Introductory Mathematical Analysis for Business Economics and the Life and Social Sciences (Canadian) 14th edition Haeussler Solutions Manual

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Chapter 2

Apply It 2.1

1. a. The formula for the area of a circle is πr^2, where r is the radius.
$a(r) = \pi r^2$

b. The domain of $a(r)$ is all real numbers.

c. Since a radius cannot be negative or zero, the domain for the function, in context, is $r > 0$.

2. a. The formula relating distance, time, and speed is $d = rt$ where d is the distance, r is the speed, and t is the time. This can also be written as $t = \frac{d}{r}$. When $d = 300$, we have $t(r) = \frac{300}{r}$.

b. The domain of $t(r)$ is all real numbers except 0.

c. Since speed is not negative, the domain for the function, in context, is $r > 0$.

d. Replacing r by x: $t(x) = \frac{300}{x}$.
Replacing r by $\frac{x}{2}$: $t\left(\frac{x}{2}\right) = \frac{300}{\frac{x}{2}} = \frac{600}{x}$.
Replacing r by
$\frac{x}{4}$: $t\left(\frac{x}{4}\right) = \frac{300}{\frac{x}{4}} = \frac{1200}{x}$.

e. When the speed is reduced (divided) by a constant, the time is scaled (multiplied) by the same constant; $t\left(\frac{r}{c}\right) = \frac{300c}{r}$.

3. a. If the price is \$18.50 per large pizza, $p = 18.5$.
$18.5 = 26 - \frac{q}{40}$
$-7.5 = -\frac{q}{40}$
$300 = q$
At a price of \$18.50 per large pizza, 300 pizzas are sold each week.

b. If 200 large pizzas are being sold each week, $q = 200$.
$p = 26 - \frac{200}{40}$
$p = 26 - 5$
$p = 21$
The price is \$21 per pizza if 200 large pizzas are being sold each week.

c. To double the number of large pizzas sold, use $q = 400$.
$p = 26 - \frac{400}{40}$
$p = 26 - 10$
$p = 16$
To sell 400 large pizzas each week, the price should be \$16 per pizza.

4. Revenue = price $\cdot$ quantity $= pq$

From the table, the weekly revenue is:

$pq = 500 \cdot 11 = 5500$
$pq = 600 \cdot 14 = 8400$
$pq = 700 \cdot 17 = 11{,}900$
$pq = 800 \cdot 20 = 16{,}000$

Problems 2.1

1. The functions are not equal because $f(x) \geq 0$ for all values of x, while $g(x)$ can be less than 0. For example, $f(-2) = \sqrt{(-2)^2} = \sqrt{4} = 2$ and $g(-2) = -2$, thus $f(-2) \neq g(-2)$.

2. The domain of G is implicitly $[-3, \infty)$ while that of H is is implicitly $(-\infty, \infty)$, so $G \neq H$. (We note, however, that for all x in the domain of G, $G(x) = H(x)$.)

3. The functions are not equal because they have different domains. $h(x)$ is defined for all nonzero real numbers, while $k(x)$ is defined for all real numbers.

4. The functions are equal. For $x = 3$ we have $f(3) = 2$ and $g(3) = 3 - 1 = 2$, hence $f(3) = g(3)$. For $x \neq 3$, we have

$$f(x) = \frac{x^2 - 4x + 3}{x - 3} = \frac{(x-3)(x-1)}{x-3} = x - 1.$$

Note that we can cancel the $x - 3$ because we are assuming $x \neq 3$ and so $x - 3 \neq 0$. Thus for $x \neq 3$, $f(x) = x - 1 = g(x)$.

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$f(x) = g(x)$ for all real numbers and they have the same domains, thus the functions are equal.

5. The denominator is zero when $x = 1$. Any other real number can be used for x.
Answer: all real numbers except 1

6. Any real number can be used for x.

Answer: all real numbers

7. We require both $\dfrac{x-2}{x+1} \geq 0$ and $x + 1 \neq 0$. $x - 2 > 0$ on $[2, \infty)$, $x - 2 = 0$ for $x = 2$, and $x - 2 < 0$ on $(-\infty, 2)$. $\dfrac{1}{x+1} > 0$ on $(-1, \infty)$, $\dfrac{1}{x+1}$ is undefined for $x = -1$, and $\dfrac{1}{x+1} < 0$ on $(-\infty, -1)$. It follows that $\dfrac{x-2}{x+1}$ being the product of $x - 2$ and $\dfrac{1}{x+1}$ is positive on $(-\infty, -1) \cup (2, \infty)$ and is 0 at $x = 2$. It follows that the domain of $\dfrac{x-2}{x+1}$ is $(-\infty, -1) \cup [2, \infty)$

8. For $\sqrt{z-1}$ to be real, $z - 1 \geq 0$, so $z \geq 1$. We exclude values of z for which $\sqrt{z-1} = 0$, so $z - 1 = 0$, thus $z = 1$.

Answer: all real numbers > 1

9. Any real number can be used for z.
Answer: all real numbers

10. We exclude values of x for which

$x + 3 = 0$

$x = -3$

Answer: all real numbers except -3

11. We exclude values of x where

$2x + 7 = 0$

$2x = -7$

$x = -\dfrac{7}{2}$

Answer: all real numbers except $-\dfrac{7}{2}$

12. We require $2 - 3x \geq 0$ equivalently $x \leq -2/3$. The domain of g is $(-\infty, -2/3]$.

13. We exclude values of y for which

$y^2 - 4y + 4 = 0$. $y^2 - 4y + 4 = (y-2)^2$, so we exclude values of y for which $y - 2 = 0$, thus

$y = 2$.

Answer: all real numbers except 2.

14. We exclude values of x for which

$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x = -3,\ 2$$

Answer: all real numbers except -3 and 2

15. We exclude all values of x for which

$$3x^2 - 5x - 2 = 0$$
$$(3x+1)(x-2) = 0$$
$$x = -\frac{1}{3}, 2$$

Answer: all real numbers except 2 and $-\dfrac{1}{3}$

16. $r^2 + 1$ is never 0.

Answer: all real numbers

17. $f(x) = 3 - 5x$;

$f(0) = 3, f(2) = 3 - 5(2) = -7,$

$f(-2) = 3 - 5(-2) = 13.$

18. $H(s) = 5s^2 - 3$

$H(4) = 5(4)^2 - 3 = 80 - 3 = 77$

$H\left(\sqrt{2}\right) = 5\left(\sqrt{2}\right)^2 - 3 = 10 - 3 = 7$

$H\left(\dfrac{2}{3}\right) = 5\left(\dfrac{2}{3}\right)^2 - 3 = \dfrac{20}{9} - 3 = -\dfrac{7}{9}$

19. $G(x) = 2 - x^2$

$G(-8) = 2 - (-8)^2 = 2 - 64 = -62$

$G(u) = 2 - u^2$

$G(u^2) = 2 - (u^2)^2 = 2 - u^4$

20. $F(x) = -7x + 1$

$F(s) = -7s + 1$

$F(t+1) = -7(t+1) + 1 = -7t - 6$

$F(x+3) = -7(x+3) + 1 = -7x - 20$

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21. $\gamma(u) = 2u^2 - u$

$\gamma(-2) = 2(-2)^2 - (-2) = 8 + 2 = 10$

$\gamma(2v) = 2(2v)^2 - (2v) = 8v^2 - 2v$

$\gamma(x + a) = 2(x + a)^2 - (x + a)$

$= 2x^2 + 4ax + 2a^2 - x - a$

22. $h(v) = \dfrac{2}{\sqrt{4v}}$; $h(36) = \dfrac{2}{\sqrt{(4)(36)}} = \dfrac{2}{(2)(6)} = \dfrac{1}{6}$,

$h\left(\dfrac{1}{4}\right) = \dfrac{2}{\sqrt{4(1/4)}} = 2/1 = 2$,

$h(1 - x) = \dfrac{2}{\sqrt{4(1 - x)}}$

23. $f(x) = x^2 + 2x + 1$

$f(1) = 1^2 + 2(1) + 1 = 1 + 2 + 1 = 4$

$f(-1) = (-1)^2 + 2(-1) + 1 = 1 - 2 + 1 = 0$

$f(x + h) = (x + h)^2 + 2(x + h) + 1$

$= x^2 + 2xh + h^2 + 2x + 2h + 1$

24. $H(x) = (x + 4)^2$

$H(0) = (0 + 4)^2 = 16$

$H(2) = (2 + 4)^2 = 6^2 = 36$

$H(t - 4) = [(t - 4) + 4]^2 = t^2$

25. $k(x) = \dfrac{x - 5}{x^2 + 1}$

$k(5) = \dfrac{5 - 5}{5^2 + 1} = 0$

$k(2x) = \dfrac{2x - 5}{(2x)^2 + 1} = \dfrac{2x - 5}{4x^2 + 1}$

$k(x + h) = \dfrac{(x + h) - 5}{(x + h)^2 + 1} = \dfrac{x + h - 5}{x^2 + 2xh + h^2 + 1}$

26. $k(x) = \sqrt{x - 3}$

$k(4) = \sqrt{4 - 3} = \sqrt{1} = 1$

$k(3) = \sqrt{3 - 3} = \sqrt{0} = 0$

$k(x + 1) - k(x) = \sqrt{(x + 1) - 3} - \sqrt{x - 3}$

$= \sqrt{x - 2} - \sqrt{x - 3}$

27. $f(x) = x^{2/5}$;

$f(0) = 0^{2/5} = 0$,

$f(243) = (243^{1/5})^2 = 3^2 = 9$,

$f\left(\dfrac{-1}{32}\right) = (-1/32)^{2/5}$

$= (-(1/32)^{1/5}))^2 = (-1/2)^2 = 1/4$

28. $g(x) = x^{2/5}$

$g(32) = 32^{2/5} = \left(\sqrt[5]{32}\right)^2 = (2)^2 = 4$

$g(-64) = (-64)^{2/5} = \left(\sqrt[5]{-64}\right)^2$

$= \left(\sqrt[5]{-32}\sqrt[5]{2}\right)^2 = \left(-2\sqrt[5]{2}\right)^2 = 4\sqrt[5]{4}$

$g(t^{10}) = (t^{10})^{2/5} = t^4$

29. $f(x) = 4x - 5$

a. $f(x + h) = 4(x + h) - 5 = 4x + 4h - 5$

b. $\dfrac{f(x + h) - f(x)}{h}$

$= \dfrac{(4x + 4h - 5) - (4x - 5)}{h} = \dfrac{4h}{h} = 4$

30. $f(x) = \dfrac{x}{3}$

a. $f(x + h) = \dfrac{x + h}{3}$

b. $\dfrac{f(x + h) - f(x)}{h} = \dfrac{\frac{x+h}{3} - \frac{x}{3}}{h} = \dfrac{\frac{h}{3}}{h} = \dfrac{1}{3}$

31. $f(x) = x^2 + 2x$

a. $f(x + h) = (x + h)^2 + 2(x + h)$

$= x^2 + 2xh + h^2 + 2x + 2h$

b. $\dfrac{f(x + h) - f(x)}{h}$

$= \dfrac{(x^2 + 2xh + h^2 + 2x + 2h) - (x^2 + 2x)}{h}$

$= \dfrac{2xh + h^2 + 2h}{h} = 2x + h + 2$

32. $f(x) = 2x^2 - 5x + 3$;

a. $f(x + h) = 2(x + h)^2 - 5(x + h) + 3$

$= 2x^2 + (4h - 5)x + (2h^2 - 5h + 3)$,

b. $\dfrac{f(x + h) - f(x)}{h} = \dfrac{4hx + (2h^2 - 5h)}{h}$

$= 4x + (2h - 5)$ for $h \neq 0$

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33. $f(x) = 3 - 2x + 4x^2$

a. $f(x+h) = 3 - 2(x+h) + 4(x+h)^2$
$= 3 - 2x - 2h + 4(x^2 + 2xh + h^2)$

b. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{3 - 2x - 2h + 4x^2 + 8xh + 4h^2 - (3 - 2x + 4x^2)}{h}$
$= \dfrac{-2h + 8xh + 4h^2}{h}$
$= -2 + 8x + 4h$

34. $f(x) = x^3$

a. $f(x+h) = (x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

b. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h} = \dfrac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$

35. $f(x) = \dfrac{1}{x-1}$

a. $f(x+h) = \dfrac{1}{x+h-1}$

b. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h} = \dfrac{\frac{x-1-(x+h-1)}{(x-1)(x+h-1)}}{h} = \dfrac{-1}{(x-1)(x+h-1)}$

36. $f(x) = \dfrac{x+8}{x}$

a. $f(x+h) = \dfrac{(x+h)+8}{x+h} = \dfrac{x+h+8}{x+h}$

b. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\frac{x+h+8}{x+h} - \frac{x+8}{x}}{h} = \dfrac{x(x+h)\left(\frac{x+h+8}{x+h} - \frac{x+8}{x}\right)}{x(x+h)h} = \dfrac{x(x+h+8) - (x+h)(x+8)}{x(x+h)h}$
$= \dfrac{x^2 + xh + 8x - x^2 - hx - 8x - 8h}{x(x+h)h} = \dfrac{-8h}{x(x+h)h} = -\dfrac{8}{x(x+h)}$

37. $f(x) = 3x + 7$;

$\dfrac{f(2+h) - f(2)}{h} = \dfrac{(3(2+h)+7) - (3(2)+7)}{h} = 3 \qquad \text{for } h \neq 0$

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38. $$\frac{f(x)-f(2)}{x-2} = \frac{2x^2-x+1-(8-2+1)}{x-2}$$
$$= \frac{2x^2-x+1-7}{x-2}$$
$$= \frac{2x^2-x-6}{x-2}$$
$$= 2x+3$$

39. $9y-3x-4=0$

The equivalent form $y=\dfrac{3x+4}{9}$ shows that for each input x there is exactly one output, $\dfrac{3x+4}{9}$. Thus y is a function of x. Solving for x gives $x=\dfrac{9y-4}{3}$. This shows that for each input y there is exactly one output, $\dfrac{9y-4}{3}$. Thus x is a function of y.

40. $x^4-1+y=0$

The equivalent form $y=-x^4+1$ shows that for each input x there is exactly one output, $-x^4+1$. Thus y is a function of x. Solving for x gives $x=\pm\sqrt[4]{1-y}$. If, for example, $y=-15$, then $x=\pm 2$, so x is not a function of y.

41. $y=7x^2$

For each input x, there is exactly one output $7x^2$. Thus y is a function of x. Solving for x gives $x=\pm\sqrt{\dfrac{y}{7}}$. If, for example, $y=7$, then $x=\pm 1$, so x is not a function of y.

42. Solving for y we get $y=\pm\sqrt{1-x^3}$. The solution is not unique so the equation does not define y as a function of x. Solving for x we get $x=\sqrt[3]{1-x^2}$. The solution is unique so the equation defines x as a function of y.

43. Yes, because corresponding to each input r there is exactly one output, πr^2.

44. **a.** $f(a)=a^2a^3+a^3a^2=a^5+a^5=2a^5$

b. $f(ab)=a^2(ab)^3+a^3(ab)^2$
$=a^2a^3b^3+a^3a^2b^2$
$=a^5b^3+a^5b^2$
$=a^5b^2(b+1)$

45. Weekly excess of income over expenses is

$7200-4900=2300.$

After t weeks the excess accumulates to $2300t$. Thus the value of V of the business at the end of t weeks is given by $V=f(t)=50{,}000+2300t$.

46. Depreciation at the end of t years is $0.02t(30{,}000)$, so value V of machine is

$V=f(t)=30{,}000-0.02t(30{,}000)$, or

$V=f(t)=30{,}000(1-0.02t)$.

47. Each (nonnegative) q determines a unique P. So P is a function of q. The dependent variable is P and the independent variable is q.

48. Charging \$600,000 per film corresponds to $p=600{,}000$.

$$600{,}000=\frac{1{,}200{,}000}{q}$$
$$q=2$$

The actor will star in 2 films per year. To star in 4 films per year the actor should charge

$$p=\frac{1{,}200{,}000}{4}=\$300{,}000 \text{ per film.}$$

49. The function can be written as $q=48p$. At \$8.39 per pound, the coffee house will supply $q=48(8.39)=402.72$ pounds per week. At \$19.49 per pound, the coffee house will supply $q=48(19.49)=935.52$ pounds per week. The amount the coffee house supplies increases as the price increases.

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50. a. $f(0) = 1 - 1 = 0$

b. $f(100) = 1 - \left(\frac{200}{300}\right)^3$
$= 1 - \left(\frac{2}{3}\right)^3$
$= 1 - \frac{8}{27}$
$= \frac{19}{27}$

c. $f(800) = 1 - \left(\frac{200}{1000}\right)^3$
$= 1 - \left(\frac{1}{5}\right)^3$
$= 1 - \frac{1}{125}$
$= \frac{124}{125}$

d. Solve

$$0.5 = 1 - \left(\frac{200}{200+t}\right)^3$$
$$\left(\frac{200}{200+t}\right)^3 = 0.5$$
$$\frac{200}{200+t} = \sqrt[3]{0.5}$$
$$200 = 200\sqrt[3]{0.5} + t\sqrt[3]{0.5}$$
$$t = \frac{200 - 200\sqrt[3]{0.5}}{\sqrt[3]{0.5}} \approx 51.98$$

Half the group was discharged after 52 days.

51. a. $f(1000) = \frac{\left(\sqrt[3]{1000}\right)^4}{2500} = \frac{10^4}{2500}$
$= \frac{10{,}000}{2500} = 4$

b. $f(2000) = \frac{\left[\sqrt[3]{1000(2)}\right]^4}{2500} = \frac{\left(10^3\sqrt{2}\right)^4}{2500}$
$= \frac{10{,}000\sqrt[3]{2^4}}{2500} = 4\sqrt[3]{2^3 \cdot 2} = 8\sqrt[3]{2}$

c. $f(2I_0) = \frac{(2I_0)^{4/3}}{2500} = \frac{2^{4/3}I_0^{4/3}}{2500}$
$= 2\sqrt[3]{2}\left[\frac{I_0^{4/3}}{2500}\right] = 2\sqrt[3]{2}f(I_0)$

Thus $f(2I_0) = 2\sqrt[3]{2}f(I_0)$, which means that doubling the intensity increases the response by a factor of $2\sqrt[3]{2}$.

52. $P(28) = 14 + \sqrt{25} = 14 + 5 = 19$;
$P(52) = 26 + \sqrt{49} = 26 + 7 = 33$

53. a. Domain: 3000, 2900, 2300, 2000
$f(2900) = 12, f(3000) = 10$

b. Domain: 10, 12, 17, 20
$g(10) = 3000, g(17) = 2300$

54. a. -18.97

b. -581.77

c. -18.51

55. a. -5.13

b. 2.64

c. -17.43

56. a. 1,997,723.57

b. 1,287,532.35

c. 2,964,247.40

57. a. $f(11.7) = 6.94$

b. $f(-73) = 40.28$

c. $f(0) = 0.67$

Apply It 2.2

5. a. Let n = the number of visits and $p(n)$ be the premium amount.
$p(n) = 125$

b. The premiums do not change regardless of the number of doctor visits.

c. This is a constant function.

6. a. $d(t) = 3t^2$ is a quadratic function.

b. The degree of $d(t) = 3t^2$ is 2.

c. The leading coefficient of $d(t) = 3t^2$ is 3.

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7. The price for n pairs of socks is given by

$$c(n) = \begin{cases} 3.5n & 0 \le n \le 5 \\ 3n & 5 < n \le 10. \\ 2.75n & 10 < n \end{cases}$$

8. Think of the bookshelf having 7 slots, from left to right. You have a choice of 7 books for the first slot. Once a book has been put in the first slot, you have 6 choices for which book to put in the second slot, etc. The number of arrangements is $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 7! = 5040$.

Problems 2.2

1. yes

2. $f(x) = \dfrac{x^3 + 7x - 3}{3} = \dfrac{1}{3}x^3 + \dfrac{7}{3}x - 1$, which is a polynomial function.

3. $g(x) = \dfrac{5}{3x + 1}$ cannot be written as a sum of multiples of nonnegative integral powers of x, so g is not a polynomial.

4. yes

5. yes

6. yes

7. no

8. $g(x) = 2x^{-5} = \dfrac{2}{x^5}$ expresses g as a quotient of polynomials and thus shows that g is rational.

9. all real numbers

10. all real numbers

11. all real numbers

12. all x such that $1 \le x \le 3$

13. F is a polynomial of degree 4 with leading coefficient 5.

14. **a.** 2

b. 9

15. **a.** 7

b. 1

16. **a.** 0

b. 9

17. $f(x) = 8$
$f(2) = 8$
$f(t + 8) = 8$
$f\left(-\sqrt{17}\right) = 8$

18. $g(20) = |2(20) + 1| = |41| = 41$;
$g(5) = |2(5) + 1| = |11| = 11$;
$g(-7) = |2(-7) + 1| = |-13| = 13$

19. $F(12) = 2$
$F\left(-\sqrt{3}\right) = -1$
$F(1) = 0$
$F\left(\dfrac{18}{5}\right) = 2$

20. $f(3) = 4$
$f(-4) = 3$
$f(0) = 4$

21. $G(8) = 8 - 1 = 7$
$G(3) = 3 - 1 = 2$
$G(-1) = 3 - (-1)^2 = 2$
$G(1) = 3 - (1)^2 = 2$

22. $F(3) = 3^2 - 3(3) + 1 = 1$
$F(-3) = 2(-3) - 5 = -11$
$F(2)$ is not defined.

23. $9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362{,}880$

24. $(3 - 3)! = 0! = 1$

25. $(4 - 2)! = 2! = 2 \cdot 1 = 2$

26. $6! \cdot 2! = (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(2 \cdot 1)$
$= (720)(2)$
$= 1440$

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27. $\dfrac{n!}{(n-1)!} = \dfrac{n\cdot(n-1)!}{(n-1)!} = n$

28. $\dfrac{9!}{4!(9-4)!} = \dfrac{9.8.7.6.5.4.3.2.1}{(4.3.2)(5.4.3.2)} = \dfrac{9.8.7.6}{4.3.2}$
$= 3.2.7.3 = 126$

29. Let i = the passenger's income and
$c(i)$ = the cost for the ticket.
$c(i) = 2.50$
This is a constant function.

30. Let w = the width of the prism, then
$w+3$ = the length of the prism, and
$2w-1$ = the height of the prism. The formula for the volume of a rectangular prism is
V = length · width · height.
$V(w) = (w+3)(w)(2w-1) = 2w^3 + 5w^2 - 3w$
This is a cubic function.

31. a. $C = 850 + 3q$

b. $1600 = 850 + 3q$
$750 = 3q$
$q = 250$

32. The interest is Prt, so principal and interest amount to $f(t) = P + Prt$, or $f(t) = P(1+rt)$. Since $f(t) = at + b$ where $a(= Pr)$ and $b(= P)$ are constants, f is a linear function of t.

33.
$$c(j) = \begin{cases} 0.075 & \text{if } 0 \le j \le 44{,}701 \\ 0.11 & \text{if } 44{,}701 < j \le 89{,}401 \\ 0.13 & \text{if } 89{,}401 < j \le 138{,}586 \\ 0.145 & \text{if } 138{,}586 < j \end{cases}$$

34. For a committee of five, there are 5 choices for who will be member A. For each choice of member A, there are 4 choices for member G. Once members A and G have been chosen, there are 3 choices for member M, two choices for member N, then one choice for member S once members A, G, M, and N have been chosen. Thus, there are $5\cdot4\cdot3\cdot2\cdot1 = 5! = 120$ ways to label the members.

35. $P(2) = \dfrac{3!\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^2}{2!(1!)} = \dfrac{6\left(\frac{1}{16}\right)\left(\frac{3}{4}\right)}{2(1)} = \dfrac{9}{64}$

36. $P(5) = \dfrac{5!\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0}{5!(0!)} = \dfrac{5!\left(\frac{1}{1024}\right)(1)}{5!(1)}$
$= \dfrac{1}{1024}$

37. a. all T such that $30 \le T \le 39$

b. $f(30) = \dfrac{1}{24}(30) + \dfrac{11}{4} = \dfrac{5}{4} + \dfrac{11}{4} = \dfrac{16}{4} = 4$
$f(36) = \dfrac{1}{24}(36) + \dfrac{11}{4} = \dfrac{6}{4} + \dfrac{11}{4} = \dfrac{17}{4}$
$F(39) = \dfrac{4}{3}(39) - \dfrac{175}{4} = 52 - \dfrac{175}{4} = \dfrac{33}{4}$

38. a. $f(2.14) = 0.11(2.14)^3 - 15.31 = -14.23$

b. $f(3.27) = 0.42(3.27)^4 - 12.31 = 35.71$

c. $f(-4) = 0.11(-4)^3 - 15.31 = -22.35$

39. a. 1182.74

b. 4985.27

c. 252.15

40. a. 19.12

b. −62.94

c. 57.69

41. a. 2.21

b. 9.98

c. −14.52

Apply It 2.3

9. The customer's price is
$(c \circ s(x) = c(s(x)) = c(x+3) = 2(x+3)$
$= 2x + 6$

10. $g(x) = (x+3)^2$ can be written as $g(x) = a(l(x)) = (a \circ l)(x)$ where $a(x) = x^2$ and $l(x) = x + 3$. Then $l(x)$ represents the length of the sides of the square, while $a(x)$ is the area of a square with side of length x.

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Problems 2.3

1. $f(x) = x + 3, g(x) = x + 5$

a. $(f+g)(x) = f(x) + g(x)$
$= (x+3) + (x+5)$
$= 2x + 8$

b. $(f+g)(0) = 2(0) + 8 = 8$

c. $(f-g)(x) = f(x) - g(x)$
$= (x+3) - (x+5)$
$= -2$

d. $(fg)(x) = f(x)g(x)$
$= (x+3)(x+5)$
$= x^2 + 8x + 15$

e. $(fg)(-2) = (-2)^2 + 8(-2) + 15 = 3$

f. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x+3}{x+5}$

g. $(f \circ g)(x) = f(g(x))$
$= f(x+5)$
$= (x+5) + 3$
$= x + 8$

h. $(f \circ g)(3) = 3 + 8 = 11$

i. $(f \circ g)(x) = f(g(x))$
$= f(x+3)$
$= (x+3) + 5$
$= x + 8$

j. $(f \circ g)(3) = 3 + 8 = 11$

2. $f(x) = 2x, g(x) = 6 + x$

a. $(f+g)(x) = f(x) + g(x)$
$= (2x) + (6+x)$
$= 3x + 6$

b. $(f-g)(x) = f(x) - g(x)$
$= (2x) - (6+x)$
$= x - 6$

c. $x(f-g)(4) = (4) - 6 = -2x$

d. $x(fg)(x) = f(x)g(x) = 2x(6+x) = 12x + 2x^2$

e. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{2x}{6+x}$

f. $\frac{f}{g}(2) = \frac{2(2)}{6+2} = \frac{4}{8} = \frac{1}{2}$

g. $(f \circ g)(x) = f(g(x))$
$= f(6+x)$
$= 2(6+x)$
$= 12 + 2x$

h. $(g \circ f)(x) = g(f(x)) = g(2x) = 6 + 2x$

i. $(g \circ f)(2) = 6 + 2(2) = 6 + 4 = 10$

3. $f(x) = x^2 - 1, \quad g(x) = x^2 + x$

a. $(f+g)(x) = f(x) + g(x)$
$= (x^2 - 1) + (x^2 + x)$
$= 2x^2 + x - 1$

b. $(f-g)(x) = f(x) - g(x)$
$= (x^2 - 1) - (x^2 + x)$
$= -x - 1$

c. $(f-g)\left(-\frac{1}{2}\right) = \frac{1}{2} - 1 = -\frac{1}{2}$

d. $(fg)(x) = f(x)g(x)$
$= (x^2 - 1)(x^2 + x)$
$= x^4 + x^3 - x^2 - x$

e. $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$
$= \frac{x^2 - 1}{x^2 + x}$
$= \frac{(x+1)(x-1)}{x(x+1)}$
$= \frac{x-1}{x}, x \neq -1$

f. $\frac{f}{g}\left(-\frac{1}{2}\right) = \frac{-\frac{1}{2} - 1}{-\frac{1}{2}} = \frac{-\frac{3}{2}}{-\frac{1}{2}} = 3$

g. $(f \circ g)(x) = f(g(x))$
$= f(x^2 + x)$
$= (x^2 + x)^2 - 1$
$= x^4 + 2x^3 + x^2 - 1$

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h. $(g \circ f)(x) = g(f(x))$
$= g(x^2 - 1)$
$= (x^2 - 1)^2 + (x^2 - 1)$
$= x^4 - 2x^2 + 1 + x^2 - 1$
$= x^4 - x^2$

i. $(g \circ f)(-3) = (-3)^4 - (-3)^2 = 72$

4. a. $(f + g)(x) = 2x^2 + 5 + 3 = 2x^2 + 8$

b. $(f+g)(1/2) = 2(1/2)^2 + 8 = 1/2 + 8 = 17/2$

c. $(f - g)(x) = 2x^2 + 5 - 3 = 2x^2 + 2$

d. $(fg)(x) = (2x^2 + 5)(3) = 6x^2 + 15$

e. $(fg)(2) = 6(2)^2 + 15 = 24 + 15 = 39$

f. $\dfrac{f}{g}(x) = \dfrac{2x^2 + 5}{3}$

g. $(f \circ g)(x) = 2(3)^2 + 5 = 23$

h. $(f \circ g)(100.003) = 23$

i. $(g \circ f)(x) = g(f(x) = 3$

5. $f(g(2)) = f(4 - 4) = f(0) = 0 + 6 = 6$

$g(f(2)) = g(12 + 6) = g(18) = 4 - 36 = -32$

6. $(f \circ g)(p) = f(g(p))$
$= f\left(\dfrac{p-2}{3}\right)$
$= \dfrac{4}{\frac{p-2}{3}}$
$= \dfrac{12}{p-2}$

$(g \circ f)(p) = g(f(p)) = g\left(\dfrac{4}{p}\right) = \dfrac{\frac{4}{p} - 2}{3}$
$= \dfrac{4 - 2p}{3p}$

7. $(F \circ G)(t) = F(G(t))$
$= F\left(\dfrac{2}{t-1}\right)$
$= \left(\dfrac{2}{t-1}\right)^2 + 7\left(\dfrac{2}{t-1}\right) + 1$
$= \dfrac{4}{(t-1)^2} + \dfrac{14}{t-1} + 1$

$(G \circ F)(t) = G(F(t))$
$= G(t^2 + 7t + 1)$
$= \dfrac{2}{(t^2 + 7t + 1) - 1}$
$= \dfrac{2}{t^2 + 7t}$

8. $(F \circ G)(t) = F(G(t))$
$= F(2t^2 - 2t + 1)$
$= \sqrt{2t^2 - 2t + 1}$

$(G \circ F)(t) = G(F(t))$
$= G\left(\sqrt{t}\right)$
$= 2\left(\sqrt{t}\right)^2 - 2\left(\sqrt{t}\right) + 1$
$= 2t - 2\sqrt{t} + 1$

9. $(f \circ g)(v) = \dfrac{2}{(\sqrt{3v+1})^2 - 3} = \dfrac{2}{3v + 1 - 3}$
$= \dfrac{2}{3v - 2}$

$(g \circ f)(v) = \sqrt{3\left(\dfrac{2}{v^2 - 3}\right) + 1} = \sqrt{\dfrac{v^2 + 3}{v^2 - 3}}$

10. $(f \circ f)(x) = f(f(x))$
$= f(x^2 + 2x - 1)$
$= (x^2 + 2x - 1)^2 + 2(x^2 + 2x - 1) - 1$
$= x^4 + 4x^3 + 4x^2 - 2$

11. Let $g(x) = 11x$ and $f(x) = x - 7$. Then
$h(x) = g(x) - 7 = f(g(x))$

12. Let $g(x) = x^2 - 2$ and $f(x) = \sqrt{x}$. Then
$h(x) = \sqrt{x^2 - 2} = \sqrt{g(x)} = f(g(x))$

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13. Let $g(x) = x^2 + x + 1$ and $f(x) = \dfrac{3}{x}$. Then

$$h(x) = \frac{3}{x^2 - x + 1} = \frac{3}{g(x)} = f(g(x))$$

14. Let $f(x) = 7x^2 - 5x + 1$ and $g(x) = 4x^2 + 7x$. Then $(f \circ g)(x) = f(g(x)) = 7(4x^2 + 7x)^2 - 5(4x^2 + 7x) + 1 = h(x)$

15. Let $g(x) = \dfrac{x^2 - 1}{x + 3}$ and $f(x) = \sqrt[4]{x}$.

Then $h(x) = \sqrt[4]{g(x)} = f(g(x))$.

16. Let $g(x) = 3x - 5$ and $f(x) = \dfrac{2 - x}{x^2 + 2}$. Then

$$h(x) = \frac{2 - (3x - 5)}{(3x - 5)^2 + 2} = f(g(x)).$$

17. a. The revenue is \$9.75 per pound of coffee sold, so $r(x) = 9.75x$.

b. The expenses are $e(x) = 4500 + 4.25x$.

c. Profit = revenue – expenses.

$$\begin{aligned}(r - e)(x) &= 9.75x - (4500 + 4.25x) \\ &= 5.5x - 4500.\end{aligned}$$

18. $v(x) = \dfrac{4}{3}\pi(3x - 1)^3$ can be written as

$v(x) = f(l(x)) = (f \circ l)(x)$ where $f(x) = \dfrac{4}{3}\pi x^3$ and $l(x) = 3x - 1$. Then $l(x)$ represents the radius of the sphere, while $f(x)$ is the volume of a sphere with radius x.

19. $r = g(q) = g(f(m) = 24\dfrac{(20m - m^2)}{2} = 12(20m - m^2)$ is revenue from output of m employees.

20.
$$\begin{aligned}(f \circ g)(E) &= f(g(E)) \\ &= f(7202 + 0.29E^{3.68}) \\ &= 0.45(7202 + 0.29E^{3.68} - 1000)^{0.53} \\ &= 0.45(6202 + 0.29E^{3.68})^{0.53}\end{aligned}$$

This represents status based on years of education.

21. a. 14.05

b. 1169.64

22. a. −0.13

b. 18.85

23. a. 194.47

b. 0.29

24. a.
$$\begin{aligned}f(g(2.17)) &= f(1/(2.17)^3 \\ &\approx f(0.097863512) \approx 2/1.097863512 \\ &\approx 1.82\end{aligned}$$

b.
$$\begin{aligned}g(f(2.17)) &= g(2/3.17) \\ &\approx g(0.630914826) \\ &\approx 1/(0.630914826)^3 \approx 3.98\end{aligned}$$

Problems 2.4

1. $f^{-1}(x) = \dfrac{x}{3} - \dfrac{7}{3}$

2. $g^{-1}(x) = \dfrac{x}{5} + \dfrac{3}{5}$

3. $F^{-1}(x) = 2x + 14$

4. $f^{-1}(x) = \dfrac{\sqrt{x}}{4} + \dfrac{5}{4}$

5. $A(r) = 4\pi r^2$, for $r \geq 0$ gives the surface area of a sphere of radius r. Solving $A = 4\pi r^2$ for r gives $r = \sqrt{\dfrac{A}{4\pi}}$. Thus if a sphere is given to us and somehow its surface area is known to be A then it's radius r is given by the last equation. Said otherwise $A^{-1}(x) = \sqrt{\dfrac{x}{4\pi}}$

6. $r(V) = \sqrt[3]{\dfrac{3V}{4\pi}}$

7. $f(x) = 5x + 12$ is one-to-one, for if $f(x_1) = f(x_2)$ then $5x_1 + 12 = 5x_2 + 12$, so $5x_1 = 5x_2$ and thus $x_1 = x_2$.

8. $g(x) = (3x + 4)^2$ is not one-to-one, because $g(x_1) = g(x_2)$ does not imply $x_1 = x_2$. For example, $g\left(-\dfrac{1}{3}\right) = g\left(-\dfrac{7}{3}\right) = 9$.

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9. $h(x) = (5x + 12)^2$, for $x \geq -\frac{5}{12}$, is one-to-one.

If $h(x_1) = h(x_2)$ then $(5x_1 + 12)^2 = (5x_2 + 12)^2$.

Since $x \geq -\frac{5}{12}$ we have $5x + 12 \geq 0$, and thus

$(5x_1 + 12)^2 = (5x_2 + 12)^2$ only if

$5x_1 + 12 = 5x_2 + 12$, and hence $x_1 = x_2$.

10. $F(-11) = |-1| = 1 = |1| = |-9+10| = F(-9)$ shows that F is not one-to-one.

11. The inverse of $f(x) = (4x - 5)^2$ for $x \geq \frac{5}{4}$ is $f^{-1}(x) = \frac{\sqrt{x}}{4} + \frac{5}{4}$, so to find the solution, we find $f^{-1}(23)$.

$$f^{-1}(23) = \frac{\sqrt{23}}{4} + \frac{5}{4}$$

The solution is $x = \frac{\sqrt{23}}{4} + \frac{5}{4}$.

12. The inverse of $f(x) = 2x^3 + 1$ is $f^{-1} = \sqrt[3]{\frac{x-1}{2}}$, so the solution is $f^{-1}(129) = 4$.

13. From $p = \frac{1{,}200{,}000}{q}$, we get $q = \frac{1{,}200{,}000}{p}$.

Since $q > 0$, p is also greater than 0, so q as a function of p is $q = q(p) = \frac{1{,}200{,}000}{p}$, $p > 0$.

$$\begin{aligned} p(q(p)) &= p\left(\frac{1{,}200{,}000}{p}\right) \\ &= \frac{1{,}200{,}000}{\frac{1{,}200{,}000}{p}} \\ &= 1{,}200{,}000 \cdot \frac{p}{1{,}200{,}000} \\ &= p \end{aligned}$$

Similarly, $q(p(q)) = q$, so the functions are inverses.

14. From $p = \frac{q}{48}$, we get $q = 48p$. Since $q > 0$, p is also greater than 0, so q as a function of p is $q = q(p) = 48p$, $p > 0$.

$$q(p(q)) = q\left(\frac{q}{48}\right) = 48 \cdot \frac{q}{48} = q$$

$$p(q(p)) = p(48p) = \frac{48p}{48} = p$$

Thus, $p(q)$ and $q(p)$ are inverses.

15. We show that $f(x) = 10^x$ is one-to-one. If $a \neq b$, we may as well assume that $b > a$, and this means, precisely, that for some $e > 0$ we have $a + e = b$. Now for such e, $10^e > 1$ and we have $10^b = 10^{a+e} = 10^a \cdot 10^e > 10^a \cdot 1 = 10^a$. So $10^a \neq 10^b$ and f is one-to-one. It follows that f has an inverse. (In fact $f^{-1}(x)$ is known as $\log(x)$ and will be studied in detail in Chapter 4.)

Apply It 2.5

11. Let $y =$ the amount of money in the account. Then, after one month, $y = 7250 - (1 \cdot 600) = \$6650$, and after two months $y = 7250 - (2 \cdot 600) = \$6050$. Thus, in general, if we let $x =$ the number of months during which Rachel spends from this account, $y = 7250 - 600x$. To identify the x-intercept, we set $y = 0$ and solve for x.

$$y = 7250 - 600x$$

$$0 = 7250 - 600x$$

$$600x = 7250$$

$$x = 12\frac{1}{12}$$

The x-intercept is $\left(12\frac{1}{12}, 0\right)$.

Therefore, after 12 months and approximately 2.5 days Rachel will deplete her savings. To identify the y-intercept, we set $x = 0$ and solve for y.

$$y = 7250 - 600x$$

$$y = 7250 - 600(0)$$

$$y = 7250$$

The y-intercept is (0, 7250).

Therefore, before any months have gone by, Rachel has \$7250 in her account.

12. Let $y =$ the cost to the customer and let $x =$ the number of rides he or she takes. Since the cost does not change, regardless of the number of rides taken, the equation $y = 24.95$ represents this situation. The graph of $y = 24.95$ is a horizontal line whose y-intercept is (0, 24.95). Since the line is parallel to the x-axis, there is no x-intercept.

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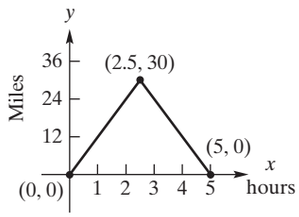
13. The formula relating distance, time, and speed is $d = rt$, where d is the distance, r is the speed, and t is the time. Let $x =$ the time spent biking (in hours). Then, $12x =$ the distance traveled. Brett bikes $12 \cdot 2.5 = 30$ miles and then turns around and bikes the same distance back to the rental shop. Therefore, we can represent the distance from the turn-around point at any time x as $|30 - 12x|$. Similarly, the distance from the rental shop at any time x can be represented by the function $y = 30 - |30 - 12x|$.

x	0	1	2	2.5	3	4	5
y	0	12	24	30	24	12	0



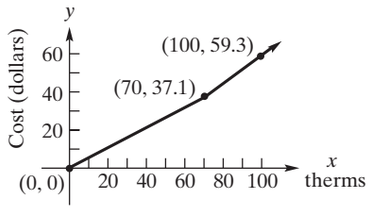
14. The monthly cost of x therms of gas is

$$y = \begin{cases} 0.53x, & \text{if } 0 \le x \le 70 \\ 0.53(70) + 0.74(x - 70), & \text{if } x > 70 \end{cases}$$

or

$$y = \begin{cases} 0.53x, & \text{if } 0 \le x \le 70 \\ 0.74x - 14.7, & \text{if } x > 70 \end{cases}$$

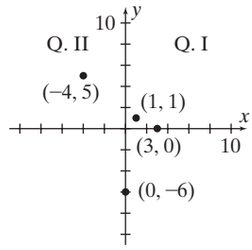
x	0	10	30	50	70	80	90	100
x	0	5.3	15.9	26.5	37.1	44.5	51.9	59.3



Problems 2.5

1. $(-1, -3)$ is in 3'rd quadrant; $(4, -2)$ is in 4'th quadrant; $\left(-\frac{2}{5}, 4\right)$ is in 2'nd quadrant; $(6, 0)$ is on the positive x-axis.

2.



3. a. $f(0) = 1, f(2) = 2, f(4) = 3, f(-2) = 0$

b. Domain: all real numbers

c. Range: all real numbers

d. $f(x) = 0$ for $x = -2$. So a real zero is -2.

4. a. $f(0) = 2, f(2) = 0$

b. Domain: all $x \ge 0$

c. Range: all $y \ge 2$

d. $f(x) = 0$ for $x = 2$. So a real zero is 2.

5. a. $f(0) = 0, f(1) = 1, f(-1) = 1$

b. Domain: all real numbers

c. Range: all nonnegative real numbers

d. $f(x) = 0$ for $x = 0$. So a real zero is 0.

6. a. $f(0) = 0, f(2) = 1, f(3) = 3, f(4) = 2$

b. Domain: all x such that $0 \le x \le 4$

c. Range: all y such that $0 \le y \le 3$

d. $f(x) = 0$ for $x = 0$. So a real zero is 0.

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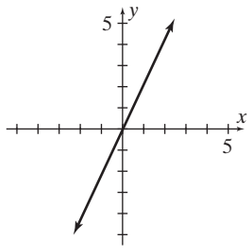
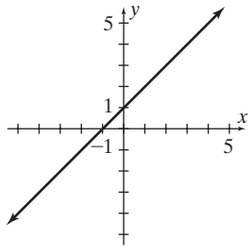
7. $y = 2x$

If $y = 0$, then $x = 0$. If $x = 0$, then $y = 0$.
Intercept: $(0, 0)$
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



8. $y = x + 1$

If $y = 0$, then $x = -1$.
If $x = 0$, then $y = 1$.
Intercepts: $(-1, 0), (0, 1)$
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers

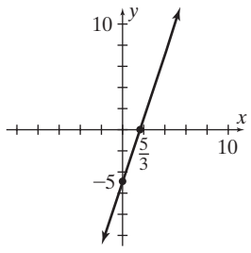


9. $y = 3x - 5$

If $y = 0$, then $0 = 3x - 5, x = \frac{5}{3}$.

If $x = 0$, then $y = -5$. Intercepts: $\left(\frac{5}{3}, 0\right), (0, -5)$

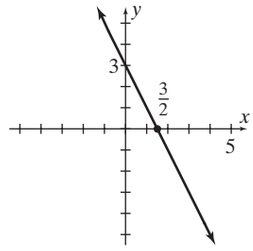
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



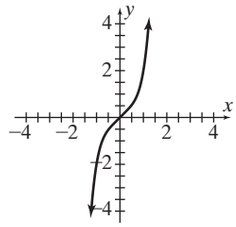
10. $y = 3 - 2x$

If $y = 0$, then $0 = 3 - 2x, x = \frac{3}{2}$.

If $x = 0$, then, $y = 3$. Intercepts: $\left(\frac{3}{2}, 0\right), (0, 3)y$ is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



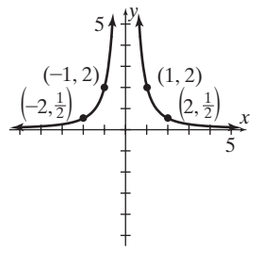
11. $(0, 0)$ is the only intercept of $y = x^5 + x$.



y is a function of x. It is a one-to-one function.
Both its domain and its range are $(-\infty, \infty)$.

12. $y = \frac{2}{x^2}$

If $y = 0$, then $0 = \frac{2}{x^2}$, which has no solution.
Thus there is no x-intercept. Because $x \neq 0$, Not one-to-one.
Domain: all real numbers except 0
Range: all real numbers > 0



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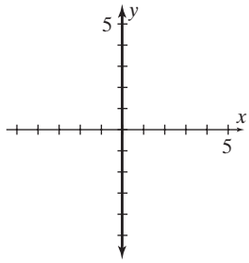
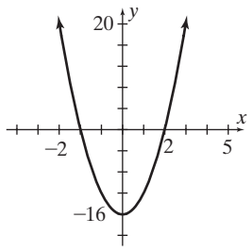
13. $x = 0$

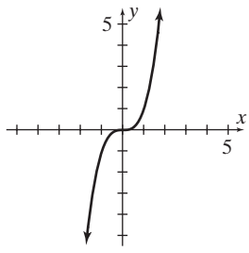
If $y = 0$, then $x = 0$. If $x = 0$, then y can be any real number. Intercepts: every point on y-axis
y is not a function of x.



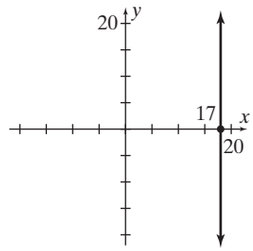
14. $y = 4x^2 - 16$
If $y = 0$, then $0 = 4x^2 - 16 = 4(x^2 - 4)$,
$0 = 4(x + 2)(x - 2)$, $x = \pm 2$.
If $x = 0$, then $y = -16$.
Intercepts: $(\pm 2, 0)$, $(0, -16)$
y is a function of x. Not one-to-one.
Domain: all real numbers
Range: all real numbers ≥ -16



15. $y = x^3$
If $y = 0$, then $0 = x^3$, $x = 0$. If $x = 0$, then $y = 0$.
Intercept: (0, 0). y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers

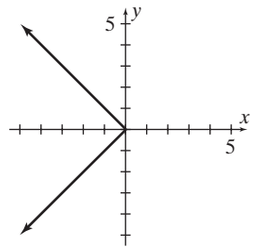


16. $(17, 0)$ is the only intercept of $x = 17$.

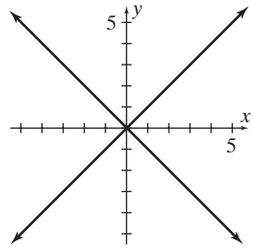


y is not a function of x.

17. $x = -|y|$
If $y = 0$, then $x = 0$. If $x = 0$, then $0 = -|y|, y = 0$.
Intercept: (0, 0)
y is not a function of x.



18. $x^2 = y^2$
If $y = 0$, then $x^2 = 0$, $x = 0$. If $x = 0$, then $0 = y^2$, $y = 0$. Intercept: $(0, 0)y$
is not a function of x.



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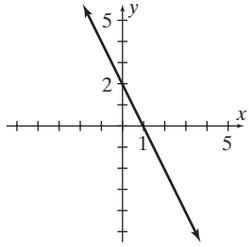
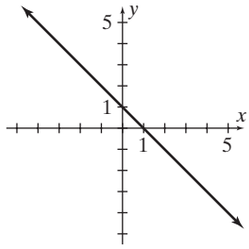
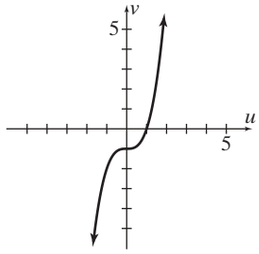
19. $2x + y - 2 = 0$
If $y = 0$, then $2x - 2 = 0$, $x = 1$. If $x = 0$, then $y - 2 = 0$, $y = 2$. Intercepts: $(1, 0)$, $(0, 2)$
Note that $y = 2 - 2x$. y is a function of x.
One-to-one.
Domain: all real numbers
Range: all real numbers



20. $x + y = 1$
If $y = 0$, then $x = 1$. If $x = 0$, then $y = 1$.
Intercepts: $(1, 0)$, $(0, 1)$
Note that $y = 1 - x$.
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



21.



has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$.
$(0, -1)$ is the only intercept.

22. $f(x) = 5 - 2x^2$. If $f(x) = 0$, then $0 = 5 - 2x^2$

$2x^2 = 5$

$x^2 = \dfrac{5}{2}$

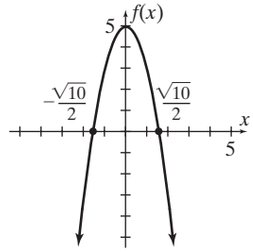
$x = \pm\sqrt{\dfrac{5}{2}} = \pm\dfrac{\sqrt{10}}{2}$.

If $x = 0$, then $f(x) = 5$.

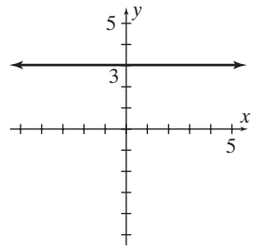
Intercepts: $\left(\pm\dfrac{\sqrt{10}}{2}, 0\right), (0, 5)$

Domain: all real numbers

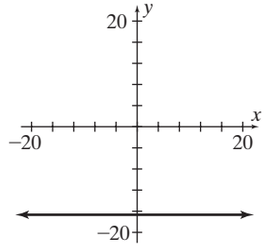
Range: all real numbers ≤ 5



23. $y = h(x) = 3$
Because y cannot be 0, there is no x-intercept. If $x = 0$, then $y = 3$. Intercept: $(0, 3)$
Domain: all real numbers
Range: 3



24. $g(s) = -17$
Because $g(s)$ cannot be 0, there is no s-intercept.
If $s = 0$, then $g(s) = -17$.
Intercept: $(0, -17)$
Domain: all real numbers
Range: -17



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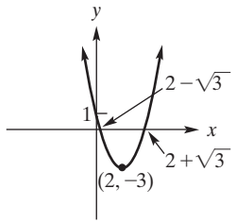
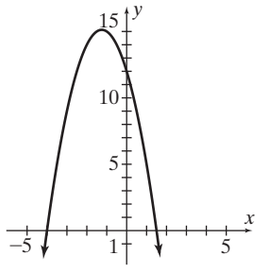
25. $y = h(x) = x^2 - 4x + 1$

If $y = 0$, then $0 = x^2 - 4x + 1$, and by the quadratic formula, $x = \dfrac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$. If $x = 0$, then $y = 1$. Intercepts: $(2 \pm \sqrt{3},\ 0)$, (0,1)
Domain: all real numbers
Range: all real numbers ≥ -3

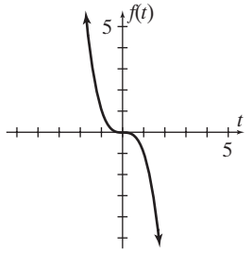


26.

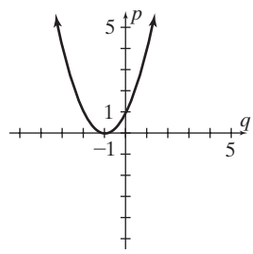


has domain $(-\infty, \infty)$ and range $(-\infty, 121/8]$.
Intercepts are $(0, 12)$, $(-4, 0)$, and $(3/2, 0)$.

27. $f(t) = -t^3$
If $f(t) = 0$, then $0 = -t^3, t = 0$.
If $t = 0$, then $f(t) = 0$. Intercept: (0, 0)
Domain: all real numbers
Range: all real number

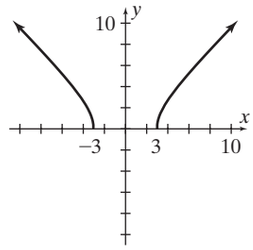


28. $p = h(q) = 1 + 2q + q^2$
If $p = 0$, then $1 + 2q + q^2 = 0, (1 + q)^2 = 0$, so $q = -1$. If $q = 0$ then $p = 1$.
Intercepts: (-1, 0), (0, 1)
Domain: all real numbers
Range: all real numbers ≥ 0



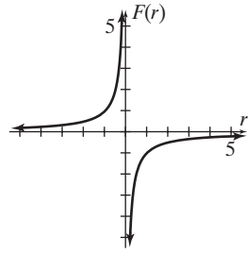
29. $s = f(t) = \sqrt{t^2 - 9}$

Note that for $\sqrt{t^2 - 9}$ to be a real number, $t^2 - 9 \geq 0$, so $t^2 \geq 9$, and $|t| \geq 3$. If $s = 0$, then $0 = \sqrt{t^2 - 9}, 0 = t^2 - 9$, or $t = \pm 3$. Because $|t| \geq 3$, we know $t \neq 0$, so no s-intercept exists.
Intercepts: $(-3, 0)$, $(3, 0)$
Domain: all real numbers $t \leq -3$ and ≥ 3
Range: all real numbers ≥ 0



30. $F(r) = -\dfrac{1}{r}$

If $F(r) = 0$, then $0 = -\dfrac{1}{r}$, which has no solution.
Because $r \neq 0$, there is no vertical-axis intercept.
Intercept: none.
Domain: all real numbers $\neq 0$
Range: all real numbers $\neq 0$



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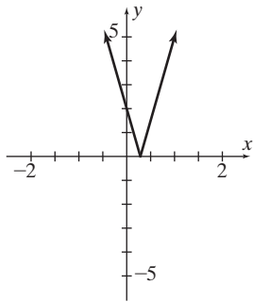
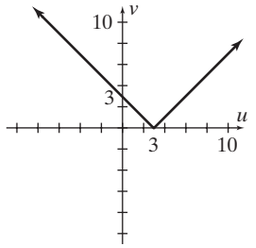
31.



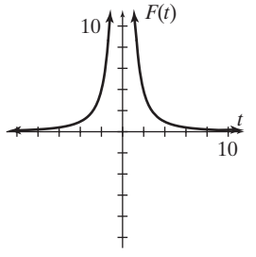
has domain $(-\infty, \infty)$ and range $[0, \infty)$.
Intercepts are $(0, 2)$ and $(2/7, 0)$.

32. $v = H(u) = |u - 3|$
If $v = 0$, then $0 = |u - 3|$, $u - 3 = 0$, so $u = 3$.
If $u = 0$, then $v = |-3| = 3$.
Intercepts: $(3, 0)$, $(0, 3)$.
Domain: all real numbers
Range: all real numbers ≥ 0



33. $F(t) = \dfrac{16}{t^2}$

If $F(t) = 0$, then $0 = \dfrac{16}{t^2}$, which has no solution.
Because $t \neq 0$, there is no vertical-axis intercept.
No intercepts
Domain: all nonzero real numbers
Range: all positive real numbers

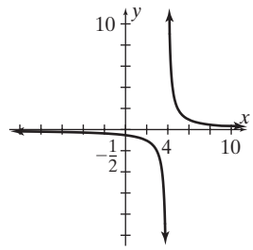


34. $y = f(x) = \dfrac{2}{x - 4}$

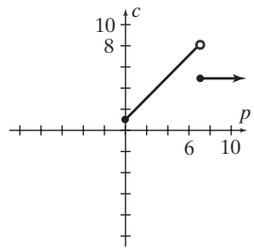
Note that the denominator is 0 when $x = 4$.
Thus $x \neq 4$. If $y = 0$, then $0 = \dfrac{2}{x - 4}$, which has no solution. If $x = 0$, then $y = -\dfrac{1}{2}$.

Intercept: $\left(0, -\dfrac{1}{2}\right)$

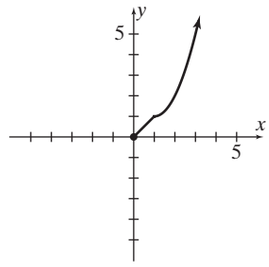
Domain: all real numbers except 4
Range: all real numbers except 0



35. Domain: all real numbers ≥ 0
Range: all real numbers $1 \leq c < 8$



36.



Both domain and range are $[0, \infty)$.

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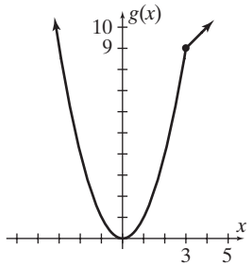
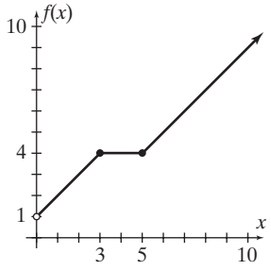
37. Domain: all real numbers
Range: all real numbers ≥ 0



38. Domain: all positive real numbers
Range: all real numbers > 1



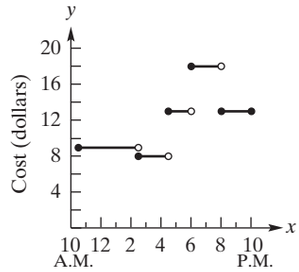
39. From the vertical-line test, the graphs that represent functions of x are (a), (b), and (d).

40. From the horizontal line test, the graphs which represent one-to-one functions of x are (c) and (d).

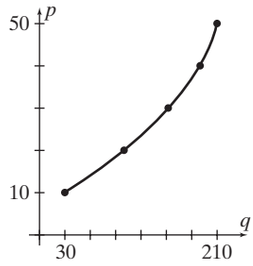
41. Write $D = D(n)$ for her debt after n payments. From the given data, $D = D(n) = 8700 - 300n$. The intercepts are $(0, 8700)$ and $(29, 0)$. The first is her initial debt load; the second is the number of months it takes her to become free of debt.

42. The cost of an item as a function of the time of day, x is

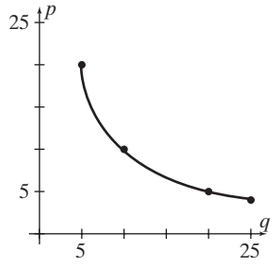
$$y = \begin{cases} 9, & \text{if } 10:30 \text{ A.M.} \leq x < 2:30 \text{ P.M.} \\ 8, & \text{if } 2:30 \text{ P.M.} \leq x < 4:30 \text{ P.M.} \\ 13, & \text{if } 4:30 \text{ P.M.} \leq x < 6:00 \text{ P.M.} \\ 18, & \text{if } 6:00 \text{ P.M.} \leq x < 8:00 \text{ P.M.} \\ 13, & \text{if } 8:00 \text{ P.M.} \leq x < 10:00 \text{ P.M.} \end{cases}$$



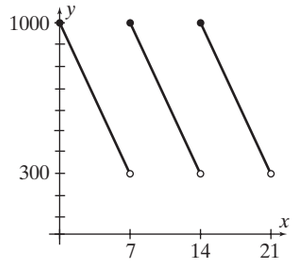
43. As price increases, quantity supplied increases; p is a function of q.



44. As price decreases, quantity increases; p is a function of q.



45.



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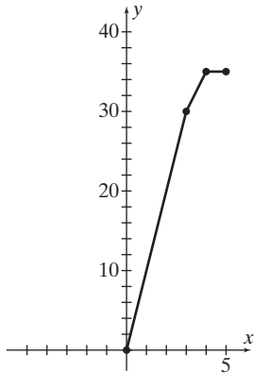
46.



No, she needs to keep training. She can run for 3 hours at a rate of 10 km/hr and can then run for another hour at a rate of 5 km/hr. She stops after a total of 4 hours, having covered 35 km which is less than the distance of a full marathon.

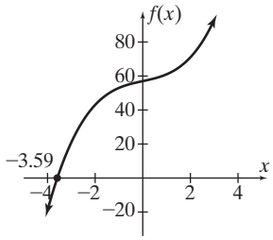
47. 0.39

48. −0.50, 0.57

49. −0.61, −0.04

50. 0.62, 1.73, 4.65

51.

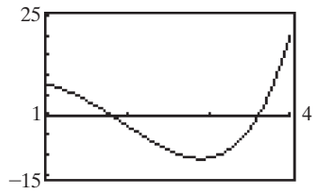


52. No real zeros

53. −1.70, 0

54. −0.49, 0.52, 1.25

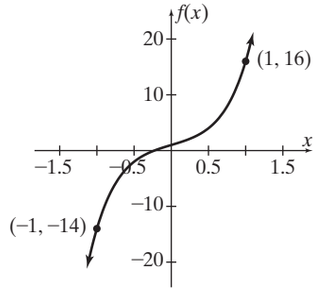
55.



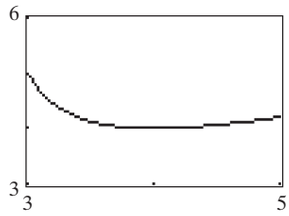
a. maximum value of $f(x)$: 19.60

b. minimum value of $f(x)$: −10.86

56.



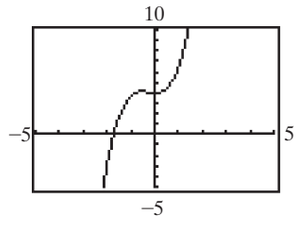
57.



a. maximum value of $f(x)$: 5

b. minimum value of $f(x)$: 4

58.



a. range: $(-\infty, \infty)$

b. intercepts: $(-1.73, 0), (0, 4)$

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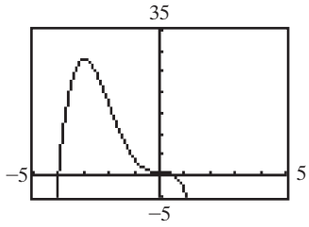
59.

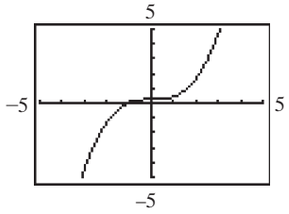


a. maximum value of $f(x)$: 28

b. range: $(-\infty, 28]$

c. real zeros: -4.02, 0.60

60.

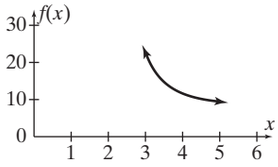


a. range: $(-\infty, \infty)$

b. intercepts: $(0, 0.29)$, $(-1.03, 0)$

c. real zero: -1.03

61.



Problems 2.6

1. $y = 5x$

Intercepts: If $y = 0$, then $5x = 0$, or $x = 0$; if $x = 0$, then $y = 5 \cdot 0 = 0$.
Testing for symmetry gives:

x-axis: $-y = 5x$

$y = -5x$

y-axis: $y = 5(-x) = -5x$

origin: $-y = 5(-x)$

$y = 5x$

line $y = x$: (a, b) on graph, then $b = 5a$, and $a = \frac{1}{5}b \neq 5b$ for all b, so (b, a) is not on the graph.

Answer: (0, 0); symmetry about origin

2. The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, -9)$. There is symmetry about the y-axis.

3. $2x^2 + y^2x^4 = 8 - y$

Intercepts: If $y = 0$, then

$2x^2 = 8$, $x^2 = 4$, or $x = \pm 2$;

if $x = 0$, then $0 = 8 - y$, so $y = 8$.

Testing for symmetry gives:

x-axis: $2x^2 + (-y)^2x^4 = 8 - (-y)$

$2x^2 + y^2x^4 = 8 + y$

y-axis: $2(-x)^2 + y^2(-x)^4 = 8 - y$

$2x^2 + y^2x^4 = 8 - y$

origin: $2(-x)^2 + (-y)^2(-x)^4 = 8 - (-y)$

$2x^2 + y^2x^4 = 8 + y$

line $y = x$: (a, b) on graph, then

$2a^2 + b^2a^4 = 8 - b$, but

$2b^2 + a^2b^4 = 8 - a$ will not

necessarily be true, so (b, a) is not on the graph.

Answer: $(\pm 2, 0)$, $(0, 8)$; symmetry about y-axis

4. $x = y^3$

Intercepts: If $y = 0$, then $x = 0$; if $x = 0$, then $0 = y^3$, so $y = 0$.

Testing for symmetry gives:

x-axis: $x = (-y)^3 = -y^3$

y-axis: $-x = y^3$

$x = -y^3$

origin: $-x = (-y)^3$

$x = y^3$

line $y = x$: (a, b) on graph, then $a = b^3$, and $b = \sqrt[3]{a} \neq a^3$ for all a, so (b, a) is not on the graph.

Answer: (0, 0); symmetry about origin

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5. $25x^2 + 144y^2 = 169$

Intercepts: If $y = 0$, then $25x^2 = 169, x^2 = \frac{169}{25}$, so $x = \pm\frac{13}{5}$;

If $x = 0$, then $144y^2 = 169 \quad y^2 = \frac{169}{144}$, and $y = \pm\frac{13}{12}$.

Testing for symmetry gives:

x-axis: $25x^2 + 144(-y)^2 = 169$

$25x^2 + 144y^2 = 169$

y-axis: $25(-x)^2 + 144y^2 = 169$

$25x^2 + 144y^2 = 169$

origin: Since the graph has symmetry about x- and y-axes, there is also symmetry about the origin.

line $y = x$: (a, b) on graph, then $25a^2 + 144b^2 = 169$, and $a^2 = \frac{1}{25}(169 - 144b^2)$.$(b, a)$ on graph, then $25b^2+144a^2=169$ and $a^2 = \frac{1}{144}(169 - 25b^2)$ $\neq \frac{1}{25}(169 - 144b^2)$ for all b, so (b, a) and (a, b) are not always both on the graph. Not symmetric about $y = x$.

Answer: $\left(\pm\frac{13}{5}, 0\right), \left(0, \pm\frac{13}{12}\right)$ symmetry about x-axis, y-axis, and origin.

6. $y = 57$

Intercepts: Because $y \neq 0$, there is no x-intercept; if $x = 0$, then $y = 57$.

Testing for symmetry gives:

x-axis: $(-y) = 57$

$y = -57$

y-axis: $y = 57$

origin: $(-y) = 57$

$y = -57$

line $y = x$: (a, b) on graph, then $b = 57$, but a can be any value, so $(b, a) = (57, a)$ is not necessarily on the graph.

Answer: (0, 57); symmetry about y-axis

7. The only intercept is $(-7, 0)$. The graph is symmetric about the x-axis.

8. $y = |2x| - 2$

Intercepts: If $y = 0$, then $|2x| = 2$, $2|x| = 2$, $|x| = 1$,so $x \pm 1$; if $x = 0$, then $y = -2$.

Testing for symmetry gives:

x-axis: $-y = |2x| - 2$

$y = -|2x| + 2$

y-axis: $y = |2(-x)| - 2$

$y = |2x| - 2$

origin: $-y = |2(-x)| - 2$

$y = -|2x| + 2$

line $y = x$: (a, b) on graph, then $b = |2a| - 2$ and $a = \pm\frac{b+2}{2} \neq |2b| - 2$ for all b, so (b, a) is not on the graph.

Answer: $(\pm1, 0), (0, -2)$; symmetry about y-axis

9. $x = -y^{-4}$

Intercepts: Because $y \neq 0$, there is no x-intercept; if $x = 0$, then $0 = -\frac{1}{y^4}$, which has no solution.

Testing for symmetry gives:

x-axis: $x = -(-y)^{-4}$

$x = -y^{-4}$

y-axis: $-x = -y^{-4}$

$x = y^{-4}$

origin: $-x = -(-y)^{-4}$

$x = y^{-4}$

line $y = x$: (a, b) on graph, then $a = -b^{-4}$ and $b = (-a)^{-1/4} \neq -a^{-4}$ for all a, so (b, a) is not on the graph.

Answer: no intercepts; symmetry about x-axis

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10. $y = \sqrt{x^2 - 36}$

Intercepts: If $y = 0$, then $\sqrt{x^2 - 36} = 0$,

$x^2 - 36 = 0, x^2 = 36$, so $x = \pm 6$;

if $x = 0$, then $y = \sqrt{-36}$, which has no real root.
Testing for symmetry gives:

x-axis: $-y = \sqrt{x^2 - 36}$

$y = -\sqrt{x^2 - 36}$

y-axis: $y = \sqrt{(-x)^2 - 36}$

$y = \sqrt{x^2 - 36}$

origin: $-y = \sqrt{(-x)^2 - 36}$

$y = -\sqrt{x^2 - 36}$

line $y = x$: (a, b) on graph, then $b = \sqrt{a^2 - 36}$

or $b^2 = a^2 - 36$ and

$a^2 = b^2 + 36 \neq b^2 - 36$ for all b,

so (b, a) is not on the graph.

Answer: $(\pm 6, 0)$; symmetry about y-axis

11. $x - 4y - y^2 + 21 = 0$

Intercepts: If $y = 0$, then $x + 21 = 0$, so $x = -21$;

if $x = 0$, then $-4y - y^2 + 21 = 0$,

$y^2 + 4y - 21 = 0, (y + 7)(y - 3) = 0$, so $y = -7$ or $y = 3$.

Testing for symmetry gives:

x-axis: $x - 4(-y) - (-y)^2 + 21 = 0$

$x + 4y - y^2 + 21 = 0$

y-axis: $(-x) - 4y - y^2 + 21 = 0$

$-x - 4y - y^2 + 21 = 0$

origin: $(-x) - 4(-y) - (-y)^2 + 21 = 0$

$-x + 4y - y^2 + 21 = 0$

line $y = x$: (a, b) on graph, then

$a - 4b - b^2 + 21 = 0$ and

$a = b^2 + 4b - 21$, but

$b = a^2 + 4a - 21$ will not necessarily

be true, so (b, a) is not on the graph.

Answer: $(-21, 0), (0, -7), (0, 3)$; no symmetry

12. The only intercept is $(0, 0)$. The graph is symmetric about $y = x$.

13. $y = f(x) = \dfrac{x^3 - 2x^2 + x}{x^2 + 1}$

Intercepts: If $y = 0$, then

$\dfrac{x^3 - 2x^2 + x}{x^2 + 1} = \dfrac{x(x-1)^2}{x^2 + 1} = 0$, so $x = 0, 1$;

if $x = 0$, then $y = 0$.

Testing for symmetry gives:

x-axis: Because f is not the zero function, there is no x-axis symmetry

y-axis: $y = \dfrac{(-x)^3 - 2(-x)^2 + (-x)}{(-x)^2 + 1}$

$y = \dfrac{-x^3 - 2x^2 - x}{x^2 + 1}$

origin: $-y = \dfrac{(-x)^3 - 2(-x)^2 + (-x)}{(-x)^2 + 1}$

$y = \dfrac{x^3 + 2x^2 + x}{x^2 + 1}$

line $y = x$: (a, b) on graph, then

$b = \dfrac{a^3 - 2a^2 + a}{a^2 + 1}$, but

$a = \dfrac{b^3 - 2b^2 + b}{b^2 + 1}$ is not necessarily

true, so (b, a) is not on the graph.

Answer: (1, 0), (0, 0); no symmetry of the given types

14. $x^2 + xy + y^2 = 0$

Intercepts: If $y = 0$, then $x^2 = 0$, so $x = 0$;

if $x = 0$, then $y^2 = 0$, so $y = 0$.

Testing for symmetry gives:

x-axis: $x^2 + x(-y) + (-y)^2 = 0$

$x^2 - xy + y^2 = 0$

y-axis: $(-x)^2 + (-x)y + y^2 = 0$

$x^2 - xy + y^2 = 0$

origin: $(-x)^2 + (-x)(-y) + (-y)^2 = 0$

$x^2 + xy + y^2 = 0$

line $y = x$: (a, b) on graph, then $a^2 + ab + b^2 = 0$ and $b^2 + ba + a^2 = 0$, so (b, a) is on the graph.

Answer: $(0, 0)$; symmetry about origin, symmetry about $y = x$

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15. $y = \dfrac{2}{x^3 + 27}$

Intercepts: If $y = 0$, then $\dfrac{2}{x^3 + 27} = 0$, which has no solution; if $x = 0$, then $y = \dfrac{2}{27}$.

Testing for symmetry gives:

x-axis: $-y = \dfrac{2}{x^3 + 27}$

$y = -\dfrac{2}{x^3 + 27}$

y-axis: $y = \dfrac{2}{(-x)^3 + 27}$

$y = \dfrac{2}{-x^3 + 27}$

origin: $-y = \dfrac{2}{(-x)^3 + 27}$

$-y = \dfrac{2}{-x^3 + 27}$

$y = \dfrac{2}{x^3 - 27}$

line $y = x$: (a, b) on graph, then $b = \dfrac{2}{a^3 + 27}$ and $a = \sqrt[3]{\dfrac{2}{b} - 27} \neq \dfrac{2}{b^3 + 27}$ for all b, so (b, a) is not on the graph.

Answer: $\left(0, \dfrac{2}{27}\right)$; no symmetry

16. $y = \dfrac{x^4}{x + y}$

Intercepts: If $y = 0$, then $\dfrac{x^4}{x} = 0$, which has no solution; if $x = 0$, then $y = \dfrac{0}{y}$, which has no solution.

Testing for symmetry gives:

x-axis: $-y = \dfrac{x^4}{x + (-y)}$

$y = \dfrac{x^4}{-x + y}$

y-axis: $y = \dfrac{(-x)^4}{(-x) + y}$

$y = \dfrac{x^4}{-x + y}$

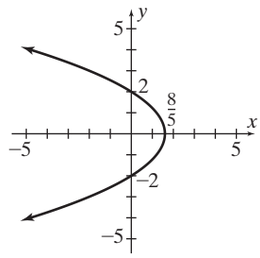
origin: $-y = \dfrac{(-x)^4}{(-x) + (-y)}$

$y = \dfrac{x^4}{x + y}$

line $y = x$: (a, b) on graph, then $b = \dfrac{a^4}{a + b}$, and $a + b = \dfrac{a^4}{b}$, but $a + b = \dfrac{b^4}{a}$ will not necessarily be true, so (b, a) is not on the graph.

Answer: no intercepts; symmetry about origin

17. The intercepts are $(0, -2)$, $(0, 2)$, and $(8/5, 0)$. The graph is symmetric about the x-axis.



18. $x - 1 = y^4 + y^2$ or $x = y^4 + y^2 + 1$

Intercepts: If $y = 0$, then $x = 1$; if $x = 0$, then $y^4 + y^2 = -1$, so no y-intercept

Testing for symmetry gives:

x-axis: $x - 1 = (-y)^4 + (-y)^2$

$x - 1 = y^4 + y^2$

y-axis: $-x = y^4 + y^2 + 1$

$x = -y^4 - y^2 - 1$

origin: $-x = (-y)^4 + (-y)^2 + 1$

$x = -y^4 - y^2 - 1$

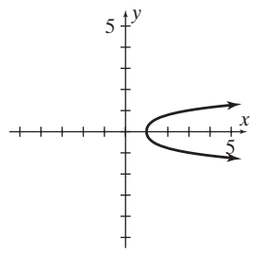
line $y = x$: (a, b) on graph, then

$a = b^4 + b^2 + 1$ and

$b \neq a^4 + a^2 + 1$

for all a so (b, a) is not on the graph.

Answer: $(1, 0)$; symmetry about x-axis.



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19. $y = f(x) = x^3 - 4x$

Intercepts: If $y = 0$, then $x^3 - 4x = 0$, $x(x + 2)(x - 2) = 0$, so $x = 0$ or $x = \pm 2$; if $x = 0$, then $y = 0$.

Testing for symmetry gives:

x-axis: Because f is not the zero function, there is no x-axis symmetry.

y-axis: $y = (-x)^3 - 4(-x)$

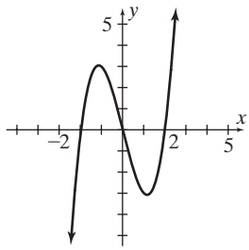
$y = -x^3 + 4x$

origin: $-y = (-x)^3 - 4(-x)$

$y = x^3 - 4x$

line $y = x$: (a, b) on graph, then $b = a^3 - 4a$, but $a = b^3 - 4b$ will not necessarily be true, so (b, a) is not on the graph.

Answer: $(0, 0)$, $(\pm 2, 0)$; symmetry about origin.



20. $2y = 5 - x^2$

Intercepts: If $y = 0$, then $5 - x^2 = 0$, so $x = \pm\sqrt{5}$. If $x = 0$, $y = \dfrac{5}{2}$.

Testing for symmetry gives:

x-axis: $2(-y) = 5 - x^2$

$2y = -5 + x^2$

y-axis: $2y = 5 - (-x)^2$

$2y = 5 - x^2$

origin: $2(-y) = 5 - (-x)^2$

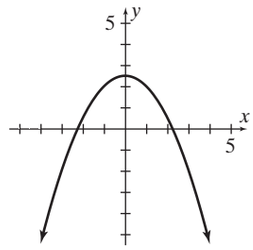
$2y = -5 + x^2$

line $y = x$: (a, b) on graph, then $2b = 5 - a^2$.

(b, a) on graph, then $2a = 5 - b^2$.

(a, b) and (b, a) are not both on the graph.

Answer: $\left(\pm\sqrt{5}, 0\right)$, $\left(0, \dfrac{5}{2}\right)$; symmetry about y-axis



21. $|x| - |y| = 0$

Intercepts: If $y = 0$, then $|x| = 0$ so $x = 0$; if $x = 0$, then $-|y| = 0$, so $y = 0$.

Testing for symmetry gives:

x-axis: $|x| - |-y| = 0$

$|x| - |y| = 0$

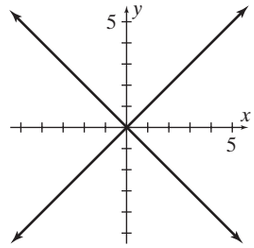
y-axis: $|-x| - |y| = 0$

$|x| - |y| = 0$

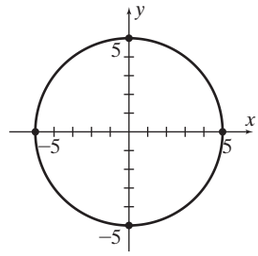
origin: Since there is symmetry about the x- and y-axes, symmetry about origin exists.

line $y = x$: (a, b) on graph, then $|a| - |b| = 0$, thus $|a| = |b|$, and $|b| - |a| = 0$, so (b, a) is on the graph.

Answer: $(0, 0)$; symmetry about x-axis, y-axis, origin, line $y = x$.



22. The intercepts are $(0, -5)$, $(0, 5)$, $(-5, 0)$, and $(5, 0)$. The graph is symmetric with respect to both the x- and y-axes and hence with respect to the origin. It is also symmetric with respect to the line $y = x$.



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23. $9x^2 + 4y^2 = 25$

Intercepts: If $y = 0$, then $9x^2 = 25, x^2 = \frac{25}{9}$, so $x = \pm\frac{5}{3}$; if $x = 0$, then $4y^2 = 25$, so $y = \pm\frac{5}{2}$.

Testing for symmetry gives:

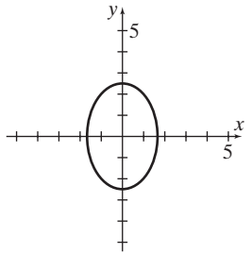
x-axis: $9x^2 + 4(-y)^2 = 25$
$9x^2 + 4y^2 = 25$

y-axis: $9(-x)^2 + 4y^2 = 25$
$9x^2 + 4y^2 = 25$

origin: Since there is symmetry about x- and y-axes, symmetry about origin exists.

line $y = x$: (a, b) on graph, then $9a^2 + 4b^2 = 25$ and $b^2 = \frac{1}{4}(25 - 9a^2)$.$(b, a)$ on graph, then $9b^2 + 4a^2 = 25$ and $b^2 = \frac{1}{9}(25 - 4a^2)$, so (a, b) and (b, a) are not always both on the graph.

Answer: $\left(\pm\frac{5}{3}, 0\right), \left(0, \pm\frac{5}{2}\right)$; symmetry about x-axis, y-axis, origin



24. $x^2 - y^2 = 4$

Intercepts: If $y = 0$, then $x^2 = 4$, so $x = \pm 2$; if $x = 0$, then $-y^2 = 4$, $y^2 = -4$, which has no real roots.

Testing for symmetry gives:

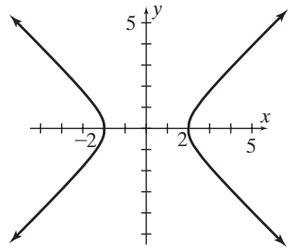
x-axis: $x^2 - (-y)^2 = 4$
$x^2 - y^2 = 4$

y-axis: $(-x)^2 - y^2 = 4$
$x^2 - y^2 = 4$

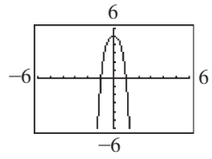
origin: Since there is symmetry about x-and y-axes, symmetry about origin exists.

line $y = x$: (a, b) on graph, then $a^2 - b^2 = 4$ and $a^2 = 4 + b^2 \neq b^2 - 4$ for all b, so (b, a) is not on the graph.

Answer: $(\pm 2, 0)$; symmetry about x-axis, y-axis, origin.



25.



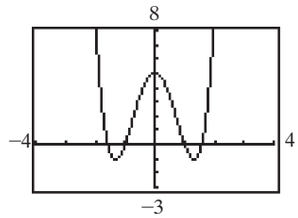
$y = f(x) = 5 - 1.96x^2 - \pi x^4$. Replacing x by $-x$ gives $y = 5 - 1.96(-x)^2 - \pi(-x)^4$ or $y = 5 - 1.96x^2 - \pi x^4$, which is equivalent to original equation. Thus the graph is symmetric about the y-axis.

a. Intercepts: $(\pm 0.99, 0), (0, 5)$

b. Maximum value of $f(x)$: 5

c. Range: $(-\infty, 5]$

26.



$y = f(x) = 2x^4 - 7x^2 + 5$. Replacing x by $-x$ gives $y = 2(-x)^4 - 7(-x)^2 + 5$ or $y = 2x^4 - 7x^2 + 5$, which is equivalent to original equation. Thus the graph is symmetric about y-axis.

Real zeros of f: $\pm 1, \pm 1.58$

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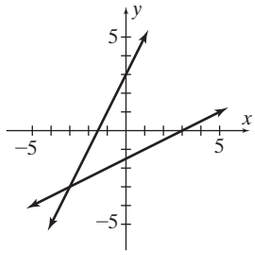
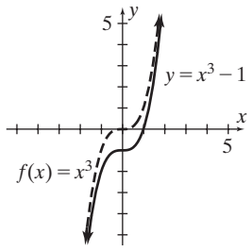
27.

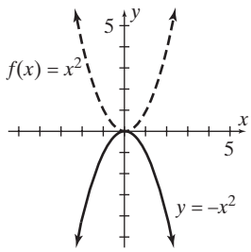


Problems 2.7

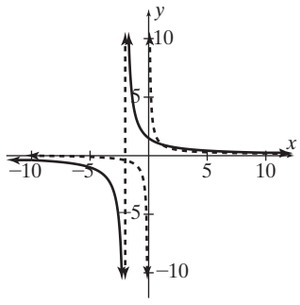
1.



2.



3. The required graph is obtained by translating the graph of $y = 1/x$ 2 units to the left and the streching the resulting graph vertically away from the x-axis by a factor of 3.



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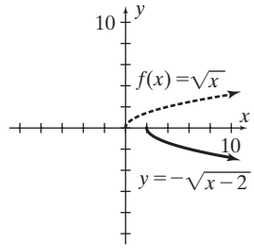

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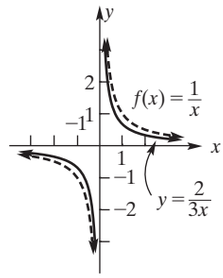

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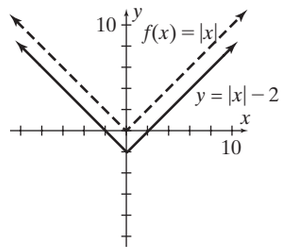

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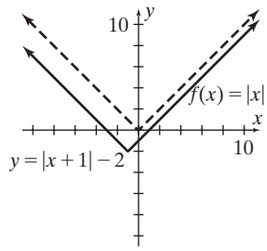

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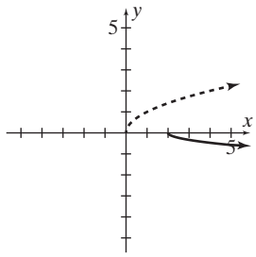
8. Translate the graph of $y = \sqrt{x}$ 2 units to the right; shrink the resulting graph vertically towards the x-axis by a factor of $1/3$; and reflect the result about the x-axis.



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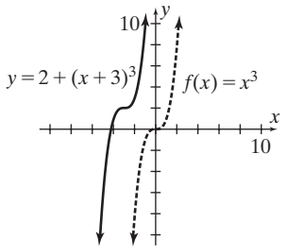

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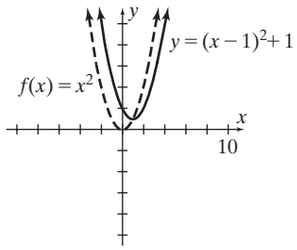

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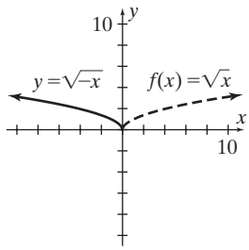

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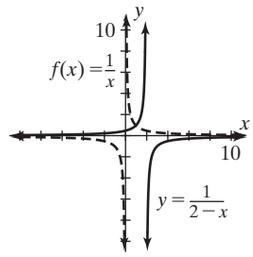


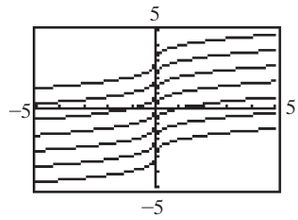
13. Translate the graph of $y = f(x)5$ units to the right and 1 unit up; shrink the result by a factor of 1/2 vertically towards the x-axis; and then reflect about the x-axis.

14. Shift one unit left, four units down, and stretch by a factor of 2 away from the x-axis.

15. Reflect about the y-axis and translate 5 units downward.

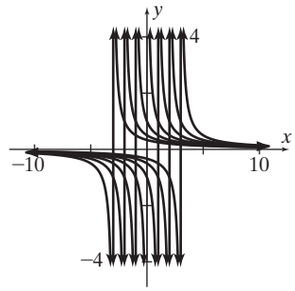
16. Shrink horizontally toward the y-axis by a factor of 3.

17.



Compared to the graph for $k = 0$, the graphs for $k = 1, 2$, and 3 are vertical shifts upward of 1, 2, and 3 units, respectively. The graphs for $k = -1, -2$, and -3 are vertical shifts downward of 1, 2, and 3 units, respectively.

18.



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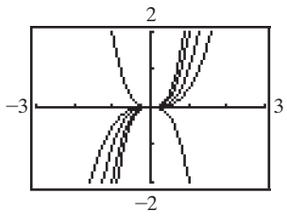
19.



Compared to the graph for $k = 1$, the graphs for $k = 2$ and 3 are vertical stretches away from the x-axis by factors of 2 and 3, respectively. The graph for $k = \frac{1}{2}$ is a vertical shrinking toward the x-axis.

Apply It 2.8

15. a. $c(500, 700) = 160 + 2(500) + 3(700) = 160 + 1000 + 2100 = 3260$

The cost of manufacturing 500 12-ounce and 700 20-ounce mugs is \$3260.

b. $c(1000, 750) = 160 + 2(1000) + 3(750) = 160 + 2000 + 2250 = 4410$

The cost of manufacturing 1000 12-ounce mugs and 750 20-ounce mugs is \$4410.

Problems 2.8

1. $f(1, 2) = 4(1) - (2)^2 + 3 = 4 - 4 + 3 = 3$

2. $f(2, \ -1) = 3(2)^2(-1) - 4(-1) = -12 + 4 = -8$

3. $g(3, 0, -1) = 2(3)[3(0) + (-1)] = -6$

4. $g(1, b) = 1 + b + b^3 = 0$

5. $h(-3, 3, 5, 4) = \dfrac{-3(3)}{5^2 - 4^2} = \dfrac{-9}{25 - 16} = \dfrac{-9}{9} = -1$

6. $h(1, 5, 3, 1) = (1)(1) = 1$

7. $g(4, 8) = 2(4)(4^2 - 5) = 2(4)(11) = 88$

8. $g(4, 9) = (4^2)\sqrt{9} + 9 = 16 \cdot 3 + 9 = 57$

9. $F(6, 0, -5) = 17$

10. $F(1, \ 0, \ 3) = \dfrac{2(1)}{(0 + 1)(3)} = \dfrac{2}{3}$

11. $f(a + h, \ b) = [(a + h) + b]^2$
$= a^2 + 2ab + 2ah + b^2 + 2bh + h^2$

12. $f(r + t, \ r) = (r + t)^2 r - 3r^3 = r\left(t^2 + 2rt - 2r^2\right)$

13. $f(200, \ 200, \ 50) = \dfrac{(200)(200)}{50} = 800$

14. We must evaluate $P(2, 7) = \dfrac{7!}{2!5!}\left(\dfrac{1}{4}\right)^2\left(\dfrac{3}{4}\right)^5 = \dfrac{7 \cdot 6}{2}\left(\dfrac{3^5}{4^7}\right) = \dfrac{7 \cdot 3^6}{4^7} = \dfrac{7 \cdot 3^6}{2^{14}} \approx 0.31146240$

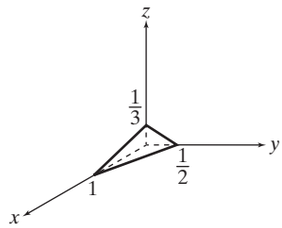
15. A plane parallel to the x, z-plane has the form $y =$ constant. Because (0, 2, 0) lies on the plane, the equation is $y = 2$.

16. A plane parallel to the y, z-plane has the form $x =$ constant. Because $(-2, 0, 0)$ lies on the plane, the equation is $x = -2$.

17. A plane parallel to the x, y-plane has the form $z =$ constant. Because (2, 7, 6) lies on the plane, the equation is $z = 6$.

18. A plane parallel to the y, z-plane has the form $x =$ constant. Because $(96, -2, 2)$ lies on the plane, the equation is $x = 96$.

19.



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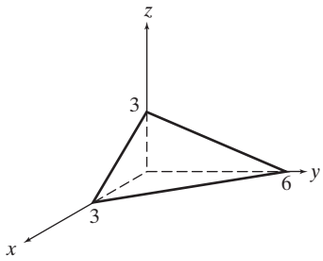
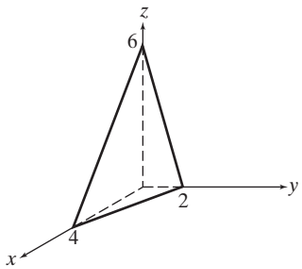
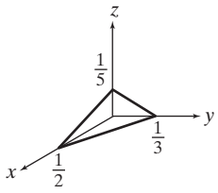
20. $2x + y + 2z = 6$ can be put in the form $Ax + By + Cz + D = 0$, so the graph is a plane. The intercepts are (3, 0, 0), (0, 6, 0), and (0, 0, 3).



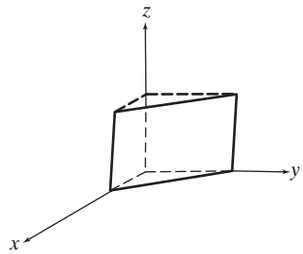
21. $3x + 6y + 2z = 12$ can be put in the form $Ax + By + Cz + D = 0$, so the graph is a plane. The intercepts are (4, 0, 0), (0, 2, 0), and (0, 0, 6).



22. $2x + 3y + 5z = 1$ can be put in the form $Ax + By + Cz + D = 0$, so the graph is a plane. The intercepts are $\left(\frac{1}{2},\ 0,\ 0\right)$, $\left(0,\ \frac{1}{3},\ 0\right)$, and $\left(0,\ 0,\ \frac{1}{5}\right)$.



23. $3x + y = 6$ can be put in the form $Ax + By + Cz + D = 0$, so the graph is a plane. There are only two intercepts: $(2, 0, 0)$ and $(0, 6, 0)$. The x, y-trace is $3x + y = 6$, which is a line. For any fixed value of z, we obtain the line $3x + y = 6$.



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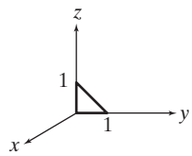
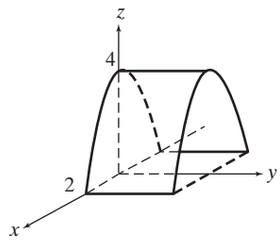
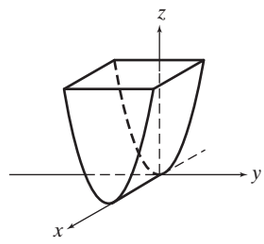


25. $z = 4 - x^2$. The x, z-trace is $z = 4 - x^2$, which is a parabola. For any fixed value of y, we obtain the parabola $z = 4 - x^2$.



26. $y = z^2$. The y, z-trace is $y = z^2$, which is a parabola. For any fixed value of x, we obtain the parabola $y = z^2$.



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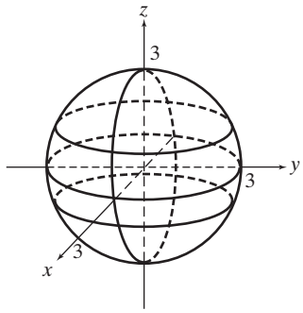
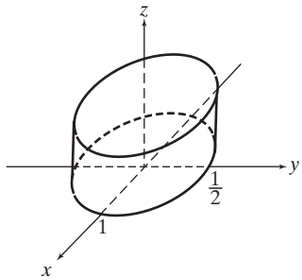
27. $x^2 + y^2 + z^2 = 9$. The x, y-trace is $x^2 + y^2 = 9$, which is a circle. The x, z-trace is $x^2 + z^2 = 9$, which is a circle. The y, z-trace is $y^2 + z^2 = 9$, which is a circle. The surface is a sphere.

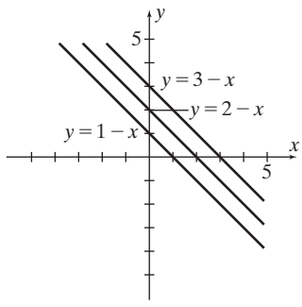


28. $x^2 + 4y^2 = 1$

The x, y-trace is $x^2 + 4y^2 = 1$, which is an ellipse. For any fixed value of z, we obtain the ellipse $x^2 + 4y^2 = 1$.

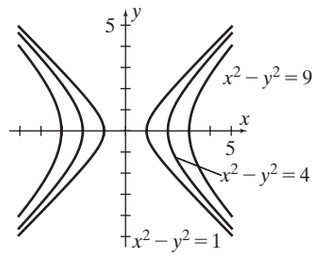


29.



30. $z = x^2 - y^2$

Choose $z = 1, 4$, and 9 for the curves.



Chapter 2 Review Problems

1. Denominator is 0 when

$$x^2 - 6x + 5 = 0$$
$$(x - 1)(x - 5) = 0$$
$$x = 1, \quad 5$$

Domain: all real numbers except 1 and 5.

2. all real numbers

3. all real numbers

4. all real numbers

5. We require x in $[2, \infty)$ for the numerator to be defined and $x \neq 3$ for the denominator to be different from 0. The domain is $[2, \infty) - \{3\}$.

6. $s - 5 \geq 0$

$s \geq 5$

Domain: all real numbers s such that $s \geq 5$.

7. $f(x) = 2x^2 - 3x + 5$

$f(0) = 2(0)^2 - 3(0) + 5 = 5$

$f(-2) = 2(-2)^2 - 3(-2) + 5 = 8 + 6 + 5 = 19$

$f(5) = 2(5)^2 - 3(5) + 5 = 50 - 15 + 5 = 40$

$f(\pi) = 2\pi^2 - 3\pi + 5$

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8. $h(x) = 7$; all function values are 7.

Answer: 7, 7, 7, 7

9. $G(x) = \sqrt[4]{x-3}$

$G(3) = \sqrt[4]{3-3} = \sqrt[4]{0} = 0$

$G(19) = \sqrt[4]{19-3} = \sqrt[4]{16} = 2$

$G(t+1) = \sqrt[4]{(t+1)-3} = \sqrt[4]{t-2}$

$G(x^3) = \sqrt[4]{x^3-3}$

10. $F(-1) = \dfrac{3(-1)+2}{(-1)-5} = \dfrac{-1}{-6} = 1/6,$

$F(0) = \dfrac{3(0)+2}{(0)-5} = \dfrac{2}{-5} = -2/5,$

$F(4) = \dfrac{3(4)+2}{(4)-5} = \dfrac{14}{-1} = -14,$

$F(x+2) = \dfrac{3(x+2)+2}{(x+2)-5} = \dfrac{3x+8}{x-3}$

11. $h(u) = \dfrac{\sqrt{u+4}}{u}$

$h(5) = \dfrac{\sqrt{5+4}}{5} = \dfrac{\sqrt{9}}{5} = \dfrac{3}{5}$

$h(-4) = \dfrac{\sqrt{-4+4}}{-4} = \dfrac{0}{-4} = 0$

$h(x) = \dfrac{\sqrt{x+4}}{x}$

$h(u-4) = \dfrac{\sqrt{(u-4)+4}}{u-4} = \dfrac{\sqrt{u}}{u-4}$

12. $H(t) = \dfrac{(t-2)^3}{5}$

$H(-1) = \dfrac{(-1-2)^3}{5} = -\dfrac{27}{5}$

$H(0) = \dfrac{(0-2)^3}{5} = -\dfrac{8}{5}$

$H\left(\dfrac{1}{3}\right) = \dfrac{\left(\frac{1}{3}-2\right)^3}{5} = \dfrac{\left(-\frac{5}{3}\right)^3}{5}$
$= \left(-\dfrac{125}{27}\right)\left(\dfrac{1}{5}\right) = -\dfrac{25}{27}$

$H(x^2) = \dfrac{(x^2-2)^3}{5}$

13. $f(4) = 4 + 16 = 20$

$f(-2) = -3$

$f(0) = -3$

$f(1)$ is not defined.

14. $f\left(-\dfrac{1}{2}\right) = -\left(-\dfrac{1}{2}\right) + 1 = \dfrac{1}{2} + 1 = \dfrac{3}{2}$

$f(0) = 0^2 + 1 = 1$

$f\left(\dfrac{1}{2}\right) = \left(\dfrac{1}{2}\right)^2 + 1 = \dfrac{1}{4} + 1 = \dfrac{5}{4}$

$f(5) = 5^3 - 99 = 125 - 99 = 26$

$f(6) = 6^3 - 99 = 216 - 99 = 117$

15. a. $f(x+h) = 1 - 3(x+h) = 1 - 3x - 3h$

b. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{(1-3x-3h)-(1-3x)}{h}$
$= \dfrac{-3h}{h} = -3 \quad \text{for } h \neq 0$

16. a. $f(x+h) = 11(x+h)^2 + 4$
$= 11x^2 + 22xh + 11h^2 + 4$

b. $\dfrac{f(x+h)-f(x)}{h}$
$= \dfrac{(11x^2 + 22xh + 11h^2 + 4) - (11x^2 + 4)}{h}$
$= \dfrac{22xh + 11h^2}{h} = 22x + 11h$

17. a. $f(x+h) = 3(x+h)^2 + (x+h) - 2$

b. $\dfrac{f(x+h)-f(x)}{h}$
$= \dfrac{3(x+h)^2 + (x+h) - 2 - (3x^2 + x - 2)}{h}$
$= \dfrac{3x^2 + 6xh + 3h^2 + x + h - 2 - 3x^2 - x + 2}{h}$
$= \dfrac{6xh + 3h^2 + h}{h}$
$= 6x + 3h + 1$

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18. a. $f(x+h) = \dfrac{7}{(x+h)+1} = \dfrac{7}{x+h+1}$

b. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{7}{x+h+1} - \frac{7}{x+1}}{h}$

$= \dfrac{\frac{7(x+1)-7(x+h+1)}{(x+h+1)(x+1)}}{h}$

$= \dfrac{-7h}{(x+h+1)(x+1)h}$

$= \dfrac{-7}{(x+h+1)(x+1)}$

19. $f(x) = 3x-1, g(x) = 2x+3$

a. $(f+g)(x) = f(x) + g(x)$
$= (3x-1) + (2x+3) = 5x+2$

b. $(f+g)(4) = 5(4) + 2 = 22$

c. $(f-g)(x) = f(x) - g(x)$
$= (3x-1) - (2x+3) = x-4$

d. $(fg)(x) = f(x)g(x) = (3x-1)(2x+3)$
$= 6x^2 + 7x - 3$

e. $(fg)(1) = 6(1)^2 = 7(1) - 3 = 10$

f. $\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x-1}{2x+3}$

g. $(f \circ g)(x) = f(g(x)) = f(2x+3)$
$= 3(2x+3) - 1 = 6x+8$

h. $(f \circ g)(5) = 6(5) + 8 = 38$

i. $(g \circ f)(x) = g(f(x)) = g(3x-1)$
$= 2(3x-1) + 3 = 6x+1$

20. a. $(f+g)(x) = x^3 + 2x + 1$

b. $(f-g)(x) = x^3 - 2x - 1$

c. $(f-g)(-6) = -216 + 12 - 1 = -205$

d. $(fg)(x) = x^3(2x+1) = 2x^4 + x^3$

e. $\dfrac{f}{g}(x) = \dfrac{x^3}{2x+1}$

f. $\dfrac{f}{g}(1) = 1/3$

g. $(f \circ g)(x) = (2x+1)^3 = 8x^3 + 12x^2 + 6x + 1$

h. $(g \circ f)(x) = 2x^3 + 1$

i. $(g \circ f)(2) = 17$

21. $f(x) = \dfrac{1}{x^2}, g(x) = x+1$

$(f \circ g)(x) = f(g(x)) = f(x+1) = \dfrac{1}{(x+1)^2}$

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{x^2}\right) = \dfrac{1}{x^2} + 1 =$

$\dfrac{1+x^2}{x^2}$

22. $f(x) = \dfrac{x-2}{3}, g(x) = \dfrac{1}{\sqrt{x}}$

$(f \circ g)(x) = f(g(x)) = \dfrac{\frac{1}{\sqrt{x}} - 2}{3} = \dfrac{1-2\sqrt{x}}{3\sqrt{x}}$

$(g \circ f)(x) = g(f(x)) = \dfrac{1}{\sqrt{\frac{x-2}{3}}} = \sqrt{\dfrac{3}{x-2}}$

23. $f(x) = \sqrt{x+2}, g(x) = x^3$

$(f \circ g)(x) = f(g(x)) = f(x^3) = \sqrt{x^3+2}$

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x+2}\right) = \left(\sqrt{x+2}\right)^3$

$= (x+2)^{3/2}$

24. $f(x) = 2, g(x) = 3$

$(f \circ g)(x) = f(g(x)) = f(3) = 2$

$(g \circ f)(x) = g(f(x)) = g(2) = 3$

25. Only intercept is $(0,0)$. $(-y) = 2(-x) + (-x)^3$ is equivalent to $-y = -2x - x^3$ is equivalent to $y = 2x + x^3$ so symmetric about the origin (but no other symmetries among those to be tested).

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26. $\dfrac{x^2y^2}{x^2+y^2+1} = 4$

Intercepts: If $y = 0$, then $0 = 4$, which is impossible; if $x = 0$, then $0 = 4$, which is impossible.

Testing for symmetry gives:

x-axis: $\dfrac{x^2(-y)^2}{x^2+(-y)^2+1} = 4$

$\dfrac{x^2y^2}{x^2+y^2+1} = 4$, which is the original equation.

y-axis: $\dfrac{(-x)^2y^2}{(-x)^2+y^2+1} = 4$

$\dfrac{x^2y^2}{x^2+y^2+1} = 4$, which is the original equation.

origin: $\dfrac{(-x)^2(-y)^2}{(-x)^2+(-y)^2+1} = 4$

$\dfrac{x^2y^2}{x^2+y^2+1} = 4$, which is the original equation.

line $y = x$: (a, b) on graph, then $\dfrac{a^2b^2}{a^2+b^2+1} = 4$ and $b^2 = \dfrac{4(a^2+1)}{a^2-4}$. (b, a) on graph, then $\dfrac{b^2a^2}{b^2+a^2+1} = 4$ and $b^2 = \dfrac{4(a^2+1)}{a^2-4}$, so (a, b) and (b, a) are both on the graph.

Answer: no intercepts; symmetry about x-axis, y-axis, origin, and $y = x$.

27. $y = 4 + x^2$

Intercepts: If $y = 0$, then $0 = 4 + x^2$, which is never true.

If $x = 0$, then $y = 4$.

Testing for symmetry gives:

x-axis: $-y = 4 + x^2$

$y = -4 - x^2$, which is not the original equation.

y-axis: $y = 4 + (-x)^2$

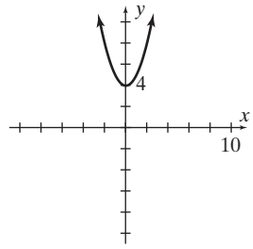
$y = 4 + x^2$, which is the original equation.

origin: $-y = 4 + (-x)^2$

$y = -4 - x^2$, which is not the original equation.

line $y = x$: (a, b) on graph, then $b = 4 + a^2$ and $a = \pm\sqrt{b-4} \neq 4 + b^2$ for all b, so (b, a) is not on the graph.

Answer: (0, 4); symmetry about y-axis.



28. $y = 3x - 7$

Intercepts: If $y = 0$, then $0 = 3x - 7$, or $x = \dfrac{7}{3}$.

If $x = 0$, then $y = -7$.

Testing for symmetry gives:

x-axis: $-y = 3x - 7$

$y = -3x + 7$, which is not the original equation.

y-axis: $y = 3(-x) - 7$

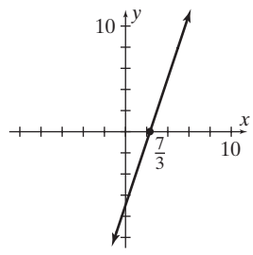
$y = -3x - 7$, which is not the original equation.

origin: $-y = 3(-x) - 7$

$y = 3x + 7$, which is not the original equation.

line $y = x$: (a, b) on graph, then $b = 3a - 7$ and $a = \dfrac{1}{3}(b + 7) \neq 3b - 7$ for all b, so (b, a) is not on the graph.

Answer: $(0, -7)$, $\left(\dfrac{7}{3}, 0\right)$; no symmetry of the given types



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29. $G(u) = \sqrt{u+4}$

If $G(u) = 0$, then $0 = \sqrt{u+4}$.

$0 = u + 4$,

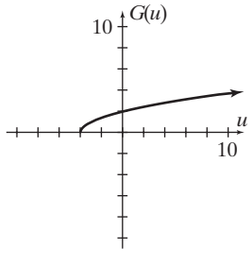
$u = -4$

If $u = 0$, then $G(u) = \sqrt{4} = 2$.

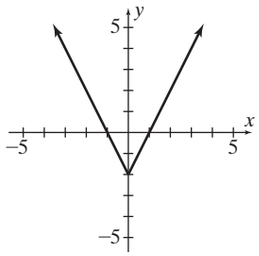
Intercepts: $(0, 2), (-4, 0)$

Domain: all real numbers u such that $u \geq -4$

Range: all real numbers ≥ 0



30.



Domain is $(-\infty, \infty)$; range is $[-2, \infty)$; intercepts are $(0, -2), (-1, 0)$ and $(1, 0)$.

31. $y = g(t) = \dfrac{2}{|t-4|}$

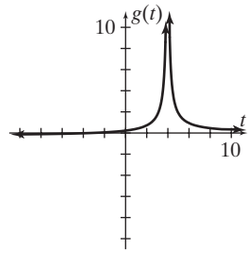
If $y = 0$, then $0 = \dfrac{2}{|t-4|}$, which has no solution.

If $t = 0$, then $y = \dfrac{2}{4} = \dfrac{1}{2}$.

Intercept: $\left(0, \dfrac{1}{2}\right)$

Domain: all real numbers t such that $t \neq 4$

Range: all real numbers > 0

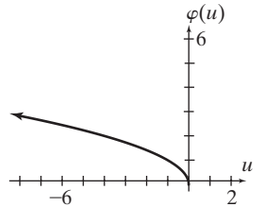


32. $v = \phi(u) = \sqrt{-u}$

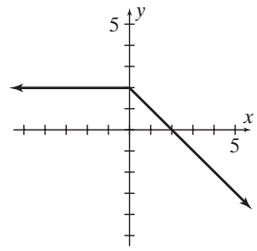
If $\phi(u) = 0$, then $0 = \sqrt{-u}$,

$u = 0$.

If $u = 0$, $\phi(u) = 0$.
Intercept: (0, 0)
Domain: all reals ≤ 0
Range: all reals ≥ 0



33. Domain: all real numbers.
Range: all real numbers $\leq$ g2



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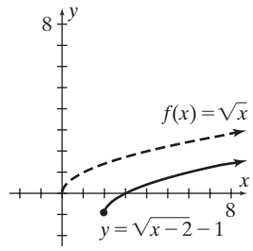

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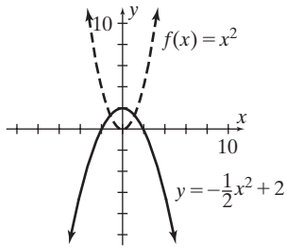
35. From the graph of $f(x) = x^2$, get the graph of $y = -\frac{1}{2}x^2 + 2$ by shrinking the graph by a factor of 1/2 towards the x-axis, reflecting the result in the x-axis, and translating that result up by 2.



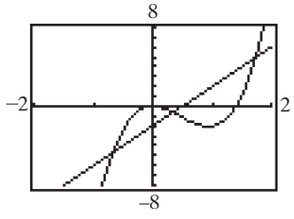
36. For 2006, $t = 5$. Hence $S = 150{,}000 + 3000(5) = \$165{,}000$. S is a function of t.

37. From the vertical-line test, the graphs that represent functions of x are (a) and (c).

38. **a.** 729

b. 359.43

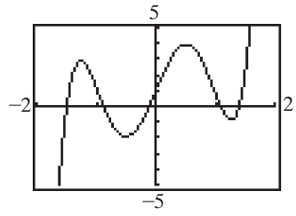
39.



$-0.67; 0.34, 1.73$

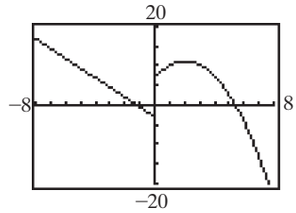
40. A graph of $f(x) = x^3 + x + 1$ will suggest that $x^3 + x + 1 = 0$ has exactly one real root and since $f(-1) = -1 < 0$ and $f(0) = 1 > 0$, the graph further suggests that the unique root, call it r lies in the interval $(-1, 0)$. Since $f(-1/2) = -1/8 - 1/2 + 1 = 3/8 > 0$, the same reasoning suggests that r lies in the interval $(-1, -1/2)$. We chose $-1/2$ as the midpoint of $(-1, 0)$. This leads us to investigate $f(-3/4)$ because $-3/4$ is the midpoint of $(-1, -1/2)$. We have $f(-3/4) = -27/64 - 3/4 + 1 = 15/64 > 0$ So r is in $(-1, -3/4)$. Now $f(-7/8) = -279/512 < 0$ So r is in $(-7/8, -3/4)$ and next we check $f(-13/16)$. Clearly, we can continue this process until the test interval has a length less than the desired degree of accuracy. The process converges rapidly!

41.



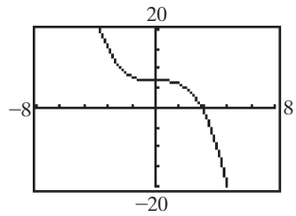
$-1.50, -0.88, -0.11, 1.09, 1.40$

42.



$(-\infty, \infty)$

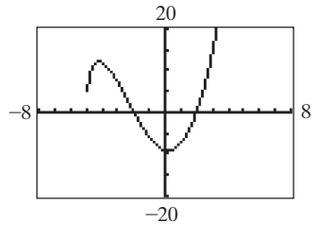
43.



a. $(-\infty, \infty)$

b. $(1.92, 0), (0, 7)$

44.



a. -9.03

b. all real numbers ≥ -9.03

c. $-5, \pm 2$.

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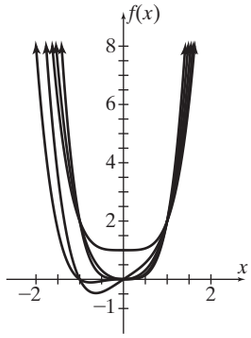
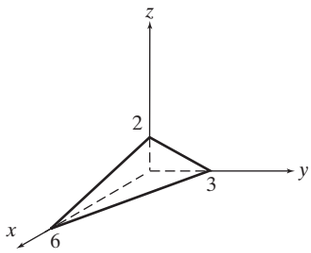
45.

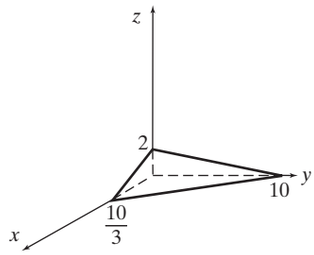


We note that if k is even then $(-x)^k = x^k$ and if k is odd then $(-x)^k = -x^k$. Assume k is even. Replacing x by $-x$ in $y = x^4 + x^k$ we get $y = (-x)^4 + (-x)^k = x^4 + x^k$ the original equation. So for k even we get symmetry about the y-axis. In this case we cannot get symmetry about the origin because that would entail symmetry about the x-axis which does not hold for any function different from the function constantly zero. Assume k is odd. Replacing x by $-x$ and y by $-y$ in the equation produces $-y = (-x)^4 + (-x)^k = x^4 - x^k$ equivalently $y = -x^4 + x^k$ which is not equivalent to the original equation. So for k odd we get no symmetries of the types under consideration.

46. $x + 2y + 3z = 6$ can be put in the form $Ax + By + Cz + D = 0$, so the graph is a plane.
Intercepts: (6, 0, 0), (0, 3, 0), (0, 0, 2)

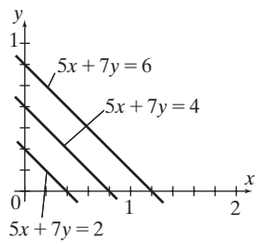


47. $3x + y + 5z = 10$ can be put in the form $Ax + By + Cz + D = 0$, so the graph is a plane.
Intercepts: $\left(\frac{10}{3},\ 0,\ 0\right)$, (0, 10, 0), (0, 0, 2)



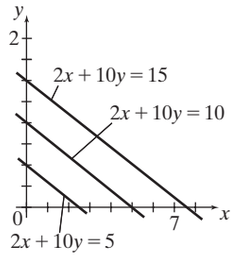
48. $P = 5x + 7y$

Choose $P = 2, 4$, and 6 for the curves.



49. $C = 2x + 10y$

Choose $C = 5$, 10, and 15 for the curves.



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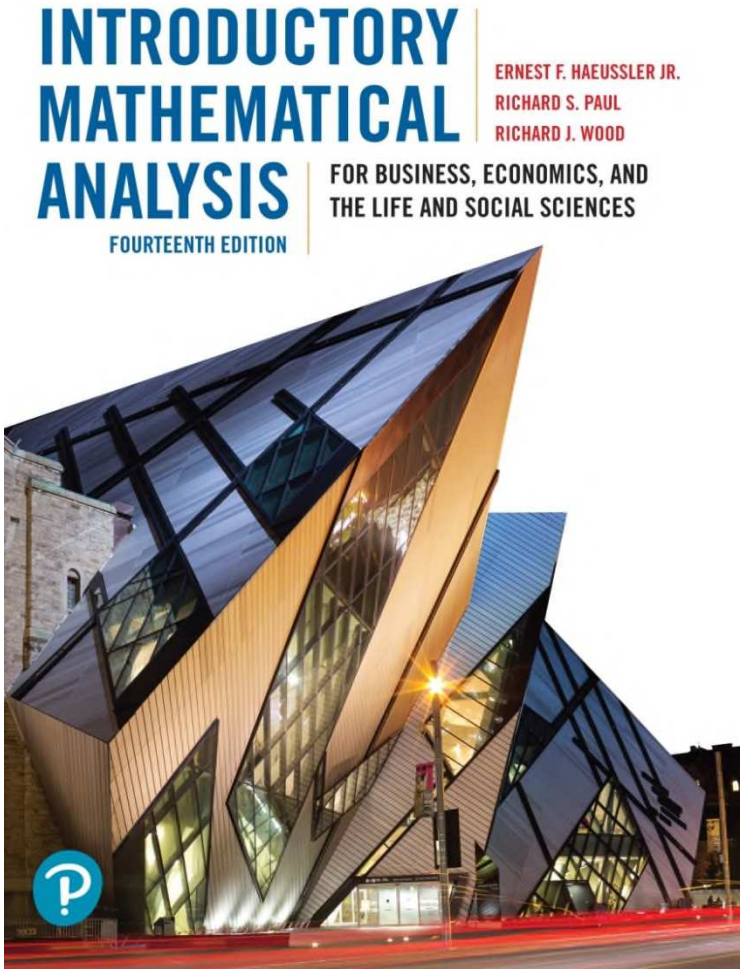

Chapter 2

Functions and Graphs

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Chapter Objectives

- To understand what functions and domains are.
- To introduce different types of functions.
- To introduce addition, subtraction, multiplication, division, and multiplication by a constant.
- To introduce inverse functions and properties.
- To graph equations and functions.
- To study symmetry about the *x*- and *y*-axis.
- To be familiar with shapes of the graphs of six basic functions.

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Chapter Outline

2.1) Functions

2.2) Special Functions

2.3) Combinations of Functions

2.4) Inverse Functions

2.5) Graphs in Rectangular Coordinates

2.6) Symmetry

2.7) Translations and Reflections

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2.1 Functions (1 of 5)

- A **function** assigns each input number to one output number.
- The set of all input numbers is the **domain** of the function.
- The set of all output numbers is the **range**.

To say that two functions $f, g : X \to Y$ are equal, denoted $f = g$, is to say that

1. The domain of f is equal to the domain of g.
2. For every x in the domain of f and g, $f(x) = g(x)$.

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2.1 Functions (2 of 5)

Example 1 – Determining Equality of Functions

Determine which of the following functions are equal.

a. $f(x) = \dfrac{(x+2)(x-1)}{(x-1)}$

b. $g(x) = x + 2$

c. $h(x) = \begin{cases} x+2 & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$

d. $k(x) = \begin{cases} x+2 & \text{if } x \neq 1 \\ 3 & \text{if } x = 1 \end{cases}$

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2.1 Functions (3 of 5)

Example 1 – Continued

Solution: Observe that the domains of g, h, and k are equal to each other, but that of f is different. So by requirement 1 for equality, $f \neq g$, $f \neq h$ and $f \neq k$.

By definition, $g(x) = h(x) = k(x)$ for all $x \neq 1$.
Note that $g(1) = 3$, $h(1) = 0$, and $k(1) = 3$.
We conclude that $g = k$ and $g \neq h$ (and $h \neq k$).

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2.1 Functions (4 of 5)

Example 3 – Finding Domain and Function Values

Let $g(x) = 3x^2 - x + 5$. Note that the domain of g is all real numbers.

a. Find $g(z)$.

Solution: $g(z) = 3z^2 - z + 5$

b. Find $g(r^2)$.

Solution: $g(r^2) = 3(r^2)^2 - r^2 + 5 = 3r^4 - r^2 + 5$

c. Find $g(x+h)$.

Solution: $g(x+h) = 3(x+h)^2 - (x+h) + 5$

$= 3(x^2 + 2xh + h^2) - x - h + 5$

$= 3x^2 + 6xh + 3h^2 - x - h + 5$

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2.1 Functions (5 of 5)

Example 5 – Demand Function

Suppose that the equation $p = 100/q$ describes the relationship between the price per unit p of a certain product and the number of units q of the product that consumers will buy (that is, demand) per week at the stated price. This equation is called a *demand equation* for the product. If q is an input, then to each value of q there is assigned at most one output p:

$$q \mapsto \frac{100}{q} = p.$$

This function is called a **demand function**.

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2.2 Special Functions (1 of 4)

Example 1 – Constant Function

Let $h:(-\infty,\infty)\to(\infty,\infty)$ be given by $h(x)=2$. The domain of h is $(-\infty,\infty)$, the set of all real numbers. All function values are 2. For example, $h(10)=2,\ h(-387)=2,\ h(x+3)=2.$

We call h a *constant function*. More generally, a function of the form $h(x)=c,$ where c is a *constant*, is called a **constant function**.

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2.2 Special Functions (2 of 4)

Example 3 – Rational Functions

a. $f(x) = \dfrac{x^2 - 6x}{x + 5}$ is a rational function, since the numerator and denominator are each polynomials.

b. $g(x) = 2x + 3$ is a rational function, since $2x + 3 = \dfrac{2x + 3}{1}$.

Example 5 – Absolute-Value Function

The function $f(x) = |x|$ is called the absolute-value function.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

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2.2 Special Functions (3 of 4)

The symbol $r!$, with r a positive integer, is read "r **factorial**".

It represents the product of the first r positive integers:

$$r! = 1 \cdot 2 \cdot 3 \cdots r$$

We also define $0! = 1$.

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2.2 Special Functions (4 of 4)

Example 7 – Genetics

Suppose two black guinea pigs are bred and produce exactly five offspring. Under certain conditions, it can be shown that the probability P that exactly r of the offspring will be brown and the others black is a function of r, $P = P(r)$, where

$$P(r) = \frac{5!\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{5-r}}{r!(5-r)!} \qquad r = 0,\ 1,\ 2,\ \ldots,\ 5$$

Find the probability that exactly three guinea pigs will be brown.

Solution: We want to find $P(3)$. We have

$$P(3) = \frac{5!\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2}{3!\ 2!} = \frac{120\left(\frac{1}{64}\right)\left(\frac{9}{16}\right)}{6(2)} = \frac{45}{512}.$$

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2.3 Combinations of Functions (1 of 5)

In general, for any functions f, $g : X \to (-\infty, \infty)$, we define the *sum* $f + g$, the *difference* $f - g$, the *product* fg, and the *quotient* $\frac{f}{g}$ as follows:

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x) \cdot g(x)$$
$$\frac{f}{g}(x) = \frac{f(x)}{g(x)} \quad \text{for } g(x) \neq 0$$

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2.3 Combinations of Functions (2 of 5)

Example 1 – Combining Functions

If $f(x) = 3x - 1$ and $g(x) = x^2 + 3x$, find a. $(f + g)(x)$,

b. $(f - g)(x)$, c. $(fg)(x)$, d. $\frac{f}{g}(x)$, e. $((1/2)f)(x)$

Solution

a. $(f + g)(x) = f(x) + g(x) = (3x - 1) + (x^2 + 3x) = x^2 + 6x - 1$

b. $(f - g)(x) = f(x) - g(x) = (3x - 1) - (x^2 + 3x) = -1 - x^2$

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2.3 Combinations of Functions (3 of 5)

Example 1 – Continued

Solution, continued

c. $(fg)(x) = f(x)g(x) = (3x-1)(x^2+3x) = 3x^3+8x^2-3x$

d. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{3x-1}{x^2+3x}$

e. $((1/2)f)(x) = (1/2)(f(x)) = (1/2)(3x-1)$

- We can also combine two functions by first applying one function to an input and then applying the other function to the output of the first.
- This is called **composition**.

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2.3 Combinations of Functions (4 of 5)

For functions $g : X \to Y$ and $f : Y \to Z$, the composite of f with g is the function $f \circ g : X \to Z$ defined by

$$(f \circ g)(x) = f(g(x))$$

Where the domain of $f \circ g$ is the set of all those x in the domain of g such that $g(x)$ is in the domain of f.

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2.3 Combinations of Functions (5 of 5)

Example 3 – Composition

If $F(p) = p^2 + 4p - 3$, $G(p) = 2p + 1$, and $H(p) = |p|$, find

a. $F(G(p))$

b. $F(G(H(p)))$

c. $G(F(1))$

Solution:

a. $F(2p+1) = (2p+1)^2 + 4(2p+1) - 3 = 4p^2 + 12p + 2$

b. $F(G(H(p))) = (F \circ (G \circ H))(p) = ((F \circ G) \circ H))(p)$

$= (F \circ G)(H(p)) = (F \circ G)(|p|) = 4|p|^2 + 12|p| + 2$

c. $G(F(1)) = G(1^2 + 4 \cdot 1 - 3) = G(2) = 2 \cdot 2 + 1 = 5$

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2.4 Inverse Functions (1 of 4)

A function that satisfies, for all a and b,
if $f(a) = f(b)$ then $a = b$,
is called a **one-to-one** function.

A function has an inverse, written $f^{-1}(x)$, precisely if it is one-to-one.
In general,
$f^{-1}(f(x)) = x$ for all x in the domain of f
and
$f(f^{-1}(y)) = y$ for all y in the range of f

Note that the range of f can be different from the domain of f.

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2.4 Inverse Functions (2 of 4)

Example 1 – Inverses of Linear Functions

Show that a linear function (a function of the form $f(x) = ax + b$, where $a \neq 0$) is one-to-one. Find the inverse of $f(x)$ and show that it is also linear).

Solution: Assume that $f(u) = f(v)$, that is, $au + b = av + b$.

This gives $au = av$, and since $a \neq 0$, it follows that $u = v$.

Thus, $f(x)$ is one-to-one.

Consider $g(x) = \dfrac{x-b}{a}$. We have

$$(f \circ g)(x) = f(g(x)) = a\frac{x-b}{a} + b = (x-b) + b = x$$

and $$(g \circ f)(x) = g(f(x)) = \frac{(ax+b)-b}{a} = \frac{ax}{a} = x$$

It follows that g is the inverse of f.

Since $g(x) = f^{-1}(x) = \dfrac{x-b}{a} = \dfrac{1}{a}x + \dfrac{-b}{a}$, we conclude that $f^{-1}(x)$ is linear.

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2.4 Inverse Functions (3 of 4)

Example 3 – Inverses Used to Solve Equations

Many equations take the form $f(x) = 0$, where f is a function. If f is a one-to-one function, then the equation has $x = f^{-1}(0)$ as its unique solution:

$f(x) = 0$

$f^{-1}(f(x)) = f^{-1}(0)$ (applying $f^{-1}(x)$ to both sides)

$x = f^{-1}(0)$ ($f^{-1}(f(x)) = x$ by definition of the inverse)

Therefore $x = f^{-1}(0)$ is the only possible solution.

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2.4 Inverse Functions (4 of 4)

Example 5 – Finding the Inverse of a Function

To find the inverse of a one-to-one function f, solve the equation $y = f(x)$ for x in terms of y, obtaining $x = g(y)$.
Then $f^{-1}(x) = g(x)$. Find $f^{-1}(x)$ if $f(x) = (x-1)^2$, for $x \geq 1$.

Solution: Let $y = (x-1)^2$, for $x \geq 1$. Then $x - 1 = \sqrt{y}$ and hence, $x = \sqrt{y} + 1$. It follows that $f^{-1}(x) = \sqrt{x} + 1$.

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2.5 Graphs in Rectangular Coordinates (1 of 5)

- A **rectangular coordinate system** allows us to specify and locate points in a plane. It also provides a geometric way to graph equations in two variables.

An x-intercept of the graph of an equation in x and y is a point where the graph intersects the x-axis.

A y-intercept is a point where the graph intersects the y-axis.

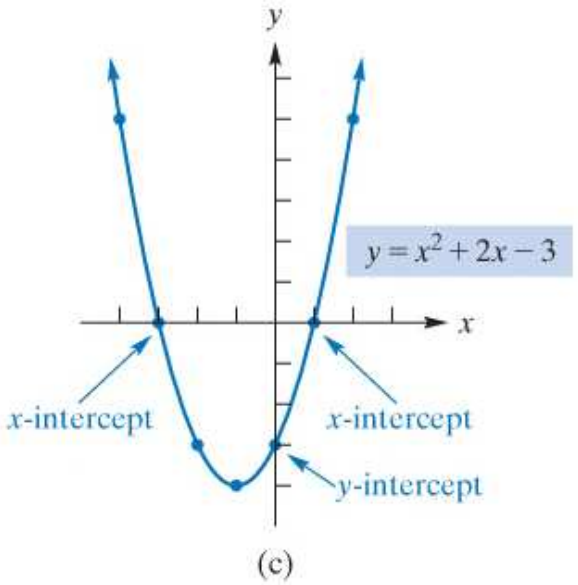

FIGURE 2.9 Graphing $y = x^2 + 2x - 3$.

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2.5 Graphs in Rectangular Coordinates (2 of 5)

Example 1 – Intercepts of a Graph

Find the x- and y-intercepts of the graph of $y = 2x + 3$, and sketch the graph.

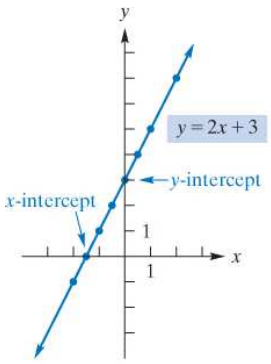
Solution: If $y = 0$,

then $0 = 2x + 3$ so that $x = -\frac{3}{2}$.

Thus, the x-intercept is $\left(-\frac{3}{2},\ 0\right)$.

If $x = 0$, then

$y = 2(0) + 3 = 3$, so the y-intercept is $(0,\ 3)$.



x	0	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1	−1	2	−2
y	3	0	4	2	5	1	7	−1

FIGURE 2.10 Graph of $y = 2x + 3$.

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2.5 Graphs in Rectangular Coordinates (3 of 5)

Example 3 – Intercepts of a Graph

Determine the intercepts of the graph $x = 3$, and sketch the graph.

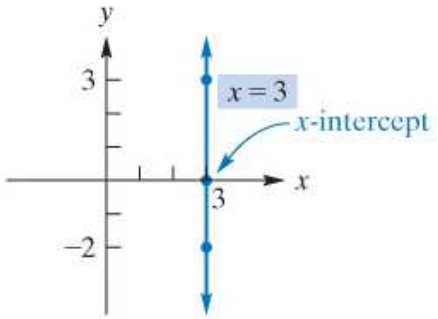
Solution: We can write $x = 3$ as $x = 3 + 0y$.

Here y can be any value, but x must be 3.

The x-intercept is $(3, 0)$,

and there is no y-intercept

because x cannot be 0.



x	3	3	3
y	0	3	−2

FIGURE 2.12 Graph of $x = 3$.

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2.5 Graphs in Rectangular Coordinates (4 of 5)

Example 5 – Graph of the Absolute-Value Function

Graph $p = G(q) = |q|$.

Solution: We use the independent variable q to label the horizontal axis. The function-value axis can be labeled either $G(q)$ or p. Note the sharp corner at the origin.

q	0	1	−1	3	−3	5	−5
p	0	1	1	3	3	5	5

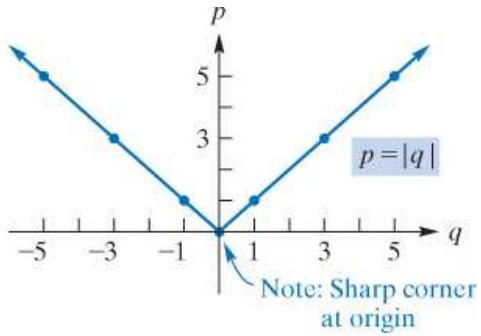

FIGURE 2.14 Graph of $p = |q|$.

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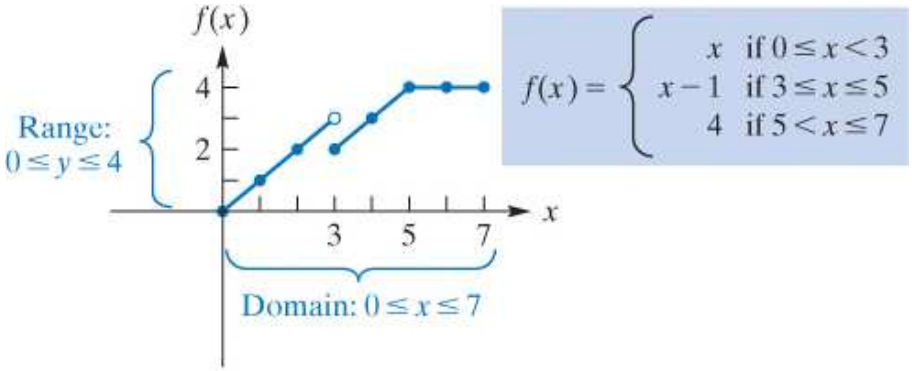
2.5 Graphs in Rectangular Coordinates (5 of 5)

Example 7 – Graph of a Case-Defined Function

Graph the case-defined function

$$f(x)=\begin{cases} x & \text{if } 0 \le x < 3 \\ x-1 & \text{if } 3 \le x \le 5 \\ 4 & \text{if } 5 < x \le 7 \end{cases}$$

Solution:



x	0	1	2	3	4	5	6	7
$f(x)$	0	1	2	2	3	4	4	4

FIGURE 2.17 Graph of a case-defined function.

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2.6 Symmetry (1 of 5)

A graph is **symmetric about the y-axis** if and only if $(-a,\ b)$ lies on the graph when $(a,\ b)$ does.

Example 1 – y-axis Symmetry

Use the preceding definition to show that the graph of $y = x^2$ is symmetric about the y-axis.

Solution: Suppose $(a,\ b)$ is any point on the graph of $y = x^2$.

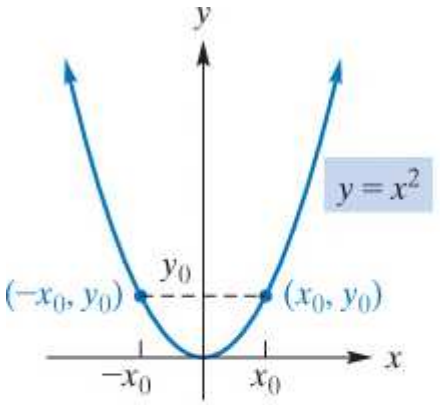

FIGURE 2.25 Symmetry about the y-axis.

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Then $b = a^2$. Moreover, consider the point $(-a,\ b)$: $(-a)^2 = a^2 = b$. This shows that $(-a,\ b)$ is also on the graph.

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2.6 Symmetry (2 of 5)

A graph is **symmetric about the x-axis** if and only if $(x, -y)$ lies on the graph when (x, y) does.

A graph is **symmetric about the origin** if and only if $(-x, -y)$ lies on the graph when (x, y) does.

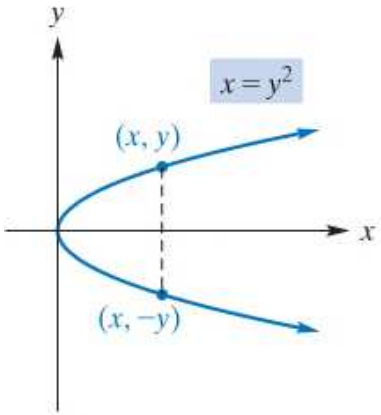

FIGURE 2.26 Symmetry about the x-axis.

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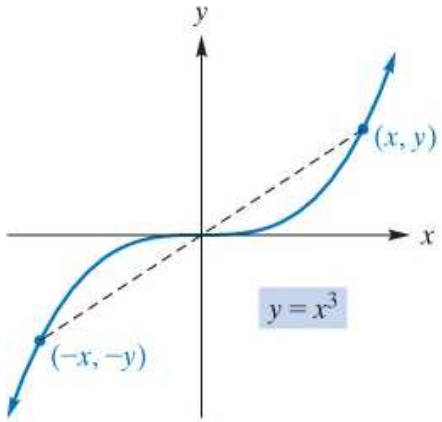

FIGURE 2.27 Symmetry about the origin.

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2.6 Symmetry (3 of 5)

Example 3 – Graphing with Intercepts and Symmetry

Test $y = f(x) = 1 - x^4$ for symmetry about the x-axis, the y-axis, and the origin. Then find the intercepts and sketch the graph.

Solution - Symmetry:

x-axis: Replacing y by $-y$ in $y = 1 - x^4$ gives $-y = 1 - x^4$, equivalently, $y = -1 + x^4$ which is not equivalent to the given equation. The graph is *not* symmetric about the x-axis.

y-axis: Replacing x by $-x$ in $y = 1 - x^4$ gives $y = 1 - (-x)^4$, equivalently, $y = 1 - x^4$ which is the given equation. The graph *is* symmetric about the y-axis.

Origin: Replacing x by $-x$ and y by $-y$ in $y = 1 - x^4$ gives $-y = 1 - (-x)^4$, equivalently, $y = -1 + x^4$ which is not equivalent to the given equation. The graph is *not* symmetric about the origin.

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2.6 Symmetry (4 of 5)

Example 3 – Continued

Solution - Intercepts:

We set $y = 0$ in $y = 1 - x^4$.

Then $1 - x^4 = 0$

$(1 - x^2)(1 + x^2) = 0$

$(1 - x)(1 + x)(1 + x^2) = 0$

$x = 1$ or $x = -1$.

x	y
0	1
$\frac{1}{2}$	$\frac{15}{16}$
$\frac{3}{4}$	$\frac{175}{256}$
1	0
$\frac{3}{2}$	$-\frac{65}{16}$

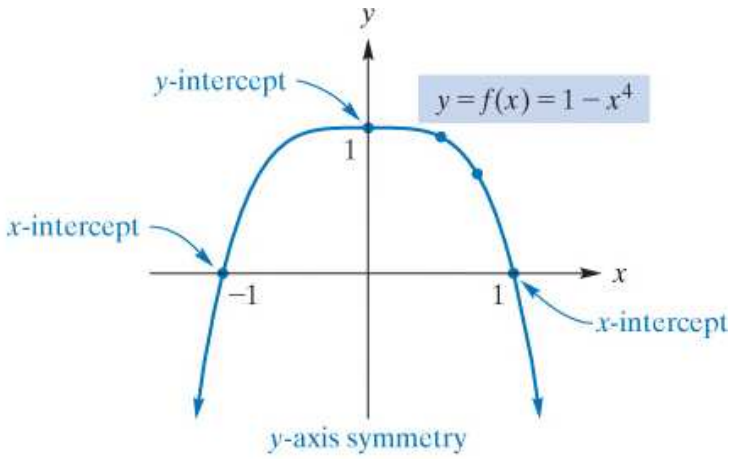

FIGURE 2.29 Graph of $y = 1 - x^4$.

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The x-intercepts are therefore $(1, 0)$ and $(-1, 0)$.

We set $x = 0$, then $y = 1$, so $(0, 1)$ is the only y-intercept.

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2.6 Symmetry (5 of 5)

A graph is **symmetric about the line** $y = x$ if and only if (b, a) lies on the graph when (a, b) does.

Example 5 – Symmetry about the Line y = x

Use the preceding definition to show that $x^2 + y^2 = 1$ is symmetric about the line $y = x$.

Solution: Interchanging the roles of x and y produces $x^2 + y^2 = 1$, which is equivalent to $x^2 + y^2 = 1$. Thus $x^2 + y^2 = 1$ is symmetric about the line $y = x$.

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2.7 Translations and Reflections (1 of 3)

- Some functions and their associated graphs occur so frequently that we find it worthwhile to memorize them. Below are six such basic functions.

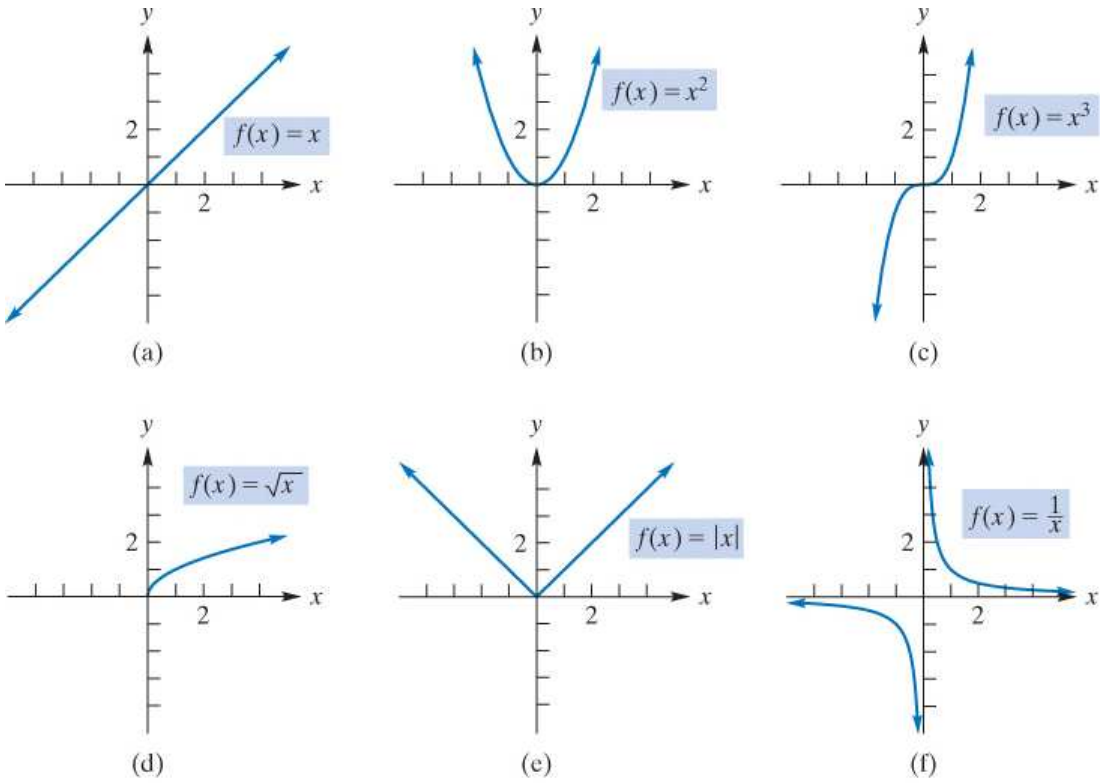

FIGURE 2.32 Six basic functions.

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2.7 Translations and Reflections (2 of 3)

- The table below gives a list of basic types of transformations:

Table 2.2 Transformations, c > 0

Equation	How to Transform Graph of $y = f(x)$ to Obtain Graph of Equation
$y = f(x) + c$	shift c units upward
$y = f(x) - c$	shift c units downward
$y = f(x - c)$	shift c units to right
$y = f(x + c)$	shift c units to left
$y = -f(x)$	reflect about x-axis
$y = f(-x)$	reflect about y-axis
$y = cf(x)$ $c > 1$	vertically stretch away from x-axis by a factor of c
$y = cf(x)$ $c < 1$	vertically shrink toward x-axis by a factor of c

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2.7 Translations and Reflections (3 of 3)

Example 1 – Horizontal Translation

Sketch the graph of $y = (x-1)^3$.

Solution: We observe that x is replaced with $x-1$. Thus, this function has the form $y = f(x-c)$, where $c = 1$.

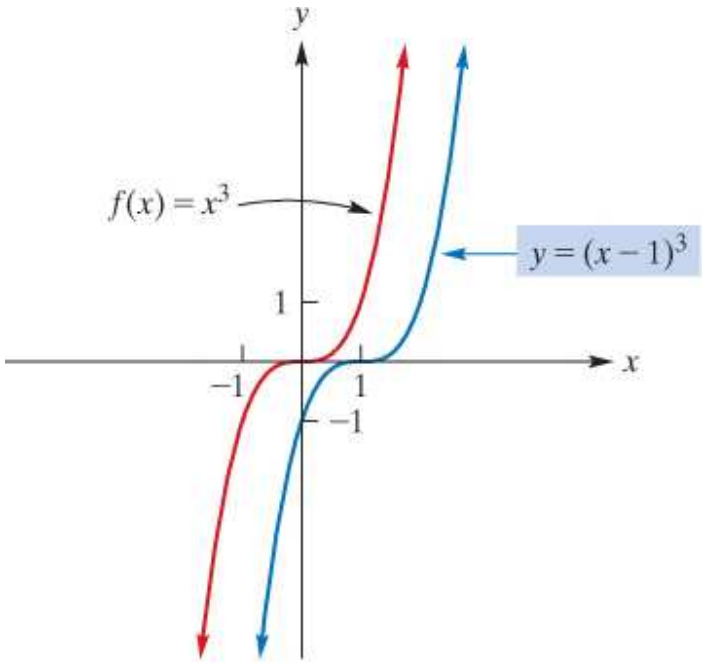

FIGURE 2.34 Graph of $y = (x-1)^3$.

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2.8 Functions of Several Variables (1 of 7)

For sets X and Y we can construct the new set $X \times Y$ whose elements are **ordered pairs** (x, y) with x in X and y in Y.

A function $f : X \times Y \to Z$ is a rule that assigns to each element (x, y) in $X \times Y$ at most one element of Z, denoted by $f((x, y))$. We agree to write $f(x, y)$.

In general, a function $f : X_1 \times X_2 \times \cdots \times X_n \to Y$ provides the notion of a Y-valued function of n-variables. An element of the domain of f is an **ordered n-tuple** $(x_1, x_2, \ldots, x_n)$, with x_i in X_i for $i = 1, 2, \ldots, n$, for which $f(x_1, x_2, \ldots, x_n)$ is defined.

The **graph** of f is the set of all ordered $n+1$-tuples of the form $(x_1, x_2, \ldots, x_n, f(x_1, x_2, \ldots, x_n))$, where $(x_1, x_2, \ldots, x_n)$ is in the domain of f.

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2.8 Functions of Several Variables (2 of 7)

Example 1 – Functions of Two Variables

a. $a(x, y) = x + y$ is a function of two variables. Some function values are

$a(1, 1) = 1 + 1 = 2$

$a(2, 3) = 2 + 3 = 5$

We have $a : (-\infty, \infty) \times (-\infty, \infty) \to (-\infty, \infty)$.

b. $m(x, y) = xy$ is a function of two variables. Some function values are

$m(2, 2) = 2 \cdot 2 = 4$

$m(3, 2) = 3 \cdot 2 = 6$

The domain of both a and m is all of $(-\infty, \infty) \times (-\infty, \infty)$.

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2.8 Functions of Several Variables (3 of 7)

Example 3 – Temperature-Humidity Index

On hot and humid days, many people tend to feel uncomfortable. In the United States, the degree of discomfort is numerically given by the temperature-humidity index, THI, which is a function of two variables, t_d and t_w: $\text{THI} = f(t_d, t_w) = 15 + 0.4(t_d + t_w)$ where t_d is the dry-bulb temperature and t_w is the wet-bulb temperature of the air. Evaluate the THI when $t_d = 90$ and $t_w = 80$.

Solution: We want to find $f(90, 80)$:

$$f(90, 80) = 15 + 0.4(90 + 80) = 15 + 68 = 83.$$

A similar measurement, called the Humidex, is used in Canada.

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2.8 Functions of Several Variables (4 of 7)

The set of all ordered triples of real numbers can be pictured as providing a 3-**dimensional rectangular coordinate system**. The three mutually perpendicular real-number lines are called the x-, y-, and z-axes, and their point of intersection is called the origin of the system.

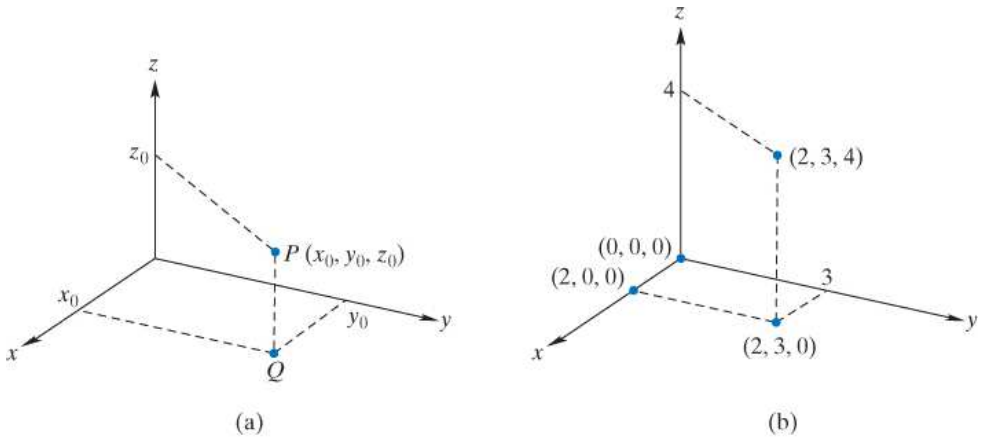

FIGURE 2.37 Points in space.

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2.8 Functions of Several Variables (5 of 7)

A "coordinate plane" is a plane containing two coordinate axes.

For example, the plane determined by the x- and y-axes is the x, y-plane.

Below are some sketches of planes parallel to coordinate planes.

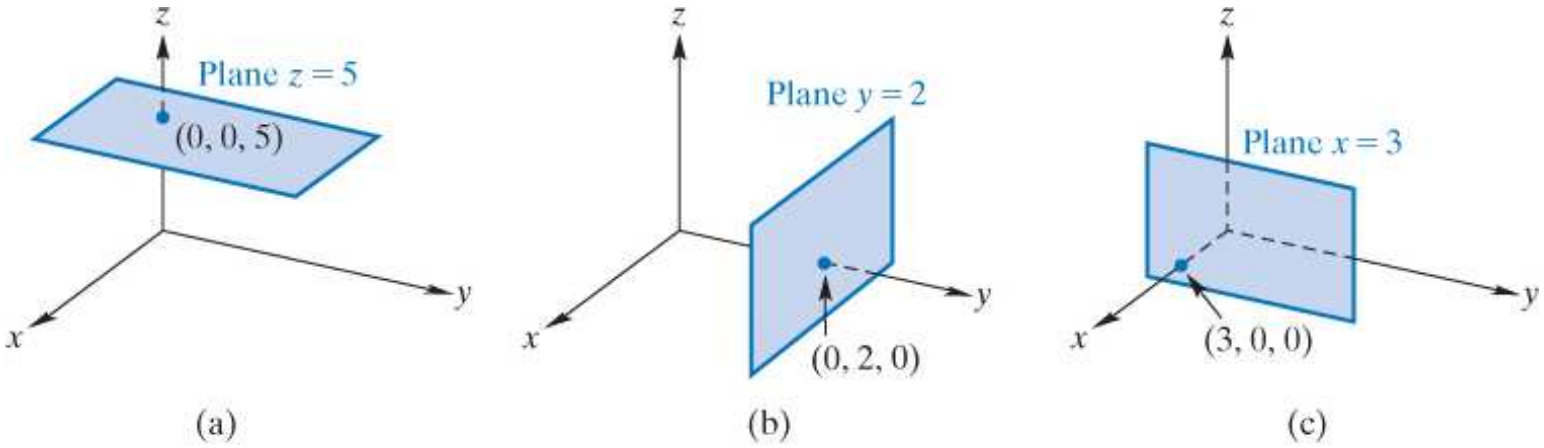

FIGURE 2.39 Planes parallel to coordinate planes.

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2.8 Functions of Several Variables (6 of 7)

Example 5 – Sketching a Surface

Sketch the surface $2x + z = 4$.

Solution: This equation has the form of a plane. The x- and z-intercepts are $(2,\ 0,\ 0)$ and $(0,\ 0,\ 4)$, and there is no y-intercept. Setting $y = 0$ gives the x, z-trace $2x + z = 4$, which is a line in the x, z-plane. The plane appears to the right.

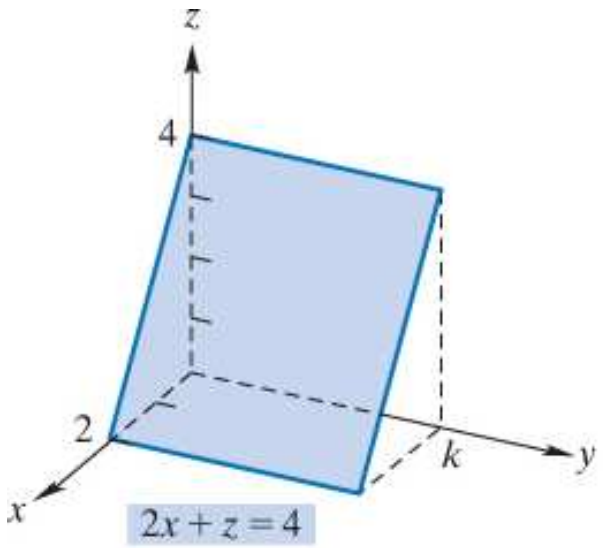

FIGURE 2.41 The plane $2x + z = 4$.

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2.8 Functions of Several Variables (7 of 7)

Example 7 – Sketching a Surface

Sketch the surface $x^2 + y^2 + z^2 = 25$.

Solution: Setting $z = 0$ gives the x, y-trace $x^2 + y^2 = 25$, which is a circle of radius 5. Similarly, the y, z- and x, z-traces are the circles $y^2 + z^2 = 25$ and $x^2 + z^2 = 25$ respectively.

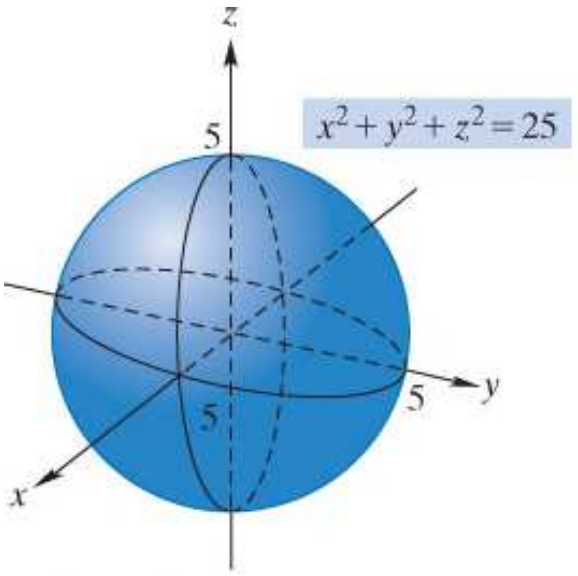

FIGURE 2.43 The surface $x^2 + y^2 + z^2 = 25$.

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The surface appears to the right; it is a sphere.

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