Engineering Mechanics Dynamics 15th edition Hibbeler Solutions Manual

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12–1.

Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6)\ \mathrm{m/s^2}$, where t is in seconds. What is the particle's velocity when $t = 6$ s, and what is its position when $t = 11$ s?

SOLUTION

$a = 2t - 6$

$dv = a\,dt$

$$\int_0^v dv = \int_0^t (2t - 6)\,dt$$

$v = t^2 - 6t$

$ds = v\,dt$

$$\int_0^s ds = \int_0^t (t^2 - 6t)\,dt$$

$$s = \frac{t^3}{3} - 3t^2$$

When $t = 6\ s$,

$v = 0$ **Ans.**

When $t = 11\ s$,

$s = 80.7\ \mathrm{m}$ **Ans.**

Ans:
$v = 0$
$s = 80.7\ \mathrm{m}$

12–2.

If a particle has an initial velocity of $v_0 = 12\text{ ft/s}$ to the right, at $s_0 = 0$, determine its position when $t = 10\text{ s}$, if $a = 2\text{ ft/s}^2$ to the left.

SOLUTION

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$= 0 + 12(10) + \frac{1}{2}(-2)(10)^2$$

$$= 20\text{ ft}$$ **Ans.**

Ans:
$s = 20\text{ ft}$

12–3.

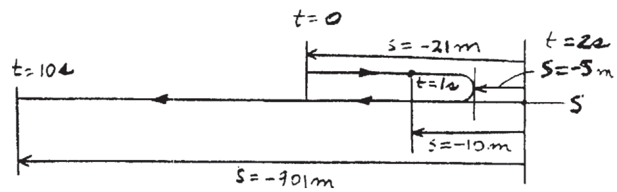
A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

SOLUTION

$$v = 12 - 3t^2 \qquad \textbf{(1)}$$

$$a = \frac{dv}{dt} = -6t\big|_{t=4} = -24 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\int_{-10}^{s} ds = \int_1^t v\,dt = \int_1^t (12 - 3t^2)\,dt$$



$$s + 10 = 12t - t^3 - 11$$

$$s = 12t - t^3 - 21$$

$$s\big|_{t=0} = -21$$

$$s\big|_{t=10} = -901$$

$$\Delta s = -901 - (-21) = -880 \text{ m} \qquad \textbf{Ans.}$$

From Eq. (1):

$v = 0$ when $t = 2s$

$$s\big|_{t=2} = 12(2) - (2)^3 - 21 = -5$$

$$s_T = (21 - 5) + (901 - 5) = 912 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$a = -24 \text{ m/s}^2$
$\Delta s = -880$ m
$s_T = 912$ m

***12–4.**

A particle travels along a straight line with a constant acceleration. When $s = 4$ ft, $v = 3$ ft/s and when $s = 10$ ft, $v = 8$ ft/s. Determine the velocity as a function of position.

SOLUTION

Velocity: To determine the constant acceleration a_c, set $s_0 = 4$ ft, $v_0 = 3$ ft/s, $s = 10$ ft and $v = 8$ ft/s and apply Eq. 12–6.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$8^2 = 3^2 + 2a_c\,(10 - 4)$$

$$a_c = 4.583 \text{ ft/s}^2$$

Using the result $a_c = 4.583 \text{ ft/s}^2$, the velocity function can be obtained by applying Eq. 12–6.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v^2 = 3^2 + 2(4.583)\,(s - 4)$$

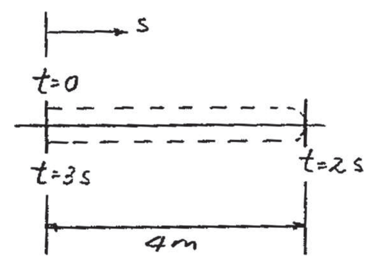
$$v = \left(\sqrt{9.17s - 27.7}\right) \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v = \left(\sqrt{9.17s - 27.7}\right)$ ft/s

12–5.

The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where t is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

SOLUTION



$$\frac{ds}{dt} = v$$

$$ds = (6t - 3t^2)\,dt$$

$$\int_0^s ds = \int_0^t (6t - 3t^2)dt$$

$$s = 3t^2 - t^3$$

At $t = 3$ s, $\quad s = 3(3)^2 - 3^3 = 0$ **Ans.**

when $v = 0$; $\quad 0 = 6t - 3t^2 \quad t = 0$ and 2 s

At $t = 2$ s, $\quad s = 3(2)^2 - 2^3 = 4$ m

From the figure $\quad s_T = 2(4) = 8$ m **Ans.**

$$v_{sp} = \frac{s_T}{\Delta t} = \frac{8}{3} = 2.67 \text{ m/s}$$ **Ans.**

$$a = \frac{dv}{dt} = 6 - 6t$$

At $t = 3$ s, $\quad a = 6 - 6(3) = -12$ m/s^2 **Ans.**

Ans:
$s = 0$
$s_T = 8$ m
$v_{sp} = 2.67$ m/s
$a = -12$ m/s^2

12–6.

A particle moving along a straight line is subjected to a deceleration $a = (-2v^3)\ \text{m/s}^2$, where v is in m/s. If it has a velocity $v = 8\ \text{m/s}$ and a position $s = 10\ \text{m}$ when $t = 0$, determine its velocity and position when $t = 4\ \text{s}$.

SOLUTION

Velocity: The velocity of the particle can be related to its position by applying Eq. 12–3.

$$ds = \frac{v\,dv}{a}$$

$$\int_{10\text{m}}^{s} ds = \int_{8\text{m/s}}^{v} -\frac{dv}{2v^2}$$

$$s - 10 = \frac{1}{2v} - \frac{1}{16}$$

$$v = \frac{8}{16s - 159} \qquad \textbf{[1]}$$

Position: The position of the particle can be related to the time by applying Eq.12–1.

$$dt = \frac{ds}{v}$$

$$\int_{0}^{t} dt = \int_{10\text{m}}^{s} \frac{1}{8}(16s - 159)\, ds$$

$$8t = 8s^2 - 159s + 790$$

When $t = 4$ s,

$$8(4) = 8s^2 - 159s + 790$$

$$8s^2 - 159s + 758 = 0$$

Choose the root greater than 10 m $s = 11.94\ \text{m} = 11.9\ \text{m}$ **Ans.**

Substitute $s = 11.94$ m into Eq. [1] yields

$$v = \frac{8}{16(11.94) - 159} = 0.250\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
$s = 11.9\ \text{m}$
$v = 0.250\ \text{m/s}$

12–7.

A particle moves along a straight line such that its position is defined by $s = (2t^3 + 3t^2 - 12t - 10)$ m. Determine the velocity, average velocity, and the average speed of the particle when $t = 3$ s.

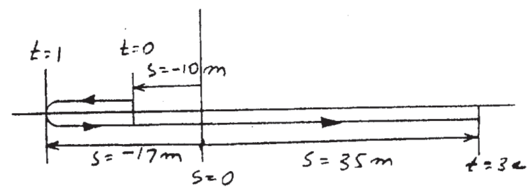
SOLUTION

$s = 2t^3 + 3t^2 - 12t - 10$

$v = \dfrac{ds}{dt} = 6t^2 + 6t - 12$

$v|_{t=3} = 6(3)^2 + 6(3) - 12 = 60 \text{ m/s}$ **Ans.**

$v = 0 \quad \text{at} \quad t = 1, \quad t = -2$



$s|_{t=0} = -10$

$s|_{t=1} = -17$

$s|_{t=3} = 35$

$v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{35 - (-10)}{3 - 0} = 15 \text{ m/s}$ **Ans.**

$(v_{sp})_{avg} = \dfrac{s_T}{\Delta t} = \dfrac{(17 - 10) + (17 + 35)}{3 - 0} = 19.7 \text{ m/s}$ **Ans.**

Ans:
$v|_{t=3s} = 60 \text{ m/s}$
$v_{avg} = 15 \text{ m/s}$
$(v_{sp})_{avg} = 19.7 \text{ m/s}$

***12–8.**

A particle is moving along a straight line such that its position is defined by $s = (10t^2 + 20)$ mm, where t is in seconds. Determine (a) the displacement of the particle during the time interval from $t = 1$ s to $t = 5$ s, (b) the average velocity of the particle during this time interval, and (c) the acceleration when $t = 1$ s.

SOLUTION

$s = 10t^2 + 20$

(a) $s|_{1\text{ s}} = 10(1)^2 + 20 = 30 \text{ mm}$

$s|_{5\text{ s}} = 10(5)^2 + 20 = 270 \text{ mm}$

$\Delta s = 270 - 30 = 240 \text{ mm}$ **Ans.**

(b) $\Delta t = 5 - 1 = 4 \text{ s}$

$v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{240}{4} = 60 \text{ mm/s}$ **Ans.**

(c) $a = \dfrac{d^2 s}{dt^2} = 20 \text{ mm/s}^2$ (for all t) **Ans.**

Ans:
$\Delta s = 240 \text{ mm}$
$v_{avg} = 60 \text{ mm/s}$
$a = 20 \text{ mm/s}^2$

12–9.

A particle moves along a straight path with an acceleration of $a = (5/s)\ \text{m/s}^2$, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it is released from rest when $s = 1$ m.

SOLUTION

$$a\,ds = v\,dv$$

$$\frac{5}{s}\,ds = v\,dv$$

$$5\int_1^2 \frac{ds}{s} = \int_0^v v\,dv$$

$$5\,(\ln s)\Big|_1^2 = \frac{v^2}{2}$$

$$v = 2.63\ \text{m/s}$$ **Ans.**

Ans:
$v = 2.63$ m/s

12–10.

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})\ \text{m/s}^2$, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use a numerical method to evaluate the integral.

SOLUTION

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

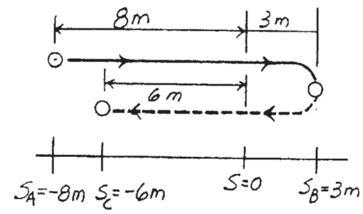
$v = 1.29$ m/s **Ans.**

Ans:
$v = 1.29$ m/s

12–11.

A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8$ m to a position $s_B = +3$ m. Then in another 5 s it moves from s_B to $s_C = -6$ m. Determine the particle's average velocity and average speed during the 9-s time interval.

SOLUTION



Average Velocity: The displacement from A to C is $\Delta s = s_C - S_A = -6 - (-8) = 2$ m.

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{2}{4 + 5} = 0.222 \text{ m/s} \qquad \textbf{Ans.}$$

Average Speed: The distances traveled from A to B and B to C are $s_{A\to B} = 8 + 3 = 11.0$ m and $s_{B\to C} = 3 + 6 = 9.00$ m, respectively. Then, the total distance traveled is $s_{Tot} = s_{A\to B} + s_{B\to C} = 11.0 + 9.00 = 20.0$ m.

$$(v_{sp})_{avg} = \frac{s_{Tot}}{\Delta t} = \frac{20.0}{4 + 5} = 2.22 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_{avg} = 0.222$ m/s
$(v_{sp})_{avg} = 2.22$ m/s

***12–12.**

The speed of a particle traveling along a straight line within a liquid is measured as a function of its position as $v = (100 - s)$ mm/s, where s is in millimeters. Determine (a) the particle's deceleration when it is located at point A, where $s_A = 75$ mm, (b) the distance the particle travels before it stops, and (c) the time needed to stop the particle.

SOLUTION

(a) $a\,ds = v\,dv$

$$a = v\frac{dv}{ds} = (100 - s)(-1) = s - 100$$

When $s = s_A = 75$ mm, $a = 75 - 100 = -25$ mm/s^2 **Ans.**

(b) $v = 100 - s$

$0 = 100 - s \qquad s = 100$ mm **Ans.**

(c) $v = \dfrac{ds}{dt}$

$$dt = \frac{ds}{v} = \frac{ds}{100 - s}$$

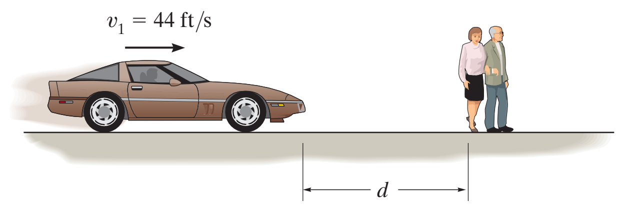
$$\int_0^t dt = \int_0^{100} \frac{ds}{100 - s}$$

$t = -\ln(100 - s)|_0^{100} \rightarrow \infty$ **Ans.**

Ans:
$a = -25$ mm/s^2
$s = 100$ mm
$t = -\ln(100 - s)|_0^{100} \rightarrow \infty$

12–13.

Tests reveal that a driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2, determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral*: If you must drink, please don't drive!



SOLUTION

Stopping Distance: For normal driver, the car moves a distance of $d' = vt = 44(0.75) = 33.0$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 33.0$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 33.0)$$

$$d = 517 \text{ ft} \qquad \textbf{Ans.}$$

For a drunk driver, the car moves a distance of $d' = vt = 44(3) = 132$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 132$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

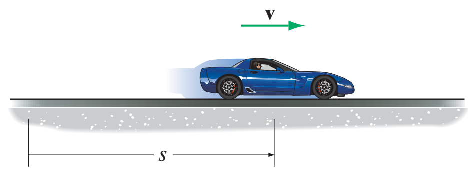
$$0^2 = 44^2 + 2(-2)(d - 132)$$

$$d = 616 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
Normal: $d = 517$ ft
Drunk: $d = 616$ ft

12–14.

The sports car travels along the straight road such that $v = 3\sqrt{100-s}$ m/s, where s is in meters. Determine the time for the car to reach $s = 60$ m. How much time does it take to stop?



SOLUTION

***s–t* Function.** This function can be determined by integrating $ds = v dt$ using the initial condition $s = 0$ at $t = 0$.

$$ds = 3\sqrt{100 - s}\, dt$$

$$\int_o^t dt = \frac{1}{3}\int_o^s \frac{ds}{\sqrt{100 - s}}$$

$$t = -\frac{2}{3}(\sqrt{100 - s})\Big|_o^s$$

$$t = \frac{2}{3}(10 - \sqrt{100 - s})$$

When $s = 60$ m,

$$t = \frac{2}{3}(10 - \sqrt{100 - 60}) = 2.4503s = 2.45s$$ **Ans.**

When the car stops, $v = 0$. Then

$$0 = 3\sqrt{100 - s} \qquad s = 100 \text{ m}$$

Thus, the required time is

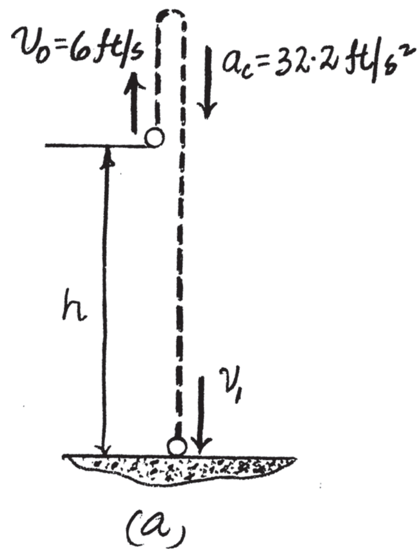
$$t = \frac{2}{3}(10 - \sqrt{100 - 100}) = 6.6667s = 6.67s$$ **Ans.**

Ans:
$t = 2.45s$
$t = 6.67s$

12–15.

A ball is released from the bottom of an elevator which is traveling upward with a velocity of 6 ft/s. If the ball strikes the bottom of the elevator shaft in 3 s, determine the height of the elevator from the bottom of the shaft at the instant the ball is released. Also, find the velocity of the ball when it strikes the bottom of the shaft.

SOLUTION



Kinematics: When the ball is released, its velocity will be the same as the elevator at the instant of release. Thus, $v_0 = 6$ ft/s. Also, $t = 3$ s, $s_0 = 0$, $s = -h$, and $a_c = -32.2$ ft/s^2.

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-h = 0 + 6(3) + \frac{1}{2}(-32.2)\left(3^2\right)$$

$$h = 127 \text{ ft}$$ **Ans.**

$(+\uparrow)$ $$v = v_0 + a_c t$$

$$v = 6 + (-32.2)(3)$$

$$= -90.6 \text{ ft/s} = 90.6 \text{ ft/s} \downarrow$$ **Ans.**

Ans:
$h = 127$ ft
$v = 90.6$ ft/s $\downarrow$

***12–16.**

A particle is moving along a straight line with an initial velocity of 6 m/s when it is subjected to a deceleration of $a = (-1.5v^{1/2})\ \text{m/s}^2$, where v is in m/s. Determine how far it travels before it stops. How much time does this take?

SOLUTION

Distance Traveled: The distance traveled by the particle can be determined by applying Eq. 12–3.

$$ds = \frac{v\,dv}{a}$$

$$\int_0^s ds = \int_{6\text{ m/s}}^{v} \frac{v}{-1.5v^{\frac{1}{2}}}\,dv$$

$$s = \int_{6\text{ m/s}}^{v} -0.6667\,v^{\frac{1}{2}}\,dv$$

$$= \left(-0.4444v^{\frac{3}{2}} + 6.532\right)\text{ m}$$

When $v = 0$, $\quad s = -0.4444\left(0^{\frac{3}{2}}\right) + 6.532 = 6.53\text{ m}$ **Ans.**

Time: The time required for the particle to stop can be determined by applying Eq. 12–2.

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = -\int_{6\text{ m/s}}^{v} \frac{dv}{1.5v^{\frac{1}{2}}}$$

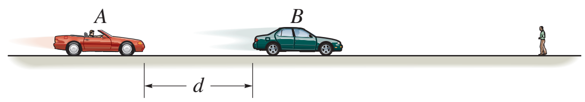
$$t = -1.333\left(v^{\frac{1}{2}}\right)\Big|_{6\text{ m/s}}^{v} = \left(3.266 - 1.333v^{\frac{1}{2}}\right)\text{s}$$

When $v = 0$, $\quad t = 3.266 - 1.333\left(0^{\frac{1}{2}}\right) = 3.27\text{ s}$ **Ans.**

Ans:
$s = 6.53\text{ m}$
$t = 3.27\text{ s}$

12–17.

Car B is traveling a distance d ahead of car A. Both cars are traveling at 60 ft/s when the driver of B suddenly applies the brakes, causing his car to decelerate at 12 ft/s^2. It takes the driver of car A 0.75 s to react (this is the normal reaction time for drivers). When he applies his brakes, he decelerates at 15 ft/s^2. Determine the minimum distance d be tween the cars so as to avoid a collision.



SOLUTION

For B:

$(\overset{+}{\rightarrow})\qquad v = v_0 + a_c t$

$$v_B = 60 - 12\,t$$

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$s_B = d + 60t - \frac{1}{2}(12)\,t^2 \qquad \textbf{(1)}$$

For A:

$(\overset{+}{\rightarrow})\qquad v = v_0 + a_c t$

$$v_A = 60 - 15(t - 0.75), \quad [t > 0.75]$$

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$s_A = 60(0.75) + 60(t - 0.75) - \frac{1}{2}(15)\,(t - 0.75)^2, \quad [t > 0.74] \qquad \textbf{(2)}$$

Require $v_A = v_B$ the moment of closest approach.

$60 - 12t = 60 - 15(t - 0.75)$

$t = 3.75$ s

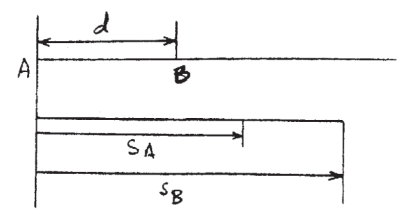
Worst case without collision would occur when $s_A = s_B$.

At $t = 3.75$ s, from Eqs. (1) and (2):

$60(0.75) + 60(3.75 - 0.75) - 7.5(3.75 - 0.75)^2 = d + 60(3.75) - 6(3.75)^2$

$157.5 = d + 140.625$

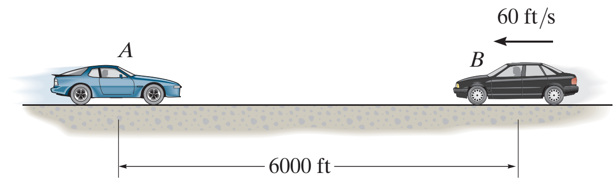
$d = 16.9$ ft **Ans.**



Ans:
$d = 16.9$ ft

12–18.

Car A starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s^2 until it reaches a speed of 80 ft/s. Afterwards it maintains this speed. Also, when $t = 0$, car B located 6000 ft down the road is traveling towards A at a constant speed of 60 ft/s. Determine the distance traveled by car A when they pass each other.



SOLUTION

Distance Traveled: Time for car A to achives $v = 80$ ft/s can be obtained by applying Eq. 12–4.

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$
$$80 = 0 + 6t$$
$$t = 13.33 \text{ s}$$

The distance car A travels for this part of motion can be determined by applying Eq. 12–6.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$
$$80^2 = 0 + 2(6)(s_1 - 0)$$
$$s_1 = 533.33 \text{ ft}$$

For the second part of motion, car A travels with a constant velocity of $v = 80$ ft/s and the distance traveled in $t' = (t_1 - 13.33)$ s (t_1 is the total time) is

$$(\overset{+}{\rightarrow}) \qquad s_2 = vt' = 80(t_1 - 13.33)$$

Car B travels in the opposite direction with a constant velocity of $v = 60$ ft/s and the distance traveled in t_1 is

$$(\overset{+}{\rightarrow}) \qquad s_3 = vt_1 = 60t_1$$

It is required that

$$s_1 + s_2 + s_3 = 6000$$
$$533.33 + 80(t_1 - 13.33) + 60t_1 = 6000$$
$$t_1 = 46.67 \text{ s}$$

The distance traveled by car A is

$$s_A = s_1 + s_2 = 533.33 + 80(46.67 - 13.33) = 3200 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$s_A = 3200$ ft

12–19.

A particle moves along a straight path with an acceleration of $a = (kt^3 + 4)$ ft/s^2, where t is in seconds. Determine the constant k, knowing that $v = 12$ ft/s when $t = 1$ s, and that $v = -2$ ft/s when $t = 2$ s.

SOLUTION

$$a = \frac{dv}{dt}$$

$$(kt^3 + 4)dt = dv$$

$$\int_1^2 (kt^3 + 4)dt = \int_{12}^{-2} dv$$

$$\frac{1}{4}kt^4 + 4t \Big|_1^2 = v\Big|_{12}^{-2}$$

$$k = -4.80 \text{ ft/s}^5$$ **Ans.**

Ans:
$k = -4.80 \text{ ft/s}^5$

***12–20.**

The velocity of a particle traveling along a straight line is $v = (3t^2 - 6t)$ ft/s, where t is in seconds. If $s = 4$ ft when $t = 0$, determine the position of the particle when $t = 4$ s. What is the total distance traveled during the time interval $t = 0$ to $t = 4$ s? Also, what is the acceleration when $t = 2$ s?

SOLUTION

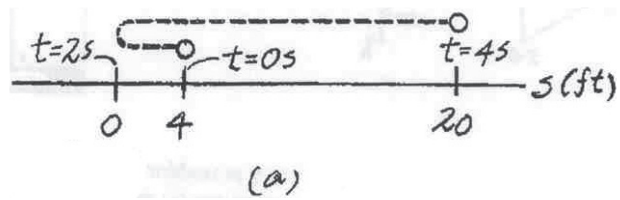
Position: The position of the particle can be determined by integrating the kinematic equation $ds = v\,dt$ using the initial condition $s = 4$ ft when $t = 0$ s. Thus,

$$(\xrightarrow{+}) \qquad ds = v\,dt$$

$$\int_{4\text{ ft}}^{s} ds = \int_0^t \left(3t^2 - 6t\right)dt$$

$$s\Big|_{4\text{ ft}}^{s} = \left(t^3 - 3t^2\right)\Big|_0^t$$

$$s = \left(t^3 - 3t^2 + 4\right)\text{ ft}$$



When $t = 4$ s,

$$s|_{t=4\text{ s}} = 4^3 - 3(4^2) + 4 = 20\text{ ft} \qquad \textbf{Ans.}$$

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. Thus,

$$v = 3t^2 - 6t = 0$$

$$t(3t - 6) = 0$$

$$t = 0 \text{ and } t = 2\text{ s}$$

The position of the particle at $t = 0$ and 2 s is

$$s|_{0\text{ s}} = 0 - 3\left(0^2\right) + 4 = 4\text{ ft}$$

$$s|_{2\text{ s}} = 2^3 - 3\left(2^2\right) + 4 = 0$$

Using the above result, the path of the particle shown in Fig. *a* is plotted. From this figure,

$$s_{\text{Tot}} = 4 + 20 = 24\text{ ft} \qquad \textbf{Ans.}$$

Acceleration:

$$(\xrightarrow{+}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}\left(3t^2 - 6t\right)$$

$$a = (6t - 6)\text{ ft/s}^2$$

When $t = 2$ s,

$$a|_{t=2\text{ s}} = 6(2) - 6 = 6\text{ ft/s}^2 \rightarrow \qquad \textbf{Ans.}$$

Ans:
$s|_{t=4\text{ s}} = 20$ ft
$s_{\text{Tot}} = 24$ ft
$a|_{t=2\text{ s}} = 6\text{ ft/s}^2 \rightarrow$

12–21.

A freight train travels at $v = 60(1 - e^{-t})$ ft/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.

SOLUTION

$v = 60(1 - e^{-t})$

$$\int_0^s ds = \int v\,dt = \int_0^3 60(1 - e^{-t})dt$$

$s = 60(t + e^{-t})\big|_0^3$

$s = 123 \text{ ft}$ **Ans.**

$$a = \frac{dv}{dt} = 60(e^{-t})$$

At $t = 3$ s

$a = 60e^{-3} = 2.99 \text{ ft/s}^2$ **Ans.**

Ans:
$s = 123$ ft
$a = 2.99 \text{ ft/s}^2$

12–22.

The acceleration of the boat is defined by $a = (1.5\, v^{1/2})$ m/s. Determine its speed when $t = 4$ s if it has a speed of 3 m/s when $t = 0$.

SOLUTION

***v*−*t* Function:** This function can be determined by integrating $dv = a\,dt$ using the initial condition $v = 3$ m/s at $t = 0$,

$$dv = \frac{3}{2}v^{\frac{1}{2}}\,dt$$

$$\int_o^t dt = \frac{2}{3}\int_{3\text{ m/s}}^{v}\frac{dv}{v^{\frac{1}{2}}}$$

$$t = \frac{4}{3}v^{\frac{1}{2}}\Big|_{3\text{ m/s}}^{v}$$

$$t = \frac{4}{3}\left(v^{\frac{1}{2}} - \sqrt{3}\right)$$

$$v = \left(\frac{3}{4}t + \sqrt{3}\right)^2 \text{ m/s}$$

When $t = 4$ s,

$$v = \left[\frac{3}{4}(4) + \sqrt{3}\right]^2 = 22.39 \text{ m/s} = 22.4 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 22.4$ m/s

12–23.

A particle is moving along a straight line such that its acceleration is defined as $a = (-2v)\ \text{m/s}^2$, where v is in meters per second. If $v = 20$ m/s when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

SOLUTION

$$a = -2v$$

$$\frac{dv}{dt} = -2v$$

$$\int_{20}^{v} \frac{dv}{v} = \int_0^t -2\,dt$$

$$\ln \frac{v}{20} = -2t$$

$$v = \left(20e^{-2t}\right)\ \text{m/s} \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = \left(-40e^{-2t}\right)\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$\int_0^s ds = v\,dt = \int_0^t \left(20e^{-2t}\right)dt$$

$$s = -10e^{-2t}\big|_0^t = -10\left(e^{-2t} - 1\right)$$

$$s = 10\left(1 - e^{-2t}\right)\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$v = \left(20e^{-2t}\right)$ m/s
$a = \left(-40e^{-2t}\right)$ m/s^2
$s = 10\left(1 - e^{-2t}\right)$ m

***12–24.**

When a particle is projected vertically upward with an initial velocity of v_0, it experiences an acceleration $a = -(g + kv^2)$, where g is the acceleration due to gravity, k is a constant, and v is the velocity of the particle. Determine the maximum height reached by the particle.

SOLUTION

Position:

$$(+\uparrow) \qquad ds = \frac{v\,dv}{a}$$

$$\int_0^s ds = \int_{v_0}^{v} -\frac{v dv}{g + kv^2}$$

$$s\big|_0^s = -\left[\frac{1}{2k}\ln\left(g + kv^2\right)\right]\Bigg|_{v_0}^{v}$$

$$s = \frac{1}{2k}\ln\left(\frac{g + k{v_0}^2}{g + kv^2}\right)$$

The particle achieves its maximum height when $v = 0$. Thus,

$$h_{\max} = \frac{1}{2k}\ln\left(\frac{g + k{v_0}^2}{g}\right)$$

$$= \frac{1}{2k}\ln\left(1 + \frac{k}{g}{v_0}^2\right) \qquad \textbf{Ans.}$$

Ans:

$h_{\max} = \frac{1}{2k}\ln\left(1 + \frac{k}{g}{v_0}^2\right)$

12–25.

If the effects of atmospheric resistance are accounted for, a freely falling body has an acceleration defined by the equation, $a = 9.81[1 - v^2(10^{-4})]\ \mathrm{m/s^2}$, where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

SOLUTION

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81}\left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)}\right]$$

$$9.81t = 50\ln\left(\frac{1 + 0.01v}{1 - 0.01v}\right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \qquad \textbf{(1)}$$

a) When $t = 5$ s, then, from Eq. (1)

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5\ \mathrm{m/s} \qquad \textbf{Ans.}$$

b) If $t \rightarrow \infty$, $\dfrac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \rightarrow 1$. Then, from Eq. (1)

$$v_{\max} = 100\ \mathrm{m/s} \qquad \textbf{Ans.}$$

Ans:
(a) $v = 45.5\ \mathrm{m/s}$
(b) $v_{\max} = 100\ \mathrm{m/s}$

12–26.

A ball is thrown with an upward velocity of 5 m/s from the top of a 10-m-high building. One second later another ball is thrown upward from the ground with a velocity of 10 m/s. Determine the height from the ground where the two balls pass each other.

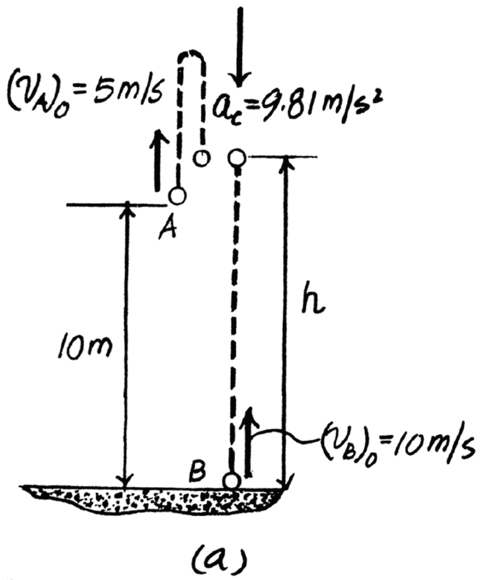
SOLUTION

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = 5$ m/s, $(s_A)_0 = 0$, $s_A = (h - 10)$m, $t_A = t'$, and $a_c = -9.81$ m/s^2. Thus,

$$(+\uparrow) \qquad s_A = (s_A)_0 + (v_A)_0 t_A + \frac{1}{2} a_c t_A^2$$

$$h - 10 = 0 + 5t' + \frac{1}{2}(-9.81)(t')^2$$

$$h = 5t' - 4.905(t')^2 + 10 \qquad \textbf{(1)}$$



Motion of ball B is with $(v_B)_0 = 10$ m/s, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - 1$ and $a_c = -9.81$ m/s^2. Thus,

$$(+\uparrow) \qquad s_B = (s_B)_0 + (v_B)_0 t_B + \frac{1}{2} a_c t_B^2$$

$$h = 0 + 10(t' - 1) + \frac{1}{2}(-9.81)(t' - 1)^2$$

$$h = 19.81t' - 4.905(t')^2 - 14.905 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$h = 4.54 \text{ m} \qquad \textbf{Ans.}$$

$$t' = 1.68 \text{ s}$$

Ans:

$h = 4.54$ m

12–27.

When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v^2{}_f)(v^2{}_f - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

SOLUTION

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)\left(v_f^2 - v^2\right)$$

$$\int_0^v \frac{dv}{v_f^2 - v^{2'}} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f + v}{v_f - v}\right)\bigg|_0^v = \frac{g}{v_f^2}\,t$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v}{v_f - v}\right)$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v_f/2}{v_f - v_f/2}\right)$$

$$t = 0.549\left(\frac{v_f}{g}\right)$$ **Ans.**

Ans:

$t = 0.549\left(\frac{v_f}{g}\right)$

***12–28.**

A train is initially traveling along a straight track at a speed of 90 km/h. For 6 s it is subjected to a constant deceleration of 0.5 m/s^2, and then for the next 5 s it has a constant deceleration a_c. Determine a_c so that the train stops at the end of the 11-s time period.

SOLUTION

First stage of motion:

$$v = v_0 + a_c t$$

$$= \frac{90(10)^3}{3600} + (-0.5)(6)$$

$$= 22 \text{ m/s}$$

Second stage of motion:

$$v = v_0 + a_c t$$

$$0 = 22 + a_c(5)$$

$$a_c = -4.40 \text{ m/s}^2$$ **Ans.**

Ans:
$a_c = -4.40$ m/s^2

12–29.

Two cars A and B start from rest at a stop line. Car A has a constant acceleration of $a_A = 8\ \text{m/s}^2$, while car B has an acceleration of $a_B = (2t^{3/2})\ \text{m/s}^2$, where t is in seconds. Determine the distance between the cars when A reaches a velocity of $v_A = 120\ \text{km/h}$.

SOLUTION

For Car A:

$(\overset{+}{\rightarrow})$
$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$\left[\frac{120(10)^3}{3600}\right]^2 = 0 + 2(8)(s_A - 0)$$

$$s_A = 69.44\ \text{m}$$

$(\overset{+}{\rightarrow})$
$$v = v_0 + a_c t$$

$$\frac{120(10)^3}{3600} = 0 + 8t \qquad t = 4.1667\ \text{s}$$

For Car B:

$(\overset{+}{\rightarrow})$
$$\frac{dv}{dt} = a$$

$$dv = 2t^{\frac{3}{2}}dt$$

$$\int_0^{vB} dv = \int_0^t 2t^{\frac{3}{2}}dt$$

$$v_B = \frac{4}{5}t^{\frac{5}{2}}$$

$(\overset{+}{\rightarrow})$
$$\frac{ds}{dt} = v$$

$$ds = \frac{4}{5}t^{\frac{5}{2}}dt$$

$$\int_0^{s_B} ds = \frac{4}{5}\int_0^{4.1667} t^{\frac{5}{2}}dt$$

$$s_B = 33.75\ \text{m}$$

The distance between cars A and B is $s_{AB} = 69.44 - 33.75 = 35.7\ \text{m}$ **Ans.**

Ans:
$s_{AB} = 35.7\ \text{m}$

12–30.

A sphere is fired downward into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)\ \text{m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

SOLUTION

Velocity: $v_0 = 27$ m/s at $t_0 = 0$ s. Applying Eq. 12–2, we have

$$(+\downarrow) \qquad dv = a\,dt$$

$$\int_{27}^{v} dv = \int_0^t -6t\,dt$$

$$v = \left(27 - 3t^2\right)\ \text{m/s} \qquad \textbf{(1)}$$

At $v = 0$, from Eq. (1)

$$0 = 27 - 3t^2 \qquad t = 3.00\ \text{s}$$

Distance Traveled: $s_0 = 0$ m at $t_0 = 0$ s. Using the result $v = 27 - 3t^2$ and applying Eq. 12–1, we have

$$(+\downarrow) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(27 - 3t^2\right) dt$$

$$s = \left(27t - t^3\right)\ \text{m} \qquad \textbf{(2)}$$

At $t = 3.00$ s, from Eq. (2)

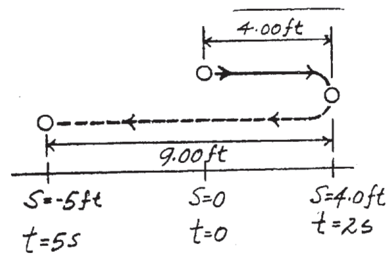
$$s = 27(3.00) - 3.00^3 = 54.0\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$s = 54.0$ m

12–31.

A particle is moving along a straight line such that its position is given by $s = (4t - t^2)$ ft, where t is in seconds. Determine the distance traveled from $t = 0$ to $t = 5$ s, the average velocity, and the average speed of the particle during this time interval.

SOLUTION



Total Distance Traveled: The velocity of the particle can be determined by applying Eq. 12–1.

$$v = \frac{ds}{dt} = 4 - 2t$$

The times when the particle stops are

$$4 - 2t = 0 \qquad t = 2\text{ s}$$

The position of the particle at $t = 0$ s, 2 s and 5 s are

$$s|_{t=0\,s} = 4(0) - \left(0^2\right) = 0$$

$$s|_{t=2\,s} = 4(2) - \left(2^2\right) = 4.00\text{ ft}$$

$$s|_{t=5\,s} = 4(5) - \left(5^2\right) = -5.00\text{ ft}$$

From the particle's path, the total distance is

$$s_{\text{tot}} = 4.00 + 9.00 = 13.0\text{ ft}$$ **Ans.**

Average Velocity: The displacement of the particle from $t = 0$ s to 5 s is $\Delta s = s|_{t=5\,t} - s|_{t=0\,s} = -5.00 - 0 = -5.00$ ft.

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{-5}{5} = -1.00\text{ ft/s}$$ **Ans.**

Average Speed:

$$\left(v_{sp}\right)_{\text{avg}} = \frac{s_{\text{Tot}}}{\Delta t} = \frac{13.0}{5} = 2.60\text{ ft/s}$$ **Ans.**

Ans:
$S_{\text{tot}} = 13.0$ ft
$v_{\text{avg}} = -1.00$ ft/s
$(v_{\text{sp}})_{\text{avg}} = 2.60$ ft/s

***12–32.**

Ball A is thrown vertically upward from the top of a 30-m-high building with an initial velocity of 5 m/s. At the same instant another ball B is thrown upward from the ground with an initial velocity of 20 m/s. Determine the height from the ground and the time at which they pass.

SOLUTION

Origin at roof:

Ball A:

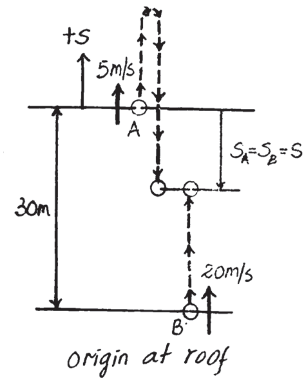
$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-s = 0 + 5t - \frac{1}{2}(9.81)t^2$$

Ball B:

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-s = -30 + 20t - \frac{1}{2}(9.81)t^2$$



Solving,

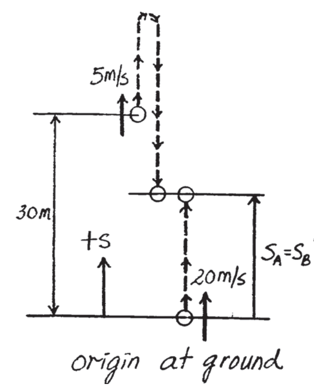
$t = 2$ s **Ans.**

$s = 9.62$ m

Distance from ground,

$d = (30 - 9.62) = 20.4$ m **Ans.**

Also, origin at ground,



$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 30 + 5t + \frac{1}{2}(-9.81)t^2$$

$$s_B = 0 + 20t + \frac{1}{2}(-9.81)t^2$$

Require

$$s_A = s_B$$

$$30 + 5t + \frac{1}{2}(-9.81)t^2 = 20t + \frac{1}{2}(-9.81)t^2$$

$t = 2$ s

$s_B = 20.4$ m

Ans:
$t = 2$ s
$d = 20.4$ m

12–33.

As a body is projected to a high altitude above the earth's surface, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81 \text{ m/s}^2$ and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint*: This requires that $v = 0$ as $y \rightarrow \infty$.

SOLUTION

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0 R^2 \int_0^\infty \frac{dy}{(R + y)^2}$$

$$\left.\frac{v^2}{2}\right|_v^0 = \left.\frac{g_0 R^2}{R + y}\right|_0^\infty$$

$$v = \sqrt{2g_0 R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167 \text{ m/s} = 11.2 \text{ km/s}$$ **Ans.**

Ans:
$v = 11.2 \text{ km/s}$

12–34.

Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–33), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–33.

SOLUTION

From Prob. 12–33,

$$(+\uparrow) \qquad a = -g_0 \frac{R^2}{(R+y)^2}$$

Since $a\,dy = v\,dv$

then

$$-g_0 R^2 \int_{y_0}^{y} \frac{dy}{(R+y)^2} = \int_0^v v\,dv$$

$$g_0 R^2 \left[\frac{1}{R+y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0 R^2 \left[\frac{1}{R+y} - \frac{1}{R+y_0}\right] = \frac{v^2}{2}$$

Thus

$$v = -R\sqrt{\frac{2g_0\,(y_0 - y)}{(R+y)(R+y_0)}} \qquad \textbf{Ans.}$$

When $y_0 = 500$ km, $\quad y = 0$,

$$v = -6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356+500)(10^6)}}$$

$$v = -3016 \text{ m/s} = 3.02 \text{ km/s} \downarrow \qquad \textbf{Ans.}$$

Ans:

$$v = -R\sqrt{\frac{2g_0(y_0 - y)}{(R+y)(R+y_0)}}$$

$$v_{\text{imp}} = 3.02 \text{ km/s}$$

12–35.

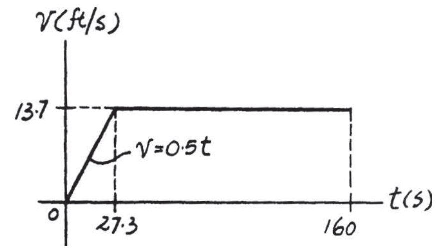
A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s^2. After a time t' it maintains a constant speed so that when $t = 160$ s it has traveled 2000 ft. Determine the time t' and draw the v–t graph for the motion.

SOLUTION

Total Distance Traveled: The distance for part one of the motion can be related to time $t = t'$ by applying Eq. 12–5 with $s_0 = 0$ and $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(t')^2 = 0.25(t')^2$$



The velocity at time t can be obtained by applying Eq. 12–4 with $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t = 0 + 0.5t = 0.5t \qquad \textbf{(1)}$$

The time for the second stage of motion is $t_2 = 160 - t'$ and the train is traveling at a constant velocity of $v = 0.5t'$ (Eq. (1)). Thus, the distance for this part of motion is

$$(\overset{+}{\rightarrow}) \qquad s_2 = vt_2 = 0.5t'(160 - t') = 80t' - 0.5(t')^2$$

If the total distance traveled is $s_{Tot} = 2000$, then

$$s_{Tot} = s_1 + s_2$$

$$2000 = 0.25(t')^2 + 80t' - 0.5(t')^2$$

$$0.25(t')^2 - 80t' + 2000 = 0$$

Choose a root that is less than 160 s, then

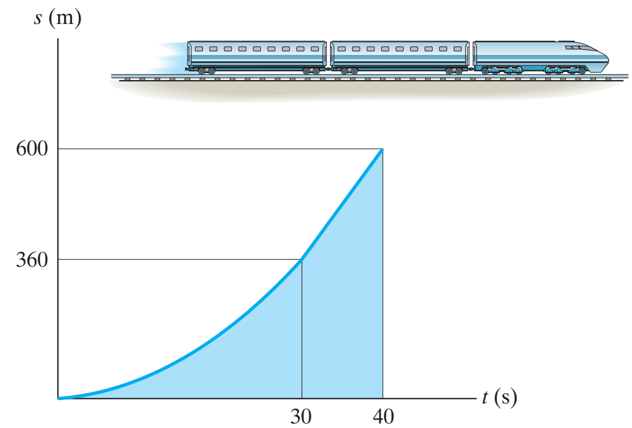
$$t' = 27.34 \text{ s} = 27.3 \text{ s} \qquad \textbf{Ans.}$$

***v* − *t* Graph:** The equation for the velocity is given by Eq. (1). When $t = t' = 27.34$ s, $v = 0.5(27.34) = 13.7$ ft/s.

Ans:
$t' = 27.3$ s.
When $t = 27.3$ s, $v = 13.7$ ft/s.

***12–36.**

The s–t graph for a train has been experimentally determined. From the data, construct the v–t and a–t graphs for the motion; $0 \le t \le 40$ s. For $0 \le t < 30$ s, the curve is $s = (0.4t^2)$ m, and then it becomes straight for $t > 30$ s.



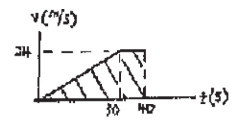
SOLUTION

$0 \le t < 30$:

$$s = 0.4t^2$$

$$v = \frac{ds}{dt} = 0.8t$$

$$a = \frac{dv}{dt} = 0.8 \text{ m/s}^2$$



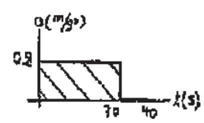
$30 < t \le 40$:

$$s - 360 = \left(\frac{600 - 360}{40 - 30}\right)(t - 30)$$

$$s = 24t - 360$$

$$v = \frac{ds}{dt} = 24 \text{ m/s}$$

$$a = \frac{dv}{dt} = 0$$



Ans:

$$v = \frac{ds}{dt} = 0.8t$$

$$a = \frac{dv}{dt} = 0.8 \text{ m/s}^2$$

$$s = 24t - 360$$

$$v = \frac{ds}{dt} = 24 \text{ m/s}$$

$$a = \frac{dv}{dt} = 0$$

12–37.

Two rockets start from rest at the same elevation. Rocket A accelerates vertically at 20 m/s^2 for 12 s and then maintains a constant speed. Rocket B accelerates at 15 m/s^2 until reaching a constant speed of 150 m/s. Construct the a–t, v–t, and s–t graphs for each rocket until $t = 20$ s. What is the distance between the rockets when $t = 20$ s?

SOLUTION

For rocket A:

For $t < 12$ s

$$+\uparrow \quad v_A = (v_A)_0 + a_A t$$

$$v_A = 0 + 20\,t$$

$$v_A = 20\,t$$

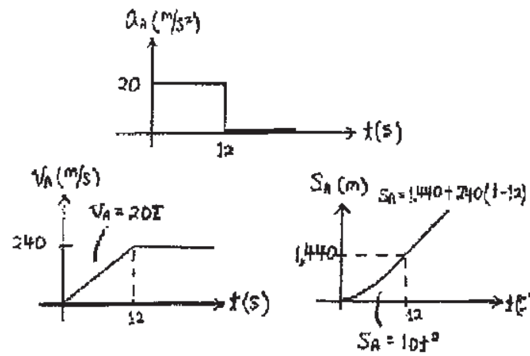
$$+\uparrow \quad s_A = (s_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(20)\,t^2$$

$$s_A = 10\,t^2$$

When $t = 12$ s, $\quad v_A = 240$ m/s

$s_A = 1440$ m



For $t > 12$ s

$v_A = 240$ m/s

$s_A = 1440 + 240(t - 12)$

For rocket B:

For $t < 10$ s

$$+\uparrow \quad v_B = (v_B)_0 + a_B t$$

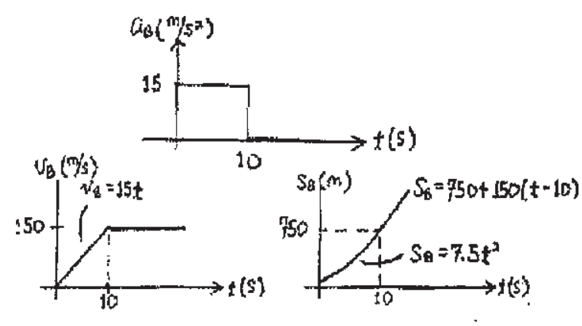
$$v_B = 0 + 15\,t$$

$$v_B = 15\,t$$

$$+\uparrow \quad s_B = (s_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(15)\,t^2$$

$$s_B = 7.5\,t^2$$



When $t = 10$ s, $\quad v_B = 150$ m/s

$s_B = 750$ m

For $t > 10$ s

$v_B = 150$ m/s

$s_B = 750 + 150(t - 10)$

When $t = 20$ s, $\quad s_A = 3360$ m, $\quad s_B = 2250$ m

$\Delta s = 1110 \text{ m} = 1.11 \text{ km}$ **Ans.**

Ans:
$\Delta s = 1.11$ km

12–38.

A particle starts from $s = 0$ and travels along a straight line with a velocity $v = (t^2 - 4t + 3)$ m/s, where t is in seconds. Construct the $v-t$ and $a-t$ graphs for the time interval $0 \le t \le 4$ s.

SOLUTION

***a–t* Graph:**

$$a = \frac{dv}{dt} = \frac{d}{dt}(t^2 - 4t + 3)$$

$$a = (2t - 4)\,\text{m/s}^2$$

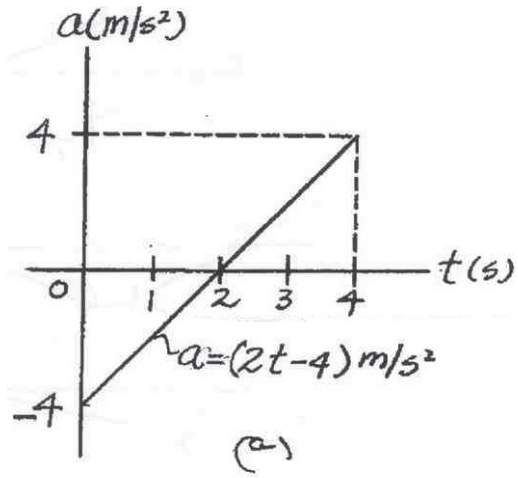
Thus,

$$a|_{t=0} = 2(0) - 4 = -4\ \text{m/s}^2$$

$$a|_{t=2} = 0$$

$$a|_{t=4\,\text{s}} = 2(4) - 4 = 4\ \text{m/s}^2$$

The $a-t$ graph is shown in Fig. a.



***v–t* Graph:** The slope of the $v-t$ graph is zero when $a = \dfrac{dv}{dt} = 0$. Thus,

$$a = 2t - 4 = 0 \qquad\qquad t = 2\ \text{s}$$

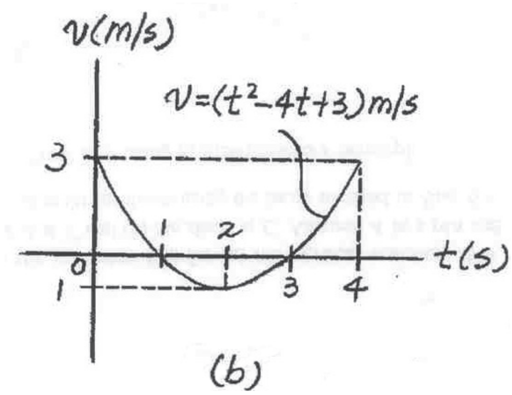
The velocity of the particle at $t = 0$ s, 2 s, and 4 s are

$$v|_{t=0\,\text{s}} = 0^2 - 4(0) + 3 = 3\ \text{m/s}$$

$$v|_{t=2\,\text{s}} = 2^2 - 4(2) + 3 = -1\ \text{m/s}$$

$$v|_{t=4\,\text{s}} = 4^2 - 4(4) + 3 = 3\ \text{m/s}$$

The $v-t$ graph is shown in Fig. b.

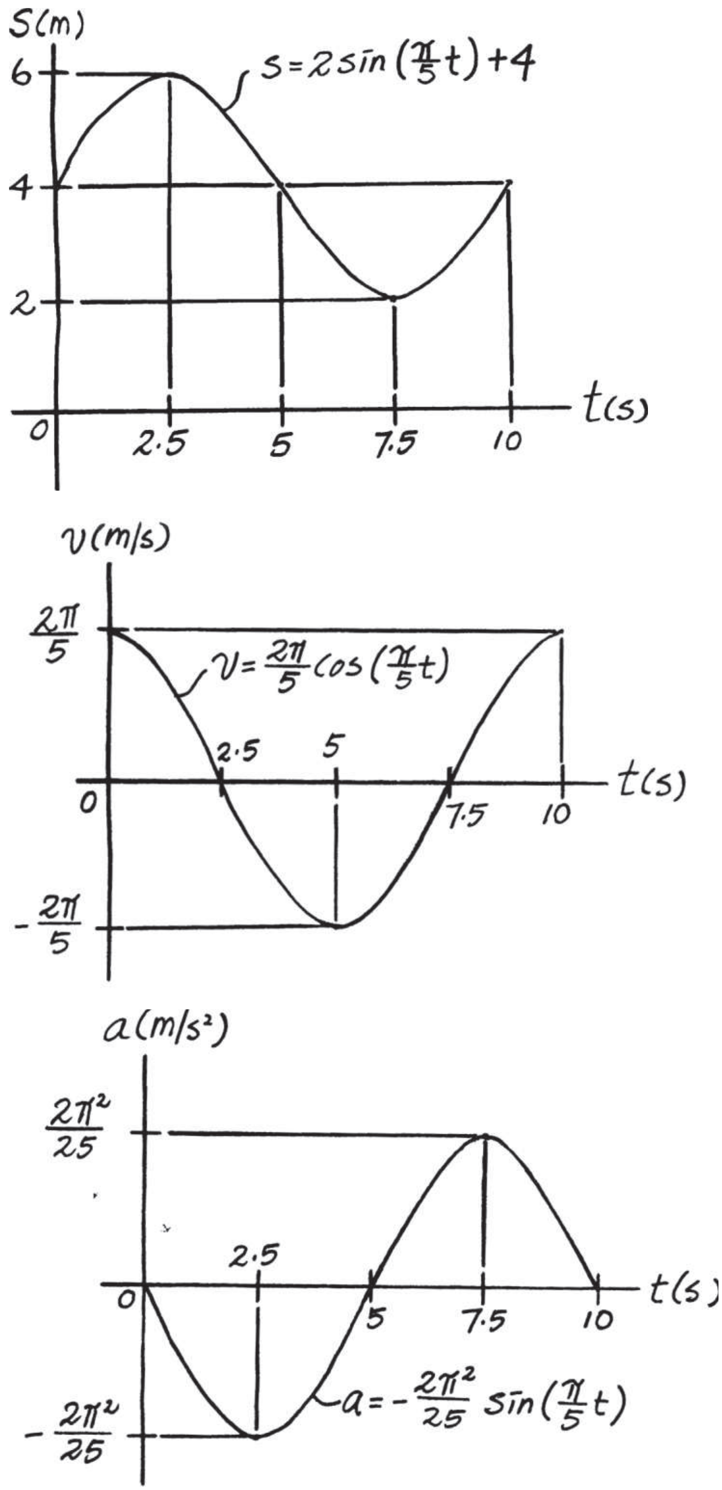


Ans:
$a|_{t=0} = -4\ \text{m/s}^2$
$a|_{t=2\,\text{s}} = 0$
$a|_{t=4\,\text{s}} = 4\ \text{m/s}^2$
$v|_{t=0} = 3\ \text{m/s}$
$v|_{t=2\,\text{s}} = -1\ \text{m/s}$
$v|_{t=4\,\text{s}} = 3\ \text{m/s}$

12–39.

If the position of a particle is defined by $s = [2 \sin (\pi/5)t + 4]$ m, where t is in seconds, construct the $s-t$, $v-t$, and $a-t$ graphs for $0 \leq t \leq 10$ s.

SOLUTION



Ans:

$s = 2 \sin\left(\frac{\pi}{5}t\right) + 4$

$v = \frac{2\pi}{5} \cos\left(\frac{\pi}{5}t\right)$

$a = -\frac{2\pi^2}{25} \sin\left(\frac{\pi}{5}t\right)$

***12–40.**

An airplane starts from rest, travels 5000 ft down a runway, and after uniform acceleration, takes off with a speed of 162 mi/h. It then climbs in a straight line with a uniform acceleration of 3 ft/s^2 until it reaches a constant speed of 220 mi/h. Draw the *s–t*, *v–t*, and *a–t* graphs that describe the motion.

SOLUTION

$v_1 = 0$

$$v_2 = 162 \frac{\text{mi}}{\text{h}} \frac{(1\text{h})\ 5280\text{ ft}}{(3600\text{ s})(1\text{ mi})} = 237.6\text{ ft/s}$$

$v_2^2 = v_1^2 + 2\,a_c(s_2 - s_1)$

$(237.6)^2 = 0^2 + 2(a_c)(5000 - 0)$

$a_c = 5.64538\text{ ft/s}^2$

$v_2 = v_1 + a_c t$

$237.6 = 0 + 5.64538\,t$

$t = 42.09 = 42.1\text{ s}$

$$v_3 = 220 \frac{\text{mi}}{\text{h}} \frac{(1\text{h})\ 5280\text{ ft}}{(3600\text{ s})(1\text{ mi})} = 322.67\text{ ft/s}$$

$v_3^2 = v_2^2 + 2a_c(s_3 - s_2)$

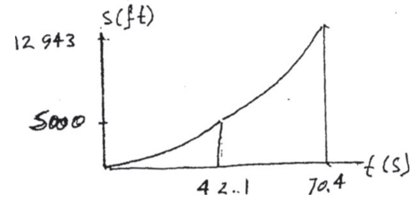
$(322.67)^2 = (237.6)^2 + 2(3)(s - 5000)$

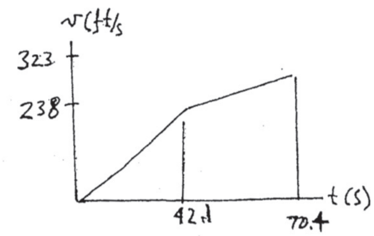
$s = 12\,943.34\text{ ft}$

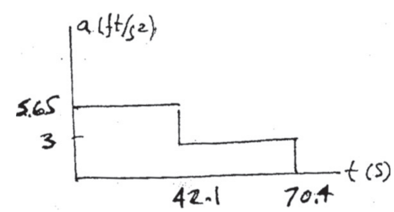
$v_3 = v_2 + a_c t$

$322.67 = 237.6 + 3\,t$

$t = 28.4\text{ s}$







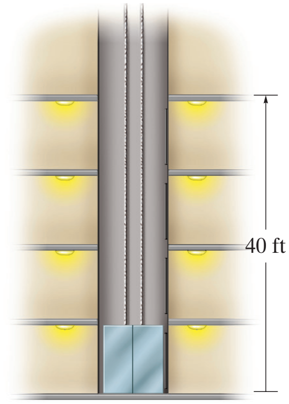
Ans:

$0 \le t < 42.1\text{ s}, a = 5.65\text{ ft/s}^2 \quad v|_{t = 42.1\text{ s}} = 238\text{ ft/s} \quad s|_{t = 42.1\text{ s}} = 5000\text{ ft}$

$42.1\text{ s} < t \le 70.4\text{ s}, a = 3\text{ ft/s}^2 \quad v|_{t = 70.4\text{ s}} = 323\text{ ft/s} \quad s|_{t = 70.4\text{ s}} = 12943\text{ ft}$

12–41.

The elevator starts from rest at the first floor of the building. It can accelerate at 5 ft/s^2 and then decelerate at 2 ft/s^2. Determine the shortest time it takes to reach a floor 40 ft above the ground. The elevator starts from rest and then stops. Draw the a–t, v–t, and s–t graphs for the motion.



SOLUTION

$+\uparrow \; v_2 = v_1 + a_c t_1$

$v_{max} = 0 + 5\,t_1$

$+\uparrow \; v_3 = v_2 + a_c\,t$

$0 = v_{max} - 2\,t_2$

Thus,

$t_1 = 0.4\,t_2$

$+\uparrow \; s_2 = s_1 + v_1 t_1 + \frac{1}{2} a_c t_1^2$

$h = 0 + 0 + \frac{1}{2}(5)(t_1^2) = 2.5\,t_1^2$

$+\uparrow \; 40 - h = 0 + v_{max} t_2 - \frac{1}{2}(2)\,t_2^2$

$+\uparrow \; v^2 = v_1^2 + 2\,a_c(s - s_1)$

$v_{max}^2 = 0 + 2(5)(h - 0)$

$v_{max}^2 = 10h$

$0 = v_{max}^2 + 2(-2)(40 - h)$

$v_{max}^2 = 160 - 4h$

Thus,

$10\,h = 160 - 4h$

$h = 11.429$ ft

$v_{max} = 10.69$ ft/s

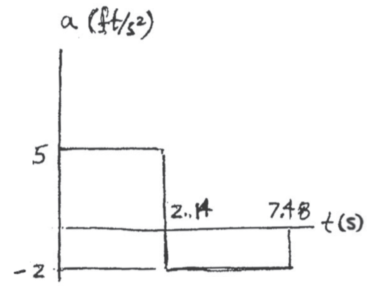
$t_1 = 2.138$ s

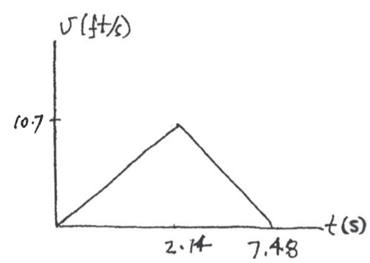
$t_2 = 5.345$ s

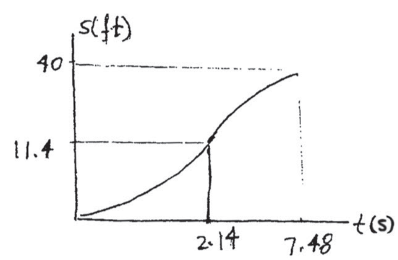
$t = t_1 + t_2 = 7.48$ s **Ans.**

When $t = 2.145$ s, $v = v_{max} = 10.7$ ft/s

and $h = 11.4$ ft.



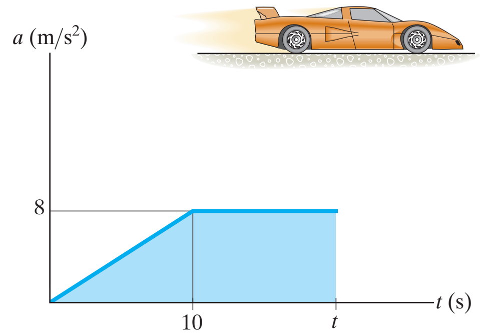




Ans:
$t = 7.48$ s. When $t = 2.14$ s,
$v = v_{max} = 10.7$ ft/s
$h = 11.4$ ft

12–42.

A car starting from rest moves along a straight track with an acceleration as shown. Determine the time t for the car to reach a speed of 50 m/s and construct the v–t graph that describes the motion until the time t.



SOLUTION

For $0 \leq t \leq 10$ s,

$$a = \frac{8}{10}t$$

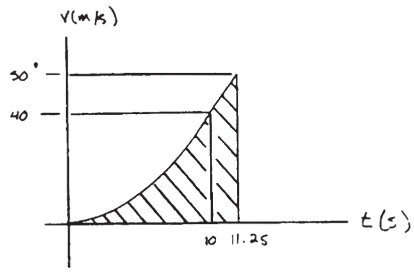
$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t \frac{8}{10}t\,dt$$

$$v = \frac{8}{20}t^2$$

At $t = 10$ s,

$$v = \frac{8}{20}(10)^2 = 40 \text{ m/s}$$



For $t > 10$ s,

$$a = 8$$

$$dv = a\,dt$$

$$\int_{40}^v dv = \int_{10}^t 8\,dt$$

$$v - 40 = 8t - 80$$

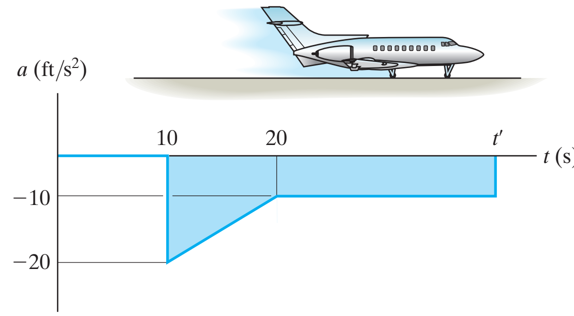
$$v = 8t - 40 \qquad \text{When } v = 50 \text{ m/s}$$

$$t = \frac{50 + 40}{8} = 11.25 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$t = 11.25$ s

12–43.

The motion of a jet plane just after landing on a runway is described by the a–t graph. Determine the time t' when the jet plane stops. Construct the v–t and s–t graphs for the motion. Here $s = 0$, and $v = 300$ ft/s when $t = 0$.



SOLUTION

***v–t* Graph:** The v–t function can be determined by integrating $dv = a\,dt$. For $0 \le t < 10$ s, $a = 0$. Using the initial condition $v = 300$ ft/s at $t = 0$,

$$\int_{300\text{ ft/s}}^{v} dv = \int_0^t 0\,dt$$

$$v - 300 = 0$$

$$v = 300\text{ ft/s} \qquad \textbf{Ans.}$$

For $10\text{ s} < t < 20\text{ s}$, $\dfrac{a-(-20)}{t-10} = \dfrac{-10-(-20)}{20-10}$, $a = (t-30)\text{ ft/s}^2$. Using the initial condition $v = 300$ ft/s at $t = 10$ s,

$$\int_{300\text{ ft/s}}^{v} dv = \int_{10\text{ s}}^{t} (t-30)\,dt$$

$$v - 300 = \left(\frac{1}{2}t^2 - 30t\right)\Bigg|_{10\text{ s}}^{t}$$

$$v = \left\{\frac{1}{2}t^2 - 30t + 550\right\}\text{ ft/s} \qquad \textbf{Ans.}$$

At $t = 20$ s,

$$v\Big|_{t=20\text{ s}} = \frac{1}{2}(20^2) - 30(20) + 550 = 150\text{ ft/s}$$

For $20\text{ s} < t < t'$, $a = -10$ ft/s. Using the initial condition $v = 150$ ft/s at $t = 20$ s,

$$\int_{150\text{ ft/s}}^{v} dv = \int_{20\text{ s}}^{t} -10\,dt$$

$$v - 150 = (-10t)\Big|_{20\text{ s}}^{t}$$

$$v = (-10t + 350)\text{ ft/s} \qquad \textbf{Ans.}$$

It is required that at $t = t'$, $v = 0$. Thus

$$0 = -10\,t' + 350$$

$$t' = 35\text{ s} \qquad \textbf{Ans.}$$

Using these results, the v–t graph shown in Fig. a can be plotted ***s-t* Graph**. The s–t function can be determined by integrating $ds = v\,dt$. For $0 \le t < 10$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t 300\,dt$$

$$s = \{300\,t\}\text{ ft} \qquad \textbf{Ans.}$$

At $= 10$ s,

$$s\big|_{t=10\text{ s}} = 300(10) = 3000\text{ ft}$$

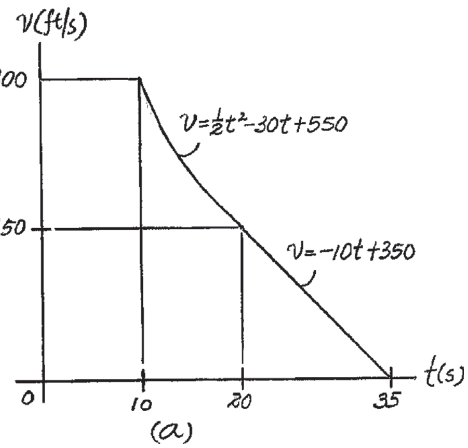
12–43. Continued

For 10 s < t < 20 s, the initial condition is $s = 3000$ ft at $t = 10$ s.

$$\int_{3000\text{ ft}}^{s} ds = \int_{10\text{ s}}^{t} \left(\frac{1}{2}t^2 - 30t + 550\right)dt$$

$$s - 3000 = \left(\frac{1}{6}t^3 - 15t^2 + 550t\right)\Big|_{10\text{ s}}^{t}$$

$$s = \left\{\frac{1}{6}t^3 - 15t^2 + 550t - 1167\right\}\text{ ft} \qquad \textbf{Ans.}$$



At $t = 20$ s,

$$s = \frac{1}{6}(20^3) - 15(20^2) + 550(20) - 1167 = 5167\text{ ft}$$

For 20 s $< t \leq 35$ s, the initial condition is $s = 5167$ ft at $t = 20$ s.

$$\int_{5167\text{ ft}}^{s} ds = \int_{20\text{ s}}^{t} (-10t + 350)\, dt$$

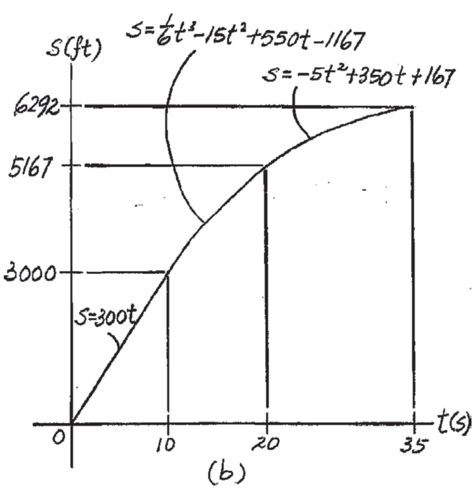
$$s - 5167 = (-5t^2 + 350t)\Big|_{20\text{ s}}^{t}$$

$$s = \{-5t^2 + 350t + 167\}\text{ ft} \qquad \textbf{Ans.}$$

At $t = 35$ s,

$$s\Big|_{t = 35\text{ s}} = -5(35^2) + 350(35) + 167 = 6292\text{ ft}$$

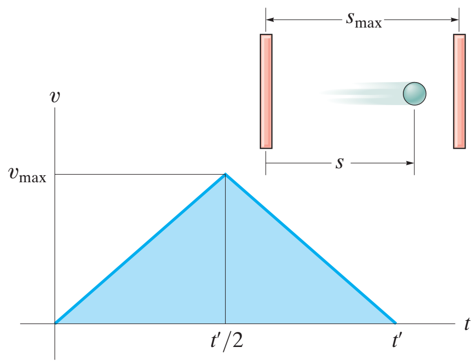
Using these results, the *s-t* graph shown in Fig. *b* can be plotted.



Ans:
$t' = 35$ s
For $0 \leq t < 10$ s,
$s = \{300t\}$ ft
$v = 300$ ft/s
For 10 s $< t < 20$ s,
$s = \left\{\frac{1}{6}t^3 - 15t^2 + 550t - 1167\right\}$ ft
$v = \left\{\frac{1}{2}t^2 - 30t + 550\right\}$ ft/s
For 20 s $< t \leq 35$ s,
$s = \{-5t^2 + 350t + 167\}$ ft
$v = (-10t + 350)$ ft/s

***12–44.**

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of $4\ \text{m/s}^2$. If the plates are spaced 200 mm apart, determine the maximum velocity v_{max} and the time t' for the particle to travel from one plate to the other. Also draw the s–t graph. When $t = t'/2$ the particle is at $s = 100$ mm.



SOLUTION

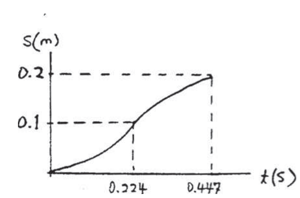
$a_c = 4\ \text{m/s}^2$

$\dfrac{s}{2} = 100\ \text{mm} = 0.1\ \text{m}$

$v^2 = v_0^2 + 2\,a_c(s - s_0)$

$v_{max}^2 = 0 + 2(4)(0.1 - 0)$

$v_{max} = 0.89442\ \text{m/s} \qquad = 0.894\ \text{m/s}$ **Ans.**



$v = v_0 + a_c\,t'$

$0.89442 = 0 + 4\left(\dfrac{t'}{2}\right)$

$t' = 0.44721\ \text{s} \qquad = 0.447\ \text{s}$ **Ans.**

$s = s_0 + v_0\,t + \dfrac{1}{2}\,a_c\,t^2$

$s = 0 + 0 + \dfrac{1}{2}\,(4)(t)^2$

$s = 2\,t^2$

When $t = \dfrac{0.44721}{2} = 0.2236 = 0.224\ \text{s},$

$s = 0.1\ \text{m}$

$\displaystyle\int_{0.894}^{v} ds = -\int_{0.2235}^{t} 4\,dt$

$v = -4\,t + 1.788$

$\displaystyle\int_{0.1}^{s} ds = \int_{0.2235}^{t} (-4t + 1.788)\,dt$

$s = -2\,t^2 + 1.788\,t - 0.2$

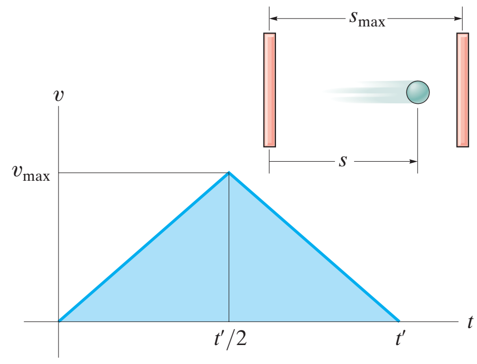
When $t = 0.447$ s,

$s = 0.2\ \text{m}$

Ans:
$t' = 0.447$ s
$s = 0.2$ m

12–45.

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



SOLUTION

For $0 < t < 0.1$ s,

$v = 100\,t$

$a = \dfrac{dv}{dt} = 100$

$ds = v\,dt$

$\displaystyle\int_0^s ds = \int_0^t 100\,t\,dt$

$s = 50\,t^2$

When $t = 0.1$ s,

$s = 0.5$ m

For $0.1\text{ s} < t < 0.2$ s,

$v = -100\,t + 20$

$a = \dfrac{dv}{dt} = -100$

$ds = v\,dt$

$\displaystyle\int_{0.5}^s ds = \int_{0.1}^t (-100t + 20)\mathrm{d}t$

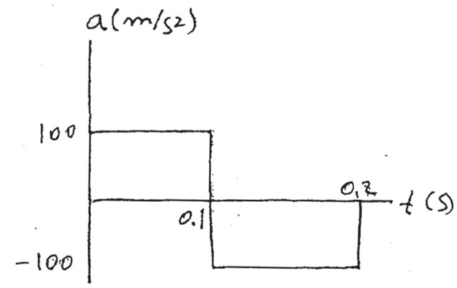
$s - 0.5 = (-50\,t^2 + 20\,t - 1.5)$

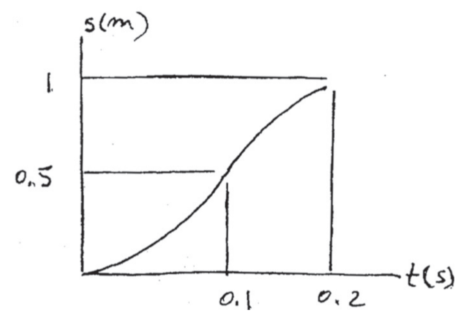
$s = -50\,t^2 + 20\,t - 1$

When $t = 0.2$ s,

$s = 1$ m

When $t = 0.1$ s, $s = 0.5$ m and a changes from 100 m/s^2 to -100 m/s^2. When t = 0.2 s, $s = 1$ m.

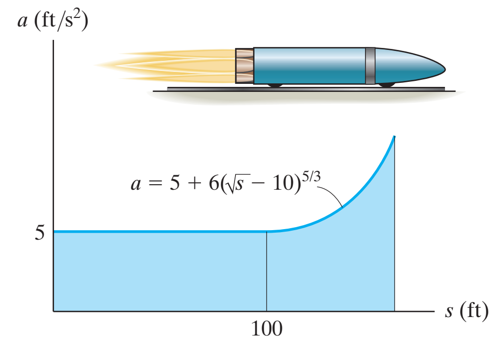




Ans:
When $t = 0.1$ s,
$s = 0.5$ m and a changes from 100 m/s^2 to -100 m/s^2. When $t = 0.2$ s, $s = 1$ m.

12–46.

The a–s graph for a rocket moving along a straight track has been experimentally determined. If the rocket starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = 75$ ft, and 125 ft, respectively. Use Simpson's rule with $n = 100$ to evaluate v at $s = 125$ ft.



SOLUTION

$0 \leq s < 100$

$$\int_0^v v\,dv = \int_0^s 5\,ds$$

$$\frac{1}{2}v^2 = 5\,s$$

$$v = \sqrt{10\,s}$$

At $s = 75$ ft, $\quad v = \sqrt{750} = 27.4$ ft/s **Ans.**

At $s = 100$ ft, $\quad v = 31.623$

$$v\,dv = a\,ds$$

$$\int_{31.623}^{v} v\,dv = \int_{100}^{125}\left[5 + 6\left(\sqrt{s} - 10\right)^{5/3}\right]ds$$

$$\frac{1}{2}v^2\Big|_{31.623}^{v} = 201.0324$$

$v = 37.4$ ft/s **Ans.**

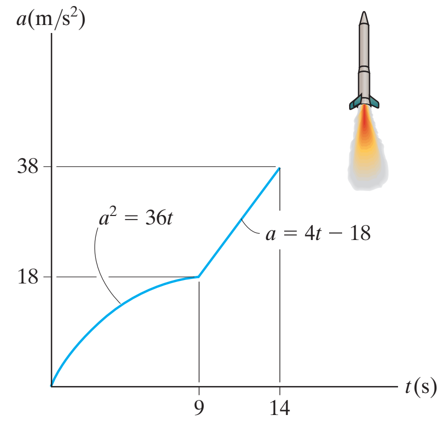
Ans:

$$v\Big|_{s=75\text{ ft}} = 27.4\text{ ft/s}$$

$$v\Big|_{s=125\text{ ft}} = 37.4\text{ ft/s}$$

12–47.

The rocket has an acceleration described by the graph. If it starts from rest, construct the $v-t$ and $s-t$ graphs for the motion for the time interval $0 \leq t \leq 14$ s.



SOLUTION

***v–t* Graph:** For the time interval $0 \leq t < 9$ s, the initial condition is $v = 0$ at $s = 0$.

$$(+\uparrow) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 6t^{1/2}\,dt$$

$$v = \left(4t^{3/2}\right)\text{m/s}$$

When $t = 9$ s,

$$v\big|_{t=9\text{ s}} = 4\left(9^{3/2}\right) = 108\text{ m/s}$$

The initial condition is $v = 108$ m/s at $t = 9$ s.

$$(+\uparrow) \qquad dv = a\,dt$$

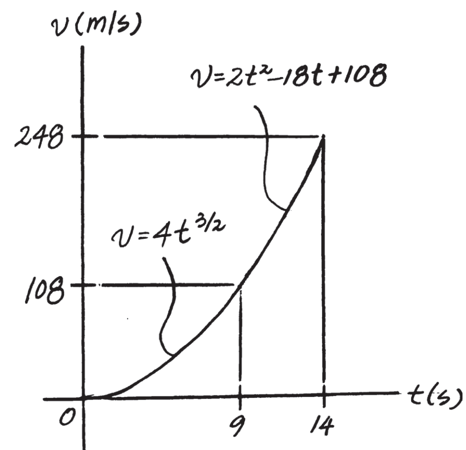
$$\int_{108\text{ m/s}}^v dv = \int_{9\text{ s}}^t (4t - 18)\,dt$$

$$v = \left(2t^2 - 18t + 108\right)\text{ m/s}$$

When $t = 14$ s,

$$v\big|_{t=14\text{ s}} = 2\left(14^2\right) - 18(14) + 108 = 248\text{ m/s}$$

The v–t graph is shown in Fig. *a*.



(a)

***s–t* Graph:** For the time interval $0 \leq t < 9$ s, the initial condition is $s = 0$ when $t = 0$.

$$(+\uparrow) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t 4t^{3/2}\,dt$$

$$s = \frac{8}{5}t^{5/2}$$

When $t = 9$ s,

$$s\big|_{t=9\text{ s}} = \frac{8}{5}\left(9^{5/2}\right) = 388.8\text{ m}$$

For the time interval $9\text{ s} < t \leq 14$ s, the initial condition is $s = 388.8$ m when $t = 9$ s.

$$(+\uparrow) \qquad ds = v\,dt$$

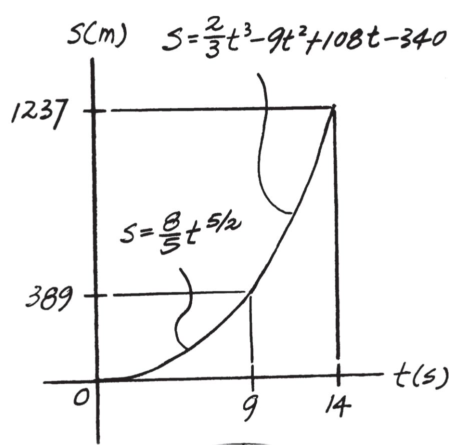
$$\int_{388.8\text{ m}}^s ds = \int_{9\text{ s}}^t \left(2t^2 - 18t + 108\right)dt$$

$$s = \left(\frac{2}{3}t^3 - 9t^2 + 108t - 340.2\right)\text{ m}$$

When $t = 14$ s,

$$s\big|_{t=14\text{ s}} = \frac{2}{3}\left(14^3\right) - 9\left(14^2\right) + 108(14) - 340.2 = 1237\text{ m}$$

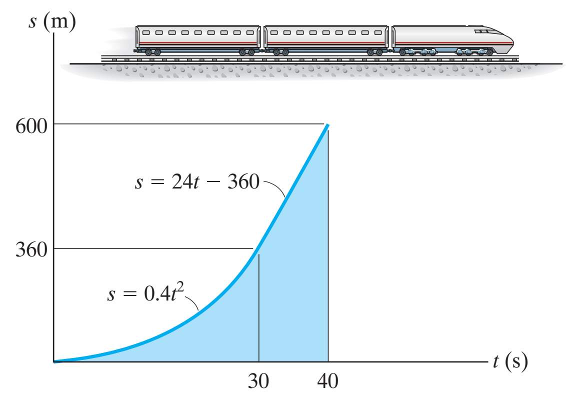
The s–t graph is shown in Fig. *b*.



(b)

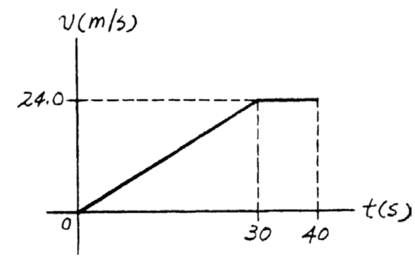
***12–48.**

The s–t graph for a train has been determined experimentally. From the data, construct the v–t and a–t graphs for the motion.



SOLUTION

***v–t* Graph:** The velocity in terms of time t can be obtained by applying $v = \dfrac{ds}{dt}$. For time interval $0 \text{ s} \leq t \leq 30 \text{ s}$,



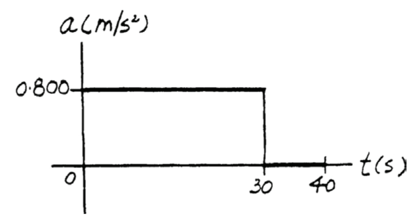
$$v = \frac{ds}{dt} = 0.8t$$

When $t = 30$ s, $\quad v = 0.8(30) = 24.0 \text{ m/s}$

For time interval $30 \text{ s} < t \leq 40 \text{ s}$,

$$v = \frac{ds}{dt} = 24.0 \text{ m/s}$$

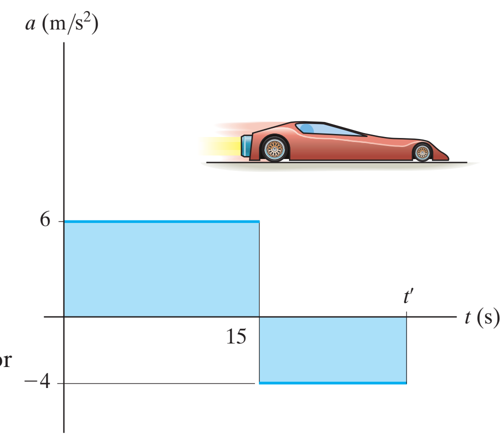
***a–t* Graph:** The acceleration in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$.



For time interval $0 \text{ s} \leq t < 30 \text{ s}$ and $30 \text{ s} < t \leq 40 \text{ s}$, $a = \dfrac{dv}{dt} = 0.800 \text{ m/s}^2$ and $a = \dfrac{dv}{dt} = 0$, respectively.

12–49.

The jet car is originally traveling at a velocity of 10 m/s when it is subjected to the acceleration shown. Determine the car's maximum velocity and the time t' when it stops. When $t = 0, s = 0$.



SOLUTION

***v–t* Function:** The $v-t$ function can be determined by integrating $dv = a\,dt$. For $0 \le t < 15$ s, $a = 6\ \text{m/s}^2$. Using the initial condition $v = 10$ m/s at $t = 0$,

$$\int_{10\text{ m/s}}^{v} dv = \int_0^t 6dt$$

$$v - 10 = 6t$$

$$v = \{6t + 10\}\ \text{m/s}$$

The maximum velocity occurs when $t = 15$ s. Then

$$v_{max} = 6(15) + 10 = 100\ \text{m/s} \qquad \textbf{Ans.}$$

For $15\text{ s} < t \le t', a = -4$ m/s, Using the initial condition $v = 100$ m/s at $t = 15$ s,

$$\int_{100\text{ m/s}}^{v} dv = \int_{15\text{ s}}^{t} -4dt$$

$$v - 100 = (-4t)\Big|_{15\text{ s}}^{t}$$

$$v = \{-4t + 160\}\ \text{m/s}$$

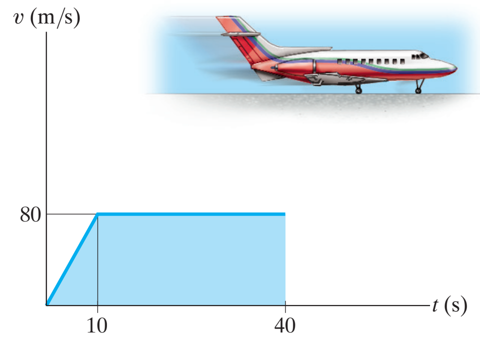
It is required that $v = 0$ at $t = t'$. Then

$$0 = -4t' + 160 \qquad t' = 40\text{ s} \qquad \textbf{Ans.}$$

Ans:
$v_{max} = 100$ m/s
$t' = 40$ s

12–50.

From experimental data, the motion of a jet plane while traveling along a runway is defined by the v–t graph shown. Construct the s–t and a–t graphs for the motion.

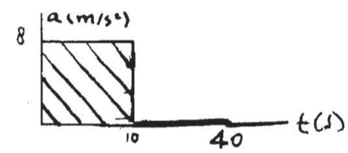


SOLUTION

From the v–t graph:

$$0 \le t \le 10\text{ s} \qquad a = \frac{\Delta v}{\Delta t} = \frac{80}{10} = 8\text{ m/s}^2$$

$$10 \le t \le 40\text{ s} \qquad a = \frac{\Delta v}{\Delta t} = \frac{(80-80)}{(40-10)} = 0$$



From the v–t graph at $t_1 = 10$ s and $t_2 = 40$ s:

$$s_1 = A_1 = \frac{1}{2}(10)(80) = 400\text{m} \qquad s_2 = A_1 + A_2 = 400 + 80(40-10) = 2800\text{ m}$$

The equations defining the s–t graph are:

$0 \le t \le 10:$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 8t\,dt$$

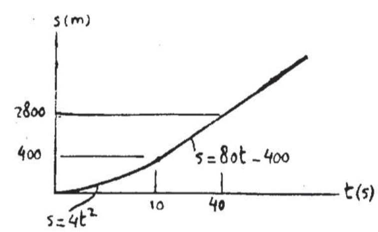
$$s = 4t^2$$

$10 \le t \le 40:$

$$ds = v\,dt$$

$$\int_{400}^s ds = \int_{10}^t 80\,dt$$

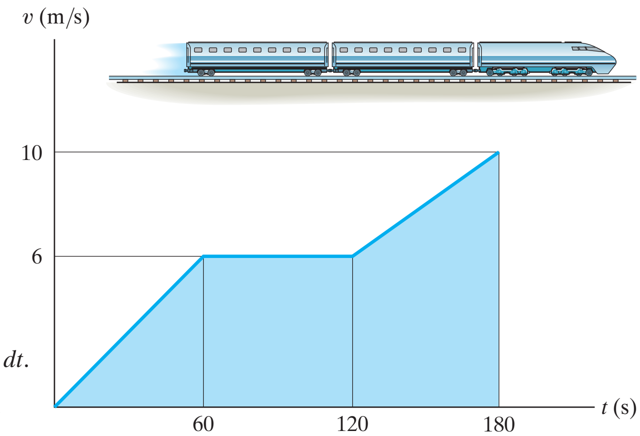
$$s = 80t - 400$$



Ans:
For $0 \le t < 10$ s
$a = 8\text{ m/s}^2$
$s = (4t^2)$ m $\qquad s|_{t=10\,s} = 400$ m
For $10\text{ s} < t \le 40$ s
$a = 0$
$s = (80t - 400)$ m $\qquad s|_{t=40\,s} = 2800$ m

12–51.

The v–t graph for a train has been experimentally determined. From the data, construct the s–t and a–t graphs for the motion for $0 \leq t \leq 180$ s. When $t = 0$, $s = 0$.



SOLUTION

***s–t* Graph:** The s–t function can be determined by integrating $ds = v\,dt$. For $0 \leq t < 60$ s, $v = \frac{6}{60}t = \left(\frac{1}{10}t\right)$ m/s. Using the initial condition $s = 0$ at $t = 0$,

$$\int_0^s ds = \int_0^t \left(\frac{1}{10}t\right)dt$$

$$s = \left\{\frac{1}{20}t^2\right\} \text{ m} \qquad \textbf{Ans.}$$

When $t = 60$ s,

$$s|_{t=60\text{ s}} = \frac{1}{20}(60^2) = 180 \text{ m}$$

For $60 \text{ s} < t < 120$ s, $v = 6$ m/s. Using the initial condition $s = 180$ m at $t = 60$ s,

$$\int_{180\text{ m}}^s ds = \int_{60\text{ s}}^t 6\,dt$$

$$s - 180 = 6t\Big|_{60\text{ s}}^t$$

$$s = \{6t - 180\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 120$ s,

$$s|_{t=120\text{ s}} = 6(120) - 180 = 540 \text{ m}$$

For $120 \text{ s} < t \leq 180$ s, $\frac{v-6}{t-120} = \frac{10-6}{180-120}$; $v = \left\{\frac{1}{15}t - 2\right\}$ m/s. Using the initial condition $s = 540$ m at $t = 120$ s,

$$\int_{540\text{ m}}^s ds = \int_{120\text{ s}}^t \left(\frac{1}{15}t - 2\right)dt$$

$$s - 540 = \left(\frac{1}{30}t^2 - 2t\right)\Big|_{120\text{ s}}^t$$

$$s = \left\{\frac{1}{30}t^2 - 2t + 300\right\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 180$ s,

$$s|_{t=180\text{ s}} = \frac{1}{30}(180^2) - 2(180) + 300 = 1020 \text{ m}$$

Using these results, s–t graph shown in Fig. a can be plotted.

12–51. Continued

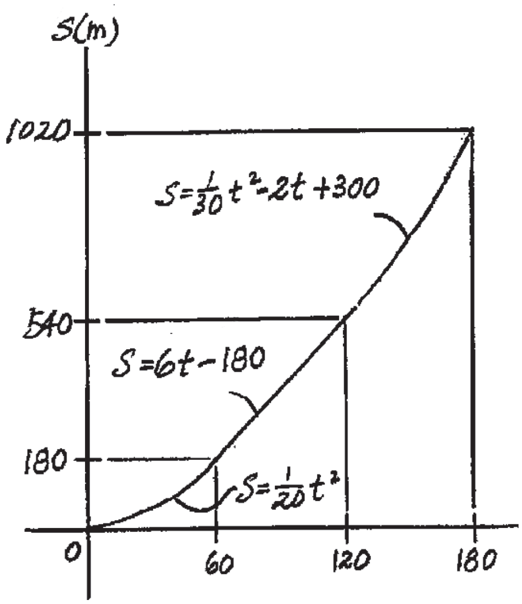
***a–t* Graph:** The *a–t* function can be determined using $a = \dfrac{dv}{dt}$.

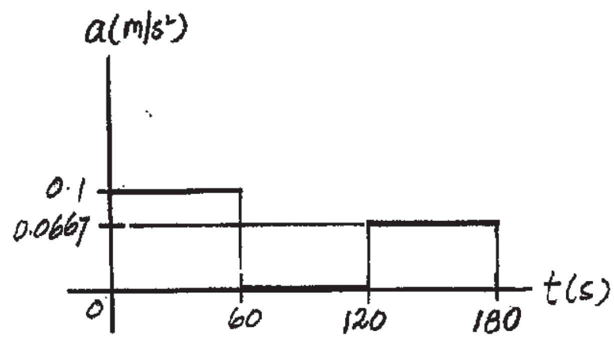
For $0 \le t < 60$ s, $\quad a = \dfrac{d\left(\frac{1}{10}t\right)}{dt} = 0.1\ \text{m/s}^2$ **Ans.**

For $60\ \text{s} < t < 120$ s, $\quad a = \dfrac{d(6)}{dt} = 0$ **Ans.**

For $120\ \text{s} < t \le 180$ s, $\quad a = \dfrac{d\left(\frac{1}{15}t - 2\right)}{dt} = 0.0667\ \text{m/s}^2$ **Ans.**

Using these results, *a–t* graph shown in Fig. *b* can be plotted.

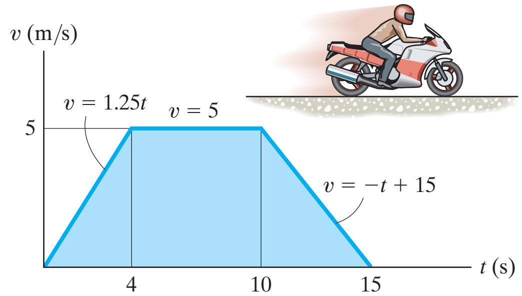




Ans:
For $0 \le t < 60$ s,
$s = \left\{\dfrac{1}{20}t^2\right\}$ m,
$a = 0.1\ \text{m/s}^2$.
For $60\ \text{s} < t < 120$ s,
$s = \{6t - 180\}$ m,
$a = 0$. For $120\ \text{s} < t \le 180$ s,
$s = \left\{\dfrac{1}{30}t^2 - 2t + 300\right\}$ m,
$a = 0.0667\ \text{m/s}^2$.

***12–52.**

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the total distance the motorcycle travels until it stops when $t = 15$ s. Also plot the a–t and s–t graphs.



SOLUTION

For $t < 4$ s

$$a = \frac{dv}{dt} = 1.25 \text{ m/s}^2$$

$$\int_0^s ds = \int_0^t 1.25\, t\, dt$$

$$s = 0.625\, t^2$$

When $t = 4$ s, $\quad s = 10$ m

For 4 s $< t <$ 10 s

$$a = \frac{dv}{dt} = 0$$

$$\int_{10}^s ds = \int_4^t 5\, dt$$

$$s = 5\, t - 10$$

When $t = 10$ s, $\quad s = 40$ m

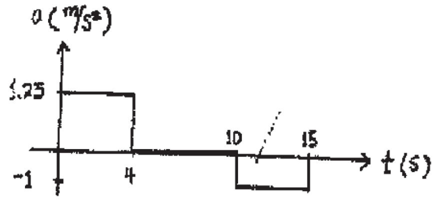
For 10 s $< t <$ 15 s

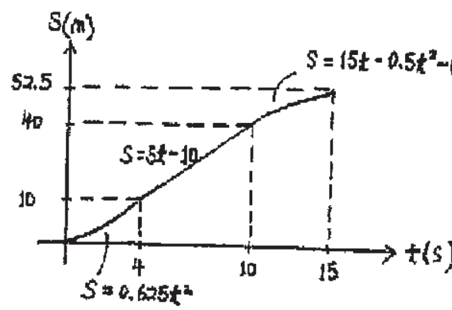
$$a = \frac{dv}{dt} = -1$$

$$\int_{40}^s ds = \int_{10}^t (15 - t)\, dt$$

$$s = 15\, t - 0.5\, t^2 - 60$$

When $t = 15$ s, $\quad s = 52.5$ m **Ans.**

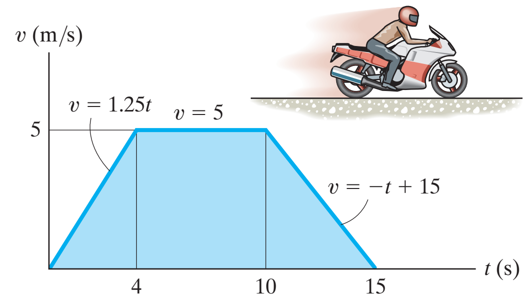




Ans:
When $t = 15$ s, $\quad s = 52.5$ m
For $0 \le t < 4$ s
$s = (0.625t^2)$ m $\quad s|_{t=4\text{ s}} = 10$ m
$a = 1.25 \text{ m/s}^2$
For 4 s $< t <$ 10 s
$s = (5\,t - 10)$ m $\quad s|_{t=10\text{ s}} = 40$ m
$a = 0$
For 10 s $< t \le$ 15 s
$s = (15t^2 - 0.5t^2 - 60)$ m $\quad s|_{t=15\text{ s}} = 52.5$ m
$a = -1 \text{ m/s}^2$

12–53.

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the motorcycle's acceleration and position when $t = 8$ s and $t = 12$ s.



SOLUTION

At $t = 8$ s

$$a = \frac{dv}{dt} = 0$$ **Ans.**

$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (8 - 4)(5) = 30$$

$$s = 30 \text{ m}$$ **Ans.**

At $t = 12$ s

$$a = \frac{dv}{dt} = \frac{-5}{5} = -1 \text{ m/s}^2$$ **Ans.**

$$\Delta s = \int v\,dt$$

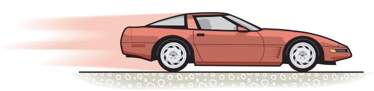
$$s - 0 = \frac{1}{2}(4)(5) + (10 - 4)(5) + \frac{1}{2}(15 - 10)(5) - \frac{1}{2}\left(\frac{3}{5}\right)(5)\left(\frac{3}{5}\right)(5)$$

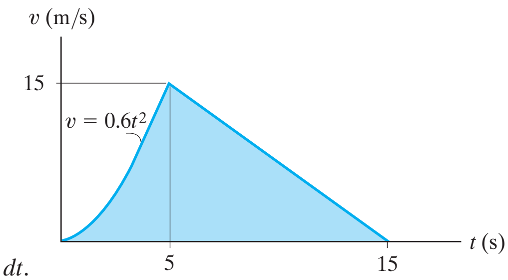
$$s = 48 \text{ m}$$ **Ans.**

Ans:
At $t = 8$ s,
$a = 0$ and $s = 30$ m.
At $t = 12$ s,
$a = -1 \text{ m/s}^2$
and $s = 48$ m.

12–54.

The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t and a–t graphs. Also determine the average speed and the distance traveled for the 15-s time interval. When $t = 0, s = 0$.





SOLUTION

***s–t* Graph:** The s–t function can be determined by integrating $ds = v\,dt$. For $0 \le t < 5$ s, $v = 0.6t^2$. Using the initial condition $s = 0$ at $t = 0$,

$$\int_0^s ds = \int_0^t 0.6t^2 dt$$

$$s = \{0.2t^3\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 5$ s,

$$s|_{t=5\text{ s}} = 0.2(5^3) = 25 \text{ m}$$

For $5 \text{ s} < t \le 15$ s, $\dfrac{v - 15}{t - 5} = \dfrac{0 - 15}{15 - 5}$; $v = \dfrac{1}{2}(45 - 3t)$. Using the initial condition $s = 25$ m at $t = 5$ s,

$$\int_{25\text{ m}}^s ds = \int_{5\text{ s}}^t \frac{1}{2}(45 - 3t)dt$$

$$s - 25 = \frac{45}{2}t - \frac{3}{4}t^2 - 93.75$$

$$s = \left\{\frac{1}{4}(90t - 3t^2 - 275)\right\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 15$ s,

$$s = \frac{1}{4}\left[90(15) - 3(15^2) - 275\right] = 100 \text{ m} \qquad \textbf{Ans.}$$

Thus the average speed is

$$v_{\text{avg}} = \frac{s_T}{t} = \frac{100 \text{ m}}{15 \text{ s}} = 6.67 \text{ m/s} \qquad \textbf{Ans.}$$

using these results, the s–t graph shown in Fig. a can be plotted.

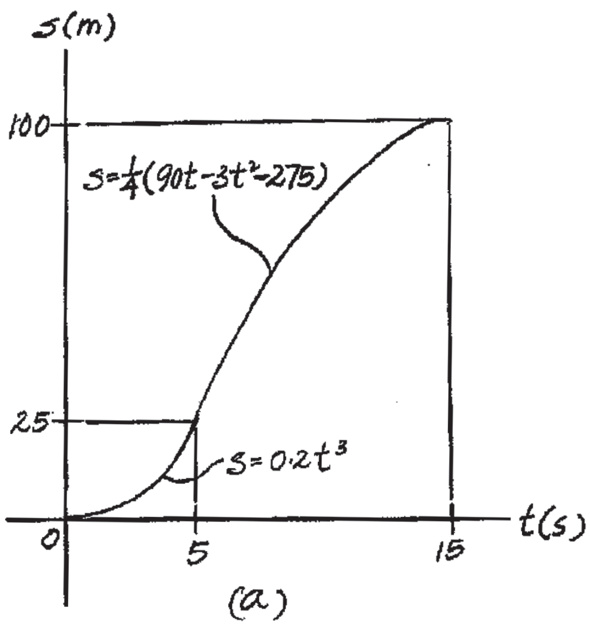
12–54. Continued

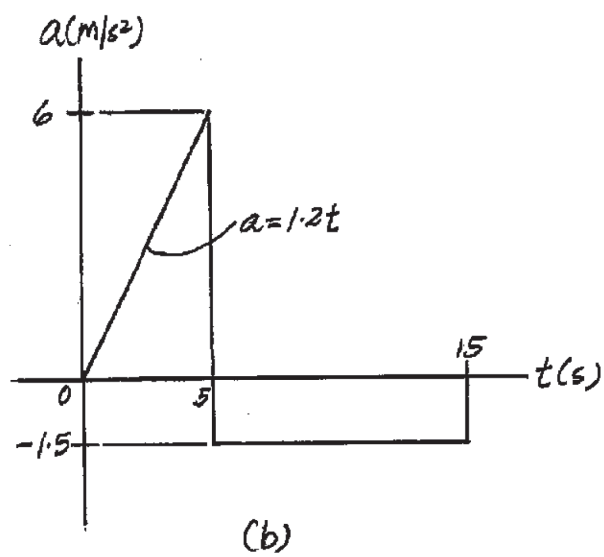
***a–t* Graph:** The *a–t* function can be determined using $a = \frac{dv}{dt}$.

For $0 \le t < 5$ s, $a = \frac{d(0.6\,t^2)}{dt} = \{1.2\,t\}$ m/s^2 **Ans.**

At $t = 5$ s, $a = 1.2(5) = 6$ m/s^2 **Ans.**

For $5\text{ s} < t \le 15$ s, $a = \frac{d\left[\frac{1}{2}(45 - 3t)\right]}{dt} = -1.5$ m/s^2 **Ans.**

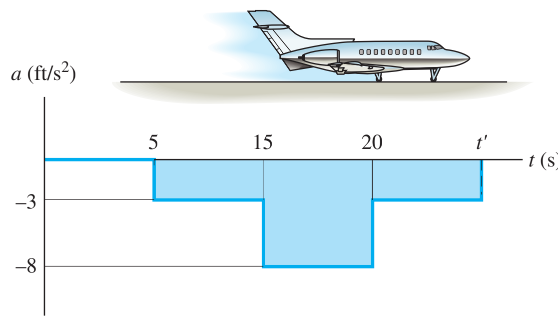




Ans:
For $0 \le t < 5$ s,
$s = \{0.2t^3\}$ m
$a = \{1.2t\}$ m/s^2
For $5\text{ s} < t \le 15$ s,
$s = \left\{\frac{1}{4}(90t - 3t^2 - 275)\right\}$ m
$a = -1.5$ m/s^2
At $t = 15$ s,
$s = 100$ m
$v_{avg} = 6.67$ m/s

12–55.

An airplane lands on the straight runway, originally traveling at 110 ft/s when $s = 0$. If it is subjected to the decelerations shown, determine the time t' needed to stop the plane and construct the s–t graph for the motion.



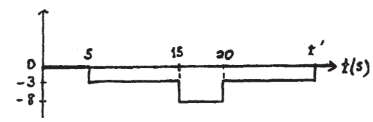
SOLUTION

$v_0 = 110 \text{ ft/s}$

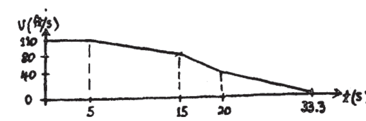
$\Delta v = \int a\, dt$

$0 - 110 = -3(15 - 5) - 8(20 - 15) - 3(t' - 20)$

$t' = 33.3 \text{ s}$ **Ans.**

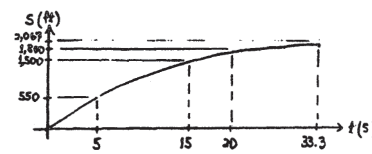


$s\big|_{t = 5\text{s}} = 550 \text{ ft}$



$s\big|_{t = 15\text{s}} = 1500 \text{ ft}$

$s\big|_{t = 20\text{s}} = 1800 \text{ ft}$



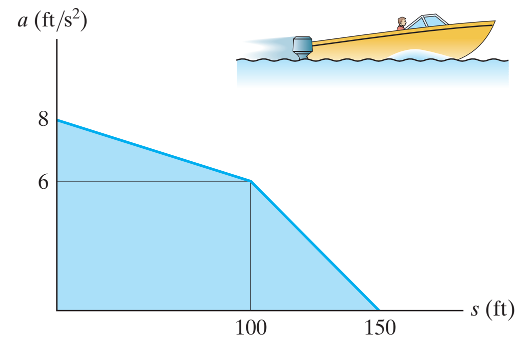
$s\big|_{t = 33.3\text{s}} = 2067 \text{ ft}$

Ans:

$t' = 33.3 \text{ s}$
$s\big|_{t=5\text{ s}} = 550 \text{ ft}$
$s\big|_{t=15\text{ s}} = 1500 \text{ ft}$
$s\big|_{t=20\text{ s}} = 1800 \text{ ft}$
$s\big|_{t=33.3\text{ s}} = 2067 \text{ ft}$

***12–56.**

Starting from rest at $s = 0$, a boat travels in a straight line with the acceleration shown by the a–s graph. Determine the boat's speed when $s = 50$ ft, 100 ft, and 150 ft.



SOLUTION

–s Function: The $v{-}s$ function can be determined by integrating $v\,dv = a\,ds$. For $0 \le s < 100$ ft, $\dfrac{a-8}{s-0} = \dfrac{6-8}{100-0}$, $a = \left\{-\dfrac{1}{50}s + 8\right\}$ ft/s^2. Using the initial condition $v{-}0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s \left(-\frac{1}{50}s + 8\right) ds$$

$$\left.\frac{v^2}{2}\right|_0^v = \left.\left(-\frac{1}{100}s^2 + 8\,s\right)\right|_0^s$$

$$\frac{v^2}{2} = 8s - \frac{1}{100}s^2$$

$$v = \left\{\sqrt{\frac{1}{50}(800\,s - s^2)}\right\} \text{ ft/s}$$

At $s = 50$ ft,

$$v|_{s=50\text{ ft}} = \sqrt{\frac{1}{50}[800\,(50) - 50^2]} = 27.39\text{ ft/s} = 27.4\text{ ft/s} \qquad \textbf{Ans.}$$

At $s = 100$ ft,

$$v|_{s=100\text{ ft}} = \sqrt{\frac{1}{50}[800\,(100) - 100^2]} = 37.42\text{ ft/s} = 37.4\text{ ft/s} \qquad \textbf{Ans.}$$

For 100 ft $< s \le 150$ ft, $\dfrac{a-0}{s-150} = \dfrac{6-0}{100-150}$; $a = \left\{-\dfrac{3}{25}s + 18\right\}$ ft/s^2. Using the initial condition $v = 37.42$ ft/s at $s = 100$ ft,

$$\int_{37.42\text{ ft/s}}^{v} v\,dv = \int_{100\text{ ft}}^{s}\left(-\frac{3}{25}s + 18\right) ds$$

$$\left.\frac{v^2}{2}\right|_{37.42\text{ ft/s}}^{v} = \left.\left(-\frac{3}{50}s^2 + 18s\right)\right|_{100\text{ ft}}^{s}$$

$$v = \left\{\frac{1}{5}\sqrt{-3s^2 + 900s - 25000}\right\} \text{ ft/s}$$

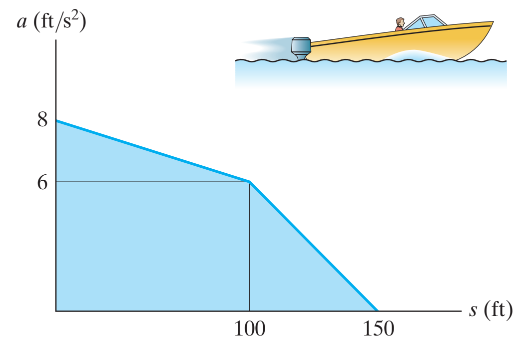
At $s = 150$ ft

$$v|_{s=150\text{ ft}} = \frac{1}{5}\sqrt{-3(150^2) + 900\,(150) - 25000} = 41.23\text{ ft/s} = 41.2\text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v|_{s=50\text{ ft}} = 27.4$ ft/s
$v|_{s=100\text{ ft}} = 37.4$ ft/s
$v|_{s=150\text{ ft}} = 41.2$ ft/s

12–57.

Starting from rest at $s = 0$, a boat travels in a straight line with the acceleration shown by the a–s graph. Construct the v–s graph.



SOLUTION

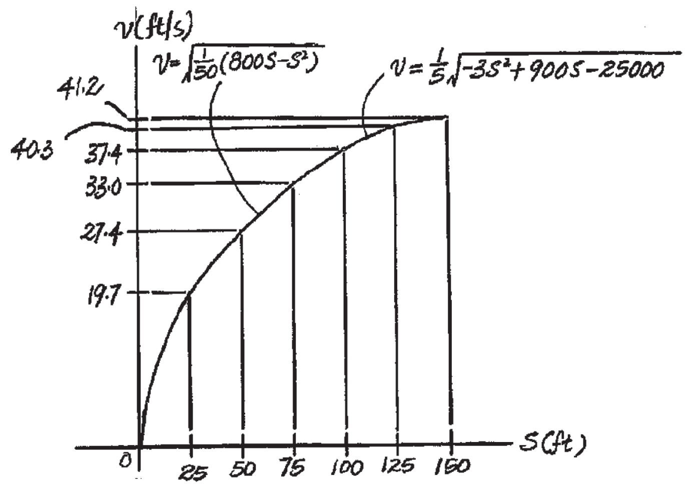
–s Graph: The $v-s$ function can be determined by integrating $v\,dv = a\,ds$. For $0 \le s < 100$ ft, $\dfrac{a-8}{s-0} = \dfrac{6-8}{100-0}$, $a = \left\{-\dfrac{1}{50}s + 8\right\}$ ft/s^2 using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s \left(-\frac{1}{50}s + 8\right)ds$$

$$\left.\frac{v^2}{2}\right|_0^v = \left.\left(-\frac{1}{100}s^2 + 8\,s\right)\right|_0^s$$

$$\frac{v^2}{2} = 8\,s - \frac{1}{100}s^2$$

$$v = \left\{\sqrt{\frac{1}{50}(800\,s - s^2)}\right\}\text{ ft/s}$$



At $s = 25$ ft, 50 ft, 75 ft and 100 ft

$$v|_{s=25\text{ ft}} = \sqrt{\frac{1}{50}[800\,(25) - 25^2]} = 19.69\text{ ft/s}$$

$$v|_{s=50\text{ ft}} = \sqrt{\frac{1}{50}[800\,(50) - 50^2]} = 27.39\text{ ft/s}$$

$$v|_{s=75\text{ ft}} = \sqrt{\frac{1}{50}[800\,(75) - 75^2]} = 32.98\text{ ft/s}$$

$$v|_{s=100\text{ ft}} = \sqrt{\frac{1}{50}[800\,(100) - 100^2]} = 37.42\text{ ft/s}$$

For 100 ft $< s \le 150$ ft, $\dfrac{a-0}{s-150} = \dfrac{6-0}{100-150}$; $a = \left\{-\dfrac{3}{25}s + 18\right\}$ ft/s^2 using the initial condition $v = 37.42$ ft/s at $s = 100$ ft,

$$\int_{37.42\text{ ft/s}}^{v} v\,dv = \int_{100\text{ ft}}^{s}\left(-\frac{3}{25}s + 18\right)ds$$

$$\left.\frac{v^2}{2}\right|_{37.42\text{ ft/s}}^{v} = \left.\left(-\frac{3}{50}s^2 + 18\,s\right)\right|_{100\text{ ft}}^{s}$$

$$v = \left\{\frac{1}{5}\sqrt{-3s^2 + 900s - 25000}\right\}\text{ ft/s}$$

At $s = 125$ ft and $s = 150$ ft

$$v|_{s=125\text{ ft}} = \frac{1}{5}\sqrt{-3(125^2) + 900\,(125) - 25000} = 40.31\text{ ft/s}$$

$$v|_{s=150\text{ ft}} = \frac{1}{5}\sqrt{-3(150^2) + 900\,(150) - 25000} = 41.23\text{ ft/s}$$

Ans:

For $0 \le s < 100$ ft,

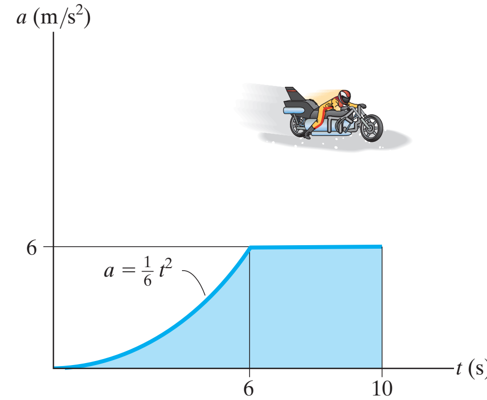
$$v = \left\{\sqrt{\frac{1}{50}(800s - s^2)}\right\}\text{ ft/s}$$

For 100 ft $< s \le 150$ ft,

$$v = \left\{\frac{1}{5}\sqrt{-3s^2 + 900s - 25\,000}\right\}\text{ ft/s}$$

12–58.

A motorcyclist starting from rest travels along a straight road and for 10 s has an acceleration as shown. Draw the $v-t$ graph that describes the motion and find the distance traveled in 10 s.



SOLUTION

For $0 \le t < 6$ $\quad dv = a\,dt$

$$\int_0^v dv = \int_0^t \frac{1}{6}t^2\,dt$$

$$v = \frac{1}{18}t^3$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{1}{18}t^3\,dt$$

$$s = \frac{1}{72}t^4$$

When $t = 6$ s, $\quad v = 12$ m/s $\quad s = 18$ m

For $6 < t \le 10$ $\quad dv = a\,dt$

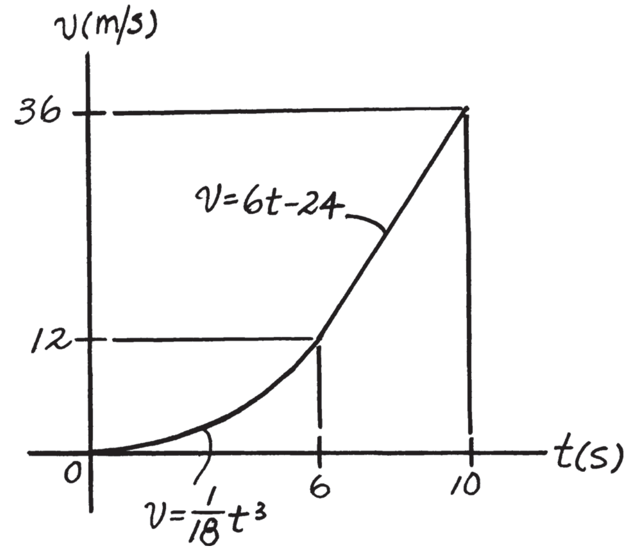
$$\int_{12}^v dv = \int_6^t 6\,dt$$

$$v = 6t - 24$$

$$ds = v\,dt$$

$$\int_{18}^s ds = \int_6^t (6t - 24)\,dt$$

$$s = 3t^2 - 24t + 54$$



When $t = 10$ s, $\quad v = 36$ m/s

$s = 114$ m **Ans.**

Ans:
For $0 < t \le 6$ s
$v = \left(\frac{1}{18}t^3\right)$ m/s $\quad s = \left(\frac{1}{72}t^4\right)$ m
For 6 s $< t \le 10$ s
$v = (6t - 24)$ m/s $\quad s = (3t^2 - 24t + 54)$ m
$s = 114$ m

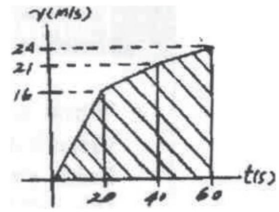
12–59.

The speed of a train during the first minute has been recorded as follows:

t (s)	0	20	40	60
v (m/s)	0	16	21	24

Plot the $v-t$ graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.

SOLUTION



The total distance traveled is equal to the area under the graph.

$$s_T = \frac{1}{2}(20)(16) + \frac{1}{2}(40 - 20)(16 + 21) + \frac{1}{2}(60 - 40)(21 + 24) = 980 \text{ m}$$ **Ans.**

Ans:
$s_T = 980$ m

***12–60.**

A man riding upward in a freight elevator accidentally drops a package off the elevator when it is 100 ft from the ground. If the elevator maintains a constant upward speed of 4 ft/s, determine how high the elevator is from the ground the instant the package hits the ground. Draw the v–t curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

SOLUTION

For package:

$(+\uparrow)\qquad v^2 = v_0^2 + 2a_c(s_2 - s_0)$

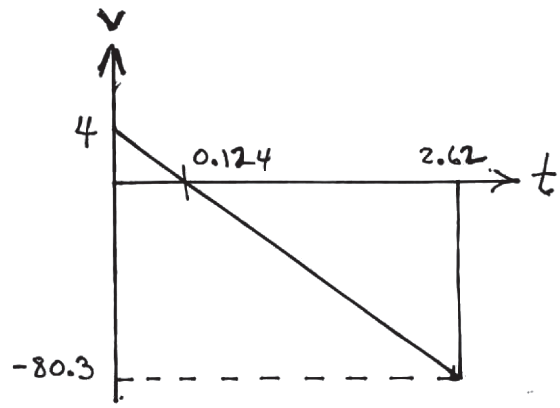
$$v^2 = (4)^2 + 2(-32.2)(0 - 100)$$

$$v = 80.35 \text{ ft/s} \downarrow$$

$(+\uparrow)\qquad v = v_0 + a_c t$

$$-80.35 = 4 + (-32.2)t$$

$$t = 2.620 \text{ s}$$



For elevator:

$(+\uparrow)\qquad s_2 = s_0 + vt$

$$s = 100 + 4(2.620)$$

$$s = 110 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$s = 110$ ft

12–61.

Two cars start from rest side by side and travel along a straight road. Car A accelerates at $4\ \text{m/s}^2$ for 10 s and then maintains a constant speed. Car B accelerates at $5\ \text{m/s}^2$ until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a–t, v–t, and s–t graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?

SOLUTION

Car A:

$$v = v_0 + a_c t$$

$$v_A = 0 + 4t$$

At $t = 10$ s, $\quad v_A = 40$ m/s

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(4)t^2 = 2t^2$$

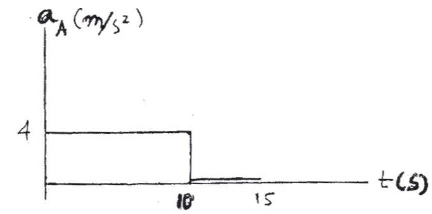
At $t = 10$ s, $\qquad s_A = 200$ m

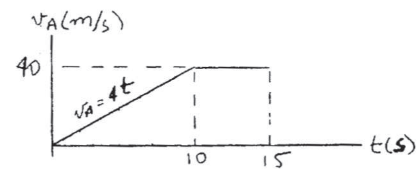
$t > 10$ s, $\qquad ds = v\,dt$

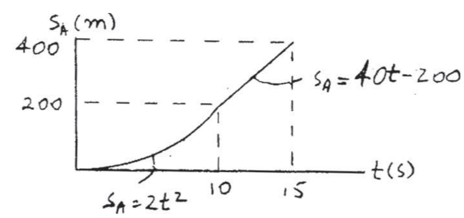
$$\int_{200}^{s_A} ds = \int_{10}^{t} 40\,dt$$

$$s_A = 40t - 200$$

At $t = 15$ s, $\qquad s_A = 400$ m







Car B:

$$v = v_0 + a_c t$$

$$v_B = 0 + 5t$$

When $v_B = 25$ m/s, $\qquad t = \dfrac{25}{5} = 5$ s

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(5)t^2 = 2.5t^2$$

When $t = 10$ s, $v_A = (v_A)_{\max} = 40$ m/s and $s_A = 200$ m.

When $t = 5$ s, $s_B = 62.5$ m.

When $t = 15$ s, $s_A = 400$ m and $s_B = 312.5$ m.

12–61. Continued

At $t = 5$ s, $\quad s_B = 62.5$ m

$t > 5$ s, $\quad ds = v\,dt$

$$\int_{62.5}^{s_B} ds = \int_{5}^{t} 25\,dt$$

$$s_B - 62.5 = 25t - 125$$

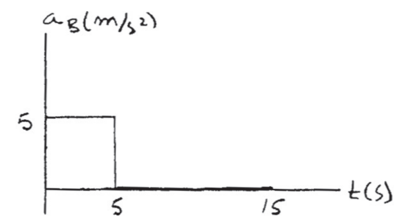
$$s_B = 25t - 62.5$$

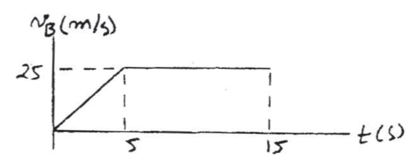
When $t = 15$ s, $\quad s_B = 312.5$

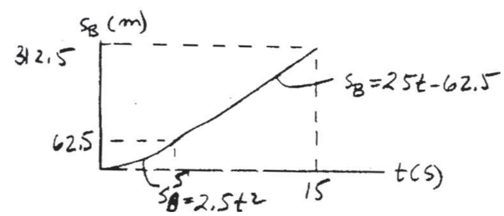
Distance between the cars is

$\Delta s = s_A - s_B = 400 - 312.5 = 87.5$ m **Ans.**

Car A is ahead of car B.







Ans:
When $t = 5$ s,
$s_B = 62.5$ m.
When $t = 10$ s,
$v_A = (v_A)_{max} = 40$ m/s and
$s_A = 200$ m.
When $t = 15$ s,
$s_A = 400$ m and $s_B = 312.5$ m.
$\Delta s = s_A - s_B = 87.5$ m

12–62.

If the position of a particle is defined as $s = (5t - 3t^2)$ ft, where t is in seconds, construct the s–t, v–t, and a–t graphs for $0 \leq t \leq 2.5\,\text{s}$.

SOLUTION

$s-t$ Graph: When $s = 0$,

$$0 = 5t - 3t^2 \qquad 0 = t(5 - 3t) \qquad t = 0 \text{ and } t = 1.67 \text{ s}$$

At $t = 2.5\text{s}$, $s|_{t=2.5\text{s}} = 5(2.5) - 3(2.5^2) = -6.25$ ft

$v-t$ Graph: The velocity function can be determined using $v = \dfrac{ds}{dt}$. Thus

$$v = \frac{ds}{dt} = \{5 - 6t\} \text{ ft/s}$$

At $t = 0$, $v|_{t=0} = 5 - 6(0) = 5$ ft/s

At $t = 2.5s$, $v|_{t=2.5s} = 5 - 6(2.5) = -10$ ft/s

When $v = 0$,

$$0 = 5 - 6t \qquad t = 0.833 \text{ s}$$

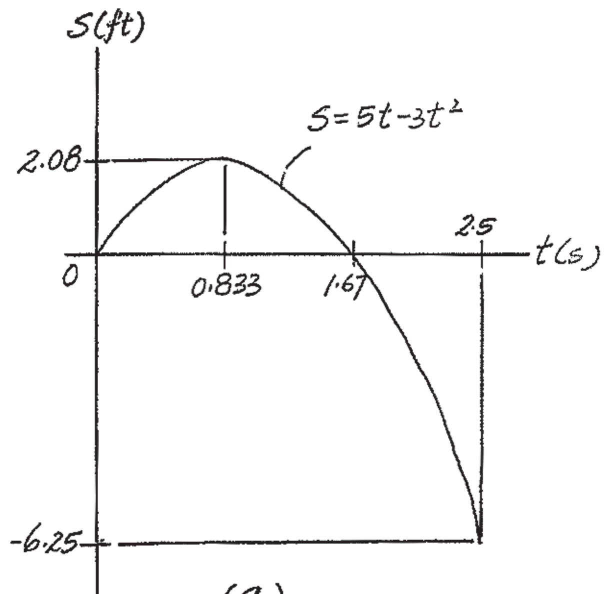
Then, at $t = 0.833\text{s}$,

$$s|_{t=0.833\text{s}} = 5(0.833) - 3(0.833^2) = 2.08 \text{ ft}$$

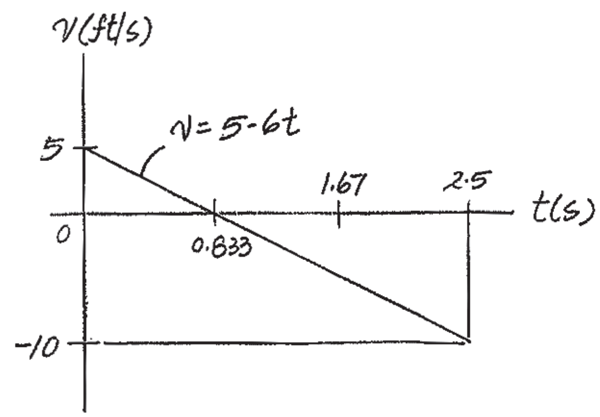
$a-t$ Graph: The acceleration function can be determined using $a = \dfrac{dv}{dt}$. Thus

$$a = \frac{dv}{dt} = -6 \text{ ft/s}^2$$

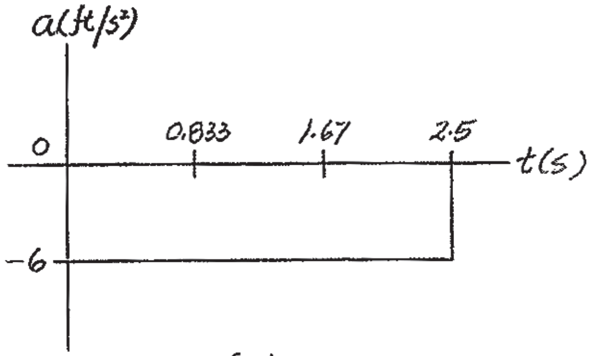
s–t, v–t and $a-t$ graph shown in Fig. a, b, and c can be plotted using these results.



(a)



(b)

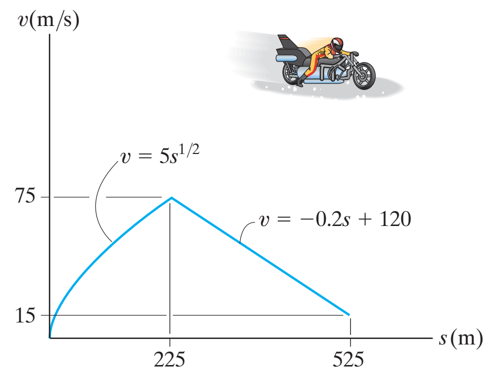


(c)

Ans:
$s|_{t\,=\,0.833\text{ s}} = 2.08$ ft
$s|_{t\,=\,1.67\text{ s}} = 0$
$s|_{t\,=\,2.5\text{ s}} = -6.25$ ft
$v = \{5 - 6t\}$ ft/s
$v|_{t\,=\,0} = 5$ ft/s
$v = 0$ at $t = 0.833$ s
$v|_{t\,=\,2.5\text{ s}} = -10$ ft/s
$a = -6$ ft/s^2 for all t

12–63.

The jet bike is moving along a straight road with the speed described by the $v-s$ graph. Construct the $a-s$ graph.



SOLUTION

***a–s* Graph:** For $0 \leq s < 225$ m,

$$(\overset{+}{\rightarrow}) \qquad a = v\frac{dv}{ds} = \left(5s^{1/2}\right)\left(\frac{5}{2}s^{-1/2}\right) = 12.5 \text{ m/s}^2$$

For 225 m $< s \leq$ 525 m,

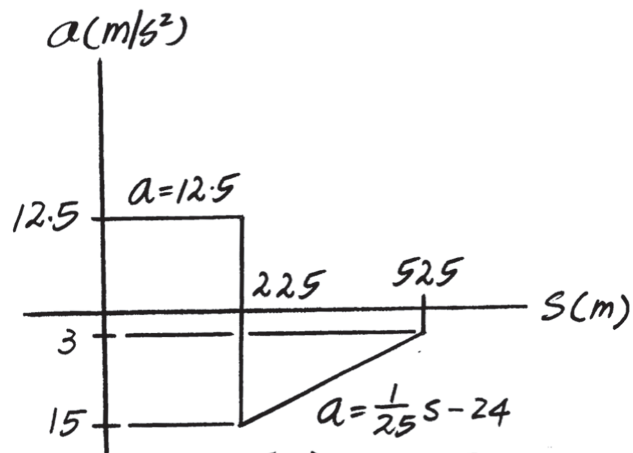
$$(\overset{+}{\rightarrow}) \qquad a = v\frac{dv}{ds} = (-0.2s + 120)(-0.2) = (0.04s - 24) \text{ m/s}^2$$

At $s = 225$ m and 525 m,

$$a\big|_{s=225\text{ m}} = 0.04(225) - 24 = -15 \text{ m/s}^2$$

$$a\big|_{s=525\text{ m}} = 0.04(525) - 24 = -3 \text{ m/s}^2$$

The a–s graph is shown in Fig. a.

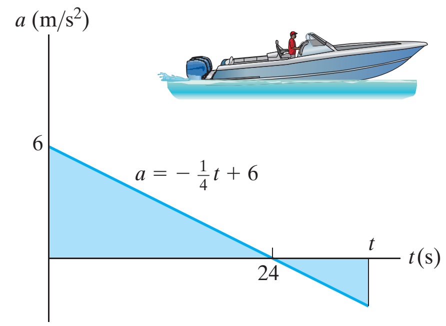


(a)

Ans:
For $0 \leq s < 225$ m
$a = 12.5 \text{ m/s}^2$
For 225 m $< s \leq$ 525 m
$a = \{0.04\,s - 24\} \text{ m/s}^2$
$a|_{s\,=\,225\text{ m}} = -15 \text{ m/s}^2 \qquad a|_{s\,=\,525\text{ m}} = -3 \text{ m/s}^2$

***12–64.**

The boat is originally traveling at a speed of 8 m/s when it is subjected to the acceleration shown in the graph. Determine the boat's maximum speed and the time t when it stops.



SOLUTION

Velocity:

$$a = -\frac{6}{24}t + 6 = -0.25t + 6$$

$$\frac{dv}{dt} = a = -\,0.25t + 6$$

$$\int_8^v dv = \int_0^t (-0.25t + 6)dt$$

$$v = -0.125t^2 + 6t + 8$$

The maximum speed occurs when $a = \dfrac{dv}{dt} = 0$ for which $t = 24$ s.

$$v_{max} = -0.125(24)^2 + 6(24) + 8 = 80 \text{ m/s}$$ **Ans.**

Time:

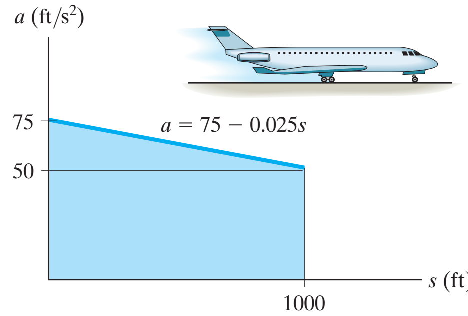
$$v = -0.125t^2 + 6t + 8 = 0$$

Take the positive root $\quad t = 49.3$ s **Ans.**

Ans:
$v_{max} = 80$ m/s
$t = 49.3$ s

12–65.

The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has traveled 1000 ft. Also, how much time is required for it to travel 1000 ft?



SOLUTION

***v–s* Function:** Here, $\dfrac{a - 75}{s - 0} = \dfrac{50 - 75}{1000 - 0}$; $a = \{75 - 0.025s\}\ \text{ft/s}^2$. The function $v_{(s)}$ can be determined by integrating $v\,dv = a\,ds$. Using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s (75 - 0.025\,s)\,ds$$

$$\frac{v^2}{2} = 75\,s - 0.0125\,s^2$$

$$v = \left\{\sqrt{150\,s - 0.025\,s^2}\right\}\ \text{ft/s}$$

At $s = 1000$ ft,

$$v = \sqrt{150\,(1000) - 0.025\left(1000^2\right)}$$

$$= 353.55\ \text{ft/s} = 354\ \text{ft/s} \qquad \textbf{Ans.}$$

Time. t as a function of s can be determined by integrating $dt = \dfrac{ds}{v}$. Using the initial condition $s = 0$ at $t = 0$;

$$\int_0^t dt = \int_0^s \frac{ds}{\sqrt{150\,s - 0.025\,s^2}}$$

$$t = \left[-\frac{1}{\sqrt{0.025}}\sin^{-1}\left(\frac{150 - 0.05\,s}{150}\right)\right]\Bigg|_0^s$$

$$t = \frac{1}{\sqrt{0.025}}\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{150 - 0.05\,s}{150}\right)\right]$$

At $s = 1000$ ft,

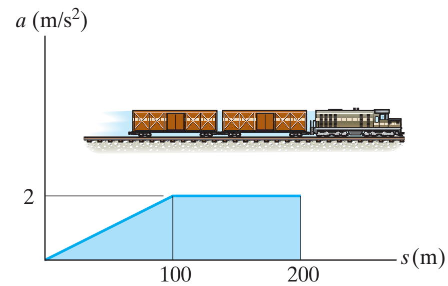
$$t = \frac{1}{\sqrt{0.025}}\left\{\frac{\pi}{2} - \sin^{-1}\left[\frac{150 - 0.05(1000)}{150}\right]\right\}$$

$$= 5.319\ \text{s} = 5.32\ \text{s} \qquad \textbf{Ans.}$$

Ans:
$v = 354$ ft/s
$t = 5.32$ s

12–66.

The a–s graph for a freight train is given for the first 200 m of its motion. Plot the v–s graph. The train starts from rest.



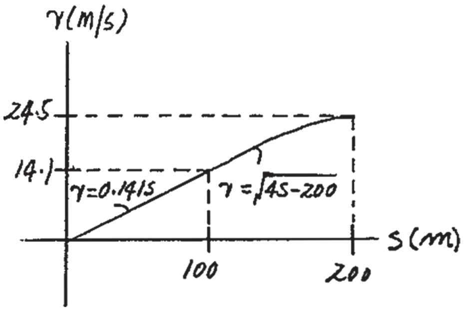
SOLUTION

For $0 \le s < 100$ m $\quad a = 0.02s \quad \int_0 v\,dv = \int_0^s 0.02s\,ds \quad v = 0.141s$

At $s = 100$ m, $\quad v = 0.141(100) = 14.1$ m/s

For $100 \text{ m} < s \le 200$ m $\quad a = 2 \quad \int_{14.1} v\,dv = \int_{100}^s 2\,ds \quad v = \sqrt{4s - 200}$

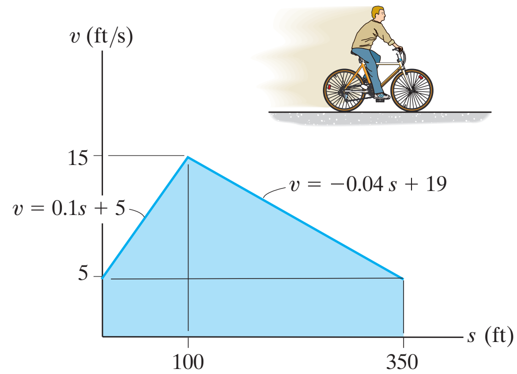
At $s = 200$ m, $\quad v = \sqrt{4(200) - 200} = 24.5$ m/s



Ans:
For $0 \le s < 100$ m
$v = \{0.141\,s\}$ m/s $\qquad v|_{s = 100\text{ m}} = 14.1$ m/s
For $100 \text{ m} < s \le 200$ m
$v = \{\sqrt{4s - 200}\}$ m/s $\qquad v|_{s = 200\text{ m}} = 24.5$ m/s

12–67.

The $v-s$ graph of a cyclist traveling along a straight road is shown. Construct the $a-s$ graph.



SOLUTION

***a–s* Graph:** For $0 \le s < 100$ ft,

$$\left(\xrightarrow{+}\right) \qquad a = v\frac{dv}{ds} = (0.1s + 5)(0.1) = (0.01s + 0.5)\ \text{ft/s}^2$$

Thus at $s = 0$ and 100 ft

$$a|_{s=0} = 0.01(0) + 0.5 = 0.5\ \text{ft/s}^2$$

$$a|_{s=100\ \text{ft}} = 0.01(100) + 0.5 = 1.5\ \text{ft/s}^2$$

For 100 ft $< s \le 350$ ft,

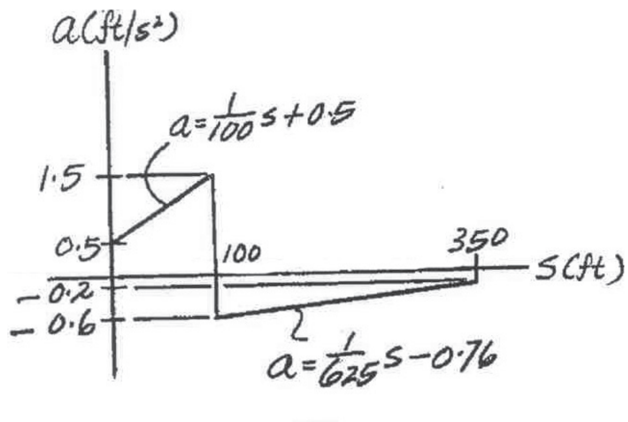
$$\left(\xrightarrow{+}\right) \qquad a = v\frac{dv}{ds} = (-0.04s + 19)(-0.04) = (0.0016s - 0.76)\ \text{ft/s}^2$$

Thus at $s = 100$ ft and 350 ft

$$a|_{s=100\ \text{ft}} = 0.0016(100) - 0.76 = -0.6\ \text{ft/s}^2$$

$$a|_{s=350\ \text{ft}} = 0.0016(350) - 0.76 = -0.2\ \text{ft/s}^2$$

The $a-s$ graph is shown in Fig. a.



(a)

Thus at $s = 0$ and 100 ft

$$a|_{s=0} = 0.01(0) + 0.5 = 0.5\ \text{ft/s}^2$$

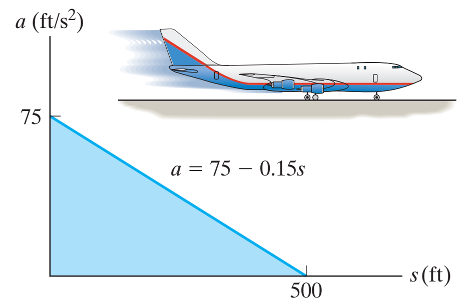
$$a|_{s=100\ \text{ft}} = 0.01(100) + 0.5 = 1.5\ \text{ft/s}^2$$

At $s = 100$ ft, a changes from $a_{\text{max}} = 1.5\ \text{ft/s}^2$ to $a_{\text{min}} = -0.6\ \text{ft/s}^2$.

Ans:
For $0 \le s < 100$ ft
$a = \{0.01\ s + 0.5\}$ ft/s $\qquad a|_{s=0} = 0.5\ \text{ft/s}^2$
$a|_{s=100\ \text{ft}} = 1.5\ \text{ft/s}^2$
For 100 ft $< s < 350$ ft
$a = \{0.0016\ s - 0.76\}\ \text{ft/s}^2$
$a|_{s=100\ \text{ft}} = -0.6\ \text{ft/s}^2$
$a|_{s=350\ \text{ft}} = -0.2\ \text{ft/s}^2$

***12–68.**

The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Construct the v–s graph and determine the time needed to travel 500 ft down the runway.



SOLUTION

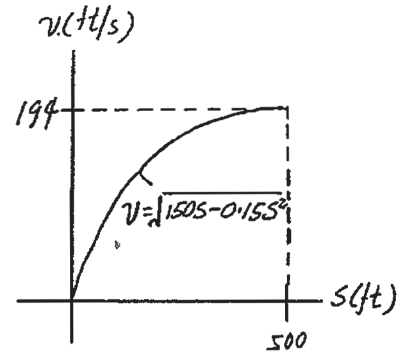
$$a = 75 - 0.15s \qquad \int_0 v\,dv = \int_0^s (75 - 0.15s)ds$$

$$v = \sqrt{150s - 0.15s^2}$$

At $s = 500$ ft, $\quad v = \sqrt{150(500) - 0.15(500)^2} = 194 \text{ ft/s}$

$$v = \frac{ds}{dt}; \qquad \int_0^t dt = \int_0^{500} \frac{ds}{\sqrt{150s - 0.15s^2}}$$

$$t = 2.582 \sin^{-1}\left(\frac{0.3x - 150}{150}\right)\Bigg|_0^{500} = 4.06 \text{ s}$$ **Ans.**



Ans:
$v = \{\sqrt{150s - 0.15s^2}\}$ ft/s $\qquad v|_{s = 500\text{ ft}} = 194$ ft/s
$t = 4.06$ s

12–69.

If the velocity of a particle is defined as $\mathbf{v}(t) = \{0.8t^2\mathbf{i} + 12t^{1/2}\mathbf{j} + 5\mathbf{k}\}$ m/s, determine the magnitude and coordinate direction angles α, β, γ of the particle's acceleration when $t = 2$ s.

SOLUTION

$$\mathbf{v}(t) = 0.8t^2\,\mathbf{i} + 12t^{1/2}\,\mathbf{j} + 5\mathbf{k}$$

$$\mathbf{a} = \frac{dv}{dt} = 1.6t\,\mathbf{i} + 6t^{-1/2}\mathbf{j}$$

When $t = 2$ s, $\quad \mathbf{a} = 3.2\mathbf{i} + 4.243\mathbf{j}$

$$a = \sqrt{(3.2)^2 + (4.243)^2} = 5.31 \text{ m/s}^2$$ **Ans.**

$$u_o = \frac{\mathbf{a}}{a} = 0.6022\mathbf{i} + 0.7984\mathbf{j}$$

$$\alpha = \cos^{-1}(0.6022) = 53.0^\circ$$ **Ans.**

$$\beta = \cos^{-1}(0.7984) = 37.0^\circ$$ **Ans.**

$$\gamma = \cos^{-1}(0) = 90.0^\circ$$ **Ans.**

Ans:
$a = 5.31 \text{ m/s}^2$
$\alpha = 53.0^\circ$
$\beta = 37.0^\circ$
$\gamma = 90.0^\circ$

12–70.

A particle moves along the curve $y = e^{2x}$ such that its velocity has a constant magnitude of $v = 4$ ft/s. Determine the x and y components of velocity when the particle is at $y = 5$ ft.

SOLUTION

Velocity: Taking the first derivative of the path $y = e^{2x}$, we have

$$\dot{y} = 2e^{2x}\dot{x} \qquad \textbf{[1]}$$

However, $\dot{x} = v_x$ and $\dot{y} = v_y$. Thus, Eq. [1] becomes

$$v_y = 2e^{2x}v_x \qquad \textbf{[2]}$$

Here, $v = 4$ ft/s. Then

$$v^2 = v_x^2 + v_y^2$$

$$v_x^2 + v_y^2 = 16 \qquad \textbf{[3]}$$

Solving Eqs. [2] and [3] yields

$$v_x = 4\sqrt{\frac{1}{1 + 4e^{4x}}} \qquad \text{and} \qquad v_y = 8\sqrt{\frac{e^{4x}}{1 + 4e^{4x}}}$$

At $y = 5$ ft, $5 = e^{2x}$, $x = 0.8047$ ft. Thus,

$$v_x = 4\sqrt{\frac{1}{1 + 4e^{4(0.8047)}}} = 0.398 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_y = 8\sqrt{\frac{e^{4(0.8047)}}{1 + 4e^{4(0.8047)}}} = 3.98 \text{ ft/s} \qquad \textbf{Ans.}$$

12–71.

A particle travels along the parabolic path $y = bx^2$. If its component of velocity along the y axis is $v_y = ct^2$, determine the x and y components of the particle's acceleration. Here b and c are constants.

SOLUTION

Velocity:

$$dy = v_y\,dt$$

$$\int_0^y dy = \int_0^t ct^2\,dt$$

$$y = \frac{c}{3}t^3$$

Substituting the result of y into $y = bx^2$,

$$\frac{c}{3}t^3 = bx^2$$

$$x = \sqrt{\frac{c}{3b}}\,t^{3/2}$$

Thus, the x component of the particle's velocity can be determined by taking the time derivative of x.

$$v_x = \dot{x} = \frac{d}{dt}\left[\sqrt{\frac{c}{3b}}\,t^{3/2}\right] = \frac{3}{2}\sqrt{\frac{c}{3b}}\,t^{1/2}$$

Acceleration:

$$a_x = \dot{v}_x = \frac{d}{dt}\left(\frac{3}{2}\sqrt{\frac{c}{3b}}t^{1/2}\right) = \frac{3}{4}\sqrt{\frac{c}{3b}}\,t^{-1/2} = \frac{3}{4}\sqrt{\frac{c}{3b}}\frac{1}{\sqrt{t}} \qquad \textbf{Ans.}$$

$$a_y = \dot{v}_y = \frac{d}{dt}\left(ct^2\right) = 2ct \qquad \textbf{Ans.}$$

Ans:

$a_x = \dfrac{3}{4}\sqrt{\dfrac{c}{3b}}\dfrac{1}{\sqrt{t}}$

$a_y = 2ct$

***12–72.**

A particle travels along the circular path $x^2 + y^2 = r^2$. If the y component of the particle's velocity is $v_y = 2r \cos 2t$, determine the x and y components of its acceleration at any instant.

SOLUTION

Velocity:

$$dy = v_y\, dt$$

$$\int_0^y dy = \int_0^t 2r \cos 2t dt$$

$$y = r \sin 2t$$

Substituting this result into $x^2 + y^2 = r^2$, we obtain

$$x^2 + r^2 \sin^2 2t = r^2$$

$$x^2 = r^2 (1 - \sin^2 2t)$$

$$x = r \cos 2t$$

Thus,

$$v_2 = \dot{x} = \frac{d}{dt}(r \cos 2t) = 2r \sin 2t$$

Acceleration:

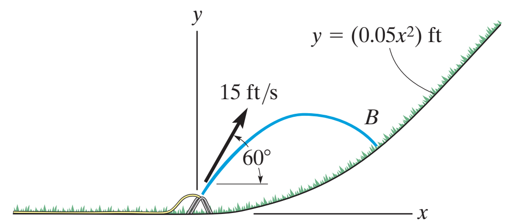
$$a_x = \dot{v}_x = \frac{d}{dt}(2r \sin 2t) = 4r \cos 2t$$ **Ans.**

$$a_y = \dot{v}_y = \frac{d}{dt}(2r \cos 2t) = -4r \sin 2t$$ **Ans.**

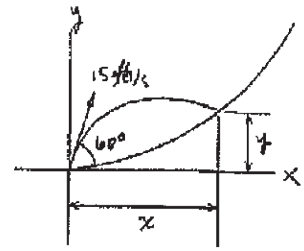
Ans:
$a_x = 4r \cos 2t$
$a_y = -4r \sin 2t$

12–73.

The water sprinkler, positioned at the base of a hill, releases a stream of water with a velocity of 15 ft/s as shown. Determine the point $B(x, y)$ where the water strikes the ground on the hill. Assume that the hill is defined by the equation $y = (0.05x^2)$ ft and neglect the size of the sprinkler.



SOLUTION



$v_x = 15 \cos 60° = 7.5 \text{ ft/s} \qquad v_y = 15 \sin 60° = 12.99 \text{ ft/s}$

$(\overset{+}{\rightarrow}) \quad s = v_0 t$

$x = 7.5t$

$(+\uparrow) \quad s = s_o + v_o t + \frac{1}{2} a_c t^2$

$y = 0 + 12.99t + \frac{1}{2}(-32.2)t^2$

$y = 1.732x - 0.286x^2$

Since $y = 0.05x^2$,

$0.05x^2 = 1.732x - 0.286x^2$

$x(0.336x - 1.732) = 0$

$x = 5.15 \text{ ft}$ **Ans.**

$y = 0.05(5.15)^2 = 1.33 \text{ ft}$ **Ans.**

Also,

$(\overset{+}{\rightarrow}) \quad s = v_0 t$

$x = 15 \cos 60° t$

$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$y = 0 + 15 \sin 60° t + \frac{1}{2}(-32.2)t^2$

Since $y = 0.05x^2$

$12.99t - 16.1t^2 = 2.8125t^2 \quad t = 0.6869 \text{ s}$

So that,

$x = 15 \cos 60° \,(0.6868) = 5.15 \text{ ft}$ **Ans.**

$y = 0.05(5.15)^2 = 1.33 \text{ ft}$ **Ans.**

Ans:
(5.15 ft, 1.33 ft)

12–74.

A particle moves along the curve $y = x - (x^2/400)$, where x and y are in ft. If the velocity component in the x direction is $v_x = 2$ ft/s and remains *constant*, determine the magnitudes of the velocity and acceleration when $x = 20$ ft.

SOLUTION

Velocity: Taking the first derivative of the path $y = x - \dfrac{x^2}{400}$, we have

$$\dot{y} = \dot{x} - \frac{1}{400}(2x\dot{x})$$

$$\dot{y} = \dot{x} - \frac{x}{200}\dot{x} \qquad \textbf{[1]}$$

However, $\dot{x} = v_x$ and $\dot{y} = v_y$. Thus, Eq.[1] becomes

$$v_y = v_x - \frac{x}{200}v_x \qquad \textbf{[1]}$$

Here, $v_x = 2\text{ft/s}$ at $x = 20$ ft. Then, From Eq. [2]

$$v_y = 2 - \frac{20}{200}(2) = 1.80 \text{ ft/s}$$

Also,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{2^2 + 1.80^2} = 2.69 \text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration: Taking the second derivative of the path $y = x - \dfrac{x^2}{400}$, we have

$$\ddot{y} = \ddot{x} - \frac{1}{200}(\dot{x}^2 + x\ddot{x}) \qquad \textbf{[3]}$$

However, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq.[3] becomes

$$a_y = a_x - \frac{1}{200}\left(v_x^2 + x a_x\right) \qquad [4]$$

Since $v_x = 2\text{ft/s}$ is constant, hence $a_x = 0$ at $x = 20$ ft. Then, From Eq. [4]

$$a_y = 0 - \frac{1}{200}\left[2^2 + 20(0)\right] = -0.020 \text{ ft/s}^2$$

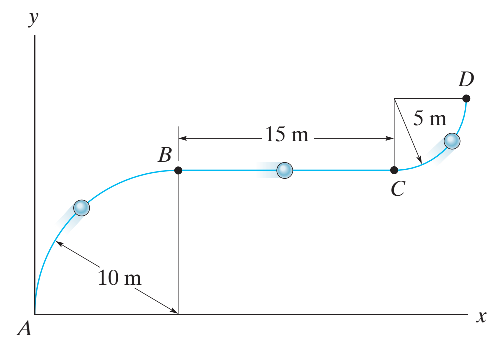
Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + (-0.020)^2} = 0.0200 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 2.69$ ft/s
$a = 0.0200$ ft/s^2

12–75.

A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



SOLUTION

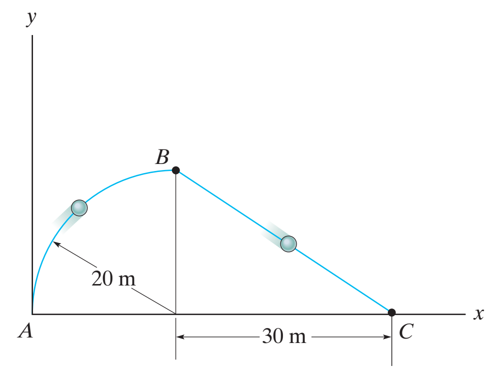
$$s_T = \frac{1}{4}(2\pi)(10)) + 15 + \frac{1}{4}(2\pi(5)) = 38.56$$

$$v_{sp} = \frac{s_T}{t_t} = \frac{38.56}{2 + 4 + 3} = 4.28 \text{ m/s}$$ **Ans.**

Ans:
$(v_{sp})_{avg} = 4.28$ m/s

***12–76.**

A particle travels along the curve from A to B in 5 s. It takes 8 s for it to go from B to C and then 10 s to go from C to A. Determine its average speed when it goes around the closed path.



SOLUTION

The total distance traveled is

$$S_{\text{Tot}} = S_{AB} + S_{BC} + S_{CA}$$

$$= 20\left(\frac{\pi}{2}\right) + \sqrt{20^2 + 30^2} + (30 + 20)$$

$$= 117.47 \text{ m}$$

The total time taken is

$$t_{\text{Tot}} = t_{AB} + t_{BC} + t_{CA}$$

$$= 5 + 8 + 10$$

$$= 23 \text{ s}$$

Thus, the average speed is

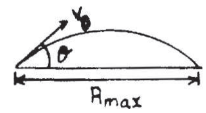
$$(v_{sp})_{\text{avg}} = \frac{S_{\text{Tot}}}{t_{\text{Tot}}} = \frac{117.47 \text{ m}}{23 \text{ s}} = 5.107 \text{ m/s} = 5.11 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$(v_{sp})_{\text{avg}} = 5.11 \text{ m/s}$

12–77.

Show that if a projectile is fired at an angle θ from the horizontal with an initial velocity v_0, the *maximum* range the projectile can travel is given by $R_{\max} = v_0^2/g$, where g is the acceleration of gravity. What is the angle θ for this condition?

SOLUTION



$(v_0)_x = v_0 \cos\theta$

$(v_0)_y = v_0 \sin\theta$

After time t,

$(\xrightarrow{+})$ $\quad s_x = (v_0)_x t; \qquad x = (v_0 \cos\theta)t$ **(1)**

$(+\uparrow)$ $\quad s_y = (v_0)_y\, t + \frac{1}{2}a_c t^2; \qquad y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$ **(2)**

Substituting Eq. (1) into Eq. (2), $y = x\tan\theta - \dfrac{gx^2}{2v_0^2\cos^2\theta}$

Set $y = 0$ to determine the range, $x = R$:

$$R = \frac{2v_0^2 \sin\theta\cos\theta}{g} = \frac{v_0^2 \sin 2\theta}{g}$$

$R_{\max}$ occurs when $\sin 2\theta = 1$ or,

$\theta = 45°$ **Ans.**

This gives: $R_{\max} = \dfrac{v_0^2}{g}$ **Q.E.D**

Ans:
$\theta = 45°$

12–78.

A rocket is fired from rest at $x = 0$ and travels along a parabolic trajectory described by $y^2 = [120(10^3)x]$ m. If the x component of acceleration is $a_x = \left(\frac{1}{4}t^2\right)$ m/s^2, where t is in seconds, determine the magnitudes of the rocket's velocity and acceleration when $t = 10$ s.

SOLUTION

Position: The parameter equation of x can be determined by integrating a_x twice with respect to t.

$$\int dv_x = \int a_x dt$$

$$\int_0^{v_x} dv_x = \int_0^t \frac{1}{4}t^2 dt$$

$$v_x = \left(\frac{1}{12}t^3\right) \text{ m/s}$$

$$\int dx = \int v_x dt$$

$$\int_0^x dx = \int_0^t \frac{1}{12}t^3 dt$$

$$x = \left(\frac{1}{48}t^4\right) \text{ m}$$

Substituting the result of x into the equation of the path,

$$y^2 = 120\left(10^3\right)\left(\frac{1}{48}t^4\right)$$

$$y = \left(50t^2\right) \text{ m}$$

Velocity:

$$v_y = \dot{y} = \frac{d}{dt}\left(50t^2\right) = \left(100t\right) \text{ m/s}$$

When $t = 10$ s,

$$v_x = \frac{1}{12}\left(10^3\right) = 83.33 \text{ m/s} \qquad v_y = 100(10) = 1000 \text{ m/s}$$

Thus, the magnitude of the rocket's velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{83.33^2 + 1000^2} = 1003 \text{ m/s}$$ **Ans.**

Acceleration:

$$a_y = \dot{v}_y = \frac{d}{dt}(100t) = 100 \text{ m/s}^2$$

When $t = 10$ s,

$$a_x = \frac{1}{4}\left(10^2\right) = 25 \text{ m/s}^2$$

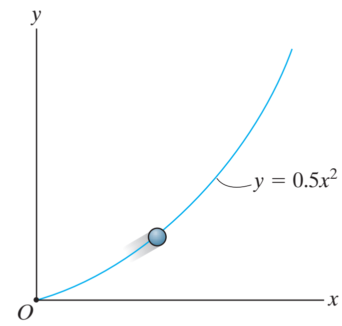
Thus, the magnitude of the rocket's acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{25^2 + 100^2} = 103 \text{ m/s}^2$$ **Ans.**

Ans:
$v = 1003$ m/s
$a = 103$ m/s^2

12–79.

The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ ft/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.



SOLUTION

Position: The x position of the particle can be obtained by applying the $v_x = \dfrac{dx}{dt}$.

$$dx = v_x dt$$

$$\int_0^x dx = \int_0^t 5t dt$$

$$x = \left(2.50t^2\right) \text{ ft}$$

Thus, $y = 0.5\left(2.50t^2\right)^2 = \left(3.125t^4\right)$ ft. At $t = 1$ s, $x = 2.5\left(1^2\right) = 2.50$ ft and $y = 3.125\left(1^4\right) = 3.125$ ft. The particle's distance from the origin at this moment is

$$d = \sqrt{(2.50 - 0)^2 + (3.125 - 0)^2} = 4.00 \text{ ft} \qquad \textbf{Ans.}$$

Acceleration: Taking the first derivative of the path $y = 0.5x^2$, we have $\dot{y} = x\dot{x}$. The second derivative of the path gives

$$\ddot{y} = \dot{x}^2 + x\ddot{x} \qquad \textbf{(1)}$$

However, $\dot{x} = v_x$, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. (1) becomes

$$a_y = v_x^2 + xa_x \qquad \textbf{(2)}$$

When $t = 1$ s, $v_x = 5(1) = 5$ ft/s $a_x = \dfrac{dv_x}{dt} = 5 \text{ ft/s}^2$, and $x = 2.50$ ft. Then, from Eq. (2)

$$a_y = 5^2 + 2.50(5) = 37.5 \text{ ft/s}^2$$

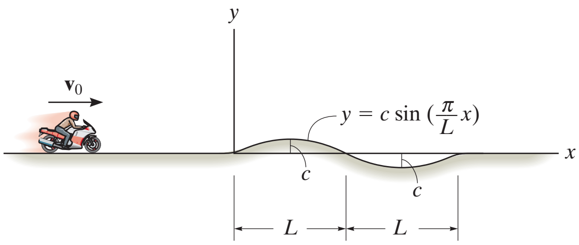
Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 37.5^2} = 37.8 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$d = 4.00$ ft
$a = 37.8 \text{ ft/s}^2$

***12–80.**

The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



SOLUTION

$$y = c\sin\left(\frac{\pi}{L}x\right)$$

$$\dot{y} = \frac{\pi}{L}c\left(\cos\frac{\pi}{L}x\right)\dot{x}$$

$$v_y = \frac{\pi}{L}c\,v_x\left(\cos\frac{\pi}{L}x\right)$$

$$v_0^2 = v_y^2 + v_x^2$$

$$v_0^2 = v_x^2\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]$$

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$ **Ans.**

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$ **Ans.**

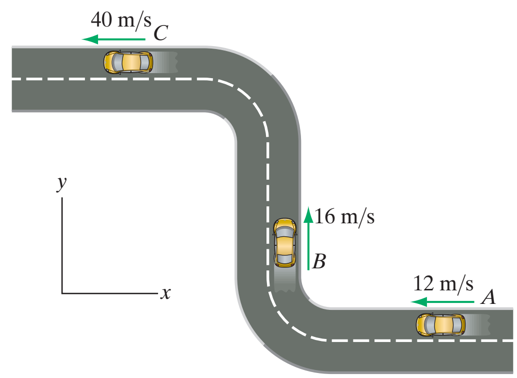
Ans:

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$

12–81.

A car traveling along the road has the velocities indicated in the figure when it arrives at points A, B, and C. If it takes 10 s to go from A to B, and then 15 s to go from B to C, determine the average acceleration between points A and B and between points A and C.



SOLUTION

$\mathbf{v}_A = \{-12\mathbf{i}\}$ m/s $\quad \mathbf{v}_B = \{16\mathbf{j}\}$ m/s $\quad \mathbf{v}_C = \{-40\mathbf{i}\}$ m/s

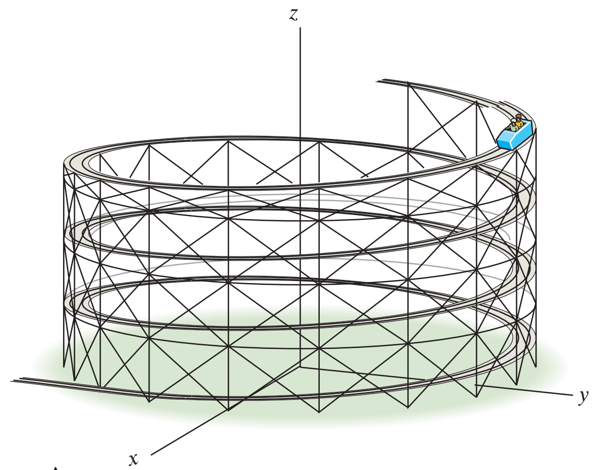
$$(\mathbf{a}_{AB})_{\text{avg}} = \frac{\mathbf{v}_B - \mathbf{v}_A}{t_{AB}} = \frac{16\mathbf{j} - (-12\mathbf{i})}{10} = \{1.2\mathbf{i} + 1.6\mathbf{j}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$(\mathbf{a}_{AC})_{\text{avg}} = \frac{\mathbf{v}_C - \mathbf{v}_A}{t_{AC}} = \frac{40\mathbf{i} - (-12\mathbf{i})}{25} = \{-1.2\mathbf{i}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$(\mathbf{a}_{AB})_{\text{avg}} = \{1.2\mathbf{i} + 1.6\mathbf{j}\}\ \text{m/s}^2$
$(\mathbf{a}_{AC})_{\text{avg}} = \{-1.2\mathbf{i}\}\ \text{m/s}^2$

12–82.

The roller coaster car travels down the helical path at constant speed such that the parametric equations that define its position are $x = c \sin kt$, $y = c \cos kt$, $z = h - bt$, where c, h, and b are constants. Determine the magnitudes of its velocity and acceleration.



SOLUTION

$x = c \sin kt \qquad \dot{x} = ck \cos kt \qquad \ddot{x} = -ck^2 \sin kt$

$y = c \cos kt \qquad \dot{y} = -ck \sin kt \qquad \ddot{y} = -ck^2 \cos kt$

$z = h - bt \qquad \dot{z} = -b \qquad \ddot{z} = 0$

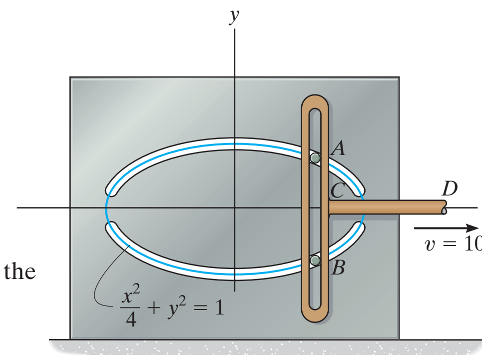
$v = \sqrt{(ck \cos kt)^2 + (-ck \sin kt)^2 + (-b)^2} = \sqrt{c^2k^2 + b^2}$ **Ans.**

$a = \sqrt{(-ck^2 \sin kt)^2 + (-ck^2 \cos kt)^2 + 0} = ck^2$ **Ans.**

Ans:
$v = \sqrt{c^2 k^2 + b^2}$
$a = ck^2$

12–83.

Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10 m/s, determine the magnitudes of the velocity and acceleration of peg A when $x = 1$ m.



SOLUTION

Velocity: The x and y components of the peg's velocity can be related by taking the first time derivative of the path's equation.

$$\frac{x^2}{4} + y^2 = 1$$

$$\frac{1}{4}(2x\dot{x}) + 2y\dot{y} = 0$$

$$\frac{1}{2}x\dot{x} + 2y\dot{y} = 0$$

or

$$\frac{1}{2}xv_x + 2yv_y = 0 \qquad \textbf{(1)}$$

At $x = 1$ m,

$$\frac{(1)^2}{4} + y^2 = 1 \qquad y = \frac{\sqrt{3}}{2}\text{ m}$$

Here, $v_x = 10$ m/s and $x = 1$. Substituting these values into Eq. (1),

$$\frac{1}{2}(1)(10) + 2\left(\frac{\sqrt{3}}{2}\right)v_y = 0 \qquad v_y = -2.887\text{ m/s} = 2.887\text{ m/s} \downarrow$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{10^2 + 2.887^2} = 10.4\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the peg's acceleration can be related by taking the second time derivative of the path's equation.

$$\frac{1}{2}(\dot{x}\dot{x} + x\ddot{x}) + 2(\dot{y}\dot{y} + y\ddot{y}) = 0$$

$$\frac{1}{2}\left(\dot{x}^2 + x\ddot{x}\right) + 2\left(\dot{y}^2 + y\ddot{y}\right) = 0$$

or

$$\frac{1}{2}\left({v_x}^2 + xa_x\right) + 2\left({v_y}^2 + ya_y\right) = 0 \qquad \textbf{(2)}$$

Since v_x is constant, $a_x = 0$. When $x = 1$ m, $y = \frac{\sqrt{3}}{2}$ m, $v_x = 10$ m/s, and $v_y = -2.887$ m/s. Substituting these values into Eq. (2),

$$\frac{1}{2}\left(10^2 + 0\right) + 2\left[(-2.887)^2 + \frac{\sqrt{3}}{2}a_y\right] = 0$$

$$a_y = -38.49\text{ m/s}^2 = 38.49\text{ m/s}^2 \downarrow$$

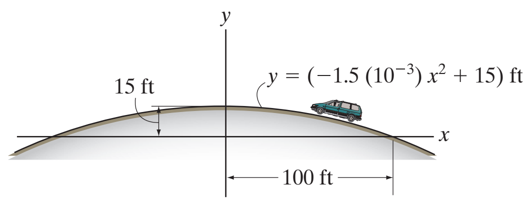
Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{{a_x}^2 + {a_y}^2} = \sqrt{0^2 + (-38.49)^2} = 38.5\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 10.4$ m/s
$a = 38.5$ m/s^2

***12–84.**

The van travels over the hill described by $y = (-1.5(10^{-3})\,x^2 + 15)$ ft. If it has a constant speed of 75 ft/s, determine the x and y components of the van's velocity and acceleration when $x = 50$ ft.



SOLUTION

Velocity: The x and y components of the van's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$y = -1.5\left(10^{-3}\right)x^2 + 15$$

$$\dot{y} = -3\left(10^{-3}\right)x\dot{x}$$

or

$$v_y = -3\left(10^{-3}\right)xv_x$$

When $x = 50$ ft,

$$v_y = -3\left(10^{-3}\right)(50)v_x = -0.15v_x \qquad \textbf{(1)}$$

The magnitude of the van's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2} \qquad \textbf{(2)}$$

Substituting $v = 75$ ft/s and Eq. (1) into Eq. (2),

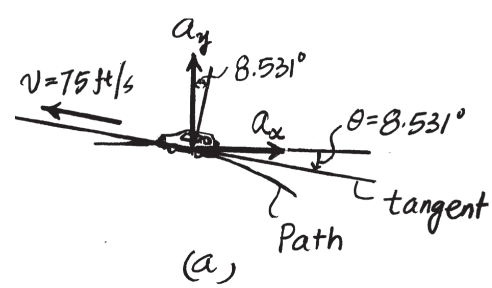
$$75 = \sqrt{v_x{}^2 + (-0.15v_x)^2}$$

$$v_x = 74.2 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

Substituting the result of v_x into Eq. (1), we obtain

$$v_y = -0.15(-74.17) = 11.12 \text{ ft/s} = 11.1 \text{ ft/s} \uparrow \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the van's acceleration can be related by taking the second time derivative of the path's equation using the chain rule.



$$\ddot{y} = -3\left(10^{-3}\right)(\dot{x}\dot{x} + x\ddot{x})$$

or

$$a_y = -3\left(10^{-3}\right)\left(v_x{}^2 + xa_x\right)$$

When $x = 50$ ft, $v_x = -74.17$ ft/s. Thus,

$$a_y = -3\left(10^{-3}\right)\left[(-74.17)^2 + 50a_x\right]$$

$$a_y = -(16.504 + 0.15a_x) \qquad \textbf{(3)}$$

Since the van travels with a constant speed along the path, its acceleration along the tangent of the path is equal to zero. Here, the angle that the tangent makes with the horizontal at $x = 50$ ft is $\theta = \tan^{-1}\left(\dfrac{dy}{dx}\right)\Big|_{x=50\text{ ft}} = \tan^{-1}\left[-3\left(10^{-3}\right)x\right]\Big|_{x=50\text{ ft}} = \tan^{-1}(-0.15) = -8.531°.$ Thus, from the diagram shown in Fig. a,

$$a_x \cos 8.531° - a_y \sin 8.531° = 0 \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

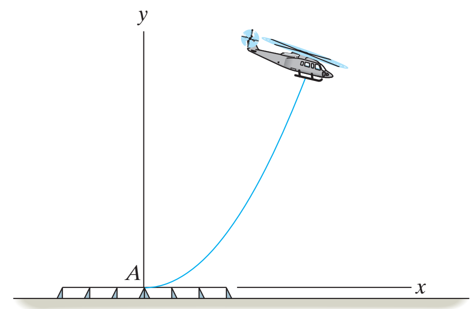
$$a_x = -2.42 \text{ ft/s} = 2.42 \text{ ft/s}^2 \leftarrow \qquad \textbf{Ans.}$$

$$a_y = -16.1 \text{ ft/s} = 16.1 \text{ ft/s}^2 \downarrow \qquad \textbf{Ans.}$$

Ans:
$v_x = 74.2$ ft/s $\leftarrow$
$v_y = 11.1$ ft/s $\uparrow$
$a_x = 2.42$ ft/s^2 $\leftarrow$
$a_y = 16.1$ ft/s^2 $\downarrow$

12–85.

The flight path of the helicopter as it takes off from A is defined by the parametric equations $x = (2t^2)$ m and $y = (0.04t^3)$ m, where t is the time in seconds. Determine the distance the helicopter is from point A and the magnitudes of its velocity and acceleration when $t = 10$ s.



SOLUTION

$x = 2t^2 \qquad y = 0.04t^3$

At $t = 10$ s, $\qquad x = 200$ m $\qquad y = 40$ m

$d = \sqrt{(200)^2 + (40)^2} = 204$ m **Ans.**

$$v_x = \frac{dx}{dt} = 4t$$

$$a_x = \frac{dv_x}{dt} = 4$$

$$v_y = \frac{dy}{dt} = 0.12t^2$$

$$a_y = \frac{dv_y}{dt} = 0.24t$$

At $t = 10$ s,

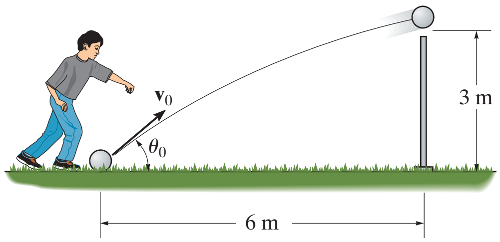
$v = \sqrt{(40)^2 + (12)^2} = 41.8$ m/s **Ans.**

$a = \sqrt{(4)^2 + (2.4)^2} = 4.66$ m/s^2 **Ans.**

Ans:
$d = 204$ m
$v = 41.8$ m/s
$a = 4.66$ m/s^2

12–86.

Determine the minimum initial velocity v_0 and the corresponding angle θ_0 at which the ball must be kicked in order for it to just cross over the 3-m-high fence.



SOLUTION

Coordinate System: The $x-y$ coordinate system will be set so that its origin coincides with the ball's initial position.

***x*-Motion:** Here, $(v_0)_x = v_0 \cos\theta$, $x_0 = 0$, and $x = 6$ m. Thus,

$$(\overset{+}{\rightarrow}) \qquad x = x_0 + (v_0)_x t$$

$$6 = 0 + (v_0 \cos\theta)t$$

$$t = \frac{6}{v_0 \cos\theta} \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_0)_x = v_0 \sin\theta$, $a_y = -g = -9.81\ \text{m/s}^2$, and $y_0 = 0$. Thus,

$$(+\uparrow) \qquad y = y_0 + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$3 = 0 + v_0(\sin\theta)t + \frac{1}{2}(-9.81)t^2$$

$$3 = v_0(\sin\theta)t - 4.905t^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$v_0 = \sqrt{\frac{58.86}{\sin 2\theta - \cos^2\theta}} \qquad \textbf{(3)}$$

From Eq. (3), we notice that v_0 is minimum when $f(\theta) = \sin 2\theta - \cos^2\theta$ is maximum. This requires $\frac{df(\theta)}{d\theta} = 0$

$$\frac{df(\theta)}{d\theta} = 2\cos 2\theta + \sin 2\theta = 0$$

$$\tan 2\theta = -2$$

$$2\theta = 116.57°$$

$$\theta = 58.28° = 58.3° \qquad \textbf{Ans.}$$

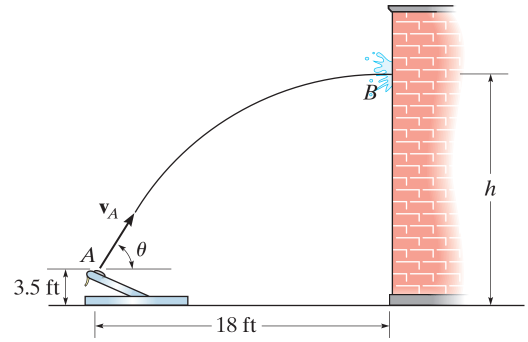
Substituting the result of θ into Eq. (2), we have

$$(v_0)_{\min} = \sqrt{\frac{58.86}{\sin 116.57° - \cos^2 58.28°}} = 9.76\ \text{m/s} \qquad \textbf{Ans.}$$

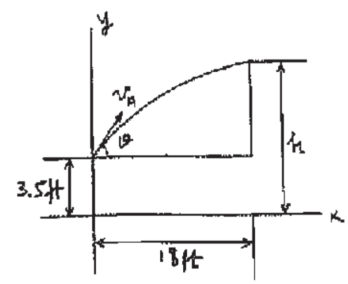
Ans:
$\theta = 58.3°$
$(v_0)_{\min} = 9.76\ \text{m/s}$

12–87.

The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes 1.5 s to travel from A to B, determine the velocity $\mathbf{v}_A$ at which it was launched, the angle of release θ, and the height h.



SOLUTION



$(\overset{+}{\rightarrow})\quad s = v_0 t$

$$18 = v_A \cos\theta(1.5) \qquad \textbf{(1)}$$

$(+\uparrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = (v_A \sin\theta)^2 + 2(-32.2)(h - 3.5)$$

$(+\uparrow)\quad v = v_0 + a_c t$

$$0 = v_A \sin\theta - 32.2(1.5) \qquad \textbf{(2)}$$

To solve, first divide Eq. (2) by Eq. (1) to get θ. Then

$$\theta = 76.0^\circ \qquad \textbf{Ans.}$$

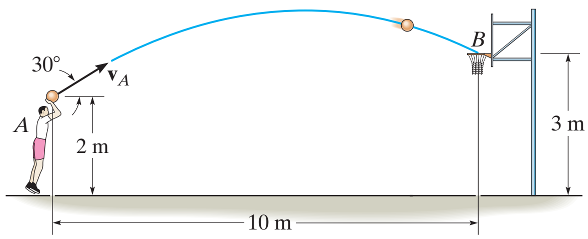
$$v_A = 49.8\text{ ft/s} \qquad \textbf{Ans.}$$

$$h = 39.7\text{ ft} \qquad \textbf{Ans.}$$

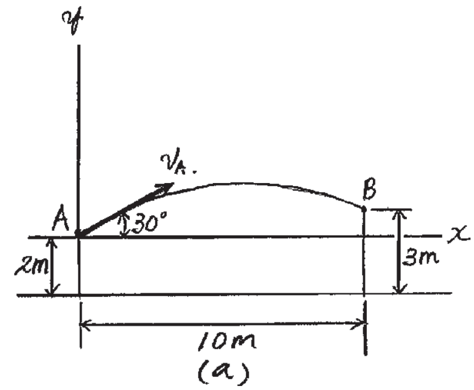
Ans:
$\theta = 76.0^\circ$
$v_A = 49.8\text{ ft/s}$
$h = 39.7\text{ ft}$

***12–88.**

Neglecting the size of the ball, determine the magnitude v_A of the basketball's initial velocity and its velocity when it passes through the basket.



SOLUTION



Coordinate System: The origin of the x-y coordinate system will be set to coinside with point A as shown in Fig. a

Horizontal Motion: Here $(v_A)_x = v_A \cos 30° \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 10\text{ m} \rightarrow$.

$$(\overset{+}{\rightarrow})\quad (s_B)_x = (s_A)_x + (v_A)_x\, t$$

$$10 = 0 + v_A \cos 30°\, t$$

$$t = \frac{10}{v_A \cos 30°} \qquad \textbf{(1)}$$

Also,

$$(\overset{+}{\rightarrow})\quad (v_B)_x = (v_A)_x = v_A \cos 30° \qquad \textbf{(2)}$$

Vertical Motion: Here, $(v_A)_y = v_A \sin 30° \uparrow$, $(s_A)_y = 0$, $(s_B)_y = 3 - 2 = 1\text{ m} \uparrow$ and $a_y = 9.81\text{ m/s}^2 \downarrow$

$$(+\uparrow)\quad (s_B)_y = (s_A)_y + (v_A)_y\, t + \frac{1}{2} a_y t^2$$

$$1 = 0 + v_A \sin 30°\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 0.5\, v_A\, t + 1 = 0 \qquad \textbf{(3)}$$

Also

$$(+\uparrow)\quad (v_B)_y = (v_A)_y + a_y\, t$$

$$(v_B)_y = v_A \sin 30° + (-9.81)t$$

$$(v_B)_y = 0.5\, v_A - 9.81t \qquad \textbf{(4)}$$

Solving Eq. (1) and (3)

$$v_A = 11.705\text{ m/s} = 11.7\text{ m/s} \qquad \textbf{Ans.}$$

$$t = 0.9865\text{ s}$$

Substitute these results into Eq. (2) and (4)

$$(v_B)_x = 11.705 \cos 30° = 10.14\text{ m/s} \rightarrow$$

$$(v_B)_y = 0.5(11.705) - 9.81(0.9865) = -3.825\text{ m/s} = 3.825\text{ m/s} \downarrow$$

Thus, the magnitude of $\mathbf{v}_B$ is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{10.14^2 + 3.825^2} = 10.83\text{ m/s} = 10.8\text{ m/s} \qquad \textbf{Ans.}$$

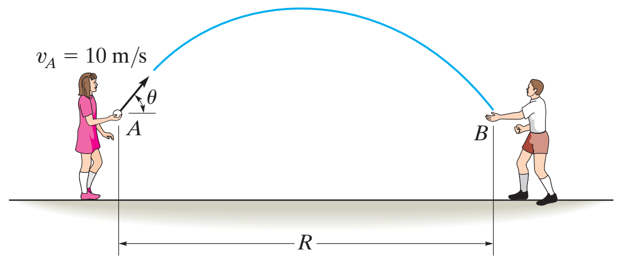
And its direction is defined by

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{3.825}{10.14}\right) = 20.67° = 20.7° \qquad \textbf{Ans.}$$

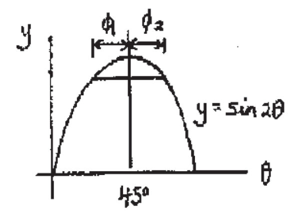
Ans:
$v_A = 11.7\text{ m/s}$
$v_B = 10.8\text{ m/s}$
$\theta = 20.7°$

12–89.

The girl at A can throw a ball at $v_A = 10$ m/s. Calculate the maximum possible range $R = R_{max}$ and the associated angle θ at which it should be thrown. Assume the ball is caught at B at the same elevation from which it is thrown.



SOLUTION



$(\xrightarrow{+})$ $s = s_0 + v_0 t$

$$R = 0 + (10 \cos\theta)t$$

$(+\uparrow)$ $v = v_0 + a_c t$

$$-10 \sin\theta = 10 \sin\theta - 9.81t$$

$$t = \frac{20}{9.81} \sin\theta$$

Thus, $R = \dfrac{200}{9.81} \sin\theta \cos\theta$

$$R = \frac{100}{9.81} \sin 2\theta \qquad \textbf{(1)}$$

Require,

$$\frac{dR}{d\theta} = 0$$

$$\frac{100}{9.81} \cos 2\theta(2) = 0$$

$$\cos 2\theta = 0$$

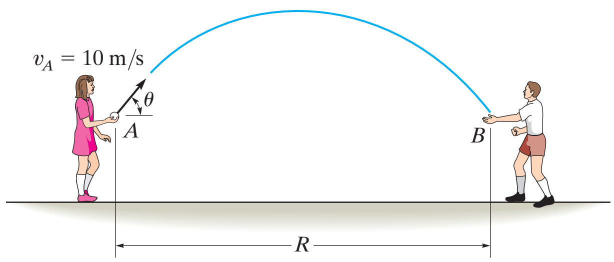
$$\theta = 45° \qquad \textbf{Ans.}$$

$$R = \frac{100}{9.81}(\sin 90°) = 10.2 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$R_{max} = 10.2$ m
$\theta = 45°$

12–90.

Show that the girl at A can throw the ball to the boy at B by launching it at equal angles measured up or down from a 45° inclination. If $v_A = 10$ m/s, determine the range R if this value is 15°, i.e., $\theta_1 = 45° - 15° = 30°$ and $\theta_2 = 45° + 15° = 60°$. Assume the ball is caught at the same elevation from which it is thrown.



SOLUTION

$(\xrightarrow{+})\quad s = s_0 + v_0 t$

$$R = 0 + (10\cos\theta)t$$

$(+\uparrow)\quad v = v_0 + a_c t$

$$-10\sin\theta = 10\sin\theta - 9.81t$$

$$t = \frac{20}{9.81}\sin\theta$$

Thus, $\quad R = \dfrac{200}{9.81}\sin\theta\cos\theta$

$$R = \frac{100}{9.81}\sin 2\theta \qquad \textbf{(1)}$$

Since the function $y = \sin 2\theta$ is symmetric with respect to $\theta = 45°$ as indicated, Eq. (1) will be satisfied if $|\phi_1| = |\phi_2|$

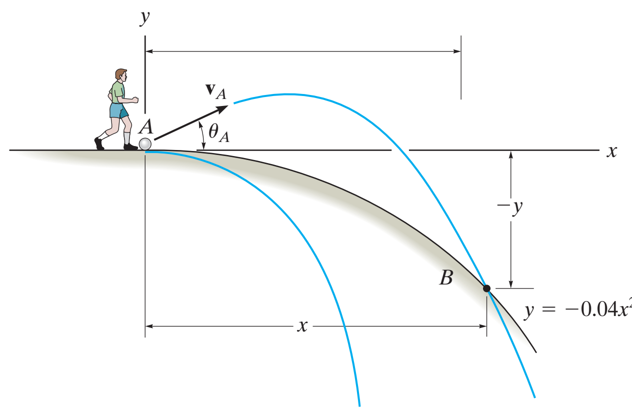
Choosing $\phi = 15°$ or $\theta_1 = 45° - 15° = 30°$ and $\theta_2 = 45° + 15° = 60°$, and substituting into Eq. (1) yields

$R = 8.83$ m **Ans.**

Ans:
$R = 8.83$ m

12–91.

The ball at A is kicked with a speed $v_A = 16$ ft/s and at an angle $\theta_A = 30°$. Determine the point $(x, -y)$ where it strikes the ground. Assume the ground has the shape of a parabola as shown.



SOLUTION

$(v_A)_x = 16 \cos 30° = 13.856$ ft/s

$(v_A)_y = 16 \sin 30° = 8$ ft/s

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$x = 0 + 13.856t$ (1)

$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$y = 0 + 8t + \frac{1}{2}(-32.2)t^2$ (2)

$y = -0.04x^2$

From Eqs. (1) and (2):

At $y = 0.5774x - 0.08386x^2$

$-0.04x^2 = 0.5774x - 0.08386x^2$

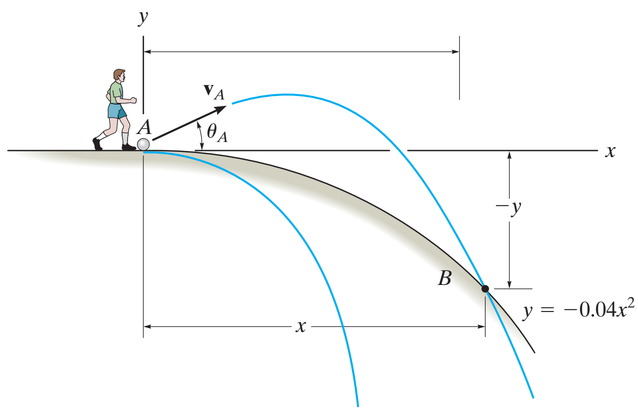
$x = 13.18 \text{ ft} = 13.2 \text{ ft}$ **Ans.**

$y = -0.04\,(13.18)^2 = -6.95 \text{ ft}$ **Ans.**

Ans:
$x = 13.2$ ft
$y = -6.95$ ft

***12–92.**

The ball at A is kicked such that $\theta_A = 30°$. If it strikes the ground at B having coordinates $x = 15$ ft, $y = -9$ ft, determine the speed at which it is kicked and the speed at which it strikes the ground.



SOLUTION

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$$15 = 0 + v_A \cos 30°\, t$$

$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-9 = 0 + v_A \sin 30°\, t + \frac{1}{2}(-32.2)t^2$$

$v_A = 16.5$ ft/s **Ans.**

$t = 1.047$ s

$(\overset{+}{\rightarrow})$ $(v_B)_x = 16.54 \cos 30° = 14.32$ ft/s

$(+\uparrow)$ $v = v_0 + a_c t$

$(v_B)_y = 16.54 \sin 30° + (-32.2)(1.047)$

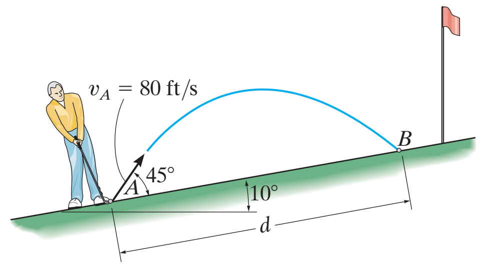
$= -25.45$ ft/s

$v_B = \sqrt{(14.32)^2 + (-25.45)^2} = 29.2$ ft/s **Ans.**

Ans:
$v_A = 16.5$ ft/s
$t = 1.047$ s
$v_B = 29.2$ ft/s

12–93.

A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.



SOLUTION

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$d \cos 10° = 0 + 80 \cos 55°\, t$

$d = 46.59\,t \qquad (1)$

$(+\uparrow)\quad s = s_0 + v_0\, t + \frac{1}{2} a_c t^2$

$d \sin 10° = 0 + 80 \sin 55°\, t - \frac{1}{2}(32.2)\left(t^2\right)$

$46.59\,t \sin 10° = 65.53\,t - 16.1\,t^2$

Solving

$t = 3.568$ s

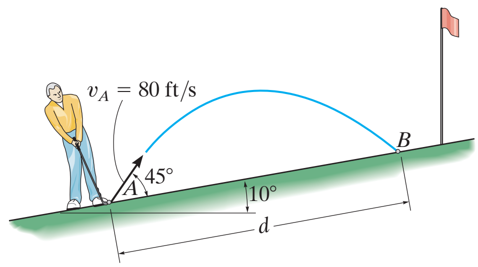
Substitute into Eq. (1),

$d = 166$ ft **Ans.**

Ans:
$d = 166$ ft

12–94.

A golf ball is struck with a velocity of 80 ft/s as shown. Determine the speed at which it strikes the ground at B and the time of flight from A to B.



SOLUTION

$(v_A)_x = 80 \cos 55° = 44.886$

$(v_A)_y = 80 \sin 55° = 65.532$

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$d \cos 10° = 0 + 45.886t$

$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$d \sin 10° = 0 + 65.532\,(t) + \frac{1}{2}(-32.2)\left(t^2\right)$

$d = 166$ ft

$t = 3.568 = 3.57$ s **Ans.**

$(v_B)_x = (v_A)_x = 45.886$

$(+\uparrow)\quad v = v_0 + a_c t$

$(v_B)_y = 65.532 - 32.2(3.568)$

$(v_B)_y = -49.357$

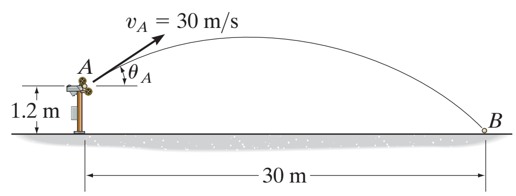
$v_B = \sqrt{(45.886)^2 + (-49.357)^2}$

$v_B = 67.4$ ft/s **Ans.**

Ans:
$t = 3.57$ s
$v_B = 67.4$ ft/s

12–95.

The pitching machine is adjusted so that the baseball is launched with a speed of $v_A = 30$ m/s. If the ball strikes the ground at B, determine the two possible angles θ_A at which it was launched.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

x-Motion: Here, $(v_A)_x = 30 \cos \theta_A$, $x_A = 0$, and $x_B = 30$ m. Thus,

$$(\xrightarrow{+}) \quad x_B = x_A + (v_A)_x t$$

$$30 = 0 + 30 \cos \theta_A t$$

$$t = \frac{1}{\cos \theta_A} \qquad \textbf{(1)}$$

y-Motion: Here, $(v_A)_y = 30 \sin \theta_A$, $a_y = -g = -9.81 \text{ m/s}^2$, and y_B Thus,

$$(+\uparrow) \quad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-1.2 = 0 + 30 \sin \theta_A t + \frac{1}{2}(-9.81) t^2$$

$$4.905 t^2 - 30 \sin \theta_A t - 1.2 = 0 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$4.905 \left(\frac{1}{\cos \theta_A} \right)^2 - 30 \sin \theta_A \left(\frac{1}{\cos \theta_A} \right) - 1.2 = 0$$

$$1.2 \cos^2 \theta_A + 30 \sin \theta_A \cos \theta_A - 4.905 = 0$$

Solving by trial and error,

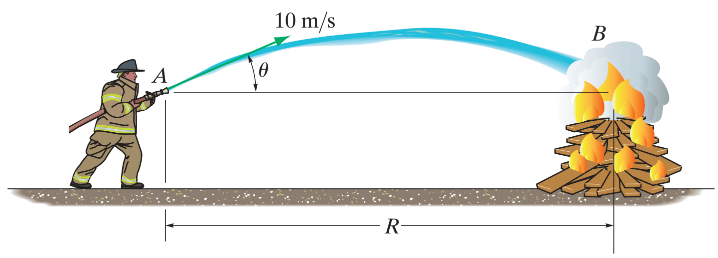
$$\theta_A = 7.19° \text{and } 80.5° \qquad \textbf{Ans.}$$

Ans:

$\theta_A = 7.19°$and $80.5°$

***12–96.**

The firefighter at A uses the hose which ejects a steady stream of water at 10 m/s. Calculate the maximum range $R = R_{max}$ to which the water particles can be "thrown" and the angle θ at which he must hold the nozzle.



SOLUTION

Vertical Motion:

$$(+\uparrow) \qquad s_y = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$0 = 0 + v_A \sin\theta\, t + \frac{1}{2}(-g)t^2$$

$$t = \frac{2v_A \sin\theta}{g}$$

Horizontal Motion:

$$(\xrightarrow{+}) \qquad s_x = v_x t$$

$$R = v_A \cos\theta\left(\frac{2v_A \sin\theta}{g}\right) = \frac{v_A^2 \sin 2\theta}{g}$$

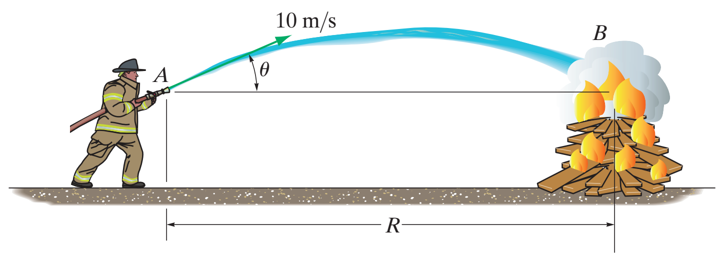
$$\frac{dR}{d\theta} = \frac{2v_A^2 \cos 2\theta}{g} = 0 \qquad \cos 2\theta = 0 \qquad \theta = 45^\circ \qquad \textbf{Ans.}$$

$$R_{max} = \frac{v_A^2 \sin 2(45^\circ)}{g} = \frac{v_A^2}{g} = \frac{10^2}{9.81} = 10.2\text{ m} \qquad \textbf{Ans.}$$

Ans:
$\theta = 45^\circ$
$R_{max} = 10.2$ m

12–97.

Show that the firefighter at A can "throw" water particles on B by aiming the nozzle at equal angles, measured up at $\theta_1 = 45° - 15° = 30°$ and $\theta_2 = 45° + 15° = 60°$. The water has a speed of 10 m/s at the nozzle.



SOLUTION

Vertical Motion:

$$(+\uparrow) \qquad s_y = (s_y)_0 + v_y t + \frac{1}{2}a_c t^2$$

$$0 = 0 + v_A \sin\theta t + \frac{1}{2}(-g)t^2$$

$$t = \frac{2v_A \sin\theta}{g}$$

Horizontal Motion:

$$(\overset{+}{\rightarrow}) \qquad s_x = v_x t$$

$$R = v_A \cos\left(\frac{2v_A \sin\theta}{g}\right) = \frac{v_A^2 \sin 2\theta}{g}$$

If $\theta = 45° + \theta_1$, then $R^{+} = \dfrac{v_A^2 \sin 2(45° + \theta_1)}{g} = \dfrac{v_A^2 \sin(90° + 2\theta_1)}{g}$

$$R^{+} = \frac{v_A^2[\sin 90° \cos 2\theta_1 + \cos 90° \sin 2\theta_1]}{g}$$

$$= \frac{v_A^2 \cos 2\theta_1}{g}$$

If $\theta = 45° - \theta_1$, then $R^{-} = \dfrac{v_A^2 \sin 2(45° - \theta_1)}{g} = \dfrac{v_A^2 \sin(90° - 2\theta_1)}{g}$

$$R^{-} = \frac{v_A^2[\sin 90° \cos 2\theta_1 - \cos 90° \sin 2\theta_1]}{g}$$

$$= \frac{v_A^2 \cos 2\theta_1}{g}$$

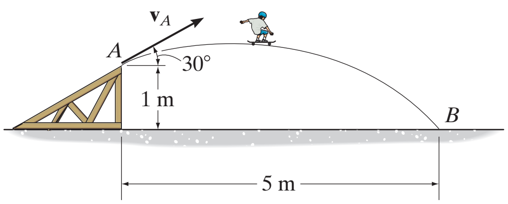
$$R^{+} = R^{-} \quad \text{(Q.E.D.)}$$

$$R = \frac{v_A^2 \cos 2\theta_1}{g} = \frac{10^2 \cos 2(15°)}{9.81} = 8.83 \text{ m}$$ **Ans.**

Ans:
$R = 8.83$ m

12–98.

The skateboard rider leaves the ramp at A with an initial velocity v_A at a 30° angle. If he strikes the ground at B determine v_A and the time of flight.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $(v_A)_x = v_A \cos 30°$, $x_A = 0$ and $x_B = 5$ m. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$5 = 0 + v_A \cos 30°\, t$$

$$t = \frac{5}{v_A \cos 30°} \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = v_A \sin 30°$, $a_y = -g = -9.81\text{ m/s}^2$, and $y_B = -1$ m. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y\, t + \frac{1}{2} a_y t^2$$

$$-1 = 0 + v_A \sin 30°\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - v_A \sin 30°\, t - 1 = 0 \qquad \textbf{(2)}$$

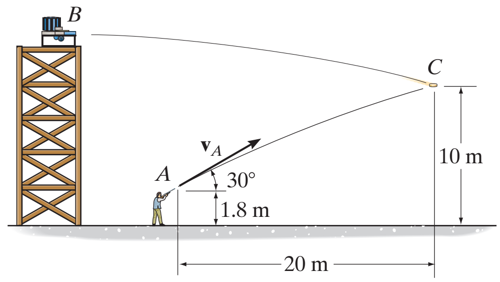
Solving Eqs. (1) and (2) yields

$$v_A = 6.49\text{ m/s} \qquad t = 0.890\text{ s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 6.49$ m/s
$t = 0.890$ s

12–99.

A projectile is fired from the platform at B. The shooter fires his gun from point A at an angle of 30°. Determine the muzzle speed of the bullet if it hits the projectile at C.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $x_A = 0$ and $x_C = 20$ m. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_C = x_A + (v_A)_x t$$

$$20 = 0 + v_A \cos 30^\circ\, t \qquad \textbf{(1)}$$

***y*-Motion:** Here, $y_A = 1.8$, $(v_A)_y = v_A \sin 30^\circ$, and $a_y = -g = -9.81 \text{ m/s}^2$. Thus,

$$(+\uparrow) \qquad y_C = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$10 = 1.8 + v_A \sin 30^\circ(t) + \frac{1}{2}(-9.81)(t)^2$$

Thus,

$$10 - 1.8 = \left(\frac{20 \sin 30^\circ}{\cos 30^\circ(t)}\right)(t) - 4.905(t)^2$$

$$t = 0.8261 \text{ s}$$

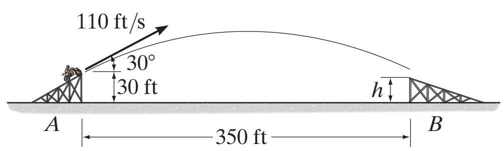
So that

$$v_A = \frac{20}{\cos 30^\circ(0.8261)} = 28.0 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 28.0$ m/s

***12–100.**

If the motorcycle leaves ramp A traveling at 110 ft/s, determine the height h ramp B must have so that the motorcycle lands safely.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with the take off point of the motorcycle at ramp A.

***x*-Motion:** Here, $x_A = 0$, $x_B = 350$ ft, and $(v_A)_x = 110 \cos 30° = 95.26$ ft/s. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$350 = 0 + 95.26t$$

$$t = 3.674 \text{ s}$$

***y*-Motion:** Here, $y_A = 0$, $y_B = h - 30$, $(v_A)_y = 110 \sin 30° = 55$ ft/s, and $a_y = -g = -32.2 \text{ ft/s}^2$. Thus, using the result of t, we have

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

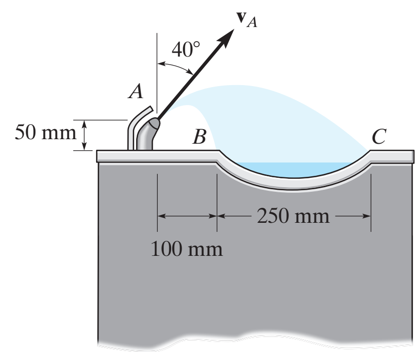
$$h - 30 = 0 + 55(3.674) + \frac{1}{2}(-32.2)\left(3.674^2\right)$$

$$h = 14.7 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$h = 14.7$ ft

12–101.

The drinking fountain is designed such that the nozzle is located a distance away from the edge of the basin as shown. Determine the maximum and minimum speed at which water can be ejected from the nozzle so that it does not splash over the sides of the basin at B and C.



SOLUTION

Horizontal Motion:

$$(\xrightarrow{+}) \qquad s = v_0 t$$

$$R = v_A \sin 40°\, t \qquad t = \frac{R}{v_A \sin 40°} \qquad \textbf{(1)}$$

Vertical Motion:

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-\,0.05 = 0 + v_A \cos 40° t + \frac{1}{2}(-9.81)t^2 \qquad \textbf{(2)}$$

Substituting Eq.(1) into (2) yields:

$$-0.05 = v_A \cos 40° \left(\frac{R}{v_A \sin 40°}\right) + \frac{1}{2}(-9.81)\left(\frac{R}{v_A \sin 40°}\right)^2$$

$$v_A = \sqrt{\frac{4.905R^2}{\sin 40° \,(R \cos 40° + 0.05 \sin 40°)}}$$

At point B, $R = 0.1$ m.

$$v_{\min} = v_A = \sqrt{\frac{4.905\,(0.1)^2}{\sin 40°\,(0.1 \cos 40° + 0.05 \sin 40°)}} = 0.838 \text{ m/s} \qquad \textbf{Ans.}$$

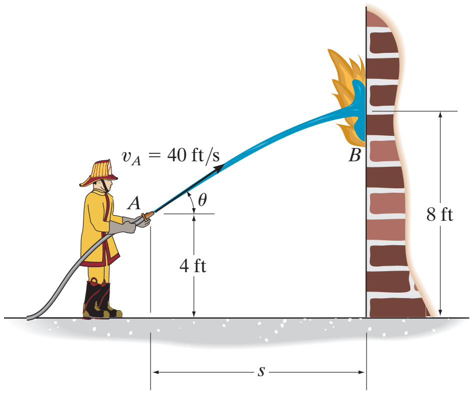
At point C, $R = 0.35$ m.

$$v_{\max} = v_A = \sqrt{\frac{4.905\,(0.35)^2}{\sin 40°\,(0.35 \cos 40° + 0.05 \sin 40°)}} = 1.76 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_{\min} = 0.838$ m/s
$v_{\max} = 1.76$ m/s

12–102.

The hose is held at an angle $\theta = 30°$ with the horizontal, and the water is discharged at A with a speed of $v_A = 40\text{ft/s}$. If the water stream strikes the building at B, determine the two possible distances s from the building.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

x-Motion: Here, $(v_A)_x = 40 \cos 30°\ \text{ft/s} = 34.64\ \text{ft/s}$, $x_A = 0$, and $x_B = s$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$s = 0 + 34.64t$$

$$s = 34.64t \qquad \textbf{(1)}$$

y-Motion: Here, $(v_A)_y = 40 \sin 30°\ \text{ft/s} = 20\ \text{ft/s}$, $a_y = -g = -32.2\ \text{ft/s}^2$, $y_A = 0$, and $y_B = 8 - 4 = 4$ ft. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$4 = 0 + 20t + \frac{1}{2}(-32.2)t^2$$

$$16.1t^2 - 20t + 4 = 0$$

$$t = 0.2505 \text{ s and } 0.9917 \text{ s}$$

Substituting these results into Eq. (1), the two possible distances are

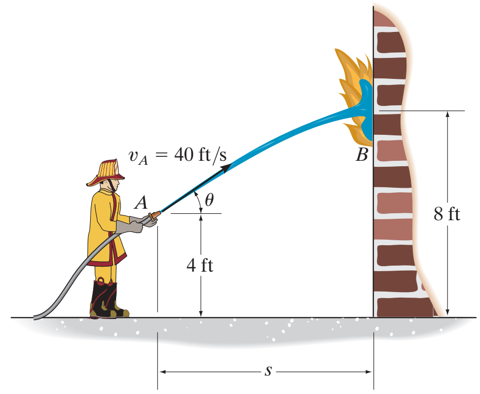
$$s = 34.64(0.2505) = 8.68 \text{ ft} \qquad \textbf{Ans.}$$

$$s = 34.64(0.9917) = 34.4 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$S = 8.68$ ft
$S = 34.4$ ft

12–103.

Water is discharged from the hose with a speed of 40 ft/s. Determine the two possible angles θ the firefighter can hold the hose so that the water strikes the building at B. Take $s = 20$ ft.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $(v_A)_x = 40\cos\theta$, $x_A = 0$, and $x_B = 20$ ft. Thus,

$$(\overset{+}{\rightarrow})\qquad x_B = x_A + (v_A)_x t$$

$$20 = 0 + 40\cos\theta t$$

$$t = \frac{1}{2\cos\theta} \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = 40\sin\theta$, $a_y = -g = -32.2\ \text{ft/s}^2$, $y_A = 0$, and $y_B = 8 - 4 = 4$ ft. Thus,

$$(+\uparrow)\qquad y_B = y_A + (v_A)_y t + \frac{1}{2}a_y t^2$$

$$4 = 0 + 40\sin\theta t + \frac{1}{2}(-32.2)t^2$$

$$16.1t^2 - 40\sin\theta t + 4 = 0 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$16.1\left(\frac{1}{2\cos\theta}\right)^2 - 40\sin\theta\left(\frac{1}{2\cos\theta}\right) + 4 = 0$$

$$20\sin\theta\cos\theta - 4\cos^2\theta = 4.025$$

$$10\sin\theta\cos\theta - 2\cos^2\theta = 2.0125$$

$$5\sin 2\theta - \left(2\cos^2\theta - 1\right) - 1 = 2.0125$$

$$5\sin 2\theta - \cos 2\theta = 3.0125$$

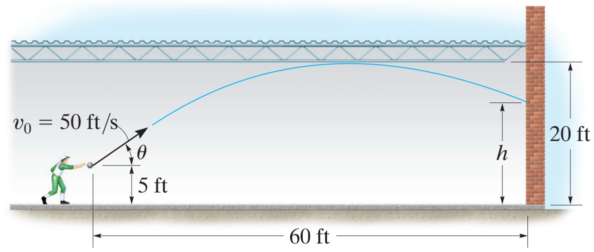
Solving by trial and error,

$$\theta = 23.8^\circ \text{ and } 77.5^\circ \qquad \textbf{Ans.}$$

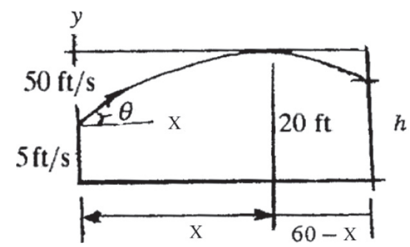
Ans:
$\theta = 23.8^\circ$ and 77.5°

***12–104.**

The man stands 60 ft from the wall and throws a ball at it with a speed of $v_0 = 50\text{ ft/s}$. Determine the angle θ at which he should release the ball so that it strikes the wall at the highest point possible. What is this height? The room has a ceiling height of 20 ft.



SOLUTION



$v_x = 50\cos\theta$

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t$

$x = 0 + 50\cos\theta\, t \qquad$ **(1)**

$(+\uparrow)\qquad v = v_0 + a_c t$

$v_y = 50\sin\theta - 32.2\,t \qquad$ **(2)**

$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$y = 0 + 50\sin\theta\, t - 16.1\,t^2 \qquad$ **(3)**

$(+\uparrow)\qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$v_y^2 = (50\sin\theta)^2 + 2(-32.2)(s - 0)$

$v_y^2 = 2500\sin^2\theta - 64.4\,s \qquad$ **(4)**

Require $v_y = 0$ at $s = 20 - 5 = 15$ ft

$0 = 2500\sin^2\theta - 64.4\,(15)$

$\theta = 38.433° = 38.4° \qquad$ **Ans.**

From Eq. (2)

$0 = 50\sin 38.433° - 32.2\,t$

$t = 0.9652$ s

From Eq. (1)

$x = 50\cos 38.433°(0.9652) = 37.8$ ft

Time for ball to hit wall

From Eq. (1),

$60 = 50(\cos 38.433°)t$

$t = 1.53193$ s

From Eq. (3)

$y = 50\sin 38.433°(1.53193) - 16.1(1.53193)^2$

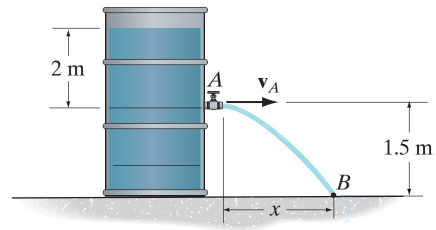
$y = 9.830$ ft

$h = 9.830 + 5 = 14.8$ ft $\qquad$ **Ans.**

Ans:
$\theta = 38.4°$
$h = 14.8$ ft

12–105.

The velocity of the water jet discharging from an orifice can be obtained from $v = \sqrt{2gh}$, where $h = 2$ m is the depth of the orifice from the free water surface. Determine the time for a particle of water leaving the orifice to reach a point B and the horizontal distance x where it hits the surface.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A. The speed of the water that the jet discharges from A is

$$v_A = \sqrt{2(9.81)(2)} = 6.264 \text{ m/s}$$

***x*-Motion:** Here, $(v_A)_x = v_A = 6.264$ m/s, $x_A = 0$, $x_B = x$, and $t = t_A$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$x = 0 + 6.264 t_A \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = 0$, $a_y = -g = -9.81 \text{ m/s}^2$, $y_A = 0$ m, $y_B = -1.5$ m, and $t = t_A$. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-1.5 = 0 + 0 + \frac{1}{2}(-9.81) t_A{}^2$$

$$t_A = 0.553 \text{ s} \qquad \textbf{Ans.}$$

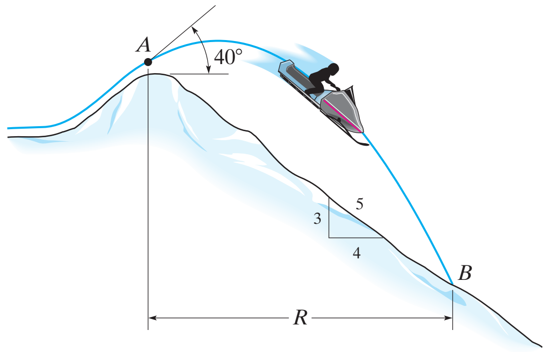
Thus,

$$x = 0 + 6.264(0.553) = 3.46 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$t_A = 0.553$ s
$x = 3.46$ m

12–106.

The snowmobile is traveling at 10 m/s when it leaves the embankment at A. Determine the time of flight from A to B and the range R of the trajectory.



SOLUTION

$(\xrightarrow{+})$ $\qquad s_B = s_A + v_A t$

$$R = 0 + 10 \cos 40° \, t$$

$(+\uparrow)$ $\qquad s_B = s_A + v_A t + \frac{1}{2} a_c t^2$

$$-R\left(\frac{3}{4}\right) = 0 + 10 \sin 40° t - \frac{1}{2}(9.81)\, t^2$$

Solving:

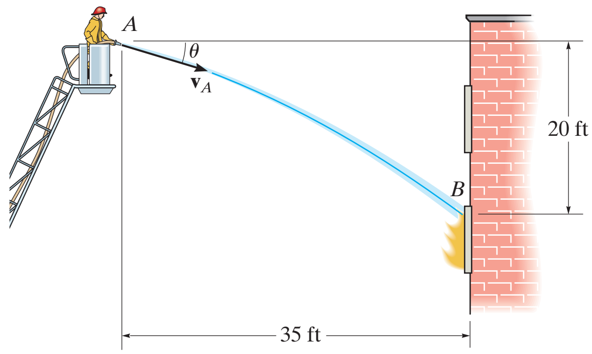
$R = 19.0 \text{ m}$ **Ans.**

$t = 2.48 \text{ s}$ **Ans.**

Ans:
$R = 19.0 \text{ m}$
$t = 2.48 \text{ s}$

12–107.

The firefighter wishes to direct the flow of water from his hose to the fire at B. Determine two possible angles θ_1 and θ_2 at which this can be done. Water flows from the hose at $v_A = 80$ ft/s.



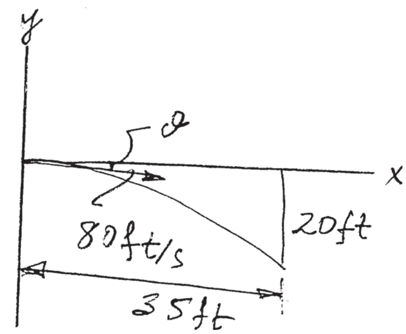
SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$35 = 0 + (80)(\cos\theta)t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-20 = 0 - 80(\sin\theta)t + \frac{1}{2}(-32.2)t^2$$



Thus,

$$20 = 80 \sin\theta \frac{0.4375}{\cos\theta} t + 16.1\left(\frac{0.1914}{\cos^2\theta}\right)$$

$$20\cos^2\theta = 17.5 \sin 2\theta + 3.0816$$

Solving,

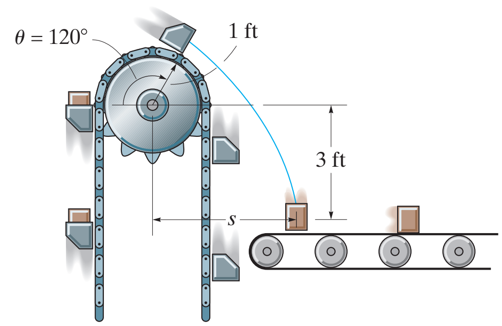
$\theta_1 = 24.9°$ (below the horizontal) **Ans.**

$\theta_2 = 85.2°$ (above the horizontal) **Ans.**

Ans:
$\theta_1 = 24.9°$ ↘
$\theta_2 = 85.2°$ ↗

***12–108.**

The buckets on the conveyor travel with a speed of 15 ft/s. Each bucket contains a block which falls out of the bucket when $\theta = 120°$. Determine the distance s to where the block strikes the conveyor. Neglect the size of the block.



SOLUTION

Vertical Motion:

$$(+\downarrow) \qquad s_y = (s_0)_y + v_y t + \frac{1}{2}a_c t^2$$

$$3 + 1\cos 30° = 0 + 15\sin 30°\, t + \frac{1}{2}(32.2)t^2$$

Take the positive root $t = 0.3096$ s

Horizontal Motion:

$$(\xrightarrow{+}) \qquad s_x = (s_0)_x + v_x t$$

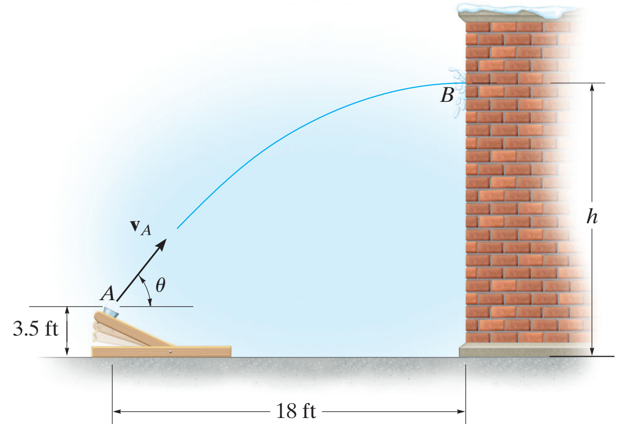
$$s - 1\sin 30° = 0 + 15\cos 30°(0.3096)$$

$$s = 4.52 \text{ ft}$$ **Ans.**

Ans:
$s = 4.52$ ft

12–109.

The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes 1.5 s to travel from A to B, determine the velocity $\mathbf{v}_A$ at which it was launched, the angle of release θ, and the height h.



SOLUTION

$(\xrightarrow{+})$ $\quad s = v_0 t$

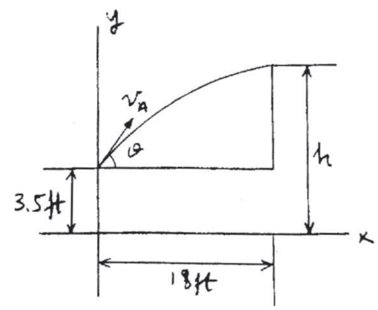
$$18 = v_A \cos\theta(1.5) \qquad \textbf{(1)}$$

$(+\uparrow)$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = (v_A \sin\theta)^2 + 2(-32.2)(h - 3.5)$$

$(+\uparrow)$ $\quad v = v_0 + a_c t$

$$0 = v_A \sin\theta - 32.2(1.5) \qquad \textbf{(2)}$$



To solve, first divide Eq. (2) by Eq. (1), to get θ. Then

$$\theta = 76.0° \qquad \textbf{Ans.}$$

$$v_A = 49.8 \text{ ft/s} \qquad \textbf{Ans.}$$

$$h = 39.7 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$\theta = 76.0°$
$v_A = 49.8 \text{ ft/s}$
$h = 39.7 \text{ ft}$

12–110.

A train travels along a circular track having a radius of 2000 ft. If the speed of the train is uniformly increased from 6 mi/h to 8 mi/h in 1 min, determine the magnitude of the acceleration when the speed of the train is 7 mi/h. 1 mi = 5280 ft.

SOLUTION

$$\frac{6\text{ mi}}{\text{hr}} \times \frac{5280\text{ ft}}{1\text{ mi}} \times \frac{1\text{ hr}}{3600\text{s}} = 8.8\text{ ft/s}$$

$$\frac{7\text{ mi}}{\text{hr}} \times \frac{5280\text{ ft}}{1\text{ mi}} \times \frac{1\text{ hr}}{3600\text{s}} = 10.27\text{ ft/s}$$

$$\frac{8\text{ mi}}{\text{hr}} \times \frac{5280\text{ ft}}{1\text{ mi}} \times \frac{1\text{ hr}}{3600\text{s}} = 11.73\text{ ft/s}$$

$$a_t = \frac{11.73 - 8.8}{60} = 0.0489\text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{10.27^2}{2000} = 0.0527\text{ ft/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.0489^2 + 0.0527^2} = 0.0719\text{ ft/s}^2$$ **Ans.**

Ans:
$a = 0.0719\text{ ft/s}^2$

12–111.

A boat has an initial speed of 16 ft/s. If it then increases its speed along a circular path of radius $\rho = 80$ ft at the rate of $\dot{v} = (1.5s)$ ft/s^2, where s is in feet, determine the normal and tangential components of the boat's acceleration at $s = 16$ ft.

SOLUTION

$$a_t = 1.5\,s$$

$$\int_0^s 1.5\,s\,ds = \int_{16}^{v} v\,dv$$

$$0.75\,s^2 = 0.5\,v^2 - 128$$

At $s = 16$ ft,

$$v = 25.30 \text{ ft/s}$$

$$a_n = \frac{25.30^2}{80} = 8.00 \text{ ft/s}^2$$ **Ans.**

$$a_\tau = 1.5(16) = 24 \text{ ft/s}^2$$ **Ans.**

Ans:
$a_n = 8.00$ ft/s^2
$a_\tau = 24$ ft/s^2

***12–112.**

A particle moves along the curve $y = 1 + \cos x$ with a constant speed $v = 4$ ft/s. Determine the normal and tangential components of its velocity and acceleration at any instant.

SOLUTION

$$y = 1 + \cos x$$

$$\frac{dy}{dx} = -\sin x$$

$$\frac{d^2y}{dx^2} = -\cos x$$

$$v_t = 4 \text{ ft/s}$$ **Ans.**

$$v_n = 0$$ **Ans.**

$$a_\tau = \frac{dv}{dt} = 0$$ **Ans.**

$$a_n = \frac{v^2}{\rho} = \frac{4^2}{\rho}$$

$$\rho = \left| \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \right| = \left| \frac{(1 + \sin^2 x)^{3/2}}{-\cos x} \right|$$

$$a_n = \frac{16 \cos x}{(1 + \sin^2 x)^{3/2}}$$ **Ans.**

Ans:
$v_t = 4$ ft/s
$v_n = 0$
$a_\tau = \frac{dv}{dt} = 0$
$a_n = \frac{16 \cos x}{(1 + \sin^2 x)^{3/2}}$

12–113.

The position of a particle is defined by $\mathbf{r} = \{4(t - \sin t)\mathbf{i} + (2t^2 - 3)\mathbf{j}\}$ m, where t is in seconds and the argument for the sine is in radians. Determine the speed of the particle and its normal and tangential components of acceleration when $t = 1$ s.

SOLUTION

$\mathbf{r} = 4(t - \sin t)\,\mathbf{i} + (2\,t^2 - 3)\mathbf{j}$

$\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = 4(1 - \cos t)\mathbf{i} + (4\,t)\mathbf{j}$

$\mathbf{v}|_{t=1} = 1.83879\mathbf{i} + 4\mathbf{j}$

$v = \sqrt{(1.83879)^2 + (4)^2} = 4.40 \text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{4}{1.83879}\right) = 65.312°$

$\mathbf{a} = 4 \sin t\mathbf{i} + 4\mathbf{j}$

$\mathbf{a}|_{t=1} = 3.3659\mathbf{i} + 4\mathbf{j}$

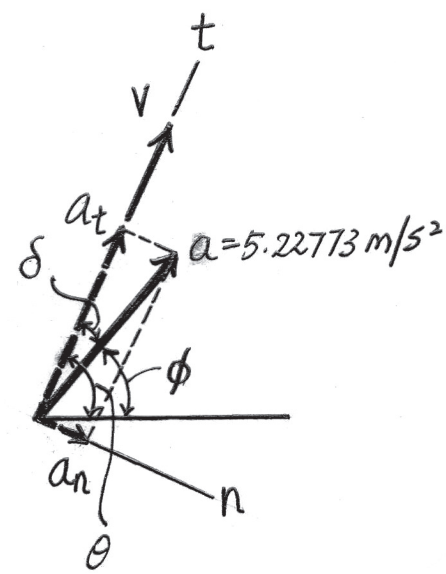
$a = \sqrt{(3.3659)^2 + (4)^2} = 5.22773 \text{ m/s}^2$

$\phi = \tan^{-1}\left(\dfrac{4}{3.3659}\right) = 49.920°$

$\delta = \theta - \phi = 15.392°$

$a_t = 5.22773 \cos 15.392° = 5.04 \text{ m/s}^2$ **Ans.**

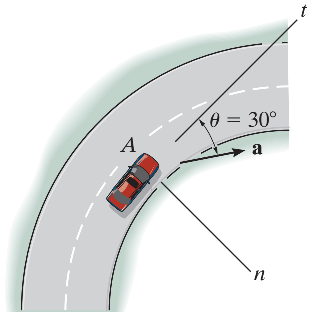
$a_n = 5.22773 \sin 15.392° = 1.39 \text{ m/s}^2$ **Ans.**



Ans:
$v = 4.40$ m/s
$a_t = 5.04$ m/s²
$a_n = 1.39$ m/s²

12–114.

The automobile has a speed of 80 ft/s at point A and an acceleration **a** having a magnitude of 10 ft/s^2, acting in the direction shown. Determine the radius of curvature of the path at point A and the tangential component of acceleration.



SOLUTION

Acceleration: The tangential acceleration is

$$a_t = a\cos 30° = 10\cos 30° = 8.66\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

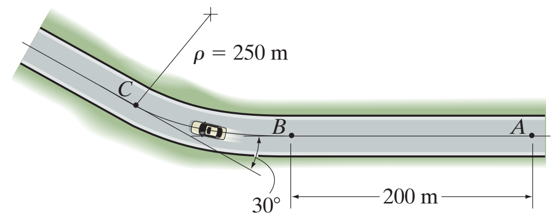
and the normal acceleration is $a_n = a\sin 30° = 10\sin 30° = 5.00\ \text{ft/s}^2$. Applying Eq. 12–20, $a_n = \dfrac{v^2}{\rho}$, we have

$$\rho = \frac{v^2}{a_n} = \frac{80^2}{5.00} = 1280\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$a_t = 8.66\ \text{ft/s}^2$
$\rho = 1280\ \text{ft}$

12–115.

When the car reaches point A it has a speed of 25 m/s. If the brakes are applied, its speed is reduced by $a_t = (-\frac{1}{4}t^{1/2})$ m/s^2. Determine the magnitude of acceleration of the car just before it reaches point C.



SOLUTION

Velocity: Using the initial condition $v = 25$ m/s at $t = 0$ s,

$$dv = a_t\,dt$$

$$\int_{25\text{ m/s}}^{v} dv = \int_0^t -\frac{1}{4}t^{1/2}\,dt$$

$$v = \left(25 - \frac{1}{6}t^{3/2}\right)\text{ m/s} \qquad \textbf{(1)}$$

Position: Using the initial conditions $s = 0$ when $t = 0$ s,

$$\int ds = \int v\,dt$$

$$\int_0^s ds = \int_0^t \left(25 - \frac{1}{6}t^{3/2}\right)dt$$

$$s = \left(25t - \frac{1}{15}t^{5/2}\right)\text{ m}$$

Acceleration: When the car reaches C, $s_C = 200 + 250\left(\frac{\pi}{6}\right) = 330.90$ m. Thus,

$$330.90 = 25t - \frac{1}{15}t^{5/2}$$

Solving by trial and error,

$$t = 15.942\text{ s}$$

Thus, using Eq. (1).

$$v_C = 25 - \frac{1}{6}(15.942)^{3/2} = 14.391\text{ m/s}$$

$$(a_t)_C = \dot{v} = -\frac{1}{4}\left(15.942^{1/2}\right) = -0.9982\text{ m/s}^2$$

$$(a_n)_C = \frac{v_C{}^2}{\rho} = \frac{14.391^2}{250} = 0.8284\text{ m/s}^2$$

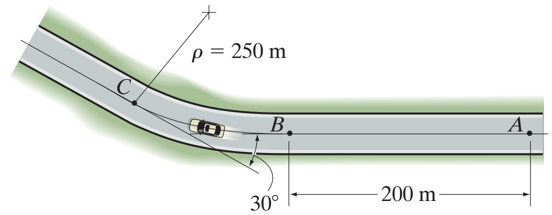
The magnitude of the car's acceleration at C is

$$a = \sqrt{(a_t)_C{}^2 + (a_n)_C{}^2} = \sqrt{(-0.9982)^2 + 0.8284^2} = 1.30\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 1.30\text{ m/s}^2$

***12–116.**

When the car reaches point A, it has a speed of 25 m/s. If the brakes are applied, its speed is reduced by $a_t = (0.001s - 1)\ \text{m/s}^2$. Determine the magnitude of acceleration of the car just before it reaches point C.



SOLUTION

Velocity: Using the initial condition $v = 25$ m/s at $t = 0$ s,

$$dv = a\,ds$$

$$\int_{25\ \text{m/s}}^{v} v\,dv = \int_0^s (0.001s - 1)\,ds$$

$$v = \sqrt{0.001s^2 - 2s + 625}$$

Acceleration: When the car is at point C, $s_C = 200 + 250\left(\frac{\pi}{6}\right) = 330.90$ m. Thus, the speed of the car at C is

$$v_C = \sqrt{0.001\left(330.90^2\right) - 2(330.90) + 625} = 8.526\ \text{m/s}^2$$

$$(a_t)_C = \dot{v} = [0.001(330.90) - 1] = -0.6691\ \text{m/s}^2$$

$$(a_n)_C = \frac{v_C{}^2}{\rho} = \frac{8.526^2}{250} = 0.2908\ \text{m/s}^2$$

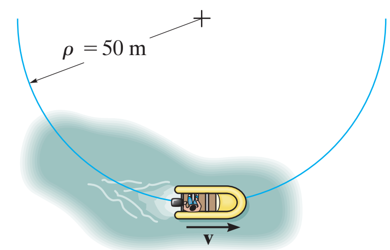
The magnitude of the car's acceleration at C is

$$a = \sqrt{(a_t)_C{}^2 + (a_n)_C{}^2} = \sqrt{(-0.6691)^2 + 0.2908^2} = 0.730\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.730\ \text{m/s}^2$

12–117.

Starting from rest, the motorboat travels around the circular path, $\rho = 50$ m, at a speed $v = (0.8t)$ m/s, where t is in seconds. Determine the magnitudes of the boat's velocity and acceleration when it has traveled 20 m.



SOLUTION

Velocity: The time for which the boat to travel 20 m must be determined first.

$$ds = vdt$$

$$\int_0^{20\text{m}} ds = \int_0^t 0.8t\,dt$$

$$t = 7.071 \text{ s}$$

The magnitude of the boat's velocity is

$$v = 0.8\,(7.071) = 5.657 \text{ m/s} = 5.66 \text{ m/s}$$ **Ans.**

Acceleration: The tangential accelerations is

$$a_t = \dot{v} = 0.8 \text{ m/s}^2$$

To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{5.657^2}{50} = 0.640 \text{ m/s}^2$$

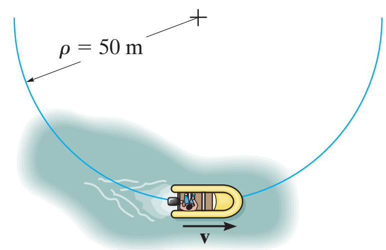
Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.8^2 + 0.640^2} = 1.02 \text{ m/s}^2$$ **Ans.**

Ans:
$v = 5.66$ m/s
$a = 1.02$ m/s^2

12–118.

Starting from rest, the motorboat travels around the circular path, $\rho = 50$ m, at a speed $v = (0.2t^2)$ m/s, where t is in seconds. Determine the magnitudes of the boat's velocity and acceleration at the instant $t = 3$ s.



SOLUTION

Velocity: When $t = 3$ s, the boat travels at a speed of

$$v = 0.2\left(3^2\right) = 1.80 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The tangential acceleration is $a_t = \dot{v} = (0.4t)$ m/s^2. When $t = 3$ s,

$$a_t = 0.4\,(3) = 1.20 \text{ m/s}^2$$

To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{1.80^2}{50} = 0.0648 \text{ m/s}^2$$

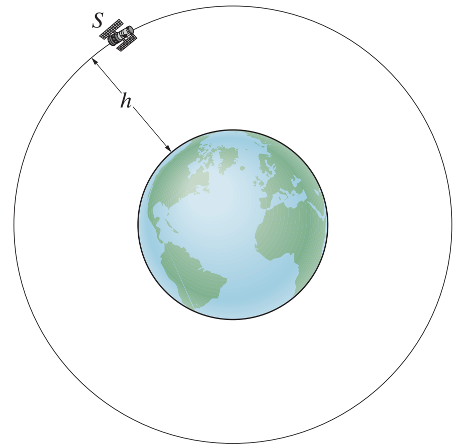
Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{1.20^2 + 0.0648^2} = 1.20 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 1.80$ m/s
$a = 1.20$ m/s^2

12–119.

The satellite S travels around the earth in a circular path with a constant speed of 20 Mm/h. If the acceleration is 2.5 m/s^2, determine the altitude h. Assume the earth's diameter to be 12 713 km.



SOLUTION

$$v = 20\ \text{Mm/h} = \frac{20(10^6)}{3600} = 5.56(10^3)\ \text{m/s}$$

Since $a_t = \dfrac{dv}{dt} = 0$, then,

$$a = a_n = 2.5 = \frac{v^2}{\rho}$$

$$\rho = \frac{(5.56(10^3))^2}{2.5} = 12.35(10^6)\ \text{m}$$

The radius of the earth is

$$\frac{12\ 713(10^3)}{2} = 6.36(10^6)\ \text{m}$$

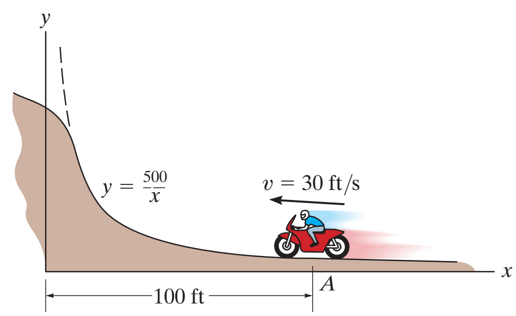
Hence,

$$h = 12.35(10^6) - 6.36(10^6) = 5.99(10^6)\ \text{m} = 5.99\ \text{Mm}$$ **Ans.**

Ans:
$h = 5.99\ \text{Mm}$

***12–120.**

The motorcyclist travels along the curve at a constant speed of 30 ft/s. Determine his acceleration when he is located at point A. Neglect the size of the motorcycle and rider for the calculation.



SOLUTION

$$\frac{dy}{dx} = -\frac{500}{x^2}\bigg|_{x=100\text{ ft}} = -0.05$$

$$\frac{d^2y}{dx^2} = \frac{1000}{x^3}\bigg|_{x=100\text{ ft}} = 0.001$$

$$\rho\bigg|_{x=100\text{ ft}} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x=100\text{ ft}}$$

$$= \frac{[1 + (-0.05)^2]^{3/2}}{|0.001|} = 1003.8\text{ ft}$$

$$a_n = \frac{v^2}{\rho} = \frac{30^2}{1003.8} = 0.897\text{ ft/s}^2$$

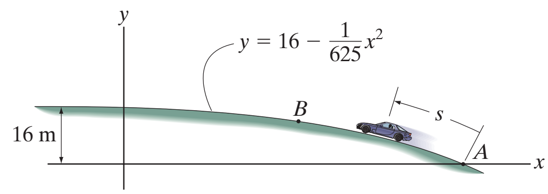
Since the motorcyclist travels with a constant speed, $a_t = 0$. Hence

$$a = a_n = 0.897\text{ ft/s}^2$$ **Ans.**

Ans:
$a = 0.897\text{ ft/s}^2$

12–121.

The car passes point A with a speed of 25 m/s after which its speed is defined by $v = (25 - 0.15s)$ m/s, where s is in meters. Determine the magnitude of the car's acceleration when it reaches point B, where $s = 51.5$ m and $x = 50$ m.



SOLUTION

Velocity: The speed of the car at B is

$$v_B = \left[25 - 0.15(51.5)\right] = 17.28 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=50\text{ m}} = 324.58 \text{ m}$$

Acceleration:

$$a_n = \frac{{v_B}^2}{\rho} = \frac{17.28^2}{324.58} = 0.9194 \text{ m/s}^2$$

$$a_t = v\frac{dv}{ds} = \left(25 - 0.15s\right)\left(-0.15\right) = \left(0.0225s - 3.75\right) \text{ m/s}^2$$

When the car is at B $\left(s = 51.5 \text{ m}\right)$

$$a_t = \left[0.0225\left(51.5\right) - 3.75\right] = -2.591 \text{ m/s}^2$$

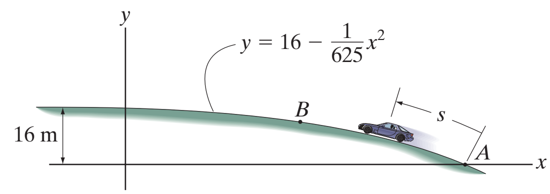
Thus, the magnitude of the car's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.591)^2 + 0.9194^2} = 2.75 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 2.75 \text{ m/s}^2$

12–122.

If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5\ \text{m/s}^2$, determine the magnitude of the car's acceleration when $s = 101.68$ m and $x = 0$.



SOLUTION

Velocity: The speed of the car at C is

$$v_C{}^2 = v_A{}^2 + 2a_t(s_C - s_A)$$

$$v_C{}^2 = 20^2 + 2(0.5)(101.68 - 0)$$

$$v_C = 22.398\ \text{m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=0} = 312.5\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = 0.5\ \text{m/s}$$

$$a_n = \frac{v_C{}^2}{\rho} = \frac{22.361^2}{312.5} = 1.60\ \text{m/s}^2$$

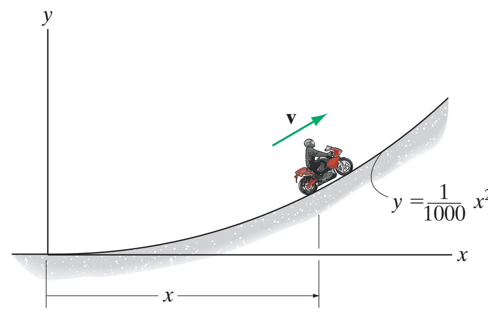
The magnitude of the car's acceleration at C is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 1.60^2} = 1.68\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 1.68\ \text{m/s}^2$

12–123.

The motorcycle travels up the hill at a constant speed of 15 m/s. Determine the magnitude of its acceleration as a function of x.



SOLUTION

Geometry: The radius of curvature of the road as a function of x must be determined first. Here

$$\frac{dy}{dx} = \frac{1}{1000}(2x) = 0.002x$$

$$\frac{d^2y}{dx^2} = 0.002$$

Then

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (0.002x)^2\right]^{3/2}}{0.002} = 500\left[1 + 40(10^{-6})\,x^2\right]^{3/2}.$$

Acceleration: since the motorcycle travels with a constant speed, the tangential component of velocity is

$$a_t = 0$$

The normal component of the acceleration is given by

$$a_n = \frac{v^2}{\rho} = \frac{15^2}{500\left[1 + 4(10^{-6})x^2\right]^{3/2}}$$

$$= \frac{9}{20\left[1 + 4(10^{-6})x^2\right]^{3/2}}$$

Then the magnitude of the acceleration is

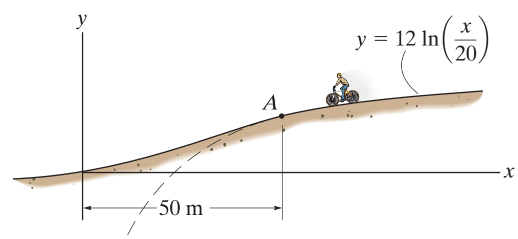
$$a = \sqrt{a_t^2 + a_n^2} = a_n = \frac{9}{20\left[1 + 4(10^{-6})x^2\right]^{3/2}} \qquad \textbf{Ans.}$$

Ans:

$$a = \frac{9}{20\left[1 + 4(10^{-6})x^2\right]^{3/2}}$$

***12–124.**

When the bicycle passes point A, it has a speed of 6 m/s, which is increasing at the rate of $\dot{v} = (0.5)\ \text{m/s}^2$. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = 12\ln\left(\frac{x}{20}\right)$$

$$\frac{dy}{dx} = 12\left(\frac{1}{x/20}\right)\left(\frac{1}{20}\right) = \frac{12}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{12}{x^2}$$

$$\rho = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\dfrac{12}{x}\right)^2\right]^{3/2}}{\left|-\dfrac{12}{x^2}\right|}\right|_{x=50\text{ m}} = 226.59\text{ m}$$

Acceleration:

$$a_t = \dot{v} = 0.5\text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{6^2}{226.59} = 0.1589\text{ m/s}^2$$

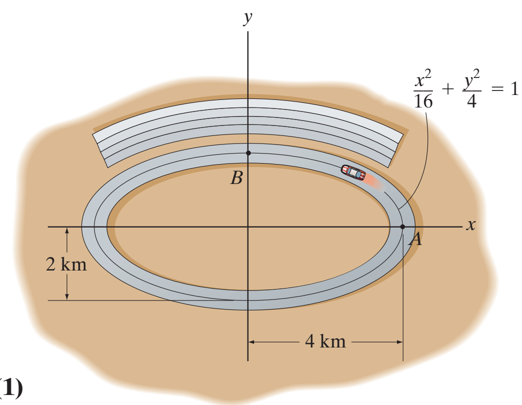
The magnitude of the bicycle's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 0.1589^2} = 0.525\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.525\text{ m/s}^2$

12–125.

A racing car travels with a constant speed of 240 km/h around the elliptical race track. Determine the acceleration experienced by the driver at A.



SOLUTION

Radius of Curvature:

$$y^2 = 4\left(1 - \frac{x^2}{16}\right)$$

$$2y\frac{dy}{dx} = -\frac{x}{2} \qquad \textbf{(1)}$$

$$\frac{dy}{dx} = -\frac{x}{4y} \qquad \textbf{(2)}$$

Differentiating Eq. (1),

$$2\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) + 2y\frac{d^2y}{dx^2} = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{1}{2} - 2\left(\frac{dy}{dx}\right)^2}{2y}$$

$$\frac{d^2y}{dx^2} = -\left[\frac{1 + 4\left(\frac{dy}{dx}\right)^2}{4y}\right] \qquad \textbf{(3)}$$

Substituting Eq. (2) into Eq. (3) yields

$$\frac{d^2y}{dx^2} = -\left[\frac{4y^2 + x^2}{16y^3}\right]$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-\frac{x}{4y}\right)^2\right]^{3/2}}{\left|\frac{4y^2 + x^2}{16y^3}\right|} = \frac{\left(1 + \frac{x^2}{16y^2}\right)^{3/2}}{\left(\frac{4y^2 + x^2}{16y^3}\right)} = \frac{\left(16y^2 + x^2\right)^{3/2}}{4\left(4y^2 + x^2\right)}$$

Acceleration: Since the race car travels with a constant speed along the track, $a_t = 0$. At $x = 4$ km and $y = 0$,

$$\rho_A = \left.\frac{\left(16y^2 + x^2\right)^{3/2}}{4\left(4y^2 + x^2\right)}\right|_{\substack{x=4\text{ km}\\ y=0}} = \frac{\left(0 + 4^2\right)^{3/2}}{4\left(0 + 4^2\right)} = 1\text{ km} = 1000\text{ m}$$

The speed of the race car is

$$v = \left(240\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{h}}{3600\text{ s}}\right) = 66.67\text{ m/s}$$

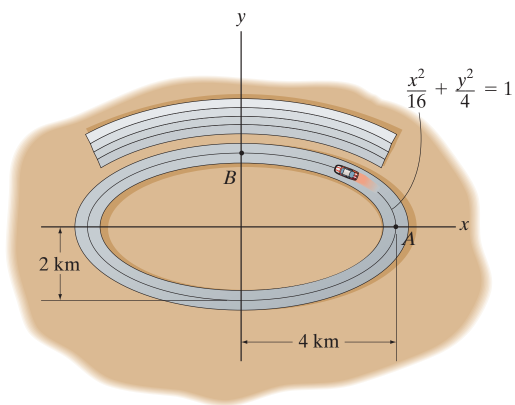
Thus,

$$a_A = \frac{v^2}{\rho_A} = \frac{66.67^2}{1000} = 4.44\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_A = 4.44\text{ m/s}^2$

12–126.

The racing car travels with a constant speed of 240 km/h around the elliptical race track. Determine the acceleration experienced by the driver at B.



SOLUTION

Radius of Curvature:

$$y^2 = 4\left(1 - \frac{x^2}{16}\right)$$

$$2y\frac{dy}{dx} = -\frac{x}{2} \qquad \textbf{(1)}$$

$$\frac{dy}{dx} = -\frac{x}{4y} \qquad \textbf{(2)}$$

Differentiating Eq. (1),

$$2\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) + 2y\frac{d^2y}{dx^2} = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{1}{2} - 2\left(\frac{dy}{dx}\right)^2}{2y}$$

$$\frac{d^2y}{dx^2} = -\left[\frac{1 + 4\left(\frac{dy}{dx}\right)^2}{4y}\right] \qquad \textbf{(3)}$$

Substituting Eq. (2) into Eq. (3) yields

$$\frac{d^2y}{dx^2} = -\left[\frac{4y^2 + x^2}{16\,y^3}\right]$$

Thus,

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+\left(-\frac{x}{4y}\right)^2\right]^{3/2}}{\left|\frac{4y^2 + x^2}{16y^3}\right|} = \frac{\left(1 + \frac{x^2}{16y^2}\right)^{3/2}}{\left(\frac{4y^2 + x^2}{16y^3}\right)} = \frac{\left(16y^2 + x^2\right)^{3/2}}{4\left(4y^2 + x^2\right)}$$

Acceleration: Since the race car travels with a constant speed along the track, $a_t = 0$. At $x = 0$ and $y = 2$ km

$$\rho_B = \left.\frac{\left(16y^2 + x^2\right)^{3/2}}{4\left(4y^2 + x^2\right)}\right|_{\substack{x=0 \\ y=2\text{ km}}} = \frac{\left[16\left(2^2\right) + 0\right]^{3/2}}{4\left[4\left(2^2\right) + 0\right]} = 8\text{ km} = 8000\text{ m}$$

The speed of the car is

$$v = \left(240\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 66.67\text{ m/s}$$

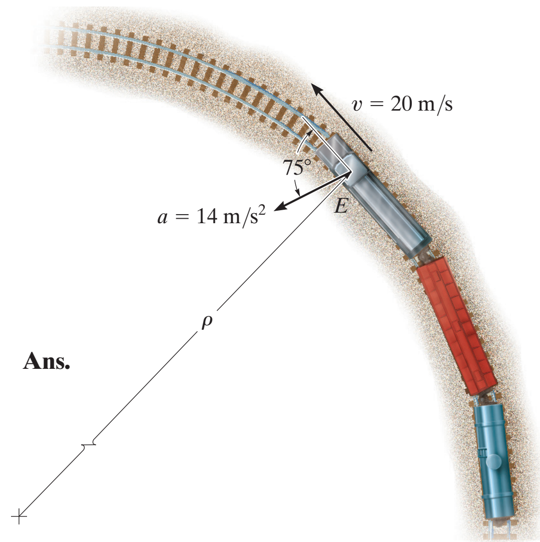
Thus,

$$a_B = \frac{v^2}{\rho_B} = \frac{66.67^2}{8000} = 0.556\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_B = 0.556\text{ m/s}^2$

12–127.

At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



SOLUTION

$a_t = 14 \cos 75° = 3.62 \text{ m/s}^2$ **Ans.**

$a_n = 14 \sin 75°$

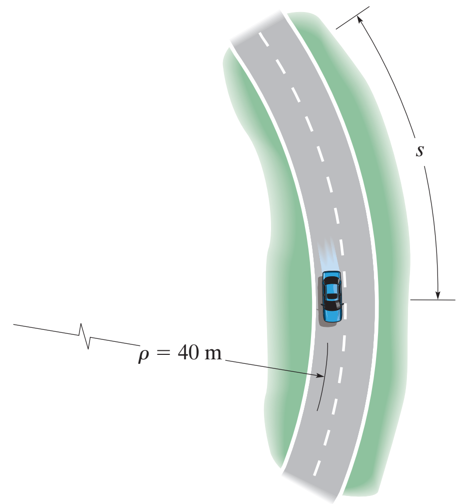
$$a_n = \frac{(20)^2}{\rho}$$

$\rho = 29.6$ m **Ans.**

Ans:
$a_t = 3.62 \text{ m/s}^2$
$\rho = 29.6$ m

***12–128.**

The car has an initial speed $v_0 = 20$ m/s at $s = 0$. If it increases its speed along the circular track $a_t = (0.8s)$ m/s^2, where s is in meters, determine the time needed for the car to travel $s = 25$ m.



SOLUTION

The distance traveled by the car along the circular track can be determined by integrating $v\,dv = a_t\,ds$. Using the initial condition $v = 20$ m/s at $s = 0$,

$$\int_{20\text{ m/s}}^{v} v\,dv = \int_0^{5} 0.8\,s\,ds$$

$$\left.\frac{v^2}{2}\right|_{20\text{ m/s}}^{v} = 0.4\,s^2$$

$$v = \left\{\sqrt{0.8\,(s^2 + 500)}\right\}\text{ m/s}$$

The time can be determined by integrating $dt = \dfrac{ds}{v}$ with the initial condition $s = 0$ at $t = 0$.

$$\int_0^{t} dt = \int_0^{25\text{ m}} \frac{ds}{\sqrt{0.8(s^2 + 500)}}$$

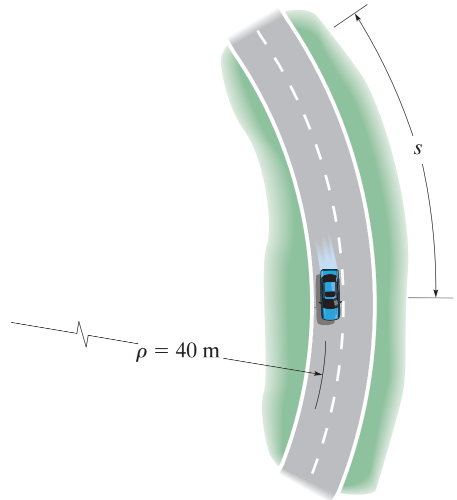
$$t = \frac{1}{\sqrt{0.8}}\left[\ln\left(s + \sqrt{s^2 + 500}\right)\right]\Big|_0^{25\text{ m}}$$

$$= 1.076\text{ s} = 1.08\text{ s}$$ **Ans.**

Ans:
$t = 1.08$ s

12–129.

The car starts from rest at $s = 0$ and increases its speed at $a_t = 4\ \text{m/s}^2$. Determine the time when the magnitude of acceleration becomes $20\ \text{m/s}^2$. At what position s does this occur?



SOLUTION

Acceleration: The normal component of the acceleration can be determined from

$$a_n = \frac{v^2}{\rho}; \qquad a_n = \frac{v^2}{40}$$

From the magnitude of the acceleration

$$a = \sqrt{a_t^2 + a_n^2}; \quad 20 = \sqrt{4^2 + \left(\frac{v^2}{40}\right)^2} \qquad v = 28.00\ \text{m/s}$$

Velocity: Since the car has a constant tangential accelaration of $a_t = 4\ \text{m/s}^2$,

$$v = v_0 + a_t t; \quad 28.00 = 0 + 4t$$

$t = 6.999\ \text{s} = 7.00\ \text{s}$ **Ans.**

$$v^2 = v_0^2 + 2a_t(s - s_0); \; 28.00^2 = 0^2 + 2(4)(s - 0)$$

$s = 97.98\ \text{m} = 98.0\ \text{m}$ **Ans.**

Ans:
$t = 7.00\ \text{s}$
$s = 98.0\ \text{m}$

12–130.

A boat is traveling along a circular curve having a radius of 100 ft. If its speed at $t = 0$ is 15 ft/s and is increasing at $\dot{v} = (0.8t)$ ft/s^2, determine the magnitude of its acceleration at the instant $t = 5$ s.

SOLUTION

$$\int_{15}^{v} dv = \int_{0}^{5} 0.8t\,dt$$

$$v = 25 \text{ ft/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{25^2}{100} = 6.25 \text{ ft/s}^2$$

At $t = 5$ s, $\quad a_t = \dot{v} = 0.8(5) = 4 \text{ ft/s}^2$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4^2 + 6.25^2} = 7.42 \text{ ft/s}^2$$ **Ans.**

Ans:
$a = 7.42$ ft/s^2

12–131.

A boat is traveling along a circular path having a radius of 20 m. Determine the magnitude of the boat's acceleration when the speed is $v = 5$ m/s and the rate of increase in the speed is $\dot{v} = 2$ m/s^2.

SOLUTION

$$a_t = 2 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{5^2}{20} = 1.25 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 1.25^2} = 2.36 \text{ m/s}^2$$ **Ans.**

Ans:

$a = 2.36$ m/s^2

***12–132.**

Starting from rest, a bicyclist travels around a horizontal circular path, $\rho = 10$ m, at a speed of $v = (0.09t^2 + 0.1t)$ m/s, where t is in seconds. Determine the magnitudes of her velocity and acceleration when she has traveled $s = 3$ m.

SOLUTION

$$\int_0^s ds = \int_0^t (0.09t^2 + 0.1t)dt$$

$$s = 0.03t^3 + 0.05t^2$$

When $s = 3$ m, $\quad 3 = 0.03t^3 + 0.05t^2$

Solving,

$t = 4.147$ s

$$v = \frac{ds}{dt} = 0.09t^2 + 0.1t$$

$$v = 0.09(4.147)^2 + 0.1(4.147) = 1.96 \text{ m/s}$$ **Ans.**

$$a_t = \frac{dv}{dt} = 0.18t + 0.1\Big|_{t=4.147\text{ s}} = 0.8465 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{1.96^2}{10} = 0.3852 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8465)^2 + (0.3852)^2} = 0.930 \text{ m/s}^2$$ **Ans.**

Ans:
$v = 1.96$ m/s
$a = 0.930$ m/s^2

12–133.

A particle travels around a circular path having a radius of 50 m. If it is initially traveling with a speed of 10 m/s and its speed then increases at a rate of $\dot{v} = (0.05\,v)\text{ m/s}^2$, determine the magnitude of the particle's acceleration four seconds later.

SOLUTION

Velocity: Using the initial condition $v = 10$ m/s at $t = 0$ s,

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{10\text{ m/s}}^{v} \frac{dv}{0.05v}$$

$$t = 20\ln\frac{v}{10}$$

$$v = (10e^{t/20})\text{ m/s}$$

When $t = 4$ s,

$$v = 10e^{4/20} = 12.214\text{ m/s}$$

Acceleration: When $v = 12.214$ m/s ($t = 4$ s),

$$a_t = 0.05(12.214) = 0.6107\text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(12.214)^2}{50} = 2.984\text{ m/s}^2$$

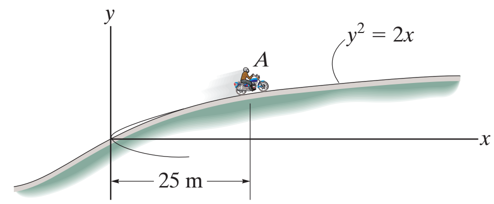
Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.6107^2 + 2.984^2} = 3.05\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 3.05\text{ m/s}^2$

12–134.

The motorcycle is traveling at a constant speed of 60 km/h. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = \sqrt{2}x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{2}x^{-1/2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}\sqrt{2}x^{-3/2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{2}\sqrt{2}x^{-1/2}\right)^2\right]^{3/2}}{\left|-\frac{1}{4}\sqrt{2}x^{-3/2}\right|}\right|_{x=25\text{ m}} = 364.21\text{ m}$$

Acceleration: The speed of the motorcycle at A is

$$v = \left(60\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 16.67\text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{364.21} = 0.7627\text{ m/s}^2$$

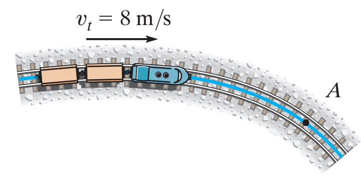
Since the motorcycle travels with a constant speed, $a_t = 0$. Thus, the magnitude of the motorcycle's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0^2 + 0.7627^2} = 0.763\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.763\text{ m/s}^2$

12–135.

When $t = 0$, the train has a speed of 8 m/s, which is increasing at 0.5 m/s^2. Determine the magnitude of the acceleration of the engine when it reaches point A, at $t = 20$ s. Here the radius of curvature of the tracks is $\rho_A = 400$ m.



SOLUTION

Velocity: The velocity of the train along the track can be determined by integrating $dv = a_t\,dt$ with initial condition $v = 8$ m/s at $t = 0$.

$$\int_{8\text{ m/s}}^{v} dv = \int_0^t 0.5\,dt$$

$$v - 8 = 0.5\,t$$

$$v = \{0.5\,t + 8\}\text{ m/s}$$

At $t = 20$ s,

$$v\big|_{t=20\,s} = 0.5(20) + 8 = 18\text{ m/s}$$

Acceleration: Here, the tangential component is $a_t = 0.5$ m/s^2. The normal component can be determined from

$$a_n = \frac{v^2}{\rho} = \frac{18^2}{400} = 0.81\text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2}$$

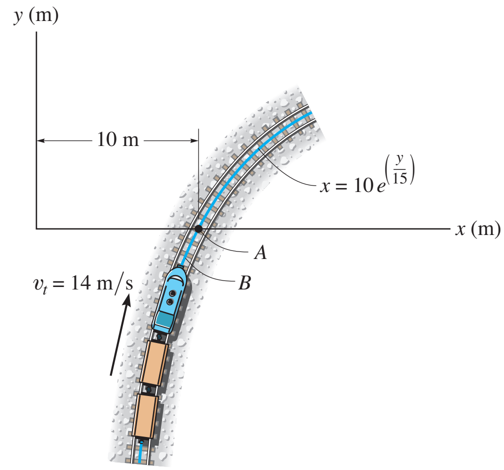
$$= \sqrt{0.5^2 + 0.81^2}$$

$$= 0.9519\text{ m/s}^2 = 0.952\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.952$ m/s^2

***12–136.**

A train is traveling with a constant speed of 14 m/s along the curved path. Determine the magnitude of the acceleration of the front of the train, B, at the instant it reaches point A ($y = 0$).



SOLUTION

$$x = 10e^{\left(\frac{y}{15}\right)}$$

$$y = 15\ln\left(\frac{x}{10}\right)$$

$$\frac{dy}{dx} = 15\left(\frac{10}{x}\right)\left(\frac{1}{10}\right) = \frac{15}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}$$

At $x = 10$,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (1.5)^2\right]^{\frac{3}{2}}}{|-0.15|} = 39.06 \text{ m}$$

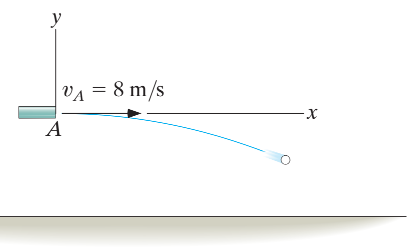
$$a_t = \frac{dv}{dt} = 0$$

$$a_n = a = \frac{v^2}{\rho} = \frac{(14)^2}{39.06} = 5.02 \text{ m/s}^2$$ **Ans.**

Ans:
$a_n = 5.02 \text{ m/s}^2$

12–137.

The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.



SOLUTION

$v_x = 8$ m/s

$(\overset{+}{\rightarrow})\qquad s = v_0 t$

$x = 8t$

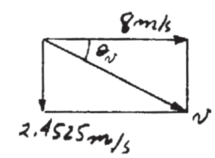
$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

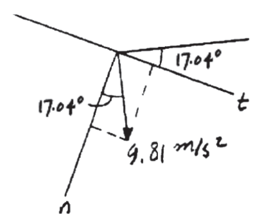
$y = 0 + 0 + \frac{1}{2}(-9.81)t^2$

$y = -4.905t^2$

$y = -4.905\left(\frac{x}{8}\right)^2$

$y = -0.0766x^2$ (Parabola) **Ans.**





$v = v_0 + a_c t$

$v_y = 0 - 9.81t$

When $t = 0.25$ s,

$v_y = -2.4525$ m/s

$v = \sqrt{(8)^2 + (2.4525)^2} = 8.37$ m/s **Ans.**

$\theta_v = \tan^{-1}\left(\frac{2.4525}{8}\right) = 17.04° = 17.0°$ ⦪ **Ans.**

$a_x = 0 \quad a_y = 9.81\ \text{m/s}^2$

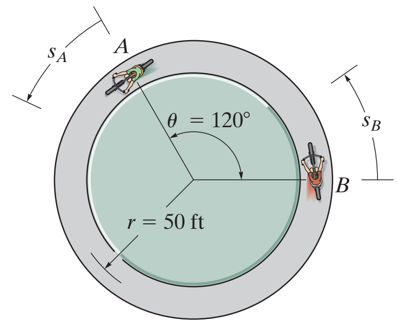
$a_n = 9.81 \cos 17.04° = 9.38\ \text{m/s}^2$ **Ans.**

$a_t = 9.81 \sin 17.04° = 2.88\ \text{m/s}^2$ **Ans.**

Ans:
$y = -0.0766x^2$
$v = 8.37$ m/s
$\theta_v = 17.0°$ ⦪
$a_n = 9.38\ \text{m/s}^2$
$a_t = 2.88\ \text{m/s}^2$

12–138.

Two cyclists, A and B, are traveling counterclockwise around a circular track at a constant speed of 8 ft/s at the instant shown. If the speed of A is increased at $(a_t)_A = (s_A)$ ft/s^2, where s_A is in feet, determine the distance measured counterclockwise along the track from B to A between the cyclists when $t = 1$ s. What is the magnitude of the acceleration of each cyclist at this instant?



SOLUTION

Distance Traveled: Initially, the distance between the cyclists is $d_0 = \rho\theta = 50\left(\frac{120°}{180°}\pi\right) = 104.72$ ft. When $t = 1$s, cyclist B travels a distance of $s_B = 8(1) = 8$ ft.The distance traveled by cyclist A can be obtained as follows

$$v_A dv_A = a_A\, ds_A$$

$$\int_{8\text{ft/s}}^{vA} v_A dv_A = \int_0^{sA} s_A ds_A$$

$$v_A = \sqrt{s_A^2 + 64} \qquad \textbf{[1]}$$

$$dt = \frac{ds_A}{v_A}$$

$$\int_0^{1s} dt = \int_0^{sA} \frac{ds_A}{\sqrt{s_A^2 + 64}}$$

$$1 = \sinh^{-1}\left(\frac{s_A}{8}\right)$$

$$s_A = 9.402 \text{ ft}$$

Thus, the distance between the two cyclists after $t = 1$is

$$d = d_0 + s_A - s_B = 104.72 + 9.402 - 8 = 106 \text{ ft} \qquad \textbf{Ans.}$$

Acceleration: The tangential acceleration for cyclist A and B at $t = 1$s is and $(a_t)_A = s_A = 9.402$ ft/s^2(cyclist B travels at constant speed), respectively. At $t = 1$s, from Eq. [1], $v_A = \sqrt{9.402^2 + 64} = 12.34$ ft/s. To determine normal acceleration, apply Eq. 12–20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{12.34^2}{50} = 3.048 \text{ ft/s}^2$$

$$(a_n)_B = \frac{v_A^2}{\rho} = \frac{8^2}{50} = 1.28 \text{ ft/s}^2$$

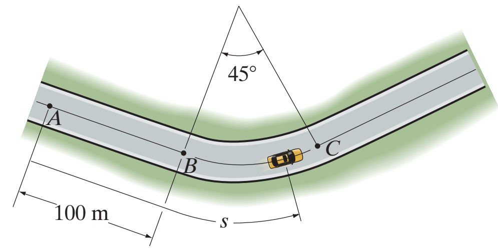
The magnitude of the acceleration for cyclist A and B are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{9.402^2 + 3.048^2} = 9.88 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{0^2 + 1.28^2} = 1.28 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–139.

The car is traveling at a constant speed of 30 m/s. The driver then applies the brakes at A and thereby reduces the car's speed at the rate of $a_t = (-0.08v)\ \text{m/s}^2$, where v is in m/s. Determine the magnitude of the acceleration of the car just before it reaches point C on the circular curve. It takes 15 s for the car to travel from A to C.



SOLUTION

Velocity: Using the initial condition $v = 30$ m/s when $t = 0$ s,

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{30\text{ m/s}}^{v} -\frac{dv}{0.08v}$$

$$t = 12.5 \ln \frac{30}{v}$$

$$v = \left(30e^{-0.08t}\right)\text{ m/s} \qquad \textbf{(1)}$$

Position: Using the initial condition $s = 0$ when $t = 0$ s,

$$ds = vdt$$

$$\int_0^s ds = \int_0^t 30e^{-0.08t}\,dt$$

$$s = \left[375\left(1 - e^{-0.08t}\right)\right]\text{ m} \qquad \textbf{(2)}$$

Acceleration:

$$s_C = 375\left(1 - e^{-0.08(15)}\right) = 262.05\text{ m}$$

$$s_{BC} = s_C - s_B = 262.05 - 100 = 162.05\text{ m}$$

$$\rho = \frac{s_{BC}}{\theta} = \frac{162.05}{\pi/4} = 206.33\text{ m}$$

$$v_C = 30e^{-0.08(15)} = 9.036\text{ m/s}$$

$$\left(a_n\right)_C = \frac{v_C{}^2}{\rho} = \frac{9.036^2}{206.33} = 0.3957\text{ m/s}^2$$

$$\left(a_t\right)_C = \dot{v} = -0.08(9.036) = -0.7229\text{ m/s}^2$$

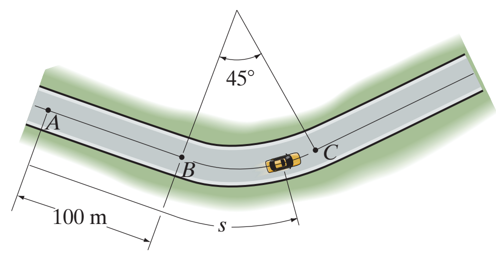
The magnitude of the car's acceleration at point C is

$$a = \sqrt{(a_t)_C{}^2 + (a_n)_C{}^2} = \sqrt{(-0.7229)^2 + 0.3957^2} = 0.824\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.824\text{ m/s}^2$

***12–140.**

The car is traveling at a constant speed of 30 m/s. The driver then applies the brakes at A and thereby reduces the speed at the rate of $a_t = \left(-\frac{1}{8}t\right)$ m/s^2, where t is in seconds. Determine the magnitude of the acceleration of the car just before it reaches point C on the circular curve. It takes 15 s for the car to travel from A to C.



SOLUTION

Velocity: Using the initial condition $v = 30$ m/s when $t = 0$ s,

$$dv = a_t\,dt$$

$$\int_{30\text{ m/s}}^{v} dv = \int_0^t -\frac{1}{8}t\,dt$$

$$v = \left(30 - \frac{1}{16}t^2\right)\text{ m/s}$$

Position: Using the initial condition $s = 0$ when $t = 0$ s,

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(30 - \frac{1}{16}t^2\right)dt$$

$$s = \left(30t - \frac{1}{48}t^3\right)\text{m}$$

Acceleration:

$$s_C = 30(15) - \frac{1}{48}\left(15^3\right) = 379.6875\text{ m}$$

$$s_{BC} = s_C - s_B = 379.6875 - 100 = 279.6875\text{ m}$$

$$\rho = \frac{s_{BC}}{\theta} = \frac{279.6875}{\pi/4} = 356.11\text{ m}$$

$$v_C = 30 - \frac{1}{16}\left(15^2\right) = 15.9375\text{ m/s}$$

$$\left(a_t\right)_C = \dot{v} = -\frac{1}{8}(15) = -1.875\text{ m/s}^2$$

$$\left(a_n\right)_C = \frac{v_C{}^2}{\rho} = \frac{15.9375^2}{356.11} = 0.7133\text{ m/s}^2$$

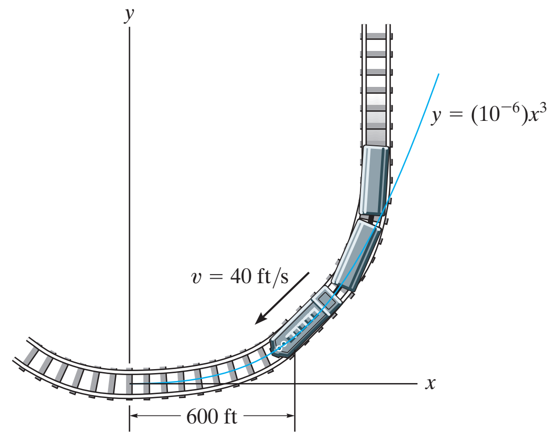
The magnitude of the car's acceleration at point C is

$$a = \sqrt{\left(a_t\right)_C{}^2 + \left(a_n\right)_C{}^2} = \sqrt{\left(-1.875\right)^2 + 0.7133^2} = 2.01\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 2.01$ m/s^2

12–141.

A spiral transition curve is used on railroads to connect a straight portion of the track with a curved portion. If the spiral is defined by the equation $y = (10^{-6})x^3$, where x and y are in feet, determine the magnitude of the acceleration of a train engine moving with a constant speed of 40 ft/s when it is at the point where $x = 600$ ft.



SOLUTION

$$y = (10)^{-6}x^3$$

$$\left.\frac{dy}{dx}\right|_{x=600\text{ ft}} = 3(10)^{-6}x^2\Big|_{x=600\text{ ft}} = 1.08$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=600\text{ ft}} = 6(10)^{-6}x\Big|_{x=600\text{ ft}} = 3.6(10)^{-3}$$

$$\rho\Big|_{x=600\text{ ft}} = \left.\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\right|_{x=600\text{ ft}} = \frac{[1 + (1.08)^2]^{3/2}}{|3.6(10)^{-3}|} = 885.7\text{ ft}$$

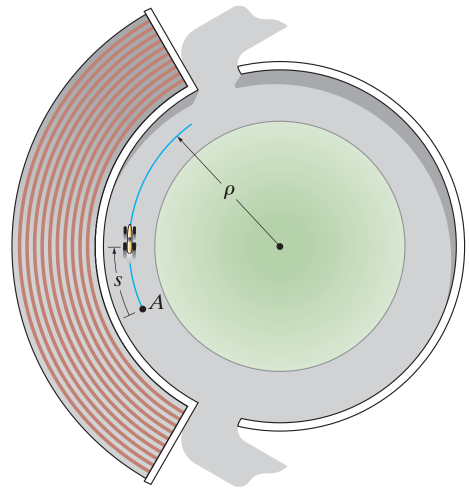
$$a_n = \frac{v^2}{\rho} = \frac{40^2}{885.7} = 1.81\text{ ft/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0 + (1.81)^2} = 1.81\text{ ft/s}^2$$ **Ans.**

Ans:
$a = 1.81\text{ ft/s}^2$

12–142.

The race car has an initial speed $v_A = 15$ m/s at A. If it increases its speed along the circular track at the rate $a_t = (0.4s)$ m/s^2, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150$ m.



SOLUTION

$$a_t = 0.4s = \frac{v\,dv}{ds}$$

$$a\,ds = v\,dv$$

$$\int_0^s 0.4s\,ds = \int_{15}^v v\,dv$$

$$\left.\frac{0.4s^2}{2}\right|_0^s = \left.\frac{v^2}{2}\right|_{15}^v$$

$$\frac{0.4s^2}{2} = \frac{v^2}{2} - \frac{225}{2}$$

$$v^2 = 0.4s^2 + 225$$

$$v = \frac{ds}{dt} = \sqrt{0.4s^2 + 225}$$

$$\int_0^s \frac{ds}{\sqrt{0.4s^2 + 225}} = \int_0^t dt$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 562.5}} = 0.632\,456t$$

$$1\text{n}\,(s + \sqrt{s^2 + 562.5})\Big|_0^s = 0.632\,456t$$

$$1\text{n}\,(s + \sqrt{s^2 + 562.5}) - 3.166\,196 = 0.632\,456t$$

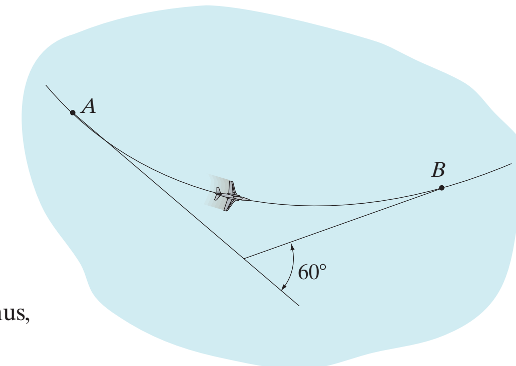
At $s = 20$ m,

$t = 1.21$ s **Ans.**

Ans:
$t = 1.21$ s

12–143.

Determine the magnitude of acceleration of the airplane during the turn. It flies along the horizontal circular path AB in 40 s, while maintaining a constant speed of 300 ft/s.



SOLUTION

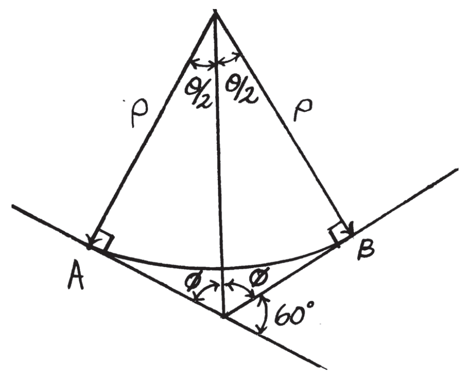
Acceleration: From the geometry in Fig. a, $2\phi + 60° = 180°$ or $\phi = 60°$. Thus, $\frac{\theta}{2} = 90° - 60°$ or $\theta = 60° = \frac{\pi}{3}$ rad.

$$s_{AB} = vt = 300(40) = 12\,000 \text{ ft}$$

Thus,

$$\rho = \frac{s_{AB}}{\theta} = \frac{12\,000}{\pi/3} = \frac{36\,000}{\pi} \text{ ft}$$

$$a_n = \frac{v^2}{\rho} = \frac{300^2}{36\,000/\pi} = 7.854 \text{ ft/s}^2$$



(a)

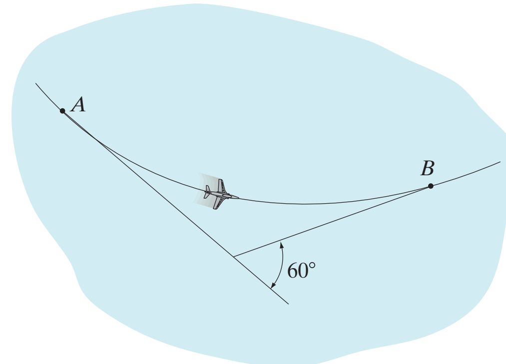
Since the airplane travels along the circular path with a constant speed, $a_t = 0$. Thus, the magnitude of the airplane's acceleration is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{0^2 + 7.854^2} = 7.85 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 7.85 \text{ ft/s}^2$

***12–144.**

The airplane flies along the horizontal circular path AB in 60 s. If its speed at point A is 400 ft/s, which decreases at a rate of $a_t = (-0.1t)$ ft/s^2, where t is in seconds, determine the magnitude of the plane's acceleration when it reaches point B.



SOLUTION

Velocity: Using the initial condition $v = 400$ ft/s when $t = 0$ s,

$$dv = a_t\, dt$$

$$\int_{400\text{ ft/s}}^{v} dv = \int_0^t -0.1t\,dt$$

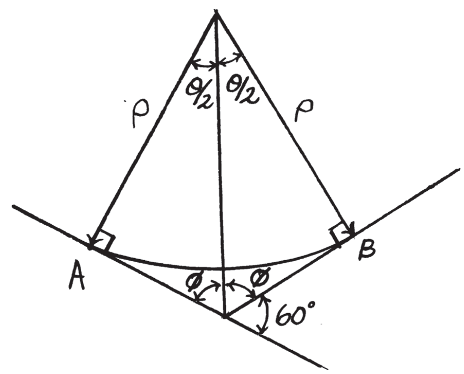
$$v = \left(400 - 0.05t^2\right)\text{ ft/s}$$

Position: Using the initial condition $s = 0$ when $t = 0$ s,

$$\int ds = \int v\,dt$$

$$\int_0^s ds = \int_0^t \left(400 - 0.05t^2\right)dt$$

$$s = \left(400t - 0.01667\,t^3\right)\text{ ft}$$



(a)

Acceleration: From the geometry in Fig. a, $2\phi + 60° = 180°$ or $\phi = 60°$. Thus, $\dfrac{\theta}{2} = 90° - 60°$ or $\theta = 60° = \dfrac{\pi}{3}$ rad.

$$s_{AB} = 400(60) - 0.01667\left(60^3\right) = 20\,400\text{ ft}$$

$$\rho = \frac{s_{AB}}{\theta} = \frac{20\,400}{\pi/3} = \frac{61200}{\pi}\text{ ft}$$

$$v_B = 400 - 0.05\left(60^2\right) = 220\text{ ft/s}$$

$$\left(a_n\right)_B = \frac{{v_B}^2}{\rho} = \frac{220^2}{61\,200/\pi} = 2.485\text{ ft/s}^2$$

$$\left(a_t\right)_B = \dot{v} = -0.1(60) = -6\text{ft/s}^2$$

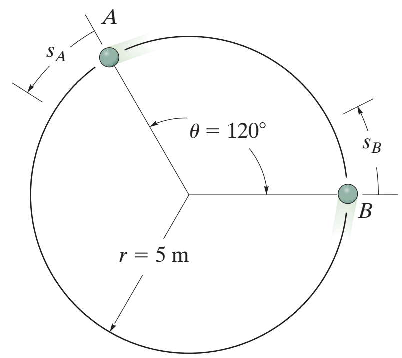
The magnitude of the airplane's acceleration is

$$a = \sqrt{{a_t}^2 + {a_n}^2} = \sqrt{(-6)^2 + 2.485^2} = 6.49\text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 6.49\text{ ft/s}^2$

12–145.

Particles A and B are traveling counterclockwise around a circular track at a constant speed of 8 m/s. If at the instant shown the speed of A begins to increase by $(a_t)_A = (0.4s_A)$ m/s^2, where s_A is in meters, determine the distance measured counterclockwise along the track from B to A when $t = 1$ s. What is the magnitude of the acceleration of each particle at this instant?



SOLUTION

Distance Traveled: Initially the distance between the particles is

$$d_0 = \rho d\theta = 5\left(\frac{120°}{180°}\right)\pi = 10.47 \text{ m}$$

When $t = 1$ s, B travels a distance of

$$d_B = 8(1) = 8 \text{ m}$$

The distance traveled by particle A is determined as follows:

$$vdv = ads$$

$$\int_{8 \text{ m/s}}^{v} vdv = \int_0^{s_A} 0.4 s_A ds_A$$

$$v = 0.6325\sqrt{s_A^2 + 160} \qquad \textbf{(1)}$$

$$dt = \frac{ds}{v}$$

$$\int_0^{1s} dt = \int_0^{s_A} \frac{ds_A}{0.6325\sqrt{s_A^2 + 160}}$$

$$1 = \frac{1}{0.6325}\left(\text{In}\left[\frac{\sqrt{s_A^2 + 160} + s_A}{\sqrt{160}}\right]\right)$$

$$s_A = 8.544 \text{ m}$$

Thus the distance between the two cyclists after $t = 1$ s is

$$d = 10.47 + 8.544 - 8 = 11.0 \text{ m} \qquad \textbf{Ans.}$$

Acceleration:

For A, when $t = 1$ s,

$$(a_t)_A = \dot{v}_A = 0.4(8.544) = 3.4176 \text{ m/s}^2$$

$$v_A = 0.6325\sqrt{8.544^2 + 160} = 9.655 \text{ m/s}$$

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{9.655^2}{5} = 18.64 \text{ m/s}^2$$

The magnitude of the A's acceleration is

$$a_A = \sqrt{3.4176^2 + 18.64^2} = 19.0 \text{ m/s}^2 \qquad \textbf{Ans.}$$

For B, when $t = 1$ s,

$$(a_t)_B = \dot{v}_A = 0$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{8^2}{5} = 12.80 \text{ m/s}^2$$

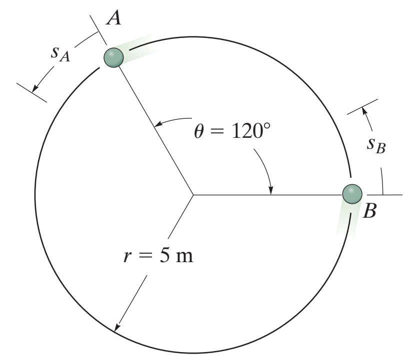
The magnitude of the B's acceleration is

$$a_B = \sqrt{0^2 + 12.80^2} = 12.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$d = 11.0$ m
$a_A = 19.0$ m/s^2
$a_B = 12.8$ m/s^2

12–146.

Particles A and B are traveling around a circular track at a speed of 8 m/s at the instant shown. If the speed of B is increasing by $(a_t)_B = 4\ \text{m/s}^2$, and at the same instant A has an increase in speed of $(a_t)_A = 0.8t\ \text{m/s}^2$, where t is in seconds, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?



SOLUTION

Distance Traveled: Initially the distance between the two particles is $d_0 = \rho\theta = 5\left(\frac{120°}{180°}\pi\right) = 10.47$ m. Since particle B travels with a constant acceleration, distance can be obtained by applying equation

$$s_B = (s_0)_B + (v_0)_B t + \frac{1}{2}a_c t^2$$

$$s_B = 0 + 8t + \frac{1}{2}(4)\,t^2 = \left(8t + 2t^2\right)\ \text{m} \qquad \textbf{[1]}$$

The distance traveled by particle A can be obtained as follows.

$$dv_A = a_A\,dt$$

$$\int_{8\ \text{m/s}}^{v_A} dv_A = \int_0^t 0.8\,t\,dt$$

$$v_A = \left(0.4t^2 + 8\right)\ \text{m/s} \qquad \textbf{[2]}$$

$$ds_A = v_A\,dt$$

$$\int_0^{s_A} ds_A = \int_0^t \left(0.4t^2 + 8\right)dt$$

$$s_A = 0.1333t^3 + 8t$$

In order for the collision to occur

$$s_A + d_0 = s_B$$

$$0.1333t^3 + 8t + 10.47 = 8t + 2t^2$$

Solving by trial and error $\quad t = 2.5074\ \text{s} = 2.51\ \text{s}$ **Ans.**

Note: If particle A strikes B then, $s_A = 5\left(\frac{240°}{180°}\pi\right) + s_B$. This equation will result in $t = 14.6\ \text{s} > 2.51\ \text{s}$.

Acceleration: The tangential acceleration for particle A and B when $t = 2.5074$ are $(a_t)_A = 0.8t = 0.8\,(2.5074) = 2.006\ \text{m/s}^2$ and $(a_t)_B = 4\ \text{m/s}^2$, respectively. When $t = 2.5074$ s, from Eq. [1], $v_A = 0.4\left(2.5074^2\right) + 8 = 10.51$ m/s and $v_B = (v_0)_B + a_c t = 8 + 4(2.5074) = 18.03$ m/s. To determine the normal acceleration, apply Eq. 12–20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{10.51^2}{5} = 22.11\ \text{m/s}^2$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{18.03^2}{5} = 65.01\ \text{m/s}^2$$

The magnitude of the acceleration for particles A and B just before collision are

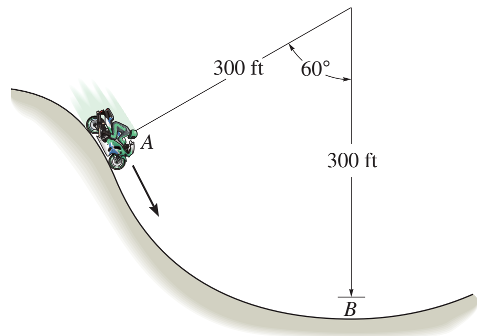
$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{2.006^2 + 22.11^2} = 22.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{4^2 + 65.01^2} = 65.1\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$t = 2.51$ s
$a_A = 22.2\ \text{m/s}^2$
$a_B = 65.1\ \text{m/s}^2$

12–147.

When the motorcyclist is at A, he increases his speed along the vertical circular path at the rate of $\dot{v} = (0.3t)$ ft/s^2, where t is in seconds and $t = 0$ at A. If he starts from rest at A, determine the magnitudes of his velocity and acceleration when he reaches B.



SOLUTION

$$\int_0^v dv = \int_0^t 0.3t\,dt$$

$$v = 0.15t^2$$

$$\int_0^s ds = \int_0^t 0.15t^2\,dt$$

$$s = 0.05t^3$$

When $s = \frac{\pi}{3}(300)$ ft, $\qquad \frac{\pi}{3}(300) = 0.05t^3 \qquad t = 18.453$ s

$v = 0.15(18.453)^2 = 51.08$ ft/s $= 51.1$ ft/s **Ans.**

$$a_t = \dot{v} = 0.3t|_{t=18.453\text{ s}} = 5.536\text{ ft/s}^2$$

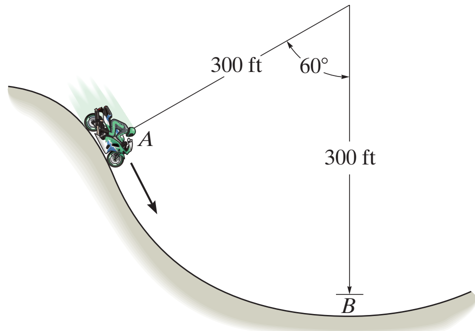
$$a_n = \frac{v^2}{\rho} = \frac{51.08^2}{300} = 8.696\text{ ft/s}^2$$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(5.536)^2 + (8.696)^2} = 10.3$ ft/s^2 **Ans.**

Ans:
$v = 51.1$ ft/s
$a = 10.3$ ft/s^2

***12–148.**

When the motorcyclist is at A, he increases his speed along the vertical circular path at the rate of $\dot{v} = (0.04s)$ ft/s^2, where s is in ft and $s = 0$ at A. If he starts at $v_A = 2$ ft/s, determine the magnitude of his velocity when he reaches B. Also, what is his initial acceleration?



SOLUTION

Velocity: At $s = 0, v = 2$. Here, $a_c = \dot{v} = 0.045$. Then

$$\int v\,dv = \int a_t\,ds$$

$$\int_2^v v\,dv = \int_0^s 0.04s\,ds$$

$$\left.\frac{v^2}{2}\right|_2^v = 0.025^2\Big|_0^s$$

$$\frac{v^2}{2} - 2 = 0.025^2$$

$$v^2 = 0.045^2 + 4 = 0.04(s^2 + 100)$$

$$v = 0.2\sqrt{s^2 + 100}$$

At B, $s = r\theta = 300\left(\frac{\pi}{3}\right) = 100\pi$ ft. Thus

$$v\Big|_{s=100\pi\text{ ft}} = 0.2\sqrt{(100\pi)^2 + 100} = 62.9\text{ ft/s}$$ **Ans.**

Acceleration: At $t = 0$, $s = 0$, and $v = 2$.

$$a_t = \dot{v} = 0.04\text{ s}$$

$$a_t\Big|_{s=0} = 0$$

$$a_n = \frac{v^2}{\rho}$$

$$a_n\Big|_{s=0} = \frac{(2)^2}{300} = 0.01333\text{ ft/s}^2$$

$$a = \sqrt{(0)^2 + (0.01333)^2} = 0.0133\text{ ft/s}^2$$ **Ans.**

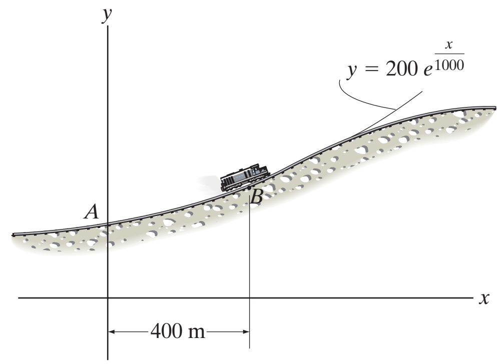
Ans:

$v\Big|_{s=100\pi\text{ ft}} = 62.9$ ft/s

$a = 0.0133$ ft/s^2

12–149.

The train passes point B with a speed of $20\ \mathrm{m/s}$ which is decreasing at $a_t = -0.5\ \mathrm{m/s^2}$. Determine the magnitude of acceleration of the train at this point.



SOLUTION

Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 200\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2e^{\frac{x}{1000}}$$

$$\frac{d^2y}{dx^2} = 0.2\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2\left(10^{-3}\right)e^{\frac{x}{1000}}.$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(0.2\,e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2\left(10^{-3}\right)e^{\frac{x}{1000}}\right|}\right|_{x=400\ \mathrm{m}} = 3808.96\ \mathrm{m}$$

Acceleration:

$$a_t = \dot{v} = -0.5\ \mathrm{m/s^2}$$

$$a_n = \frac{v^2}{\rho} = \frac{20^2}{3808.96} = 0.1050\ \mathrm{m/s^2}$$

The magnitude of the train's acceleration at B is

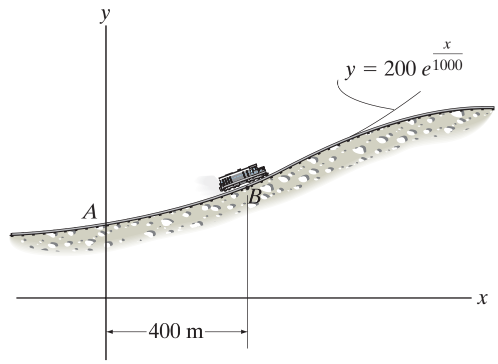
$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{\left(-0.5\right)^2 + 0.1050^2} = 0.511\ \mathrm{m/s^2} \qquad \textbf{Ans.}$$

Ans:

$a = 0.511\ \mathrm{m/s^2}$

12–150.

The train passes point A with a speed of 30 m/s and begins to decrease its speed at a constant rate of $a_t = -0.25 \text{ m/s}^2$. Determine the magnitude of the acceleration of the train when it reaches point B, where $s_{AB} = 412$ m.



SOLUTION

Velocity: The speed of the train at B can be determined from

$$v_B{}^2 = v_A{}^2 + 2a_t(s_B - s_A)$$

$$v_B{}^2 = 30^2 + 2(-0.25)(412 - 0)$$

$$v_B = 26.34 \text{ m/s}$$

Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 0.2e^{\frac{x}{1000}}$$

$$\frac{d^2y}{dx^2} = 0.2\left(10^{-3}\right)e^{\frac{x}{1000}}$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(0.2e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2\left(10^{-3}\right)e^{\frac{x}{1000}}\right|}\right|_{x=400\text{ m}} = 3808.96 \text{ m}$$

Acceleration:

$$a_t = \dot{v} = -0.25 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{26.34^2}{3808.96} = 0.1822 \text{ m/s}^2$$

The magnitude of the train's acceleration at B is

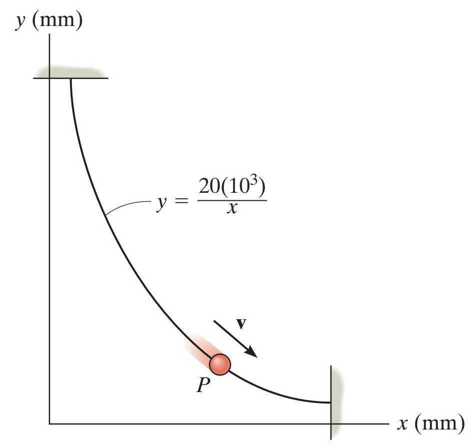
$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.5)^2 + 0.1822^2} = 0.309 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$a = 0.309 \text{ m/s}^2$

12–151.

The particle travels with a constant speed of 300 mm/s along the curve. Determine the particle's acceleration when it is located at point (200 mm, 100 mm) and sketch this vector on the curve.



SOLUTION

$v = 300 \text{ mm/s}$

$$a_t = \frac{dv}{dt} = 0$$

$$y = \frac{20(10^3)}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=200} = -\frac{20(10^3)}{x^2} = -0.5$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=200} = \frac{40(10^3)}{x^3} = 5(10^{-3})$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (-0.5)^2\right]^{\frac{3}{2}}}{\left|5(10^{-3})\right|} = 279.5 \text{ mm}$$

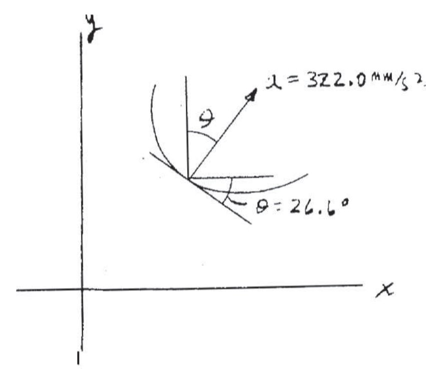
$$a_n = \frac{v^2}{\rho} = \frac{(300)^2}{279.5} = 322 \text{ mm/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2}$$

$$= \sqrt{(0)^2 + (322)^2} = 322 \text{ mm/s}^2 \qquad \textbf{Ans.}$$

Since $\frac{dy}{dx} = -0.5$,

$$\theta = \tan^{-1}(-0.5) = 26.6° \qquad \textbf{Ans.}$$

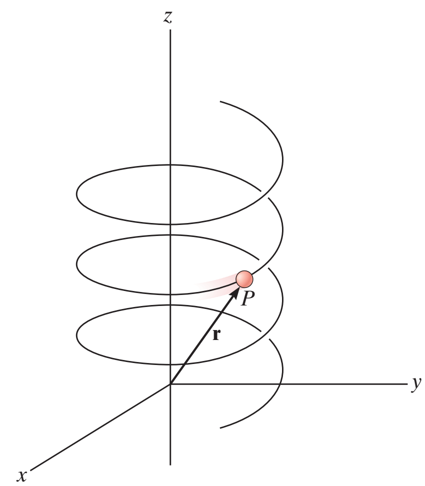


Ans:

$a = 322 \text{ mm/s}^2$
$\theta = 26.6°$

***12–152.**

A particle P travels along an elliptical spiral path such that its position vector $\mathbf{r}$ is defined by $\mathbf{r} = \{2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + (2t)\mathbf{k}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. When $t = 8$ s, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint*: Solve for the velocity $\mathbf{v}_P$ and acceleration $\mathbf{a}_P$ of the particle in terms of their $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?



SOLUTION

$$\mathbf{r}_P = 2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + 2t\mathbf{k}$$

$$\mathbf{v}_P = \dot{\mathbf{r}} = -0.2\sin(0.1t)\mathbf{i} + 0.15\cos(0.1t)\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = \ddot{\mathbf{r}} = -0.02\cos(0.1t)\mathbf{i} - 0.015\sin(0.1t)\mathbf{j}$$

When $t = 8$ s,

$$\mathbf{v}_P = -0.2\sin(0.8\text{ rad})\mathbf{i} + 0.15\cos(0.8\text{ rad})\mathbf{j} + 2\mathbf{k} = -0.143\,47\mathbf{i} + 0.104\,51\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = -0.02\cos(0.8\text{ rad})\mathbf{i} - 0.015\sin(0.8\text{ rad})\mathbf{j} = -0.013\,934\mathbf{i} - 0.010\,76\mathbf{j}$$

Since the binormal vector is perpendicular to the plane containing the n–t axis, and $\mathbf{a}_p$ and $\mathbf{v}_p$ are in this plane, then by the definition of the cross product,

$$\mathbf{b} = \mathbf{v}_P \times \mathbf{a}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.14\,347 & 0.104\,51 & 2 \\ -0.013\,934 & -0.010\,76 & 0 \end{vmatrix} = 0.021\,52\mathbf{i} - 0.027\,868\mathbf{j} + 0.003\mathbf{k}$$

$$b = \sqrt{(0.02152)^2 + (-0.027868)^2 + (0.003)^2} = 0.035\,338$$

$$\mathbf{u}_b = 0.608\,99\mathbf{i} - 0.788\,62\mathbf{j} + 0.085\mathbf{k}$$

$\alpha = \cos^{-1}(0.608\,99) = 52.5°$ **Ans.**

$\beta = \cos^{-1}(-0.788\,62) = 142°$ **Ans.**

$\gamma = \cos^{-1}(0.085) = 85.1°$ **Ans.**

Note: The direction of the binormal axis may also be specified by the unit vector $\mathbf{u}_{b'} = -\mathbf{u}_b$, which is obtained from $\mathbf{b}' = \mathbf{a}_p \times \mathbf{v}_p$.

For this case, $\alpha = 128°$, $\beta = 37.9°$, $\gamma = 94.9°$ **Ans.**

Ans:

$\alpha = 52.5°$

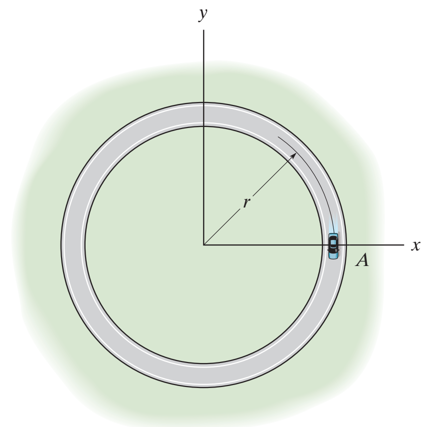
$\beta = 142°$

$\gamma = 85.1°$

$\alpha = 128°$, $\beta = 37.9°$, $\gamma = 94.9°$

■12–153.

A car travels around a circular track having a radius of $r = 300$ m such that when it is at point A it has a velocity of 5 m/s, which is increasing at the rate of $\dot{v} = (0.06t)$ m/s^2, where t is in seconds. Determine the magnitudes of its velocity and acceleration when it has traveled one-third the way around the track.



SOLUTION

$a_t = v = 0.06t$

$dv = a_t\, dt$

$$\int_s^v dv = \int_0^t 0.06t\, dt$$

$v = 0.03t^2 + 5$

$ds = v\, dt$

$$\int_0^s ds = \int_0^t (0.03t^2 + 5)\, dt$$

$s = 0.01t^3 + 5t$

$$s = \frac{1}{3}(2\pi(300)) = 628.3185$$

$0.01t^3 + 5t - 628.3185 = 0$

Solve for the positive root,

$t = 35.58$ s

$v = 0.03(35.58)^2 + 5 = 42.978 \text{ m/s} = 43.0 \text{ m/s}$ **Ans.**

$$a_n = \frac{v^2}{\rho} = \frac{(42.978)^2}{300} = 6.157 \text{ m/s}^2$$

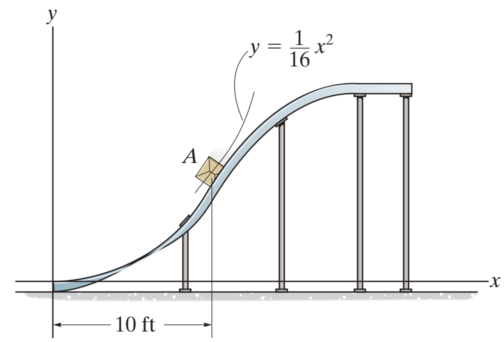
$a_t = 0.06(35.58) = 2.135 \text{ m/s}^2$

$a = \sqrt{(6.157)^2 + (2.135)^2} = 6.52 \text{ m/s}^2$ **Ans.**

Ans:
$v = 43.0$ m/s
$a = 6.52$ m/s^2

12–154.

If the speed of the crate at A is 15 ft/s, which is increasing at a rate $\dot{v} = 3\text{ ft/s}^2$, determine the magnitude of the acceleration of the crate at this instant.



SOLUTION

Radius of Curvature:

$$y = \frac{1}{16}x^2$$

$$\frac{dy}{dx} = \frac{1}{8}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{8}$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{8}x\right)^2\right]^{3/2}}{\left|\frac{1}{8}\right|}\right|_{x=10\text{ ft}} = 32.82\text{ ft}$$

Acceleration:

$$a_t = \dot{v} = 3\text{ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{15^2}{32.82} = 6.856\text{ ft/s}^2$$

The magnitude of the crate's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 6.856^2} = 7.48\text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:

$a = 7.48\text{ ft/s}^2$

12–155.

If a particle's position is described by the polar coordinates $r = 4(1 + \sin t)$ m and $\theta = (2e^{-t})$ rad, where t is in seconds and the argument for the sine is in radians, determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

When $t = 2$ s,

$r = 4(1+\sin t) = 7.637$

$\dot{r} = 4 \cos t = -1.66459$

$\ddot{r} = -4 \sin t = -3.6372$

$\theta = 2\, e^{-t}$

$\dot{\theta} = -2\, e^{-t} = -0.27067$

$\ddot{\theta} = 2\, e^{-t} = 0.270665$

$v_r = \dot{r} = -1.66$ m/s **Ans.**

$v_\theta = r\dot{\theta} = 7.637(-0.27067) = -2.07$ m/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -3.6372 - 7.637(-0.27067)^2 = -4.20\text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.637(0.270665) + 2(-1.66459)(-0.27067) = 2.97\text{ m/s}^2$ **Ans.**

Ans:
$v_r = -1.66$ m/s
$v_\theta = -2.07$ m/s
$a_r = -4.20\text{ m/s}^2$
$a_\theta = 2.97\text{ m/s}^2$

***12–156.**

A particle moves along a path defined by polar coordinates $r = (2e^t)$ ft and $\theta = (8t^2)$ rad, where t is in seconds. Determine the components of its velocity and acceleration when $t = 1$ s.

SOLUTION

When $t = 1$ s,

$r = 2e^t = 5.4366$

$\dot{r} = 2e^t = 5.4366$

$\ddot{r} = 2e^t = 5.4366$

$\theta = 8t^2$

$\dot{\theta} = 16t = 16$

$\ddot{\theta} = 16$

$v_r = r = 5.44$ ft/s **Ans.**

$v_\theta = r\dot{\theta} = 5.4366(16) = 87.0$ ft/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 5.4366 - 5.4366(16)^2 = -1386$ ft/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5.4366(16) + 2(5.4366)(16) = 261$ ft/s^2 **Ans.**

Ans:

$v_r = 5.44$ ft/s
$v_\theta = 87.0$ ft/s
$a_r = -1386$ ft/s^2
$a_\theta = 261$ ft/s^2

12–157.

A particle travels around a limaçon, defined by the equation $r = b - a\cos\theta$, where a and b are constants. Determine the particle's radial and transverse components of velocity and acceleration as a function of θ and its time derivatives.

SOLUTION

$$r = b - a\cos\theta$$

$$\dot{r} = a\sin\theta\dot{\theta}$$

$$\dot{r} = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta}$$

$$v_r = \dot{r} = a\sin\theta\dot{\theta} \qquad \textbf{Ans.}$$

$$v_\theta = r\theta = (b - a\cos\theta)\dot{\theta} \qquad \textbf{Ans.}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta} - (b - a\cos\theta)\dot{\theta}^2$$

$$= (2a\cos\theta - b)\,\dot{\theta}^2 + a\sin\theta\,\ddot{\theta} \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\,\dot{\theta} = (b - a\cos\theta)\ddot{\theta} + 2\left(a\sin\theta\dot{\theta}\right)\dot{\theta}$$

$$= (b - a\cos\theta)\ddot{\theta} + 2a\dot{\theta}^2\sin\theta \qquad \textbf{Ans.}$$

Ans:
$v_r = a\sin\theta\dot{\theta}$
$v_\theta = (b - a\cos\theta)\dot{\theta}$
$a_r = (2a\cos\theta - b)\dot{\theta}^2 + a\sin\theta\ddot{\theta}$
$a_\theta = (b - a\cos\theta)\ddot{\theta} + 2a\dot{\theta}^2\sin\theta$

12–158.

An airplane is flying in a straight line with a velocity of 200 mi/h and an acceleration of 3 mi/h². If the propeller has a diameter of 6 ft and is rotating at a constant angular rate of 120 rad/s, determine the magnitudes of velocity and acceleration of a particle located on the tip of the propeller.

SOLUTION

$$v_{Pl} = \left(\frac{200 \text{ mi}}{\text{h}}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 293.3 \text{ ft/s}$$

$$a_{Pl} = \left(\frac{3 \text{ mi}}{\text{h}^2}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2 = 0.001\,22 \text{ ft/s}^2$$

$$v_{Pr} = 120(3) = 360 \text{ ft/s}$$

$$v = \sqrt{v_{Pl}^2 + v_{Pr}^2} = \sqrt{(293.3)^2 + (360)^2} = 464 \text{ ft/s}$$ **Ans.**

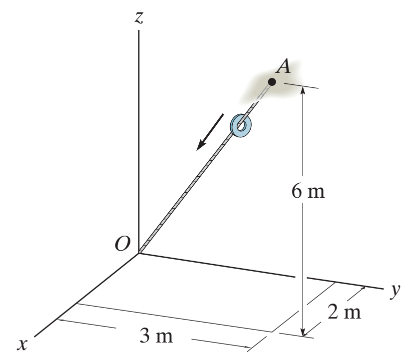
$$a_{Pr} = \frac{v_{Pr}^2}{\rho} = \frac{(360)^2}{3} = 43\,200 \text{ ft/s}^2$$

$$a = \sqrt{a_{Pl}^2 + a_{Pr}^2} = \sqrt{(0.001\,22)^2 + (43\,200)^2} = 43.2(10^3) \text{ ft/s}^2$$ **Ans.**

Ans:
$v = 464$ ft/s
$a = 43.2(10^3)$ ft/s²

12–159.

The small washer is sliding down the cord OA. When it is at the midpoint, its speed is 28 m/s and its acceleration is 7 m/s^2. Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.



SOLUTION

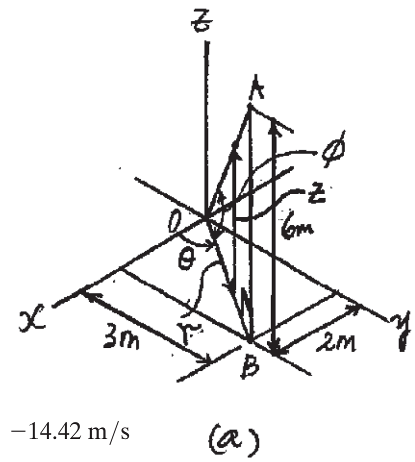
The position of the washer can be defined using the cylindrical coordinate system (r, θ and z) as shown in Fig. a. Since θ is constant, there will be no transverse component for **v** and **a**. The velocity and acceleration expressed as Cartesian vectors are

$$\mathbf{v} = v\left(\frac{\mathbf{r}_{AO}}{r_{AO}}\right) = 28\left[\frac{(0-2)\mathbf{i} + (0-3)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-2)^2 + (0-3)^2 + (0-6)^2}}\right] = \{-8\mathbf{i} - 12\mathbf{j} - 24\mathbf{k}\}\text{ m/s}$$

$$\mathbf{a} = a\left(\frac{\mathbf{r}_{AO}}{r_{AO}}\right) = 7\left[\frac{(0-2)\mathbf{i} + (0-3)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-2)^2 + (0-3)^2 + (0-6)^2}}\right] = \{-2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}\}\text{ m}^2/\text{s}$$

$$\mathbf{u}_r = \frac{\mathbf{r}_{OB}}{r_{OB}} = \frac{2\mathbf{i} + 3\mathbf{j}}{\sqrt{2^2 + 3^2}} = \frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}$$

$$\mathbf{u}_z = \mathbf{k}$$



Using vector dot product

$$v_r = \mathbf{v}\cdot\mathbf{u}_r = (-8\,\mathbf{i} - 12\,\mathbf{j} - 24\,\mathbf{k})\cdot\left(\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}\right) = -8\left(\frac{2}{\sqrt{13}}\right) + \left[-12\left(\frac{3}{\sqrt{13}}\right)\right] = -14.42\text{ m/s}$$

$$v_z = \mathbf{v}\cdot\mathbf{u}_z = (-8\,\mathbf{i} - 12\,\mathbf{j} - 24\,\mathbf{k})\cdot(\mathbf{k}) = -24.0\text{ m/s}$$

$$a_r = \mathbf{a}\cdot\mathbf{u}_r = (-2\,\mathbf{i} - 3\,\mathbf{j} - 6\,\mathbf{k})\cdot\left(\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}\right) = -2\left(\frac{2}{\sqrt{13}}\right) + \left[-3\left(\frac{3}{\sqrt{13}}\right)\right] = -3.606\text{ m/s}^2$$

$$a_z = a\cdot\mathbf{u}_z = (-2\,\mathbf{i} - 3\,\mathbf{j} - 6\,\mathbf{k})\cdot\mathbf{k} = -6.00\text{ m/s}^2$$

Thus, in vector form

$\mathbf{v} = \{-14.4\,\mathbf{u}_r - 24.0\,\mathbf{u}_z\}$ m/s **Ans.**

$a = \{-3.61\,\mathbf{u}_r - 6.00\,\mathbf{u}_z\}$ m/s^2 **Ans.**

These components can also be determined using trigonometry by first obtain angle ϕ shown in Fig. a.

$$OA = \sqrt{2^2 + 3^2 + 6^2} = 7\text{ m} \qquad OB = \sqrt{2^2 + 3^2} = \sqrt{13}$$

Thus,

$\sin\phi = \dfrac{6}{7}$ and $\cos\phi = \dfrac{\sqrt{13}}{7}$. Then

$$v_r = -v\cos\phi = -28\left(\frac{\sqrt{13}}{7}\right) = -14.42\text{ m/s}$$

$$v_z = -v\sin\phi = -28\left(\frac{6}{7}\right) = -24.0\text{ m/s}$$

$$a_r = -a\cos\phi = -7\left(\frac{\sqrt{13}}{7}\right) = -3.606\text{ m/s}^2$$

$$a_z = -a\sin\phi = -7\left(\frac{6}{7}\right) = -6.00\text{ m/s}^2$$

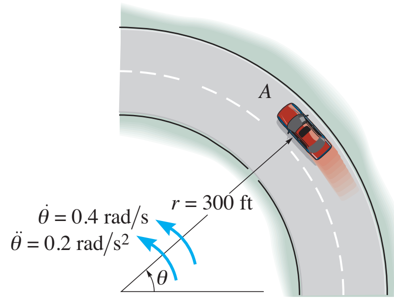
Ans:

$\mathbf{v} = \{-14.4\mathbf{u}_r - 24.0\mathbf{u}_z\}$ m/s

$\mathbf{a} = \{-3.61\mathbf{u}_r - 6.00\mathbf{u}_z\}$ m/s^2

***12–160.**

A car is traveling along the circular curve of radius $r = 300$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.4$ rad/s, which is increasing at the rate of $\ddot{\theta} = 0.2$ rad/s^2. Determine the magnitudes of the car's velocity and acceleration at this instant.



SOLUTION

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 300(0.4) = 120 \text{ ft/s}$$

Thus, the magnitude of the velocity of the car is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 120^2} = 120 \text{ ft/s}$$ **Ans.**

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300\left(0.4^2\right) = -48.0 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300(0.2) + 2(0)(0.4) = 60.0 \text{ ft/s}^2$$

Thus, the magnitude of the acceleration of the car is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-48.0)^2 + 60.0^2} = 76.8 \text{ ft/s}^2$$ **Ans.**

Ans:
$v = 120$ ft/s
$a = 76.8$ ft/s^2

12–161.

If a particle moves along a path such that $r = (2\cos t)$ ft and $\theta = (t/2)$ rad, where t is in seconds, plot the path $r = f(\theta)$ and determine the particle's radial and transverse components of velocity and acceleration.

SOLUTION

$t = 2\theta \qquad r = (2\cos 2\theta)$ ft **Ans.**

$r = 2\cos t \quad \dot{r} = -2\sin t \quad \ddot{r} = -2\cos t$

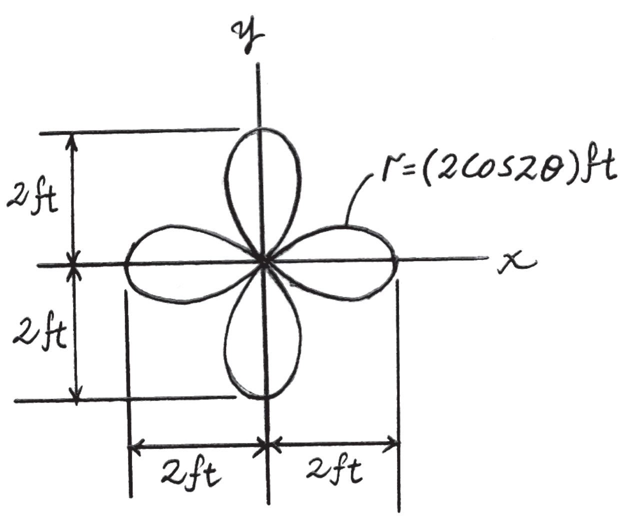
$$\theta = \frac{t}{2} \qquad \dot{\theta} = \frac{1}{2} \qquad \ddot{\theta} = 0$$

$v_r = \dot{r} = -2\sin t$ **Ans.**

$v_\theta = r\dot{\theta} = (2\cos t)\left(\frac{1}{2}\right) = \cos t$ **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = -2\cos t - (2\cos t)\left(\frac{1}{2}\right)^2 = -\frac{5}{2}\cos t$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2\cos t(0) + 2(-2\sin t)\left(\frac{1}{2}\right) = -2\sin t$ **Ans.**



Ans:

$r = (2\cos 2\theta)$ ft

$v_r = -2\sin t$

$v_\theta = \cos t$

$a_r = -\frac{5}{2}\cos t$

$a_\theta = -2\sin t$

12–162.

If a particle's position is described by the polar coordinates $r = (2 \sin 2\theta)$ m and $\theta = (4t)$ rad, where t is in seconds, determine the radial and transverse components of its velocity and acceleration when $t = 1$ s.

SOLUTION

When $t = 1$ s

$\theta = 4\,t = 4$

$\dot{\theta} = 4$

$\ddot{\theta} = 0$

$r = 2 \sin 2\theta = 1.9787$

$\dot{r} = 4 \cos 2\theta\, \dot{\theta} = -2.3280$

$\ddot{r} = -8 \sin 2\theta(\dot{\theta})^2 + 4 \cos 2\theta\, \ddot{\theta} = -126.638$

$v_r = \dot{r} = -2.33 \text{ m/s}$ **Ans.**

$v_\theta = r\dot{\theta} = 1.9787(4) = 7.91 \text{ m/s}$ **Ans.**

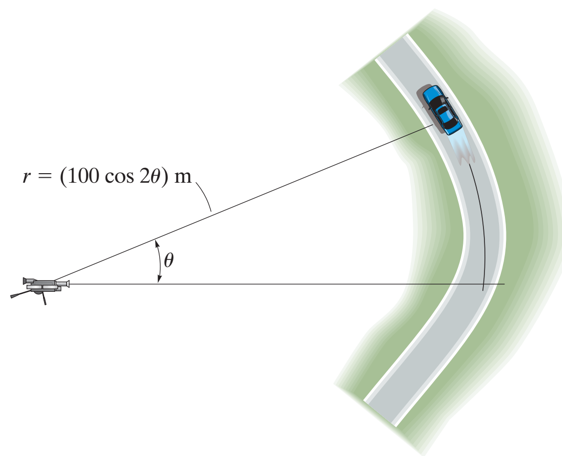
$a_r = \ddot{r} - r(\dot{\theta})^2 = -126.638 - (1.9787)(4)^2 = -158 \text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.9787(0) + 2(-2.3280)(4) = -18.6 \text{ m/s}^2$ **Ans.**

Ans:
$v_r = -2.33 \text{ m/s}$
$v_\theta = 7.91 \text{ m/s}$
$a_r = -158 \text{ m/s}^2$
$a_\theta = -18.6 \text{ m/s}^2$

12–163.

The driver of the car maintains a constant speed of 40 m/s. Determine the angular velocity of the camera tracking the car when $\theta = 15°$.



SOLUTION

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 (\sin 2\theta)\dot{\theta}) \text{ m/s}$$

At $\theta = 15°$,

$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60 \text{ m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30°\dot{\theta} = -100\dot{\theta} \text{ m/s}$$

Velocity: Referring to Fig. a, $v_r = -40 \cos \phi$ and $v_\theta = 40 \sin \phi$.

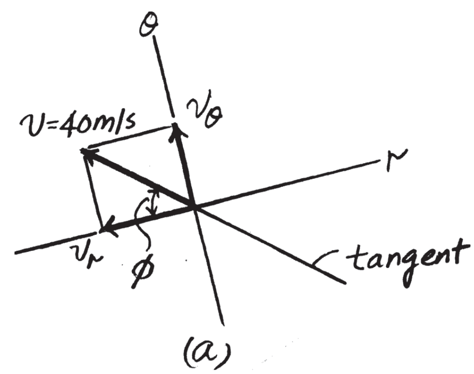
$$v_r = \dot{r}$$

$$-40 \cos \phi = -100\dot{\theta} \quad \textbf{(1)}$$

and

$$v_\theta = r\dot{\theta}$$

$$40 \sin \phi = 86.60\dot{\theta} \quad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

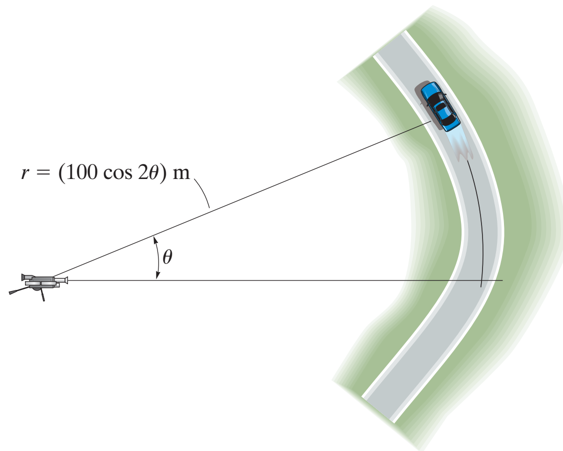
$$\phi = 40.89°$$

$$\dot{\theta} = 0.3024 \text{ rad/s} = 0.302 \text{ rad/s} \quad \textbf{Ans.}$$

Ans:
$\dot{\theta} = 0.302$ rad/s

***12–164.**

When $\theta = 15°$, the car has a speed of 50 m/s which is increasing at 6 m/s^2. Determine the angular velocity of the camera tracking the car at this instant.



SOLUTION

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200\,(\sin 2\theta)\dot{\theta})\ \text{m/s}$$

$$\ddot{r} = -200\left[(\sin 2\theta)\ddot{\theta}) + 2(\cos 2\theta)\dot{\theta}^2\right]\ \text{m/s}^2$$

At $\theta = 15°$,

$$r\Big|_{\theta = 15°} = 100 \cos 30° = 86.60\ \text{m}$$

$$\dot{r}\Big|_{\theta = 15°} = -200 \sin 30°\dot{\theta} = -100\dot{\theta}\ \text{m/s}$$

$$\dot{r}\Big|_{\theta = 15°} = -200\left[\sin 30°\ddot{\theta} + 2\cos 30°\,\dot{\theta}^2\right] = (-100\ddot{\theta} - 346.41\dot{\theta}^2)\ \text{m/s}^2$$

Velocity: Referring to Fig. a, $v_r = -50 \cos \phi$ and $v_\theta = 50 \sin \phi$.Thus,

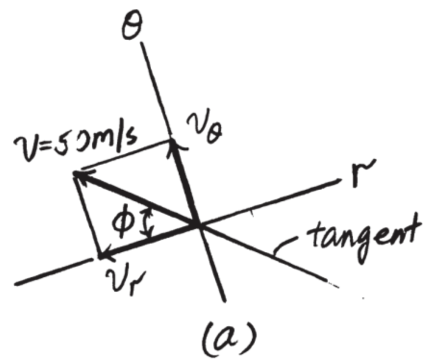
$$v_r = \dot{r}$$

$$-50 \cos \phi = -100\dot{\theta} \qquad \textbf{(1)}$$

and

$$v_\theta = r\dot{\theta}$$

$$50 \sin \phi = 86.60\dot{\theta} \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$\phi = 40.89°$$

$$\dot{\theta} = 0.378\ \text{rad/s} \qquad \textbf{Ans.}$$

Ans:
$\dot{\theta} = 0.378$ rad/s

12–165.

The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, $\dot{\mathbf{a}}$, in terms of its cylindrical components, using Eq. 12–32.

SOLUTION

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{u}_\theta + \ddot{z}\,\mathbf{u}_z$$

$$\dot{\mathbf{a}} = \left(\dddot{r} - \dot{r}\dot{\theta}^2 - 2r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(\ddot{r} - r\dot{\theta}^2\right)\dot{\mathbf{u}}_r + \left(\dot{r}\ddot{\theta} + r\dddot{\theta} + 2\ddot{r}\dot{\theta} + 2\dot{r}\ddot{\theta}\right)\mathbf{u}_\theta + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\dot{\mathbf{u}}_\theta + \dddot{z}\,\mathbf{u}_z + \ddot{z}\dot{\mathbf{u}}_z$$

But, $\dot{\mathbf{u}}_r = \dot{\theta}\mathbf{u}_\theta \quad \dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r \quad \dot{\mathbf{u}}_z = 0$

Substituting and combining terms yields

$$\dot{\mathbf{a}} = \left(\dddot{r} - 3\dot{r}\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(3\dot{r}\ddot{\theta} + r\dddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3\right)\mathbf{u}_\theta + \left(\dddot{z}\right)\mathbf{u}_z \qquad \textbf{Ans.}$$

Ans:
$\dot{\mathbf{a}} = (\dddot{r} - 3\dot{r}\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta})\mathbf{u}_r$
$+ (3\dot{r}\ddot{\theta} + r\dddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3)\mathbf{u}_\theta + (\dddot{z})\mathbf{u}_z$

12–166.

The position of a particle is described by $r = (300e^{-0.5t})$ mm and $\theta = (0.3t^2)$ rad, where t is in seconds. Determine the magnitudes of the particle's velocity and acceleration at the instant $t = 1.5$ s.

SOLUTION

Time Derivatives: The first and second time derivative of r and θ when $t = 1.5$ s are

$$r = 300e^{-0.5t}\Big|_{t-1.5\text{ s}} = 141.77\text{ mm} \qquad \theta = 0.3t^2\text{ rad}$$

$$\dot{r} = -150e^{-0.5t}\Big|_{t-1.5\text{ s}} = -70.85\text{ mm/s} \qquad \dot{\theta} = 0.6t\Big|_{t-1.5\text{ s}} = 0.9\text{ rad/s}$$

$$\ddot{r} = 75e^{-0.5t}\Big|_{t-1.5\text{ s}} = 35.43\text{ mm/s}^2 \qquad \ddot{\theta} = 0.6\text{ rad/s}^2$$

Velocity:

$$v_r = \dot{r} = -70.85\text{ mm/s} \qquad v_\theta = r\dot{\theta} = 141.71(0.9) = 127.54\text{ mm/s}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{v_r^{\,2} + v_\theta^{\,2}} = \sqrt{(-70.85)^2 + 127.54^2} = 146\text{ mm/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 35.43 - 141.71\left(0.9^2\right) = -79.36\text{ mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 141.71(0.6) + 2(-70.85)(0.9) = -42.51\text{ mm/s}^2$$

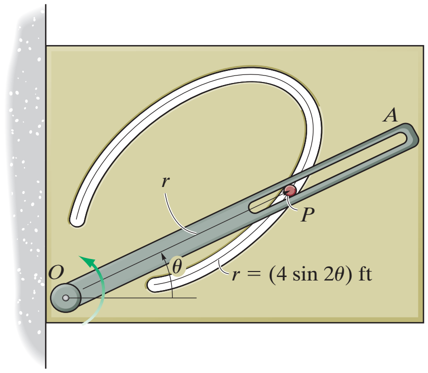
Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \sqrt{(-79.36)^2 + (-42.51)^2} = 90.0\text{ mm/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 146$ mm/s
$a = 90.0$ mm/s^2

12–167.

The pin P is constrained to move along the curve defined by the lemniscate $r = (4 \sin 2\theta)$ ft. If the angular position of the slotted arm OA is defined by $\theta = (3t^{3/2})$ rad, where t is in seconds, determine the magnitudes of the velocity and acceleration of the pin P when $\theta = 60°$.



SOLUTION

Time Derivatives:

$$r = 4 \sin 2\theta$$

$$\dot{r} = (8(\cos 2\theta)\dot{\theta})\ \text{ft/s}$$

$$\ddot{r} = 8[(\cos 2\theta)\ddot{\theta} - 2(\sin 2\theta)\dot{\theta}^2]\ \text{ft/s}^2$$

When $\theta = 60° = \dfrac{\pi}{3}$ rad,

$$\frac{\pi}{3} = 3t^{3/2} \qquad t = 0.4958\,\text{s}$$

Thus, the angular velocity and angular acceleration of arm OA when $\theta = \dfrac{\pi}{3}$ rad $(t = 0.4958\,\text{s})$ are

$$\dot{\theta} = \frac{9}{2}t^{1/2}\bigg|_{t=0.4958\,\text{s}} = 3.168\ \text{rad/s}$$

$$\ddot{\theta} = \frac{9}{4t^{1/2}}\bigg|_{t=0.4958\,\text{s}} = 3.196\ \text{rad/s}^2$$

Thus,

$$r|_{\theta=60°} = 4 \sin 120° = 3.464\ \text{ft}$$

$$\dot{r}|_{\theta=60°} = 8 \cos 120°(3.168) = -12.67\ \text{ft/s}$$

$$\ddot{r}|_{\theta=60°} = 8[\cos 120°(3.196) - 2 \sin 120°(3.168^2)] = -151.89\ \text{ft/s}^2$$

Velocity:

$$v_r = \dot{r} = -12.67\ \text{ft/s} \qquad v_\theta = r\dot{\theta} = 3.464(3.168) = 10.98\ \text{ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-12.67)^2 + 10.98^2} = 16.8\ \text{ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -151.89 - 3.464(3.168^2) = -186.67\ \text{ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3.464(3.196) + 2(-12.67)(3.168) = -69.24\ \text{ft/s}^2$$

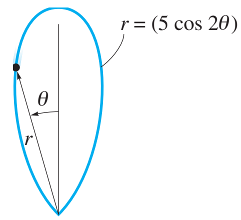
Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-186.67)^2 + (-69.24)^2} = 199\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 16.8$ ft/s
$a = 199$ ft/s^2

***12–168.**

A particle travels along the portion of the "four-leaf rose" defined by the equation $r = (5\cos 2\theta)$ m. If the angular velocity of the radial coordinate line is $\dot{\theta} = (3t^2)$ rad/s, where t is in seconds, determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = 30°$. When $t = 0$, $\theta = 0$.



SOLUTION

$\dot{\theta} = 3\,t^2$

$\ddot{\theta} = 6\,t$

$$\int_0^\theta d\theta = \int_0^t 3\,t^2\,dt$$

$\theta = t^3$

At $\theta = 30° = \dfrac{\pi}{6}$

$t = (\pi/6)^{1/3} = 0.806$

$\dot{\theta} = 1.95$

$\ddot{\theta} = 4.84$

$r = 5\cos 2\theta$

$\dot{r} = -10\sin 2\theta\,\dot{\theta}$

$\ddot{r} = -10(2\cos 2\theta)(\dot{\theta})^2 + \sin 2\theta\,\ddot{\theta})$

At $\theta = 30°$

$r = 2.5$

$\dot{r} = -16.88$

$\ddot{r} = -79.87$

$v_r = \dot{r} = -16.9$ m/s **Ans.**

$v_\theta = r\dot{\theta} = 2.5(1.95) = 4.87$ m/s **Ans.**

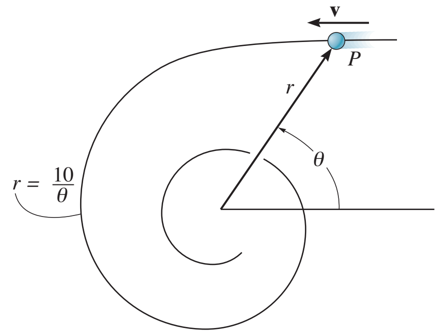
$a_r = \ddot{r} - r(\dot{\theta})^2 = -79.86 - 2.5(1.95)^2 = -89.4$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 2.5(4.84) + 2(-16.88)(1.95) = -53.7$ m/s^2 **Ans.**

Ans:
$v_r = -16.9$ m/s
$v_\theta = 4.87$ m/s
$a_r = -89.4$ m/s^2
$a_\theta = -53.7$ m/s^2

12–169.

A particle P moves along the spiral path $r = (10/\theta)$ ft, where θ is in radians. If it maintains a constant speed of $v = 20$ ft/s, determine v_r and v_θ as functions of θ and evaluate each at $\theta = 1$ rad.



SOLUTION

$$r = \frac{10}{\theta}$$

$$\dot{r} = -\left(\frac{10}{\theta^2}\right)\dot{\theta}$$

Since $v^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)\dot{\theta}^2 + \left(\frac{10^2}{\theta^2}\right)\dot{\theta}^2$$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)(1 + \theta^2)\dot{\theta}^2$$

Thus, $\dot{\theta} = \dfrac{2\theta^2}{\sqrt{1 + \theta^2}}$

$$v_r = \dot{r} = -\left(\frac{10}{\theta^2}\right)\left(\frac{2\theta^2}{\sqrt{1 + \theta^2}}\right) = -\frac{20}{\sqrt{1 + \theta^2}} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = \left(\frac{10}{\theta}\right)\left(\frac{2\theta^2}{\sqrt{1 + \theta^2}}\right) = \frac{20\theta}{\sqrt{1 + \theta^2}} \qquad \textbf{Ans.}$$

When $\theta = 1$ rad

$$v_r = \left(-\frac{20}{\sqrt{2}}\right) = -14.1 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = \left(\frac{20}{\sqrt{2}}\right) = 14.1 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:

$v_r = -\dfrac{20}{\sqrt{1 + \theta^2}}$

$v_\theta = \dfrac{20\theta}{\sqrt{1 + \theta^2}}$

$v_r = -14.1$ ft/s

$v_\theta = 14.1$ ft/s

12–170.

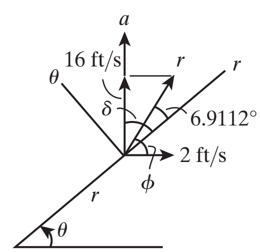
A particle moves in the x–y plane such that its position is defined by $\mathbf{r} = \{2t\mathbf{i} + 4t^2\mathbf{j}\}$ ft, where t is in seconds. Determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

$\mathbf{r} = 2t\mathbf{i} + 4\,t^2\mathbf{j}|_{t=2} = 4\mathbf{i} + 16\mathbf{j}$

$\mathbf{v} = 2\mathbf{i} + 8\,t\mathbf{j}|_{t=2} = 2\mathbf{i} + 16\mathbf{j}$

$\mathbf{a} = 8\mathbf{j}$



$$\theta = \tan^{-1}\left(\frac{16}{4}\right) = 75.964°$$

$$v = \sqrt{(2)^2 + (16)^2} = 16.1245 \text{ ft/s}$$

$$\phi = \tan^{-1}\left(\frac{16}{2}\right) = 82.875°$$

$a = 8 \text{ ft/s}^2$

$\phi - \theta = 6.9112°$

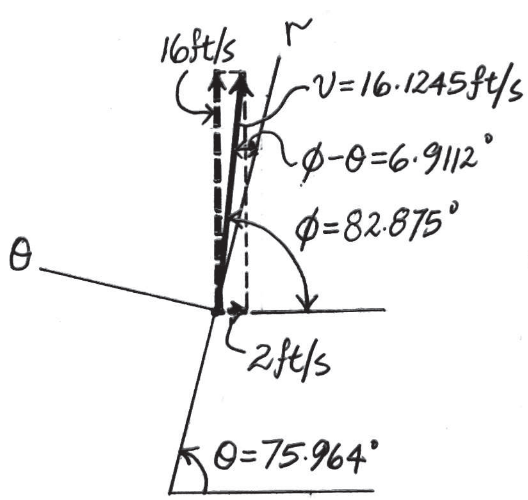
$v_r = 16.1245 \cos 6.9112° = 16.0 \text{ ft/s}$ **Ans.**

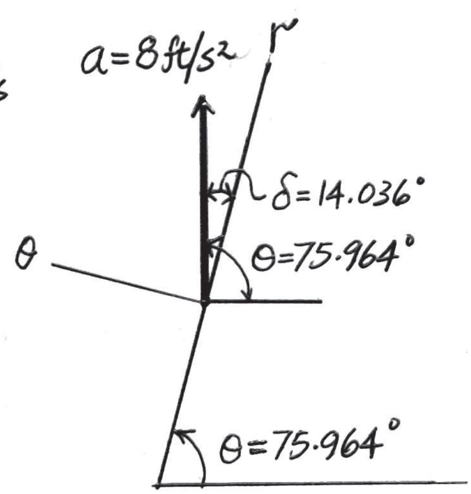
$v_\theta = 16.1245 \sin 6.9112° = 1.94 \text{ ft/s}$ **Ans.**

$\delta = 90° - \theta = 14.036°$

$a_r = 8 \cos 14.036° = 7.76 \text{ ft/s}^2$ **Ans.**

$a_\theta = 8 \sin 14.036° = 1.94 \text{ ft/s}^2$ **Ans.**

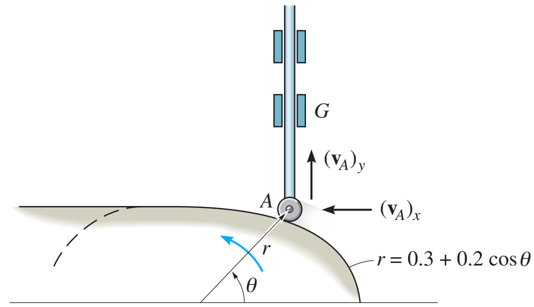




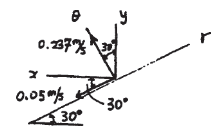
Ans:
$v_r = 16.0$ ft/s
$v_\theta = 1.94$ ft/s
$a_r = 7.76$ ft/s^2
$a_\theta = 1.94$ ft/s^2

12–171.

The mechanism of a machine is constructed so that the roller at A follows the surface of the cam described by the equation $r = (0.3 + 0.2\cos\theta)$ m. If $\dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$, determine the magnitudes of the roller's velocity and acceleration when $\theta = 30°$. Neglect the size of the roller. Also determine the velocity components $(\mathbf{v}_A)_x$ and $(\mathbf{v}_A)_y$ of the roller at this instant. The rod to which the roller is attached remains vertical and can slide up or down along the guides while the guides translate horizontally to the left.



SOLUTION



$\dot{\theta} = 0.5$

$\ddot{\theta} = 0$

$r = (0.3 + 0.2\cos\theta)$

$\dot{r} = -0.2\sin\theta\,\dot{\theta}$

$\ddot{r} = -0.2(\cos\theta\,\dot{\theta}^2 + \sin\theta\,\ddot{\theta})$

At $\theta = 30°$

$r = 0.473$

$\ddot{r} = -0.0433$

$v_r = \dot{r} = -0.05$

$v_\theta = r\dot{\theta} = 0.473(0.5) = 0.237$

$v = \sqrt{(-0.05)^2 + (0.237)^2} = 0.242$ m/s **Ans.**

$a_r = \ddot{r} - r\,\dot{\theta}^2 = -0.0433 - 0.473(0.5)^2$

$a_r = -0.162$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.05)(0.5)$

$a_\theta = -0.05$ m/s^2

$a = \sqrt{(-0.162)^2 + (-0.5)^2} = 0.169$ m/s^2 **Ans.**

$(\overset{+}{\leftarrow})$ $(v_A)_x = 0.05\cos 30° + 0.237\sin 30°$

$(v_A)_x = 0.162$ m/s $\leftarrow$ **Ans.**

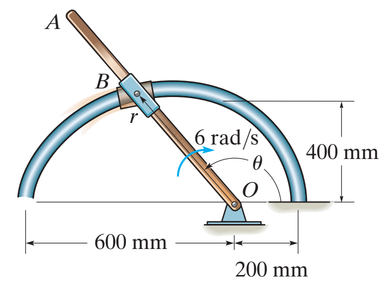
$(+\uparrow)$ $(v_A)_y = -0.05\sin 30° + 0.237\cos 30°$

$(v_A)_y = 0.180$ m/s $\uparrow$ **Ans.**

Ans:
$v = 0.242$ m/s
$a = 0.169$ m/s^2
$(v_A)_x = 0.162$ m/s $\leftarrow$
$(v_A)_y = 0.180$ m/s $\uparrow$

***12–172.**

The rod OA rotates clockwise with a constant angular velocity of 6 rad/s. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 200(2 - \cos\theta)$ mm. Determine the speed of the slider blocks at the instant $\theta = 150°$.



SOLUTION

Velocity: Using the chain rule, the first and second time derivatives of r can be determined.

$$r = 200(2 - \cos\theta)$$

$$\dot{r} = 200(\sin\theta)\,\dot{\theta} = \{200(\sin\theta)\,\dot{\theta}\}\ \text{mm/s}$$

$$\ddot{r} = \{200[(\cos\theta)\dot{\theta}^2 + (\sin\theta)\ddot{\theta}]\}\ \text{mm/s}^2$$

The radial and transverse components of the velocity are

$$v_r = \dot{r} = \{200(\sin\theta)\dot{\theta}\}\ \text{mm/s}$$

$$v_\theta = r\dot{\theta} = \{200(2 - \cos\theta)\dot{\theta}\}\ \text{mm/s}$$

Since $\dot{\theta}$ is in the opposite sense to that of positive θ, $\dot{\theta} = -6$ rad/s. Thus, at $\theta = 150°$,

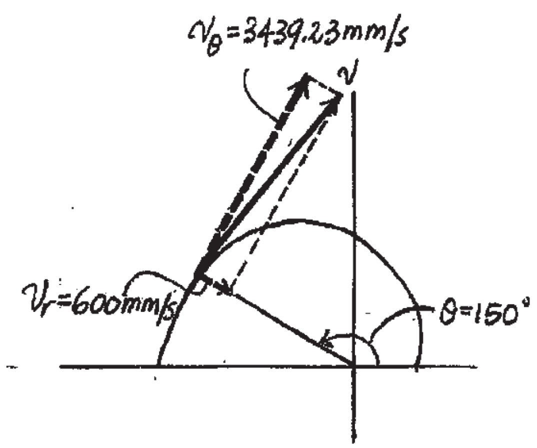
$$v_r = 200(\sin 150°)(-6) = -600\ \text{mm/s}$$

$$v_\theta = 200(2 - \cos 150°)(-6) = -3439.23\ \text{mm/s}$$

Thus, the magnitude of the velocity is

$$v = \sqrt{v_r^2 + v_\theta^2}\ \ \sqrt{(-600)^2 + (-3439.23)^2} = 3491\ \text{mm/s} = 3.49\ \text{m/s} \qquad \textbf{Ans.}$$

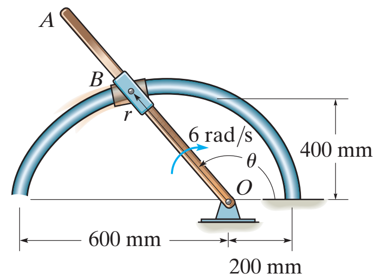
These components are shown in Fig. *a*



Ans:
$v = 3.49$ m/s

12–173.

Determine the magnitude of the acceleration of the slider blocks in Prob. 12–172 when $\theta = 150°$.



SOLUTION

Acceleration: Using the chain rule, the first and second time derivatives of r can be determined

$$r = 200(2 - \cos\theta)$$

$$\dot{r} = 200(\sin\theta)\dot{\theta} = \{200(\sin\theta)\dot{\theta}\}\text{ mm/s}$$

$$\ddot{r} = \{200[(\cos\theta)\dot{\theta}^2 + (\sin\theta)\ddot{\theta}]\}\text{ mm/s}^2$$

Here, since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$. Since $\dot{\theta}$ is in the opposite sense to that of positive θ, $\dot{\theta} = -6$ rad/s. Thus, at $\theta = 150°$

$$r = 200(2 - \cos 150°) = 573.21\text{ mm}$$

$$\dot{r} = 200(\sin 150°)(-6) = -600\text{ mm/s}$$

$$\ddot{r} = 200\left[(\cos 150°)(-6)^2 + \sin 150°(0)\right] = -6235.38\text{ mm/s}^2$$

The radial and transverse components of the acceleration are

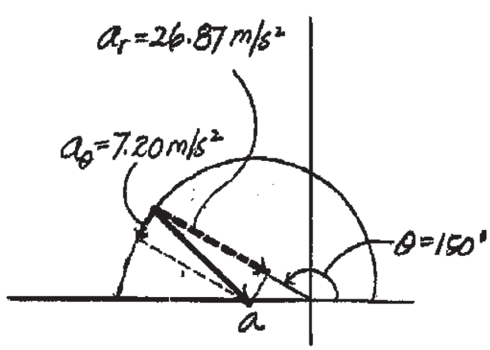
$$a_r = \ddot{r} - r\dot{\theta}^2 = -6235.38 - 573.21\,(-6)^2 = -26870.77\text{ mm/s}^2 = -26.87\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 573.21(0) + 2(-600)(-6) = 7200\text{ mm/s}^2 = 7.20\text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-26.87)^2 + 7.20^2} = 27.82\text{ m/s}^2 = 27.8\text{ m/s}^2 \qquad \textbf{Ans.}$$

These components are shown in Fig. *a*.

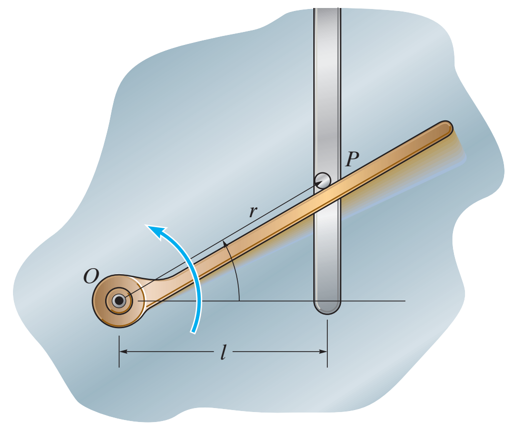


(a)

Ans:
$a = 27.8\text{ m/s}^2$

12–174.

The link is pinned at O, and as a result of its rotation it drives the peg P along the vertical guide. Calculate the magnitudes of the velocity and acceleration of P if $\theta = ct$ rad, where c is a constant.



SOLUTION

$$\theta = ct \qquad \dot{\theta} = c \qquad \ddot{\theta} = 0$$

$$r = \frac{l}{\cos\theta} = l\sec\theta = l\sec ct$$

$$\dot{r} = l\sec\theta\tan\theta\dot{\theta} = lc\sec ct\tan ct$$

$$\ddot{r} = l\{[(\sec\theta\tan\theta\dot{\theta})\tan\theta + \sec\theta(\sec^2\theta\dot{\theta})]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\}$$

$$= l[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}]$$

$$= l[c^2\sec ct\tan^2 ct + c^2\sec^3 ct]$$

Velocity:

$$v_r = \dot{r} = lc\sec ct\tan ct$$

$$v_\theta = r\dot{\theta} = lc\sec ct(c) = lc\sec ct$$

$$v = \sqrt{v_r^2 + v_\theta^2}$$

$$= \sqrt{(lc\sec ct\tan ct)^2 + (lc\sec ct)^2}$$

$$= lc\sec ct\sqrt{\tan^2 ct + 1} \qquad \text{Where} \qquad \tan^2 ct + 1 = \sec^2 ct$$

$$= lc\sec^2 ct \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$= l[c^2\sec ct\tan^2 ct + c^2\sec^3 ct] - l\sec ct\,(c)^2$$

$$= lc^2\sec ct[\tan^2 ct + \sec^2 ct - 1] \qquad \text{where} \quad \sec^2 ct - 1 = \tan^2 ct$$

$$= 2lc^2\sec ct\tan^2 ct$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$= 0 + 2(lc\sec ct\tan ct)(c)$$

$$= 2lc^2\sec ct\tan ct$$

$$a = \sqrt{a_r^2 + a_\theta^2}$$

$$= \sqrt{(2lc^2\sec ct\tan^2 ct)^2 + (2lc^2\sec ct\tan ct)^2}$$

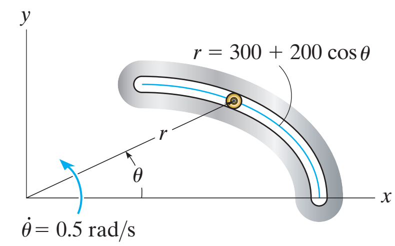
$$= 2lc^2\sec ct\tan ct\sqrt{1 + \tan^2 ct} \qquad \text{Where} \quad \tan^2 ct + 1 = \sec^2 ct$$

$$= 2lc^2\sec^2 ct\tan ct \qquad \textbf{Ans.}$$

Ans:
$v = lc\sec^2 ct$
$a = 2lc^2\sec^2 ct\tan ct$

12–175.

The mechanism within a machine is constructed so that the pin follows the path described by the equation $r = (300 + 200\cos\theta)$ mm. If $\dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$, determine the magnitudes of the pin's velocity and acceleration when $\theta = 30°$. Neglect the size of the pin. Also, calculate the velocity components $(v_A)_x$ and $(v_A)_y$ of the pin at this instant.



SOLUTION

$$r = 300 + 200\cos\theta = 300 + 200\cos 30° = 473.2 \text{ mm}$$

$$\dot{r} = -200\sin\theta\dot{\theta} = -200\sin 30°(0.5) = -50 \text{ mm/s}$$

$$\ddot{r} = -200\left[\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right] = -200\left[\cos 30°(0.5)^2 + \sin 30°(0)\right] = -43.30 \text{ mm/s}^2$$

Velocity:

$$v_r = \dot{r} = -50 \text{ mm/s}$$

$$v_\theta = r\dot{\theta} = 473.2(0.5) = 236.6 \text{ mm/s}$$

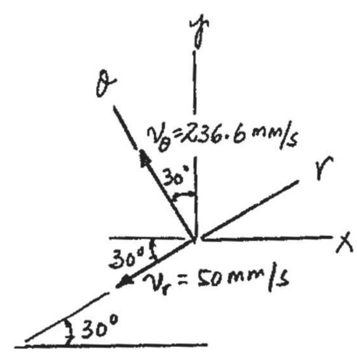
$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-50)^2 + 236.6^2} = 242 \text{ mm/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -43.30 - 473.2(0.5)^2 = -161.6 \text{ mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 473.2(0) + 2(-50)(0.5) = -50 \text{ mm/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-161.6)^2 + (-50)^2} = 169 \text{ mm/s}^2 \qquad \textbf{Ans.}$$



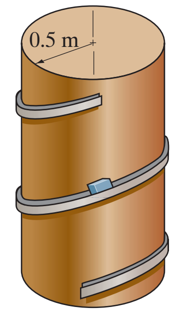
$(\overset{+}{\leftarrow})\qquad (v_A)_x = 236.6\sin 30° + 50\cos 30° = 162 \text{ mm/s} \leftarrow$ **Ans.**

$(+\uparrow)\qquad (v_A)_y = 236.6\cos 30° - 50\sin 30° = 180 \text{ mm/s} \uparrow$ **Ans.**

Ans:
$v = 242$ mm/s
$a = 169$ mm/s^2
$(v_A)_x = 162$ mm/s $\leftarrow$
$(v_A)_y = 180$ mm/s $\uparrow$

***12–176.**

The box slides down the helical ramp with a constant speed of $v = 2$ m/s. Determine the magnitude of its acceleration. The ramp descends a vertical distance of 1 m for every full revolution. The mean radius of the ramp is $r = 0.5$ m.



SOLUTION

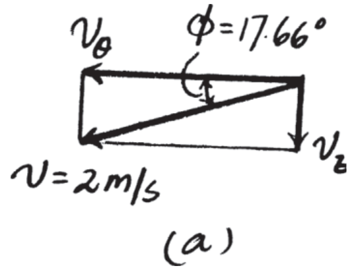
Velocity: The inclination angle of the ramp is $\phi = \tan^{-1}\dfrac{L}{2\pi r} = \tan^{-1}\left[\dfrac{1}{2\pi(0.5)}\right] = 17.66°$.

Thus, from Fig. a, $v_\theta = 2\cos 17.66° = 1.906$ m/s and $v_z = 2\sin 17.66° = 0.6066$ m/s. Thus,

$$v_\theta = r\dot{\theta}$$

$$1.906 = 0.5\dot{\theta}$$

$$\dot{\theta} = 3.812 \text{ rad/s}$$



Acceleration: Since $r = 0.5$ m is constant, $\dot{r} = \ddot{r} = 0$. Also, $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$. Using the above results,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(3.812)^2 = -7.264 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(0) + 2(0)(3.812) = 0$$

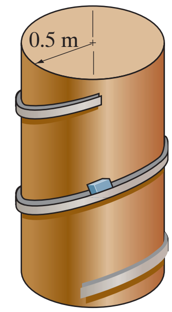
Since $\boldsymbol{v}_z$ is constant $a_z = 0$. Thus, the magnitude of the box's acceleration is

$$a = \sqrt{{a_r}^2 + {a_\theta}^2 + {a_z}^2} = \sqrt{(-7.264)^2 + 0^2 + 0^2} = 7.26 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 7.26$ m/s^2

12–177.

The box slides down the helical ramp such that $r = 0.5$ m, $\theta = (0.5t^3)$ rad, and $z = (2 - 0.2t^2)$ m, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the box at the instant $\theta = 2\pi$ rad.



SOLUTION

Time Derivatives:

$$r = 0.5 \text{ m}$$

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = \left(1.5t^2\right) \text{ rad/s} \qquad \ddot{\theta} = (3t) \text{ rad/s}^2$$

$$z = 2 - 0.2t^2$$

$$\dot{z} = (-0.4t) \text{ m/s} \qquad \ddot{z} = -0.4 \text{ m/s}^2$$

When $\theta = 2\pi$ rad,

$$2\pi = 0.5t^3 \qquad t = 2.325 \text{ s}$$

Thus,

$$\dot{\theta}\big|_{t=2.325\text{ s}} = 1.5(2.325)^2 = 8.108 \text{ rad/s}$$

$$\ddot{\theta}\big|_{t=2.325\text{ s}} = 3(2.325) = 6.975 \text{ rad/s}^2$$

$$\dot{z}\big|_{t=2.325\text{ s}} = -0.4(2.325) = -0.92996 \text{ m/s}$$

$$\ddot{z}\big|_{t=2.325\text{ s}} = -0.4 \text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 0.5(8.108) = 4.05385 \text{ m/s}$$

$$v_z = \dot{z} = -0.92996 \text{ m/s}$$

Thus, the magnitude of the box's velocity is

$$v = \sqrt{{v_r}^2 + {v_\theta}^2 + {v_z}^2} = \sqrt{0^2 + 4.05385^2 + (-0.92996)^2} = 4.16 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(8.108)^2 = -32.867 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(6.975) + 2(0)(8.108)^2 = 3.487 \text{ m/s}^2$$

$$a_z = \ddot{z} = -0.4 \text{ m/s}^2$$

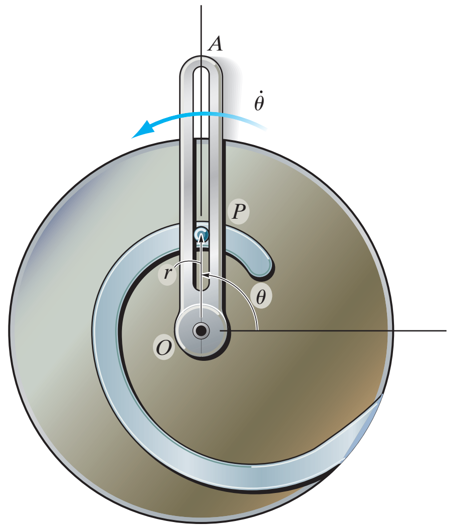
Thus, the magnitude of the box's acceleration is

$$a = \sqrt{{a_r}^2 + {a_\theta}^2 + {a_z}^2} = \sqrt{(-32.867)^2 + 3.487^2 + (-0.4)^2} = 33.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 4.16$ m/s
$a = 33.1$ m/s^2

12–178.

The motion of the pin P is controlled by the rotation of the grooved link OA. If the link is rotating at a constant angular rate of $\dot{\theta} = 6$ rad/s, determine the magnitudes of the velocity and acceleration of P at the instant $\theta = \pi/2$ rad. The spiral path is defined by the equation $r = (40\,\theta)$ mm, where θ is in radians.



SOLUTION

$$r = 40\theta|_{\theta=\pi/2} = 20\pi \text{ mm}$$

$$\dot{r} = 40\dot{\theta} = 40(6) = 240 \text{ mm/s}$$

$$\ddot{r} = 40\ddot{\theta} = 40(0) = 0$$

Velocity:

$$v_r = \dot{r} = 240 \text{ mm/s}$$

$$v_\theta = r\dot{\theta} = 20\pi(6) = 120\pi \text{ mm/s}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{240^2 + (120\pi)^2} = 447 \text{ mm/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 20\pi(6)^2 = -720\pi \text{ mm/s}^2$$

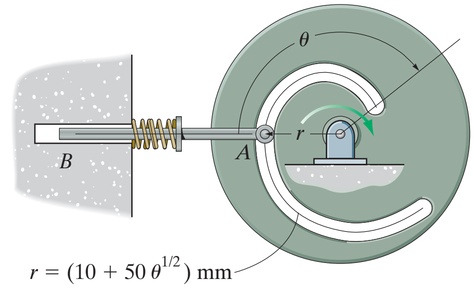
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 20\pi(0) + 2(240)(6) = 2880 \text{ mm/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-720\pi)^2 + 2880^2} = 3662 \text{ mm/s}^2 = 3.66 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 447$ mm/s
$a = 3.66$ m/s^2

12–179.

If the circular plate rotates clockwise with a constant angular velocity of $\dot{\theta} = 1.5$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod AB when $\theta = (2/3\pi)$ rad.



SOLUTION

Time Derivatives:

$$r = \left(10 + 50\theta^{1/2}\right) \text{mm}$$

$$\dot{r} = 25\theta^{-1/2}\dot{\theta} \text{ mm/s}$$

$$\ddot{r} = 25\left[\theta^{-1/2}\ddot{\theta} - \frac{1}{2}\theta^{-3/2}\dot{\theta}^2\right] \text{mm/s}^2$$

When $\theta = \dfrac{2\pi}{3}$ rad,

$$r|_{\theta=\frac{2\pi}{3}} = \left[10 + 50\left(\frac{2\pi}{3}\right)^{1/2}\right] = 82.36 \text{ mm}$$

$$\dot{r}|_{\theta=\frac{2\pi}{3}} = 25\left(\frac{2\pi}{3}\right)^{-1/2}(1.5) = 25.91 \text{ mm/s}$$

$$\ddot{r}|_{\theta=\frac{2\pi}{3}} = 25\left[0 - \frac{1}{2}\left(\frac{2\pi}{3}\right)^{-3/2}(1.5^2)\right] = -9.279 \text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

$$v_{AB} = \dot{r} = 25.9 \text{ mm/s}$$ **Ans.**

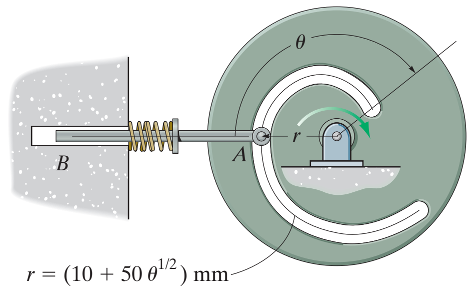
Acceleration: The rod's acceleration is given by

$$a_{AB} = \ddot{r} = -9.279 \text{ mm/s}^2 = -9.28 \text{ mm/s}^2$$ **Ans.**

Ans:
$v_{AB} = 25.9$ mm/s
$a_{AB} = -9.28$ mm/s^2

***12–180.**

When $\theta = (2/3\pi)$ rad, the angular velocity and angular acceleration of the circular plate are $\dot{\theta} = 1.5$ rad/s and $\ddot{\theta} = 3$ rad/s^2, respectively. Determine the magnitudes of the velocity and acceleration of the rod AB at this instant.



SOLUTION

Time Derivatives:

$$r = (10 + 50\theta^{1/2})\text{ mm}$$

$$\dot{r} = 25\theta^{-1/2}\dot{\theta}\text{ mm/s}$$

$$\ddot{r} = 25\left[\theta^{-1/2}\ddot{\theta} - \frac{1}{2}\theta^{-3/2}\dot{\theta}^2\right]\text{ mm/s}^2$$

When $\theta = \dfrac{2\pi}{3}$ rad,

$$r|_{\theta=\frac{2\pi}{3}} = \left[10 + 50\left(\frac{2\pi}{3}\right)^{1/2}\right] = 82.36\text{ mm}$$

$$\dot{r}|_{\theta=\frac{2\pi}{3}} = 25\left(\frac{2\pi}{3}\right)^{-1/2}(1.5) = 25.91\text{ mm/s}$$

$$\ddot{r}|_{\theta=\frac{2\pi}{3}} = 25\left[\left(\frac{2\pi}{3}\right)^{-1/2}(3) - \frac{1}{2}\left(\frac{2\pi}{3}\right)^{-3/2}(1.5^2)\right] = 42.54\text{ mm/s}^2$$

For the rod,

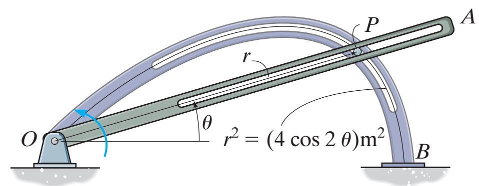
$$v_{AB} = \dot{r} = 25.91\text{ mm/s} = 25.9\text{ mm/s}$$ **Ans.**

$$a_{AB} = \ddot{r} = 42.54\text{ mm/s}^2 = 42.5\text{ mm/s}^2$$ **Ans.**

Ans:
$v_{AB} = 25.9$ mm/s
$a_{AB} = 42.5$ mm/s^2

12–181.

The motion of peg P is constrained by the lemniscate curved slot in OB and by the slotted arm OA. If OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 3$ rad/s, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30°$.



SOLUTION

Time Derivatives:

$$r^2 = 4\cos 2\theta$$

$$2r\dot{r} = -8\sin 2\theta\dot{\theta}$$

$$\dot{r} = \left(\frac{-4\sin 2\theta\dot{\theta}}{r}\right)\text{ m/s} \qquad \dot{\theta} = 3\text{ rad/s}$$

$$2\left(r\ddot{r} + \dot{r}^2\right) = -8\left(\sin 2\theta\ddot{\theta} + 2\cos 2\theta\,\dot{\theta}^2\right)$$

$$\ddot{r} = \left[\frac{-4\left(\sin 2\theta\ddot{\theta} + 2\cos 2\theta\dot{\theta}^2\right) - \dot{r}^2}{r}\right]\text{ m/s}^2 \qquad \ddot{\theta} = 0$$

$\dot{\theta} = 3$ rad/s. Thus, when $\theta = 30°$,

$$r\big|_{\theta=30°} = \sqrt{4\cos 60°} = \sqrt{2}\text{ m}$$

$$\dot{r}\big|_{\theta=30°} = \frac{-4\sin 60°(3)}{\sqrt{2}} = -7.348\text{ m/s}$$

$$\ddot{r}\big|_{\theta=30°} = \frac{-4\left[0 + 2\cos 60°(3)^2\right] - (-7.348)^2}{\sqrt{2}} = -63.64\text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = -7.348\text{ m/s} \qquad v_\theta = r\dot{\theta} = \sqrt{2}(3) = 4.243\text{ m/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^{\,2} + v_\theta^{\,2}} = \sqrt{7.348^2 + (-4.243)^2} = 8.49\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -63.64 - \sqrt{2}(3)^2 = -76.37\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-7.348)(3) = -44.09\text{ m/s}^2$$

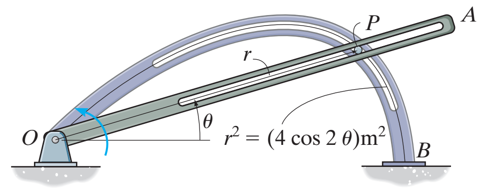
Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \sqrt{(-76.37)^2 + (-44.09)^2} = 88.2\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 8.49$ m/s
$a = 88.2$ m/s^2

12–182.

The motion of peg P is constrained by the lemniscate curved slot in OB and by the slotted arm OA. If OA rotates counterclockwise with an angular velocity of $\dot{\theta} = (3t^{3/2})$ rad/s, where t is in seconds, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30°$. When $t = 0$, $\theta = 0°$.



SOLUTION

Time Derivatives:

$$r^2 = 4\cos 2\theta$$

$$2r\dot{r} = -8\sin 2\theta\dot{\theta}$$

$$\dot{r} = \left(\frac{-4\sin 2\theta\dot{\theta}}{r}\right)\text{ m/s}$$

$$2\left(r\ddot{r} + \dot{r}^2\right) = -8\left(\sin 2\theta\ddot{\theta} + 2\cos 2\theta\,\dot{\theta}^2\right)$$

$$\ddot{r} = \left[\frac{-4\left(\sin 2\theta\ddot{\theta} + 2\cos 2\theta\dot{\theta}^2\right) - \dot{r}^2}{r}\right]\text{ m/s}^2$$

$$\frac{d\theta}{dt} = \dot{\theta} = 3t^{3/2}$$

$$\int_{0°}^{\theta} d\theta = \int_0^t 3t^{3/2}dt$$

$$\theta = \left(\frac{6}{5}t^{5/2}\right)\text{ rad}$$

At $\theta = 30° = \dfrac{\pi}{6}$ rad,

$$\frac{\pi}{6} = \frac{6}{5}t^{5/2} \qquad t = 0.7177\text{ s}$$

$$\dot{\theta} = 3t^{3/2}\Big|_{t=0.7177\text{ s}} = 1.824\text{ rad/s}$$

$$\ddot{\theta} = \frac{9}{2}t^{1/2}\Big|_{t=0.7177\text{ s}} = 3.812\text{ rad/s}^2$$

Thus,

$$r\big|_{\theta=30°} = \sqrt{4\cos 60°} = \sqrt{2}\text{ m}$$

$$\dot{r}\big|_{\theta=30°} = \frac{-4\sin 60°(1.824)}{\sqrt{2}} = -4.468\text{ m/s}$$

$$\ddot{r}\big|_{\theta=30°} = \frac{-4\left[\sin 60°(3.812) + 2\cos 60°(1.824)^2\right] - (-4.468)^2}{\sqrt{2}} = -32.86\text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = -4.468\text{ m/s} \qquad v_\theta = r\dot{\theta} = \sqrt{2}(1.824) = 2.579\text{ m/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^{\,2} + v_\theta^{\,2}} = \sqrt{(-4.468)^2 + 2.579^2} = 5.16\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -32.86 - \sqrt{2}(1.824)^2 = -37.57\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \sqrt{2}(3.812) + 2(-4.468)(1.824) = -10.91\text{ m/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \sqrt{(-37.57)^2 + (-10.91)^2} = 39.1\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 5.16$ m/s
$a = 39.1$ m/s^2

12–183.

The car travels along a road, which for a short distance is defined by $r = (200/\theta)$ ft, where θ is in radians. If it maintains a constant speed of $v = 35$ ft/s, determine the radial and transverse components of its velocity when $\theta = \pi/3$ rad.

SOLUTION

$$r = \frac{200}{\theta}\bigg|_{\theta=\pi/3\text{ rad}} = \frac{600}{\pi}\text{ ft}$$

$$\dot{r} = -\frac{200}{\theta^2}\dot{\theta}\bigg|_{\theta=\pi/3\text{ rad}} = -\frac{1800}{\pi^2}\dot{\theta}$$

$$v_r = \dot{r} = -\frac{1800}{\pi^2}\dot{\theta} \qquad v_\theta = r\dot{\theta} = \frac{600}{\pi}\dot{\theta}$$

$$v^2 = v_r^2 + v_\theta^2$$

$$35^2 = \left(-\frac{1800}{\pi^2}\dot{\theta}\right)^2 + \left(\frac{600}{\pi}\dot{\theta}\right)^2$$

$$\dot{\theta} = 0.1325\text{ rad/s}$$

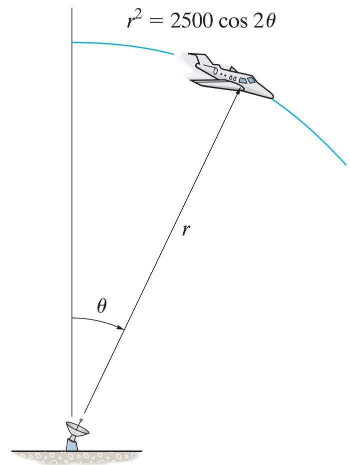
$$v_r = -\frac{1800}{\pi^2}(0.1325) = -24.2\text{ ft/s}$$ **Ans.**

$$v_\theta = \frac{600}{\pi}(0.1325) = 25.3\text{ ft/s}$$ **Ans.**

Ans:
$v_r = -24.2$ ft/s
$v_\theta = 25.3$ ft/s

***12–184.**

For a short time the jet plane moves along a path in the shape of a lemniscate, $r^2 = (2500 \cos 2\theta)\ \text{km}^2$. At the instant $\theta = 30°$, the radar tracking device is rotating at $\dot{\theta} = 5(10^{-3})$ rad/s with $\ddot{\theta} = 2(10^{-3})$ rad/s^2. Determine the radial and transverse components of velocity and acceleration of the plane at this instant.



SOLUTION

Time Derivatives: Here, $\dot{\theta} = 5(10^{-3})$ rad/s and $\ddot{\theta} = 2(10^{-3})$ rad/s^2.

$$r^2 = 2500 \cos 2\theta \qquad r = 50\sqrt{\cos 2\theta}$$

$$2r\dot{r} = -5000 \sin 2\theta\dot{\theta} \qquad \dot{r} = -\frac{2500 \sin 2\theta}{r}\dot{\theta}$$

$$2r\ddot{r} + 2\dot{r}^2 = -5000\left(2 \cos 2\theta\dot{\theta}^2 + \sin 2\theta\ddot{\theta}\right)$$

$$\ddot{r} = \frac{-2500(2 \cos 2\theta\dot{\theta}^2 + \sin 2\theta\ddot{\theta}) - \dot{r}^2}{r}$$

At $\theta = 30°$, $r = 50\sqrt{\cos 60°} = 35.36$ km, $\dot{r} = -\dfrac{2500 \sin 60°}{35.36}\left[5\left(10^{-3}\right)\right] = -0.3062$ km/s

and $\ddot{r} = \dfrac{-2500\{2 \cos 60°\left[5(10^{-3})\right]^2 + \sin 60°\left[2(10^{-3})\right]\} - (-0.3062)^2}{35.36} = -0.1269\ \text{km/s}^2$.

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = -0.3062\ \text{km/s} = 306\ \text{m/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 35.36\left[5\left(10^{-3}\right)\right] = 0.1768\ \text{km/s} = 177\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1269 - 35.36\left[5\left(10^{-3}\right)\right]^2$$

$$= -0.1278\ \text{km/s}^2 = -128\ \text{m/s}^2 \qquad \textbf{Ans.}$$

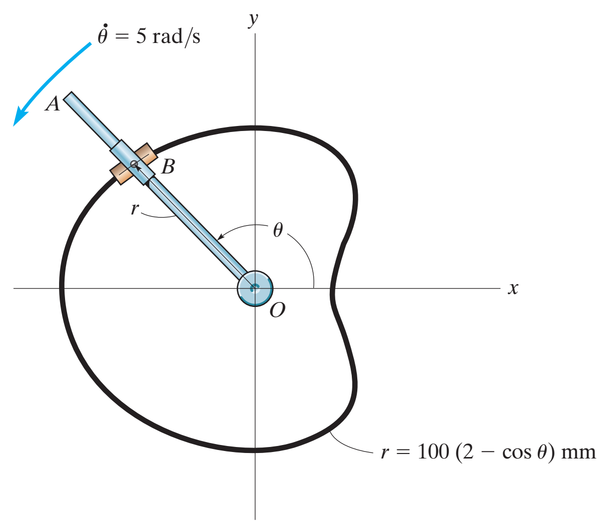
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 35.36\left[2\left(10^{-3}\right)\right] + 2(-0.3062)\left[5\left(10^{-3}\right)\right]$$

$$= 0.06765\ \text{km/s}^2 = 67.7\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_r = 306$ m/s
$v_\theta = 177$ m/s
$a_r = -128$ m/s^2
$a_\theta = 67.7$ m/s^2

12–185.

The rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 100(2 - \cos\theta)$ mm. Determine the speed of the slider blocks at the instant $\theta = 120°$.



SOLUTION

$\dot{\theta} = 5$

$r = 100(2 - \cos\theta)$

$\dot{r} = 100 \sin\theta\dot{\theta} = 500 \sin\theta$

At $\theta = 120°$,

$v_r = \dot{r} = 500 \sin 120° = 433.013$

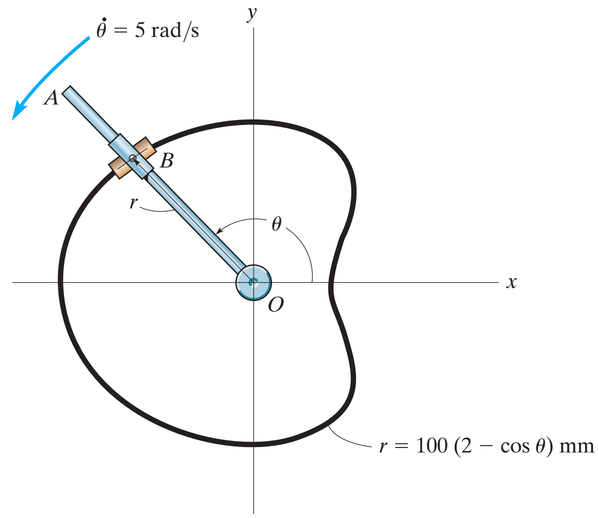
$v_\theta = r\dot{\theta} = 100\,(2 - \cos 120°)(5) = 1250$

$v = \sqrt{(433.013)^2 + (1250)^2} = 1322.9 \text{ mm/s} = 1.32 \text{ m/s}$ **Ans.**

Ans:
$v = 1.32$ m/s

12–186.

Determine the magnitude of the acceleration of the slider blocks in Prob. 12–185 when $\theta = 120°$.



SOLUTION

$\dot{\theta} = 5$

$\dot{\theta} = 0$

$r = 100(2 - \cos\theta)$

$\dot{r} = 100\sin\theta\dot{\theta} = 500\sin\theta$

$\ddot{r} = 500\cos\theta\dot{\theta} = 2500\cos\theta$

$a_r = \ddot{r} - r\dot{\theta}^2 = 2500\cos\theta - 100(2 - \cos\theta)(5)^2 = 5000(\cos 120° - 1) = -7500\ \text{mm/s}^2$

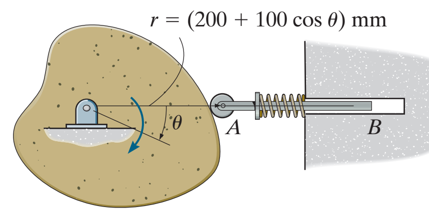
$a_s = r\dot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(500\sin\theta)(5) = 5000\sin 120° = 4330.1\ \text{mm/s}^2$

$a = \sqrt{(-7500)^2 + (4330.1)^2} = 8660.3\ \text{mm/s}^2 = 8.66\ \text{m/s}^2$ **Ans.**

Ans:
$a = 8.66\ \text{m/s}^2$

12–187.

If the cam rotates clockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod AB at the instant $\theta = 30°$. The surface of the cam has a shape of limaçon defined by $r = (200 + 100\cos\theta)$ mm.



SOLUTION

Time Derivatives:

$$r = (200 + 100\cos\theta)\text{ mm}$$

$$\dot{r} = (-100\sin\theta\dot{\theta})\text{ mm/s} \qquad \dot{\theta} = 5\text{ rad/s}$$

$$\ddot{r} = -100\left[\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right]\text{ mm/s}^2 \qquad \ddot{\theta} = 0$$

When $\theta = 30°$,

$$r\Big|_{\theta=30°} = 200 + 100\cos 30° = 286.60\text{ mm}$$

$$\dot{r}\Big|_{\theta=30°} = -100\sin 30°(5) = -250\text{ mm/s}$$

$$\ddot{r}\Big|_{\theta=30°} = -100\left[0 + \cos 30°\left(5^2\right)\right] = -2165.06\text{ mm/s}^2$$

For the rod,

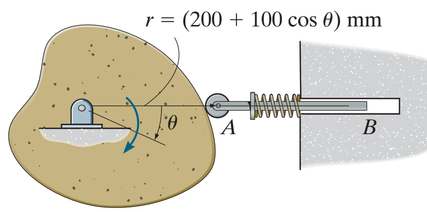
$$v_{AB} = \dot{r} = -250\text{ mm/s} \qquad \textbf{Ans.}$$

$$a_{AB} = \ddot{r} = -2165.06\text{ mm/s}^2 = -2.17\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_{AB} = -250$ mm/s
$a_{AB} = -2.17$ m/s^2

***12–188.**

At the instant $\theta = 30°$, the cam rotates with a clockwise angular velocity of $\dot{\theta} = 5\text{ rad/s}$ and angular acceleration of $\ddot{\theta} = 6\text{ rad/s}^2$. Determine the magnitudes of the velocity and acceleration of the follower rod AB at this instant. The surface of the cam has the shape of a limaçon defined by $r = (200 + 100\cos\theta)$ mm.



SOLUTION

Time Derivatives:

$$r = (200 - 100\cos\theta)\text{ mm}$$

$$\dot{r} = (-100\sin\theta\dot{\theta})\text{ mm/s}$$

$$\ddot{r} = -100\left[\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right]\text{ mm/s}^2$$

When $\theta = 30°$,

$$r\big|_{\theta=30°} = 200 + 100\cos 30° = 286.60\text{ mm}$$

$$\dot{r}\big|_{\theta=30°} = -100\sin 30°(5) = -250\text{ mm/s}$$

$$\ddot{r}\big|_{\theta=30°} = -100\left[\sin 30°(6) + \cos 30°\left(5^2\right)\right] = -2465.06\text{ mm/s}^2$$

For the rod

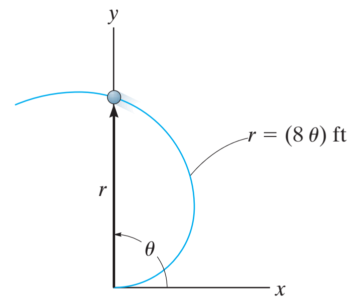
$$v_{AB} = \dot{r} = -250\text{ mm/s}$$ **Ans.**

$$a_{AB} = \ddot{r} = -2465.06\text{ mm/s}^2 = -2.47\text{ m/s}^2$$ **Ans.**

Ans:
$v_{AB} = -250$ mm/s
$a_{AB} = -2.47$ m/s^2

12–189.

A particle moves along an Archimedean spiral $r = (8\theta)$ ft, where θ is in radians. If $\dot{\theta} = 4$ rad/s (constant), determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = \pi/2$ rad. Sketch the curve and show the components on the curve.



SOLUTION

Time Derivatives: Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi \text{ ft} \qquad \dot{r} = 8\dot{\theta} = 8(4) = 32.0 \text{ ft/s} \qquad \ddot{r} = 8\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

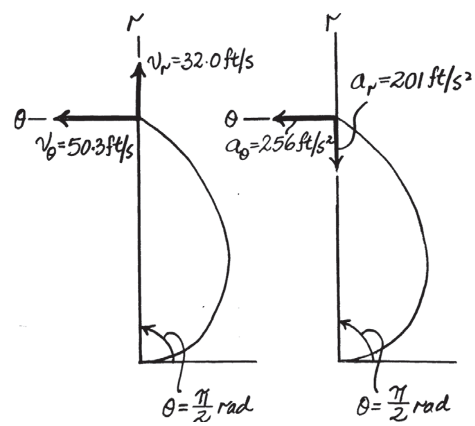
$$v_r = \dot{r} = 32.0 \text{ ft/s}$$ **Ans.**

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3 \text{ ft/s}$$ **Ans.**

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4\pi\left(4^2\right) = -201 \text{ ft/s}^2$$ **Ans.**

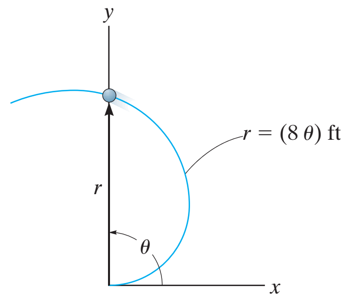
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(32.0)(4) = 256 \text{ ft/s}^2$$ **Ans.**



Ans:
$a_r = -201 \text{ ft/s}^2$
$a_\theta = 256 \text{ ft/s}^2$

12–190.

Solve Prob. 12–189 if the particle has an angular acceleration $\ddot{\theta} = 5\ \text{rad/s}^2$ when $\dot{\theta} = 4\ \text{rad/s}$ at $\theta = \pi/2$ rad.



SOLUTION

Time Derivatives: Here,

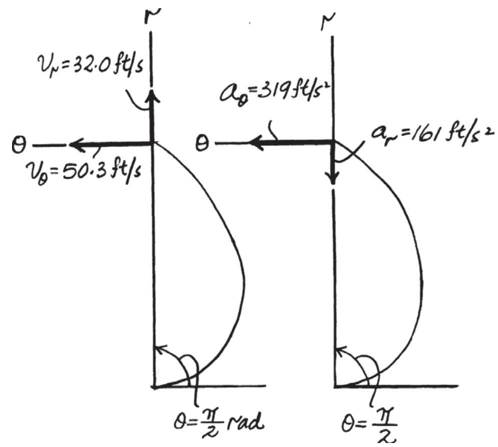
$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi\ \text{ft} \qquad \dot{r} = 8\dot{\theta} = 8(4) = 32.0\ \text{ft/s}$$

$$\ddot{r} = 8\ddot{\theta} = 8(5) = 40\ \text{ft/s}^2$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 32.0\ \text{ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3\ \text{ft/s} \qquad \textbf{Ans.}$$



Acceleration: Applying Eq. 12–29, we have

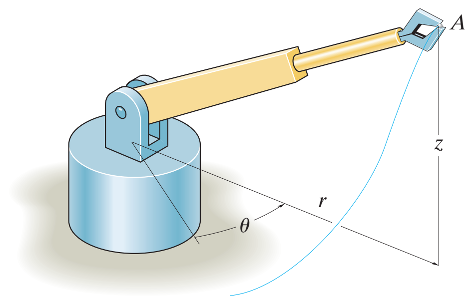
$$a_r = \ddot{r} - r\dot{\theta}^2 = 40 - 4\pi\left(4^2\right) = -161\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4\pi(5) + 2(32.0)(4) = 319\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_r = 32.0\ \text{ft/s}$
$v_\theta = 50.3\ \text{ft/s}$
$a_r = -161\ \text{ft/s}^2$
$a_\theta = 319\ \text{ft/s}^2$

12–191.

The arm of the robot moves so that $r = 3$ ft is constant, and its grip A moves along the path $z = (3 \sin 4\theta)$ ft, where θ is in radians. If $\theta = (0.5t)$ rad, where t is in seconds, determine the magnitudes of the grip's velocity and acceleration when $t = 3$ s.



SOLUTION

$\theta = 0.5\,t \qquad r = 3 \qquad z = 3 \sin 2t$

$\dot{\theta} = 0.5 \qquad \dot{r} = 0 \qquad \dot{z} = 6 \cos 2t$

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = -12 \sin 2t$

At $t = 3$ s,

$z = -0.8382$

$\dot{z} = 5.761$

$\ddot{z} = 3.353$

$v_r = 0$

$v_\theta = 3(0.5) = 1.5$

$v_z = 5.761$

$v = \sqrt{(0)^2 + (1.5)^2 + (5.761)^2} = 5.95 \text{ ft/s}$ **Ans.**

$a_r = 0 - 3(0.5)^2 = -0.75$

$a_\theta = 0 + 0 = 0$

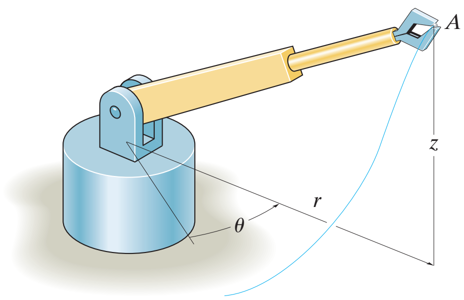
$a_z = 3.353$

$a = \sqrt{(-0.75)^2 + (0)^2 + (3.353)^2} = 3.44 \text{ ft/s}^2$ **Ans.**

Ans:
$v = 5.95 \text{ ft/s}$
$a = 3.44 \text{ ft/s}^2$

***12–192.**

For a short time the arm of the robot is extending such that $\dot{r} = 1.5$ ft/s when $r = 3$ ft, $z = (4t^2)$ ft, and $\theta = 0.5t$ rad, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the grip A when $t = 3$ s.



SOLUTION

$\theta = 0.5\,t$ rad $\qquad r = 3$ ft $\qquad z = 4\,t^2$ ft

$\dot{\theta} = 0.5$ rad/s $\qquad \dot{r} = 1.5$ ft/s $\qquad \dot{z} = 8\,t$ ft/s

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = 8$ ft/s^2

At $t = 3$ s,

$\theta = 1.5 \qquad r = 3 \qquad z = 36$

$\dot{\theta} = 0.5 \qquad \dot{r} = 1.5 \qquad \dot{z} = 24$

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = 8$

$v_r = 1.5$

$v_\theta = 3(0.5) = 1.5$

$v_z = 24$

$v = \sqrt{(1.5)^2 + (1.5)^2 + (24)^2} = 24.1$ ft/s **Ans.**

$a_r = 0 - 3(0.5)^2 = -0.75$

$a_\theta = 0 + 2(1.5)(0.5) = 1.5$

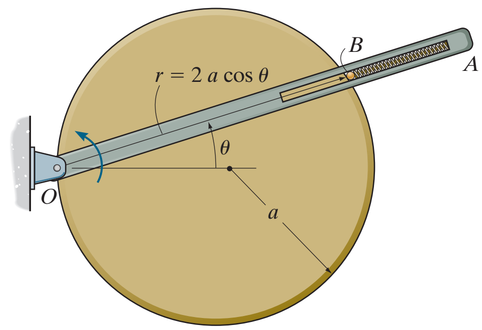
$a_z = 8$

$a = \sqrt{(-0.75)^2 + (1.5)^2 + (8)^2} = 8.17$ ft/s^2 **Ans.**

Ans:
$v = 24.1$ ft/s
$a = 8.17$ ft/s^2

12–193.

The slotted arm OA rotates counterclockwise about O with a constant angular velocity of $\dot{\theta}$. The motion of pin B is constrained such that it moves on the fixed circular surface and along the slot in OA. Determine the magnitudes of the velocity and acceleration of pin B as a function of θ.



SOLUTION

Time Derivatives:

$$r = 2a\cos\theta$$

$$\dot{r} = -2a\sin\theta\dot{\theta}$$

$$\ddot{r} = -2a\left[\cos\theta\dot{\theta}\dot{\theta} + \sin\theta\ddot{\theta}\right] = -2a\left[\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right]$$

Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$. Thus,

$$\ddot{r} = -2a\cos\theta\dot{\theta}^2$$

Velocity:

$$v_r = \dot{r} = -2a\sin\theta\dot{\theta} \qquad v_\theta = r\dot{\theta} = 2a\cos\theta\dot{\theta}$$

Thus, the magnitude of the pin's velocity is

$$v = \sqrt{{v_r}^2 + {v_\theta}^2} = \sqrt{\left(-2a\sin\theta\dot{\theta}\right)^2 + \left(2a\cos\theta\dot{\theta}\right)^2}$$

$$= \sqrt{4a^2\dot{\theta}^2\left(\sin^2\theta + \cos^2\theta\right)} = 2a\dot{\theta} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2a\cos\theta\dot{\theta}^2 - 2a\cos\theta\dot{\theta}^2 = -4a\cos\theta\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2\left(-2a\sin\theta\dot{\theta}\right)\dot{\theta} = -4a\sin\theta\dot{\theta}^2$$

Thus, the magnitude of the pin's acceleration is

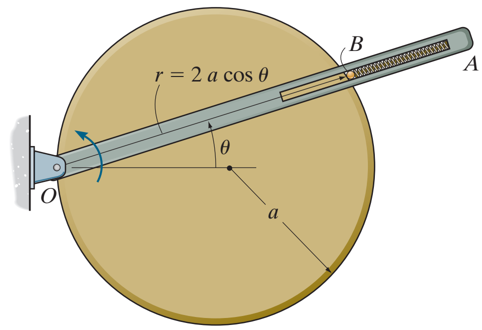
$$a = \sqrt{{a_r}^2 + {a_\theta}^2} = \sqrt{\left(-4a\cos\theta\dot{\theta}^2\right)^2 + \left(-4a\sin\theta\dot{\theta}^2\right)^2}$$

$$= \sqrt{16a^2\dot{\theta}^4\left(\cos^2\theta + \sin^2\theta\right)} = 4a\dot{\theta}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 2a\dot{\theta}$
$a = 4a\dot{\theta}^2$

12–194.

The slotted arm OA rotates counterclockwise about O such that when $\theta = \pi/4$, arm OA is rotating with an angular velocity of $\dot{\theta}$ and an angular acceleration of $\ddot{\theta}$. Determine the magnitudes of the velocity and acceleration of pin B at this instant. The motion of pin B is constrained such that it moves on the fixed circular surface and along the slot in OA.



SOLUTION

Time Derivatives:

$$r = 2a\cos\theta$$

$$\dot{r} = -2a\sin\theta\dot{\theta}$$

$$\ddot{r} = -2a\left[\cos\theta\dot{\theta}\dot{\theta} + \sin\theta\ddot{\theta}\right] = -2a\left[\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right]$$

When $\theta = \dfrac{\pi}{4}$ rad,

$$r\big|_{\theta} = \tfrac{\pi}{4} = 2a\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}a$$

$$\dot{r}\big|_{\theta} = \tfrac{\pi}{4} = -2a\left(\frac{1}{\sqrt{2}}\right)\dot{\theta} = -\sqrt{2}a\dot{\theta}$$

$$\ddot{r}\big|_{\theta} = \tfrac{\pi}{4} = -2a\left(\frac{1}{\sqrt{2}}\dot{\theta}^2 + \frac{1}{\sqrt{2}}\ddot{\theta}\right) = -\sqrt{2}a\left(\dot{\theta}^2 + \ddot{\theta}\right)$$

Velocity:

$$v_r = \dot{r} = -\sqrt{2}a\dot{\theta} \qquad v_\theta = r\dot{\theta} = \sqrt{2}a\dot{\theta}$$

Thus, the magnitude of the pin's velocity is

$$v = \sqrt{v_r^{\,2} + v_\theta^{\,2}} = \sqrt{\left(-\sqrt{2}a\dot{\theta}\right)^2 + \left(\sqrt{2}a\dot{\theta}\right)^2} = 2a\dot{\theta}$$ **Ans.**

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\sqrt{2}a\left(\dot{\theta}^2 + \ddot{\theta}\right) - \sqrt{2}a\dot{\theta}^2 = -\sqrt{2}a\left(2\dot{\theta}^2 + \ddot{\theta}\right)$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \sqrt{2}a\ddot{\theta} + 2\left(-\sqrt{2}a\dot{\theta}\right)\left(\dot{\theta}\right) = \sqrt{2}a\left(\ddot{\theta} - 2\dot{\theta}^2\right)$$

Thus, the magnitude of the pin's acceleration is

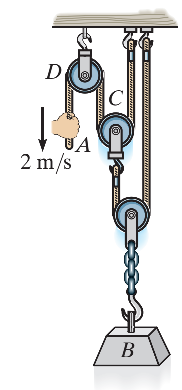
$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \sqrt{\left[-\sqrt{2}a\left(2\dot{\theta}^2 + \ddot{\theta}\right)\right]^2 + \left[\sqrt{2}a\left(\ddot{\theta} - 2\dot{\theta}^2\right)\right]^2}$$

$$= 2a\sqrt{4\dot{\theta}^4 + \ddot{\theta}^2}$$ **Ans.**

Ans:
$v = 2a\dot{\theta}$
$a = 2a\sqrt{4\dot{\theta}^4 + \ddot{\theta}^2}$

12–195.

If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.

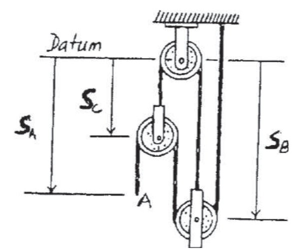


SOLUTION

Position-Coordinate Equation: Datum is established at fixed pulley D. The position of point A, block B and pulley C with respect to datum are s_A, s_B, and s_C respectively. Since the system consists of two cords, two position-coordinate equations can be derived.

$$(s_A - s_C) + (s_B - s_C) + s_B = l_1 \qquad \textbf{(1)}$$

$$s_B + s_C = l_2 \qquad \textbf{(2)}$$



Eliminating s_C from Eqs. (1) and (2) yields

$$s_A + 4s_B = l_1 = 2l_2$$

Time Derivative: Taking the time derivative of the above equation yields

$$v_A + 4v_B = 0 \qquad \textbf{(3)}$$

Since $v_A = 2$ m/s, from Eq. (3)

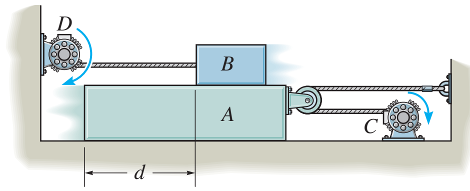
$(+\downarrow)$ $$2 + 4v_B = 0$$

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

Ans:
$v_B = 0.5$ m/s

***12–196.**

The motor at C pulls in the cable with an acceleration $a_C = (3t^2)\ \text{m/s}^2$, where t is in seconds. The motor at D draws in its cable at $a_D = 5\ \text{m/s}^2$. If both motors start at the same instant from rest when $d = 3$ m, determine (a) the time needed for $d = 0$, and (b) the velocities of block A relative to block B when this occurs.



SOLUTION

For A:

$s_A + (s_A - s_C) = l$

$2v_A = v_C$

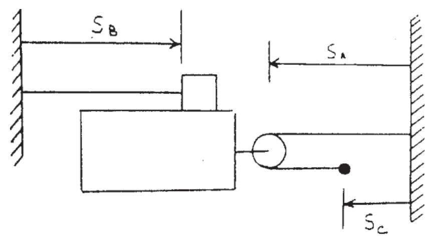
$2a_A = a_C = -3t^2$

$a_A = -1.5t^2 = 1.5t^2 \quad \rightarrow$

$v_A = 0.5t^3 \quad \rightarrow$

$s_A = 0.125t^4 \quad \rightarrow$

For B:



$a_B = 5\ \text{m/s}^2 \quad \leftarrow$

$v_B = 5t \quad \leftarrow$

$s_B = 2.5t^2 \quad \leftarrow$

Require $s_A + s_B = d$

$0.125t^4 + 2.5t^2 = 3$

Set $u = t^2 \qquad 0.125u^2 + 2.5u = 3$

The positive root is $u = 1.1355$. Thus,

$t = 1.0656 = 1.07$ s **Ans.**

$v_A = 0.5(1.0656)^3 = 0.6050\ \text{m/s} \quad \rightarrow$

$v_B = 5(1.0656) = 5.3281\ \text{m/s} \quad \leftarrow$

$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

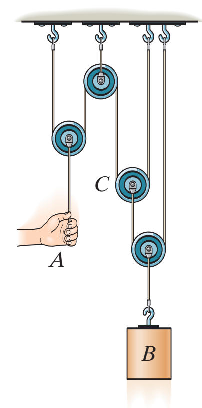
$0.6050\mathbf{i} = -5.3281\mathbf{i} + v_{A/B}\,\mathbf{i}$

$v_{A/B} = 5.93\ \text{m/s} \quad \rightarrow$ **Ans.**

Ans:
$t = 1.07$ s
$v_{A/B} = 5.93\ \text{ms/s} \rightarrow$

12–197.

Determine the displacement of the block at B if A is pulled down 4 ft.



SOLUTION

$2s_A + 2s_C = l_1$

$\Delta s_A = -\Delta s_C$

$s_B - s_C + s_B = l_2$

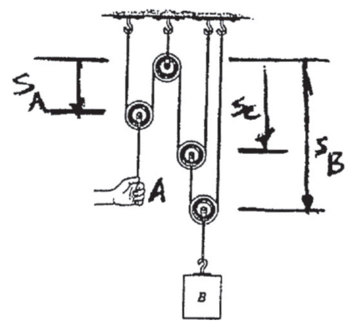
$2\,\Delta s_B = \Delta s_C$

Thus,

$2\,\Delta s_B = -\Delta s_A$

$2\,\Delta s_B = -4$

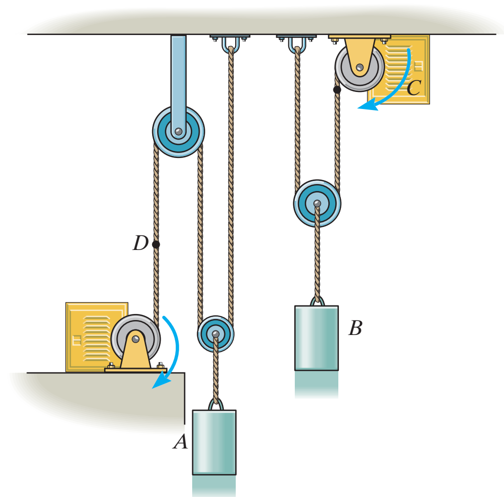
$\Delta s_B = \;-2\text{ ft} = 2\text{ ft}\uparrow$ **Ans.**



Ans:
$\triangle s_B = 2\text{ ft}\uparrow$

12–198.

The motor draws in the cable at C with a constant velocity of $v_C = 4\ \text{m/s}$. The motor draws in the cable at D with a constant acceleration of $a_D = 8\ \text{m/s}^2$. If $v_D = 0$ when $t = 0$, determine (a) the time needed for block A to rise 3 m, and (b) the relative velocity of block A with respect to block B when this occurs.



SOLUTION

(a) $a_D = 8\ \text{m/s}^2$

$v_D = 8\,t$

$s_D = 4\,t^2$

$s_D + 2s_A = l$

$\Delta s_D = -2\Delta s_A$ **(1)**

$\Delta s_A = -2\,t^2$

$-3 = -2\,t^2$

$t = 1.2247 = 1.22\ \text{s}$ **Ans.**

(b) $v_A = \dot{s}_A = -4\,t = -4(1.2247) = -4.90\ \text{m/s} = 4.90\ \text{m/s}\uparrow$

$s_B + (s_B - s_C) = l'$

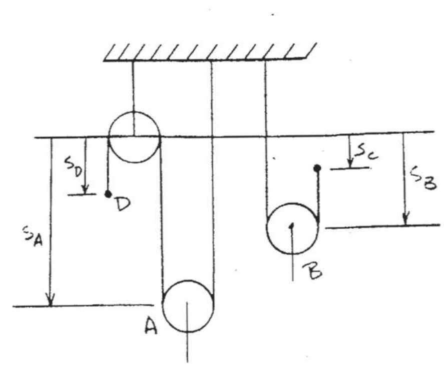
$2v_B = v_C = -4$

$v_B = -2\ \text{m/s} = 2\ \text{m/s}\uparrow$

$(+\downarrow)$ $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$-4.90 = -2 + v_{A/B}$

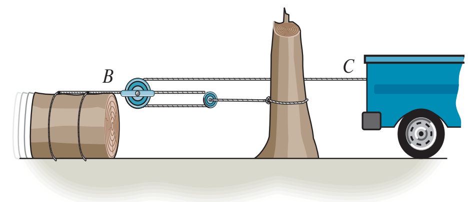
$v_{A/B} = -2.90\ \text{m/s} = 2.90\ \text{m/s}\uparrow$ **Ans.**



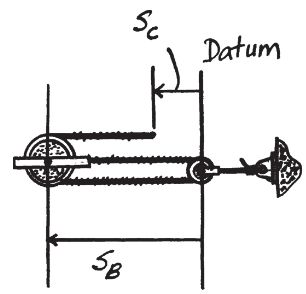
Ans:
$t = 1.22\ \text{s}$
$v_{A/B} = 2.90\ \text{m/s}\uparrow$

12–199.

Determine the displacement of the log if the truck at C pulls the cable 4 ft to the right.



SOLUTION



$2s_B + (s_B - s_C) = l$

$3s_B - s_C = l$

$3\Delta s_B - \Delta s_C = 0$

Since $\Delta s_C = -4$, then

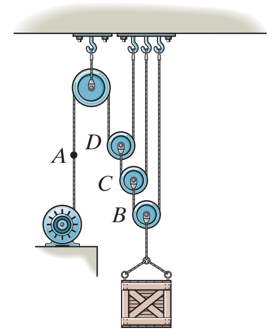
$3\Delta s_B = -4$

$\Delta s_B = -1.33 \text{ ft} = 1.33 \text{ ft} \rightarrow$ **Ans.**

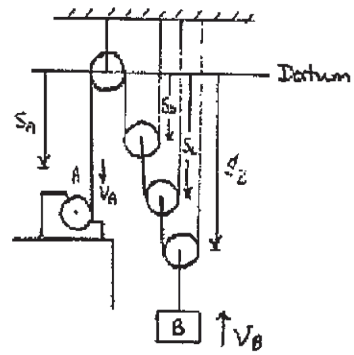
Ans:
$\Delta s_B = 1.33 \text{ ft} \rightarrow$

***12–200.**

Determine the constant speed at which the cable at A must be drawn in by the motor in order to hoist the load 6 m in 1.5 s.



SOLUTION



$$v_B = \frac{6}{1.5} = 4 \text{ m/s} \uparrow$$

$$s_B + (s_B - s_C) = l_1$$

$$s_C + (s_C - s_D) = l_2$$

$$s_A + 2\,s_D = l_3$$

Thus,

$$2s_B - s_C = l_1$$

$$2s_C - s_D = l_2$$

$$s_A + 2s_D = l_3$$

$$2v_A = v_C$$

$$2v_C = v_D$$

$$v_A = -2v_D$$

$$2(2v_B) = v_D$$

$$v_A = -2(4v_B)$$

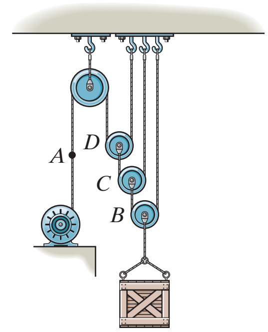
$$v_A = -8v_B$$

$$v_A = -8(-4) = 32 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

Ans:
$v_A = 32 \text{ m/s} \downarrow$

12–201.

Starting from rest, the cable can be wound onto the drum of the motor at a rate of $v_A = (3t^2)$ m/s, where t is in seconds. Determine the time needed to lift the load 7 m.



SOLUTION

$s_B + (s_B - s_C) = l_1$

$s_C + (s_C - s_D) = l_2$

$s_A + 2s_D = l_3$

Thus,

$2s_B - s_C = l_1$

$2s_C - s_D = l_2$

$s_A + 2s_D = l_3$

$2v_B = v_C$

$2v_C = v_D$

$v_A = -2v_D$

$v_A = -8v_B$

$3\,t^2 = -8v_B$

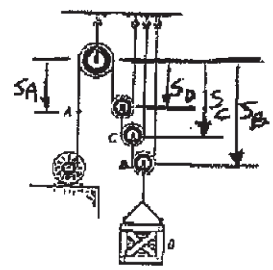
$v_B = \dfrac{-3}{8}t^2$

$s_B = \displaystyle\int_0^t \frac{-3}{8}t^2\,dt$

$s_B = \dfrac{-1}{8}t^3$

$-7 = \dfrac{-1}{8}t^3$

$t = 3.83$ s **Ans.**

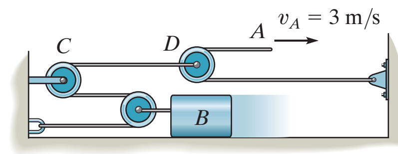


Ans:

$t = 3.83$ s

12–202.

If the end A of the cable is moving at $v_A = 3$ m/s, determine the speed of block B.



SOLUTION

Position Coordinates: The positions of pulley B, D and point A are specified by position coordinates s_B, s_D and s_A respectively as shown in Fig. *a*. The pulley system consists of two cords which give

$$2\,s_B + s_D = l_1 \qquad \textbf{(1)}$$

and

$$(s_A - s_D) + (b - s_D) = l_2$$

$$s_A - 2\,s_D = l_2 - b \qquad \textbf{(2)}$$

Time Derivative: Taking the time derivatives of Eqs. (1) and (2), we get

$$2v_B + v_D = 0 \qquad \textbf{(3)}$$

$$v_A - 2v_D = 0 \qquad \textbf{(4)}$$

Eliminate v_0 from Eqs. (3) and (4),

$$v_A + 4v_B = 0 \qquad \textbf{(5)}$$

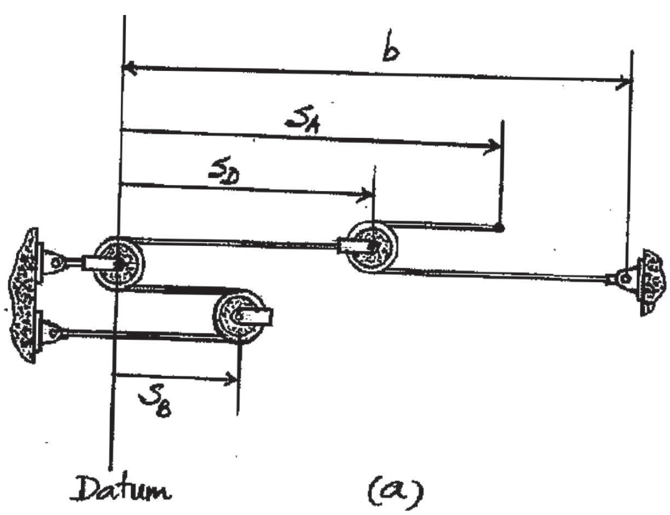
Here $v_A = +3$ m/s since it is directed toward the positive sense of s_A.

Thus

$$3 + 4v_B = 0$$

$$v_B = -0.75 \text{ m/s} = 0.75 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_D$ is directed toward the negative sense of $\mathbf{s}_B$.

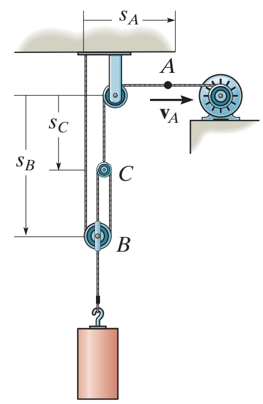


(a)

Ans:
$v_B = 0.75$ m/s

12–203.

Determine the time needed for the load at B to attain a speed of 10 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 3 m/s^2.



SOLUTION

Position Coordinates: The position of pulleys B, C and point A are specified by position coordinates s_B, s_C and s_A respectively as shown in Fig. *a*. The pulley system consists of two cords which gives

$$s_B + 2(s_B - s_C) = l_1$$

$$3s_B - 2s_C = l_1 \qquad \textbf{(1)}$$

And

$$s_C + s_A = l_2 \qquad \textbf{(2)}$$

Time Derivative: Taking the time derivative twice of Eqs. (1) and (2),

$$3a_B - 2a_C = 0 \qquad \textbf{(3)}$$

And

$$a_C + a_A = 0 \qquad \textbf{(4)}$$

Eliminate a_C from Eqs. (3) and (4)

$$3a_B + 2a_A = 0$$

Here, $a_A = +3\text{ m/s}^2$ since it is directed toward the positive sense of s_A. Thus,

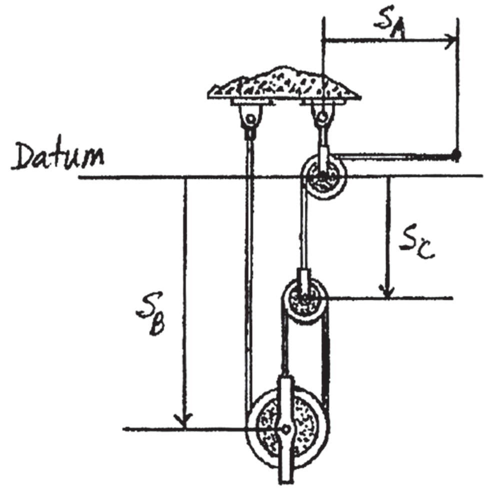
$$3a_B + 2(3) = 0 \qquad a_B = -2\text{ m/s}^2 = 2\text{ m/s}^2 \uparrow$$

The negative sign indicates that $\mathbf{a}_B$ is directed toward the negative sense of s_B. Applying kinematic equation of constant acceleration,

$$+\uparrow \qquad v_B = (v_B)_0 + a_B t$$

$$10 = 0 + 2t$$

$$t = 5.00\text{ s} \qquad \textbf{Ans.}$$

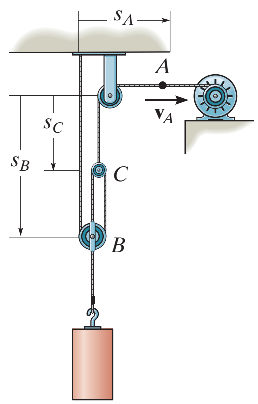


(a)

Ans:
$t = 5.00$ s

***12–204.**

The cable at A is being drawn toward the motor at $v_A = 8$ m/s. Determine the velocity of the block.



SOLUTION

Position Coordinates: The position of pulleys B, C and point A are specified by position coordinates s_B, s_C and s_A respectively as shown in Fig. *a*. The pulley system consists of two cords which give

$$s_B + 2(s_B - s_C) = l_1$$

$$3s_B - 2s_C = l_1 \quad \textbf{(1)}$$

And

$$s_C + s_A = l_2 \quad \textbf{(2)}$$

Time Derivative: Taking the time derivatives of Eqs. (1) and (2), we get

$$3v_B - 2v_C = 0 \quad \textbf{(3)}$$

And

$$v_C + v_A = 0 \quad \textbf{(4)}$$

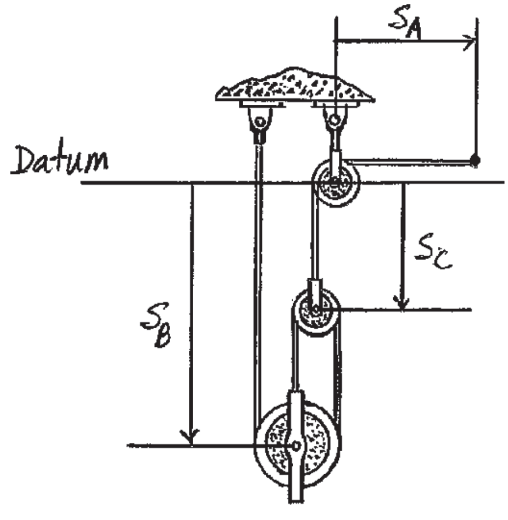
Eliminate v_C from Eqs. (3) and (4),

$$3v_B + 2v_A = 0$$

Here $v_A = +8$ m/s since it is directed toward the positive sense of s_A. Thus,

$$3v_B + 2(8) = 0 \qquad v_B = -5.33 \text{ m/s} = 5.33 \text{ m/s}\uparrow \quad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_B$ is directed toward the negative sense of s_B.

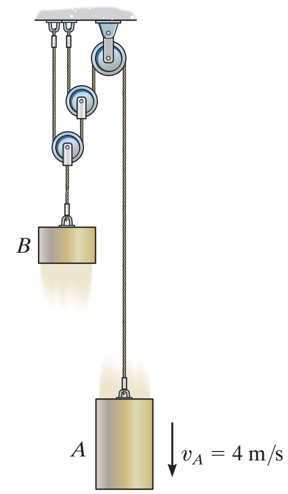


(a)

Ans:
$v_B = 5.33$ m/s$\uparrow$

12–205.

Determine the speed of B if A is moving downward with a speed of $v_A = 4$ m/s at the instant shown.



SOLUTION

Position Coordinates: By referring to Fig. *a*, the length of the two ropes written in terms of the position coordinates s_A, s_B, and s_C are

$$s_A + 2s_C = l_1 \quad \textbf{(1)}$$

and

$$s_B + (s_B - s_C) = l_2 \quad \textbf{(2)}$$

Eliminating s_C from Eqs. (1) and (2),

$$4s_B + s_A = 2l_2 + l_1 \quad \textbf{(3)}$$

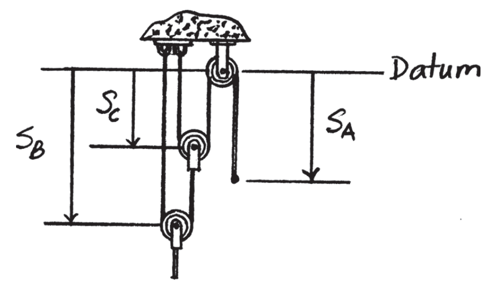
Time Derivative: Taking the time derivative of Eq. (3),

$$(+\downarrow) \qquad 4v_B + v_A = 0$$

Here, $v_A = 4$ m/s. Thus,

$$4v_B + 4 = 0$$

$$v_B = -1 \text{ m/s} = 1 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

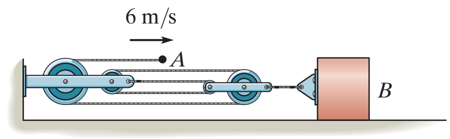


(a)

Ans:
$v_B = 1$ m/s $\uparrow$

12–206.

Determine the speed of the block at B.



SOLUTION

Position Coordinate: The positions of pulley B and point A are specified by position coordinates s_B and s_A respectively as shown in Fig. *a*. This is a single cord pulley system. Thus,

$$s_B + 2(s_B - a - b) + (s_B - a) + s_A = l$$

$$4s_B + s_A = l + 3a + 2b \qquad \textbf{(1)}$$

Time Derivative: Taking the time derivative of Eq. (1),

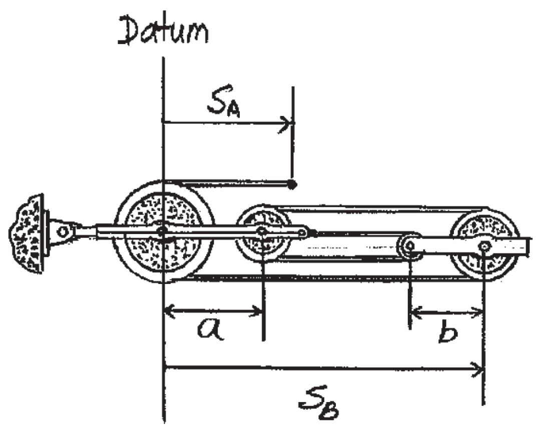
$$4v_B + v_A = 0 \qquad \textbf{(2)}$$

Here, $v_A = +6$ m/s since it is directed toward the positive sense of s_A. Thus,

$$4v_B + 6 = 0$$

$$v_B = -1.50 \text{ m/s} = 1.50 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_B$ is directed towards negative sense of s_B.

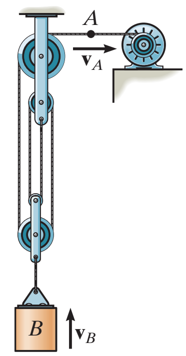


(a)

Ans:
$v_B = 1.50$ m/s

12–207.

Determine the constant speed at which the cable at A must be drawn in by the motor in order to hoist the load at B 15 ft in 5 s.



SOLUTION

$$v_B = \frac{-15}{5} = -3 \text{ ft/s} = 3 \text{ ft/s} \uparrow$$

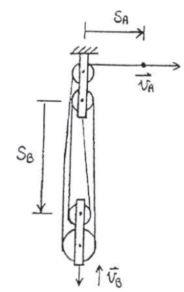
$$4s_B + s_A = l$$

$$4\Delta s_B = -\Delta s_A$$

$$4v_B = -v_A$$

$$4(-3) = -v_A$$

$$v_A = 12 \text{ ft/s} \rightarrow$$ **Ans.**

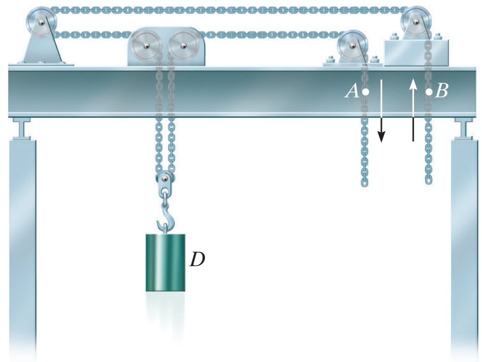


Ans:

$v_A = 12 \text{ ft/s} \rightarrow$

***12–208.**

The hoist is used to lift the load at D. If the end A of the chain is traveling downward at $v_A = 5$ ft/s and the end B is traveling upward at $v_B = 2$ ft/s, determine the velocity of the load at D.



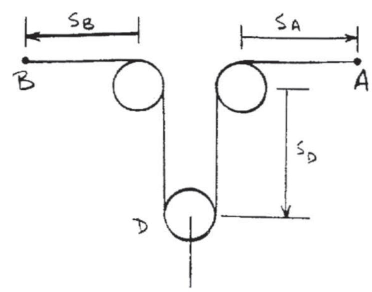
SOLUTION

$s_A + s_B + 2s_D = l$

$v_A + v_B + 2v_D = 0$

$5 - 2 + 2v_D = 0$

$v_D = -1.5 \text{ ft/s} = 1.5 \text{ ft/s} \uparrow$ **Ans.**

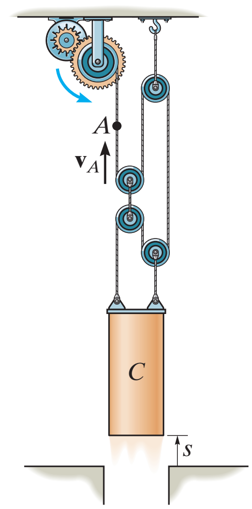


Ans:

$v_D = 1.5 \text{ ft/s} \uparrow$

12–209.

The cylinder C is being lifted using the cable and pulley system shown. If point A on the cable is being drawn toward the drum with a speed of 2 m/s, determine the speed of the cylinder.



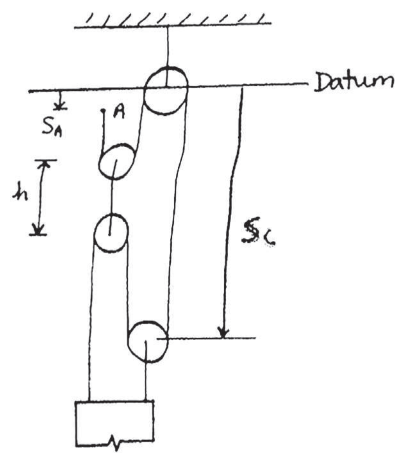
SOLUTION

$$l = s_C + (s_C - h) + (s_C - h - s_A)$$

$$l = 3\,s_C - 2\,h - s_A$$

$$0 = 3\,v_C - v_A$$

$$v_C = \frac{v_A}{3} = \frac{-2}{3} = -0.667\text{ m/s} = 0.667\text{ m/s}\uparrow \qquad \textbf{Ans.}$$

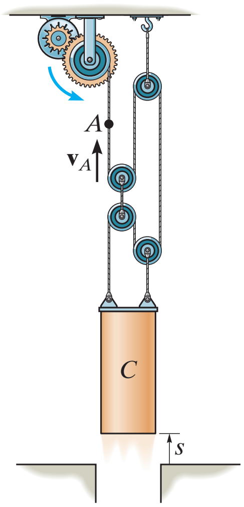


Ans:

$v_C = 0.667\text{ m/s}\uparrow$

12–210.

The cylinder C can be lifted with a maximum acceleration of $a_C = 3\ \text{m/s}^2$ without causing the cables to fail. Determine the speed at which point A is moving toward the drum when $s = 4$ m if the cylinder is lifted from rest in the shortest time possible.



SOLUTION

$$v^2 = v_0^2 + 2\,a_c\,(s - s_0)$$

$$v_C^2 = 0 + 2(3)\,(4 - 0)$$

$$v_C = 4.899\ \text{m/s}$$

$$l = s_C + (s_C - h) + (s_C - h - s_A)$$

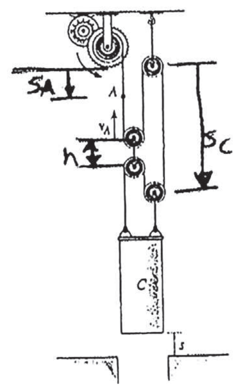
$$l = 3\,s_C - 2\,h - s_A$$

$$0 = 3\,v_C - v_A$$

$$3\,v_C = v_A$$

$$3(4.899) = v_A$$

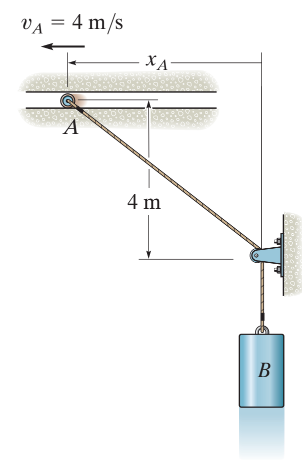
$$v_A = 14.7\ \text{m/s} \qquad \textbf{Ans.}$$



Ans:
$v_A = 14.7\ \text{m/s}$

12–211.

The roller at A is moving with a velocity of $v_A = 4\ \text{m/s}$ and has an acceleration of $a_A = 2\ \text{m/s}^2$ when $x_A = 3\ \text{m}$. Determine the velocity and acceleration of block B at this instant.



SOLUTION

Position Coordinates: The position of roller A and block B are specified by position coordinates x_A and y_B respectively as shown in Fig. a. We can relate these two position coordinates by considering the length of the cable, which is constant

$$\sqrt{x_A^2 + 4^2} + y_B = l$$

$$y_B = l - \sqrt{x_A^2 + 16} \qquad \textbf{(1)}$$

Velocity: Taking the time derivative of Eq. (1) using the chain rule,

$$\frac{dy_B}{dt} = 0 - \frac{1}{2}\left(x_A^2 + 16\right)^{-\frac{1}{2}}(2x_A)\frac{dx_A}{dt}$$

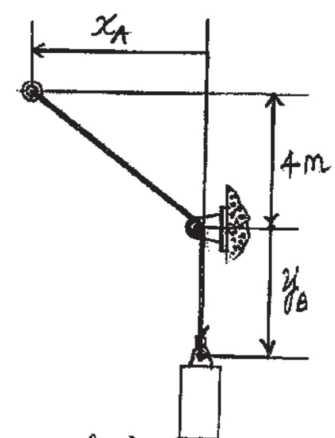
$$\frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 16}}\frac{dx_A}{dt}$$

However, $\dfrac{dy_B}{dt} = v_B$ and $\dfrac{dx_A}{dt} = v_A$. Then

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 16}}v_A \qquad \textbf{(2)}$$

At $x_A = 3\ \text{m}$, $v_A = +4\ \text{m/s}$ since v_A is directed toward the positive sense of x_A. Then Eq. (2) give

$$v_B = -\frac{3}{\sqrt{3^2 + 16}}(4) = -2.40\ \text{m/s} = 2.40\ \text{m/s}\uparrow \qquad \textbf{Ans.}$$



(a)

The negative sign indicates that v_B is directed toward the negative sense of y_B.

Acceleration: Taking the time derivative of Eq. (2),

$$\frac{dv_B}{dt} = -\left[x_A\left(-\frac{1}{2}\right)(x_A^2 + 16)^{-3/2}(2x_A)\frac{dx_A}{dt} + (x_A^2 + 16)^{-1/2}\frac{dx_A}{dt}\right]$$

$$v_A - x_A(x_A^2 + 16)^{-1/2}\frac{dv_A}{dt}$$

However, $\dfrac{dv_B}{dt} = a_B$, $\dfrac{dv_A}{dt} = a_A$ and $\dfrac{dx_A}{dt} = v_A$. Then

$$a_B = \frac{x_A^2 v_A^2}{\left(x_A^2 + 16\right)^{3/2}} - \frac{v_A^2}{\left(x_A^2 + 16\right)^{1/2}} - \frac{x_A a_A}{\left(x_A^2 + 16\right)^{1/2}}$$

$$a_B = -\frac{16\,v_A^2 + a_A x_A\left(x_A^2 + 16\right)}{\left(x_A^2 + 16\right)^{3/2}}$$

At $x_A = 3\ \text{m}$, $v_A = +4\ \text{m/s}$, $a_A = +2\ \text{m/s}^2$ since v_A and a_A are directed toward the positive sense of x_A.

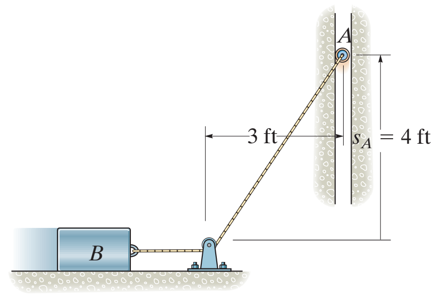
$$a_B = -\frac{16(4^2) + 2(3)(3^2 + 16)}{(3^2 + 16)^{3/2}} = -3.248\ \text{m/s}^2 = 3.25\ \text{m/s}^2\uparrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{a}_B$ is directed toward the negative sense of y_B.

Ans:
$v_B = 2.40\ \text{m/s}\uparrow$
$a_B = 3.25\ \text{m/s}^2\uparrow$

***12–212.**

The roller at A is moving upward with a velocity of $v_A = 3$ ft/s and has an acceleration of $a_A = 4$ ft/s^2 when $s_A = 4$ ft. Determine the velocity and acceleration of block B at this instant.



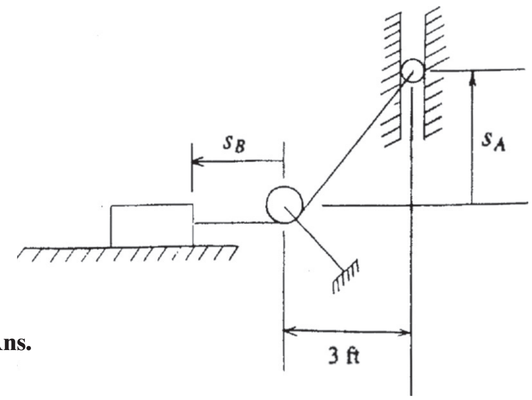
SOLUTION

$$s_B + \sqrt{(s_A)^2 + 3^2} = l$$

$$\dot{s}_B + \frac{1}{2}\left[(s_A)^2 + 3^2\right]^{-\frac{1}{2}}(2s_A)\,\dot{s}_A = 0$$

$$\dot{s}_B + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(s_A\dot{s}_A\right) = 0$$

$$\ddot{s}_B - \left[(s_A)^2 + 9\right]^{-\frac{3}{2}}\left(s_A^2\,\dot{s}_A^2\right) + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(s_A\ddot{s}_A\right) = 0$$



At $s_A = 4$ ft, $\dot{s}_A = 3$ ft/s, $\ddot{s}_A = 4$ ft/s^2

$$\dot{s}_B + \left(\frac{1}{5}\right)(4)(3) = 0$$

$v_B = -2.4$ ft/s $= 2.40$ ft/s $\rightarrow$ **Ans.**

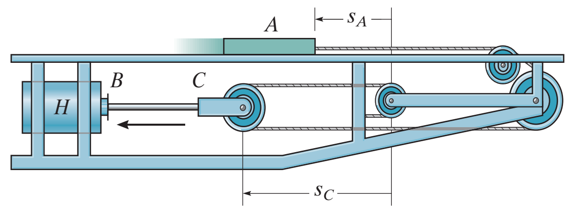
$$\ddot{s}_B - \left(\frac{1}{5}\right)^3(4)^2(3)^2 + \left(\frac{1}{5}\right)(3)^2 + \left(\frac{1}{5}\right)(4)(4) = 0$$

$a_B = -3.85$ ft/s^2 $= 3.85$ ft/s$^2 \rightarrow$ **Ans.**

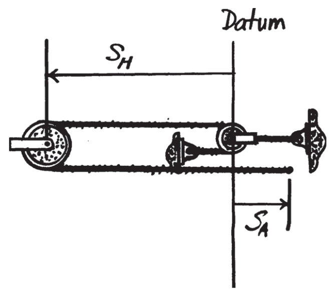
Ans:
$v_B = 2.40$ ft/s $\rightarrow$
$a_B = 3.85$ ft/s^2 $\rightarrow$

12–213.

If the hydraulic cylinder H draws in rod BC at 2 ft/s, determine the speed of slider A.



SOLUTION



$$2s_H + s_A = l$$

$$2v_H = -v_A$$

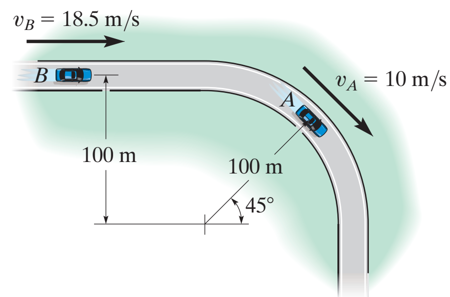
$$2(2) = -v_A$$

$$v_A = -4 \text{ ft/s} = 4 \text{ ft/s} \leftarrow$$ **Ans.**

Ans:
$v_A = 4$ ft/s

12–214.

At the instant shown, the car at A is traveling at 10 m/s around the curve while increasing its speed at 5 m/s^2. The car at B is traveling at 18.5 m/s along the straightaway and increasing its speed at 2 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this instant.



SOLUTION

$$v_A = 10\cos 45°\mathbf{i} - 10\sin 45°\mathbf{j} = \{7.071\mathbf{i} - 7.071\mathbf{j}\}\ \text{m/s}$$

$$v_B = \{18.5\mathbf{i}\}\ \text{m/s}$$

$$v_{A/B} = v_A - v_B$$

$$= (7.071\mathbf{i} - 7.071\mathbf{j}) - 18.5\mathbf{i} = \{-11.429\mathbf{i} - 7.071\mathbf{j}\}\ \text{m/s}$$

$$v_{A/B} = \sqrt{(-11.429)^2 + (-7.071)^2} = 13.4\ \text{m/s}$$ **Ans.**

$$\theta = \tan^{-1}\frac{7.071}{11.429} = 31.7°$$ **Ans.**

$$(a_A)_n = \frac{v_A^2}{\rho} = \frac{10^2}{100} = 1\ \text{m/s}^2 \qquad (a_A)_t = 5\ \text{m/s}^2$$

$$\mathbf{a}_A = (5\cos 45° - 1\cos 45°)\mathbf{i} + (-1\sin 45° - 5\sin 45°)\mathbf{j}$$

$$= \{2.828\mathbf{i} - 4.243\mathbf{j}\}\ \text{m/s}^2$$

$$\mathbf{a}_B = \{2\mathbf{i}\}\ \text{m/s}^2$$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{u}_B$$

$$= (2.828\mathbf{i} - 4.243\mathbf{j}) - 2\mathbf{i} = \{0.828\mathbf{i} - 4.24\mathbf{j}\}\ \text{m/s}^2$$

$$a_{A/B} = \sqrt{0.828^2 + (-4.243)^2} = 4.32\ \text{m/s}^2$$ **Ans.**

$$\theta = \tan^{-1}\frac{4.243}{0.828} = 79.0°$$ **Ans.**

Ans:

$v_{A/B} = 13.4$ m/s

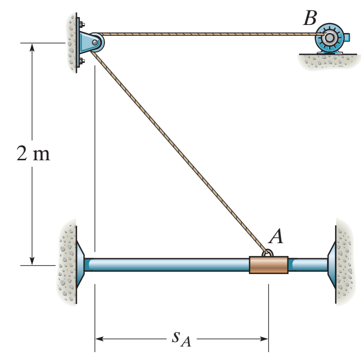
$\theta_v = 31.7°$

$a_{A/B} = 4.32$ m/s^2

$\theta_a = 79.0°$

12–215.

The motor draws in the cord at B with an acceleration of $a_B = 2\ \text{m/s}^2$. When $s_A = 1.5$ m, $v_B = 6$ m/s. Determine the velocity and acceleration of the collar at this instant.



SOLUTION

Position Coordinates: The position of collar A and point B are specified by s_A and s_C respectively as shown in Fig. *a*. We can relate these two position coordinates by considering the length of the cable, which is constant.

$$s_B + \sqrt{s_A^2 + 2^2} = l$$

$$s_B = l - \sqrt{s_A^2 + 4} \qquad \textbf{(1)}$$

Velocity: Taking the time derivative of Eq. (1),

$$\frac{ds_B}{dt} = 0 - \frac{1}{2}\left(s_A^2 + 4\right)^{-1/2}\left(2s_A\frac{ds_A}{dt}\right)$$

$$\frac{ds_B}{dt} = -\frac{s_A}{\sqrt{s_A^2 + 4}}\frac{ds_A}{dt}$$

However, $\dfrac{ds_B}{dt} = v_B$ and $\dfrac{ds_A}{dt} = v_A$. Then this equation becomes

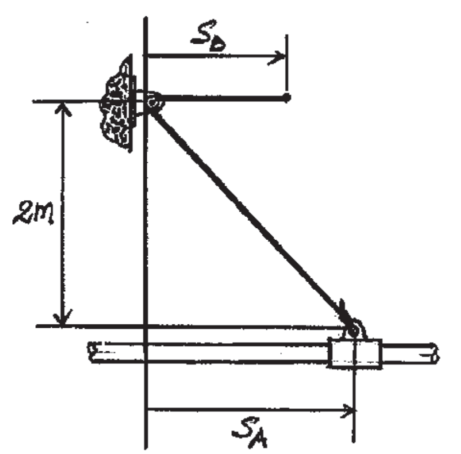
$$v_B = -\frac{s_A}{\sqrt{s_A^2 + 4}}v_A \qquad \textbf{(2)}$$

At the instant $s_A = 1.5$ m, $v_B = +6$ m/s. v_B is positive since it is directed toward the positive sense of s_B.

$$6 = -\frac{1.5}{\sqrt{1.5^2 + 4}}v_A$$

$$v_A = -10.0\ \text{m/s} = 10.0\ \text{m/s} \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that v_A is directed toward the negative sense of s_A.



(a)

Acceleration: Taking the time derivative of Eq. (2),

$$\frac{dv_B}{dt} = -\left[s_A\left(-\frac{1}{2}\right)\left(s_A^2 + 4\right)^{-3/2}\left(2s_A\frac{ds_A}{dt}\right) + \left(s_A^2 + 4\right)^{-1/2}\frac{ds_A}{dt}\right]$$

$$v_A - s_A\left(s_A^2 + 4\right)^{-1/2}\frac{dv_A}{dt}$$

However, $\dfrac{dv_B}{dt} = a_B$, $\dfrac{dv_A}{dt} = a_A$ and $\dfrac{ds_A}{dt} = v_A$. Then

$$a_B = \frac{s_A^2 v_A^2}{\left(s_A^2 + 4\right)^{3/2}} - \frac{v_A^2}{\left(s_A^2 + 4\right)^{1/2}} - \frac{a_A s_A}{\left(s_A^2 + 4\right)^{1/2}}$$

$$a_B = -\frac{4v_A^2 + a_A s_A\left(s_A^2 + 4\right)}{\left(s_A^2 + 4\right)^{3/2}}$$

At the instant $s_A = 1.5$ m, $a_B = +2\ \text{m/s}^2$. $\mathbf{a}_B$ is positive since it is directed toward the positive sense of s_B. Also, $v_A = -10.0$ m/s. Then

$$2 = -\left[\frac{4(-10.0)^2 + a_A(1.5)(1.5^2 + 4)}{(1.5^2 + 4)^{3/2}}\right]$$

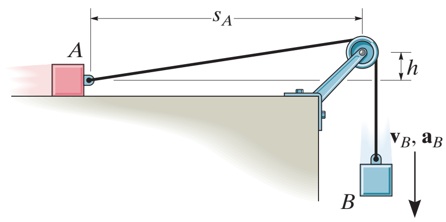
$$a_A = -46.0\ \text{m/s}^2 = 46.0\ \text{m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{a}_A$ is directed toward the negative sense of s_A.

Ans:
$v_A = 10.0\ \text{m/s} \leftarrow$
$a_A = 46.0\ \text{m/s}^2 \leftarrow$

***12–216.**

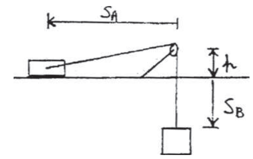
If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.



SOLUTION

$$l = s_B + \sqrt{s_A^2 + h^2}$$

$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2}\,2s_A\,\dot{s}_A$$



$$v_A = \dot{s}_A = \frac{-\dot{s}_B(s_A^2 + h^2)^{1/2}}{s_A}$$

$$v_A = -v_B\left[1 + \left(\frac{h}{s_A}\right)^2\right]^{1/2}$$ **Ans.**

$$a_A = \dot{v}_A = -\dot{v}_B\left[1 + \left(\frac{h}{s_A}\right)^2\right]^{1/2} - v_B\left(\frac{1}{2}\right)\left[1 + \left(\frac{h}{s_A}\right)^2\right]^{-1/2}(h^2)(-2)(s_A)^{-3}\dot{s}_A$$

$$a_A = -a_B\left[1 + \left(\frac{h}{s_B}\right)^2\right]^{1/2} + \frac{v_A v_B h^2}{s_A^3}\left[1 + \left(\frac{h}{s_A}\right)^2\right]^{-1/2}$$ **Ans.**

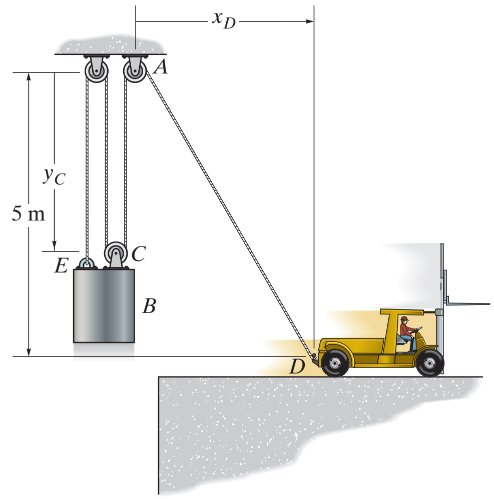
Ans:

$$v_A = -v_B\left[1 + \left(\frac{h}{s_A}\right)^2\right]^{1/2}$$

$$a_A = -a_B\left[1 + \left(\frac{h}{s_B}\right)^2\right]^{1/2} + \frac{v_A v_B h^2}{s_A^3}\left[1 + \left(\frac{h}{S_A}\right)^2\right]^{-1/2}$$

12–217.

The block B is suspended from a cable that is attached to the block at E, wraps around three pulleys, and is tied to the back of a truck. If the truck starts from rest when x_D is zero, and moves forward with a constant acceleration of $a_D = 0.5\ \text{m/s}^2$, determine the speed of the block at the instant $x_D = 2$ m. Neglect the size of the pulleys in the calculation. When $x_D = 0$, $y_C = 5$ m, so that points C and D are at the same elevation.



SOLUTION

Geometry:

$$3y_C + \sqrt{5^2 + x_D^2} = l$$

Velocity:

$$3\dot{y}_C + \frac{x_D}{\sqrt{25 + x_D^2}}\dot{x}_D = 0 \qquad \text{where } \dot{y}_C = v_C \text{ and } \dot{x}_D = v_D$$

$$3v_C + \frac{x_D}{\sqrt{25 + x_D^2}}v_D = 0 \qquad [1]$$

For the truck

$$(\overset{+}{\rightarrow}) \qquad v_x^2 = (v_0^2)_x + 2a_c\left[s_x - (s_0)_x\right]$$

$$v_D^2 = 0 + 2(0.5)(2 - 0)$$

$$v_D = 1.414\ \text{m/s}$$

From Eq. [1] $\qquad 3v_C + \dfrac{2}{\sqrt{25 + 2^2}}(1.414) = 0$

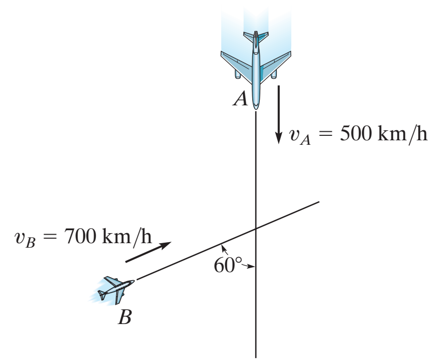
$$v_C = -0.175\ \text{m/s} = 0.175\ \text{m/s}\uparrow \qquad \textbf{Ans.}$$

The negative sign indicates that v_C is in the opposite direction of positive y_C.

Ans:
$v_C = 0.175\ \text{m/s}\uparrow$

12–218.

Two planes, A and B, are flying at the same altitude. If their velocities are $v_A = 500$ km/h and $v_B = 700$ km/h such that the angle between their straight-line courses is $\theta = 60°$, determine the velocity of plane B with respect to plane A.



SOLUTION

Relative Velocity. Express $\mathbf{v}_A$ and $\mathbf{v}_B$ in Cartesian vector form,

$$\mathbf{v}_A = \{-500\,\mathbf{j}\} \text{ km/h}$$

$$\mathbf{v}_B = \{700 \sin 60°\mathbf{i} + 700 \cos 60°\mathbf{j}\} \text{ km/h} = \left\{350\sqrt{3}\mathbf{i} + 350\mathbf{j}\right\} \text{ km/h}$$

Applying the relative velocity equation.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$350\sqrt{3}\mathbf{i} + 350\mathbf{j} = -500\mathbf{j} + \mathbf{v}_{B/A}$$

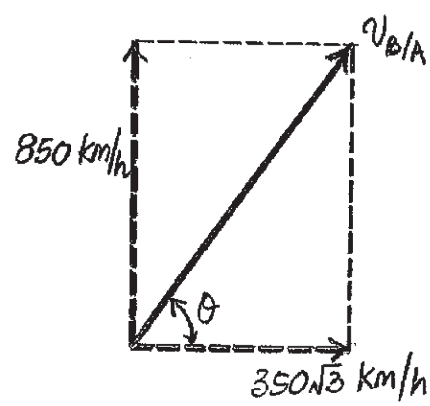
$$\mathbf{v}_{B/A} = \left\{350\sqrt{3}\mathbf{i} + 850\mathbf{j}\right\} \text{km/h}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{(350\sqrt{3})^2 + 850^2} = 1044.03 \text{ km/h} = 1044 \text{ km/h} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig. *a*.

$$\theta = \tan^{-1}\left(\frac{850}{350\sqrt{3}}\right) = 54.50° = 54.5° \measuredangle \qquad \textbf{Ans.}$$

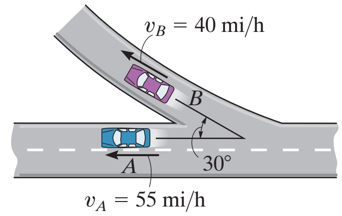


(a)

Ans:
$v_{B/A} = 1044$ km/h
$\theta = 54.5° \measuredangle$

12–219.

At the instant shown, cars A and B are traveling at speeds of 55 mi/h and 40 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.5 mi.



SOLUTION

$$\mathbf{v}_B = -40 \cos 30°\mathbf{i} + 40 \sin 30°\mathbf{j} = \{-34.64\mathbf{i} + 20\mathbf{j}\} \text{ mi/h}$$

$$\mathbf{v}_A = \{-55\mathbf{i}\} \text{ mi/h}$$

$$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A$$

$$= (-34.64\mathbf{i} + 20\mathbf{j}) - (-55\mathbf{i}) = \{20.36\mathbf{i} + 20\mathbf{j}\} \text{ mi/h}$$

$$v_{B/A} = \sqrt{20.36^2 + 20^2} = 28.5 \text{ mi/h} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{20}{20.36} = 44.5° \qquad \textbf{Ans.}$$

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{40^2}{0.5} = 3200 \text{ mi/h}^2 \; 60° \qquad (a_B)_t = 1200 \text{ mi/h}^2 \; 30°$$

$$\mathbf{a}_B = (3200 \cos 60° - 1200 \cos 30°)\mathbf{i} + (3200 \sin 60° + 1200 \sin 30°)\mathbf{j}$$

$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\} \quad \text{mi/h}^2$$

$$\mathbf{a}_A = 0$$

$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$$

$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\} - 0 = \{560.77\mathbf{i} + 3371.28\mathbf{j}\} \quad \text{mi/h}^2$$

$$a_{B/A} = \sqrt{(560.77)^2 + (3371.28)^2} = 3418 \text{ mi/h}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{3371.28}{560.77} = 80.6° \qquad \textbf{Ans.}$$

Ans:

$v_{B/A} = 28.5 \text{ mi/h}$

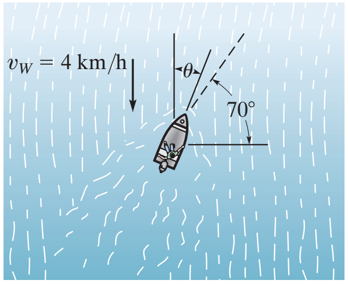
$\theta_v = 44.5°$

$a_{B/A} = 3418 \text{ mi/h}^2$

$\theta_a = 80.6°$

***12–220.**

The boat can travel with a speed of 16 km/h in still water. The point of destination is located along the dashed line. If the water is moving at 4 km/h, determine the bearing angle θ at which the boat must travel to stay on course.



SOLUTION

$\mathbf{v}_B = \mathbf{v}_W + \mathbf{v}_{B/W}$

$v_B \cos 70°\mathbf{i} + v_B \sin 70°\mathbf{j} = -4\mathbf{j} + 16 \sin\theta\mathbf{i} + 16\cos\theta\mathbf{j}$

$(\overset{+}{\rightarrow})$ $\quad v_B \cos 70° = 0 + 16\sin\theta$

$(+\uparrow)$ $\quad v_B \sin 70° = -4 + 16\cos\theta$

$2.748 \sin\theta - \cos\theta + 0.25 = 0$

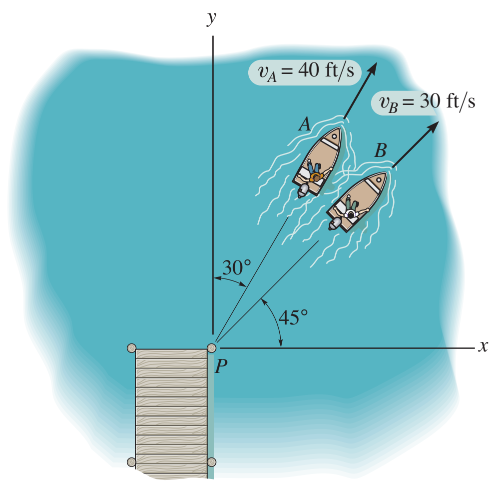
Solving,

$\theta = 15.1°$ **Ans.**

Ans:
$\theta = 15.1°$

12–221.

Two boats leave the pier P at the same time and travel in the directions shown. If $v_A = 40$ ft s and $v_B = 30$ ft/s, determine the magnitude of the velocity of boat A relative to boat B. How long after leaving the pier will the boats be 1500 ft apart?



SOLUTION

Relative Velocity:

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$40 \sin 30°\mathbf{i} + 40 \cos 30°\mathbf{j} = 30 \cos 45°\mathbf{i} + 30 \sin 45°\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-1.213\mathbf{i} + 13.43\mathbf{j}\} \text{ ft/s}$$

Thus, the magnitude of the relative velocity $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-1.213)^2 + 13.43^2} = 13.48 \text{ ft/s} = 13.5 \text{ ft/s} \qquad \textbf{Ans.}$$

One can obtain the time t required for boats A and B to be 1500 ft apart by noting that boat B is at rest and boat A travels at the relative speed $v_{A/B} = 13.48$ ft/s for a distance of 1500 ft. Thus

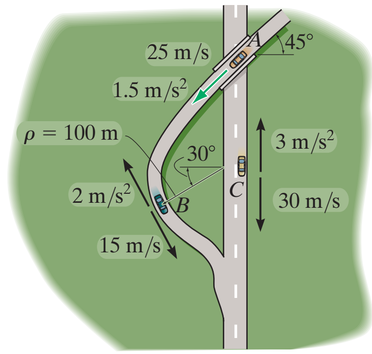
$$t = \frac{1500}{v_{A/B}} = \frac{1500}{13.48} = 111.26 \text{ s} = 1.85 \text{ min} \qquad \textbf{Ans.}$$

Ans:

$v_{A/B} = 13.5$ ft/s
$t = 1.85$ min

12–222.

Car A travels along a straight road at a speed of 25 m/s while accelerating at 1.5 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car A relative to car C.



SOLUTION

Velocity: The velocity of cars A and C expressed in Cartesian vector form are

$$\mathbf{v}_A = [-25\cos 45°\mathbf{i} - 25\sin 45°\mathbf{j}]\text{ m/s} = [-17.68\mathbf{i} - 17.68\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}]\text{ m/s}$$

Applying the relative velocity equation, we have

$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$-17.68\mathbf{i} - 17.68\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{A/C}$$

$$\mathbf{v}_{A/C} = [-17.68\mathbf{i} + 12.32\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{A/C}$ is given by

$$v_{A/C} = \sqrt{(-17.68)^2 + 12.32^2} = 21.5\text{ m/s}$$ **Ans.**

and the direction angle θ_v that $\mathbf{v}_{A/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{12.32}{17.68}\right) = 34.9°$$ **Ans.**

Acceleration: The acceleration of cars A and C expressed in Cartesian vector form are

$$\mathbf{a}_A = [-1.5\cos 45°\mathbf{i} - 1.5\sin 45°\mathbf{j}]\text{ m/s}^2 = [-1.061\mathbf{i} - 1.061\mathbf{j}]\text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}]\text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$-1.061\mathbf{i} - 1.061\mathbf{j} = 3\mathbf{j} + \mathbf{a}_{A/C}$$

$$\mathbf{a}_{A/C} = [-1.061\mathbf{i} - 4.061\mathbf{j}]\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{A/C}$ is given by

$$a_{A/C} = \sqrt{(-1.061)^2 + (-4.061)^2} = 4.20\text{ m/s}^2$$ **Ans.**

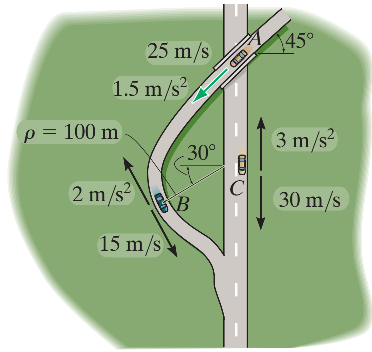
and the direction angle θ_a that $\mathbf{a}_{A/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{4.061}{1.061}\right) = 75.4°$$ **Ans.**

Ans:
$v_{A/C} = 21.5$ m/s
$\theta_v = 34.9°$
$a_{A/C} = 4.20$ m/s^2
$\theta_a = 75.4°$

12–223.

Car B is traveling along the curved road with a speed of 15 m/s while decreasing its speed at 2 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car B relative to car C.



SOLUTION

Velocity: The velocity of cars B and C expressed in Cartesian vector form are

$$\mathbf{v}_B = [15 \cos 60° \, \mathbf{i} - 15 \sin 60° \, \mathbf{j}] \text{ m/s} = [7.5\mathbf{i} - 12.99\mathbf{j}] \text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}] \text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$7.5\mathbf{i} - 12.99\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{B/C}$$

$$\mathbf{v}_{B/C} = [7.5\mathbf{i} + 17.01\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/C}$ is given by

$$v_{B/C} = \sqrt{7.5^2 + 17.01^2} = 18.6 \text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{B/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{17.01}{7.5}\right) = 66.2° \qquad \textbf{Ans.}$$

Acceleration: The normal component of car B's acceleration is $(a_B)_n = \frac{v_B{}^2}{\rho} = \frac{15^2}{100} = 2.25 \text{ m/s}^2$. Thus, the tangential and normal components of car B's acceleration and the acceleration of car C expressed in Cartesian vector form are

$$(\mathbf{a}_B)_t = [-2 \cos 60° \, \mathbf{i} + 2 \sin 60° \mathbf{j}] = [-1\mathbf{i} + 1.732\mathbf{j}] \text{ m/s}^2$$

$$(\mathbf{a}_B)_n = [2.25 \cos 30° \, \mathbf{i} + 2.25 \sin 30° \, \mathbf{j}] = [1.9486\mathbf{i} + 1.125\mathbf{j}] \text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}] \text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

$$(-1\mathbf{i} + 1.732\mathbf{j}) + (1.9486\mathbf{i} + 1.125\mathbf{j}) = 3\mathbf{j} + \mathbf{a}_{B/C}$$

$$\mathbf{a}_{B/C} = [0.9486\mathbf{i} - 0.1429\mathbf{j}] \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/C}$ is given by

$$a_{B/C} = \sqrt{0.9486^2 + (-0.1429)^2} = 0.959 \text{ m/s}^2 \qquad \textbf{Ans.}$$

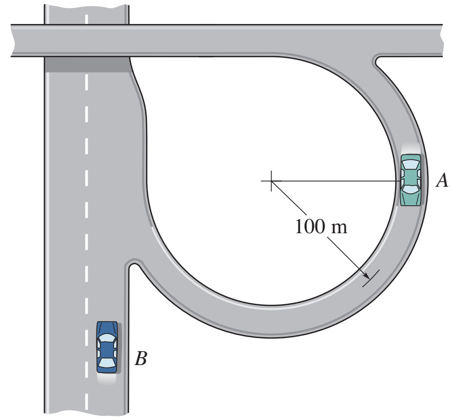
and the direction angle θ_a that $\mathbf{a}_{B/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{0.1429}{0.9486}\right) = 8.57° \qquad \textbf{Ans.}$$

Ans:
$v_{B/C} = 18.6$ m/s
$\theta_v = 66.2°$
$a_{B/C} = 0.959$ m/s^2
$\theta_a = 8.57°$

***12–224.**

At the instant shown, car A has a speed of 20 km/h, which is being increased at the rate of 300 km/h^2 as the car enters the expressway. At the same instant, car B is decelerating at 250 km/h^2 while traveling forward at 100 km/h. Determine the velocity and acceleration of A with respect to B.



SOLUTION

$\mathbf{v}_A = \{-20\mathbf{j}\}$ km/h $\qquad \mathbf{v}_B = \{100\mathbf{j}\}$ km/h

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= (-20\mathbf{j} - 100\mathbf{j}) = \{-120\mathbf{j}\} \text{ km/h}$$

$$v_{A/B} = 120 \text{ km/h} \downarrow \qquad \textbf{Ans.}$$

$$(a_A)_n = \frac{v_A^2}{\rho} = \frac{20^2}{0.1} = 4000 \text{ km/h}^2 \qquad (a_A)_t = 300 \text{ km/h}^2$$

$$\mathbf{a}_A = -4000\mathbf{i} + (-300\mathbf{j})$$

$$= \{-4000\mathbf{i} - 300\mathbf{j}\} \text{ km/h}^2$$

$$\mathbf{a}_B = \{-250\mathbf{j}\} \text{ km/h}^2$$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B$$

$$= (-4000\mathbf{i} - 300\mathbf{j}) - (-250\mathbf{j}) = \{-4000\mathbf{i} - 50\mathbf{j}\} \text{ km/h}^2$$

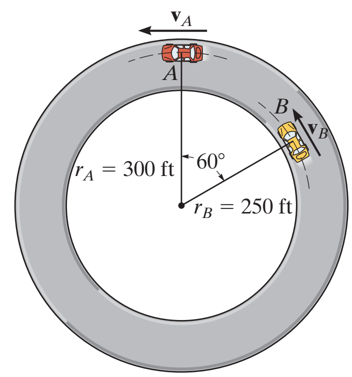
$$a_{A/B} = \sqrt{(-4000)^2 + (-50)^2} = 4000 \text{ km/h}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{50}{4000} = 0.716^\circ \qquad \textbf{Ans.}$$

Ans:
$v_{A/B} = 120$ km/h $\downarrow$
$a_{A/B} = 4000$ km/h^2
$\theta = 0.716^\circ$

12–225.

Cars A and B are traveling around the circular race track. At the instant shown, A has a speed of 90 ft/s and is increasing its speed at the rate of 15 ft/s^2, whereas B has a speed of 105 ft/s and is decreasing its speed at 25 ft/s^2. Determine the relative velocity and relative acceleration of car A with respect to car B at this instant.



SOLUTION

$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$-90\mathbf{i} = -105 \sin 30°\,\mathbf{i} + 105 \cos 30°\mathbf{j} + \mathbf{v}_{A/B}$

$\mathbf{v}_{A/B} = \{-37.5\mathbf{i} - 90.93\mathbf{j}\}$ ft/s

$v_{A/B} = \sqrt{(-37.5)^2 + (-90.93)^2} = 98.4$ ft/s **Ans.**

$\theta = \tan^{-1}\left(\dfrac{90.93}{37.5}\right) = 67.6°$ **Ans.**

$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$

$-15\mathbf{i} - \dfrac{(90)^2}{300}\mathbf{j} = 25 \cos 60°\mathbf{i} - 25 \sin 60°\mathbf{j} - 44.1 \sin 60°\mathbf{i} - 44.1 \cos 60°\mathbf{j} + \mathbf{a}_{A/B}$

$\mathbf{a}_{A/B} = \{10.69\mathbf{i} + 16.70\mathbf{j}\}$ ft/s^2

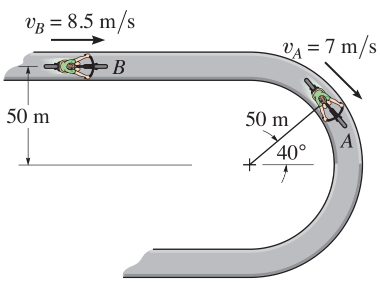
$a_{A/B} = \sqrt{(10.69)^2 + (16.70)^2} = 19.8$ ft/s^2 **Ans.**

$\theta = \tan^{-1}\left(\dfrac{16.70}{10.69}\right) = 57.4°$ **Ans.**

Ans:
$v_{A/B} = 98.4$ ft/s
$\theta_v = 67.6°$
$a_{A/B} = 19.8$ ft/s^2
$\theta_a = 57.4°$

12–226.

At the instant shown, the bicyclist at A is traveling at 7 m/s around the curve on the race track while increasing the bicycle's speed at 0.5 m/s ^{2}. The bicyclist at B is traveling at 8.5 m/s along the straightaway and increasing the bicycle's speed at 0.7 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this istant.



SOLUTION

$v_A = v_B + v_{A/B}$

$[7 \searrow_{40^\circ}] = [8.5 \rightarrow] + [(v_{A/B})_x \rightarrow] + [(v_{A/B})_y \downarrow]$

$(\xrightarrow{+})\qquad 7 \sin 40^\circ = 8.5 + (v_{A/B})_x$

$(+\downarrow)\qquad 7 \cos 40^\circ = (v_{A/B})_y$

Thus,

$(v_{A/B})_x = 4.00 \text{ m/s} \leftarrow$

$(v_{A/B})_y = 5.36 \text{ m/s} \downarrow$

$(v_{A/B}) = \sqrt{(4.00)^2 + (5.36)^2}$

$v_{A/B} = 6.69 \text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{5.36}{4.00}\right) = 53.3^\circ\ \theta \measuredangle$ **Ans.**

$(a_A)_n = \dfrac{7^2}{50} = 0.980 \text{ m/s}^2$

$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$

$[0.980]\ {}^{\theta}_{40^\circ}\measuredangle + [0.5]\searrow_{40^\circ} = [0.7 \rightarrow] + [(a_{A/B})_x \rightarrow] + [(a_{A/B})_y \downarrow]$

$(+\rightarrow)\qquad -0.980 \cos 40^\circ + 0.5 \sin 40^\circ = 0.7 + (a_{A/B})_x$

$(a_{A/B})_x = 1.129 \text{ m/s}^2 \leftarrow$

$(+\downarrow)\qquad 0.980 \sin 40^\circ + 0.5 \cos 40^\circ = (a_{A/B})_y$

$(a_{A/B})_y = 1.013 \text{ m/s}^2 \downarrow$

$(a_{A/B}) = \sqrt{(1.129)^2 + (1.013)^2}$

$a_{A/B} = 1.52 \text{ m/s}^2$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{1.013}{1.129}\right) = 41.9^\circ\ \theta \measuredangle$ **Ans.**

Ans:

$v_{B/A} = 6.69 \text{ m/s}$

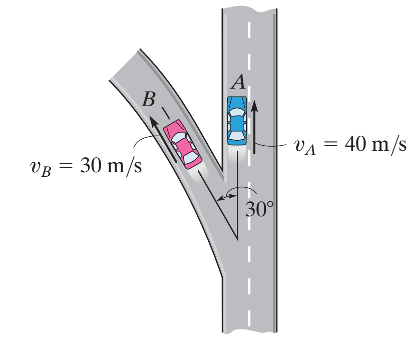
$\theta = 53.3^\circ\ \theta \measuredangle$

$a_{B/A} = 1.52 \text{ m/s}^2$

$\theta = 41.9^\circ\ \theta \measuredangle$

12–227.

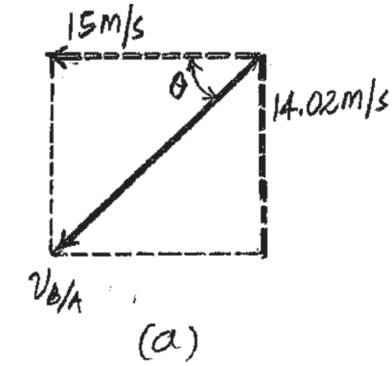
At the instant shown, cars A and B are traveling at velocities of 40 m/s and 30 m/s, respectively. If B is increasing its velocity by 2 m/s^2, while A maintains a constant velocity, determine the velocity and acceleration of B with respect to A. The radius of curvature at B is $\rho_B = 200$ m.



SOLUTION

Relative Velocity: Express v_A and v_B as Cartesian vectors.

$$v_A = \{40\mathbf{j}\}\text{ m/s} \quad v_B = \{-30\sin 30°\mathbf{i} + 30\cos 30°\mathbf{j}\}\text{ m/s} = \{-15\mathbf{i} + 15\sqrt{3}\mathbf{j}\}\text{ m/s}$$



Applying the relative velocity equation,

$$v_B = v_A + v_{B/A}$$

$$-15\mathbf{i} + 15\sqrt{3}\mathbf{j} = 40\mathbf{j} + v_{B/A}$$

$$v_{B/A} = \{-15\mathbf{i} - 14.02\mathbf{j}\}\text{ m/s}$$

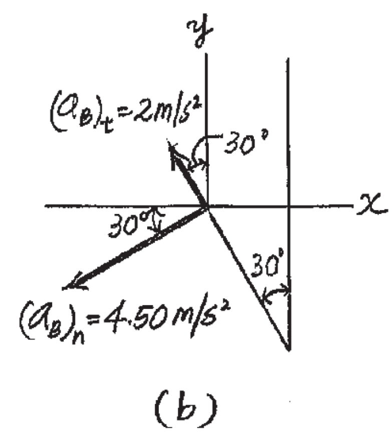
Thus, the magnitude of $v_{B/A}$ is

$$v_{B/A} = \sqrt{(-15)^2 + (-14.02)^2} = 20.53\text{ m/s} = 20.5\text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig. *a*

$$\theta = \tan^{-1}\left(\frac{14.02}{15}\right) = 43.06° = 43.1° \measuredangle \qquad \textbf{Ans.}$$

Relative Acceleration: Here, $(a_B)_t = 2\text{ m/s}^2$ and $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{30^2}{200} = 4.50\text{ m/s}^2$ and their directions are shown in Fig. *b*. Then, express $\mathbf{a}_B$ as a Cartesian vector,



$$\mathbf{a}_B = (-2\sin 30° - 4.50\cos 30°)\mathbf{i} + (2\cos 30° - 4.50\sin 30°)\mathbf{j}$$

$$= \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\}\text{ m/s}^2$$

Applying the relative acceleration equation with $\mathbf{a}_A = \mathbf{0}$,

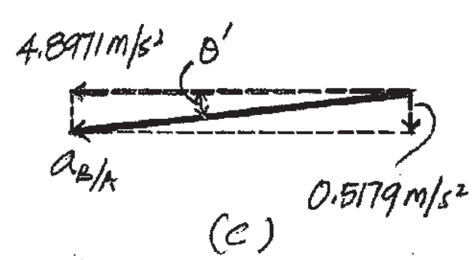
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$-4.8971\mathbf{i} - 0.5179\mathbf{j} = \mathbf{0} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\}\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/A}$ is

$$\mathbf{a}_{B/A} = \sqrt{(-4.8971)^2 + (-0.5179)^2} = 4.9244\text{ m/s}^2 = 4.92\text{ m/s}^2 \qquad \textbf{Ans.}$$



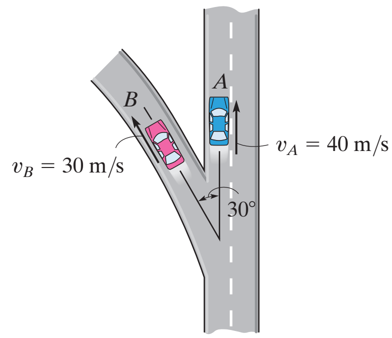
And its direction is defined by angle θ', Fig. *c*,

$$\theta' = \tan^{-1}\left(\frac{0.5179}{4.8971}\right) = 6.038° = 6.04° \measuredangle \qquad \textbf{Ans.}$$

Ans:
$v_{B/A} = 20.5$ m/s
$\theta_v = 43.1° \measuredangle$
$a_{B/A} = 4.92$ m/s^2
$\theta_a = 6.04° \measuredangle$

***12–228.**

At the instant shown, cars A and B are traveling at velocities of 40 m/s and 30 m/s, respectively. If A is increasing its speed at 4 m/s^2, whereas the speed of B is decreasing at 3 m/s^2, determine the velocity and acceleration of B with respect to A. The radius of curvature at B is $\rho_B = 200$ m.



SOLUTION

Relative Velocity: Express v_A and v_B as Cartesian vector.

$$v_A = \{40\mathbf{j}\}\text{ m/s} \quad v_B = \{-30 \sin 30°\mathbf{i} + 30 \cos 30°\mathbf{j}\}\text{ m/s} = \{-15\mathbf{i} + 15\sqrt{3}\mathbf{j}\}\text{ m/s}$$

Applying the relative velocity equation,

$$v_B = v_A + v_{B/A}$$

$$-15\mathbf{i} + 15\sqrt{3}\mathbf{j} = 40\mathbf{j} + v_{B/A}$$

$$v_{B/A} = \{-15\mathbf{i} - 14.02\}\text{ m/s}$$

Thus the magnitude of $_{B/A}$ is

$$v_{B/A} = \sqrt{(-15)^2 + (-14.02)^2} = 20.53\text{ m/s} = 20.5\text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig a.

$$\theta = \tan^{-1}\left(\frac{14.02}{15}\right) = 43.06° = 43.1° \qquad \textbf{Ans.}$$

Relative Acceleration: Here $(a_B)_t = 3\text{ m/s}^2$ and $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{30^2}{200} = 4.5\text{ m/s}^2$ and their directions are shown in Fig. b. Then express $\mathbf{a}_B$ as a Cartesian vector,

$$\mathbf{a}_B = (3 \sin 30° - 4.50 \cos 30°)\mathbf{i} + (-3 \cos 30° - 4.50 \sin 30°)\mathbf{j}$$

$$= \{-2.3971\mathbf{i} - 4.8481\mathbf{j}\}\text{ m/s}^2$$

Applying the relative acceleration equation with $\mathbf{a}_A = \{4\mathbf{j}\}\text{ m/s}^2$,

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$-2.3971\mathbf{i} - 4.8481\mathbf{j} = 4\mathbf{j} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{-2.3971\mathbf{i} - 8.8481\mathbf{j}\}\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/A}$ is

$$\mathbf{a}_{B/A} = \sqrt{(-2.3971)^2 + (-8.8481)^2} = 9.167\text{ m/s}^2 = 9.17\text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by angle θ', Fig. c

$$\theta' = \tan^{-1}\left(\frac{8.8481}{2.3971}\right) = 74.84° = 74.8° \qquad \textbf{Ans.}$$

Ans:

$v_{B/A} = 20.5$ m/s

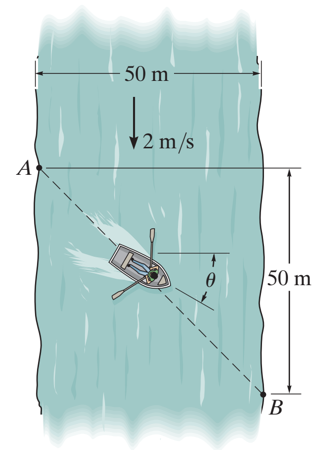
$\theta = 43.1°$

$\mathbf{a}_{B/A} = 9.17\text{ m/s}^2$

$\theta' = 74.8°$

12–229.

A man can row a boat at 5 m/s in still water. He wishes to cross a 50-m-wide river to point B, 50 m downstream. If the river flows with a velocity of 2 m/s, determine the speed of the boat and the time needed to make the crossing.



SOLUTION

Relative Velocity:

$$v_b = v_r + v_{b/r}$$

$$v_b \sin 45°\mathbf{i} - v_b \cos 45°\mathbf{j} = -2\mathbf{j} + 5\cos\theta\mathbf{i} - 5\sin\theta\mathbf{j}$$

Equating **i** and **j** component, we have

$$v_b \sin 45° = 5\cos\theta \qquad \textbf{[1]}$$

$$-v_b \cos 45° = -2 - 5\sin\theta \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$\theta = 28.57°$$

$$v_b = 6.210 \text{ m/s} = 6.21 \text{ m/s} \qquad \textbf{Ans.}$$

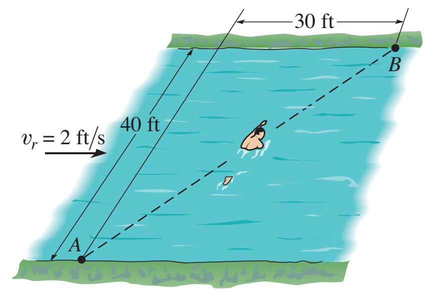
Thus, the time t required by the boat to travel from point A to B is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{50^2 + 50^2}}{6.210} = 11.4 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$v_b = 6.21$ m/s
$t = 11.4$ s

12–230.

A man can swim at 4 ft/s in still water. He wishes to cross the 40-ft-wide river to point B, 30 ft downstream. If the river flows with a velocity of 2 ft/s, determine the speed of the man and the time needed to make the crossing. *Note*: While in the water he must not direct himself toward point B to reach this point. Why?

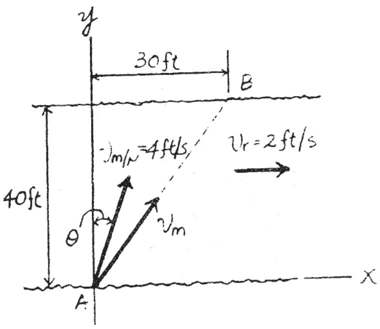


SOLUTION

Relative Velocity:

$$v_m = v_r + v_{m/r}$$

$$\frac{3}{5}v_m\mathbf{i} + \frac{4}{5}v_m\mathbf{j} = 2\mathbf{i} + 4\sin\theta\mathbf{i} + 4\cos\theta\mathbf{j}$$



Equating the i and j components, we have

$$\frac{3}{5}v_m = 2 + 4\sin\theta \qquad \textbf{(1)}$$

$$\frac{4}{5}v_m = 4\cos\theta \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\theta = 13.29°$$

$$v_m = 4.866 \text{ ft/s} = 4.87 \text{ ft/s} \qquad \textbf{Ans.}$$

Thus, the time t required by the boat to travel from points A to B is

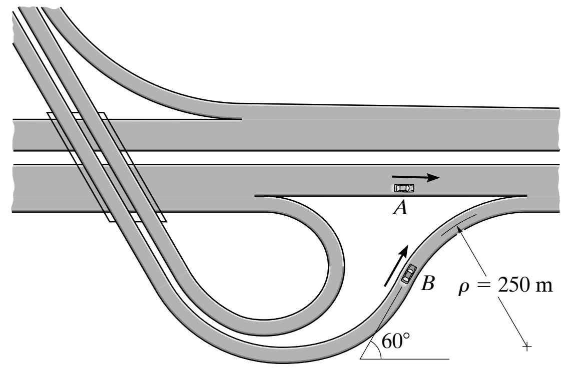
$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{40^2 + 30^2}}{4.866} = 10.3 \text{ s} \qquad \textbf{Ans.}$$

In order for the man to reached point B, the man has to direct himself at an angle $\theta = 13.3°$ with y axis.

Ans:
$v_m = 4.87$ ft/s
$t = 10.3$ s

12–231.

At the instant shown car A is traveling with a velocity of 30 m/s and has an acceleration of 2 m/s^2 along the highway. At the same instant B is traveling on the trumpet interchange curve with a speed of 15 m/s, which is decreasing at 0.8 m/s^2. Determine the relative velocity and relative acceleration of B with respect to A at this instant.



SOLUTION

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

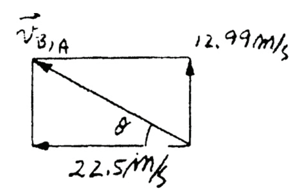
$15\cos 60°\mathbf{i} + 15\sin 60°\mathbf{j} = 30\mathbf{i} + (v_{B/A})_x\mathbf{i} + (v_{B/A})_y\mathbf{j}$

$15\cos 60° = 30 + (v_{B/A})_x$

$15\sin 60° = 0 + (v_{B/A})_y$

$(v_{B/A})_x = -22.5 = 22.5\text{ m/s} \leftarrow$

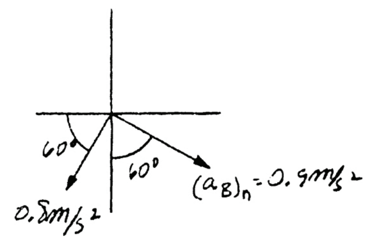
$(v_{B/A})_y = 12.99\text{ m/s}\uparrow$



$v_{B/A} = \sqrt{(22.5)^2 + (12.99)^2} = 26.0\text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{12.99}{22.5}\right) = 30°$ **Ans.**

$(a_B)_n = \dfrac{v^2}{\rho} = \dfrac{15^2}{250} = 0.9\text{ m/s}^2$



$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

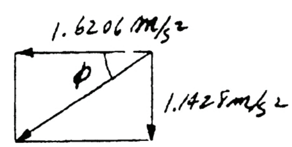
$-0.8\cos 60°\mathbf{i} - 0.8\sin 60°\mathbf{j} + 0.9\sin 60°\mathbf{i} - 0.9\cos 60°\mathbf{j} = 2\mathbf{i} + (a_{B/A})_x\mathbf{i} + (a_{B/A})_y\mathbf{j}$

$-0.8\cos 60° + 0.9\sin 60° = 2 + (a_{B/A})_x$

$-0.8\sin 60° - 0.9\cos 60° = (a_{B/A})_y$

$(a_{B/A})_x = -1.6206\text{ ft/s}^2 = 1.6206\text{ m/s}^2 \leftarrow$

$(a_{B/A})_y = -1.1428\text{ ft/s}^2 = 1.1428\text{ m/s}^2\downarrow$



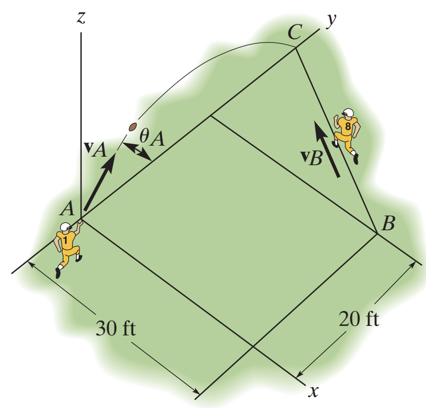
$a_{B/A} = \sqrt{(1.6206)^2 + (1.1428)^2} = 1.98\text{ m/s}^2$ **Ans.**

$\phi = \tan^{-1}\left(\dfrac{1.1428}{1.6260}\right) = 35.2°$ **Ans.**

Ans:
$v_{B/A} = 26.0\text{ m/s}$
$\theta = 30°$
$a_{B/A} = 1.98\text{ m/s}^2$
$\phi = 35.2°$

***12–232.**

The football player at A throws the ball in the y–z plane at a speed $v_A = 50$ ft/s and an angle $\theta_A = 60°$ with the horizontal. At the instant the ball is thrown, the player is at B and is running with constant speed along the line BC in order to catch it. Determine this speed, v_B, so that he makes the catch at the same elevation from which the ball was thrown.



SOLUTION

$$(\xrightarrow{+}) \quad s = s_0 + v_0 t$$

$$d_{AC} = 0 + (50 \cos 60°)\, t$$

$$(+\uparrow) v = v_0 + a_c t$$

$$-50 \sin 60° = 50 \sin 60° - 32.2\, t$$

$$t = 2.690 \text{ s}$$

$$d_{AC} = 67.24 \text{ ft}$$

$$d_{BC} = \sqrt{(30)^2 + (67.24 - 20)^2} = 55.96 \text{ ft}$$

$$v_B = \frac{55.96}{2.690} = 20.8 \text{ ft/s}$$ **Ans.**

Ans:
$v_B = 20.8$ ft/s