Integrated Microelectronic Devices 1st edition Alamo Solutions Manual

SKU: 9780134670904-SOLUTIONS

Integrated Microelectronic Devices: Physics and Modeling

J. A. del Alamo

Pearson, 2017

Problem Solutions

Chapter 1: Electrons, Photons and Phonons

updated: January 9, 2018

INSTRUCTORS: Please do not distribute these solutions to students in electronic form!

Problem 1.1

This is an interesting exercise in units.

a) For the tennis ball:

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 57 \times 10^{-3}\ kg \times (\frac{160 \times 10^3}{3600}\ m/s)^2 \times \frac{1}{1.6 \times 10^{-19}\ J/eV} = 3.52 \times 10^{20}\ eV$$

$$\lambda_B = \frac{h}{p} = \frac{h}{\sqrt{2mE_K}} = \frac{4.14 \times 10^{-15}\ eV.s}{\sqrt{2 \times 57 \times 10^{-3}\ kg \times 6.24 \times 10^{14}\ (eV.s^2/cm^2) \times 3.52 \times 10^{20}\ eV}} = 2.62 \times 10^{-32}\ cm$$

This wavelength is much smaller than the dimension of the tennis court so a particle-type description in this case is appropriate.

b) For the car, in a similar way:

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 3000 \times 0.454\ kg \times (\frac{65 \times 1.61 \times 10^3}{3600}\ m/s)^2 \times \frac{1}{1.6 \times 10^{-19}\ J/eV} = 3.60 \times 10^{24}\ eV$$

$$\lambda_B = \frac{h}{\sqrt{2mE_K}} = \frac{4.14 \times 10^{-15}\ eV.s}{\sqrt{2 \times 3000 \times 0.454\ kg \times 6.24 \times 10^{14}\ (eV.s^2/cm^2) \times 3.60 \times 10^{24}\ eV}} = 1.67 \times 10^{-36}\ cm$$

Again we find here that λ_B is much smaller than the dimensions of the road. Hence, a particle description is appropriate.

c) For the electron:

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 9.11 \times 10^{-31}\ kg \times (10^7\ m/s)^2 \times \frac{1}{1.6 \times 10^{-19}\ J/eV} = 285\ eV$$

$$\lambda_B = \frac{h}{\sqrt{2mE_K}} = \frac{4.14 \times 10^{-15}\ eV.s}{\sqrt{2 \times 5.69 \times 10^{-16}\ (eV.s^2/cm^2) \times 3.60 \times 10^{24}\ eV}} = 7.24 \times 10^{-9}\ cm$$

In a 1-inch (2.5 cm) diameter pipe, this electron can also be adequately described as a particle.

Problem 1.2

According to Sec. 1.1.3, the energy needed to free up a Si 1s electron is 1839 eV. Then, from $E = h\nu$, we have:

$$\nu = \frac{E}{h} = \frac{1839\ eV}{4.14 \times 10^{-15}\ eV.s} = 4.44 \times 10^{17}\ s^{-1}$$

The wavelength is:

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^{8}\ m/s}{4.44 \times 10^{17}\ s^{-1}} = 6.76 \times 10^{-10}\ m = 0.68\ nm$$

Problem 1.3

To solve this exercise, we have to compare the bandgap of the semiconductor and the photon energy of each beam of light. So, first, we have to calculate the photon energy. This is best done using Eq. 1.2:

$$E_{ph1}(eV) = \frac{1.24}{\lambda_1(\mu m)} = \frac{1.24}{0.85} = 1.46\ eV$$

Similarly, $E_{ph2} = 0.95\ eV$ and $E_{ph3} = 0.8\ eV$.

When light hits on a semiconductor, if the photon energy is higher than the semiconductor bandgap, the light is absorbed. If, on the other hand, the photon energy is smaller than the bandgap, then the light passes through.

For the case on the left, beam 1 with a photon energy of 1.46 eV is absorbed in the GaAs wafer since the bandgap of GaAs is 1.42 eV. Beams 2 and 3 have photon energies smaller than the bandgaps of GaAs and Si, hence they go through both wafers. However, both beams have an energy higher than the Ge bandgap, hence they are both absorbed by the Ge wafer.

For the case on the right, all three beams are absorbed in the Ge wafer since its bandgap is smaller than the photon energy of each of them.

Problem 1.4

We will estimate the temperature by equating the kinetic energy to the thermal energy, that is:

$$\frac{p^2}{2m} = \frac{3}{2}kT$$

p is obtained from:

$$p = \frac{h}{\lambda} = \frac{4.14 \times 10^{-15}\ eV.s}{15 \times 10^{-4}\ cm} = 2.76 \times 10^{-12}\ eV.s/cm$$

To calculate the mass of the Na atom, we need to know the atomic weight $A = 23\ g/mol$, and Avogadro's number $N_A = 6.02 \times 10^{23}\ atoms/mol$. Then:

$$m = \frac{A}{N_A} = \frac{23\ g/mol}{6.02 \times 10^{23}\ atoms/mol} = 3.8 \times 10^{-23}\ g = 2.38 \times 10^{-11}\ eV.s^2/cm^2$$

We can now calculate T:

$$T = \frac{p^2}{3mk} = \frac{(2.76 \times 10^{-12}\ eV.s/cm)^2}{3 \times 2.38 \times 10^{-11}\ eV.s^2/cm^2 \times 8.62 \times 10^{-5}\ eV/k} = 1.24 \times 10^{-9}\ K$$

This is in the nK range. Small as it is, this is a reachable temperature using special techniques.

The de Broglie wavelength of the Na atom at room temperature is:

$$\lambda_B = \frac{h}{\sqrt{3mkT}} = \frac{4.14 \times 10^{-15}\ eV.s}{\sqrt{3 \times 2.38 \times 10^{-11}\ (eV.s^2/cm^2) \times 0.026\ eV}} = 3.03 \times 10^{-9}\ cm$$

This is an extremely small dimension that makes observing wave phenomena in Na atoms at room temperature impossible.

Problem 1.5

a) The energy of a photon with $\lambda = 0.6\ \mu m$ is:

$$E_{ph} = \frac{1.24}{0.6} = 2.1\ eV = 2.1\ \times 1.6 \times 10^{-19}\ J/eV = 3.4 \times 10^{-19}\ J$$

At $\lambda = 0.6\ \mu m$, the spectral irradiance is $1,300\ W \cdot m^{-2} \cdot \mu m^{-1}$. The spectral density of the photon flux is:

$$n_{ph} = \frac{1,300\ W \cdot m^{-2} \cdot \mu m^{-1}}{3.4 \times 10^{-19}\ J} = 4.2 \times 10^{21}\ m^{-2} \cdot s^{-1} \cdot \mu m^{-1}$$

b) Assuming constant spectral density over the relatively narrow range of wavelengths that is specified, the photon flux is:

$$N_{ph} \simeq n_{ph}(\lambda_1 - \lambda_2) = 4.2 \times 10^{21}\ m^{-2} \cdot s^{-1} \cdot \mu m^{-1} \times 0.02\ \mu m = 8.4 \times 10^{19}\ m^{-2} \cdot s^{-1}$$

c) Semiconductors with a bandgap above $E_{ph} = 2.1\ eV$ do not absorb photons of this wavelength.

Problem 1.6

According to the Appendix, the atomic density of Si is $N_a = 5 \times 10^{22}\ cm^{-3}$ and the interatomic distance is 0.235 nm.

The densest way of packing hard spherical balls is in the Face-Centered Cubic arrangement. In this approach, there are four atoms at the four corners of a cube and six atoms in the middle of each of the six facets. In a cube of volume a^3, where a is the length of the cube side, there are $8 \times 1/8 + 6 \times 1/2 = 4$ atoms. The side length of the cube is, then:

$$a = (\frac{4}{N_a})^{1/3} = (\frac{4}{5 \times 10^{22}\ cm^{-3}})^{1/3} = 4.3 \times 10^{-8}\ cm = 0.43\ nm$$

The interatomic distance is:

$$d = \frac{a}{\sqrt{2}} = 0.31\ nm$$

If we assume that the solid is made of hard cubes, then, they are simply stacked on top of each other. The atomic distance is then:

$$d = (\frac{1}{N_a})^{1/3} = (\frac{1}{5 \times 10^{22}\ cm^{-3}})^{1/3} = 0.27\ nm$$

Both estimates, give an atomic distance that is somehow larger than the actual value.

2. Problem 1.8

The point of this question is to show that the Maxwell-Boltzmann approximation to the Fermi-Dirac statistics fail horribly when the energy is near the Fermi level. There's two different equations for the Maxwell-Boltzmann approximation, one valid when energy is less than the Fermi energy and one valid when energy is greator than the Fermi energy.

Relative Error for E-Ef >> kT

$$f_{MB_{high}} := \exp\left(-\frac{E-E_F}{kT}\right):$$

$$f_{FD} := \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}:$$

Let's let $\epsilon = \exp\left(\frac{E-E_F}{kT}\right)$ to simplify the math

$$RelativeError_{high} := \frac{f_{FD}-f_{MB_{high}}}{f_{FD}} = \frac{\frac{1}{1+\epsilon}-\epsilon^{-1}}{\frac{1}{1+\epsilon}} = \frac{1-\epsilon^{-1}(1+\epsilon)}{1} = 1-\epsilon^{-1}+1 = \epsilon^{-1}$$

$$= \exp\left(-\frac{E-E_F}{kT}\right)$$

The relative error for the high energy limit becomes larger the smaller E gets

Relative Error for E-Ef << kT

$$f_{MB_{low}} := 1-\exp\left(\frac{E-E_F}{kT}\right):$$

$$RelativeError_{low} := \frac{f_{FD}-f_{MB_{low}}}{f_{FD}} = \frac{\frac{1}{1+\epsilon}-(1-\epsilon)}{\frac{1}{1+\epsilon}} = \frac{1-(1-\epsilon)(1+\epsilon)}{1} = 1-1+\epsilon^2 = \epsilon^2$$

$$= \exp\left(\frac{2(E-E_F)}{kT}\right)$$

The relative error for the low energy limit becomes larger the larger E gets

Relative Error Plot

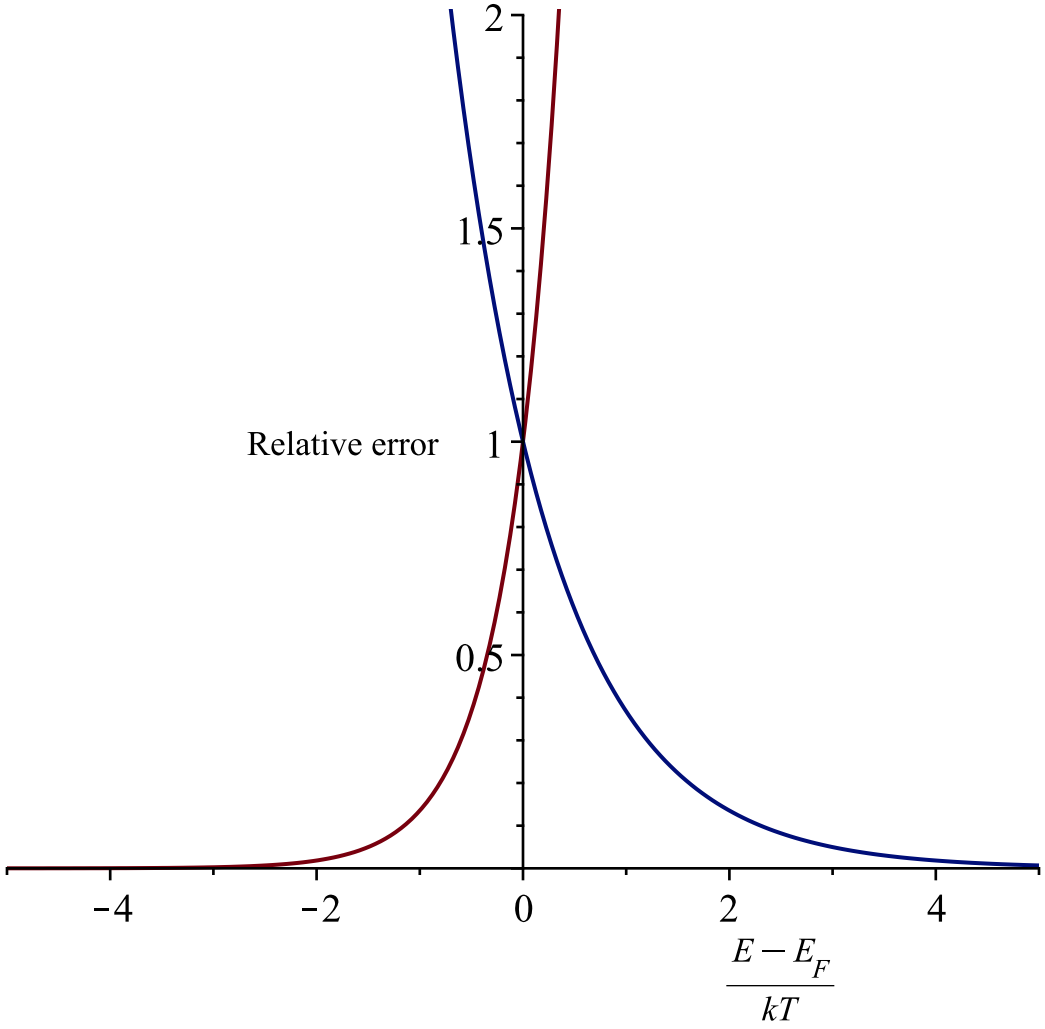
We can make our x axis be (E-Ef)/kT since we don't have values for Ef and kT. Then,

$$RelativeError_{low} := \frac{f_{FD}-f_{MB_{low}}}{f_{FD}} = \exp(2x)$$

$$RelativeError_{high} := \frac{f_{FD}-f_{MB_{high}}}{f_{FD}} = \exp(-x)$$

We can now plot the evolution of the relative error versus energy

$$plot\left([\exp(2x), \exp(-x)], x = -5..5, y = -0..2, labels = \left[\frac{E - E_F}{kT}, \text{"Relative error"} \right] \right)$$



We can see that the Maxwell-Boltzmann approximation to the Fermi Dirac distribution is very poor when E - Ef is between -2kT to 2kT.

Problem 1.9

First, the mass of a Rb atom is:

$$m_{Rb} = \frac{85.5\ g/mol}{6.02 \times 10^{23}\ atoms/mol} \times 10^{-3}\ kg/g = 1.42 \times 10^{-25}\ kg$$

In "microelectronic units":

$$m_{Rb} = 1.42 \times 10^{-25}\ kg \times 6.24 \times 10^{14}\ eV.s^2.cm^{-2}.kg^{-1} = 8.86 \times 10^{-11} eV.s^2/cm^2$$

The average kinetic energy of an atom is:

$$< E_k >= \frac{3}{2}kT$$

Also,

$$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Setting these equal and solving for p yields:

$$p = \sqrt{3mkT}$$

de Broglie's wavelength is then:

$$\lambda_B = \frac{h}{p} = \frac{h}{\sqrt{3mkT}} = \frac{4.14 \times 10^{-15}\ eV.s}{\sqrt{3(8.86 \times 10^{-11}\ eV.s^2/cm^2)(8.62 \times 10^{-5}\ eV/K)(290 \times 10^{-9}\ K)}} = 5.08 \times 10^{-5}\ cm = 0.508\ \mu m$$

Problem 1.10

The probability of the state being occupied is:

$$f(E) = 1 - 0.15 = 0.85$$

Since this is higher than 0.5, the state is *below* the Fermi level.

To get the relative location of the state with respect to the Fermi level, solve from the Fermi-Dirac distribution function:

$$E - E_F = kT \ln[\frac{1}{f(E)} - 1] = 0.026 \ln(\frac{1}{0.85} - 1) = -0.045\ eV$$

The negative sign means that the state in question is indeed below the Fermi level.

Problem 1.11

The photon spectral density is given by:

$$N_{ph}(\nu) = \frac{E(\nu)}{h\nu} = \frac{2\pi\nu^2}{c^2}\frac{1}{\exp\frac{h\nu}{kT} - 1}$$

The frequency corresponding to the bandgap of Si is:

$$\nu = \frac{E_g}{h} = \frac{1.124\ eV}{4.14 \times 10^{-15}\ eV.s} = 2.7 \times 10^{14}\ Hz$$

Plugging this into the equation above yields:

$$N_{ph} = \frac{2\pi(2.7 \times 10^{14}\ s^{-1})^2}{(3 \times 10^{10}\ cm/s)^2}\frac{1}{\exp\frac{1.124}{0.026} - 1} = 8.5 \times 10^{-11}\ cm^{-2}$$

Note that the units are correct: $cm^{-2} = cm^{-2}.s^{-1}.s$. So it is N_{ph} photons per unit time, per unit area, per unit frequency.

Problem 1.12

a) A semiconductor only absorbs photons with energy bigger than its bandgap. However, all the extra energy in excess of the bandgap is dissipated in the form of heat and is not delivered as electrical energy to the load. So, an ideal solar cell is one that is composed of multiple layers of different semiconductors of different bandgaps in a way that photons are absorbed in a semiconductor with a bandgap as close to (but below) the photon energy. For this concept to work well, the wider bandgap material should come first to absorb the high energy photons while letting through the medium and low energy photons. Then one should place the medium gamdgap material that absorbs medium energy photons but lets low energy photons through. Finally, the low energy material is placed at the bottom. So, the optimum arrangement is, from top to bottom, InGaP, InGaAs and Ge.

b) We can convert from energy to wavelength using the following expression:

$$E_{ph}(eV) = \frac{1.24}{\lambda(\mu m)}$$

For InGaP, the bandgap corresponds to:

$$\lambda_g(InGaP) = \frac{1.24}{1.86} = 0.67\ \mu m$$

Similarly, $\lambda_g(InGaAs) = 0.89\ \mu m$ and $\lambda_g(Ge) = 1.77\ \mu m$.

Then, InGaP absorbs any light with $\lambda < 0.67\ \mu m$, InGaAs absorbs photons with λ between 0.67 and 0.89 μm, and Ge absorbs photons with λ between 0.89 and 1.77 μm. Photons with $\lambda > 1.77\ \mu m$ go through the entire stack without being absorbed.