Intermediate Algebra Functions and Authentic Applications 6th edition Lehmann Solutions Manual

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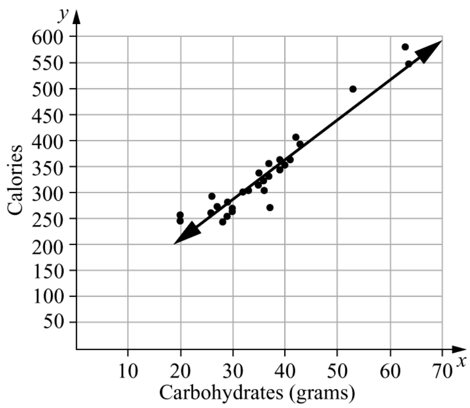
Chapter 2
Modeling with Linear Functions

Homework 2.1

2. a. The variables x and y are approximately linearly related. The points in the scatterplot lie close to a line.

b. Draw a line that comes close to the points to construct the linear model as shown.

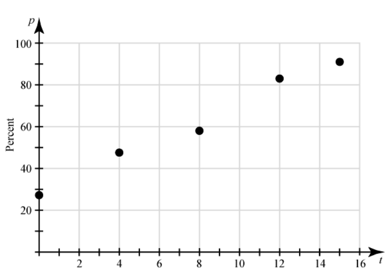


c. A pizza with 30 carbohydrates has about 290 calories. We performed interpolation because we used a part of the model whose x-coordinates are between the x-coordinates of two data points.

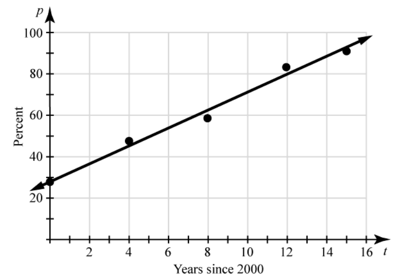
d. A pizza with 450 calories has about 51 carbohydrates. We performed interpolation because we used a part of the model whose x-coordinates are between the x-coordinates of two data points.

e. The line in the scatterplot goes up from left to right. Answers may vary. Example: A line going up from left to right makes sense because it shows that an increase in the number of carbohydrates results in an increase in the number of calories.

4. a.



b.



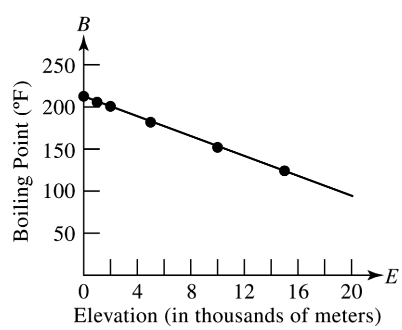
c. When half of tax returns were filed online, p equals 50. When p equals 50, t is approximately 5. Therefore, when half of tax returns were filed online, the year was approximately 2005.

d. When p equals 80, t equals approximately 12, which represents the year 2012. Therefore, it is estimated that the 80% tax return percentage was achieved in the year 2012. The goal was not achieved.

e. The goal was not reached by 2011, according to the model, the percentage of tax returns filed online in 2011 was about 75%.

6. Responses for this exercise are the same for parts (c) and (d) of Exercise 5. Responses for this exercise are different for parts (a), (b), and (e) of Exercise 5. Parts (a) and (b) will be different since we graph values of t since 1985 rather than 1990. This means that the values of t shift to the right by 5 for all coordinates in the scatterplot Part (e) will also be different since again, the graph will have shifted to the right by 5. Note that even though the t-intercept of this model is (43, 0), the predicted year in which there will be no collisions does not change since the calculation is based on years since 1985. (That is, 1985 + 43 = 2028.)

8. a.



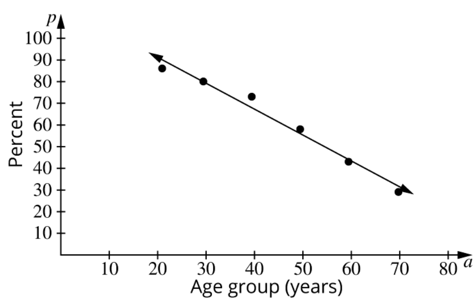
b. Draw a line through the points. See the graph in part (a).

c. The elevation 8850 meters is represented by $E = 8.85$. Locating this on the linear model, note that B is approximately 160. Therefore, the boiling point of water is 160°F at the peak of Mount Everest. Interpolation was used, owing to our examination of predicted data in between points.

d. According to the linear model, when $B = 98.6$, the elevation is approximately 19 thousand meters. Therefore, at an elevation of 19,000 meters, boiling water feels lukewarm. Extrapolation was used, owing to the examination of predicted data outside of the data points.

e. Answers may vary. Example:
Keep in mind that the higher the boiling point (lower elevation), the less time it takes for an egg to cook. Likewise, the lower the boiling point (higher elevation), the more time it takes for an egg to cook. The linear model should be increasing.

10. a.



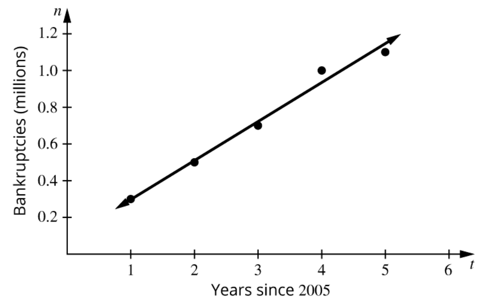
b. Draw a line that comes close to the points to construct the linear model. See the graph in part (a).

c. According to the model, 70% of Americans have a profile page around 38 years of age.

d. The p-intercept of the model is (0, 115). This means 115% of newborns have a personal profile page. Model breakdown has occurred.

e. The a-intercept of the model is (96, 0). This means no 96-year-olds have a personal profile page. Model breakdown is likely since there may be some 96-year-olds with personal profile pages.

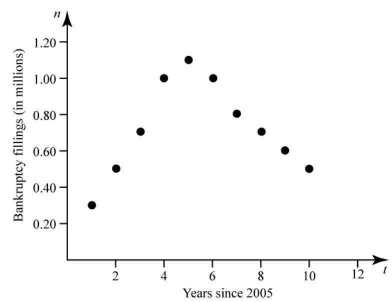
12. a.



b. Draw a line that comes close to the points to construct the linear model. See the graph in part (a).

c. According to the model, the number of bankruptcy filings in 2015 was about 2.2 million.

d.

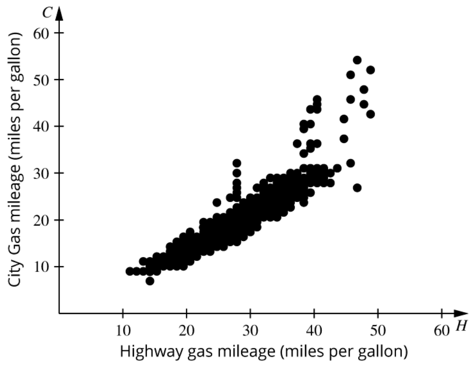


e. According to the table, in 2015, there were actually 0.5 million bankruptcy filings. The error is therefore $2.2 - 0.5 = 1.7$ million filings. The error is likely so large since we extrapolated the number of bankruptcies in 2015 using the model.

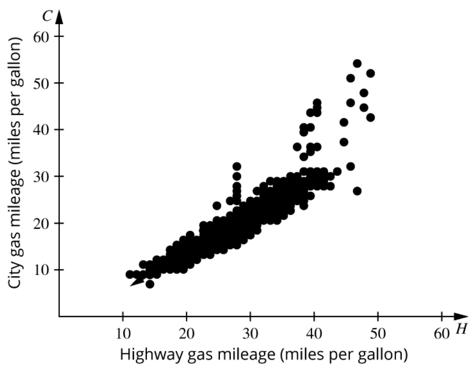
14. a. To find the t-intercept, estimate what the temperature will be when $w = 0$. This estimate is approximately (13, 0). This means that if the wind speed is 10 mph and the temperature is 13 degrees Fahrenheit, it will feel like it is 0 degrees Fahrenheit.

b. To find the w-intercept, estimate what the windchill will be when $t = 0$. This estimate is approximately (0, –16). This means that if the wind speed is 10 mph and the temperature is 0 degrees Fahrenheit, it will feel like it is 16 degrees Fahrenheit below 0.

16. a.



b.



c. According to the model, the city gas mileage for a car with a highway gas mileage of 30 miles per gallon is about 22 miles per gallon.

d. According to the model, the highway gas mileage of a car with a city gas mileage of 14 miles per gallon is about 20 miles per gallon.

e. The *H*-intercept is (3.3, 0). This means that a car with a highway gas mileage of 3.3 miles per gallon has a city gas mileage of 0 miles per gallon. This seems unlikely since a car must use some amount of gas to run, and we assume that the cars in question are cars that run on gas. Model breakdown occurs in this case.

f. Going back to the original data, if you sort the data numerically, you will see that there are cars for which the highway and city gas mileages match. For example, there are several cars that have a city gas mileage of 13 and a highway gas mileage of 19. In other cases, there are cars whose city and gas mileages are unique and therefore don't match other cars. When the data is graphed on a scatterplot, we cannot show how a point like (13, 19) represents multiple cars; we see just one point at (13, 19). We may see other points like this that represent multiple cars, but there is no way for us to tell looking at the scatterplot alone. In other cases, a single point represents just one car since there are cars whose city and gas mileages are unique. Because a single data point might possibly represent gas mileages of several cars and another data point might represent only a single car, we shouldn't give the same weight to each data point. In other words, using a scatterplot, we cannot know which data values occur most frequently. Therefore, the line we draw is not necessarily the best one since it may not trend toward the points where the data is actually the most frequent.

g. The data points that are farthest from the model are mostly above the trendline. The type of car that most of these points represent is the hybrid, which makes sense because a hybrid vehicle tends to get better city mileage than highway mileage.

18. We usually have more faith in a result obtained by interpolation.
Answers may vary.

20. Answers may vary. Example:
All points will probably not describe the situation exactly, but they are likely to describe the situation quite well.

22. Answers may vary.

Homework 2.2

2. We first find the slope using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Substitute the values (0, 8.7) and (14, 14.7) into the formula:
$\dfrac{14.7 - 8.7}{14 - 0} = \dfrac{6}{14} \approx 0.43$. Now we can use the formula $y = mx + b$ to find the equation. Using the variables n and t we have $n = 0.43t + b$. Since we have two sets of coordinates, we can choose one and substitute values for t and n. If we choose (0, 8.7) we have:

$$n = 0.43t + b$$
$$8.7 = 0.43 \cdot 0 + b$$
$$8.7 = b$$

Substituting 8.7 for b in the equation $n = 0.43t + b$ gives us $n = 0.43t + 8.7$.

4. We first find the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Substitute the values (4, 34) and (14, 30) into the formula:

$\frac{30-34}{14-4} = \frac{-4}{10} = -0.4$. Now we can use the formula $y = mx + b$ to find the equation. Using the variables E and t we have $E = -0.4t + b$. Since we have two sets of coordinates, we can choose one and substitute values for t and E. If we choose (4, 34) we have:

$$E = -0.4t + b$$
$$34 = -0.4 \cdot 4 + b$$
$$35.6 = b$$

Substituting 35.6 for b in the equation $E = -0.4t + b$ gives us $E = -0.4t + 35.6$.

6. We first find the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Substitute the values (3, 6922) and (6, 8014) into the formula:

$\frac{8014 - 6922}{6-3} = \frac{1092}{3} = 364$. Now we can use the formula $y = mx + b$ to find the equation. Using the variables s and t we have $s = 364t + b$. Since we have two sets of coordinates, we can choose one and substitute values for t and s. If we choose (3, 6922) we have:

$$s = 364t + b$$
$$6922 = 364 \cdot 3 + b$$
$$5830 = b$$

Substituting 5830 for b in the equation $s = 364t + b$ gives us $s = 364t + 5830$.

8. Use (4, 0.10) and (10, 0.19) to write the equation of the line. First, find the slope.

$$m = \frac{0.19 - 0.10}{10 - 4} = \frac{0.09}{6} = 0.015$$

So, $c = 0.015w + b$. Substitute 4 for t and 0.10 for c since the line contains (4, 0.10) and then solve for b.

$$c = mw + b$$
$$0.10 = 0.015(4) + b$$
$$0.10 = 0.06 + b$$
$$0.04 = b$$

The equation of the line is $c = 0.015w + 0.04$.

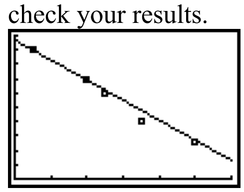
10. Use (1, 18) and (10, 5) to write the equation of the line. First, find the slope.

$$m = \frac{5 - 18}{10 - 1} = \frac{-13}{9} \approx -1.44$$

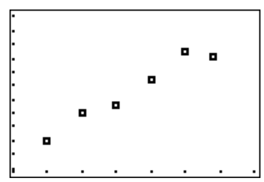
So, $y = -1.44x + b$. Substitute 1 for x and 18 for y since the line contains (1, 18) and then solve for b.

$$y = mx + b$$
$$18 = -1.44(1) + b$$
$$18 = -1.44 + b$$
$$19.44 = b$$

The equation of the line is $y = -1.44x + 19.44$. (Your equation may be slightly different if you chose different points, or if you used the linear regression feature of your graphing calculator.) Use the graphing calculator to check your results.



12. a.



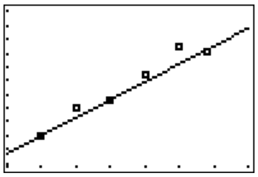
b. We first find the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Substitute the values (90, 28.0) and (100, 33.2) into the formula:

$\frac{33.2 - 28.0}{100 - 90} = \frac{5.2}{10} = 0.52$. Now, we can use the formula $y = mx + b$ to find the equation. Using the variables t and p, we have $p = 0.52t + b$. Since we have two sets of coordinates, we can choose one and substitute values for t and p. If we choose (100, 33.2), we have:

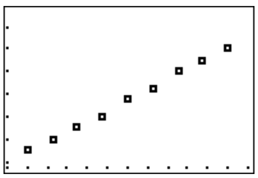
$$p = 0.52t + b$$
$$33.2 = 0.52 \cdot 100 + b$$
$$33.2 = 52 + b$$
$$-18.8 = b$$

Substituting -18.8 for b in the equation $p = 0.52t + b$ gives us $p = 0.52t - 18.8$. Note that the slope of this equation matches the slope of the equation in Exercise 17. However, the p-intercepts are not the same. We would expect this since the change in t-coordinates impacts the positions of the lines, which means each line crosses the p-axis at a different value of p. The slope stays the same because the change from one value of t to another stays the same and the change from one value of p to another also stays the same.

c.



14. a.

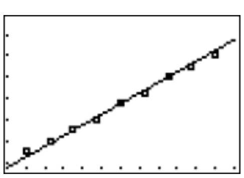


b. We first find the slope using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Substitute the values (42, 10) and (62, 20) into the formula:
$\dfrac{20-10}{62-42} = \dfrac{10}{20} = 0.5$. Now, we can use the formula $y = mx + b$ to find the equation. Using the variables a and p, we have $p = 0.5a + b$. Since we have two sets of coordinates, we can choose one and substitute values for a and p. If we choose (42, 10), we have:

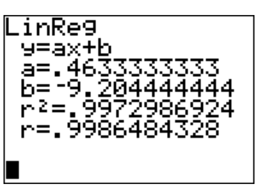
$$
\begin{aligned}
p &= 0.5a + b \\
10 &= 0.5 \cdot 42 + b \\
10 &= 21 + b \\
-11 &= b
\end{aligned}
$$

Substituting -11 for b in the equation $p = 0.5a + b$ gives us $p = 0.5a - 11$.

c.

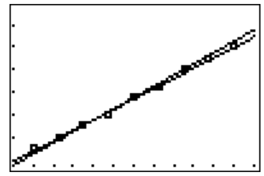


d.



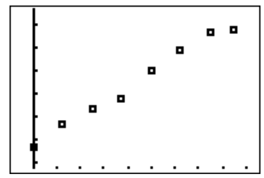
Given the screen, the regression equation may be written as $p = 0.46a - 9.20$.

e.



The two graphs are very close. They are both good models of the data.

16. a.

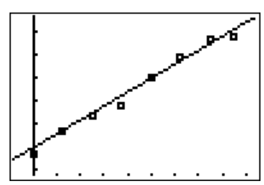


b. We first find the slope using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Substitute the values (5, 74.7) and (20, 77) into the formula:
$\dfrac{77-74.7}{20-5} = \dfrac{2.3}{15} \approx 0.15$. Now, we can use the formula $y = mx + b$ to find the equation. Using the variables t and L, we have $L = 0.15t + b$. Since we have two sets of coordinates, we can choose one and substitute values for t and L. If we choose (20, 77), we have:

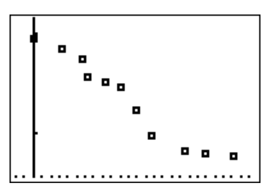
$$
\begin{aligned}
L &= 0.15t + b \\
77 &= 0.15 \cdot 20 + b \\
77 &= 3 + b \\
74 &= b
\end{aligned}
$$

Substituting 74 for b in the equation $L = 0.15t + b$ gives us $L = 0.15t + 74$.

c.



18. a.

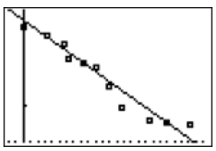


b. We first find the slope using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Substitute the values (0, 47.8) and (99, 43.18) into the formula:

$\dfrac{43.18-47.8}{99-0}=\dfrac{-4.62}{99}\approx -0.05$. Now, we can use the formula $y = mx + b$ to find the equation. Using the variables t and r, we have $r=-0.05t+b$. Since we have two sets of coordinates, we can choose one and substitute values for t and r. If we choose (99, 43.18), we have:

$$r=-0.05t+b$$
$$43.18=-0.05\cdot 99+b$$
$$43.18=-4.95+b$$
$$48.13=b$$

Substituting 48.13 for b in the equation $r=-0.05t+b$ gives us $r=-0.05t+48.13$.

c. 

The line comes close to all the data points and so the model $r=-0.05t+48.13$ is a good approximation.

20. **a.** The equation of the line is $r=0.080s+1.766$.

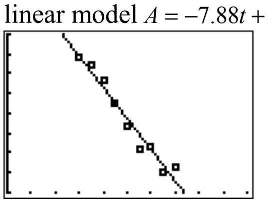
b. The equation of the line is $r=0.079s+1.657$.

c. The women's equation has the larger r-coordinate (1.766). This tells you that the women's regression line is above the men's line when $s = 0$.

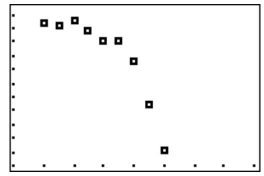
d. The women's model has a slightly larger slope. This indicates the line rises more quickly in comparison to the men's line.

e. The women's line begins above the men's line and also rises faster than the men's line, so the two lines can never intersect in quadrant I. Women, in general, have faster stride rates, regardless of speed.

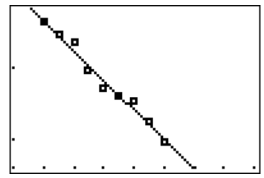
22. **a.** Yes, the data can be modeled well by the linear model $A=-7.88t+336$.



b. Since no line will fit most of the data points, the data is not modeled well by a linear model.



c. Yes, the data can be modeled well by the linear model $C=-103.28t+4439.06$.

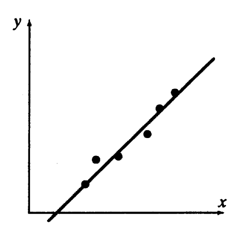


d. **i.** Aggravated assaults continued to decline even though the economy did poorly, which supports the theory that crime and the economy are not related.

ii. Burglaries were somewhat level during the period 2006–2008 when the economy performed well and dropped fairly dramatically during 2012–2014 when the economy was doing fairly well.

iii. All types of crime declined linearly, despite the ups and downs of the economy.

24. Student C made the best choice. A line that comes close to all the data points is best modeled by a line that goes through the points for the data in the years 2003 and 2014.

26. 

In this model, m needs to be greater (steeper slope) than it was in the original model. The value of b should decrease.

28. Answers may vary.

Homework 2.3

2. $f(-2)=6(-2)-4=-12-4=-16$

4. $f\left(\frac{5}{2}\right)=6\left(\frac{5}{2}\right)-4=\frac{30}{2}-4=15-4=11$

6. $f(a-3)=6(a-3)-4=6a-18-4=6a-22$

8. $g(3)=2(3^2)-5(3)=2(9)-5(3)=18-15=3$

10. $g(-2)=2((-2)^2)-5(-2)=2(4)-5(-2)$
$=8-(-10)=18$

12. $h(-4)=\frac{3(-4)-4}{5(-4)+2}=\frac{-12-4}{-20+2}=\frac{-16}{-18}=\frac{8}{9}$

14. $h(3a)=\frac{3(3a)-4}{5(3a)+2}=\frac{9a-4}{15a+2}$

16. $g(-1)=-3(-1)^2+2(-1)=-3-2=-5$

18. $f(-4)=-2(-4)+7=8+7=15$

20. $h(-9)=-4$

22. $f(-3a)=-4(-3a)-7=12a-7$

24. $f\left(\frac{3a}{2}\right)=-4\left(\frac{3a}{2}\right)-7=-\frac{12a}{2}-7=-6a-7$

26. $f(a-4)=-4(a-4)-7$
$=-4a+16-7$
$=-4a+9$

28. Substituting $a + h$ wherever there is an x in $f(x)$:
$f(a+h)=-4(a+h)-7=-4a-4h-7$

30. To find x when $f(x)=0$, substitute 0 for $f(x)$ and solve for x:

$$\begin{aligned}0&=-3x+7\\3x&=7\\x&=\frac{7}{3}\end{aligned}$$

32. To find x when $f(x)=\frac{4}{3}$, substitute $-\frac{4}{3}$ for $f(x)$ and solve for x:

$$\begin{aligned}-\frac{4}{3}&=-3x+7\\-4&=-9x+21\\9x-4&=21\\9x&=25\\x&=\frac{25}{9}\end{aligned}$$

34. To find x when $f(x)=a+2$, substitute $a+2$ for $f(x)$ and solve for x:

$$\begin{aligned}a+2&=-3x+7\\a-5&=-3x\\\frac{a-5}{-3}&=x\\\frac{5-a}{3}&=x\end{aligned}$$

36. $f(17.28)=-5.95(17.28)+183.22$
$=-102.816+183.22=80.404$
≈ 80.40

38. To find x when $f(x)=72.06$, substitute 72.06 for $f(x)$ and solve for x:

$$\begin{aligned}72.06&=-5.95x+183.22\\0&=-5.95x+183.22-72.06\\5.95x&=111.16\\x&\approx 18.68\end{aligned}$$

40. $f(4)=0$

42. When $f(x)=4$, x is 2.

44. Since the line contains the point (0, 2), $f(0)=2$.

46. Since the line contains the point $\left(-\frac{11}{2},\frac{23}{6}\right)$, $f\left(-\frac{11}{2}\right)=\frac{23}{6}$.

48. Since the line contains the point (3, 1), $x = 3$ when $f(x)$ or $y = 1$.

50. Since the line contains the point (–4.5, 3.5), $x = -4.5$ when $f(x)$ or $y = 3.5$.

52. Since the line contains the point $\left(-\frac{3}{2}, \frac{5}{2}\right)$,

$x = -\frac{3}{2}$ when $f(x)$ or $y = \frac{5}{2}$.

54. The range is $-\infty < f(x) < \infty$.

56. Since the function contains the point (2, 3), $x = 2$ when $g(x)$ or $y = 3$.

58. $-2 \le g(x) \le 4$

60. Since the function contains the point $(-2, -1)$,

$x = -2$ when $h(x)$ or $y = -1$.

62. $-4 \le h(x) \le 1$

64. $f(x)$ or $y = 4x + 2$

To find the y-intercept, set $x = 0$ and solve for y.

$y = 4(0) + 2 = 0 + 2 = 2$

The y-intercept is (0, 2).

To find the x-intercept, set $y = 0$ and solve for x.

$$0 = 4x + 2$$
$$-2 = 4x$$
$$x = -\frac{2}{4} = -\frac{1}{2}$$

The x-intercept is $\left(-\frac{1}{2}, 0\right)$.

66. $f(x)$ or $y = -7x$

To find the y-intercept, set $x = 0$ and solve for y.

$y = -7(0) = 0$

The y-intercept is (0, 0).

To find the x-intercept, set $y = 0$ and solve for x.

$$0 = -7x$$
$$0 = x$$

The x-intercept is (0, 0).

68. $f(x)$ or $y = -2$ is the equation of a horizontal line. The y-intercept of the line will be (0, –2). There is no x-intercept.

70. $f(x)$ or $y = -\frac{3}{4}x + \frac{1}{2}$

To find the y-intercept, set $x = 0$ and solve for y.

$y = -\frac{3}{4}(0) + \frac{1}{2} = 0 + \frac{1}{2} = \frac{1}{2}$

The y-intercept is $\left(0, \frac{1}{2}\right)$.

To find the x-intercept, set $y = 0$ and solve for x.

$$0 = -\frac{3}{4}x + \frac{1}{2}$$
$$-\frac{1}{2} = -\frac{3}{4}x$$
$$x = -\frac{1}{2}\left(-\frac{4}{3}\right) = \frac{2}{3}$$

The x-intercept is $\left(\frac{2}{3}, 0\right)$.

72. $f(x)$ or $y = -4.29x + 37.58$

To find the y-intercept, set $x = 0$ and solve for y.

$y = -4.29(0) + 37.58 = 37.58$

The y-intercept is (0, 37.58).

To find the x-intercept, set $y = 0$ and solve for x.

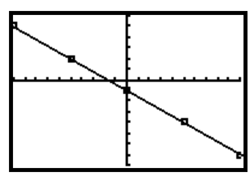
$$0 = -4.29x + 37.58$$
$$4.29x = 37.58$$
$$x \approx 8.76$$

The x-intercept is (8.76, 0).

74. a. Your table may be different if you chose different x values.

x	$f(x)$
–10	5
–5	2
0	–1
5	–4
10	–7

b.



c. For each input-output pair, the output is 1 less than $-\frac{3}{5}$ times the input.

76. a. Substitute $f(t)$ for p in the equation $p = 0.54t - 19.56$:

$f(t) = 0.54t - 19.56$

b. Since t is now years since 1900 instead of 1990, we must add 90 to the value 31. Then we evaluate the function. Substitute 121 for t in the function $f(t) = 0.54t - 19.56$:

$f(121) = 0.54 \cdot 121 - 19.56 = 45.78$

This means in 2021 (1900 + 121), about 46% of births will be outside marriage.

This is essentially the same result as the one we obtained in Exercise 1, part (b).

c. Set the function, $f(t)$ equal to 31:

$$31 = 0.54t - 19.56$$
$$\frac{50.56}{0.54} = \frac{0.54t}{0.54}$$
$$93.63 \approx t$$

This means that in 1994 (1900 + 94), 31% of births were outside marriage. This is the same result as the one we obtained in Exercise 1, part (c).

d. To find in what year all births will be outside of marriage, substitute 100 for $f(t)$ in the function $f(t) = 0.54t - 19.56$:

$$f(t) = 0.54t - 19.56$$
$$100 = 0.54t - 19.56$$
$$\frac{119.56}{0.54} = \frac{0.54t}{0.54}$$
$$221.41 \approx t$$

This means all births, 100%, will be outside marriage in 2121 (1900 + 221). Model breakdown has likely occurred. This is essentially the same result as the one we obtained in Exercise 1, part (d).

e. Using $f(t) = 0.54t - 19.56$ we can estimate the percentage of births outside marriage in 2005. Substitute 105 for t in the function:

$$f(t) = 0.54t - 19.56$$
$$f(105) = 0.54 \cdot 105 - 19.56$$
$$= 56.7 - 19.56$$
$$= 37.14$$

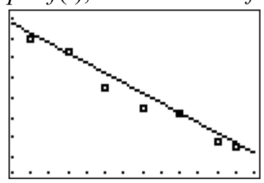
We estimate the percentage of births outside marriage in 2005 to be about 37.1%. The actual percentage is 36.9%. To find the error, subtract: $37.1\% - 36.9\% = 0.2\%$. Since the estimate of the percentage of births outside marriage is greater than the actual, the error is 0.2%. This is not the same result as the one we obtained in Exercise 1, part (e). In this case, the model overestimated the percentage, whereas the model in Exercise 1 underestimated the percentage.

78. a. We select two points from the table and use the slope using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Substitute the values (20, 69) and (50, 57) into the formula:

$\dfrac{57-69}{50-20} = \dfrac{-12}{30} = -0.4$. Now, we can use the formula $y = mx + b$ to find the equation. Using the variables p and t, we have $p = -0.4t + b$. Since we have two sets of coordinates, we can choose one and substitute values for t and p. If we choose (20, 69), we have:

$$p = -0.4t + b$$
$$69 = -0.4 \cdot 20 + b$$
$$69 = -8 + b$$
$$77 = b$$

Substituting 77 for b in the equation $p = -0.4t + b$ gives us $p = -0.4t + 77$. Since $p = f(t)$, we can write $f(t) = -0.4t + 77$.



We can see that the model does fit the data well.

b. To evaluate $f(70)$, substitute 70 for t in $f(t) = -0.4t + 77$:

$$f(70) = -0.4 \cdot 70 + 77$$
$$= -28 + 77$$
$$= 49$$

This means that in 2020 (1950 + 70), 49 percent of Americans 18 and older will be married.

c. Set $f(t)$ equal to 47:

$$47 = -0.4t + 77$$
$$\frac{-30}{-0.4} = \frac{-0.4t}{-0.4}$$
$$75 = t$$

This means that in 2025 (1950 + 75), 47 percent of Americans 18 and older will be married.

d. The p-intercept is (0, 77). This means in 1950, 77% of Americans 18 and older were married.

e. To find the t-intercept of the model, set $f(t) = -0.4t + 77$ equal to 0:

$$0 = -0.4t + 77$$
$$\frac{-77}{-0.4} = \frac{-0.4t}{-0.4}$$
$$192.5 = t$$

The t-intercept of the model is (192.5, 0). This means in 2143 (1950 + 193), 0 percent of Americans 18 and older will be married. Model breakdown has occurred.

80. a. To predict the life expectancy of an American born in 2021, substitute 41 for t in the equation $L = 0.16t + 73.79$:

$L = 0.16 \cdot 41 + 73.79$

$= 80.35$

The life expectancy of an American born in 2021 is estimated to be about 80.

b. To find the birth year in which the life expectancy of an American will be 80 years, substitute 80 for L in the equation $L = 0.16t + 73.79$:

$80 = 0.16t + 73.79$

$\frac{6.21}{0.16} = \frac{0.16t}{0.16}$

$38.81 \approx t$

The birth year in which the life expectancy will be 80 years is 2019 (1980 + 39).

c. Answers may vary. Example: If your year of birth was 1998, substitute 18 for t in the equation $L = 0.16t + 73.79$

$L = 0.16 \cdot 18 + 73.79$

$L = 2.88 + 73.79$

$L = 76.67$

This means your life expectancy at birth was estimated to be about 77 years.

d. To find the t-intercept, substitute 0 for L in the equation $L = 0.16t + 73.79$:

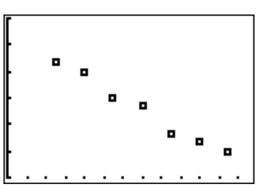
$0 = 0.16t + 73.79$

$\frac{-73.79}{0.16} = \frac{0.16t}{0.16}$

$-461.19 \approx t$

The t-intercept is $(-461.19, 0)$. This means in 1519 $(1980 - 461)$, the life expectancy of an American was 0 years. Model breakdown has occurred.

82. a.



b. We first find the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Choose two points from the table and substitute the values into the formula. We choose (5, 28) and (20, 19):

$\frac{19 - 28}{20 - 5} = \frac{-9}{15} = -0.6$. Now, we can use the formula $y = mx + b$ to find the equation. Using the variables t and A, we have $A = -0.6t + b$. Since we have two sets of coordinates, we can choose one and substitute values for t and A. If we choose (20, 19) we have:

$A = -0.6t + b$

$19 = -0.6 \cdot 20 + b$

$19 = -12 + b$

$31 = b$

Substituting 31 for b in the equation $A = -0.6t + b$ gives us $A = -0.6t + 31$. Since $A = f(t)$, we can write $f(t) = -0.6t + 31$.

Yes, this model fits the data very well.

c. To find the t-intercept, substitute 0 for $f(t)$ in $f(t) = -0.6t + 31$:

$0 = -0.6t + 31$

$-31 = -0.6t$

$\frac{-31}{-0.6} = \frac{-0.6t}{-0.6}$

$51.7 \approx t$

This means in 2042 (1990 + 52), the average gasoline tax per 1000 miles driven will be 0. Model breakdown has occurred.

d. Answers may vary. Example: Inflation would cause the average gasoline tax per 1000 miles driven to increase, since tax is a percent of the amount paid. This increase is offset by the increased gas mileage of cars. Since cars go farther on a gallon of gas, less gas is needed to go 1000 miles, and less tax is collected.

e. **i.** To find the tax paid for 1000 miles driven in 2019, substitute 29 for t in $f(t) = -0.6t + 31$:

$f(29) = -0.6 \cdot 29 + 31$

$f(29) = -17.4 + 31$

$f(29) = 13.6$

Now, the total tax paid is found by multiplying the tax with the mileage:

$7.624 \cdot 13.6 \approx 103.69$.

To find the tax paid for1000 miles driven in 2020, substitute 30 for t in $f(t) = -0.6t + 31$:

$f(30) = -0.6 \cdot 30 + 31$

$f(30) = -18 + 31$

$f(30) = 13$

Now, the total tax paid is found by multiplying the tax with the mileage:

$7.624 \cdot 13 \approx 99.11$.

To find the tax paid for1000 miles driven in 2021, substitute 31 for t in

$f(t) = -0.6t + 31$:
$f(31) = -0.6 \cdot 31 + 31$
$f(31) = -18.6 + 31$
$f(31) = 12.4$

Now, the total tax paid is found by multiplying the tax with the mileage:
$7.624 \cdot 12.4 \approx 94.54$.

Therefore, during the period 2019–2021, a teenager will pay 103.69 + 99.11 + 94.54 = 297.34 dollars in gasoline taxes.

ii. The tax paid for 3 years driving 7624 miles per year at a rate of \$28 per 1000 miles driven is:
$3 \cdot 7.624 \cdot 28 \approx 640.42$.
The additional tax a teenager would pay is $640.42 - 297.34 = 343.08$ dollars.

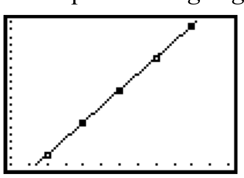
84. a. Two of the data points are (50, 43) and (90, 215). To find the equation for *g*, find the slope using these points.

$$m = \frac{215 - 43}{90 - 50} = \frac{172}{40} = 4.3$$

So, $g(F) = 4.3F + b$. To solve for *b*, substitute 50 for *F* and 43 for *g* since the line contains (50, 43).

$$43 = 4.3(50) + b$$
$$43 = 215 + b$$
$$-172 = b$$

The equation for *g* is $g(F) = 4.3F - 172$.



b. $g(73) = 4.3(73) - 172 = 313.9 - 172 = 141.9$

When the temperature is 73°F, crickets will chirp approximately 142 times per minute.

c.
$$100 = 4.3F - 172$$
$$272 = 4.3F$$
$$63.26 \approx F$$

When the temperature is 63°F, crickets will chirp approximately 100 times per minute.

d. When crickets are not chirping, $g(F) = 0$.

We need to find *F*, when $g(F) = 0$.

$$0 = 4.3F - 172$$
$$172 = 4.3F$$
$$40 = F$$

When the temperature is 40°F or below, crickets will not chirp.

86. a. Substitute *f*(*a*) for *p* in the equation $p = 0.46a - 9.20$:
$f(a) = 0.46a - 9.20$.

b. Substitute 40 for *a* in $p = 0.46a - 9.20$:
$p = 0.46 \cdot 40 - 9.20$
$p = 18.4 - 9.20$
$p = 9.2$

We estimate the percentage of 40-year-old Americans diagnosed with diabetes to be about 9%.

c. Substitute 16 for *p* in $p = 0.46a - 9.20$:

$$16 = 0.46a - 9.20$$
$$25.2 = 0.46a$$
$$\frac{25.2}{0.46} = \frac{0.46a}{0.46}$$
$$54.78 \approx a$$

We estimate the age at which 16% of Americans are diagnosed with diabetes to be about 55 years old.

d. Substitute 0 for *p* in $p = 0.46a - 9.20$:

$$0 = 0.46a - 9.20$$
$$9.20 = 0.46a$$
$$\frac{9.20}{0.46} = \frac{0.46a}{0.46}$$
$$20 = a$$

The *a*-intercept is (20, 0). This means the age at which 0% of Americans are diagnosed with diabetes is 20 years old. Model breakdown has likely occurred.

e. Substitute 77 (the average of 75 and 79) for *a* in $p = 0.46a - 9.20$:
$p = 0.46 \cdot 77 - 9.20$
$p = 35.42 - 9.20$
$p = 26.22$

This means about 26% of Americans are diagnosed with diabetes at the age of 77. So, $0.26 \cdot 8{,}367{,}895 = 2{,}175{,}652.7$ of these Americans are diagnosed with diabetes.

88. We need to find an equation. We first find the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Let *x* be years since 1955. Use the two given values (0, 1.4) and (60, 10.1) and substitute them into

the formula: $\frac{10.1-1.4}{60-0}=\frac{8.7}{60}\approx 0.15$.
Now, we can use the formula $y = mx + b$ to find the equation. Substitute 0.15 for m: $y=0.15x+b$. Since we have two sets of coordinates, we can choose one and substitute values for x and y. If we choose (60, 10.1), we have:

$$\begin{aligned} y&=0.15x+b\\ 10.1&=0.15\cdot 60+b\\ 10.1&=9.0+b\\ 1.1&=b \end{aligned}$$

Substituting 1.1 for b gives us $y=0.15x+1.1$. Now, we substitute 66 (2021 – 1955) for x in the equation $y=0.26x+2.22$:

$$\begin{aligned} y&=0.15x+1.1\\ y&=0.15\cdot 66+1.1\\ y&=9.9+1.1\\ y&=11 \end{aligned}$$

We predict that the number of words in the federal tax code in 2021 will be 11 million.

90. We need to find an equation. We first find the slope using the formula $m=\frac{y_2-y_1}{x_2-x_1}$. Let x be years since 1975. Use the two given values (0, 7) and (39, 20) and substitute them into the formula: $\frac{20-7}{39-0}=\frac{13}{39}\approx 0.33$.
Now, we can use the formula $y = mx + b$ to find the equation. Substitute 0.33 for m: $y=0.33x+b$. Since we have two sets of coordinates, we can choose one and substitute values for x and y. If we choose (39, 20), we have:

$$\begin{aligned} 20&=0.33\cdot 39+b\\ 20&=12.87+b\\ 7.13&=b \end{aligned}$$

Substituting 7.13 for b gives us $y=0.33x+7.13$. Now, we substitute 22 for y in the equation $y=0.33x+7.13$

$$\begin{aligned} 22&=0.33x+7.13\\ 14.87&=0.33x\\ \frac{14.87}{0.33}&=\frac{0.33x}{0.33}\\ 45.06&\approx x \end{aligned}$$

We predict that 22% of workers will prefer a female boss in 2020 (1975 + 45).

92. a. First, we need to find an equation. Find the slope using the formula $m=\frac{y_2-y_1}{x_2-x_1}$. Let x be years since 2006. Use the two given values (0, 518) and (9, 511) and substitute them into the formula:
$\frac{511-518}{9-0}=\frac{-7}{9}\approx -0.78$. Now, we can use the formula $y = mx + b$ to find the equation. Substitute -0.78 for m: $y=-0.78x+b$. Since we have two sets of coordinates, we can choose one and substitute values for x and y. If we choose (9, 511), we have:

$$\begin{aligned} 511&=-0.78\cdot 9+b\\ 511&=-7.02+b\\ 518.02&=b \end{aligned}$$

Substituting 518.02 for b gives us $y=-0.78x+518.02$. Now, substitute 16 for x:

$$\begin{aligned} y&=-0.78x+518.02\\ y&=-0.78\cdot 16+518.02\\ y&=-12.48+518.02\\ y&=505.54 \end{aligned}$$

So, in 2022, we predict the average math score on the SAT will be 506 points.

b. Substitute 507 for y in the equation $y=-0.78x+518.02$:

$$\begin{aligned} 507&=-0.78x+518.02\\ -11.02&=-0.78x\\ \frac{-11.02}{-0.78}&=\frac{-0.78x}{-0.78}\\ 14.13&=x \end{aligned}$$

We predict that the average math score on the SAT will be 507 points in 2020 (2006 + 14).

94. First, we need to find an equation. Find the slope using the formula $m=\frac{y_2-y_1}{x_2-x_1}$. Use the two given values (4, 4233) and (6, 5675) and substitute them into the formula:
$\frac{5675-4233}{6-4}=\frac{1442}{2}=721$. Now, we can use the formula $y = mx + b$ to find the equation. Substitute 721 for m: $y=721x+b$. Since we have two sets of coordinates, we can choose one and substitute values for x and y. If we choose (6, 5675), we have:

$5675 = 721 \cdot 6 + b$
$5675 = 4326 + b$
$1349 = b$

Substituting 1349 for b gives us $y = 721x + 1349$. Now, substitute 7 for x:

$y = 721 \cdot 7 + 1349$
$y = 5047 + 1349$
$y = 6396$

So, the income limit for a family of seven is \$6396.

96. a. From the given information, we have the data points (0, 12) and (5, 0). Use these points to find the slope of the equation for f.

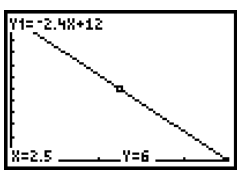
$$m = \frac{0-12}{5-0} = \frac{-12}{5} = -2.4$$

So, $f(t) = -2.4t + b$. Since the line contains (0, 12), substitute 0 for t and 12 for $f(t)$ and solve for b.

$12 = -2.4(0) + b$
$12 = b$

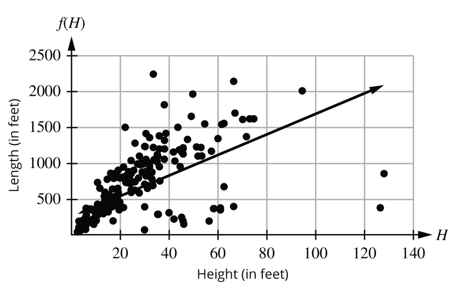
The linear model is $f(t) = -2.4t + 12$.

b.



c. Since it takes 5 minutes to eat the ice cream, the domain is $0 \le t \le 5$. Since there are 12 ounces of ice cream, the range is $0 \le f(t) \le 12$.

98. a.



b. To find an equation, we can use a regression feature that yields $f(H) = 14.314H + 257.33$.

c. See graph in part (a).

d. Substitute 30 for H in $f(H) = 14.314H + 257.33$:

$f(H) = 14.314 \cdot 30 + 257.33$
$f(H) = 429.42 + 257.33$
$f(H) = 686.75$

So, we estimate the length of a wooden roller coaster with a height of 30 feet is 686.75 feet.

e. Substitute 1600 for $f(H)$ in $f(H) = 14.314H + 257.33$:

$1600 = 14.314H + 257.33$
$1342.67 = 14.314H$
$$\frac{1342.67}{14.314} = \frac{14.314H}{14.314}$$
$93.80 \approx H$

So, we estimate the height of a wooden roller coaster with a length of 1600 feet to be about 93.80 feet.

f. The slope of the graph is 14.314. This means that the length of a wooden roller coaster is 14.314 feet longer than one that is 1 foot shorter.

g. The L-intercept is (0, 257.33). This means that a wooden roller coaster with a height of 0 feet has a length of 257.33 feet.

100. Instead of taking the opposite of 5 squared, the student should find –5 squared, since the function should be evaluated at –5. The correct procedure is:

$g(-5) = (-5)^2 = 25$

102. a. $f(5) = 3(5) + 2 = 15 + 2 = 17$
$f(4) = 3(4) + 2 = 12 + 2 = 14$
$f(5) - f(4) = 17 - 14 = 3$

This value and the slope are the same.

b. $f(7) = 2(7) + 5 = 14 + 5 = 19$
$f(6) = 2(6) + 5 = 12 + 5 = 17$
$f(7) - f(6) = 19 - 17 = 2$

This value and the slope are the same.

c. $f(3) = 4(3) + 1 = 12 + 1 = 13$
$f(2) = 4(2) + 1 = 8 + 1 = 9$
$f(3) - f(2) = 13 - 9 = 4$

This value and the slope are the same.

d. $f(a+1) = m(a+1) + b = ma + m + b$
$f(a) = m(a) + b = ma + b$
$f(a+1) - f(a) = ma + m + b - ma - b = m$

This value and the slope are the same. This indicates that for every increment of one in a linear function, the next value will have a y value increased by the slope value.

104. No. Answers may vary. Example:
$f(4)$ means that you should evaluate the function f at 4, that is, substitute 4 for the explanatory variable and solve.

106. Answers may vary. Example:
The four steps are: **1.** Construct a scatterplot of the data to determine whether there is a nonvertical line that comes close to the data points. If so, choose two points (not necessarily data points) that you can use to find the equation of a linear model. **2.** Find an equation of your model. **3.** Verify your equation by checking that the graph of your model contains the two chosen points and comes close to all the data points. **4.** Use the equation of your model to make estimates, make predictions, and draw conclusions.

Homework 2.4

2. The approximate rate of change of the amount of labor union campaign spending is given by dividing the change in the amount of spending by the change in years.

$$\begin{aligned} m &= \frac{\text{change in spending}}{\text{change in years}} \\ &= \frac{108.2-37.2}{2016-1992} \\ &= \frac{71}{24} \\ &\approx 2.96 \end{aligned}$$

This means that the approximate rate of change of the amount of labor union campaign spending in presidential election cycles was \$2.96 million per year.

4. The approximate rate of change in the percentage of American adults who have quite a lot of trust in newspapers is given by dividing the change in the percentage of American adults who have quite a lot of trust in newspapers by the change in years.

$$\begin{aligned} m &= \frac{\text{change in percentage}}{\text{change in years}} \\ &= \frac{20-28}{2016-2011} \\ &= \frac{-8}{5} \\ &= -1.6 \end{aligned}$$

This means that the approximate rate of change in the percentage of American adults who have quite a lot of trust in newspapers was a decrease of 1.6% per year.

6. The approximate rate of change in the enrollment at ITT Educational Services is given by dividing the change in the enrollment by the change in years.

$$\begin{aligned} m &= \frac{\text{change in number of students}}{\text{change in years}} \\ &= \frac{40-65}{2016-2012} \\ &= \frac{-25}{4} \\ &= -6.25 \end{aligned}$$

This means that the approximate rate of change in the number of students enrolled at ITT Educational Services is –6.25 thousand students per year, or a decrease of 6.25 thousand students per year.

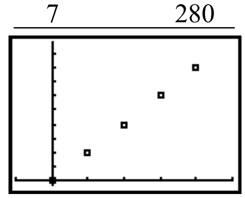
8. We find the approximate rate of change in the median price (\$2,500,000 – \$1,635,000) of a house in San Francisco to the change in number of bedrooms (4 – 2) as follows:

$$\begin{aligned} &\frac{\text{change in median price}}{\text{number of bedrooms}} \\ &= \frac{\$2,500,000-\$1,635,000}{4-2 \text{ bedrooms}} \\ &= \frac{\$865,000}{2 \text{ bedrooms}} \\ &= \frac{\$432,500}{1 \text{ bedroom}} \end{aligned}$$

So, the median price of a house in San Francisco increases at a rate of \$432,500 per bedroom.

10. a.

t **(hr)**	***d*** **(mi)**
1	40
2	80
3	120
4	160
5	200
6	240
7	280



b. There is a linear relationship because the train is moving at a constant speed. The slope is 40 miles per hour, so the distance increases 40 miles for each hour of travel.

12. a. Yes, there is an approximate linear relationship between t and p; the rate of increase is constant. The slope of the model is 3, which means the amount a player on a winning team receives for playing in the NFL Pro Bowl increases by about $3 thousand per year.

b. The p-intercept of the model is (0, 58). This means that at $t = 0$, or 2015, the amount a player on a winning team received for playing in the NFL Pro Bowl was $58 thousand.

c. $p = 3t + 58$

d. thousands of dollars

$$= \frac{\text{thousands of dollars}}{\text{year}}\text{year} + \text{thousands of dollars}$$
$$= \text{thousands of dollars} + \text{thousands of dollars}$$
$$= \text{thousands of dollars}$$

This unit analysis shows that this model uses the correct units.

e. Since 2022 – 2015 = 7, the year 2022 corresponds to $t = 7$.

$p = 3(7) + 58 = 21 + 58 = 79$

The model predicts that in 2022, the amount a player on a winning team will receive for playing in the NFL Pro Bowl will be about $79 thousand.

14. a. The slope of the graph of f is –0.9.

b. $f(t) = -0.9t + 51$

c. To find this quantity, we substitute 7 for t in $f(t)$ and solve.

$f(t) = -0.9(7) + 51$
$f(t) = 44.7$

The model predicts that 44.7% of American adults will say they are upper middle class or middle class in 2022.

d. To find this quantity, we substitute 46 for $f(t)$ and solve.

$$46 = -0.9(t) + 51$$
$$-5 = -0.9t$$
$$5.56 \approx t$$

The model predicts that 46% of American adults will say they are upper middle class or middle class in about $2015 + 6 = 2021$.

e. The t-intercept of the model is given by solving for t when $f(t) = 0$.

$$0 = -0.9(t) + 51$$
$$-51 = -0.9t$$
$$\frac{-51}{-0.9} = t$$
$$56.67 \approx t$$

The t-intercept is (56.67, 0). The model predicts that no American adults will say they are upper middle class or middle class in 2015 + 56.67 $\approx$ 2072. Model breakdown has occurred.

16. a. The slope of the graph is $102.91 + 5.25 = 108.16$, or $108.16 per credit a student is taking.

b. $g(c) = 108.16c + 15$

c. $$\text{dollars} = \frac{\text{dollars}}{\text{credit}}\text{credits} + \text{dollars}$$
$$= \text{dollars} + \text{dollars}$$
$$= \text{dollars}$$

This unit analysis shows that this model uses the correct units.

d. To find $g(6)$, substitute 6 for c in the equation we found and compute.

$$g(6) = 108.16(6) + 15$$
$$= 648.96 + 15$$
$$= 663.96$$

$g(6)$ means that the total one-quarter cost of tuition and fees for 6 credits is $663.96.

e. $$988.44 = 108.16(c) + 15$$
$$973.44 = 108.16c$$
$$9 = c$$

This means that a student who pays $988.44 for one-quarter is taking 9 courses.

f. The domain of g are the possible values of c. In this case, $c > 0$ and $c < 10$ since the problem states that g is true for up to 10 credits.

18. a. The slope is 0.0303 atm/ft. This means that for every foot you descend, the pressure increases by 0.0303 atm.

b. $f(d) = 0.0303d + 1$

c. $\text{atm} = \dfrac{\text{atm}}{\text{ft}}(\text{ft}) + \text{atm}$

$\text{atm} = \text{atm} + \text{atm}$

$\text{atm} = \text{atm}$

This unit analysis shows that this model uses the correct units.

d. Twice the pressure at sea level would be 2 atm. We must find the depth where this pressure occurs. To do this, substitute 2 for $f(d)$ and solve for d.

$$2 = 0.0303d + 1$$
$$1 = 0.0303d$$
$$33.0 \approx d$$

The depth at which the pressure is 2 atm is 33 ft.

e. To find the pressure at 1943 feet, substitute 1943 for d and compute.

$$f(1943) = 0.0303(1943) + 1$$
$$f(1943) \approx 58.87 + 1$$
$$f(1943) \approx 59.87$$

The pressure at Crater Lake's greatest depth is about 60 atm.

20. a. Let n be the number of U.S. brewpubs in the year that is t years since 2015.

$n = 67.6t + 1650$

b. $\text{no. of brewpubs} = \dfrac{\text{no. of brewpubs}}{\text{year}}\text{years} + \text{no. of brewpubs}$

$\text{no. of brewpubs} = \text{no. of brewpubs} + \text{no. of brewpubs}$

$\text{no. of brewpubs} = \text{no. of brewpubs}$

This unit analysis shows that this model uses the correct units.

c. The slope of the model is 67.6. This means each year the number of brewpubs increases by 67.6.

d. Let $n = 41$ and solve for t.

$$41 = 67.6t + 1650$$
$$-1609 = 67.6t$$
$$-23.80 \approx t$$

This means that in $2015 - 24 \approx 1991$, there was an average of 41 brewpubs per state.

22. a. Let c be the total cost of n teenagers traveling to Disneyland Resort in a bus.

$c = 99n + 30$

b. To find n when c is 2307, substitute 2307 for c and solve for n.

$$2307 = 99n + 30$$
$$2277 = 99n$$
$$23 = n$$

This means there are 23 teenagers traveling to Disneyland Resort in a bus.

24. Let p be the percentage of Americans who have ever listened to an audio podcast in the year that is t years since 2015.

$p = 1.8t + 33$

To find this quantity, we substitute 50 for p and solve for t.

$$50 = 1.8t + 33$$
$$17 = 1.8t$$
$$9.44 \approx t$$

In $2015 + 9 = 2024$, 50% of Americans will have listened to an audio podcast.

26. The customer paid for n chocolate bars and 1 half-gallon of milk, with a total of \$19.17 spent. Since we know the prices for these items, we can set up an equation to find n.

$$19.17 = 1.29n + 2.40$$
$$16.77 = 1.29n$$
$$13 = n$$

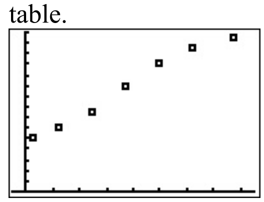
The person bought 13 chocolate bars.

28. The slope of the model is -0.40. This means that each year after 1950, the percentage of Americans 18 years or older who are married is decreases by 0.4 percent.

30. The slope of this model is –0.48, indicating that the percentage of adult Americans who smoke decreases by 0.48 per year.

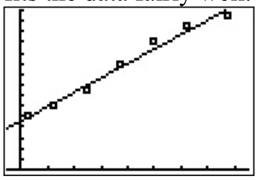
32. The slope of this model is 4.3, indicating that for every degree Fahrenheit increase in temperature, the number of cricket chirps increases by 4.3 per minute.

34. a. Construct a scatterplot using the data in the table.



The regression equation is $f(t) = 0.068t + 0.91$.

(Your equation may differ slightly if you used two data points to form a model.) Graphing the equation on the same graph as the scatterplot, we see that the equation fits the data fairly well.



b. The slope of the model is 0.068, indicating that the population of Nevada increases by 0.068 million per year.

c. To predict the population in 2021, we substitute $t = 2021 - 1985 = 36$ into the model and calculate.

$$\begin{aligned} f(36) &= 0.068(36) + 0.91 \\ &= 2.448 + 0.91 \\ &= 3.358 \end{aligned}$$

In 2021, Nevada's population will be about 3.4 million.

d. To find the *t*-intercept of the model, set $f(t)$ to 0 and solve for *t*.

$$\begin{aligned} 0 &= 0.068t + 0.91 \\ -0.91 &= 0.068t \\ -13.38 &\approx t \end{aligned}$$

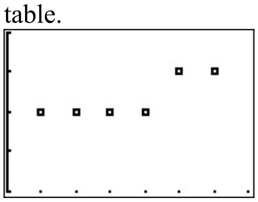
The model indicates that Nevada's population was zero in $1985 - 13.38 \approx 1972$. This is false. Model breakdown has occurred.

e. To find this quantity, substitute 3.6 for $f(t)$ and solve for *t*.

$$\begin{aligned} 3.6 &= 0.068t + 0.91 \\ 2.69 &= 0.068t \\ 39.56 &\approx t \end{aligned}$$

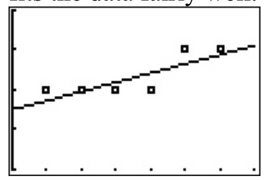
Nevada's population will reach 3.6 million in about $1985 + 40 = 2025$.

36. a. Construct a scatterplot using the data in the table.



The data is approximately linear, so it can be modeled using a linear model. The regression equation that models this data is $g(t) = 0.023t + 1.15$. (Your equation may differ slightly if you used two data points to form a model.)

Graphing the equation on the same graph as the scatterplot, we see that the equation fits the data fairly well.

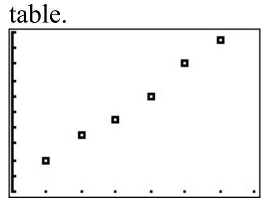


b. Use the data points (1, 1.2) and (4, 1.2) to find the slope.

$$m = \frac{1.2 - 1.2}{4 - 1} = \frac{0}{3} = 0$$

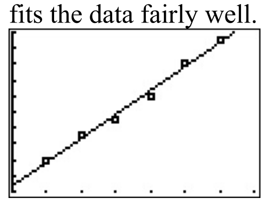
This rate of change is very similar to the one we found in part (a) using linear regression. Note that the data points nearly resemble a horizontal line which reflect in the fact that the rate of change is close to zero. (We know that the slope of a horizontal line is in fact zero.)

c. Construct a scatterplot using the data in the table.



The data is approximately linear, so it can be modeled using a linear model. The regression equation that models this data is $p(t) = 0.3t + 5.07$. (Your equation may differ slightly if you used two data points to form a model.)

Graphing the equation on the same graph as the scatterplot, we see that the equation fits the data fairly well.



d. The rate of change of the total annual personal income from private industries is about \$0.3 trillion per year.

e. To find the year, substitute 8.5 for $p(t)$ and solve for *t*.

$$\begin{aligned} 8.5 &= 0.3t + 5.07 \\ 3.43 &= 0.3t \\ 11.4 &\approx t \end{aligned}$$

The model predicts that in $2010+11.4 \approx 2021$, total annual personal income from private industries will be \$8.5 trillion.

38. a. The regression equation for the data is $f(p)=0.11p+69.48$. Your answer may differ slightly if you used two data points to model the data.

b. The slope of the model is 0.11. The slope tells us that for every increase of one in the relative humidity there will be a 0.11 degree increase in the heat index.

c. To find the average rate of change of the heat index with respect to relative humidity, divide the differences in the heat index by the differences in the humidity.

i. $m=\dfrac{80-69}{100-0}=\dfrac{11}{100}=0.11$

This is the same rate of change that we found in part (b).

ii. $m=\dfrac{78-72}{80-20}=\dfrac{6}{60}=0.1$

This is a slightly shallower rate of change than we found in part (b).

iii. $m=\dfrac{76-74}{60-40}=\dfrac{2}{20}=0.1$

This is a slightly shallower rate of change than we found in part (b).

d. To find the relative humidity, substitute 77.6 for $f(p)$ and solve for p.

$$77.6=0.11p+69.48$$
$$8.12=0.11p$$
$$73.82 \approx p$$

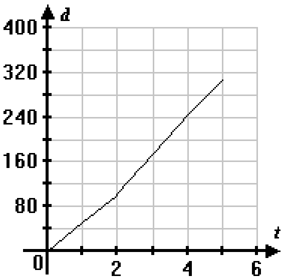
The relative humidity is approximately 74%.

40. a.

Accelerating from 50 mph to 70 mph in One Minute

Time	t (hours)	d (in miles)
12:00	0	50(0) = 0
1:00	1	50(1) = 50
2:00	2	50(2) = 100
2:01	2.017	100 + 1 = 101
3:01	3.017	101 + 70(1) = 171
4:01	4.017	101 + 70(2) = 241
5:01	5.017	101 + 70(3) = 311

b.



42. The points (5, 40) and (25, 30) are on the line and can be used to find the slope.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{30-40}{25-5}=\frac{-10}{20}=-0.5$$

The population is decreasing by about 0.5 thousand or 500 people per year.

44. The points (1, 60) and (5, 240) are on the line and can be used to find the slope.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{240-60}{5-1}=\frac{180}{4}=45$$

The total cost increases \$45 for each additional person.

46. To find the slope of the model, choose two points (choices may vary; we will choose (65, 76) and (101, 122)) from the graph and use the formula for slope:

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{122-76}{101-65}=\frac{46}{36}\approx 1.28.$$

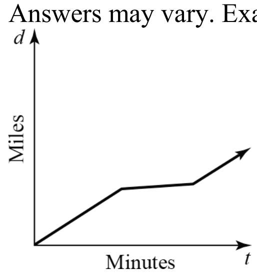
This means that the annual mining of rare earth metals increases worldwide by about 1.28 thousand metric tons for an annual revenue increase of 1 billion dollars by U.S. electronics and appliance stores.

48. To find the slope of the model, choose two points (choices may vary; we will choose (12.0, 4.9) and (450, 3)) from the graph and use the formula for slope:

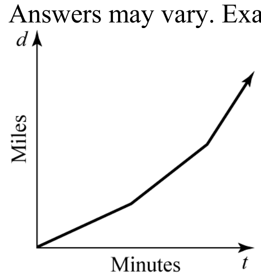
$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{3-4.9}{450-12}=\frac{-1.9}{438}\approx -0.004.$$

This means that between 1939 and 2013, for every 1 million pound of herbicide used in the United States, the number of bee colonies dropped by 0.004 million.

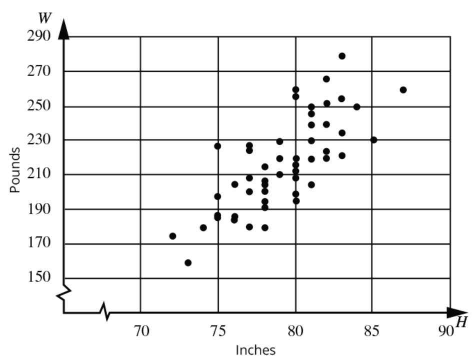
50. Answers may vary. Example:



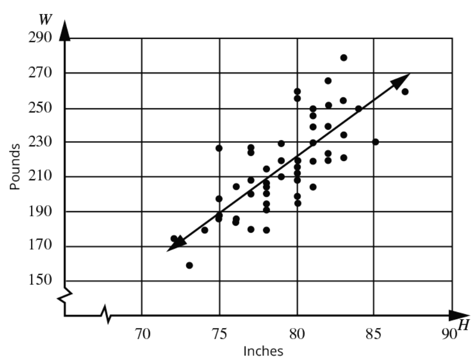
52. Answers may vary. Example:



54. a.



b.



c. To estimate the slope of the linear model, choose two points (choices may vary; we will choose (75, 188) and (84, 250)) from the graph and use the formula for slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{250 - 188}{84 - 75} = \frac{62}{9} \approx 6.89.$$

This means that a player who is 1 inch taller than another weighs about 6.89 pounds more.

d. Answers may vary. Using the graph to estimate, the weight of an 80-inch-tall player is about 220 pounds.

e. Answers may vary. Using the graph to estimate, the height of a 200-pound player is about 77 inches.

f. Based on the linear model, the player who was heavier for his height was Marcus Smart. Looking at the linear model, a player whose height is 75 inches has a typical weight of about 189 pounds. Marcus Smart's weight was 227pounds, which was 38 pounds away from the typical weight. Looking at the linear model, a player whose height is 82 inches has a typical weight of about 230 pounds. Cameron Bairstow's weight was 252 pounds, which was 22 pounds away from the typical weight. Therefore, Marcus Smart appeared to be heavier for his height than Cameron Bairstow.

56. Answers may vary. Example:
The rate of change is the quotient of the change in the response variable and the change in the explanatory variable. If one change is negative and the other is positive, the quotient is negative.

58. Answers may vary.

Chapter 2 Review Exercises

1. $f(3) = 3(3)^2 - 7 = 3(9) - 7 = 27 - 7 = 20$

2. $f(-3) = 3(-3)^2 - 7 = 3(9) - 7 = 27 - 7 = 20$

3. $g(2) = \dfrac{2(2)+5}{3(2)+6} = \dfrac{4+5}{6+6} = \dfrac{9}{12} = \dfrac{3}{4}$

4. $h\left(\dfrac{3}{5}\right) = -10\left(\dfrac{3}{5}\right) - 3 = -2(3) - 3 = -6 - 3 = -9$

5.
$$\begin{aligned} h(a+3) &= -10(a+3) - 3 \\ &= -10a - 30 - 3 \\ &= -10a - 33 \end{aligned}$$

6.
$$\begin{aligned} -6 &= 2x + 3 \\ -6 - 3 &= 2x + 3 - 3 \\ -9 &= 2x \\ \frac{-9}{2} &= x \end{aligned}$$

7.
$$\frac{2}{3}=2x+3$$
$$\frac{2}{3}-3=2x+3-3$$
$$\frac{-7}{3}=2x$$
$$\frac{1}{2}\left(\frac{-7}{3}\right)=\frac{1}{2}(2x)$$
$$\frac{-7}{6}=x$$

8.
$$a+7=2x+3$$
$$a+7-3=2x+3-3$$
$$a+4=2x$$
$$\frac{a+4}{2}=\frac{2x}{2}$$
$$\frac{a}{2}+2=x$$

9. Since the graph includes the point (2, 0), $f(2)=0$.

10. Since the graph includes the point (0, 1), $f(0)=1$.

11. Since the graph includes a point close to (–3, 3.5), $f(-3)\approx 3.5$.

12. Since the graph includes the point (–2, 3), $x=-2$ when $f(x)=3$.

13. Since the graph includes the point (2, 0), $x=2$ when $f(x)=0$.

14. Since the graph includes the point (4, –1), $x=4$ when $f(x)=-1$.

15. The domain of f is $-5\le x\le 6$.

16. The range of f is $-2\le y\le 4$.

17. Since x is 0 when $f(x)=1$, $f(0)=1$.

18. Since x is 2 when $f(x)=4$, $f(2)=4$.

19. When $f(x)=0$, $x=4$.

20. When $f(x)=2$, $x=1$.

21. $f(x)$ or $y=-7x+3$

To find the x-intercept, set $y=0$ and solve for x.
$$0=-7x+3$$
$$-3=-7x$$
$$x=\frac{-3}{-7}=\frac{3}{7}$$
The x-intercept is $\left(\frac{3}{7},0\right)$.

To find the y-intercept, set $x=0$ and solve for y.
$$y=-7(0)+3=0+3=3$$
The y-intercept is (0, 3).

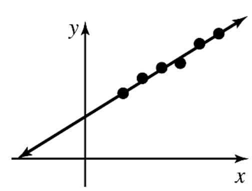
22. $f(x)$ or $y=4$ is the equation of a horizontal line. The y-intercept of the line is (0, 4). There is no x-intercept.

23. $f(x)$ or $y=-\frac{4}{7}x+2$

To find the y-intercept, set $x=0$ and solve for y.
$$y=-\frac{4}{7}(0)+2=0+2=2$$
The y-intercept is (0, 2).
To find the x-intercept, set $y=0$ and solve for x.
$$0=-\frac{4}{7}x+2$$
$$-2=-\frac{4}{7}x$$
$$-2\left(-\frac{7}{4}\right)=x$$
$$x=\frac{7}{2}$$
The x-intercept is $\left(\frac{7}{2},0\right)$.

24. To find the x-intercept, set y equal to zero and solve for x;
$$2.56x-9.41(0)=78.25$$
$$2.56x=78.25$$
$$x\approx 30.57$$
The x-intercept is (30.57, 0).
To find the y-intercept, set x equal to zero and solve for y;
$$2.56(0)-9.41y=78.25$$
$$-9.41y=78.25$$
$$y\approx -8.32$$
The y-intercept is (0, –8.32).

25.



The new line has an increased slope since it is steeper than the model. The y-intercept is lower in the new line as it intersects the y-axis at a lower point than the model.

26. **a.** The student's car has 13 gallons of gas in the tank when no hours have been driven, so when $t = 0$, $b = 13$. Since the car uses 1.8 gallons of gas per hour, the slope is -1.8. An equation for f is $f(t) = -1.8t + 13$.

b. The slope of f is -1.8. This means that the amount of gasoline decreases at a constant rate of 1.8 gallons per hour of driving.

c. To find the A-intercept of f, let $t = 0$.

$A = f(t) = -1.8(0) + 13 = 0 + 13 = 13$

The A-intercept is (0, 13). This represents the amount of gas in a full tank before any time has been spent driving ($t = 0$). At that time, the car has 13 gallons of gas in the tank.

d.
$$\begin{aligned} \text{gallons} &= \text{gallons} - \frac{\text{gallons}}{\text{hour}}\text{hour} \\ &= \text{gallons} - \text{gallons} \\ &= \text{gallons} \end{aligned}$$

This unit analysis shows that this model uses the correct units.

e. To find the t-intercept of f, let $f(t) = 0$.

$$\begin{aligned} 0 &= -1.8t + 13 \\ 0 + 1.8t &= -1.8t + 13 + 1.8t \\ 1.8t &= 13 \\ t &= \frac{13}{1.8} \\ t &\approx 7.22 \end{aligned}$$

The t-intercept is (7.22, 0). This means that the student can drive for 7.22 hours before running out of gas.

f. Since the car can be driven for 7.22 hours before running out of gas, the domain is $0 \le t \le 7.22$. Since the gas tank has between 0 and 13 gallons of gas, the range is $0 \le A \le 13$.

27. **a.** The slope of the graph of g is 8.5. This means the annual revenue from gift cards is increasing by \$8.5 billion per year.

b. We can write an equation where $g(t)$ is the annual revenue from gift cards (in billions of dollars) and t is year since 2014.

$g(t) = 8.5t + 124$.

c. To find $g(6)$, substitute 6 for t in the equation $g(t) = 8.5t + 124$ and solve:

$$\begin{aligned} g(6) &= 8.5(6) + 124 \\ g(6) &= 175 \end{aligned}$$

This means in 2020, the annual revenue from gift cards will be \$175 billion.

d. Let $g(t) = 180$ and solve for t.

$$\begin{aligned} 180 &= 8.5t + 124 \\ 56 &= 8.5t \\ 6.59 &\approx t \end{aligned}$$

This means in $2014 + 6.59 \approx 2021$ the annual revenue from gift cards will be \$180 billion.

28. The approximate rate of change in the annual sales of anti-aging skin care in the United States is given by dividing the change in the annual sales by the change in years.

$$\begin{aligned} m &= \frac{\text{change in annual sales}}{\text{change in years}} \\ &= \frac{2.05 - 2.2}{2015 - 2011} \\ &= \frac{-0.15}{4} \\ &= -0.0375 \end{aligned}$$

This means that the approximate rate of change in the annual sales of anti-aging skin care in the United States was a decrease of about \$0.04 billion per year.

29. We first find the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Substitute the values (0, 16,363) and (25, 43,327) into the formula:

$$\frac{43{,}327 - 16{,}363}{25 - 0} = \frac{26{,}964}{25} = 1078.56.$$

Now we can use the formula $y = mx + b$ to find the equation. Using the variables p and t we have $p = 1078.56t + b$. Since we have two sets of coordinates, we can choose one and substitute values for t and p. If we choose (0, 16,363) we have:

$$\begin{aligned} p &= 1078.56t + b \\ 16{,}363 &= 1078.56(0) + b \\ 16{,}363 &= b \end{aligned}$$

Substituting 16,363 for b in the equation $p = 1078.56t + b$ gives us $p = 1078.56t + 16{,}363$.

To estimate the median personal income in 2010, substitute 20 for t in

$p = 1078.56t + 16,363$ and solve for b.
$p = 1078.56(20) + 16,363$
$p = 37,934.20$
We estimate the median personal income in 2010 to be about \$37,934. This is an overestimate since the actual median personal income in that year was \$34,397.
Answers may vary. Example: Given that personal incomes, given the years 1990 to 2015, were rising based on the model, the estimate reflects an approximately steady increase year to year. The model does not take into account variations that may occur year to year, such as economic slumps as was the case in 2010. We therefore do not see this decline in income in the model for that year.

30. **a.** We first find the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Choose two values from the table. We choose (0, 32.5) and (10, 50). Then substitute them into the formula:
$\frac{50-32.5}{10-0} = \frac{17.5}{10} = 1.75$. Now, we can use the formula $y = mx + b$ to find the equation. Using the variables M and t we have $M = 1.75t + b$. Since we have two sets of coordinates, we can choose one and substitute values for t and M. If we choose (10, 50) we have:
$50 = 1.75t + b$
$50 = 1.75 \cdot 10 + b$
$50 = 17.5 + b$
$32.5 = b$
Substituting 32.5 for b gives us $M = 1.75t + 32.5$. Since $M = f(t)$ we can write $f(t) = 1.75t + 32.5$.

b. The slope is 1.75. This means that each year, the IRS standard mileage rate for businesses increases by about 1.75 cents per mile.

c. The M-intercept is (0, 32.5). This means in 2000, the standard mileage rate was about 32.5 cents per mile.

d. Substitute 68 for M in $M = 1.75t + 32.5$:
$68 = 1.75t + 32.5$
$35.5 = 1.75t$
$\frac{35.5}{1.75} = \frac{1.75t}{1.75}$
$20.3 = t$
We predict the standard mileage rate will be 68 cents per mile in 2020 (2000 + 20).

e. Substitute 21 for t in $M = 1.75t + 32.5$:
$M = 1.75 \cdot 21 + 32.5$
$M = 36.75 + 32.5$
$M = 69.25$
We predict the standard mileage rate will be about 69 cents per mile in 2021. Therefore, if a person drives 12, 500 miles on business trips, she can deduct $12,500 \cdot 0.69 = \$8625$ for driving expenses.

f. Since two different rates, 50.5 and 58.5, were used in 2008, the average of the two, 54.5, was calculated.

31. **a.** We first find the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Choose two values from the table. We choose (3, 34) and (9, 19). Then substitute them into the formula:
$\frac{19-34}{9-3} = \frac{-15}{6} = -2.5$. Now we can use the formula $f(t) = mt + b$ to find the equation. Substitute -2.5 for m:
$f(t) = -2.5t + b$. Since we have two sets of coordinates, we can choose one and substitute values for t and $f(t)$. If we choose (3, 34), we have:
$f(t) = -2.5t + b$
$34 = -2.5 \cdot 3 + b$
$34 = -7.5 + b$
$41.5 = b$
Substituting 41.5 for b gives us $f(t) = -2.5t + 41.5$.

b. The slope of the equation is -2.5. This means that the percentage of Americans who think the First Amendment goes too far in the rights it guarantees decreases by 2.5 percentage points per year.

c. Substitute 8 for t in $f(t) = -2.5t + 41.5$:
$f(t) = -2.5t + 41.5$
$f(8) = -2.5 \cdot 8 + 41.5$
$f(8) = -20 + 41.5$
$f(8) = 21.5$
This means that in 2008 (2000 + 8), about 22% of Americans thought the First Amendment went too far in the rights it guarantees.

d. Substitute 30 for $f(t)$ in $f(t) = -2.5t + 41.5$:

$$30 = -2.5t + 41.5$$
$$-11.5 = -2.5t$$
$$\frac{-11.5}{-2.5} = \frac{-2.5t}{-2.5}$$
$$4.6 = t$$

This means in 2005 (2000 + 5), 30% of Americans thought the First Amendment went too far in the rights it guarantees.

e. Substitute 14 for t in $f(t) = -2.5t + 41.5$:

$$f(14) = -2.5 \cdot 14 + 41.5$$
$$f(14) = -35 + 41.5$$
$$f(14) = 6.5$$

So we estimate in 2014 that about 7% of Americans thought the First Amendment went too far in the rights it guarantees. The actual percentage, 38%, is so much greater since sentiment over public safety was heightened in the aftermath of the bombings at the Boston Marathon.

Chapter 2 Test

1. Since the line includes the point (–3, –2), $f(-3) = -2$.

2. Since the line includes the point (3, 0), $f(3) = 0$.

3. Since the line includes the point (0, –1), $f(0) = -1$.

4. Since the line includes the point $\left(-5, \frac{-8}{3}\right)$,

$f(-5) = -\frac{8}{3}$ or approximately –2.7.

5. Since the line includes the point (–6, –3), $x = -6$ when $f(x) = -3$.

6. Since the line includes the point (–3, –2), $x = -3$ when $f(x) = -2$.

7. Since the line includes the point (3, 0), $x = 3$ when $f(x) = 0$.

8. Since the line includes the point (4.5, 0.5), $x = 4.5$ when $f(x) = 0.5$.

9. The domain of f is all the x-coordinates of the points in the graph. In this case f has a domain of $-6 \le x \le 6$.

10. The range of f is all the y-coordinates of the points in the graph. In this case f has a range of $-3 \le y \le 1$.

11. To find $f(-3)$, substitute –3 for x.

$$f(-3) = -4(-3) + 7 = 12 + 7 = 19$$

12. To find $f(a-5)$, substitute $a - 5$ for x.

$$f(a-5) = -4(a-5) + 7$$
$$= -4a + 20 + 7$$
$$= -4a + 27$$

13. To find x when $f(x) = 2$, substitute a for $f(x)$.

$$2 = -4x + 7$$
$$-5 = -4x$$
$$\frac{5}{4} = x$$

14. To find x when $f(x) = a$, substitute 2 for $f(x)$.

$$a = -4x + 7$$
$$a - 7 = -4x$$
$$\frac{a-7}{-4} = x \quad \text{or} \quad x = \frac{-a+7}{4}$$

15. To find the x-intercept, let $f(x) = 0$ and solve for x.

$$0 = 3x - 7$$
$$7 = 3x$$
$$\frac{7}{3} = x$$

The x-intercept is $\left(\frac{7}{3}, 0\right)$.

To find the y-intercept, let $x = 0$ and solve for y or $f(x)$ since $y = f(x)$.

$$y = 3(0) - 7 = -7$$

The y-intercept is (0, –7).

16. To find the x-intercept, let $g(x) = 0$ and solve for x.

$$0 = -2x$$
$$0 = x$$

The x-intercept is (0, 0).
To find the y-intercept, let $x = 0$ and solve for y or $g(x)$ since $y = g(x)$.

$$y = -2(0) = 0$$

The y-intercept is (0, 0).

17. To find the x-intercept, let $k(x)=0$ and solve for x.

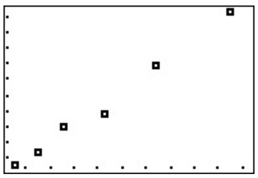
$$0=\frac{1}{3}x-8$$
$$8=\frac{1}{3}x$$
$$24=x$$

The x-intercept is (24, 0).
To find the y-intercept, let $x=0$ and solve for y or $k(x)$ since $y=k(x)$.

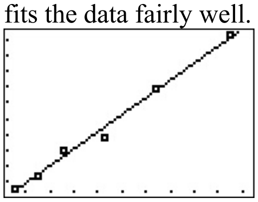
$$y=\frac{1}{3}(0)-8=-8$$

The y-intercept is (0, –8).

18. a. Construct a scatterplot using the data in the table.



The data is approximately linear, so it can be modeled using a linear model. The regression equation that models this data is $f(I)=9.73I+545.71$. (Your equation may differ slightly if you used two data points to form a model.)
Graphing the equation on the same graph as the scatterplot, we see that the equation fits the data fairly well.



b. The slope of the model is 9.73 This means the average annual credit card cost for a household is \$9.73 more than for a household whose income is \$1 thousand less.

c. The C-intercept is found when $I=0$.

$$f(0)=9.73(0)+545.71$$
$$f(0)=545.71$$

The C-intercept is (0, 545.71).
This means for households with average annual income of 0 dollars, the average annual credit card cost is \$545.71.

d. Substitute 56.5 for I in $f(I)=9.73I+545.71$ and solve.

$$f(56.5)=9.73(56.5)+545.71$$
$$f(56.5)\approx 1095.46$$

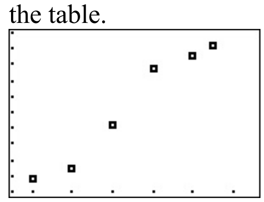
The model estimates that the annual credit card cost for a household with income \$56.5 thousand is \$1095.46.

e. Let $f(I)$= 1500 and solve.

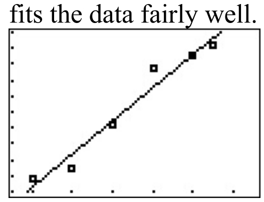
$$1500=9.73I+545.71$$
$$954.29=9.73I$$
$$98.08\approx I$$

The model estimates the income of a household that has an annual credit card cost of \$1500 is \$98.1 thousand.

19. a. Use the first three steps of the modeling process to find the equation. Begin by constructing a scatterplot using the data in the table.



The data is approximately linear, so it can be modeled using a linear model. The regression equation that models this data is $f(t)=0.52t+1.03$. (Your equation may differ slightly if you used two data points to form a model.)
Graphing the equation on the same graph as the scatterplot, we see that the equation fits the data fairly well.



b. The slope of the model is 0.52. This means the number of farmers markets is increasing by 0.52 thousand (510) markets per year.

c. $f(22)=0.52(22)+1.03=12.47$
This means in 2022, there were about 12.47 thousand farmers markets.

d. Let $f(t)$ = 12 and solve for t.

$$12=0.52t+1.03$$
$$10.97=0.52t$$
$$21.10\approx t$$

This means in $2000+21.1\approx 2021$, there will be 12 thousand farmers markets.

e. To find the t-intercept, set $f(t)$ equal to 0 and solve for t.

$$0 = 0.52t + 1.03$$
$$-1.03 = 0.52t$$
$$-1.98 \approx t$$

The t-intercept is $(-1.98, 0)$ According to the model, in $2000 - 1.98 \approx 1998$, there were no farmers markets. Model breakdown has occurred.

20. We first find the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Substitute the values (0, 0.5) and (9, 1.19) into the formula:

$\frac{1.19 - 0.5}{9 - 0} = \frac{0.69}{9} \approx 0.077$. Now we can use the formula $y = mx + b$ to find the equation. Using the variables s and t we have $s = 0.077t + b$. Since we have two sets of coordinates, we can choose one and substitute values for t and s. If we choose (0, 0.5) we have:

$$s = 0.077t + b$$
$$0.5 = 0.077(0) + b$$
$$0.5 = b$$

Substituting 0.5 for b in the equation $s = 0.077t + b$ gives us $s = 0.077t + 0.5$.

To find the total annual ad spending for March Madness in 2022, let $t = 16$ and solve:

$$s = 0.077(16) + 0.5$$
$$s \approx 1.73$$

The model estimates the total annual ad spending for March Madness in 2022 will be about $1.73 billion.

21. a. Yes, there is an approximate linear relationship between t and c. Since the worldwide economic cost of violence is reported to increase at a constant rate ($0.23 trillion since 2014), the relationship between t and c is approximately linear. The slope is 0.23 which means the worldwide economic cost of violence is increasing by $0.23 trillion per year.

b. The c-intercept is (0, 14.3). This means in 2014, the worldwide economic cost of violence was $14.3 trillion.

c. We can write $c = 0.23t + 14.3$, where c represents the worldwide economic cost of violence and t represents years since 2014.

d. Let $t = 6$ and solve.

$$c = 0.23(6) + 14.3$$
$$c = 15.68$$

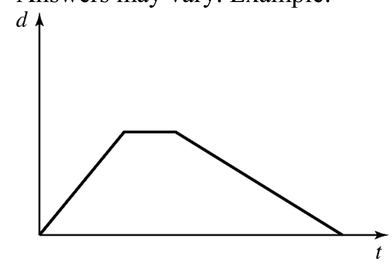
This means in 2020, the worldwide economic cost of violence will be $15.68 trillion.

e. Set c equal to 16 and solve for t.

$$16 = 0.23t + 14.3$$
$$1.7 = 0.23t$$
$$7.39 \approx t$$

The model predicts in $2014 + 7.39 \approx 2021$, the worldwide economic cost of violence will be $16 trillion.

22. Answers may vary. Example:



Chapter 2

Lecture Notes, Detailed Comments, and Additional Explorations

This chapter includes chapter overviews, section-by-section lecture notes, comments about the lecture notes, and additional explorations. The lecture notes include short and medium homework assignments.

The comments about the lecture notes describe typical student difficulties and what an instructor can do to help students overcome these obstacles. This chapter contains many explorations that do not appear in the textbook. For each exploration in this manual and the textbook, there is a discussion of what to do before, during, and after it.

CHAPTER 1 OVERVIEW

In Section 1.1 students sketch qualitative graphs (graphs without scaling on the axes). In sections 1.2—1.4 they learn about slope and how to graph linear equations in two variables. Students learn to find an equation of a line that contains two given points in Section 1.5. In Section 1.6, students learn fundamental concepts related to function. Sections 1.2, 1.3, and 1.5 are likely to contain review material for your students and Sections 1.1, 1.4, and 1.6 are likely to contain new material.

SECTION 1.1 LECTURE NOTES

Objectives

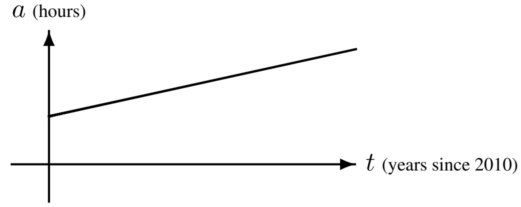
1. Describe the meaning of qualitative graphs.
2. Identify explanatory variables and response variables.
3. Describe the meaning of an *intercept* of a curve.
4. Sketch qualitative graphs.
5. Identify increasing and decreasing curves.
6. Describe a concept or procedure.

Main point: Sketch qualitative graphs.

OBJECTIVE 1

A **qualitative graph** is a graph without scaling (tick marks and their numbers) on the axes.

1. Let a be the average number of hours per day that Americans watch television in the year that is t years since 2010. The qualitative graph that follows describes the relationship between t and a. What does the graph tell us?



OBJECTIVE 2

Definition *Explanatory and response variables*

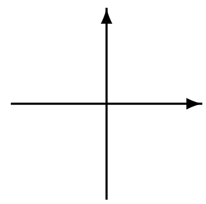
Assume that an authentic situation can be described by using the variables t and p and assume that t affects (explains) p. Then

- We call t the **explanatory variable** (or **independent variable**).
- We call p the **response variable** (or **dependent variable**).

For each situation, identify the explanatory variable and the response variable.

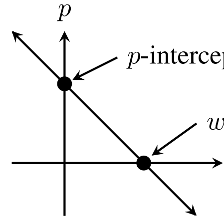
2. Let c be the total cost (in dollars) of renting a car for n days.
3. Let T be the time (in minutes) it takes a person to drive to work if the person travels at s miles per hour.
4. Let p be the percentage of high school students with a SAT score of s points who are accepted to Harvard.
5. Let p be the percentage of adults with income I dollars who qualify for welfare assistance.

For graphs, we describe the values of the explanatory variable along the horizontal axis and the values of the response variable along the vertical axis.



OBJECTIVE 3

An **intercept** of a curve is any point where the curve and an axis (or axes) intersect.



OBJECTIVE 4

Sketch a qualitative graph that describes the relationship between the two given variables.

6. Let v be the volume of air (in liters) in a balloon after a person has blown n times into the balloon.

7. A person decides to get in shape by running three miles each day. Let T be the amount of time (in minutes) it takes the person to run three miles after training for n days.

8. Let s be a person's salary (in dollars) after working at a company for t years.

9. Let T be the number of hours a furnace in a house is on in one day if the average outside temperature that day is F degrees Fahrenheit.

OBJECTIVE 5

- If a curve goes upward from left to right, we say it is an **increasing curve**.
- If a curve goes downward from left to right, we say it is a **decreasing curve**.

Determine whether the curve sketched in the given problem is increasing, decreasing, or neither.

10. Problem 6 **11.** Problem 7 **12.** Problem 8 **13.** Problem 9

OBJECTIVE 6

Discuss the guidelines included on page 4 of the textbook. Describe your expectations of your student's written explanations of concepts and procedures.

SHORT HW 1, 5, 7, 13, 21, 29, 33, 37, 39

MEDIUM HW 1, 3, 5, 7, 13, 19, 21, 23, 29, 33, 37, 39

SECTION 1.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn how to sketch qualitative graphs. They will use the skill of sketching qualitative graphs throughout the course. For example, students may find a model that describes a situation well for a span of years. Beyond that span of years, if model breakdown obviously occurs, students will be asked to sketch a qualitative graph that better describes what happens beyond that span of years.

General Comments

Because it is not necessary to perform symbolic work to sketch qualitative graphs, you will be free to give an overview of some of the types of models which students will use in the course. I like to give examples of linear, exponential, and quadratic models because these three types of models are the focus of the first seven chapters of the textbook and are important models for many math courses.

For my students, it has worked well to begin the course with this section. Students get the idea that something new and exciting will be happening in the course and we often have fun discussing which parts of a curve should be increasing, decreasing, concave up, and/or concave down (although I don't use concavity terminology).

However, some students may be mildly intimidated by the generality of the section. To address this concern, I emphasize that the qualitative graphs can be sketched based on what information is given in the problem and on common sense.

OBJECTIVE 1 COMMENTS

When discussing Problem 1 in the lecture notes, I take a moment to discuss the meaning of "Years since 2010." I point out that if a child was born in 2010, then t would describe the child's age. Then I discuss the meaning of a couple of values of t such as $t = 0$ and $t = 3$.

Workbook Exploration

One way to begin this section is to assign the following 10-minute exploration in the student workbook, *Concepts and Explorations Notebook: Working with Authentic Data*. See Sections 1.4 and 1.5 in this manual for a discussion about explorations and collaborative learning, respectively. Because the exploration is similar to Example 5 in Section 1.1 of the textbook, it is best if students complete the exploration with their textbooks closed.

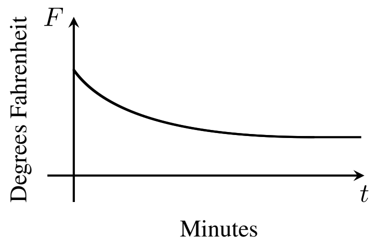
Once groups complete an exploration, I like to have a class discussion about it. For Problem 2, I generate a discussion about why it makes sense that the curve decreases at the greatest rate at $t = 0$.

You may want to assign the Cooling Water Lab (see pages 248 and 249 of the textbook) as a follow-up to this exploration when your students complete Chapter 4.

Group Exploration

Section Opener: Interpreting a qualitative graph

Some hot coffee is poured into a cup at room temperature. Let F be the temperature (in degrees Fahrenheit) of the coffee and t be the time (in minutes) elapsed since the coffee was poured. The following graph describes how F depends on t.



1. Explain in terms of time and the temperature of the coffee, why it makes sense that the curve is going downward from left to right.

2. Where is the curve decreasing at the greatest rate? Explain why you would expect this to happen.

3. Explain why the curve is almost horizontal for large values of t.

OBJECTIVE 2 COMMENTS

Although many students have a tough time initially learning this concept, even weaker students eventually learn this concept because it is addressed so often in the course.

OBJECTIVE 3 COMMENTS

You may want to define *vertical intercept* and *horizontal intercept*. The textbook avoids doing so, to keep the terminology to a minimum; referring to the intercepts within the context of a situation (such as "t-intercept" and "p-intercept" seems to work fine). However, using the terminology "vertical intercept" does come in handy when discussing how to sketch qualitative graphs (see Objective 4 comments).

OBJECTIVE 4 COMMENTS

To give students an idea of how to sketch qualitative curves, it is best to provide at least a couple of examples. Many students are not sure where to begin. I suggest to my students that it is good practice to first determine which variable is the explanatory variable and which variable is the response variable. Then, we determine whether there is a vertical intercept and, if so, determine whether the intercept is above, at, or below the origin. Next, we decide what happens to the value of the response variable as the explanatory variable increases. A similar strategy can then be used to sketch portions of the graph in quadrants II or III if the graph lies in these quadrants.

I often discuss the textbook's Exercise 23 of Homework 1.1 in class. It is interesting to note that many students think that the h-intercept is at the origin.

It is a good idea to have groups work on at least one example where the explanatory variable represents some quantity other than time (such as Problems 6 and 9 in the lecture notes). It is with this type of qualitative graph that students need the most coaching to consider what happens to the value of the response variable as the explanatory variable increases.

Textbook and Workbook Exploration

The 10-minute "Sketching a qualitative graph" exploration serves as a nice collaborative activity that can give closure to Objective 4. Groups will be quick to agree that the curve is decreasing, but they will not be as quick to decide on the curve's concavity. If one group finishes early, check that all group members have the same graph. If group members do not have the same graph, I point this out and encourage them to compare their work and decide which graph is correct.

When groups are done, I have them sketch their graphs on the board. (Or have them work on butcher paper and have them post their work when all groups are done.) Usually, some groups will sketch a decreasing line, other groups will sketch a decreasing concave-up curve, and yet other groups will sketch a decreasing concave-down curve. I then have students make compelling arguments about which curve is correct. Sometimes this discussion is quite lively and if so, I make a point of assigning the Water-Flow Lab (see pages 481 and 482 in the textbook) once we complete Chapter 7.

For Problem 3 in the exploration, some students may believe that the volume of the water is directly proportional to the height of the water. Some students will come up with the inefficient method of draining the bathtub many times by first draining it of one gallon of water, then draining it of two gallons of water, and so on. Few students will come up with the more efficient method of only having to drain the water once by marking the side of the bathtub each time a gallon of water is added and waiting to drain the water until it is full.

I also ask my students whether the curve has intercepts. This can lead to a discussion about initial conditions and also serve as a precursor to domain and range, which are defined for functions in Section 1.6 and described for models in Section 2.3.

OBJECTIVE 5 COMMENTS

Students have an easy time with this objective.

OBJECTIVE 6 COMMENTS

Most Homework Sections contain exercises in which students must describe concepts and explain why procedures work. This is essential because most students will tend to memorize without understanding, if they think they can get away with it.

At least once per chapter, a conceptual question includes a page reference to the guidelines on page 4 to remind students of all the facets of a good response. I tell my students my expectations for their explanations on homework exercises, quizzes, and exams. For example, I tell my students that in many cases they should provide an example and that they must always write complete sentences.

In addition to containing all the conceptual questions in the textbook, the student workbook, *Concepts and Explorations Notebook: Working with Authentic Data* contains additional conceptual questions called "Mini-Essay Questions." These questions have merit for all students, but especially for students who will use MyLab Math because it is much more challenging and, hence, beneficial to write responses, rather than respond to a multiple-choice question as is required by MyLab Math.

SECTION 1.2 LECTURE NOTES

Objectives

1. Describe the meaning of *solution*, *satisfy*, and *solution set*.
2. Describe the meaning of a *graph* of an equation.
3. Sketch graphs of linear equations.
4. Find intercepts of graphs of linear equations.
5. Graph equations of the form $x = a$ and $y = b$.

Main point: Graph linear equations.

OBJECTIVE 1

- Show that when $x = 5$, then $y = 7$ for the equation $y = 2x - 3$.
- The *ordered pair* $(5, 7)$ represents that $x = 5$ when $y = 7$.
- For an **ordered pair** (a, b), we write the value of the explanatory variable in the first (left) position and the value of the response variable in the second (right) position. The numbers a and b are called **coordinates**.

Definition *Solution, satisfy, and solution set of an equation in two variables*

An ordered pair (a, b) is a **solution** of an equation in x and y if the equation becomes a true statement when a is substituted for x and b is substituted for y. We say (a, b) **satisfies** the equation. The **solution set** of an equation is the set of all solutions of the equation.

OBJECTIVE 2

Find five solutions of the equation $y = 2x - 3$, and plot them. Draw the line $y = 2x - 3$. Show that points that lie on the line represent solutions and points that do not lie on the line do not represent solutions.

Definition *Graph*

The **graph** of an equation in two variables is the set of points that correspond to all solutions of the equation.

The equation $y = 2x - 3$ is of the form $y = mx + b$ and the graph of the equation is a line.

Graphs of Equations That Can Be Put into $y = mx + b$ Form

If an equation can be put into the form $y = mx + b$, where m and b are constants, then the graph of the equation is a line.

The graphs of the equations $y = -3x + 7$ and $y = 3$ ($y = 0x + 3$) are lines.

OBJECTIVE 3

Graph the equation by hand.

1. $y = -2x + 4$

2. $y = -3x$

3. $6x - 2y = 10$

4. $4x - 5y + 2 = -2x - 3y + 4$

5. $3(2x - y) = 4(2x + 3) - 5x$

6. $y = \dfrac{2}{3}x - 1$

OBJECTIVE 4

Show that the line $y = -2x + 4$ has x-intercept $(2, 0)$ and y-intercept $(0, 4)$.

Intercepts of the Graph of an Equation

For an equation containing the variables x and y,

- To find the x-coordinate of each x-intercept, substitute 0 for y and solve for x.
- To find the y-coordinate of each y-intercept, substitute 0 for x and solve for y.

For Problems 7 and 8, find all x-intercepts and y-intercepts. [Optional: Graph each equation.]

7. $y = -4x + 8$

8. $3x - 4y = 12$

9. Assuming that the graph of the equation $a(bx + y) = c$ has an x-intercept and a y-intercept, find both intercepts.

OBJECTIVE 5

Find five solutions of the given equation. Then graph the equation by hand.

10. $x = 1$

11. $y = 4$

Equations of Vertical and Horizontal Lines

If a and b are constants, then

- An equation that can be put into the form $x = a$ has a vertical line as its graph [Show graph.]
- An equation that can be put into the form $y = b$ has a horizontal line as its graph [Show graph.]

Graph the equation by hand.

12. $x = -2$

13. $y = -3$

14. $x = 0$

15. $y = 0$

Linear Equations in Two Variables

If an equation can be put into either form $y = mx + b$ or $x = a$, where m, a, and b are constants, then the graph of the equation is a line. We call such an equation a **linear equation in two variables**.

Here are some examples of linear equations in two variables:

$$y = 4x - 7 \qquad 5x - 6y = 10 \qquad y = -5 \qquad x = 6$$

SHORT HW 5, 25, 37, 43, 53, 57, 61, 67, 79, 81, 83, 85, 89, 102, 107

MEDIUM HW 5, 11, 23, 25, 33, 37, 39, 43, 53, 57, 59, 61, 67, 79, 81, 83, 85, 87, 89, 91, 95, 97, 102, 107

SECTION 1.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students review how to sketch graphs of linear equations. The meanings of a solution and a graph should be emphasized because these concepts are needed to find equations of linear, exponential, quadratic, and radical equations in the course. The technique of sketching graphs can be somewhat deemphasized because students will learn the more efficient method of using slope and y-intercept to sketch graphs of linear equations in Section 1.4. Students will need the skill of graphing linear equations throughout Chapters 1–3.

OBJECTIVE 1 COMMENTS

These terminologies are important and will be used throughout the course.

OBJECTIVE 2 COMMENTS

Knowing the meaning of a graph is a key concept of the course. This concept is crucial both for solving systems of equations by graphing as well as for finding values of parameters of a function whose graph contains some given points. Unfortunately, students tend to think of a graph only as an end product of a task. In fact, even calculus students tend to lose sight of the meaning of a graph, even though they know how to graph many relations.

One way to begin is to graph an equation of the form $x + y = k$, such as $x + y = 5$. On the one hand, this is a great way to introduce solutions of an equation because students can easily suggest values of x and y whose sum is 5. On the other hand, this is not an ideal way to begin because the equation is not of the form $y = mx + b$. It is nice to come out of the blocks with an example that suggests that the graph of an equation in slope-intercept form is a line. Because one of the main goals of this section is to drive home the meaning of a graph, beginning with graphing an equation such as $x + y = 5$ may be the better way to go.

Workbook Exploration

The following 15-minute exploration supports students in discovering the meaning of a graph. This exploration can be a gentle introduction to technology such as a graphing calculator. One difficulty, however, is that if you do not provide students with technology, many students may not have purchased a graphing calculator by, say, the second day of class. One option is to go low tech and have students do the exploration using graph paper.

Another option is to postpone assigning the exploration until students have purchased a graphing calculator. Students will have much to gain from this exploration even if it is assigned many days after you discuss Section 1.2.

As the mathematics for this exploration is straightforward, most groups' questions will revolve around graphing calculator usage (unless they do the graphing by hand). In particular, groups may have questions about ZDecimal.

When debriefing the activity, be sure to emphasize the main point of the exploration.

EXPLORATION *Solutions of an equation*

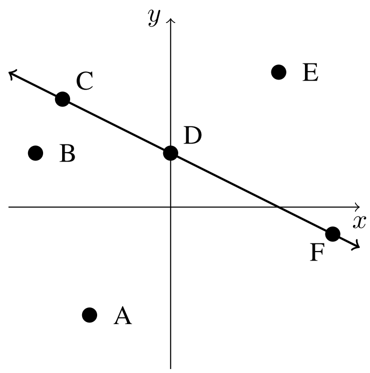
1. Sketch the graph of $y = x + 2$.

2. Pick three points that lie on the graph of $y = x + 2$. Do the coordinates of these points satisfy the equation $y = x + 2$?

3. Pick three points that do not lie on the graph of $y = x + 2$. Do the coordinates of these points satisfy the equation $y = x + 2$?

4. Which ordered pairs satisfy the equation $y = x + 2$? There are too many to list, but describe them in words. [**Hint:** You should say something about the points that do or do not lie on the line.]

5. The graph of an equation is sketched below. Which of the points A, B, C, D, E, and F represent ordered pairs that satisfy the equation?



OBJECTIVE 3 COMMENTS

Although there are various ways to graph linear equations, I emphasize the method of isolating y to one side of such an equation. This lays the foundation for using slope to graph linear equations in Section 1.4. Students can get extra practice in Section A.11.

Workbook Exploration

Most students will take for granted that for equations of the form $y = mx + b$, the graphs are lines because they were told so in elementary algebra. In the following 25-minute exploration, students see that graphs of some linear equations are lines *and* that the graphs of some nonlinear equations are not lines.

If you are worried about getting bogged down in Chapter 1, this is a good exploration to skip and just lecture about the concepts because you can communicate the concepts in much less time than it takes groups to do the exploration.

One advantage with having students do the exploration is that they get a preview of the types of curves with which they will be working in subsequent chapters. They'll also see a fuller range of what their graphing calculator can do. And they'll get hands-on experience of the concepts addressed in this exploration.

Group Exploration

Section Opener: Forms of equations whose graphs are lines

1. Use a graphing calculator to determine which of the equations that follow have graphs that are lines. (To view these graphs, use ZStandard followed by ZSquare.)

a. $y = x$

b. $y = x^2$ (For "x^2": Press $\boxed{\text{X,T,}\Theta,n}$ $\boxed{\wedge}$ **2** or $\boxed{\text{X,T,}\Theta,n}$ $\boxed{x^2}$.)

c. $y = 2^x$ (For "2^x": Press **2** $\boxed{\wedge}$ $\boxed{\text{X,T,}\Theta,n}$.)

d. $y = \dfrac{2}{x}$

e. $y = 3x$

f. $y = 2x + 4$

g. $y = -2x - 8$ (For "$-2x - 8$": Press $\boxed{(-)}$ **2** $\boxed{\text{X,T,}\Theta,n}$ $\boxed{-}$ **8**.)

h. $y = 4$

2. Make up your own example of an equation whose graph is a line. Use a graphing calculator to verify that you are right.

3. Here are some examples of equations whose graphs are lines.

$$y = 3x + 5 \qquad y = 2x - 8 \qquad y = -4x + 7 \qquad y = -7x - 6 \qquad y = 5x \qquad y = -3$$

   Form a theory about how to recognize whether an equation has a graph that is a line without graphing the equation. Test your theory. You can do this by creating equations you think have graphs that are lines and then using a graphing calculator to graph each equation to see if you are correct.

4. Is the graph of $3x^2 + y = 3x^2 + 2x + 8$ a line? Explain. Modify your theory in Problem 3, if necessary.

OBJECTIVE 4 COMMENTS

I sum up finding intercepts by saying "To find an intercept, we substitute 0 for the *other* variable." Most students lose sight that an intercept is represented by an ordered pair, not a number.

OBJECTIVE 5 COMMENTS

Sometimes students are thrown because only one variable appears in these equations. To find solutions of the equation $x = 1$ (Problem 10), I refer to the scenario that follows:

> Suppose that a younger sibling must return home by 1 A.M. but an older sibling can return home at any time. Assuming that the younger sibling always returns home at 1 A.M., discuss possible times when the siblings will return home.

This story helps students see that the equation $x = 1$ is restricting the value of x to be 1, but the value of y can be any real number. (I'd pick an earlier curfew, but I'd rather not graph an equation such as $x = 10$ for an introductory example; plus a curfew of 10 P.M. is a setup for having to contend with the issue that 1 follows 12 with time of day.)

For students who confuse the graphs of equations of the forms $x = a$ and $y = b$, I suggest that they build a table of (at least two) solutions to tip them off whether they should graph a horizontal or vertical line.

Workbook Exploration

Not only does the following 10-minute exploration get across the idea of how to graph equations of the forms $x = a$ and $y = b$, it also encourages students to reflect on the nature of the coordinates of points on a horizontal or vertical line.

Group Exploration

Section Opener: Equations of horizontal and vertical lines

1. Sketch a horizontal line on the given coordinate system.

2. List the ordered pairs of five points that lie on the horizontal line you sketched in Problem 1.

3. What do the coordinates of the ordered pairs you listed in Problem 2 have in common?

4. Translate the response your wrote in Problem 3 into an equation by filling in the blank: $y =$ _____.

5. Graph $y = -2$.

6. Graph $x = 4$.

SECTION 1.3 LECTURE NOTES

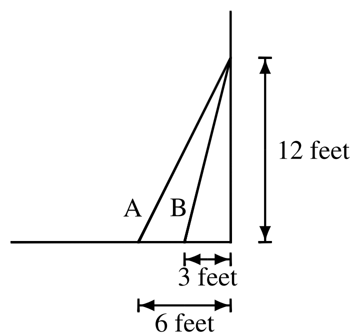
Objectives

1. Compare the steepness of two objects.
2. Describe the meaning of, and how to calculate, the *slope* of a nonvertical line.
3. Determine the sign of the slope of an increasing or a decreasing line.
4. Know the slopes of horizontal lines and vertical lines.
5. Describe the relationship between slopes of parallel lines.
6. Describe the relationship between slopes of perpendicular lines.

Main point: Know the meaning of, and how to calculate, the *slope* of a line.

OBJECTIVE 1

1. Ladder B is steeper than ladder A. Why?



Compare $\dfrac{\text{vertical distance}}{\text{horizontal distance}}$ for ladders A and B.

Comparing the Steepness of Two Objects

To compare the steepness of two objects, compute the unit ratio $\frac{\text{vertical distance}}{\text{horizontal distance}}$ for each object. The object with the larger ratio is the steeper object.

2. While taking off, airplane A steadily climbs for 2100 feet over a horizontal distance of 7600 feet. Airplane B steadily climbs for 2850 feet over a horizontal distance of 9900 feet. Which plane is climbing at a greater incline? Explain.

OBJECTIVE 2

Definition *Slope of a Nonvertical Line*

Let (x_1, y_1) and (x_2, y_2) be two distinct points of a nonvertical line. The **slope** of the line is

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Find the slope of the line that contains the given points.

3. $(2, 4)$ and $(5, 10)$

4. $(-4, 3)$ and $(2, -1)$

5. $(3, 1)$ and $(-6, -2)$

6. $(-5, -4)$ and $(-1, -6)$

OBJECTIVE 3

For Problems 3 and 5, the *increasing* lines have *positive* slopes. For Problems 4 and 6, the *decreasing* lines have *negative* slopes.

Slopes of Increasing and Decreasing Lines

- An increasing line has positive slope.
- A decreasing line has negative slope.

Explain this property in general by discussing the signs of rises and runs for an increasing line and for a decreasing line.

Measuring the Steepness of a Line

The absolute value of the slope of a line measures the steepness of the line. The steeper the line, the larger the absolute value of its slope will be.

OBJECTIVE 4

- Draw a horizontal line and find its slope $\left(\frac{0}{\text{run}} = 0\right)$.
- Draw a vertical line and explain why its slope, $\frac{\text{rise}}{0}$, is undefined.

Slopes of Horizontal and Vertical Lines

- A horizontal line has slope equal to zero.
- A vertical line has undefined slope.

For Problems 7–10, find the slope of the line that contains the given points. Determine whether the line is increasing, decreasing, horizontal, or vertical.

7. $(3, -1)$ and $(3, 4)$

8. $(-5, 2)$ and $(1, -6)$

9. $(3, -2)$ and $(7, -2)$

10. $(-4, -7)$ and $(-1, -5)$

11. Sketch qualitative graphs of an increasing line, a decreasing line, a horizontal line, and a vertical line, and determine whether the line's slope is positive, negative, zero, or undefined (Exercise 51).

OBJECTIVE 5

Sketch two parallel lines and compare their slopes.

Slopes of Parallel Lines

If lines l_1 and l_2 are nonvertical parallel lines on the same coordinate system, then the slopes of the lines are equal: $m_1 = m_2$. Also, if two distinct lines have equal slope, then the lines are parallel.

OBJECTIVE 6

Sketch two perpendicular lines and compare their slopes.

Slopes of Perpendicular Lines

If lines l_1 and l_2 are nonvertical perpendicular lines, then the slope of one line is the opposite of the reciprocal of the slope of the other line: $m_2 = -\dfrac{1}{m_1}$. Also, if the slope of one line is the opposite of the reciprocal of another line's slope, then the lines are perpendicular.

Using the indicated slopes of lines l_1 and l_2, determine whether the lines are parallel, perpendicular, or neither. Assume that the lines are not the same.

12. $m_1 = \dfrac{2}{5}, m_2 = -\dfrac{5}{2}$

13. $m_1 = 7, m_2 = 7$

14. $m_1 = \dfrac{3}{8}, m_2 = \dfrac{8}{3}$

SHORT HW 1, 11, 15, 19, 21, 27, 29, 31, 35, 41, 45, 51, 61, 63, 74

MEDIUM HW 1, 7, 11, 15, 19, 21, 27, 29, 31, 35, 41, 45, 51, 53, 55, 61, 63, 65, 67, 69, 74

SECTION 1.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

This section reviews slope concepts that will lay the groundwork for more in-depth concepts related to slope in Section 1.4. Because Section 1.4 contains more information that will be new to students, Section 1.4 should be emphasized more than Section 1.3. The concepts of slope will be needed throughout Chapters 1–3. Students will have an easy time with Section 1.3.

OBJECTIVE 1 COMMENTS

When discussing slope, I explain why it is defined in terms of both run and rise of two points on a line and not just the rise of the two points. Students appreciate it when I discuss Problem 2 in the lecture notes. Otherwise, they will have trouble with Exercises 1–4 in Homework 1.3.

Discussing the grade of a road is a good motivator of slope (see page 18 of the textbook). It can help to explain why truckers need to know the grade of a road in mountainous areas.

Workbook Exploration

The following 20-minute exploration emphasizes that when measuring the steepness of an object, it does not suffice to consider the rise of the object. Problem 2 is a challenging open-ended question. Resist the urge to help students too much. If a group's measure is not valid, point out why that's the case and challenge them to improve their measure or come up with a different one. Remember, even if a group's measure is not valid, putting thought into the concept of steepness will mentally prepare them to then learn about slope and help them appreciate how slope is defined.

Group Exploration

Section Opener: Steepness

Two ladders lean against a building. The foot of ladder A is 3 feet from the building and the top reaches a point at height 12 feet on the building. The foot of ladder B is 6 feet from the building and it reaches a point at height 18 feet on the building.

1. Draw a figure of the situation.
2. Which ladder is steeper? Explain by using calculations as well as words.
3. A student says ladder B is steeper than ladder A because ladder B reaches a point on the building higher than ladder A does. What would you tell the student?
4. For ladder A, find the unit ratio of the vertical distance to the horizontal distance. Find the unit ratio for ladder B, too. What do the unit ratios mean in this situation? Compare the two ladders' unit ratios. What does your comparison mean in this situation?
5. A student says it would be better to find the unit ratio of the horizontal distance to the vertical distance for each ladder. What would you tell the student?
6. A portion of road A climbs steadily for 105 feet over a horizontal distance of 2950 feet. A portion of road B climbs steadily for 130 feet over a horizontal distance of 4325 feet. Which road is steeper?

OBJECTIVE 2 COMMENTS

I show students that when we use the slope formula with two points on a line, it doesn't matter which point we choose to be (x_1, y_1) and which we choose to be (x_2, y_2). You can use specific points to illustrate, or show students that

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-(y_1 - y_2)}{-(x_1 - x_2)} = \frac{y_1 - y_2}{x_1 - x_2}$$

Although students can follow these steps, they may not see the point of this, so it may be better to use specific points (and use the slope formula both ways).

I discuss the warning near the top of page 20 in the textbook.

When some or all coordinates are negative, a common student error is to omit at least one subtraction symbol from the slope formula or negative sign from the coordinates.

Workbook Exploration

There are many concepts addressed in the following 30-minute exploration. Start by demonstrating how to construct a scatterplot of the women's data and then sketch a linear model. Then groups can work with the men's data and continue working on the exploration.

Another option would be to provide groups with the data but forego giving them the questions. Instead, you could simply ask, "Which gender is improving faster?" This option probably has more value because groups must wrestle with many concepts that are crucial for the course. Some groups might take a numerical approach and other groups might take a graphical approach. The debrief should include groups comparing and contrasting their approaches.

Because the concept of rate of change will first be addressed in Section 2.4, yet another option would be to delay using this exploration until then. But using the exploration while discussing Section 1.3 might be wise because it's a really organic, effective way to motivate slope.

EXPLORATION *Rate of change*

1. World record times for the 200-meter run are listed in the following table for various years.

Women		Men	
Year	Record Time (seconds)	Year	Record Time (seconds)
1973	22.38	1951	20.6
1974	22.21	1963	20.3
1978	22.06	1967	20.14
1984	21.71	1979	19.72
1988	21.34	1996	19.32
		2009	19.19

Source: *IAAF Statistics Handbook*

1. Which gender's record times are decreasing the most per year? Use calculations and words to explain.

2. Construct scatterplots by hand for the women's data and the men's data on the same coordinate system. Make it clear which data points are for which gender.

3. On the scatterplots you constructed in Problem 2, sketch a linear model for each gender.

4. Calculate the slope of each of your linear models. What do your results mean in this situation? Do your results support the claim you made in Problem 1? Explain.

5. A student says the men's record times are decreasing the most per year because the decrease in their record times from 1951 to 2009 is greater than the decrease in the women's record times from 1973 to 1988. What would you tell the student

6. A student uses the 1974 and 1978 data points to find a linear model for women's record times and the 1963 and 1967 data points to find a linear model for the men's record times. She finds that both gender's record times decreased by about 0.04 second per year. The student concludes that the record times for each gender are decreasing at the same rate. What would you tell the student?

Textbook and Workbook Exploration

For the 10-minute "Using different pairs of points to calculate slope" exploration, students discover that for a line, the ratio of the rise to the run is constant for any distinct pair of points on the line. Although your students probably already know how to calculate the slope of a line, they may be surprised that the value of the slope is independent of which two points are chosen.

OBJECTIVES 3 and 4 COMMENTS

Performing a sign analysis of an increasing line and a decreasing line will lead to the meanings of the signs of slope (see Figs. 50 and 51 on page 20 of the textbook). Discussing the meanings of the signs of slope will help students understand the meanings of the signs of rates of change in Section 2.4.

OBJECTIVES 5 and 6 COMMENTS

Because slope is a measure of the steepness of a line, students should be able to deduce that the slopes of two parallel lines are equal. Both parallel and perpendicular lines will continue to be addressed in Sections 1.4 and 1.5, but only parallel lines will be needed after Section 1.5; parallel lines will be relevant for inconsistent systems in Sections 3.1 and 3.2.

SECTION 1.4 LECTURE NOTES

Objectives

1. Find the slope of a nonvertical line from an equation in *slope-intercept form*.
2. Describe the *vertical change property*.
3. Find the y-intercept of a nonvertical line from an equation in slope-intercept form.
4. Use slope and y-intercept to sketch the graph of an equation in slope-intercept form.
5. Apply the *slope addition property*.

Main point: Use slope and y-intercept to graph an equation of the form $y = mx + b$.

OBJECTIVE 1

1. Use the graph of $y = 3x + 1$ to find the slope of the line $y = 3x + 1$.
2. Use the graph of $y = -2x + 6$ to find the slope of the line $y = -2x + 6$.

Finding the Slope from a Linear Equation of the Form $y = mx + b$

For a linear equation of the form $y = mx + b$, m is the slope of the line.

Determine whether the given pair of lines is parallel, perpendicular, or neither. Explain.

3. $y = 3x - 7$ and $y = 3x + 5$
4. $y = \frac{2}{3}x - 2$ and $3x + 2y = 8$
5. $x = 2$ and $y = 4$

OBJECTIVE 2

Use the fact $m = \frac{m}{1}$ to explain the property that follows. Draw lines to illustrate.

Vertical Change Property

For a line $y = mx + b$, if the run is 1, then the rise is the slope m.

OBJECTIVE 3

Substitute 0 for x to show that the line $y = mx + b$ has y-intercept $(0, b)$.

Finding the y-Intercept from a Linear Equation of the Form $y = mx + b$

For a linear equation of the form $y = mx + b$, the y-intercept is $(0, b)$.

Find the y-intercepts of the given lines.

6. $y = 3x + 6$
7. $y = 2x - 5$.

> **Definition** *Slope-intercept form*
>
> If an equation is of the form $y = mx + b$, we say it is in **slope-intercept form**.

Find the slopes and the y-intercepts of the lines $y = 5x - 9$ and $y = -8x + 3$.

OBJECTIVE 4

Determine the slope and y-intercept of the graph of the linear equation. Use the slope and y-intercept to graph the equation by hand.

8. $y = 2x - 3$

9. $y = -\dfrac{3}{2}x + 4$

10. $y = -\dfrac{2}{7}x - 1$

11. $3x + 5y = 10$

12. $2(x - 2y) + 1 = 9$

13. $5 - 2(3x - y) = 1 - 3(x - 2y)$

14. Determine the slope and the y-intercept of the graph of $b(y + a) = cx$, where a, b, and c are constants and $b \neq 0$.

OBJECTIVE 5

Use tables of solutions of $y = 2x + 3$ and $y = -3x + 20$ to motivate the slope addition property.

> **Slope Addition Property**
>
> For a linear equation of the form $y = mx + b$, if the value of the explanatory variable increases by 1, then the value of the response variable changes by the slope m.

15. Four sets of points are described in the following table. For each set, decide whether there is a line that passes through every point. If so, find the slope of that line. If not, decide whether there is a line that comes close to every point.

Set 1		Set 2		Set 3		Set 4	
x	y	x	y	x	y	x	y
0	5	0	73	2	13	5	2
1	9	1	67	3	18	6	2
2	13	2	61	4	22	7	2
3	17	3	55	5	27	8	2
4	21	4	49	6	31	9	2

16. Some values of four linear equations are provided in the following table. Complete the table.

Equation 1		Equation 2		Equation 3		Equation 4	
x	y	x	y	x	y	x	y
0	5	0	83	21	14	43	92
1	9	1	76	22		44	
2		2		23		45	
3		3		24	23	46	
4		4		25		47	76

SHORT HW 3, 9, 15, 19, 29, 37, 41, 45, 47, 53, 61, 63 , 65, 81, 85

MEDIUM HW 3, 9, 13, 15, 19, 21, 29, 37, 41, 43, 45, 47, 53, 61, 63, 65, 69, 71, 75, 81, 85

SECTION 1.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

This section discusses slope concepts that will likely be new for most of your students. After completing this section, students should be able to think of slope in terms of graphs, tables, and equations.

Thinking of slope in terms of tables can lead to the slope addition property, which is a key property of this course. This property is related to using unit ratios to show that slope is a rate of change (Section 2.4). The slope addition property can be used to find an equation of a linear equation when working with certain tables of solutions. The property will help students see the connection between an arithmetic sequence and the related linear function. Finally, the slope addition property will be compared with the base multiplier property in Section 4.4.

In Section 1.4, students will learn to graph linear equations using slope and y-intercept. This is an important skill used throughout Chapters 1—3.

OBJECTIVE 1 COMMENTS

Students will likely remember that m is the slope of a line $y = mx + b$. Doing Problem 1 and skipping Problem 2 will probably suffice in reminding students of this concept.

Workbook Exploration

The following 10-minute exploration is probably the most convincing way for students to learn that the slope of the line $y = mx + b$ is m.

Group Exploration

Section Opener: The meaning of m in the equation $y = mx + b$

1. **a.** Carefully sketch a graph of the line $y = 2x - 1$.

b. Using the formula $m = \frac{\text{rise}}{\text{run}}$, find the slope of the line you sketched.

c. What number is multiplied by x in the equation $y = 2x - 1$? How does it compare with the slope you found in part (b)?

2. **a.** Carefully sketch a graph of the line $y = -3x + 5$.

b. Using the formula $m = \frac{\text{rise}}{\text{run}}$, find the slope of the line you sketched.

c. What number is multiplied by x in the equation $y = -3x + 5$? How does it compare with the slope you found in part (b)?

3. Describe what you have learned in this exploration so far.

4. Without graphing, determine the slope of each line.

a. $y = 4x - 7$ **b.** $y = -2x + 4$ **c.** $y = \frac{2}{5}x - 3$ **d.** $y = x - 2$ **e.** $y = 3$

Textbook and Workbook Exploration

The 15-minute "Graphical significance of m and b" exploration assists students in discovering the graphical significance of m and b for an equation of the form $y = mx + b$, which are key concepts of the course. After students complete the exploration, I have a short classroom discussion where students share their responses for

Problem 4 of the exploration. There is a lot of information in this exploration, and students have trouble seeing how it all fits together.

Textbook and Workbook Exploration

Students really enjoy the "Drawing lines with various slopes" exploration, which encourages students to reflect on the graphical significance of m and b of an equation of the form $y = mx + b$. Most students restrict their choices of slopes to positive integers at first and then realize that they need to use negative integers, too. Most students need hints to get them thinking about using slopes between -1 and 1.

OBJECTIVE 2 COMMENTS

Most students will never have seen the vertical change property before. This property serves as nice preparation for graphing equations of the form $y = mx + b$ where m is an integer.

OBJECTIVE 3 COMMENTS

The slope-intercept form will be used heavily throughout Chapters 1—3. In fact, the textbook uses the slope-intercept form rather than the slope-point form to find linear equations containing two given points because students have better success with this method. Lighter treatment of the point-slope form is included in Section 1.5 to accommodate instructors who prefer this form.

OBJECTIVE 4 COMMENTS

For Problem 8, I emphasize that we can write the slope 2 in the form $\frac{2}{1}$. From such a tactic, some students think that a slope must *always* be expressed in fraction form; this may be a good time to clear up that misconception.

For Problem 9, I emphasize that we can write the slope as $\frac{-3}{2}$. Some students would write the slope as $\frac{-3}{-2}$; I point out that is incorrect because $\frac{-3}{-2}$ is positive and the slope is negative.

For Problem 11, I point out that the coefficient of x, 3, is not the slope of the line $3x + 5y = 10$. I emphasize that we must isolate y to one side of the equation before we can determine the slope and the y-intercept.

For Problems 8–10, I have students use their graphing calculators to verify the graphs. For at least one of the Problems 11–13, I verify the graph by checking that an ordered pair corresponding to a point on the graph satisfies the *original* equation. This is a nice reminder of the meaning of a graph.

OBJECTIVE 5 COMMENTS

I make sure that students see the connection between the vertical change property, the slope addition property, and the fact $m = \frac{m}{1}$.

Workbook Exploration

The following 15-minute exploration can serve as a good exploration of the slope addition property. You could show students how to use graphing calculator tables to complete the table for Problem 1. Although students should certainly be able to complete the table without technology, a small arithmetic error might prevent a group from discovering the concept.

EXPLORATION *Slope addition property*

1. Complete the following table.

$y = 2x + 1$		$y = 3x - 5$		$y = -2x + 6$	
x	y	x	y	x	y
0		0		0	
1		1		1	
2		2		2	
3		3		3	
4		4		4	

2. In the table, the x-coordinates increase by 1 each time. For each equation, what do you notice about the y-coordinates? Compare what you notice with the coefficient of x in each equation.

3. Describe what the pattern from Problem 2 would be in general for any equation of the form $y = mx + b$.

4. Create an equation of the form $y = mx + b$, and check whether it behaves as you described in Problem 3.

5. Substitute 1 for x in the equation $y = mx + b$. Then substitute 2 for x. Then substitute 3. Explain why these results suggest that your description in Problem 3 is correct.

SECTION 1.5 LECTURE NOTES

Objectives

1. Use the slope-intercept form to find an equation of a line.
2. Use the *point-slope form* to find an equation of a line.

Main point: Find an equation of a line.

OBJECTIVE 1

Use the slope-intercept form $y = mx + b$ to find an equation of the line that has the given slope and contains the given point.

1. $m = 2$, $(-4, 7)$
2. $m = -3$, $(-2, -5)$
3. $m = \frac{2}{7}$; $(-3, 1)$
4. $m = -\frac{3}{5}$; $(2, -7)$

Use the slope-intercept form to find an equation of the line that contains the two given points.

5. $(4, 2)$ and $(7, 11)$ [slope is an integer]
6. $(-2, 4)$ and $(1, -5)$ [slope is an integer]
7. $(-3, -1)$, and $(9, 5)$ [slope is a noninteger]
8. $(-7, -3)$ and $(-2, -5)$ [slope is a noninteger]

Finding an Equation of the Line That Contains Two Given Points

To find an equation of the line that passes through two given points whose x-coordinates are different,

1. Use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to find the slope of the line.
2. Substitute the m value you found in step 1 into the equation $y = mx + b$.
3. Substitute the coordinates of one of the given points into the equation you found in step 2, and solve for b.
4. Substitute the m value you found in step 1 and the b value you found in step 3 into the equation $y = mx + b$.
5. Use a graphing calculator to check that the graph of your equation contains the two given points.

9. Find an approximate equation of the line that contains the points $(-5.71, -2.98)$ and $(3.22, -9.68)$. Round the slope and the constant term to two decimal places.
10. Find an equation of a line that has undefined slope and contains the point $(2, 5)$.
11. Find an equation of the line that contains $(2, -5)$ and is parallel to the line $y = -4x + 1$.
12. Find an equation of the line that contains $(1, -3)$ and is perpendicular to the line $2x + 5y = 15$.

OBJECTIVE 2 (optional)

Let (x_1, y_1) and (x, y) be two distinct points on a line with slope m. Use the true statement $\frac{y - y_1}{x - x_1} = m$ to derive the point-slope form.

Point-Slope Form

If a nonvertical line has slope m and contains the point (x_1, y_1), then an equation of the line is

$$y - y_1 = m(x - x_1)$$

Repeat any of Problems 1–9, 11, and 12, but now use the point-slope form. Compare your results to your earlier results.

SHORT HW 1, 9, 23, 33, 35, 37, 43, 47, 57, 61, 69, 73, 75, 79, 87

MEDIUM HW 1, 5, 9, 17, 19, 23, 33, 35, 37, 43, 47, 57, 61, 69, 71, 73, 75, 77, 79, 81, 87

SECTION 1.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

This section discusses how to find equations of lines, both by using the slope-intercept form and the point-slope form. In subsequent sections, only the slope-intercept form will be used to find such equations. So, the point-slope form is optional. However, if some of your students will eventually take calculus, you should spend some time on the point-slope form because this concept will likely be used in that course.

I have chosen to emphasize finding linear equations using the slope-intercept form for several reasons:

- My students have better success with this method than with using the point-slope form, especially when the slope is a noninteger.
- The notion of substituting a point to find the parameter b of $y = mx + b$ generalizes to finding parameters for exponential, quadratic, rational, and radical functions.
- Students have a better sense of why this method “works” than with using the point-slope form.
- Using two forms for linear equations in Chapters 1—3 confuses the issue for many students.

Students will find equations of linear models in Chapters 2 and 3. In particular, this skill is the primary practice of Section 2.2.

The skills required for Problems 11 and 12 will not be needed in the rest of the textbook. However, the concepts about parallel lines addressed in Sections 1.3 and 1.4 will be needed in Sections 3.1 and 3.2.

OBJECTIVE 1 COMMENTS

When I substitute an ordered pair into an equation (to find b of $y = mx + b$), I remind students that an ordered pair that corresponds to a point on a graph satisfies an equation of the graph. So, making such a substitution will give a true statement, one that will allow us to find the value of b.

I make sure that students understand that they should use fractions, not decimals, when finding linear equations. Some students will have trouble working with fractions. Such work is a good primer for work with rational expressions and equations in Chapter 8. When solving an equation with fractions (to find b of $y = mx + b$), I prefer to have students multiply both sides of the equation by the LCD because that is how students will solve rational equations in Chapter 8; also, a majority of my students prefer this approach.

For Problems 5–8, I show students how to use their graphing calculators to verify the equations.

Problem 9 is a good primer for Section 2.2, where students will find equations of linear models.

Students will be challenged by Problems 11 and 12. One of students’ difficulties is trying to track what is true for one line and what is true for the other. When doing Problem 11, it can help to use one color for all words and symbols in the problem statement and solution that describe one line and to use another color for such things that describe the other line. Similar color coding can be used for Problem 12.

Discussing Fig. 100 on page 41 of the textbook is a nice way to summarize Sections 1.2—1.5.

Textbook and Workbook Exploration

The 10-minute "Finding an equation of a line" serves as a good bridge from Section 1.4 to this section. In this exploration, students use the slope addition property to find equations. Although this skill is not necessary for this section, it does have groups reflect on how to go from tables to equations for linear functions. After completing this section, students should be able to go in all six directions between equations, graphs, and tables for linear functions.

Workbook Exploration

The skill of finding an equation using two given points is introduced in the following 15-minute exploration. A few groups will have trouble with Problem 2 of the exploration. In particular, some groups will ignore Problem 2 and write $1 = m(2) + b$ in Problem 3.

Group Exploration

Section Opener: Finding linear equations

1. Find the slope of the line that contains the points $(4,\ 5)$ and $(6,\ 8)$.
2. Plot the points $(4,\ 5)$ and $(6,\ 8)$ on the same coordinate system. Then draw the line that contains the points. Finally, find the y-intercept of the line.
3. Recall that a line of the form $y = mx + b$ has slope m and y-intercept $(0,\ b)$. Use the results you found in Problems 1 and 2 to find an equation of the line that contains the points $(4,\ 5)$ and $(6,\ 8)$.
4. Substitute the coordinates of the point $(4,\ 5)$ into the equation $y = \frac{3}{2}x + b$ and solve for b. Then substitute the value for b that you found into the equation $y = \frac{3}{2}x + b$.
5. Explain why it makes sense that the equations you found in Problems 3 and 4 are the same.

Textbook and Workbook Exploration

The 15-minute "Deciding which points to use to find an equation of a line" exploration responds to the frequent student query, "Does it matter which ordered pair is used to find b?" and "Does it matter which two distinct points I choose from the graph of a line to find its equation?" I make sure that students understand the directions for Problem 2 of the exploration, and that they imagine a line that is *not* parallel to either axis. This is an easy exploration for most students, and it could be assigned as homework.

Workbook Exploration

The following 20-minute exploration is a fun way for students to practice finding equations of lines. It also challenges students to think geometrically as well as analytically when finding the equations.

Group Exploration
Finding an equation of a line

Some solutions of four linear equations are provided in the following table. Find an equation of each of the four lines.

Equation 1		Equation 2		Equation 3		Equation 4	
x	y	x	y	x	y	x	y
0	3	3	40	2	3	1	22
1	7	4	37	4	15	3	18
2	11	5	34	6	27	5	14
3	15	6	31	8	39	7	10

OBJECTIVE 2 COMMENTS
This concept is optional. See my comments made earlier under "Main points."

SECTION 1.6 LECTURE NOTES

Objectives

1. Describe the meanings of *relation*, *domain*, *range*, and *function*.
2. Identify functions by using the *vertical line test*.
3. Describe a *linear function*.
4. Describe the Rule of Four for functions.
5. Use the graph of a function to find the function's domain and range.

Main point: Know the meaning of *function* and identify functions.

OBJECTIVE 1

For the ordered pairs $(1, 4)$, $(2, 3)$, $(2, 5)$, and $(3, 1)$:

- Construct a table and a scatterplot.
- The set of ordered pairs are called a *relation*.
- Describe the domain and range of the previous relation.
- A **relation** is a set of ordered pairs. The **domain** of a relation is the set of all values of the explanatory variable and the **range** of the relation is the set of all values of the response variable.
- Discuss inputs and outputs for the previous relation.
- Each member of the domain is an **input**. Each member of the range is an **output**.
- For the previous relation, observe that the input $x = 2$ has *two* outputs: $y = 3$ and $y = 5$.

Definition *Function*

A **function** is a relation in which each input leads to exactly one output.

Determine whether the given relation is a function.

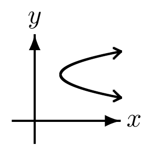
1. $y = x + 5$ **2.** $y = 2x$ **3.** $y = \pm x$

4. Some ordered pairs of four relations are listed in the following table. Which of these relations could be functions? Explain.

Relation 1		Relation 2		Relation 3		Relation 4	
x	y	x	y	x	y	x	y
0	13	2	20	5	3	7	1
1	18	3	18	6	3	7	2
2	23	3	16	7	3	7	3
3	28	4	14	8	3	7	4
4	33	5	12	9	3	7	5

OBJECTIVE 2

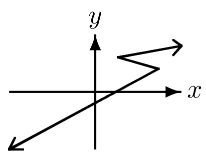
Use arrows to show that there is an input that has two outputs for the relation graphed below. Conclude that the relation is not a function.

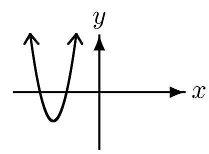


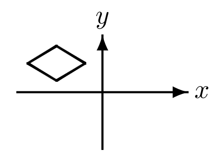
Vertical Line Test

A relation is a function if and only if each vertical line intersects the graph of the relation at no more than one point. We call this requirement the **vertical line test**.

Determine whether the following graph represents a function. Explain.







OBJECTIVE 3

Use the vertical line test to deduce that any nonvertical line is the graph of a function.

Definition *Linear Function*

A **linear function** is a relation whose equation can be put into the form $y = mx + b$, where m and b are constants.

Determine whether the given relation is a function.

5. $6x + 9y = 27$

6. $x = 2$

7. $y = x^2$

8. $x = y^2$

OBJECTIVE 4

Refer to your previous work to motivate the Rule of Four for functions.

Rule of Four for Functions

We can describe some or all of the input-output pairs of a function by means of

1. an equation,
2. a graph,
3. a table, or
4. words.

These four ways to describe input-output pairs of a function are known as the **Rule of Four** for functions.

OBJECTIVE 5

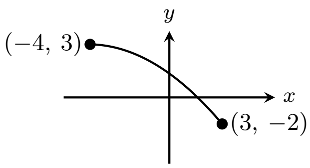
Describe the meaning of the notations $\leq$ and $\geq$.

Describe the given inequality in words.

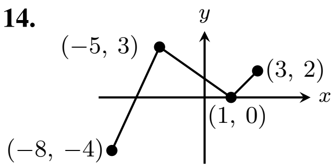
9. $x \leq 7$ **10.** $x \geq 4$ **11.** $5 \leq x$ **12.** $1 \leq x \leq 9$

Use the graph of the function to determine the function's domain and range.

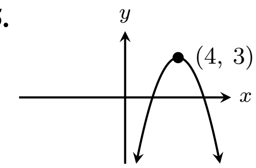
13.



14.



15.



SHORT HW 1, 3, 5, 11, 13, 17, 21, 29, 31, 35, 39, 45, 47, 51, 57

MEDIUM HW 1, 3, 5, 7, 9, 11, 13, 17, 21, 25, 29, 31, 35, 39, 45, 47, 49, 51, 57

SECTION 1.6 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students learn about the concepts of relation and function in this section. Concepts related to functions will be needed throughout the rest of the textbook.

Functional notation will first be discussed in Section 2.3. It is easier to motivate such notation there because we can discuss how such notation can be used to distinguish between two or more models that are used to describe an authentic situation. Delaying discussion of functional notation also frees up Section 1.6 to emphasize the meaning of a function.

OBJECTIVE 1 COMMENTS

To get across the definition of a function, I like to draw lots of "function machines" such as the ones in Figs. 106 and 107 on pages 45 and 46 of the textbook as well as tables such as Table 24 on page 47 in the textbook. I make sure that students understand that a function may have two inputs that share the same output.

Workbook Exploration

In the following 20-minute exploration, students practice determining whether a relation is a function. It will be more challenging and beneficial to students if you simply define a relation and function and then have groups work on the exploration, rather than you first giving examples.

Group Exploration

Section Opener: Identifying functions

1. If we think of Americans as inputs to a machine where the outputs are the current ages of the Americans, then each input has exactly one output because each American has exactly one current age. For each situation, determine whether each input has exactly one output.

a. Inputs: Americans; Outputs: current heights (in inches) of Americans

b. Inputs: Americans; Outputs: names of siblings of Americans

c. Inputs: phones; Outputs: names of apps on phones

d. Inputs: homes in the United States; outputs: numbers of bedrooms in homes

2. For each table, think of the values of x as inputs and the values of y as outputs. Does each input have exactly one output?

a.

x	y
0	6
1	2
2	5
3	3
4	4

b.

x	y
0	6
1	2
2	5
2	3
3	4

c.

x	y
0	3
1	3
2	3
3	3
4	3

3. For each equation, think of the values of x as inputs and the values of y as outputs. Does each input have exactly one output?

a. $y = 3x$

b. $y = \pm x$

c. $y = x^2$

d. $x = y^2$ [**Hint:** Substitute 9 for x and solve for y.]

OBJECTIVE 2 COMMENTS

Before discussing the vertical line test, I first use arrows similar to those in Fig. 109 on page 47 of the textbook to get students accustomed to thinking visually of inputs being sent to outputs. I find that even calculus students lack this essential perspective. I also use arrows with tables and arrows with graphs to help students see the connection between tables and graphs in terms of inputs being sent to outputs.

Once students see how to draw such arrows with graphs, students are quick to identify graphs of functions. They can do this without having been told about the vertical line test. I prefer having students draw arrows because the vertical line test strips away the key idea of a function. Once students have mastered using arrows to identify graphs of functions, I'll sometimes briefly discuss the vertical line test. Most students persist in drawing arrows on quizzes and exams, which I take to be a good sign.

Textbook and Workbook Exploration

The vertical line test is nicely motivated by the 15-minute "Vertical line test" exploration. By completing the exploration, students will see not only how to use the vertical line test, but also *why* it works.

OBJECTIVE 3 COMMENTS

Students have an easy time with this objective.

OBJECTIVE 4 COMMENTS

There are exercises scattered throughout the textbook that refer to the Rule of Four. For example, see Exercises 29 and 30 Homework 1.6.

OBJECTIVE 5 COMMENTS

To solve Problem 13, draw arrows indicating around eight inputs being sent to corresponding outputs. Students will then be quick to pick up on the domain and range of the function. Apply a similar approach to Problems 14 and 15. For Problem 15, students will wonder whether the domain is really the set of all real numbers. It will help to explain the meaning of the two displayed arrows.

Students will be quick to pick up on how to use inequality notation to describe the domain and range of a function. If you would rather postpone discussion about inequalities until they are fully developed in Section 3.5, you could have students describe the domain and range of a function using words. Because the answers to such exercises in Homework 1.6 use inequality notation, you should supply verbal answers for such exercises if you want to go that route.

CHAPTER 2 OVERVIEW

In Section 2.1, students use graphical methods to model situations with linear functions. They use hand-drawn lines to make estimates and predictions and learn that model breakdown may occur. Section 2.1 provides a graphical overview of the symbolic work to be done in Sections 2.2 and 2.3. In Section 2.2, students use technology to draw scatterplots and then find equations of linear models by hand; they also learn how to perform linear regression, although finding linear models by hand (using two "good" points) is emphasized. In Section 2.3, students use linear models in conjunction with symbolic work to make estimates and predictions. In Section 2.4, students learn that the slope of a linear model is a rate of change.

SECTION 2.1 LECTURE NOTES

Objectives

1. Construct a scatterplot.
2. Describe the meaning of a *linear model*.
3. Use a linear model to make estimates and predictions.
4. Determine when to use a linear function to model data.
5. Find intercepts of a linear model.
6. Describe the meaning of *interpolate*, *extrapolate*, and *model breakdown*.
7. Modify a model.

Main point: Use lines to make estimates and predictions about authentic situations.

OBJECTIVES 1 and 2

1. The numbers of bank robberies per 100 branches are shown in the following table for various years. Let n be the number of bank robberies per 100 branches at t years since 1990. Describe the data with a graph. Then sketch a line that comes close to the points.

Table 2.1: Numbers of Bank Robberies per 100 Branches

Year	Robberies per 100 Branches	Year	Robberies per 100 Branches
1997	10.6	2007	6.0
1999	8.8	2009	6.0
2001	10.3	2011	6.0
2003	8.7	2013	5.0
2005	7.7	2015	5.2

Source: *FBI*

A graph of plotted ordered pairs is called a **scatterplot**. If the points in a scatterplot of data lie close to (or on) a line, then we say that the relevant variables are **approximately linearly related**.

Definition *Model*

A **model** is a mathematical description of an authentic situation. We say that the description *models* the situation.

Definition *Linear Model*

A **linear model** is a linear function, or its graph, that describes the relationship between two quantities for an authentic situation.

Our goal is to find a line that comes close to *all* the data points.

OBJECTIVE 3

2. Use the robbery model to estimate the number of bank robberies per 100 branches in 2001. Find the error. [interpolation]

3. Use the robbery model to estimate the number of bank robberies per 100 branches in 2007. Find the error. [interpolation]

4. Use the model to predict when there will be 2.5 bank robberies per 100 branches. [extrapolation]

OBJECTIVE 4

5. Consider the scatterplots of data shown in Figs. 2.1, 2.2, and 2.3 for situations 1, 2, and 3, respectively. For each situation, determine whether a linear model would model it well.

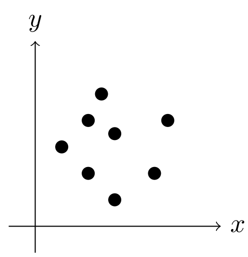


Figure 2.1: Scatterplot for situation 1

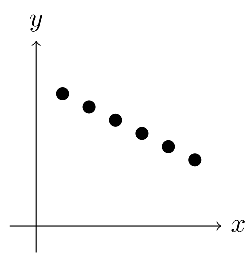


Figure 2.2: Scatterplot for situation 2

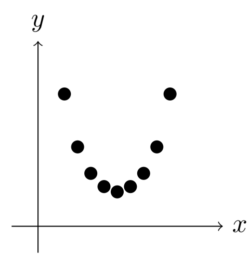


Figure 2.3: Scatterplot for situation 3

OBJECTIVE 5

6. Find the n-intercept of the robbery model. What does it mean in this situation? [extrapolation]

7. Find the t-intercept of the robbery model. What does it mean in this situation? [extrapolation]

OBJECTIVE 6

Refer to the estimates and predictions that you made in Problems 3–7 as interpolations or extrapolations.

Definition *Interpolation, extrapolation*

For a situation that can be described by a model whose explanatory variable is x,

- We perform **interpolation** when we use a part of the model whose x-coordinates are between the x-coordinates of two data points.
- We perform **extrapolation** when we use a part of the model whose x-coordinates are not between the x-coordinates of any two data points.

8. Discuss whether the t-intercept of the robbery model is realistic.

Definition *Model breakdown*

When a model gives a prediction that does not make sense or an estimate that is not a good approximation, we say that **model breakdown** has occurred.

9. For what values of t does model breakdown occur for the robbery model?

OBJECTIVES 1–6 (revisited)

10. The numbers of Oregonians who have committed suicide under Oregon's Death with Dignity Act (DWDA) are shown in the following table for various years.

Table 2.2: Numbers of DWDA Deaths in Oregon

Year	Deaths
2011	71
2012	85
2013	73
2014	105
2015	135
2016	133

Source: *Oregon Public Health Division*

Let n be the number of Oregonians who have committed suicide under Oregon's Death with Dignity Act (DWDA) in the year that is t years since 2010.

a. Construct a scatterplot of the data.

b. Draw a line that comes close to the data points.

c. Use your line to estimate the number of DWDA deaths in Oregon in 2015. What is the error?

d. Predict in which year there will be 200 DWDA deaths in Oregon.

e. Find the t-intercept of the model. What does it mean in this situation? Has model breakdown occurred? [**Hint:** DWDA was enacted late in 1997 and there were 16 DWDA deaths in Oregon in 1998.]

OBJECTIVE 7

11. Additional research yields the data shown in the following table. Use this data as well as the fact that DWDA was enacted late in 1997 to modify the model you found in Problem 10.

Table 2.3: Numbers of DWDA Deaths in Oregon

Year	Deaths
1998	16
2000	27
2002	38
2004	37
2006	46
2008	60
2010	65

Source: *Oregon Public Health Division*

SHORT HW 1, 3, 5, 11, 13, 15, 19, 21

MEDIUM HW 1, 3, 5, 6, 7, 9, 11, 13, 15. 17, 19, 21

SECTION 2.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use pencil and paper to construct a scatterplot and an appropriate linear model. They use such a model to make estimates and predictions. Students learn the meaning of interpolation, extrapolation, and model breakdown. They will use linear modeling throughout Chapters 2 and 3, and they will use the concepts of interpolation, extrapolation, and model breakdown throughout the rest of the textbook.

OBJECTIVES 1 and 2 COMMENTS

For Problem 1, many students will likely need reminders about the concepts of explanatory variable and response variable and about which type of variable is represented by which axis. Some students will likely need a reminder about the meaning of the phrase "at t years since 1990."

When some of my students construct scatterplots, they fail to use uniform scaling on the axes (even though they've seen me use uniform scaling repeatedly on the board) unless this issue is discussed. For example, if a table of data displays the years 2010, 2012, 2014, 2016, and 2017, and t is the number of years since 2010, students would write the numbers 0, 2, 4, 6, and 7 equally spaced on the t-axis.

In Problem 1, I demonstrate that there are many acceptable linear models. Many students try to find a line that contains the greatest number of data points, rather than a line that comes close to all the data points. To clarify this confusion, I compare a reasonable linear model that comes close to all the data points, but does not contain any data points, with the terribly inaccurate linear model that contains the data points $(9, 8.8)$ and $(11, 10.3)$.

When describing the meaning of a model, I give intuitive examples such as the textbook's example about airplane models (see the middle of page 62).

OBJECTIVE 3 COMMENTS

When doing Problems 2–4, I draw input-output arrows similar to those in Fig. 4 on page 63 of the textbook. I require my students to use such arrows so that I can better evaluate their work on quizzes and exams. It is difficult to award any partial credit if no arrows are included.

For many students, the distinction between a model and reality is foggy. For example, many students would think that the answers to Problems 2 and 3 are the actual 10.3 robberies per 100 branches and 6.0 robberies per 100 branches, respectively, even if the linear model does not contain the data points $(11,\ 10.3)$ and $(17,\ 6.0)$. Making such a distinction while doing Problems 2 and 3 pays big dividends both in the short and long run. For an intuitive explanation, I tell my students that a model is fiction, which we hope is very close to the truth.

I warn my students that even though their answers may differ from those in the answers section of the textbook, their answers may be correct. I explain that it's possible to draw reasonable linear models that are different than the reasonable models that the author used. Although I want my students to carefully construct scatterplots and linear models, I don't want them to be anxious about getting the exact results.

Workbook Exploration

The following 20-minute exploration could be assigned at the start of class before you even discuss how to construct a scatterplot. Some groups may use a numerical approach and others may use a graphical approach. For groups that come up with simplistic approaches, challenge them to develop more sophisticated ones. For example, if a group uses the increase in Snapchat users only from 2017 to 2018 to predict the number of users in 2022, you could nudge them toward using the average of the increases from one year to the next. It may seem that a numerical approach is too far askew from the main point of this section, but this open-ended exploration prevents mimicry, supports the powerful learning process of productive struggle, and has students consider the key concept of trend. During the debrief, comparing and contrasting various groups methods can lay a great foundation for the entire chapter.

Group Exploration

Section Opener: Making a prediction

The number of daily active Snapchat users are shown in the following table for the first quarter of various years. Predict the number of daily active Snapchat users in 2022.

Year	Number of Daily Active Snapchat Users (millions)
2014	46
2015	80
2016	122
2017	166
2018	191

Source: *Snap Inc.*

Textbook and Workbook Exploration

The 20-minute "Linear modeling" exploration has much more structure than the "Making a prediction" exploration in the workbook, and for that reason might be less valuable. However, it does have students go deeper into modeling with Problems 5–8. So, this exploration could be used directly after using the "Making a prediction" exploration.

In Problem 1, many students can complete Table 7 correctly but without understanding that t represents the number of years since 2010. It's a good idea to go about the classroom and make sure groups understand this concept.

Despite the instruction to use a straight line in Problem 4, a few groups may sketch a "zigzag" curve. Waiting for students to make this error and then clarifying the meaning of "line" might make a more lasting impression on students. In exercises, the clarification "(straight)" will not be included.

Textbook and Workbook Exploration

The 20-minute "How defining the explanatory variable affects a model" exploration is not essential for subsequent sections. However, unless students do this exploration, Exercise 6 of Homework 2.1, Exercise 12 of Homework 2.2, or some such activity, most of them will never understand how the definition of the explanatory variable impacts estimations, predictions, intercepts, slope, and so on.

OBJECTIVE 4 COMMENTS

Problem 5 helps students learn the importance of a scatterplot. And these figures also suggest that it is not a good idea to use a linear model for every situation.

OBJECTIVE 5 COMMENTS

Remind your students to include both coordinates of an intercept. They may need a reminder about which variable is represented by which coordinate.

OBJECTIVE 6 COMMENTS

The concept of model breakdown is important because most models do "break down." I tell my students that when model breakdown occurs, they must say so on quizzes and exams. Even if a student believes that no bank robberies will occur in the year represented by the t-intercept, the student should explain why that is reasonable (see Problem 8).

OBJECTIVE 7 COMMENTS

When I discuss the model breakdown in Problem 9, I ask my students for suggestions of graphs of (nonlinear) curves that might turn out to be better models. I do the same in Problem 11. In general, such questions can generate some fun and interesting discussions.

Textbook and Workbook Exploration

If you do not plan to assign most (or all) of the explorations in the textbook, it would probably be good to omit assigning the 20-minute "Identifying types of modeling errors" exploration or to assign it for homework. Many of these ideas can be touched on as you proceed through Chapter 2.

SECTION 2.2 LECTURE NOTES

Objectives

1. Find an equation of a linear model by using data described in words.
2. Find an equation of a linear model by using data displayed in a table.

Main point: Find an equation of a linear model.

OBJECTIVE 1

Recall from Section 1.6 that a linear function can be put in the form $y = mx + b$.

1. About 23% of 30-year-old Americans use wearable devices and about 11% of 60-year-old Americans use wearable devices (Source: *eMarketer*). Let p be the percentage of Americans at age a years who use wearable devices. The variables a and p are approximately linearly related. Find an equation of a linear model to describe the data.

2. Faculty Regalia® sells 16 graduation sets (cap, gown, and tassel) for \$31 per set and sells 75 sets for \$24 per set (Source: *Faculty Regalia*). Let p be the price (in dollars) per set for purchasing n sets. The variables n and p are approximately linearly related. Find an equation of a linear model to describe the data.

OBJECTIVE 2

3. The total U.S. consumptions (in billions of pounds) of avocados are shown in the following table for various years.

Table 2.4: Total U.S. Consumption of Avocados

Year	Consumption (billions of pounds)
2009	1.15
2010	1.32
2011	1.24
2012	1.59
2013	1.79
2014	1.97
2015	2.28
2016	2.39

Source: *USDA*

Let A be the annual total U.S. consumption (in billions of pounds) of avocados at t years since 2000. Find an equation of a linear model to describe the data. Use a graphing calculator to verify your equation.

Finding an Equation of a Linear Model

To find an equation of a linear model, given some data:

1. Construct a scatterplot of the data.
2. Determine whether there is a line that comes close to the data points. If so, choose two points (not necessarily data points) that you can use to find an equation of a linear model.
3. Find an equation of the line you identified in step 2.
4. Use a graphing calculator to verify the graph of your equation comes close to the points of the scatterplot.

4. Annual U.S. new-car and light-truck sales are shown in the following table for various years.

Table 2.5: U.S. New-Car and Light-Truck Sales

Year	Sales (millions)
2009	10.5
2010	11.6
2011	12.8
2012	14.7
2013	15.6
2014	16.7
2015	17.5

Source: *Automotive News*

Let n be the U.S. annual new-car and light-truck sales (in millions) at t years since 2000. Find an equation of a linear model to describe the data. Use a graphing calculator to verify your equation.

SHORT HW 1, 7, 9, 15, 17, 21, 23, 25, 28

MEDIUM HW 1, 7, 5, 9, 11, 15, 17, 18, 19, 21, 23, 25, 28

SECTION 2.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students learn to use technology to draw a scatterplot of data in this section. Then they use two "good points" to find an equation of a linear model. In Section 2.3, they will use these models to make estimates and predictions. These skills will be used in Sections 2.3 and 2.4 and in Chapter 3.

OBJECTIVE 1 COMMENTS

For Problems 1 and 2, I remind my students once more of the roles of the explanatory variable and the response variable. It is helpful to organize the information in a table such as Table 23 on page 73 of the textbook. I write the equation $y = mx + b$ and then replace the variables x and y with the appropriate variables. Some of my students become stressed because we are no longer working with the more familiar variables x and y. I reassure them that they will get used to using other variable names with practice. I also point out that using the first letter of the names of quantities will make it easier to know which variable to make substitutions for when making estimates and predictions.

I tell my students that when modeling, we round the values of m and b to the second decimal place, unless values are close to 0, such as $m = 0.003$.

Finally, I use a graphing calculator to verify an equation that we have found. The most efficient way to do this is to use a graphing calculator table in “Ask” mode. However, it is important that students can visualize what they have accomplished, so it’s probably better to use TRACE. If you want to avoid discussing window settings, you can enter the two data points in the STAT editor and then use ZoomStat.

OBJECTIVE 2 COMMENTS

I begin solving Problem 3 by showing my students how to use their graphing calculators (or other technology) to draw a scatterplot of the avocado data. It is from this point on that students will have graphing-calculator invalid-dimension errors crop up. I describe the meaning of the message “ERR:INVALID DIM” to my students so that they know what to do when the message is displayed (see the third bulleted comment in Section B.26 of the textbook). When my students accidentally delete a list in the STAT editor, I explain how to use “SetUpEditor” to get back the list. (See the first margin note in Section B.8 of the textbook.)

I then number the data points (or use letters) and ask my students which pairs of data points lie on a line that comes close to the other data points and which pairs do not. Many students tend to skip constructing a scatterplot and blindly pick two points, often the first two points or the first and last points. I advise quickly constructing scatterplots to illustrate how both of these practices will not always generate reasonable models.

Once the class has determined two “good points” from the avocado scatterplot, I derive an equation of the line that contains the two points. I emphasize that it’s okay to use decimals; many students think they need to use fractions because they were required to do so when finding equations of (non-model) lines in Section 1.5. I explain that it’s okay to round when finding a model because there are already various types of errors built into the process, anyway (see the exploration "Identifying types of modeling errors" on page 68).

When verifying a model’s equation, I explain that the line should contain the two chosen points (and come close to the other data points). I encourage my students to take the time to distinguish between their chosen two points and the other data points. (By the way, most students do not realize that a point that lies close to, but not on a line, may appear to lie *on* the line on a graphing calculator screen.) For quizzes and exams, I tell my students to copy their graphing calculator screen which displays how well their model fits the data; if I don’t require this (by making it worth points), some students will resist ever learning how to use a graphing calculator to draw scatterplots and to verify their models.

Students could use two nondata points to find a linear model, but I don’t mention this option for four reasons. First, because using data points works just fine. Second, it’s easier to use coordinates of data points than to estimate coordinates of nondata points. Third, skipping this option saves me a bit of class time. Fourth, by not discussing how to use nondata points, I reduce the amount of time I spend grading exams because there is less variation in my students’ derived equations for models.

Although the skill of using a graphing calculator to obtain a linear regression equation is described in Section 2.2 of the textbook, discussion of this topic can be postponed until a later point or it can be avoided entirely. Skipping this topic would mean that you would have to be careful to avoid assigning a small number of exercises throughout the textbook that ask students to use regression. The only such exercise in Chapter 2 is Exercise 20 of Homework 2.2.

I have included regression equations as answers to exercises simply because I can’t anticipate which two “good” points students will choose. So, when assigning this section’s homework, I warn my students that their answers will vary from those in the textbook. I remind them that they can verify their work by using a graphing calculator.

I require my students to use pencil and paper to derive the equations of all types of models in the course. Such skills are foundational to algebra, and students need to do lots of symbolic work in order to prepare for subsequent courses. However, I find regression to be a useful student tool for the four “Topic of Your Choice” labs (see pages 115–116, 249-250, 318, and 482—483) in which students may need to find equations of many models while searching through data sets.

Workbook Exploration

The following 15-minute exploration about electric vehicles could be assigned at the start of class. After all, students have already learned how to find an equation of a line using two points in Section 1.5. Most, if not all, groups will forgo constructing a scatterplot and simply choose two data pairs in the table to use to find an equation. This creates the opportunity to point out the danger in doing so during the debrief. In fact, if a group uses the data pairs for 2016 and 2017, you can point to how poorly the model fits all the data points. The textbook exploration "Choosing 'good points' to find a model" also attends to this.

Most, if not all, groups will ignore that t is defined to be years since 2010 and will in effect, define t to be the year. During the debrief, comparing and contrasting the equations and graphs of the models for the two definitions of t can be very instructive.

Group Exploration

Section Opener: Finding an equation of a linear model

The numbers of electric vehicle models available to consumers in North America are shown in the following table for various years. Let n be the number of electric vehicle models available to consumers in North America at t year since 2010. Find an equation of a linear model.

Year	Number of Electric Vehicle Models in North America
2011	10
2012	17
2013	24
2014	29
2015	39
2016	44
2017	54

Source: *Bloomberg New Energy Finance*

Workbook Exploration

The following 20-minute exploration about airline passengers provides more structure than the one about electric vehicles, so it's probably less valuable. But it does outline the desired approach. So, having students work on it directly after the electric-vehicle exploration can work well.

Group Exploration

Section Opener: Finding an equation of a linear model

The number of airline passengers worldwide are shown in the following table for various years.

Year	Number of Airline Passengers Worldwide (millions)
2009	2.5
2010	2.7
2011	2.9
2012	3.0
2013	3.2
2014	3.3
2015	3.6
2016	3.8

Source: *ICAO; IATA*

Let n be the number (in millions) of airline passengers worldwide in the year that is t years since 2000.

1. Use a graphing calculator to construct a scatterplot of the data. Copy the screen.
2. Imagine a line that comes close to the data points. Estimate the coordinates of two points that lie on the line. Use the coordinates to find the slope of the line.
3. Substitute the slope you found in Problem 2 for m in the equation $n = mt + b$.
4. Substitute the coordinates of one of the points you identified in Problem 2 into the equation you found in Problem 3 and solve for b.
5. Substitute the value of b you found in Problem 4 into the equation you found in Problem 3. We call such an equation a *linear model*.
6. Predict the number of airline passengers worldwide in 2018 by substituting 18 for t in your linear model.
7. International Air Transport Association predicts that there will be 4.3 million airline passengers worldwide in 2018. Is this prediction greater than, equal to, or less than the prediction you made in Problem 6?

Textbook and Workbook Exploration

When finding a linear model, groups can discover the impact of their selection of a pair of points by completing the 15-minute "Choosing 'good points' to find a model" exploration. It is best, however, to first demonstrate how to derive an equation of a linear model before assigning this exploration, so students can focus on the main point of this exploration, which is the selection of a pair of points, not finding the equation.

You may want your students to skip Problem 7 in the exploration to save class time and/or to save time when you grade exams (as I explained before).

Workbook Exploration

The following 15-minute exploration is a nice way to tie the concepts in this section to the graphical significance of slope and vertical intercept discussed in Chapter 1.

Group Exploration

Adjusting the fit of a model

The winning times for the men's Olympic 100-meter freestyle swimming event are shown in the following table for various years.

Year	Swimmer	Country	Winning Time (seconds)
1980	Jörg Woithe	E. Germany	50.40
1984	Rowdy Gaines	USA	49.80
1988	Matt Biondi	USA	48.63
1992	Aleksandr Popov	Unified Team	49.02
1996	Aleksandr Popov	Russia	48.74
2000	Pieter van den Hoogenband	Netherlands	48.30
2004	Pieter van den Hoogenband	Netherlands	48.17
2008	Alain Bernard	France	47.21
2012	Nathan Adrian	USA	47.52
2016	Kyle Chalmers	Australia	47.58

Source: *The New York Times Almanac*

Let w be the winning time (in seconds) at t years since 1980.

1. Use a graphing calculator to draw a scatterplot of the data. Copy the screen. Do the variables t and w appear to be approximately linearly related?

2. The linear model $w = -0.0325t + 48.95$ can be found by using the data points $(20,\ 48.30)$ and $(24,\ 48.17)$. Draw the line and the scatterplot in the same viewing window. Copy the screen. Check that the line contains these two points.

3. The model $w = -0.0325t + 48.95$ does not fit all the data points very well. Adjust the equation by increasing or decreasing the slope -0.0325 and/or the constant term 48.95 so that your new model will fit the data better. Keep adjusting the model until it fits the data points reasonably well.

4. Use your improved model to predict the winning time in the 2020 Olympics.

SECTION 2.3 LECTURE NOTES

Objectives

1. Use *function notation*.
2. Find inputs and outputs of a function.
3. Use function notation to make estimates and predictions.
4. Find intercepts of a model.
5. Use data described in words to make estimates and predictions.
6. Find the *domain* and *range* of a model.

Main point: Use function notation and make estimates and predictions about authentic situations.

OBJECTIVE 1

To use "f" as the name of the function $y = 3x - 5$, we use "$f(x)$" to represent y: $y = f(x)$. We can substitute $f(x)$ for y in the equation $y = 3x - 5$: $f(x) = 3x - 5$. Warning: "$f(x)$" does not mean f times x.

OBJECTIVE 2

Evaluate $f(x) = 3x - 5$ at the given values of x.

1. $f(4)$ **2.** $f\left(-\frac{1}{3}\right)$ **3.** $f(a-3)$ **4.** $f(a+k)$

5. Evaluate $g(x) = 3x^2 - 4x$ at $x = -2$.

Let $h(x) = -2x + 1$.

6. Find $h(4)$. **7.** Find x when $h(x) = 4$.

Let $f(x) = \frac{3}{5}x + 2$.

8. Find $f(10)$. **9.** Find x when $f(x) = 10$.

Some input-output pairs of a function f are shown in the table.

x	y
1	4
2	3
3	2
4	3
5	4

10. Find $f(3)$.

11. Find x when $f(x) = 3$.

12. Find the intercepts of the graph of $f(x) = \frac{2}{3}x - 7$.

OBJECTIVES 3 and 4

Definition *Function notation*

The response variable of a function f can be represented by the expression formed by writing the explanatory variable name within the parentheses of $f(\ )$:

$$\text{Response variable} = f(\text{explanatory variable})$$

13. In Section 2.2, we found an equation close to $A = 0.19t - 0.65$, where A is the total U.S. consumption (in billions of pounds) of avocados at t years since 2000. [Use the non-regression equation that you found in class.]

a. Rewrite the equation $A = 0.19t - 0.65$ with the function name f.

b. Find $f(22)$. What does it mean in this situation?

c. Find t when $f(t) = 3$. What does it mean in this situation?

d. Find the t-intercept. What does it mean in this situation?

e. Find the A-intercept. What does it mean in this situation?

Using an Equation of a Linear Model to Make Predictions

- To make a prediction about the response variable of a linear model, substitute a chosen value for the explanatory variable in the model's equation. Then solve for the response variable.
- To make a prediction about the explanatory variable of a linear model, substitute a chosen value for the response variable in the model's equation. Then solve for the response variable.

Four-Step Modeling Process

To find a linear model and make estimates and predictions,

1. Construct a scatterplot of the data to determine whether there is a nonvertical line that comes close to the data points. If so, choose two points (not necessarily data points) that you can use to find the equation of a linear model.
2. Find an equation of your model.
3. Verify your equation by checking that the graph of your model contains the two chosen points and comes close to all the data points.
4. Use the equation of your model to make estimates, make predictions, and draw conclusions.

Using an Equation of a Linear Model to Find Intercepts

If a function of the form $p = mt + b$, where $m \neq 0$, is used to model a situation, then

- The p-intercept is $(0, b)$.
- To find the t-coordinate of the t-intercept, substitute 0 for p in the model's equation and solve for t.

OBJECTIVE 5

14. U.S. sales of refrigerated ready-to-drink coffee increased approximately linearly from \$116 million in 2012 to \$249 million in 2016 (Source: *IRI*). Predict when the annual sales will reach \$400 million.

OBJECTIVE 6

Recall from Section 1.6 that the domain of a function is the set of all inputs and the range is the set of all outputs. For the **domain** and **range** of a model, we consider input-output pairs only when both the input and the output make sense in the situation.

15. An airplane at an altitude of 36 thousand feet begins to descend steadily. Ten minutes later, the altitude of the airplane is 12 thousand feet. Let $g(t)$ be the altitude (in thousands of feet) of the airplane after t minutes of descending.

a. Find an equation of g.

b. Graph g by hand.

c. What are the domain and range of g?

SHORT HW 1, 9, 23, 29, 39, 41, 43, 49, 57, 69, 73, 77, 81, 89, 95, 97, 101

MEDIUM HW 1, 9, 13, 23, 27, 29, 37, 39, 41, 43, 49, 57, 58, 69, 73, 77, 81, 85, 89, 93, 95, 97, 99, 101, 105

SECTION 2.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn about functional notation. They will use this notation both with models and functions that are not models throughout the rest of the textbook.

OBJECTIVE 1 COMMENTS

Many students wonder why we would want to represent "y" by the more complicated notation "$f(x)$." I emphasize that functional notation allows us to conveniently name a function. I explain that being able to name a function is especially helpful when working with two models such as the salary models described near the top of page 87 in the textbook.

Many of my students are reassured when I tell them that $f(x)$ simply stands for y. Later (when discussing Objective 3) I say that, in general, response variable $= f$(explanatory variable).

OBJECTIVE 2 COMMENTS

I spend a good chunk of time discussing how to find values of x or $f(x)$ where a function f is described by a table, a graph, or an equation. I also tie in function notation with function-machine figures such as the one in Fig. 30 on page 82 of the textbook.

Some students have difficulty with Problems 3 and 4. I compare the work for these problems with the work for Problems 1 and 2. I also emphasize that when substituting an expression for x, parentheses must be used.

Many students think that the notation $f(4)$ means that y is 4 or that the equation $f(x) = 4$ means that x is 4. Comparing the work for Problems 6 and 7, 8 and 9, and 13b and 13c should help address these issues.

Workbook Exploration

The following 15-minute exploration is a great way for students to think about the numerical aspect of a function. Symbolic work with functions can distract students from the main point, which has to do with inputs and outputs. Problem 2 has students work with two functions, which is good preparation for linear systems in Sections 3.1–3.4, linear inequalities in Sections 3.5 and 3.6, and algebra of functions in Sections 6.1, 6.2, and 8.1.

EXPLORATION *Using a table to evaluate a function*

1. Input–output pairs of a function f are shown in the following table.

x	$f(x)$
0	5
1	2
2	4
3	2
4	3
5	0

a. Find $f(4)$.

b. Find $f(0)$.

c. Find x when $f(x) = 4$.

d. Find x when $f(x) = 2$.

2. All Input–output pairs of functions f and g are shown in the following table.

x	$f(x)$	$g(x)$
0	3	1
1	0	4
2	4	3
3	5	5
4	2	0
5	1	2

a. Find $f(1)$.

b. Find $g(2)$.

c. Find x when $f(x) = 2$.

d. Find x when $g(x) = 1$.

e. Which is larger, $f(2)$ or $g(2)$? Explain.

f. Which is larger, $f(5)$ or $g(5)$? Explain.

g. Find $f(0) - g(0)$.

h. Find $f(1) - g(1)$.

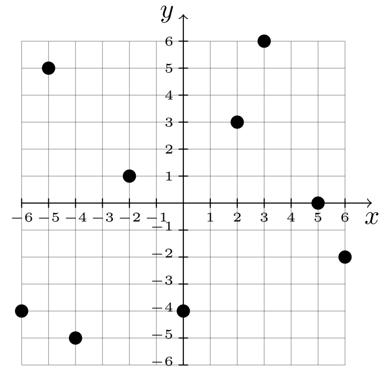
i. Find x when $f(x) = g(x)$.

Workbook Exploration

The following 30-minute exploration has students consider the graphical aspect of a function, which is what they tend to be weakest in doing. Problem 3 has students work with two functions, which is good preparation for linear systems in Sections 3.1–3.4, linear inequalities in Sections 3.5 and 3.6, and algebra of functions in Sections 6.1, 6.2, and 8.1.

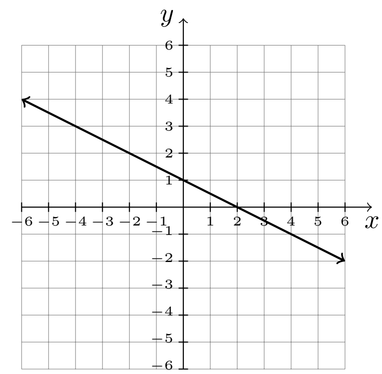
EXPLORATION *Using a graph to evaluate a function*

1. The graph of a function f is displayed below.



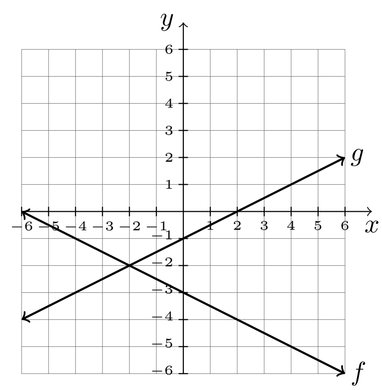
a. Find $f(-2)$. **b.** Find $f(5)$. **c.** Find $f(0)$.
d. Find x when $f(x) = 6$. **e.** Find x when $f(x) = -4$. **f.** Find x when $f(x) = 0$

2. The graph of a function f is displayed below.



a. Find $f(-4)$. **b.** Find $f(0)$. **c.** Estimate $f(-4.7)$.
d. Find x when $f(x) = 3$. **e.** Find x when $f(x) = 0$. **f.** Estimate x when $f(x) = -1.2$

3. The graphs of functions f and g are displayed below.



a. Find $f(-4)$.
b. Find $g(2)$.
c. Find $f(0)$.
d. Find x when $f(x) = -4$.
e. Find x when $g(x) = -1$.
f. Find x when $g(x) = 0$.
g. Which is larger, $f(6)$ or $g(6)$? Explain.
h. Which is larger, $f(-5)$ or $g(-5)$? Explain.
i. Find $f(4) - g(4)$.
j. Find $f(-4) - g(-4)$.
k. Find x when $f(x) = g(x)$.

Workbook Exploration

By completing the following 30-minute exploration, students can see one of the benefits of function notation: naming function is a convenient way to refer to them. The exploration nicely motivates linear systems in Sections 3.1–3.4.

Group Exploration

Comparing linear models

Average U.S. per-person consumptions of chicken and red meat are described in the following table for various years.

	Average Consumption (pounds per person)	
Year	Chicken	Red Meat
1970	40.3	145.8
1980	48.0	136.8
1990	61.5	120.0
2000	78.0	120.7
2010	83.7	108.7
2015	90.0	104.8

Source: *U.S. Department of Agriculture*

Let $C(t)$ be the average annual per-person consumption of chicken and $R(t)$ be the average annual per-person consumption of red meat, both in pounds per person, in the year that is t years since 1970.

1. Find equations of C and R. Use a graphing calculator to verify that your models fit the data points well. Copy the screen.
2. Compare the values of $C(0)$ and $R(0)$. What does your comparison mean in this situation?
3. Compare the slopes of the two models. What does your comparison mean in this situation?
4. The results you found in Problems 2 and 3 should suggest an event that will happen in the future. Describe that event.
5. Use "intersect" on a graphing calculator to find the point where the graphs of the models intersect. Copy the screen. In terms of consumption, what does it mean that the models intersect at this point? State the year and consumption for this event.
6. For the period 1970–2015, has the *total* average consumption of chicken and red meat generally increased, decreased, or neither? Use calculations and words to explain.

Workbook Exploration

The following 25-minute exploration provides a nice summary of Sections 2.1–2.3.

Group Exploration

Using a linear model to make predictions

The percentages of workers enrolled in high-deductible health-insurance plans are shown in the following table for various years.

Year	Percentage of Workers Enrolled in a High-Deductible Health-Insurance Plan
2006	4
2008	8
2010	13
2012	19
2014	20
2016	29

Source: *Kaiser Family Foundation*

Let $f(t)$ be the percentage of workers enrolled in high-deductible health-insurance plans at t years since 2000.

1. Find an equation of f. Use a graphing calculator to verify that your model fits the data points well. Copy the screen.
2. Find $f(20)$. What does it mean in this situation?
3. Find t when $f(t) = 42$. What does it mean in this situation?
4. What is the slope of the graph of f? What does it mean in this situation?
5. Find the t-intercept. What does it mean in this situation?
6. For what years is there model breakdown for certain?

Textbook and Workbook Exploration

The 10-minute "Formula for slope" exploration is good to assign if a sizable number of your students will eventually take calculus.

OBJECTIVES 3 and 4 COMMENTS

Inform your students to what extent they can use regression for homework exercises, as most problems do not specify how students should find a model's equation. I usually do not tell my students about regression until I know that they are proficient at finding equations for linear, exponential, and quadratic models because I believe these skills are important ones for students to learn. A good time to introduce regression is when you assign projects. If you do choose to introduce regression now, you can prevent your students from becoming too dependent on this tool by requiring them to show their symbolic work when they find equations of models on exams.

In Problem 13, compare part (b) with part (c). For parts (d) and (e), I remind my students that we begin to find an intercept by substituting 0 for the *other* variable into the model's equation.

Workbook Exploration

In the following 30-minute exploration, students perform the four-step modeling process. In Problem 1 of the exploration, students are to find an equation of a linear model by choosing two points. Many students need to revisit this skill (from Section 2.2) before they become proficient at it. Problems 2 and 3 have students perform symbolic work as well as help them get used to using function notation. After the exploration, I compare the work for Problem 2 with the work for Problem 3.

Group Exploration

Section Opener: Using a linear model to make estimates

The percentages of Americans who currently have a personal profile page on a social networking website such as Facebook are shown in the following table for various age groups.

Age Group (years)	Age Used to Represent Age Group (years)	Percent
18–24	21.0	86
25–34	29.5	80
35–44	39.5	73
45–54	49.5	58
55–64	59.5	43
over 64	70.0	29

Source: *Edison Research and Arbitron*

Let p be the percentage of Americans at age a years who have a personal profile page.

1. Construct a scatterplot of the data.
2. Draw a *straight* line on your scatterplot that comes close to all the data points.
3. Use your line to estimate at what age 40% of Americans have a profile page.
4. Use your line to estimate the percentage of 40-year-old Americans who have a profile page.
5. Find the p-intercept. What does it mean in this situation? Is your result reasonable?
6. Find the t-intercept. What does it mean in this situation? Is your result reasonable?
7. For what ages do you trust your line to make reasonable estimates? Explain.
8. For what ages do you definitely *not* trust your line to make reasonable estimates? Explain.
9. For what ages do you maybe trust your line to make reasonable estimates? Explain.
10. On the scatterplot you constructed, modify your model so it might make sense for more ages than your linear model. [**Hint:** Your new model should *not* be a line.]

OBJECTIVE 5 COMMENTS

I tell my students that for exercises similar to Problem 14, it is left to them to define the variables.

OBJECTIVE 6 COMMENTS

On page 89 of the textbook, there is a discussion about the domain and range of a linear model. For Example 9 on that page, I point out that both the domain and range of the function $f(t) = 10t$ is the set of (all) real numbers if f is not a model. Such comparisons help students learn about the domain and range of functions not being used as models as well as the domain and range of models.

SECTION 2.4 LECTURE NOTES

Objectives

1. Calculate the *rate of change* of a quantity.
2. Explain why slope is a rate of change.
3. Use rate of change to help find an equation of a linear model.
4. Perform a *unit analysis* of a linear model.
5. Describe the meaning of slope when two variables are approximately linearly related.

Main point: Understand why slope is a rate of change.

OBJECTIVE 1

A ratio of a to b is $\frac{a}{b}$. A **unit ratio** is a ratio written as $\frac{a}{b}$ with $b = 1$.

1. If the temperature increased by 6°F over 3 hours, estimate how much the temperature increased per hour.
2. If temperature increased from 60°F at 7 A.M. to 72°F at 11 A.M., estimate how much the temperature increased per hour.

Formula for Rate of Change

Suppose a quantity y changes steadily from y_1 to y_2 as a quantity x changes steadily from x_1 to x_2. Then the **rate of change** of y with respect to x is the ratio of the change in y to the change in x:

$$\frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

3. The per-person consumption of orange juice in the United States declined approximately steadily from 4.0 gallons in 2009 to 2.6 gallons in 2016 (Source: *Florida Citrus Outlook 2016–2017 Season*). Find the approximate rate of change of per-person consumption of orange juice.
4. For the 48 contiguous states and the District of Columbia, the federal poverty level of a household with 3 people is $20,780 and the Federal poverty level of a household with 5 people is $29,420 (Source: *Federal Poverty Income Guidelines*). Find the approximate rate of change of the poverty level of a household with respect to the number of people.

Signs of Rate of Change

Suppose a quantity t affects (explains) a quantity p:

- If p increases steadily as t increases steadily, then the rate of change of p with respect to t is positive.
- If p decreases steadily as t increases steadily, then the rate of change of p with respect to t is negative.

OBJECTIVE 2

We use the formula $\dfrac{y_2 - y_1}{x_2 - x_1}$ to find rate of change and also to find the slope of a line. So, slope is a rate of change.

5. Let d be the distance (in miles) that a student can drive in t hours (see the table).

t (hours)	d (miles)
0	0
1	50
2	100
3	150
4	200
5	250

a. Construct a scatterplot by hand. Then draw a linear model.

b. Find the slope of the linear model.

c. Find the rate of change of distance traveled.

Slope Is a Rate of Change

If there is a linear relationship between quantities t and p, and if t affects (explains) p, then the slope of the linear model is equal to the rate of change of p with respect to t.

Constant Rate of Change

Suppose a quantity t affects (explains) a quantity p:

- If there is a linear relationship between t and p, then the rate of change of p with respect to t is constant.
- If the rate of change of p with respect to t is constant, then there is a linear relationship between t and p.

OBJECTIVE 3

6. The revenue of Smith & Wesson was \$552 million in 2015 and has increased by \$58.6 million per year (Source: *Smith & Wesson*). Let $r = f(t)$ be the annual revenue (in millions of dollars) at t years since 2015.

a. What is the slope of the graph of f? What does it mean in this situation?

b. What is the r-intercept of the graph of f? What does it mean in this situation?

c. Find an equation of f.

d. Predict the revenue in 2022.

7. The U.S. property crime rates was \$2501 in 2015 and has decreased by about \$107 per year (Source: *FBI*). Predict when the U.S. property crime rate will be \$1850.

OBJECTIVE 4

Definition *Unit analysis*

We perform a **unit analysis** of a model's equation by determining the units of the expressions on both sides of the equation. The simplified units of the expressions on both sides of the equation should be the same.

8. In Problem 2 of the lecture notes of Section 2.2, we found the model $p = -0.12n + 32.90$, where p is the price (in dollars) per graduation set when purchasing n sets.

a. What is the slope? What does it mean in this situation?

b. What is the p-intercept? What does it mean in this situation?

c. Perform a unit analysis of the equation $p = -0.12n + 32.90$.

9. The number of birds that collided with aircraft in the United States was 12.98 thousand birds in 2015 and has increased by about 0.84 thousand birds per year (Source: *Federal Aviation Administration*). Let $f(t)$ be the annual number (in thousands) of birds that collided with aircraft in the United States at t years since 2015.

a. Find an equation of f.

b. Perform a unit analysis of your equation of f.

c. Predict in which year 19 thousand birds will collide with aircraft in the United States.

OBJECTIVE 5

10. The percentages of Fortune 1000 board seats filled by women are shown in the following table for various years.

Table 2.6: Percentages of Fortune 1000 Board Seats Filled by Women

Year	Percent
2011	14.6
2012	15.6
2013	16.6
2014	17.7
2015	18.8
2016	19.7

Source: *2020 Women on Boards*

Let $f(t)$ be the percentage of Fortune 1000 board seats that are filled by women at t years since 2010.

a. Find an equation of f.

b. What is the slope of the model? What does it mean in this situation?

c. Find the rates of change of the percentage of seats filled by women from one year to the next. Compare the rates of change with the result you found in part (b).

d. Find the average of the rates of change you found in part (c) by dividing the sum of the five rates of change by the number 5. How does this average compare with the slope of your model?

e. 2020 Women on Boards has set a goal that women will fill 20% of Fortune 1000 board seats by the year 2020. Does the model predict that the goal will be reached?

SHORT HW 1, 5, 7, 11, 17, 19, 23, 29, 31, 33, 43, 47, 51, 53, 57

MEDIUM HW 1, 5, 7, 9, 11, 17, 19, 23, 27, 29, 31, 33, 35, 37, 43, 47, 51, 53, 55, 57

SECTION 2.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn that slope is a rate of change. They will use this concept in many parts of the textbook including Sections 3.1, 3.3, 3.5, 4.4, 10.1, and 10.3. In some cases, students can use this concept to quickly find an equation of a linear model. By identifying the slope of a linear model, students will be able to determine the rate of change of one quantity with respect to another quantity.

OBJECTIVE 1 COMMENTS

Problems 1 and 2 are good primers for the formula of rate of change. When doing Problems 1–4, I emphasize we should include units when computing changes in the response variable and changes in the explanatory variable. Including units throughout the calculations will help students find the correct units for their result.

I emphasize that a rate of change is "something per something," such as miles per hour.

Workbook Exploration

You could start class by having students work on the following 15-minute exploration. This is a great way to motivate the concept rate of change. Some groups may use a numerical approach and other groups may use a graphical approach. Some groups will likely compute changes in the response variable but not take into account changes in the explanatory variable; in fact, I selected the data set so that this approach will yield the wrong conclusion. During the debrief, comparing and contrasting groups' results should generate a productive discussion.

Group Exploration

Section Opener: Rate of change

For which type of car, domestic or imported, has fuel efficiency improved the fastest? Explain.

Year	Domestic (miles per gallon)	Year	Imported (miles per gallon)
2010	33.1	2007	32.2
2011	32.7	2009	33.8
2012	34.8	2011	33.7
2013	36.0	2013	36.6
2014	36.7	2014	36.0

Source: *U.S. Department of Transportation*

OBJECTIVE 2 COMMENTS

I point to the difference quotients used to find the results for Problems 2–4 and ask students whether the difference quotients remind them of something. Usually at least one student makes the connection, but if no one is reminded of slope, I refer to the boxed formula

$$\frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

and conclude that slope is a rate of change. This concept can be reinforced by doing Problem 5.

Textbook and Workbook Exploration

The 20-minute "Significance of the slope and the response variable's intercept of a model" exploration is an excellent way for students to discover that for a linear function, slope is a rate of change. Students will likely gain a deeper understanding of this concept by completing the exploration than by watching a lecture (which is probably true of most of the explorations, but especially this one.) This concept might be the most important one of the course for those students who will take calculus.

Students have an easy time with this exploration. To save time, I usually assign only Problems 1 and 2. Once most groups are done, I emphasize that slope is a rate of change. I remind my students of the graphical and numerical interpretations of slope we've discussed so far.

Workbook Exploration

In the following 20-minute exploration, students explore the connection between rate of change and computing the slope of a linear model where the change in the explanatory variable is different than 1. Many groups have difficulty with Problem 3 in the exploration. They often are unsure what "ratio" means. Once students are over this hurdle, the rest of the exploration flows nicely.

Group Exploration

Section Opener: Slope is a ratio

Here you will work with the women's 400-meter run model $r = -0.27t + 70.45$, where r is the record time (in seconds) at t years since 1900.

1. What is the slope of the graph of $r = -0.27t + 70.45$? What does the slope mean in this situation? [**Hint:** Consider the slope addition property.]
2. According to the model, by how much should the record time change each year?
3. By how much should the record time change in two years? What is the ratio of the change in the record time to the change in calendar time?
4. By how much does the 400-meter record change in three years? What is the ratio of the change in the record time to the change in calendar time?
5. For any period, what is the ratio of the change in the record time to the corresponding change in calendar time? Explain.

OBJECTIVE 3 COMMENTS

A few students will be to apply concepts in this section and previous sections to determine the values of m and b of a linear model of the form $r = mt + b$. Most students will have much better success if they first sketch a linear model and refer to its slope and r-intercept to determine m and b.

Background Information for the Exploration That Follows

Another way that students can find an equation of a linear model is by recognizing a numerical pattern as suggested by the 15-minute exploration that follows. This approach displays a nice connection between algebra and arithmetic. Before assigning this exploration, I emphasize to my students that they should avoid performing arithmetic so that a pattern is more apparent.

EXPLORATION *Finding linear equations using pattern recognition*

A U-Haul rental company charges a flat daily fee of \$40 plus \$0.29 per mile for a 24-foot truck rental. Let c be the one-day cost (in dollars) of renting a 24-foot truck that is driven d miles.

1. Some values of d and the corresponding values of c are listed in the table that follows. For example, if $d = 2$, then two miles were driven, so the cost would be two times \$0.29 plus the flat fee of \$40. Thus, c would be $40 + 2(0.29)$. Notice that the arithmetic for the c values has not been carried out so that a pattern is more apparent. Complete the table.

Distance, d (miles)	Cost, c (dollars)
1	$40 + 1(0.29)$
2	$40 + 2(0.29)$
3	
4	
d	

2. Refer to the bottom row of your completed table to find an equation for d and c.

3. Use a graphing calculator to draw a scatterplot of the data in your completed table. Then draw a graph of your equation and the scatterplot in the same viewing window. Is your equation correct?

OBJECTIVE 4 COMMENTS

Performing a unit analysis of a model is a great way to verify an equation found by the method discussed in this section. Students often transpose the values of m and b when finding a linear model's equation, and a unit analysis of the faulty equation would reveal this type of error.

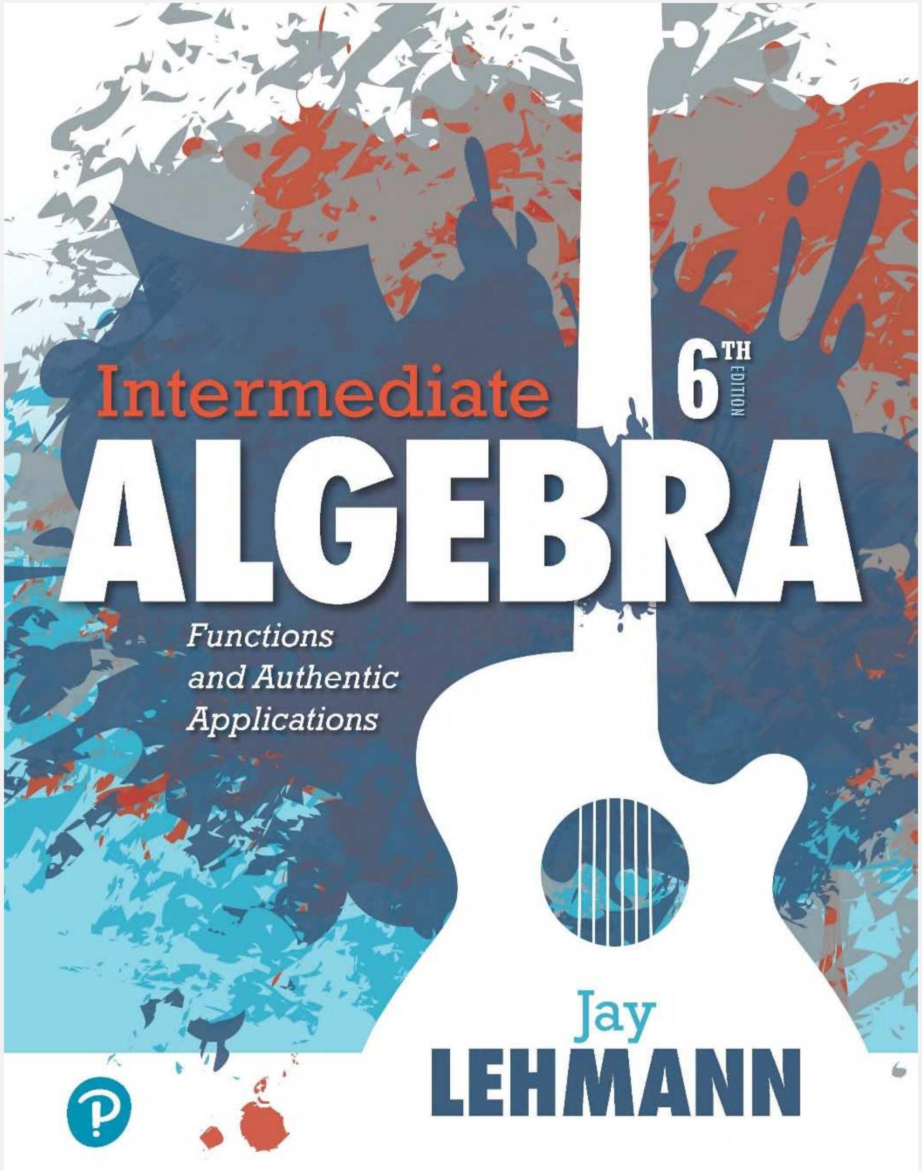
Problem 8 is useful because it's the first time students must determine a rate of change from an equation.

OBJECTIVE 5 COMMENTS

Part (a) of Problem 10 can remind students of the other method that yields an equation of a linear model. I compare and contrast the two methods and discuss when each one is useful. Part (c) reminds students that a model does not describe a situation exactly; students tend to struggle with what is fact and what is fiction.

Chapter 2

Modeling with
Linear Functions



2.2 Finding Equations of Linear Models

Example: Finding an Equation of a Linear Model (1 of 4)

The number of bird species involved in airline bird strikes has increased approximately linearly from 230 species in 2008 to 330 species in 2014 (Source: *Federal Aviation Administration*). Let n be the number of bird species involved in airline bird strikes in the year that is t years since 2000. Find an equation of a linear model.

Example: Finding an Equation of a Linear Model (2 of 4)

Values of t and n are shown in the table. Use the data points (8, 230) and (14, 330) to find the slope of the model.

Years since 2000 t	Number of Birds Species n
8	230
14	330

$$m = \frac{330 - 230}{14 - 8} = \frac{100}{6} \approx 16.67$$

Example: Finding an Equation of a Linear Model (3 of 4)

Substitute 16.67 for m in the equation $n = mt + b$:

$$n = 16.67t + b$$

Find the constant b by substituting the coordinates of the point (8, 230) into the equation $n = 16.67t + b$:

$$230 = 16.67(8) + b$$

$$230 = 133.36 + b$$

$$230 - 133.36 = 133.36 - 133.36$$

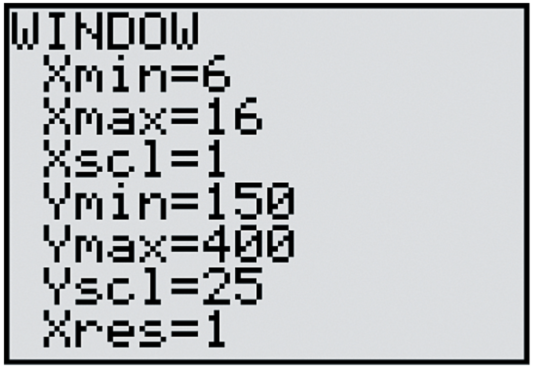
$$96.64 = b$$

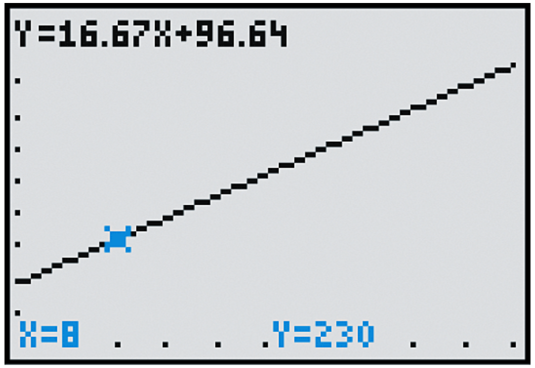
Example: Finding an Equation of a Linear Model (4 of 4)

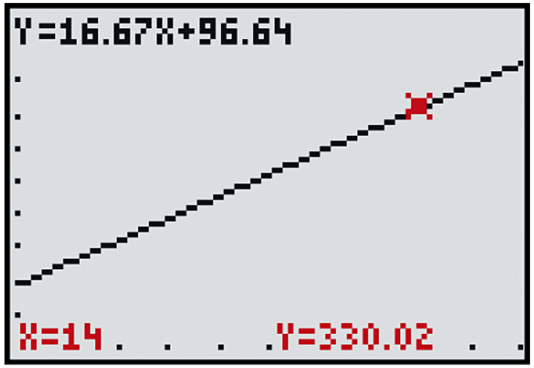
Substitute 96.64 for b in the equation $n = 16.67t + b$:

$$n = 16.67t + 96.64$$

Verify the equation using TRACE on a graphing calculator to check that the line approximately contains the points (8, 230) and (14, 330).







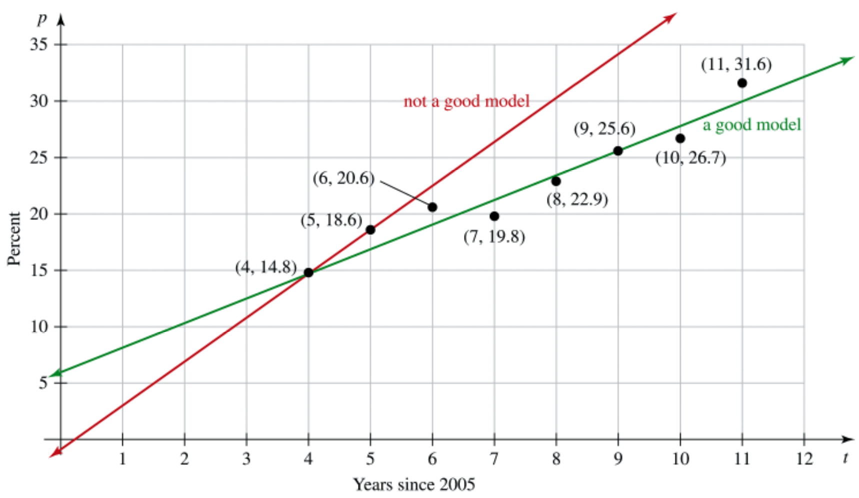
Example: Finding an Equation of Linear Models (1 of 6)

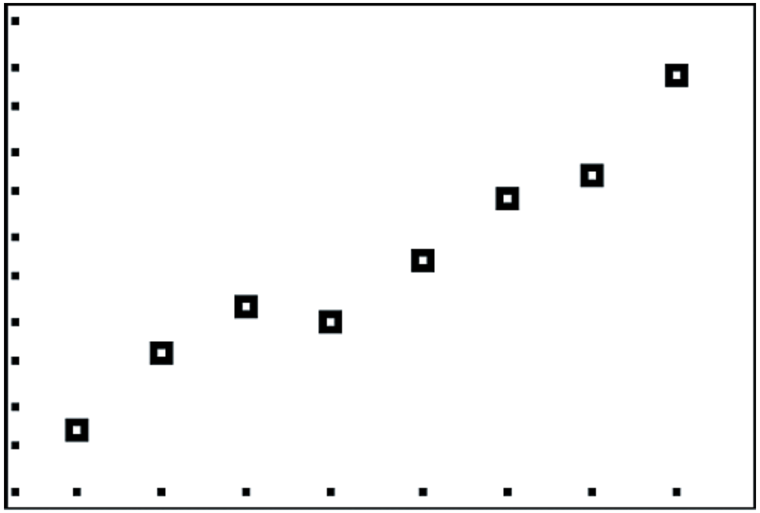
The percentages of new vehicle transactions that are leases are shown for various years. Let p be the percentage of new vehicle transactions that are leases at t years since 2005. Find an equation of a line that comes close to the points in the scatterplot of the data.

Year	**Percent**
2009	14.8
2010	18.6
2011	20.6
2012	19.8
2013	22.9
2014	25.6
2015	26.7
2016	31.6

Example: Finding an Equation of Linear Models (2 of 6)

View the scatterplot using a graphing calculator to save time and improve accuracy in plotting the points.





Example: Finding an Equation of Linear Models (3 of 6)

We want to find an equation of a line that comes close to the data points.

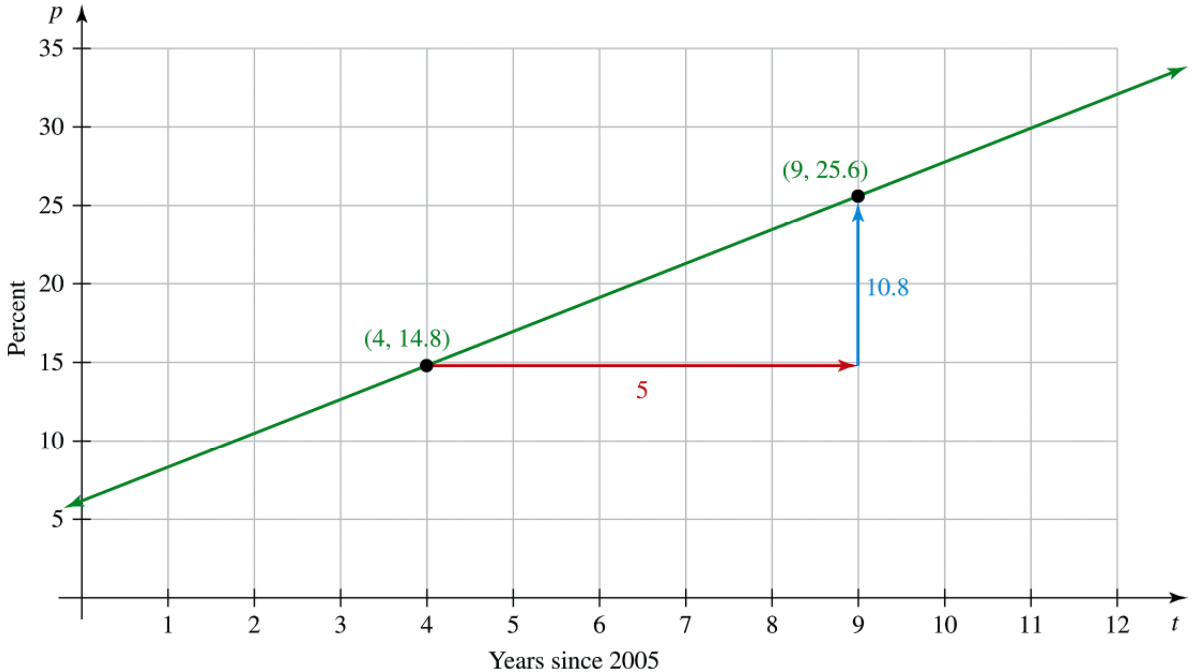
Recall that it is not necessary to use two *data* points to find an equation, although it is often convenient and satisfactory to do so.

The red line that contains the points (4, 14.8) and (5, 18.6) does *not* come close to the other data points. However, the green line that passes through the points (4, 14.8) and (9, 25.6) appears to come close to the rest of the points. We will find the equation of this line.

Example: Finding an Equation of Linear Models (4 of 6)

Use the points (4, 14.8) and (9, 25.6) to find m:

$$m = \frac{25.6 - 14.8}{9 - 4} = \frac{10.8}{5} = 21.6$$



Example: Finding an Equation of Linear Models (5 of 6)

Substitute 2.16 for m in the equation $p = mt + b$:

$$p = 2.16t + b$$

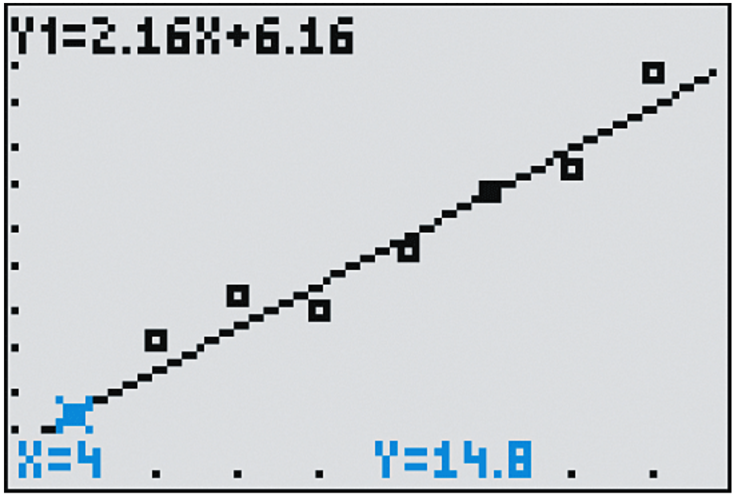
To find b, substitute (4, 14.8) into the equation:

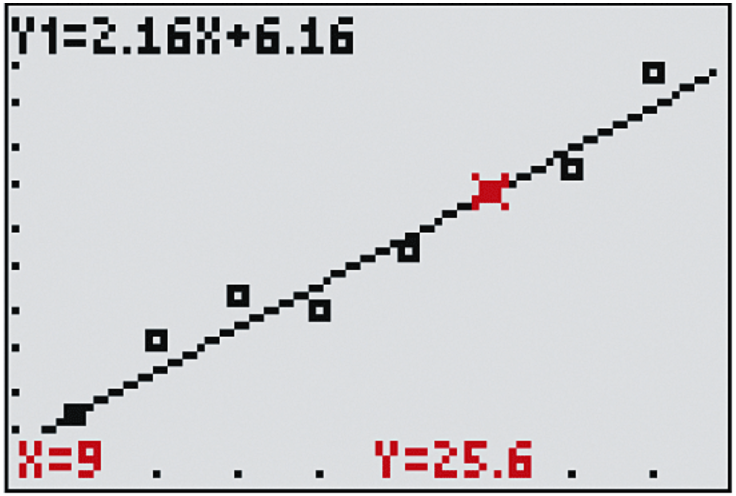
$$\begin{aligned} 14.8 &= 2.16(4) + b \\ 14.8 &= 8.64 + b \\ 14.8 - 8.64 &= 8.64 + b - 8.64 \\ 6.16 &= b \end{aligned}$$

Substitute 6.16 for b in the equation: $p = 2.16t + 6.16$

Example: Finding an Equation of Linear Models (6 of 6)

Check the correctness of our equation using a graphing calculator to verify that our line approximately contains the points (4, 14.8) and (9, 25.6).





Constructing a Scatterplot

Warning

It is a common error to skip creating a scatterplot. However there are many benefits:

1. We can determine whether the data are approximately linearly related.
2. If so, it can help us choose two good points with which to find a linear model.
3. By graphing, we can assess whether the data fits the model well.

Finding an Equation of a Linear Model

To find an equation of a linear model, given some data,

1. Construct a scatterplot of the data.

2. Determine whether there is a line that comes close to the data points. If so, choose two points (not necessarily data points) you can use to find the equation of a linear model.

3. Find an equation of the line you identified in step 2

4. Use a graphing calculator to verify the graph of your equation comes close to the points of the scatterplot.

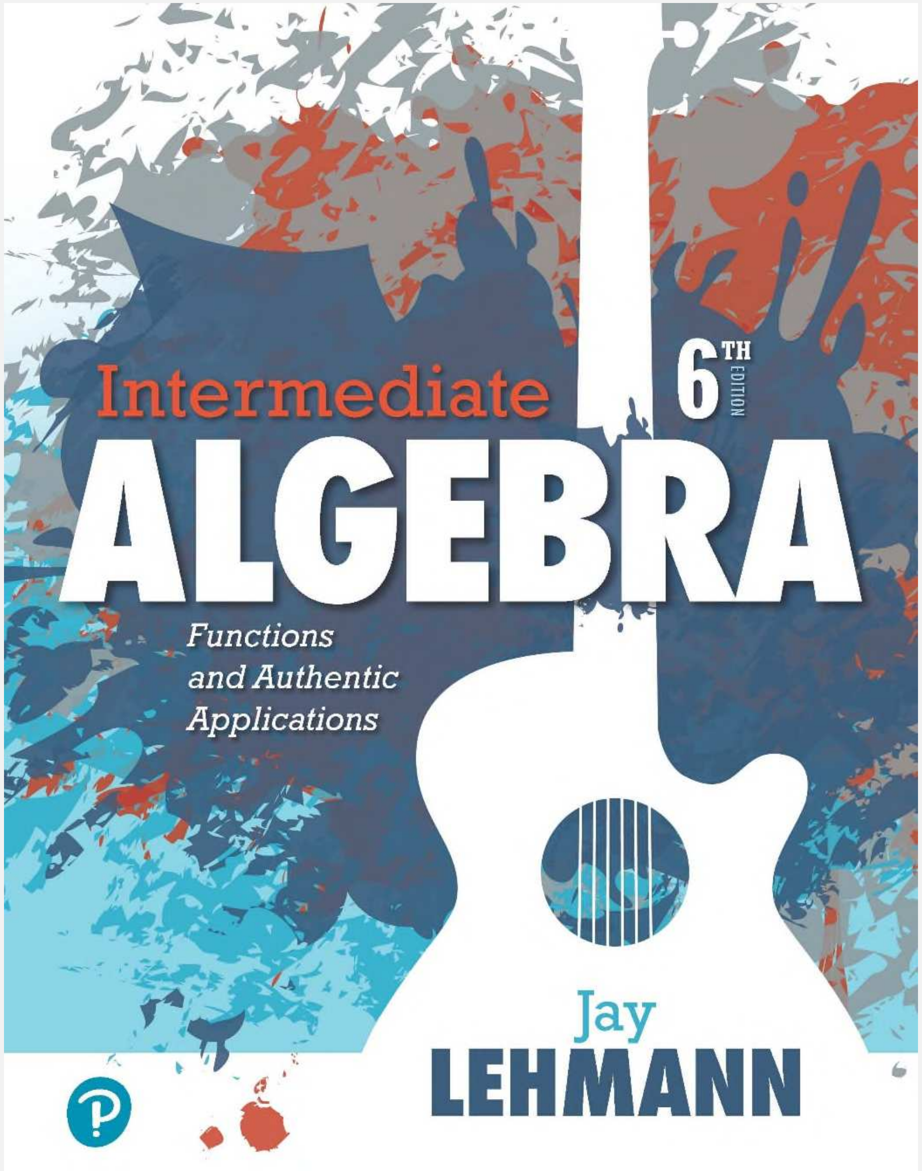
Linear Regression

Most graphing calculators have a built-in **linear regression** feature for finding an equation of a linear model.

A linear equation found by linear regression is called a **linear regression equation**, and the function described by the equation is called a **linear regression function**. The graph is called a **regression line.**

Chapter 2

Modeling with Linear Functions



2.3 Function Notation and Making Predictions

Function Notation (1 of 2)

Rather than use an equation, table, graph, or words to refer to a function, we name the function. We use $f(x)$, read "f of x" to represent y:

$$y = f(x)$$

We refer to $f(x)$ as *function notation*. To rewrite the equation $y = 2x + 1$ using function notation, substitute $f(x)$ for y:

$$f(x) = 2x + 1$$

Function Notation (2 of 2)

The notation "$f(x)$" does *not* mean f times x. It is another variable name for y.

Think of a function as a machine that send inputs to outputs.

$$f(\text{input}) = \text{output}$$

This is true for any function. To find $f(4)$, we say we **evaluate** the function f at $x = 4$.

Example: Evaluating a Function

Evaluate $f(x) = –4x + 2$ at 5.

$$f(x) = –4x + 2$$

$$f(5) = –4(5) + 2$$

$$f(5) = –18$$

Example: Evaluating Functions (1 of 3)

For $f(x) = 2x^2 - 3x$, $g(x) = \dfrac{4x-2}{5x-1}$, and $h(x) = 3x - 5$, find the following.

1. $f(-2)$

2. $g(3)$

3. $h(a)$

4. $h(a-2)$

Example: Evaluating Functions (2 of 3)

1. $f(x) = 2x^2 - 3x$

$$f(-2) = 2(-2)^2 - 3(-2)$$
$$= 2(4) - 3(-2)$$
$$= 8 + 6$$
$$= 14$$

2. $g(x) = \dfrac{4x-2}{5x-1}$

$$g(3) = \frac{4 \cdot 3 - 2}{5 \cdot 3 - 1}$$
$$= \frac{12-2}{15-1}$$
$$= \frac{10}{14}$$
$$= \frac{5}{7}$$

Example: Evaluating Functions (3 of 3)

3. To find $h(a)$, we substitute a for x in the equation:

$$h(a) = 3a - 5$$

4. $h(a - 2)$

$$\begin{aligned} h(a-2) &= 3(a-2)-5 \\ &= 3a-6-5 \\ &= 3a-11 \end{aligned}$$

Example: Using a Table to Find an Output and an Input (1 of 2)

Some input-output pairs of a function *g* are shown in the table.

1. Find $g(7)$.

2. Find x when $g(x) = 9$.

x	y
3	12
4	9
5	8
6	9
7	12

Example: Using a Table to Find an Output and an Input (2 of 2)

1. From the table, we see the input $x = 7$ leads to the output $y = 12$.
So, $g(7) = 12$.

2. To find x when $g(x) = 9$, we need to find all inputs in the table that lead to the output $y = 9$. We see both inputs $x = 4$ and $x = 6$ lead to the output $y = 9$. So, the values of x are 4 and 6.

x	y
3	12
4	9
5	8
6	9
7	12

Example: Using an Equation to Find an Output and an Input (1 of 3)

Let $f(x) = \frac{3}{2}x - 1.$

1. Find $f(4)$.

$$f(x) = \frac{3}{2}x - 1$$

$$f(4) = \frac{3}{2}(4) - 1 = 6 - 1 = 5$$

Example: Using an Equation to Find an Output and an Input (2 of 3)

Let $f(x) = \dfrac{3}{2}x - 1.$

2. Find x when $f(x) = -4$.

Substitute -4 for $f(x)$ in $f(x) = \dfrac{3}{2}x - 1$ and solve for x:

$$-4 = \frac{3}{2}x - 1$$

$$2(-4) = 2 \cdot \frac{3}{2}x - 2 \cdot 1$$

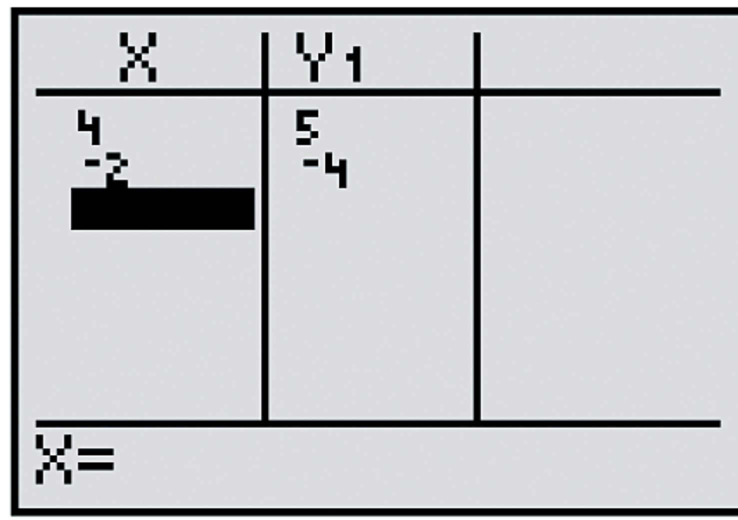
$$-8 = 3x - 2$$

$$-6 = 3x$$

$$-2 = x$$

Example: Using an Equation to Find an Output and an Input (3 of 3)

We can verify our work in Problems 1 and 2 by putting a graphing calculator table into Ask mode.



Function Notation

Warning

Be sure you know which value you need to find. When you are asked for $f(x)$, what you are looking for is a value of y, not a value of x.

Example: Using a Graph to Find Value of *x* or *f* (*x*) (1 of 3)

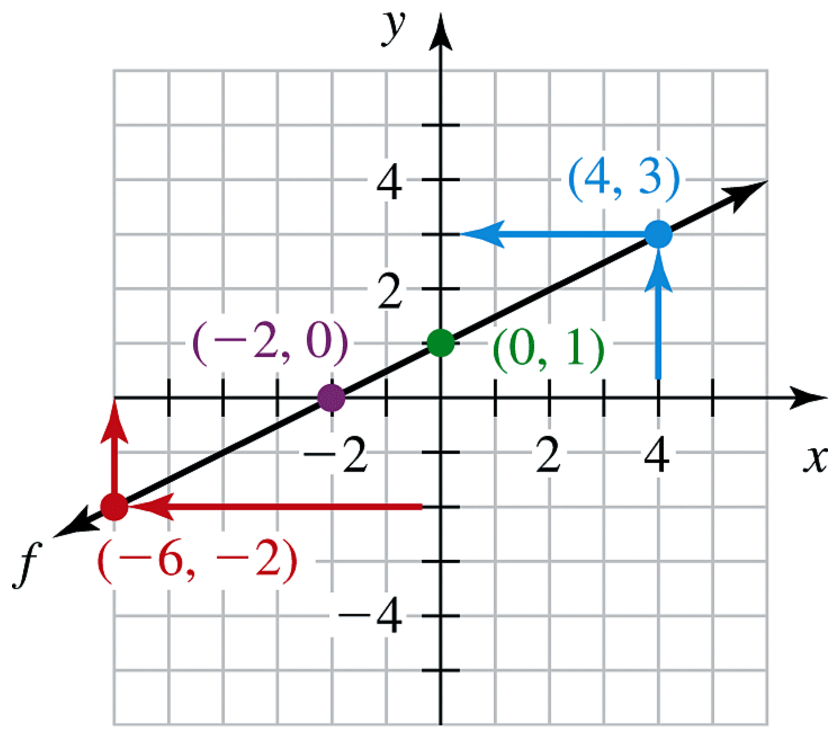
A graph of a function *f* is sketched below.

1. Find $f(4)$.

2. Find $f(0)$.

3. Find x when $f(x) = -2$.

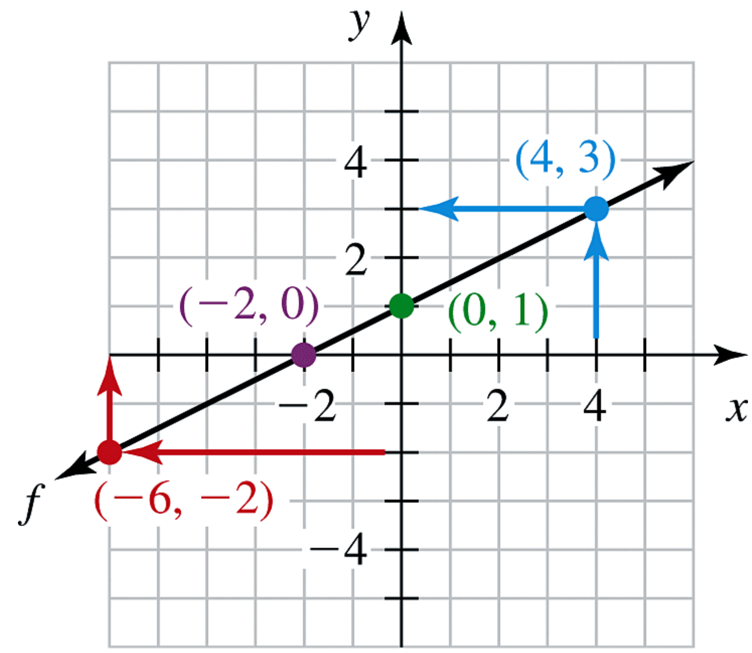
4. Find x when $f(x) = 0$.



Example: Using a Graph to Find Value of *x* or *f* (*x*) (2 of 3)

1. The notation $f(4)$ refers to $f(x)$ when $x = 4$. So, we want the value of y when $x = 4$. Hence, $f(4) = 3$.

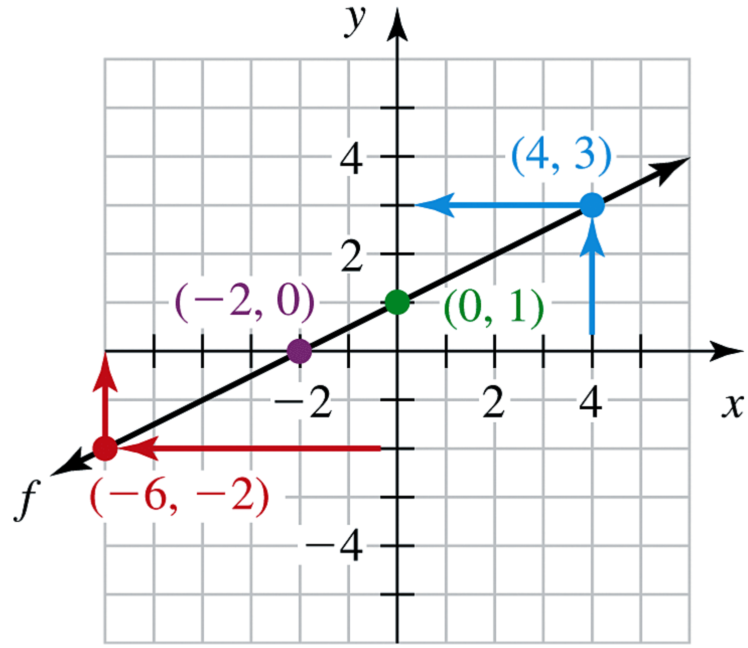
2. To find $f(0)$, we want the value of y when $x = 0$. The line contains the point $(0, 1)$, so $f(0) = 1$.



Example: Using a Graph to Find Value of x or $f(x)$ (3 of 3)

3. We have $y = f(x) = -2$. Thus, $y = -2$. So, we want original values of x when $y = -2$. See the red arrows in the graph. Hence, $x = -6$.

4. We have $y = f(x) = 0$. Thus, $y = 0$. The line contains the point $(-2, 0)$ so, $x = -2$.



Function Notation (1 of 2)

The response variable of a function f can be represented by the expression formed by writing the explanatory variable name within the parentheses of $f(\ )$:

$$\text{Response variable} = f\,(\text{explanatory variable})$$

We call this representation **function notation.**

Function Notation (2 of 2)

response variable

explanatory variable

$$y = f(x)$$

function name

Using an Equation of a Linear Model to Make Predictions

- To make a prediction about the response variable of a linear model, substitute a chosen value for the explanatory variable in the model's equation. Then solve for the response variable.

- To make a prediction about the explanatory variable of a linear model, substitute a chosen value for the response variable in the model's equation. Then solve for the explanatory variable.

Four-Step Modeling Process (1 of 2)

To find a linear model and make estimates and predictions,

1. Construct a scatterplot of the data to determine whether there is a nonvertical line that comes close to the data points. If so, choose two points (not necessarily data points) that you can use to find the equation of a linear model.

2. Find an equation of your model.

Four-Step Modeling Process (2 of 2)

3. Verify your equation by checking that the graph of your model contains the two chosen points and comes close to all of the data points.

4. Use the equation of your model to make estimates, make predictions, and draw conclusions.

Example: Using Function Notation; Finding Intercepts (1 of 8)

Let $p = -0.48t + 71$ be the equation that models the percentage p of American adults who smoke at t years since 1900. The table below shows the data.

Year	Percent
1970	37.4
1980	33.2
1990	25.3
2000	23.1
2001	19.4
2015	15.1

Example: Using Function Notation; Finding Intercepts (2 of 8)

1. Rewrite the equation $p = -0.48t + 71$ with the function name g.

2. Find $g(121)$. What does the result mean in this situation?

3. Find the value of t when $g(t) = 30$. What does it mean in this situation?

4. Find the p-intercept of the model. What does it mean in this situation?

5. Find the t-intercept of the model. What does it mean in this situation?

Example: Using Function Notation; Finding Intercepts (3 of 8)

1. To use the name g, we substitute $g(t)$ for p in the equation:

$$g(t) = -0.48t + 71$$

2. To find $g(121)$, substitute 121 for t in the equation:

$$g(t) = -0.48t + 71$$

$$g(121) = -0.48(121) + 71$$

$$= 12.92$$

According to the model, about 13.0% of American adults will smoke in 2021.

Example: Using Function Notation; Finding Intercepts (4 of 8)

3. Substitute 30 for $g(t)$ in the equation and solve for t:

$$g(t) = -0.48t + 71$$

$$30 = -0.48t + 71$$

$$30 - 71 = -0.48t + 71 - 71$$

$$-41 = -0.48t$$

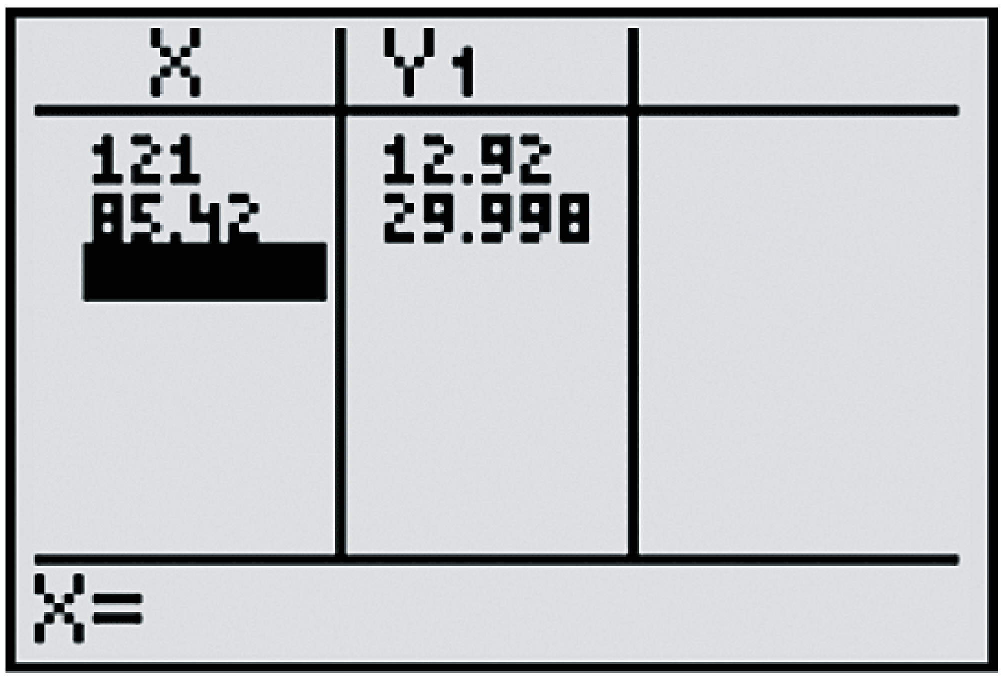
$$\frac{-41}{-0.48} = \frac{-0.48t}{-0.48}$$

$$85.42 \approx t$$

The model estimates 30% of Americans smoked in $1900 + 85.42 \approx 1985$.

Example: Using Function Notation; Finding Intercepts (5 of 8)

We can verify our work for Problems 2 and 3 by using a graphing calculator table.



Example: Using Function Notation; Finding Intercepts (6 of 8)

4. Since the model $g(t) = -0.48t + 71$ is in slope-intercept form, the p-intercept is $(0, 71)$.

So, the model estimates 71% of American adults smoked in 1900. Research would show this estimate is too high; model breakdown has occurred.

Example: Using Function Notation; Finding Intercepts (7 of 8)

5. To find the t-intercept, we substitute 0 for $g(t)$ and solve for t:

$$0 = -0.48t + 71$$

$$0 + 0.48t = -0.48t + 71 + 0.48t$$

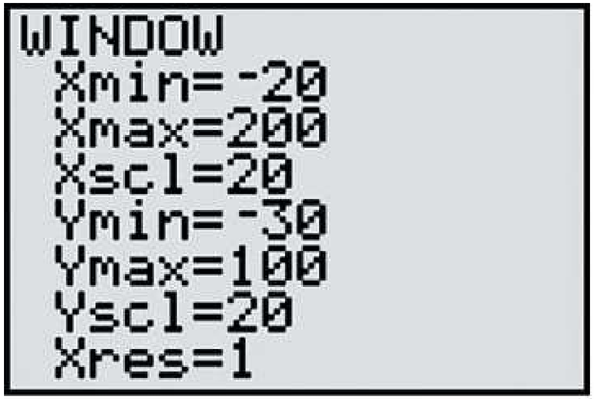
$$0.48t = 71$$

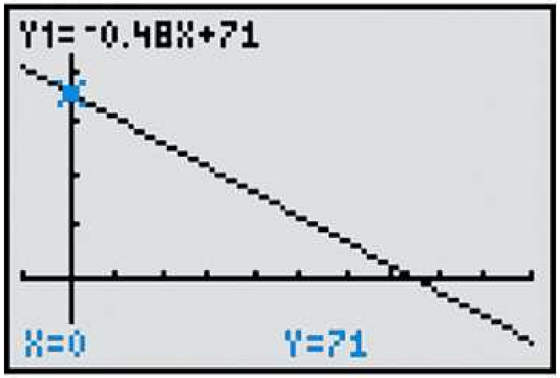
$$\frac{0.48t}{0.48} = \frac{71}{0.48} \qquad t \approx 147.92$$

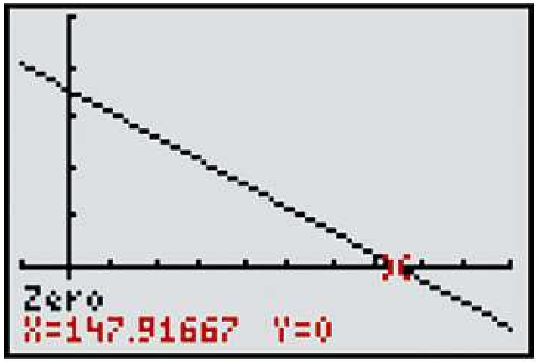
The t-intercept is (147.92, 0). So, the model predicts that no American adults will smoke in 1900 + 147.92 ≈ 2048. However, common sense suggests this event probably won't occur.

Example: Using Function Notation; Finding Intercepts (8 of 8)

We can use TRACE to verify the p-intercept and the "zero" option to verify the t-intercept.







Using an Equation of a Linear Model to Find Intercepts

If a function of the form $p = mt + b$, where $m \neq 0$, is used to model a situation, then

- The p-intercept is $(0, b)$.

- To find the t-coordinate of the t-intercept, substitute 0 for p in the model's equation and solve for t.

Example: Finding the Domain and Range of a Model (1 of 3)

A store is open from 9 A.M. to 5 P.M., Mondays through Saturdays. Let $I = f(t)$ be an employee's weekly income (in dollars) from working t hours each week at \$10 per hour.

1. Find the equation of the model f.

The employee's weekly income (in dollars) is equal to the pay per hour times the number of hours worked per week:

$$f(t) = 10t$$

Example: Finding the Domain and Range of a Model (2 of 3)

2. Find the domain and range of the model *f*.

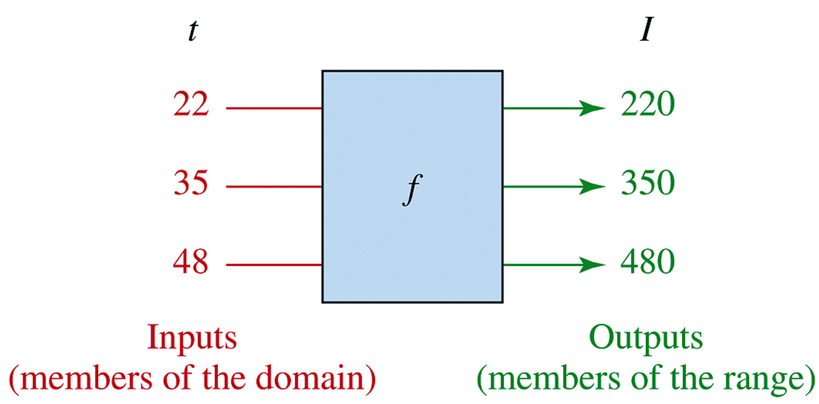
To find the domain and range of the model *f*, we consider input-output pairs only when both the input and output make sense in this situation.

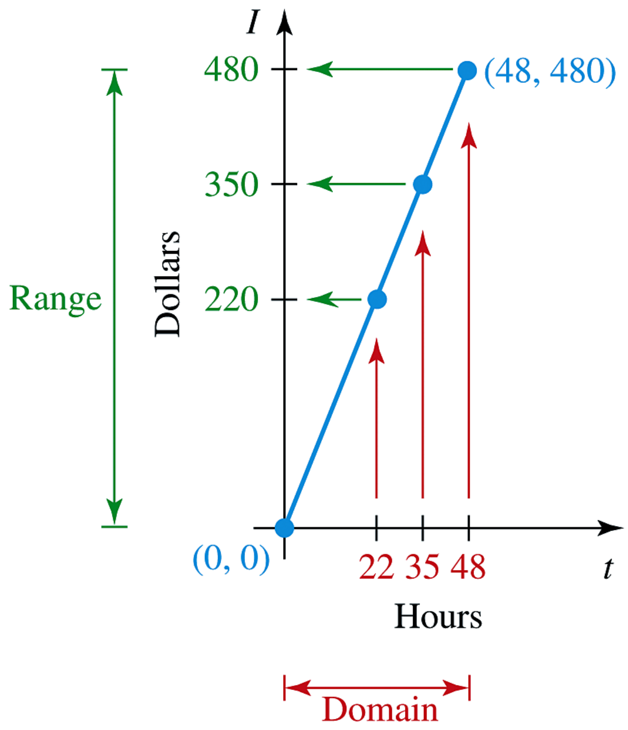
Time is the input. Since the store is open 8 hours a day, 6 days a week, the employee can work up to 48 hours each week.

So, the domain is the set of numbers between 0 and 48 inclusive: $0 \le t \le 48$.

Example: Finding the Domain and Range of a Model (3 of 3)

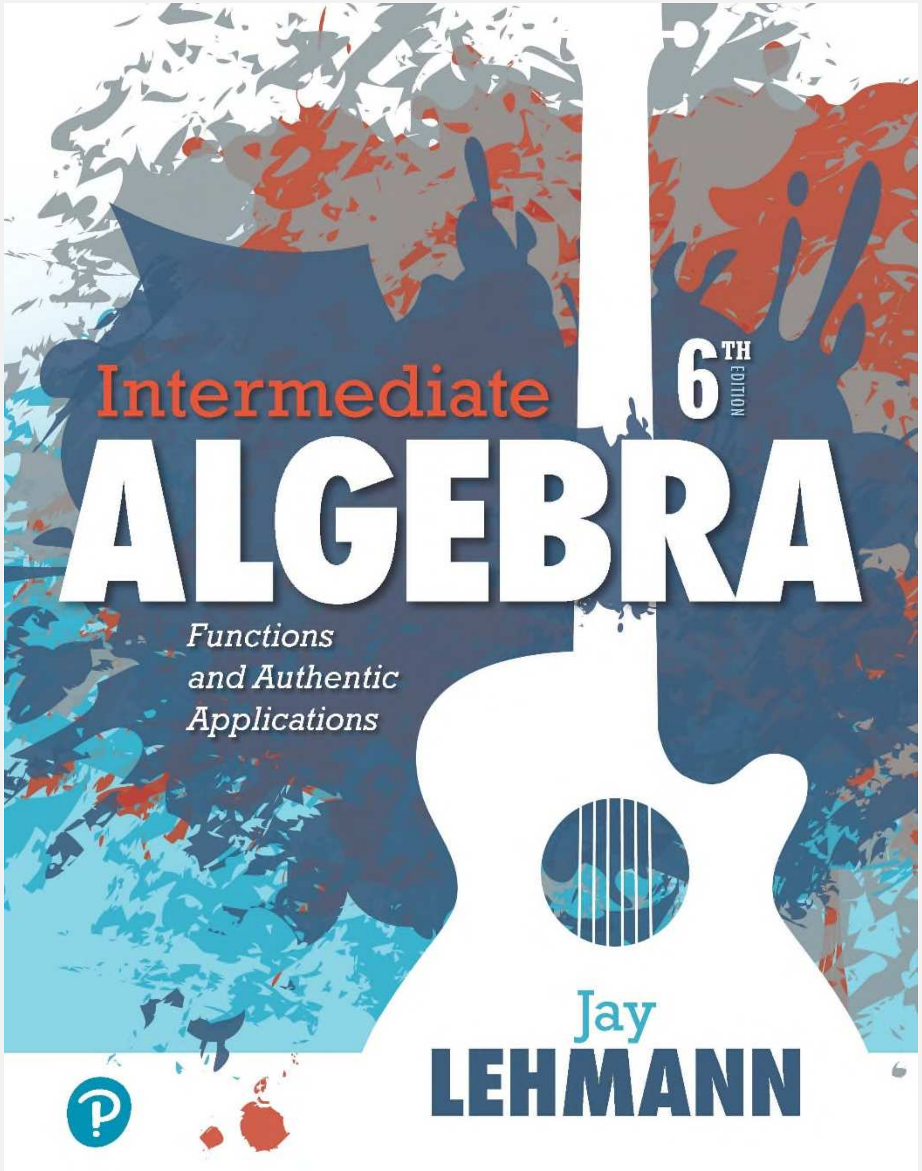
2. The graph illustrates the domain and range of the model *f*.





Chapter 2

Modeling with Linear Functions



2.4 Slope Is a Rate of Change

Ratio and Unit Ratio

The **ratio** of a to b is the fraction $\frac{a}{b}$.

A **unit ratio** is a ratio written as $\frac{a}{b}$ with $b = 1$.

Formula for Rate of Change and Average Rate of Change

Suppose a quantity y changes steadily from y_1 to y_2 as a quantity x changes steadily from x_1 to x_2. Then the **rate of change** of y with respect to x is the ratio of the change in y to the change in x:

$$\frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Finding Rates of Change (1 of 3)

1. The enrollment at DeVry University declined approximately steadily from 82 thousand students in 2012 to 37 thousand students in 2016 (Source: *DeVry University*). Find the approximate rate of change of enrollment.

$$\frac{\text{change in enrollment}}{\text{change in time}} = \frac{37\text{thousand students} - 82 \text{ thousand students}}{\text{year } 2016 \ - \ \text{year } 2012}$$

$$= \frac{-45 \text{ thousand students}}{4 \text{ years}}$$

$$= \frac{-11.25 \text{ thousand students}}{1 \text{ year}}$$

The rate of change of enrollment was about –11.25 thousand students per year. The enrollment declined yearly by about 11.25 thousand students.

Example: Finding Rates of Change (2 of 3)

2. The *median* of a group of numbers is the middle number, where the numbers are listed from smallest to largest. In Phoenix, Arizona, the median value of a two-bedroom home is $151 thousand and the median value of a four-bedroom home is $274 thousand (Source: *Trulia*). Find the approximate rate of change of the median value of a home with respect to the number of bedrooms.

To be consistent in finding the signs of the changes, assume that the number of bedrooms increases from two to four and that the median value increases from $151 thousand to $274 thousand.

Example: Finding Rates of Change (3 of 3)

$$\frac{\text{change in median value}}{\text{change in number of bedrooms}} = \frac{\$274 \text{ thousand } - 151 \text{ thousand dollars}}{4 \text{ bedrooms } - 2 \text{ bedrooms}}$$

$$= \frac{123 \text{ thousand dollars}}{2 \text{ bedrooms}} = \frac{61.5 \text{ thousand dollars}}{1 \text{ bedroom}}$$

The rate of change of the median value with respect to the number of bedrooms is about \$61.5 thousand per bedroom. So, the median value increases by \$61.5 thousand per bedroom.

Signs of Rate of Change

Suppose a quantity t affects (explains) a quantity p:

- If p increases steadily as t increases steadily, then the rate of change of p with respect to t is positive.

- If p decreases steadily as t increases steadily, then the rate of change of p with respect to t is negative.

Example: Comparing Slope with a Rate of Change (1 of 6)

Suppose a student drives at a constant rate. Let d be the distance (in miles) the student can drive in t hours. Some values of t and d are shown in the table.

Time (hours) t	Distance (miles) d
0	0
1	60
2	120
3	180
4	240
5	300

Example: Comparing Slope with a Rate of Change (2 of 6)

1. Create a scatterplot. Then draw a linear model.

2. Find the slope of the linear model.

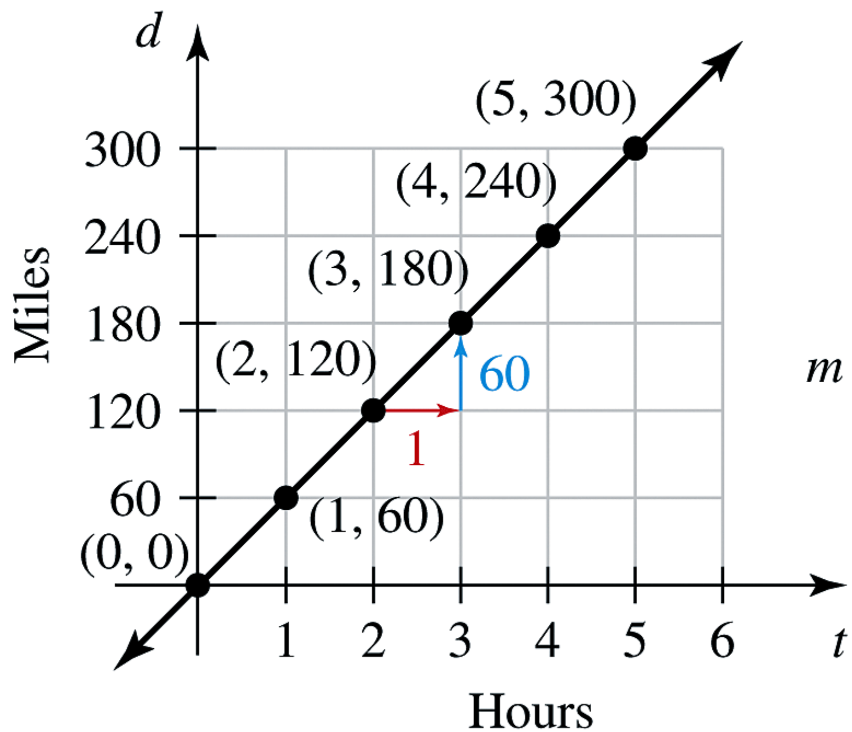
3. Find the rate of change of distance traveled for each given period. Compare each result with the slope of the linear model.

a. From $t = 2$ to $t = 3$

b. From $t = 0$ to $t = 4$

Example: Comparing Slope with a Rate of Change (3 of 6)

1. Draw the scatterplot and then draw a line that contains the data points.



$$m = \frac{60}{1} = 60$$

Example: Comparing Slope with a Rate of Change (4 of 6)

2. The slope formula is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. So, with the variables t and d, we have $\dfrac{d_2 - d_1}{t_2 - t_1}$.

Arbitrarily use the points (2, 120) and (3, 180) to calculate the slope:

$$\frac{180 - 120}{3 - 2} = \frac{60}{1} = 60$$

So, the slope is 60.

Example: Comparing Slope with a Rate of Change (5 of 6)

3. a. Calculate the rate of change of the distance traveled from $t = 2$ to $t = 3$:

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{180 \text{ miles} - 120 \text{ miles}}{3 \text{ hours} - 2 \text{ hours}}$$

$$= \frac{60 \text{ miles}}{1 \text{ hour}} = 60 \text{ miles per hour}$$

The rate of change (60 miles per hour) is equal to the slope (60).

Example: Comparing Slope with a Rate of Change (6 of 6)

3. b. Calculate the rate of change of the distance traveled from $t = 0$ to $t = 4$:

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{240 \text{ miles} - 0 \text{ miles}}{4 \text{ hours} - 0 \text{ hours}}$$

$$= \frac{240 \text{ miles}}{4 \text{ hours}} = \frac{60 \text{ miles}}{1 \text{ hour}} = 60 \text{ miles per hour}$$

The rate of change (60 miles per hour) is equal to the slope (60).

Slope Is a Rate of Change

If there is a linear relationship between quantities t and p, and if t affects (explains) p, then the slope of the linear model is equal to the rate of change of p with respect to t.

Constant Rate of Change

Suppose a quantity t affects (explains) a quantity p:

- If there is a linear relationship between t and p, then the rate of change of p with respect to t is constant.

- If the rate of change of p with respect to t is constant, then there is a linear relationship between t and p.

Example: Find a Model (1 of 3)

A company's profit was \$10 million in 2010 and has increased by \$3 million per year. Let p be the annual profit (in millions of dollars) at t years since 2010.

1. Is there a linear relationship between t and p? Explain.

2. Find the p-intercept of the linear model.

3. Find the slope of the linear model.

4. Find an equation of the linear model.

Example: Find a Model (2 of 3)

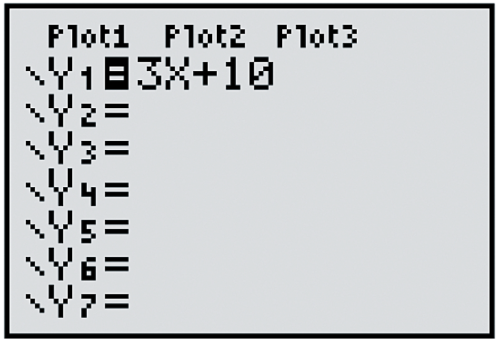
1. Because the rate of change of annual profit is a *constant* \$3 million per year, the variables t and p are linearly related.

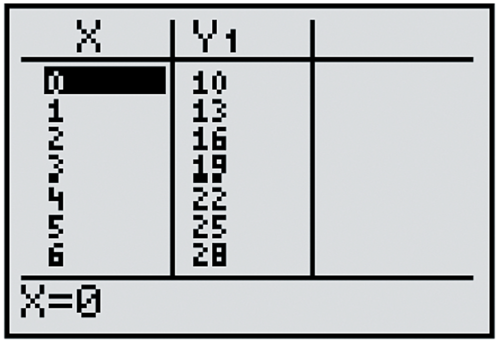
2. The profit was \$10 million in 2010. Because 2010 is 0 years since 2010, we can represent this relationship by the ordered pair (0, 10). So, the p-intercept of the linear model is (0, 10).

3. The rate of change of annual profit is \$3 million per year. So, the slope of the linear model is 3.

Example: Find a Model (3 of 3)

4. An equation of the linear model can be written in the form $p = mt + b$. Because the slope is 3 and the p-intercept is (0, 10), we have $p = 3t + 10$. The ordered pair (0, 10) shown in the first row of the table means the profit was $10 million in 2010, which checks. Also, as the input increases by 1, the output increases by 3. This means the annual profit increases by $3 million per year, which also checks.





Unit Analysis

We perform a **unit analysis** of a model's equation by determining the units of the expressions on both sides of the equation. The simplified units of the expressions on both sides of the equation should be the same.

Example: Finding a Model (1 of 4)

A driver fills her car's 12-gallon gasoline tank and drives at a constant speed. The car consumes 0.04 gallon per mile. Let G be the number of gallons of gasoline remaining in the tank after she has driven d miles since filling up.

1. Is there a linear relationship between d and G? Explain.

2. Find the G-intercept of a linear model.

3. Find the slope of the linear model.

4. Find an equation of the linear model.

5. Perform a unit analysis of the equation.

Example: Finding a Model (2 of 4)

1. Since the rate of change of gallons remaining with respect to distance traveled is a *constant* (–0.04 gallon per mile), the variable d and G are linearly related.

2. When the tank was filled, it contained 12 gallons of gasoline. We can represent this by the ordered pair (0, 12), which is the G-intercept.

Example: Finding a Model (3 of 4)

3. The rate of change of gasoline remaining in the tank with respect to distance traveled is –0.04 gallon per mile. So, the slope is –0.04.

4. An equation of the linear model can be written in the form $G = md + b$. Since slope is –0.04 and the G-intercept is (0, 12) we have $G = -0.04d + 12$.

Example: Finding a Model (4 of 4)

5. $$\underbrace{G}_{\text{gallons}} = \underbrace{-0.04}_{\text{gallons/mile}} \cdot \underbrace{d}_{\text{miles}} + \underbrace{12}_{\text{gallons}}$$

$$\frac{\text{gallon}}{\text{mile}} \cdot \text{miles} + \text{gallons} = \text{gallon} + \text{gallons}$$

$$= \text{gallons}$$

So, the units on both sides of the equation are gallons, which suggests the equation is correct.

Example: Analyzing a Model (1 of 6)

The percentages of Fortune 1000 board seats filled by women are shown for various years. Let p be the percentage of the seats filled by women at t years since 2010. A model of the situation is $p = 1.03t + 13.55$

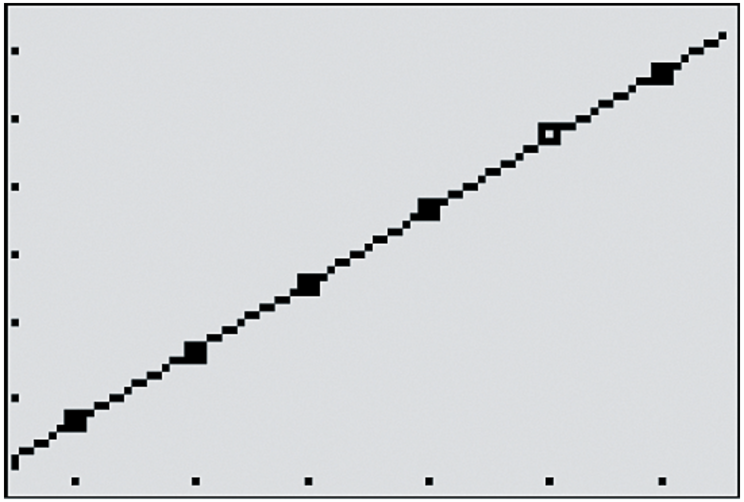
Year	Percent
2011	14.6
2012	15.6
2013	16.6
2014	17.7
2015	18.8
2016	19.7

Example: Analyzing a Model (2 of 6)

1. Use a graphing calculator to draw a scatterplot and the model in the same viewing window. Check whether the line comes close to the data points.
2. What is the slope of the model? What does it mean in this situation?
3. Find the rates of change of the percentage of seats filled by women from one year to the next. Compare the rates of change with the result found in Problem 2.
4. 2020 Women on Boards has set a goal that women will fill 20% of Fortune 1000 board seats by the year 2020. Does the model predict that the goal will be reached?

Example: Analyzing a Model (3 of 6)

1. Draw a scatterplot and the model in the same viewing window. The line comes quite close to the data points, so the model is a reasonable one.



Example: Analyzing a Model (4 of 6)

2. The slope is 1.03 because $p = 1.03t + 13.55$ is of the form $y = mx + b$ and $m = 1.03$. According to the model, the percentage of seats filled by women are increasing by 1.03 percentage points per year.

Example: Analyzing a Model (5 of 6)

3. The rates of change of the percentage of seats filled by women are shown. All the rates of change are close to 1.03 percentage points per year.

Year	**Percent**	**Rate of Change of Percent from Previous Year (percent points per year)**
2011	14.6	
2012	15.6	$(15.6 - 14.6) \div (2012 - 2011) = 1.0$
2013	16.6	$(16.6 - 15.6) \div (2013 - 2012) = 1.0$
2014	17.7	$(17.7 - 16.6) \div (2014 - 2013) = 1.1$
2015	18.8	$(18.8 - 17.7) \div (2015 - 2014) = 1.1$
2016	19.7	$(19.7 - 18.8) \div (2016 - 2015) = 0.9$

Example: Analyzing a Model (6 of 6)

4. We substitute the input 10 for t in the equation

$$p = 1.03t + 13.55:$$

$$p = 1.03(10) + 13.55 = 23.85$$

According to the model, about 23.9% of Fortune 1000 board seats will be filled by women in 2020, so the goal will be surpassed.

Describing the Meaning of Slope

Warning

It is a common error to be vague in describing the meaning of the slope of a model.

For example, a description such as

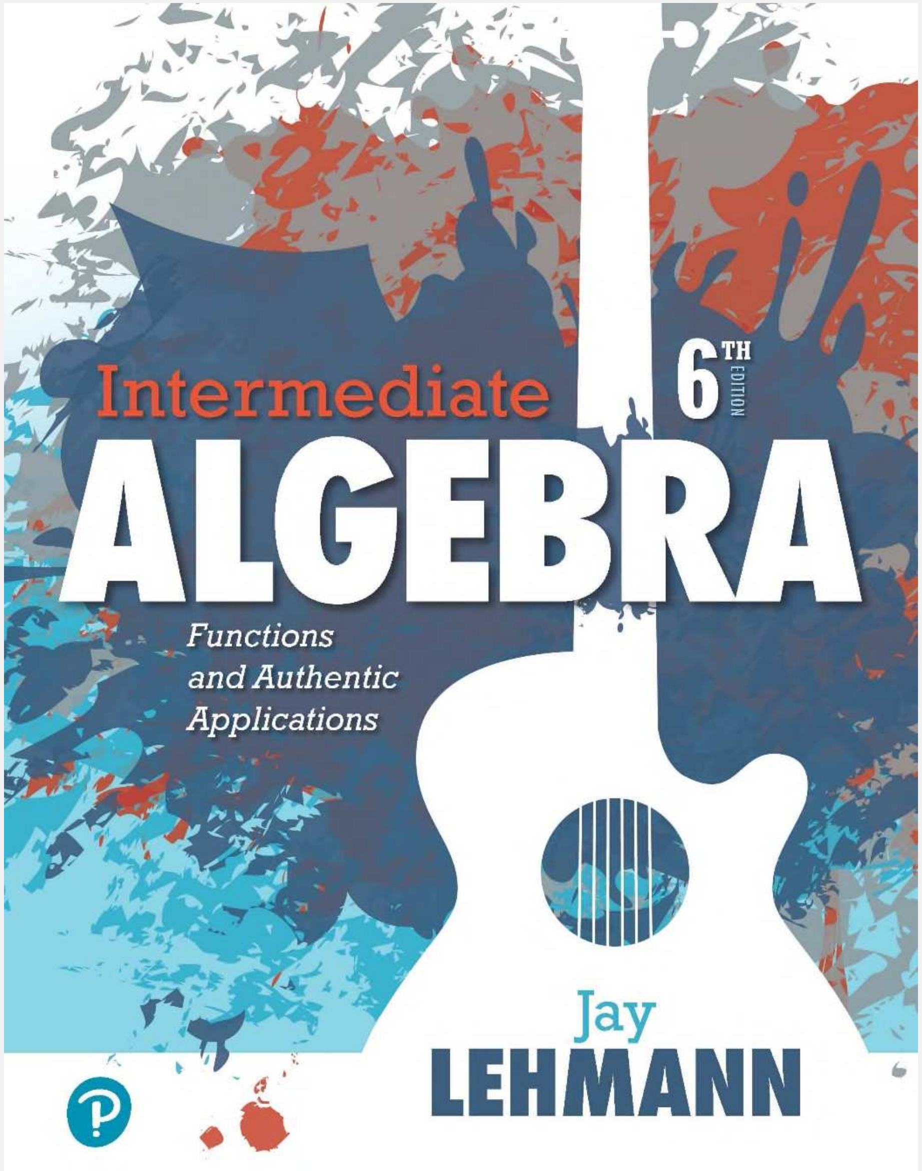
The slope means it is increasing.

neither specifies the quantity that is increasing nor the rate of increase. The following statement includes the missing information:

The slope of 1.03 means the percentage of Fortune 1000 board seats filled by women increased by 1.03 percentage points per year.

Chapter 2

Modeling with Linear Functions



2.1 Using Lines to Model Data

Example: Using a Graph to Describe an Authentic Situation (1 of 3)

The Grand Canyon is a beautiful landmark, yet the difficulty of finding a parking spot can detract from visitors' enjoyment. The numbers of Grand Canyon visitors are listed in the table for various years. Describe the data with a graph.

Year	**Number of Visitors (millions**
1960	1.2
1970	2.3
1980	2.6
1990	3.8
2000	4.8
2010	4.4
2015	5.5

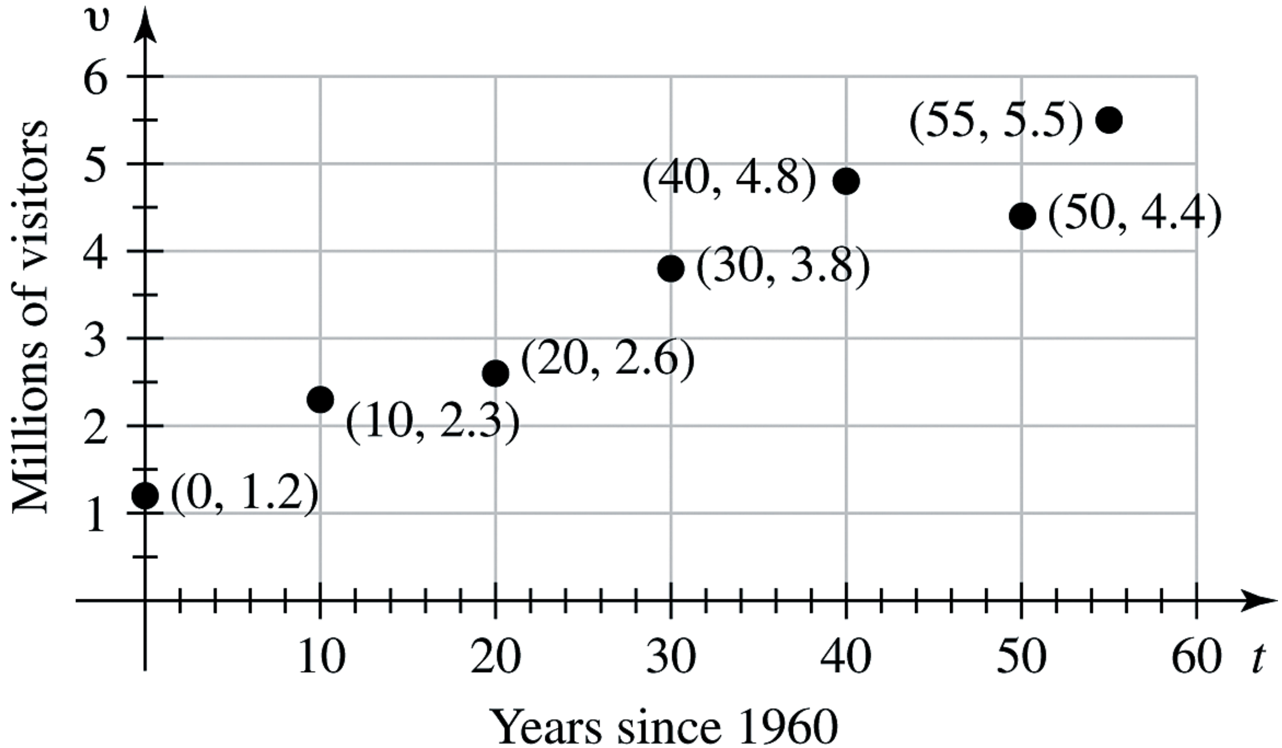
Example: Using a Graph to Describe an Authentic Situation (2 of 3)

Let v be the number (in millions) of visitors in the year that is t years since 1960. We can describe the data with a table of values for v and t.

Number of Years since 1960 t	**Number of Visitors (millions)** v
0	1.2
10	2.3
20	2.6
30	3.8
40	4.8
50	4.4
55	5.5

Example: Using a Graph to Describe an Authentic Situation (3 of 3)

Plot the (t, v) data points shown in the table. Let the horizontal axis be the t-axis and the vertical axis be the v-axis.



Scatterplot

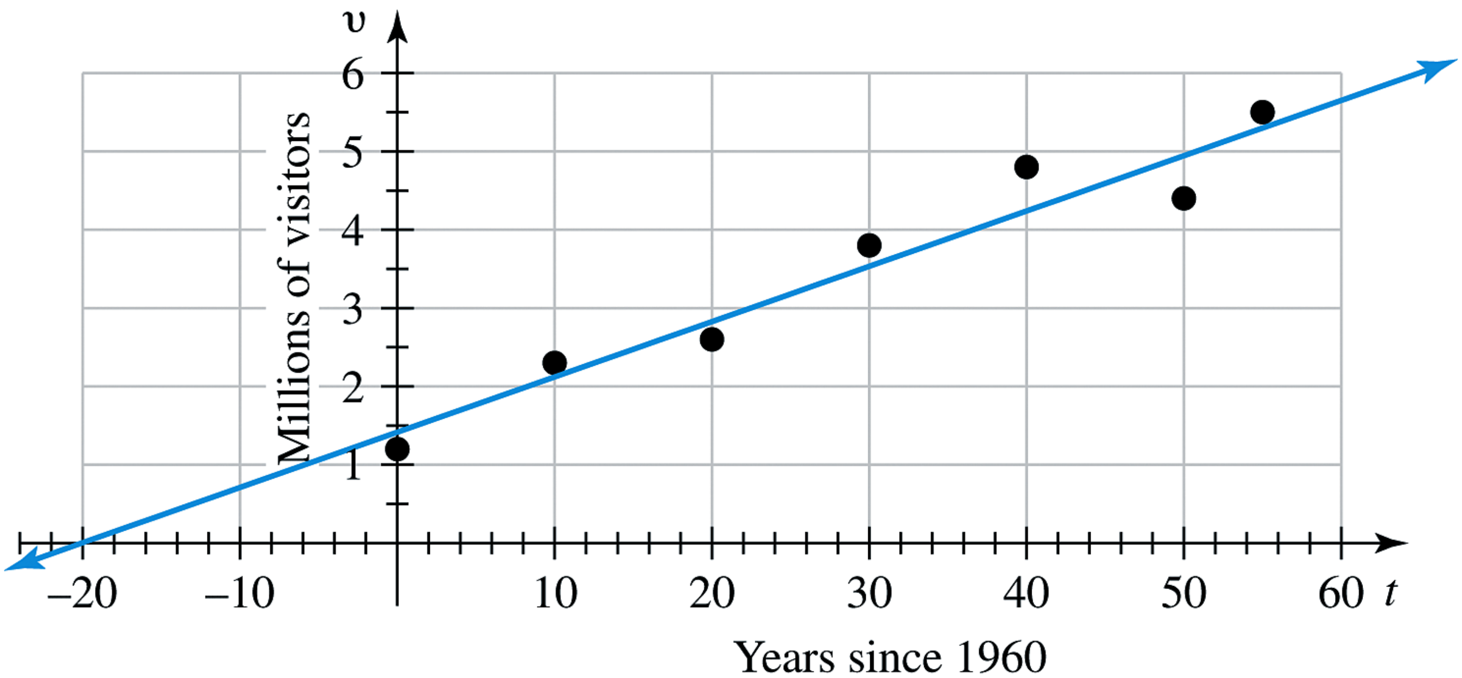
A graph of plotted ordered pairs is called a **scatterplot. A scatterplot should have scaling on both axes and labels indicating the variable names and scale units.**

Approximately Linearly Related (1 of 3)

We can sketch a line that comes close to (or on) the data points of the graph from the previous example. Since the points in the scatterplot lie close to (or on) a line, then we say that the relevant variables are **approximately linearly related**.

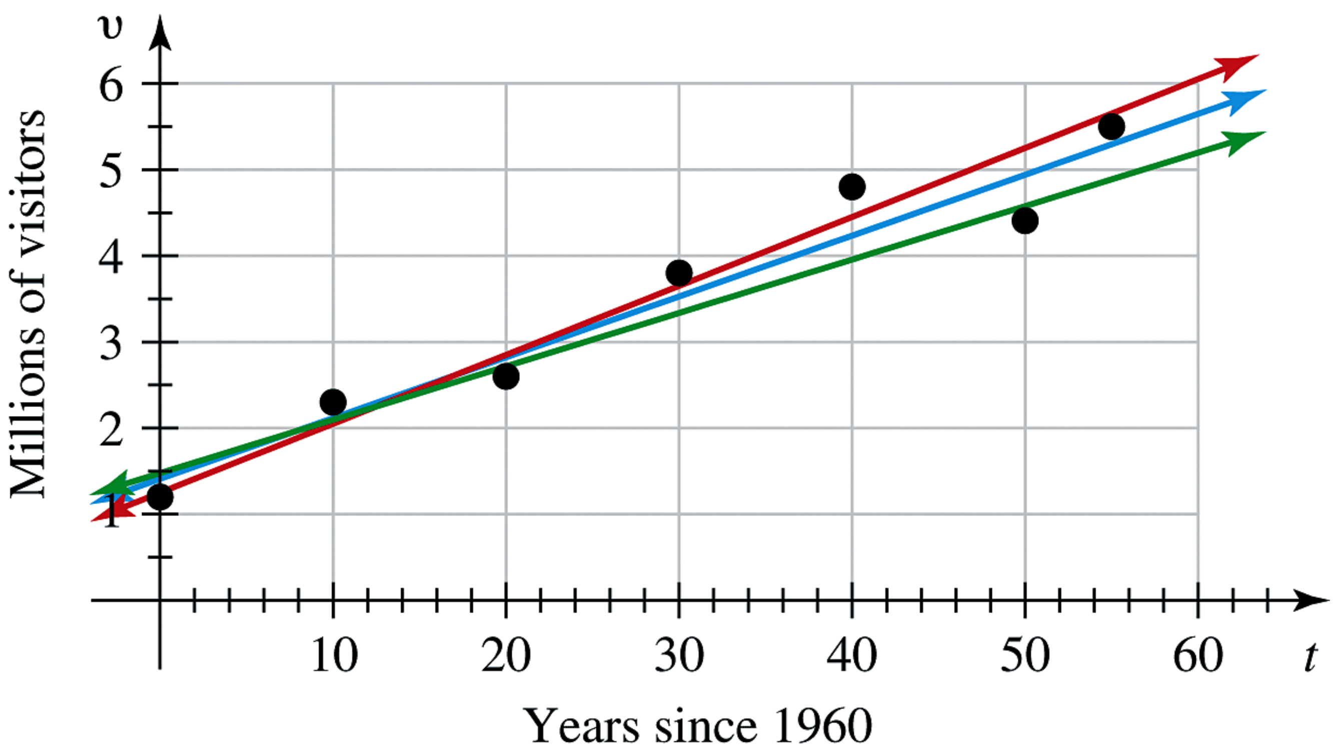
Approximately Linearly Related (2 of 3)

One possible line that comes close to (or on) the data points.



Approximately Linearly Related (3 of 3)

A few of the many lines that come close to (or on) the data points.



Model

A **model** is a mathematical description of an authentic situation. We say the description *models* the situation.

Linear model

A **linear model** is a linear function, or its graph, that describes the relationship between two quantities for an authentic situation.

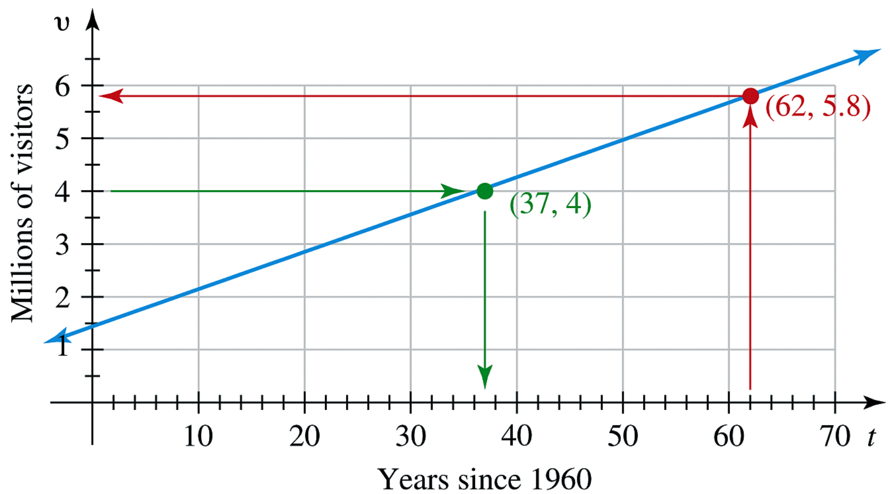
Example: Using a Linear Model to Make a Prediction and an Estimate (1 of 3)

1. Use the linear model shown on the next slide to predict the number of visitors in 2022.

2. Use the linear model to estimate in what year there were 4 million visitors.

Example: Using a Linear Model to Make a Prediction and an Estimate (2 of 3)

1. 2022 – 1960 = 62, so $t = 62$. To estimate the number of visitors, locate the point on the linear model where the t-coordinate is 62. The corresponding v-coordinate is about 5.8.



So there will be 5.8 million visitors in 2022.

Example: Using a Linear Model to Make a Prediction and an Estimate (3 of 3)

2. To find the year when there were 4 million visitors, locate the point on the linear model where the v-coordinate is 4. The coordinating t-coordinate is about 37. So, *according to the linear model*, there were 4 million visitors in $1960 + 37 = 1997$.

Using a Linear Function to Model Data

Warning

We create a scatterplot of data to determine whether the relevant variables are approximately linearly related. If they are, we draw a line that comes close to (or on) the data points and use the line to make estimates and predictions. It is a common error to try to find a line that contains the greatest number of data points. **Our goal is to find a line that comes close to *all* of the data points.**

Example: Intercepts of a Model; Model Breakdown (1 of 5)

The percentages of cell phone users who send or receive text messages multiple times per day are shown in the table for various age groups.

Age Group (Years)	Age Used to Represent Age Group (years)	Percent
18–24	21.0	76
25–34	29.5	63
35–44	39.5	42
45–54	49.5	37
55–64	59.5	17

Example: Intercepts of a Model; Model Breakdown (2 of 5)

Let p be the percentage of cell phone users at age a who send or receive text messages multiple times per day.

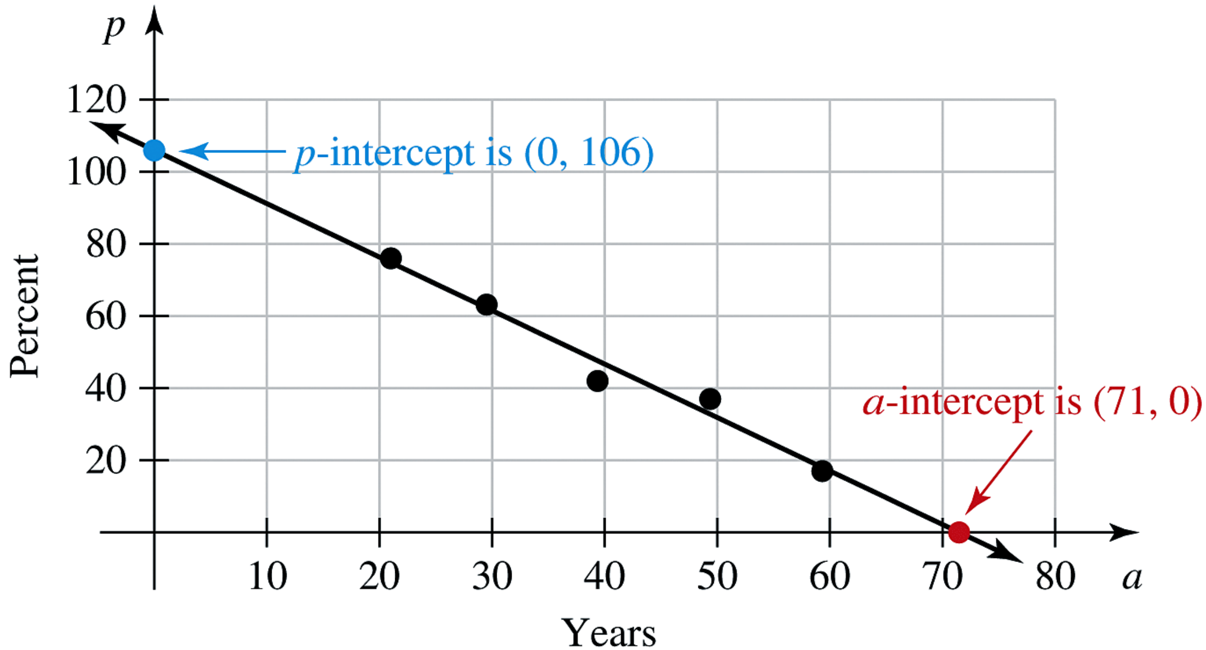
1. Find a linear model that describes the relationship between a and p.

2. Find the p-intercept. What does it mean in this situation?

3. Find the a-intercept. What does it mean in this situation?

Example: Intercepts of a Model; Model Breakdown (3 of 5)

View the positions of the points in the scatterplot. It appears that *a* and *p* are approximately linearly related, so we sketch a line that comes close to the data points.



Example: Intercepts of a Model; Model Breakdown (4 of 5)

2. The p-intercept is (0, 106), or $p = 106$, when $a = 0$.

According to the model, 106% of newborns who use cell phones send or receive text messages multiple times per day.

Our model gives a false estimate for two reasons: Percentages cannot be larger than 100% in this situation, and newborns cannot send or receive text messages.

Example: Intercepts of a Model; Model Breakdown (5 of 5)

3. The a-intercept is (71, 0), or $p = 0$, when $a = 71$.

According to the model, no 71-year-old cell phone users send or receive text messages multiple times per day. This is a false statement.

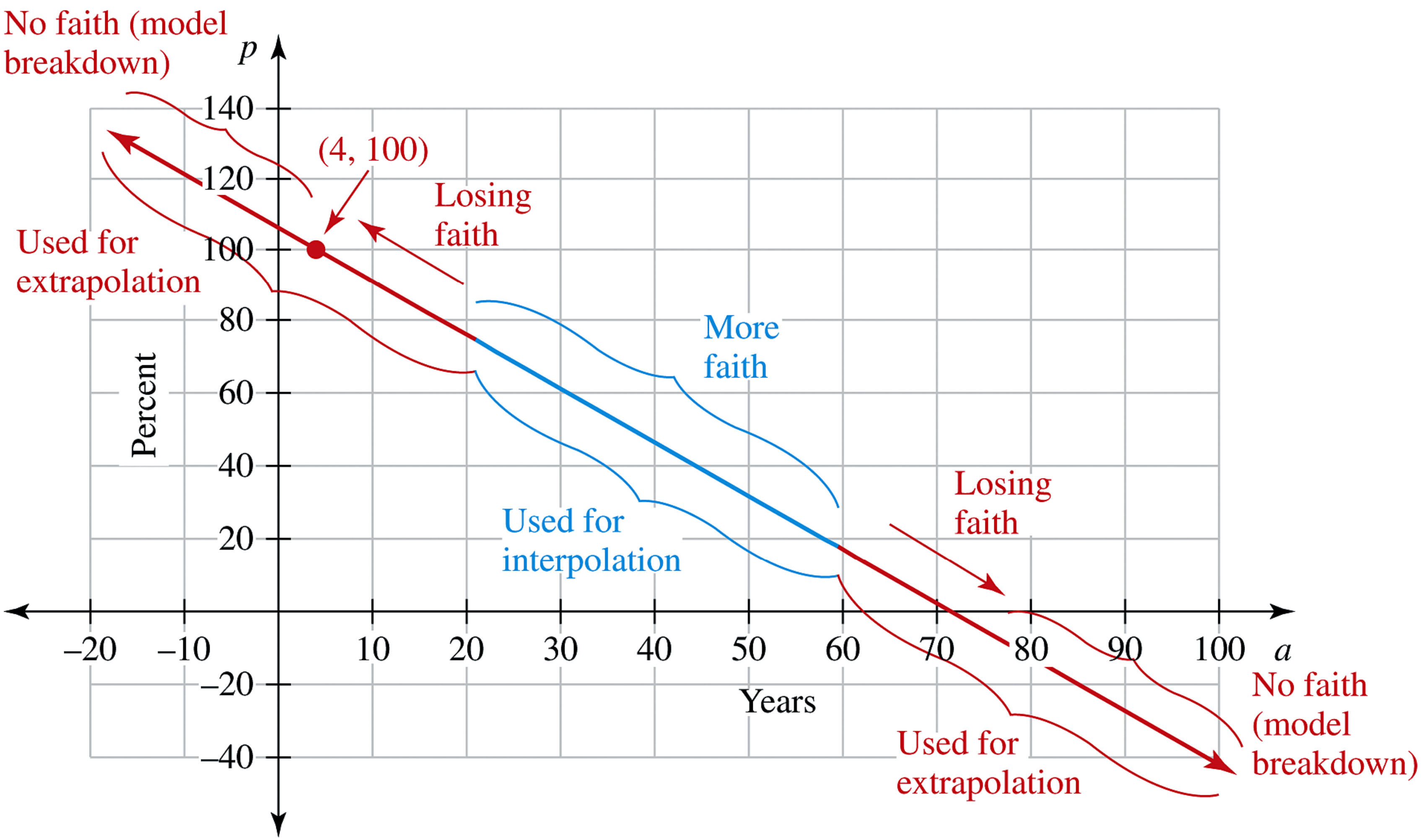
A little research would show some 71-year-old cell phone users send or receive text messages multiple times per day.

Interpolation, extrapolation (1 of 2)

For a situation that can be described by a model by a function whose explanatory variable is x,

- We perform **interpolation** when we use a part of the model whose x-coordinates are between the x-coordinates of two data points.
- We perform **extrapolation** when we use a part of the model whose x-coordinates are not between the x-coordinates of any two data points.

Interpolation, extrapolation (2 of 2)



Model Breakdown

When a model gives a prediction that does not make sense or an estimate that is not a good approximation, we say **model breakdown** has occurred.

Example: Modifying a Model (1 of 3)

Additional research yields the data shown in the first and last rows of the table. Use the data and the following assumptions to modify the model found in the previous example.

- Children 3 years old and younger do not send or receive text message multiple times per day.
- The percentage of cell phone users who send or receive text messages multiple times per day levels off at 5% for users over 80 years of age.
- The age of the oldest cell phone user is 116 years.

Example: Modifying a Model (2 of 3)

Age Group (Years)	Age Used to Represent Age Group (years)	Percent
12–17	14.5	75
18–24	21.0	76
25–34	29.5	63
35–44	39.5	42
45–54	49.5	37
55–64	59.5	17
over 64	70.0	7

Example: Modifying a Model (3 of 3)

Recall that p is the percentage of cell phone users at age a years who send or receive multiple text messages per day. Taking into account the three assumptions, we sketch a scatterplot and draw a model that comes close to the data points.