

***Thinking Mathematically 7th edition Blitzer Solutions  
Manual***

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## Chapter 2

### Set Theory

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#### Check Points 2.1

1. Set  $L$  is the set of the first six lowercase letters in the English alphabet.
2.  $M = \{\text{April, August}\}$
3.  $O = \{1, 3, 5, 7, 9\}$
4.
  - a. not the empty set; Many numbers meet the criteria to belong to this set.
  - b. the empty set; No numbers meet the criteria, thus this set is empty
  - c. not the empty set; “nothing” is not a set.
  - d. not the empty set; This is a set that contains one element, that element is a set.
5.
  - a. true; 8 is an element of the given set.
  - b. true;  $r$  is not an element of the given set.
  - c. false;  $\{\text{Monday}\}$  is a set and the set  $\{\text{Monday}\}$  is not an element of the given set.
6.
  - a.  $A = \{1, 2, 3\}$
  - b.  $B = \{15, 16, 17, \dots\}$
  - c.  $O = \{1, 3, 5, \dots\}$
7.
  - a.  $\{1, 2, 3, 4, \dots, 199\}$
  - b.  $\{51, 52, 53, 54, \dots, 200\}$
8.
  - a.  $n(A) = 5$ ; the set has 5 elements
  - b.  $n(B) = 1$ ; the set has only 1 element
  - c.  $n(C) = 8$ ; Though this set lists only five elements, the three dots indicate 12, 13, and 14 are also elements.
  - d.  $n(D) = 0$  because the set has no elements.
9. No, the sets are not equivalent. Set A has 4 distinct elements yet set B has only 3 distinct elements.

10.
  - a. true;  $\{O, L, D\} = \{D, O, L\}$  because the sets contain exactly the same elements.
  - b. false; The two sets do not contain exactly the same elements.

#### Concept and Vocabulary Check 2.1

1. roster; set builder
2. empty;  $\emptyset$
3. is an element
4. natural numbers
5. cardinal;  $n(A)$
6. equivalent
7. equal

#### Exercise Set 2.1

1. This is well defined and therefore it is a set.
2. This is well defined and therefore it is a set.
3. This is a matter of opinion and not well defined, thus it is not a set.
4. This is a matter of opinion and not well defined, thus it is not a set.
5. This is well defined and therefore it is a set.
6. This is well defined and therefore it is a set.
7. The set of known planets in our Solar System.  
Note to student: This exercise did not forget Pluto. In 2006, based on the requirement that a planet must dominate its own orbit, the International Astronomical Union removed Pluto from the list of planets.
8. The set of weekend days.
9. The set of months that begin with J.
10. The set of months that begin with A.
11. The set of natural numbers greater than 5.

12. The set of natural numbers greater than 8.
13. The set of natural numbers between 6 and 20, inclusive.
14. The set of natural numbers between 9 and 25, inclusive.
15. {winter, spring, summer, fall}
16. {April, June, September, November}
17. {September, October, November, December}
18. {e, f, g, h, i}
19. {1, 2, 3}
20. {1, 2, 3, 4, 5, 6}
21. {1, 3, 5, 7, 9, 11}
22. {2, 4, 6, 8}
23. {1, 2, 3, 4, 5}
24. {1, 2, 3, 4}
25. {6, 7, 8, 9, ...}
26. {5, 6, 7, 8, ...}
27. {7, 8, 9, 10}
28. {8, 9, 10, 11}
29. {10, 11, 12, 13, ..., 79}
30. {15, 16, 17, 18, ..., 59}
31. {2}
32. {6}
33. not the empty set
34. not the empty set
35. empty set
36. empty set
37. not the empty set  
Note that the number of women who served as U.S. president before 2016 is 0. Thus the number 0 is an element of the set.
38. not the empty set  
Note that the number of living U.S. presidents born before 1200 is 0. Thus the number 0 is an element of the set.
39. empty set
40. empty set
41. empty set
42. empty set
43. not the empty set
44. not the empty set
45. not the empty set
46. not the empty set
47. true  
3 is a member of the set.
48. true  
6 is a member of the set.
49. true  
12 is a member of the set.
50. true  
10 is a member of the set.
51. false  
5 is *not* a member of the set.
52. false  
8 is *not* a member of the set.
53. true  
11 is *not* a member of the set.
54. true.  
17 is *not* a member of the set.
55. false  
37 is a member of the set.
56. false  
26 is a member of the set.
57. false  
4 is a member of the set.
58. false  
2 is *not* a member of the set.
59. true  
13 is *not* a member of the set.

60. true  
20 is *not* a member of the set.
61. false  
16 is a member of the set.
62. false  
19 is a member of the set.
63. false  
The set  $\{3\}$  is *not* a member of the set.
64. false  
The set  $\{7\}$  is *not* a member of the set.
65. true  
-1 is *not* a natural number.
66. true  
-2 is *not* a natural number.
67.  $n(A) = 5$ ; There are 5 elements in the set.
68.  $n(A) = 6$ ; There are 6 elements in the set.
69.  $n(B) = 15$ ; There are 15 elements in the set.
70.  $n(B) = 11$ ; There are 11 elements in the set.
71.  $n(C) = 0$ ; There are *no* days of the week beginning with A.
72.  $n(C) = 0$ ; There are no such months.
73.  $n(D) = 1$ ; There is 1 element in the set.
74.  $n(D) = 1$ ; There is 1 element in the set.
75.  $n(A) = 4$ ; There is 4 elements in the set.
76.  $n(A) = 3$ ; There is 3 elements in the set.
77.  $n(B) = 5$ ; There is 5 elements in the set.
78.  $n(B) = 7$ ; There is 7 elements in the set.
79.  $n(C) = 0$ ; There are no elements in the set.
80.  $n(C) = 0$ ; There are no elements in the set.
81. a. Not equivalent  
The number of elements is not the same.
- b. Not equal  
The two sets contain different elements.
82. a. Equivalent  
The number of elements is the same.
- b. Not equal  
The two sets contain different elements.
83. a. Equivalent  
The number of elements is the same.
- b. Not equal  
The elements are not exactly the same.
84. a. Equivalent  
The number of elements is the same.
- b. Not equal  
The elements are not exactly the same.
85. a. Equivalent  
The number of elements is the same.
- b. Equal  
The elements are exactly the same.
86. a. Equivalent  
Number of elements is the same.
- b. Equal  
The elements are exactly the same.
87. a. Equivalent  
Number of elements is the same.
- b. Not equal  
The two sets contain different elements.
88. a. Equivalent  
Number of elements is the same.
- b. Not equal  
The two sets contain different elements.
89. a. Equivalent  
Number of elements is the same.
- b. Equal  
The elements are exactly the same.
90. a. Equivalent  
Number of elements is the same.
- b. Equal  
The elements are exactly the same.
91. infinite
92. infinite
93. finite
94. finite
95. finite
96. finite

97.  $\{x | x \in \mathbb{N} \text{ and } x \geq 61\}$
98.  $\{x | x \in \mathbb{N} \text{ and } x \geq 36\}$
99.  $\{x | x \in \mathbb{N} \text{ and } 61 \leq x \leq 89\}$
100.  $\{x | x \in \mathbb{N} \text{ and } 36 \leq x \leq 59\}$
101. Answers will vary; an example is:  $\{0, 1, 2, 3\}$  and  $\{1, 2, 3, 4\}$ .
102. Answers will vary; an example is:  $\{x | x \in \mathbb{N} \text{ and } x < 5\}$  and  $\{1, 2, 3, 4\}$ .
103. Impossible. Equal sets have exactly the same elements. This would require that there also must be the same number of elements.
104. Answers will vary; an example is:  $\{1\}$  and  $\{1, 2\}$ .
105.  $\{\text{Engineering, Accounting}\}$
106.  $\{\text{Engineering}\}$
107.  $\{\text{Journalism, Nursing, Philosophy}\}$
108.  $\{\text{Nursing, Philosophy, Social Work}\}$
109.  $\{ \}$
110.  $\{\text{Group 1, Group 2, Group 3, Group 4, Group 5, Group 6}\}$
111.  $\{\text{Group 7}\}$
112.  $\{\text{Group 1, Group 2, Group 3, Group 5, Group 6}\}$
113.  $\{12, 19\}$
114.  $\{10, 14, 16\}$
115.  $\{20, 21\}$
116.  $\{20, 21, 22\}$
117. There is not a one-to-one correspondence. These sets are not equivalent.
124. makes sense
125. does not make sense; Explanations will vary.  
Sample explanation: The natural numbers do not include negative numbers. Since the temperature will be below zero, a set that includes negative numbers would be necessary.
126. does not make sense; Explanations will vary.  
Sample explanation: There is not a one-to-one correspondence for men because two ages sleep for 8.3 hours.
127. makes sense
128. false; Changes to make the statement true will vary.  
A sample change is: If two sets are equal, they must be equivalent.
129. false; Changes to make the statement true will vary.  
A sample change is: If a roster set contains three dots, it is finite if there is an ending value after the three dots.
130. false; Changes to make the statement true will vary.  
A sample change is: The cardinality of the empty set is 0.
131. true
132. true
133. false; Changes to make the statement true will vary.  
A sample change is: Though that set has many values, it is still a finite set.
134. false; Changes to make the statement true will vary.  
A sample change is: Some finite sets could have so many elements that it would take longer than a trillion years to count them.
135. false; Changes to make the statement true will vary.  
A sample change is: If 0 is removed from a set, it will lower the cardinality of that set by one.
136. This question contains a paradox. Sweeney Todd cannot shave himself because he does not shave any men who shave themselves. That suggests that  $s \notin A$  which implies  $s \in B$ . However, if Sweeney Todd does not shave himself, the question states he shaves all such men who do not shave themselves. That suggests that he does shave himself, giving  $s \in A$  which implies  $s \notin B$ . Therefore, paradoxically,  $s$  belongs and does not belong in both sets.
- a. no
- b. no

## Check Points 2.2

1. a.  $\not\subseteq$ ; because 6, 9, and 11 are not in set  $B$ .  
 b.  $\subseteq$ ; because all elements in set  $A$  are also in set  $B$ .  
 c.  $\subseteq$ ; because all elements in set  $A$  are also in set  $B$ .
2. a. Both  $\subseteq$  and  $\subset$  are correct.  
 b. Both  $\subseteq$  and  $\subset$  are correct.
3. Yes, the empty set is a subset of any set.
4. a. 16 subsets, 15 proper subsets  
 There are 4 elements, which means there are  $2^4$  or 16 subsets. There are  $2^4 - 1$  proper subsets or 15.  
 b. 64 subsets, 63 proper subsets  
 There are 6 elements, which means there are  $2^6$  or 64 subsets. There are  $2^6 - 1$  proper subsets or 63.

## Concept and Vocabulary Check 2.2

1.  $A \subseteq B$ ; every element in set  $A$  is also an element in set  $B$
2.  $A \subset B$ ; sets  $A$  and  $B$  are not equal
3. the empty; subset
4.  $2^n$
5.  $2^n - 1$

## Exercise Set 2.2

1.  $\subseteq$
2.  $\subseteq$
3.  $\not\subseteq$
4.  $\not\subseteq$
5.  $\not\subseteq$
6.  $\not\subseteq$

7.  $\not\subseteq$   
 Subset cannot be larger than the set.
8.  $\not\subseteq$   
 Subset cannot be larger than the set.
9.  $\subseteq$
10.  $\subseteq$
11.  $\not\subseteq$
12.  $\not\subseteq$
13.  $\subseteq$
14.  $\subseteq$
15.  $\not\subseteq$
16.  $\not\subseteq$
17.  $\subseteq$
18.  $\subseteq$
19.  $\subseteq$  or  $\subset$
20.  $\subseteq$  or  $\subset$
21.  $\subseteq$
22.  $\subseteq$
23. neither
24. neither
25. both
26. both
27.  $\subseteq$
28.  $\subseteq$
29.  $\subseteq$
30.  $\subseteq$
31. both
32. both
33. both

34. neither
35. both
36. both
37. neither
38. neither
39.  $\subseteq$
40.  $\subseteq$
41. true
42. true
43. false  
    {Ralph} is a subset, not Ralph.
44. false  
    {Canada} is a subset, not Canada.
45. true
46. true
47. false  
    The symbol " $\emptyset$ " is not a member of the set.
48. true
49. true
50. true
51. false  
    All elements of {1, 4} are members of {4, 1}
52. true
53. true
54. true
55. { } {Border Collie} {Poodle} {Border Collie, Poodle}
56. { } {Romeo} {Juliet} {Romeo, Juliet}
57. { } {t} {a} {b} {t, a} {t, b} {a, b} {t, a, b}
58. { } {I} {II} {III} {I, II} {I, III} {II, III} {I, II, III}
59. { } {0}
60.  $\emptyset$
61. 16 subsets, 15 proper subsets  
    There are 4 elements, which means there are  $2^4$  or 16 subsets. There are  $2^4 - 1$  proper subsets or 15.
62. 16 subsets, 15 proper subsets  
    There are 4 elements, which means there are  $2^4$  or 16 subsets. There are  $2^4 - 1$  proper subsets or 15.
63. 64 subsets, 63 proper subsets  
    There are 6 elements, which means there are  $2^6$  or 64 subsets. There are  $2^6 - 1$  proper subsets or 63.
64. 64 subsets, 63 proper subsets  
    There are 6 elements, which means there are  $2^6$  or 64 subsets. There are  $2^6 - 1$  proper subsets or 63.
65. 128 subsets, 127 proper subsets  
    There are 7 elements, which means there are  $2^7$  or 128 subsets. There are  $2^7 - 1$  proper subsets or 127.
66. 32 subsets, 31 proper subsets  
    There are 5 elements, which means there are  $2^5$  or 32 subsets. There are  $2^5 - 1$  proper subsets or 31.
67. 8 subsets, 7 proper subsets  
    There are 3 elements, which means there are  $2^3$  or 8 subsets. There are  $2^3 - 1$  proper subsets or 7.
68. 32 subsets, 31 proper subsets  
    There are 5 elements, which means there are  $2^5$  or 32 subsets. There are  $2^5 - 1$  proper subsets or 31.
69. false; The set {1, 2, 3, ..., 1000} has  $2^{1000} - 1$  proper subsets.
70. false; The set {1, 2, 3, ..., 10,000} has  $2^{10,000} - 1$  proper subsets.
71. true
72. true
73. false;  $\emptyset \subseteq \{\emptyset, \{\emptyset\}\}$
74. false;  $\{\emptyset\} \subseteq \{\emptyset, \{\emptyset\}\}$
75. true
76. true
77. true

78. true
79. true
80. true
81. false; The set of subsets of  $\{a, e, i, o, u\}$  contains  $2^5$  or 32 elements.
82. false; The set of subsets of  $\{a, b, c, d, e, f\}$  contains  $2^6$  or 64 elements.
83. false;  $D \subseteq T$
84. false;  $R \subseteq T$
85. true
86. true
87. false; If  $x \in W$ , then  $x \in D$ .
88. false; If  $x \in M$ , then  $x \in D$ .
89. true
90. true
91. true
92. true
93.  $2^5 = 32$  option combinations
94.  $2^9 = 512$  topping combinations
95.  $2^6 = 64$  viewing combinations
96.  $2^4 = 16$  response options
97.  $2^8 = 256$  city combinations
98.  $2^7 = 128$  viewing combinations
105. does not make sense; Explanations will vary.  
Sample explanation: The set's elements are not members of the other set.
106. makes sense
107. does not make sense; Explanations will vary.  
Sample explanation: The same formulas are used for each of the mentioned problems.
108. makes sense
109. false; Changes to make the statement true will vary.  
A sample change is: The set has one element and has  $2^1 = 2$  subsets.
110. true
111. false; Changes to make the statement true will vary.  
A sample change is: The empty set does not have a proper subset.
112. false; Changes to make the statement true will vary.  
A sample change is: The set has two elements and has  $2^2 = 4$  subsets.
113. 0, 5¢, 10¢, 25¢, 40¢, 15¢, 30¢, 35¢  
Since there are 3 elements or coins, there are  $2^3$  or 8 different coin combinations.
114. Number of proper subsets is  $2^n - 1$ , which means there are 128 total subsets ( $127 + 1$ ).  
128 is  $2^7$ , so there are 7 elements.

### Check Points 2.3

- $\{1, 5, 6, 7, 9\}$
  - $\{1, 5, 6\}$
  - $\{7, 9\}$
- $\{a, b, c, d\}$
  - $\{e\}$
  - $\{e, f, g\}$
  - $\{f, g\}$
- $A' = \{b, c, e\}$ ; those are the elements in  $U$  but not in  $A$ .
- $\{1, 3, 5, \underline{7}, \underline{10}\} \cap \{6, \underline{7}, \underline{10}, 11\} = \{7, 10\}$
  - $\{1, 2, 3\} \cap \{4, 5, 6, 7\} = \emptyset$
  - $\{1, 2, 3\} \cap \emptyset = \emptyset$
- $\{1, 3, 5, 7, 10\} \cup \{6, 7, 10, 11\} = \{1, 3, 5, 6, 7, 10, 11\}$
  - $\{1, 2, 3\} \cup \{4, 5, 6, 7\} = \{1, 2, 3, 4, 5, 6, 7\}$
  - $\{1, 2, 3\} \cup \emptyset = \{1, 2, 3\}$

6. a.  $A \cup B = \{b, c, e\}$   
 $(A \cup B)' = \{a, d\}$

b.  $A' = \{a, d, e\}$   
 $B' = \{a, d\}$   
 $A' \cap B' = \{a, d\}$

7. a.  $\{5\}$ ; region II

b.  $\{2, 3, 7, 11, 13, 17, 19\}$ ; the complement of region II

c.  $\{2, 3, 5, 7, 11, 13\}$ ; regions I, II, and III

d.  $\{17, 19\}$ ; the complement of regions I, II, and III

e.  $\{5, 7, 11, 13, 17, 19\}$ ; the complement of  $A$  united with  $B$

f.  $\{2, 3\}$ ;  $A$  intersected with the complement of  $B$

8.  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$   
 $= 26 + 11 - 9$   
 $= 28$

### Concept and Vocabulary Check 2.3

1. Venn diagrams

2. complement;  $A'$

3. intersection;  $A \cap B$

4. union;  $A \cup B$

5.  $n(A) + n(B) - n(A \cap B)$

6. true

7. false

8. true

9. false

4.  $U$  is the set of all models of automobiles.

5.  $A' = \{c, d, e\}$

6.  $B' = \{a, b, f, g\}$

7.  $C' = \{b, c, d, e, f\}$

8.  $D' = \{g\}$

9.  $A' = \{6, 7, 8, \dots, 20\}$

10.  $B' = \{1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

11.  $C' = \{2, 4, 6, 8, \dots, 20\}$

12.  $D' = \{1, 3, 5, 7, \dots, 19\}$

13.  $A' = \{21, 22, 23, 24, \dots\}$

14.  $B' = \{51, 52, 53, 54, \dots\}$

15.  $C' = \{1, 3, 5, 7, \dots\}$

16.  $D' = \{2, 4, 6, 8, \dots\}$

17.  $A = \{1, 3, 5, 7\}$

$B = \{1, 2, 3\}$

$A \cap B = \{1, 3\}$

18.  $B = \{1, 2, 3\}$

$C = \{2, 3, 4, 5, 6\}$

$B \cap C = \{2, 3\}$

19.  $A = \{1, 3, 5, 7\}$

$B = \{1, 2, 3\}$

$A \cup B = \{1, 2, 3, 5, 7\}$

20.  $B = \{1, 2, 3\}$

$C = \{2, 3, 4, 5, 6\}$

$B \cup C = \{1, 2, 3, 4, 5, 6\}$

21.  $A = \{1, 3, 5, 7\}$

$U = \{1, 2, 3, 4, 5, 6, 7\}$

$A' = \{2, 4, 6\}$

22.  $B = \{1, 2, 3\}$

$U = \{1, 2, 3, 4, 5, 6, 7\}$

$B' = \{4, 5, 6, 7\}$

### Exercise Set 2.3

1.  $U$  is the set of all composers.

2.  $U$  is the set of all writers.

3.  $U$  is the set of all brands of soft drinks.

23.  $A' = \{2, 4, 6\}$   
 $B' = \{4, 5, 6, 7\}$   
 $A' \cap B' = \{4, 6\}$
24.  $B' = \{4, 5, 6, 7\}$   
 $C = \{2, 3, 4, 5, 6\}$   
 $B' \cap C = \{4, 5, 6\}$
25.  $A = \{1, 3, 5, 7\}$   
 $C' = \{1, 7\}$   
 $A \cup C' = \{1, 3, 5, 7\}$
26.  $B = \{1, 2, 3\}$   
 $C' = \{1, 7\}$   
 $B \cup C' = \{1, 2, 3, 7\}$
27.  $A = \{1, 3, 5, 7\}$   
 $C = \{2, 3, 4, 5, 6\}$   
 $A \cap C = \{3, 5\}$   
 $(A \cap C)' = \{1, 2, 4, 6, 7\}$
28.  $A = \{1, 3, 5, 7\}$   
 $B = \{1, 2, 3\}$   
 $A \cap B = \{1, 3\}$   
 $(A \cap B)' = \{2, 4, 5, 6, 7\}$
29.  $A = \{1, 3, 5, 7\}$        $C = \{2, 3, 4, 5, 6\}$   
 $A' = \{2, 4, 6\}$        $C' = \{1, 7\}$   
 $A' \cup C' = \{1, 2, 4, 6, 7\}$
30.  $A = \{1, 3, 5, 7\}$   
 $B = \{1, 2, 3\}$   
 $A' = \{2, 4, 6\}$   
 $B' = \{4, 5, 6, 7\}$   
 $A' \cup B' = \{2, 4, 5, 6, 7\}$
31.  $A = \{1, 3, 5, 7\}$      $B = \{1, 2, 3\}$   
 $(A \cup B) = \{1, 2, 3, 5, 7\}$   
 $(A \cup B)' = \{4, 6\}$
32.  $A = \{1, 3, 5, 7\}$   
 $C = \{2, 3, 4, 5, 6\}$   
 $A \cup C = \{1, 2, 3, 4, 5, 6, 7\}$   
 $(A \cup C)' = \emptyset$
33.  $A = \{1, 3, 5, 7\}$   
 $A \cup \emptyset = \{1, 3, 5, 7\}$
34.  $C = \{2, 3, 4, 5, 6\}$   
 $C \cup \emptyset = \{2, 3, 4, 5, 6\}$
35.  $A \cap \emptyset = \emptyset$
36.  $C \cap \emptyset = \emptyset$
37.  $A \cup U = U$   
 $U = \{1, 2, 3, 4, 5, 6, 7\}$
38.  $B \cup U = U$   
 $U = \{1, 2, 3, 4, 5, 6, 7\}$
39.  $A \cap U = A$   
 $A = \{1, 3, 5, 7\}$
40.  $B \cap U = B$   
 $B = \{1, 2, 3\}$
41.  $A = \{a, g, h\}$   
 $B = \{b, g, h\}$   
 $A \cap B = \{g, h\}$
42.  $B = \{b, g, h\}$   
 $C = \{b, c, d, e, f\}$   
 $B \cap C = \{b\}$
43.  $A = \{a, g, h\}$   
 $B = \{b, g, h\}$   
 $A \cup B = \{a, b, g, h\}$
44.  $B = \{b, g, h\}$   
 $C = \{b, c, d, e, f\}$   
 $B \cup C = \{b, c, d, e, f, g, h\}$
45.  $A = \{a, g, h\}$   
 $U = \{a, b, c, d, e, f, g, h\}$   
 $A' = \{b, c, d, e, f\}$
46.  $B = \{b, g, h\}$   
 $U = \{a, b, c, d, e, f, g, h\}$   
 $B' = \{a, c, d, e, f\}$
47.  $A' = \{b, c, d, e, f\}$   
 $B' = \{a, c, d, e, f\}$   
 $A' \cap B' = \{c, d, e, f\}$
48.  $B' = \{a, c, d, e, f\}$   
 $C = \{b, c, d, e, f\}$   
 $B' \cap C = \{c, d, e, f\}$
49.  $A = \{a, g, h\}$   
 $C' = \{a, g, h\}$   
 $A \cup C' = \{a, g, h\}$

$$\begin{aligned} 50. \quad & B = \{b, g, h\} \\ & C = \{b, c, d, e, f\} \\ & C' = \{a, g, h\} \\ & B \cup C' = \{a, b, g, h\} \end{aligned}$$

$$\begin{aligned} 51. \quad & A = \{a, g, h\} \\ & C = \{b, c, d, e, f\} \\ & A \cap C = \emptyset \\ & (A \cap C)' = \{a, b, c, d, e, f, g, h\} \end{aligned}$$

$$\begin{aligned} 52. \quad & A = \{a, g, h\} \\ & B = \{b, g, h\} \\ & A \cap B = \{g, h\} \\ & (A \cap B)' = \{a, b, c, d, e, f\} \end{aligned}$$

$$\begin{aligned} 53. \quad & A' = \{b, c, d, e, f\} \\ & C' = \{a, g, h\} \\ & A' \cup C' = \{a, b, c, d, e, f, g, h\} \end{aligned}$$

$$\begin{aligned} 54. \quad & A' = \{b, c, d, e, f\} \\ & B' = \{a, c, d, e, f\} \\ & A' \cup B' = \{a, b, c, d, e, f\} \end{aligned}$$

$$\begin{aligned} 55. \quad & A = \{a, g, h\} \\ & B = \{b, g, h\} \\ & A \cup B = \{a, b, g, h\} \\ & (A \cup B)' = \{c, d, e, f\} \end{aligned}$$

$$\begin{aligned} 56. \quad & A = \{a, g, h\} \\ & C = \{b, c, d, e, f\} \\ & A \cup C = \{a, b, c, d, e, f, g, h\} \\ & (A \cup C)' = \emptyset \end{aligned}$$

$$\begin{aligned} 57. \quad & A \cup \emptyset = A \\ & A = \{a, g, h\} \end{aligned}$$

$$\begin{aligned} 58. \quad & C \cup \emptyset = C \\ & C = \{b, c, d, e, f\} \end{aligned}$$

$$59. \quad A \cap \emptyset = \emptyset$$

$$60. \quad C \cap \emptyset = \emptyset$$

$$\begin{aligned} 61. \quad & A = \{a, g, h\} \\ & U = \{a, b, c, d, e, f, g, h\} \\ & A \cup U = \{a, b, c, d, e, f, g, h\} \end{aligned}$$

$$\begin{aligned} 62. \quad & B = \{b, g, h\} \\ & U = \{a, b, c, d, e, f, g, h\} \\ & B \cup U = \{a, b, c, d, e, f, g, h\} \end{aligned}$$

$$\begin{aligned} 63. \quad & A = \{a, g, h\} \\ & U = \{a, b, c, d, e, f, g, h\} \\ & A \cap U = \{a, g, h\} \end{aligned}$$

$$\begin{aligned} 64. \quad & B = \{b, g, h\} \\ & U = \{a, b, c, d, e, f, g, h\} \\ & B \cap U = \{b, g, h\} \end{aligned}$$

$$\begin{aligned} 65. \quad & A = \{a, g, h\} \\ & B = \{b, g, h\} \\ & B' = \{a, c, d, e, f\} \\ & A \cap B = \{g, h\} \\ & (A \cap B) \cup B' = \{a, c, d, e, f, g, h\} \end{aligned}$$

$$\begin{aligned} 66. \quad & A = \{a, g, h\} \\ & B = \{b, g, h\} \\ & B' = \{a, c, d, e, f\} \\ & A \cup B = \{a, b, g, h\} \\ & (A \cup B) \cap B' = \{a\} \end{aligned}$$

$$67. \quad A = \{1, 3, 4, 7\}$$

$$68. \quad B = \{2, 3, 5, 6, 7\}$$

$$69. \quad U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$70. \quad A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$$

$$71. \quad A \cap B = \{3, 7\}$$

$$72. \quad A' = \{2, 5, 6, 8, 9\}$$

$$73. \quad B' = \{1, 4, 8, 9\}$$

$$74. \quad (A \cap B)' = \{1, 2, 4, 5, 6, 8, 9\}$$

$$75. \quad (A \cup B)' = \{8, 9\}$$

$$\begin{aligned} 76. \quad & A' = \{2, 5, 6, 8, 9\} \\ & B = \{2, 3, 5, 6, 7\} \\ & A' \cap B = \{2, 5, 6\} \end{aligned}$$

$$\begin{aligned} 77. \quad & A = \{1, 3, 4, 7\} \\ & B' = \{1, 4, 8, 9\} \\ & A \cap B' = \{1, 4\} \end{aligned}$$

$$\begin{aligned} 78. \quad & A = \{1, 3, 4, 7\} \\ & B' = \{1, 4, 8, 9\} \\ & A \cup B' = \{1, 3, 4, 7, 8, 9\} \end{aligned}$$

$$79. \quad B = \{\triangle, \text{two}, \text{four}, \text{six}\}$$

80.  $A = \{\Delta, \#, \$\}$

81.  $A \cup B = \{\Delta, \#, \$, \text{two, four, six}\}$

82.  $A \cap B = \{\Delta\}$

83.  $n(A \cup B) = n(\{\Delta, \#, \$, \text{two, four, six}\}) = 6$

84.  $n(A \cap B) = n(\{\Delta\}) = 1$

85.  $n(A') = 5$

86.  $n(B') = 4$

87.  $(A \cap B)' = \{\#, \$, \text{two, four, six, 10, 01}\}$

88.  $(A \cup B)' = \{10, 01\}$

89.  $A' \cap B = \{\text{two, four, six}\}$

90.  $A \cap B' = \{\#, \$\}$

91.  $n(U) - n(B) = 8 - 4 = 4$

92.  $n(U) - n(A) = 8 - 3 = 5$

93. 
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 17 + 20 - 6 \\ &= 31 \end{aligned}$$

94. 
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 30 + 18 - 5 \\ &= 43 \end{aligned}$$

95. 
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 17 + 17 - 7 \\ &= 27 \end{aligned}$$

96. 
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 30 + 24 - 7 \\ &= 47 \end{aligned}$$

97. 
$$\begin{aligned} A &= \{1, 3, 5, 7\} \\ B &= \{2, 4, 6, 8\} \\ A \cup B &= \{1, 2, 3, 4, 5, 6, 7, 8\} \end{aligned}$$

98. 
$$\begin{aligned} B &= \{2, 4, 6, 8\} \\ C &= \{2, 3, 4, 5\} \\ B \cup C &= \{2, 3, 4, 5, 6, 8\} \end{aligned}$$

99.  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A = \{1, 3, 5, 7\}$

$A \cap U = \{1, 3, 5, 7\}$

100.  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A = \{1, 3, 5, 7\}$

$A \cup U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

101.  $A = \{1, 3, 5, 7\}$

$C' = \{1, 6, 7, 8\}$

$A \cap C' = \{1, 7\}$

102.  $A = \{1, 3, 5, 7\}$

$B' = \{1, 3, 5, 7\}$

$A \cap B' = \{1, 3, 5, 7\}$

103.  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 4, 6, 8\}$

$C = \{2, 3, 4, 5\}$

$B \cap C = \{2, 4\}$

$(B \cap C)' = \{1, 3, 5, 6, 7, 8\}$

104.  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A = \{1, 3, 5, 7\}$

$C = \{2, 3, 4, 5\}$

$A \cap C = \{3, 5\}$

$(A \cap C)' = \{1, 2, 4, 6, 7, 8\}$

105.  $A \cup (A \cup B)'$

$= \{23, 29, 31, 37, 41, 43, 53, 59, 61, 67, 71\}$

106.  $(A' \cap B) \cup (A \cap B) = \{41, 43, 47\}$

107. 
$$\begin{aligned} n(U)[n(A \cup B) - n(A \cap B)] &= 12[7 - 2] \\ &= 12(5) = 60 \end{aligned}$$

108.  $n(A \cap B)[n(A \cup B) - n(A')] = 2[7 - 6] = 2(1) = 2$

109.  $\{\text{Ashley, Mike, Josh}\}$

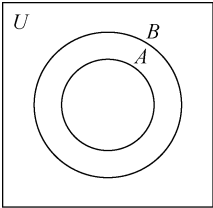
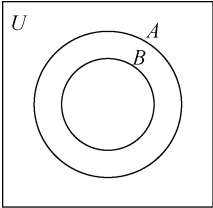
110.  $\{\text{Mike, Josh, Emily, Hannah, Ethan}\}$

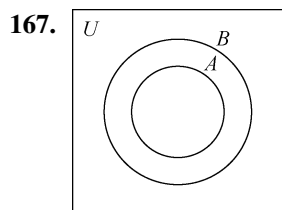
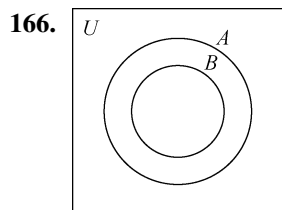
111.  $\{\text{Ashley, Mike, Josh, Emily, Hannah, Ethan}\}$

112.  $\{\text{Mike, Josh}\}$

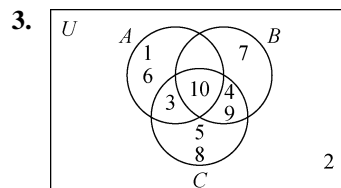
113.  $\{\text{Ashley}\}$

114.  $\{\text{Emily, Hannah, Ethan}\}$

115. {Jacob}
116. {Ashley, Mike, Josh, Emily, Hannah, Ethan, Jacob}
117. Region III, *elementary school teacher* is in set  $B$  but not set  $A$ .
118. Region I, *police officer* is in set  $A$  but not set  $B$ .
119. Region I, *surgeon* is in set  $A$  but not set  $B$ .
120. Region III, *banker* is in set  $B$  but not set  $A$ .
121. Region II, *family doctor* is in set  $A$  and set  $B$ .
122. Region II, *lawyer* is in set  $A$  and set  $B$ .
123. Region I, 11 is in set  $A$  but not set  $B$ .
124. Region II, 22 is in set  $A$  and set  $B$ .
125. Region IV, 15 is in neither set  $A$  nor set  $B$ .
126. Region IV, 17 is in neither set  $A$  nor set  $B$ .
127. Region II, 454 is in set  $A$  and set  $B$ .
128. Region I, 101 is in set  $A$  but not set  $B$ .
129. Region III, 9558 is in set  $B$  but not set  $A$ .
130. Region III, 9778 is in set  $B$  but not set  $A$ .
131. Region I, 9559 is in set  $A$  but not set  $B$ .
132. Region I, 9779 is in set  $A$  but not set  $B$ .
133.  $\{\underline{2001}, 2015\} \cap \{1969, 1985, \underline{2001}\} = \{2001\}$
134.  $\{\underline{1985}, 2001, 2015\} \cap \{1969, \underline{1985}\} = \{1985\}$
135.  $\{\underline{2001}, 2015\} \cup \{1969, 1985, \underline{2001}\} = \{1969, 1985, 2001, 2015\}$
136.  $\{\underline{1985}, 2001, 2015\} \cup \{1969, \underline{1985}\} = \{1969, 1985, 2001, 2015\}$
137.  $\{2015\} \cap \{1969, 1985, 2001\} = \emptyset$
138.  $\{1990, 2000, 2010\} \cup \{1980\} = \{1969, 1985, 2001, 2015\}$
139.  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$   
 $= 178 + 154 - 49$   
 $= 283$  people
140.  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$   
 $= 96 + 97 - 29$   
 $= 164$  people
152. does not make sense; Explanations will vary.  
 Sample explanation: Even with only one common element, the sets intersection will be shown by overlapping circles.
153. makes sense
154. does not make sense; Explanations will vary.  
 Sample explanation: The given expression indicates that you should find the union of set  $A$  and set  $B$ , and then find the complement of the resulting set.
155. makes sense
156. false; Changes to make the statement true will vary.  
 A sample change is:  
 $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
157. true
158. false; Changes to make the statement true will vary.  
 A sample change is:  $A \subseteq (A \cup B)$
159. false; Changes to make the statement true will vary.  
 A sample change is: If  $A \subseteq B$ , then  $A \cup B = B$ .
160. false; Changes to make the statement true will vary.  
 A sample change is:  $A \cup U = U$
161. false; Changes to make the statement true will vary.  
 A sample change is:  $A \cap \emptyset = \emptyset$
162. false; Changes to make the statement true will vary.  
 A sample change is: If  $A \subseteq B$ , then  $A \cap B = A$ .
163. true
164. 
165. 



- e.  $A \cup B \cup C$  is represented by regions I, II, III, IV, V, VI, and VII.  
Thus,  
 $A \cup B \cup C = \{1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12\}$



4. a.  $A \cup B$  is represented by regions I, II, and III.  
Therefore  $(A \cup B)'$  is represented by region IV.
- b.  $A'$  is represented by regions III and IV.  
 $B'$  is represented by regions I and IV.  
Therefore  $A' \cap B'$  is represented by region IV.
- c.  $(A \cup B)' = A' \cap B'$  because they both represent region IV.
5. a.  $B \cup C$  is represented by regions II, III, IV, V, VI, and VII.  
Therefore  $A \cap (B \cup C)$  is represented by regions II, IV, and V.
- b.  $A \cap B$  is represented by regions II and V.  
 $A \cap C$  is represented by regions IV and V.  
Therefore  $(A \cap B) \cup (A \cap C)$  is represented by regions II, IV, and V.
- c.  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$  because they both represent region IV.

### Check Points 2.4

1. a.  $A \cup (B \cap C) = \{a, b, c, d\} \cup \{b, f\}$   
 $= \{a, b, c, d, f\}$
- b.
- $$(A \cup B) \cap (A \cup C) = \{a, b, c, d, f\} \cap \{a, b, c, d, f\}$$
- $$= \{a, b, c, d, f\}$$
- c.
- $$A \cap (B \cup C) = \{a, b, c, d\} \cap (\{a, b, d, f\} \cup \{a, d, e\})$$
- $$= \{a, b, c, d\} \cap \{a, b, d, e, f\}$$
- $$= \{a, b, d\}$$
2. a.  $C$  is represented by regions IV, V, VI, and VII.  
Thus,  $C = \{5, 6, 7, 8, 9\}$
- b.  $B \cup C$  is represented by regions II, III, IV, V, VI, and VII.  
Thus,  $B \cup C = \{1, 2, 5, 6, 7, 8, 9, 10, 12\}$
- c.  $A \cap C$  is represented by regions IV and V.  
Thus,  $A \cap C = \{5, 6, 7\}$
- d.  $B'$  is represented by regions I, IV, VII, and VIII.  
Thus,  $B' = \{3, 4, 6, 8, 11\}$

### Concept and Vocabulary Check 2.4

- inside parentheses
- eight
- false
- true

## Exercises 2.4

1.  $B \cap C = \{2, 3\}$   
 $A \cup (B \cap C) = \{1, 2, 3, 5, 7\}$
2.  $B \cup C = \{1, 2, 3, 4, 5, 6\}$   
 $A \cap (B \cup C) = \{1, 3, 5\}$
3.  $A \cup B = \{1, 2, 3, 5, 7\}$   
 $A \cup C = \{1, 2, 3, 4, 5, 6, 7\}$   
 $(A \cup B) \cap (A \cup C) = \{1, 2, 3, 5, 7\}$
4.  $(A \cap B) = \{1, 3\}$   
 $(A \cap C) = \{3, 5\}$   
 $(A \cap B) \cup (A \cap C) = \{1, 3, 5\}$
5.  $A' = \{2, 4, 6\}$        $C' = \{1, 7\}$   
 $B \cup C' = \{1, 2, 3, 7\}$   
 $A' \cap (B \cup C') = \{2\}$
6.  $B' = \{4, 5, 6, 7\}$   
 $C' = \{1, 7\}$   
 $A \cup B' = \{1, 3, 4, 5, 6, 7\}$   
 $C' \cap (A \cup B') = \{1, 7\}$
7.  $A' = \{2, 4, 6\}$        $C' = \{1, 7\}$   
 $A' \cap B = \{2\}$   
 $A' \cap C' = \emptyset$   
 $(A' \cap B) \cup (A' \cap C') = \{2\}$
8.  $B' = \{4, 5, 6, 7\}$   
 $C' = \{1, 7\}$   
 $(C' \cap A) = \{1, 7\}$   
 $(C' \cap B') = \{7\}$   
 $(C' \cap A) \cup (C' \cap B') = \{1, 7\}$
9.  $A = \{1, 3, 5, 7\}$   
 $B = \{1, 2, 3\}$   
 $C = \{2, 3, 4, 5, 6\}$   
 $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7\}$   
 $(A \cup B \cup C)' = \emptyset$
10.  $A = \{1, 3, 5, 7\}$   
 $B = \{1, 2, 3\}$   
 $C = \{2, 3, 4, 5, 6\}$   
 $A \cap B \cap C = \{3\}$   
 $(A \cap B \cap C)' = \{1, 2, 4, 5, 6, 7\}$
11.  $A = \{1, 3, 5, 7\}$   
 $B = \{1, 2, 3\}$   
 $A \cup B = \{1, 2, 3, 5, 7\}$   
 $(A \cup B)' = \{4, 6\}$   
 $C = \{2, 3, 4, 5, 6\}$   
 $(A \cup B)' \cap C = \{4, 6\}$
12.  $B \cup C = \{1, 2, 3, 4, 5, 6\}$   
 $(B \cup C)' = \{7\}$   
 $(B \cup C)' \cap A = \{7\}$
13.  $B \cap C = \{b\}$   
 $A \cup (B \cap C) = \{a, b, g, h\}$
14.  $B \cup C = \{b, c, d, e, f, g, h\}$   
 $A \cap (B \cup C) = \{g, h\}$
15.  $A \cup B = \{a, b, g, h\}$   
 $A \cup C = \{a, b, c, d, e, f, g, h\}$   
 $(A \cup B) \cap (A \cup C) = \{a, b, g, h\}$
16.  $A \cap B = \{g, h\}$   
 $A \cap C = \emptyset$   
 $(A \cap B) \cup (A \cap C) = \{g, h\}$
17.  $A' = \{b, c, d, e, f\}$   
 $C' = \{a, g, h\}$   
 $B \cup C' = \{a, b, g, h\}$   
 $A' \cap (B \cup C') = \{b\}$
18.  $C' = \{a, g, h\}$   
 $B' = \{a, c, d, e, f\}$   
 $A \cup B' = \{a, c, d, e, f, g, h\}$   
 $C' \cap (A \cup B') = \{a, g, h\}$
19.  $A' = \{b, c, d, e, f\}$   
 $A' \cap B = \{b\}$   
 $C' = \{a, g, h\}$   
 $A' \cap C' = \emptyset$   
 $(A' \cap B) \cup (A' \cap C') = \{b\}$
20.  $C' = \{a, g, h\}$   
 $B' = \{a, c, d, e, f\}$   
 $C' \cap A = \{a, g, h\}$   
 $C' \cap B' = \{a\}$   
 $(C' \cap A) \cup (C' \cap B') = \{a, g, h\}$

21.  $A \cup B \cup C = \{a, b, c, d, e, f, g, h\}$   
 $(A \cup B \cup C)' = \emptyset$

22.  $A \cap B \cap C = \emptyset$   
 $(A \cap B \cap C)' = \{a, b, c, d, e, f, g, h\}$

23.  $A \cup B = \{a, b, g, h\}$   
 $(A \cup B)' = \{c, d, e, f\}$   
 $(A \cup B)' \cap C = \{c, d, e, f\}$

24.  $B \cup C = \{b, c, d, e, f, g, h\}$   
 $(B \cup C)' = \{a\}$   
 $(B \cup C)' \cap A = \{a\}$

25. II, III, V, VI

26. IV, V, VI, VII

27. I, II, IV, V, VI, VII

28. II, III, IV, V, VI, VII

29. II, V

30. IV, V

31. I, IV, VII, VIII

32. I, II, III, VIII

33.  $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

34.  $B = \{4, 5, 6, 9, 10, 11\}$

35.  $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

36.  $B = \{4, 5, 6, 9, 10, 11\}$   
 $C = \{6, 7, 8, 9, 12\}$   
 $B \cup C = \{4, 5, 6, 7, 8, 9, 10, 11, 12\}$

37.  $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$   
 $B = \{4, 5, 6, 9, 10, 11\}$   
 $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$   
 $(A \cup B)' = \{12, 13\}$

38.  $B = \{4, 5, 6, 9, 10, 11\}$   
 $C = \{6, 7, 8, 9, 12\}$   
 $B \cup C = \{4, 5, 6, 7, 8, 9, 10, 11, 12\}$   
 $(B \cup C)' = \{1, 2, 3, 13\}$

39. The set contains the elements in the two regions where the circles representing sets  $A$  and  $B$  overlap.  
 $A \cap B = \{4, 5, 6\}$

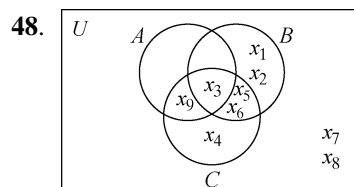
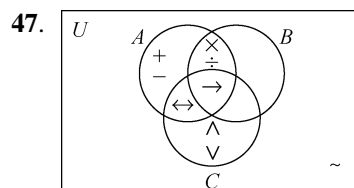
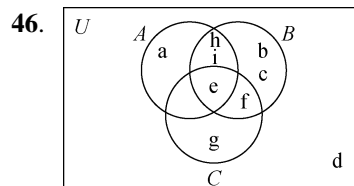
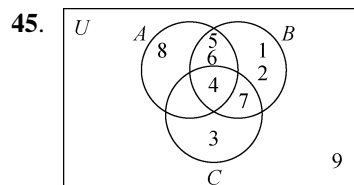
40. The set contains the elements in the two regions where the circles representing sets  $A$  and  $C$  overlap.  
 $A \cap C = \{6, 7, 8\}$

41. The set contains the element in the center region where the circles representing sets  $A$ ,  $B$ , and  $C$  overlap.  
 $A \cap B \cap C = \{6\}$

42. The set contains the elements in the seven regions of the circles representing sets  $A$ ,  $B$ , and  $C$ . Only element 13 lies outside these regions.  
 $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

43.  $A \cap B \cap C = \{6\}$   
 $(A \cap B \cap C)' = \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13\}$

44.  $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$   
 $(A \cup B \cup C)' = \{13\}$



49. a. II  
b. II  
c.  $A \cap B = B \cap A$
50. a. I, II, III  
b. I, II, III  
c.  $A \cup B = B \cup A$
51. a. I, III, IV  
b. IV  
c. No,  $(A \cap B)' \neq A' \cap B'$
52. a. IV  
b. I, III, IV  
c. No,  $(A \cup B)' \neq A' \cup B'$
53. Set  $A$  is represented by regions I and II.  
Set  $A'$  is represented by regions III and IV.  
Set  $B$  is represented by regions II and III.  
Set  $B'$  is represented by regions I and IV.  
 $A' \cup B$  is represented by regions II, III, and IV.  
 $A \cap B'$  is represented by region I.  
Thus,  $A' \cup B$  and  $A \cap B'$  are not equal for all sets  $A$  and  $B$ .
54. Set  $A$  is represented by regions I and II.  
Set  $A'$  is represented by regions III and IV.  
Set  $B$  is represented by regions II and III.  
Set  $B'$  is represented by regions I and IV.  
 $A' \cap B$  is represented by region III.  
 $A \cup B'$  is represented by regions I, II, and IV.  
Thus,  $A' \cap B$  and  $A \cup B'$  are not equal for all sets  $A$  and  $B$ .
55. Set  $A$  is represented by regions I and II.  
Set  $B$  is represented by regions II and III.  
 $(A \cup B)'$  is represented by region IV.  
 $(A \cap B)'$  is represented by regions I, III, and IV.  
Thus,  $(A \cup B)'$  and  $(A \cap B)'$  are not equal for all sets  $A$  and  $B$ .
56. Set  $A$  is represented by regions I and II.  
Set  $B$  is represented by regions II and III.  
 $(A \cup B)'$  is represented by region IV.  
 $A' \cap B$  is represented by region III.  
Thus,  $(A \cup B)'$  and  $A' \cap B$  are not equal for all sets  $A$  and  $B$ .
57. Set  $A$  is represented by regions I and II.  
Set  $A'$  is represented by regions III and IV.  
Set  $B$  is represented by regions II and III.  
Set  $B'$  is represented by regions I and IV.  
 $(A' \cap B)'$  is represented by regions I, II, and IV.  
 $A \cup B'$  is represented by regions I, II, and IV.  
Thus,  $A' \cap B$  and  $A \cup B'$  are equal for all sets  $A$  and  $B$ .
58. Set  $A$  is represented by regions I and II.  
Set  $A'$  is represented by regions III and IV.  
Set  $B$  is represented by regions II and III.  
Set  $B'$  is represented by regions I and IV.  
 $(A \cup B)'$  is represented by region III.  
 $A' \cap B$  is represented by region III.  
Thus,  $A' \cap B$  and  $A \cup B'$  are equal for all sets  $A$  and  $B$ .
59. a. II, IV, V, VI, VII  
b. II, IV, V, VI, VII  
c.  $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
60. a. IV, V, VI  
b. IV, V, VI  
c.  $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
61. a. II, IV, V  
b. I, II, IV, V, VI  
c. No  
The results in **a** and **b** show  $A \cap (B \cup C) \neq A \cup (B \cap C)$  because of the different regions represented.
62. a. II, IV, V, VI, VII  
b. IV, V, VI  
c. No  
The results in **a** and **b** show  $C \cup (B \cap A) \neq C \cap (B \cup A)$  because of the different regions represented.
63. The left expression is represented by regions II, IV, and V. The right expression is represented by regions II, IV, V, VI, and VII. Thus this statement is not true.

64. The left expression is represented by regions I, II, IV, V, and VI. The right expression is represented by regions IV, V, and VI. Thus this statement is not true.
65. Both expressions are represented by regions II, III, IV, V, and VI. Thus this statement is true and is a theorem.
66. Both expressions are represented by regions II, V, and VI. Thus this statement is true and is a theorem.
67. Both expressions are represented by region I. Thus this statement is true and is a theorem.
68. Both expressions are represented by regions I, II, III, IV, V, VII, and VIII. Thus this statement is true and is a theorem.
69. a.  $A \cup (B' \cap C') = \{c, e, f\}$   
 $(A \cup B') \cap (A \cup C') = \{c, e, f\}$
- b.  $A \cup (B' \cap C') = \{1, 3, 5, 7, 8\}$   
 $(A \cup B') \cap (A \cup C') = \{1, 3, 5, 7, 8\}$
- c.  $A \cup (B' \cap C') = (A \cup B') \cap (A \cup C')$
- d.  $A \cup (B' \cap C')$  and  $(A \cup B') \cap (A \cup C')$  are both represented by regions I, II, IV, V, and VIII. Thus, the conjecture in part c is a theorem.
70. a.  $(A \cup B)' \cap C = \{4\}$   
 $A' \cap (B' \cap C) = \{4\}$
- b.  $(A \cup B)' \cap C = \{e\}$   
 $A' \cap (B' \cap C) = \{e\}$
- c.  $(A \cup B)' \cap C = A' \cap (B' \cap C)$
- d.  $(A \cup B)' \cap C$  and  $A' \cap (B' \cap C)$  are both represented by regions IV, V, and VI. Thus, the conjecture in part c is a theorem.
71.  $(A \cap B') \cap (A \cup B)$
72.  $(A \cup B)'$
73.  $A' \cup B$
74.  $A' \cap B$
75.  $(A \cap B) \cup C$
76.  $A \cap (B \cup C)$
77.  $A' \cap (B \cup C)$
78.  $(A \cup B)' \cap C$
79. {Ann, Jose, Al, Gavin, Amy, Ron, Grace}
80. {Jose, Ron, Grace, Lee, Maria}
81. {Jose}
82. {Ann, Jose}
83. {Lily, Emma}
84. {Ron, Grace, Lee, Maria}
85. {Lily, Emma, Ann, Jose, Lee, Maria, Fred, Ben, Sheila, Ellen, Gary}
86. {Jose, Ron, Grace, Lee, Maria, Al, Gavin, Amy, Fred, Ben, Sheila, Ellen, Gary}
87. {Lily, Emma, Al, Gavin, Amy, Lee, Maria}
88. {Ann, Jose, Ron, Grace}
89. {Al, Gavin, Amy}
90. {Lily, Emma}
91. The set of students who scored 90% or above on exam 1 and exam 3 but not on exam 2 is the empty set.
92. The set of students who did not score 90% or higher on any of the exams.
93. Region I
94. Region III
95. Region V
96. Region VII
97. Region VI
98. Region VIII
99. Region III
100. Region VI
101. Region IV

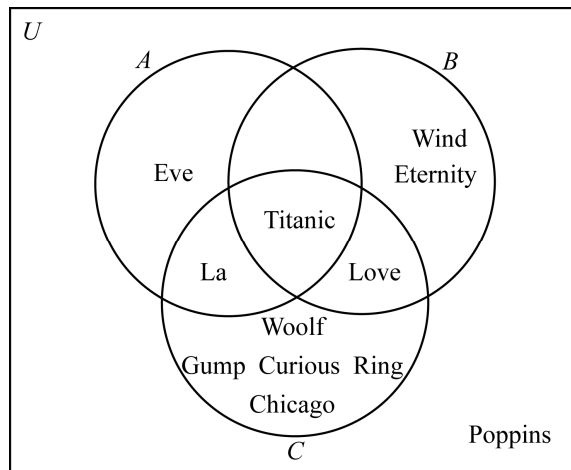
102. Region VIII

103. Region VI

104. Region I

105. Venn diagram:

106.



109. does not make sense; Explanations will vary.  
Sample explanation: You should begin by placing elements in the innermost region.

110. makes sense

111. makes sense

112. does not make sense; Explanations will vary.  
Sample explanation: Finding examples, even many examples, is not enough to prove a conjecture.

113.  $AB^+$

114.  $O^-$

115. no

116. yes

### Check Points 2.5

1. a.  $55 + 20 = 75$   
b.  $20 + 70 = 90$   
c. 20  
d.  $55 + 20 + 70 = 145$

e. 55

f. 70

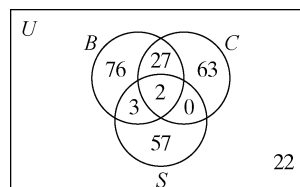
g. 30

h.  $55 + 20 + 70 + 30 = 175$

2. Start by placing 290 in region II.  
Next place  $1040 - 290$  or 750 in region III.  
Next place  $810 - 290$  or 520 in region I.  
Finally, place  $1700 - 520 - 290 - 750$  or 140 in region IV.

- a. 750 French adults agreed with the statement and are represented by region III.
- b. 140 French adults disagreed with the statement and are represented by region IV.

3. Since 2 people collect all three items, begin by placing a 2 in region V.  
Since 29 people collect baseball cards and comic books,  $29 - 2$  or 27 should be placed in region II.  
Since 5 people collect baseball cards and stamps,  $5 - 2$  or 3 should be placed in region IV.  
Since 2 people collect comic books and stamps,  $2 - 2$  or 0 should be placed in region VI.  
Since 108 people collect baseball cards,  $108 - 27 - 3 - 2$  or 76 should be placed in region I.  
Since 92 people collect comic books,  $92 - 27 - 2 - 0$  or 63 should be placed in region III.  
Since 62 people collect stamps,  $62 - 3 - 2 - 0$  or 57 should be placed in region VII.  
Since there were 250 people surveyed, place  $250 - 76 - 27 - 63 - 3 - 2 - 0 - 57 = 22$  in region VIII.



4. a. 63 as represented by region III.  
b. 3 as represented by region IV.  
c. 136 as represented by regions I, IV, and VII.  
d. 30 as represented by regions II, IV, and VI.  
e. 228 as represented by regions I through VII.  
f. 22 as represented by region VIII.

**Concept and Vocabulary Check 2.5**

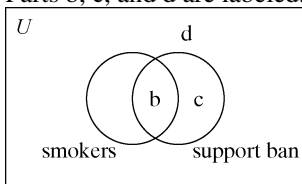
1. and/but
2. or
3. not
4. innermost; subtraction
5. true
6. true
7. false
8. true

**Exercise Set 2.5**

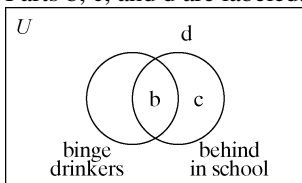
1. 26
2. 20
3. 17
4. 11
5. 37
6. 9
7. 7
8. 44
9. Region I has  $21 - 7$  or 14 elements.  
Region III has  $29 - 7$  or 22 elements.  
Region IV has  $48 - 14 - 7 - 22$  or 5 elements.
10. Region I has  $23 - 7$  or 16 elements.  
Region III has  $27 - 7$  or 20 elements.  
Region IV has  $53 - 16 - 7 - 20$  or 10 elements.
11. 17 as represented by regions II, III, V, and VI.
12. 15 as represented by regions I, II, IV, and V.
13. 6 as represented by regions I and II.
14. 9 as represented by regions III and VI.
15. 28 as represented by regions I, II, IV, V, VI, and VII.
16. 24 as represented by regions I through VI.
17. 9 as represented by regions IV and V.
18. 8 as represented by regions II and V.
19. 3 as represented by region VI.
20. 2 as represented by region IV.
21. 19 as represented by regions III, VI, and VII.
22. 17 as represented by regions I, IV, and VII.
23. 21 as represented by regions I, III, and VII.
24. 6 as represented by regions II, IV, and VI.
25. 34 as represented by regions I through VII.
26. 13 as represented by regions II, IV, V, and VI.
27. Since  $n(A \cap B) = 3$ , there is 1 element in region II.  
Since  $n(A \cap C) = 5$ , there are 3 elements in region IV.  
Since  $n(B \cap C) = 3$ , there is 1 element in region VI.  
Since  $n(A) = 11$ , there are 5 elements in region I.  
Since  $n(B) = 8$ , there are 4 elements in region III.  
Since  $n(C) = 14$ , there are 8 elements in region VII.  
Since  $n(U) = 30$ , there are 6 elements in region VIII.
28. Since  $n(A \cap B) = 6$ , there are 4 elements in region II.  
Since  $n(A \cap C) = 7$ , there are 5 elements in region IV.  
Since  $n(B \cap C) = 8$ , there are 6 elements in region VI.  
Since  $n(A) = 21$ , there are 10 elements in region I.  
Since  $n(B) = 15$ , there are 3 elements in region III.  
Since  $n(C) = 14$ , there is 1 element in region VII.  
Since  $n(U) = 32$ , there is 1 element in region VIII.

- 29.** Since  $n(A \cap B \cap C) = 7$ , there are 7 elements in region V.  
 Since  $n(A \cap B) = 17$ , there are 10 elements in region II.  
 Since  $n(A \cap C) = 11$ , there are 4 elements in region IV.  
 Since  $n(B \cap C) = 8$ , there is 1 element in region VI.  
 Since  $n(A) = 26$ , there are 5 elements in region I.  
 Since  $n(B) = 21$ , there are 3 elements in region III.  
 Since  $n(C) = 18$ , there are 6 elements in region VII.  
 Since  $n(U) = 38$ , there are 2 elements in region VIII.
- 30.** Since  $n(A \cap B \cap C) = 5$ , there are 5 elements in region V.  
 Since  $n(A \cap B) = 17$ , there are 12 elements in region II.  
 Since  $n(A \cap C) = 11$ , there are 6 elements in region IV.  
 Since  $n(B \cap C) = 9$ , there are 4 elements in region VI.  
 Since  $n(A) = 26$ , there are 3 elements in region I.  
 Since  $n(B) = 22$ , there is 1 element in region III.  
 Since  $n(C) = 25$ , there are 10 elements in region VII.  
 Since  $n(U) = 42$ , there is 1 element in region VIII.
- 31.** Since  $n(A \cap B \cap C) = 2$ , there are 2 elements in region V.  
 Since  $n(A \cap B) = 6$ , there are 4 elements in region II.  
 Since  $n(A \cap C) = 9$ , there are 7 elements in region IV.  
 Regions II, IV, and V contain a total of 13 elements, yet set A is stated to contain a total of only 10 elements. That is impossible.
- 32.** Since  $n(A \cap B \cap C) = 5$ , there are 5 elements in region V.  
 Since  $n(A \cap B) = 6$ , there is 1 element in region II.  
 Since  $n(A \cap C) = 9$ , there are 4 elements in region IV.  
 Regions II, IV, and V contain a total of 10 elements, yet set A is stated to contain a total of only 8 elements. That is impossible.
- 33.**  $4 + 5 + 2 + 7 = 18$  respondents agreed with the statement.
- 34.**  $8 + 2 + 3 + 9 = 22$  respondents disagreed with the statement.
- 35.**  $2 + 7 = 9$  women agreed with the statement.
- 36.**  $4 + 2 = 6$  people who are not African American agreed with the statement.
- 37.** 9 women who are not African American disagreed with the statement.
- 38.** 8 men who are not African American agreed with the statement.
- 39.** Start by placing 270 in region II.  
 Next place  $370 - 270$  or 100 in region III.  
 Next place  $780 - 270$  or 510 in region I.  
 Finally, place  $1620 - 510 - 270 - 100$  or 740 in region IV.
- a.** 100 British adults agreed with the statement and are represented by region III.
- b.** 740 British adults disagreed with the statement and are represented by region IV.
- 40.** Start by placing 260 in region II.  
 Next place  $440 - 260$  or 180 in region III.  
 Next place  $770 - 260$  or 510 in region I.  
 Finally, place  $1510 - 510 - 260 - 180$  or 560 in region IV.
- a.** 180 German adults agreed with the statement and are represented by region III.
- b.** 560 German adults disagreed with the statement and are represented by region IV.

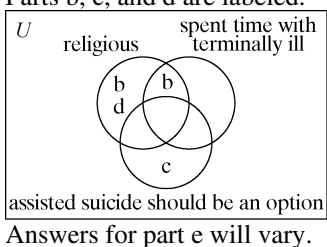
41. Parts b, c, and d are labeled.



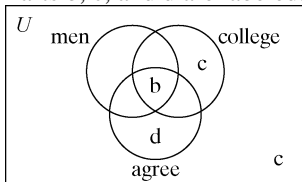
42. Parts b, c, and d are labeled.



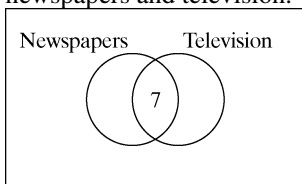
43. Parts b, c, and d are labeled.



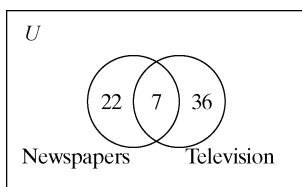
44. Parts b, c, and d are labeled.



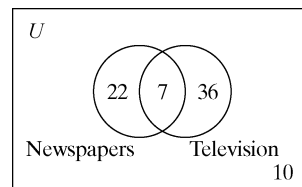
45. Begin by placing 7 in the region that represents both newspapers and television.



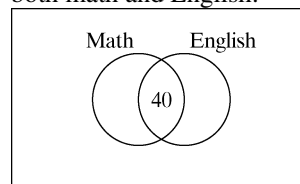
- a. Since 29 students got news from newspapers,  $29 - 7 = 22$  got news from only newspapers.
- b. Since 43 students got news from television,  $43 - 7 = 36$  got news from only television.



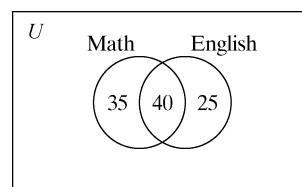
- c.  $22 + 7 + 36 = 65$  students who got news from newspapers or television.
- d. Since 75 students were surveyed,  $75 - 65 = 10$  students who did not get news from either.



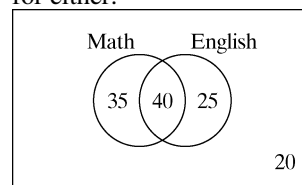
46. Begin by placing 40 in the region that represents both math and English.



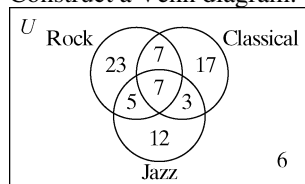
- a. Since 75 students registered for math,  $75 - 40 = 35$  registered for only math.
- b. Since 65 students registered for English,  $65 - 40 = 25$  registered for only English.



- c.  $35 + 40 + 25 = 100$  students who registered for math or English.
- d. Since 120 students were surveyed,  $120 - 100 = 20$  students who did not register for either.

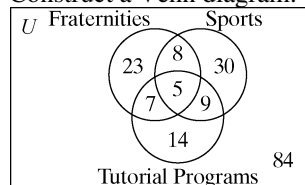


47. Construct a Venn diagram.



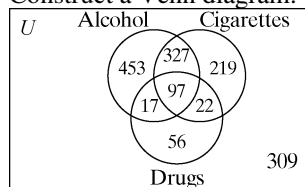
- 23
- 3
- $17 + 3 + 12 = 32$
- $23 + 17 + 12 = 52$
- $7 + 3 + 5 + 7 = 22$
- 6

48. Construct a Venn diagram.



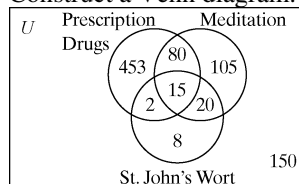
- 30
- 8
- $23 + 8 + 30 = 61$
- $23 + 30 + 14 = 67$
- $8 + 7 + 9 + 5 = 29$
- 84

49. Construct a Venn diagram.



- 1500 (all eight regions)
- 1135 (the six regions of sets A and C)
- 56 (region VII)
- 327 (region II)
- 526 (regions I, IV, and VII)
- 366 (regions II, IV, and VI)
- 1191 (regions I through VII)

50. Construct a Venn diagram.



- 833 (all eight regions)
  - 675 (the six regions of sets A and C)
  - 8 (region VII)
  - 80 (region II)
  - 463 (regions I, IV, and VII)
  - 102 (regions II, IV, and VI)
  - 683 (regions I through VII)
53. does not make sense; Explanations will vary. Sample explanation: A survey problem could present the information in any order.
54. makes sense
55. does not make sense; Explanations will vary. Sample explanation: Since there is a circle to represent smokers, then nonsmokers are represented by being placed outside that circle, not in a separate circle.
56. does not make sense; Explanations will vary. Sample explanation: The bar graph does not indicate how the various ailments intersect.
57. false; Changes to make the statement true will vary. A sample change is: It is possible that some students are taking more than one of these courses. If so, then the number surveyed is less than 220.
58. true
59. false; Changes to make the statement true will vary. A sample change is: Then innermost region is the first region to be filled in.
60. true
61. a. 0; This would assume none of the psychology students were taking mathematics.  
b. 30; This would assume all 30 students taking psychology were taking mathematics.  
c. 60;  $U = 150$  so with 90 taking mathematics, if we assume all the psychology students are taking mathematics courses,  $U - 90 = 60$ .

62. Under the conditions given concerning enrollment in math, chemistry, and psychology courses, the total number of students is 100, not 90

## Chapter 2 Review Exercises

1. the set of days of the week beginning with the letter T
2. the set of natural numbers between 1 and 10, inclusive
3. {m, i, s}
4. {8, 9, 10, 11, 12}
5. {1, 2, 3, ..., 30}
6. not empty
7. empty set
8.  $\in$   
93 is an element of the set.
9.  $\notin$   
{d} is a subset, not a member; "d" would be a member.
10. 12  
12 months in the year.
11. 15
12.  $\neq$   
The two sets do not contain exactly the same elements.
13.  $\neq$   
One set is infinite. The other is finite.
14. Equivalent  
Same number of elements, but different elements.
15. Equal and equivalent  
The two sets have exactly the same elements.
16. finite
17. infinite
18.  $\subseteq$
19.  $\subsetneq$
20.  $\subseteq$
21.  $\subseteq$
22. both
23. false; Texas is not a member of the set.
24. false; 4 is not a subset. {4} is a subset.
25. true
26. false; It is a subset but not a proper subset.
27. true
28. false; The set {six} has only one element so it has  $2^1 = 2$  subsets.
29. true
30.  $\emptyset$  {1} {5} {1, 5}  
{1, 5} is not a proper subset.
31. There are 5 elements. This means there are  $2^5 = 32$  subsets.  
There are  $2^5 - 1 = 31$  proper subsets.
32. {January, June, July}  
There are 3 elements. This means there are  $2^3 = 8$  subsets.  
There are  $2^3 - 1 = 7$  proper subsets.
33.  $A \cap B = \{1, 2, 4\}$
34.  $A \cup B' = \{1, 2, 3, 4, 6, 7, 8\}$
35.  $A' \cap B = \{5\}$
36.  $(A \cup B)' = \{6, 7, 8\}$
37.  $A' \cap B' = \{6, 7, 8\}$
38. {4, 5, 6}
39. {2, 3, 6, 7}
40. {1, 4, 5, 6, 8, 9}
41. {4, 5}
42. {1, 2, 3, 6, 7, 8, 9}
43. {2, 3, 7}
44. {6}
45. {1, 2, 3, 4, 5, 6, 7, 8, 9}

$$\begin{aligned} 46. \quad n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 25 + 17 - 9 \\ &= 33 \end{aligned}$$

$$\begin{aligned} 47. \quad B \cap C &= \{1, 5\} \\ A \cup (B \cap C) &= \{1, 2, 3, 4, 5\} \end{aligned}$$

$$\begin{aligned} 48. \quad A \cap C &= \{1\} \\ (A \cap C)' &= \{2, 3, 4, 5, 6, 7, 8\} \\ (A \cap C)' \cup B &= \{1, 2, 3, 4, 5, 6, 7, 8\} \end{aligned}$$

$$49. \quad \{c, d, e, f, k, p, r\}$$

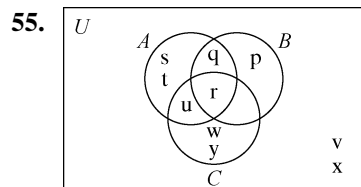
$$50. \quad \{f, p\}$$

$$51. \quad \{c, d, f, k, p, r\}$$

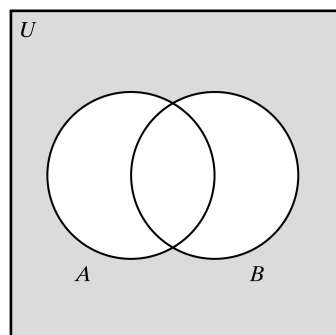
$$52. \quad \{c, d, e\}$$

$$53. \quad \{a, b, c, d, e, g, h, p, r\}$$

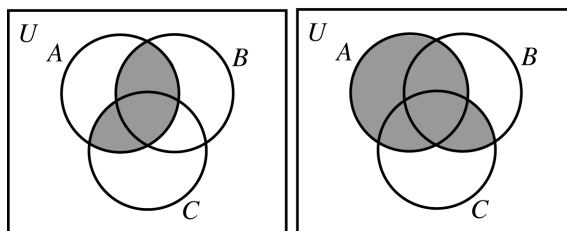
$$54. \quad \{f\}$$



56. The shaded regions are the same for  $(A \cup B)'$  and  $A' \cap B'$ . Therefore  $(A \cup B)' = A' \cap B'$



57. The statement is false because the shaded regions are different.

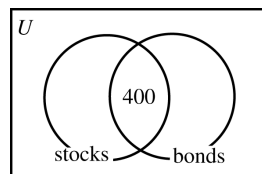


58. United States is in V; Italy is in IV; Turkey is in VIII; Norway is in V; Pakistan is in VIII; Iceland is in V; Mexico is in I

59. Start by placing 700 in region II.  
Next place  $1190 - 700$  or 490 in region III.  
Since half of those surveyed were women, place  $1000 - 700$  or 300 in region I.  
Finally, place  $2000 - 300 - 700 - 490$  or 510 in region IV.

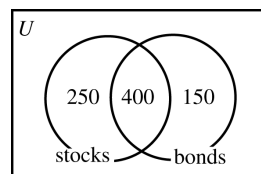
- 490 men agreed with the statement and are represented by region III.
- 510 men disagreed with the statement and are represented by region IV.

60. Begin by placing 400 in the region that represents both stocks and bonds.



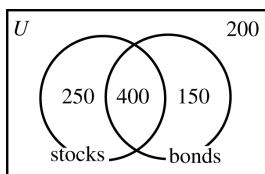
- Since 650 respondents invested in stocks,  $650 - 400 = 250$  invested in only stocks.

Furthermore, since 550 respondents invested in bonds,  $550 - 400 = 150$  invested in only bonds. Place this data in the Venn diagram.

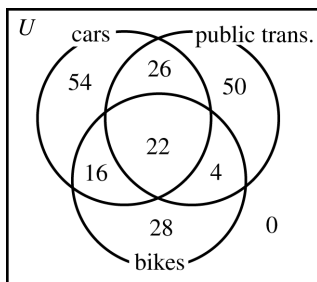


- $250 + 400 + 150 = 800$  respondents invested in stocks or bonds.

- c. Since 1000 people were surveyed,  
 $1000 - 800 = 200$  respondents who did not  
invest in either.



61. Construct a Venn diagram.



- a. 50  
b. 26  
c.  $54 + 26 + 50 = 130$   
d.  $26 + 16 + 4 = 46$   
e. 0

### Chapter 2 Test

- $\{18, 19, 20, 21, 22, 23, 24\}$
- false,  $\{6\}$  is not an element of the set, but 6 is an element.
- true, both sets have seven elements.
- true
- false,  $g$  is not an element in the larger set.
- true
- false, 14 is an element of the set.
- false, Number of subsets:  $2^N$  where  $N$  is the number of elements. There are 5 elements.  $2^5 = 32$  subsets
- false,  $\emptyset$  is *not* a proper subset of itself.

10.  $\emptyset \{6\} \{9\} \{6, 9\}$   
 $\{6, 9\}$  is not a proper subset.

11.  $\{a, b, c, d, e, f\}$

12.  $B \cap C = \{e\}$   
 $(B \cap C)' = \{a, b, c, d, f, g\}$

13.  $C' = \{b, c, d, f\}$   
 $A \cap C' = \{b, c, d\}$

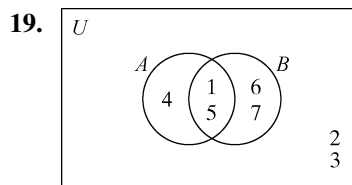
14.  $A \cup B = \{a, b, c, d, e, f\}$   
 $(A \cup B) \cap C = \{a, e\}$

15.  $B' = \{a, b, g\}$   
 $A \cup B' = \{a, b, c, d, g\}$   
 $n(A \cup B') = 5$

16.  $\{b, c, d, i, j, k\}$

17.  $\{a\}$

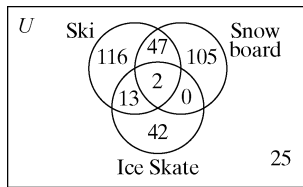
18.  $\{a, f, h\}$



20. Both expressions are represented by regions III, VI, and VII. Thus this statement is true and is a theorem.

21. a. region V  
b. region VII  
c. region IV  
d. region I  
e. region VI

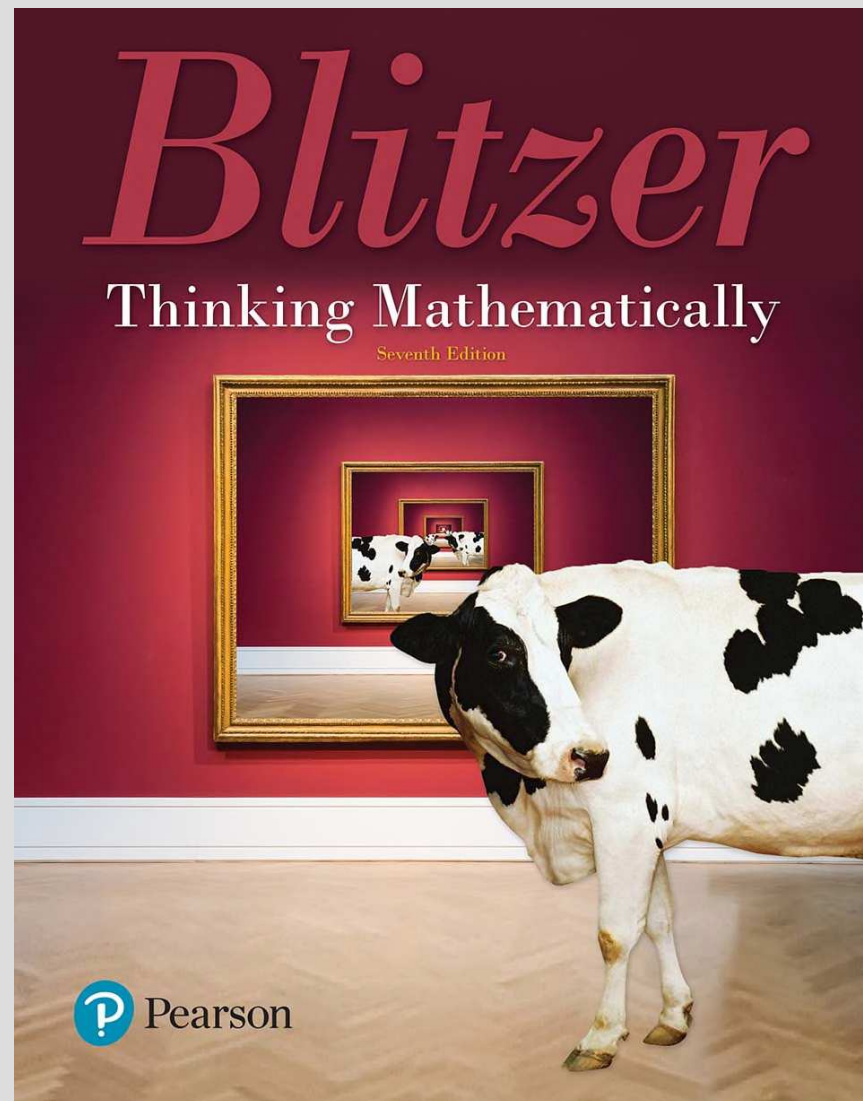
22. a.



- b. 263 (regions I, III, and VII)
- c. 25 (region VIII)
- d. 62 (regions II, IV, V, and VI)
- e. 0 (region VI)
- f. 147 (regions III, VI, and VII)
- g. 116 (region I)\*/

# CHAPTER 1

## Problem Solving and Critical Thinking



# 1.2

## Estimation, Graphs and Mathematical Models

# Objectives

1. Use estimation techniques to arrive at an approximate answer to a problem.
2. Apply estimation techniques to information given by graphs.
3. Develop mathematical models that estimate relationships between variables.

# Estimation

The process of arriving at an approximate answer to a question.

## Methods of Estimation

Rounding Numbers

Using Graphs

Using Mathematical Models

# Rounding Whole Numbers

1. Look at the digit to the right of the digit where rounding is to occur.
2. a. If the digit to the right is 5 or greater, add 1 to the digit to be rounded.

Replace all digits to the right with zeros.

- b. If the digit to the right is less than 5, do not change the digit to be rounded.

Replace all digits to the right with zeros.

The symbol  $\approx$  means “is approximately equal to”

# Example: Rounding A Whole Number

a. Round world population 7,476,242,056

To the nearest hundred million:

$$7,476,242,056 \approx 7,500,000,000$$

World population to the nearest hundred million is seven billion, five-hundred million.

# Example: Rounding A Whole Number

b. Round world population 7,476,242,056

To the nearest million:

$$7,476,242,056 \approx 7,476,000,000$$

World population to the nearest million is seven billion, four-hundred seventy-six million.

# Example: Rounding A Whole Number

c. Round world population 7,476,242,056

To the nearest hundred thousand:

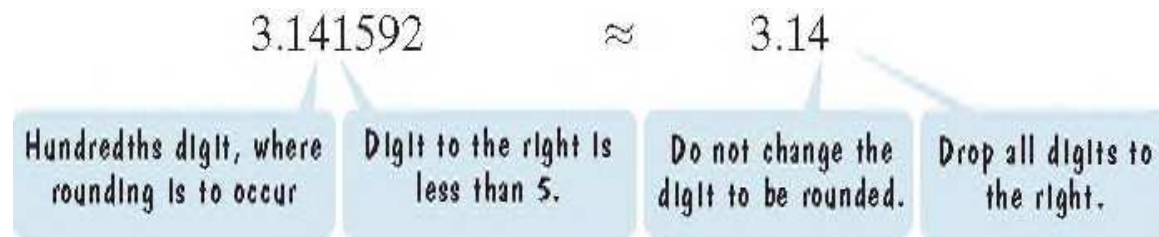
$$7,476,242,056 \approx 7,476,200,000$$

World population to the nearest hundred thousand is seven billion, four hundred seventy-six million, two hundred thousand.

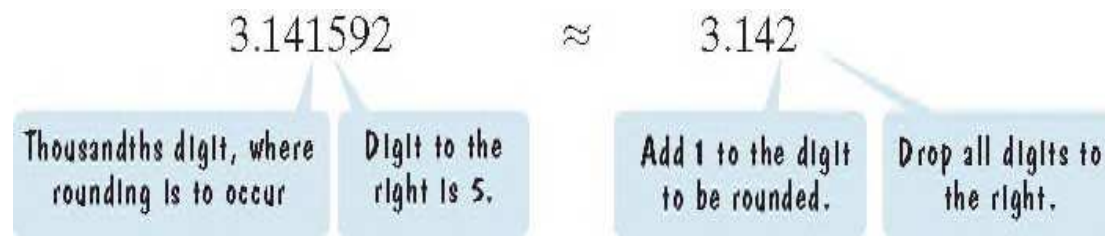
# Example: Rounding The Decimal Part of a Number

The first seven digits of  $\pi$  are: 3.141592.  
Round  $\pi$ :

To the nearest hundredth: 3.141592



To the nearest thousandth: 3.141592.



# Example: Estimation by Rounding

You purchased bread for \$2.59, detergent for \$5.17, a sandwich for \$3.65, an apple for \$0.47 and coffee for \$8.79. The total bill was given as \$24.67. Is this amount reasonable?

To check, round the cost of each item to the nearest dollar.

This bill of \$24.67 seems high compared to the \$21.00 estimate. Adding the prices of all five items gives the true total bill of \$20.67

Round to the nearest dollar.

Use digits in the tenths place to do the rounding.

|           |        |           |         |
|-----------|--------|-----------|---------|
| Bread     | \$2.59 | $\approx$ | \$3.00  |
| Detergent | \$5.17 | $\approx$ | \$5.00  |
| Sandwich  | \$3.65 | $\approx$ | \$4.00  |
| Apple     | \$0.47 | $\approx$ | \$0.00  |
| Coffee    | \$8.79 | $\approx$ | \$9.00  |
|           |        |           | <hr/>   |
|           |        |           | \$21.00 |

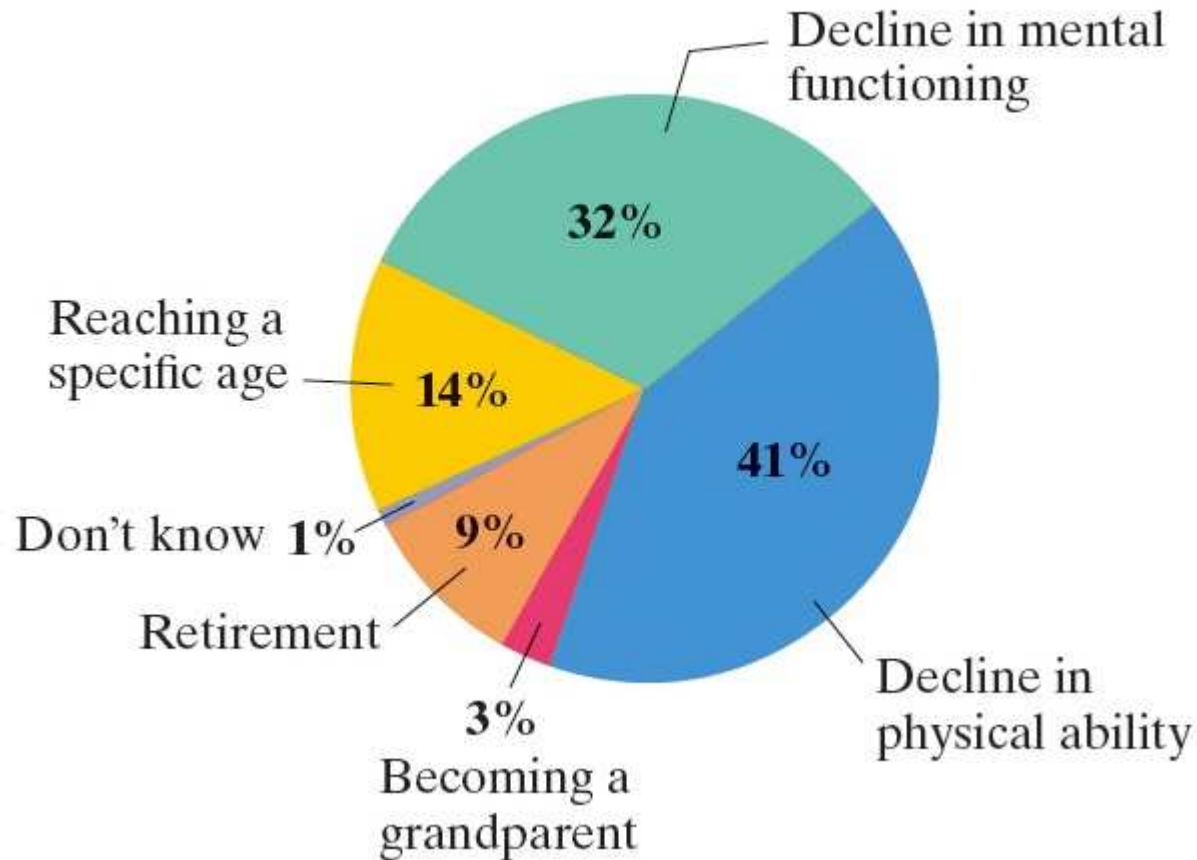
# Circle Graphs

**Circle graphs**, also called **pie charts**, show how a whole quantity is divided into parts. Circle graphs are divided into pieces, called **sectors**.

The figure (on the next slide) shows a circle graph that indicates how Americans disagree as to when “old age” begins.

# Circle Graphs

**Americans' Definition of Old Age**



# Example: Applying Estimation Techniques to a Circle Graph

According to the U.S. Census Bureau, in 2016, there were 219,345,624 Americans 25 years and older. Assuming the circle graph on the previous slide is representative of this age group,

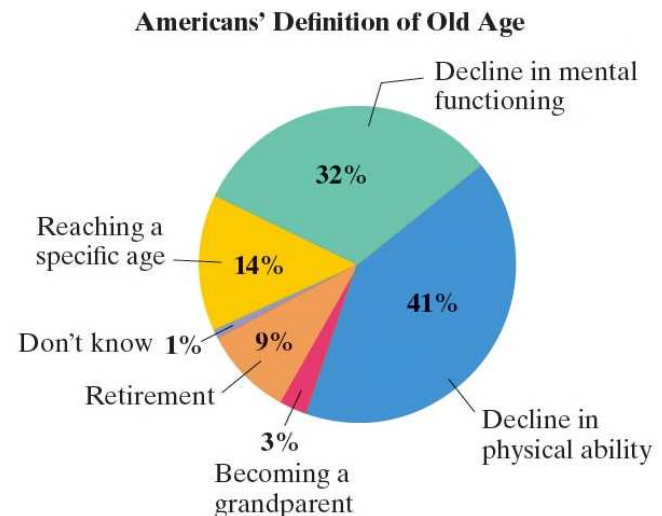
- a.** Use the appropriate information displayed by the graph to determine a calculation that shows the number of Americans 25 years and older who define old age by a decline in physical ability.
- b.** Use rounding to find a reasonable estimate for this calculation.

# Example: Applying Estimation Techniques to a Circle Graph

a. Use the to determine a calculation that shows the number of Americans 25 years and older who define old age by a decline in physical ability

The circle graph indicates that 41% of Americans define old age by a decline in physical ability.

$$0.41 \times 219,345,624$$



# Example: Applying Estimation Techniques to a Circle Graph

**b.** Use rounding to find a reasonable estimate for this calculation.

$$0.41 \times 219,345,624$$

Round 0.41 to the nearest tenth. **0.4**

Round 219,345,624 to the nearest ten million.

**220,000,000**

$$\mathbf{0.4} \times \mathbf{220,000,000} = 88,000,000$$

Approximately 88,000,000 Americans 25 years and older define old age by a decline in physical ability.

# Estimation with Graphs: Bar Graphs

Bar graphs use a vertical or horizontal bar to represent each item.

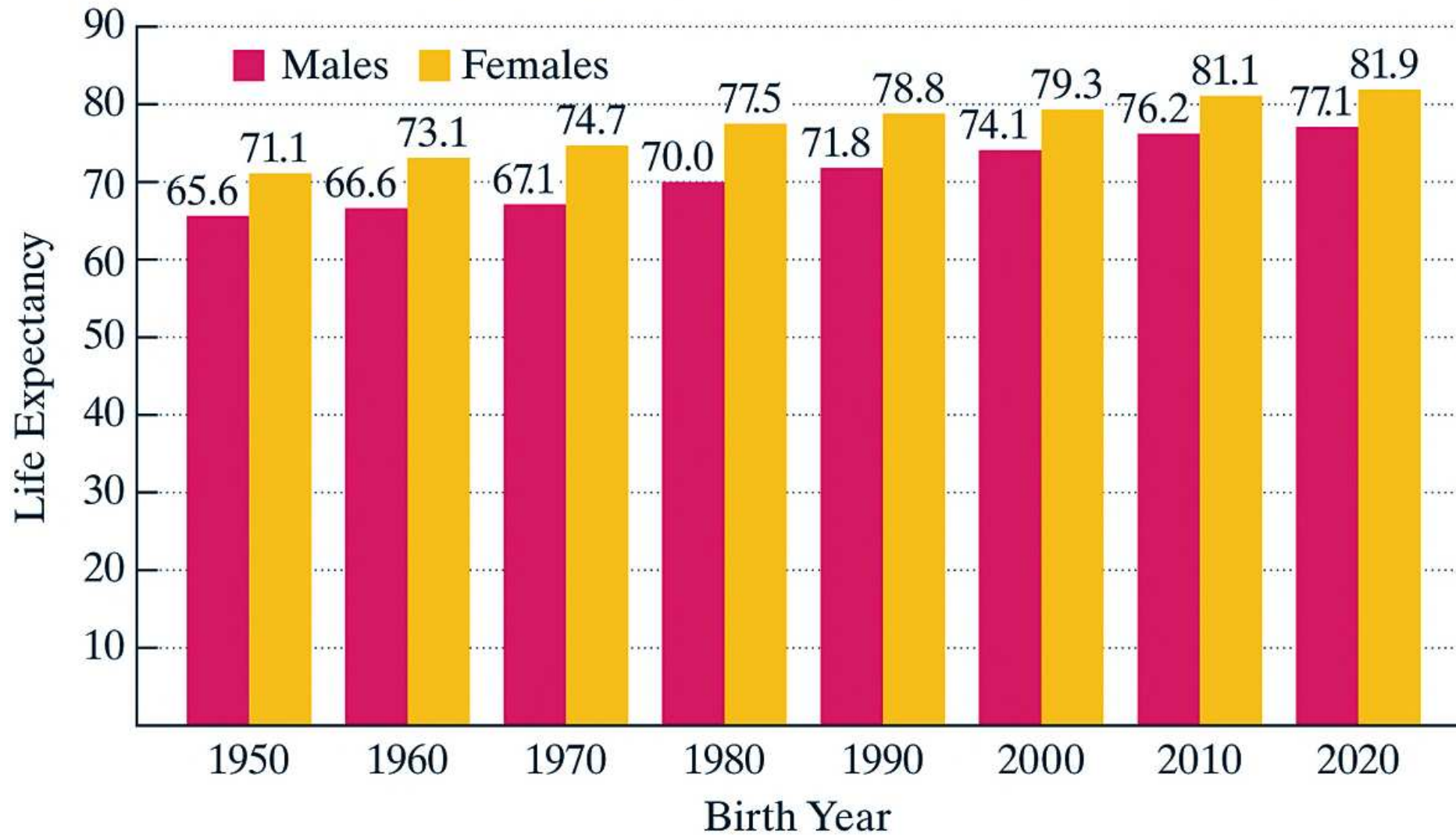
The length of the bar determines the amount.

We can use bar graphs to make predictions.

The bar graph on the next slide shows the life expectancy in the United States by Year of Birth.



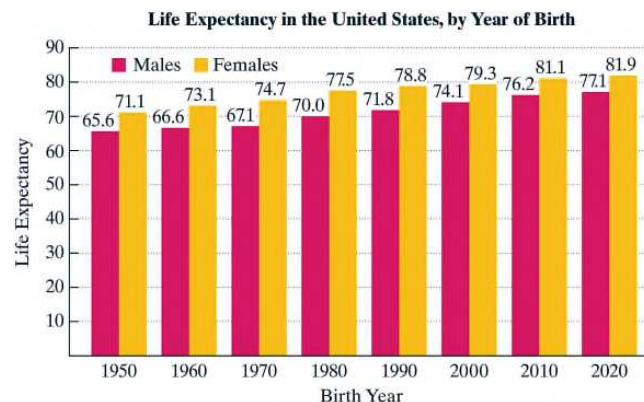
## Life Expectancy in the United States, by Year of Birth



# Example: Applying Estimation and Inductive Reasoning to Data in a Bar Graph

a. Estimate a man's increased life expectancy, rounded to the nearest hundredth of a year, for each subsequent birth year.

$$\frac{77.1 - 65.6}{2020 - 1950} \approx 0.16$$

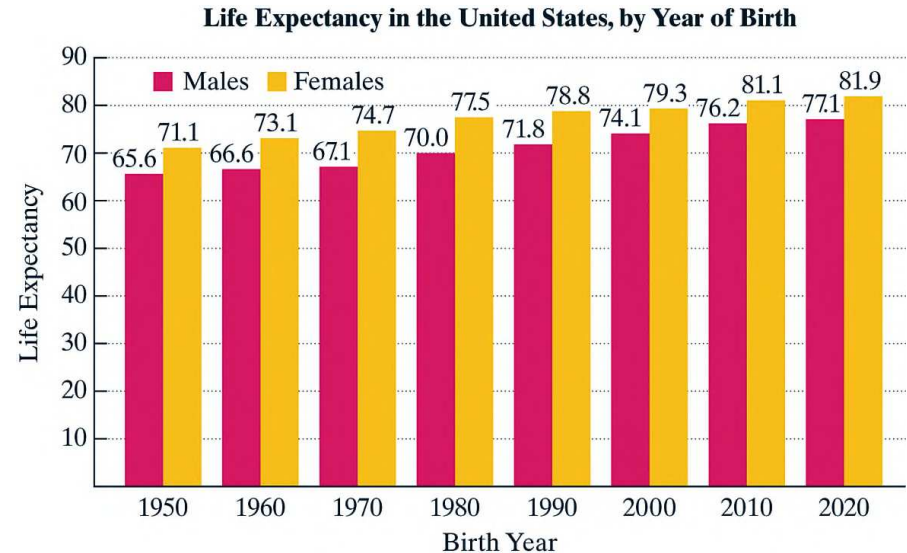


For each subsequent birth year, a man's life expectancy is increasing by approximately 0.16 year.

# Example: Applying Estimation and Inductive Reasoning to Data in a Bar Graph

b. Estimate the life expectancy of a man born in 2030.

$$\begin{aligned} &\approx 65.6 + 0.16 \times 70 \\ &= 65.6 + 12.8 \\ &= 78.4 \end{aligned}$$



An American man born in 2030 will have a life expectancy of approximately 78.4 years.

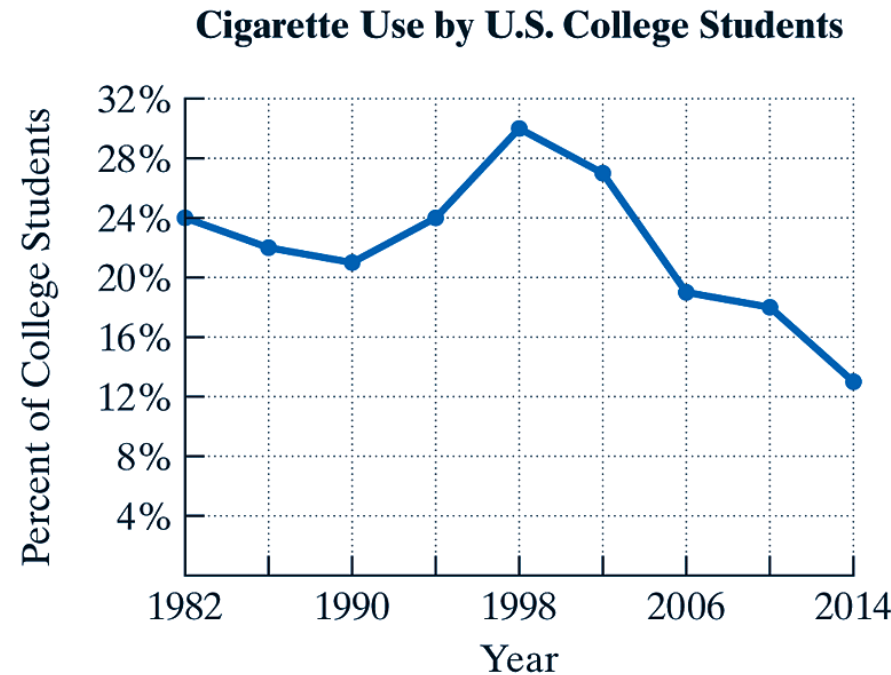
# Estimation with Graphs: Line Graphs

**Line graphs** are often used to illustrate trends over time. Some measure of time, such as months or years, frequently appears on the horizontal axis. Amounts are generally listed on the vertical axis. Points are drawn to represent the given information. The graph is formed by connecting the points with line segments.

# Example: Using a Line Graph

The line graph shows the percentage of U.S. college students who smoked cigarettes from 1982 through 2014.

a. Find an estimate for the percentage of college students who smoked cigarettes in 2010.



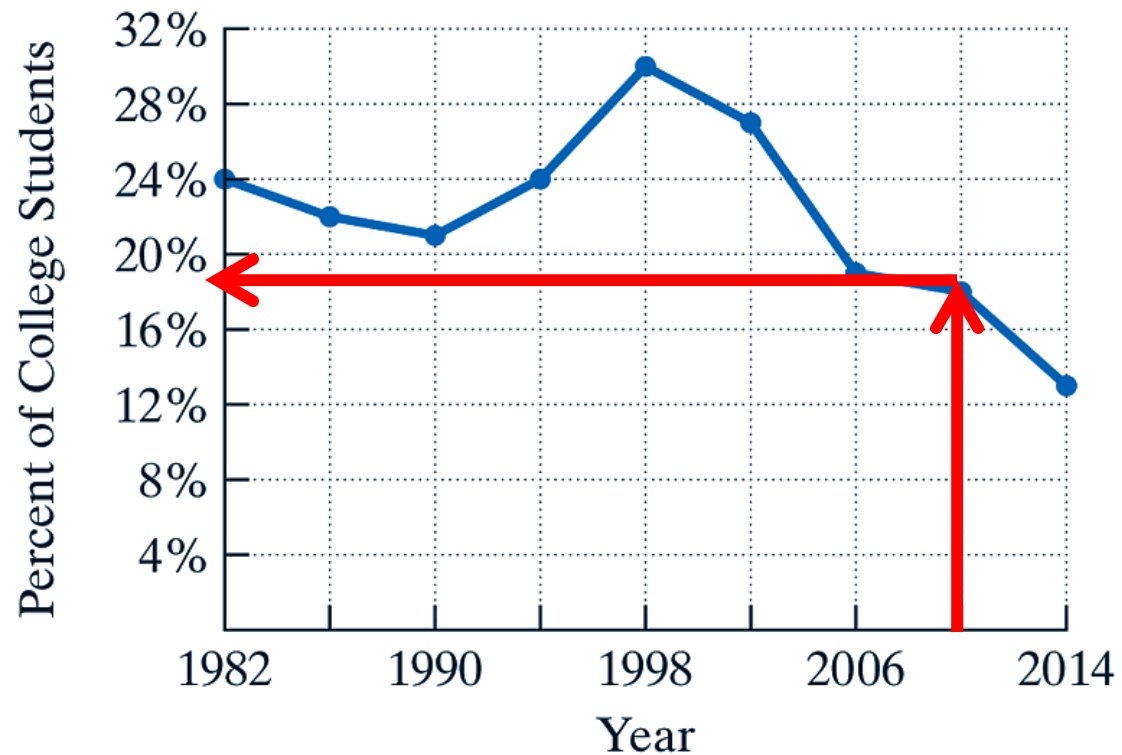
# Example: Using a Line Graph

Look for 2010  
on the horizontal  
axis.

Value on the  
vertical axis is  
approximately  
midway between  
16% and 20%

In 2010, approximately 18% of college students  
smoked cigarettes.

**Cigarette Use by U.S. College Students**



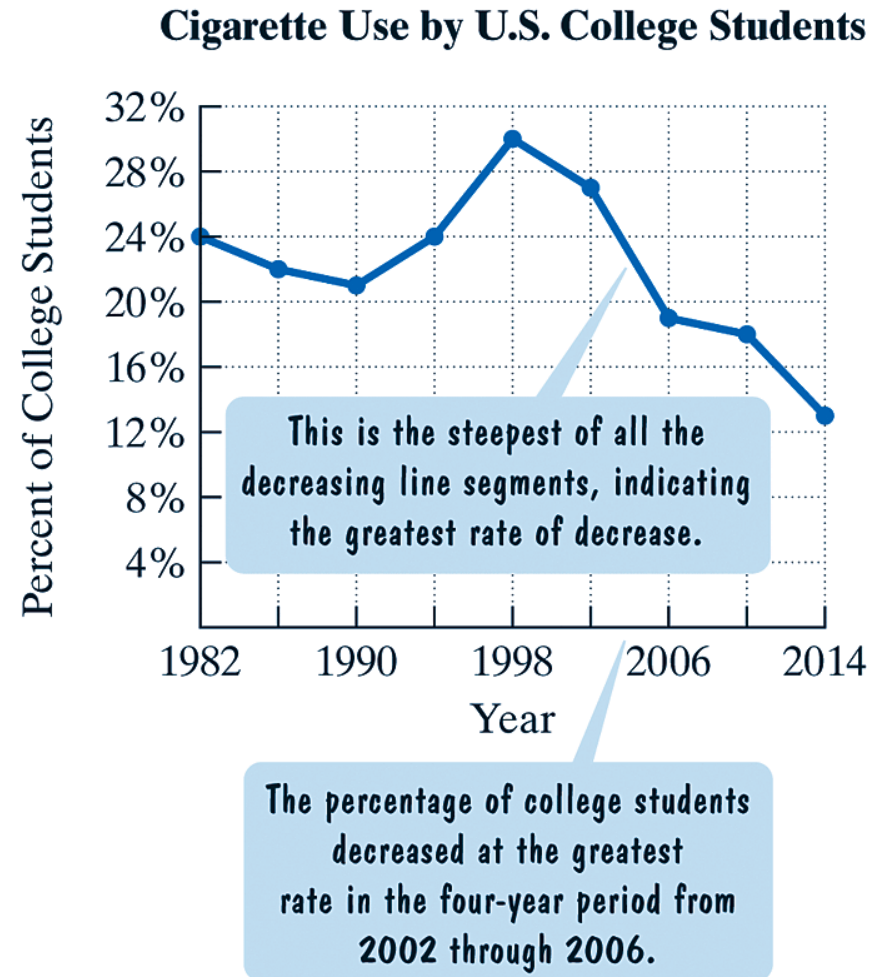
# Example: Using a Line Graph

The line graph shows the percentage of U.S. college students who smoked cigarettes from 1982 through 2014.

**b.** In which four-year period did the percentage of college students who smoked cigarettes decrease at the greatest rate?

# Example: Using a Line Graph

This is the steepest of all the decreasing line segments, indicating the greatest rate of decrease; years 2002 through 2006.



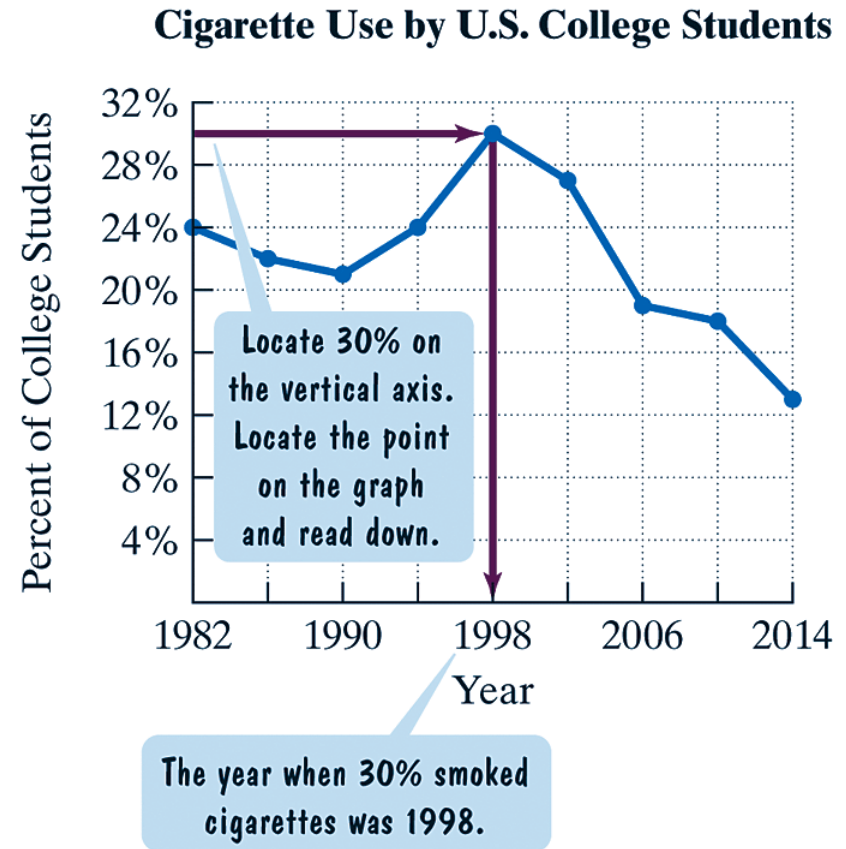
# Example: Using a Line Graph

The line graph shows the percentage of U.S. college students who smoked cigarettes from 1982 through 2010.

c. In which year did 30% of college students smoke cigarettes?

# Example: Using a Line Graph

Locate 30% on the vertical axis.  
Locate the point on the graph and read down.

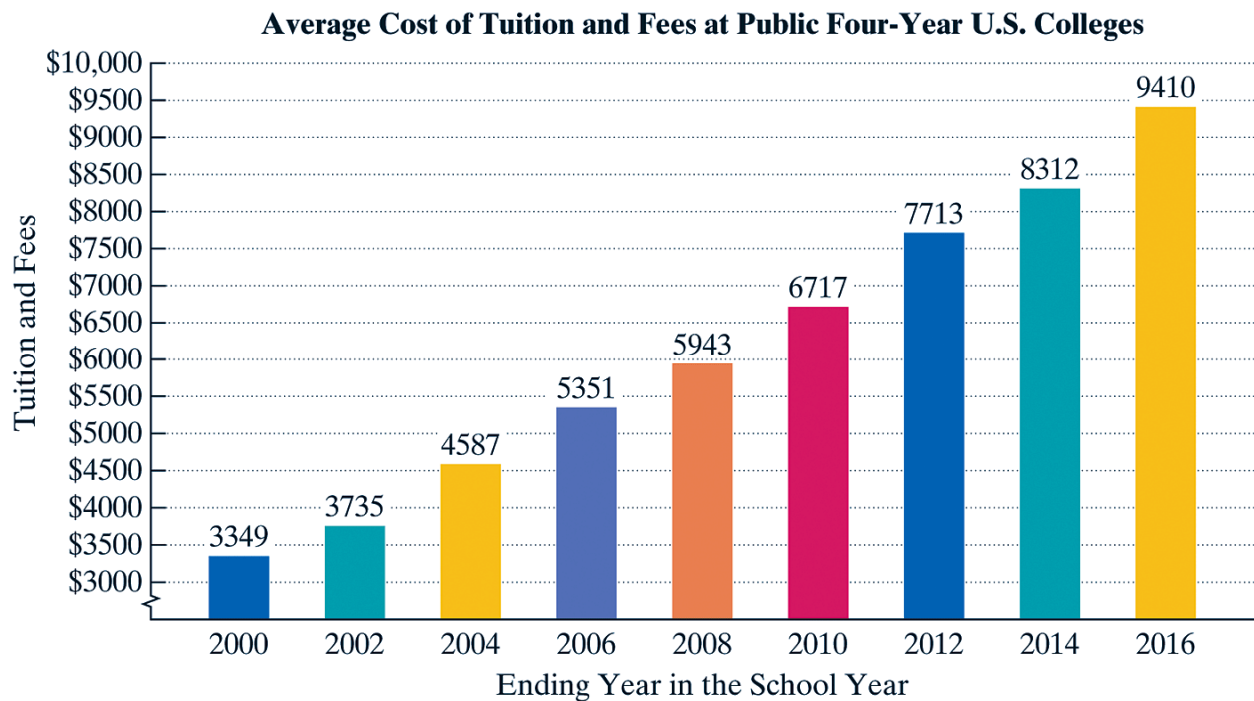


The year when 30% smoked cigarettes was 1998.

# Example: Modeling the Cost of Attending a Public College

a. Estimate the yearly increase in tuition and fees.

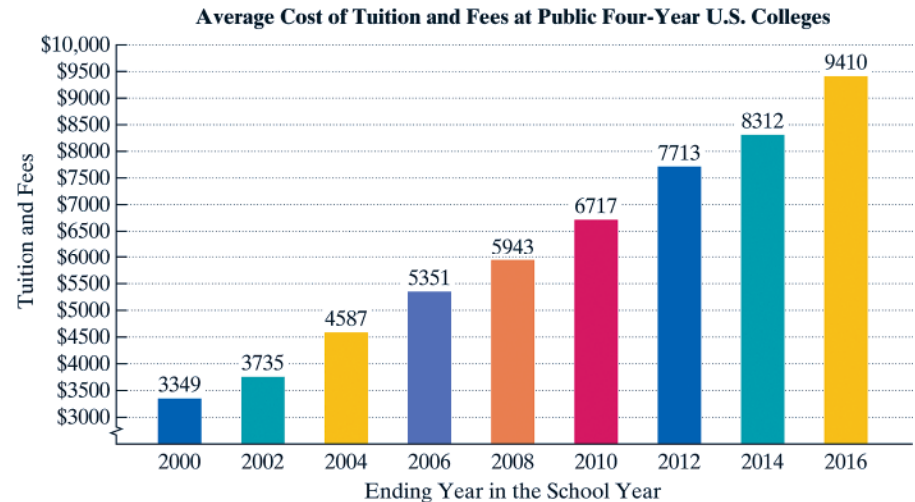
$$\begin{aligned} &\approx \frac{9410 - 3349}{2016 - 2000} \\ &\approx \frac{6061}{16} \\ &= 378.8 \approx 379 \end{aligned}$$



The average yearly increase is about \$379.

## Example: continued

b. Write a mathematical model that estimates the average cost of tuition and fees,  $T$ , at public four-year colleges for the school year ending  $x$  years after 2000.



$$T = 3349 + 379x$$

## Example: continued

c. Use the mathematical model to project the average cost of tuition and fees for the school year ending in 2020.

$$T = 3349 + 379x$$

$$T = 3349 + 379(20)$$

$$= 3349 + 7580$$

$$= 10,929$$

The model projects that the average cost of tuition and fees for the school year ending in 2020 will be \$10,929.

