

***Linear Algebra 5th edition Friedberg Solutions
Manual***

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Linear Transformations and Matrices

2.1 LINEAR TRANSFORMATIONS, NULL SPACES, AND RANGES

3. The nullity is 0, and the rank is 2. Thus T is one-to-one, but not onto.
6. The nullity is $n - 1$, and the rank is 1. Thus T is not one-to-one unless $n = 1$, and T is not onto unless $n = 1$.
11. $T(8, 11) = (5, -3, 16)$
18. $T(a, b) = (b, 0)$. $N(T) = \text{span}\{(1, 0)\} = R(T)$.
19. Let $V = W = R$, and define $T = I$ and $U = 2I$.
23. All of R^3 or a plane in R^3 through the origin
25. (a) $T(a, b) = (0, b)$ (b) $T(a, b) = (0, b - a)$
26. (b) $T(a, b, c) = (0, 0, c)$
28. (b) See (a) and (b) of Exercise 25.
32. (c) Let $V = P(F)$ and $W = \text{span}(\{1\})$. Define T first on the standard basis of V by $T(1) = T(x) = 0$, and $T(x^k) = x^{k-1}$ for $k \geq 2$. Now extend T to a linear transformation from V to V . Then $N(T) = \text{span}(\{1, x\})$, and $R(T) = \text{span}(\{x^k: k \geq 1\})$. So $V = R(T) \oplus W$, but $W \neq N(T)$.

2.2 THE MATRIX REPRESENTATION OF A LINEAR TRANSFORMATION

2. (b) $\begin{pmatrix} 2 & 3 & -1 \\ 1 & 0 & 1 \end{pmatrix}$ (e) $\begin{pmatrix} 1 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 1 & 0 & \dots & 0 \end{pmatrix}$
4. $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \end{pmatrix}$
5. (c) $(1 \ 0 \ 0 \ 1)$ (d) $(1 \ 2 \ 4)$ (f) $\begin{pmatrix} 3 \\ -6 \\ 1 \end{pmatrix}$ (g) (a)

2.3 COMPOSITION OF LINEAR TRANSFORMATIONS AND MATRIX MULTIPLICATION

2. (a) $(AB)D = \begin{pmatrix} 29 \\ -26 \end{pmatrix}$

(b) $A^t = \begin{pmatrix} 2 & -3 & 4 \\ 5 & 1 & 2 \end{pmatrix}$, $BC^t = \begin{pmatrix} 12 \\ 16 \\ 29 \end{pmatrix}$, $CA = \begin{pmatrix} 20 \\ 26 \end{pmatrix}$

3. (b) $[h(x)]_\beta = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$, $[U(h(x))]_\gamma = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$

4. (b) $\begin{pmatrix} -6 \\ 2 \\ 0 \\ 6 \end{pmatrix}$ (d) (12)

9. $T(a_1, a_2) = (0, a_1 + a_2)$, $U(a_1, a_2) = (0, a_1)$

$A = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

21. (a) $B = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$, $B^3 = \begin{pmatrix} 0 & 2 & 0 & 3 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 3 & 0 & 2 & 0 \end{pmatrix}$ There are no cliques.

(b) $B = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$, $B^3 = \begin{pmatrix} 2 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 \\ 3 & 0 & 2 & 3 \\ 3 & 0 & 3 & 2 \end{pmatrix}$

Persons 1, 3, and 4 belong to a clique.

24. $\frac{n^2 - n}{2}$

2.4 INVERTIBILITY AND ISOMORPHISMS

14. $T \begin{pmatrix} a & a+b \\ 0 & c \end{pmatrix} = (a, b, c)$

19. (a) $[T]_\beta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2.5 THE CHANGE OF COORDINATE MATRIX

2. (b) $\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & -1 \\ 5 & -4 \end{pmatrix}$
3. (b) $\begin{pmatrix} a_0 & b_0 & c_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 3 & 1 \end{pmatrix}$ (f) $\begin{pmatrix} -2 & 1 & 2 \\ 3 & 4 & 1 \\ -1 & 5 & 2 \end{pmatrix}$
6. (b) $Q = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $[L_A]_\beta = \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$
- (d) $Q = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -2 & 0 & 1 \end{pmatrix}$, $[L_A]_\beta = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 18 \end{pmatrix}$
7. (b) $T(x, y) = \frac{1}{m^2+1}(x + my, mx + m^2y)$

2.6 DUAL SPACES

3. (b) $f_1(a + bx + cx^2) = a$, $f_2(a + bx + cx^2) = b$, $f_3(a + bx + cx^2) = c$
4. The basis for V is $\{(.4, -.3, -.1), (.6, .3, .1), (.2, .1, -.3)\}$
6. (a) $T^t(f)(x, y) = 7x + 4y$ (b) $[T^t]_{\beta^*} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}$
- (c) $[T]_\beta = \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}$ and $([T]_\beta)^t = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}$

2.7 HOMOGENEOUS LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

3. (b) $\{1, e^t, e^{-t}\}$ (d) $\{e^{-t}, te^{-t}\}$
4. (b) $\{e^t, te^t, t^2e^t\}$
16. (a) $\theta(t) = c_1 \cos\left(\sqrt{\frac{g}{l}}t\right) + c_2 \sin\left(\sqrt{\frac{g}{l}}t\right)$
- (b) $\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{l}}t\right)$
17. $y(t) = c_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + c_2 \sin\left(\sqrt{\frac{k}{m}}t\right)$
18. (a) Case 1: $r^2 = 4km$. $y(t) = e^{-(r/2m)t}[c_1 + c_2t]$
Case 2: $r^2 > 4km$. $y(t) = c_1e^{at} + c_2e^{bt}$, where

$$a = \frac{-r}{2m} + \frac{\sqrt{r^2 - 4mk}}{2m}, \quad b = \frac{-r}{2m} - \frac{\sqrt{r^2 - 4mk}}{2m}$$

Case 3: $r^2 < 4km$. $y(t) = e^{at}[c_1 \cos bt + c_2 \sin bt]$, where

$$a = \frac{-r}{2m}, \quad b = \frac{\sqrt{4mk - r^2}}{2m}$$

(b) Referring to the three cases listed in (a):

Case 1: $y(t) = v_0 t e^{-(r/2m)t}$

Case 2: $y(t) = \frac{v_0 m}{\sqrt{r^2 - 4mk}}[e^{at} - e^{bt}]$

Case 3: $y(t) = \frac{v_0}{b} e^{at} \sin bt$