Using and Understanding Mathematics A Quantitative Reasoning Approach 7th edition Bennett Solutions Manual

SKU: 9780134705187-SOLUTIONS

UNIT 2A: UNDERSTAND, SOLVE, AND EXPLAIN

THINK ABOUT IT

Pg. 77. Answers will vary. This question should help the student think about the units associated with everyday numbers. Even if they pick a page number from the newspaper or magazine, note that this has units of "pages." This would be a good topic for a discussion either during or outside of class.

Pg. 80. Using $\frac{60\text{ s}}{1\text{ min}}$, the solution would be $3000\text{ s}\times\frac{60\text{ s}}{1\text{ min}}=18{,}000\text{ s}^2/\text{min}$. Since the question asked for minutes, you would know there was an error since the units of s^2/min are incorrect.

Pg. 81. Understand: You need to find the area of a room in square feet and then convert that answer to square yards. Solve: Find the area of the room by multiplying the length and width. Use the appropriate conversion factor to change the solution of 120 ft^2 to 13.3 yd^2. Explain: 13.3 square yards are needed to carpet the room, so you will need to buy 14 square yards to complete the project.

Pg. 82. $\frac{1}{1.221}\approx 0.819$, $\frac{1}{0.7586}\approx 1.318$, $\frac{1}{1.058}\approx 0.9452$, $\frac{1}{0.008658}\approx 115.5$, and $\frac{1}{0.04574}\approx 21.86$. The values in the two columns are reciprocals of one another (at least approximately). For example, the conversion factor used to convert from euros to U.S. dollars is $\frac{\$1.058}{1\text{ euro}}$ and the conversion factor to change from U.S. dollars to euros is $\frac{1\text{ euro}}{\$1.058}$. These numbers are reciprocals.

Pg. 89. Answers will vary, but this question should help the student make the connection between abstract ideas of measurement and the ancient origin of units as body measures. This would be a good topic for a discussion either during or outside of class.

Pg. 91. Literally, a megabuck is a million dollars. But it is used colloquially to mean "a lot of money." Another example of the use of metric prefixes is "nanotechnology" for very small machines. This would be a good topic for a discussion either during or outside of class.

QUICK QUIZ

1. **b**. Speed is described by distance per unit of time, so dividing a distance by a time is the correct choice.
2. **a**. Think of the unit *miles per hour*; the unit of *mile* is divided by the unit of *hour*.
3. **b**. "Of" implies multiplication.
4. **a**. Take dollars per gallon and divide it by miles per gallon, and you get $\frac{\$}{\text{gal}}\div\frac{\text{mi}}{\text{gal}}=\frac{\$}{\text{gal}}\times\frac{\text{gal}}{\text{mi}}=\frac{\$}{\text{mi}}$.
5. **b**. The area of a square is its length multiplied by its width (these are, of course, equal for squares), and thus a square of side length 3 mi has area of $3\text{ mi}\times 3\text{ mi}=9\text{ mi}^2$.
6. **c**. When multiplying quantities that have units, the units are also multiplied, so $\text{ft}^2\times\text{ft}=\text{ft}^3$.
7. **b**. $1\text{ mi}^3=(1760\text{ yd})^3=1760^3\text{ yd}^3$
8. **c**. $1\text{ ft}^2=12\text{ in}\times 12\text{ in}=144\text{ in}^2$
9. **a**. Apples are most likely to be sold by units of weight (or more accurately, mass), and thus euros per kilogram is the best answer.
10. **b**. \$1.058 per euro means 1 euro = \$1.058, which is more than \$1.

REVIEW QUESTIONS

1. See the Information Box on Page 72. Examples will vary.

2. Units associated with a number describe what we are measuring or counting, such as hours, miles, or pounds. If the expected units for a final solution are known, that can help determine how to combine the various values in a problem. Also, if the results of a calculation match those expected units, you can be more confident that the solution was found correctly.

3. *Per* means "for every," *of* implies multiplication, *square* implies the second power, and *cube* implies the third power. A hyphen represents the product of different units.

4. In unit conversion, the fraction involves a numerator and denominator that represent the same value, so you are in essence dividing a value by itself, which results in a value of the fraction of 1. The two forms of 1 that are equivalent to $1 \text{ lb} = 16 \text{ oz}$ are $\frac{16 \text{ oz}}{1 \text{ lb}}$ and $\frac{1 \text{ lb}}{16 \text{ oz}}$.

5. A square yard is three feet wide by three feet high, so it contains $3 \text{ ft} \times 3 \text{ ft} = 9 \text{ ft}^2$. A cubic yard is three feet wide by three feet high by three feet deep, so it contains $3 \text{ ft} \times 3 \text{ ft} \times 3 \text{ ft} = 27 \text{ ft}^3$. To find the conversion factor for squares and cubes, we square or cube the equivalent measure of the unit not raised to an exponent. In the preceding example, to find square yards and cubic yards, the number of feet in a yard was squared and cubed, respectively.

6. For a given foreign currency, the "Dollars per Foreign" column gives the value of a single unit of the foreign currency in dollars. To determine the dollar value of a foreign currency, you multiply the amount of foreign currency by this conversion value. The "Foreign per Dollar" column gives the foreign currency value of a single dollar. To determine the foreign value of a given dollar amount, you multiply the dollar amount of currency by this conversion value.

DOES IT MAKE SENSE?

7. Does not make sense. 35 miles is a distance, not a speed.

8. Does not make sense. One yen would be worth about 1/115th of $1.

9. Makes sense. Using the familiar formula $distance = rate \times time$, one can see that dividing the distance by the rate (or speed) will result in time.

10. Does not make sense. Two ft^2 describes an area, not a volume.

11. Does not make sense. $\frac{\$3}{1 \text{ ft}^2} \times \frac{9 \text{ ft}^2}{1 \text{ yd}^2} = \$27/\text{yd}^2$, which is more than the price at Y-mart.

12. Does not make sense. For this to be true, the car would have to be moving at less than 10 feet per mile.

BASIC SKILLS AND CONCEPTS

13. a. $\frac{3}{4} \times \frac{1}{2} = \frac{3 \cdot 1}{4 \cdot 2} = \frac{3}{8}$

b. $\frac{2}{3} \times \frac{3}{5} = \frac{2 \cdot 3}{3 \cdot 5} = \frac{2}{5}$

c. $\frac{1}{2} + \frac{3}{2} = \frac{1+3}{2} = \frac{4}{2} = 2$

d. $\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{4+1}{6} = \frac{5}{6}$

e. $\frac{2}{3} \times \frac{1}{4} = \frac{2 \cdot 1}{3 \cdot 4} = \frac{2}{12} = \frac{1}{6}$

f. $\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{2+3}{8} = \frac{5}{8}$

g. $\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{5-2}{8} = \frac{3}{8}$

h. $\frac{3}{2} \times \frac{2}{3} = \frac{3 \cdot 2}{3 \cdot 2} = 1$

14. a. $\frac{1}{3}+\frac{1}{5}=\frac{5}{15}+\frac{3}{15}=\frac{8}{15}$

b. $\frac{10}{3}\times\frac{3}{7}=\frac{10\cdot 3}{3\cdot 7}=\frac{10}{7}=1\frac{3}{7}$

c. $\frac{3}{4}-\frac{1}{8}=\frac{6}{8}-\frac{1}{8}=\frac{5}{8}$

d. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}=\frac{6}{12}+\frac{8}{12}+\frac{9}{12}=\frac{23}{12}=1\frac{11}{12}$

e. $\frac{6}{5}+\frac{4}{15}=\frac{18}{15}+\frac{4}{15}=\frac{22}{15}=1\frac{7}{15}$

f. $\frac{3}{5}\times\frac{2}{7}=\frac{3\cdot 2}{5\cdot 7}=\frac{6}{35}$

g. $\frac{1}{3}+\frac{13}{6}=\frac{2}{6}+\frac{13}{6}=\frac{15}{6}=\frac{5}{2}=2\frac{1}{2}$

h. $\frac{3}{5}\times\frac{10}{3}\times\frac{3}{2}=\frac{3\cdot 10\cdot 3}{5\cdot 3\cdot 2}=\frac{3}{1}=3$

15. Answers may vary depending on whether fractions are reduced.

a. $3.5=\frac{35}{10}=\frac{7}{2}$

b. $0.3=\frac{3}{10}$

c. $0.05=\frac{5}{100}=\frac{1}{20}$

d. $4.1=\frac{41}{10}$

e. $2.15=\frac{215}{100}=\frac{43}{20}$

f. $0.35=\frac{35}{100}=\frac{7}{20}$

g. $0.98=\frac{98}{100}=\frac{49}{50}$

h. $4.01=\frac{401}{100}$

16. Answers may vary depending on whether fractions are reduced.

a. $2.75=\frac{275}{100}=\frac{11}{4}$

b. $0.45=\frac{45}{100}=\frac{9}{20}$

c. $0.005=\frac{5}{1000}=\frac{1}{200}$

d. $1.16=\frac{116}{100}=\frac{29}{25}$

e. $6.5=\frac{65}{10}=\frac{13}{2}$

f. $4.123=\frac{4123}{1000}$

g. $0.0003=\frac{3}{10{,}000}$

h. $0.034=\frac{34}{1000}=\frac{17}{500}$

17. a. $\frac{1}{4}=0.25$

b. $\frac{3}{8}=0.375$

c. $\frac{2}{3}\approx 0.667$

d. $\frac{3}{5}=0.6$

e. $\frac{13}{2}=6.5$

f. $\frac{23}{6}\approx 3.833$

g. $\frac{103}{50}=2.06$

h. $\frac{42}{26}\approx 1.615$

18. a. $\frac{1}{5}=0.2$

b. $\frac{4}{9}\approx 0.444$

c. $\frac{4}{11}\approx 0.364$

d. $\frac{12}{7}\approx 1.714$

e. $\frac{28}{9}\approx 3.111$

f. $\frac{56}{11}\approx 5.091$

g. $\frac{102}{49}\approx 2.082$

h. $\frac{15}{4}=3.75$

19. $28 \text{ ft}^3 \times \frac{64.2 \text{ lb}}{1 \text{ ft}^3} = 1797.6 \text{ lb}$

20. $\frac{25{,}000 \text{ mi}}{24 \text{ hr}} = 1041.7 \text{ mi/hr}$

21. $2.5 \text{ lb} \times \frac{\$1.25}{1 \text{ lb}} = \$3.13$

22. $11 \text{ basketballs} \times \frac{22 \text{ oz}}{1 \text{ basketball}} = 242 \text{ oz}$

23. $\frac{\$420}{23 \text{ hours}} = \$18.26 \text{ \$/hr}$

24. $495 \text{ people} \times \frac{1 \text{ bus}}{45 \text{ people}} = 11 \text{ buses}$

25. a. The area of the storage pod's floor is $12 \text{ ft} \times 20 \text{ ft} = 240 \text{ ft}^2$, and the volume of the storage pod is $12 \text{ ft} \times 20 \text{ ft} \times 8 \text{ f} = 1920 \text{ ft}^3$.

b. The surface area of the pool is $25 \text{ yd} \times 20 \text{ yd} = 500 \text{ yd}^2$, and the volume of water it holds is $25 \text{ yd} \times 20 \text{ yd} \times 2 \text{ yd} = 1000 \text{ yd}^3$.

c. The area of the bed is $30 \text{ ft} \times 6 \text{ ft} = 180 \text{ ft}^2$, and the volume of soil it holds is $30 \text{ ft} \times 6 \text{ ft} \times 1.2 \text{ ft} = 216 \text{ ft}^3$.

26. a. The area of the warehouse floor is $60 \text{ yd} \times 30 \text{ yd} = 1800 \text{ yd}^2$ and the volume of the cartons is $\frac{1}{2} \times 60 \text{ yd} \times 30 \text{ yd} \times 6 \text{ yd} = 5400 \text{ yd}^3$.

b. The area of the room's floor is $24 \text{ ft} \times 16 \text{ ft} = 384 \text{ ft}^2$ and the volume of air the room holds is $24 \text{ ft} \times 15 \text{ ft} \times 8 \text{ ft} = 3072 \text{ ft}^3$.

c. The volume of the silo is the area of its base multiplied by its height, which is $260 \text{ ft}^2 \times 22 \text{ ft} = 5720 \text{ ft}^3$.

27. Speed has units of miles per hour, or mi/hr.

28. The per-mile price of airline tickets has units of dollars per mile, or \$/mi.

29. The flow rate has units of cubic inches per second, or cfs (or in^3/s).

30. The price of a bottle of perfume has units of euros.

31. The cost of a car trip has units of dollars, or \$.

32. The units for the number of bagels would be bagels.

33. Convert 1.2 cubic yards to dollars by using the conversion $1 \text{ cubic yard} = \$24$: $1.2 \text{ yd}^3 \times \frac{\$24}{\text{yd}^3} = \$28.80$.

34. Convert 400 gallons to minutes by using the conversion $4.5 \text{ gallons} = 1 \text{ minute}$: $400 \text{ gal} \times \frac{1 \text{ min}}{4.5 \text{ gal}} = 88.9 \text{ min}$.

35. Convert 2.5 ounces to dollars by using the conversion $1 \text{ oz} = \$1200$: $2.5 \text{ oz} \times \frac{\$1200}{\text{oz}} = \$3000$.

36. You would earn $24 \text{ days} \times \frac{8 \text{ hr}}{1 \text{ day}} \times \frac{\$10.30}{\text{hr}} = \$1977.60$ in that month.

37. First, note that 321 million people is 3210 groups of 100,000 people each. The mortality rate is $\frac{595{,}700 \text{ deaths}}{321{,}000{,}000 \text{ people}} \times \frac{321{,}000{,}000 \text{ people}}{3210 \text{ groups of } 100{,}000} = 185.6 \text{ deaths/100,000 people}.$.

38. The population density for Manila is about $\frac{1{,}800{,}000 \text{ people}}{16.6 \text{ mi}^2} = 108{,}434 \ \frac{\text{people}}{\text{mi}^2}$.

39. The cost to drive 30 miles is $30 \text{ mi} \times \frac{1 \text{ gal}}{32 \text{ mi}} \times \frac{\$2.55}{1 \text{ gal}} = \$2.39.$

40. He is paid $\frac{\$30,700,000}{30 \text{ games}} \approx \$1,023,333/\text{game},$ or about $1.02 million per game.

41. The solution is wrong. It's helpful to include units on your numerical values. This would have signaled your error, as illustrated: (*incorrect*) $0.11 \text{ lb} \div \frac{\$7.70}{\text{lb}} = \frac{0.11 \text{ lb}^2}{\$7.70} = 0.014 \frac{\text{lb}^2}{\$}.$ Notice that your solution, when the correct units are added, produces units of square pounds per dollar, which isn't very helpful, and doesn't answer the question. The correct solution is $0.11 \text{ lb} \times \frac{\$7.70}{\text{lb}} = \$0.85.$

42. The solution is wrong. Attach units to your numerical quantities so that you can determine whether the answer has correct units. When it doesn't, this is an indication there's an error. Here's your incorrect solution with units attached: $\frac{5 \text{ mi}}{\text{hr}} \div 3 \text{ hr} = \frac{5 \text{ mi}}{\text{hr}} \times \frac{1}{3 \text{ hr}} = 1.7 \frac{\text{mi}}{\text{hr}^2}.$ It is clear this is incorrect as the question asked how far did you go, and thus you should expect an answer with units of miles. The correct solution is $3 \text{ hr} \times \frac{5 \text{ mi}}{\text{hr}} = 15 \text{ mi}.$

43. The solution is wrong. Units should be included with all quantities. When dividing by $11, the unit of dollars goes with the 11 into the denominator (as shown below). Also, while it's reasonable to round an answer to the nearest tenth, it's more useful to round to the nearest hundredth in a problem like this, as you'll be comparing the price per pound of the large bag to the price per pound of the small bag, which is 39¢ per pound. Here's your solution with all units attached, division treated as it should be, and rounded to the hundredth-place:

$$50 \text{ lb} \div \$11 = \frac{50 \text{ lb}}{\$11} = 4.55 \frac{\text{lb}}{\$}.$$

This actually produces some useful information, as you can see a dollar buys 4.55 pounds of flour. This is a better buy than 39¢ per pound, which is roughly 3 pounds for a dollar. But it's better to find the price per pound for the large bag:

$$\$11 \div 50 \text{ lb} = \frac{\$11}{50 \text{ lb}} = 0.22 \frac{\$}{\text{lb}} = 22¢ \text{ per pound.}$$

Now you can compare this value to the price per pound of the small bag (39¢/lb). The large bag is the better buy.

44. The solution is wrong. Always include units with numerical quantities; this helps in deciding whether the correct procedure was followed. Here's your solution with units attached:

$$\frac{1500 \text{ Cal}}{\text{d}} \times \frac{140 \text{ Cal}}{\text{Coke}} = 210,000 \frac{\text{Cal}^2}{\text{d-Coke}}.$$

It's clear this calculation won't answer the question (note that as shown, there's nothing mathematically incorrect about the above calculation – it just doesn't provide useful information). The correct solution looks like this: $\frac{1500 \text{ Cal}}{\text{d}} \times \frac{1 \text{ Coke}}{140 \text{ Cal}} = 10.7 \frac{\text{Cokes}}{\text{d}}.$

Thus you would need about 11 Cokes per day to meet your caloric needs.

45. $32 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} = 384 \text{ in}$

46. $16 \text{ ft} \times \frac{1 \text{ yd}}{3 \text{ ft}} = 5\frac{1}{3} \text{ yd},$ or 5.33 yd

47. $35 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} = 2100 \text{ s}$

48. $17 \text{ year} \times \frac{365 \text{ day}}{1 \text{ year}} = 6205 \text{ days}$

49. $4.2 \text{ hr} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ s}}{1 \text{ min}} = 15,120 \text{ s}$

50. $17{,}200\ \frac{\text{mi}}{\text{hr}} \times \frac{1\ \text{hr}}{60\ \text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} \approx 4.78\ \frac{\text{mi}}{\text{s}}$

51. $4\ \text{yr} \times \frac{365\ \text{day}}{1\ \text{yr}} \times \frac{24\ \text{hr}}{1\ \text{day}} = 35{,}040\ \text{hr}$

52. $45{,}789\ \text{in} \times \frac{1\ \text{ft}}{12\ \text{in}} \times \frac{1\ \text{yd}}{3\ \text{ft}} \times \frac{1\ \text{mi}}{1760\ \text{yd}} \approx 0.72\ \text{mi}$

53. Note that $1\ \text{ft} = 12\ \text{in}$, and thus $(1\ \text{ft})^2 = (12\ \text{in})^2$, which means $1\ \text{ft}^2 = 144\ \text{in}^2$. This can also be written as $\frac{1\ \text{ft}^2}{144\ \text{in}^2} = 1$, or $\frac{144\ \text{in}^2}{1\ \text{ft}^2} = 1$.

54. The area of the yard is $20\ \text{yd} \times 12\ \text{yd} = 240\ \text{yd}^2$. Since $1\ \text{yd} = 3\ \text{ft}$, we have $(1\ \text{yd})^2 = (3\ \text{ft})^2$, or $1\ \text{yd}^2 = 9\ \text{ft}^2$. This can be used to convert to square feet: $240\ \text{yd}^2 \times \frac{9\ \text{ft}^2}{1\ \text{yd}^2} = 2160\ \text{ft}^2$.

55. $3.5\ \text{acres} \times \frac{43{,}560\ \text{ft}^2}{1\ \text{acre}} = 152{,}460\ \text{ft}^2$

56. $100\ \text{yd} \times 60\ \text{yd} \times \left(\frac{3\ \text{ft}}{1\ \text{yd}}\right)^2 = 54{,}000\ \text{ft}^2$

57. Since $1\ \text{m} = 100\ \text{cm}$, we have $(1\ \text{m})^3 = (100\ \text{cm})^3$, which means $1\ \text{m}^3 = 1{,}000{,}000\ \text{cm}^3$. This can also be written as $\frac{1\ \text{m}^3}{1{,}000{,}000\ \text{cm}^3} = 1$, or $\frac{1{,}000{,}000\ \text{cm}^3}{1\ \text{m}^3} = 1$.

58. The volume of the sidewalk is $4\ \text{ft} \times 150\ \text{ft} \times 0.5\ \text{ft} = 300\ \text{ft}^3$. Since $1\ \text{yd} = 3\ \text{ft}$, we have $(1\ \text{yd})^3 = (3\ \text{ft})^3$, or $1\ \text{yd}^3 = 27\ \text{ft}^3$. This can be used to convert to cubic yards: $300\ \text{ft}^3 \times \frac{1\ \text{yd}^3}{27\ \text{ft}^3} = 11.1\ \text{yd}^3$.

59. Since $1\ \text{yd} = 3\ \text{ft}$, we have $(1\ \text{yd})^3 = (3\ \text{ft})^3$, or $1\ \text{yd}^3 = 27\ \text{ft}^3$. Therefore $350\ \text{ft}^3 \times \frac{1\ \text{yd}^3}{27\ \text{ft}^3} = 13.0\ \text{yd}^3$.

60. Since $1\ \text{ft} = 12\ \text{in}$, $(1\ \text{ft})^3 = (12\ \text{in})^3$, which means $1\ \text{ft}^3 = 1728\ \text{in}^3$. Therefore, $3.5\ \text{ft}^3 \times \frac{1728\ \text{in}^3}{1\ \text{ft}^3} = 6048\ \text{in}^3$.

61. $82\ \text{pounds} \times \frac{\$1.221}{1\ \text{pound}} = \$100.12$ or $82\ \text{Pounds} \times \frac{\$1}{0.8191\ \text{Pound}} = \$100.11$

62. $45{,}000\ \text{Yen} \times \frac{\$0.008658}{1\ \text{Yen}} = \$389.61$ or $45{,}000\ \text{Yen} \times \frac{\$1}{115.5\ \text{Yen}} = \$389.61$

63. $320\ \text{euros} \times \frac{\$1.058}{1\ \text{euro}} = \$338.56$ or $320\ \text{euros} \times \frac{\$1}{0.9449\ \text{euro}} = \$338.66$

64. $2500\ \text{pesos} \times \frac{\$0.04574}{1\ \text{peso}} \approx \$114.35$ or $2500\ \text{pesos} \times \frac{\$1}{21.86\ \text{peso}} = \$114.36$

65. $\frac{1.5\ \text{euro}}{1\ \text{L}} \times \frac{3.785\ \text{L}}{1\ \text{gal}} \times \frac{\$1.058}{1\ \text{euro}} = \frac{\$6.01}{\text{gal}}$

66. $\frac{28\ \text{pesos}}{1\ \text{kg}} \times \frac{\$1}{21.86\ \text{pesos}} \times \frac{1\ \text{kg}}{2.205\ \text{lb}} = \frac{\$0.58}{\text{lb}}$

FURTHER APPLICATIONS

67. The train is traveling with speed $\frac{45\text{ mi}}{34\text{ min}}$, and this needs to be converted to mi/hr.

$$\frac{45\text{ mi}}{34\text{ min}} \times \frac{60\text{ min}}{1\text{ hr}} = 79.4\ \frac{\text{mi}}{\text{hr}}$$

68. The skydiver is traveling with speed 614 mi/hr, and this needs to be converted to ft/s.

$$\frac{614\text{ mi}}{1\text{ hr}} \times \frac{1\text{ hr}}{60\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{5280\text{ ft}}{1\text{ mi}} = 900.5\ \frac{\text{ft}}{\text{s}}$$

69. Convert 3,998,000 births per year into births per minute. $\frac{3{,}998{,}000\text{ births}}{1\text{ yr}} \times \frac{1\text{ yr}}{365\text{ d}} \times \frac{1\text{ d}}{24\text{ hr}} \times \frac{1\text{ hr}}{60\text{ min}}$ $= 7.6\frac{\text{births}}{\text{min}}$. Note that 321 million people is 321,000 groups of 1000 people each. The annual birth rate is $\frac{3{,}998{,}000\text{ births}}{321{,}000{,}000\text{ people}} \times \frac{321{,}000{,}000\text{ people}}{321{,}000\text{ groups of 1000}} = 12.5\text{ births/100,000 people.}$

70. Convert one year into hours of sleep: $1\text{ yr} \times \frac{365\text{ d}}{\text{yr}} \times \frac{7.5\text{ hr (of sleep)}}{\text{d}} = 2737.5\text{ hr.}$

71. Convert one lifetime into heart beats: $1\text{ lifetime} \times \frac{80\text{ yr}}{\text{lifetime}} \times \frac{365\text{ d}}{\text{yr}} \times \frac{24\text{ hr}}{1\text{ d}} \times \frac{60\text{ min}}{1\text{ hr}} \times \frac{65\text{ beats}}{1\text{ min}}$ $= 2{,}733{,}120{,}000\text{ beats}$, or about 2.7 billions heart beats.

72. Convert 2000 miles into gallons by using the conversion $32\text{ miles} = 1\text{ gallon}$: $2000\text{ mi} \times \frac{1\text{ gal}}{32\text{ mi}} = 62.5\text{ gal.}$ Yes; a car that has half the gas mileage would need twice as much gas. Halving the value of the denominator has the same effect as doubling the value of the fraction.

73. a. One full tank of gas for Car A costs $12\text{ gal} \times \frac{\$2.55}{1\text{ gal}} = \$30.60$ and one full tank of gas for Car B costs $20\text{ gal} \times \frac{\$2.55}{1\text{ gal}} = \$51.00.$

b. Car A will use $3000\text{ mi} \times \frac{1\text{ gal}}{40\text{ mi}} \times \frac{1\text{ tank}}{12\text{ gal}} = 6.25$ tanks of gas and Car B will use $3000\text{ mi} \times \frac{1\text{ gal}}{30\text{ mi}} \times \frac{1\text{ tank}}{20\text{ gal}}$ $= 5$ tanks of gas.

c. The driver of Car A will spend $6.25\text{ tanks} \times \frac{\$30.60}{1\text{ tank}} = \$191.50$ and the driver of Car B will spend $5\text{ tanks} \times \frac{\$51.00}{1\text{ tank}} = \$255.00.$

74. a. The driving time when traveling at 55 miles per hour is $2000\text{ mi} \times \frac{1\text{ hr}}{55\text{ mi}} = 36.4\text{ hr}$, while the time at 70 miles per hour is $2000\text{ mi} \times \frac{1\text{ hr}}{70\text{ mi}} = 28.6\text{ hr.}$

74. (continued)

b. Your car gets 38 miles to the gallon when driving at 55 mph, so the cost is: $2000 \text{ mi} \times \frac{1 \text{ gal}}{38 \text{ mi}} \times \frac{\$2.55}{\text{gal}}$ $= \$134.21.$

Your car gets 32 miles to the gallon when driving at 70 mph, so the cost is: $2000 \text{ mi} \times \frac{1 \text{ gal}}{32 \text{ mi}} \times \frac{\$2.55}{\text{gal}}$ $= \$159.38.$

75. a. The driving time when traveling at 60 miles per hour is $1500 \text{ mi} \times \frac{1 \text{ hr}}{60 \text{ mi}} = 25 \text{ hr},$ while the time at 75 miles per hour is $1500 \text{ mi} \times \frac{1 \text{ hr}}{75 \text{ mi}} = 20 \text{ hr}.$

b. Your car gets 32 miles to the gallon when driving at 60 mph, so the cost is: $1500 \text{ mi} \times \frac{1 \text{ gal}}{32 \text{ mi}} \times \frac{\$2.55}{\text{gal}}$ $= \$119.53.$

Your car gets 25 miles to the gallon when driving at 75 mph, so the cost is: $1500 \text{ mi} \times \frac{1 \text{ gal}}{25 \text{ mi}} \times \frac{\$2.55}{\text{gal}}$ $= \$153.00.$

76. a. Convert \$31,000,000 per 80 games into dollars per game: $\frac{\$31,000,000}{80 \text{ games}} = \$387,500/\text{game}.$

b. Convert \$387,500 per game into dollars per minute: $\frac{\$387,500}{1 \text{ game}} \times \frac{1 \text{ game}}{48 \text{ min}} \approx \$8073/\text{min}.$

c. His total hours spent training and playing were $\frac{40 \text{ hrs}}{1 \text{ game}} \times 80 \text{ games} = 3200 \text{ hours}.$ Convert \$31,000,000 per 3200 hours into dollars per hour: $\frac{\$31,000,000}{3200 \text{ hr}} = \$9688/\text{hr}.$

77. Convert 6 breaths per minute into liters of warmed air per day using 1 minute = 6 breaths, and 1 breath = 0.5 L.

$$\frac{6 \text{ breaths}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{0.5 \text{ L}}{1 \text{ breath}} \times \frac{24 \text{ hr}}{1 \text{ day}} = 4320 \frac{\text{L}}{\text{day}}$$

78. a. Convert 16 gigabytes into pages, using the fact that 2000 bytes are need to store 1 page of text.

$$16 \text{ Gbyte} \times \frac{1 \text{ billion bytes}}{1 \text{ Gbyte}} \times \frac{1 \text{ page}}{2000 \text{ bytes}} = 8,000,000 \text{ pages}$$

b. $8,000,000 \text{ pages} \times \frac{1 \text{ book}}{500 \text{ pages}} = 16,000 \text{ books}$

79. Answers will vary depending on prices. The area of the yard is $60 \text{ ft} \times 35 \text{ ft} = 2100 \text{ ft}^2.$

a. The cost to plant the region with grass seed would be $\frac{\text{Cost of grass seed}}{1 \text{ ft}^2} \times 2100 \text{ ft}^2.$

b. The cost to cover the region with sod would be $\frac{\text{Cost of sod}}{1 \text{ ft}^2} \times 2100 \text{ ft}^2.$

79. (continued)

c. The cost to cover the region with topsoil and plant bulbs would be $\frac{\text{Cost of topsoil}}{\text{ft}^2} \times 2100 \text{ ft}^2$

$+\frac{\text{Cost}}{1 \text{ bulb}} \times \frac{2 \text{ bulbs}}{1 \text{ ft}^2} \times 2100 \text{ ft}^2.$

USING TECHNOLOGY

85. Answers will vary.

UNIT 2B: EXTENDING UNIT ANALYSIS

THINK ABOUT IT

Pg. 96. Even without carrying out the conversions, it should be obvious that this is a Fahrenheit temperature; $59°\text{C}$ is more than halfway between the freezing and boiling points of water, which means it is well over 100°F (a precise conversion shows it is $138.2°\text{F}$). No populated place on Earth gets this hot, so the forecast of 59° must refer to a Fahrenheit temperature.

Pg. 98(1st). Answers will vary. Note: many utility bills do not give the price per kilowatt-hour explicitly, but it can be calculated by dividing the total price charged for electricity by the metered usage in kilowatt-hours, both of which are almost always shown clearly.

Pg. 98(2nd). You have a greater total volume but essentially the same mass (except for the weight of air) when your lungs are filled with air, so your density is lower and you float better.

Pg. 101. Two beers contain enough alcohol to put you at 12 times the legal limit if all of it were in your bloodstream at once. Given the metabolic absorption rate, you should wait at least 3 hours before driving.

QUICK QUIZ

1. **a**. Divide both sides of $1 \text{ L} = 1.057 \text{ qt}$ by 1 L.
2. **a**. Since one meter is a little longer than one yard, 1200 meters will be longer than 3600 feet.
3. **b**. Intuitively, it makes sense to divide a volume (with units of, say, ft^3) by a depth (with units of ft) to arrive at surface area (which has units of ft^2).
4. **b**. A watt is defined to be one joule per second, which is a unit of power, not energy.
5. **c**. Energy used by an appliance is computed by multiplying the power rating by the amount of time the appliance is used, so you need to know how long the light bulb is on in order to compute the energy it uses in that time span.
6. **a**. Multiplying the population density, which has units of $\frac{\text{people}}{\text{mi}^2}$, by the area, which has units of mi^2, will result in an answer with units of people.
7. **c**. Water boils at $100°\text{C}$ (at sea level), so $110°\text{C}$ is boiling hot.
8. **b**. Concentrations of gases are often stated in parts per million (and the other two answers make no sense).
9. **c**. Multiplying by 30 kg will give the daily dose, which should be divided by three to determine the dose for one eight hour period.
10. **b**. $4 \text{ L} \times \frac{0.08 \text{ gm}}{100 \text{ mL}} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 0.08 \text{ gm} \times 40 = 3.2 \text{ gm}$

REVIEW QUESTIONS

1. Standardized units have the same meaning for everyone. Standardized units allow measurements to be used by people with no misunderstanding of what they mean or represent.
2. The basic metric units are: the meter for length, abbreviated m, the kilogram for mass, abbreviated kg, the second for time, abbreviated s, and the liter for volume, abbreviated L. Each of these basic units can be combined with a prefix that indicates multiplication by a power of 10.

3. Energy is what makes matter move or heat up. The international metric unit of energy is the joule. Calories are used to measure the energy our bodies can draw from food, where 1 Calorie = 4184 joules. Electrical energy is usually measured in kilowatt-hours, where 1 kilowatt-hour = 3.6 million joules.

4. $95^\circ\text{ F} = \frac{5}{9}(80-32) = 35^\circ\text{C}$ $35^\circ\text{C} = \frac{9}{5}(35)+32 = 95^\circ\text{ F}$

   $35^\circ\text{C} = 35+273.12 = 308.15\text{ K}$ $308.15\text{ K} = 308.15-273.12 = 35^\circ\text{C}$

   $95^\circ\text{ F} = \frac{5}{9}(80-32) = 35^\circ\text{C} = 35+273.12 = 308.15\text{ K}$

   $308.15\text{ K} = 308.15-273.12 = 35^\circ\text{C} = \frac{9}{5}(35)+32 = 95^\circ\text{ F}$

5. Power is the rate at which energy is used. The international metric unit of power is the watt, defined as $1\text{ watt} = 1\ \dfrac{\text{joule}}{\text{s}}$.

6. Density is generally used to describe compactness or crowding. Material density is usually measured in units of mass per unit volume, such as g/cm^3. Population density is measured by the number of people per unit area, such as $\text{people}/\text{mi}^2$ or $\text{people}/\text{km}^2$. Information density is used to describe how much memory can be stored by digital media. It is usually measured in byte, megabytes, or gigabytes.

   Concentration is generally used to describe the amount of one substance that is mixed with another. Air pollutants are usually measured by the number of molecules of the pollutant per million molecules of air or ppm. Medicine dosages are usually measured in $\text{mg of drug}/\text{ml of solution}$ or $\text{mg of drug/kg of body weight}$. Blood alcohol content is measures in $\text{g of drug}/100\text{ ml of blood}$.

DOES IT MAKE SENSE?

7. Makes sense. Liquids are measured in liters, and since one liter is about a quart, drinking two liters is a reasonable thing to do.

8. Does not make sense. First of all, we use the unit of kilogram to measure mass, not weight, though mass and weight are often used interchangeably in everyday conversation. Even so, a bicyclist with a mass of 300 kg would weigh more than 650 pounds (on the surface of the earth), which is an unheard of weight for a professional cyclist.

9. Does not make sense. The unit of meter measures length, not volume.

10. Makes sense. There are 4184 joules in a Calorie, and when 10,000,000 joules is converted into Calories, the result is about 2400 Calories, which is a typical caloric intake for an active person.

11. Does not make sense. The volume of a sphere is $V = \dfrac{4}{3}\pi r^3$, so the volume of a beach ball with a radius of 20 cm (about 8 inches) would be more than $32{,}000\text{ cm}^3$. This translates into a mass of 320,000 grams if the density is 10 grams per cm^3, which is 320 kg. The mass of a beach ball isn't anywhere near that large.

12. Does not make sense. $\dfrac{15\text{ people}}{1\text{ km}^2} \times \left(\dfrac{1\text{ km}}{1000\text{ m}}\right)^2 = 0.000015$ people per square meter, which is very small.

BASIC SKILLS AND CONCEPTS

13. a. $10^4 \times 10^7 = 10^{4+7} = 10^{11}$

b. $10^5 \times 10^{-3} = 10^{5-3} = 10^2$

c. $10^6 \div 10^2 = 10^{6-2} = 10^4$

d. $\dfrac{10^8}{10^{-4}} = 10^{8-(-4)} = 10^{12}$

e. $\dfrac{10^{12}}{10^{-4}} = 10^{12-(-4)} = 10^{16}$

f. $10^{23} \times 10^{-23} = 10^{23-23} = 10^0 = 1$

g. $10^4 + 10^2 = 10{,}000 + 100 = 10{,}100$

h. $10^{15} \div 10^{-5} = 10^{15-(-5)} = 10^{20}$

14. a. $10^{-2} \times 10^{-6} = 10^{-2+(-6)} = 10^{-8}$

b. $\dfrac{10^{-6}}{10^{-8}} = 10^{-6-(-8)} = 10^2$

c. $10^{12} \times 10^{23} = 10^{12+23} = 10^{35}$

d. $\dfrac{10^{-4}}{10^5} = 10^{-4-5} = 10^{-9}$

e. $\dfrac{10^{25}}{10^{15}} = 10^{25-15} = 10^{10}$

f. $10^1 + 10^0 = 10 + 1 = 11$

g. $10^2 + 10^{-1} = 100 + 0.1 = 100.1$

h. $10^2 - 10^1 = 100 - 10 = 90$

15. a. $10 \text{ furlongs} \times \dfrac{1 \text{ mi}}{8 \text{ furlongs}} = 1.25 \text{ miles}$

b. $10 \text{ furlongs} \times \dfrac{1 \text{ mi}}{8 \text{ furlongs}} \times \dfrac{5280 \text{ ft}}{1 \text{ mi}} \times \dfrac{1 \text{ yd}}{3 \text{ ft}} = 2200 \text{ yards}$

16. a. $36{,}198 \text{ ft} \times \dfrac{1 \text{ fathom}}{6 \text{ ft}} = 6033 \text{ fathoms}$

b. $36{,}198 \text{ ft} \times \dfrac{1 \text{ naut. mi}}{6076.1 \text{ ft}} \times \dfrac{1 \text{ marine league}}{3 \text{ naut. mi}} \approx 1.99 \text{ marine leagues}$

17. Convert cubic feet of water into pounds: $6 \text{ ft}^3 \text{ (of water)} \times \dfrac{7.48 \text{ gal}}{1 \text{ ft}^3} \times \dfrac{8.33 \text{ lb}}{1 \text{ gal}} = 373.9 \text{ lb.}$

Now convert that answer into tons: $373.9 \text{ lb} \times \dfrac{1 \text{ ton}}{2000 \text{ lb}} = 0.19 \text{ ton.}$

18. $15 \text{ bags} \times \dfrac{40 \text{ lb}}{1 \text{ bag}} \times \dfrac{1 \text{ kg}}{2.205 \text{ lb}} \approx 272 \text{ kg}$; $15 \text{ bags} \times \dfrac{2 \text{ ft}^3}{1 \text{ bag}} \times \dfrac{1 \text{ yd}^3}{27 \text{ ft}^3} \approx 1.1 \text{ yd}^3$

19. a. $30 \text{ knots} = \dfrac{30 \text{ naut. mi}}{\text{hr}} \times \dfrac{6076.1 \text{ ft}}{1 \text{ naut. mi}} \times \dfrac{1 \text{ mi}}{5280 \text{ ft}} = 34.5 \dfrac{\text{mi}}{\text{hr}}$

b. $102{,}000 \text{ tons} \times \dfrac{2000 \text{ lb}}{1 \text{ ton}} \times \dfrac{1 \text{ kg}}{2.205 \text{ lb}} \times \dfrac{1 \text{ metric tonne}}{1000 \text{ kg}} = 92{,}517 \text{ metric tonnes}$

20. $\dfrac{1 \text{ mi}}{4 \text{ min}} \times \dfrac{60 \text{ min}}{1 \text{ hr}} = 15 \dfrac{\text{mi}}{\text{hr}}$; $\dfrac{1 \text{ mi}}{4 \text{ min}} \times \dfrac{1 \text{ min}}{60 \text{ s}} \times \dfrac{5280 \text{ ft}}{1 \text{ mi}} = 22 \dfrac{\text{f}}{\text{s}}$

21. 6-ounce bottle

$\dfrac{\$3.99}{6 \text{ oz}} = \dfrac{\$0.665}{1 \text{ oz}}$

14-ounce bottle

$\dfrac{\$9.49}{14 \text{ oz}} \approx \dfrac{\$0.678}{1 \text{ oz}}$

The 6-ounce bottle is the better deal.

22. Box of pens

$$\frac{\$165}{150 \text{ pens}} = \$1.10/\text{pen}$$

dozen pens

$$\frac{\$13.50}{12 \text{ pens}} = \$1.13/\text{pen}$$

The box of Sharpie pens is the better deal.

23. The 15-gallon tank will cost $\frac{\$35.25}{15 \text{ gal}} = \frac{\$2.35}{1 \text{ gal}}$ so the 15-gallon tank option is the better deal.

24. $\$30/\text{yd}^2$ per month: $\dfrac{\frac{\$30}{1 \text{ yd}^2} \times \left(\frac{1 \text{ yd}}{3 \text{ ft}}\right)^2}{30 \text{ days}} \approx \$0.11/\text{ft}^2/\text{day}$ (Assuming 30 days per month.)

$\$1.90/\text{ft}^2$ per week: $\dfrac{\$1.90/\text{ft}^2}{7 \text{ days}} \approx \$0.27/\text{ft}^2/\text{day}$

The $\$30/\text{yd}^2$ per week option is the better deal.

25. You can tell the factor by which the first unit is larger than the second by dividing the second into the first. Rewrite metric prefixes as powers of 10, and then simplify, as shown below.

$\frac{1 \text{ km}}{1 \text{ m}} = \frac{10^3 \text{ m}}{10^0 \text{ m}} = \frac{10^3}{10^0} = 10^{3-0} = 10^3$. This means a kilometer is 1000 times as large as a meter.

26. You can tell the factor by which the first unit is larger than the second by dividing the second into the first. Rewrite metric prefixes as powers of 10, and then simplify, as shown below.

$\frac{1 \text{ kg}}{1 \ \mu\text{g}} = \frac{10^3 \text{ g}}{10^{-6} \text{ g}} = 10^{3-(-6)} = 10^9$, so a kilogram is 1,000,000,000 times as large as a microgram.

27. You can tell the factor by which the first unit is larger than the second by dividing the second into the first. Rewrite metric prefixes as powers of 10, and then simplify, as shown below.

$\frac{1 \text{ L}}{1 \ \mu\text{L}} = \frac{10^0 \text{ L}}{10^{-6} \text{ L}} = 10^{0-(-6)} = 10^6$, so a liter is 1,000,000 times larger than a microliter.

28. You can tell the factor by which the first unit is larger than the second by dividing the second into the first. Rewrite metric prefixes as powers of 10, and then simplify, as shown below.

$\frac{1 \text{ km}}{1 \text{ nm}} = \frac{10^3 \text{ m}}{10^{-9} \text{ m}} = 10^{3-(-9)} = 10^{12}$, so a kilometer is 1,000, 000,000,000 times as large as a micrometer.

29. You can tell the factor by which the first unit is larger than the second by dividing the second into the first. Rewrite metric prefixes as powers of 10, and then simplify, as shown below.

$\frac{1 \text{ m}^2}{1 \text{ mm}^2} = \frac{10^0 \text{ m}^2}{(10^{-3} \text{ m})^2} = \frac{10^0 \text{ m}^2}{10^{-6} \text{ m}^2} = 10^{0-(-6)} = 10^6$, so a square meter is 1,000,000 times as large as a square millimeter.

30. You can tell the factor by which the first unit is larger than the second by dividing the second into the first. Rewrite metric prefixes as powers of 10, and then simplify, as shown below.

$\frac{1 \text{ m}^3}{1 \text{ cm}^3} = \frac{10^0 \text{ m}^3}{(10^{-2} \text{ m})^3} = \frac{10^0 \text{ m}^3}{10^{-6} \text{ m}^3} = 10^{0-(-6)} = 10^6$, so a cubic meter is 1,000,000 times as large as a cubic centimeter.

31. $13 \text{ L} \times \frac{1.057 \text{ qt}}{1 \text{ L}} = 13.7 \text{ qt}$

32. $3.5 \text{ m} \times \frac{3.28 \text{ ft}}{1 \text{ m}} = 11.5 \text{ ft}$

33. $34 \text{ lb} \times \frac{1 \text{ kg}}{2.205 \text{ lb}} = 15.4 \text{ kg}$

34. $8.6 \text{ mi} \times \frac{1.6093 \text{ km}}{1 \text{ mi}} = 13.8 \text{ km}$

35. Square both sides of the conversion $1 \text{ km} = 0.6214 \text{ mi}$ to find the conversion between square miles and square kilometers: $(1 \text{ km})^2 = (0.6214 \text{ mi})^2 \Rightarrow 1 \text{ km}^2 = 0.38614 \text{ mi}^2$.

Now use this to complete the problem: $3 \text{ km}^2 \times \frac{0.38614 \text{ mi}^2}{1 \text{ km}^2} = 1.2 \text{ mi}^2$.

36. $\frac{70 \text{ km}}{\text{hr}} \times \frac{1 \text{ mi}}{1.6093 \text{ km}} = 43.5 \frac{\text{mi}}{\text{hr}}$

37. $47 \frac{\text{mi}}{\text{hr}} \times \frac{1.6093 \text{ km}}{1 \text{ mi}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 21.0 \frac{\text{m}}{\text{s}}$

38. Cube both sides of the conversion $2.54 \text{ cm} = 1 \text{ in}$ to find the conversion between cubic centimeters and cubic inches: $(2.54 \text{ cm})^3 = (1 \text{ in})^3 \Rightarrow 16.387 \text{ cm}^3 = 1 \text{ in}^3$.

Now use this to complete the problem: $200 \text{ cm}^3 \times \frac{1 \text{ in}^3}{16.387 \text{ cm}^3} = 12.2 \text{ in}^3$.

39. $\frac{23 \text{ g}}{1 \text{ cm}^3} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{2.205 \text{ lb}}{1 \text{ kg}} \times \left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)^3 = 0.8 \frac{\text{lb}}{\text{in}^3}$

40. $2500 \text{ acres} \times \frac{43,560 \text{ ft}^2}{1 \text{ acre}} \times \left(\frac{1 \text{ m}}{3.28 \text{ ft}}\right)^2 \times \left(\frac{1 \text{ km}}{1000 \text{ m}}\right)^2 = 10.1 \text{ km}^2$

41. The rise in sea level is found by dividing the volume of water by the Earth's surface area, so

sea level rise $= \frac{2.5 \times 10^6 \text{ km}^3}{340 \times 10^6 \text{ km}^2} = 0.007 \text{ km}$ or about 7 meters.

42. If the volume of ash is divided by the area it covers, you'll get the average depth of the ash (see Exercise 39):

average depth $= \frac{\text{volume}}{\text{area}} = \frac{100 \text{ km}^3}{600 \text{ km}^2} = 0.167 \text{ km} = 167 \text{ km}$.

43. a. $C = \frac{45 - 32}{1.8} = 7.2°\text{C}$

b. $F = 1.8(20) + 32 = 68.0°\text{F}$

c. $F = 1.8(-15) + 32 = 5.0°\text{F}$

d. $F = 1.8(-30) + 32 = -22.0°\text{F}$

e. $C = \frac{70 - 32}{1.8} = 21.1°\text{C}$

44. a. $F = 1.8(-8) + 32 = 17.6°\text{F}$

b. $C = \frac{15 - 32}{1.8} = -9.4°\text{C}$

c. $F = 1.8(15) + 32 = 59.0°\text{F}$

d. $C = \frac{75 - 32}{1.8} = 23.9°\text{C}$

e. $C = \frac{20 - 32}{1.8} = -6.7°\text{C}$

45. a. $C = 50 - 273.15 = -223.15°\text{C}$

b. $C = 240 - 273.15 = -33.15°\text{C}$

c. $K = 10 + 273.15 = 283.15 \text{ K}$

46. a. $K = -40 + 273.15 = 233.15 \text{ K}$

b. $C = 400 - 273.15 = 126.85°\text{C}$

c. $K = 125 + 273.15 = 398.15 \text{ K}$

47. Power is the rate at which energy is used, so the power is 150 Calories per mile. Convert this to joules per second, or watts. $\frac{150 \text{ Cal}}{1 \text{ mi}} \times \frac{1 \text{ mi}}{6 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{4184 \text{ J}}{1 \text{ Cal}} = 1743 \text{ W}$

48. Power is the rate at which energy is used, so the power is 100 Calories per mile. Convert this to joules per second, or watts. $\frac{100\text{ Cal}}{1\text{ mi}}\times\frac{20\text{ mi}}{1\text{ hr}}\times\frac{1\text{ hr}}{60\text{ min}}\times\frac{1\text{ min}}{60\text{ s}}\times\frac{4184\text{ J}}{1\text{ Cal}}=2324\text{ W}$

49. In order to compute the cost, we need to find the energy used by the bulbs in kW-hr, and then convert to cents. This can be done in a single chain of unit conversions, beginning with the idea that $\text{energy}=\text{power}\times\text{time}$.

100 watt bulb: One day will cost $100\text{ W}\times\frac{12\text{ hr}}{1\text{ day}}\times\frac{1\text{ kW}}{1000\text{ W}}\times\frac{\$0.13}{1\text{ kW-hr}}=\$0.156,$ or about \$0.16.

25 watt bulb: One day will cost $25\text{ W}\times\frac{12\text{ hr}}{1\text{ day}}\times\frac{1\text{ kW}}{1000\text{ W}}\times\frac{\$0.13}{1\text{ kW-hr}}=\$0.039,$ or about \$0.04.

You will save $(\$0.156-\$0.039)\times 365\text{ days}=\$42.71$ in one year.

50. In order to compute the cost, we need to find the energy used by the clothes dryer in kW-hr, and then convert to cents. This can be done in a single chain of unit conversions, beginning with the idea that energy $=\text{power}\times\text{time}$.

4000 watt clothes dryer: Average daily cost is $4000\text{ W}\times 1\text{ hr}\times\frac{1\text{ kW}}{1000\text{ W}}\times\frac{\$0.14}{1\text{ kW-hr}}=\$0.56.$

2000 watt clothes dryer: Average daily cost is $2000\text{ W}\times 1\text{ hr}\times\frac{1\text{ kW}}{1000\text{ W}}\times\frac{\$0.14}{1\text{ kW-hr}}=\$0.28.$

You will save $(\$0.56-\$0.28)\times 365=\$102.20$ in one year.

51. Density is mass per unit volume, so the density of the block is $\frac{0.12\text{ kg}}{200\text{ cm}^3}\times\frac{1000\text{ g}}{1\text{ kg}}=0.6\ \frac{\text{g}}{\text{cm}^3}$. It will float in water because the density of water is $1\frac{\text{g}}{\text{cm}^3}$.

52. Density is mass per unit volume, so the density of the sample of Styrofoam is $\frac{0.03\text{ oz}}{1\text{ in}^3}\times\frac{(1\text{ in})^3}{(2.540\text{ cm})^3}\times\frac{28.35\text{ g}}{1\text{ oz}}$ $=0.052\ \frac{\text{g}}{\text{cm}^3}$. It will float in water because the density of water is $1\frac{\text{g}}{\text{cm}^3}$.

53. County A has more people.

County A: $100\text{ mi}^2\times\left(\frac{1.6093\text{ km}}{1\text{ mi}}\right)^2\times\frac{25\text{ people}}{1\text{ km}^2}=6474\text{ people}$

County B: $25\text{ km}^2\times\left(\frac{1\text{ mi}}{1.6093\text{ km}}\right)^2\times\frac{100\text{ people}}{1\text{ mi}^2}=965\text{ people}$

54. The population density for Monaco is about $\frac{38{,}000}{1.95\text{ km}^2}=19{,}487\ \frac{\text{people}}{\text{km}^2}$, which is about $\frac{19{,}487\ \frac{\text{people}}{\text{km}^2}}{\frac{35\text{ people}}{1\text{ km}^2}}$ $=557$ times greater than that of the United States.

55. The population density for New Jersey is $\frac{9{,}000{,}000\text{ people}}{7417\text{ mi}^2}=1213\ \frac{\text{people}}{\text{mi}^2}$. The population density for Alaska is $\frac{738{,}000\text{ people}}{571{,}951\text{ mi}^2}=1.3\ \frac{\text{people}}{\text{mi}^2}$, which is smaller than New Jersey's population density.

56. Information density is bytes per unit area, so, depending on the formatting, the DVD's information density is:

Single sided: $\dfrac{4.7 \text{ Gbyte}}{134 \text{ cm}^2} \times \dfrac{10^9 \text{ byte}}{1 \text{ Gbyte}} = 35{,}000{,}000 \ \dfrac{\text{byte}}{\text{cm}^2}$

Double sided: $\dfrac{9.4 \text{ Gbyte}}{134 \text{ cm}^2} \times \dfrac{10^9 \text{ byte}}{1 \text{ Gbyte}} = 70{,}000{,}000 \ \dfrac{\text{byte}}{\text{cm}^2}$

57. a. In one week, a 100-pound person would take $1 \text{ week} \times \dfrac{7 \text{ days}}{1 \text{ week}} \times \dfrac{25 \text{ mg}}{6 \text{ hr}} \times \dfrac{24 \text{ hr}}{1 \text{ day}} = 700 \text{ mg},$ so should take

$700 \text{ mg} \times \dfrac{1 \text{ tablet}}{12.5 \text{ mg}} = 56 \text{ tablets}.$

b. A 100-pound person should take $700 \text{ mg} \times \dfrac{5 \text{ mL}}{12.5 \text{ mg}} = 280 \text{ mL}$ of liquid Benadryl.

58. a. The dose is $\dfrac{9000 \text{ units}}{1 \text{ kg}} \times \dfrac{250 \text{ mg}}{400{,}000 \text{ units}} = 5.625 \text{ mg}/\text{kg}.$

b. A child would take 4 doses in a day, so a 20-kg child would receive $4 \times 20 \text{ kg} \times \dfrac{5.625 \text{ mg}}{1 \text{ kg}} = 450 \text{ mg}.$

59. a. BAC is usually measured in units of grams of alcohol per 100 milliliters of blood. A woman who drinks two glasses of wine, each with 20 grams of alcohol, has consumed 40 grams of alcohol. If she has 4000 milliliters of blood, her BAC is $\dfrac{40 \text{ g}}{4000 \text{ mL}} = \dfrac{0.01 \text{ g}}{\text{mL}} \times \dfrac{100}{100} = \dfrac{1.0 \text{ g}}{100 \text{ mL}}.$ It is fortunate that alcohol is not absorbed immediately, because if it were, the woman would most likely die – a BAC above 0.4 g/mL is typically enough to induce coma or death.

b. If alcohol is eliminated from the body at a rate of 10 grams per hour, then after 3 hours, 30 grams would have been eliminated. This leaves 10 grams in the woman's system, which means her BAC is $\dfrac{10 \text{ g}}{4000 \text{ mL}} = \dfrac{0.0025 \text{ g}}{\text{mL}} \times \dfrac{100}{100} = \dfrac{0.25 \text{ g}}{100 \text{ mL}}$. This is well above the legal limit for driving, so it is not safe to drive. Of course this solution assumes the woman survives 3 hours of lethal levels of alcohol in her body, because we have assumed all the alcohol is absorbed immediately. In reality, the situation is somewhat more complicated.

60. a. BAC is usually measured in units of grams of alcohol per 100 milliliters of blood. A man who drinks 8 ounces of hard liquor has consumed 70 grams of alcohol, and with 6000 milliliters of blood, his BAC is $\dfrac{70 \text{ g}}{6000 \text{ mL}} = \dfrac{0.0117 \text{ g}}{\text{mL}} \times \dfrac{100}{100} = \dfrac{1.17 \text{ g}}{100 \text{ mL}}$. It is fortunate that alcohol is not absorbed immediately, because if it were, the man would be dead – a BAC above 0.4g/mL is typically enough to induce coma or death.

b. If alcohol is eliminated from the body at a rate of 15 grams per hour, then after 4 hours, 60 grams would have been eliminated. This leaves 10 grams of alcohol in the man's system, which means his BAC is $\dfrac{10 \text{ g}}{6000 \text{ mL}} = \dfrac{0.0017 \text{ g}}{\text{mL}} \times \dfrac{100}{100} = \dfrac{0.17 \text{ g}}{100 \text{ mL}}.$ This is well above the legal limit for driving, so it is not safe to drive. Of course this solution assumes the man survives 4 hours of lethal levels of alcohol in his body, because we have assumed all the alcohol is absorbed immediately. In reality, the situation is somewhat more complicated.

FURTHER APPLICATIONS

61. a. A metric mile is $1500\text{ m}\times\frac{3.28\text{ ft}}{1\text{ m}}=4920\text{ ft}$. A USCS mile is 5280 ft. Since $4920/5280=0.932$, the metric mile is 93.2% of the USCS mile.

b. Men's mile: $\frac{1\text{ mi}}{223.13\text{ s}}\times\frac{60\text{ s}}{1\text{ min}}\times\frac{60\text{ min}}{1\text{ hr}}=16.13\text{ mi/hr}$; $(3:43:13=223.13\text{ seconds})$

Men's metric mile: $\frac{4920\text{ ft}}{206\text{ s}}\times\frac{1\text{ mi}}{5280\text{ ft}}\times\frac{60\text{ s}}{1\text{ min}}\times\frac{60\text{ min}}{1\text{ hr}}=16.28\text{ mi/hr}$; $(3:26:00=206\text{ seconds})$

The record holder in the metric mile ran at a faster pace.

c. Women's mile: $\frac{1\text{ mi}}{252.56\text{ s}}\times\frac{60\text{ s}}{1\text{ min}}\times\frac{60\text{ min}}{1\text{ hr}}=14.25\text{ mi/hr}$; $(4:12:56=252.56\text{ seconds})$

Women's metric mile: $\frac{4920\text{ ft}}{230.07\text{ s}}\times\frac{1\text{ mi}}{5280\text{ ft}}\times\frac{60\text{ s}}{1\text{ min}}\times\frac{60\text{ min}}{1\text{ hr}}=14.58\text{ mi/hr}$; $(3:50:7=230.07\text{ seconds})$

The record holder in the metric mile ran at a faster pace.

d. This would be true in both cases since, for both men and women, the record holder in the metric mile ran at a faster pace.

62. A league refers to distance traveled. $20{,}000\text{ leagues}\times\frac{3\text{ naut. miles}}{1\text{ league}}=60{,}000$ nautical miles, and

$60{,}000\text{ naut. miles}\times\frac{6076.1\text{ ft}}{1\text{ naut. mile}}\times\frac{1\text{ mi}}{5280\text{ ft}}\approx 70{,}000$ miles, which is far to large too be a measure of ocean depth on Earth.

63. $\frac{1}{2}\text{gal}\times\frac{3.785\text{ L}}{1\text{ gal}}\times\frac{100\text{ pesos}}{0.8\text{ L}}\times\frac{\$1}{21.86\text{ pesos}}=\$10.82$

64. $\frac{16\text{ pounds}}{1\text{ m}^2}\times\left(\frac{1\text{ m}}{1.094\text{ yd}}\right)^2\times\frac{\$1.221}{1\text{ pound}}=\$16.32/\text{yd}^2$

65. Monte Carlo costs $\frac{1150\text{ euro}}{80\text{ m}^2}\times\left(\frac{1\text{ m}}{3.28\text{ ft}}\right)^2\times\frac{\$1.058}{1\text{ euro}}=\$1.41/\text{ft}^2$ and Santa Fe costs $\frac{\$800}{500\text{ ft}^2}=\$1.60/\text{ft}^2$, so Monte Carlo is less expensive.

66. $\frac{1.40\text{ euro}}{1\text{ L}}\times\frac{\$1.058}{1\text{ euro}}\times\frac{3.785\text{ L}}{1\text{ gal}}=\$5.61/\text{gal}$

67 The Hope diamond weighs 45.52 carats, which is 0.32 oz.

$$45.52\text{ carats}\times\frac{0.2\text{ g}}{1\text{ carat}}=9.1\text{ g and }9.1\text{ g}\times\frac{1\text{ kg}}{1000\text{ g}}\times\frac{2.205\text{ lb}}{1\text{ kg}}\times\frac{16\text{ oz}}{1\text{ lb}}=0.32\text{ oz.}$$

68. 14-karat gold is 58% of pure gold since $\frac{14}{24}=0.58$.

69. 2.2 ounces of 16-karat gold is $2.2\text{ oz}\times\frac{16}{24}=1.47\text{ oz}$ of pure gold.

70. A 0.15 ounce diamond will weigh $0.15\text{ oz}\times\frac{437.5\text{ grain}}{1\text{ oz}}\times\frac{0.0648\text{ g}}{1\text{ grain}}\times\frac{1\text{ carat}}{0.2\text{ g}}=21.26$ carat.

71. The Cullinan diamond weighs 3106 carats.

$$3106\text{ carats}\times\frac{0.2\text{ g}}{1\text{ carat}}\times\frac{1000\text{ mg}}{1\text{ g}}=621{,}200\text{ mg} \text{ and } 3106\text{ carats}\times\frac{0.2\text{ g}}{1\text{ carat}}\times\frac{1\text{ kg}}{1000\text{ g}}\times\frac{2.205\text{ lb}}{1\text{ kg}}=1.37\text{ lb}$$

72. The Star of Africa weighs 530.2 carats, which is 106,040 mg, and 0.23 lb (the conversions being identical to those shown in Exercise 71).

73. a. The volume of the bath is 6 ft $\times$ 3 ft $\times$ 2.5 ft = 45 ft^3. Fill it to the halfway point, and you'll use 22.5 ft^3 of water (half of 45 is 22.5). (Interesting side note: it doesn't matter which of the bathtub's three dimensions you regard as the height – fill it halfway, and it's always 22.5 ft^3 of water). When you take a shower, you use

$$1\text{ shower}\times\frac{10\text{ min}}{\text{shower}}\times\frac{1.75\text{ gal}}{\text{min}}\times\frac{1\text{ ft}^3}{7.5\text{ gal}}=2.33\text{ ft}^3$$

of water, and thus you use considerably more water when taking a bath.

b. Convert 22.5 ft^3 of water (the water used in a bath) into minutes.

$$22.5\text{ ft}^3\times\frac{7.5\text{ gal}}{1\text{ ft}^3}\times\frac{1\text{ min}}{1.75\text{ gal}}=96\text{ min}$$

c. Plug the drain in the bathtub, and mark the depth to which you would normally fill the tub when taking a bath. Take a shower, and note how long your shower lasts. Step out and towel off, but keep the shower running. When the water reaches your mark (you used a crayon, and not a pencil, right?), note the time it took to get there. You now have a sense of how many showers it takes to use the same amount of water as a bath. For example, suppose your shower took 12 minutes, and it takes a full hour (60 minutes) for the water to reach your mark. That would mean every bath uses as much water as five showers.

74. a. Convert 300,000 long tons into kilograms. $300{,}000\text{ long ton}\times\frac{2240\text{ lb}}{1\text{ long ton}}\times\frac{1\text{kg}}{2.205\text{ lb}}=305{,}000{,}000\text{ kg}$

b. Convert 305,000,000 kilograms to cubic meters. $305{,}000{,}000\text{ kg}\times\frac{1\text{ m}^3}{850\text{ kg}}=359{,}000\text{ m}^3$

c. Convert 359,000 cubic meters to barrels of oil. $359{,}000\text{ m}^3\times\frac{1000\text{ L}}{1\text{ m}^3}\times\frac{1\text{ gal}}{3.785\text{ L}}\times\frac{1\text{ barrel}}{42\text{ gal}}$

$=2{,}260{,}000$ barrels

d. Answers will vary. Assuming \$100/barrel, the value of oil in a full supertanker would be $\frac{\$100}{1\text{ barrel}}$

$\times 2{,}260{,}000\text{ barrels}=\$226{,}000{,}000$ or about \$226 million.

75. a. Convert 9 billion gallons per day into cfs: $\frac{9{,}000{,}000{,}000\text{ gal}}{\text{d}}\times\frac{1\text{ ft}^3}{7.5\text{ gal}}\times\frac{1\text{ d}}{24\text{ hr}}\times\frac{1\text{ hr}}{60\text{ min}}\times\frac{1\text{ min}}{60\text{ s}}$

$=13{,}889\frac{\text{ft}^3}{\text{s}}$, or 13,889 cfs, which is about 46% of the flow rate of the Colorado River.

b. The volume of water entering the city over the course of a day, in cubic feet, was

$9{,}000{,}000{,}000\text{ gal}\times\frac{1\text{ ft}^3}{7.5\text{ gal}}=1{,}200{,}000{,}000\text{ ft}^3.$

The area of the flooded portion of the city, in square feet, was $6\text{ mi}^2\times\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)^2=167{,}270{,}400\text{ ft}^2.$

If the volume of water is divided by the area it covers, you'll get the average depth of the water (see Exercise 59): $\text{average depth}=\frac{\text{volume}}{\text{area}}=\frac{1{,}200{,}000{,}000\text{ ft}^3}{167{,}270{,}400\text{ ft}^2}=7.2\text{ ft}.$

Thus the water level rose about 7 feet in one day, assuming the entire 6 square miles was covered in that day.

76. Convert 1 week into cubic feet of water. $1\text{ wk}\times\frac{7\text{ d}}{1\text{ wk}}\times\frac{24\text{ hr}}{1\text{ d}}\times\frac{60\text{ min}}{1\text{ hr}}\times\frac{60\text{ s}}{1\text{ min}}\times\frac{25{,}800\text{ ft}^3}{\text{s}}$

$=15{,}603{,}840{,}000\text{ ft}^3$, which is about 15.6 billion cubic feet. Since $\frac{15{,}603{,}840{,}000}{1{,}200{,}000{,}000{,}000}=0.013$, about 1.3% of the water in the reservoir was released.

77. a. The volume of one hemlock tree would be about $\pi(15\text{ in})^2(120\text{ ft})=84{,}823\text{ in}\times\text{in}\times\text{ft}$ and

$84{,}823\text{ in}\times\text{in}\times\text{ft}\times\frac{1\text{ ft}}{12\text{ in}}=7069\text{ ft}\times\text{ft}\times\text{in}$, or 7069 fbm.

b. One 8 ft 2-by-4 contains about $8\text{ ft}\times1.5\text{ in}\times3.5\text{ in}\times\frac{1\text{ ft}}{12\text{ in}}=3.5\text{ ft}\times\text{ft}\times\text{in}=3.5$ fbm, so you could cut

$\frac{150\text{ fbm}}{3.5\text{ fbm}}=42.9$, or 42 whole boards.

c. One 12 ft 2-by-6 is $12\text{ ft}\times1.5\text{ in}\times5.5\text{ in}\times\frac{1\text{ ft}}{12\text{ in}}=8.25\text{ ft}\times\text{ft}\times\text{in}=8.25$ fbm, so the project would require

$75\times8.25\text{ fbm}\approx619$ fbm.

78. The stand would contain $60\text{ acres}\times\frac{200\text{ trees}}{20\text{ acres}}\times\frac{400\text{ fbm}}{1\text{ tree}}=240{,}000$ fbm. So there would be $0.1\times240{,}000=24{,}000$ fbm if one-tenth of the trees were cut.

79. Case A:

Income: $50\text{ acres}\times\frac{60\text{ bushels}}{1\text{ acre}}\times\frac{\$3.50}{1\text{ bushel}}=\$10{,}500$

Cost: $50\text{ acres}\times\frac{100\text{ lb}}{1\text{ acre}}\times\frac{\$0.25}{1\text{ lb}}=\$1250$

Revenue: $\$10{,}500-\$1250=\$9250$

Case B:

Income: $50\text{ acres}\times\frac{50\text{ bushels}}{1\text{ acre}}\times\frac{\$4.50}{1\text{ bushel}}=\$11{,}250$

Cost: $50\text{ acres}\times\frac{70\text{ lb}}{1\text{ acre}}\times\frac{\$0.50}{1\text{ lb}}=\$1750$

Revenue: $\$11{,}250-\$1750=\$9500$

Case B has the higher revenue.

80 a. $1\text{ hectares}\times\frac{100\text{ ares}}{1\text{ hectares}}\times\frac{100\text{ m}^2}{1\text{ are}}=10{,}000\text{ m}^2=10^4\text{ m}^2$

b. $1\text{ km}^2\times\left(\frac{1000\text{ m}}{1\text{ km}}\right)^2\times\frac{1\text{ are}}{100\text{ m}^2}\times\frac{1\text{ ha}}{100\text{ are}}=100\text{ ha}$

c. $1\text{ acre}=43{,}560\text{ ft}^2$, so $1\text{ ha}\times\frac{100\text{ are}}{1\text{ ha}}\times\frac{100\text{ m}^2}{1\text{ are}}\times\left(\frac{3.28\text{ ft}}{1\text{ m}}\right)^2\times\frac{1\text{ acre}}{43{,}560\text{ ft}^2}\approx2.47$ acres.

d. $\frac{10{,}000\text{ euro}}{1\text{ ha}}\times\frac{\$1.058}{1\text{ euro}}\times\frac{1\text{ ha}}{2.47\text{ acre}}\approx\$4283/\text{acre}$, so 10,000 euro/ha is the better option.

81. a. Convert kilowatt-hours into joules: $900 \text{ kW-hr} \times \dfrac{3{,}600{,}000 \text{ J}}{1 \text{ kW-hr}} = 3{,}240{,}000{,}000 \text{ J}.$

b. May has 31 days, so the average power is $\dfrac{3{,}240{,}000{,}000 \text{ J}}{31 \text{ d}} \times \dfrac{1 \text{ d}}{24 \text{ hr}} \times \dfrac{1 \text{ hr}}{60 \text{ min}} \times \dfrac{1 \text{ min}}{60 \text{ s}} = \dfrac{1210 \text{ J}}{\text{s}} = 1210 \text{ W}.$

One could also begin with 900 kW-hr per 31 days, and convert that into watts, using 1 watt = 1 joule per second.

c. First, convert joules into liters: $3{,}240{,}000{,}000 \text{ J} \times \dfrac{1 \text{ L}}{12{,}000{,}000 \text{ J}} = 270 \text{ L}.$

Now convert liters into gallons: $270 \text{ L} \times \dfrac{1 \text{ gal}}{3.785 \text{ L}} = 71.33 \text{ gal}.$

Thus it would take 270 liters = 71.33 gallons of oil to provide the energy shown on the bill (assuming all the energy released by the burning oil could be captured and delivered to your home with no loss).

82. a. Convert kilowatt-hours into joules: $1050 \text{ kW-hr} \times \dfrac{3{,}600{,}000 \text{ J}}{1 \text{ kW-hr}} = 3{,}780{,}000{,}000 \text{ J}.$

b. October has 31 days, so the average power is $\dfrac{3{,}780{,}000{,}000 \text{ J}}{31 \text{ d}} \times \dfrac{1 \text{ d}}{24 \text{ hr}} \times \dfrac{1 \text{ hr}}{60 \text{ min}} \times \dfrac{1 \text{ min}}{60 \text{ s}} = \dfrac{1411 \text{ J}}{\text{s}} = 1411 \text{ W}.$ One could also begin with 1050 kW-hr per 31 days, and convert that into watts, using 1 watt = 1 joule per second.

c. First, convert joules into liters: $3{,}780{,}000{,}000 \text{ J} \times \dfrac{1 \text{ L}}{12{,}000{,}000 \text{ J}} = 315 \text{ L}.$ Now convert liters into gallons: $315 \text{ L} \times \dfrac{1 \text{ gal}}{3.785 \text{ L}} = 83.22 \text{ gal}.$ Thus it would take 315 liters = 83.22 gallons of oil to provide the energy shown on the bill (assuming all the energy released by the burning oil could be captured and delivered to your home with no loss).

83. a. Your power is the rate at which you use energy, and thus your average power is 2500 Calories per day. Convert this to watts: $\dfrac{2500 \text{ Cal}}{1 \text{ d}} \times \dfrac{1 \text{ d}}{24 \text{ hr}} \times \dfrac{1 \text{ hr}}{60 \text{ min}} \times \dfrac{1 \text{ min}}{60 \text{ s}} \times \dfrac{4184 \text{ J}}{1 \text{ Cal}} = 121 \dfrac{\text{J}}{\text{s}}.$ Since $1 \text{ W} = 1 \text{ J/s}$, this is 121 watts.

b. $\dfrac{2500 \text{ Cal}}{1 \text{ d}} \times \dfrac{365 \text{ d}}{1 \text{ yr}} \times \dfrac{4184 \text{ J}}{1 \text{ Cal}} = 3{,}817{,}900{,}000 \ \dfrac{\text{J}}{\text{yr}},$ so you need about 3.8 billion joules each year from food, which is very close to 1% of your total energy consumption (3.8 billion/400 billion = 0.0095).

84. In order to compute the cost, we need to find the energy used by the outdoor spa in kW-hr, and then convert to dollars. This can be done in a single chain of unit conversions, beginning with the idea that $\text{energy} = \text{power} \times \text{time},$ (this is true because power is the rate at which energy is consumed; that is, $\text{power} = \text{energy} / \text{time}$) and assuming a 30-day month.

$$1500 \text{ W} \times 4 \text{ months} \times \frac{1 \text{ kW}}{1000 \text{ W}} \times \frac{30 \text{ d}}{1 \text{ month}} \times \frac{24 \text{ hr}}{1 \text{ d}} \times \frac{\$0.10}{1 \text{ kW-hr}} = \$432$$

85. Since $\text{energy} = \text{power} \times \text{time},$ the power plant can produce (assuming a 30-day month) $1190 \text{ MW} \times 1 \text{ month} \times \dfrac{30 \text{ d}}{1 \text{ month}} \times \dfrac{24 \text{ hr}}{1 \text{ d}} \times \dfrac{1000 \text{ kW}}{1 \text{ MW}} \approx 8.57 \times 10^8 \text{ kW-hr}$ of energy each month. The amount of uranium used each month is given by $8.57 \times 10^8 \ \dfrac{\text{kW-hr}}{\text{month}} \times \dfrac{1 \text{ kg}}{16 \times 10^6 \text{ kW-hr}} \approx 53.6 \ \dfrac{\text{kg}}{\text{month}}.$ The plant can serve about 857,000 homes, because $8.57 \times 10^8 \text{ kW-hr} \times \dfrac{1 \text{ home}}{1000 \text{ kW-hr}} = \ 857{,}000 \text{ homes}.$

86. Since $\text{energy} = \text{power} \times \text{time}$, the power plant can produce (assuming a 30-day month)

$$1.5\times10^9\ \text{W}\times1\ \text{month}\times\frac{30\ \text{d}}{\text{month}}\times\frac{24\ \text{hr}}{\text{d}}\times\frac{1\ \text{kW}}{1000\ \text{W}}=1.08\times10^9\ \text{kW-hr}\text{=1.08 billion kW-hr}$$

of energy every month. The plant needs

$1.08\times10^9\ \dfrac{\text{kW-hr}}{1\ \text{month}}\times\dfrac{1\ \text{kg}}{450\ \text{kW-hr}}=2.4\times10^6\ \dfrac{\text{kg}}{\text{month}}$, or 2.4 million kg of coal every month. Because $1.08\times10^9\ \text{kW-hr}\times\dfrac{1\ \text{home}}{1000\ \text{kW-hr}}=1.08\times10^6\ \text{homes}$, the plant can serve 1.08 million homes.

87. At 20% efficiency, this solar panel can generate 200 watts of power when exposed to direct sunlight. Since $\text{energy} = \text{power} \times \text{time}$, and because the panel receives the equivalent of 6 hours of direct sunlight, the panel can produce $200\ \text{W}\times6\ \text{hr}\times\dfrac{1\ \text{kW}}{1000\ \text{W}}\times\dfrac{3{,}600{,}000\ \text{j}}{\text{kW-hr}}=4{,}320{,}000\ \text{j}$. This occurs over the span on one day, so the panel produces an average power of $\dfrac{4{,}320{,}000\ \text{j}}{\text{d}}\times\dfrac{1\ \text{d}}{24\ \text{hr}}\times\dfrac{1\ \text{hr}}{60\ \text{min}}\times\dfrac{1\ \text{min}}{60\ \text{s}}=50\ \dfrac{\text{j}}{\text{s}}=50\ \text{W}$.

88. In Exercise 87, each solar panel covers one square meter. You would need

$1\ \text{kW}\times\dfrac{1000\ \text{W}}{1\ \text{kW}}\times\dfrac{1\ \text{panel}}{50\ \text{W}}=20\ \text{panels}$, which is 20 square meters of solar panels.

89. $\text{Energy} = \text{power}\times\text{time}$, so the energy produced by a wind turbine over the course of a year is

$2.5\ \text{MW}\times\dfrac{1000\ \text{kW}}{1\ \text{MW}}\times1\ \text{yr}\times\dfrac{365\ \text{d}}{1\ \text{yr}}\times\dfrac{24\ \text{hr}}{1\ \text{d}}=21{,}900{,}000\ \text{kW-hr}$. This is enough energy to serve

$21{,}900{,}000\ \text{kW-hr}\times\dfrac{1\ \text{household}}{10{,}000\ \text{kW-hr}}=2190\ \text{households}$.

90. a. $\text{Energy} = \text{power}\times\text{time}$, so the energy capacity of the wind farms over the span of a year is

$6.1\ \text{GW}\times\dfrac{1000\ \text{MW}}{1\ \text{GW}}\times\dfrac{1000\ \text{kW}}{1\ \text{MW}}\times1\ \text{yr}\times\dfrac{365\ \text{d}}{1\ \text{yr}}\times\dfrac{24\ \text{hr}}{1\ \text{d}}=5.34\times10^{10}\ \text{kW-hr}$. Since the wind farms produce 30% of their capacity, on average, the energy produced is $0.30\times5.34\times10^{10}\ \text{kW-hr}=1.6\times10^{10}\ \text{kW-hr}$ or 16 billion kW-hr. This is enough energy to serve $1.6\times10^{10}\ \text{kW-hr}\times\dfrac{1\ \text{household}}{10{,}000\ \text{kW-hr}}=1.6\times10^6$ households, or 1.6 million households.

b. If the $1.6\times10^{10}\ \text{kW-hr}$ of energy produced by wind farms were instead produced from fossil fuels, there would be $1.6\times10^{10}\ \text{kW-hr}\times\dfrac{1.5\ \text{lb}}{1\ \text{kw-hr}}=2.4\times10^{10}$ lb, or about 24 billion pounds of carbon dioxide entering the atmosphere each year.

91. a. There are $750\ \text{mL}\times\dfrac{5\ \text{mg}}{100\ \text{mL}}=37.5\ \text{mg}$ of dextrose in 750 mL of D5W. The patient should be given

$50\ \text{mg}\times\dfrac{750\ \text{mL}}{37.5\ \text{mg}}=1000\ \text{mL}$ of D5W.

b. There are $1.2\ \text{L}\times\dfrac{1000\ \text{mL}}{1\ \text{L}}\times\dfrac{0.9\ \text{mg}}{100\ \text{mL}}=10.8\ \text{mg}$ of dextrose in 1.2 L of NS. The patient should be given

$15\ \text{mg}\times\dfrac{1.2\ \text{L}}{10.8\ \text{mg}}\times\dfrac{1000\ \text{mL}}{1\ \text{L}}=1667\ \text{mL}$ of NS.

92. a. The flow rates are $\frac{1.5\text{ L}}{12\text{ hr}}\times\frac{1000\text{ mL}}{1\text{ L}}=125\frac{\text{mL}}{\text{hr}}$ and $125\frac{\text{mL}}{\text{hr}}\times\frac{5\text{ mg}}{100\text{ L}}=6.25\frac{\text{mg}}{\text{hr}}$.

b. The rate is $125\frac{\text{mL}}{\text{hr}}\times\frac{15\text{ gtt}}{1\text{ mL}}=1875\frac{\text{gtt}}{\text{hr}}$.

c. $12\text{ hr}\times 6.25\frac{\text{mg}}{\text{hr}}=75\text{ mg}$ of dextrose is delivered.

93. a. The flow rates are $\frac{0.5\text{ L}}{4\text{ hr}}\times\frac{1000\text{ mL}}{1\text{ L}}=125\frac{\text{mL}}{\text{hr}}$ and $125\frac{\text{mL}}{\text{hr}}\times\frac{0.9\text{ mg}}{100\text{ L}}=1.125\frac{\text{mg}}{\text{hr}}$.

b. The rate is $125\frac{\text{mL}}{\text{hr}}\times\frac{20\text{ gtt}}{1\text{ mL}}=2500\frac{\text{gtt}}{\text{hr}}$.

c. $4\text{ hr}\times 1.125\frac{\text{mg}}{\text{hr}}=4.5\text{ mg}$ of sodium chloride is delivered.

94. a. The flow rate is $\frac{10\text{ mL}}{1\text{ hr}}\times\frac{300\text{ mg}}{200\text{ mL}}=15\frac{\text{mg}}{\text{hr}}$.

b. The infusion should last $60\text{ mg}\times\frac{1\text{ hr}}{15\text{ mg}}=4$ hours.

95. a. $40\text{ kg}\times\frac{25\text{ mg/kg}}{1\text{ day}}=1000\text{ mg/day}$ and $\frac{1000\text{ mg}}{1\text{ day}}\times\frac{1\text{ capsule}}{250\text{ mg}}=4$ capsules/day, so the patient should take 1 capsule every six hours.

b. From part (a), you would need 250 mg in 6 hours, which requires 5 mL of solution in 6 hours, so the rate would be $\frac{5\text{ mL}}{6\text{ hr}}\times\frac{60\text{ gtt}}{1\text{ mL}}=50\text{ gtt/hr}$.

96. a. $36\text{ kg}\times\frac{50\text{ mg/kg}}{1\text{ day}}=1800\text{ mg/day}$ and $\frac{1800\text{ mg}}{1\text{ day}}\times\frac{1\text{ tablet}}{300\text{ mg}}=6$ tablets/day, so the patient should take 1 tablet every four hours.

b. From part (a), you would need 2 tablets or 600 mg in 8 hours, so $600\text{ mg}\times\frac{5\text{ mL}}{200\text{ mg}}=15\text{ ml}$ of solution in 8 hours, so the rate would be $\frac{15\text{ mL}}{8\text{ hr}}\times\frac{15\text{ gtt}}{1\text{ mL}}=28.13\text{ gtt/hr}$, or about 28 gtt/hr.

97 a. $0.5\text{ L}\times\frac{20\text{ gtt}}{1\text{ mL}}\times\frac{1000\text{ mL}}{1\text{ L}}=10{,}000\text{ gtt}$

b. $1\text{ L}\times\frac{60\text{ gtt}}{1\text{ mL}}\times\frac{1000\text{ mL}}{1\text{ L}}=60{,}000\text{ gtt}$

c. $1\text{ L}\times\frac{15\text{ gtt}}{1\text{ mL}}\times\frac{1000\text{ mL}}{1\text{ L}}=15{,}000\text{ gtt}$; The rate of infusion is $\frac{15{,}000\text{ gtt}}{5\text{ hr}}\times\frac{1\text{ hr}}{60\text{ min}}=50$ gtt/min.

98 The child is receiving $\frac{200\text{ mg}}{8\text{ hr}}\times\frac{24\text{ hr}}{1\text{ day}}=600$ mg/day, which is $\frac{600\text{ mg/day}}{20\text{ lb}}\times\frac{2.205\text{ lb}}{1\text{ kg}}\approx 66$ mg/kg/day. This is not within the guidelines.

UNIT 2C: PROBLEM-SOLVING HINTS

THINK ABOUT IT

Pg. 108. The combination of 5 child tickets and 4 adult tickets totals to $\$130(\$10 \times 5 + \$20 \times 4 = \$130)$. Similar calculations will verify the remained child/ticket combinations. There cannot be an even number of child tickets since the amount of money left for adult tickets would not be evenly divisible by \$20. For example, if there were 4 child tickets, the adult tickets would have to be worth $\$1304 \times \$10 = \$90,$ which is not divisible by \$20, so there could not be a whole number of tickets.

Pg. 112. Answers will vary. This would be a good topic for a discussion either during or outside of class.

Pg. 114. This question simply checks that students followed Example 6. This is a subjective question designed to spur debate.

QUICK QUIZ

1. **c**. Look at example 1 (*Box Office Receipts*) in this unit.
2. **a**. Many mathematical problems have more than one solution.
3. **b**. Common experience tells us that batteries can power a flashlight for many hours, not just a few minutes nor several years.
4. **b**. An elevator that carried only 10 kg couldn't accommodate even one person (a 150 lb person weighs about 75 kg), and hotel elevators aren't designed to carry hundreds of people (you'd need at least 100 people to reach 10,000 kg).
5. **b**. Refer to the discussion of Zeno's paradox in the text. There, it was shown that the sum of an infinite number of ever-smaller fraction is equal to 2, and thus we can eliminate answers **a** and **c**. This leaves **b**.
6. **a**. If you cut the cylinder along its length, and lay it flat, it will form a rectangle with width equal to the circumference of the cylinder (i.e. 10 in), and with length equal to the length of the cylinder.
7. **c**. Solving a simpler problem will many times guide you to the solution to a more complex problem.
8. **c**. It could happen that the first 20 balls selected are odd, in which case the next two would have to be even, and this is the first time one can be certain of selecting two even balls.
9. **b**. The most likely explanation is that the A train always arrives 10 minutes after the B train. Suppose the A train arrives on the hour (12:00, 1:00, 2:00,…), while the B train arrives ten minutes before the hour (11:50, 12:50, 1:50,…). If Karen gets to the station in the first 50 minutes of the hour, she'll take the B train; otherwise she'll take the A train. Since an hour is 60 minutes long, 5/6 of the time, Karen will take the B train to the beach. Note that 5/6 of 30 days is 25 days, which is the number of times Karen went to the beach. The other scenarios *could* happen, but they aren't nearly as likely as the scenario in answer **b**.
10. **b**. Label the hamburgers as A, B, and C. Put burgers A and B on the grill. After 5 minutes, turn burger A, take burger B and put it on a plate, and put burger C on the grill. After 10 minutes, burger A is cooked, while burgers B and C are half-cooked. Finish off burgers B and C in the final 5 minutes, and you've cooked all three in 15 minutes.

REVIEW QUESTIONS

1. Examples will vary.

   Hint 1: There May Be More Than One Answer: Some problems may have more than one mathematically correct solution. Multiple solutions often occur because not enough information is available to distinguish among a variety of possibilities.

   Hint 2: There May Be More Than One Method: Although there may be more than one method for finding an answer, not all methods are equally efficient. An efficient method can save a lot of time and work.

   Hint 3: Use Appropriate Tools: You usually will have a choice of tools to use in any problem. Choosing the tools most suited to the job will make your task much easier.

   Hint 4: Consider Simpler, Similar Problems: Solving a simpler, but similar, problem may provide insight to help you understand the original problem.

1. (continued)

   Hint 5: Consider Equivalent Problems with Simpler Solutions: An equivalent problem will have the same numerical answer but may be easier to solve.

   Hint 6: Approximations Can Be Useful: Approximations may reveal the essential character of a problem, making it easier to reach an exact solution. Approximations also provide a useful check: If you come up with an "exact solution" that isn't close to the approximate one, something may have gone wrong.

   Hint 7: Try Alternative Patterns of Thought: Approach every problem with an openness that allows innovative ideas to percolate.

   Hint 8: Do Not Spin Your Wheels: Often the best strategy in problem solving is to put a problem aside for a few hours or days.

2. When a ball is thrown into the, it may attain a certain height at two different times. Then the ball reaches that height on its upward trajectory and when it passes through that height on its way back towards the ground.

DOES IT MAKE SENSE?

3. Does not make sense. There is no problem-solving recipe that can be applied to all problems.

4. Makes sense. It's generally a waste of time to blindly dive into the solution of a problem if you don't take the time to understand its nature.

5. Does not make sense. Approximations can used to simplify calculations and can result in solutions to problems that are accurate enough for real world use.

6. Does not make sense. A fundamental process in problem solving is the ability to recognize when a chosen solution plan is not working.

BASIC SKILLS AND CONCEPTS

7. You won't be able to determine the exact number of cars and buses that passed through the toll booth, but the method of trial-and-error leads to the following possible solutions: (16 cars, 0 buses), (13 cars, 2 buses), (10 cars, 4 buses), (7 cars, 6 buses), (4 cars, 8 buses), (1 car, 10 buses). Along the way, you may have noticed that the number of buses must be an even number, because when it is odd, the remaining money cannot be divided evenly into $2 (car) tolls.

8. The method of trial-and-error produces the following possible recipes, the ordered pairs are (Level 1, Level 2): (14, 0), (12, 1), (10, 2), (8, 3), (6, 4), (4, 5), (2, 6), (0, 7).

9. a. Based on the first race data, Jordan runs 200 meters in the time it takes Amari to run 190 meters. In the second race, Jordan will catch up to Amari 10 meters from the finish line (because Jordan has covered 200 meters at that point, and Amari has covered 190 meters). Jordan will win the race in the last ten meters, because Jordan runs faster than Amari.

   b. Jordan will be 5 meters from the finish line when Amari is 10 meters from the finish line (because Jordan has covered 200 meters at that point, and Amari has covered 190 meters). Jordan will win the race in the because Jordan runs faster than Amari and is in the lead.

   c. Jordan will be 15 meters from the finish line when Amari is 10 meters from the finish line (because Jordan has covered 200 meters at that point, and Amari has covered 190 meters). In the time Amari runs 10 meters, Jordan will only run $10\ \text{m}_{\text{Amari}} \times \frac{200\ \text{m}_{\text{Jordan}}}{190\ \text{m}_{\text{Amari}}} = 10.53\ \text{m}_{\text{Jordan}}$, so Amari wins the race.

   d. From part (c), Jordan must start 10.53 meters behind the starting line, since that is how far she would run in the same time Amari finishes the last 10 meters of the race.

10. a. Roadrunner: $100\ \text{m} \times \frac{1\ \text{s}}{5\ \text{m}} = 20\ \text{s}$; Coyote: $20\ \text{s} \times \frac{1\ \text{m}}{1\ \text{s}} = 20\ \text{m}$

    b. Roadrunner: $20\ \text{m} \times \frac{1\ \text{s}}{5\ \text{m}} = 4\ \text{s}$; Coyote: $4\ \text{s} \times \frac{1\ \text{m}}{1\ \text{s}} = 4\ \text{m}$

10. (continued)

c. Roadrunner: $4 \text{ m} \times \frac{1 \text{ s}}{5 \text{ m}} = 0.8 \text{ s}$; Coyote: $0.8 \text{ s} \times \frac{1 \text{ m}}{1 \text{ s}} = 0.8 \text{ m}$

d. The roadrunner will have traveled for at least $20 \text{ s} + 4 \text{ s} + 0.8 \text{ s} = 24.8$ seconds. Continuing the pattern from parts (a), (b), and (c), it will take no longer than 26 seconds for the roadrunner to catch the coyote.

e. $25 \text{ s} \times \frac{5 \text{ m}}{1 \text{ s}} = 125 \text{ m}$

11. On the second transfer, there are four possibilities to consider. A) All three marbles are black. B) Two are black, one is white. C) One is black, two are white. D) All three are white. In case A), after the transfer, there are no black marbles in the white pile, and no white marbles in the black pile. In case B), one white marble is transferred to the black pile, and one black marble is left in the white pile. In case C), two white marbles are transferred to the black pile, and two black marbles are left in the white pile. In case D), three white marbles are transferred to the black pile, and three black marbles are left in the white pile. In all four cases, there are as many white marbles in the black pile as there are black marbles in the white pile after the second transfer.

12. Cut the pipe along its length, and press it flat – this results in a rectangle with height of 20 cm. The width, which is the circumference of pipe, is 6 cm. As shown in example 5, a wrap of wire becomes the hypotenuse of a right triangle, whose base is 6 cm, and whose height, because there are 8 turns of wire, is 1/8 of 20 cm, or 2.5 cm. By the Pythagorean theorem, the length of one turn of the wire is $h = \sqrt{6^2 + 2.5^2} = 6.5 \text{ cm}$, and thus the total length of wire required is 52 cm (8 turns of wire, each 6.5 cm long).

13. Proceeding as in example 6, the circular arc formed by the bowed track can be approximated by two congruent right triangles. Since 1 km is 100,000 cm, the base of one of the triangles is 50,000 cm, and its hypotenuse is 50,005 cm. The height of the triangle can be computed as $h = \sqrt{50{,}005^2 - 50{,}000^2} = 707 \text{ cm}$, which is about 7.1 meters.

14. Answers will vary,

15. Yes, imagine that at the same time the monk leaves the monastery to walk up the mountain, his twin brother leaves the temple and walks down the mountain. Clearly, the two must pass each other somewhere along the path.

16. The knight should lay one plank across a corner of the moat and then lay the other plank between the first plank and the other side of the moat.

FURTHER APPLICATIONS

17. a. Yes; $P = 2(7 \text{ m}) + 2(3 \text{ m}) = 20 \text{ m}$; $A = (7 \text{ m}) \times (3 \text{ m}) = 21 \text{ m}^2$

b. Yes; $P = 2(8 \text{ m}) + 2(2 \text{ m}) = 20 \text{ m}$; $A = (8 \text{ m}) \times (2 \text{ m}) = 16 \text{ m}^2$

c. By trial and error, possible dimensions (l, w) are:

$(9 \text{ m}, 1 \text{ m})$; $P = 2(9 \text{ m}) + 2(1 \text{ m}) = 20 \text{ m}$; $A = (9 \text{ m}) \times (1 \text{ m}) = 9 \text{ m}^2$

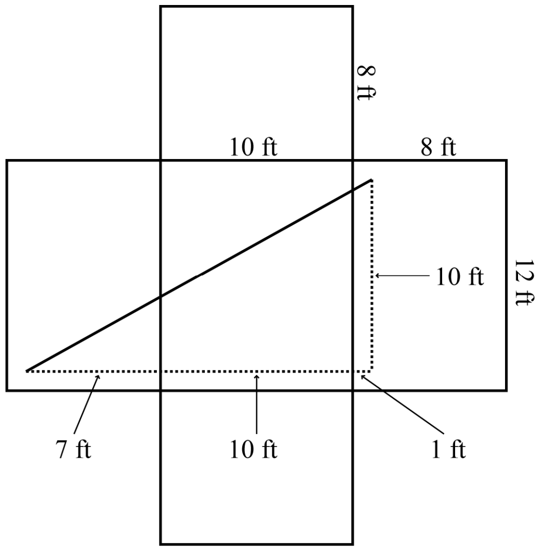
$(6 \text{ m}, 4 \text{ m})$; $P = 2(6 \text{ m}) + 2(4 \text{ m}) = 20 \text{ m}$; $A = (6 \text{ m}) \times (4 \text{ m}) = 24 \text{ m}^2$

$(5 \text{ m}, 5 \text{ m})$; $P = 2(5 \text{ m}) + 2(5 \text{ m}) = 20 \text{ m}$; $A = (5 \text{ m}) \times (5 \text{ m}) = 25 \text{ m}^2$

Note that reversing the length and width will result in the same areas, so the maximum area is when the yard is a square with sides of length 5 meters.

18. Note that at least one truck must have passed over the counter as there are an odd number of counts, and we must have an odd number of trucks for the same reason. Beginning with one truck, followed by 3, 5, 7, etc. trucks, and computing the number of cars that go along with these possible truck solutions, we arrive at the following answers: (1 truck, 16 cars), (3 t, 13 c), (5 t, 10 c), (7 t, 7 c), (9 t, 4 c), and (11 t, 1 c).

19. Following the hint given in the problem, the figure below shows the room cut along its vertical corners and laid flat with the ceiling in the center. The shortest route for the ant is shown as the solid line, which is the hypotenuse of a right triangle with sides shown as dashed lines. The length of the hypotenuse (using the Pythagorean Theorem) is $\sqrt{(12\text{ ft}-1\text{ ft}-1\text{ ft})^2+(7\text{ ft}+10\text{ ft}+1\text{ ft})^2}=\sqrt{(10\text{ ft})^2+(18\text{ ft})^2}=\sqrt{424\text{ ft}^2}\approx 20.6\text{ ft}.$



20. a. The time spent running is $2\text{ mi}\div 4\dfrac{\text{mi}}{\text{hr}}=2\text{ mi}\times\dfrac{1\text{ hr}}{4\text{ mi}}=\dfrac{1}{2}\text{ hr}.$

    b. The time spent walking is $2\text{ mi}\div 2\dfrac{\text{mi}}{\text{hr}}=2\text{ mi}\times\dfrac{1\text{ hr}}{2\text{ mi}}=1\text{ hr}.$

    c. Not true; more time is spent walking than running.

    d. The average speed for the trip is $4\text{ mi}\div\dfrac{3}{2}\text{hr}=4\text{ mi}\times\dfrac{2}{3\text{ h}}=\dfrac{8}{3}\text{ mi/hr},$ or about 2.7 mi/hr.

21. In going from the first floor to the third floor, you have gone up two floors. Since it takes 30 seconds to do that, your pace is 15 seconds per floor. Walking from the first floor to the sixth floor means walking up five floors, and at 15 seconds per floor, this will take 75 seconds.

22. It is possible. Reuben's birthday is December 31, and he's talking to you on January 1. Thus two days ago, he was 20 years old on December 30. He turned 21 the next day. "Later *next* year," refers to almost two years later, when he will turn 23 (*this* year on December 31, he will turn 22).

23. You might pick one of each kind of apple on the first three draws. On the fourth draw, you are guaranteed to pick an apple that matches one of the first three, so four draws are required.

24. The woman gained $100 in each transaction, so she gained $200 overall.

25. Select a fruit from the box labeled *Apples and Oranges*. If it's an apple, that box must be the *Apple* box (because its original label is incorrect, leaving only *Apples* or *Oranges* as its correct label). The correct label for the box labeled *Oranges* is either *Apples* or *Apples and Oranges*, but we just determined the box containing apples, so the correct label for this box is *Apples and Oranges*. There's only one choice left for the box labeled *Apples*, and that's *Oranges*. A similar argument allows one to determine the correct labeling for each box if an orange is selected first.

26. Select one ball from the first barrel, two balls from the second barrel, three from the third, and so on, ending with ten balls chosen from the tenth barrel. Find the weight of all 55 balls thus chosen. If all the balls weighed one ounce, the weight would be 55 ounces. But we know one of the barrels contains two-ounce balls. Suppose the first barrel contained the two-ounce balls – then the weighing would reveal a result of 56 ounces, and we'd know the first barrel was the one that contains two-ounce balls. If, in fact, the second barrel contained the two-ounce balls, the combined weight would be 57 ounces. A combined weight of 58 ounces means the third barrel contained the heavy balls. Continuing in this fashion, we see that a combined weight of n ounces more than 55 ounces corresponds to the nth barrel containing the two-ounce balls.

27. Seven crossings are required. Trip 1: the woman crosses with the goose. Trip 2: the woman returns to the other shore by herself. Trip 3: the woman takes the mouse across. Trip 4: the woman returns with the goose. Trip 5: the woman brings the wolf across. Trip 6: the woman returns to the other shore by herself. Trip 7: the woman brings the goose over. In this way, the goose is never left alone with the wolf, nor the mouse with the goose. The foursome could get to the other side of the river in five crossings if you give rowing abilities to the animals, but the statement of the problem (and common sense) probably forbids it (the boat "…will hold only herself and one other animal"). Try it anyway for an interesting twist to the problem.

28. A balance scale is one that displays a needle in the middle when both sides of the scale are loaded with equal weights. Put six coins on each side of the scale. The side with the heavy coin will be lower, so discard the other six. Now put three coins of the remaining six on each side of the scale, and as before, discard the three that are in the light group. Finally, select two of the remaining three coins, and put one on each side of the scale. The scale will be even if the heavy coin is not on the scale. The scale will tip to one side if the heavy coin was in the final selection. This is just one solution – another begins by dividing the 12 coins into three groups of four coins. See if you can supply the logic necessary to find the heavy coin in that scenario.

29. It's possible you'll pick a sock of each color on the first two draws, but on the third draw, you are assured of making a match.

30. Since the gray, orange, and pink books are consecutive and the pink book cannot be rightmost, the only possible orders are *gray, orange, pink, other, other* or *other, gray, orange, pink, other*. The only way to add the brown and gold books so that the gold book is not leftmost and there are two books between the brown and gold books is the order *brown, gray, orange, pink, gold*.

31. When a clock chimes five times, there are four pauses between the chimes. Since it takes five seconds to chime five times, each pause lasts 1.25 seconds. There are nine pauses that need to be accounted for when the clock chimes 10 times, and thus it takes $9 \times 1.25 = 11.25$ seconds to chime 10 times. This solution assumes it takes no time for the clock to chime once (we'd need to know the duration of the chime to solve it otherwise).

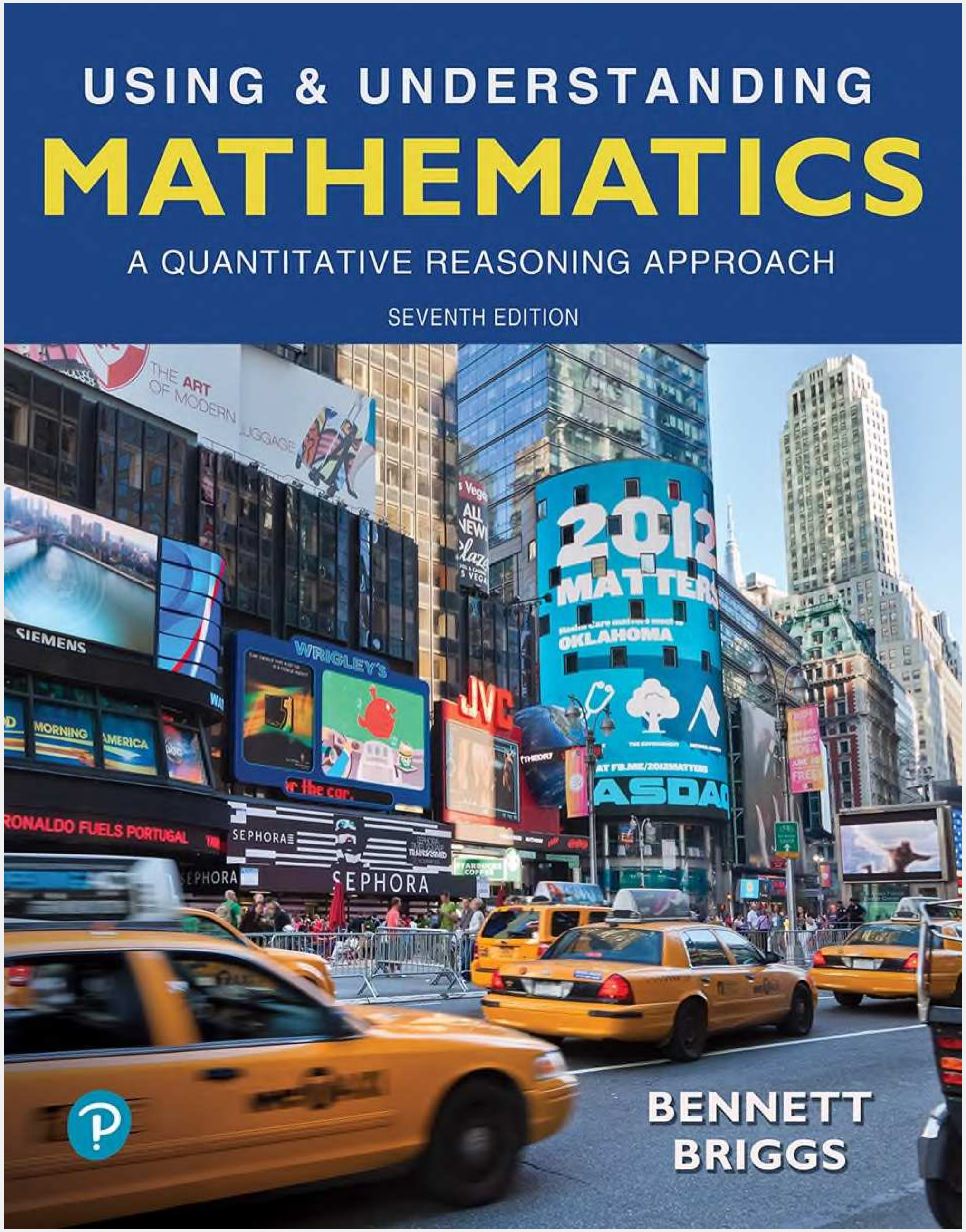
32. a. Assume the babies are named A, B, C, and D. If two babies are correctly tagged, the other two labels must be switched for their labels to be incorrect. Thus the problem boils down to counting the number of ways two of the babies can be correctly tagged. Listing all the options is the easiest way to count them. Babies AB could be correctly labeled (leaving babies C and D labeled incorrectly), and the other options for correct labels are AC, AD, BC, BD, and CD. This makes six different ways.

    b. If three of the babies have correct labels, the other baby must also have a correct label, which means there is no way to label three of the babies correctly while the fourth is labeled incorrectly.

33. – 40. Answers will vary.

Chapter 2

Approaches to Problem Solving



Unit 2A

Understand, Solve, and Explain

Three-Step Problem Solving: Understand-Solve-Explain (U-S-E)

Step 1: UNDERSTAND the problem.

- Think about what the problem asks you to do.
- Draw a picture or diagram to help make sense of the problem.
- Ask yourself what the solution should look like.
- Try to map a path (either mentally or in writing) that will lead you from your understanding of the problem to its solution.
- Continually revisit your understanding of the problem.

Three-Step Problem Solving: Understand-Solve-Explain (U-S-E)

Step 2: SOLVE the problem.

- Obtain any needed information or data.
- For multi-step problems, be sure to keep an organized, neatly written record of your work.
- Double-check each step as you work to avoid carrying errors through to the end of your solution.
- Constantly reevaluate your plan as you work.

Three-Step Problem Solving: Understand-Solve-Explain (U-S-E)

Step 3: EXPLAIN your result.

- Be sure that your result makes sense.
- Recheck your calculations once more or, even better, find an independent way to check your result.
- Identify and understand any potential sources of uncertainty in your result. If you made assumptions, were they reasonable?
- Write your solution clearly and concisely, using complete sentences to make sure the context and meaning are clear.

Units

The **units** of a quantity describe what that quantity measures or counts.

Unit analysis is the process of working with units to help solve problems.

Example: Using Key Words

You are buying 30 acres of farm land at $12,000 per acre. What is the total cost?

Solution

Understand. The question asks about total cost, and one of the given units is dollars, so we expect an answer in *dollars.*

Example

Solve: We carry out the calculation; note that the price is given in dollars per acre, so we write the division by acres in fraction form. That allows "acres" to cancel, leaving the final answer in dollars:

$$30\text{ acres} \times \frac{\$12{,}000}{\text{acre}} = \$360{,}000$$

Explain: We have found that purchasing 30 acres of farmland at a price of $12,000 per acre will cost a total of $360,000.

Example

Show operations and units clearly to answer the question: What is the total distance traveled when you run 7 laps around a 400-meter track?

Solution We could express the same idea as "7 laps *of* a 400-meter track." Therefore, this problem requires multiplying 7 laps by the 400 meters you run per lap:

$$7\ \cancel{\text{laps}} \times \frac{400\ \text{m}}{\cancel{\text{lap}}} = 2800\ \text{m}$$

Key Words and Operations with Units

Key word or symbol	Operation	Example
per	Division	Read miles ÷ hours as "miles per hour."
of or hyphen	Multiplication	Read kilowatts × hours as "kilowatt-hours."
square	Raising to second power	Read ft × ft, or ft^2, as "square feet" or "feet squared."
cube or *cubic*	Raising to third power	Read ft × ft × ft, or ft^3, as "cubic feet" or "feet cubed."

Conversion Factors

A **conversion factor** is a statement of equality that is used to convert between units.

Some conversion factors:

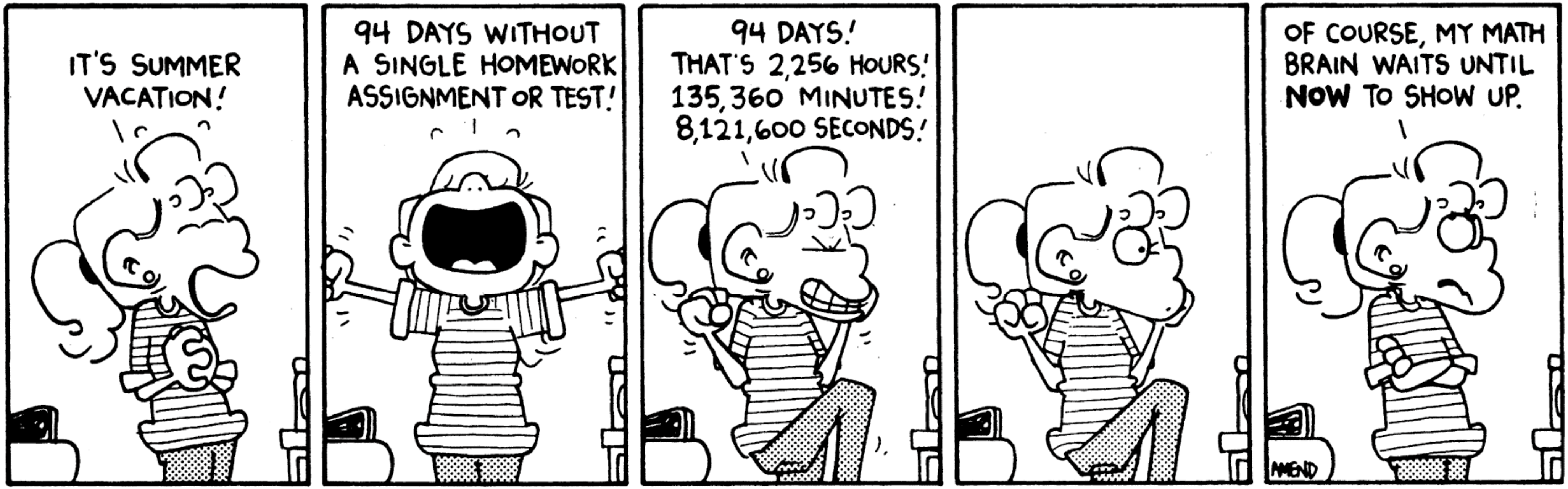
$$12\text{ in.} = 1\text{ ft} \quad \text{or} \quad \frac{12\text{ in.}}{1\text{ ft}} = 1 \quad \text{or} \quad \frac{1\text{ ft}}{12\text{ in.}} = 1$$

$$24\text{ hr} = 1\text{ day} \quad \text{or} \quad \frac{24\text{ hr}}{1\text{ day}} = 1 \quad \text{or} \quad \frac{1\text{ day}}{24\text{ hr}} = 1$$

Unit Conversions

Convert a distance of 9 feet into inches.

$$9\ \text{ft} = 9\ \cancel{\text{ft}} \times \frac{12\ \text{in.}}{1\ \cancel{\text{ft}}} = 108\ \text{in.}$$

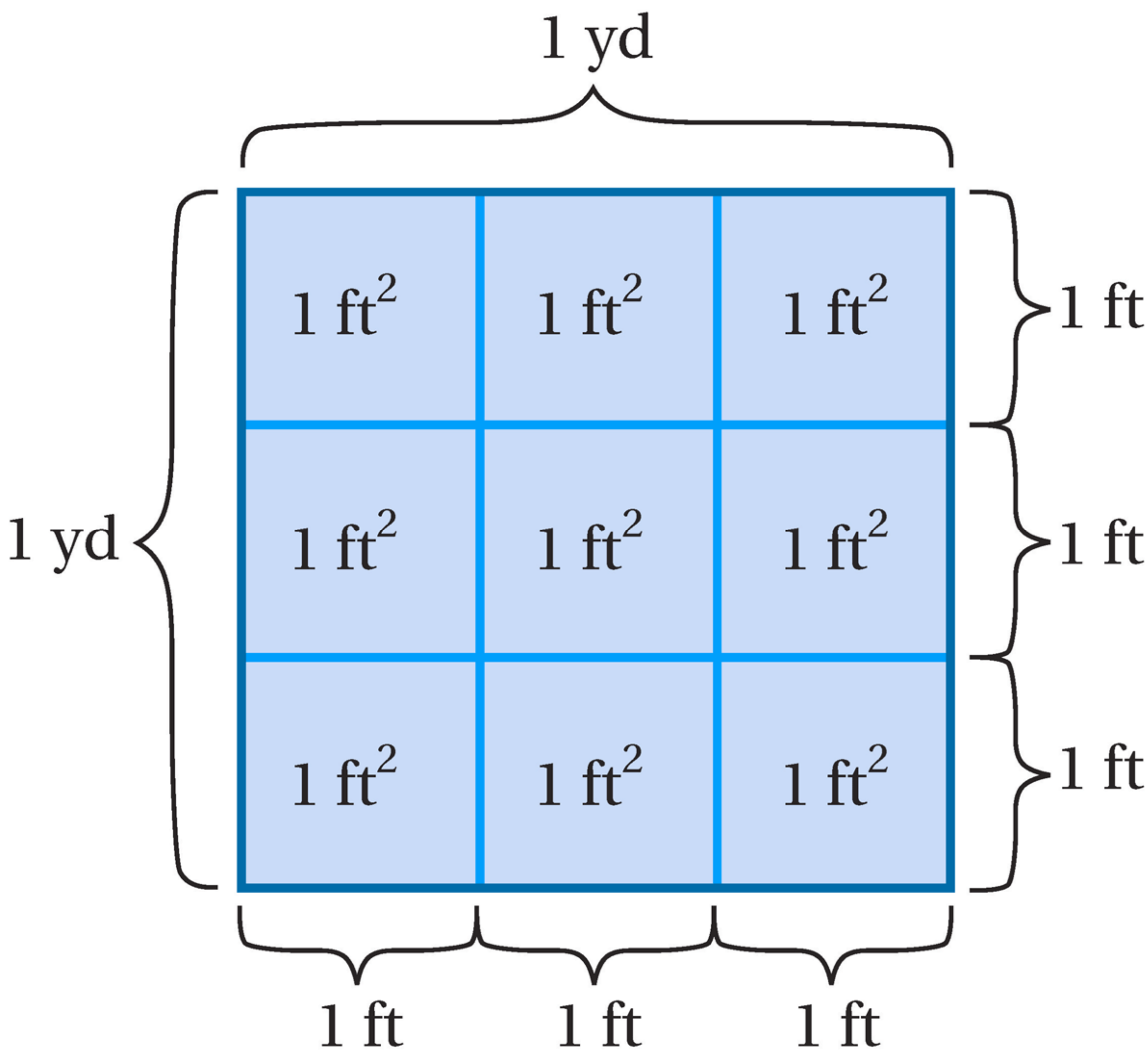


$$94\ \cancel{\text{days}} \times \frac{24\ \cancel{\text{hr}}}{1\ \cancel{\text{day}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \times \frac{60\ \text{sec}}{1\ \cancel{\text{min}}} = 8{,}121{,}600\ \text{sec}$$

Conversions with Units Raised to Powers

$1 \text{ yd} = 3 \text{ ft}$

$$\begin{aligned} 1 \text{ yd}^2 &= 1 \text{ yd} \times 1 \text{ yd} \\ &= 3 \text{ ft} \times 3 \text{ ft} \\ &= 9 \text{ ft}^2 \end{aligned}$$



Example

How many cubic yards of soil are needed to fill a planter that is 20 feet long by 3 feet wide by 4 feet tall?

The volume is $20 \text{ ft} \times 3 \text{ ft} \times 4 \text{ ft} = 240 \text{ ft}^3$

$1 \text{ yd} = 3 \text{ ft}$, so $(1 \text{ yd})^3 = (3 \text{ ft})^3 = 27 \text{ ft}^3$

$$240 \text{ ft}^3 \times \frac{1 \text{ yd}^3}{27 \text{ ft}^3} \approx 8.9 \text{ yd}^3$$

Example

The length of the Kentucky Derby horse race is 10 furlongs. How long is the race in miles?

Solution

See page 78 for Table 2.1, 1 furlong = 1/8 mi which is the same as 0.125 mile. We can write the conversion factor in two other equivalent forms:

$$\frac{1\text{ furlong}}{0.125\text{ m}} = 1 \text{ or } \frac{0.125\text{ mi}}{1\text{ furlong}} = 1$$

Example (cont)

The length of the Kentucky Derby horse race is 10 furlongs. How long is the race in miles?

$$10\ \cancel{\text{furlongs}} \times \frac{0.125\ \text{mi}}{1\ \cancel{\text{furlong}}} = 1.25\ \text{mi}$$

The Kentucky Derby is a race of 1.25 miles.

Metric System

The international metric system was invented in France late in the 18th century for two primary reasons: (1) to replace many customary units with just a few basic units and (2) to simplify conversions through use of a decimal (base 10) system. The basic units of length, mass, time, and volume in the metric system are

- the **meter** for length, abbreviated m
- the **kilogram** for mass, abbreviated kg
- the **second** for time, abbreviated s
- the **liter** for volume, abbreviated L

Common Metric Prefixes

TABLE 2.2 Common Metric Prefixes

Small Values			Large Values		
Prefix	**Abbrev.**	**Value**	**Prefix**	**Abbrev.**	**Value**
deci	d	10^{-1} (one-tenth)	deca	da	10^{1} (ten)
centi	c	10^{-2} (one-hundredth)	hecto	h	10^{2} (hundred)
milli	m	10^{-3} (one-thousandth)	kilo	k	10^{3} (thousand)
micro	μ or mc*	10^{-6} (one-millionth)	mega	M	10^{6} (million)
nano	n	10^{-9} (one-billionth)	giga	G	10^{9} (billion)
pico	p	10^{-12} (one-trillionth)	tera	T	10^{12} (trillion)

* Micro is usually abbreviated with μ (the Greek letter mu), but in medical fields it is common to use "mc" instead.

Example

Convert 2759 centimeters to meters.

Solution

Table 2.2 shows that *centi* means 10^{-2} so
$1 \text{ cm} = 10^{-2} \text{ m}$ or, equivalently, $1 \text{ m} = 100 \text{ cm}$.
Therefore, 2759 centimeters is the same as

$$2759 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} = 27.59 \text{ m}$$

Example

The marathon running race is about 26.2 miles. About how far is it in kilometers?

Solution

Table 2.3 shows that 1 mi = 1.6093 km. We use the conversion in the form with miles in the denominator to find

$$26.2\text{ mi} \times \frac{1.6093\text{ km}}{1\text{ mi}} = 42.2\text{ km}$$

Temperature Units

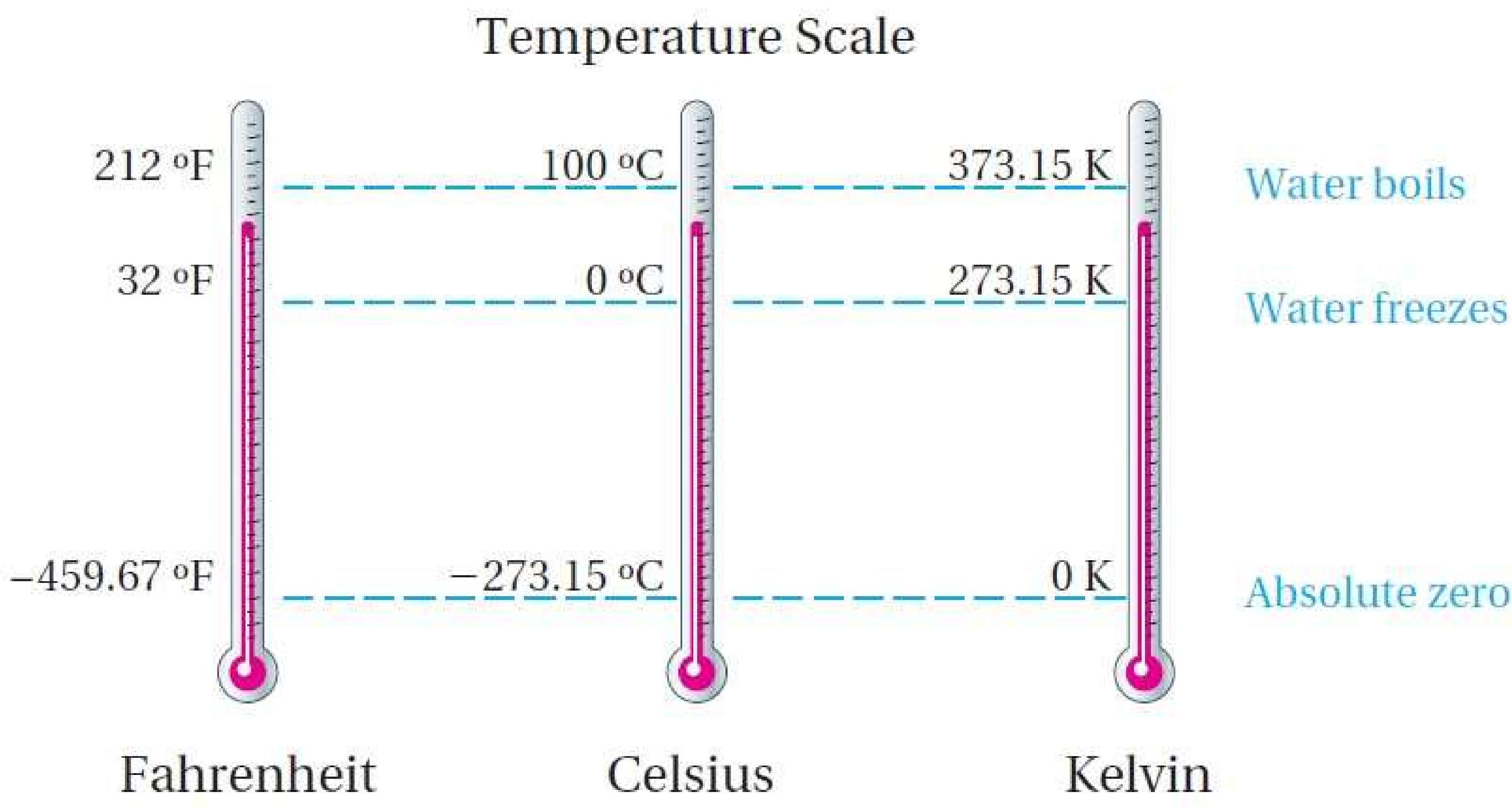
The **Fahrenheit** scale, commonly used in the United States, is defined so water freezes at 32° F and boils at 212° F.

The rest of the world uses the **Celsius** scale, which places the freezing point of water at 0° C and the boiling point at 100° C.

Temperature Units

In science, we use the **Kelvin** scale, which is the same as the Celsius scale except for its zero point, which corresponds to -273.15° C. A temperature of 0 K is known as **absolute zero**, because it is the coldest possible temperature. (The degree symbol [°] is not used on the Kelvin scale.)

Temperature Units



Temperature Conversions

The conversions are given both in words and with formulas in which C, F, and K are Celsius, Fahrenheit, and Kelvin temperatures, respectively.

To Convert from	Conversion in Words	Conversion Formula
Celsius to Fahrenheit	Multiply by 1.8 (or $\frac{9}{5}$). Then add 32.	$F = 1.8C + 32$
Fahrenheit to Celsius	Subtract 32. Then divide by 1.8, which is $\frac{9}{5}$, or equivalently multiply by $\frac{5}{9}$.	$C = \frac{F - 32}{1.8}$
Celsius to Kelvin	Add 273.15.	$K = C + 273.15$
Kelvin to Celsius	Subtract 273.15.	$C = K - 273.15$

Example

Average human body temperature is 98.6° F. What is it in Celsius and Kelvin?

Solution

Convert from Fahrenheit to Celsius by subtracting 32 and then dividing by 1.8:

$$C = \frac{F-32}{1.8} = \frac{98.6-32}{1.8} = \frac{66.6}{1.8} = 37.0^\circ\mathrm{C}$$

We find the Kelvin equivalent by adding 273.15 to the Celsius temperature:

$$K = C + 273.15 = 37 + 273.15 = 310.15 \text{ K}$$

Example

At a French department store, the price for a pair of jeans is 45 euros. What is the price in U.S. dollars? Use the exchange rates in Table 2.4.

Solution

From the *Dollars per Foreign* column in Table 2.4, we see that 1 euro = \$1.320. As usual, we can write this conversion factor in two other equivalent forms:

$$\frac{1\text{ euro}}{\$1.320} = 1 \quad \text{or} \quad \frac{\$1.320}{1\text{ euro}} = 1$$

$$45\text{ euro} \times \frac{\$1.320}{1\text{ euro}} \approx \$59.40$$

Currency Conversions

Converting between currencies is a unit conversion problem in which the conversion factors are known as the *exchange rates*. Table 2.4 shows a typical table of currency exchange rates:

TABLE 2.4 Sample Currency Exchange Rates (February 2013)

Currency	Dollars per Foreign	Foreign per Dollar
British pound	1.624	0.6158
Canadian dollar	1.005	0.9950
European euro	1.320	0.7576
Japanese yen	0.0120	83.33
Mexican peso	0.07855	12.73

Example

A gas station in Canada sells gasoline for CAD 1.34 per liter. (CAD is an abbreviation for Canadian dollars.) What is the price in dollars per gallon? Use the currency exchange rate in Table 2.4.

Solution

We use a chain of conversions to convert from CAD to dollars and then from liters to gallons. From Table 2.4, the currency conversion is $1.005 per CAD, and from Table 2.3, there are 3.785 liters per gallon.

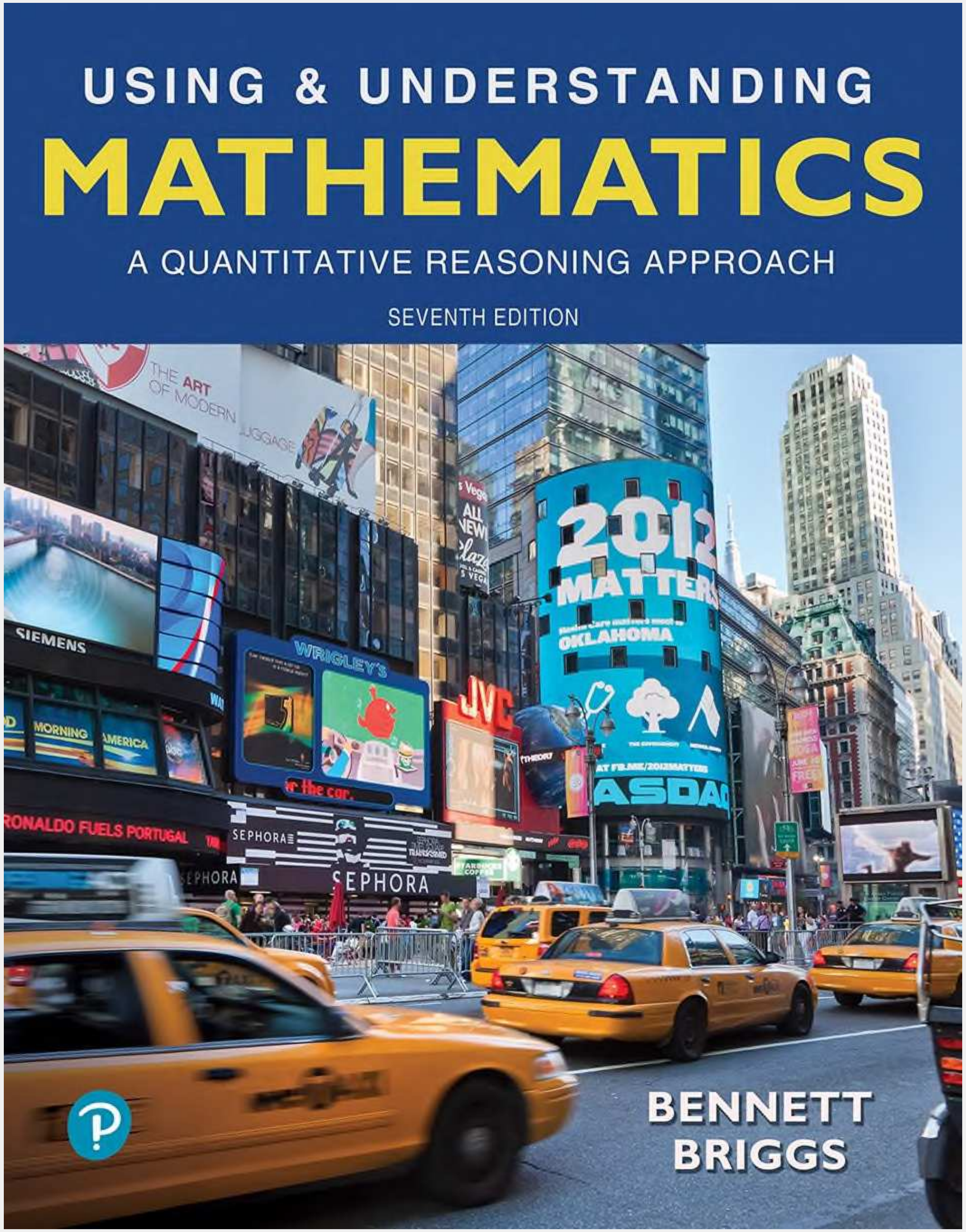
Example

A gas station in Canada sells gasoline for CAD 1.34 per liter. (CAD is an abbreviation for Canadian dollars.) What is the price in dollars per gallon? Use the currency exchange rate in Table 2.4.

$$\frac{1.34\ \cancel{\text{CAD}}}{1\ \cancel{\text{L}}} \times \frac{\$1.005}{1\ \cancel{\text{CAD}}} \times \frac{3.785\ \cancel{\text{L}}}{1\ \text{gal}} \approx \frac{\$5.10}{1\ \text{gal}}$$

Chapter 2

Approaches to Problem Solving



Unit 2B

Problem-Solving with Units

Unit Analysis in Problem Solving

Step 1. Identify the units involved in the problem and the units that you expect for the answer.

Step 2. Use the given units and the expected answer units to help you find a strategy for solving the problem. Be sure to perform all operations (such as multiplication or division) on both the numbers and their associated units.

Unit Analysis in Problem Solving

Remember:

• You cannot add or subtract numbers with different units, but you can combine different units through multiplication, division, or raising to powers.

• It is easier to keep track of units if you replace division with multiplication by the reciprocal. For example, instead of dividing by 60 s/min, multiply by 1 min/60 s.

Step 3. When you complete your calculations, make sure that your answer has the units you expected. If it doesn't, then you've done something wrong.

Example

You are planning to make pesto and need to buy basil. At the grocery store, you can buy small containers of basil priced at $2.99 for each 2/3-ounce container. At the farmer's market, you can buy basil in bunches for $12 per pound. Which is the better deal?

Solution

To compare the prices, we need them both in the same units.

Convert the small container price to a price per pound.

Example

The container price is $2.99 *per* 2/3 ounce, which means we need to divide. We then multiply by the conversion of 16 ounces per pound:

$$\frac{\$2.99}{\frac{2}{3}\text{ oz}} \times \frac{16\text{ oz}}{1\text{ lb}} = \frac{\$71.76}{\text{lb}}$$

The small containers are priced at almost $72 per pound, which is six times as much as the farmer's market price.

Example

Your destination is 90 miles away, and your fuel gauge shows that your gas tank is one-quarter full. Your tank holds 12 gallons of gas, and your car averages about 25 miles per gallon. Do you need to stop for gas?

Solution

$$90\text{ mi} \div \frac{25\text{ mi}}{1\text{ gal}} = 90\text{ mi} \times \frac{1\text{ gal}}{25\text{ mi}} = 3.6\text{ gal}$$

You will need 3.6 gallons of gas for the 90-mile trip, but one-quarter of a 12-gallon tank is only 3 gallons. Therefore, you should stop for gas.

Units of Energy and Power

Energy is what makes matter move or heat up. International metric unit is the **joule.**

Power is the rate at which energy is used. International metric unit is the **watt.**

$$1 \text{ watt} = 1 \frac{\text{joule}}{\text{s}}$$

A **kilowatt-hour** is a unit of energy.

1 kilowatt-hour = 3.6 million joules

Operating Cost of a Light Bulb

A utility company charges 15¢ per kilowatt-hour of electricity. How much does it cost to keep a 100-watt light bulb on for a week? How much will you save in a year if you replace the bulb with an LED bulb that provides the same amount of light for only 25 watts of power?

Solution

$$100\ \cancel{\text{watt}} \times \frac{1\ \text{kilowatt}}{1000\ \cancel{\text{watt}}} \times 1\ \cancel{\text{week}} \times \frac{7\ \cancel{\text{day}}}{1\ \cancel{\text{week}}} \times \frac{24\ \text{hr}}{1\ \cancel{\text{day}}} =$$

$$= 16.8\ \text{kilowatt-hour}$$

Operating Cost of a Light Bulb

Now find the cost.

$$16.8 \text{ kilo}\cancel{\text{watt-hour}} \times 15 \frac{\text{cents}}{\text{kilo}\cancel{\text{watt-hr}}} = 252 \qquad = \$2.52$$

The electricity for the bulb costs \$2.52 per week. If you replace the 100-watt bulb with a 25-watt LED, you'll use only 1/4 as much energy, which means your weekly cost will be only 63¢. In other words, your savings will be \$2.52 - \$0.63 = \$1.89 per week, so in a year you'll save about:

$$\frac{\$1.89}{\text{wk}} \times \frac{52 \text{ wk}}{\text{yr}} \approx \$98 / \text{yr}$$

Units of Density and Concentration

Density describes compactness or crowding.

- *Material density* is given in units of mass per unit volume.
  - e.g., grams per cubic centimeter (g/cm^3)
- *Population density* is given by the number of people per unit area.
  - e.g., people per square mile ($people/mi^2$)
- *Information density* is given in units of mass per unit volume.
  - e.g., gigabytes per square inch ($GB/in.^2$)

Pearson

Units of Density and Concentration

Concentration describes the amount of one substance mixed with another.

- The *concentration of an air pollutant* is often measured by the number of molecules of the pollutant per million molecules of air.
  - e.g., parts per million (ppm)
- *Blood alcohol content (BAC)* describes the concentration of alcohol in a person’s body.
  - e.g., grams of alcohol per 100 milliliters of blood

Example

A child weighing 15 kilograms has a bacterial ear infection. A physician orders treatment with amoxicillin at a dosage based on 30 milligrams per kilogram of body weight per day, divided into doses every 12 hours.

a. How much amoxicillin should the child be prescribed every 12 hours?

b. If the medicine is to be taken in a liquid suspension with concentration 25 mg/ml, how much should the child take every 12 hours?

Example

a. How much amoxicillin should the child be prescribed every 12 hours?

The prescribed dosage is 30 mg/kg of body weight per day, but because it will be given in two doses (every 12 hours), each dose will be based on half of the total, or 15 mg/kg of body weight. Therefore, for a child weighing 15 kilograms, the dosage should be

$$12\text{ hrs} = \frac{15\text{ mg}}{\text{kg}} \times 15\text{ kg} = 225\text{ mg}$$

Example

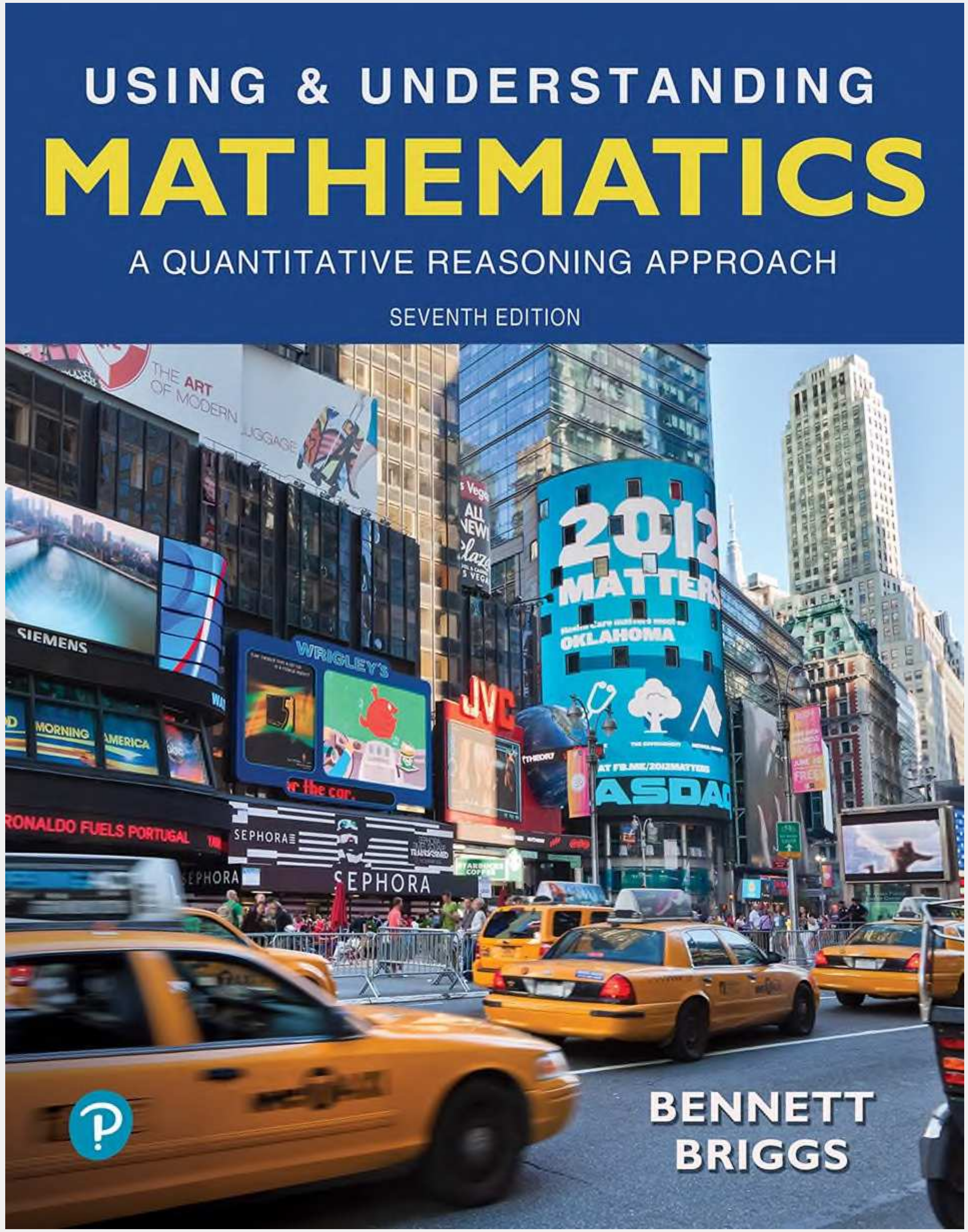
b. If the medicine is to be taken in a liquid suspension with concentration 25 mg/ml, how much should the child take every 12 hours?

The liquid suspension contains 25 milligrams of amoxicillin per milliliter (ml) of liquid, and from part (a) we know the total amount of amoxicillin in each dose should be 225 mg. We are looking for the total amount of liquid that the child should be given for each dose, so the answer should have units of milliliters.

$$225 \text{ mg} \div \frac{25 \text{ mg}}{\text{ml}} = 225 \text{ mg} \times \frac{1 \text{ ml}}{25 \text{ mg}} = 9 \text{ ml}$$

Chapter 2

Approaches to Problem Solving



Unit 2C

Problem-Solving Hints

Understand-Solve-Explain Process

Step 1: Understand the problem.

Step 2: Solve the problem.

Step 3: Explain your result.

Problem Solving Guidelines and Hints

Hint 1: There may be more than one answer.

Hint 2: There may be more than one method/

Hint 3: Use appropriate tools.

Hint 4: Consider simpler, similar problems.

Hint 5: Consider equivalent problems with simpler solutions.

Hint 6: Approximations can be useful.

Hint 7: Try alternative patterns of thought.

Hint 8: Do not spin your wheels.

Example

Tickets for a fundraising event were priced at \$10 for children and \$20 for adults. Shauna worked the first shift at the box office, selling a total of \$130 worth of tickets. However, she did not keep a careful count of how many tickets she sold for children and adults. How many tickets of each type (child and adult) did she sell?

Solution

Try trial and error. Suppose Shauna sold just one \$10 child ticket. In that case, she would have sold \$130 - \$10 = \$120 worth of adult tickets.

Example (cont)

Because the adult tickets cost \$20 apiece, this means she would have sold \$120 ÷ (\$20 per adult ticket) = 6 adult tickets.

We have found an answer to the question: Shauna could have collected \$130 by selling 1 child and 6 adult tickets. But it is not the only answer, as we can see by testing other values.

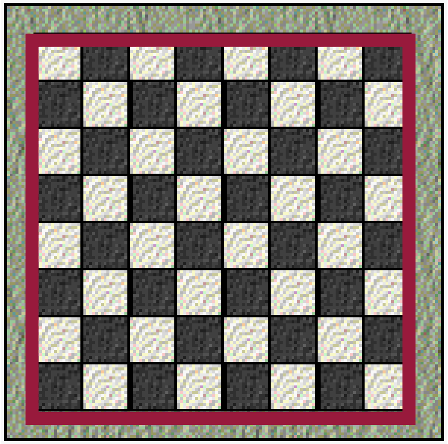
For example, suppose she sold three of the \$10 child tickets, for a total of \$30. Then she would have sold \$130 - \$30 = \$100 worth of adult tickets, which means 5 of the \$20 adult tickets.

Example (cont)

We have a second possible answer—3 child and 5 adult tickets—and have no way to know which answer is the actual number of tickets sold. In fact, there are seven possible answers to the question. In addition to the two answers we've already found, other possible answers are 5 child tickets and 4 adult tickets; 7 child tickets and 3 adult tickets; 9 child tickets and 2 adult tickets; 11 child tickets and 1 adult ticket; and 13 child tickets with 0 adult tickets. Without further information, we do not know which combination represents the actual ticket sales.

Example

Find the total number of possible squares on the chessboard by looking for a pattern.



Solution

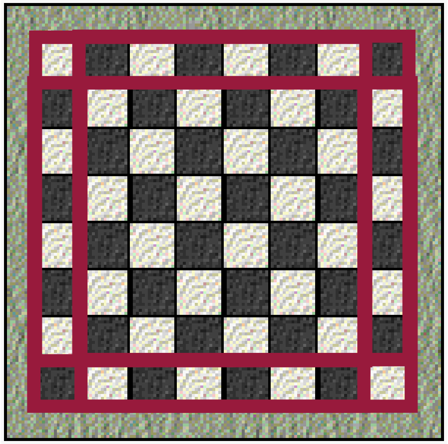
Start with the largest possible square:

There is only **one** way to make an

8 x 8 square.

Example (cont)

Find the total number of possible squares on the chessboard by looking for a pattern.

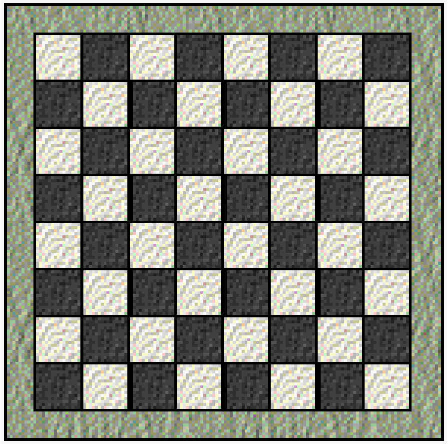


Now, look for the number of ways to make a 7 x 7 square.

There are only **four** ways.

Example (cont)

Find the total number of possible squares on the chessboard by looking for a pattern.



If you continue looking at 6 x 6, then 5 x 5 squares, and so on, you will see the **perfect square** pattern as indicated in the following table for this chessboard problem:

Example (cont)

Find the total number of possible squares on the chessboard by looking for a pattern.

n x *n* squares	#
8 x 8	1
7 x 7	4
6 x 6	9
5 x 5	16
4 x 4	25
3 x 3	36
2 x 2	49
1 x 1	64
Total:	**204**