

***Basic College Mathematics 10th edition Lial
Solutions Manual***

SKU: 9780134467795-SOLUTIONS

CHAPTER 2 MULTIPLYING AND DIVIDING FRACTIONS

2.1 Basics of Fractions

2.1 Margin Exercises

- The figure has 4 equal parts.
Three parts are shaded:
One part is unshaded: $\frac{1}{4}$
- An area equal to 8 of the $\frac{1}{7}$ parts is shaded.

$$\frac{8}{7}$$

- $\frac{2}{3}$ ← Numerator
 $\frac{2}{3}$ ← Denominator



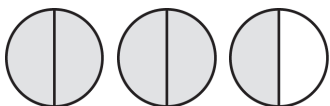
- $\frac{1}{4}$ ← Numerator
 $\frac{1}{4}$ ← Denominator



- $\frac{8}{5}$ ← Numerator
 $\frac{8}{5}$ ← Denominator



- $\frac{5}{2}$ ← Numerator
 $\frac{5}{2}$ ← Denominator



- Proper fractions: numerator *smaller* than denominator.

$$\frac{2}{3}, \frac{3}{4}, \frac{1}{3}$$

- Improper fractions: numerator *greater than* or *equal to* denominator.

$$\frac{4}{3}, \frac{8}{8}, \frac{3}{1}$$

2.1 Section Exercises

- $\frac{4}{5}$ ← Numerator
 $\frac{4}{5}$ ← Denominator
- $\frac{5}{6}$ ← Numerator
 $\frac{5}{6}$ ← Denominator

- $\frac{9}{8}$ ← Numerator
 $\frac{9}{8}$ ← Denominator

- $\frac{7}{5}$ ← Numerator
 $\frac{7}{5}$ ← Denominator

- The fraction $\frac{3}{8}$ represents 3 of the 8 equal parts into which a whole is divided.
- The fraction $\frac{7}{16}$ represents 7 of the 16 equal parts into which a whole is divided.
- The fraction $\frac{5}{24}$ represents 5 of the 24 equal parts into which a whole is divided.
- The fraction $\frac{24}{32}$ represents 24 of the 32 equal parts into which a whole is divided.

- The figure has 4 equal parts.
Three parts are shaded: $\frac{3}{4}$
One part is unshaded: $\frac{1}{4}$

- The figure has 8 equal parts.
Five parts are shaded: $\frac{5}{8}$
Three parts are unshaded: $\frac{3}{8}$

- The figure has 3 equal parts.
One part is shaded: $\frac{1}{3}$
Two parts are unshaded: $\frac{2}{3}$

- An area equal to 5 of the $\frac{1}{3}$ parts is shaded: $\frac{5}{3}$
One part is unshaded: $\frac{1}{3}$

- Each of the two figures is divided into 5 parts and 7 are shaded: $\frac{7}{5}$
Three are unshaded: $\frac{3}{5}$

- An area equal to 11 of the $\frac{1}{6}$ parts is shaded: $\frac{11}{6}$
One part is unshaded: $\frac{1}{6}$

- Five of the 6 bills have a lifespan of 2 years or greater: $\frac{5}{6}$
Four of the 6 bills have a lifespan of 4 years or less: $\frac{4}{6}$
Two of the 6 bills have a lifespan of 9 years: $\frac{2}{6}$

- Four of the 9 coins are pennies: $\frac{4}{9}$
Three of the 9 coins are nickels: $\frac{3}{9}$
Two of the 9 coins are dimes: $\frac{2}{9}$

- There are 25 students, and 8 are hearing impaired.

$$\frac{8}{25} \leftarrow \begin{array}{l} \text{hearing impaired students (numerator)} \\ \text{total students (denominator)} \end{array}$$

18. There are 215 songs of which 76 are the rock 'n roll. $215 - 76 = 139$ are pop songs.

$$\text{Fraction of pop songs: } \frac{139}{215}$$

19. There are 520 rooms. 217 are for guests with pets, and $520 - 217 = 303$ are for guests without pets.

$$\text{Fraction for guests without pets: } \frac{303}{520}$$

20. There are 46 employees. $46 - 15 = 31$ are part-time.

$$\text{Fraction of part-time employees: } \frac{31}{46}$$

21. Proper fractions: numerator *less* than denominator.

$$\frac{1}{3}, \frac{5}{8}, \frac{7}{16}$$

Improper fractions: numerator *greater than or equal to* denominator.

$$\frac{8}{5}, \frac{6}{6}, \frac{12}{2}$$

22. Proper fractions: numerator *smaller* than denominator.

$$\frac{1}{3}, \frac{3}{8}, \frac{3}{4}$$

Improper fractions: numerator *greater than or equal to* denominator.

$$\frac{16}{12}, \frac{10}{8}, \frac{6}{6}$$

23. Proper fractions: numerator *smaller* than denominator.

$$\frac{3}{4}, \frac{9}{11}, \frac{7}{15}$$

Improper fractions: numerator *greater than or equal to* denominator.

$$\frac{3}{2}, \frac{5}{5}, \frac{19}{18}$$

24. Proper fractions: numerator *smaller* than denominator.

none

Improper fractions: numerator *greater than or equal to* denominator.

$$\frac{12}{12}, \frac{15}{11}, \frac{13}{12}, \frac{11}{8}, \frac{17}{17}, \frac{19}{12}$$

25. (a) Three pieces are missing, so $\frac{3}{8}$ of the pie was eaten.

(b) Five pieces are remaining, so $\frac{5}{8}$ of the pie remains.

26. (a) Four pieces have been removed, so $\frac{4}{10}$ of the pack was taken.

(b) $10 - 4 = 6$
Six pieces are remaining, so $\frac{6}{10}$ of the pack remains.

27. Answers will vary. One possibility is

$$\frac{3}{4} \leftarrow \begin{array}{l} \text{Numerator} \\ \text{Denominator} \end{array}$$

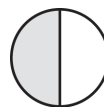


The denominator shows the number of equal parts in the whole and the numerator shows how many of the parts are being considered.

28. An example is $\frac{1}{2}$ as a proper fraction and $\frac{3}{2}$ as an improper fraction.

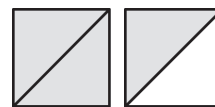
A proper fraction has a numerator *smaller* than the denominator.

An improper fraction has a numerator that is *greater than or equal to* the denominator.



$$\frac{1}{2}$$

Proper fraction



$$\frac{3}{2}$$

Improper fraction

2.2 Mixed Numbers

2.2 Margin Exercises

1. (a) The figure shows 1 whole object with 3 equal parts, all shaded, and a second whole with 2 parts shaded, so 5 parts are shaded in all.

$$1\frac{2}{3} = \frac{5}{3}$$

(b) Since each of these diagrams is divided into 4 pieces, the denominator will be 4. The number of pieces shaded is 9.

$$2\frac{1}{4} = \frac{9}{4}$$

2. (a) $6\frac{1}{2}$ $6 \cdot 2 = 12$ Multiply 6 and 2.

$$12 + 1 = 13 \quad \text{Add 1.}$$

$$6\frac{1}{2} = \frac{13}{2}$$

(b) $7\frac{3}{4}$ $7 \cdot 4 = 28$ Multiply 7 and 4.

$28 + 3 = 31$ Add 3.

$$7\frac{3}{4} = \frac{31}{4}$$

(c) $4\frac{7}{8}$ $4 \cdot 8 = 32$ Multiply 4 and 8.

$32 + 7 = 39$ Add 7.

$$4\frac{7}{8} = \frac{39}{8}$$

(d) $8\frac{5}{6}$ $8 \cdot 6 = 48$ Multiply 8 and 6.

$48 + 5 = 53$ Add 5.

$$8\frac{5}{6} = \frac{53}{6}$$

3. (a) $\frac{6}{5}$ Divide 6 by 5.

1 ← Whole number part

$$5 \overline{)6}$$

$$\frac{5}{1}$$

1 ← Remainder

$$\frac{6}{5} = 1\frac{1}{5}$$

(b) $\frac{9}{4}$ Divide 9 by 4.

2 ← Whole number part

$$4 \overline{)9}$$

$$\frac{8}{1}$$

1 ← Remainder

$$\frac{9}{4} = 2\frac{1}{4}$$

(c) $\frac{35}{5}$ Divide 35 by 5.

7 ← Whole number part

$$5 \overline{)35}$$

$$\frac{35}{0}$$

0 ← Remainder

$$\frac{35}{5} = 7$$

(d) $\frac{78}{7}$ Divide 78 by 7.

11 ← Whole number part

$$7 \overline{)78}$$

$$\frac{7}{8}$$

$$\frac{7}{1}$$

1 ← Remainder

$$\frac{78}{7} = 11\frac{1}{7}$$

2.2 Section Exercises

1. $\frac{12}{12}$ is an improper fraction since the numerator is *greater than or equal to* the denominator. The statement is *true*.

2. $\frac{2}{3}$ is a proper fraction since the numerator is *smaller than* the denominator. The statement is *true*.

3. $7\frac{2}{5}$ $7 \cdot 5 = 35$ Multiply 7 and 5.

$35 + 2 = 37$ Add 2.

$$7\frac{2}{5} = \frac{37}{5}$$

The mixed number $7\frac{2}{5}$ can be changed to the improper fraction $\frac{37}{5}$, not $\frac{14}{5}$. The statement is *false*.

4. The statement "Some mixed number cannot be changed to an improper fraction" is *false* since any mixed number *can* be changed to an improper fraction.

5. $6\frac{1}{2}$ $6 \cdot 2 = 12$ Multiply 6 and 2.

$12 + 1 = 13$ Add 1.

$$6\frac{1}{2} = \frac{13}{2}$$

The mixed number $6\frac{1}{2}$ can be changed to the improper fraction $\frac{13}{2}$, not $\frac{12}{2}$. The statement is *false*.

6. $5\frac{5}{6}$ $5 \cdot 6 = 30$ Multiply 5 and 6.

$30 + 5 = 35$ Add 5.

$$5\frac{5}{6} = \frac{35}{6}$$

The statement is *true*.

7. $1\frac{1}{4}$ $1 \cdot 4 = 4$ Multiply 1 and 4.

$4 + 1 = 5$ Add 1.

$$1\frac{1}{4} = \frac{5}{4}$$

8. $2\frac{1}{2}$ $2 \cdot 2 = 4$ Multiply 2 and 2.

$4 + 1 = 5$ Add 1.

$$2\frac{1}{2} = \frac{5}{2}$$

9. $4\frac{3}{5}$ $4 \cdot 5 = 20$ Multiply 4 and 5.

$20 + 3 = 23$ Add 3.

$4\frac{3}{5} = \frac{23}{5}$

10. $8\frac{1}{4}$ $8 \cdot 4 = 32$ Multiply 8 and 4.

$32 + 1 = 33$ Add 1.

$8\frac{1}{4} = \frac{33}{4}$

11. $8\frac{1}{2}$ $8 \cdot 2 = 16$ Multiply 8 and 2.

$16 + 1 = 17$ Add 1.

$8\frac{1}{2} = \frac{17}{2}$

12. $1\frac{7}{11}$ $1 \cdot 11 = 11$ Multiply 1 and 11.

$11 + 7 = 18$ Add 7.

$1\frac{7}{11} = \frac{18}{11}$

13. $10\frac{1}{8}$ $10 \cdot 8 = 80$ Multiply 10 and 8.

$80 + 1 = 81$ Add 1.

$10\frac{1}{8} = \frac{81}{8}$

14. $12\frac{2}{3}$ $12 \cdot 3 = 36$ Multiply 12 and 3.

$36 + 2 = 38$ Add 2.

$12\frac{2}{3} = \frac{38}{3}$

15. $10\frac{3}{4}$ $10 \cdot 4 = 40$ Multiply 10 and 4.

$40 + 3 = 43$ Add 3.

$10\frac{3}{4} = \frac{43}{4}$

16. $3\frac{3}{8}$ $3 \cdot 8 = 24$ Multiply 3 and 8.

$24 + 3 = 27$ Add 3.

$3\frac{3}{8} = \frac{27}{8}$

17. Write $5\frac{4}{5}$ as an improper fraction.

$5 \cdot 5 = 25$ Multiply 5 and 5.

$25 + 4 = 29$ Add 4. The numerator is 29.

$5\frac{4}{5} = \frac{29}{5}$ Use the same denominator.

18. $2\frac{8}{9}$ $2 \cdot 9 = 18$ Multiply 2 and 9.

$18 + 8 = 26$ Add 8.

$2\frac{8}{9} = \frac{26}{9}$

19. $8\frac{3}{5}$ $8 \cdot 5 = 40$ Multiply 8 and 5.

$40 + 3 = 43$ Add 3.

$8\frac{3}{5} = \frac{43}{5}$

20. $3\frac{4}{7}$ $3 \cdot 7 = 21$ Multiply 3 and 7.

$21 + 4 = 25$ Add 4.

$3\frac{4}{7} = \frac{25}{7}$

21. $4\frac{10}{11}$ $4 \cdot 11 = 44$ Multiply 4 and 11.

$44 + 10 = 54$ Add 10.

$4\frac{10}{11} = \frac{54}{11}$

22. $11\frac{5}{8}$ $11 \cdot 8 = 88$ Multiply 11 and 8.

$88 + 5 = 93$ Add 5.

$11\frac{5}{8} = \frac{93}{8}$

23. $32\frac{3}{4}$ $32 \cdot 4 = 128$ Multiply 32 and 4.

$128 + 3 = 131$ Add 3.

$32\frac{3}{4} = \frac{131}{4}$

24. $15\frac{3}{10}$ $15 \cdot 10 = 150$ Multiply 15 and 10.

$150 + 3 = 153$ Add 3.

$15\frac{3}{10} = \frac{153}{10}$

25. $18\frac{5}{12}$ $18 \cdot 12 = 216$ Multiply 18 and 12.

$216 + 5 = 221$ Add 5.

$18\frac{5}{12} = \frac{221}{12}$

26. $19\frac{8}{11}$ $19 \cdot 11 = 209$ Multiply 19 and 11.

$209 + 8 = 217$ Add 8.

$19\frac{8}{11} = \frac{217}{11}$

27. $17\frac{14}{15}$ $17 \cdot 15 = 255$ Multiply 17 and 15.

$255 + 14 = 269$ Add 14.

$17\frac{14}{15} = \frac{269}{15}$

28. $9\frac{5}{16}$ $9 \cdot 16 = 144$ Multiply 9 and 16.

$144 + 5 = 149$ Add 5.

$9\frac{5}{16} = \frac{149}{16}$

80 Chapter 2 Multiplying and Dividing Fractions

29. $7\frac{19}{24}$ $7 \cdot 24 = 168$ Multiply 7 and 24.
 $168 + 19 = 187$ Add 19.
 $7\frac{19}{24} = \frac{187}{24}$

30. $9\frac{7}{12}$ $9 \cdot 12 = 108$ Multiply 9 and 12.
 $108 + 7 = 115$ Add 7.
 $9\frac{7}{12} = \frac{115}{12}$

31. The improper fraction $\frac{4}{3}$ can be changed to the mixed number $1\frac{1}{3}$, not $1\frac{1}{4}$. The statement is *false*.

32. The statement "An improper fraction cannot always be written as a whole number or a mixed number" is *false* since a mixed number always has a value equal to or greater than a whole number.

33. The statement "Some improper fractions can be written as a whole number with no fraction part" is *true*. For example, $\frac{6}{2} = 3$.

34. The statement "The improper fraction $\frac{48}{6}$ can be written as the whole number 8" is *true*.

35. $\frac{4}{3}$

$$\begin{array}{r} 1 \leftarrow \text{Whole number part} \\ 3 \overline{)4} \\ \underline{3} \\ 1 \leftarrow \text{Remainder} \end{array}$$

$$\frac{4}{3} = 1\frac{1}{3}$$

36. $\frac{11}{9}$

$$\begin{array}{r} 1 \leftarrow \text{Whole number part} \\ 9 \overline{)11} \\ \underline{9} \\ 2 \leftarrow \text{Remainder} \end{array}$$

$$\frac{11}{9} = 1\frac{2}{9}$$

37. $\frac{9}{4}$

$$\begin{array}{r} 2 \leftarrow \text{Whole number part} \\ 4 \overline{)9} \\ \underline{8} \\ 1 \leftarrow \text{Remainder} \end{array}$$

$$\frac{9}{4} = 2\frac{1}{4}$$

38. $\frac{7}{2}$

$$\begin{array}{r} 3 \leftarrow \text{Whole number part} \\ 2 \overline{)7} \\ \underline{6} \\ 1 \leftarrow \text{Remainder} \end{array}$$

$$\frac{7}{2} = 3\frac{1}{2}$$

39. $\frac{54}{6}$

$$\begin{array}{r} 9 \leftarrow \text{Whole number part} \\ 6 \overline{)54} \\ \underline{54} \\ 0 \leftarrow \text{Remainder} \end{array}$$

$$\frac{54}{6} = 9$$

40. $\frac{63}{9}$

$$\begin{array}{r} 7 \leftarrow \text{Whole number part} \\ 9 \overline{)63} \\ \underline{63} \\ 0 \leftarrow \text{Remainder} \end{array}$$

$$\frac{63}{9} = 7$$

41. $\frac{38}{5}$

$$\begin{array}{r} 7 \leftarrow \text{Whole number part} \\ 5 \overline{)38} \\ \underline{35} \\ 3 \leftarrow \text{Remainder} \end{array}$$

$$\frac{38}{5} = 7\frac{3}{5}$$

42. $\frac{33}{7}$

$$\begin{array}{r} 4 \leftarrow \text{Whole number part} \\ 7 \overline{)33} \\ \underline{28} \\ 5 \leftarrow \text{Remainder} \end{array}$$

$$\frac{33}{7} = 4\frac{5}{7}$$

43. $\frac{63}{4}$

Divide 63 by 4.

$$\begin{array}{r} 15 \leftarrow \text{Whole number part} \\ 4 \overline{)63} \\ \underline{4} \\ 23 \\ \underline{20} \\ 3 \leftarrow \text{Remainder} \end{array}$$

The quotient 15 is the whole number part of the mixed number. The remainder 3 is the numerator of the fraction, and the denominator remains as 4.

$$\frac{63}{4} = 15\frac{3}{4}$$

44. $\frac{19}{5}$

$$\begin{array}{r} 3 \leftarrow \text{Whole number part} \\ 5 \overline{)19} \\ \underline{15} \\ 4 \leftarrow \text{Remainder} \end{array}$$

$$\frac{19}{5} = 3\frac{4}{5}$$

45. $\frac{47}{9}$

$$\begin{array}{r} 5 \leftarrow \text{Whole number part} \\ 9 \overline{)47} \\ \underline{45} \\ 2 \leftarrow \text{Remainder} \end{array}$$

$$\frac{47}{9} = 5\frac{2}{9}$$

46. $\frac{65}{9}$

$$\begin{array}{r} 7 \leftarrow \text{Whole number part} \\ 9 \overline{)65} \\ \underline{63} \\ 2 \leftarrow \text{Remainder} \end{array}$$

$$\frac{65}{9} = 7\frac{2}{9}$$

47. $\frac{65}{8}$

$$\begin{array}{r} 8 \leftarrow \text{Whole number part} \\ 8 \overline{)65} \\ \underline{64} \\ 1 \leftarrow \text{Remainder} \end{array}$$

$$\frac{65}{8} = 8\frac{1}{8}$$

48. $\frac{37}{6}$

$$\begin{array}{r} 6 \leftarrow \text{Whole number part} \\ 6 \overline{)37} \\ \underline{36} \\ 1 \leftarrow \text{Remainder} \end{array}$$

$$\frac{37}{6} = 6\frac{1}{6}$$

49. $\frac{84}{5}$

$$\begin{array}{r} 16 \leftarrow \text{Whole number part} \\ 5 \overline{)84} \\ \underline{5} \\ 34 \\ \underline{30} \\ 4 \leftarrow \text{Remainder} \end{array}$$

$$\frac{84}{5} = 16\frac{4}{5}$$

50. $\frac{92}{3}$

$$\begin{array}{r} 30 \leftarrow \text{Whole number part} \\ 3 \overline{)92} \\ \underline{9} \\ 2 \\ \underline{0} \\ 2 \leftarrow \text{Remainder} \end{array}$$

$$\frac{92}{3} = 30\frac{2}{3}$$

51. $\frac{112}{4}$

$$\begin{array}{r} 28 \leftarrow \text{Whole number part} \\ 4 \overline{)112} \\ \underline{8} \\ 32 \\ \underline{32} \\ 0 \leftarrow \text{Remainder} \end{array}$$

$$\frac{112}{4} = 28$$

52. $\frac{117}{9}$

$$\begin{array}{r} 13 \leftarrow \text{Whole number part} \\ 9 \overline{)117} \\ \underline{9} \\ 27 \\ \underline{27} \\ 0 \leftarrow \text{Remainder} \end{array}$$

$$\frac{117}{9} = 13$$

82 Chapter 2 Multiplying and Dividing Fractions

53. $\frac{183}{7}$

$$\begin{array}{r} 26 \leftarrow \text{Whole number part} \\ 7 \overline{) 183} \\ \underline{14} \\ 43 \\ \underline{42} \\ 1 \leftarrow \text{Remainder} \end{array}$$

$$\frac{183}{7} = 26\frac{1}{7}$$

54. $\frac{212}{11}$

$$\begin{array}{r} 19 \leftarrow \text{Whole number part} \\ 11 \overline{) 212} \\ \underline{22} \\ 102 \\ \underline{99} \\ 3 \leftarrow \text{Remainder} \end{array}$$

$$\frac{212}{11} = 19\frac{3}{11}$$

55. Multiply the denominator by the whole number and add the numerator. The result becomes the new numerator, which is placed over the original denominator.

$$\begin{array}{c} \downarrow \quad \downarrow \\ 2\frac{1}{2} \quad (2 \cdot 2) + 1 = 5 \quad \frac{5}{2} \\ \uparrow \quad \uparrow \end{array}$$

56. Divide the numerator by the denominator. The quotient is the whole number of the mixed number and the remainder is the numerator of the fraction part. The denominator is unchanged.

$$\begin{array}{c} \downarrow \quad \downarrow \\ \frac{5}{2} \quad 2\frac{1}{2} \leftarrow \text{Remainder} \\ \uparrow \quad \uparrow \\ \frac{4}{1} \leftarrow \text{Remainder} \end{array}$$

57. $250\frac{1}{2}$ $250 \cdot 2 = 500$

$$500 + 1 = 501$$

$$250\frac{1}{2} = \frac{501}{2}$$

58. $185\frac{3}{4}$ $185 \cdot 4 = 740$

$$740 + 3 = 743$$

$$185\frac{3}{4} = \frac{743}{4}$$

59. $333\frac{1}{3}$ $333 \cdot 3 = 999$

$$999 + 1 = 1000$$

$$333\frac{1}{3} = \frac{1000}{3}$$

60. $138\frac{4}{5}$ $138 \cdot 5 = 690$

$$690 + 4 = 694$$

$$138\frac{4}{5} = \frac{694}{5}$$

61. Write $522\frac{3}{8}$ as an improper fraction.

$$522 \cdot 8 = 4176$$

$$4176 + 3 = 4179 \quad \text{Add 3. The numerator is 4179.}$$

$$522\frac{3}{8} = \frac{4179}{8} \quad \text{Use the same denominator.}$$

62. $622\frac{1}{4}$ $622 \cdot 4 = 2488$

$$2488 + 1 = 2489$$

$$622\frac{1}{4} = \frac{2489}{4}$$

63. $\frac{617}{4}$

$$\begin{array}{r} 154 \leftarrow \text{Whole number part} \\ 4 \overline{) 617} \\ \underline{4} \\ 21 \\ \underline{20} \\ 17 \\ \underline{16} \\ 1 \leftarrow \text{Remainder} \end{array}$$

$$\frac{617}{4} = 154\frac{1}{4}$$

64. $\frac{760}{8}$

$$\begin{array}{r} 95 \leftarrow \text{Whole number part} \\ 8 \overline{) 760} \\ \underline{72} \\ 40 \\ \underline{40} \\ 0 \leftarrow \text{Remainder} \end{array}$$

$$\frac{760}{8} = 95$$

65. The commands used will vary. The following is from a TI-83 Plus:

```
int(2565/15) 171
fPart(2565/15) F
rac 0
```

$$\frac{2565}{15} = 171$$

66. The commands used will vary. The following is from a TI-83 Plus:

```
int(2915/16) 182
fPart(2915/16) F
rac 3/16
```

$$\frac{2915}{16} = 182\frac{3}{16}$$

Note: You can use the following procedure on any calculator. Divide 2915 by 16 to get 182.1875. Subtract 182. Multiply by 16 to get 3. The mixed number is $182\frac{3}{16}$.

67. The commands used will vary. The following is from a TI-83 Plus:

```
int(3917/32) 122
fPart(3917/32) F
rac 13/32
```

$$\frac{3917}{32} = 122\frac{13}{32}$$

Note: You can use the following procedure on any calculator. Divide 3917 by 32 to get 122.40625. Subtract 122. Multiply by 32 to get 13. The mixed number is $122\frac{13}{32}$.

68. The commands used will vary. The following is from a TI-83 Plus:

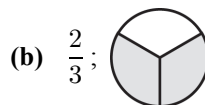
```
int(5632/64) 88
fPart(5632/64) F
rac 0
```

$$\frac{5632}{64} = 88$$

69. The following fractions are proper fractions.

$$\frac{2}{3}, \frac{4}{5}, \frac{3}{4}, \frac{7}{10}$$

70. (a) The proper fractions in Exercise 69 are the ones where the numerator is less than the denominator.

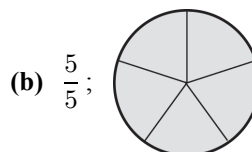


- (c) The proper fractions in Exercise 69 are all less than 1.

71. The following fractions are improper fractions.

$$\frac{5}{5}, \frac{10}{3}, \frac{6}{5}$$

72. (a) The improper fractions in Exercise 71 are the ones where the numerator is equal to or greater than the denominator.



- (c) The improper fractions in Exercise 71 are all equal to or greater than 1.

73. The following fractions can be written as whole or mixed numbers.

$$\frac{5}{3}$$

$$\begin{array}{r} 1 \leftarrow \text{Whole number part} \\ 3 \overline{)5} \\ \underline{3} \\ 2 \leftarrow \text{Remainder} \end{array}$$

$$\frac{5}{3} = 1\frac{2}{3}$$

$$\frac{7}{7}$$

$$\begin{array}{r} 1 \leftarrow \text{Whole number part} \\ 7 \overline{)7} \\ \underline{7} \\ 0 \leftarrow \text{Remainder} \end{array}$$

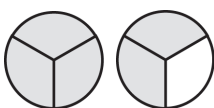
$$\frac{7}{7} = 1$$

$$\frac{11}{6}$$

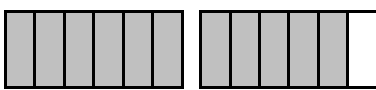
$$\begin{array}{r} 1 \leftarrow \text{Whole number part} \\ 6 \overline{)11} \\ \underline{6} \\ 5 \leftarrow \text{Remainder} \end{array}$$

$$\frac{11}{6} = 1\frac{5}{6}$$

74. (a) The fractions that can be written as whole or mixed numbers in Exercise 73 are improper fractions, and their value is always greater than or equal to 1.

(b) $\frac{5}{3} = 1\frac{2}{3}$; 

$\frac{7}{7} = 1$; 

$\frac{11}{6} = 1\frac{5}{6}$; 

2.3 Factors

2.3 Margin Exercises

- (a) Factorizations of 18:
 $1 \cdot 18 = 18$ $2 \cdot 9 = 18$ $3 \cdot 6 = 18$
 The factors of 18 are 1, 2, 3, 6, 9, and 18.
 (b) Factorizations of 16:
 $1 \cdot 16 = 16$ $2 \cdot 8 = 16$ $4 \cdot 4 = 16$
 The factors of 16 are 1, 2, 4, 8, and 16.
 (c) Factorizations of 80:
 $1 \cdot 80 = 80$ $2 \cdot 40 = 80$ $4 \cdot 20 = 80$
 $5 \cdot 16 = 80$ $8 \cdot 10 = 80$
 The factors of 80 are 1, 2, 4, 5, 8, 10, 16, 20, 40, and 80.
- 4, 7, 9, 13, 17, 19, 29, 33
 7, 13, 17, 19, and 29 are prime because they are divisible only by themselves and 1.
- 2, 4, 5, 6, 8, 10, 11, 13, 19, 21, 27, 28, 33, 36, 42
 2, 5, 11, 13, and 19 each have no factor other than themselves or 1.
 4, 6, 8, 10, 28, 36, and 42 each have a factor of 2.
 21, 27, and 33 have a factor of 3.
 So 4, 6, 8, 10, 21, 27, 28, 33, 36, and 42 are composite.
- (a) $8 \div 2 = 4$
 $4 \div 2 = 2$ *prime*
 $8 = 2 \cdot 2 \cdot 2$
 (b) $28 \div 2 = 14$
 $14 \div 2 = 7$ *prime*
 $28 = 2 \cdot 2 \cdot 7$
 (c) $18 \div 2 = 9$
 $9 \div 3 = 3$ *prime*
 $18 = 2 \cdot 3 \cdot 3$
 (d) $40 \div 2 = 20$
 $20 \div 2 = 10$
 $10 \div 2 = 5$ *prime*
 $40 = 2 \cdot 2 \cdot 2 \cdot 5$
- (a) This division is done from the "bottom-up."

$$\begin{array}{r} 1 \text{ Quotient is } 1. \\ 3 \overline{)3} \text{ Divide 3 by 3.} \\ 3 \overline{)9} \text{ Divide 9 by 3.} \\ 2 \overline{)18} \text{ Divide 18 by 2.} \\ 2 \overline{)36} \text{ Divide 36 by 2.} \end{array}$$

(continued)

This division is done from the "top-down."

$$\begin{array}{r} 18 \\ 2 \overline{)36} \quad \text{Divide 36 by 2.} \\ \underline{36} \\ 0 \end{array}$$

$$\begin{array}{r} 9 \\ 2 \overline{)18} \quad \text{Divide 18 by 2.} \\ \underline{18} \\ 0 \end{array}$$

$$\begin{array}{r} 3 \\ 3 \overline{)9} \quad \text{Divide 9 by 3.} \\ \underline{9} \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ 3 \overline{)3} \quad \text{Divide 3 by 3.} \\ \underline{3} \\ 0 \end{array}$$

Quotient is 1.

Either method is correct and yields the prime factorization as follows:

$$36 = 2 \cdot 2 \cdot 3 \cdot \underline{3} = 2^2 \cdot \underline{3^2}$$

(b) $\begin{array}{r} 27 \\ 2 \overline{)54} \quad \text{Divide 54 by 2.} \\ \underline{54} \\ 0 \end{array}$

$$\begin{array}{r} 9 \\ 3 \overline{)27} \quad \text{Divide 27 by 3.} \\ \underline{27} \\ 0 \end{array}$$

$$\begin{array}{r} 3 \\ 3 \overline{)9} \quad \text{Divide 9 by 3.} \\ \underline{9} \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ 3 \overline{)3} \quad \text{Divide 3 by 3.} \\ \underline{3} \\ 0 \end{array}$$

Quotient is 1.

$$54 = 2 \cdot 3 \cdot 3 \cdot 3 = 2 \cdot 3^3$$

(c) $\begin{array}{r} 30 \\ 2 \overline{)60} \quad \text{Divide 60 by 2.} \\ \underline{60} \\ 0 \end{array}$

$$\begin{array}{r} 15 \\ 2 \overline{)30} \quad \text{Divide 30 by 2.} \\ \underline{30} \\ 0 \end{array}$$

$$\begin{array}{r} 5 \\ 3 \overline{)15} \quad \text{Divide 15 by 3.} \\ \underline{15} \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ 5 \overline{)5} \quad \text{Divide 5 by 5.} \\ \underline{5} \\ 0 \end{array}$$

Quotient is 1.

$$60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$$

(d) $\begin{array}{r} 27 \\ 3 \overline{)81} \quad \text{Divide 81 by 3.} \\ \underline{81} \\ 0 \end{array}$

$$\begin{array}{r} 9 \\ 3 \overline{)27} \quad \text{Divide 27 by 3.} \\ \underline{27} \\ 0 \end{array}$$

$$\begin{array}{r} 3 \\ 3 \overline{)9} \quad \text{Divide 9 by 3.} \\ \underline{9} \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ 3 \overline{)3} \quad \text{Divide 3 by 3.} \\ \underline{3} \\ 0 \end{array}$$

Quotient is 1.

$$81 = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4$$

6. (a) $\begin{array}{r} 24 \\ 2 \overline{)48} \quad \text{Divide 48 by 2.} \\ \underline{48} \\ 0 \end{array}$

$$\begin{array}{r} 12 \\ 2 \overline{)24} \quad \text{Divide 24 by 2.} \\ \underline{24} \\ 0 \end{array}$$

$$\begin{array}{r} 6 \\ 2 \overline{)12} \quad \text{Divide 12 by 2.} \\ \underline{12} \\ 0 \end{array}$$

$$\begin{array}{r} 3 \\ 2 \overline{)6} \quad \text{Divide 6 by 2.} \\ \underline{6} \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ 3 \overline{)3} \quad \text{Divide 3 by 3.} \\ \underline{3} \\ 0 \end{array}$$

Quotient is 1.

$$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 2^4 \cdot 3$$

(b) $\begin{array}{r} 22 \\ 2 \overline{)44} \quad \text{Divide 44 by 2.} \\ \underline{44} \\ 0 \end{array}$

$$\begin{array}{r} 11 \\ 2 \overline{)22} \quad \text{Divide 22 by 2.} \\ \underline{22} \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ 11 \overline{)11} \quad \text{Divide 11 by 11.} \\ \underline{11} \\ 0 \end{array}$$

Quotient is 1.

$$44 = 2 \cdot 2 \cdot 11 = 2^2 \cdot 11$$

(c) $\begin{array}{r} 45 \\ 2 \overline{)90} \quad \text{Divide 90 by 2.} \\ \underline{90} \\ 0 \end{array}$

$$\begin{array}{r} 15 \\ 3 \overline{)45} \quad \text{Divide 45 by 3.} \\ \underline{45} \\ 0 \end{array}$$

$$\begin{array}{r} 5 \\ 3 \overline{)15} \quad \text{Divide 15 by 3.} \\ \underline{15} \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ 5 \overline{)5} \quad \text{Divide 5 by 5.} \\ \underline{5} \\ 0 \end{array}$$

Quotient is 1.

$$90 = 2 \cdot 3 \cdot 3 \cdot 5 = 2 \cdot 3^2 \cdot 5$$

(d) $\begin{array}{r} 60 \\ 2 \overline{)120} \quad \text{Divide 120 by 2.} \\ \underline{120} \\ 0 \end{array}$

$$\begin{array}{r} 30 \\ 2 \overline{)60} \quad \text{Divide 60 by 2.} \\ \underline{60} \\ 0 \end{array}$$

$$\begin{array}{r} 15 \\ 2 \overline{)30} \quad \text{Divide 30 by 2.} \\ \underline{30} \\ 0 \end{array}$$

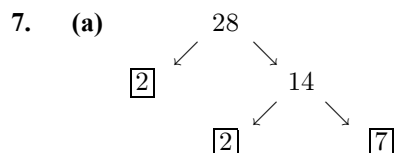
$$\begin{array}{r} 5 \\ 3 \overline{)15} \quad \text{Divide 15 by 3.} \\ \underline{15} \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ 5 \overline{)5} \quad \text{Divide 5 by 5.} \\ \underline{5} \\ 0 \end{array}$$

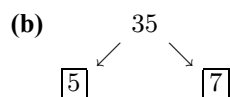
Quotient is 1.

$$120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 = 2^3 \cdot 3 \cdot 5$$

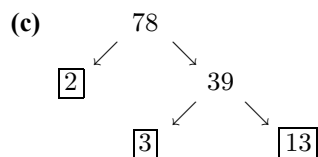
(e)
$$\begin{array}{r} 90 \\ 2 \overline{)180} \\ \underline{45} \\ 2 \overline{)90} \\ \underline{15} \\ 3 \overline{)45} \\ \underline{5} \\ 3 \overline{)15} \\ \underline{1} \\ 5 \overline{)5} \end{array}$$
 Divide 180 by 2.
Divide 90 by 2.
Divide 45 by 3.
Divide 15 by 3.
Divide 5 by 5.
Quotient is 1.
 $180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 2^2 \cdot 3^2 \cdot 5$



$$28 = 2 \cdot 2 \cdot 7 = 2^2 \cdot 7$$



$$35 = 5 \cdot 7$$



$$78 = 2 \cdot 3 \cdot 13$$

2.3 Section Exercises

1. Factorizations of 8:

$$1 \cdot 8 = 8 \quad 2 \cdot 4 = 8$$

The factors of 8 are 1, 2, 4, and 8. The statement is *false* (missing 1 and includes 6).

2. Factorizations of 12:

$$1 \cdot 12 = 12 \quad 2 \cdot 6 = 12 \quad 3 \cdot 4 = 12$$

The factors of 12 are 1, 2, 3, 4, 6, and 12. The statement is *true*.

3. Factorizations of 15:

$$1 \cdot 15 = 15 \quad 3 \cdot 5 = 15$$

The factors of 15 are 1, 3, 5, and 15. The statement is *true*.

4. Factorizations of 28:

$$1 \cdot 28 = 28 \quad 2 \cdot 14 = 28 \quad 4 \cdot 7 = 28$$

The factors of 28 are 1, 2, 4, 7, 14, and 28. The statement is *false* (missing 1 and 28).

5. Factorizations of 48:

$$1 \cdot 48 = 48 \quad 2 \cdot 24 = 48 \quad 3 \cdot 16 = 48 \\ 4 \cdot 12 = 48 \quad 6 \cdot 8 = 48$$

The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.

6. Factorizations of 30:

$$1 \cdot 30 = 30 \quad 2 \cdot 15 = 30 \quad 3 \cdot 10 = 30 \quad 5 \cdot 6 = 30$$

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

7. Factorizations of 56:

$$1 \cdot 56 = 56 \quad 2 \cdot 28 = 56 \quad 4 \cdot 14 = 56 \quad 7 \cdot 8 = 56$$

The factors of 56 are 1, 2, 4, 7, 8, 14, 28, and 56.

8. Factorizations of 72:

$$1 \cdot 72 = 72 \quad 2 \cdot 36 = 72 \quad 3 \cdot 24 = 72 \\ 4 \cdot 18 = 72 \quad 6 \cdot 12 = 72 \quad 8 \cdot 9 = 72$$

The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.

9. Factorizations of 36:

$$1 \cdot 36 = 36 \quad 2 \cdot 18 = 36 \quad 3 \cdot 12 = 36 \quad 4 \cdot 9 = 36 \\ 6 \cdot 6 = 36$$

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

10. Factorizations of 20:

$$1 \cdot 20 = 20 \quad 2 \cdot 10 = 20 \quad 4 \cdot 5 = 20$$

The factors of 20 are 1, 2, 4, 5, 10, and 20.

11. Factorizations of 40:

$$1 \cdot 40 = 40 \quad 2 \cdot 20 = 40 \quad 4 \cdot 10 = 40 \quad 5 \cdot 8 = 40$$

The factors of 40 are 1, 2, 4, 5, 8, 10, 20, and 40.

12. Factorizations of 60:

$$1 \cdot 60 = 60 \quad 2 \cdot 30 = 60 \quad 3 \cdot 20 = 60 \\ 4 \cdot 15 = 60 \quad 5 \cdot 12 = 60 \quad 6 \cdot 10 = 60$$

The factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.

13. Factorizations of 64:

$$1 \cdot 64 = 64 \quad 2 \cdot 32 = 64 \quad 4 \cdot 16 = 64 \quad 8 \cdot 8 = 64$$

The factors of 64 are 1, 2, 4, 8, 16, 32, and 64.

14. Factorizations of 84:

$$1 \cdot 84 = 84 \quad 2 \cdot 42 = 84 \quad 3 \cdot 28 = 84 \\ 4 \cdot 21 = 84 \quad 6 \cdot 14 = 84 \quad 7 \cdot 12 = 84$$

The factors of 84 are 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, and 84.

15. Factorizations of 82:

$$1 \cdot 82 = 82 \quad 2 \cdot 41 = 82$$

The factors of 82 are 1, 2, 41, and 82.

16. Factorizations of 39:

$$1 \cdot 39 = 39 \quad 3 \cdot 13 = 39$$

The factors of 39 are 1, 3, 13, and 39.

17. 6 is divisible by 2 and 3, so 6 is composite.
 18. 9 is divisible by 3, so 9 is composite.
 19. 5 is only divisible by itself and 1, so it is prime.
 20. 16 is divisible by 2, so 16 is composite.
 21. 10 is divisible by 2 and 5, so 10 is composite.
 22. 13 is only divisible by itself and 1, so it is prime.
 23. 19 is only divisible by itself and 1, so it is prime.
 24. 17 is only divisible by itself and 1, so it is prime.
 25. 25 is divisible by 5, so 25 is composite.
 26. 48 is divisible by 2 and 3, so 48 is composite.
 27. 47 is only divisible by itself and 1, so it is prime.
 28. 45 is divisible by 3 and 5, so 45 is composite.
 29. 40

$$\begin{array}{r} 20 \\ 2 \overline{)40} \end{array} \quad \text{Divide 40 by 2.}$$

$$\begin{array}{r} 10 \\ 2 \overline{)20} \end{array} \quad \text{Divide 20 by 2.}$$

$$\begin{array}{r} 5 \\ 2 \overline{)10} \end{array} \quad \text{Divide 10 by 2.}$$

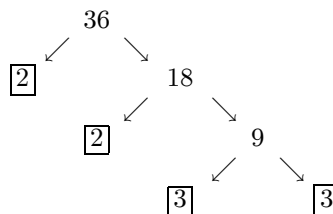
$$\begin{array}{r} 1 \\ 5 \overline{)5} \end{array} \quad \text{Divide 5 by 5.}$$

Quotient is 1.

$$40 = 2 \cdot 2 \cdot 2 \cdot 5 = 2^3 \cdot 5$$

The correct choice is **(b)**.

- 30.



$$36 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2$$

The correct choice is **(c)**.

31. 100

$$\begin{array}{r} 50 \\ 2 \overline{)100} \end{array} \quad \text{Divide 100 by 2.}$$

$$\begin{array}{r} 25 \\ 2 \overline{)50} \end{array} \quad \text{Divide 50 by 2.}$$

$$\begin{array}{r} 5 \\ 5 \overline{)25} \end{array} \quad \text{Divide 25 by 5.}$$

$$\begin{array}{r} 1 \\ 5 \overline{)5} \end{array} \quad \text{Divide 5 by 5.}$$

Quotient is 1.

$$100 = 2 \cdot 2 \cdot 5 \cdot 5 = 2^2 \cdot 5^2$$

The correct choice is **(a)**.

32. 8

$$\begin{array}{r} 4 \\ 2 \overline{)8} \end{array} \quad \text{Divide 8 by 2.}$$

$$\begin{array}{r} 2 \\ 2 \overline{)4} \end{array} \quad \text{Divide 4 by 2.}$$

$$\begin{array}{r} 1 \\ 2 \overline{)2} \end{array} \quad \text{Divide 2 by 2.}$$

Quotient is 1.

$$8 = 2 \cdot 2 \cdot 2 = 2^3$$

33. 6

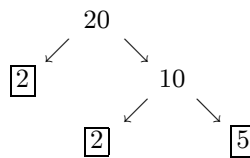
$$\begin{array}{r} 3 \\ 2 \overline{)6} \end{array} \quad \text{Divide 6 by 2.}$$

$$\begin{array}{r} 1 \\ 3 \overline{)3} \end{array} \quad \text{Divide 3 by 3.}$$

Quotient is 1.

$$6 = 2 \cdot 3$$

34. We can also use a factor tree.



$$20 = 2 \cdot 2 \cdot 5 = 2^2 \cdot 5$$

35.
$$\begin{array}{c} 25 \\ \swarrow \quad \searrow \\ \boxed{5} \quad \boxed{5} \end{array}$$

 $25 = 5 \cdot \underline{5} = 5^2$

36. 56

$$\begin{array}{r} 28 \\ 2 \overline{)56} \end{array}$$
 Divide 56 by 2.

$$\begin{array}{r} 14 \\ 2 \overline{)28} \end{array}$$
 Divide 28 by 2.

$$\begin{array}{r} 7 \\ 2 \overline{)14} \end{array}$$
 Divide 14 by 2.

$$\begin{array}{r} 1 \\ 7 \overline{)7} \end{array}$$
 Divide 7 by 7.
Quotient is 1.
 $56 = \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot 7 = 2^3 \cdot 7$

37.
$$\begin{array}{c} 68 \\ \swarrow \quad \searrow \\ \boxed{2} \quad 34 \\ \quad \swarrow \quad \searrow \\ \quad \boxed{2} \quad \boxed{17} \end{array}$$

 $68 = 2 \cdot 2 \cdot 17 = 2^2 \cdot 17$

38. 70

$$\begin{array}{r} 35 \\ 2 \overline{)70} \end{array}$$
 Divide 70 by 2.

$$\begin{array}{r} 7 \\ 5 \overline{)35} \end{array}$$
 Divide 35 by 5.

$$\begin{array}{r} 1 \\ 7 \overline{)7} \end{array}$$
 Divide 7 by 7.
Quotient is 1.
 $70 = 2 \cdot 5 \cdot 7$

39. 72

$$\begin{array}{r} 36 \\ 2 \overline{)72} \end{array}$$
 Divide 72 by 2.

$$\begin{array}{r} 18 \\ 2 \overline{)36} \end{array}$$
 Divide 36 by 2.

$$\begin{array}{r} 9 \\ 2 \overline{)18} \end{array}$$
 Divide 18 by 2.

$$\begin{array}{r} 3 \\ 3 \overline{)9} \end{array}$$
 Divide 9 by 3.

$$\begin{array}{r} 1 \\ 3 \overline{)3} \end{array}$$
 Divide 3 by 3.
Quotient is 1.
 $72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$

40.
$$\begin{array}{c} 64 \\ \swarrow \quad \searrow \\ \boxed{2} \quad 32 \\ \quad \swarrow \quad \searrow \\ \quad \boxed{2} \quad 16 \\ \quad \quad \swarrow \quad \searrow \\ \quad \quad \boxed{2} \quad 8 \\ \quad \quad \quad \swarrow \quad \searrow \\ \quad \quad \quad \boxed{2} \quad 4 \\ \quad \quad \quad \quad \swarrow \quad \searrow \\ \quad \quad \quad \quad \boxed{2} \quad \boxed{2} \end{array}$$

 $64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6$

41. 44

$$\begin{array}{r} 1 \\ 11 \overline{)11} \end{array}$$
 Divide 11 by 11.

$$\begin{array}{r} 11 \\ 2 \overline{)22} \end{array}$$
 Divide 22 by 2.

$$\begin{array}{r} 22 \\ 2 \overline{)44} \end{array}$$
 Divide 44 by 2.
Quotient is 1.

Because all factors (divisors) are prime, the prime factorization of 44 is $2 \cdot 2 \cdot 11 = 2^2 \cdot 11$.

42.
$$\begin{array}{c} 104 \\ \swarrow \quad \searrow \\ \boxed{2} \quad 52 \\ \quad \swarrow \quad \searrow \\ \quad \boxed{2} \quad 26 \\ \quad \quad \swarrow \quad \searrow \\ \quad \quad \boxed{2} \quad \boxed{13} \end{array}$$

 $104 = 2 \cdot 2 \cdot 2 \cdot 13 = 2^3 \cdot 13$

43. 100

$$\begin{array}{r} 50 \\ 2 \overline{)100} \end{array}$$
 Divide 100 by 2.

$$\begin{array}{r} 25 \\ 2 \overline{)50} \end{array}$$
 Divide 50 by 2.

$$\begin{array}{r} 5 \\ 5 \overline{)25} \end{array}$$
 Divide 25 by 5.

$$\begin{array}{r} 1 \\ 5 \overline{)5} \end{array}$$
 Divide 5 by 5.
Quotient is 1.
 $100 = 2 \cdot 2 \cdot 5 \cdot 5 = 2^2 \cdot 5^2$

44.
$$\begin{array}{c} 112 \\ \swarrow \quad \searrow \\ \boxed{2} \quad 56 \\ \quad \swarrow \quad \searrow \\ \quad \boxed{2} \quad 28 \\ \quad \quad \swarrow \quad \searrow \\ \quad \quad \boxed{2} \quad 14 \\ \quad \quad \quad \swarrow \quad \searrow \\ \quad \quad \quad \boxed{2} \quad \boxed{7} \end{array}$$

 $112 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7 = 2^4 \cdot 7$

45. 125

$$\begin{array}{r} 25 \\ 5 \overline{)125} \end{array} \quad \text{Divide 125 by 5.}$$

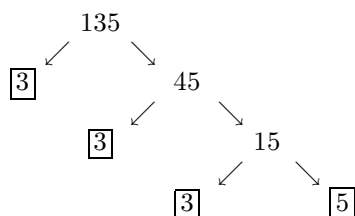
$$\begin{array}{r} 5 \\ 5 \overline{)25} \end{array} \quad \text{Divide 25 by 5.}$$

$$\begin{array}{r} 1 \\ 5 \overline{)5} \end{array} \quad \text{Divide 5 by 5.}$$

Quotient is 1.

$$125 = 5 \cdot 5 \cdot 5 = 5^3$$

46.



$$135 = 3 \cdot 3 \cdot 3 \cdot 5 = 3^3 \cdot 5$$

47. 180

$$\begin{array}{r} 90 \\ 2 \overline{)180} \end{array} \quad \text{Divide 180 by 2.}$$

$$\begin{array}{r} 45 \\ 2 \overline{)90} \end{array} \quad \text{Divide 90 by 2.}$$

$$\begin{array}{r} 15 \\ 3 \overline{)45} \end{array} \quad \text{Divide 45 by 3.}$$

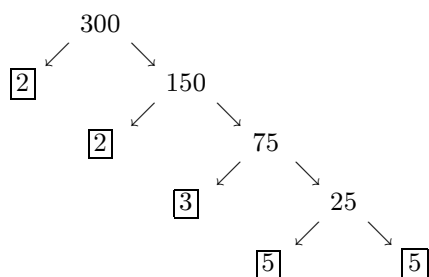
$$\begin{array}{r} 5 \\ 3 \overline{)15} \end{array} \quad \text{Divide 15 by 3.}$$

$$\begin{array}{r} 1 \\ 5 \overline{)5} \end{array} \quad \text{Divide 5 by 5.}$$

Quotient is 1.

$$180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 2^2 \cdot 3^2 \cdot 5$$

48.



$$300 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 = 2^2 \cdot 3 \cdot 5^2$$

49. 320

$$\begin{array}{r} 160 \\ 2 \overline{)320} \end{array} \quad \text{Divide 320 by 2.}$$

$$\begin{array}{r} 80 \\ 2 \overline{)160} \end{array} \quad \text{Divide 160 by 2.}$$

$$\begin{array}{r} 40 \\ 2 \overline{)80} \end{array} \quad \text{Divide 80 by 2.}$$

$$\begin{array}{r} 20 \\ 2 \overline{)40} \end{array} \quad \text{Divide 40 by 2.}$$

$$\begin{array}{r} 10 \\ 2 \overline{)20} \end{array} \quad \text{Divide 20 by 2.}$$

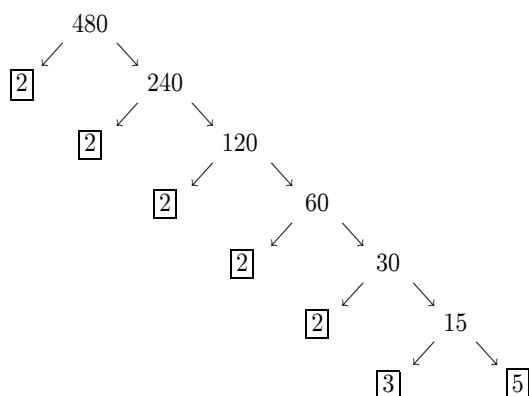
$$\begin{array}{r} 5 \\ 2 \overline{)10} \end{array} \quad \text{Divide 10 by 2.}$$

$$\begin{array}{r} 1 \\ 5 \overline{)5} \end{array} \quad \text{Divide 5 by 5.}$$

Quotient is 1.

$$320 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 = 2^6 \cdot 5$$

50.



$$480 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 = 2^5 \cdot 3 \cdot 5$$

51. 360

$$\begin{array}{r} 180 \\ 2 \overline{)360} \end{array} \quad \text{Divide 360 by 2.}$$

$$\begin{array}{r} 90 \\ 2 \overline{)180} \end{array} \quad \text{Divide 180 by 2.}$$

$$\begin{array}{r} 45 \\ 2 \overline{)90} \end{array} \quad \text{Divide 90 by 2.}$$

$$\begin{array}{r} 15 \\ 3 \overline{)45} \end{array} \quad \text{Divide 45 by 3.}$$

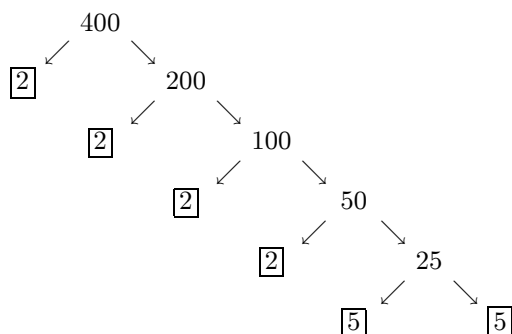
$$\begin{array}{r} 5 \\ 3 \overline{)15} \end{array} \quad \text{Divide 15 by 3.}$$

$$\begin{array}{r} 1 \\ 5 \overline{)5} \end{array} \quad \text{Divide 5 by 5.}$$

Quotient is 1.

$$360 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 2^3 \cdot 3^2 \cdot 5$$

52.



$$400 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 = 2^4 \cdot 5^2$$

53. Answers will vary. A sample answer follows. A prime number is a whole number that has exactly two *different* factors, itself and 1. Examples include 2, 3, 5, 7, and 11. A composite number has a factor(s) other than itself or 1. Examples include 4, 6, 8, 9, and 10. The numbers 0 and 1 are neither prime nor composite.

54. No even number other than 2 is prime because all even numbers have 2 as a factor. Many odd numbers are multiples of prime numbers and are not prime. For example, 9, 21, 33, and 45 are all multiples of 3.

55. All the possible factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. This list includes both prime numbers and composite numbers. The prime factors of 24 include only prime numbers. The prime factorization of 24 is

$$24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3.$$

56. Yes, you can divide by 3s before you divide by 2. No, the order of division does not matter. As long as you use only prime numbers, your answers will be correct. However, it does seem easier to always start with 2 and then use progressively greater prime numbers. The prime factorization of 36 is

$$36 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2.$$

57. 350

$$\begin{array}{r} 175 \\ 2 \overline{)350} \end{array} \quad \text{Divide 350 by 2.}$$

$$\begin{array}{r} 35 \\ 5 \overline{)175} \end{array} \quad \text{Divide 175 by 5.}$$

$$\begin{array}{r} 7 \\ 5 \overline{)35} \end{array} \quad \text{Divide 35 by 5.}$$

$$\begin{array}{r} 1 \\ 7 \overline{)7} \end{array} \quad \text{Divide 7 by 7.}$$

Quotient is 1.

$$350 = 2 \cdot 5 \cdot 5 \cdot 7 = 2 \cdot 5^2 \cdot 7$$

58. 640

$$\begin{array}{r} 320 \\ 2 \overline{)640} \end{array} \quad \text{Divide 640 by 2.}$$

$$\begin{array}{r} 160 \\ 2 \overline{)320} \end{array} \quad \text{Divide 320 by 2.}$$

$$\begin{array}{r} 80 \\ 2 \overline{)160} \end{array} \quad \text{Divide 160 by 2.}$$

$$\begin{array}{r} 40 \\ 2 \overline{)80} \end{array} \quad \text{Divide 80 by 2.}$$

$$\begin{array}{r} 20 \\ 2 \overline{)40} \end{array} \quad \text{Divide 40 by 2.}$$

$$\begin{array}{r} 10 \\ 2 \overline{)20} \end{array} \quad \text{Divide 20 by 2.}$$

$$\begin{array}{r} 5 \\ 2 \overline{)10} \end{array} \quad \text{Divide 10 by 2.}$$

$$\begin{array}{r} 1 \\ 5 \overline{)5} \end{array} \quad \text{Divide 5 by 5. Quotient is 1.}$$

$$640 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 = 2^7 \cdot 5$$

59. 960

$$\begin{array}{r} 480 \\ 2 \overline{)960} \end{array} \quad \text{Divide 960 by 2.}$$

$$\begin{array}{r} 240 \\ 2 \overline{)480} \end{array} \quad \text{Divide 480 by 2.}$$

$$\begin{array}{r} 120 \\ 2 \overline{)240} \end{array} \quad \text{Divide 240 by 2.}$$

$$\begin{array}{r} 60 \\ 2 \overline{)120} \end{array} \quad \text{Divide 120 by 2.}$$

$$\begin{array}{r} 30 \\ 2 \overline{)60} \end{array} \quad \text{Divide 60 by 2.}$$

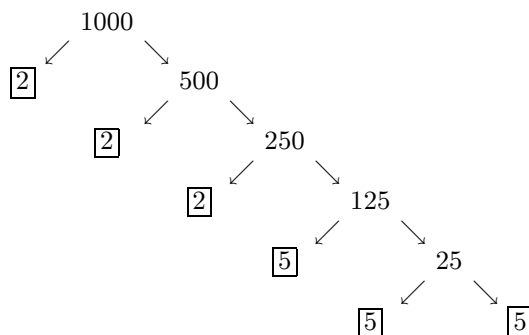
$$\begin{array}{r} 15 \\ 2 \overline{)30} \end{array} \quad \text{Divide 30 by 2.}$$

$$\begin{array}{r} 5 \\ 3 \overline{)15} \end{array} \quad \text{Divide 15 by 3.}$$

$$\begin{array}{r} 1 \\ 5 \overline{)5} \end{array} \quad \text{Divide 5 by 5. Quotient is 1.}$$

$$960 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 = 2^6 \cdot 3 \cdot 5$$

60.



$$1000 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 = 2^3 \cdot 5^3$$

61. 1560

$$\begin{array}{r} 780 \\ 2 \overline{)1560} \end{array}$$

Divide 1560 by 2.

$$\begin{array}{r} 390 \\ 2 \overline{)780} \end{array}$$

Divide 780 by 2.

$$\begin{array}{r} 195 \\ 2 \overline{)390} \end{array}$$

Divide 390 by 2.

$$\begin{array}{r} 65 \\ 3 \overline{)195} \end{array}$$

Divide 195 by 3.

$$\begin{array}{r} 13 \\ 5 \overline{)65} \end{array}$$

Divide 65 by 5.

$$\begin{array}{r} 1 \\ 13 \overline{)13} \end{array}$$

Divide 13 by 13.

Quotient is 1.

$$1560 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 13 = 2^3 \cdot 3 \cdot 5 \cdot 13$$

62. 2000

$$\begin{array}{r} 1000 \\ 2 \overline{)2000} \end{array}$$

Divide 2000 by 2.

$$\begin{array}{r} 500 \\ 2 \overline{)1000} \end{array}$$

Divide 1000 by 2.

$$\begin{array}{r} 250 \\ 2 \overline{)500} \end{array}$$

Divide 500 by 2.

$$\begin{array}{r} 125 \\ 2 \overline{)250} \end{array}$$

Divide 250 by 2.

$$\begin{array}{r} 25 \\ 5 \overline{)125} \end{array}$$

Divide 125 by 5.

$$\begin{array}{r} 5 \\ 5 \overline{)25} \end{array}$$

Divide 25 by 5.

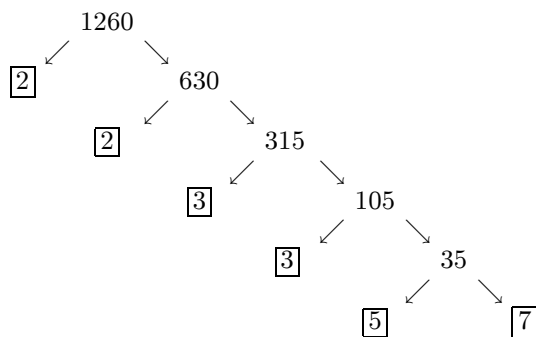
$$\begin{array}{r} 1 \\ 5 \overline{)5} \end{array}$$

Divide 5 by 5.

Quotient is 1.

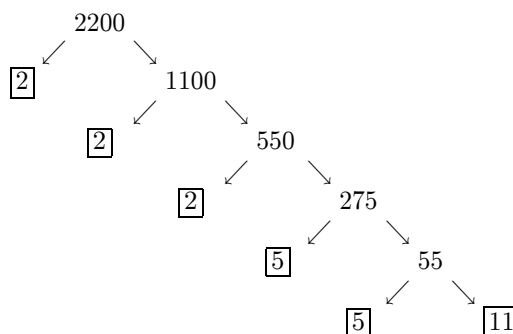
$$2000 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 = 2^4 \cdot 5^3$$

63.



$$1260 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7 = 2^2 \cdot 3^2 \cdot 5 \cdot 7$$

64.



$$2200 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 11 = 2^3 \cdot 5^2 \cdot 11$$

65. The prime numbers less than 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.

66. A prime number is a whole number that is evenly divisible by itself and 1 only.

67. No. Every other even number is divisible by 2 in addition to being divisible by itself and 1.

68. No. A multiple of a prime number can never be prime because it will always be divisible by the prime number.

69. 2100

$$\begin{array}{r} 1050 \\ 2 \overline{)2100} \end{array}$$

Divide 2100 by 2.

$$\begin{array}{r} 525 \\ 2 \overline{)1050} \end{array}$$

Divide 1050 by 2.

$$\begin{array}{r} 175 \\ 3 \overline{)525} \end{array}$$

Divide 525 by 3.

$$\begin{array}{r} 35 \\ 5 \overline{)175} \end{array}$$

Divide 175 by 5.

$$\begin{array}{r} 7 \\ 5 \overline{)35} \end{array}$$

Divide 35 by 5.

$$\begin{array}{r} 1 \\ 7 \overline{)7} \end{array}$$

Divide 7 by 7.

Quotient is 1.

$$2100 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 7$$

$$70. \quad 2100 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 7 = 2^2 \cdot 3 \cdot 5^2 \cdot 7$$

2.4 Writing a Fraction in Lowest Terms

2.4 Margin Exercises

1. (a) 6, 12; 2
 $6 = 2 \cdot 3$ $12 = 2 \cdot 6$
 Yes, 2 is a common factor of 6 and 12.

(b) 32, 64; 8
 $32 = 8 \cdot 4$ $64 = 8 \cdot 8$
 Yes, 8 is a common factor of 32 and 64.

(c) 32, 56; 16

 $32 = 16 \cdot 2$, but 16 is not a factor of 56.

No.

(d) 75, 81; 1

 $75 = 75 \cdot 1$ $81 = 81 \cdot 1$

Yes, 1 is a common factor of 75 and 81.

2. (a) $\frac{4}{5}$

4 and 5 have no common factor other than 1.

Yes, it is in lowest terms.

(b) $\frac{6}{18}$

6 and 18 have a common factor of 6.

No, it is not in lowest terms.

(c) $\frac{9}{15}$

9 and 15 have a common factor of 3.

No, it is not in lowest terms.

(d) $\frac{17}{46}$

17 and 46 have no common factor other than 1.

Yes, it is in lowest terms.

3. (a) $\frac{8}{16} = \frac{8 \div 8}{16 \div 8} = \frac{1}{2}$

(b) $\frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$

(c) $\frac{28}{42} = \frac{28 \div 14}{42 \div 14} = \frac{2}{3}$

(d) $\frac{30}{80} = \frac{30 \div 10}{80 \div 10} = \frac{3}{8}$

(e) $\frac{16}{40} = \frac{16 \div 8}{40 \div 8} = \frac{2}{5}$

4. (a) $\frac{12}{36} = \frac{2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 3}$
 $= \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}}}{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot 3} = \frac{1 \cdot 1 \cdot 1}{1 \cdot 1 \cdot 1 \cdot 3} = \frac{1}{3}$

(b) $\frac{32}{56} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 7}$
 $= \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2}{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 7} = \frac{1 \cdot 1 \cdot 1 \cdot 2 \cdot 2}{1 \cdot 1 \cdot 1 \cdot 7} = \frac{4}{7}$

(c) $\frac{74}{111} = \frac{2 \cdot \overset{1}{\cancel{37}}}{3 \cdot \overset{1}{\cancel{37}}} = \frac{2 \cdot 1}{3 \cdot 1} = \frac{2}{3}$

(d) $\frac{124}{340} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 31}{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 5 \cdot 17} = \frac{1 \cdot 1 \cdot 31}{1 \cdot 1 \cdot 5 \cdot 17} = \frac{31}{85}$

5. (a) $\frac{24}{48}$ and $\frac{36}{72}$

$\frac{24}{48} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}}}{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2} = \frac{1 \cdot 1 \cdot 1 \cdot 1}{1 \cdot 1 \cdot 1 \cdot 2 \cdot 1} = \frac{1}{2}$

$\frac{36}{72} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}}}{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2} = \frac{1 \cdot 1 \cdot 1 \cdot 1}{1 \cdot 1 \cdot 2 \cdot 1 \cdot 1} = \frac{1}{2}$

The fractions are *equivalent* ($\frac{1}{2} = \frac{1}{2}$).

(b) $\frac{45}{60}$ and $\frac{50}{75}$

$\frac{45}{60} = \frac{3 \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}}}{2 \cdot 2 \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}}} = \frac{3 \cdot 1 \cdot 1}{2 \cdot 2 \cdot 1 \cdot 1} = \frac{3}{4}$

$\frac{50}{75} = \frac{2 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}}}{3 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}}} = \frac{2 \cdot 1 \cdot 1}{3 \cdot 1 \cdot 1} = \frac{2}{3}$

The fractions are *not equivalent* ($\frac{3}{4} \neq \frac{2}{3}$).

(c) $\frac{20}{4}$ and $\frac{110}{22}$

$\frac{20}{4} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 5}{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}}} = \frac{1 \cdot 1 \cdot 5}{1 \cdot 1} = 5$

$\frac{110}{22} = \frac{\overset{1}{\cancel{2}} \cdot 5 \cdot \overset{1}{\cancel{11}}}{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{11}}} = \frac{1 \cdot 5 \cdot 1}{1 \cdot 1} = 5$

The fractions are *equivalent* ($5 = 5$).

(d) $\frac{120}{220}$ and $\frac{180}{320}$

$\frac{120}{220} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 3 \cdot \overset{1}{\cancel{5}}}{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{5}} \cdot 11} = \frac{1 \cdot 1 \cdot 2 \cdot 3 \cdot 1}{1 \cdot 1 \cdot 1 \cdot 11} = \frac{6}{11}$

$\frac{180}{320} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3 \cdot 3 \cdot \overset{1}{\cancel{5}}}{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{5}}}$
 $= \frac{1 \cdot 1 \cdot 3 \cdot 3 \cdot 1}{1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 1} = \frac{9}{16}$

The fractions are *not equivalent* ($\frac{6}{11} \neq \frac{9}{16}$).

2.4 Section Exercises

1. A number can be divided by 2 if the number is an
- even
- number.

2. A number can be divided by 5 if the number ends in 5 or 0.
3. Any number can be divided by 10 if the number ends in 0.
4. If the sum of a number's digits is divisible by 3, the number is divisible by 3.

	2	3	5	10
5. 60	✓	✓	✓	✓
6. 90	✓	✓	✓	✓
7. 48	✓	✓	X	X
8. 36	✓	✓	X	X
9. 160	✓	X	✓	✓
10. 175	X	X	✓	X
11. 138	✓	✓	X	X
12. 150	✓	✓	✓	✓

13. $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$, *true*

14. $\frac{7}{12}$ is in lowest terms, so the fractions $\frac{7}{12}$ and $\frac{1}{2}$ are not equivalent. *false*

15. $\frac{5}{16}$ is in lowest terms, so the fractions $\frac{5}{16}$ and $\frac{3}{8}$ are not equivalent. *false*

16. $\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$, *true*

17. $\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}$

18. $\frac{32}{48} = \frac{32 \div 16}{48 \div 16} = \frac{2}{3}$

19. $\frac{36}{42} = \frac{36 \div 6}{42 \div 6} = \frac{6}{7}$

20. $\frac{22}{33} = \frac{22 \div 11}{33 \div 11} = \frac{2}{3}$

21. Because the greatest common factor of 56 and 64 is 8, divide both numerator and denominator by 8.

$$\frac{56}{64} = \frac{56 \div 8}{64 \div 8} = \frac{7}{8}$$

22. $\frac{21}{35} = \frac{21 \div 7}{35 \div 7} = \frac{3}{5}$

23. $\frac{180}{210} = \frac{180 \div 30}{210 \div 30} = \frac{6}{7}$

24. $\frac{72}{80} = \frac{72 \div 8}{80 \div 8} = \frac{9}{10}$

25. $\frac{72}{126} = \frac{72 \div 18}{126 \div 18} = \frac{4}{7}$

26. $\frac{73}{146} = \frac{73 \div 73}{146 \div 73} = \frac{1}{2}$

27. $\frac{12}{600} = \frac{12 \div 12}{600 \div 12} = \frac{1}{50}$

28. $\frac{8}{400} = \frac{8 \div 8}{400 \div 8} = \frac{1}{50}$

29. $\frac{96}{132} = \frac{96 \div 12}{132 \div 12} = \frac{8}{11}$

30. $\frac{165}{180} = \frac{165 \div 15}{180 \div 15} = \frac{11}{12}$

31. $\frac{60}{108} = \frac{60 \div 12}{108 \div 12} = \frac{5}{9}$

32. $\frac{112}{128} = \frac{112 \div 16}{128 \div 16} = \frac{7}{8}$

33. Write the prime factorization of both numerator and denominator. Then divide both numerator and denominator by any common factors, and write a 1 by each factor that has been divided. Finally, multiply the remaining factors in both numerator and denominator.

$$\frac{18}{24} = \frac{\overset{1}{2} \cdot \overset{1}{3} \cdot 3}{\underset{1}{2} \cdot 2 \cdot 2 \cdot \underset{1}{3}} = \frac{1 \cdot 1 \cdot 3}{1 \cdot 2 \cdot 2 \cdot 1} = \frac{3}{4}$$

34. $\frac{16}{64} = \frac{\overset{1}{2} \cdot \overset{1}{2} \cdot \overset{1}{2} \cdot \overset{1}{2}}{\underset{1}{2} \cdot \underset{1}{2} \cdot \underset{1}{2} \cdot \underset{1}{2} \cdot 2 \cdot 2} = \frac{1 \cdot 1 \cdot 1 \cdot 1}{1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 \cdot 2} = \frac{1}{4}$

35. $\frac{35}{40} = \frac{\overset{1}{5} \cdot 7}{2 \cdot 2 \cdot 2 \cdot \underset{1}{5}} = \frac{1 \cdot 7}{2 \cdot 2 \cdot 2 \cdot 1} = \frac{7}{8}$

36. $\frac{20}{32} = \frac{\overset{1}{2} \cdot \overset{1}{2} \cdot 5}{\underset{1}{2} \cdot \underset{1}{2} \cdot 2 \cdot 2 \cdot 2} = \frac{1 \cdot 1 \cdot 5}{1 \cdot 1 \cdot 2 \cdot 2 \cdot 2} = \frac{5}{8}$

37. $\frac{90}{180} = \frac{\overset{1}{2} \cdot \overset{1}{3} \cdot \overset{1}{3} \cdot \overset{1}{2}}{\underset{1}{2} \cdot \underset{1}{2} \cdot \underset{1}{3} \cdot \underset{1}{3} \cdot \underset{1}{2} \cdot \underset{1}{2}} = \frac{1 \cdot 1 \cdot 1 \cdot 1}{1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 1} = \frac{1}{2}$

38. $\frac{36}{48} = \frac{\overset{1}{2} \cdot \overset{1}{2} \cdot \overset{1}{3} \cdot 3}{\underset{1}{2} \cdot \underset{1}{2} \cdot 2 \cdot 2 \cdot \underset{1}{3}} = \frac{1 \cdot 1 \cdot 1 \cdot 3}{1 \cdot 1 \cdot 2 \cdot 2 \cdot 1} = \frac{3}{4}$

39. $\frac{36}{12} = \frac{\overset{1}{2} \cdot \overset{1}{2} \cdot \overset{1}{3} \cdot 3}{\underset{1}{2} \cdot \underset{1}{2} \cdot \underset{1}{3}} = \frac{1 \cdot 1 \cdot 1 \cdot 3}{1 \cdot 1 \cdot 1} = 3$

40. $\frac{192}{48} = \frac{\overset{1}{2} \cdot \overset{1}{2} \cdot \overset{1}{2} \cdot \overset{1}{2} \cdot 2 \cdot 2 \cdot \overset{1}{3}}{\underset{1}{2} \cdot \underset{1}{2} \cdot \underset{1}{2} \cdot \underset{1}{2} \cdot \underset{1}{3}} = \frac{1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1}{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1} = 4$

$$41. \frac{72}{225} = \frac{2 \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}} \cdot 5 \cdot 5} = \frac{2 \cdot 2 \cdot 2 \cdot 1 \cdot 1}{1 \cdot 1 \cdot 5 \cdot 5} = \frac{8}{25}$$

$$42. \frac{3}{8} \text{ and } \frac{27}{72}$$

$$\frac{3}{8} = \frac{3}{2 \cdot 2 \cdot 2} = \frac{3}{8}$$

$$\frac{27}{72} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}} \cdot 3}{2 \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}}} = \frac{3}{8}$$

The fractions are *equivalent* ($\frac{3}{8} = \frac{3}{8}$).

$$43. \frac{3}{6} \text{ and } \frac{18}{36} \text{ Write each fraction in lowest terms.}$$

$$\frac{3}{6} = \frac{1 \cdot \overset{1}{\cancel{3}}}{2 \cdot \underset{1}{\cancel{3}}} = \frac{1}{2}$$

$$\frac{18}{36} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}}} = \frac{1}{2}$$

The fractions are *equivalent* ($\frac{1}{2} = \frac{1}{2}$).

$$44. \frac{10}{24} \text{ and } \frac{12}{30}$$

$$\frac{10}{24} = \frac{\overset{1}{\cancel{2}} \cdot 5}{\underset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot 3} = \frac{5}{12}$$

$$\frac{12}{30} = \frac{\overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 5} = \frac{2}{5}$$

The fractions are *not equivalent* ($\frac{5}{12} \neq \frac{2}{5}$).

$$45. \frac{15}{35} \text{ and } \frac{18}{40}$$

$$\frac{15}{35} = \frac{3 \cdot \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \cdot 7} = \frac{3}{7}$$

$$\frac{18}{40} = \frac{\overset{1}{\cancel{2}} \cdot 3 \cdot 3}{\underset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot 5} = \frac{9}{20}$$

The fractions are *not equivalent* ($\frac{3}{7} \neq \frac{9}{20}$).

$$46. \frac{15}{24} \text{ and } \frac{35}{52}$$

$$\frac{15}{24} = \frac{\overset{1}{\cancel{3}} \cdot 5}{2 \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{3}}} = \frac{5}{8}$$

$$\frac{35}{52} = \frac{5 \cdot 7}{2 \cdot 2 \cdot 13} = \frac{35}{52}$$

The fractions are *not equivalent* ($\frac{5}{8} \neq \frac{35}{52}$).

$$47. \frac{14}{16} \text{ and } \frac{35}{40}$$

$$\frac{14}{16} = \frac{\overset{1}{\cancel{2}} \cdot 7}{\underset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot 2} = \frac{7}{8}$$

$$\frac{35}{40} = \frac{\overset{1}{\cancel{5}} \cdot 7}{2 \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{5}}} = \frac{7}{8}$$

The fractions are *equivalent* ($\frac{7}{8} = \frac{7}{8}$).

$$48. \frac{27}{90} \text{ and } \frac{24}{80}$$

$$\frac{27}{90} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}} \cdot 3}{2 \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}} \cdot 5} = \frac{3}{10}$$

$$\frac{24}{80} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot 5} = \frac{3}{10}$$

The fractions are *equivalent* ($\frac{3}{10} = \frac{3}{10}$).

$$49. \frac{48}{6} \text{ and } \frac{72}{8}$$

$$\frac{48}{6} = \frac{2 \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}}} = 8$$

$$\frac{72}{8} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3 \cdot 3}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}}} = 9$$

The fractions are *not equivalent* ($8 \neq 9$).

$$50. \frac{33}{11} \text{ and } \frac{72}{24}$$

$$\frac{33}{11} = \frac{3 \cdot \overset{1}{\cancel{11}}}{\underset{1}{\cancel{11}}} = 3$$

$$\frac{72}{24} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}}} = 3$$

The fractions are *equivalent* ($3 = 3$).

51. A fraction is in lowest terms when the numerator and the denominator have no common factors other than 1. Some examples are $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{2}{3}$.

52. Two fractions are equivalent when they represent the same portion of a whole. For example, the fractions $\frac{10}{15}$ and $\frac{8}{12}$ are equivalent.

$$\frac{10}{15} = \frac{2 \cdot \cancel{5}}{3 \cdot \cancel{5}} = \frac{2 \cdot 1}{3 \cdot 1} = \frac{2}{3}$$

$$\frac{8}{12} = \frac{\cancel{2} \cdot \cancel{2} \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot 3} = \frac{1 \cdot 1 \cdot 2}{1 \cdot 1 \cdot 3} = \frac{2}{3}$$

The fractions are *equivalent* ($\frac{2}{3} = \frac{2}{3}$).

53.
$$\frac{160}{256} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 5}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2} = \frac{5}{8}$$

54.
$$\frac{363}{528} = \frac{\cancel{3} \cdot \cancel{11} \cdot 11}{2 \cdot 2 \cdot 2 \cdot 2 \cdot \cancel{3} \cdot \cancel{11}} = \frac{11}{16}$$

55.
$$\frac{238}{119} = \frac{2 \cdot \cancel{7} \cdot \cancel{17}}{\cancel{7} \cdot \cancel{17}} = \frac{2}{1} = 2$$

56.
$$\frac{570}{95} = \frac{2 \cdot 3 \cdot \cancel{5} \cdot \cancel{19}}{\cancel{5} \cdot \cancel{19}} = \frac{6}{1} = 6$$

57.
$$\frac{65}{234} = \frac{5 \cdot \cancel{13}}{2 \cdot 3 \cdot 3 \cdot \cancel{13}} = \frac{5}{18}$$

58.
$$\frac{105}{180} = \frac{\cancel{3} \cdot 7 \cdot \cancel{5}}{2 \cdot 2 \cdot 3 \cdot \cancel{3} \cdot \cancel{5}} = \frac{7}{12}$$

59.
$$\frac{19}{171} = \frac{\cancel{19}}{3 \cdot 3 \cdot \cancel{19}} = \frac{1}{9}$$

60.
$$\frac{36}{468} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{3}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{3} \cdot 13} = \frac{1}{13}$$

Summary Exercises *Fraction Basics*

- The figure has 6 equal parts.
Five parts are shaded: $\frac{5}{6}$
One part is unshaded: $\frac{1}{6}$
- The figure has 3 equal parts.
One part is shaded: $\frac{1}{3}$
Two parts are unshaded: $\frac{2}{3}$

- The figure has 8 equal parts.
Five parts are shaded: $\frac{5}{8}$

Three parts are unshaded: $\frac{3}{8}$

- $\frac{3}{4}$ ← Numerator
 $\frac{3}{4}$ ← Denominator

- $\frac{8}{5}$ ← Numerator
 $\frac{8}{5}$ ← Denominator

- Proper fractions: numerator *smaller* than denominator.

$$\frac{3}{5}, \frac{4}{25}, \frac{1}{32}$$

Improper fractions: numerator *greater than or equal to* denominator.

$$\frac{8}{2}, \frac{16}{7}, \frac{8}{8}$$

- There are 40 winners in total (12 + 10 + 8 + 4 + 3 + 3). Eight of the 40 winners were from Switzerland. $\frac{8}{40} = \frac{1}{5}$
- Since 3 + 10 = 13 of the winners *were* from either France or South Africa, 40 - 13 = 27 were not. $\frac{27}{40}$
- 3 + 12 = 15 of the winners were from either Japan or the United States. $\frac{15}{40} = \frac{3}{8}$
- Since 4 of the winners *were* from Canada, 40 - 4 = 36 were not. $\frac{36}{40} = \frac{9}{10}$

11. $\frac{5}{2}$

$\frac{2}{2}$ ← Whole number part

$$2 \overline{) 5}$$

$\frac{4}{1}$ ← Remainder

$$\frac{5}{2} = 2 \frac{1}{2}$$

12. $\frac{11}{8}$

$\frac{1}{8}$ ← Whole number part

$$8 \overline{) 11}$$

$\frac{3}{3}$ ← Remainder

$$\frac{11}{8} = 1 \frac{3}{8}$$

13. $\frac{9}{7}$

$$\begin{array}{r} 1 \leftarrow \text{Whole number part} \\ 7 \overline{)9} \\ \underline{7} \\ 2 \leftarrow \text{Remainder} \end{array}$$

$$\frac{9}{7} = 1\frac{2}{7}$$

14. $\frac{8}{3}$

$$\begin{array}{r} 2 \leftarrow \text{Whole number part} \\ 3 \overline{)8} \\ \underline{6} \\ 2 \leftarrow \text{Remainder} \end{array}$$

$$\frac{8}{3} = 2\frac{2}{3}$$

15. $\frac{64}{8}$

$$\begin{array}{r} 8 \leftarrow \text{Whole number part} \\ 8 \overline{)64} \\ \underline{64} \\ 0 \leftarrow \text{Remainder} \end{array}$$

$$\frac{64}{8} = 8$$

16. $\frac{45}{9}$

$$\begin{array}{r} 5 \leftarrow \text{Whole number part} \\ 9 \overline{)45} \\ \underline{45} \\ 0 \leftarrow \text{Remainder} \end{array}$$

$$\frac{45}{9} = 5$$

17. $\frac{36}{5}$

$$\begin{array}{r} 7 \leftarrow \text{Whole number part} \\ 5 \overline{)36} \\ \underline{35} \\ 1 \leftarrow \text{Remainder} \end{array}$$

$$\frac{36}{5} = 7\frac{1}{5}$$

18. $\frac{47}{10}$

$$\begin{array}{r} 4 \leftarrow \text{Whole number part} \\ 10 \overline{)47} \\ \underline{40} \\ 7 \leftarrow \text{Remainder} \end{array}$$

$$\frac{47}{10} = 4\frac{7}{10}$$

19. $3\frac{1}{3}$ $3 \cdot 3 = 9$ Multiply 3 and 3.
 $9 + 1 = 10$ Add 1.
 $3\frac{1}{3} = \frac{10}{3}$

20. $5\frac{3}{8}$ $5 \cdot 8 = 40$ Multiply 5 and 8.
 $40 + 3 = 43$ Add 3.
 $5\frac{3}{8} = \frac{43}{8}$

21. $6\frac{4}{5}$ $6 \cdot 5 = 30$ Multiply 6 and 5.
 $30 + 4 = 34$ Add 4.
 $6\frac{4}{5} = \frac{34}{5}$

22. $10\frac{3}{5}$ $10 \cdot 5 = 50$ Multiply 10 and 5.
 $50 + 3 = 53$ Add 3.
 $10\frac{3}{5} = \frac{53}{5}$

23. $12\frac{3}{4}$ $12 \cdot 4 = 48$ Multiply 12 and 4.
 $48 + 3 = 51$ Add 3.
 $12\frac{3}{4} = \frac{51}{4}$

24. $4\frac{10}{13}$ $4 \cdot 13 = 52$ Multiply 4 and 13.
 $52 + 10 = 62$ Add 10.
 $4\frac{10}{13} = \frac{62}{13}$

25. $11\frac{5}{6}$ $11 \cdot 6 = 66$ Multiply 11 and 6.
 $66 + 5 = 71$ Add 5.
 $11\frac{5}{6} = \frac{71}{6}$

26. $23\frac{5}{8}$ $23 \cdot 8 = 184$ Multiply 23 and 8.
 $184 + 5 = 189$ Add 5.
 $23\frac{5}{8} = \frac{189}{8}$

27.

$$\begin{array}{ccc} & 10 & \\ \swarrow & & \searrow \\ \boxed{2} & & \boxed{5} \end{array}$$

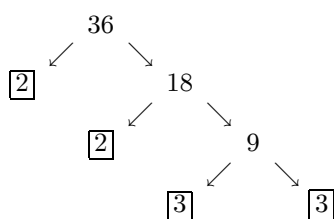
$$10 = 2 \cdot 5$$

28.

$$\begin{array}{ccc} & 55 & \\ \swarrow & & \searrow \\ \boxed{5} & & \boxed{11} \end{array}$$

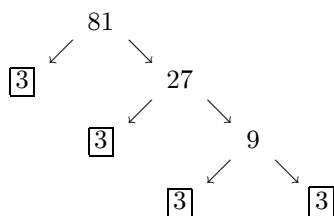
$$55 = 5 \cdot 11$$

29.



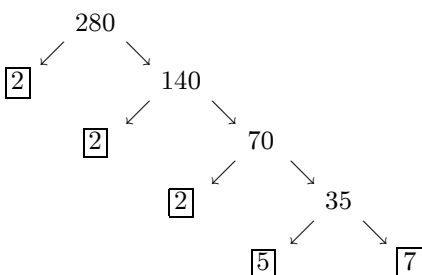
$$36 = 2^2 \cdot 3^2$$

30.



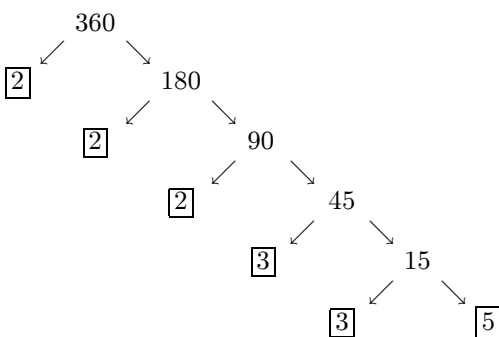
$$81 = 3^4$$

31.



$$280 = 2^3 \cdot 5 \cdot 7$$

32.



$$360 = 2^3 \cdot 3^2 \cdot 5$$

33.
$$\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$$

34.
$$\frac{7}{14} = \frac{7 \div 7}{14 \div 7} = \frac{1}{2}$$

35.
$$\frac{4}{16} = \frac{4 \div 4}{16 \div 4} = \frac{1}{4}$$

36.
$$\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}$$

37.
$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

38.
$$\frac{30}{36} = \frac{30 \div 6}{36 \div 6} = \frac{5}{6}$$

39.
$$\frac{56}{64} = \frac{56 \div 8}{64 \div 8} = \frac{7}{8}$$

40.
$$\frac{6}{300} = \frac{6 \div 6}{300 \div 6} = \frac{1}{50}$$

41.
$$\frac{25}{200} = \frac{25 \div 25}{200 \div 25} = \frac{1}{8}$$

42.
$$\frac{125}{225} = \frac{125 \div 25}{225 \div 25} = \frac{5}{9}$$

43.
$$\frac{88}{154} = \frac{88 \div 22}{154 \div 22} = \frac{4}{7}$$

44.
$$\frac{70}{126} = \frac{70 \div 14}{126 \div 14} = \frac{5}{9}$$

45.
$$\frac{24}{36} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 3} = \frac{2}{3}$$

46.
$$\frac{80}{160} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{5}} \cdot 2} = \frac{1}{2}$$

47.
$$\frac{126}{42} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}} \cdot 7}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{7}}} = \frac{3}{1} = 3$$

48.
$$\frac{96}{112} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 3}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 7} = \frac{6}{7}$$

2.5 Multiplying Fractions

2.5 Margin Exercises

1. $\frac{1}{4}$ of $\frac{1}{2}$ as read from the figure is the darker shaded part of the second figure. One of eight equal parts is shaded, or $\frac{1}{8}$.

$$\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

2. (a)
$$\frac{1}{2} \cdot \frac{3}{4} = \frac{1 \cdot 3}{2 \cdot 4} = \frac{3}{8}$$

(b)
$$\frac{3}{5} \cdot \frac{1}{3} = \frac{\overset{1}{\cancel{3}} \cdot 1}{5 \cdot \underset{1}{\cancel{3}}} = \frac{1}{5}$$

(c)
$$\frac{5}{6} \cdot \frac{1}{2} \cdot \frac{1}{8} = \frac{5 \cdot 1 \cdot 1}{6 \cdot 2 \cdot 8} = \frac{5}{96}$$

(d)
$$\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{3}{8} = \frac{1 \cdot 3 \cdot 3}{2 \cdot 4 \cdot 8} = \frac{9}{64}$$

3. (a)
$$\frac{\overset{3}{\cancel{2}}}{\underset{2}{\cancel{2}}} \cdot \frac{1}{5} = \frac{3 \cdot 1}{2 \cdot 5} = \frac{3}{10}$$

$$(b) \frac{\frac{2}{\cancel{11}} \cdot \frac{3}{\cancel{21}}}{1} = \frac{2 \cdot 3}{1 \cdot 7} = \frac{6}{7}$$

$$(c) \frac{\frac{1}{\cancel{20}} \cdot \frac{1}{\cancel{40}} \cdot \frac{1}{\cancel{2}}}{1} = \frac{1 \cdot 1 \cdot 1}{4 \cdot 2 \cdot 1} = \frac{1}{8}$$

$$(d) \frac{\frac{1}{\cancel{17}} \cdot \frac{1}{\cancel{36}} \cdot \frac{1}{\cancel{2}}}{1} = \frac{1 \cdot 1 \cdot 1}{17 \cdot 1 \cdot 3} = \frac{1}{51}$$

$$4. (a) 8 \cdot \frac{1}{8} = \frac{8}{1} \cdot \frac{1}{8} = \frac{1 \cdot 1}{1 \cdot 1} = \frac{1}{1} = 1$$

$$(b) \frac{1}{4} \cdot 5 \cdot \frac{5}{\cancel{2}} = \frac{1 \cdot 5 \cdot 5}{4 \cdot 1} = \frac{25}{4} \text{ or } 6\frac{1}{4}$$

$$(c) \frac{3}{5} \cdot 40 = \frac{3}{\cancel{5}} \cdot \frac{40}{1} = \frac{3 \cdot 8}{1 \cdot 1} = \frac{24}{1} = 24$$

$$(d) \frac{3}{25} \cdot \frac{5}{11} \cdot 99 = \frac{3}{\cancel{25}} \cdot \frac{5}{\cancel{11}} \cdot \frac{99}{1} \\ = \frac{3 \cdot 1 \cdot 9}{5 \cdot 1 \cdot 1} \\ = \frac{27}{5} = 5\frac{2}{5}$$

$$5. (a) \text{Area} = \text{length} \cdot \text{width}$$

$$= \frac{1}{\cancel{2}} \cdot \frac{1}{\cancel{2}} \\ = \frac{1}{4} \text{yd}^2$$

$$(b) \text{Area} = \text{length} \cdot \text{width}$$

$$= \frac{1}{\cancel{8}} \cdot \frac{1}{\cancel{8}} \\ = \frac{1}{8} \text{mi}^2$$

$$(c) \text{Area} = \text{length} \cdot \text{width}$$

$$= \frac{3}{\cancel{7}} \cdot \frac{1}{\cancel{12}} = \frac{3 \cdot 1}{1 \cdot 4} \\ = \frac{3}{4} \text{mi}^2$$

2.5 Section Exercises

- To multiply two or more fractions, you multiply the numerators and you multiply the denominators.
- To write a fraction in lowest terms, you must divide both the numerator and denominator by a common factor.
- A shortcut when multiplying fractions is to divide both a numerator and a denominator by the same number.
- Using the shortcut when multiplying fractions should result in an answer that is in lowest terms.

$$5. \frac{1}{3} \cdot \frac{3}{4} = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{4} = \frac{1 \cdot 1}{1 \cdot 4} = \frac{1}{4}$$

$$6. \frac{2}{5} \cdot \frac{3}{4} = \frac{\cancel{2}}{5} \cdot \frac{3}{\cancel{4}} = \frac{1 \cdot 3}{5 \cdot 2} = \frac{3}{10}$$

$$7. \frac{2}{7} \cdot \frac{1}{5} = \frac{2 \cdot 1}{7 \cdot 5} = \frac{2}{35}$$

$$8. \frac{2}{3} \cdot \frac{1}{2} = \frac{\cancel{2}}{3} \cdot \frac{1}{\cancel{2}} = \frac{1 \cdot 1}{3 \cdot 1} = \frac{1}{3}$$

$$9. \frac{8}{5} \cdot \frac{15}{32} = \frac{\cancel{8}}{5} \cdot \frac{15}{\cancel{32}} = \frac{1 \cdot 3}{1 \cdot 4} = \frac{3}{4}$$

$$10. \frac{5}{9} \cdot \frac{4}{3} = \frac{5 \cdot 4}{9 \cdot 3} = \frac{20}{27}$$

- Divide a numerator and denominator by the same number to use the multiplication shortcut.

$$\frac{2}{3} \cdot \frac{7}{12} \cdot \frac{9}{14} = \frac{\cancel{2}}{3} \cdot \frac{\cancel{7}}{\cancel{12}} \cdot \frac{\cancel{9}}{\cancel{14}} = \frac{1 \cdot 1 \cdot 1}{1 \cdot 2 \cdot 2} = \frac{1}{4}$$

$$12. \frac{7}{8} \cdot \frac{16}{21} \cdot \frac{1}{2} = \frac{\cancel{7}}{8} \cdot \frac{\cancel{16}}{\cancel{21}} \cdot \frac{1}{\cancel{2}} = \frac{1 \cdot 1 \cdot 1}{1 \cdot 3 \cdot 1} = \frac{1}{3}$$

$$13. \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{2}{3} = \frac{\cancel{3}}{4} \cdot \frac{5}{\cancel{6}} \cdot \frac{\cancel{2}}{\cancel{3}} = \frac{1 \cdot 5 \cdot 1}{2 \cdot 6 \cdot 1} = \frac{5}{12}$$

$$14. \frac{5}{8} \cdot \frac{16}{25} = \frac{\cancel{5}}{8} \cdot \frac{\cancel{16}}{\cancel{25}} = \frac{1 \cdot 2}{1 \cdot 5} = \frac{2}{5}$$

$$15. \quad \frac{6}{11} \cdot \frac{22}{15} = \frac{\cancel{6}^2}{\cancel{11}_1} \cdot \frac{\cancel{22}^2}{\cancel{15}_5} = \frac{2 \cdot 2}{1 \cdot 5} = \frac{4}{5}$$

$$16. \quad \frac{14}{25} \cdot \frac{65}{48} \cdot \frac{15}{28} = \frac{\cancel{14}^1}{\cancel{25}_5} \cdot \frac{\cancel{65}^{13}}{\cancel{48}_{16}} \cdot \frac{\cancel{15}^1}{\cancel{28}_2} = \frac{1 \cdot 13 \cdot 1}{1 \cdot 16 \cdot 2} = \frac{13}{32}$$

$$17. \quad \frac{35}{64} \cdot \frac{32}{15} \cdot \frac{27}{72} = \frac{\cancel{35}^7}{\cancel{64}_{2^6}} \cdot \frac{\cancel{32}^1}{\cancel{15}_3} \cdot \frac{\cancel{27}^3}{\cancel{72}_{2^3 \cdot 3}} = \frac{7 \cdot 1 \cdot 1}{2 \cdot 1 \cdot 8} = \frac{7}{16}$$

$$18. \quad \frac{16}{25} \cdot \frac{35}{32} \cdot \frac{15}{64} = \frac{\cancel{16}^1}{\cancel{25}_5} \cdot \frac{\cancel{35}^7}{\cancel{32}_2} \cdot \frac{\cancel{15}^3}{\cancel{64}_{2^6}} = \frac{1 \cdot 7 \cdot 3}{1 \cdot 2 \cdot 64} = \frac{21}{128}$$

$$19. \quad \frac{39}{42} \cdot \frac{7}{13} \cdot \frac{7}{24} = \frac{\cancel{39}^1}{\cancel{42}_{2 \cdot 3}} \cdot \frac{\cancel{7}^1}{\cancel{13}_1} \cdot \frac{7}{24} = \frac{1 \cdot 1 \cdot 7}{2 \cdot 1 \cdot 24} = \frac{7}{48}$$

20. The statement "When multiplying a fraction by a whole number, the whole number should be rewritten as the number over 1." is *true*.

$$21. \quad \frac{4}{5} \cdot 8 = \frac{\cancel{4}^1}{\cancel{5}_2} \cdot \frac{1}{10} = \frac{1}{10} \text{ is false.}$$

The correct method is as follows:

$$\frac{4}{5} \cdot 8 = \frac{4}{5} \cdot \frac{8}{1} = \frac{32}{5} = 6\frac{2}{5}$$

$$22. \quad 5 \cdot \frac{4}{5} = \frac{\cancel{5}^1}{\cancel{1}_1} \cdot \frac{\cancel{4}^1}{\cancel{5}_1} = \frac{4}{1} = 4$$

$$23. \quad 20 \cdot \frac{3}{4} = \frac{\cancel{20}^5}{\cancel{1}_1} \cdot \frac{\cancel{3}_1}{\cancel{4}_1} = \frac{15}{1} = 15$$

$$24. \quad 36 \cdot \frac{2}{3} = \frac{\cancel{36}^{12}}{\cancel{1}_1} \cdot \frac{\cancel{2}_1}{\cancel{3}_1} = \frac{24}{1} = 24$$

25. Write 36 as $\frac{36}{1}$ and multiply.

$$36 \cdot \frac{5}{8} \cdot \frac{9}{15} = \frac{\cancel{36}^9}{\cancel{1}_1} \cdot \frac{\cancel{5}^1}{\cancel{8}_2} \cdot \frac{\cancel{9}^3}{\cancel{15}_5} = \frac{27}{2} = 13\frac{1}{2}$$

$$26. \quad 30 \cdot \frac{3}{10} = \frac{\cancel{30}^3}{\cancel{10}_1} \cdot \frac{\cancel{3}_1}{\cancel{10}_1} = \frac{9}{1} = 9$$

$$27. \quad 100 \cdot \frac{21}{50} \cdot \frac{3}{4} = \frac{\cancel{100}^2}{\cancel{50}_1} \cdot \frac{\cancel{21}^1}{\cancel{50}_1} \cdot \frac{\cancel{3}_1}{\cancel{4}_2} = \frac{63}{2} = 31\frac{1}{2}$$

$$28. \quad 400 \cdot \frac{7}{8} = \frac{\cancel{400}^{50}}{\cancel{8}_1} \cdot \frac{\cancel{7}_1}{\cancel{8}_1} = \frac{350}{1} = 350$$

$$29. \quad \frac{2}{5} \cdot 200 = \frac{\cancel{2}_1}{\cancel{5}_1} \cdot \frac{\cancel{200}^{40}}{\cancel{1}_1} = \frac{80}{1} = 80$$

$$30. \quad \frac{6}{7} \cdot 245 = \frac{\cancel{6}_1}{\cancel{7}_1} \cdot \frac{\cancel{245}^{35}}{\cancel{1}_1} = \frac{210}{1} = 210$$

$$31. \quad 142 \cdot \frac{2}{3} = \frac{\cancel{142}^1}{\cancel{1}_1} \cdot \frac{\cancel{2}_1}{\cancel{3}_1} = \frac{284}{3} = 94\frac{2}{3}$$

$$32. \quad \frac{12}{25} \cdot 430 = \frac{\cancel{12}_2}{\cancel{25}_5} \cdot \frac{\cancel{430}^{86}}{\cancel{1}_1} = \frac{1032}{5} = 206\frac{2}{5}$$

$$33. \quad \frac{28}{21} \cdot 640 \cdot \frac{15}{32} = \frac{\cancel{28}^4}{\cancel{21}_3} \cdot \frac{\cancel{640}^{20}}{\cancel{1}_1} \cdot \frac{\cancel{15}^5}{\cancel{32}_1} = \frac{4 \cdot 20 \cdot 5}{1 \cdot 1 \cdot 1} = \frac{400}{1} = 400$$

$$34. \quad \frac{21}{13} \cdot 520 \cdot \frac{7}{20} = \frac{\cancel{21}_3}{\cancel{13}_1} \cdot \frac{\cancel{520}^{20}}{\cancel{1}_1} \cdot \frac{\cancel{7}_1}{\cancel{20}_4} = \frac{21 \cdot 2 \cdot 7}{1 \cdot 1 \cdot 1} = \frac{294}{1} = 294$$

$$35. \quad \frac{54}{38} \cdot 684 \cdot \frac{5}{6} = \frac{\cancel{54}^9}{\cancel{38}_2} \cdot \frac{\cancel{684}^{18}}{\cancel{1}_1} \cdot \frac{\cancel{5}_1}{\cancel{6}_3} = \frac{9 \cdot 18 \cdot 5}{1 \cdot 1 \cdot 1} = \frac{810}{1} = 810$$

$$\begin{aligned}
 36. \quad \frac{76}{43} \cdot 473 \cdot \frac{5}{19} &= \frac{\cancel{76}^4}{\cancel{43}_1} \cdot \frac{\cancel{473}^{11}}{1} \cdot \frac{5}{\cancel{19}_1} \\
 &= \frac{4 \cdot 11 \cdot 5}{1 \cdot 1 \cdot 1} \\
 &= \frac{220}{1} = 220
 \end{aligned}$$

$$\begin{aligned}
 37. \quad \text{Area} &= \text{length} \cdot \text{width} \\
 &= \frac{3}{4} \cdot \frac{1}{3} = \frac{\cancel{3}^1}{4} \cdot \frac{1}{\cancel{3}_1} = \frac{1}{4} \text{ mi}^2
 \end{aligned}$$

$$\begin{aligned}
 38. \quad \text{Area} &= \text{length} \cdot \text{width} \\
 &= \frac{7}{8} \cdot \frac{1}{4} = \frac{7 \cdot 1}{8 \cdot 4} = \frac{7}{32} \text{ ft}^2
 \end{aligned}$$

$$\begin{aligned}
 39. \quad \text{Area} &= \text{length} \cdot \text{width} \\
 &= 12 \cdot \frac{3}{4} = \frac{\cancel{12}^3}{1} \cdot \frac{3}{\cancel{4}_1} = \frac{9}{1} = 9 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 40. \quad \text{Area} &= \text{length} \cdot \text{width} \\
 &= 8 \cdot \frac{3}{8} = \frac{\cancel{8}^1}{1} \cdot \frac{3}{\cancel{8}_1} = \frac{3}{1} = 3 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 41. \quad \text{Area} &= \text{length} \cdot \text{width} \\
 \text{Area} &= \frac{5}{6} \cdot \frac{3}{10} \\
 &= \frac{\cancel{5}^1}{\cancel{6}_2} \cdot \frac{\cancel{3}_1}{\cancel{10}_2} \quad \begin{array}{l} \text{Divide 5 and 10 by 5.} \\ \text{Divide 3 and 6 by 3.} \end{array} \\
 &= \frac{1}{2} \text{ mi}^2
 \end{aligned}$$

$$\begin{aligned}
 42. \quad \text{Area} &= \text{length} \cdot \text{width} \\
 &= \frac{7}{5} \cdot \frac{3}{8} = \frac{21}{40} \text{ mi}^2
 \end{aligned}$$

43. Multiply the numerators and multiply the denominators. An example is

$$\frac{3}{4} \cdot \frac{1}{2} = \frac{3 \cdot 1}{4 \cdot 2} = \frac{3}{8}$$

44. You must divide a numerator and a denominator by the same number. If you do all possible divisions, your answer will be in lowest terms. Examples will vary. One possible example is shown.

$$\frac{3}{4} \cdot \frac{2}{3} = \frac{3 \cdot 2}{4 \cdot 3} = \frac{\cancel{3}^1 \cdot \cancel{2}_2}{\cancel{4}_2 \cdot \cancel{3}_1} = \frac{1}{2}$$

45. Area = length • width

$$\begin{aligned}
 \text{Area} &= 2 \cdot \frac{3}{4} \\
 &= \frac{\cancel{2}^1}{1} \cdot \frac{3}{\cancel{4}_2} \quad \begin{array}{l} \text{Divide numerator and} \\ \text{denominator by 2.} \end{array} \\
 &= \frac{3}{2} \quad \text{Write as a mixed number.} \\
 &= 1 \frac{1}{2} \text{ yd}^2 \quad \text{Square yards}
 \end{aligned}$$

46. Area = width • height

$$\begin{aligned}
 &= 2 \cdot \frac{15}{16} = \frac{\cancel{2}^1}{1} \cdot \frac{15}{\cancel{16}_8} = \frac{15}{8} = 1 \frac{7}{8} \text{ yd}^2
 \end{aligned}$$

47. Area = length • width

$$\begin{aligned}
 &= 4 \cdot \frac{7}{8} = \frac{\cancel{4}^1}{1} \cdot \frac{7}{\cancel{8}_2} = \frac{7}{2} = 3 \frac{1}{2} \text{ mi}^2
 \end{aligned}$$

48. Area = length • width

$$\begin{aligned}
 &= 7 \cdot \frac{3}{4} = \frac{7}{1} \cdot \frac{3}{4} = \frac{21}{4} = 5 \frac{1}{4} \text{ mi}^2
 \end{aligned}$$

49. **Sunny Side Soccer Park** **Creekside Soc. Park**

$$\begin{array}{ll}
 \text{Area} = \text{length} \cdot \text{width} & \text{Area} = \text{length} \cdot \text{width} \\
 = \frac{1}{4} \cdot \frac{3}{16} & = \frac{3}{8} \cdot \frac{1}{8} \\
 = \frac{3}{64} \text{ mi}^2 & = \frac{3}{64} \text{ mi}^2
 \end{array}$$

They are both the same size.

50. **Rocking Horse Ranch** **Silver Spur Ranch**

$$\begin{array}{ll}
 \text{Area} = \text{length} \cdot \text{width} & \text{Area} = \text{length} \cdot \text{width} \\
 = \frac{3}{4} \cdot \frac{2}{3} & = \frac{5}{8} \cdot \frac{4}{5} \\
 = \frac{\cancel{3}^1}{\cancel{4}_2} \cdot \frac{\cancel{2}_2}{\cancel{3}_1} = \frac{1}{2} \text{ mi}^2 & = \frac{\cancel{5}^1}{\cancel{8}_2} \cdot \frac{\cancel{4}_4}{\cancel{5}_1} = \frac{1}{2} \text{ mi}^2
 \end{array}$$

Neither ranch is larger in area. They are both the same size.

51. $4000 + 2000 + 1000 + 800 + 400 + 200$

$$+ 100 + 80 = \underline{8580 \text{ supermarkets}}$$

The estimate of the total number of supermarkets in these states is 8580.

52. $3753 + 2202 + 1349 + 826 + 441 + 163$

$$+ 98 + 80 = \underline{8912 \text{ supermarkets}}, \text{ which is the exact total number of supermarkets in these states.}$$

53. An estimate of the number of supermarkets in medium to large population areas in New York is

$$\frac{4}{5} \cdot 2000 = \frac{4}{\cancel{5}^{\frac{400}{1}}} \cdot \frac{\cancel{2000}}{1} = 1600.$$

The exact value is

$$\frac{4}{5} \cdot 2202 = \frac{4}{5} \cdot \frac{2202}{1} = \frac{8808}{5} = 1761\frac{3}{5}.$$

Rounding gives us 1762 supermarkets.

54. An estimate of the number of supermarkets in New Hampshire which are in shopping centers, is

$$\frac{3}{8} \cdot 200 = \frac{3}{\cancel{8}^{\frac{25}{1}}} \cdot \frac{\cancel{200}}{1} = 75.$$

The exact value is

$$\frac{3}{8} \cdot 163 = \frac{3}{8} \cdot \frac{163}{1} = \frac{489}{8} = 61\frac{1}{8}.$$

Rounding gives us 61 supermarkets.

55. We need a multiple of 5 with *two* nonzero digits that is close to 2202. A reasonable choice is 2200 and an estimate is

$$\frac{4}{5} \cdot 2200 = \frac{4}{\cancel{5}^{\frac{440}{1}}} \cdot \frac{\cancel{2200}}{1} = 1760 \text{ supermarkets.}$$

This value is closer to the exact value because using 2200 as a rounded guess is closer to 2202 than using 2000 as a rounded guess.

56. We need a multiple of 8 with *two* nonzero digits that is close to 163. A reasonable choice is 160 and an estimate is

$$\frac{3}{8} \cdot 160 = \frac{3}{\cancel{8}^{\frac{20}{1}}} \cdot \frac{\cancel{160}}{1} = 60 \text{ supermarkets.}$$

This value is closer to the exact value because using 160 as a rounded guess is closer to 163 than using 200 as a rounded guess.

2.6 Applications of Multiplication

2.6 Margin Exercises

1. (a) *Step 1*

The problem asks us to find the amount of money they can save in a year.

Step 2

Find the amount they can save by multiplying $\frac{3}{8}$ and 81,576.

Step 3

We can estimate this amount using $\frac{1}{2}$ and 82,000.

$$\frac{1}{2} \cdot 82,000 = \frac{1}{\cancel{2}^{\frac{41,000}{1}}} \cdot \frac{\cancel{82,000}}{1} = \underline{41,000}$$

Step 4

Now solve the problem using the original values.

$$\frac{3}{8} \cdot 81,576 = \frac{3}{\cancel{8}^{\frac{10,197}{1}}} \cdot \frac{\cancel{81,576}}{1} = \underline{30,591}$$

Step 5

They can save \$30,591 in a year.

Step 6

The answer is reasonably close to our estimate.

(b) *Step 1*

The problem asks us to find the amount of money she will receive as retirement income.

Step 2

To find her retirement income, multiply $\frac{5}{8}$ and \$62,504.

Step 3

We can estimate this amount using $\frac{1}{2}$ and 63,000.

$$\frac{1}{2} \cdot 63,000 = \frac{1}{\cancel{2}^{\frac{31,500}{1}}} \cdot \frac{\cancel{63,000}}{1} = 31,500$$

Step 4

Now solve the problem using the original values.

$$\frac{5}{8} \cdot 62,504 = \frac{5}{\cancel{8}^{\frac{7813}{1}}} \cdot \frac{\cancel{62,504}}{1} = 39,065$$

Step 5

She will receive \$39,065 as retirement income.

Step 6

The answer is reasonably close to our estimate.

2. *Step 1*

The problem asks for the number of prescriptions paid for by a third party.

Step 2

A third party pays for $\frac{5}{16}$ of the total number of prescriptions, 3696.

Step 3

$$\text{An estimate is } \frac{1}{4} \cdot 4000 = \frac{1}{\cancel{4}^{\frac{1000}{1}}} \cdot \frac{\cancel{4000}}{1} = 1000.$$

(continued)

Step 4

The exact value is

$$\frac{5}{16} \cdot 3696 = \frac{5}{\cancel{16}_1} \cdot \frac{3696}{1} = 1155.$$

Step 5

A third party pays for 1155 prescriptions.

Step 6

The answer is reasonably close to our estimate.

3. Step 1

The problem asks for the fraction of students who speak Spanish.

Step 2

 $\frac{3}{4}$ of the $\frac{1}{3}$ of the students who speak a foreign language, speak Spanish.

Step 3

$$\text{An estimate is } \frac{3}{4} \cdot \frac{1}{3} = \frac{\cancel{3}}{4} \cdot \frac{1}{\cancel{3}_1} = \frac{1}{4}.$$

Step 4

The exact value is $\frac{1}{4}$, which is the same as the estimate since we didn't round.

Step 5

The fraction of students who speak Spanish is $\frac{1}{4}$.

Step 6

The answer, $\frac{1}{4}$, matches our estimate.4. (a) From the circle graph, the fraction is $\frac{1}{5}$.(b) Multiply $\frac{1}{5}$ by the number of people in the survey, 2500. Since we can estimate the answer using the exact values, our estimated answer will be the same as the exact answer.

$$\frac{1}{5} \cdot 2500 = \frac{1}{\cancel{5}_1} \cdot \frac{2500}{1} = \frac{500}{1} = 500$$

500 children buy food from vending machines.

(c) From the circle graph, the fraction is $\frac{1}{10}$.(d) Multiply $\frac{1}{10}$ by 2500. Since we can estimate the answer using the exact values, our estimated answer will be the same as the exact answer.

$$\frac{1}{10} \cdot 2500 = \frac{1}{\cancel{10}_1} \cdot \frac{2500}{1} = \frac{250}{1} = 250$$

250 children buy food from a convenience store or street vendor.

2.6 Section Exercises

- The words that are indicator words for multiplication are *of*, *times*, *twice*, *triple*, *product*, and *twice as much*.
- The final step when solving an application problem is to check your work.
- When you multiply length by width you are finding the area of a rectangular surface.
- When calculating area, the length and the width must be in the same units of measurement. If the measurements are both in miles, the answer will be in square miles and shown as mi^2 .
- Step 1:* The problem asks for the area of the oil painting.

Step 2: To find area, multiply the length of $\frac{3}{4}$ foot by the width of $\frac{2}{3}$ foot.Step 3: An estimate is $\frac{6}{12}$ square foot. (ft^2)

$$\frac{3}{4} \cdot \frac{2}{3} = \frac{\cancel{3} \cdot \cancel{2}}{\cancel{4}_2 \cdot \cancel{3}_1} = \frac{1}{2}$$

Step 4: The exact value is $\frac{1}{2}$, which is the same as the estimate since we didn't round.Step 5: The area of the oil painting is $\frac{1}{2}$ square foot. (ft^2)Step 6: The answer, $\frac{1}{2}$ square foot (ft^2), matches our estimate.

6. Multiply the length and width.

$$\frac{14}{15} \cdot \frac{3}{4} = \frac{\cancel{14} \cdot \cancel{3}}{\cancel{15}_5 \cdot \cancel{4}_2} = \frac{7}{10}$$

The area of the frame is $\frac{7}{10}$ yd^2 .

7. Multiply the length and the width.

$$\frac{4}{3} \cdot \frac{2}{3} = \frac{8}{9}$$

The area of the cookie sheet is $\frac{8}{9}$ ft^2 .8. Multiply $\frac{2}{5}$ by 16,000,000.

$$\frac{2}{5} \cdot 16,000,000 = \frac{2}{\cancel{5}_1} \cdot \frac{3,200,000}{1} = 6,400,000$$

6,400,000 people who shop at flea markets on a daily basis purchase produce.

9. Multiply the length and the width.

$$\frac{4}{5} \cdot \frac{3}{8} = \frac{\cancel{4} \cdot 3}{5 \cdot \cancel{8}_2} = \frac{3}{10}$$

The area of the top of the table is $\frac{3}{10}$ yd².

10. Multiply the number of patients by the fraction of patients that are children.

$$\frac{3}{10} \cdot 160 = \frac{3}{\cancel{10}_2} \cdot \frac{\cancel{160}^{16}}{1} = 48$$

The hygienist treats 48 children each month.

11. Multiply $\frac{3}{8}$ by 6848.

$$\frac{3}{8} \cdot 6848 = \frac{3}{\cancel{8}_1} \cdot \frac{\cancel{6848}^{856}}{1} = 2568$$

He earned \$2568 on his job.

12. Multiply $\frac{3}{8}$ by 400.

$$\frac{3}{8} \cdot 400 = \frac{3}{\cancel{8}_1} \cdot \frac{\cancel{400}^{50}}{1} = 150$$

The average household does 150 loads of wash in the winter months.

13. Multiply the daily parking fee by the fraction.

$$40 \cdot \frac{7}{8} = \frac{\cancel{40}^5}{1} \cdot \frac{7}{\cancel{8}_1} = 35$$

The daily parking fee in Boston is \$35.

14. Multiply the daily parking fee by the fraction.

$$35 \cdot \frac{4}{5} = \frac{\cancel{35}^7}{1} \cdot \frac{4}{\cancel{5}_1} = 28$$

The daily parking fee in San Francisco is \$28.

15. (a) *Step 1:* The problem asks for the number of runners who are women.

Step 2: $\frac{7}{12}$ of the 1560 runners are women, so multiply $\frac{7}{12}$ by 1560 to find the number of runners who are women.

Step 3: Round the number of runners to 1600; then, $\frac{1}{2}$ of 1600 is 800. Since $\frac{7}{12}$ is more than $\frac{1}{2}$, our estimate is that "more than 800 runners" are women.

$$\text{Step 4: } \frac{7}{12} \cdot 1560 = \frac{7 \cdot \cancel{1560}^{130}}{\cancel{12}_4 \cdot 1} = 910$$

Step 5: 910 runners are women.

Step 6: The exact answer, 910, fits our estimate of "more than 800."

(b) The number of runners that are men is

$$1560 \text{ total runners} - 910 \text{ women} = 650 \text{ men.}$$

16. (a) Multiply the fraction of reclining seats by the number of seats.

$$\frac{9}{17} \cdot 408 = \frac{9}{\cancel{17}_1} \cdot \frac{\cancel{408}^{24}}{1} = 216$$

There are 216 reclining seats.

(b) The number of not reclining seats is

$$408 - 216 = 192.$$

17. The smallest sector of the circle graph is the 4 hours group, so this response was given by the least number of people. To find how many people gave this response, multiply $\frac{3}{20}$ by the total number of people, 1020.

$$\frac{3}{20} \cdot 1020 = \frac{3 \cdot \cancel{1020}^{51}}{\cancel{20}_4 \cdot 1} = 153$$

153 people gave this response.

18. The largest sector of the circle graph is the 2 hours or less group, so this response was given by the greatest number of people. To find how many people gave this response, multiply $\frac{3}{5}$ by the total number of people, 1020.

$$\frac{3}{5} \cdot 1020 = \frac{3 \cdot \cancel{1020}^{204}}{\cancel{5}_1 \cdot 1} = 612$$

612 people gave this response.

19. The only group that is *not* willing to wait 4 hours or less is the 8 hours group, and the fraction corresponding to that group is $\frac{1}{4}$. Thus, the fraction willing to wait 4 hours or less is

$$1 - \frac{1}{4} = \frac{4}{4} - \frac{1}{4} = \frac{3}{4}.$$

The total number of people willing to wait 4 hours or less is

$$\frac{3}{4} \cdot 1020 = \frac{3 \cdot \cancel{1020}^{255}}{\cancel{4}_1 \cdot 1} = 765.$$

20. The only group that is *not* willing to wait 4 hours or more is the *2 hours or less* group, and the fraction corresponding to that group is $\frac{3}{5}$. Thus, the fraction willing to wait 4 hours or more is

$$1 - \frac{3}{5} = \frac{5}{5} - \frac{3}{5} = \frac{2}{5}.$$

The total number of people willing to wait 4 hours or more is

$$\frac{2}{5} \cdot 1020 = \frac{2 \cdot \cancel{1020}^{204}}{\cancel{5} \cdot 1} = 408.$$

21. Because everyone is included and fractions are given for *all* groups, the sum of the fractions must be 1, or *all* of the people.
22. Answers will vary. Some possibilities are
1. You made an addition error.
 2. The fractions on the circle graph are incorrect.
 3. The fraction errors were caused by rounding.
23. Add the income for all twelve months to find the income for the year.

$$6075 + 5812 + 6488 + 6030 + 5820 + 6398 \\ + 7040 + 5232 + 5670 + 7012 + 6465 + 7958 \\ = 76,000$$

The Owens family had income of \$76,000 for the year.

24. Multiply the fraction $\frac{1}{4}$ by the total income (\$76,000).

$$\frac{1}{4} \cdot 76,000 = \frac{1}{\cancel{4}} \cdot \frac{\cancel{76,000}^{19,000}}{1} = 19,000$$

Their taxes were \$19,000.

25. From Exercise 23, the total income is \$76,000. The circle graph shows that $\frac{1}{5}$ of the income is for rent.

$$\frac{1}{5} \cdot 76,000 = \frac{1}{\cancel{5}} \cdot \frac{\cancel{76,000}^{15,200}}{1} = 15,200$$

The amount of their rent is \$15,200.

26. Multiply the fraction $\frac{5}{16}$ by the total income.

$$\frac{5}{16} \cdot 76,000 = \frac{5}{\cancel{16}} \cdot \frac{\cancel{76,000}^{4750}}{1} = 23,750$$

They spent \$23,750 on food.

27. Multiply the total income by the fraction saved.

$$\frac{1}{16} \cdot 76,000 = \frac{1}{\cancel{16}} \cdot \frac{\cancel{76,000}^{4750}}{1} = 4750$$

The Owens family saved \$4750 for the year.

28. Multiply the fraction $\frac{1}{8}$ by the total income.

$$\frac{1}{8} \cdot 76,000 = \frac{1}{\cancel{8}} \cdot \frac{\cancel{76,000}^{9500}}{1} = 9500$$

They spent \$9500 on clothing.

29. The error was made when dividing 21 by 3 and writing 3 instead of 7. The correct solution is

$$\frac{9}{10} \times \frac{20}{21} = \frac{\cancel{9}}{\cancel{10}} \times \frac{\cancel{20}^2}{\cancel{21}_7} = \frac{6}{7}.$$

30. Yes, the statements are true. Since whole numbers are 1 or greater, when you multiply, the product will always be greater than either of the numbers multiplied. But, when you multiply two proper fractions, you are finding a fractional part of a fraction, and the product will be smaller than either of the two proper fractions.

31. We want $\frac{1}{128}$ of the actual length.

$$\frac{1}{128} \cdot 256 = \frac{1}{\cancel{128}} \cdot \frac{\cancel{256}^2}{1} = 2$$

The length of the scale model is 2 feet.

32. Multiply the purchase price by $\frac{3}{50}$.

$$36,300 \cdot \frac{3}{50} = \frac{\cancel{36,300}^{726}}{1} \cdot \frac{3}{\cancel{50}_1} = 2178$$

The cost of gasoline for one year is \$2178.

33. Multiply the number of views by $\frac{3}{8}$.

$$2000 \cdot \frac{3}{8} = \frac{\cancel{2000}^{250}}{1} \cdot \frac{3}{\cancel{8}_1} = 750$$

750 dancers viewed the YouTube video.

34. First multiply $\frac{1}{10}$ and 10,000 to find the number of pounds saved.

$$\frac{1}{10} \cdot 10,000 = \frac{1}{\cancel{10}} \cdot \frac{\cancel{10,000}^{1000}}{1} = 1000$$

To find the weight of the test truck, subtract:

$$10,000 - 1000 = 9000 \text{ pounds.}$$

The test truck weighs 9000 pounds.

35. First multiply $\frac{2}{3}$ and 27,000 to find the number of votes she needs from senior citizens.

$$\frac{2}{3} \cdot 27,000 = \frac{2}{3} \cdot \frac{27,000}{1} = 18,000$$

To find the votes needed from voters other than the senior citizens, subtract:

$$27,000 - 18,000 = 9000 \text{ votes.}$$

She needs 9000 votes from voters other than the senior citizens.

36. Multiply the fraction $\frac{5}{16}$ by the cost (\$25,600).

$$\frac{5}{16} \cdot 25,600 = \frac{5}{16} \cdot \frac{25,600}{1} = 8000$$

To find the amount borrowed in the first years, subtract:

$$\$25,600 - \$8000 = \$17,600$$

\$17,600 was borrowed in the first years.

37. To find the remaining amount of the estate, subtract $\frac{7}{8}$ from 1.

$$1 - \frac{7}{8} = \frac{8}{8} - \frac{7}{8} = \frac{1}{8}$$

Multiply the remaining $\frac{1}{8}$ of the estate by the fraction going to the American Cancer Society.

$$\frac{1}{4} \cdot \frac{1}{8} = \frac{1}{32}$$

$\frac{1}{32}$ of the estate goes to the American Cancer Society.

38. Multiply the remaining $\frac{3}{5}$ of their total investments by the fraction invested in bonds.

$$\frac{3}{5} \cdot \frac{1}{3} = \frac{3}{5} \cdot \frac{1}{3} = \frac{1}{5}$$

The couple invested $\frac{1}{5}$ of their total investment in bonds.

2.7 Dividing Fractions

2.7 Margin Exercises

1. (a) $\frac{4}{5} \times \frac{5}{4}$; The reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$ because

$$\frac{4}{5} \cdot \frac{5}{4} = \frac{20}{20} = 1.$$

- (b) $\frac{3}{8} \times \frac{8}{3}$; The reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$ because

$$\frac{3}{8} \cdot \frac{8}{3} = \frac{24}{24} = 1.$$

- (c) The reciprocal of $\frac{9}{4}$ is $\frac{4}{9}$ because

$$\frac{9}{4} \cdot \frac{4}{9} = \frac{36}{36} = 1.$$

- (d) The reciprocal of 16 is $\frac{1}{16}$ because

$$\frac{16}{1} \cdot \frac{1}{16} = \frac{16}{16} = 1.$$

2. (a) $\frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \cdot \frac{3}{2} = \frac{1 \cdot 3}{4 \cdot 2} = \frac{3}{8}$

(b) $\frac{3}{8} \div \frac{5}{8} = \frac{3}{8} \cdot \frac{8}{5} = \frac{3 \cdot \cancel{8}}{\cancel{8} \cdot 5} = \frac{3}{5}$

(c) $\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \cdot \frac{5}{4} = \frac{1 \cdot 5}{3 \cdot 2} = \frac{5}{6}$

(d) $\frac{\frac{5}{6}}{\frac{7}{12}} = \frac{5}{6} \div \frac{7}{12}$

$$= \frac{5}{6} \cdot \frac{12}{7} = \frac{5 \cdot 2}{1 \cdot 7} = \frac{10}{7} = 1\frac{3}{7}$$

3. (a) $10 \div \frac{1}{2} = 10 \cdot \frac{2}{1} = \frac{10 \cdot 2}{1 \cdot 1} = \frac{20}{1} = 20$

(b) $6 \div \frac{6}{7} = \frac{6}{1} \div \frac{6}{7} = \frac{6}{1} \cdot \frac{7}{6} = \frac{7}{1} = 7$

(c) $\frac{4}{5} \div 6 = \frac{4}{5} \div \frac{6}{1} = \frac{4}{5} \cdot \frac{1}{6} = \frac{2 \cdot 1}{5 \cdot 3} = \frac{2}{15}$

(d) $\frac{3}{8} \div 4 = \frac{3}{8} \div \frac{4}{1} = \frac{3}{8} \cdot \frac{1}{4} = \frac{3 \cdot 1}{8 \cdot 4} = \frac{3}{32}$

4. (a) *Step 1*

The problem asks for the number of $\frac{5}{6}$ -ounce dispensers that can be filled using 40 ounces of eye drops.

Step 2

Divide the total number of ounces of eye drops by the fraction of an ounce each dispenser holds.

Step 3

An estimate is $40 \div 1 = 40$.

(continued)

Step 4

Solving gives us

$$40 \div \frac{5}{6} = \frac{40}{1} \div \frac{5}{6}$$

$$= \frac{40}{1} \cdot \frac{6}{5} = \frac{8 \cdot 6}{1 \cdot 1} = \frac{48}{1} = 48$$

*Step 5*48 $\frac{5}{6}$ -ounce dispensers can be filled.*Step 6*

The answer is reasonably close to our estimate.

(b) Step 1The problem asks for the number of $\frac{4}{5}$ -quart bottles that can be filled from a 120-quart cask.*Step 2*

Divide the total number of quarts in the cask by the size of the bottles.

*Step 3*An estimate is $120 \div 1 = 120$.*Step 4*

Solving gives us

$$120 \div \frac{4}{5} = \frac{120}{1} \div \frac{4}{5}$$

$$= \frac{120}{1} \cdot \frac{5}{4} = \frac{30 \cdot 5}{1 \cdot 1} = \frac{150}{1} = 150$$

*Step 5*150 $\frac{4}{5}$ -quart bottles can be filled.*Step 6*

The answer is reasonably close to our estimate.

5. (a) Step 1

The problem asks for the fraction of the bonus money that each employee will receive.

*Step 2*Divide the fraction of the bonus money, $\frac{3}{4}$, by the number of employees, 12.*Step 3*An estimate is $1 \div 12 = \frac{1}{12}$.*Step 4*

Solving gives us

$$\frac{3}{4} \div 12 = \frac{3}{4} \div \frac{12}{1} = \frac{3}{4} \cdot \frac{1}{12} = \frac{1 \cdot 1}{4 \cdot 4} = \frac{1}{16}$$

*Step 5*Each employee will receive $\frac{1}{16}$ of the bonus money.*Step 6*

The answer is reasonably close to our estimate.

(b) Step 1

The problem asks for the fraction of the prize money that each employee will receive.

*Step 2*Since they donate $\frac{1}{5}$ of the winnings, they have

$$1 - \frac{1}{5} = \frac{5}{5} - \frac{1}{5} = \frac{4}{5}$$

of the winnings left to divide. Divide the fraction of the winnings that remain, $\frac{4}{5}$, by the number of employees, 8.*Step 3*An estimate is $1 \div 8 = \frac{1}{8}$.*Step 4*

Solving gives us

$$\frac{4}{5} \div 8 = \frac{4}{5} \div \frac{8}{1} = \frac{4}{5} \cdot \frac{1}{8} = \frac{1}{10}$$

*Step 5*Each employee will receive $\frac{1}{10}$ of the prize money.*Step 6*

The answer is reasonably close to our estimate.

2.7 Section Exercises

- When you invert or flip a fraction, you have the reciprocal of the fraction.
- To find the reciprocal of a whole number, you must first write the whole number over 1, and then invert it.
- To divide by a fraction, you must first invert the divisor and then change division to multiplication.
- After completing a fraction division problem, it is best to write the answer in lowest terms.
- The reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$ because $\frac{3}{8} \cdot \frac{8}{3} = \frac{24}{24} = 1$.
- The reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$ because $\frac{2}{5} \cdot \frac{5}{2} = \frac{10}{10} = 1$.

7. The reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$ because
 $\frac{5}{6} \cdot \frac{6}{5} = \frac{30}{30} = 1.$

8. The reciprocal of $\frac{12}{7}$ is $\frac{7}{12}$ because
 $\frac{12}{7} \cdot \frac{7}{12} = \frac{84}{84} = 1.$

9. The reciprocal of $\frac{8}{5}$ is $\frac{5}{8}$ because
 $\frac{8}{5} \cdot \frac{5}{8} = \frac{40}{40} = 1.$

10. The reciprocal of $\frac{13}{20}$ is $\frac{20}{13}$ because
 $\frac{13}{20} \cdot \frac{20}{13} = \frac{260}{260} = 1.$

11. The reciprocal of 4 is $\frac{1}{4}$ because $\frac{4}{1} \cdot \frac{1}{4} = \frac{4}{4} = 1.$

12. The reciprocal of 10 is $\frac{1}{10}$ because
 $\frac{10}{1} \cdot \frac{1}{10} = \frac{10}{10} = 1.$

13. $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$

14. $\frac{5}{8} \div \frac{7}{8} = \frac{5}{8} \cdot \frac{8}{7} = \frac{5}{7}$

15. $\frac{7}{8} \div \frac{1}{3} = \frac{7}{8} \cdot \frac{3}{1} = \frac{21}{8} = 2\frac{5}{8}$

16. $\frac{7}{8} \div \frac{3}{4} = \frac{7}{8} \cdot \frac{4}{3} = \frac{7}{6} = 1\frac{1}{6}$

17. $\frac{3}{4} \div \frac{5}{3} = \frac{3}{4} \cdot \frac{3}{5} = \frac{9}{20}$

18. $\frac{4}{5} \div \frac{9}{4} = \frac{4}{5} \cdot \frac{4}{9} = \frac{16}{45}$

19. $\frac{7}{9} \div \frac{7}{36} = \frac{7}{9} \cdot \frac{36}{7} = \frac{4}{1} = 4$

20. $\frac{5}{8} \div \frac{5}{16} = \frac{5}{8} \cdot \frac{16}{5} = 2$

21. $\frac{15}{32} \div \frac{5}{64} = \frac{15}{32} \cdot \frac{64}{5} = \frac{6}{1} = 6$

22. $\frac{7}{12} \div \frac{14}{15} = \frac{7}{12} \cdot \frac{15}{14} = \frac{5}{4}$

23. $\frac{\frac{13}{20}}{\frac{4}{5}} = \frac{13}{20} \div \frac{4}{5} = \frac{13}{\cancel{20}_4} \cdot \frac{5}{4} = \frac{13}{16}$

24. $\frac{\frac{9}{10}}{\frac{3}{5}} = \frac{9}{10} \div \frac{3}{5} = \frac{\cancel{9}_3}{\cancel{10}_2} \cdot \frac{5}{3} = \frac{3}{2} = 1\frac{1}{2}$

25. $\frac{\frac{5}{6}}{\frac{25}{24}} = \frac{5}{6} \div \frac{25}{24} = \frac{5}{\cancel{6}_2} \cdot \frac{\cancel{24}_4}{25} = \frac{4}{5}$

26. $\frac{\frac{28}{15}}{\frac{21}{5}} = \frac{28}{15} \div \frac{21}{5} = \frac{\cancel{28}_4}{\cancel{15}_3} \cdot \frac{5}{\cancel{21}_3} = \frac{4}{9}$

27. $12 \div \frac{2}{3} = \frac{12}{1} \div \frac{2}{3} = \frac{\cancel{12}_6}{1} \cdot \frac{3}{2} = \frac{18}{1} = 18$

28. $7 \div \frac{1}{4} = \frac{7}{1} \div \frac{1}{4} = \frac{7}{1} \cdot \frac{4}{1} = 28$

29. $\frac{18}{\frac{3}{4}} = 18 \div \frac{3}{4}$ Rewrite using the $\div$ symbol for division.

$= \frac{18}{1} \div \frac{3}{4}$ Write 18 as $\frac{18}{1}$.

$= \frac{\cancel{18}_6}{1} \cdot \frac{4}{3}$ Change " $\div$ " to " $\cdot$ ".
Divide the numerator and denominator by 3.

$= \frac{6 \cdot 4}{1 \cdot 1}$ Multiply.
 $= \frac{24}{1} = 24$

30. $\frac{12}{\frac{3}{4}} = 12 \div \frac{3}{4} = \frac{12}{1} \div \frac{3}{4} = \frac{\cancel{12}_4}{1} \cdot \frac{4}{3} = 16$

31. $\frac{\frac{4}{7}}{\frac{8}{7}} = \frac{4}{7} \div \frac{8}{7} = \frac{4}{7} \div \frac{8}{1} = \frac{\cancel{4}_2}{7} \cdot \frac{1}{8} = \frac{1}{14}$

32. $\frac{\frac{7}{10}}{\frac{3}{10}} = \frac{7}{10} \div \frac{3}{10} = \frac{7}{10} \div \frac{3}{1} = \frac{7}{10} \cdot \frac{1}{3} = \frac{7}{30}$

33. $\frac{8}{9}$ of a quart divided into 4 parts:

$$\frac{8}{9} \div 4 = \frac{8}{9} \div \frac{4}{1} = \frac{8}{9} \cdot \frac{1}{4} = \frac{2}{9}$$

Each horse will get $\frac{2}{9}$ of a quart.

34. Divide the number of quarts of shampoo by the fraction of a quart each container holds.

$$15 \div \frac{3}{8} = \frac{15}{1} \cdot \frac{8}{3} = 40$$

Harold can fill 40 containers.

35. *Step 1:* The problem asks for the number of times a measuring cup needs to be filled.

Step 2: Solve the problem by dividing the total number of cups (5) by the size of the measuring cup ($\frac{1}{3}$).

Step 3: Because there are three $\frac{1}{3}$ -cups in one cup, multiply 3 by 5 to get 15. So, our estimate is 15.

$$\text{Step 4: } 5 \div \frac{1}{3} = \frac{5}{1} \div \frac{1}{3} = \frac{5}{1} \cdot \frac{3}{1} = 15$$

Step 5: They need to fill the measuring cup 15 times.

Step 6: The exact answer, 15, is the same as our estimate.

36. Divide the total number of pounds of jelly beans by the size of the bag.

$$408 \div \frac{3}{8} = \frac{408}{1} \div \frac{3}{8} = \frac{408}{1} \cdot \frac{8}{3} = 1088$$

1088 $\frac{3}{8}$ -pound bags can be filled.

37. Divide the total number of cups of sugar by the amount of sugar in each loaf.

$$11 \div \frac{1}{4} = \frac{11}{1} \div \frac{1}{4} = \frac{11}{1} \cdot \frac{4}{1} = 44$$

You can make 44 loaves.

38. Divide the number of pounds of peanuts by the fraction of pounds of peanuts each person will likely eat.

$$10 \div \frac{5}{16} = \frac{10}{1} \cdot \frac{16}{5} = 32$$

32 guests can be served with 10 pounds of peanuts.

39. Divide the total weight of a carton by the weight per fastener.

$$25 \div \frac{5}{32} = \frac{25}{1} \cdot \frac{32}{5} = \frac{160}{1} = 160$$

There are 160 $\frac{5}{32}$ -pound fasteners in each carton.

40. Divide the total acreage by the acreage per lot.

$$210 \div \frac{3}{4} = \frac{210}{1} \cdot \frac{4}{3} = \frac{280}{1} = 280$$

There are 280 $\frac{3}{4}$ -acre lots in the subdivision.

41. Answers will vary. A sample answer follows:

You can divide two fractions by multiplying the first fraction by the reciprocal of the second fraction (divisor).

42. Sometimes the answer is less and sometimes it is greater.

$$\frac{1}{4} \div \frac{7}{8} = \frac{1}{4} \cdot \frac{8}{7} = \frac{2}{7} \quad (\text{less than } \frac{7}{8}, \text{ but not less than } \frac{1}{4})$$

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \cdot \frac{4}{1} = 2 \quad (\text{greater})$$

43. Each loafcake requires $\frac{3}{4}$ pound of jellybeans, so to make 16 loafcakes, use multiplication.

$$\frac{3}{4} \cdot 16 = \frac{3}{4} \cdot \frac{16}{1} = 12$$

12 pounds will be needed.

44. We want $\frac{7}{8}$ of 1520 patients—use multiplication.

$$\frac{7}{8} \cdot 1520 = \frac{7}{8} \cdot \frac{1520}{1} = \frac{1330}{1} = 1330$$

1330 patients were still taking their drugs.

45. Divide the 156 cans of compound by the fraction of a can needed for each new home.

$$156 \div \frac{3}{4} = \frac{156}{1} \cdot \frac{4}{3} = \frac{52}{1} \cdot \frac{4}{1} = 208$$

208 homes can be plumbed.

46. Divide the 50 gallons of differential fluid by the fraction of a gallon needed for each car serviced.

$$50 \div \frac{2}{3} = \frac{50}{1} \cdot \frac{3}{2} = \frac{\cancel{50}^{25}}{1} \cdot \frac{3}{\cancel{2}_1} = 75$$

75 cars can be serviced.

47. (a) In $\frac{2}{3}$ of the 186 visits, doctors failed to discuss the issues—use multiplication.

$$\frac{2}{3} \cdot 186 = \frac{2}{\cancel{3}_1} \cdot \frac{\cancel{186}^{62}}{1} = 124$$

The doctors failed to discuss the issues in 124 visits.

- (b) The doctors *did* discuss the issues in $186 - 124 = 62$ visits.

48. (a) $\frac{4}{5}$ of the 285 miles have been completed—use multiplication.

$$\frac{4}{5} \cdot 285 = \frac{4}{\cancel{5}_1} \cdot \frac{\cancel{285}^{57}}{1} = 228$$

He has gone 228 miles.

- (b) The number of miles that remain is $285 - 228 = 57$ miles.

49. *Step 1:* The problem asks for the number of towels that can be made.

Step 2: Solve the problem by dividing the 912 yards of fabric by the fraction of a yard ($\frac{3}{8}$) needed for each dish towel.

Step 3: Round 912 yards to 900 yards. Round $\frac{3}{8}$ to $\frac{1}{2}$.

$$\text{Step 4: } 912 \div \frac{3}{8} = \frac{912}{1} \cdot \frac{8}{3} = \frac{\cancel{912}^{304}}{1} \cdot \frac{8}{\cancel{3}_1} = 2432$$

Step 5: 2432 towels can be made.

Step 6: The exact answer, 2432, is close to our estimate, 1800.

50. Multiply the number of actors by the fraction of actors who land a role.

$$300 \cdot \frac{1}{60} = \frac{\cancel{300}^5}{1} \cdot \frac{1}{\cancel{60}_{10}} = 5$$

There are 5 lead roles being offered.

51. The indicator words for multiplication are underlined below.

more than	per
<u>double</u>	<u>twice</u>
<u>times</u>	<u>product</u>
less than	difference
equals	<u>twice as much</u>

52. The indicator words for division are underlined below.

fewer	sum of
<u>goes into</u>	<u>divide</u>
<u>per</u>	<u>quotient</u>
equals	double
loss of	<u>divided by</u>

53. Two numbers are reciprocals of each other if their product is 1.

54. The number 0 has no reciprocal because there is no number that can be multiplied by 0 to get 1.

55. To divide two fractions, multiply the first fraction by the reciprocal of the second fraction.

56. The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$ because

$$\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12} = 1.$$

The reciprocal of $\frac{7}{8}$ is $\frac{8}{7}$ because

$$\frac{7}{8} \cdot \frac{8}{7} = \frac{56}{56} = 1.$$

The reciprocal of 5 is $\frac{1}{5}$ because $\frac{5}{1} \cdot \frac{1}{5} = \frac{5}{5} = 1.$

The reciprocal of $\frac{12}{19}$ is $\frac{19}{12}$ because

$$\frac{12}{19} \cdot \frac{19}{12} = \frac{228}{228} = 1.$$

57. (a) To find the perimeter of any flat equal-sided 3-, 4-, 5-, or 6-sided figure, multiply the length of one side by 3, 4, 5, or 6, respectively.

- (b) The stamp has four sides, so multiply $\frac{15}{16}$ by 4.

$$\frac{15}{16} \times 4 = \frac{15}{\cancel{16}_4} \times \frac{\cancel{4}^1}{1} = \frac{15}{4} = 3\frac{3}{4} \text{ in.}$$

The perimeter of the stamp is $3\frac{3}{4}$ inches.

58. Area = length • width

$$= \frac{15}{16} \cdot \frac{15}{16} = \frac{225}{256}$$

The area is $\frac{225}{256}$ in.². Multiply the length by the width to find the area of any rectangle.

2.8 Multiplying and Dividing Mixed Numbers

2.8 Margin Exercises

1. (a) $4\frac{2}{3} \leftarrow$ Half of 3 is $1\frac{1}{2}$.
2 is more than half of 3.
 $4\frac{2}{3}$ rounds up to 5.

- (b) $3\frac{2}{5} \leftarrow$ Half of 5 is $2\frac{1}{2}$.
2 is less than of 5.
 $3\frac{2}{5}$ rounds down to 3.

- (c) $5\frac{3}{4} \leftarrow$ Half of 4 is 2.
3 is more than half of 4.
 $5\frac{3}{4}$ rounds up to 6.

- (d) $4\frac{7}{12} \leftarrow$ Half of 12 is 6.
7 is more than half of 12.
 $4\frac{7}{12}$ rounds up to 5.

- (e) $1\frac{1}{2} \leftarrow$ Half of 2 is 1.
1 is the same as half of 2.
 $1\frac{1}{2}$ rounds up to 2.

- (f) $8\frac{4}{9} \leftarrow$ Half of 9 is $4\frac{1}{2}$.
4 is less than half of 9.
 $8\frac{4}{9}$ rounds down to 8.

2. (a) $3\frac{1}{4} \cdot 6\frac{2}{3}$
Estimate: $3\frac{1}{4}$ rounds to 3. $6\frac{2}{3}$ rounds to 7.
 $\underline{3 \cdot 7 = 21}$

Exact: $3\frac{1}{4} \cdot 6\frac{2}{3} = \frac{13}{4} \cdot \frac{20}{3} = \frac{65}{3} = 21\frac{2}{3}$

- (b) $4\frac{2}{3} \cdot 2\frac{3}{4}$
Estimate: $4\frac{2}{3}$ rounds to 5. $2\frac{3}{4}$ rounds to 3.
 $\underline{5 \cdot 3 = 15}$

Exact:

$$4\frac{2}{3} \cdot 2\frac{3}{4} = \frac{14}{3} \cdot \frac{11}{4} = \frac{14}{3} \cdot \frac{11}{\cancel{4}_2} = \frac{77}{6} = 12\frac{5}{6}$$

- (c) $3\frac{3}{5} \cdot 4\frac{4}{9}$

Estimate: $3\frac{3}{5}$ rounds to 4. $4\frac{4}{9}$ rounds to 4.

$\underline{4 \cdot 4 = 16}$

Exact:

$$3\frac{3}{5} \cdot 4\frac{4}{9} = \frac{18}{5} \cdot \frac{40}{9} = \frac{18}{\cancel{5}_1} \cdot \frac{\cancel{40}_{10}}{\cancel{9}_3} = 16$$

- (d) $5\frac{1}{4} \cdot 3\frac{3}{5}$

Estimate: $5\frac{1}{4}$ rounds to 5. $3\frac{3}{5}$ rounds to 4.

$\underline{5 \cdot 4 = 20}$

Exact:

$$5\frac{1}{4} \cdot 3\frac{3}{5} = \frac{21}{4} \cdot \frac{18}{5} = \frac{21}{\cancel{4}_2} \cdot \frac{\cancel{18}_9}{5} = \frac{189}{10} = 18\frac{9}{10}$$

3. (a) $3\frac{1}{8} \div 6\frac{1}{4}$

Estimate: $3\frac{1}{8}$ rounds to 3. $6\frac{1}{4}$ rounds to 6.

$\underline{3 \div 6 = \frac{1}{2}}$

Exact:

$$3\frac{1}{8} \div 6\frac{1}{4} = \frac{25}{8} \div \frac{25}{4} = \frac{25}{8} \cdot \frac{4}{25} = \frac{\cancel{25}_5}{8} \cdot \frac{\cancel{4}_2}{\cancel{25}_5} = \frac{1}{2}$$

- (b) $10\frac{1}{3} \div 2\frac{1}{2}$

Estimate: $10\frac{1}{3}$ rounds to 10. $2\frac{1}{2}$ rounds to 3.

$\underline{10 \div 3 = 3\frac{1}{3}}$

Exact:

$$10\frac{1}{3} \div 2\frac{1}{2} = \frac{31}{3} \div \frac{5}{2} = \frac{31}{3} \cdot \frac{2}{5} = \frac{62}{15} = 4\frac{2}{15}$$

- (c) $8 \div 5\frac{1}{3}$

Estimate: 8 rounds to 8. $5\frac{1}{3}$ rounds to 5.

$\underline{8 \div 5 = 1\frac{3}{5}}$

Exact:

$$8 \div 5\frac{1}{3} = \frac{8}{1} \div \frac{16}{3} = \frac{8}{1} \cdot \frac{3}{\cancel{16}_8} = \frac{3}{2} = 1\frac{1}{2}$$

(d) $13\frac{1}{2} \div 18$

Estimate: $13\frac{1}{2}$ rounds to 14. 18 rounds to 18.

$$\frac{14}{18} \div \frac{18}{18} = \frac{14}{18} = \frac{7}{9}$$

Exact:

$$13\frac{1}{2} \div 18 = \frac{27}{2} \div \frac{18}{1} = \frac{27}{2} \cdot \frac{1}{\cancel{18}_2} = \frac{3}{4}$$

4. The problem asks how many quarts of paint are needed to paint 16 cars.

Multiply the amount of paint needed for each car by the number of cars.

Estimate: $2\frac{5}{8}$ rounds to 3. 16 rounds to 16.

$$3 \cdot 16 = 48$$

$$\text{Exact: } 2\frac{5}{8} \cdot 16 = \frac{21}{8} \cdot \frac{\cancel{16}^2}{1} = \frac{42}{1} = 42$$

42 quarts are needed for 16 cars. The answer is reasonably close to the estimate.

5. (a) The problem asks how many propellers can be manufactured from 57 pounds of brass.

Divide the total pounds of brass by the number of pounds needed for one engine.

Estimate: 57 rounds to 57. $4\frac{3}{4}$ rounds to 5.

$$57 \div 5 \approx 11$$

Exact:

$$57 \div 4\frac{3}{4} = \frac{57}{1} \div \frac{19}{4} = \frac{57}{1} \cdot \frac{4}{\cancel{19}_1} = \frac{12}{1} = 12$$

12 propellers can be manufactured from 57 pounds of brass. The answer is reasonably close to the estimate.

- (b) The problem asks the number of oil changes that can be made with 609 quarts of oil.

Divide the total number of quarts by the number of quarts needed for each oil change.

Estimate: 609 rounds to 600. $21\frac{3}{4}$ rounds to 22.

$$600 \div 22 \approx 27$$

Exact:

$$609 \div 21\frac{3}{4} = 609 \div \frac{87}{4} = \frac{609}{1} \cdot \frac{4}{\cancel{87}_3} = 28$$

28 oil changes can be made with 609 quarts of oil. The answer is reasonably close to the estimate.

2.8 Section Exercises

- The statement "When multiplying two mixed numbers, the reciprocal of the second mixed number must be used." is *false*. A reciprocal is used when *dividing* fractions, not *multiplying* fractions.
- The statement "If you were dividing a mixed number by the whole number 10, the reciprocal of 10 would be $\frac{10}{1}$." is *false*. The reciprocal of 10 is $\frac{1}{10}$.
- The statement "When rounding a mixed number before estimating the answer, round up to the next whole number if the numerator of the fraction part is half of the denominator or more" is *true*.
- The statement "When you round mixed numbers to estimate the answer to a problem, the estimated answer can vary quite a bit from the exact answer. However, it can still show whether the exact answer is reasonable." is *true*.

5. $4\frac{1}{2} \cdot 1\frac{3}{4}$

Estimate: $5 \cdot 2 = 10$

To find the exact answer, change each mixed number to an improper fraction and then multiply.

$$\text{Exact: } 4\frac{1}{2} \cdot 1\frac{3}{4} = \frac{9}{2} \cdot \frac{7}{4} = \frac{63}{8} = 7\frac{7}{8}$$

6. $2\frac{1}{2} \cdot 2\frac{1}{4}$

Estimate: $3 \cdot 2 = 6$

$$\text{Exact: } 2\frac{1}{2} \cdot 2\frac{1}{4} = \frac{5}{2} \cdot \frac{9}{4} = \frac{45}{8} = 5\frac{5}{8}$$

7. $1\frac{2}{3} \cdot 2\frac{7}{10}$

Estimate: $2 \cdot 3 = 6$

$$\text{Exact: } 1\frac{2}{3} \cdot 2\frac{7}{10} = \frac{5}{3} \cdot \frac{27}{10} = \frac{\cancel{5}_1}{3} \cdot \frac{\cancel{27}^9}{\cancel{10}_2} = \frac{9}{2} = 4\frac{1}{2}$$

8. $4\frac{1}{2} \cdot 2\frac{1}{4}$

Estimate: $5 \cdot 2 = 10$

$$\text{Exact: } 4\frac{1}{2} \cdot 2\frac{1}{4} = \frac{9}{2} \cdot \frac{9}{4} = \frac{81}{8} = 10\frac{1}{8}$$

9. $3\frac{1}{9} \cdot 1\frac{2}{7}$

Estimate: $3 \cdot 1 = 3$

Exact: $3\frac{1}{9} \cdot 1\frac{2}{7} = \frac{28}{9} \cdot \frac{9}{7} = \frac{\cancel{28}^4}{\cancel{9}_1} \cdot \frac{\cancel{9}_1}{7} = \frac{4}{1} = 4$

10. $6\frac{1}{4} \cdot 3\frac{1}{5}$

Estimate: $6 \cdot 3 = 18$

Exact: $6\frac{1}{4} \cdot 3\frac{1}{5} = \frac{25}{4} \cdot \frac{16}{5} = \frac{20}{1} = 20$

11. $8 \cdot 6\frac{1}{4}$

Estimate: $8 \cdot 6 = 48$

Exact: $8 \cdot 6\frac{1}{4} = \frac{8}{1} \cdot \frac{25}{4} = \frac{\cancel{8}^2}{1} \cdot \frac{25}{\cancel{4}_1} = \frac{50}{1} = 50$

12. $6 \cdot 2\frac{1}{3}$

Estimate: $6 \cdot 2 = 12$

Exact: $6 \cdot 2\frac{1}{3} = \frac{6}{1} \cdot \frac{7}{3} = \frac{14}{1} = 14$

13. $4\frac{1}{2} \cdot 2\frac{1}{5} \cdot 5$

Estimate: $5 \cdot 2 \cdot 5 = 10 \cdot 5 = 50$

Exact: $4\frac{1}{2} \cdot 2\frac{1}{5} \cdot 5 = \frac{9}{2} \cdot \frac{11}{5} \cdot \frac{\cancel{5}_1}{1} = \frac{99}{2} = 49\frac{1}{2}$

14. $5\frac{1}{2} \cdot 1\frac{1}{3} \cdot 2\frac{1}{4}$

Estimate: $6 \cdot 1 \cdot 2 = 12$

Exact: $5\frac{1}{2} \cdot 1\frac{1}{3} \cdot 2\frac{1}{4} = \frac{11}{2} \cdot \frac{\cancel{4}_1}{3} \cdot \frac{\cancel{3}_1}{2} = \frac{33}{2} = 16\frac{1}{2}$

15. $3 \cdot 1\frac{1}{2} \cdot 2\frac{2}{3}$

Estimate: $3 \cdot 2 \cdot 3 = 6 \cdot 3 = 18$

Exact: $3 \cdot 1\frac{1}{2} \cdot 2\frac{2}{3} = \frac{\cancel{3}_1}{1} \cdot \frac{\cancel{3}_1}{2} \cdot \frac{\cancel{4}_1}{3} = \frac{12}{1} = 12$

16. $\frac{2}{3} \cdot 3\frac{2}{3} \cdot \frac{6}{11}$

Estimate: $1 \cdot 4 \cdot 1 = 4$

Exact: $\frac{2}{3} \cdot 3\frac{2}{3} \cdot \frac{6}{11} = \frac{2}{3} \cdot \frac{\cancel{11}_1}{3} \cdot \frac{\cancel{6}_1}{\cancel{11}_1} = \frac{4}{3} = 1\frac{1}{3}$

17. $3\frac{1}{4} \cdot 7\frac{5}{8}$ *Estimate:* $3 \cdot 8 = 24$

The best estimate is choice **(d)**.

18. $5\frac{3}{4} \cdot 2\frac{2}{5}$ *Estimate:* $6 \cdot 2 = 12$

The best estimate is choice **(a)**.

19. $2\frac{1}{8} \div 1\frac{3}{4}$ *Estimate:* $2 \div 2 = 1$

The best estimate is choice **(b)**.

20. $8\frac{3}{5} \div 2\frac{3}{8}$ *Estimate:* $9 \div 2 = 4\frac{1}{2}$

The best estimate is choice **(c)**.

21. $1\frac{1}{4} \div 3\frac{3}{4}$

Estimate: $1 \div 4 = \frac{1}{4}$

To find the exact answer, first change each mixed number to an improper fraction. Then, use the reciprocal of the divisor (the second fraction) and multiply.

Exact: $1\frac{1}{4} \div 3\frac{3}{4} = \frac{5}{4} \div \frac{15}{4} = \frac{\cancel{5}_1}{4} \cdot \frac{\cancel{4}_1}{\cancel{15}_3} = \frac{1}{3}$

22. $1\frac{1}{8} \div 2\frac{1}{4}$

Estimate: $1 \div 2 = \frac{1}{2}$

Exact: $1\frac{1}{8} \div 2\frac{1}{4} = \frac{9}{8} \div \frac{9}{4} = \frac{\cancel{9}_1}{8} \cdot \frac{\cancel{4}_1}{\cancel{9}_1} = \frac{1}{2}$

23. $2\frac{1}{2} \div 3$

Estimate: $3 \div 3 = 1$

Exact: $2\frac{1}{2} \div 3 = \frac{5}{2} \div \frac{3}{1} = \frac{5}{2} \cdot \frac{1}{3} = \frac{5}{6}$

24. $2\frac{3}{4} \div 2$

Estimate: $3 \div 2 = \frac{3}{2} = 1\frac{1}{2}$

Exact:

$2\frac{3}{4} \div 2 = \frac{11}{4} \div \frac{2}{1} = \frac{11}{4} \cdot \frac{1}{2} = \frac{11}{8} = 1\frac{3}{8}$

25. $9 \div 2\frac{1}{2}$

Estimate: $9 \div 3 = 3$

Exact: $9 \div 2\frac{1}{2} = \frac{9}{1} \div \frac{5}{2} = \frac{9}{1} \cdot \frac{2}{5} = \frac{18}{5} = 3\frac{3}{5}$

26. $5 \div 1\frac{7}{8}$

Estimate: $5 \div 2 = \frac{5}{2} = 2\frac{1}{2}$

Exact: $5 \div 1\frac{7}{8} = \frac{5}{1} \div \frac{15}{8} = \frac{5}{1} \cdot \frac{8}{15} = \frac{8}{3} = 2\frac{2}{3}$

27. $\frac{5}{8} \div 1\frac{1}{2}$

Estimate: $1 \div 2 = \frac{1}{2}$

Exact: $\frac{5}{8} \div 1\frac{1}{2} = \frac{5}{8} \div \frac{3}{2} = \frac{5}{8} \cdot \frac{2}{3} = \frac{5}{12}$

28. $\frac{3}{4} \div 2\frac{1}{2}$

Estimate: $1 \div 3 = \frac{1}{3}$

Exact: $\frac{3}{4} \div 2\frac{1}{2} = \frac{3}{4} \div \frac{5}{2} = \frac{3}{4} \cdot \frac{2}{5} = \frac{3}{10}$

29. $1\frac{7}{8} \div 6\frac{1}{4}$

Estimate: $2 \div 6 = \frac{2}{6} = \frac{1}{3}$

Exact: $1\frac{7}{8} \div 6\frac{1}{4} = \frac{15}{8} \div \frac{25}{4} = \frac{15}{8} \cdot \frac{4}{25} = \frac{3}{10}$

30. $8\frac{2}{5} \div 3\frac{1}{2}$

Estimate: $8 \div 4 = 2$

Exact:

$$8\frac{2}{5} \div 3\frac{1}{2} = \frac{42}{5} \div \frac{7}{2} = \frac{42}{5} \cdot \frac{2}{7} = \frac{12}{5} = 2\frac{2}{5}$$

31. $5\frac{2}{3} \div 6$

Estimate: $6 \div 6 = 1$

Exact: $5\frac{2}{3} \div 6 = \frac{17}{3} \div \frac{6}{1} = \frac{17}{3} \cdot \frac{1}{6} = \frac{17}{18}$

32. $5\frac{3}{4} \div 2$

Estimate: $6 \div 2 = 3$

Exact:

$$5\frac{3}{4} \div 2 = \frac{23}{4} \div \frac{2}{1} = \frac{23}{4} \cdot \frac{1}{2} = \frac{23}{8} = 2\frac{7}{8}$$

33. Multiply each amount by $2\frac{1}{2}$.

(a) Blueberries: $\frac{2}{3}$ cup

Estimate: $1 \cdot 3 = 3$ cups

Exact: $\frac{2}{3} \cdot 2\frac{1}{2} = \frac{2}{3} \cdot \frac{5}{2} = \frac{5}{3} = 1\frac{2}{3}$ cups

(b) Salt: $\frac{1}{2}$ tsp.

Estimate: $1 \cdot 3 = 3$ tsp.

Exact: $\frac{1}{2} \cdot 2\frac{1}{2} = \frac{1}{2} \cdot \frac{5}{2} = \frac{5}{4} = 1\frac{1}{4}$ tsp.

(c) Flour: $1\frac{1}{4}$ cups

Estimate: $1 \cdot 3 = 3$ cups

Exact: $1\frac{1}{4} \cdot 2\frac{1}{2} = \frac{5}{4} \cdot \frac{5}{2} = \frac{25}{8} = 3\frac{1}{8}$ cups

34. Multiply each amount by $1\frac{1}{2}$.

(a) Flour: $1\frac{1}{4}$ cups

Estimate: $1 \cdot 2 = 2$ cups

Exact: $1\frac{1}{4} \cdot 1\frac{1}{2} = \frac{5}{4} \cdot \frac{3}{2} = \frac{15}{8} = 1\frac{7}{8}$ cups

(b) Blueberries: $\frac{2}{3}$ cup

Estimate: $1 \cdot 2 = 2$ cups

Exact: $\frac{2}{3} \cdot 1\frac{1}{2} = \frac{2}{3} \cdot \frac{3}{2} = 1$ cup

(c) Almond Extract: $\frac{1}{2}$ teaspoon

Estimate: $1 \cdot 2 = 2$ teaspoons

Exact: $\frac{1}{2} \cdot 1\frac{1}{2} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$ teaspoon

35. Divide each amount by 2.

(a) Vanilla extract: $\frac{1}{2}$ tsp.

Estimate: $1 \div 2 = \frac{1}{2}$ tsp.

Exact: $\frac{1}{2} \div 2 = \frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ tsp.

(b) Blueberries: $\frac{2}{3}$ cup*Estimate:* $1 \div 2 = \frac{1}{2}$ cup*Exact:* $\frac{2}{3} \div 2 = \frac{2}{3} \div \frac{2}{1} = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$ cup**(c) Flour:** $1\frac{1}{4}$ cups*Estimate:* $1 \div 2 = \frac{1}{2}$ cup*Exact:* $1\frac{1}{4} \div 2 = \frac{5}{4} \div \frac{2}{1} = \frac{5}{4} \cdot \frac{1}{2} = \frac{5}{8}$ cup**36.** Divide each amount by 3.**(a) Flour:** $1\frac{1}{4}$ cups*Estimate:* $1 \div 3 = \frac{1}{3}$ cup*Exact:* $1\frac{1}{4} \div 3 = \frac{5}{4} \div \frac{3}{1} = \frac{5}{4} \cdot \frac{1}{3} = \frac{5}{12}$ cup**(b) Salt:** $\frac{1}{2}$ tsp.*Estimate:* $1 \div 3 = \frac{1}{3}$ teaspoon*Exact:* $\frac{1}{2} \div 3 = \frac{1}{2} \div \frac{3}{1} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$ teaspoon**(c) Blueberries:** $\frac{2}{3}$ cup*Estimate:* $1 \div 3 = \frac{1}{3}$ cup*Exact:* $\frac{2}{3} \div 3 = \frac{2}{3} \div \frac{3}{1} = \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$ cup**37.** Divide the number of gallons available by the number of gallons needed for each unit.*Estimate:* $329 \div 12 \approx 27$ units*Exact:*

$$329 \div 11\frac{3}{4} = \frac{329}{1} \div \frac{47}{4} = \frac{329}{1} \cdot \frac{4}{47} = 28$$

28 units can be painted with 329 gallons of paint.

38. Divide the number of total minutes by the number of minutes per student.*Estimate:* $480 \div 8 = 60$ students*Exact:*

$$480 \div 7\frac{1}{2} = \frac{480}{1} \div \frac{15}{2} = \frac{480}{1} \cdot \frac{2}{15} = \frac{64}{1} = 64$$

There are 64 students in the class.

39. Each handle requires $19\frac{1}{2}$ inches of steel tubing. Use multiplication.*Estimate:* $45 \cdot 20 = 900$ in.*Exact:* $45 \cdot 19\frac{1}{2} = \frac{45}{1} \cdot \frac{39}{2} = \frac{1755}{2} = 877\frac{1}{2}$ in. $877\frac{1}{2}$ inches of steel tubing is needed to make 45 jacks.**40.** Assume that the $61\frac{1}{2}$ inch length listed in the overall dimensions is the length of the handle. Use multiplication.*Estimate:* $182 \cdot 62 = 11,284$ in.*Exact:* $182 \cdot 61\frac{1}{2} = \frac{182}{1} \cdot \frac{123}{2} = 11,193$ in.

The amount of wood that is necessary to make 182 handles is 11,193 inches.

41. The answer should include:*Step 1*

Change mixed numbers to improper fractions.

Step 2

Multiply the fractions.

Step 3

Write the answer in lowest terms, changing to mixed or whole numbers where possible.

42. The additional step is to use the reciprocal of the second fraction (divisor).**43.** Multiply the amount of money for each cell phone times the number of cell phones to get the total amount of money from the sale of gold.*Estimate:* $\$1 \cdot 130$ million = \$130 million*Exact:*

$$1\frac{2}{5} \cdot 130 \text{ million} = \frac{7}{5} \cdot \frac{130}{1} = \frac{910}{5} = 182 = 182 \text{ million}$$

You would have \$182 million from the sale of the gold.

44. Divide the number of square yards of carpet by the amount of carpet needed for each apartment unit.*Estimate:* $6750 \div 63 \approx 107$ units*Exact:*

$$6750 \div 62\frac{1}{2} = \frac{6750}{1} \div \frac{125}{2} = \frac{6750}{1} \cdot \frac{2}{125} = 108$$

108 units can be carpeted.

45. Divide the total amount of firewood to be delivered by the amount of firewood that can be delivered per trip.

Estimate: $140 \div 1 = 140$ trips

$$\text{Exact: } 140 \div 1\frac{1}{4} = \frac{140}{1} \div \frac{5}{4} = \frac{140}{1} \cdot \frac{4}{5} = 112$$

112 trips will be needed to deliver 140 cords of firewood.

46. Divide the total amount of roofing material by the amount of roofing material needed for each roof.

Estimate: $1827 \div 32 \approx 57$ homes

Exact:

$$1827 \div 31\frac{1}{2} = \frac{1827}{1} \div \frac{63}{2} = \frac{1827}{1} \cdot \frac{2}{63} = 58$$

58 homes can be re-roofed with 1827 squares of roofing material.

47. (a) The maximum height of the standard jack is $17\frac{3}{4}$ inches. Use multiplication.

Estimate: $18 \cdot 4 = 72$ in.

$$\text{Exact: } 17\frac{3}{4} \cdot 4 = \frac{71}{4} \cdot \frac{4}{1} = \frac{71}{1} = 71 \text{ in.}$$

The hydraulic lift must raise the car 71 inches.

(b) There are 12 inches in a foot, so the 6-foot-tall mechanic is $6 \times 12 = 72$ inches tall. So no, the mechanic can not stand under the car without bending.

48. (a) The maximum height of the low-profile jack is $15\frac{1}{16}$ inches. Use division.

Estimate: $15 \div 3 = 5$ in.

$$\text{Exact: } 15\frac{1}{16} \div 3 = \frac{241}{16} \cdot \frac{1}{3} = \frac{241}{48} = 5\frac{1}{48} \text{ in.}$$

The low-profile lift must raise the car $5\frac{1}{48}$ inches.

(b) No, because 6 in. is greater than $5\frac{1}{48}$ in.

49. Multiply the swimming speed of the person times the number of times faster that a shark can swim than a person.

Estimate: $6 \cdot 6 = 36$ miles per hour

$$\text{Exact: } 6\frac{1}{8} \cdot 5\frac{1}{2} = \frac{49}{8} \cdot \frac{11}{2} = \frac{539}{16} = 33\frac{11}{16}$$

The shark can swim $33\frac{11}{16}$ miles per hour.

50. Multiply the numbers of cubic yards of dirt per truck times the number of truckloads.

Estimate: $13 \cdot 237 = 3081$ boxes

$$\text{Exact: } 13\frac{1}{3} \cdot 237 = \frac{40}{3} \cdot \frac{237}{1} = \frac{3160}{1} = 3160$$

3160 cubic yards of dirt are needed.

51. (a) The recipe makes four Alaska burgers, so divide the $\frac{1}{3}$ -pound by 4.

$$\frac{1}{3} \div 4 = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$$

Each $\frac{1}{2}$ -inch slice of cheese weighs $\frac{1}{12}$ pound.

(b) There are four $\frac{1}{2}$ -inch slices in the $\frac{1}{3}$ -pound brick, so multiply 4 times $\frac{1}{2}$.

$$4 \cdot \frac{1}{2} = \frac{4}{1} \cdot \frac{1}{2} = \frac{2}{1} = 2$$

A $\frac{1}{3}$ -pound brick of smoked cheddar cheese is 2 inches thick.

52. (a) One good pinch of allspice is equal to $\frac{1}{4}$ teaspoon, so multiply 8 times $\frac{1}{4}$.

$$8 \cdot \frac{1}{4} = \frac{8}{1} \cdot \frac{1}{4} = \frac{2}{1} = 2$$

There are 2 teaspoons of allspice in 8 good pinches.

(b) Two good pinches of ground cumin is equal to $\frac{1}{2}$ teaspoon, so first divide 7 by 2 to find the number of "two-pinches" there are.

$$7 \div 2 = \frac{7}{2} \text{ "two-pinches"}$$

Now multiply $\frac{7}{2}$ by $\frac{1}{2}$ to find the number of teaspoons.

$$\frac{7}{2} \cdot \frac{1}{2} = \frac{7}{4} = 1\frac{3}{4} \text{ teaspoons}$$

There are $1\frac{3}{4}$ teaspoons of ground cumin in 7 good pinches.

53. The recipe makes four Alaska burgers, so to prepare 18 burgers, multiply the ingredient amounts by $\frac{18}{4} = \frac{9}{2} = 4\frac{1}{2}$.

54. (a) The five large eaters will eat

$$5 \cdot 1\frac{1}{2} = \frac{5}{1} \cdot \frac{3}{2} = \frac{15}{2} = 7\frac{1}{2} \text{ burgers.}$$

The five children will eat

$$5 \cdot \frac{1}{2} = \frac{5}{1} \cdot \frac{1}{2} = \frac{5}{2} = 2\frac{1}{2} \text{ burgers.}$$

There are $15 - (5 + 5) = 5$ guests who will eat 1 burger each, for a total of 5 burgers. So, you will need

$$7\frac{1}{2} + 2\frac{1}{2} + 5 = 15 \text{ burgers.}$$

(b) The recipe makes four Alaska burgers, so to prepare 15 burgers, multiply the ingredient amounts by $\frac{15}{4} = 3\frac{3}{4}$.

55. To find the ingredient amounts for 9 servings, multiply each amount in the recipe by
- $\frac{9}{4}$
- .

Lean ground beef: $1 \cdot \frac{9}{4} = \frac{9}{4} = 2\frac{1}{4}$ pounds

Spanish onions: $\frac{1}{2} \cdot \frac{9}{4} = \frac{9}{8} = 1\frac{1}{8}$ onions

Worcestershire sauce:

$$4 \cdot \frac{9}{4} = \frac{4}{1} \cdot \frac{9}{4} = \frac{9}{1} = 9 \text{ shakes}$$

Allspice: $\frac{1}{4} \cdot \frac{9}{4} = \frac{9}{16}$ tsp

Ground cumin: $\frac{1}{2} \cdot \frac{9}{4} = \frac{9}{8} = 1\frac{1}{8}$ tsp

Cheddar cheese: $\frac{1}{3} \cdot \frac{9}{4} = \frac{3}{4}$ pound

56. One pound of beef makes 4 Alaska burgers, so
- $5\frac{3}{4}$
- pounds make

$$4 \cdot 5\frac{3}{4} = \frac{4}{1} \cdot \frac{23}{4} = 23 \text{ burgers}$$

Improper fractions have numerator (top) larger than or equal to the denominator (bottom). They are $\frac{4}{3}, \frac{5}{5}$.

5. Proper fractions have numerator (top) smaller than denominator (bottom). They are
- $\frac{15}{16}, \frac{1}{8}$
- .

Improper fractions have numerator (top) larger than or equal to the denominator (bottom). They are $\frac{6}{5}, \frac{16}{13}, \frac{5}{3}$.

$$6. \quad 4\frac{3}{4} \quad 4 \cdot 4 = 16$$

$$16 + 3 = 19$$

$$4\frac{3}{4} = \frac{19}{4}$$

$$7. \quad 9\frac{5}{6} \quad 9 \cdot 6 = 54$$

$$54 + 5 = 59$$

$$9\frac{5}{6} = \frac{59}{6}$$

$$8. \quad \frac{27}{8}$$

$$\begin{array}{r} 3 \leftarrow \text{Whole number part} \\ 8 \overline{) 27} \\ \underline{24} \\ 3 \leftarrow \text{Remainder} \end{array}$$

$$\frac{27}{8} = 3\frac{3}{8}$$

$$9. \quad \frac{63}{5}$$

$$\begin{array}{r} 12 \leftarrow \text{Whole number part} \\ 5 \overline{) 63} \\ \underline{5} \\ 13 \\ \underline{10} \\ 3 \leftarrow \text{Remainder} \end{array}$$

$$\frac{63}{5} = 12\frac{3}{5}$$

10. Factorizations of 6:

$$1 \cdot 6 = 6 \quad 2 \cdot 3 = 6$$

The factors of 6 are 1, 2, 3, and 6.

11. Factorizations of 24:

$$1 \cdot 24 = 24 \quad 2 \cdot 12 = 24 \quad 3 \cdot 8 = 24 \quad 4 \cdot 6 = 24$$

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

12. Factorizations of 55:

$$1 \cdot 55 = 55 \quad 5 \cdot 11 = 55$$

The factors of 55 are 1, 5, 11, and 55.

Chapter 2 Review Exercises

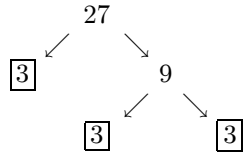
- $\frac{1}{3}$ and 1 is shaded.
- $\frac{5}{8}$ and 5 are shaded.
- $\frac{2}{4}$ and 2 are shaded.
- Proper fractions have numerator (top) smaller than denominator (bottom). They are $\frac{1}{8}, \frac{3}{4}, \frac{2}{3}$.

13. Factorizations of 90:

$$1 \cdot 90 = 90 \quad 2 \cdot 45 = 90 \quad 3 \cdot 30 = 90 \\ 5 \cdot 18 = 90 \quad 6 \cdot 15 = 90 \quad 9 \cdot 10 = 90$$

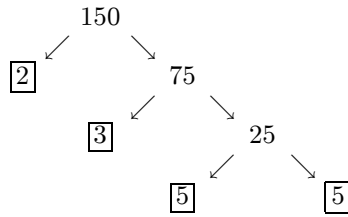
The factors of 90 are 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, and 90.

- 14.



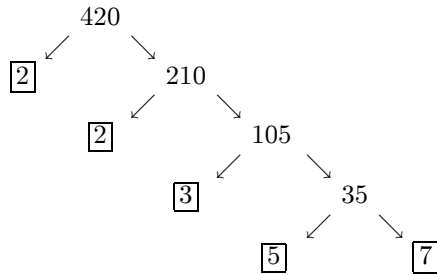
$$27 = 3 \cdot 3 \cdot 3 = 3^3$$

- 15.



$$150 = 2 \cdot 3 \cdot 5 \cdot 5 = 2 \cdot 3 \cdot 5^2$$

- 16.



$$420 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7 = 2^2 \cdot 3 \cdot 5 \cdot 7$$

17. $5^2 = 5 \cdot 5 = 25$

18. $6^2 \cdot 2^3 = 36 \cdot 8 = 288$

19. $8^2 \cdot 3^3 = 64 \cdot 27 = 1728$

20. $4^3 \cdot 2^5 = 64 \cdot 32 = 2048$

21. All 24 parts out of a possible 24 parts are gold.

$$\frac{24}{24} = 1$$

22. 18 of the possible $18 + 6 = 24$ parts are gold.

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

23. 14 of the possible $14 + 10 = 24$ parts are gold.

$$\frac{14}{24} = \frac{14 \div 2}{24 \div 2} = \frac{7}{12}$$

24. 10 of the possible $10 + 14 = 24$ parts are gold.

$$\frac{10}{24} = \frac{10 \div 2}{24 \div 2} = \frac{5}{12}$$

25. $\frac{25}{60} = \frac{\frac{1}{\cancel{3}} \cdot 5}{2 \cdot 2 \cdot 3 \cdot \cancel{3}} = \frac{5}{12}$

26. $\frac{384}{96} = \frac{\frac{1}{\cancel{2}} \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot 2 \cdot \frac{1}{\cancel{3}}}{\frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{3}}{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1}} = \frac{4}{1} = 4$

27. $\frac{3}{4}$ and $\frac{48}{64}$

$$\frac{48}{64} = \frac{48 \div 16}{64 \div 16} = \frac{3}{4}$$

The fractions are equivalent ($\frac{3}{4} = \frac{3}{4}$).

28. $\frac{5}{8}$ and $\frac{70}{120}$

$$\frac{70}{120} = \frac{70 \div 10}{120 \div 10} = \frac{7}{12}$$

The fractions are not equivalent ($\frac{5}{8} \neq \frac{7}{12}$).

29. $\frac{2}{3}$ and $\frac{360}{540}$

$$\frac{360}{540} = \frac{360 \div 180}{540 \div 180} = \frac{2}{3}$$

The fractions are equivalent ($\frac{2}{3} = \frac{2}{3}$).

30. $\frac{4}{5} \cdot \frac{3}{4} = \frac{\cancel{4}}{5} \cdot \frac{3}{\cancel{4}} = \frac{1 \cdot 3}{5 \cdot 1} = \frac{3}{5}$

31. $\frac{3}{10} \cdot \frac{5}{8} = \frac{3}{\cancel{10}} \cdot \frac{\frac{1}{\cancel{2}}}{8} = \frac{3 \cdot 1}{2 \cdot 8} = \frac{3}{16}$

32. $\frac{70}{175} \cdot \frac{5}{14} = \frac{\frac{1}{\cancel{5}}}{\frac{\cancel{70}}{7}} \cdot \frac{\frac{1}{\cancel{2}}}{\frac{\cancel{14}}{1}} = \frac{1 \cdot 1}{7 \cdot 1} = \frac{1}{7}$

33. $\frac{44}{63} \cdot \frac{3}{11} = \frac{\frac{4}{\cancel{11}}}{\frac{\cancel{63}}{21}} \cdot \frac{\frac{1}{\cancel{3}}}{\cancel{11}} = \frac{4 \cdot 1}{21 \cdot 1} = \frac{4}{21}$

34. $\frac{5}{16} \cdot 48 = \frac{5}{\cancel{16}} \cdot \frac{3}{\cancel{16}} = \frac{5 \cdot 3}{1 \cdot 1} = \frac{15}{1} = 15$

35. $\frac{5}{8} \cdot 1000 = \frac{5}{\frac{8}{1}} \cdot \frac{125}{1} = \frac{5 \cdot 125}{1 \cdot 1} = \frac{625}{1} = 625$

36. $\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \cdot \frac{2}{1} = \frac{2 \cdot 2}{3 \cdot 1} = \frac{4}{3} = 1\frac{1}{3}$

$$37. \quad \frac{5}{6} \div \frac{1}{2} = \frac{5}{\cancel{6}_3} \cdot \frac{2}{1} = \frac{5}{3} = 1\frac{2}{3}$$

$$38. \quad \frac{\frac{15}{18}}{\frac{10}{30}} = \frac{15}{18} \div \frac{10}{30}$$

$$= \frac{\cancel{15}^5}{\cancel{18}_6} \cdot \frac{\cancel{30}^3}{\cancel{10}_2} = \frac{5 \cdot 3}{6 \cdot 1} = \frac{5 \cdot 1}{2 \cdot 1} = \frac{5}{2} = 2\frac{1}{2}$$

$$39. \quad \frac{\frac{3}{4}}{\frac{3}{8}} = \frac{3}{4} \div \frac{3}{8} = \frac{\cancel{3}^1}{\cancel{4}_2} \cdot \frac{8}{\cancel{3}_1} = \frac{1 \cdot 2}{1 \cdot 1} = 2$$

$$40. \quad 7 \div \frac{7}{8} = \frac{7}{1} \div \frac{7}{8} = \frac{\cancel{7}^1}{1} \cdot \frac{8}{\cancel{7}_1} = \frac{1 \cdot 8}{1 \cdot 1} = 8$$

$$41. \quad 18 \div \frac{3}{4} = \frac{\cancel{18}^6}{1} \cdot \frac{4}{\cancel{3}_1} = \frac{6 \cdot 4}{1 \cdot 1} = 24$$

$$42. \quad \frac{5}{8} \div 3 = \frac{5}{8} \div \frac{3}{1} = \frac{5}{8} \cdot \frac{1}{3} = \frac{5 \cdot 1}{8 \cdot 3} = \frac{5}{24}$$

$$43. \quad \frac{2}{3} \div 5 = \frac{2}{3} \div \frac{5}{1} = \frac{2}{3} \cdot \frac{1}{5} = \frac{2 \cdot 1}{3 \cdot 5} = \frac{2}{15}$$

$$44. \quad \frac{\frac{12}{13}}{\frac{3}{1}} = \frac{12}{13} \div 3 = \frac{12}{13} \div \frac{3}{1} = \frac{\cancel{12}^4}{13} \cdot \frac{1}{\cancel{3}_1} = \frac{4 \cdot 1}{13 \cdot 1} = \frac{4}{13}$$

45. To find the area, multiply the length and the width.

$$2\frac{3}{4} \cdot \frac{1}{2} = \frac{11}{4} \cdot \frac{1}{2} = \frac{11}{8} = 1\frac{3}{8}$$

The area is $1\frac{3}{8}$ ft².

46. To find the area, multiply the length and the width.

$$4\frac{1}{2} \cdot \frac{7}{8} = \frac{9}{2} \cdot \frac{7}{8} = \frac{63}{16} = 3\frac{15}{16}$$

The area is $3\frac{15}{16}$ yd².

47. Multiply the length and width.

$$108 \cdot 72\frac{3}{4} = \frac{\cancel{108}^{27}}{1} \cdot \frac{291}{\cancel{4}_1} = 7857$$

The area is 7857 ft².

48. Multiply the length and width.

$$6 \cdot \frac{11}{12} = \frac{\cancel{6}^1}{1} \cdot \frac{11}{\cancel{12}_2} = \frac{11}{2}$$

The area is $5\frac{1}{2}$ ft².

$$49. \quad 5\frac{1}{2} \cdot 1\frac{1}{4}$$

Estimate: $6 \cdot 1 = 6$

$$\text{Exact: } 5\frac{1}{2} \cdot 1\frac{1}{4} = \frac{11}{2} \cdot \frac{5}{4} = \frac{55}{8} = 6\frac{7}{8}$$

$$50. \quad 2\frac{1}{4} \cdot 7\frac{1}{8} \cdot 1\frac{1}{3}$$

Estimate: $2 \cdot 7 \cdot 1 = 14$

$$\text{Exact: } 2\frac{1}{4} \cdot 7\frac{1}{8} \cdot 1\frac{1}{3} = \frac{9}{4} \cdot \frac{57}{8} \cdot \frac{\cancel{4}^1}{\cancel{3}_1} = \frac{171}{8} = 21\frac{3}{8}$$

$$51. \quad 15\frac{1}{2} \div 3$$

$$\text{Estimate: } 16 \div 3 = \frac{16}{3} = 5\frac{1}{3}$$

$$\text{Exact: } 15\frac{1}{2} \div 3 = \frac{31}{2} \cdot \frac{1}{3} = \frac{31}{6} = 5\frac{1}{6}$$

$$52. \quad 4\frac{3}{4} \div 6\frac{1}{3}$$

$$\text{Estimate: } 5 \div 6 = \frac{5}{6}$$

$$\text{Exact: } 4\frac{3}{4} \div 6\frac{1}{3} = \frac{19}{4} \div \frac{19}{3} = \frac{\cancel{19}^1}{4} \cdot \frac{3}{\cancel{19}_1} = \frac{3}{4}$$

53. Divide the total tons of almonds by the size of the bins.

$$\text{Estimate: } 300 \div \frac{1}{2} = 300 \cdot 2 = 600 \text{ bins}$$

$$\text{Exact: } 320 \div \frac{5}{8} = \frac{320}{1} \cdot \frac{8}{\cancel{5}_1} = 512$$

512 bins will be needed to store the almonds.

54. The 4 other equal partners own

$$1 - \frac{2}{5} = \frac{3}{5}$$

of the business. Divide that amount by 4.

$$\frac{3}{5} \div 4 = \frac{3}{5} \div \frac{4}{1} = \frac{3}{5} \cdot \frac{1}{4} = \frac{3}{20}$$

Each of the other partners owns $\frac{3}{20}$ of the business.

55. Divide the total yardage by the amount needed for each pull cord.

Estimate: $158 \div 4 \approx 40$ pull cords

Exact:

$$157\frac{1}{2} \div 4\frac{3}{8} = \frac{315}{2} \div \frac{35}{8} = \frac{315}{2} \cdot \frac{8}{35} = 36$$

36 pull cords can be made.

56. Multiply the weight per gallon times the number of aquariums times the gallons per aquarium.

Estimate: $8 \cdot 2 \cdot 50 = 800$

$$\text{Exact: } 8\frac{1}{3} \cdot 2 \cdot 50 = \frac{25}{3} \cdot \frac{2}{1} \cdot \frac{50}{1} = \frac{2500}{3}$$

The weight of the water is $\frac{2500}{3}$, or $833\frac{1}{3}$ pounds.

57. Ebony gave $\frac{1}{4}$ of 100 cookies to her neighbor.

$$\frac{1}{4} \cdot 100 = \frac{1}{4} \cdot \frac{100}{1} = \frac{1 \cdot 25}{1 \cdot 1} = \frac{25}{1} = 25 \text{ cookies}$$

Thus, $100 - 25 = 75$ cookies remain. She used $\frac{2}{3}$ of 75 cookies for the bake sale.

$$\frac{2}{3} \cdot 75 = \frac{2}{3} \cdot \frac{75}{1} = \frac{2 \cdot 25}{1 \cdot 1} = \frac{50}{1} = 50 \text{ cookies}$$

Ebony used 50 cookies for the bake sale. She has $75 - 50 = 25$ cookies left for her family.

58. Sheila paid $\frac{3}{8}$ of \$2976 for taxes, social security, and a retirement plan.

$$\frac{3}{8} \cdot 2976 = \frac{3}{8} \cdot \frac{372}{1} = 1116$$

She paid \$1116 for taxes, social security, and a retirement plan.

She paid $\frac{9}{10}$ of the remainder,
 $\$2976 - \$1116 = \$1860$, for basic living expenses.

$$\frac{9}{10} \cdot 1860 = \frac{9}{10} \cdot \frac{186}{1} = \frac{9 \cdot 186}{1 \cdot 1} = 1674$$

She has $\$1860 - \$1674 = \$186$ left.

59. $\frac{7}{8}$ must be divided by 6.

$$\frac{7}{8} \div 6 = \frac{7}{8} \div \frac{6}{1} = \frac{7}{8} \cdot \frac{1}{6} = \frac{7 \cdot 1}{8 \cdot 6} = \frac{7}{48}$$

Each school will receive $\frac{7}{48}$ of the amount raised.

60. $\frac{4}{5}$ of the catch must be divided evenly among 5 fishermen.

$$\frac{4}{5} \div 5 = \frac{4}{5} \div \frac{5}{1} = \frac{4}{5} \cdot \frac{1}{5} = \frac{4}{25}$$

Each fisherman receives $\frac{4}{25}$ ton.

Chapter 2 Mixed Review Exercises

1. $\frac{1}{2} \cdot \frac{3}{4} = \frac{1 \cdot 3}{2 \cdot 4} = \frac{3}{8}$

2. $\frac{2}{3} \cdot \frac{3}{5} = \frac{2}{3} \cdot \frac{1}{5} = \frac{2 \cdot 1}{1 \cdot 5} = \frac{2}{5}$

3. $12\frac{1}{2} \cdot 2\frac{1}{2} = \frac{25}{2} \cdot \frac{5}{2} = \frac{125}{4} = 31\frac{1}{4}$

4. $8\frac{1}{3} \cdot 3\frac{2}{5} = \frac{25}{3} \cdot \frac{17}{5} = \frac{425}{15} = \frac{85}{3} = 28\frac{1}{3}$

5. $\frac{4}{5} = \frac{4}{5} \div 8 = \frac{4}{5} \div \frac{8}{1} = \frac{4}{5} \cdot \frac{1}{8} = \frac{1 \cdot 1}{5 \cdot 2} = \frac{1}{10}$

6. $\frac{5}{8} = \frac{5}{8} \div \frac{4}{1} = \frac{5}{8} \cdot \frac{1}{4} = \frac{5}{32}$

7. $\frac{15}{31} \cdot 62 = \frac{15}{31} \cdot \frac{62}{1} = \frac{15 \cdot 2}{1 \cdot 1} = \frac{30}{1} = 30$

8. $3\frac{1}{4} \div 1\frac{1}{4} = \frac{13}{4} \div \frac{5}{4} = \frac{13}{4} \cdot \frac{4}{5} = \frac{13 \cdot 1}{1 \cdot 5} = \frac{13}{5} = 2\frac{3}{5}$

9. $\frac{8}{5} = 1\frac{3}{5}$

$\frac{1}{8} \leftarrow$ Whole number part
 $5 \overline{)8}$
 $\frac{5}{3} \leftarrow$ Remainder

10. $\frac{153}{4} = 38\frac{1}{4}$

$38 \leftarrow$ Whole number part
 $4 \overline{)153}$
 $\frac{12}{33}$
 $\frac{32}{1} \leftarrow$ Remainder

$$11. \quad 5\frac{2}{3} \cdot 5 \cdot 3 = 15$$

$$15 + 2 = 17$$

$$5\frac{2}{3} = \frac{17}{3}$$

$$12. \quad 38\frac{3}{8} \cdot 8 = 304$$

$$304 + 3 = 307$$

$$38\frac{3}{8} = \frac{307}{8}$$

$$13. \quad \frac{8}{12} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 3} = \frac{1 \cdot 1 \cdot 2}{1 \cdot 1 \cdot 3} = \frac{2}{3}$$

$$14. \quad \frac{108}{210} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot 3 \cdot 3}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 5 \cdot 7} = \frac{1 \cdot 2 \cdot 1 \cdot 3 \cdot 3}{1 \cdot 1 \cdot 5 \cdot 7} = \frac{18}{35}$$

$$15. \quad \frac{75}{90} = \frac{75 \div 15}{90 \div 15} = \frac{5}{6}$$

$$16. \quad \frac{48}{72} = \frac{48 \div 24}{72 \div 24} = \frac{2}{3}$$

$$17. \quad \frac{44}{110} = \frac{44 \div 22}{110 \div 22} = \frac{2}{5}$$

$$18. \quad \frac{87}{261} = \frac{87 \div 87}{261 \div 87} = \frac{1}{3}$$

19. Multiply $2\frac{1}{2}$ ounces per gallon by the number of gallons.

Estimate: $3 \cdot 50 = 150$ ounces

$$\text{Exact: } 2\frac{1}{2} \cdot 50 = \frac{5}{\underset{1}{\cancel{2}}} \cdot \frac{\overset{25}{\cancel{50}}}{1} = \frac{5 \cdot 25}{1 \cdot 1} = \frac{125}{1} = 125$$

125 ounces of the product are needed.

20. Multiply the number of tanks by the number of quarts needed for each tank.

Estimate: $7 \cdot 9 = 63$ qt

$$\text{Exact: } 7\frac{1}{4} \cdot 9\frac{1}{3} = \frac{29}{4} \cdot \frac{28}{3} = \frac{812}{12} = \frac{203}{3} = 67\frac{2}{3}$$

$67\frac{2}{3}$ quarts are needed.

21. To find the area, multiply the length and the width.

$$1\frac{3}{4} \cdot \frac{7}{8} = \frac{7}{4} \cdot \frac{7}{8} = \frac{49}{32} = 1\frac{17}{32}$$

The area of the stamp is $1\frac{17}{32}$ in.².

22. To find the area, multiply the length and the width.

$$\frac{7}{8} \cdot 2\frac{1}{4} = \frac{7}{8} \cdot \frac{9}{4} = \frac{63}{32} = 1\frac{31}{32}$$

The area of the patio table top is $1\frac{31}{32}$ yd².

Chapter 2 Test

- $\frac{5}{6}$ There are 6 parts, and 5 are shaded.
- $\frac{3}{8}$ There are 8 parts, and 3 are shaded.
- Proper fractions have the numerator (top) smaller than the denominator (bottom).

$$\frac{2}{3}, \frac{6}{7}, \frac{1}{4}, \frac{5}{8}$$

$$4. \quad 3\frac{3}{8} \cdot 8 = 24$$

$$24 + 3 = 27$$

$$3\frac{3}{8} = \frac{27}{8}$$

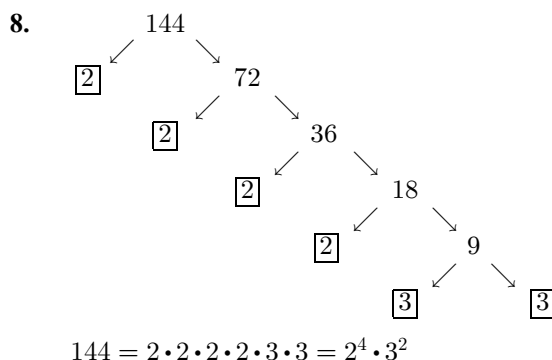
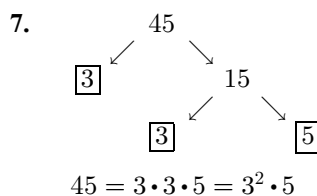
$$5. \quad \frac{123}{4} = 30\frac{3}{4}$$

$$\begin{array}{r} 30 \leftarrow \text{Whole number part} \\ 4 \overline{) 123} \\ \underline{12} \\ 3 \leftarrow \text{Remainder} \end{array}$$

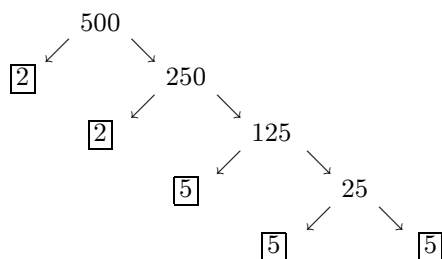
6. Factorizations of 18:

$$1 \cdot 18 = 18 \quad 2 \cdot 9 = 18 \quad 3 \cdot 6 = 18$$

The factors of 18 are 1, 2, 3, 6, 9, and 18.



9.



$$500 = 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 = 2^2 \cdot 5^3$$

$$10. \quad \frac{36}{48} = \frac{36 \div 12}{48 \div 12} = \frac{3}{4}$$

$$11. \quad \frac{60}{72} = \frac{60 \div 12}{72 \div 12} = \frac{5}{6}$$

12. Write the prime factorization of both numerator and denominator. Divide the numerator and denominator by any common factors. Multiply the remaining factors in the numerator and denominator.

$$\frac{56}{84} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 3 \cdot \underset{1}{\cancel{7}}} = \frac{2}{3}$$

13. Multiply fractions by multiplying the numerators and multiplying the denominators. Divide two fractions by using the reciprocal of the divisor (the second fraction) and then changing division to multiplication.

$$14. \quad \frac{3}{4} \cdot \frac{4}{9} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}} \cdot \underset{3}{9}} = \frac{1}{3}$$

$$15. \quad 54 \cdot \frac{2}{3} = \frac{18}{\cancel{54}} \cdot \frac{2}{\overset{3}{\cancel{6}}} = 36$$

16. Multiply the length and the width.

$$\frac{15}{16} \cdot \frac{4}{9} = \frac{\overset{5}{\cancel{15}} \cdot \overset{1}{\cancel{4}}}{\underset{4}{\cancel{16}} \cdot \underset{3}{9}} = \frac{5 \cdot 1}{4 \cdot 3} = \frac{5}{12}$$

The area of the grill is $\frac{5}{12}$ yd².

17. First, find the number of seedlings that don't survive.

$$8760 \cdot \frac{3}{8} = \frac{1095}{\cancel{8760}} \cdot \frac{3}{\underset{1}{\cancel{8}}} = 3285$$

Next, subtract to find the number that do survive.

$$8760 - 3285 = 5475$$

5475 seedlings do survive.

$$18. \quad \frac{3}{4} \div \frac{5}{6} = \frac{3}{\cancel{4}} \cdot \frac{\overset{3}{6}}{5} = \frac{3 \cdot 3}{2 \cdot 5} = \frac{9}{10}$$

$$19. \quad \frac{7}{\frac{4}{9}} = 7 \div \frac{4}{9} = \frac{7}{1} \div \frac{4}{9} = \frac{7}{1} \cdot \frac{9}{4} = \frac{63}{4} = 15\frac{3}{4}$$

20. Divide the total length by the length of the pieces.

$$54 \div 2\frac{1}{4} = \frac{54}{1} \div \frac{9}{4} = \frac{\overset{6}{\cancel{54}}}{1} \cdot \frac{4}{\underset{1}{\cancel{9}}} = 24$$

24 pieces can be cut.

$$21. \quad 4\frac{1}{8} \cdot 3\frac{1}{2}$$

Estimate: $4 \cdot 4 = 16$

$$\text{Exact: } 4\frac{1}{8} \cdot 3\frac{1}{2} = \frac{33}{8} \cdot \frac{7}{2} = \frac{231}{16} = 14\frac{7}{16}$$

$$22. \quad 1\frac{5}{6} \cdot 4\frac{1}{3}$$

Estimate: $2 \cdot 4 = 8$

$$\text{Exact: } 1\frac{5}{6} \cdot 4\frac{1}{3} = \frac{11}{6} \cdot \frac{13}{3} = \frac{143}{18} = 7\frac{17}{18}$$

$$23. \quad 9\frac{3}{5} \div 2\frac{1}{4}$$

Estimate: $10 \div 2 = 5$

Exact:

$$9\frac{3}{5} \div 2\frac{1}{4} = \frac{48}{5} \div \frac{9}{4} = \frac{\overset{16}{\cancel{48}}}{5} \cdot \frac{4}{\underset{3}{\cancel{9}}} = \frac{64}{15} = 4\frac{4}{15}$$

$$24. \quad \frac{8\frac{1}{2}}{1\frac{3}{4}}$$

$$\text{Estimate: } 9 \div 2 = \frac{9}{2} = 4\frac{1}{2}$$

Exact:

$$\frac{8\frac{1}{2}}{1\frac{3}{4}} = 8\frac{1}{2} \div 1\frac{3}{4} = \frac{17}{2} \div \frac{7}{4}$$

$$= \frac{17}{2} \cdot \frac{4}{7} = \frac{17 \cdot 4}{2 \cdot 7} = \frac{68}{14} = \frac{34}{7} = 4\frac{6}{7}$$

25. If $2\frac{1}{2}$ grams can be synthesized per day, multiply to find the amount synthesized in $12\frac{1}{4}$ days.

Estimate: $3 \cdot 12 = 36$ grams

Exact:

$$2\frac{1}{2} \cdot 12\frac{1}{4} = \frac{5}{2} \cdot \frac{49}{4} = \frac{5 \cdot 49}{2 \cdot 4} = \frac{245}{8} = 30\frac{5}{8}$$

$30\frac{5}{8}$ grams can be synthesized.

Cumulative Review Exercises (Chapters 1–2)

1. $\overline{783}$

hundreds: 7

tens: 8

2. $\overline{8,621,785}$

millions: 8

ten-thousands: 2

$$\begin{array}{r} 1 \\ 71 \\ 23 \\ 47 \\ + 36 \\ \hline 177 \end{array}$$

$$\begin{array}{r} 1 11 \\ 82, \overline{121} \\ 5 \ 468 \\ 316 \\ + 61,294 \\ \hline 149,199 \end{array}$$

$$\begin{array}{r} 413 \\ 6 \overline{537} \\ - 2085 \\ \hline 4452 \end{array}$$

$$\begin{array}{r} 7118 \ 1510 \\ 4, \overline{810}, \overline{600} 4 \\ - 1,597,783 \\ \hline 3,221,821 \end{array}$$

$$\begin{array}{r} 2 \\ 83 \\ \times 9 \\ \hline 747 \end{array}$$

8. $9 \cdot 4 \cdot 2 = (9 \cdot 4) \cdot 2 = 36 \cdot 2 = 72$

- 9.

$$\begin{array}{r} 3784 \cdot 573 \\ \hline 2168232 \end{array}$$

$$\begin{array}{r} 563 \\ \times 800 \\ \hline \end{array} \quad \begin{array}{r} 5 \ 2 \\ 5 \ 63 \\ \times 8 \\ \hline 45 \ 04 \end{array} \quad \begin{array}{r} 563 \\ \times 800 \\ \hline 450,400 \end{array} \text{ Attach } 00.$$

$$\frac{63}{7} \quad 7 \overline{)63} \quad \frac{63}{7} = 9$$

$$\begin{array}{r} 136458 \overline{)18} \\ \hline 7581 \end{array}$$

$$\begin{array}{r} 33,886 \div 4 \quad \begin{array}{r} 8 \ 4 \ 7 \ 1 \ R2 \\ 4 \overline{)33,886} \\ \underline{32} \\ 18 \\ \underline{16} \\ 28 \\ \underline{28} \\ 06 \\ \underline{04} \\ 2 \end{array} \end{array}$$

Check: 8471

$$\begin{array}{r} \times 4 \\ 33,884 \\ + 2 \\ \hline 33,886 \end{array}$$

14. $\begin{array}{r} 2 \ 2 \ R26 \\ 492 \overline{)10,850} \\ \underline{984} \\ 1010 \\ \underline{984} \\ 26 \end{array}$

Check: 492

$$\begin{array}{r} \times 22 \\ 984 \\ \hline 984 \\ 10,824 \\ + 26 \\ \hline 10,850 \end{array}$$

15. **To the nearest ten:** $\underline{658}3$

Next digit is 4 or less.

Tens place does not change.

All digits to the right of the underlined place change to zero. **6580**

To the nearest hundred: $\underline{65}83$

Next digit is 5 or more.

Hundreds place changes ($5 + 1 = 6$).

All digits to the right of the underlined place change to zero. **6600**

(continued)

To the nearest thousand: 6583

Next digit is 5 or more.

Thousands place changes ($6 + 1 = 7$).

All digits to the right of the underlined place change to zero. **7000**

16. **To the nearest ten:** 76,271

Next digit is 4 or less.

Tens place does not change.

All digits to the right of the underlined place change to zero. **76,270**

To the nearest hundred: 76,271

Next digit is 5 or more.

Hundreds place changes ($2 + 1 = 3$).

All digits to the right of the underlined place change to zero. **76,300**

To the nearest thousand: 76,271

Next digit is 4 or less.

Thousands place does not change.

All digits to the right of the underlined place change to zero. **76,000**

17. $2^5 - 6(4)$ *Exponent*
 $= 32 - 6(4)$ *Multiply.*
 $= 32 - 24 = 8$ *Subtract.*

18. $\sqrt{36} - 2 \cdot 3 + 5$ *Square root*
 $= 6 - 2 \cdot 3 + 5$ *Multiply.*
 $= 6 - 6 + 5$ *Subtract.*
 $= 0 + 5 = 5$ *Add.*

19. Multiply to find the amount used for the half-day and full-day tours; then add to find the total.

$$\begin{array}{r} 26 \\ \times 9 \\ \hline 234 \end{array} \quad \begin{array}{r} 18 \\ \times 17 \\ \hline 126 \\ 18 \\ \hline 306 \end{array} \quad \begin{array}{r} 234 \\ + 306 \\ \hline 540 \end{array}$$

540 gallons of fuel are needed.

20. Subtract to find the difference in cases.

$$\begin{array}{r} 110 \ 128 \ 11 \\ \cancel{2} \ \cancel{1}, \ \cancel{2} \ \cancel{0} \ \cancel{1} \\ - \quad 4, \ 5 \ 1 \ 9 \\ \hline 1 \ 6, \ 7 \ 7 \ 2 \end{array}$$

There were 16,772 more cases of pertussis than mumps.

21. Find the number of hairs lost in 2 years and subtract to find the hairs remaining.

$$\begin{array}{r} 365 \\ \times 100 \\ \hline 36,500 \end{array} \quad \begin{array}{r} 36,500 \\ \times 2 \\ \hline 73,000 \end{array} \quad \begin{array}{r} 120,000 \\ - \quad 73,000 \\ \hline 47,000 \end{array}$$

47,000 hairs remain.

22. Divide the total number of hours by the number of workers.

$$\begin{array}{r} 188 \\ 22 \overline{)4136} \\ \underline{22} \\ 193 \\ \underline{176} \\ 176 \\ \underline{176} \\ 0 \end{array}$$

Each health care worker will work 188 hours.

23. Multiply the number of flushes and the amount of water used per flush to find the number of gallons of water used.

$$160 \cdot 1\frac{3}{5} = \frac{160}{1} \cdot \frac{8}{5} = \frac{32}{1} \cdot \frac{8}{\cancel{5}_1} = 256$$

256 gallons of water are used in 160 flushes.

24. Divide the total length by the length of the pieces.

$$70 \div 3\frac{1}{3} = \frac{70}{1} \div \frac{10}{3} = \frac{7}{1} \cdot \frac{3}{\cancel{10}_1} = 21$$

21 pieces can be cut.

25. $\frac{2}{3}$ is *proper* because the numerator (2) is smaller than the denominator (3).

26. $\frac{6}{6}$ is *improper* because the numerator (6) is larger than or the same as the denominator (6).

27. $\frac{9}{18}$ is *proper* because the numerator (9) is smaller than the denominator (18).

28. $3\frac{3}{8}$ $3 \cdot 8 = 24$

$$24 + 3 = 27$$

$$3\frac{3}{8} = \frac{27}{8}$$

29. $6\frac{2}{5}$ $6 \cdot 5 = 30$

$$30 + 2 = 32$$

$$6\frac{2}{5} = \frac{32}{5}$$

30. $\frac{14}{7}$

$$\begin{array}{r} 2 \leftarrow \text{Whole number part} \\ 7 \overline{)14} \\ \underline{14} \\ 0 \leftarrow \text{Remainder} \end{array}$$

$$\frac{14}{7} = 2$$

$$31. \quad \frac{103}{8}$$

$$\begin{array}{r} 12 \leftarrow \text{Whole number part} \\ 8 \overline{)103} \\ \underline{8} \\ 23 \\ \underline{16} \\ 7 \leftarrow \text{Remainder} \end{array}$$

$$\frac{103}{8} = 12\frac{7}{8}$$

$$32. \quad 72$$

$$\begin{array}{r} 36 \\ 2 \overline{)72} \\ \underline{18} \\ 36 \\ \underline{9} \\ 18 \\ \underline{3} \\ 9 \\ \underline{1} \\ 3 \\ \underline{3} \end{array}$$

$$72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$$

$$33. \quad \begin{array}{c} 126 \\ \swarrow \quad \searrow \\ \boxed{2} \quad 63 \\ \quad \swarrow \quad \searrow \\ \quad \boxed{3} \quad 21 \\ \quad \quad \swarrow \quad \searrow \\ \quad \quad \boxed{3} \quad \boxed{7} \end{array}$$

$$126 = 2 \cdot 3 \cdot 3 \cdot 7 = 2 \cdot 3^2 \cdot 7$$

$$34. \quad \begin{array}{c} 350 \\ \swarrow \quad \searrow \\ \boxed{2} \quad 175 \\ \quad \swarrow \quad \searrow \\ \quad \boxed{5} \quad 35 \\ \quad \quad \swarrow \quad \searrow \\ \quad \quad \boxed{5} \quad \boxed{7} \end{array}$$

$$350 = 2 \cdot 5 \cdot 5 \cdot 7 = 2 \cdot 5^2 \cdot 7$$

$$35. \quad 4^2 \cdot 2^2 = 16 \cdot 4 = 64$$

$$36. \quad 2^3 \cdot 6^2 = 8 \cdot 36 = 288$$

$$37. \quad 2^3 \cdot 4^2 \cdot 5 = 8 \cdot 16 \cdot 5$$

$$= 128 \cdot 5$$

$$= 640$$

$$38. \quad \frac{42}{48} = \frac{42 \div 6}{48 \div 6} = \frac{7}{8}$$

$$39. \quad \frac{24}{36} = \frac{24 \div 12}{36 \div 12} = \frac{2}{3}$$

$$40. \quad \frac{30}{54} = \frac{30 \div 6}{54 \div 6} = \frac{5}{9}$$

$$41. \quad \frac{1}{2} \cdot \frac{3}{4} = \frac{1 \cdot 3}{2 \cdot 4} = \frac{3}{8}$$

$$42. \quad 30 \cdot \frac{2}{3} \cdot \frac{3}{5} = \frac{30}{1} \cdot \frac{2}{3} \cdot \frac{3}{5} = \frac{\overset{6}{\cancel{30}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{5}}} = 12$$

$$43. \quad 7\frac{1}{2} \cdot 3\frac{1}{3} = \frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{2}}} \cdot \frac{\overset{5}{\cancel{10}}}{\underset{1}{\cancel{3}}} = \frac{25}{1} = 25$$

$$44. \quad \frac{3}{5} \div \frac{5}{8} = \frac{3}{5} \cdot \frac{8}{5} = \frac{24}{25}$$

$$45. \quad \frac{7}{8} \div 1\frac{1}{2} = \frac{7}{8} \div \frac{3}{2} = \frac{7}{8} \cdot \frac{\overset{1}{\cancel{2}}}{\underset{4}{\cancel{3}}} = \frac{7 \cdot 1}{4 \cdot 3} = \frac{7}{12}$$

$$46. \quad 3 \div 1\frac{1}{4} = \frac{3}{1} \div \frac{5}{4} = \frac{3}{1} \cdot \frac{4}{5} = \frac{12}{5} = 2\frac{2}{5}$$

Chapter 2: Multiplying and Dividing Fractions

SECTION 2.1 Basics of Fractions

- 1 Use a fraction to show how many parts of a whole are shaded.
- 2 Identify the numerator and denominator.
- 3 Identify proper and improper fractions.

In this chapter, use hands-on manipulatives as often as possible.

Terry Joe Collins, *Hinds Community College*

I use a handout sheet with this section that makes students match a list of unfinished pictures with a list of fractions. They must choose the correct fraction and then complete the picture by shading or circling to illustrate the correct numerator and denominator. They are surprised how difficult this is..

Nancy Ketchum, *Moberly Area Community College*

Students are often confused when an improper fraction is represented graphically, as in Example 2. Explain that one unit is divided into whatever number is in the denominator of a fraction.

Betty Linneman, *Jefferson College*

SECTION 2.2 Mixed Numbers

- 1 Identify mixed numbers.
- 2 Write mixed numbers as improper fractions.
- 3 Write improper fractions as mixed numbers.

Stress that $3\frac{1}{2}$ means $3 + \frac{1}{2}$, not $3 \cdot \frac{1}{2}$.

Catherine Kunicki, *Idaho State University*

SECTION 2.3 Factors

- 1 Find factors of a number.
- 2 Identify prime numbers and composite numbers.
- 3 Find prime factorizations.

Reiterate to students that 0 and 1 are neither prime nor composite and that all prime and composite numbers are whole numbers greater than 1. Mention to students that prime factorization only applies to composite numbers. There isn't a prime factorization of a prime number.

Tell students that there is a method to identify prime numbers up to a certain point called the Sieve of Eratosthenes. A good group exercise would be to use the Sieve of Eratosthenes to find all the prime numbers smaller than 300.

Vernon Bridges, *Durham Technical Community College*

When finding all factors of a number, I require that my students write them in *pairs* underneath the number to be factored, in an organized manner. Very seldom would you want to know a factor of a number without needing to know the other factor to go with it. (This is also good practice in anticipation of factoring trinomials.) A continuous ascending list of individual factors is of little value. The method of writing pairs of factors will cut your time in half.

Nancy Ketchum, *Moberly Area Community College*

Tell your students, “On a test, you may be asked to give the prime factorization of certain numbers. How would you like a guarantee that you will get it right? When giving the prime factorization of a number, you can always check your answer. Simply multiply it out. Also, be sure all the numbers in your answer are prime.”

Point out the difference to students between the prime factorization of a number and listing all the factors of a number. This is a common point of confusion.

Betty Linneman, *Jefferson College*

SECTION 2.4 Writing a Fraction in Lowest Terms

- 1** Tell whether a fraction is written in lowest terms.
- 2** Write a fraction in lowest terms using common factors.
- 3** Write a fraction in lowest terms using prime factors.
- 4** Determine whether two fractions are equivalent.

I strongly emphasize using prime factors to write fractions in lowest terms, telling my students that this is a more algebraic approach.

Nancy Ketchum, *Moberly Area Community College*

Review the Tests for Divisibility. Show students how these tests can be applied when looking for common factors in the numerator and denominator of a fraction.

If students are using the method of dividing by a common factor, they may be worried that they haven’t found the *greatest* common factor. Explain that if they divide by a common factor that is not the greatest common factor, they will still be able to succeed, but it will take more steps. Emphasize that they must check to see that their final fraction is in fact in lowest terms.

Abby Tanenbaum

SECTION 2.5 Multiplying Fractions

- 1** Multiply fractions.
- 2** Use a multiplication shortcut.
- 3** Multiply a fraction and a whole number.
- 4** Find the area of a rectangle.

When discussing the application problems involving the area of a rectangle, make sure that students understand the meaning of square units and get in the habit of always including the correct units in their answers. This will be important for more extensive work with area in Chapter 8.

Abby Tanenbaum

SECTION 2.6 Applications of Multiplication

- 1** Solve fraction application problems using multiplication.

Emphasize that the keyword “of” often signifies multiplication.

Betty Linneman, *Jefferson College*

Emphasize use of the six-step method for solving application problems that was introduced in Section 1.10.

Abby Tanenbaum

SECTION 2.7 Dividing Fractions

- 1** Find the reciprocal of a fraction.
- 2** Divide fractions.
- 3** Solve application problems in which fractions are divided.

With fractions, students have difficulty deciding which fraction needs to be the dividend and which one needs to be the divisor. So I repeatedly say that division asks the question, “How many of the divisor is in the dividend?” If they know that they should have an answer larger than 1, then the dividend needs to be the larger number, but if their answer should be less than 1, then the dividend needs to be the smaller number. Side Problems 4 and 5 are good for practicing this.

Nancy Ketchum, *Moberly Area Community College*

When working with students on division problems, remind them that whatever is being divided up is listed first. In multiplication, order does not matter. In division, order is everything.

Betty Linneman, *Jefferson College*

Students often have a very difficult time determining whether to multiply or divide in an application problem involving fractions. Encourage them to picture the problem. Try to demonstrate that dividing by a proper fraction actually makes a larger answer, whereas multiplying by a proper fraction creates a smaller number.

Kelly A. Sanchez, *Columbus State Community College*

SECTION 2.8 Multiplying and Dividing Mixed Numbers

- 1** Estimate the answer and multiply mixed numbers.
- 2** Estimate the answer and divide mixed numbers.
- 3** Solve application problems with mixed numbers.

When multiplying and dividing mixed numbers, remind students that they must always turn the mixed numbers into improper fractions first.

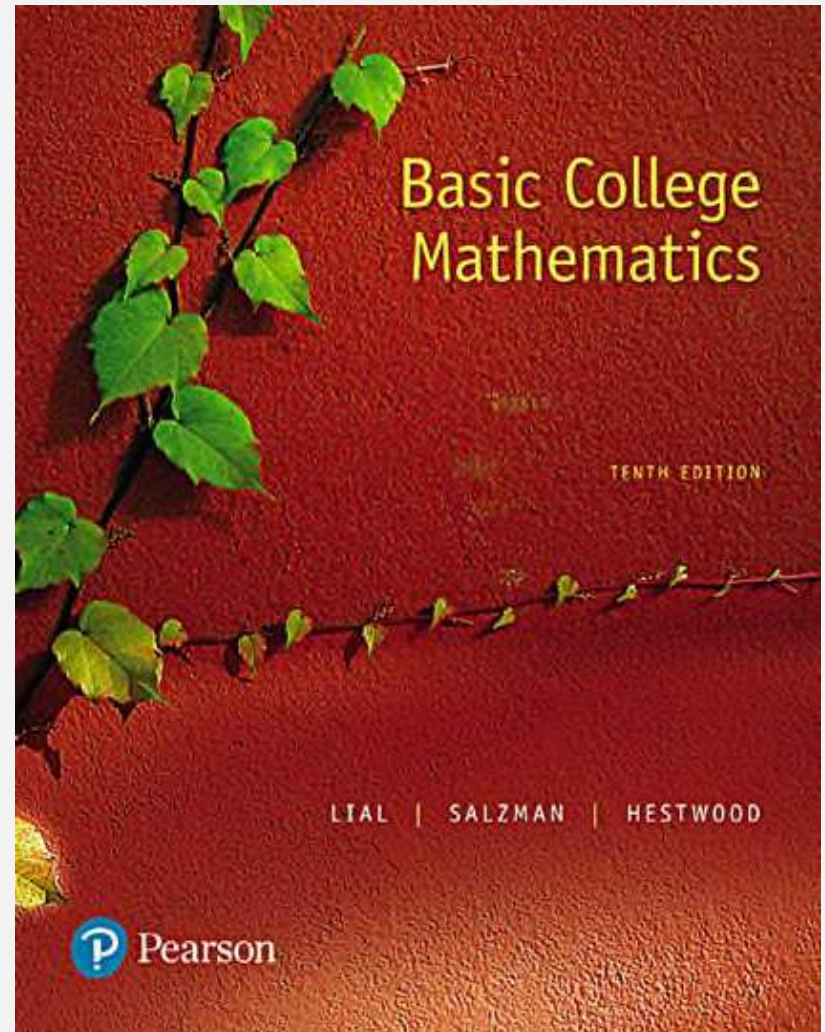
Betty Linneman, *Jefferson College*

Emphasize the importance of good estimation skills. Talk about real-life situations in which an estimated answer is good enough and those in which an exact answer is essential. Stress that even when an exact answer is required, estimation gives you a quick and easy way to check that your answer is reasonable.

Abby Tanenbaum

1

Whole Numbers



1.2 Adding Whole Numbers

Objectives

1. Add two single-digit numbers.
2. Add more than two numbers.
3. Add when regrouping (carrying) is not required.
4. Add with regrouping (carrying).
5. Use addition to solve application problems.
6. Check the answers in addition.



Commutative Property of Addition

By the **commutative property of addition**, changing the **order** of two addends in an addition problem does not change the sum.



Adding Two Single-Digit Numbers

Add, and then change the order of numbers to write another addition problem.

a. $7 + 6 = 13$ and $6 + 7 = 13$

b. $9 + 6 = 15$ and $6 + 9 = 15$

c. $3 + 7 = 10$ and $7 + 3 = 10$

Associative Property of Addition

By the **associative property of addition**, changing the **grouping** of the addends in an addition problem does not change the sum.

The sum of $6 + 7 + 8$ may be found as follows.

$$(6 + 7) + 8 = 13 + 8 = 21$$

Another way to add the same numbers is

$$6 + (7 + 8) = 6 + 15 = 21$$



Example

Adding More Than Two Numbers

Add 3, 4, 6, 2, and 5.

$$\begin{array}{r} 3 \\ 4 \\ \hline 6 \\ 2 \\ 5 \\ + \\ \hline 20 \end{array}$$

3 + 4 = 7

7 + 6 = 13

13 + 2 = 15

5 + 15 = 20

Example

Adding without Regrouping

Add $411 + 13 + 124 + 11$.

4	1	1	Hundreds, tens and ones in a column	
	1	3		
	1	2		4
		1		1
5	5	9		
↑	↑	↑	Sum of ones	
	↑		Sum of tens	
↑			Sum of hundreds	

Example

Adding with Regrouping

Add 48 and 36.

$$\begin{array}{r} 1 \\ 48 \\ + 36 \\ \hline 84 \end{array}$$

↑ 8 ones and 6 ones = 14 ones

Regroup 14 ones as 1 ten and 4 ones. Write 4 ones in the ones column and write 1 ten in the tens column.

Add the digits in the tens column, including the regrouped 1.

Example

Adding with Regrouping (Carrying)

Add $226 + 7756 + 24 + 8 + 85$.

$$\begin{array}{r} 1\ 1\ 2 \\ 226 \\ 7756 \\ 24 \\ 8 \\ +\ 85 \\ \hline 8099 \end{array}$$

Sum the ones column (29). Write 9 in the ones column and write 2 in the tens column.

Sum the tens column (19). Write 9 in the tens column and write 1 in the hundreds column.

Sum the hundreds column, including the regrouped 1.

Sum the thousands column, including the regrouped 1.

The sum is 8099.

Example

Applying Addition Skills

On this map of the Walt Disney World area in Florida, the distance in miles from one location to another is written alongside of the road. Find the shortest route from Orlando to Belle Isle.

Solution

Add the mileage along various routes to determine the distances from Orlando to Belle Isle.

Orlando to Resort	9
Resort to Pine	6
Pine to Belle	<u>3</u>
	18 miles



Example

Applying Addition Skills (cont)

On this map of the Walt Disney World area in Florida, the distance in miles from one location to another is written alongside of the road. Find the shortest route from Orlando to Belle Isle.

Solution

Another possible route:

Orlando to Clear Lake	8
Clear Lake to Shadow	11
Shadow to Conway	7
Conway to Belle	<u>6</u>
	32 miles



The shortest route is from Orlando to Resort Area to Pine Castle to Belle Island, 18 miles

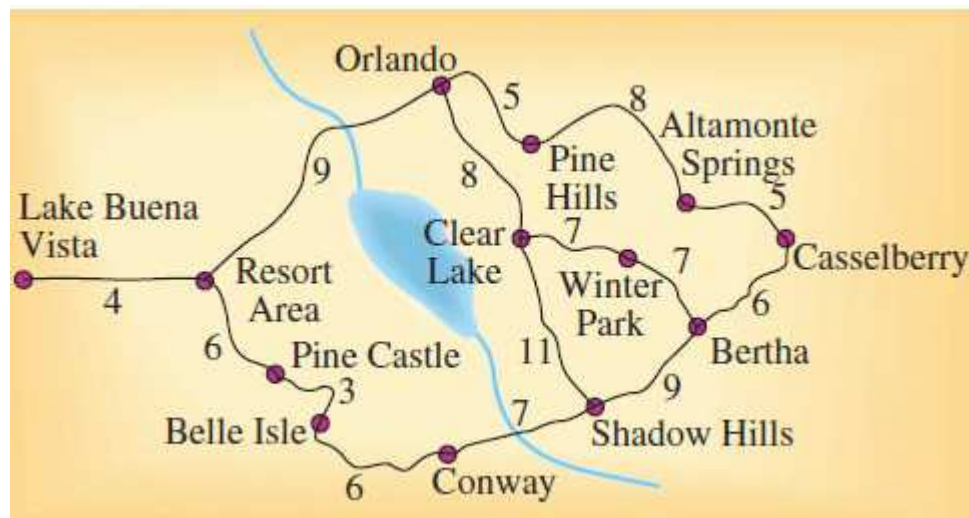
Example

Finding a Total Distance

Use the map to find the total distance from Bertha to Clear Lake to Shadow Hills and back to Bertha.

Solution

Add the mileage from Bertha to Clear Lake to Shadow Hills to Bertha.



Bertha to Winter Park	7
Winter Park to Clear Lake	7
Clear Lake to Shadow Hills	11
Shadow Hills to Bertha	<u>9</u>
	34 miles

Example

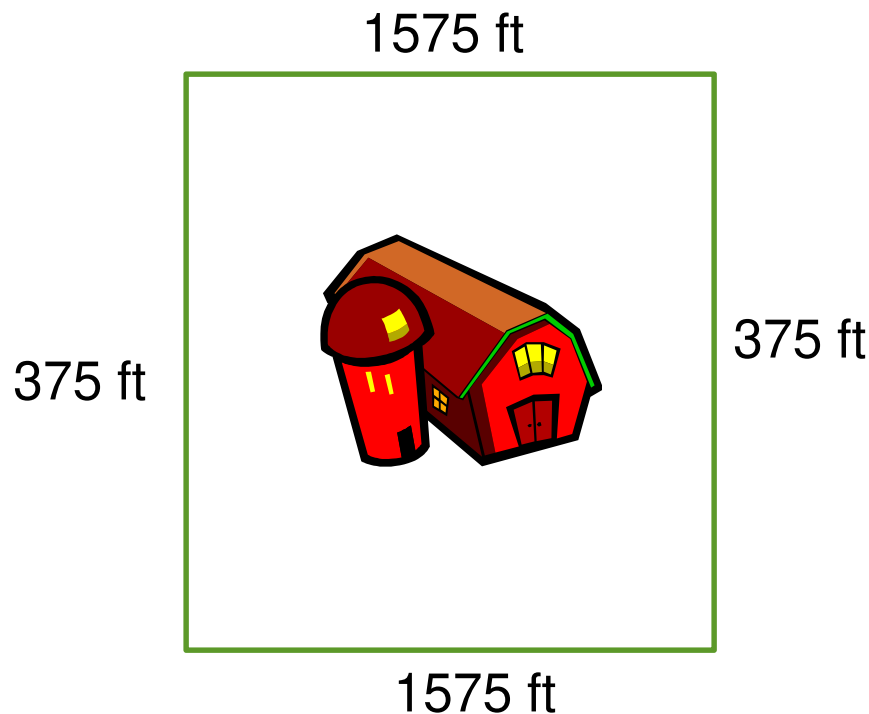
Finding a Perimeter

Find the number of feet of fencing needed to enclose the farm pasture shown.

Solution

Find the **perimeter**, or total distance around the pasture by adding the lengths of all the sides.

$$\begin{array}{r} 1575 \\ 375 \\ 1575 \\ + 375 \\ \hline 3900 \text{ miles} \end{array}$$



The amount of fencing needed is 3900 ft.



Example

Checking Addition

Check the following addition.

Add down.

$$\begin{array}{r} 766 \\ 448 \\ 246 \\ + \quad 72 \\ \hline 766 \end{array}$$

↑ To check, add up.