Trigonometry 11th edition Sullivan Solutions Manual

SKU: 9780135181133-SOLUTIONS

Chapter 1

Graphs and Functions

Section 1.1

1. 0

2. $|5-(-3)| = |8| = 8$

3. $\sqrt{3^2+4^2} = \sqrt{25} = 5$

4. $11^2+60^2 = 121+3600 = 3721 = 61^2$
Since the sum of the squares of two of the sides of the triangle equals the square of the third side, the triangle is a right triangle.

5. $\frac{1}{2}bh$

6. true

7. x-coordinate or abscissa; y-coordinate or ordinate

8. quadrants

9. midpoint

10. False; the distance between two points is never negative.

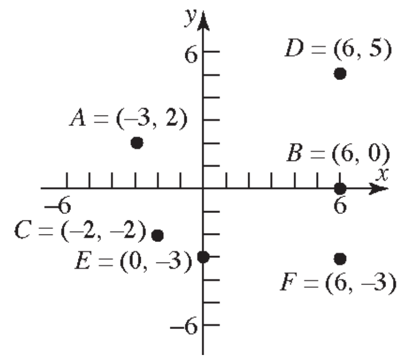
11. False; points that lie in quadrant IV will have a positive x-coordinate and a negative y-coordinate. The point $(-1, 4)$ lies in quadrant II.

12. True; $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

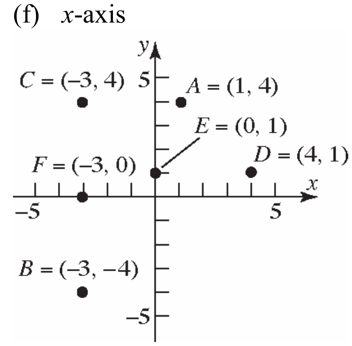
13. b

14. a

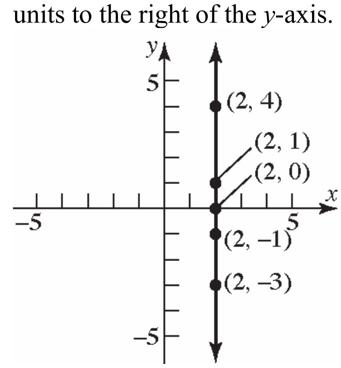
15. (a) Quadrant II
(b) x-axis
(c) Quadrant III
(d) Quadrant I
(e) y-axis
(f) Quadrant IV



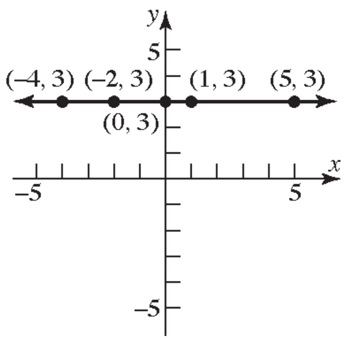
16. (a) Quadrant I
(b) Quadrant III
(c) Quadrant II
(d) Quadrant I
(e) y-axis
(f) x-axis



17. The points will be on a vertical line that is two units to the right of the y-axis.



18. The points will be on a horizontal line that is three units above the x-axis.



19. $d(P_1,P_2)=\sqrt{(2-0)^2+(1-0)^2}$
$=\sqrt{2^2+1^2}=\sqrt{4+1}=\sqrt{5}$

20. $d(P_1,P_2)=\sqrt{(-2-0)^2+(1-0)^2}$
$=\sqrt{(-2)^2+1^2}=\sqrt{4+1}=\sqrt{5}$

21. $d(P_1,P_2)=\sqrt{(-2-1)^2+(2-1)^2}$
$=\sqrt{(-3)^2+1^2}=\sqrt{9+1}=\sqrt{10}$

22. $d(P_1,P_2)=\sqrt{\left(2-(-1)\right)^2+(2-1)^2}$
$=\sqrt{3^2+1^2}=\sqrt{9+1}=\sqrt{10}$

23. $d(P_1,P_2)=\sqrt{(5-3)^2+\left(4-(-4)\right)^2}$
$=\sqrt{2^2+(8)^2}=\sqrt{4+64}=\sqrt{68}=2\sqrt{17}$

24. $d(P_1,P_2)=\sqrt{\left(2-(-1)\right)^2+(4-0)^2}$
$=\sqrt{(3)^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$

25. $d(P_1,P_2)=\sqrt{\left(4-(-7)\right)^2+(0-3)^2}$
$=\sqrt{11^2+(-3)^2}=\sqrt{121+9}=\sqrt{130}$

26. $d(P_1,P_2)=\sqrt{(4-2)^2+\left(2-(-3)\right)^2}$
$=\sqrt{2^2+5^2}=\sqrt{4+25}=\sqrt{29}$

27. $d(P_1,P_2)=\sqrt{(6-5)^2+\left(1-(-2)\right)^2}$
$=\sqrt{1^2+3^2}=\sqrt{1+9}=\sqrt{10}$

28. $d(P_1,P_2)=\sqrt{\left(6-(-4)\right)^2+\left(2-(-3)\right)^2}$
$=\sqrt{10^2+5^2}=\sqrt{100+25}$
$=\sqrt{125}=5\sqrt{5}$

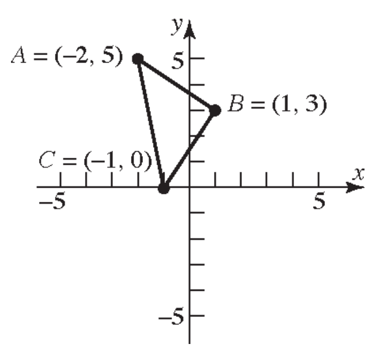
29. $d(P_1,P_2)=\sqrt{\left(2.3-(-0.2)\right)^2+\left(1.1-(0.3)\right)^2}$
$=\sqrt{2.5^2+0.8^2}=\sqrt{6.25+0.64}$
$=\sqrt{6.89}\approx 2.62$

30. $d(P_1,P_2)=\sqrt{\left(-0.3-1.2\right)^2+\left(1.1-2.3\right)^2}$
$=\sqrt{(-1.5)^2+(-1.2)^2}=\sqrt{2.25+1.44}$
$=\sqrt{3.69}\approx 1.92$

31. $d(P_1,P_2)=\sqrt{(0-a)^2+(0-b)^2}$
$=\sqrt{(-a)^2+(-b)^2}=\sqrt{a^2+b^2}$

32. $d(P_1,P_2)=\sqrt{(0-a)^2+(0-a)^2}$
$=\sqrt{(-a)^2+(-a)^2}$
$=\sqrt{a^2+a^2}=\sqrt{2a^2}=|a|\sqrt{2}$

33. $A=(-2,5),\ \ B=(1,3),\ \ C=(-1,0)$
$d(A,B)=\sqrt{\left(1-(-2)\right)^2+(3-5)^2}$
$=\sqrt{3^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$
$d(B,C)=\sqrt{\left(-1-1\right)^2+(0-3)^2}$
$=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$
$d(A,C)=\sqrt{\left(-1-(-2)\right)^2+(0-5)^2}$
$=\sqrt{1^2+(-5)^2}=\sqrt{1+25}=\sqrt{26}$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

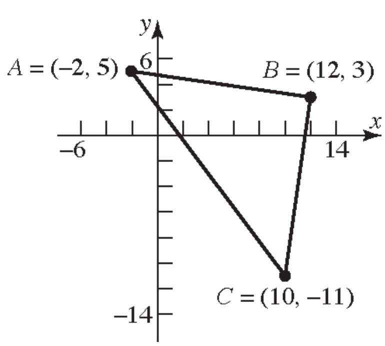
$$[d(A,B)]^2+[d(B,C)]^2=[d(A,C)]^2$$
$$\left(\sqrt{13}\right)^2+\left(\sqrt{13}\right)^2=\left(\sqrt{26}\right)^2$$
$$13+13=26$$
$$26=26$$

The area of a triangle is $A=\frac{1}{2}\cdot bh$. In this problem,

$$A=\frac{1}{2}\cdot[d(A,B)]\cdot[d(B,C)]$$
$$=\frac{1}{2}\cdot\sqrt{13}\cdot\sqrt{13}=\frac{1}{2}\cdot 13$$
$$=\frac{13}{2}\text{ square units}$$

34. $A=(-2,5),\ B=(12,3),\ C=(10,-11)$

$$d(A,B)=\sqrt{(12-(-2))^2+(3-5)^2}$$
$$=\sqrt{14^2+(-2)^2}$$
$$=\sqrt{196+4}=\sqrt{200}$$
$$=10\sqrt{2}$$
$$d(B,C)=\sqrt{(10-12)^2+(-11-3)^2}$$
$$=\sqrt{(-2)^2+(-14)^2}$$
$$=\sqrt{4+196}=\sqrt{200}$$
$$=10\sqrt{2}$$
$$d(A,C)=\sqrt{(10-(-2))^2+(-11-5)^2}$$
$$=\sqrt{12^2+(-16)^2}$$
$$=\sqrt{144+256}=\sqrt{400}$$
$$=20$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

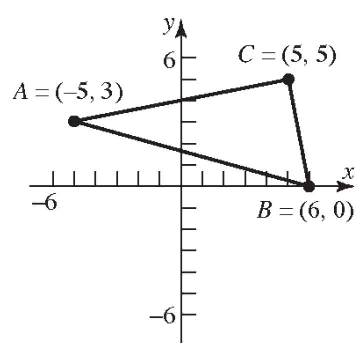
$$[d(A,B)]^2+[d(B,C)]^2=[d(A,C)]^2$$
$$\left(10\sqrt{2}\right)^2+\left(10\sqrt{2}\right)^2=(20)^2$$
$$200+200=400$$
$$400=400$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

$$A=\frac{1}{2}\cdot[d(A,B)]\cdot[d(B,C)]$$
$$=\frac{1}{2}\cdot 10\sqrt{2}\cdot 10\sqrt{2}$$
$$=\frac{1}{2}\cdot 100\cdot 2=100\text{ square units}$$

35. $A=(-5,3),\ B=(6,0),\ C=(5,5)$

$$d(A,B)=\sqrt{(6-(-5))^2+(0-3)^2}$$
$$=\sqrt{11^2+(-3)^2}=\sqrt{121+9}$$
$$=\sqrt{130}$$
$$d(B,C)=\sqrt{(5-6)^2+(5-0)^2}$$
$$=\sqrt{(-1)^2+5^2}=\sqrt{1+25}$$
$$=\sqrt{26}$$
$$d(A,C)=\sqrt{(5-(-5))^2+(5-3)^2}$$
$$=\sqrt{10^2+2^2}=\sqrt{100+4}$$
$$=\sqrt{104}$$
$$=2\sqrt{26}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

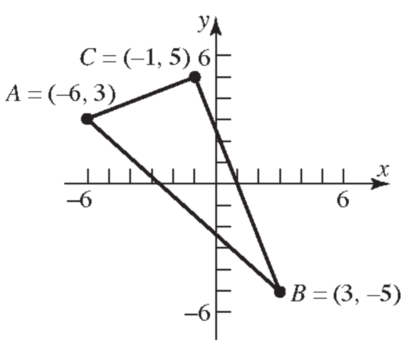
$$[d(A,C)]^2 + [d(B,C)]^2 = [d(A,B)]^2$$
$$\left(\sqrt{104}\right)^2 + \left(\sqrt{26}\right)^2 = \left(\sqrt{130}\right)^2$$
$$104 + 26 = 130$$
$$130 = 130$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2}\cdot[d(A,C)]\cdot[d(B,C)]$$
$$= \frac{1}{2}\cdot\sqrt{104}\cdot\sqrt{26}$$
$$= \frac{1}{2}\cdot 2\sqrt{26}\cdot\sqrt{26}$$
$$= \frac{1}{2}\cdot 2\cdot 26$$
$$= 26 \text{ square units}$$

36. $A = (-6, 3),\ B = (3, -5),\ C = (-1, 5)$

$$d(A,B) = \sqrt{(3-(-6))^2 + (-5-3)^2}$$
$$= \sqrt{9^2 + (-8)^2} = \sqrt{81+64}$$
$$= \sqrt{145}$$
$$d(B,C) = \sqrt{(-1-3)^2 + (5-(-5))^2}$$
$$= \sqrt{(-4)^2 + 10^2} = \sqrt{16+100}$$
$$= \sqrt{116} = 2\sqrt{29}$$
$$d(A,C) = \sqrt{(-1-(-6))^2 + (5-3)^2}$$
$$= \sqrt{5^2 + 2^2} = \sqrt{25+4}$$
$$= \sqrt{29}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

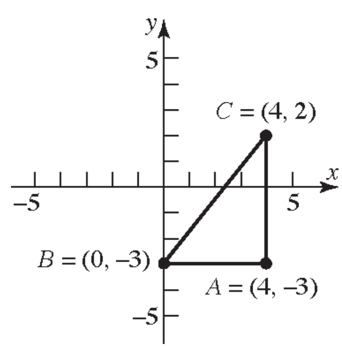
$$[d(A,C)]^2 + [d(B,C)]^2 = [d(A,B)]^2$$
$$\left(\sqrt{29}\right)^2 + \left(2\sqrt{29}\right)^2 = \left(\sqrt{145}\right)^2$$
$$29 + 4\cdot 29 = 145$$
$$29 + 116 = 145$$
$$145 = 145$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2}\cdot[d(A,C)]\cdot[d(B,C)]$$
$$= \frac{1}{2}\cdot\sqrt{29}\cdot 2\sqrt{29}$$
$$= \frac{1}{2}\cdot 2\cdot 29$$
$$= 29 \text{ square units}$$

37. $A = (4, -3),\ B = (0, -3),\ C = (4, 2)$

$$d(A,B) = \sqrt{(0-4)^2 + (-3-(-3))^2}$$
$$= \sqrt{(-4)^2 + 0^2} = \sqrt{16+0}$$
$$= \sqrt{16}$$
$$= 4$$
$$d(B,C) = \sqrt{(4-0)^2 + (2-(-3))^2}$$
$$= \sqrt{4^2 + 5^2} = \sqrt{16+25}$$
$$= \sqrt{41}$$
$$d(A,C) = \sqrt{(4-4)^2 + (2-(-3))^2}$$
$$= \sqrt{0^2 + 5^2} = \sqrt{0+25}$$
$$= \sqrt{25}$$
$$= 5$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$[d(A,B)]^2 + [d(A,C)]^2 = [d(B,C)]^2$$
$$4^2 + 5^2 = \left(\sqrt{41}\right)^2$$
$$16 + 25 = 41$$
$$41 = 41$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

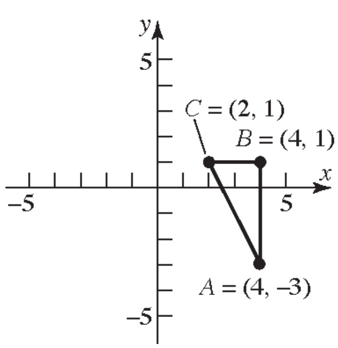
$$A = \frac{1}{2}\cdot[d(A,B)]\cdot[d(A,C)]$$
$$= \frac{1}{2}\cdot 4\cdot 5$$
$$= 10 \text{ square units}$$

38. $A = (4,-3),\ B = (4,1),\ C = (2,1)$

$$d(A,B) = \sqrt{(4-4)^2 + (1-(-3))^2}$$
$$= \sqrt{0^2 + 4^2}$$
$$= \sqrt{0+16}$$
$$= \sqrt{16}$$
$$= 4$$

$$d(B,C) = \sqrt{(2-4)^2 + (1-1)^2}$$
$$= \sqrt{(-2)^2 + 0^2} = \sqrt{4+0}$$
$$= \sqrt{4}$$
$$= 2$$

$$d(A,C) = \sqrt{(2-4)^2 + (1-(-3))^2}$$
$$= \sqrt{(-2)^2 + 4^2} = \sqrt{4+16}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$[d(A,B)]^2 + [d(B,C)]^2 = [d(A,C)]^2$$
$$4^2 + 2^2 = \left(2\sqrt{5}\right)^2$$
$$16 + 4 = 20$$
$$20 = 20$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2}\cdot[d(A,B)]\cdot[d(B,C)]$$
$$= \frac{1}{2}\cdot 4\cdot 2$$
$$= 4 \text{ square units}$$

39. The coordinates of the midpoint are:

$$(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{3+5}{2}, \frac{-4+4}{2}\right)$$
$$= \left(\frac{8}{2}, \frac{0}{2}\right)$$
$$= (4,0)$$

40. The coordinates of the midpoint are:

$$(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{-2+2}{2}, \frac{0+4}{2}\right)$$
$$= \left(\frac{0}{2}, \frac{4}{2}\right)$$
$$= (0,2)$$

41. The coordinates of the midpoint are:

$$(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{-1+8}{2}, \frac{4+0}{2}\right)$$
$$= \left(\frac{7}{2}, \frac{4}{2}\right)$$
$$= \left(\frac{7}{2}, 2\right)$$

42. The coordinates of the midpoint are:

$$\begin{aligned}(x,y)&=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\&=\left(\frac{2+4}{2},\frac{-3+2}{2}\right)\\&=\left(\frac{6}{2},\frac{-1}{2}\right)\\&=\left(3,-\frac{1}{2}\right)\end{aligned}$$

43. The coordinates of the midpoint are:

$$\begin{aligned}(x,y)&=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\&=\left(\frac{7+9}{2},\frac{-5+1}{2}\right)\\&=\left(\frac{16}{2},\frac{-4}{2}\right)\\&=(8,-2)\end{aligned}$$

44. The coordinates of the midpoint are:

$$\begin{aligned}(x,y)&=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\&=\left(\frac{-4+2}{2},\frac{-3+2}{2}\right)\\&=\left(\frac{-2}{2},\frac{-1}{2}\right)\\&=\left(-1,-\frac{1}{2}\right)\end{aligned}$$

45. The coordinates of the midpoint are:

$$\begin{aligned}(x,y)&=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\&=\left(\frac{a+0}{2},\frac{b+0}{2}\right)\\&=\left(\frac{a}{2},\frac{b}{2}\right)\end{aligned}$$

46. The coordinates of the midpoint are:

$$\begin{aligned}(x,y)&=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\&=\left(\frac{a+0}{2},\frac{a+0}{2}\right)\\&=\left(\frac{a}{2},\frac{a}{2}\right)\end{aligned}$$

47. The x coordinate would be $2+3=5$ and the y coordinate would be $5-2=3$. Thus the new point would be $(5,3)$.

48. The new x coordinate would be $-1-2=-3$ and the new y coordinate would be $6+4=10$. Thus the new point would be $(-3,10)$

49. **a.** If we use a right triangle to solve the problem, we know the hypotenuse is 13 units in length. One of the legs of the triangle will be 2+3=5. Thus the other leg will be:

$$\begin{aligned}5^2+b^2&=13^2\\25+b^2&=169\\b^2&=144\\b&=12\end{aligned}$$

Thus the coordinates will have an y value of $-1-12=-13$ and $-1+12=11$. So the points are $(3,11)$ and $(3,-13)$.

b. Consider points of the form $(3,y)$ that are a distance of 13 units from the point $(-2,-1)$.

$$\begin{aligned}d&=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\\&=\sqrt{(3-(-2))^2+(-1-y)^2}\\&=\sqrt{(5)^2+(-1-y)^2}\\&=\sqrt{25+1+2y+y^2}\\&=\sqrt{y^2+2y+26}\\13&=\sqrt{y^2+2y+26}\\13^2&=\left(\sqrt{y^2+2y+26}\right)^2\\169&=y^2+2y+26\\0&=y^2+2y-143\\0&=(y-11)(y+13)\end{aligned}$$

$$y-11=0 \quad\text{or}\quad y+13=0$$
$$y=11 \qquad\qquad y=-13$$

Thus, the points $(3,11)$ and $(3,-13)$ are a distance of 13 units from the point $(-2,-1)$.

50. **a.** If we use a right triangle to solve the problem, we know the hypotenuse is 17 units in length. One of the legs of the triangle will be 2+6=8. Thus the other leg will be:

$$\begin{aligned} 8^2+b^2 &= 17^2 \\ 64+b^2 &= 289 \\ b^2 &= 225 \\ b &= 15 \end{aligned}$$

Thus the coordinates will have an x value of $1-15=-14$ and $1+15=16$. So the points are $(-14,-6)$ and $(16,-6)$.

b. Consider points of the form $(x,-6)$ that are a distance of 17 units from the point $(1,2)$.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(1-x)^2+(2-(-6))^2} \\ &= \sqrt{x^2-2x+1+(8)^2} \\ &= \sqrt{x^2-2x+1+64} \\ &= \sqrt{x^2-2x+65} \\ 17 &= \sqrt{x^2-2x+65} \\ 17^2 &= \left(\sqrt{x^2-2x+65}\right)^2 \\ 289 &= x^2-2x+65 \\ 0 &= x^2-2x-224 \\ 0 &= (x+14)(x-16) \end{aligned}$$

$$x+14=0 \quad \text{or} \quad x-16=0$$
$$x=-14 \qquad\qquad x=16$$

Thus, the points $(-14,-6)$ and $(16,-6)$ are a distance of 13 units from the point $(1,2)$.

51. Points on the x-axis have a y-coordinate of 0. Thus, we consider points of the form $(x,0)$ that are a distance of 6 units from the point $(4,-3)$.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(4-x)^2+(-3-0)^2} \\ &= \sqrt{16-8x+x^2+(-3)^2} \\ &= \sqrt{16-8x+x^2+9} \\ &= \sqrt{x^2-8x+25} \\ 6 &= \sqrt{x^2-8x+25} \\ 6^2 &= \left(\sqrt{x^2-8x+25}\right)^2 \\ 36 &= x^2-8x+25 \\ 0 &= x^2-8x-11 \\ x &= \frac{-(-8)\pm\sqrt{(-8)^2-4(1)(-11)}}{2(1)} \\ &= \frac{8\pm\sqrt{64+44}}{2} = \frac{8\pm\sqrt{108}}{2} \\ &= \frac{8\pm 6\sqrt{3}}{2} = 4\pm 3\sqrt{3} \end{aligned}$$

$$x=4+3\sqrt{3} \quad \text{or} \quad x=4-3\sqrt{3}$$

Thus, the points $\left(4+3\sqrt{3},0\right)$ and $\left(4-3\sqrt{3},0\right)$ are on the x-axis and a distance of 6 units from the point $(4,-3)$.

52. Points on the y-axis have an x-coordinate of 0. Thus, we consider points of the form $(0,y)$ that are a distance of 6 units from the point $(4,-3)$.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(4-0)^2+(-3-y)^2} \\ &= \sqrt{4^2+9+6y+y^2} \\ &= \sqrt{16+9+6y+y^2} \\ &= \sqrt{y^2+6y+25} \end{aligned}$$

$$6 = \sqrt{y^2 + 6y + 25}$$
$$6^2 = \left(\sqrt{y^2 + 6y + 25}\right)^2$$
$$36 = y^2 + 6y + 25$$
$$0 = y^2 + 6y - 11$$
$$y = \frac{(-6) \pm \sqrt{(6)^2 - 4(1)(-11)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{36+44}}{2} = \frac{-6 \pm \sqrt{80}}{2}$$
$$= \frac{-6 \pm 4\sqrt{5}}{2} = -3 \pm 2\sqrt{5}$$
$$y = -3 + 2\sqrt{5} \quad \text{or} \quad y = -3 - 2\sqrt{5}$$

Thus, the points $\left(0, -3 + 2\sqrt{5}\right)$ and $\left(0, -3 - 2\sqrt{5}\right)$ are on the y-axis and a distance of 6 units from the point $(4, -3)$.

53. **a.** To shift 3 units left and 4 units down, we subtract 3 from the x-coordinate and subtract 4 from the y-coordinate.
$$\left(2-3, 5-4\right) = \left(-1, 1\right)$$

b. To shift left 2 units and up 8 units, we subtract 2 from the x-coordinate and add 8 to the y-coordinate.
$$\left(2-2, 5+8\right) = \left(0, 13\right)$$

54. Let the coordinates of point B be (x, y). Using the midpoint formula, we can write
$$(2,3) = \left(\frac{-1+x}{2}, \frac{8+y}{2}\right).$$
This leads to two equations we can solve.
$$\frac{-1+x}{2} = 2 \qquad \frac{8+y}{2} = 3$$
$$-1 + x = 4 \qquad 8 + y = 6$$
$$x = 5 \qquad y = -2$$
Point B has coordinates $(5, -2)$.

55. $M = (x, y) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

$P_1 = (x_1, y_1) = (-3, 6)$ and $(x, y) = (-1, 4)$, so
$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$
$$-1 = \frac{-3 + x_2}{2} \qquad 4 = \frac{6 + y_2}{2}$$
$$-2 = -3 + x_2 \qquad 8 = 6 + y_2$$
$$1 = x_2 \qquad 2 = y_2$$
Thus, $P_2 = (1, 2)$.

56. $M = (x, y) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

$P_2 = (x_2, y_2) = (7, -2)$ and $(x, y) = (5, -4)$, so
$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$
$$5 = \frac{x_1 + 7}{2} \qquad -4 = \frac{y_1 + (-2)}{2}$$
$$10 = x_1 + 7 \qquad -8 = y_1 + (-2)$$
$$3 = x_1 \qquad -6 = y_1$$
Thus, $P_1 = (3, -6)$.

57. The midpoint of AB is: $D = \left(\dfrac{0+6}{2}, \dfrac{0+0}{2}\right) = (3, 0)$

The midpoint of AC is: $E = \left(\dfrac{0+4}{2}, \dfrac{0+4}{2}\right) = (2, 2)$

The midpoint of BC is: $F = \left(\dfrac{6+4}{2}, \dfrac{0+4}{2}\right) = (5, 2)$

$$d(C, D) = \sqrt{(0-4)^2 + (3-4)^2}$$
$$= \sqrt{(-4)^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}$$
$$d(B, E) = \sqrt{(2-6)^2 + (2-0)^2}$$
$$= \sqrt{(-4)^2 + 2^2} = \sqrt{16+4}$$
$$= \sqrt{20} = 2\sqrt{5}$$
$$d(A, F) = \sqrt{(2-0)^2 + (5-0)^2}$$
$$= \sqrt{2^2 + 5^2} = \sqrt{4+25}$$
$$= \sqrt{29}$$

58. Let $P_1 = (0, 0)$, $P_2 = (0, 4)$, $P = (x, y)$

$$d(P_1, P_2) = \sqrt{(0-0)^2 + (4-0)^2} = \sqrt{16} = 4$$

$$d(P_1, P) = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2} = 4 \rightarrow x^2 + y^2 = 16$$

$$d(P_2, P) = \sqrt{(x-0)^2 + (y-4)^2} = \sqrt{x^2 + (y-4)^2} = 4 \rightarrow x^2 + (y-4)^2 = 16$$

Therefore,

$$y^2 = (y-4)^2$$
$$y^2 = y^2 - 8y + 16$$
$$8y = 16$$
$$y = 2$$

which gives

$$x^2 + 2^2 = 16$$
$$x^2 = 12$$
$$x = \pm 2\sqrt{3}$$

Two triangles are possible. The third vertex is $\left(-2\sqrt{3}, 2\right)$ or $\left(2\sqrt{3}, 2\right)$.

59.
$$d(P_1, P_2) = \sqrt{(-4-2)^2 + (1-1)^2} = \sqrt{(-6)^2 + 0^2} = \sqrt{36} = 6$$

$$d(P_2, P_3) = \sqrt{\left(-4-(-4)\right)^2 + (-3-1)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$$

$$d(P_1, P_3) = \sqrt{(-4-2)^2 + (-3-1)^2} = \sqrt{(-6)^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

Since $\left[d(P_1, P_2)\right]^2 + \left[d(P_2, P_3)\right]^2 = \left[d(P_1, P_3)\right]^2$, the triangle is a right triangle.

60.
$$d(P_1, P_2) = \sqrt{\left(6-(-1)\right)^2 + (2-4)^2} = \sqrt{7^2 + (-2)^2} = \sqrt{49+4} = \sqrt{53}$$

$$d(P_2, P_3) = \sqrt{\left(4-6\right)^2 + (-5-2)^2} = \sqrt{(-2)^2 + (-7)^2} = \sqrt{4+49} = \sqrt{53}$$

$$d(P_1, P_3) = \sqrt{\left(4-(-1)\right)^2 + (-5-4)^2} = \sqrt{5^2 + (-9)^2} = \sqrt{25+81} = \sqrt{106}$$

Since $\left[d(P_1, P_2)\right]^2 + \left[d(P_2, P_3)\right]^2 = \left[d(P_1, P_3)\right]^2$, the triangle is a right triangle.

Since $d\left(P_1, P_2\right) = d\left(P_2, P_3\right)$, the triangle is isosceles.

Therefore, the triangle is an isosceles right triangle.

61.
$$d(P_1, P_2) = \sqrt{\left(0-(-2)\right)^2 + \left(7-(-1)\right)^2} = \sqrt{2^2 + 8^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$$

$$d(P_2, P_3) = \sqrt{\left(3-0\right)^2 + (2-7)^2} = \sqrt{3^2 + (-5)^2} = \sqrt{9+25} = \sqrt{34}$$

$$d(P_1, P_3) = \sqrt{\left(3-(-2)\right)^2 + \left(2-(-1)\right)^2} = \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}$$

Since $d(P_2, P_3) = d(P_1, P_3)$, the triangle is isosceles.

Since $\left[d(P_1, P_3)\right]^2 + \left[d(P_2, P_3)\right]^2 = \left[d(P_1, P_2)\right]^2$, the triangle is also a right triangle.

Therefore, the triangle is an isosceles right triangle.

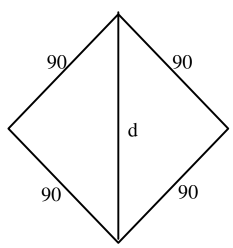
62. $d(P_1,P_2)=\sqrt{(-4-7)^2+(0-2)^2}$
$=\sqrt{(-11)^2+(-2)^2}$
$=\sqrt{121+4}=\sqrt{125}$
$=5\sqrt{5}$

$d(P_2,P_3)=\sqrt{(4-(-4))^2+(6-0)^2}$
$=\sqrt{8^2+6^2}=\sqrt{64+36}$
$=\sqrt{100}$
$=10$

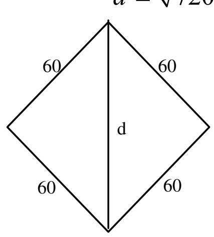
$d(P_1,P_3)=\sqrt{(4-7)^2+(6-2)^2}$
$=\sqrt{(-3)^2+4^2}=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

Since $\left[d(P_1,P_3)\right]^2+\left[d(P_2,P_3)\right]^2=\left[d(P_1,P_2)\right]^2$, the triangle is a right triangle.

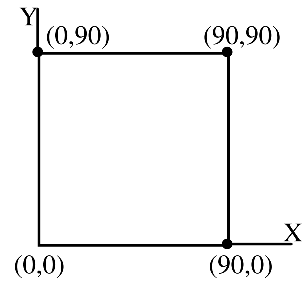
63. Using the Pythagorean Theorem:
$90^2+90^2=d^2$
$8100+8100=d^2$
$16200=d^2$
$d=\sqrt{16200}=90\sqrt{2}\approx 127.28$ feet



64. Using the Pythagorean Theorem:
$60^2+60^2=d^2$
$3600+3600=d^2\rightarrow 7200=d^2$
$d=\sqrt{7200}=60\sqrt{2}\approx 84.85$ feet



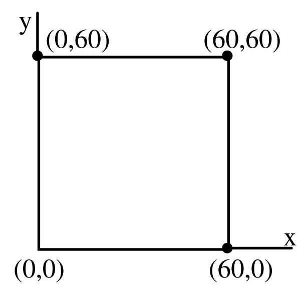
65. a. First: (90, 0), Second: (90, 90), Third: (0, 90)



b. Using the distance formula:
$d=\sqrt{(310-90)^2+(15-90)^2}$
$=\sqrt{220^2+(-75)^2}=\sqrt{54025}$
$=5\sqrt{2161}\approx 232.43$ feet

c. Using the distance formula:
$d=\sqrt{(300-0)^2+(300-90)^2}$
$=\sqrt{300^2+210^2}=\sqrt{134100}$
$=30\sqrt{149}\approx 366.20$ feet

66. a. First: (60, 0), Second: (60, 60) Third: (0, 60)

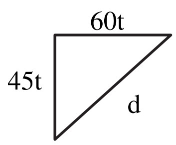


b. Using the distance formula:
$d=\sqrt{(180-60)^2+(20-60)^2}$
$=\sqrt{120^2+(-40)^2}=\sqrt{16000}$
$=40\sqrt{10}\approx 126.49$ feet

c. Using the distance formula:
$d=\sqrt{(220-0)^2+(220-60)^2}$
$=\sqrt{220^2+160^2}=\sqrt{74000}$
$=20\sqrt{185}\approx 272.03$ feet

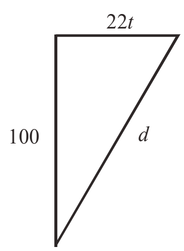
67. The Focus heading east moves a distance $60t$ after t hours. The truck heading south moves a distance $40t$ after t hours. Their distance apart after t hours is:

$$\begin{aligned} d &= \sqrt{(60t)^2 + (45t)^2} \\ &= \sqrt{3600t^2 + 2025t^2} \\ &= \sqrt{5625t^2} \\ &= 75t \text{ miles} \end{aligned}$$



68. $\dfrac{15 \text{ miles}}{1 \text{ hr}} \cdot \dfrac{5280 \text{ ft}}{1 \text{ mile}} \cdot \dfrac{1 \text{ hr}}{3600 \text{ sec}} = 22 \text{ ft/sec}$

$$\begin{aligned} d &= \sqrt{100^2 + (22t)^2} \\ &= \sqrt{10000 + 484t^2} \text{ feet} \end{aligned}$$



69. **a.** The shortest side is between $P_1 = (2.6, 1.5)$ and $P_2 = (2.7, 1.7)$. The estimate for the desired intersection point is:

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{2.6 + 2.7}{2}, \frac{1.5 + 1.7}{2}\right) \\ &= \left(\frac{5.3}{2}, \frac{3.2}{2}\right) \\ &= (2.65, 1.6) \end{aligned}$$

b. Using the distance formula:

$$\begin{aligned} d &= \sqrt{(2.65 - 1.4)^2 + (1.6 - 1.3)^2} \\ &= \sqrt{(1.25)^2 + (0.3)^2} \\ &= \sqrt{1.5625 + 0.09} \\ &= \sqrt{1.6525} \\ &\approx 1.285 \text{ units} \end{aligned}$$

70. Let $P_1 = (2013, 102.87)$ and $P_2 = (2017, 126.17)$. The midpoint is:

$$\begin{aligned} (x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{2013 + 2017}{2}, \frac{102.87 + 126.17}{2}\right) \\ &= \left(\frac{4030}{2}, \frac{229.04}{2}\right) \\ &= (2015, 114.52) \end{aligned}$$

The estimate for 2010 is \$114.52 billion. The estimate net sales of Costco Wholesale Corporation in 2015 is \$0.85 billion off from the reported value of \$113.67 billion.

71. For 2009 we have the ordered pair $(2009, 21756)$ and for 2017 we have the ordered pair $(2017, 24858)$. The midpoint is

$$\begin{aligned} (\text{year}, \$) &= \left(\frac{2009 + 2017}{2}, \frac{21756 + 24858}{2}\right) \\ &= \left(\frac{4026}{2}, \frac{46614}{2}\right) \\ &= (2013, 23307) \end{aligned}$$

Using the midpoint, we estimate the poverty level in 2013 to be \$23,307. This is lower than the actual value.

72. Let $P_1 = (0, 0)$, $P_2 = (a, 0)$, and $P_3 = \left(\dfrac{a}{2}, \dfrac{\sqrt{3}\,a}{2}\right)$. Then

$$\begin{aligned} d(P_1, P_2) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(a - 0)^2 + (0 - 0)^2} = \sqrt{a^2} = |a| \\ d(P_2, P_3) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(\frac{a}{2} - a\right)^2 + \left(\frac{\sqrt{3}\,a}{2} - 0\right)^2} \\ &= \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{4a^2}{4}} = \sqrt{a^2} = |a| \end{aligned}$$

$$d(P_1,P_3)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{\left(\frac{a}{2}-0\right)^2+\left(\frac{\sqrt{3}\,a}{2}-0\right)^2}$$
$$=\sqrt{\frac{a^2}{4}+\frac{3a^2}{4}}=\sqrt{\frac{4a^2}{4}}=\sqrt{a^2}=|a|$$

Since the lengths of the three sides are all equal, the triangle is an equilateral triangle.
The midpoints of the saids are

$$M_{P_1P_2}=\left(\frac{0+a}{2},\frac{0+0}{2}\right)=\left(\frac{a}{2},0\right)$$

$$M_{P_2P_3}=\left(\frac{a+\frac{a}{2}}{2},\frac{0+\frac{\sqrt{3}\,a}{2}}{2}\right)=\left(\frac{3a}{4},\frac{\sqrt{3}\,a}{4}\right)$$

$$M_{P_1P_3}=\left(\frac{0+\frac{a}{2}}{2},\frac{0+\frac{\sqrt{3}a}{2}}{2}\right)=\left(\frac{a}{4},\frac{\sqrt{3}\,a}{4}\right)$$

Then,

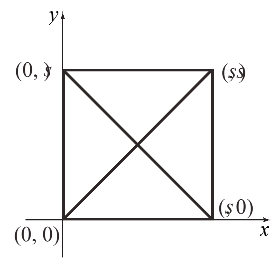
$$d\left(M_{P_1P_2},M_{P_2P_3}\right)=\sqrt{\left(\frac{3a}{4}-\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}\,a}{4}-0\right)^2}$$
$$=\sqrt{\left(\frac{a}{4}\right)^2+\left(\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$=\sqrt{\frac{a^2}{16}+\frac{3a^2}{16}}=\frac{|a|}{2}$$

$$d\left(M_{P_2P_3},M_{P_1P_3}\right)=\sqrt{\left(\frac{3a}{4}-\frac{a}{4}\right)^2+\left(\frac{\sqrt{3}\,a}{4}-\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$=\sqrt{\left(\frac{a}{2}\right)^2+0^2}$$
$$=\sqrt{\frac{a^2}{4}}=\frac{|a|}{2}$$

$$d\left(M_{P_1P_2},M_{P_1P_3}\right)=\sqrt{\left(\frac{a}{4}-\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}\,a}{4}-0\right)^2}$$
$$=\sqrt{\left(-\frac{a}{4}\right)^2+\left(\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$=\sqrt{\frac{a^2}{16}+\frac{3a^2}{16}}=\frac{|a|}{2}$$

Since the lengths of the sides of the triangle formed by the midpoints are all equal, the triangle is equilateral.

73. Let $P_1=(0,0)$, $P_2=(0,s)$, $P_3=(s,0)$, and $P_4=(s,s)$ be the vertices of the square.



The points P_1 and P_4 are endpoints of one diagonal and the points P_2 and P_3 are the endpoints of the other diagonal.

$$M_{P_1P_4}=\left(\frac{0+s}{2},\frac{0+s}{2}\right)=\left(\frac{s}{2},\frac{s}{2}\right)$$
$$M_{P_2P_3}=\left(\frac{0+s}{2},\frac{s+0}{2}\right)=\left(\frac{s}{2},\frac{s}{2}\right)$$

The midpoints of the diagonals are the same. Therefore, the diagonals of a square intersect at their midpoints.

74. Let $P=(a,2a)$. Then

$$\sqrt{(a+5)^2+(2a-1)^2}=\sqrt{(a-4)^2+(2a+4)^2}$$
$$(a+5)^2+(2a-1)^2=(a-4)^2+(2a+4)^2$$
$$5a^2+6a+26=5a^2+8a+32$$
$$6a+26=8a+32$$
$$-2a=6$$
$$a=-3$$

Then $P=(-3,-6)$.

75. Arrange the parallelogram on the coordinate plane so that the vertices are
$P_1=(0,0), P_2=(a,0), P_3=(a+b,c)$ and $P_4=(b,c)$
Then the lengths of the sides are:

$$d(P_1,P_2)=\sqrt{(a-0)^2+(0-0)^2}$$
$$=\sqrt{a^2}=|a|$$

$$d(P_2,P_3)=\sqrt{[(a+b)-a]^2+(c-0)^2}$$
$$=\sqrt{b^2+c^2}$$

$$d(P_3,P_4)=\sqrt{[b-(a+b)]^2+(c-c)^2}$$
$$=\sqrt{a^2}=|a|$$

and

$$d(P_1,P_4)=\sqrt{(b-0)^2+(c-0)^2}$$
$$=\sqrt{b^2+c^2}$$

P_1 and P_3 are the endpoints of one diagonal, and P_2 and P_4 are the endpoints of the other diagonal. The lengths of the diagonals are

$$d(P_1,P_3)=\sqrt{[(a+b)-0]^2+(c-0)^2}$$
$$=\sqrt{a^2+2ab+b^2+c^2}$$

and

$$d(P_2,P_4)=\sqrt{(b-a)^2+(c-0)^2}\ .$$
$$=\sqrt{a^2-2ab+b^2+c^2}$$

Sum of the squares of the sides:

$$a^2+(\sqrt{b^2+c^2})^2+a^2+(\sqrt{b^2+c^2})^2$$
$$=2a^2+2b^2+2c^2$$

Sum of the squares of the diagonals:

$$\left(\sqrt{a^2+2ab+b^2+c^2}\right)^2+\left(\sqrt{a^2-2ab+b^2+c^2}\right)^2$$
$$=2a^2+2b^2+2c^2$$

76. Answers will vary.

Section 1.2

1. add; $\left(\frac{1}{2}\cdot 10\right)^2=25$

2. $(x-2)^2=9$

$$x-2=\pm\sqrt{9}$$
$$x-2=\pm 3$$
$$x=2\pm 3$$
$$x=5 \quad \text{or} \quad x=-1$$

The solution set is $\{-1, 5\}$.

3. intercepts

4. y-axis

5 $(-3,4)$

6. True

7. False

8. radius

9. True; $r^2=9\rightarrow r=3$

10. d

11. False; the center of the circle $(x+3)^2+(y-2)^2=13$ is $(-3,2)$.

12. a

13. $y=x^4-\sqrt{x}$

$0=0^4-\sqrt{0}$ $\quad 1=1^4-\sqrt{1}$ $\quad 4=(2)^4-\sqrt{2}$

$0=0$ $\quad 1\neq 0$ $\quad 4\neq 16-\sqrt{2}$

The point (0, 0) is on the graph of the equation.

14. $y=x^3-2\sqrt{x}$

$0=0^3-2\sqrt{0}$ $\quad 1=1^3-2\sqrt{1}$ $\quad -1=1^3-2\sqrt{1}$

$0=0$ $\quad 1\neq -1$ $\quad -1=-1$

The points (0, 0) and (1, –1) are on the graph of the equation.

15. $y^2=x^2+9$

$3^2=0^2+9$ $\quad 0^2=3^2+9$ $\quad 0^2=(-3)^2+9$

$9=9$ $\quad 0\neq 18$ $\quad 0\neq 18$

The point (0, 3) is on the graph of the equation.

16. $y^3=x+1$

$2^3=1+1$ $\quad 1^3=0+1$ $\quad 0^3=-1+1$

$8\neq 2$ $\quad 1=1$ $\quad 0=0$

The points (0, 1) and (–1, 0) are on the graph of the equation.

17. $x^2+y^2=4$

$0^2+2^2=4$ $\quad (-2)^2+2^2=4$ $\quad \left(\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2=4$

$4=4$ $\quad 8\neq 4$ $\quad 4=4$

(0, 2) and $\left(\sqrt{2},\sqrt{2}\right)$ are on the graph of the equation.

18. $x^2 + 4y^2 = 4$

$0^2 + 4 \cdot 1^2 = 4$ $\quad$ $2^2 + 4 \cdot 0^2 = 4$ $\quad$ $2^2 + 4\left(\frac{1}{2}\right)^2 = 4$

$4 = 4$ $\quad$ $4 = 4$ $\quad$ $5 \neq 4$

The points (0, 1) and (2, 0) are on the graph of the equation.

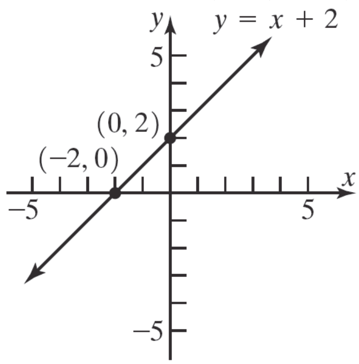
19. $y = x + 2$

x-intercept: $\quad$ y-intercept:

$0 = x + 2$ $\quad$ $y = 0 + 2$

$-2 = x$ $\quad$ $y = 2$

The intercepts are $(-2, 0)$ and $(0, 2)$.



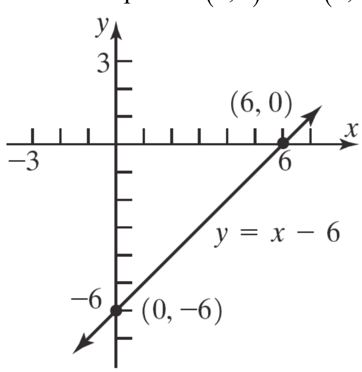
20. $y = x - 6$

x-intercept: $\quad$ y-intercept:

$0 = x - 6$ $\quad$ $y = 0 - 6$

$6 = x$ $\quad$ $y = -6$

The intercepts are $(6, 0)$ and $(0, -6)$.



21. $y = 2x + 8$

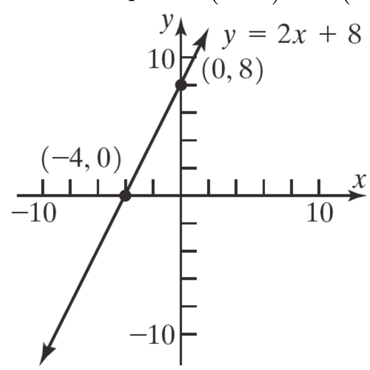
x-intercept: $\quad$ y-intercept:

$0 = 2x + 8$ $\quad$ $y = 2(0) + 8$

$2x = -8$ $\quad$ $y = 8$

$x = -4$

The intercepts are $(-4, 0)$ and $(0, 8)$.



22. $y = 3x - 9$

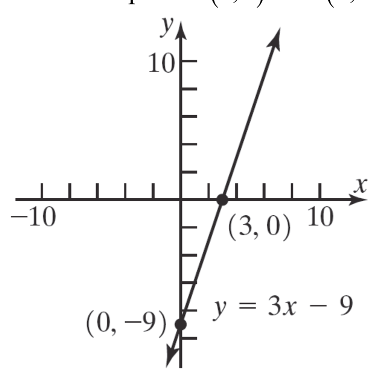
x-intercept: $\quad$ y-intercept:

$0 = 3x - 9$ $\quad$ $y = 3(0) - 9$

$3x = 9$ $\quad$ $y = -9$

$x = 3$

The intercepts are $(3, 0)$ and $(0, -9)$.



23. $y = x^2 - 1$

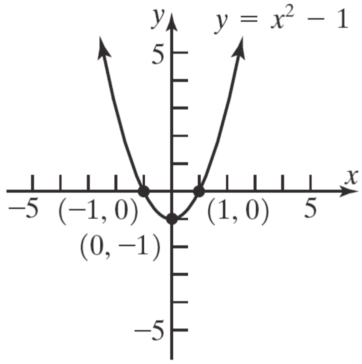
x-intercepts: $\quad$ y-intercept:

$0 = x^2 - 1$ $\quad$ $y = 0^2 - 1$

$x^2 = 1$ $\quad$ $y = -1$

$x = \pm 1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,-1)$.

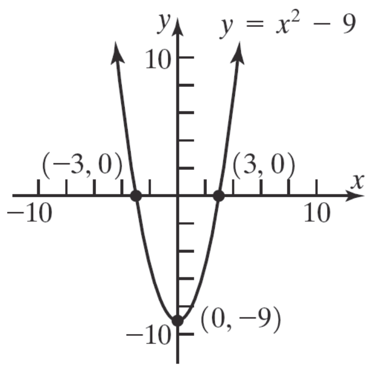


24. $y = x^2 - 9$

x-intercepts:
$$0 = x^2 - 9$$
$$x^2 = 9$$
$$x = \pm 3$$

y-intercept:
$$y = 0^2 - 9$$
$$y = -9$$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,-9)$.

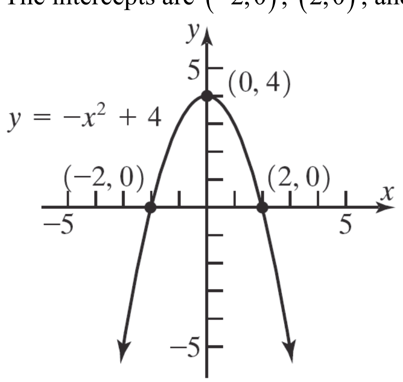


25. $y = -x^2 + 4$

x-intercepts:
$$0 = -x^2 + 4$$
$$x^2 = 4$$
$$x = \pm 2$$

y-intercepts:
$$y = -(0)^2 + 4$$
$$y = 4$$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,4)$.

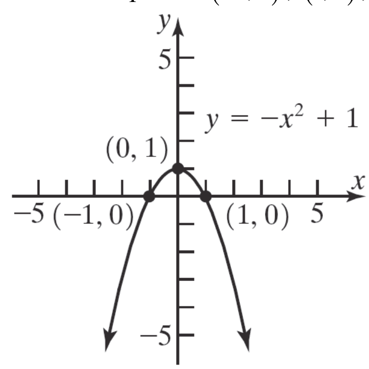


26. $y = -x^2 + 1$

x-intercepts:
$$0 = -x^2 + 1$$
$$x^2 = 1$$
$$x = \pm 1$$

y-intercept:
$$y = -(0)^2 + 1$$
$$y = 1$$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,1)$.

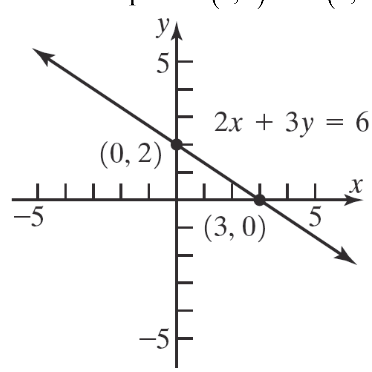


27. $2x + 3y = 6$

x-intercepts:
$$2x + 3(0) = 6$$
$$2x = 6$$
$$x = 3$$

y-intercept:
$$2(0) + 3y = 6$$
$$3y = 6$$
$$y = 2$$

The intercepts are $(3,0)$ and $(0,2)$.

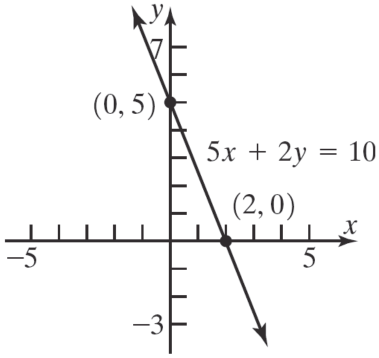


28. $5x + 2y = 10$

x-intercepts:
$$5x + 2(0) = 10$$
$$5x = 10$$
$$x = 2$$

y-intercept:
$$5(0) + 2y = 10$$
$$2y = 10$$
$$y = 5$$

The intercepts are $(2,0)$ and $(0,5)$.



29. $9x^2 + 4y = 36$

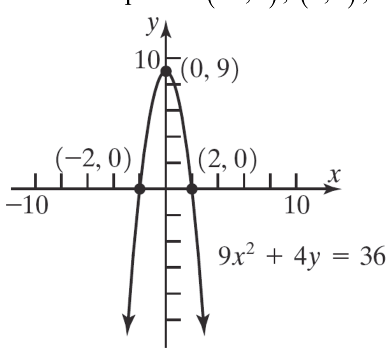
x-intercepts:

$$9x^2 + 4(0) = 36$$
$$9x^2 = 36$$
$$x^2 = 4$$
$$x = \pm 2$$

y-intercept:

$$9(0)^2 + 4y = 36$$
$$4y = 36$$
$$y = 9$$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,9)$.



30. $4x^2 + y = 4$

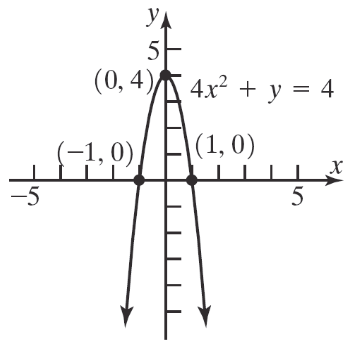
x-intercepts:

$$4x^2 + 0 = 4$$
$$4x^2 = 4$$
$$x^2 = 1$$
$$x = \pm 1$$

y-intercept:

$$4(0)^2 + y = 4$$
$$y = 4$$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,4)$.



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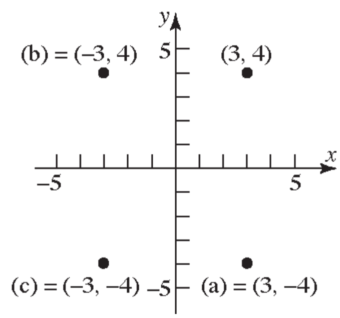

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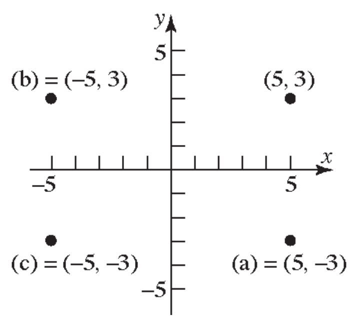

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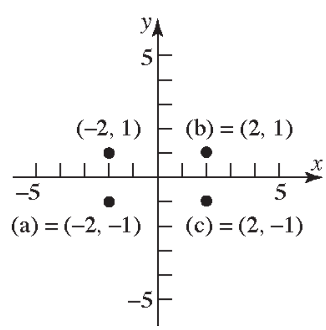

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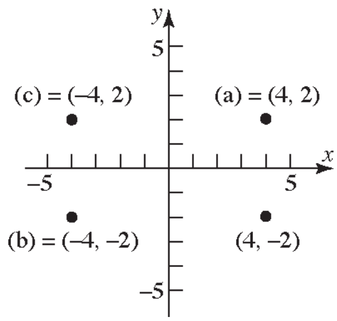

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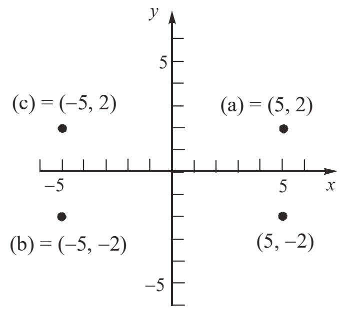

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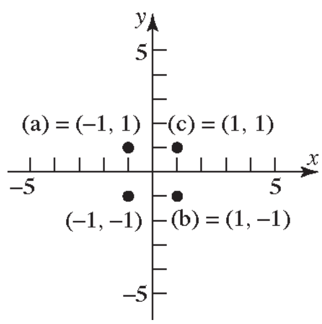

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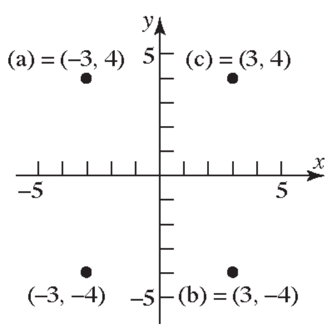

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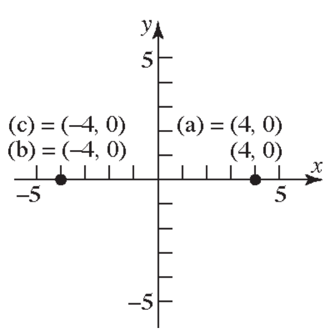

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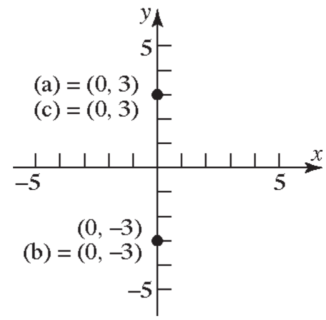

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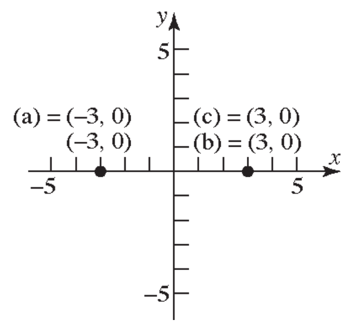


41. **a.** Intercepts: $(-1,0)$ and $(1,0)$

b. Symmetric with respect to the *x*-axis, *y*-axis, and the origin.

42. **a.** Intercepts: $(0,1)$

b. Not symmetric to the *x*-axis, the *y*-axis, nor the origin

43. **a.** Intercepts: $\left(-\frac{\pi}{2},0\right)$, $(0,1)$, and $\left(\frac{\pi}{2},0\right)$

b. Symmetric with respect to the *y*-axis.

44. **a.** Intercepts: $(-2,0)$, $(0,-3)$, and $(2,0)$

b. Symmetric with respect to the *y*-axis.

45. **a.** Intercepts: $(0,0)$

b. Symmetric with respect to the *x*-axis.

46. **a.** Intercepts: $(-2,0)$, $(0,2)$, $(0,-2)$, and $(2,0)$

b. Symmetric with respect to the *x*-axis, *y*-axis, and the origin.

47. **a.** Intercepts: $(-2,0)$, $(0,0)$, and $(2,0)$

b. Symmetric with respect to the origin.

48. **a.** Intercepts: $(-4,0)$, $(0,0)$, and $(4,0)$

b. Symmetric with respect to the origin.

49. a. x-intercept: $[-2,1]$, y-intercept 0

b. Not symmetric to x-axis, y-axis, or origin.

50. a. x-intercept: $[-1,2]$, y-intercept 0

b. Not symmetric to x-axis, y-axis, or origin.

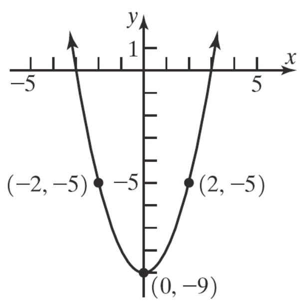
51. a. Intercepts: none

b. Symmetric with respect to the origin.

52. a. Intercepts: none

b. Symmetric with respect to the x-axis.

53.



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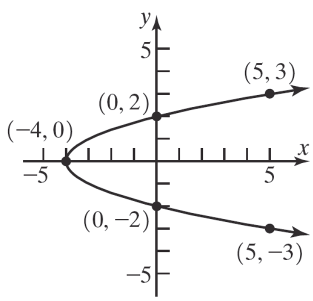

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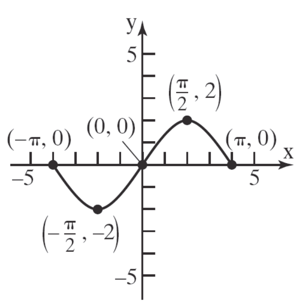

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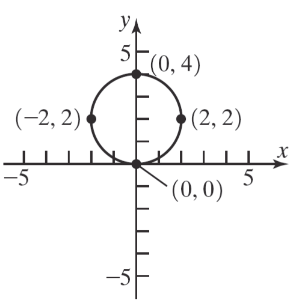


57. $y^2 = x + 4$

x-intercepts:
$0^2 = x + 4$
$-4 = x$

y-intercepts:
$y^2 = 0 + 4$
$y^2 = 4$
$y = \pm 2$

The intercepts are $(-4,0)$, $(0,-2)$ and $(0,2)$.

Test x-axis symmetry: Let $y = -y$

$(-y)^2 = x + 4$
$y^2 = x + 4$ same

Test y-axis symmetry: Let $x = -x$

$y^2 = -x + 4$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-y)^2 = -x + 4$
$y^2 = -x + 4$ different

Therefore, the graph will have x-axis symmetry.

58. $y^2 = x + 9$

x-intercepts:
$(0)^2 = -x + 9$
$0 = -x + 9$
$x = 9$

y-intercepts:
$y^2 = 0 + 9$
$y^2 = 9$
$y = \pm 3$

The intercepts are $(-9,0)$, $(0,-3)$ and $(0,3)$.

Test x-axis symmetry: Let $y = -y$

$(-y)^2 = x + 9$
$y^2 = x + 9$ same

Test y-axis symmetry: Let $x = -x$

$y^2 = -x + 9$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-y)^2 = -x + 9$
$y^2 = -x + 9$ different

Therefore, the graph will have x-axis symmetry.

59. $y = \sqrt[3]{x}$

x-intercepts:
$0 = \sqrt[3]{x}$
$0 = x$

y-intercepts:
$y = \sqrt[3]{0} = 0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \sqrt[3]{x}$ different

Test y-axis symmetry: Let $x=-x$

$y=\sqrt[3]{-x}=-\sqrt[3]{x}$ different

Test origin symmetry: Let $x=-x$ and $y=-y$

$-y=\sqrt[3]{-x}=-\sqrt[3]{x}$

$y=\sqrt[3]{x}$ same

Therefore, the graph will have origin symmetry.

60. $y=\sqrt[5]{x}$

x-intercepts: $0=\sqrt[3]{x}$, $0=x$

y-intercepts: $y=\sqrt[5]{0}=0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y=-y$

$-y=\sqrt[5]{x}$ different

Test y-axis symmetry: Let $x=-x$

$y=\sqrt[5]{-x}=-\sqrt[5]{x}$ different

Test origin symmetry: Let $x=-x$ and $y=-y$

$-y=\sqrt[5]{-x}=-\sqrt[5]{x}$

$y=\sqrt[5]{x}$ same

Therefore, the graph will have origin symmetry.

61. $x^2+y-9=0$

x-intercepts:

$x^2-9=0$

$x^2=9$

$x=\pm 3$

y-intercepts:

$0^2+y-9=0$

$y=9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,9)$.

Test x-axis symmetry: Let $y=-y$

$x^2-y-9=0$ different

Test y-axis symmetry: Let $x=-x$

$(-x)^2+y-9=0$

$x^2+y-9=0$ same

Test origin symmetry: Let $x=-x$ and $y=-y$

$(-x)^2-y-9=0$

$x^2-y-9=0$ different

Therefore, the graph will have y-axis symmetry.

62. $x^2-y-4=0$

x-intercepts:

$x^2-0-4=0$

$x^2=4$

$x=\pm 2$

y-intercept:

$0^2-y-4=0$

$-y=4$

$y=-4$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,-4)$.

Test x-axis symmetry: Let $y=-y$

$x^2-(-y)-4=0$

$x^2+y-4=0$ different

Test y-axis symmetry: Let $x=-x$

$(-x)^2-y-4=0$

$x^2-y-4=0$ same

Test origin symmetry: Let $x=-x$ and $y=-y$

$(-x)^2-(-y)-4=0$

$x^2+y-4=0$ different

Therefore, the graph will have y-axis symmetry.

63. $9x^2+4y^2=36$

x-intercepts:

$9x^2+4(0)^2=36$

$9x^2=36$

$x^2=4$

$x=\pm 2$

y-intercepts:

$9(0)^2+4y^2=36$

$4y^2=36$

$y^2=9$

$y=\pm 3$

The intercepts are $(-2,0),(2,0)$, $(0,-3)$, and $(0,3)$.

Test x-axis symmetry: Let $y=-y$

$9x^2+4(-y)^2=36$

$9x^2+4y^2=36$ same

Test y-axis symmetry: Let $x=-x$

$9(-x)^2+4y^2=36$

$9x^2+4y^2=36$ same

Test origin symmetry: Let $x=-x$ and $y=-y$

$9(-x)^2+4(-y)^2=36$

$9x^2+4y^2=36$ same

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

64. $4x^2+y^2=4$

x-intercepts:

$4x^2+0^2=4$

$4x^2=4$

$x^2=1$

$x=\pm 1$

y-intercepts:

$4(0)^2+y^2=4$

$y^2=4$

$y=\pm 2$

The intercepts are $(-1,0)$, $(1,0)$, $(0,-2)$, and $(0,2)$.

Test x-axis symmetry: Let $y=-y$

$4x^2+(-y)^2=4$

$4x^2+y^2=4$ same

Test y-axis symmetry: Let $x=-x$

$4(-x)^2+y^2=4$

$4x^2+y^2=4$ same

Test origin symmetry: Let $x=-x$ and $y=-y$

$4(-x)^2+(-y)^2=4$

$4x^2+y^2=4$ same

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

65. $y=x^3-27$

x-intercepts:	y-intercepts:
$0=x^3-27$	$y=0^3-27$
$x^3=27$	$y=-27$
$x=3$	

The intercepts are $(3,0)$ and $(0,-27)$.

Test x-axis symmetry: Let $y=-y$

$-y=x^3-27$ different

Test y-axis symmetry: Let $x=-x$

$y=(-x)^3-27$

$y=-x^3-27$ different

Test origin symmetry: Let $x=-x$ and $y=-y$

$-y=(-x)^3-27$

$y=x^3+27$ different

Therefore, the graph has none of the indicated symmetries.

66. $y=x^4-1$

x-intercepts:	y-intercepts:
$0=x^4-1$	$y=0^4-1$
$x^4=1$	$y=-1$
$x=\pm 1$	

The intercepts are $(-1,0)$, $(1,0)$, and $(0,-1)$.

Test x-axis symmetry: Let $y=-y$

$-y=x^4-1$ different

Test y-axis symmetry: Let $x=-x$

$y=(-x)^4-1$

$y=x^4-1$ same

Test origin symmetry: Let $x=-x$ and $y=-y$

$-y=(-x)^4-1$

$-y=x^4-1$ different

Therefore, the graph will have y-axis symmetry.

67. $y=x^2-3x-4$

x-intercepts:	y-intercepts:
$0=x^2-3x-4$	$y=0^2-3(0)-4$
$0=(x-4)(x+1)$	$y=-4$
$x=4$ or $x=-1$	

The intercepts are $(4,0)$, $(-1,0)$, and $(0,-4)$.

Test x-axis symmetry: Let $y=-y$

$-y=x^2-3x-4$ different

Test y-axis symmetry: Let $x=-x$

$y=(-x)^2-3(-x)-4$

$y=x^2+3x-4$ different

Test origin symmetry: Let $x=-x$ and $y=-y$

$-y=(-x)^2-3(-x)-4$

$-y=x^2+3x-4$ different

Therefore, the graph has none of the indicated symmetries.

68. $y=x^2+4$

x-intercepts:	y-intercepts:
$0=x^2+4$	$y=0^2+4$
$x^2=-4$	$y=4$
no real solution	

The only intercept is $(0,4)$.

Test x-axis symmetry: Let $y=-y$

$-y=x^2+4$ different

Test y-axis symmetry: Let $x=-x$

$y=(-x)^2+4$

$y=x^2+4$ same

Test origin symmetry: Let $x=-x$ and $y=-y$

$-y=(-x)^2+4$

$-y=x^2+4$ different

Therefore, the graph will have y-axis symmetry.

69. $y = \dfrac{3x}{x^2+9}$

x-intercepts:

$$0 = \frac{3x}{x^2+9}$$
$$3x = 0$$
$$x = 0$$

y-intercepts:

$$y = \frac{3(0)}{0^2+9} = \frac{0}{9} = 0$$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{3x}{x^2+9} \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{3(-x)}{(-x)^2+9}$$
$$y = -\frac{3x}{x^2+9} \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{3(-x)}{(-x)^2+9}$$
$$-y = -\frac{3x}{x^2+9}$$
$$y = \frac{3x}{x^2+9} \quad \text{same}$$

Therefore, the graph has origin symmetry.

70. $y = \dfrac{x^2-4}{2x}$

x-intercepts:

$$0 = \frac{x^2-4}{2x}$$
$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = \pm 2$$

y-intercepts:

$$y = \frac{0^2-4}{2(0)} = \frac{-4}{0}$$

undefined

The intercepts are $(-2,0)$ and $(2,0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{x^2-4}{2x} \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{(-x)^2-4}{2(-x)}$$
$$y = -\frac{x^2-4}{2x} \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{(-x)^2-4}{2(-x)}$$
$$-y = \frac{x^2-4}{-2x}$$
$$y = \frac{x^2-4}{2x} \quad \text{same}$$

Therefore, the graph has origin symmetry.

71. $y = \dfrac{-x^3}{x^2-9}$

x-intercepts:

$$0 = \frac{-x^3}{x^2-9}$$
$$-x^3 = 0$$
$$x = 0$$

y-intercepts:

$$y = \frac{-0^3}{0^2-9} = \frac{0}{-9} = 0$$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{-x^3}{x^2-9}$$
$$y = \frac{x^3}{x^2-9} \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{-(-x)^3}{(-x)^2-9}$$
$$y = \frac{x^3}{x^2-9} \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{-(-x)^3}{(-x)^2-9}$$
$$-y = \frac{x^3}{x^2-9}$$
$$y = \frac{-x^3}{x^2-9} \quad \text{same}$$

Therefore, the graph has origin symmetry.

72. $y = \dfrac{x^4+1}{2x^5}$

x-intercepts:

$0 = \dfrac{x^4+1}{2x^5}$

$x^4 = -1$

no real solution

y-intercepts:

$y = \dfrac{0^4+1}{2(0)^5} = \dfrac{1}{0}$

undefined

There are no intercepts for the graph of this equation.

<u>Test x-axis symmetry:</u> Let $y = -y$

$-y = \dfrac{x^4+1}{2x^5}$ different

<u>Test y-axis symmetry:</u> Let $x = -x$

$y = \dfrac{(-x)^4+1}{2(-x)^5}$

$y = \dfrac{x^4+1}{-2x^5}$ different

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$

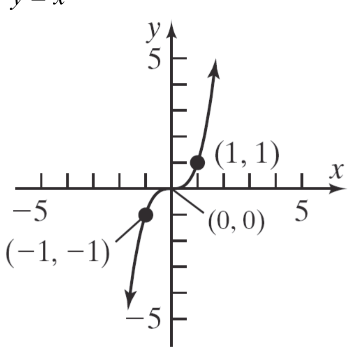
$-y = \dfrac{(-x)^4+1}{2(-x)^5}$

$-y = \dfrac{x^4+1}{-2x^5}$

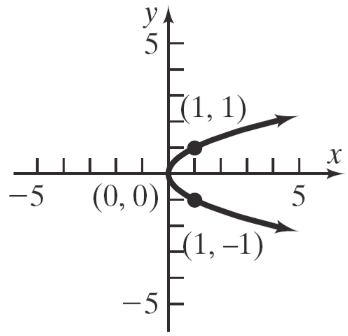
$y = \dfrac{x^4+1}{2x^5}$ same

Therefore, the graph has origin symmetry.

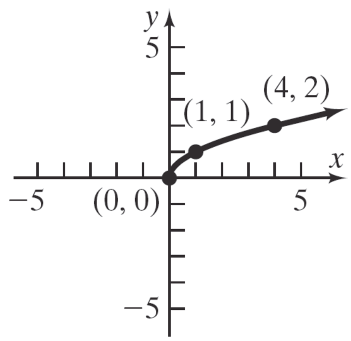
73. $y = x^3$



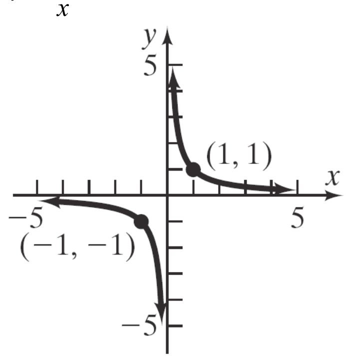
74. $x = y^2$



75. $y = \sqrt{x}$



76. $y = \dfrac{1}{x}$



77. If the point $(a, 4)$ is on the graph of $y = x^2 + 3x$, then we have

$4 = a^2 + 3a$

$0 = a^2 + 3a - 4$

$0 = (a+4)(a-1)$

$a + 4 = 0$ or $a - 1 = 0$

$a = -4 \qquad a = 1$

Thus, $a = -4$ or $a = 1$.

78. If the point $(a,-5)$ is on the graph of $y = x^2 + 6x$, then we have

$$-5 = a^2 + 6a$$
$$0 = a^2 + 6a + 5$$
$$0 = (a+5)(a+1)$$
$$a+5=0 \quad \text{or} \quad a+1=0$$
$$a=-5 \qquad\quad a=-1$$

Thus, $a=-5$ or $a=-1$.

79. Center = (2, 1)
Radius = distance from (0,1) to (2,1)

$$= \sqrt{(2-0)^2 + (1-1)^2} = \sqrt{4} = 2$$

Equation: $(x-2)^2 + (y-1)^2 = 4$

80. Center = (1, 2)
Radius = distance from (1,0) to (1,2)

$$= \sqrt{(1-1)^2 + (2-0)^2} = \sqrt{4} = 2$$

Equation: $(x-1)^2 + (y-2)^2 = 4$

81. Center = midpoint of (1, 2) and (4, 2)

$$= \left(\frac{1+4}{2}, \frac{2+2}{2}\right) = \left(\frac{5}{2}, 2\right)$$

Radius = distance from $\left(\frac{5}{2}, 2\right)$ to (4,2)

$$= \sqrt{\left(4-\frac{5}{2}\right)^2 + (2-2)^2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

Equation: $\left(x-\frac{5}{2}\right)^2 + (y-2)^2 = \frac{9}{4}$

82. Center = midpoint of (0, 1) and (2, 3)

$$= \left(\frac{0+2}{2}, \frac{1+3}{2}\right) = (1, 2)$$

Radius = distance from $(1,2)$ to (2,3)

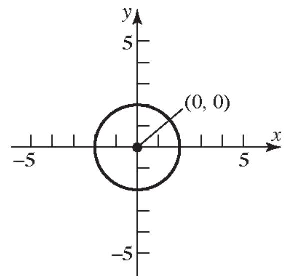
$$= \sqrt{(2-1)^2 + (3-2)^2} = \sqrt{2}$$

Equation: $(x-1)^2 + (y-2)^2 = 2$

83. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + (y-0)^2 = 2^2$$
$$x^2 + y^2 = 4$$

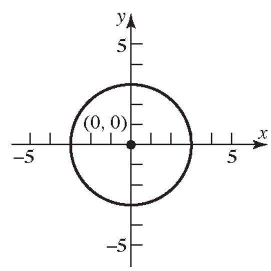
General form: $x^2 + y^2 - 4 = 0$



84. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + (y-0)^2 = 3^2$$
$$x^2 + y^2 = 9$$

General form: $x^2 + y^2 - 9 = 0$

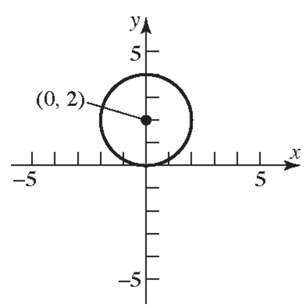


85. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + (y-2)^2 = 2^2$$
$$x^2 + (y-2)^2 = 4$$

General form: $x^2 + y^2 - 4y + 4 = 4$

$$x^2 + y^2 - 4y = 0$$



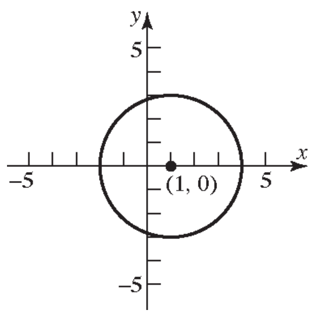
86. $(x-h)^2+(y-k)^2=r^2$

$(x-1)^2+(y-0)^2=3^2$

$(x-1)^2+y^2=9$

General form: $x^2-2x+1+y^2=9$

$x^2+y^2-2x-8=0$



87. $(x-h)^2+(y-k)^2=r^2$

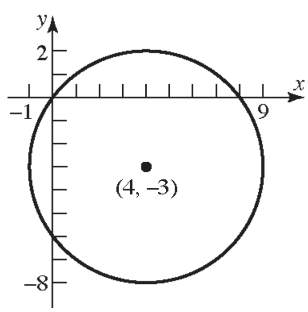
$(x-4)^2+(y-(-3))^2=5^2$

$(x-4)^2+(y+3)^2=25$

General form:

$x^2-8x+16+y^2+6y+9=25$

$x^2+y^2-8x+6y=0$



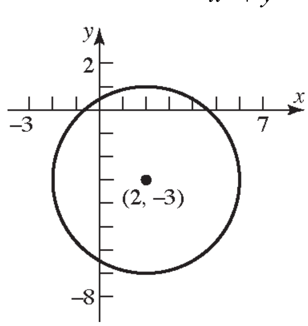
88. $(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y-(-3))^2=4^2$

$(x-2)^2+(y+3)^2=16$

General form: $x^2-4x+4+y^2+6y+9=16$

$x^2+y^2-4x+6y-3=0$



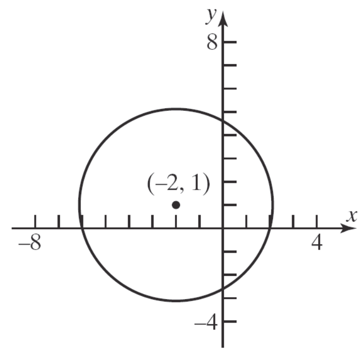
89. $(x-h)^2+(y-k)^2=r^2$

$(x-(-2))^2+(y-1)^2=4^2$

$(x+2)^2+(y-1)^2=16$

General form: $x^2+4x+4+y^2-2y+1=16$

$x^2+y^2+4x-2y-11=0$



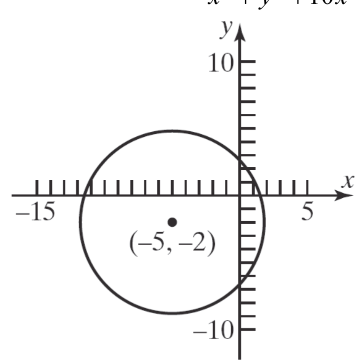
90. $(x-h)^2+(y-k)^2=r^2$

$(x-(-5))^2+(y-(-2))^2=7^2$

$(x+5)^2+(y+2)^2=49$

General form: $x^2+10x+25+y^2+4y+4=49$

$x^2+y^2+10x+4y-20=0$



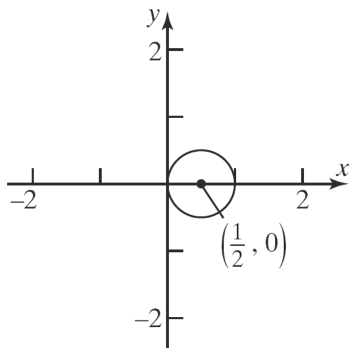
91. $(x-h)^2+(y-k)^2=r^2$

$\left(x-\dfrac{1}{2}\right)^2+(y-0)^2=\left(\dfrac{1}{2}\right)^2$

$\left(x-\dfrac{1}{2}\right)^2+y^2=\dfrac{1}{4}$

General form: $x^2-x+\dfrac{1}{4}+y^2=\dfrac{1}{4}$

$x^2+y^2-x=0$



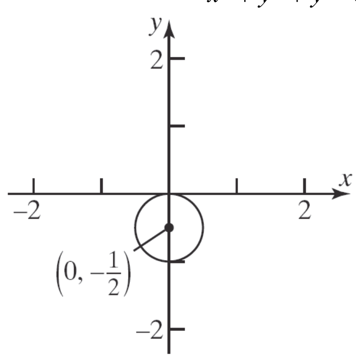
92. $(x-h)^2+(y-k)^2=r^2$

$$(x-0)^2+\left(y-\left(-\frac{1}{2}\right)\right)^2=\left(\frac{1}{2}\right)^2$$

$$x^2+\left(y+\frac{1}{2}\right)^2=\frac{1}{4}$$

General form: $x^2+y^2+y+\frac{1}{4}=\frac{1}{4}$

$$x^2+y^2+y=0$$

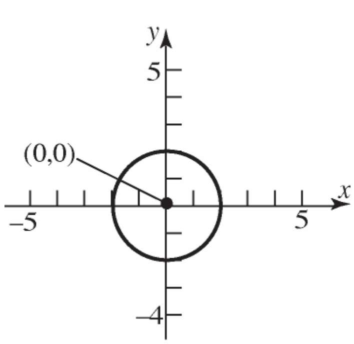


93. $x^2+y^2=4$

$x^2+y^2=2^2$

a. Center: $(0,0)$; Radius $=2$

b.



c. x-intercepts: $x^2+(0)^2=4$

$$x^2=4$$

$$x=\pm\sqrt{4}=\pm2$$

y-intercepts: $(0)^2+y^2=4$

$$y^2=4$$

$$y=\pm\sqrt{4}=\pm2$$

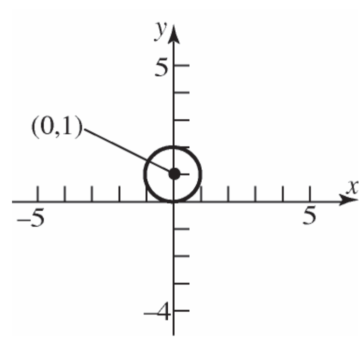
The intercepts are $(-2,0)$, $(2,0)$, $(0,-2)$, and $(0,2)$.

94. $x^2+(y-1)^2=1$

$x^2+(y-1)^2=1^2$

a. Center:$(0,1)$; Radius $=1$

b.



c. x-intercepts: $x^2+(0-1)^2=1$

$$x^2+1=1$$

$$x^2=0$$

$$x=\pm\sqrt{0}=0$$

y-intercepts: $(0)^2+(y-1)^2=1$

$$(y-1)^2=1$$

$$y-1=\pm\sqrt{1}$$

$$y-1=\pm1$$

$$y=1\pm1$$

$$y=2 \quad \text{or} \quad y=0$$

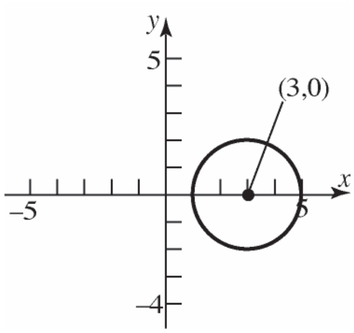
The intercepts are $(0,0)$ and $(0,2)$.

95. $2(x-3)^2+2y^2=8$

$(x-3)^2+y^2=4$

a. Center: $(3,0)$; Radius $=2$

b.



c. x-intercepts: $(x-3)^2+(0)^2=4$

$$(x-3)^2=4$$
$$x-3=\pm\sqrt{4}$$
$$x-3=\pm 2$$
$$x=3\pm 2$$
$$x=5 \quad \text{or} \quad x=1$$

y-intercepts: $(0-3)^2+y^2=4$

$$(-3)^2+y^2=4$$
$$9+y^2=4$$
$$y^2=-5$$

No real solution.

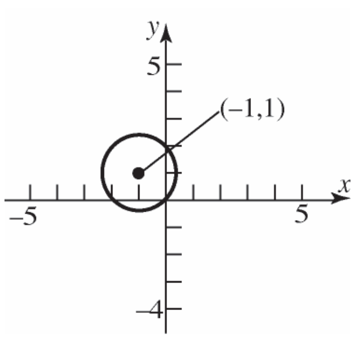
The intercepts are $(1,0)$ and $(5,0)$.

96. $3(x+1)^2+3(y-1)^2=6$

$$(x+1)^2+(y-1)^2=2$$

a. Center: $(-1,1)$; Radius $=\sqrt{2}$

b.



c. x-intercepts: $(x+1)^2+(0-1)^2=2$

$$(x+1)^2+(-1)^2=2$$
$$(x+1)^2+1=2$$
$$(x+1)^2=1$$
$$x+1=\pm\sqrt{1}$$
$$x+1=\pm 1$$
$$x=-1\pm 1$$
$$x=0 \quad \text{or} \quad x=-2$$

y-intercepts: $(0+1)^2+(y-1)^2=2$

$$(1)^2+(y-1)^2=2$$
$$1+(y-1)^2=2$$
$$(y-1)^2=1$$
$$y-1=\pm\sqrt{1}$$
$$y-1=\pm 1$$
$$y=1\pm 1$$
$$y=2 \quad \text{or} \quad y=0$$

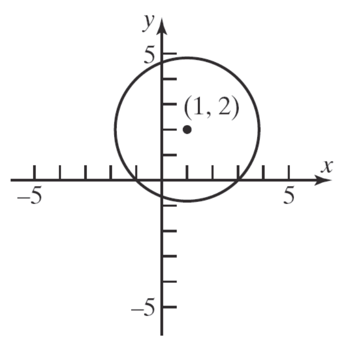
The intercepts are $(-2,0)$, $(0,0)$, and $(0,2)$.

97. $x^2+y^2-2x-4y-4=0$

$$x^2-2x+y^2-4y=4$$
$$(x^2-2x+1)+(y^2-4y+4)=4+1+4$$
$$(x-1)^2+(y-2)^2=3^2$$

a. Center: $(1, 2)$; Radius $=3$

b.



c. x-intercepts: $(x-1)^2+(0-2)^2=3^2$

$$(x-1)^2+(-2)^2=3^2$$
$$(x-1)^2+4=9$$
$$(x-1)^2=5$$
$$x-1=\pm\sqrt{5}$$
$$x=1\pm\sqrt{5}$$

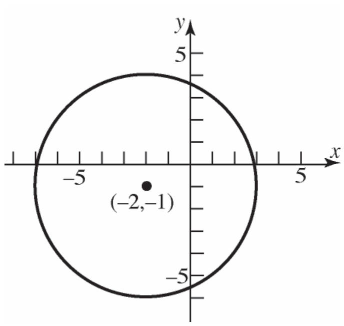
$$\begin{aligned} y\text{-intercepts: } (0-1)^2+(y-2)^2 &= 3^2 \\ (-1)^2+(y-2)^2 &= 3^2 \\ 1+(y-2)^2 &= 9 \\ (y-2)^2 &= 8 \\ y-2 &= \pm\sqrt{8} \\ y-2 &= \pm 2\sqrt{2} \\ y &= 2\pm 2\sqrt{2} \end{aligned}$$

The intercepts are $\left(1-\sqrt{5},\,0\right)$, $\left(1+\sqrt{5},\,0\right)$, $\left(0,\,2-2\sqrt{2}\right)$, and $\left(0,\,2+2\sqrt{2}\right)$.

98.
$$\begin{aligned} x^2+y^2+4x+2y-20 &= 0 \\ x^2+4x+y^2+2y &= 20 \\ (x^2+4x+4)+(y^2+2y+1) &= 20+4+1 \\ (x+2)^2+(y+1)^2 &= 5^2 \end{aligned}$$

a. Center: (–2,–1); Radius = 5

b.



c.
$$\begin{aligned} x\text{-intercepts: } (x+2)^2+(0+1)^2 &= 5^2 \\ (x+2)^2+1 &= 25 \\ (x+2)^2 &= 24 \\ x+2 &= \pm\sqrt{24} \\ x+2 &= \pm 2\sqrt{6} \\ x &= -2\pm 2\sqrt{6} \end{aligned}$$

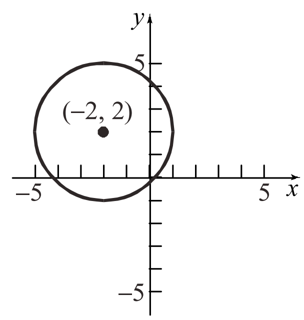
$$\begin{aligned} y\text{-intercepts: } (0+2)^2+(y+1)^2 &= 5^2 \\ 4+(y+1)^2 &= 25 \\ (y+1)^2 &= 21 \\ y+1 &= \pm\sqrt{21} \\ y &= -1\pm\sqrt{21} \end{aligned}$$

The intercepts are $\left(-2-2\sqrt{6},\,0\right)$, $\left(-2+2\sqrt{6},\,0\right)$, $\left(0,\,-1-\sqrt{21}\right)$, and $\left(0,\,-1+\sqrt{21}\right)$.

99.
$$\begin{aligned} x^2+y^2+4x-4y-1 &= 0 \\ x^2+4x+y^2-4y &= 1 \\ (x^2+4x+4)+(y^2-4y+4) &= 1+4+4 \\ (x+2)^2+(y-2)^2 &= 3^2 \end{aligned}$$

a. Center: (–2, 2); Radius = 3

b.



c.
$$\begin{aligned} x\text{-intercepts: } (x+2)^2+(0-2)^2 &= 3^2 \\ (x+2)^2+4 &= 9 \\ (x+2)^2 &= 5 \\ x+2 &= \pm\sqrt{5} \\ x &= -2\pm\sqrt{5} \end{aligned}$$

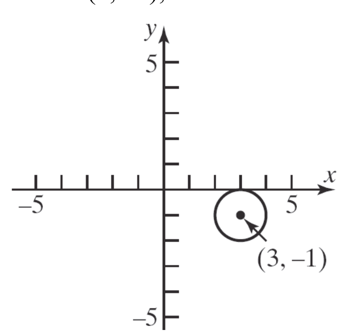
$$\begin{aligned} y\text{-intercepts: } (0+2)^2+(y-2)^2 &= 3^2 \\ 4+(y-2)^2 &= 9 \\ (y-2)^2 &= 5 \\ y-2 &= \pm\sqrt{5} \\ y &= 2\pm\sqrt{5} \end{aligned}$$

The intercepts are $\left(-2-\sqrt{5},\,0\right)$, $\left(-2+\sqrt{5},\,0\right)$, $\left(0,\,2-\sqrt{5}\right)$, and $\left(0,\,2+\sqrt{5}\right)$.

100.
$$\begin{aligned} x^2+y^2-6x+2y+9 &= 0 \\ x^2-6x+y^2+2y &= -9 \\ (x^2-6x+9)+(y^2+2y+1) &= -9+9+1 \\ (x-3)^2+(y+1)^2 &= 1^2 \end{aligned}$$

a. Center: (3, –1); Radius = 1

b.



c. x-intercepts:
$$\begin{aligned}(x-3)^2+(0+1)^2&=1^2\\(x-3)^2+1&=1\\(x-3)^2&=0\\x-3&=0\\x&=3\end{aligned}$$

y-intercepts:
$$\begin{aligned}(0-3)^2+(y+1)^2&=1^2\\9+(y+1)^2&=1\\(y+1)^2&=-8\end{aligned}$$

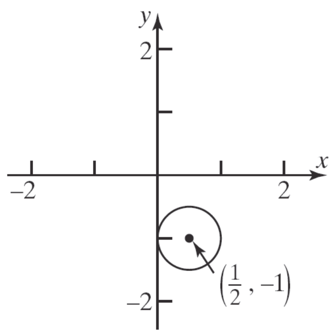
No real solution.

The intercept only intercept is $(3,0)$.

101.
$$\begin{aligned}x^2+y^2-x+2y+1&=0\\x^2-x+y^2+2y&=-1\\\left(x^2-x+\frac{1}{4}\right)+(y^2+2y+1)&=-1+\frac{1}{4}+1\\\left(x-\frac{1}{2}\right)^2+(y+1)^2&=\left(\frac{1}{2}\right)^2\end{aligned}$$

a. Center: $\left(\frac{1}{2},-1\right)$; Radius $=\frac{1}{2}$

b.



c. x-intercepts:
$$\begin{aligned}\left(x-\frac{1}{2}\right)^2+(0+1)^2&=\left(\frac{1}{2}\right)^2\\\left(x-\frac{1}{2}\right)^2+1&=\frac{1}{4}\\\left(x-\frac{1}{2}\right)^2&=-\frac{3}{4}\end{aligned}$$

No real solutions

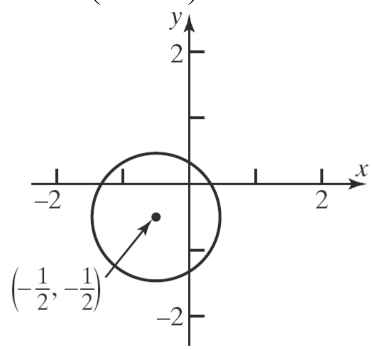
y-intercepts:
$$\begin{aligned}\left(0-\frac{1}{2}\right)^2+(y+1)^2&=\left(\frac{1}{2}\right)^2\\\frac{1}{4}+(y+1)^2&=\frac{1}{4}\\(y+1)^2&=0\\y+1&=0\\y&=-1\end{aligned}$$

The only intercept is $(0,-1)$.

102.
$$\begin{aligned}x^2+y^2+x+y-\frac{1}{2}&=0\\x^2+x+y^2+y&=\frac{1}{2}\\\left(x^2+x+\frac{1}{4}\right)+\left(y^2+y+\frac{1}{4}\right)&=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\\\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2&=1^2\end{aligned}$$

a. Center: $\left(-\frac{1}{2},-\frac{1}{2}\right)$; Radius = 1

b.



c. x-intercepts:
$$\begin{aligned}\left(x+\frac{1}{2}\right)^2+\left(0+\frac{1}{2}\right)^2&=1^2\\\left(x+\frac{1}{2}\right)^2+\frac{1}{4}&=1\\\left(x+\frac{1}{2}\right)^2&=\frac{3}{4}\\x+\frac{1}{2}&=\pm\frac{\sqrt{3}}{2}\\x&=\frac{-1\pm\sqrt{3}}{2}\end{aligned}$$

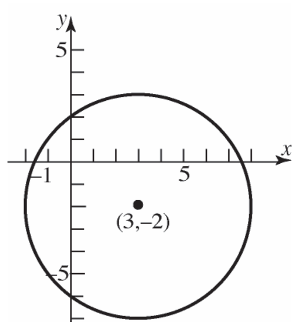
y-intercepts:
$$\begin{aligned}\left(0+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2&=1^2\\\frac{1}{4}+\left(y+\frac{1}{2}\right)^2&=1\\\left(y+\frac{1}{2}\right)^2&=\frac{3}{4}\\y+\frac{1}{2}&=\pm\frac{\sqrt{3}}{2}\\y&=\frac{-1\pm\sqrt{3}}{2}\end{aligned}$$

The intercepts are $\left(\frac{-1-\sqrt{3}}{2},0\right)$, $\left(\frac{-1+\sqrt{3}}{2},0\right)$, $\left(0,\frac{-1-\sqrt{3}}{2}\right)$, and $\left(0,\frac{-1+\sqrt{3}}{2}\right)$.

103.
$$2x^2+2y^2-12x+8y-24=0$$
$$x^2+y^2-6x+4y=12$$
$$x^2-6x+y^2+4y=12$$
$$(x^2-6x+9)+(y^2+4y+4)=12+9+4$$
$$(x-3)^2+(y+2)^2=5^2$$

a. Center: (3,–2); Radius = 5

b.



c. x-intercepts: $(x-3)^2+(0+2)^2=5^2$
$$(x-3)^2+4=25$$
$$(x-3)^2=21$$
$$x-3=\pm\sqrt{21}$$
$$x=3\pm\sqrt{21}$$

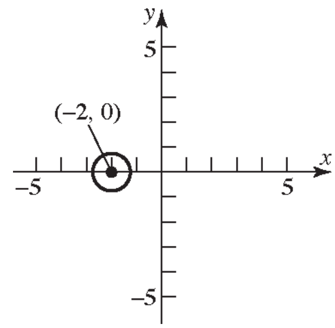
y-intercepts: $(0-3)^2+(y+2)^2=5^2$
$$9+(y+2)^2=25$$
$$(y+2)^2=16$$
$$y+2=\pm 4$$
$$y=-2\pm 4$$
$$y=2 \quad\text{or}\quad y=-6$$

The intercepts are $\left(3-\sqrt{21},0\right)$, $\left(3+\sqrt{21},0\right)$, $(0,-6)$, and $(0,2)$.

104. a.
$$2x^2+2y^2+8x+7=0$$
$$2x^2+8x+2y^2=-7$$
$$x^2+4x+y^2=-\frac{7}{2}$$
$$(x^2+4x+4)+y^2=-\frac{7}{2}+4$$
$$(x+2)^2+y^2=\frac{1}{2}$$
$$(x+2)^2+y^2=\left(\frac{\sqrt{2}}{2}\right)^2$$

Center: (–2, 0); Radius = $\dfrac{\sqrt{2}}{2}$

b.



c. x-intercepts: $(x+2)^2+(0)^2=\dfrac{1}{2}$
$$(x+2)^2=\frac{1}{2}$$
$$x+2=\pm\sqrt{\frac{1}{2}}$$
$$x+2=\pm\frac{\sqrt{2}}{2}$$
$$x=-2\pm\frac{\sqrt{2}}{2}$$

y-intercepts: $(0+2)^2+y^2=\dfrac{1}{2}$
$$4+y^2=\frac{1}{2}$$
$$y^2=-\frac{7}{2}$$

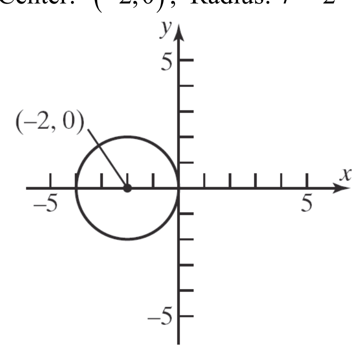
No real solutions.

The intercepts are $\left(-2-\dfrac{\sqrt{2}}{2},0\right)$ and $\left(-2+\dfrac{\sqrt{2}}{2},0\right)$.

105.
$$2x^2+8x+2y^2=0$$
$$x^2+4x+y^2=0$$
$$x^2+4x+4+y^2=0+4$$
$$(x+2)^2+y^2=2^2$$

a. Center: $(-2,0)$; Radius: $r=2$

b.



c. x-intercepts: $(x+2)^2+(0)^2=2^2$

$$(x+2)^2=4$$
$$(x+2)^2=\pm\sqrt{4}$$
$$x+2=\pm 2$$
$$x=-2\pm 2$$
$$x=0 \quad \text{or} \quad x=-4$$

y-intercepts: $(0+2)^2+y^2=2^2$

$$4+y^2=4$$
$$y^2=0$$
$$y=0$$

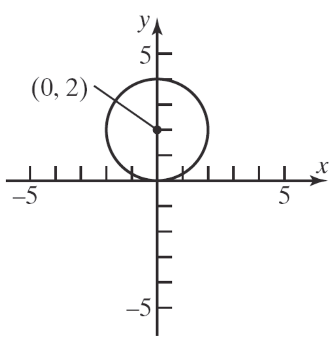
The intercepts are $(-4,0)$ and $(0,0)$.

106. $3x^2+3y^2-12y=0$

$$x^2+y^2-4y=0$$
$$x^2+y^2-4y+4=0+4$$
$$x^2+(y-2)^2=4$$

a. Center: $(0,2)$; Radius: $r=2$

b.



c. x-intercepts: $x^2+(0-2)^2=4$

$$x^2+4=4$$
$$x^2=0$$
$$x=0$$

y-intercepts: $0^2+(y-2)^2=4$

$$(y-2)^2=4$$
$$y-2=\pm\sqrt{4}$$
$$y-2=\pm 2$$
$$y=2\pm 2$$
$$y=4 \quad \text{or} \quad y=0$$

The intercepts are $(0,0)$ and $(0,4)$.

107. Center at (0, 0); containing point (–2, 3).

$$r=\sqrt{(-2-0)^2+(3-0)^2}=\sqrt{4+9}=\sqrt{13}$$

Equation: $(x-0)^2+(y-0)^2=\left(\sqrt{13}\right)^2$

$$x^2+y^2=13$$

108. Center at (1, 0); containing point (–3, 2).

$$r=\sqrt{(-3-1)^2+(2-0)^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$$

Equation: $(x-1)^2+(y-0)^2=\left(\sqrt{20}\right)^2$

$$(x-1)^2+y^2=20$$

109. Endpoints of a diameter are (1, 4) and (–3, 2). The center is at the midpoint of that diameter:

Center: $\left(\dfrac{1+(-3)}{2},\dfrac{4+2}{2}\right)=(-1,3)$

Radius: $r=\sqrt{(1-(-1))^2+(4-3)^2}=\sqrt{4+1}=\sqrt{5}$

Equation: $(x-(-1))^2+(y-3)^2=\left(\sqrt{5}\right)^2$

$$(x+1)^2+(y-3)^2=5$$

110. Endpoints of a diameter are (4, 3) and (0, 1). The center is at the midpoint of that diameter:

Center: $\left(\dfrac{4+0}{2},\dfrac{3+1}{2}\right)=(2,2)$

Radius: $r=\sqrt{(4-2)^2+(3-2)^2}=\sqrt{4+1}=\sqrt{5}$

Equation: $(x-2)^2+(y-2)^2=\left(\sqrt{5}\right)^2$

$$(x-2)^2+(y-2)^2=5$$

111. Center at (2, –4); circumference 16π.
This means that the radius of the circle is:
$2\pi r = 16\pi$
$r = 8$
Equation: $(x-2)^2 + (y+4)^2 = (8)^2$
$(x-2)^2 + (y+4)^2 = 64$

112. Center at (–5 ,6); area 49π.
This means that the radius of the circle is:
$\pi r^2 = 49\pi$
$r = 7$
Equation: $(x+5)^2 + (y-6)^2 = (7)^2$
$(x+5)^2 + (y-6)^2 = 49$

113. For a graph with origin symmetry, if the point (a,b) is on the graph, then so is the point $(-a,-b)$. Since the point $(1,2)$ is on the graph of an equation with origin symmetry, the point $(-1,-2)$ must also be on the graph.

114. For a graph with y-axis symmetry, if the point (a,b) is on the graph, then so is the point $(-a,b)$. Since 6 is an x-intercept in this case, the point $(6,0)$ is on the graph of the equation. Due to the y-axis symmetry, the point $(-6,0)$ must also be on the graph. Therefore, -6 is another x-intercept.

115. For a graph with origin symmetry, if the point (a,b) is on the graph, then so is the point $(-a,-b)$. Since -4 is an x-intercept in this case, the point $(-4,0)$ is on the graph of the equation. Due to the origin symmetry, the point $(4,0)$ must also be on the graph. Therefore, 4 is another x-intercept.

116. For a graph with x-axis symmetry, if the point (a,b) is on the graph, then so is the point $(a,-b)$. Since 2 is a y-intercept in this case, the point $(0,2)$ is on the graph of the equation. Due to the x-axis symmetry, the point $(0,-2)$ must also be on the graph. Therefore, -2 is another y-intercept.

117. a. $\left(x^2+y^2-x\right)^2 = x^2+y^2$

x-intercepts:

$$\left(x^2+(0)^2-x\right)^2 = x^2+(0)^2$$
$$\left(x^2-x\right)^2 = x^2$$
$$x^4-2x^3+x^2 = x^2$$
$$x^4-2x^3 = 0$$
$$x^3(x-2) = 0$$
$$x^3 = 0 \quad \text{or} \quad x-2=0$$
$$x = 0 \qquad\qquad x = 2$$

y-intercepts:

$$\left((0)^2+y^2-0\right)^2 = (0)^2+y^2$$
$$\left(y^2\right)^2 = y^2$$
$$y^4 = y^2$$
$$y^4-y^2 = 0$$
$$y^2\left(y^2-1\right) = 0$$
$$y^2 = 0 \quad \text{or} \quad y^2-1=0$$
$$y = 0 \qquad\qquad y^2 = 1$$
$$y = \pm 1$$

The intercepts are $(0,0)$, $(2,0)$, $(0,-1)$, and $(0,1)$.

b. Test x-axis symmetry: Let $y=-y$

$$\left(x^2+(-y)^2-x\right)^2 = x^2+(-y)^2$$
$$\left(x^2+y^2-x\right)^2 = x^2+y^2 \quad \text{same}$$

Test y-axis symmetry: Let $x=-x$

$$\left((-x)^2+y^2-(-x)\right)^2 = (-x)^2+y^2$$
$$\left(x^2+y^2+x\right)^2 = x^2+y^2 \quad \text{different}$$

Test origin symmetry: Let $x=-x$ and $y=-y$

$$\left((-x)^2+(-y)^2-(-x)\right)^2 = (-x)^2+(-y)^2$$
$$\left(x^2+y^2+x\right)^2 = x^2+y^2 \quad \text{different}$$

Thus, the graph will have x-axis symmetry.

118. **a.** $16y^2 = 120x - 225$

x-intercepts:

$16y^2 = 120(0) - 225$

$16y^2 = -225$

$y^2 = -\dfrac{225}{16}$

no real solution

y-intercepts:

$16(0)^2 = 120x - 225$

$0 = 120x - 225$

$-120x = -225$

$x = \dfrac{-225}{-120} = \dfrac{15}{8}$

The only intercept is $\left(\dfrac{15}{8}, 0\right)$.

b. Test x-axis symmetry: Let $y = -y$

$16(-y)^2 = 120x - 225$

$16y^2 = 120x - 225$ same

Test y-axis symmetry: Let $x = -x$

$16y^2 = 120(-x) - 225$

$16y^2 = -120x - 225$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$16(-y)^2 = 120(-x) - 225$

$16y^2 = -120x - 225$ different

Thus, the graph will have x-axis symmetry.

119. Let the upper-right corner of the square be the point (x, y). The circle and the square are both centered about the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square. Therefore, we get

$x^2 + y^2 = 9$

$x^2 + x^2 = 9$

$2x^2 = 9$

$x^2 = \dfrac{9}{2}$

$x = \sqrt{\dfrac{9}{2}} = \dfrac{3\sqrt{2}}{2}$

The length of one side of the square is $2x$. Thus, the area is

$A = s^2 = \left(2 \cdot \dfrac{3\sqrt{2}}{2}\right)^2 = \left(3\sqrt{2}\right)^2 = 18$ square units.

120. The area of the shaded region is the area of the circle, less the area of the square. Let the upper-right corner of the square be the point (x, y). The circle and the square are both centered about the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square. Therefore, we get

$x^2 + y^2 = 36$

$x^2 + x^2 = 36$

$2x^2 = 36$

$x^2 = 18$

$x = 3\sqrt{2}$

The length of one side of the square is $2x$. Thus, the area of the square is $\left(2 \cdot 3\sqrt{2}\right)^2 = 72$ square units. From the equation of the circle, we have $r = 6$. The area of the circle is

$\pi r^2 = \pi(6)^2 = 36\pi$ square units.

Therefore, the area of the shaded region is $A = 36\pi - 72$ square units.

121. The diameter of the Ferris wheel was 250 feet, so the radius was 125 feet. The maximum height was 264 feet, so the center was at a height of $264 - 125 = 139$ feet above the ground. Since the center of the wheel is on the y-axis, it is the point (0, 139). Thus, an equation for the wheel is:

$(x - 0)^2 + (y - 139)^2 = 125^2$

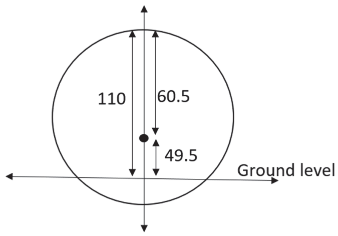
$x^2 + (y - 139)^2 = 15{,}625$

122. The diameter of the wheel is 520 feet, so the radius is 260 feet. The maximum height is 550 feet, so the center of the wheel is at a height of $550 - 260 = 290$ feet above the ground. Since the center of the wheel is on the y-axis, it is the point (0, 290). Thus, an equation for the wheel is:

$(x - 0)^2 + (y - 290)^2 = 260^2$

$x^2 + (y - 290)^2 = 67{,}600$

123.



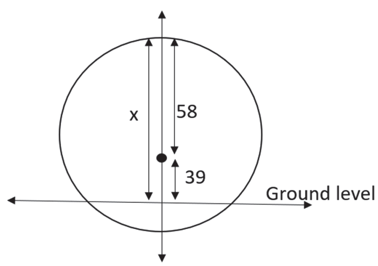
Refer to figure. Since the radius of the building is 60.5 m and the height of the building is 110 m, then the center of the building is 49.5 m above the ground, so the y-coordinate of the center is 49.5. The equation of the circle is given by

$x^2+(y-49.5)^2=60.5^2=3660.25$

124. Complete the square to find the equation of the circle representing the formula for the building.

$x^2+y^2-78y+1521=1843+1521=3364$

$x^2+(y-39)^2=58^2$



Refer to figure. The y coordinate of the center is 39. The radius is 58. Thus the height of the building is 58 + 39 = 97 m.

125. Center at (2, 3); tangent to the x-axis.

$r=3$

Equation: $(x-2)^2+(y-3)^2=3^2$

$(x-2)^2+(y-3)^2=9$

126. Center at (–3, 1); tangent to the y-axis.

$r=3$

Equation: $(x+3)^2+(y-1)^2=3^2$

$(x+3)^2+(y-1)^2=9$

127. Center at (–1, 3); tangent to the line $y = 2$. This means that the circle contains the point (–1, 2), so the radius is $r = 1$.

Equation: $(x+1)^2+(y-3)^2=(1)^2$

$(x+1)^2+(y-3)^2=1$

128. Center at (4, –2); tangent to the line $x = 1$. This means that the circle contains the point (1, –2), so the radius is $r = 3$.

Equation: $(x-4)^2+(y+2)^2=(3)^2$

$(x-4)^2+(y+2)^2=9$

129. a. Substitute $y=mx+b$ into $x^2+y^2=r^2$:

$$x^2+(mx+b)^2=r^2$$
$$x^2+m^2x^2+2bmx+b^2=r^2$$
$$(1+m^2)x^2+2bmx+b^2-r^2=0$$

This equation has one solution if and only if the discriminant is zero.

$$(2bm)^2-4(1+m^2)(b^2-r^2)=0$$
$$4b^2m^2-4b^2+4r^2-4b^2m^2+4m^2r^2=0$$
$$-4b^2+4r^2+4m^2r^2=0$$
$$-b^2+r^2+m^2r^2=0$$
$$r^2(1+m^2)=b^2$$

b. From part (a) we know $(1+m^2)x^2+2bmx+b^2-r^2=0$. Using the quadratic formula, since the discriminant is zero, we get:

$$x=\frac{-2bm}{2(1+m^2)}=\frac{-bm}{\left(\frac{b^2}{r^2}\right)}=\frac{-bmr^2}{b^2}=\frac{-mr^2}{b}$$

$$y=m\left(\frac{-mr^2}{b}\right)+b$$
$$=\frac{-m^2r^2}{b}+b=\frac{-m^2r^2+b^2}{b}=\frac{r^2}{b}$$

The point of tangency is $\left(\frac{-mr^2}{b},\frac{r^2}{b}\right)$.

c. The slope of the tangent line is m.

The slope of the line joining the point of tangency and the center (0,0) is:

$$\frac{\left(\frac{r^2}{b}-0\right)}{\left(\frac{-mr^2}{b}-0\right)}=\frac{r^2}{b}\cdot\frac{b}{-mr^2}=-\frac{1}{m}$$

The two lines are perpendicular.

130. Let (h, k) be the center of the circle.

$$x-2y+4=0$$
$$2y=x+4$$
$$y=\frac{1}{2}x+2$$

The slope of the tangent line is $\frac{1}{2}$. The slope from (h, k) to (0, 2) is –2.

$$\frac{2-k}{0-h}=-2$$
$$2-k=2h$$

The other tangent line is $y=2x-7$, and it has slope 2.

The slope from (h, k) to (3, –1) is $-\frac{1}{2}$.

$$\frac{-1-k}{3-h}=-\frac{1}{2}$$
$$2+2k=3-h$$
$$2k=1-h$$
$$h=1-2k$$

Solve the two equations in h and k:

$$2-k=2(1-2k)$$
$$2-k=2-4k$$
$$3k=0$$
$$k=0$$
$$h=1-2(0)=1$$

The center of the circle is (1, 0).

---

131. The center of the circle is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ and the radius is $\frac{1}{2}\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$. Then the equation of the circle is $\left(x-\frac{x_1+x_2}{2}\right)^2+\left(y-\frac{y_1+y_2}{2}\right)^2=\frac{1}{4}\left[(x_1-x_2)^2+(y_1-y_2)^2\right]$. Expanding, gives

$$x^2-x(x_1+x_2)+\frac{(x_1+x_2)^2}{4}+y^2-y(y_1+y_2)+\frac{(y_1+y_2)^2}{4}=\frac{1}{4}\left[x_1^2-2x_1x_2+x_2^2+y_1^2-2y_1y_2+y_2^2\right]$$
$$4x^2-4x_1x-4x_2x+x_1^2+2x_1x_2+x_2^2+4y^2-4y_1y-4y_2y+y_1^2+2y_1y_2+y_2^2=x_1^2-2x_1x_2+x_2^2+y_1^2-2y_1y_2+y_2^2$$
$$4x^2-4x_1x-4x_2x+4x_1x_2+4y^2-4y_1y-4y_2y+4y_1y_2=0$$
$$x^2-x_1x-x_2x+x_1x_2+y^2-y_1y-y_2y+y_1y_2=0$$
$$x(x-x_1)-x_2(x+x_1)+y(y-y_1)-y_2(y+y_1)=0$$
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

132. Complete the square to get $\left(x+\frac{d}{2}\right)^2+\left(y+\frac{e}{2}\right)^2=\frac{d^2+e^2-4f}{4}$. The slope of the line between the center $\left(-\frac{d}{2},-\frac{e}{2}\right)$ and the point of tangency (x_0, y_0) is $m=\frac{y_0+\frac{e}{2}}{x_0+\frac{d}{2}}$. So the slope of the tangent line is $m_{\tan}=-\frac{x_0+\frac{d}{2}}{y_0+\frac{e}{2}}$.

Therefore, the equation of the tangent line is $y-y_0=-\frac{x_0+\frac{d}{2}}{y_0+\frac{e}{2}}(x-x_0)$ which is equivalent to

$$(x-x_0)\left(x_0+\tfrac{d}{2}\right)+(y-y_0)\left(y_0+\tfrac{e}{2}\right)=0$$
$$x_0x+\frac{d}{2}x-x_0^2-\frac{d}{2}x_0+y_0y+\frac{e}{2}y-y_0^2-\frac{e}{2}y_0=0$$
$$x_0x+y_0y+\frac{d}{2}x+\frac{e}{2}y-\left(x_0^2+y_0^2+\frac{d}{2}x_0+\frac{e}{2}y_0\right)=0$$

Because (x_0, y_0) is on the circle, $x_0{}^2 + y_0{}^2 + dx_0 + ey_0 + f = 0$, and

$x_0{}^2 + y_0{}^2 + \frac{d}{2}x_0 + \frac{e}{2}y_0 = -\frac{d}{2}x_0 - \frac{e}{2}y_0 - f$ Substituting this result gives

$$x_0{}^2 + y_0{}^2 + \frac{d}{2}x_0 + \frac{e}{2}y_0 - \left(-\frac{d}{2}x_0 - \frac{e}{2}y_0 - f\right) = 0$$

$$x_0{}^2 + y_0{}^2 + \frac{d}{2}x_0 + \frac{e}{2}y_0 + \frac{d}{2}x_0 + \frac{e}{2}y_0 + f = 0$$

$$x_0{}^2 + y_0{}^2 + d\left(\frac{x + x_0}{2}\right) + e\left(\frac{y + y_0}{2}\right) + f = 0$$

133. Let $y = 0$.

$$(x^2 + 0^2)^2 = a^2(x^2 - 0^2)$$
$$x^4 = a^2(x^2)$$
$$x^4 - a^2x^2 = 0$$
$$x^2(x^2 - a^2) = 0$$
$$x^2 = 0 \quad \text{or} \quad (x^2 - a^2) = 0$$
$$x = 0 \quad \text{or} \quad x^2 = a^2$$
$$x = -a, a$$

Let $x = 0$.

$$(0^2 + y^2)^2 = a^2(0^2 - y^2)$$
$$y^4 = a^2(-y^2)$$
$$y^4 + a^2y^2 = 0$$
$$y^2(y^2 + a^2) = 0$$
$$y = 0$$

(Note that the solutions to $y^2 + a^2 = 0$ are not real)

So the intercepts are are (0,0), (a,0) and (-a,0).

Test x-axis symmetry: Replace y by -y

$$(x^2 + (-y)^2)^2 = a^2(x^2 - (-y)^2)$$
$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \text{ equivalent}$$

Test y-axis symmetry: replace x by -x

$$((-x)^2 + y^2)^2 = a^2((-x)^2 - y^2)$$
$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \text{ equivalent}$$

Test origin symmetry: replace x by -x and y by -y

$$((-x)^2 + (-y)^2)^2 = a^2((-x)^2 - (-y)^2)$$
$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \text{ equivalent}$$

The graph is symmetric by respect to the x-axis, the y-axis, and the origin.

134. Let $y = 0$.

$$(x^2 + 0^2 - ax)^2 = b^2(x^2 + 0^2)$$
$$x^4 - 2ax^3 + a^2x^2 - b^2x^2 = 0$$
$$x^2\left[(x - (a + b)\right]\left[(x - (a - b)\right] = 0$$
$$x = 0 \quad \text{or} \quad x = a + b$$
$$\text{or} \quad x = a - b$$

Let $x = 0$.

$$(0^2 + y^2 - a \cdot 0)^2 = b^2(0^2 + y^2)$$
$$y^4 - b^2y^2 = 0$$
$$y^2(y + b)(y - b) = 0$$
$$y = 0, y = -b, y = b$$

So the intercepts are (0,0), (a-b,0), (a+b,0), (0,-b), (0, b).

Test x-axis symmetry: replace y by -y

$$\left[x^2 + (-y)^2 - ax\right]^2 = b^2\left[x^2 + (-y)^2\right]$$
$$(x^2 + y^2 - ax)^2 = b^2(x^2 + y^2) \text{ Equivalent}$$

Test y-axis symmetry: replace x by -x

$$\left[(-x)^2 + y^2 - a(-x)\right]^2 = b^2\left[(-x)^2 + y^2\right]$$
$$(x^2 + y^2 + ax)^2 = b^2(x^2 + y^2) \text{ Not equivalent}$$

Test origin symmetry: replace x by -x and y by -y

$$\left[(-x)^2 + (-y)^2 - a(-x)\right]^2 = b^2\left[(-x)^2 + (-y)^2\right]$$
$$(x^2 + y^2 + ax)^2 = b^2(x^2 + y^2) \text{ No equivalent}$$

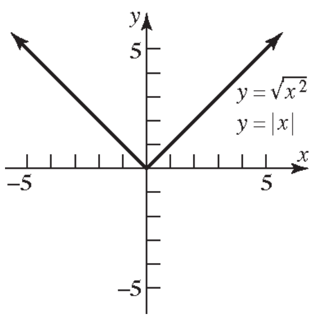
The graph is symmetric with respect to the x-axis only.

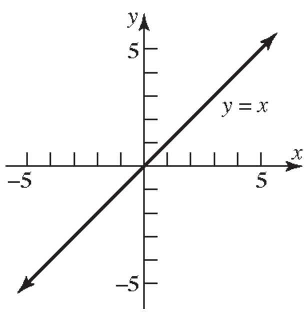
135. (b), (c), (e) and (g)
We need $h, k > 0$ and $(0,0)$ on the graph.

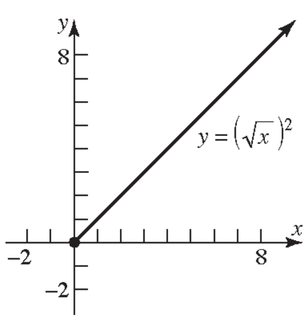
136. (b), (e) and (g)
We need $h < 0$, $k = 0$, and $|h| > r$.

Thus, the graph will have x-axis symmetry.

137. **a.**





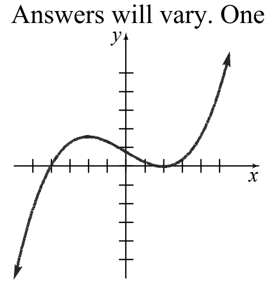


b. Since $\sqrt{x^2} = |x|$ for all x, the graphs of $y = \sqrt{x^2}$ and $y = |x|$ are the same.

c. For $y = \left(\sqrt{x}\right)^2$, the domain of the variable x is $x \geq 0$; for $y = x$, the domain of the variable x is all real numbers. Thus, $\left(\sqrt{x}\right)^2 = x$ only for $x \geq 0$.

d. For $y = \sqrt{x^2}$, the range of the variable y is $y \geq 0$; for $y = x$, the range of the variable y is all real numbers. Also, $\sqrt{x^2} = x$ only if $x \geq 0$. Otherwise, $\sqrt{x^2} = -x$.

138. Answers will vary. One example:



139. Answers will vary

140. Answers will vary

141. Answers will vary.
Case 1: Graph has x-axis and y-axis symmetry, show origin symmetry.
(x, y) on graph $\rightarrow (x, -y)$ on graph
(from x-axis symmetry)
$(x, -y)$ on graph $\rightarrow (-x, -y)$ on graph
(from y-axis symmetry)
Since the point $(-x, -y)$ is also on the graph, the graph has origin symmetry.

Case 2: Graph has x-axis and origin symmetry, show y-axis symmetry.
(x, y) on graph $\rightarrow (x, -y)$ on graph
(from x-axis symmetry)
$(x, -y)$ on graph $\rightarrow (-x, y)$ on graph
(from origin symmetry)
Since the point $(-x, y)$ is also on the graph, the graph has y-axis symmetry.

Case 3: Graph has y-axis and origin symmetry, show x-axis symmetry.
(x, y) on graph $\rightarrow (-x, y)$ on graph
(from y-axis symmetry)

$(-x, y)$ on graph $\rightarrow (x, -y)$ on graph
$($from origin symmetry$)$
Since the point $(x, -y)$ is also on the graph, the graph has x-axis symmetry.

142. Answers may vary. The graph must contain the points $(-2,5)$, $(-1,3)$, and $(0,2)$. For the graph to be symmetric about the y-axis, the graph must also contain the points $(2,5)$ and $(1,3)$ (note that (0, 2) is on the y-axis).

For the graph to also be symmetric with respect to the x-axis, the graph must also contain the points $(-2,-5)$, $(-1,-3)$, $(0,-2)$, $(2,-5)$, and $(1,-3)$. Recall that a graph with two of the symmetries (x-axis, y-axis, origin) will necessarily have the third. Therefore, if the original graph with y-axis symmetry also has x-axis symmetry, then it will also have origin symmetry.

143. Answers will vary.

144. The student has the correct radius, but the signs of the coordinates of the center are incorrect. The student needs to write the equation in the standard form $(x-h)^2 + (y-k)^2 = r^2$.

$(x+3)^2 + (y-2)^2 = 16$

$(x-(-3))^2 + (y-2)^2 = 4^2$

Thus, $(h,k) = (-3,2)$ and $r = 4$.

Section 1.3

1. $(-1,3)$

2. $3(-2)^2 - 5(-2) + \dfrac{1}{(-2)} = 3(4) - 5(-2) - \dfrac{1}{2}$

$= 12 + 10 - \dfrac{1}{2}$

$= \dfrac{43}{2}$ or $21\tfrac{1}{2}$ or 21.5

3. We must not allow the denominator to be 0.
$x + 4 \neq 0 \Rightarrow x \neq -4$; Domain: $\{x | x \neq -4\}$.

4. $3 - 2x > 5$

$-2x > 2$

$x < -1$

Solution set: $\{x \mid x < -1\}$ or $(-\infty, -1)$

5. $\sqrt{5} + 2$

6. radicals

7. independent; dependent

8. False; every function is a relation, but not every relation is a function. For example, the relation $x^2 + y^2 = 1$ is not a function.

9. False

10. False; $\{x \mid x \neq 0\}$

11. a

12. c

13. d

14. a

15. vertical

16. -3

17. $f(x) = ax^2 + 4$

$a(-1)^2 + 4 = 2 \Rightarrow a = -2$

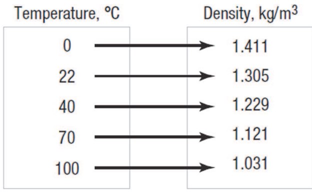
18. False. The graph must pass the vertical line test in order to be the graph of a function.

19. True

20. a

21. a. Domain: {0,22,40,70,100}
Range: {1.031, 1.121, 1.229, 1.305, 1.411}

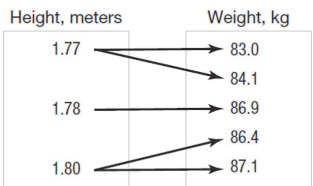
b.



c. {(0, 1.411), (22, 1.305), (40, 1.229), (70, 1.121), (100, 1.031)}

22. a. Domain: {1.80, 1.78, 1.77}
Range: {87.1, 86.9, 92.0, 84.1, 86.4}

b.



c. {(1.80, 87.1), (1.78, 86.9), (1.77, 83.0), (1.77, 84.1), (1.80, 86.4)}

23. Function
Domain: {Elvis, Colleen, Kaleigh, Marissa}
Range: {Jan. 8, Mar. 15, Sept. 17}

24. Domain: {Bob, John, Chuck}
Range: {Beth, Diane, Linda, Marcia}
Not a function

25. Domain: {20, 30, 40}
Range: {200, 300, 350, 425}
Not a function

26. Function
Domain: {Less than 9th grade, 9th-12th grade, High School Graduate, Some College, College Graduate}
Range: {\$18,120, \$23,251, \$36,055, \$45,810, \$67,165}

27. Domain: {-3, 2, 4}
Range: {6, 9, 10}
Not a function

28. Function
Domain: {–2, –1, 3, 4}
Range: {3, 5, 7, 12}

29. Function
Domain: {1, 2, 3, 4}
Range: {3}

30. Function
Domain: {0, 1, 2, 3}
Range: {–2, 3, 7}

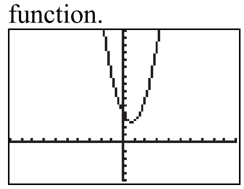
31. Domain: {-2, 0, 3}
Range: {3, 4, 6, 7}
Not a function

32. Domain: {-4, -3, -2, -1}
Range: {0, 1, 2, 3, 4}
Not a function

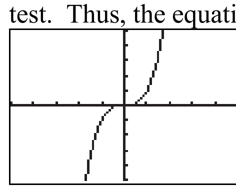
33. Function
Domain: {–2, –1, 0, 1}
Range: {0, 1, 4}

34. Function
Domain: {–2, –1, 0, 1}
Range: {3, 4, 16}

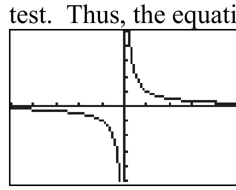
35. Graph $y = 2x^2 - 3x + 4$. The graph passes the vertical line test. Thus, the equation represents a function.



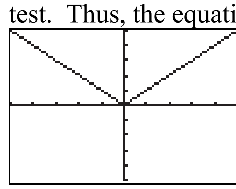
36. Graph $y = x^3$. The graph passes the vertical line test. Thus, the equation represents a function.



37. Graph $y = \dfrac{1}{x}$. The graph passes the vertical line test. Thus, the equation represents a function.



38. Graph $y = |x|$. The graph passes the vertical line test. Thus, the equation represents a function.



39. $y^2 = 4 - x^2$

Solve for y: $y = \pm\sqrt{4 - x^2}$

For $x = 0$, $y = \pm 2$. Thus, (0, 2) and (0, –2) are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

40. $y = \pm\sqrt{1-2x}$

For $x = 0$, $y = \pm 1$. Thus, (0, 1) and (0, –1) are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

41. $x = y^2$

Solve for y: $y = \pm\sqrt{x}$

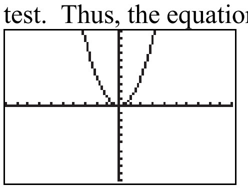
For $x = 1$, $y = \pm 1$. Thus, (1, 1) and (1, –1) are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

42. $x + y^2 = 1$

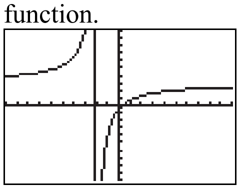
Solve for y: $y = \pm\sqrt{1-x}$

For $x = 0$, $y = \pm 1$. Thus, (0, 1) and (0, –1) are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

43. Graph $y = x^2$. The graph passes the vertical line test. Thus, the equation represents a function.



44. Graph $y = \dfrac{3x-1}{x+2}$. The graph passes the vertical line test. Thus, the equation represents a function.



45. $2x^2 + 3y^2 = 1$

Solve for y:
$$\begin{aligned} 2x^2 + 3y^2 &= 1 \\ 3y^2 &= 1 - 2x^2 \\ y^2 &= \frac{1-2x^2}{3} \\ y &= \pm\sqrt{\frac{1-2x^2}{3}} \end{aligned}$$

For $x = 0$, $y = \pm\sqrt{\dfrac{1}{3}}$. Thus, $\left(0, \sqrt{\dfrac{1}{3}}\right)$ and $\left(0, -\sqrt{\dfrac{1}{3}}\right)$ are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

46. $x^2 - 4y^2 = 1$

Solve for y:
$$\begin{aligned} x^2 - 4y^2 &= 1 \\ 4y^2 &= x^2 - 1 \\ y^2 &= \frac{x^2-1}{4} \\ y &= \frac{\pm\sqrt{x^2-1}}{2} \end{aligned}$$

For $x = \sqrt{2}$, $y = \pm\dfrac{1}{2}$. Thus, $\left(\sqrt{2}, \dfrac{1}{2}\right)$ and $\left(\sqrt{2}, -\dfrac{1}{2}\right)$ are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

47. $f(x) = 3x^2 + 2x - 4$

a. $f(0) = 3(0)^2 + 2(0) - 4 = -4$

b. $f(1) = 3(1)^2 + 2(1) - 4 = 3 + 2 - 4 = 1$

c. $f(-1) = 3(-1)^2 + 2(-1) - 4 = 3 - 2 - 4 = -3$

d. $f(-x) = 3(-x)^2 + 2(-x) - 4 = 3x^2 - 2x - 4$

e. $-f(x) = -\left(3x^2 + 2x - 4\right) = -3x^2 - 2x + 4$

f.
$$\begin{aligned} f(x+1) &= 3(x+1)^2 + 2(x+1) - 4 \\ &= 3\left(x^2 + 2x + 1\right) + 2x + 2 - 4 \\ &= 3x^2 + 6x + 3 + 2x + 2 - 4 \\ &= 3x^2 + 8x + 1 \end{aligned}$$

g. $f(2x) = 3(2x)^2 + 2(2x) - 4 = 12x^2 + 4x - 4$

h.
$$\begin{aligned} f(x+h) &= 3(x+h)^2 + 2(x+h) - 4 \\ &= 3\left(x^2 + 2xh + h^2\right) + 2x + 2h - 4 \\ &= 3x^2 + 6xh + 3h^2 + 2x + 2h - 4 \end{aligned}$$

4. $f(x) = -2x^2 + x - 1$

a. $f(0) = -2(0)^2 + 0 - 1 = -1$

b. $f(1)=-2(1)^2+1-1=-2$

c. $f(-1)=-2(-1)^2+(-1)-1=-4$

d. $f(-x)=-2(-x)^2+(-x)-1=-2x^2-x-1$

e. $-f(x)=-\left(-2x^2+x-1\right)=2x^2-x+1$

f. $$\begin{aligned} f(x+1)&=-2(x+1)^2+(x+1)-1\\ &=-2\left(x^2+2x+1\right)+x+1-1\\ &=-2x^2-4x-2+x\\ &=-2x^2-3x-2 \end{aligned}$$

g. $f(2x)=-2(2x)^2+(2x)-1=-8x^2+2x-1$

h. $$\begin{aligned} f(x+h)&=-2(x+h)^2+(x+h)-1\\ &=-2\left(x^2+2xh+h^2\right)+x+h-1\\ &=-2x^2-4xh-2h^2+x+h-1 \end{aligned}$$

49. $f(x)=\dfrac{x}{x^2+1}$

a. $f(0)=\dfrac{0}{0^2+1}=\dfrac{0}{1}=0$

b. $f(1)=\dfrac{1}{1^2+1}=\dfrac{1}{2}$

c. $f(-1)=\dfrac{-1}{(-1)^2+1}=\dfrac{-1}{1+1}=-\dfrac{1}{2}$

d. $f(-x)=\dfrac{-x}{(-x)^2+1}=\dfrac{-x}{x^2+1}$

e. $-f(x)=-\left(\dfrac{x}{x^2+1}\right)=\dfrac{-x}{x^2+1}$

f. $$\begin{aligned} f(x+1)&=\frac{x+1}{(x+1)^2+1}\\ &=\frac{x+1}{x^2+2x+1+1}\\ &=\frac{x+1}{x^2+2x+2} \end{aligned}$$

g. $f(2x)=\dfrac{2x}{(2x)^2+1}=\dfrac{2x}{4x^2+1}$

h. $f(x+h)=\dfrac{x+h}{(x+h)^2+1}=\dfrac{x+h}{x^2+2xh+h^2+1}$

50. $f(x)=\dfrac{x^2-1}{x+4}$

a. $f(0)=\dfrac{0^2-1}{0+4}=\dfrac{-1}{4}=-\dfrac{1}{4}$

b. $f(1)=\dfrac{1^2-1}{1+4}=\dfrac{0}{5}=0$

c. $f(-1)=\dfrac{(-1)^2-1}{-1+4}=\dfrac{0}{3}=0$

d. $f(-x)=\dfrac{(-x)^2-1}{-x+4}=\dfrac{x^2-1}{-x+4}$

e. $-f(x)=-\left(\dfrac{x^2-1}{x+4}\right)=\dfrac{-x^2+1}{x+4}$

f. $$\begin{aligned} f(x+1)&=\frac{(x+1)^2-1}{(x+1)+4}\\ &=\frac{x^2+2x+1-1}{x+5}=\frac{x^2+2x}{x+5} \end{aligned}$$

g. $f(2x)=\dfrac{(2x)^2-1}{2x+4}=\dfrac{4x^2-1}{2x+4}$

h. $f(x+h)=\dfrac{(x+h)^2-1}{(x+h)+4}=\dfrac{x^2+2xh+h^2-1}{x+h+4}$

51. $f(x)=|x|+4$

a. $f(0)=|0|+4=0+4=4$

b. $f(1)=|1|+4=1+4=5$

c. $f(-1)=|-1|+4=1+4=5$

d. $f(-x)=|-x|+4=|x|+4$

e. $-f(x)=-(|x|+4)=-|x|-4$

f. $f(x+1)=|x+1|+4$

g. $f(2x)=|2x|+4=2|x|+4$

h. $f(x+h)=|x+h|+4$

52. $f(x)=\sqrt{x^2+x}$

a. $f(0)=\sqrt{0^2+0}=\sqrt{0}=0$

b. $f(1)=\sqrt{1^2+1}=\sqrt{2}$

c. $f(-1)=\sqrt{(-1)^2+(-1)}=\sqrt{1-1}=\sqrt{0}=0$

d. $f(-x)=\sqrt{(-x)^2+(-x)}=\sqrt{x^2-x}$

e. $-f(x)=-\left(\sqrt{x^2+x}\right)=-\sqrt{x^2+x}$

f. $$f(x+1)=\sqrt{(x+1)^2+(x+1)}$$
$$=\sqrt{x^2+2x+1+x+1}$$
$$=\sqrt{x^2+3x+2}$$

g. $f(2x)=\sqrt{(2x)^2+2x}=\sqrt{4x^2+2x}$

h. $$f(x+h)=\sqrt{(x+h)^2+(x+h)}$$
$$=\sqrt{x^2+2xh+h^2+x+h}$$

53. $f(x)=\dfrac{2x+1}{3x-5}$

a. $f(0)=\dfrac{2(0)+1}{3(0)-5}=\dfrac{0+1}{0-5}=-\dfrac{1}{5}$

b. $f(1)=\dfrac{2(1)+1}{3(1)-5}=\dfrac{2+1}{3-5}=\dfrac{3}{-2}=-\dfrac{3}{2}$

c. $f(-1)=\dfrac{2(-1)+1}{3(-1)-5}=\dfrac{-2+1}{-3-5}=\dfrac{-1}{-8}=\dfrac{1}{8}$

d. $f(-x)=\dfrac{2(-x)+1}{3(-x)-5}=\dfrac{-2x+1}{-3x-5}=\dfrac{2x-1}{3x+5}$

e. $-f(x)=-\left(\dfrac{2x+1}{3x-5}\right)=\dfrac{-2x-1}{3x-5}$

f. $f(x+1)=\dfrac{2(x+1)+1}{3(x+1)-5}=\dfrac{2x+2+1}{3x+3-5}=\dfrac{2x+3}{3x-2}$

g. $f(2x)=\dfrac{2(2x)+1}{3(2x)-5}=\dfrac{4x+1}{6x-5}$

h. $f(x+h)=\dfrac{2(x+h)+1}{3(x+h)-5}=\dfrac{2x+2h+1}{3x+3h-5}$

54. $f(x)=1-\dfrac{1}{(x+2)^2}$

a. $f(0)=1-\dfrac{1}{(0+2)^2}=1-\dfrac{1}{4}=\dfrac{3}{4}$

b. $f(1)=1-\dfrac{1}{(1+2)^2}=1-\dfrac{1}{9}=\dfrac{8}{9}$

c. $f(-1)=1-\dfrac{1}{(-1+2)^2}=1-\dfrac{1}{1}=0$

d. $f(-x)=1-\dfrac{1}{(-x+2)^2}$

e. $-f(x)=-\left(1-\dfrac{1}{(x+2)^2}\right)=\dfrac{1}{(x+2)^2}-1$

f. $f(x+1)=1-\dfrac{1}{(x+1+2)^2}=1-\dfrac{1}{(x+3)^2}$

g. $f(2x)=1-\dfrac{1}{(2x+2)^2}=1-\dfrac{1}{4(x+1)^2}$

h. $f(x+h)=1-\dfrac{1}{(x+h+2)^2}$

55. $f(x)=-5x+4$

Domain: $\{x \mid x \text{ is any real number}\}$

56. $f(x)=x^2+2$

Domain: $\{x \mid x \text{ is any real number}\}$

57. $f(x)=\dfrac{x}{x^2+1}$

Domain: $\{x \mid x \text{ is any real number}\}$

58. $f(x)=\dfrac{x^2}{x^2+1}$

Domain: $\{x \mid x \text{ is any real number}\}$

59. $g(x)=\dfrac{x}{x^2-16}$

$x^2-16\neq 0$

$x^2\neq 16 \Rightarrow x\neq \pm 4$

Domain: $\{x \mid x\neq -4,\ x\neq 4\}$

60. $h(x)=\dfrac{2x}{x^2-4}$

$x^2-4\neq 0$

$x^2\neq 4 \Rightarrow x\neq \pm 2$

Domain: $\{x \mid x\neq -2,\ x\neq 2\}$

61. $F(x)=\dfrac{x-2}{x^3+x}$

$x^3+x\neq 0$

$x(x^2+1)\neq 0$

$x\neq 0,\quad x^2\neq -1$

Domain: $\{x \mid x\neq 0\}$

62. $G(x)=\dfrac{x+4}{x^3-4x}$

$x^3-4x\neq 0$

$x(x^2-4)\neq 0$

$x\neq 0,\quad x^2\neq 4$

$x\neq 0,\quad x\neq \pm 2$

Domain: $\{x \mid x\neq -2,\ x\neq 0,\ x\neq 2\}$

63. $h(x)=\sqrt{3x-12}$

$3x-12\geq 0$

$3x\geq 12$

$x\geq 4$

Domain: $\{x \mid x\geq 4\}$

64. $G(x)=\sqrt{1-x}$

$1-x\geq 0$

$-x\geq -1$

$x\leq 1$

Domain: $\{x \mid x\leq 1\}$

65. $p(x)=\sqrt{\dfrac{2}{x-1}}=\dfrac{\sqrt{2}}{\sqrt{x-1}}$

$x-1>0$

$x>1$

Domain: $\{x \mid x>1\}$

66. $f(x)=\dfrac{4}{\sqrt{x-9}}$

$x-9>0$

$x>9$

Domain: $\{x \mid x>9\}$

67. $f(x)=\dfrac{x}{\sqrt{x-4}}$

$x-4>0$

$x>4$

Domain: $\{x \mid x>4\}$

68. $q(x)=\frac{-x}{\sqrt{-x-2}}$

$-x-2>0$

$-x>2$

$x<-2$

Domain: $\{x \mid x<-2\}$

69. $P(t)=\dfrac{\sqrt{t-4}}{3t-21}$

$t-4\geq 0$

$t\geq 4$

Also $3t-21\neq 0$

$3t-21\neq 0$

$3t\neq 21$

$t\neq 7$

Domain: $\{t \mid t\geq 4, t\neq 7\}$

70. $h(z)=\dfrac{\sqrt{z+3}}{z-2}$

$z+3\geq 0$

$z\geq -3$

Also $z-2\neq 0$

$z\neq 2$

Domain: $\{z \mid z\geq -3, z\neq 2\}$

71. $f(x)=4x+3$

$$\frac{f(x+h)-f(x)}{h}=\frac{4(x+h)+3-(4x+3)}{h}=\frac{4x+4h+3-4x-3}{h}=\frac{4h}{h}=4$$

72. $f(x)=-3x+1$

$$\frac{f(x+h)-f(x)}{h}=\frac{-3(x+h)+1-(-3x+1)}{h}=\frac{-3x-3h+1+3x-1}{h}=\frac{-3h}{h}=-3$$

73. $f(x)=x^2-4$

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2-4-(x^2-4)}{h}=\frac{x^2+2xh+h^2-4-x^2+4}{h}=\frac{2xh+h^2}{h}=2x+h$$

74. $f(x)=3x^2+2$

$$\frac{f(x+h)-f(x)}{h}=\frac{3(x+h)^2+2-(3x^2+2)}{h}=\frac{3x^2+6xh+3h^2+2-3x^2-2}{h}=\frac{6xh+3h^2}{h}=6x+3h$$

75. $f(x)=x^2-x+4$

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2-(x+h)+4-(x^2-x+4)}{h}=\frac{x^2+2xh+h^2-x-h+4-x^2+x-4}{h}=\frac{2xh+h^2-h}{h}=2x+h-1$$

76. $f(x)=3x^2-2x+6$

$$\frac{f(x+h)-f(x)}{h}=\frac{\left[3(x+h)^2-2(x+h)+6\right]-\left[3x^2-2x+6\right]}{h}=\frac{3\left(x^2+2xh+h^2\right)-2x-2h+6-3x^2+2x-6}{h}=\frac{3x^2+6xh+3h^2-2h-3x^2}{h}=\frac{6xh+3h^2-2h}{h}=6x+3h-2$$

77. $f(x)=\dfrac{1}{x^2}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}=\frac{\dfrac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}=\frac{\dfrac{x-\left(x^2+2xh+h^2\right)}{x^2(x+h)^2}}{h}=\left(\frac{1}{h}\right)\frac{-2xh-h^2}{x^2(x+h)^2}=\left(\frac{1}{h}\right)\frac{h(-2x-h)}{x^2(x+h)^2}=\frac{-2x-h}{x^2(x+h)^2}=\frac{-(2x+h)}{x^2(x+h)^2}$$

78. $f(x)=\dfrac{1}{x+3}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{x+h+3}-\frac{1}{x+3}}{h}$$

$$=\frac{\frac{x+3-(x+3+h)}{(x+h+3)(x+3)}}{h}$$

$$=\left(\frac{x+3-x-3-h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)$$

$$=\left(\frac{-h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)$$

$$=\frac{-1}{(x+h+3)(x+3)}$$

79. $f(x)=\dfrac{2x}{x+3}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{2(x+h)}{x+h+3}-\frac{2x}{x+3}}{h}$$

$$=\frac{\frac{2(x+h)(x+3)-2x(x+3+h)}{(x+h+3)(x+3)}}{h}$$

$$=\frac{\frac{2x^2+6x+2hx+6h-2x^2-6x-2xh}{(x+h+3)(x+3)}}{h}$$

$$=\left(\frac{6h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)$$

$$=\frac{6}{(x+h+3)(x+3)}$$

80. $f(x)=\dfrac{5x}{x-4}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{5(x+h)}{x+h-4}-\frac{5x}{x-4}}{h}$$

$$=\frac{\frac{5(x+h)(x-4)-5x(x-4+h)}{(x+h-4)(x-4)}}{h}$$

$$=\frac{\frac{5x^2-20x+5hx-20h-5x^2+20x-5xh}{(x+h-4)(x-4)}}{h}$$

$$=\left(\frac{-20h}{(x+h-4)(x-4)}\right)\left(\frac{1}{h}\right)$$

$$=-\frac{20}{(x+h-4)(x-4)}$$

81. $f(x)=\sqrt{x-2}$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{\sqrt{x+h-2}-\sqrt{x-2}}{h}$$

$$=\frac{\sqrt{x+h-2}-\sqrt{x-2}}{h}\cdot\frac{\sqrt{x+h-2}+\sqrt{x-2}}{\sqrt{x+h-2}+\sqrt{x-2}}$$

$$=\frac{x+h-2-x+2}{h\left(\sqrt{x+h-2}+\sqrt{x-2}\right)}$$

$$=\frac{h}{h\left(\sqrt{x+h-2}+\sqrt{x-2}\right)}$$

$$=\frac{1}{\sqrt{x+h-2}+\sqrt{x-2}}$$

82. $f(x)=\sqrt{x+1}$

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}$$
$$=\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\cdot\frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}$$
$$=\frac{x+h+1-(x+1)}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}=\frac{h}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$
$$=\frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$$

83. **a.** $f(0)=3$ since $(0,3)$ is on the graph.
$f(-6)=-3$ since $(-6,-3)$ is on the graph.

b. $f(6)=0$ since $(6, 0)$ is on the graph.
$f(11)=1$ since $(11, 1)$ is on the graph.

c. $f(3)$ is positive since $f(3)\approx 3.7$.

d. $f(-4)$ is negative since $f(-4)\approx -1$.

e. $f(x)=0$ when $x=-3$, $x=6$, and $x=10$.

f. $f(x)>0$ when $-3<x<6$, and $10<x\le 11$.

g. The domain of f is $\{x\mid -6\le x\le 11\}$ or $[-6, 11]$.

h. The range of f is $\{y\mid -3\le y\le 4\}$ or $[-3, 4]$.

i. The x-intercepts are -3, 6, and 10.

j. The y-intercept is 3.

k. The line $y=\frac{1}{2}$ intersects the graph 3 times.

l. The line $x=5$ intersects the graph 1 time.

m. $f(x)=3$ when $x=0$ and $x=4$.

n. $f(x)=-2$ when $x=-5$ and $x=8$.

84. **a.** $f(0)=0$ since $(0,0)$ is on the graph.
$f(6)=0$ since $(6,0)$ is on the graph.

b. $f(2)=-2$ since $(2,-2)$ is on the graph.
$f(-2)=1$ since $(-2, 1)$ is on the graph.

c. $f(3)$ is negative since $f(3)\approx -1$.

d. $f(-1)$ is positive since $f(-1)\approx 1.0$.

e. $f(x)=0$ when $x=0$, $x=4$, and $x=6$.

f. $f(x)<0$ when $0<x<4$.

g. The domain of f is $\{x\mid -4\le x\le 6\}$ or $[-4, 6]$.

h. The range of f is $\{y\mid -2\le y\le 3\}$ or $[-2, 3]$.

i. The x-intercepts are 0, 4, and 6.

j. The y-intercept is 0.

k. The line $y=-1$ intersects the graph 2 times.

l. The line $x=1$ intersects the graph 1 time.

m. $f(x)=3$ when $x=5$.

n. $f(x)=-2$ when $x=2$.

85. Not a function since vertical lines will intersect the graph in more than one point.

86. Function

a. Domain: $\{x\mid x \text{ is any real number}\}$;
Range: $\{y\mid y>0\}$

b. Intercepts: $(0,1)$

c. None

87. Function

a. Domain: $\{x\mid -\pi\le x\le\pi\}$;
Range: $\{y\mid -1\le y\le 1\}$

b. Intercepts: $\left(-\frac{\pi}{2},0\right)$, $\left(\frac{\pi}{2},0\right)$, $(0,1)$

c. Symmetry about y-axis.

88. Function

a. Domain: $\{x\mid -\pi\le x\le\pi\}$;
Range: $\{y\mid -1\le y\le 1\}$

b. Intercepts: $(-\pi, 0)$, $(\pi, 0)$, $(0, 0)$

c. Symmetry about the origin.

89. Not a function since vertical lines will intersect the graph in more than one point.

90. Not a function since vertical lines will intersect the graph in more than one point.

91. Function

a. Domain: $\{x \mid 0 < x < 3\}$;

Range: $\{y \mid y < 2\}$

b. Intercepts: (1, 0)

c. None

92. Function

a. Domain: $\{x \mid 0 \le x < 4\}$;

Range: $\{y \mid 0 \le y < 3\}$

b. Intercepts: (0, 0)

c. None

93. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \le 2\}$

b. Intercepts: (–3, 0), (3, 0), (0,2)

c. Symmetry about *y*-axis.

94. Function

a. Domain: $\{x \mid x \ge -3\}$;

Range: $\{y \mid y \ge 0\}$

b. Intercepts: (–3, 0), (2,0), (0,2)

c. None

95. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \ge -3\}$

b. Intercepts: (1, 0), (3,0), (0,9)

c. None

96. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \le 5\}$

b. Intercepts: (–1, 0), (2,0), (0,4)

c. None

97. $f(x) = 2x^2 - x - 1$

a. $f(-1) = 2(-1)^2 - (-1) - 1 = 2$

The point $(-1, 2)$ is on the graph of *f*.

b. $f(-2) = 2(-2)^2 - (-2) - 1 = 9$

The point $(-2, 9)$ is on the graph of *f*.

c. Solve for x:

$$-1 = 2x^2 - x - 1$$
$$0 = 2x^2 - x$$
$$0 = x(2x-1) \Rightarrow x = 0, x = \tfrac{1}{2}$$

(0, –1) and $\left(\tfrac{1}{2}, -1\right)$ are on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. *x*-intercepts:

$$f(x)=0 \Rightarrow 2x^2 - x - 1 = 0$$
$$(2x+1)(x-1) = 0 \Rightarrow x = -\frac{1}{2}, x = 1$$

$\left(-\dfrac{1}{2}, 0\right)$ and $(1, 0)$

f. *y*-intercept:

$$f(0)=2(0)^2 - 0 - 1 = -1 \Rightarrow (0, -1)$$

98. $f(x) = -3x^2 + 5x$

a. $f(-1) = -3(-1)^2 + 5(-1) = -8 \ne 2$

The point $(-1, 2)$ is not on the graph of *f*.

b. $f(-2) = -3(-2)^2 + 5(-2) = -22$

The point $(-2, -22)$ is on the graph of *f*.

c. Solve for x:

$$-2 = -3x^2 + 5x \Rightarrow 3x^2 - 5x - 2 = 0$$
$$(3x+1)(x-2) = 0 \Rightarrow x = -\tfrac{1}{3}, x = 2$$

(2, –2) and $\left(-\tfrac{1}{3}, -2\right)$ on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. *x*-intercepts:

$$f(x)=0 \Rightarrow -3x^2 + 5x = 0$$
$$x(-3x+5) = 0 \Rightarrow x = 0, x = \tfrac{5}{3}$$

$(0,0)$ and $\left(\tfrac{5}{3}, 0\right)$

f. *y*-intercept:

$$f(0) = -3(0)^2 + 5(0) = 0 \Rightarrow (0,0)$$

99. $f(x)=\dfrac{x+2}{x-6}$

a. $f(3)=\dfrac{3+2}{3-6}=-\dfrac{5}{3}\neq 14$

The point $(3,14)$ is not on the graph of *f*.

b. $f(4)=\dfrac{4+2}{4-6}=\dfrac{6}{-2}=-3$

The point $(4,-3)$ is on the graph of *f*.

c. Solve for x:

$$2=\frac{x+2}{x-6}$$
$$2x-12=x+2$$
$$x=14$$

(14, 2) is a point on the graph of f.

d. The domain of *f* is $\{x \mid x\neq 6\}$.

e. *x*-intercepts:

$f(x)=0\Rightarrow\dfrac{x+2}{x-6}=0$

$x+2=0\Rightarrow x=-2\Rightarrow(-2,0)$

f. *y*-intercept: $f(0)=\dfrac{0+2}{0-6}=-\dfrac{1}{3}\Rightarrow\left(0,-\dfrac{1}{3}\right)$

100. $f(x)=\dfrac{x^2+2}{x+4}$

a. $f(1)=\dfrac{1^2+2}{1+4}=\dfrac{3}{5}$

The point $\left(1,\dfrac{3}{5}\right)$ is on the graph of *f*.

b. $f(0)=\dfrac{0^2+2}{0+4}=\dfrac{2}{4}=\dfrac{1}{2}$

The point $\left(0,\dfrac{1}{2}\right)$ is on the graph of *f*.

c. Solve for x:

$$\frac{1}{2}=\frac{x^2+2}{x+4}\Rightarrow x+4=2x^2+4$$
$$0=2x^2-x$$

$x(2x-1)=0\Rightarrow x=0$ or $x=\dfrac{1}{2}$

$\left(0,\dfrac{1}{2}\right)$ and $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$ are on the graph of f.

d. The domain of *f* is $\{x \mid x\neq -4\}$.

e. *x*-intercepts:

$f(x)=0\Rightarrow\dfrac{x^2+2}{x+4}=0\Rightarrow x^2+2=0$

This is impossible, so there are no x-intercepts.

f. *y*-intercept:

$f(0)=\dfrac{0^2+2}{0+4}=\dfrac{2}{4}=\dfrac{1}{2}\Rightarrow\left(0,\dfrac{1}{2}\right)$

101. $f(x)=\dfrac{2x^2}{x^4+1}$

a. $f(-1)=\dfrac{2(-1)^2}{(-1)^4+1}=\dfrac{2}{2}=1$

The point $(-1,1)$ is on the graph of *f*.

b. $f(2)=\dfrac{2(2)^2}{(2)^4+1}=\dfrac{8}{17}$

The point $\left(2,\dfrac{8}{17}\right)$ is on the graph of *f*.

c. Solve for x:

$$1=\frac{2x^2}{x^4+1}$$
$$x^4+1=2x^2$$
$$x^4-2x^2+1=0$$
$$(x^2-1)^2=0$$
$$x^2-1=0\Rightarrow x=\pm1$$

(1,1) and (–1,1) are on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. *x*-intercept:

$f(x)=0\Rightarrow\dfrac{2x^2}{x^4+1}=0$

$2x^2=0\Rightarrow x=0\Rightarrow(0,0)$

f. *y*-intercept:

$f(0)=\dfrac{2(0)^2}{0^4+1}=\dfrac{0}{0+1}=0\Rightarrow(0,0)$

102. $f(x)=\dfrac{2x}{x-2}$

a. $f\left(\dfrac{1}{2}\right)=\dfrac{2\left(\dfrac{1}{2}\right)}{\dfrac{1}{2}-2}=\dfrac{1}{-\dfrac{3}{2}}=-\dfrac{2}{3}$

The point $\left(\dfrac{1}{2},-\dfrac{2}{3}\right)$ is on the graph of *f*.

b. $f(4)=\dfrac{2(4)}{4-2}=\dfrac{8}{2}=4$

The point $(4,4)$ is on the graph of *f*.

c. Solve for x:

$1=\dfrac{2x}{x-2}\Rightarrow x-2=2x\Rightarrow -2=x$

$(-2,1)$ is a point on the graph of f.

d. The domain of *f* is $\{x \mid x\neq 2\}$.

e. *x*-intercept:

$f(x)=0\Rightarrow \dfrac{2x}{x-2}=0\Rightarrow 2x=0$

$\Rightarrow x=0\Rightarrow (0,0)$

f. *y*-intercept: $f(0)=\dfrac{0}{0-2}=0\Rightarrow (0,0)$

103. $11=x^2-2x+3$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x-4=0$ or $x+2=0$

$x=4 \qquad x=-2$

The solution set is $\{-2,4\}$

104. $-\dfrac{7}{16}=\dfrac{5}{6}x-\dfrac{3}{4}$

$\dfrac{5}{16}=\dfrac{5}{6}x$

$\dfrac{6}{5}\cdot\dfrac{5}{16}=x$

$\dfrac{3}{8}=x$

The solution set is $\left\{\dfrac{3}{8}\right\}$

105. $f(x)=2x^3+Ax^2+4x-5$ and $f(2)=5$

$f(2)=2(2)^3+A(2)^2+4(2)-5$

$5=16+4A+8-5$

$5=4A+19$

$-14=4A$

$A=\dfrac{-14}{4}=-\dfrac{7}{2}$

106. $f(x)=3x^2-Bx+4$ and $f(-1)=12$:

$f(-1)=3(-1)^2-B(-1)+4$

$12=3+B+4$

$B=5$

107. $f(x)=\dfrac{3x+8}{2x-A}$ and $f(0)=2$

$f(0)=\dfrac{3(0)+8}{2(0)-A}$

$2=\dfrac{8}{-A}$

$-2A=8$

$A=-4$

108. $f(x)=\dfrac{2x-B}{3x+4}$ and $f(2)=\dfrac{1}{2}$

$f(2)=\dfrac{2(2)-B}{3(2)+4}$

$\dfrac{1}{2}=\dfrac{4-B}{10}$

$5=4-B$

$B=-1$

109. Let *x* represent the length of the rectangle.

Then, $\dfrac{x}{2}$ represents the width of the rectangle since the length is twice the width. The function for the area is: $A(x)=x\cdot\dfrac{x}{2}=\dfrac{x^2}{2}=\dfrac{1}{2}x^2$

110. Let *x* represent the length of one of the two equal sides. The function for the area is:

$A(x)=\dfrac{1}{2}\cdot x\cdot x=\dfrac{1}{2}x^2$

111. Let *x* represent the number of hours worked. The function for the gross salary is:

$G(x)=14x$

112. Let x represent the number of items sold. The function for the gross salary is: $G(x) = 10x + 100$

113. **a.**
$$H(1) = 20 - 4.9(1)^2 = 20 - 4.9 = 15.1 \text{ meters}$$
$$H(1.1) = 20 - 4.9(1.1)^2 = 20 - 4.9(1.21) = 20 - 5.929 = 14.071 \text{ meters}$$
$$H(1.2) = 20 - 4.9(1.2)^2 = 20 - 4.9(1.44) = 20 - 7.056 = 12.944 \text{ meters}$$
$$H(1.3) = 20 - 4.9(1.3)^2 = 20 - 4.9(1.69) = 20 - 8.281 = 11.719 \text{ meters}$$

b. $H(x) = 15$:
$$15 = 20 - 4.9x^2$$
$$-5 = -4.9x^2$$
$$x^2 \approx 1.0204$$
$$x \approx 1.01 \text{ seconds}$$

$H(x) = 10$:
$$10 = 20 - 4.9x^2$$
$$-10 = -4.9x^2$$
$$x^2 \approx 2.0408$$
$$x \approx 1.43 \text{ seconds}$$

$H(x) = 5$:
$$5 = 20 - 4.9x^2$$
$$-15 = -4.9x^2$$
$$x^2 \approx 3.0612$$
$$x \approx 1.75 \text{ seconds}$$

c. $H(x) = 0$
$$0 = 20 - 4.9x^2$$
$$-20 = -4.9x^2$$
$$x^2 \approx 4.0816$$
$$x \approx 2.02 \text{ seconds}$$

114. **a.**
$$H(1) = 20 - 13(1)^2 = 20 - 13 = 7 \text{ meters}$$
$$H(1.1) = 20 - 13(1.1)^2 = 20 - 13(1.21) = 20 - 15.73 = 4.27 \text{ meters}$$
$$H(1.2) = 20 - 13(1.2)^2 = 20 - 13(1.44) = 20 - 18.72 = 1.28 \text{ meters}$$

b. $H(x) = 15$
$$15 = 20 - 13x^2$$
$$-5 = -13x^2$$
$$x^2 \approx 0.3846$$
$$x \approx 0.62 \text{ seconds}$$

$H(x) = 10$
$$10 = 20 - 13x^2$$
$$-10 = -13x^2$$
$$x^2 \approx 0.7692$$
$$x \approx 0.88 \text{ seconds}$$

$H(x) = 5$
$$5 = 20 - 13x^2$$
$$-15 = -13x^2$$
$$x^2 \approx 1.1538$$
$$x \approx 1.07 \text{ seconds}$$

c. $H(x) = 0$
$$0 = 20 - 13x^2$$
$$-20 = -13x^2$$
$$x^2 \approx 1.5385$$
$$x \approx 1.24 \text{ seconds}$$

115. $C(x) = 100 + \dfrac{x}{10} + \dfrac{36{,}000}{x}$

a.
$$C(500) = 100 + \frac{500}{10} + \frac{36{,}000}{500} = 100 + 50 + 72 = \$222$$

b.
$$C(450) = 100 + \frac{450}{10} + \frac{36{,}000}{450} = 100 + 45 + 80 = \$225$$

c. $C(600) = 100 + \frac{600}{10} + \frac{36,000}{600}$
$= 100 + 60 + 60$
$= \$220$

d. $C(400) = 100 + \frac{400}{10} + \frac{36,000}{400}$
$= 100 + 40 + 90$
$= \$230$

116. $A(x) = 4x\sqrt{1-x^2}$

a. $A\left(\frac{1}{3}\right) = 4 \cdot \frac{1}{3}\sqrt{1-\left(\frac{1}{3}\right)^2} = \frac{4}{3}\sqrt{\frac{8}{9}} = \frac{4}{3} \cdot \frac{2\sqrt{2}}{3}$
$= \frac{8\sqrt{2}}{9} \approx 1.26 \text{ ft}^2$

b. $A\left(\frac{1}{2}\right) = 4 \cdot \frac{1}{2}\sqrt{1-\left(\frac{1}{2}\right)^2} = 2\sqrt{\frac{3}{4}} = 2 \cdot \frac{\sqrt{3}}{2}$
$= \sqrt{3} \approx 1.73 \text{ ft}^2$

c. $A\left(\frac{2}{3}\right) = 4 \cdot \frac{2}{3}\sqrt{1-\left(\frac{2}{3}\right)^2} = \frac{8}{3}\sqrt{\frac{5}{9}} = \frac{8}{3} \cdot \frac{\sqrt{5}}{3}$
$= \frac{8\sqrt{5}}{9} \approx 1.99 \text{ ft}^2$

117. $h(x) = -\frac{44x^2}{v^2} + x + 6$

a. $h(8) = -\frac{44(8)^2}{28^2} + (8) + 6$
$= -\frac{2816}{784} + 14 \approx 10.4$ feet

b. $h(12) = -\frac{44(12)^2}{28^2} + (12) + 6$
$= -\frac{6336}{784} + 18 \approx 9.9$ feet

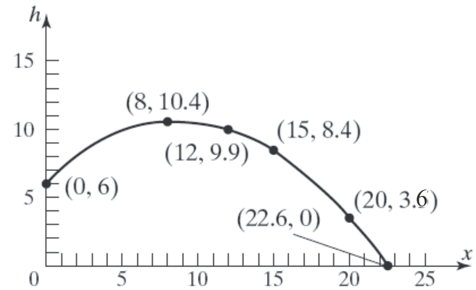
c. From part (a) we know the point $(8, 10.4)$ is on the graph and from part (b) we know the point $(12, 9.9)$ is on the graph. We could evaluate the function at several more values of x (e.g. $x = 0$, $x = 15$, and $x = 20$) to obtain additional points.

$h(0) = -\frac{44(0)^2}{28^2} + (0) + 6 = 6$

$h(15) = -\frac{44(15)^2}{28^2} + (15) + 6 \approx 8.4$

$h(20) = -\frac{44(20)^2}{28^2} + (20) + 6 \approx 3.6$

Some additional points are $(0,6)$, $(15, 8.4)$ and $(20, 3.6)$. The complete graph is given below.



d. $h(15) = -\frac{44(15)^2}{28^2} + (15) + 6 \approx 8.4$ feet

No; when the ball is 15 feet in front of the foul line, it will be below the hoop. Therefore it cannot go through the hoop.

In order for the ball to pass through the hoop, we need to have $h(15) = 10$.

$$10 = -\frac{44(15)^2}{v^2} + (15) + 6$$
$$-11 = -\frac{44(15)^2}{v^2}$$
$$v^2 = 4(225) = 900$$
$$v = 30 \text{ ft/sec}$$

The ball must be shot with an initial velocity of 30 feet per second in order to go through the hoop.

118. $h(x) = -\frac{136x^2}{v^2} + 2.7x + 3.5$

a. We want $h(15) = 10$.

$$-\frac{136(15)^2}{v^2} + 2.7(15) + 3.5 = 10$$

$$-\frac{30,600}{v^2} = -34$$

$$v^2 = 900$$

$$v = 30 \text{ ft/sec}$$

The ball needs to be thrown with an initial velocity of 30 feet per second.

b. $h(x) = -\frac{126x^2}{30^2} + 2.7x + 3.5$

which simplifies to

$h(x) = -\frac{34}{225}x^2 + 2.7x + 3.5$

c. Using the velocity from part (b),

$h(9) = -\frac{34}{225}(9)^2 + 2.7(9) + 3.5 = 15.56 \text{ ft}$

The ball will be 15.56 feet above the floor when it has traveled 9 feet in front of the foul line.

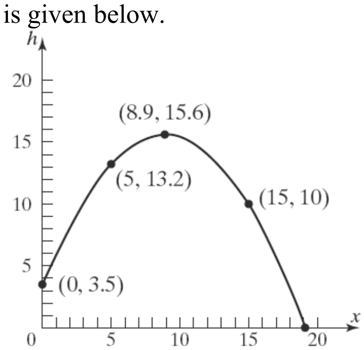
d. Select several values for x and use these to find the corresponding values for h. Use the results to form ordered pairs (x, h). Plot the points and connect with a smooth curve.

$h(0) = -\frac{34}{225}(0)^2 + 2.7(0) + 3.5 = 3.5 \text{ ft}$

$h(5) = -\frac{34}{225}(5)^2 + 2.7(5) + 3.5 \approx 13.2 \text{ ft}$

$h(15) = -\frac{24}{225}(15)^2 + 2.7(15) + 3.5 \approx 10 \text{ ft}$

Thus, some points on the graph are $(0, 3.5)$, $(5, 13.2)$, and $(15, 10)$. The complete graph is given below.



119. $h(x) = \frac{-32x^2}{130^2} + x$

a. $h(100) = \frac{-32(100)^2}{130^2} + 100$

$= \frac{-320,000}{16,900} + 100 \approx 81.07 \text{ feet}$

b. $h(300) = \frac{-32(300)^2}{130^2} + 300$

$= \frac{-2,880,000}{16,900} + 300 \approx 129.59 \text{ feet}$

c. $h(500) = \frac{-32(500)^2}{130^2} + 500$

$= \frac{-8,000,000}{16,900} + 500 \approx 26.63 \text{ feet}$

d. Solving $h(x) = \frac{-32x^2}{130^2} + x = 0$

$$\frac{-32x^2}{130^2} + x = 0$$

$$x\left(\frac{-32x}{130^2} + 1\right) = 0$$

$$x = 0 \quad \text{or} \quad \frac{-32x}{130^2} + 1 = 0$$

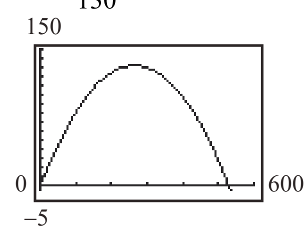
$$1 = \frac{32x}{130^2}$$

$$130^2 = 32x$$

$$x = \frac{130^2}{32} = 528.13 \text{ feet}$$

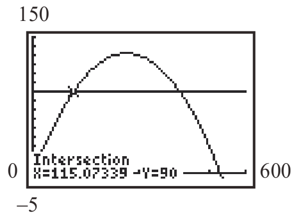
Therefore, the golf ball travels 528.13 feet.

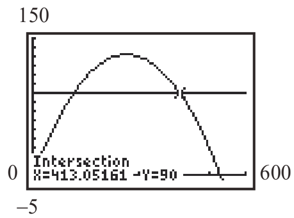
e. $y_1 = \frac{-32x^2}{130^2} + x$



f. Use INTERSECT on the graphs of

$$y_1 = \frac{-32x^2}{130^2} + x \quad \text{and} \quad y_2 = 90.$$





The ball reaches a height of 90 feet twice. The first time is when the ball has traveled approximately 115.07 feet, and the second time is when the ball has traveled about 413.05 feet.

g. The ball travels approximately 275 feet before it reaches its maximum height of approximately 131.8 feet.

X	Y_1
200	124.26
225	129.14
250	131.66
275	131.8
300	129.59
325	125
350	118.05

X=275

h. The ball travels approximately 264 feet before it reaches its maximum height of approximately 132.03 feet.

X	Y_1
260	132
261	132.01
262	132.02
263	132.03
264	132.03
265	132.03
266	132.02

Y_1=132.029112426

X	Y_1
260	132
261	132.01
262	132.02
263	132.03
264	132.03
265	132.03
266	132.02

Y_1=132.031242604

X	Y_1
260	132
261	132.01
262	132.02
263	132.03
264	132.03
265	132.03
266	132.02

Y_1=132.029585799

120. a. $C(0) = 5000$

This represents the fixed overhead costs. That is, the company will incur costs of \$5000 per day even if no computers are manufactured.

b. $C(10) = 19{,}000$

It costs the company \$19,000 to produce 10 computers in a day.

c. $C(50) = 51{,}000$

It costs the company \$51,000 to produce 50 computers in a day.

d. The domain is $\{q \mid 0 \le q \le 80\}$. This indicates that production capacity is limited to 80 computers in a day.

e. The graph is curved down and rises slowly at first. As production increases, the graph becomes rises more quickly and changes to being curved up.

f. The inflection point is where the graph changes from being curved down to being curved up.

121. a. $C(0) = \$50$

It costs \$50 if you use 0 gigabytes.

b. $C(5) = \$50$

It costs \$50 if you use 5 gigabytes.

c. $C(15) = \$150$

It costs \$150 if you use 15 gigabytes.

d. The domain is $\{g \mid 0 \le g \le 30\}$. This indicates that there are at most 30 gigabytes in a month.

e. The graph is flat at first and then rises in a straight line.

122. a. $h(x) = 2x$

$$h(a+b) = 2(a+b) = 2a + 2b$$
$$= h(a) + h(b)$$

$h(x) = 2x$ has the property.

b. $g(x) = x^2$

$$g(a+b) = (a+b)^2 = a^2 + 2ab + b^2$$

Since

$$a^2 + 2ab + b^2 \ne a^2 + b^2 = g(a) + g(b),$$

$g(x) = x^2$ does not have the property.

c. $F(x) = 5x - 2$

$F(a+b) = 5(a+b) - 2 = 5a + 5b - 2$

Since

$5a + 5b - 2 \neq 5a - 2 + 5b - 2 = F(a) + F(b)$,

$F(x) = 5x - 2$ does not have the property.

d. $G(x) = \dfrac{1}{x}$

$G(a+b) = \dfrac{1}{a+b} \neq \dfrac{1}{a} + \dfrac{1}{b} = G(a) + G(b)$

$G(x) = \dfrac{1}{x}$ does not have the property.

---

123. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} =$

$$= \frac{(x+h)^{1/3} - x^{1/3}}{h}$$

$$= \frac{(x+h)^{1/3} - x^{1/3}}{h} \cdot \frac{(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}}{(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}}$$

$$= \frac{h}{(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}}$$

$$= \frac{x+h-x}{h\left[(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}\right]} = \frac{h}{h\left[(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}\right]}$$

$$= \frac{1}{(x+h)^{2/3} + x^{1/3}(x+h)^{1/3} + x^{2/3}}$$

124. $f\left(\dfrac{x+4}{5x-4}\right) = 3x^2 - 2$

Solve $\dfrac{x+4}{5x-4} = 1$.

$\dfrac{x+4}{5x-4} = 1$

$x + 4 = 5x - 4$

$x = 2$

Therefore, $f(1) = 3(2)^2 - 2 = 10$

125. $g(-2) = 5 = f(-2) = 4$

Since $f(-2) = (-2)^2 - 4(-2) + c$

$= 12 + c$

we have

$\dfrac{12+c}{3} - 4 = 5$

$\dfrac{12+c}{3} = 9$

$12 + c = 27$

$c = 15$

$f(3) = 3^2 - 4 \cdot 3 + 15 = 12$

126. $g(5) = 5^2 + n = 25 + n$

$f(g(5)) = f(25+n) = \sqrt{25+n} + 2 = 4$

so, $\sqrt{25+n} = 2$

$25 + n = 4$

$n = -21$

$g(n) = n^2 + n = (-21)^2 + (-21) = 420$.

127. Answers will vary. From a graph, the domain can be found by visually locating the x-values for which the graph is defined. The range can be found in a similar fashion by visually locating the y-values for which the function is defined.

If an equation is given, the domain can be found by locating any restricted values and removing them from the set of real numbers. The range can be found by using known properties of the graph of the equation, or estimated by means of a table of values.

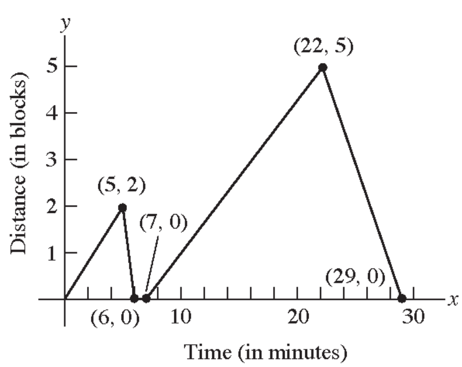
128. The graph of a function can have any number of x-intercepts. The graph of a function can have at most one y-intercept (otherwise the graph would fail the vertical line test).

129. Yes, the graph of a single point is the graph of a function since it would pass the vertical line test. The equation of such a function would be something like the following: $f(x)=2$, where $x=7$.

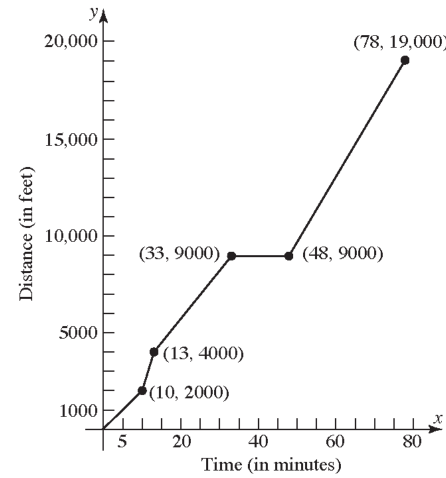
130. (a) III; (b) IV; (c) I; (d) V; (e) II

131. (a) II; (b) V; (c) IV; (d) III; I I

132.



133.



134. **a.** 2 hours elapsed; Kevin was between 0 and 3 miles from home.

b. 0.5 hours elapsed; Kevin was 3 miles from home.

c. 0.3 hours elapsed; Kevin was between 0 and 3 miles from home.

d. 0.2 hours elapsed; Kevin was at home.

e. 0.9 hours elapsed; Kevin was between 0 and 2.8 miles from home.

f. 0.3 hours elapsed; Kevin was 2.8 miles from home.

g. 1.1 hours elapsed; Kevin was between 0 and 2.8 miles from home.

h. The farthest distance Kevin is from home is 3 miles.

i. Kevin returned home 2 times.

135. **a.** Michael travels fastest between 7 and 7.4 minutes. That is, $(7,7.4)$.

b. Michael's speed is zero between 4.2 and 6 minutes. That is, $(4.2,6)$.

c. Between 0 and 2 minutes, Michael's speed increased from 0 to 30 miles/hour.

d. Between 4.2 and 6 minutes, Michael was stopped (i.e, his speed was 0 miles/hour).

e. Between 7 and 7.4 minutes, Michael was traveling at a steady rate of 50 miles/hour.

f. Michael's speed is constant between 2 and 4 minutes, between 4.2 and 6 minutes, between 7 and 7.4 minutes, and between 7.6 and 8 minutes. That is, on the intervals (2, 4), (4.2, 6), (7, 7.4), and (7.6, 8).

136. Answers (graphs) will vary. Points of the form $(5, y)$ and of the form $(x, 0)$ cannot be on the graph of the function.

137. The only such function is $f(x)=0$ because it is the only function for which $f(x)=-f(x)$. Any other such graph would fail the vertical line test.

138. Answers may vary.

Section 1.4

1. $2<x<5$

2. slope $=\dfrac{\Delta y}{\Delta x}=\dfrac{8-3}{3-(-2)}=\dfrac{5}{5}=1$

3. x-axis: $y\to -y$

$(-y)=5x^2-1$

$-y=5x^2-1$

$y=-5x^2+1$ different

y-axis: $x\to -x$

$y=5(-x)^2-1$

$y=5x^2-1$ same

origin: $x\to -x$ and $y\to -y$

$(-y)=5(-x)^2-1$

$-y=5x^2-1$

$y=-5x^2+1$ different

The equation has symmetry with respect to the y-axis only.

4. $y-y_1=m(x-x_1)$

$y-(-2)=5(x-3)$

$y+2=5(x-3)$

5. $y=x^2-9$

x-intercepts:

$0=x^2-9$

$x^2=9\to x=\pm 3$

y-intercept:

$y=(0)^2-9=-9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,-9)$.

6. increasing

7. even; odd

8. True

9. True

10. False; odd functions are symmetric with respect to the origin. Even functions are symmetric with respect to the y-axis.

11. c

12. d

13. Yes

14. No, it is increasing.

15. No

16. Yes

17. f is increasing on the intervals $[-8,-2],[0,2],[5,7]$.

18. f is decreasing on the intervals: $[-10,-8],[-2,0],[2,5]$.

19. Yes. The local maximum at $x=2$ is 10.

20. No. There is a local minimum at $x=5$; the local minimum is 0.

21. f has local maxima at $x=-2$ and $x=2$. The local maxima are 6 and 10, respectively.

22. f has local minima at $x=-8$, $x=0$ and $x=5$. The local minima are –4, 0, and 0, respectively.

23. f has absolute minimum of -4 at $x=-8$.

24. f has absolute maximum of 10 at $x=2$.

25. **a.** Intercepts: (–2, 0), (2, 0), and (0, 3).

b. Domain: $\{x\mid -4\le x\le 4\}$ or $[-4, 4]$;

Range: $\{y\mid 0\le y\le 3\}$ or $[0, 3]$.

c. Increasing: [–2, 0] and [2, 4];
Decreasing: [–4, –2] and [0, 2].

d. Since the graph is symmetric with respect to the y-axis, the function is <u>even</u>.

26. a. Intercepts: (–1, 0), (1, 0), and (0, 2).

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;

Range: $\{y \mid 0 \le y \le 3\}$ or $[0, 3]$.

c. Increasing: [–1, 0] and [1, 3];
Decreasing: [–3, –1] and [0, 1].

d. Since the graph is symmetric with respect to the y-axis, the function is <u>even</u>.

27. a. Intercepts: (0, 1).

b. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y > 0\}$ or $(0, \infty)$.

c. Increasing: $(-\infty, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is <u>neither</u> even nor odd.

28. a. Intercepts: (1, 0).

b. Domain: $\{x \mid x > 0\}$ or $(0, \infty)$;

Range: $\{y \mid y \text{ is any real number}\}$.

c. Increasing: $[0, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is <u>neither</u> even nor odd.

29. a. Intercepts: $(-\pi, 0)$, $(\pi, 0)$, and $(0, 0)$.

b. Domain: $\{x \mid -\pi \le x \le \pi\}$ or $[-\pi, \pi]$;

Range: $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

c. Increasing: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$;

Decreasing: $\left[-\pi, -\frac{\pi}{2}\right]$ and $\left[\frac{\pi}{2}, \pi\right]$.

d. Since the graph is symmetric with respect to the origin, the function is <u>odd</u>.

30. a. Intercepts: $\left(-\frac{\pi}{2}, 0\right)$, $\left(\frac{\pi}{2}, 0\right)$, and $(0, 1)$.

b. Domain: $\{x \mid -\pi \le x \le \pi\}$ or $[-\pi, \pi]$;

Range: $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

c. Increasing: $[-\pi, 0]$; Decreasing: $[0, \pi]$.

d. Since the graph is symmetric with respect to the y-axis, the function is <u>even</u>.

31. a. Intercepts: $\left(\frac{1}{3}, 0\right)$, $\left(\frac{5}{2}, 0\right)$, and $\left(0, \frac{1}{2}\right)$.

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;

Range: $\{y \mid -1 \le y \le 2\}$ or $[-1, 2]$.

c. Increasing: $[2, 3]$; Decreasing: $[-1, 1]$;

Constant: $[-3, -1]$ and $[1, 2]$

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is <u>neither</u> even nor odd.

32. a. Intercepts: $(-2.3, 0)$, $(3, 0)$, and $(0, 1)$.

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;

Range: $\{y \mid -2 \le y \le 2\}$ or $[-2, 2]$.

c. Increasing: $[-3, -2]$ and $[0, 2]$;

Decreasing: $[2, 3]$; Constant: $[-2, 0]$.

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is <u>neither</u> even nor odd.

33. a. f has a local maximum value of 3 at $x = 0$.

b. f has a local minimum value of 0 at both $x = -2$ and $x = 2$.

34. a. f has a local maximum value of 2 at $x = 0$.

b. f has a local minimum value of 0 at both $x = -1$ and $x = 1$.

35. a. f has a local maximum value of 1 at $x = \frac{\pi}{2}$.

b. f has a local minimum value of –1 at $x = -\frac{\pi}{2}$.

36. a. f has a local maximum value of 1 at $x = 0$.

b. f has a local minimum value of –1 both at $x = -\pi$ and $x = \pi$.

37. $f(x)=4x^3$

$f(-x)=4(-x)^3=-4x^3=-f(x)$

Therefore, f is odd.

38. $f(x)=2x^4-x^2$

$f(-x)=2(-x)^4-(-x)^2=2x^4-x^2=f(x)$

Therefore, f is even.

39. $g(x)=10-x^2$

$g(-x)=10-(-x)^2=10-x^2=g(x)$

Therefore, g is even.

40. $h(x)=3x^3+5$

$h(-x)=3(-x)^3+5=-3x^3+5$

h is neither even nor odd.

41. $F(x)=\sqrt[3]{4x}$

$F(-x)=\sqrt[3]{-4x}=-\sqrt[3]{4x}=-F(x)$

Therefore, F is odd.

42. $G(x)=\sqrt{x}$

$G(-x)=\sqrt{-x}$

G is neither even nor odd.

43. $f(x)=x+|x|$

$f(-x)=-x+|-x|=-x+|x|$

f is neither even nor odd.

44. $f(x)=\sqrt[3]{2x^2+1}$

$f(-x)=\sqrt[3]{2(-x)^2+1}=\sqrt[3]{2x^2+1}=f(x)$

Therefore, f is even.

45. $g(x)=\dfrac{1}{x^2+8}$

$g(-x)=\dfrac{1}{(-x)^2+8}=\dfrac{1}{x^2+8}=g(x)$

Therefore, g is even.

46. $h(x)=\dfrac{x}{x^2-1}$

$h(-x)=\dfrac{-x}{(-x)^2-1}=\dfrac{-x}{x^2-1}=-h(x)$

Therefore, h is odd.

47. $h(x)=\dfrac{-x^3}{3x^2-9}$

$h(-x)=\dfrac{-(-x)^3}{3(-x)^2-9}=\dfrac{x^3}{3x^2-9}=-h(x)$

Therefore, h is odd.

48. $F(x)=\dfrac{2x}{|x|}$

$F(-x)=\dfrac{2(-x)}{|-x|}=\dfrac{-2x}{|x|}=-F(x)$

Therefore, F is odd.

49. f has an absolute maximum of 4 at $x=1$.
f has an absolute minimum of 1 at $x=5$.
f has an local maximum value of 3 at $x=3$.
f has an local minimum value of 2 at $x=2$.

50. f has an absolute maximum of 4 at $x=4$.
f has an absolute minimum of 0 at $x=5$.
f has an local maximum value of 4 at $x=4$.
f has an local minimum value of 1 at $x=1$.

51. f has an absolute minimum of 1 at $x=1$.
f has an absolute maximum of 4 at $x=3$.
f has an local minimum value of 1 at $x=1$.
f has an local maximum value of 4 at $x=3$.

52. f has an absolute minimum of 1 at $x=0$.
f has no absolute maximum.
f has no local minimum.
f has no local maximum.

53. f has an absolute minimum of 0 at $x=0$.
f has no absolute maximum.
f has an local minimum value of 0 at $x=0$.
f has an local minimum value of 2 at $x=3$.
f has an local maximum value of 3 at $x=2$.

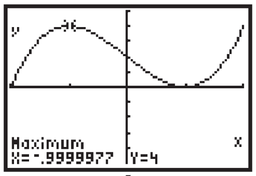
54. f has an absolute maximum of 4 at $x=2$.
f has no absolute minimum.
f has an local maximum value of 4 at $x=2$.
f has an local minimum value of 2 at $x=0$.

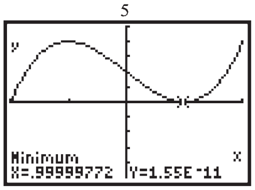
55. f has no absolute maximum or minimum.
f has no local maximum or minimum.

56. f has no absolute maximum or minimum.
f has no local maximum or minimum.

57. $f(x) = x^3 - 3x + 2$ on the interval $(-2, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = x^3 - 3x + 2$.





local maximum: $f(-1) = 4$

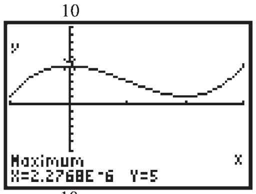
local minimum: $f(1) = 0$

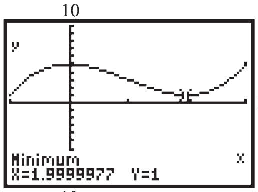
f is increasing on: $[-2, -1]$ and $[1, 2]$;

f is decreasing on: $[-1, 1]$

58. $f(x) = x^3 - 3x^2 + 5$ on the interval $(-1, 3)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = x^3 - 3x^2 + 5$.





local maximum: $f(0) = 5$

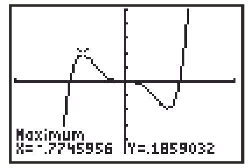
local minimum: $f(2) = 1$

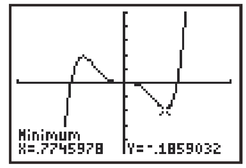
f is increasing on: $[-1, 0]$ and $[2, 3]$;

f is decreasing on: $[0, 2]$

59. $f(x) = x^5 - x^3$ on the interval $(-2, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = x^5 - x^3$.





local maximum: $f(-0.77) = 0.19$

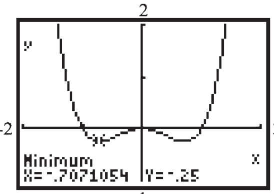
local minimum: $f(0.77) = -0.19$

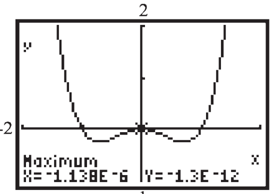
f is increasing on: $[-2, -0.77]$ and $[0.77, 2]$;

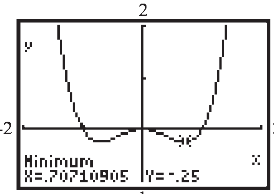
f is decreasing on: $[-0.77, 0.77]$

60. $f(x) = x^4 - x^2$ on the interval $(-2, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = x^4 - x^2$.







local maximum: $f(0) = 0$

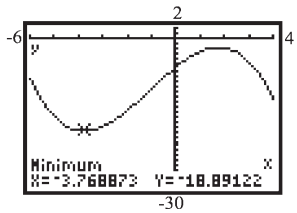
local minimum: $f(-0.71) = -0.25$; $f(0.71) = -0.25$

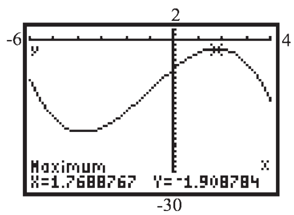
f is increasing on: $[-0.71, 0]$ and $[0.71, 2]$;

f is decreasing on: $[-2, -0.71]$ and $[0, 0.71]$

61. $f(x) = -0.2x^3 - 0.6x^2 + 4x - 6$ on the interval $(-6, 4)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.2x^3 - 0.6x^2 + 4x - 6$.





local maximum: $f(1.77) = -1.91$

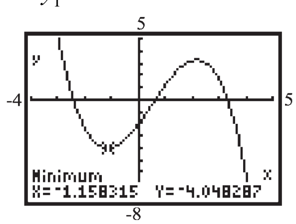
local minimum: $f(-3.77) = -18.89$

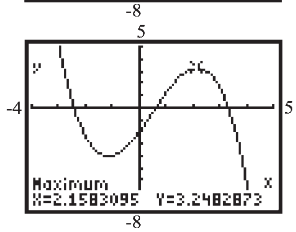
f is increasing on: $[-3.77, 1.77]$;

f is decreasing on: $[-6, -3.77]$ and $[1.77, 4]$

62. $f(x) = -0.4x^3 + 0.6x^2 + 3x - 2$ on the interval $(-4, 5)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^3 + 0.6x^2 + 3x - 2$.





local maximum: $f(2.16) = 3.25$

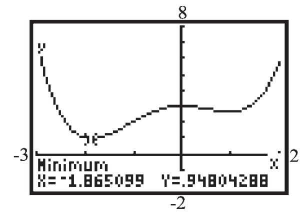
local minimum: $f(-1.16) = -4.05$

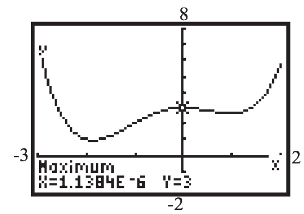
f is increasing on: $[-1.16, 2.16]$;

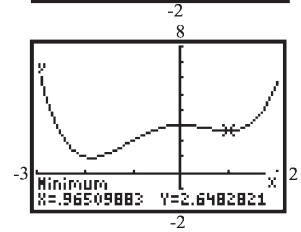
f is decreasing on: $[-4, -1.16]$ and $[2.16, 5]$

63. $f(x) = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$ on the interval $(-3, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$.







local maximum: $f(0) = 3$

local minimum:

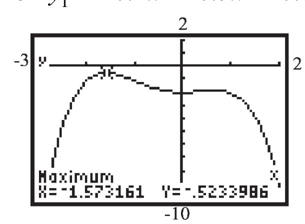
$f(-1.87) = 0.95$, $f(0.97) = 2.65$

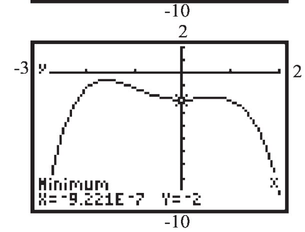
f is increasing on: $[-1.87, 0]$ and $[0.97, 2]$;

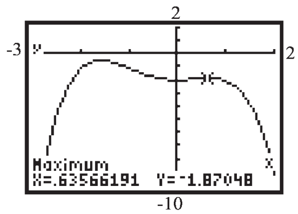
f is decreasing on: $[-3, -1.87]$ and $[0, 0.97]$

64. $f(x) = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2$ on the interval $(-3, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2$.







local maxima: $f(-1.57) = -0.52$,
$f(0.64) = -1.87$

local minimum: $(0,-2)$ $f(0) = -2$

f is increasing on: $[-3,-1.57]$ and $[0,0.64]$;

f is decreasing on: $[-1.57,0]$ and $[0.64,2]$

65. $f(x) = -2x^2 + 4$

a. Average rate of change of *f* from $x = 0$ to $x = 2$

$$\frac{f(2)-f(0)}{2-0} = \frac{\left(-2(2)^2+4\right)-\left(-2(0)^2+4\right)}{2} = \frac{(-4)-(4)}{2} = \frac{-8}{2} = -4$$

b. Average rate of change of *f* from $x = 1$ to $x = 3$:

$$\frac{f(3)-f(1)}{3-1} = \frac{\left(-2(3)^2+4\right)-\left(-2(1)^2+4\right)}{2} = \frac{(-14)-(2)}{2} = \frac{-16}{2} = -8$$

c. Average rate of change of *f* from $x = 1$ to $x = 4$:

$$\frac{f(4)-f(1)}{4-1} = \frac{\left(-2(4)^2+4\right)-\left(-2(1)^2+4\right)}{3} = \frac{(-28)-(2)}{3} = \frac{-30}{3} = -10$$

66. $f(x) = -x^3 + 1$

a. Average rate of change of *f* from $x = 0$ to $x = 2$:

$$\frac{f(2)-f(0)}{2-0} = \frac{\left(-(2)^3+1\right)-\left(-(0)^3+1\right)}{2} = \frac{-7-1}{2} = \frac{-8}{2} = -4$$

b. Average rate of change of *f* from $x = 1$ to $x = 3$:

$$\frac{f(3)-f(1)}{3-1} = \frac{\left(-(3)^3+1\right)-\left(-(1)^3+1\right)}{2} = \frac{-26-(0)}{2} = \frac{-26}{2} = -13$$

c. Average rate of change of *f* from $x = -1$ to $x = 1$:

$$\frac{f(1)-f(-1)}{1-(-1)} = \frac{\left(-(1)^3+1\right)-\left(-(-1)^3+1\right)}{2} = \frac{0-2}{2} = \frac{-2}{2} = -1$$

67. $g(x) = x^3 - 4x + 7$

a. Average rate of change of *g* from $x = -3$ to $x = -2$:

$$\frac{g(-2)-g(-3)}{-2-(-3)} = \frac{\left[(-2)^3-4(-2)+7\right]-\left[(-3)^3-4(-3)+7\right]}{1} = \frac{(7)-(-8)}{1} = \frac{15}{1} = 15$$

b. Average rate of change of *g* from $x = -1$ to $x = 1$:

$$\frac{g(1)-g(-1)}{1-(-1)} = \frac{\left[(1)^3-4(1)+7\right]-\left[(-1)^3-4(-1)+7\right]}{2} = \frac{(4)-(10)}{2} = \frac{-6}{2} = -3$$

c. Average rate of change of *g* from $x = 1$ to $x = 3$:

$$\frac{g(3)-g(1)}{3-1} = \frac{\left[(3)^3-4(3)+7\right]-\left[(1)^3-4(1)+7\right]}{2} = \frac{(22)-(4)}{2} = \frac{18}{2} = 9$$

68. $h(x) = x^2 - 2x + 3$

a. Average rate of change of h from $x = -1$ to $x = 1$:

$$\frac{h(1)-h(-1)}{1-(-1)}$$
$$=\frac{\left[(1)^2-2(1)+3\right]-\left[(-1)^2-2(-1)+3\right]}{2}$$
$$=\frac{(2)-(6)}{2}=\frac{-4}{2}=-2$$

b. Average rate of change of h from $x = 0$ to $x = 2$:

$$\frac{h(2)-h(0)}{2-0}$$
$$=\frac{\left[(2)^2-2(2)+3\right]-\left[(0)^2-2(0)+3\right]}{2}$$
$$=\frac{(3)-(3)}{2}=\frac{0}{2}=0$$

c. Average rate of change of h from $x = 2$ to $x = 5$:

$$\frac{h(5)-h(2)}{5-2}$$
$$=\frac{\left[(5)^2-2(5)+3\right]-\left[(2)^2-2(2)+3\right]}{3}$$
$$=\frac{(18)-(3)}{3}=\frac{15}{3}=5$$

69. $f(x) = 5x - 2$

a. Average rate of change of f from 1 to 3:

$$\frac{\Delta y}{\Delta x}=\frac{f(3)-f(1)}{3-1}=\frac{13-3}{3-1}=\frac{10}{2}=5$$

Thus, the average rate of change of f from 1 to 3 is 5.

b. From (a), the slope of the secant line joining $(1, f(1))$ and $(3, f(3))$ is 5. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 3 = 5(x-1)$$
$$y - 3 = 5x - 5$$
$$y = 5x - 2$$

70. $f(x) = -4x + 1$

a. Average rate of change of f from 2 to 5:

$$\frac{\Delta y}{\Delta x}=\frac{f(5)-f(2)}{5-2}=\frac{-19-(-7)}{5-2}$$
$$=\frac{-12}{3}=-4$$

Therefore, the average rate of change of f from 2 to 5 is -4.

b. From (a), the slope of the secant line joining $(2, f(2))$ and $(5, f(5))$ is -4. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y-(-7) = -4(x-2)$$
$$y + 7 = -4x + 8$$
$$y = -4x + 1$$

71. $g(x) = x^2 - 2$

a. Average rate of change of g from -2 to 1:

$$\frac{\Delta y}{\Delta x}=\frac{g(1)-g(-2)}{1-(-2)}=\frac{-1-2}{1-(-2)}=\frac{-3}{3}=-1$$

Therefore, the average rate of change of g from -2 to 1 is -1.

b. From (a), the slope of the secant line joining $(-2, g(-2))$ and $(1, g(1))$ is -1. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 2 = -1(x-(-2))$$
$$y - 2 = -x - 2$$
$$y = -x$$

72. $g(x) = x^2 + 1$

a. Average rate of change of g from -1 to 2:

$$\frac{\Delta y}{\Delta x}=\frac{g(2)-g(-1)}{2-(-1)}=\frac{5-2}{2-(-1)}=\frac{3}{3}=1$$

Therefore, the average rate of change of g from -1 to 2 is 1.

b. From (a), the slope of the secant line joining $(-1, g(-1))$ and $(2, g(2))$ is 1. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 2 = 1(x - (-1))$$
$$y - 2 = x + 1$$
$$y = x + 3$$

73. $h(x) = x^2 - 2x$

a. Average rate of change of h from 2 to 4:

$$\frac{\Delta y}{\Delta x} = \frac{h(4) - h(2)}{4 - 2} = \frac{8 - 0}{4 - 2} = \frac{8}{2} = 4$$

Therefore, the average rate of change of h from 2 to 4 is 4.

b. From (a), the slope of the secant line joining $(2, h(2))$ and $(4, h(4))$ is 4. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 0 = 4(x - 2)$$
$$y = 4x - 8$$

74. $h(x) = -2x^2 + x$

a. Average rate of change from 0 to 3:

$$\frac{\Delta y}{\Delta x} = \frac{h(3) - h(0)}{3 - 0} = \frac{-15 - 0}{3 - 0}$$
$$= \frac{-15}{3} = -5$$

Therefore, the average rate of change of h from 0 to 3 is -5.

b. From (a), the slope of the secant line joining $(0, h(0))$ and $(3, h(3))$ is -5. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 0 = -5(x - 0)$$
$$y = -5x$$

75. a. $g(x) = x^3 - 27x$

$$g(-x) = (-x)^3 - 27(-x)$$
$$= -x^3 + 27x$$
$$= -(x^3 - 27x)$$
$$= -g(x)$$

Since $g(-x) = -g(x)$, the function is odd.

b. Since $g(x)$ is odd then it is symmetric about the origin so there exist a local maximum at $x = -3$.

$$g(-3) = (-3)^3 - 27(-3) = -27 + 81 = 54$$

So there is a local maximum of 54 at $x = -3$.

76. $f(x) = -x^3 + 12x$

a.
$$f(-x) = -(-x)^3 + 12(-x)$$
$$= x^3 - 12x$$
$$= -(-x^3 + 12x)$$
$$= -f(x)$$

Since $f(-x) = -f(x)$, the function is odd.

b. Since $f(x)$ is odd then it is symmetric about the origin so there exist a local maximum at $x = -3$.

$$f(-2) = -(-2)^3 + 12(-2) = 8 - 24 = -16$$

So there is a local minimum value of -16 at $x = -2$.

77. $F(x) = -x^4 + 8x^2 + 9$

a.
$$F(-x) = -(-x)^4 + 8(-x)^2 + 9$$
$$= -x^4 + 8x + 9$$
$$= F(x)$$

Since $F(-x) = F(x)$, the function is even.

b. Since the function is even, its graph has y-axis symmetry. The second local maximum value is 25 and occurs at $x = -2$.

c. Because the graph has y-axis symmetry, the area under the graph between $x = 0$ and $x = 3$ bounded below by the x-axis is the same as the area under the graph between $x = -3$ and $x = 0$ bounded below the x-axis. Thus, the area is 50.4 square units.

78. $G(x) = -x^4 + 32x^2 + 144$

a. $G(-x) = -(-x)^4 + 32(-x)^2 + 144$
$= -x^4 + 32x^2 + 144$
$= G(x)$

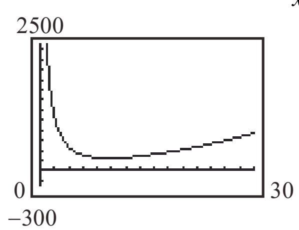
Since $G(-x) = G(x)$, the function is even.

b. Since the function is even, its graph has y-axis symmetry. The second local maximum is in quadrant II and is 400 and occurs at $x = -4$.

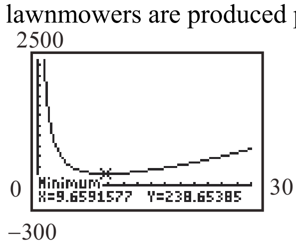
c. Because the graph has y-axis symmetry, the area under the graph between $x = 0$ and $x = 6$ bounded below by the x-axis is the same as the area under the graph between $x = -6$ and $x = 0$ bounded below the x-axis. Thus, the area is 1612.8 square units.

79. $\overline{C}(x) = 0.3x^2 + 21x - 251 + \dfrac{2500}{x}$

a. $y_1 = 0.3x^2 + 21x - 251 + \dfrac{2500}{x}$



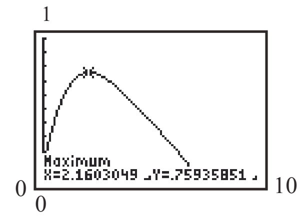
b. Use MINIMUM. Rounding to the nearest whole number, the average cost is minimized when approximately 10 lawnmowers are produced per hour.



c. The minimum average cost is approximately \$239 per mower.

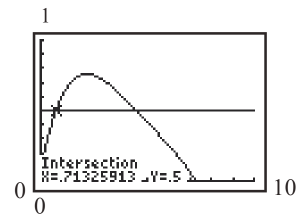
80. a. $C(t) = -.002t^4 + .039t^3 - .285t^2 + .766t + .085$

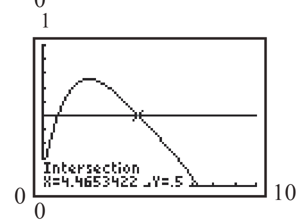
Graph the function on a graphing utility and use the Maximum option from the CALC menu.



The concentration will be highest after about 2.16 hours.

b. Enter the function in Y1 and 0.5 in Y2. Graph the two equations in the same window and use the Intersect option from the CALC menu.





After taking the medication, the woman can feed her child within the first 0.71 hours (about 42 minutes) or after 4.47 hours (about 4hours 28 minutes) have elapsed.

81. a. avg. rate of change $= \dfrac{P(2.5) - P(0)}{2.5 - 0}$
$= \dfrac{0.18 - 0.09}{2.5 - 0}$
$= \dfrac{0.09}{2.5}$
$= 0.036$ gram per hour

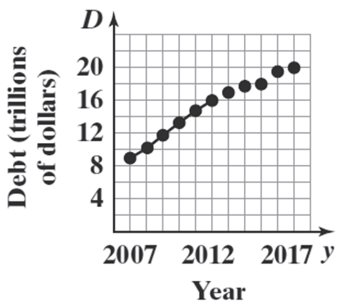
On average, the population is increasing at a rate of 0.036 gram per hour from 0 to 2.5 hours.

b. avg. rate of change $= \dfrac{P(6) - P(4.5)}{6 - 4.5}$
$= \dfrac{0.50 - 0.35}{6 - 4.5}$
$= \dfrac{0.15}{1.5}$
$= 0.1$ gram per hour

On average, the population is increasing at a rate of 0.1 gram per hour from 4.5 to 6 hours.

c. The average rate of change is increasing as time passes. This indicates that the population is increasing at an increasing rate.

82. a.



b. The slope represents the average rate of change of the debt from 2007 to 2012.

c. $$\text{avg. rate of change} = \frac{P(2010)-P(2008)}{2010-2008} = \frac{13562-10025}{2} = \frac{3537}{2} = \$\ 1{,}768.5 \text{ billion/yr}$$

d. $$\text{avg. rate of change} = \frac{P(2013)-P(2011)}{2013-2011} = \frac{16738-14790}{2} = \frac{1948}{2} = \$\ 974 \text{ billion/yr}$$

e. $$\text{avg. rate of change} = \frac{P(2016)-P(2014)}{2016-2014} = \frac{19573-17824}{2} = \frac{1749}{2} = \$\ 874.5 \text{ billion}$$

f. The average rate of change is decreasing as time passes.

83. $f(x)=x^2$

a. Average rate of change of *f* from $x=0$ to $x=1$:

$$\frac{f(1)-f(0)}{1-0}=\frac{1^2-0^2}{1}=\frac{1}{1}=1$$

b. Average rate of change of *f* from $x=0$ to $x=0.5$:

$$\frac{f(0.5)-f(0)}{0.5-0}=\frac{(0.5)^2-0^2}{0.5}=\frac{0.25}{0.5}=0.5$$

c. Average rate of change of *f* from $x=0$ to $x=0.1$:

$$\frac{f(0.1)-f(0)}{0.1-0}=\frac{(0.1)^2-0^2}{0.1}=\frac{0.01}{0.1}=0.1$$

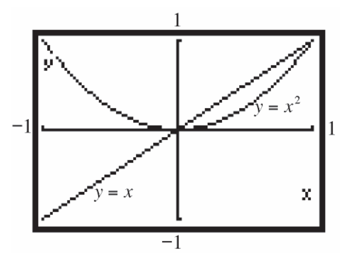
d. Average rate of change of *f* from $x=0$ to $x=0.01$:

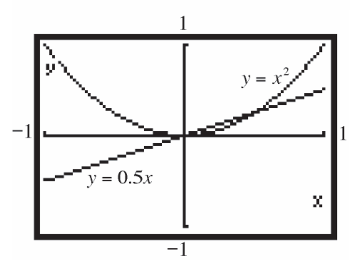
$$\frac{f(0.01)-f(0)}{0.01-0}=\frac{(0.01)^2-0^2}{0.01}=\frac{0.0001}{0.01}=0.01$$

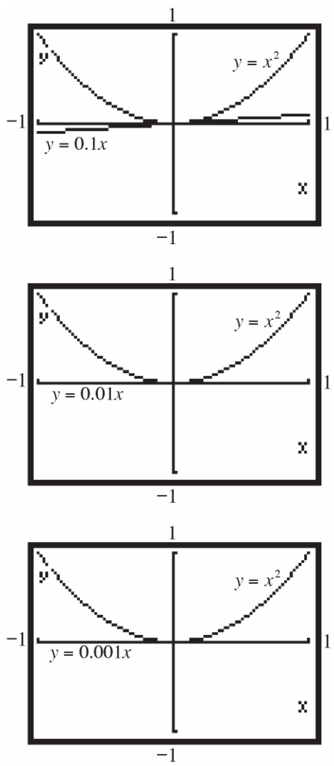
e. Average rate of change of *f* from $x=0$ to $x=0.001$:

$$\frac{f(0.001)-f(0)}{0.001-0}=\frac{(0.001)^2-0^2}{0.001}=\frac{0.000001}{0.001}=0.001$$

f. Graphing the secant lines:







g. The secant lines are beginning to look more and more like the tangent line to the graph of *f* at the point where $x = 0$.

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number zero.

84. $f(x) = x^2$

a. Average rate of change of *f* from $x = 1$ to $x = 2$:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{2^2 - 1^2}{1} = \frac{3}{1} = 3$$

b. Average rate of change of *f* from $x = 1$ to $x = 1.5$:

$$\frac{f(1.5) - f(1)}{1.5 - 1} = \frac{(1.5)^2 - 1^2}{0.5} = \frac{1.25}{0.5} = 2.5$$

c. Average rate of change of *f* from $x = 1$ to $x = 1.1$:

$$\frac{f(1.1) - f(1)}{1.1 - 1} = \frac{(1.1)^2 - 1^2}{0.1} = \frac{0.21}{0.1} = 2.1$$

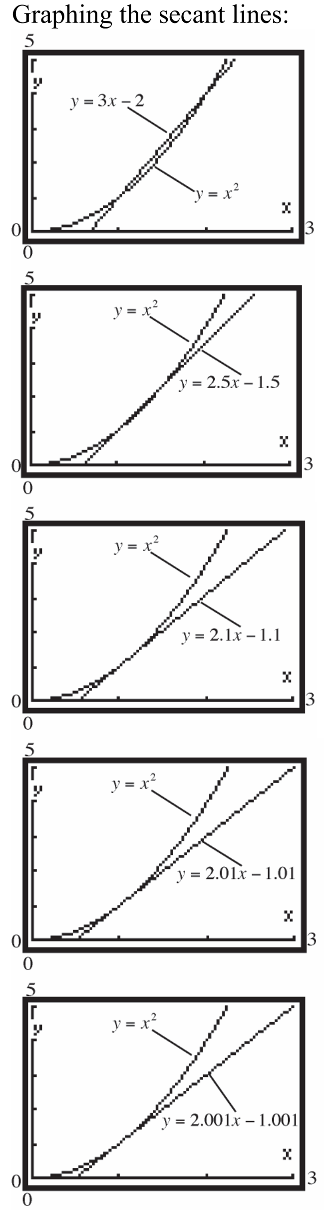
d. Average rate of change of *f* from $x = 1$ to $x = 1.01$:

$$\frac{f(1.01) - f(1)}{1.01 - 1} = \frac{(1.01)^2 - 1^2}{0.01} = \frac{0.0201}{0.01} = 2.01$$

e. Average rate of change of *f* from $x = 1$ to $x = 1.001$:

$$\frac{f(1.001) - f(1)}{1.001 - 1} = \frac{(1.001)^2 - 1^2}{0.001}$$
$$= \frac{0.002001}{0.001} = 2.001$$

f. Graphing the secant lines:



g. The secant lines are beginning to look more and more like the tangent line to the graph of *f* at the point where $x = 1$.

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number 2.

85. $f(x) = 2x + 5$

a. $$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{2(x+h)+5-2x-5}{h} = \frac{2h}{h} = 2$$

b. When $x = 1$:

$h = 0.5 \Rightarrow m_{\text{sec}} = 2$

$h = 0.1 \Rightarrow m_{\text{sec}} = 2$

$h = 0.01 \Rightarrow m_{\text{sec}} = 2$

as $h \to 0,\ m_{\text{sec}} \to 2$

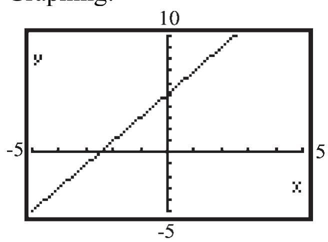
c. Using the point $(1, f(1)) = (1, 7)$ and slope, $m = 2$, we get the secant line:

$y - 7 = 2(x-1)$

$y - 7 = 2x - 2$

$y = 2x + 5$

d. Graphing:



The graph and the secant line coincide.

86. $f(x) = -3x + 2$

a. $$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{-3(x+h)+2-(-3x+2)}{h} = \frac{-3h}{h} = -3$$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = -3$

$h = 0.1 \Rightarrow m_{\text{sec}} = -3$

$h = 0.01 \Rightarrow m_{\text{sec}} = -3$

as $h \to 0,\ m_{\text{sec}} \to -3$

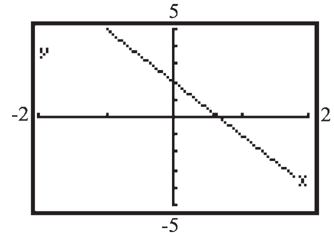
c. Using point $(1, f(1)) = (1, -1)$ and slope $= -3$, we get the secant line:

$y - (-1) = -3(x-1)$

$y + 1 = -3x + 3$

$y = -3x + 2$

d. Graphing:



The graph and the secant line coincide.

87. $f(x) = x^2 + 2x$

a. $$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h}$$
$$= \frac{(x+h)^2 + 2(x+h) - (x^2+2x)}{h}$$
$$= \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$$
$$= \frac{2xh + h^2 + 2h}{h}$$
$$= 2x + h + 2$$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = 2\cdot 1 + 0.5 + 2 = 4.5$

$h = 0.1 \Rightarrow m_{\text{sec}} = 2\cdot 1 + 0.1 + 2 = 4.1$

$h = 0.01 \Rightarrow m_{\text{sec}} = 2\cdot 1 + 0.01 + 2 = 4.01$

as $h \to 0,\ m_{\text{sec}} \to 2\cdot 1 + 0 + 2 = 4$

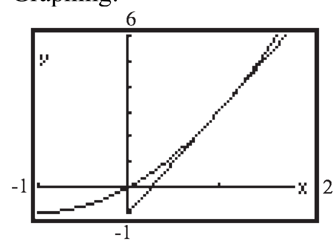
c. Using point $(1, f(1)) = (1, 3)$ and slope $= 4.01$, we get the secant line:

$y - 3 = 4.01(x-1)$

$y - 3 = 4.01x - 4.01$

$y = 4.01x - 1.01$

d. Graphing:



88. $f(x)=2x^2+x$

a. $m_{\sec}=\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{2(x+h)^2+(x+h)-(2x^2+x)}{h}$

$=\dfrac{2(x^2+2xh+h^2)+x+h-2x^2-x}{h}$

$=\dfrac{2x^2+4xh+2h^2+x+h-2x^2-x}{h}$

$=\dfrac{4xh+2h^2+h}{h}$

$=4x+2h+1$

b. When $x=1$,

$h=0.5 \Rightarrow m_{\sec}=4\cdot1+2(0.5)+1=6$

$h=0.1 \Rightarrow m_{\sec}=4\cdot1+2(0.1)+1=5.2$

$h=0.01 \Rightarrow m_{\sec}=4\cdot1+2(0.01)+1=5.02$

as $h\to0$, $m_{\sec}\to4\cdot1+2(0)+1=5$

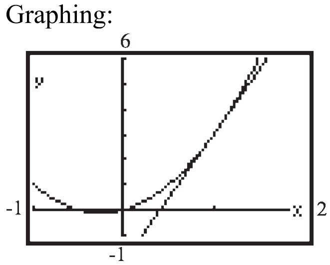
c. Using point $(1,f(1))=(1,3)$ and slope = 5.02, we get the secant line:

$y-3=5.02(x-1)$

$y-3=5.02x-5.02$

$y=5.02x-2.02$

d. Graphing:



89. $f(x)=2x^2-3x+1$

a. $m_{\sec}=\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{2(x+h)^2-3(x+h)+1-\left(2x^2-3x+1\right)}{h}$

$=\dfrac{2(x^2+2xh+h^2)-3x-3h+1-2x^2+3x-1}{h}$

$=\dfrac{2x^2+4xh+2h^2-3x-3h+1-2x^2+3x-1}{h}$

$=\dfrac{4xh+2h^2-3h}{h}$

$=4x+2h-3$

b. When $x=1$,

$h=0.5 \Rightarrow m_{\sec}=4\cdot1+2(0.5)-3=2$

$h=0.1 \Rightarrow m_{\sec}=4\cdot1+2(0.1)-3=1.2$

$h=0.01 \Rightarrow m_{\sec}=4\cdot1+2(0.01)-3=1.02$

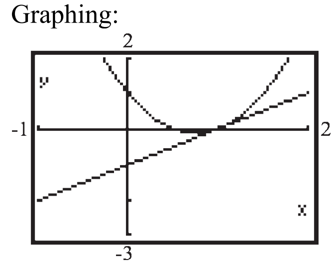
as $h\to0$, $m_{\sec}\to4\cdot1+2(0)-3=1$

c. Using point $(1,f(1))=(1,0)$ and slope = 1.02, we get the secant line:

$y-0=1.02(x-1)$

$y=1.02x-1.02$

d. Graphing:



90. $f(x)=-x^2+3x-2$

a. $m_{\sec}=\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{-(x+h)^2+3(x+h)-2-\left(-x^2+3x-2\right)}{h}$

$=\dfrac{-(x^2+2xh+h^2)+3x+3h-2+x^2-3x+2}{h}$

$=\dfrac{-x^2-2xh-h^2+3x+3h-2+x^2-3x+2}{h}$

$=\dfrac{-2xh-h^2+3h}{h}$

$=-2x-h+3$

b. When $x=1$,

$h=0.5 \Rightarrow m_{\sec}=-2\cdot1-0.5+3=0.5$

$h=0.1 \Rightarrow m_{\sec}=-2\cdot1-0.1+3=0.9$

$h=0.01 \Rightarrow m_{\sec}=-2\cdot1-0.01+3=0.99$

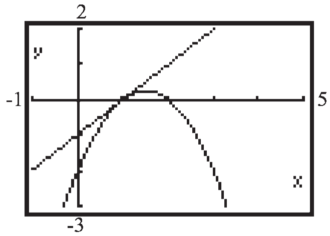
as $h\to0$, $m_{\sec}\to-2\cdot1-0+3=1$

c. Using point $(1,f(1))=(1,0)$ and slope = 0.99, we get the secant line:

$y-0=0.99(x-1)$

$y=0.99x-0.99$

d. Graphing:



91. $f(x)=\dfrac{1}{x}$

a. $$m_{\text{sec}}=\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(\frac{1}{x+h}-\frac{1}{x}\right)}{h}=\frac{\left(\frac{x-(x+h)}{(x+h)x}\right)}{h}$$
$$=\left(\frac{x-x-h}{(x+h)x}\right)\left(\frac{1}{h}\right)=\left(\frac{-h}{(x+h)x}\right)\left(\frac{1}{h}\right)$$
$$=-\frac{1}{(x+h)x}$$

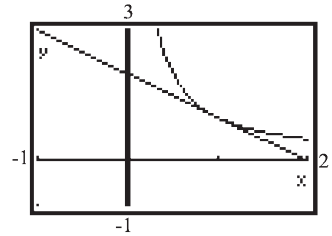
b. When $x=1$,

$$h=0.5\Rightarrow\ m_{\text{sec}}=-\frac{1}{(1+0.5)(1)}$$
$$=-\frac{1}{1.5}=-\frac{2}{3}\approx-0.667$$
$$h=0.1\Rightarrow\ m_{\text{sec}}=-\frac{1}{(1+0.1)(1)}$$
$$=-\frac{1}{1.1}=-\frac{10}{11}\approx-0.909$$
$$h=0.01\Rightarrow\ m_{\text{sec}}=-\frac{1}{(1+0.01)(1)}$$
$$=-\frac{1}{1.01}=-\frac{100}{101}\approx-0.990$$
$$\text{as } h\to0,\quad m_{\text{sec}}\to-\frac{1}{(1+0)(1)}=-\frac{1}{1}=-1$$

c. Using point $\left(1,f(1)\right)=(1,1)$ and slope $=-\dfrac{100}{101}$, we get the secant line:

$$y-1=-\frac{100}{101}(x-1)$$
$$y-1=-\frac{100}{101}x+\frac{100}{101}$$
$$y=-\frac{100}{101}x+\frac{201}{101}$$

d. Graphing:



92. $f(x)=\dfrac{1}{x^2}$

a. $$m_{\text{sec}}=\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(\frac{1}{(x+h)^2}-\frac{1}{x^2}\right)}{h}$$
$$=\frac{\left(\frac{x^2-(x+h)^2}{(x+h)^2x^2}\right)}{h}$$
$$=\left(\frac{x^2-\left(x^2+2xh+h^2\right)}{(x+h)^2x^2}\right)\left(\frac{1}{h}\right)$$
$$=\left(\frac{-2xh-h^2}{(x+h)^2x^2}\right)\left(\frac{1}{h}\right)$$
$$=\frac{-2x-h}{(x+h)^2x^2}=\frac{-2x-h}{\left(x^2+2xh+h^2\right)x^2}$$

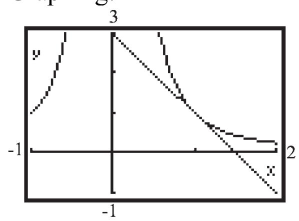
b. When $x=1$,

$$h=0.5\Rightarrow m_{\text{sec}}=\frac{-2\cdot1-0.5}{(1+0.5)^2\,1^2}=-\frac{10}{9}\approx-1.1111$$
$$h=0.1\Rightarrow m_{\text{sec}}=\frac{-2\cdot1-0.1}{(1+0.1)^2\,1^2}=-\frac{210}{121}\approx-1.7355$$
$$h=0.01\Rightarrow m_{\text{sec}}=\frac{-2\cdot1-0.01}{(1+0.01)^2\,1^2}$$
$$=-\frac{20{,}100}{10{,}201}\approx-1.9704$$
$$\text{as } h\to0,\quad m_{\text{sec}}\to\frac{-2\cdot1-0}{(1+0)^2\,1^2}=-2$$

c. Using point $\left(1, f\left(1\right)\right) = \left(1,1\right)$ and slope $= -1.9704$, we get the secant line:

$$y - 1 = -1.9704\left(x - 1\right)$$
$$y - 1 = -1.9704x + 1.9704$$
$$y = -1.9704x + 2.9704$$

d. Graphing:



93. $f(2) = 12$ and $f(-1) = 8$, so

$$\frac{f(2) - f(-1)}{2 - (-1)} = \frac{12}{3} = 4$$
$$f'(x) = 4 \rightarrow$$
$$3x^2 + 4x - 1 = 4$$
$$3x^2 + 4x - 5 = 0$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-5)}}{2(3)} = \frac{-4 \pm \sqrt{76}}{6}$$
$$= \frac{-4 \pm 2\sqrt{19}}{6} = \frac{-2 \pm \sqrt{19}}{3}$$
$$x = \frac{-2 - \sqrt{19}}{3} \approx -2.1 \text{ is outside the interval}$$
$$x = \frac{-2 + \sqrt{19}}{3} \approx .079 \text{ is in the interval.}$$

The only such number is $\dfrac{-2 + \sqrt{19}}{3}$.

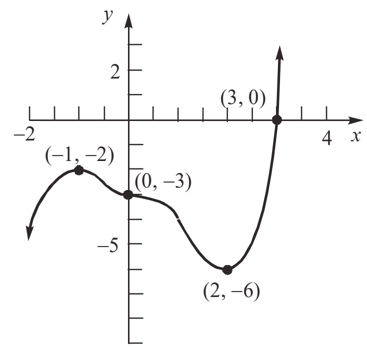
94. $g(-x) = -2f\left(-\dfrac{-x}{3}\right) = -2f\left(\dfrac{x}{3}\right)$. Since f is odd,

$$f\left(-\frac{x}{3}\right) = -f\left(\frac{x}{3}\right)$$
$$g(-x) = -2f\left(\frac{x}{3}\right) = 2f\left(-\frac{x}{3}\right) = -g(x)$$

so g is odd.

So g is odd.

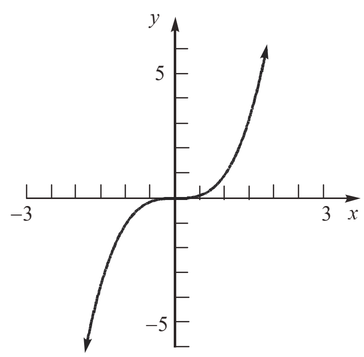
95. Answers will vary. One possibility follows:



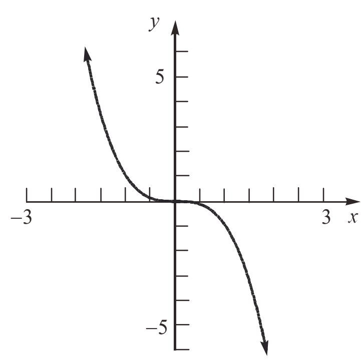
96. Answers will vary. See solution to Problem 89 for one possibility.

97. A function that is increasing on an interval can have at most one x-intercept on the interval. The graph of f could not "turn" and cross it again or it would start to decrease.

98. An increasing function is a function whose graph goes up as you read from left to right.

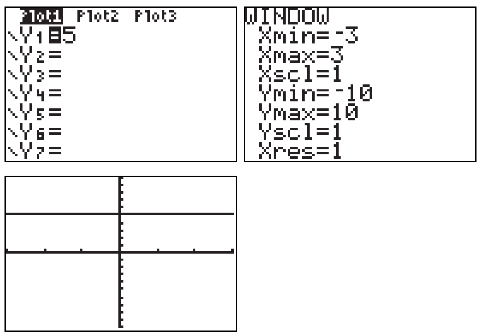


A decreasing function is a function whose graph goes down as you read from left to right.



99. To be an even function we need $f(-x)=f(x)$ and to be an odd function we need $f(-x)=-f(x)$. In order for a function be both even and odd, we would need $f(x)=-f(x)$. This is only possible if $f(x)=0$.

100. The graph of $y=5$ is a horizontal line.



The local maximum is $y=5$ and it occurs at each x-value in the interval.

101. Not necessarily. It just means $f(5)>f(2)$. The function could have both increasing and decreasing intervals.

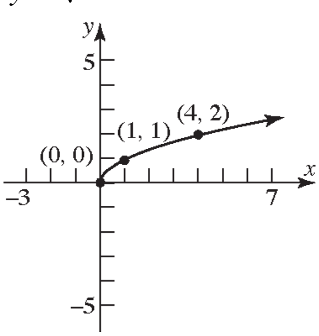
102. $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}=\dfrac{b-b}{x_2-x_1}=0$

$\dfrac{f(2)-f(-2)}{2-(-2)}=\dfrac{0-0}{4}=0$

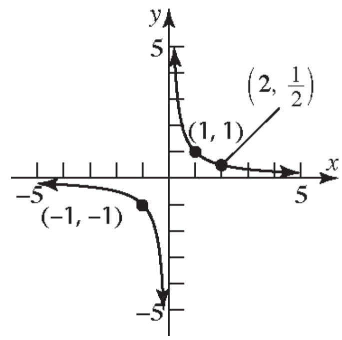
The solution set is $\{-2,1\}$.

Section 1.5

1. $y=\sqrt{x}$



2. $y=\dfrac{1}{x}$



3. $y=x^3-8$

<u>y-intercept:</u>

Let $x=0$, then $y=(0)^3-8=-8$.

<u>x-intercept:</u>

Let $y=0$, then $0=x^3-8$

$x^3=8$

$x=2$

The intercepts are $(0,-8)$ and $(2,0)$.

4. $(-\infty,0)$

5. piecewise-defined

6. True

7. False; the cube root function is odd and increasing on the interval $(-\infty,\infty)$.

8. False; the domain and range of the reciprocal function are both the set of real numbers except for 0.

9. b

10. a

11. C

12. A

13. E

14. G

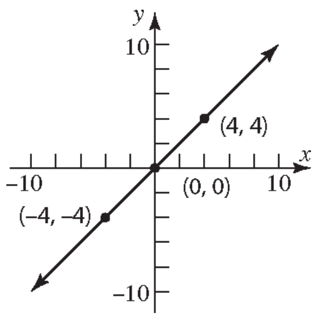
15. B

16. D

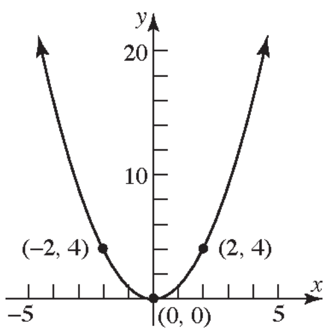
17. F

18. H

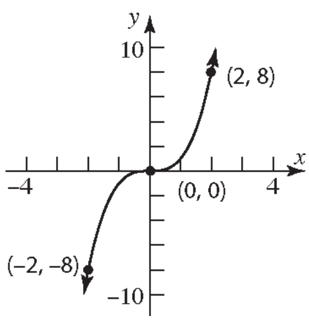
19. $f(x) = x$



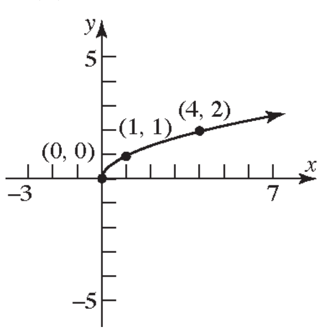
20. $f(x) = x^2$



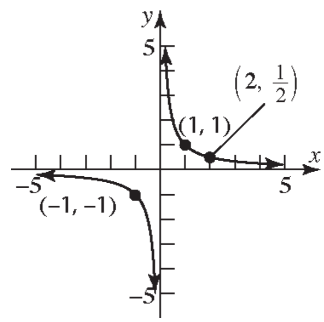
21. $f(x) = x^3$



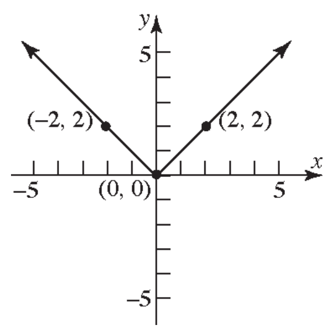
22. $f(x) = \sqrt{x}$



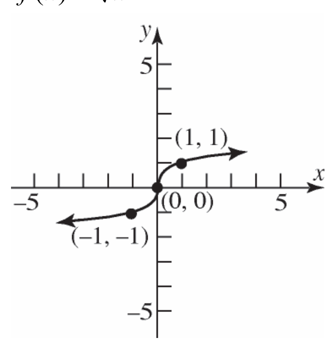
23. $f(x) = \frac{1}{x}$



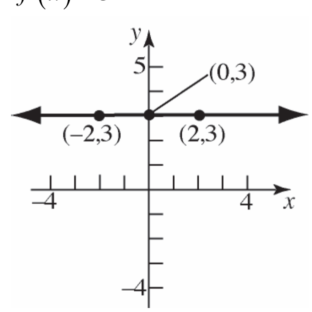
24. $f(x) = |x|$



25. $f(x) = \sqrt[3]{x}$



26. $f(x) = 3$



27. a. $f(-3) = -(-3)^2 = -9$

b. $f(0) = 4$

c. $f(2)=3(3)-2=7$

28. a. $f\left(-2\right)=-3\left(-2\right)=6$

b. $f\left(-1\right)=0$

c. $f\left(0\right)=2\left(0\right)^2+1=1$

29. a. $f\left(-2\right)=2\left(-2\right)+4=0$

b. $f\left(0\right)=2\left(0\right)+4=4$

c. $f\left(1\right)=2\left(1\right)+4=6$

d. $f\left(3\right)=\left(3\right)^3-1=26$

30. a. $f(-1)=(-1)^3=-1$

b. $f(0)=\left(0\right)^3=0$

c. $f(1)=3(1)+2=5$

d. $f\left(3\right)=3\left(3\right)+2=11$

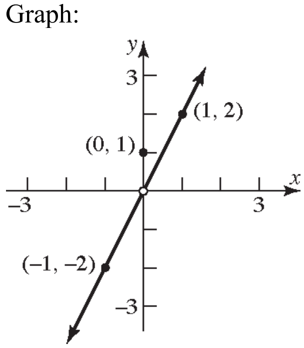
31. $f(x)=\begin{cases}2x & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$

a. Domain: $\left\{x \mid x \text{ is any real number}\right\}$

b. x-intercept: none
y-intercept:
$f\left(0\right)=1$

The only intercept is $\left(0,1\right)$.

c. Graph:



d. Range: $\left\{y \mid y\neq 0\right\}$; $\left(-\infty,0\right)\cup\left(0,\infty\right)$

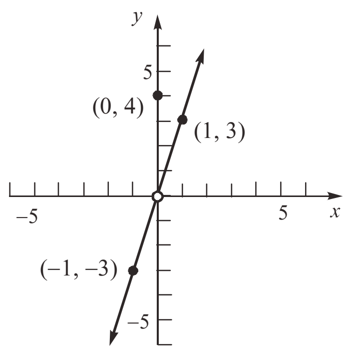
32. $f(x)=\begin{cases}3x & \text{if } x\neq 0\\ 4 & \text{if } x=0\end{cases}$

a. Domain: $\left\{x \mid x \text{ is any real number}\right\}$

b. x-intercept: none
y-intercept: $f\left(0\right)=4$

The only intercept is $\left(0,4\right)$.

c. Graph:



d. Range: $\left\{y \mid y\neq 0\right\}$; $\left(-\infty,0\right)\cup\left(0,\infty\right)$

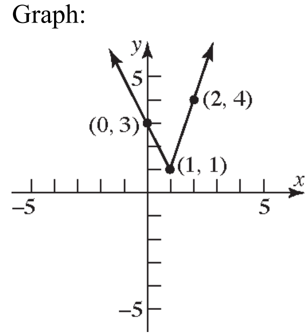
33. $f(x)=\begin{cases}-2x+3 & \text{if } x<1\\ 3x-2 & \text{if } x\geq 1\end{cases}$

a. Domain: $\left\{x \mid x \text{ is any real number}\right\}$

b. x-intercept: none
y-intercept: $f\left(0\right)=-2\left(0\right)+3=3$

The only intercept is $\left(0,3\right)$.

c. Graph:



d. Range: $\left\{y \mid y\geq 1\right\}$; $\left[1,\infty\right)$

34. $f(x)=\begin{cases}x+3 & \text{if } x<-2\\ -2x-3 & \text{if } x\geq -2\end{cases}$

a. Domain: $\left\{x \mid x \text{ is any real number}\right\}$

b. $x+3=0$ $\quad$ $-2x-3=0$

$x=-3$ $\quad$ $-2x=3$

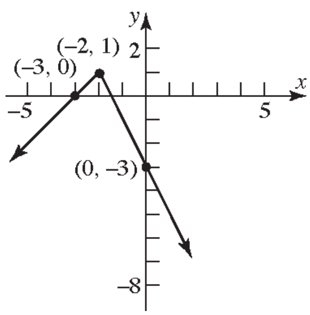
$x=-\dfrac{3}{2}$

x-intercepts: $-3,-\dfrac{3}{2}$

y-intercept: $f(0)=-2(0)-3=-3$

The intercepts are $(-3,0)$, $\left(-\dfrac{3}{2},0\right)$, and $(0,-3)$.

c. Graph:



d. Range: $\{y\,|\,y\le 1\}$; $(-\infty,1]$

35. $f(x)=\begin{cases} x+3 & \text{if } -2\le x<1 \\ 5 & \text{if } x=1 \\ -x+2 & \text{if } x>1 \end{cases}$

a. Domain: $\{x\,|\,x\ge -2\}$; $[-2,\infty)$

b. $x+3=0$ $\quad$ $-x+2=0$

$x=-3$ $\quad$ $-x=-2$

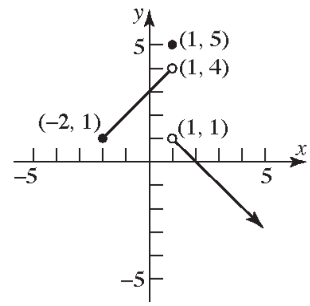
(not in domain) $\quad$ $x=2$

x-intercept: 2

y-intercept: $f(0)=0+3=3$

The intercepts are $(2,0)$ and $(0,3)$.

c. Graph:



d. Range: $\{y\,|\,y<4,\ y=5\}$; $(-\infty,4)\cup\{5\}$

36. $f(x)=\begin{cases} 2x+5 & \text{if } -3\le x<0 \\ -3 & \text{if } x=0 \\ -5x & \text{if } x>0 \end{cases}$

a. Domain: $\{x\,|\,x\ge -3\}$; $[-3,\infty)$

b. $2x+5=0$ $\quad$ $-5x=0$

$2x=-5$ $\quad$ $x=0$

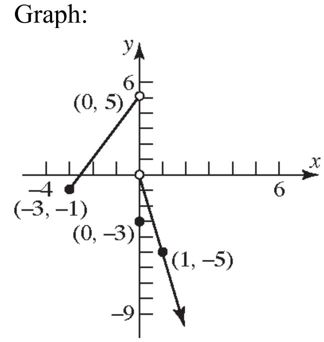
$x=-\dfrac{5}{2}$ $\quad$ (not in domain of piece)

x-intercept: $-\dfrac{5}{2}$

y-intercept: $f(0)=-3$

The intercepts are $\left(-\dfrac{5}{2},0\right)$ and $(0,-3)$.

c. Graph:



d. Range: $\{y\,|\,y<5\}$; $(-\infty,5)$

37. $f(x)=\begin{cases} 1+x & \text{if } x<0 \\ x^2 & \text{if } x\ge 0 \end{cases}$

a. Domain: $\{x\,|\,x \text{ is any real number}\}$

b. $1+x=0$ $\quad$ $x^2=0$

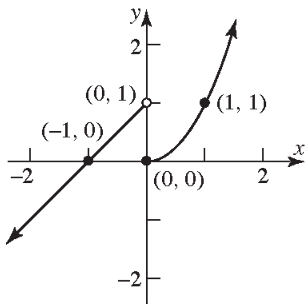
$x=-1$ $\quad$ $x=0$

x-intercepts: $-1,0$

y-intercept: $f(0) = 0^2 = 0$

The intercepts are $(-1, 0)$ and $(0, 0)$.

c. Graph:



d. Range: $\{y \mid y \text{ is any real number}\}$

38. $f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ \sqrt[3]{x} & \text{if } x \geq 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

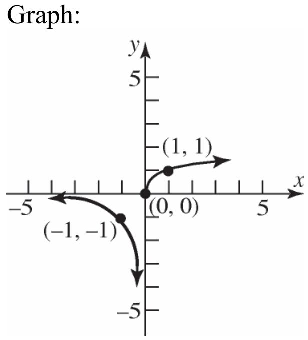
b. $\dfrac{1}{x} = 0$ (no solution) $\sqrt[3]{x} = 0$, $x = 0$

x-intercept: 0

y-intercept: $f(0) = \sqrt[3]{0} = 0$

The only intercept is $(0, 0)$.

c. Graph:



d. Range: $\{y \mid y \text{ is any real number}\}$

39. $f(x) = \begin{cases} |x| & \text{if } -2 \leq x < 0 \\ x^3 & \text{if } x > 0 \end{cases}$

a. Domain: $\{x \mid -2 \leq x < 0 \text{ and } x > 0\}$ or $\{x \mid x \geq -2, x \neq 0\}$; $[-2, 0) \cup (0, \infty)$.

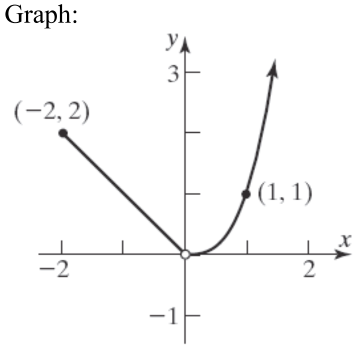
b. x-intercept: none

There are no x-intercepts since there are no values for x such that $f(x) = 0$.

y-intercept:

There is no y-intercept since $x = 0$ is not in the domain.

c. Graph:



d. Range: $\{y \mid y > 0\}$; $(0, \infty)$

40. $f(x) = \begin{cases} 2 - x & \text{if } -3 \leq x < 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$

a. Domain: $\{x \mid -3 \leq x < 1 \text{ and } x > 1\}$ or $\{x \mid x \geq -3, x \neq 1\}$; $[-3, 1) \cup (1, \infty)$.

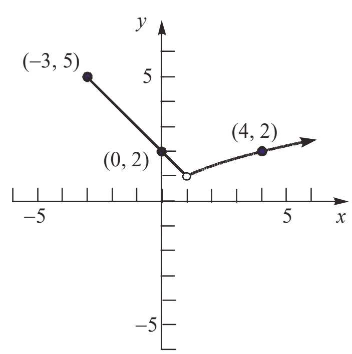
b. $2 - x = 0$, $x = 2$ $\sqrt{x} = 0$, $x = 0$ (not in domain of piece)

no x-intercepts

y-intercept: $f(0) = 2 - 0 = 2$

The intercept is $(0, 2)$.

c. Graph:



d. Range: $\{y \mid y > 1\}$; $(1, \infty)$

41. $f(x)=\begin{cases} x^2 & \text{if } 0<x\le 2 \\ x+2 & \text{if } 2<x<5 \\ 7 & \text{if } \ge 5 \end{cases}$

a. Domain: $\{x \mid x>0\}$; $(0,\infty)$.

b. $x^2=0$

$x=0$

(not in domain of piece)

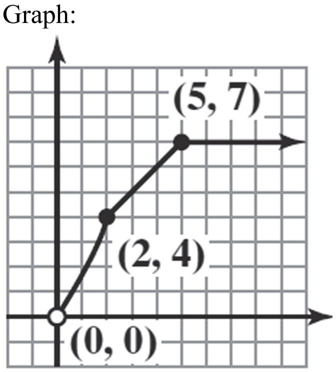
$x+2=0$ $\quad$ $7=0$

$x=-2$ $\quad$ (not possible)

(not in domain of piece)

No intercepts.

c. Graph:



d. Range: $\{y \mid 0<y\le 7\}$; $(0,7]$

42. $f(x)=\begin{cases} 3x+5 & \text{if } -3\le x<0 \\ 5 & \text{if } 0\le x\le 2 \\ x^2+1 & \text{if } x>2 \end{cases}$

a. Domain: $\{x \mid x\ge -3\}$; $[3,\infty)$.

b. $3x+5=0$ $\quad$ $5=0$ $\quad$ $x^2+1=0$

$x=-\dfrac{5}{3}$ $\quad$ (not possible) $\quad$ $x^2=-1$

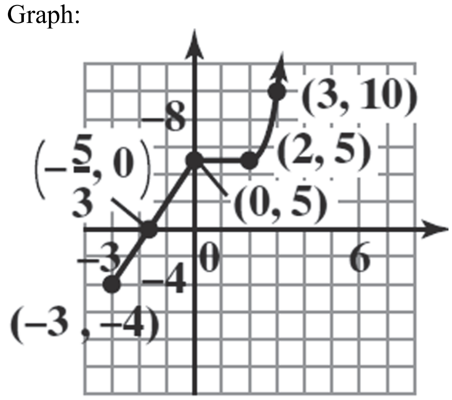
(not possible)

x-intercept: $-\dfrac{5}{3}$

y-intercept: $f(0)=5$

The intercepts are $(0,5)$ and $\left(-\dfrac{5}{3},0\right)$.

c. Graph:



d. Range: $[-4,\infty)$

43. Answers may vary. One possibility follows:

$f(x)=\begin{cases} -x & \text{if } -1\le x\le 0 \\ \frac{1}{2}x & \text{if } 0<x\le 2 \end{cases}$

44. Answers may vary. One possibility follows:

$f(x)=\begin{cases} x & \text{if } -1\le x\le 0 \\ 1 & \text{if } 0<x\le 2 \end{cases}$

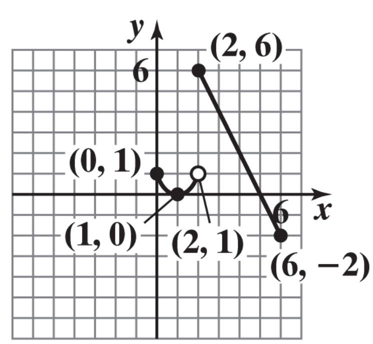
45. Answers may vary. One possibility follows:

$f(x)=\begin{cases} -x & \text{if } x\le 0 \\ -x+2 & \text{if } 0<x\le 2 \end{cases}$

46. Answers may vary. One possibility follows:

$f(x)=\begin{cases} 2x+2 & \text{if } -1\le x\le 0 \\ x & \text{if } x>0 \end{cases}$

47. **a.**

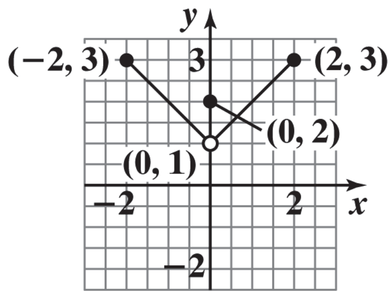


b. The domain is $[0,6]$.

c. Absolute max: $f(2)=6$

Absolute min: $f(6)=-2$

48. a.



b. The domain is $[-2, 2]$.

c. Absolute max: $f(-2) = f(2) = 3$

Absolute min: none

49. $C = \begin{cases} 34.99 & \text{if } 0 < x \le 3 \\ 15x - 10.01 & \text{if } x > 3 \end{cases}$

a. $C(2) = \$34.99$

b. $C(5) = 15(5) - 10.01 = \$64.99$

c. $C(13) = 15(13) - 10.01 = \$184.99$

50. $F(x) = \begin{cases} 3 + x & 0 < x \le 4 \\ 4x + 10 & 4 < x \le 9 \\ 74 & 9 < x \le 24 \end{cases}$

a. $F(2) = 3 + 2 = 5$

Parking for 2 hours costs \$5.

b. $F(7) = 4(7) + 10 = 38$

Parking for 7 hours costs \$38.

c. $F(15) = 74$

Parking for 15 hours costs \$74.

d. $F(9) = 4(9) + 1 = 46$

Parking for 8 hours and 24 minutes costs \$46.

51. a. Charge for 20 therms:

$C = 23.44 + 0.91686(20) + 0.26486(20)$
$= \$47.07$

b. Charge for 150 therms:

$C = 23.44 + 0.91686(30) + 0.26486(30)$
$+ 0.50897(120)$
$= \$119.97$

c. For $0 \le x \le 30$:

$C = 23.44 + 0.91686x + 0.26486x$
$= 1.18172x + 23.44$

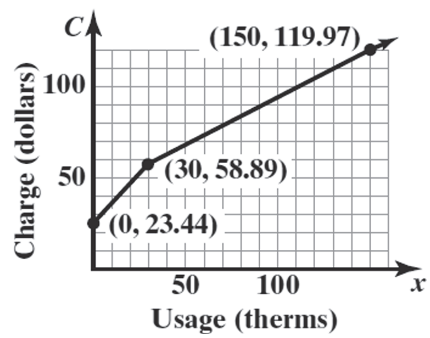
For $x > 30$:

$C = 23.44 + 0.91686(30) + 0.50897(x - 30)$
$+ 0.26486(30)$
$= 23.44 + 27.5058 + 0.50897x - 15.2691$
$+ 7.9458$
$= 0.50897x + 43.6225$

The monthly charge function:

$C = \begin{cases} 1.18172x + 23.44 & \text{if } 0 \le x \le 30 \\ 0.50897x + 43.6225 & \text{if } x > 30 \end{cases}$

d. Graph:



52. a. Charge for 1000 therms:

$C = 90.00 + 0.1201(150) + 0.0549(850)$
$+ 0.35(1000)$
$= \$504.68$

b. Charge for 6000 therms:

$C = 90.00 + 0.1201(150) + 0.0549(4850)$
$+ 0.0482(1000) + 0.35(6000)$
$= \$2522.48$

c. For $0 \le x \le 150$:

$C = 90.00 + 0.1201x + 0.35x$
$= 0.4701x + 90.00$

For $150 < x \le 5000$:

$C = 90.00 + 0.1201(150) + 0.0549(x - 150)$
$+ 0.35x$
$= 90.00 + 18.015 + 0.0549x - 8.235$
$+ 0.35x$
$= 0.4049x + 99.78$

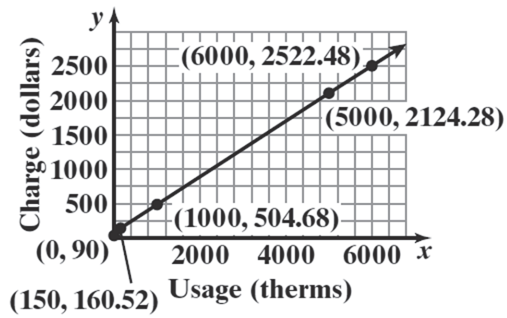
For $x > 5000$:

$$\begin{aligned} C &= 90.00 + 0.1201(150) + 0.0549(4850) \\ &\quad + 0.0482(x - 5000) + 0.35x \\ &= 90.00 + 18.015 + 266.265 + 0.0482x - 241 \\ &\quad + 0.35x \\ &= 0.3982x + 133.28 \end{aligned}$$

The monthly charge function:

$$C(x) = \begin{cases} 0.4701x + 90.00 & \text{if } \ 0 \le x \le 150 \\ 0.4049x + 82.38 & \text{if } \ 150 < x \le 5000 \\ 0.3982x + 115.88 & \text{if } \ x > 5000 \end{cases}$$

d.



53. For Schedule X:

$$f(x) = \begin{cases} 0.10x & \text{if } \ 0 < x \le 9525 \\ 952.50 + 0.12(x - 9525) & \text{if } \ 9525 < x \le 38{,}700 \\ 4453.50 + 0.22(x - 38{,}700) & \text{if } \ 38{,}700 < x \le 82{,}500 \\ 14{,}089.50 + 0.24(x - 82{,}500) & \text{if } \ 82{,}500 < x \le 157{,}500 \\ 32{,}089.75 + 0.32(x - 157{,}500) & \text{if } \ 157{,}500 < x \le 200{,}000 \\ 45{,}689.50 + 0.35(x - 200{,}000) & \text{if } \ 200{,}000 < x \le 500{,}000 \\ 150{,}689.50 + 0.37(x - 500{,}000) & \text{if } \ x > 500{,}000 \end{cases}$$

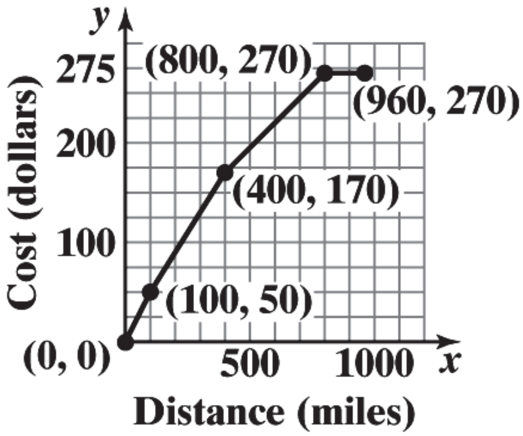
54. For Schedule $\text{Y}-1$:

$$f(x) = \begin{cases} 0.10x & \text{if } \ 0 < x \le 19{,}050 \\ 1905.00 + 0.12(x - 19{,}050) & \text{if } \ 19{,}050 < x \le 77{,}400 \\ 8{,}907.00 + 0.22(x - 77{,}400) & \text{if } \ 77{,}400 < x \le 165{,}000 \\ 28{,}179.00 + 0.24(x - 165{,}000) & \text{if } \ 165{,}000 < x \le 315{,}000 \\ 64{,}179.00 + 0.32(x - 315{,}000) & \text{if } \ 315{,}000 < x \le 400{,}000 \\ 91{,}379.00 + 0.35(x - 400{,}000) & \text{if } \ 400{,}000 < x \le 600{,}000 \\ 161{,}379.00 + 0.37(x - 600{,}000) & \text{if } \ x > 600{,}000 \end{cases}$$

55. a. Let x represent the number of miles and C be the cost of transportation.

$$C(x) = \begin{cases} 0.50x & \text{if } 0 \le x \le 100 \\ 0.50(100) + 0.40(x - 100) & \text{if } 100 < x \le 400 \\ 0.50(100) + 0.40(300) + 0.25(x - 400) & \text{if } 400 < x \le 800 \\ 0.50(100) + 0.40(300) + 0.25(400) + 0(x - 800) & \text{if } 800 < x \le 960 \end{cases}$$

$$C(x) = \begin{cases} 0.50x & \text{if } 0 \le x \le 100 \\ 10 + 0.40x & \text{if } 100 < x \le 400 \\ 70 + 0.25x & \text{if } 400 < x \le 800 \\ 270 & \text{if } 800 < x \le 960 \end{cases}$$

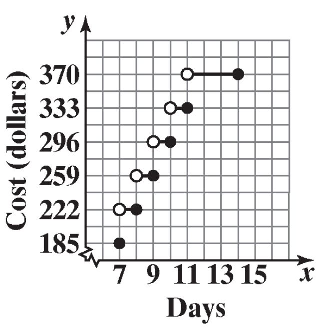


b. For hauls between 100 and 400 miles the cost is: $C(x) = 10 + 0.40x$.

c. For hauls between 400 and 800 miles the cost is: $C(x) = 70 + 0.25x$.

56. Let x = number of days car is used. The cost of renting is given by

$$C(x) = \begin{cases} 185 & \text{if } x = 7 \\ 222 & \text{if } 7 < x \le 8 \\ 259 & \text{if } 8 < x \le 9 \\ 296 & \text{if } 9 < x \le 10 \\ 333 & \text{if } 10 < x \le 11 \\ 370 & \text{if } 11 < x \le 14 \end{cases}$$



57. a. Let s = the credit score of an individual who wishes to borrow \$300,000 with an 80% LTV ratio. The adverse market delivery charge is given by

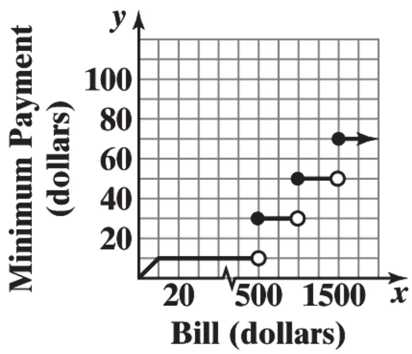
$$C(s) = \begin{cases} 9000 & \text{if } s \le 659 \\ 8250 & \text{if } 660 \le s \le 679 \\ 5250 & \text{if } 680 \le s \le 699 \\ 3750 & \text{if } 700 \le s \le 719 \\ 2250 & \text{if } 720 \le s \le 739 \\ 1500 & \text{if } s \ge 740 \end{cases}$$

b. 725 is between 720 and 739 so the charge would be \$2250.

c. 670 is between 660 and 679 so the charge would be \$8250.

58. Let x = the amount of the bill in dollars. The minimum payment due is given by

$$f(x) = \begin{cases} x & \text{if } 0 \le x < 10 \\ 10 & \text{if } 10 \le x < 500 \\ 30 & \text{if } 500 \le x < 1000 \\ 50 & \text{if } 1000 \le x < 1500 \\ 70 & \text{if } x \ge 1500 \end{cases}$$



59. a. $W = 10°C$

b. $W = 33 - \frac{(10.45 + 10\sqrt{5} - 5)(33-10)}{22.04} \approx 4°C$

c. $W = 33 - \frac{(10.45 + 10\sqrt{15} - 15)(33-10)}{22.04} \approx -3°C$

d. $W = 33 - 1.5958(33-10) = -4°C$

e. When $0 \le v < 1.79$, the wind speed is so small that there is no effect on the temperature.

f. When the wind speed exceeds 20, the wind chill depends only on the air temperature.

60. a. $W = -10°C$

b. $W = 33 - \frac{\left(10.45 + 10\sqrt{5} - 5\right)\left(33-(-10)\right)}{22.04}$

$\approx -21°C$

c. $W = 33 - \frac{\left(10.45 + 10\sqrt{15} - 15\right)\left(33-(-10)\right)}{22.04}$

$\approx -34°C$

d. $W = 33 - 1.5958\left(33-(-10)\right) = -36°C$

61. Let x = the number of ounces and $C(x)$ = the postage due.

For $0 < x \le 1$: $C(x) = \$1.00$

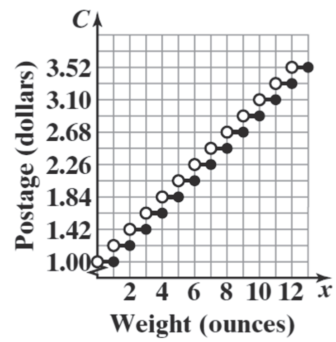
For $1 < x \le 2$: $C(x) = 1.00 + 0.21 = \$1.21$

For $2 < x \le 3$: $C(x) = 1.00 + 2(0.21) = \$1.42$

For $3 < x \le 4$: $C(x) = 1.0 + 3(0.21) = \$1.63$

$\vdots$

For $12 < x \le 13$: $C(x) = 1.00 + 12(0.21) = \$3.52$



62. Use intervals $(0,8),[8,16),[16,32),[32,38)$ (exclude 0 and 38 since those would be the walls). Depth for the intervals $[8,16)$ and $[32,38)$ are constant (8 ft and 3 ft respectively). The other two are linear functions. On $(0,8)$ the endpoint coordinates can be thought or as $(0,3)$ and $(8,8)$.

$m = \frac{8-3}{8-0} = \frac{5}{8} \quad y = \frac{5}{8}x + 3$

On $[16,32)$ the endpoint coordinates can be thought of as $(16,8)$ and $(32,3)$.

$m = \frac{3-8}{32-16} = -\frac{5}{16}$

$8 = -\frac{5}{16}(16) + b$

$13 = b$

$y = -\frac{5}{16}x + 13$

Therefore,

$$d(x) = \begin{cases} \frac{5}{8}x + 3 & \text{if } 0 < x < 8 \\ 8 & \text{if } 8 \le x < 16 \\ -\frac{5}{16}x + 13 & \text{if } 16 \le x < 32 \\ 3 & \text{if } 32 \le x < 38 \end{cases}$$

63. The function f changes definition at 2 and the function g changes definition at 0. Combining these together, the sum function will change definitions at 0 and 2.

On the interval $(-\infty, 0]$:

$$\begin{aligned}(f+g)(x) &= f(x)+g(x) = (2x+3)+(-4x+1) \\ &= -2x+4\end{aligned}$$

On the interval $(0, 2)$:

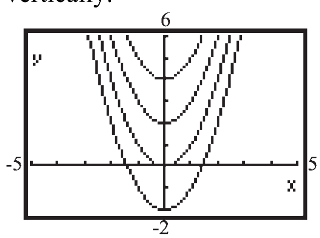
$$\begin{aligned}(f+g)(x) &= f(x)+g(x) = (2x+3)+(x-7) \\ &= 3x-4\end{aligned}$$

On the interval $[2, \infty)$:

$$\begin{aligned}(f+g)(x) &= f(x)+g(x) = (x^2+5x)+(x-7) \\ &= x^2+6x-7\end{aligned}$$

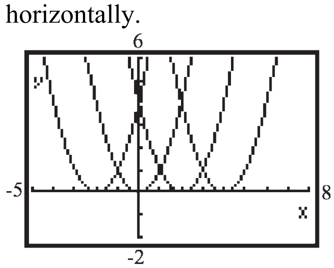
So, $(f+g)(x) = \begin{cases} -2x+4 & \text{if } x \le 0 \\ 3x-4 & \text{if } 0 < x \le 2 \\ x^2+6x-7 & \text{if } x \ge 2 \end{cases}$

64. Each graph is that of $y = x^2$, but shifted vertically.



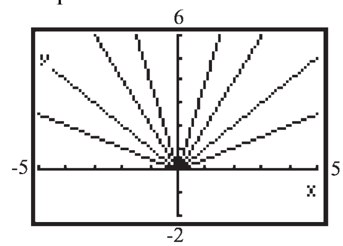
If $y = x^2 + k$, $k > 0$, the shift is up k units; if $y = x^2 - k$, $k > 0$, the shift is down k units. The graph of $y = x^2 - 4$ is the same as the graph of $y = x^2$, but shifted down 4 units. The graph of $y = x^2 + 5$ is the graph of $y = x^2$, but shifted up 5 units.

65. Each graph is that of $y = x^2$, but shifted horizontally.



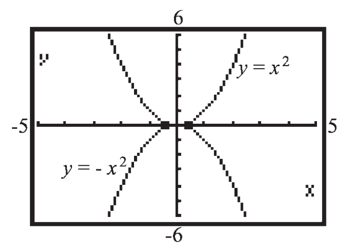
If $y = (x-k)^2$, $k > 0$, the shift is to the right k units; if $y = (x+k)^2$, $k > 0$, the shift is to the left k units. The graph of $y = (x+4)^2$ is the same as the graph of $y = x^2$, but shifted to the left 4 units. The graph of $y = (x-5)^2$ is the graph of $y = x^2$, but shifted to the right 5 units.

66. Each graph is that of $y = |x|$, but either compressed or stretched vertically.

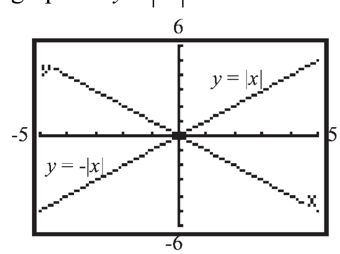


If $y = k|x|$ and $k > 1$, the graph is stretched vertically; if $y = k|x|$ and $0 < k < 1$, the graph is compressed vertically. The graph of $y = \frac{1}{4}|x|$ is the same as the graph of $y = |x|$, but compressed vertically. The graph of $y = 5|x|$ is the same as the graph of $y = |x|$, but stretched vertically.

67. The graph of $y = -x^2$ is the reflection of the graph of $y = x^2$ about the x-axis.

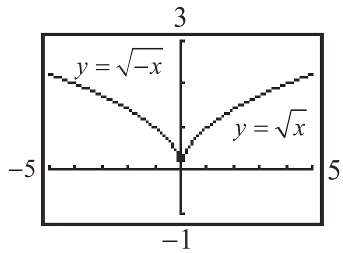


The graph of $y = -|x|$ is the reflection of the graph of $y = |x|$ about the x-axis.

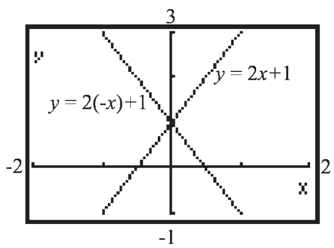


Multiplying a function by -1 causes the graph to be a reflection about the x-axis of the original function's graph.

68. The graph of $y = \sqrt{-x}$ is the reflection about the y-axis of the graph of $y = \sqrt{x}$.

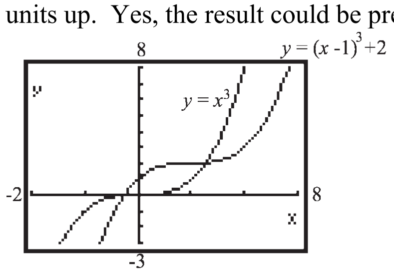


The same type of reflection occurs when graphing $y = 2x + 1$ and $y = 2(-x) + 1$.

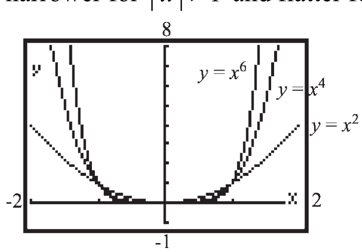


The graph of $y = f(-x)$ is the reflection about the y-axis of the graph of $y = f(x)$.

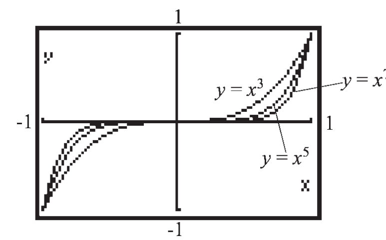
69. The graph of $y = (x-1)^3 + 2$ is a shifting of the graph of $y = x^3$ one unit to the right and two units up. Yes, the result could be predicted.



70. The graphs of $y = x^n$, n a positive even integer, are all U-shaped and open upward. All go through the points $(-1, 1)$, $(0, 0)$, and $(1, 1)$. As n increases, the graph of the function is narrower for $|x| > 1$ and flatter for $|x| < 1$.



71. The graphs of $y = x^n$, n a positive odd integer, all have the same general shape. All go through the points $(-1, -1)$, $(0, 0)$, and $(1, 1)$. As n increases, the graph of the function increases at a greater rate for $|x| > 1$ and is flatter around 0 for $|x| < 1$.



72. $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

Yes, it is a function.

Domain = $\{x \mid x \text{ is any real number}\}$ or $(-\infty, \infty)$

Range = $\{0, 1\}$ or $\{y \mid y = 0 \text{ or } y = 1\}$

y-intercept: $x = 0 \Rightarrow x$ is rational $\Rightarrow y = 1$

So the y-intercept is $y = 1$.

x-intercept: $y = 0 \Rightarrow x$ is irrational

So the graph has infinitely many x-intercepts, namely, there is an x-intercept at each irrational value of x.

$f(-x) = 1 = f(x)$ when x is rational;

$f(-x) = 0 = f(x)$ when x is irrational.

Thus, f is even.

The graph of f consists of 2 infinite clusters of distinct points, extending horizontally in both directions. One cluster is located 1 unit above the x-axis, and the other is located along the x-axis.

73. Answers will vary.

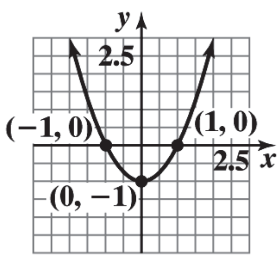
Section 1.6

1. horizontal; right
2. y
3. False
4. True; the graph of $y = -f(x)$ is the reflection about the x-axis of the graph of $y = f(x)$.
5. False
6. d
7. b
8. True
9. B
10. E
11. H
12. D
13. I
14. A
15. L
16. C
17. F
18. J
19. G
20. K
21. $y = (x-4)^3$
22. $y = (x+4)^3$
23. $y = x^3 + 4$
24. $y = x^3 - 4$
25. $y = (-x)^3 = -x^3$
26. $y = -x^3$
27. $y = 5x^3$
28. $y = \left(\frac{1}{4}x\right)^3 = \frac{1}{64}x^3$
29. $y = (2x)^3 = 8x^3$
30. $y = \frac{1}{4}x^3$
31. (1) $y = \sqrt{x} + 2$
    (2) $y = -\left(\sqrt{x} + 2\right)$
    (3) $y = -\left(\sqrt{-x} + 2\right) = -\sqrt{-x} - 2$
32. (1) $y = -\sqrt{x}$
    (2) $y = -\sqrt{x-3}$
    (3) $y = -\sqrt{x-3} - 2$
33. (1) $y = 3\sqrt{x}$
    (2) $y = 3\sqrt{x} + 4$
    (3) $y = 3\sqrt{x+5} + 4$
34. (1) $y = \sqrt{x} + 2$
    (2) $y = \sqrt{-x} + 2$
    (3) $y = \sqrt{-(x+3)} + 2 = \sqrt{-x-3} + 2$
35. (c); To go from $y = f(x)$ to $y = -f(x)$ we reflect about the x-axis. This means we change the sign of the y-coordinate for each point on the graph of $y = f(x)$. Thus, the point $(3, 6)$ would become $(3, -6)$.
36. (d); To go from $y = f(x)$ to $y = f(-x)$, we reflect each point on the graph of $y = f(x)$ about the y-axis. This means we change the sign of the x-coordinate for each point on the graph of $y = f(x)$. Thus, the point $(3, 6)$ would become $(-3, 6)$.
37. (c); To go from $y = f(x)$ to $y = 2f(x)$, we stretch vertically by a factor of 2. Multiply the y-coordinate of each point on the graph of $y = f(x)$ by 2. Thus, the point $(1, 3)$ would become $(1, 6)$.

38. (c); To go from $y = f(x)$ to $y = f(2x)$, we compress horizontally by a factor of 2. Divide the x-coordinate of each point on the graph of $y = f(x)$ by 2. Thus, the point $(4,2)$ would become $(2,2)$.

39. $f(x) = x^2 - 1$

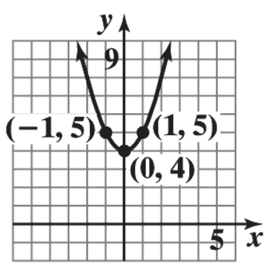
Using the graph of $y = x^2$, vertically shift downward 1 unit.



The domain is $(-\infty, \infty)$ and the range is $[-1, \infty)$.

40. $f(x) = x^2 + 4$

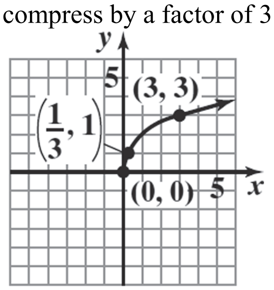
Using the graph of $y = x^2$, vertically shift upward 4 units.



The domain is $(-\infty, \infty)$ and the range is $[4, \infty)$.

41. $g(x) = \sqrt{3x}$

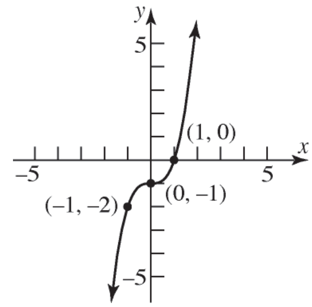
Using the graph of $y = \sqrt{x}$, horizontally compress by a factor of 3.



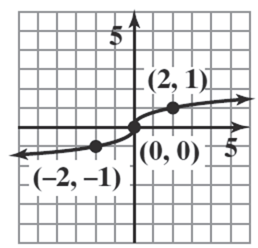
The domain is $[0, \infty)$ and the range is $[0, \infty)$.

42. $g(x) = \sqrt[3]{\frac{1}{2}x}$

Using the graph of $y = \sqrt[3]{x}$, horizontally stretch by a factor of ½.

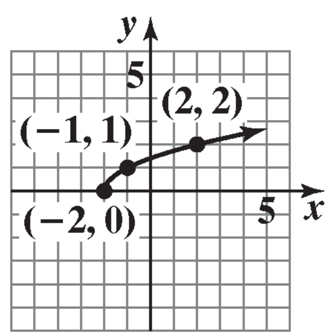


The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.



43. $h(x) = \sqrt{x + 2}$

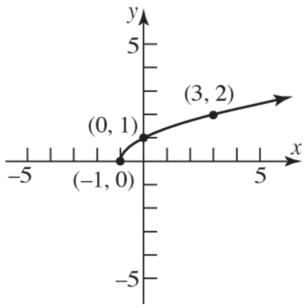
Using the graph of $y = \sqrt{x}$, horizontally shift to the left 2 units.



The domain is $[-2, \infty)$ and the range is $[0, \infty)$.

44. $h(x) = \sqrt{x + 1}$

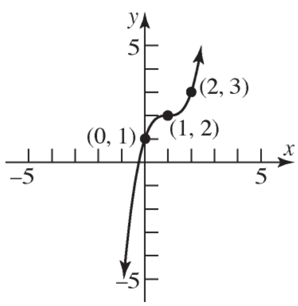
Using the graph of $y = \sqrt{x}$, horizontally shift to the left 1 unit.



The domain is $[-1,\infty)$ and the range is $[0,\infty)$.

45. $f(x)=(x-1)^3+2$

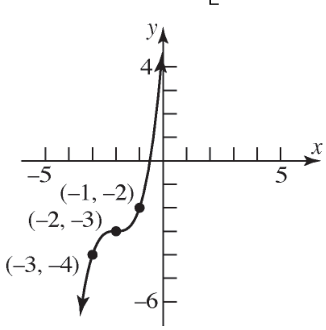
Using the graph of $y=x^3$, horizontally shift to the right 1 unit $\left[y=(x-1)^3\right]$, then vertically shift up 2 units $\left[y=(x-1)^3+2\right]$.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

46. $f(x)=(x+2)^3-3$

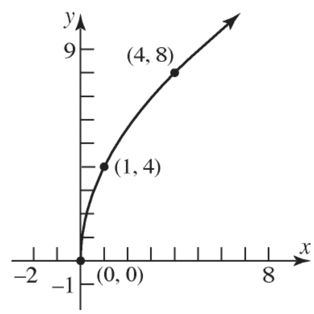
Using the graph of $y=x^3$, horizontally shift to the left 2 units $\left[y=(x+2)^3\right]$, then vertically shift down 3 units $\left[y=(x+2)^3-3\right]$.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

47. $g(x)=4\sqrt{x}$

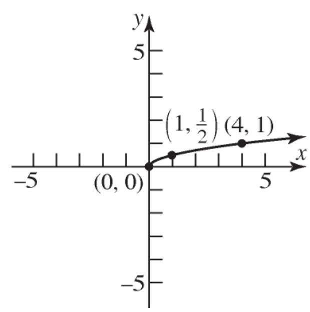
Using the graph of $y=\sqrt{x}$, vertically stretch by a factor of 4.



The domain is $[0,\infty)$ and the range is $[0,\infty)$.

48. $g(x)=\dfrac{1}{2}\sqrt{x}$

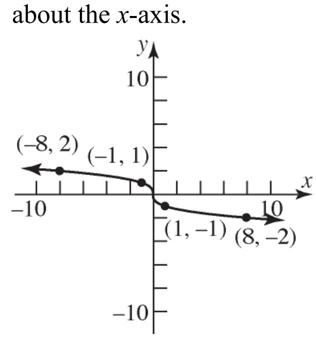
Using the graph of $y=\sqrt{x}$, vertically compress by a factor of $\dfrac{1}{2}$.



The domain is $[0,\infty)$ and the range is $[0,\infty)$.

49. $f(x)=-\sqrt[3]{x}$

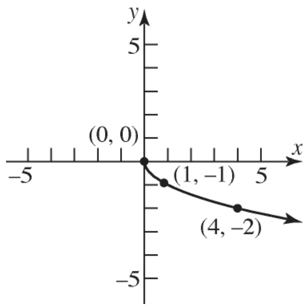
Using the graph of $y=\sqrt[3]{x}$, reflect the graph about the x-axis.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

50. $f(x) = -\sqrt{x}$

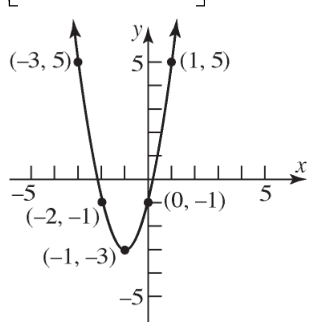
Using the graph of $y = \sqrt{x}$, reflect the graph about the x-axis.



The domain is $[0, \infty)$ and the range is $(-\infty, 0]$.

51. $f(x) = 2(x+1)^2 - 3$

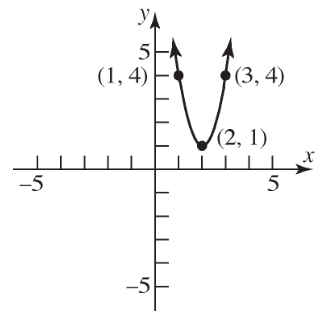
Using the graph of $y = x^2$, horizontally shift to the left 1 unit $\left[y = (x+1)^2\right]$, vertically stretch by a factor of 2 $\left[y = 2(x+1)^2\right]$, and then vertically shift downward 3 units $\left[y = 2(x+1)^2 - 3\right]$.



The domain is $(-\infty, \infty)$ and the range is $[-3, \infty)$.

52. $f(x) = 3(x-2)^2 + 1$

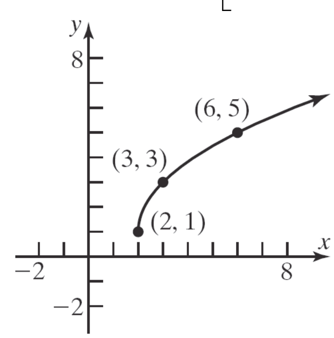
Using the graph of $y = x^2$, horizontally shift to the right 2 units $\left[y = (x-2)^2\right]$, vertically stretch by a factor of 3 $\left[y = 3(x-2)^2\right]$, and then vertically shift upward 1 unit $\left[y = 3(x-2)^2 + 1\right]$.



The domain is $(-\infty, \infty)$ and the range is $[1, \infty)$.

53. $g(x) = 2\sqrt{x-2} + 1$

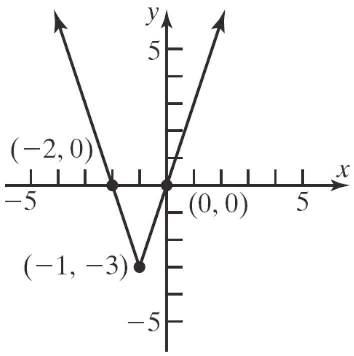
Using the graph of $y = \sqrt{x}$, horizontally shift to the right 2 units $\left[y = \sqrt{x-2}\right]$, vertically stretch by a factor of 2 $\left[y = 2\sqrt{x-2}\right]$, and vertically shift upward 1 unit $\left[y = 2\sqrt{x-2} + 1\right]$.



The domain is $[2, \infty)$ and the range is $[1, \infty)$.

54. $g(x) = 3|x+1| - 3$

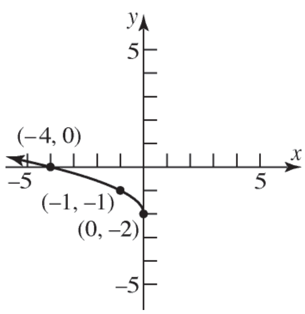
Using the graph of $y = |x|$, horizontally shift to the left 1 unit $\left[y = |x+1|\right]$, vertically stretch by a factor of 3 $\left[y = 3|x+1|\right]$, and vertically shift downward 3 units $\left[y = 3|x+1| - 3\right]$.



The domain is $(-\infty, \infty)$ and the range is $[-3, \infty)$.

55. $h(x) = \sqrt{-x} - 2$

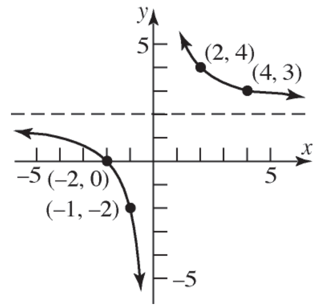
Using the graph of $y = \sqrt{x}$, reflect the graph about the y-axis $\left[y = \sqrt{-x}\right]$ and vertically shift downward 2 units $\left[y = \sqrt{-x} - 2\right]$.



The domain is $(-\infty, 0]$ and the range is $[-2, \infty)$.

56. $h(x) = \dfrac{4}{x} + 2 = 4\left(\dfrac{1}{x}\right) + 2$

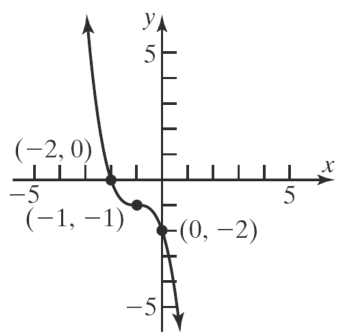
Stretch the graph of $y = \dfrac{1}{x}$ vertically by a factor of 4 $\left[y = 4 \cdot \dfrac{1}{x} = \dfrac{4}{x}\right]$ and vertically shift upward 2 units $\left[y = \dfrac{4}{x} + 2\right]$.



The domain is $(-\infty, 0) \cup (0, \infty)$ and the range is $(-\infty, 2) \cup (2, \infty)$.

57. $f(x) = -(x+1)^3 - 1$

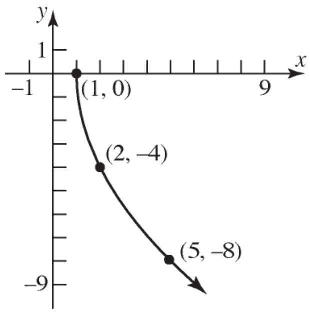
Using the graph of $y = x^3$, horizontally shift to the left 1 unit $\left[y = (x+1)^3\right]$, reflect the graph about the x-axis $\left[y = -(x+1)^3\right]$, and vertically shift downward 1 unit $\left[y = -(x+1)^3 - 1\right]$.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

58. $f(x) = -4\sqrt{x-1}$

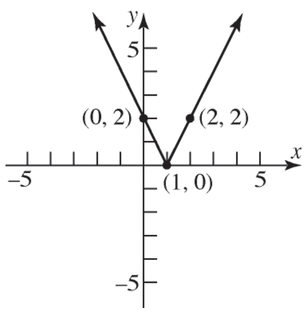
Using the graph of $y = \sqrt{x}$, horizontally shift to the right 1 unit $\left[y = \sqrt{x-1}\right]$, reflect the graph about the x-axis $\left[y = -\sqrt{x-1}\right]$, and stretch vertically by a factor of 4 $\left[y = -4\sqrt{x-1}\right]$.



The domain is $[1,\infty)$ and the range is $(-\infty,0]$.

59. $g(x)=2|1-x|=2|-(-1+x)|=2|x-1|$

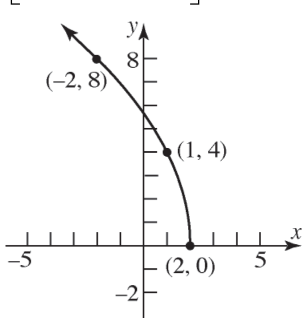
Using the graph of $y=|x|$, horizontally shift to the right 1 unit $\left[y=|x-1|\right]$, and vertically stretch by a factor or 2 $\left[y=2|x-1|\right]$.



The domain is $(-\infty,\infty)$ and the range is $[0,\infty)$.

60. $g(x)=4\sqrt{2-x}=4\sqrt{-(x-2)}$

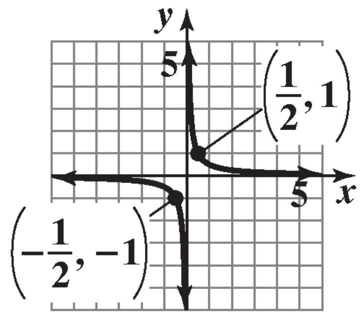
Using the graph of $y=\sqrt{x}$, reflect the graph about the y-axis $\left[y=\sqrt{-x}\right]$, horizontally shift to the right 2 units $\left[y=\sqrt{-(x-2)}\right]$, and vertically stretch by a factor of 4 $\left[y=4\sqrt{-(x-2)}\right]$.



The domain is $(-\infty,2]$ and the range is $[0,\infty)$.

61. $h(x)=\dfrac{1}{2x}$

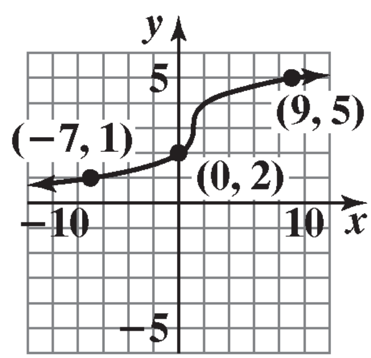
Using the graph of $y=\dfrac{1}{x}$, vertically compress by a factor of $\dfrac{1}{2}$.



The domain is $(-\infty,0)\cup(0,\infty)$ and the range is $(-\infty,0)\cup(0,\infty)$.

62. $f(x)=\sqrt[3]{x-1}+3$

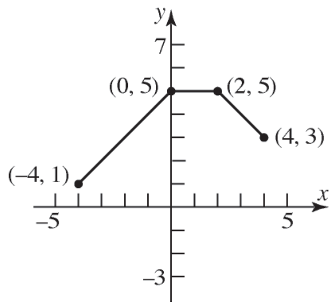
Using the graph of $f(x)=\sqrt[3]{x}$, horizontally shift to the right 1 unit $\left[y=\sqrt[3]{x-1}\right]$, then vertically shift up 3 units $\left[y=\sqrt[3]{x-1}+3\right]$.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

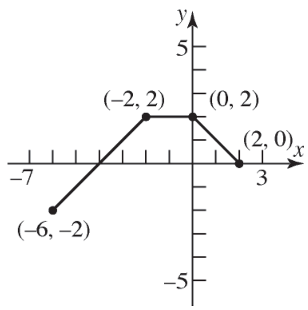
63. a. $F(x) = f(x) + 3$

Shift up 3 units.



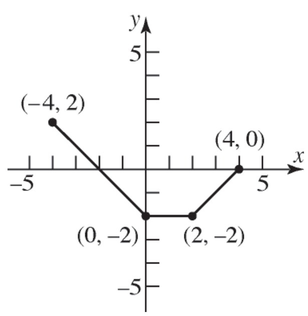
b. $G(x) = f(x+2)$

Shift left 2 units.



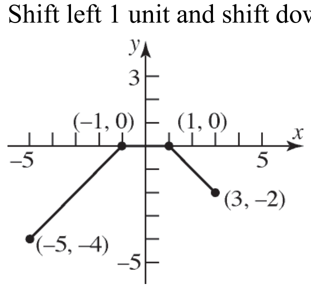
c. $P(x) = -f(x)$

Reflect about the *x*-axis.



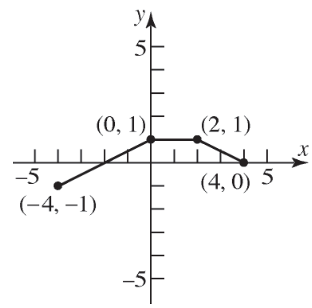
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



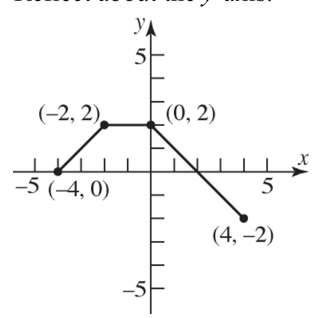
e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



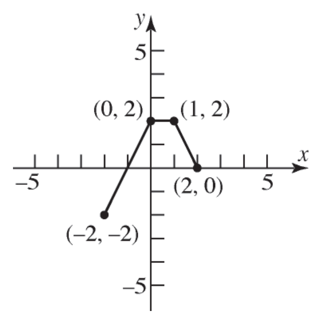
f. $g(x) = f(-x)$

Reflect about the *y*-axis.



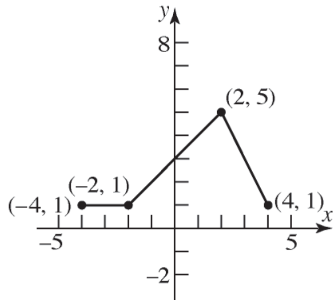
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



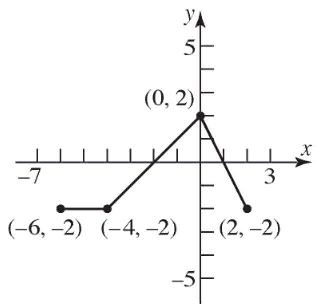
64. a. $F(x) = f(x) + 3$

Shift up 3 units.



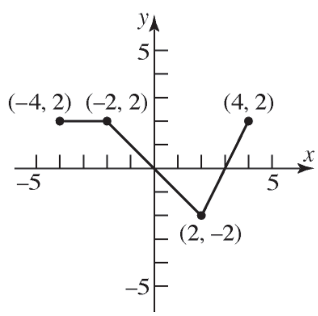
b. $G(x) = f(x+2)$

Shift left 2 units.



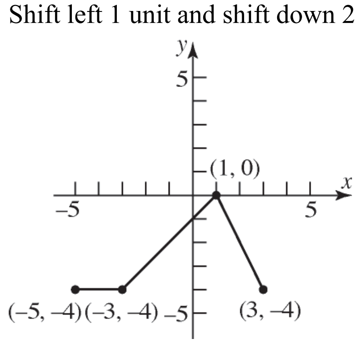
c. $P(x) = -f(x)$

Reflect about the x-axis.



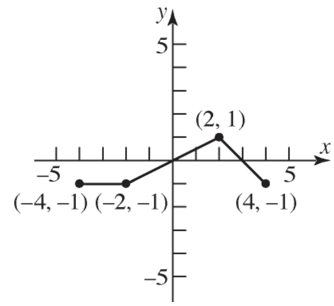
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



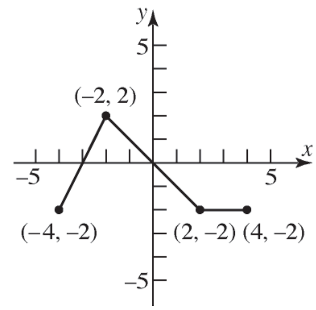
e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



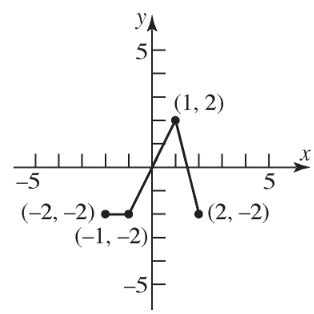
f. $g(x) = f(-x)$

Reflect about the y-axis.



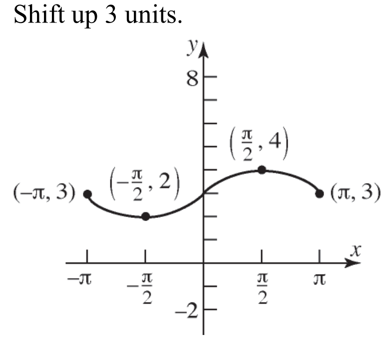
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



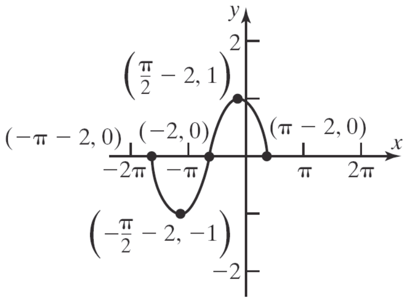
65. a. $F(x) = f(x) + 3$

Shift up 3 units.



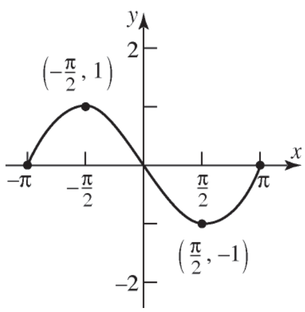
b. $G(x) = f(x+2)$

Shift left 2 units.



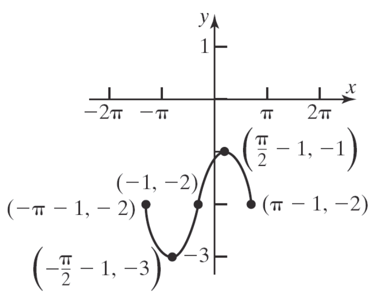
c. $P(x) = -f(x)$

Reflect about the x-axis.



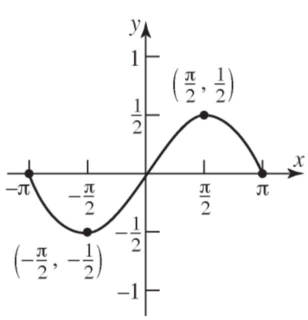
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



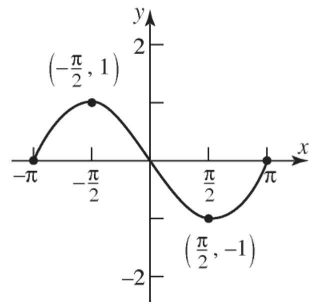
e. $Q(x) = \dfrac{1}{2} f(x)$

Compress vertically by a factor of $\dfrac{1}{2}$.



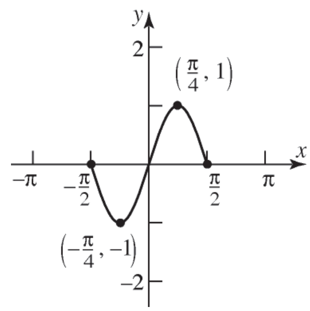
f. $g(x) = f(-x)$

Reflect about the y-axis.



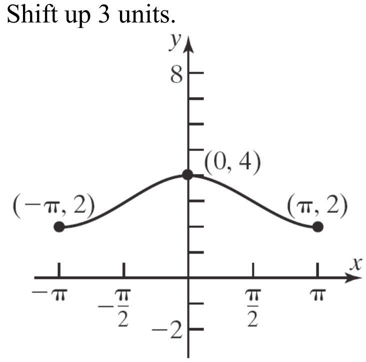
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\dfrac{1}{2}$.



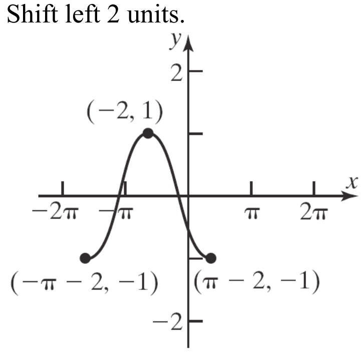
66. a. $F(x) = f(x) + 3$

Shift up 3 units.



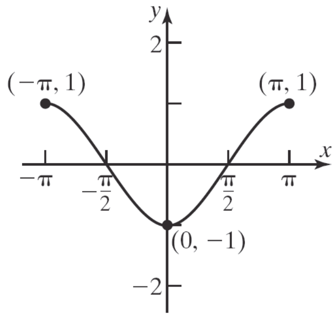
b. $G(x) = f(x+2)$

Shift left 2 units.



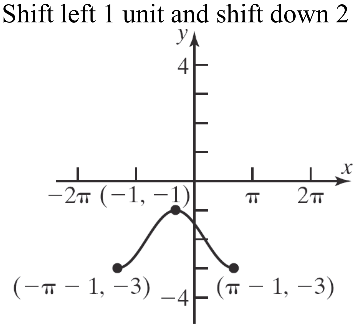
c. $P(x) = -f(x)$

Reflect about the x-axis.



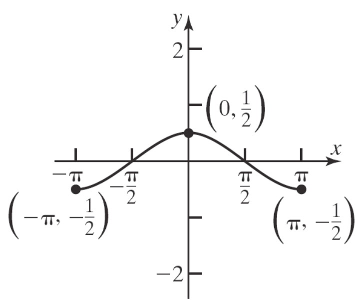
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



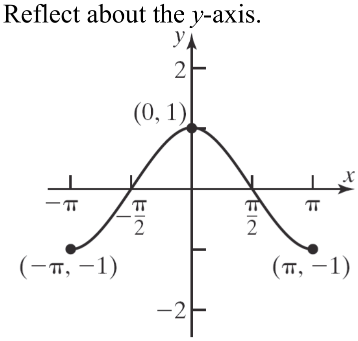
e. $Q(x) = \dfrac{1}{2} f(x)$

Compress vertically by a factor of $\dfrac{1}{2}$.



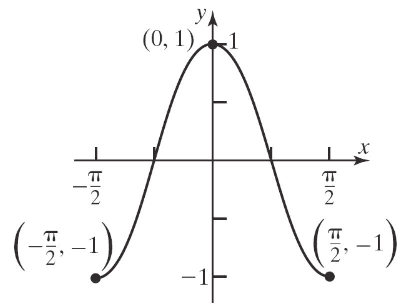
f. $g(x) = f(-x)$

Reflect about the y-axis.



g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.

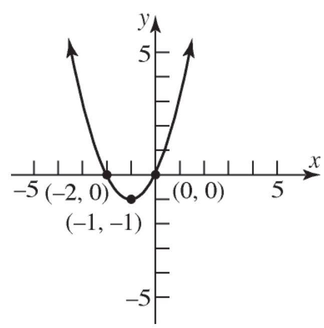


67. $f(x) = x^2 + 2x$

$f(x) = (x^2 + 2x + 1) - 1$

$f(x) = (x+1)^2 - 1$

Using $f(x) = x^2$, shift left 1 unit and shift down 1 unit.

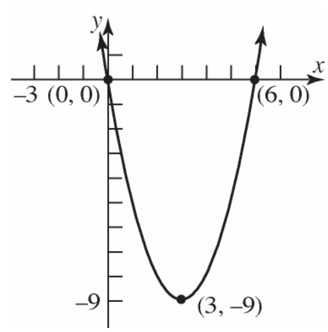


68. $f(x) = x^2 - 6x$

$f(x) = (x^2 - 6x + 9) - 9$

$f(x) = (x-3)^2 - 9$

Using $f(x) = x^2$, shift right 3 units and shift down 9 units.

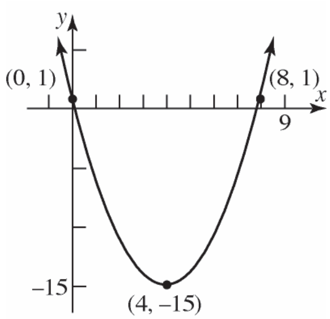


69. $f(x) = x^2 - 8x + 1$

$f(x) = (x^2 - 8x + 16) + 1 - 16$

$f(x) = (x - 4)^2 - 15$

Using $f(x) = x^2$, shift right 4 units and shift down 15 units.

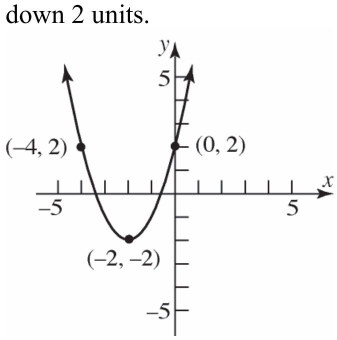


70. $f(x) = x^2 + 4x + 2$

$f(x) = (x^2 + 4x + 4) + 2 - 4$

$f(x) = (x + 2)^2 - 2$

Using $f(x) = x^2$, shift left 2 units and shift down 2 units.



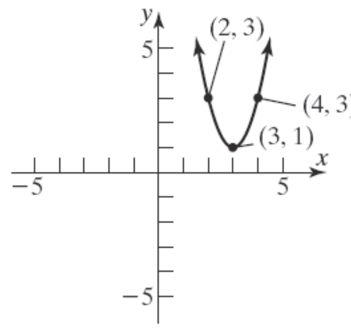
71. $f(x) = 2x^2 - 12x + 19$

$= 2(x^2 - 6x) + 19$

$= 2(x^2 - 6x + 9) + 19 - 18$

$= 2(x - 3)^2 + 1$

Using $f(x) = x^2$, shift right 3 units, vertically stretch by a factor of 2, and then shift up 1 unit.



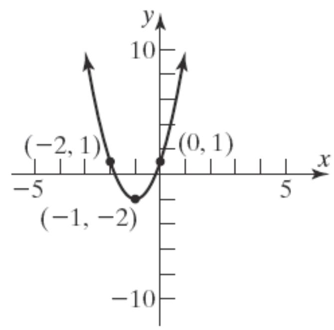
72. $f(x) = 3x^2 + 6x + 1$

$= 3(x^2 + 2x) + 1$

$= 3(x^2 + 2x + 1) + 1 - 3$

$= 3(x + 1)^2 - 2$

Using $f(x) = x^2$, shift left 1 unit, vertically stretch by a factor of 3, and shift down 2 units.



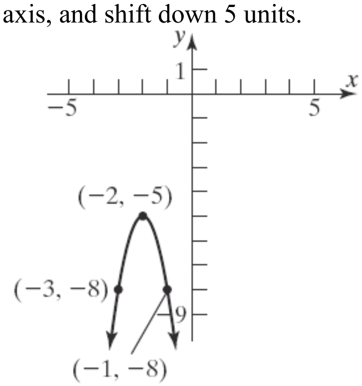
73. $f(x) = -3x^2 - 12x - 17$

$= -3(x^2 + 4x) - 17$

$= -3(x^2 + 4x + 4) - 17 + 12$

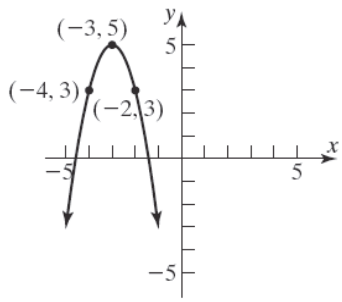
$= -3(x + 2)^2 - 5$

Using $f(x) = x^2$, shift left 2 units, stretch vertically by a factor of 3, reflect about the x-axis, and shift down 5 units.



74. $f(x) = -2x^2 - 12x - 13$
$= -2(x^2 + 6x) - 13$
$= -2(x^2 + 6x + 9) - 13 + 18$
$= -2(x+3)^2 + 5$

Using $f(x) = x^2$, shift left 3 units, stretch vertically by a factor of 2, reflect about the x-axis, and shift up 5 units.



75. **a.** The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the x-intercepts are -7 and 1.

b. The graph of $y = f(x-2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the right. Therefore, the x-intercepts are -3 and 5.

c. The graph of $y = 4f(x)$ is the same as the graph of $y = f(x)$, but stretched vertically by a factor of 4. Therefore, the x-intercepts are still -5 and 3 since the y-coordinate of each is 0.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, the x-intercepts are 5 and -3.

76. **a.** The graph of $y = f(x+4)$ is the same as the graph of $y = f(x)$, but shifted 4 units to the left. Therefore, the x-intercepts are -12 and -3.

b. The graph of $y = f(x-3)$ is the same as the graph of $y = f(x)$, but shifted 3 units to the right. Therefore, the x-intercepts are -5 and 4.

c. The graph of $y = 2f(x)$ is the same as the graph of $y = f(x)$, but stretched vertically by a factor of 2. Therefore, the x-intercepts are still -8 and 1 since the y-coordinate of each is 0.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, the x-intercepts are 8 and -1.

77. **a.** The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is increasing on the interval $[-3, 3]$.

b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is increasing on the interval $[4, 10]$.

c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x-axis. Therefore, we can say that the graph of $y = -f(x)$ must be *decreasing* on the interval $[-1, 5]$.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, we can say that the graph of $y = f(-x)$ must be *decreasing* on the interval $[-5, 1]$.

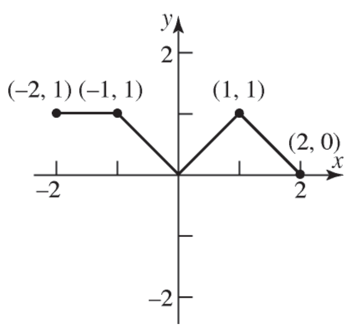
78. **a.** The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is decreasing on the interval $[-4, 5]$.

b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is decreasing on the interval $[3,12]$.

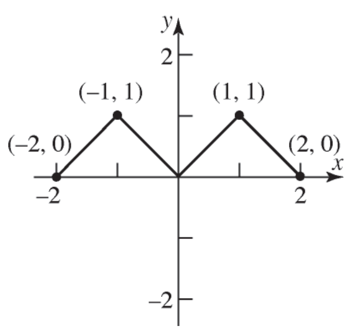
c. The graph of $y=-f(x)$ is the same as the graph of $y=f(x)$, but reflected about the x-axis. Therefore, we can say that the graph of $y=-f(x)$ must be *increasing* on the interval $[-2,7]$.

d. The graph of $y=f(-x)$ is the same as the graph of $y=f(x)$, but reflected about the y-axis. Therefore, we can say that the graph of $y=f(-x)$ must be *increasing* on the interval $[-7,2]$.

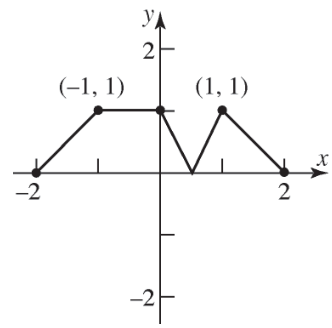
79. a. $y=|f(x)|$



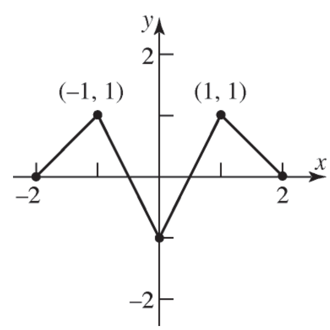
b. $y=f(|x|)$



80. a. To graph $y=|f(x)|$, the part of the graph for f that lies in quadrants III or IV is reflected about the x-axis.



b. To graph $y=f(|x|)$, the part of the graph for f that lies in quadrants II or III is replaced by the reflection of the part in quadrants I and IV reflected about the y-axis.



81. a. The graph of $y=f(x+3)-5$ is the graph of $y=f(x)$ but shifted left 3 units and down 5 units. Thus, the point $(1,3)$ becomes the point $(-2,-2)$.

b. The graph of $y=-2f(x-2)+1$ is the graph of $y=f(x)$ but shifted right 2 units, stretched vertically by a factor of 2, reflected about the x-axis, and shifted up 1 unit. Thus, the point $(1,3)$ becomes the point $(3,-5)$.

c. The graph of $y=f(2x+3)$ is the graph of $y=f(x)$ but shifted left 3 units and horizontally compressed by a factor of 2. Thus, the point $(1,3)$ becomes the point $(-1,3)$.

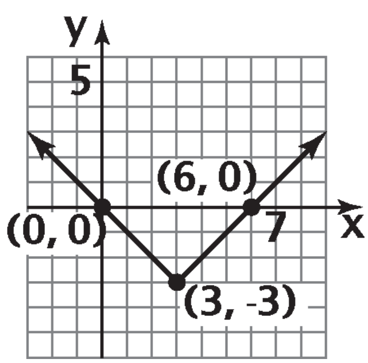
82. a. The graph of $y = g(x+1) - 3$ is the graph of $y = g(x)$ but shifted left 1 unit and down 3 units. Thus, the point $(-3,5)$ becomes the point $(-4,2)$.

b. The graph of $y = -3g(x-4)+3$ is the graph of $y = g(x)$ but shifted right 4 units, stretched vertically by a factor of 3, reflected about the x-axis, and shifted up 3 units. Thus, the point $(-3,5)$ becomes the point $(1,-12)$.

c. The graph of $y = g(3x+9)$ is the graph of $y = f(x)$ but shifted left 9 units and horizontally compressed by a factor of 3. Thus, the point $(-3,5)$ becomes the point $(-4,5)$.

83. a. $f(x) = |x-3| - 3$

Using the graph of $y = |x|$, horizontally shift to the right 3 units $[y = |x-3|]$ and vertically shift downward 3 units $[y = |x-3| - 3]$.



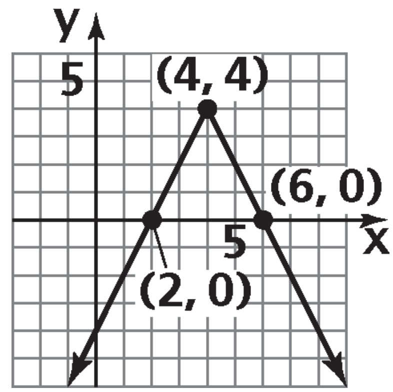
b. $A = \frac{1}{2}bh$

$= \frac{1}{2}(6)(3) = 9$

The area is 9 square units.

84. a. $f(x) = -2|x-4| + 4$

Using the graph of $y = |x|$, horizontally shift to the right 4 units $[y = |x-4|]$, vertically stretch by a factor of 2 and flip on the x-axis $[y = -2|x-4|]$, and vertically shift upward 4 units $[y = -2|x-4| + 4]$.



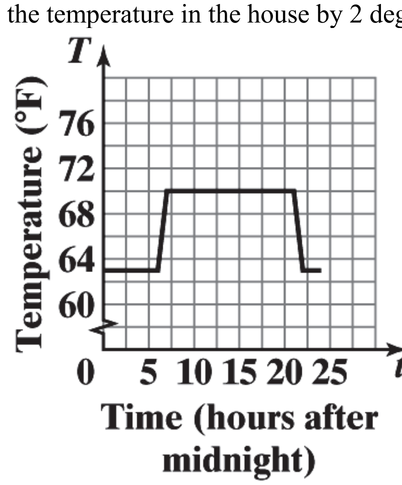
b. $A = \frac{1}{2}bh$

$= \frac{1}{2}(4)(4) = 8$

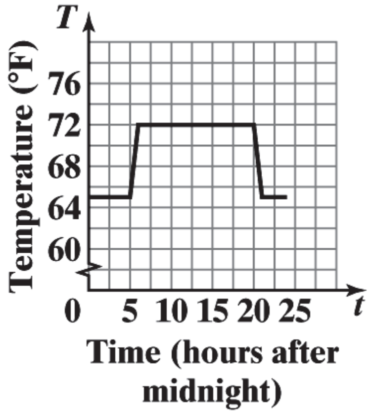
The area is 8 square units.

85. a. From the graph, the thermostat is set at 72°F during the daytime hours. The thermostat appears to be set at 65°F overnight.

b. To graph $y = T(t) - 2$, the graph of $T(t)$ is shifted down 2 units. This change will lower the temperature in the house by 2 degrees.



c. To graph $y = T(t+1)$, the graph of $T(t)$ should be shifted left one unit. This change will cause the program to switch between the daytime temperature and overnight temperature one hour sooner. The home will begin warming up at 5am instead of 6am and will begin cooling down at 8pm instead of 9pm.



86. a. $R(0)=0.15(0)^2-0.03(0)+5.46=5.46$

The estimated worldwide music revenue for 2012 is \$5.46 billion.

$R(3)=0.15(3)^2-0.03(3)+5.46$
$=6.72$

The estimated worldwide music revenue for 2015 is \$6.72 billion.

$R(5)=0.15(5)^2-0.03(5)+5.46$
$=9.06$

The estimated worldwide music revenue for 2017 is \$5.46 billion.

b. $r(x)=R(x-2)$

$=0.15(x-2)^2-0.03(x-2)+5.46$

$=0.15(x^2-4x+4)-0.03(x-2)$
$\quad+5.46$

$=0.15x^2-0.6x+0.6-0.03x$
$\quad+0.06+5.46$

$=0.15x^2-0.63x+6.12$

c. The graph of $r(x)$ is the graph of $R(x)$ shifted 2 units to the left. Thus, $r(x)$ represents the estimated worldwide music revenue, x years after 2010.

$r(2)=0.15(2)^2-0.63(2)+6.12=5.46$

The estimated worldwide music revenue for 2012 is \$5.46 billion.

$r(5)=0.15(5)^2-0.63(5)+6.12$
$=6.72$

The estimated worldwide music revenue for 2015 is \$6.72 billion.

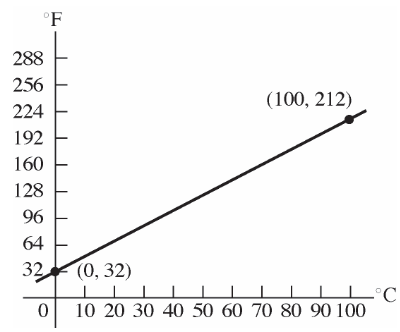
$r(7)=0.15(7)^2-0.63(7)+6.12$
$=9.06$

The estimated worldwide music revenue for 2017 is \$9.06 billion.

d. In $r(x)$, x represents the number of years after 2010 (see the previous part).

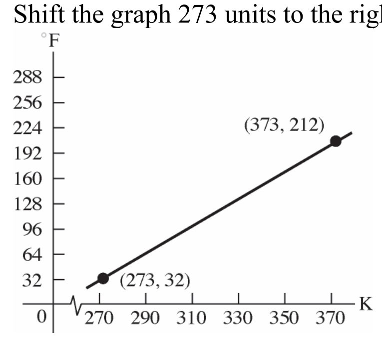
e. Answers will vary. One advantage might be that it is easier to determine what value should be substituted for x when using $r(x)$ instead of $R(x)$ to estimate worldwide music revenue.

87. $F=\dfrac{9}{5}C+32$

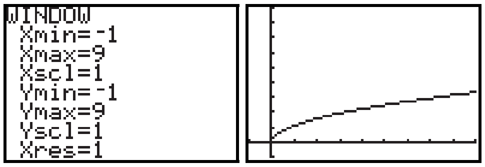


$F=\dfrac{9}{5}(K-273)+32$

Shift the graph 273 units to the right.

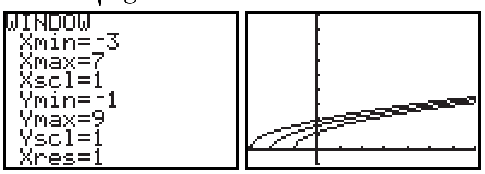


88. **a.** $T = 2\pi\sqrt{\dfrac{l}{g}}$



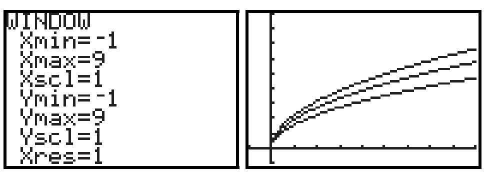
b. $T_1 = 2\pi\sqrt{\dfrac{l+1}{g}}$; $T_2 = 2\pi\sqrt{\dfrac{l+2}{g}}$;

$T_3 = 2\pi\sqrt{\dfrac{l+3}{g}}$



c. As the length of the pendulum increases, the period increases.

d. $T_1 = 2\pi\sqrt{\dfrac{2l}{g}}$; $T_2 = 2\pi\sqrt{\dfrac{3l}{g}}$; $T_3 = 2\pi\sqrt{\dfrac{4l}{g}}$



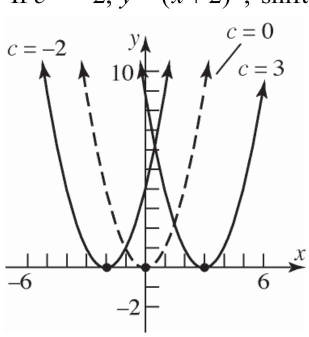
e. If the length of the pendulum is multiplied by k, the period is multiplied by $\sqrt{k}$.

89. $y = (x-c)^2$

If $c = 0$, $y = x^2$.

If $c = 3$, $y = (x-3)^2$; shift right 3 units.

If $c = -2$, $y = (x+2)^2$; shift left 2 units.

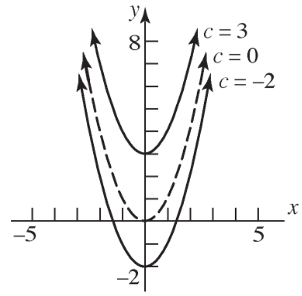


90. $y = x^2 + c$

If $c = 0$, $y = x^2$.

If $c = 3$, $y = x^2 + 3$; shift up 3 units.

If $c = -2$, $y = x^2 - 2$; shift down 2 units.



91. $f(x-5)$ is a shift right 5 units; increasing on $[2,8]$ and $[16,24]$; decreasing of $[8,16]$.

$f(2x-5)$ compresses horizontally by a factor of ½; increasing on $[1,4]$ and $[8,12]$; decreasing on $[4,8]$. $-f(2x-5)$ reflects about the x-axis; increasing on $[4,8]$; decreasing on $[1,4]$ and $[8,12]$. $-3f(2x-5)$ stretches vertically by a factor or 3 but does not affect increasing/decreasing. Therefore $-3f(2x-5)$ is increasing on $[4,8]$.

92. Write the general normal density as

$$f(x) = \frac{1}{\sigma}\cdot\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{\left(\frac{x-\mu}{\sigma}\right)^2}{2}\right].$$ Starting

with the standard normal density,

$f(x) = \dfrac{1}{\sqrt{2\pi}}\exp\left[-\dfrac{x^2}{2}\right]$, stretch/compress

horizontally by a factor of σ to get

$$f(x) = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{\left(\frac{x}{\sigma}\right)^2}{2}\right]$$

$\left[\text{multiply all the } x\text{-coordinates by } \dfrac{1}{\sigma}\right]$, then shift the graph horizontally $|\mu|$ units (left if $\mu < 0$ and right if $\mu > 0$) to get

$f(x)=\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{\left(\frac{x-\mu}{\sigma}\right)^2}{2}\right]$. Then stretch/compress

vertically by a factor of $\frac{1}{\sigma}$ to get

$f(x)=\frac{1}{\sigma}\cdot\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{\left(\frac{x-\mu}{\sigma}\right)^2}{2}\right]$

[multiply all the y-coordinates by $\frac{1}{\sigma}$].

1. Stretch/compress horizontally by a factor or σ (stretch if $\sigma>1$)
2. Shift horizontally $|\mu|$ units (left if $\mu<0$ and right if $\mu>0$).
3. Stretch/compress vertically by a factor of $\frac{1}{\sigma}$ (compress if $\sigma>1$)

93. The graph of $y=4f(x)$ is a vertical stretch of the graph of *f* by a factor of 4, while the graph of $y=f(4x)$ is a horizontal compression of the graph of *f* by a factor of $\frac{1}{4}$.

94. The graph of $y=f(x)-2$ will shift the graph of $y=f(x)$ down by 2 units. The graph of $y=f(x-2)$ will shift the graph of $y=f(x)$ to the right by 2 units.

95. The graph of $y=\sqrt{-x}$ is the graph of $y=\sqrt{x}$ but reflected about the y-axis. Therefore, our region is simply rotated about the y-axis and does not change shape. Instead of the region being bounded on the right by $x=4$, it is bounded on the left by $x=-4$. Thus, the area of the second region would also be $\frac{16}{3}$ square units.

96. The range of $f(x)=x^2$ is $[0,\infty)$. The graph of $g(x)=f(x)+k$ is the graph of f shifted up k units if k > 0 and shifted down $|k|$ units if k < 0, so the range of g is $[k,\infty)$.

97. The domain of $g(x)=\sqrt{x}$ is $[0,\infty)$. The graph of $g(x-k)$ is the graph of g shifted k units to the right, so the domaine of g is $[k,\infty)$.

Section 1.7

1. $f\left(\frac{x+2}{4}\right)=4\left(\frac{x+2}{4}\right)-2=x+2-2=x$.

2. The function $f(x)=x^2$ is increasing on the interval $[0,\infty)$. It is decreasing on the interval $(-\infty,0]$.

3. The function is not defined when $x^2+3x-18=0$.
Solve: $x^2+3x-18=0$
$(x+6)(x-3)=0$
$x=-6$ or $x=3$
The domain is $\{x\mid x\neq-6,\ x\neq3\}$.

4.
$$\begin{aligned}x-3y&=-8\\-3y&=-x-8\\y&=\frac{-x-8}{-3}\\y&=\frac{x+8}{3}=\frac{1}{3}x+\frac{8}{3}\end{aligned}$$

5. $f(x_1)\neq f(x_2)$

6. one-to-one

7. 3

8. $y=x$

9. $[4,\infty)$

10. True

11. a

12. d

13. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

14. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

15. The function is not one-to-one because there are two different inputs, 20 Hours and 50 Hours, that correspond to the same output, \$200.

16. The function is not one-to-one because there are two different inputs, John and Chuck, that correspond to the same output, Phoebe.

17. The function is not one-to-one because there are two distinct inputs, 2 and -3, that correspond to the same output.

18. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

19. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

20. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

21. The function *f* is one-to-one because every horizontal line intersects the graph at exactly one point.

22. The function *f* is one-to-one because every horizontal line intersects the graph through at most one point.

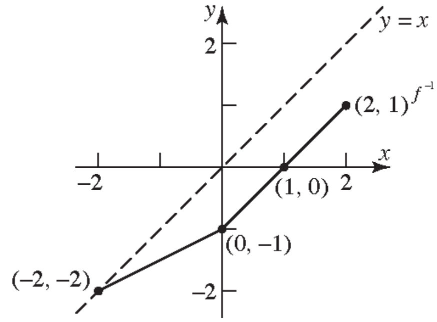
23. The function *f* is not one-to-one because there are horizontal lines (for example, $y = 1$) that intersect the graph at more than one point.

24. The function *f* is not one-to-one because there are horizontal lines (for example, $y = 1$) that intersect the graph at more than one point.

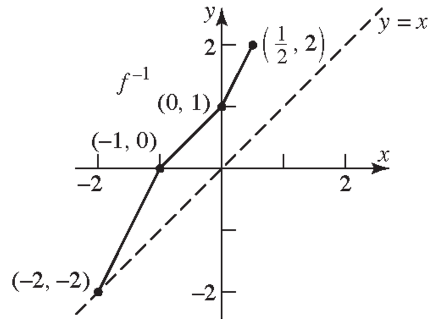
25. The function *f* is one-to-one because every horizontal line intersects the graph through at most one point.

26. The function *f* is not one-to-one because the horizontal line $y = 2$ intersects the graph at more than one point.

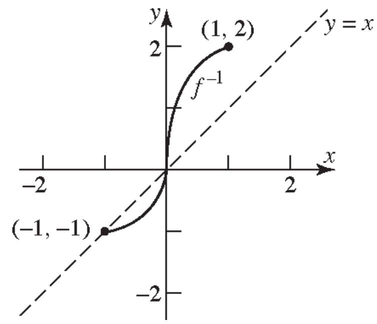
27. Graphing the inverse:



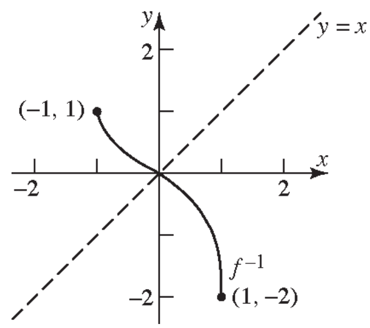
28. Graphing the inverse:



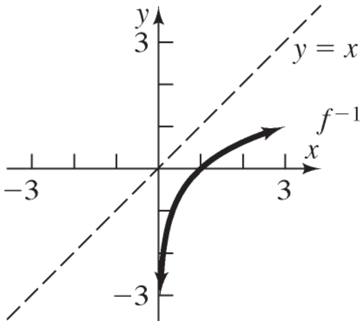
29. Graphing the inverse:



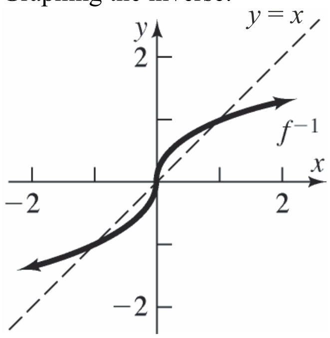
30. Graphing the inverse:



31. Graphing the inverse:



32. Graphing the inverse:



33. $f(x)=3x+4; \qquad g(x)=\frac{1}{3}(x-4)$

$$\begin{aligned} f(g(x)) &= f\left(\frac{1}{3}(x-4)\right) \\ &= 3\left(\frac{1}{3}(x-4)\right)+4 \\ &= (x-4)+4 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(3x+4) \\ &= \frac{1}{3}((3x+4)-4) \\ &= \frac{1}{3}(3x)=x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

34. $f(x)=3-2x; \qquad g(x)=-\frac{1}{2}(x-3)$

$$\begin{aligned} f(g(x)) &= f\left(-\frac{1}{2}(x-3)\right) \\ &= 3-2\left(-\frac{1}{2}(x-3)\right) \\ &= 3+(x-3) \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(3-2x) \\ &= -\frac{1}{2}((3-2x)-3) \\ &= -\frac{1}{2}(-2x) \\ &= x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

35. $f(x)=4x-8; \qquad g(x)=\frac{x}{4}+2$

$$\begin{aligned} f(g(x)) &= f\left(\frac{x}{4}+2\right) \\ &= 4\left(\frac{x}{4}+2\right)-8 \\ &= x+8-8 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(4x-8) \\ &= \frac{4x-8}{4}+2 \\ &= x-2+2 \\ &= x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

36. $f(x)=2x+6; \qquad g(x)=\frac{1}{2}x-3$

$$\begin{aligned} f(g(x)) &= f\left(\frac{1}{2}x-3\right) \\ &= 2\left(\frac{1}{2}x-3\right)+6=x-6+6 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(2x+6) \\ &= \frac{1}{2}(2x+6)-3=x+3-3 \\ &= x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

37. $f(x)=x^3-8; \qquad g(x)=\sqrt[3]{x+8}$

$$\begin{aligned} f(g(x)) &= f\left(\sqrt[3]{x+8}\right) \\ &= \left(\sqrt[3]{x+8}\right)^3-8 \\ &= x+8-8 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(x^3-8) \\ &= \sqrt[3]{(x^3-8)+8} \\ &= \sqrt[3]{x^3} \\ &= x \end{aligned}$$

Thus, f and g are inverses of each other.

38. $f(x)=(x-2)^2,\ x \ge 2; \qquad g(x)=\sqrt{x}+2$

$$\begin{aligned} f(g(x)) &= f\left(\sqrt{x}+2\right) \\ &= \left(\sqrt{x}+2-2\right)^2 \\ &= \left(\sqrt{x}\right)^2 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g\left((x-2)^2\right) \\ &= \sqrt{(x-2)^2}+2 \\ &= x-2+2 \\ &= x \end{aligned}$$

Thus, f and g are inverses of each other.

39. $f(x)=\dfrac{1}{x}; \qquad g(x)=\dfrac{1}{x}$

$$f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1\cdot\frac{x}{1} = x$$

$$g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1\cdot\frac{x}{1} = x$$

Thus, f and g are inverses of each other.

40. $f(x)=x; \qquad g(x)=x$

$$f(g(x)) = f(x) = x$$

$$g(f(x)) = g(x) = x$$

Thus, f and g are inverses of each other.

41. $f(x)=\dfrac{2x+3}{x+4}; \quad g(x)=\dfrac{4x-3}{2-x}$

$$\begin{aligned} f(g(x)) &= f\left(\frac{4x-3}{2-x}\right),\ x \ne 2 \\ &= \frac{2\left(\frac{4x-3}{2-x}\right)+3}{\frac{4x-3}{2-x}+4} \\ &= \frac{\left(2\left(\frac{4x-3}{2-x}\right)+3\right)(2-x)}{\left(\frac{4x-3}{2-x}+4\right)(2-x)} \\ &= \frac{2(4x-3)+3(2-x)}{4x-3+4(2-x)} \\ &= \frac{8x-6+6-3x}{4x-3+8-4x} \\ &= \frac{5x}{5} \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g\left(\frac{2x+3}{x+4}\right),\quad x \ne -4 \\ &= \frac{4\left(\frac{2x+3}{x+4}\right)-3}{2-\frac{2x+3}{x+4}} \\ &= \frac{\left(4\left(\frac{2x+3}{x+4}\right)-3\right)(x+4)}{\left(2-\frac{2x+3}{x+4}\right)(x+4)} \\ &= \frac{4(2x+3)-3(x+4)}{2(x+4)-(2x+3)} \\ &= \frac{8x+12-3x-12}{2x+8-2x-3} \\ &= \frac{5x}{5} \\ &= x \end{aligned}$$

Thus, f and g are inverses of each other.

42. $f(x)=\dfrac{x-5}{2x+3}$; $g(x)=\dfrac{3x+5}{1-2x}$

$$\begin{aligned}
f\left(g(x)\right) &= f\left(\frac{3x+5}{1-2x}\right),\ x\neq\frac{1}{2}\\
&= \frac{\frac{3x+5}{1-2x}-5}{2\left(\frac{3x+5}{1-2x}\right)+3}\\
&= \frac{\left(\frac{3x+5}{1-2x}-5\right)(1-2x)}{\left(2\left(\frac{3x+5}{1-2x}\right)+3\right)(1-2x)}\\
&= \frac{3x+5-5(1-2x)}{2(3x+5)+3(1-2x)}\\
&= \frac{3x+5-5+10x}{6x+10+3-6x}\\
&= \frac{13x}{13}\\
&= x
\end{aligned}$$

$$\begin{aligned}
g\left(f(x)\right) &= g\left(\frac{x-5}{2x+3}\right),\ x\neq-\frac{3}{2}\\
&= \frac{3\left(\frac{x-5}{2x+3}\right)+5}{1-2\left(\frac{x-5}{2x+3}\right)}\\
&= \frac{\left(3\left(\frac{x-5}{2x+3}\right)+5\right)(2x+3)}{\left(1-2\left(\frac{x-5}{2x+3}\right)\right)(2x+3)}\\
&= \frac{3(x-5)+5(2x+3)}{1(2x+3)-2(x-5)}\\
&= \frac{3x-15+10x+15}{2x+3-2x+10}\\
&= \frac{13x}{13}\\
&= x
\end{aligned}$$

Thus, *f* and *g* are inverses of each other.

43. **a**. $f(x)=3x$

$y=3x$

$x=3y$ Inverse

$y=\dfrac{x}{3}$

$f^{-1}(x)=\dfrac{1}{3}x$

Verifying: $f\left(f^{-1}(x)\right)=f\left(\frac{1}{3}x\right)=3\left(\frac{1}{3}x\right)=x$

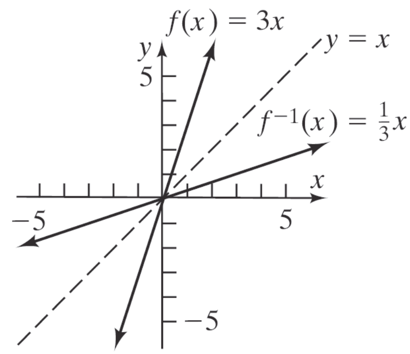
$f^{-1}\left(f(x)\right)=f^{-1}(3x)=\frac{1}{3}(3x)=x$

b.

domain of f = range of f^{-1} = all real numbers c/

range of f = domain of f^{-1} = all real numbers

c.



44. **a.** $f(x)=-4x$

$y=-4x$

$x=-4y$ Inverse

$y=\dfrac{x}{-4}$

$f^{-1}(x)=-\dfrac{1}{4}x$

Verifying:

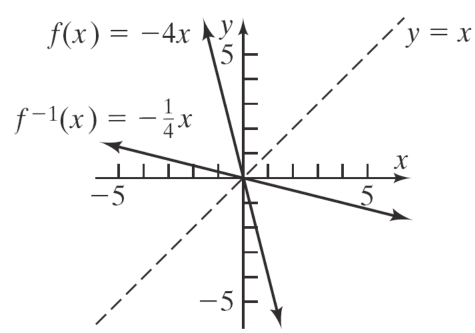
$f\left(f^{-1}(x)\right)=f\left(-\frac{1}{4}x\right)=-4\left(-\frac{1}{4}x\right)=x$

$f^{-1}\left(f(x)\right)=f^{-1}(-4x)=-\frac{1}{4}(-4x)=x$

b. domain of f = range of f^{-1} = all real numbers

range of f = domain of f^{-1} = all real numbers

c.



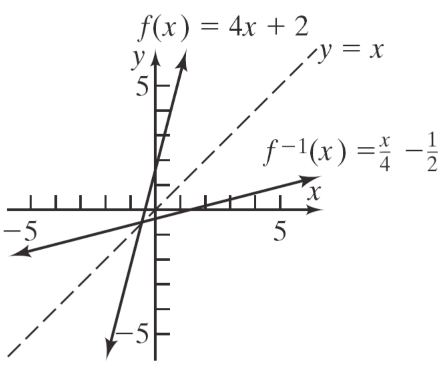
45. a.
$$f(x) = 4x + 2$$
$$y = 4x + 2$$
$$x = 4y + 2 \quad \text{Inverse}$$
$$4y = x - 2$$
$$y = \frac{x-2}{4}$$
$$y = \frac{x}{4} - \frac{1}{2}$$
$$f^{-1}(x) = \frac{x}{4} - \frac{1}{2}$$

Verifying:
$$f\left(f^{-1}(x)\right) = f\left(\frac{x}{4} - \frac{1}{2}\right) = 4\left(\frac{x}{4} - \frac{1}{2}\right) + 2$$
$$= x - 2 + 2 = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}(4x+2) = \frac{4x+2}{4} - \frac{1}{2}$$
$$= x + \frac{1}{2} - \frac{1}{2} = x$$

b. domain of f = range of f^{-1} = all real numbers

range of f = domain of f^{-1} = all real numbers

c.



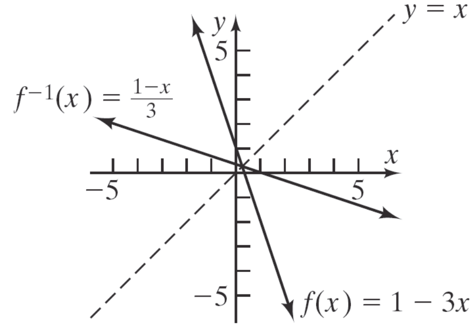
46. a.
$$f(x) = 1 - 3x$$
$$y = 1 - 3x$$
$$x = 1 - 3y \quad \text{Inverse}$$
$$3y = 1 - x$$
$$y = \frac{1-x}{3}$$
$$f^{-1}(x) = \frac{1-x}{3}$$

Verifying:
$$f\left(f^{-1}(x)\right) = f\left(\frac{1-x}{3}\right) = 1 - 3\left(\frac{1-x}{3}\right)$$
$$= 1 - (1-x) = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}(1-3x) = \frac{1-(1-3x)}{3} = \frac{3x}{3} = x$$

b. domain of f = range of f^{-1} = all real numbers

range of f = domain of f^{-1} = all real numbers

c.



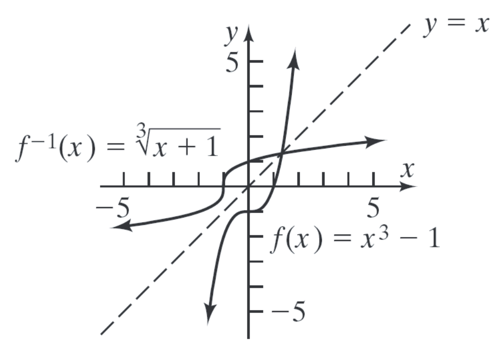
47. a.
$$f(x) = x^3 - 1$$
$$y = x^3 - 1$$
$$x = y^3 - 1 \quad \text{Inverse}$$
$$y^3 = x + 1$$
$$y = \sqrt[3]{x+1}$$
$$f^{-1}(x) = \sqrt[3]{x+1}$$

Verifying: $f\left(f^{-1}(x)\right) = f\left(\sqrt[3]{x+1}\right) = \left(\sqrt[3]{x+1}\right)^3 - 1$
$$= x + 1 - 1 = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(x^3 - 1\right) = \sqrt[3]{\left(x^3-1\right)+1}$$
$$= \sqrt[3]{x^3} = x$$

b. domain of f = range of f^{-1} = all real numbers

range of f = domain of f^{-1} = all real numbers

c.



48. a. $f(x)=x^3+1$

$y=x^3+1$

$x=y^3+1$ Inverse

$y^3=x-1$

$y=\sqrt[3]{x-1}$

$f^{-1}(x)=\sqrt[3]{x-1}$

Verifying:

$f\left(f^{-1}(x)\right)=f\left(\sqrt[3]{x-1}\right)=\left(\sqrt[3]{x-1}\right)^3+1$

$=x-1+1=x$

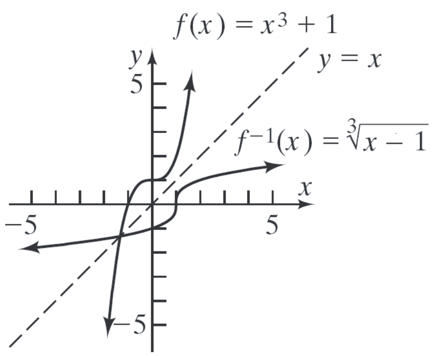
$f^{-1}\left(f(x)\right)=f^{-1}\left(x^3+1\right)=\sqrt[3]{\left(x^3+1\right)-1}$

$=\sqrt[3]{x^3}=x$

b. domain of f = range of f^{-1} = all real numbers

range of f = domain of f^{-1} = all real numbers

c.



49. a. $f(x)=x^2+4,\ x\ge 0$

$y=x^2+4,\ x\ge 0$

$x=y^2+4,\ y\ge 0$ Inverse

$y^2=x-4,\quad x\ge 4$

$y=\sqrt{x-4},\ x\ge 4$

$f^{-1}(x)=\sqrt{x-4},\ x\ge 4$

Verifying: $f\left(f^{-1}(x)\right)=f\left(\sqrt{x-4}\right)$

$=\left(\sqrt{x-4}\right)^2+4$

$=x-4+4=x$

$f^{-1}\left(f(x)\right)=f^{-1}\left(x^2+4\right)$

$=\sqrt{\left(x^2+4\right)-4}$

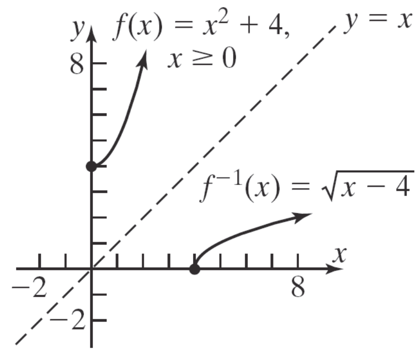
$=\sqrt{x^2}=|x|$

$=x,\ x\ge 0$

b. domain of f = range of $f^{-1}=\{x\,|\,x\ge 0\}$

range of f = domain of $f^{-1}=\{x\,|\,x\ge 4\}$

c.



50. a. $f(x)=x^2+9,\ x\ge 0$

$y=x^2+9,\ x\ge 0$

$x=y^2+9,\ y\ge 0$ Inverse

$y^2=x-9,\quad x\ge 9$

$y=\sqrt{x-9},\ x\ge 9$

$f^{-1}(x)=\sqrt{x-9},\ x\ge 9$

Verifying: $f\left(f^{-1}(x)\right)=f\left(\sqrt{x-9}\right)$

$=\left(\sqrt{x-9}\right)^2+9$

$=x-9+9$

$=x$

$f^{-1}\left(f(x)\right)=f^{-1}\left(x^2+9\right)$

$=\sqrt{\left(x^2+9\right)-9}$

$=\sqrt{x^2}$

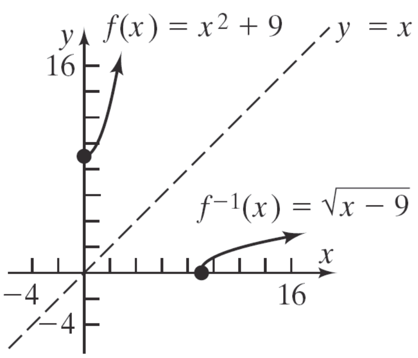
$=|x|$

$=x,\ x\ge 0$

b. domain of f = range of $f^{-1}=\{x\,|\,x\ge 0\}$

range of f = domain of $f^{-1}=\{x\,|\,x\ge 9\}$

c.



51. a. $f(x) = \dfrac{4}{x}$

$$y = \frac{4}{x}$$

$$x = \frac{4}{y} \quad \text{Inverse}$$

$$xy = 4$$

$$y = \frac{4}{x}$$

$$f^{-1}(x) = \frac{4}{x}$$

Verifying:

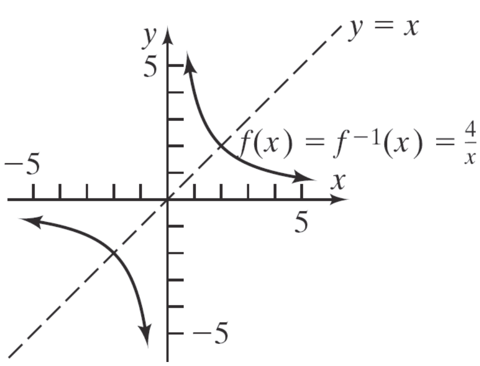
$$f\left(f^{-1}(x)\right) = f\left(\frac{4}{x}\right) = \frac{4}{\frac{4}{x}} = 4 \cdot \left(\frac{x}{4}\right) = x$$

$$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{4}{x}\right) = \frac{4}{\frac{4}{x}} = 4 \cdot \left(\frac{x}{4}\right) = x$$

b. domain of f = range of $f^{-1} = \{x \mid x \neq 0\}$

range of f = domain of $f^{-1} = \{x \mid x \neq 0\}$

c.



52. a. $f(x) = -\dfrac{3}{x}$

$$y = -\frac{3}{x}$$

$$x = -\frac{3}{y} \quad \text{Inverse}$$

$$xy = -3$$

$$y = -\frac{3}{x}$$

$$f^{-1}(x) = -\frac{3}{x}$$

Verifying:

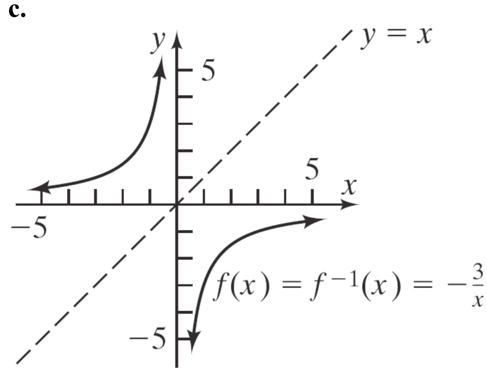
$$f\left(f^{-1}(x)\right) = f\left(-\frac{3}{x}\right) = -\frac{3}{-\frac{3}{x}} = -3 \cdot \left(-\frac{x}{3}\right) = x$$

$$f^{-1}\left(f(x)\right) = f^{-1}\left(-\frac{3}{x}\right) = -\frac{3}{-\frac{3}{x}} = -3 \cdot \left(-\frac{x}{3}\right) = x$$

b. domain of f = range of $f^{-1} = \{x \mid x \neq 0\}$

range of f = domain of $f^{-1} = \{x \mid x \neq 0\}$

c.



53. a. $f(x) = \dfrac{1}{x-2}$

$$y = \frac{1}{x-2}$$

$$x = \frac{1}{y-2} \quad \text{Inverse}$$

$$xy - 2x = 1$$

$$xy = 2x + 1$$

$$y = \frac{2x+1}{x}$$

$$f^{-1}(x) = \frac{2x+1}{x}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{2x+1}{x}\right)=\frac{1}{\frac{2x+1}{x}-2}$$

$$=\frac{1\cdot x}{\left(\frac{2x+1}{x}-2\right)x}=\frac{x}{2x+1-2x}$$

$$=\frac{x}{1}=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{1}{x-2}\right)=\frac{2\left(\frac{1}{x-2}\right)+1}{\frac{1}{x-2}}$$

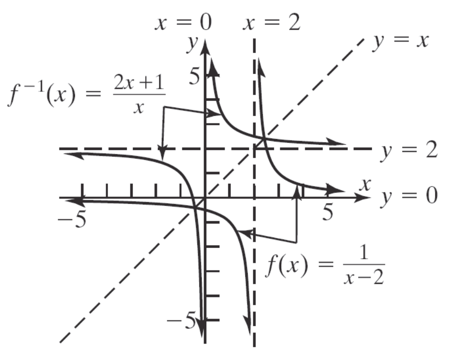
$$=\frac{\left(2\left(\frac{1}{x-2}\right)+1\right)(x-2)}{\left(\frac{1}{x-2}\right)(x-2)}$$

$$=\frac{2+(x-2)}{1}=\frac{x}{1}=x$$

b. domain of f = range of $f^{-1}=\{x\mid x\neq 2\}$

range of f = domain of $f^{-1}=\{x\mid x\neq 0\}$

c.



54. a. $f(x)=\dfrac{4}{x+2}$

$$y=\frac{4}{x+2}$$

$$x=\frac{4}{y+2}\quad \text{Inverse}$$

$$x(y+2)=4$$

$$xy+2x=4$$

$$xy=4-2x$$

$$y=\frac{4-2x}{x}$$

$$f^{-1}(x)=\frac{4-2x}{x}\quad\text{or}\quad f^{-1}(x)=\frac{4}{x}-2$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{4-2x}{x}\right)=\frac{4}{\frac{4-2x}{x}+2}$$

$$=\frac{4\cdot x}{\left(\frac{4-2x}{x}+2\right)x}=\frac{4x}{4-2x+2x}=\frac{4x}{4}=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{4}{x+2}\right)=\frac{4-2\left(\frac{4}{x+2}\right)}{\frac{4}{x+2}}$$

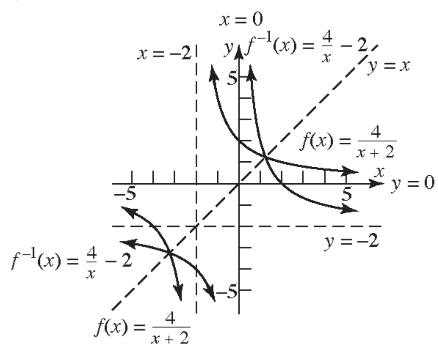
$$=\frac{\left(4-2\left(\frac{4}{x+2}\right)\right)(x+2)}{\left(\frac{4}{x+2}\right)(x+2)}=\frac{4(x+2)-2(4)}{4}$$

$$=\frac{4x+8-8}{4}=\frac{4x}{4}=x$$

b. domain of f = range of $f^{-1}=\{x\mid x\neq -2\}$

range of f = domain of $f^{-1}=\{x\mid x\neq 0\}$

c.



55. **a.** $f(x)=\dfrac{2}{3+x}$

$$y=\frac{2}{3+x}$$

$$x=\frac{2}{3+y}\quad\text{Inverse}$$

$$x(3+y)=2$$

$$3x+xy=2$$

$$xy=2-3x$$

$$y=\frac{2-3x}{x}$$

$$f^{-1}(x)=\frac{2-3x}{x}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{2-3x}{x}\right)=\frac{2}{3+\frac{2-3x}{x}}$$

$$=\frac{2\cdot x}{\left(3+\frac{2-3x}{x}\right)x}=\frac{2x}{3x+2-3x}$$

$$=\frac{2x}{2}=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{2}{3+x}\right)=\frac{2-3\left(\frac{2}{3+x}\right)}{\frac{2}{3+x}}$$

$$=\frac{\left(2-3\left(\frac{2}{3+x}\right)\right)(3+x)}{\left(\frac{2}{3+x}\right)(3+x)}$$

$$=\frac{2(3+x)-3(2)}{2}=\frac{6+2x-6}{2}$$

$$=\frac{2x}{2}=x$$

b. domain of f = range of $f^{-1}=\{x\mid x\neq -3\}$

range of f = domain of $f^{-1}=\{x\mid x\neq 0\}$

56. **a**. $f(x)=\dfrac{4}{2-x}$

$$y=\frac{4}{2-x}$$

$$x=\frac{4}{2-y}\quad \text{Inverse}$$

$$x(2-y)=4$$

$$2x-xy=4$$

$$xy=2x-4$$

$$y=\frac{2x-4}{x}$$

$$y=2-\frac{4}{x}$$

$$f^{-1}(x)=2-\frac{4}{x}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(2-\frac{4}{x}\right)=\frac{4}{2-\left(2-\frac{4}{x}\right)}$$

$$=\frac{4}{2-2+\frac{4}{x}}=\frac{4}{\frac{4}{x}}=4\cdot\frac{x}{4}=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{4}{2-x}\right)=2-\frac{4}{\left(\frac{4}{2-x}\right)}$$

$$=2-4\left(\frac{2-x}{4}\right)=2-\left(2-x\right)$$

$$=2-2+x=x$$

b. domain of f = range of $f^{-1}=\{x\mid x\neq 2\}$

range of f = domain of $f^{-1}=\{x\mid x\neq 0\}$

57. **a**. $f(x)=\dfrac{3x}{x+2}$

$$y=\frac{3x}{x+2}$$

$$x=\frac{3y}{y+2}\quad \text{Inverse}$$

$$x(y+2)=3y$$

$$xy+2x=3y$$

$$xy-3y=-2x$$

$$y(x-3)=-2x$$

$$y=\frac{-2x}{x-3}$$

$$f^{-1}(x)=\frac{-2x}{x-3}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{-2x}{x-3}\right)$$

$$=\frac{3\left(\frac{-2x}{x-3}\right)}{\frac{-2x}{x-3}+2}=\frac{\left(3\left(\frac{-2x}{x-3}\right)\right)(x-3)}{\left(\frac{-2x}{x-3}+2\right)(x-3)}$$

$$=\frac{-6x}{-2x+2x-6}=\frac{-6x}{-6}=x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{3x}{x+2}\right)$$
$$= \frac{-2\left(\frac{3x}{x+2}\right)}{\frac{3x}{x+2}-3} = \frac{\left(-2\left(\frac{3x}{x+2}\right)\right)(x+2)}{\left(\frac{3x}{x+2}-3\right)(x+2)}$$
$$= \frac{-6x}{3x-3x-6} = \frac{-6x}{-6} = x$$

b. domain of f = range of $f^{-1} = \{x \mid x \neq -2\}$

range of f = domain of $f^{-1} = \{x \mid x \neq 3\}$

58. a. $f(x) = -\frac{2x}{x-1}$

$$y = -\frac{2x}{x-1}$$
$$x = -\frac{2y}{y-1} \quad \text{Inverse}$$
$$x(y-1) = -2y$$
$$xy + -x = -2y$$
$$xy + 2y = x$$
$$y(x+2) = x$$
$$y = \frac{x}{x+2}$$
$$f^{-1}(x) = \frac{x}{x+2}$$

Verifying:

$$f(f^{-1}(x)) = f\left(\frac{x}{x+2}\right) = -\frac{2\left(\frac{x}{x+2}\right)}{\frac{x}{x+2}-1}$$
$$= -\frac{\left(2\left(\frac{x}{x+2}\right)\right)(x+2)}{\left(\frac{x}{x+2}-1\right)(x+2)}$$
$$= \frac{-2x}{x-(x+2)}$$
$$= \frac{-2x}{-2}$$
$$= x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{-2x}{x-1}\right) = \frac{-\frac{2x}{x-1}}{-\frac{2x}{x-1}+2}$$
$$= \frac{\left(-\frac{2x}{x-1}\right)(x-1)}{\left(-\frac{2x}{x-1}+2\right)(x-1)}$$
$$= \frac{-2x}{-2x+2x-2}$$
$$= \frac{-2x}{-2}$$
$$= x$$

b. domain of f = range of $f^{-1} = \{x \mid x \neq 1\}$

range of f = domain of $f^{-1} = \{x \mid x \neq -2\}$

59. a. $f(x) = \frac{2x}{3x-1}$

$$y = \frac{2x}{3x-1}$$
$$x = \frac{2y}{3y-1} \quad \text{Inverse}$$
$$3xy - x = 2y$$
$$3xy - 2y = x$$
$$y(3x-2) = x$$
$$y = \frac{x}{3x-2}$$
$$f^{-1}(x) = \frac{x}{3x-2}$$

Verifying:

$$f(f^{-1}(x)) = f\left(\frac{x}{3x-2}\right) = \frac{2\left(\frac{x}{3x-2}\right)}{3\left(\frac{x}{3x-2}\right)-1}$$
$$= \frac{\left(2\left(\frac{x}{3x-2}\right)\right)(3x-2)}{\left(3\left(\frac{x}{3x-2}\right)-1\right)(3x-2)}$$
$$= \frac{2x}{3x-(3x-2)} = \frac{2x}{2} = x$$

$$f^{-1}\left(f(x)\right)=f\left(\frac{2x}{3x-1}\right)=\frac{\frac{2x}{3x-1}}{3\left(\frac{2x}{3x-1}\right)-2}$$
$$=\frac{\left(\frac{2x}{3x-1}\right)(3x-1)}{\left(3\left(\frac{2x}{3x-1}\right)-2\right)(3x-1)}$$
$$=\frac{2x}{3(2x)-2(3x-1)}$$
$$=\frac{2x}{6x-6x+2}=\frac{2x}{2}=x$$

b. domain of f = range of $f^{-1}=\left\{x\mid x\neq\frac{1}{3}\right\}$

range of f = domain of $f^{-1}=\left\{x\mid x\neq\frac{2}{3}\right\}$

60. a. $f(x)=-\frac{3x+1}{x}$

$$y=-\frac{3x+1}{x}$$
$$x=-\frac{3y+1}{y}\quad \text{Inverse}$$
$$xy=-(3y+1)$$
$$xy=-3y-1$$
$$xy+3y=-1$$
$$y(x+3)=-1$$
$$y=\frac{-1}{x+3}$$
$$f^{-1}(x)=\frac{-1}{x+3}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{-1}{x+3}\right)$$
$$=\frac{3\left(-\frac{1}{x+3}\right)+1}{-\left(-\frac{1}{x+3}\right)}=\frac{\frac{-3}{x+3}+1}{\frac{1}{x+3}}$$
$$=\left(\frac{-3}{x+3}+1\right)\cdot\frac{x+3}{1}$$
$$=-3+(x+3)$$
$$=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{3x+1}{-x}\right)$$
$$=\frac{-1}{\left(\frac{3x+1}{-x}\right)+3}=\frac{1}{\frac{3x+1}{x}-3}$$
$$=\frac{1\cdot x}{\left(\frac{3x+1}{x}-3\right)x}=\frac{x}{3x+1-3x}$$
$$=\frac{x}{1}=x$$

b. domain of f = range of $f^{-1}=\{x\mid x\neq 0\}$

range of f = domain of $f^{-1}=\{x\mid x\neq -3\}$

61. a. $f(x)=\frac{3x+4}{2x-3}$

$$y=\frac{3x+4}{2x-3}$$
$$x=\frac{3y+4}{2y-3}\quad \text{Inverse}$$
$$x(2y-3)=3y+4$$
$$2xy-3x=3y+4$$
$$2xy-3y=3x+4$$
$$y(2x-3)=3x+4$$
$$y=\frac{3x+4}{2x-3}$$
$$f^{-1}(x)=\frac{3x+4}{2x-3}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{3x+4}{2x-3}\right)=\frac{3\left(\frac{3x+4}{2x-3}\right)+4}{2\left(\frac{3x+4}{2x-3}\right)-3}$$
$$=\frac{\left(3\left(\frac{3x+4}{2x-3}\right)+4\right)(2x-3)}{\left(2\left(\frac{3x+4}{2x-3}\right)-3\right)(2x-3)}$$
$$=\frac{3(3x+4)+4(2x-3)}{2(3x+4)-3(2x-3)}$$
$$=\frac{9x+12+8x-12}{6x+8-6x+9}=\frac{17x}{17}=x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{3x+4}{2x-3}\right) = \frac{3\left(\frac{3x+4}{2x-3}\right)+4}{2\left(\frac{3x+4}{2x-3}\right)-3}$$

$$= \frac{\left(3\left(\frac{3x+4}{2x-3}\right)+4\right)(2x-3)}{\left(2\left(\frac{3x+4}{2x-3}\right)-3\right)(2x-3)}$$

$$= \frac{3(3x+4)+4(2x-3)}{2(3x+4)-3(2x-3)}$$

$$= \frac{9x+12+8x-12}{6x+8-6x+9} = \frac{17x}{17} = x$$

b. domain of f = range of $f^{-1} = \left\{x \mid x \neq \frac{3}{2}\right\}$

range of f = domain of $f^{-1} = \left\{x \mid x \neq \frac{3}{2}\right\}$

62. **a**. $f(x) = \dfrac{2x-3}{x+4}$

$$y = \frac{2x-3}{x+4}$$

$$x = \frac{2y-3}{y+4} \quad \text{Inverse}$$

$$x(y+4) = 2y-3$$

$$xy+4x = 2y-3$$

$$xy-2y = -4x-3$$

$$y(x-2) = -(4x+3)$$

$$y = \frac{-(4x+3)}{x-2} = \frac{4x+3}{2-x}$$

$$f^{-1}(x) = \frac{4x+3}{2-x}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{4x+3}{2-x}\right) = \frac{2\left(\frac{4x+3}{2-x}\right)-3}{\frac{4x+3}{2-x}+4}$$

$$= \frac{2(4x+3)-3(2-x)}{4x+3+4(2-x)}$$

$$= \frac{8x+6-6+3x}{4x+3+8-4x}$$

$$= \frac{11x}{11}$$

$$= x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2x-3}{x+4}\right)$$

$$= \frac{4\left(\frac{2x-3}{x+4}\right)+3}{2-\frac{2x-3}{x+4}}$$

$$= \frac{4(2x-3)+3(x+4)}{2(x+4)-(2x-3)}$$

$$= \frac{8x-12+3x+12}{2x+8-2x+3}$$

$$= \frac{11x}{11}$$

$$= x$$

b. domain of f = range of $f^{-1} = \{x \mid x \neq -4\}$

range of f = domain of $f^{-1} = \{x \mid x \neq 2\}$

63. **a**. $f(x) = \dfrac{2x+3}{x+2}$

$$y = \frac{2x+3}{x+2}$$

$$x = \frac{2y+3}{y+2} \quad \text{Inverse}$$

$$xy+2x = 2y+3$$

$$xy-2y = -2x+3$$

$$y(x-2) = -2x+3$$

$$y = \frac{-2x+3}{x-2}$$

$$f^{-1}(x) = \frac{-2x+3}{x-2}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{-2x+3}{x-2}\right) = \frac{2\left(\frac{-2x+3}{x-2}\right)+3}{\frac{-2x+3}{x-2}+2}$$

$$= \frac{\left(2\left(\frac{-2x+3}{x-2}\right)+3\right)(x-2)}{\left(\frac{-2x+3}{x-2}+2\right)(x-2)}$$

$$= \frac{2(-2x+3)+3(x-2)}{-2x+3+2(x-2)}$$

$$= \frac{-4x+6+3x-6}{-2x+3+2x-4} = \frac{-x}{-1} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2x+3}{x+2}\right) = \frac{-2\left(\frac{2x+3}{x+2}\right)+3}{\frac{2x+3}{x+2}-2}$$
$$= \frac{\left(-2\left(\frac{2x+3}{x+2}\right)+3\right)(x+2)}{\left(\frac{2x+3}{x+2}-2\right)(x+2)}$$
$$= \frac{-2(2x+3)+3(x+2)}{2x+3-2(x+2)}$$
$$= \frac{-4x-6+3x+6}{2x+3-2x-4} = \frac{-x}{-1} = x$$

b. domain of f = range of $f^{-1} = \{x \mid x \neq -2\}$

range of f = domain of $f^{-1} = \{x \mid x \neq 2\}$

64. **a**. $f(x) = \dfrac{-3x-4}{x-2}$

$$y = \frac{-3x-4}{x-2}$$
$$x = \frac{-3y-4}{y-2} \quad \text{Inverse}$$
$$x(y-2) = -3y-4$$
$$xy-2x = -3y-4$$
$$xy+3y = 2x-4$$
$$y(x+3) = 2x-4$$
$$y = \frac{2x-4}{x+3}$$
$$f^{-1}(x) = \frac{2x-4}{x+3}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{2x-4}{x+3}\right)$$
$$= \frac{-3\left(\frac{2x-4}{x+3}\right)-4}{\frac{2x-4}{x+3}-2}$$
$$= \frac{-3(2x-4)-4(x+3)}{2x-4-2(x+3)}$$
$$= \frac{-6x+12-4x-12}{2x-4-2x-6}$$
$$= \frac{-10x}{-10}$$
$$= x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{-3x-4}{x-2}\right)$$
$$= \frac{2\left(\frac{-3x-4}{x-2}\right)-4}{\frac{-3x-4}{x-2}+3}$$
$$= \frac{2(-3x-4)-4(x-2)}{-3x-4+3(x-2)}$$
$$= \frac{-6x-8-4x+8}{-3x-4+3x-6}$$
$$= \frac{-10x}{-10}$$
$$= x$$

b. domain of f = range of $f^{-1} = \{x \mid x \neq 2\}$

range of f = domain of $f^{-1} = \{x \mid x \neq -3\}$

65. **a**. $f(x) = \dfrac{x^2-4}{2x^2}, \; x > 0$

$$y = \frac{x^2-4}{2x^2}, \quad x > 0$$
$$x = \frac{y^2-4}{2y^2}, \quad y > 0 \quad \text{Inverse}$$
$$2xy^2 = y^2-4, \quad x < \frac{1}{2}$$
$$2xy^2 - y^2 = -4, \quad x < \frac{1}{2}$$
$$y^2(2x-1) = -4, \quad x < \frac{1}{2}$$
$$y^2(1-2x) = 4, \quad x < \frac{1}{2}$$
$$y^2 = \frac{4}{1-2x}, \quad x < \frac{1}{2}$$
$$y = \sqrt{\frac{4}{1-2x}}, \quad x < \frac{1}{2}$$
$$y = \frac{2}{\sqrt{1-2x}}, \quad x < \frac{1}{2}$$
$$f^{-1}(x) = \frac{2}{\sqrt{1-2x}}, \quad x < \frac{1}{2}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{2}{\sqrt{1-2x}}\right)=\frac{\left(\frac{2}{\sqrt{1-2x}}\right)^2-4}{2\left(\frac{2}{\sqrt{1-2x}}\right)^2}$$

$$=\frac{\frac{4}{1-2x}-4}{2\left(\frac{4}{1-2x}\right)}=\frac{\left(\frac{4}{1-2x}-4\right)(1-2x)}{\left(2\left(\frac{4}{1-2x}\right)\right)(1-2x)}$$

$$=\frac{4-4(1-2x)}{2(4)}=\frac{4-4+8x}{8}=\frac{8x}{8}=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{x^2-4}{2x^2}\right)=\frac{2}{\sqrt{1-2\left(\frac{x^2-4}{2x^2}\right)}}$$

$$=\frac{2}{\sqrt{1-\frac{x^2-4}{x^2}}}=\frac{2}{\sqrt{1-1+\frac{4}{x^2}}}$$

$$=\frac{2}{\sqrt{\frac{4}{x^2}}}=\frac{2}{\frac{2}{|x|}}=2\cdot\frac{|x|}{2}$$

$$=|x|=x,\quad x>0$$

b. domain of f = range of $f^{-1}=\{x\mid x>0\}$

range of f = domain of $f^{-1}=\left\{x\mid x<\frac{1}{2}\right\}$

66. **a**.

$$f(x)=\frac{x^2+3}{3x^2},\quad x>0$$

$$y=\frac{x^2+3}{3x^2},\quad x>0$$

$$x=\frac{y^2+3}{3y^2},\quad y>0 \quad \text{Inverse}$$

$$3xy^2=y^2+3,\quad x>\frac{1}{3}$$

$$3xy^2-y^2=3,\quad x>\frac{1}{3}$$

$$y^2(3x-1)=3,\quad x>\frac{1}{3}$$

$$y^2=\frac{3}{3x-1},\quad x>\frac{1}{3}$$

$$y=\sqrt{\frac{3}{3x-1}},\quad x>\frac{1}{3}$$

$$f^{-1}(x)=\sqrt{\frac{3}{3x-1}},\quad x>\frac{1}{3}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\sqrt{\frac{3}{3x-1}}\right)=\frac{\left(\sqrt{\frac{3}{3x-1}}\right)^2+3}{3\left(\sqrt{\frac{3}{3x-1}}\right)^2}$$

$$=\frac{\frac{3}{3x-1}+3}{3\left(\frac{3}{3x-1}\right)}=\frac{\left(\frac{3}{3x-1}+3\right)(3x-1)}{\left(3\left(\frac{3}{3x-1}\right)\right)(3x-1)}$$

$$=\frac{3+3(3x-1)}{3(3)}$$

$$=\frac{3+9x-3}{9}$$

$$=\frac{9x}{9}$$

$$=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{x^2+3}{3x^2}\right)=\sqrt{\frac{3}{3\left(\frac{x^2+3}{3x^2}\right)-1}}$$

$$=\sqrt{\frac{3}{\frac{x^2+3}{x^2}-1}}=\sqrt{\frac{3}{1+\frac{3}{x^2}-1}}$$

$$=\sqrt{\frac{3}{\frac{3}{x^2}}}=\sqrt{(3)\left(\frac{x^2}{3}\right)}=\sqrt{x^2}$$

$$=|x|=x,\quad x>0$$

b. domain of f = range of $f^{-1}=\{x\mid x>0\}$

range of f = domain of $f^{-1}=\left\{x\mid x>\frac{1}{3}\right\}$

67. **a**.

$$f(x)=x^{\frac{2}{3}}-4$$

$$y=x^{\frac{2}{3}}-4$$

$$x=y^{\frac{2}{3}}-4 \quad \text{Inverse}$$

$$y^{\frac{2}{3}}=x+4$$

$$y=(x+4)^{\frac{3}{2}},x\ge -4$$

$$f^{-1}(x)=(x+4)^{\frac{3}{2}},x\ge -4$$

Verifying: $f\left(f^{-1}(x)\right)=f\left((x+4)^{\frac{3}{2}}\right)$

$$=\left((x+4)^{\frac{3}{2}}\right)^{\frac{2}{3}}-4$$

$$=x+4-4=x$$

$$f^{-1}(f(x)) = f^{-1}\left(x^{\frac{2}{3}} - 4\right)$$
$$= \left(\left(x^{\frac{2}{3}} - 4\right) + 4\right)^{\frac{3}{2}}$$
$$= \left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = x, \text{since } x \ge 0$$

b. domain of f = range of $f^{-1} = \{x \mid x \ge 0\}$

range of f = domain of $f^{-1} = \{x \mid x \ge -4\}$

68. a. $f(x) = x^{\frac{3}{2}} + 5$

$$y = x^{\frac{3}{2}} + 5$$
$$x = y^{\frac{3}{2}} + 5 \quad \text{Inverse}$$
$$y^{\frac{3}{2}} = x - 5$$
$$y = (x-5)^{\frac{2}{3}}, x \ge 5$$
$$f^{-1}(x) = (x-5)^{\frac{2}{3}}, x \ge 5$$

Verifying: $f\left(f^{-1}(x)\right) = f\left((x-5)^{\frac{2}{3}}\right)$

$$= \left((x-5)^{\frac{2}{3}}\right)^{\frac{3}{2}} + 5$$
$$= x - 5 + 5 = x, \text{since } x \ge 5$$

$$f^{-1}(f(x)) = f^{-1}\left(x^{\frac{3}{2}} + 5\right)$$
$$= \left(\left(x^{\frac{3}{2}} + 5\right) - 5\right)^{\frac{2}{3}}$$
$$= \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = x, \text{since } x \ge 0$$

b. domain of f = range of $f^{-1} = \{x \mid x \ge 0\}$

range of f = domain of $f^{-1} = \{x \mid x \ge 5\}$

69. a. $f(x) = \sqrt[3]{x^5 - 2}$

$$y = \sqrt[3]{x^5 - 2}$$
$$x = \sqrt[3]{y^5 - 2} \quad \text{Inverse}$$
$$y^5 = x^3 + 2$$
$$y = \sqrt[5]{x^3 + 2}$$
$$f^{-1}(x) = \sqrt[5]{x^3 + 2}$$

Verifying: $f\left(f^{-1}(x)\right) = f\left(\sqrt[5]{x^3 + 2}\right)$

$$= \sqrt[3]{\left(\sqrt[5]{x^3 + 2}\right)^5 - 2}$$
$$= \sqrt[3]{\left(x^3 + 2\right) - 2} = \sqrt[3]{x^3} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\sqrt[3]{x^5 - 2}\right)$$
$$= \sqrt[5]{\left(\sqrt[3]{x^5 - 2}\right)^3 + 2}$$
$$= \sqrt[5]{\left(x^5 - 2\right) + 2} = \sqrt[5]{x^5} = x$$

b. domain of f = range of f^{-1} = all real numbers

range of f = domain of f^{-1} = all real numbers

70. a. $f(x) = \sqrt[5]{x^3 + 13}$

$$y = \sqrt[5]{x^3 + 13}$$
$$x = \sqrt[5]{y^3 + 13} \quad \text{Inverse}$$
$$y^3 = x^5 - 13$$
$$y = \sqrt[3]{x^5 - 13}$$
$$f^{-1}(x) = \sqrt[3]{x^5 - 13}$$

Verifying: $f\left(f^{-1}(x)\right) = f\left(\sqrt[3]{x^5 - 13}\right)$

$$= \sqrt[5]{\left(\sqrt[3]{x^5 - 13}\right)^3 + 13}$$
$$= \sqrt[5]{\left(x^5 - 13\right) + 13} = \sqrt[5]{x^5} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\sqrt[5]{x^3 + 13}\right)$$
$$= \sqrt[3]{\left(\sqrt[5]{x^3 + 13}\right)^5 + 2}$$
$$= \sqrt[3]{\left(x^3 + 13\right) - 13} = \sqrt[3]{x^3} = x$$

b. domain of f = range of f^{-1} = all real numbers

range of f = domain of f^{-1} = all real numbers

71. a. $f(x) = \frac{1}{9}(x-1)^2 + 2$

$$y = \tfrac{1}{9}(x-1)^2 + 2$$
$$x = \tfrac{1}{9}(y-1)^2 + 2 \quad \text{Inverse}$$
$$x - 2 = \tfrac{1}{9}(y-1)^2$$
$$\sqrt{9(x-2)} + 1 = y$$
$$y = 3\sqrt{x-2} + 1, x \ge 2$$
$$f^{-1}(x) = 3\sqrt{x-2} + 1, x \ge 2$$

Verifying: $f\left(f^{-1}(x)\right)=f\left(3\sqrt{x-2}+1\right)$

$$=\tfrac{1}{9}\left[(3\sqrt{x-2}+1)-1\right]^2+2$$
$$=\tfrac{1}{9}\left[(3\sqrt{x-2}\right]^2+2$$
$$=\tfrac{1}{9}\left[(9(x-2)\right]+2$$
$$=(x-2)+2=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\tfrac{1}{9}(x-1)^2+2\right)$$
$$=3\sqrt{\left(\tfrac{1}{9}(x-1)^2+2\right)-2}+1$$
$$=3\sqrt{\tfrac{1}{9}(x-1)^2}+1=3\left(\tfrac{1}{3}\right)(x-1)+1$$
$$=(x-1)+1=x,\text{since } x\ge 1$$

b. domain of f = range of $f^{-1}=\{x\mid x\ge 1\}$

range of f = domain of $f^{-1}=\{x\mid x\ge 2\}$

72. a. $f(x)=2\sqrt{x+3}-5$

$y=2\sqrt{x+3}-5$

$x=2\sqrt{y+3}-5$ Inverse

$x+5=2\sqrt{y+3}$

$$\left(\frac{x+5}{2}\right)^2=y+3$$

$$y=\left(\frac{x+5}{2}\right)^2-3, x\ge -5$$

$$f^{-1}(x)=\left(\frac{x+5}{2}\right)^2-3, x\ge -5$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\left(\frac{x+5}{2}\right)^2-3\right)$$
$$=2\sqrt{\left(\left(\frac{x+5}{2}\right)^2-3\right)+3}-5$$
$$=2\sqrt{\left(\frac{x+5}{2}\right)^2}-5$$
$$=2\left(\frac{x+5}{2}\right)-5$$
$$=(x+5)-5=x,\text{since } x\ge -5$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(2\sqrt{x+3}-5\right)$$
$$=\left(\frac{\left(2\sqrt{x+3}-5\right)+5}{2}\right)^2-3$$
$$=\left(\frac{2\sqrt{x+3}}{2}\right)^2-3$$
$$=(x+3)-3=x$$

b. domain of f = range of $f^{-1}=\{x\mid x\ge -3\}$

range of f = domain of $f^{-1}=\{x\mid x\ge -5\}$

73. a. Because the ordered pair $(-1,0)$ is on the graph, $f(-1)=0$.

b. Because the ordered pair $(1,2)$ is on the graph, $f(1)=2$.

c. Because the ordered pair $(0,1)$ is on the graph, $f^{-1}(1)=0$.

d. Because the ordered pair $(1,2)$ is on the graph, $f^{-1}(2)=1$.

74. a. Because the ordered pair $\left(2,\dfrac{1}{2}\right)$ is on the graph, $f(2)=\dfrac{1}{2}$.

b. Because the ordered pair $(1,0)$ is on the graph, $f(1)=0$.

c. Because the ordered pair $(1,0)$ is on the graph, $f^{-1}(0)=1$.

d. Because the ordered pair $(0,-1)$ is on the graph, $f^{-1}(-1)=0$.

75. Since $f(7)=13$, we have $f^{-1}(13)=7$; the input of the function is the output of the inverse when the output of the function is the input of the inverse.

76. Since $g(-5)=3$, we have $g^{-1}(3)=-5$; the input of the function is the output of the inverse when the output of the function is the input of the inverse.

77. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for f^{-1}:

Domain: $[-2,\infty)$ Range: $[5,\infty)$

78. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for f^{-1}:
Domain: $[5,\infty)$ Range: $[0,\infty)$

79. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for g^{-1}:
Domain: $[0,\infty)$ Range: $(-\infty,0]$

80. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for g^{-1}:
Domain: $(0,8)$ Range: $[0,15]$

81. Since $f(x)$ is increasing on the interval $(0,5)$, it is one-to-one on the interval and has an inverse, $f^{-1}(x)$. In addition, we can say that $f^{-1}(x)$ is increasing on the interval $[f(0), f(5)]$.

82. Since $f(x)$ is decreasing on the interval $(0,5)$, it is one-to-one on the interval and has an inverse, $f^{-1}(x)$. In addition, we can say that $f^{-1}(x)$ is decreasing on the interval $[f(5), f(0)]$.

83.
$$\begin{aligned} f(x) &= mx+b, \quad m \neq 0 \\ y &= mx+b \\ x &= my+b \quad \text{Inverse} \\ x-b &= my \\ y &= \frac{1}{m}(x-b) \\ f^{-1}(x) &= \frac{1}{m}(x-b), \quad m \neq 0 \end{aligned}$$

84.
$$\begin{aligned} f(x) &= \sqrt{r^2-x^2}, \quad 0 \le x \le r \\ y &= \sqrt{r^2-x^2} \\ x &= \sqrt{r^2-y^2} \quad \text{Inverse} \\ x^2 &= r^2-y^2 \\ y^2 &= r^2-x^2 \\ y &= \sqrt{r^2-x^2} \\ f^{-1}(x) &= \sqrt{r^2-x^2}, \quad 0 \le x \le r \end{aligned}$$

85. If (a,b) is on the graph of *f*, then (b,a) is on the graph of f^{-1}. Since the graph of f^{-1} lies in quadrant I, both coordinates of (a,b) are positive, which means that both coordinates of (b,a) are positive. Thus, the graph of f^{-1} must lie in quadrant I.

86. If (a,b) is on the graph of *f*, then (b,a) is on the graph of f^{-1}. Since the graph of *f* lies in quadrant II, *a* must be negative and *b* must be positive. Thus, (b,a) must be a point in quadrant IV, which means the graph of f^{-1} lies in quadrant IV.

87. Answers may vary. One possibility follows:
$f(x) = |x|, x \ge 0$ is one-to-one.
Thus,
$$\begin{aligned} f(x) &= x, x \ge 0 \\ y &= x, x \ge 0 \\ f^{-1}(x) &= x, x \ge 0 \end{aligned}$$

88. Answers may vary. One possibility follows:
$f(x) = x^4, x \ge 0$ is one-to-one.
Thus,
$$\begin{aligned} f(x) &= x^4, \quad x \ge 0 \\ y &= x^4, \quad x \ge 0 \\ x &= y^4 \quad \text{Inverse} \\ y &= \sqrt[4]{x}, \quad x \ge 0 \\ f^{-1}(x) &= \sqrt[4]{x}, \quad x \ge 0 \end{aligned}$$

89. a.
$$\begin{aligned} d &= 6.97r - 90.39 \\ d + 90.39 &= 6.97r \\ \frac{d+90.39}{6.97} &= r \end{aligned}$$
Therefore, we would write
$$r(d) = \frac{d+90.39}{6.97}$$

b.
$$\begin{aligned} r(d(r)) &= \frac{(6.97r-90.39)+90.39}{6.97} \\ &= \frac{6.97r+90.39-90.39}{6.97} = \frac{6.97r}{6.97} \\ &= r \end{aligned}$$

$$\begin{aligned} d(r(d)) &= 6.97\left(\frac{d+90.39}{6.97}\right) - 90.39 \\ &= d + 90.39 - 90.39 \\ &= d \end{aligned}$$

c. $r(300)=\dfrac{300+90.39}{6.97}\approx 56.01$

If the distance required to stop was 300 feet, the speed of the car was roughly 56 miles per hour.

90. a.
$$H(C)=2.15C-10.53$$
$$H=2.15C-10.53$$
$$H+10.53=2.15C$$
$$\frac{H+10.53}{2.15}=C$$
$$C(H)=\frac{H+10.53}{2.15}$$

b.
$$H(C(H))=2.15\left(\frac{H+10.53}{2.15}\right)-10.53$$
$$=H+10.53-10.53$$
$$=H$$
$$C(H(C))=\frac{(2.15C-10.53)+10.53}{2.15}$$
$$=\frac{2.15C-10.53+10.53}{2.15}$$
$$=\frac{2.15C}{2.15}=C$$

c. $C(26)=\dfrac{26+10.53}{2.15}\approx 16.99$

The head circumference of a child who is 26 inches tall is about 17 inches.

91. a. 6 feet = 72 inches
$$W(72)=50+2.3(72-60)$$
$$=50+2.3(12)=50+27.6=77.6$$

The ideal weight of a 6-foot male is 77.6 kilograms.

b.
$$W=50+2.3(h-60)$$
$$W-50=2.3h-138$$
$$W+88=2.3h$$
$$\frac{W+88}{2.3}=h$$

Therefore, we would write
$$h(W)=\frac{W+88}{2.3}$$

c.
$$h(W(h))=\frac{(50+2.3(h-60))+88}{2.3}$$
$$=\frac{50+2.3h-138+88}{2.3}=\frac{2.3h}{2.3}=h$$
$$W(h(W))=50+2.3\left(\frac{W+88}{2.3}-60\right)$$
$$=50+W+88-138=W$$

d. $h(80)=\dfrac{80+88}{2.3}=\dfrac{168}{2.3}\approx 73.04$

The height of a male who is at his ideal weight of 80 kg is roughly 73 inches.

92. a.
$$F=\frac{9}{5}C+32$$
$$F-32=\frac{9}{5}C$$
$$\frac{5}{9}(F-32)=C$$

Therefore, we would write
$$C(F)=\frac{5}{9}(F-32)$$

b.
$$C(F(C))=\frac{5}{9}\left(\left(\frac{9}{5}C+32\right)-32\right)$$
$$=\frac{5}{9}\cdot\frac{9}{5}C=C$$
$$F(C(F))=\frac{9}{5}\left(\frac{5}{9}(F-32)\right)+32$$
$$=F-32+32=F$$

c. $C(70)=\dfrac{5}{9}(70-32)=\dfrac{5}{9}(38)\approx 21.1°\text{C}$

93. a. From the restriction given in the problem statement, the domain is $\{g\mid 38{,}700\le g\le 82{,}500\}$ or $[38700, 82500]$.

b.
$$T(38{,}700)=4453.50+0.22(38{,}700-38{,}700)$$
$$=4453.50$$
$$T(82{,}500)=4453.50+0.22(82{,}500-38{,}700)$$
$$=14{,}089.50$$

Since T is linear and increasing, we have that the range is $\{T\mid 4453.50\le T\le 14{,}089.50\}$ or $[4453.50, 14089.50]$.

c. $T = 4453.50 + 0.22(g - 38,700)$
$$T - 4453.50 = 0.22(g - 38,700)$$
$$\frac{T - 4453.50}{0.22} = g - 38,700$$
$$\frac{T - 4453.50}{0.22} + 38,700 = g$$
Therefore, we would write
$$g(T) = \frac{T - 4453.50}{0.22} + 38,700$$
Domain: $\{T \mid 4453.50 \le T \le 14,089.50\}$

Range: $\{g \mid 38,700 \le g \le 82,500\}$

94. a. From the restriction given in the problem statement, the domain is $\{g \mid 19,050 \le g \le 77,400\}$ or $[19050, 77400]$.

b. $T(19,050) = 1905 + 0.12(19,050 - 19,050)$
$$= 1905$$
$$T(77,400) = 1905 + 0.12(77,400 - 19,050)$$
$$= 8907$$
Since T is linear and increasing, we have that the range is $\{T \mid 1905 \le T \le 8907\}$ or $[1905, 8907]$.

c. $T = 1905 + 0.12(g - 19,050)$
$$T - 1905 = 0.12(g - 19,050)$$
$$\frac{T - 1905}{0.12} = g - 19,050$$
$$\frac{T - 1905}{0.12} + 19,050 = g$$
We would write $g(T) = \dfrac{T - 1905}{0.12} + 19,050$.

Domain: $\{T \mid 1905 \le T \le 8907\}$

Range: $\{g \mid 19,050 \le g \le 77,400\}$

95. a. The graph of H is symmetric about the y-axis. Since t represents the number of seconds *after* the rock begins to fall, we know that $t \ge 0$. The graph is strictly decreasing over its domain, so it is one-to-one.

b. $H = 100 - 4.9t^2$
$$H + 4.9t^2 = 100$$
$$4.9t^2 = 100 - H$$
$$t^2 = \frac{100 - H}{4.9}$$
$$t = \sqrt{\frac{100 - H}{4.9}}$$
Therefore, we would write $t(H) = \sqrt{\dfrac{100 - H}{4.9}}$.

(Note: we only need the principal square root since we know $t \ge 0$)
$$H(t(H)) = 100 - 4.9\left(\sqrt{\frac{100 - H}{4.9}}\right)^2$$
$$= 100 - 4.9\left(\frac{100 - H}{4.9}\right)$$
$$= 100 - 100 + H$$
$$= H$$
$$t(H(t)) = \sqrt{\frac{100 - (100 - 4.9t^2)}{4.9}}$$
$$= \sqrt{\frac{4.9t^2}{4.9}} = \sqrt{t^2} = t \quad (\text{since } t \ge 0)$$

c. $t(80) = \sqrt{\dfrac{100 - 80}{4.9}} \approx 2.02$

It will take the rock about 2.02 seconds to fall 80 meters.

96. a. $T(l) = 2\pi\sqrt{\dfrac{l}{32.2}}$
$$T = 2\pi\sqrt{\frac{l}{32.2}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{l}{32.2}}$$
$$\left(\frac{T}{2\pi}\right)^2 = \frac{l}{32.2}$$
$$l = 32.2 \cdot \left(\frac{T}{2\pi}\right)^2$$
$$l(T) = 32.2 \cdot \left(\frac{T}{2\pi}\right)^2, \quad T > 0$$

b. $l(3) = 32.2 \cdot \left(\dfrac{3}{2\pi}\right)^2 \approx 7.34$

A pendulum whose period is 3 seconds will be about 7.34 feet long.

97. $f(x)=\dfrac{ax+b}{cx+d} \Rightarrow y=\dfrac{ax+b}{cx+d}$

Interchange x and y, and then solve for y:

$$x=\frac{ay+b}{cy+d}$$
$$x(cy+d)=ay+b$$
$$cxy+dx=ay+b$$
$$cxy-ay=b-dx$$
$$y(cx-a)=b-dx$$
$$y=\frac{b-dx}{cx-a}$$

So $f^{-1}(x)=\dfrac{-dx+b}{cx-a}$

Now,

$f=f^{-1}\Rightarrow$

$$\frac{ax+b}{cx+d}=\frac{-dx+b}{cx-a}$$
$$(ax+b)(cx-a)=(-dx+b)(cx+d)$$
$$acx^2-a^2x+bcx-ab=-cdx^2-d^2x+bcx+bd$$
$$acx^2-\left(a^2+bc\right)x-ab=-cdx^2+\left(-d^2+bc\right)x+bd$$

So $ac=-cd; -a^2+bc=-d^2+bc; -ab=bd$, which means $a=-d$.

98. $h(x)=f(g(x))$.

$y=f(g(x))$

Interchange x and y, and then solve for y:

$x=f(g(y))$

$f^{-1}(x)=g(y)\Rightarrow g^{-1}(f^{-1}(x))=y$

So, $h^{-1}(x)=g^{-1}(f^{-1}(x))$

99. **a.** The domain of f is $(-\infty,\infty)$. From $y=2x+3$, $x<0$, then $y<3$, From $y=3x+4$, $x\ge 0$, then $y\ge 4$. The range of f is $(-\infty,3)\cup[4,\infty)$.

b. Consider the piece

$f(x)=2x+3, x<0 \Rightarrow y=2x+3, x<0$

Interchange x and y:

$x=2y+3, y<0 \leftrightarrow$ *Note*: If $y<0$ then $x<3$ so

$x-3=2y, x<3 \Rightarrow \dfrac{x-3}{2}=y, x<3$

$f^{-1}(x)=\dfrac{x-3}{2}, x<3$

Now consider the piece

$f(x)=3x+4, x\ge 0 \Rightarrow y=3x+4, x\ge 0$.

Interchange x and y:

$x=3y+4, y\ge 0 \leftrightarrow$ *Note*: If $y\ge 0$, then $x\ge 4$ so

$x-4=3y, x\ge 4 \Rightarrow \dfrac{x-4}{3}=y, x\ge 4$

$f^{-1}(x)=\dfrac{x-4}{3}, x\ge 4$

Putting the pieces together we have

$$f^{-1}(x)=\begin{cases}\dfrac{x-3}{2}, x<3\\[2mm] \dfrac{x-4}{3}, x\ge 4\end{cases}$$

c. The domain of f^{-1} is $(-\infty,3)\cup[4,\infty)$, the range of *f*.

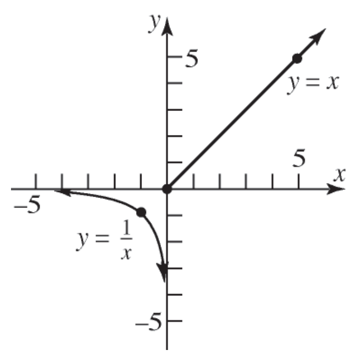
The range is f^{-1} is $(-\infty,\infty)$, the domain of *f*.

100. Yes. In order for a one-to-one function and its inverse to be equal, its graph must be symmetric about the line $y=x$. One such example is the function $f(x)=\dfrac{1}{x}$.

101. Answers will vary.

102. Answers will vary. One example is

$$f(x)=\begin{cases}\dfrac{1}{x}, & \text{if } x<0\\ x, & \text{if } x\ge 0\end{cases}$$



This function is one-to-one since the graph passes the Horizontal Line Test. However, the function is neither increasing nor decreasing on its domain.

103. No, not every odd function is one-to-one. For example, $f(x)=x^3-x$ is an odd function, but it is not one-to-one.

104. $C^{-1}(800,000)$ represents the number of cars manufactured for \$800,000.

105. If a horizontal line passes through two points on a graph of a function, then the y value associated with that horizontal line will be assigned to two different x values which violates the definition of one-to-one.

106. Answers may vary.

Chapter 1 Review Exercises

1. $P_1=(0,0)$ and $P_2=(4,2)$

a. $d(P_1,P_2)=\sqrt{(4-0)^2+(2-0)^2}$
$=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$

b. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{0+4}{2},\frac{0+2}{2}\right)=\left(\frac{4}{2},\frac{2}{2}\right)=(2,1)$$

2. $P_1=(1,-1)$ and $P_2=(-2,3)$

a. $d(P_1,P_2)=\sqrt{(-2-1)^2+(3-(-1))^2}$
$=\sqrt{9+16}=\sqrt{25}=5$

b. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{1+(-2)}{2},\frac{-1+3}{2}\right)$$
$$=\left(\frac{-1}{2},\frac{2}{2}\right)=\left(-\frac{1}{2},1\right)$$

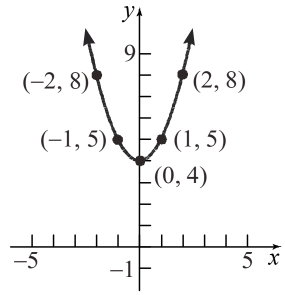
3. $P_1=(4,-4)$ and $P_2=(4,8)$

a. $d(P_1,P_2)=\sqrt{(4-4)^2+(8-(-4))^2}$
$=\sqrt{0+144}=\sqrt{144}=12$

b. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{4+4}{2},\frac{-4+8}{2}\right)=\left(\frac{8}{2},\frac{4}{2}\right)=(4,2)$$

4. $y=x^2+4$



5. x-intercepts: $-4, 0, 2$; y-intercepts: $-2, 0, 2$
Intercepts: $(-4, 0), (0, 0), (2,0), (0,-2), (0, 2)$

6. $2x=3y^2$

x-intercepts:
$2x=3(0)^2$
$2x=0$
$x=0$

y-intercepts:
$2(0)=3y^2$
$0=y^2$
$y=0$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y=-y$
$2x=3(-y)^2$
$2x=3y^2$ same

Test y-axis symmetry: Let $x=-x$
$2(-x)=3y^2$
$-2x=3y^2$ different

Test origin symmetry: Let $x=-x$ and $y=-y$.
$2(-x)=3(-y)^2$
$-2x=3y^2$ different

Therefore, the graph will have x-axis symmetry.

7. $x^2+4y^2=16$

x-intercepts:
$x^2+4(0)^2=16$
$x^2=16$
$x=\pm 4$

y-intercepts:
$(0)^2+4y^2=16$
$4y^2=16$
$y^2=4$
$y=\pm 2$

The intercepts are $(-4,0), (4,0), (0,-2)$, and $(0,2)$.

Test x-axis symmetry: Let $y=-y$
$x^2+4(-y)^2=16$
$x^2+4y^2=16$ same

Test y-axis symmetry: Let $x = -x$

$(-x)^2 + 4y^2 = 16$

$x^2 + 4y^2 = 16$ same

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-x)^2 + 4(-y)^2 = 16$

$x^2 + 4y^2 = 16$ same

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

8. $y = x^4 + 2x^2 + 1$

x-intercepts:

$0 = x^4 + 2x^2 + 1$

$0 = (x^2 + 1)(x^2 + 1)$

$x^2 + 1 = 0$

$x^2 = -1$

no real solutions

y-intercepts:

$y = (0)^4 + 2(0)^2 + 1$

$= 1$

The only intercept is (0, 1).

Test x-axis symmetry: Let $y = -y$

$-y = x^4 + 2x^2 + 1$

$y = -x^4 - 2x^2 - 1$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^4 + 2(-x)^2 + 1$

$y = x^4 + 2x^2 + 1$ same

Test origin symmetry: Let $x = -x$ and $y = -y$.

$-y = (-x)^4 + 2(-x)^2 + 1$

$-y = x^4 + 2x^2 + 1$

$y = -x^4 - 2x^2 - 1$ different

Therefore, the graph will have y-axis symmetry.

9. $y = x^3 - x$

x-intercepts:

$0 = x^3 - x$

$0 = x(x^2 - 1)$

$0 = x(x+1)(x-1)$

$x = 0, x = -1, x = 1$

y-intercepts:

$y = (0)^3 - 0$

$= 0$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(1, 0)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^3 - x$

$y = -x^3 + x$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^3 - (-x)$

$y = -x^3 + x$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$-y = (-x)^3 - (-x)$

$-y = -x^3 + x$

$y = x^3 - x$ same

Therefore, the graph will have origin symmetry.

10. $x^2 + x + y^2 + 2y = 0$

x-intercepts: $x^2 + x + (0)^2 + 2(0) = 0$

$x^2 + x = 0$

$x(x+1) = 0$

$x = 0, x = -1$

y-intercepts: $(0)^2 + 0 + y^2 + 2y = 0$

$y^2 + 2y = 0$

$y(y+2) = 0$

$y = 0, y = -2$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(0, -2)$.

Test x-axis symmetry: Let $y = -y$

$x^2 + x + (-y)^2 + 2(-y) = 0$

$x^2 + x + y^2 - 2y = 0$ different

Test y-axis symmetry: Let $x = -x$

$(-x)^2 + (-x) + y^2 + 2y = 0$

$x^2 - x + y^2 + 2y = 0$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-x)^2 + (-x) + (-y)^2 + 2(-y) = 0$

$x^2 - x + y^2 - 2y = 0$ different

The graph has none of the indicated symmetries.

11. $(x-h)^2 + (y-k)^2 = r^2$

$(x-(-2))^2 + (y-3)^2 = 4^2$

$(x+2)^2 + (y-3)^2 = 16$

12. $(x-h)^2 + (y-k)^2 = r^2$

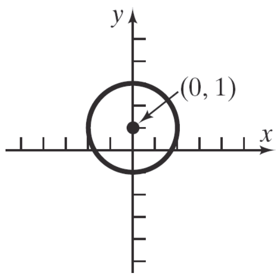
$(x-(-1))^2 + (y-(-2))^2 = 1^2$

$(x+1)^2 + (y+2)^2 = 1$

13. $x^2 + (y-1)^2 = 4$

$x^2 + (y-1)^2 = 2^2$

Center: (0,1); Radius = 2



x-intercepts: $x^2+(0-1)^2=4$

$$x^2+1=4$$
$$x^2=3$$
$$x=\pm\sqrt{3}$$

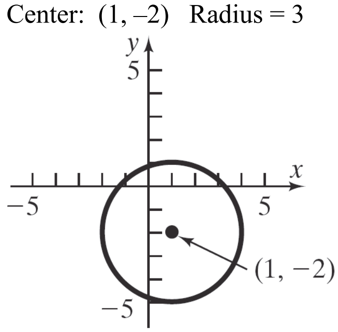
y-intercepts: $0^2+(y-1)^2=4$

$$(y-1)^2=4$$
$$y-1=\pm 2$$
$$y=1\pm 2$$
$$y=3 \text{ or } y=-1$$

The intercepts are $\left(-\sqrt{3},0\right)$, $\left(\sqrt{3},0\right)$, $(0,-1)$, and $(0,3)$.

14.
$$x^2+y^2-2x+4y-4=0$$
$$x^2-2x+y^2+4y=4$$
$$\left(x^2-2x+1\right)+\left(y^2+4y+4\right)=4+1+4$$
$$(x-1)^2+(y+2)^2=3^2$$

Center: (1, –2) Radius = 3



x-intercepts: $(x-1)^2+(0+2)^2=3^2$

$$(x-1)^2+4=9$$
$$(x-1)^2=5$$
$$x-1=\pm\sqrt{5}$$
$$x=1\pm\sqrt{5}$$

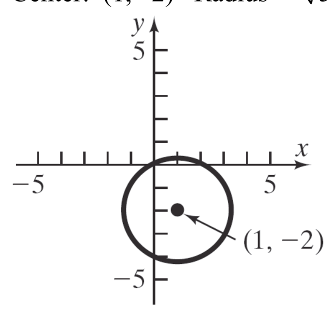
y-intercepts: $(0-1)^2+(y+2)^2=3^2$

$$1+(y+2)^2=9$$
$$(y+2)^2=8$$
$$y+2=\pm\sqrt{8}$$
$$y+2=\pm 2\sqrt{2}$$
$$y=-2\pm 2\sqrt{2}$$

The intercepts are $\left(1-\sqrt{5},0\right)$, $\left(1+\sqrt{5},0\right)$, $\left(0,-2-2\sqrt{2}\right)$, and $\left(0,-2+2\sqrt{2}\right)$.

15.
$$3x^2+3y^2-6x+12y=0$$
$$x^2+y^2-2x+4y=0$$
$$x^2-2x+y^2+4y=0$$
$$\left(x^2-2x+1\right)+\left(y^2+4y+4\right)=1+4$$
$$(x-1)^2+(y+2)^2=\left(\sqrt{5}\right)^2$$

Center: (1, –2) Radius = $\sqrt{5}$



x-intercepts: $(x-1)^2+(0+2)^2=\left(\sqrt{5}\right)^2$

$$(x-1)^2+4=5$$
$$(x-1)^2=1$$
$$x-1=\pm 1$$
$$x=1\pm 1$$
$$x=2 \text{ or } x=0$$

y-intercepts: $(0-1)^2+(y+2)^2=\left(\sqrt{5}\right)^2$

$$1+(y+2)^2=5$$
$$(y+2)^2=4$$
$$y+2=\pm 2$$
$$y=-2\pm 2$$
$$y=0 \text{ or } y=-4$$

The intercepts are $(0,0)$, $(2,0)$, and $(0,-4)$.

16. Find the distance between each pair of points.

$d_{A,B} = \sqrt{(1-3)^2 + (1-4)^2} = \sqrt{4+9} = \sqrt{13}$

$d_{B,C} = \sqrt{(-2-1)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$

$d_{A,C} = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{25+1} = \sqrt{26}$

Since $AB = BC$, triangle ABC is isosceles.

17. Endpoints of the diameter are (–3, 2) and (5,–6). The center is at the midpoint of the diameter:

Center: $\left(\frac{-3+5}{2}, \frac{2+(-6)}{2}\right) = (1,-2)$

Radius: $r = \sqrt{(1-(-3))^2 + (-2-2)^2}$

$= \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$

Equation: $(x-1)^2 + (y+2)^2 = (4\sqrt{2})^2$

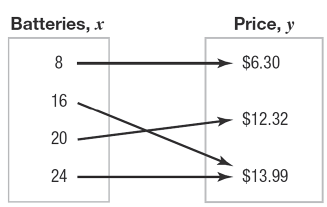
$(x-1)^2 + (y+2)^2 = 32$

18. a. Domain {8, 16, 20, 24}

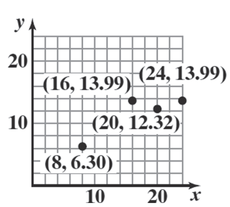
Range {\$6.30, \$12.32, \$13.99}

b. {(8,\$6.30), (16,\$13.99), (20,\$12.32), (24,\$13.99)}

c.



d.



19. This relation represents a function. Domain = {–1, 2, 4}; Range = {0, 3}.

20. This relation does not represent a function, since 4 is paired with two different values.

21. $f(x) = \dfrac{3x}{x^2-1}$

a. $f(2) = \dfrac{3(2)}{(2)^2-1} = \dfrac{6}{4-1} = \dfrac{6}{3} = 2$

b. $f(-2) = \dfrac{3(-2)}{(-2)^2-1} = \dfrac{-6}{4-1} = \dfrac{-6}{3} = -2$

c. $f(-x) = \dfrac{3(-x)}{(-x)^2-1} = \dfrac{-3x}{x^2-1}$

d. $-f(x) = -\left(\dfrac{3x}{x^2-1}\right) = \dfrac{-3x}{x^2-1}$

e. $f(x-2) = \dfrac{3(x-2)}{(x-2)^2-1}$

$= \dfrac{3x-6}{x^2-4x+4-1} = \dfrac{3(x-2)}{x^2-4x+3}$

f. $f(2x) = \dfrac{3(2x)}{(2x)^2-1} = \dfrac{6x}{4x^2-1}$

22. $f(x) = \sqrt{x^2-4}$

a. $f(2) = \sqrt{2^2-4} = \sqrt{4-4} = \sqrt{0} = 0$

b. $f(-2) = \sqrt{(-2)^2-4} = \sqrt{4-4} = \sqrt{0} = 0$

c. $f(-x) = \sqrt{(-x)^2-4} = \sqrt{x^2-4}$

d. $-f(x) = -\sqrt{x^2-4}$

e. $f(x-2) = \sqrt{(x-2)^2-4}$

$= \sqrt{x^2-4x+4-4}$

$= \sqrt{x^2-4x}$

f. $f(2x) = \sqrt{(2x)^2-4} = \sqrt{4x^2-4}$

$= \sqrt{4(x^2-1)} = 2\sqrt{x^2-1}$

23. $f(x) = \dfrac{x^2-4}{x^2}$

a. $f(2) = \dfrac{2^2-4}{2^2} = \dfrac{4-4}{4} = \dfrac{0}{4} = 0$

b. $f(-2) = \dfrac{(-2)^2-4}{(-2)^2} = \dfrac{4-4}{4} = \dfrac{0}{4} = 0$

c. $f(-x) = \dfrac{(-x)^2-4}{(-x)^2} = \dfrac{x^2-4}{x^2}$

d. $-f(x) = -\left(\dfrac{x^2-4}{x^2}\right) = \dfrac{4-x^2}{x^2} = -\dfrac{x^2-4}{x^2}$

e. $f(x-2) = \dfrac{(x-2)^2-4}{(x-2)^2} = \dfrac{x^2-4x+4-4}{(x-2)^2}$
$= \dfrac{x^2-4x}{(x-2)^2} = \dfrac{x(x-4)}{(x-2)^2}$

f. $f(2x) = \dfrac{(2x)^2-4}{(2x)^2} = \dfrac{4x^2-4}{4x^2}$
$= \dfrac{4(x^2-1)}{4x^2} = \dfrac{x^2-1}{x^2}$

24. $f(x) = \dfrac{x}{x^2-9}$
The denominator cannot be zero:
$x^2-9 \neq 0$
$(x+3)(x-3) \neq 0$
$x \neq -3$ or 3
Domain: $\{x \mid x \neq -3, x \neq 3\}$

25. $f(x) = \sqrt{2-x}$
The radicand must be non-negative:
$2-x \geq 0$
$x \leq 2$
Domain: $\{x \mid x \leq 2\}$ or $(-\infty, 2]$

26. $g(x) = \dfrac{|x|}{x}$
The denominator cannot be zero:
$x \neq 0$
Domain: $\{x \mid x \neq 0\}$

27. $f(x) = \dfrac{x}{x^2+2x-3}$
The denominator cannot be zero:
$x^2+2x-3 \neq 0$
$(x+3)(x-1) \neq 0$
$x \neq -3$ or 1
Domain: $\{x \mid x \neq -3,\ x \neq 1\}$

28. $f(x) = \dfrac{\sqrt{x+1}}{x^2-4}$
The denominator cannot be zero:
$x^2-4 \neq 0$
$(x+2)(x-2) \neq 0$
$x \neq -2$ or 2
Also, the radicand must be non-negative:
$x+1 \geq 0$
$x \geq -1$
Domain: $[-1,2) \cup (2,\infty)$

29. $f(x) = \dfrac{x}{\sqrt{x+8}}$
The radicand must be non-negative and not zero:
$x+8 > 0$
$x > -8$
Domain: $\{x \mid x > -8\}$

30. $f(x) = -2x^2 + x + 1$

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{-2(x+h)^2 + (x+h) + 1 - (-2x^2+x+1)}{h}$$
$$= \frac{-2(x^2+2xh+h^2) + x + h + 1 + 2x^2 - x - 1}{h}$$
$$= \frac{-2x^2 - 4xh - 2h^2 + x + h + 1 + 2x^2 - x - 1}{h}$$
$$= \frac{-4xh - 2h^2 + h}{h} = \frac{h(-4x-2h+1)}{h}$$
$$= -4x - 2h + 1$$

31. **a.** Domain: $\{x \mid -4 \leq x \leq 3\}$; $[-4, 3]$
Range: $\{y \mid -3 \leq y \leq 3\}$; $[-3, 3]$

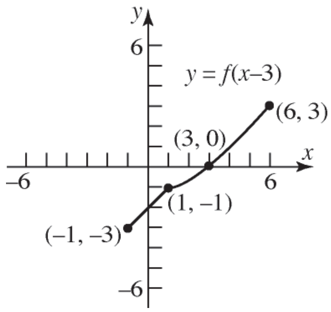
b. Intercept: $(0,0)$

c. $f(-2) = -1$

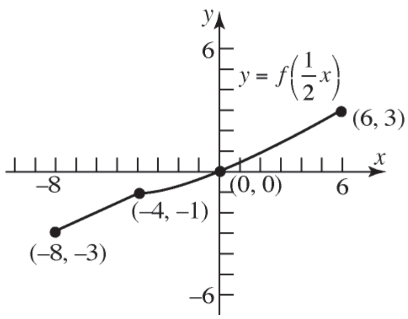
d. $f(x) = -3$ when $x = -4$

e. $f(x) > 0$ when $0 < x \leq 3$
$\{x \mid 0 < x \leq 3\}$

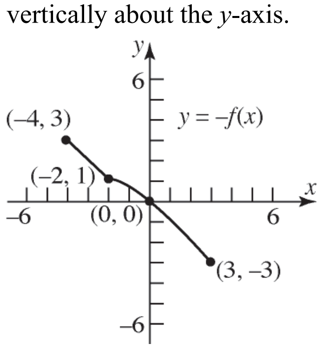
f. To graph $y = f(x-3)$, shift the graph of f horizontally 3 units to the right.



g. To graph $y = f\left(\frac{1}{2}x\right)$, stretch the graph of f horizontally by a factor of 2.



h. To graph $y = -f(x)$, reflect the graph of f vertically about the y-axis.



32. a. Domain: $(-\infty, 4]$

Range: $(-\infty, 3]$

b. Increasing: $(-\infty, -2)$ and $(2, 4)$;

Decreasing: $(-2, 2)$

c. Local minimum is -1 at $x = 2$;
Local maximum is 1 at $x = -2$

d. No absolute minimum;
Absolute maximum is 3 at $x = 4$

e. The graph has no symmetry.

f. The function is neither.

g. x-intercepts: $-3, 0, 3$;
y-intercept: 0

33. $f(x) = x^3 - 4x$

$f(-x) = (-x)^3 - 4(-x) = -x^3 + 4x$

$= -\left(x^3 - 4x\right) = -f(x)$

f is odd.

34. $g(x) = \dfrac{4 + x^2}{1 + x^4}$

$g(-x) = \dfrac{4 + (-x)^2}{1 + (-x)^4} = \dfrac{4 + x^2}{1 + x^4} = g(x)$

g is even.

35. $G(x) = 1 - x + x^3$

$G(-x) = 1 - (-x) + (-x)^3$

$= 1 + x - x^3 \neq -G(x)$ or $G(x)$

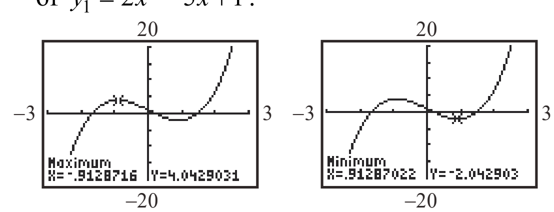
G is neither even nor odd.

36. $f(x) = \dfrac{x}{1 + x^2}$

$f(-x) = \dfrac{-x}{1 + (-x)^2} = \dfrac{-x}{1 + x^2} = -f(x)$

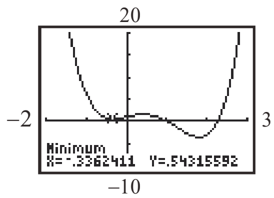
f is odd.

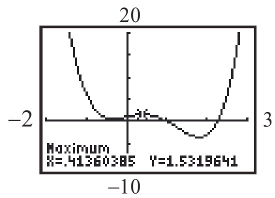
37. $f(x) = 2x^3 - 5x + 1$ on the interval $(-3, 3)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = 2x^3 - 5x + 1$.

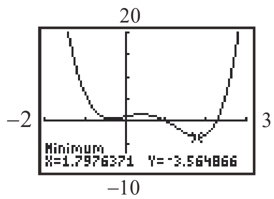


local maximum: 4.04 when $x \approx -0.91$
local minimum: -2.04 when $x = 0.91$
f is increasing on: $[-3, -0.91]$ and $[0.91, 3]$;
f is decreasing on: $[-0.91, 0.91]$.

38. $f(x) = 2x^4 - 5x^3 + 2x + 1$ on the interval $(-2, 3)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = 2x^4 - 5x^3 + 2x + 1$.







local maximum: 1.53 when $x = 0.41$
local minima: 0.54 when $x = -0.34$, -3.56 when $x = 1.80$
f is increasing on: $[-0.34, 0.41]$ and $[1.80, 3]$;
f is decreasing on: $[-2, -0.34]$ and $[0.41, 1.80]$.

39. $f(x) = 8x^2 - x$

a. $$\frac{f(2)-f(1)}{2-1} = \frac{8(2)^2 - 2 - [8(1)^2 - 1]}{1} = 32 - 2 - (7) = 23$$

b. $$\frac{f(1)-f(0)}{1-0} = \frac{8(1)^2 - 1 - [8(0)^2 - 0]}{1} = 8 - 1 - (0) = 7$$

c. $$\frac{f(4)-f(2)}{4-2} = \frac{8(4)^2 - 4 - [8(2)^2 - 2]}{2} = \frac{128 - 4 - (30)}{2} = \frac{94}{2} = 47$$

40. $f(x) = 2 - 5x$

$$\frac{f(3)-f(2)}{3-2} = \frac{[2-5(3)]-[2-5(2)]}{3-2} = \frac{(2-15)-(2-10)}{1} = -13-(-8) = -5$$

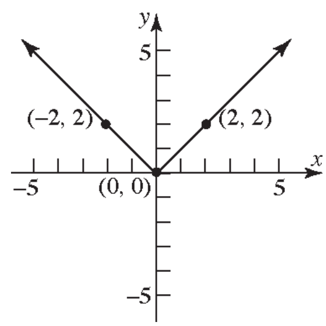
41. $f(x) = 3x - 4x^2$

$$\frac{f(3)-f(2)}{3-2} = \frac{\left[3(3)-4(3)^2\right]-\left[3(2)-4(2)^2\right]}{3-2} = \frac{(9-36)-(6-16)}{1} = -27+10 = -17$$

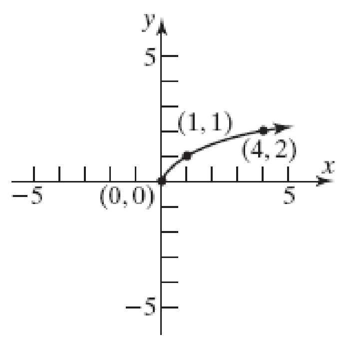
42. The graph does not pass the Vertical Line Test and is therefore not a function.

43. The graph passes the Vertical Line Test and is therefore a function.

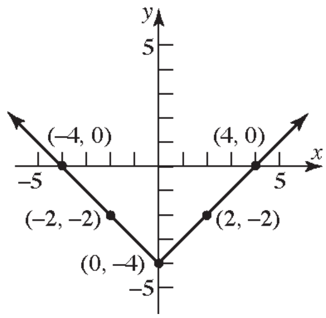
44. $f(x) = |x|$



45. $f(x) = \sqrt{x}$



46. $F(x) = |x| - 4$. Using the graph of $y = |x|$, vertically shift the graph downward 4 units.

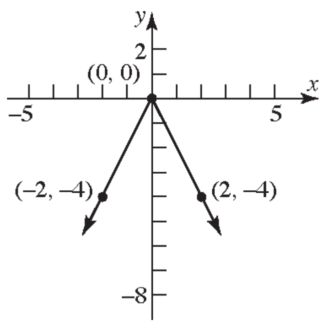


Intercepts: (–4,0), (4,0), (0,–4)

Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \geq -4\}$ or $[-4, \infty)$

47. $g(x) = -2|x|$. Reflect the graph of $y = |x|$ about the x-axis and vertically stretch the graph by a factor of 2.

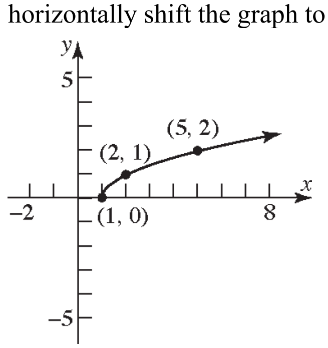


Intercepts: (0, 0)

Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \leq 0\}$ or $(-\infty, 0]$

48. $h(x) = \sqrt{x-1}$. Using the graph of $y = \sqrt{x}$, horizontally shift the graph to the right 1 unit.

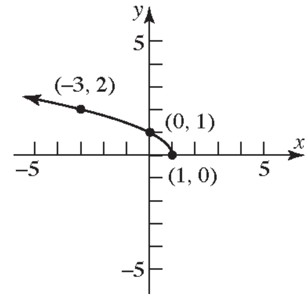


Intercept: (1, 0)

Domain: $\{x \mid x \geq 1\}$ or $[1, \infty)$

Range: $\{y \mid y \geq 0\}$ or $[0, \infty)$

49. $f(x) = \sqrt{1-x} = \sqrt{-(x-1)}$. Reflect the graph of $y = \sqrt{x}$ about the y-axis and horizontally shift the graph to the right 1 unit.

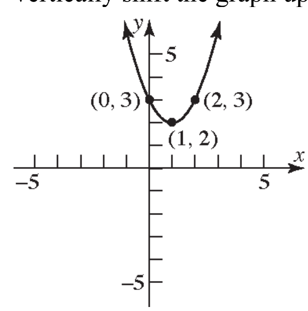


Intercepts: (1, 0), (0, 1)

Domain: $\{x \mid x \leq 1\}$ or $(-\infty, 1]$

Range: $\{y \mid y \geq 0\}$ or $[0, \infty)$

50. $h(x) = (x-1)^2 + 2$. Using the graph of $y = x^2$, horizontally shift the graph to the right 1 unit and vertically shift the graph up 2 units.



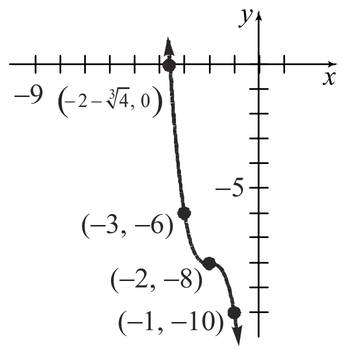
Intercepts: (0, 3)

Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \geq 2\}$ or $[2, \infty)$

51. $g(x) = -2(x+2)^3 - 8$

Using the graph of $y = x^3$, horizontally shift the graph to the left 2 units, vertically stretch the graph by a factor of 2, reflect about the x-axis, and vertically shift the graph down 8 units.



Intercepts: (0,–24), $\left(-2-\sqrt[3]{4}, 0\right) \approx (-3.6, 0)$

Domain: $\{x \mid x \text{ is any real number}\}$

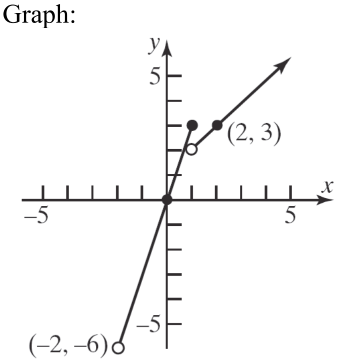
Range: $\{y \mid y \text{ is any real number}\}$

52. $f(x)=\begin{cases} 3x & \text{if } -2<x\le 1 \\ x+1 & \text{if } x>1 \end{cases}$

a. Domain: $\{x \mid x>-2\}$ or $(-2, \infty)$

b. Intercept: $(0,0)$

c. Graph:



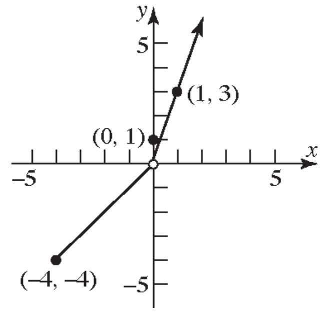
d. Range: $\{y \mid y>-6\}$ or $(-6, \infty)$

53. $f(x)=\begin{cases} x & \text{if } -4\le x<0 \\ 1 & \text{if } x=0 \\ 3x & \text{if } x>0 \end{cases}$

a. Domain: $\{x \mid x\ge -4\}$ or $[-4, \infty)$

b. Intercept: (0, 1)

c. Graph:



d. Range: $\{y \mid y\ge -4,\ y\ne 0\}$

54. $f(x)=\dfrac{Ax+5}{6x-2}$ and $f(1)=4$

$$\frac{A(1)+5}{6(1)-2}=4$$

$$\frac{A+5}{4}=4$$

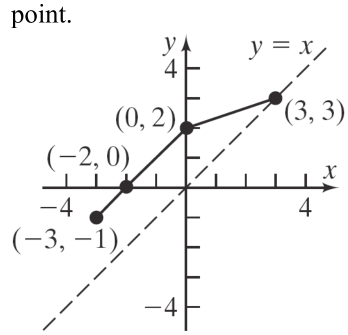
$$A+5=16$$

$$A=11$$

55. a. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

b. The inverse is $\{(2,1),(5,3),(8,5),(10,6)\}$.

56. The function *f* is one-to-one because every horizontal line intersects the graph at exactly one point.



57. $f(x)=5x-10;\ g(x)=\frac{1}{5}x+2$

$$f(g(x))=f\left(\frac{1}{5}x+2\right)$$
$$=5\left(\frac{1}{5}x+2\right)-10=x+10-10=x$$
$$g(f(x))=g(5x-10)$$
$$=\frac{1}{5}(5x-10)+2=x-2+2=x$$

Thus, f and g are inverses of each other.

58. $f(x)=\frac{x-4}{x};\ g(x)=\frac{4}{1-x}$

$$f(g(x))=f\left(\frac{4}{1-x}\right)$$
$$=\frac{\frac{4}{1-x}-4}{\frac{4}{1-x}}=\frac{4-4(1-x)}{4}$$
$$=\frac{4-4+4x}{4}=x,\ x\neq 1$$
$$g(f(x))=g\left(\frac{x-4}{x}\right)$$
$$=\frac{4}{1-\frac{x-4}{x}}=\frac{4x}{x-(x-4)}$$
$$=\frac{4x}{4}=x,\ x\neq 0$$

Thus, f and g are inverses of each other.

59. $f(x)=\frac{2x+3}{5x-2}$

$$y=\frac{2x+3}{5x-2}$$
$$x=\frac{2y+3}{5y-2}\quad \text{Inverse}$$
$$x(5y-2)=2y+3$$
$$5xy-2x=2y+3$$
$$5xy-2y=2x+3$$
$$y(5x-2)=2x+3$$
$$y=\frac{2x+3}{5x-2}$$
$$f^{-1}(x)=\frac{2x+3}{5x-2}$$

Domain of f = Range of f^{-1}

= All real numbers except $\frac{2}{5}$.

Range of f = Domain of f^{-1}

= All real numbers except $\frac{2}{5}$.

60. $f(x)=\frac{1}{x-1}$

$$y=\frac{1}{x-1}$$
$$x=\frac{1}{y-1}\quad \text{Inverse}$$
$$x(y-1)=1$$
$$xy-x=1$$
$$xy=x+1$$
$$y=\frac{x+1}{x}$$
$$f^{-1}(x)=\frac{x+1}{x}$$

Domain of f = Range of f^{-1}

= All real numbers except 1

Range of f = Domain of f^{-1}

= All real numbers except 0

61. $f(x)=\sqrt{x-2}$

$$y=\sqrt{x-2}$$
$$x=\sqrt{y-2}\quad \text{Inverse}$$
$$x^2=y-2\quad x\geq 0$$
$$y=x^2+2\quad x\geq 0$$
$$f^{-1}(x)=x^2+2\quad x\geq 0$$

Domain of f = Range of f^{-1} $=\{x\mid x\geq 2\}$ or $[2,\infty)$

Range of f = Domain of f^{-1} $=\{x\mid x\geq 0\}$ or $[0,\infty)$

62. $f(x)=x^{1/3}+1$

$$y=x^{1/3}+1$$
$$x=y^{1/3}+1\quad \text{Inverse}$$
$$y^{1/3}=x-1$$
$$y=(x-1)^3$$
$$f^{-1}(x)=(x-1)^3$$

Domain of f = Range of f^{-1}

= All real numbers or $(-\infty,\infty)$

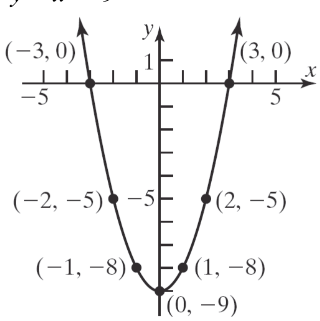
Range of f = Domain of f^{-1}
= All real numbers or $(-\infty, \infty)$

Chapter 1 Test

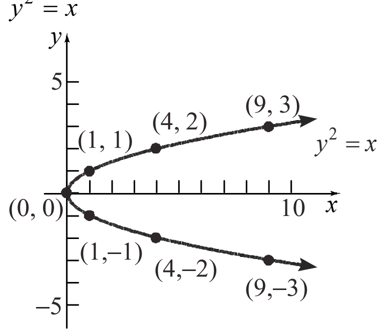
1. $d(P_1, P_2) = \sqrt{(5-(-1))^2 + (-1-3)^2}$
$= \sqrt{6^2 + (-4)^2}$
$= \sqrt{36+16}$
$= \sqrt{52} = 2\sqrt{13}$

2. The coordinates of the midpoint are:
$(x, y) = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$
$= \left(\dfrac{-1+5}{2}, \dfrac{3+(-1)}{2}\right)$
$= \left(\dfrac{4}{2}, \dfrac{2}{2}\right)$
$= (2, 1)$

3. $y = x^2 - 9$



4. $y^2 = x$



5. $x^2 + y = 9$

x-intercepts: y-intercept:

$x^2 + 0 = 9$ $\quad\quad (0)^2 + y = 9$
$x^2 = 9$ $\quad\quad y = 9$
$x = \pm 3$

The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, 9)$.

Test x-axis symmetry: Let $y = -y$
$x^2 + (-y) = 9$
$x^2 - y = 9$ different

Test y-axis symmetry: Let $x = -x$
$(-x)^2 + y = 9$
$x^2 + y = 9$ same

Test origin symmetry: Let $x = -x$ and $y = -y$
$(-x)^2 + (-y) = 9$
$x^2 - y = 9$ different

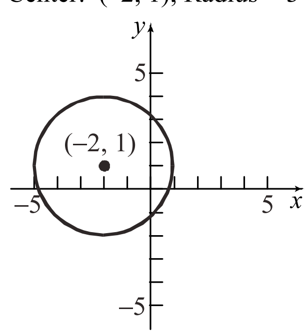
Therefore, the graph will have y-axis symmetry.

6. $(x-h)^2 + (y-k)^2 = r^2$
$(x-4)^2 + (y-(-3))^2 = 5^2$
$(x-4)^2 + (y+3)^2 = 25$

General form: $(x-4)^2 + (y+3)^2 = 25$
$x^2 - 8x + 16 + y^2 + 6y + 9 = 25$
$x^2 + y^2 - 8x + 6y = 0$

7. $x^2 + y^2 + 4x - 2y - 4 = 0$
$x^2 + 4x + y^2 - 2y = 4$
$(x^2 + 4x + 4) + (y^2 - 2y + 1) = 4 + 4 + 1$
$(x+2)^2 + (y-1)^2 = 3^2$

Center: (–2, 1); Radius = 3



8. a. $\{(2,5), (4,6), (6,7), (8,8)\}$
This relation is a function because there are no ordered pairs that have the same first element and different second elements.

Domain: $\{2,4,6,8\}$

Range: $\{5,6,7,8\}$

b. $\{(1,3),(4,-2),(-3,5),(1,7)\}$
This relation is not a function because there are two ordered pairs that have the same first element but different second elements.

c. This relation is not a function because the graph fails the vertical line test.

d. This relation is a function because it passes the vertical line test.
Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \geq 2\}$ or $[2, \infty)$

9. $f(x)=\sqrt{4-5x}$
The function tells us to take the square root of $4-5x$. Only nonnegative numbers have real square roots so we need $4-5x \geq 0$.

$$4-5x \geq 0$$
$$4-5x-4 \geq 0-4$$
$$-5x \geq -4$$
$$\frac{-5x}{-5} \leq \frac{-4}{-5}$$
$$x \leq \frac{4}{5}$$

Domain: $\left\{x \,\middle|\, x \leq \frac{4}{5}\right\}$ or $\left(-\infty, \frac{4}{5}\right]$

$f(-1)=\sqrt{4-5(-1)}=\sqrt{4+5}=\sqrt{9}=3$

10. $g(x)=\dfrac{x+2}{|x+2|}$

The function tells us to divide $x+2$ by $|x+2|$. Division by 0 is undefined, so the denominator can never equal 0. This means that $x \neq -2$.
Domain: $\{x \mid x \neq -2\}$

$$g(-1)=\frac{(-1)+2}{|(-1)+2|}=\frac{1}{|1|}=1$$

11. $h(x)=\dfrac{x-4}{x^2+5x-36}$
The function tells us to divide $x-4$ by $x^2+5x-36$. Since division by 0 is not defined, we need to exclude any values which make the denominator 0.

$$x^2+5x-36=0$$
$$(x+9)(x-4)=0$$
$$x=-9 \text{ or } x=4$$

Domain: $\{x \mid x \neq -9, x \neq 4\}$
(note: there is a common factor of $x-4$ but we must determine the domain prior to simplifying)

$$h(-1)=\frac{(-1)-4}{(-1)^2+5(-1)-36}=\frac{-5}{-40}=\frac{1}{8}$$

12. a. To find the domain, note that all the points on the graph will have an *x*-coordinate between -5 and 5, inclusive. To find the range, note that all the points on the graph will have a *y*-coordinate between -3 and 3, inclusive.
Domain: $\{x \mid -5 \leq x \leq 5\}$ or $[-5, 5]$

Range: $\{y \mid -3 \leq y \leq 3\}$ or $[-3, 3]$

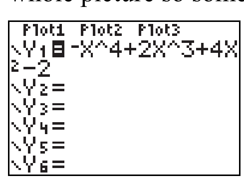
b. The intercepts are $(0,2)$, $(-2,0)$, and $(2,0)$.
x-intercepts: -2, 2
y-intercept: 2

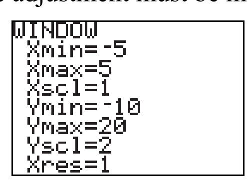
c. $f(1)$ is the value of the function when $x=1$. According to the graph, $f(1)=3$.

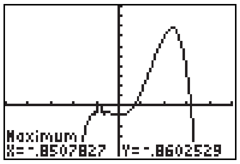
d. Since $(-5,-3)$ and $(3,-3)$ are the only points on the graph for which $y=f(x)=-3$, we have $f(x)=-3$ when $x=-5$ and $x=3$.

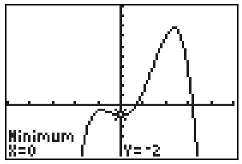
e. To solve $f(x)<0$, we want to find *x*-values such that the graph is below the x-axis. The graph is below the x-axis for values in the domain that are less than -2 and greater than 2. Therefore, the solution set is $\{x \mid -5 \leq x < -2 \text{ or } 2 < x \leq 5\}$. In interval notation we would write the solution set as $[-5,-2) \cup (2,5]$.

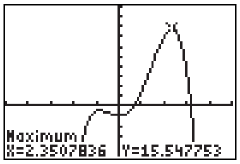
13. $f(x)=-x^4+2x^3+4x^2-2$
We set Xmin = -5 and Xmax = 5. The standard Ymin and Ymax will not be good enough to see the whole picture so some adjustment must be made.











We see that the graph has a local maximum of -0.86 (rounded to two places) when $x=-0.85$ and another local maximum of 15.55 when $x=2.35$. There is a local minimum of -2 when $x=0$. Thus, we have

Local maxima: $f(-0.85)\approx -0.86$

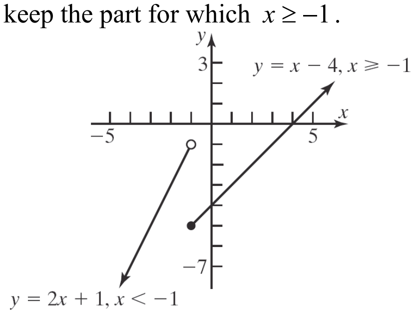
$f(2.35)\approx 15.55$

Local minima: $f(0)=-2$

The function is increasing on the intervals $(-5,-0.85)$ and $(0,2.35)$ and decreasing on the intervals $(-0.85,0)$ and $(2.35,5)$.

14. **a.** $f(x)=\begin{cases} 2x+1 & x<-1 \\ x-4 & x\geq -1 \end{cases}$

To graph the function, we graph each "piece". First we graph the line $y=2x+1$ but only keep the part for which $x<-1$. Then we plot the line $y=x-4$ but only keep the part for which $x\geq -1$.



b. To find the intercepts, notice that the only piece that hits either axis is $y=x-4$.

$y=x-4$	$y=x-4$
$y=0-4$	$0=x-4$
$y=-4$	$4=x$

The intercepts are $(0,-4)$ and $(4,0)$.

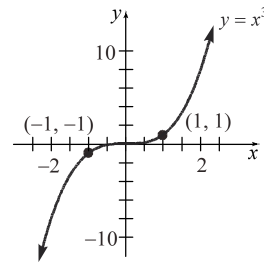
c. To find $g(-5)$ we first note that $x=-5$ so we must use the first "piece" because $-5<-1$.
$g(-5)=2(-5)+1=-10+1=-9$

d. To find $g(2)$ we first note that $x=2$ so we must use the second "piece" because $2\geq -1$.
$g(2)=2-4=-2$

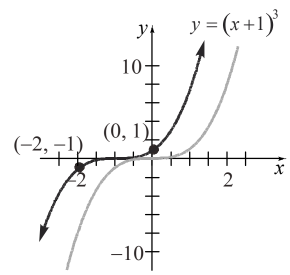
15. The average rate of change from 3 to 4 is given by

$$\frac{\Delta y}{\Delta x}=\frac{f(4)-f(3)}{4-3}$$
$$=\frac{\left(3(4)^2-2(4)+4\right)-\left(3(3)^2-2(3)+4\right)}{4-3}$$
$$=\frac{44-25}{4-3}=\frac{19}{1}=19$$

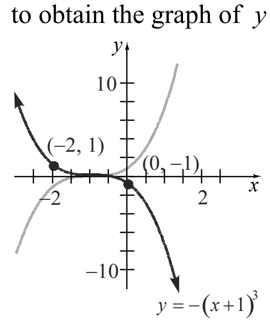
16. **a.** The basic function is $y=x^3$ so we start with the graph of this function.



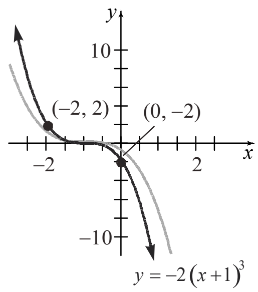
Next we shift this graph 1 unit to the left to obtain the graph of $y=(x+1)^3$.



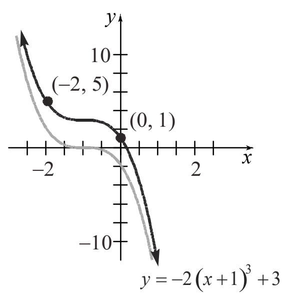
Next we reflect this graph about the x-axis to obtain the graph of $y=-(x+1)^3$.



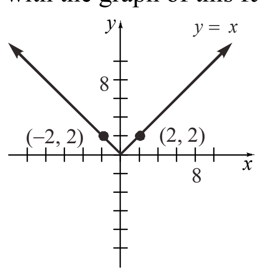
Next we stretch this graph vertically by a factor of 2 to obtain the graph of $y = -2(x+1)^3$.



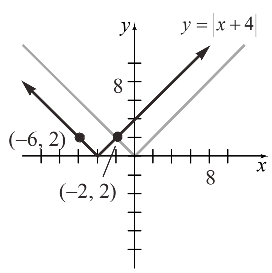
The last step is to shift this graph up 3 units to obtain the graph of $y = -2(x+1)^3 + 3$.



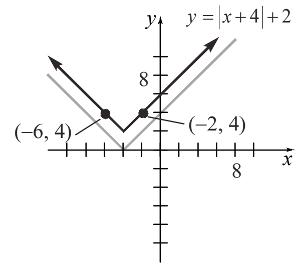
b. The basic function is $y = |x|$ so we start with the graph of this function.



Next we shift this graph 4 units to the left to obtain the graph of $y = |x+4|$.



Next we shift this graph up 2 units to obtain the graph of $y = |x+4| + 2$.



17.
$$f(x) = \frac{2}{3x-5}$$
$$y = \frac{2}{3x-5}$$
$$x = \frac{2}{3y-5} \quad \text{Inverse}$$
$$x(3y-5) = 2$$
$$3xy - 5x = 2$$
$$3xy = 5x + 2$$
$$y = \frac{5x+2}{3x}$$
$$f^{-1}(x) = \frac{5x+2}{3x}$$

Domain of f = Range of f^{-1}
$= \left\{x \mid x \neq \frac{5}{3}\right\}$.

Range of f = Domain of f^{-1}
$= \{x \mid x \neq 0\}$

18. If the point $(3, -5)$ is on the graph of *f*, then the point $(-5,\ 3)$ must be on the graph of f^{-1}.

Chapter 1 Projects

Project I – Internet-based Project – Answers will vary

Project II

1. Silver: $C(x) = 20 + 0.16(x - 200) = 0.16x - 12$

$$C(x) = \begin{cases} 20 & 0 \le x \le 200 \\ 0.16x - 12 & x > 200 \end{cases}$$

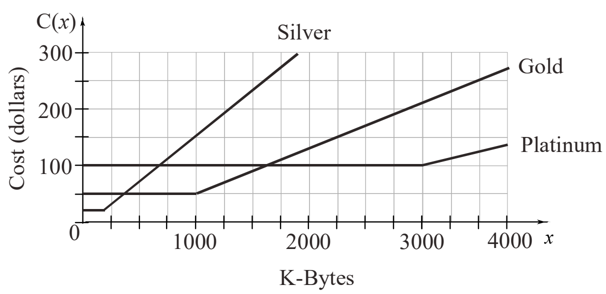
Gold: $C(x) = 50 + 0.08(x - 1000) = 0.08x - 30$

$$C(x) = \begin{cases} 50.00 & 0 \le x \le 1000 \\ 0.08x - 30 & x > 1000 \end{cases}$$

Platinum: $C(x) = 100 + 0.04(x - 3000)$

$$= 0.04x - 20$$

$$C(x) = \begin{cases} 100.00 & 0 \le x \le 3000 \\ 0.04x - 20 & x > 3000 \end{cases}$$



3. Let y = #K-bytes of service over the plan minimum.

Silver: $20 + 0.16y \le 50$

$$0.16y \le 30$$

$$y \le 187.5$$

Silver is the best up to $187.5 + 200 = 387.5$ K-bytes of service.

Gold: $50 + 0.08y \le 100$

$$0.08y \le 50$$

$$y \le 625$$

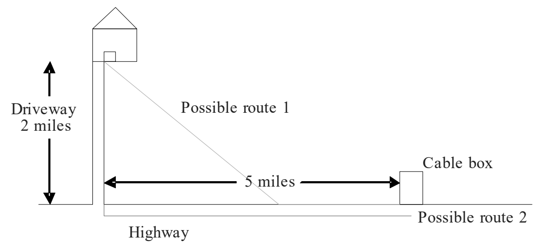
Gold is the best from 387.5 K-bytes to $625 + 1000 = 1625$ K-bytes of service.

Platinum: Platinum will be the best if more than 1625 K-bytes is needed.

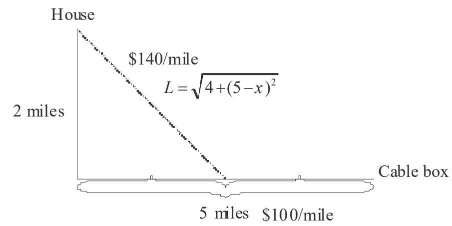
4. Answers will vary.

Project III

1.



2.



$C(x) = 100x + 140L$

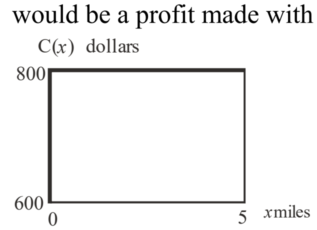
$C(x) = 100x + 140\sqrt{4 + (5 - x)^2}$

3.

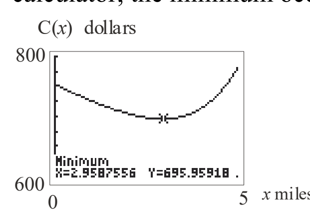
x	$C(x)$
0	$100(0) + 140\sqrt{4+25} \approx \$753.92$
1	$100(1) + 140\sqrt{4+16} \approx \$726.10$
2	$100(2) + 140\sqrt{4+9} \approx \$704.78$
3	$100(3) + 140\sqrt{4+4} \approx \$695.98$
4	$100(4) + 140\sqrt{4+1} \approx \$713.05$
5	$100(5) + 140\sqrt{4+0} = \$780.00$

The choice where the cable goes 3 miles down the road then cutting up to the house seems to yield the lowest cost.

4. Since all of the costs are less than \$800, there would be a profit made with any of the plans.



Using the MINIMUM function on a graphing calculator, the minimum occurs at $x \approx 2.96$.



The minimum cost occurs when the cable runs for 2.96 mile along the road.

6. $C(4.5) = 100(4.5) + 140\sqrt{4+(5-4.5)^2}$
   $\approx \$738.62$
   The cost for the Steven's cable would be $738.62.

7. $5000(738.62) = \$3{,}693{,}100$ State legislated
   $5000(695.96) = \$3{,}479{,}800$ cheapest cost
   It will cost the company $213,300 more.

Project IV

1. $A = \pi r^2$

2. $r = 2.2t$

3. $r = 2.2(2) = 4.4$ ft
   $r = 2.2(2.5) = 5.5$ ft

4. $A = \pi(4.4)^2 = 60.82\text{ ft}^2$
   $A = \pi(5.5)^2 = 95.03\text{ ft}^2$

5. $A = \pi(2.2t)^2 = 4.84\pi t^2$

6. $A = 4.84\pi(2)^2 = 60.82\text{ ft}^2$
   $A = 4.84\pi(2.5)^2 = 95.03\text{ ft}^2$

7. $\dfrac{A(2.5)-A(2)}{2.5-2} = \dfrac{95.03-60.82}{0.5} = 68.42$ ft/hr

8. $\dfrac{A(3.5)-A(3)}{3.5-3} = \dfrac{186.27-136.85}{0.5} = 98.84$ ft/hr

9. The average rate of change is increasing.

10. 150 yds = 450 ft
    $r = 2.2t$
    $t = \dfrac{450}{2.2} = 204.5$ hours

11. 6 miles = 31680 ft
    Therefore, we need a radius of 15,840 ft.
    $t = \dfrac{15{,}840}{2.2} = 7200$ hours