Mathematics in Action 6th edition Consortium for Foundation Mathematics Solutions Manual

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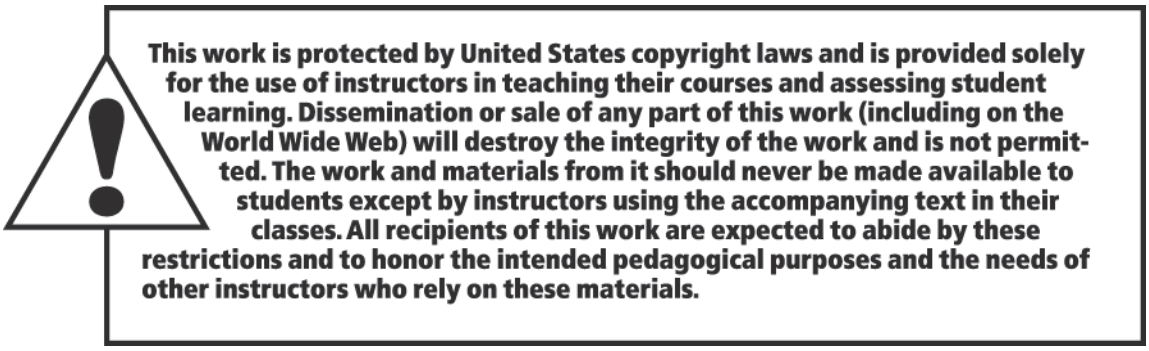
INSTRUCTOR'S RESOURCE MANUAL WITH TESTS

MATHEMATICS IN ACTION:
AN INTRODUCTION TO ALGEBRAIC, GRAPHICAL, AND NUMERICAL PROBLEM SOLVING
SIXTH EDITION

The Consortium for Foundation Mathematics

Pearson



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FOREWORD

Mathematics in Action: An Introduction to Algebraic, Graphical, and Numerical Problem Solving, Sixth Edition, aims to empower underprepared college students by providing them with a true mathematical literacy in the real world, and simultaneously, helping them build a secure foundation for future study in mathematics and other quantitative disciplines. The *Mathematics in Action* series grew out of a National Science Foundation grant (DUE 9455638) that enabled fourteen faculty, primarily from the State University of New York and the City University of New York systems, to work together over a three-year period to design a new curriculum for the traditional elementary, intermediate and college algebra sequence. The rationale for such a project had already been articulated by national organizations and professional societies from the National Research Council's *Everybody Counts* to the NCTM's *Standard*s and AMATYC's *Crossroads*. In addition, NSF was funding a massive reform effort on the calculus level that would necessarily affect the courses that feed into calculus and those which follow.

The need for reform was also evident to those of us who teach foundation college mathematics. The diversity of our student body – returning adults, newly-arrived immigrants with English language deficiencies, high-school graduates with a history of poor performance – coupled with the higher level quantitative literacy required of them in the workplace called for an honest and thorough *re-evaluation* of desired student outcomes. The fourteen co-authors agreed to formulate a wish list of topics to be included in building a curriculum from scratch. One basic principle to which we would adhere was that students learn mathematics best by actively *doing* mathematics within a clear and meaningful context.

The original two textbooks in the *Mathematics in Action* series*: An Introduction to Algebraic, Graphical, and Numerical Problem Solving* and *Algebraic, Graphical, and Trigonometric Problem Solving* were the results of a five-year cycle of writing, meeting, piloting, and revising. Because of the success of these textbooks, we have now released fifth editions with updated data and features intended to further help students develop problem-solving skills.

The best one-word description of these texts is *interactive* – short on narrative and templates, and long on structured prompts and thoughtful questions. The authors have carefully attempted to bridge the gap between abstraction and application by encouraging students to construct, apply, and describe their own mathematical models as they solve meaningful problems. Students are challenged to approach the material from multiple perspectives and to share their insights and strategies with one another. The language and varied contexts are geared to adults and build upon their familiarity and sophistication with the world in which we live. You will also discover that the unique "activity" structure of the text is unparalleled in supporting your efforts to foster active learning in a stimulating and cooperative environment.

Activities are presented in "clusters" within each chapter. Each cluster contains a series of learning activities that introduce or reinforce key concepts and skills. Each cluster concludes with two sections: What Have I Learned? and How Can I Practice? The What Have I Learned? exercises are designed to help students synthesize the main ideas of the cluster. The How Can I Practice? exercises are designed to provide additional reinforcement of the numeric and algebraic skills of the cluster. Taken as a whole, these exercises give students the tools they need to bridge the gaps between abstraction, skills, and application. Each chapter also ends with a Gateway review, providing students with an opportunity to check their understanding of the chapter's concepts and skills.

Features that make definitions, procedures and properties more visible to the student include:

- A list of objectives that opens each activity. The objectives also appear in the Table of Contents.
- Activities containing examples that are worked out in great detail.
- Chapter Summaries that list the concepts and skills covered in the chapter with a corresponding description and example. Each concept or skill includes an activity number for reference.
- Activity questions that are numbered for easy reference.
- Definitions, properties, and procedures that are offset and identified.
- Exercise sets at the end of each activity that reinforce topics and provide an appropriate level of difficulty for an elementary or intermediate algebra course.
- Geometry problems that are integrated throughout the text.

CHANGES FROM THE FIFTH EDITION

The Sixth Edition retains all the features of the previous edition, with the following content changes:

- All the data-based activities and exercises have been updated to reflect the most recent information and/or replaced with more relevant topics.
- The introductory scenarios in several activities have been replaced with more robust, up-to-date situations.
- Several new real-world exercises have been added throughout.
- The exposition and treatment of topics has been carefully reviewed and revised/rewritten where necessary to provide students with a more clear and easy to understand presentation.
- New PowerPoint presentations have been developed to support instructors looking to implement the contextual approach to introductory algebra.
- New videos have been developed to support students looking for more assistance on a concept.
- Learning Catalytics questions have been developed for nearly every activity, providing an opportunity for instructors to quickly assess the progress on a given concept and give students an opportunity to use technology as an interactive learning tool.

STUDENT OUTCOMES

In creating a curriculum for an introductory or intermediate algebra course, we found that most of our desired student outcomes fell naturally into five main areas:

- Developing a number sense
- Developing a symbolic sense
- Developing a general function sense
- Developing a thorough linear function sense
- Introducing a sense of nonlinear (especially quadratic) relationships

The word “sense” in each of these areas conveys far more than requisite skills development. It also entails developing mathematical intuition and building techniques of reasoning. Continuously interpreting results demonstrates the relevance of mathematics in everyday life and requires active

participation by every student. Practice with expressing mathematical ideas in words will develop skills that will be very valuable when the student leaves the classroom.

PERFORMANCE OBJECTIVES

At the conclusion of the course and in terms of specific mathematical performance, we expect students will be able to:

- Extract relevant data and solve real-life problems that involve computations with whole numbers, fractions, decimals, and percents.
- Analyze and interpret graphical and tabular data.
- Estimate and determine the magnitude of quantities before formal computation and compare the estimated and computed values for consistency.
- Express numerical quantities, including variables, in meaningful units.
- Evaluate formulas from a variety of disciplines, using calculators when appropriate.
- Interpret an algebraic expression in one variable as an explicit sequence of arithmetic operations to be performed on that variable.
- Recognize and express verbally, numerically, graphically, and symbolically the patterns displayed by linear data.
- Recognize equivalent linear relationships numerically, algebraically, and graphically and translate from one representation to any of the others.
- Solve linear equations and simple systems of linear equations.
- Recognize polynomial functions of degree two or higher, exponential functions, and inverse variation functions as nonlinear.
- Solve quadratic equations by factoring and the quadratic formula.
- Identify and define the basic vocabulary and notation of functions: input, output, domain, range, increasing function, decreasing function, rate of change.

GENERAL EDUCATION COMPETENCIES

Mathematics in Action promotes the development of general competencies because it presents mathematics in realistic contextual situations. By the end of the course, we expect that students will be able to:

- Use proportional reasoning appropriately in contextual applications.
- Read technical material with facility to determine information relevant to the solution of a problem.
- Recognize that solution processes are multifaceted, and be able to use the necessary logic and formulas in a path to a solution.
- Communicate solutions to problems verbally and in writing, using logical and clear sequences of thought.
- Express solutions in writing using appropriate vocabulary, sentence structure, and persuasion.
- Demonstrate an improved comfort level when applying critical thinking and problem solving skills.
- Work collaboratively with others to enhance mathematical understanding through problem solving.
- Appreciate the importance of mathematics in everyday life and in the workplace.

SECTION 1 OVERVIEW

RESOURCES IN THIS GUIDE

We hope that the content and format of this *Instructor's Resource Manual with Tests* will be of great help to you in teaching with *Mathematics in Action*. This section of the *Manual* is an annotated directory to the resources addressed at length in the other sections. What is available to you in this *Guide* is described under the following headings.

- Textbook Content and Structure
- Supplements available with the textbook
- Pedagogy: Learning in Groups and Writing to Learn
- Technology: Scientific Calculators, Graphing Calculators, and Computer Programs
- Suggestions for Syllabi: Annotated List of the Major Mathematical Topics Covered in the Text
- Sample Course Outlines
- Assessment: Group Oral and Written Tests, Journals, Formal Writing Assignments, Traditional Testing, Basic Skills, Portfolios

Mathematics in Action is adaptable to the various needs of courses at the introductory foundation level and we offer you an annotated list of the major mathematical topics covered in the text as the best guide to determining the content of your course. We also offer sample course syllabi, one for an elementary algebra course and one for an intermediate level algebra course. Please note that in planning your course, supplemental teaching and learning aids are available to you and your students. These supplements are listed in this Overview Section for your convenience.

We organized and formatted this *Instructor's Resource Manual with Tests* so, if you choose, you can use the sample graphing calculator experiments, journal assignments, writing assignments, skills worksheets, tests, and exams as presented.

TEXTBOOK CONTENT AND STRUCTURE

The text is divided into four chapters followed by appendices and a glossary.

- Chapter 1: Number Sense
- Chapter 2: Variable Sense
- Chapter 3: Function Sense and Linear Functions
- Chapter 4: An Introduction to Nonlinear Problem Solving

Each chapter is subdivided into sections we call clusters. Each cluster focuses on a major mathematical topic within the context of a chapter. For example, Chapter 1 Cluster 3 focuses on comparisons and proportional reasoning and Chapter 4 Cluster 2 concentrates on problem solving using quadratic equations and functions.

There are thirteen clusters in all; four in Chapter 1, three in Chapter 2, three in Chapter 3 and three in Chapter 4. The clusters are listed here to give you a convenient overview.

Chapter 1 Number Sense

- Cluster 1 Introduction to Problem Solving
- Cluster 2 Problem Solving with Fractions and Decimals (Rational Numbers)
- Cluster 3 Comparisons and Proportional Reasoning
- Cluster 4 Problem Solving with Signed Numbers

Chapter 2 Variable Sense

- Cluster 1 Symbolic Rules and Expressions
- Cluster 2 Solving Equations
- Cluster 3 More Problem Solving using Algebra

Chapter 3 Function Sense and Linear Functions

- Cluster 1 Function Sense
- Cluster 2 Introduction to Linear Functions
- Cluster 3 Linear Regression, Systems, and Inequalities

Chapter 4 An Introduction to Nonlinear Problem Solving

- Cluster 1 Mathematical Modeling Involving Polynomials
- Cluster 2 Problem Solving with Quadratic Equations and Functions
- Cluster 3 Other Nonlinear Functions

Each cluster is further divided into sections named

- *Activities* (with Exercises)
- *What Have I Learned*?
- *How Can I Practice*?

Activities are the essence of *Mathematics in Action.* They are where mathematical concepts and skills are introduced and developed in meaningful contextual problems and situations. We expect that students will see for themselves the need to master arithmetic and algebra skills, and be able to answer their own frequently asked questions, "Why do I need to learn this?" and "When will I ever use this outside this class?"

In *Activities*, students respond to a series of structured prompts and questions that direct them to recall and apply mathematical knowledge they already have, and that lead them to acquire the other concepts and skills they must have to be successful problem solvers. Basic arithmetic and algebra skills are presented as they are needed in the contextual problems. *Activities* also include exercises that may be assigned as additional practice in class or for homework.

A set of *Activities* in a cluster is followed by a section entitled *What Have I Learned?* As the name implies, students stop here to think about what they have learned before continuing to the next cluster. They have the opportunity to analyze and synthesize the ideas they studied in the cluster, and to test their knowledge by applying what they have learned to solve similar problems. You may want to use these sections to assess your students' progress. Journal and writing assignments are good assessment tools for the questions in *What Have I Learned?* In addition, you will find suggestions for assessment in Sections

4, 5 and 7 of this guide. Students can also be encouraged to make these assignments part of their portfolios where they will have easy access to what they have learned when they are preparing for exams.

How Can I Practice? sections follow *What Have I Learned?* sections to provide further opportunities for practice.

Our textbook also addresses the need for plentiful skills practice and preparation for gateway exams that may be required in various programs. Appendix C includes two *Skills Check* sections specifically aimed to reinforce arithmetic skills.

Each chapter concludes with

- *Chapter Summary*
- *Gateway Review*

The *Chapter Summary* lists important concepts and skills from the chapter, together with a brief description and example of the concept or skill. The *Chapter Summary* provides reinforcement of what has been learned in the chapter and is helpful to students studying for exams.

Following each *Chapter Summary* are *Gateway Reviews,* composed of exercises that test the fundamental concepts and skills in their respective chapters. Please note that some additional skills worksheets are provided in Section 6 of this *Guide.* You will also find several appendices in *Mathematics in Action* that provide just-in-time instruction on basic arithmetic and algebraic skills and the use of a graphing calculator.

- Appendix A: Fractions
- Appendix B: Decimals
- Appendix C: Skills Checks
- Appendix D: Algebraic Extensions
- Appendix E: Getting Started with the TI-84 Plus Family of Graphing Calculators

The textbook also contains the following:

- Answers to selected problems and gateway questions (the Instructor's Edition contains answers to every problem within the text itself)
- Glossary
- Index

TEXTBOOK SUPPLEMENTS

A number of supplemental instructional and learning aids are available for users of *Mathematics in Action* and they are described here.

Instructor Supplements

Annotated Instructor's Edition
ISBN 10 – 0-13-498931-7
ISBN 13 – 978-0-13-498931-0
Contains all the content found in the student edition, plus answers to all exercises directly beneath each problem and Learning Catalytics instructor annotations.

Instructor's Resource Manual with Tests
ISBN 10 – 0-13-498933-3
ISBN 13 – 978--0-13-498933-4
This valuable teaching resource includes the following materials:

- Sample syllabi suggesting ways to structure the course around core and supplemental activities.
- Sample course outlines containing timelines for covering topics.
- Teaching notes for each chapter—ideal for using the text for the first time.
- Extra skills practice worksheets for difficult topics.
- Sample chapter tests and final exams.
- Information about incorporating technology in the classroom, such as graphing calculators.

TestGen®
ISBN 10 – 0-13-516330-7
ISBN 13 – 978-0-13-516330-6
TestGen® enables instructors to build, edit, print and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions.

Instructor's Training Videos
From author Ernie Danforth, the videos provide instructors with advice ranging from the Consortium teaching philosophy to tips for implementing group-work.

New! PowerPoint Lectures
These slides present key concepts and definitions from the text. These have been created to support instructors looking to implement this contextual approach in the classroom, and can also be used as a student study aid.

Student Supplements

Worksheets for Classroom or Lab Practice
ISBN 10 – 0-13-516714-0
ISBN 13 – 978-0-13-516714-4

- Provide extra practice to ensure that students have many opportunities to work problems related to the concepts learned in every activity.
- Concept Connections, a feature unique to these worksheets, offer students an opportunity to show in words that they understand the mathematical concepts they have just practiced.

Supplements for Instructors and Students

MyLab Math® Online Course (access code required)

MyLab Math from Pearson is the world's leading online resource in mathematics, integrating interactive homework, assessment, and media in a flexible, easy to use format. It provides engaging experiences that

personalize, stimulate, and measure learning for each student. Moreover, it comes from an experienced partner with educational expertise and an eye on the future.

MyLab Math is a complete online course that provides interactive, multimedia instruction correlated to this textbook content. A MyLab course provides the full eText with a multimedia library to provide additional support for students when they need it. For this revision, a new video series and PowerPoint series are available, in addition to MyLab's exercises with learning aids and feedback, a personalized study plan, and more.

Instructor's Training Video available in MyLab Math® (access code required)

The Instructor's Training Video discusses effective ways to implement the teaching pedagogy of the *Mathematics in Action* series, focusing on how to make collaborative learning, discovery learning, and alternative means of assessment work in the classroom. (Available in the Instructor Resources tab in MyLab Math®)

PowerPoint Lectures are available in MyLab Math. These slides present key concepts and definitions from the text. These have been created to support instructors looking to implement this contextual approach in the classroom, and can also be used as a student study aid.

To learn more about how MyLab Math combines proven learning applications with powerful assessment, visit **www.MyLabMath.com** or contact your Pearson representative.

PEDAGOGY

The pedagogical theme of *Mathematics in Action* is active learning, facilitated by emphasis on:

- Writing to Learn Mathematics
- Strategies for Problem Solving
- Collaborative Learning in Groups
- Whole-Class Interactive Discussions
- Use of Technology

Writing-to-learn strategies are built into the structure of the textbook, so students are consistently helped to think by writing, a time-honored way to learn mathematics. Section 5 of this *Guide* presents additional writing-to-learn techniques to share with your students.

Students need to recognize and assimilate thinking strategies for problem solving. In Section 2, we describe our insights on the purposes and goals of individual activities, problems, and exercise sets, and the problem-solving strategies they elicit.

In the classroom, we found, as many have, that the most active learning takes place in group settings where the teacher is a guide who helps students find their own correct understanding of concepts. Small groups promote attention to individual needs in learning, increase peer support for puzzling out solutions, and provide experience in teamwork. Whole-class interactive discussions allow an instructor to efficiently explain key concepts and clear up common misunderstandings, give students the opportunity to develop self-reliance in learning quantitative and algebraic skills for problem solving, and encourage the sharing and discussing of ideas in a large group setting. In Section 3, we share our experiences and offer

suggestions for using collaborative learning and interactive discussion with the *Activities* and other components in the textbook.

TECHNOLOGY

Calculators and graphing programs are not only tools for doing mathematics and solving problems; they have fundamentally changed the way we teach and learn mathematics. We developed our textbook recognizing this modern pedagogical truth and you will find suggestions and ideas for using technology with the textbook in Section 4 of this *Guide*.

Technology is an integral part of the textbook and a number of technical tools are compatible with the activities in the course.

- Scientific Calculators
- Graphing Calculators
- Graphing Software
- Tutorial Software for Skills Practice

Scientific calculators are required and sufficient for the course, although graphing calculators are useful and are occasionally referenced for visual representation of functions in Chapters 3 and 4. Tutorial software can serve as an adjunct for practicing basic skills. You will find further discussion, suggestions, and examples regarding these tools in Section 4.

SUGGESTIONS FOR SYLLABI

Mathematics in Action: *An Introduction to Algebraic, Graphical, and Numerical Problem Solving* can be used as the text in a variety of foundation level courses. Among the authors, we have adopted all or part of the book for courses in arithmetic, elementary algebra, intermediate algebra, and introduction to functions. The number of contact hours for each of these courses varies considerably from three to six hours, with the latter often including a computer laboratory component.

We compiled the following annotated list of the major mathematical topics covered in the textbook to serve as an effective guide for you to construct a syllabus that best suits your course. We have attempted to identify those activities that provide the essence of each major topic. In addition to these core activities, we have suggested other highly recommended activities that expand or reinforce the mathematical theme. We encourage you to use the many remaining activities for further exploration and practice.

MATHEMATICAL TOPICS COVERED IN THE TEXTBOOK

Below is a list of the major topics covered in *Mathematics in Action.* We then present an annotated list that indicates both the core activities and recommended supplemental activities that explore each of these topics.

- Arithmetic Operations using Whole Numbers, Fractions (Rational Numbers) and Decimals
- Numerical Literacy through Proportional Reasoning
- Arithmetic Operations using Signed Numbers

- Introduction to Variable Quantities; Numerical, Symbolic and Graphical Representations of Relationships Between Input and Output Variables
- Symbolic Representation of Variable Quantities
- Constructing, Evaluating, and Solving Single-Operation Algebraic Models
- Constructing, Evaluating, and Solving Two-Operation Algebraic (Linear) Models
- Equivalent Algebraic Expressions: Numerically, Graphically, and Algebraically
- Solving More Complex (Linear) Equations
- Function Definition and Notation
- Rates of Change
- Linear Functions: Numerical, Graphical, and Algebraic Features
- Scatterplots and Linear Regression Equations
- Problem Solving with Linear Functions
- Systems of Linear Equations
- Linear Inequalities
- Polynomials and Properties of Exponents
- Problem Solving with Quadratic Equations and Functions
- A Brief Glimpse at Other Function Families

ANNOTATED LIST OF CORE AND RECOMMENDED SUPPLEMENTAL ACTIVITIES

Arithmetic Operations using Whole Numbers, Fractions and Decimals (Rational Numbers)

Recommended: Activity 1.1 The Bookstore
Steps in Problem Solving
Develop communication skills and problem-solving skills; organize information.

Recommended: Activity 1.2 The Classroom
Problem-Solving Strategies
Develop communication skills and problem-solving skills; organize information.

Core: Activity 1.3 Properties of Arithmetic
Properties and Vocabulary for Arithmetic Calculations
Use the order of operations convention to evaluate arithmetic expressions containing positive integers; use the Distributive and Commutative Properties; exponential notation; use Scientific Notation for large numbers.

Recommended: Appendix A Fractions
Templates and skill practice.

Recommended: Appendix B Decimals
Templates and skill practice.

Core: Activity 1.4 Top Chef
Operations with Fractions and Mixed Numbers
Add and subtract fractions; multiply and divide fractions.

Core: Project 1.5 Course Grades and Your GPA
Problem Solving Using Fractions and Decimals
Solve problems with fractions and decimals.

Numerical Literacy through Proportional Reasoning

Core: Activity 1.6 Everything is Relative
Ratios as Fractions, Decimals, and Percents
Relative versus actual measures; express ratios as fractions, percents, and decimals; ratios as comparison.

Core: Activity 1.7 The Opioid Epidemic
Proportional Reasoning
Proportional reasoning using ratios.

Core: Activity 1.8 Who Really Did Better?
Actual and Relative Change, Percent Increase and Decrease
Investigate relative (percent) versus actual change – percent increase or percent decrease.

Core: Activity 1.9 Going Shopping?
Growth and Decay Factors
Define and explore growth and decay factors.

Core: Activity 1.10 Take an Additional 20% Off
Consecutive Growth and Decay Factors
Apply growth or decay factors consecutively.

Core: Activity 1.11 Fuel Economy
Rates and Unit Analysis
Solve problems involving unit-of-measure conversions by Unit (Dimensional) Analysis.

Arithmetic Operations using Signed Numbers

Recommended: Activity 1.12 Celsius Thermometers
Addition and Subtraction of Integers
Add, subtract, and compare signed numbers; compute absolute values.

Recommended: Activity 1.13 Shedding the Extra Pounds
Multiplication and Division of Integers
Multiply and divide signed numbers.

Core: Activity 1.14 Order of Operations Revisited
Negative Exponents and Scientific Notation
Evaluate arithmetic expressions containing positive and negative numbers using the order of operations convention; evaluate expressions with negative exponents; Scientific Notation for very small numbers.

Introduction to Variable Quantities; Numerical, Graphical and Symbolic Representations of Relationships Between Input and Output Variables

Core: Activity 2.1 Symbolizing Arithmetic
Formulas and Algebraic Expressions
Generalize from an arithmetic calculation to a symbolic representation by utilizing variables.

Core: Activity 2.2 Blood Alcohol Levels
Represent a Two-Variable Relationship Algebraically, Numerically, and Graphically
Identify input and output in situations involving two variables; interpret relationships numerically and graphically.

Recommended: Activity 2.3 College Expenses
Symbolic Rules
Identify input and output variables from a graph; write verbal rules and translate into symbolic rules.

Symbolic Representation of Variable Quantities

Core: Activity 2.4 Are they the Same?
Equivalent Expressions and Grouping Symbols
Identify equivalent algebraic expressions by examining their outputs and comparing the graphs.

Constructing, Evaluating and Solving Single-Operation Algebraic Models

Core: Activity 2.5 Let's Go Shopping
Solve an Equation Containing One Operation
Construct, evaluate and solve equations of the form $ax = b$ and $x + a = b$.

Constructing, Evaluating, and Solving Two-Operation (Linear) Algebraic Models

Core: Activity 2.6 Leasing a Copier
Solve an Equation Containing Two or More Operations
Construct, evaluate and solve equations of the form $ax + b = c$, $a \neq 0$.

Core: Activity 2.7 The Algebra of Weather
Solve a Formula for a Specified Variable
Solve a formula (literal equation) for a specified variable.

Core: Activity 2.8 Four Out of Five Dentists Prefer Crest
Proportions
Construct and solve proportion equations using equivalent fractions and cross multiplication.

Equivalent Algebraic Expressions: Numerically, Graphically and Algebraically

Core: Activity 2.9 Do It Two Ways
Distributive Property, Greatest Common Factor, and Combining Like Terms
Use the distributive property to transform one algebraic expression into an equivalent one. Expand and factor expressions; combine like terms.

Recommended: Activity 2.10 Decoding
Simplifying Algebraic Expressions
Recognize an algebraic expression as a code of instruction; simplify algebraic expressions.

Solving More Complex (Linear) Equations

Core: Activity 2.11 Comparing Energy Costs
Mathematical Models, General Strategy for Solving Linear Equations
Model and solve equations of the form $ax + b = cx + d$.

Recommended: Project Activity 2.12 Summer Job Opportunities
Problem Solving Using Linear Equations
Use critical thinking skills to make decisions based on solutions of systems of two linear equations.

Function Definition and Notation

Core: Activity 3.1 Summer Olympics
Functions, Numerical and Graphical Representation of Functions
Introduction to functions; function notation; graphical and numerical representations; expansion of rectangular coordinate system to include all 4 quadrants in the plane

Core: Project 3.3 Comparing Symbolically Defined Functions and their Graphs
Define functions by symbolic rules.

Core: Activity 3.4 Course Grade
Representing Functions Symbolically
Determine symbolic rule that defines a function as well as practical domain and range of a function.

Rates of Change

Core: Activity 3.2 How Fast Did You Lose?
Average Rate of Change
Average rate of change—definition, contextual interpretations

Linear Functions: Numerical, Graphical, and Algebraic Features

Core: Activity 3.5 The Snowy Tree Cricket
Slope and Intercepts of a Line
Define linear functions by their constant rate of change and graph consisting of a single line; identify slope as the constant rate of change.

Core: Activity 3.6 Software Sales
Slope-Intercept Equation of a Line
Write equation of line in slope intercept form; graph linear function using y-intercept and slope; use intercepts to graph linear equation.

Recommended: Activity 3.7 Predicting Population
Problem Solving Using Slope-Intercept Equation of a Line
The slope-intercept form of a linear function; relative error in measurement using a linear model

Core: Activity 3.8 College Tuition
Point-Slope Equation of a Line

Determine equation for linear function when given two points. Identify lines having negative, zero or undefined slopes; equations of horizontal and vertical lines.

Scatterplots and Regression Lines

Recommended: Activity 3.9 Education Pays
Line of Best Fit and Regression Equations

Construct scatterplots from sets of data; use a scatterplot to estimate a line of best fit; generate linear regression models using technology.

Recommended: Lab Activity 3.10 Body Parts
Problem Solving Using Regression Equations

Collect sets of data by measurement; use a scatterplot to estimate a line of best fit; generate linear regression models using technology.

Systems of Linear Equations

Core: Activity 3.11 Smartphone Plan Options
Systems of Linear Equations in Two Variables

Solve a system of two linear equations numerically, graphically and symbolically by the substitution method; interpret the solution.

Recommended: Appendix D Addition Method for Solving a System of Two Linear Equations

Core: Activity 3.12 Healthy Lifestyle
Solving a System of Linear Equations in Two Variables Using the Addition Method

Solve a system of two linear equations algebraically by the substitution method and by the addition method.

Recommended: Project Activity 3.13 Modeling a Business
Problem Solving Using Systems of Linear Equations in Two Variables

Solve a system of two linear equations by any method; determine and interpret the break-even point; interpret break-even points in contextual situations.

Linear Inequalities

Recommended: Activity 3.14 How Long Can You Live?
Linear Inequalities

Solve linear inequalities in one variable algebraically and graphically.

Polynomials and Properties of Exponents

Core: Activity 4.1 Fatal Crashes
Polynomials

Identify, classify and simplify polynomials; add and subtract polynomials; evaluate and interpret polynomial models.

Core: Activity 4.2 Volume of a Storage Box
Properties of Exponents

Properties of exponents; identify greatest common factor in an algebraic expression; factor an algebraic expression completely; use the distributive property to expand an algebraic expression in factored form.

Core: Activity 4.3 Room for Work
Operations with Polynomials
Products of polynomials including difference of squares and perfect-square trinomials

Problem Solving with Quadratic Equations and Functions

Core: Activity 4.4 The Amazing Property of Gravity
Solving Quadratic Equations
Evaluate, graph and interpret functions of the form $y = ax^2$; solve equations of the form $ax^2 = c$ algebraically by taking square roots and graphically; solve an equation of the form $(x \pm a)^2 = c$ algebraically by taking square roots.

Core: Activity 4.5 What Goes Up, Comes Down
Quadratic Functions and Their Graphs
Evaluate and graph quadratic functions of the form $y = ax^2 + bx$; identify and interpret the x-intercepts; factor a binomial to solve an equation of the form $ax^2 + bx = 0$ using the zero-product property.

Core: Activity 4.6 How High Did It Go?
Solving Quadratic Equations by Factoring
Recognize and write a quadratic equation in standard form $ax^2 + bx + c = 0$; solve a factorable quadratic equation of form $x^2 + bx + c = 0$ using the zero-product property; identify a quadratic function from its algebraic form.

Core: Activity 4.7 More Ups and Downs
Solving Quadratic Equations Using the Quadratic Formula
Solve quadratic equations using the quadratic formula; identify the solutions of a quadratic equation with points on the corresponding graph.

Recommended: Appendix D Factoring Trinomials with Leading Coefficient $a \neq 1$
Solve quadratic equations of the form $ax^2 + bx + c = 0$ ***by factoring.***

A Brief Glimpse at Other Function Families

Core: Activity 4.8 Inflation
Exponential Functions
Recognize an exponential function as a rule for repeatedly applying the same growth or decay factor; graph exponential functions from numerical data and symbolic rules; recognize and graph exponential functions from symbolic rules.

Core: Activity 4.9 A Thunderstorm
Direct Variation
Explore direct variation functions $y = k \cdot x$. and $y = k \cdot x^n$, for n a positive integer.

Core: Activity 4.10 Diving Under Pressure, or Don't Hold Your Breath
Inverse Variation

Explore inverse variation functions of the form $y = \frac{k}{x}$.

Core: Activity 4.11 Hang Time
Square Root Functions

Explore the square root function $y = a\sqrt{x}$.

SAMPLE COURSE OUTLINES

The course outlines contained in this section represent several syllabi for which the textbook is appropriate. In all cases, the outlines should be viewed as suggestions and should be used in conjunction with individual activity notes found in Section 2 of this manual. We encourage you to adapt them as best suits your curriculum and students. The **core activities** in each Cluster appear in bold font.

COURSE OUTLINE FOR ELEMENTARY ALGEBRA CLASS: CHAPTERS 1, 2, 3
(MEETING 4 HOURS PER WEEK FOR 15 WEEKS)

Chapter 1 Number Sense

Cluster 1 Introduction to Problem Solving (4 hours)

Activity 1.1 The Bookstore
Steps in Problem Solving
Activity 1.2 The Classroom
Problem-Solving Strategies
Activity 1.3 Properties of Arithmetic
Properties and Vocabulary for Arithmetic Calculations
What Have I Learned?
How Can I Practice?

Cluster 2 Problem Solving with Fractions and Decimals (Rational Numbers) (2 hours)

Activity 1.4 Top Chef
Operations with Fractions and Mixed Numbers
Project 1.5 Course Grades and Your GPA
Problem Solving Using Fractions and Decimals
What Have I Learned?
How Can I Practice?

Cluster 3 Comparisons and Proportional Reasoning (7 hours)

Activity 1.6 Everything is Relative
Ratios as Fractions, Decimals, and Percents
Activity 1.7 The Opioid Epidemic
Proportional Reasoning
Activity 1.8 Who Really Did Better?
Actual and Relative Change, Percent Increase and Decrease

Activity 1.9 Going Shopping?
Growth and Decay Factors
Activity 1.10 Take an Additional 20% Off
Consecutive Growth and Decay Factors
Activity 1.11 Fuel Economy
Rates and Unit Analysis
What Have I Learned?
How Can I Practice?
Skills Check 1 Appendix C

Cluster 4 Problem Solving With Signed Numbers (3 hours)
Activity 1.12 Celsius Thermometers
Addition and Subtraction of Integers
Activity 1.13 Shedding the Extra Pounds
Multiplication and Division of Integers
Activity 1.14 Order of Operations Revisited
Negative Exponents and Scientific Notation
What Have I Learned?
How Can I Practice?

Skills Check 2 Appendix C

Chapter 1 Summary
Chapter 1 Gateway Review (1 hour)

Chapter 2 Variable Sense

Cluster 1: Symbolic Rules and Expressions (6 hours)
Activity 2.1 Symbolizing Arithmetic
Formulas and Algebraic Expressions
Activity 2.2 Blood Alcohol Levels
Represent a Two-Variable Relationship Algebraically, Numerically, and Graphically
Activity 2.3 College Expenses
Symbolic Rules
Activity 2.4 Are They the Same?
Equivalent Expressions and Grouping Symbols
What Have I Learned?
How Can I Practice?

Cluster 2: Solving Equations (6 hours)
Activity 2.5 Let's Go Shopping
Solve an Equation Containing One Operation

Activity 2.6 Leasing a Copier
Solve an Equation Containing Two or More Operations
Activity 2.7 Algebra of Weather
Solve a Formula for a Specified Variable
Activity 2.8 Four Out of Five Dentists Prefer Crest
Proportions
What Have I Learned?
How Can I Practice?
Cluster 3: More Problem Solving Using Algebra (5 hours)
Activity 2.9 Do It Two Ways
Distributive Property, Greatest Common Factor, and Combining Like Terms
Activity 2.10 Decoding
Simplifying Algebraic Expressions
Activity 2.11 Comparing Energy Costs
Mathematical Models, General Strategy for Solving Linear Equations
Project 2.12 Summer Job Opportunities
Problem Solving Using Linear Equations
What Have I Learned?
How Can I Practice?

Chapter 2 Summary
Chapter 2 Gateway Review (1 hour)

Chapter 3 Function Sense and Linear Functions

Cluster 1 Function Sense (4 hours)
Activity 3.1 Summer Olympics
Functions, Numerical and Graphical Representation of Functions
Activity 3.2 How Fast Did You Lose?
Average Rate of Change
Project 3.3 Symbolically Defined Functions and their Graphs
Activity 3.4 Course Grade
Representing Functions Symbolically
What Have I Learned?
How Can I Practice?

Cluster 2 Introduction to Linear Functions (7 hours)
Activity 3.5 The Snowy Tree Cricket
Slope and Intercepts of a Line
Activity 3.6 Software Sales
Slope-Intercept Equation of a Line

Activity 3.7 Predicting Populations
Problem Solving Using Slope-Intercept Equation of a Line
Activity 3.8 College Tuition
Point-Slope Equation of a Line
What Have I Learned?
How Can I Practice?

Cluster 3 Linear Regression, Systems and Inequalities (2 hours)
Activity 3.9 Education Pays
Line of Best Fit and Regression Equations
Lab 3.10 Body Parts
Problem Solving Using Regression Equations
What Have I Learned?
How Can I Practice?

Four Hourly Exams (4 hours)
Weekly Quizzes (4 hours)
Worksheets/Review (4 hours)

Total Course Hours 60 hours

COURSE OUTLINE FOR IMMEDIATE ALGEBRA CLASS: CHAPTERS 2, 3, 4 (MEETING 4 HOURS PER WEEK FOR 15 WEEKS)

Chapter 2 Variable Sense

Cluster 1: Symbolic Rules and Expressions (4 hours)
Activity 2.1 Symbolizing Arithmetic
Formulas and Algebraic Expressions
Activity 2.2 Blood Alcohol Levels
Represent a Two-Variable Relationship Algebraically, Numerically, and Graphically
Activity 2.3 College Expenses
Symbolic Rules
Activity 2.4 Are They the Same?
Equivalent Expressions and Grouping Symbols
What Have I Learned?
How Can I Practice?

Cluster 2: Solving Equations (5 hours)
Activity 2.5 Let's Go Shopping
Solve an Equation Containing One Operation
Activity 2.6 Leasing a Copier
Solve an Equation Containing Two or More Operations

Activity 2.7 Algebra of Weather
Solve a Formula for a Specified Variable
Activity 2.8 Four Out of Five Dentists Prefer Crest
Proportions
What Have I Learned?
How Can I Practice?

Cluster 3: More Problem Solving Using Algebra (4 hours)
Activity 2.9 Do It Two Ways
Distributive Property, Greatest Common Factor, and Combining Like Terms
Activity 2.10 Decoding
Simplifying Algebraic Expressions
Activity 2.11 Comparing Energy Costs
Mathematical Models, General Strategy for Solving Linear Equations
Project 2.12 Summer Job Opportunities
Problem Solving Using Linear Equations
What Have I Learned?
How Can I Practice?

Chapter 2 Summary
Chapter 2 Gateway Review (1 hour)

Chapter 3 Function Sense and Linear Functions

Cluster 1 Function Sense (4 hours)
Activity 3.1 Summer Olympics
Functions, Numerical and Graphical Representation of Functions
Activity 3.2 How Fast Did You Lose?
Average Rate of Change
Project 3.3 Symbolically Defined Functions and their Graphs
Activity 3.4 Course Grade
Representing Functions Symbolically
What Have I Learned?
How Can I Practice?

Cluster 2 Introduction to Linear Functions (5 hours)
Activity 3.5 The Snowy Tree Cricket
Slope and Intercepts of a Line
Activity 3.6 Software Sales
Slope-Intercept Equation of a Line
Activity 3.7 Predicting Populations
Problem Solving Using Slope-Intercept Equation of a Line

Chapter 4 An Introduction to Nonlinear Problem Solving

Activity 4.5 What Goes Up, Comes Down
Quadratic Functions and Their Graphs
Activity 4.6 How High Did It Go?
Solving Quadratic Equations by Factoring
Activity 4.7 More Ups and Downs
Solving Quadratic Equations Using the Quadratic Formula
What Have I Learned?
How Can I Practice?

Cluster 3 Other Nonlinear Functions (5 hours)

Activity 4.8 Inflation
Exponential Functions
Activity 4.9 A Thunderstorm
Direct Variation
Activity 4.10 Diving Under Pressure, or Don't Hold Your Breath
Inverse Variation
Activity 4.11 Hang Time
Square Root Functions
What Have I Learned?
How Can I Practice?

Chapter 4 Summary
Chapter 4 Gateway Review (1 hour)

Four Hourly Exams (4 hours)
Weekly Quizzes (4 hours)
Worksheets/Review (4 hours)

Total Course Hours 60 hours

ASSESSMENT

With *Mathematics in Action*, you will be able to use a variety of assessment tools to evaluate the progress of your students and to suggest to them useful tools for self-assessment. You will find samples of the following assessment tools and/or suggestions for using them in the places indicated. Most assessment tools are flexible to be used for either individual or group assessment as you see fit.

Group Exams: Sections 3, 4, 5, 7

Oral Presentations: Section 3

Technology Labs: Section 4

Projects: Section 4, Section 7

Journal Writing/Formal Writing Assignments: Section 5

Basic Skills: Section 6

Exams and Quizzes: Section 7

Gateway Exams: Section 7

Portfolios: Section 7

Section 3 offers ways to assign and handle group exams and oral presentations of group work results. Section 4 consists of sample technology labs specifically for use with the Graphing Calculator. Section 5 describes journal assignments and formal writing assignments and how they can be used for assessment. Section 6 contains additional skills worksheets that you can also use for traditional assessment. Section 7 has sample quizzes, tests, projects and exams including gateway exams.

Learning Catalytics is a "bring your own device" student engagement, assessment, and classroom intelligence tool, now integrated within MyLab Math courses. As an instructor, you can pose a variety of open-ended questions that help your students develop critical thinking skills, while monitoring responses with real-time analytics to find out where they're struggling. With this information, you can adjust your instructional strategy in real time and try additional ways of engaging your students during class. Learning Catalytics also lets you manage student interactions by automatically grouping students for discussion, team-based learning, and peer-to-peer learning.

Pre-made Learning Catalytics questions specific to each text have been developed for nearly every activity, providing an opportunity for instructors to quickly assess the progress on a given concept and give students an opportunity to use technology as an interactive learning tool.

Instructors can access Learning Catalytics from the Course Home screen in MyLab Math. From there, instructors can create modules in Learning Catalytics using their own questions that they create, or by pulling from a library of pre-made questions. Questions from other instructors can be accessed and searched, or search for MathInActionGreen# - where # is the chapter number - to find the Consortium-specific questions. These Consortium-specific questions have also been noted in the Annotated Instructor's Edition margins.

SECTION 2 CHAPTER NOTES

These notes provide a brief description of the theme and focus of the individual activities and sections in the textbook. We have tried to share with you many of our classroom experiences with this material, including topics you might wish to emphasize, and areas where many students encounter difficulty that you want to address. However, the best advice we can give you is to carefully read each *activity* before class. You will certainly have your own ideas on how best to explore a given topic with your class – as a whole class activity, in peer groups, in pairs. You may also anticipate conceptual or computational difficulties and choose to head them off by addressing them first in a pre-activity discussion or in a supplemental question, worksheet, or assignment. Your students will be best served when you work in partnership with the text, permitting your own emphasis and enthusiasm to help guide their discovery.

PROJECT AND LAB ACTIVITIES

Some activities have been designated as lab activities or project activities and you can use them either as required or as optional activities as you see fit in your course.

Lab activities are meant to give students hands on experience to develop intuitive understanding of certain mathematical concepts and procedures. These activities help students develop investigative and research skills in exploring mathematical and scientific ideas. They are ideal for group activities in the classroom. Some can be started in class to give students some guidance and then completed as homework assignments to prepare for discussion of results during the next class session. Students can prepare summary reports of their findings and you can assess the reports as writing assignments. See Section 5 for a method of holistic evaluation of writing assignments.

- ***Lab Activity* 3.10 Body Parts**

Project activities provide students with opportunities to synthesize the mathematical goals of a cluster or a chapter by applying the concepts and skills they learned to a realistic situation. Applications used in the projects were selected so students could personally relate to them. Students can work on projects individually or with one or two partners and you can assess their efforts by requiring a written or oral summary of their results.

- ***Project Activity* 2.12 Summer Job Opportunities**
- ***Project Activity* 3.13 Modeling a Business**

CHAPTER 1 NUMBER SENSE

CLUSTER 1 INTRODUCTION TO PROBLEM SOLVING

Activity 1.1 The Bookstore
Problem-Solving Strategies

This is an excellent problem with which to begin the course−it presents a very familiar situation and is ideal for group work. You may want to put students into groups of three or four and have them introduce themselves to one another. This problem will not be easy for them. Emphasize that the bookstore is initially full of people. Expect many different solutions, some of which will be incorrect. To hold each group accountable, you may want to call on one group to explain the problem to the class. Then ask each group to add to the explanation, especially if they approached the problem differently. To make sure that each individual understands the concepts, and to have students practice communication skills, ask them to write out in sentences the solution of the problem to hand in either at the end of class or at the next meeting. Your comments on content as well as communication skills will give the students an idea of what is expected of them.

> ***Example of student work*:** *Today I had to go and purchase my books at the college bookstore. I got to the bookstore and it was 11:00 A.M. There were 38 people in front of me so as I was waiting I decided to see how long I had to wait because my next class started at 12:00 noon. The lady in the bookstore only let 6 people in after 6 left the store. Since there were 38 people in front of me, I divided them into groups of six and I was in the 7th group. People were leaving the store at the rate of 2 students per minute. Now, I gave every 2 students in each group one minute. So all together, each group took 3 minutes. There are 7 groups, times 3 minutes, which gave me 21 minutes until I got into the bookstore. It is now 11:21 A.M. Each student spends 15 minutes getting his books. I added another 15 minutes. Now it's 11:36 and I still have to buy my books. I go to buy my books and it took me 10 minutes to wait. Now it is 11:46 A.M. It takes me 10 minutes to get to class. I add 11:46 and 10 minutes and I arrive to class at 11:56 A.M. I make it to class with 4 minutes to spare.*

Activity 1.2 The Classroom
Properties and Vocabulary for Arithmetic Calculations

This *activity* also involves a nice icebreaker – shaking hands. After forming groups of three, four or five, students introduce themselves to every other student in the group. Students are guided through some problem solving strategies – drawing a picture, recognizing a pattern, taking a simpler problem and generalizing to a more difficult setting. Have students discuss the usefulness of George Polya's four-step problem solving process. To make sure that each individual understands the concept of detecting patterns, the activity concludes with a visual problem involving the arrangement of tables in a classroom. Encourage students to do the exercises at the end of this activity with a learning partner.

Activity 1.3 Properties of Arithmetic
Properties and Vocabulary for Arithmetic Calculations

This *activity* presents a brief review of arithmetic vocabulary and operations involving natural numbers. Students use the commutative and associative properties of addition and multiplication to regroup and evaluate arithmetic expressions. Positive exponent and square root notation are introduced. In discussing order of operations, students are reminded that the operations within the numerator or denominator of a fraction or inside a radical sign must be treated as though they are grouped within a pair of parentheses. The *activity* also challenges students to use appropriate keystrokes in evaluating algebraic expressions containing fractions or radicals on a calculator. Finally, scientific notation is motivated by asking students to enter a number such as the national debt into their calculators or by requiring a calculation

whose result is too large to be displayed on their calculator. Scientific notation using negative exponents will be covered at the end of Chapter 1 Cluster 4, Problem Solving with Signed Numbers.

WHAT HAVE I LEARNED?

It is good to assign the problems in this section for homework so that students have an opportunity to think about the answers before class discussion. Using this section for an open class discussion helps the students tie together what they have learned.

HOW CAN I PRACTICE?

These problems provide an overall review of the material from Cluster 1.

CLUSTER 2 PROBLEM SOLVING WITH FRACTIONS AND DECIMALS (RATIONAL NUMBERS)

Activity 1.4 Top Chef
Operations with Fractions and Mixed Numbers

The purpose of this *activity* is to provide a conceptual understanding of fractions and arithmetic operations involving fractions—the idea that a fraction is classified by its denominator, the denominator indicating the total number of equal parts into which a "unit" or "whole" has been split. This unit can often be visualized as a length (such as a ruler) or as a circle (such as the pizza pie context of the *activity*). The numerator indicates the specific number of parts being counted. Fractions with the same denominators are considered "like" fractions, and adding like fractions means counting the total number of parts contained in all the numerators. The *activity* also reminds students that a fraction can be deconstructed as follows:

The fraction $\frac{3}{4}$ indicates that the unit has been split into 4 equal parts, each representing $\frac{1}{4}$ of the unit, and that we are counting 3 of these parts. So $\frac{3}{4}$ can also be thought of as $3 \cdot \frac{1}{4}$

This can be useful in discussing multiplication and division of fractions in this activity.

Depending on the level of your students, you might want to spend some class time working on Appendix A in groups, or you might prefer to designate several Appendix A exercises as a pre-activity assignment.

Project 1.5 Course Grades and Your GPA
Problem Solving Using Fractions and Decimals

The focus of this project is the relevant and often misunderstood concept of weighted averages in the context of student GPAs. Students will encounter fractions and decimals in forming the weights and computing the weighted averages. If necessary, students can be directed to Appendix B for more practice interpreting and using decimal formats.

WHAT HAVE I LEARNED?

In a discussion of operations with fractions, students usually have difficulty expressing themselves in words or in written form. Exercise 1 is an opportunity for students to discuss operations with fractions to develop some insight into what probably has been a very mechanical process. In Exercise 2, students should conclude that in the first meaning the size of the group is three; in the second, the size of each group is eight.

HOW CAN I PRACTICE?

Students are often relieved to tackle the arithmetic drill in Problem 6. Many have struggled with the contextual problems in the first five problems, especially in terms of which operation to use in a solution.

CLUSTER 3 COMPARISONS AND PROPORTIONAL REASONING

The objective of this cluster is to provide students with the conceptual understanding and manipulative skills for basic literacy involving percent change. The concepts are presented using familiar contexts in which relative measures and relative change are encountered in daily life.

Activity 1.6 Everything is Relative

Ratios as Fractions, Decimals, and Percents

In this *activity* students examine in context the distinction between relative and actual measures. The percent concept is introduced as the natural language for describing ratios as relative values. The *activity* provides many interesting and familiar contextual problems as well as drill and practice exercises in converting among verbal, fraction, decimal and percent formats. The daily newspaper is usually a great source of additional problems.

Activity 1.7 The Opioid Epidemic

Proportional Reasoning

This *activity* is an introduction to proportional reasoning in which students must apply a known ratio—expressed in % format—to a given piece of data. Students may first need to set up the appropriate "formula"

$$part = total \cdot (\% \text{ in decimal form}) \quad \text{or} \quad total = part \div (\% \text{ in decimal form})$$

but fairly quickly will understand when to multiply or divide by the given percent. The exercises are of critical importance for student mastery. Students are forced to read, extract pertinent data, plan a solution strategy, perform one or more calculations, and verify that their final answer makes sense. You will probably want to assign all of the exercises, using some in class and the rest for homework.

Activity 1.8 Who *Really* Did Better?

Actual and Relative Change, Percent Increase and Decrease

The concept of actual versus relative measure is extended here to the notion of change. Actual change and relative change provide distinct perspectives on the size of a change. Emphasize the fact that relative change quantifies the *significance* of the change, and is usually described as a percent increase or decrease.

Activity 1.9 Going Shopping

Growth and Decay Factors

Percent increases and percent decreases are associated with respective *growth* and *decay factors*. Although these terms are traditionally defined when studying exponential growth and decay in a precalculus setting, this is the natural place to introduce them. Students should be encouraged to calculate the "new" value by multiplying the "original" value by the corresponding growth factor. In a similar vein, they will begin to understand that when a quantity has already increased by a given percent, the "original" value can be directly calculated from the "new" value only by dividing by the growth factor. That is, if a quantity has increased by 20%, say to 180, its original value can be found only by dividing 180 by 1.20 obtaining the original value 150. Too often, people are tempted to subtract 20% of 180 from 180 which would yield an incorrect original value of 144. Further illustrate that a 20% increase is not balanced out by a 20% decrease. Mastery of this material is crucial to developing numerical literacy. You should not hesitate to devote an additional class hour to this topic

Activity 1.10 Take an Additional 20% Off
Consecutive Growth and Decay Factors

The material here builds on the growth and decay factor concepts developed in the previous *activity,* and should be explored in a similar manner. The focus of this *activity* is on how to calculate the cumulative effect of consecutively applied percent changes. Again, the natural (incorrect) temptation is to simply add or subtract the cumulative percents. The correct procedure is to multiply the cumulative growth or decay factors.

Activity 1.11 Fuel Economy
Rates and Unit Analysis

The approach in this a*ctivity* parallels the multiplication approach of the previous activity. Guide students to recognize that solving problems involving consecutive rates is most efficiently done by unit analysis−setting up the correct sequence of rates to be multiplied directly. This holds true for unit conversions too. It is hoped that this activity will convince students that the direct method of applying rates is, in general, the most efficient way to handle rate problems. The metric system is briefly introduced in this *activity* to show conversion of one measurement unit to another by applying unit analysis. Students may refer to the conversion tables on the inside back cover of the textbook for conversion equivalencies.

WHAT HAVE I LEARNED?

This section is a good opportunity to review the important concepts and vocabulary of Cluster 3.

HOW CAN I PRACTICE?

This is a good set of problems to assess overall mastery of proportional reasoning skills.

CLUSTER 4 PROBLEM SOLVING WITH SIGNED NUMBERS

Activity 1.12 Celsius Thermometers
Addition and Subtraction of Integers

This *activity* provides some visual aids to help students add and subtract signed numbers. If you choose not to use the thermometer models, many problems and exercises still offer plenty of drill and practice in adding and subtracting. Students are prompted to use common sense to reinforce their answers; adding a positive number to a given number will always yield a number greater than the given number, adding a negative number to a given number will always yield a number less than the given number. The concept of subtracting a negative is important to discuss. Calculators help to convince students of the correct answers when they are skeptical. Adding and subtracting signed numbers using absolute values is also discussed. It is often helpful for students to write two or three problems on an index card, and to exchange cards with a classmate. The classmate does the calculation and returns it to the first student for checking.

Activity 1.13 Shedding the Extra Pounds
Multiplication and Division of Integers

Students use weight gain and loss to reinforce addition and subtraction of signed numbers. Repeated addition is extended to the multiplication of signed numbers. This *activity* takes students through the mechanics of multiplying and dividing signed numbers. This *activity* ends with a discussion of divisions involving zero. The number "0" had confounded even the best mathematicians until well into the first millennium, so it is not surprising that students still struggle with the concept today. Students should understand that 0 is a perfectly acceptable numerical

value and that an expression such as $\frac{0}{4}$ has the value 0. Similarly, they should understand why expressions such as $\frac{0}{0}$ and $\frac{4}{0}$ have no values.

Activity 1.14 Order of Operations Revisited
Negative Exponents and Scientific Notation

The beginning of this *activity* is usually easy for students; however, the meaning of a symbol such as -3^2 is often difficult for students to grasp. A class discussion involving several similar calculations is essential. This is also the *activity* where negative exponents are introduced for the first time- not an easy topic for students to comprehend. Scientific notation is revisited to include negative powers of 10. The scientific calculator should be used here to facilitate these calculations.

WHAT HAVE I LEARNED?

In this section are questions to enhance understanding of operations with signed numbers.

HOW CAN I PRACTICE?

Exercises to assess the students' mastery of operations with signed numbers are found here.

END OF CHAPTER SUMMARY

GATEWAY REVIEW

This section provides questions for review before a Gateway Exam. Skills from all of Chapter 1 are included.

CHAPTER 2 VARIABLE SENSE

In this chapter, students will explore the concept of variables through many contextual encounters with related variables. The notion of input and output variables is introduced. The variables describe real quantities that are familiar and often meaningful to students. The relationships among variables are examined in multiple representations: verbally, numerically, symbolically and graphically.

Algebraic expressions are presented conceptually as shorthand codes for listing the specific sequence of operations which transforms the input value into the corresponding output value. One evaluates an algebraic expression by performing this sequence directly. One solves an associated equation by reversing this sequence and replacing each operation by its inverse. Algebraic equivalence is first explored numerically and graphically. The distributive property is presented as a means of transforming one algebraic expression (i.e., a specific sequence of operations) into an equivalent expression (i.e., a new, often streamlined sequence that produces identical outputs). Students are provided with the algebraic skills necessary to model, analyze, and perform all calculations involving linear relationships between two variables.

CLUSTER 1 SYMBOLIC RULES AND EXPRESSIONS

Activity 2.1 Symbolizing Arithmetic
Formulas and Algebraic Expressions

This *activity* serves as the important bridge between arithmetic and algebra. Formulas such as $A = b \cdot h$ are presented as mathematical statements that describe the arithmetic relationship between specific quantities. Students are asked to identify the variables, the quantities they represent and the

operations described by this and other familiar formulas. "Algebraic expressions" are presented as compact ways of describing the arithmetic operations to be performed on the "input variables." The result of performing these operations produces the "output." Students will also encounter contextual situations in which they must identify the quantities, represent them by symbols and construct the corresponding algebraic expression. Encourage students to understand that whereas arithmetic involves performing a sequence of operations to obtain a numerical result, algebra focuses on the sequence of operations, not on the numerical result.

Activity 2.2 Blood-Alcohol Levels
Represent a Two-Variable Relationship Algebraically, Numerically, and Graphically

This *activity* is an extension of 2.1 using blood alcohol concentration as the contextual example to introduce input and output variables. After determining appropriate input and output variables and replacement values, the blood alcohol formula is used to determine a numerical representation and then a graphical representation of the data. In the numerical representation, explain that the input variable is listed first, either in the top row of a horizontal table, or in the left column of a vertical one. Stress that input data is referenced on the horizontal axis, and output data on the vertical axis. The first relationship is presented numerically and graphically, with the graph pre-scaled. It is a good idea to point out the uniformity and independence of the scaling on each axis. Discuss the advantages and disadvantages of presenting data values in a table versus in a graph. The exercises should be assigned to give students practice in creating the graphical representation.

Activity 2.3 College Expenses
Symbolic Rules

In this *activity,* students encounter relationships between input and output variables that can be described by arithmetic operations. Students are prompted to identify the arithmetic operation (or sequence of two operations) by which every output value is obtained from its corresponding input value. This activity encourages students to develop their pattern recognition skills. After determining the replacement values for the number of credit hours, students create a table of input/output values. They are then asked to write the verbal rule in symbolic form and finally to create a graph using the table that they created. The TI84 Plus C graphing calculator is introduced in this activity. The exercises here are so important for homework or working in groups in class.

Activity 2.4 Are They the Same?
Equivalent Expressions and Grouping Symbols

In this *activity,* students are first prompted to model a contextual situation numerically and then to construct the accompanying algebraic expression. The appropriate sequence of operations is explored numerically and symbolically, and will require the use of parentheses. Students are NOT asked to expand these expressions—the distributive law will be explored in *activity* 2.9. The focus here is to have students understand the sequence of operations indicated by the algebraic expression, and to view parentheses as an "override" of the normal order of operations convention. Equivalency of algebraic expressions is presented numerically by comparing output values. You might also want to take a close look at the expressions x^2, $-x^2$ and $(-x)^2$ that are in the exercises. Remind students about the meaning of the arithmetic expression -5^2 . Here the two operations, negation and squaring, are exerting a "tug-of-war" on 5. Which wins and goes first, and why? Many students will stumble when evaluating these algebraic expressions for negative values of x.

WHAT HAVE I LEARNED?

This section is an opportunity to reflect on the key topics of the cluster: contextual variables, numerical and graphical representations of relationships between two variables, symbolic representation of variables.

HOW CAN I PRACTICE?

Additional opportunities for practice interpreting graphs, as well as labeling and scaling axes, and plotting points are given in this section. Many students will assign the outputs in the table in Exercise 2 to equally spaced grid marks. Also, watch for students who will label the vertical axis from bottom to top with a decreasing scale. Those who make both errors will discover a perfectly linear, increasing graph!

CLUSTER 2 SOLVING EQUATIONS

Activity 2.5 Let's Go Shopping
Solve an Equation Containing One Operation

Here, students construct/encounter algebraic rules containing a single operation: $y = a \cdot x$ or $y = x + b$. They will first evaluate the rule for a given value of x. Then they will be given a value of y and be prompted to set up and solve the associated equation. Emphasize the fact that solving an equation involves performing an inverse operation. This concept is revisited and reinforced throughout the chapter. The exercises offer lots of skills practice.

Activity 2.6 Leasing a Copier
Solve an Equation Containing Two or More Operations

This *activity* carefully guides students through an entire process of modeling a contextual situation with symbolic rules of the form $y = ax + b$. Students are asked to identify the input and output variables, write a verbal rule based on the situation described, and translate the verbal rule into a symbolic rule. Emphasis is placed on the sequence of operations that must be followed to determine an output value for a given input value. Reversing this process is exactly how students will solve an equation of the form (note the revised form of this equation) $ax + b = c$ for the input variable x. Focusing on the sequence of operations helps to make solving equations more intuitive and less mechanical for the student. This activity is core to this course and any subsequent math courses.

Activity 2.7 The Algebra of Weather
Solve a Formula for a Specified Variable

Solving a formula for a specified variable is an overwhelming task for many students. This *activity* begins with the familiar task of evaluating formulas for specified input (x) values and solving an equation for a specified (y) output value. Doing this in context provides a foundation for students to better understand the underlying concepts. After solving an equation for x given a numerical y-value, students transition to solving for x in terms of y itself. Time is well spent on this *activity* because it further develops problem solving skills in a somewhat abstract setting. The exercises also gradually build to solving an equation for a specified variable.

Activity 2.8 Four out of Five Dentists Prefer Crest
Proportions

This *activity* explores proportional reasoning by setting up and solving proportion equations. Here students encounter the given ratio in fraction form whereas in Activity 1.7 the ratio was given as a percent. In Activity 1.7 students obtained the "missing" piece of information by a direct multiplication or division by a decimal quantity. Here students are prompted to use a symbol, such as x, to represent the

unknown value, to form a proportion equation and to solve the equation using algebraic techniques from this Cluster.

WHAT HAVE I LEARNED?
Here students are asked to identify the input and output variables in an algebraic rule, and then to describe the operations indicated by the rule and the order in which they are to be performed.

HOW CAN I PRACTICE?
Opportunities to construct algebraic rules, evaluate the rules, and solve associated equations are provided in this section. Prompt students to remember to indicate units where appropriate, and to check their answers for reasonableness.

CLUSTER 3 MORE PROBLEM SOLVING USING ALGEBRA

Activity 2.9 Do It Two Ways
Distributive Property, Greatest Common Factor, and Combining Like Terms

The distributive property is presented numerically, algebraically and geometrically. The distributive property then becomes the algebraic mechanism for transforming a sequence of operations into an equivalent, often simplified, expression. Students will use the distributive property to expand and to factor expressions. Common factors and greatest common factors are introduced here. Several of these examples are designed to lead naturally to a discussion of like terms. The exercises offer plenty of opportunity for skills practice.

Activity 2.10 Decoding
Simplifying Algebraic Expressions

This optional and somewhat whimsical *activity* enhances student awareness that an algebraic expression is just a shorthand notation for the order in which to perform a sequence of arithmetic operations. If desired this activity may be skipped with no loss of coverage. Students will first explore the numerical sequence, and then they are prompted to express the process algebraically. Based on specific examples, the difference between inductive and deductive reasoning is explained.

Activity 2.11 Comparing Energy Costs
Mathematical Models, General Strategy for Solving Linear Equations

This *activity* focuses on writing mathematical models that lead to solving equations of the form $ax + b = cx + d$. A detailed example is given, illustrating the steps involved in solving $ax + b = cx + d$ for x. Solving formulas for a specified variable is also revisited. In total, this activity contains a great deal of practice in the algebraic skills of solving linear equations. Students may have been exposed to these skills in the past, but they will need practice in determining the models from the real life situations. Working in groups here may be helpful.

Project 2.12 Summer Job Opportunities
Problem Solving Using Linear Equations

This is a wonderful *activity* to assign as an out-of-class group or individual project. The context is meaningful, requires careful reading, and uses the new linear solving skills that allow students to make an informed decision regarding the merits and disadvantages of three summer jobs.

WHAT HAVE I LEARNED?
The main ideas of this cluster, common factors, like terms, and algebraic equivalencies are reinforced.

HOW CAN I PRACTICE?
These exercises provide additional opportunities for simplifying and expanding algebraic expressions using the distributive property and properties of exponents.

END OF CHAPTER SUMMARY

GATEWAY REVIEW
This section offers practice in applying the skills and methods that were studied in this chapter.

CHAPTER 3 FUNCTION SENSE AND LINEAR FUNCTIONS

Chapter 2 has prepared students for an introduction to functions by examining many input/output relationships numerically, graphically and algebraically. In Chapter 3, we formally define a function and introduce function notation. After discussing average rate of change of a function, we devote the remainder of the chapter to linear functions and their numerical, graphical and algebraic features.

CLUSTER 1 FUNCTION SENSE

Activity 3.1. Summer Olympics
Functions, Numerical and Graphical Representation of Functions
The concept of function is introduced in this activity as an input-output relationship presented in tabular and then graphical formats. The opening context involves a discrete function whose input-output pairs are displayed in a table. In discussing function notation such as $M(25)$ make sure to emphasize that the number inside the parentheses is always an input value and the entire symbol "$M(25)$" represents the corresponding output (whose value can be determined from the table in the activity to be 763). If desired, you can return to page 165 in Activity 2.2 to review the basic features of a first quadrant graph when discussing scaling issues in plotting the tabular data. This *activity* is also designed to review the basic ideas and vocabulary of a two-dimensional rectangular coordinate system—labeling of the quadrants, identifying the origin, and noting the common characteristics among the coordinates of points in a given quadrant or on a given axis. The coordinates of a point can also be viewed as the point's "address" with respect to the origin. Remind students that a good rule for choosing a scale for an axis is to determine an interval that contains all the data values, divide the length of that interval by the number of grid marks on the axis, and then use that value to set a "friendly" scale. It is important for students to understand that there is no single correct scaling. Any reasonable scale is acceptable. Of course, they must also realize that identical data plotted on differently scaled grids will not produce identical-looking graphs.

Activity 3.2 How Fast Did You Lose?
Average Rate of Change
Students are asked to examine the notion of change. You may want to use the analogy that evaluating or solving an equation is like looking at a single frame of a movie. To discuss rate of change we focus on the output trends that occur as the inputs increase. This is more like viewing the movie. Discussion should concentrate on connections between graphical change, i.e., rising, falling, or horizontal line segments, and their corresponding numerical rates of change, i.e., positive, negative and zero. Also very important are the units of rate of change, *output unit* per *input unit*. Insist that students express all rates of change with appropriate sign and units.

Project 3.3 Symbolically Defined Functions and their Graphs

This *project* expands the discussion of functions (already introduced by tables and graphs in Activities 3.1 and 3.2) to include symbolic representations. The goal is for students to actively construct the connections among a function's symbolic representation, table of values and shape of its graph. By plotting the ordered pairs and calculating rates of change students can begin to truly understand how a symbolic rule is associated with a specific graph with a particular shape. Without using the vocabulary of concavity, students will encounter graphs that bend upward (increasing rates of change), downward (decreasing rates of change) and linear graphs (constant rate of change). Once the connections are established, students will use the symbolic rules and their graphs to evaluate functions and solve associated equations.

Activity 3.4 Course Grade
Representing Functions Symbolically

In this *activity* students are presented with a familiar context and asked to complete a table of values, plot the ordered pairs and then construct the symbolic rule using function notation. Nothing is really new here, but a good opportunity to reinforce function notation.

WHAT HAVE I LEARNED?

This section allows students to focus on the abstract concepts of function and rate of change. A word of caution here: the average rate of change of a function over an interval *only* considers the endpoint values of the function and ignores the function's behavior on the interior. Therefore, a positive rate of change just indicates that the function's right endpoint value is greater than its initial value and does not imply that the function has been increasing over the entire interval. Similar observations should be made for negative and zero rates of change.

WHAT HAVE I LEARNED?
HOW CAN I PRACTICE?

Students have the opportunity to do some more exercises on evaluating functions, constructing algebraic models of contextual situations and calculating a function's rates of change.

CLUSTER 2 INTRODUCTION TO LINEAR FUNCTIONS

Activity 3.5 The Snowy Tree Cricket
Slope and Intercepts of a Line

This *activity* begins by noting the constant of rate of change in the chirp data. Note that in this problem the units are a complex fraction (chirps per minute per degree), although that generally doesn't seem to bother students. So no big deal need be made of it (except to insist the units be correct). We recommend that you encourage students to de-emphasize simply memorizing the traditional slope formula (which some students will recall from previous courses). Students need to internalize the slope concept as a constant rate of change: geometrically and numerically. In contextual situations, the practical interpretation of slope is the key to understanding the behavior of the associated linear model. In order to interpret slope, encourage students to first write the slope as a fraction with denominator 1, in this context, $\frac{4}{1}$. The practical meaning should be stated explicitly: for each 1 degree increase in temperature, the chirp rate increases by 4 chirps per minute. In this activity, we begin characterizing some of the features of a linear relationship—*numerical* feature (constant rate of change: equally spaced inputs correspond to equally spaced outputs) and the *graphical* feature (the graph is a single line). Horizontal and vertical intercepts are defined. It should be stressed that the intercepts are actual points on the graph.

Activity 3.6 Software Sales
Slope-Intercept Equation of a Line

In this *activity*, students will use their newly acquired linear functions skills to model and identify linear function rules, interpret slopes and intercepts in context, and connect the algebraic (symbolic) rule with its graph. Students are then prompted to construct the algebraic form of the linear function and to note the significance of the two numerical constants in their model; hence, the name "slope-intercept" form of the equation of a line. Students will be asked to sketch the line by plotting the vertical intercept and using the slope to identify a second point on the line. They will also be asked to determine the horizontal intercept algebraically and sketch the line by plotting the two intercepts.

Exercises will reinforce understanding of the connections between the algebraic, graphical, and numerical representations of linear functions.

Activity 3.7 Predicting Population
Problem Solving Using Slope-Intercept Equation of a Line

This brief activity presents two data points from which students will construct a linear population model. Discussion can include the notions of interpolation and extrapolation in judging the validity of a model, however these terms are not introduced formally until *activity* 3.9.

The relative error in a prediction is defined. *Exercises* use a similar context to that in the activity, and students construct additional linear population models in slope-intercept form.

Activity 3.8 College Tuition
Point-Slope Equation of a Line

In this *activity*, students will investigate how to construct a linear function rule from two points, neither of which is the *y*-intercept. The point–slope formula is presented in this activity as a useful alternative to the slope-intercept form. Students discover how to determine the equations and the slopes of horizontal and vertical lines in the context of situations.

Exercises are primarily practice in determining linear equations given a point and a slope, or given two points.

WHAT HAVE I LEARNED?

Several questions are provided to reinforce the main concepts of the cluster: slope, intercepts, and connections among algebraic, graphical, and numerical representations of linear functions.

HOW CAN I PRACTICE?

This is a fine opportunity for linking all representations (contextual, numerical, graphical and algebraic) of a linear function. Students often do not recognize the crucial connection between the algebraic (symbolic) rule and the points on the graph. In discussing Problem 1b,c, ask students how the *y*-intercept can be recognized from the table. It is surprising how many students cannot recognize the *y*-intercept from a table of values – they often choose the first output value in the table, regardless of the corresponding input value.

CLUSTER 3 LINEAR REGRESSION, SYSTEMS, AND INEQUALITIES

Activity 3.9 Education Pays
Line of Best Fit and Regression Equations

This *activity* is a hands-on introduction to regression equations and lines of best fit. To "eyeball" the line, several techniques can be used to help students who might otherwise have a difficult time visualizing a line. Suggest using a clear ruler, a thread or a dry strand of spaghetti to move around until it looks "best." In Problems 8 and 10, point out the absolute value in the last column that some students will overlook.

Ask groups why they think absolute value is used. If your students are using a TI-84 Plus C calculator, you can refer them to the guided instructions in **Appendix E** to help them complete Problems 9 and 10. *Exercises* mimic the problems in the *activity* using technology.

Lab 3.10 Body Parts
Problem Solving Using Regression Equations

This *lab activity* requires students to measure and collect physical data on arm lengths, foot lengths, neck circumference, etc. Students will record the measurements, create a scatterplot, and use technology to obtain a regression equation. Anthropology models are presented for comparison.

Activity 3.11 Smartphone Plan Options
Problem Solving Using Regression Equations

This activity flows rather well for the students and can be completed in groups or for homework. . Students may need help with the equations in Problems 2 and 3. Emphasis should be placed on all three methods of obtaining the solution (graphically, numerically (table), and algebraically (substitution)). This is a good opportunity to review using the graphing calculator to solve a linear equation numerically and graphically. The visual reinforcement helps the student make connections with the algebraic skill. Be sure students notice that the substitution method is appropriate when one or both of the two linear equations is written in the form of one variable expressed in terms of the other.

Activity 3.12 Healthy Lifestyle
Solving a System of Linear Equations in Two Variables Using the Addition Method

In this *activity* students will model linear equations of the form $ax + by = c$ from a contextual situation. Students will be guided to solve the system by substitution and by the addition method. We strongly suggest that students write the final answer to a solution in sentence form, including the interpretation of the result in context. This requirement should go a long way in dissuading students from the common notion that an answer to a math problem is simply a number. Be sure students notice that the substitution method is appropriate when one or both of the two linear equations is written in the form of one variable expressed in terms of the other.

Project 3.13 Modeling a Business
Problem Solving Using Systems of Linear Equations in Two Variables

In this *project activity* students use their understanding of linear functions to recognize linearity and to algebraically model a cost function, a revenue function, and a profit function. They are asked to identify and interpret the practical meaning of each function's intercepts and slope, and to solve a system of linear equations graphically and algebraically to determine the break-even point.

Activity 3.14 How Long Can You Live?
Linear Inequalities

Here students work with linear inequalities. Emphasize that the process of solving an inequality algebraically is very similar to solving an equation and that the strategy is to isolate the variable on one side of the inequality sign. The process of multiplying or dividing an inequality by a negative number is carefully developed using inequalities that involve numbers only. This *activity* also guides students through the graphical solution to inequalities of the form $f(x) > g(x)$.

WHAT HAVE I LEARNED?

In this section students are asked to synthesize what they have learned.

HOW CAN I PRACTICE?

There are plenty of practice exercises in this section.

END OF CHAPTER SUMMARY

GATEWAY REVIEW

This review covers all aspects of Chapter 3.

CHAPTER 4 AN INTRODUCTION TO NONLINEAR PROBLEM SOLVING

CLUSTER 1 MATHEMATICAL MODELING INVOLVING POLYNOMIALS

Activity 4.1 Fatal Crashes
Polynomials

This *activity* defines a polynomial, its degree and its classification according to the number of terms. Students will also combine like terms and add and subtract polynomials. In a contextual setting, students evaluate polynomials using function notation. Assign all exercises to further strengthen algebraic skills.

Activity 4.2 Volume of a Storage Box
Properties of Exponents

Students encounter contexts in which properties of exponents are motivated and then formally stated. We also present zero exponents and define negative exponents as reciprocals.

Activity 4.3 Room for Work
Operations with Polynomials

Multiplication of binomials is presented contextually, geometrically, and algebraically. The four sections of the expanded rectangle help to serve as a concrete visual reminder of the four terms that are obtained in the formal multiplication of binomials. In context, students multiply conjugate binomials and square binomials resulting in the difference of squares and perfect-square trinomials respectively. *Exercises* 1, 2 and 3 offer plenty of traditional skills practice. *Exercises* 4 and 5 are good contextual examples; students become aware that the factored form of a polynomial is sometimes simpler to work with than is the expanded form.

WHAT HAVE I LEARNED?

Students are asked questions about polynomials, negation, exponents and the distributive property that require some in-depth reflection.

HOW CAN I PRACTICE?

Here there are plenty of skills-based problems to reinforce the material presented in this cluster. As an extension of the cluster, in exercise 6j, students are asked to multiply two trinomials

CLUSTER 2 PROBLEM SOLVING WITH QUADRATIC EQUATIONS AND FUNCTIONS

Activity 4.4 The Amazing Property of Gravity
Solving Quadratic Equations

This *activity* begins with an algebraic approach to quadratic functions. Students compute the average rate of change over different intervals. Be sure the students notice that the average rate of change is not constant. The students plot points and sketch the curve, then verify the points on the graph of the quadratic function using the Trace or Table feature of the graphing calculator. Given an output value, students solve for the corresponding input value. Students solve equations of the form $ax^2 = c$ and $(x \pm a)^2 = c$. Finally the graphical features of a parabola are explored. The exercises are mainly skill-based with contextual problems that involve the Pythagorean Theorem.

Activity 4.5 What Goes Up, Comes Down
Quadratic Functions and Their Graphs

Quadratic functions become a little more complicated in this *activity*. Students evaluate and graph functions of the form $y = ax^2 + bx$. The meaning of the intercepts is investigated in context. Students will factor binomials of the form $ax^2 + bx$ and solve equations of the form $ax^2 + bx = 0$ using the Zero-Product Property. The exercises reinforce the content of the activity and all exercises should be assigned.

Activity 4.6 How High Did It Go?
Solving Quadratic Equations by Factoring

The standard form of a quadratic equation is presented along with the procedure for solving such an equation. In this activity, students factor trinomials of the form $x^2 + bx + c$, and use this technique to solve the factored form of a quadratic equation using the zero-product property. After writing a quadratic equation in standard form, students must identify *a, b* and *c.* Students will also encounter the situation where the coefficient of x^2 is -1. Students should identify the *x*-intercepts graphically and algebraically. Encourage students to explain the meaning of the *x*-intercepts in context. All exercises should be assigned.

Activity 4.7 More Ups and Downs
Solving Quadratic Equations Using the Quadratic Formula

In this *activity*, students use the quadratic formula to solve quadratic equations. In context, students are challenged by a quadratic equation that is not factorable. However, graphically students can see that the equation has two solutions. It is pointed out that most quadratic equations (in standard form) are not factorable – hence the quadratic formula. Exercises are straightforward.

WHAT HAVE I LEARNED?

This set of questions probes the student's understanding of quadratic functions in some depth.

HOW CAN I PRACTICE?

These exercises provide additional reinforcement for solving quadratic equations and investigating the graphical features of quadratic functions.

CLUSTER 3 OTHER NON-LINEAR FUNCTIONS

Activity 4.8 Inflation
Exponential Functions

This is an interesting topic for class discussion – what is inflation, and how it affects income, consumer prices and investments. Exponential growth is a natural extension of the growth factor concepts that were introduced in Chapter 1, Cluster 3, e.g., **Activities 1.9** and **1.10**. You might have the class participate together in completing the table in Problem 2. If your students have graphing calculators, they should use the graphing feature in exploring Problems 4 – 6. If not, and you have a calculator/view screen for demonstration you might want to explore these problems together as a class. Use your judgment in deciding how much or little discussion of exponential functions is appropriate at this time.

Activity 4.9 A Thunderstorm
Direct Variation

Although direct variation functions of the form $y = k\,x$ are certainly linear, we introduce them here as a counterpoint to inverse variation which is covered in the next *activity*. Students should note the proportionality in the increases or decreases in both variables.

Activity 4.10 Diving Under Pressure, or Don't Hold Your Breath
Inverse Variation

This is an engaging *activity* for students. Inverse variation is presented numerically, algebraically and graphically in a contextual application of Boyle's Law. Students see how the mathematics of inverse variation explains the necessity of proper breathing techniques when scuba diving.

Activity 4.11 Hang Time
Square Root Functions

Many students have indicated that this is one of their favorite *activities*. The context is engaging and really helps to motivate the mathematics. Here students are first asked to identify the sequence of operations indicated by an algebraic rule involving squares and square roots. They evaluate an expression and they are then prompted to solve an equation by reversing direction and replacing each operation by its inverse. They are required to walk through a specific solution verbally and algebraically.

WHAT HAVE I LEARNED?

Students are prompted to review the nonlinear functions presented in this cluster.

HOW CAN I PRACTICE?

These exercises provide additional opportunities for solving equations, manipulating formulas, and modeling contextual situations.

END OF CHAPTER SUMMARY

GATEWAY REVIEW

This section offers practice in applying the skills and methods that were studied in this chapter.

SECTION 3 LEARNING IN GROUPS

We believe that when students are guided to take an active role in the classroom they are more likely to develop a sense of independence and take responsibility for their own learning. We also believe that to stimulate active student interest and cooperation in the learning process, the mathematics should come from problems that arise from realistic contexts and appeal to a diverse population of students.

Representatives from various businesses and industries, whom we consulted, told us that their companies need team workers. Many workplace projects are too complex for one individual, and increasingly, employers look for people who will be productive in both small and large group situations. The ability to work well with others and to express one's ideas verbally and in written form is a crucial skill in today's job market.

With the economy and technology ever changing, few people actually do the job they are trained for in school. As technologies are rapidly superseded, workers need to be continually learning on the job.

As teachers, we would like to empower our students with mathematical problem-solving skills for life-long learning and for securing economic, political, social, and personal well-being. We hope to help our students become self-learners able to discover things on their own, able to deal creatively with new situations, possessing confidence to face the new challenges.

We have a tall order, but shifting the emphasis from a lecture-dominated classroom to a more interactive classroom will help us and our students achieve these goals. The format of *Mathematics in Action* was developed to facilitate learning in small groups and in whole class interactive discussions. Keep in mind, however, that the textbook allows considerable flexibility in teaching and learning modes. As instructors, we are in the best position to judge what learning strategies work best for our students. We believe you will discover, as did we, that you and your students will employ a number of teaching and learning modes, sometimes in just one lesson!

We expect that your students will frequently work in small groups and you will serve as a facilitator, responding to questions, providing hints, etc. On occasion, you might lecture to efficiently present mathematical concepts that you decide your students need to hear from you. Many times, you will find that interactive discussions with the whole class are very productive for presenting concepts, for clearing up misconceptions, and for analyzing and synthesizing the concepts and skills being learned.

As mathematics instructors, we were familiar with whole class interactive lessons and lecturing, and indeed, we found that a brief lecture on a topic was sometimes needed or that a large group or whole class interactive discussion worked best. But we also undertook to integrate small group learning with other instructional modes so we could reach as many students as possible. Thus, you will see that many pedagogical answers we sought were with respect to what is commonly called collaborative or cooperative learning.

We share our insights with you in a question and answer format that we trust is helpful.

QUESTIONS AND ANSWERS ABOUT LEARNING IN GROUPS

What is collaborative learning?

The *basic idea* of collaborative learning is that a group undertakes an assignment, brainstorms about the issues, works efficiently by using each individual's strengths in a division of tasks, and brings the assignment to closure in a suitable form. Collaborative learning is customarily done in small groups, usually of size three or four. At times, a lesson is best served by collaborative learning in the whole class.

An *example* of cooperative learning that uses both small groups and the whole class is a simple lesson on counting principles for use on the first day of class.

Randomly assign students to groups of three and ask them to shake hands with each member of their group.

- Have students introduce themselves to each other in the group, and tell each other a little bit about themselves.
- Let the students know that you will call on one member of the group to introduce the group members to the entire class. This helps the class become a cooperative learning alliance right from the start.
- After the introductions are made, ask each group to determine how many handshakes they made in their group, and how they arrived at their count.
- After a few minutes, ask a member from each group for their result. You will probably be surprised that most groups do not get the right answer and you should have a lively discussion in your class.
- If you have time and the desire, extend the lesson. For example, ask how many handshakes the nine Supreme Court justices would make at an opening of a court session. The small groups work to find a pattern for solving the problem and to determine the answer, later sharing results in a whole class discussion.
- Similar questions for homework will help all students learn and practice the solution methods.

What skills do students gain through active learning in a collaborative environment?

We found that students have many opportunities to gain a variety of valuable skills while learning in a collaborative environment.

- Students propose and argue their ideas in both small and large group settings, providing oral presentation practice.
- The time students take to organize their thoughts for presentation helps them internalize their knowledge and catch flaws in their thinking.
- Group members learn how to help each other refresh knowledge of concepts and skills they need, but have forgotten.
- Group learning fosters permanent learning based on personal exploration. It is easy to forget information that you are told, but almost impossible to forget something you have discovered on your own. Active learning promotes freer thinking. Students become better problem solvers because they do not merely mimic the teacher's solutions.