Calculus for Business Economics Life Sciences and Social Sciences 14th edition Barnett Solutions Manual

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Calculus for Business Economics Life Sciences and Social Sciences 14th edition Barnett Solutions Manual

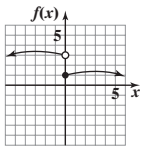
ANSWERS

Chapter 2

Exercises 2.1

1. $(x-9)(x+9)$ **2.** $(x-8)(x+8)$ **3.** $(x-7)(x+3)$
4. $(x+9)(x-4)$ **5.** $x(x-3)(x-4)$ **6.** $x(x+5)(x+10)$
7. $(2x-1)(3x+1)$ **8.** $(4x+3)(5x-1)$ **9.** 2 **10.** 2 **11.** 1.25
12. 2.25 **13.** (A) 2 (B) 2 (C) 2 (D) 2 **14.** (A) 2 (B) 2 (C) 2 (D) 2
15. (A) 1 (B) 2 (C) Does not exist (D) 2 **16.** (A) 4 (B) 4 (C) 4
(D) Does not exist **17.** 2 **18.** 1.9 **19.** 0.5 **20.** 1.5 **21.** (A) 1 (B) 2
(C) Does not exist (D) Does not exist **22.** (A) 2 (B) 2 (C) 2 (D) 2
23. (A) 1 (B) 1 (C) 1 (D) 3 **24.** (A) 0 (B) 0 (C) 0 (D) 0
25. (A) -2 (B) -2 (C) -2 (D) 1 **26.** (A) 3 (B) -3 (C) Does not exist
(D) -3 **27.** (A) 2 (B) 2 (C) 2 (D) Does not exist **28.** (A) -3 B) 3
(C) Does not exist (D) 3 **29.** 12 **30.** -6 **31.** 1 **32.** 2 **33.** -4
34. -2 **35.** -1.5 **36.** 1/2 **37.** 3 **38.** 4 **39.** 15 **40.** 8 **41.** -6
42. 19 **43.** $\frac{7}{5}$ **44.** $-8/15$ **45.** 3 **46.** -2

47.



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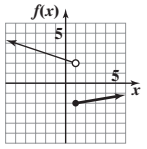

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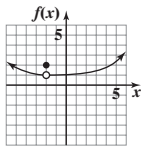

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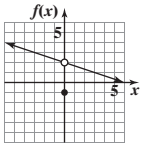
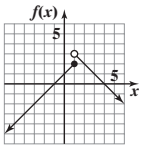
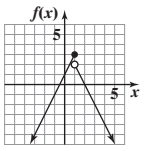


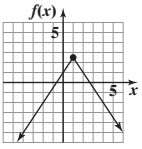
51. (A) 1 (B) 1 (C) 1 (D) 1 **52.** (A) 2 (B) 2
(C) 2 (D) 2 **53.** (A) 2 (B) 1 (C) Does not exist
(D) Does not exist **54.** (A) 0 (B) 1 (C) Does not exist
(D) Does not exist **55.** (A) -6 (B) Does not exist
(C) 6 **56.** (A) Does not exist (B) 0 (C) Does not exist

57. (A) 1 (B) -1 (C) Does not exist (D) Does not exist **58.** (A) 1
(B) -1 (C) Does not exist (D) Does not exist **59.** (A) Does not exist
(B) $\frac{1}{2}$ (C) $\frac{1}{4}$ **60.** (A) $-1/3$ (B) Does not exist (C) 1/3 **61.** (A) -5
(B) -3 (C) 0 **62.** (A) -5 (B) -2 (C) 0 **63.** (A) 0 (B) -1
(C) Does not exist **64.** (A) Does not exist (B) -1 (C) 0 **65.** (A) 1
(B) $\frac{1}{3}$ (C) $\frac{3}{4}$ **66.** (A) 10 (B) -4 (C) 5/4 **67.** False **68.** True **69.** False
70. False **71.** True **72.** False **73.** Yes; 2 **74.** No; Does not exist
75. No; Does not exist **76.** Yes; -4 **77.** Yes; $7/5$ **78.** Yes; $1/2$
79. No; 0 **80.** No; 9 **81.** 3 **82.** 5 **83.** 4 **84.** 4 **85.** -7 **86.** -4
87. 1 **88.** -3

89. (A) $\lim_{x\to 1^-} f(x) = 2$, $\lim_{x\to 1^+} f(x) = 3$ (B) $\lim_{x\to 1^-} f(x) = 3$, $\lim_{x\to 1^+} f(x) = 2$ (C) $m = 1.5$

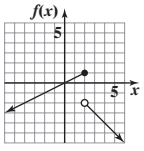


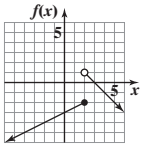


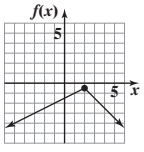


(D) The graph in (A) is broken when it jumps from $(1, 2)$ up to $(1, 3)$. The graph in (B) is also broken when it jumps down from $(1, 3)$ to $(1, 2)$. The graph in (C) is one continuous piece, with no breaks or jumps.

90. (A) $\lim_{x\to 2^-} f(x) = 1$, $\lim_{x\to 2^+} f(x) = -1$ (B) $\lim_{x\to 2^-} f(x) = -2$, $\lim_{x\to 2^+} f(x) = 1$ (C) $m = 0.5$

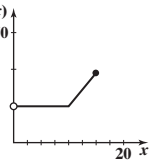






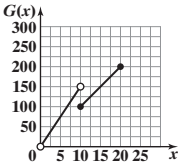
(D) The graph in (A) is broken when it jumps from $(2,1)$ down to $(2, -2)$, the graph in (B) is also broken when it jumps from $(2, -2)$ up to $(2, 1)$, while the graph in (C) is one continuous piece with no jumps or breaks.

91. (A) $F(x) = \begin{cases} 50 & \text{if } 0 \le x \le 10 \\ 9x - 40 & \text{if } x > 10 \end{cases}$ (B)



(C) All 3 limits are 50.

92. (A) $G(x) = \begin{cases} 15x, & 0 \le x \le 10 \\ 10x, & x > 10 \end{cases}$ (B)



(C) \$150; \$100; does not exist

95. (A) $D(x) = \begin{cases} x & \text{if } 0 \le x < 300 \\ 0.97x & \text{if } 300 \le x < 1{,}000 \\ 0.95x & \text{if } 1{,}000 \le x < 3{,}000 \\ 0.93x & \text{if } 3{,}000 \le x < 5{,}000 \\ 0.9x & \text{if } x \ge 5{,}000 \end{cases}$

(B) $\lim_{x\to 1{,}000} D(x)$ does not exist because
$\lim_{x\to 1{,}000^-} D(x) = 970$ and $\lim_{x\to 1{,}000^+} D(x) = 950$;
$\lim_{x\to 3{,}000^-} D(x)$ does not exist because
$\lim_{x\to 3{,}000^-} D(x) = 2{,}850$ and $\lim_{x\to 3{,}000^+} D(x) = 2{,}790$

96. (A)

$$P(x) = \begin{cases} x & \text{if } 0 \le x < 300 \\ 300 + 0.97(x - 300) = 0.97x + 9 & \text{if } 300 \le x < 1{,}000 \\ 0.97(1{,}000) + 9 + 0.95(x - 1{,}000) = 0.95x + 29 & \text{if } 1{,}000 \le x < 3{,}000 \\ 0.95(3{,}000) + 29 + 0.93(x - 3{,}000) = 0.93x + 89 & \text{if } 3{,}000 \le x < 5{,}000 \\ 0.93(5{,}000) + 89 + 0.90(x - 5{,}000) = 0.90x + 239 & \text{if } x \ge 5{,}000 \end{cases}$$

(B) $\lim_{x\to 1{,}000} P(x) = 979$; $\lim_{x\to 3{,}000} = P(x) = 2{,}879$

97. $F(x) = \begin{cases} 20x & \text{if } 0 < x \le 4{,}000 \\ 80{,}000 & \text{if } x \ge 4{,}000 \end{cases}$
$\lim_{x\to 4{,}000} F(x) = 80{,}000$; $\lim_{x\to 8{,}000} F(x) = 80{,}000$

98. $A(x) = \begin{cases} 20 & \text{if } 0 < x \le 4{,}000 \\ \frac{80{,}000}{x} & \text{if } x \ge 4{,}000 \end{cases}$
$\lim_{x\to 4{,}000} A(x) = 20$; $\lim_{x\to 8{,}000} A(x) = 10$

99. $\lim_{x\to 5^-} f(x)$ does not exist; $\lim_{x\to 10} f(x) = 0$;
$\lim_{x\to 5^-} g(x) = 0$; $\lim_{x\to 10} g(x) = 1$

Exercises 2.2

1. $y = 4$ **2.** $x = 5$ **3.** $x = -6$ **4.** $y = 1$ **5.** $2x - y = -13$
6. $3x + y = 20$ **7.** $7x + 9y = 63$ **8.** $-5x + y = 25$ **9.** -2 **10.** ∞
11. $-\infty$ **12.** ∞ **13.** Does not exist **14.** ∞ **15.** 0 **16.** Does not exist
17. (A) $-\infty$ (B) ∞ (C) Does not exist **18.** (A) $-\infty$ (B) ∞
(C) Does not exist **19.** (A) ∞ (B) ∞ (C) ∞ **20.** (A) $-\infty$ (B) $-\infty$
(C) $-\infty$ **21.** (A) 3 (B) 3 (C) 3 **22.** (A) $-\infty$ (B) ∞ (C) Does not exist
23. (A) $-\infty$ (B) ∞ (C) Does not exist **24.** (A) -3 (B) -3 (C) -3
25. (A) $-5x^3$ (B) $-\infty$ (C) ∞ **26.** (A) $-x^6$ (B) $-\infty$ (C) $-\infty$
27. (A) $-6x^4$ (B) $-\infty$ (C) $-\infty$ **28.** (A) $-x^5$ (B) $-\infty$ (C) ∞
29. (A) x^2 (B) ∞ (C) ∞ **30.** (A) x^3 (B) ∞ (C) $-\infty$
31. (A) $2x^5$ (B) ∞ (C) $-\infty$ **32.** (A) $4x^4$ (B) ∞ (C) ∞
33. (A) $\frac{47}{41} \approx 1.146$ (B) $\frac{407}{491} \approx 0.829$ (C) $\frac{4}{5} = 0.8$
34. (A) $-\frac{373}{507} \approx -0.736$ (B) $-\frac{2{,}998}{4{,}007} \approx -0.748$ (C) $-\frac{3}{4} = -0.75$
35. (A) $\frac{2{,}011}{138} \approx 14.572$ (B) $\frac{12{,}511}{348} \approx 35.951$ (C) ∞

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36. (A) $\frac{29}{3{,}586} \approx 0.008$ (B) $\frac{69}{28{,}674} \approx 0.002$ (C) 0

37. (A) $-\frac{8{,}568}{46{,}653} \approx -0.184$ (B) $-\frac{143{,}136}{1{,}492{,}989} \approx -0.096$ (C) 0

38. (A) $-\frac{8{,}724}{567} \approx -15.386$ (B) $-\frac{1{,}119{,}696}{8{,}100} \approx -138.234$ (C) $-\infty$

39. (A) $-\frac{7{,}010}{996} \approx -7.038$ (B) $-\frac{56{,}010}{7{,}996} \approx -7.005$ (C) -7

40. (A) $\frac{47}{195} \approx 0.241$ (B) $\frac{97}{395} \approx 0.246$ (C) $\frac{1}{4} = 0.25$

41. $\lim_{x\to 2^-} f(x) = -\infty$; $\lim_{x\to 2^+} f(x) = \infty$; $x = 2$ is a vertical asymptote

42. $\lim_{x\to 5^-} f(x) = -\infty$; $\lim_{x\to 5^+} f(x) = \infty$; $x = 5$ is a vertical asymptote

43. $\lim_{x\to -1} f(x) = -0.5$; $\lim_{x\to 1^-} f(x) = -\infty$; $\lim_{x\to 1^+} f(x) = \infty$; $x = 1$ is a vertical asymptote **44.** No zeros of denominator; no vertical asymptotes

45. No zeros of denominator; no vertical asymptotes

46. $\lim_{x\to -4^-} f(x) = -\infty$; $\lim_{x\to -4^+} f(x) = \infty$; $\lim_{x\to 4^-} f(x) = \infty$; $\lim_{x\to 4^+} f(x) = -\infty$; $x = -4$ and $x = 4$ are vertical asymptotes

47. $\lim_{x\to -2^-} f(x) = -\infty$; $\lim_{x\to -2^+} f(x) = \infty$; $\lim_{x\to 5^-} f(x) = \infty$; $\lim_{x\to 5^+} f(x) = -\infty$; $x = -2$ and $x = 5$ are vertical asymptotes

48. $\lim_{x\to -2^-} f(x) = -\infty$; $\lim_{x\to -2^+} f(x) = \infty$; $\lim_{x\to -1} f(x) = 2$; $\lim_{x\to 0^-} f(x) = \infty$; $\lim_{x\to 0^+} f(x) = -\infty$; $x = -2$ and $x = 0$ are vertical asymptotes

49. $\lim_{x\to -2} f(x) = -\frac{2}{3}$; $\lim_{x\to 0^-} f(x) = -\infty$; $\lim_{x\to 0^+} f(x) = \infty$; $\lim_{x\to 1^-} f(x) = \infty$; $\lim_{x\to 1^+} f(x) = -\infty$; $x = 0$ and $x = 1$ are vertical asymptotes

50. $\lim_{x\to -4^-} f(x) = -\infty$; $\lim_{x\to -4^+} f(x) = \infty$; $\lim_{x\to 2} f(x) = \infty$; $\lim_{x\to 2^-} f(x) = -\infty$; $x = -4$ and $x = 2$ are vertical asymptotes

51. Horizontal asymptote: $y = 2$; vertical asymptote: $x = -2$

52. Horizontal asymptote: $y = 3$; vertical asymptote: $x = 4$

53. Horizontal asymptote: $y = 1$; vertical asymptotes: $x = -1$ and $x = 1$

54. Horizontal asymptote: $y = 1$; no vertical asymptotes

55. No horizontal asymptotes; no vertical asymptotes

56. Horizontal asymptote: $y = 0$; vertical asymptotes: $x = -2$, $x = 2$

57. Horizontal asymptote: $y = 0$; no vertical asymptotes

58. No horizontal asymptotes; vertical asymptote: $x = 0$

59. No horizontal asymptotes; vertical asymptote: $x = 3$

60. Horizontal asymptote: $y = 0$; vertical asymptote: $x = 0$

61. Horizontal asymptote: $y = 2$; vertical asymptotes: $x = -1$ and $x = 2$

62. Horizontal asymptote: $y = 1$; vertical asymptotes: $x = -4$ and $x = \frac{3}{2}$

63. Horizontal asymptote: $y = 2$; vertical asymptote: $x = -1$

64. Horizontal asymptote: $y = \frac{1}{2}$; vertical asymptotes: $x = -4$ and $x = \frac{3}{2}$

65. $\lim_{x\to\infty} f(x) = 0$ **66.** $\lim_{x\to\infty} f(x) = -\infty$ **67.** $\lim_{x\to\infty} f(x) = \infty$

68. $\lim_{x\to\infty} f(x) = \frac{4}{5}$ **69.** $\lim_{x\to -\infty} f(x) = -\frac{1}{4}$ **70.** $\lim_{x\to -\infty} f(x) = 0$

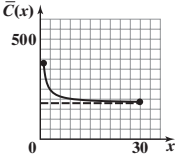
71. $\lim_{x\to -\infty} f(x) = -\infty$ **72.** $\lim_{x\to -\infty} f(x) = \infty$ **73.** False **74.** False

75. False **76.** True **77.** True **78.** False

79. If $n \geq 1$ and $a_n > 0$, then the limit is ∞. If $n \geq 1$ and $a_n < 0$, then the limit is $-\infty$.

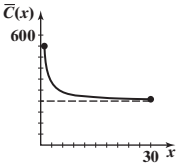
80. If $n \geq 1$ is odd and $a_n < 0$, or $n \geq 1$ is even and $a_n > 0$, then the limit is ∞. If $n \geq 1$ is odd and $a_n > 0$, or $n \geq 1$ is even and $a_n < 0$, then the limit is $-\infty$.

81. (A) $C(x) = 180x + 200$ (B) $\overline{C}(x) = \frac{180x + 200}{x}$

(C) 

(D) \$180 per board

82. (A) $C(x) = 240x + 300$ (B) $\overline{C}(x) = \frac{240x + 300}{x}$

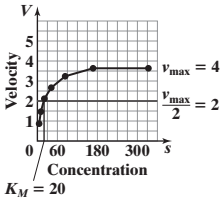
(C) 

(D) \$240 per board

83. (A) 20%; 50%; 80% (B) $P(t) \to 100\%$ **84.** (A) 33%; 66%; 88% (B) $P(t) \to 99\%$ **85.** The long-term drug concentration is 5 mg/ml.

86. The long-term drug concentration is 0 mg/ml. **87.** (A) \$18 million (B) \$38 million (C) $\lim_{x\to 1^-} P(x) = \infty$ **88.** (A) 40 components/day (B) 21 days (C) $\lim_{x\to\infty} N(t) = 100$

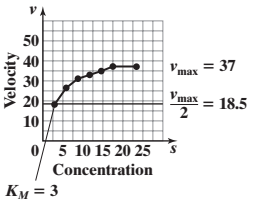
89. (C) $V_{\max} = 4$, $K_M = 20$



(D) $v(s) = \frac{4s}{20 + s}$

(E) $v = \frac{12}{7}$ when $s = 15$; $s = 60$ when $v = 3$

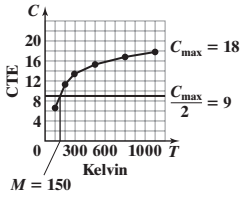
90. (A) $V_{\max} = 37$, $K_M = 3$



(B) $v(s) = \frac{37s}{3 + s}$

(C) $v = 27.75$ when s is 9; $s = 19.2$ when $v = 32$

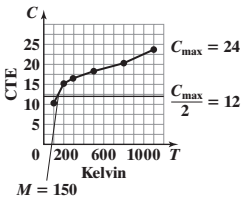
91. (A) $C_{\max} = 18$, $M = 150$



(B) $C(T) = \frac{18T}{150 + T}$

(C) $C = 14.4$ when $T = 600$ K; $T = 300$ K when $C = 12$

92. (A) $C_{\max} = 24$, $M = 150$



(B) $C(T) = \frac{24T}{150 + T}$

(C) $C = 19.2$ at $T = 600$ K; $T = 150$ K when $C = 12$

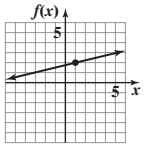
Exercises 2.3

1. $[-3, 5]$ **2.** $(-8, -4]$ **3.** $(-10, 100)$ **4.** $[0.1, 0.3]$

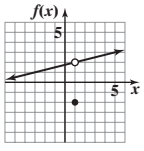
5. $(-\infty, -5) \cup (5, \infty)$ **6.** $(-\infty, -4] \cup [4, \infty)$

7. $(-\infty, -1) \cup (2, \infty)$ **8.** $(-\infty, 6) \cup [9, \infty)$

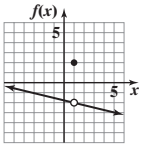
9. f is continuous at $x = 1$, since $\lim_{x\to 1} f(x) = f(1)$.



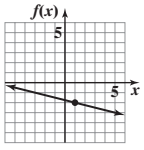
10. f is discontinuous at $x = 1$, since $\lim_{x\to 1} f(x) \neq f(1)$



11. f is discontinuous at $x = 1$, since $\lim_{x\to 1} f(x) \neq f(1)$.



12. f is continuous at $x = 1$, since $\lim_{x\to 1} f(x) = f(1)$



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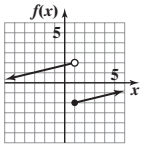
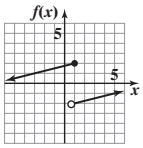
13. f is discontinuous at $x = 1$, since $\lim_{x\to 1} f(x)$ does not exist.

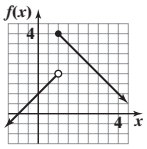


14. f is discontinuous at $x = 1$, since $\lim_{x\to 1} f(x)$ does not exist

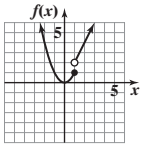


15. 1.9 **16.** 1 **17.** 0.1 **18.** 0.9 **19.** (A) 2 (B) 1 (C) Does not exist (D) 1 (E) No **20.** (A) 2 (B) 2 (C) 2 (D) Does not exist (E) No **21.** (A) 1 (B) 1 (C) 1 (D) 3 (E) No **22.** (A) 0 (B) 0 (C) 0 (D) 0 (E) Yes **23.** -0.1 **24.** 0.9 **25.** 0.1 **26.** 2.95 **27.** (A) 0 (B) 0 (C) 0 (D) 2 (E) No **28.** (A) 1 (B) 3 (C) Does not exist (D) Does not exist (E) No **29.** (A) 1 (B) -2 (C) Does not exist (D) 1 (E) No **30.** (A) 0 (B) 0 (C) 0 (D) 0 (E) Yes **31.** All x **32.** All x **33.** All x, except $x = -2$ **34.** All x, except $x = 4$ **35.** All x, except $x = -4$ and $x = 1$ **36.** All x, except $x = 3$ and $x = -1$ **37.** All x **38.** All x **39.** All x, except $x = \pm\frac{3}{2}$ **40.** All x, except $x = \pm\frac{2}{5}$ **41.** $-\frac{8}{3}, 4$ **42.** $-\frac{7}{2}, \frac{1}{5}$ **43.** $-1, 1$ **44.** $-3, 3$ **45.** $-9, -6, 0, 5$ **46.** $-6, 0, 7$ **47.** $-3 < x < 4$; $(-3, 4)$ **48.** $-2 < x < 4$; $(-2, 4)$ **49.** $x < 3$ or $x > 7$; $(-\infty, 3) \cup (7, \infty)$ **50.** $x < -5$ or $x > -2$; $(-\infty, -5) < (-2, \infty)$ **51.** $x < -2$ or $0 < x < 2$; $(-\infty, -2) \cup (0, 2)$ **52.** $x < -3$ or $x > 3$; $(-\infty, -3) < (3, \infty)$ **53.** $-5 < x < 0$ or $x > 3$; $(-5, 0) \cup (3, \infty)$ **54.** $x < -2$ or $0 < x < 4$; $(-\infty, -2) < (0, 4)$ **55.** (A) $(-4, -2) \cup (0, 2) \cup (4, \infty)$ (B) $(-\infty, -4) \cup (-2, 0) \cup (2, 4)$ **56.** (A) $(-\infty, -4) < (4, \infty)$ (B) $(-4, 1) < (1, 4)$ **57.** (A) $(-\infty, -2.5308) \cup (-0.7198, \infty)$ (B) $(-2.5308, -0.7198)$ **58.** (A) $(-\infty, 0.5113) \cup (2.1209, \infty)$ (B) $(0.5113, 2.1209)$ **59.** (A) $(-\infty, -2.1451) \cup (-1, -0.5240) \cup (1, 2.6691)$ (B) $(-2.1451, -1) \cup (-0.5240, 1) \cup (2.6691, \infty)$ **60.** (A) $(-2.3301, -1) \cup (0.2016, 1) \cup (2.1284, \infty)$ (B) $(-\infty, -2.3301) \cup (-1, 0.2016) \cup (1, 2.1284)$ **61.** $[6, \infty)$ **62.** $(-\infty, 7]$ **63.** $(-\infty, \infty)$ **64.** $(-\infty, \infty)$ **65.** $(-\infty, -3] \cup [3, \infty)$ **66.** $[-2, 2]$ **67.** $(-\infty, \infty)$ **68.** $(-\infty, \infty)$

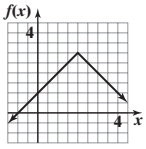
69. Since $\lim_{x\to 1^-} f(x) = 2$ and $\lim_{x\to 1^+} f(x) = 4$, $\lim_{x\to 1} f(x)$ does not exist and f is not continuous at $x = 1$.



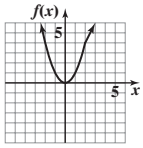
70. Since $\lim_{x\to 1^-} f(x) = 1$ and $\lim_{x\to 1^+} f(x) = 2$, $\lim_{x\to 1} f(x)$ does not exist and f is not continuous at $x = 1$.



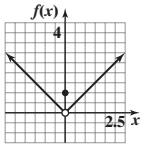
71. This function is continuous for all x.



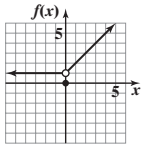
72. This function is continuous for all x.



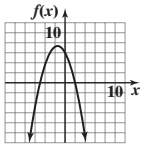
73. Since $\lim_{x\to 0} f(x) = 0$ and $f(0) = 1$, $\lim_{x\to 0} f(x) \neq f(0)$ and f is not continuous at $x = 0$.



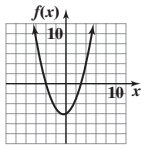
74. Since $\lim_{x\to 0} f(x) = 1 \neq f(0) = 0$, f is not continuous at $x = 0$.



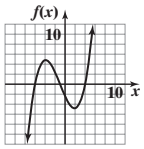
75. (A) Yes (B) No (C) Yes(D) No (E) Yes
76. (A) Yes (B) No (C) Yes (D) No (E) Yes
77. True **78.** True **79.** False **80.** True **81.** True **82.** False
83. x int.: $-5, 2$



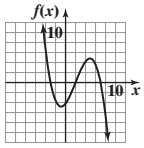
84. x int.: $x = -4, 3$



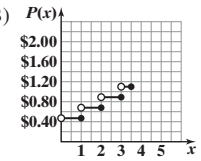
85. x int.: $x = -6, -1, 4$



86. x int.: $x = -3, 2, 7$

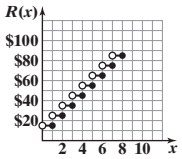


87. No, but this does not contradict Theorem 2, since f is discontinuous at $x = 1$.
88. No, but this does not contradict Theorem 2 since f is discontinuous at $x = 4$.
89. (A)
$$P(x) = \begin{cases} 0.47 & \text{if } 0 < x \le 1 \\ 0.68 & \text{if } 1 < x \le 2 \\ 0.89 & \text{if } 2 < x \le 3 \\ 1.10 & \text{if } 3 < x \le 3.5 \end{cases}$$
(B)



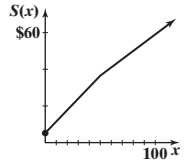
(C) Yes; no

90. (A)
$$R(x) = \begin{cases} 15 & \text{if } 0 < x \le 1 \\ 25 & \text{if } 1 < x \le 2 \\ 35 & \text{if } 2 < x \le 3 \\ 45 & \text{if } 3 < x \le 4 \\ 55 & \text{if } 4 < x \le 5 \\ 65 & \text{if } 5 < x \le 6 \\ 75 & \text{if } 6 < x \le 7 \\ 85 & \text{if } 7 < x \le 8 \end{cases}$$
(B)



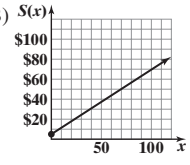
(C) Yes; no

93. (A)
$$S(x) = \begin{cases} 5 + 0.63x & \text{if } 0 \le x \le 50 \\ 14 + 0.45x & \text{if } 50 < x \end{cases}$$
(B)



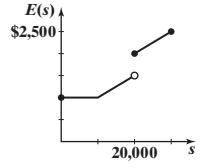
(C) Yes

94. (A)
$$S(x) = \begin{cases} 5 + 0.69x & \text{if } 0 \le x < 5 \\ 5.2 + 0.65x & \text{if } 5 < x \le 50 \\ 6.2 + 0.63x & \text{if } 50 < x \end{cases}$$
(B)



(C) Yes; yes

95. (A)



(B) $\lim_{x\to 10{,}000} E(s) = \$1{,}000$; $E(10{,}000) = \$1{,}000$
(C) $\lim_{x\to 20{,}000} E(s)$ does not exist; $E(20{,}000) = \$2{,}000$
(D) Yes; no

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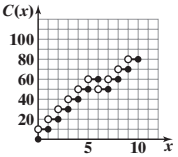
96. (A) 

(B) $\lim_{x\to 4.5} C(x) = 50$; $C(4.5) = 50$

(C) $\lim_{x\to 8} C(x)$ does not exist; $C(8) = 60$

(D) Yes; no

97. (A) t_2, t_3, t_4, t_6, t_7

(B) $\lim_{t\to t_5} N(t) = 7$; $N(t_5) = 7$

(C) $\lim_{t\to t_3} N(t)$ does not exist; $N(t_3) = 4$

98. (A) t_2, t_4

(B) $\lim_{t\to t_1} p(t) = 10$; $p(t_1) = 10$

(C) $\lim_{t\to t_2} p(t) = 30$; $p(t_2) = 10$

(D) $\lim_{t\to t_4} p(t)$ does not exist; $p(t_4) = 80$

Exercises 2.4

1. $\frac{9}{4} = 2.25$ **2.** $-\frac{3}{2} = 21.5$ **3.** $-\frac{27}{5} = -5.4$ **4.** $\frac{3}{8} = 0.375$

5. $\frac{1}{3}\sqrt{3}$ **6.** $\frac{2}{5}\sqrt{5}$ **7.** $\frac{15}{2} - \frac{5}{2}\sqrt{7}$ **8.** $\frac{7}{23} - \frac{6}{23}\sqrt{2}$

9. (A) -3; slope of the secant line through $(1, f(1))$ and $(2, f(2))$
(B) $-2 - h$; slope of the secant line through $(1, f(1))$ and $(1 + h, f(1 + h))$ (C) -2; slope of the tangent line at $(1, f(1))$

10. (A) 3; slope of the secant line through $(-2, f(-2))$ and $(-1, f(-1))$
(B) $4 - h$; slope of the secant line through $(-2, f(-2))$ and $(-2 + h, f(-2 + h))$ (C) 4; slope of the tangent line at $(-2, f(-2))$

11. (A) 15 (B) $6 + 3h$ (C) 6 **12.** (A) 21 (B) $12 + 3h$ (C) 12

13. (A) 40 km/hr (B) 40 (C) $y - 80 = 45(x - 2)$ or $y = 45x - 10$ **14.** (A) 132 mi/hr (B) 132 (C) $y - 528 = 150(x - 4)$ or $y = 150x - 72$ **15.** $y - \frac{1}{2} = -\frac{1}{2}(x - 1)$ or $y = -\frac{x}{2} + 1$

16. $y - 0.2 = -0.16(x - 2)$ or $y = -0.16x + 0.52$

17. $y - 16 = -32(x + 2)$ or $y = -32x - 48$

18. $y - 1 = -4(x + 1)$ or $y = -4x - 3$

19. $f'(x) = 0$; $f'(1) = 0$, $f'(2) = 0$, $f'(3) = 0$

20. $f'(x) = 0$; $f'(1) = 0$, $f'(2) = 0$, $f'(3) = 0$

21. $f'(x) = 3$; $f'(1) = 3$, $f'(2) = 3$, $f'(3) = 3$

22. $f'(x) = -6$; $f'(1) = -6$, $f'(2) = -6$, $f'(3) = -6$

23. $f'(x) = -6x$; $f'(1) = -6$, $f'(2) = -12$, $f'(3) = -18$

24. $f'(x) = 4x$; $f'(1) = 4$, $f'(2) = 8$, $f'(3) = 12$

25. $f'(x) = 2x - 2$; $f'(1) = 0$, $f'(2) = 2$, $f'(3) = 4$

26. $f'(x) = 6x + 2$; $f'(1) = 8$; $f'(2) = 14$; $f'(3) = 20$

27. $f'(x) = 8x + 3$; $f'(1) = 11$, $f'(2) = 19$, $f'(3) = 27$

28. $f'(x) = 2x - 4$; $f'(1) = -2$; $f'(2) = 0$; $f'(3) = 2$

29. $f'(x) = -2x + 5$; $f'(1) = 3$, $f'(2) = 1$, $f'(3) = -1$

30. $f'(x) = 12x - 3$; $f'(1) = 9$; $f'(2) = 21$; $f'(3) = 33$

31. $f'(x) = 20x - 9$; $f'(1) = 11$, $f'(2) = 31$, $f'(3) = 51$

32. $f'(x) = -2x + 3$; $f'(1) = 1$; $f'(2) = -1$; $f'(3) = -3$

33. $f'(x) = 6x^2$; $f'(1) = 6$, $f'(2) = 24$, $f'(3) = 54$

34. $f'(x) = -6x^2$; $f'(1) = -6$, $f'(2) = -24$, $f'(3) = -54$

35. $f'(x) = -\frac{4}{x^2}$; $f'(1) = -4$, $f'(2) = -1$, $f'(3) = -\frac{4}{9}$

36. $f'(x) = -\frac{6}{x^2}$; $f'(1) = -6$, $f'(2) = -\frac{3}{2}$, $f'(3) = -\frac{2}{3}$

37. $f'(x) = \frac{3}{2\sqrt{x}}$; $f'(1) = \frac{3}{2}$, $f'(2) = \frac{3}{2\sqrt{2}}$ or $\frac{3\sqrt{2}}{4}$, $f'(3) = \frac{3}{2\sqrt{3}}$ or $\frac{\sqrt{3}}{2}$ **38.** $f'(x) = \frac{-7}{2\sqrt{x}}$; $f'(1) = -\frac{7}{2}$, $f'(2) = -\frac{7}{2\sqrt{2}}$ or $-\frac{7\sqrt{2}}{4}$, $f'(3) = -\frac{7}{2\sqrt{3}}$ or $-\frac{7\sqrt{3}}{6}$ **39.** $f'(x) = \frac{5}{\sqrt{x+5}}$; $f'(1) = \frac{5}{\sqrt{6}}$ or $\frac{5\sqrt{6}}{6}$, $f'(2) = \frac{5}{\sqrt{7}}$ or $\frac{5\sqrt{7}}{7}$, $f'(3) = \frac{5}{2\sqrt{2}}$ or $\frac{5\sqrt{2}}{4}$ **40.** $f'(x) = \frac{8}{\sqrt{x+9}}$; $f'(1) = \frac{8}{\sqrt{10}}$ or $\frac{4\sqrt{10}}{5}$, $f'(2) = \frac{8}{\sqrt{11}}$ or $\frac{8\sqrt{11}}{11}$, $f'(3) = \frac{8}{\sqrt{12}}$ or $\frac{4\sqrt{3}}{3}$ **41.** $f'(x) = -\frac{1}{(x-4)^2}$; $f'(1) = -\frac{1}{9}$; $f'(2) = -\frac{1}{4}$; $f'(3) = -1$

42. $f'(x) = -\frac{1}{(x+4)^2}$; $f'(1) = -\frac{1}{25}$; $f'(2) = -\frac{1}{36}$; $f'(3) = -\frac{1}{49}$

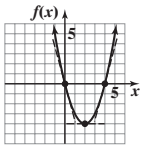
43. $f'(x) = \frac{1}{(x+1)^2}$; $f'(1) = \frac{1}{4}$; $f'(2) = \frac{1}{9}$; $f'(3) = \frac{1}{16}$

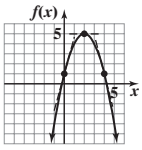
44. $f'(x) = \frac{2}{(x+2)^2}$; $f'(1) = \frac{2}{9}$; $f'(2) = \frac{1}{8}$; $f'(3) = \frac{2}{25}$

45. (A) 5 (B) $3 + h$ (C) 3 (D) $y = 3x - 1$ **46.** (A) 7 (B) $5 + h$ (C) 5 (D) $y = 5x - 4$ **47.** (A) 5 m/s (B) $3 + h$ m/s (C) 3 m/s

48. (A) 7 m/s (B) $5 + h$ m/s (C) 5 m/s **49.** Yes **50.** No **51.** No **52.** Yes

53. Yes **54.** No **55.** Yes **56.** No

57. (A) $f'(x) = 2x - 4$
(B) $-4, 0, 4$
(C) 

58. (A) $f'(x) = 4 - 2x$
(B) $4, 0, -4$
(C) 

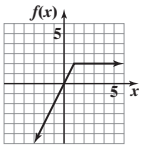
59. $v = f'(x) = 8x - 2$; 6 ft/s, 22 ft/s, 38 ft/s

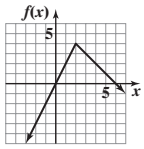
60. $v = f'(x) = 16x - 4$; 12 ft/s, 44 ft/s, 76 ft/s

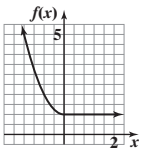
61. (A) The graphs of g and h are vertical translations of the graph of f. All three functions should have the same derivative. (B) $2x$

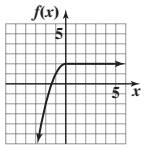
62. (A) The graphs of g and h are vertical translations of the graph of f. All three functions should have the same derivative. (B) $-2x$

63. True **64.** True **65.** False **66.** True **67.** False **68.** False

69. f is nondifferentiable at $x = 1$


70. f is not differentiable at $x = 2$


71. f is differentiable for all real numbers


72. f is differentiable for all real numbers


73. No **74.** No **75.** No **76.** No **77.** $f'(0) = 0$

78. $f'(0) = 0$ **79.** 6 s; 192 ft/s **80.** 8 s; 256 ft/s **81.** (A) \$8.75 (B) $R'(x) = 60 - 0.05x$ (C) $R(1{,}000) = 35{,}000$; $R'(1{,}000) = 10$; At a production level of 1,000 car seats, the revenue is \$35,000 and is increasing at the rate of \$10 per seat.

82. (A) \$3.75 (B) $P'(x) = 45 - 0.05x$ (C) $P(800) = 15{,}000$; $P'(800) = 5$; At a production level of 800 car seats, the revenue is \$15,000 and is increasing at the rate of \$5 per seat. **83.** (A) $S'(t) = 1/(2\sqrt{t})$ (B) $S(4) = 6$; $S'(4) = 0.25$; After 4 months, the total sales are \$6 million and are increasing at the rate of \$0.25 million per month. (C) \$6.25 million; \$6.5 million **84.** (A) $S'(t) = \frac{1}{2\sqrt{t}}$ (B) $S(9) = 11$; $S'(9) = 1/6$; After 9 months, the total sales are \$11 million and are increasing at the rate of \$0.167 million per month. (C) \$11.167 million; \$11.333 million

85. (A) $p'(t) = 276t + 1{,}072$ (B) $p(15) = 62{,}047$, $p'(15) = 5{,}212$; In 2025, 62,047 metric tons of tungsten are consumed and this quantity is increasing at the rate of 5,212 metric tons per year.

86. (A) $p'(t) = 96t - 37$ (B) $p(17) = 14{,}941$; $p'(17) = 1{,}595$; In 2027, 14,941 thousand metric tons of copper are consumed and this quantity is increasing at the rate of 1,595 thousand metric tons per year.

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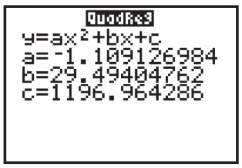
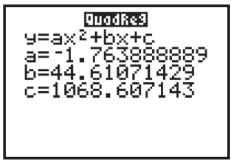
87. (A)



(B) $R(30) \approx 1083.6$ billion kilowatts, $R'(30) \approx -37.1$ billion kilowatts per year. In 2030, 1083.6 billion kilowatts will be sold and the amount sold is decreasing at the rate of 37.1 billion kilowatts per year.

88. (A)



(B) $C(30) \approx 819.4$, $C'(30) \approx -61.2$ In 2030, 819.4 billion kilowatts will be sold and the amount sold is decreasing at the rate of 61.2 billion kilowatts per year.

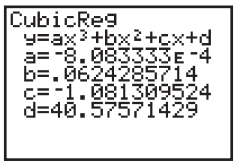
89. (A) $P'(t) = 12 - 2t$ (B) $P(3) = 107$; $P'(3) = 6$. After 3 hours, the ozone level is 107 ppb and is increasing at the rate of 6 ppb per hour.

90. (A) $F'(t) = -\dfrac{4}{(t+1)^2}$ (B) $F(3) = 99$; $F'(3) = -0.25$. After 3 hours, the body temperature is 99°F and is decreasing at the rate of 0.25°F per hour.

Exercises 2.5

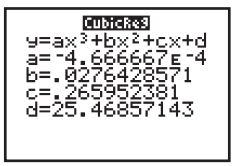
1. $x^{1/2}$ **2.** $x^{1/3}$ **3.** x^{-5} **4.** x^{-1} **5.** x^{12} **6.** x^{-10} **7.** $x^{-1/4}$ **8.** $x^{-1/5}$
9. 0 **10.** 0 **11.** $7x^6$ **12.** $8x^7$ **13.** $4x^3$ **14.** $9x^8$ **15.** $-3x^{-4}$ **16.** $-5x^{-6}$
17. $\dfrac{4}{3}x^{1/3}$ **18.** $\dfrac{5}{2}x^{3/2}$ **19.** $-\dfrac{9}{x^{10}}$ **20.** $-\dfrac{7}{x^8}$ **21.** $6x^2$ **22.** $-6x$
23. $1.8x^5$ **24.** $2.1x^2$ **25.** $\dfrac{x^3}{3}$ **26.** $\dfrac{x^2}{3}$ **27.** 12 **28.** -5 **29.** 2 **30.** -4
31. 9 **32.** -17 **33.** 2 **34.** -4 **35.** $4t - 3$ **36.** $5 - 24t^2$
37. $-10x^{-3} - 9x^{-2}$ **38.** $-35x^{-8} + 8x^{-5}$ **39.** $1.5u^{-0.7} - 8.8u^{1.2}$
40. $9u^{3.5} - 3.1$ **41.** $0.5 - 3.3t^2$ **42.** $0.6t^2 - 3.1$ **43.** $-\dfrac{8}{5}x^{-5}$
44. $-\dfrac{14}{5u^3}$ **45.** $3x + \dfrac{14}{5}x^{-3}$ **46.** $\dfrac{15x^2}{4} + \dfrac{6}{5x^4}$ **47.** $-\dfrac{20}{9}\omega^{-5} + \dfrac{5}{3}\omega^{-2/3}$
48. $-\dfrac{30}{w^7} - \dfrac{1}{\sqrt{2}}$ **49.** $2u^{-1/3} - \dfrac{5}{3}u^{-2/3}$ **50.** $6u^{-1/4} - u^{-5/4}$
51. $-\dfrac{9}{5}t^{-8/5} + 3t^{-3/2}$ **52.** $-t^{-6/5} + 12t^{-5/2}$ **53.** $-\dfrac{1}{3}x^{-4/3}$ **54.** $-2u^{-6/5}$
55. $-0.6x^{-3/2} + 6.4x^{-3} + 1$ **56.** $-8.4x^{-4} + 0.4x^{-5/3}$
57. (A) $f'(x) = 6 - 2x$ (B) $f'(2) = 2$; $f'(4) = -2$ (C) $y = 2x + 4$; $y = -2x + 16$ (D) $x = 3$ **58.** (A) $f'(x) = 4x + 8$ (B) $f'(2) = 16$; $f'(4) = 24$ (C) $y = 16x - 8$; $y = 24x - 32$ (D) $x = 22$
59. (A) $f'(x) = 12x^3 - 12x$ (B) $f'(2) = 72$; $f'(4) = 720$ (C) $y = 72x - 127$; $y = 720x - 2{,}215$ (D) $x = -1, 0, 1$
60. (A) $f'(x) = 4x^3 - 64x$ (B) $f'(2) = -96$; $f'(4) = 0$ (C) $y = -96x + 90$; $y = -246$ (D) $x = -4, 0, 4$
61. (A) $v = f'(x) = 176 - 32x$ (B) $f'(0) = 176$ ft/s; $f'(3) = 80$ ft/s (C) 5.5 s **62.** (A) $v = f'(x) = 80 - 20x$ (B) $f'(0) = 80$ ft/s; $f'(3) = 20$ ft/s (C) 4 s **63.** (A) $v = f'(x) = 3x^2 - 18x + 15$ (B) $f'(0) = 15$ ft/s; $f'(3) = -12$ ft/s (C) $x = 1$ s, $x = 5$ s
64. (A) $v = f'(x) = 3x^2 - 18x + 24$ (B) $f'(0) = 24$ ft/s; $f'(3) = -3$ ft/s (C) $x = 2$ s, $x = 4$ s
65. $f'(x) = 2x - 3 - 2x^{-1/2} = 2x - 3 - \dfrac{2}{x^{1/2}}$; $x = 2.1777$
66. $f'(x) = 2x + 1 - 5x^{-1/2} = 2x + 1 - \dfrac{5}{x^{1/2}}$; $x = 1.5247$
67. $f'(x) = 4\sqrt[3]{x} - 3x - 3$; $x = -2.9018$
68. $f'(x) = 4\sqrt[3]{x} - 4x + 4$; $x = 2.3247$
69. $f'(x) = 0.2x^3 + 0.3x^2 - 3x - 1.6$; $x = -4.4607, -0.5159, 3.4765$
70. $f'(x) = 0.08x^3 - 0.18x^2 - 1.56x + 0.94$; $x = -3.7626, 0.5742, 5.4384$
71. $f'(x) = 0.8x^3 - 9.36x^2 + 32.5x - 28.25$; $x = 1.3050$
72. $f'(x) = x^3 - 7.8x^2 - 16.2x - 10$; $x = 1.2391, 1.6400, 4.9209$
76. No **77.** $8x - 4$ **78.** $8x - 20$ **79.** $-20x^{-2}$ **80.** $-50x^{-3}$
81. $-\dfrac{1}{4}x^{-2} + \dfrac{2}{3}x^{-3}$ **82.** $4x - 4x^{-3}$ **83.** False **84.** False **85.** True
86. False **89.** (A) $S'(t) = 0.09t^2 + t + 2$ (B) $S(5) = 29.25$, $S'(5) = 9.25$. After 5 months, sales are \$29.25 million and are increasing at the rate of \$9.25 million per month. (C) $S(10) = 103$, $S'(10) = 21$. After 10 months, sales are \$103 million and are increasing at the rate of \$21 million per month. **90.** (A) $S'(t) = 0.06t^3 + 1.2t^2 + 6.8t + 10$ (B) $S(4) = 120.84$, $S'(4) = 60.24$. After 4 months, sales are \$120.84 million and are increasing at the rate of \$60.24 million per month. (C) $S(8) = 560.84$, $S'(8) = 171.92$. After 8 months, sales are \$560.84 million and are increasing at the rate of \$171.92 million per month. **91.** (A) $N'(x) = 3{,}780/x^2$ (B) $N'(10) = 37.8$. At the \$10,000 level of advertising, sales are increasing at the rate of 37.8 boats per \$1,000 spent on advertising. $N'(20) = 9.45$. At the \$20,000 level of advertising, sales are increasing at the rate of 9.45 boats per \$1,000 spent on advertising. **92.** $x(5) = 46$; $x'(5) = -7.2$. When the price is \$5 per quarter pound, the demand is 46 pounds and is decreasing at a rate of 7.2 pounds per dollar.

93. (A)



(B) In 2025, 35.5% of male high school graduates enroll in college and the percentage is decreasing at the rate of 1.5% per year.

94. (A)

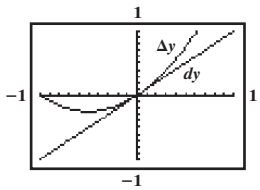


(B) In 2025, 46.1% of female high school graduates enroll in college and the percentage is decreasing at the rate of 0.9% per year.

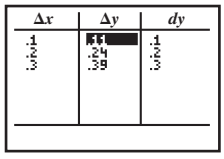
95. (A) -1.37 beats/min (B) -0.58 beats/min **96.** (A) -0.2 parts per million per mile (B) -0.025 parts per million per mile **97.** (A) 25 items/hr (B) 8.33 items/hr **98.** (A) 14 items per hour (B) 7 items per hour

Exercises 2.6

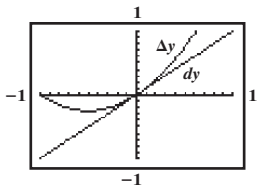
1. 3; 3.01 **2.** 3.7; 3.71 **3.** 2.8; 2.799 **4.** 2; 1.999 **5.** 0; 0.01
6. 1; 1.21 **7.** 100; 102.01 **8.** 25; 24.01 **9.** $\Delta x = 3$; $\Delta y = 75$; $\Delta y/\Delta x = 25$ **10.** $\Delta x = 3$; $\Delta y = 105$; $\Delta y/\Delta x = 35$ **11.** 20
12. 25 **13.** 20 **14.** 25 **15.** $dy = (24x - 3x^2)dx$
16. $dy = \left(200 - \dfrac{x}{15}\right)dx$ **17.** $dy = \left(2x - \dfrac{x^2}{3}\right)dx$
18. $dy = (180x^2 - 4x^3)dx$ **19.** $dy = -\dfrac{295}{x^{3/2}}dx$
20. $dy = (26x^{-1/2})dx$ **21.** (A) $12 + 3\Delta x$ (B) 12 **22.** (A) $18 + 3\Delta x$ (B) 18
23. $dy = 6(3x - 1)dx$ **24.** $dy = 4(2x + 3)dx$ **25.** $dy = \left(\dfrac{x^2 + 5}{x^2}\right)dx$
26. $dy = \dfrac{18}{x^3}dx$ **27.** $dy = 1.4$; $\Delta y = 1.44$ **28.** $dy = 3.6$; $\Delta y = 3.66$
29. $dy = -3$; $\Delta y = -\dfrac{10}{3}$ **30.** $dy = -20$; $\Delta y = -20.803$
31. 120 in.^{3} **32.** 31.4 cm^3
33. (A) $\Delta y = \Delta x + (\Delta x)^2$; $dy = \Delta x$
(B)



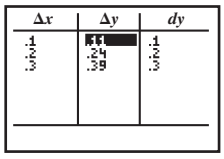
(C)



34. (A) $\Delta y = -2\Delta x + (\Delta x)^2$; $dy = -2dx$
(B)



(C)



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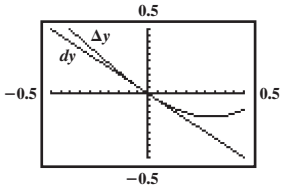
35. (A) $\Delta y = -\Delta x + (\Delta x)^2 + (\Delta x)^3$; $dy = -\Delta x$

(B)

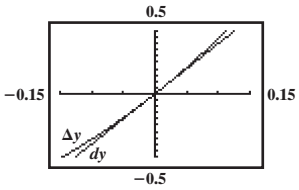


(C)

Δx	Δy	dy
.05	.0474	.05
.1	.089	.1
.15	.1241	.15

36. (A) $\Delta y = 4\Delta x + 4(\Delta x)^2 + (\Delta x)^3$; $dy = 4dx$

(B)



(C)

Δx	Δy	dy
.15	.5134	.6
.1	.384	.4
.05	.2406	.2

Y1=-.190125

37. True **38.** False **39.** False **40.** True

41. $dy = \left(\frac{2}{3}x^{-1/3} - \frac{10}{3}x^{2/3}\right)dx$ **42.** $dy = \left(\frac{2}{3}x^{-1/3} - \frac{10}{3}x^{2/3}\right)dx$

43. $dy = 3.9$; $\Delta y = 3.83$ **44.** $dy = -0.576$; $\Delta y = -0.57$

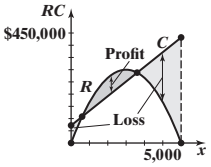
45. 40-unit increase; 20-unit increase **46.** 48-lb decrease **47.** −$2.50; $1.25 **48.** $1,000, $400; −$1,000, −$1,600 **49.** −1.37 beats/min; −0.58 beats/min **50.** 94.2 mm^3 **51.** 1.26 mm^2 **52.** (A) 0.27°F (B) 0.3°F (C) 0.27°F **53.** 3 wpm **54.** 2.6 item increase; 1.3 item increase **55.** (A) 2,100 increase (B) 4,800 increase (C) 2,100 increase

Exercises 2.7

1. $22,889.80 **2.** $23,000.00 **3.** $110.20 **4.** $31,929.80 **5.** $32,000.00 **6.** $70.20 **7.** $230.00 **8.** $160.00 **9.** $C'(x) = 0.7$ **10.** $C'(x) = 6$ **11.** $C'(x) = -0.2(0.1x - 23)$ **12.** $C'(x) = 12 - 0.2x$ **13.** $R'(x) = 4 - 0.02x$ **14.** $R'(x) = 36 - 0.06x$ **15.** $R'(x) = 12 - 0.08x$ **16.** $R'(x) = 25 - 0.1x$ **17.** $P'(x) = 3.3 - 0.02x$ **18.** $P'(x) = 30 - 0.06x$ **19.** $P'(x) = 7.4 - 0.06x$ **20.** $P'(x) = 13 + 0.1x$ **21.** $\bar{C}(x) = 1.1 + \frac{145}{x}$ **22.** $\bar{R}'(x) = 5 - 0.02x$ **23.** $\bar{C}'(x) = -\frac{145}{x^2}$ **24.** $\bar{R}'(x) = -0.02$ **25.** $P(x) = 3.9x - 0.02x^2 - 145$ **26.** $P'(x) = 3.9 - 0.04x$ **27.** $\bar{P}(x) = 3.9 - 0.02x - \frac{145}{x}$ **28.** $\bar{P}'(x) = 20.02 + \frac{145}{x^2}$ **29.** True **30.** True **31.** False **32.** False **33.** (A) $29.50 (B) $30 **34.** (A) $74.75 (B) $75 **35.** (A) $420 (B) $\bar{C}'(500) = -0.24$. At a production level of 500 frames, average cost is decreasing at the rate of 24¢ per frame. (C) Approximately $419.76 **36.** (A) $\bar{C}(1{,}000) = \$30$. (B) $\bar{C}'(1{,}000) = -1$¢. At a production level of 1,000 board games, average cost is decreasing at the rate of 1¢ per game. (C) Approximately $29.99 **37.** (A) $14.70 (B) $15 **38.** (A) $5.80 (B) $6 **39.** (A) $P'(450) = 0.5$. At a production level of 450 sweatshirts, profit is increasing at the rate of 50¢ per sweatshirt. (B) $P'(750) = -2.5$. At a production level of 750 sweatshirts, profit is decreasing at the rate of $2.50 per sweatshirt. **40.** (A) $P'(200) = \$4$. At a production level of 200 cameras, profit is increasing at the rate of $4 per camera. (B) $P'(350) = -\$2$. At a production level of 350 cameras, profit is decreasing at the rate of $2 per camera. **41.** (A) $13.50 (B) $\bar{P}'(50) = \$0.27$. At a production level of 50 mowers, the average profit per mower is increasing at the rate of $0.27 per mower. (C) Approximately $13.77 **42.** (A) $11.20 (B) $\bar{P}'(40) = \$0.18$. At a production level of 40 grills, the average profit per grill is increasing at the rate of $0.18 per grill. (C) Approximately $11.38 **43.** (A) $p = 100 - 0.025x$, domain: $0 \le x \le 4{,}000$ (B) $R(x) = 100x - 0.025x^2$, domain: $0 \le x \le 4{,}000$ (C) $R'(1{,}600) = 20$. At a production level of 1,600 pairs of running shoes, revenue is increasing at the rate of $20 per pair. (D) $R'(2{,}500) = -25$. At a production level of 2,500 pairs of running shoes, revenue is decreasing at the rate of $25 per pair.

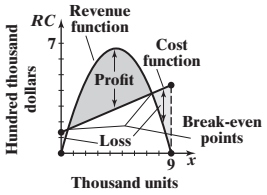
44. (A) $p = 50 - 0.05x$, domain: $0 \le x \le 1{,}000$ (B) $R(x) = 50x - 0.05x^2$, domain: $0 \le x \le 1{,}000$ (C) $R'(400) = 10$. At a production level of 400 steam irons, revenue is increasing at the rate of $10 per steam iron. (D) $R'(650) = -15$. At a production level of 650 steam irons, revenue is decreasing at the rate of $15 per steam iron.

45. (A) $p = 200 - \frac{1}{30}x$, domain: $0 \le x \le 6{,}000$ (B) $C'(x) = 60$ (C) $R(x) = 200x - (x^2/30)$, domain: $0 \le x \le 6{,}000$ (D) $R'(x) = 200 - (x/15)$ (E) $R'(1{,}500) = 100$. At a production level of 1,500 saws, revenue is increasing at the rate of $100 per saw. $R'(4{,}500) = -100$. At a production level of 4,500 saws, revenue is decreasing at the rate of $100 per saw. (F) Break-even points: (600, 108,000) and (3,600, 288,000)

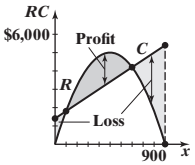


(G) $P(x) = -(x^2/30) + 140x - 72{,}000$ (H) $P'(x) = -(x/15) + 140$ (I) $P'(1{,}500) = 40$. At a production level of 1,500 saws, profit is increasing at the rate of $40 per saw. $P'(3{,}000) = -60$. At a production level of 3,000 saws, profit is decreasing at the rate of $60 per saw.

46. (A) $p = 300 - \frac{1}{30}x$, domain: $0 \le x \le 9{,}000$ (B) $C'(x) = 30$ (C) $R(x) = 300x - \frac{1}{30}x^2$, domain: $0 \le x \le 9{,}000$ (D) $R'(x) = 300 - \frac{1}{15}x$ (E) $R'(3{,}000) = 100$. At a production level of 3,000 TVs, revenue is increasing at the rate of $100 per TV. $R'(6000) = -100$. At a production level of 6,000 TVs, revenue is decreasing at the rate of $100 per TV. (F) Break-even points: (600, 168,000) and (7,500, 375,000)



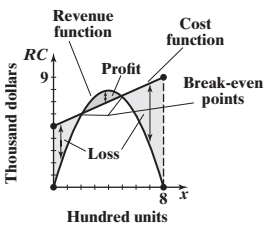
(G) $P(x) = -\frac{1}{30}x^2 + 270x - 150{,}000$ (H) $P'(x) = -\frac{1}{15}x + 270$ (I) $P'(1{,}500) = 170$. At a production level of 1,500 sets, profit is increasing at the rate of $170 per set. $P'(4{,}500) = -30$. At a production level of 4,500 sets, profit is decreasing at the rate of $30 per set. **47.** (A) $p = 20 - 0.02x$, domain: $0 \le x \le 1{,}000$ (B) $R(x) = 20x - 0.02x^2$, domain: $0 \le x \le 1{,}000$ (C) $C(x) = 4x + 1{,}400$



(D) Break-even points: (100, 1,800) and (700, 4,200)

(E) $P(x) = 16x - 0.02x^2 - 1{,}400$ (F) $P'(250) = 6$. At a production level of 250 toasters, profit is increasing at the rate of $6 per toaster. $P'(475) = -3$. At a production level of 475 toasters, profit is decreasing at the rate of $3 per toaster.

48. (A) $p = 40 - \frac{x}{20}$, domain: $0 \le x \le 800$ (B) $R(x) = 40x - \frac{x^2}{20}$, domain: $0 \le x \le 800$ (C) $C(x) = 5x + 5{,}000$ (D) Break-even points: (200, 6,000) and (500, 7,500)



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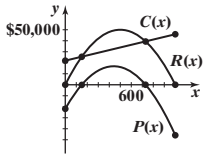
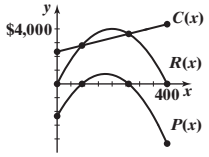
(E) $P(x) = 35x - \frac{x^2}{20} - 5{,}000$

(F) $P'(325) = 2.5$. At a production level of 325 toasters, profit is increasing at the rate of \$2.50 per toaster. $P'(425) = -7.5$. At a production level of 425 toasters, profit is decreasing at the rate of \$7.50 per toaster.

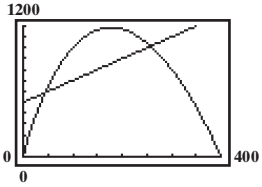
49. (A) $x = 500$ (B) $P(x) = 176x - 0.2x^2 - 21{,}900$ (C) $x = 440$
(D) Break-even points: $(150, 25, 500)$ and $(730, 39, 420)$; x intercepts for $P(x)$: $x = 150$ and $x = 730$



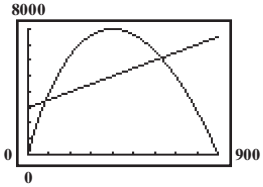
50. (A) $x = 200$ (B) $P(x) = 35x - 0.1x^2 - 2{,}340$ (C) $x = 175$
(D) Break-even points: $(90, 2{,}790)$ and $(260, 3{,}640)$; x intercepts for $P(x)$: $x = 90$ and $x = 260$



51. (A) $R(x) = 20x - x^{3/2}$
(B) Break-even points: $(44,588)$, $(258,1{,}016)$



52. (A) $R(x) = 60x - 2x^{3/2}$
(B) Break-even points: $(81, 3{,}405)$, $(631, 6{,}155)$



53. (A)

```
QuadReg
 y=ax²+bx+c
 a=1.4101002E-5
 b=-.2732556676
 c=1320.924694
```

(B) Fixed costs $\approx$ \$721,680; variable costs $\approx$ \$121

```
LinReg
 y=ax+b
 a=120.7047281
 b=721680.1282
 r=.9934384133
```

(C) $(713, 807{,}703)$, $(5{,}423, 1{,}376{,}227)$ (D) $\$254 \le p \le \$1{,}133$

54. (A)

```
LinReg
 y=ax+b
 a=-.1985715253
 b=1996.678966
 r=-.982877241
```

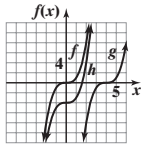
(B) Fixed costs: \$2,832,085; variable costs: \$292

```
LinReg
 y=ax+b
 a=292.126464
 b=2832084.659
 r=.9956751513
```

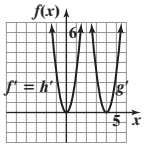
(C) Break-even points: $(2{,}253, 3{,}490{,}130)$, $(6{,}331, 4{,}681{,}675)$
(D) $\$740 \le p \le \$1{,}549$

Chapter 2 Review Exercises

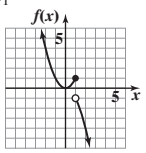
1. (A) 16 (B) 8 (C) 8 (D) 4 (E) 4 (F) 4 *(2.2)* **2.** $f'(x) = -3$ *(2.2)*
3. (A) 22 (B) 8 (C) 2 (D) -5 *(2.1)* **4.** 1.5 *(2.1)* **5.** 3.5 *(2.1)*
6. 3.75 *(2.1)* **7.** 3.75 *(2.1)* **8.** (A) 1 (B) 1 (C) 1 (D) 1 *(2.1)*
9. (A) 2 (B) 3 (C) Does not exist (D) 3 *(2.1)* **10.** (A) 4 (B) 4
(C) 4 (D) Does not exist *(2.1)* **11.** (A) Does not exist (B) 3
(C) No *(2.3)* **12.** (A) 2 (B) Not defined (C) No *(2.3)* **13.** (A) 1
(B) 1 (C) Yes *(2.3)* **14.** 10 *(2.2)* **15.** 5 *(2.2)* **16.** ∞ *(2.2)* **17.** $-\infty$ *(2.2)*
18. ∞ *(2.2)* **19.** ∞ *(2.2)* **20.** ∞ *(2.2)* **21.** $x = 2$; $x = 6$ *(2.2)*
22. $y = 5$; $y = 10$ *(2.2)* **23.** $x = 2$; $x = 6$ *(2.3)* **24.** $f'(x) = 6x$ *(2.4)*
25. (A) -3 (B) 6 (C) -2 (D) 3 (E) -11 **26.** $x^2 - 10x$ *(2.5)*
27. $x^{-1/2} - 3 = \dfrac{1}{x^{1/2}} - 3$ *(2.5)* **28.** 0 *(2.5)*
29. $-\frac{3}{2}x^{-2} + \frac{15}{4}x^2 = \dfrac{-3}{2x^2} + \dfrac{15x^2}{4}$ *(2.5)*
30. $-2x^{-5} + x^3 = \dfrac{-2}{x^5} + x^3$ *(2.5)*
31. $f'(x) = 12x^3 + 9x^2 - 2$ *(2.5)* **32.** $\Delta x = 2$, $\Delta y = 10$, $\Delta y/\Delta x = 5$ *(2.6)*
33. 5 *(2.6)* **34.** 6 *(2.6)* **35.** $\Delta y = 0.64$; $dy = 0.6$ *(2.6)*
36. (A) 4 (B) 6 (C) Does not exist (D) 6 (E) No *(2.3)*
37. (A) 3 (B) 3 (C) 3 (D) 3 (E) Yes *(2.3)* **38.** (A) $(8, \infty)$
(B) $[0, 8]$ *(2.3)* **39.** $(-3, 4)$ *(2.3)* **40.** $(-3, 0) \cup (5, \infty)$ *(2.3)*
41. $(-2.3429, -0.4707) \cup (1.8136, \infty)$ *(2.3)* **42.** (A) 3
(B) $2 + 0.5h$ (C) 2 *(2.4)* **43.** $-x^{-4} + 10x^{-3}$ *(2.4)*
44. $\frac{3}{4}x^{-1/2} - \frac{5}{6}x^{-3/2} = \dfrac{3}{4\sqrt{x}} - \dfrac{5}{6\sqrt{x^3}}$ *(2.5)*
45. $0.6x^{-2/3} - 0.3x^{-4/3} = \dfrac{0.6}{x^{2/3}} - \dfrac{0.3}{x^{4/3}}$ *(2.4)*
46. $-\frac{3}{5}(-3)x^{-4} = \dfrac{9}{5x^4}$ *(2.5)* **47.** (A) $m = f'(1) = 2$
(B) $y = 2x + 3$ *(2.4, 2.5)* **48.** $x = 5$ *(2.4)* **49.** $x = -5$, $x = 3$ *(2.5)*
50. $x = -1.3401, 0.5771, 2.2630$ *(2.4)* **51.** ± 2.4824 *(2.5)*
52. (A) $v = f'(x) = 16x - 4$ (B) 44 ft/sec *(2.5)*
53. (A) $v = f'(x) = -10x + 16$ (B) $x = 1.6$ sec *(2.5)*
54. (A) The graph of g is the graph of f shifted 4 units to the right, and the graph of h is the graph of f shifted 4 units down:



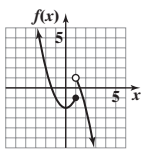
(B) The graph of g' is the graph of f' shifted 4 units to the right, and the graph of h' is the graph of f':



55. $(-\infty, \infty)$ *(2.3)* **56.** $(-\infty, 2) \cup (2, \infty)$ *(2.3)*
57. $(-\infty, -4) \cup (-4, 1) \cup (1, \infty)$ *(2.3)* **58.** $(-\infty, \infty)$ *(2.3)*
59. $[-2, 2]$ *(2.3)* **60.** (A) -1 (B) Does not exist (C) $-\frac{2}{3}$ *(2.1)*
61. (A) $\frac{1}{2}$ (B) 0 (C) Does not exist *(2.1)* **62.** (A) -1 (B) 1
(C) Does not exist *(2.1)*
63. (A) $-\frac{1}{6}$ (B) Does not exist (C) $-\frac{1}{3}$ *(2.1)* **64.** (A) 0 (B) -1
(C) Does not exist *(2.1)* **65.** (A) $\frac{2}{3}$ (B) $\frac{2}{3}$ (C) Does not exist *(2.3)*
66. (A) ∞ (B) $-\infty$ (C) ∞ *(2.3)* **67.** (A) 0 (B) 0
(C) Does not exist *(2.2)* **68.** 4 *(2.1)* **69.** $\dfrac{-1}{(x+2)^2}$ *(2.1)*
70. $2x - 1$ *(2.4)* **71.** $1/(2\sqrt{x})$ *(2.4)* **72.** Yes *(2.4)* **73.** No *(2.4)*
74. No *(2.4)* **75.** No *(2.4)* **76.** Yes *(2.4)* **77.** Yes *(2.4)*
78. Horizontal asymptote: $y = 5$; vertical asymptote: $x = 7$ *(2.2)*
79. Horizontal asymptote: $y = 0$; vertical asymptote: $x = 4$ *(2.2)*
80. No horizontal asymptote; vertical asymptote: $x = 3$ *(2.2)*
81. Horizontal asymptote: $y = 1$; vertical asymptotes: $x = -2$, $x = 1$ *(2.2)*
82. Horizontal asymptote: $y = 1$; vertical asymptotes: $x = -1$, $x = 1$ *(2.2)*
83. The domain of $f'(x)$ is all real numbers except $x = 0$. At $x = 0$, the graph of $y = f(x)$ is smooth, but it has a vertical tangent. *(2.4)*
84. (A) $\lim_{x \to 1^-} f(x) = 1$; $\lim_{x \to 1} f(x) = -1$



(B) $\lim_{x \to 1^-} f(x) = -1$; $\lim_{x \to 1^+} f(x) = 1$



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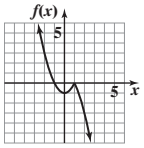
(C) $m = 1$

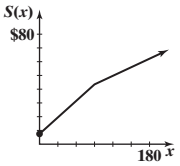


(D) The graphs in (A) and (B) have discontinuities at $x = 1$; the graph in (C) does not. *(2.2)*

85. (A) 1 (B) -1 (C) Does not exist (D) No *(2.4)*

86. (A) $S(x) = \begin{cases} 7.47 + 0.4x & \text{if } 0 \le x \le 90 \\ 24.786 + 0.2076x & \text{if } 90 < x \end{cases}$

(B)



(C) Yes *(2.2)*

87. (A) \$179.90 (B) \$180 *(2.7)* **88.** (A) $C(100) = 9{,}500$; $C'(100) = 50$. At a production level of 100 bicycles, the total cost is \$9,500, and cost is increasing at the rate of \$50 per bicycle.

(B) $\overline{C}(100) = 95$; $\overline{C}'(100) = -0.45$. At a production level of 100 bicycles, the average cost is \$95, and average cost is decreasing at a rate of \$0.45 per bicycle. *(2.7)*

89. The approximate cost of producing the 201st printer is greater than that of the 601st printer. Since these marginal costs are decreasing, the manufacturing process is becoming more efficient. *(2.7)*

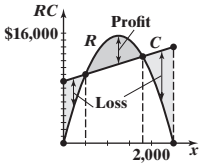
90. (A) $C'(x) = 2$; $\overline{C}(x) = 2 + \dfrac{9{,}000}{x}$; $\overline{C}'(x) = \dfrac{-9{,}000}{x^2}$

(B) $R(x) = xp = 25x - 0.01x^2$; $R'(x) = 25 - 0.02x$; $\overline{R}(x) = 25 - 0.01x$; $\overline{R}'(x) = -0.01$ (C) $P(x) = R(x) - C(x) = 23x - 0.01x^2 - 9{,}000$; $P'(x) = 23 - 0.02x$; $\overline{P}(x) = 23 - 0.01x - \dfrac{9{,}000}{x}$; $\overline{P}'(x) = -0.01 + \dfrac{9{,}000}{x^2}$

(D) (500, 10,000) and (1,800, 12,600)

(E) $P'(1{,}000) = 3$. Profit is increasing at the rate of \$3 per umbrella.
$P'(1{,}150) = 0$. Profit is flat.
$P'(1{,}400) = -5$. Profit is decreasing at the rate of \$5 per umbrella.

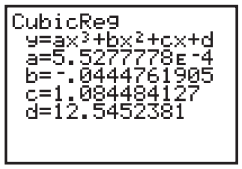
(F)



91. (A) 8 (B) 20 *(2.5)* (C) Long-term employees should near 40 components per day since as $t \to \infty$, $N(t) \to 40$. *(2.2)*

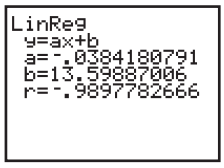
92. $N(9) = 27$; $N'(9) = 3.5$; After 9 months, 27,000 pools have been sold and the total sales are increasing at the rate of 3,500 pools per month. *(2.5)*

93. (A)

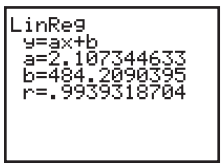


(B) $N(60) = 36.9$; $N'(60) = 1.7$. In 2020, natural-gas consumption is 36.9 trillion cubic feet and is increasing at the rate of 1.7 trillion cubic feet per year *(2.4)*

94. (A)



(B) Fixed costs: \$484.21; variable costs per kringle: \$2.11



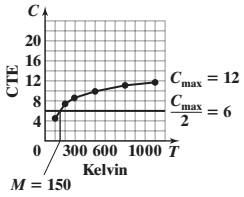
(C) (51, 591.15), (248, 1,007.62) (D) \$4.07 $< p <$ \$11.64 *(2.7)*

95. $C'(10) = -1$; $C'(100) = -0.001(2.5)$

96. $F(4) = 98.16$; $F'(4) = -0.32$; After 4 hours the patient's temperature is 98.16°F and is decreasing at the rate of 0.32°F per hour. *(2.5)*

97. (A) 10 items/h (B) 5 items/h *(2.5)*

98. (A)



(B) $C(T) = \dfrac{12T}{150 + T}$

(C) $C = 9.6$ at $T = 600$ K; $T = 750$ K when $C = 10$ *(2.3)*

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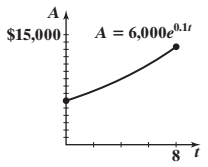
ANSWERS

Chapter 3

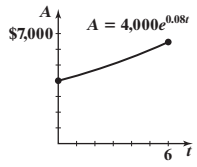
Exercises 3.1

1. $A = 1{,}465.68$ **2.** $A = 6{,}041.26$ **3.** $P = 9{,}117.21$ **4.** $P = 34{,}261.53$
5. $t = 5.61$ **6.** $r = 3.38$ **7.** $r = 0.04$ **8.** $r = 0.06$
9. \$11,051.71; \$12,214.03; \$13,498.59 **10.** \$27,182.82; \$73,890.56; \$200,855.37

11.



12.



13. $t = 5.78$ **14.** $t = 7.70$ **15.** $r = 0.08$ **16.** $r = 0.04$ **17.** $t = 36.62$ **18.** $t = 13.73$

19. $r = 0.09$ **20.** $r = 0.05$

21.

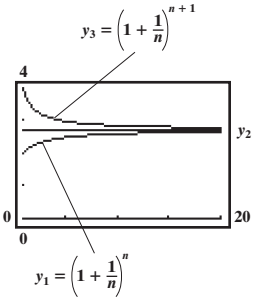
n	$[1 + (1/n)]^n$
10	2.593 74
100	2.704 81
1,000	2.716 92
10,000	2.718 15
100,000	2.718 27
1,000,000	2.718 28
10,000,000	2.718 28
↓	↓
∞	$e = 2.718\,281\,828\,459\ldots$

22.

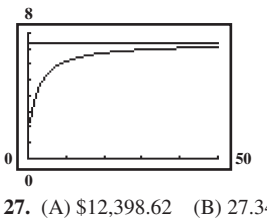
s	$(1 + s)^{1/s}$
0.01	2.70481
−0.01	2.73200
0.001	2.71692
−0.001	2.71964
0.0001	2.71815
−0.0001	2.71842
0.00001	2.71827
−0.00001	2.71830
↓	↓
0	$e = 2.7182818\ldots$

23. $\lim_{n\to\infty}(1+n)^{1/n} = 1$ **24.** $\lim_{s\to 0^+}\left(1+\frac{1}{s}\right)^s = 1$

25.

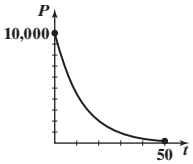


26. $y_1 = \left(1+\frac{2}{n}\right)^n$, $y_2 = 7.389056099 \approx e^2$



27. (A) \$12,398.62 (B) 27.34 yr
28. (A) \$10,504.30 (B) 5.81 yr
29. \$11,890.41 **30.** \$36,307.45 **31.** 8.11% **32.** 4.52%

33. (A)

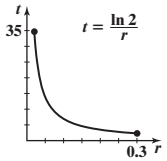


(B) $\lim_{t\to\infty} 10{,}000e^{-0.08t} = 0$

34. 8.66 yr **35.** 17.33 yr
36. 13.86 yr **37.** 8.66%
38. 6.93% **39.** 7.3 yr **40.** 5.7 yr

41. (A) $A = Pe^{rt}$
$2P = Pe^{rt}$
$2 = e^{rt}$
$rt = \ln 2$
$t = \dfrac{\ln 2}{r}$

(B)

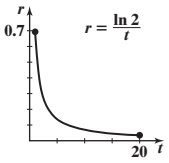


Although r could be any positive number, the restrictions on r are reasonable in the sense that most investments would be expected to earn a return of between 2% and 30%.

(C) The doubling times (in years) are 13.86, 6.93, 4.62, 3.47, 2.77, and 2.31, respectively.

42. (A) $A = Pe^{rt}$; $2P = Pe^{rt}$
$2 = e^{rt}$;
$rt = \ln 2$;
$r = \dfrac{\ln 2}{t}$

(B)



Although t could be any positive number, the restrictions on t are reasonable in the sense that the doubling times for most investments would be expected to be between 1 and 20 years.

(C) The doubling times (in years) are 13.86, 6.93, 4.62, 3.47, 2.77, and 2.31, respectively. **43.** $t = -(\ln 0.5)/0.000\,433\,2 \approx 1{,}600$ yr

44. $t = \dfrac{-(\ln 0.5)}{0.0001238} \approx 5{,}599$ yr **45.** $r = (\ln 0.5)/30 \approx -0.0231$

46. $r = (\ln 0.5)/90 \approx -0.0077$ **47.** 61.3 yr **48.** 88.9 yr **49.** 1.39%
50. 0.35%

Exercises 3.2

1. $y = 4$ **2.** $y = 3$ **3.** $x = 1/5$ **4.** $x = 0.001$ **5.** $y = 1/3$ **6.** $x = e^2$
7. $\ln x - \ln y$ **8.** x **9.** $5 \ln x$ **10.** $\ln x + \ln y$ **11.** $\ln u + 2\ln v - \ln w$
12. $2\ln u - 3\ln v - \ln w$ **13.** $5e^x + 3$ **14.** $-7e^x - 2$ **15.** $-\dfrac{2}{x} + 2x$
16. $\dfrac{6}{x} - 3x^2$ **17.** $3x^2 - 6e^x$ **18.** $9e^x + 4x$ **19.** $e^x + 1 - \dfrac{1}{x}$
20. $\dfrac{1}{x} + 2e^x - 6x$ **21.** $\dfrac{3}{x}$ **22.** $\dfrac{8}{x}$ **23.** $5 - \dfrac{5}{x}$ **24.** $\dfrac{9}{x}$ **25.** $\dfrac{2}{x} + 4e^x$
26. $\dfrac{12}{x}$ **27.** $f'(x) = e^x + ex^{e-1}$ **28.** $f'(x) = 3ex^{e-1} - 2e^x$
29. $f'(x) = (e+1)x^e$ **30.** $f'(x) = ee^x = e^{x+1}$
31. $f'(x) = \dfrac{1}{x}$; $y = x + 2$ **32.** $f'(x) = \dfrac{2}{x}$; $y = 2x - 2$
33. $f'(x) = 3e^x$; $y = 3x + 3$ **34.** $f'(x) = e^x$; $y = x + 2$
35. $f'(x) = \dfrac{3}{x}$; $y = \dfrac{3x}{e}$ **36.** $f'(x) = \dfrac{4}{x}$; $y = (4e^{-1})x + 1$
37. $f'(x) = e^x$; $y = ex + 2$ **38.** $f'(x) = 5e^x$; $y = (5e)x$ **39.** Yes; yes
40. Yes; no **41.** No; no **42.** Yes; no
43. $f(x) = 10x + \ln 10 + \ln x$; $f'(x) = 10 + \dfrac{1}{x}$
44. $f(x) = 2 - 3\ln x$; $f'(x) = -\dfrac{3}{x}$ **45.** $f(x) = \ln 4 - 3\ln x$; $f'(x) = -\dfrac{3}{x}$
46. $f(x) = x + 5\ln 6 + 5\ln x$; $f'(x) = \dfrac{x+5}{x}$ **47.** $\dfrac{1}{x \ln 2}$ **48.** $\dfrac{3}{x \ln 5}$
49. $3^x \ln 3$ **50.** $4^x \ln 4$ **51.** $2 - \dfrac{1}{x \ln 10}$ **52.** $8x + \dfrac{1}{x \ln 10}$
53. $1 + 10^x \ln 10$ **54.** $5x^4 - 5^x \ln 5$ **55.** $\dfrac{3}{x} + \dfrac{2}{x \ln 3}$ **56.** $\left(10 - \dfrac{1}{\ln 2}\right)\dfrac{1}{x}$

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57. $2^x \ln 2$ **58.** $-3^x \ln 3$ **59.** $(-0.82, 0.44), (1.43, 4.18), (8.61, 5503.66)$ **60.** (1.30, 3.65), (12.71, 332,105.11) **61.** (0.49, 0.49) **62.** (6.41, 6.41), (93.35, 93.35) **63.** (3.65, 1.30), (332,105.11, 12.71) **64.** (4.18, 1.43), (5,503.66, 8.61) **67.** \$28,447/yr; \$18,664/yr; \$11,021/yr **68.** \$2,594/yr; \$2,231/yr; \$1,919/yr **69.** $A'(t) = 5{,}000(\ln 4)4^t$; $A'(1) = 27{,}726$ bacteria/hr (rate of change at the end of the first hour); $A'(5) = 7{,}097{,}827$ bacteria/hr (rate of change at the end of the fifth hour) **70.** $A'(t) = 1000 \cdot 16^t \ln 16$; $A'(1) = 44{,}361$ bacteria/hr (rate of change at the end of the first hour); $A'(5) = 1000 \cdot (16)^5 \ln 16 \approx 2{,}907{,}269{,}992$ bacteria/hr (rate of change at the end of the fifth hour) **71.** At the 40-lb weight level, blood pressure would increase at the rate of 0.44 mm of mercury per pound of weight gain. At the 90-lb weight level, blood pressure would increase at the rate of 0.19 mm of mercury per pound of weight gain. **72.** 58 lb **73.** $dR/dS = k/S$ **74.** After 10 hours of instruction and practice, the rate of learning is 0.6 words/minute per hour of instruction and practice. After 100 hours of instruction and practice, the rate of learning is 0.06 words/minute per hour of instruction and practice. **75.** (A) \$808.41 per year (B) \$937.50 per year **76.** (A) \$2,484.17/yr (B) \$2,520.00/yr

Exercises 3.3

1. (A) $5x^4$ (B) $4x^3$ **2.** (A) $6x^5$ (B) $9x^4$ **3.** (A) $15x^{14}$ (B) $50x^{13}$
4. (A) $9x^8 + 8x^7$ (B) $8x^7$ **5.** (A) $3x^2$ (B) $2x^3$ **6.** (A) $6x^5$ (B) $4x^6$
7. (A) $-5x^{-6}$ (B) $\frac{2}{7}x^{-5}$ **8.** (A) $-9x^{-10}$ (B) 0
9. $2x^3(2x) + (x^2 - 2)(6x^2) = 10x^4 - 12x^2$
10. $5x^2(3x^2) + (x^3 + 2)(10x) = 25x^4 + 20x$
11. $(x - 3)(2) + (2x - 1)(1) = 4x - 7$
12. $(3x + 2)(4) + (4x - 5)(3) = 24x - 7$
13. $\dfrac{(x-3)(1) - x(1)}{(x-3)^2} = \dfrac{-3}{(x-3)^2}$
14. $\dfrac{(2x+1)(3) - 3x(2)}{(2x+1)^2} = \dfrac{3}{(2x+1)^2}$
15. $\dfrac{(x-2)(2) - (2x+3)(1)}{(x-2)^2} = \dfrac{-7}{(x-2)^2}$
16. $\dfrac{(2x+3)(3) - (3x-4)(2)}{(2x+3)^2} = \dfrac{17}{(2x+3)^2}$
17. $3xe^x + 3e^x = 3(x+1)e^x$ **18.** $x^2e^x + e^x(2x) = x(x+2)e^x$
19. $x^3\left(\dfrac{1}{x}\right) + 3x^2 \ln x = x^2(1 + 3 \ln x)$
20. $5x\left(\dfrac{1}{x}\right) + (\ln x)(5) = 5(1 + \ln x)$
21. $(x^2 + 1)(2) + (2x - 3)(2x) = 6x^2 - 6x + 2$
22. $(3x + 5)(2x) + (x^2 - 3)(3) = 9x^2 + 10x - 9$
23. $(0.4x + 2)(0.5) + (0.5x - 5)(0.4) = 0.4x - 1$
24. $(0.5x - 4)(0.2) + (0.2x + 1)(0.5) = 0.20x - 0.30$
25. $\dfrac{(2x-3)(2x) - (x^2+1)(2)}{(2x-3)^2} = \dfrac{2x^2 - 6x - 2}{(2x-3)^2}$
26. $\dfrac{(x^2-3)(3) - (3x+5)(2x)}{(x^2-3)^2} = \dfrac{-3x^2 - 10x - 9}{(x^2-3)^2}$
27. $(x^2 + 2)2x + (x^2 - 3)2x = 4x^3 - 2x$
28. $(x^2 - 4)(2x) + (x^2 + 5)(2x) = 4x^3 + 2x$
29. $\dfrac{(x^2-3)2x - (x^2+2)2x}{(x^2-3)^2} = \dfrac{-10x}{(x^2-3)^2}$
30. $\dfrac{(x^2+5)(2x) - (x^2-4)(2x)}{(x^2+5)^2} = \dfrac{18x}{(x^2+5)^2}$
31. $\dfrac{(x^2+1)e^x - e^x(2x)}{(x^2+1)^2} = \dfrac{(x-1)^2e^x}{(x^2+1)^2}$
32. $\dfrac{(1+e^x)(-e^x) - (1-e^x)e^x}{(1+e^x)^2} = \dfrac{-2e^x}{(1+e^x)^2}$
33. $\dfrac{(1+x)\left(\frac{1}{x}\right) - \ln x}{(1+x)^2} = \dfrac{1 + x - x \ln x}{x(1+x)^2}$
34. $\dfrac{(1 + \ln x)(2) - 2x\left(\frac{1}{x}\right)}{(1 + \ln x)^2} = \dfrac{2 \ln x}{(1 + \ln x)^2}$
35. $xf'(x) + f(x)$ **36.** $x^2f'(x) + 2xf(x)$ **37.** $x^3f'(x) + 3x^2f(x)$
38. $\dfrac{xf'(x) - f(x)}{x^2}$ **39.** $\dfrac{x^2f'(x) - 2xf(x)}{x^4}$ **40.** $\dfrac{x^3f'(x) - 3x^2f(x)}{x^6}$
41. $\dfrac{f(x) - xf'(x)}{[f(x)]^2}$ **42.** $\dfrac{2xf(x) - x^2f'(x)}{f(x)^2}$
43. $e^xf'(x) + f(x)e^x = e^x[f'(x) + f(x)]$
44. $\dfrac{f(x)e^x - e^xf'(x)}{f(x)^2} = \dfrac{e^x(f(x) - f'(x))}{f(x)^2}$
45. $\dfrac{f(x)\left(\frac{1}{x}\right) - (\ln x)f'(x)}{f(x)^2} = \dfrac{f(x) - (x \ln x)f'(x)}{xf(x)^2}$
46. $\dfrac{(\ln x)f'(x) - f(x) \cdot \frac{1}{x}}{(\ln x)^2} = \dfrac{f'(x)x \ln x - f(x)}{x(\ln x)^2}$
47. $(2x + 1)(2x - 3) + (x^2 - 3x)(2) = 6x^2 - 10x - 3$
48. $(x^3 + 2x^2)(3) + (3x - 1)(3x^2 + 4x) = 12x^3 + 15x^2 - 4x$
49. $(2.5t - t^2)(4) + (4t + 1.4)(2.5 - 2t) = -12t^2 + 17.2t + 3.5$
50. $(3 - 0.4t^3)(t - 2) + (0.5t^2 - 2t)(-1.2t^2) = -t^4 + 3.2t^3 + 3t - 6$
51. $\dfrac{(x^2+2x)(5) - (5x-3)(2x+2)}{(x^2+2x)^2} = \dfrac{-5x^2 + 6x + 6}{(x^2+2x)^2}$
52. $\dfrac{(2x-1)(6x) - 3x^2(2)}{(2x-1)^2} = \dfrac{6x^2 - 6x}{(2x-1)^2}$
53. $\dfrac{(w^2-1)(2w-3) - (w^2-3w+1)(2w)}{(w^2-1)^2} = \dfrac{3w^2 - 4w + 3}{(w^2-1)^2}$
54. $\dfrac{(3w-1)(4w^3 - 3w^2) - (w^4 - w^3)(3)}{(3w-1)^2} = \dfrac{9w^4 - 10w^3 + 3w^2}{(3w-1)^2}$
55. $(1 + x - x^2)e^x + e^x(1 - 2x) = (2 - x - x^2)e^x$
56. $(1 + e^t)\left(\dfrac{1}{t}\right) + e^t \ln t = \dfrac{1 + e^t + (t \ln t)e^t}{t}$
57. (A) $f'(x) = \dfrac{x \cdot 0 - 1 \cdot 1}{x^2} = -\dfrac{1}{x^2}$ (B) Note that $f(x) = x^{-1}$ and use the power rule: $f'(x) = -x^{-2} = -\dfrac{1}{x^2}$
58. (A) $f'(x) = \dfrac{x^2 \cdot 0 - (-1) \cdot 2x}{x^4} = \dfrac{2}{x^3}$ (B) Note that $f(x) = -x^{-2}$ and use the power rule: $f'(x) = 2x^{-3} = \dfrac{2}{x^3}$
59. (A) $f'(x) = \dfrac{x^4 \cdot 0 - (-3) \cdot 4x^3}{x^8} = \dfrac{12}{x^5}$ (B) Note that $f(x) = -3x^{-4}$ and use the power rule: $f'(x) = 12x^{-5} = \dfrac{12}{x^5}$
60. (A) $f'(x) = \dfrac{x^3 \cdot 0 - (2) \cdot 3x^2}{x^6} = -\dfrac{6}{x^4}$ (B) Note that $f(x) = 2x^{-3}$ and use the power rule: $f'(x) = -6x^{-4} = -\dfrac{6}{x^4}$
61. $f'(x) = (1 + 3x)(-2) + (5 - 2x)(3); y = -11x + 29$
62. $f'(x) = (7 - 3x)(2) + (1 + 2x)(-3); y = -13x + 31$

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63. $f'(x) = \dfrac{(3x-4)(1)-(x-8)(3)}{(3x-4)^2}; y = 5x - 13$

64. $f'(x) = \dfrac{(2x-3)(2)-(2x-5)(2)}{(2x-3)^2}; y = 4x - 9$

65. $f'(x) = \dfrac{2^x - x(2^x \ln 2)}{2^{2x}}; y = \left(\dfrac{1-2\ln 2}{4}\right)x + \ln 2$

66. $f'(x) = (x-2)\left(\dfrac{1}{x}\right) + \ln x; y = (\ln 2)x - 2\ln 2$

67. $f'(x) = (2x-15)(2x) + (x^2+18)(2) = 6(x-2)(x-3); x = 2, x = 3$ **68.** $f'(x) = (2x-3)(2x) + (x^2-6)(2) = 6x^2 - 6x - 12; x = -1, 2$

69. $f'(x) = \dfrac{(x^2+1)(1)-x(2x)}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}; x = -1, x = 1$

70. $f'(x) = \dfrac{x^2+9-x(2x)}{(x^2+9)^2} = \dfrac{9-x^2}{(x^2+9)^2}; x = -3, 3$

71. $7x^6 - 3x^2$ **72.** $7x^6 - 4x^3$ **73.** $-27x^{-4} = -\dfrac{27}{x^4}$ **74.** $-16x^{-5} = -\dfrac{16}{x^5}$

75. $(w+1)2^w \ln 2 + 2^w = [(w+1)\ln 2 + 1]2^w$

76. $(w-5)\left(\dfrac{1}{\ln 3}\cdot\dfrac{1}{w}\right) + (1)\log_3 w = \dfrac{w - 5 + w\ln w}{w \ln 3}$

77. $9x^{1/3}(3x^2) + (x^3+5)(3x^{-2/3}) = \dfrac{30x^3+15}{x^{2/3}}$

78. $(4x^{1/2}-1)x^{-2/3} + (3x^{1/3}+2)(2x^{-1/2}) = \dfrac{10x + 4x^{2/3} - x^{1/2}}{x^{7/6}}$

79. $\dfrac{(1+x^2)\dfrac{1}{x\ln 2} - 2x\log_2 x}{(1+x^2)^2} = \dfrac{1+x^2-2x^2\ln x}{x(1+x^2)^2\ln 2}$

80. $\dfrac{(1+x^4)10^x(\ln 10) - 10^x(4x^3)}{(1+x^4)^2} = \dfrac{10^x[(1+x^4)\ln 10 - 4x^3]}{(1+x^4)^2}$

81. $\dfrac{(x^2-3)(2x^{-2/3}) - 6x^{1/3}(2x)}{(x^2-3)^2} = \dfrac{-10x^2-6}{(x^2-3)^2x^{2/3}}$

82. $\dfrac{(x^2-3x+1)(x^{-1/2}) - 2\sqrt{x}(2x-3)}{(x^2-3x+1)^2} = \dfrac{-3x^2+3x+1}{(x^2-3x+1)^2x^{1/2}}$

83. $g'(t) = \dfrac{(3t^2-1)(0.2)-(0.2t)(6t)}{(3t^2-1)^2} = \dfrac{-0.6t^2-0.2}{(3t^2-1)^2}$

84. $h'(t) = \dfrac{(2t+1)(-0.1t)-(-0.05t^2)(2)}{(2t+1)^2} = \dfrac{-0.1t^2-0.1t}{(2t+1)^2}$

85. $(20x)\dfrac{1}{x\ln 10} + 20\log x = \dfrac{20(1+\ln x)}{\ln 10}$

86. $10^t\left(\dfrac{1}{\ln 10}\cdot\dfrac{1}{t}\right) + (\log t)(10^t \ln 10) = \dfrac{10^t(1+t\ln t(\ln 10))}{t\ln 10}$

87. $(x-1)(2x+1) + (x^2+x+1)(1) = 3x^2$

88. $(x^4+x^2+1)(2x) + (x^2-1)(4x^3+2x) = 6x^5$

89. $(x^2+x+1)(2x-1) + (x^2-x+1)(2x+1) = 4x^3 + 2x$

90. $(t+1)(4t^3-3t^2+2t-1) + (t^4-t^3+t^2-t+1)(1) = 5t^4$

91. $\dfrac{e^t(1+\ln t)-(t\ln t)e^t}{e^{2t}} = \dfrac{1+\ln t - t\ln t}{e^t}$

92. $\dfrac{(1+\ln u)(u^2e^u+2ue^u) - ue^u}{(1+\ln u)^2} = \dfrac{ue^u[(u+2)\ln u + u + 1]}{(1+\ln u)^2}$

93. (A) $S'(t) = \dfrac{(t^2+50)(180t) - 90t^2(2t)}{(t^2+50)^2} = \dfrac{9{,}000t}{(t^2+50)^2}$

(B) $S(10) = 60$; $S'(10) = 4$. After 10 months, the total sales are 60,000 video games, and sales are increasing at the rate of 4,000 video games per month. (C) Approximately 64,000 video games

94. (A) $N'(t) = \dfrac{180(t+4) - 180t}{(t+4)^2} = \dfrac{720}{(t+4)^2}$

(B) $N(16) = 144; N'(16) = 1.8$. After 16 months, the total number of subscribers is 144,000 and is increasing at a rate of 1,800 subscribers per month. (C) Approximately 145,800 subscribers

95. (A) $\dfrac{dx}{dp} = \dfrac{(0.1p+1)(0) - 4{,}000(0.1)}{(0.1p+1)^2} = \dfrac{-400}{(0.1p+1)^2}$

(B) $x = 800$; $dx/dp = -16$. At a price level of \$40, the demand is 800 DVD players per week, and demand is decreasing at the rate of 16 players per dollar. (C) Approximately 784 DVD players

96. (A) $\dfrac{dx}{dp} = \dfrac{100(0.1p+1) - 0.1(100p)}{(0.1p+1)^2} = \dfrac{100}{(0.1p+1)^2}$

(B) $x(40) = 800$; when $p = \$40, \dfrac{dx}{dp} = 4$. At a price level of \$40, the supply is 800 DVD players and is increasing at the rate of 4 players per dollar. (C) Approximately 804 DVD players

97. (A) $C'(t) = \dfrac{(t^2+1)(0.14) - 0.14t(2t)}{(t^2+1)^2} = \dfrac{0.14 - 0.14t^2}{(t^2+1)^2}$

(B) $C'(0.5) = 0.0672$. After 0.5 hr, concentration is increasing at the rate of 0.0672 mg/cm^3 per hour. $C'(3) = -0.0112$. After 3 hr, concentration is decreasing at the rate of 0.0112 mg/cm^3 per hour.

98. (A) $T'(x) = 2x\left(1 - \dfrac{x}{9}\right) + x^2\left(-\dfrac{1}{9}\right) = 2x - \dfrac{1}{3}x^2$

(B) $T'(1) = \left(\dfrac{5}{3}\right)^\circ$ per mg of drug; $T'(3) = 3^\circ$ per mg of drug; $T'(6) = 0^\circ$ per mg of drug.

Exercises 3.4

1. $f'(x) = 9x^8 + 10$ **2.** $f'(x) = -30x^4$ **3.** $f'(x) = \dfrac{7}{2}x^{-1/2} - 6x^{-3}$

4. $f'(x) = -45x^{-4} + \dfrac{4}{3}x^{-2/3}$ **5.** $f'(x) = 8e^x$

6. $f'(x) = 12e^x - 11ex^{e-1}$ **7.** $f'(x) = \dfrac{4}{x} + 8x$ **8.** $f'(x) = \ln 3 - \dfrac{3}{x}$

9. 3 **10.** (-2) **11.** $(-4x)$ **12.** $6x$ **13.** $2x$ **14.** 4 **15.** $4x^3$

16. $1 - 3x^2$ **17.** $-8(5-2x)^3$ **18.** $-10(9-5x)$

19. $5(4+0.2x)^4(0.2) = (4+0.2x)^4$ **20.** $-2(6-0.5x)^3$

21. $30x(3x^2+5)^4$ **22.** $60x(5x^2-3)^5$ **23.** $5e^{5x}$ **24.** $-12e^{-2x}$ **25.** $-18e^{-6x}$

26. $(2x+3)e^{x^2+3x+1}$ **27.** $(2x-5)^{-1/2} = \dfrac{1}{(2x-5)^{1/2}}$

28. $2(4x+3)^{-1/2} = \dfrac{2}{(4x+3)^{1/2}}$ **29.** $-8x^3(x^4+1)^{-3} = \dfrac{-8x^3}{(x^4+1)^3}$

30. $-15x^4(x^5+2)^{-4} = -\dfrac{15x^4}{(x^5+2)^4}$ **31.** $\dfrac{6x}{1+x^2}$

32. $\dfrac{2(2x-3)}{x^2-3x+4}$ **33.** $\dfrac{3(1+\ln x)^2}{x}$ **34.** $\dfrac{4(x-2)(x-2\ln x)^3}{x}$

35. $f'(x) = 6(2x-1)^2; y = 6x - 5; x = \frac{1}{2}$

36. $f'(x) = 12(3x-1)^3; y = 96x - 80; x = \dfrac{1}{3}$

37. $f'(x) = 2(4x-3)^{-1/2} = \dfrac{2}{(4x-3)^{1/2}}; y = \dfrac{2}{3}x + 1$; none

38. $f'(x) = \dfrac{1}{(2x+8)^{1/2}}; y = \dfrac{1}{4}x + 3$; none

39. $f'(x) = 10(x-2)e^{x^2-4x+1}; y = -20ex + 5e; x = 2$

40. $f'(x) = \dfrac{2x(4x^2-1)}{2x^4-x^2+1}; y = 3(x-1) + \ln 2; x = 0, \pm\dfrac{1}{2}$

41. $12(x^2-2)^3(2x) = 24x(x^2-2)^3$

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42. $2(5)(x^3+6)^4(3x^2) = 30x^2(x^3+6)^4$

43. $-6(t^2+3t)^{-4}(2t+3) = \dfrac{-6(2t+3)}{(t^2+3t)^4}$

44. $3(-2)(t^3+t^2)^{-3}(3t^2+2t) = \dfrac{-6(3t^2+2t)}{(t^3+t^2)^3}$

45. $\dfrac{1}{2}(w^2+8)^{-1/2}(2w) = \dfrac{w}{\sqrt{w^2+8}}$

46. $\dfrac{1}{3}(3w-7)^{-2/3}(3) = \dfrac{1}{\sqrt[3]{(3w-7)^2}}$

47. $12xe^{3x} + 4e^{3x} = 4(3x+1)e^{3x}$

48. $\dfrac{2e^{2x}(x^2+9) - 2xe^{2x}}{(x^2+9)^2} = \dfrac{2e^{2x}(x^2-x+9)}{(x^2+9)^2}$

49. $\dfrac{3x\left(\dfrac{1}{1+x^2}\right)2x - 3\ln(1+x^2)}{9x^2} = \dfrac{2x^2-(1+x^2)\ln(1+x^2)}{3x^2(1+x^2)}$

50. $x\left(\dfrac{1}{1+e^x}\right)e^x + \ln(1+e^x) = \dfrac{xe^x+(1+e^x)\ln(1+e^x)}{1+e^x}$

51. $6te^{3(t^2+1)}$ **52.** $4e^{2t}(e^{2t}-1)$ **53.** $\dfrac{3x}{x^2+3}$

54. $\dfrac{3x[\ln(x^2+3)]^{1/2}}{x^2+3}$ **55.** $-5(w^3+4)^{-6}(3w^2) = \dfrac{-15w^2}{(w^3+4)^6}$

56. $(-6)\cdot(w^2-2)^{-7}(2w) = \dfrac{-12w}{(w^2-2)^7}$

57. $f'(x) = (4-x)^3 - 3x(4-x)^2 = 4(4-x)^2(1-x)$; $y = -16x+48$

58. $f'(x) = 2x(1-x)^4 + x^2[4(1-x)^3(-1)] = 2x(1-3x)(1-x)^3$; $y = 20x-36$

59. $f'(x) = \dfrac{(2x-5)^3 - 6x(2x-5)^2}{(2x-5)^6} = \dfrac{-4x-5}{(2x-5)^4}$; $y = -17x+54$

60. $f'(x) = \dfrac{4x^3(3x-8)^2 - [2(3x-8)(3)]x^4}{(3x-8)^4} = \dfrac{2x^3(3x-16)}{(3x-8)^3}$; $y = -8x+48$

61. $f'(x) = \dfrac{1}{2x\sqrt{\ln x}}$; $y = \dfrac{x}{2e} + \dfrac{1}{2}$ **62.** $f(x) = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$; $y = \dfrac{e}{2}(x+1)$

63. $f'(x) = 2x(x-5)^3 + 3x^2(x-5)^2 = 5x(x-5)^2(x-2)$; $x = 0, 2, 5$

64. $f'(x) = 3x^2(x-7)^4 + x^3[4(x-7)^3(1)] = 7x^2(x-3)(x-7)^3$; $x = 0, 3, 7$

65. $f'(x) = \dfrac{(2x+5)^2 - 4x(2x+5)}{(2x+5)^4} = \dfrac{5-2x}{(2x+5)^3}$; $x = \frac{5}{2}$

66. $f'(x) = \dfrac{(x-3)^3 - [3(x-3)^2(1)](x-1)}{(x-3)^6} = \dfrac{-2x}{(x-3)^4}$; $x = 0$

67. $f'(x) = (x^2-8x+20)^{-1/2}(x-4) = \dfrac{x-4}{(x^2-8x+20)^{1/2}}$; $x = 4$

68. $f'(x)\dfrac{1}{2}(x^2+4x+5)^{-1/2}(2x+4) = \dfrac{x+2}{(x^2+4x+5)^{1/2}}$; $x = -2$

69. No; yes **70.** No; no **71.** Domain of f: $(0,\infty)$; domain of g: $[0,\infty)$; domain of m: $(0,\infty)$ **72.** Domain of f: $(-\infty,\infty)$; domain of g: $[0,\infty)$; domain of m: $[0,\infty)$ **73.** Domain of f: $[0,\infty)$; domain of g: $(0,\infty)$; domain of m: $[1,\infty)$ **74.** Domain of f: $[0,\infty)$; domain of g: $(-\infty,\infty)$; domain of m: $(-\infty,\infty)$ **75.** Domain of f: $(0,\infty)$; domain of g: $(-\infty,\infty)$; domain of m: $(-2,2)$ **76.** Domain of f: $(0,\infty)$; domain of g: $(-\infty,\infty)$; domain of m: $(-5,\infty)$ **77.** Domain of f: all real numbers except ±1; domain of g: $(0,\infty)$; domain of m: all positive real numbers except e and $\frac{1}{e}$ **78.** Domain of f: all real numbers except 0; domain of g: all real numbers; domain of m: all real numbers except ±3

79. $18x^2(x^2+1)^2 + 3(x^2+1)^3 = 3(x^2+1)^2(7x^2+1)$

80. $4x(x^3-3)^4 + 2x^2[4(x^3-3)^3(3x^2)] = 4x(x^3-3)^3(7x^3-3)$

81. $\dfrac{24x^5(x^3-7)^3 - (x^3-7)^4 6x^2}{4x^6} = \dfrac{3(x^3-7)^3(3x^3+7)}{2x^4}$

82. $\dfrac{6x(x^2+5)^3 - [3(x^2+5)^2(2x)](3x^2)}{(x^2+5)^6} = \dfrac{30x-12x^3}{(x^2+5)^4}$

83. $\dfrac{1}{\ln 2}\left(\dfrac{6x}{3x^2-1}\right)$ **84.** $\dfrac{1}{\ln 10}\cdot\dfrac{3x^2}{x^3-1}$ **85.** $(2x+1)(10^{x^2+x})(\ln 10)$

86. $-4x8^{1-2x^2}(\ln 8)$ **87.** $\dfrac{12x^2+5}{(4x^3+5x+7)\ln 3}$ **88.** $2x$

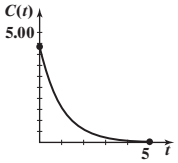
89. $2^{x^3-x^2+4x+1}(3x^2-2x+4)\ln 2$ **90.** $\dfrac{\ln 10}{x}10^{\ln x}$

91. (A) $C'(x) = (2x+16)^{-1/2} = \dfrac{1}{(2x+16)^{1/2}}$ (B) $C'(24) = \frac{1}{8}$, or \$12.50. At a production level of 24 cell phones, total cost is increasing at the rate of \$12.50 per cell phone and the cost of producing the 25th cell phone is approximately \$12.50. $C'(42) = \frac{1}{10}$, or \$10.00. At a production level of 42 cell phones, total cost is increasing at the rate of \$10.00 per cell phone and the cost of producing the 43rd cell phone is approximately \$10.00.

92. (A) $C'(x) = \dfrac{1}{2}(4x+4)^{-1/2}(4) = \dfrac{2}{(4x+4)^{1/2}}$ (B) $C'(15) = \dfrac{1}{4}$, or \$25. At a production level of 15 cameras, total costs are increasing at the rate of \$25 per camera and the cost of producing the 16th camera is approximately \$25. $C'(24) = \dfrac{2}{10}$, or \$20. At a production level of 24 cameras, total costs are increasing at the rate of \$20 per camera and the cost of producing the 25th camera is approximately \$20. **93.** (A) $\dfrac{dx}{dp} = 40(p+25)^{-1/2} = \dfrac{40}{(p+25)^{1/2}}$ (B) $x = 400$ and $dx/dp = 4$. At a price of \$75, the supply is 400 bicycle helmets per week, and supply is increasing at the rate of 4 bicycle helmets per dollar. **94.** (A) $\dfrac{dx}{dp} = -60\left(\dfrac{1}{2}\right)(p+25)^{-1/2} = \dfrac{-30}{(p+25)^{1/2}}$ (B) $x = 400$ and $\dfrac{dx}{dp} = -3$. At a price of \$75, the demand is 400 bicycle helmets and is decreasing at the rate of 3 bicycle helmets per dollar.

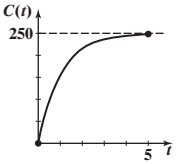
95. (A) After 1 hr, the concentration is decreasing at the rate of 1.60 mg/mL per hour; after 4 hr, the concentration is decreasing at the rate of 0.08 mg/mL per hour.

(B)



96. (A) After 1 minute, the concentration is increasing at the rate of 92 mg/mL per minute; after 4 minutes, the concentration is increasing at the rate of 4.6 mg/mL per minute.

(B)



97. 2.27 mm of mercury/yr; 0.81 mm of mercury/yr; 0.41 mm of mercury/yr

98. −9.74°F per hr; −1.71°F per hr

Exercises 3.5

1. $y = -\dfrac{3}{2}x + 10$ **2.** $y = \dfrac{4}{3}x^2 - 4$ **3.** $y = \pm\dfrac{4}{3}\sqrt{9-x^2}$

4. $y = \pm\dfrac{1}{2}\sqrt{36+x^2}$ **5.** $y = \dfrac{-x \pm \sqrt{4-3x^2}}{2}$ **6.** Impossible

7. Impossible **8.** $y = \dfrac{-e^x \pm \sqrt{e^{2x}-4x^3}}{2}$ **9.** $y' = 4/3$

10. $y' = -2/9$ **11.** $y' = -\dfrac{4x}{y^2}$ **12.** $y' = -\dfrac{x^2}{y^2}$ **13.** $y' = -\dfrac{1}{e^y}$

14. $y' = \dfrac{8x}{e^y}$ **15.** $y' = 2xy$ **16.** $y' = -y$ **17.** $y' = 10x$; 10 **18.** $y' = 15x^2$; 15

19. $y' = \dfrac{2x}{3y^2}$; $\dfrac{4}{3}$ **20.** $y' = -\dfrac{3x^2}{2y}$; -3 **21.** $y' = -\dfrac{3}{2y+2}$; $-\dfrac{3}{4}$

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22. $y' = \dfrac{4}{2y-1}; 4$ **23.** $y' = -\dfrac{y}{x}; -\dfrac{3}{2}$ **24.** $y' = \dfrac{2-3y}{3x}; -\dfrac{1}{6}$

25. $y' = -\dfrac{2y}{2x+1}; 4$ **26.** $y' = -\dfrac{y}{x+2}; -1$ **27.** $y' = \dfrac{6-2y}{x}; -1$

28. $y' = \dfrac{3(1-2y)}{2x}; \dfrac{15}{2}$ **29.** $y' = \dfrac{2x}{e^y - 2y}; 2$ **30.** $y' = \dfrac{2x}{1+4e^{y^2}}; \dfrac{4}{5}$

31. $y' = \dfrac{3x^2y}{y+1}; \dfrac{3}{2}$ **32.** $y' = \dfrac{y}{4y^2-1}; \dfrac{1}{3}$ **33.** $y' = \dfrac{6x^2y - y\ln y}{x+2y}; 2$

34. $y' = \dfrac{2x-e^y}{xe^y-1}; 3$ **35.** $x' = \dfrac{2tx-3t^2}{2x-t^2}; 8$ **36.** $x' = \dfrac{x}{3x-2t}$; not defined

37. $y'|_{(1.6,1.8)} = -\dfrac{3}{4}; y'|_{(1.6,0.2)} = \dfrac{3}{4}$ **38.** $y'\Big|_{(0.2,1.6)} \dfrac{4}{3}; y'_{(0.2,0.4)}\Big| = -\dfrac{4}{3}$

39. $y = -x + 5$ **40.** $y = x - 1$ **41.** $y = \frac{2}{5}x - \frac{12}{5}; y = \frac{3}{5}x + \frac{12}{5}$

42. $y = -\dfrac{4}{3}x + \dfrac{10}{3}, y = \dfrac{1}{3}x - \dfrac{4}{3}$ **43.** $y' = -\dfrac{1}{x}$ **44.** $y' = -\dfrac{e^y + 3x^2}{xe^y + 1}; -e$

45. $y' = \dfrac{1}{3(1+y)^2+1}; \dfrac{1}{13}$ **46.** $y' = \dfrac{1}{4(y-3)^3-1}; \dfrac{1}{3}$

47. $y' = \dfrac{3(x-2y)^2}{6(x-2y)^2+4y}; \dfrac{3}{10}$ **48.** $y' = \dfrac{8(2x-y)^3}{4(2x-y)^3+3y^2}; 0$

49. $y' = \dfrac{3x^2(7+y^2)^{1/2}}{y}; 16$ **50.** $y' = \dfrac{x^{1/2}(y^3+1)^{1/2}}{3y^2}; \dfrac{1}{2}$

51. $y' = \dfrac{y}{2xy^2 - x}; 1$ **52.** $y' = \dfrac{2 - ye^{xy}}{xe^{xy} - 1}; -2$ **53.** $y = 0.63x + 1.04$

54. $y = 1.58x + 1.64$ **55.** $\dfrac{dp}{dx} = \dfrac{1}{p-3}$ **56.** $\dfrac{dp}{dx} = -\dfrac{5}{p}$

57. $\dfrac{dp}{dx} = -\dfrac{\sqrt{p+25}}{15}$ **58.** $\dfrac{dp}{dx} = \dfrac{\sqrt{p+50}}{30}$ **59.** $\dfrac{dL}{dV} = \dfrac{-(L+m)}{V+n}$

60. $\dfrac{dV}{dL} = -\dfrac{(V+n)}{(L+m)}$ **61.** $\dfrac{dT}{dv} = \dfrac{2}{k}\sqrt{T}$ **62.** $\dfrac{dr}{dF} = -\dfrac{r}{2F} = -\dfrac{r^3}{2Gm_1m_2}$

63. $\dfrac{dv}{dT} = \dfrac{k}{2\sqrt{T}}$ **64.** $\dfrac{dF}{dr} = -\dfrac{2F}{r} = -\dfrac{2Gm_1m_2}{r^3}$

Exercises 3.6

1. 19.5 ft **2.** 502,655 m^2 **3.** 46 m **4.** 88 ft **5.** 34.5 ft **6.** 113.1 m^3

7. 32 ft **8.** $r = 5.42$ m; $h = 10.84$ m **9.** 30 **10.** -24 **11.** $-\dfrac{16}{3}$

12. $\dfrac{20}{3}$ **13.** $-\dfrac{16}{7}$ **14.** $-\dfrac{1}{3}$ **15.** Decreasing at 9 units/sec **16.** Does not change at that moment **17.** Approx. 3.03 ft/sec **18.** Approx. 2.83 ft/sec

19. Approx. 126 ft^2/sec **20.** $\dfrac{dC}{dt} = 12.56$ ft/sec **21.** 3,770 cm^3/min

22. 754.0 cm^2/min **23.** 6 lb/in.^{2}/hr **24.** 0.2 lbs/in.^{2}/sec **25.** $\dfrac{9}{4}$ ft/sec

26. 4 m/sec **27.** $\dfrac{20}{3}$ ft/sec **28.** $\dfrac{5}{3}$ ft/sec **29.** 0.0214 ft/sec; 0.0135 ft/sec; yes, at $t = 0.000\,19$ sec **30.** (0.436, 0), (1.333, 0); the distance from (0, 1) is never increasing at a rate of 5 units per second or 10 units per second

31. 3.835 units/sec **32.** $(0.875, -0.574)$ and $(-2, -3)$

33. (A) $\dfrac{dC}{dt} = \$15{,}000$/wk (B) $\dfrac{dR}{dt} = -\$50{,}000$/wk

(C) $\dfrac{dP}{dt} = -\$65{,}000$/wk **34.** (A) $\dfrac{dC}{dt} = \$30{,}000$/wk

(B) $\dfrac{dR}{dt} = \$50{,}000$/wk (C) $\dfrac{dP}{dt} = \$20{,}000$/wk **35.** $\dfrac{ds}{dt} = \$2{,}207$/wk

36. $\dfrac{dS}{dt} \approx \$1{,}078$/wk **37.** $\dfrac{dx}{dt} = 165$ headphones/wk

38. $\dfrac{dx}{dt} = -4.5$ ovens/wk **39.** $R(p) = p(6{,}000 - 0.15p^2)$;

$\dfrac{dR}{dt} = -\$2{,}775$/wk **40.** $R(p) = p(800 - 36\sqrt{p+20})$; $\dfrac{dR}{dt} = \$546$/wk

41. $\dfrac{dx}{dt} = 160$ lbs/wk **42.** $\dfrac{dx}{dt} = -339$ bushels/wk

43. $\dfrac{dx}{dt} = \$3$ million/wk **44.** $\dfrac{dy}{dt} = 1.2$ percentage points/wk

45. (A) $\dfrac{dx}{dt} = -12.73$ units/month (B) $\dfrac{dp}{dt} = \$1.53$/month

46. (A) $\dfrac{dx}{dt} \approx -8.26$ units/month (B) $\dfrac{dp}{dt} \approx \$0.99$/month

47. Approx. 100 ft^3/min **48.** $-\dfrac{1}{12}$ min/hr

Exercises 3.7

1. $x = f(p) = 105 - 2.5p, 0 \le p \le 42$

2. $x = f(p) = 6{,}250 - 50p, 0 \le p \le 125$

3. $x = f(p) = \sqrt{100 - 2p}, 0 \le p \le 50$

4. $x = f(p) = \sqrt{225 - 1.25p}, 0 \le p \le 180$

5. $x = f(p) = 20(\ln 25 - \ln p), 25/e \approx 9.2 \le p \le 25$

6. $x = f(p) = 4\ln(45 - p), 45 - e^3 \approx 24.9 \le p \le 44$

7. $x = f(p) = e^{8-0.1p}, 80 - 10\ln 30 \approx 46.0 \le p \le 80$

8. $x = f(p) = 100 - 0.2e^p, \ln 50 \approx 3.91 \le p \le 6.21 \approx \ln 500$

9. $\dfrac{35 - 0.8x}{35x - 0.4x^2}$ **10.** $\dfrac{60 - 2.4x}{60x - 1.2x^2}$ **11.** $-\dfrac{4e^{-x}}{7 + 4e^{-x}}$ **12.** $\dfrac{1.5e^{-0.5x}}{15 - 3e^{-0.5x}}$

13. $\dfrac{5}{x(12 + 5\ln x)}$ **14.** $\dfrac{2}{x(25 - 2\ln x)}$ **15.** 0 **16.** 0 **17.** -0.017

18. -0.023 **19.** -0.034 **20.** -0.12 **21.** 1.013 **22.** 0.341 **23.** 0.405 **24.** 0.156 **25.** 11.8% **26.** 21.4% **27.** 5.4% **28.** 6.0% **29.** -14.7%

30. -10.4% **31.** -431.6% **32.** -70.6% **33.** $E(p) = \dfrac{450p}{25{,}000 - 450p}$

34. $E(p) = \dfrac{190p}{10{,}000 - 190p}$ **35.** $E(p) = \dfrac{8p^2}{4{,}800 - 4p^2}$

36. $E(p) = \dfrac{14p^2}{8{,}400 - 7p^2}$ **37.** $E(p) = \dfrac{0.6pe^p}{98 - 0.6e^p}$

38. $E(p) = \dfrac{35}{160 - 35\ln p}$ **39.** 0.07 **40.** 0.052 **41.** 0.15 **42.** 0.24

43. $\dfrac{x+1}{x}$ **44.** $\dfrac{x+2}{x}$ **45.** $\dfrac{1}{x\ln x}$ **46.** $\dfrac{1+\ln x}{x\ln x}$

47. (A) Inelastic (B) Unit elasticity (C) Elastic **48.** (A) Inelastic (B) Unit elasticity (C) Elastic **49.** (A) Inelastic (B) Unit elasticity (C) Elastic **50.** (A) Inelastic (B) Unit elasticity (C) Elastic

51. $E(12) = 0.6$; 2.4% decrease **52.** $E(28) = 7$; 42% increase

53. $E(22) = 2.2$; 11% increase **54.** $E(16) = 1$; 9% decrease

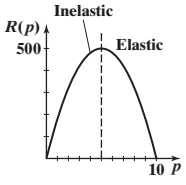
55. Elastic on $(16, 32)$ **56.** Inelastic on $(0, 16)$ **57.** Decrease **58.** Increase

59. Elastic on $(3.5, 7)$; inelastic on $(0, 3.5)$ **60.** Elastic on $(30, 60)$; inelastic on $(0, 30)$ **61.** Elastic on $(25/\sqrt{3}, 25)$; inelastic on $(0, 25/\sqrt{3})$

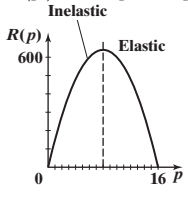
62. Elastic on $(20/\sqrt{3}, 20)$; inelastic on $(0, 20/\sqrt{3})$ **63.** Elastic on $(48, 72)$; inelastic on $(0, 48)$ **64.** Elastic on $(108, 162)$; inelastic on $(0, 108)$

65. Elastic on $(25, 25\sqrt{2})$; inelastic on $(0, 25)$ **66.** Elastic on $(30, 30\sqrt{2})$; inelastic on $(0, 30)$

67. $R(p) = 20p(10 - p)$



68. $R(p) = 160p - 10p^2$



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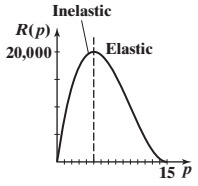
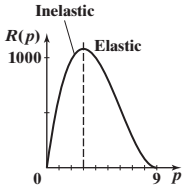
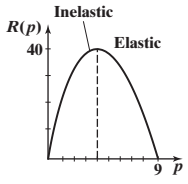
69. $R(p) = 40p(p - 15)^2$



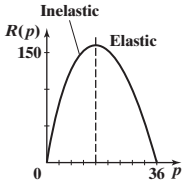
70. $R(p) = 10p(p - 9)^2$



71. $R(p) = 30p - 10p\sqrt{p}$

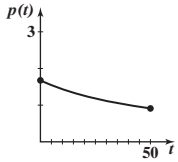


72. $R(p) = 30p - 5p^{3/2}$

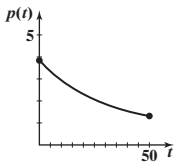


73. $\frac{3}{2}$ **74.** $\frac{1}{2}$ **75.** $\frac{1}{2}$ **76.** 2 **77.** Elastic on $(0, 300)$; inelastic on $(300, 600)$ **78.** Elastic on $(0, 800)$; inelastic on $(800, 1{,}600)$ **79.** Elastic on $(0, 10\sqrt{3})$; inelastic on $(10\sqrt{3}, 30)$ **80.** Elastic on $(0, 30)$; inelastic on $(30, 30\sqrt{3})$ **81.** k **82.** kp **83.** \$75 per day **84.** \$36 per day **85.** Increase **86.** Decrease **87.** Decrease **88.** Increase **89.** \$3.75 **90.** \$1.25

91. $p(t) = \dfrac{31}{0.31t + 18.5}$



92. $p(t) = \dfrac{149}{1.49t + 38.8}$



93. -0.028 **94.** -0.016

Chapter 3 Review Exercises

1. \$3,136.62; \$4,919.21; \$12,099.29 *(3.1)* **2.** $\frac{2}{x} + 3e^x$ *(3.2)* **3.** $2e^{2x-3}$ *(3.4)*

4. $\dfrac{2}{2x + 7}$ *(3.4)* **5.** $\dfrac{e^x}{3 + e^x}$ *(3.4)* **6.** $y' = \dfrac{9x^2}{4y}$; $\dfrac{9}{8}$ *(3.5)* **7.** $dy/dt = 216$ *(3.6)*

8. $x = 5{,}000 - 200p$ *(3.7)* **9.** $E(p) = \dfrac{p}{25 - p}$ *(3.7)* **10.** $E(15) = 1.5$; 7.5% decrease *(3.7)* **11.** Elastic on (12.5, 25) *(3.7)* **12.** Increase *(3.7)*

13. -10 *(3.2)* **14.** $\lim\limits_{n\to\infty}\left(1 + \dfrac{2}{n}\right)^n = e^2 \approx 7.389\,06$ *(3.1)*

15. $\dfrac{7[(\ln z)^6 + 1]}{z}$ *(3.4)* **16.** $x^5(1 + 6 \ln x)$ *(3.3)* **17.** $\dfrac{e^x(x - 6)}{x^7}$ *(3.3)*

18. $\dfrac{6x^2 - 3}{2x^3 - 3x}$ *(3.4)* **19.** $(3x^2 - 2x)e^{x^3 - x^2}$ *(3.4)* **20.** $\dfrac{1 - 2x \ln 5x}{xe^{2x}}$ *(3.4)*

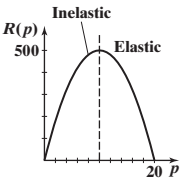
21. $y = -x + 2$; $y = -ex + 1$ *(3.4)* **22.** $y' = \dfrac{3y - 2x}{8y - 3x}$; $\dfrac{8}{19}$ *(3.5)*

23. $x' = \dfrac{4tx}{3x^2 - 2t^2}$; -4 *(3.5)* **24.** $y' = \dfrac{1}{e^y + 2y}$; 1 *(3.5)*

25. $y' = \dfrac{2xy}{1 + 2y^2}$; $\dfrac{2}{3}$ *(3.5)* **26.** 0.049 *(3.7)* **27.** $-\dfrac{3}{100 - 3p}$ *(3.7)*

28. $\dfrac{2x}{1 + x^2}$ *(3.7)* **29.** $dy/dt = -2$ units/sec *(3.6)* **30.** 0.27 ft/sec *(3.6)*

31. $dR/dt = 1/\pi \approx 0.318$ in./min *(3.6)* **32.** Elastic for $5 < p < 15$; inelastic for $0 < p < 5$ *(3.7)* **33.**



34. (A) $y = [\ln(4 - e^x)]^3$ (B) $\dfrac{dy}{dx} = \dfrac{-3e^x[\ln(4 - e^x)]^2}{4 - e^x}$ *(3.4)*

35. $2x(5^{x^2-1})(\ln 5)$ *(3.4)* **36.** $\left(\dfrac{1}{\ln 5}\right)\dfrac{2x - 1}{x^2 - x}$ *(3.4)*

37. $\dfrac{2x + 1}{2(x^2 + x)\sqrt{\ln(x^2 + x)}}$ *(3.4)* **38.** $y' = \dfrac{2x - e^{xy}y}{xe^{xy} - 1}$; 0 *(3.5)*

39. The rate of increase of area is proportional to the radius R, so the rate is smallest when $R = 0$, and has no largest value. *(3.6)* **40.** Yes, for $-\sqrt{3}/3 < x < \sqrt{3}/3$ *(3.6)* **41.** (A) 14.2 yr (B) 13.9 yr *(3.1)*

42. $A'(t) = 10e^{0.1t}$; $A'(1) = \$11.05$/yr; $A'(10) = \$27.18$/yr *(3.1)*

43. \$987.50/yr *(3.2)* **44.** $R'(x) = (1{,}000 - 20x)e^{-0.02x}$ *(3.4)*

45. $p' = -\dfrac{x}{3p^2} = \dfrac{-(5{,}000 - 2p^3)^{1/2}}{3p^2}$ *(3.5)* **46.** $dR/dt = \$2{,}242$/day *(3.6)*

47. Decrease price *(3.7)* **48.** 0.02125 *(3.7)* **49.** -1.111 mg/mL per hour; -0.335 mg/mL per hour *(3.4)* **50.** $dR/dt = -3/(2\pi)$; approx. 0.477 mm/day *(3.6)* **51.** (A) Increasing at the rate of 2.68 units/ day at the end of 1 day of training; increasing at the rate of 0.54 unit/day after 5 days of training (B) 7 days *(3.4)* **52.** $dT/dt = -1/27 \approx -0.037$ min/hr *(3.6)*

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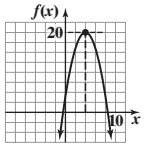
ANSWERS

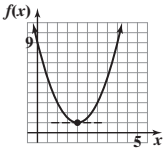
Chapter 4

Exercises 4.1

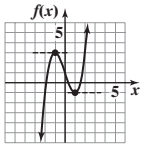
1. Decreasing **2.** Increasing **3.** Increasing **4.** Decreasing
5. Increasing **6.** Decreasing **7.** Decreasing **8.** Increasing
9. $(a, b); (d, f); (g, h)$ **10.** $(b, d); (f, g)$ **11.** $(b, c); (c, d); (f, g)$
12. $(a, b); (d, f); (g, h)$ **13.** c, d, f **14.** b, g **15.** b, f **16.** d, g
17. Local maximum at $x = a$; local minimum at $x = c$; no local extrema at $x = b$ and $x = d$ **18.** Local maximum at $x = d$; local minimum at $x = b$; no local extrema at $x = a$ or $x = c$ **19.** $f(3) = 5$ is a local maximum; e
20. $f(3) = 1$ is a local minimum; b **21.** No local extremum; d **22.** No local extremum; h **23.** $f(3) = 5$ is a local maximum; f **24.** No local extremum; g **25.** No local extremum; c **26.** $f(3) = 1$ is a local minimum; a
27. (A) $f'(x) = 3x^2 - 12$ (B) $-2, 2$ (C) $-2, 2$
28. (A) $f'(x) = 3x^2 - 27$ (B) $-3, 3$ (C) $-3, 3$
29. (A) $f'(x) = -\dfrac{6}{(x+2)^2}$ (B) -2 (C) 0
30. (A) $f'(x) = -\dfrac{5}{(x-4)^2}$ (B) 4 (C) None
31. (A) $f'(x) = \dfrac{1}{3}x^{-2/3}$ (B) 0 (C) 0
32. (A) $f'(x) = \dfrac{2}{3}x^{-1/3}$ (B) 0 (C) 0 **33.** Decreasing on $(-\infty, 2)$; increasing on $(2, \infty)$; $f(2) = -10$ is a local minimum **34.** Decreasing on $(-\infty, 1)$; increasing on $(1, \infty)$; $f(1) = -8$ is a local minimum
35. Increasing on $(-\infty, -4)$; decreasing on $(-4, \infty)$; $f(-4) = 7$ is a local maximum **36.** Increasing on $(-\infty, 2)$; decreasing on $(2, \infty)$; $f(2) = 7$ is a local maximum **37.** Increasing for all x; no local extrema
38. Decreasing for all x; no local extrema **39.** Increasing on $(-\infty, -1)$ and $(1, \infty)$; decreasing on $(-1, 1)$; $f(-1) = 7$ is a local maximum, $f(1) = 3$ is a local minimum **40.** Decreasing on $(-\infty, -1)$ and $(1, \infty)$; increasing on $(-1, 1)$; $f(-1) = 5$ is a local minimum, $f(1) = 9$ is a local maximum
41. Decreasing on $(-\infty, -4)$ and $(2, \infty)$; increasing on $(-4, 2)$; $f(-4) = -220$ is a local minimum and $f(2) = 104$ is a local maximum **42.** Increasing on $(-\infty, -10)$ and $(8, \infty)$; decreasing on $(-10, 8)$; $f(-10) = 5{,}085$ is a local maximum, $f(8) = -3{,}663$ is a local minimum **43.** Decreasing on $(-\infty, -3)$; increasing on $(-3, \infty)$; $f(-3) = 3$ is a local minimum **44.** Decreasing on $(-\infty, 6)$; increasing on $(6, \infty)$; $f(6) = -400$ is a local minimum **45.** Decreasing on $(-\infty, -4)$; increasing on $(-4, \infty)$; $f(-4) = -e^{-4} \approx -0.0183$ is a local minimum **46.** Decreasing on $(-\infty, -3)$; increasing on $(-3, \infty)$; $f(-3) = -e^{-3} \approx -0.0498$ is a local minimum **47.** Decreasing on $(-\infty, -2)$ and $(0, 2)$; increasing on $(-2, 0)$ and $(2, \infty)$; $f(-2) = 0$ and $f(2) = 0$ are local minima; $f(0) = \sqrt[3]{16} \approx 2.5198$ is a local maximum **48.** Decreasing on $(-\infty, -2)$ and $(-2, 0)$; increasing on $(0, 2)$ and $(2, \infty)$; $f(0) = \sqrt[3]{-4} \approx -1.5874$ is a local minimum
49. Increasing on $(-\infty, 4)$
Decreasing on $(4, \infty)$
Horizontal tangent at $x = 4$



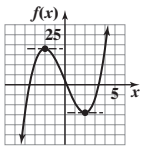
50. Increasing on $(2, \infty)$
Decreasing on $(-\infty, 2)$
Horizontal tangent $x = 2$



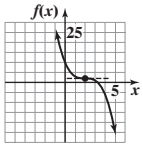
51. Increasing on $(-\infty, -1), (1, \infty)$
Decreasing on $(-1, 1)$
Horizontal tangents at $x = -1, 1$



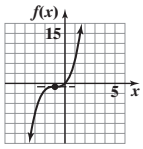
52. Increasing on $(-\infty, -2), (2, \infty)$
Decreasing on $(-2, 2)$
Horizontal tangents $x = -2, 2$



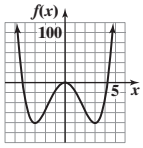
53. Decreasing for all x
Horizontal tangent at $x = 2$



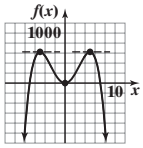
54. Increasing for all x
Horizontal tangent at $x = -1$



55. Decreasing on $(-\infty, -3)$ and $(0, 3)$; increasing on $(-3, 0)$ and $(3, \infty)$ Horizontal tangents at $x = -3, 0, 3$



56. Increasing on $(-\infty, -5), (0, 5)$
Decreasing on $(-5, 0), (5, \infty)$
Horizontal tangents at $x = -5, 0, 5$



57. Critical numbers: $x = -0.77, 1.08, 2.69$; decreasing on $(-\infty, -0.77)$ and $(1.08, 2.69)$; increasing on $(-0.77, 1.08)$ and $(2.69, \infty)$; $f(-0.77) = -4.75$ and $f(2.69) = -1.29$ are local minima; $f(1.08) = 6.04$ is a local maximum **58.** Critical numbers: $x = -3.43, -1.22, 0.90$; decreasing on $(-\infty, -3.43)$ and $(-1.22, 0.90)$; increasing on $(-3.43, -1.22)$ and $(0.90, \infty)$; $f(-3.43) = -11.91$ and $f(0.90) = -9.20$ are local minima; $f(-1.22) = 11.44$ is a local maximum **59.** Critical numbers: $x = -2.83, -0.20$; decreasing on $(-\infty, -2.83)$ and $(-0.20, \infty)$; increasing on $(-2.83, -0.20)$; $f(-2.83) = -7.08$ is a local minimum; $f(-0.20) = 1.10$ is a local maximum **60.** Critical numbers: 0.36, 2.15; increasing on $(-\infty, 0.36)$ and $(2.15, \infty)$; decreasing on $(0.36, 2.15)$; $f(0.36) = 1.17$ is a local maximum; $f(2.15) = -0.66$ is a local minimum

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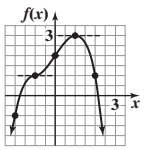

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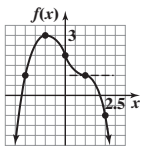

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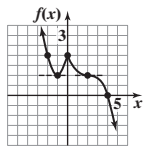

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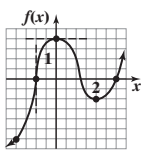

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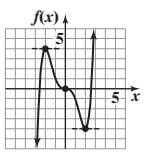

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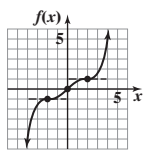

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2 LIMITS AND THE DERIVATIVE

EXERCISE 2-1

2. $x^2 - 64 = (x-8)(x+8)$ **4.** $x^2 + 5x - 36 = (x+9)(x-4)$

6. $x^3 + 15x^2 + 50x = x(x^2 + 15x + 50) = x(x+5)(x+10)$ **8.** $20x^2 + 11x - 3 = (4x+3)(5x-1)$

10. $f(0.5) = 2$ **12.** $f(2.25) = 2.25$

14. (A) $\lim\limits_{x\to 1^-} f(x) = 2$ (B) $\lim\limits_{x\to 1^+} f(x) = 2$ (C) $\lim\limits_{x\to 1} f(x) = 2$ (D) $f(1) = 2$

16. (A) $\lim\limits_{x\to 4^-} f(x) = 4$ (B) $\lim\limits_{x\to 4^+} f(x) = 4$ (C) $\lim\limits_{x\to 4} f(x) = 4$ (D) $f(4)$ does not exist

18. $g(2.1) = 1.9$ **20.** $g(2.5) = 1.5$

22. (A) $\lim\limits_{x\to 2^-} g(x) = 2$ (B) $\lim\limits_{x\to 2^+} g(x) = 2$ (C) $\lim\limits_{x\to 2} g(x) = 2$ (D) $g(2) = 2$

24. (A) $\lim\limits_{x\to 4^-} g(x) = 0$ (B) $\lim\limits_{x\to 4^+} g(x) = 0$ (C) $\lim\limits_{x\to 4} g(x) = 0$ (D) $g(4) = 0$

26. (A) $\lim\limits_{x\to -2^+} f(x) = 3$ (B) $\lim\limits_{x\to -2^-} f(x) = -3$

(C) Since $\lim\limits_{x\to -2^+} f(x) \neq \lim\limits_{x\to -2^-} f(x)$, $\lim\limits_{x\to -2} f(x)$ does not exist.

(D) $f(-2) = -3$

28. (A) $\lim\limits_{x\to 2^+} f(x) = -3$ (B) $\lim\limits_{x\to 2^-} f(x) = 3$

(C) $\lim\limits_{x\to 2} f(x)$ does not exist since $\lim\limits_{x\to 2^+} f(x) \neq \lim\limits_{x\to 2^-} f(x)$

(D) $f(2) = 3$

30. $3x \to -6$ as $x \to -2$; thus $\lim\limits_{x\to -2} 3x = -6$

32. $x - 3 \to 5 - 3 = 2$ as $x \to 5$; thus $\lim\limits_{x\to 5} (x - 3) = 2$

34. $x(x + 3) \to (-1)(-1 + 3) = -2$ as $x \to -1$; thus $\lim\limits_{x\to -1} x(x + 3) = -2$

36. $x - 2 \to 4 - 2 = 2$ as $x \to 4$; thus $\lim\limits_{x\to 4} \dfrac{x-2}{x} = \dfrac{2}{4} = \dfrac{1}{2}$

38. $\sqrt{16-7x} \to \sqrt{16-7(0)} = \sqrt{16} = 4$ as $x \to 0$; thus $\lim\limits_{x\to 0} \sqrt{16-7x} = 4$

40. $\lim\limits_{x\to 1} 2g(x) = 2 \lim\limits_{x\to 1} g(x) = 2(4) = 8$

42. $\lim\limits_{x\to 1} [g(x) - 3f(x)] = \lim\limits_{x\to 1} g(x) - 3 \lim\limits_{x\to 1} f(x) = 4 - 3(-5) = 19$

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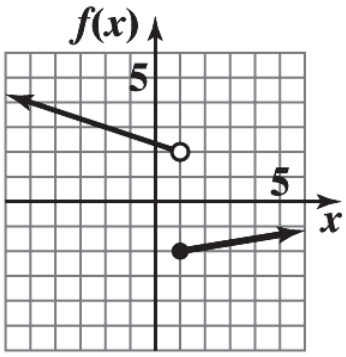
44. $\lim\limits_{x\to 1}\dfrac{3-f(x)}{1-4g(x)} = \dfrac{\lim\limits_{x\to 1}[3-f(x)]}{\lim\limits_{x\to 1}[1-4g(x)]} = \dfrac{3-\lim\limits_{x\to 1}f(x)}{1-4\lim\limits_{x\to 1}g(x)} = \dfrac{3-(-5)}{1-4(4)} = -\dfrac{8}{15}$

46. $\lim\limits_{x\to 1}\sqrt[3]{2x+2f(x)} = \sqrt[3]{\lim\limits_{x\to 1}[2x+2f(x)]}$

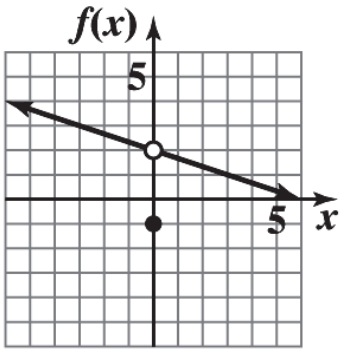
$= \sqrt[3]{2\lim\limits_{x\to 1}x+2\lim\limits_{x\to 1}f(x)}$

$= \sqrt[3]{2-10} = -2$

48.



50.



52. $f(x) = \begin{cases} 2+x & \text{if} \quad x \le 0 \\ 2-x & \text{if} \quad x > 0 \end{cases}$

(A) $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+}(2-x) = 2$

(B) $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-}(2+x) = 2$

(C) $\lim\limits_{x\to 0} f(x) = 2$ since $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^-} f(x) = 2$

(D) $f(0) = 2 + 0 = 2$

54. $f(x) = \begin{cases} x+3 & \text{if} \quad x < -2 \\ \sqrt{x+2} & \text{if} \quad x > -2 \end{cases}$

(A) $\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2^+}\sqrt{x+2} = 0$

(B) $\lim\limits_{x\to -2^-} f(x) = \lim\limits_{x\to -2^-}(x+3) = 1$

(C) $\lim\limits_{x\to -2} f(x)$ does not exist since $\lim\limits_{x\to -2^+} f(x) \ne \lim\limits_{x\to -2^-} f(x)$

(D) $f(-2)$ does not exist; f is not defined at $x = -2$.

56. $f(x) = \begin{cases} \dfrac{x}{x+3} & \text{if} \quad x < 0 \\ \dfrac{x}{x-3} & \text{if} \quad x > 0 \end{cases}$

(A) $\lim\limits_{x\to -3} f(x) = \lim\limits_{x\to -3}\dfrac{x}{x+3}$ does not exist since $x = -3$ is a non-removable zero of the denominator.

(B) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0^-}\dfrac{x}{x+3} = \lim\limits_{x\to 0^+}\dfrac{x}{x+3} = 0$

(C) $\lim\limits_{x\to 3} f(x)$ does not exist, since $\lim\limits_{x\to 3^+} f(x)$ does not exist.

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58. $f(x) = \dfrac{x-3}{|x-3|} = \begin{cases} \dfrac{x-3}{-(x-3)} = -1 & \text{if} \quad x < 3 \\ \dfrac{x-3}{x-3} = 1 & \text{if} \quad x > 3 \end{cases}$

(Note: Observe that for $x < 3$, $|x-3| = 3 - x = -(x-3)$
and for $x > 3$, $|x-3| = x - 3$)

(A) $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} 1 = 1$ (B) $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} (-1) = -1$

(C) $\lim\limits_{x\to 3} f(x)$ does not exist, since $\lim\limits_{x\to 3^+} f(x) \neq \lim\limits_{x\to 3^-} f(x)$

(D) $f(3)$ does not exist; f is not defined at $x = 3$.

60. $f(x) = \dfrac{x+3}{x^2+3x} = \dfrac{x+3}{x(x+3)}$

(A) $\lim\limits_{x\to -3} \dfrac{x+3}{x(x+3)} = \lim\limits_{x\to -3} \dfrac{1}{x} = -\dfrac{1}{3}$ (B) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{1}{x}$ does not exist. (C) $\lim\limits_{x\to 3} \dfrac{1}{x} = \dfrac{1}{3}$

62. $f(x) = \dfrac{x^2+x-6}{x+3} = \dfrac{(x+3)(x-2)}{(x+3)}$

(A) $\lim\limits_{x\to -3} f(x) = \lim\limits_{x\to -3} \dfrac{(x+3)(x-2)}{(x+3)} = \lim\limits_{x\to -3} (x-2) = -5$

(B) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{x^2+x-6}{x+3} = \dfrac{-6}{3} = -2$

(C) $\lim\limits_{x\to 2} f(x) = \lim\limits_{x\to 2} \dfrac{x^2+x-6}{x+3} = \dfrac{0}{5} = 0$

64. $f(x) = \dfrac{x^2-1}{(x+1)^2} = \dfrac{(x-1)(x+1)}{(x+1)^2}$

(A) $\lim\limits_{x\to -1} f(x) = \lim\limits_{x\to -1} \dfrac{(x-1)(x+1)}{(x+1)^2} = \lim\limits_{x\to -1} \dfrac{x-1}{x+1}$ does not exist since
$\lim\limits_{x\to -1} (x-1) = -2$ but $\lim\limits_{x\to -1} (x+1) = 0$.

(B) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{x^2-1}{(x+1)^2} = \dfrac{-1}{1} = -1$

(C) $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \dfrac{x^2-1}{(x+1)^2} = \dfrac{0}{4} = 0$

66. $f(x) = \dfrac{3x^2+2x-1}{x^2+3x+2} = \dfrac{(3x-1)(x+1)}{(x+2)(x+1)}$

(A) $\lim\limits_{x\to -3} f(x) = \lim\limits_{x\to -3} \dfrac{3x^2+2x-1}{x^2+3x+2} = \dfrac{20}{2} = 10$

(B) $\lim\limits_{x\to -1} f(x) = \lim\limits_{x\to -1} \dfrac{(3x-1)(x+1)}{(x+2)(x+1)} = \lim\limits_{x\to -1} \dfrac{3x-1}{x+2} = \dfrac{-4}{1} = -4$

(C) $\lim\limits_{x\to 2} f(x) = \lim\limits_{x\to 2} \dfrac{3x^2+2x-1}{x^2+3x+2} = \dfrac{15}{12} = \dfrac{5}{4}$

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68. True: $\lim\limits_{x\to 1}\dfrac{f(x)}{g(x)}=\dfrac{\lim\limits_{x\to 1}f(x)}{\lim\limits_{x\to 1}g(x)}=\dfrac{1}{1}=1$

70. Not always true. For example, the statement is false for $f(x)=\begin{cases}-1 & x\le 0\\ 1 & x>0\end{cases}$

72. Not always true. For example, the statement is false for $f(x)=\dfrac{1}{x}$.

74. $\lim\limits_{x\to -3}\dfrac{x-2}{x+3}$ does not have the form $\dfrac{0}{0}$; the limit does not exist since

$\lim\limits_{x\to -3^-}\dfrac{x-2}{x+3}=\infty,\ \lim\limits_{x\to -3^+}\dfrac{x-2}{x+3}=-\infty.$

76. $\lim\limits_{x\to 3}\dfrac{(x+1)(x-3)}{(x-3)(x-4)}$ has the form $\dfrac{0}{0}$; $\dfrac{(x+1)(x-3)}{(x-3)(x-4)}=\dfrac{x+1}{x-4}$ provided $x\ne 3$.

Therefore $\lim\limits_{x\to 3}\dfrac{(x+1)(x-3)}{(x-3)(x-4)}=\lim\limits_{x\to 3}\dfrac{x+1}{x-4}=-4.$

78. $\lim\limits_{x\to 5}\dfrac{x^2-7x+10}{x^2-4x-5}$ has the form $\dfrac{0}{0}$; $\dfrac{x^2-7x+10}{x^2-4x-5}=\dfrac{(x-5)(x+2)}{(x-5)(x+1)}=\dfrac{x-5}{x+1}$, provided $x\ne 5$.

Therefore $\lim\limits_{x\to 5}\dfrac{x^2-7x+10}{x^2-4x-5}=\lim\limits_{x\to 5}\dfrac{x-2}{x+1}=\dfrac{1}{2}.$

80. $\lim\limits_{x\to 2}\dfrac{x^2+2x+1}{x^2-2x+1}=\lim\limits_{x\to 2}\dfrac{(x+1)^2}{(x-1)^2}$ does not have the form $\dfrac{0}{0}$; $\lim\limits_{x\to 2}\dfrac{x^2+2x+1}{x^2-2x+1}=9.$

82. $f(x)=5x-1$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{5(2+h)-1-(10-1)}{h}=\lim_{h\to 0}\frac{10+5h-1-9}{h}=\lim_{h\to 0}\frac{5h}{h}=\lim_{h\to 0}5=5$$

84. $f(x)=x^2-2$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{(2+h)^2-2-(4-2)}{h}=\lim_{h\to 0}\frac{4+4h+h^2-2-2}{h}$$

$$=\lim_{h\to 0}\frac{4h+h^2}{h}=\lim_{h\to 0}(4+h)=4$$

86. $f(x)=-4x+13$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{-4(2+h)+13-[-4(2)+13]}{h}=\lim_{h\to 0}\frac{-4h}{h}=-4$$

88. $f(x)=-3|x|$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{-3|2+h|-[-3(2)]}{h}=\lim_{h\to 0}\frac{-3(2+h)+6}{h}=\lim_{h\to 0}\frac{-3h}{h}=-3$$

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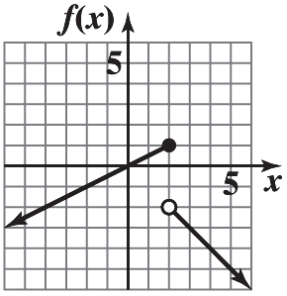
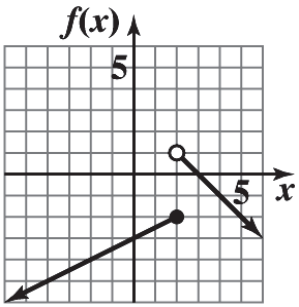
90. (A) $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} (0.5x) = 1$

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} (-x) = -2$



(B) $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} (-3 + 0.5x) = -2$

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} (3 - x) = 1$



(C) $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} (-3m + 0.5x) = -3m + 1$

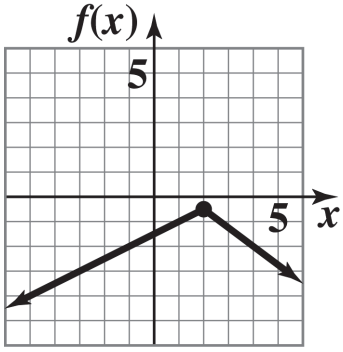
$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} (3m - x) = 3m - 2$

$-3m + 1 = 3m - 2$

$6m = 3$

$m = \dfrac{1}{2} = 0.5$

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) = -0.5$

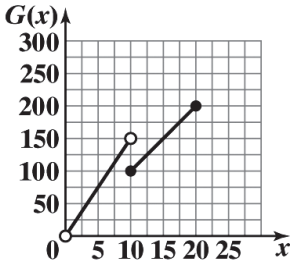


(D) The graph in (A) is broken when it jumps from (2, 1) down to (2, –2), the graph in (B) is also broken when it jumps from (2, –2) up to (2, 1), while the graph in (C) is one continuous piece with no jumps or breaks.

92. (A) For car sharing of not more than 10 hours, the charge per hour is $15x$. The charge per hour for sharing more than 10 hours is $10x$. Thus,

$$G(x) = \begin{cases} 15x & \text{if} \quad 0 < x \le 10 \\ 10x & \text{if} \quad x > 10 \end{cases}$$

(B)



(C) As x approaches 10 from the left, $G(x)$ approaches 150, thus, the left limit of $G(x)$ at $x = 10$ exists, $\lim\limits_{x\to 10^-} G(x) = 150$.

Similarly, $\lim\limits_{x\to 10^+} G(x) = 100$. However, $\lim\limits_{x\to 10} G(x)$ does not exist, since $\lim\limits_{x\to 10^-} G(x) \ne \lim\limits_{x\to 10^+} G(x)$.

94. For car sharing of more more than 10 hours per month, the charge for the service given in Problem 91 is $9x$-40 while the charge for in Problem 92 is $10x$. It is clear that the latter is more expensive than the former.

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96. (A) Let x be the volume of a purchase before the discount is applied. Then $P(x)$ is given by:

$$P(x) = \begin{cases} x & \text{if } 0 \le x < 300 \\ 300 + 0.97(x - 300) = 0.97x + 9 & \text{if } 300 \le x < 1{,}000 \\ 0.97(1{,}000) + 9 + 0.95(x - 1{,}000) = 0.95x + 29 & \text{if } 1{,}000 \le x < 3{,}000 \\ 0.95(3{,}000) + 29 + 0.93(x - 3{,}000) = 0.93x + 89 & \text{if } 3{,}000 \le x < 5{,}000 \\ 0.93(5{,}000) + 89 + 0.90(x - 5{,}000) = 0.90x + 239 & \text{if } x \ge 5{,}000 \end{cases}$$

(B) $\lim\limits_{x \to 1{,}000^-} P(x) = 0.97(1{,}000) + 9 = 979$

$\lim\limits_{x \to 1{,}000^+} P(x) = 0.95(1{,}000) + 29 = 979$

Thus, $\lim\limits_{x \to 1{,}000} P(x) = 979$

$\lim\limits_{x \to 3{,}000^-} P(x) = 0.95(3{,}000) + 29 = 2{,}879$

$\lim\limits_{x \to 3{,}000^+} P(x) = 0.93(3{,}000) + 89 = 2{,}879$

Thus, $\lim\limits_{x \to 3{,}000} P(x) = 2{,}879$

(C) For $0 \le x < 300$, they produce the same price. For $x \ge 300$, the one in Problem 95 produces a lower price.

98. From Problem 97, we have:

$$F(x) = \begin{cases} 20x & \text{if } 0 < x \le 4{,}000 \\ 80{,}000 & \text{if } x > 4{,}000 \end{cases}$$

Thus

$$A(x) = \frac{F(x)}{x} = \begin{cases} 20 & \text{if } 0 < x \le 4{,}000 \\ \dfrac{80{,}000}{x} & \text{if } x > 4{,}000 \end{cases}$$

$$\lim_{x \to 4{,}000^-} A(x) = \lim_{x \to 4{,}000^+} A(x) = 20 = \lim_{x \to 4{,}000} A(x)$$

$$\lim_{x \to 8{,}000^-} A(x) = \lim_{x \to 8{,}000^+} A(x) = \frac{80{,}000}{8{,}000} = 10 = \lim_{x \to 8{,}000} A(x)$$

EXERCISE 2-2

2. $x = 5$

4. $y = 1$

6. $y + 4 = -3(x - 8)$ (point-slope form); $3x + y = 20$

8. Slope: $m = \dfrac{30 - 20}{1 - (-1)} = 5$; $y - 20 = 5[x - (-1)]$ (point-slope form); $-5x + y = 25$

10. $\lim\limits_{x \to -\infty} f(x) = \infty$

12. $\lim\limits_{x \to -2^-} f(x) = \infty$

14. $\lim\limits_{x \to 2^+} f(x) = \infty$

16. $\lim\limits_{x \to 2} f(x)$ does not exist

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18. $f(x) = \dfrac{x^2}{x+3}$

(A) $\lim\limits_{x\to -3^-} \dfrac{x^2}{x+3} = -\infty$; as x approaches –3 from the left, the denominator is negatively approaching 0 and the numerator is positively approaching $(-3)^2 = 9$.

(B) $\lim\limits_{x\to -3^+} \dfrac{x^2}{x+3} = \infty$; numerator approaches $(-3)^2 = 9$ and denominator is positively approaching 0.

(C) Since left and right limits at –3 are not equal, $\lim\limits_{x\to -3} f(x)$ does not exist.

20. $f(x) = \dfrac{2x+2}{(x+2)^2}$

(A) $\lim\limits_{x\to -2^-} \dfrac{2x+2}{(x+2)^2} = -\infty$; as x approaches –2 from the left, the denominator is positively approaching 0 and the numerator is negatively approaching $2(-2) + 2 = -2$.

(B) $\lim\limits_{x\to -2^+} \dfrac{2x+2}{(x+2)^2} = -\infty$; as x approaches –2 from the right, the denominator is positively approaching 0 and the numerator is negatively approaching $2(-2) + 2 = -2$.

(C) Since $\lim\limits_{x\to -2^-} f(x) = \lim\limits_{x\to -2^+} f(x) = -\infty$, we can say that $\lim\limits_{x\to -2} f(x) = -\infty$.

22. $f(x) = \dfrac{x^2+x+2}{x-1}$

(A) $\lim\limits_{x\to 1^-} \dfrac{x^2+x+2}{x-1} = -\infty$; as x approaches 1, the numerator approaches 4 and the denominator negatively approaches 0.

(B) $\lim\limits_{x\to 1^+} \dfrac{x^2+x+2}{x-1} = \infty$; in this case the denominator positively approaches 0.

(C) $\lim\limits_{x\to 1} \dfrac{x^2+x+2}{x-1}$ does not exist.

24. $f(x) = \dfrac{x^2+x-2}{(x+2)}$

$f(x) = \dfrac{(x-1)(x+2)}{(x+2)}$

(A) $\lim\limits_{x\to -2^-} \dfrac{(x-1)(x+2)}{(x+2)} = \lim\limits_{x\to -2^-} (x-1) = -3$

(B) $\lim\limits_{x\to -2^+} \dfrac{(x-1)(x+2)}{(x+2)} = \lim\limits_{x\to -2^+} (x-1) = -3$

(C) $\lim\limits_{x\to -2} \dfrac{(x-1)(x+2)}{(x+2)} = \lim\limits_{x\to -2} (x-1) = -3$ or we can say that left and right limits at $x = -2$ exist and are equal, therefore $\lim\limits_{x\to -2} f(x)$ exists and is equal to the common value –3.

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26. $p(x) = 10 - x^6 + 7x^3 = -x^6 + 7x^3 + 10$

(A) Leading term: $-x^6$ (B) $\lim\limits_{x\to\infty} p(x) = \lim\limits_{x\to\infty}(-x^6) = -\infty$ (C) $\lim\limits_{x\to-\infty} p(x) = \lim\limits_{x\to-\infty}(-x^6) = -\infty$

28. $p(x) = -x^5 + 2x^3 + 9x$

(A) Leading term: $-x^5$ (B) $\lim\limits_{x\to\infty} p(x) = \lim\limits_{x\to\infty}(-x^5) = -\infty$ (C) $\lim\limits_{x\to-\infty} p(x) = \lim\limits_{x\to-\infty}(-x^5) = \infty$

30. $p(x) = 5x + x^3 - 8x^2 = x^3 - 8x^2 + 5x$

(A) Leading term: x^3 (B) $\lim\limits_{x\to\infty} p(x) = \lim\limits_{x\to\infty}(x^3) = \infty$ (C) $\lim\limits_{x\to-\infty} p(x) = \lim\limits_{x\to-\infty}(x^3) = -\infty$

32. $p(x) = 1 + 4x^2 + 4x^4 = 4x^4 + 4x^2 + 1$

(A) Leading term: $4x^4$ (B) $\lim\limits_{x\to\infty} p(x) = \lim\limits_{x\to\infty}(4x^4) = \infty$ (C) $\lim\limits_{x\to-\infty} p(x) = \lim\limits_{x\to-\infty}(4x^4) = \infty$

34. (A) $f(5) = \dfrac{2-3(5)^3}{7+4(5)^3} = -\dfrac{373}{507} \approx -0.736$

(B) $f(10) = \dfrac{2-3(10)^3}{7+4(10)^3} = -\dfrac{2,998}{4,007} \approx -0.748$

(C) $\lim\limits_{x\to\infty} \dfrac{2-3x^3}{7+4x^3} = \lim\limits_{x\to\infty} \dfrac{-3x^3}{4x^3} = \lim\limits_{x\to\infty} \dfrac{\frac{2}{x^3} - 3}{\frac{7}{x^3} + 4}$ (Divide numerator and denominator by x^3.)

$$= \frac{0-3}{0+4} = \frac{-3}{4}.$$

36. (A) $f(-8) = \dfrac{5(-8)+11}{7(-8)^3 - 2} = \dfrac{-29}{-3,586} = \dfrac{29}{3,586} \approx 0.008$

(B) $f(-16) = \dfrac{5(-16)+11}{7(-16)^3 - 2} = \dfrac{-69}{-28,674} = \dfrac{69}{28,674} \approx 0.002$

(C) $\lim\limits_{x\to\infty} \dfrac{5x+11}{7x^3 - 2} = \lim\limits_{x\to\infty} \dfrac{\frac{5}{x^2} + \frac{11}{x^3}}{7 - \frac{2}{x^3}}$ (Divide numerator and denominator by x^3.)

$$= \frac{0+0}{7-0} = 0$$

38. (A) $f(-3) = \dfrac{4(-3)^7 - 8(-3)}{6(-3)^4 + 9(-3)^2} = -\dfrac{8,724}{567} \approx -15.386$

(B) $f(-6) = \dfrac{4(-6)^7 - 8(-6)}{6(-6)^4 + 9(-6)^2} = -\dfrac{1,119,696}{8,100} \approx -138.234$

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(C) $\lim\limits_{x\to-\infty}\dfrac{4x^7-8x}{6x^4+9x^2}=\lim\limits_{x\to-\infty}\dfrac{4x^3-\dfrac{8}{x^3}}{6+\dfrac{9}{x^2}}$ (Divide numerator and denominator by x^4.)

As $x\to-\infty$, $4x^3-\dfrac{8}{x^3}\to-\infty$ and $6+\dfrac{9}{x^2}\to 6$. Therefore, $\lim\limits_{x\to-\infty}\dfrac{4x^7-8x}{6x^4+9x^2}=-\infty$.

40. (A) $f(-50)=\dfrac{3+(-50)}{5+4(-50)}=\dfrac{47}{195}\approx 0.241$

(B) $f(-100)=\dfrac{3+(-100)}{5+4(-100)}=\dfrac{97}{395}\approx 0.246$

(C) $\lim\limits_{x\to-\infty}\dfrac{3+x}{5+4x}=\lim\limits_{x\to\infty}\dfrac{\dfrac{3}{x}+1}{\dfrac{5}{x}+4}$ (Divide numerator and denominator by x.)

$=\dfrac{0+1}{0+4}=\dfrac{1}{4}$

42. $f(x)=\dfrac{2x}{x-5}$; $\lim\limits_{x\to5^-}f(x)=-\infty$, $\lim\limits_{x\to5^+}f(x)=\infty$; $x=5$ is a vertical asymptote.

44. $f(x)=\dfrac{x+2}{x^2+3}$, the denominator has no zeros; no vertical asymptotes.

46. $f(x)=\dfrac{x-5}{x^2-16}=\dfrac{x-5}{(x+4)(x-4)}$; $\lim\limits_{x\to-4^-}f(x)=-\infty$, $\lim\limits_{x\to-4^+}f(x)=\infty$,
$\lim\limits_{x\to4^-}f(x)=\infty$, $\lim\limits_{x\to4^+}f(x)=-\infty$; $x=-4$ and $x=4$ are vertical asymptotes.

48. $f(x)=\dfrac{x^2-1}{x^3+2x^2+3x}=\dfrac{(x+1)(x-1)}{x(x+2)(x+1)}=\dfrac{x-1}{x(x+2)}$, $x\neq-1$
$\lim\limits_{x\to-2^-}f(x)=-\infty$, $\lim\limits_{x\to-2^+}f(x)=\infty$, $\lim\limits_{x\to-1}f(x)=2$, $\lim\limits_{x\to0^-}f(x)=\infty$, $\lim\limits_{x\to0^+}f(x)=-\infty$;
$x=-2$ and $x=0$ are vertical asymptotes.

50. $f(x)=\dfrac{x^2+2x-15}{x^2+2x-8}=\dfrac{(x+5)(x-3)}{(x+4)(x-2)}$;
$\lim\limits_{x\to-4^-}f(x)=-\infty$, $\lim\limits_{x\to-4^+}f(x)=\infty$, $\lim\limits_{x\to2^-}f(x)=\infty$, $\lim\limits_{x\to2^+}f(x)=-\infty$;
$x=-4$ and $x=2$ are vertical asymptotes.

52. $f(x)=\dfrac{3x+2}{x-4}$

$\lim\limits_{x\to\infty}f(x)=\lim\limits_{x\to\infty}\dfrac{3x+2}{x-4}=\lim\limits_{x\to\infty}\dfrac{3+\dfrac{2}{x}}{1-\dfrac{4}{x}}=\dfrac{3+0}{1-0}=3$

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So $y = 3$ is the horizontal asymptote.
Vertical asymptote: $x = 4$ (since $n(4) = 14$, $d(4) = 0$).

54. $f(x) = \dfrac{x^2 - 1}{x^2 + 2}$.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2 - 1}{x^2 + 2} = \lim_{x\to\infty} \frac{1 - \frac{1}{x^2}}{1 + \frac{2}{x^2}}$$ (Dividing the numerator and denominator by x^2.)

$$= \frac{1 - 0}{1 + 0} = 1$$

So, the horizontal asymptote is: $y = 1$.
$d(x) = x^2 + 2 > 0$ so, there are no vertical asymptotes.

56. $f(x) = \dfrac{x}{x^2 - 4} = \dfrac{x}{(x-2)(x+2)}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x}{x^2 - 4} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1 - \frac{4}{x^2}} = \frac{0}{1 - 0} = 0,$$

so the horizontal asymptote is: $y = 0$.
Since $n(-2) = -2$, $n(2) = 2$, $d(-2) = d(2) = 0$, we have two vertical asymptotes: $x = -2$, $x = 2$.

58. $f(x) = \dfrac{x^2 + 9}{x}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2 + 9}{x} = \lim_{x\to\infty} \frac{1 + \frac{9}{x^2}}{\frac{1}{x}} = \frac{1 + 0}{0} = \infty$$

So, there are no horizontal asymptotes. Since $n(0) = 9$, $d(0) = 0$,
$x = 0$ is the only vertical asymptote.

60. $f(x) = \dfrac{x + 5}{x^2}$.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x + 5}{x^2} = \lim_{x\to\infty} \frac{\frac{1}{x} + \frac{5}{x^2}}{1} = \frac{0 + 0}{1} = 0,$$

so the horizontal asymptote is: $y = 0$.
Since $n(0) = 5$, $d(0) = 0$, $x = 0$ is the vertical asymptote.

62. $f(x) = \dfrac{2x^2 + 7x + 12}{2x^2 + 5x - 12}$.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x^2 + 7x + 12}{2x^2 + 5x - 12} = \lim_{x\to\infty} \frac{2x^2}{2x^2} = 1,$$

so, $y = 1$ is the horizontal asymptote.
Since $n(-4) = 16$, $n\left(\dfrac{3}{2}\right) = 27$, $d(-4) = d\left(\dfrac{3}{2}\right) = 0$, $x = -4$ and $x = \dfrac{3}{2}$ are the vertical asymptotes.

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64. $f(x) = \dfrac{x^2 - x - 12}{2x^2 + 5x - 12}$

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x^2 - x - 12}{2x^2 + 5x - 12} = \lim\limits_{x\to\infty} \dfrac{x^2}{2x^2} = \dfrac{1}{2}$, so $y = \dfrac{1}{2}$ is the horizontal asymptote. Since $n(-4) = 8$, $n\left(\dfrac{3}{2}\right) = -11.25$, $d(-4) = d\left(\dfrac{3}{2}\right) = 0$, $x = -4$ and $x = \dfrac{3}{2}$ are the vertical asymptotes.

66. $f(x) = \dfrac{3 + 4x + x^2}{5 - x}$; $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{3 + 4x + x^2}{5 - x} = \lim\limits_{x\to\infty} \dfrac{x^2}{-x} = \lim\limits_{x\to\infty} (-x) = -\infty$

68. $f(x) = \dfrac{4x + 1}{5x - 7}$; $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{4x + 1}{5x - 7} = \lim\limits_{x\to\infty} \dfrac{4x}{5x} = \dfrac{4}{5}$

70. $f(x) = \dfrac{2x + 3}{x^2 - 1}$; $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{2x + 3}{x^2 - 1} = \lim\limits_{x\to-\infty} \dfrac{2x}{x^2} = 0$

72. $f(x) = \dfrac{6 - x^4}{1 + 2x}$; $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{6 - x^4}{1 + 2x} = \lim\limits_{x\to-\infty} \dfrac{-x^4}{2x} = \infty$

74. False: $f(x) = \dfrac{1}{(x-2)(x+2)} = \dfrac{1}{x^2 - 4}$ has two vertical asymptotes.

76. True: Theorem 4 gives three possible cases, two of which give exactly one horizontal asymptote and one of which gives no horizontal asymptote.

78. False: $f(x) = \dfrac{x^2 + 2x}{x^2 + x + 2}$ crosses the horizontal asymptote $y = 1$ at $x = 2$.

80. $\lim\limits_{x\to-\infty} (a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0) = \infty$ if $a_n > 0$ and n an even positive integer, or $a_n < 0$ and n an odd positive integer.

$\lim\limits_{x\to-\infty} (a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0) = -\infty$ if $a_n > 0$ and n is an odd positive integer or $a_n < 0$ and n is an even positive integer.

82. (A) Since $C(x)$ is a linear function of x, it can be written in the form

$C(x) = mx + b$

Since the fixed costs are \$300, $b = 300$.

Also, $C(20) = 5100$, so

$5100 = m(20) + 300$

$20m = 4800$

$m = 240$

Therefore, $C(x) = 240x + 300$

(B) $\overline{C}(x) = \dfrac{C(x)}{x}$

$= \dfrac{240x + 300}{x}$

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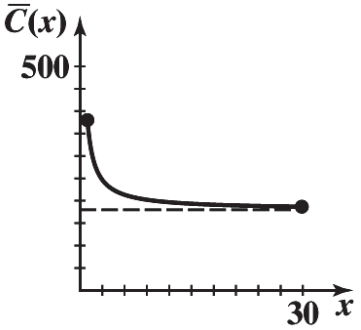
(C)



(D) $\overline{C}(x) = \dfrac{240x + 300}{x}$

$= \dfrac{240 + \dfrac{300}{x}}{1}$

As x increases, the numerator tends to 240 and the denominator is 1. Therefore, $\overline{C}(x)$ tends to 240 or \$240 per board. Therefore, $\overline{C}(x)$ tends to \$240 per board.

$$\lim_{x\to\infty} \overline{C}_c(x) = \lim_{x\to\infty}\left(\frac{2,700}{x} + 1,332\right) = 0 + 1,332 = 1,332$$

84. $P(t) = \dfrac{99t^2}{t^2 + 50}$

(A) $P(5) = \dfrac{99(5)^2}{5^2 + 50} = \dfrac{2475}{75} = 33$ or 33%; $P(10) = \dfrac{99(10)^2}{10^2 + 50} = \dfrac{9900}{150} = 66$ or 66%

$P(20) = \dfrac{99(20)^2}{20^2 + 50} = \dfrac{39,600}{450} = 88$ or 88%

(B) $\lim\limits_{t\to\infty} P(t) = \lim\limits_{t\to\infty} \dfrac{99t^2}{t^2 + 50} = \lim\limits_{t\to\infty} \dfrac{99}{1 + \dfrac{50}{t^2}}$ (divide numerator and denominator by t^2)

$= \dfrac{99}{1 + 0} = 99, \quad P(t) \to 99\%$

86. $C(t) = \dfrac{5t(t + 50)}{t^3 + 100}$

$\lim\limits_{t\to\infty} C(t) = \lim\limits_{t\to\infty} \dfrac{5t^2 + 250t}{t^3 + 100}$ (Divide numerator and denominator by t^3.)

$= \lim\limits_{t\to\infty} \dfrac{\dfrac{5}{t} + \dfrac{250}{t^2}}{1 + \dfrac{100}{t^3}} = \dfrac{0 + 0}{1 + 0} = 0$

The long term drug concentration is 0 mg/ml.

88. $N(t) = \dfrac{100t}{t + 9}, t \geq 0$

(A) $N(6) = \dfrac{100(6)}{6 + 9} = \dfrac{600}{15} \approx 40$ components/day

(B) $70 = \dfrac{100t}{t + 9}$ or

$70t + 630 = 100t$

$30t = 630$

$t = \dfrac{630}{30} = 21$ days

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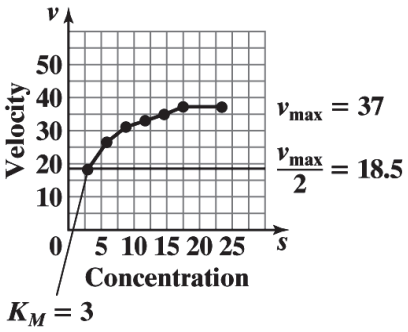
(C) $\lim_{t\to\infty} N(t) = \lim_{t\to\infty} \dfrac{100t}{t+9} = \lim_{t\to\infty} \dfrac{100}{1+\dfrac{9}{t}} = \dfrac{100}{1+0} = 100$

The maximum number of components an employee can produce in consecutive days is 100 components.

90. (A) $v_{\max} = 37, K_M = 3$



(B) $v(s) = \dfrac{37s}{3+s}$

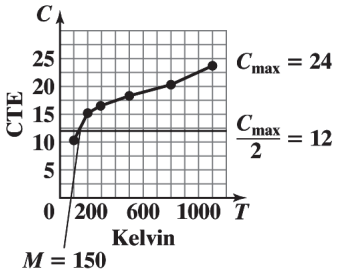
(C) For $s = 9$, $v = \dfrac{37(9)}{3+9} = 27.75$

For $v = 32$, $32 = \dfrac{37s}{3+s}$

or $96 + 32s = 37s$

and $s = \dfrac{96}{5} = 19.2$

92. (A) $C_{\max} = 24, M = 150$



(B) $C(T) = \dfrac{24T}{150+T}$

(C) For $T = 600$,

$C = \dfrac{(24)(600)}{150+600} = 19.2$

For $C = 12$, $12 = \dfrac{24T}{150+T}$ or

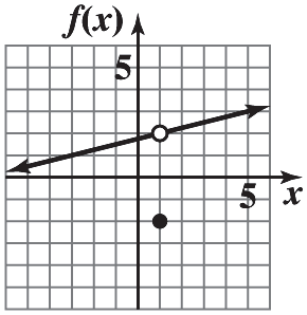
$1800 + 12T = 24T$,

$T = 150$.

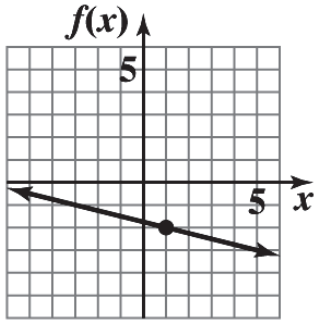
EXERCISE 2-3

2. $(-8,-4]$ **4.** $[0.1,0.3]$ **6.** $(-\infty,-4]\cup[4,\infty)$ **8.** $(-\infty,-6)\cup[9,\infty)$

10. f is discontinuous at $x = 1$ since $\lim_{x\to 1} f(x) \neq f(1)$



12. f is discontinuous at $x = 1$ since $\lim_{x\to 1} f(x) = f(1)$



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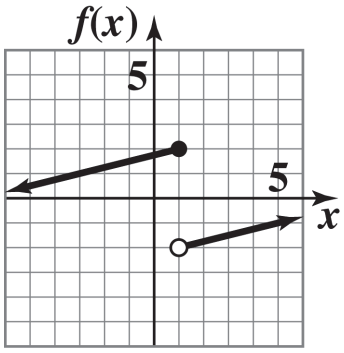
14. f is discontinuous at $x = 1$, since $\lim_{x\to 1} f(x)$ does not exist



16. $f(-2.1) = 1$

18. $f(-1.9) = 0.9$

20. (A) $\lim_{x\to 2^-} f(x) = 2$ (B) $\lim_{x\to 2^+} f(x) = 2$ (C) $\lim_{x\to 2} f(x) = 2$

$\left(\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 2\right)$

(D) $f(2)$ does not exist; f is not defined at $x = 2$.(E) No, since f is not even defined at $x = 2$.

22. (A) $\lim_{x\to -1^-} f(x) = 0$

(B) $\lim_{x\to -1^+} f(x) = 0$

(C) $\lim_{x\to -1} f(x) = 0$ $\left(\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) = 0\right)$

(D) $f(-1) = 0$

(E) Yes, since $\lim_{x\to -1} f(x) = f(0)$.

24. $g(-2.1) = 0.9$

26. $g(-1.9) = 2.95$

28. (A) $\lim_{x\to -2^-} g(x) = 1$

(B) $\lim_{x\to -2^+} g(x) = 3$

(C) $\lim_{x\to -2} g(x)$ does not exist, since $\lim_{x\to -2^-} g(x) \neq \lim_{x\to -2^+} g(x)$

(D) $g(-2)$ does not exist; g is not defined at $x = -2$. (E) No, since g is not even defined at $x = -2$.

30. (A) $\lim_{x\to 4^-} g(x) = 0$ (B) $\lim_{x\to 4^+} g(x) = 0$ (C) $\lim_{x\to 4} g(x) = 0$

$\left(\lim_{x\to 4^-} g(x) = \lim_{x\to 4^+} g(x) = 0\right)$

(D) $g(4) = 0$. (E) Yes, since $\lim_{x\to 4} f(x) = f(4)$

32. $h(x) = 4 - 2x$ is a polynomial function. Therefore, f is continuous for all x [Theorem 1(C)].

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34. $k(x) = \dfrac{2x}{x-4}$ is a rational function and the denominator $x - 4$ is 0 when $x = 4$. Thus, k is continuous for all x except $x = 4$ [Theorem 1(D)].

36. $n(x) = \dfrac{x-2}{(x-3)(x+1)}$ is a rational function and the denominator $(x - 3)(x + 1)$ is 0 when $x = 3$ or $x = -1$. Thus, n is continuous for all x except $x = 3$, $x = -1$ [Theorem 1(D)].

38. $G(x) = \dfrac{1-x^2}{x^2+1}$

$G(x)$ is a rational function and its denominator is never zero, hence by Theorem 1(D), $G(x)$ is continuous for all x.

40. $N(x) = \dfrac{x^2+4}{4-25x^2}$

$N(x)$ is a rational function and according to Theorem 1(D), $N(x)$ is continuous for all x except $x = \pm\dfrac{2}{5}$ which make the denominator 0.

42. $f(x) = \dfrac{2x+7}{5x-1}$; f is discontinuous at $x = \dfrac{1}{5}$; $f(x) = 0$ at $x = \dfrac{-7}{2}$. Partition numbers $\dfrac{1}{5}, \dfrac{-7}{2}$.

44. $f(x) = \dfrac{x^2+4}{x^2-9}$; f is discontinuous at $x = 3, -3$; $f(x) \neq 0$ for all x. Partition numbers $3, -3$.

46. $f(x) = \dfrac{x^3+x}{x^2-x-42} = \dfrac{x(x^2+1)}{(x-7)(x+6)}$; f is discontinuous at $x = 7, -6$; $f(x) = 0$ at $x = 0$. Partition numbers $-6, 0, 7$.

48. $x^2 - 2x - 8 < 0$

Let $f(x) = x^2 - 2x - 8 = (x - 4)(x + 2)$.
Then f is continuous for all x and $f(-2) = f(4) = 0$.
Thus, $x = -2$ and $x = 4$ are partition numbers.

Test Numbers

x	$f(x)$
-3	$7(+)$
0	$-8(-)$
5	$7(+)$

Thus, $x^2 - 2x - 8 < 0$ for: $-2 < x < 4$ (inequality notation), $(-2, 4)$ (interval notation)

50. $x^2 + 7x > -10$ or $x^2 + 7x + 10 > 0$

Let $f(x) = x^2 + 7x + 10 = (x + 2)(x + 5)$.
Then f is continuous for all x and $f(-5) = f(-2) = 0$.

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Thus, $x = -5$ and $x = -2$ are partition numbers.

Test Numbers

x	$f(x)$
-6	$4(+)$
-4	$-2(-)$
0	$10(+)$

Thus, $x^2 + 7x + 10 > 0$ for: $x < -5$ or $x > -2$ (inequality notation), $(-\infty, -5) \cup (-2, \infty)$ (interval notation)

52. $x^4 - 9x^2 > 0$

$x^4 - 9x^2 = x^2(x^2 - 9)$

Since $x^2 > 0$ for $x \neq 0$, then $x^4 - 9x^2 > 0$ if $x^2 - 9 > 0$ or $x^2 > 9$

or "$x < -3$ or $x > 3$" or $(-\infty, -3) \cup (3, \infty)$.

54. $\dfrac{x-4}{x^2+2x} < 0$

Let $f(x) = \dfrac{x-4}{x^2+2x} = \dfrac{x-4}{x(x+2)}$. Then f is discontinuous at $x = 0$ and

$x = -2$ and $f(4) = 0$. Thus, $x = -2$, $x = 0$, and $x = 4$ are partition numbers.

Test Numbers

x	$f(x)$
-3	$-\frac{7}{3}(-)$
-1	$5(+)$
1	$-1(-)$
5	$\frac{1}{35}(+)$

Thus, $\dfrac{x-4}{x^2+2x} < 0$ for: $x < -2$ or $0 < x < 4$ (inequality notation), $(-\infty, -2) \cup (0, 4)$(interval notation)

56. (A) $g(x) > 0$ for $x < -4$ or $x > 4$; $(-\infty, -4) \cup (4, \infty)$.

(B) $g(x) < 0$ for $-4 < x < 1$ or $1 < x < 4$; $(-4, 1) \cup (1, 4)$.

58. $f(x) = x^4 - 4x^2 - 2x + 2$. Partition numbers: $x_1 \approx 0.5113$, $x_2 \approx 2.1209$

(A) $f(x) > 0$ on $(-\infty, 0.5113) \cup (2.1209, \infty)$

(B) $f(x) < 0$ on $(0.5113, 2.1209)$

60. $f(x) = \dfrac{x^3 - 5x + 1}{x^2 - 1}$. Partition numbers: $x_1 \approx -2.3301$, $x_2 \approx -1$, $x_3 \approx 0.2016$, $x_4 = 1$, $x_5 \approx 2.1284$

(A) $f(x) > 0$ on $(-2.3301, -1) \cup (0.2016, 1) \cup (2.1284, \infty)$.

(B) $f(x) < 0$ on $(-\infty, -2.3301) \cup (-1, 0.2016) \cup (1, 2.1284)$.

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62. $\sqrt{7-x}$

Let $f(x) = 7 - x$. Then $\sqrt{7-x} = \sqrt[2]{f(x)}$ is continuous whenever $f(x)$ is continuous and nonnegative [Theorem 1(F)]. Since $f(x) = 7 - x$ is continuous for all x [Theorem 1(C)] and $f(x) \geq 0$ for $x \leq 7$, $\sqrt{7-x}$ is continuous on $(-\infty, 7]$.

64. $\sqrt[3]{x-8}$

Let $f(x) = x - 8$. Then $\sqrt[3]{x-8} = \sqrt[3]{f(x)}$ is continuous whenever $f(x)$ is continuous [Theorem 1(E)]. Since $f(x) = x - 8$ is continuous for all x [Theorem 1(C)], $\sqrt[3]{x-8}$ is continuous on $(-\infty, \infty)$.

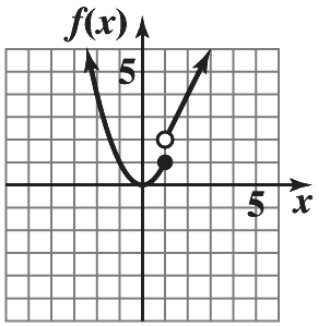
66. $\sqrt{4-x^2}$

Let $f(x) = 4 - x^2$. Then $\sqrt{4-x^2} = \sqrt[2]{f(x)}$ is continuous whenever $f(x)$ is continuous and nonnegative [Theorem 1(F)]. Since $f(x) = 4 - x^2$ is continuous for all x [Theorem 1(C)] and $f(x)$ is nonnegative on $[-2, 2]$, $\sqrt{4-x^2}$ is continuous on $[-2, 2]$.

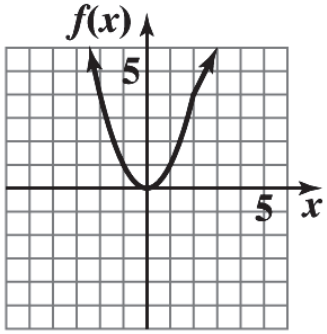
68. $\sqrt[3]{x^2+2}$

Let $f(x) = x^2 + 2$. Then $\sqrt[3]{x^2+2} = \sqrt[3]{f(x)}$ is continuous whenever $f(x)$ is continuous [Theorem 1(E)]. Since $f(x) = x^2 + 2$ is continuous for all x [Theorem 1(C)], $\sqrt[3]{x^2+2}$ is continuous on $(-\infty, \infty)$.

70. The graph of f is shown at the right. This function is discontinuous at $x = 1$.
$[\lim_{x \to 1^-} f(x) = 1$ and $\lim_{x \to 1^+} f(x) = 2$;
Thus, $\lim_{x \to 1} f(x)$ does not exist.]



72. The graph of f is shown at the right.
This function is continuous for all x. $[\lim_{x \to 2} f(x) = f(2) = 4]$



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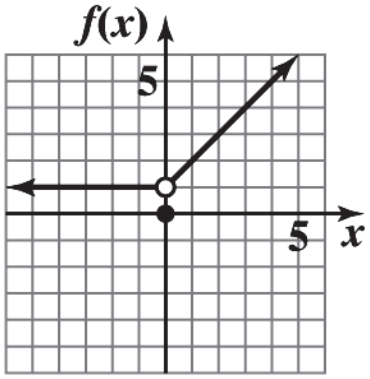
74. The graph of f is shown at the right.

This function is discontinuous at $x = 0$, since $\lim_{x\to 0} f(x) = 1 \neq f(0) = 0$



76. (A) Since $\lim_{x\to 2^+} f(x) = f(2) = 2$, f is continuous from the right at $x = 2$.

(B) Since $\lim_{x\to 2^-} f(x) = 1 \neq f(2) = 2$, f is not continuous from the left at $x = 2$.

(C) f is continuous on the open interval (1, 2).

(D) f is *not* continuous on the closed interval [1, 2] since $\lim_{x\to 2^-} f(x) = 1 \neq f(2) = 2$, i.e., f is not continuous from the left at $x = 2$.

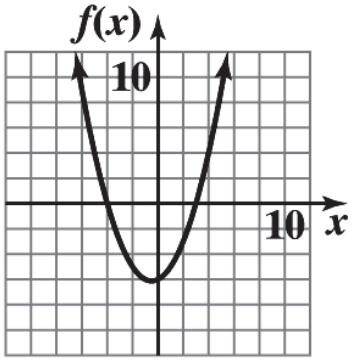
(E) f is continuous on the half-closed interval [1, 2).

78. True: If $r(x) = \dfrac{n(x)}{d(x)}$ is a rational function and $d(x)$ has degree n, then $r(x)$ has at most n points of discontinuity.

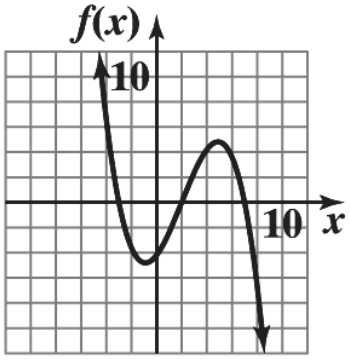
80. True: Continuous on (0, 2) means continuous at every real number x in (0, 2), including $x = 1$.

82. False. The greatest integer function has infinitely many points of discontinuity. See Prob. 75.

84. x intercepts: $x = -4, 3$



86. x intercepts: $x = -3, 2, 7$



88. $f(x) = \dfrac{6}{x-4} \neq 0$ for all x. This does not contradict Theorem 2 because f is not continuous on (2, 7); f is discontinuous at $x = 4$.

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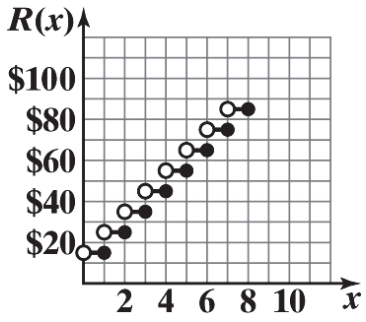
90. (A)
$$\begin{cases} 15, & 0 \le x < 1 \\ 25, & 1 \le x < 2 \\ 35, & 2 \le x < 3 \\ 45, & 3 \le x < 4 \\ 55, & 4 \le x < 5 \\ 65, & 5 \le x < 6 \\ 75, & 6 \le x < 7 \\ 85, & 7 \le x < 8 \end{cases}$$

(B)



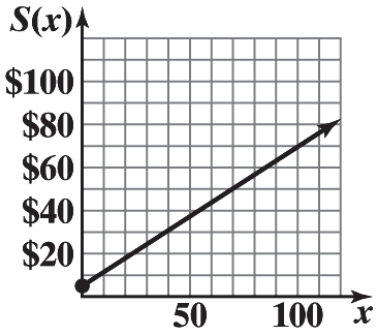
(C) $\lim\limits_{x \to 3.5} R(x) = 45 = R(3.5)$; thus, $R(x)$ is continuous at $x = 3.5$.

$\lim\limits_{x \to 4} R(x)$ does not exist; thus, $R(x)$ is not continuous at $x = 4$.

92. $S(x) = R(x)$.

94. (A) $S(x) = \begin{cases} 5 + 0.69x \text{ if } & 0 \le x \le 5 \\ 5.2 + 0.65x \text{ if } & 5 < x \le 50 \\ 6.2 + 0.63x \text{ if } & 50 < x \end{cases}$

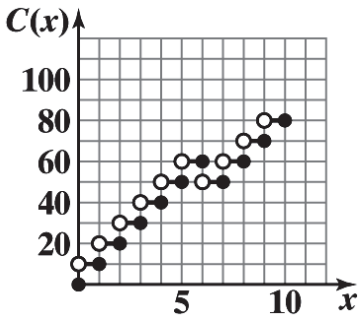
(B) The graph of S is:



(C) $\lim\limits_{x \to 5} S(x) = 8.45 = S(5)$; thus, $S(x)$ is continuous at $x = 5$.

$\lim\limits_{x \to 50} S(x) = 37.7 = S(50)$; thus, $S(x)$ is continuous at $x = 50$.

96. (A) The graph of $C(x)$ is:



(B) From the graph, $\lim\limits_{x \to 4.5} C(x) = 50$ and $C(4.5) = 50$.

(C) From the graph, $\lim\limits_{x \to 8} C(x)$ does not exist; $C(8) = 60$.

(D) Since $\lim\limits_{x \to 4.5} C(x) = 50 = C(4.5)$, $C(x)$ is continuous at $x = 4.5$.

Since $\lim\limits_{x \to 8} C(x)$ does not exist and $C(8) = 60$, $C(x)$ is not continuous at $x = 8$.

98. (A) From the graph, p is discontinuous at $t = t_2$, and $t = t_4$.

(B) $\lim\limits_{t \to t_1} p(t) = 10$; $p(t_1) = 10$.

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(C) $\lim_{t\to t_2} p(t) = 30, p(t_2) = 10.$

(D) $\lim_{t\to t_4} p(t)$ does not exist; $p(t_4) = 80.$

EXERCISE 2-4

2. Slope $m = \dfrac{8-11}{1-(-1)} = \dfrac{-3}{2}, \ -1.5$

4. Slope $m = \dfrac{3-(-3)}{4-(-12)} = \dfrac{6}{16} = \dfrac{3}{8}; \ 0.375$

6. $\dfrac{2}{\sqrt{5}} = \dfrac{1}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$

8. $\dfrac{1-\sqrt{2}}{5+\sqrt{2}} = \dfrac{1-\sqrt{2}}{5+\sqrt{2}} \cdot \dfrac{5-\sqrt{2}}{5-\sqrt{2}} = \dfrac{7-6\sqrt{2}}{23} = \dfrac{7}{23} - \dfrac{6}{23}\sqrt{2}$

10. (A) $\dfrac{f(-1)-f(-2)}{-1-(-2)} = \dfrac{4-1}{1} = 3$ is the slope of the secant line through $(-2, f(-2))$ and $(-1, f(-1))$.

(B) $$\frac{f(-2+h)-f(-2)}{h} = \frac{5-(-2+h)^2-1}{h} = \frac{5-[4-4h+h^2]-1}{h}$$
$$= \frac{5-4+4h-h^2-1}{h} = \frac{4h-h^2}{h} = 4-h;$$

slope of the secant line through $(-2, f(-2))$ and $(-2+h, f(-2+h))$

(C) $\lim_{h\to 0} \dfrac{f(-2+h)-f(-2)}{h} = \lim_{h\to 0} (4-h) = 4;$

slope of the tangent line at $(-2, f(-2))$

12. $f(x) = 3x^2$

(A) Slope of secant line through $(2, f(2))$ and $(5, f(5))$:

$$\frac{f(5)-f(2)}{5-2} = \frac{3(5)^2-3(2)^2}{5-2} = \frac{75-12}{3} = \frac{63}{3} = 21$$

(B) Slope of secant line through $(2, f(2))$ and $(2+h, f(2+h))$:

$$\frac{3(2+h)^2-3(2)^2}{2+h-2} = \frac{3(4+4h+h^2)-12}{h} = \frac{12+12h+3h^2-12}{h} = \frac{12h+3h^2}{h} = 12+3h$$

(C) Slope of the graph at $(2, f(2))$: $\lim_{h\to 0} \dfrac{f(2+h)-f(2)}{h} = \lim_{h\to 0} (12+3h) = 12.$

14. (A) Distance traveled for $0 \le t \le 4$: $352(1.5) = 528;$ average velocity: $v = \dfrac{528}{4} = 132$ mph.

(B) $\dfrac{f(4)-f(0)}{4-0} = \dfrac{528}{4} = 132.$

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(C) Slope at $x=4$: $m=150$. Equation of tangent line at $(4, f(4))$: $y-528=150(x-4)$ or $y=150x-72$.

16. $f(x)=\dfrac{1}{1+x^2}$; $f(2)=\dfrac{1}{5}=0.2$. Equation of tangent line: $y-0.2=-0.16(x-2)$ or $y=-0.16x+3.4$.

18. $f(x)=x^4$; $f(-1)=1$. Equation of tangent line: $y-1=-4(x+1)$ or $y=-4x-3$.

20. $f(x)=9$

Step 1. Find $f(x+h)$.

$f(x+h)=9$

Step 2. Find $f(x+h)-f(x)$.

$f(x+h)-f(x)=9-9=0$

Step 3. Find $\dfrac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h}=\frac{0}{h}=0$$

Step 4. Find $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}(0)=0$

Thus, if $f(x)=9$, then $f'(x)=0$, $f'(1)=0$, $f'(2)=0$, $f'(3)=0$.

22. $f(x)=4-6x$

Step 1. $f(x+h)=4-6(x+h)=4-6x-6h$

Step 2. $f(x+h)-f(x)=(4-6x-6h)-(4-6x)$

$=4-6x-6h-4+6x=-6h$

Step 3. $$\frac{f(x+h)-f(x)}{h}=\frac{-6h}{h}=-6$$

Step 4. $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}(-6)=-6$

$f'(1)=-6$, $f'(2)=-6$, $f'(3)=-6$

24. $f(x)=2x^2+8$

Step 1. $f(x+h)=2(x+h)^2+8=2(x^2+2xh+h^2)+8$

$=2x^2+4xh+2h^2+8$

Step 2. $f(x+h)-f(x)=(2x^2+4xh+2h^2+8)-(2x^2+8)$

$=2x^2+4xh+2h^2+8-2x^2-8$

$=4xh+2h^2$

Step 3. $$\frac{f(x+h)-f(x)}{h}=\frac{4xh+2h^2}{h}=4x+2h$$

Step 4. $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}(4x+2h)=4x$

$f'(1)=4$, $f'(2)=8$, $f'(3)=12$

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26. $f(x) = 3x^2 + 2x - 10$

Step 1. $f(x+h) = 3(x+h)^2 + 2(x+h) - 10 = 9x^2 + 6xh + 3h^2 + 2x + 2h - 10$

Step 2. $f(x+h) - f(x) = (3x^2 + 6xh + 3h^2 + 2x + 2h - 10) - (3x^2 + 2x - 10)$
$= 6xh + 3h^2 + 2h = h(6x + 3h + 2)$

Step 3. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(6x+3h+2)}{h} = 6x + 3h + 2$

Step 4. $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (6x + 3h + 2) = 6x + 2$

$f'(1) = 8, \quad f'(2) = 14, \quad f'(3) = 20$

28. $f(x) = x^2 - 4x + 7$

Step 1. $f(x+h) = (x+h)^2 - 4(x+h) + 7 = x^2 + 2xh + h^2 - 4x - 4h + 7$

Step 2. $f(x+h) - f(x) = x^2 + 2xh + h^2 - 4x - 4h + 7 - (x^2 - 4x + 7)$
$= 2xh + h^2 - 4h = h(2x + h - 4)$

Step 3. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(2x+h-4)}{h} = 2x + h - 4$

Step 4. $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (2x + h - 4) = 2x - 4$

$f'(1) = -2, \quad f'(2) = 0, \quad f'(3) = 2$

30. $f(x) = 6x^2 - 3x + 4$

Step 1. $f(x+h) = 6(x+h)^2 - 3(x+h) + 4 = 6x^2 + 12xh + 6h^2 - 3x - 3h + 4$

Step 2. $f(x+h) - f(x) = (6x^2 + 12xh + h^2 - 3x - 3h + 4) - (6x^2 - 3x + 4)$
$= 12xh + h^2 - 3h = h(12x + h - 3)$

Step 3. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(12x+h-3)}{h} = 12x + h - 3$

Step 4. $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (12x + h - 3) = 12x - 3$

$f'(1) = 9, \quad f'(2) = 21, \quad f'(3) = 33$

32. $f(x) = -x^2 + 3x + 2$

Step 1. $f(x+h) = -(x+h)^2 + 3(x+h) + 2 = -x^2 - 2xh - h^2 + 3x + 3h + 2$

Step 2. $f(x+h) - f(x) = (-x^2 - 2xh - h^2 + 3x + 3h + 2) - (-x^2 + 3x + 2)$
$= -2xh - h^2 + 3h = h(-2x - h + 3)$

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Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{h(-2x-h+3)}{h} = -2x-h+3$

Step 4. $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = = \lim\limits_{h\to 0}(-2x-h+3) = -2x+3$

$f'(1) = 1, \quad f'(2) = -1, \quad f'(3) = -3$

34. $f(x) = -2x^3 + 5$

Step 1. $f(x+h) = -2(x+h)^3 + 5 = -2(x^3 + 3x^2h + 3xh^2 + h^3) + 5$

$= -2x^3 - 6x^2h - 6xh^2 - 2h^3 + 5$

Step 2. $f(x+h) - f(x) = -2x^3 - 6x^2h - 6xh^2 - 2h^3 + 5 - (-2x^3 + 5)$

$= -6x^2h - 6xh^2 - 2h^3$

$= -2h(3x^2 + 3xh + h^2)$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-2h(3x^2+3xh+h^2)}{h} = -2(3x^2 + 3xh + h^2)$

Step 4. $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\{-2(3x^2 + 3xh + h^2)\} = -6x^2$

$f'(1) = -6, \quad f'(2) = -24, \quad f'(3) = -54$

36. $f(x) = \dfrac{6}{x} - 2$

Step 1. $f(x+h) = \dfrac{6}{x+h} - 2$

Step 2. $f(x+h) - f(x) = \left(\dfrac{6}{x+h} - 2\right) - \left(\dfrac{6}{x} - 2\right)$

$= \dfrac{6}{x+h} - \dfrac{6}{x} = \dfrac{6x - 6x - 6h}{x(x+h)} = \dfrac{-6h}{x(x+h)}$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\dfrac{-6h}{x(x+h)}}{h} = -\dfrac{6}{x(x+h)}$

Step 4. $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{-6}{x(x+h)} = -\dfrac{6}{x^2}$

$f'(1) = -6, \quad f'(2) = -\dfrac{6}{4} = -\dfrac{3}{2}, \quad f'(3) = -\dfrac{6}{9} = -\dfrac{2}{3}$

38. $f(x) = 3 - 7\sqrt{x}$

Step 1. $f(x+h) = 3 - 7\sqrt{x+h}$

Step 2. $f(x+h) - f(x) = (3 - 7\sqrt{x+h}) - (3 - 7\sqrt{x}) = 7(\sqrt{x} - \sqrt{x+h})$

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Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{7(\sqrt{x}-\sqrt{x+h})}{h} = \dfrac{7(\sqrt{x}-\sqrt{x+h})}{h} \cdot \dfrac{(\sqrt{x}+\sqrt{x+h})}{(\sqrt{x}+\sqrt{x+h})}$

$= \dfrac{7(x-(x+h))}{h(\sqrt{x}+\sqrt{x+h})} = \dfrac{7(x-x-h)}{h(\sqrt{x}+\sqrt{x+h})}$

$= \dfrac{-7h}{h(\sqrt{x}+\sqrt{x+h})} = \dfrac{-7}{\sqrt{x}+\sqrt{x+h}}$

Step 4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\left(\dfrac{-7}{\sqrt{x}+\sqrt{x+h}}\right) = \dfrac{-7}{2\sqrt{x}}$

$f'(1) = -\dfrac{7}{2}, \quad f'(2) = -\dfrac{7}{2\sqrt{2}} = -\dfrac{7\sqrt{2}}{4}, \quad f'(3) = -\dfrac{7}{2\sqrt{3}} = -\dfrac{7\sqrt{3}}{6}$

40. $f(x) = 16\sqrt{x+9}$

Step 1. $f(x+h) = 16\sqrt{x+h+9}$

Step 2. $f(x+h) - f(x) = 16\sqrt{x+h+9} - 16\sqrt{x+9}$

$= 16(\sqrt{x+h+9} - \sqrt{x+9})$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{16(\sqrt{x+h+9}-\sqrt{x+9})}{h}$

$= \dfrac{16(\sqrt{x+h+9}-\sqrt{x+9})}{h} \cdot \dfrac{(\sqrt{x+h+9}+\sqrt{x+9})}{(\sqrt{x+h+9}+\sqrt{x+9})}$

$= \dfrac{16((x+h+9)-(x+9))}{h(\sqrt{x+h+9}+\sqrt{x+9})}$

$= \dfrac{16h}{h(\sqrt{x+h+9}+\sqrt{x+9})} = \dfrac{16}{\sqrt{x+h+9}+\sqrt{x+9}}$

Step 4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{16}{\sqrt{x+h+9}+\sqrt{x+9}} = \dfrac{16}{2\sqrt{x+9}} = \dfrac{8}{\sqrt{x+9}}$

$f'(1) = \dfrac{8}{\sqrt{10}} = \dfrac{4\sqrt{10}}{5}, \quad f'(2) = \dfrac{8}{\sqrt{11}} = \dfrac{8\sqrt{11}}{11}, \quad f'(3) = \dfrac{8}{\sqrt{12}} = \dfrac{4\sqrt{3}}{3}$

42. $f(x) = \dfrac{1}{x+4}$.

Step 1. $f(x+h) = \dfrac{1}{x+4+h}$

Step 2. $f(x+h) - f(x) = \dfrac{1}{x+4+h} - \dfrac{1}{x+4} = \dfrac{x+4-(x+4+h)}{(x+4+h)(x+4)} = \dfrac{-h}{(x+4+h)(x+4)}$

Step3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-h}{h(x+4+h)(x+4)} = \dfrac{-1}{(x+4+h)(x+4)}$

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Step 4. $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\dfrac{-1}{(x+4+h)(x+4)}=\dfrac{-1}{(x+4)^2}.$

$f'(1)=\dfrac{-1}{25},\quad f'(2)=\dfrac{-1}{36},\quad f'(3)=\dfrac{-1}{49}$

44. $f(x)=\dfrac{x}{x+2}$

Step1. $f(x+h)=\dfrac{x+h}{x+2+h}$

Step 2. $f(x+h)-f(x)=\dfrac{x+h}{x+2+h}-\dfrac{x}{x+2}=\dfrac{(x+h)(x+2)-x(x+2+h)}{(x+2+h)(x+2)}=\dfrac{2h}{(x+2+h)(x+2)}$

Step 3. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{2h}{h(x+2+h)(x+2)}=\dfrac{2}{(x+2+h)(x+2)}$

Step 4. $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\dfrac{2}{(x+2+h)(x+2)}=\dfrac{2}{(x+2)^2}.$

$f'(1)=\dfrac{2}{9},\quad f'(2)=\dfrac{2}{16}=\dfrac{1}{8},\quad f'(3)=\dfrac{2}{25}$

46. $y=f(x)=x^2+x$

(A) $f(2)=2^2+2=6, f(4)=4^2+4=20$

Slope of secant line: $\dfrac{f(4)-f(2)}{4-2}=\dfrac{20-6}{2}=\dfrac{14}{2}=7$

(B) $f(2)=6, f(2+h)=(2+h)^2+(2+h)=4+4h+h^2+2+h$

$=6+5h+h^2$

Slope of secant line: $\dfrac{f(2+h)-f(2)}{h}=\dfrac{6+5h+h^2-6}{h}$

$=\dfrac{5h+h^2}{h}=5+h$

(C) Slope of tangent line at $(2, f(2))$:

$\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h}=\lim_{h\to 0}(5+h)=5$

(D) Equation of tangent line at $(2, f(2))$:

$y-f(2)=f'(2)(x-2)$ or $y-6=5(x-2)$ and $y=5x-4$.

48. $f(x)=x^2+x$

(A) Average velocity: $\dfrac{f(4)-f(2)}{4-2}=\dfrac{(4)^2+4-((2)^2+2)}{2}=\dfrac{16+4-6}{2}=7$ meters per second

(B) Average velocity: $\dfrac{f(2+h)-f(2)}{h}=\dfrac{(2+h)^2+(2+h)-6}{h}=\dfrac{4+4h+h^2+2+h-6}{h}$

$=\dfrac{5h+h^2}{h}=5+h$ meters per second

(C) Instantaneous velocity: $\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h}=\lim_{h\to 0}(5+h)=5$ meters per second

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50. $F(x)$ does not exist at $x = b$.

52. $F(x)$ does exist at $x = d$.

54. $F(x)$ does not exist at $x = f$.

56. $F(x)$ does not exist at $x = h$.

58. (A) To find f' use the two step process for the given function $f(x) = 4x - x^2 + 1$.

Step 1.

$$\frac{f(x+h)-f(x)}{h} = \frac{[4(x+h)-(x+h)^2+1]-[4x-x^2+1]}{h}$$

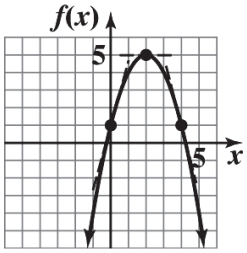
$$= \frac{(4x+4h-x^2-2xh-h^2+1)-(4x-x^2+1)}{h}$$

$$= \frac{4h-2xh-h^2}{h} = 4-2x-h$$

Step 2. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(4-2x-h) = 4-2x.$

(B) Slopes: at $x = 0$, $f'(0) = 4$; $x = 2$, $f'(2) = 0$; $x = 4$, $f'(4) = -4$

(C)



60. To find $v = f'(x)$, use the two-step process for the given distance function, $f(x) = 8x^2 - 4x$.

Step 1. $$\frac{f(x+h)-f(x)}{h} = \frac{8(x+h)^2-4(x+h)-(8x^2-4x)}{h}$$

$$= \frac{8(x^2+2xh+h^2)-4x-4h-8x^2+4x}{h}$$

$$= \frac{8x^2+16xh+8h^2-4x-4h-8x^2+4x}{h}$$

$$= \frac{16xh-4h+8h^2}{h} = 16x - 4 + 8h$$

Step 2. $$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(16x - 4 + 8h) = 16x - 4$$

Thus, the velocity, $v = f'(x) = 16x - 4$

$f'(1) = 12$ feet per second, $f'(3) = 44$ feet per second, $f'(5) = 76$ feet per second

62. (A) The graphs of g and h are vertical translations of the graph of f. All Three functions should have the same derivatives; they differ from each other by a constant.

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(B) $m(x) = -x^2 + c$

Step 1. $m(x+h) = -(x+h)^2 + c = -x^2 - 2xh - h^2 + c$

Step 2. $m(x+h) - m(x) = (-x^2 - 2xh - h^2 + c) - (-x^2 + c)$
$= -x^2 - 2xh - h^2 + c + x^2 - c = -2xh - h^2$

Step 3. $\dfrac{m(x+h)-m(x)}{h} = \dfrac{-2xh-h^2}{h} = -2x - h$

Step 4. $m'(x) = \lim\limits_{h\to 0} \dfrac{m(x+h)-m(x)}{h} = \lim\limits_{h\to 0}(-2x-h) = -2x$

64. True: $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{m(x+h)+b-(mx+b)}{h}$

$= \lim\limits_{h\to 0} \dfrac{mx+mh+b-mx-b}{h} = \lim\limits_{h\to 0} \dfrac{mh}{h} = \lim\limits_{h\to 0} m = m$

66. Let $c \in (a,b)$. We wish to show that $\lim\limits_{x\to c} f(x) = f(c)$. If we let $h = x - c$, then $x = h + c$, and this statement is equivalent to $\lim\limits_{h\to 0} f(c+h) = f(c)$, which is in turn equivalent to $\lim\limits_{h\to 0}\big(f(c+h) - f(c)\big) = 0$. Since $f'(x)$ exists at every point in the interval, we know that $f'(c)$ is defined and

$\lim\limits_{h\to 0} \dfrac{f(c+h)-f(c)}{h} = f'(c)$

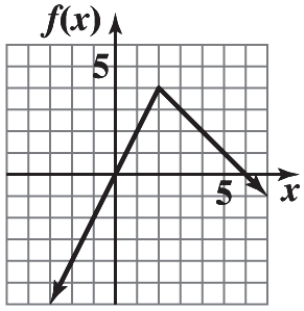
$\left(\lim\limits_{h\to 0} h\right)\left(\lim\limits_{h\to 0} \dfrac{f(c+h)-f(c)}{h}\right) = \left(\lim\limits_{h\to 0} h\right) f'(c)$

$\lim\limits_{h\to 0} h\left(\dfrac{f(c+h)-f(c)}{h}\right) = 0$

$\lim\limits_{h\to 0}\big(f(c+h) - f(c)\big) = 0$

68. False. For example, $f(x) = |x|$ has a sharp corner at $x = 0$, but is continuous there.

70. The graph of $f(x) = \begin{cases} 2x & \text{if } x < 2 \\ 6 - x & \text{if } x \geq 2 \end{cases}$ is:



f is not differentiable at $x = 2$ because the graph of f has a sharp corner at this point.

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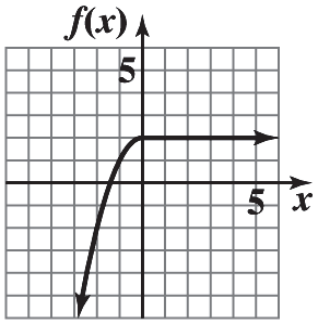
72. $f(x) = \begin{cases} 2 - x^2 & \text{if } x \le 0 \\ 2 & \text{if } x > 0 \end{cases}$



It is clear that $f'(x) = \begin{cases} -2x & \text{if } x < 0 \\ 0 & \text{if } x > 2 \end{cases}$

Thus, the only question is $f'(0)$.

Since $\lim\limits_{x\to 0^-} f'(x) = \lim\limits_{x\to 0^-} (-2x) = 0$ and $\lim\limits_{x\to 0^+} f'(x) = \lim\limits_{x\to 0^+} (0) = 0$, f is differentiable at 0 as well;

f is differentiable for all real numbers.

74. $f(x) = 1 - |x|$

$$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{1 - |0+h| - (1 - |0|)}{h} = \lim_{h\to 0} -\frac{|h|}{h}$$

The limit does not exist. Thus, f is not differentiable at $x = 0$.

76. $f(x) = x^{2/3}$

$$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{(0+h)^{2/3} - 0^{2/3}}{h} = \lim_{h\to 0} \frac{h^{2/3}}{h} = \lim_{h\to 0} \frac{1}{h^{1/3}}$$

The limit does not exist. Thus, f is not differentiable at $x = 0$.

78. $f(x) = \sqrt{1+x^2}$

$$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{\sqrt{1+(0+h)^2} - \sqrt{1+0^2}}{h} = \lim_{h\to 0} \frac{\sqrt{1+h^2} - 1}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+h^2} - 1}{h} \cdot \frac{\sqrt{1+h^2} + 1}{\sqrt{1+h^2} + 1} = \lim_{h\to 0} \frac{1+h^2-1}{h\left[\sqrt{1+h^2} + 1\right]} = \lim_{h\to 0} \frac{h}{\sqrt{1+h^2} + 1} = \frac{0}{2} = 0$$

f is differentiable at $x = 0$ and $f'(0) = 0$.

80. $y = 16x^2$

Now, if $y = 1{,}024$ ft, then

$$16x^2 = 1{,}024$$
$$x^2 = \frac{1{,}024}{16} = 64$$
$$x = 8 \text{ sec.}$$

$y' = 32x$ and at $x = 8$, $y' = 32(8) = 256$ ft/sec.

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82. $P(x) = 45x - 0.025x^2 - 5{,}000,\ 0 \le x \le 2{,}400.$

(A) Average change $= \dfrac{P(850) - P(800)}{850 - 800}$

$= \dfrac{[45(850) - 0.025(850)^2 - 5{,}000] - [45(800) - 0.025(800)^2 - 5{,}000]}{50}$

$= \dfrac{45(850) - 0.025(850)^2 - 45(800) + 0.025(800)^2}{50}$

$= \dfrac{54{,}250 - 54{,}062.5}{50} = \dfrac{187.5}{50} = \$3.75$

(B) $P(x) = 45x - 0.025x^2 - 5{,}000$

Step 1. $P(x + h) = 45(x + h) - 0.025(x + h)^2 - 5{,}000$

$= 45x + 45h - 0.025x^2 - 0.05xh - 0.025h^2 - 5{,}000$

Step 2. $P(x + h) - P(x) = (45x + 45h - 0.025x^2 - 0.05xh - 0.025h^2 - 5{,}000) - (45x - 0.025x^2 - 5{,}000)$

$= 45h - 0.05xh - 0.025h^2$

Step 3. $\dfrac{P(x+h) - P(x)}{h} = \dfrac{45h - 0.05xh - 0.025h^2}{h} = 45 - 0.05x - 0.025h$

Step 4. $P'(x) = \lim\limits_{h \to 0} \dfrac{P(x+h) - P(x)}{h} = \lim\limits_{h \to 0} (45 - 0.05x - 0.025h) = 45 - 0.05x$

(C) $P(800) = 45(800) - 0.025(800)^2 - 5{,}000 = 15{,}000$

$P'(800) = 45 - 0.05(800) = 5;$

At a production level of 800 car seats, the profit is \$15,000 and is increasing at the rate of \$5 per seat.

84. $S(t) = \sqrt{t} + 8$

(A) Step 1. $S(t + h) = \sqrt{t + h} + 8$

Step 2. $S(t + h) - S(t) = \left(\sqrt{t + h} + 8\right) - \left(\sqrt{t} + 8\right) = \sqrt{t + h} - \sqrt{t}$

$= \left(\sqrt{t + h} - \sqrt{t}\right) \cdot \dfrac{\sqrt{t + h} + \sqrt{t}}{\sqrt{t + h} + \sqrt{t}}$

$= \dfrac{(t + h) - t}{\sqrt{t + h} + \sqrt{t}} = \dfrac{h}{\sqrt{t + h} + \sqrt{t}}$

Step 3. $\dfrac{S(t+h) - S(t)}{h} = \dfrac{\dfrac{h}{\sqrt{t + h} + \sqrt{t}}}{h} = \dfrac{1}{\sqrt{t + h} + \sqrt{t}}$

Step 4. $S'(t) = \lim\limits_{h \to 0} \dfrac{S(t+h) - S(t)}{h} = \lim\limits_{h \to 0} \dfrac{1}{\sqrt{t + h} + \sqrt{t}} = \dfrac{1}{2\sqrt{t}}$

(B) $S(9) = \sqrt{9} + 8 = 11\ ;\quad S'(9) = \dfrac{1}{2\sqrt{9}} = \dfrac{1}{6} \approx 0.167$

After 9 months, the total sales are \$11 million and are increasing at the rate of \$0.167 million = \$167,000 per month.

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(C) The estimated total sales are \$11.167 million after 10 months and \$11.334 million after 11 months.

86. (A) $p(t) = 48t^2 - 37t + 1{,}698$

Step 1. $p(t+h) = 48(t+h)^2 - 37(t+h) + 1{,}698$

$= 48\left(t^2 + 2th + h^2\right) - 37t - 37h + 1{,}698$

$= 48t^2 + 96th + 48h^2 - 37t - 37h + 1{,}698$

Step 2. $p(t+h) - p(t) = 48t^2 + 96th + 48h^2 - 37t - 37h + 1{,}698 - (48t^2 - 37t + 1{,}698)$

$= 96th + 48h^2 - 37h$

Step 3. $\dfrac{p(t+h) - p(t)}{h} = \dfrac{96th + 48h^2 - 37h}{h} = 96t + 48h - 37$

Step 4. $p'(t) = \lim\limits_{h \to 0} \dfrac{p(t+h) - p(t)}{h} = \lim\limits_{h \to 0} (96t + 48h - 37) = 96t - 37$

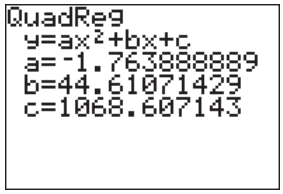
(B) 2027 corresponds to $t = 17$. Thus

$p(17) = 48(17)^2 - 37(17) + 1{,}698 = 14{,}941$

$p'(17) = 96(17) - 37 = 1{,}595$

In 2027, 14,941 thousand tons of copper will be consumed and this quantity is increasing at the rate of 1,595 thousand tons/year.

88. (A) Quadratic regression model



$C(x) \approx -1.764x^2 + 44.611x + 1068.607, \quad C'(x) \approx -3.528x + 44.611.$

(B) $C(30) \approx -1.764(30)^2 + 44.611(30) + 1068.607 \approx 819.337$;

$C'(30) \approx -3.526(30) + 44.611 = -61.169$

In 2030, 819.3 billion kilowatts will be sold and the amount sold is decreasing at the rate of 61.2 billion kilowatts per year.

90. (A) $F(t) = 98 + \dfrac{4}{t+1}$

Step 1. $F(t+h) = 98 + \dfrac{4}{t+h+1}$

Step 2. $F(t+h) - F(t) = \left(98 + \dfrac{4}{t+h+1}\right) - \left(98 + \dfrac{4}{t+1}\right) = \dfrac{4}{t+h+1} - \dfrac{4}{t+1}$

$= 4\left[\dfrac{(t+1) - (t+h+1)}{(t+h+1)(t+1)}\right] = \dfrac{-4h}{(t+h+1)(t+1)}$

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Step 3. $\dfrac{F(t+h)-\mathrm{F}(t)}{h}=\dfrac{\dfrac{-4h}{(t+h+1)(t+1)}}{h}=\dfrac{-4}{(t+h+1)(t+1)}$

Step 4. $F'(t)=\lim\limits_{h\to 0}\dfrac{F(t+h)-\mathrm{F}(t)}{h}=\lim\limits_{h\to 0}\dfrac{-4}{(t+h+1)(t+1)}=\dfrac{-4}{(t+1)^2}$

(B) $F(3)=99,\quad F'(3)=\dfrac{-4}{16}=\dfrac{-1}{4}$. The body temperature 3 hours after taking the medicine is 99° and is decreasing at the rate of 0.25° per hour.

EXERCISE 2-5

2. $\sqrt[3]{x}=x^{1/3}$

4. $\dfrac{1}{x}=x^{-1}$

6. $\dfrac{1}{(x^5)^2}=\dfrac{1}{x^{10}}=x^{-10}$

8. $\dfrac{1}{\sqrt[5]{x}}=\dfrac{1}{x^{1/5}}=x^{-1/5}$

10. $\dfrac{d}{dx}(5)=0$ (Derivative of a constant rule.)

12. $y=x^8$

$y'=8x^{8-1}=8x^7$ (Power rule)

14. $g(x)=x^9$

$g'(x)=9x^{9-1}=9x^8$ (Power rule)

16. $y=x^{-5}$

$\dfrac{dy}{dx}=-5x^{-5-1}=-5x^{-6}$ (Power rule)

18. $f(x)=x^{5/2}$

$f'(x)=\dfrac{5}{2}x^{5/2-1}=\dfrac{5}{2}x^{3/2}$ (Power rule)

20. $y=\dfrac{1}{x^7}=x^{-7}$

$y'=-7x^{-7-1}=-7x^{-8}=\dfrac{-7}{x^8}$ (Power rule)

22. $\dfrac{d}{dx}(-3x^2)=-3(2x)=-6x$ (constant times a function rule)

24. $f(x)=0.7x^3$

$f'(x)=0.7(3x^2)=2.1x^2$

26. $y=\dfrac{x^3}{9}$

$y'=\dfrac{1}{9}(3x^2)=\dfrac{x^2}{3}$

28. $h(x) = 5g(x)$; $h'(2) = 5g'(2) = 5(-1) = -5$

30. $h(x) = g(x) - f(x)$; $h'(2) = g'(2) - f'(2) = -1 - 3 = -4$

32. $h(x) = -4f(x) + 5g(x) - 9$; $h'(2) = -4f'(2) + 5g'(2) = -4(3) + 5(-1) = -17$

34. $\dfrac{d}{dx}(-4x + 9) = \dfrac{d}{dx}(-4x) + \dfrac{d}{dx}(9) = -4 + 0 = -4$

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36. $y = 2 + 5t - 8t^3$

$\dfrac{dy}{dt} = 0 + 5 - 24t^2 = 5 - 24t^2$

38. $g(x) = 5x^{-7} - 2x^{-4}$

$g'(x) = (5) \cdot (-7)x^{-8} - (2) \cdot (-4)x^{-5}$

$= -35x^{-8} + 8x^{-5}$

40. $\dfrac{d}{du}(2u^{4.5} - 3.1u + 13.2) = (2) \cdot (4.5)u^{3.5} - 3.1 + 0 = 9u^{3.5} - 3.1$

42. $F(t) = 0.2t^3 - 3.1t + 13.2$

$F'(t) = (0.2) \cdot (3)t^2 - 3.1 + 0 = 0.6t^2 - 3.1$

44. $w = \dfrac{7}{5u^2} = \dfrac{7}{5}u^{-2}$

$w' = \left(\dfrac{7}{5}\right) \cdot (-2)u^{-3} = -\dfrac{14}{5}u^{-3}$

46. $\dfrac{d}{dx}\left(\dfrac{5x^3}{4} - \dfrac{2}{5x^3}\right) = \dfrac{d}{dx}\left(\left(\dfrac{5}{4}\right)x^3 - \left(\dfrac{2}{5}\right)x^{-3}\right) = \left(\dfrac{5}{4}\right) \cdot (3)x^2 - \left(\dfrac{2}{5}\right) \cdot (-3)x^{-4} = \dfrac{15}{4}x^2 + \dfrac{6}{5}x^{-4}$

48. $H(w) = \dfrac{5}{w^6} - 2\sqrt{w} = 5w^{-6} - 2w^{1/2}$

$H'(w) = (5) \cdot (-6)w^{-7} - (2) \cdot \left(\dfrac{1}{2}\right)w^{-1/2} = -30w^{-7} - w^{-1/2}$

50. $\dfrac{d}{du}(8u^{3/4} + 4u^{-1/4}) = (8) \cdot \left(\dfrac{3}{4}\right)u^{-1/4} + (4) \cdot \left(-\dfrac{1}{4}\right)u^{-5/4} = 6u^{-1/4} - u^{-5/4}$

52. $F(t) = \dfrac{5}{t^{1/5}} - \dfrac{8}{t^{3/2}} = 5t^{-1/5} - 8t^{-3/2}$

$F'(t) = (5) \cdot \left(-\dfrac{1}{5}\right)t^{-6/5} - (8) \cdot \left(-\dfrac{3}{2}\right)t^{-5/2} = -t^{-6/5} + 12t^{-5/2}$

54. $w = \dfrac{10}{\sqrt[5]{u}} = 10u^{-1/5}$

$w' = (10) \cdot \left(-\dfrac{1}{5}\right)u^{-6/5} = -2u^{-6/5}$

56. $\dfrac{d}{dx}\left(2.8x^{-3} - \dfrac{0.6}{\sqrt[3]{x^2}} + 7\right) = \dfrac{d}{dx}(2.8x^{-3} - 0.6x^{-2/3} + 7) = (2.8) \cdot (-3)x^{-4} - (0.6) \cdot \left(-\dfrac{2}{3}\right)x^{-5/3} + 0$

$= -8.4x^{-4} + 0.4x^{-5/3}$

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58. $f(x) = 2x^2 + 8x$

(A) $f'(x) = 4x + 8$

(B) Slope of the graph of f at $x = 2$: $f'(2) = 4(2) + 8 = 16$
Slope of the graph of f at $x = 4$: $f'(4) = 4(4) + 8 = 24$

(C) Tangent line at $x = 2$: $y - y_1 = m(x - x_1)$

$x_1 = 2$

$y_1 = f(2) = 2(2)^2 + 8(2) = 24$

$m = f'(2) = 16$

Thus, $y - 24 = 16(x - 2)$ or $y = 16x - 8$

Tangent line at $x = 4$: $y - y_1 = m(x - x_1)$

$x_1 = 4$

$y_1 = f(4) = 2(4)^2 + 8(4) = 64$

$m = f'(4) = 24$

Thus, $y - 64 = 24(x - 4)$ or $y = 24x - 32$

(D) The tangent line is horizontal at the values $x = c$ such that $f'(c) = 0$. Thus, we must solve the following:

$$f'(x) = 4x + 8 = 0$$
$$4x = -8$$
$$x = -2$$

60. $f(x) = x^4 - 32x^2 + 10$

(A) $f'(x) = 4x^3 - 64x$

(B) Slope of the graph of f at $x = 2$: $f'(2) = 4(2)^3 - 64(2) = -96$
Slope of the graph of f at $x = 4$: $f'(4) = 4(4)^3 - 64(4) = 0$

(C) Tangent line at $x = 2$: $y - y_1 = m(x - x_1)$, where

$x_1 = 2$, $y_1 = f(2) = (2)^4 - 32(2)^2 + 10 = -102$, $m = -96$

$y + 102 = -96(x - 2)$ or $y = -96x + 90$

Tangent line at $x = 4$ is a horizontal line since the slope $m = 0$. Therefore, the equation of the tangent line at $x = 4$ is: $y = f(4) = (4)^4 - 32(4)^2 + 10 = -246$

(D) Solve $f'(x) = 0$ for x:

$$4x^3 - 64x = 0$$
$$4x(x^2 - 16) = 0$$
$$4x(x + 4)(x - 4) = 0$$
$$x = -4, \quad x = 0, \quad x = 4$$

62. $f(x) = 80x - 10x^2$

(A) $v = f'(x) = 80 - 20x$

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(B) $v\big|_{x=0} = f'(0) = 80$ ft/sec.

$v\big|_{x=3} = f'(3) = 80 - 20(3) = 20$ ft/sec.

(C) Solve $v = f'(x) = 0$ for x:

$80 - 20x = 0$

$20x = 80$

$x = 4$ seconds

64. $f(x) = x^3 - 9x^2 + 24x$

(A) $v = f'(x) = 3x^2 - 18x + 24$

(B) $v\big|_{x=0} = f'(0) = 24$ ft/sec.

$v\big|_{x=3} = f'(3) = 3(3)^2 - 18(3) + 24 = -3$ ft/sec.

(C) Solve $v = f'(x) = 0$ for x:

$3x^2 - 18x + 24 = 0$ or $x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

$x = 2,\ x = 4$ seconds

66. $f'(x) = 2x + 1 - \dfrac{5}{\sqrt{x}}$; $f'(x) = 0$ at $x \approx 1.5247$.

68. $f'(x) = 4x^{1/3} - 4x + 4$; $f'(x) = 0$ at $x \approx 2.3247$.

70. $f'(x) = 0.08x^3 - 0.18x^2 - 1.56x + 0.94$; $f'(x) = 0$ at $x \approx -3.7626,\ 0.5742,\ 5.4384$.

72. $f'(x) = x^3 - 7.8x^2 + 16.2x - 10$; $f'(x) = 0$ at $x \approx 1.2391,\ 1.6400,\ 4.9209$.

74. The tangent line to the graph of a parabola at the vertex is a horizontal line. Therefore, to find the x coordinate of the vertex,
we solve $f'(x) = 0$ for x.

76. No. The derivative is a quadratic function which can have at most two zeros.

78. $y = (2x - 5)^2$; $y' = (2)(2x - 5)(2) = 8x - 20$

80. $y = \dfrac{x^2 + 25}{x^2} = 1 + \dfrac{25}{x^2} = 1 + 25x^{-2}$; $\dfrac{dy}{dx} = 0 + (25) \cdot (-2)x^{-3} = -50x^{-3}$

82. $f(x) = \dfrac{2x^5 - 4x^3 + 2x}{x^3} = \dfrac{2x^5}{x^3} - \dfrac{4x^3}{x^3} + \dfrac{2x}{x^3} = 2x^2 - 4 + 2x^{-2}$; $f'(x) = 4x - 4x^{-3}$

84. False: The function $f(x) = \dfrac{1}{x}$ is a counter-example.

86. False: The function $f(x) = 2x$ is a counter-example.

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88. $f(x) = u(x) - v(x)$

Step1. $f(x+h) = u(x+h) - v(x+h)$

Step 2. $f(x+h) - f(x) = u(x+h) - v(x+h) - [u(x) - v(x)] = u(x+h) - u(x) - [v(x+h) - v(x)]$

Step 3. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{u(x+h) - u(x) - [v(x+h) - v(x)]}{h} = \dfrac{u(x+h) - u(x)}{h} - \dfrac{v(x+h) - v(x)}{h}$

Step 4. $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0}\left[\dfrac{u(x+h) - u(x)}{h} - \dfrac{v(x+h) - v(x)}{h}\right]$

$= \lim\limits_{h\to 0} \dfrac{u(x+h) - u(x)}{h} - \lim\limits_{h\to 0} \dfrac{v(x+h) - v(x)}{h} = u'(x) - v'(x)$

90. $S(t) = 0.015t^4 + 0.4t^3 + 3.4t^2 + 10t - 3$

(A) $S'(t) = (0.015)\cdot(4)t^3 + (0.4)\cdot(3)t^2 + (3.4)(2)t + 10 - 0 = 0.06t^3 + 1.2t^2 + 6.8t + 10$

(B) $S(4) = 0.015(4)^4 + 0.4(4)^3 + 3.4(4)^2 + 10(4) - 3 = 120.84,$
$S'(4) = 0.06(4)^3 + 1.2(4)^2 + 6.8(4) + 10 = 60.24.$

After 4 months, sales are \$120.84 million and are increasing at the rate of \$60.24 million per month.

(C) $S(8) = 0.015(8)^4 + 0.4(8)^3 + 3.4(8)^2 + 10(8) - 3 = 560.84,$
$S'(8) = 0.06(8)^3 + 1.2(8)^2 + 6.8(8) + 10 = 171.92.$

After 8 months, sales are \$560.84 million and are increasing at the rate of \$171.92 million per month.

92. $x = 10 + \dfrac{180}{p}, 2 \le p \le 10$

For $p = 5$, $x = 10 + \dfrac{180}{5} = 10 + 36 = 46$

$x = 10 + \dfrac{180}{p} = 10 + 180p^{-1}$

$\dfrac{dx}{dp} = -180p^{-2} = -\dfrac{180}{p^2}$

For $p = 5$, $\left.\dfrac{dx}{dp}\right|_{p=5} = -\dfrac{180}{25} = -7.2$

At the \$5 price level, the demand is 46 pounds and is decreasing at the rate of 7.2 pounds per dollar increase in price.

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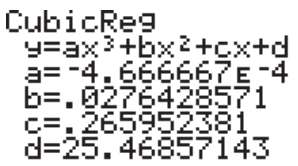
94 . (A) Cubic Regression model



$F(x) \approx -0.000467x^3 + 0.027643x^2 + 0.265952x + 25.468751$

(B) $F'(x) \approx -0.001401x^2 + 0.055286x + 0.265952$

$F(55) \approx 46.1, \quad F'(55) \approx -0.9$

In 2025, 46.1% of female high-school graduates enroll in college and the percentage is decreasing at the rate of 0.9% per year.

96. $C(x) = \dfrac{0.1}{x^2} = 0.1x^{-2}$

$C'(x) = -0.2x^{-3} = -\dfrac{0.2}{x^3}$, the instantaneous rate of change of concentration at x miles.

(A) At $x = 1$, $C'(1) = -0.2$ parts per million per mile.

(B) At $x = 2$, $C'(2) = -\dfrac{0.2}{8} = -0.025$ parts per million per mile.

98. $y = 21\sqrt[3]{x^2}$, $0 \le x \le 8$.

First, find $y = 21\sqrt[3]{x^2} = 21x^{2/3}$.

Then $y' = 21\left(\dfrac{2}{3}x^{-1/3}\right) = 14x^{-1/3} = \dfrac{14}{x^{1/3}} = \dfrac{14}{\sqrt[3]{x}}$, is the rate of learning at the end of x hours.

(A) Rate of learning at the end of 1 hour:

$\dfrac{14}{\sqrt[3]{1}} = 14$ items per hour.

(B) Rate of learning at the end of 8 hours:

$\dfrac{14}{\sqrt[3]{8}} = \dfrac{14}{2} = 7$ items per hour.

EXERCISE 2-6

2. $f(x) = 0.1x + 3; \quad f(7) = 0.1(7) + 3 = 3.7, \quad f(7.1) = 0.1(7.1) + 3 = 3.71$

4. $f(x) = 0.1x + 3; \quad f(-10) = 0.1(-10) + 3 = 2, \quad f(-10.1) = 0.1(-10.1) + 3 = 1.99$

6. $g(x) = x^2; \quad g(1) = 1^2 = 1, \quad g(1.1) = (1.1)^2 = 1.21$

8. $g(x) = x^2; \quad g(5) = 5^2 = 25, \quad g(4.9) = (4.9)^2 = (5 - 0.1)^2 = 24.01$

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10. $\Delta x = x_2 - x_1 = 5 - 2 = 3, \quad \Delta y = f(x_2) - f(x_1) = 5(5)^2 - 5(2)^2 = 125 - 20 = 105$

$$\frac{\Delta y}{\Delta x} = \frac{105}{3} = 35$$

12. $$\frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x} = \frac{f(2+1) - f(2)}{1} = \frac{f(3) - f(2)}{1} = \frac{5(3)^2 - 5(2)^2}{1} = 45 - 20 = 25$$

14. $\Delta y = f(x_2) - f(x_1) = f(3) - f(2) = 5(3)^2 - 5(2)^2 = 45 - 20 = 25$

$\Delta x = x_2 - x_1 = 3 - 2 = 1; \quad \dfrac{\Delta y}{\Delta x} = \dfrac{25}{1} = 25$

16. $y = 200x - \dfrac{x^2}{30}, \quad dy = \left(200x - \dfrac{x^2}{30}\right)' dx = \left(200 - \dfrac{x}{15}\right) dx$

18. $y = x^3(60 - x) = 60x^3 - x^4, \quad dy = (180x^2 - 4x^3)\,dx$

20. $y = 52\sqrt{x} = 52x^{1/2}, \quad dy = (52x^{1/2})'dx = (26x^{-1/2})dx$

22. (A) $$\frac{f(3+\Delta x) - f(3)}{\Delta x} = \frac{3(3+\Delta x)^2 - 3(3)^2}{\Delta x} = \frac{3(9 + 6\Delta x + (\Delta x)^2) - 27}{\Delta x}$$

$$= \frac{27 + 18\Delta x + 3(\Delta x)^2 - 27}{\Delta x} = \frac{18\Delta x + 3(\Delta x)^2}{\Delta x} = 18 + 3\Delta x$$

(B) As Δx tends to zero, then, clearly, $18 + 3\Delta x$ tends to 18.
Note the values in the following table:

Δx	$18 + 3\Delta x$
1	21
0.1	18.3
0.01	18.03
0.001	18.003

24. $y = (2x + 3)^2 = 4x^2 + 12x + 9, \quad dy = (8x + 12)dx = 4(2x + 3)dx$

26. $y = \dfrac{x^2 - 9}{x^2} = 1 - \dfrac{9}{x^2} = 1 - 9x^{-2}, \quad dy = 18x^{-3}\,dx = \dfrac{18}{x^3}dx.$

28. $y = f(x) = 30 + 12x^2 - x^3$

$\Delta y = f(2 + 0.1) - f(2) = f(2.1) - f(2) = [30 + 12(2.1)^2 - (2.1)^3] - [(30 + 12(2)^2 - 2^3]$

$= 30 + 52.92 - 9.261 - 30 - 48 + 8 = 3.66$

$dy = (30 + 12x^2 - x^3)'\big|_{x=2}\ dx = (24x - 3x^2)\big|_{x=2}\ (0.1) = (24(2) - 3(2)^2)(0.1) = (48 - 12)(0.1) = 3.6$

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30. $y = f(x) = 100\left(x - \frac{4}{x^2}\right)$

$$\Delta y = f(2 - 0.1) - f(2) = f(1.9) - f(2) = 100\left(1.9 - \frac{4}{(1.9)^2}\right) - 100\left(2 - \frac{4}{2^2}\right) = 79.197 - 100 = -20.803$$

$$dy = \left(100\left(x - \frac{4}{x^2}\right)\right)' \Big|_{x=2} dx = 100\left(1 + \frac{8}{x^3}\right) \Big|_{x=2} (-0.1) = 100\left(1 + \frac{8}{2^3}\right)(-0.1) = -20$$

32. $V = \frac{4}{3}\pi r^3$, $r = 5$ cm, $dr = \Delta r = 0.1$ cm.

$$dV = \left(\frac{4}{3}\pi r^3\right)' \Big|_{r=5} dr = 4\pi r^2 \Big|_{r=5} (0.1) = 31.4 \text{ cm}^3.$$

34. $f(x) = x^2 + 2x + 3$; $f'(x) = 2x + 2$; $x = -2$; $\Delta x = dx$ (C)

(A) $\Delta y = f(-2 + \Delta x) - f(-2)$

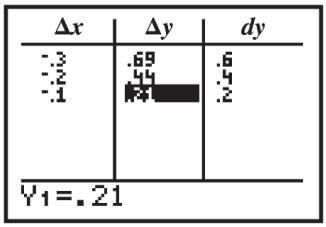
$= [(-2 + \Delta x)^2 + 2(-2 + \Delta x) + 3]$

$\quad - [(-2)^2 + 2(-2) + 3]$

$= 4 - 4\Delta x + (\Delta x)^2 - 4 + 2\Delta x + 3 - 4 + 4 - 3$

$= -2\Delta x + (\Delta x)^2$

$dy = f'(-2)dx = -2\,dx$



(B) $\Delta y(-0.1) = -2(-0.1) + (-0.1)^2 = 0.21$

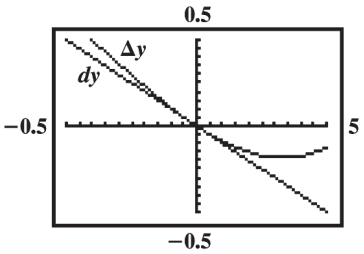
$dy(-0.1) = -2(-0.1) = 0.2$

$\Delta y(-0.2) = -2(-0.2) + (-0.2)^2 = 0.44$

$dy(-0.2) = -2(-0.2) = 0.4$

$\Delta y(-0.3) = -2(-0.3) + (-0.3)^2 = 0.69$

$dy(-0.3) = -2(-0.3) = 0.6$



36. $f(x) = x^3 - 2x^2$; $f'(x) = 3x^2 - 4x$; $x = 2$, $\Delta x = dx$ (C)

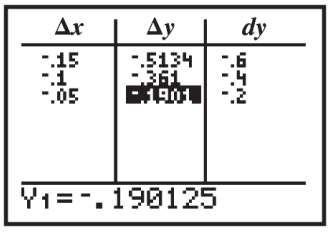
(A) $\Delta y = f(2 + \Delta x) - f(2)$

$= [(2 + \Delta x)^3 - 2(2 + \Delta x)^2] - [2^3 - 2(2)^2]$

$= 8 + 12\Delta x + 6(\Delta x)^2 + (\Delta x)^3 - 8 - 8\Delta x - 2(\Delta x)^2 - 8 + 8$

$= 4\Delta x + 4(\Delta x)^2 + (\Delta x)^3$

$dy = f'(2)dx = 4\,dx$



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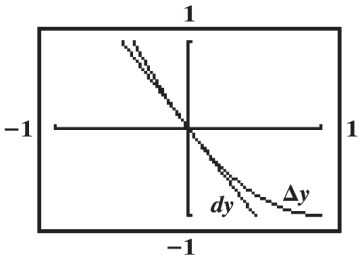
(B) $\Delta y(-0.05) = 4(-0.05) + 4(-0.05)^2 + (-0.05)^3$
$= -0.1901$
$dy(-0.05) = 4(-0.05) = -0.2$
$\Delta y(-0.10) = 4(-0.10) + 4(-0.10)^2 + (-0.10)^3$
$= -0.361$
$dy(-0.10) = 4(-0.10) = -0.4$
$\Delta y(-0.15) = 4(-0.15) + 4(-0.15)^2 + (-0.15)^3 = -0.5134$
$dy(-0.15) = 4(-0.15) = -0.6$



38. False.
Example. Let $y = f(x) = x^2 + 1$. Then
$\Delta y = f(0 + \Delta x) - f(0) = f(\Delta x) - f(0) = (\Delta x)^2 + 1 - 1 = (\Delta x)^2$
$dy = f'(0)dx = 0\ dx = 0.$

40. True.
$\Delta y = f(2 + \Delta x) - f(2) = 0$ implies that
$f(2 + \Delta x) = f(2)$
Since this is true for every increment and since the right-hand side of this equation is a constant, the function $f(x)$ must be a constant function.

42. $y = (2x^2 - 4)\sqrt{x} = (2x^2 - 4)(x)^{1/2} = 2x^{5/2} - 4x^{1/2}, \quad dy = (5x^{3/2} - 2x^{-1/2})\,dx.$

44. $y = f(x) = \dfrac{590}{\sqrt{x}} = 590x^{-1/2};\ x = 64,\ \Delta x = dx = 1.$

$\Delta y = f(x + \Delta x) - f(x) = f(64 + 1) - f(64) = f(65) - f(64) = \dfrac{590}{\sqrt{65}} - \dfrac{590}{\sqrt{64}} = -0.57$

$y = f(x) = \dfrac{590}{\sqrt{x}} = 590x^{-1/2}, \qquad f'(x) = -295x^{-3/2}$

$dy = f'(64)dx = f'(64)(1) = -295(64)^{-3/2} = -\dfrac{295}{512} = -0.576$

46. Given $D(x) = 1{,}000 - 40x^2$, $1 \le x \le 5$. Then, $D'(x) = -80x$.
The approximate change in demand dD corresponding to a change $\Delta x = dx$ in the price x is:
$dD = D'(x)dx$
Thus, letting $x = 3$ and $dx = 0.20$, we get
$dD = D'(3)(0.20) = -80(3)(0.20) = -48.$
There will be a 48-pound decrease in demand (approximately) when the price is increased from \$3.00 to \$3.20.

48. $R(x) = 200x - \dfrac{x^2}{30}; \quad R'(x) = 200 - \dfrac{x}{15}$

Profit $P(x) = R(x) - C(x) = 200x - \dfrac{x^2}{30} - 72{,}000 - 60x = 140x - \dfrac{x^2}{30} - 72{,}000$

$P'(x) = 140 - \dfrac{x}{15}$

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Now, for $x = 1{,}500$, $\Delta x = dx = 10$, we get

$$dR = R'(1{,}500)(10) = \left(200 - \frac{1{,}500}{15}\right)(10) = 1{,}000$$

$$dP = P'(1{,}500)(10) = \left(140 - \frac{1{,}500}{15}\right)(10) = 400$$

Thus, the approximate change in revenue is \$1,000 and the approximate change in profit is \$400 if the production is increased from 1,500 to 1,510 televisions.

For $x = 4{,}500$, $\Delta x = dx = 10$, we have:

$$dR = R'(4{,}500)(10) = \left(200 - \frac{4{,}500}{15}\right)(10) = -1{,}000$$

$$dP = P'(4{,}500)(10) = \left(140 - \frac{4{,}500}{15}\right)(10) = -1{,}600.$$

Thus, the approximate change in revenue is –\$1,000 and and the approximate change in profit is –\$1,600 if the production is increased from 4,500 to 4,510 televisions.

50. $V = \frac{4}{3}\pi r^3$; $V' = 4\pi r^2$.

The approximate volume of the shell for a radius change from 5 mm to 5.3 mm is given by:

$dV = 4\pi r^2\big|_{r=5}\ dx = 4\pi(5)^2(0.3)$ (Note: $\Delta x = dx = 0.3$ mm)

$= 94.2$ cubic millimeters

52. $T = x^2\left(1 - \frac{x}{9}\right) = x^2 - \frac{x^3}{9}, \quad 0 \le x \le 6; \quad T' = 2x - \frac{x^2}{3}.$

(A) For $x = 2$, $\Delta x = dx = 0.1$,

$$dT = \left(2x - \frac{x^2}{3}\right)\Big|_{x=2}\ dx = \left(2(2) - \frac{2^2}{3}\right)(0.1) = 0.27 \text{ degrees}$$

(B) For $x = 3$, $\Delta x = dx = 0.1$

$$dT = \left(2x - \frac{x^2}{3}\right)\Big|_{x=3}\ dx = \left(2(3) - \frac{3^2}{3}\right)(0.1) = 0.3 \text{ degrees}$$

(C) For $x = 4$, $\Delta x = dx = 0.1$

$$dT = \left(2x - \frac{x^2}{3}\right)\Big|_{x=4}\ dx = \left(2(4) - \frac{4^2}{3}\right)(0.1) = 0.27 \text{ degrees}$$

54. $y = 52\sqrt{x}$, $0 \le x \le 9$; $y = 52x^{1/2}$ and hence $y' = \frac{52}{2}x^{-1/2} = 26x^{-1/2}$.

For $x = 1$ and $\Delta x = dx = 0.1$ the approximate increase in the number of items learned is given by

$dy = y'\big|_{x=1}\ dx = 26(1)^{-1/2}(0.1) = 2.6$ items.

Similarly, for $x = 4$, $\Delta x = dx = 0.1$, we have

$dy = y'\big|_{x=4}\ dx = 26(4)^{-1/2}(0.1) = 1.3$ items.

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EXERCISE 2-7

In Problems 2 – 8, $C(x) = 10{,}000 + 150x - 0.2x^2$.

2. $C(100) = 10{,}000 + 150(100) - 0.2(100)^2 = 25{,}000 - 2{,}000 = 23{,}000$, \$23,000

4. $C(199) = 10{,}000 + 150(199) - 0.2(199)^2 = 39{,}850 - 7{,}920.20 = 31{,}929.80$, \$31,929.80

6. Using the results in Problems 4 and 5, $C(200) - C(199) = 32{,}000 - 31{,}929.80 = 70.20$, \$70.20

8. Average cost of producing 200 bicycles: $\dfrac{C(200)}{200} = \dfrac{32{,}000}{200} = 160$, \$160

10. $C'(x) = 6$

12. $C'(x) = 12 - 0.2x$

14. $R'(x) = 36 - 0.06x$

16. $R'(x) = 25 - 0.10x$

18. $P'(x) = (36 - 0.06x) - 6 = 30 - 0.06x$

20. $P'(x) = (25 - 0.1x) - (12 - 0.2x) = 13 + 0.1x$

22. $\bar{R}(x) = \dfrac{5x - 0.02x^2}{x} = 5 - 0.02x$

24. $\bar{R}'(x) = -0.02$

26. $P'(x) = 3.9 - 0.04x$

28. $\bar{P}'(x) = -0.02 + \dfrac{145}{x^2}$

30. True: If $p = b - mx$ then $R(x) = xp = bx - mx^2$, and $R'(x) = b - 2mx$.

32. False: If $C(x) = 5x + 10$, then the marginal cost is $C'(x) = 5$. In this case, the average marginal cost over any interval is 5. However, the average cost is $\bar{C}(x) = 5 + \dfrac{10}{x}$ so the marginal average cost is $\bar{C}'(x) = -\dfrac{10}{x^2}$, which is not equal to 5 over the interval [1,2], for example.

34. $C(x) = 1{,}000 + 100x - 0.25x^2$

(A) The exact cost of producing the 51st guitar is:

$C(51) - C(50)$

$= 1{,}000 + 100(51) - 0.25(51)^2 - [1{,}000 + 100(50) - 0.25(50)^2]$

$= 100 - 0.25(51)^2 + 0.25(50)^2 = 74.75$ or \$74.75

(B) $C'(x) = 100 - 0.5x$

$C'(50) = 100 - 0.5(50) = 75$ or \$75.

36. $C(x) = 10{,}000 + 20x$

(A) $\bar{C}(x) = \dfrac{10{,}000 + 20x}{x} = \dfrac{10{,}000}{x} + 20$

$\bar{C}(1{,}000) = \dfrac{10{,}000}{1000} + 20 = 30 \quad = 30$ or \$30

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(B) $\overline{C}\,'(x) = -10{,}000x^{-2} = \dfrac{-10{,}000}{x^2}$

$\overline{C}\,'(1{,}000) = \dfrac{-10{,}000}{(1{,}000)^2} = -0.01$ or $-1¢$

At a production level of 1,000 dictionaries, average cost is decreasing at the rate of 1¢ per game.

(C) The average cost per game if 1,001 are produced is approximately \$30.00 – \$0.01 = \$29.99.

38. $P(x) = 22x - 0.2x^2 - 400,\ 0 \le x \le 100$

(A) The exact profit from the sale of the 41st calendar is

$P(41) - P(40) = 22(41) - 0.2(41)^2 - 400 - [22(40) - 0.2(40)^2 - 400]$

$= 22 - 0.2(41)^2 + 0.2(40)^2 = 5.80$ or \$5.80

(B) $P'(x) = 22 - 0.4x$

$P'(40) = 22 - 0.4(40) = 22 - 16 = 6$ or \$6

40. $P(x) = 12x - 0.02x^2 - 1{,}000,\ 0 \le x \le 600;\quad P'(x) = 12 - 0.04x$

(A) $P'(200) = 12 - 0.04(200) = 12 - 8 = 4$ or \$4;
at a production level of 200 cameras, profit is increasing at the rate of \$4 per camera.

(B) $P'(350) = 12 - 0.04(350) = 12 - 14 = -2$ or –\$2;
at a production level of 350 cameras, profit is decreasing at the rate of \$2 per camera.

42. $P(x) = 20x - 0.02x^2 - 320,\ 0 \le x \le 1{,}000$

Average profit: $\overline{P}\,(x) = \dfrac{P(x)}{x} = 20 - 0.02x - \dfrac{320}{x} = 20 - 0.02x - 320x^{-1}$

(A) At $x = 40$, $\overline{P}\,(40) = 20 - 0.02(40) - \dfrac{320}{40} = 11.20$ or \$11.20.

(B) $\overline{P}\,'(x) = -0.02 + 320x^{-2} = -0.02 + \dfrac{320}{x^2}$

$\overline{P}\,'(40) = -0.02 + \dfrac{320}{(40)^2} = 0.18$ or \$0.18;

at a production level of 40 grills, the average profit per grill is increasing at the rate of \$0.18 per grill.

(C) The average profit per grill if 41 grills are produced is approximately \$11.20 + \$0.18 = \$11.38.

44. $x = 1{,}000 - 20p$

(A) $20p = 1{,}000 - x,\ p = 50 - 0.05x,\ 0 \le x \le 1{,}000$

(B) $R(x) = x(50 - 0.05x) = 50x - 0.05x^2,\ 0 \le x \le 1{,}000$

(C) $R'(x) = 50 - 0.10x$

$R'(400) = 50 - 0.10(400) = 50 - 40 = 10$;
at a production level of 400 steam irons, revenue is increasing at the rate of \$10 per steam iron.

(D) $R'(650) = 50 - 0.10(650) = 50 - 65 = -15$;
at a production level of 650 steam irons, revenue is decreasing at the rate of \$15 per steam iron.

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46. $x = 9{,}000 - 30p$ and $C(x) = 150{,}000 + 30x$

(A) $30p = 9{,}000 - x, \quad p = 300 - \frac{1}{30}x, \; 0 \le x \le 9{,}000$

(B) $C'(x) = 30$

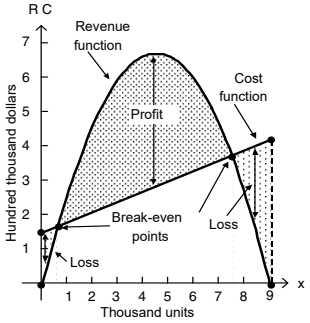
(C) $R(x) = x\left(300 - \frac{1}{30}x\right) = 300x - \frac{1}{30}x^2, \; 0 \le x \le 9{,}000$

(D) $R'(x) = 300 - \frac{1}{15}x$

(E) $R'(3{,}000) = 300 - \frac{1}{15}(3{,}000) = 100$; at a production level of 3,000 sets, revenue is increasing at the rate of \$100 per set.

$R'(6000) = 300 - \frac{1}{15}(6{,}000) = 300 - 400 = -100$; at a production level of 6,000 sets, revenue is decreasing at the rate of \$100 per set.

(F) The graphs of $C(x)$ and $R(x)$ are shown at the right.



To find the break-even points, set $C(x) = R(x)$:

$$150{,}000 + 30x = 300x - \frac{1}{30}x^2$$

$$x^2 - 8{,}100x + 4{,}500{,}000 = 0$$

$$(x - 600)(x - 7{,}500) = 0$$

$$x = 600 \quad \text{or} \quad x = 7{,}500$$

Now, $C(600) = 150{,}000 + 30(600) = 168{,}000$;

$C(7{,}500) = 150{,}000 + 30(7{,}500) = 375{,}000$

Thus, the break-even points are:
(600, 168,000) and (7,500, 375,000).

(G) $P(x) = R(x) - C(x) = 300x - \frac{1}{30}x^2 - (150{,}000 + 30x)$

$$= -\frac{1}{30}x^2 + 270x - 150{,}000$$

(H) $P'(x) = -\frac{1}{15}x + 270$

(I) $P'(1{,}500) = -\frac{1}{15}(1{,}500) + 270 = 170$; at a production level of 1,500 sets, profit is increasing at the rate of \$170 per set.

$P'(4{,}500) = -\frac{1}{15}(4{,}500) + 270 = -30$; at a production level of 4,500 sets, profit is decreasing at the rate of \$30 per set.

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48. (A) We are given $p = 25$ when $x = 300$ and $p = 20$ when $x = 400$. Thus, we have the pair of equations:

$25 = 300m + b$

$20 = 400\ m + b$

Subtracting the second equation from the first, we get $-100m = 5$. Thus, $m = -\frac{1}{20}$.

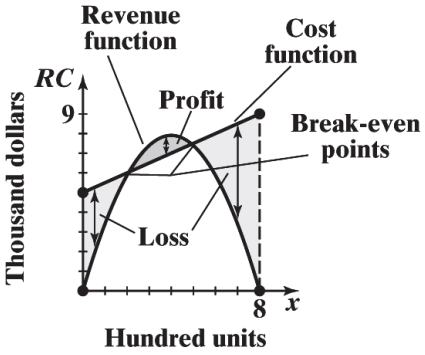
Substituting this into either equation yields $b = 40$. Therefore,

$$p = -\frac{1}{20}x + 40 = 40 - \frac{x}{20},\ 0 \le x \le 800$$

(B) $$R(x) = x\left(40 - \frac{x}{20}\right) = 40x - \frac{x^2}{20},\ 0 \le x \le 800$$

(C) From the financial department's estimates, $m = 5$ and $b = 5{,}000$. Thus, $C(x) = 5x + 5{,}000$.

(D) The graphs of $R(x)$ and $C(x)$ are shown at the right.



To find the break-even points, set $C(x) = R(x)$:

$$\begin{aligned} 5x + 5{,}000 &= 40x - \frac{x^2}{20} \\ x^2 - 700x + 100{,}000 &= 0 \\ (x - 200)(x - 500) &= 0 \\ x = 200 &\quad \text{or } x = 500 \end{aligned}$$

Now, $C(200) = 5(200) + 5{,}000 = 6{,}000$ and

$C(500) = 5(500) + 5{,}000 = 7{,}500$

Thus, the break-even points are: (200, 6,000) and (500, 7,500).

(E) $$P(x) = R(x) - C(x) = 40x - \frac{x^2}{20} - (5x + 5{,}000)$$

$$= 35x - \frac{x^2}{20} - 5{,}000$$

(F) $$P'(x) = 35 - \frac{x}{10}$$

$P'(325) = 35 - \frac{325}{10} = 2.5$; at a production level of 325 toasters, profit is increasing at the rate of \$2.50 per toaster.

$P'(425) = 35 - \frac{425}{10} = -7.5$; at a production level of 425 toasters, profit is decreasing at the rate of \$7.50 per toaster.

50. Total cost: $C(x) = 5x + 2{,}340$

Total revenue: $R(x) = 40x - 0.1x^2,\ 0 \le x \le 400$

(A) $R'(x) = 40 - 0.2x$

The graph of R has a horizontal tangent line at the value(s) of x where $R'(x) = 0$, i.e.

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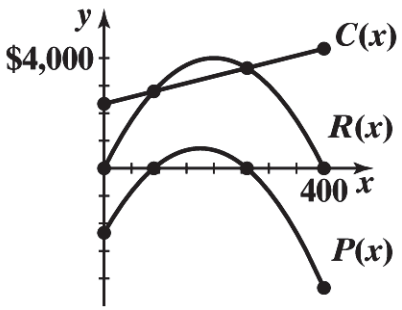
$40 - 0.2x = 0$
or $x = 200$

(B) $P(x) = R(x) - C(x) = 40x - 0.1x^2 - (5x + 2{,}340)$
$= 35x - 0.1x^2 - 2{,}340$

(C) $P'(x) = 35 - 0.2x$. Setting $P'(x) = 0$, we have
$35 - 0.2x = 0$
or $x = 175$

(D) The graphs of $C(x)$, $R(x)$ and $P(x)$ are shown at the right.



Break-even points: $R(x) = C(x)$

$40x - 0.1x^2$	$= 5x + 2{,}340$
$x^2 - 350x + 23{,}400$	$= 0$
$(x - 90)(x - 260)$	$= 0$
$x = 90$	or $x = 260$

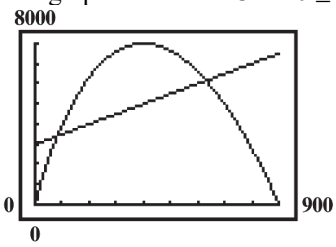
Thus, the break-even points are:
(90, 2,790) and (260, 3,640).

x intercepts for P: $-0.1x^2 + 35x - 2{,}340 = 0$ or
$x^2 - 350x + 23{,}400 = 0$
which is the same as the above equation. Thus, $x = 90$ and $x = 260$ are x intercepts of P.

52. Demand equation: $p = 60 - 2\sqrt{x} = 60 - 2x^{1/2}$
Cost equation: $C(x) = 3{,}000 + 5x$

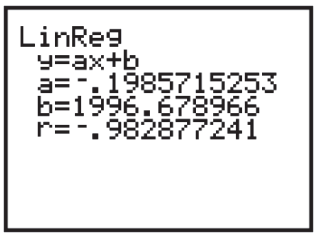
(A) Revenue $R(x) = xp = x(60 - 2x^{1/2})$
$= 60x - 2x^{3/2}$

(B) The graphs for R and C for $0 \le x \le 900$ are shown below:

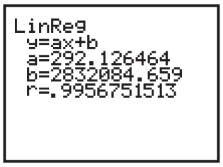


Break-even points: (81, 3,405), (631, 6,155)

54. (A)



(B) Fixed costs: \$2,832,085; variable cost: \$292



(C) Let $y = p(x)$ be the linear regression equation found in part (A) and let $y = C(x)$ be the linear regression equation found in part (B). Then revenue $R(x) = xp(x)$, and the break-even points are

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$R(x) = C(x)$.

Break-even points: (2,253, 3,490,130), (6,331, 4,681,675).

(D) The company will make a profit when $2{,}253 \le x \le 6{,}331$. From part A), $p(2{,}253) = 740$ and $p(6{,}331) = 1{,}549$. Thus, the company will make a profit for the price range $\$740 \le p \le \$1{,}549$.

CHAPTER 2 REVIEW

1. $f(x) = 2x^2 + 5$

(A) $f(3) - f(1) = 2(3)^2 + 5 - [2(1)^2 + 5] = 16$

(B) Average rate of change: $\dfrac{f(3)-f(1)}{3-1} = \dfrac{16}{2} = 8$

(C) Slope of secant line: $\dfrac{f(3)-f(1)}{3-1} = \dfrac{16}{2} = 8$

(D) Instantaneous rate of change at $x = 1$:

Step 1. $\dfrac{f(1+h)-f(1)}{h} = \dfrac{2(1+h)^2+5-[2(1)^2+5]}{h} = \dfrac{2(1+2h+h^2)+5-7}{h} = \dfrac{4h+2h^2}{h} = 4+2h$

Step 2. $\lim\limits_{h\to 0}\dfrac{f(1+h)-f(1)}{h} = \lim\limits_{h\to 0}(4+2h) = 4$

(E) Slope of the tangent line at $x = 1$: 4

(F) $f'(1) = 4$ (2-2)

2. $f(x) = -3x + 2$

Step 1. Find $f(x+h)$

$f(x+h) = -3(x+h)+2 = -3x-3h+2$

Step 2. Find $f(x+h)-f(x)$

$f(x+h)-f(x) = -3x-3h+2-(-3x+2) = -3x-3h+2+3x-2 = -3h$

Step 3. Find $\dfrac{f(x+h)-f(x)}{h}$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{-3h}{h} = -3$

Step 4. Find $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

$\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}(-3) = -3$ (2-2)

3. (A) $\lim\limits_{x\to 1}(5f(x)+3g(x)) = 5\lim\limits_{x\to 1}f(x) + 3\lim\limits_{x\to 1}g(x) = 5\cdot 2 + 3\cdot 4 = 22$

(B) $\lim\limits_{x\to 1}[f(x)g(x)] = [\lim\limits_{x\to 1}f(x)][\lim\limits_{x\to 1}g(x)] = 2\cdot 4 = 8$

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(C) $\lim\limits_{x\to 1}\dfrac{g(x)}{f(x)} = \dfrac{\lim\limits_{x\to 1} g(x)}{\lim\limits_{x\to 1} f(x)} = \dfrac{4}{2} = 2$

(D) $\lim\limits_{x\to 1}[5 + 2x - 3g(x)] = \lim\limits_{x\to 1} 5 + \lim\limits_{x\to 1} 2x - 3\lim\limits_{x\to 1} g(x) = 5 + 2 - 3(4) = -5$ (2-1)

4. $f(1.5) \approx 1.5$ (2-1)

5. $f(2.5) \approx 3.5$ (2-1)

6. $f(2.75) \approx 3.75$ (2-1)

7. $f(3.25) \approx 3.75$ (2-1)

8. (A) $\lim\limits_{x\to 1^-} f(x) = 1$ (B) $\lim\limits_{x\to 1^+} f(x) = 1$ (C) $\lim\limits_{x\to 1} f(x) = 1$ (D) $f(1) = 1$ (2-1)

9. (A) $\lim\limits_{x\to 2^-} f(x) = 2$ (B) $\lim\limits_{x\to 2^+} f(x) = 3$ (C) $\lim\limits_{x\to 2} f(x)$ does not exist (D) $f(2) = 3$ (2-1)

10. (A) $\lim\limits_{x\to 3^-} f(x) = 4$ (B) $\lim\limits_{x\to 3^+} f(x) = 4$ (C) $\lim\limits_{x\to 3} f(x) = 4$ (D) $f(3)$ does not exist (2-1)

11. (A) From the graph, $\lim\limits_{x\to 1} f(x)$ does not exist since

$\lim\limits_{x\to 1^-} f(x) = 2 \neq \lim\limits_{x\to 1^+} f(x) = 3.$

(B) $f(1) = 3$

(C) f is NOT continuous at $x = 1$, since $\lim\limits_{x\to 1} f(x)$ does not exist. (2-3)

12. (A) $\lim\limits_{x\to 2} f(x) = 2$ (B) $f(2)$ is not defined

(C) f is NOT continuous at $x = 2$ since $f(2)$ is not defined. (2-3)

13. (A) $\lim\limits_{x\to 3} f(x) = 1$ (B) $f(3) = 1$

(C) f is continuous at $x = 3$ since $\lim\limits_{x\to 3} f(x) = f(3)$. (2-3)

14. $\lim\limits_{x\to\infty} f(x) = 10$ (2-2)

15. $\lim\limits_{x\to-\infty} f(x) = 5$ (2-2)

16. $\lim\limits_{x\to 2^+} f(x) = \infty$ (2-2)

17. $\lim\limits_{x\to 2^-} f(x) = -\infty$ (2-2)

18. $\lim\limits_{x\to 6^-} f(x) = \infty$ (2-2)

19. $\lim\limits_{x\to 6^+} f(x) = \infty$ (2-2)

20. $\lim\limits_{x\to 6} f(x) = \infty$ (2-2)

21. $x = 2$ and $x = 6$ (2-2)

22. $y = 5$ and y = 10 (2-2)

23. x = 2 and x = 6 (2-3)

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24. $f(x) = 3x^2 - 5$

Step 1. Find $f(x + h)$:

$$f(x + h) = 3(x + h)^2 - 5 = 3x^2 + 6xh + 3h^2 - 5$$

Step 2. Find $f(x + h) - f(x)$:

$$f(x + h) - f(x) = 3x^2 + 6xh + 3h^2 - 5 - (3x^2 - 5) = 6xh + 3h^2$$

Step 3. Find $\dfrac{f(x+h) - f(x)}{h}$:

$$\frac{f(x + h) - f(x)}{h} = \frac{6xh + 3h^2}{h} = \frac{h(6x + 3h)}{h} = 6x + 3h,\ h \neq 0$$

Step 4. Find $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$\lim_{h\to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h\to 0}(6x + 3h) = 6x$$

Thus, $f'(x) = 6x$. (2-4)

25. (A) $h'(x) = (3f(x))' = 3f'(x);\ h'(5) = 3f'(5) = 3(-1) = -3$

(B) $h'(x) = (-2g(x))' = -2g'(x);\ h'(5) = -2g'(5) = -2(-3) = 6$

(C) $h'(x) = 2f'(x);\ h'(5) = 2(-1) = -2$

(D) $h'(x) = -g'(x);\ h'(5) = -(-3) = 3$

(E) $h'(x) = 2f'(x) + 3g'(x);\ h'(5) = 2(-1) + 3(-3) = -11$ (2-5)

26. $f(x) = \dfrac{1}{3}x^3 - 5x^2 + 1;\ f'(x) = x^2 - 10$ (2-5)

27. $f(x) = 2x^{1/2} - 3x;\ f'(x) = 2 \cdot \dfrac{1}{2}x^{-1/2} - 3 = \dfrac{1}{x^{1/2}} - 3$ (2-5)

28. $f(x) = 5$

$f'(x) = 0$ (2-5)

29. $f(x) = \dfrac{3}{2x} + \dfrac{5x^3}{4} = \dfrac{3}{2}x^{-1} + \dfrac{5}{4}x^3;$

$$f'(x) = -\frac{3}{2}x^{-2} + \frac{15}{4}x^2 = -\frac{3}{2x^2} + \frac{15}{4}x^2$$ (2-5)

30. $f(x) = \dfrac{0.5}{x^4} + 0.25x^4 = 0.5x^{-4} + 0.25x^4$

$$f'(x) = 0.5(-4)x^{-5} + 0.25(4x^3) = -2x^{-5} + x^3 = -\frac{2}{x^5} + x^3$$ (2-5)

31. $f(x) = (3x^3 - 2)(x + 1) = 3x^4 + 3x^3 - 2x - 2$

$f'(x) = 12x^3 + 9x^2 - 2$ (2-5)

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For Problems 32 – 35, $f(x) = x^2 + x$.

32. $\Delta x = x_2 - x_1 = 3 - 1 = 2$, $\Delta y = f(x_2) - f(x_1) = 12 - 2 = 10$,

$$\frac{\Delta y}{\Delta x} = \frac{10}{2} = 5. \qquad (2\text{-}6)$$

33. $$\frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x} = \frac{f(1+2) - f(1)}{2} = \frac{f(3) - f(1)}{2} = \frac{12 - 2}{2} = 5 \qquad (2\text{-}6)$$

34. $dy = f'(x)dx = (2x + 1)dx$. For $x_1 = 1$, $x_2 = 3$,

$dx = \Delta x = 3 - 1 = 2$, $dy = (2 \cdot 1 + 1) \cdot 2 = 3 \cdot 2 = 6$ (2-6)

35. $\Delta y = f(x + \Delta x) - f(x)$; at $x = 1$, $\Delta x = 0.2$,

$\Delta y = f(1.2) - f(1) = 0.64$

$dy = f'(x)dx$ where $f'(x) = 2x + 1$; at $x = 1$

$dy = 3(0.2) = 0.6$ (2-6)

36. From the graph:

(A) $\lim\limits_{x \to 2^-} f(x) = 4$ (B) $\lim\limits_{x \to 2^+} f(x) = 6$

(C) $\lim\limits_{x \to 2} f(x)$ does not exist since $\lim\limits_{x \to 2^-} f(x) \neq \lim\limits_{x \to 2^+} f(x)$

(D) $f(2) = 6$ (E) No, since $\lim\limits_{x \to 2} f(x)$ does not exist. (2-3)

37. From the graph:

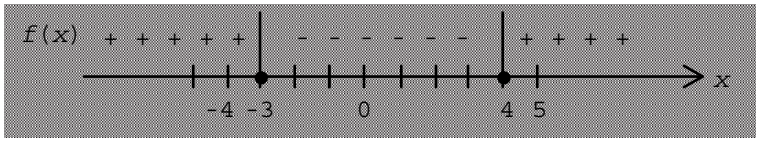
(A) $\lim\limits_{x \to 5^-} f(x) = 3$ (B) $\lim\limits_{x \to 5^+} f(x) = 3$ (C) $\lim\limits_{x \to 5} f(x) = 3$ (D) $f(5) = 3$

(E) Yes, since $\lim\limits_{x \to 5} f(x) = f(5) = 3$. (2-3)

38. (A) $f(x) < 0$ on $(8, \infty)$ (B) $f(x) \geq 0$ on $[0, 8]$ (2-3)

39. $x^2 - x < 12$ or $x^2 - x - 12 < 0$

Let $f(x) = x^2 - x - 12 = (x + 3)(x - 4)$. Then f is continuous for all x and $f(-3) = f(4) = 0$. Thus, $x = -3$ and $x = 4$ are partition numbers.



Test Numbers

x	$f(x)$
−4	8(+)
0	−12 (−)
5	8(+)

Thus, $x^2 - x < 12$ for: $-3 < x < 4$ or $(-3, 4)$. (2-3)

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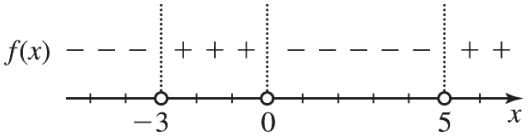
40. $\dfrac{x-5}{x^2+3x} > 0$ or $\dfrac{x-5}{x(x+3)} > 0$

Let $f(x) = \dfrac{x-5}{x(x+3)}$. Then f is discontinuous at $x = 0$ and $x = -3$, and $f(5) = 0$. Thus, $x = -3$, $x = 0$, and $x = 5$ are partition numbers.



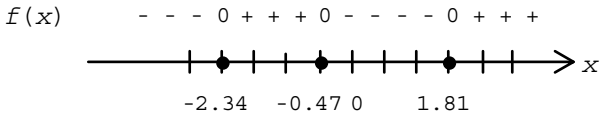
Test Numbers

x	$f(x)$
-4	$-\frac{9}{4}(-)$
-1	$3(+)$
1	$-1(-)$
6	$\frac{1}{54}(+)$

Thus, $\dfrac{x-5}{x^2+3x} > 0$ for $-3 < x < 0$ or $x > 5$, or $(-3, 0) \cup (5, \infty)$. (2-3)

41. $x^3 + x^2 - 4x - 2 > 0$

Let $f(x) = x^3 + x^2 - 4x - 2$. Then f is continuous for all x and $f(x) = 0$ at $x = -2.3429$, -0.4707 and 1.8136.



Thus, $x^3 + x^2 - 4x - 2 > 0$ for $-2.3429 < x < -0.4707$ or $1.8136 < x < \infty$, or $(-2.3429, -0.4707) \cup (1.8136, \infty)$. (2-3)

42. $f(x) = 0.5x^2 - 5$

(A) $\dfrac{f(4)-f(2)}{4-2} = \dfrac{0.5(4)^2 - 5 - [0.5(2)^2 - 5]}{2} = \dfrac{8-2}{2} = 3$

(B) $\dfrac{f(2+h)-f(2)}{h} = \dfrac{0.5(2+h)^2 - 5 - [0.5(2)^2 - 5]}{h} = \dfrac{0.5(4+4h+h^2) - 5 + 3}{h}$

$$= \frac{2h + 0.5h^2}{h} = \frac{h(2+0.5h)}{h} = 2 + 0.5h$$

(C) $\lim\limits_{h\to 0} \dfrac{f(2+h)-f(2)}{h} = \lim\limits_{h\to 0} (2 + 0.5h) = 2$ (2-4)

43. $y = \dfrac{1}{3}x^{-3} - 5x^{-2} + 1;$

$\dfrac{dy}{dx} = \dfrac{1}{3}(-3)x^{-4} - 5(-2)x^{-3} = -x^{-4} + 10x^{-3}$ (2-5)

44. $y = \dfrac{3\sqrt{x}}{2} + \dfrac{5}{3\sqrt{x}} = \dfrac{3}{2}x^{1/2} + \dfrac{5}{3}x^{-1/2};$

$y' = \dfrac{3}{2}\left(\dfrac{1}{2}x^{-1/2}\right) + \dfrac{5}{3}\left(-\dfrac{1}{2}x^{-3/2}\right) = \dfrac{3}{4x^{1/2}} - \dfrac{5}{6x^{3/2}} = \dfrac{3}{4\sqrt{x}} - \dfrac{5}{6\sqrt{x^3}}$ (2-5)

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45. $g(x) = 1.8\sqrt[3]{x} + \dfrac{0.9}{\sqrt[3]{x}} = 1.8x^{1/3} + 0.9x^{-1/3}$

$g'(x) = 1.8\left(\dfrac{1}{3}x^{-2/3}\right) + 0.9\left(-\dfrac{1}{3}x^{-4/3}\right) = 0.6x^{-2/3} - 0.3x^{-4/3} = \dfrac{0.6}{x^{2/3}} - \dfrac{0.3}{x^{4/3}}$ (2-5)

46. $y = \dfrac{2x^3 - 3}{5x^3} = \dfrac{2}{5} - \dfrac{3}{5}x^{-3}; y' = -\dfrac{3}{5}(-3x^{-4}) = \dfrac{9}{5x^4}$ (2-5)

47. $f(x) = x^2 + 4$

$f'(x) = 2x$

(A) The slope of the graph at $x = 1$ is $m = f'(1) = 2$.

(B) $f(1) = 1^2 + 4 = 5$

The tangent line at (1, 5), where the slope $m = 2$, is:

$(y - 5) = 2(x - 1)$ [Note: $(y - y_1) = m(x - x_1)$.]

$y = 5 + 2x - 2$

$y = 2x + 3$ (2-4, 2-5)

48. $f(x) = 10x - x^2$

$f'(x) = 10 - 2x$

The tangent line is horizontal at the values of x such that

$f'(x) = 0$:

$10 - 2x = 0$

$x = 5$ (2-4)

49. $f(x) = x^3 + 3x^2 - 45x - 135$

$f'(x) = 3x^2 + 6x - 45$

Set $f'(x) = 0$:

$3x^2 + 6x - 45 = 0$

$x^2 + 2x - 15 = 0$

$(x - 3)(x + 5) = 0$

$x = 3, \quad x = -5$ (2-5)

50. $f(x) = x^4 - 2x^3 - 5x^2 + 7x$

$f'(x) = 4x^3 - 6x^2 - 10x + 7$

Set $f'(x) = 4x^3 - 6x^2 - 10x + 7 = 0$ and solve for x using a root-approximation routine on a graphing utility:

$f'(x) = 0$ at $x = -1.34, \quad x = 0.58, \quad x = 2.26$ (2-5)

51. $f(x) = x^5 - 10x^3 - 5x + 10$

$f'(x) = 5x^4 - 30x^2 - 5 = 5(x^4 - 6x^2 - 1)$

Let $f'(x) = 5(x^4 - 6x^2 - 1) = 0$ and solve for x using a root-approximation routine on a graphing utility; that is, solve $x^4 - 6x^2 - 1 = 0$ for x.

$f'(x) = 0$ at $x = \pm 2.4824$ (2-5)

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52. $y = f(x) = 8x^2 - 4x + 1$

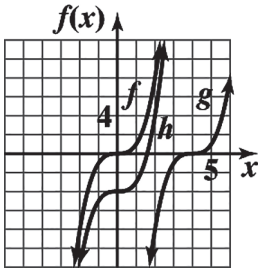
(A) Instantaneous velocity function; $v(x) = f'(x) = 16x - 4$.
(B) $v(3) = 16(3) - 4 = 44$ ft/sec. (2-5)

53. $y = f(x) = -5x^2 + 16x + 3$

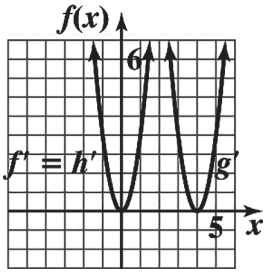
(A) Instantaneous velocity function: $v(x) = f'(x) = -10x + 16$.
(B) $v(x) = 0$ when $-10x + 16 = 0$

$$10x = 16$$
$$x = 1.6 \text{ sec}$$ (2-5)

54. (A) The graph of g is the graph of f shifted 4 units to the right, and the graph of h is the graph of f shifted 4 units down.



(B) The graph of g' is the graph of f' shifted 4 units to the right, and the graph of h' is the graph of f'.



(2-4)

55. $f(x) = x^2 - 4$ is a polynomial function; f is continuous on $(-\infty, \infty)$. (2-3)

56. $f(x) = \dfrac{x+1}{x-2}$ is a rational function and the denominator $x - 2$ is 0 at $x = 2$. Thus f is continuous for all x such that $x \neq 2$, i.e., on $(-\infty, 2) \cup (2, \infty)$. (2-3)

57. $f(x) = \dfrac{x+4}{x^2+3x-4}$ is a rational function and the denominator $x^2 + 3x - 4 = (x + 4)(x - 1)$ is 0 at $x = -4$ and $x = 1$. Thus, f is continuous for all x except $x = -4$ and $x = 1$, i.e., on $(-\infty, -4) \cup (-4, 1) \cup (1, \infty)$. (2-2)

58. $f(x) = \sqrt[3]{4 - x^2}$; $g(x) = 4 - x^2$ is continuous for all x since it is a polynomial function. Therefore, $f(x) = \sqrt[3]{g(x)}$ is continuous for all x, i.e., on $(-\infty, \infty)$. (2-3)

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59. $f(x) = \sqrt{4-x^2}$; $g(x) = 4 - x^2$ is continuous for all x and $g(x)$ is nonnegative for $-2 \le x \le 2$.
Therefore, $f(x) = \sqrt{g(x)}$ is continuous for $-2 \le x \le 2$, i.e., on $[-2, 2]$. (2-3)

60. $f(x) = \dfrac{2x}{x^2-3x} = \dfrac{2x}{x(x-3)} = \dfrac{2}{x-3}, x \ne 0$

(A) $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \dfrac{2}{x-3} = \dfrac{\lim\limits_{x\to 1} 2}{\lim\limits_{x\to 1}(x-3)} = \dfrac{2}{-2} = -1$

(B) $\lim\limits_{x\to 3} f(x) = \lim\limits_{x\to 3} \dfrac{2}{x-3}$ does not exist since $\lim\limits_{x\to 3} 2 = 2$ and $\lim\limits_{x\to 3} (x-3) = 0$

(C) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{2}{x-3} = -\dfrac{2}{3}$ (2-1)

61. $f(x) = \dfrac{x+1}{(3-x)^2}$

(A) $\lim\limits_{x\to 1} \dfrac{x+1}{(3-x)^2} = \dfrac{\lim\limits_{x\to 1}(x+1)}{\lim\limits_{x\to 1}(3-x)^2} = \dfrac{2}{2^2} = \dfrac{1}{2}$

(B) $\lim\limits_{x\to -1} \dfrac{x+1}{(3-x)^2} = \dfrac{\lim\limits_{x\to -1}(x+1)}{\lim\limits_{x\to -1}(3-x)^2} = \dfrac{0}{4^2} = 0$

(C) $\lim\limits_{x\to 3} \dfrac{x+1}{(3-x)^2}$ does not exist since $\lim\limits_{x\to 3}(x+1) = 4$ and $\lim\limits_{x\to 3}(3-x)^2 = 0$ (2-1)

62. $f(x) = \dfrac{|x-4|}{x-4} = \begin{cases} -1 & \text{if } x < 4 \\ 1 & \text{if } x > 4 \end{cases}$

(A) $\lim\limits_{x\to 4^-} f(x) = -1$ (B) $\lim\limits_{x\to 4^+} f(x) = 1$ (C) $\lim\limits_{x\to 4} f(x)$ does not exist. (2-1)

63. $f(x) = \dfrac{x-3}{9-x^2} = \dfrac{x-3}{(3+x)(3-x)} = \dfrac{-(3-x)}{(3+x)(3-x)} = \dfrac{-1}{3+x}, x \ne 3$

(A) $\lim\limits_{x\to 3} f(x) = \lim\limits_{x\to 3} \dfrac{-1}{3+x} = -\dfrac{1}{6}$

(B) $\lim\limits_{x\to -3} f(x) = \lim\limits_{x\to -3} \dfrac{-1}{3+x}$ does not exist

(C) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{-1}{3+x} = -\dfrac{1}{3}$ (2-1)

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64. $f(x) = \dfrac{x^2 - x - 2}{x^2 - 7x + 10} = \dfrac{(x-2)(x+1)}{(x-2)(x-5)} = \dfrac{x+1}{x-5}, x \neq 2$

(A) $\lim\limits_{x \to -1} f(x) = \lim\limits_{x \to -1} \dfrac{x+1}{x-5} = 0$

(B) $\lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2} \dfrac{x+1}{x-5} = \dfrac{3}{-3} = -1$

(C) $\lim\limits_{x \to 5} f(x) = \lim\limits_{x \to 5} \dfrac{x+1}{x-5}$ does not exist (2-1)

65. $f(x) = \dfrac{2x}{3x-6} = \dfrac{2x}{3(x-2)}$

(A) $\lim\limits_{x \to \infty} \dfrac{2x}{3x-6} = \lim\limits_{x \to \infty} \dfrac{2x}{3x} = \dfrac{2}{3}$

(B) $\lim\limits_{x \to -\infty} \dfrac{2x}{3x-6} = \lim\limits_{x \to -\infty} \dfrac{2x}{3x} = \dfrac{2}{3}$

(C) $\lim\limits_{x \to 2^-} \dfrac{2x}{3x-6} = \lim\limits_{x \to 2^-} \dfrac{2x}{3(x-2)} = -\infty$

$\lim\limits_{x \to 2^+} \dfrac{2x}{3(x-2)} = \infty$; $\lim\limits_{x \to 2} \dfrac{2x}{3x-6}$ does not exist. (2-2)

66. $f(x) = \dfrac{2x^3}{3(x-2)^2} = \dfrac{2x^3}{3x^2 - 12x + 12}$

(A) $\lim\limits_{x \to \infty} \dfrac{2x^3}{3x^2 - 12x + 12} = \lim\limits_{x \to \infty} \dfrac{2x^3}{3x^2} = \lim\limits_{x \to \infty} \dfrac{2x}{3} = \infty$

(B) $\lim\limits_{x \to -\infty} \dfrac{2x^3}{3x^2 - 12x + 12} = \lim\limits_{x \to -\infty} \dfrac{2x^3}{3x^2} = \lim\limits_{x \to -\infty} \dfrac{2x}{3} = -\infty$

(C) $\lim\limits_{x \to 2^-} \dfrac{2x^3}{3(x-2)^2} = \lim\limits_{x \to 2^+} \dfrac{2x^3}{3(x-2)^2} = \infty$; $\lim\limits_{x \to 2} \dfrac{2x^3}{3(x-2)^2} = \infty$ (2-2)

67. $f(x) = \dfrac{2x}{3(x-2)^3}$

(A) $\lim\limits_{x \to \infty} \dfrac{2x}{3(x-2)^3} = \lim\limits_{x \to \infty} \dfrac{2x}{3x^3} = \lim\limits_{x \to \infty} \dfrac{2}{3x^2} = 0$

(B) $\lim\limits_{x \to -\infty} \dfrac{2x}{3(x-2)^3} = \lim\limits_{x \to -\infty} \dfrac{2x}{3x^3} = \lim\limits_{x \to -\infty} \dfrac{2}{3x^2} = 0$

(C) $\lim\limits_{x \to 2^-} \dfrac{2x}{3(x-2)^3} = -\infty$, $\lim\limits_{x \to 2^+} \dfrac{2x}{3(x-2)^3} = \infty$; $\lim\limits_{x \to 2} \dfrac{2x}{3(x-2)^3}$ does not exist. (2-2)

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68. $f(x) = x^2 + 4$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{[(2+h)^2+4]-[2^2+4]}{h} = \lim_{h\to 0}\frac{4+4h+h^2+4-8}{h} = \lim_{h\to 0}\frac{4h+h^2}{h}$$
$$= \lim_{h\to 0}(4+h) = 4 \qquad (2\text{-}1)$$

69. Let $f(x) = \dfrac{1}{x+2}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\dfrac{1}{(x+h)+2}-\dfrac{1}{x+2}}{h} = \lim_{h\to 0}\frac{x+2-(x+h+2)}{h(x+h+2)(x+2)} = \lim_{h\to 0}\frac{-h}{h(x+h+2)(x+2)}$$
$$= \lim_{h\to 0}\frac{-1}{(x+h+2)(x+2)} = \frac{-1}{(x+2)^2} \qquad (2\text{-}1)$$

70. $f(x) = x^2 - x$

Step 1. Find $f(x+h)$.

$$f(x+h) = (x+h)^2 - (x+h) = x^2 + 2xh + h^2 - x - h$$

Step 2. Find $f(x+h) - f(x)$

$$f(x+h) - f(x) = x^2 + 2xh + h^2 - x - h - (x^2 - x) = x^2 + 2xh + h^2 - x - h - (x^2 - x)$$
$$= x^2 + 2xh + h^2 - x - h - x^2 + x = 2xh + h^2 - h$$

Step 3. Find $\dfrac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2-h}{h} = 2x + h - 1$$

Step 4. Find $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(2x + h - 1) = 2x - 1$$

Thus, $f'(x) = 2x - 1$. (2-4)

71. $f(x) = \sqrt{x} - 3$

Step 1. Find $f(x+h)$.

$$f(x+h) = \sqrt{x+h} - 3$$

Step 2. Find $f(x+h) - f(x)$

$$f(x+h) - f(x) = \sqrt{x+h} - 3 - (\sqrt{x} - 3) = \sqrt{x+h} - 3 - \sqrt{x} + 3 = \sqrt{x+h} - \sqrt{x}$$

Step 3. Find $\dfrac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

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Step 4. Find $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}} \qquad (2\text{-}4)$$

72. Yes, $f'(-1)=0$. (2-4)

73. No. f is not differentiable at $x=0$ since it is not continuous at $x=0$. (2-4)

74. No. f has a vertical tangent at $x=1$. (2-4)

75. No. f is not differentiable at $x=2$; the curve has a "corner" at this point. (2-4)

76. Yes. f is differentiable at $x=3$. In fact, $f'(3)=0$. (2-4)

77. Yes. f is differentiable at $x=4$. (2-4)

78. $f(x)=\dfrac{5x}{x-7}$; f is discontinuous at $x=7$

$\lim\limits_{x\to 7^-}\dfrac{5x}{x-7}=-\infty$, $\lim\limits_{x\to 7^+}\dfrac{5x}{x-7}=\infty$; $x=7$ is a vertical asymptote

$\lim\limits_{x\to\infty}f(x)=\lim\limits_{x\to\infty}\dfrac{5x}{x-7}=\lim\limits_{x\to\infty}\dfrac{5x}{x}=5$; $y=5$ is a horizontal asymptote. (2-2, 2-3)

79. $f(x)=\dfrac{-2x+5}{(x-4)^2}$; f is discontinuous at $x=4$.

$\lim\limits_{x\to 4^-}\dfrac{-2x+5}{(x-4)^2}=-\infty$, $\lim\limits_{x\to 4^+}\dfrac{-2x+5}{(x-4)^2}=-\infty$; $x=4$ is a vertical asymptote.

$\lim\limits_{x\to\infty}\dfrac{-2x+5}{(x-4)^2}=\lim\limits_{x\to\infty}\dfrac{-2x}{x^2}=\lim\limits_{x\to\infty}\dfrac{-2}{x}=0$; $y=0$ is a horizontal asymptote. (2-2)

80. $f(x)=\dfrac{x^2+9}{x-3}$; f is discontinuous at $x=3$.

$\lim\limits_{x\to 3^-}\dfrac{x^2+9}{x-3}=-\infty$, $\lim\limits_{x\to 3^+}\dfrac{x^2+9}{x-3}=\infty$; $x=3$ is a vertical asymptote.

$\lim\limits_{x\to\infty}\dfrac{x^2+9}{x-3}=\lim\limits_{x\to\infty}\dfrac{x^2}{x}=\lim\limits_{x\to\infty}x=\infty$; no horizontal asymptotes. (2-2)

81. $f(x)=\dfrac{x^2-9}{x^2+x-2}=\dfrac{x^2-9}{(x+2)(x-1)}$; f is discontinuous at $x=-2$, $x=1$.

At $x=-2$:

$\lim\limits_{x\to -2^-}\dfrac{x^2-9}{(x+2)(x-1)}=-\infty$, $\lim\limits_{x\to -2^+}\dfrac{x^2-9}{(x+2)(x-1)}=\infty$; $x=-2$ is a vertical asymptote.

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At $x = 1$

$\lim\limits_{x\to 1^-} \dfrac{x^2-9}{(x+2)(x-1)} = \infty$, $\lim\limits_{x\to 1^+} \dfrac{x^2-9}{(x+2)(x-1)} = -\infty$; $x = 1$ is a vertical asymptote.

$\lim\limits_{x\to\infty} \dfrac{x^2-9}{x^2+x-2} = \lim\limits_{x\to\infty} \dfrac{x^2}{x^2} = \lim\limits_{x\to\infty} 1 = 1$; $y = 1$ is a horizontal asymptote. (2-2)

82. $f(x) = \dfrac{x^3-1}{x^3-x^2-x+1} = \dfrac{(x-1)(x^2+x+1)}{(x-1)(x^2-1)} = \dfrac{(x-1)(x^2+x+1)}{(x-1)^2(x+1)} = \dfrac{x^2+x+1}{(x-1)(x+1)}, x \neq 1.$

f is discontinuous at $x = 1, x = -1$.

At $x = 1$:

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} \dfrac{x^2+x+1}{(x-1)(x+1)} = -\infty$, $\lim\limits_{x\to 1^+} f(x) = \infty$; $x = 1$ is a vertical asymptote.

At $x = -1$:

$\lim\limits_{x\to -1^-} \dfrac{x^2+x+1}{(x-1)(x+1)} = \infty$, $\lim\limits_{x\to -1^+} \dfrac{x^2+x+1}{(x-1)(x+1)} = -\infty$; $x = -1$ is a vertical asymptote.

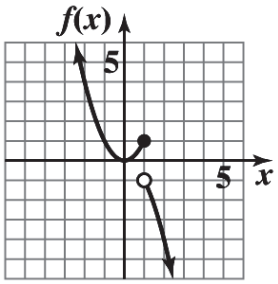
$\lim\limits_{x\to\infty} \dfrac{x^3-1}{x^3-x^2-x+1} = \lim\limits_{x\to\infty} \dfrac{x^3}{x^3} = \lim\limits_{x\to\infty} 1 = 1$; $y = 1$ is a horizontal asymptote. (2-2)

83. $f(x) = x^{1/3}$; $f'(x) = \dfrac{1}{3}x^{-2/3} = \dfrac{1}{3x^{2/3}}$

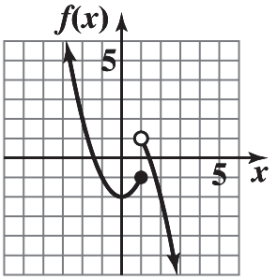
The domain of $f'(x)$ is all real numbers except $x = 0$. At $x = 0$, the graph of $f(x)$ is smooth, but it has a vertical tangent. (2-4)

84. $f(x) = \begin{cases} x^2 - m & \text{if } x \le 1 \\ -x^2 + m & \text{if } x > 1 \end{cases}$

(A)



$\lim\limits_{x\to 1^-} f(x) = 1$, $\lim\limits_{x\to 1^+} f(x) = -1$



$\lim\limits_{x\to 1^-} f(x) = -1$, $\lim\limits_{x\to 1^+} f(x) = 1$

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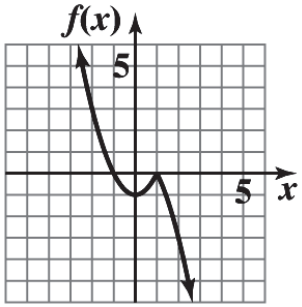
(C) $\lim\limits_{x\to 1^-} f(x) = 1 - m$, $\lim\limits_{x\to 1^+} f(x) = -1 + m$

We want $1 - m = -1 + m$ which implies $m = 1$.



(D) The graphs in (A) and (B) have jumps at $x = 1$; the graph in (C) does not. (2-2)

85. $f(x) = 1 - |x - 1|$, $0 \le x \le 2$

(A) $\lim\limits_{h\to 0^-} \dfrac{f(1+h)-f(1)}{h} = \lim\limits_{h\to 0^-} \dfrac{1-|1+h-1|-1}{h} = \lim\limits_{h\to 0^-} \dfrac{-|h|}{h} = \lim\limits_{h\to 0^-} \dfrac{h}{h} = 1$ ($|h| = -h$ if $h < 0$)

(B) $\lim\limits_{h\to 0^+} \dfrac{f(1+h)-f(1)}{h} = \lim\limits_{h\to 0^+} \dfrac{1-|1+h-1|-1}{h} = \lim\limits_{h\to 0^+} \dfrac{-|h|}{h} = \lim\limits_{h\to 0^+} \dfrac{-h}{h} = -1$ ($|h| = h$ if $h > 0$)

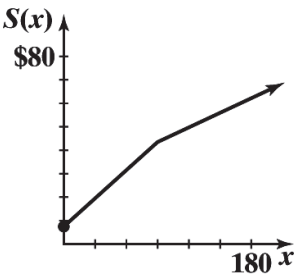
(C) $\lim\limits_{h\to 0} \dfrac{f(1+h)-f(1)}{h}$ does not exist, since the left limit and the right limit are not equal.

(D) $f'(1)$ does not exist. (2-4)

86. (A) $S(x) = 7.47 + 0.4000x$ for $0 \le x \le 90$; $S(90) = 43.47$;
$S(x) = 43.47 + 0.2076\,(x - 90) = 24.786 + 0.2076x$, $x > 90$
Therefore,

$$S(x) = \begin{cases} 7.47 + 0.4000x & \text{if } 0 \le x \le 90 \\ 24.786 + 0.2076x & \text{if } x > 90 \end{cases}$$

(B)



(C) $\lim\limits_{x\to 90^-} S(x) = \lim\limits_{x\to 90^+} S(x) = 43.47 = S(90)$;
$S(x)$ is continuous at $x = 90$.

(2-2)

87. $C(x) = 10{,}000 + 200x - 0.1x^2$

(A) $C(101) - C(100) = 10{,}000 + 200(101) - 0.1(101)^2 - [10{,}000 + 200(100) - 0.1(100)^2]$
$= 29{,}179.90 - 29{,}000 = \$179.90$

(B) $C'(x) = 200 - 0.2x$
$C'(100) = 200 - 0.2(100) = 200 - 20 = \$180$ (2-7)

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88. $C(x) = 5{,}000 + 40x + 0.05x^2$

(A) Cost of producing 100 bicycles:

$C(100) = 5{,}000 + 40(100) + 0.05(100)^2 = 9{,}000 + 500 = 9{,}500$

Marginal cost:

$C'(x) = 40 + 0.1x$

$C'(100) = 40 + 0.1(100) = 40 + 10 = 50$

Interpretation: At a production level of 100 bicycles, the total cost is \$9,500 and is increasing at the rate of \$50 per additional bicycle.

(B) Average cost: $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{5{,}000}{x} + 40 + 0.05x$

$\overline{C}(100) = \dfrac{5{,}000}{100} + 40 + 0.05(100) = 50 + 40 + 5 = 95$

Marginal average cost: $\overline{C}'(x) = -\dfrac{5{,}000}{x^2} + 0.05$ and

$\overline{C}'(100) = -\dfrac{5{,}000}{(100)^2} + 0.05 = -0.5 + 0.05 = -0.45$

Interpretation: At a production level of 100 bicycles, the average cost is \$95 and the average cost is decreasing at a rate of \$0.45 per additional bicycle. (2-7)

89. The approximate cost of producing the 201st printer is greater than that of producing the 601st printer (the slope of the tangent line at $x = 200$ is greater than the slope of the tangent line at $x = 600$). Since the marginal costs are decreasing, the manufacturing process is becoming more efficient. (2-7)

90. $p = 25 - 0.01x$, $C(x) = 2x + 9{,}000$

(A) Marginal cost: $C'(x) = 2$

Average cost: $\overline{C}(x) = \dfrac{C(x)}{x} = 2 + \dfrac{9{,}000}{x}$

Marginal average cost: $\overline{C}' = -\dfrac{9{,}000}{x^2}$

(B) Revenue: $R(x) = xp = 25x - 0.01x^2$

Marginal revenue: $R'(x) = 25 - 0.02x$

Average revenue: $\overline{R}(x) = \dfrac{R(x)}{x} = 25 - 0.01x$

Marginal average revenue: $\overline{R}'(x) = -0.01$

(C) Profit: $P(x) = R(x) - C(x) = 25x - 0.01x^2 - (2x + 9{,}000) = 23x - 0.01x^2 - 9{,}000$

Marginal profit: $P'(x) = 23 - 0.02x$

Average profit: $\overline{P}(x) = \dfrac{P(x)}{x} = 23 - 0.01x - \dfrac{9{,}000}{x}$

Marginal average profit: $\overline{P}'(x) = -0.01 + \dfrac{9{,}000}{x^2}$

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(D) Break-even points: $R(x) = C(x)$

$$25x - 0.01x^2 = 2x + 9{,}000$$
$$0.01x^2 - 23x + 9{,}000 = 0$$
$$x^2 - 2{,}300x + 900{,}000 = 0$$
$$(x - 500)(x - 1{,}800) = 0$$

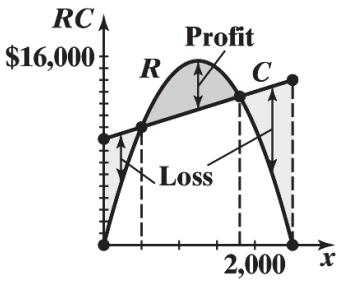
Thus, the break-even points are at $x = 500$, $x = 1{,}800$;
break-even points: (500, 10,000), (1,800, 12,600).

(E) $P'(1{,}000) = 23 - 0.02(1000) = 3$; profit is increasing at the rate of \$3 per umbrella.

$P'(1{,}150) = 23 - 0.02(1{,}150) = 0$; profit is flat.

$P'(1{,}400) = 23 - 0.02(1{,}400) = -5$; profit is decreasing at the rate of \$5 per umbrella.

(F)



(2-7)

91. $N(t) = \dfrac{40t - 80}{t} = 40 - \dfrac{80}{t}, t \geq 2$

(A) Average rate of change from $t = 2$ to $t = 5$:

$$\frac{N(5) - N(2)}{5 - 2} = \frac{\dfrac{40(5) - 80}{5} - \dfrac{40(2) - 80}{2}}{3} = \frac{120}{15} = 8 \text{ components per day.}$$

(B) $N(t) = 40 - \dfrac{80}{t} = 40 - 80t^{-1}$; $N'(t) = 80t^{-2} = \dfrac{80}{t^2}$.

$N'(2) = \dfrac{80}{4} = 20$ components per day. (2-5)

(C) $\lim\limits_{t\to\infty} \dfrac{40t - 80}{t} = \lim\limits_{t\to\infty}\left(\dfrac{40t}{t} - \dfrac{80}{t}\right) = \lim\limits_{t\to\infty}\left(40 - \dfrac{80}{t}\right) = 40$

Long-term employees should near 40 components per day. (2-2)

92. $N(t) = 2t + \dfrac{1}{3}t^{3/2}$, $N'(t) = 2 + \dfrac{1}{2}t^{1/2} = \dfrac{4 + \sqrt{t}}{2}$

$N(9) = 18 + \dfrac{1}{3}(9)^{3/2} = 27$, $N'(9) = \dfrac{4 + \sqrt{9}}{2} = \dfrac{7}{2} = 3.5$

After 9 months, 27,000 pools have been sold and the total sales are increasing at the rate of 3,500 pools per month. (2-5)

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93. (A)

```
CubicReg
 y=ax³+bx²+cx+d
 a=5.5277778E-4
 b=-.0444761905
 c=1.084484127
 d=12.5452381
```

(B) $N(x) \approx 0.0005528x^3 - 0.044x^2 + 1.084x + 12.545$

$N'(x) \approx 0.0016584x^2 - 0.088x + 1.084$

$N(60) \approx 36.9$, $N'(60) \approx 1.7$. In 2020, natural gas consumption will be 36.9 trillion cubic feet and will be INCREASING at the rate of 1.7 trillion cubic feet per year. (2-4)

94. (A)

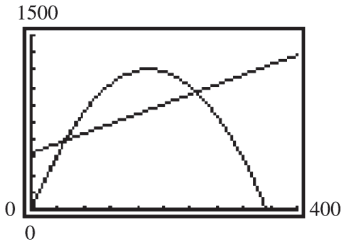
```
LinReg
 y=ax+b
 a=-.0384180791
 b=13.59887006
 r=-.9897782666
```

(B) Fixed costs: \$484.21; variable cost per kringle: \$2.11.

```
LinReg
 y=ax+b
 a=2.107344633
 b=484.2090395
 r=.9939318704
```

(C) Let $p(x)$ be the linear regression equation found in part (A) and let $C(x)$ be the linear regression equation found in part (B). Then revenue $R(x) = xp(x)$ and the break-even points are the points where $R(x) = C(x)$.

Using an intersection routine on a graphing utility, the break-even points are: (51, 591.15) and (248, 1,007.62).



(D) The bakery will make a profit when $51 < x < 248$. From the regression equation in part (A), $p(51) = 11.64$ and $p(248) = 4.07$. Thus, the bakery will make a profit for the price range \$4.07 < p < \$11.64. (2-7)

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95. $C(x) = \dfrac{500}{x^2} = 500x^{-2}, x \geq 1.$

The instantaneous rate of change of concentration at x meters is:

$C'(x) = 500(-2)x^{-3} = \dfrac{-1,000}{x^3}$

The rate of change of concentration at 10 meters is:

$C'(10) = \dfrac{-1,000}{10^3} = -1$ parts per million per meter

The rate of change of concentration at 100 meters is:

$C'(100) = \dfrac{-1,000}{(100)^3} = \dfrac{-1,000}{100,000,000} = -\dfrac{1}{1,000} = -0.001$ part per million per meter. (2-5)

96. $F(t) = 0.16t^2 - 1.6t + 102, F'(t) = 0.32t - 1.6$

$F(4) = 98.16, F'(4) = -0.32.$

After 4 hours the patient's temperature is 98.16°F and is decreasing at the rate of 0.32°F per hour. (2-5)

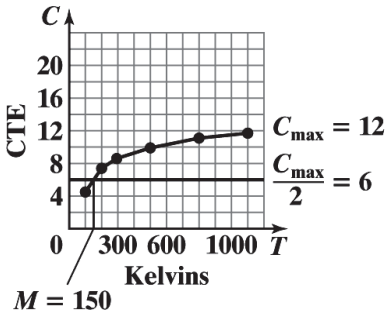
97. $N(t) = 20\sqrt{t} = 20t^{1/2}$

The rate of learning is $N'(t) = 20\left(\dfrac{1}{2}\right)t^{-1/2} = 10t^{-1/2} = \dfrac{10}{\sqrt{t}}.$

(A) The rate of learning after one hour is $N'(1) = \dfrac{10}{\sqrt{1}} = 10$ items per hour.

(B) The rate of learning after four hours is $N'(4) = \dfrac{10}{\sqrt{4}} = \dfrac{10}{2} = 5$ items per hour. (2-5)

98. (A)



(B) $C(T) = \dfrac{12T}{150 + T}$

(C) $C(600) = \dfrac{12(600)}{150 + 600} = 9.6$

To find T when $C = 10$, solve $\dfrac{12T}{150 + T} = 10$ for T.

$$\frac{12T}{150 + T} = 10$$
$$12T = 1500 + 10T$$
$$2T = 1500$$
$$T = 750$$

$T = 750$ when $C = 10$. (2-3)

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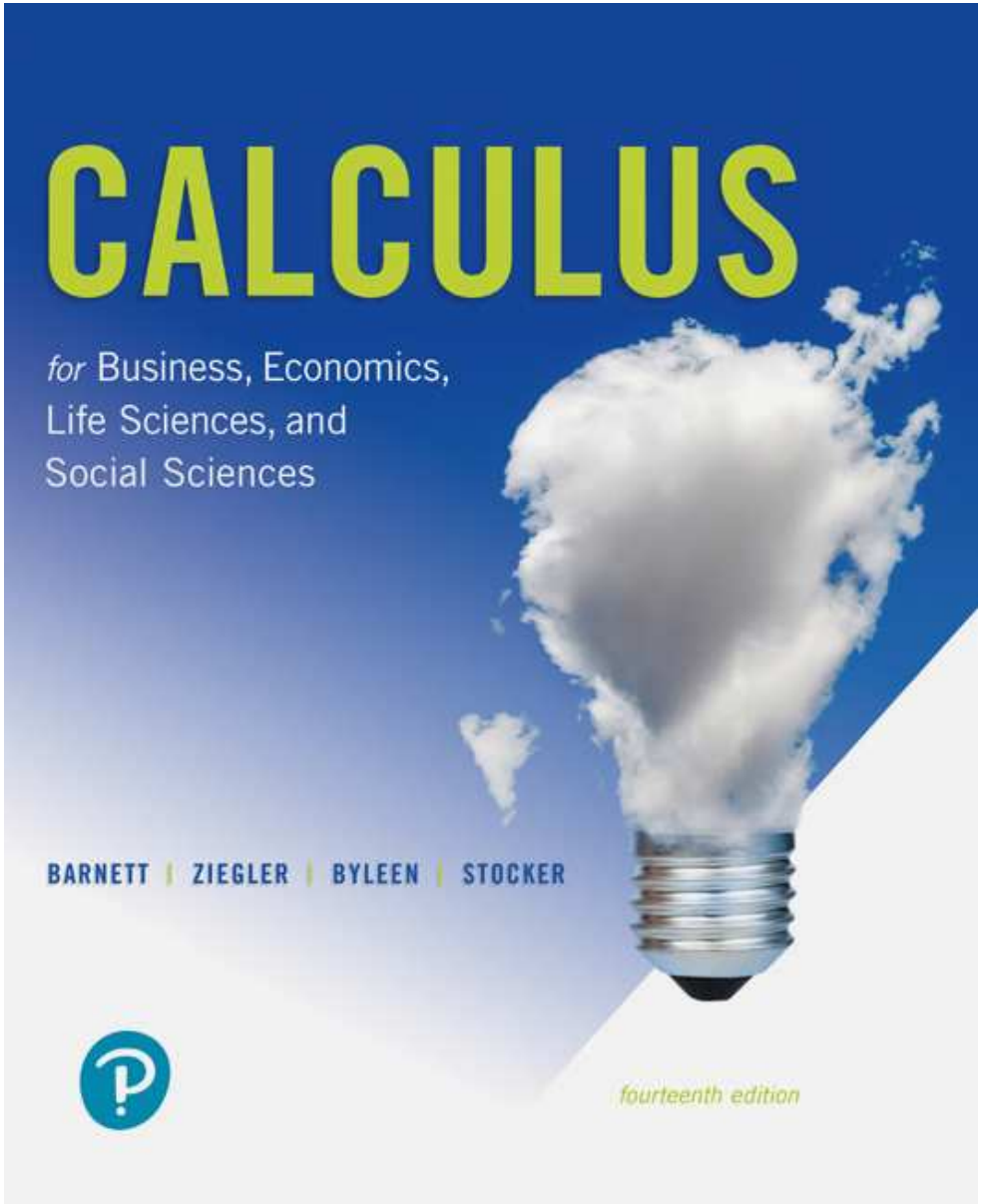

Chapter 2

Limits and The Derivative

Section 1

Introduction to Limits

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Introduction

As we embark on a journey into the study of calculus, it is reasonable to consider how calculus differs from algebra.

The words *static* and *dynamic* capture the essence of the difference between these two mathematics disciplines.

In algebra, we solve equations for a particular value of a variable—a static notion.

In calculus, we are interested in how a change in one variable affects another variable—a dynamic notion.

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Historical Background

Concepts of calculus were developed independently by Isaac Newton (1642–1727) of England and Gottfried Wilhelm von Leibniz (1646–1716) of Germany. They sought methods to solve problems concerning motion.

Calculus today is used in the physical sciences, in business, economics, life sciences, and social sciences—any discipline that seeks to understand dynamic phenomena.

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Key Concepts of Calculus

In this chapter, we introduce the *derivative*, one of the two key concepts in calculus.

In a later chapter, we introduce the *integral*, the second key concept in calculus.

Each of these concepts depend on the notion of a *limit*, which is explained in Sections 2.1 and 2.2.

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Functions and Graphs: Brief Review

The graph of a function is the graph of the set of all ordered pairs that satisfy the function.

Example: The graph of the function with equation $f(x) = 2x - 1$ is the set of all ordered pairs $(x, f(x))$.

When $x = 2$, $f(2) = 3$ and $(x, f(x)) = (2, f(2)) = (2, 3)$ is a point on the graph of f.

For $x = 0$, $f(0) = -1$ and $(0, f(0)) = (0, -1)$ is on the graph of f.

For $x = -1$, $f(-1) = -3$ and $(-1, f(-1)) = (-1, -3)$ is on the graph of f.

The domain values 2, 0, and -1 are associated with the x axis and the range values $f(2) = 3$, $f(0) = -1$, and $f(-1) = -3$ are associated with the y axis.

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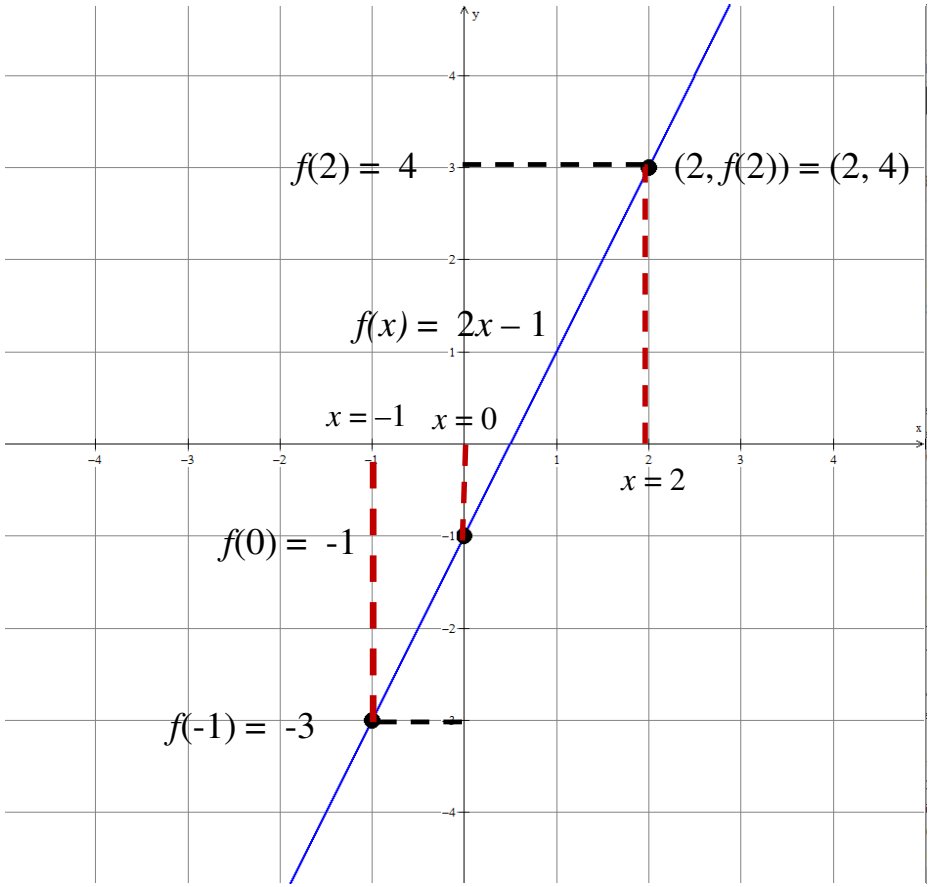
Functions and Graphs: Brief Review (continued)

For $f(x) = 2x - 1$, the graph shows the three points found on the previous slide along with the graph of additional points on the function.

For each point, the domain value x is located on the horizontal axis and the function value, $f(x)$ is located on the vertical axis.



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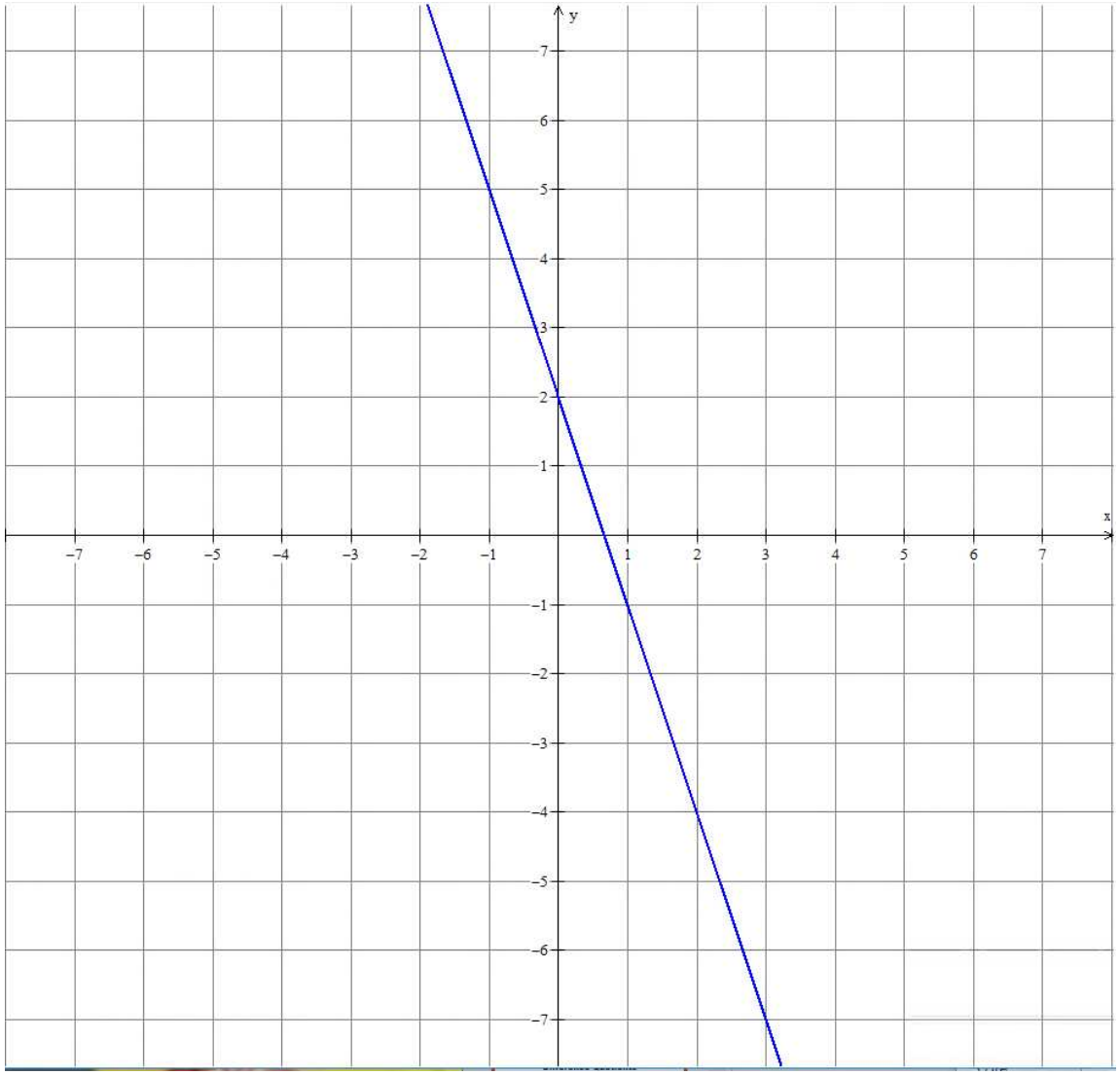
Example: Finding Function Values from Its Graph

Complete the following table using the graph.

x	$g(x)$
–1	
0	
1	
2	



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Analyzing a Limit

The a limit of a function at some domain value $x = a$ involves examining the behavior of the output values of the function as the input values get increasingly closer to the domain value a. The chart gives function values for the function $f(x) = 2x - 1$ for domain values near $x = 2$.

x	1.5	1.9	1.99	1.999	2	2.001	2.01	2.1	2.5
$f(x)$	2	2.8	2.98	2.998	?	3.002	3.02	3.2	4

Notice that the closer x is to 2, the closer the function $f(x)$ is to 3.

We symbolize this limit concept with the notation

$$\lim_{x \to 2} f(x) = 3 \text{ for } f(x) = 2x - 1.$$

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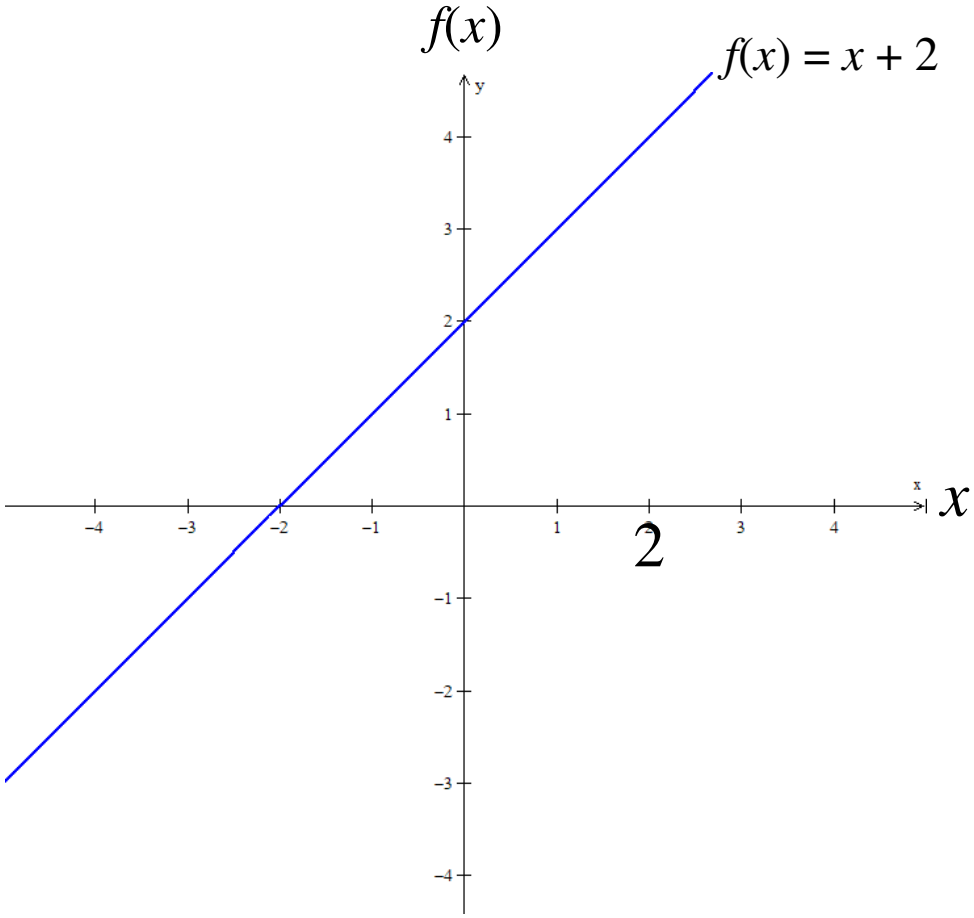
Example: Analyzing a Limit

Let $f(x) = x + 2$. Discuss the behavior of the values of $f(x)$ when x is close to 2.

Solution: We begin by drawing a graph of f that includes the domain value $x = 2$.



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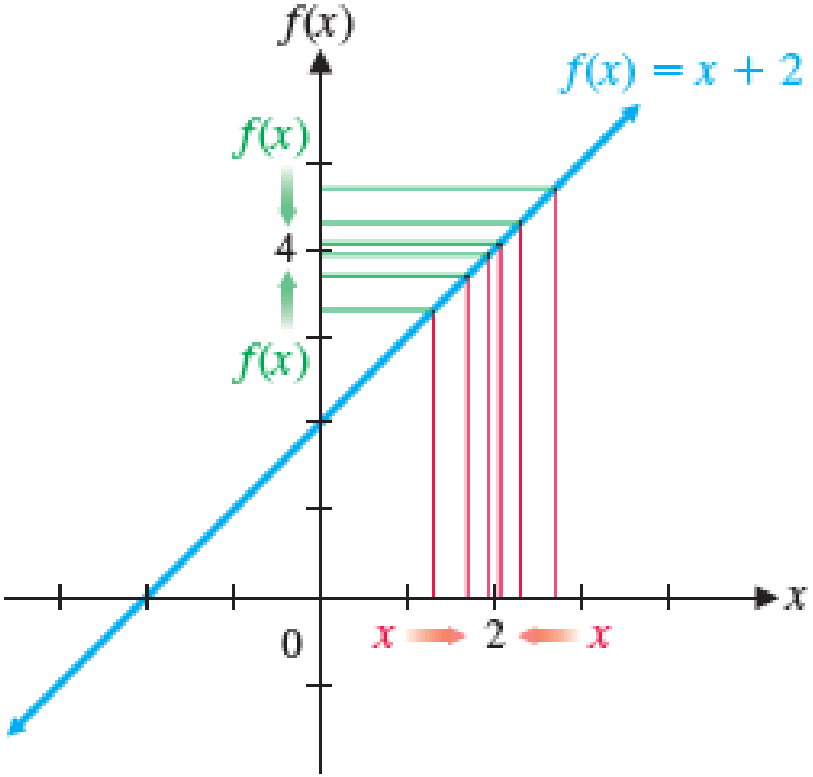
Analyzing a Limit (continued)



The thin vertical lines represent values of x that are close to 2. The corresponding horizontal lines identify the value of $f(x)$ associated with each value of x.

The graph in the figure indicates that as the values of x get closer to 2 on either side of 2, the corresponding values of $f(x)$ get closer and closer to 4.

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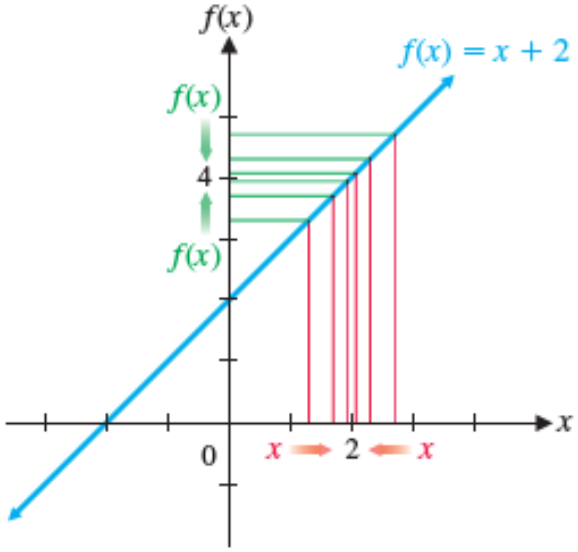
Analyzing a Limit (continued)



Symbolically, we write $\lim_{x \to 2} f(x) = 4.$

This equation is read as "The limit of $f(x)$ as x approaches 2 is 4." In this example, the value of the function at 2 and the limit of the function as x approaches 2 are the same.

In this example, $\lim_{x \to 2} f(x) = f(2) = 4$. Graphically, this means that there is no hole or break in the graph of f at $x = 2$.

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Definition: Limit

We write $\lim_{x \to c} f(x) = L$ or $f(x) \to L$ as $x \to c$ if the functional value $f(x)$ is close to the single real number L whenever x is close, but not equal, to c (on either side of c).

Note: The existence of a limit at c is independent of the value of the function at c. In fact, c may not even be in the domain of f. However, the function must be defined on both sides of c.

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Try this Limit Example With the Absolute Value Function in a Limit

The **absolute value function** is defined as:

$$f(x) = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

Example: Let $h(x) = \dfrac{|x|}{x}$. Explore the behavior of $h(x)$ near, but not equal to, 0. Find $\lim\limits_{x \to 0} h(x)$ if it exists.

Solution: Hint: Evaluate $h(x)$ for positive values of x near 0. Then evaluate $h(x)$ for negative values of x near 0.

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Definition One-Sided Limits

We write $\lim_{x \to c^-} f(x) = K$ and call K the **limit from the left** if $f(x)$ is close to K whenever x is close to, but to the left of, c on the real number line.

We write $\lim_{x \to c^+} f(x) = K$ and call K the **limit from the right** if $f(x)$ is close to K whenever x is close to, but to the right of, c on the real number line.

If no direction is specified in a limit statement, assume that the limit is **two-sided** or **unrestricted**.

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Theorem 1
On the Existence of a Limit

For a (two-sided) limit to exist, the limit from the left and the limit from the right must exist and be equal.

That is $\lim_{x \to c} f(x) = L$ if and only if $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$.

In an earlier example, we found $\lim_{x \to 0^-} \frac{|x|}{x} = -1$ and $\lim_{x \to 0^+} \frac{|x|}{x} = 1$.

Since the left- and right-hand limits are *not* the same,

$\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

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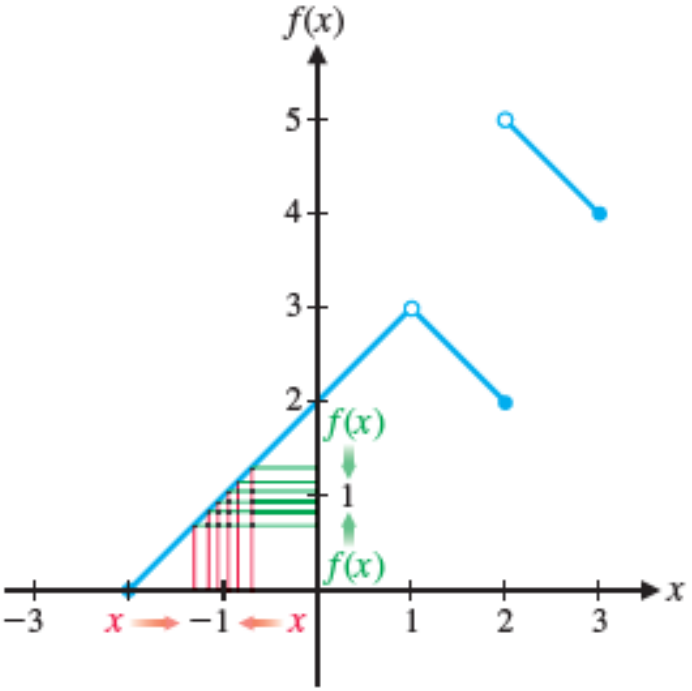
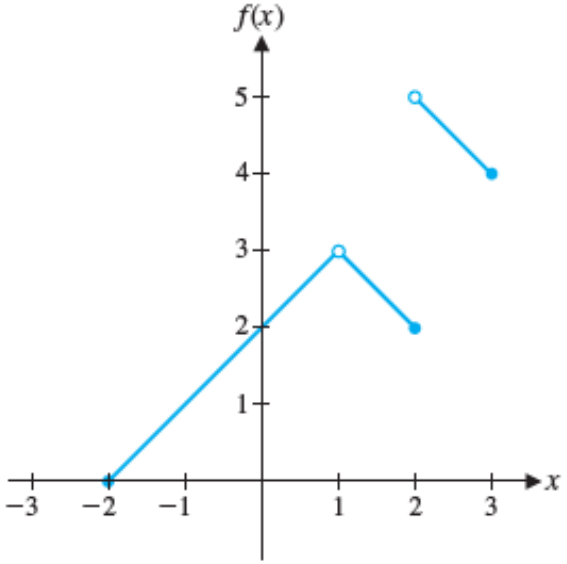
Example: Analyze Limits Graphically

Use the graph to discuss the behavior of $f(x)$ for x near -1.





For x near -1 on either side of -1, the function value $f(x)$ is close to 1. The left- and right-hand limits are equal and agree with the function value.

Therefore, $\lim_{x \to -1} f(x) = f(-1) = 1.$

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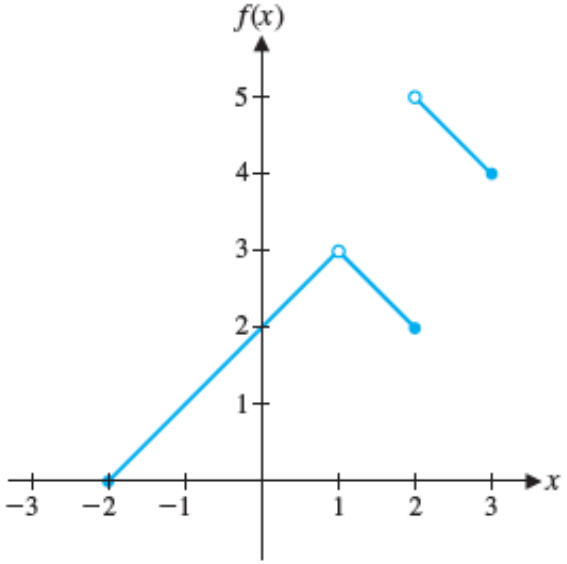
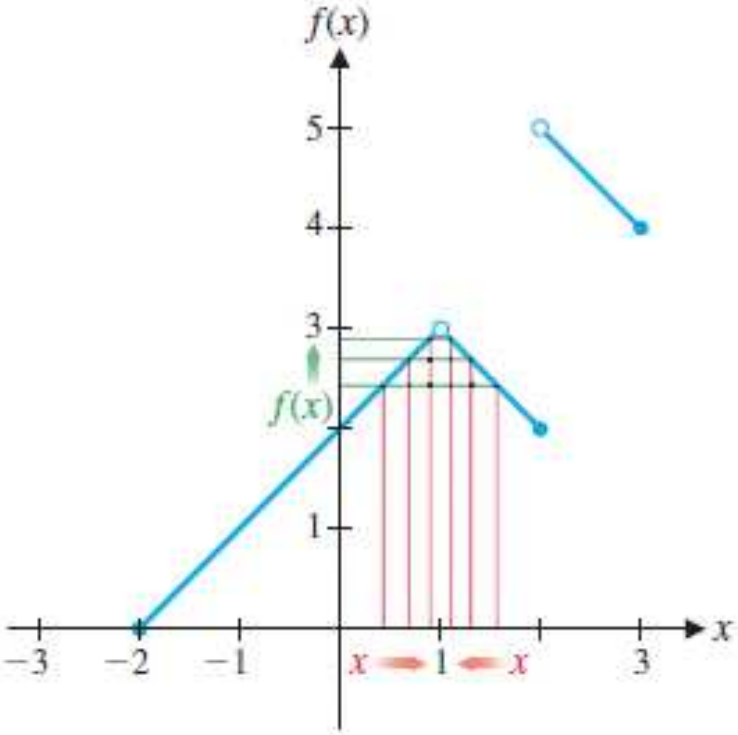
Example: Analyze Limits Graphically

Use the graph to discuss the behavior of $f(x)$ for x near 1.





For x near 1 on either side of 1, the function value $f(x)$ is close to 3. The left- and right-hand limits are equal. Therefore, $\lim_{x \to 1} f(x) = 3$.

$f(1)$ is not defined.

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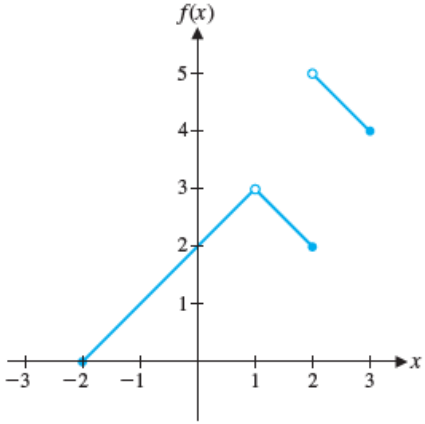
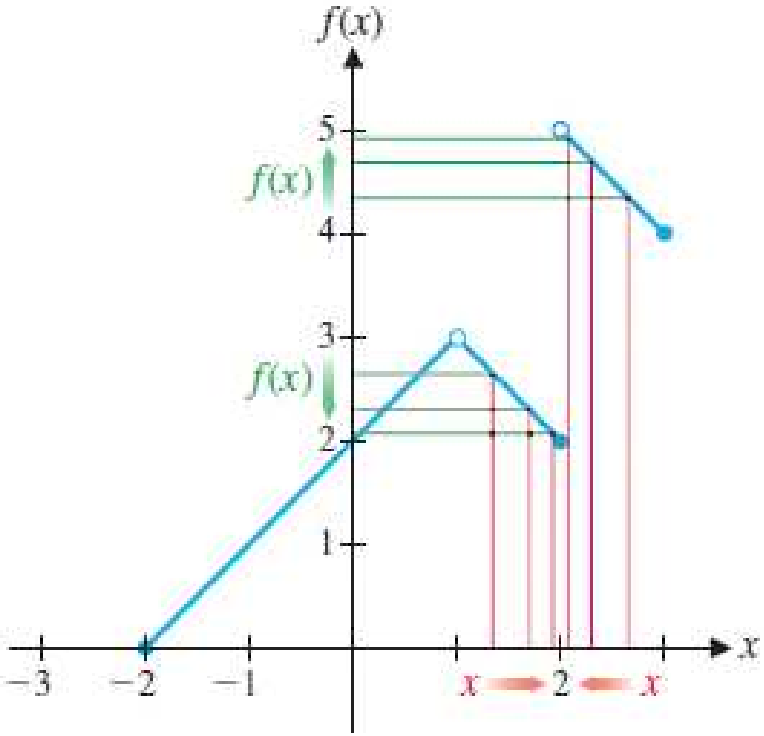
Example: Analyze Limits Graphically

Use the graph to discuss the behavior of $f(x)$ for x near 2.





The abrupt break in the graph at $x = 2$ indicates that the left- and right-hand limits are not equal.

$$\lim_{x \to 2^-} f(x) = 2 \qquad \lim_{x \to 2^+} f(x) = 5$$

$\lim_{x \to 2} f(x)$ does not exist.

$f(2) = 2$

The function exists at $x = 2$ but the limit does not exist at $x = 2$.

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Theorem 2 Properties of Limits

Let f and g be two functions, and assume that

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = M$$

Where L and M are real numbers (both limits exist). Then

1. $\lim_{x \to c} k = k$ for any constant k
2. $\lim_{x \to c} x = c$
3. $\lim_{x \to c}[f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x) = L + M$
4. $\lim_{x \to c}[f(x) - g(x)] = \lim_{x \to c} f(x) - \lim_{x \to c} g(x) = L - M$
5. $\lim_{x \to c} kf(x) = k \lim_{x \to c} f(x) = kL$ for any constant k

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Theorem 2 Properties of Limits (continued)

Let f and g be two functions, and assume that

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = M$$

Where L and M are real numbers (both limits exist). Then

6. $\lim_{x \to c}[f(x) \cdot g(x)] = [\lim_{x \to c} f(x)][\lim_{x \to c} g(x)] = LM$

7. $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \dfrac{L}{M}$ if $M \neq 0$

8. $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)} = \sqrt[n]{L}$ if $L > 0$ or n is odd

Each property in Theorem 2 is valid for left and right limits.

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Examples: Use Theorem 2 to Evaluate Limits

$$\lim_{x\to 2} x^2 - 3x = \lim_{x\to 2} x^2 - \lim_{x\to 2} 3x = 4 - 6 = -2$$

$$\lim_{x\to 4} \frac{2x}{3x+1} = \frac{\lim_{x\to 4} 2x}{\lim_{x\to 4} 3x+1} = \frac{8}{13}$$

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Theorem 3 Limits of Polynomial and Rational Functions

1. $\lim_{x \to c} f(x) = f(c)$ for f any polynomial function.

2. $\lim_{x \to c} r(x) = r(c)$ for r any rational function with a nonzero denominator at $x = c$.

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Examples: Evaluate Limits

Find each limit.

(A) $\lim_{x\to 2}(2x^4 - 3x^2 + 1)$ (B) $\lim_{x\to 1}\sqrt[3]{2x^4 - 3x^2 + 1}$

Solution: (A) $\lim_{x\to 2}(2x^4 - 3x^2 + 1) = 2\cdot 2^4 - 3\cdot 2^2 + 1 = 21$

(B) $\lim_{x\to 1}\sqrt[3]{2x^4 - 3x^2 + 1} = \sqrt[3]{\lim_{x\to 1}(2x^4 - 3x^2 + 1)}$

$= \sqrt[3]{(2\cdot 1^4 - 3\cdot 1^2 + 1)}$

$= \sqrt[3]{(2 - 3 + 1)} = \sqrt[3]{0} = 0$

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Examples: Evaluate Limits

Find each limit.

(A) $\lim_{x\to 2}\dfrac{2x+3}{3x+2}$ (B) $\lim_{x\to 4}\dfrac{|x|}{x}$

Solution: (A) $\lim_{x\to 2}\dfrac{2x+3}{3x+2}=\dfrac{\lim_{x\to 2}(2x+3)}{\lim_{x\to 2}(3x+2)}=\dfrac{(2\cdot 2+3)}{(3\cdot 2+2)}=\dfrac{7}{8}$

(B) $\lim_{x\to 4}\dfrac{|x|}{x}=\dfrac{\lim_{x\to 4}|x|}{\lim_{x\to 4}x}=\dfrac{4}{4}=1$

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Examples: Evaluate Limits

$$\text{Let } f(x) = \begin{cases} x^2 + 1 & \text{if } x < 2 \\ x - 1 & \text{if } x > 2 \end{cases}$$

Find (A) $\lim_{x \to 2^-} f(x)$ (B) $\lim_{x \to 2^+} f(x)$ (C) $\lim_{x \to 2} f(x)$ (D) $f(2)$

Solution: (A) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x^2 + 1) = 2^2 + 1 = 5$

(B) $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x - 1) = 2 - 1 = 1$

(C) Since the one-sided limits are not equal, $\lim_{x \to 2} f(x)$ does not exist.

(D) Because the definition of the function does not assign a value to f for $x = 2$, $f(2)$ does not exist.

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Definition: Indeterminant Form

If $\lim_{x \to c} f(x) = 0$ and $\lim_{x \to c} g(x) = 0$, then $\lim_{x \to c} \frac{f(x)}{g(x)}$

is said to be **indeterminate**, or, more specifically, a **0 / 0 indeterminate form**.

The term *indeterminate* is used because the limit of an indeterminate form may or may not exist.

Note that the expression 0/0 does not represent a real number and is never used as a value of a limit.

As in the following examples, if a limit has an indeterminate form, further investigation is required to determine if the limit exists and to find its value if it does exist.

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Example: Indeterminate Form

Find the value of the limit if it exists.

$$\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x+2)(x-2)}{x-2}=\lim_{x\to 2}(x+2)=4$$

In this example, both numerator and denominator approach zero as x approaches 2. This limit is an indeterminate form.

Algebraic simplification is used to reduce the fraction by dividing the common factor $x - 2$ (which happens to be the factor in the numerator and denominator that approaches zero when x approaches 2.)

Algebraic simplification is often useful when the numerator and denominator are both approaching 0.

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Theorem 4 Limit of a Quotient

If $\lim_{x \to c} f(x) = L,\ L \neq 0,$ and $\lim_{x \to c} g(x) = 0,$ then

$$\lim_{x \to c} \frac{f(x)}{g(x)} \text{ does not exist.}$$

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Example

Determine if the limit is a 0/0 indeterminate form. Find the limit or explain why the limit does not exist.

$$\lim_{x\to 2}\frac{x-2}{x^2+4}$$

Solution: $\lim_{x\to 2}(x-2)=0$ but $\lim_{x\to 2}(x^2+4)=8.$

This limit is not a 0/0 indeterminate form.

$$\lim_{x\to 2}\frac{x-2}{x^2+4}=\frac{\lim_{x\to 2}(x-2)}{\lim_{x\to 2}(x^2+4)}=\frac{0}{2}=0$$

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Example

Determine if the limit is a 0/0 indeterminate form. Find the limit or explain why the limit does not exist.

$$\lim_{x\to 2}\frac{x-2}{x^2-4}$$

Solution: $\lim_{x\to 2}(x-2)=0$ and $\lim_{x\to 2}(x^2-4)=0.$

This limit is a 0/0 indeterminate form.

$$\lim_{x\to 2}\frac{x-2}{x^2-4}=\lim_{x\to 2}\frac{x-2}{(x-2)(x+2)}=\lim_{x\to 2}\frac{1}{x+2}=\frac{1}{4}$$

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Examples:

Determine if the limit is a 0/0 indeterminate form. Find the limit or explain why the limit does not exist.

$$\lim_{x \to 2} \frac{x+2}{x^2-4}$$

Solution: $\lim_{x \to 2}(x+2) = 4$ and $\lim_{x \to 2}(x^2-4) = 0.$

This limit is not a 0/0 indeterminate form.

By Theorem 4, this limit does not exist.

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Limits of Difference Quotients

Suppose $f(x)$ is a function defined in an open interval containing the number a. One of the most important limits in calculus is the limit of the **difference quotient**,

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}.$$

If $\lim_{h\to 0}[f(a+h)-f(a)]=0$ as it often does,

then $\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ is an indeterminate form.

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Example: Limit of a Difference Quotient-You Try It!

$$\text{Find } \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \text{ for } f(x) = 2x - 3.$$

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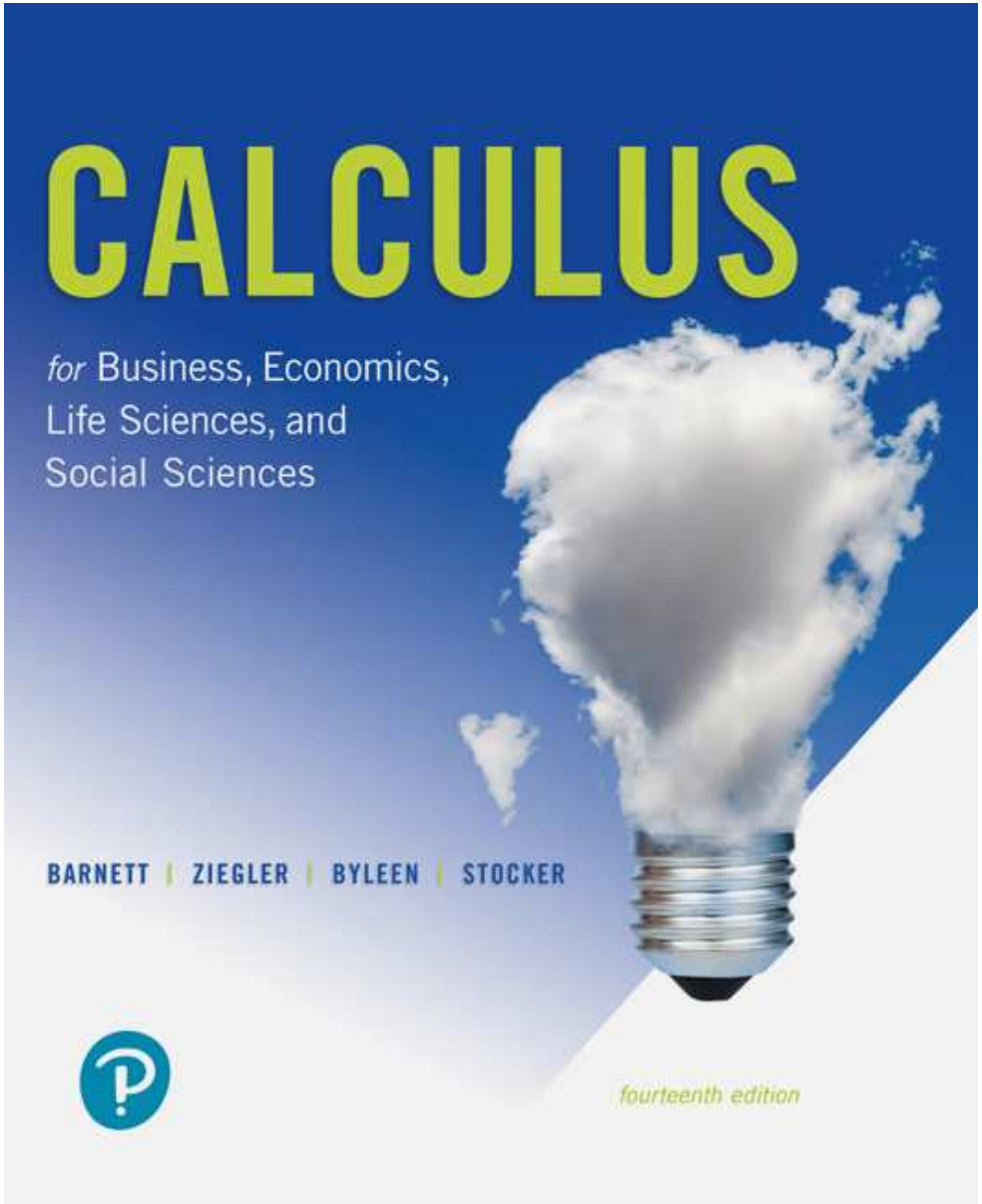

Chapter 2

Limits and the Derivative

Section 2

Infinite Limits

And

Limits at Infinity

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Infinite Limits

Infinite limits and vertical asymptotes help describe the behavior of functions that are unbounded near $x = a$.

We illustrate the concept of infinite limits using the function,

$$f(x) = \frac{1}{x-1}$$

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Example: Infinite Limit

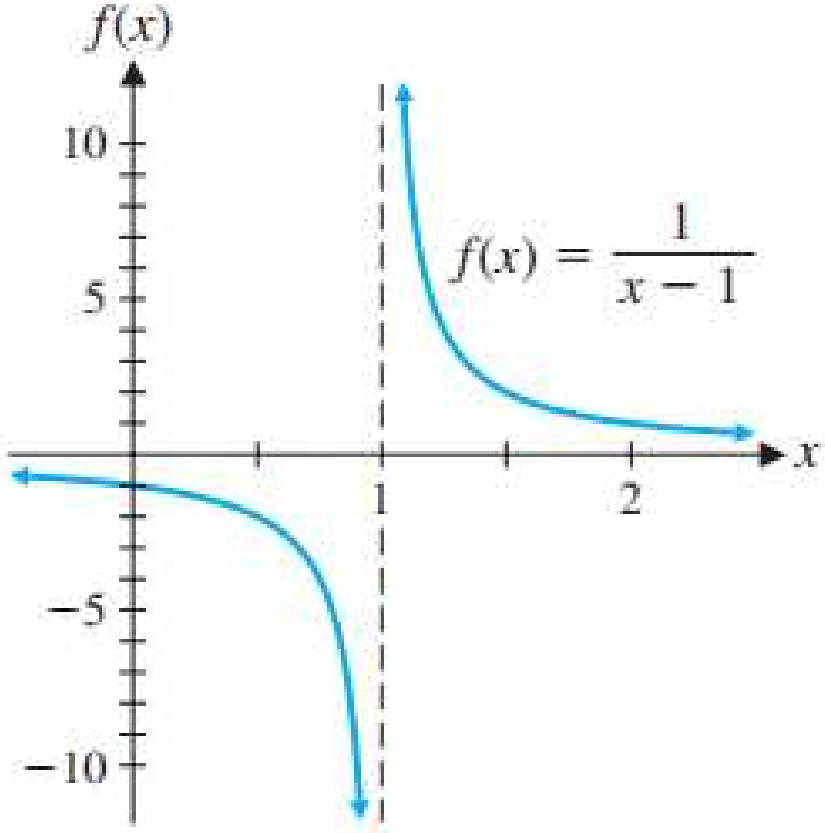
$$f(x) = \frac{1}{x-1}$$

The graph of $f(x)$ is shown.

The graph indicates that there is no real number L that the values of $f(x)$ approach as x approaches 1.

The dashed vertical line at $x = 1$ illustrates the infinite limit as x approaches 1 from the right and left.

Such a line is called a *vertical asymptote*.



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Example: Infinite Limit

$$f(x) = \frac{1}{x-1}$$

In Table 1, as x approaches 1 from the right (values greater than 1), the function values grow positively without bound.

In Table 2, as x approaches 1 from the left (values less than 1), The function values decrease negatively without bound.

Table 1

x	$f(x) = \frac{1}{x-1}$
1.1	10
1.01	100
1.001	1,000
1.0001	10,000
1.00001	100,000
1.000001	1,000,000

Table 2

x	$f(x) = \frac{1}{x-1}$
0.9	−10
0.99	−100
0.999	−1,000
0.9999	−10,000
0.99999	−100,000
0.999999	−1,000,000

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Example: Infinite Limit

$$\text{Let } f(x) = \frac{1}{x-1}$$

The function is discontinuous at $x = 1$. ($f(1)$ does not exist.)

As x approaches 1 from the right, the values of $f(x)$ are positive and become increasingly large.

We say that $f(x)$ increases without bound.

We write this symbolically as

$$\lim_{x \to 1^+} \frac{1}{x-1} = \infty \text{ or } f(x) = \frac{1}{x-1} \to \infty \text{ as } x \to 1^+.$$

Since ∞ is not a real number, *this limit does not exist.*

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Example: Infinite Limit

$$\text{Let } f(x) = \frac{1}{x-1}$$

As x approaches 1 from the left, the values of $f(x)$ are negative and become increasingly large in a negative sense.

We say that $f(x)$ decreases through negative values without bound.

We write this symbolically as

$$\lim_{x \to 1^-} \frac{1}{x-1} = -\infty \text{ or } f(x) = \frac{1}{x-1} \to -\infty \text{ as } x \to 1^-.$$

Since $-\infty$ is not a real number, *this limit does not exist.*

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The Infinity Symbol

The infinity symbol ∞ is used to describe the manner in which these limits fail to exist.

Such limits are called **infinite limits**.

The symbol ∞ is used to describe positive growth without bound and the symbol $-\infty$ is used to describe negative growth without bound.

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Definition Infinite Limits and Vertical Asymptotes

The vertical line $x = a$ is a **vertical asymptote** for the graph of $y = f(x)$ if $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a^+$ or $x \to a^-$.

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Locating Vertical Asymptotes

If f is a polynomial function, then $\lim_{x \to a} f(x) = f(a)$.

Polynomial functions have no vertical asymptotes.

A *vertical asymptote of a rational function can occur only at a zero of its denominator.*

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Theorem 1 Locating Vertical Asymptotes of Rational Functions

If $f(x) = \dfrac{n(x)}{d(x)}$ is a rational function, $d(c) = 0$ and $n(c) \neq 0$,

then the line $x = c$ is a vertical asymptote of the graph of f.

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Example: Locating Vertical Asymptotes

$$\text{Let } f(x) = \frac{x^2 + x - 2}{x^2 - 1}.$$

Describe the behavior of f at each zero of the denominator. Use ∞ or $-\infty$ when appropriate.

Solution: The denominator factors, $x^2 - 1 = (x - 1)(x + 1)$ and has two zeroes, $x = -1$ and $x = 1$.

When $x = -1$, the numerator evaluates to -2 which is non-zero.

According to Theorem 1, the line $x = -1$ is a vertical asymptote.

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Example: Locating Vertical Asymptotes (continued)

$$\text{Let } f(x) = \frac{x^2 + x - 2}{x^2 - 1}.$$

Solution: For value, $x = 1$, (which is a denominator zero), the numerator evaluates to 0

This gives the indeterminate 0/0 form and Theorem 1 does not apply.

We use algebraic simplification to investigate the behavior of the function at $x = 1$.

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Example: Locating Vertical Asymptotes (continued)

$$\text{For } f(x) = \frac{x^2 + x - 2}{x^2 - 1}, \lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1}$$

This is the 0/0 indeterminate form.

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1}$$

$$= \lim_{x \to 1} \frac{(x-1)(x+2)}{(x-1)(x+1)}$$

$$= \lim_{x \to 1} \frac{(x+2)}{(x+1)} = \frac{3}{2}$$

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Limits at Infinity

Limits at infinity and horizontal asymptotes are used to describe the behavior of functions as x assumes arbitrarily large positive values or arbitrarily large negative values.

The symbol ∞ is used to indicate that an independent variable is increasing without bound through positive values.

The symbol $-\infty$ is used to indicate that an independent variable is decreasing without bound through negative values.

Limits at infinity are limits where either $x \to \infty$ or $x \to -\infty$.

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Theorem 2 Limits of Power Functions at Infinity

If p is a positive real number and k is any real number except 0, then

1. $\lim\limits_{x \to -\infty} \dfrac{k}{x^p} = 0$

2. $\lim\limits_{x \to \infty} \dfrac{k}{x^p} = 0$

3. $\lim\limits_{x \to -\infty} k x^p = \infty$ or $-\infty$

4. $\lim\limits_{x \to \infty} k x^p = \infty$ or $-\infty$

provided that x^p is a real number for negative values of x. The limits in 3 and 4 will either be $-\infty$ or ∞, depending on k and p.

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Example: Limit of a Polynomial Function at Infinity

Let $p(x) = 5x^3 - 2x^2 - 3x + 6$.
Find the limit of $p(x)$ as x approaches ∞ and as x approaches $-\infty$.

We factor the term with highest power, $5x^3$ from $p(x)$ and write $p(x)$ as

$$p(x) = 5x^3\left(1 - \frac{2}{5x} - \frac{3}{5x^2} + \frac{6}{5x^3}\right).$$

$$\lim_{x\to\infty}\left(1 - \frac{2}{5x} - \frac{3}{5x^2} + \frac{6}{5x^3}\right) = 1 - 0 - 0 + 0 = 1$$

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Example: Limit of a Polynomial Function at Infinity (continued)

For large values of x,

$$\left(1-\frac{2}{5x}-\frac{3}{5x^2}+\frac{6}{5x^3}\right)\approx 1 \text{ and } p(x)=5x^3\left(1-\frac{2}{5x}-\frac{3}{5x^2}+\frac{6}{5x^3}\right)\approx 5x^3$$

Since $5x^3 \to \infty$ as $x \to \infty$, it follows that

$$\lim_{x\to\infty} p(x)=\lim_{x\to\infty} 5x^3=\infty.$$

Similarly $5x^3 \to -\infty$ as $x \to -\infty$, implies that

$$\lim_{x\to-\infty} p(x)=\lim_{x\to-\infty} 5x^3=-\infty.$$

The behavior of $p(x)$ for large values is the same as the behavior of the highest degree term, $5x^3$.

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Theorem 3 Limits of Polynomial Functions at Infinity

If $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,\ a_n \neq 0,\ n \geq 1$, for integer values n then

$$\lim_{x \to \infty} p(x) = \lim_{x \to \infty} a_n x^n = \infty \text{ or } -\infty$$

and

$$\lim_{x \to -\infty} p(x) = \lim_{x \to -\infty} a_n x^n = \infty \text{ or } -\infty$$

Each limit will be either $-\infty$ or ∞, depending on a_n and n.

A polynomial of degree 0 is a constant function $p(x) = a_0$, and its limit as x approaches ∞ or $-\infty$ is the number a_0.

Polynomial functions of degree 1 or greater never have horizontal asymptotes.

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End Behavior of a Function

A pair of limit expressions of the form

$$\lim_{x \to \infty} f(x) = A, \quad \lim_{x \to -\infty} f(x) = B$$

where A and B are $\infty, -\infty,$ or real numbers, describes the **end behavior** of the function f.

The first of the two limits describes the **right end behavior** and the second describes the **left end behavior**.

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Example: Find the End Behavior of a Function

Give a pair of limit expressions that describe the end behavior of the function $p(x) = 7x^3 - 2x^2 + 3x - 4$.

Solution: Using Theorem 3,

$$\lim_{x\to\infty}(7x^3 - 2x^2 + 3x - 4) = \lim_{x\to\infty}(7x^3) = \infty \quad \textit{Right end behavior}$$

$$\lim_{x\to-\infty}(7x^3 - 2x^2 + 3x - 4) = \lim_{x\to-\infty}(7x^3) = -\infty \quad \textit{Left end behavior}$$

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Theorem 4 Limits of Rational Functions at Infinity/Horizontal Asymptotes of Rational Functions

A) If $f(x) = \dfrac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0}$, $a_m \neq 0, b_n \neq 0$

then $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{a_m x^m}{b_n x^n}$ and $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{a_m x^m}{b_n x^n}$

B) There are three possible cases for these limits:

1. If $m < n$, then $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to-\infty} f(x) = 0$, and the line $y = 0$ (the x axis) is a horizontal asymptote of $f(x)$.

2. If $m = n$, then $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to-\infty} f(x) = \dfrac{a_m}{b_n}$, and the line $y = \dfrac{a_m}{b_n}$ is a horizontal asymptote of $f(x)$.

3. If $m > n$, then each limit will be ∞ or $-\infty$, depending on m, n, a_m, and b_n and $f(x)$ does not have a horizontal asymptote.

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Example: Finding Horizontal Asymptotes

Find all horizontal asymptotes, if any, of the function.

$$f(x) = \frac{3x^4 - 5x^2 + 2}{4x^3 + 2x + 7}$$

Solution: Using Theorem 4,

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{3x^4 - 5x^2 + 2}{4x^3 + 2x + 7} = \lim_{x\to\infty} \frac{3x^4}{4x^3} = \lim_{x\to\infty} \frac{3x}{4} = \infty$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{3x^4 - 5x^2 + 2}{4x^3 + 2x + 7} = \lim_{x\to-\infty} \frac{3x^4}{4x^3} = \lim_{x\to-\infty} \frac{3x}{4} = -\infty$$

The function $f(x)$ has no horizontal asymptotes.

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Example: Finding Horizontal Asymptotes

Find all horizontal asymptotes, if any, of the function.

$$f(x) = \frac{5x^3 - 2x^2 + 2}{4x^4 + 7x + 3}$$

Solution: Using Theorem 4,

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{5x^3 - 2x^2 + 2}{4x^4 + 7x + 3} = \lim_{x\to\infty} \frac{5x^3}{4x^4} = \lim_{x\to\infty} \frac{5}{4x} = 0$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{5x^3 - 2x^2 + 2}{4x^4 + 7x + 3} = \lim_{x\to-\infty} \frac{5x^3}{4x^4} = \lim_{x\to-\infty} \frac{5}{4x} = 0$$

The x axis ($y = 0$) is a horizontal asymptote of $f(x)$.

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Example: Finding Horizontal Asymptotes

Find all horizontal asymptotes, if any, of the function.

$$f(x) = \frac{7x^3 - 10x^2 + 4}{3x^3 + 9x - 11}$$

Solution: Using Theorem 4,

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{7x^3 - 10x^2 + 4}{3x^3 + 9x - 11} = \lim_{x\to\infty} \frac{7x^3}{3x^3} = \frac{7}{3}$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{7x^3 - 10x^2 + 4}{3x^3 + 9x - 11} = \lim_{x\to-\infty} \frac{7x^3}{3x^3} = \frac{7}{3}$$

The line $y = \frac{7}{3}$ is a horizontal asymptote of $f(x)$.

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Example: Find the End Behavior of a Function

A newly released smart-phone operating system gives users an update notice when they download a new app.

The percent P of users that have installed the new update after t days is given by $P(t) = \dfrac{100t^2}{t^2 + 100}$.

A) What percentage of users have installed the new update after 15 days? After 25 days? (Round answers to the nearest tenth of a percent.)

B) What happens to $P(t)$ as $t \to \infty$?

Solution: A) $P(15) = \dfrac{100t^2}{t^2 + 100} = \dfrac{100 \cdot 15^2}{15^2 + 100} = 69.2\%.$

$$P(25) = \frac{100t^2}{t^2 + 100} = \frac{100 \cdot 25^2}{25^2 + 100} = 86.2\%.$$

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Example: Find the End Behavior of a Function

A newly released smart-phone operating system gives users an update notice when they download a new app.

The percent P of users that have installed the new update after t days is given by $P(t) = \dfrac{100t^2}{t^2 + 100}$.

B) What happens to $P(t)$ as $t \to \infty$?

By Theorem 4, since the degrees of the numerator and denominator are equal, the horizontal asymptote is the line $y = \dfrac{100}{1} = 100$. It follows that,

$$\lim_{t\to\infty} P(t) = \lim_{t\to\infty} \frac{100t^2}{t^2 + 100} = 100\%.$$

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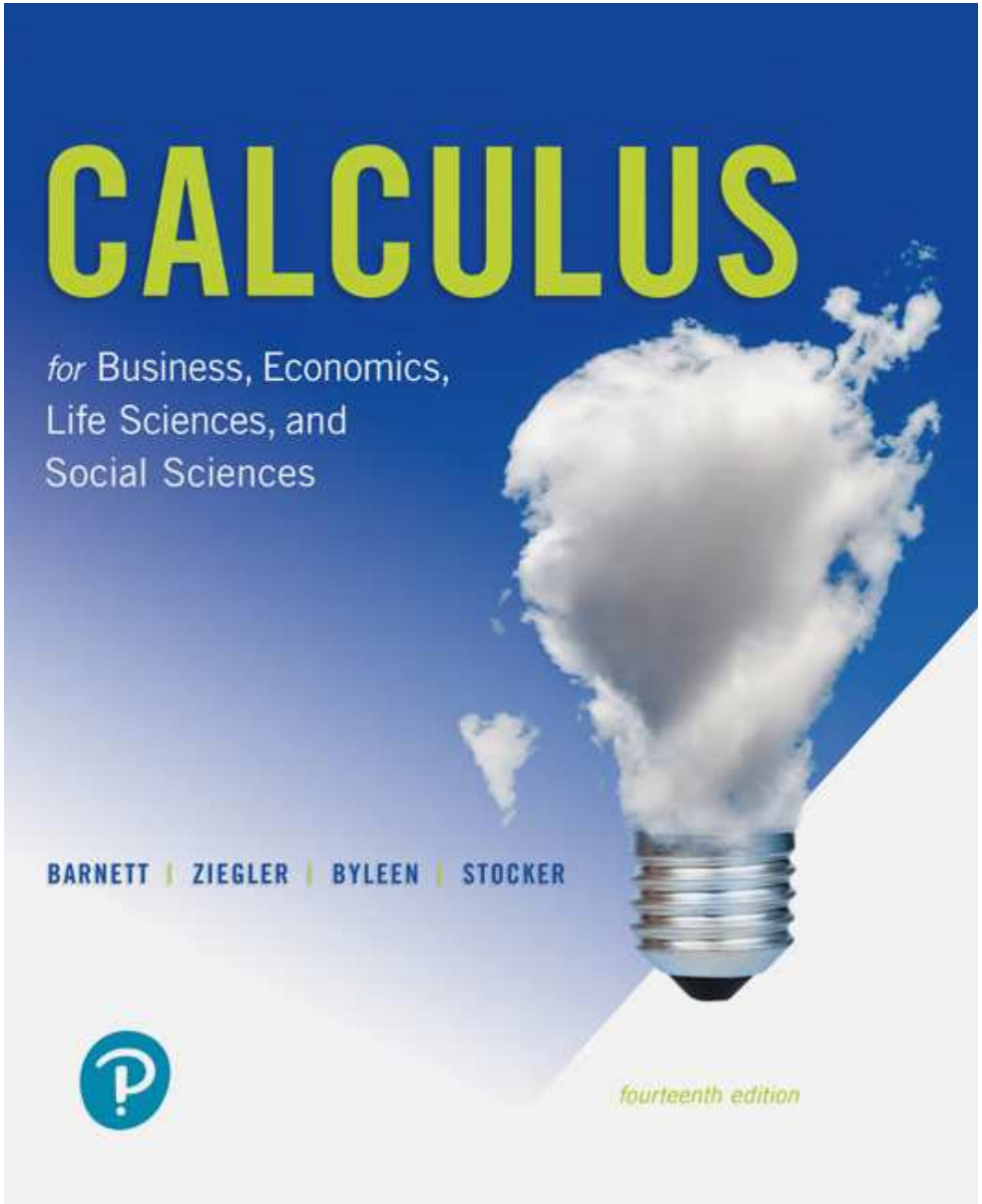

Chapter 2

Limits and the Derivative

Section 3

Continuity

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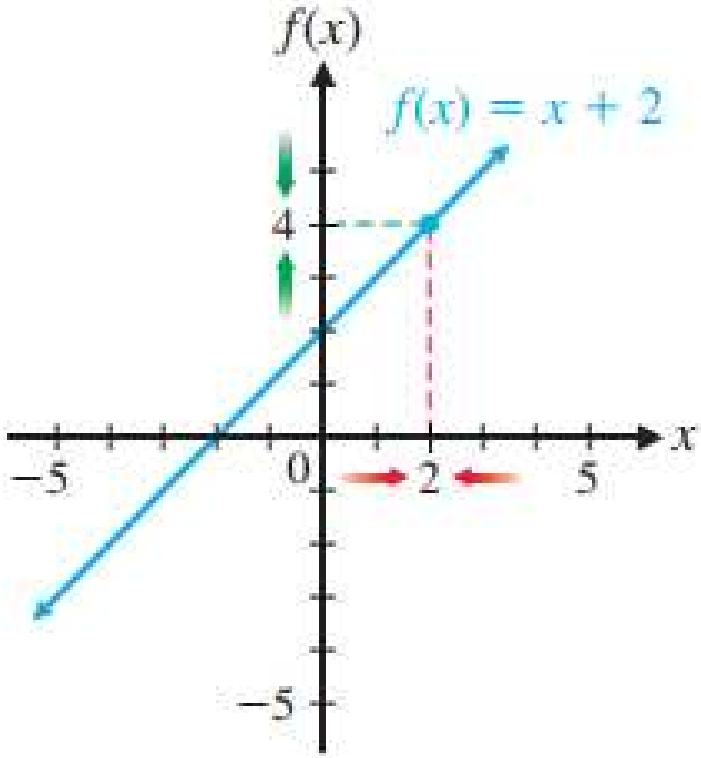
Continuity

Informally, a function is *continuous over an interval* if its graph can be drawn without removing a pen from the paper.

The function f in Figure A is continuous for all x.



(A) $\lim_{x \to 2} f(x) = 4$

$f(2) = 4$

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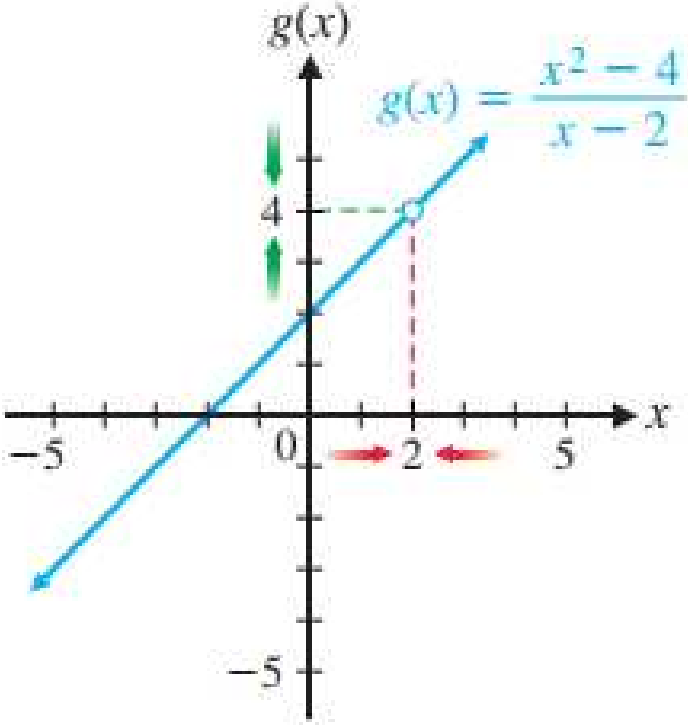
Continuity

When a graph is broken, or disconnected, at a point $x = c$, the function is said to be *discontinuous* at $x = c$.

The function g in Figure B is continuous for any interval that does not contain $x = 2$.



(B) $\lim_{x \to 2} g(x) = 4$

g(2) is not defined

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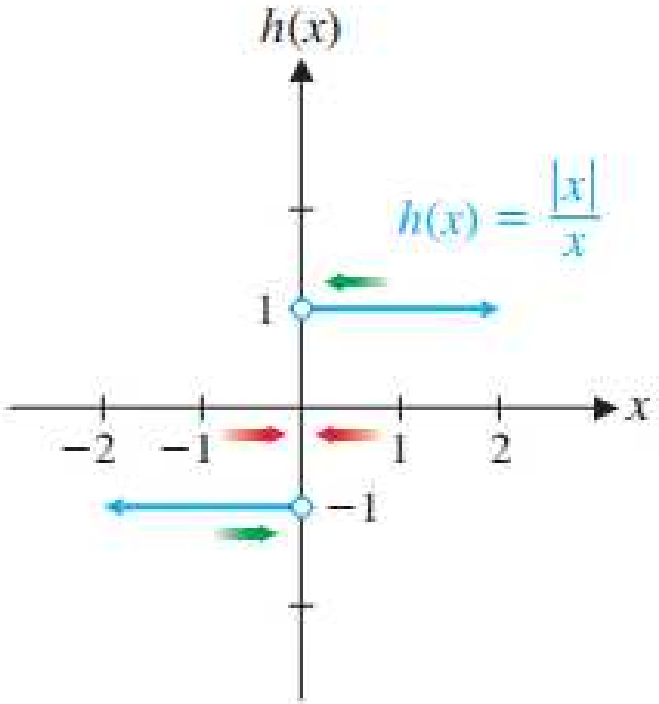
Continuity

When a graph is broken, or disconnected, at a point $x = c$, the function is said to be *discontinuous* at $x = c$.

The function h in Figure C is discontinuous at $x = 0$ but is continuous over any interval that does not contain $x = 0$.



(C) $\lim_{x \to 0} h(x)$ does not exist
$h(0)$ is not defined

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Definition Continuity

A function f is **continuous at the point $x = c$ if**

1. $\lim_{x \to c} f(x)$ exists
2. $f(c)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

A function is **continuous on the open interval (a, b)** if it continuous at each point on the interval.

If one or more of the three conditions in the definition fails, then the function is **discontinuous** at $x = c$.

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Continuity of a Function Defined by a Graph

Use the definition of continuity to discuss the continuity of the function $f(x)$ shown in the graph.

Discontinuity at $x = -4$:

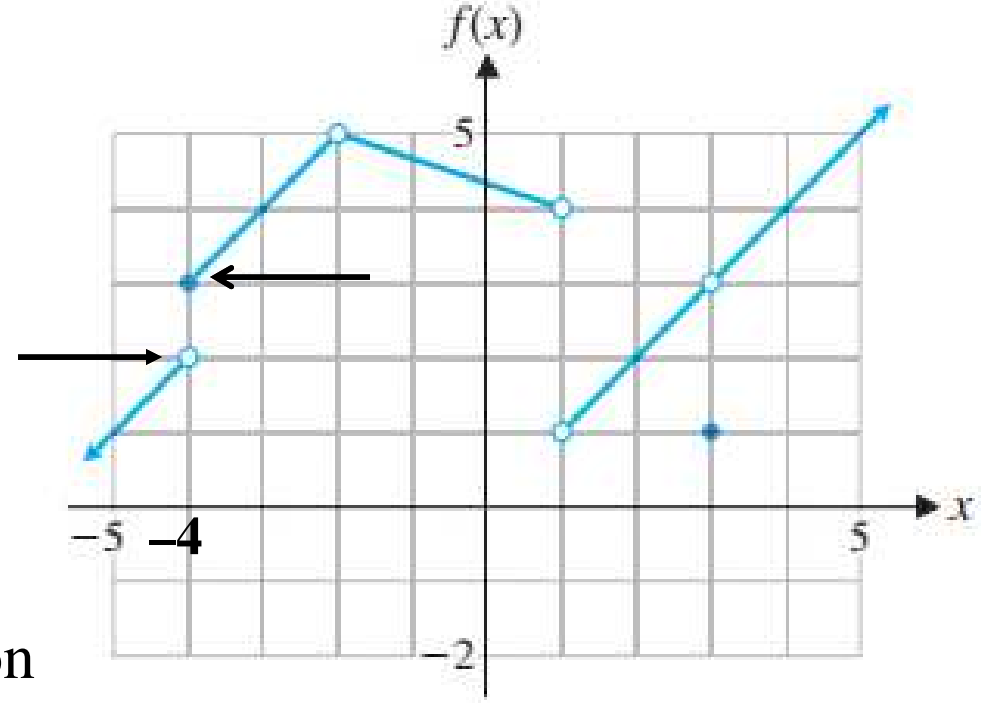
$\lim_{x \to -4^-} f(x) = 2$

$\lim_{x \to -4^+} f(x) = 3$

$\lim_{x \to -4} f(x)$ does not exist.

$f(-4) = 3$

Condition 1 in the definition of continuity fails for $x = -4$.



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Continuity of a Function Defined by a Graph

Use the definition of continuity to discuss the continuity of the function $f(x)$ shown in the graph.

Discontinuity at $x = -2$:

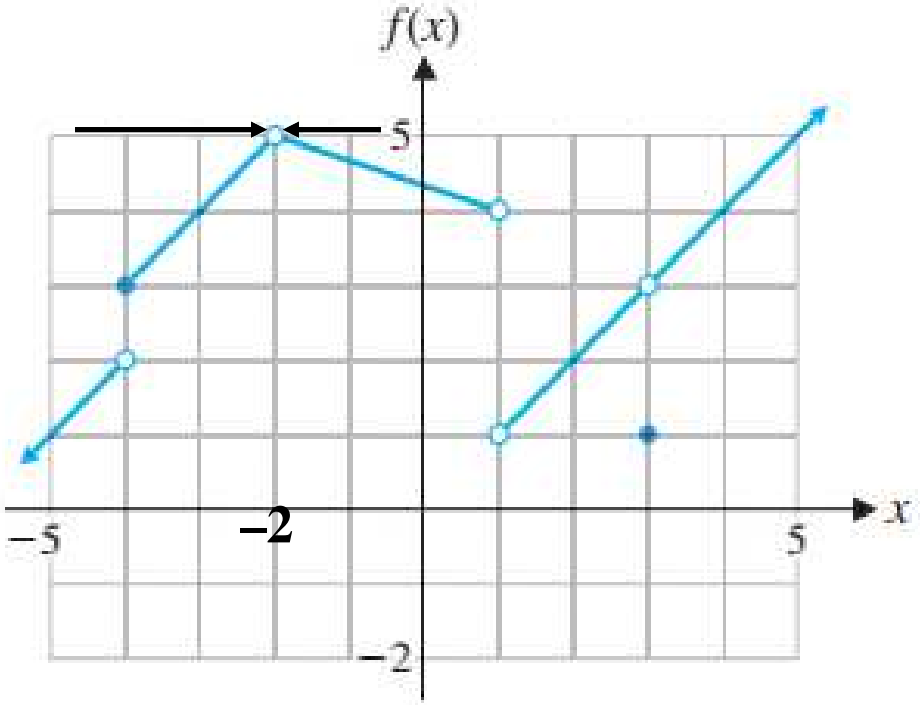
$$\lim_{x \to -2^-} f(x) = 5$$

$$\lim_{x \to -2^+} f(x) = 5$$

$$\lim_{x \to 2} f(x) = 5$$

$f(-2)$ = does not exist.

Condition 2 in the definition of continuity fails for $x = -2$.



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Continuity of a Function Defined by a Graph

Use the definition of continuity to discuss the continuity of the function $f(x)$ shown in the graph.

Discontinuity at $x = 1$:

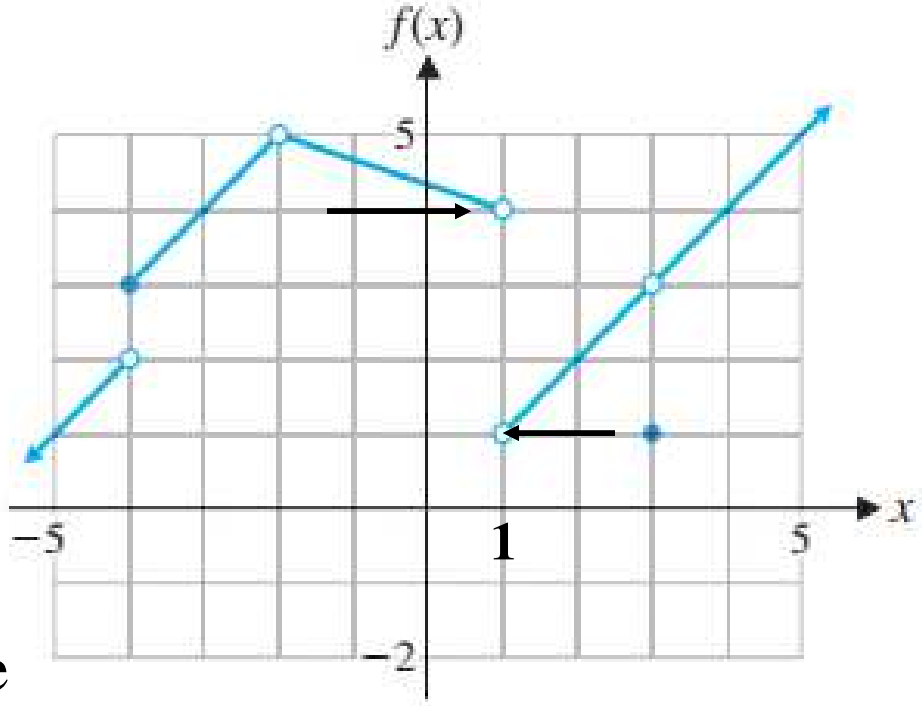
$\lim_{x \to 1^-} f(x) = 5$

$\lim_{x \to 1^+} f(x) = 1$

$\lim_{x \to 1} f(x)$ does not exist.

$f(1)$ = does not exist.

Conditions 1 and 2 in the definition of continuity fail for $x = 1$.



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Continuity of a Function Defined by a Graph

Use the definition of continuity to discuss the continuity of the function $f(x)$ shown in the graph.

Discontinuity at $x = 3$:

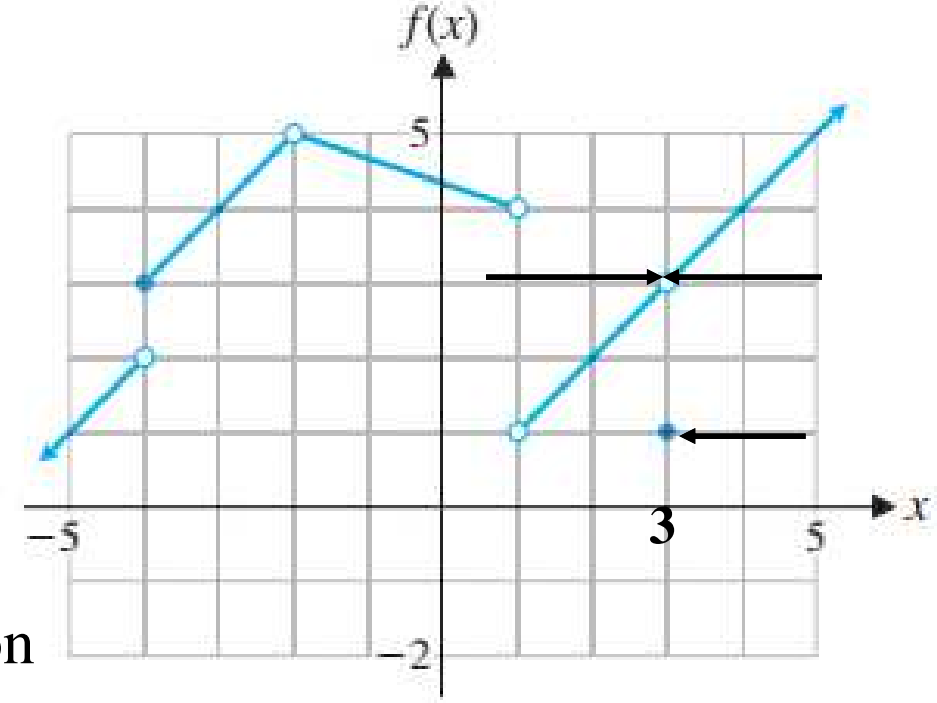
$$\lim_{x \to 3^-} f(x) = 3$$

$$\lim_{x \to 3^+} f(x) = 3$$

$$\lim_{x \to 3} f(x) = 3.$$

$$f(3) = 1$$

Condition 3 in the definition of continuity fails for $x = 3$.



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Continuity of Functions Defined by Equations

Use the definition of continuity to discuss the continuity of the function at the indicated point.

$$f(x) = x + 3 \text{ at } x = 4$$

Solution: f is continuous at $x = 4$, since $\lim_{x \to 4} f(x) = 7 = f(4)$.

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Continuity of Functions Defined by Equations

Use the definition of continuity to discuss the continuity of the function at the indicated point.

$$g(x) = \frac{x^2 - 9}{x + 3} \text{ at } x = -3$$

Solution: g is not continuous at $x = -3$, since $g(-3) = 0/0$ is not defined.

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Continuity of Functions Defined by Equations

Use the definition of continuity to discuss the continuity of the function at the indicated point.

$$h(x) = \frac{|x-1|}{x-1} \text{ at } x = 1$$

Solution: h is not continuous at $x = 1$.

$h(1) = 0/0$ is not defined.

$\lim_{x \to 1} h(x)$ does not exist.

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One-sided Continuity

A function is said to be **continuous on the right** at $x = c$ if $\lim_{x \to c^+} f(x) = f(c)$.

A function is said to be **continuous on the left** at $x = c$ if $\lim_{x \to c^-} f(x) = f(c)$.

A function is said to be **continuous on the closed interval $[a, b]$** at $x = c$ if it is continuous on the open interval (a, b) and is continuous both on the right at a and on the left at b.

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General Continuity Properties

If two functions are continuous on the same interval, then their sum, difference, product, and quotient are continuous on the same interval except for values of x that make a denominator 0.

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Theorem 1: Continuity Properties of Some Specific Functions

(A) A constant function $f(x) = k$, where k is a constant, is continuous for all x.

(B) For n a positive integer, $f(x) = x^n$ is continuous for all x.

(C) A polynomial function is continuous for all x.

(D) A rational function is continuous for all x except those values that make a denominator 0.

(E) For n an odd positive integer greater than 1, $\sqrt[n]{f(x)}$ is continuous wherever $f(x)$ is continuous.

(F) For n an even positive integer greater than 1, $\sqrt[n]{f(x)}$ is continuous wherever $f(x)$ is continuous and nonnegative.

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Example: Using Continuity Properties

Determine where the function is continuous.

$$f(x) = 3x^2 - 5x + 8$$

Solution: Since f is a polynomial function, f is continuous for all x.

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Example: Using Continuity Properties

Determine where the function is continuous.

$$f(x) = \frac{x}{(x-3)(x+1)}$$

Solution: Since f is a rational function, f is continuous for all x except 3 and –1 (values that make the denominator 0).

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Example: Using Continuity Properties

Determine where the function is continuous.

$$f(x) = \sqrt[3]{x^2 - 9}$$

Solution: The polynomial function $x^2 - 9$ is continuous for all x. Since $n = 3$ is odd, f is continuous for all x.

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Example: Using Continuity Properties

Determine where the function is continuous.

$$f(x) = \sqrt{x+1}$$

Solution: The polynomial function $x + 1$ is continuous for all x. Since the index of this radical, $n = 2$ is even, f is continuous for values of $x \geq -1$, which in interval form is $[-1, \infty)$.

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Solving Inequalities Using Continuity Properties

A *sign chart* is a basic tool for analyzing graphs in calculus. These will be used extensively in later sections. We use continuity properties to develop a simple efficient procedure for constructing sign charts.

Suppose that a function f is continuous over the interval (1, 8) and $f(x) \neq 0$ for any x in (1, 8).

Further suppose that $f(2) = 5$.

Can $f(x)$ be negative for any x in the interval (1, 8)?

The answer is "no." For example, if $f(7)$ were –5, it would not be possible to join the points (2, 5) and (7, –5) without crossing the x axis at least once.

Crossing the x axis would violate the given condition that f is not zero for any value of x in the interval.

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Theorem 2 Sign Properties on an Interval (a, b)

If f is continuous on (a, b) and $f(x) \neq 0$ for all x in (a, b), then either $f(x) > 0$ for all x in (a, b) or $f(x) < 0$ for all x in (a, b).

Theorem 2 provides the basis for an effective method of solving many types of inequalities.

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Example: Solving an Inequality

Solve the inequality $\dfrac{x+1}{x-2} > 0$

Solution: Let $f(x) = \dfrac{x+1}{x-2}$.

The denominator is 0 when $x = 2$, and the numerator is 0 when $x = -1$.

f is not continuous at $x = 2$, and $f(x) = 0$ for $x = -1$.

We plot $x = 2$ and $x = -1$ (called *partition numbers*) on a real number line as shown in the figure.

The dot at 2 is open because the function is not defined at $x = 2$.

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Example: Solving an Inequality (continued)

The partition numbers 2 and –1 determine three open intervals: $(-\infty, -1)$, $(-1, 2)$, and $(2, \infty)$.

The function f is continuous and nonzero on each of these intervals and from Theorem 2 does not change sign on any of these intervals.

The sign of $f(x)$ on each interval is found using a **test number** and evaluating the function for that test number to determine the sign of $f(x)$ on the interval.

Any value in each subinterval is a suitable choice for a test number. We choose numbers that are easy to evaluate: –2, 0, and 3.

$f(-2) = \frac{1}{4}$ (positive), $f(0) = -\frac{1}{2}$ (negative), and $f(3) = 4$ (positive).

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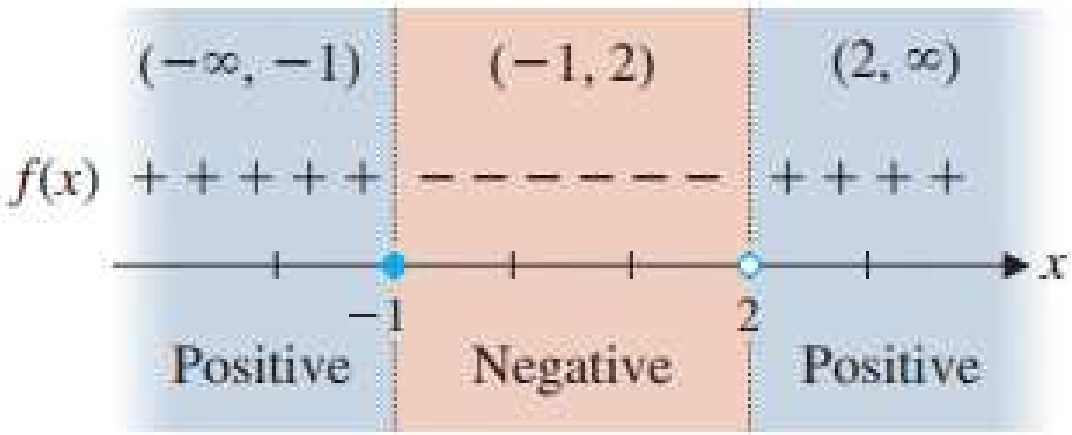
Example: Solving an Inequality (continued)

The function f is positive on $(-\infty, -1)$, negative on $(-1, 2)$, and positive on $(2, \infty)$.

Using this information, we construct a **sign chart** to summarize these results.



We use the sign chart to write the solution to the inequality: $f(x) > 0$ for $x < -1$ or $x > 2$. Note that $f(x) < 0$ for $-1 < x < 2$.

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DEFINITION A real number x is a **partition number** for a function f if f is discontinuous at x or $f(x) = 0$.

Partition numbers determine open intervals on which f does not change sign.

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Procedure: Constructing Sign Charts

Given a function f,

- Step 1 Find all partition numbers of f:
  - A) Find all numbers x such that f is discontinuous at x.
  - B) Find all numbers x such that $f(x) = 0$.
- Step 2 Plot the numbers found in step 1 on a real number line, dividing the number line into intervals.
- Step 3 Select a test number in each open interval determined in step 2 and evaluate $f(x)$ at each test number to determine whether $f(x)$ is positive or negative in each interval.
- Step 4 Construct a sign chart, using the real number line in step 2 to show the sign of $f(x)$ on each open interval.

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Example: Positive Profit

A bakery estimates its annual profits from the production and sale of x loaves of bread per year to be $P(x)$ dollars where the profit $P(x) = 6x - 0.001x^2 - 5000$.

For what values of x does the bakery make a profit selling bread?

Solution: Construct a sign chart for $P(x)$.

The polynomial $P(x)$ is continuous everywhere.

There are no values of x where $P(x)$ is undefined.

Factor $P(x)$ to find where $P(x) = 0$.

$P(x) = 6x - 0.001x^2 - 5000 = -0.001(x - 1000)(x - 5000)$

$P(x) = 0$ when $x = 1000$ and $x = 5000$.

$x = 1000$ and $x = 5000$ are the two partition numbers.

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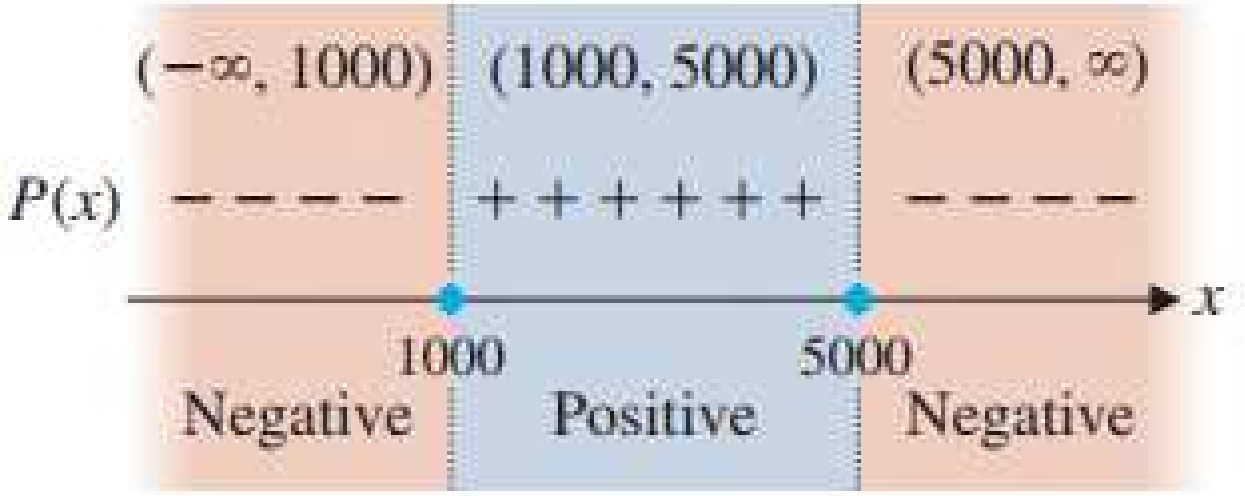
Example: Positive Profit (continued)

Plot the partition numbers on the real number line and test points ($x = 0$, $x = 2000$, and $x = 10000$) in each interval to determine the sign of $P(x)$ in each interval.

Record that information on the sign chart.



The profit $P(x)$ is positive if $1000 < x < 5000$.
The bakery should make between 1000 and 5000 loaves of bread to have a positive profit.

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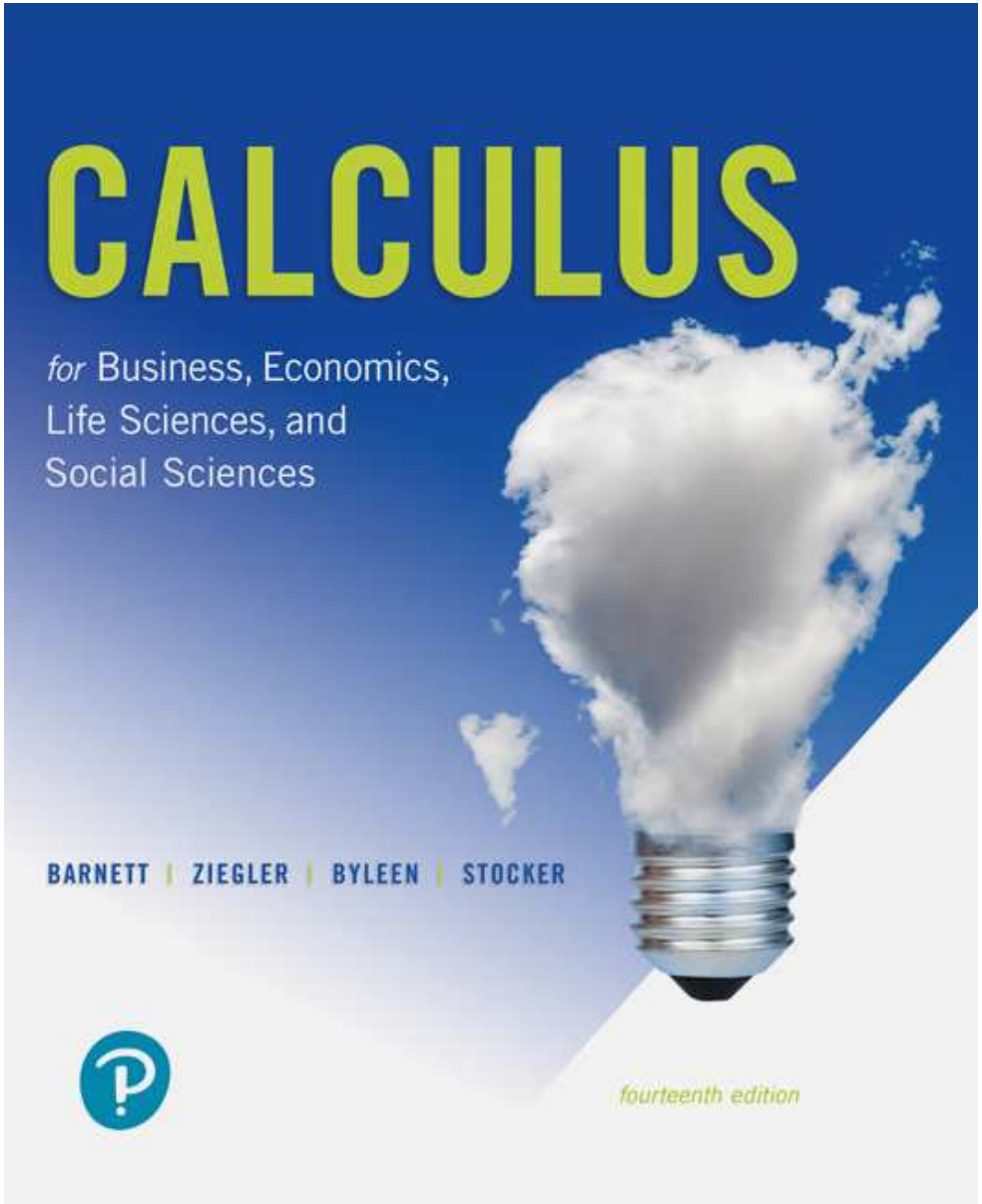

Chapter 2

Limits and the Derivative

Section 4

The Derivative

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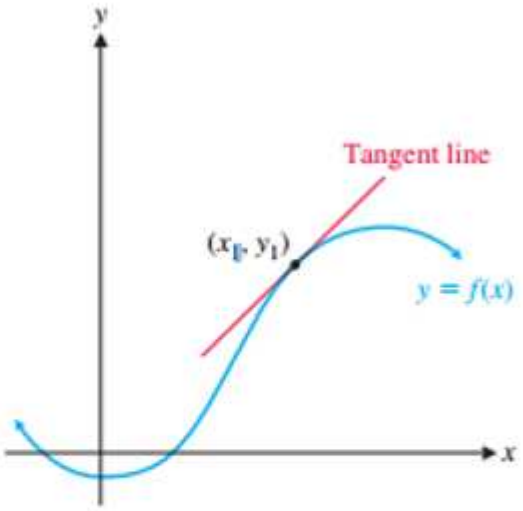
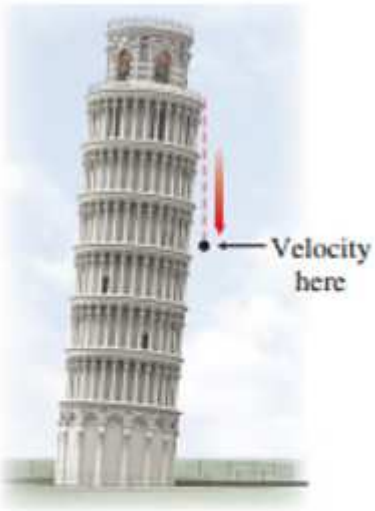
Introduction to Derivatives

- Concepts of limits help solve two basic calculus problems
  - (A) finding the equation of the tangent line to a function at a specified point
  - (B) finding the instantaneous velocity of an object in motion



(A) Find the equation of the tangent line at (x_1, y_1) given $y = f(x)$



(B) Find the instantaneous velocity of a falling object

Figure 1 Two basic problems of calculus

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Rate of Change

On a trip you pass mile marker 120 on the interstate highway at 9 AM and pass mile marker 250 at 11 AM.

The ***average rate of change*** of distance with respect to time, also known as ***average velocity***, is the distance traveled divided by the elapsed time.

In this example, the ***average rate of change*** is

$$\frac{(250-120)\text{ miles}}{(11-9)\text{ hours}} = \frac{130\text{ miles}}{2\text{ hours}} = 65\text{ miles per hour}$$

During this trip, it is likely that your speed varied from 65 mph, sometimes faster, sometimes slower.
Your speedometer gives the *instantaneous rate of change*, or *instantaneous velocity* for any moment in time during the trip.

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Definition Average Rate of Change

For $y = f(x)$, the **average rate of change from $x = a$ to $x = a + h$**

is $$\frac{f(a+h)-f(a)}{(a+h)-a} = \frac{f(a+h)-f(a)}{h} \quad h \neq 0$$

Recall from a previous section, this is known at a **difference quotient**.

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Example Revenue Analysis

A company manufactures plastic planter boxes. The revenue (in dollars) generated from selling x of these boxes is given by $R(x) = 20x - 0.02x^2$ whenever $0 \leq x \leq 1{,}000$.

Find the change in revenue if production changes from 100 planters to 400 planters?

Solution: The change in revenue is found by subtracting the revenue generated by selling 100 planters from the revenue generated from selling 400 planters

$$\begin{aligned} R(400) - R(100) &= 20(400) - 0.02(400)^2 - [20(100) - 0.02(100)^2] \\ &= 4{,}800 - 1{,}800 \\ &= \$3{,}000 \end{aligned}$$

Revenue increases by \$3,000 when production increases from 100 planters to 400 planters.

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Example Revenue Analysis (continued)

Solution: The average rate of change in revenue is found by dividing the change in revenue by the change in production.

We found the change in revenue to be \$3,000 as production changed from 100 to 400 planters.

The change in production is 300.

The average rate of change (AROC) is

$$\frac{R(400) - R(100)}{400 - 100} = \frac{3,000}{300} = \$10$$

When production is increased from 100 planters to 400 planters, the revenue increases an average of \$10 per planter.

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Example Revenue Analysis (continued) - Visual

A company manufactures plastic planter boxes. The revenue (in dollars) generated from selling x of these boxes is given by $R(x) = 20x - 0.02x^2$ whenever $0 \leq x \leq 1{,}000$.

The graph of the revenue function is shown with points corresponding to $x = 100$ and $x = 400$ marked.

The line connecting these points is called a **secant line**.

The average rate of change in the revenue between these points is the slope of this secant line.

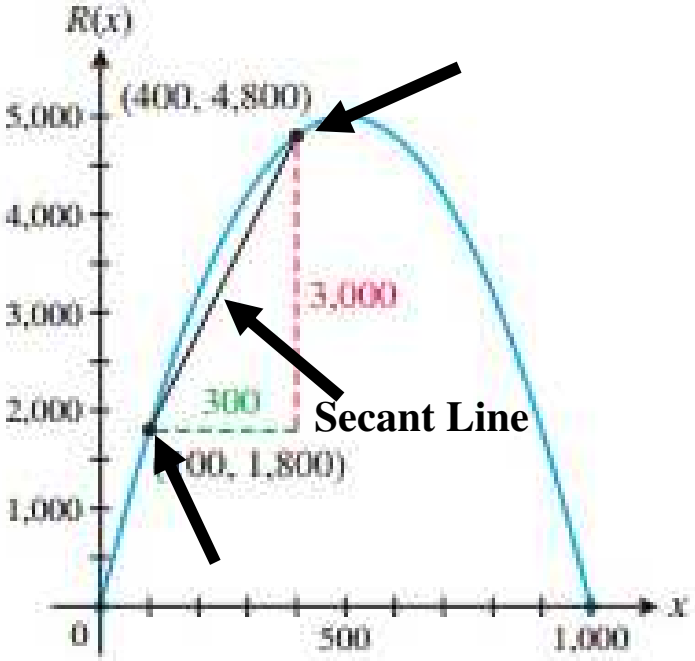

Figure 2 $R(x) = 20x - 0.02x^2$

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Example Velocity

A small steel ball dropped from a tower will fall a distance of y feet in x seconds, approximated by the formula,

$$y = f(x) = 16x^2$$

The figure shows the position of the ball on a coordinate line (with positive direction down) at the end of 0, 1, 2, and 3 seconds.

At one second, the ball fell 16 feet.
At two seconds, the ball fell 64 feet.
At three seconds, the ball fell 144 feet.

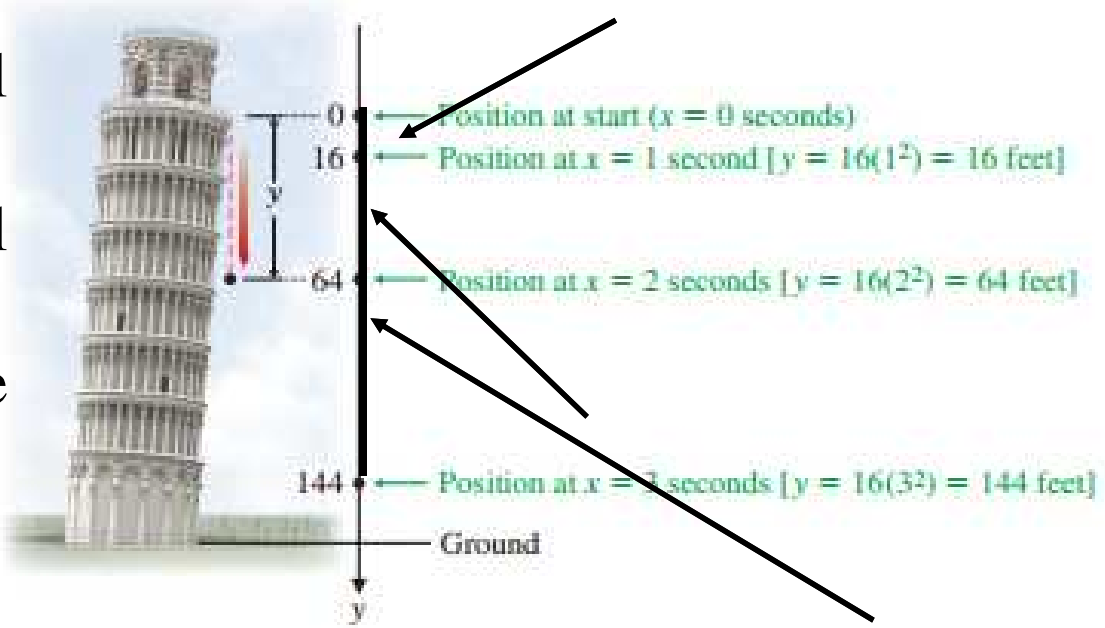

Figure 3

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Example Velocity (continued)

A small steel ball dropped from a tower will drop a distance of y feet in x seconds, approximated by the formula,

$$y = f(x) = 16x^2$$

Find the average velocity from $x = 2$ seconds to $x = 3$ seconds.

Solution: Average velocity is found by using an equivalent form of the formula $d = r \cdot t$ (distance equals rate times time)**.**

$$r = \frac{d}{t} = \frac{\text{Distance covered}}{\text{Elapsed time}} = \text{Average velocity}$$

$$\text{Average velocity} = \frac{(f(3) - f(2))\text{ feet}}{(3-2)\text{ seconds}}$$

$$= \frac{(16(3)^2 - 16(2)^2)\text{ feet}}{1\text{ second}} = 80\text{ feet per second}$$

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Example Velocity (continued)

A small steel ball dropped from a tower will drop a distance of y feet in x seconds, is approximated by the formula,

$$y = f(x) = 16x^2$$

Find and simplify the average rate of change from $x = 2$ seconds to $x = 2 + h$ seconds, $h \neq 0$.

Solution: Average velocity $= \dfrac{\text{Distance covered}}{\text{Elapsed time}}$

$$= \frac{f(2+h) - f(2)}{h}$$

$$= \frac{64 + 64h + 16h^2 - 64}{h}$$

$$= \frac{h(64 + 16h)}{h} = 64 + 16h \quad h \neq 0$$

Evaluating this general form for average velocity for $h = 1$ agrees with the value found earlier in this example.

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Example Velocity (continued)

A small steel ball dropped from a tower will drop a distance of y feet in x seconds, is approximated by the formula,

$$y = f(x) = 16x^2$$

Find the limit of the general expression for average velocity found in the previous slide as h approaches zero.

Solution: The expression for average velocity was found to be

$$\text{Average velocity} = 64 + 16h$$

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} (64 + 16h) = 64 \text{ feet per second}$$

What are some possible interpretations of the limit of this average velocity?

Generalize this for times other than 2 seconds.

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Definition Instantaneous Rate of Change

For $y = f(x)$, the **instantaneous rate of change at $x = a$** is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if the limit exists.

In further discussions the adjective *instantaneous* will be omitted with the understanding that **rate of change** will always refer to the instantaneous rate of change and not average rate of change.

Similarly, **velocity** always refers to the instantaneous rate of change in distance with respect to time.

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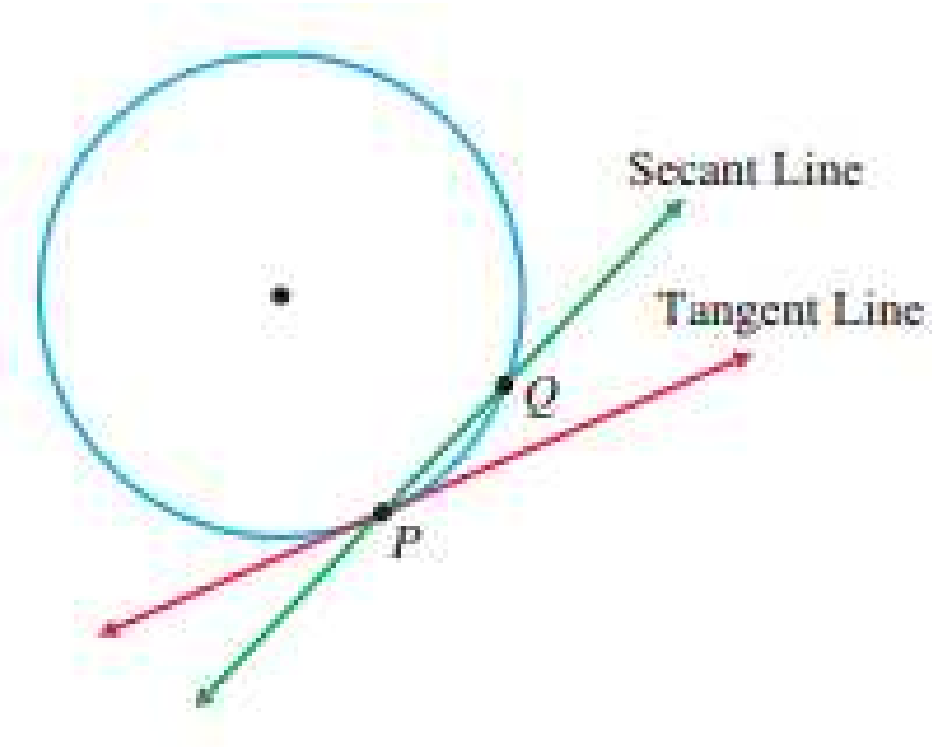
Slope of the Tangent Line

In geometry, a line that intersects a circle in two points is called a secant line, and a line that intersects a circle in exactly one point is called a tangent line.

The figure illustrates these two lines.

If point Q on the secant line is moved closer and closer to point P, the angle between the secant line and the tangent line gets increasingly small.

We generalize this concept and use slopes of secant lines to approximate slopes of tangent lines.



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Secant Lines

A line containing two points on the graph of a function is called a **secant line**.

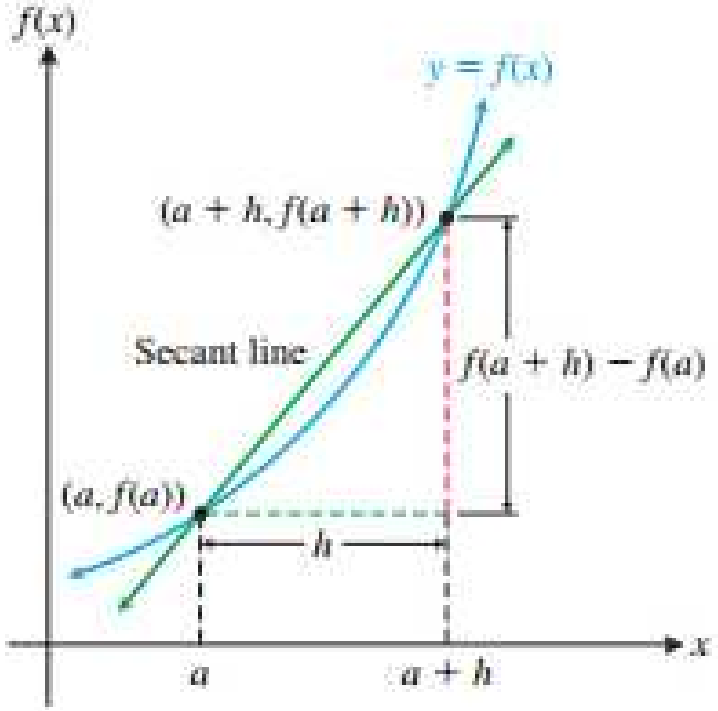
If $(a, f(a))$ and $(a + h, f(a + h))$ are two points on the graph of $y = f(x)$, we use the slope of a line formula to find the slope of the secant line through these two points.

$$\text{Slope of secant line} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(a+h) - f(a)}{(a+h) - h}$$

$$= \frac{f(a+h) - f(a)}{h}$$

This difference quotient is the slope of the secant line and can be interpreted as the average rate of change.



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Definition Slope of a Graph and Tangent Line

Given $y = f(x)$, the **slope of the graph** at the point $(a, f(a))$ is given by $\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

provided the limit exists.

In this case, the **tangent line** to the graph is the line through $(a, f(a))$ with slope given by the limit.

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Definition The Derivative

For $y = f(x)$, we define the **derivative of f at x**, denoted $f'(x)$, by

$$f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \text{ if the limit exists.}$$

If $f'(x)$ exists for each x in the open interval (a,b), then f is said to be **differentiable** over (a,b).

The process for finding the derivative of a function is called **differentiation**.

The derivative of a function is obtained by **differentiating** the function.

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Summary Interpretations of the Derivative

The derivative of a function f is a new function f'.
The domain of f' is a subset of the domain of f.
The derivative has various applications and interpretations, including:

1. *Slope of the tangent line*. For each x in the domain of f', $f'(x)$ is the slope of the line tangent to the graph of f at the point $(x, f(x))$.
2. *Instantaneous rate of change*. For each x in the domain of f', $f'(x)$ is the instantaneous rate of change of $y = f(x)$ with respect to x.
3. *Velocity*. If $f(x)$ is the position of a moving object at time x, then $v = f'(x)$ is the velocity of the object at that time.

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Procedure: The Four-step process for finding the derivative of a function *f*.

Step 1: Find $f(x+h)$.

Step 2: Find $f(x+h)-f(x)$.

Step 3 Find $\dfrac{f(x+h)-f(x)}{h}$.

Step 4 Find $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

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Example Find a Derivative

Use the four-step process to find $f'(x)$, the derivative of f at x, for $f(x) = -16x^2 + 80x + 6$.

Solution: Step 1 Find $f(x + h)$.

$$\begin{aligned} f(x+h) &= -16(x+h)^2 + 80(x+h) + 6 \\ &= -16(x^2 + 2xh + h^2) + 80x + 80h + 6 \\ &= -16x^2 - 32xh - 16h^2 + 80x + 80h + 6 \end{aligned}$$

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Example Find a Derivative (continued)

Use the four-step process to find $f'(x)$, the derivative of f at x, for $f(x) = -16x^2 + 80x + 6$.

Step 2 Find $f(x + h) - f(x)$.

$$f(x+h) - f(x) =$$

$$(-16x^2 - 32xh - 16h^2 + 80x + 80h + 6) - (-16x^2 + 80x + 6)$$

$$= -16x^2 - 32xh - 16h^2 + 80x + 80h + 6 + 16x^2 - 80x - 6$$

$$f(x+h) - f(x) = -32xh - 16h^2 + 80h$$

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Example Find a Derivative (continued)

Use the four-step process to find $f'(x)$, the derivative of f at x, for $f(x) = -16x^2 + 80x + 6$.

Step 3 Find $\dfrac{f(x+h) - f(x)}{h}$.

$$\frac{f(x+h) - f(x)}{h} = \frac{-32xh - 16h^2 + 80h}{h} = -32x - 16h + 80$$

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Example Find a Derivative (continued)

Use the four-step process to find $f'(x)$, the derivative of f at x, for $f(x) = -16x^2 + 80x + 6$.

Step 4 Find $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-32xh - 16h^2 + 80h}{h}$$

$$= \lim_{h \to 0} (-32x - 16h + 80)$$

$$= -32x + 80$$

For $f(x) = -16x^2 + 80x + 6$,

the derivative of f at x, is $f'(x) = -32x + 80$.

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Nonexistence of the Derivative

The existence of the derivative at $x = a$ depends on the existence of a limit at $x = a$.

A derivative $f'(x)$ exists whenever $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ exists.

If the limit does not exist at $x = a$, we say that the function f is **nondifferentiable at $x = a$**, or **$f'(x)$ does not exist**.

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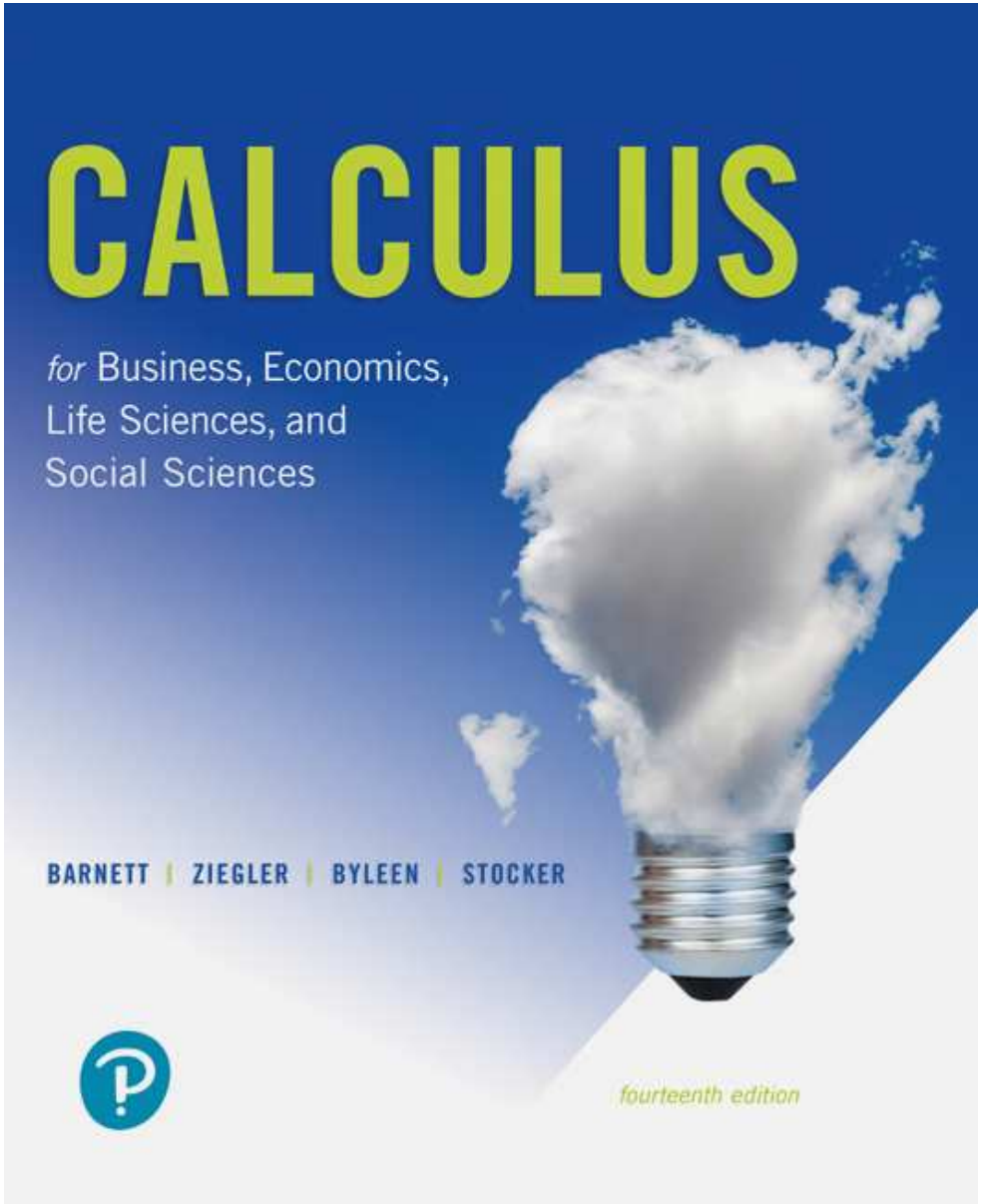

Chapter 2

Limits and the Derivative

Section 5

Basic Differentiation Properties

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Brief Review and Derivative Notation

We defined the derivative of f at x as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \text{ if the limit exists.}$$

Derivative Notation

If $y = f(x)$, then $f'(x)$, y', $\frac{dy}{dx}$, and $\frac{d}{dx} f(x)$

all represent the derivative of f at x.

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Theorem 1 Constant Function Rule

$$\text{If } y = f(x) = C, \text{ then } f'(x) = 0.$$

$$\text{Also, } y' = 0 \text{ and } \frac{dy}{dx} = 0.$$

The derivative of any constant function is 0.

Example: Suppose $f(x) = 10\pi^2$. Find $f'(x)$.

Solution: This function has a constant value, $10\pi^2$
According to Theorem 1, its derivative $f'(x) = 0$.

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Power Rule

A function of the form $f(x) = x^k$, where k is a real number, is called a **power function.**

Note that the exponent k does not have to be an integer, it can be an irrational number or a rational number in decimal or fractional form.

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Theorem 2 Power Rule

If $y = f(x) = x^n$, where n is a real number, then

$f'(x) = n \cdot x^{n-1}$.

Also, $y' = n \cdot x^{n-1}$ and $\dfrac{dy}{dx} = n \cdot x^{n-1}$.

Example: Suppose $f(x) = x^6$. Find $f'(x)$.

Solution:

According to Theorem 2, the derivative $f'(x) = 6x^{6-1} = 6x^5$.

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Example Differentiating Power Functions

$$\text{Find } f'(x) \text{ for } f(x) = \frac{1}{x^5}.$$

Solution: Write the function as a power function.

$$f(x) = \frac{1}{x^5} = x^{-5}$$

Use Theorem 2 to find the derivative $f'(x) = (x^{-5})' = -5x^{-5-1}$

$$= -5x^{-6} = -\frac{5}{x^6}$$

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Example Differentiating Power Functions

Find $f'(x)$ for $f(x) = \sqrt[4]{x^3}$.

Solution: Write the function as a power function.

$$f(x) = \sqrt[4]{x^3} = x^{\frac{3}{4}}$$

Use Theorem 2 to find the derivative $f'(x) = \left(x^{\frac{3}{4}} \right)'$

$$f'(x) = \frac{3}{4} x^{\frac{3}{4}-1} = \frac{3}{4} x^{-\frac{1}{4}}$$

$$= \frac{3}{4x^{\frac{1}{4}}} = \frac{3}{4\sqrt[4]{x}}$$

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Theorem 3 Constant Multiple Property

If $y = f(x) = k \cdot u(x)$, where k is a real number, and $u(x)$ is a function, then

$f'(x) = k \cdot u'(x)$.

Also, $y' = k \cdot u'$ and $\dfrac{dy}{dx} = k \cdot \dfrac{du}{dx}$.

Example: Suppose $f(x) = 12x^4$. Find $f'(x)$.

Solution:

With $k = 12$ and $u(x) = x^4$ According to Theorems 2 and 3, the derivative $f'(x) = 12(x^4)' = 12 \cdot 4(x^3) = 48x^3$.

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Theorem 4 Sum and Difference Property

If $y = f(x) = u(x) \pm v(x)$,

where $u(x)$ and $v(x)$ are functions, then

$f'(x) = u'(x) \pm v'(x)$

Also, $y' = u' + v'$ and $\dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$.

Example: Suppose $f(x) = 3x^3 + 11x^2$. Find $f'(x)$.

Solution:

$f'(x) = (3x^3 + 11x^2)' = (3x^3)' + (11x^2)' = 9x^2 + 22x.$

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Example Rewrite before Differentiating

$$\text{Find } f'(x) \text{ for } f(x) = \frac{3 - x^3}{x^6}.$$

Solution: Rewrite the function as a sum of terms.

$$f(x) = \frac{3 - x^3}{x^6} = \frac{3}{x^6} - \frac{x^3}{x^6} = \frac{3}{x^6} - \frac{1}{x^3}$$

Use Theorem 4 and find the derivative of each term

$$f'(x) = \left(\frac{3}{x^6} - \frac{1}{x^3}\right)' = \left(\frac{3}{x^6}\right)' - \left(\frac{1}{x^3}\right)'$$

$$= (3x^{-6})' - (x^{-3})' = -18x^{-7} + 3x^{-4}$$

$$= -\frac{18}{x^7} + \frac{3}{x^4} = \frac{3x^3 - 18}{x^7}$$

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Application Instantaneous Velocity

An object moves along the y axis (marked in feet) so that its position at time x (in seconds) is $f(x) = x^3 - 6x^2 + 9x$

Find the instantaneous velocity function v.

Solution: The instantaneous velocity function is the derivative of the position function with respect to time.

$$v(x) = f'(x) = (x^3)' - (6x^2)' + (9x)' = 3x^2 - 12x + 9$$

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Application Instantaneous Velocity

An object moves along the y axis (marked in feet) so that its position at time x (in seconds) is $f(x) = x^3 - 6x^2 + 9x$

Find the velocity at $x = 2$ and $x = 5$ seconds.

Solution: The instantaneous velocity was found to be

$$v(x) = 3x^2 - 12x + 9$$

Evaluate $v(2) = 3 \cdot 2^2 - 12 \cdot 2 + 9 = 12 - 24 + 9 = -3$

The instantaneous velocity at $x = 2$ seconds is -3 feet per second.

Evaluate $v(5) = 3 \cdot 5^2 - 12 \cdot 5 + 9 = 75 - 60 + 9 = 24$

The instantaneous velocity at $x = 5$ seconds is 24 feet per second.

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Application Instantaneous Velocity

An object moves along the y axis (marked in feet) so that its position at time x (in seconds) is $f(x) = x^3 - 6x^2 + 9x$

Find the time when the velocity is 0.

Solution: The instantaneous velocity was found to be.

$$v(x) = 3x^2 - 12x + 9$$

We solve $v(x) = 3x^2 - 12x + 9 = 0$

$$3(x^2 - 4x + 3) = 0$$

$$3(x - 1)(x - 3) = 0$$

$$x = 1, 3$$

The velocity of this object is zero at $x = 1$ and $x = 3$ seconds.

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Example Equation of Tangent Line

Let $f(x) = 3x^3 - 2x^2 + 5$.

1.) Find the derivative $f'(x)$.

Solution: 1.) $f'(x) = (3x^3)' - (2x^2)' + (5)'$

$= 9x^2 - 4x$

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Example Equation of Tangent Line

Let $f(x) = 3x^3 - 2x^2 + 5$.

1.) The derivative $f'(x)$ was found to be $f'(x) = 9x^2 - 4x$.

2.) Find the equation of the tangent line for $x = 1$.

Solution: 2.) $f(1) = 3 \cdot 1^3 - 2 \cdot 1^2 + 5 = 6$.

We find the tangent line at the point $(1, 6)$

The slope of the tangent for $x = 1$ is $f'(1) = 9 - 4 = 5$

The point-slope form for a line is $y - y_1 = m(x - x_1)$

$y - 6 = 5(x - 1)$ which simplifies to $y = 5x + 1$.

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Example Equation of Tangent Line

Let $f(x) = 3x^3 - 2x^2 + 5$.

1.) The derivative $f'(x)$ was found to be $f'(x) = 9x^2 - 4x$.

2.) The tangent line equation was found to be $y = 5x + 1$.

3.) Find the values of x where the tangent line is horizontal.

Solution: 3.) $f'(x) = 9x^2 - 4x$

A line is horizontal when its slope is zero.

We solve the equation $f'(x) = 9x^2 - 4x = 0$.

$$9x^2 - 4x = 0$$

$$x(9x - 4) = 0$$

$$x = 0, x = 4/9$$

The tangent line to the function $f(x)$ is horizontal when either $x = 0$ or $x = 4/9$.

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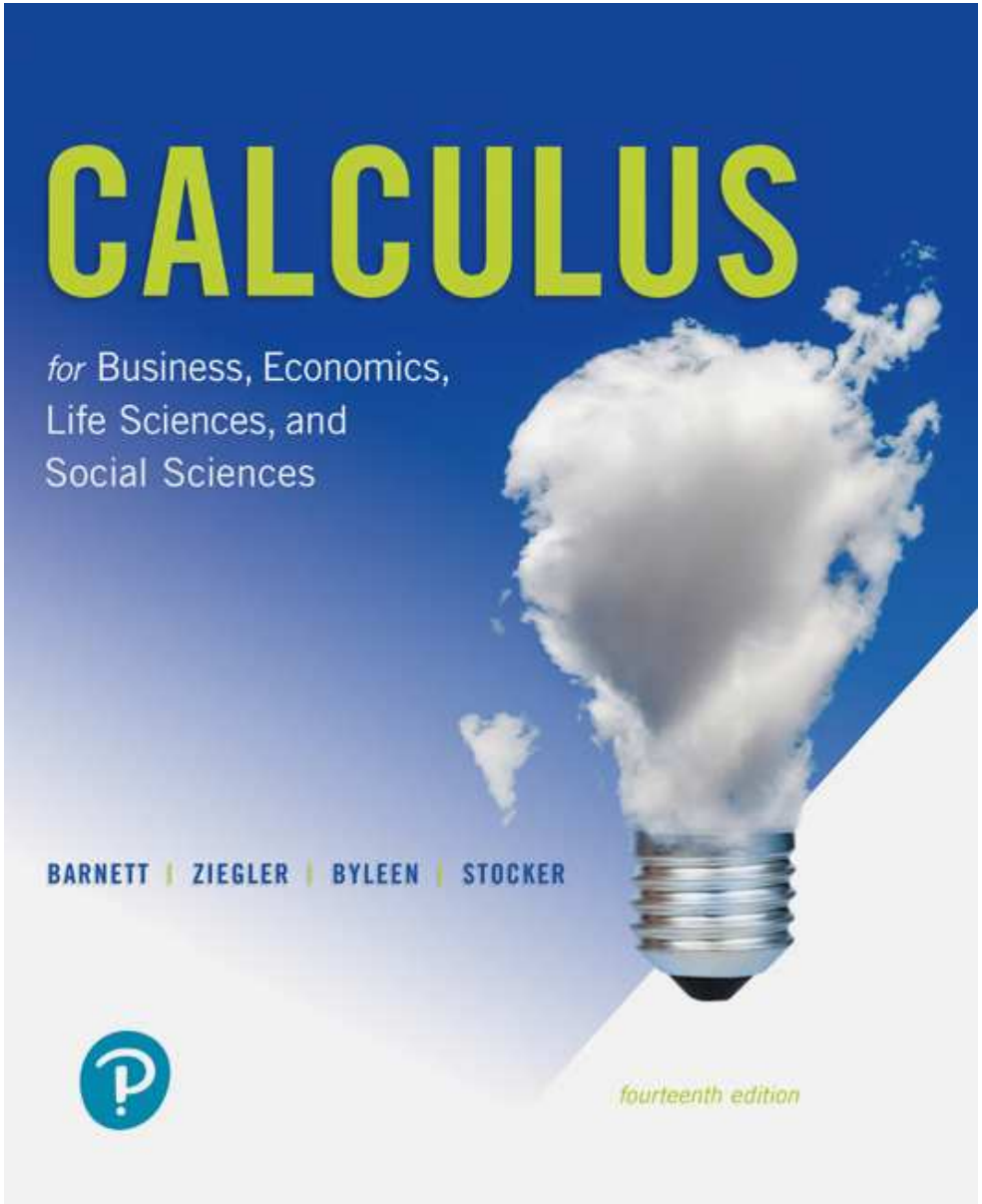

Chapter 2

Limits and the Derivative

Section 6
Differentials

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Increments

The derivative of f at x is the limit of the difference quotient:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Increment notation allows interpreting the numerator and the denominator of the difference quotient separately.

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Example Concept of Increment

For $y = f(x) = x^3$, a change in x from 2 to 2.1 corresponds to a change in y from $y = f(2) = 8$ to $y = f(2.1) = 9.261$.

Increment Notation

Change in x (the **increment in x**) is denoted by Δx.

The Greek letter delta, in mathematics stands for a *difference* or *change*.

Change in y (the **increment in y**) is denoted by Δy.

In the example, $\Delta x = 2.1 - 2 = 0.1$

$$\Delta y = f(2.1) - f(2) = 9.261 - 8 = 1.261.$$

The difference quotient can be written as

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x+h) - f(x)}{h} = \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x} \text{ where } h = \Delta x$$

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Increments Interpreted Graphically

For $y = f(x)$, $\Delta x = x_2 - x_1$, so $x_2 = x_1 + \Delta x$, and

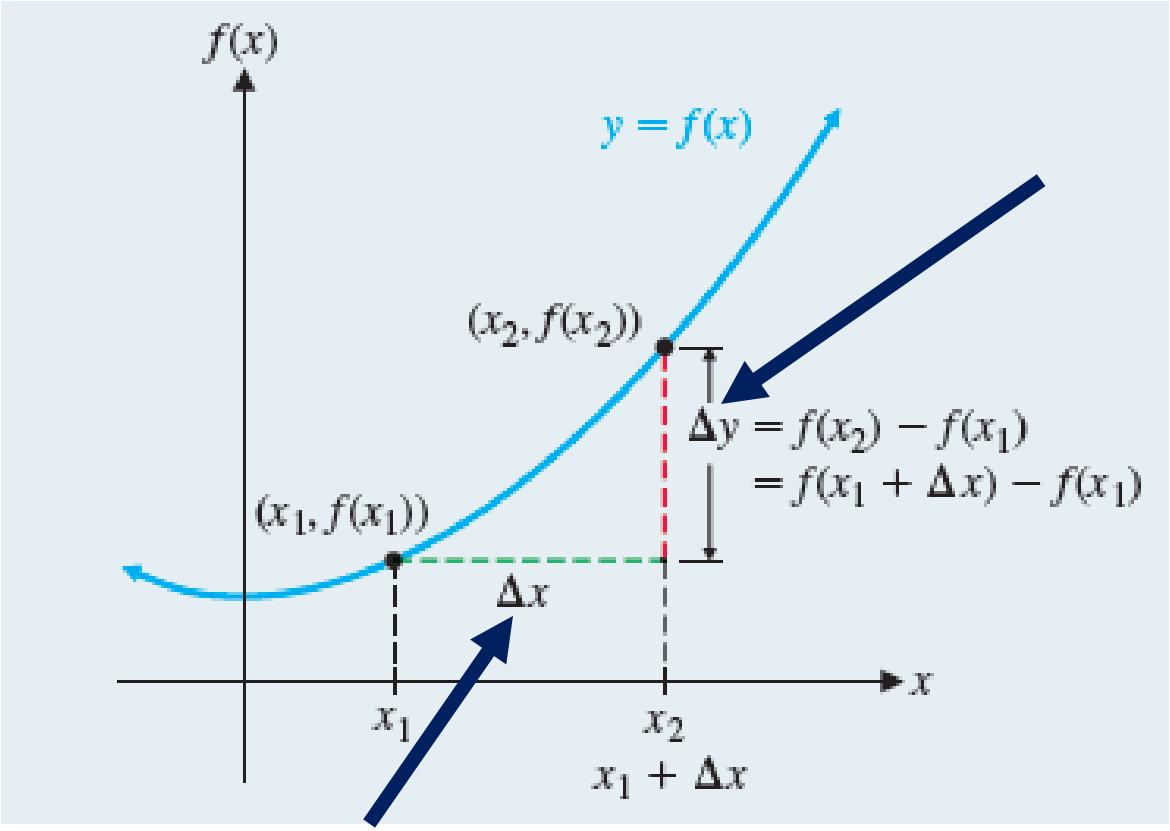
$$\Delta y = y_2 - y_1 = f(x_2) - f(x_1) = f(x_1 + \Delta x) - f(x_1)$$

Δx can be either positive or negative.

Δy represents the change in y corresponding to a Δx change in x.

$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

gives the slope of the secant line connecting the points with first coordinates x_1 and $x_2 = x_1 + \Delta x$



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Example Increments

Given the function $y = f(x) = \dfrac{x^2}{2}$,

(A) Find $\Delta x, \Delta y,$ and $\dfrac{\Delta y}{\Delta x}$ for $x_1 = 1$ and $x_2 = 2$.

Solution:

(A) $\Delta x = x_2 - x_1 = 2 - 1 = 1$

$$\Delta y = f(x_2) - f(x_1) = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$$

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_2} = \frac{\frac{3}{2}}{1} = \frac{3}{2}$$

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Example Increments

Given the function $y = f(x) = \dfrac{x^2}{2}$,

(B) Find $\dfrac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$ for $x_1 = 1$ and $\Delta x = 2$.

Solution (B) $\dfrac{f(x_2) - f(x_1)}{\Delta x} = \dfrac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$

$$= \frac{f(1+2) - f(1)}{2}$$

$$= \frac{f(3) - f(1)}{2} = \frac{\frac{9}{2} - \frac{1}{2}}{2} = \frac{4}{2} = 2$$

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Differentials

Suppose that the limit $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ exists.

For small Δx, $f'(x) \approx \frac{\Delta y}{\Delta x}$

and $\Delta y \approx \Delta x \cdot f'(x)$

The differential $dy = f'(x) \cdot \Delta x$ or $df = f'(x) \cdot \Delta x$

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Definition Differentials

If $y = f(x)$ defines a differentiable function, then the **differential** dy, or df, is defined as the product of $f'(x)$ and dx, where $dx = \Delta x$.

Symbolically, $dy = f'(x) \cdot dx$, or

$df = f'(x) \cdot dx$ where $dx = \Delta x$

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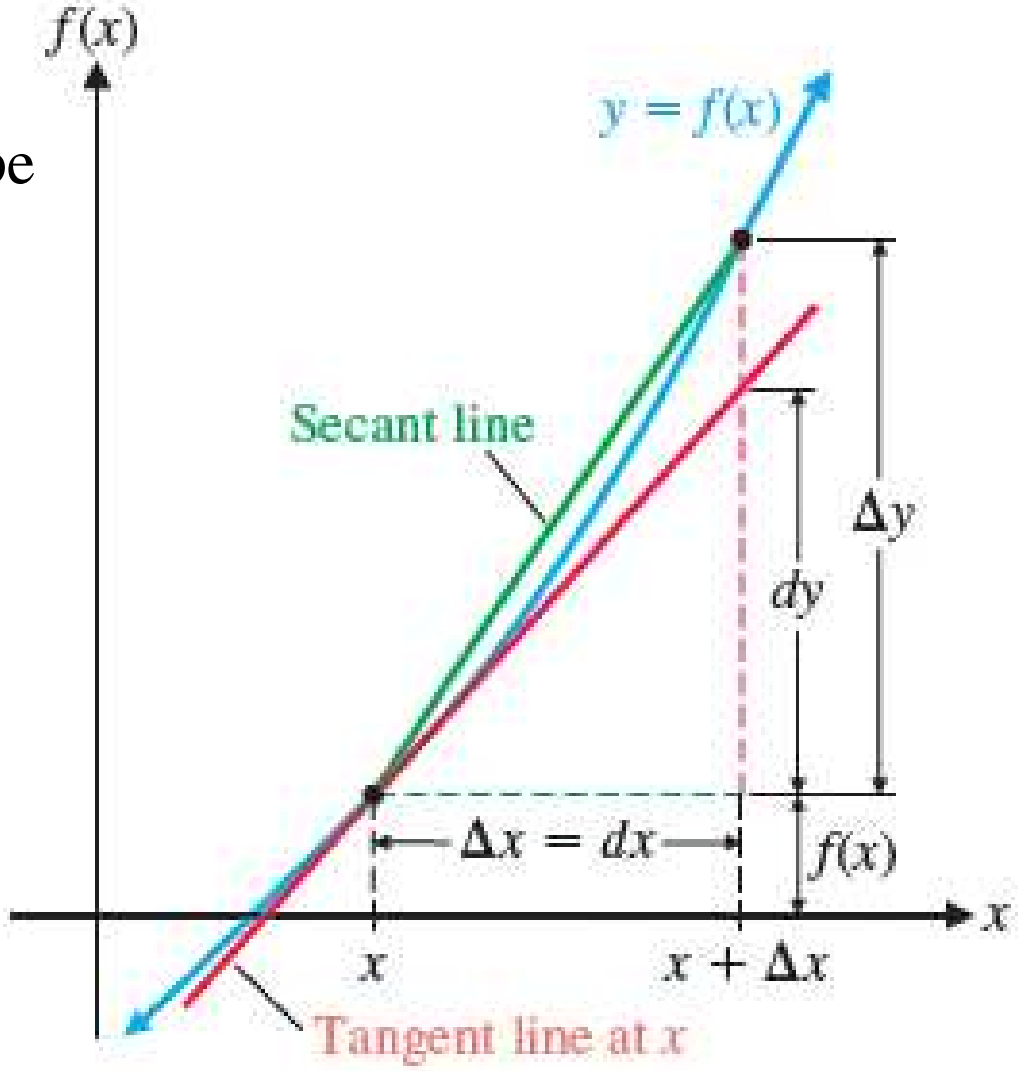
Differentials

The tangent line has slope $f'(x)$ with horizontal change dx.

The vertical change is given by $dy = f'(x)\,dx$.



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Interpretation of Differentials

Δx and dx both represent change in x.

The increment Δy stands for the **actual change** in y corresponding to the change in x.

The differential dy stands for the **approximate change** in y, estimated by using derivatives.

$$\Delta y \approx dy = f'(x)\,dx$$

In applications, we use dy to estimate Δy.

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Example Differentials

Find dy for $f(x) = x^2 + 3x$ and evaluate dy for $x = 2$ and $dx = 0.1$.

Solution:

$$dy = f'(x)\,dx$$

$$= (2x + 3)\,dx$$

For $x = 2$ and $dx = 0.1$, $dy = (2(2) + 3)\,0.1 = 0.7$

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Example Comparing Increments and Differentials

Let $y = f(x) = 6x - x^2$.

(A) Find Δy and dy when $x = 2$.

Solution

(A) $\Delta y = f(2 + \Delta x) - f(2)$

$$= 6(2 + \Delta x) - (2 + \Delta x)^2 - (6 \cdot 2 - 2^2)$$

$$= 12 + 6\Delta x - 4 - 4\Delta x - \Delta x^2 - 12 + 4$$

$$= 2\Delta x - \Delta x^2$$

Since $f'(x) = 6 - 2x$, $f'(2) = 2$, and $dx = \Delta x$,

then $dy = f'(2)dx = 2\Delta x$

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Example Comparing Increments and Differentials

Let $y = f(x) = 6x - x^2$.

(B) Find Δy and dy from part (A) for $\Delta x = 0.1$, 0.2, and 0.3

Solution

(B) The table compares the values of Δy and dy for the indicated values of Δx.

Δx	Δy	dy
0.1	0.19	0.2
0.2	0.36	0.4
0.3	0.51	0.6

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Example Cost-Revenue

A company manufactures and sells x transistor radios per week. If the weekly cost and revenue equations are

$$C(x) = 5,000 + 2x \qquad R(x) = 10x - \frac{x^2}{1,000} \qquad 0 \le x \le 8,000$$

use differentials to approximate changes in revenue and profit if production is increased from 2,000 to 2,010 units/week.

Solution Profit $P(x) = R(x) - C(x) = 8x - \frac{x^2}{1,000} - 5,000$

Change in revenue dR

$$dR = R'(x)\,dx = (10 - \frac{x}{500})\,dx$$

$$= \left(10 - \frac{2,000}{500}\right) \cdot 10 = \$60 \quad \text{per week}$$

Change in profit dP

$$dP = P'(x)\,dx = (8 - \frac{x}{500})\,dx$$

$$= (8 - \frac{2,000}{500})\,10 = \$40 \quad \text{per week}$$

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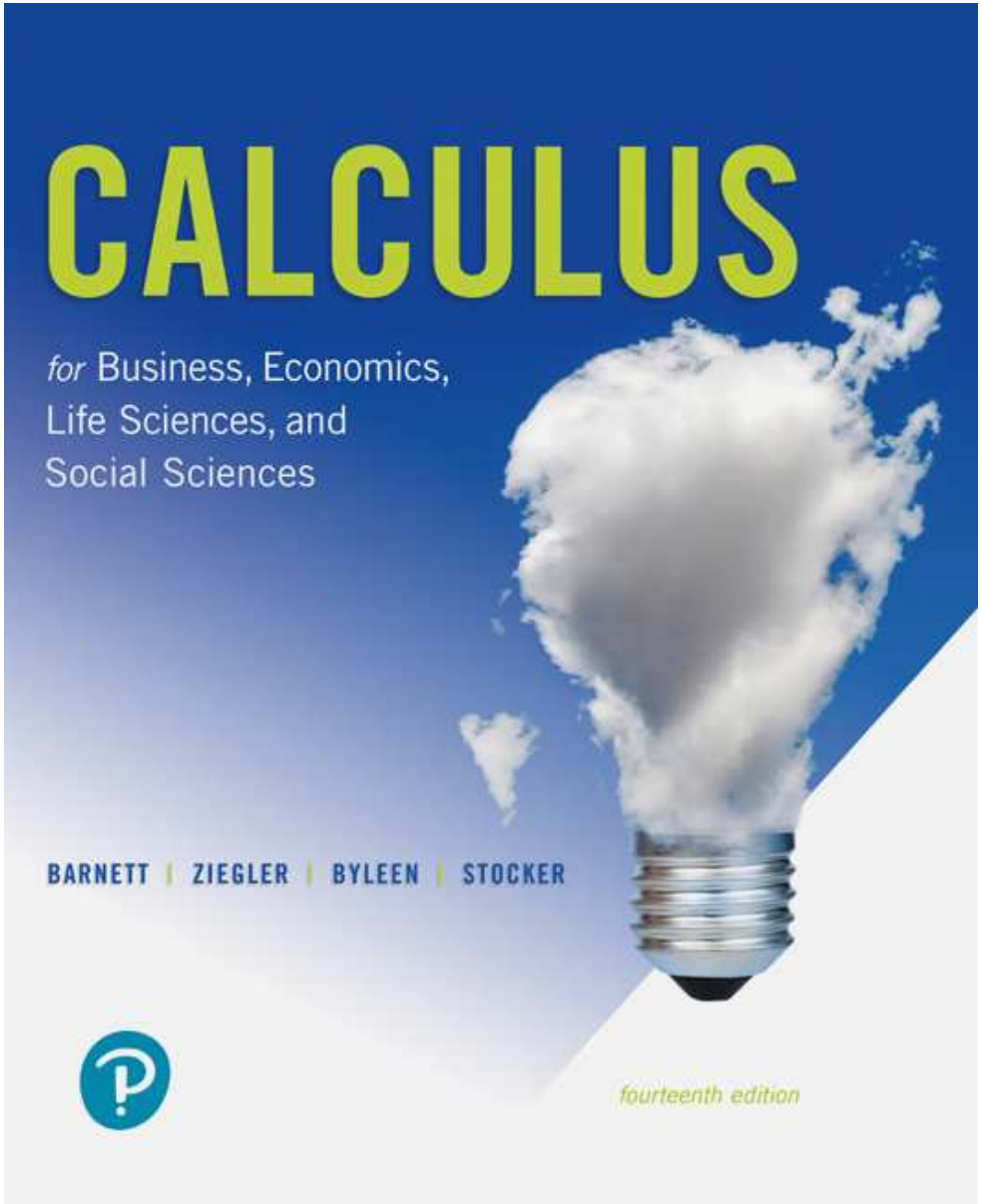

Chapter 2

Limits and the Derivative

Section 7
Marginal Analysis in Business and Economics

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Definition Marginal Cost, Revenue, and Profit

If x is the number of units of a product produced in some time interval, then for

$$\text{total cost} = C(x),$$

$$\textbf{marginal cost} = C'(x)$$

$$\text{total revenue} = R(x),$$

$$\textbf{marginal revenue} = R'(x)$$

$$\text{total profit} = P(x) = R(x) - C(x),$$

$$\textbf{marginal profit} = P'(x) = R'(x) - C'(x)$$

$$= (\text{marginal revenue}) - (\text{marginal cost})$$

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Marginals and Derivatives

Marginal cost (or revenue or profit) is the instantaneous rate of change of cost (or revenue or profit) relative to production at a given production level.

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Marginal Cost and Exact Cost

Suppose $C(x)$ is the total cost of producing x items.

The exact *additional* cost of producing one more item is

$$C(x + 1) - C(x).$$

This is also true for revenue and profit.

(additional revenue) $R(x + 1) - R(x)$

(additional profit) $P(x + 1) - P(x)$

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Example Cost Analysis

A company manufactures fuel tanks for cars. The total weekly cost (in dollars) of producing x tanks is given by

$$C(x) = 10{,}000 + 90x - 0.05x^2$$

(A) Find the marginal cost function.

Solution

(A) Marginal cost function is $C'(x) = 90 - 0.1x$

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Example Cost Analysis

A company manufactures fuel tanks for cars. The total weekly cost (in dollars) of producing x tanks is given by

$$C(x) = 10{,}000 + 90x - 0.05x^2$$

(B) Find the marginal cost at a production level of 500 tanks per week.
(C) Interpret the results of part (B).

Solution (B) $C'(500) = 90 - 0.1(500) = \$40$ Marginal cost

(C) At a production level of 500 tanks per week, the total production costs are increasing at the rate of \$40 per tank.

The 501st tank should cost about \$40 to make.

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Example Cost Analysis

A company manufactures fuel tanks for cars. The total weekly cost (in dollars) of producing x tanks is given by

$$C(x) = 10{,}000 + 90x - 0.05x^2$$

(D) Find the exact cost of producing the 501st tank.

Solution

$C(501) = 10{,}000 + 90(501) - 0.05(501)^2$

$= \$42{,}539.95$ Total cost of making 501 tanks

$C(500) = 10{,}000 + 90(500) - 0.05(500)^2$

$= \$42{,}500.00$ Total cost of making 500 tanks

$C(501) - C(500) = 42{,}539.95 - 42{,}500.00$

$= \$39.95$ Exact cost of producing the 501st tank

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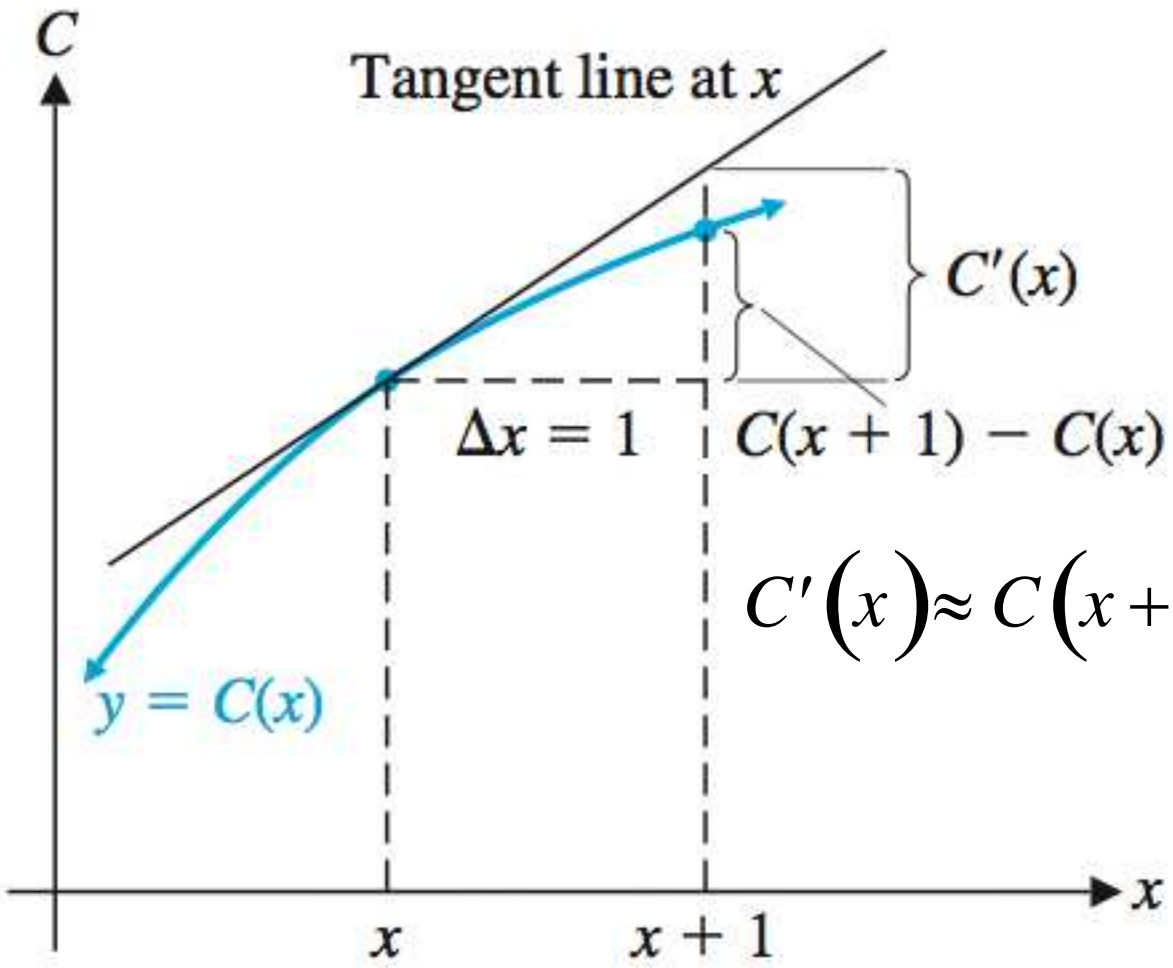
Marginal Cost and Exact Cost



$$C'(x) \approx C(x+1) - C(x)$$

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Theorem 1 Marginal Cost and Exact Cost

If $C(x)$ is the total cost of producing x items, then the marginal cost function approximates the exact cost of producing the $(x + 1)$st item.

$$C'(x) \approx C(x + 1) - C(x).$$

Similar statements are true for total revenue and total cost functions.

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Example Compare Exact Cost and Marginal Cost

The total cost of producing x bicycles is

$$C(x) = 10{,}000 + 150x - 0.2x^2.$$

(A) Find the exact cost of producing the 121st bicycle.

Solution The cost of producing 121 bicycles is

$$C(121) = 10{,}000 + 150(121) - 0.2(121)^2 = \$25{,}221.80$$

The cost of producing 120 bicycles is

$$C(120) = 10{,}000 + 150(120) - 0.2(120)^2 = \$25{,}120.00$$

The exact cost of producing the 121st bicycle is

$$C(121) - C(120) = \$25{,}221.80 - \$25{,}120.00 = \$101.80$$

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Example Compare Exact Cost and Marginal Cost (continued)

The total cost of producing x bicycles is

$$C(x) = 10{,}000 + 150x - 0.2x^2.$$

(B) Use the marginal cost to approximate the cost of producing the 121st bicycle.

Solution By Theorem 1, $C'(x)$, evaluated for $x = 120$, approximates the cost of producing the 121st bicycle.

$$C'(x) = 150 - 0.4x$$

$$C'(120) = 150 - 0.4(120) = \$102.00$$

The marginal cost, \$102.00, at a production level of 120 bicycles, is a good approximation to the exact cost, \$101.80, of producing the 121st bicycle.

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Example Production Strategy

A company's market research department recommends the manufacture and marketing of a new headphone. The research department presents the following **price-demand equation:**

$$x = 10{,}000 - 1{,}000p \quad (x \text{ is demand at price } p.)$$

Solve for price as a function of demand:

$$p = 10 - 0.001x$$

where x is the number of headphones that retailers are likely to buy at price $\$p$ per set.

(A) Find the domain of the function defined by the price-demand equation.

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Example Production Strategy (continued)

Solve for price as a function of demand:

$$p = 10 - 0.001x$$

where x is the number of headphones that retailers are likely to buy at price $\$p$ per set.

(A) Find the domain of the function defined by the price-demand equation.

Solution

(A) Both price p and demand x must be nonnegative, so $x \geq 0$.

$$p = 10 - 0.001x \geq 0$$

$$10 \geq 0.001x$$

$$10{,}000 \geq x$$

The permissible values of x are $0 \leq x \leq 10{,}000$.

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Example Production Strategy (continued)

A company's market research department recommends the manufacture and marketing of a new headphone.

The finance department provides the **cost function**

$$C(x) = 7{,}000 + 2x$$

where \$7,000 is the estimate of fixed costs and \$2 is the estimate of variable costs per headphone.

(B) Find and interpret the marginal cost function $C'(x)$.

Solution The marginal cost $C'(x) = 2$. Since this is a constant, it costs an additional \$2 to produce one more headphone at any production level.

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Example Production Strategy

A company's market research department recommends the manufacture and marketing of a new headphone.
Price as a function of demand is:

$$p = 10 - 0.001x$$

where x is the number of headphones that retailers are likely to buy at price $\$p$ per set.

(C) Find the revenue function of x and find its domain.

Solution Revenue is the money received from sales and is given by $R = x\,p$ where x is the number sold at price p.

$$R(x) = x\,p = x(10 - 0.001x)$$
$$= 10x - 0.001x^2$$

The domain is the same as the price function $0 \leq x \leq 10{,}000$.

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Example Production Strategy (continued)

The revenue function was found to be

$$R(x) = 10x - 0.001x^2$$

(D) Find the marginal revenue at $x = 2{,}000$, 5,000, and 7,000. Interpret these results.

Solution The **marginal revenue** is $R'(x) = 10 - 0.002x$

For production levels $x = 2{,}000$, 5,000, and 7,000,

$R'(2{,}000) = 6$ $R'(5{,}000) = 0$ $R'(7{,}000) = -4$

For 2,000 produced, revenue increases for higher production.
For 5,000 produced, revenue does not change for higher production.
For 7,000 produced, revenue decreases for higher production.

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Example Production Strategy (continued)

The revenue function is $R(x) = 10x - 0.001x^2$ and the cost function is $C(x) = 7{,}000 + 2x$

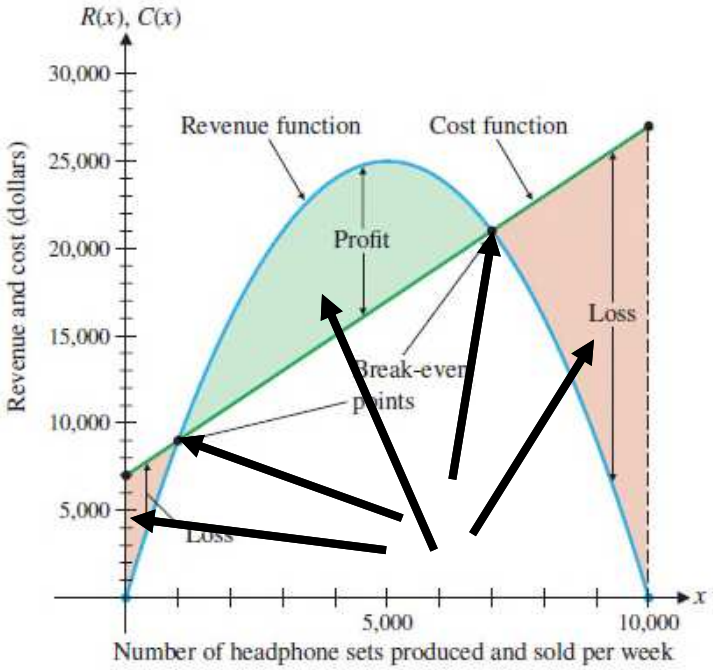
(E) Graph the cost and revenue function in the same axes. Find the intersection points and interpret the results.

Solution (E) The graphs are shown.

Using a graphing calculator, the intersection points are (1,000, 9,000) and (7,000, 21,000).

Cost exceeds revenue for production less than 1,000 and greater than 7,000 (up to 10,000 due to initial conditions.)

Revenue exceeds cost for production between 1,000 and 7,000.



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Example Production Strategy (continued)

Revenue, $R(x) = 10x - 0.001x^2$ and cost, $C(x) = 7{,}000 + 2x$

(F) Find the profit function, its domain, and graph.

Solution (F) The **profit function** is $P(x) = R(x) - C(x)$.

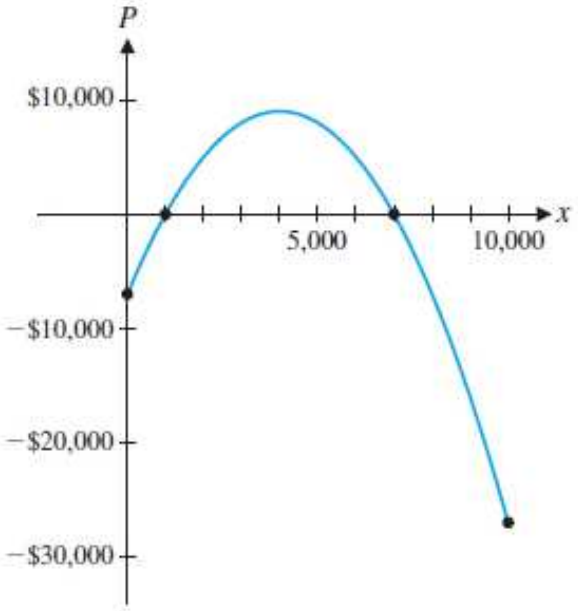
$P(x) = 10x - 0.001x^2 - (7{,}000 + 2x) = -0.001x^2 + 8x - 7{,}000$

The domain of the cost function is $x \geq 0$, and the domain of the revenue function is $0 \leq x \leq 10{,}000$.

The domain of the profit function is $0 \leq x \leq 10{,}000$.

The graph of the profit function is shown.

$P(x) \geq 0$ when $1{,}000 < x < 7{,}000$.



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Example Production Strategy (continued)

The **profit function** is $P(x) = -0.001x^2 + 8x - 7{,}000$

(G) Find the marginal profit at $x = 1{,}000$, 4,000, and 6,000

Solution (G) The **marginal profit function** is

$$P'(x) = -0.002x + 8$$

$P'(1{,}000) = 6$ $P'(4{,}000) = 0$ $P'(6{,}000) = -4$

Production level 1,000, profit increases if production increases

Production level 4,000, profit does not change if production increases

Production level 6,000, profit decreases if production increases

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Definition Marginal Average Cost

Definition:

If x is the number of units of a product produced in some time interval, then

Cost per unit:

$$\textbf{average cost} = \overline{C}(x) = \frac{\overline{C}(x)}{x}$$

$$\textbf{marginal average cost} = \overline{C}'(x) = \frac{d}{dx}\overline{C}(x)$$

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Definition Marginal Average Revenue

Definition:

If x is the number of units of a product produced in some time interval, then

Revenue per unit:

$$\textbf{average revenue} = \overline{R}(x) = \frac{R(x)}{x}$$

$$\textbf{marginal average revenue} = \overline{R}'(x) = \frac{d}{dx}\overline{R}(x)$$

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Definition Marginal Average Profit

Definition:

If x is the number of units of a product produced in some time interval, then

Profit per unit:

$$\textbf{average profit} = \overline{P}(x) = \frac{P(x)}{x}$$

$$\textbf{marginal average profit} = \overline{P}'(x) = \frac{d}{dx}\overline{P}(x)$$

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Example Cost Analysis

The total cost of printing x dictionaries is

$$C(x) = 20{,}000 + 10x$$

(A) Find the average cost per unit if 1,000 dictionaries are produced.

Solution
$$\overline{C}(x) = \frac{C(x)}{x} = \frac{20{,}000 + 10x}{x} = \frac{20{,}000}{x} + 10$$

$$\overline{C}(1{,}000) = \frac{20{,}000}{1{,}000} + 10 = 30$$

When 1,000 dictionaries are produced they cost an average of \$30 each.

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Example Cost Analysis (continued)

(B) Find the marginal average cost at a production level of 1,000 dictionaries, and interpret the results.

$$\textbf{marginal average cost} = \overline{C}'(x) = \frac{d}{dx}\left(\frac{20{,}000 + 10x}{x}\right) = -\frac{20{,}000}{x^2}$$

$$\textbf{marginal average cost} = \overline{C}'(1000) = -\frac{20{,}000}{1000^2} = -0.02$$

At production level 1,000 dictionaries, each additional dictionary produced reduces the per unit cost by about 2 cents.

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Example Cost Analysis (continued)

(C) Use the previous results to estimate the average cost per dictionary if 1,001 dictionaries are produced.

Average cost for 1000 dictionaries = \$30.00
Marginal average cost = – 0.02

The average cost per dictionary for 1001 dictionaries would be the average for 1000, plus the marginal average cost, or

$$\$30.00 + \$(-0.02) = \$29.98$$

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Caution

The marginal average cost function is computed by first finding the average cost and second finding the derivative.

If the order of these steps is reversed a different function with no useful interpretation is found.

Two interpretations for the marginal cost function:
(1) The derivative as the instantaneous rate of change in the cost function
(2) An approximate of exact cost of producing the $(x + 1)$st item.
This does not apply to the marginal average cost function.

The previous remarks apply to revenue and profit functions.

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