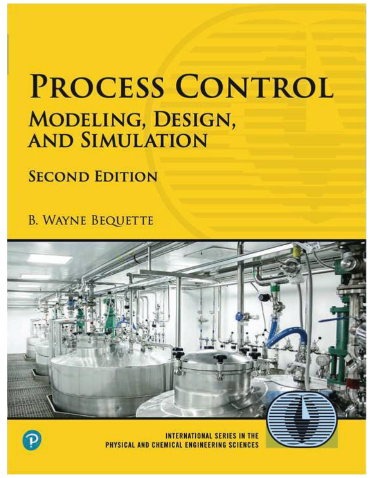
Process Control 2nd edition Bequette Solutions Manual

SKU: 9780134033839-SOLUTIONS



Bequette, B.W. *Process Control: Modeling, Design and Simulation*, Pearson, Boston, MA. 2nd edition (2024).

Ancillary Material Available

- Powerpoint slides for each chapter (material for roughly two lectures for each chapter)
- On-line quizzes (could be used before each lecture)
- MATLAB Livescripts for selected Textbook Examples and Student Exercises

Textbook Errata

Student exercises 6.23 and 8.5 have the same error. The equations should be

$$\frac{dx_{c1}}{dt} = e$$

$$\frac{dx_{c2}}{dt} = \frac{k_c(\propto -1)}{\alpha^2 \tau_I \tau_D} x_{c1} - \frac{1}{\alpha \tau_D} x_{c2} + \frac{k_c(\alpha - 1)}{\alpha^2 \tau_D} e$$

$$u = \frac{k_c}{\alpha \tau_I} x_{c1} + x_{c2} + \frac{k_c}{\alpha} e$$

page 292. The phase margin requirement should be PM > 0.

Solutions Manual

version 1.2, 31 December 2023.

Solutions Manual co-authors: Cesar C. Palerm, Lucky E. Yerimah, Jacinta Okpanum, Mrunal Sontakke, Andreas Rebmann.

There remain a few unsolved student exercises. For questions and comments, contact Professor Bequette at bequette@rpi.edu.

Cesar C. Palerm, Ph.D.

Dr. Palerm is a systems thinker with extensive experience in medical device R&D, from early-stage research to commercialization. He has a technical background in engineering and scientific research, including data science, engineering design, modeling, simulation, and control algorithms. He has leveraged model-based design (MBD) for embedded medical devices and has led technical and cross-functional teams, establishing processes and structures based on an agile/lean framework.

With a strong background in human physiology, particularly in glucose metabolism and hemodynamics, Dr. Palerm thoroughly understands design control, risk assessment, and regulatory requirements for medical devices. He also has clinical research expertise, including developing clinical research protocols and clinical trial support.

Dr. Palerm's interests include systems engineering, design thinking, modeling and control, data science, lean development methodologies, lean six sigma, model-based design (MBD), change management, innovation, safety engineering, and the application of systems theory, modeling, and control to medical, biological, and other complex systems. He has been honored with the Star of Excellence Award and the Medtronic Diabetes Inventure Award and has participated in the National Academy of Engineering's 2011 U.S. Frontiers of Engineering Symposium. He is an elected full member of Sigma Xi and an IEEE Senior Member.

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Lucky E. Yerimah, Ph.D.

Lucky E. Yerimah is a Ph.D. candidate in Chemical Engineering at Rensselaer Polytechnic Institute (RPI), with a focus on data-driven process monitoring and control for smart manufacturing. His research interests include process modeling, control systems, reinforcement learning, and optimization. Yerimah has a strong background in chemical engineering, with a Bachelor Engineering degree from the University of Benin, Nigeria, and a Master of Engineering degree from Rensselaer Polytechnic Institute.

Yerimah has a proven track record of research success in developing novel AI models for fault detection, and feedback control applications using model-based reinforcement learning. He has also received several awards during his time at the University of Benin and Rensselaer Polytechnic Institute. In addition to his research experience, Yerimah also has extensive industry experience. He has interned at GE Global Research and Applied Materials, where he worked on developing advanced control algorithms for wind turbines and plasma etch chambers.

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Jacinta Okpanum

Jacinta Okpanum is currently a Ph.D. student in the Department of Chemical and Biological Engineering at Rensselaer Polytechnic Institute. She earned her bachelor's degree in Chemical Engineering from the Federal University of Petroleum Resources (FUPRE) in 2018, specializing in process instrumentation, design, modeling, control, and automation.

In her role as a teaching and research assistant, Jacinta focuses on various areas, including process control, modeling, design and simulation, computational chemical engineering, and chemical engineering design. She actively contributes to an academia-industry collaborative project titled "Controlled Protein Capture via Precipitation and Crystallization for the Downstream Processing of Monoclonal Antibodies (mAbs)." This project aims to advance continuous purification processes, addressing the global demand for high-purity, high-volume, and high-dose protein therapeutics.

Jacinta's important contributions to the project encompass expertise in data acquisition, modeling, and process control. She successfully designed a proportional integral controller to regulate protein precipitate particle morphology, playing a crucial role in transitioning from batch to continuous processing. Her efforts contribute to the overall transformation of the manufacturing process in alignment with Industry 4.0 paradigms.

Jacinta's interests lie in first principle modeling, data-driven modeling/ system Identification, design, control, and optimization of complex processes. She is also interested in Advanced manufacturing, Manufacturing system Integrations, lean development methodologies, lean six sigma, model-based design (MBD).

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Mrunal Sontakke

Mrunal Sontakke is currently a Ph.D. Candidate in the Department of Chemical and Biological Engineering at Rensselaer Polytechnic Institute. Her research focuses on developing techniques to assist the Human-in-the-Loop in Smart Manufacturing and Type-1 Diabetes. Prior to pursuing her doctorate, she earned her bachelor's degree in chemical engineering from the Institute of Chemical Technology (ICT), Mumbai, India.

As a research associate at RPI, Mrunal has contributed to several projects. She has worked on developing a particle-filter-based state estimation algorithm to detect events for patients with Type-1 Diabetes. Additionally, she is an active member of a project to create educational tools to teach undergraduate Chemical Engineering students the basics of Smart Manufacturing and Machine learning. Mrunal is an active member of a group project aimed at making Smart Manufacturing simple and user-friendly for Small and Medium-scale manufacturers. Currently, she is leading the design of a human-machine interface for a project in collaboration with a group from the Department of Cognitive Science.

Mrunal is passionate about developing software packages that are accessible to the end user. She is particularly interested in the design of Human-Machine interfaces and their impact on the human decision-making process, with applications in Education, Healthcare, and Process Industries.

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Andreas Rebmann

Andreas Rebmann is currently a Ph.D. Student in the Department of Chemical and Biological Engineering at Rensselaer Polytechnic Institute. His research focuses on using low-rank techniques for process modeling and control and Smart Manufacturing. Prior to pursuing his doctorate, he earned his bachelor's degree in chemical engineering from RPI.

As a research associate at RPI, Andreas has contributed to multiple projects. He started by helping set up the heat-exchanger experiment in the senior unit operations lab at RPI. He then moved on to continuous steel casting, where he worked on processing and using industrial data from steel manufacturing plants to detect and predict clogging in the casters. This work also resulted in a project where he developed tutorials for engineers to pass on the knowledge and techniques used in clogging detection and prediction. He is currently working with multiple small manufacturers to demonstrate affordable and approachable modernization of sensors, data storage, and process modeling for these manufacturers. He also works on low-rank methods of analysis and process modeling using robust principal component analysis, reduced-order models, and model approximation with neural networks.

Andreas is interested in making interpretable data models and process control using low-rank methods. He also wants to develop tools and methods for educating workers on smart manufacturing techniques to make them usable and engaging for all workers.

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Chapter 1 Solutions

1.1

a) *Driving a car*
Please see either jogging, cycling, stirred tank heater, or household thermostat for a representative answer.

b) Two sample favorite activities:
Jogging

i. Objectives:
- Jog intensely (heart rate at 180bpm) for 30 min.
- Smooth changes in jogging intensity and speed.

ii. Input Variables:
- Jogging Rate – Manipulated input
- Shocking surprises (dogs, cars, etc.)–Disturbance

iii. Output Variable
- Blood Oxygen level – unmeasured
- Heartbeat – measured.
- Breathing rate – unmeasured

iv. Constraints:
- Hard: Max Heart Rate (to avoid heart attack! death)
- Hard: Blood oxygen minimum and maximum
- Soft: Time spent jogging

v. Operating characteristics: Continuous during period, Semi Batch when viewed over larger time periods.

vi. Safety, environmental, economic factors: Potential for injury, overexertion

vii. Control: Feedback/Feedforward system. Oxygen level, heartbeat, fatigue all part of determining action after the fact. Path, weather is part of feedforward system.

Cycling

i. Objectives:
- Ensure stability (don't crash)
- Enjoy ride.
- Prevent mechanical failure.

ii. Input Variables – Manipulated
- Body Position
- Steering
- Braking Force
- Gear Selection

Input Variables – Disturbance

- Weather
- Path Conditions
- Other people, animals

iii. Output Variables – Measured:
- Speed
- Direction
- Caloric Output (Via electron monitor)

Output Variables – Unmeasured:
- Level of enjoyment
- Mechanical integrity of person and bicycle
- Aesthetics (smoothness of ride)

iv. Constraints – Hard:
- Turning radius
- Mechanical limits of bike and person
- Maximum fatigue limit of person

Constraints – Soft:

- Steering dynamics that lead to instability before mechanical failure (i.e. you crash, the bike doesn't break)
- Terrain and weather can limit enjoyment level.

v. Operation: Continuous: Steering, weight distribution, terrain selection within a path, pedal force Semi batch: Gear selection, braking force Batch: Tire pressure, bike selection, path selection

vi. Safety, environment, economics: Safety: Stability and mechanical limits prevent injury to rider and others Environment: Trail erosion, noise Economics: Health costs, maintenance costs.

vii. Control Structure: Feedback: Levels of exertion, bike performance is monitored and ride is adjusted after the fact Feedforward Path is seen ahead and ride is adjusted accordingly.

c) A stirred tank heater
   i. Objectives
      - Maintain Operating Temperature
      - Maintain °ow rate at desired level.
   ii. Input Variables:
      - Manipulated: Added heat to system
      - Disturbance: Upstream Flow rate and conditions
   iii. Output Variables – Measured: Tank fluid temperature, Outflow.
   iv. Constraints:
      - Hard: Max inflow and outflow as per pipe size and valve limitations
      - Soft: Fluid temperature for operating objective
   v. Operating conditions: Continuous fluid flow adjustment, continuous heating adjustment
   vi. Safety, Environmental, Economic considerations: Safety: Tank overflow, failure could cause injury Economics: Heating costs, spill costs, process quality costs Environmental: Energy consumption, contamination due to spills of hot water.
   vii. Control System: Feedback: Temperature is monitored, heating rate is adjusted Feedforward: Upstream flow velocity is used to predict future tank state and input is adjusted accordingly.

d) Beer Fermentation
   Please see either jogging, cycling, stirred tank heater, or household thermostat for a representative answer.

e) An activated sludge process.
   Please see either jogging, cycling, stirred tank heater, or household thermostat for a representative answer.

f) A household thermostat
   i. Objectives:
      - Maintain comfortable temperature.
      - Minimize energy consumption.
   ii. Input Variables:
      - Manipulated: Temperature setting
      - Disturbance: Outside temperature, energy transmission between house and environment
   iii. Output Variables:
      - Measured: Thermostat reading
      - Unmeasured: Comfort level
   iv. Constraints:
      - Hard: Max heating or cooling duty of system
      - Soft: Max or minimum temperature for comfort
   v. Operating conditions: Continuous heating adjustment, continuous temperature reading.
   vi. Safety, Environmental, Economic considerations: Safety: heater may be an electrical or burning hazard Economics: Heating costs Environmental: Energy consumption.
   vii. Control System: Feedback; temperature is monitored, heating rate is adjusted after the fact.

g) Air traffic control
   Please see either jogging, cycling, stirred tank heater, or household thermostat for a representative answer.

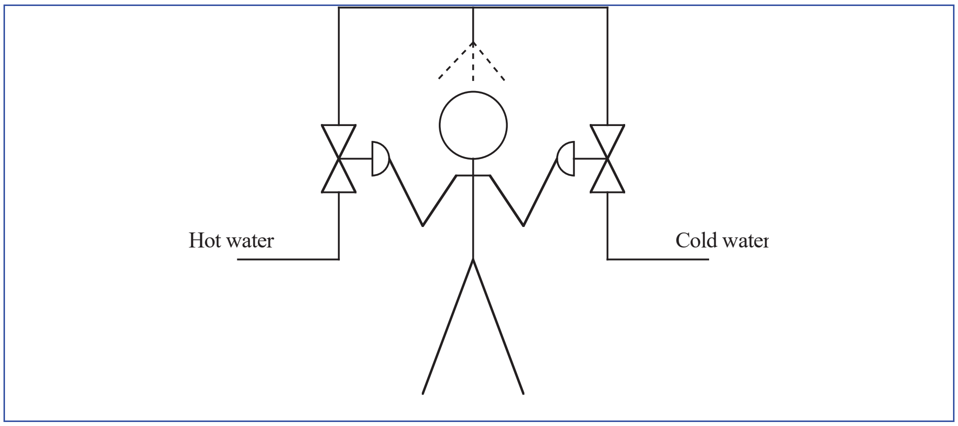
h) Taking a shower
   A common multivariable control problem that we face every day is taking a shower. A simplified process schematic is shown in the Figure below. We analyze this process step by step.
   i. Objectives:
      - To become clean
      - To be comfortable (correct temperature and water velocity as it contacts the body)
      - To "look good" (clean hair, etc)
      - To become refreshed

   To simplify our analysis, for the rest of the problem we discuss how we can satisfy the second objective (to maintain water temperature and flow rate at comfortable

1.1h Shower problem.

A common multivariable control problem that we face every day is taking a shower. A simplified process schematic is shown in the Figure below. We analyze this process step by step.

- **i.** Control objectives: Control objectives when taking a shower include the following:
  - **a.** to become clean
  - **b.** to be comfortable (correct temperature and water velocity as it contacts the body)
  - **c.** to “look good” (clean hair, etc.)
  - **d.** to become refreshed



  To simplify our analysis, for the rest of the problem we discuss how we can satisfy the second objective (to maintain water temperature and flow rate at comfortable values). Similar analysis can be performed for the other objectives.
- **ii.** Input variables: The manipulated input variables are hot-water and cold-water valve positions. Some showers can also vary the velocity by adjustment of the shower head. Another input is body position—you can move into and out of the shower stream. Disturbance inputs include a drop in water pressure (say, owing to a toilet flushing) and changes in hot water temperature owing to “using up the hot water from the heater.”
- **iii.** Output variables: The “measured” output variables are the temperature and flow rate (or velocity) of the mixed stream as it contacts your body.
- **iv.** Constraints: There are minimum and maximum valve positions (and therefore flow rates) on both streams. The maximum mixed temperature is equal to the hot water temperature and the minimum mixed temperature is equal to the cold water temperature. The previous constraints were hard constraints—they cannot be physically violated. An example of a soft constraint is the mixed-stream water temperature—you do not want it to be above a certain value because you may get scalded. This is a soft constraint because it can physically happen, although you do not want it to happen.
- **v.** Operating characteristics: This process is continuous while you are taking a shower but is most likely viewed as a batch process, since it is a small part of your day. It could easily be called a semicontinuous (semibatch) process.
- **vi.** Safety, environmental, and economic considerations: Too high of a temperature can scald you—this is certainly a safety consideration. Economically, if your showers are too long, more energy is consumed to heat the water, costing money. Environmentally (and economically), more water consumption means that more water and wastewater must be treated. An economic objective might be to minimize the shower time. However, if the shower time is too short, or not frequent enough, your clothes will become dirty and must be washed more often—increasing your clothes-cleaning bill.
- **vii.** Control structure: This is a multivariable control problem because adjusting either valve affects both temperature and flow rate. Control manipulations must be “coordinated,” that is, if the hot-water flow rate is increased to increase the temperature, the cold-water flow rate must be decreased to maintain the

same total flow rate. The measurement signals are continuous, but the manipulated variable changes are likely to be discrete (unless your hands are continuously varying the valve positions).
Feedback control: As the body feels the temperature changing, adjustments to one or both valves is made. As the body senses a flow rate or velocity change, one or both valves are adjusted.
Feed-forward control: If you hear the toilet flush, you move your body out of the stream to avoid the higher temperature that you anticipate. Notice that you are making a manipulated variable change (moving your body) before the effect of an output (temperature or flow rate) change is actually detected.

Some showers may have a relatively large time delay (or dead time) between when a manipulated variable change is made and when the actual output change is measured. This could happen, for example, if there was a large pipe run between the mixing point and the shower head (this would be considered an input time delay). Another type of time delay is measurement dead time, for example if your body takes a while to detect a change in the temperature of the stream contacting your body.

Notice that the control strategy used has more manipulated variables (two valve positions and body movement) than measured outputs (total mixed-stream flow rate and temperature).

In the shower example, the individual taking the shower served as the controller. The measurements and manipulations for this example are somewhat qualitative (you do not know the exact temperature or flow rate, for example). Most of the rest of the textbook consists of quantitative controller design procedures, that is, a mathematical model of the process is used to develop the control algorithm.

1.2l AP problem. Automated insulin delivery (closed-loop artificial pancreas) for individuals with type 1 diabetes. See Figure 1-8 and Module 12.

Literature Review: Bequette BW. Automated Insulin Dosing for Type 1 Diabetes. Chapter in Encyclopedia of Systems and Control. Baillieul J and Samad T (eds). Springer (2020). https://doi.org/10.1007/978-1-4471-5102-9 100131-1

i. This encyclopedia article provides an overview of automated insulin delivery devices. It is interesting because these algorithms and devices represent an excellent opportunity for individuals with type 1 diabetes to no longer need to be constant monitoring their glucose values and making frequent insulin (and/or meal) adjustments, but can let the closed-loop algorithm make these decisions.

While neither the model nor the algorithms are directly presented, a number of plot stress the important steady-state and dynamic problems involved. A number of algorithms are suggested, including PID and MPC.

ii. Words and concepts: PID, MPC, fuzzy logic, fault detection, simulation models.

1.3h CGM. Instrumentation Search. Select one of the following measurement devices and use Internet resources to learn more about it. Determine what types of signals are input to or output from the device.

The following NIDDK source provides a solid background on continuous glucose monitoring: https://www.niddk.nih.gov/health-information/diabetes/overview/managing-diabetes/continuous-glucose-monitoring

The following description is directly from the website:

" How does a continuous glucose monitor work?

A continuous glucose monitor (CGM) estimates what your glucose level is every few minutes and keeps track of it over time.

A CGM has three parts. First, there is a tiny sensor that can be inserted under your skin, often the skin on your belly or arm, with a sticky patch that helps it stay there. These sensors are called disposable sensors. Another type of CGM sensor—called an implantable sensor—may be placed inside your body. CGM sensors estimate the glucose level in the fluid between your cells, which is very similar to the glucose level in your blood. Sensors must be replaced at specific times, such as every few weeks, depending on the type of sensor you have.

The second part of the CGM is a transmitter. The transmitter sends the information, without using wires, to the third part, a software program that is stored on a smartphone, on an insulin pump, or on a separate device called a receiver."

While not discussed, it should be clear that the sensors are measuring the interstitial fluid glucose, and that there may be a time lag between the capillary blood and interstitial fluid glucose values.

The article goes on to discuss "real time" vs. "intermittent-scan" monitors. The real time monitors are more usually for implementation in a automated insulin delivery device, while the intermittent-scan monitors are useful for someone that simply wants to scan their values an infrequent intervals. In both cases the monitors are indirectly inferring the blood glucose by sending signals that are related to the interstitial fluid glucose values.

1.5 Surge Drum Problem. Consider Example 1.1, the liquid surge vessel, which assumed that the outlet stream flowrate was manipulated. Now, perform a complete analysis assuming that the inlet stream flowrate is manipulated.

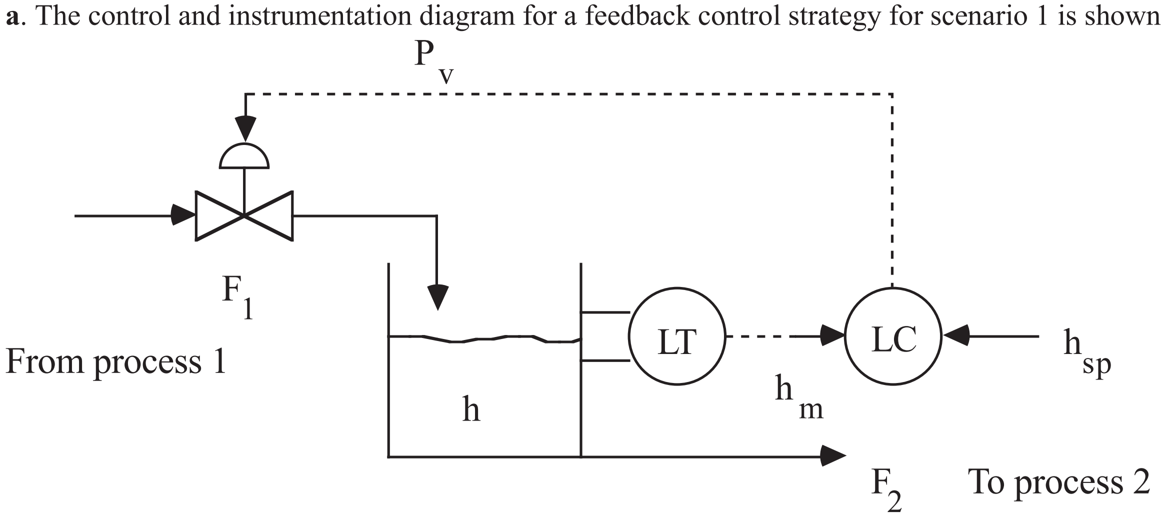
a. Draw the instrumentation diagram for this problem.

b. Should the control valve on the inlet stream be fail-open or fail-closed? Why?

c. Draw the feedback block diagram for this problem.

d. Develop the instrumentation diagram for combined feedforward and feedback control for this problem.

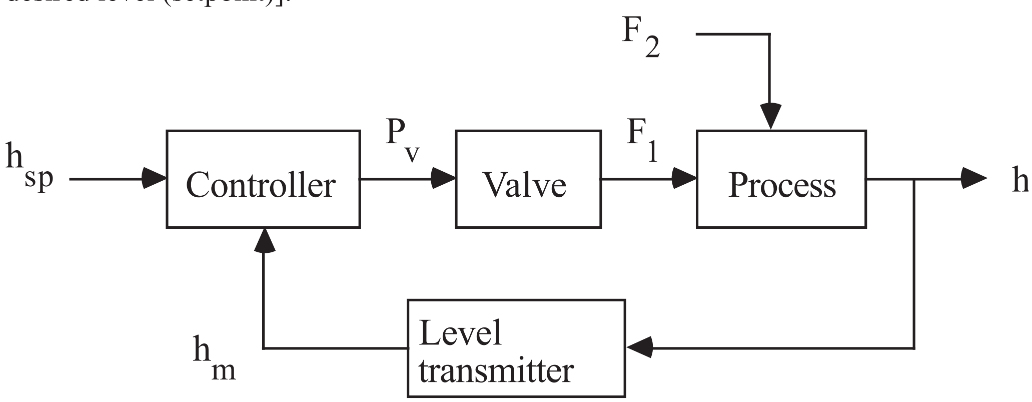
a. The control and instrumentation diagram for a feedback control strategy for scenario 1 is shown below.



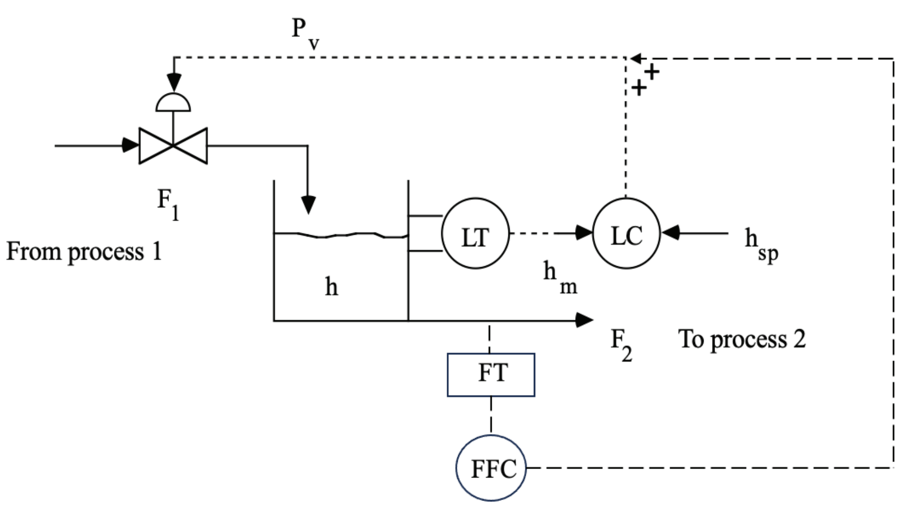
Notice that the level transmitter (LT) sends the measured height of liquid in the tank (h_m) to the level controller (LC). The LC compares the measured level with the desired level (h_{sp}, the height setpoint) and sends a pressure signal (P_v) to the valve. This valve top pressure moves the valve stem up and down, changing the flow rate through the valve (F_1). If the controller is designed properly, the flow rate changes to bring the tank height close to the desired setpoint. In this process and instrumentation diagram we use dashed lines to indicate signals between different pieces of instrumentation.

b. The control valve should be specified as fail-closed or air-to-open, so that the tank will not overflow on loss of instrument air or other valve failure.

c. A simplified block diagram representing this system is shown in below. Each signal and device (or process) is shown on the block diagram. We use a slightly different form for block diagrams when we use transfer function notation for control system analysis in Chapter 6. Note that each block represents a dynamic element. We expect that the valve and LT dynamics will be much faster than the process dynamics. We also see clearly from the block diagram why this is known as a *feedback* control "loop." The controller "decides" on the valve position, which affects the inlet flow rate (the manipulated input), which affects the level; the outlet flow rate (the disturbance input) also affects the level. The level is measured, and that value is fed back to the controller [which compares the measured level with the desired level (setpoint)].



d. Feedforward and Feedback control is shown below



1.2

a. Fluidized Catalytic Cracking Unit
   i. Summary of paper:
   A fluidized catalytic cracking unit (FCCU) is one of the typical and complex processes in petroleum refining. Its principal components are a reactor and a generator. The reactor executes catalytic cracking to produce lighter petro-oil products. The regenerator recharges the catalyst and feeds it back to the reactor. In this paper, the authors test their control schemes on an FCCU model. The model is a nonlinear multi-input/multi-output (MIMO) which couples time varying and stochastic processes. Considerable computation is needed to use model predictive process control algorithms (MPC). Standard PID control gives inferior performance. A simplified MPC algorithm can reduce the number of parameters and computational load while still performing better than a PID control method.
   ii. Familiar Terms: Constraint, nonlinearity, control performance, MPC, unmeasured disturbance rejection, modeling, simulation.

b. Reactive Ion Etching
   Please see FCCU for a representative answer.

c. Rotary Lime Kiln
   Please see FCCU for a representative answer.

d. Continuous Drug Infusion
   Please see FCCU for a representative answer.

e. Anaerobic Digester
   Please see FCCU for a representative answer

f. Distillation
   Please see FCCU for a representative answer.

g. Polymerization reactor
   Please see FCCU for a representative answer

h. pH
   Please see FCCU for a representative answer.

i. Beer Production
   Please see FCCU for a representative answer.

j. Paper Machine Headbox
   Please see FCCU for a representative answer.

k. Batch Chemical Reactor
   Please see FCCU for a representative answer.

l. AP problem.
   Automated insulin delivery (closed-loop artificial pancreas) for individuals with type 1 diabetes. See Figure 1-8 and Module 12.
   Literature Review: Bequette BW. Automated Insulin Dosing for Type 1 Diabetes. Chapter in Encyclopedia of Systems and Control. Baillieul J and Samad T (eds). Springer (2020). https://doi.org/10.1007/978-1-4471-5102-9 100131-1
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   While neither the model nor the algorithms are directly presented, a number of plots stress the important steady-state and dynamic problems involved. A number of algorithms are suggested, including PID and MPC.
   ii. Words and concepts: PID, MPC, fuzzy logic, fault detection, simulation models.

1.3

a. *Vortex-shedding flow meters*
   The principal of vortex shedding can be seen in the curling motion of a flag waving in the breeze, or the eddies created by a fast-moving stream. The flag outlines the shape of air vortices as the flow past the pole. Van Karman produced a formula describing the phenomena in 1911. In the late 1960's the first vortex shedding meters appeared on the market. Turbulent flow causes vortex formation in a fluid. The frequency of vortex detachment is directly proportional to fluid velocity in moderate to high flow regions.

At low velocity, algorithms exist to account for nonlinearity. Vortex frequency is an input, fluid velocity is an output.

b. *Orifice-plate flow meters*
   Please see vortex–shedding flow meters for a representative answer.
c. Mass flow meters
   Please see vortex–shedding flow meters for a representative answer.
d. *Thermocouple based temperature measurements.*
   Please see vortex–shedding flow meters for a representative answer.
e. *Differential pressure measurement*
   Please see vortex–shedding flow meters for a representative answer.
f. Control Valves
   Please see vortex–shedding flow meters for a representative answer.
g. pH
   Please see vortex–shedding flow meters for a representative answer.
h. Continuous Glucose Monitor (CGM) to measure blood glucose level.
   Instrumentation Search. Select one of the following measurement devices and use Internet resources to learn more about it. Determine what types of signals are input to or output from the device.

The following NIDDK source provides a solid background on continuous glucose monitoring:
https://www.niddk.nih.gov/health-information/diabetes/overview/managing-diabetes/continuous-glucose-monitoring

The following description is directly from the website:
" How does a continuous glucose monitor work?
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A CGM has three parts. First, there is a tiny sensor that can be inserted under your skin, often the skin on your belly or arm, with a sticky patch that helps it stay there. These sensors are called disposable sensors. Another type of CGM sensor—called an implantable sensor—may be placed inside your body. CGM sensors estimate the glucose level in the fluid between your cells, which is very similar to the glucose level in your blood. Sensors must be replaced at specific times, such as every few weeks, depending on the type of sensor you have.

The second part of the CGM is a transmitter. The transmitter sends the information, without using wires, to the third part, a software program that is stored on a smartphone, on an insulin pump, or on a separate device called a receiver."

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The article goes on to discuss "real time" vs. "intermittent-scan" monitors. The real time monitors are more usually for implementation in a automated insulin delivery device, while the intermittent-scan monitors are useful for someone that simply wants to scan their values an infrequent intervals. In both cases the monitors are indirectly inferring the blood glucose by sending signals that are related to the interstitial fluid glucose values.

1.4

No solutions are required to work through Module 1

1.5

1.6

a. The main objective is to maintain the process fluid outlet temperature at a desired setpoint of 300 C.
b. The measured output is the process fluid outlet temperature.
c. The manipulated input is the fuel gas flowrate, specifically the valve position of the fuel gas control valve.
d. Possible disturbances include process fluid flowrate, process fluid inlet temperature, fuel gas quality, and fuel gas upstream pressure.
e. This is a continuous process.
f. This is a feedback controller.

g. The control valve should be fail-closed. Increasing air pressure to the valve will then increase the valve position and lead to an increase in flowrate. Loss of air to the valve will cause it to close. The gain of the valve is positive because an increase in the signal to the valve results in an increase in flow.

h. It is important from a safety perspective to have a fail-closed valve. If the valve failed to open, there might not be enough combustion air, causing a loss of the flame - this could cause the furnace firebox to fill with fuel gas, which could then re-ignite under certain conditions. Although the combustion air is not shown, it should be supplied with a small stoichiometric excess. If there is too much excess combustion air, energy is wasted in heating up air that is not combusted. If there is too little excess air, combustion will not be complete, causing fuel gas to be wasted and pollution to the atmosphere. The process fluid is flowing to another unit. If the process fluid is not at the desired setpoint temperature, the performance of the unit (reactor, etc) may not be as good as desired, and therefore not as profitable.

i. The control block diagram for the process furnace is shown below in Figure 1. Where the signals as

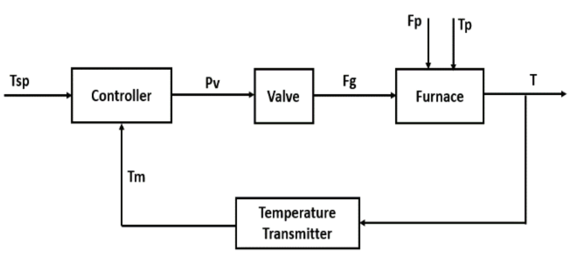


Figure 1-1: Control block diagram of process furnace

Follows:

- Tsp: Temperature setpoint
- pv: Valve top pressure
- Fg: Fuel gas flowrate
- Fp: Process fluid flowrate
- Tp: Process fluid inlet temperature
- T: Temperature of process fluid outlet
- Tm: Measured temperature

1.7

The problem statement tells us that the gasoline is worth $500,000 a day. A 2% increase in value is:

$$\frac{\$500{,}000}{\text{day}} \text{ x } 0.02 = \frac{\$10{,}000}{\text{day}}$$

We are also given that revamping will cost $2,000,000. We can now calculate the time required to pay back the control system investment.

$$\frac{\$2{,}000{,}000}{\frac{\$10{,}000}{\text{days}}} = 200 \text{ days}$$

Therefore, we know it will take 200 days to pay off the investment.

1.8

$$2\text{yrs} \cdot 4.4\text{million } \$/\text{yr} \cdot 0.2\% = \$176{,}000$$

1.9

We know from the problem statement that the inlet (F_{in}) and outlet (F_{out}) flowrates can be represented with the following equations:

$$F_{in} = 50 + 10\sin(0.1t)$$
$$F_{out} = 50$$

The change in volume as a function of time is:

$$\frac{dV}{dt} = F_{in} - F_{out}$$

Substituting what we know:

$$\frac{dV}{dt} = 50 + 10\sin(0.1t) - 50$$

Simplifying

$$\frac{dV}{dt} = 10\sin(0.1t)$$

Rearranging the equation

$$dV = 10\sin(0.1t)dt$$

Taking the integral of both sides

$$\int dV = \int 10\sin(0.1t)dt$$

Using basic calculus to solve

$$V - V_0 = \frac{-10}{0.1}\cos(0.1t)\,|_{t=0}^{t}$$

We know the initial tank volume is 500 liters.

$$V - 500 = -100\cos(0.1t) - 100\cos(0)$$
$$V - 500 = -100\cos(0.1t) + 100$$
$$V(t) = 600 - 100\cos(0.1t)$$

The equation tells us how the volume of the tank will vary with time. This can also be seen visually in Figure 2 below.

1.10

a. The objective is to maintain a desired blood glucose concentration by insulin injection. Insulin is the manipulated input and blood glucose is the measured output. As performed by injection, the input is really discrete and not continuous. Also, glucose is not continuously measured, so the measured output is discrete. Disturbances include meal consumption and exercise. Feedforward action is used when a diabetic administers an injection to compensate for a meal. Feedback action occurs when a diabetic administers more or less insulin based on a blood glucose measurement. It is important not to administer too much insulin, because this could lead to too low of a blood glucose level, resulting in hypoglycemia.

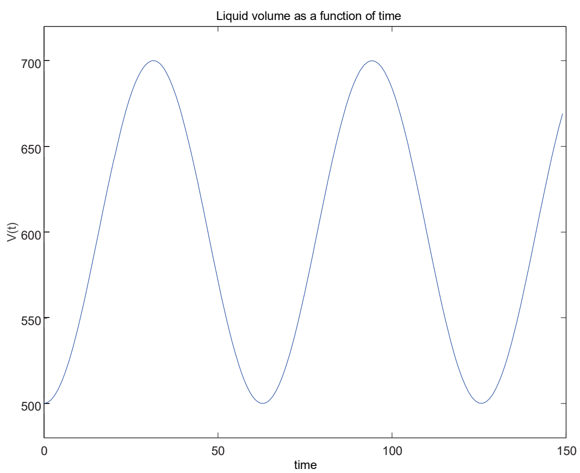


Figure 1-2: Liquid volume as a function of time

b. A process and instrumentation diagram of an automated closed-loop system is shown in Figure 3 below. For simplicity, this is shown as a pump and valve arrangement. In practice, the pump speed would be varied. The associated control block diagram is shown in figure 4 below.

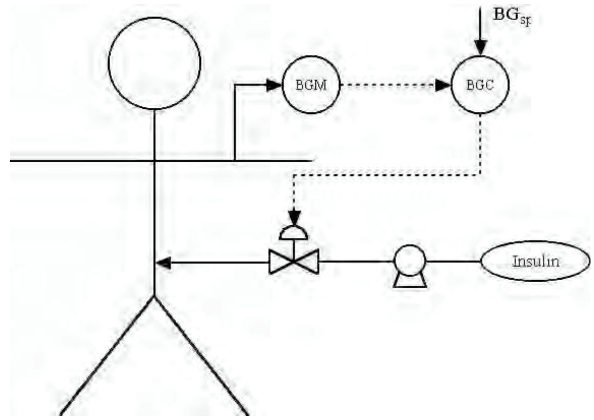


Figure 1-3: P&ID of closed-loop insulin infusion

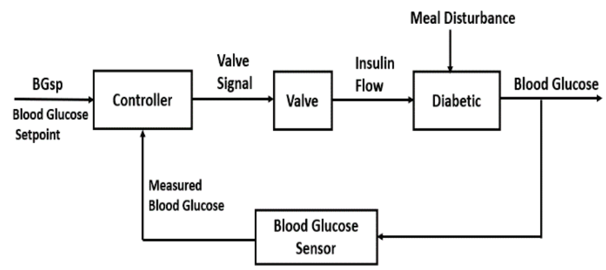


Figure 1-4: control block diagram of closed-loop insulin infusion

1.11

a. An increase in the hot stream flowrate leads to an increase in the cold stream outlet temperature, so the gain is positive. A fail-closed valve should be specified. If the valve failed-open, the cold stream outlet temperature could become too high.

b. An increase in the hot by-pass flow leads to a decrease in the cold stream outlet temperature, so the gain is negative. A fail-open valve should be specified.

c. An increase in the cold by-pass flowrate leads to a decrease in the outlet temperature, so the gain is negative. A fail-open valve should be specified, so that the outlet temperature is not too high, or the air pressure is lost.

d. Strategy (c), cold bypass, will have the fastest dynamic behavior because the effect of changing the bypass flow will be almost instantaneous. The other strategies have a dynamic lag through the heat exchanger.

1.12

The anesthesiologist attempts to maintain a desired setpoint for blood pressure. This is done by manipulating the drug flowrate. A major disturbance is the effect of an anesthetic on blood pressure.

The control block diagram for the automated system is show below in Figure 5, where, for simplicity, the drug is shown being changed by a valve.

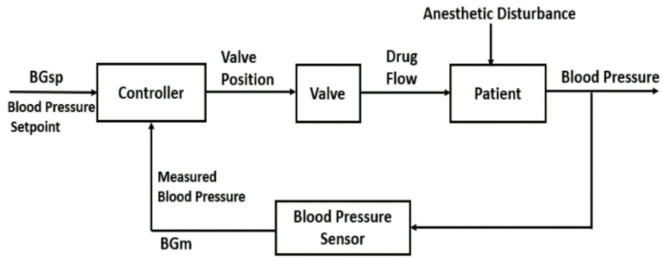
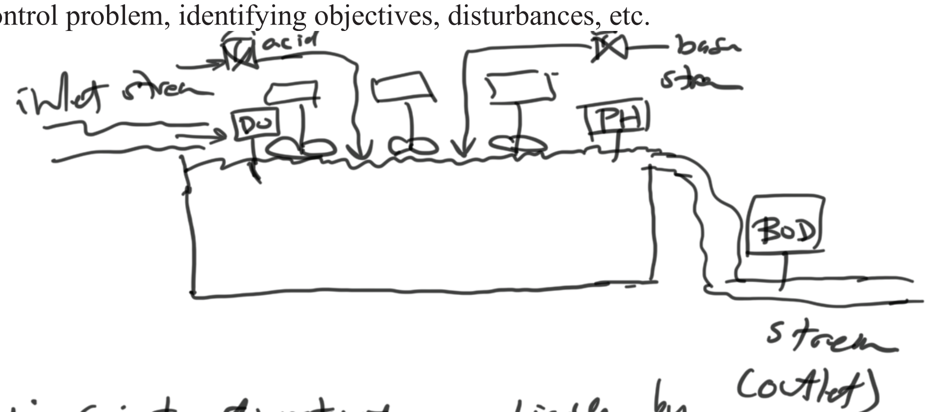


Figure 1-5: Control block diagram of drug delivery

1.13. An aeration basin (a very large "vessel") is used to continually treat a wastewater stream aerobically. A number of mechanical agitators at the surface can be used to "pump" air into the basin; these can be independently turned on or off to change the level of dissolved oxygen in the basin. The microbes that "eat" the waste function best in a small pH range. Acid and base stream flowrates into the basin can be manipulated. An analytical device that can measure the concentration of waste material in terms of BOD (biochemical oxygen demand) is placed in the outlet stream of the basin. The liquid level in the basin is relatively constant, since the effluent flows over a weir. The outlet flowrate can be estimated by the height of liquid over the weir. Provide a sketch of this process and discuss it in the context of a control problem, identifying objectives, disturbances, etc.



Objectives: treat wastewater aerobically, by controlling BOD below a specified value

manipulated inputs include: mechanical agitation (more air into basin), acid & base stream flowrates

disturbance inputs include: feed concentration of wastes, flowrate of feed waste stream

Measured outputs include: dissolved oxygen, pH, BOD, outlet flowrate (inferred from weir height)

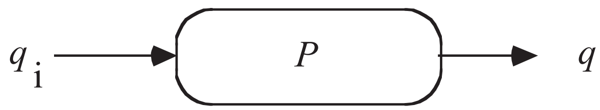
constraints include: maximum mechanical agitation, max flowrates of acid & base streams

operation: continuous

safety: avoid too low or high pH. Keep operators from falling into the basin!

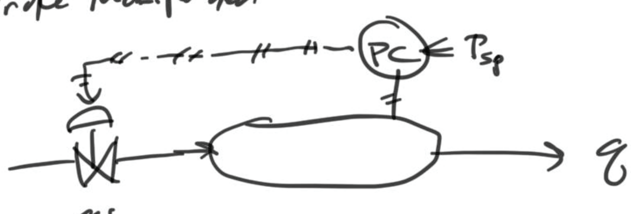
Control: maintain pH by manipulating acid & base flows
maintain dissolved O_2 by mechanical agitation

1.14. A gas surge drum, shown below, is used to buffer the effect of gas flow between two operating used. It is important that the gas drum pressure be controlled not to exceed low and high pressure limits. Consider two possible scenarios: (a) the inlet flowrate (qi) is manipulated, and (b) the outlet flowrate (q) is manipulated. In each case, sketch the process and instrumentation diagram with a pressure sensor, controller and control valve. Discuss whether the control valve should be fail-open or fail-closed.



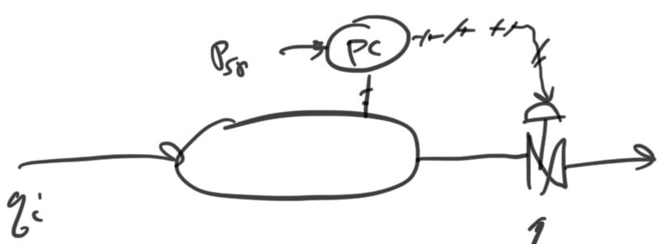
1.14 Gas Drum

(a) inlet flowrate manipulated



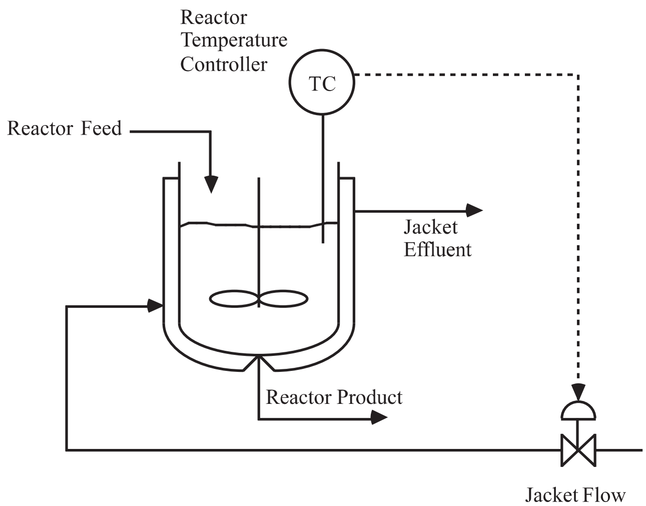
valve should fail-closed to avoid over-pressure

(b) outlet valve manipulated



valve should fail open to avoid over pressure

1.15 Consider the temperature control strategy shown below, under two different scenarios: (a) an endothermic reaction, and (b) and exothermic reactor. For each case, discuss whether the jacket fluid is a hot or a cold stream. Also discuss whether the control valve should be designed to fail-open or fail-closed.



a. Endothermic Reaction. The jacket fluid will be a hot stream and the control valve should fail closed to make certain that the reactor does not overheat.

b. Exothermic Reaction. The jacket fluid will be a cold stream and the control valve should fail open to make certain that the reactor does not overheat, possibly causing an explosion.

Chapter 2 Solution

2.1

The modeling equation is

$$\frac{dP}{dt} = \frac{RT}{V} q_i - \frac{RT}{V} \beta \sqrt{P - P_h}$$

At steady state

$$\frac{dP}{dt} = \frac{RT}{V} q_i - \frac{RT}{V} \beta \sqrt{P - P_h} = 0$$

$$\frac{RT}{V} \beta \sqrt{P_s - P_{hs}} = \frac{RT}{V} q_{is}$$

$$P_s = P_{hs} + \frac{q_{is}^2}{\beta^2}$$

Thus, we can conclude that it is a self–regulating system, as for a change in input it will attain a new steady–state.

The sketch of the steady–state input–output curve should look like figure 2-1

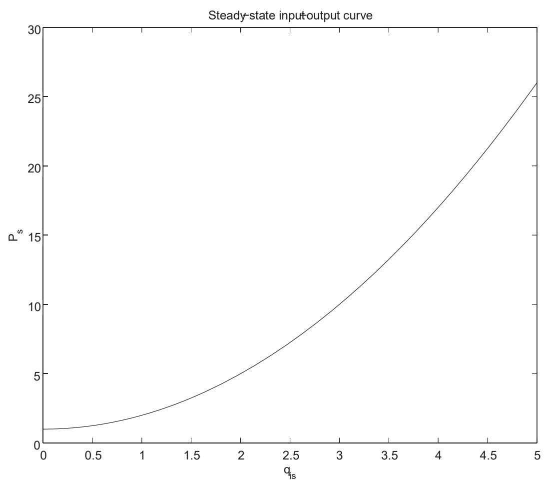


Figure 2-1: Plot for 2.1

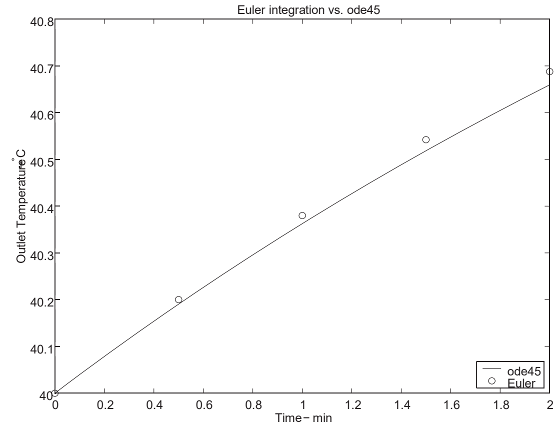
2.2

The model equations are:

$$\frac{dV}{dt} = F_i - F$$

$$\frac{dT}{dt} = \frac{F_i}{V}(T_i - T) + Q$$

a. At steady–state, the volume will not change, as



The inlet and outlet flow are the same. Thus

$$\frac{F_i}{V}(T_i - T) + Q = 0$$

$$\frac{100}{500}(20 - 40) + Q = 0$$

$$Q = 4^0 c/_{min}$$

b.

For this part we need to integrate

$$\frac{dT}{dt} = \frac{100}{500}(22 - T) + 4$$

From the initial state of T=$40^0 c$. The Euler formula is:

$$xk+1 = xk + \Delta xk'_k$$

$$x'_k = f(xk)$$

Where $f(\cdot)$ is the right hand side of the differential equation, and x is the state, in this case T. using $\Delta t = 0.5$, and for a total of 2 minutes, we have

$$x_0 = 40$$

$$x_1 = 40 + 0.5\left(\frac{1}{5}(22 - 40) + 4\right) = 40.2$$

$$x_2 = 40.2 + 0.5\left(\frac{1}{5}(22 - 40.2 + 4\right) = 40.38$$

$$x_3 = 40.38 + 0.5\left(\frac{1}{5}(22 - 40.38) + 4\right) = 40.542$$

$$x_4 = 40.542 + 0.5\left(\frac{1}{5}(22 - 40.542) + 4\right) = 40.6878$$

Figure 2-2 shows the curve of the solution found using MATLAB'S ode45, with the circles marking the point of the Euler solution.

2.3

Since the model equation have only two states, V_l and P, we have to assume the following are constant:

Density of the liquid *(p)*, temperature *(T)*, the ideal gas constant *(R)* and the molecular weight of the gas *(MW)*.

Starting with the balance of the liquid mass in the system, we have

$$\frac{dM_t}{dt} = \frac{dV_{lp}}{dt} = F_{fp} - F_p$$
$$\frac{dV_t}{dt} = F_f - F$$

For the balance of the mass of gas

$$\frac{dnMW}{dt} = qiMW_i - qMW$$
$$\frac{dn}{dt} = qi - q$$

From the ideal gas law $PV_g = nRT$, where the volume of gas is $V_g = V - V_i$ then,

$$\frac{d(PV_g)}{dt} = \frac{nRT}{dt}$$
$$P\frac{d(V - V_l)}{dt} + (V - V_l)\frac{dP}{dt} = RT\frac{dn}{dt}$$

And using the previously derived expressions for $\frac{dn}{dt}$ and $\frac{v_i}{dt}$

$$P\frac{d(V - V_l)}{dt} + (V - V_l)\frac{dP}{dt} = RT\frac{dn}{dt}$$

$$-P\frac{dV_l}{dt} + (V - v_l)\frac{dP}{dt} = RT(q_i - q)$$

$$\frac{dP}{dt} = \frac{P}{V - V_l}(F_f - F) + \frac{RT}{V - V_l}(q_i - q)$$

Thus, our model equations are.

$$\frac{dV_l}{dt} = F_f - F$$
$$\frac{dP}{dt} = \frac{P}{V - V_l}(F_f - F) + \frac{RT}{V - V_l}(q_i - q)$$

2.4

Since we have a larger volume than the example, we have to calculate the flow rate for a single reactor as well. Our volume is V =106.9444ft^3 the equations for the first tank are

$$\frac{dC_{A1}}{dt} = \frac{F}{V}(C_{Ai} - C_{A1}) - kC_{A1}$$

$$\frac{dC_{A1}}{dt} = -\frac{F}{V}C_{P1} + kC_{A1}$$

Solving at steady-state, we get

$$C_{P1_s} = \frac{C_{Ai_s}}{\frac{F_s}{kV}+1}$$

We need to meet a yearly production, so our final constraint is

$$F_s C_{P1_s} S = 100x10^6\ 1b/yr$$

Where S = 62lb/lbmol · 504000min /yr is our conversion factor, assuming 350 days of operation in a year. then,

$$F_s \frac{C_{Ai_s}}{\frac{F_s}{kV}+1} S = 100x10^6 Ib/yr$$

Solving for the flowrate, we get $F_s = 7.9256ft^3/min$.

Now we need to consider the second reactor in series, which will also change the flowrate needed to meet production levels. The equations for the second reactor are

$$\frac{dC_{A2}}{dt} = \frac{F}{V}(C_{A1} - C_{A2}) - kC_{A2}$$
$$\frac{dC_{P1}}{dt} = \frac{F}{V}(C_{P1} - C_{P2}) + kC_{A2}$$

Solving at steady-state, we get

$$C_{P2_s} = \frac{\left(\frac{kv}{F_s}\right)\left(\frac{kv}{F_s} + 2\right)C_{Ai_s}}{\left(\frac{kv}{F_s} + 1\right)^2}$$

Again, we need to meet production levels, so

$$F_s C_{P2_s} S = 100x10^6 Ib/yr$$

$$\frac{kV\left(\frac{kv}{F_s} + 2\right)C_{Ai_s}}{\left(\frac{kv}{F_s} + 1\right)^2} S = 100x10^6 Ib/yr$$

Solving the flowrate, we get $F_s = 6.5799ft^3/min$.

Thus we have a savings of 16.98% using the two reactors in series over a single one.

2.5

The resulting graph should be the same as figure 2-5 except that the time range from -1 to 0 will not appear.

2.6

$$\frac{d(VC_a)}{dt} = F_{in}C_{A_{in}} - kVC_A$$
$$\frac{dV}{dt} = F_{in}$$

We need equations whose states are V and C_A, then

$$\frac{d(VC_a)}{dt} = F_{in}C_{A_{in}} - kVC_A$$
$$V\frac{dC_a}{dt} + C_A\frac{d(V)}{dt} = F_{in}C_{A_{in}} - kVC_A$$
$$V\frac{dC_a}{dt} = -C_A\frac{d(V)}{dt} + F_{in}C_{A_{in}} - kVC_A$$
$$V\frac{dC_a}{dt} = -C_A F_{in} + F_{in}C_{A_{in}} - kVC_A$$
$$\frac{dC_a}{dt} = \frac{F_{in}}{V}(C_{Ain} - C_A) - kC_A$$

Our two equations are

$$\frac{dV}{dt} = F_{in}$$
$$\frac{dC_a}{dt} = \frac{F_{in}}{V}(C_{Ain} - C_A) - kC_A$$

2.7

a. The modeling equations are

$$\frac{dC_{w1}}{dt} = \frac{F}{V_1}(C_{wi} - C_{w1}) - kC_{w1}^2$$
$$\frac{dC_{w2}}{dt} = \frac{F}{V_2}(C_{w1} - C_{w2}) - kC_{w2}^2$$

b. At steady-state we can solve the following equations

$$\frac{F_s}{V_1}(C_{wis} - C_{w1s}) - kC_{w1s}^2 = 0$$
$$\frac{F_s}{V_2}(C_{w1s} - C_{w2s}) - kC_{w2s}^2 = 0$$

Rearranging the first equation, we have the quadratic

$$kC_{w1s}^2 + \frac{F_s}{V_1}C_{w1s} - \frac{F_s}{V_1}C_{w1s} = 0$$

The positive root gives us

$$C_{w2s}^2 = 0.33333 \text{ mol/liter}$$

Rearranging the second equation, we get the quadratic

$$kC_{w2s}^2 + \frac{F_s}{V_2}C_{w2s} - \frac{F_s}{V_2}C_{w1s} = 0$$

The positive root gives us

$$C_{w2s} = 0.0900521 \text{ mol/liter}$$

c. To linearize, we have the functions

$$f_1 = \frac{dC_{w1}}{dt} = \frac{F}{V_1}(C_{wi} - C_{w1}) - kC_{w1}^2$$
$$f_2 = \frac{dC_{w2}}{dt} = \frac{F}{V_2}(C_{w1} - C_{w2}) - kC_{w2}^2$$

And using the state and input variables as defined, we have

$$a_{11} = \left.\frac{\delta f_1}{\delta x_1}\right|_{ss} = \frac{\delta}{\delta C_{w1}}\left(\frac{F}{V_1}(C_{wi} - C_{w1}) - kC_{w1}^2\right)\Big|_{ss}$$
$$= -\frac{F_s}{V_1} - 2kC_{w1s}$$

$$a_{12} = \left.\frac{\delta f_1}{\delta x_2}\right|_{ss} = \frac{\delta}{\delta C_{w2}}\left(\frac{F}{V_1}(C_{wi} - C_{w1}) - kC_{w1}^2\right)\Big|_{ss}$$
$$= 0$$

$$a_{21} = \left.\frac{\delta f_2}{\delta x_1}\right|_{ss} = \frac{\delta}{\delta C_{w1}}\left(\frac{F}{V_2}(C_{w1} - C_{w2}) - kC_{w1}^2\right)\Big|_{ss}$$
$$= \frac{F_s}{V_2}$$

$$a_{22} = \left.\frac{\delta f_2}{\delta x_2}\right|_{ss} = \frac{\delta}{\delta C_{w2}}\left(\frac{F}{V_2}(C_{w1} - C_{w2}) - kC_{w2}^2\right)\Big|_{ss}$$
$$= -\frac{F_s}{V_2} - 2kC_{w2s}$$

$$b_{11} = \left.\frac{\delta f_1}{\delta u_1}\right|_{ss} = \frac{\delta}{\delta F}\left(\frac{F}{V_1}(C_{wi} - C_{w1}) - kC_{w1}^2\right)\Big|_{ss}$$
$$= \frac{1}{V_1}(C_{wis} - C_{w1s})$$

$$b_{12} = \left.\frac{\delta f_1}{\delta u_2}\right|_{ss} = \frac{\delta}{\delta C_{wi}}\left(\frac{F}{V_1}(C_{wi} - C_{w1}) - kC_{w1}^2\right)\Big|_{ss} = \frac{F_s}{V_1}$$

$$b_{21} = \left.\frac{\delta f_2}{\delta u_1}\right|_{ss} = \frac{\delta}{\delta F}\left(\frac{F}{V_2}(C_{wi} - C_{w2}) - kC_{w2}^2\right)\Big|_{ss}$$
$$= \frac{1}{V_2}(C_{w1s} - C_{w2s})$$

$$b_{22} = \left.\frac{\delta f_2}{\delta u_2}\right|_{ss} = \frac{\delta}{\delta C_{wi}}\left(\frac{F}{V_2}(C_{w1} - C_{w2}) - kC_{w2}^2\right)\Bigg|_{ss} = 0$$

d. Evaluating these coefficients at our steady-state, we have

$$A = \begin{bmatrix} -1.25\ \mathrm{hr}^{-1} & 0 \\ 0.05\ \mathrm{hr}^{-1} & -0.320156\ \mathrm{hr}^{-1} \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0016667\ \mathrm{mol/I}^2 & 0.25\ hr^{-1} \\ 0.000121641\ \mathrm{mol/I}^2 & 0 \end{bmatrix}$$

e. Since $\vec{y} = \vec{x}$, it is straightforward to show that.

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

f. Using MATLAB, the eigenvalues are -0.320156 and -1.25.

Analytically, we have

$$\det(\lambda I - A) = 0$$

$$\det\begin{bmatrix} \lambda + 1.25 & 0 \\ -0.05 & \lambda + 0.320156 \end{bmatrix} = 0$$
$$(\lambda + 1.25)(\lambda + 0.320156) = 0$$

Thus, the eigenvalues are $\lambda_1 = -1.25$ and $\lambda_2 = -0.320156$

g. Figure 2-3 shows the plot for the linear and non-linear responses; as it can be seen, the extraction requirements are still met.

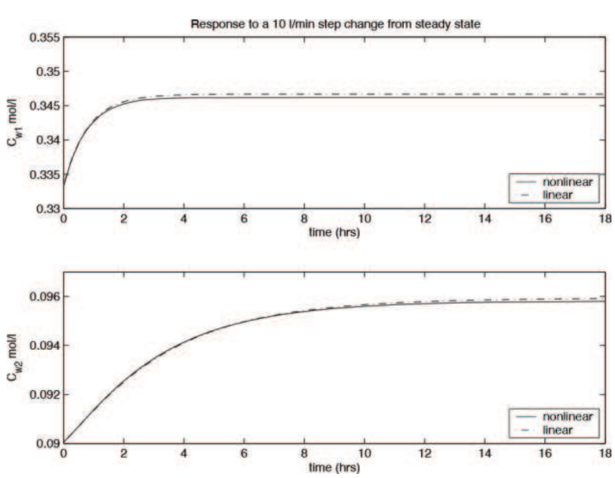


Figure 2-3: Plot for 2.7g

h. if the order of the reaction vessels is reversed, the steady-state equations we have to solve are

$$\frac{F_s}{V_2}(C_{wis} - C_{w1s}) - kC_{w1s}^2 = 0$$
$$\frac{F_s}{V_1}(C_{w1s} - C_{w2s}) - kC_{w2s}^2 = 0$$

Solving then we have $C_{w1s} = \frac{1}{6}\mathrm{mol/I}$ and $C_{w2s} = 0.10301\ \mathrm{mol/I}$. thus, the extraction requirements are no longer met.

2.8

a. solve the following simultaneous equations using the parameters and steady-state values provided:

$$\frac{F_s}{V}(T_{is} - T_s) + \frac{UA}{V_{\rho c_p}}(T_{js} - T_s) = 0$$
$$\frac{F_{js}}{V_j}(T_{jins} - T_{js}) - \frac{UA}{V_{j\rho j c_{pj}}}(T_{js} - T_s) = 0$$

Then

$$UA = 183.9\ \mathrm{Btu/(°F.\,min)}$$
$$F_{js} = 1.5\ ft^3/min$$

b. applying the equations for the elements of the linearization matrices

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -0.4 & 0.3 \\ 1.2 & -1.8 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -7.5 & 0.1 & 0 \\ 20 & 0 & 0 & 0.6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

c. heater .m file should be like the example in the book (p.73)

d. using delJ = 0, run code 45to solve the equations defined in heater. m, then plot the two states vs. time. The result should be constant values that match the steady states for all time.

e. To get the desired plots for the two step responses, the m–file shown in pages 74-75 can be used, starting with the definition of the state space linear model. Since the model is linear, the output of the step response command can be scaled accordingly for steps of different sizes by just multiplying by delFj. Figures 4(a) and 4(b) show the responses for a small (0.2% change in F_j) and a large (10% change in F_j) steps, respectively.

f. Since we know UA for the small vessel, and we are assuming that U remains constant, we can find the value of UA for a larger volume as

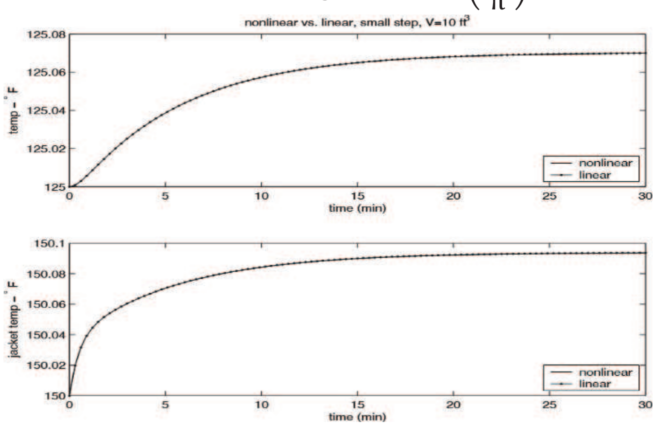
$$UA_{small} \cdot \frac{A_{large}}{A_{small}} = UA_{large}$$

Modeling the vessel as a cylinder, the volume is $V = \frac{\pi}{2} D^3$, and the area is $A = 2.25\pi D^2$. We can then calculate the area of the small vessel as a function of its volume, for which we get

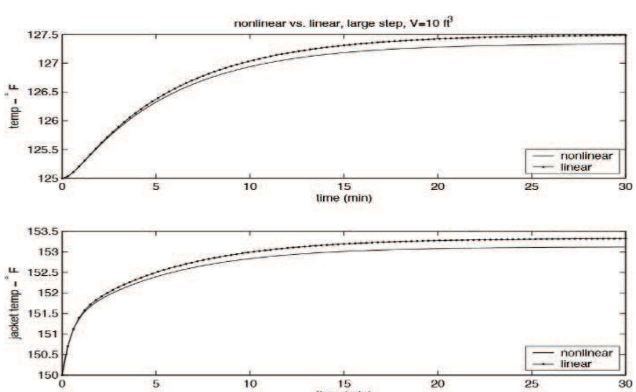
$$A_{small} = 2.25\pi \left(\frac{20}{\pi}\right)^{\frac{2}{3}}$$

Similarly, we have the area of the larger vessel in terms of its volume.

$$A_{large} = 2.25\pi \left(\frac{2V}{\pi}\right)^{\frac{2}{3}}$$



(a)



(b)

Figure 2-4: Plot for 2.8e (a) small step of 0.2% (b) large step of 10%

and the ratio are is

$$\frac{A_{large}}{A_{small}} = \left(\frac{V}{10}\right)^{\frac{2}{3}}$$

Applying this, we find that

$$UA_{large} = 853.588\ Btu/(°F \cdot \min)$$

g. Solve the following two simultaneous equations using the value of UA calculated for the large vessel

$$\frac{F_s}{V}(T_{is} - T_s) + \frac{UA}{V_{\rho c_p}}(T_{js} - T_s) = 0$$

$$F_{js}(T_{jins} - T_{js}) - \frac{UA}{\rho_j c_{pj}}(T_{js} - T_s) = 0$$

Then

$$T_{js} = 178.86°F$$
$$F_{js} = 35.478\ ft^3/min$$

We can already see that the steady–state jacket temperature has gone up, so we want to know how large we can make the vessel before the jacket temperature approaches the inlet jacket temperature. We can solve the following equation, using $\frac{F_s}{V}=0.1\ min^{-1}$,

as we want to maintain the residence time. We also use the expression in terms of the volume that we found in part f

$$\frac{F_s}{V}(T_{is} - T_s) + \frac{UA}{V_{\rho c_p}}(T_{js} - T_s) = 0$$

$$0.1(T_{is} - T_s) + \frac{UA_{small}\left(\frac{V}{10}\right)^{\frac{2}{3}}}{V_{\rho c_p}}(T_{js} - T_s) = 0$$

Then $V = 270 ft^3$.

h. Applying the equations for the elements of the linearization matrices using the steady state for the larger vessel, we have

$$A = \begin{bmatrix} -0.2392 & 0.1392 \\ 0.5570 & -1.9761 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -0.75 & 0.1 & 0 \\ 0.8456 & 0 & 0 & 1.4191 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The eigenvalues for the system with the large vessel are at $\lambda_1 = -0.1957$ and$\lambda_2 = -2.0197$. While for the system with the smaller vessel, they are $\lambda_1 = -0.1780$ and $\lambda_2 = -2.0220$. They are very close to each other; thus the speed of the response will be similar for both vessels

i. Figure 2-5 shows the response to a step of $0.1\ ft^3/min$. Comparing this to figure 4(a), we can see that both linear model responses are practically the same as the nonlinear model response. The change in temperatures is also minor, as the change in jacket flow rate is small (0.2% of the steady state value in both cases).

j. Figure 2-6 shows the response to a step of 10% of the steady state jacket flow rate. Comparing this to figure 4(b), we can see that both linear model.

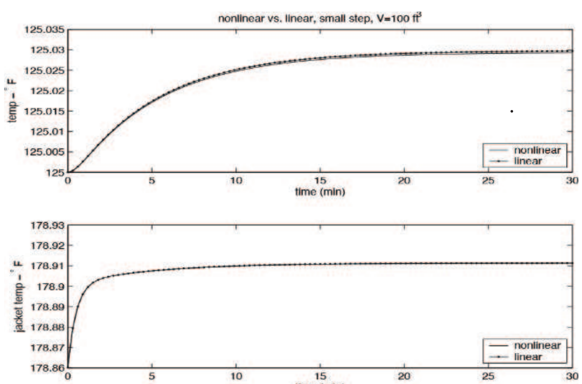


Figure 2-5: Plot for 2.8i

responses are close to the nonlinear model response, but there is an offset in the steady state. The change in temperatures is also more marked, and larger in the case of the smaller reactor, as would be expected due to the smaller volume.

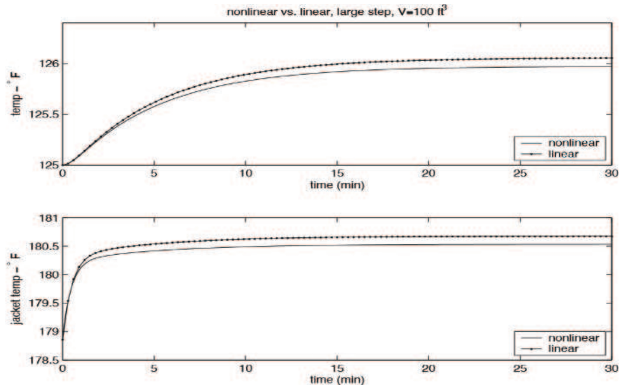
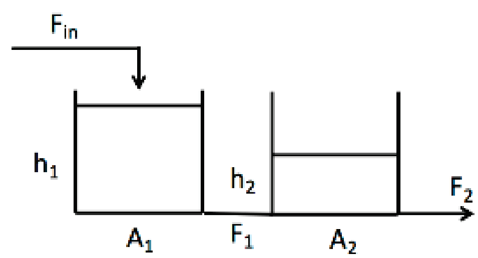


Figure 2-6: Plot for 2.8j

2.10

Consider two interacting tanks in series, with a specified inlet volumetric flowrate (F_{in}) to the first tank, and outlet flowrate from tank 1 into tank 2 proportional to the *square root of the tank height differences,* and the outlet flowrate from tank 2 proportional to the square root height of liquid in tank 2 (flow coefficients β_1 & β_2). Notice that, if the height of tank 2 is initially higher than that in tank 1, the flow could be from tank 2 back to tank 1; that is, F_1 could be negative. Assume that the cross-sectional areas (A_1 and A_2) of the tanks are constant but may differ for the two tanks.



a. state your assumption about fluid density and write the modeling equation using tank height as the state variables.

$$F_1 = \beta_1\sqrt{h_1 - h_2}$$

$$\begin{array}{c|c} assume & constant\ \rho \\ \hline conservation\ of\ mass & C.S\ area \\ & A_1 + A_2 \end{array}$$

$$F_2 = \beta_2\sqrt{h_2}$$

<u>Material balance around tank 1</u>

$$\frac{d(A_1 h_1 e)}{dt} = F_{in} e - F_1 e$$

$$\frac{A_1 dh_1}{dt} = F_{in} - F_1 = F_{in} - \beta_1\sqrt{h_1 - h_2}$$

$$\frac{dh_1}{dt} = \frac{1}{A_1}\left(F_{in} - \beta_1\sqrt{h_1 - h_2}\right) \cdots\cdots\cdots (1)$$

<u>Material balance around tank 2</u>

$$\frac{d(A_2 h_2 e)}{dt} = F_1 e - F_2 e$$

$$A_2\frac{dh_2}{dt} = F_1 - F_2 = \beta_1\sqrt{h_1} - h_2 - \beta_2\sqrt{h_2}$$

$$\frac{dh_2}{dt} = \frac{1}{A_1}\left(\beta_1\sqrt{h_1} - h_2 - \beta_2\sqrt{h_2}\right) \cdots\cdots (2)$$

b. Solve for the steady-state tank heights, as a function of the parameters and inlet flowrate

• <u>Steady-state</u>, $\frac{dh_1}{dt} = \frac{dh_2}{dt} = 0$

From (1) $F_{ins} = \beta_1\sqrt{h_{1s} - h_{2s}}$

From (2) $\beta_2\sqrt{h_{2s}} = \beta_1\sqrt{h_{1s} - h_{2s}} = F_{ins}$

$$=> \beta_2\sqrt{h_{2s}} = F_{ins} => \sqrt{h_{2s}} = \frac{F_{ins}}{\beta_2}$$

$$=> h_{2s} = \left(\frac{F_{ins}}{\beta_2}\right)^2$$

Similarly, $\sqrt{h_{1s} - h_{2s}} = \frac{F_{ins}}{\beta_1}$

$=> h_{1s} - h_{2s} = \left(\frac{F_{ins}}{\beta_1}\right)^2$

$=> h_{1s} = h_{2s} + \left(\frac{F_{ins}}{\beta_1}\right)^2 = \left(\frac{F_{ins}}{\beta_2}\right)^2 + \left(\frac{F_{ins}}{\beta_1}\right)^2$

$=> h_{is} = \left(\frac{F_{ins}}{\beta_1}\right)^2 + \left(\frac{F_{ins}}{\beta_2}\right)^2$

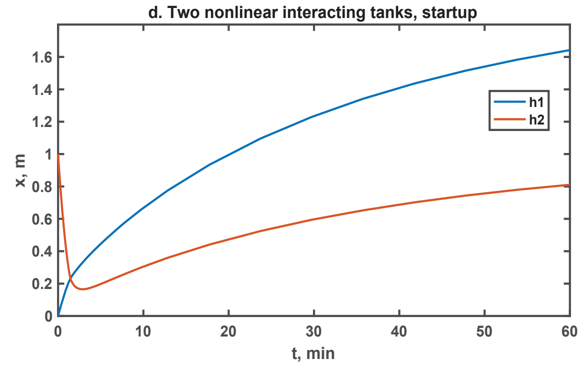
c. Write a *MATLAB* function file to be used by ode23s; you may wish to use the abs and sign functions. Use the following values $A_1 = 2.5m^2, \beta_1 = \beta_2 = 1\ m^3/min/m^{0.5}$.

```
function xdot = TwoIntTnkNL(t,x,par,u)
% two interacting tank problem with nonlinear height-flow relationship
% dh1/dt = (1/A1)*(Fin - beta1*sqrt(h1-h2))
% dh2/dt = (1/A2)*(beta1*sqrt(h1-h2) - beta2*sqrt(h2))
% 14 September 2021 -- Homework 2 due 10 Sept (revised to 14 Sept)
% parameter vector
 area1 = par(1);
 area2 = par(2);
 beta1 = par(3);
 beta2 = par(4);
 Fin  = u(1);
 h1   = x(1);
 h2   = x(2);
 dh1dt = (1/area1)*(Fin - sign(h1-h2)*beta1*sqrt(abs(h1-h2)));
 dh2dt = (1/area2)*(sign(h1-h2)*beta1*sqrt(abs(h1-h2)) - beta2*sqrt(h2));
 xdot  = [dh1dt;dh2dt];
```

d. For the start-up problem, assume the initial condition for tank height 1 is 0 meters, tank height 2 is 1 meter, and inlet flowrate (F_{in}) is 1 cubic $meter/min$ plot both heights as a function of time, for $t = 0\ to\ 60\ minutes$.

```
% Homework 2 - CPDC 2021 -- Two Interacting Tanks
% Script file for ode23s & ode45 to solve the nonlinear and linear problems
% 14 September 2021
% Nonlinear file used: TwoIntTnk_NL.m (problem c)
% Linear file used: linearmod.m (parts f and g)
% modeling equations:
% dh1dt = (1/area1)*(Fin - beta1*sqrt(h1-h2))
% dh2dt = (1/area2)*(beta1*sqrt(h1-h2) - beta2*sqrt(h2))
```

```
% parameter values
  area1 = 10;   % square meters
  area2 =  2.5; % square meters
  beta1 =  1;   % cubic meters/min/meter of height
  beta2 =  1;   % cubic meters/min/meter of height
  Fin   =  1.0; % cubic meters/min
% put the parameters into a parameter vector
  par = [area1;area2;beta1;beta2]; % will be passed
into function
  u = Fin; % input is inlet volumetric flowrate
%
  tinit  = 0;  % initial time, min
  tfinal = 60; % simulate for 60 minutes
% *** Problem d
*****************************************
*************
% initial heights
  x0 = [0;1]; % startup with tank 1 empty and tank 2
at 1 meter
%
  tspan = [tinit tfinal];
%  the following is an optional formulation to pass
parameters and inputs
%   into the ode function file. Notice that the open
square brackets []
%   indicates that default simulation parameters are
used
  [t,x] = ode23s(@TwoIntTnkNL,tspan,x0,[],par,u);
%
  figure(1)
  plot(t,x)
  title('d. Two nonlinear interacting tanks, startup')
  xlabel('t, min')
  ylabel('x, m')
  legend('h1','h2')
```



e. Solve (analytically) for the steady-state heights assuming the inlet flowrate is 1 $cubic\ meter/min$, and use these heights at the initials for your next simulation.

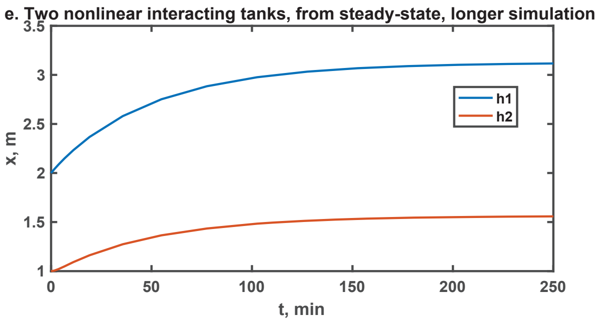
The initial steady-state values are (since beta1=beta2=1) h1s = 1^2 + 1^2 = 2m and h2s = 1^2 = 1m.

Change the inlet flowrate from 1 cubic meter/min to 1.25 cubic meters/min. Plot the tank heights as a function of time. Are the heights at t = 0 and t = 60 minutes consistent with steady-state solutions? Should the simulation time be longer to achieve the steady state?

```
% *** Problem e
********************************************
***
% now, new simulation, using initial heights of 1 meter.
Change
% the inlet flowrate to 1.25 cubic meters/min
  Fin = 1.25;
  u = Fin;
  x20 = [2;1]; % heights initially at steady-state for an
input
% flowrate of 1 cubic meters/min
  tfinal2 = 60;% final simulation time of 60 minutes
  tspan = [tinit tfinal2];
  [t2,x2] = ode23s(@TwoIntTnkNL,tspan,x20,[],par,u);
  figure(2)
  plot(t2,x2)
  title('e. Two nonlinear interacting tanks, from steady-
state')
  xlabel('t, min')
  ylabel('x, m')
  legend('h1','h2')
% not really near steady-state at t = 60 min. Integrate to t
= 250 min
  tfinal3 = 250;
```

```
tspan = [tinit tfinal3];

[t3,x3] = ode23s(@TwoIntTnkNL,tspan,x20,[],par,u);

figure(3)

plot(t3,x3)

title('e. Two nonlinear interacting tanks, from steady-state, longer simulation')

xlabel('t, min')

ylabel('x, m')

legend('h1','h2')
```

As shown below, the simulation time must be longer than 60 min to achieve steady-state values of h1s = 1.25^2 + 1.25^ = 3.125m and h2s = 1.25^2 = 1.5625m.



f. Linearize your model into state space form based on the steady-states at $F_{in} = 1\ cubic\ meter/min$. For the specified parameter values, find A, B, C, and D matrices, and calculate the eigenvalue of the A matrix

```
% linear state space model

h1s = 2; h2s = 1;  % steady-state heights

a11 = -0.5*beta1/(area1*sqrt(h1s-h2s));  % df1/dx1

a12 =  0.5*beta1/(area1*sqrt(h1s-h2s));  % df1/dx2

a21 = 0.5*beta1/(area2*sqrt(h1s-h2s));       % df2/dx1

a22 = -a21-0.5*beta2/(area2*sqrt(h2s));  % df2/dx2

b11 = 1/area1;                    % df1/du1

b21 = 0;                          % df2/du1

A = [a11 a12;a21 a22]    % state space A matrix

B = [b11;b21]            % state space B matrix

%

A_eig = eig(A)

%
```

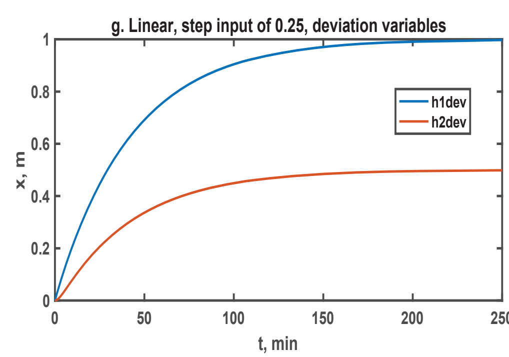
$$A = \begin{bmatrix} -0.0500 & 0.0500 \\ 0.2000 & -0.4000 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.10 \\ 0 \end{bmatrix}$$

$$A_{eigen} = \begin{bmatrix} -0.0234 \\ -0.4266 \end{bmatrix}$$

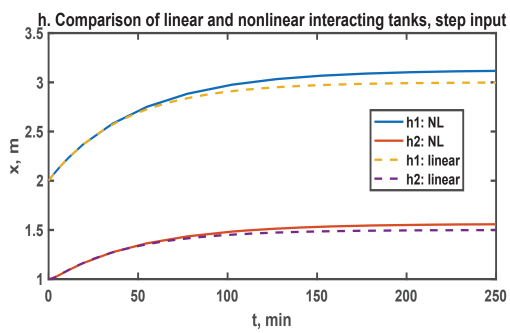
g. For the linear model based on deviation variables, change the input flow rate from 0 to 0.25 cubic$meter/min$ and plot the tank heights as a function of time.

```
% ** Part g, change input by 0.25 ***************************

x0lin = [0;0]; % initial condition, deviation variables

[tlin,xlin] = ode45(@linearmod,tspan,x0lin,[],A,B,0.25);

%  plot of linear result in deviation variables

figure(4)

plot(tlin,xlin)

legend('h1dev','h2dev')

title('g. Linear, step input of 0.25, deviation variables')

xlabel('t, min')

ylabel('x, m')
```



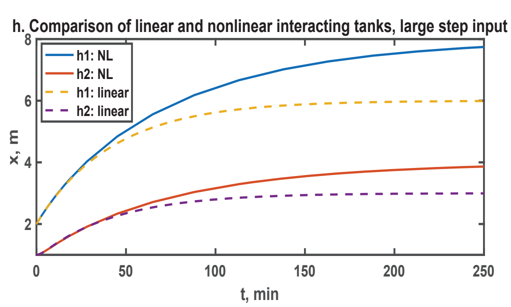
h. Convert the linear deviation variable from part *g* into physical variables by adding in the initial steady-state values. compare these tank heights with those from part *e* and describe the differences between the nonlinear and linear simulation results.

```
% comparison of nonlinear and linear step responses

  figure (5)

  plot (t3,x3,tlin,xlin+[2 1],'--') % dashed line for linear responses

  title('h. Comparison of linear and nonlinear interacting tanks, step input')

  xlabel('t, min')

  ylabel('x, m')

  legend('h1: NL','h2: NL','h1: linear','h2: linear')
```



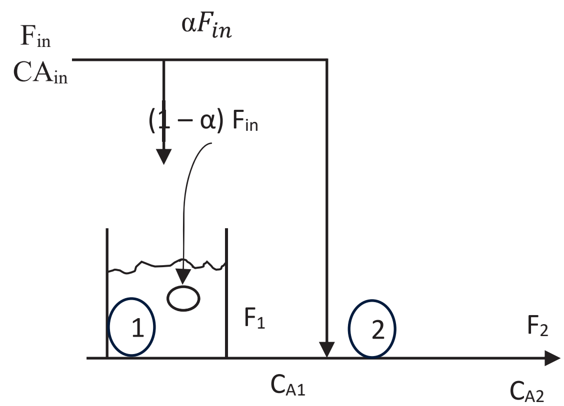
There is a relatively minor difference between the linear and nonlinear responses.

When there is much larger step input change (1 to 2 cubic meters/min), there is a more substantial difference between the linear and nonlinear models



2.11

mixing tank with by-pass problem



a. Model

Balance around tank

Around: $\frac{dv_1e_1}{dt} = (1-\alpha)F_{in}e_{in} - F_1e_1,\ \ let\ e_1 = e_\alpha = constant\rho$

$e\frac{dv_1}{dt} = (1-\alpha)F_{in}e - F_1e \qquad assume\ v_1 = constant$

$0 = \ (1-\alpha)\ F_{in} - F_1$

$F_1 = \ (1-\alpha)F_{in}$

Comput A:

$$\frac{dV_1C_{A1}}{dt} = V_1\frac{dC_{A1}}{dt} = (1-\alpha)F_{in}C_{Ain} - F_1C_{A1}$$

$$\frac{dC_{A1}}{dt} = \frac{1}{V_1}(1-\alpha)F_{in}(C_{Ain} - C_{A1})$$

$$\frac{dC_{A1}}{dt} = \frac{-(1-\alpha)F_{in}}{V_1}C_{A1} + \frac{(1-\alpha)}{V_1}C_{Ain} \text{ .... } (1)$$

Balanced around mixinnng ρt(rho dynamic)

Aroun:

$$F_2e_2 = \ F_1e_1 + \alpha F_{in}e_{in} = (1-\alpha)F_{in} + \alpha F_{in}$$

$$F_2 = F_{in}$$

Compp A: