}{2gA^2D}=(10)^2\frac{(0.0332)(65)}{2(9.807)(4.418\times10^{-3})^2(0.075)}=7.515\times10^6\ \mathrm{s^2/m}$$

Applying the energy equation between the pump and the reservoir gives

$$\frac{p}{\gamma}-\frac{fL}{D}\frac{Q^2}{2gA^2}=4+h \tag{1}$$

and the continuity equation requires that

$$Q = A_\mathrm{t}\frac{\mathrm{d}h}{\mathrm{d}t} \tag{2}$$

Combining Equations 1 and 1 to eliminate Q gives

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \sqrt{\frac{\frac{p}{\gamma} - 4 - h}{A_\mathrm{t}^2\frac{fL}{2gA^2D}}} \quad \rightarrow \quad \frac{\mathrm{d}h}{\mathrm{d}t} = \sqrt{\frac{a-h}{b}}$$

To integrate, use the substitution:

$$x = \frac{a-h}{b}, \qquad \mathrm{d}h = -b\,\mathrm{d}x$$

Therefore, the requisite integral in terms of x is

$$-b\int_{x_1}^{x_2}\frac{\mathrm{d}x}{\sqrt{x}} = \int_{t_1}^{t_2}\mathrm{d}t \quad \rightarrow \quad (t_2 - t_1) = -2b\left[\sqrt{x_2} - \sqrt{x_1}\right]$$

The given initial and final conditions of $h_1 = 0.5\,\mathrm{m}$ and $h_2 = 3\,\mathrm{m}$ translate into $x_1 = 6.88 \times 10^{-6}\ \mathrm{m}^2/\mathrm{s}^2$ and $x_2 = 6.54 \times 10^{-6}\ \mathrm{m}^2/\mathrm{s}^2$ and so

$$(t_2 - t_1) = -2b\left[\sqrt{6.54\times10^{-6}} - \sqrt{6.88\times10^{-6}}\right] = 965\,\mathrm{s} = \boxed{16.1\,\mathrm{min}} \tag{3}$$

2.44. From the given data: L = 3 km = 3000 m, Q_ave = 0.0175 m^3/s, and Q_peak = 0.578 m^3/s. If the velocity, V_peak, during peak flow conditions is 2.5 m/s, then

$$2.5 = \frac{Q_\mathrm{peak}}{\pi D^2/4} = \frac{0.578}{\pi D^2/4}$$

which gives

$$D = \sqrt{\frac{0.578}{\pi(2.5)/4}} = 0.543\,\mathrm{m}$$

Rounding to the nearest 25 mm gives

$$\boxed{D = 550\,\mathrm{mm}}$$

with a cross-sectional area, A, given by

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.550)^2 = 0.238\,\mathrm{m}^2$$

During average demand conditions, the head, h_ave, at the suburban development is given by

$$h_\mathrm{ave} = \frac{p_\mathrm{ave}}{\gamma} + \frac{V_\mathrm{ave}^2}{2g} + z_0 \tag{1}$$

where p_{ave} = 340 kPa, γ = 9.79 kN/m^3, $V_{\text{ave}} = Q_{\text{ave}}/A = 0.0175/0.238 = 0.0735\,\text{m/s}$, and z_0 = 8.80 m. Substituting into Equation 1 gives

$$h_{\text{ave}} = \frac{340}{9.79} + \frac{0.0735^2}{2(9.81)} + 8.80 = 43.5\,\text{m}$$

For ductile-iron pipe, k_s = 0.26 mm, $k_s/D = 0.26/550 = 4.73 \times 10^{-4}$, at 20°C $\nu = 1.00 \times 10^{-6}$ m^2/s, and therefore

$$\text{Re} = \frac{V_{\text{ave}}D}{\nu} = \frac{(0.0735)(0.550)}{1.00 \times 10^{-6}} = 4.04 \times 10^4$$

and the Swamee-Jain equation gives

$$\frac{1}{\sqrt{f_{\text{ave}}}} = -2\log\left[\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}}\right] = -2\log\left[\frac{4.73 \times 10^{-4}}{3.7} + \frac{5.74}{(4.04 \times 10^4)^{0.9}}\right]$$

and yields

$$f_{\text{ave}} = 0.0234$$

The head loss between the water treatment plant and the suburban development is therefore given by

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = (0.0234)\frac{3000}{0.550}\frac{0.0735^2}{2(9.81)} = 0.035\,\text{m}$$

Since the head at the water treatment plant is 10.00 m, the pump head, h_p, that must be added is

$$h_p = (43.5 + 0.035) - 10.00 = 33.5\,\text{m}$$

and the power requirement, P, is given by

$$P = \gamma Q h_p = (9.79)(0.0175)(33.5) = \boxed{5.74\,\text{kW}}$$

During peak demand conditions, the head, h_{peak}, at the suburban development is given by

$$h_{\text{peak}} = \frac{p_{\text{peak}}}{\gamma} + \frac{V_{\text{peak}}^2}{2g} + z_0 \tag{2}$$

where p_{peak} = 140 kPa, γ = 9.79 kN/m^3, $V_{\text{peak}} = Q_{\text{peak}}/A = 0.578/0.238 = 2.43\,\text{m/s}$, and z_0 = 8.80 m. Substituting into Equation 2 gives

$$h_{\text{peak}} = \frac{140}{9.79} + \frac{2.43^2}{2(9.81)} + 8.80 = 23.4\,\text{m}$$

For pipe, $k_s/D = 4.73 \times 10^{-4}$, and

$$\text{Re} = \frac{V_{\text{peak}}D}{\nu} = \frac{(2.43)(0.550)}{1.00 \times 10^{-6}} = 1.34 \times 10^6$$

and the Swamee-Jain equation gives

$$\frac{1}{\sqrt{f_{\text{peak}}}} = -2\log\left[\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}}\right] = -2\log\left[\frac{4.73 \times 10^{-4}}{3.7} + \frac{5.74}{(1.34 \times 10^6)^{0.9}}\right]$$

and yields

$$f_{\text{peak}} = 0.0170$$

The head loss between the water treatment plant and the suburban development is therefore given by

$$h_{\text{f}} = f\frac{L}{D}\frac{V^2}{2g} = (0.0170)\frac{3000}{0.550}\frac{2.43^2}{2(9.81)} = 27.9\,\text{m}$$

Since the head at the water treatment plant is 10.00 m, the pump head, h_{p}, that must be added is

$$h_{\text{p}} = (23.4 + 27.9) - 10.00 = 41.3\,\text{m}$$

and the power requirement, P, is given by

$$P = \gamma Q h_{\text{p}} = (9.79)(0.578)(41.3) = \boxed{234\,\text{kW}}$$

2.45. From the given data: $Q = 240\,\text{L/min} = 4\times10^{-3}\,\text{m}^3/\text{s}$, $D = 50\,\text{mm}$, $L = 15\,\text{m}$, $k_{\text{s}} = 0.5\,\text{mm}$, and $p_{\text{atm}} = 101.3\,\text{kPa}$. For water at 15°C, $\rho = 999.1\,\text{kg/m}^3$, $\mu = 1.139\,\text{mPa·s}$, $\nu = \mu/\rho = 1.140\times10^{-6}\,\text{m}^2/\text{s}$, $\gamma = \rho g = 9.798\,\text{kN/m}^3$, and $p_{\text{vap}} = 1.704\,\text{kPa}$. The head-loss coefficient for the reentrant intake pipe can be conservatively taken as $K_{\text{e}} = 1.0$. The following preliminary calculations are useful,

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.05)^2}{4} = 1.963\times10^{-3}\,\text{m}^2, \qquad V = \frac{Q}{A} = \frac{4\times10^{-3}}{1.963\times10^{-3}} = 2.037\,\text{m/s}$$

$$h_{\text{v}} = \frac{V^2}{2g} = \frac{2.037^2}{2(9.807)} = 0.2116\,\text{m}, \qquad \frac{k_{\text{s}}}{D} = \frac{0.5}{50} = 0.01$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(2.037)(0.05)}{1.140\times10^{-6}} = 8.935\times10^4$$

Substituting the given and derived parameters into the Colebrook equation (Equation 2.28) gives

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_{\text{s}}/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right] \quad\rightarrow\quad \frac{1}{\sqrt{f}} = -2\log\left[\frac{0.01}{3.7} + \frac{2.51}{8.935\times10^4\sqrt{f}}\right]$$

which yields $f = 0.0386$.

(a) Applying the energy equation between the water in the well and the suction side of the pump gives

$$\frac{p_{\text{atm}}}{\gamma} - \left[K_{\text{e}} + \frac{fL}{D}\right]h_{\text{v}} = \frac{p_{\text{vap}}}{\gamma} + h_{\text{v}} + h$$

$$\frac{101.3}{9.798} - \left[1.0 + \frac{(0.0386)(15)}{(0.05)}\right](0.2116) = \frac{1.704}{9.798} + 0.2116 + h \quad\rightarrow\quad h = \boxed{7.72\,\text{m}}$$

(b) Since h increases as Q decreases, the maximum value of h occurs when $Q = 0$. In this case, the energy equation degenerates into the hydrostatic equation as follows

$$\frac{p_{\text{atm}}}{\gamma} = \frac{p_{\text{vap}}}{\gamma} + h_{\max} \quad\rightarrow\quad \frac{101.3}{9.798} = \frac{1.704}{9.798} + h_{\max} \quad\rightarrow\quad h_{\max} = \boxed{10.2\,\text{m}}$$

(c) For any pump pumping water at 15°C from any reservoir, the suction side of the pump should not be more that $\boxed{10.2\text{ m}}$ above the surface of the reservoir. Otherwise, the pump will not work.

2.46. From the given data: $D = 0.05$ m, $A = \pi D^2/4 = 0.001963$ m^2, $Q = 4$ L/s $= 0.004$ m^3/s, $V = Q/A = 2.038$ m/s, $L = 125$ m, $\nu = 1.00 \times 10^{-6}$ m^2/s, $\gamma = 9.79$ kN/m^3, $k_\mathrm{s} = 0.23$ mm, Re $= VD/\nu = 1.02 \times 10^5$. Using the Swamee-Jain equation,

$$f = \frac{0.25}{\left[\log\left(\dfrac{k_s}{3.7D} + \dfrac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\dfrac{0.23}{3.7(50)} + \dfrac{5.74}{(1.02\times 10^5)^{0.9}}\right)\right]^2} = 0.0308$$

For a sharp-edged entrance, $K_\mathrm{e} = 0.5$, for an open globe valve, $K_\mathrm{v} = 10.0$. The energy equation applied between the reservoir and the outlet is given by

$$40 - \left[K_\mathrm{e} + \frac{fL}{D} + K_\mathrm{v}\right]\frac{V^2}{2g} - h_\mathrm{t} = \frac{V^2}{2g} \quad \rightarrow \quad h_\mathrm{t} = 40 - \left[K_\mathrm{e} + \frac{fL}{D} + K_\mathrm{v} + 1\right]\frac{V^2}{2g} \tag{1}$$

Substituting the given and derived data into Equation 1 gives

$$h_\mathrm{t} = 40 - \left[0.5 + \frac{(0.0308)(125)}{0.05} + 10.0 + 1\right]\frac{(2.038)^2}{2(9.81)} = 21.27\,\mathrm{m}$$

Therefore, the power extracted by the turbine is given by

$$P = \gamma Q h_\mathrm{t} = (9.79)(0.004)(21.27) = \boxed{0.833\,\mathrm{kW}}$$

A similar problem would be encountered in calculating the power output at a $\boxed{\text{hydroelectric facility}}$.

2.47. From the given data: $L = 150$ m, $D = 350$ mm, $k_\mathrm{s} = 1$ mm, $P = 60$ kW, and $\Delta z = 45$ m. For water at 20°C, $\gamma = 9.79\,\mathrm{kN/m^3}$ and $\nu = 1.00 \times 10^{-6}\,\mathrm{m^2/s}$. The following preliminary calculations are useful,

$$A = \frac{\pi D^2}{4} = \frac{\pi 0.350^2}{4} = 0.09621\,\mathrm{m}^2, \qquad \frac{k_\mathrm{s}}{D} = \frac{1}{350} = 2.857 \times 10^{-3}$$

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{QD}{\nu A} \quad \rightarrow \quad Q = 2.749 \times 10^{-7}\mathrm{Re}$$

$$P = \gamma Q h_\mathrm{t} \quad \rightarrow \quad h_\mathrm{t} = \frac{6.129}{Q}$$

Applying the energy equation between the upper and lower reservoir gives,

$$\Delta z - \left[\frac{fL}{D} + K_\mathrm{in} + K_\mathrm{out}\right]\frac{Q^2}{2gA^2} - h_\mathrm{t} = 0$$

$$45 - \left[\frac{f(150)}{0.350} + 0.8 + 1.0\right]\frac{Q^2}{2(9.807)(0.09621)^2} - \frac{6.129}{Q} = 0$$

Solving this equation simultaneously with the Colebrook equation gives $\boxed{Q = 0.141\,\mathrm{m^3/s}}$.

2.48. From the given data: $L = 1$ km, $Q = 2\,\mathrm{m^3/s}$, $k_\mathrm{s} = 1.5\,\mathrm{mm}$, $\Delta z = 23\,\mathrm{m}$, and $T = 20°\mathrm{C}$. For water at 20°C, $\nu = 1.00 \times 10^{-6}\,\mathrm{m^2/s}$. From the given data,

$$\mathrm{Re} = \frac{(Q/A)D}{\nu} = \left[\frac{4Q}{\pi\nu}\right]\frac{1}{D} = \left[\frac{4(2)}{\pi(1.00\times10^{-6})}\right]\frac{1}{D} = \frac{2.547\times10^6}{D}$$

(a) Neglecting local losses, the energy equation gives

$$\Delta z = \frac{fL}{D}\frac{Q^2}{2gA^2} \rightarrow \Delta z = \frac{8fLQ^2}{g\pi^2D^5} \rightarrow 23 = \frac{8f(1000)(2)^2}{(9.807)\pi^2D^5}$$

which simplifies to

$$f = 0.0696\,D^5$$

Solving this equation simultaneously with the Colebrook equation (e.g., using MATLAB) gives $D = 0.802$ m $= \boxed{802\text{ mm}}$.

(b) Taking local losses into account with $K = 15.3$, the energy equation gives

$$\Delta z = \left[\frac{fL}{D} + K\right]\frac{Q^2}{2gA^2} \rightarrow \Delta z = \left[\frac{fL}{D} + K\right]\frac{8Q^2}{\pi^2D^4} \rightarrow 23 = \left[\frac{f(1000)}{D} + 15.3\right]\frac{8(2)^2}{\pi^2D^4}$$

Solving this equation simultaneously with the Colebrook equation (e.g., using MATLAB) gives $D = 0.802$ m $= \boxed{802\text{ mm}}$. Therefore, local losses have $\boxed{\text{no impact}}$ on the tunnel diameter that is required to meet the design objective.

2.49. The head loss is calculated using Equation 2.72. The hydraulic diameter, D_h, is given by

$$D_\mathrm{h} = \frac{4A}{P} = \frac{4(2)(1)}{2(2+1)} = 1.333\,\mathrm{m}$$

and the mean velocity, V, is given by

$$V = \frac{Q}{A} = \frac{5}{(2)(1)} = 2.5\,\mathrm{m/s}$$

At 20°C, $\rho = 998.2$ kg/m^3, $\mu = 1.002 \times 10^{-3}$ N·s/m^2, and therefore the Reynolds number, Re, is given by

$$\mathrm{Re} = \frac{\rho V D_\mathrm{h}}{\nu} = \frac{(998.2)(2.5)(1.333)}{1.002\times10^{-3}} = 3.32\times10^6$$

A median equivalent sand roughness for concrete can be taken as $k_\mathrm{s} = 1.6$ mm (Table 2.1), and therefore the relative roughness, $k_\mathrm{s}/D_\mathrm{h}$, is given by

$$\frac{k_\mathrm{s}}{D_\mathrm{h}} = \frac{1.6\times10^{-3}}{1.333} = 0.00120$$

Substituting Re and $k_\mathrm{s}/D_\mathrm{h}$ into the Swamee-Jain equation (Equation 2.31) for the friction factor yields

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_\mathrm{s}/D_\mathrm{h}}{3.7} + \frac{5.74}{\mathrm{Re}^{0.9}}\right] = -2\log\left[\frac{0.00120}{3.7} + \frac{5.74}{(3.32\times10^6)^{0.9}}\right] = 6.96$$

which yields

$$f = 0.0206$$

The frictional head loss in the culvert, h_f, is therefore given by the Darcy-Weisbach equation as

$$h_\mathrm{f} = \frac{fL}{D_\mathrm{h}}\frac{V^2}{2g} = \frac{(0.0206)(100)}{1.333}\frac{2.5^2}{2(9.81)} = \boxed{0.493\,\mathrm{m}}$$

2.50. The frictional head loss is calculated using Equation 2.72. The hydraulic diameter, D_h, is given by

$$D_\mathrm{h} = \frac{4A}{P} = \frac{4(2)(2)}{2(2+2)} = 2.000\,\mathrm{m}$$

and the mean velocity, V, is given by

$$V = \frac{Q}{A} = \frac{10}{(2)(2)} = 2.5\,\mathrm{m/s}$$

At 20°C, ρ = 998 kg/m^3, μ = 1.00 × 10^{-3} N·s/m^2, and therefore the Reynolds number, Re, is given by

$$\mathrm{Re} = \frac{\rho V D_\mathrm{h}}{\mu} = \frac{(998)(2.5)(2.000)}{1.00\times10^{-3}} = 4.99\times10^6$$

A median equivalent sand roughness for concrete can be taken as k_s = 1.6 mm (Table 2.1), and therefore the relative roughness, $k_\mathrm{s}/D_\mathrm{h}$, is given by

$$\frac{k_\mathrm{s}}{D_\mathrm{h}} = \frac{1.6\times10^{-3}}{2.000} = 0.0008$$

Substituting Re and $k_\mathrm{s}/D_\mathrm{h}$ into the Swamee-Jain equation (Equation 2.32) for the friction factor yields

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_\mathrm{s}/D_\mathrm{h}}{3.7} + \frac{5.74}{\mathrm{Re}^{0.9}}\right] = -2\log\left[\frac{0.0008}{3.7} + \frac{5.74}{(4.99\times10^6)^{0.9}}\right] = 7.31$$

which yields

$$f = 0.0187$$

The frictional head loss in the culvert, h_f, is therefore given by the Darcy-Weisbach equation as

$$h_\mathrm{f} = \frac{fL}{D_\mathrm{h}}\frac{V^2}{2g} = \frac{(0.0187)(500)}{2.500}\frac{2.5^2}{2(9.81)} = 1.49\,\mathrm{m}$$

Applying the energy equation between the upstream and downstream sections (Sections 1 and 2 respectively),

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_\mathrm{f}$$

which gives

$$\frac{p_1}{9.79} + \frac{2.5^2}{2(9.81)} + (0.002)(500) = \frac{p_2}{9.79} + \frac{2.5^2}{2(9.81)} + 0 + 1.49$$

Re-arranging this equation gives

$$\boxed{p_1 - p_2 = 4.80\,\mathrm{kPa}}$$

2.51. The Hazen-Williams formula is given by

$$V = 0.849 C_H R^{0.63} S_f^{0.54}, \qquad S_f = \frac{h_f}{L}$$

Combining the above two equations to eliminate S_f, taking $R = D/4$, and simplifying gives

$$V = 0.849 C_H \left(\frac{D}{4}\right)^{0.63} \left(\frac{h_f}{L}\right)^{0.54} \quad \rightarrow \quad \boxed{h_f = 6.82 \frac{L}{D^{1.17}} \left(\frac{V}{C_H}\right)^{1.85}}$$

2.52. Comparing the Hazen–Williams and Darcy–Weisbach equations for head loss gives

$$(h_f =)\ 6.82 \frac{L}{D^{1.17}} \left(\frac{V}{C_H}\right)^{1.85} = f \frac{L}{D} \frac{V^2}{2g} \quad \rightarrow \quad \boxed{f = \frac{134}{C_H^{1.85} D^{0.17}} \frac{1}{V^{0.15}}}$$

For laminar flow, Equation 2.29 gives $f \sim 1/\mathrm{Re} \sim 1/V$, and for fully-turbulent flow Equation 2.28 gives $f \sim 1/V^0$. Since the Hazen–Williams formula requires that $f \sim 1/V^{0.15}$, this indicates that the flow must be in the $\boxed{\text{transition regime}}$.

2.53. From the given data: $p_1 = 450$ kPa, $p_2 = 150$ kPa, $k_s = 0.002$ mm, $Q = 150$ L/min $= 0.0025$ m^3/s, $\Delta z_1 = 27$ m, $\Delta z_2 = 10$ m, and $C_H = 140$. For water at 20°C, $\gamma = 9.789$ kN/m^3.

(a) Applying the energy and Darcy-Weisbach equations between the water-supply pipe and the location of fixture F gives

$$p_2 = p_1 - \gamma \left[\frac{f \Delta z_1}{D} \frac{2Q^2}{g \pi D^2} + \Delta z_1 \right] \quad \rightarrow \quad p_2 = 450 - (9.789) \left[\frac{f(27)}{D} \frac{2(0.0025)^2}{(9.807)\pi D^2} + 27 \right]$$

where p_2 is the pressure at fixture F. Calculating p_2 for the given commercial pipe sizes yields the results shown in the following table:

D (mm)	p_2 (kPa)
32	111
38	153
51	177

Since the minimum allowable pressure at F is 150 kPa, a pipe diameter of $\boxed{38\text{ mm}}$ should be used for the supply line.

(b) Applying the energy and Hazen–Williams equations between the water-supply pipe and the location of fixture F gives

$$p_2 = p_1 - \gamma \left[85.4 \left(\frac{\Delta z_1}{C_H^{1.85} D^{4.87}} \right) Q^{1.85} + \Delta z_1 \right]$$

$$\rightarrow \quad p_2 = 450 - (9.789) \left[85.4 \left(\frac{27}{140^{1.85} D^{4.87}} \right) (150)^{1.85} + 27 \right]$$

where p_2 is the pressure at fixture F, D is in cm, and Q is in L/min. Calculating p_2 for the given commercial pipe sizes yields the results shown in the following table:

D (mm)	p_2 (kPa)
32	97
38	147
51	176

Since the minimum allowable pressure at F is 150 kPa, a pipe diameter of $\boxed{51\,\text{mm}}$ should be used for the supply line.

(c) If the fixture F is supplied by both the water-supply line and the roof tank, and $D =$ 25 mm, and the flow rate extracted at F is 150 L/min, then the following equations must hold:

$$Q = Q_1 + Q_2$$

$$p_2 = p_1 - \gamma \left[85.4 \left(\frac{\Delta z_1}{C_\mathrm{H}^{1.85} D^{4.87}}\right) Q_1^{1.85} + \Delta z_1\right]$$

$$p_2 = 0 - \gamma \left[85.4 \left(\frac{\Delta z_2}{C_\mathrm{H}^{1.85} D^{4.87}}\right) Q^{1.85} - \Delta z_2\right]$$

Substituting known quantities yields:

$$150 = Q_1 + Q_2$$

$$p_2 = 450 - (9.789) \left[85.4 \left(\frac{27}{140^{1.85} 2.5^{4.87}}\right) Q_1^{1.85} + 27\right]$$

$$p_2 = 0 - (9.789) \left[85.4 \left(\frac{10}{140^{1.85} 2.5^{4.87}}\right) Q_2^{1.85} - 10\right]$$

Solving these equations yields $\boxed{Q_1 = 42.3\,\text{L/s},\ Q_2 = 107.7\,\text{L/s, and } p_2 = 157\,\text{kPa}}$.

2.54. The Manning equation is given by

$$V \left(= \frac{1}{n} R^{\frac{2}{3}} \mathrm{S}_\mathrm{f}^{\frac{1}{2}}\right) = \frac{1}{n} \left(\frac{D}{4}\right)^{\frac{2}{3}} \left(\frac{h_\mathrm{f}}{L}\right)^{\frac{1}{2}} \quad \rightarrow \quad \boxed{h_\mathrm{f} = 6.35 \frac{n^2 L V^2}{D^{\frac{4}{3}}}}$$

2.55. Comparing the Manning and Darcy–Weisbach equations gives

$$(h_\mathrm{f}) \ = 6.35 \frac{n^2 L V^2}{D^{\frac{4}{3}}} = f \frac{L}{D} \frac{V^2}{2g} \quad \rightarrow \quad \boxed{f = 125 \frac{n^2}{D^{\frac{1}{3}}}}$$

For laminar flow, Equation 2.29 gives $f \sim 1/\mathrm{Re} \sim 1/V$, and for fully-turbulent flow Equation 2.28 gives $f \sim 1/V^0$. Since the Manning equation requires that $f \sim 1/V^0$, this indicates that the flow must be fully turbulent or $\boxed{\text{rough}}$.

2.56. Equating the Hazen–Williams and Manning head loss expressions

$$6.82 \frac{L}{D^{1.17}} \left(\frac{V}{C_\mathrm{H}}\right)^{1.85} = 6.35 \frac{n^2 L V^2}{D^{\frac{4}{3}}} \quad \rightarrow \quad \boxed{n = \left(1.04 \frac{D^{0.082}}{V^{0.075}}\right) \frac{1}{C_\mathrm{H}^{0.93}}}$$

2.57. Choose the $\boxed{\text{Darcy-Weisbach equation}}$ since this equation is applicable in all flow regimes. The Hazen-Williams and Manning equations are limited to particular flow conditions (transition and fully turbulent respectively).

2.58. (a) The Hazen-Williams roughness coefficient, C_H, can be taken as 130 (Table 2.2), $L =$ 500 m, $D = 0.300$ m, $V = 2$ m/s, and therefore the head loss, h_f, is given by Equation 2.78 as

$$h_\mathrm{f} = 6.82 \frac{L}{D^{1.17}} \left(\frac{V}{C_\mathrm{H}}\right)^{1.85} = 6.82 \frac{500}{(0.30)^{1.17}} \left(\frac{2}{130}\right)^{1.85} = \boxed{6.17\,\mathrm{m}}$$

(b) The Manning roughness coefficient, n, can be taken as 0.013 (approximation from Table 2.2), and therefore the head loss, h_f, is given by Equation 2.81 as

$$h_\mathrm{f} = 6.35 \frac{n^2 L V^2}{D^{\frac{4}{3}}} = 6.35 \frac{(0.013)^2(500)(2)^2}{(0.30)^{\frac{4}{3}}} = \boxed{10.7\,\mathrm{m}}$$

(c) The equivalent sand roughness, k_s, can be taken as 0.26 mm (Table 2.1), and the Reynolds number, Re, is given by

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(2)(0.30)}{1.00 \times 10^{-6}} = 6.00 \times 10^5$$

where $\nu = 1.00 \times 10^{-6}\,\mathrm{m^2/s}$ at 20°C. Substituting k_s, D, and Re into the Colebrook equation yields the friction factor, f, where

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{k_\mathrm{s}}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right] = -2 \log \left[\frac{0.26}{3.7(300)} + \frac{2.51}{6.00 \times 10^5 \sqrt{f}}\right] \quad \rightarrow \quad f = 0.0195$$

The head loss, h_f, is therefore given by the Darcy-Weisbach equation as

$$h_\mathrm{f} = f \frac{L}{D} \frac{V^2}{2g} = 0.0195 \frac{500}{0.30} \frac{2^2}{2(9.81)} = \boxed{6.63\,\mathrm{m}}$$

It is reasonable to assume that the Darcy-Weisbach equation yields the most accurate estimate of the head loss. In this case, the Hazen-Williams formula gives a head loss that is 7% too low, and the Manning formula yields a head loss that is 61% too high. Problem 2.52 has demonstrated that the relationship between the friction factor, f, and the Hazen-Williams coefficient, C_H, is given by

$$f = \frac{134}{C_\mathrm{H}^{1.85} D^{0.17}} \frac{1}{V^{0.15}}$$

which can be re-arranged to give

$$C_\mathrm{H} = \frac{14.2}{f^{0.54} D^{0.092} V^{0.081}} = \frac{14.2}{(0.0195)^{0.54}(0.30)^{0.092}(2)^{0.081}} = \boxed{126}$$

which is 3% lower than the assumed value of C_H (130).

Problem 2.55 has demonstrated that the relationship between the friction factor, f, and the Manning coefficient, n, is given by

$$f = 125\frac{n^2}{D^{\frac{1}{3}}}$$

which can be re-arranged to give

$$n = 0.0894 f^{0.5} D^{0.17} = 0.0894(0.0195)^{0.5}(0.30)^{0.17} = \boxed{0.010}$$

which is 23% lower than the assumed value of n (= 0.013).

2.59. The pressure at the midpoint of the pipe as a function of time is shown in Figure 2.2.

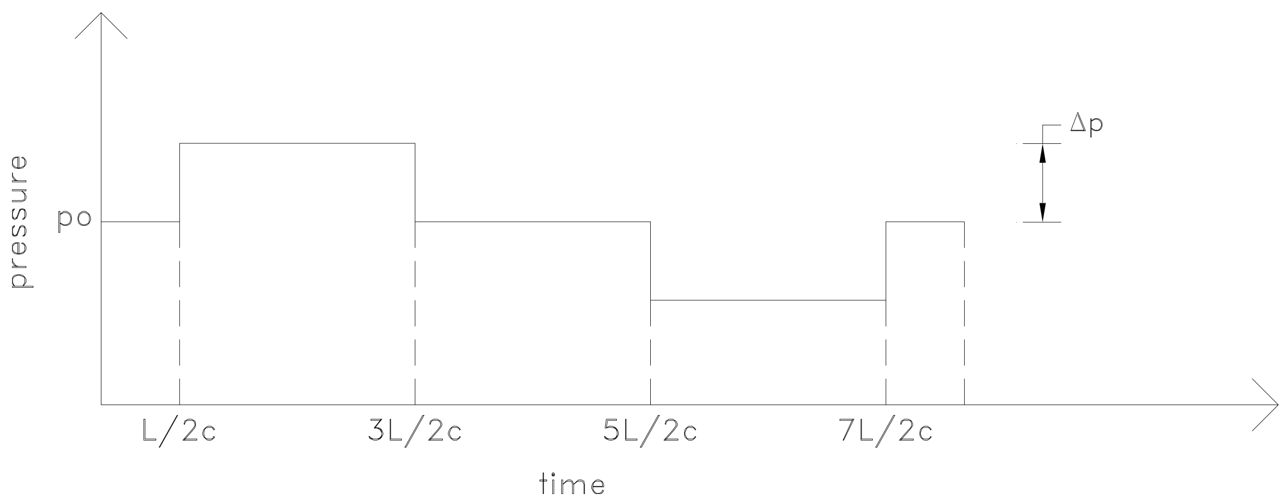

Figure 2.2: Problem 2.59.

2.60. From the given data: $L = 100$ m, $V = 3$ m/s, and $T = 20$°C. The critical time, t_c, for valve closure is

$$t_c = \frac{2L}{c} \tag{1}$$

At 20°C, $E_v = 2.15 \times 10^9$ Pa, and $\rho_0 = 998$ kg/m^3 (Appendix B, Table B.1), therefore

$$c = \sqrt{\frac{E_v}{\rho_0}} = \sqrt{\frac{2.15 \times 10^9}{998}} = 1470\,\text{m/s}$$

Substituting into Equation 1 gives

$$t_c = \frac{2(100)}{1470} = \boxed{0.14\,\text{s}}$$

The maximum water-hammer pressure that can occur is given by

$$\Delta p = \rho c V = (998)(1470)(3) = 4.40 \times 10^6\ \text{Pa} = \boxed{4400\,\text{kPa}}$$

If the water temperature is 10°C, $E_v = 2.10 \times 10^9$ Pa, $\rho_0 = 999.7$ kg/m^3 (Appendix B, Table B.1), and

$$c = \sqrt{\frac{2.10 \times 10^9}{999.7}} = 1450\,\text{m/s}$$

and the maximum water-hammer pressure is given by

$$\Delta p = \rho c V = (999.7)(1450)(3) = 4.35 \times 10^6\,\text{Pa} = \boxed{4350\,\text{kPa}}$$

2.61. $\boxed{\text{Yes}}$, in cases of rapid valve closure.

2.62. From the given data: T = 20°C, L = 150 m, D = 50 mm = 0.050 m, V = 4 m/s, e = 1.5 mm = 0.0015 m, and $E = 1.655 \times 10^5$ MN/m^2 = 1.655×10^{11} N/m^2. Taking $\rho_0 = 998$ kg/m^3 and $E_v = 2.15 \times 10^9$ Pa, the speed of the pressure wave, c, is given by Equation 2.92 as

$$c = \sqrt{\frac{E_v/\rho_0}{1 + (E_v D/eE)}} = \sqrt{\frac{(2.15 \times 10^9)/(998)}{1 + (2.15 \times 10^9 \times 0.050)/(0.0015 \times 1.655 \times 10^{11})}} = 1226\,\text{m/s}$$

Hence, the pressure increase, Δp is given by Equation 2.86 as

$$\Delta p = \rho_0 c V = (998)(1226)(4) = 4.89 \times 10^6\,\text{Pa} = \boxed{4890\,\text{kPa}}$$

2.63. From the given data: T = 20°C, L = 150 m, D = 50 mm = 0.050 m, V = 4 m/s, e = 2.0 mm = 0.0020 m, and $E = 1.7 \times 10^4$ MN/m^2 = 1.7×10^{10} N/m^2. Taking $\rho_0 = 998$ kg/m^3 and $E_v = 2.15 \times 10^9$ Pa, the speed of the pressure wave, c, is given by Equation 2.92 as

$$c = \sqrt{\frac{E_v/\rho_0}{1 + (E_v D/eE)}} = \sqrt{\frac{(2.15 \times 10^9)/(998)}{1 + (2.15 \times 10^9 \times 0.050)/(0.0020 \times 1.7 \times 10^{10})}} = 719\,\text{m/s}$$

Hence, the pressure increase, Δp is given by Equation 2.86 as

$$\Delta p = \rho_0 c V = (998)(719)(4) = 2.87 \times 10^6\,\text{Pa} = \boxed{2870\,\text{kPa}}$$

2.64. From the given data: Q = 0.06 m^3/s, D = 1000 mm = 1 m, L = 3.2 km = 3200 m, z_1 = 10.06 m, z_2 = 11.52 m, and Δz_p =1.5 m. For DIP, k_s = 0.26 mm, and at 20°C, $\nu = 1.00 \times 10^{-6}$ m^2/s.

(a) The friction factor in the pipe is obtained using the Swamee-Jain equation from the following sequence of calculations:

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(1)^2 = 0.7854\,\text{m}^2$$

$$V = \frac{Q}{A} = \frac{0.6}{0.7854} = 0.764\,\text{m/s}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(0.764)(1)}{1.00 \times 10^6} = 7.64 \times 10^5$$

$$f = \frac{0.25}{\left[\log\left(\frac{k_\text{s}}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{0.26}{3.7(1000)} + \frac{5.74}{(7.64 \times 10^5)^{0.9}}\right)\right]^2} = 0.01449$$

The energy equation gives

$$z_1 + h_\mathrm{p} - f\frac{L}{D}\frac{V^2}{2g} = \frac{p_2}{\gamma} + \frac{V^2}{2g} + z_2$$

$$z_1 + h_\mathrm{p} - \left(f\frac{L}{D} + 1\right)\frac{V^2}{2g} = \frac{p_2}{\gamma} + z_2$$

$$10.06 + h_\mathrm{p} - \left(0.01449\frac{3200}{1} + 1\right)\frac{0.764^2}{2(9.81)} = \frac{350}{9.79} + 11.52$$

which gives $h_\mathrm{p} = 38.62$ m. Hence the required power, P, of the pump is given by

$$P = \gamma Q h_\mathrm{p} = (9.79)(0.6)(38.62) = \boxed{227\,\mathrm{kW}}$$

(b) At the pump, with subscripts 1 and 2 referring to just before and just after the pump, respectively:

$$z_1 + h_\mathrm{p} = \frac{p_2}{\gamma} + \frac{V^2}{2g} + z_2$$

$$10.06 + 38.62 = \frac{p_2}{9.79} + \frac{0.764^2}{2(9.81)} + (10.06 - 1.5)$$

which gives $\boxed{p_2 = 392\text{ kPa}}$.

(c) For each of the two pipelines: $D = 0.6$ m and $Q = 0.3$ m^3/s and the friction factor in the pipe can be obtained as follows:

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.6)^2 = 0.283\,\mathrm{m}^2$$

$$V = \frac{Q}{A} = \frac{0.3}{0.283} = 1.06\,\mathrm{m/s}$$

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(1.06)(0.6)}{1.00\times 10^6} = 6.36\times 10^5$$

$$f = \frac{0.25}{\left[\log\left(\frac{k_\mathrm{s}}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{0.26}{3.7(600)} + \frac{5.74}{(6.36\times 10^5)^{0.9}}\right)\right]^2} = 0.0171$$

The energy equation, with subscripts 1 and 2 referring to the source reservoir and the location 3.2 km downstream of the source, respectively, gives

$$z_1 + h_\mathrm{p} - \left(f\frac{L}{D} + 1\right)\frac{V^2}{2g} = \frac{p_2}{\gamma} + z_2$$

$$10.06 + h_\mathrm{p} - \left(0.0171\frac{3200}{0.6} + 1\right)\frac{1.06^2}{2(9.81)} = \frac{350}{9.79} + 11.52$$

which gives $h_\mathrm{p} = 42.49$ m. Hence the required power, P, of the pump is given by

$$P = \gamma Q h_\mathrm{p} = (9.79)(0.6)(42.49) = \boxed{250\,\mathrm{kW}}$$

2.65. From the given data: $Q = 30\,\mathrm{L/s}$, $k_\mathrm{s} = 0.05\,\mathrm{mm}$, $D_1 = 50\,\mathrm{mm}$, $\Delta z_1 = 4\,\mathrm{m}$, $L_1 = 10\,\mathrm{m}$, $D_2 = 75\,\mathrm{mm}$, $L_2 = 10\,\mathrm{m}$, $D_3 = 85\,\mathrm{mm}$, and $L_3 = 7\,\mathrm{m}$. The friction factors and pipe areas can be calculated as follows:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_\mathrm{s}/D}{3.7}\right), \quad f_1 = 0.01964, \quad f_2 = 0.01733, \quad f_3 = 0.01783$$

$$A = \frac{\pi D^2}{4}, \quad A_1 = 1.963\times10^{-3}\,\mathrm{m}^2, \quad A_2 = 5.675\times10^{-3}\,\mathrm{m}^2, \quad A_3 = 4.418\times10^{-3}\,\mathrm{m}^2$$

The energy equation between A and C gives:

$$\Delta z_1 - f_1\frac{L_1}{D_1}\frac{Q_1^2}{2gA_1^2} - f_3\frac{L_3}{D_3}\frac{Q_3^2}{2gA_3^2} = 0$$

$$4 - (0.01964)\frac{10}{0.050}\frac{Q_1^2}{2(9.807)(1.963\times10^{-3})^2} - (0.01783)\frac{7}{0.086}\frac{(0.030)^2}{2(9.807)(4.418\times10^{-3})^2} = 0$$

which gives $Q_1 = 0.00323\,\mathrm{m^3/s} = 3.23\,\mathrm{L/s}$. By continuity, $Q_2 = Q_3 - Q_1 = 30 - 3.23 = 26.77\,\mathrm{L/s} = 0.02677\,\mathrm{m^3/s}$. The energy equation between B and C gives:

$$h - f_2\frac{L_2}{D_2}\frac{Q_2^2}{2gA_2^2} - f_3\frac{L_3}{D_3}\frac{Q_3^2}{2gA_3^2} = 0$$

$$h - (0.01733)\frac{10}{0.075}\frac{(0.02677)^2}{2(9.807)(5.675\times10^{-3})^2} - (0.01783)\frac{7}{0.086}\frac{(0.030)^2}{2(9.807)(4.418\times10^{-3})^2} = 0$$

which gives $h = \boxed{0.992\,\mathrm{m}}$.

2.66. $D = 0.85$ m, $A = \pi D^2/4 = \pi(0.85)^2/4 = 0.567$ m^2, $k_\mathrm{s} = 0.26$ mm (ductile iron), $k_\mathrm{s}/D = 0.26/850 = 0.000306$, $T = 20°$C, $\rho = 998$ kN/m^3, $\mu = 1.00\times10^{-3}$ N·s/m^2. Assuming fully turbulent flow, then

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_\mathrm{s}/D}{3.7}\right) = -2\log\left(\frac{0.000306}{3.7}\right) = 8.16$$

which leads to

$$f = 0.0150$$

for each pipe. Writing the energy equations,

$$100 - f\frac{L_\mathrm{AJ}}{D}\frac{Q_\mathrm{AJ}^2}{2gA^2} = 80 - f\frac{L_\mathrm{BJ}}{D}\frac{Q_\mathrm{BJ}^2}{2gA^2} \tag{1}$$

and

$$100 - f\frac{L_\mathrm{AJ}}{D}\frac{Q_\mathrm{AJ}^2}{2gA^2} = 60 + f\frac{L_\mathrm{CJ}}{D}\frac{Q_\mathrm{CJ}^2}{2gA^2} \tag{2}$$

and the continuity equation is

$$Q_\mathrm{AJ} + Q_\mathrm{BJ} = Q_\mathrm{CJ} \tag{3}$$

Equations 1 and 3 assume that the flow is out of reservoir B. Substituting known values into Equation 1 gives

$$100 - 0.0150\frac{900}{0.85}\frac{Q_{\rm AJ}^2}{2(9.81)(0.567)^2} = 80 - 0.0150\frac{800}{0.85}\frac{Q_{\rm BJ}^2}{2(9.81)(0.567)^2}$$

which leads to

$$Q_{\rm BJ}^2 = 1.12Q_{\rm AJ}^2 - 8.93 \tag{4}$$

Substituting known values into Equation 2 gives

$$100 - 0.0150\frac{900}{0.85}\frac{Q_{\rm AJ}^2}{2(9.81)(0.567)^2} = 60 + 0.0150\frac{700}{0.85}\frac{Q_{\rm CJ}^2}{2(9.81)(0.567)^2}$$

which leads to

$$Q_{\rm CJ}^2 = 20.4 - 1.29Q_{\rm AJ}^2 \tag{5}$$

Substituting Equations 4 and 5 into Equation 3 leads to

$$Q_{\rm AJ} + \sqrt{1.12Q_{\rm AJ}^2 - 8.93} - \sqrt{20.4 - 1.29Q_{\rm AJ}^2} = 0$$

Solving for $Q_{\rm AJ}$ by trial and error gives

$$Q_{\rm AJ} = 2.84\,\mathrm{m^3/s}$$

and Equations 4 and 5 give

$$Q_{\rm BJ} = 0.32\,\mathrm{m^3/s}$$

$$Q_{\rm CJ} = 3.16\,\mathrm{m^3/s}$$

Recalculating f for each pipe (using Re and $k_{\rm s}/D$) and repeating the process until the assumed f is equal to the calculated f leads to

$$\boxed{\begin{aligned} Q_{\rm AJ} &= 2.81\,\mathrm{m^3/s} \\ Q_{\rm BJ} &= 0.33\,\mathrm{m^3/s} \\ Q_{\rm CJ} &= 3.14\,\mathrm{m^3/s} \end{aligned}} \tag{6}$$

These results do not differ by much from the initial result that assumes complete turbulence.

2.67. From the given data: $L_{\rm AB} = 1.5\,\mathrm{km}$, $L_{\rm BC} = 1.5\,\mathrm{km}$, $L_{\rm BD} = 1.6\,\mathrm{km}$, $k_1 = 1\,\mathrm{mm}$, $k_2 = 0.5\,\mathrm{mm}$, $D_1 = 150\,\mathrm{mm}$, $D_2 = 200\,\mathrm{mm}$, and $\Delta z = 8\,\mathrm{m}$. For water at 20°C, $\nu = 1.00 \times 10^{-6}\,\mathrm{m^2/s}$. The following preliminary calculations are useful,

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi(0.15)^2}{4} = 1.767 \times 10^{-2}\,\mathrm{m^2}, \qquad A_2 = \frac{\pi D_2^2}{4} = \frac{\pi(0.2)^2}{4} = 3.142 \times 10^{-2}\,\mathrm{m^2}$$

$$\frac{k_1}{D_1} = \frac{1}{150} = 6.667 \times 10^{-3}, \qquad \frac{k_2}{D_2} = \frac{0.5}{200} = 2.5 \times 10^{-3}$$

$$f_1 = f_{\rm CE}\left(\frac{k_1}{D_1}\right) = 0.0332, \qquad f_2 = f_{\rm CE}\left(\frac{k_2}{D_2}\right) = 0.0249$$

$$\alpha_{AB} = \frac{f_1 L_{AB}}{2gA_1^2 D_1} = 5.420 \times 10^4, \qquad \alpha_{BC} = \frac{f_1 L_{BC}}{2gA_1^2 D_1} = 5.420 \times 10^4$$

$$\alpha_{BD} = \frac{f_2 L_{BD}}{2gA_2^2 D_2} = 1.028 \times 10^4$$

where f_{CE} represents the friction factor given by the Colebrook equation.

(a) Under the existing system, the energy equation applied between reservoirs requires that

$$\Delta z - 2\frac{f_1 L_{AB}}{D_1}\frac{Q_{AB}^2}{2gA_1^2} = 0 \quad \rightarrow \quad \Delta z - 2\alpha_{AB} Q_{AB}^2 = 0 \quad \rightarrow \quad 8 - 2(5.420 \times 10^4)Q_{AB}^2 = 0$$

which yields $Q_{AB} = 8.591 \times 10^{-3}\,\text{m}^3/\text{s} = \boxed{8.59\ \text{L/s}}$. When the new pipeline is added, the following three equations must be satisfied:

$$Q_{AB} = Q_{BC} + Q_{BD} \tag{1}$$

$$\Delta z - \frac{f_1 L_{AB}}{D_1}\frac{Q_{AB}^2}{2gA_1^2} - \frac{f_1 L_{BC}}{D_1}\frac{Q_{BC}^2}{2gA_1^2} = 0 \tag{2}$$

$$\frac{f_1 L_{BC}}{D_1}\frac{Q_{BC}^2}{2gA_1^2} = \frac{f_2 L_{BD}}{D_2}\frac{Q_{BD}^2}{2gA_2^2} \tag{3}$$

Equations 1 to 3 can be more compactly expressed as

$$Q_{AB} = Q_{BC} + Q_{BD} \tag{4}$$

$$\Delta z - \alpha_{AB} Q_{AB}^2 - \alpha_{BC} Q_{BC}^2 = 0 \tag{5}$$

$$\alpha_{BC} Q_{BC}^2 = \alpha_{BD} Q_{BD}^2 \tag{6}$$

Combining Equations 4 to 6 gives

$$Q_{AB} - \sqrt{\frac{\Delta z - \alpha_{AB} Q_{AB}^2}{\alpha_{BC}}} - \left(\sqrt{\frac{\alpha_{BD}}{\alpha_{BC}}}\right)^{-1} Q_{AB} = 0 \tag{7}$$

which yields $Q_{AB} = 11.62 \times 10^{-3}\,\text{m}^3/\text{s} = \boxed{11.62\ \text{L/s}}$. Substituting this result back into Equations 4 to 6 gives $Q_{BC} = \boxed{3.52\ \text{L/s}}$ and $Q_{BD} = \boxed{8.10\ \text{L/s}}$. Therefore, the flow from the upper to the lower reservoir increases from 8.59 L/s to 11.62 L/s, an increase of approximately $\boxed{35\%}$.

(b) Flow in the existing pipe will change from 8.59 L/s to 3.52 L/s, which is a change of $\boxed{-59\%}$.

(c) The friction factors can be determined using the calculated flows and compared to the originally assumed friction factors. The results of this analysis are as follows:

Pipe	Assumed f	Recalculated f	Difference (%)
AB	0.0332	0.0340	2.4
BC	0.0332	0.0356	7.2
BD	0.0249	0.0275	10.4

Based on these results, the assumption of fully turbulent flow in all pipes is $\boxed{\text{not validated}}$ and the determination of the flows should be repeated using the updated friction factors. This calculation loop should continue until the calculated and assumed friction factors are the same.

2.68. From the given data: $L_{\rm AB} = 3.0\,\rm km$, $L_{\rm AC} = 2.0\,\rm km$, $L_{\rm AD} = 2.5\,\rm km$, $L_{\rm BC} = 1.8\,\rm km$, $D_{\rm AB} = 300\,\rm mm$, $D_{\rm AC} = 200\,\rm mm$, $D_{\rm AD} = 250\,\rm mm$, $D_{\rm BC} = 150\,\rm mm$, $z_{\rm A} = 3.40\,\rm m$, $z_{\rm B} = 2.80\,\rm m$, $z_{\rm C} = 3.20\,\rm m$, $z_{\rm D} = 2.60\,\rm m$, $p_{\rm A} = 450\,\rm kPa$, $p_{\rm B} = 330\,\rm kPa$, $p_{\rm C} = 300\,\rm kPa$, $p_{\rm D} = 360\,\rm kPa$, and $k_{\rm s} = 0.2\,\rm mm$. For water at 20°C, $\gamma = 9.79\,\rm kN/m^3$, and $\nu = 1.00 \times 10^{-6}\,\rm m^2/s$. The following preliminary calculations are useful,

$$A_{\rm AB} = \frac{\pi D_{\rm AB}^2}{4} = 0.0707\,\rm m^2, \qquad A_{\rm AC} = \frac{\pi D_{\rm AC}^2}{4} = 0.0314\,\rm m^2$$

$$A_{\rm AD} = \frac{\pi D_{\rm AD}^2}{4} = 0.0491\,\rm m^2, \qquad A_{\rm BC} = \frac{\pi D_{\rm BC}^2}{4} = 0.0177\,\rm m^2$$

$$h_{\rm A} = \frac{p_{\rm A}}{\gamma} + z_{\rm A} = 49.37\,\rm m, \qquad h_{\rm B} = \frac{p_{\rm B}}{\gamma} + z_{\rm B} = 36.51\,\rm m$$

$$h_{\rm C} = \frac{p_{\rm C}}{\gamma} + z_{\rm C} = 33.84\,\rm m, \qquad h_{\rm D} = \frac{p_{\rm D}}{\gamma} + z_{\rm D} = 39.37\,\rm m$$

$$\beta_{\rm AB} = \frac{L_{\rm AB}}{2gD_{\rm AB}A_{\rm AB}^2} = 1.020 \times 10^5, \qquad \beta_{\rm AC} = \frac{L_{\rm AC}}{2gD_{\rm AC}A_{\rm AC}^2} = 5.166 \times 10^5$$

$$\beta_{\rm AD} = \frac{L_{\rm AD}}{2gD_{\rm AD}A_{\rm AD}^2} = 2.116 \times 10^5, \qquad \beta_{\rm BC} = \frac{L_{\rm BC}}{2gD_{\rm BC}A_{\rm BC}^2} = 1.959 \times 10^6$$

$$\frac{k_{\rm AB}}{D_{\rm AB}} = 6.667 \times 10^{-4}, \qquad \frac{k_{\rm AC}}{D_{\rm AC}} = 1.000 \times 10^{-3}$$

$$\frac{k_{\rm AD}}{D_{\rm AD}} = 8.000 \times 10^{-4}, \qquad \frac{k_{\rm BC}}{D_{\rm BC}} = 0.0013$$

The following functions of the flow rates can be defined,

$$\mathrm{Re}_{\rm AB}(Q_{\rm AB}) = \frac{Q_{\rm AB}D_{\rm AB}}{\nu} = \frac{Q_{\rm AB}(0.3)}{1.00 \times 10^{-6}}, \qquad \mathrm{Re}_{\rm AC}(Q_{\rm AC}) = \frac{Q_{\rm AC}D_{\rm AC}}{\nu} = \frac{Q_{\rm AC}(0.2)}{1.00 \times 10^{-6}}$$

$$\mathrm{Re}_{\rm AD}(Q_{\rm AD}) = \frac{Q_{\rm AD}D_{\rm AD}}{\nu} = \frac{Q_{\rm AD}(0.25)}{1.00 \times 10^{-6}}, \qquad \mathrm{Re}_{\rm BC}(Q_{\rm BC}) = \frac{Q_{\rm BC}D_{\rm BC}}{\nu} = \frac{Q_{\rm AB}(0.15)}{1.00 \times 10^{-6}}$$

$$f_{\rm AB}(Q_{\rm AB}) = f_{\rm CE}\left[\mathrm{Re}(Q_{\rm AB}), \frac{k_{\rm AB}}{D_{\rm AB}}\right], \qquad f_{\rm AC}(Q_{\rm AC}) = f_{\rm CE}\left[\mathrm{Re}(Q_{\rm AC}), \frac{k_{\rm AC}}{D_{\rm AC}}\right]$$

$$f_{\rm AD}(Q_{\rm AD}) = f_{\rm CE}\left[\mathrm{Re}(Q_{\rm AD}), \frac{k_{\rm AD}}{D_{\rm AD}}\right], \qquad f_{\rm BC}(Q_{\rm BC}) = f_{\rm CE}\left[\mathrm{Re}(Q_{\rm BC}), \frac{k_{\rm BC}}{D_{\rm BC}}\right]$$

where $f_{\rm CE}$ is the value of the friction factor given by the Colebrook equation.

(a) For the existing system, applying the energy equation to each of the pipe segments gives

$$h_{\rm A} - \beta_{\rm AB}f_{\rm AB}(Q_{\rm AB})Q_{\rm AB}^2 = h_{\rm B} \quad \rightarrow \quad Q_{\rm AB} = Q_{\rm A} = 0.0816\,\rm m^3/s = \boxed{81.6\,\rm L/s}$$

$$h_\mathrm{A} - \beta_\mathrm{AC} f_\mathrm{AC}(Q_\mathrm{AC}) Q_\mathrm{AC}^2 = h_\mathrm{C} \quad \rightarrow \quad Q_\mathrm{AC} = Q_\mathrm{B} = 0.0380\,\mathrm{m^3/s} = \boxed{38.0\,\mathrm{L/s}}$$

$$h_\mathrm{A} - \beta_\mathrm{AD} f_\mathrm{AD}(Q_\mathrm{AD}) Q_\mathrm{AD}^2 = h_\mathrm{D} \quad \rightarrow \quad Q_\mathrm{AD} = Q_\mathrm{C} = 0.0487\,\mathrm{m^3/s} = \boxed{48.7\,\mathrm{L/s}}$$

(b) For the new system,

$$h_\mathrm{B} - \beta_\mathrm{BC}(Q_\mathrm{BC}) f_\mathrm{BC} Q_\mathrm{BC}^2 = h_\mathrm{C} \quad \rightarrow \quad Q_\mathrm{BC} = 0.0075\,\mathrm{m^3/s} = 7.5\,\mathrm{L/s}$$

$$Q_\mathrm{A} = 81.6\,\mathrm{L/s} - 7.5\,\mathrm{L/s} = \boxed{74.1\,\mathrm{L/s}}$$

$$Q_\mathrm{B} = 38.0\,\mathrm{L/s} + 7.5\,\mathrm{L/s} = \boxed{45.5\,\mathrm{L/s}}$$

$$Q_\mathrm{C} = \boxed{48.7\,\mathrm{L/s}}$$

2.69. For ductile-iron pipe,

$$h_\mathrm{f} = f \frac{L}{D} \frac{Q^2}{2gA^2} = \left(\frac{L}{D} \frac{1}{2gA^2} \right) Q^2 f = rQ^2$$

Assuming fully turbulent flow:

Pipe	k_s/D	f	r
AD	0.000650	0.0177	143
BC	0.000867	0.0190	517
BD	0.000743	0.0183	345
AC	0.00104	0.0198	1173

From the given data: $h_\mathrm{A} = 25$ m, and $h_\mathrm{B} = 20$ m. Since the flow directions in the pipe network are not known in advance, flow directions can be assumed and validated. Assumed flow directions are validated when the calculated flows are real and positive.

Assuming that the flow in pipe AC is from C to A and that the flow in pipe BC is from C to B (i.e. $h_\mathrm{A} < h_\mathrm{C} > h_\mathrm{B}$), the Darcy–Weisbach equation gives

$$h_\mathrm{C} = 25 + 1173 Q_\mathrm{CA}^2 \tag{1}$$

and for flow from C to B,

$$h_\mathrm{C} = 20 + 517 Q_\mathrm{CB}^2 \tag{2}$$

Combining Equations 1 and 2 gives

$$25 + 1173 Q_\mathrm{CA}^2 = 20 + 517 Q_\mathrm{CB}^2 \quad \rightarrow \quad Q_\mathrm{CA}^2 - 0.441 Q_\mathrm{CB}^2 + 0.00426 = 0 \tag{3}$$

The continuity equation requires that

$$Q_\mathrm{CA} + Q_\mathrm{CB} = 0.2 \quad \rightarrow \quad Q_\mathrm{CA} = 0.2 - Q_\mathrm{CB} \tag{4}$$

Combining Equations 3 and 4 gives

$$(0.2 - Q_\mathrm{CB})^2 - 0.441 Q_\mathrm{CB}^2 + 0.00426 = 0 \quad \rightarrow \quad 0.559 Q_\mathrm{CB}^2 - 0.4 Q_\mathrm{CB} + 0.04426 = 0 \tag{5}$$

Equation 5 has two real solutions that are given by

$$Q_{\rm CB} = 0.579\,{\rm m^3/s} \quad \text{and} \quad Q_{\rm CB} = 0.137\,{\rm m^3/s}$$

and substituting these results into Equation 4 gives

$$Q_{\rm CA} = -0.379\,{\rm m^3/s} \quad \text{and} \quad Q_{\rm CA} = 0.063\,{\rm m^3/s}$$

Since only real and positive flows are acceptable, $Q_{\rm CB} = 0.137\,{\rm m^3/s}$, $Q_{\rm CA} = 0.063\,{\rm m^3/s}$, and the assumed flow directions are validated. [If invalid flow directions were assumed, then there would be no real and positive simultaneous solutions to the corresponding Darcy–Weisbach and continuity equations.]

Assuming that the flow in pipe AD is from A to D and that the flow in pipe BD is from B to D (i.e. $h_{\rm A} > h_{\rm D} < h_{\rm B}$), the Darcy–Weisbach equation gives

$$h_{\rm D} = 25 - 143Q_{\rm AD}^2 \tag{6}$$

and for the flow from B to D,

$$h_{\rm D} = 20 - 345Q_{\rm BD}^2 \tag{7}$$

Combining Equations 6 and 7 gives

$$25 - 143Q_{\rm AD}^2 = 20 - 345Q_{\rm BD}^2 \quad \rightarrow \quad Q_{\rm BD}^2 - 0.414Q_{\rm AD}^2 + 0.0145 = 0 \tag{8}$$

The continuity equation requires that

$$Q_{\rm AD} + Q_{\rm BD} = 0.2 \quad \rightarrow \quad Q_{\rm BD} = 0.2 - Q_{\rm AD} \tag{9}$$

Combining Equations 8 and 9 gives

$$(0.2 - Q_{\rm AD})^2 - 0.414Q_{\rm AD}^2 + 0.0145 = 0 \quad \rightarrow \quad 0.586Q_{\rm AD}^2 - 0.4Q_{\rm AD} + 0.0545 = 0 \tag{10}$$

Equation 10 has two real solutions that are given by

$$Q_{\rm AD} = 0.495\,{\rm m^3/s} \quad \text{and} \quad Q_{\rm AD} = 0.188\,{\rm m^3/s}$$

and substituting into Equation 9 gives

$$Q_{\rm BD} = -0.295\,{\rm m^3/s} \quad \text{and} \quad Q_{\rm BD} = 0.012\,{\rm m^3/s}$$

Since only real and positive flows are consistent with the assumed flow directions, $Q_{\rm AD} = 0.188\,{\rm m^3/s}$, $Q_{\rm BD} = 0.012\,{\rm m^3/s}$, and the assumed flow directions are validated.

Therefore, based on these results,

$$\text{flow out of reservoir at A} = Q_{\rm AD} - Q_{\rm CA} = 0.188 - 0.063 = \boxed{0.125\,{\rm m^3/s}}$$

$$\text{flow into reservoir at B} = Q_{\rm CB} - Q_{\rm BD} = 0.137 - 0.012 = \boxed{0.125\,{\rm m^3/s}}$$

The flows into and out of the reservoirs (0.125 $\rm m^3/s$) are equal as expected.

2.70. From the given data: $z_E = z_F = z_G = 100$ m, $z_A = z_B = 0$ m, and $V_{AC} = V_{BC} = 2.5$ m/s. Since pipes AC and BC are identical, they have the same flow. If Q is the flow through the pump, then

$$\frac{Q}{2} = V_{AC}A_{AC} = 2.5\frac{\pi}{4}(0.5)^2 = 0.4909\,\mathrm{m^3/s} \Rightarrow \boxed{Q = 0.9817\,\mathrm{m^3/s}}$$

and so $V_{CD} = Q/A_{CD} = 2.222$ m/s. If h_p is the head added by the pump, applying the energy equation gives

$$0 + \frac{2.5^2}{2(9.81)} + 0 - \frac{0.0055(100)}{0.50}\frac{2.5^2}{2(9.81)} - \frac{0.0050(300)}{0.75}\frac{(2.222)^2}{2(9.81)} + h_\mathrm{p} - \frac{0.0060(500)}{0.30}\frac{Q_{DE}^2}{2(9.81)\left[\frac{\pi}{4}(0.3)^2\right]^2} = \frac{Q_{DE}^2}{2(9.81)\left[\frac{\pi}{4}(0.3)^2\right]^2} + 100$$

which simplifies to

$$h_\mathrm{p} = 112.2Q_{DE}^2 + 100.5 \tag{1}$$

For lines F and G,

$$\frac{Q_{DF}^2}{2(9.81)\left[\frac{\pi}{4}(0.25)^2\right]^2} + 100 + \frac{0.0060(400)}{0.25}\frac{Q_{DF}^2}{2(9.81)\left[\frac{\pi}{4}(0.25)^2\right]^2} = \frac{Q_{DG}^2}{2(9.81)\left[\frac{\pi}{4}(0.30)^2\right]^2} + 100 + \frac{0.0060(500)}{0.30}\frac{Q_{DG}^2}{2(9.81)\left[\frac{\pi}{4}(0.30)^2\right]^2}$$

which simplifies to

$$Q_{DF} = 0.7074Q_{DG} \tag{2}$$

Since E and G are identical pipes, then

$$Q_{DE} = Q_{DG} \tag{3}$$

and by continuity

$$0.9817 = Q_{DE} + Q_{DF} + Q_{DG} \tag{4}$$

Combining Equations 2 to 4 gives

$$0.9817 = Q_{DE} + 0.7074Q_{DE} + Q_{DE}$$

which yields $Q_{DE} = 0.3626$ m^3/s. Substituting into Equation 1 gives

$$h_\mathrm{p} = 112.2(0.3626)^2 + 100.5 = 115.5\,\mathrm{m}$$

Therefore, the pressure difference across the pump is $9.79(115.6) = \boxed{1128\ \mathrm{kPa}}$ and the power consumed by the pump is $\gamma Q h_\mathrm{p}/\eta = (9.79)(0.9817)(115.6)/0.76 = \boxed{1461\ \mathrm{kW}}$.

2.71. Using the Darcy-Weisbach head loss equation,

$$r = \frac{fL}{2gA^2D} = \frac{fL}{2(9.81)(\pi D^2/4)^2D} = 0.0826\frac{fL}{D^5}$$

For ductile-iron, $k_s = 0.26$ mm, and for fully turbulent flow the friction factor, f, is estimated using

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_s/D}{3.7}\right)$$

Using the given values of D and L in Equations 2 and 2 yields

Pipe	f	L (m)	D (mm)	r
AB	0.0190	1000	300	646
BC	0.0186	750	325	318
CD	0.0210	800	200	4337
DE	0.0198	700	250	1172
EF	0.0190	900	300	581
FA	0.0198	900	250	1507
BE	0.0183	950	350	273

2.72. Using the Darcy-Weisbach equation to calculate frictional head losses in each pipe, the Hardy-Cross parameters are:

$$r = \left(\frac{fL}{D} + K\right)\frac{1}{2gA^2}, \quad n = 2$$

Assuming the flow is fully turbulent, the friction factor, f, is given by both the Colebrook and Swamee-Jain equations as

$$f = \frac{0.25}{\left[\log\left(\frac{k_s}{3.7D}\right)\right]^2}$$

Using the given pipe characteristics and the above equations for r and f yields the following results:

Pipe	f	r
AB	0.00965	51.0
BC	0.01014	69.7
AD	0.00965	69.0
CD	0.01014	105.3

Applying the Hardy Cross method to any assumed flow distribution and using the derived values of r and n ($= 2$) yields the flow distribution shown in Figure 2.3. Taking $Q_{AB} = 31.9$ L/min $= 0.000532$ m^3/s, the pressure at B is determined using the energy equations as follows:

$$\left(\frac{p_B}{\gamma} + \cancel{\frac{V_B^2}{2g}} + z_B\right) = \left(\frac{p_A}{\gamma} + \cancel{\frac{V_A^2}{2g}} + z_A\right) - r_{AB}Q^2$$

$$\left(\frac{p_B}{\gamma} + 2\right) = \left(\frac{340}{9.79} + 1\right) - (51.0)(0.000532)^2$$

which yields $p_B = \boxed{330 \text{ kPa}}$.

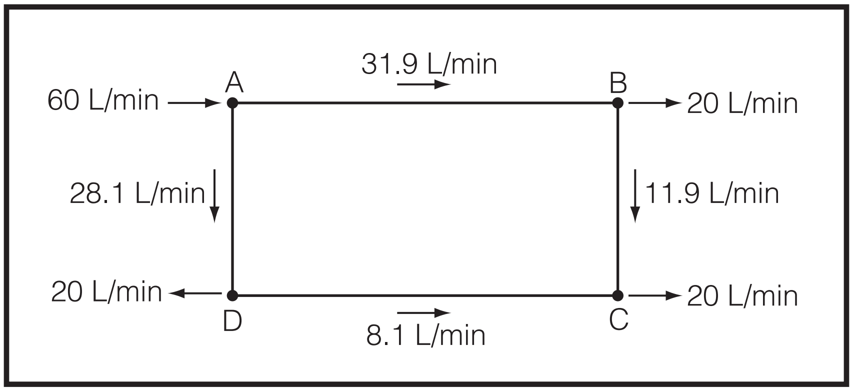

Figure 2.3: Flow distribution

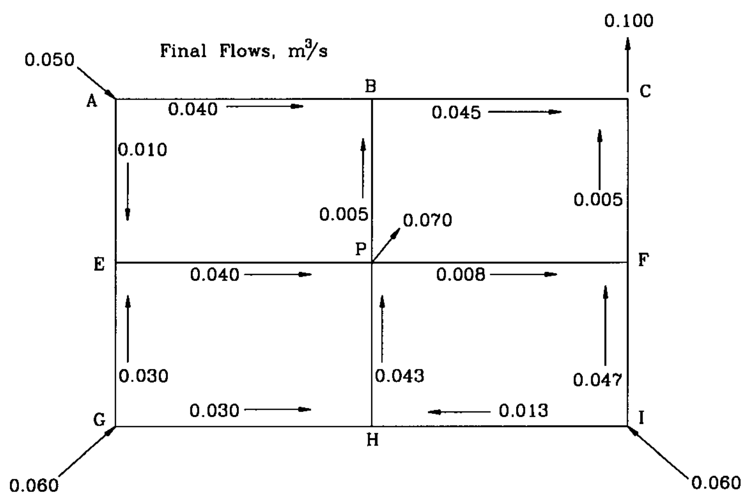

Figure 2.4: Final Flows

2.73. The final flows using the Hardy Cross method are shown in Figure 2.4. The results of intermediate calculations will depend on initial flow assumptions. Since the pressure at P is 500 kPa, and the network is on flat terrain, then the head at other intersections can be calculated by accounting for the frictional head losses. For each pipe, the pressure, p, is given by

$$p = 500 \pm \gamma h_\mathrm{f} \text{ kPa} \tag{1}$$

where h_f is the frictional head loss, and the $\pm$ accounts for whether the flow is towards P (+) or away from P ($-$). The frictional head loss, h_f, is given by the Darcy-Weisbach equation as

$$h_f = f\frac{L}{D}\frac{Q^2}{2gA^2}$$

Since $D = 0.3$ m and $A = \pi D^2/4 = \pi(0.3)^2/4 = 0.0707$ m^2, then

$$h_\text{f} = f\frac{L}{0.3}\frac{Q^2}{2(9.81)(0.0707)^2} = 33.99 f L Q^2 \tag{2}$$

where L is given for each pipe, Q has been calculated, and f can be calculated using the Swamee-Jain equation

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_\text{s}/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}}\right] \tag{3}$$

where $k_\text{s}/D = 0.26/300 = 0.000867$, and Re is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{\rho Q D}{A\mu} = \frac{(998)(Q)(0.3)}{(0.0707)(1.00\times 10^{-3})} = 4.23\times 10^6 Q \tag{4}$$

Combining Equations 3 and 4 gives the following expression for f in terms of the flow rate, Q,

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{0.000867}{3.7} + \frac{5.74}{(4.23\times 10^6 Q)^{0.9}}\right]$$

which simplifies to

$$f = \frac{0.25}{\left\{\log\left[2.34\times 10^{-4} + \frac{6.22\times 10^{-6}}{Q^{0.9}}\right]\right\}^2} \tag{5}$$

Combining Equations 2 and 5 gives the following expression for the frictional head loss in each pipe

$$h_f = \frac{8.50 L Q^2}{\left\{\log\left[2.34\times 10^{-4} + \frac{6.22\times 10^{-6}}{Q^{0.9}}\right]\right\}^2} \tag{6}$$

The pressure drop, Δp, in each pipe is equal to γh_f, $= 9.79 h_\text{f}$ kPa, and therefore

$$\Delta p = \frac{83.2 L Q^2}{\left\{\log\left[2.34\times 10^{-4} + \frac{6.22\times 10^{-6}}{Q^{0.9}}\right]\right\}^2}$$

Calculating the pressure changes in each pipe, and adding/subtracting from the pressure at P (= 500 kPa) yields the following results:

Pipe	L (m)	Q (m^3/s)	Δp (kPa)	Node	Pressure (kPa)
PB	100	0.005	0.02	B	500.0
PF	150	0.008	0.08	F	499.9
PH	100	0.043	1.28	H	501.3
PE	150	0.040	1.67	E	501.7
AB	150	0.040	1.67	A	501.6
BC	150	0.045	2.10	C	497.9
FI	100	0.047	1.52	I	502.8
GH	150	0.030	0.96	G	502.2

2.74. This problem can be solved using the Hardy Cross method. From the given data, k_s = 0.05 mm and D = 1120 mm. Hence,

$$f = \frac{0.25}{\left[\log\left(\frac{k_s/D}{3.7}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{4.464\times10^{-5}}{3.7}\right)\right]^2} = 0.01033$$

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(1.12)^2 = 0.9852\,\text{m}^2$$

and the Hardy Cross parameters corresponding to the Darcy-Weisbach equation are $n = 2$ and

$$r = f\frac{L}{D}\frac{1}{2gA^2} = (0.01033)\frac{L}{1.12}\frac{1}{2(9.81)(0.9852)^2} = 0.0004843L$$

Using this formulation with the given pipe lengths, the r values for each of the pipes are: CD = 2.42, DE = 5.81, EF = 4.36, FC = 2.91, FG = 3.39, GH = 3.87, and HC = 4.84. The intermediate flow values will depend on the initial assumed flow distribution. The final flow values are shown in Figure 2.5.

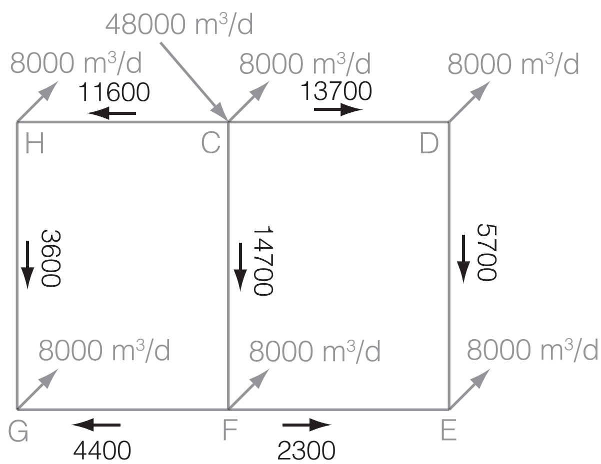

Figure 2.5: Flow distribution.

2.75. From the given data: $\omega_1 = \omega_2 = 1200$ rpm, $D_1 = 500$ mm, $D_2 = 250$ mm, $Q_1 = 250$ L/s, h_{p1} = 63.7 m, and $\eta_1 = 81\%$. Applying the affinity relationships given by Equation 2.116 requires that

$$\left[\frac{Q_1}{(\omega_1)(D_1)^3}\right] = \left[\frac{Q_2}{(\omega_2)(D_2)^3}\right]$$

$$\left[\frac{250}{(1200)(500)^3}\right] = \left[\frac{Q_2}{(1200)(250)^3}\right]$$

which yields $Q_2 = 31$ L/s. Also,

$$\left[\frac{h_{p1}}{(\omega_1)^2(D_1)^2}\right] = \left[\frac{h_{p2}}{(\omega_2)^2(D_2)^2}\right]$$

$$\left[\frac{63.7}{(1200)^2(500)^2}\right] = \left[\frac{h_{p2}}{(1200)^2(250)^2}\right]$$

which yields $h_{p2} = 15.9$ m. Therefore, the best-efficiency operating point for a 250-mm pump in the given homologous series is at $Q = \boxed{31 \text{ L/s}}$ and $h_\text{p} = \boxed{50.3 \text{ m}}$. The efficiency at this operating point can be estimated using Equation 2.120 which gives

$$\frac{1-\eta_2}{1-\eta_1} = \left(\frac{D_1}{D_2}\right)^{\frac{1}{4}}$$
$$\frac{1-\eta_2}{1-0.81} = \left(\frac{500}{250}\right)^{\frac{1}{4}}$$

which yields $\eta_2 = 0.77$. The efficiency can also be estimated by Equation 2.121 which gives

$$\frac{0.94-\eta_2}{0.94-\eta_1} = \left(\frac{Q_1}{Q_2}\right)^{0.32}$$
$$\frac{0.94-\eta_2}{0.94-0.81} = \left(\frac{250}{31}\right)^{0.32}$$

which yields $\eta_2 = 0.69$. Based on these results, it is estimated that the 250-mm pump will have an efficiency somewhere in the range of $\boxed{69\text{–}77\%}$.

2.76. (a) Three types of pumps: centrifugal flow, axial flow, mixed flow. A homologous pump series has common characteristics. Two properties in common are geometric similarity and the same specific speed.

(b) From the given data: $k_\text{s} = 0.9\,\text{mm}$, $D = 1\,\text{m}$, $p_\text{v} = 2.337\,\text{kPa}$, $Q = 2\,\text{m}^3/\text{s}$, $L = 20\,\text{m}$, $K = 5$, $\Delta z = 3.5\,\text{m} - 3\,\text{m} = 0.5\,\text{m}$, $\Delta h = 2\,\text{m}$, and $\omega = 1800\,\text{rpm} = 188.5\,\text{rad/s}$. For water at 20°C, $\nu = 1.0 \times 10^{-6}\,\text{m}^2/\text{s}$ and $\gamma = 9.789\,\text{kN/m}^3$. Assume $p_\text{atm} = 101.3\,\text{kPa}$. The following preliminary calculations are useful:

$$A = \tfrac{1}{4}\pi D^2 = 0.7854\,\text{m}^2, \qquad V = \frac{Q}{A} = 5.547\,\text{m/s}$$

$$\frac{k_\text{s}}{D} = 9.0 \times 10^{-4}, \qquad \text{Re} = \frac{VD}{\nu} = 2.547 \times 10^6$$

Using the Swamee-Jain equation,

$$f = \frac{0.25}{\left[\log\left(\frac{k_\text{s}/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{9\times10^{-4}}{3.7} + \frac{5.74}{(2.547\times10^6)^{0.9}}\right)\right]^2} = 0.01933$$

The following calculations give the head added by the pump, h_p, and the head at the inlet of the pump, h_i, as:

$$h_\text{p} = \Delta z + \left(f\frac{L}{D} + K\right)\frac{V^2}{2g} = 0.5 + \left(0.01933\frac{20}{1} + 5\right)\frac{2.547^2}{2(9.807)} = 2.281\,\text{m}$$

$$h_{\mathrm{i}} = \frac{p_{\mathrm{atm}}}{\gamma} - 0.5\left(f\frac{L}{D} + K\right)\frac{V^2}{2g} = \frac{101.3}{9.807} - \left(0.01933\frac{10}{1} + 2.5\right)\frac{2.547^2}{2(9.807)} = 9.458\,\mathrm{m}$$

So, the specific speed and the net positive suction head are given by:

$$n_{\mathrm{s}} = \frac{\omega Q^{\frac{1}{2}}}{(gh_{\mathrm{p}})^{\frac{3}{4}}} = \frac{(188.5)(2)^{\frac{1}{2}}}{[(9.807)(2.281)]^{\frac{3}{4}}} = \boxed{25.9}$$

$$\mathrm{NPSH} = h_{\mathrm{i}} - \left(\frac{p_{\mathrm{v}}}{\gamma} + \Delta h\right) = 9.458 - \left(\frac{2.337}{9.807} + 2\right) = \boxed{7.22\,\mathrm{m}}$$

The specific speed would be used in selecting the $\boxed{\text{pump type}}$, and the NPSH would be used in determining the $\boxed{\text{pump placement}}$.

2.77. From the given data: $\omega = 3500\,\mathrm{rpm} = 366.5\,\mathrm{rad/s}$. The points in the following table can be read from the given dimensional performance curves:

Q	Head, h_{p} for given impeller diameter, D				
($\mathrm{m^3/h}$)	178 mm	165 mm	152 mm	140 mm	127 mm
0	68.2	55.4	46.4	37.0	29.0
5	69.0	55.6	46.2	37.5	28.8
10	68.0	54.6	45.0	36.4	27.8
15	66.6	52.4	42.8	33.8	25.5
20	63.7	49.5	39.5	29.1	19.5
25	59.4	44.7	34.0	–	–
30	54.1	39.9	–	–	–

(a) Converting Q from $\mathrm{m^3/s}$ to $\mathrm{m^3/s}$ and plotting $gh_{\mathrm{p}}/\omega^2D^2$ versus $Q/\omega D^3$ gives the dimensionless performance curves shown in Figure 2.6.

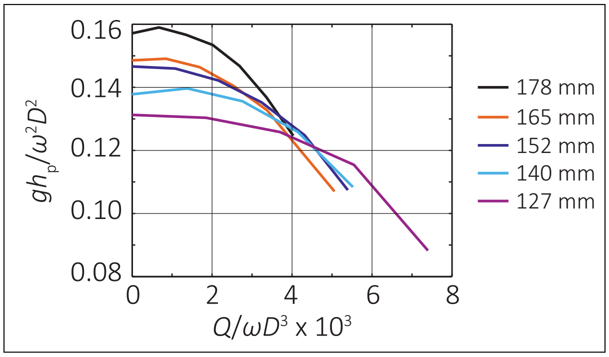

Figure 2.6: Nondimensional performance curves.

(b) Since these nondimensional performance curves do not coincide, the given pump sizes are $\boxed{\text{not from a homologous series}}$.

(c) The maximum efficiency, flow rate, and head at the best-efficiency point (BEP) for each pump size, along with the derived spacific speed are given in the following table.

D (mm)	$\eta_{\rm max}$ (%)	Q (m^3/h)	$h_{\rm p}$ (m)	$n_{\rm s}$ (–)
178	59.5	23.5	60.6	0.25
165	58.0	21.9	47.9	0.28
152	56.0	18.7	40.4	0.30
140	54.0	16.9	32.0	0.34
127	51.5	16.0	24.6	0.40

The specific speed is calculated using the relation: $n_{\rm s} = \omega Q^{\frac{1}{2}}/(gh_{\rm p})^{\frac{3}{4}}$. The efficiencies are in the range $\boxed{51.5\text{–}59.5\%}$ and the specific speeds are in the range $\boxed{0.25\text{–}0.40}$. The fact that the BEP efficiency and the specific speed is not the same for all pump sizes further indicates that the pumps are not homologous.

2.78. From the given data: $Q = 20\,\text{L/s} = 0.02\,\text{m}^3/\text{s}$, $P = 5\,\text{kW}$, and $\eta = 0.80$. For water at 15°C, $\gamma = 9798\,\text{N/m}^3$. The head, $h_{\rm p}$, added by the pump can be determined from the relationship between power, P, and head as follows

$$P = \frac{\gamma Q h_{\rm p}}{\eta} \quad \rightarrow \quad 5 \times 10^3 = \frac{(9798)(0.02)h_{\rm p}}{0.8} \quad \rightarrow \quad \boxed{h_{\rm p} = 20.4\,\text{m}}$$

2.79. From the given data: $D_1 = 300\,\text{mm}$, $\omega_1 = 1800\,\text{rpm} = 188.5\,\text{rad/s}$, $Q_1 = 300\,\text{L/s} = 0.3\,\text{m}^3/\text{s}$, $h_{\rm p1} = 65\,\text{m}$, $P_1 = 240\,\text{kW}$, $D_2 = 250\,\text{mm}$, and $\omega_2 = 1400\,\text{rpm} = 146.6\,\text{rad/s}$. For water at 20°C, $\rho = 998.2\,\text{kg/m}^3$.

(a) Using the definitions of C_Q, C_H, and C_P gives

$$C_Q = \frac{Q_1}{\omega_1 D_1^3} = \frac{0.3}{(188.5)(0.3)^3} = \boxed{0.0589}, \qquad C_H = \frac{gh_{\rm p1}}{\omega_1^2 D_1^2} = \frac{(9.807)(65)}{(188.5)^2(0.3)^2} = \boxed{0.199}$$

$$C_P = \frac{P_1}{\rho\omega_1^3 D_1^5} = \frac{240 \times 10^3}{(998.2)(188.5)^3(0.3)^5} = \boxed{0.0148}$$

(b) Under optimal operating conditions, the 250-mm pump has the same values of C_Q, C_H, and C_P as the 300-mm pump, so

$$\frac{Q_2}{\omega_2 D_2^3} = C_Q \quad \rightarrow \quad \frac{Q_2}{(146.6)(0.25)^3} = 0.0589 \quad \rightarrow \quad Q_2 = 0.135\,\text{m}^3/\text{s} = \boxed{135\,\text{L/s}}$$

$$\frac{gh_{\rm p2}}{\omega_2^2 D_2^2} = C_H \quad \rightarrow \quad \frac{(9.807)h_{\rm p2}}{(146.6)^2(0.25)^2} = 0.199 \quad \rightarrow \quad h_{\rm p2} = \boxed{27.3\,\text{m}}$$

$$\frac{P_2}{\rho\omega_2^3 D_2^5} = C_P \rightarrow \frac{P_2}{(998.2)(146.6)^3(0.25)^5} = 0.0148 \rightarrow P_2 = 45.4 \times 10^3\,\text{W} = \boxed{45.4\,\text{kW}}$$

2.80. From the given data: $C_Q = 0.035$, $C_H = 0.14$, $C_P = 0.006$, $D = 600$ mm, and $\omega = 1140$ rpm = 119.4 rad/s. For water at 20°C, $\rho = 998.2\,\text{kg/m}^3$, and $\gamma = 9.789\,\text{kN/m}^3$ (Appendix B.1). Using the definitions of the coefficients given in Equation 2.117, the best-efficiency conditions are calculated as follows:

$$\frac{Q}{\omega D^3} = 0.035 \quad \rightarrow \quad \frac{Q}{(119.4)(0.6)^3} = 0.035 \quad \rightarrow \quad Q = \boxed{0.903\,\text{m}^3/\text{s}}$$

$$\frac{gh_\text{p}}{\omega^2 D^2} = 0.14 \quad \rightarrow \quad \frac{(9.807)h_\text{p}}{(119.4)^2(0.6)^2} = 0.14 \quad \rightarrow \quad h_\text{p} = \boxed{73.2\,\text{m}}$$

$$\frac{\text{BHP}}{\rho\omega^3 D^5} = 0.006 \quad \rightarrow \quad \frac{\text{BHP}}{(998.2)(119.4)^3(0.6)^5} = 0.006 \quad \rightarrow \quad \text{BHP} = 7.93 \times 10^5\,\text{W} = \boxed{793\,\text{kW}}$$

Using these results, the efficiency of the pump, η_p, is given by

$$\eta_\text{p} = \frac{\gamma Q h_\text{p}}{\text{BHP}} = \frac{(9.789)(0.903)(73.2)}{793} = 0.816 \approx \boxed{82\%}$$

2.81. From the given data: $C_Q = 0.035$, $C_H = 0.14$, $C_P = 0.006$, $Q = 200$ L/s, and $\omega = 850$ rpm = 89.01 rad/s. For water at 20°C, $\rho = 998.2\,\text{kg/m}^3$, and $\gamma = 9.789\,\text{kN/m}^3$ (Appendix B.1). Using the definition of the flow coefficient given in Equation 2.117,

$$\frac{Q}{\omega D^3} = 0.035 \quad \rightarrow \quad \frac{0.2}{(89.01)D^3} = 0.035 \quad \rightarrow \quad D = 0.400\,\text{m} = \boxed{400\,\text{mm}}$$

2.82. From the given data: $D_1 = 400$ mm, $Q_1 = 200$ L/s, $h_1 = 24$ m, $P_1 = 50$ kW, $\eta_1 = 85\%$, and $D_2 = 350$ mm. It is also implicit in the problem statement that $\omega_1 = \omega_2$.

(a) Assume that viscous and scale effects are negligible. Since both pumps are geometrically similar and have the same efficiency, Equations **??** and 2.116 require that

$$\left[\frac{Q}{\omega D^3}\right]_1 = \left[\frac{Q}{\omega D^3}\right]_2 \quad \rightarrow \quad Q_2 = \left(\frac{D_2}{D_1}\right)^3 \left(\frac{\omega_2}{\omega_1}\right) Q_1 = \left(\frac{350}{400}\right)^3 (1)(200) = \boxed{134\,\text{L/s}}$$

$$\left[\frac{gh}{\omega^2 D^2}\right]_1 = \left[\frac{gh}{\omega^2 D^2}\right]_2 \quad \rightarrow \quad h_2 = \left(\frac{D_2}{D_1}\right)^2 \left(\frac{\omega_2}{\omega_1}\right)^2 h_1 = \left(\frac{350}{400}\right)^2 (1)^2(24) = \boxed{18.4\,\text{m}}$$

$$\left[\frac{P}{\rho\omega^3 D^5}\right]_1 = \left[\frac{P}{\rho\omega^3 D^5}\right]_2 \quad \rightarrow \quad P_2 = \left(\frac{D_2}{D_1}\right)^5 \left(\frac{\omega_2}{\omega_1}\right)^3 P_1 = \left(\frac{350}{400}\right)^5 (1)^3(50) = \boxed{25.6\,\text{kW}}$$

(b) The efficiency with the 350-mm impeller can be estimated using Equation 2.120 which gives

$$\frac{1-\eta_2}{1-\eta_1} = \left(\frac{D_1}{D_2}\right)^{\frac{1}{4}} \quad \rightarrow \quad \frac{1-\eta_2}{1-0.85} = \left(\frac{400}{350}\right)^{\frac{1}{4}} \quad \rightarrow \quad \eta_2 = 0.84$$

The efficiency can also be estimated by Equation 2.121 which gives

$$\frac{0.94-\eta_2}{0.94-\eta_1} = \left(\frac{Q_1}{Q_2}\right)^{0.32} \quad \rightarrow \quad \frac{0.94-\eta_2}{0.94-0.85} = \left(\frac{200}{134}\right)^{0.32} \quad \rightarrow \quad \eta_2 = 0.84$$

Based on these results, it is estimated that the pump with the 350-mm diameter impeller will have an efficiency of approximately $\boxed{84\%}$. This scale effect in relatively small and can be characterized as $\boxed{\text{not significant}}$.

2.83. From the given data: $\omega_1 = \omega_2 = 2400$ rpm, $D_1 = 400$ mm, $D_2 = 300$ mm, $Q_1 = 500$ L/s, $h_{p1} = 89.5$ m, and $\eta_1 = 85\%$. Applying the affinity relationships given by Equation 2.116 requires that

$$\left[\frac{Q_1}{(\omega_1)(D_1)^3}\right] = \left[\frac{Q_2}{(\omega_2)(D_2)^3}\right] \quad \rightarrow \quad \left[\frac{500}{(2400)(400)^3}\right] = \left[\frac{Q_2}{(2400)(300)^3}\right] \quad \rightarrow \quad Q_2 = 210\,\text{L/s}$$

$$\left[\frac{h_{p1}}{(\omega_1)^2(D_1)^2}\right] = \left[\frac{h_{p2}}{(\omega_2)^2(D_2)^2}\right] \quad \rightarrow \quad \left[\frac{89.5}{(2400)^2(400)^2}\right] = \left[\frac{h_{p2}}{(2400)^2(300)^2}\right] \quad \rightarrow \quad h_{p2} = 50.3\,\text{m}$$

Therefore, the best-efficiency operating point for a 300-mm pump in the given homologous series is at $Q = 210$ L/s and $h_p = 50.3$ m. The efficiency at this operating point can be estimated using Equation 2.120 which gives

$$\frac{1-\eta_2}{1-\eta_1} = \left(\frac{D_1}{D_2}\right)^{\frac{1}{4}} \quad \rightarrow \quad \frac{1-\eta_2}{1-0.85} = \left(\frac{400}{300}\right)^{\frac{1}{4}} \quad \rightarrow \quad \eta_2 = 0.84$$

The efficiency can also be estimated by Equation 2.121 which gives

$$\frac{0.94-\eta_2}{0.94-\eta_1} = \left(\frac{Q_1}{Q_2}\right)^{0.32} \quad \rightarrow \quad \frac{0.94-\eta_2}{0.94-0.85} = \left(\frac{500}{210}\right)^{0.32} \quad \rightarrow \quad \eta_2 = 0.82$$

Based on these results, it is estimated that the 300-mm pump will have an efficiency somewhere in the range of 82%–84%.

2.84. From the given data: $D = 250$ mm, $\omega = 1200$ rpm $= 125.7$ rad/s, $h_p = 8.2$ m, and $Q = 20$ L/s. Using the definition of specific speed given by Equation 2.124 yields

$$n_s = \frac{\omega Q^{\frac{1}{2}}}{(gh_p)^{\frac{3}{4}}} = \frac{(125.7)(0.020)^{\frac{1}{2}}}{[(9.807)(8.2)]^{\frac{3}{4}}} = \boxed{0.662}$$

For a specific speed of 0.662, Table 2.3 indicates that the pump is likely to be a $\boxed{\text{centrifugal pump}}$.

2.85. The specific speed in SI units is given by Equation 2.124 as

$$n_s = \frac{\omega Q^{\frac{1}{2}}}{(gh_p)^{\frac{3}{4}}}$$

and the specific speed in English units is given by Equation 2.125 as

$$N_s = \frac{\omega Q^{\frac{1}{2}}}{h_p^{\frac{3}{4}}}$$

The unit conversions are as follows:

$$\omega \text{ (rad/s)} = \frac{2\pi}{60}\omega \text{ (rpm)} = 0.1047\omega \text{ (rpm)}$$
$$Q \text{ (m}^3\text{/s)} = \frac{3.785 \times 10^{-3}}{60} Q \text{ (gpm)} = 6.308 \times 10^{-5} Q \text{ (gpm)}$$
$$h_\text{p} \text{ (m)} = 0.3048 h_\text{p} \text{ (ft)}$$

Taking $g = 9.81$ m/s^2 gives n_s in English units as

$$n_s = \frac{(0.1047\omega)(6.308 \times 10^{-5}Q)^{\frac{1}{2}}}{(9.81)^{\frac{3}{4}}(0.3048h_\text{p})^{\frac{3}{4}}} = 0.000366\frac{\omega Q^{\frac{1}{2}}}{h_\text{p}^{\frac{3}{4}}} = 0.000366 N_s$$

which can also be written as

$$N_s = 2730 n_s$$

Therefore, the constant used to convert the specific speed in SI units to the specific speed in English units is $\boxed{2730}$.

2.86. From the given data: $n_\text{s} = 4.2$, $Q = 300\,\text{L/s} = 0.3\,\text{m}^3\text{/s}$, and $h_\text{p} = 10\,\text{m}$. The required rotational speed can be derived from the the specific-speed equation (Equation 2.124) as follows

$$n_\text{s} = \frac{\omega Q^{\frac{1}{2}}}{[gh_\text{p}]^{\frac{3}{4}}} \quad \rightarrow \quad \omega = n_\text{s}\frac{[gh_\text{p}]^{\frac{3}{4}}}{Q^{\frac{1}{2}}} = (4.2)\frac{[(9.807)(10)]^{\frac{3}{4}}}{(0.3)^{\frac{1}{2}}} = 239\,\text{rad/s} = \boxed{2300\,\text{rad/s}}$$

According to Table 2.3, since $3.7 \leq n_\text{s} \leq 5.5$ the pump is an $\boxed{\text{axial-flow pump}}$.

2.87. The specific speed n_s, flow coefficient, C_Q, and head coefficient, C_H, are defined by

$$n_\text{s} = \frac{\omega Q^{\frac{1}{2}}}{(gh_\text{p})^{\frac{3}{4}}}, \qquad C_Q = \frac{Q}{\omega D^3}, \qquad C_H = \frac{gh_\text{p}}{\omega^2 D^2}$$

The relationship between n_s, C_Q, and C_H, can be derived directly from Equation 2.123 which gives

$$\boxed{n_\text{s} = \frac{C_Q^{\frac{1}{2}}}{C_H^{\frac{3}{4}}}}$$

2.88. From the given data: $Q = 500\,\text{L/s} = 0.5\,\text{m}^3\text{/s}$, $h_\text{p} = 80\,\text{m}$, and $\omega = 1800\,\text{rpm} = 188.5\,\text{rad/s}$. Application of Equation 2.124 gives the specific speed, n_s, as

$$n_\text{s} = \frac{\omega Q^{\frac{1}{2}}}{[gh_\text{p}]^{\frac{3}{4}}} = \frac{(188.5)(0.5)^{\frac{1}{2}}}{[(9.807)(80)]^{\frac{3}{4}}} = \boxed{0.90}$$

According to Table 2.3, since $0.15 \leq n_\text{s} \leq 1.5$ a $\boxed{\text{centrifugal pump}}$ is required.

2.89. The synchronous speed is given by Equation 2.126 as

$$\text{synchronous speed} = \frac{3600}{\text{No. of pairs of poles}}$$

The maximum speed occurs when there is 1 pair of poles, which gives a speed of $\boxed{3600 \text{ rpm}}$.

2.90. From the given data: $Q = 100\,\text{L/s}$, and $h_\text{p} = 50\,\text{m}$. The available specific speeds are: 0.95, 0.81, 0.61, 0.36, and 0.25. For the angular speed ω_rpm, the specific speed, n_s, is given by

$$n_\text{s} = \frac{\omega Q^{\frac{1}{2}}}{(gh_\text{p})^{\frac{3}{4}}} = \frac{\left(\dfrac{\omega_\text{rpm}2\pi}{60}\right)(0.1)^{\frac{1}{2}}}{[(9.807)(50)]^{\frac{3}{4}}} = 3.178 \times 10^{-4}\omega_\text{rpm}$$

Using this relationship yields the following results,

ω_rpm (rpm)	n_s (–)
3450	1.096
1725	0.548
1140	0.362
850	0.270

The closest match with the available specific speeds is 0.362, which matches the available specific speed of $\boxed{0.36}$. A motor speed of $\boxed{1140 \text{ rpm}}$ should be specified with this pump.

2.91. From the given data: $D = 250\,\text{mm}$, $\omega_1 = 1200\,\text{rpm}$, $h_\text{so} = 8.2\,\text{m}$, $h_\text{bep} = 6.5\,\text{m}$, and $Q_\text{bep} = 20\,\text{L/s}$.

(a) Apply the parabolic equation at the shutoff and best-efficiency points:

$$\text{shutoff:} \quad 8.2 = a - b(0)^2 \quad \rightarrow \quad a = 8.2\,\text{m}$$

$$\text{bep:} \quad 6.5 = a - b(20)^2 \quad \rightarrow \quad 6.5 = 8.2 - b(20)^2 \quad \rightarrow \quad b = 0.0045$$

Combining the above results gives the following performance curve:

$$\boxed{h = 8.2 - 0.00425Q^2} \tag{1}$$

(b) Applying the homologous relations given by Equation 2.116 with $\omega_1/\omega_2 = 1200/1800 = 0.6667$ yields

$$Q_1 = \frac{\omega_1}{\omega_2}Q_2 = 0.6667Q_2, \qquad h_1 = \left(\frac{\omega_1}{\omega_1}\right)^2 h_2 = 0.4444h_2$$

Substituting these relations into Equation 1 yields

$$(0.444h_2) = 8.2 - 0.00425(0.6667Q_2)^2 \quad \rightarrow \quad h_2 = 18.45 - 0.00425Q_2^2$$

$$\rightarrow \quad \boxed{h = 18.45 - 0.00425Q^2}$$

2.92. From the given data: $n_\mathrm{s} = 1.2$, $Q_\mathrm{p} = 5\,\mathrm{L/s}$, $h_\mathrm{p} = 10\,\mathrm{m}$, $\nu_\mathrm{r} = 5$–10, and $D_\mathrm{r} = \frac{1}{5}$. From the gives specific speed,

$$n_\mathrm{s} = \frac{\omega Q^{\frac{1}{2}}}{[gh_\mathrm{p}]^{\frac{3}{4}}} \quad \rightarrow \quad 1.2 = \frac{\omega(0.05)^{\frac{1}{2}}}{[(9.807)(10)]^{\frac{3}{4}}} \quad \rightarrow \quad \omega = 167.2\,\mathrm{rad/s} = 1597\,\mathrm{rpm}$$

In accordance with Equation **??** the following similarity requirements are applicable if viscous effects are to be reproduced in the model,

$$\frac{Q_\mathrm{r}}{\omega_\mathrm{r} D_\mathrm{r}^3} = 1, \qquad \frac{\omega_\mathrm{r} D_\mathrm{r}^2}{\nu_\mathrm{r}} = 1, \qquad \frac{h_\mathrm{r}}{\omega_\mathrm{r}^2 D_\mathrm{r}^2} = 1$$

For Reynolds number similarity, and noting that $D_\mathrm{r} = \frac{1}{5}$ and $\nu_\mathrm{r} = 5$–10, gives,

$$\omega_\mathrm{r} = \frac{\nu_\mathrm{r}}{D_\mathrm{r}^2} = 5^2\nu_\mathrm{r} \rightarrow \omega_\mathrm{r} = 125 \text{ to } 250 \rightarrow \omega_\mathrm{m} = 125(1597) \text{ to } 250(1597)\,\mathrm{rpm}$$

$$\rightarrow \omega_\mathrm{m} = \boxed{1.99 \times 10^5 \text{ to } 3.99 \times 10^5\,\mathrm{rpm}}$$

For flow rate similarity, and using the requirement for Reynolds similarity,

$$Q_\mathrm{r} = \omega_\mathrm{r} D_\mathrm{r}^3 = \left(\frac{\nu_\mathrm{r}}{D_\mathrm{r}^2}\right) D_\mathrm{r}^3 = \nu_\mathrm{r} D_\mathrm{r} = \frac{\nu_\mathrm{r}}{5} \rightarrow Q_\mathrm{r} = 1 \text{ to } 2 \rightarrow Q_\mathrm{m} = 1(5) \text{ to } 2(5)\,\mathrm{L/s}$$

$$\rightarrow Q_\mathrm{m} = \boxed{5 \text{ to } 10\,\mathrm{L/s}}$$

For head similarity, and using the requirement for Reynolds similarity,

$$h_\mathrm{r} = \omega_\mathrm{r}^2 D_\mathrm{r}^2 = \left(\frac{\nu_\mathrm{r}}{D_\mathrm{r}^2}\right)^2 D_\mathrm{r}^2 = \frac{\nu_\mathrm{r}^2}{D_\mathrm{r}^2} = 5^2\nu_\mathrm{r}^2 \rightarrow h_\mathrm{r} = 125 \text{ or } 250 \rightarrow h_\mathrm{m} = 125(10) \text{ to } 250(10)\,\mathrm{m}$$

$$\rightarrow h_\mathrm{m} = \boxed{1250 \text{ to } 2500\,\mathrm{m}}$$

These model requirements are clearly unrealistic. In reality, Reynolds similarity is usually not required as long as fully turbulent flow is maintained in both the model and the prototype.

2.93. From the given data: $Q = 10\,\mathrm{L/s} = 0.01\,\mathrm{m^3/s}$, $p_1 = -20\,\mathrm{kPa}$, $p_2 = 100\,\mathrm{kPa}$, $\Delta z = 0.83\,\mathrm{m}$, $D_1 = 150\,\mathrm{mm}$, and $D_2 = 75\,\mathrm{mm}$. For water at 10°C, $\rho = 999.7\,\mathrm{kg/m^3}$ and $\gamma = 9.784\,\mathrm{kN/m^3}$. The following preliminary calculations are useful,

$$A_1 = \frac{\pi D_1^2}{2g} = \frac{\pi(0.15)^2}{2(9.807)} = 1.767 \times 10^{-2}\,\mathrm{m}^2, \qquad V_1 = \frac{Q}{A_1} = \frac{0.02}{1.767 \times 10^{-2}} = 0.5659\,\mathrm{m/s}$$

$$A_2 = \frac{\pi D_2^2}{2g} = \frac{\pi(0.075)^2}{2(9.807)} = 4.418 \times 10^{-3}\,\mathrm{m}^2, \qquad V_1 = \frac{Q}{A_1} = \frac{0.02}{4.418 \times 10^{-3}} = 2.264\,\mathrm{m/s}$$

Using the given and derived data, the head, h_p, added by the pump is given by the energy equation as

$$h_\mathrm{p} = \left[\frac{p_2}{\gamma} - \frac{V_2^2}{2g} + \Delta z\right] - \left[\frac{p_1}{\gamma} - \frac{V_1^2}{2g}\right]$$

$$= \left[\frac{100}{9.784} - \frac{2.264^2}{2(9.807)} + 0.83\right] - \left[\frac{-20}{9.784} - \frac{0.5659^2}{2(9.807)}\right] = \boxed{13.3\,\mathrm{m}}$$

2.94. From the given data: $\Delta z = 0.38\,\text{m}$. Corresponding values of Q, $\Delta p = p_2 - p_1$, and P are given in tabulated form. For water at 20°C, $\gamma = 9.79\,\text{kN/m}^3$.

(a) Using the given data, h_p and η are derived using the following relationships (derived from the energy equation and definition of efficiency, respectively):

$$h_\text{p} = \frac{\Delta p}{\gamma} + \Delta z, \qquad \eta = \frac{\gamma Q h_\text{p}}{P}$$

Using these equations yields the following tabulated data:

Q (L/s)	1.26	2.52	3.79	5.05	6.31	7.57	8.83
h_p (m)	68.9	68.2	67.1	64.6	61.0	57.3	51.2
η (%)	24	40	50	57	60	59	57

Plotting these data gives the following performance graph:

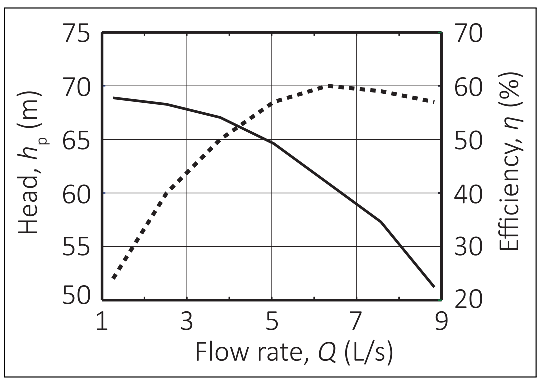

Figure 2.7: Performance curves.

(b) From the transformed data (tabulated above) the maximum efficiency is $\boxed{60\%}$ and the corresponding flow rate is $\boxed{7.57\ \text{L/s}}$.

2.95. From the given data: $D = 300\,\text{mm} = 0.3\,\text{m}$, $\Delta z = 1.5\,\text{m} + 2\,\text{m} = 3.5\,\text{m}$, and $L = 300\,\text{m}$. For water at 20°C, $\nu = 1.00 \times 10^{-6}\,\text{m}^2/\text{s}$. The following preliminary calculations are useful:

$$A = \pi\frac{D^2}{4} = \pi\frac{0.3^2}{4} = 0.07069\,\text{m}^2, \qquad \text{Re} = \frac{VD}{\nu} = \frac{QD}{\nu A} = \frac{Q(0.3)}{(10^{-6})(0.07069)} = 4.244 \times 10^6 Q$$

The energy equation requires that

$$h_\text{p} - f\frac{L}{D}\frac{V^2}{2g} = \Delta z \quad \rightarrow \quad h_\text{p} = 3.5 + f\frac{300}{0.3}\frac{Q^2}{2(9.807)(0.07069)^2} \quad \rightarrow \quad h_\text{p} = 3.5 + 1.020 \times 10^4 f Q^2 \quad (1)$$

The friction factor, f, can be approximated by the Swamee-Jain equation (smooth pipe, $k_\text{s} \approx 0$)

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{5.74}{\text{Re}^{0.9}}\right] = -2\log_{10}\left[\frac{5.74}{(4.244 \times 10^6 Q)^{0.9}}\right] \quad \rightarrow \quad f = \left[2\log_{10}\left(\frac{Q^{0.9}}{6.222 \times 10^{-6}}\right)\right]^{-2} \quad (2)$$

Combining Equations (1) and (2) gives the system curve as

$$h_\mathrm{p} = 3.5 + 1.020 \times 10^4 \left[2\log_{10}\left(\frac{Q^{0.9}}{6.222 \times 10^{-6}} \right) \right]^{-2} Q^2 \tag{3}$$

The pump curve is given as

$$h_\mathrm{p} = 6 - 6.67 \times 10^{-5}(Q \times 10^3)^2 \quad \rightarrow \quad h_\mathrm{p} = 6 - 66.7Q^2 \tag{4}$$

where Q is in m^3/s. Solving Equations (3) and (4) simultaneously gives

$$Q = 0.111\,\mathrm{m^3/s} = 111\,\mathrm{L/s}$$

Since the flow rate (111 L/s) is not within 10% of the desired flow rate of 150 L/s, the suggested pump is $\boxed{\text{inadequate}}$.

2.96. From the given data: D = 100 mm = 0.1 m. At 20°C, $\nu = 1 \times 10^6$ m^2/s. Using these data:

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.1)^2 = 0.00785\,\mathrm{m}^2$$

$$V = \frac{Q}{A} = \frac{Q}{0.00785} = 127Q$$

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(127Q)(0.1)}{1 \times 10^6} = 1.27 \times 10^7 Q$$

$$\frac{5.74}{\mathrm{Re}^{0.9}} = \frac{5.74}{(1.27 \times 10^7 Q)^{0.9}} = 2.32 \times 10^{-6} Q^{-0.9}$$

$$f = \frac{0.25}{\left[\log\left(\frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{[\log(2.32 \times 10^{-6}Q^{-0.9})]^2} \tag{1}$$

The energy equation to be satisfied is given by:

$$h_\mathrm{p} = (z_2 - z_1) + f\frac{L}{D}\frac{Q^2}{2gA^2} = 9 + f\frac{100}{0.1}\frac{Q^2}{2(9.81)(0.00785)^2}$$

which simplifies to

$$h_\mathrm{p} = 9 + 8.27 \times 10^5 fQ^2 \tag{2}$$

Combining Equations 1 and 2 yields the following system equation

$$h_\mathrm{p} = 9 + \frac{2.07 \times 10^5 Q^2}{[\log(2.32 \times 10^{-6}Q^{-0.9})]^2}$$

This curve can be plotted on the pump performance curve using the following points:

Q		h_p
(L/min)	(m^3/s)	(m)
200	0.00333	9.20
400	0.00667	9.68
600	0.01	10.41
800	0.0133	11.36

The system curve intersects the pump performance curve at $Q = \boxed{600 \text{ L/min}}$ and the corresponding efficiency is approximately $\boxed{55\%}$. Since the maximum pump efficiency is approximately 59%, the pump will be not be operating at its maximum efficiency and so this pump is $\boxed{\text{not optimal}}$.

2.97. From the given data: $\Delta z = 10$ m, $D = 200$ mm $= 0.200$ m, $A = \pi D^2/4 = 0.0314$ m^2, $L =$ 2 km $=$ 2000 m, and $K_\text{m} = 6.2$. For DIP, $k_\text{s} = 0.26$ mm. Assuming fully-turbulent flow, f is given by the Colebrook equation as

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_\text{s}/D}{3.7}\right)$$
$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{0.26/200}{3.7}\right)$$

which yields $f = 0.021$.

(a) The energy equation gives

$$0 - \left(\frac{fL}{D} + K_\text{m}\right)\frac{Q^2}{2gA^2} + h_\text{p} = \Delta z$$
$$0 - \left(\frac{(0.021)(2000)}{0.200} + 6.2\right)\frac{Q^2}{2(9.81)(0.0314)^2} + h_\text{p} = 10$$

which simplifies to

$$h_\text{p} = 10 + 11176Q^2$$

where Q is in m^3/s. If Q is in L/s, then the energy equation becomes

$$h_\text{p} = 10 + 11176\left(\frac{Q}{10^3}\right)^2 \tag{1}$$

Solving iteratively with the pump characteristics is shown in the following table,

Q (L/s)	h_p (m) (Equation 1)	h_p (m) (Table 15.3)
10	11.1	23.2
20	14.5	20.8
30	20.1	17.0
26.7	18.0	18.2
26.9	18.1	18.2
27.0	18.1	18.1

Therefore, the flow rate delivered by the pump is $\boxed{27.0 \text{ L/s}}$.

(b) For the given pump data, the efficiency, η, of the pump is 69.2%, $h_\text{p} = 18.1$ m, $Q =$ 27.0 L/s $= 0.027$ m^3/s, hence the power required from the pump motor is given by

$$P = \frac{\gamma Q h_\text{p}}{\eta} = \frac{(9.79)(0.027)(18.1)}{0.692} = \boxed{6.91\,\text{kW}}$$

2.98. The system curve is given by

$$0 - f\frac{L}{D}\frac{V^2}{2g} + h_\mathrm{p} = 61 \rightarrow h_\mathrm{p} = 61 + f\frac{L}{D}\frac{V^2}{2g} \tag{1}$$

For DIP it can be assumed that $k_\mathrm{s} = 0.12$ mm, and for the system:

$$A = \frac{\pi}{4}D^2 = 1.169\,\mathrm{m}^2$$
$$V = \frac{Q}{A} = 1.711\,\mathrm{m/s}$$
$$\nu = 1 \times 10^{-6}\ \mathrm{m}^2/\mathrm{s}$$
$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(1.169)(1.220)}{1 \times 10^{-6}} = 2.087 \times 10^6$$
$$f = \frac{0.25}{\left[\log\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} = 0.0128$$

Substituting the above results into Equation 1 gives

$$h_\mathrm{p} = 61 + 0.0128\frac{3200}{1.22}\frac{(1.711)^2}{2(9.81)} = 61 + 5.01 = 66.01\,\mathrm{m}$$
$$\text{power delivered to water} = \gamma Q h_\mathrm{p} = (9.79)(2)(66.01) = 1292\,\mathrm{kW}$$
$$\text{power consumed by pump} = \frac{1292}{0.85} = 1520\,\mathrm{kW}$$
$$\text{energy consumed in 8 h} = 1520 \times 8 = 12160\,\mathrm{kWh}$$
$$\text{cost of electricity} = 12160 \times 0.06 = \$730 \text{ per day}$$

When the system is operating in power-generation (turbine) mode,

$$h_t = 61 - 5.01 = 55.99\,\mathrm{m}$$
$$\text{power extracted from water} = \gamma Q h_t = (9.79)(2)(55.99) = 1096\,\mathrm{kW}$$
$$\text{power delivered as electricity} = 0.90(1096) = 986\,\mathrm{kW}$$
$$\text{energy delivered in 8 h} = 986 \times 8 = 7888\,\mathrm{kWh}$$
$$\text{revenue from sale of electricity} = 7888 \times 0.12 = \$947 per day$$

Based on these results, the profit is \$947 − \$730 = \$217/day = $\boxed{\text{\$79,200/year}}$

When the elevation difference is 65 m and $h_\mathrm{p} = 80 - 3.5Q^2$, then the combination of the pump curve and system curve gives

$$0 - f\frac{L}{D}\frac{Q^2}{2gA^2} + 80 - 3.5Q^2 = 65\,\mathrm{m}$$

and for fully turbulent flow,

$$f = \frac{0.25}{\left[\log\left(\frac{k_s}{3.7D}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{0.12}{3.7(1220)}\right)\right]^2} = 0.0119$$

Substituting into the pump/system curve equation gives

$$-(0.0119)\frac{3200}{1.22}\frac{Q^2}{2(9.81)(1.169)^2} + 80 - 3.5Q^2 = 65$$

which yields

$$\begin{aligned}
Q &= 3.20\,\mathrm{m^3/s} \\
h_\mathrm{p} &= 80 - 3.5Q^2 = 80 - 3.5(3.20)^2 = 44.16\,\mathrm{m} \\
h_\mathrm{f} &= 11.92\,\mathrm{m} \\
h_t &= 65 - 11.92 = 53.08\,\mathrm{m} \\
\text{cost of electricity} &= 8 \times \frac{\gamma Q h_\mathrm{p}}{0.85} \times 0.06 = \$781 \text{ per day} \\
\text{revenue} &= \gamma Q h_t \times 0.9 \times 8 \times 0.12 = \$1437 \text{ per day}
\end{aligned}$$

Based on these results, the profit is \$1437 − \$781 = \$656/day = $\boxed{\$239{,}440/\text{year}}$.

2.99. From the given data: $L = 300\,\mathrm{m}$, $k_\mathrm{s} = 0.26\,\mathrm{mm}$, $D = 800\,\mathrm{mm}$, $A = \pi D^2/4 = \pi(0.8)^2/4 = 0.503\,\mathrm{m^2}$, $k_\mathrm{s}/D = 0.26/800 = 0.000325$, $T = 20^\circ\mathrm{C}$, $\rho = 998.2\,\mathrm{kg/m^3}$, $\gamma = 9.789\,\mathrm{kN{\cdot}m^3}$, $\mu = 1.002 \times 10^{-3}\,\mathrm{N{\cdot}s/m^2}$, $z_1 = -5\,\mathrm{m}$, $z_\mathrm{p} = 0.5\,\mathrm{m}$, $z_2 = 4\,\mathrm{m}$, and pump performance curve given by

$$h_\mathrm{p} = 12 - 0.1Q^2$$

The energy equation between well and reservoir

$$h_\mathrm{p} = 9 + h_\mathrm{f} = 9 + f\frac{L}{D}\frac{Q^2}{2gA^2}$$

For fully turbulent flow, and $k_\mathrm{s}/D = 0.000325$, then $f = 0.015$ and the head loss equation becomes

$$h_\mathrm{p} = 9 + 0.015\frac{300}{0.8}\frac{Q^2}{2(9.81)(0.503)^2} = 9 + 1.13Q^2$$

Combining the energy equation with the pump equation gives

$$9 + 1.13Q^2 = 12 - 0.1Q^2 \quad \rightarrow \quad Q = 1.56\,\mathrm{m^3/s}$$

Check the assumption of $f = 0.015$,

$$\mathrm{Re} = \frac{\rho(Q/A)D}{\mu} = \frac{(998.2)(1.56/0.503)(0.8)}{1.002 \times 10^{-3}} = 2.47 \times 10^6$$

The Swamee-Jain equation confirms that $f = 0.015$ and therefore $\boxed{\mathrm{Q} = 1.56\,\mathrm{m^3/s}}$. The specific speed, N_s, is given by

$$N_\mathrm{s} = \frac{\omega Q^{\frac{1}{2}}}{h_\mathrm{p}^{\frac{3}{4}}} \tag{1}$$

where

$Q = 1.56\,\mathrm{m^3/s} = 24{,}727$ gpm, $\quad h_\mathrm{p} = 12 - 0.1(1.56)^2 = 11.8\,\mathrm{m} = 38.6$ ft, $\quad \omega = 1200$ rpm

Substituting into Equation (1) gives

$$N_s = \frac{(1200)(24727)^{\frac{1}{2}}}{(38.6)^{\frac{3}{4}}} = \boxed{12{,}185}$$

Therefore the type of pump required is an $\boxed{\text{axial flow pump}}$.

2.100. From the given data: $Q = 1500\,\text{L/s}$, $h_\ell = 2.3\,\text{m}$, and $\text{NPSH}_\text{R} = 2.9\,\text{m}$. Under standard sea-level atmospheric conditions, $p_0 = 101.3\,\text{kPa}$. For water at 25°C, $p_\text{v} = 3.167\,\text{kPa}$ and $\gamma = 9.778\,\text{kN/m}^3$ (from Appendix B.1). Substituting these data into Equation 2.133 gives

$$z_\text{max} = \frac{p_0 - p_\text{v}}{\gamma} - h_\ell - \text{NPSH}_\text{R} = \frac{101.3 - 3.167}{9.778} - 2.3 - 2.9 = \boxed{4.84\,\text{m}}$$

2.101. From the given data: $Q = 20\,\text{L/s} = 0.02\,\text{m}^3\text{/s}$, $D = 150\,\text{mm}$, $k_\text{s} = 0.3\,\text{mm}$, $K = 12$, and $\text{NPSH}_\text{R} = 5.5\,\text{m}$. For water at 20°C, $\gamma = 9.79\,\text{kN/m}^3$, $\nu = 1.00 \times 10^{-6}\,\text{m}^2\text{/s}$, and $p_\text{v} = 2.337\,\text{kPa}$. Under standard atmospheric conditions, $p_\text{atm} = 101.3\,\text{kPa}$. The following preliminary calculations are useful:

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.15)^2}{4} = 1.767 \times 10^{-2}\,\text{m}^2, \qquad V = \frac{Q}{A} = \frac{0.02}{1.767 \times 10^{-2}} = 1.132\,\text{m/s}$$

$$h_\text{v} = \frac{V^2}{2g} = \frac{1.132^2}{2(9.807)} = 0.06531\,\text{m}, \qquad \frac{k_\text{s}}{D} = \frac{0.3}{150} = 0.002$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.132)(0.15)}{1.00 \times 10^{-6}} = 1.698 \times 10^5, \qquad f = f_\text{CE}\left(\text{Re}, \frac{k_\text{s}}{D}\right) = 0.0245$$

where f_CE denotes the friction factor given by the Colebrook equation.

(a) When the available net positive suction head is equal to the required net positive suction head, Equation 2.131 gives

$$\text{NPSH}_\text{R} = \frac{p_0}{\gamma} - \Delta z - h_\ell - \frac{p_\text{v}}{\gamma} \quad \rightarrow \quad \text{NPSH}_\text{R} = \frac{p_\text{atm}}{\gamma} - \Delta z - \left[\frac{f\Delta z}{D} + K\right] h_\text{v} - \frac{p_\text{v}}{\gamma}$$

Substituting the given and derived parameters yields

$$5.5 = \frac{101.3}{9.79} - \Delta z - \left[\frac{(0.0245)\Delta z}{0.15} + 12\right](0.06531) - \frac{2.337}{9.79} \quad \rightarrow \quad \Delta z = \boxed{3.78\,\text{m}}$$

(b) If K=1.2, then

$$5.5 = \frac{101.3}{9.79} - \Delta z - \left[\frac{(0.0245)\Delta z}{0.15} + 1.2\right](0.06531) - \frac{2.337}{9.79} \quad \rightarrow \quad \Delta z = \boxed{4.48\,\text{m}}$$

Therefore, by reducing the local head loss coefficient to 1.2 the pump can be places $4.48 - 3.78 = \boxed{0.70\ \text{m}}$ higher.

2.102. From the given data: $D_1 = 225\,\text{mm}$, $\omega_1 = 1725\,\text{rpm}$, $Q = 50\,\text{L/s}$, $p_\text{s} = 80\,\text{kPa}$, $V_\text{s} = 5\,\text{m/s}$, $\omega_2 = 1140\,\text{rpm}$, and $D_2 = 675\,\text{mm}$. For water at 80°C, $\gamma = 9.530\,\text{kN/m}^3$, and $p_\text{v} = 47.367\,\text{kPa}$ (from Appendix B.1).

(a) Using the definition of NPSH given by Equation 2.130 gives

$$\text{NPSH} = \frac{p_\text{s} - p_\text{v}}{\gamma} + \frac{V_\text{s}^2}{2g} = \frac{80 - 47.367}{9.530} + \frac{5^2}{2(9.807)} = \boxed{4.70\,\text{m}}$$

(b) Using the NPSH scaling relationship given by Equation 2.134 gives

$$\left[\frac{(\text{NPSH})}{\omega^2 D^2}\right]_1 = \left[\frac{(\text{NPSH})}{\omega^2 D^2}\right]_2 \rightarrow \frac{4.70}{(1725)^2(225)^2} = \frac{\text{NPSH}_2}{(1140)^2(675)^2} \rightarrow \text{NPSH}_2 = \boxed{18.5\,\text{m}}$$

2.103. From the given data: $\Delta z = 3\,\text{m} + 19.3\,\text{m} = 22.3\,\text{m}$, $L = 100\,\text{m}$, $D = 50\,\text{mm} = 0.05\,\text{m}$, $A = \pi/4D^2 = 0.00196\,\text{m}^2$, and $Q = 370\,\text{L/min} = 0.00617\,\text{m}^3/\text{s}$. For galvanized iron, $k_\text{s} = 0.15\,\text{mm}$. Taking $Q = 370\,\text{L/min}$ as the operating point gives $V = Q/A = 3.15\,\text{m/s}$, $\text{Re} = VD/\nu = 1.58 \times 10^5$ (where $\nu = 10^{-6}\,\text{m}^2/\text{s}$ at 20°C) and

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_\text{s}}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f}}\right] = -2\log\left[\frac{0.15}{3.7(50)} + \frac{2.51}{1.58 \times 10^5\sqrt{f}}\right]$$

which gives $f = 0.0270$. The energy equation for the system is given by

$$0 - \left[\frac{fL}{D} + 1.8\right]\frac{Q^2}{2gA^2} + h_\text{p} = \Delta z + \frac{Q^2}{2gA^2}$$

$$0 - \left[\frac{(0.0270)(100)}{0.05} + 1.8\right]\frac{Q^2}{2(9.81)(0.00196^2)} + h_\text{p} = 22.3 + \frac{Q^2}{2(9.81)(0.00196^2)}$$

which simplifies to

$$h_\text{p} = 22.3 + 7.54 \times 10^5 Q^2$$

This equation is appropriate for h_p in meters and Q in m^3/s. For h_p in ft and Q in gpm, the system equation becomes

$$\left(\frac{h_\text{p}}{3.281}\right) = 22.3 + 7.54 \times 10^5(Q \times 6.309 \times 10^{-5})^2 \quad \rightarrow \quad h_\text{p} = 73.2 + 0.00985Q^2$$

Plotting the system curve on the pump performance curve indicates that $\boxed{\text{7-inch pump}}$ will be required, with a maximum flow rate of around 112 gpm = 424 L/min. The next smaller pump size of 6.5-inch will yield a maximum flow of around 95 gpm = 360 L/min, which is slightly less than the minimum requirement of 370 L/min.

Under the pump operating conditions, $\text{NPSH}_\text{R} = 15\,\text{ft} = 4.57\,\text{m}$, $p_\text{v} = 2.34\,\text{kPa}$ (at 20°C), $p_0 = 101\,\text{kPa}$, $\gamma = 9.79\,\text{kPa}$, $Q = 424\,\text{L/min} = 0.00707\,\text{m}^3/\text{s}$, $V = Q/A = 3.61\,\text{m/s}$, and the NPSH requirement is that

$$\text{NPSH}_\text{R} = \frac{p_0}{\gamma} - \Delta z_\text{s} - \frac{f\Delta z_\text{s}}{D}\frac{V^2}{2g} - \frac{p_\text{v}}{\gamma}$$

$$4.57 = \frac{101}{9.79} - \Delta z_\mathrm{s} - \frac{0.0270\Delta z_\mathrm{s}}{0.05}\frac{3.61^2}{2(9.81)} - \frac{2.34}{9.79}$$

which gives $\Delta z_\mathrm{s} = 4.05$ m. Since the water is 3 m below ground and the pump must be placed 4.05 m above the water, the pump must be placed a maximum of 4.05 m − 3 m = $\boxed{1.05\,\mathrm{m}}$ above ground.

2.104. The following points are derived from the performance curve of the 6.5-in. (165-mm) pump in Figure 2.64:

Q (gpm)	0	10	20	30	40	50	60	70	80	90	100	110	120	130
h_p (ft)	182	184	184	182	180	178	176	172	168	160	156	148	140	126
η (%)	–	–	25	35	42	47.5	52	55	57	57.5	58	57.5	56	53

The above points correspond to a rotational speed of 3500 rpm.

(a) Using the affinity laws, the following conversions are applied to adjust for the change in rotational speed to 2500 rpm:

$$h_{\mathrm{p}2} = \left(\frac{\omega_2}{\omega_1}\right)^2 h_{\mathrm{p}1}, \qquad Q_2 = \left(\frac{\omega_2}{\omega_1}\right) Q_1$$

Taking $\omega_2/\omega_1 = 2500/3500 = 0.7143$ and applying the unit conversions (1 gpm = 0.06309 L/s, 1 ft = 0.3048 m) gives the following points on the performance curve of the modified pump:

Q (L/s)	0	0.451	0.901	1.35	1.80	2.25	2.70	3.15	3.61	4.06	4.51	4.96	5.41	5.86
h_p (m)	28.3	28.6	28.6	28.3	28.0	27.7	27.4	26.7	26.1	24.9	24.3	23.0	21.8	19.6
η (%)	–	–	25	35	42	47.5	52	55	57	57.5	58	57.5	56	53

This performance curve is plotted in Figure 2.8.

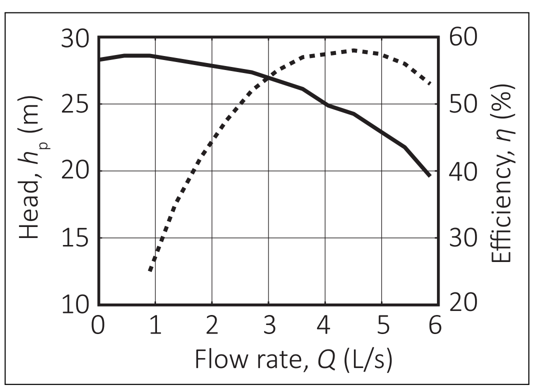

Figure 2.8: Performance curve of modified pump.

(b) It is apparent from the points on the performance curve derived in Part (a) that at the best efficiency point the efficiency is $\boxed{58\%}$, the flow rate is $\boxed{4.51\ \mathrm{L/s}}$, and the head added is $\boxed{24.3\ \mathrm{m}}$.

(c) At the BEP of the existing pump: $\omega = 3500\,\text{rpm} = 366.5\,\text{rad/s}$, $Q = 100\,\text{gpm} = 6.31 \times 10^{-3}\,\text{m}^3/\text{s}$, and $h_\text{p} = 156\,\text{ft} = 47.5\,\text{m}$. Using these values the specific speed, n_s, is given by Equation 2.124 as

$$n_\text{s} = \frac{\omega Q^{\frac{1}{2}}}{[gh_\text{p}]^{\frac{3}{4}}} = \frac{(366.5)(6.31 \times 10^{-3})^{\frac{1}{2}}}{[(9.807)(47.5)]^{\frac{3}{4}}} = \boxed{0.29}$$

At the BEP of the modified pump: $\omega = 2500\,\text{rpm} = 261.8\,\text{rad/s}$, $Q = 4.51 \times 10^{-3}\,\text{m}^3/\text{s}$, and $h_\text{p} = 24.3\,\text{m}$. The specific speed is given by

$$n_\text{s} = \frac{\omega Q^{\frac{1}{2}}}{[gh_\text{p}]^{\frac{3}{4}}} = \frac{(261.8)(4.51 \times 10^{-3})^{\frac{1}{2}}}{[(9.807)(24.3)]^{\frac{3}{4}}} = \boxed{0.29}$$

As expected, both the existing and modified pumps have the same specific speed, since they are from the same homologous series. According to Table 2.3, since $0.15 \le n_\text{s} \le 1.5$, the pump is most likely a $\boxed{\text{centrifugal pump}}$.

2.105. From the given data: $\Delta z = 2\,\text{m}$, $L = 3\,\text{m}$, $D = 150\,\text{mm}$, $A = \pi D^2/4 = 1.767 \times 10^{-2}\,\text{m}^2$, $k_\text{s} = 0.25\,\text{mm}$, $K = 50$, $\text{NPSH}_\text{R} = 4\,\text{m}$, and $p_\text{atm} = 101.3\,\text{kPa}$. For water at 25°C, $\gamma = 9.778\,\text{kN/m}^3$, $\nu = 8.927 \times 10^{-7}\,\text{m}^2/\text{s}$, and $p_\text{v} = 3.167\,\text{kPa}$ (from Appendix B.1). The following preliminary calculations are useful,

$$\frac{k_\text{s}}{D} = \frac{0.25}{150} = 1.667 \times 10^{-3}, \qquad V(Q) = \frac{Q}{A} = \frac{Q}{1.767 \times 10^{-2}}$$

$$\text{Re}(Q) = \frac{V(Q)D}{\nu} = \frac{V(Q)(0.150)}{8.927 \times 10^{-7}}, \qquad f(Q) = f_\text{CE}(\text{Re}(Q), 1.667 \times 10^{-3})$$

The the energy equation and the NPSH requirement are stated as follows,

$$\Delta z - \left[K + \frac{fL}{D}\right]\frac{Q^2}{2gA^2} = \frac{p_\text{s}}{\gamma} + \frac{V_\text{s}^2}{2g} \tag{1}$$

$$\text{NPSH}_\text{A} = \frac{p_\text{s}}{\gamma} + \frac{V_\text{s}^2}{2g} - \frac{p_\text{v}}{\gamma} \tag{2}$$

Combining Equations 1 and 2 with the requirement that $\text{NPSH}_\text{R} = \text{NPSH}_\text{A}$ yields the following equation for the maximum allowable flow rate, Q^*,

$$\text{NPSH}_\text{R} = \Delta z - \left[K + \frac{fL}{D}\right]\frac{Q^{*2}}{2gA^2} - \frac{p_\text{v}}{\gamma}$$

$$\rightarrow \quad 4 = 2 - \left[50 + \frac{f(Q^*)(3)}{0.15}\right]\frac{Q^{*2}}{2(9.807)(1.767 \times 10^{-2})^2} - \frac{3.167 - 101.3}{9.778}$$

$$\rightarrow \quad Q^* = 0.0312\,\text{m}^3/\text{s} = \boxed{31.2\,\text{L/s}}$$

2.106. The system curve is derived from the energy equation:

$$0 - \frac{V^2}{2g} - h_\text{f} + h_\text{p} - \frac{V^2}{2g} = 10 \tag{1}$$

where the $V^2/2g$ terms account for the entrance and exit losses. The head loss, h_f, is given by the Darcy-Weisbach equation as

$$h_\mathrm{f} = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{Q^2}{2gA^2}$$

The energy equation can therefore be written as

$$h_\mathrm{p} = 10 + \left[2 + f\frac{L}{D}\right]\frac{Q^2}{2gA^2}$$

For ductile iron, $k_\mathrm{s} = 0.26\,\mathrm{mm}$, therefore $k_\mathrm{s}/D = 0.26/100 = 0.0026$. Assuming the pipe is hydraulically rough,

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_\mathrm{s}/D}{3.7}\right] = -2\log\left[\frac{0.0026}{3.7}\right] = 6.306 \quad\rightarrow\quad f = 0.0251$$

For the pipeline, $L = 104\,\mathrm{m}$, $D = 0.100\,\mathrm{mm}$, $A = \pi D^2/4 = 0.00785\,\mathrm{m}^2$, and therefore the system curve given by Equation (1) becomes

$$h_\mathrm{p} = 10+\left[2 + f\frac{L}{D}\right]\frac{Q^2}{2gA^2} = 10+\left[2 + (0.0251)\frac{104}{0.1}\right]\frac{Q^2}{2(9.81)(0.00785)^2} \quad\rightarrow\quad h_\mathrm{p} = 10+23250Q^2$$

This equation is for Q in m^3/s. For Q in L/s, the system equation becomes

$$h_\mathrm{p} = 10 + 0.0233Q^2$$

Solving the system curve and the pump curve yields

$$10 + 0.0233Q^2 = 15 - 0.1Q^2 \quad\rightarrow\quad \boxed{Q = 6.37\,\mathrm{L/s}}$$

which leads to $h_\mathrm{p} = 10.9\,\mathrm{m}$. Under this operating condition, the (given) required net positive suction head, $\mathrm{NPSH_R}$, is 1.5 m. Putting $\mathrm{NPSH_R} = \mathrm{NPSH_A}$ requires that

$$1.5 = \frac{p_0}{\gamma} - \Delta z_\mathrm{s} - h_\ell - \frac{p_\mathrm{v}}{\gamma} \qquad (2)$$

In this case, $p_0 = 101\,\mathrm{kPa}$, $\gamma = 9.79\,\mathrm{kN/m}^3$, $p_\mathrm{v} = 2.34\,\mathrm{kPa}$ (at 20°C), and

$$h_\ell = \left[1 + f\frac{L}{D}\right]\frac{Q^2}{2gA^2}$$

where $L = \Delta z_\mathrm{s} + 1$, $f = 0.0251$, $D = 0.1\,\mathrm{m}$, $Q = 0.00637\,\mathrm{m}^3/\mathrm{s}$, $A = 0.00785\,\mathrm{m}^2$, which gives

$$h_\ell = \left[1 + 0.0251\frac{(\Delta z_\mathrm{s} + 1)}{0.1}\right]\frac{0.00637^2}{2(9.81)(0.00785)^2} \quad\rightarrow\quad h_\ell = 0.0420 + 0.00843\Delta z_\mathrm{s} \qquad (3)$$

Combining Equations (2) to (3) gives

$$1.5 = \frac{101}{9.79} - \Delta z_\mathrm{s} - 0.0420 - 0.00843\Delta z_\mathrm{s} - \frac{2.34}{9.79} \quad\rightarrow\quad \boxed{\Delta z_\mathrm{s} = 8.40\,\mathrm{m}}$$

2.107. From the given data: $L_1 = 8\,\text{m}$, $L_2 = 50\,\text{m}$, $D = 100\,\text{mm}$, $A = \pi D^2/4 = 7.854 \times 10^{-3}\,\text{m}^2$, $k_\text{s} = 0.5\,\text{mm}$, $\text{NPSH}_\text{A} = 2.8\,\text{m}$, $Q_\text{min} = 6\,\text{L/s} = 0.006\,\text{m}^3/\text{s}$, $z_\text{p} = 0.8\,\text{m}$, and $z_\text{e} = 2\,\text{m}$. Assuming that the flow is fully turbulent, the friction factor, f, can be calculated as follows:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_\text{s}/D}{3.7}\right) = -2\log\left(\frac{0.5/100}{3.7}\right) \quad \rightarrow \quad f = 0.03037$$

The system (energy) equation for the irrigation system is determined as follows:

$$0 - f\frac{L_1 + L_2}{D}\frac{Q^2}{2gA^2} + h_\text{p} = 2 + z + \frac{Q^2}{2gA^2}$$

$$0 - 0.03037\frac{8+50}{0.1}\frac{Q^2}{2(9.807)(7.854\times10^{-3})^2} + h_\text{p} = 2 + z + \frac{Q^2}{2(9.807)(7.854\times10^{-3})^2}$$

$$\rightarrow \quad h_\text{p} = 2 + z + 15386Q^2 \tag{1}$$

where h_p is in m, and Q is in m^3/s. Putting the pump performance curve in a form having the same units gives:

$$h_\text{p} = 40 - 0.9(10^3 \times Q)^2 \quad \rightarrow \quad h_\text{p} = 40 - 9\times10^5 Q^2 \tag{2}$$

(a) Solving Equations (1) and (2) for $Q = 0.006\,\text{m}^3/\text{s}$ gives

$$2 + z + 15386Q^2 = 40 - 9\times10^5Q^2 \quad \rightarrow \quad 2 + z + 15386(0.006)^2 = 40 - 9\times10^5(0.006)^2$$

$$\rightarrow \quad z = 5.05\,\text{m}$$

Therefore, the system delivers $Q \geq 6\,\text{L/s}$ when $\boxed{z \leq 5.05\,\text{m}}$.

(b) For water at 20°C, take $p_\text{svp} = 2.34\,\text{kPa}$ and $\gamma = 9.789\,\text{kN·m}^3$. Under limiting cavitation conditions, when $z = z^*$,

$$\text{NPSH}_\text{A} = \frac{p_\text{atm} - p_\text{svp}}{\gamma} - z_\text{s} - f\frac{L_1}{D}\frac{Q^2}{2gA^2}$$

$$2.8 = \frac{101.3 - 2.34}{9.789} - (z^* + 0.8) - 0.03037\frac{8}{0.1}\frac{Q^2}{2(9.807)(7.854\times10^{-3})^2}$$

$$\rightarrow \quad z^* = 9.31 - 2008Q^2 \tag{3}$$

Combining Equations (1) and (2) (eliminating h_p) gives the following condition at the operating point:

$$z = 38 - 9.154\times10^5Q^2 \tag{4}$$

Solving Equations (3) and (4) for $z = z^*$ gives $Q = 0.0056\,\text{m}^3/\text{s}$ and $z = z^* = 9.25\,\text{m}$. For $Q \geq 6\,\text{L/s}$, $z < z^*$ and $\boxed{\text{cavitation will not be a problem}}$.

2.108. From the given data: $L = 30\,\text{m}$, $D = 0.15\,\text{m}$, $A = \pi D^2/4 = \pi(.15)^2/4 = 0.01767\,\text{m}^2$, and specific speed = 3000.

(a) Energy equation:

$$z_\mathrm{A} + h_\mathrm{p} = z_\mathrm{F} + \left(f\frac{L}{D} + K_\mathrm{A} + K_\mathrm{B} + K_\mathrm{C} + K_\mathrm{D} + K_\mathrm{E} + K_\mathrm{F}\right)\frac{Q^2}{2gA^2}$$

For PVC pipe, $k_\mathrm{s} \approx 0$, and the friction factor is a function of the (unknown) Reynolds number according to the relation

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{1}$$

Substituting known values into the energy equation gives

$$0 + h_\mathrm{p} = 10 + \left(f\frac{30}{0.15} + 1 + 0.9 + 0.2 + 0.9 + 0.9 + 1\right)\frac{Q^2}{2(9.81)(0.01767)^2}$$

$$\rightarrow \quad \boxed{h_\mathrm{p} = 10 + 163.2Q^2(4.9 + 200f)}$$

(b) The pump performance curve is given by

$$h_\mathrm{p} = 20 - 4713Q^2$$

Simultaneous solution of the energy equation and the pump performance curve gives

$$10 + 163.2Q^2(4.9 + 200f) = 20 - 4713Q^2 \quad \rightarrow \quad Q = \sqrt{\frac{10}{5513 + 32640f}} \tag{2}$$

This equation must be solved simultaneously with the friction factor equation, Equation (1), where Re is given by

$$\mathrm{Re} = \frac{\rho QD}{A\mu} = \frac{(998.2)(Q)(0.15)}{(0.01767)(1.002 \times 10^{-3})} = 8.46 \times 10^6 Q$$

which combined with Equation (2) gives

$$\mathrm{Re} = 2.68 \times 10^7(5513 + 32640f)^{-\frac{1}{2}} \tag{3}$$

Substituting Equation (3) into Equation (1) gives the following implicit equation for f,

$$\frac{1}{\sqrt{f}} = -2\log\left[9.37 \times 10^{-8}\sqrt{\frac{5513 + 32640f}{f}}\right] \quad \rightarrow \quad f = 0.014$$

from which Equation (2) gives

$$\boxed{Q = 0.0409\,\mathrm{m^3/s}} \quad \text{and} \quad V = \frac{Q}{A} = \frac{0.0409}{0.01767} = \boxed{2.31\,\mathrm{m/s}}$$

(c) The available net positive suction head, NPSH_A, is given by

$$\mathrm{NPSH_A} = \frac{p_0}{\gamma} - \Delta z_\mathrm{s} - h_\ell - \frac{p_\mathrm{v}}{\gamma} = \frac{101}{9.79} - 3 - 0 - \frac{2.34}{9.79} = 7.08\,\mathrm{m}$$

Since the required net positive suction head, $\mathrm{NPSH_R}$ at the pump operating point is given (by the pump manufacturer) as 3.0 m, and $\mathrm{NPSH_A}(= 7.08\,\mathrm{m}) > \mathrm{NPSH_R}(= 3.0\,\mathrm{m})$, cavitation is not expected to occur in the pump.

(d) According to the affinity laws,

$$Q_1 = \frac{\omega_1}{\omega_2} Q_2 = \frac{800}{1600} Q_2 = 0.5 Q_2$$

and

$$h_{\text{p1}} = \frac{\omega_1^2}{\omega_2^2} h_{\text{p2}} = \frac{800^2}{1600^2} h_{\text{p2}} = 0.25 h_{\text{p2}}$$

Using the affinity relations in the pump performance curve gives

$$0.25 h_{\text{p2}} = 20 - 4713(0.5Q_2)^2 \quad \rightarrow \quad \boxed{h_{\text{p2}} = 80 - 4713 Q_2^2}$$

2.109. (a) From the given data: D = 6 cm = 0.06 m, L = 107 m, k_s = 0.01 mm, z_1 = 10.00 m, z_2 = 15.00 m, and $\Delta z = z_2 - z_1$ = 15.00 m − 10.00 m = 5.00 m. The system curve is given by

$$h_\text{p} = \Delta z + \frac{fL}{D} \frac{Q^2}{2gA^2} \tag{1}$$

where

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.06)^2 = 0.002827\,\text{m}^2 \tag{2}$$

and the Swamee-Jain formula gives

$$f = \frac{0.25}{\left[\log\left(\frac{k_\text{s}}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \tag{3}$$

where

$$\text{Re} = \frac{VD}{\nu} = \frac{QD}{A\nu} \tag{4}$$

Taking $\nu = 1.00 \times 10^{-6}$ m^2/s (at 20°C) and combining Equations 2 to 4 yields

$$f = \frac{0.25}{\left[\log\left(\frac{0.01}{3.7(60)} + \frac{5.74}{\left(\frac{Q \times 0.06}{0.002827 \times 1.00 \times 10^{-6}}\right)^{0.9}}\right)\right]^2}$$

which simplifies to

$$f = \frac{0.25}{[\log(4.505 \times 10^{-5} + 1.461 \times 10^{-6} Q^{-0.9})]^2} \tag{5}$$

Combining Equations 1 and 5, and substituting other known data yields

$$h_\text{p} = 5 + \frac{0.25}{[\log(4.505 \times 10^{-5} + 1.461 \times 10^{-6} Q^{-0.9})]^2} \frac{107}{0.06} \frac{Q^2}{2(9.81)(0.002827)^2}$$

which simplifies to

$$h_\text{p} = 5 + \frac{2.84 \times 10^6 Q^2}{[\log(4.505 \times 10^{-5} + 1.461 \times 10^{-6} Q^{-0.9})]^2} \tag{6}$$

These system curve is plotted on the pump curve in Figure 2.9 using the following tabulated values derived from Equation 6:

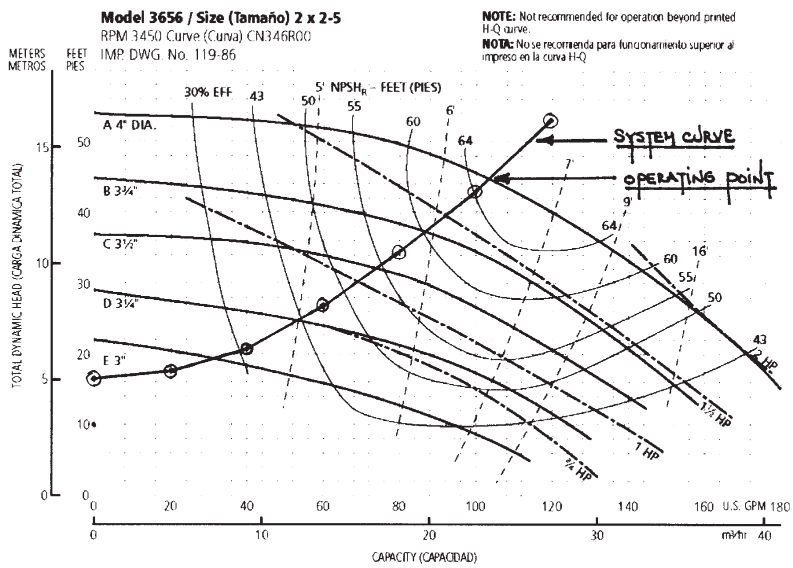

Figure 2.9: System Curve on Pump Curve.

Q (gpm)	Q (m^3/s)	h_p (m)	h_p (ft)
0	0	5.00	16.4
20	0.001262	5.44	17.9
40	0.002524	6.53	21.4
60	0.003785	8.18	26.9
80	0.005047	10.38	34.1
100	0.006309	13.11	43.0
120	0.007571	16.35	53.7

The intersection of the system curve with the pump curve indicates that $\boxed{\text{size A}}$ pump should be used. This yields an operating point of Q = 390 L/min (= 103 gpm) which is slightly greater than the desired flow rate of 380 L/min. The maximum flow rate can be throttled down with a valve. The head added by the pump, h_p, at the operating point is 13.6 m (= 44.5 ft).

(b) The efficiency, η, of the pump at the operating point is approximately 64%, and hence the power, P, of the required motor is given by

$$P = \frac{\gamma Q h_p}{\eta} = \frac{(9.79)(390 \times 10^{-3}/60)(13.6)}{0.64} = \boxed{1.35\,\text{kW}} \quad (= 1.8\text{ HP})$$

(c) The available net positive suction head, NPSH$_A$, can be estimated by (neglecting pipe-entrance losses)

$$\text{NPSH}_A = \frac{p_0}{\gamma} - \Delta z_s - h_L - \frac{p_v}{\gamma} = \frac{p_0}{\gamma} - \left(1 + \frac{f}{D}\frac{V^2}{2g}\right)\Delta z_s - \frac{p_v}{\gamma} \tag{7}$$

From the pump curve at the operating point, $\text{NPSH}_A = 6.5$ ft $= 1.98$ m, $p_0 = 101$ kPa, $\gamma = 9.79$ kN/m^3, $p_v = 2.34$ kPa (at 20°C), $Q = 390$ L/min $= 0.00650$ m^3/s, $D = 0.06$ m, $A = 0.002827$ m^2, $V = Q/A = 2.30$ m/s, and Equation 5 gives $f = 0.0178$. Substituting these data into Equation 7 gives

$$1.98 = \frac{101}{9.79} - \left[1 + \frac{0.0178}{0.06}\frac{2.30^2}{2(9.81)}\right]\Delta z_s - \frac{2.34}{9.79}$$

which yields $\Delta z_s = 7.50$ m. Therefore, the pump can be placed up to $\boxed{7.50\text{ m}}$ above the pond.

2.110. If H_p is the head added by the pump system, and Q is the flow through the system, then for n pumps in series

$$\frac{H_\text{p}}{n} = 30 - 0.05Q^2$$

or

$$\boxed{H_\text{p} = 30n - 0.05nQ^2}$$

For n pumps in parallel,

$$H_\text{p} = 30 - 0.05\left(\frac{Q}{n}\right)^2$$

or

$$\boxed{H_\text{p} = 30 - \frac{0.05}{n^2}Q^2}$$

2.111. The flow rate is given by the simultaneous solution of the system curve and the pump curve, hence

$$15 + 0.03Q^2 = 20 - 0.08Q^2$$

which gives

$$\boxed{Q = 6.74\,\text{L/s}}$$

The pump curve for two identical pumps in series is given by

$$\frac{H_\text{p}}{2} = 20 - 0.08Q^2$$

or

$$H_\text{p} = 40 - 0.16Q^2 \tag{1}$$

Solving Equation 1 with the system curve gives

$$15 + 0.03Q^2 = 40 - 0.16Q^2$$

which leads to

$$\boxed{Q = 11.5\,\text{L/s}}$$

The pump curve for two identical pumps in parallel is given by

$$H_\text{p} = 20 - 0.08\left(\frac{Q}{2}\right)^2$$

or

$$H_p = 20 - 0.02Q^2 \qquad (2)$$

Solving Equation 2 with the system curve gives

$$15 + 0.03Q^2 = 40 - 0.02Q^2$$

which leads to

$$\boxed{Q = 10\,\mathrm{L/s}}$$

2.112. From the given data: $Q = 1000\,\mathrm{L/s}$, $h_p = 9\,\mathrm{m}$, $P_0 = 35\,\mathrm{kW}$, $\eta = 0.62$, and $n_s = 1.5$. Assume $\gamma = 9.789\,\mathrm{kN/m^3}$.

(a) From the definition of overall pump efficiency, the flow rate through each pump can be estimated using the relation

$$h_p = \frac{\eta P_0}{\gamma Q_p} \quad \rightarrow \quad 9 = \frac{(0.62)(35)}{(9.789)Q_p} \quad \rightarrow \quad Q_p = 0.246\,\mathrm{m^3/s} = 246\,\mathrm{L/s}$$

Therefore, the number of pump units required is given by

$$\text{number of pumps} = \frac{Q}{Q_p} = \frac{1000}{246} = 4.06 \approx \boxed{5 \text{ units}}$$

(b) Since the specific speed, n_s, of the pump is 1.5, the motor speed can be estimated as follows:

$$n_s = \frac{\omega Q^{\frac{1}{2}}}{[gh_p]^{\frac{3}{4}}} \quad \rightarrow \quad 1.5 = \frac{\omega(0.246)^{\frac{1}{2}}}{[(9.807)(9)]^{\frac{3}{4}}} \quad \rightarrow \quad \omega = 87.09\,\mathrm{rad/s} = \boxed{832\,\mathrm{rpm}}$$

This could be rounded to the nearest rated speed of available motors.

2.113. From the given data: $N = 9$, $P_0 = 40\,\mathrm{kW}$, $h_p = 35\,\mathrm{m}$, and $\eta = 0.60$. For water at 20°C, $\gamma = 9.789\,\mathrm{kN/m^3}$. Using these data gives the flow from each pump unit,

$$\eta = \frac{\gamma Q h_p}{P} \quad \rightarrow \quad 0.60 = \frac{(9.789)Q(35)}{40} \quad \rightarrow \quad Q = 7.005 \times 10^{-2}\,\mathrm{m^3/s}$$

For all 9 units,

$$Q_{\text{total}} = NQ = (9)(7.005 \times 10^{-2}) = \boxed{0.630\,\mathrm{m^3/s}}$$

2.114. From the given data: $D = 250$ mm, $k_s = 0.1$ mm, $L = 5000$ m, and $\Delta z = 70$ m. Find the system curve with Q in L/s:

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D}\right)\right]} = \frac{0.25}{\left[\log_{10}\left(\frac{0.1}{3.7(250)}\right)\right]} = 0.01589$$

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.250)^2 = 0.04909\,\mathrm{m^2}$$

$$h_p = \Delta z + f\frac{L}{D}\frac{Q^2}{2gA^2} = 70 + (0.01589)\frac{5000}{0.250}\frac{(Q \times 10^{-3})^2}{2(9.81)(0.04909)^2} = 70 + 0.006722Q^2$$

The points on the system curve are as follows:

Q (L/s)	10	20	30	40	50	60	70
h_p (m)	70.7	72.7	76.0	80.8	86.8	94.2	102.9

(a) Plotting the system curve on the given pump performance curves shows that the $\boxed{\text{225 mm pump}}$ will have an operating point with a flow that is closest to (but greater than) the desired pumping rate. The desired flow rate can then be obtained by installing a (partially closed) valve in the pipeline system.

(b) Reading directly from the performance curve of the 225-mm pump, the efficiency at an operating flow of 35 L/s with the 225 mm pump is $\boxed{73\%}$.

(c) For a flow of 35 L/s through the 225-mm pump, the given performance characteristics indicate that (approximately) $\text{NPSH}_\text{r} = 5.0$ m. Neglecting friction and entrance losses in the suction pipe and taking $p_0 = 101$ kPa, $p_\text{v} = 2.34$ kPa, and $\gamma = 9.79$ kN/m^3 gives

$$\text{NPSH}_\text{r} = \frac{p_0}{\gamma} - \Delta z_\text{s} - \frac{p_\text{v}}{\gamma}$$

$$\Delta z_\text{s} = \left(\frac{p_0 - p_\text{v}}{\gamma}\right) - \text{NPSH}_\text{r} = \left(\frac{101 - 2.34}{9.79}\right) - 5.0 = 5.1\,\text{m}$$

Therefore, the pump should be no more than $\boxed{\text{5.1 m}}$ above the water surface in the river.

(d) For 2-225 mm pumps (approximately):

Q (L/s)	60	88
h_p (m)	100	80

Intersection with system curve = 65 L/s

For 3-225 mm pumps (approximately):

Q (L/s)	60	90	132
h_p (m)	105	100	80

Intersection with system curve = 77 L/s

Based on the above results $\boxed{\text{3 pumps}}$ will be needed to provide a flow rate of (at least) 70 L/s.

2.115. From the given data: $D = 600$ mm, $k_\text{s} = 0.26$ mm (for DIP), and $p_\text{B} = 350$ kPa. It will be assumed that $\nu = 10^{-6}$ m^2/s (@$T = 20$°C). The flow area and relative roughness are given by

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.600)^2 = 0.2827\,\text{m}^2$$

$$\frac{k_\text{s}}{D} = \frac{0.26}{600} = 4.33 \times 10^{-4}$$

For a flow of 200 L/s, the following variables can be calculated:

$$V = \frac{Q}{A} = \frac{0.200}{0.2827} = 0.7074\,\text{m/s}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(0.7074)(0.600)}{10^{-6}} = 4.24 \times 10^5$$

$$f = \frac{0.25}{\left[\log\left(\frac{k_\text{s}}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{4.33 \times 10^{-4}}{3.7} + \frac{5.74}{(4.24 \times 10^5)^{0.9}}\right)\right]^2} = 0.0175$$

$$h_\text{f} = f\frac{L}{D}\frac{V^2}{2g} = (0.0175)\frac{2000}{0.600}\frac{0.7074^2}{2(9.81)} = 1.47\,\text{m}$$

Applying the energy equation between A and B gives

$$z_\text{A} + h_\text{p} - h_\text{f} = \frac{p_\text{B}}{\gamma} + \frac{V_\text{B}^2}{2g} + z_\text{B}$$

$$20 + h_\text{p} - 1.47 = \frac{350}{9.79} + \frac{0.7074^2}{2(9.81)} + 3$$

which yields h_p = 20.2 m. For a flow of 400 L/s, similar calculations yield V = 1.415 m/s, Re = 8.49×10^5, f = 0.0169, h_f = 5.70 m, and the energy equation gives

$$20 + h_\text{p} - 5.70 = \frac{350}{9.79} + 3$$

which yields h_p = 24.4 m. Hence, when one pump is operating a delivering 200 L/s the desirable head added is $\boxed{20.2\text{ m}}$, and when two pumps are operating and delivering 400 L/s (each pump delivering 200 L/s) the desirable head added is $\boxed{24.4\text{ m}}$.

Selection of the pump model must be based on each pump having an operating point of (200 L/s, 22.4 m), with some throttling of the flow necessary when 200 L/s is being delivered. Using the given pump curve,

$$h_\text{p} = (0.293D - 24.3) - 5.72 \times 10^{-4}Q^2$$
$$22.4 = (0.293D - 24.3) - 5.72 \times 10^{-4}(200)^2$$

which yields D = 237 mm. Therefore, the pump model with an impeller diameter of $\boxed{237\text{ mm}}$ should be selected.

2.116. From the given data: L = 20 km = 20000 m, D = 1120 mm = 1.120 m, k_s = 0.05 mm, z_A = z_B = 5 m, h_A = 2 m, z_C = 15 m, Q = 48000 m^3/d = 0.556 m^3/s, and p_C = 448 kPa. At T = 20°C, $\nu = 1 \times 10^{-6}$ m^2/s and γ = 9.79 kN/m^3.

(a) Using the given data,

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(1.120)^2 = 0.9852\,\text{m}^2$$
$$V = \frac{Q}{A} = \frac{0.556}{0.9852} = 0.564\,\text{m/s}$$
$$\text{Re} = \frac{VD}{\nu} = \frac{(0.564)(1.120)}{1.0 \times 10^{-6}} = 6.317 \times 10^5$$

$$\frac{k_s}{D} = \frac{0.05}{1120} = 4.464 \times 10^{-5}$$

$$f = \frac{0.25}{\left[\log\left(\frac{k_s/D}{3.7} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{4.464\times10^{-5}}{3.7} + \frac{5.74}{(6.317\times10^5)^{0.9}}\right)\right]^2} = 0.01333$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = (0.01333)\frac{20000}{1.120}\frac{0.564^2}{2(9.81)} = 3.858\,\text{m}$$

The system curve derived from the energy equation as follows:

$$\frac{p_A}{\gamma} + z_A + h_p - h_f = \frac{p_C}{\gamma} + z_C$$

$$2 + 5 + h_p - 3.858 = \frac{448}{9.79} + 15$$

which yields $h_p = 57.62$ m. For N pumps in parallel the pump-system performance is given by

$$h_p = 65 - 7.6 \times 10^{-8}\left(\frac{Q}{N}\right)^2$$

which gives

$$N = \sqrt{\frac{7.6\times10^{-8}}{65 - h_p}}\,Q = \sqrt{\frac{7.6\times10^{-8}}{65 - 57.62}}\,(48000) = 4.87 \approx 5$$

Hence $\boxed{\text{5 pumps}}$ are needed to deliver the required flow at the required pressure.

(b) To determine the actual operating point, the system curve must be expressed in terms of the flow Q. Assuming fully turbulent flow,

$$f = \frac{0.25}{\left[\log\left(\frac{k_s/D}{3.7}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{4.464\times10^{-5}}{3.7}\right)\right]^2} = 0.01033$$

$$h_f = f\frac{L}{D}\frac{Q^2}{2gA^2} = (0.01033)\frac{20000}{1.12}\frac{Q^2}{2(9.81)(0.9852)^2} = 9.686Q^2$$

The system curve is derived from the energy equation as follows:

$$\frac{p_A}{\gamma} + z_A + h_p - h_f = \frac{p_C}{\gamma} + z_C$$

$$2 + 5 + h_p - 9.686Q^2 = \frac{448}{9.79} + 15$$

which simplifies to

$$h_p = 53.76 + 9.686Q^2$$

This equation is for Q in m^3/s. For Q in m^3/d,

$$h_p = 53.76 + 9.686\left(\frac{Q}{86400}\right)^2 = 53.76 + 1.298 \times 10^{-9}Q^2 \qquad (1)$$

For 5 pumps, the performance curve of the pump system is given by

$$h_\mathrm{p} = 65 - 7.6 \times 10^{-8} \left(\frac{Q}{5}\right)^2 = 65 - 3.04 \times 10^{-9} Q^2 \tag{2}$$

Combining Equations 1 and 2 gives

$$65 - 3.04 \times 10^{-9} Q^2 = 53.76 + 1.298 \times 10^{-9} Q^2$$

which yields $\boxed{50{,}900\ \mathrm{m^3/d}}$.

2.117. (a) The energy equation between the supply reservoir and any delivery location, X, is given by

$$3 - \frac{f_{AB}L_{AB}}{D_{AB}}\frac{V_{AB}^2}{2g} + h_\mathrm{p} - \frac{f_{BX}L_{BX}}{D_{BX}}\frac{V_{BX}^2}{2g} = \frac{p_X}{\gamma} + \frac{V_X^2}{2g} + z_X$$

which can be conveniently written as

$$h_\mathrm{p} = \left[\frac{f_{AB}L_{AB}}{D_{AB}}\frac{V_{AB}^2}{2g} - 3\right] + \left[\frac{f_{BX}L_{BX}}{D_{BX}}\frac{V_{BX}^2}{2g} + \frac{p_X}{\gamma} + \frac{V_X^2}{2g} + z_X\right] \tag{1}$$

where the first term in brackets on the right-hand side is the same for both pipe destinations.

`For pipe AB:`

$$\begin{aligned}
Q_{AB} &= 27\,\mathrm{L/s} = 0.027\,\mathrm{m^3/s} \\
L_{AB} &= 1050\,\mathrm{m} \\
D_{AB} &= 0.200\,\mathrm{m} \\
A_{AB} &= \frac{\pi}{4}D_{AB}^2 = \frac{\pi}{4}(0.200)^2 = 0.03142\,\mathrm{m^2} \\
V_{AB} &= \frac{Q_{AB}}{A_{AB}} = \frac{0.027}{0.03142} = 0.859\,\mathrm{m/s} \\
\mathrm{Re}_{AB} &= \frac{V_{AB}D_{AB}}{\nu} = \frac{(0.859)(0.200)}{10^{-6}} = 171800 \\
f_{AB} &= \frac{0.25}{\left[\log\left(\frac{k_s}{3.7D_{AB}} + \frac{5.74}{\mathrm{Re}_{AB}^{0.9}}\right)\right]^2}\frac{0.25}{\left[\log\left(\frac{0.26}{3.7(200)} + \frac{5.74}{(171800)^{0.9}}\right)\right]^2} = 0.02248
\end{aligned}$$

which gives

$$\left[\frac{f_{AB}L_{AB}}{D_{AB}}\frac{V_{AB}^2}{2g} - 3\right] = \left[\frac{(0.02248)(1050)}{(0.200)}\frac{0.859^2}{2(9.81)} - 3\right] = 1.439\,\mathrm{m} \tag{2}$$

`For pipe BC:`

$$\begin{aligned}
Q_{BC} &= 12\,\mathrm{L/s} = 0.012\,\mathrm{m^3/s} \\
L_{BC} &= 2800\,\mathrm{m}
\end{aligned}$$

$$
\begin{aligned}
D_{BC} &= 0.150\,\mathrm{m} \\
A_{BC} &= \frac{\pi}{4}D_{BC}^2 = \frac{\pi}{4}(0.150)^2 = 0.01767\,\mathrm{m}^2 \\
V_{BC} &= \frac{Q_{BC}}{A_{BC}} = \frac{0.012}{0.01767} = 0.679\,\mathrm{m/s} \\
\mathrm{Re}_{BC} &= \frac{V_{BC}D_{BC}}{\nu} = \frac{(0.679)(0.150)}{10^{-6}} = 101850 \\
f_{BC} &= \frac{0.25}{\left[\log\left(\frac{0.26}{3.7(150)} + \frac{5.74}{(101850)^{0.9}}\right)\right]^2} = 0.0246 \\
z_C &= 2\,\mathrm{m}
\end{aligned}
$$

which gives

$$
\begin{aligned}
\left[\frac{f_{BC}L_{BC}}{D_{BC}}\frac{V_{BC}^2}{2g} + \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C\right] &= \left[\frac{(0.0246)(2800)}{(0.150)}\frac{0.679^2}{2(9.81)} + \frac{350}{9.79} + \frac{0.679^2}{2(9.81)} + 2\right] \\
&= 48.567\,\mathrm{m} \qquad (3)
\end{aligned}
$$

`For pipe BD:`

$$
\begin{aligned}
Q_{BD} &= 15\,\mathrm{L/s} = 0.015\,\mathrm{m^3/s} \\
L_{BD} &= 2500\,\mathrm{m} \\
D_{BD} &= 0.150\,\mathrm{m} \\
A_{BD} &= \frac{\pi}{4}D_{BD}^2 = \frac{\pi}{4}(0.150)^2 = 0.01767\,\mathrm{m}^2 \\
V_{BD} &= \frac{Q_{BD}}{A_{BD}} = \frac{0.015}{0.01767} = 0.849\,\mathrm{m/s} \\
\mathrm{Re}_{BD} &= \frac{V_{BD}D_{BD}}{\nu} = \frac{(0.849)(0.150)}{10^{-6}} = 127350 \\
f_{BD} &= \frac{0.25}{\left[\log\left(\frac{0.26}{3.7(150)} + \frac{5.74}{(127350)^{0.9}}\right)\right]^2} = 0.0242 \\
z_D &= 5\,\mathrm{m}
\end{aligned}
$$

which gives

$$
\begin{aligned}
\left[\frac{f_{BD}L_{BD}}{D_{BD}}\frac{V_{BD}^2}{2g} + \frac{p_D}{\gamma} + \frac{V_D^2}{2g} + z_D\right] &= \left[\frac{(0.0242)(2500)}{(0.150)}\frac{0.849^2}{2(9.81)} + \frac{350}{9.79} + \frac{0.849^2}{2(9.81)} + 5\right] \\
&= 55.605\,\mathrm{m} \qquad (4)
\end{aligned}
$$

Assessing the results in Equations 1 to 4, it is apparent that the required conditions at location D will yield the maximum value of h_p such that

$$h_\mathrm{p} = [1.439] + [55.605] = 57.04\,\mathrm{m}$$

and the required pump power, P, delivered to the water is given by

$$P = \gamma Q_{AB}h_\mathrm{p} = (9.79)(0.027)(57.04) = \boxed{15.1\,\mathrm{kW}}$$

(b) If n pumps are used in series and the pump size, D, can be adjusted to meet the desired operating conditions, then

$$\frac{h_\mathrm{p}}{n} = 0.455D + 4000Q_{AB}^2$$
$$\frac{57.04}{n} = 0.455D + 4000(0.027)^2$$

which can be put in the form

$$D = \frac{125.4}{n} - 6.4$$

Assuming different values for n yields the following results

n	D (cm)
1	119
2	69
3	48
4	38

Since the pump manufacturer requires that 40 cm $< D <$ 50 cm, use $\boxed{\text{3 pumps}}$ with $\boxed{D = 48\text{ cm}}$.

2.118. At $\omega = 600$ rpm, the operating point is determined by simultaneous solution of the pump and system curves,

$$6 - 0.05Q^2 = 3 + 0.042Q^2$$

which yields $Q = \boxed{5.7\text{ m}^3/\text{min}}$.

From the given data: $\omega_1 = 600$ rpm, $\omega_2 = 1200$ rpm, and the affinity laws (Equation 2.144) give that

$$Q_1 = \frac{\omega_1}{\omega_2} Q_2 = \frac{600}{1200} Q_2 = 0.5Q_2$$

$$h_1 = \frac{\omega_1^2}{\omega_2^2} h_2 = \frac{600^2}{1200^2} h_2 = 0.25h_2$$

Since the performance curve of the pump at speed ω_1 is given by

$$h_1 = 6 - 0.05Q_1^2$$

then the performance curve at speed ω_2 is given by

$$0.25h_2 = 6 - 0.05(0.5Q_2)^2$$

which leads to

$$h_2 = 24 - 0.05Q_2^2$$

The new operating point is determined by simultaneous solution of the pump and system curves,

$$24 - 0.05Q^2 = 3 + 0.042Q^2$$

which yields $Q = \boxed{15.1\text{ m}^3/\text{min}}$.