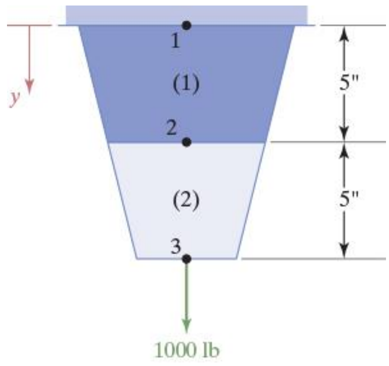
Finite Element Analysis 5th edition Moaveni Solutions Manual

SKU: 9780135212103-SOLUTIONS

Chapter 1

1.1

(a) Using two elements



$$A(y) = 0.25 - 0.0125y$$

$$A_1 = 0.25 \text{ in}^2 \qquad A_2 = 0.1875 \text{ in}^2$$

$$A_3 = 0.125 \text{ in}^2$$

$$K_{eq} = \frac{(A_{i+1} + A_i)E}{2\ell}$$

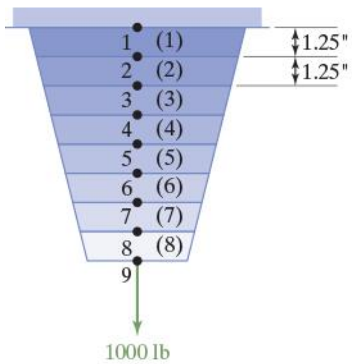
$$K_1 = \frac{(0.25 + 0.1875)(10.4 \times 10^6)}{2(5)} = 455{,}000 \frac{\text{lb}}{\text{in}}$$

$$K_2 = \frac{(0.1875 + 0.125)\,(10.4 \times 10^6)}{2(5)} = 325{,}000 \frac{\text{lb}}{\text{in}}$$

$$10^3 \begin{bmatrix} \overset{1}{\cancel{455}} & \overset{0}{-\cancel{455}} & \\ \underset{0}{-\cancel{455}} & 455 + 325 & -325 \\ & -325 & 325 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1000 \end{Bmatrix}$$

$$u_1 = 0, \quad \underline{\underline{u_2 = 0.002197 \text{ in}}} \quad \underline{\underline{u_3 = 0.005274 \text{ in}}} \quad \Leftarrow$$

(b)



$$A_1 = 0.25 \text{ in}^2$$

$$A_2 = 0.234375 \text{ in}^2$$

$$A_3 = 0.21875 \text{ in}^2$$

$$A_4 = 0.203125 \text{ in}^2$$

$$A_5 = 0.1875 \text{ in}^2$$

$$A_6 = 0.171875 \text{ in}^2$$

$$A_7 = 0.15625 \text{ in}^2$$

$$A_8 = 0.140625 \text{ in}^2$$

$$A_9 = 0.125 \text{ in}^2$$

1.1 Cont.

$$K_1 = 2015000 \frac{\text{lb}}{\text{in}} \qquad K_2 = 1885000 \frac{\text{lb}}{\text{in}} \qquad K_3 = 1755000 \frac{\text{lb}}{\text{in}}$$

$$K_4 = 1625000 \frac{\text{lb}}{\text{in}} \qquad K_5 = 1495000 \frac{\text{lb}}{\text{in}} \qquad K_6 = 1365000 \frac{\text{lb}}{\text{in}}$$

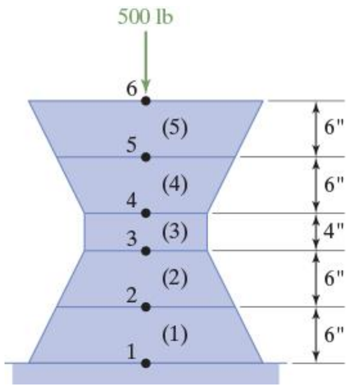
$$K_7 = 1235000 \frac{\text{lb}}{\text{in}} \qquad K_8 = 1105000 \frac{\text{lb}}{\text{in}}$$

$$\begin{bmatrix} \overset{1}{\cancel{2015000}} & \overset{0}{\cancel{-2015000}} & & & & & & & \\ \underset{0}{\cancel{-2015000}} & 2015000+1885000 & -1885000 & & & & & & \\ & -1885000 & 1885000+1755000 & -1755000 & & & & & \\ & & -1755000 & 1755000+1625000 & -1625000 & & & & \\ & & & -1625000 & 1625000+1495000 & -1495000 & & & \\ & & & & -1495000 & 1495000+1365000 & -1365000 & & \\ & & & & & -1365000 & 1365000+1235000 & -1235000 & \\ & & & & & & -1235000 & 1235000+1105000 & -1105000 \\ & & & & & & & -1105000 & 1105000 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1000 \end{Bmatrix}$$

$u_1 = 0,\ u_2 = 0.00049628,\ u_3 = 0.0010268,\ u_4 = 0.0015965,\ u_5 = 0.0022119,$
$u_6 = 0.0028808,\ u_7 = 0.0036134,\ u_8 = 0.0044232,\ u_9 = 0.0053281$ in.

y (in)	exact defl.	Two-element	eight element
0	0	0	0
1.25	0.00049645		0.00049628
2.5	0.0010272		0.0010268
3.75	0.0015972		0.0015965
5.0	0.0022129	0.002197	0.0022119
6.25	0.0028822		0.0028808
7.5	0.0036154		0.0036134
8.75	0.0044259		0.0044232
10.0	0.0053319	0.005274	0.0053281

1.2



$$K = \frac{A_{\text{avg.}}E}{\ell}$$

$$K_1 = K_5 = \frac{\left(\dfrac{10.5+12}{2}\right)(3)(3.27\times 10^6)}{6}$$

$$K_1 = K_5 = 18,393,750\,\frac{\text{lb}}{\text{in}}$$

$$K_2 = K_4 = \frac{\left(\dfrac{10.5+9}{2}\right)(3)(3.27\times 10^6)}{6} = 15,941,250\,\frac{\text{lb}}{\text{in}}$$

$$K_3 = \frac{(9)(3)(3.27\times 10^6)}{4} = 22,072,500\,\frac{\text{lb}}{\text{in}}$$

$$\begin{bmatrix} \overset{1}{\cancel{18393750}} & \overset{0}{\cancel{-18393750}} & & & & \\ \underset{0}{\cancel{-18393750}} & \begin{matrix}18393750\\+15941250\end{matrix} & -15941250 & & & \\ & -15941250 & \begin{matrix}15941250\\+22072500\end{matrix} & -22072500 & & \\ & & -22072500 & \begin{matrix}22072500\\+15941250\end{matrix} & -15941250 & \\ & & & -15941250 & \begin{matrix}15941250\\+18393750\end{matrix} & -18393750 \\ & & & & -18393750 & 18393750 \end{bmatrix} \begin{Bmatrix} u_1\\ u_2\\ u_3\\ u_4\\ u_5\\ u_6 \end{Bmatrix} = \begin{Bmatrix} 0\\ 0\\ 0\\ 0\\ 0\\ -500 \end{Bmatrix}$$

$$\{u\} = \begin{Bmatrix} 0 \\ -2.71831\times 10^{-5} \\ -5.85483\times 10^{-5} \\ -8.12009\times 10^{-5} \\ -1.12566\times 10^{-4} \\ -1.39749\times 10^{-4} \end{Bmatrix} \text{in} \quad \Leftarrow$$

$$\sigma^{(3)} = \frac{(-8.12009\times 10^{-5} + 5.85483)\times 10^{-5} \times (3.27\times 10^6)}{4}$$

$$\sigma^{(3)} = \underline{\underline{18.5 \text{ Psi C}}}$$

1.2 Cont.

as a check:

$$\sigma^{(3)} = \frac{F}{A} = \frac{500}{(9)(3)} = 18.5\text{ Psi}$$

$$\sigma = \frac{(u_{i+1} - u_i)}{\ell}E$$

$$\sigma^{(1)} = \frac{(-2.71831\times10^{-5} - 0)(3.27\times10^{6})}{6} = \underline{\underline{14.8\text{ Psi C}}}$$

$$\sigma^{(5)} = \frac{(-1.39749 + 1.12566)\times10^{-4}\times(3.27\times10^{6})}{6} = \underline{\underline{14.8\text{ Psi C}}}$$

as a check:

$$\sigma^{(1)} = \sigma^{(5)} = \frac{F}{A} = \frac{500}{\left(\frac{10.5+12}{2}\right)(3)} = 14.8\text{ Psi C}$$

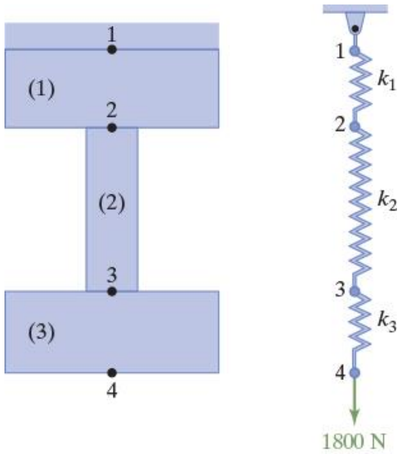
$$\sigma^{(2)} = \frac{(-5.85483 + 2.71831)\times10^{-5}\times(3.27\times10^{6})}{6} = \underline{\underline{17.1\text{ Psi C}}}$$

$$\sigma^{(4)} = \frac{(-1.12566\times10^{-4} + 8.12009\times10^{-5})(3.27\times10^{6})}{6} = \underline{\underline{17.1\text{ Psi C}}}$$

as a check:

$$\sigma^{(2)} = \sigma^{(4)} = \frac{F}{A} = \frac{500}{\left(\frac{10.5+9}{2}\right)(3)} = 17.1\text{ Psi}$$

1.3



$$K = \frac{AE}{\ell}$$

$$K_1 = K_3 = \frac{(0.08)(0.006)(68.9\times 10^9)}{0.025}$$

$$K_1 = K_3 = 1.32288\times 10^9\ \frac{\text{N}}{\text{m}}$$

$$K_2 = \frac{(0.02)(0.006)(68.9\times 10^9)}{0.1}$$

$$K_2 = 0.08268\times 10^9\ \frac{\text{N}}{\text{m}}$$

$$10^9\begin{bmatrix} \overset{1}{\cancel{1.32288}} & \overset{0}{\cancel{-1.32288}} & & \\ \overset{0}{\cancel{-1.32288}} & 1.32288+0.08268 & -0.08268 & \\ & -0.08268 & 0.08268+1.32288 & -1.32288 \\ & & -1.32288 & 1.32288 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1800 \end{Bmatrix}$$

$$\begin{bmatrix} 1.40556 & -0.08268 & 0 \\ -0.08268 & 1.40556 & -1.32288 \\ 0 & -1.32288 & 1.32288 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1800\times 10^{-9} \end{Bmatrix}$$

$$\underline{\underline{u_2 = 1.360668\times 10^{-6}\ \text{m}}} \qquad \underline{\underline{u_3 = 2.313135\times 10^{-5}\ \text{m}}} \qquad \underline{\underline{u_4 = 2.449202\times 10^{-5}\ \text{m}}} \quad \Leftarrow$$

$$\sigma^{(1)} = E\frac{u_2 - u_1}{\ell} = (68.9\times 10^9)\frac{(1.360668\times 10^{-6} - 0)}{0.025} = 3750000 = \underline{\underline{3.75\ \text{MPa}}}$$

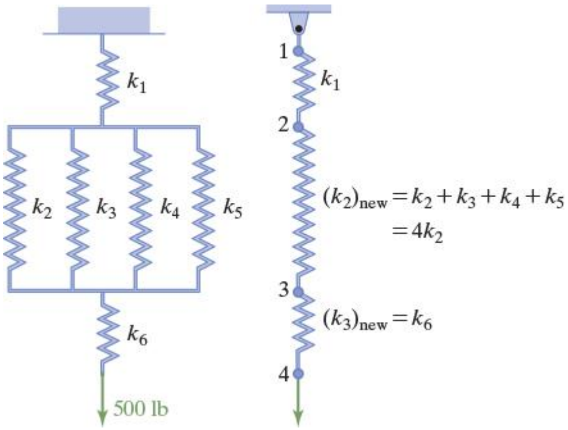
as a check: $\sigma^{(1)} = \dfrac{F}{A} = \dfrac{1800\ \text{N}}{(0.08)(0.006)} = \underline{\underline{3.75\ \text{MPa}}}$

$$\sigma^{(2)} = E\frac{u_3 - u_2}{\ell} = (68.9\times 10^9)\frac{(2.313135\times 10^{-5} - 1.360668\times 10^{-6})}{0.1} = \underline{\underline{15\ \text{MPa}}} \quad \Leftarrow$$

as a check: $\sigma^{(2)} = \dfrac{1800\ \text{N}}{(0.02)(0.006)} = \underline{\underline{15\ \text{MPa}}}$

$$\sigma^{(3)} = E\frac{u_4 - u_3}{\ell} = (68.9\times 10^9)\frac{(2.449202 - 2.313135)\times 10^{-5}}{0.025} = \underline{\underline{3.75\ \text{MPa}}}$$

1.4



$$K_1 = K_6 = \frac{(4)(0.125)(28\times10^6)}{2}$$

$$K_1 = K_6 = 7\times10^6\ \frac{\text{lb}}{\text{in}}$$

$$K_2 = K_3 = K_4 = K_5 = \frac{(0.625)(0.125)(28\times10^6)}{8}$$

$$K_2 = 273438\ \frac{\text{lb}}{\text{in}}$$

$$(K_2)_{\text{new}} = (4)(273438) = 1093750\ \frac{\text{lb}}{\text{in}}$$

$$\begin{bmatrix} \overset{1}{\cancel{7\times10^6}} & \overset{0}{\cancel{-7\times10^6}} & & \\ \overset{0}{\cancel{-7\times10^6}} & 7\times10^6 + 1093750 & -1093750 & \\ & -1093750 & 1093750 + 7\times10^6 & -7\times10^6 \\ & & -7\times10^6 & 7\times10^6 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 500 \end{Bmatrix}$$

$$\begin{bmatrix} 8093750 & -1093750 & 0 \\ -1093750 & 8093750 & -7\times10^6 \\ 0 & -7\times10^6 & 7\times10^6 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 500 \end{Bmatrix}$$

$$u_2 = \underline{\underline{7.142857\times10^{-5}\ \text{in}}} \quad u_3 = \underline{\underline{5.285714\times10^{-4}\ \text{in}}} \quad u_4 = \underline{\underline{6\times10^{-4}\ \text{in}}} \quad \Leftarrow$$

$$\sigma^{(1)} = E\left(\frac{u_2 - u_1}{\ell}\right) = 28\times10^6\left(\frac{7.142857\times10^{-5}\ \text{in}}{2}\right) = \underline{\underline{1000\ \frac{\text{lb}}{\text{in}^2}}}$$

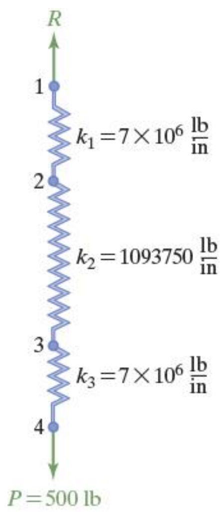
as a check: $\sigma^{(1)} = \dfrac{F}{A} = \dfrac{500}{(4)(0.125)} = 1000\ \dfrac{\text{lb}}{\text{in}^2}$

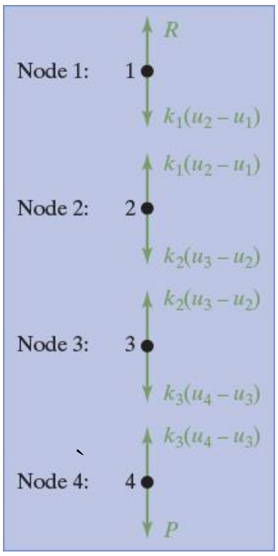
$$\sigma^{(2)} = E\left(\frac{u_3 - u_2}{\ell}\right) = 28\times10^6\left(\frac{5.285714\times10^{-4} - 7.142857\times10^{-5}}{8}\right) = \underline{\underline{1600\ \frac{\text{lb}}{\text{in}^2}}} \quad \Leftarrow$$

as a check: $\sigma^{(2)} = \dfrac{500}{(2.5)(0.125)} = 1600\ \dfrac{\text{lb}}{\text{in}^2}$

$$\sigma^{(3)} = E\left(\frac{u_4 - u_3}{\ell}\right) = 28\times10^6\left(\frac{6\times10^{-4} - 5.285714\times10^{-4}}{2}\right) = \underline{\underline{1000\ \frac{\text{lb}}{\text{in}^2}}}$$

1.5





$$R - K_1(u_2 - u_1) = 0$$

$$K_1(u_2 - u_1) - K_2(u_3 - u_2) = 0$$

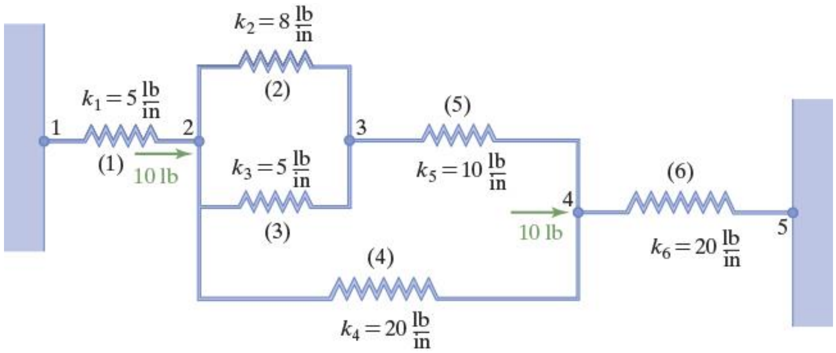
$$K_2(u_3 - u_2) - K_3(u_4 - u_3) = 0$$

$$K_3(u_4 - u_3) - P = 0$$

In matrix form:

$$\begin{bmatrix} K_1 & -K_1 & & \\ -K_1 & K_1 + K_2 & -K_2 & \\ & -K_2 & K_2 + K_3 & -K_3 \\ & & K_3 & K_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} -R \\ 0 \\ 0 \\ P \end{Bmatrix}$$

1.6



Size of the global matrix: 5 × 5

$$[K]^{(1)} = \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array} \qquad [K]^{(2)} = \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ 3 \end{array}$$

$$[K]^{(3)} = \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ 3 \end{array} \qquad [K]^{(4)} = \begin{array}{c} \begin{array}{cc} 2 & 4 \end{array} \\ \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ 4 \end{array}$$

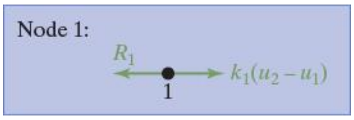
$$[K]^{(5)} = \begin{array}{c} \begin{array}{cc} 3 & 4 \end{array} \\ \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \end{array} \begin{array}{c} 3 \\ 4 \end{array} \qquad [K]^{(6)} = \begin{array}{c} \begin{array}{cc} 4 & 5 \end{array} \\ \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix} \end{array} \begin{array}{c} 4 \\ 5 \end{array}$$

$$[K]^{(6)} = \begin{bmatrix} 5 & -5 & & & \\ -5 & 5+8+5+20 & -8-5 & -20 & \\ & -8-5 & 8+5+10 & -10 & \\ & -20 & -10 & 20+10+20 & -20 \\ & & & -20 & 20 \end{bmatrix}$$

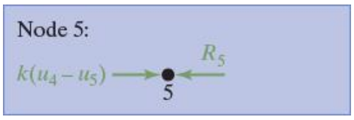
applying B.Cs and loads; $u_1 = u_5 = 0 \quad F_2 = 10\text{ lb}, \quad F_4 = 10\text{ lb}$

$$\begin{bmatrix} 38 & -13 & -20 \\ -13 & 23 & -10 \\ -20 & -10 & 50 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 10 \\ 0 \\ 10 \end{Bmatrix}$$

$u_2 = \underline{\underline{0.962\text{ in}}} \quad u_3 = \underline{\underline{0.874\text{ in}}} \quad u_4 = \underline{\underline{0.760\text{ in}}}$ ⇐



$R_1 = 5(0.962 - 0) = \underline{\underline{4.8\text{ lb}}}$ ⇐



$R_5 = 20(0.760 - 0) = \underline{\underline{15.2\text{ lb}}}$

1.6 Cont.

$K(u_4 - u_5)$

note also as a check, $R_1 + R_5 = \sum F_{\text{external}}$ 4.8 + 15.2 = 10 +10

1.7

$$150\begin{bmatrix} \overset{\frac{1}{150}}{\cancel{5.88}} & \overset{0}{\cancel{5.88}} & & & & & & \\ -5.88 & 5.88+2.27 & -2.27 & & & & & \\ & -2.27 & 2.27+10 & -10 & & & & \\ & & -10 & 10+0.581 & -0.581 & & & \\ & & & -0.581 & 0.581+0.781 & -0.781 & & \\ & & & & -0.781 & 0.781+2.22 & -2.22 & \\ & & & & & -2.22 & 2.22+1.47 & -1.47 \\ & & & & & & \underset{0}{\cancel{-1.47}} & \underset{\frac{1}{150}}{\cancel{1.47}} \end{bmatrix}\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \end{Bmatrix}=\begin{Bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 68 \end{Bmatrix}$$

$$\{T\}=\begin{Bmatrix} 10 \\ 12.04 \\ 17.31 \\ 18.51 \\ 39.12 \\ 54.46 \\ 59.85 \\ 68 \end{Bmatrix}{}^\circ\text{F}$$

$$q = UA\Delta T$$

$$q=(1.47)(150)(68-59.85)=\underline{\underline{1800\,\frac{\text{Btu}}{\text{hr}}}}$$

or as another example:

$$q=(0.781)(150)(54.46-39.12)=1800\,\frac{\text{Btu}}{\text{hr}}$$

or

$$q=\frac{1}{\sum \text{Resistance}}A(T_{\text{in}}-T_{\text{out}})=\frac{(150)(68-10)}{0.17+0.44+0.1+1.72+1.28+0.45+0.68}=1800\,\frac{\text{Btu}}{\text{hr}}$$

1.8

$$150\begin{bmatrix} \overset{\frac{1}{150}}{\cancel{5.88}} & \overset{0}{\cancel{-5.88}} & & & & & \\ -5.88 & 5.88+1.23 & -1.23 & & & & \\ & -1.23 & 1.23+0.76 & -0.76 & & & \\ & & -0.76 & 0.76+0.053 & -0.053 & & \\ & & & -0.053 & 0.053+2.22 & -2.22 & \\ & & & & -2.22 & 2.22+1.47 & -1.47 \\ & & & & & \underset{0}{\cancel{-1.47}} & \underset{\frac{1}{150}}{\cancel{1.47}} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 68 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 20 \\ 20.37 \\ 22.12 \\ 24.95 \\ 65.57 \\ 66.54 \\ 68 \end{Bmatrix} {}^\circ\text{F}$$

$$q = UA\Delta T$$

as an example:

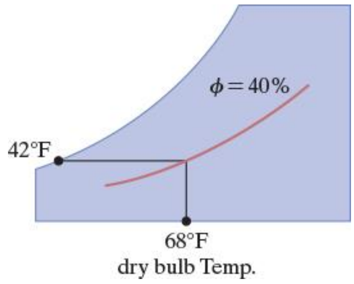
$$q = (1.47)(150)(68 - 66.54) = \underline{\underline{320\ \frac{\text{Btu}}{\text{hr}}}}$$

or

$$q = \frac{1}{\sum \text{Resistance}} A(T_{\text{in}} - T_{\text{out}}) = \frac{(150)(68-20)}{0.17+0.81+1.32+19+0.45+0.68} = 320\ \frac{\text{Btu}}{\text{hr}}$$

1.9

with the help of a psychometric chart, using a dry bulb Temp. of $68\,°\text{F}$ and $\phi = 40\%,$ we identify condensation temperature to be $42\,°\text{F}.$ Thus condensation will occur between surfaces 4 and 5.



1.10

$$1000\begin{bmatrix} \overset{\frac{1}{1000}}{\cancel{1.47}} & \overset{0}{\cancel{-1.47}} & & & \\ -1.47 & 1.47+0.053 & -0.053 & & \\ & -0.053 & 0.053+2.22 & -2.22 & \\ & & -2.22 & 2.22+1.47 & -1.47 \\ & & & \underset{0}{\cancel{-1.47}} & \underset{\frac{1}{1000}}{\cancel{1.47}} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix} = \begin{Bmatrix} 15 \\ 0 \\ 0 \\ 0 \\ 70 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 15 \\ 16.81 \\ 66.99 \\ 68.19 \\ 70 \end{Bmatrix} {}^\circ\mathrm{F}$$

$$q = UA\Delta T$$

as an example:

$$q = (1.47)(1000)(70 - 68.19) = \underline{\underline{2660\ \frac{\text{Btu}}{\text{hr}}}}$$

or

$$q = (1.47)(1000)(16.81 - 15) = 2660\ \frac{\text{Btu}}{\text{hr}}$$

1.11

$$22.5\begin{bmatrix} \overset{\frac{1}{22.5}}{\cancel{5.88}} & \overset{0}{\cancel{-5.88}} & & \\ -5.88 & 5.88+2.56 & -2.56 & \\ & -2.56 & 2.56+1.47 & -1.47 \\ & & \underset{0}{\cancel{-1.47}} & \underset{\frac{1}{22.5}}{\cancel{1.47}} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 0 \\ 0 \\ 70 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 20 \\ 26.85 \\ 42.59 \\ 70 \end{Bmatrix} °\text{F}$$

$$q = U_1 A \Delta T = (5.88)(22.5)(26.85 - 20) = \underline{\underline{910 \, \frac{\text{Btu}}{\text{hr}}}}$$

or as another example:

$$q = U_2 A \Delta T = (2.56)(22.5)(42.59 - 26.85) = \underline{\underline{910 \, \frac{\text{Btu}}{\text{hr}}}}$$

or

$$q = \frac{1}{\sum \text{Resistance}} A(T_{\text{in}} - T_{\text{out}}) = \frac{1}{(0.17 + 0.39 + 0.68)} (22.5)(70 - 20)$$

$$\underline{\underline{q = 910 \, \frac{\text{Btu}}{\text{hr}}}}$$

1.12

$$(A_1)_c = (A_5)_c = \left(\frac{12+10.5}{2}\right)(3) - (3)\frac{\pi}{4}(0.5)^2 = 33.161\,\text{in}^2$$

$$(A_2)_c = (A_4)_c = \frac{(10.5+9)}{2}(3) - 3\frac{\pi}{4}(0.5)^2 = 28.661\,\text{in}^2$$

$$(A_3)_c = (9)(3) - 3\frac{\pi}{4}(0.5)^2 = 26.411\,\text{in}^2$$

$$(K_1)_c = (K_5)_c = \frac{(33.161)(3.27\times 10^6)}{6} = 18{,}072{,}745\,\frac{\text{lb}}{\text{in}}$$

$$(K_2)_c = (K_4)_c = \frac{(28.661)(3.27\times 10^6)}{6} = 15{,}620{,}245\,\frac{\text{lb}}{\text{in}}$$

$$(K_3)_c = \frac{(26.411)(3.27\times 10^6)}{4} = 21{,}590{,}992\,\frac{\text{lb}}{\text{in}}$$

$$(K_1)_s = (K_2)_s = (K_4)_s = (K_5)_s = \frac{(3)\left(\frac{\pi}{4}\right)(0.5)^2(29\times 10^6)}{6} = 2{,}847{,}068\,\frac{\text{lb}}{\text{in}}$$

$$(K_3)_s = \frac{(3)\left(\frac{\pi}{4}\right)(0.5)^2(29\times 10^6)}{4} = 4{,}270{,}602\,\frac{\text{lb}}{\text{in}}$$

The Combined stiffnesses:

$$K_1 = K_5 = 18{,}072{,}745 + 2{,}847{,}068 = 20{,}919{,}813$$

$$K_2 = K_4 = 15{,}620{,}245 + 2{,}847{,}068 = 18{,}467{,}313$$

$$K_3 = 21{,}590{,}992 + 4{,}270{,}602 = 25{,}861{,}594$$

$$\begin{bmatrix}
\overset{1}{\cancel{20{,}919{,}813}} & \overset{0}{\cancel{-20{,}919{,}813}} & & & & \\
\underset{0}{\cancel{-20{,}919{,}813}} & \begin{array}{c}20{,}919{,}813\\+18{,}467{,}313\end{array} & -18{,}467{,}313 & & & \\
 & -18{,}467{,}313 & \begin{array}{c}18{,}467{,}313\\+25{,}861{,}594\end{array} & -25{,}861{,}594 & & \\
 & & -25{,}861{,}594 & \begin{array}{c}25{,}861{,}594\\+18{,}467{,}313\end{array} & -18{,}467{,}313 & \\
 & & & -18{,}467{,}313 & \begin{array}{c}18{,}467{,}313\\+20{,}919{,}813\end{array} & -20{,}919{,}813 \\
 & & & & -20{,}919{,}813 & 20{,}919{,}813
\end{bmatrix}
\begin{Bmatrix} u_1\\ u_2\\ u_3\\ u_4\\ u_5\\ u_6 \end{Bmatrix}
=
\begin{Bmatrix} 0\\ 0\\ 0\\ 0\\ 0\\ -1000 \end{Bmatrix}$$

$u_1 = 0$ $\qquad u_2 = -4.78016\times 10^{-5}$ in $\qquad u_3 = -1.01951\times 10^{-4}$ in

$u_4 = -1.40618\times 10^{-4}$ $\qquad u_5 = -1.94768\times 10^{-4}$ in $\qquad u_6 = -2.42570\times 10^{-4}$ in

1.12 Cont.

$$\sigma^{(1)}_{\text{Concrete}} = E_c \frac{(u_2 - u_1)}{\ell} = \frac{(3.27\times 10^6)(-4.78016\times 10^{-5})}{6} = \underline{\underline{26\,\frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

$$\sigma^{(11}_{\text{Steel}} = E_s \frac{(u_2 - u_1)}{\ell} = \underline{\underline{231\,\frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

as a check:

$$\sigma^{(1)}_{\text{Concrete}}(A_1)_c + \sigma_s^{(1)}(A_1)_s = (26)(33.161) + (231)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = \underline{\underline{1000\ \text{lb}}}$$

$$\sigma_c^{(2)} = E_c\left(\frac{u_3 - u_2}{\ell}\right) = \frac{(3.27\times 10^6)(-1.01951\times 10^{-4} + 4.78016\times 10^{-5})}{6} = \underline{\underline{30\,\frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

$$\sigma_s^{(2)} = 231\,\frac{\text{lb}}{\text{in}^2}\ \text{C}$$

Check: $\sigma_c^{(2)}(A_2)_c + \sigma_s^{(2)}(A_2)_s = (30)(28.661) + (231)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = 1000\ \text{lb}$

$$\sigma_c^{(3)} = E_c \frac{(u_4 - u_3)}{\ell} = \frac{(3.27\times 10^6)(-1.40618\times 10^{-4} + 1.01951\times 10^{-4})}{4} = \underline{\underline{32\,\frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

$$\sigma_s^{(3)} = E_s \frac{(u_4 - u_3)}{\ell} = \frac{(29\times 10^6)(-1.40618\times 10^{-4} + 1.01951\times 10^{-4})}{4} = \underline{\underline{280\,\frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

Check: $\sigma_c^{(3)}(A_3)_c + \sigma_s^{(3)}(A_3)_s = (32)(26.411) + (280)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = 1000\ \text{lb}$

$$\sigma_c^{(4)} = E_c \frac{(u_5 - u_4)}{\ell} = \frac{(3.27\times 10^6)(-1.94768\times 10^{-4} + 1.40618\times 10^{-4})}{6} = \underline{\underline{30\,\frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

$$\sigma_s^{(4)} = 231\,\frac{\text{lb}}{\text{in}^2}\ \text{C}$$

$$\sigma_c^{(5)} = E_c \frac{(u_6 - u_5)}{\ell} = \frac{(3.27\times 10^6)(-2.42570 + 1.94768)\times 10^{-4}}{6} = \underline{\underline{26\,\frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

$$\sigma_s^{(5)} = 231\,\frac{\text{lb}}{\text{in}^2}\ \text{C}$$

note $\sigma_c^{(1)} = \sigma_c^{(5)}$ and $\sigma_c^{(2)} = \sigma_c^{(4)}$ as expected.

1.13

$$\Lambda^{(e)}=\frac{A_{\text{avg}}\,E}{2\ell}\left(u_{i+1}-u_i\right)^2$$

$$\Lambda_{\text{total}}=\Lambda^{(1)}+\Lambda^{(2)}+\Lambda^{(3)}+\Lambda^{(4)}+\Lambda^{(5)}$$

$$\Lambda^{(1)}=\Lambda^{(5)}=\frac{(A_1)_c E_c}{2\ell}(u_{i+1}-u_i)^2+\frac{(A_1)_s E_s}{2\ell}(u_{i+1}-u_i)^2$$

$$=\frac{(A_1)_c E_c+(A_1)_s E_s}{2\ell}(u_2-u_1)^2$$

$$\Lambda^{(1)}=\left[\frac{(33.161)(3.27\times10^6)+3\left(\frac{\pi}{4}\right)(0.5)^2(29\times10^6)}{(2)(6)}\right](-4.78016\times10^{-5})^2=0.024\ \text{lb}\cdot\text{in}$$

or

$$\Lambda^{(5)}=\left[\frac{(33.161)(3.27\times10^6)+3\left(\frac{\pi}{4}\right)(0.5)^2(29\times10^6)}{(2)(6)}\right]=[(-2.4257+1.94768)\times10^{-4}]^2=0.024\ \checkmark$$

$$\Lambda^{(2)}=\frac{(A_2)_c E_c+(A_2)_s E_s}{2\ell}(u_3-u_2)$$

$$=\left[\frac{(28.661)(3.27\times10^6)+3\frac{\pi}{4}(0.5)^2(29\times10^6)}{2(6)}\right][(-1.01951\times10^{-4}+4.78016\times10^{-5})]^2=0.027$$

or

$$\Lambda^{(4)}=\left[\frac{(28.661)(3.27\times10^6)+3\frac{\pi}{4}(0.5)^2(29\times10^6)}{2(6)}\right]=[(-1.94768+1.40618)\times10^{-4}]^2=0.027\ \checkmark$$

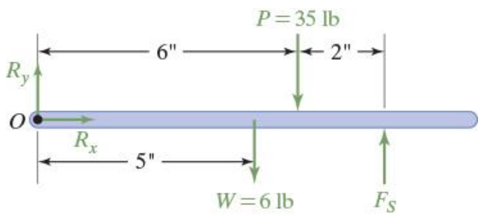
$$\Lambda^{(3)}=\frac{(A_3)_c E_c+(A_3)_s E_s}{2\ell}(u_4-u_3)$$

$$\Lambda^{(3)}=\left[\frac{(26.411)(3.27\times10^6)+3\frac{\pi}{4}(0.5)^2(29\times10^6)}{2(4)}\right][(-1.40618+1.01951\times10^{-4}]^2$$

$$=0.019\ \text{lb}\cdot\text{in}$$

$$\Lambda_{\text{total}}=(2)(0.024)+0.019+2(0.027)=\underline{\underline{0.121\ \text{lb}\cdot\text{in}}}\quad\Leftarrow$$

1.14



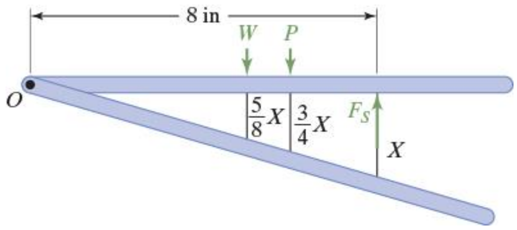
(a) $\circlearrowleft + \sum M_0 = 0$

$$(-6\text{ lb})(5\text{ in}) + (35\text{ lb})(6\text{ in}) + F_s(8\text{ in}) = 0$$

$$F_s = 30\text{ lb}$$

$$F_s = KX \quad 30\text{ lb} = 60\,X \quad \underline{\underline{X = 0.5\text{ in}}} \quad \Leftarrow$$

(b)



$$\frac{\partial \pi}{\partial u_i} = \frac{\partial}{\partial u_i}\sum_{i=1}^{n}\Lambda^{(e)} - \frac{\partial}{\partial u_i}\sum_{i=1}^{m}F_i u_i = 0$$

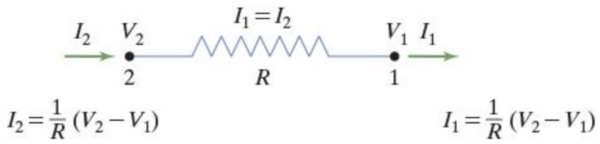
$$\frac{\partial}{\partial X}\left(\frac{1}{2}KX^2\right) - \frac{\partial}{\partial X}\left(35\left(\frac{3}{4}X\right)\right) - (6)\left(\frac{5}{8}X\right) = 0$$

$$KX - (35)\left(\frac{3}{4}\right) - (6)\left(\frac{5}{8}\right) = 0$$

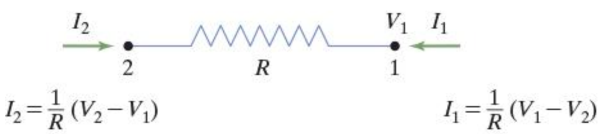
$$X = \frac{(35)\left(\frac{3}{4}\right) + (6)\left(\frac{5}{8}\right)}{60}$$

$$\underline{\underline{X = 0.5\text{ in}}} \quad \Leftarrow$$

1.15



(or)



Because of the fact that charge is conserved in a circuit (Kirchhoff's current law), at any time, the algebraic sum of the currents entering any node must be zero. Thus we can write

$$I_1 = \frac{1}{R}(V_1 - V_2)$$

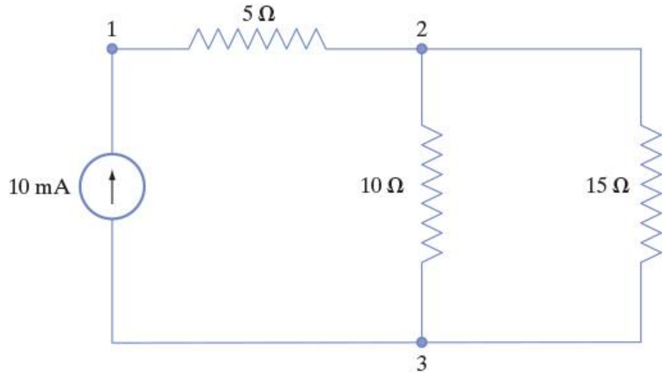
$$I_2 = \frac{1}{R}(V_2 - V_1)$$

Note $I_1 + I_2 = 0$

In matrix form:

$$\frac{1}{R}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \begin{Bmatrix} I_1 \\ I_2 \end{Bmatrix}$$

1.16



$$[K]^{(1)} = \frac{1}{5}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K]^{(2)} = \frac{1}{10}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K]^{(3)} = \frac{1}{15}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}; \text{ Apply } V_3 = 0 \text{ as a boundary condition}$$

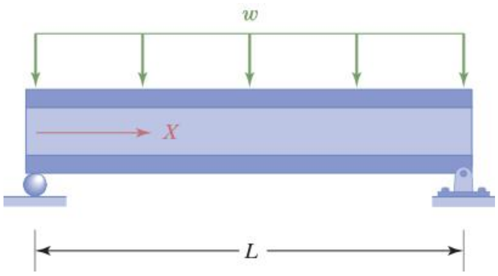
$$\begin{bmatrix} \frac{1}{5} & -\frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{10}+\frac{1}{15} & -\frac{1}{10}-\frac{1}{15} \\ 0 & \underset{0}{\cancel{-\frac{1}{10}-\frac{1}{15}}} & \underset{1}{\cancel{\frac{1}{10}+\frac{1}{15}}} \end{bmatrix}\begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} 0.01 \\ 0 \\ 0 \end{Bmatrix}$$

$V_1 = 0.11$ $\qquad$ $V_2 = 0.06$

$\underline{\underline{V_1 - V_2 = 0.05 \text{ Volts}}}$ ⇦

$\underline{\underline{V_2 - V_3 = 0.06 \text{ Volts}}}$ ⇦

1.17



$$\frac{d^2Y}{dX^2}=\frac{M(X)}{EI}=\frac{wX(L-X)}{2EI}$$

$$\frac{dY}{dX}=\frac{1}{2EI}=\left(\frac{wX^2L}{2}-\frac{wX^3}{3}\right)+C_1$$

$$Y=\frac{1}{2EI}\left(\frac{wX^3L}{6}-\frac{wX^4}{12}\right)+C_1X+C_2$$

Applying Boundary Conditions:

$Y=0 \quad @ \quad X=0 \quad C_2=0$

$Y=0 \quad @ \quad X=L \quad C_1=-\dfrac{wL^3}{24EI}$

$$\underline{\underline{Y_{\text{exact}}=\frac{-wX}{24EI}(X^3-2LX^2+L^3)}} \quad \Leftarrow$$

(a) Note that the assumed Solution satisfies the boundary conditions. $Y=C_1\left[\left(\dfrac{X}{L}\right)^2-\left(\dfrac{X}{L}\right)\right]$

$$\frac{dY}{dX}=C_1\left[\frac{2X}{L^2}-\frac{1}{L}\right]$$

$$\frac{d^2Y}{dX^2}=C_1\left(\frac{2}{L^2}\right)$$

$$\frac{2C_1}{L^2}-\frac{wX(L-X)}{2EI}=\mathrm{R}$$

We may force the error function to equal zero at $X=\dfrac{L}{2}$

$$\frac{2C_1}{L^2}-\frac{w\frac{L}{2}\left(L-\frac{L}{2}\right)}{2EI}=0\rightarrow C_1=\frac{wL^4}{16EI}$$

$$\underline{\underline{Y=\frac{wL^4}{16EI}\left[\left(\frac{X}{L}\right)^2-\left(\frac{X}{L}\right)\right]}} \quad \Leftarrow$$

1.17 Cont.

(b) $\int_0^L R\,dX = 0$

$$\int_0^L \left[\frac{2c_1}{L^2} - \frac{wX(L-X)}{2EI}\right] dX = 0$$

$$\frac{2c_1}{L^2}L - \frac{w}{2EI}\left(\frac{L^3}{2} - \frac{L^3}{3}\right) = 0 \rightarrow c_1 = \frac{wL^4}{24EI}$$

$$\underline{\underline{Y = \frac{wL^4}{24EI}\left(\left(\frac{X}{L}\right)^2 - \left(\frac{X}{L}\right)\right)}} \Leftarrow$$

For $W24 \times 104$ Beam $I = 3100\,\text{in}^4$; $E = 29 \times 10^6\,\dfrac{\text{lb}}{\text{in}^2}$

Exact

$$Y_{\max} = \frac{-5wL^4}{384EI} = \frac{-5\left(5000\,\dfrac{\text{lb}}{\text{ft}} \times \dfrac{1\,\text{ft}}{12\,\text{in}}\right)\left(20\,\text{ft} \times \dfrac{12\,\text{in}}{\text{ft}}\right)^4}{(384)\left(29 \times 10^6\,\dfrac{\text{lb}}{\text{in}^2}\right)\left(3100\,\text{in}^4\right)} = \underline{\underline{-0.20\,\text{in}}}$$

Collocation

$$Y_{\max} = \frac{-wL^4}{64EI} = \frac{-\left(5000\,\dfrac{\text{lb}}{\text{ft}} \times \dfrac{1\,\text{ft}}{12\,\text{in}}\right)\left(20\,\text{ft} \times \dfrac{12\,\text{in}}{\text{ft}}\right)^4}{(64)\left(29 \times 10^6\,\dfrac{\text{lb}}{\text{in}^2}\right)(3100\,\text{in}^4)} = \underline{\underline{-0.24\,\text{in}}} \Leftarrow$$

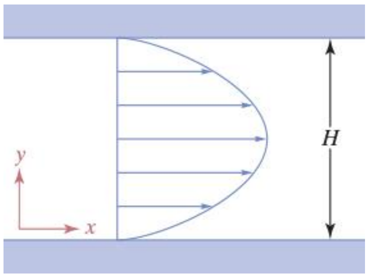
Subdomain

$$Y_{\max} = \frac{-wL^4}{96EI} = \frac{-\left(5000\,\dfrac{\text{lb}}{\text{ft}} \times \dfrac{1\,\text{ft}}{12\,\text{in}}\right)\left(20\,\text{ft} \times \dfrac{12\,\text{in}}{\text{ft}}\right)^4}{(96)\left(29 \times 10^6\,\dfrac{\text{lb}}{\text{in}^2}\right)\left(3100\,\text{in}^4\right)} = \underline{\underline{-0.16\,\text{in}}}$$

1.18 See Section 1.7

1.19

$$\mu \frac{d^2u}{dy^2} = \frac{dP}{dX}$$



$$\frac{du}{dy} = \frac{1}{\mu}\frac{dP}{dX}y + c_1$$

$$u(y) = \frac{1}{\mu}\frac{dP}{dX}\frac{y^2}{2} + c_1 y + c_2$$

applying B.Cs $u(0) = 0$ and $u(H) = 0$

$$0 = 0 + 0 + c_2 \quad \text{and} \quad 0 = \frac{1}{\mu}\frac{dP}{dX}\frac{H^2}{2} + c_1 H$$

$$c_2 = 0 \quad \text{and} \quad c_1 = -\frac{H}{2\mu}\frac{dP}{dX}$$

$$\underline{\underline{u(y) = -\frac{1}{2\mu}\frac{dP}{dX}(Hy - y^2)}}$$

note because Pressure drops in the direction of flow, $\frac{dP}{dX} < 0$ in velocity equation.

(a) $\mu \frac{d^2u}{dy^2} = \frac{dP}{dX}$ $\qquad u_{\text{assumed}} = c_1 \sin\left(\frac{\pi y}{H}\right)$

note the assumed solution Satisfies the boundary conditions.

$$\frac{du}{dy} = \frac{d}{dy}\left(c_1 \sin\left(\frac{\pi y}{H}\right)\right) = c_1 \frac{\pi}{H}\cos\left(\frac{\pi y}{H}\right)$$

$$\frac{d^2u}{dy^2} = \frac{d}{dy}\left(c_1 \frac{\pi}{H}\cos\left(\frac{\pi y}{H}\right)\right) = -c_1\left(\frac{\pi}{H}\right)^2 \sin\left(\frac{\pi y}{H}\right)$$

$$\mu\left[-c_1\left(\frac{\pi}{H}\right)^2 \sin\left(\frac{\pi y}{H}\right)\right] - \frac{dP}{dX} = \mathrm{R}$$

We may force the error function to equal zero at $y = \frac{H}{2}$

$$\mu\left[-c_1\left(\frac{\pi}{H}\right)^2\right]\sin\left(\frac{\pi}{H}\frac{H}{2}\right) - \frac{dP}{dX} = 0$$

$$c_1 = -\frac{H^2}{\mu\pi^2}\frac{dP}{dX} \quad \text{then,} \quad \underline{\underline{u(y) = -\frac{H^2}{\mu\pi^2}\frac{dP}{dX}\left(\sin\left(\frac{\pi y}{H}\right)\right)}} \Leftarrow$$

1.19 Cont.

(b) $\int_0^H R\,dy = 0$

$$\int_0^H \left[\mu\left[-c_1\left(\frac{\pi}{H}\right)^2 \sin\frac{\pi y}{H}\right] - \frac{dP}{dX}\right] dy = 0$$

$$\mu c_1\left(\frac{\mu}{H}\right)^2 \left[\frac{1}{\frac{\pi}{H}}\cos\frac{\pi y}{H}\right]_0^H - \frac{dP}{dX}H = 0$$

$$\mu c_1 \frac{\pi}{H}(-2) = H\frac{dP}{dX}$$

$$c_1 = -\frac{H^2}{2\mu\pi}\frac{dP}{dX} \quad \text{then,} \quad \underline{\underline{u(y) = -\frac{H^2}{2\mu\pi}\frac{dP}{dX}\left[\sin\left(\frac{\pi y}{H}\right)\right]}} \quad \Leftarrow$$

For Comparison, let's use:

$$\mu = 0.02\,\frac{\mathrm{N\cdot S}}{\mathrm{m}^2}; \quad H = 0.01\ \mathrm{mm} = 1\times10^{-5}\ \mathrm{m}; \quad \frac{dP}{dX} = -1\times10^8\ \frac{\mathrm{Pa}}{\mathrm{m}}$$

evaluating max vel. @ $y = \frac{H}{2}$

Exact

$$u_{\max} = -\frac{1}{2\mu}\frac{dP}{dX}\left(H\left(\frac{H}{2}\right) - \left(\frac{H}{2}\right)^2\right) = -\frac{1}{2\mu}\frac{dP}{dX}\frac{H^2}{4} = -\frac{(-1\times10^8)(1\times10^{-5})^2}{(8)(0.02)} = \underline{\underline{0.06\ \frac{\mathrm{m}}{\mathrm{s}}}}$$

Collocation method

$$u_{\max} = -\frac{H^2}{\mu\pi^2}\frac{dP}{dX}\left(\sin\left(\frac{\pi\frac{H}{2}}{H}\right)\right) = -\frac{H^2}{\mu\pi^2}\frac{dP}{dX} = -\frac{(1\times10^{-5})^2(-1\times10^8)}{(0.02)\pi^2} = \underline{\underline{0.05\ \frac{\mathrm{m}}{\mathrm{s}}}}$$

Subdomain method

$$u_{\max} = -\frac{H^2}{2\mu\pi}\frac{dP}{dX}\left(\sin\left(\frac{\pi\frac{H}{2}}{H}\right)\right) = -\frac{H^2}{2\mu\pi}\frac{dP}{dX} = -\frac{(1\times10^{-5})^2(-1\times10^8)}{(2)(0.02)\pi} = \underline{\underline{0.08\ \frac{\mathrm{m}}{\mathrm{s}}}}$$

1.20

(a) *Galerkin method*

$$\int_0^H \sin\left(\frac{\pi y}{H}\right)\left[\mu\left(-C_1\left(\frac{\pi}{H}\right)^2 \sin\frac{\pi y}{H}\right)-\frac{dP}{dX}\right]dy=0$$

$$-\mu C_1\left(\frac{\pi}{H}\right)^2\int_0^H \sin^2\left(\frac{\pi y}{H}\right)dy=\frac{dP}{dX}\int_0^H \sin\left(\frac{\pi y}{H}\right)dy$$

$$-\mu C_1\left(\frac{\pi}{H}\right)^2\left[\frac{1}{2}y-\frac{1}{4\left(\frac{\pi}{H}\right)}\sin\left(\frac{2\pi}{H}y\right)\right]_0^H=\frac{dP}{dX}\left[-\frac{1}{\frac{\pi}{H}}\cos\left(\frac{\pi}{H}y\right)\right]_0^H$$

$$-\mu C_1\left(\frac{\pi}{H}\right)^2\left(\frac{1}{2}H\right)=-\frac{H}{\pi}\frac{dP}{dX}(-2)$$

$$C_1=\frac{-4H^2}{\mu\pi^3}\frac{dP}{dX}\quad \text{then,}\quad \underline{\underline{u(y)=-\frac{4H^2}{\mu\pi^3}\frac{dP}{dX}\sin\left(\frac{\pi y}{H}\right)}}\quad \Leftarrow$$

(b) *least-squares method*

$$\int_0^H R\frac{\partial R}{\partial C_1}dy=\int_0^H \overbrace{\sin\left(\frac{\pi y}{H}\right)\left[\mu\left(-C_1\left(\frac{\pi}{H}\right)^2\right)\sin\frac{\pi y}{H}-\frac{dP}{dX}\right]}^{R}\overbrace{\left[-\mu\left(\frac{\pi}{H}\right)^2\sin\frac{\pi y}{H}\right]}^{\frac{\partial R}{\partial c_1}}dy=0$$

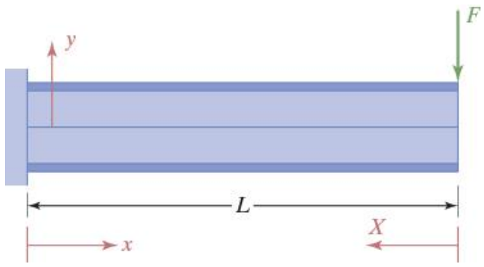
results in:

$$C_1=-\frac{4H^2}{\mu\pi^3}\frac{dP}{dX}\quad \text{and}\quad \underline{\underline{u(y)=-\frac{4H^2}{\mu\pi^3}\frac{dP}{dX}\sin\left(\frac{\pi y}{H}\right)}}\quad \Leftarrow$$

using values from Problem 1.19 for comparison

$$u_{\max}=\frac{-4H^2}{\mu\pi^3}\frac{dP}{dX}=\frac{-4(1\times10^{-5})^2(-1\times10^{8})}{(0.02)\pi^3}=\underline{\underline{0.06\ \frac{\text{m}}{\text{s}}}}\quad \Leftarrow$$

1.21



$$\frac{d^2y}{dX^2} = -\frac{PX}{EI}$$

$$\frac{dy}{dX} = \frac{-p}{EI}\frac{X^2}{2} + c_1$$

$$y = -\frac{p}{EI}\frac{X^3}{6} + c_1 X + c_2$$

B.Cs ① $y(L) = 0$, ② $\left.\frac{dy}{dX}\right|_{X=L} = 0$

② $\quad 0 = \frac{-P}{EI}\frac{L^2}{2} + c_1 \qquad c_1 = \frac{P}{EI}\frac{L^2}{2}$

① $\quad 0 = -P\frac{L^3}{6} + \frac{P}{EI}\frac{L^2}{2}L + C_2 \qquad C_2 = \frac{P}{EI}\left(-\frac{1}{3}L^3\right)$

$$\underline{\underline{y_{\text{exact}} = \frac{P}{6EI}(-X^3 + 3L^2X - 2L^3)}} \quad \Leftarrow$$

In terms of x: Substitute for $X = L - x$ and simplify

$$y_{\text{exact}} = \frac{P}{6EI}(-(L-x)^3 + 3L^2(L-x) - 2L^3)$$

$$\underline{\underline{y_{\text{exact}} = \frac{P}{6EI}\left(x^3 - 3L^2x\right)}} \quad \Leftarrow$$

$$EI\frac{d^2y}{dx^2} + P\left(L - x\right) = 0$$

B.Cs $y(0) = 0$ and $\left.\frac{dy}{dx}\right|_{x=0} = 0$

Let us assume: $y_{\text{assumed}} = c_1x^2 + c_2x^3$

note, the assumed solution satisfies the boundary conditions.

1.21 Cont.

$$\frac{dy}{dx} = 2c_1 x + 3c_2 x^2$$

$$\frac{d^2y}{dx^2} = 2c_1 + 6c_2 x$$

then Residual R becomes:

$$R = EI(2c_1 + 6c_2 x) + P(L - x)$$

Subdomain method:

$$\int_0^{\frac{L}{2}} R\,dx = 0 \qquad \int_0^{\frac{L}{2}} [EI(2c_1 + 6c_2 x) + P(L - x)]dx = 0 \qquad (1)$$

$$\int_{\frac{L}{2}}^{L} R\,dx = 0 \qquad \int_{\frac{L}{2}}^{L} [EI(2c_1 + 6c_2 x) + P(L - x)]\,dx = 0 \qquad (2)$$

Integrating (1):

$$EI\left(2c_1\left(\frac{L}{2}\right) + 6c_2\frac{\left(\frac{L}{2}\right)^2}{2}\right) + P\left(L\left(\frac{L}{2}\right) - \frac{\left(\frac{L}{2}\right)^2}{2}\right) = 0$$

Simplifying:

$$2c_1 + \frac{3}{2}c_2 L = -\frac{3PL}{4EI} \qquad (3)$$

Integrating (2) and Simplifying:

$$c_1 + \frac{9}{4}c_2 L = -\frac{PL}{8EI} \qquad (4)$$

Solving (3) and (4) Simultaneously:

$$\underline{\underline{c_1 = -\frac{PL}{2EI}}} \quad \text{and} \quad \underline{\underline{c_2 = \frac{P}{6EI}}} \qquad \Leftarrow$$

$$y = \frac{P}{6EI}(x^3 - 3Lx^2)$$

note, because the assumed solution has the same functional form as the exact solution, the coefficients are exact.

1.21 Cont.

Galerkin method:

$$\int_0^L x^2 R\,dx = 0 \qquad \int_0^L x^2[EI(2c_1+6c_2)+P(L-x)]dx = 0 \qquad (1)$$

$$\int_0^L x^3 R\,dx = 0 \qquad \int_0^L x^3[EI(2c_1+6c_2)+P(L-x)]dx = 0 \qquad (2)$$

Integrating and Simplifying (1) and (2), we get:

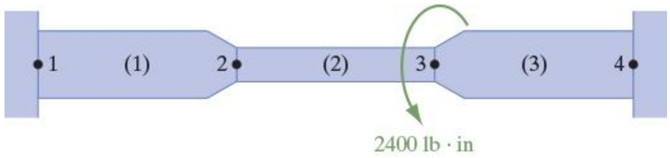
$$\begin{cases} \dfrac{2}{3}c_1 + \dfrac{3}{2}c_2 L = -\dfrac{PL}{12EI} & (3) \\ \dfrac{1}{2}c_1 + \dfrac{6}{5}c_2 L = -\dfrac{PL}{20EI} & (4) \end{cases}$$

Solving (3) and (4) Simultaneously

$$c_1 = -\frac{PL}{2EI} \quad \text{and} \quad c_2 = \frac{P}{6EI} \quad \Leftarrow$$

$$y = \frac{P}{6EI}(x^3 - 3Lx^2)$$

1.22



$$J_1 = J_3 = \frac{1}{2}\pi r^4$$

$$J_1 = J_3 = \frac{1}{2}\pi\left(\frac{1.5}{2}\right)^4 = 0.497\ \text{in}^4$$

$$J_2 = \frac{1}{2}\pi\left(\frac{1}{2}\right)^4 = 0.0982\ \text{in}^4$$

$$[K]^{(1)} = [K]^{(3)} = \frac{JG}{\ell}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{(0.497)(9.8\times 10^6)}{(2)(12)}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 202942 & -202942 \\ -202942 & 202942 \end{bmatrix}$$

$$[K]^{(2)} = \frac{(0.0982)(11.2\times 10^6)}{(1.5)(12)}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 61102 & -61102 \\ -61102 & 61102 \end{bmatrix}$$

$$\begin{bmatrix} \overset{1}{\cancel{202942}} & \overset{0}{\cancel{-202942}} & & \\ \underset{0}{\cancel{-202942}} & 202942+61102 & -61102 & \\ & -61102 & 61102+202942 & \underset{0}{\cancel{-202942}} \\ & & \underset{0}{\cancel{-202942}} & \underset{1}{\cancel{202942}} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 2400 \\ 0 \end{Bmatrix}$$

$\theta_1 = 0 \quad \underline{\underline{\theta_2 = 0.002222\ \text{rad}}} \quad \underline{\underline{\theta_3 = 0.009604\ \text{rad}}} \quad \theta_4 = 0$

$R_4 = K_3(\theta_3 - 0) = 202942 \times 0.009604 = \underline{\underline{1949\ \text{lb}\cdot\text{in}}}$

$R_1 = K_1(\theta_2 - 0) = 202942 \times 0.002222 = \underline{\underline{451\ \text{lb}\cdot\text{in}}}$

note they
add up to 2400 lb·in

1.23

$$\begin{bmatrix} 264044 & -61102 \\ -61102 & 264044 \end{bmatrix} \begin{Bmatrix} \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 1200 \\ 1200 \end{Bmatrix}$$

$$\underline{\underline{\theta_2 = \theta_3 = 0.005913 \text{ rad}}} \quad \Leftarrow$$

$$\underline{\underline{R_1 = R_4}} = K_1(\theta_2 - 0) = K_3(\theta_3 - 0) = 202942 \times 0.005913 \underline{\underline{= 1200 \text{ lb} \cdot \text{in}}}$$

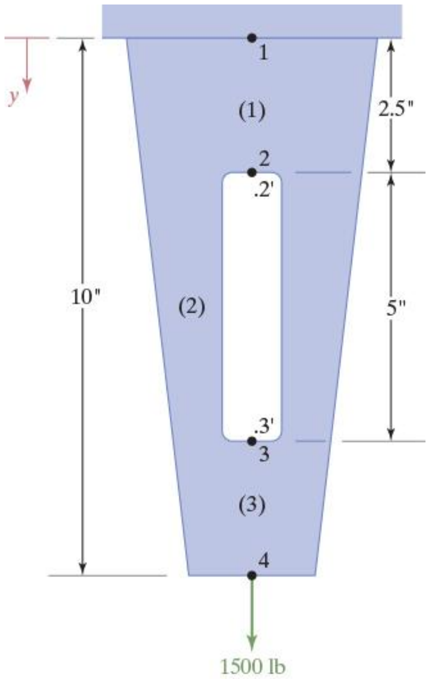
1.24

$$A_1 = 0.5\ \text{in}^2$$

$$A_2 = 0.4375\ \text{in}^2 \qquad A_{2'} = 0.4375.\,(0.5)(0.125)$$
$$= 0.375\ \text{in}^2$$

$$A_3 = 0.3125\ \text{in}^2 \qquad A_{3'} = 0.3125 - (0.5)(0.125)$$
$$= 0.25\ \text{in}^2$$

$$A_4 = 0.25\ \text{in}^2$$



without the whole:

$$A(y) = (4 - 0.2y)(0.125)$$
$$= 0.5 - 0.025y$$

$$K = \frac{A_{\text{avg}} E}{\ell}$$

$$K_1 = \frac{\left(\frac{0.5 + 0.4375}{2}\right)(10.6 \times 10^6)}{2.5} = 1{,}987{,}500\ \frac{\text{lb}}{\text{in}}$$

$$K_2 = \frac{\left(\frac{0.375 + 0.25}{2}\right)(10.6 \times 10^6)}{5} = 662{,}500\ \frac{\text{lb}}{\text{in}}$$

$$K_3 = \frac{\left(\frac{0.3125 + 0.25}{2}\right)(10.6 \times 10^6)}{2.5} = 1{,}192{,}500\ \frac{\text{lb}}{\text{in}}$$

$$\begin{bmatrix} \overset{1}{\cancel{1987500}} & \overset{0}{\cancel{-1,987,500}} & & \\ \underset{0}{\cancel{-1,987,500}} & 1,987,500 + 662500 & -662500 & \\ & -662500 & 662500 + 1192500 & -1192500 \\ & & -1192500 & 1192500 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1500 \end{Bmatrix}$$

$u_1 = 0 \quad \underline{\underline{u_2 = 7.5472 \times 10^{-4}\ \text{in}}} \quad \underline{\underline{u_3 = 0.003020\ \text{in}}} \quad \underline{\underline{u_4 = 0.004277\ \text{in}}}$

$$\sigma^{(1)} = (10.6 \times 10^6)\left(\frac{7.5472 \times 10^{-4}}{2.5}\right) = 3200\ \frac{\text{lb}}{\text{in}^2}$$

as a check: $\sigma^{(1)} = \dfrac{1500}{\frac{0.5 + 0.4375}{2}} = 3200\ \dfrac{\text{lb}}{\text{in}^2}$

$$\sigma^{(2)} = (10.6 \times 10^6)\left(\frac{0.00302 - 7.5472 \times 10^{-4}}{5}\right) = 4800\ \frac{\text{lb}}{\text{in}^2}$$

check: $\sigma^{(2)} = \dfrac{1500}{\left(\frac{0.375 + 0.25}{2}\right)} = 4800\ \dfrac{\text{lb}}{\text{in}^2}$

1.24 Cont.

$$\sigma^{(3)} = (10.6 \times 10^6)\left(\frac{0.004277 - 0.00302}{2.5}\right) = 5300\,\frac{\text{lb}}{\text{in}^2}$$

check: $\sigma^{(3)} = \dfrac{1500}{\dfrac{0.3125 + 0.25}{2}} = 5300\,\text{Psi}$

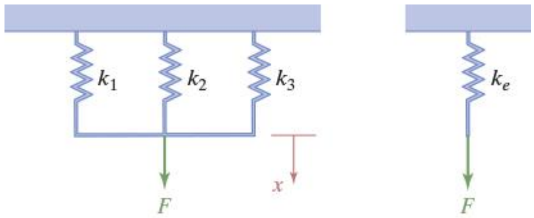
1.25

Springs in Parallel

$$F = F_1 + F_2 + F_3$$

$$k_e x = k_1 x + k_2 x + k_3 x$$

$$\underline{\underline{k_e = k_1 + k_2 + k_3}}$$



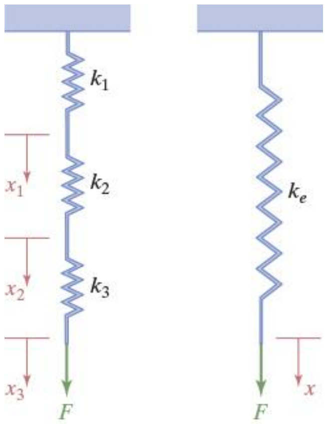
Springs in Series

$$x = x_1 + x_2 + x_3$$

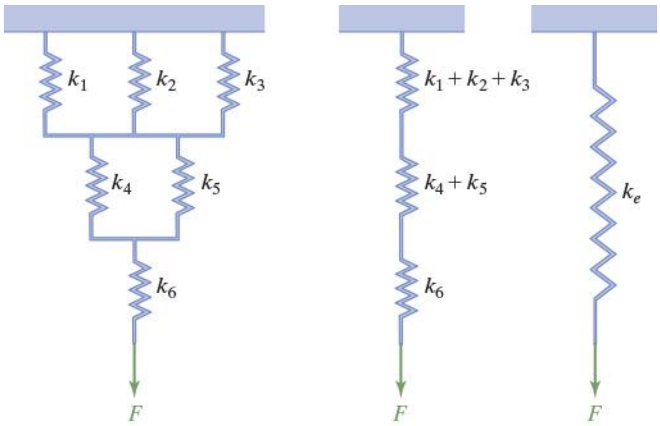
$$\frac{F}{k_e} = \frac{F}{k_1} + \frac{F}{k_2} + \frac{F}{k_3}$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

$$\underline{\underline{k_e = \cfrac{1}{\cfrac{1}{k_1} + \cfrac{1}{k_2} + \cfrac{1}{k_3}}}}$$

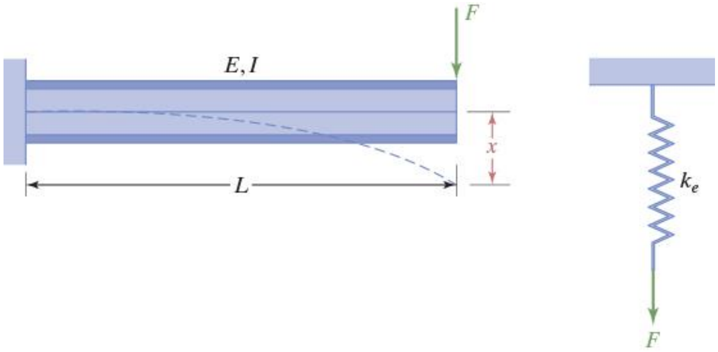


1.26



$$k_e = \frac{1}{\dfrac{1}{k_1 + k_2 + k_3} + \dfrac{1}{k_4 + k_5} + \dfrac{1}{k_6}} \quad \Leftarrow$$

1.27



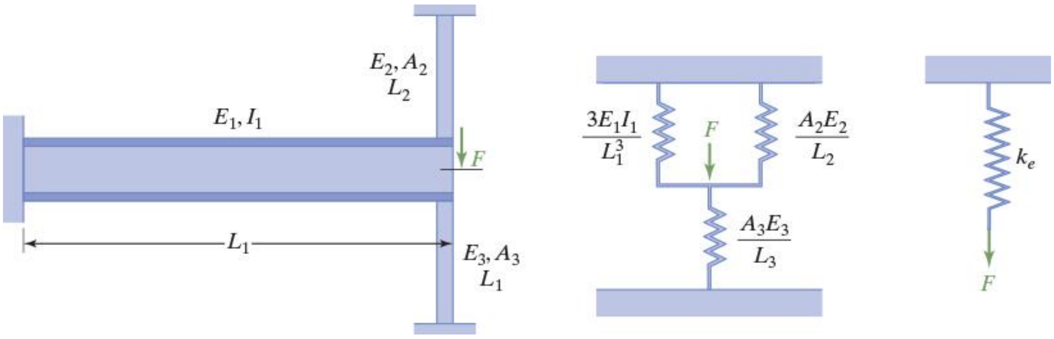
From Table 4.1

$$X = \frac{FL^3}{3EI}$$

$$F = \frac{3EI}{L^3}x$$

$$\underline{\underline{K_e = \frac{3EI}{L^3}}} \Leftarrow$$

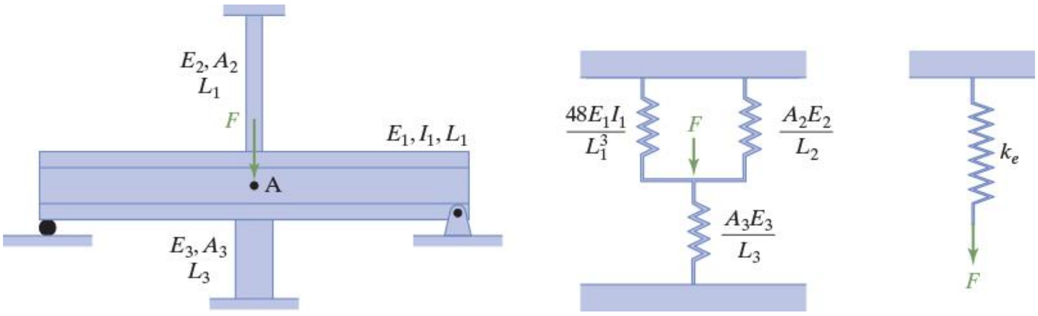
1.28



$$\underline{\underline{K_e = \frac{3E_1I_1}{L_1^3} + \frac{A_2E_2}{L_2} + \frac{A_3E_3}{L_3}}} \quad \Leftarrow$$

Note, deflection of each spring is the same, therefore, springs are in Parallel.

1.29



From Table 4.1, the K_e for the beam is:

$$(K_e)_{\text{beam}} = \frac{48E_1I_1}{L_1^3}$$

$$K_e = \frac{48E_1I_1}{L_1^3} + \frac{A_2E_2}{L_2} + \frac{A_3E_3}{L_3}$$

Note, deflection of each spring is the same, therefore, springs are in parallel.

1.30

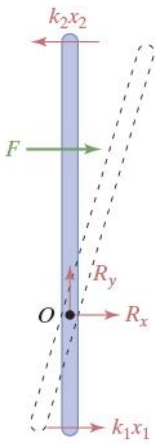
$$\sum M_0 = 0$$

$$-FL + (K_2 x_2)(2L) + (K_1 x_1)(L) = 0$$

Note $x_2 = 2x_1$ and simplify

$$(2K_2 x_1)(2L) + K_1 x_1 L = FL$$

$$\underline{\underline{x_1 = \frac{F}{4K_2 + K_1}}} \quad \Leftarrow$$



$$\pi = \sum_{e=1}^{n} \Lambda^{(e)} - \sum F_i u_i$$

$$\pi = \frac{1}{2}K_2 x_2^2 + \frac{1}{2}K_1 x_1^2 - Fx_1 = \frac{1}{2}K_2 (2x_1)^2 + \frac{1}{2}K_1 x_1^2 - Fx_1$$

$$\pi = 2K_2 x_1^2 + \frac{1}{2}K_1 x_1^2 - Fx_1$$

$$\frac{\partial \pi}{\partial x_1} = 4K_2 x_1 + K_1 x_1 - F = 0$$

$$\underline{\underline{x_1 = \frac{F}{4K_2 + K_1}}} \quad \Leftarrow$$